INTEGRATED PROCUREMENT AND TRANSPORTATION PLANNING FOR PURCHASED COMPONENTS: A CASE STUDY

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I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

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ABSTRACT

INTEGRATED PROCUREMENT AND TRANSPORTATION PLANNING FOR PURCHASED COMPONENTS: A CASE STUDY

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This study is about an integrated procurement and transportation planning system for purchased components of a consumer-durables manufacturer. Due to transportation cost structures and demand characteristics our problem can be classified as a variant of the dynamic-demand joint replenishment problem. The problem is to determine the replenishment policy using the advantages of coordinated transportation of items that will minimize the sum of total inventory holding and tranportation costs over a finite planning horizon. A mathematical model is formulated for purchasing and transportation decisions for the purchased items using the advantage of joint transportation costs. A two-phased solution method is proposed in order to obtain a “good solution” for the problem. The proposed solution method is compared with the current practice for different problem instances using retrospective data and created data. As a result it is shown that proposed method decrease the total inventory and transportation cost of the system even though the first aggregate problem can not be solved to optimality.

Keywords: transportation planning, joint replenishment, dynamic demand.

ÖZ

SATINALINAN KOMPONENTLER İÇİN ENTEGRE TEDARİK VE TAŞIMA PLANLAMASI: BİR VAKA ÇALIŞMASI

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Bu çalışma dayanıklı tüketim malları sektöründe faaliyet gösteren bir üreticiye satın alınan parçalar için entegre tedarik ve taşıma planlaması sistemi kurmayı amaçlamaktadır. Nakliye maliyetleri ve talebin yapısı nedeniyle problemimiz dinamik talep toplu sipariş probleminin değişik bir şeklidir. Problem, toplu taşıma avantajlarını kullanarak sınırlı bir planlama dönemi içinde oluşan tüm envanter tutma ve nakliye maliyetlerinin toplamını en azlayacak bir sipariş politikası belirlemektir. Satınalınan kalemlerin satınalma ve nakliye kararları için toplu taşıma maliyetlerinin faydalarını kullanacak bir matematiksel model oluşturulmuştur. Problemimize “iyi çözüm” elde etmek için iki fazli bir çözüm yöntemi önerilmektedir. Önerilen çözüm yöntemi geçmiş dönemlerdeki talep verileri ve yapay veriler kullanılarak mevcut durum ile karşılaştırılmıştır. Sonuç olarak ilk faz problemi optimal olarak çözülememesine rağmen önerilen yöntemin toplam taşıma ve envanter tutma maliyetlerini düşürdüğü gösterilmektedir.

Anahtar kelimeler: taşıma planlaması, toplu sipariş, dinamik talep.

To everyone who trust me more than myself
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TABLE OF CONTENTS

CHAPTER

APPENDICES

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

This study aims to propose an integrated procurement and transportation planning approach for the purchased components of a consumer-durables manufacturer. Among the purchased components, the ones that are procured from foreign suppliers are the main focus of the study. Due to transportation cost structures and demand characteristics, the problem addressed can be classified as a variant of the dynamic-demand joint replenishment problem (DJRP).

The consumer-durables manufacturing company in concern manufactures cooking appliances. Ovens and hobs are the main end-product types manufactured in the company. 25 % of the end-products are sold in local market, while 75 % of them are sold in foreign markets.

The supply chain of the company consists of more than 120 suppliers located in about 25 countries all around the world, 20-25 local suppliers, two manufacturing plants at the same location, one central warehouse, one warehouse at each plant site, about 15 transportation companies, 25 forwarding agencies of these transportation companies and 2 companies responsible for customs procedures before components are transferred to the manufacturing plants.

Since the market for the cooking appliances is a highly competitive one, the variety in the product mix of the company is very high and the product mix changes very frequently. Depending on the end-product variety, there exists a high proliferation of the imported components over time.

There are about 3,200 stock keeping units (SKUs) of imported components currently defined in the system. Due to changing product mix, only 1,000-1,500 of them are actively used in a year. Among the active imported components that are used in a given year, 650 to 750 of them are actively used in a given month. There are 95 raw material classes defined based on their functionality. Imported components belong to 50 different raw material classes. Since they are purchased from foreign suppliers, the procurement lead-times are high, changing from 4 to 20 weeks. Since these parts are mostly functional parts, they are relatively expensive. Although only 7% of the raw materials SKUs are the imported components, they constitute 30 % of the total raw material stock value. Moreover, transportation cost and the other logistics costs of the imported components add up to a value which is about 4 % of the associated input material stock value of the imported components.

The demand for components arises from two sources: the demand for the end-products due to BOM structures and the service part requirements for after-sale service activities. In reality, the latter source of demand depends on the past sales of the company. The components that are not used in the manufacturing of the current product mix can be demanded as spare parts if they were used in the previous product mixes. However, the relationship between the end-product sales and demand for the associated spare parts can become too complicated to be modeled. Besides, there is no sufficient data for the reliability of the components and the products that are in use by the customers. Therefore, we can treat the demand of components that are used as spare parts as the independent demand, whereas we refer to the demand of components used to build the end-products as the dependent demand. In general, the demand for components as spare parts is lower compared to the demand for components to be used in end-product manufacturing.

Since the end-product demands can not be known with certainty in advance, the biggest proportion of the component demand is not known with certainty in advance, either. Purchasing orders for the imported components are placed based on the end-product demand forecasts far in advance of demand realization, since the procurement lead time of imported components is very long. After the demand forecasts are finalized by the marketing department, the production schedule is prepared considering the shipment schedule of the end-products. The shipment schedule of the end-products, via backward scheduling, defines the time point that each component is required in manufacturing.

The company is currently subject to high forecast errors; usually demand is underestimated. This leads to extra costs in both procurement and transportation, since the procurement lead-time for the unplanned component requirements should be short. Expedited transportation means can be used as a remedy in decreasing the transportation lead-time, and additional logistics cost may be incurred in order to decrease the lead-time after the arrival of the components to the customs zone. Procurement lead-time can be decreased further if the component manufacturers make extra shifts in order to produce the components earlier. In order to cope with the uncertainty in the end-product demand, the company can hold safety stock for the components. All of these remedies are the factors increasing the transportation costs or inventory holding costs, even the purchasing costs. However, in order to prevent the risk of lost sales, a project has already been started in the company in order to decrease the deviations between the actual demand and the end-product demand forecasts.

Currently, procurement and transportation decisions are made sequentially. First, ordering decisions are made considering the timing of component demand and the transportation lead-times. At this stage the following cost saving opportunities are lost: possible transportation economies of scale that may result from both consolidation of orders associated with different periods of the planning horizon (temporal aggregation), and consolidation of shipments of suppliers that are close to each other. The main objective of this stage is solely to minimize inventory holding cost. Once the orders are ready at the suppliers' facilities, the third party carrier (transportation company) picks up the orders. At this stage the company chooses the minimum transportation cost alternative, given the procurement plan.

Our objective is to propose an integrated transportation and inventory planning system for the imported components. We try to use the advantage of the coordinated replenishment and transportation of the imported components which may decrease the sum of inventory holding and transportation costs of the company, but with a possible increase in inventory holding costs. Since our objective is to exploit the economies of scale in transportation costs, by following an integrated approach for procurement and transportation decisions, we ignore the uncertainty in demand, i.e., we assume that forecasts are perfectly accurate. Hence, the shipment schedule of the end-products together with the manufacturing lead times of the end-products define the timing of dependent demand for each component.

In Chapter 2, the literature on the joint replenishment problem (JRP) is reviewed. First JRP is introduced. Then traditional JRP is discussed with the associated solution approaches. Two main extensions, the studies on stochastic joint replenishment problem and our main focus, dynamic demand-joint replenishment problems are discussed. Finally position of our work in the JRP literature is described in this chapter.

In Chapter 3, the general characteristics of the environment are presented and current procurement and transportation planning practices in the company are discussed; finally the problem is defined in this chapter.

The proposed model for the integrated procurement and transportation decisions is stated and explained in Chapter 4. Verification and validation of the mathematical model is presented at the end of this chapter. Small-sized problem instances are iteratively created based on one initial instance and the differences such as changes in solution time and number of iterations, whether an optimal solution is found or not are observed and presented for these trial problems.

In Chapter 5, solution approach, i.e., decomposition approach, which is proposed to solve our problem in reasonable times, is presented. The approach is discussed in two phases. The modifications done in the original model in order to obtain mathematical model for Phase-1 problem are stated at the end of this chapter.

In Chapter 6, computational study is presented. The effect of working with a number of alternative suppliers on the total cost is analyzed first. Then the proposed solution approach is evaluated. The proposed approach is compared with the originally developed mathematical model and the performance of the proposed system is compared with the current system in terms of total transportation and inventory holding cost. Finally the results are summarized at the end of this chapter.

Finally in Chapter 7 conclusion of the study and remarks for further research are presented.

CHAPTER 2

LITERATURE REVIEW

Our study focuses on purchasing and inventory decisions for the imported components of a manufacturing company. Due to the transportation cost and demand characteristics, our problem can be classified as a variant of dynamic-demand joint replenishment problem (DJRP). In this chapter, we will focus on the DJRP literature. We will review the joint replenishment problem (JRP) literature and traditional joint replenishment problem first. Then DJRP is reviewed. Stochastic demand joint replenishment problem (SJRP), another variant of JRP is not included in the scope of the literature review.

Joint replenishment problem (JRP) is introduced in Section 2.1. Traditional joint replenishment problem where the customer demand is assumed to be stationary over time, and the associated heuristics solution approaches to this problem are discussed in Section 2.2. Our main focus dynamic demand joint replenishment problem is discussed in Sections 2.3. Finally, positioning of our work in JRP literature is made in Section 2.4.

2.1 Introduction to Joint Replenishment Problem

The aim of joint replenishment is to use the advantage of coordinated replenishment of items. Joint replenishment problem models can be used whenever a number of items are involved in a single replenishment and they share a fixed cost associated with replenishment (Goyal and Satir (1989)). In such problems, the objective generally is to minimize total cost while satisfying the demand. Some of the advantages of coordinated replenishment are listed by Silver et al. (1998) are as follows:

- **Decrease in unit purchase or transportation cost**: When a group of items is procured from the same supplier purchase cost may be discounted. Similar case applies to transportation cost. Utilization of full truckload may be justified.
- **Decrease in ordering costs**: If the fixed ordering cost of items is high, coordinated replenishment of items decreases total annual fixed cost.
- **Ease of scheduling:** Coordinated replenishment of items makes the scheduling of receiving and inspection activities easier. Moreover, companies tend to think in terms of vendors or suppliers instead of individual stock keeping units.

There may be some disadvantages of joint replenishment problem such as increase in inventory level and decrease in flexibility because of not working with item basis i.e. possible decrease in service level on individual item basis.

There are three main cost components in JRP:

- **Fixed setup or ordering cost:** Fixed cost terms, which are incurred whenever replenishment is made, are independent of order size. They include setup of machinery and equipment, cost of receiving and preparing the order and transportation cost. There are two main types of fixed ordering costs:
    - **Major ordering cost** is incurred whenever an item is ordered.
    - **Minor ordering cost** is incurred in addition to the major ordering cost whenever a specific item is ordered.

- **Variable ordering costs:** These costs are incurred per item basis. They include unit manufacturing, unit handling and unit transportation costs.
- **Holding cost:** Inventory holding cost is incurred per unit time basis for each item held in inventory. It includes opportunity cost of capital tied up in inventory, and all out-of-pocket costs associated with carrying stock.

Joint replenishment problem has been studied since the early work of Starr and Miller (1962), Balintfy (1964) and Shu (1971). Reviews of work done until 1989, are provided by Goyal and Satir (1989), and Aksoy and Erenguc (1988). Khouja and Goyal (2008) provide a review of the JRP literature between 1989 and 2005.

2.2 Traditional Joint Replenishment Problem

Traditional JRP is known as JRP under deterministic stationary demand. The problem is also known as deterministic joint replenishment problem.

Basic assumptions of the problem are as follows (Silver (1976)):

- The demand is constant and known.
- The planning horizon is infinite.
- Replenishment rate of each item is infinite.
- Unit costs are constant; there is no quantity discount.
- Lead-time is constant.
- Shortages are not allowed.
- Delivery of entire quantity is made at once in any replenishment.

The objective of the problem is to minimize the total cost per unit time. Three elements are considered in the cost function of the problem; fixed replenishment cost, item dependent replenishment and inventory holding cost for individual items (Goyal and Satir (1989)).

Under these assumptions, in traditional JRP, replenishments are made at equal time intervals (Goyal and Satir (1989)). This time interval is referred to as basic replenishment cycle time and the approach is known as basic cycle approach in the literature. Individual items are replenished at equal, integer multiplier of replenishment cycle time, which is referred to as replenishment cycle time of the item.

Basic notation and parameters of the problem is as follows:

i : Item index

n : Number of items ($i=1, 2\ldots n$)

S : Major set up or ordering cost (\$/order)

s_i : Minor ordering cost of item i (\$/order)

h_i : Annual holding cost of item i (\$/unit/year)

D_i: Annual demand of item i (units/year).

Decision variables of the traditional JRP are as follows:

T : Time interval, in years, between replenishments, which is assumed to be a continuous variable. This decision variable is denoted as basic cycle time.

T_i : Time interval, in years, between replenishments of item i. This variable is referred to as replenishment cycle time of items i.

k_i : Number of cycles between successive replenishments of item i: This decision variable is integer multiplier of basic cycle time which is used to calculate cycle time of item i.

K : Set of integer multipliers, $[k_1, k_2\ldots k_n]$.

Q_i : Replenishment quantity of items i in units.

In traditional JRP the objective function is to minimize the total annual holding and ordering costs for all of the products ($/year), which is denoted by $TC(T, K)$.

The total annual cost $TC(T, K)$ is given by:

$$TC(T,K) = C_h + C_o = \frac{T}{2}\sum_{i=1}^{n} k_i D_i h_i + \frac{\left(S + \sum_{i=1}^{n} s_i / k_i\right)}{T}$$

where;

$C_o = S/T + \sum_{i}^{n} s_i / k_i T = \frac{\left(S + \sum_{i=1}^{n} s_i / k_i\right)}{T}$ represents total annual ordering costs, and $C_h = \sum_{i}^{n} Q_i h_i / 2 = \frac{T}{2}\sum_{i=1}^{n} k_i D_i h_i$ represents total annual holding costs.

The mathematical problem is to select *T* and *K* in order to minimize total annual cost *TC (T, K)* (Silver (1976)). Cycle time of individual item *i* is an integer, k_i multiplier of replenishment cycle time *T*, $T_i = k_i T$ Individual cycle time determines the corresponding order quantity for each item; $Q_i = T_i D_i = T k_i D_i$.

Arkin et al. (1989) prove that JRP is NP-hard, that is not solvable in polynomial time.

The core point of the solution of traditional JRP is that vector *K* cannot be determined without knowing *T*; and *T* cannot be known without knowing *K*. The interdependence between *T* and *K* makes the determination of optimal solution difficult. Further studies on heuristics approaches of traditional JRP mainly deal with basic replenishment cycle time; *T* and set of multipliers; *K*. These are known as cyclic policies. There are two types of the cyclic policies: cyclic policy and strict cyclic policy. If there is at least one item included in every order, i.e., $k_i = 1$ for at least one of the items, the policy is referred to as strict cyclic policy otherwise as cyclic policy in the literature.

Goyal (1974) develops an enumeration algorithm to identify all local minima and so global minimum under strict cyclic policy. The algorithm does not guarantee to generate an exact solution for large problems.

There are 2 starting points for the heuristic approaches for the solution of the traditional JRP (Goyal and Satir (1989)): to assume k_i is equal to 1 for all items which is adopted by Brown (1967) and Goyal (1973) and to assume first trial value of T which is first suggested by Silver (1976).

Silver (1976) develops an efficient heuristic algorithm for the problem. Derivative of total annual cost is taken with respect to T in order to find the best T for a particular set K. Substituting T in total cost equation a new equation is obtained which is a function of k_i's. Integer restriction of set K is ignored and partial derivatives with respect to k_i's are taken. It is shown that the item i with the lowest value of $s_i / D_i h_i$ should have the smallest value of k_i which is 1. Setting this particular item i's k_i equal to 1, the other k_i values are determined. Values of k_i's are rounded to the nearest integer greater than 0. Integer restriction on k_i variables results in that more than one item may have k value, which is equal to the unity.

Goyal and Belton (1979) and then Kaspi and Rosenblatt (1983) improve heuristic approach developed by Silver (1976). Common element in these studies is the identification of the first item, which is replenished in each period and then, based on it, deciding on the basic cycle for each item (Kaspi and Rosenblatt (1991)).

Kaspi and Rosenblatt (1991) develop a heuristic solution approach, which is called the RAND heuristic based on Silver (1976) heuristic. The basic idea is that this heuristic compares possible values of T between maximum and minimum cycle times; $T_{\max}$ and $T_{\min}$. These possible values are taken as equally spaced. The algorithm by Kaspi and Rosenblatt (1983) is applied for each T. The T value with the minimum total cost is selected.

Goyal and Desmukh (1993) propose a modified RAND method whose difference is a new and tighter lower bound on $T_{\min}$. This modified RAND showed a 3.5% increase in the number of problems that achieved the optimum when compared with RAND method.

Van Eijs (1993) proposes that in strict cyclic policies total cost will be higher than that of cyclic policies if major ordering cost is low relative to minor cost. This algorithm is also a complete enumeration algorithm identifying all local minima. The difference of Van Eijs (1993) approach from Goyal (1974) is the new lower bound T.

Viswanathan (1996) proposes an algorithm to determine optimal strict cyclic policy as well as optimal cyclic policy. The main focus of Viswanathan's algorithm is to tighten the bounds of T. Computational effort to determine optimal cyclic policy is reduced. Computational examples show that the proposed algorithm performs better than existing ones with the moderate values of the major setup cost. When the major ordering and setup cost is very low, Goyal's method for optimal strict-cyclic policy requires less CPU time than the proposed algorithm for determining the strict-cyclic policy. Van Eijs's method is quite marginal for high values major setup cost. The JRP is relevant only when the major setup cost is moderate (Viswanathan (1996)).

Fung and Ma (2001) improve lower and upper bounds for cyclic and strict cyclic policies when the major cost is high.

Viswanathan (2002) shows that the strict cyclic policy proposed by Fung and Ma (2002) does not guarantee optimal solution. Viswanathan (2002) also gives a counter example to show why results of Fung and Ma's algorithm do not guarantee optimal solution. He proposes a modification to their algorithm, which obtains optimal strict cylic policy. Moreover, the study compares modified Fung and Ma (2002) algorithm and Viswanathan (1996) approach concluding that the latter is computationally more efficient than the former one.

Hariga (1994) develops two efficient heuristics, which generate near optimal replenishment policy iteratively. In the first heuristic all items have the same reorder interval. This reorder interval is cycle time, which is 1. The main idea behind this heuristic procedure is to decrease cycle time so increase ordering frequency. In the second heuristic some items are in a set and cycle time of them is equal to L_i, where L_i is the first integer, which satisfies the condition $L_i(L_i - 1) \leq (T_i / T_0)^2 \leq L_i(L_i + L_{i+1})$. The other items had cycle time value of 1. Frequencies derived from the solution of relaxed problem, in which the assumption that the order intervals are an integer multiple of the cycle time is relaxed, are used for other items. This method is based on the method of Goyal (1974) where starting frequencies are derived from solution of relaxed problem as well. Cycle time is decreased again iteratively but in 2 steps, first for the items in the set and for the others. The procedure continues until ordering frequencies do not change further. Both heuristics provide optimal solutions to the seven problems presented in JRP literature. Number of variables changes from 2 to 20 in these problems.

Ben-Daya and Hariga (1995) test the performance of second heuristic of Hariga (1994) against the modified RAND method of Goyal and Deshmukh (1993) in their numerical experimentation. Hariga (1994)'s algorithm gives lower total cost for 86.9% of the problems studied. In addition, Hariga's algorithm is 21 times faster for 10-product problems and 40 times faster for the 20-product problems.

Wildeman et al. (1997) present an alternative optimal approach based on global optimization theory. They propose that by applying Lipschitz optimization, a solution can be found with small deviation from the optimal. Running time grows only linearly with the number of items while there is an exponential growth in running time in previous studies.

There are also some special solution approaches of the traditional JRP, which are based on different calculation methods of k_i variables.

Power-of-two (PoT) policy of Lee and Yao (2003) and evolutionary computing method are among these special approaches. In PoT policy, replenishment frequency k_i of each item needs to be PoT integer i.e. $k_i=2^p$. The method of (Lee and Yao (2003)) gives shorter search range using powerful bounds.

Evolutionary computing and genetic algorithms (GA) are process of natural evolution and the best solution after some number of iterations represents the near optimal. Khouja et al. (2000) develop a GA and compare it with RAND algorithm. They show that GA's are easy to implement, have an easy code to understand and modify; and allow dealing with constrained JRP easily. Olsen (2005) develops an evolutionary algorithm (EA), compares the results with those of best available algorithm and shows that the new method finds a lower total cost than the best available method for certain parameters.

2.3 Dynamic Demand Joint Replenishment Problem

Dynamic demand JRP is known as the JRP under dynamic-deterministic demand. The problem is also called as dynamic demand coordinated replenishment problem.

In DJRP, demand is deterministic as in classical JRP, but it changes over time. Since the parameter changes with time and decision variables are time dependent, a subscript, *t*, is added to the parameters and decision variables. The objective of the problem is to find a replenishment schedule in order to minimize the sum of common ordering cost, item dependent ordering costs and inventory holding costs over a finite planning horizon while satisfying all demands without backlogging.

Boctor et al. (2004) presents the basic assumptions of the DJRP as follows:

- Replenishments are made at the beginning of each period.
- There is no quantity discount.
- Backlogging is not allowed.
- No limit is imposed on order sizes and inventory levels.

Basic notation, parameters and decision variables for classical DJRP problem are as follows (Boctor et al. (2004)):

- **Notation:**

i : Item index

t : Time index

n : Number of items ($i=1, 2\ldots n$)

T : Number of time periods in planning horizon ($t=1, 2\ldots T$)

- **Parameters:**

S_t : Common ordering cost in period *t*,

s_{it} : Individual ordering cost of item i in period *t*,

h_{it} : Unit inventory holding cost for item i during period *t*,

d_{it} : Demand for item i for period *t*,

M : A large number.

- **Decision variables:**

I_{it} : Inventory level of item i at the end of period *t*,

x_{it} : Replenishment quantity of item i at the beginning of period *t*,

y_{it} :Binary variable that is equal to 1 if item i is replenished at the beginning of period *t*,

z_t : Binary variable that is equal to 1 if an order is placed at the beginning of period *t*,

Mathematical programming formulation is as follows (Boctor et al. (2004)):

$$\text{(DJRP1) Min } \sum_{t=1}^{T}\left[S_t z_t + \sum_{i=1}^{n}\{s_{it}y_{it} + h_{it}I_{it}\}\right] \qquad (1)$$

Subject to

$$I_{i,t-1} + x_{it} - I_{it} = d_{it} \quad (i = 1\ldots, n, t = 1\ldots, T) \qquad (2)$$

$$x_{it} \le My_{it} \quad (i = 1\ldots, n, t = 1\ldots, T) \qquad (3)$$

$$\sum_{i=1}^{n} y_{it} \le nz_t \quad (t = 1\ldots, T) \qquad (4)$$

$$I_{it} \ge 0 \quad (i = 1\ldots, n, t = 1\ldots, T) \qquad (5)$$

$$x_{it} \ge 0 \quad (i = 1\ldots, n, t = 1\ldots, T) \qquad (6)$$

$$y_{it} = 0 \text{ or } 1 \;\; (i = 1\ldots, n, t = 1\ldots, T) \qquad (7)$$

$$z_t = 0 \text{ or } 1 \;\; (t = 1\ldots, T) \qquad (8)$$

Objective function equation (1) gives the total item independent ordering cost, item dependent ordering costs and inventory holding costs over the planning horizon.

Constraint set (2) maintains the inventory balance for each item and for each period. Constraint set (3) guarantees the existence of order quantity for item *i* in case of placing order for item *i* at the beginning of period *t*. Constraint set (4) guarantees that order for item *i* is placed only if an order is given at the beginning of period *t*. Constraint sets (5), (6), (7) shows non-negative and binary variables.

DJRP is proved to be NP-hard (Arkin et al., 1989).

Boctor et al. (2004) classify the exact algorithms for the DJRP: dynamic programming, branch-and-bound, branch-and-cut and Dantzig–Wolfe decomposition.

The problem can be solved in polynomial time using dynamic programming approach for a fixed number of items or periods. Zangwill (1966) is the first to use dynamic programming approach to solve DJRP. He analyses a deterministic multi-product, multi-facility, multi-period production planning model. The objective is to determine an optimal production schedule specifying how much each facility should produce in order to minimize the total cost. Backlogging is allowed in this model; the model considers concave production costs, which can depend upon the production in several different facilities and piecewise concave inventory costs. Then by the theory of concave functions the total cost considered as a function on a particular bounded polyhedral set and is minimized at an extreme point of that set.

Veinott (1967) searches extreme points in order to find an optimal point using dynamic programming recursions. He only gives algorithms whose computational effort increases algebraically with the size of the problem. The important contribution is that broad class of problems can be formulated as minimization of a concave function over the solution set of a Leontief substitution system. A matrix A is called Leonlief if it has exactly one positive element in each column and there is a nonnegative (column) vector x for which Ax is positive (i.e., has all positive components). If there is an optimal solution, then the simplex method will produce a basic optimal solution and the associated optimal basis matrix which is Leontief and has a nonnegative inverse.

Kao (1977) considers a multi-product dynamic lot-sizing problem and presents a dynamic programming formulation in order to find the optimal ordering policy, which calls for a smaller state space than that proposed by Zangwill (1966). Kao (1977) also introduces a very simple heuristic procedure (basic iterative procedure) and two of its variants. The procedure involves iterating between two independent single-product dynamic lot-size problems. First variant of the problem aggregates set-up costs and demands weighted by holding costs. Then resultant one-product problem is solved. Variant 1 and basic iterative procedure are combined in variant 2.

The solution procedures of Zangwill (1966) and Veinott (1967) for finding optimal ordering policies are constrained by the number of products and the number of planning periods involved since the state space is a limiting factor for computer solution. Kao (1979) proposes two alternative solution procedures which call for smaller state space to overcome the difficulty; first one requires a smaller state space than the one under Zangwill's formulation; the other one is a very simple heuristic approach which either produce an optimal solution or a solution close to optimal.

The major drawback of dynamic programming approach is that computational effort grows exponentially with the increasing problem size (Robinson and Gao (1996)) although Veinott (1967) only gives examples where computational load does not grow exponentially.

Erenguc (1988) develops a branch-and-bound approach to solve DJRP. The algorithm branches on major setup time periods to establish when individual item setups may occur. A *T*-period single-product lot sizing problem is solved for each of the *K* items at each node in the branch-and-bound tree. This approach gives solution to the problems up to 12 periods and 20 items. Solution times are approximately linear. However, the solution times are sensitive to cost structures and number of major setup patterns, and increase exponentially with the length of the planning horizon.

Federgruen and Tzur (1994), develop an exact branch-and-bound method for the JRP with time-varying parameters. This method differs from the approach of Erenguc (1988) in the choice of branching rules, upper and lower bounds.

Federgruen and Tzur (1994) solve moderate size problems, i.e., problems with 20-30 periods and 20-30 items.

Kirca (1995) considers multi-item dynamic lot-sizing problem with joint set-up costs (LPJS). A tight formulation of the problem and the dual of the linear relaxation of this formulation are presented. A procedure to solve dual problem is developed where the solution provides a strong lower bound for the LPJS. The lower bound is used in a branch and bound procedure to find an optimal solution to the problem. Difficult class of LPJS problems that are with 50 items and 24 periods can be solved with the proposed algorithm.

Robinson and Gao (1996) develop dual ascent based branch-and-bound algorithm for multi-product dynamic demand coordinated replenishment problem with backlogging. There exists single sourcing property, i.e., each time period's demand is replenished from one replenishment setup time period. The hierarchical structure of the fixed-charge and continuous variables in the formulation yield an extremely tight linear programming relaxation for the problem. Computational results indicate that the new procedures find optimal solutions in less than five percent of the computational time of the most efficient previous algorithm. The heuristic performance of the procedures also demonstrates their superiority over existing approaches. They solve all problems on IBM 3090- 120E computers. The problems with 12 time periods and 20 products are solved in 0.41 CPU seconds. Heuristic solutions with a worst-case three-percent optimality gap are found in 0.068 CPU seconds.

Boctor et al. (2004) propose two new mathematical formulations of the problem based on the properties of classical formulation. These properties are as follows:

Property 1: Any optimal DJRP solution is such that: $x_{it}^{*} \times I_{it-1}^{*} = 0 \quad (i = 1,\dots,n; t = 1,\dots,T)$. If an item is replenished at the beginning of period *t*, it does not pay to hold the item in stock during period $t-1$ (Wagner and Within (1958)).

Property 2: Optimal order x_{it}^{*} takes one of the values: $d_{it}, d_{it} + d_{i,t+1}, \dots, \sum_{q=t}^{T} d_{iq}$ (Wagner and Within (1958)).

Property 3: Optimal inventory level I_{it-1}^{*} takes one of the values: $0, d_{it}, d_{it} + d_{i,t+1}, \dots, \sum_{q=t}^{T} d_{iq}$ (Wagner and Within (1958)).

Property 4: If $d_{iq}\sum_{r=t}^{q-1}h_{ir} > S_q + s_{iq}$ for any $q > t$, then it is not optimal to replenish the demand of item i for period q at the beginning of period t (Silver (1979).

Boctor et al., (2004) propose the following formulation, DJPR2, based on property 2:

$$\text{(DJRP2) Min } \sum_{t=1}^{T}\left[S_t z_t + \sum_{i=1}^{n}\sum_{q=t}^{T} c_{itq} w_{itq}\right] \tag{9}$$

Subject to

$$\sum_{q=1}^{t}\sum_{r=t}^{T} w_{iqr} = 1 \quad (i = 1\ldots,n;\ t = 1\ldots,T) \tag{10}$$

$$\sum_{i=1}^{n}\sum_{q=t}^{T} w_{itq} \le n z_t \quad (t = 1\ldots,T) \tag{11}$$

$$w_{itq} = 0 \text{ or } 1 \ \ (i = 1\ldots,n;\ t = 1\ldots,T;\ q = 1\ldots,T) \tag{12}$$

$$z_t = 0 \text{ or } 1 \ \ (t = 1\ldots,T). \tag{13}$$

Objective function equation (9) gives the total item independent ordering cost, and the sum of item dependent ordering and inventory holding costs over the planning horizon.

Constraint set (10) guarantees that in each period only one order is placed and demand of a period is satisfied only by an order placed in one of the previous period or by an order placed at the beginning of that period. Constraint set (11) guarantees that order for item *i* which covers the demand until period *q* is placed only if an order is given at the beginning of period *t*. Constraint set (12) and (13) shows binary variables.

Binary variable w_{itq} takes the value of 1 if the order of item i at the beginning of t covers the demand of the item until period *q*. Moreover c_{itq} is defined as the sum of individual holding and ordering costs of item *i*, i.e. $c_{itq} = s_{it} + \sum_{r=t+1}^{q}\left(\sum_{k=t}^{r-1} h_{ik}\right) d_{ir}$. Property 4 can also be used to reduce the number of w_{itq} variables in the above model.

Second new formulation of Boctor et al. (2004) defines again a binary variable u_{itq} which takes the value of 1 if and only if the demand of item i for period q is included in the replenishment made at the beginning of period *t*. The formulation is as follows:

$$\text{(DJRP3) Min } \sum_{t=1}^{T} S_t z_t + \sum_{i=1}^{n}\sum_{t=1}^{T} s_{it} u_{itt} + \sum_{i=1}^{n}\sum_{t=2}^{T}\sum_{q=1}^{t-1}\left(\sum_{r=q}^{t-1} h_{ir}\right) d_{it} u_{iqt} \tag{14}$$

Subject to

$$\sum_{q=1}^{t} u_{iqt} = 1 \ \ (i = 1\ldots,n;\ t = 1\ldots,T) \tag{15}$$

$$\sum_{i=1}^{n} u_{itt} \le nz_t \ \ (t = 1\ldots,T) \tag{16}$$

$$u_{itq} \le u_{itt} \ \ (i = 1\ldots,n;\ t = 1\ldots,T-1;\ q = t+1\ldots,T) \tag{17}$$

$$u_{itq} = 0 \text{ or } 1 \ \ (i = 1\ldots,n;\ t = 1\ldots,T;\ q = 1\ldots,T) \tag{18}$$

$$z_t = 0 \text{ or } 1 \ \ (t = 1\ldots,T). \tag{19}$$

Objective function equation (14) gives the total item independent ordering cost, item dependent ordering costs and inventory holding costs over the planning horizon.

Constraint set (15) guarantees that demand of a specific period is satisfied by an order placed in only one period. Constraint set (16) guarantees the existence of order quantity for item i which is placed in period t to satisfy demand of t in case of placing order for item i at the beginning of period t. Constraint set (17) guarantees that if the demand of a specific period is not satisfied by an order placed in that period no successive period's demand is not satisfied by that order. Constraint set (18) and (19) shows binary variables. Property 4 is used in this model as well.

Large DJRP problems can not be solved by exact algorithms. In the literature, there are heuristics proposed in order to deal with larger problem instances.

Fogarty and Barringer (1987) develop one of the earliest heuristic approaches of DJRP. The major point of this heuristic is that a replenishment covers all the demands until the next replenishment.

Federgruen and Tzur (1994) propose a greedy add heuristic for the DJRP. The solution of this greedy heuristic is used as an upper bound for branch-and-bound procedure developed by Federgruen and Tzur (1994). The heuristic approach starts with an empty set. The set is incremented by a single period at each iteration whose addition results in the largest cost saving for the DJRP. The heuristic terminates when replenishment is made in each period or when the cost cannot be reduced by the addition of a period.

Greedy drop heuristic is the counterpart of greedy adds heuristic. Replenishment is made in each period at the beginning and they are iteratively removed as long as saving is obtained.

There are also two extensions of Silver and Meal (1973) heuristic to the DJRP (Boctor et al. (2004)). Atkins and Iyogun (1988) extend the Silver-Meal heuristic of the single item model to joint replenishment problem with time-varying demands but constant cost parameters. The second extension is developed by Iyogun (1991). The difference between these two extensions is that the former one uses period *t* in order to test for replenishment but the latter one uses period *t-1*.

Iyogun (1991) develops two generalization of part-period heuristic of De Matteis and Mendoza (1968). The part period balancing heuristic minimizes the absolute difference between the set-up cost and the inventory holding cost until the next set-up. Iyogun (1991) shows that when part-period balancing heuristic with worst case performance of 1/3 is generalized to multi-product dynamic lot-size problem, worst-case performance can not be less than 1/3. Iyogun (1991) also shows that when simple variant of part-period balancing heuristic is used this bound will be 1/2 and is preserved when it is generalized to multiple-product problem.

Silver and Kelle (1988) proposes an improvement heuristic which can be applied to any feasible solution. It is looked for whether there is a cost saving or not by adding the order quantity of the item to the previous replenishment.

Robinson and Lawrence (2004) describe a mixed-integer programming formulation and Lagrange relaxation procedure for the single-family coordinated capacitated lot-sizing problem with dynamic demand. The results indicate the superiority of the dual-based heuristic over linear programming-based approaches to the problem. The quality of the Lagrange heuristic solution improved in most instances with increases in problem size. Heuristic solutions averaged 2.52% above optimal.

Boctor et al. (2004) provide a systematic and unified description of the available DJRP heuristics and fill the gap of performing computational comparison of the proposed algorithms. They also propose a new improvement procedure that can be combined with any heuristic. The proposed improvement procedure of Boctor et al. (2004) is a local search procedure that uses the solution perturbation principle introduced by Storer et al. (1992) to attempt escaping local optima. This heuristic is referred to as perturbation heuristic. Small random change is done to a feasible solution while maintaining feasibility and post optimization of perturbed solution. First, a feasible solution is found by Fogarty and Barringer (1987) heuristic. Then a random t is chosen in the planning horizon. If there is replenishment at period t, it is cancelled and combined with the previous replenishment but if there is no replenishment at period t, previous replenishment is cancelled and replenishment at period t is added. After that, greedy drop heuristic and Silver-Kelle improvement heuristic is applied. If the solution is improved a new random t is chosen and same steps are applied until no more improvement in the solution.

Boctor et al. (2004) additionally compare classical mathematical formulation and the new two mathematical formulations of the problem. In order to compare mathematical formulations and heuristics 720 instances are generated randomly and divided into four equal-sized subgroups of different instant sizes: n=10, T=13; n=10, T=26; n=20, T=13; and n=20, T=26. Then each group is made up of six equal subgroups called S1, S2, S3, S4, S5 and S6. Inventory holding costs and ordering costs are taken as constant to simplify the problem. All instances are solved to optimality by CPLEX 8.0 (on a Pentium III, 1.26GHz with a Windows 2000 Server) using three mathematical formulations mentioned above. The computation times associated with first new mathematical model are three times smaller than those of classical mathematical formulation. Similarly, second new mathematical model cuts the time of classical model by half.

Boctor et al. (2004) also compare some heuristic approaches of DJRP and the new perturbation heuristics. These heuristics are: Fogarty and Barringer (1987) heuristic with Silver and Kelle (1988) improvement heuristic, greedy add heuristic proposed by Federgruen and Tzur (1994), Atkins and Iyogun's (1988) extension of Silver-Meal's (Silver and Meal, 1973) heuristic, Iyogun's (1991) modification of part period balancing method that is first proposed by De Matteis and Mendoza (1968). All heuristic solution values are compared with each other and with the optimal values. It is shown that the Fogarty–Barringer heuristic with Silver–Kelle improvement procedure, while being relatively simple, offers the best performance. Applying new improvement heuristic, perturbation method, after Fogarty and Barringer heuristic with Silver-Kelle improvement procedure reduces the average deviation from 0.028 to 0.014 on the test problems. It also produces the smallest maximal deviation from the optimum and largest number of optimal solutions. All computational times are negligible (<1 second on the average). The method identifies the global minimum for maximum number of problem instances; 646 out of 1000 instances, and the smallest average deviation when it does not give optimal.

Narayanan and Robinson (2006) investigate the tightness of the two mathematical formulations of Boctor et al. (2004). They propose disaggregating the variable upper bound constraints to obtain the convex envelope relaxations of DJRP. This relaxation represents the joint set-up variables more tightly to take on integer values in the linear programming (LP) relaxation. They also evaluate the performance of the perturbation heuristic of Boctor at al. (2004) and dual-ascent heuristic of Robinson and Gao (1996) for solving DJRP. They show that the perturbation heuristic is superior at relatively low set-up cost ratios and high joint set-up cost levels, whereas the dual-ascent heuristic strongly dominates at relatively high set-up cost ratios and low joint set-up costs. Narayanan and Robinson (2006) suggest solving the problem with both of the heuristics and to implement the best found solution since both methods is found to be efficient.

Robinson et al. (2007) present two forward-pass heuristics; a two-phase heuristic and a simulated annealing metaheuristic (SAM) to solve DJRP. The new heuristics consider economic interaction among joint setup costs and items setup costs explicitly. Forward-pass heuristics find high-quality solutions averaging 1.42% and 1.53% from the optimality. Two-phase heuristic improves the optimality gap of Fogarty and Barringer (1987) heuristic with Silver and Kelle (1988) improvement procedure from 0.92% to 0.56%. SAM with 0.2% optimality gap improves 0.87% optimality gap of perturbation heuristic of Boctor et al. (2004) (Robinson et al. (2007)).

2.4 Position of the Study in JRP Literature

Studies on optimal solutions for classical JRP reached to a saturation point. Decreasing the solution time of the problem takes more attention than finding the optimal solution in current studies. Moreover, practical extensions deal with constrained versions of the JRP such as storage, transportation capacity, budget and other resource constraints (Hoque (2006), Moon and Cha (2006) Porras and Decker (2006)) and variable production costs (Bayindir et al. (2006)). Moreover it is commented that the trend in the literature of JRP evolved into developing models applicable for real cases (Khouja and Goyal (2008)).

Our study will contribute to the need for real life applications for the least studied extension of the problem DJRP.

Joint costs as well as the demand structures of the items make our problem a variant of dynamic demand joint replenishment problem. Joint costs occur in the form of transportation costs in our study. Inventory planning activities are also critical as a result of high input material stock value of the chosen items. Our aim is to propose an integrated inventory and transportation planning for these purchased parts using the advantages of joint transportation costs within the supplier clusters defined.

CHAPTER 3

DESCRIPTION OF THE PROBLEM ENVIRONMENT

In this chapter we describe the problem environment. The physical environment that the problem is associated with and an overview of the current purchasing practice are provided in Section 3.1. Demand characteristics of the components are discussed in Section 3.2. The supplier characteristics and constraints in ordering are stated in Section 3.3, while transportation processes are discussed in detail in Section 3.4. Then operations conducted after the goods arrive at customs are described in Section 3.5. Finally, operations at the manufacturing plant are explained in Section 3.6.

3.1 Physical Environment and Current Purchasing Practice

There are two manufacturing plants within the campus of the company. In this study we represent the two plants as a single manufacturing plant. Two main end products -ovens and hobs- are produced in the two manufacturing plants.

80-85 % of the ovens (free standing ovens, range cookers, some types of built-in ovens) are produced in the first plant. Hobs (built-in and table top hobs), mini/midi ovens, and several other ones like the special branded built-in ovens are produced in the second plant. There are many common components in the bill-of-material (BOM) structures of the products. Purchasing and material requirements planning for the components are made centrally.

There are two main material categories in the BOM structures of the end-products: raw materials and semi-products. Raw materials are procured from both local and foreign suppliers, whereas semi-products are manufactured both by the manufacturing departments in the company and subcontractors. In our study we focus only on the raw materials' procurement decisions.

In the company, raw materials that are directly procured from the suppliers are classified into four main material groups:

- **Sheet steel, raw materials used in enameling, painting, plastics manufacturing operations and fabrication of styrofoam**: This group consists of raw materials which are used for manufacturing semi-products: metal, enameled, painted and plastic parts as well as styrofoam that is used as a raw material itself. This group of materials is procured from both local and foreign suppliers.
- **Adhesive substances:** They are used in manufacturing processes of semi-products. 95 % of the stock keeping units (SKUs) of these adhesive substances are procured from foreign suppliers.
- **The parts used to build the product structure, accessories and printed materials:** This group of materials consists of metal parts, packing materials, accessories and printed materials. Metal parts are both produced by the manufacturing departments and the local suppliers. These parts are used to build the structure of the end products and the structures where the components are assembled. Packing materials are both used for the end-products and the spare parts for end-of-sale services. Except the packaging stripe of end-product boxes, all of the packing materials are procured from local suppliers. Accessories such as buttons and handles are the parts which are used to activate the product functions or designed to allow the end

users to manage and control the products; these parts are produced both by the manufacturing departments and local suppliers. Lastly, printed materials consist of etiquettes, stickers to indicate some product characteristics and operating manuals of the end-products. Printed materials are procured from local suppliers.

- **Components:** These parts are the core parts of ovens and hobs that are used to build the working mechanism of the end-products. Components are made up of metal, plastic, ceramic or glass-ceramic materials and they may have mechanical, electrical, electronic, electromechanical functions. Components are procured both from local and foreign suppliers but locally supplied components have a small proportion. 90 % of the components are imported.

There are more than 40,000 stock keeping units (SKUs) within the four groups of raw materials defined above. Every month about 200 new SKUs are defined and added in the system due to the dynamic nature of the market and the product mix. There are about 17,000 SKUs, about 3,500 of which are semi-products stocked in the plant at any time instance.

Among the four main raw material groups, the group of imported components is the one with the most functional variety of SKUs among the raw materials in the plant. There are 205 raw materials classes defined in the system for these four main raw material groups. Classification criteria of these materials in the plant are the functional and structural similarities of the materials. Moreover, the type of the supplier for the raw material is a classification criterion (local, foreign, both local and foreign); similarly, the plant where the raw material is used is a classification criterion. used as an additional. 95 of these 205 raw material classes are related with the imported components. In our study we consider 50 classes of the imported components out of 95 imported raw material classes. Although the imported components make up about only 7 % of the raw material SKUs, they account for 30-35 % of total direct material cost of the end-products, and about 30 % of the input material stock value is associated with the imported components.

The procurement lead times of the imported components are longer than those of the locally supplied raw materials. Depending on the location of the supplier the orders are received within 4-20 weeks. Imported components are usually treated as critical components, since their unavailability gives rise to idleness in the manufacturing plants.

Inventory planning activities, time and cost considerations of these activities for the imported components are critical for the company. This group is the one with the highest material variety for the given number of SKUs. The associated procurement lead times are very long for the imported components. Decision on the replenishment quantities and the timeliness of the arrivals of these components at the manufacturing plant are very crucial in order to meet the demand on time. Moreover, about one third of direct material cost of products and input material stock value are related to these components; hence decreasing the inventory level while meeting the demand on time is essential.

Any procurement or quality problem about one of the raw material SKUs may have tremendous effect for the procurement process of the other SKUs. If there is a procurement problem for a material which is particular to one group of products, none of the products in the group can be produced. In order not to stop the production lines, production program has to be revised and this may cause other SKUs to be procured in advance with extra expenses. While sustaining the production processes; inventory holding cost should be kept at the minimum level.

Sheet steel and raw materials of enameling, painting, plastics is out of consideration in such cases and therefore in our study. Their inventory level should cover at least 3 months' production requirements, since a problem in the procurement process of these raw materials causes the manufacturing plant to be closed until the problem is solved.

The materials that are procured from the local suppliers have little effect on both production continuity and inventory holding cost, since their procurement lead time is about 2-3 days and their input material stock value is about 8 % of the total value.

Supply chain of the company regarding transportation and inventory planning for the imported components consists of the two manufacturing plants at the same location, one central warehouse, more than 120 suppliers located in about 25 countries all around the world, about 15 transportation companies, 25 forwarding agencies of the transportation companies and 2 companies responsible for customs procedure before the transfer of the components to the manufacturing plant.

A purchase order is placed and delivered in the supply chain as follows: first, the order is placed to the supplier considering timing and quantity of the imported component demand. The supplier processes the order and assigns a delivery date for this order. When the lot is ready in the supplier's plant, the transportation company is informed so that they can pick up the ready lot. After the lot arrives at the customs zone, the required activities are completed and the component is transferred to the central warehouse. Upon completion of the whole customs procedure, the lot is transferred to the manufacturing plant where it is either used directly in production or preprocessed.

From the time the orders are placed to the suppliers till the components are used in the main assembly of the end-products, there are 3 different locations where the components are stored:

1. The central warehouse where the components are stored before being transferred to the manufacturing plant.
2. The manufacturing plant's warehouse.
3. The subcontractor's warehouse if they are not directly assembled to the end-products but assembled to other components by subcontractors.

The interaction of the actors and entities of the supply chain are maintained by the two main actors of the chain: engineers that are responsible for materials requirement planning in the production planning department and logistics staff that are responsible for import logistics. Production planning department is located in the manufacturing plant, whereas logistics department is in the Headquarters.

Determining the components demand and their timing is the starting point regarding the procurement activities. Demand of these imported components has two parts: dependent demand and independent demand. Dependent demand part of a component comes from the end-product demand and BOM structure of the products, while the independent demand part comes from the after-sale service operations. In general, the demand for components as spare parts is lower compared to the demand for components to be used in end-product manufacturing.

Dependent demand for the imported components can be determined based on the end-product production quantities and BOM structure. Dependency between end-product demand and imported components demand, and effect of end-product variety on the imported component variety is discussed in Section 3.2.

In reality, independent demand depends on the past sales of the company. The components that are not used in the manufacturing of the current product mix can be demanded as spare parts if they were used in the products in the previous product mixes. However, this relation is highly complex and there is no sufficient reliability data of the components and the products that are in use by the customers. Therefore, we treat the demand of components which are spare parts as the independent demand.

Figure 3.1 depicts, on a timeline, the events associated with the procurement and preprocessing of imported materials from the order placement till the usage in the main assembly of end-products.

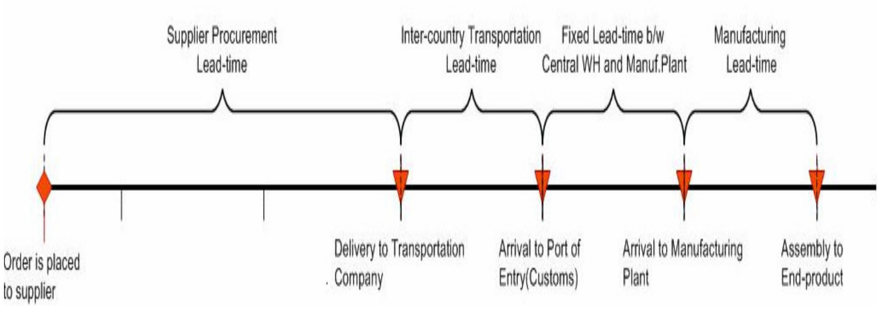

Figure 3.1: Timeline of procurement and preprocessing of imported materials

As it is seen in Figure 3.1, first, orders are placed to suppliers from the manufacturing plant by the engineers that are responsible for materials requirement planning. Suppliers need time to deliver the materials to the transportation company. During this time period, first orders are processed in the suppliers' system, and then a delivery date is confirmed. After the components are manufactured by the supplier, they are delivered to the transportation company. The time period from the order placement till the delivery to the transportation company is called the **supplier procurement lead-time.**

Transportation company delivers the materials to the port of entry after completing some official customs procedures. Then the vehicle is dispatched. In the company's current practice, inter-country transportation activities can be conducted by either the supplier or the company. But here in this study we restrict ourselves to the case where all transportation activities are arranged by the company in order to fully exploit the cost saving opportunities in transportation. As for air-freight transportation, the transportation company takes the components from the supplier's location by its own trucks, and then delivers the materials to the airplane after completing the local customs procedure in the country of origin.

The inter-country transportation activity ends when the vehicle reaches the port of entry, i.e. customs zone. This time period depends both on the distance and the mode of transportation used and is referred to as the **inter-country transportation lead-time.**

After arrival at the port of entry, local customs procedures are completed. All the documents are collected and filed in the logistics department of the company. Then logistics operators record the receipt of the materials in the ERP system. It is decided whether or not to unload the components at the central warehouse of the company depending on the component characteristics and urgency of need. In our study we assume that all the components are first unloaded at the central warehouse before being transferred to the manufacturing plant. Unloading time changes with respect to the mode of transportation that is used, as it is discussed in Section 3.5. After a temporary storage period and the required customs procedures, the components are sent to the manufacturing plant by a local transportation company. Customs procedures are completed in different time periods depending on whether customs incentives are used for the suppliers or not. The time period between the arrival of the components at the port of entry and arrival of the components at the manufacturing plant is referred to as the **fixed lead-time between customs zone and manufacturing plant** which is affected by both the mode of transportation and the supplier the components are procured from.

As the last step, after the components arrive at the manufacturing plant, the materials are unloaded, necessary quantity-control procedures are completed, and the receipt of materials is recorded in the ERP system. Then quality control department performs the necessary tests on the components. Upon approval, the components are stored in the warehouse. In our study, we assume that all components conform to the quality standards. After the quality control process; according to where the components are used, they are sent to either the subcontractors or the associated manufacturing department in the plant. Eventually, all components are assembled to the end-products. The time period from the arrival of the components at the manufacturing plant till the main assembly process is referred to as the **manufacturing lead-time**

3.2 Characteristics of Component Demand

Demand of a component has two parts: dependent demand due to requirements of end-product production and independent demand due to requirements as spare parts.

End-products have multilevel BOM structures. Per-unit quantity of components used in the end-products and the level of component in the BOM structures define the dependent demand part of the total demand for components.

There exists high variety in the product mix of the cooking appliance plant. Moreover, the product mix changes continuously. While some of the products turn to be out of style, some new products are introduced in the market. Sometimes, design changes yield a new product model, and as a result product variety increases. However, new products are to be introduced to the market mostly by structural changes, functional changes and innovations. Such structural and functional differences among products usually require component changes. New components are possibly introduced by the new product models. The variety in the end-product mix brings about variety in the components used; the fluctuations in the demands of end-products bring about fluctuations in the component demand.

The shipment schedule of the end-products from the company together with the lead times defines the timing of demand for each component. The shipment plan for the end-products is generated by the marketing department. Production plan of the manufacturing plant is prepared taking this shipment plan into account. But the time that the requirements become certain does not allow for the order planning of the components based on the realized demand figures of the end-products. Forecasts of end-product demand are used instead of the actual demand for inventory planning. Possible discrepancies between forecasts and actual demand may cause stockouts or high inventory in the manufacturing plant. Marketing department works in order to prevent fluctuations between forecasts and actual demand; so forecasts of demand for the end-products are assumed to be the real values in our study.

Currently, the components are classified based on their functions. Functional classes are referred to as “component classes” each of which is given a code is in the system. Material planning of each component class is the responsibility of one person in order to make planning easier because demands of components in the same cluster are believed to be somehow correlated. Component classes are used in supplier quota calculations, which are discussed in Section 3.3.

Independent part of component demand that stems from the requirements of spare parts for end-of-sale services is treated as external demand for the components in our approach. Independent part of component demand is determined based on the quantities ordered by the end-of-sale service department in the Headquarters.

3.3 Supplier Characteristics and Ordering Constraints

A certain supplier may supply a number of different components that may possibly belong to different component clusters. Similarly, there may be multiple suppliers for a certain component. Unit price, supplier procurement lead-time, customer service quality differ among the suppliers. Supplier selection has already been made in order to increase bargaining power on purchasing by the company; hence, we do not have the supplier selection problem. Moreover, since there is a tendency of the company to purchase from the far-eastern countries, currently there is likely to be more components with multiple suppliers than there were before.

Orders for a certain component are placed among the alternative suppliers according to the predetermined quotas; total purchase quantity is rationed among the available suppliers. Purchasing quota of a supplier is defined as the ratio of the planned purchase amount of a component from that supplier in a specific time period to the total planned purchase amount of the component in that time period. The time period in quota calculations generally is taken as the planning horizon. Although purchasing quota is defined for each component separately, the performance measure is defined as the conformance to the total planned purchase amount of the component class, but not to the total planned purchase amount of components individually. The total realized purchase amount of the component class is compared with the total planned purchase amount of the component class.

Decision problems about supplier selection and setting of the purchasing quotas are excluded in this study. We restrict ourselves to the current suppliers and their purchasing quotas.

Another restriction on ordering decisions comes from the box sizes: order quantity of a component should be multiples of its box size. Different suppliers of a component usually use non-identical boxes. Therefore, this restriction should be taken care of for each supplier-component combination.

All of the suppliers work on a make-to-order basis and need time to manufacture the components upon receipt of the order. Immediate procurement occasions are sometimes observed, when the supplier has on-hand inventory especially for the components that they supply to other customers as well. Such occasions are exceptional and supplier procurement lead times are defined in the purchasing agreements as well. In practice, procurement lead time of an order is equal to at most the lead time defined in the agreement. However, in our study, the supplier procurement lead time for a component is assumed to be constant and independent of the order quantity.

There is no fixed ordering cost incurred, since the orders are placed to the suppliers via e-mail.

In the current system each supplier gets in touch with the related transportation company before the deliveries. The deliveries are not under the control of the logistics department of the company, so that consolidation possibilities of the deliveries of suppliers that are in the same country can not be taken into account. In our study, each country is considered as a supplier cluster in order to take the opportunity of consolidated shipments into account.

3.4 Transportation Processes

There are both inter-country and domestic transportation activities within the system.

3.4.1 Inter-country Transportation Activities

As for inter-country transportation activities, there are about 15 transportation companies that the company works with for both inbound and outbound logistics. These transportation companies have different agencies in different regions of the countries.

In our study we assume that all transportation activities are arranged not by the suppliers, but by a third party transportation company which is under the control of the logistics department. In order to see the advantages of consolidated shipments, we additionally assume that there is a single transportation company to work with in each supplier cluster. The goods that are transported from the same supplier cluster can be consolidated. Transportation company delivers the components through direct shipment if the load is full-truckload, otherwise use the consolidation terminals.

The possible modes of transportation that the transportation companies utilize are trucks, ship and air. Mode of transportation is chosen taking into account the required delivery time and transportation cost. Sometimes expedited modes of transportation, e.g. using air-freight instead of truck, using truck with two-drivers, transporting the components as full-truckload even if it is less-than-truckload, can be used. Moreover, it is assumed that there is no restriction on the number of vehicles departing from a country in a given period. Moreover, the vehicles are equally sized and there are no special loading principles used for the components, like loading onto the truck with two layers.

In this study, our aim is to choose the best mode among the possible transportation modes while decreasing the sum of inventory holding and transportation costs.

There are five alternative transportation modes used for inter-country transportation activities as follows:

1- Full-truckload transportation
2- Less-than-truckload transportation
3- Full-truckload with two drivers
4- Less-than-truckload with two drivers
5- Air-freight

Transportation costs and inter-country transportation lead-times differ among the above transportation modes. Moreover, transportation mode used affects the fixed lead-time between customs zone and the manufacturing plant.

Transportation costs depend on both weight and volume of components transported. Since both measures are almost directly proportional to each other for the components in concern, we only include the dependency of costs on weight.

The inter-country transportation lead-time and cost structures of these modes are discussed in the following sections.

3.4.1.1 Full-truckload Transportation

Components are directly transported from the supplier to the port-of-entry when full-truckload (FTL) mode is utilized. Transportation lead-time is less for FTL than less-than-truckload (LTL), since additional time is needed for consolidation of the loads from several customers in LTL mode. A fixed cost is incurred for each vehicle utilized as it can be seen below in Figure 3.2. Moreover, FTL transportation can be used even when the truck is not loaded in full capacity in order to decrease the transportation lead-time while still having a cost advantage. In the current system FTL transportation is used directly from one supplier to the port-of-entry.

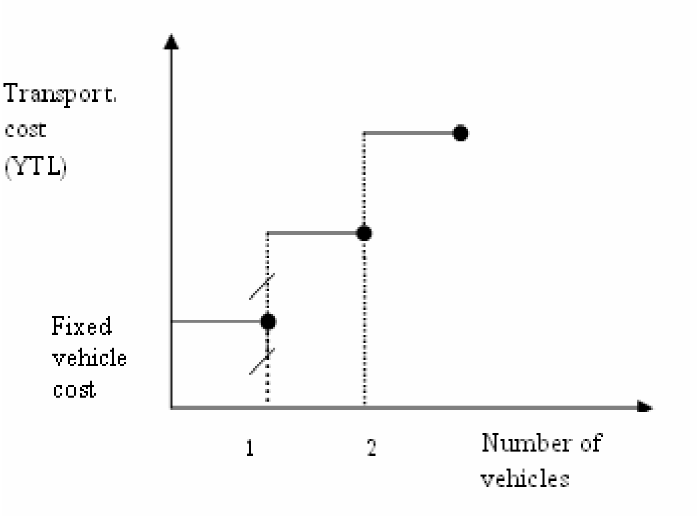

Figure 3.2: Cost structure for full-truckload shipment

3.4.1.2 Less-than-truckload Transportation

Components are not directly transported from the supplier to the port-of-entry in LTL shipments. Vehicles have to wait for consolidation of components of other customers the transportation company is working with. The vehicle is loaded in full capacity after consolidation of the customers' loads. However, for the company in concern, this mode of transportation is LTL transportation.

General cost structure of LTL transportation is provided in Figure 3.3. A fixed cost is incurred up to a certain threshold value for the weight. In general, this threshold value for the weight is different for different countries. When this threshold weight is exceeded, a variable cost is incurred for each kilogram of weight transported. This variable cost is calculated basically by dividing the fixed cost by the threshold weight.

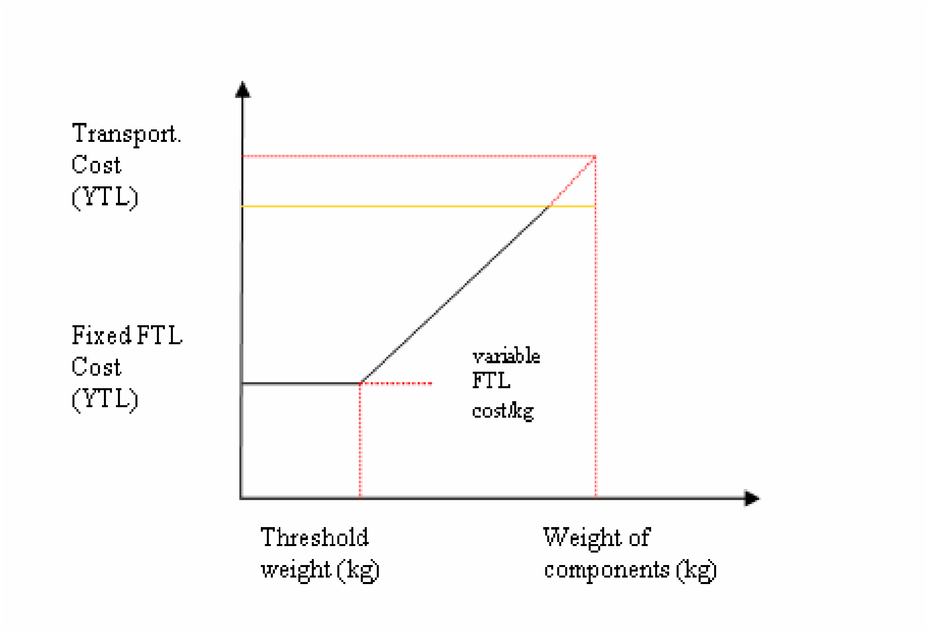

Figure 3.3: Cost structure for less-than-truckload shipment

3.4.1.3 Full-truckload with Two Drivers

Two-driver trucks are used to expedite the delivery. Cost structure is the same as that of FTL transportation mode; however, fixed costs are higher compared to regular FTL.

3.4.1.4 Less-than-truckload with Two Drivers

As in FTL transportation, less-than-truckload shipments can also be made by two drivers. Again the cost structure is the same as that of LTL except an additional fixed two-driver cost is incurred.

3.4.1.5 Air-freight

Cost structure is the same as in LTL transportation. A fixed cost is incurred up to a certain threshold weight. In general, this threshold weight is different for different countries.

3.4.2 Domestic Transportation Activities

Domestic transportation occurs first from the port of entry to the central warehouse, then from the central warehouse to the manufacturing plant. When the components are urgently needed for production, in order to decrease the fixed lead time between the customs zone and the manufacturing plant, domestic transportation may occur from the port of entry directly to the manufacturing plant which means extra domestic transportation cost for the manufacturing plant. We ignore this possibility and assume that all incoming materials are temporarily stored at the central warehouse.

3.5 Operations after the Goods Arrive at Customs

After the vehicle reaches the port of entry, the transportation company prepares the related official documents and delivers them to customs broker. All the documents are delivered to the logistics department in order for the receipt of the components to be recorded into the ERP system of the company.

If the components are needed by the manufacturing plant urgently, the components can be imported directly from the port of entry. Otherwise the components are sent to the central warehouse. In order to send the components to the central warehouse, some documents are prepared before the transportation to the central warehouse. Then the goods are unloaded at the central warehouse. In this study it is assumed that all the components are unloaded at the central warehouse after arrival at the port of entry in Turkey.

Even all the components are assumed to be unloaded at the central warehouse first, this time period is not the same for all transportation modes. It differs depending on the mode of transportation used. Moreover, the completion time of customs procedure differs among the suppliers, that is the result of using customs incentives for some suppliers. Hence, the fixed lead time between customs zone and manufacturing plant is affected by both the mode of transportation used and the supplier.

Wherever the components are imported directly from the port of entry, either to the central warehouse or the manufacturing plant, after the customs procedures, the components are sent to the manufacturing plant by an internal transportation activity.

This time period from the arrival of the components to the port of entry till the arrival of the components to the manufacturing plant is the fixed lead-time. This lead-time depends on both the supplier and the mode of transportation used for each component.

The components may be stored at the central warehouse for a while after being recorded at the central warehouse stock records. However, in this study it is assumed that all the arriving components are immediately sent to the manufacturing plant after being unloaded at the central warehouse. The components are assumed to be held as pipeline inventory in the time interval between the material receipt of the components being recorded at the central warehouse stocks and arriving at the manufacturing plant, since inventory holding cost is incurred upon receipt at the central warehouse stocks. In our study, pipeline inventory represents the inventory that is held at the central warehouse. The period of time the components are held as pipeline inventory is also affected by both the mode of transportation and the supplier. Customs incentives are used for payments to some of the suppliers which affect the completion time of customs procedures.

3.6 Operations in the Manufacturing Plant

After the truck arrives at the warehouse of the manufacturing plant, the components are unloaded and checked for quantity and package quality. Then component receipt is recorded at the plant’s stocks in the ERP system.

At the same time incoming quality control department is informed about the incoming components by the system. Components are controlled by the incoming quality control department. If they are approved, they are stored at the warehouse; if not, they are kept in the “rejected components” area. In this study we ignore the probable quality problems; and assume that all the components are approved by the incoming quality control department.

After the components are unloaded at the warehouse in the manufacturing plant and approved by the incoming quality control department, they may be brought directly to the assembly lines or first stored at the warehouse, depending on the urgency of the need for the component. If the components are in the lower levels in BOM structure, they are sent to the manufacturing units in the plant or the subcontractors to be used in the subassemblies.

The time period from the arrival of the components at the manufacturing plant to the time the component or preassembled part is used in the assembly lines is defined as the manufacturing lead-time of the component. Although there may be some problems at any step of manufacturing/assembly processes in practice, we assume that the manufacturing lead-time is constant.

CHAPTER 4

THE MATHEMATICAL MODEL

In this chapter we develop the mathematical programming formulation for the integrated procurement and transportation decisions. In Section 4.1, first the mathematical model and its assumptions are presented. Then the model is described in detail. In Section 4.2, model verification and validation are presented. Computational requirements for the optimal solution of the model are searched by gradually increasing the model size and changing the model parameters.

4.1 Model Formulation

We consider a supply chain that consists of $|J|$ imported components that are procured from $|S|$ suppliers being located all around the world and grouped into $|C|$ supplier clusters where a cluster represents a country. In order to cope with the ordering constraints and purchasing quota, these components are grouped into $|F|$ imported component classes. Each component is procured from $|S_{jc}|$ suppliers in a given supplier cluster. A single third party transportation company, with a fleet consisting of vehicles for full-truckload, less-than-truckload and air-freight transportation modes, is assumed to conduct the inter-country transportation activities after the suppliers make the components ready for shipment. Also there exist a central warehouse, a plant warehouse and a national transportation company operating between these warehouses.

In this study, we aim to formulate the multiple-item dynamic-demand joint replenishment problem as a mathematical model in order to use the advantages of joint ordering and transportation strategies within supplier clusters to decrease the total inventory holding and transportation costs over the planning horizon. Purchasing costs are not included in the objective function, since the purchasing cost of a component with alternative suppliers is almost the same over all suppliers.

There are more than one supplier in almost every country, since the total number of suppliers is more than 120; 25 clusters. Moreover, there is more than one SKU procured from almost every supplier. This characteristic of the system brings the possibility for the consolidated transportation of the components delivered from the same supplier or several suppliers in the same country. Coordination of procurement and transportation of different SKUs coming from the same or different suppliers may justify utilization of full trucks or containers, decrease the fixed transportation cost, decrease the number of expedited deliveries, and decrease in total transportation costs. Consolidation of orders that correspond to different periods of the planning horizon has similar effects on transportation with a probable increase in inventory holding costs.

In our proposed integrated model, we make procurement and transportation decisions in coordination with each other. Specifically, the decisions made for each period of the planning horizon are:

- Quantity to order for each SKU from each supplier cluster
- Quantity to transport for each SKU from each supplier with each mode of transportation
- Transportation modes to be used for each supplier cluster

- Number of trucks to be used for each transportation mode in each supplier cluster
- Inventory held at the central warehouse and at the plant's warehouse at the end of each period for each component

We have made the following assumptions in our model formulation as follows:

- Planning horizon is finite.
- Demand of the components is deterministic and dynamic (time varying over time).
- The orders are placed to the suppliers via e-mail. The associated fixed costs are insignificant, so they are ignored.
- Orders are placed as multiples of the minimum package sizes.
- Cost parameters do not change during the planning horizon.
- Purchasing quotas do not change during the planning horizon.
- Purchasing quota requirements are satisfied based on the total procurement quantity over the planning horizon.
- Replenishment lead time of the components is composed of four phases: supplier procurement lead-time, inter-country transportation lead-time, fixed lead-time between the central warehouse and the plant, and the manufacturing lead-time of the components.
- All lead time parameters are assumed to be constant.
- Supplier procurement lead-time of the components depends only on the suppliers.
- Inter-country transportation lead time depends on mode of transportation and the supplier cluster.
- All components are first unloaded at the central warehouse before being transferred to the manufacturing plant. After the components are unloaded here, their payments are made; they are recorded in

company's books. Therefore, inventory holding cost is incurred starting from the unloading time.

- Fixed lead time between the arrival of the components at the customs zone and arrival of the components at the manufacturing plant depends on the mode of transportation and the supplier from which the components are procured.
- Manufacturing lead-time depends only on the component and includes both quality control and possible preprocessing time before its use in the main assembly.
- Transportation cost parameters change with mode and supplier cluster.
- All the components arriving at the manufacturing plant conform to the quality standards.
- There is no restriction on the number of vehicles.
- The trucks are identical in terms of capacity and there is no special loading principles used for the components, e.g. loading the truck with two layers.
- Full-truckload transportation can be used even if the truck is not loaded in full capacity, i.e., partially loaded trucks can be treated as FTL in terms of associated costs and lead times.
- An average unit holding cost is incurred for the components procured from more than one supplier. Each alternative supplier's price is included in the average in proportion to respective supplier quota.

4.1.1 The Model

Our objective is to construct a mathematical model for inventory, purchasing decisions of 1074 components, 962 of which are used in 1792 end-products of the current product-mix. The rest of the components may have end-of-sale service demand. Each of these 1074 imported component SKUs belongs to one of the 50 component classes. 886 of 962 components, which are used in the current product mix, are procured from only one supplier, 75 of them are procured from 2 suppliers and only one SKU is procured from 3 suppliers. Including the components that are not used for the current product mix, 774 of 1074 components are procured from 1 supplier, 187 of them are procured from 2 suppliers and only one of them is procured from 3 suppliers. The company works with 99 suppliers located in 20 countries. 99 suppliers manufacture components belonging to one ore more component class. In real life, the number of supplier-component class pairs is equal to 171. The countries are referred to as supplier clusters in our study.

The indices and sets used in the formulation are provided in Table 4.1.

4.1.1.1 Parameters

- **Demand Parameters:**

$D_{i,t}$: Demand of end-product i in period t.

$a_{j,i}$: Number of component j used in one unit of product i.

$E_{j,t}$: External demand of component j in period t.

Table 4.1: Indices and sets

Index	Definition	Set	Definition
i	Product index, $i=1, 2, \ldots, \lvert I \rvert$	I	Set of end-products
j	Component index, $j=1,2,\ldots, \lvert J \rvert$	J	Set of imported components
s	Supplier index, $s=1, 2, \ldots, \lvert S \rvert$	S	Set of suppliers
c	Supplier cluster index, $c=1, 2, \ldots, \lvert C \rvert$	C	Set of supplier clusters
f	Component class index, $f=1, 2, \ldots, \lvert F \rvert$	F	Set of component classes
m	Transportation mode index 1: FTL, 2: LTL, 3: Two drivers for FTL, 4: Two drivers for LTL, 5: Air-freight	S_{jc}	Set of suppliers in cluster c supplying component j
t	Time index, $t= 1, 2,\ldots, T$	J_f	Set of components in class f
		J_{fs}	Set of components in class f procured from supplier s
		F_s	Set of component classes of supplier s

- **Ordering Constraints Related Parameters:**

$O_{j,s}$: Number of components per box for component j coming from supplier s.

$q_{s,j}$: Purchasing quota of supplier s for component j.

- **Lead-time Parameters:**

$l_{s,j}$: Procurement lead-time of supplier s for component j.

$L_{c,m}$: Inter-country transportation lead-time of cluster c for mode of transportation m.

$\lambda_{s,m}$: Fixed lead-time for component j coming from supplier s by mode of transportation m between arrival in Turkey and arrival at the manufacturing plant.

Λ_j: Manufacturing lead-time of component j in the manufacturing plant.

$z_{s,m}$: Number of days that components procured from supplier s and transported by mode of transportation m stay at the central warehouse after they are unloaded there.

- **Transportation Cost Parameters:**

w_j: Unit weight of item j.

K_m: Capacity of a vehicle in mode of transportation m.

$V_{m,c}$: Fixed cost of utilizing a vehicle for mode of transportation m for cluster c.

$g_{m,c}$: Variable cost for mode of transportation $m = 2,4,5$ per unit weight in cluster c.

$G_{m,c}$: Threshold weight for mode of transportation $m = 2, 4, 5$ for cluster c up to which only fixed vehicle cost is incurred.

Note that $G_{m,c} = \dfrac{V_{m,c}}{g_{m,c}}$

Φ_c: Additional fixed cost of having two drivers for cluster c.

- **Inventory Related Parameters:**

$I_{j,0}$: Initial inventory level of component j in the manufacturing plant.

$u_{j,s}$: Unit cost of component j procured from supplier s.

r: Inventory carrying rate in YTL/YTL/period.

$h_{j,s}$: Unit inventory holding cost for component j procured from supplier s per period; $h_{j,s} = u_{j,s} r$.

H_j: Average unit inventory holding cost of component j per period in the manufacturing plant.

For the components procured from more than one supplier, supplier of the inventory held in the manufacturing plant can not be determined with certainty. In order to deal with this issue this average unit inventory holding cost parameter is defined. This parameter is calculated by the assumption that, at any instance inventory level of a component procured from a certain supplier is proportional to the associated purchasing quota. Therefore, $H_j = r\sum_{s\in S_{jc}} q_{s,j} \cdot u_{j,s}$

4.1.1.2 Decision Variables

$n_{j,s,t,m}$: Number of boxes of component j which is ordered in period t and transported from supplier s by mode of transportation m.

The variable is restricted to be integer in order to force order quantity to be a multiple of quantity per box. Order sizes are multiplication of number of boxes by the parameter $O_{j,s}$.

$N_{c,t,m}$: Number of full trucks used for modes of transportation $m = 1,3$ from cluster c in period t.

This variable is not restricted to be integer since FTL transportation can be used even when the truck is not fully loaded. In the current system FTL transportation is only used when the load from a single supplier justifies it. The proposed model enables consolidation of orders from different suppliers.

$NIV_{c,t,m}$: Number of trucks used for modes of transportation $m = 1,3$ from cluster c in period t.

This variable is used to include fixed cost of FTL transportation in the objective function.

$I_{j,t}$: Inventory level of item j in the manufacturing plant at the end of period *t*.

$W_{c,t,m}$: Weight of components assigned to be dispatched by mode of transportation m in cluster c in period *t*.

$k_{c,t,m}$: Surplus of weight (over the threshold weight) when mode of transportation $m = 2, 4$ is used for cluster *c*, in period t.

$$y_{c,t,m} = \begin{cases} 1; \text{ if mode } m = 2, 4 \text{ is used at time } t \\ 0; \text{ otherwise} \end{cases}$$

It is an indicator variable taking value of 1 if mode *m* =2, 4 is used in period t in cluster c.

The number of non-negative, integer and binary decision variables are provided in Table 4.2.

Table 4.2: Total number of non-negative, integer and binary decision variables

Variable	**Type**	**Number of variables**	**Variable**	**Type**	**Number of variables**
$N_{c,t,m}$	non-negative	$\lvert C\rvert \times T \times 2$	$n_{j,s,t,m}$	integer	$\left(\sum_{1}^{\lvert C\rvert} \lvert S_{jc}\rvert\right) \times T \times 5$
$W_{c,t,m}$	non-negative	$\lvert C\rvert \times T \times 5$	$NIV_{c,t,m}$	integer	$\lvert C\rvert \times T \times 2$
$I_{j,t}$	non-negative	$\lvert J\rvert \times T$	$y_{c,t,m}$	binary	$\lvert C\rvert \times T \times 3$
$k_{c,t,m}$	non-negative	$\lvert C\rvert \times T \times 3$			

Let us take the planning horizon T as 180 days. Five transportation modes are considered for the deliveries of 99 suppliers belonging to one of the 20 supplier clusters. Among the 1074 components that are procured from one or more of 99 suppliers, 774 of them are procured from 1 supplier, 187 of them are procured from 2 suppliers and only one of them is procured from 3 suppliers.

There exists 1151= (774 + $187\times2+$ 3) supplier-component pairs leading total number of $n_{j,s,t,m}$ variables to be 1,035,900 = ($1151\times5\times180$).

Two transportation modes are considered for FTL transportation for each of the 20 supplier clusters in each period *t* of 180 periods. In our model which 7,200= ($20\times180\times2$) $N_{c,t,m}$ and $NIV_{c,t,m}$ variables are to be decided in order to find the number of full truck used.

There are 193,320 variables in the system representing the end-of-period inventory levels of the 1074 components for 180 periods.

Weight of components assigned to the supplier clusters in period t to be dispatched by one of the transportation modes is represented by 18,000= ($20\times180\times5$) variables. Surplus weight is calculated for less-than-truckload transportation and represented by 7,200= ($20\times180\times2$) variables. Moreover since it is also represent less-than-truckload transportation, there are 7,200= ($20\times180\times2$) decisions to be made by our model in order to decide whether using less-than-truckload transportation for a given supplier cluster in a given time period.

4.1.1.3 Objective Function and Constraints

The proposed mathematical model can be stated as follows:

$$\text{Minimize} \sum_{j\in J}\sum_{t=1}^{T} H_j \cdot I_{j,t} \quad + \quad \sum_{j\in J}\sum_{s\in S}\sum_{t=1}^{T}\sum_{m=1}^{5} z_{s,m} \cdot h_{j,s} \cdot O_{j,s} \cdot n_{j,s,t,m} \quad +$$

$$\sum_{c\in C}\sum_{t=1}^{T}\{\left[V_{1,c} \cdot NIV_{c,t,1}\right] + \left[g_{2,c} \cdot k_{c,t,2} + V_{2,c} \cdot y_{c,t,2}\right] + \left[(V_{3,c} + \Phi_c) \cdot NIV_{c,t,3}\right] + \qquad \textbf{(4.1)}$$

$$\left[g_{4,c} \cdot k_{c,t,4} + (V_{4,c} + \Phi_c) \cdot y_{c,t,4}\right] + \left[g_{5,c} \cdot k_{c,t,5} + V_{5,c} \cdot y_{c,t,5})\right]\}$$

Subject to

$$I_{j,t-1} + \sum_{s\in S}\sum_{m=1}^{5} O_{j,s} \cdot n_{j,s,t-l_{j,s}-L_{c,m}-\lambda_{s,m},m} - I_{j,t} = \sum_{i\in I} D_{i,t+\Lambda_j} \cdot a_{ji} + E_{j,t} \quad \forall j \in J;\ t = 1,2,..,T \qquad \textbf{(4.2)}$$

$$W_{c,t,m} - \sum_{s\in S_{jc}}\sum_{j\in J} O_{j,s} \cdot w_j \cdot n_{j,s,t-l_{j,s},m} = 0 \qquad \forall c \in C;\ m = 1,2,3,4,5;\ t = 1,2,\ldots,T \qquad \textbf{(4.3)}$$

$$W_{c,t,m} \le K_m \cdot N_{c,t,m} \quad \forall\ t = 1,2,\ldots,T;\ m = 1,3;\ c \in C \qquad \textbf{(4.4)}$$

$$N_{c,t,m} \le NIV_{c,t,m} \quad \forall\ t = 1,2,\ldots,T;\ m = 1,3;\ c \in C \qquad \textbf{(4.5)}$$

$$W_{c,t,m} \le K_m \cdot y_{c,t,m} \quad \forall\ c \in C;\ t = 1,2,\ldots,T;\ m = 2,4,5 \qquad \textbf{(4.6)}$$

$$W_{c,t,m} - G_{m,c} - k_{c,t,m} \le 0 \qquad \forall c \in C;\ t = 1,2,\ldots,T;\ m = 2,4,5 \qquad \textbf{(4.7)}$$

$$\sum_{j\in J_f}\sum_{t=1}^{T}\sum_{m=1}^{5} O_{j,s} \cdot n_{j,s,t,m} \ge \sum_{j\in J_f} q_{s,j} \sum_{t=1}^{T}\left(\sum_{i\in I} D_{i,t} \cdot a_{ji} + E_{j,t}\right) \quad \forall s \in S;\ f \in F_s \qquad \textbf{(4.8)}$$

$$N_{c,t,m},\ k_{c,t,m},\ I_{j,t},\ W_{c,t,m} \ge 0 \qquad \forall c,t,m,j$$

$$NIV_{c,t,m}, n_{j,s,m,t} \in \{0,1,2,\ldots\} \qquad \forall c,t,m,j,s$$

$$y_{c,t,m} \in \{0,1\} \qquad \forall c,t,m \tag{4.9}$$

The objective function (4.1) is the sum of inventory holding cost and transportation costs over the planning horizon.

The term $\sum_{j\in J}\sum_{t=1}^{T} H_j \cdot I_{j,t}$ in (4.1) represents the total inventory holding cost in the manufacturing plant throughout the planning horizon, T. The term $\sum_{j\in J}\sum_{c\in C}\sum_{s\in S_{jc}}\sum_{t=1}^{T}\sum_{m=1}^{5} z_{s,m} \cdot h_{j,s} \cdot O_{j,s} \cdot n_{j,s,t,m}$ represents the inventory holding cost incurred for the pipeline inventory held at the central warehouse. The term $\sum_{c\in C}\sum_{t=1}^{T}\left[V_{1,c} \cdot NIV_{c,t,1}\right]$ represents the FTL transportation cost during the planning horizon. The term $\sum_{c\in C}\sum_{t=1}^{T}\left[(V_{3,c} + \Phi_c) \cdot NIV_{c,t,3}\right]$ represents FTL cost with two drivers which differs from the cost term of FTL transportation by the additional fixed two-driver cost term Φ_c. The term $\sum_{c\in C}\sum_{t=1}^{T}\left[g_{2,c} \cdot k_{c,t,2} + V_{2,c} \cdot y_{c,t,2}\right]$ represents the LTL transportation cost during the planning horizon. If LTL transportation is used, then first a fixed cost is incurred up to the threshold weight. Then for each kilogram above the threshold weight, a variable cost is incurred. The difference of the term $\sum_{c\in C}\sum_{t=1}^{T}\left[g_{4,c} \cdot k_{c,t,4} + (V_{4,c} + \Phi_c) \cdot y_{c,t,4}\right]$ in (4.1) which represents the LTL transportation with two drivers cost with respect to LTL transportation cost is the additional fixed cost term Φ_c incurred for two drivers. Finally the term $\sum_{c\in C}\sum_{t=1}^{T}\left[g_{5,c} \cdot k_{c,t,5} + V_{5,c} \cdot y_{c,t,5}\right]$ which has the same form as LTL mode gives air-freight costs during the planning horizon.

Constraint set (4.2) gives the inventory balance equations for each component at each period in the planning horizon. The term in the right hand side gives the total demand of component j at time t. The term $\sum_{i \in I} D_{i,t+\Lambda_j} \cdot a_{ji}$ is the dependent part of the demand. Product requirement in period $t+\Lambda_j$ becomes component requirement in period t. Inflows to the plant inventory in period t is $O_{j,s} \cdot n_{j,s,t-l_{j,s}-L_{c,m}-\lambda_{s,m},m}$, since in the definition of this decision variable, t represents the period to place the order.

Constraint set (4.3) gives total weight of items to be dispatched from cluster c by the mode of transportation m, in period t.

Constraint set (4.4) determines number of trucks to be dispatched in each supplier cluster in each period for transportation modes 1 and 3.

Constraint set (4.5) is used to find the integer number of full trucks utilized. Since full truck may be used even if the the truck is not fully loaded, we have to define a new variable $NIV_{c,t,m}$ in order to include full truck cost in the objective function. So variable $N_{c,t,m}$ is rounded up to the nearest integer value, $NIV_{c,t,m}$.

Constraint set (4.6) represents the decision of whether to use LTL transportation for each cluster and period, or not. If the weight of LTL transportation is bigger than 0, then binary variable $y_{c,t,m}$ takes the value of 1; fixed cost of LTL transportation is incurred in the objective function. If there is no weight assigned to LTL transportation, then binary variable $y_{c,t,m}$ will take the value of 0 to decrease the objective function value.

Constraint set (4.7) is used to find weight surplus over the threshold weight for less-than-truckload transportation. If the weight of the components to be transported by LTL is less than the threshold weight, the surplus weight $k_{c,t,m}$ will take the value of 0 since it is nonnegative. However, if the total weight is more than the threshold weight and since the left hand side of the constraint set (4.7) should be less than or equal to 0, then surplus weight $k_{c,t,m}$ will be at least be equal to the difference of total weight and the threshold weight. Because of the associated objective function coefficient of surplus weight $k_{c,t,m}$, the value of this variable is guaranteed to be equal to this minimum value.

Constraint set (4.8) represents the need to guarantee purchasing quota for all component clusters through the planning horizon. Constraint set (4.8) is defined for each supplier-component class pairs.

Constraint set (4.9) gives decision variables restrictions. Since FTL cost should be reflected in the objective function as the multiple of number of trucks but the weight assigned may not load the truck in full capacity, variable $NIV_{c,t,m}$ is restricted to be integer but not the variable $N_{c,t,m}$.

The number of constraints is provided in Table 4.3.

Table 4.3: Total number of constraints

Constraint	**Number of constraints**	**Constraint**	**Number of constraints**
(4.2)	$\lvert J\rvert \times T$	(4.6)	$\lvert C\rvert \times T \times 3$
(4.3)	$\lvert C\rvert \times T \times 5$	(4.7)	$\lvert C\rvert \times T \times 3$
(4.4)	$\lvert C\rvert \times T \times 2$	(4.8)	$\lvert S\rvert \times \lvert F_s\rvert$
(4.5)	$\lvert C\rvert \times T \times 2$	(4.9)	$\left(\sum_{1}^{\lvert C\rvert} \lvert S_{jc}\rvert\right) \times T \times 10 + \lvert C\rvert \times T \times 15$ $+ \lvert J\rvert \times \lvert T\rvert$

4.2 Verification and Validation of the Proposed Model

Verification and validation of our model are performed for small-sized problems solving the mathematical model developed.

The problem instances are solved in GAMS using CPLEX 10.0. solver on 32 bit Windows XP, 2 GB RAM computer.

In the basic initial instance, 3 components all of which are procured from only a single supplier from single supplier is considered. Initial inventories and all lead times are set to 0. Only one transportation mode is considered. Number of components per package is 1. Planning horizon is 4 periods in that problem.

The basic instance is extended to 15 other instances. The modifications for each trial run and the solution times are provided in Table 4.4. “0” in Table 4.4 represents the basic problem instance. The first column in the table, i.e., model name, shows the base model name which are modified to obtain new model as well (0-M1 shows that M1 is obtained based on model 0). In these problems such changes as increasing number of suppliers, changing modes of transportation allowed to be used, increasing number of components, adding lead time, extending the planning horizon, defining components that are procured from more than one suppliers so applying purchasing quota, changing the package size, changing the unit price of the components between their alternative suppliers, changing number of components with alternative suppliers are integrated. Moreover average transportation cost for the unit weight is compared for the trial problems. The changes made at each problem instance the changes in number of iterations, solution time and average unit weight transportation cost is provided in Table 4.4.

Increase in the number of variables and planning horizon without any other change increase both number of iterations and the solution time. Increasing the parameter values increases the number of iterations. However this may even decrease the solution time (between M1 and M2). Moreover, adding purchasing quota may not change the solution time as well (M1 and M2). Other factors such as unit price, proportion of purchasing quota affect the solution time. Although the last problem is much smaller than our problem it gives out of memory error when the relative gap between optimal solution and found solution is % 6.09 which shows that our problem can not be solved to by the model we developed.

Table 4.4: Problem instances - Associated solution times - Average unit eight transportation costs

MODEL NAME	Component	Suplier Cluster	Component Class	Mode	Time	Periods with demand	Lead Time /equality.	Purchasing quota (yes/no, number, equal/not equal)	Unit cost of alternative suppliers	Quantity /Box	Demand	Average unit weight transport. cost	SOLUTION	Gap	Solution time (sec.)
0 M1	3	1	1	1	4	4	0	no	-	1	initial	3.333	optimal	0	0.125
M1 M2	3	1	1	1	4	4	0	no	-	1	*2	3.125	optimal	0	0.109
M1 M3	3	1	1	1	4	4	0	no	-	≠1	initial	3.331	optimal	0	0.171
M1 M4	3	2	1	1	4	4	0	yes,1, equal	same	1	initial	3.333	optimal	0	0.125
M1 M5	3	2	1	1	4	4	0	yes,1, equal	same	≠1	initial	3.331	optimal	0	0.171
M1 M6	10	1	1	1	4	4	0	no		1	initial	3.113	optimal	0	0.234
M6 M7	10	2	1	1	4	4	0	yes,5, equal	same	1	initial	3.115	optimal	0	74.078
M7 M8	10	2	1	1	4	4	0	yes,5, not equal	same	1	initial	3.113	integer	2,90%	803.703
M8 M9	10	2	1	1	4	4	0	yes,5, not equal	different	1	initial	3.113	optimal	0	142.015
M9 M10	10	2	1	1	6	4	Non-zero, identical	yes,5, not equal	different	1	initial	3.055	optimal	0	47.203
M10 M11	10	2	1	1	6	4	Non-zero, identical	yes,5, not equal	Different	1	İnitial	3.113	İnteger	3,10%	975.921
M11 M12	10	2	1	2	6	4	Non-zero, identical	yes,5, not equal	different	1	initial	2.949	integer	0,08%	1225,44
M12 M13	10	2	1	2	6	4	Non-zero, identical	yes,5, not equal	different	≠1	initial	2.991	integer	0,27%	500.640
M13 M14	10	2	1	2	10	8	Non-zero, identical	yes,5, not equal	different	≠1	initial	3.127	integer	0,40%	1196,53
M14 M15	10	3	1	2	10	8	Non-zero, identical	3 alt. Supl., 5, not equal	different	≠1	initial	-	out of memory	-	-

CHAPTER 5

PROPOSED SOLUTION APPROACH

In this chapter we propose a solution approach to solve our problem which can not be directly solved by using the model developed in Chapter 4 in reasonable times. We aim to decompose our problem into small-sized independent sub-problems in order to obtain a “good” solution.

Our solution approach is a two-phase method. In the first phase, an aggregate problem is constructed and solved in order to obtain order quantities from each supplier for the components with alternative suppliers. In this phase, while allocating the demand for such components to different suppliers, in order to include possible economies exist in consolidation of orders at each supplier cluster, the orders for components procured from a sole supplier are aggregated. Given the purchasing quantities from each alternative supplier, the problem of one cluster is independent of others. In the second phase, we solve the original model (with several modifications) for given allocated demands to alternative suppliers obtained in phase 1 for each supplier cluster.

In Section 5.1, we discuss our solution approach in general. The proposed solution method for the problem is stated and discussed step by step in Section 5.2.

5.1 Solution Approach

In our problem environment, many of the components are procured from only one supplier and some of them are procured from more than one supplier. Suppliers of such components belong either to the same supplier cluster or to different supplier clusters.

A schematic representation of supplier clusters is provided in Figure 5.1. The clusters are made up of one or more suppliers. Intersections of the supplier clusters represent the components with alternative suppliers. These intersections may occur both within the supplier cluster and between the supplier clusters. The former structure does not affect the independency of the clusters, that is the purchasing and transportation decisions associated with the cluster can be made independent of others. However, the latter one cause the supplier clusters to be dependent.

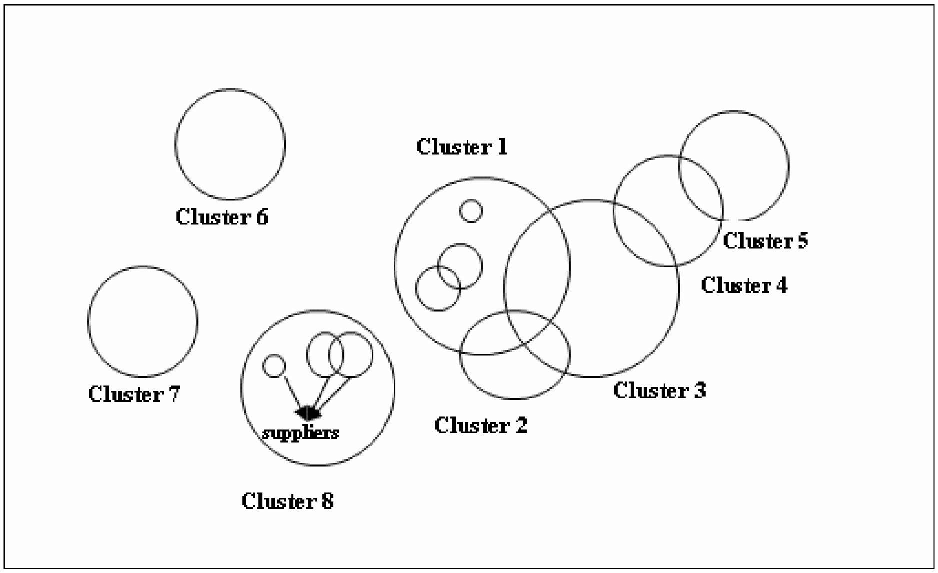

Figure 5.1: Schematic representation of supplier clusters

The supplier clusters in which all suppliers are sole suppliers are naturally decomposed from the problem and can be solved individually by the model that we develop in Chapter 4. Therefore, for the components with alternative suppliers in different clusters, the clusters need to be made independent first. Common components that are supplied from these clusters make them dependent because of the purchasing quota constraint. Quantities purchased from alternative suppliers in each period affect both transportation and inventory decisions. As the number of components with alternative supplier clusters increase the purchasing decision becomes more important. Current purchasing strategy of heading towards Far Eastern countries makes purchasing quota constraint especially important in our study since there will be more alternatives for European suppliers located in Far East in the future.

Our proposed approach depends on the idea that the independent supplier clusters' problems can be solved independent of others. The proposed procedure is composed of two phase. In the first phase, the purchasing quantities of components with alternative suppliers located in different supplier clusters are allocated among alternative suppliers. In this phase the demand of components with as single supplier is aggregated based on their weights so that they are considered as a single item. By aggregation the number of decision variables, i.e. the size of, in the mathematical model is decreased. Possible transportation economies are tried to be captured by representing these components in the model. In the second phase, our initially developed model is solved for each cluster independently. Purchasing quantities of components with alternative supplier clusters that are obtained in the first phase are used as input demand figures in the second phase problems.

5.2 Proposed Solution Method

Step 1: Natural decomposition

Supplier clusters in which all suppliers are sole suppliers are determined. The model developed in Chapter 4 is solved for each cluster.

Step 2: Preprocessing for aggregation

For the components that are procured from single supplier cluster;

- Demands of the components are converted into demand figures in boxes over the entire planning horizon taking integrity condition into account. Let $R_{j,t}$ the requirement for component j in period t in unit of boxes.

$$R_{j,t} = \max\left\{0, \left\lceil \left(\sum_{i \in I} \left(D_{i,t+\Lambda_j} a_{ji} \right) + E_{j,t} \right) \Big/ O_{j,s} - \sum_{t=1}^{t-1} R_{j,t-1} + \sum_{t=1}^{t-1} \left(\sum_{i \in I} \left(D_{i,t+\Lambda_j} a_{ji} \right) + E_{j,t} \right) \Big/ O_{j,s} \right\rceil \right\} \quad \textbf{(5.1)}$$

- If requirement $R_{j,t}$ is more than the demand of period t because of integrity constraint of number of boxes, the excess amount is subtracted from the demand of following period or periods depending on the excess quantity and demands of the following periods.

- Maximum transportation lead time, that is the lead time under the slowest transportation mode, is determined for each of these components based on the supplier clusters. Let $LT_{\max}^{j} = LT_{\max,c} = \max_m \{L_{c,m}\}$ (where $L_{c,m}$ is the inter-country transportation lead time of component j under mode of transportation m) be the lead time for cluster j under the slowest mode of transportation. In our real-life case the slowest mode is less-than-truckload transportation for all clusters so $LT_{\max,c} = L_{c,2}$ for all supplier clusters.

- The requirements of the components are back shifted by $LT^{j}_{\max}$ in order to obtain the latest time period that the order can be dispatched under the slowest mode, $R_{j,t} \leftarrow R_{j,t+LT^{j}_{\max}}$. The new time period t will be denoted as a new index time k in the new model that is a new parameter $\overleftarrow{R}_{j,k}$ is defined. $\overleftarrow{R}_{j,k} = R_{j,t+LT^{j}_{\max}}$.
- Let wb_j be the weight of one box of component j, total weight to be dispatched by the slowest transportation mode from cluster c in period t is determined. Let $WA_{c,t}$ is that total weight. $WA_{c,t} = \sum_{J} wb_j \overleftarrow{R}_{j,k}$. This weight should be dispatched latest in period t. $WA_{c,t}$ is to be defined as a new parameter for the new model.
- Difference between the transportation lead time of mode m and the slowest mode is determined for each supplier cluster. Let $\Delta_{c,m} = LT_{\max,c} - L_{c,m}$ be is that difference for mode m, and cluster c.

Step 3: Formulation and the solution of the mathematical model for the aggregate problem

New parameters, decision variables, objective function terms and constraints are added to the mathematical model developed in Chapter 4 after the aggregation process. Moreover some of the constraints are modified. The new model is referred to as Phase-1 model.

First a new decision variable, $\Psi_{c,t,k,m}$ is defined in accordance with the new parameter aggregate weight $WA_{c,t}$ of cluster c which is to be dispatched latest in period t *under the slowest mode of transportation.* $\Psi_{c,t,k,m}$ denotes the weight of components to be dispatched from cluster c by mode m in period t to satisfy the requirement of period k. Index k represents the latest dispatch time if the slowest transportation mode is used.

If the slowest mode of transportation is used, the dispatch period t cannot be later than period k. Quicker transportation modes should be used for the dispatches later than k. In such conditions the difference between t and k should be at most the lead time difference between the chosen transportation mode and the slowest mode. The difference $|t-k|$ and its relation to lead time difference affect the possible values of decision variable $\Psi_{c,t,k,m}$ and inventory accumulated. There are three different cases:

Case 1: If $t-k > \Delta_{c,m}$, then $\Psi_{c,t,k,m} = 0$. The arrival time is later than the production time so there is no need to define these variables under this condition. Figure 5.2 depicts condition 1 on the time-line.

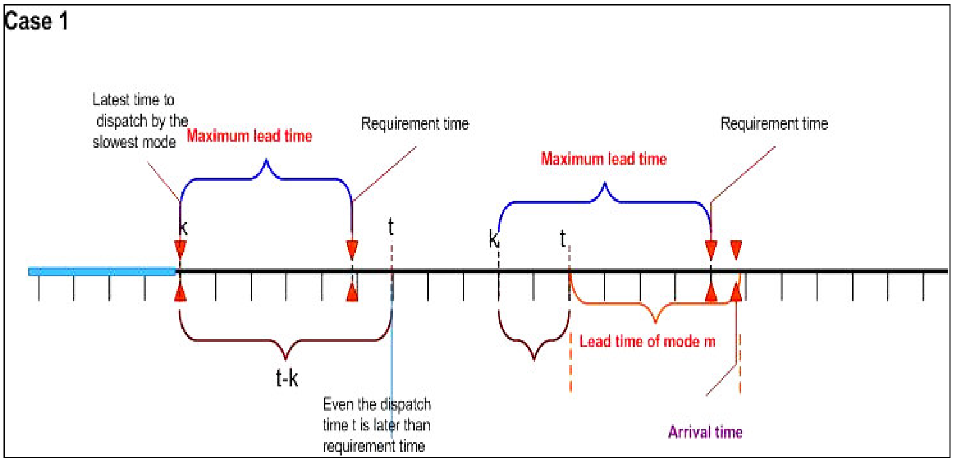

Figure 5.2: Representation of Case 1 on the timeline

Case 2: If $0 < t - k \leq \Delta_{c,m}$, holding cost is incurred for $\Delta_{c,m} - (t - k)$ periods.

Case 3: If $t - k \leq 0$, holding cost is incurred for $k - t + \Delta_{c,m}$ periods.

Figure 5.3 depicts case 2 and case 3. $\Psi_{c,t,k,m}$ variables need to be defined in those cases. Components are held in the inventory for $\Delta_{c,m} - t + k$ periods for dispatch period t such that $t - k \leq \Delta_{c,m}$.

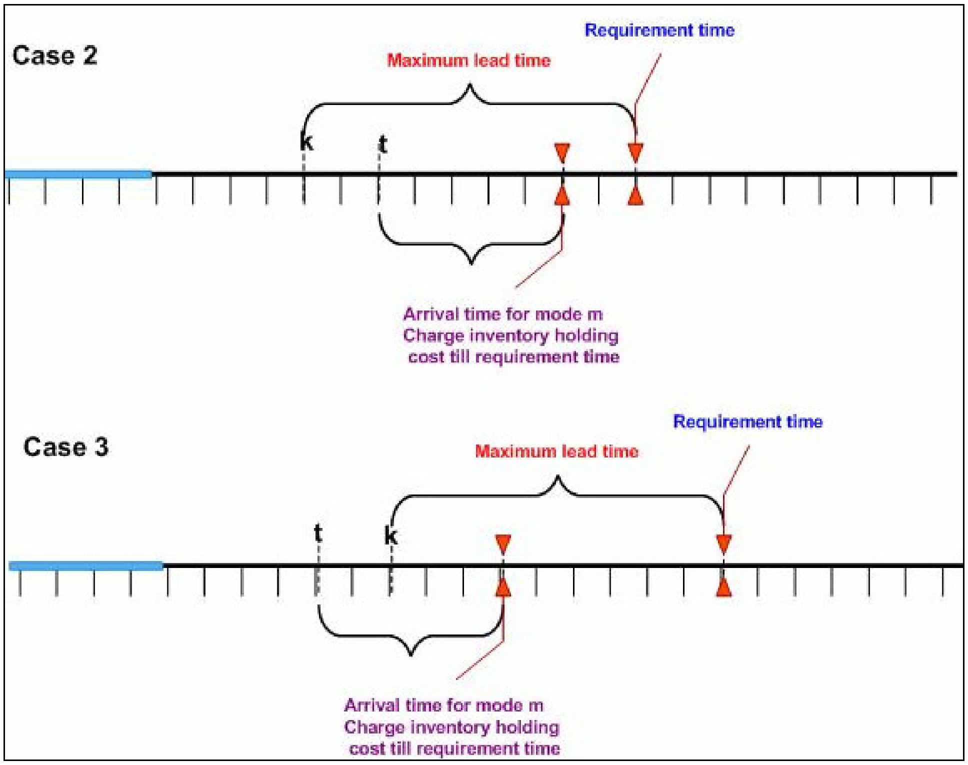

Figure 5.3: Representation of Case 2 and Case 3 on the timeline

$\sum_{m} \sum_{t \ni t-k \leq \Delta_{c,m}} \Psi_{c,t,k,m} = WA_{c,k}$ constraints are added instead of inventory balance equations for the components with single supplier cluster. Moreover, holding cost is charged for $\Psi_{c,t,k,m}$ variables in the objective function if $k - t + \Delta_{c,m}$ is positive. In order to charge the holding cost unit cost for unit weight is calculated for each supplier cluster for each *k*. Holding cost is determined as the cost charged for the unit weight transported for the requirement in period *t*. Let $ha_{c,k}$ be the unit cost of the unit weight transported for the requirement of period k, it is calculated as it is follows:

$$ha_{c,k} = \sum_{j \in Sjc} u_j R_{j,k} \Big/ w_j R_{j,k} \ .$$

The term $\sum_{c \in C} \sum_{t \in T} \sum_{k \in T} \sum_{m} (\Delta_{c,m} - t + k) rha_{c,k} \Psi_{c,t,k,m}$ is added in the objective function for the components with single supplier clusters in order to charge holding cost for these components.

The mathematical model for the aggregate problem is as follows:

$$\begin{aligned}
\mathit{Minimize} \quad & \sum_{j \in J} \sum_{t=1}^{T} H_j I_{j,t} \quad + \quad \sum_{j \in J} \sum_{s \in S} \sum_{t=1}^{T} \sum_{m=1}^{5} z_{s,m} h_{j,s} O_{j,s} n_{j,s,t,m} \quad + \\
& \sum_{c \in C} \sum_{t \in T} \sum_{k \in T} \sum_{m} (\Delta_{c,m} - t + k) rha_{c,k} \Psi_{c,t,k,m} + \\
& \sum_{c \in C} \sum_{t=1}^{T} ((V_{1,c} \cdot NIV_{c,t,1}) + (g_{2,c} k_{c,t,2} + V_{2,c} y_{c,t,2}) + \\
& ((V_{3,c} + \Phi_c) NIV_{c,t,3}) + \\
& (g_{4,c} k_{c,t,4} + (V_{4,c} + \Phi_c) y_{c,t,4}) + (g_{5,c} k_{c,t,5} + V_{5,c} y_{c,t,5}))
\end{aligned} \qquad \textbf{(5.2)}$$

Subject to

$$I_{j,t-1} + \sum_{s \in S} \sum_{m=1}^{5} O_{j,s} n_{j,s,t-L_{c,m},m} - I_{j,t} = \sum_{i \in I} D_{i,t+\Lambda_j} a_{ji} + E_{j,t} \quad \forall j \in J, t = 1,2,..,T \qquad \textbf{(5.3)}$$

$$\sum_{m} \sum_{t \ni t-k \le \Delta_{c,m}} \Psi_{c,t,k,m} = WA_{c,k} \quad \forall m = 1,2,3,4,5,\ t = 1,2,\ldots,T, \tag{5.4}$$

$$W_{c,t,m} - \sum_{s \in S_{jc}} \sum_{j \in J} O_{j,s} \cdot n_{j,s,t,m} \cdot w_j - \sum_{k \in T} \Psi_{c,t,k,m} = 0 \quad \forall c \in C,\ m = 1,2,3,4,5,\ t = 1,2,..,T \tag{5.5}$$

$$W_{c,t,m} \le K_1 N_{c,t,m} \quad \forall\ t = 1,2,\ldots,T,\ m = 1,3,\ c \in C \tag{5.6}$$

$$N_{c,t,m} \le NIV_{c,t,m} \quad \forall\ t = 1,2,\ldots,T,\ m = 1,3,\ c \in C \tag{5.7}$$

$$W_{c,t,m} \le K_m y_{c,t,m} \quad \forall\ c \in C,\ t = 1,2,\ldots,T\ m = 2,4,5 \tag{5.8}$$

$$W_{c,t,m} - G_{m,c} - k_{c,t,m} \le 0 \qquad \forall c \in C,\ t = 1,2,\ldots,T \quad m = 2,4,5 \tag{5.9}$$

$$\sum_{j \in J_f} \sum_{t=1}^{T} \sum_{m=1}^{5} O_{j,s} n_{j,s,t,m} \ge \sum_{j \in J_f} q_{s,j} \sum_{t=1}^{T} \Big(\sum_{i \in I} D_{i,t} a_{ji} + E_{j,t} \Big) \quad \forall s \in S,\ f \in F_s \tag{5.10}$$

$$N_{c,t,m},\ k_{c,t,m},\ I_{j,t},\ W_{c,t,m}, \Psi_{c,t,k,m} \ge 0$$

$$NIV_{c,t,m}, n_{j,s,m,t} \in \{0,1,2,\ldots\}$$

$$y_{c,t,m} \in \{0,1\} \tag{5.11}$$

The decision variables with index *j*; $n_{j,s,m,t}$ and $I_{j,t}$ are defined only for the components with alternative suppliers in different supplier clusters.

$\Psi_{c,t,k,m}$ is the new variable defined for the aggregate problem which represents the weight dispatch from cluster *c* in period *t* by mode *m* in order to be used in period $k + LT_{\max,c}$. The requirements of the components with one supplier are back shifted as disccussed in Step 1 of the porposed method, *k* is referred to as the requirement period in the new problem and model. The term $\sum_{c\in C}\sum_{t\in T}\sum_{k\in T}\sum_{m}(\Delta_{c,m}-t+k)rha_{c,k}\Psi_{c,t,k,m}$ in (5.2) represents the inventory holding cost for the aggregate weights that are transported in order to be used in period k. In this model different from the original one, inter-country transportation lead time $L_{c,m}$ and the manufacturing lead time Λ_j are the considered in constraints (5.3) and (5.5). Constraint set (5.4) represents the inventory balance for the aggregated weights $\sum_{m}\sum_{t\ni t-k\le\Delta_{c,m}}\Psi_{c,t,k,m}=WA_{c,k}$. The term $\sum_{k\in T}\Psi_{c,t,k,m}$ in constraint set (5.5) represents the weight of components with one supplier cluster which is dispatched in period t for all possible k.

Step 4: Phase-2 problem

Phase-2 problem is solved for each supplier cluster separately after the order quantities are determined for the components with more than one supplier. Solution of the aggregate problem gives the purchasing quantities $O_{j,s}\cdot n_{j,c,t,m}$ of the components with more than one supplier clusters. These purchasing quantities are used as demand figures in Phase-2 problem. The quantities are taken as the demand figures of these components in period when they arrive in Phase-1 model (dispatch time determined by Phase-1 model + lead time of mode m which is determined by Phase-1 model). These demand figures given by the solution of the aggregate problem are integer multiples of the quantity per box for each component. The demand quantities for the components with single supplier are modified in order to make them integer multiples of the quantity per box as well. Phase-2 problem is solved by the original model without the following purchasing quota constraint;

$$\sum_{j\in J_f}\sum_{t=1}^{T}\sum_{m=1}^{5}O_{j,s}n_{j,s,t,m}\ge\sum_{j\in J_f}q_{s,j}\sum_{t=1}^{T}\left(\sum_{i\in I}D_{i,t}a_{ji}+E_{j,t}\right)\quad\forall s\in S,\ f\in F_s \qquad \textbf{(4.8)}$$

Even though the demands are integer multiples of the quantity per box, integrity constraint for the number of boxes still need to be satisfied by the model.

Step 5: Final solution:

In order to obtain a final total cost, Phase-2 problem is solved for each supplier cluster.

CHAPTER 6

COMPUTATIONAL STUDY

In this chapter we first analyze the effect of working with a number of alternative suppliers on the total cost incurred in the system. We investigate the conditions under which it is advantageous to have multiple suppliers for the same product. Smaller problem instances are studied in order to discuss multi-supplier effect on the optimized costs.

In the second part, we evaluate the performance of the proposed solution approach. First the proposed two-phase solution method is compared with the originally developed mathematical model for small-sized problem instances. Then, the behavior of the proposed system under different settings is discussed. Performance of the proposed method is compared with the current practice of the company for both real and generated problem instances in which retrospective and random data are used, respectively.

6.1 Multi-Supplier Effect on the Behavior of the System

Effect of having alternative suppliers for some components, i.e., multi-supplier effect is discussed for the smaller-sized problem instances based on the mathematical model developed in Chapter 4. The effect of working with alternative suppliers on the optimized total costs is presented first. Then, the conditions under which it is advantageous to have alternative suppliers are investigated.

The problem instances are formulated based on the following two base cases.

Base case 1: A problem instance with two independent supplier clusters is taken as the base case. Several scenarios are generated adding an alternative supplier from the second cluster for one of the current items procured from the first cluster. Mainly the effects of unit cost and lead-time changes are investigated based on those variations.

Characteristics that are common for all these problem instances are provided in Table 6.1.

Table 6.1: Common characteristics of the Case 1 instances

Number of components	7
Number of supplier clusters	2
Number of suppliers	2
Number of transportation modes	5
Planning horizon	10
Initial inventory	0
Number of periods with component demand	6

Problem instances are derived by the unit cost or lead-time change of alternative supplier with respect to the current supplier's cost or lead-time. The problems with the same unit cost change or same lead-time change are referred to as a trial set. The problems within each set are differentiated with respect to purchasing quotas. Table 6.2 provides the characteristics of the problems in first two trial sets.

Table 6.2: Characteristics of the problems in first 2 trial sets in case-1

	TRIAL SET 1					**TRIAL SET 2**					
Characteristics	**II**	**III**	**IV**	**V**	**VI**	**VII**	**VIII**	**IX**	**X**	**XI**	**XII**
Number of components with alternative suppliers	1	1	1	1	1	1	1	1	1	1	1
Unit cost change of alternative supplier	-	-	-	-	-	-10%	-10%	-10%	-10%	-10%	-10%
Purchasing quota of alternative supplier	10%	25%	50%	75%	90%	10%	25%	50%	75%	90%	99%
Lead-time change of alternative supplier	-	-	-	-	-	-	-	-	-	-	-

Problem I is referred to as the base problem. In the first set of problems, an alternative supplier is added for one of the components with the same unit with the current supplier. In set 2 the unit cost of the alternative supplier is reduced by 10%. The associated inventory holding costs and transportation costs of these problems in the optimal solution are provided in Table 6.3.

Table 6.3: Cost components of trial set 1-2 in Base case-1

	Problem Instance	**Inventory Holding Cost (Central WH)**	**Inventory Holding Cost (Plant WH)**	**Transportation Cost**	**Total Cost**	**% change from base case**
BASE	**I**	1,521.6	60.4	42,454.8	44,036.8	**-**
TRIAL SET 1	**II**	1,511.5	125.1	42,262.6	43,899.2	**-0,31%**
	III	1,559.4	173.0	41,806.8	43,539.2	**-1,13%**
	IV	1,511.5	125.2	41,601.6	43,238.3	**-1,81%**
	V	1,511.5	127	41,601.6	43,240.1	**-1,81%**
	VI	1,511.5	133.5	41,601.6	43,246.6	**-1,79%**
TRIAL SET 2	**VII**	1,493.2	125.2	42,262.5	43,880.9	**-0,35%**
	VIII	1,541.1	171.8	41,806.8	43,519.7	**-1,17%**
	IX	1,484.5	125.2	41,601.6	43,211.2	**-1,87%**
	X	1,471	126.9	41,601.6	43,199.5	**-1,90%**
	XI	1,462.9	133.4	41,601.6	43,197.9	**-1,90%**
	XII	1,457.9	154.1	41,601.6	43,213.6	**-1,87%**

Inventory holding cost of both central warehouse and manufacturing plant increase with the addition of alternative supplier which result from the minimum order quantity constraint and minimum planned amount of purchase through the planning horizon, i.e., purchasing quota constraint. Some orders of current supplier are consolidated in order not to increase the transportation cost and to use transportation economies which result in increase in inventory holding cost.

Table 6.4: Characteristics and costs obtained for trial sets 3-6 in case-1

	Problem	Unit cost decrease	Lead-time change	Purchasing quota	Inventory Holding Cost Central Warehouse	Inventory Holding Cost Plant Warehouse	Transportation Cost	Total Cost	% change from base case
TRIAL SET 3	**XIII**	10%	+1	10%	1,493.2	155.7	42,964.5	44,613.4	**1,31%**
	XIV	10%	+1	25%	1,541.1	202.3	42,508.8	44,252.2	**0,49%**
	XV	10%	+1	50%	1,484.5	150.9	42,303.6	43,939	**-0,22%**
	XVI	10%	+1	75%	1,470.9	149.6	42,303.6	43,924.2	**-0,26%**
	XVII	10%	+1	90%	1,462.9	150.1	42,303.6	43,916.6	**-0,27%**
	XVIII	10%	+1	99%	1,458	167.2	42,303.6	43,928.8	**-0,25%**
TRIAL SET 4	**XIX**	25%		10%	1,465.8	125.1	42,262.6	43,853.5	**-0,42%**
	XX	25%		25%	1,513.6	170.0	41,806.8	43,490.4	**-1,24%**
	XXI	25%		50%	1,443.9	125.1	41,601.6	43,170.6	**-1,97%**
	XXII	25%		75%	1,410.1	126.9	41,601.6	43,138.6	**-2,04%**
	XXIII	25%		90%	1,389.8	133.4	41,601.6	43,124.8	**-2,07%**
TRIAL SET 5	**XXIV**	50%		10%	1,420	125.2	42,262.5	43,807.7	**-0,52%**
	XXV	50%		25%	1,467.9	167	41,806.8	43,441.7	**-1,35%**
	XXVI	50%		50%	1,376	125.1	41,601.6	43,103	**-2,12%**
	XXVII	50%		75%	1,308.7	126.9	41,601.6	43,037.2	**-2,27%**
	XXVIII	50%		90%	1,243.8	144.5	41,601.6	42,989.9	**-2,38%**
TRIAL SET 6	**XXIX**	50%	+1	10%	1,419.9	155.6	42,964.5	44,540.2	**1,14%**
	XXX	50%	+1	25%	1,462.4	192.4	42,485.6	44,140.4	**0,24%**
	XXXI	50%	+1	50%	1,376.3	151	42,303.6	43,830.8	**-0,47%**
	XXXII	50%	+1	75%	1,308.7	149.6	42,303.6	43,761.9	**-0,62%**
	XXXIII	50%	+1	90%	1,268.1	150.1	42,303.6	43,721.8	**-0,72%**

The other trials sets of the problem are created with the decrease of the unit cost of alternative supplier by 25%, 50%, and 75% respectively. Purchasing quota of the alternative supplier ranges from 10% to 90% in these problems as well. Table 6.4 provides characteristics and costs obtained for the rest of the trial problems.

Table 6.4 shows that increase in the lead-time of alternative supplier increases the total cost. The solution of trial set 3 gives worse total cost than the base problem until the purchasing quota of alternative supplier increases to 50%. The optimal solution of the problems shows that in the cases where the purchasing quota is relatively small, the purchase quantity is more than the planned total purchase quantity in order to use transportation economies and since the inventory holding cost is much smaller than transportation cost. Table 6.4 provides an insight for determining the optimum purchasing quota for alternative suppliers depending on the lead-time and unit cost for these trial problem instances.

Base case 2: A problem with only one supplier cluster is solved first. Then an alternative supplier from a different cluster is added for one of the items. Effects of both unit cost changes and lead time changes are observed with the solution of derived problems.

The base problem of case 2 is same as the base problem of case one, except there is only one supplier cluster and all of the components are procured prom this cluster initially.

Table 6.5 provides both the characteristics of the problems trial sets and associated costs obtained by the solution of these problems.

Table 6.5: Characteristics and costs obtained for trial in base case-2

	Problem	Unit cost change	Lead-time change	Purchasing quota	Inventory Holding Cost Central Warehouse	Inventory Holding Cost Plant Warehouse	Transportation Cost	Total Cost	% change from base case
Base	**I**			N/A	1,511.3	103.4	44,491.5	46,106.4	
TRIAL SET 1	**II**	-	-	10%	1,511.5	87.4	44,409.6	46,008.5	**-0,21%**
	III	-	-	15%	1,511.5	79.1	44,409.6	46,000.2	**-0,23%**
	IV	-	-	20%	1,511.55	78.7	44,409.6	45,999.8	**-0,23%**
	V	-	-	25%	1,511.5	73.2	44,409.6	45,994.3	**-0,24%**
	VI	-	-	26%	1,511.5	77.4	44,409.6	45,998.5	**-0,23%**
	VII	-	-	30%	1,511.5	95.9	44,409.6	46,017	**-0,19%**
	VII	-	-	35%	1,511.5	119	44,409.6	46,040.1	**-0,14%**
	VIII	-	-	50%	1,511.5	66.8	44,517.6	46,095.9	**-0,02%**
	IX	-	-	75%	1,511.5	185.3	44,517.7	46,214.5	**0,23%**
	XIX	-	-	90%	1,511.5	192.2	44,625.6	46,329.3	**0,48%**
TRIAL SET 2	**XX**	+10%	-	10%	1,516.9	87.4	44,409.6	46,013.9	**-0,20%**
	XXI	+10%	-	25%	1,525.1	73.2	44,409.6	46,007.9	**-0,21%**
	XXII	+10%	-	50%	1,538.6	66.8	44,517.6	46,123	**0,04%**
	XXIII	+10%	-	75%	1,552.1	87.2	44,625.6	46,264.9	**0,34%**
TRIAL SET 2	**XXIV**	-10%	-	10%	1,506.1	87.4	44,409.6	46,003.1	**-0,22%**
	XXV	-10%	-	25%	1,498	73.2	44,409.6	45,980.8	**-0,27%**
	XXVI	-10%	-	50%	1,484.5	66.6	44,517.6	46,068.7	**-0,08%**
	XXVII	-10%	-	75%	1,471	175	44,517.6	46,163.6	**0,12%**
TRIAL SET 3	**XXVIII**	-25%	-	10%	1,498	87.4	44,409.6	45,995	**-0,24%**
	XXIX	-25%	-	25%	1,477.7	73.1	44,409.6	45,960.4	**-0,32%**
	XXX	-25%	-	50%	1,443.9	66.5	44,517.6	46,028	**-0,17%**
	XXXI	-25%	-	75%	1,410.1	159.6	44,517.6	46,087.3	**-0,04%**
TRIAL SET 4	**XXXII**	-25%	+1	10%	1,498	87.4	44,409.6	45,995	**-0,24%**
	XXXIII	-25%	+1	25%	1,477.7	73.1	44,409.6	45,960.4	**-0,32%**
	XXXIV	-25%	+1	50%	1,443.9	66.5	44,517.6	46,028	**-0,17%**
	XXXV	-25%	+1	75%	1,410.1	159.6	44,517.6	46,087.3	**-0,04%**
TRIAL SET 5	**XXXVI**	-50%	-	10%	1,484.5	87.3	44,409.6	45,981.4	**-0,27%**
	XXXVII	-50%	-	25%	1,443.9	73.1	44,409.6	45,926.6	**-0,39%**
	XXXVIII	-50%	-	50%	1,376.3	66.3	44,517.6	45,960.2	**-0,32%**
	XXXIX	-50%	-	75%	1,308.6	134	44,517.6	45,960.2	**-0,32%**

Trial set-1 is tested in detail as provided in Table 6.5 in order to see the optimum purchasing quota to obtain minimum cost. Problem V in which the purchasing quota of alternative supplier is equal to 25% gives the smallest total cost. Moreover the purchasing quota of 25% seems to be optimal quota for alternative suppliers for all other sets as well for case-2 setting. Inventory holding cost in central warehouse is the same for all the problems in set 1 since minimum order quantity is taken equal to 1. One important observation can be made by comparing set 3 and set 4; all the problem instances give same solution even the lead time of alternative supplier is longer for the problems in set 4.

All of these problem instances are also important regarding to the future strategic decisions. As the unit cost of alternative suppliers decrease inventory holding cost will decrease as well. As the number of alternative suppliers increase, orders of the new component mix will probably be consolidated in order to continue to use transportation economies from the current supplier clusters (where the total purchase quantity is very high),. This will probably result in some increase in inventory holding cost. Moreover this may result in procurement of a component from the supplier with shorter lead-time for the foremost periods and from the alternative supplier with longer lead-time for the rest of the periods.

6.2 Evaluation of the Proposed System

The proposed solution method is first compared with the solution of the original mathematical model developed in Chapter 4 in order to check its quality compared to the optimal solution. The comparison is made for small-sized artificial problem instances. Then, the proposed approach is tested on the problem instances created with both retrospective demand data and randomly generated demand data.

Comparison of proposed method and original mathematical model

Comparison of the two-phased method with the original mathematical model precedes this final comparison of proposed method and current method. Small-sized problem instances are created in order to compare two methods since the original model cannot be solved for optimality in reasonable times for realistic instances. Table 6.6 provides the characteristics of these problems.

Table 6.6: Characteristics of the problems

	PROBLEM INSTANCES						
	I	II	III	IV	V	VI	VII
Number of clusters	2	2	2	2	2	2	2
Number of items	10	10	10	10	10	12	10
items with alternative suppliers	4	4	4	4	4	4	3
items procured solely from cluster 1	6	6	3	3	3	3	5
items procured solely from cluster 2	0	0	3	3	3	5	1
demands of items with sole supplier	multiple of quantiy/ box	multiple of quantiy /box	random	random	random	random	random
Purchasing quota	identical	non-identical	identical	non-identical	identical	non-identical	identical
Planning horizon	7	7	9	10	10	8	15

All of the problems include two supplier clusters. These problems are created by changing total number of items; number of items with sole suppliers; length of the planning horizon; purchasing quota; and demand data. 64 bit XP 4 GB RAM PC is used to use more memory so solve the problems. While problems V, VI and VII can not be solved optimally by 32 bit Windows XP, 2 GB RAM PC, 6 bit XP solves the problems to the optimal. Table 6.7 provides the total cost obtained both by the solution of original model and proposed two-phased method for the trial problems together with associated solution times.

Table 6.7: Solution times and total cost of the problems

		Optimal Solution		**Phase-1 Problem (Aggregate Problem)**		**Phase-2 Problem**		
		Solution time (seconds)	**Total Cost (YTL)**	**Solution time (seconds)**	**Total Cost (YTL)**	**Solution time (seconds**	**Total Cost (YTL)**	**% cost change from optimal**
PROBLEM INSTANCE	**I**	28.1	58,205	1.7	58,205	0.187	58,205	same
	II	27.7	58,138.8	4.4	58,138.8	0.264	58,138.8	same
	III	960.5	22,895.6	5.6	23,318.9	1.312	22,940.5	0.20%
	IV	1,701.1	37,478.8	731	37,482.1	0.656	37,520.2	0.11%
	V	1,884.2	46,184.1	1,093.7	46,236.3	632,765	46,764.7	1.26%
	VI	8,195.9	2,542.4	0.4	4614	0.7	3564.5	40.2%
	VII	9,357.8	125,844.7	1,187.2	125,975.4	365,203	126481.6	0.51%

Even the solution of Phase-1 problem of Problems I and II gives the same solution as the original model developed because the demand values of the components with sole suppliers are multiples of box sizes and all parameters are the same for the components with sole suppliers ($ha_{c,k}$ be the unit cost of the unit weight transported for the requirement of period k is equal to unit cost of the items).

The proposed two-phase method is proved to be efficient method because of the small difference between the total costs which are obtained by solving the problems by the original mathematical model and solving them with proposed two-phase method. . Moreover there is a significant improvement in the solution time for the proposed method. Even though the number of decision variables is much less compared to the original problem instances the mathematical model developed in chapter 4 does not give solution in reasonable times.

Difficulties of the proposed method

Phase-1 problem can not be solved optimally for the realistic problem instances in terms of number of components and the length of planning horizon, Table 6.8 provides summary results for some of the pilot runs for Phase-1 problem.

Table 6.8: Summary of results for pilot runs

CHARACTERISTICS	**A**	**B**	**C**	**D**	**E**	**F**
Number of clusters in Europe	8	8	8	8	8	8
Number of clusters in Far East or Japan	2	2	2	2	2	0
Number of components with alternative supplier	28	44	44	44	44	16
Time bucket (days)	3	3	3	2	2	1
Number of transportation modes	5	5	5	5	5	5
Number of periods	18	18	25	45	45	38
SOLUTION TIME (seconds)	5,493.5	74,931.9	258,194.1	49,591.6	3,141.2	46,884.1
ITERATION COUNT	6,244,418	5,410,575	9,903,860	1,960,934	931,376	7,616,855
Relative gap (%)	5.75%	6.10%	3.99%	9.90%	19.71%	4.01%

Initial inventory of the components is set to 0 for all problems. Table 6.8 shows that an exact solution can not be obtained by the solution of Phase-1 problem. Problem C is a good illustration to show these characteristic. Even after 71 hours, the problem can not be solved optimally. The gap between the best integer solution found and the optimal solution of LP relaxation is 3.99%. These pilot runs also give an idea about the problem settings which are used for further tests and for the comparison of the proposed method with the current practice. First of all, the planning horizon of the problems should be long enough to see the advantages of consolidation of orders and transportations. . Since initial inventory is set to zero there should be some periods without any demand as well. To overcome this difficulty time bucket of the problems are taken as 3-days first. In problem instances A, B, C the time bucket is taken as 3 days. The planning horizon of problem instances A and B, and of problem instance C, covers one month's and two months' demand, respectively. The solution of the problems with time bucket of 3-days does not reflect the lead time differences. Time bucket of 2-days is decided to use for following problems reflecting the real case.

The only difference between Problem D and E is the time elapsed before the solution run is interrupted. The gap between the obtained integer solution optimal solution of LP relaxation in Problem E decreases by 10% in 12 hours in Problem D as seen in Table 6.8.

Even though problems with 2-months' demand is preferred to be used for comparison, some trials are performed taking time bucket of 1 day as well. Even the sample problem instances without longer lead-times, like problem instance F, cannot be solved for optimality. For problem instance F, the gap decreases to 4.01% in 12.7 hours.

The pilot runs show that expedited means of transportation are not preferred for certain cases. These can be used in determining the conditions where expedited transportation modes are advantageous in advance for the mathematical model of Phase-1 problem. This preprocessing is based on the breakpoint of the costs between expedited means and standard means of transportation, i.e., FTL and LTL and deterministic demand. The aim is to find a certain weight above which expedited transportation means are more expensive. Moreover, since the demand is deterministic the orders can be placed to the supplier earlier than the case in which expedited transportation means are used.

The pilot runs show that given all other parameters are the same, mainly the planning period, number of components with alternative suppliers, the rest of the components that are procured from those alternative suppliers and inventory carrying charge affect the quality of the solution, i.e., decrease in the relative gap for a given time-interval.

- As the planning horizon increases the solution time increases as well.
- As the holding cost decreases the solution time decreases as well.
- As the holding cost decreases less dispatches i.e., more consolidated shipments, are preferred.
- As the number of components with alternative suppliers decrease the solution time of Phase-1 problems decrease dramatically.
- If only several components are procured from a supplier and if there is at least one component with alternative supplier among those components, order of that component is placed only several times through the planning horizon, i.e., only at once in for the problems provided by Table 6.8..

Some trials in which integer variables are relaxed are performed as well. The integer variables representing number of trucks used for FTL transportation no matter it is with two-driver or not are not relaxed in the trials because these variables directly affect the transportation cost which constitutes almost 90-95% of the total cost. Integer variables representing the number of boxes are relaxed in the problem instances which are first solved without relaxing $n_{j,s,t,m}$ variables. Table 6.6 provides the solution times and the relative gap for these problems in comparison with those of the un-relaxed problems.

Table 6.9: Solution times and relative gaps for relaxed and un-relaxed problems

	$n_{j,s,t,m}$ **constrained to be integer**		$n_{j,s,t,m}$ **is not constrained to be integer**	
	Solution time (seconds)	**Relative gap (%)**	**Solution time (seconds)**	**Relative gap (%)**
Problem I	8,274.4	27.99%	8,270.8	4.61%
Problem 2	3,163.9	27.50%	3,561.5	8.21%
Problem 3	752.6	6.67%	740.9	0%
Problem 4	9,252.9	12.53%	10,369	2.12%
Problem 5	8,849.1	0.66%	1,379.7	0%
Problem 6	7638.4	19.07%	8343.2	4.81%
Problem 7	7884.7	2.90%	7098.5	1.94%
Problem 8	534	13.20%	528	0.65%

Table 6.9 shows the noteworthy difference between relaxed and un-relaxed problems regarding the relative gaps obtained in the same solution time. This difference leads us to test the proposed method relaxing the integrity constraint of variables representing number of boxes as well while solving Phase-1 problems.

6.2.1. Environmental Settings

The common environmental settings for which the performance of the proposed method is compared against the current practice are as follows:

- Initial inventory of each component is set to zero.
- The planning horizon is 44 or 31 days depending on whether two-month demand data or one-month demand data is used as input. Since we have positive lead times and zero initial inventories, demand only occur on the last 26 and 13 periods of the planning horizon respectively.
- A time bucket of 2 days is considered.

- In order to solve problems with longer demand period, inter-country transportation lead time is the only lead-time component considered solving the mathematical models. The effect of other lead times is reflected with an initial preprocessing of timing of the demands, i.e., manufacturing lead-time is implicitly taken into account in retrospective component demand data, demand of each component is shifted backwards by the time equal to fixed lead time, dispatch time that is obtained by the solution is back shifted by supplier manufacturing lead-time in order to obtain the latest time to place order for the components.
- Although there is an improvement in the total cost of the rest of the supplier clusters, the comparison is made only for the clusters which are alternative suppliers for some components because of time restriction.
- The number of intersecting clusters is 10 for our trials. Table 6.1 shows the intersections of suppliers.

Table 6.10: Intersections of supplier clusters

	Germany	**UK**	**Spain**	**Italy**	**France**	**Slovenia**	**China**	**Czech Rep.**	**Holland**	**Japan**
Germany		X	X			X	X	X	X	X
UK	X		X							
Spain	X	X		X						
Italy			X							
France						X	X			
Slovenia					X					
China					X					
Czech Rep.	X									
Holland	X									
Japan	X									

- Transportation modes 1, 2 and 3 are used while solving the aggregate problem in order to make the model more efficient. The initial trial runs give the idea for the base environmental settings as discussed previously. During the performance of trial runs, transportation mode 4, LTL with two-drivers, is not selected in any case. Moreover mode 5, airfreight is found not to be selected to transport the components. The limit weight for airfreight transportation is constrained for all clusters comparing the cost with that of less-than-truckload transportation. Breakpoint for the weight is found and set as limit weight for the airfreight dispatch. Above that weight the less-than-truckload cost is more than the airfreight cost. The main reason for that initial decision is to obtain a solution. However even for smaller sized problems the model did not prefer airfreight in any case. Considering the quantity/box constraint as well, almost none of the components are proved not to be transported by airfreight as well. Then such restriction for mode 5 as well is made while solving the sample problems.

6.2.2. Comparison of Proposed Method with the Current System

The proposed method is tested on the problems with both retrospective and random demand data.

The following pieces of data are obtained or generated:

- **Demand:** Retrospective daily usage rates of the components are obtained from ERP reports for year 20005-2008. Manufacturing lead-time is implicitly taken into account in these data. Randomly generated demand data are used for some problems.
- **Quantity per Box, Purchasing Quota, Unit price:** Year 2008 data are used.
- **Transportation costs:** Year 2008 data are used.

- **Inventory carrying charge:** 2-day interest rate, 0.028 %, is used, since each period represents 2-days in our comparison.

Total of inventory holding costs and transportation costs in current settings are calculated based on the current inventory and transportation decisions which are stated in general in Chapter 3, which is the description of the problem environment. The decision criteria of current activities in order to compare with the proposed method are stated first in following part.

Current Inventory and Transportation Decisions

Main considerations regarding order planning are as follows:

- Lead-time components that are mentioned in Chapter 4 (maximum inter-country transportation lead-time for the related supplier cluster is considered as for transportation lead time).
- Minimum order quantity constraint.
- Standard dispatch date of the vehicles for the related supplier.
- Purchasing quota of the suppliers (purchasing quota is taken into account in every period).

Transportation decisions are made independent of the ordering decisions. Each supplier is considered independently. A certain standard transportation mode is defined for each supplier in the current system. The decision of standard transportation mode depends on the country of origin, unit weight of the component and some special component characteristics. Some important remarks for current transportation decisions are as follows:

- Less-than-truckload is the standard transportation mode for the suppliers that are located in Europe. Dispatches are made weekly from those suppliers. The dispatch day is fixed for most of the suppliers. Only in cases of the truck is loaded in full capacity by chance, FTL cost is incurred.

- Vessel is the standard transportation vehicle for the parts that are procured from the Far East and Japan. Containers are loaded in full capacity for the components which are large both in weight and volume when the standard vehicle is vessel. Containers are loaded in full capacity for the dispatches from Japan so the orders are placed considering this constraint.
- The fragile components such as electronic parts with relatively smaller weight are preferred to be transported by airfreight.

Dispatch frequency, dispatch date, standard transportation mode is set for each suppliers. Current order and transportation activities are as follows:

- The demand figures are considered as the multiples of number of boxes as in the proposed system. $R_{j,t}$, demand of component *j* in period *t* in terms of number of boxes, is used to obtain these demand figures.

  $$R_{j,t} = \max\left\{0, \left\lceil \left(\sum_{i \in I}\left(D_{i,t+\Lambda_j} a_{ji}\right) + E_{j,t}\right) \Big/ O_{j,s} - \sum_{t=1}^{t-1} R_{j,t-1} + \sum_{t=1}^{t-1}\left(\sum_{i \in I}\left(D_{i,t+\Lambda_j} a_{ji}\right) + E_{j,t}\right) \Big/ O_{j,s} \right\rceil\right\}$$

  (5.1)
- Demands of the components with alternative suppliers are allocated to each alternative supplier in each period.
- Dispatches are made weekly in the current system which makes the demand of some periods to arrive in advance.

Figure 6.1 depicts the current practice of inventory and transportation decisions.

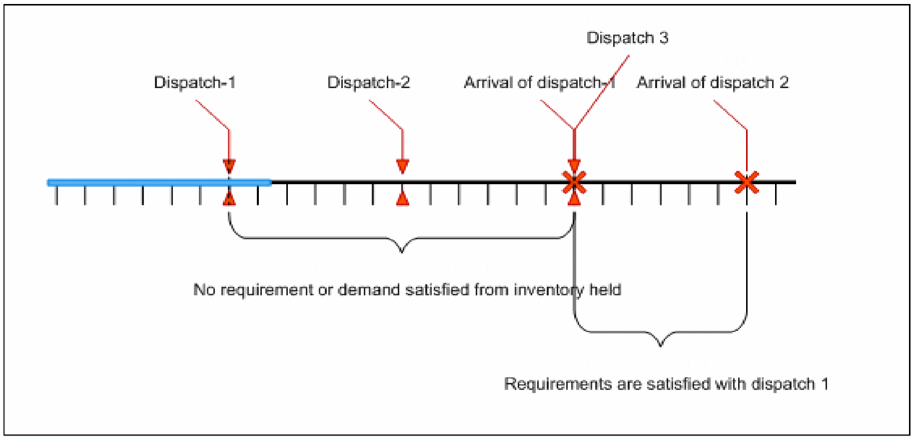

Figure 6.1: Current practice of inventory and transportation decisions

One dispatch is made from each supplier located in Europe in a specific day of the week. Daily requirements between the arrivals of two consecutive dispatches are satisfied by the components that are procured with the preceding one. Current practice in Japan and Far Eastern countries is different. There is no specific dispatch date in these countries. Full container is loaded in every dispatch so some orders have to be placed in advance and those components have higher inventory holding costs.

Total inventory holding and transportation cost of the current system is calculated by an iterative approach considering current decision making strategies. Detailed cost calculation steps for the suppliers located in Europe are given in Appendix A.

Comparison based on Realistic Problem Instances

The comparison is made for both real and generated problems instances. Transportation and inventory holding costs are determined as the main performance measure for the comparison. Regarding the transportation cost, additional costs such as the payment for custom procedures, national transportation, are not included in transportation costs.

Both in the solution of Phase-2 problems of the proposed method and in the solution of the problem with the current decisions, the demand figures are converted into multiples of the quantity per box. Actual inventory holding cost is not calculated as a result of this conversion. That cost is excluded in the comparison since this excluded component of inventory holding cost is the same in for current practice and proposed procedure.

Phase-1 problem as stated before in this section can not be solved optimally which leads us to propose some procedures to obtain approximate solution for Phase-1 problem.

The approximate solution approaches which are used to solve the realistic problems, with retrospective or random demand, are as follows:

- **Approach 1:** Taking two-month demand data as an input for Phase-1 problem and using the output obtained in three-hour solution time to solve Phase-2 problems.
- **Approach 2:** Taking two-month demand data as an input for Phase-1 problem and using the output obtained in two-hour solution time to solve Phase-2 problems.

Addition to these two approaches some other approaches which are not included in the scope of our study are as follows:

- Converting Phase-1 problem with two-month demand data into two separate problems with one-month demand and using the output obtained in two-hour solution time to solve two distinct set of Phase-2 problems.
- Integrity constraint for number of boxes is relaxed in Phase-1 problem. Phase-2 problems are solved using the output of the optimal solution or

integer solution obtained in 3 hours (depending on the problem size optimal solution for relaxed problems can be obtained in 3 hours)

There are three modes of transportation i.e., FTL, LTL and FTL with two-drivers, defined in the mathematical model of the Phase-1 problems for all problem instances discussed. However all of the five transportation modes are defined in the model of Phase-2 problems. Components are classified under nine different classes in all problems. Problem characteristics that are not common are provided in Table 6.11. Number of suppliers has no importance for the solution of the problem with the proposed approach. This number has importance for the solution under current settings since transportation decisions are made separately for each supplier and as this number increase total cost obtained by current method increases because of the fixed transportation costs.

Table 6.11: Characteristics for realistic problem instances

	PROBLEM												
	I	**II**	**III**	**IV**	**V**	**VI**	**VII**	**VIII**	**IX**	**X**	**XI**	**XII**	**XIII**
Planning period	45	44	37	46	42	44	42	44	45	36	42	44	44
Demand	real	real	real	real	real	real	real	real	created	real	created	real	created
Periods with demand	27	26	19	28	24	26	24	26	27	18	22	26	26
Number of components with alternative suppliers	27	37	26	34	41	35	37	38	41	33	20	36	36
Number of components with sole suppliers	466	297	355	435	554	392	523	312	479	496	377	466	466
Number of suppliers	58	39	51	55	43	53	38	68	36	58	37	53	53
Number of supplier clusters	10	10	10	10	10	10	10	10	10	10	10	10	10
Number of clusters for components with sole supplier	10	10	9	10	8	9	5	9	5	10	5	9	9

In Table 6.11 planning period from problems III and X are shorter than the others because of the planned closure for one week in each. Changes in the number of suppliers, number of components, and number of clusters from which the components with sole suppliers are procured show the variations of number of active components in a specific time period as stated and discussed in Chapters 1 and 2.

Phase-1 problems for the above problem instances are solved both for three and two hours to give input for Phase-2 problems. The obtained solutions are compared with the solution of the problem with current method. Table 6.12 provides the relative gap between the best integer solution found by solving Phase-1 problem under the two approaches stated above and the optimal solution of LP relaxation.

Table 6.12: Relative gap between Phase-1 and relaxed LP solution

		Gap obtained Solving Phase-1 problem	
		Elapsed time: 3 hours	**Elapsed time: 2 hours**
PROBLEM	**I**	13.84%	13.88%
	II	12.67%	12.74%
	III	2.12%	4.6%
	IV	12.31%	12.36%
	V	12.90%	12.91%
	VI	1.63%	1.69%
	VII	34.23%	35.04%
	VIII	27.41%	27.49%
	IX	12.49%	12.49%
	X	1.48%	2.06%
	XI	7.31%	9.08%
	XII	15.67%	15.73%
	XIII	12.45%	12.68%

Table 6.13 provides the comparison of final solution obtained by Phase-2 problems and current practice. Both transportation and holding costs are provided in Table 6.13. Decrease in both transportation and inventory holding costs are observed as well. The percentage of these cost decreases are referred to as percentage improvement and provided in Table 6.13 as well. No matter the cost component the percentage improvement in cost component is found as follows:

$$Improvement\ \% = (cost_{current} - cost_{proposed}) \big/ cost_{current} \cdot 100$$

Table: 6.13: Transportation and inventory holding costs for realistic problems obtained by current and proposed approaches

	Current Method		**Proposed Method Approach 1**				**Proposed Method Approach 2**			
	Transport. cost	**Inventory holding cost**	**Transportation cost**	**Inventory holding cost**	**Improvement percentage (transportation)**	**Improvement percentage (holding)**	**Transport. cost**	**Inventory holding cost**	**Improvement percentage (transportation)**	**Improvement percentage (holding)**
I	182,163	2642	157,445	1704	13.57%	35.50%	157,445	1704	13.57%	35.50%
II	161,430	4569	149,066	2544	7.66%	44.32%	149,066	2544	7.66%	44.32%
III	135,450	4013	112,544	1888	16.91%	52.95%	113,178	1866	16.44%	53.50%
IV	191,065	7822	171,617	2785	10.18%	64.40%	171,617	2785	10.18%	64.40%
V	111,355	4807	71,238	1757	36.03%	63.45%	71,238	1757	36.03%	63.45%
VI	138,381	7634	120,320	2247	13.05%	70.57%	120,320	2247	91.31%	70.57%
VII	117,759	5141	91,454	1764	22.34%	65.69%	92,307	1656	21.61%	67.79%
VIII	153,546	14218	123201	3622	19.76%	74.53%	123201	3622	19.76%	74.53%
IX	139,036	4610	115,685	1940	16.79%	57.92%	115,685	1940	16.79%	57.92%
X	155,103	8671	127,672	2115	17.69%	75.61%	128,881	2084	16.91%	75.97%
XI	134,370	9161	100,295	2556	25.36%	72.10%	101,548	2366	24.43%	74.17%
XII	113,973	7400	99,269	2860	12.90%	61.35%	99,269	2860	12.90%	61.35%
XIII	93,176	5188	82,826	392	11.11%	92.44%	82,826	392	11.11%	92.44%

Improvement level for inventory holding cost is relatively high compared to the improvement observed for the transportation cost. The main reason of this is that the proposed method allows dispatch in every day of the week since the goods of different suppliers can be consolidated instead of waiting for one week to consolidate the components that are procured from a sole supplier.

Total cost obtained by the proposed solution approaches and current method and improvement percentage in total cost for all of the problems are provided in Table 6.14.

Table: 6.14: Total cost for realistic problems obtained by current and proposed approaches

	Current Method	Proposed Method Approach 1		Proposed Method Approach 2	
	Total Cost	Total Cost	Improvement percentage (total cost)	Total Cost	Improvement percentage (total cost)
I	184,805	159,149	13.88%	159,149	13.88%
II	165,999	151,610	8.67%	151,610	8.67%
III	139,463	114,428	17.95%	115,044	17.51%
IV	198,887	174,402	12.31%	174,402	12.31%
V	116,162	72,995	37.16%	72,995	37.16%
VI	146,015	122,567	16.06%	122,567	90.22%
VII	122,900	93,218	24.15%	93,963	23.55%
VIII	167,764	126,823	24.40%	126,823	24.40%
IX	143,646	117,625	18.11%	117,625	18.11%
X	163,774	129,787	20.75%	130,965	20.03%
XI	143,531	102,851	28.34%	103,914	27.60%
XII	121,373	102,129	15.86%	102,129	15.86%
XIII	98,364	83,218	15.40%	83,218	15.40%

Both Table 6.13 and Table 6.14 shows that the both the cost components and the total costs are approximate for the two approaches of proposed solution method. As the difference of relative gaps, gap that is between solution of the first approach and LP relaxation and the difference between solutions of Approach-2 and LP relaxation decrease, total cost obtained by solving the problem with Approach-1 and Approach2 of the proposed method are found as the same. The small difference of the costs obtained by solving Phase-1 problem results generally from the cost terms associated with the aggregate problems or the demand differences obtained by solving Phase-1 problems have no effect for the solutions of Phase-2 problems. However for the cases where the gap difference is relatively high, the order quantities are different for Phase-model in Approach 1 and 2 which affects the Phase-2 problem solutions depending on the size of Phase1 problem order quantities and the weights of the associated components. As the proportion of the orders of these components get higher the difference of the total cost obtained under the two approaches become higher.

Full data set of the realistic problem instances are provided in Appendix B.

6.2.3. Summary of Results

- The system, first of all proposes an integrated method for purchasing and transportation decisions of the components.
- As the unit cost of alternative suppliers decrease inventory holding cost will decrease as well. As the number of alternative suppliers increase, orders of the new component mix will probably be consolidated in order to continue to use transportation economies from the current supplier clusters (where the total purchase quantity is very high),. This will probably result in some increase in inventory holding cost. Moreover this may result in procurement of a component from the supplier with

shorter lead-time for the foremost periods and from the alternative supplier with longer lead-time for the rest of the periods.

- A purchasing plan is proposed for the components with more than one supplier clusters.
- The solution of the realistic problem instances shows that consolidation of load of different suppliers in the same cluster saves in fixed transportation cost.
- Consolidation of the load of same supplier is taken into account at the same time since inventory and transportation decisions are integrated in the proposed system.
- Order consolidation is preferred for the supplier clusters from where relatively less number of components are procured.
- If inventory holding cost is high purchasing quota is guaranteed by the orders which are placed in the latest periods.
- Cost advantage is more for the components currently purchased in small lots because of the joint transportation costs.
- Consolidation decisions bring some other fixed transportation cost advantages as well which are not taken in the scope of the study.

CHAPTER 7

CONCLUSION AND FUTURE DIRECTIONS

In this study an integrated procurement and transportation planning approach for the purchased components of a consumer-durables company is proposed. Proposed method use the advantage of the coordinated replenishment and transportation of the imported components which decrease the sum of inventory holding and transportation costs of the company.

There are more than 120 suppliers located in about 20 countries all around the world so that there is more than one supplier in almost every country. Moreover, there is more than one type of component that is procured from almost every supplier. This characteristic of the system brings the possibility for the consolidated transportation of the goods delivered from the suppliers within the same country or consolidated replenishments of SKUs from a given supplier. A mathematical model is formulated in order to solve this problem.

The proposed method is turned to be a two-phased prolem in order obtain a good solution for the mathematical model developed. The supplier clusters in which all suppliers are sole suppliers are naturally decomposed from the problem and the original mathematical model is solved for those clusters. Therefore, for the components with alternative suppliers in different clusters, the clusters are made independent first since purchasing amounts from alternative suppliers affect the both transportation and inventory decisions. Current purchasing strategy of heading towards Far Eastern makes this issue especially important in our study since there is a need to have alternative suppliers with for the suppliers in Far East. The amounts are determined by solving an a first phase problem where the components with single clusters are aggregated with respect to the weight transported in order to consider consolidation decisions in this first problem as well. Then the original model is solved for each cluster separately and final solution is obtained.

Performance of the proposed inventory and transportation decisions is compared with the current decisions of the company for different instances in terms of the total transportation and inventory holding costs using retrospective data. Uncertainty of demand affects purchasing decisions in the current system. In our study demand is assumed to be constant so the inventory holding cost of the two systems is compared in the condition where the demand is deterministic. Proposed method is proved to decrease the total inventory and transportation cost of the system even though the first aggregate problem can not be solved to optimality.

Some more remarks for future directions are as follows:

- For the solution of the proposed method may result in the changes of the standard transportation mode some of the suppliers.
- Consolidation of the goods from Far East which is important especially to decrease the long transportation lead-time will become more important in the future because of the new trend of increasing purchased amounts from Far Eastern countries.
- The proposed method is useful indirectly while making budget for transportation costs. The optimal ratio of transportation cost to total

value of the components can be determined in advance using the demand figures.

- Consolidation decisions bring some other cost advantages which are not taken in the scope of the study. Transportation cost is limited only with the charter fee cost. As the number of dispatches decrease the fixed costs related with the number of dispatches decrease as well.

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APPENDIX A

Step 1: A new parameter $\Xi_{j,s,t}$ demand of component *j* in boxes that is procured from supplier *s* in period *t* is defined. $\Xi_{j,s,t} = q_{s,j} \cdot R_{j,t}$.

Step 2: Inter-country transportation lead time of every supplier is set equal to the LTL transportation lead-time of corresponding supplier cluster.

Step 3: A standard dispatch day is fixed for every supplier. *t*=1 is considered as that standard date for every supplier in order to make the calculations easier.

Step 4: The latest dispatch time of each period's demand is calculated considering that there is only one dispatch per week for each supplier. A new parameter $LTD_{j,s,t}$ latest time for dispatch is defined for each component *j* procured from supplier *s* for the demand in period *t*. Order sizes are defined for each dispatch period as well.

$$LTD_{j,s,t} = \left\{ \begin{array}{l} 1 \;\; if \;\; t - L_{c,m=2} < 7 \\ 7 \;\; if \;\; t - L_{c,m=2} \geq 7 \; and \; |t| - L_{c,m=2} < 14 \\ 14 \;\; if \;\; t - L_{c,m=2} \geq 14 \; and \; |t| - L_{c,m=2} < 21 \\ 21 \;\; if \;\; t - L_{c,m=2} \geq 21 \; and \; |t| - L_{c,m=2} < 28 \\ \ldots \\ \ldots \end{array} \right\}$$

Step 5: Demands of some periods are dispatched in advance because of the weekly deliveries. The number of periods that the items are held in inventory is determined as the next step. Let $\hbar_{j,s,t}$ number of days of holding the demand of component *j* procured from supplier *s* in period *t* as inventory. $\hbar_{j,s,t} = |t| - L_{c,m=2} - LTD_{j,s,t}$

Step 6: Inventory holding cost in the manufacturing plant is calculated for each component. Let $C^{inv}_{j,s,t}$ inventory holding cost in the manufacturing plant for component *j* that is procured from supplier *s* for demand in period *t*. $C^{inv}_{j,s,t} = r \cdot u_{j,s} \cdot O_{j,s} \cdot \hbar_{j,s,t} \cdot \Xi_{j,s,t}$. Total inventory holding cost is calculated summing the inventory holding costs for all components, all suppliers and for all periods.

Step 7: Weight to be dispatched in period *t*=1, 7, 14 ... is calculated for each supplier in the next step. Let $\varpi_{s,t}$ represents this total weight.

Step 8: Total transportation cost is calculated for each $\varpi_{s,c,t}$ where *t*=1, 7, 14 ... If the total weight loads the truck in full capacity LTL cost is incurred. Otherwise the transportation cost is calculated using LTL transportation cost structure. Let $C^{transport}_{s,t}$ transportation cost of components dispatched by supplier *s* in period *t*.

$$C^{transport}_{s,t} = \left\{ \begin{array}{l} 0, if\ \varpi_{s,t} = 0 \\ V_{2,c}, if\ \varpi_{s,t} \leq G_{2,c} \\ g_{2,c} \cdot \varpi_{s,t}, if\ \ \varpi_{s,t} > G_{2,c} \end{array} \right\} \text{ where supplier } s \text{ is located in cluster } c.$$

Finally the total transportation cost is calculated summing $C^{transport}_{s,t}$ for all suppliers and dispatch periods.

APPENDIX B

(Provided in the CD attached to the front cover)