A LINEAR PROGRAMMING APPROACH TO QUALITY IMPROVEMENT PROJECT AND PRODUCT MIX SELECTION UNDER INSPECTION ERROR AND REWORK

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NEDRET ŞARBAK

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Prof. Dr. Canan Özgen
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

Prof. Dr. Çağlar Güven
Head of Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

Prof. Dr. Gülser Köksal
Supervisor

Examining Committee Members

Assistant Prof. Dr. Seçil Savaşaneril (METU, IE)

Prof. Dr. Gülser Köksal (METU, IE)

Assistant Prof. Dr. Pelin Bayındır (METU, IE)

Assoc. Prof. Dr. Murat Caner Testik (Hacettepe Unv., IE)

Benin Mertoğlu (M.S.) (ASELSAN)

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name : Nedret ŞARBAK

Signature :

ABSTRACT

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Şarbak, Nedret

MS., Department of Industrial Engineering

Supervisor : Prof. Dr. Gülser Köksal

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In this study, the effect of inspection error on the product mix and quality projects selection in a manufacturing environment where rework and inspection errors exist is examined. It is assumed that the products (items) for which rework is necessary are reprocessed at a separate work center and 100% inspection is performed for the products both after rework and processing operations. Markov chain approach is used to compute yield and rework rates. In addition, nominal-the-best type of a quality loss function is used in computing quality loss due to products shipped to the customers. A linear programming (LP) model is developed to support the product mix and quality improvement project selection decisions. The use of LP model is demonstrated on an example problem. The results obtained under different experimental conditions are compared with solutions of a naive QI project selection method, improving the least capable process. The analysis shows that developed LP model is relatively better than process capability approach. Besides, according to the results obtained under different experimental conditions, the factors that have significant effect on throughput and QI project selection are being determined.

Keywords: Product mix, quality improvement, inspection error, rework, %100 inspection

ÖZ

MUAYENE HATASI VE YENİDEN İŞLEME OLDUĞUNDA KALİTE İYİLEŞTİRME PROJELERİ VE ÜRÜN KARMASI SEÇİMİNE DOĞRUSAL PROGRAMLAMA YAKLAŞIMI

Şarbak, Nedret
Yüksek Lisans, Endüstri Mühendisliği Bölümü
Danışman : Prof. Dr. Gülser Köksal

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Bu çalışmada, yeniden işlemenin olduğu üretim ortamında muayene hatalarının ürün karması ve kalite iyileştirme (Kİ) projelerinin seçimi üzerindeki etkisi incelenmiştir. Yeniden işlenmesi gereken ürünlerin (parçaların) asıl işleme istasyonundan ayrı bir yerde yeniden işlem gördüğü ve ürünlerin hem işleme hem de yeniden işleme istasyonlarında %100 muayene edildiği varsayılmıştır. Üretim oranları ve yeniden işleme oranlarının hesaplanmasında Markov zincirleri yöntemi kullanılmıştır. Ayrıca, bu imalat sürecinde işlem görüp müşteriye gönderilen ürünlerin sebep olacağı kalite kaybının hesaplanmasında Taguchi'nin hedef-en-iyi kalite kayıp fonksiyonundan yararlanılmıştır. Çalışmada, ürün karması seçimine ve öncelikle iyileştirilmesi gereken iş istasyonuna karar vermede kullanılabilecek bir doğrusal programlama (DP) modeli geliştirilmiştir. DP modelinin kullanımı bir örnek üzerinde gösterilmiştir. Farklı deneysel durumlar altındaki sonuçlar basit bir kalite iyileştirme proje seçim metodu olan süreç yeterlilik yaklaşımı ile karşılaştırılmıştır. Analizler DP modelinin süreç yeterlilik yaklaşımına göre çok daha iyi olduğunu göstermiştir. Ayrıca, bu deneysel tasarım sonuçlarına göre, satış geliri ve Kİ proje seçimi üzerinde önemli etkiye sahip olan faktörler belirlenmeye çalışılmıştır.

Anahtar kelimeler: Ürün karması, kalite iyileştirme, muayene hatası, yeniden işleme, %100 muayene

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CHAPTER 1

INTRODUCTION

Most of the quality improvement (QI) studies fail to achieve satisfactory quality levels and financial results, because of their lack of focus on the results. When there are more than one QI projects, all of these may not be applied at the same time due to time and budget constraints. Therefore, only the vital few projects need to be isolated from trivial many. Accordingly, decision makers should make a selection between the potential projects.

Manufacturing sector is focused in this study. In QI studies performed in production systems, during the selection of QI projects, product mix is generally assumed to be constant. However, the product mix quantities show changes with respect to the improved process. Atwater and Chakravorty (1995) showed that product mix affects QI projects and vice versa. Therefore, the two problems, quality improvement project selection and product mix determination should be resolved simultaneously. These two problems were studied together and an LP model aiming to maximize profit and also minimize quality loss was constructed by Köksal (2004). For the examined system by Köksal (2004), it is assumed that there is no inspection error and no rework. However, the problem can be made more realistic since highly advanced measurement systems may contain errors. In addition, when a product characteristic value does not conform to specification limits, the product may not be scrapped in practice. If rework is possible this item is sent to rework and recovered. In this thesis, a production environment where inspection error exists and rework is possible is studied.

In this study, the effect of inspection errors on the product mix and QI project selection in a manufacturing environment where rework and inspection errors exist is examined. It is assumed that the products (items) for which rework is necessary are reprocessed at a separate rework center and 100% inspection is performed for the products both after rework and main processing operations. A Markov chain approach is used to compute yield and rework rates. In addition, nominal-the-best type of a quality loss function is used in computing quality loss due to products shipped to the customers.

By modifying the mathematical model in Köksal (2004), a linear programming model is developed to support the product mix and quality improvement project selection decisions. By using the constructed model, it is aimed to help the decision maker for the QI project selection. Using this model, product mix and QI project priorities can be identified for a given planning period.

The thesis consists of seven chapters. In the first part of the Chapter 2, background information is given about the basic concepts used in this research. The latter part is a summary of the relevant literature. In Chapter 3, the examined production system, derivation of the probability formulas and the validation of these formulas are explained. In Chapter 4, the QI project and product mix selection algorithm using the developed linear programming (LP) model is explained on a sample problem. On the same problem, the performance of a naive QI project selection method, improving the least capable process, is tested and compared with the LP model, in the latter part of the Chapter 4. In Chapter 5, a design of experiment is constructed and the effects of the terms in the Throughput-Loss model are aimed to investigate. Conclusions and future research directions are given in Chapter 7.

CHAPTER 2

LITERATURE REVIEW

2.1 Background

2.1.1 Quality Loss

The quality of a product is measured in terms of its characteristics. These characteristics determine the products performance with respect to customer requirements or expectations (Ross, 1996). According to the traditional view of quality, all products measured as between the lower and upper specification limits are of high quality while those that are outside the specification limits are defective. But, it is not sufficient to manufacture a product that conforms to specification limits to satisfy customers and to keep the position in the competitive market.

Some manufacturers tend to measure quality in terms of the rate of the total number of defective items to the total number of produced items, i.e., fraction defective (Phadke, 1989). This method implies that the products are not different and equally good as long as they are within the specification limits. This approach is called as 'goal-post' syndrome (Ross, 1996). In the goal-post model, no loss is incurred unless the quality characteristic of the product is outside the specification limits. An item, which is very close to a limit but within the specifications and another one, which is very close to or at the target, are treated as the same. However, an item, which is close to limit but out of the specification, is labeled as defective and therefore, consumers incur a quality loss.

Taguchi disagrees with the traditional goal-post model (Taguchi et al., 1989). Today, the quality level of a product is determined according to product's total loss to the society 'due to the failure of the product to deliver the target performance and due to harmful side effects of the product, including its operating cost' (Phadke, 1989). The important thing ignored in the goal-post approach is that, from customers' point of view, there is not much difference between the product barely meeting the limits and the product barely being out of the limits (Ross, 1996). Hence, a product, which conforms to the producer specifications, determined with respect to customers' needs and expectations (Ross, 1996), inflict a quality loss to the consumer. This loss adversely affects the sales of the product and name of the manufacturer (Phadke, 1989).

According to the modern philosophy developed by Taguchi, every product sent to a customer reflects a loss even if its quality characteristic value is defined as conforming. This loss can generally be defined as the loss caused by deformation of the product functionality or properties during its life cycle. Thus, if a product does not perform as expected, consumers may sense some loss. Hence, quality loss function must also be defined for the products that meet specifications (Ross, 1996).

Taguchi declared that a customer is fully satisfied only when quality characteristic of the product is at the ideal value (target). The loss to the consumer increases as the quality characteristic value deviates from the nominal level. He emphasizes the importance of the quality performance that aims to reach target value on the average with the minimum deviation from this average value (Summers, 2000). For this reason, he focuses more on the process rather than product and develops a quality measure, which is a function of the deviation of the process from the target and variation in the process.

In Figure 2.1, loss function for goal-post syndrome and quadratic loss function of Taguchi are plotted for a nominal-the-best type quality characteristic, X. Here, LSL and USL represent lower and upper specification limits, respectively.

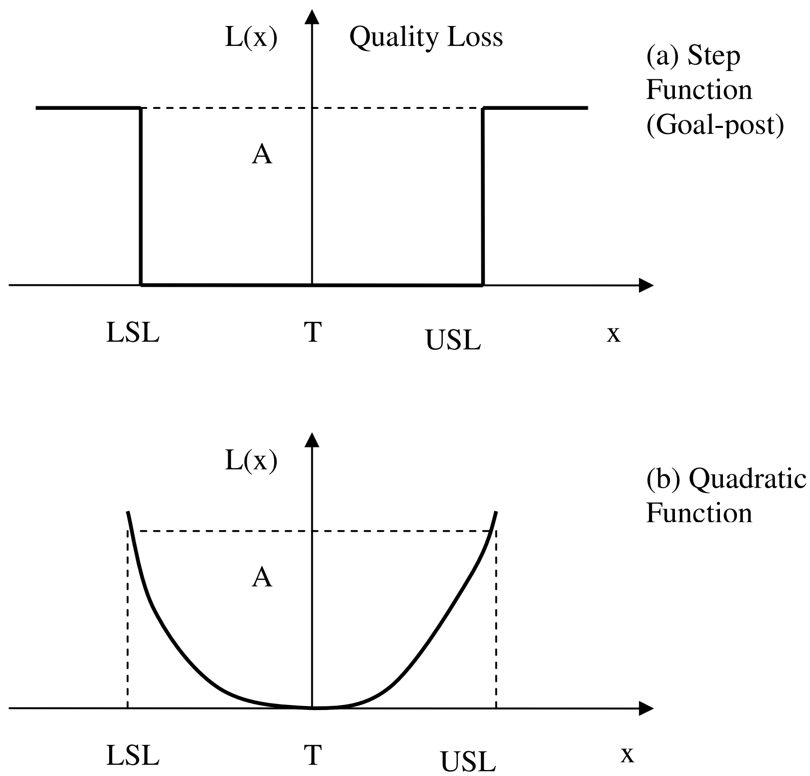


Figure 2.1 Step and Quadratic Loss Functions (Source: Adapted from Ross, 1989)

In Figure 2.1, USL and LSL are the upper and lower specification limits, respectively and T is the target value of the process.

Various loss functions, such as linear, quadratic and 0-1 loss functions, have been discussed in the statistical decision theory literature (Berger 1985, DeGroot 1970, Phadke 1989, Taguchi et al. 1989). In all types, loss is approximated via evaluating the deviation from a target value. Four general quadratic functions that express Taguchi philosophy about the relationship between quality and variability of the process are

- Nominal-the-best type,
- Smaller-the-better type,
- Larger-the-better type,
- Asymmetric type.

Nominal-the-best Type: For this type of loss function target value is the nominal value. This type of quality characteristics can take less or greater than the target values. The loss is incurred when quality characteristic value deviates from the target to either direction. Taguchi derives quadratic loss function using Taylor series expansion and eliminating the higher order terms (Taguchi, 1989). Afterwards, the loss function is approximated as follows:

$$L(x) = k(x - T)^2 \quad (2.1)$$

where x is the quality characteristic, k is the quality loss coefficient and T is the target.

In order to find k, specification limits and the loss at these limits should be determined. This loss consists of all the losses such as repair and replacement of the product. It also includes loss to consumer due to the lack of the product during repair operations and transportation cost (Phadke, 1989).

Then, the quality loss coefficient is computed as

$$k = \frac{A_0}{{\Delta_0}^2}, \quad (2.2) \quad \text{where } \Delta_0 = USL - T .$$

Hereafter, A_0 and T denote the expected quality loss incurred at the specification limits and process target value, respectively.

Smaller-the-better Type: Some quality characteristics can never take negative values. Also, their target value is zero and as their value increases their performance becomes worse. Such characteristics are called smaller-the-better type characteristics. For instance, waiting time for order delivery at a fast food restaurant is a smaller-the-better type characteristic. The quality loss function in such situations can be estimated by the following function:

$$L(x) = kx^2 \quad (2.3)$$

The quality loss coefficient k can be computed from functional limit, Δ_0, and the quality loss, A_0, in the same way as it is computed for nominal-the-best type characteristics.

Larger-the-better Type: Some quality characteristics can never take negative values and also their ideal value is infinity. Hence, zero is their worst value and as their values increase their performances get better and better. Such characteristics are called larger-the-better type quality characteristic. Bond strength of adhesives is a larger-the-better type characteristic. The quality loss function in such situations can be estimated by the following function:

$$L(x) = k\frac{1}{x^2} \quad (2.4).$$

By substituting functional limit, Δ_0, and the quality loss, A_0, in the equation above we can compute k as $k = A_0 \Delta_0^{\ 2}$.

Asymmetric Loss Function: In some situations, same amount of deviation from the target to one side can cause more loss than to another side. Then, different loss coefficient values should be used. The quality loss function can be approximated by the following asymmetric function:

$$L(x) = \begin{cases} k_1(x-T)^2, & x > T \\ k_2(x-T)^2, & x \le T \end{cases} \quad (2.5)$$

The types of quadratic loss functions are plotted in Figure 2.2.

Quantifying quality loss is difficult because of the different customers and applications and different environments in which it is used (Phadke, 1989). However, using average unit loss can approximate it (Kolarik, 1999). The average quality loss for nominal-the-best type is computed by Phadke (1989). The average quality loss function formulations for all types of quadratic loss functions are given in Table 2.1 where μ and σ^2 are the mean and the variance of the process, respectively.

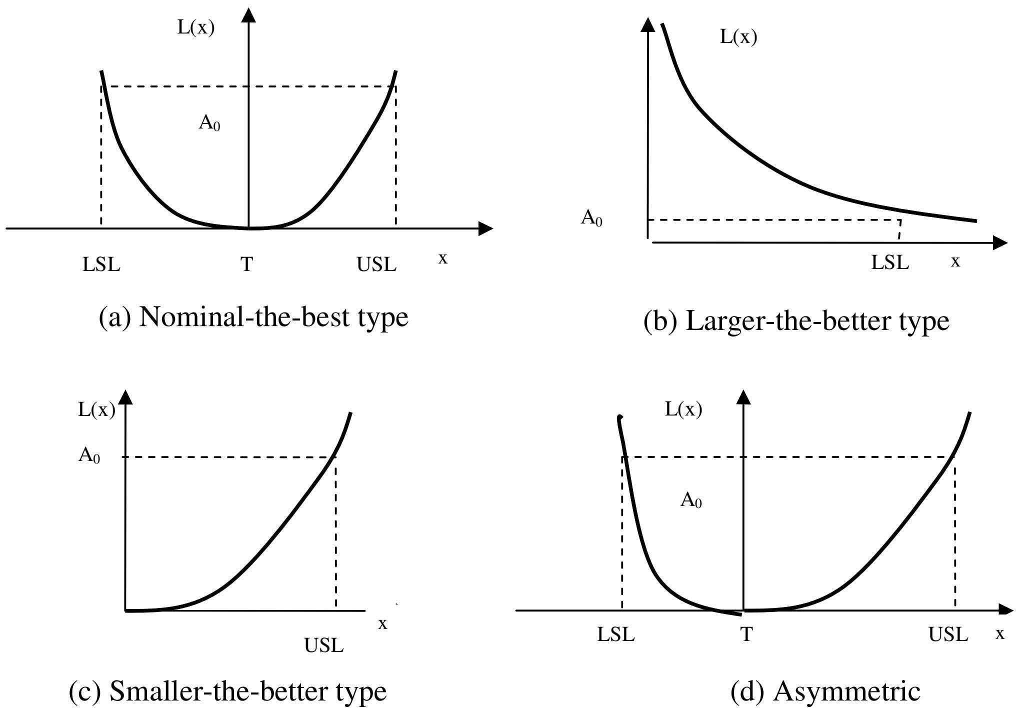


Figure 2.2. Types of Quadratic Loss Functions. (Source: Phadke, 1989)

Table 2.1. Expected value of Quality Loss Functions.

Type	Average Loss Function
Nominal-the-best	$\mathrm{E[L]} = k[(\mu - T)^2 + \sigma^2]$
Smaller-the-better	$\mathrm{E[L]} = k[\mu^2 + \sigma^2]$
Larger-the-better	$\mathrm{E[L]} = k(1/\mu)^2[1 + 3\sigma^2/\mu^2]$
Asymmetric	$\mathrm{E[L]} = \begin{cases} k_1[(\mu - T)^2 + \sigma^2], x > T \\ k_2[(\mu - T)^2 + \sigma^2], x \le T. \end{cases}$

On the other hand, Raiman and Case (1990) [cited in Kapur and Cho (1996)] discuss multi dimensional loss function in order to monitor product and improve process over time. Tang and Tang (1989) study the economic specification limits for multiple quality characteristics. They assume that the quality characteristics are independent of each other and computed the total loss by adding losses caused by quality characteristics.

Kapur and Cho (1996) think that the quality characteristics may be dependent in the real life and developed a multivariate quality loss function to evaluate such cases. In this loss function, $x_1, x_2, x_3, ..., x_n$ are the quality characteristics of the product and $t_1, t_2, t_3, ..., t_n$ are the target values of $x_1, x_2, x_3, ..., x_n$. They derive the loss function from the 2nd order Taylor series. Then, multivariate quality loss function can be approximated as the following:

$$\mathrm{L(x,t)} = \sum_{\mathrm{i=1}}^{\mathrm{m}} \sum_{\mathrm{j=1}}^{\mathrm{i}} k_{ij} (x_i - t_i)^2 (x_j - t_j)^2 \quad (2.6)$$

where k_{ij} is the proportionality constant depending on the losses at the specification limits. k_{ij} can be determined by using regression method (Chen and Kapur 1989, Neter et al. 1983)[cited in Kapur and Cho, 1996].

Then the expected quality loss is as follows:

$$\mathrm{E[L(}X,\mathrm{T)]} = \sum_{i=1}^{m} \mathrm{k_{ij}}[(\mu_i - T_i)^2 + \sigma^2] + \sum_{i=2}^{m} \sum_{j=1}^{i-1} \mathrm{k_{ij}}[\sigma_{ij} + (\mu_i - T_i)^2 (\mu_j - T_j)^2] \quad (2.7)$$

where k_i and k_j are the loss coefficients, and μ_i and μ_j are the mean values of the i^{th} and j^{th} quality characteristics x_i and x_j, respectively.

Moreover, Chen and Chou (2003) use multivariate quality loss function as well. They proposed a cost model with bivariate quality characteristics and quadratic asymmetrical quality loss function to determine the optimum process mean.

Taguchi Loss Functions do not consider the decrement in quality during the use of the product and are restricted to only the shipment time of the product. However, Teran et al. (1996) study the degradation effect of usage over time on the product. By considering product deterioration on quality, they introduced the present worth of expected quality losses and translated quality performance measures into financial measures.

As previously stated, the loss function L(x) explains the product characteristic deviation at an arbitrary time. However, as the inherent result of usage, such deviation may change during the time and hence, its loss. Thus, x and the corresponding loss function is a function of time and denoted as x(t) and L(x;t), respectively. The average loss $E[L(x;t)]$ is expressed by Teran et al. (1996) as follows:

$$E[L(x;t)] = k[\sigma_x^2(t) + (\mu(t) - T)^2] \quad (2.8)$$

where T denotes the target value and $\mu(t)$ and $\sigma_x^2(t)$ represent the mean and variance of *x* at time *t*, respectively.

The present worth of expected quality losses introduced by Teran et al. (1996) takes into account only one quality characteristic. By incorporating multivariate quality loss function approach of Kapur and Cho (1996) into Teran et al. (1996)'s loss function, Chou and Chen (2001) develop the present worth for expected multivariate quality loss (PWEMQL). They decompose PWEMQL into these three additive components:

1. present worth of expected multivariate quality loss due to variances
2. present worth of expected multivariate quality loss due to means
3. present worth of expected multivariate quality loss due to covariance

2.1.2 Measurement System Analysis

In any production process, even it is well designed and carefully maintained; a certain amount of inherent variability will always exist (Montgomery, 2001).

Hence, any time the results of a process are measured, some variability will be seen between the measurement values. The two major sources of this variation are the differences between parts and imperfectness of the measurement. So, the variability of the measurable values can be defined as

$$\sigma^2_{\text{measured}-\text{values}} = \sigma^2_{\text{product}} + \sigma^2_{\text{measurement}-\text{error}}$$

As Montgomery and Runger (1993) declare, the quality of the recorded data relies very much on the gauge (gage) capability [cited in Pearn and Liao, 2005] and measurement error has great impact on the decisions depending on the measurement. Therefore, achieving an adequate gauge capability is one of the aspects that should to be considered in process control and quality improvement studies (Montgomery, 2001).

Measurement system errors can be classified into two categories: accuracy and precision.

- *Accuracy* describes the difference between the part's actual value and the average value of the measurements on the same characteristics of the same part.

- *Precision* describes the variation observed when the same part is repeatedly measured with the same device. Variability in measurements has two major concerns:

1) *Gauge Repeatability:* expresses the variation in measurements observed when the same operator measures the same part repeatedly with the same device. This is the variation due to measurement device.

2) *Operator Reproducibility:* expresses the variation in measurements recorded by different operators using the same measurement instrument on the same part. (Minitab, Release 14, 2004)

Kolarik (1999) defines the variance of the measurable values as

$$\sigma^2_{\text{measured values}} = \sigma^2_{\text{product}} + \sigma^2_{\text{gauge repeatability}} + \sigma^2_{\text{operator reproducibility}}$$

It is usually assumed that the three sources of variation in this equation are independent. If either the gauge repeatability or the producer reproducibility is large, with respect to the true dimensional variance, a certain action should be taken to reduce it (Kolarik, 1999).

The precision to tolerance ratio (P/T) is sometimes used to assess gauge capability. The P/T value is the ratio between the measurement precision estimate and the tolerances of the quality characteristic being measured. According to Kolarik (1999),

$$P/T = \frac{6\sigma_{\text{measurement error}}}{\text{USL - LSL}} \quad (2.9)$$

If P/T ratio is less than 0.10, then the gauge or measurement system is typically considered adequate (Montgomery, 2001). Otherwise, faulty measurement process will limit the ability to assess products or processes. Corrective actions that should be taken to eliminate measurement error can be more precise gauging (better instrumentation), operator training programs, or both (Kolarik, 1999).

In addition, according to P/T ratio values, measurement system is assumed to be conforming or need adjusting. After consulting to Tümer Arıtürk (SPAC Six Sigma Consulting Company, personal communication, 2006), we have learned that measurement capability studies are performed in practice as follows:

- if P/T ratio is less than 0.05, there is no problem in the measurement system,
- if P/T ratio is between the values of 0.05 and 0.10, the measurement system is accepted as conformable,
- if P/T ratio is between the values of 0.10 and 0.30, the measurement system is assumed to be conformable or needs to be adjusted relative to the second gage capability rate, Gage-Repeatability & Reproducbility (Gage-R&R). The Gage-R&R value is the ratio between the measurement precision estimate and the measured values standard deviation. This rate is estimated as

$$\text{Gage R \& R} = \frac{\sigma_{\text{measurement error}}}{\sigma_{\text{measured values}}} \quad (2.10)$$

In practice, Gage-R&R value is aimed not to exceed 0.30 for the conformed measurement system. However, if it reaches a value close to 0.90, measurement is assumed not performed well and therefore, measurement study is executed once more. On the other hand, in this study, we assumed that the values of Gage R&R exceeding 0.90 are based on the high standard deviation values of the measurement system rather than wrong measurements.

2.1.3 Process Capability Analysis

Process capability indices are critical performance measures, which are used to express the relationship between technical specifications and process abilities on the production floor (Kolarik, 1999). They have been widely used in industry, to provide a numerical measure on how well a process is capable of producing items meeting the quality requirements preset in the factory (Pearn et al., 2001).

The most commonly used process capability indices are C_p and C_{pk}. C_p measures an inherent or potential capability of production process that meets the specifications (Kolarik, 1999). C_p is defined as

$$C_p = \frac{USL - LSL}{6\sigma} \quad (2.11)$$

where USL and LSL are upper and lower specification limits, respectively and σ is the standard deviation of the process.

C_p measures the spread of the specifications relative to the 6σ spread in the process provided that process is centered at the target value. Hence, it does not take the location of the process mean relative to specifications into consideration (Montgomery, 2001). Thus, C_p does not detect the off centering condition of the process. Therefore, for the situations, where the process does not produce at the target, it is more appropriate to use another measure that considers shifts from the target. That measure is C_{pk} and defined as

$$C_{pk} = \text{minimum}\left\{\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma}\right\} \quad (2.12)$$

While C_p simply compares the tolerance spread to the natural spread of the process, C_{pk} also considers the location of the process (Rodriguez, 1992). Thus, C_{pk} index is a realized measure of actual production (Kolarik, 1999).

In some cases production specifications are set only one side. At that time, the following two measures can be used to evaluate the process capability:

$C_{pU} = \dfrac{USL-\mu}{3\sigma}$ for processes that have only an upper specification limit

$C_{pL} = \dfrac{\mu-LSL}{3\sigma}$ for processes that have only a lower specification limit.

C_{pk} was initially developed to compensate the deficiency of C_p about determining off centered mean processes. However, C_{pk} alone is still an inadequate measure of process centering (Montgomery, 2001).

C_{pm} was developed independently by Hsiang and Taguchi (1985) and Chan et al. (1988) (cited in Rodriguez, 1992). This new index has the advantage of applicability to a process where the target is not located in the middle of the interval (for the processes that have asymmetrical intervals) and also the ability to show the shift of the process mean from the target. C_{pm} is defined as

$$Cpm = \frac{USL-LSL}{6\sqrt{(\mu - T)^2 + \sigma^2}} \quad (2.13)$$

pm index measures the degree of the process output location with respect to the target (Kolarik, 1999).

Pearn et al. (1992) described a more advanced capability index C_{pmk} by combining C_{pm} and C_{pk} .

$$C_{pmk} = \text{Minimum}\left\{\frac{\text{USL} - \mu}{3\sqrt{(\mu - \text{T})^2 + \sigma^2}}, \frac{\mu - \text{LSL}}{3\sqrt{(\mu - \text{T})^2 + \sigma^2}}\right\} \quad (2.14)$$

C_{pmk} is also defined by Kolarik (1999) as,

$$\text{Cpmk} = \frac{\text{Cpk}}{\sqrt{1 + (\frac{\mu - \text{T}}{\sigma})^2}} \quad (2.15)$$

When process mean is not on the target value, the C_{pmk} index provides more assurance than other process capability indices. Because it is more sensitive to departures of the process mean from the target value. The rank of four indices from the most sensitive to the least sensitive with regard to departures of the process mean from target value is as follows: C_{pmk}, C_{pm}, C_{pk} and C_p (Pearn et al., 1992).

2.1.4 Throughput Accounting and Theory of Constraints

Since first introduced by Goldratt and Cox (1984), TOC has been widely studied by practitioners and researchers. Two main review sources of TOC are Rahman (1998), and Mabin and Balderstone (2000). Rahman (1998) classifies TOC literature and proposes guidance for future research. Mabin and Balderstone (2000) introduce a comprehensive catalog of TOC literature. Gupta (2003) provides a brief historical background and basic concepts of the TOC and points out potential research issues, which may have substantial impact on the future TOC research.

Gupta (2003) provides a brief summary about TOC's business system perspective based on the mindset of the organization, the measures that drive the organization and the methods utilized within the organization. One of the main assumptions of the TOC is that every business has the primary goal "making money now and in the future as well" (Goldratt 1990).

Common to all Theory of Constraints applications are the following five focusing steps, which are used to manage constraints and continuously improve an organization (Goldratt and Cox, 1992):

Step 1 Identify the systems constraint or bottleneck
Step 2 Decide how to exploit the bottleneck
Step 3 Subordinate everything else to the above decision
Step 4 Elevate the system's bottlenecks
Steps 5 If, in a previous step, a bottleneck has been broken go back to step 1, but do not let inertia cause a system constraint. According to Umble and Srikanth (1995), 'a constraint is any element that prevents the system from achieving the goal of making money'.

A set of global operational measures (i.e. throughput (T), inventory (I), operating expenses (OE)) is used in TOC measurement system to determine the extent to which the organization is achieving its goal. Throughput is defined as 'the rate at which the system generates money through sales' (Goldratt and Cox, 1984), in other words, revenue through sales minus totally variable costs (Balderstone and Keef, 1999). Totally variable costs include raw material content of the products and any other variable costs that increase with the production simultaneously. Inventory is defined as 'all the money invested in purchasing things the system intends to sell' (Goldratt and Cox, 1984), which includes investments such as machines and equipment. Operating expenses is defined as 'all the money the system spends in turning inventory into throughput' (Goldratt and Cox, 1984), which includes wages, salaries, depreciation and maintenance.

TOC suggests that an organization must focus on three fundamental questions which are concerning change to accelerate its improvement process: (1) What to change, i.e., how do organizations identify the weakest link, the constraints? (2) To what to change, i.e., how should organizations strengthen the constraint by developing practical and good solutions? (3) How to cause the change, i.e., how should organizations should implement the solutions? (Gupta, 2003).

Management accounting systems affect the product mix decisions when a firm has demand that exceeds its production capacity. The management accounting system affects product mix decision through the product cost calculation and product's contribution margin (the difference between the selling price and the product cost). If the calculated product cost is not correct, a product mix may be determined which result in a less profitable product (Lea and Fredendall, 2002). Lea and Fredendall (2002) compare traditional costing, activity-based-costing (ABC) and throughput accounting. They state that traditional accounting is appropriate for the production environment in which labor and material costs are predominant factors, technology is stable and product range is narrow. ABC was introduced to response increased overhead costs as a result of higher technology usage, larger product range and decreased work force necessity. ABC allocates overhead to products based on their activity usage (Köksal, 2004). In addition to these two accounting systems, TOC offers throughput accounting to answer to the need for performance measures (Balderstone and Keef, 1999). In throughput accounting, overhead is treated as a corporate cost rather than a product cost. Throughput accounting computes product costs as the sum of the totally variable costs of production. All other costs are taken as operating expenses (Goldratt and Cox, 1992). Lea (1998) studies which accounting system is appropriate under which production conditions and evaluates accounting systems as well. He finds that throughput accounting does not perform adequately when there are significant overhead and labor costs. Kee and Schmidt (2000) conclude that the relative performance of throughput accounting and ABC accounting in terms of the product mix decisions depends on the extent of management's control over labor and overhead within the specified planning horizon.

The product mix problem is one that has been discussed in the management science literature for decades. Several algorithms were developed based on TOC to be used in product mix selection problems (Goldratt and Cox 1992, Luebbe and Finch 1992, Patterson 1992, Lee and Plenert 1993, Plenert 1993, Atwater and Chakravorty 1995, Lea and Fredendall 1997, Hsu and Chung 1998, Aryanezhad and Komijan 2004). The TOC heuristic supplied by Goldratt and Cox (1992) finds throughput per constraint hour for each product sorts them in a descending order, allocates resources starting from the end of the list until any one of the resources is reduced. The resource fully utilized is the bottleneck resource. Luebbe and Finch (1992) and Patterson (1992) explain in detail product mix determination heuristic of TOC methodology and create examples in which the traditional algorithm could lead to the optimum solution. They agree that the TOC heuristic is simpler to use than an ILP. However, traditional algorithm does not generate optimal solutions under a multiple constrained resources environment (Plenert 1993, Lee and Plenert 1993). They provide a multiple resource-constrained example where one product did not use the bottleneck. Under these circumstances, TOC heuristic does not result in an optimal solution, because it exhausts the capacity of the non-dominate bottleneck while the dominant bottleneck still had remaining capacity. They study on sample problems and conclude that the integer linear programming (ILP) is a better tool than TOC heuristic. ILP formulation identifies a product mix that more fully utilized the bottleneck and was more profitable than the TOC heuristic's solution. On the other hand, Maday (1994) and Posnack (1994) argue that the TOC heuristic's solution would be optimal if an integer solution was not necessary. Lea and Fredendall (1997) and Hsu and Chung (1998) propose an explicit algorithm to cope with multiple constraints. Mabin (2001) discusse that TOC and LP can be used effectively in synergy. Aryanezhad and Komijan (2004) put forward an algorithm, which they called as the improved algorithm, and compared the efficiency of this algorithm in reaching the optimum solution with the ILP method through an example. They state that the improved algorithm lets every bottleneck contribute to the decision-making process by its priority sequence and reaches an initial MPS and under the guidance of all bottlenecks, finds the best path to reach the optimum solution. They claim that improved algorithm is efficient in reaching the optimum solution either in single or multiple bottleneck problems.

Atwater and Chakravorty (1995) study TOC heuristic and proposed ways to determine quality improvement (QI) projects priorities. They show that product mix affects the QI projects and vice versa. They also point out that selecting bottleneck or the succeeding work center priority is wise to increase throughput.

Köksal (2004) improves the TOC-based algorithm in Atwater and Chakravorty (1995) and by incorporating quality loss developed a linear programming (LP) model to guide for the QI projects selection and product mix determination. In Köksal (2004), the production environment, where rework and measurement error does not exist, is studied. However, in practice, the parts for which rework is necessary are reprocessed and if possible, recovered. In addition, a measurement system, even highly advanced, may contain inspection error. Therefore, these two criteria are aimed to incorporate into the examined production system.

In this study, throughput accounting with a special treatment of quality costs (quality loss) is assumed under a manufacturing environment in which rework and measurement error exists. Long-term effects of customer satisfaction are considered on the short-term decisions and the product mix algorithm of TOC is used to determine QI projects and product mix.

2.1.5 Markov Chains

Let X_t be a random variable denoting the value of the system characteristic at time t and taking values in a set S. A stochastic process $\{Xt, t \geq 0\}$ is called a discrete time Markov chain with state space S if,

$$\text{-}\ \forall t \geq 0, X_t \in S \text{ with probability 1,}$$

$$\begin{aligned}\text{-}\ &P(X_{t+1} = i_{t+1} \mid X_t = i_t, X_{t-1} = i_{t-1}, \ldots, X_1 = i_1, X_0 = i_0) \\ &\quad = P(X_{t+1} = i_{t+1} \mid X_t = i_t)\end{aligned}$$

Basically, the probability distribution of the state at time t+1 only depends on the state at time t, not on the previous (past) states that the chain passed through until reaching to the current state. The probability of the system moving from state i during one period to state j during next period is called as transition probability for the Markov chain. The Markov chain, for which the transition probability does not change over time, hence $P(X_{t+1} = i_{t+1} \mid X_t = i_t) = p_{ij} \quad \forall t$ is often called as a stationary (time homogeneous) Markov chain (Winston, 2004). The transition probability matrix is often written as

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1s} \\ p_{21} & p_{22} & \cdots & p_{2s} \\ \cdots & \cdots & & \cdots \\ p_{s1} & p_{s2} & \cdots & p_{ss} \end{bmatrix}$$

All entries in the transition probability matrix are nonnegative and the entries in each row must sum to 1. Thus, given that the state at time t is i, the process must be somewhere at time t+1.

Given two states i and j, a path from i to j is a sequence of transitions that begins in state i and ends in state j. A state is reachable from state i if there is a path leading from i to j.

If the system at time t+1 is absolutely in state i given that the state at time t is again in state i, in other words $p_{ii} = 1$, the state i is an absorbing state. A state i is a transient state if there exist a state j that is reachable from i, but the state i is not reachable from state j.

A Markov chain is said to be an absorbing chain if and only if it contains at least one absorbing state and it is possible to go from any non-absorbing state to an absorbing state in one or more stages. The states, which are not absorbing, are transient states and when we begin in a transient state, eventually we leave it and end up in an absorbing state (Winston, 2004). If there are m absorbing states and n transient states, the transition matrix will have the following canonical form

$$
\begin{array}{cc}
 & \begin{array}{cc} \text{TR} & \text{ABS} \end{array} \\
P = \begin{array}{c} \text{TR} \\ \text{ABS} \end{array} & \left(\begin{array}{c|c} Q & B \\ \hline 0 & I \end{array}\right)
\end{array}
$$

where *I* is an $m \times m$ identity matrix, *0* is a $m \times n$ zero matrix, *B* is a nonzero $n \times m$ matrix, and *Q* is a $n \times n$ matrix. In addition, TR and ABS denote the transition and absorbing states, respectively.

For an absorbing Markov chain the matrix *I-Q* has an inverse M, which is often referred as Markov chain's fundamental matrix (Winston, 2004). The ij^{th} entry m_{ij} of the matrix M is the expected number of times the chain is in transient state j, given that it starts in another transient state *i* before absorption.

On the other hand, the absorption probability of a chain can be found using fundamental matrix and R matrix of the chain. If the chain is in transient state i at present time, the probability that it will eventually be absorbed in absorbing state k is the ik^{th} element of the matrix $(I - Q)^{-1} B$.

Markov chains are used to analyze optimum process target levels in literature (Deliman and Feldman 1996, Bowling et al. 2004). Deliman and Feldman (1996) study a serial manufacturing system, which is under inspection error effect. Since inspector may not perfectly identify a defective item but conforming items are always correctly classified; hence, only type II error exists in the system. 100 % inspection is performed at the processes. They develop a model determining which process stations should be improved first, the amount of improvement and the location of the inspection stations with the objective of minimizing expected per unit total cost. They derive a nonlinear processing cost function with the probabilities calculated with respect to Markov renewal process approach and use a gradient search method to optimize process improvement.

Bowling et al., (2004) consider a production system where products are produced continuously and all produced item are screened (100% inspection) for conformance with their specification limits. They assume that when a product performance falls below a lower specification limit, product is scrapped and above an upper specification limit, product is reworked. If product performance falls within the limits, the product goes on to the next stage. In addition, they also assume that each quality characteristic is governed by a normal distribution. They aim to determine optimal process target levels by employing Markovian properties in order to maximize the total profit associated with a multi-stage serial production system, in which lower and upper specification limits are given at each stage.

2.2. Related Work

In this master thesis, we only investigate production environments in which measurements are not perfect and the items, whose quality characteristic value is out of the specification limits, are classified as either scrap or rework regarding their quality characteristic (QC) values. In the examined production environment 100 % inspection is applied.

When a system incurs measurement errors, it is possible to observe two types of inspection error: Type I and Type II errors. Type I error is the error of rejecting a conforming item, whereas Type II error is the error of accepting a nonconforming item. These two types of errors are assumed to exist in the studied production system.

Taşeli (2004) study a production environment where rework exists and 100% inspection is performed. At the end of the inspection, items are classified as conforming, rework or scrap with respect to their QC values. The items, for which rework is necessary, are reprocessed at a separate rework center and afterwards, all reprocessed items are inspected as a succeeding operation. After rework and inspection operations, if an item recovered and classified as conforming, it is sent to succeeding work center. Expected values of the quality loss to the consumer are computed in Taşeli (2004) with respect to the changes in the process capability (C_p) and inspection error at work (ε_p) and rework centers (ε_r).

Atwater and Chakravorty (1995) propose a TOC algorithm to guide in quality improvement studies in production systems. However, the point that they ignored in this study is that even if the product is within the specification limits, it may not satisfy customer expectations. Thus, they did not take into consideration the quality loss to the consumer. Köksal (2004) put forward an improvement of the TOC-based algorithm by incorporating quality loss with it. A linear programming (LP) model is developed to guide for the QI projects selection and product mix determination. But, this study assumes no rework and no inspection error for the production system. These assumptions are not realistic. In practice, even with very good measurement systems, the parts may be measured with some deviation from the real value. In addition, the parts for which rework is necessary are not discarded, on the contrary, they are reworked and tried to become such a quality characteristic which can be identified as conforming. Therefore, in this research, we aim to study on a more realistic production system with more relaxed assumptions.

CHAPTER 3

ANALYSIS OF A SINGLE ITEM, SINGLE WORK AND REWORK SYSTEM

3.1. Problem Definition

The effect of inspection errors on the product mix and QI project selection in a manufacturing environment where rework and inspection errors exist is examined in this study. It is assumed that the products (items) for which rework is necessary are reprocessed at a separate rework center and 100% inspection is performed for the products both after rework and main processing operations. Under these circumstances, a Markov chain approach is used to compute yield and rework rates. In addition, nominal-the-best type of a quality loss function is used in computing quality loss due to products shipped to the customers.

The measurement system in the examined production environment is under the measurement error effect. Therefore, the parts are measured with some deviation from the real value.

In our production system, it is assumed that only one quality characteristic is produced at each work (processing or rework) center, and the distribution of the quality characteristic is not affected by the succeeding work centers. The quality characteristic, X_p, produced in the processing center is assumed to be normally distributed with mean μ_p and variance σ_p^2 ($X_p \sim N(\mu_p, \sigma_p^2)$).

In this production environment, 100% inspection is performed rather than an acceptance sampling. Hence, all items are measured one by one after both processing and rework operations. At the end of the inspection, items are classified as conforming, rework or scrap. If the quality characteristic of the item is within lower and upper specification limits, it is defined as conforming and sent to the succeeding work center. On the other hand, if the product's quality characteristic value is out of the specification limits but within the scrap limits, it is qualified as rework and sent to the specific rework station of that processing center. Lastly, the item with the quality characteristic value out of the scrap limits is called as scrap and discarded. These inspection rules are valid for the items coming from both processing and rework operations.

An item sent to a rework station is assigned a new quality characteristic value, X_r, after the rework operation. As in process, this new quality characteristic value is also assumed to have a normal distribution with parameters μ_r and σ_r^2 ($X_r \sim N(\mu_r, \sigma_r^2)$). In this production environment, it is assumed that the items are reprocessed under a more skillful operation in rework centers. Therefore, standard deviation of the reworked items, σ_r, is less than that of the items processed in work center ($\sigma_r < \sigma_p$). In addition, the mean of the items reprocessed at rework centers is assumed to be at the target ($\mu_r = T$).

Moreover, the system is assumed to have a two-sided symmetric specification and scrap limits around the target. Also, the instruments used in inspection are calibrated. Hence, there is no accuracy problem in the measurement system.

The general picture of this production environment can be summarized as in Figure 3.1 for a work center. Solely the items accepted in both process and rework centers are sent to the succeeding work center to be processed.

In this thesis, the effects of measurement system on the quality improvement project and product mix selection and eventually on the throughput are investigated.

The examined system is under the measurement error effect. Thus, the real process quality characteristic value, X_p, is observed as Y_p with E_p amount of deviation. It is also assumed that E_p is normally distributed with mean 0 (because system is unbiased) and variance $\varepsilon_p{}^2 \, (E_p \sim N(0, \varepsilon_p{}^2))$.

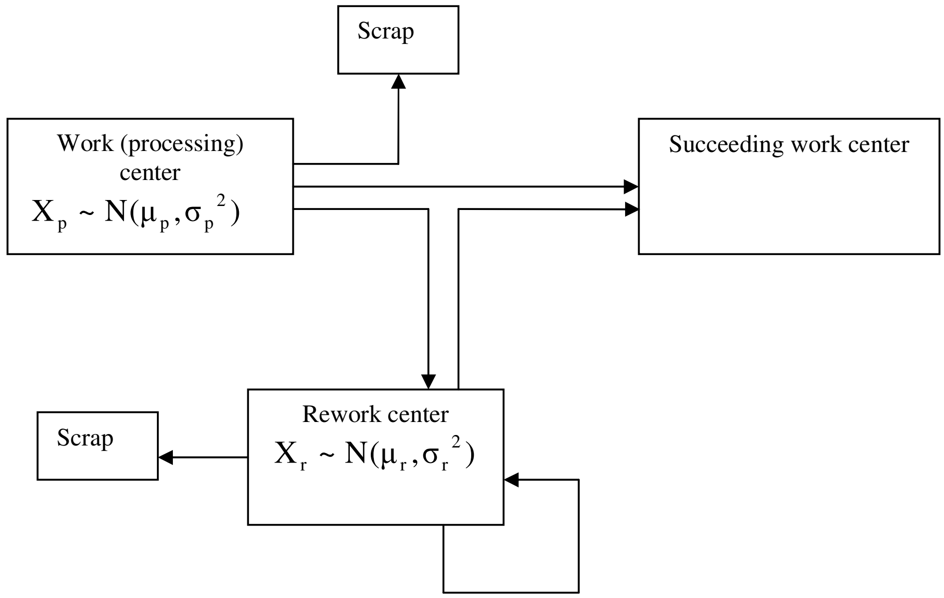


Figure 3.1 A summary of production environment

$$Y_p = X_p + E_p$$

Accordingly, the conditional distribution of the observed quality characteristic, given the real quality characteristic, is normally distributed with mean x_p and variance $\varepsilon_p{}^2 \, (Y_p \mid X_p \sim N(x_p, \varepsilon_p{}^2))$. Here, X_p and E_p are assumed to be stochastically independent. (Chen and Chung, 1994)

In addition to the process, the inspection error also exists in rework stations. Similar to the processing center, the real quality characteristic, X_r is observed as Y_r with E_r amount of deviation. Here, E_r is assumed to have a normal distribution with mean *0* and variance $\varepsilon_r{}^2 \, (E_r \sim N(0, \varepsilon_r{}^2))$. The conditional distribution of the observed quality characteristic, Y_r, given the real quality characteristic value is normally distributed with parameters x_r and $\varepsilon_r{}^2$ $(Y_r \mid X_r \sim N(x_r, \varepsilon_r{}^2))$. Same as in the process, X_r and E_r are assumed to be stochastically independent.

As previously stated, the items produced in the system have a two-sided symmetric specification and scrap limits. Therefore, Taguchi's Nominal-the-best type quality function is used since it is thought to be the best-conformed type for the analyzed production system. In order to compute the quality loss of the accepted items that are sent to the next processing station, the truncated variance formula derived in Taşeli (2004) is used. Taşeli (2004) derived mean and the variance of the accepted items by using the first and second moments of the distribution of the QC of the accepted items. She calculated these parameters for no rework-no inspection error; rework-no inspection error; no rework-inspection error and rework-inspection error cases. The formulas derived for the production environment where rework and two types of inspection error exist both in the process and the rework are utilized in this thesis. The exploited mixture distribution of the accepted QC value, first and second moments of this distribution are given in Appendix A.

Table 3.1 summarizes the assumptions of the examined production environment.

Table 3.1. Production system assumptions

$X_p \sim N(\mu_p, \sigma_p^{\,2})$
$X_r \sim N(\mu_r, \sigma_r^{\,2})$
Two-sided symmetric specification and scrap limits. LSL=Lower specification limit, USL=Upper specification limit. LLs=Lower scrap limit, ULs=Upper scrap limit. LLs<LSL<USL<ULs.
Only one operation is performed at each work center and the distribution of the quality characteristic is not affected by the succeeding work station.
100% inspection.
The measurement tools are calibrated, so that the tools are accurate.
The processes are under statistical control.
Rework operations are performed at a separate rework unit of work station.
X_p and X_r are independent.
The observed quality characteristic value of an item processed in a work center is distributed normally with parameters x_p and $\varepsilon_p^{\,2}$.$(Y_p / X_p \sim N(x_p, \varepsilon_p^{\,2}))$
The observed quality characteristic value of an item reworked in a work center is distributed normally with parameters x_r and $\varepsilon_r^{\,2}$ $(Y_r / X_r \sim N(x_r, \varepsilon_r^{\,2}))$
Y_p and Y_r are independent.
Rework operation is more skillful than processing. $\sigma_r < \sigma_p$ and the mean value of X_r is on the target.

3.2. Markovian Approach

The production environment constructed under the assumptions declared in Table 3.1 is thought to be a Markov Chain in terms of the observed Quality Characteristic (QC) value Y.

Observed QCs can be one of the previously defined quality type; (C) conforming, (R) rework or (S) scrap. Accordingly, system consists of 3 states as follows,

State space: S = {(C), (R), (S)}.

In this system, future states, given the present state, depends only upon the current state and independent of the past states. Since the jumps from the present to any future state are actualized at specific time frames, this system is called as a discrete time Markov chain. Furthermore, in this system, steps from the current state to a future state are independent of time. Consequently, it is also a time-homogenous Markov chain.

The states (C) and (S) are called absorbing states. Thus, when an item ever enters any of these states, it is impossible to leave that state. The state except absorbing states- (R)- is called as transient state. As it is possible to go from each transient state to an absorbing state (not necessarily in one step), this Markov chain is called as absorbing Markov chain (Winston, 2004).

Under these conditions, the transition probability of the chain is formed as follows;

$$P = \begin{array}{c} \\ R \\ C \\ S \end{array} \begin{array}{c} \begin{array}{ccc} R & C & S \end{array} \\ \left(\begin{array}{c|cc} u_1 & u_2 & u_3 \\ \hline 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right) \end{array}$$

Once an item is sent to rework center, after reprocessing, it jumps to transient state R with probability u_1 and to absorbing states C and S with probabilities u_2 and u_3, respectively.

Transition diagram for the Markov chain of the production system is summarized as in Figure 3.2.

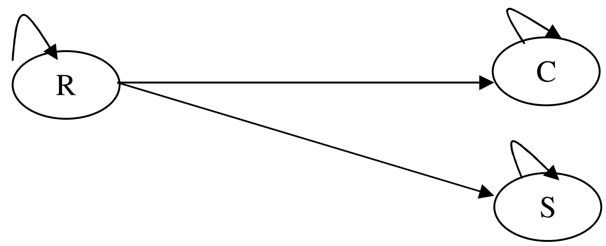


Figure 3.2. Transition diagram for the Markov chain of the production system

When a Markov chain has only finitely many transient states, it ultimately will leave and afterwards never returns to these transient states (Çinlar, 1975). Hence, in our production system, the Markov chain will ultimately leave the transient state, R, and be absorbed by one of the states (C) and (S).

The flow of items and the probabilities of the jumping into the states are shown in Figure 3.3. "p_i" probabilities are the probabilities that the item is at state i at time zero. "v_{ij}" probabilities in Figure 3.3 are the probabilities that the absorbing chain will be absorbed in the absorbing state s_j if it starts in the transient state s_i. For instance, v_{12} is the probability that the items will be absorbed in the absorbing state (S) given that it starts in the transient state (R). Absorbing probabilities are acquired by using the matrix $(I - Q)^{-1} B$.

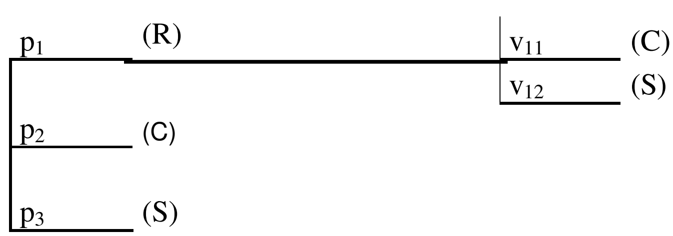


Figure 3.3. The flow of the items in the production system

v_{ij} probabilities are computed as in Section 4.2.1. In addition, p_i probabilities are the rework, yield and scrap rates of the process. p_1, p_2 and p_3 are the rework, yield and scrap rates of the process, which are computed as in Section 4.2.1, respectively.

CHAPTER 4

AN INTEGRATED APPROACH FOR PRODUCT MIX AND QUALITY IMPROVEMENT PROJECT SELECTION

4.1. The Production System

In production systems, each product may not be processed at each work center; thus, may not pass through all work centers. While a product only needs fraising and afterwards drilling operations, another product may need cutting operation as well. Therefore, the products may follow different routes during their manufacturing. However, for the examined production system, we assume that the manufactured products pass through all work centers and are processed by a specific operation at each.

In practice, an operation performed at a work center may be spoiled or be affected by a former operation completed at a preceding work or rework center. For instance, when an item is drilled larger or smaller from the target value, a screw cannot be fixed through this hole well. On the other hand, a spinning operation may not be performed well and spoil the part drilled at the former work center. Nevertheless, in the examined production system, it is assumed that the operations performed at work centers are independent of each other and besides, an operation performed at a succeeding work center does not influence the characteristics processed at the previous work centers. Hence, the distribution of the quality characteristic processed at a work center is not affected by the succeeding workstations.

In practice, inspection operations may be performed at the same or a separate station. Moreover, inspection may be executed after the specific operations, which have critical effect on the product. On the other hand, 100% inspection or acceptance sampling technique may be used with respect to the location of the inspection station in the production. In this system, it is assumed that the products are inspected one by one (100% inspection is committed) at the same station that the processing or reprocessing operations are implemented.

In throughput accounting, overhead is treated as a corporate cost. Throughput accounting computes product costs as the sum of the totally variable costs due to the production. In the examined system, it is assumed that highly automation is not involved in the system and also, overhead costs are not significant. Hence, it is thought that throughput accounting is appropriate for the studied system. In addition, the wages of the labors are assumed to compute over working hours. Therefore, labor costs are taken as variable costs and incorporated into the totally variable costs.

In six sigma studies performed in manufacturing, impressive bottom-line results normally flow from Six Sigma quality improvement projects. All projects cannot be applied at the same time. It is not possible because of the time and budget constraints. Therefore, vital few projects that matter most should be isolated from trivial few ones. Accordingly, decision maker should make a selection between the potential projects.

Six sigma focuses on defects and variations. It begins with the identification of the critical-to-quality (CTQ) elements of a process, the attributes most important to the customer (Brue, 2002). Thus, mostly, the processes that have the most scrap and rework rates, longer cycle time and produce fewer items than expected can be thought as candidate projects.

In general, six sigma studies follow the DMAIC (Define, Measure, Analyze, Improve and Control) phases. In the Define phase, candidate projects' aims, expected profit of their execution and their requirements are clarified. The scrap rate and capability of the processes (C_p and C_{pk} are used mostly) and the variance of the inspection gauge on the work centers are measured in the Measure phase. In the Analyze phase, the data derived in the Measure phase and process maps are examined to characterize the nature and extent of the defect. In the Improve phase, the ways to eliminate the defects in both quality and process velocity are investigated and implemented. When the process has achieved the required quality level, the tools of the Control phase are employed to lock in the benefits (George, 2002).

In quality improvement studies, mostly, process capability values, scrap and rework rates of the candidate processes are investigated. In addition to these criteria, the Decision maker uses expected time and estimated monetary values of the project, such as estimated profit and the budget needed for the projects to make his selection. We assume that in the examined production system, the implementation costs of quality improvement projects do not significantly differ from each other. Hence, improvement cost is ignored and excluded from improvement studies.

Besides, measurement systems, even highly advanced, may contain errors. However, if the system is under the measurement error effect, we measure a fewer process capability value than the true process capability and thus, the true process capability is understated (Pearn and Liao, 2005). Therefore, the measurement error is taken into consideration in the measurement phase. As aforementioned in Section 2.1.2, the variability in the measured results of a process may be caused by the differences between parts and imperfectness of the measurement. In the examined production system, only the imperfectness in the measurement system is investigated and incorporated into the studied production system.

In the Analyze phase, the product mix is assumed given. Whereas, the product mix quantities show changes with respect to the improved process. As noted before, Atwater and Chakravorty (1995) showed that product mix affects the QI project selection and vice versa. Therefore, the two problems, quality improvement project selection and product mix determination should be resolved at the same time. These two problems are studied together and an LP model, which aims to maximize profit and also minimize quality loss, is developed by Köksal (2004). But, this study ignores rework and inspection error. In practice, even with very good measurement systems, the parts may be measured with some deviation from the real value. In addition, the parts for which reworking is necessary are not discarded, on the contrary, they are reworked to make them conforming. Therefore, in this research, rework and inspection error are also taken into consideration and the T-L model in Köksal (2004) is improved. In addition, this LP model is compared with a process capability based approach since it is one of the mostly used traditional methods to determine the priorities of the quality improvement projects. But, C_{pm} is used rather than C_p because the former is more sensitive to departures of the mean from the target value.

In this production system, if an item is identified as rework due to its QC value processed at a work center, it is sent to a separate rework center of that work center to be reprocessing.

In the examined production environment, it is assumed that one period is required to complete the improvement. The selected improvement project is started at the beginning of the current period, proceeded until the end of this period and will have been completed in the beginning of the next period. In practice, if the improvement periods are not completed in one period, the improvement values can be estimated from the measured values gathered through inspection and put into the Linear Programming model as an input. The studied production system can be summarized as follows:

- All of the products in the system pass through the all work centers.
- The items for which rework is necessary are reprocessed at a separate station.
- Inspection error exists in the system.
- Selected improvement project is started at the beginning of the current period and terminated at the end of this period.
- Inspection operations are performed at the work and rework centers after the processing and reprocessing operations, respectively.

- Quality improvement cost of a work center is not significantly different from that of another work center.
- No overtime is used.
- Processes are in a state of statistical control. (constant variance and mean)
- Overall demand in the market is much larger than the production capacity of the company. Thus, what is produced can be sold.

4.2 Throughput-Loss (T-L) Linear Programming Approach

4.2.1 T-L Linear Programming Model

In order to find the product mix for such a production environment defined in Section 4.1, an LP model was developed by using Throughput-Loss (T-L) mathematical model in Köksal (2004). However, the LP model in Köksal (2004) was constructed for the production environment where rework does not exist and under the assumption of no inspection error in the system. Rework and inspection error are incorporated into T-L model and the following LP formulation is constructed:

$$\text{Max}\left[\sum_{i=1}^{m}\ (SP_i\ S_i - \sum_{k=1}^{n} U_{ik}\ TVCP_{ik} - \sum_{i=1}^{m}\ \sum_{t=n+1}^{2n}(RT_{it} R_{it} TVCR_{it}\ )\right] - \left[\sum_{i=1}^{m}\sum_{k=1}^{n}(S_i\ \overline{L}_{ik}\ )\right]$$

S.t. $U_{ik} RR_{ik} = R_{i,k+n} \qquad i = 1,...,m, \quad k = 1,...,n \qquad (1)$

$$R_{i,t} YRR_{i,t} + U_{ik} YR_{ik} = U_{i,k+1} \qquad i = 1,...,m, \quad k = 1,...,n-1, \; t = k+n \qquad (2)$$

$$R_{i,2n} YRR_{i,2n} + U_{in} YR_{in} = S_i \qquad i = 1,...,m \qquad (3)$$

$$S_i \le D_i \qquad i = 1,...,m \qquad (4)$$

$$\sum_{i=1}^{m} (U_{ik} PT_{ik}) \le CAP_k \qquad k = 1,...,n \qquad (5)$$

$$\sum_{i=1}^{m} (R_{it} RET_{it} RT_{it}) \le CAR_t \qquad t = n+1,...,2n \qquad (6)$$

$$U_{ik}, S_i, R_{it} \ge 0 \qquad i = 1,...,m, \quad k = 1,...,n, \; t = n+1,\ldots,2n \qquad (7)$$

The model parameters

SP_i : Selling price of product i,

$TVCP_{ik}$: Totally variable cost due to process of product i at work center k,

$\overline{L}_{ik}$: Average loss incurred per product i due to its processing at work center k and rework center k+n,

RT_{it} : Expected number of rework operations for product i at rework center t

$TVCR_{it}$: Totally variable cost due to rework operation of product i at rework center t,

RR_{ik} : The probability that a product i is sent to rework center k+n from work center k (Rework rate).

YRR_{it} : Yield rate of rework center t of product i. The probability that a product i is sent to work center k+1 from rework center t=k+n,

YR_{ik} : Yield rate of work center k of product i. The probability that a product i is sent to work center k+1 from work center k,

D_i : Market demand of product i,

PT_{ik} : Processing time of product i at work center k,

CAP_k : Capacity of work center k,

RET_{it} : Rework time of product i at rework center t,

CAR_t : Capacity of rework center t.

i: product type,

k: work center number,

t: rework center type which is numbered from n+1 to 2n.

The decision variables

S_i : Number of sold units of product i produced within the specified time frame.

U_{ik} : Total number of parts to be processed at work center k to satisfy production requirements of product i,

R_{it} : Total number of parts to be reprocessed at rework center t to satisfy production requirements of product i,

In this T-L model, there exist *n* work centers and each work center has a separate rework center. Rework centers are installed close to their work centers and numbered from *n+1* to *2n*. The number of the parts of product i processed in work center k is U_{ik} and reprocessed in rework center, station *t=k+n*, is R_{it}. Some parts of U_{ik} are identified as conforming with rate YR_{ik} and sent to the succeeding work center directly; some parts are classified as rework with rate RR_{ik} and sent to its rework center (expressed in constraint (1)) and reprocessed; the rest of the parts are defined as scrap and discarded. On the other hand, some of the products i reworked in rework center *t*, are identified as conforming with probability YRR_{it} and sent to the next work center *k+1*. By this way, the parts passed to the $k+1^{th}$ work center can be found by summation of the conforming parts coming from work and rework center as in constraint (2). No more products than demand are produced (constraint (4)). Therefore, in the examined system, what is produced, is sold. The products lastly processed in work center n and rework center 2n are sent to customer with a specific selling price (constraint (3)).

All products are processed at the work and rework centers for a PT_{ik} and RET_{ik} amount of time, respectively. The capacity needed to process the products at a work center should be less than its capacity (constraint (5)). An item sent to a rework center is reprocessed for an expected number of times. For each rework operation, this item requires to be reprocessed for a RET_{ik} amount of time. Therefore, the capacity needed to reprocess items at a rework center is computed as in constraint (6) and this capacity cannot exceed the capacity of that rework center.

In practice, the yield and rework rates are approximated by using the previous period's production data. However, since we have not any data about production rates, in the LP model, the yield and rework rates of the processes are computed by using the following formula:

$$YR_{ik} = \int_{LSL}^{USL} m_{Ypik}(y)dy\,, \qquad RR_{ik} = \int_{USL}^{ULs} m_{Ypik}(y)dy + \int_{LLs}^{LSL} m_{Ypik}(y)dy$$

where m(y) is the marginal distribution of the observed Quality Characteristic (QC) value of product i at work center k, Y_{pik}.

Yield rates and also the expected number of reworks for the rework operations are found by using Markov chain properties. Yield rate of work center t for product i, YRR_{it}, is found from $(I - Q)^{-1}B$ matrix. Transition matrix for the rework operations can be written as:

$$P = \begin{array}{c|ccc} & R & C & S \\ \hline R & u_1 & u_2 & u_3 \\ C & 0 & 1 & 0 \\ S & 0 & 0 & 1 \end{array}$$

where R , C and S denote the classification of the items as mentioned before; rework, conforming and scrap, respectively. However, here, Q matrix consists of only the probability u_1, and B matrix comprises from the matrix $[\mathrm{u}_2 \quad \mathrm{u}_3]$. Then, the transition probabilities can be computed as:

$$u_1 = \int_{USL}^{ULs} m_{YRit}(y)dy + \int_{LLs}^{LSL} m_{YRit}(y)dy\,, \quad u_2 = \int_{LSL}^{USL} m_{YRit}(y)dy \quad \text{and} \quad u_3 = 1 - u_1 - u_2$$

where $m_{YRit}(y)$ is the marginal distribution of the QC value of product i at rework center t, Y_{Rit}. Then, $(\mathrm{I} - \mathrm{Q})^{-1}\mathrm{R}$ is a 1×2 matrix such as,

$$(I - Q)^{-1}B = \begin{array}{c|cc} & C & S \\ \hline R & v_1 & v_2 \end{array}$$

$\mathrm{YRR}_{\mathrm{it}}$ is the value of the probability v_1 for product i at rework center t.

In addition, expected number of reworks of product i in rework center t, $\mathrm{RT}_{\mathrm{it}}$, is the row summation of the $(\mathrm{I} - \mathrm{Q})^{-1}$ matrix.

The first part of the objective function consisting of three terms is the throughput and the second part is the total expected quality loss incurred form all products sold. The average quality loss for a key QC of product i, assigned at work centre k, $\overline{\mathrm{L}}_{\mathrm{ik}}$, used to compute total quality loss is determined by using the equation,

$$\overline{L}_{ik} = A_{ik}[(\mu_{ik} - T_{ik})^2 + \sigma_{Tik}{}^2]$$

where A_{ik} is the loss coefficient, which can be computed by using the equation (2.2) in Section 2.1.1, $\sigma_{Tik}{}^2$ is the variance of the accepted QC processed at work center k and rework center t=k+n of product i, which can be estimated using the formulas in Taşeli (2004), which are given in Appendix A, and μ_{ik} is the mean value of the accepted products of i coming from work center k and rework center t=k+n.

In this LP model, yield (YRR_{it}) and expected number of rework operations (RT_{it}) are computed by using Markov chains and $\overline{L}_{ik}$ is estimated by using the formulas in Taşeli (2004), which are also given in Appendix A. Selling price (SP_i), totally variable cost due to process and rework ($TVCP_{ik}$ and $TVCR_{it}$), processing and reprocessing times (PT_{ik} and RET_{it}) are given parameters and put into the model as an input. The inputs and outputs of the model can be summarized as in Figure 4.1.

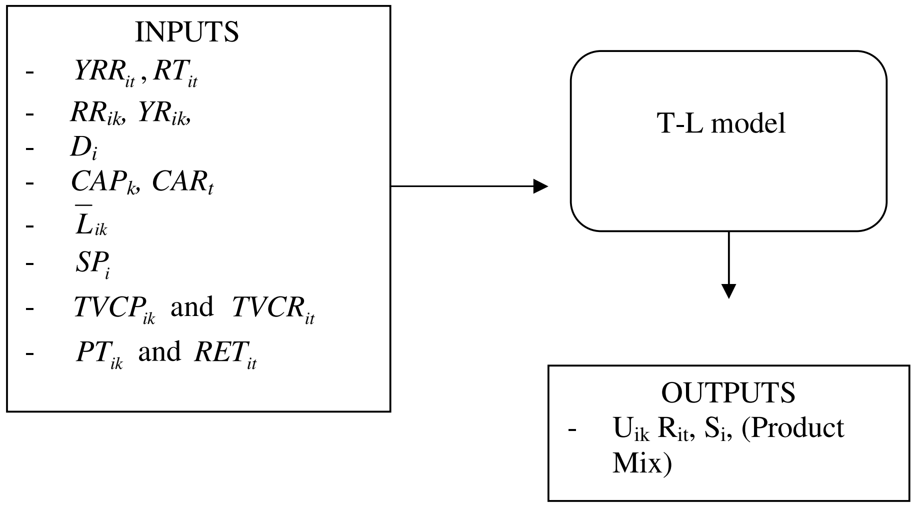


Figure 4.1 Input and Outputs of the model

4.2.2 Algorithm of the Throughput-Loss (T-L) Model

QI project selection and product mix determination studies are performed by using Throughput–Loss model given in Section 4.2.1 as in Figure 4.2. In quality improvement algorithm, estimated demand of the succeeding period is used for the selection of the improvement studies and real demand of the current period is used to determine product mix. In the current period, production planning department determines product mix by using Throughput-Loss (T-L) LP model subject to real demand and capacity constraints for the current period. The production is realized with respect to this production plan. On the other hand, quality department starts improvement study at the beginning of this current period. Each candidate work centers are searched for improvement one by one. In this examined system, all work and rework centers are thought as candidate projects. While a process is investigated and the values of the process are changed to their improvement rates, all other process values are remained fixed to their first value in the current period. In practice, these improvement rates are given with the proposal at the beginning of the QI studies. In the Define phase, in addition to candidate projects' aims, expected profit of their execution, expected quality improvement rates are given by the project proposals.

In improvement studies C_{pm} is used rather than C_p because it is more sensitive to departures of the process mean from the target value. As noted in Section 2.1.3, the other process capability indice, C_{pmk} is more responsive than C_{pm} to the departures of the mean from the target. However, when shift of the mean from the target is between 1.5σ - 2σ, the difference between these two indices decreases as sample size increases (see Figure 4 and 5, in Pearn et al., 1992). Since 100% inspection is performed, it is assumed that C_{pm} is sufficient to determine the process capability in this production system.

When a process is improved, it is thought that the process values of the selected station will reach to their improvement values at the end of the period. T-L model for improvement phase is run and search the answer of this question: if a work center is improved, what will be the gain of the system? After T-L model is run for assumed improvement value of the studied work or rework center, it terminates to an objective function value. This objective function value is saved and the investigations for the other work and rework centers are started. At this time, standard deviation of the measurement device and process standard deviation of the other stations except the examined station are again fixed to their previous values at the beginning of the current period. After all work and rework stations are explored one by one, the station that gives the largest objective function value is selected and improvement process is performed on this station. By this way, quality improvement study for the first period is completed.

At the beginning of the second period, product mix is determined by using T-L model regarding the real demand quantities and improved values of the selected station. The terminated objective function value of the T-L model of this product mixture is the first improvement T-L value. After that, if it is decided to proceed QI studies, quality improvement study search is started as in the previous period. However, at this time the improved values of the selected process are used. Work and rework centers are investigated as in the previous period. The station, which has the maximum objective function value, is selected and improved.

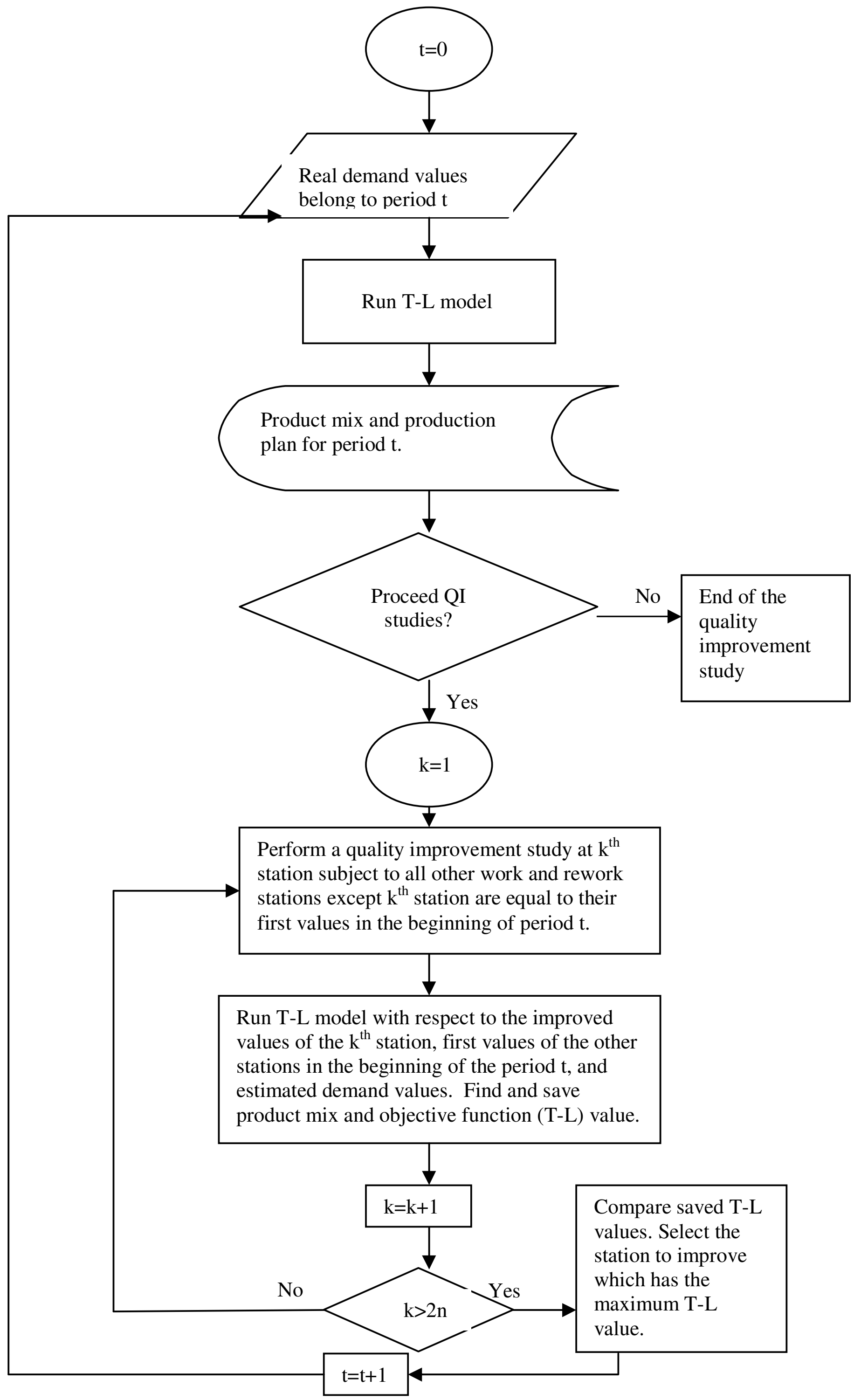


Figure 4.2 Flow of the quality improvement studies with T-L method

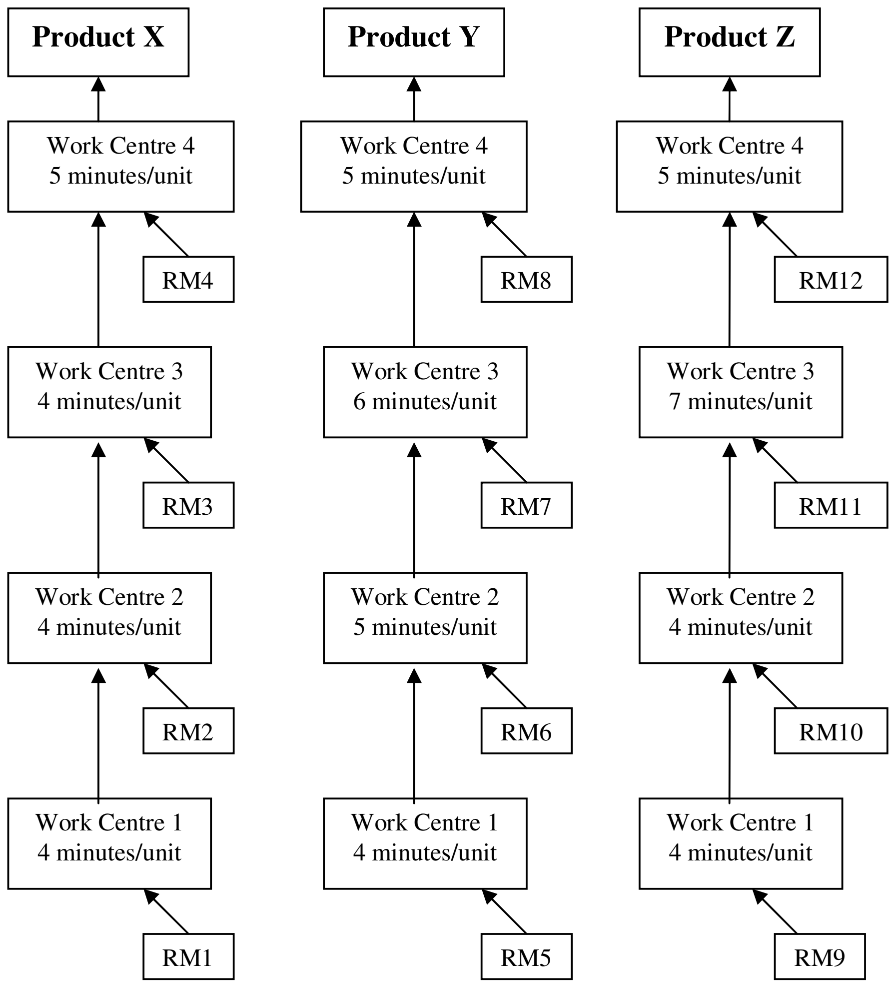
Improvement studies in the third and the succeeding periods are performed as the same manner in the former periods. At the end of these periods, the sum of improvement T-L values of the periods is the net profit of the yearly quality improvement studies.

4.2.3. Application of T-L Method on an Example Problem

In order to clarify the application of the algorithm, an example problem is constructed which is adapted from the case in Atwater and Chakravorty (1995). This imaginative production environment consists of three products all of which are processed at four work centers and reprocessed at separate four rework centers, if necessary. This production environment is given in Figure 4.3. In addition to the conditions of the case in Atwater and Chakravorty (1995), in this sample problem, rework centers are assumed to be close to the work centers as in Figure 3.1. However, to avoid the confusion in figure, rework centers are not displayed in Figure 4.3. A detailed production environment for Product X is given in Figure 4.4 to explain the production system well.

Different raw materials are used at each work center to process products. Products have distinct processing times, selling prices and constant demand during the period. Furthermore, other given parameters used in the algorithm are loss coefficients, specification limits, standard deviation of the measurement error and totally variable costs due to processes and reworks.

In practice, standard deviation of the process should take value regarding the specification limits of the process. In other words, when the specification limits are expressed in the micrometer units, it is meaningless that the standard deviation of the process to be in the meter unit. Therefore, the standard deviation is defined a constant term of the upper specification limit of the process, in the example problem.



Information:

- Each work center uses only one resource for each product type.
- RM: raw metarial
- The plant operates
  8h/shift, 2 shifts/day, 5 days/week, 4 weeks/month.
- An improvement period proceeds 4 months; period capacity 76.800 minutes.

Figure 4.3 Process Flow of the numerical example

(Adapted from the case in Atwater and Chakravorty (1995))

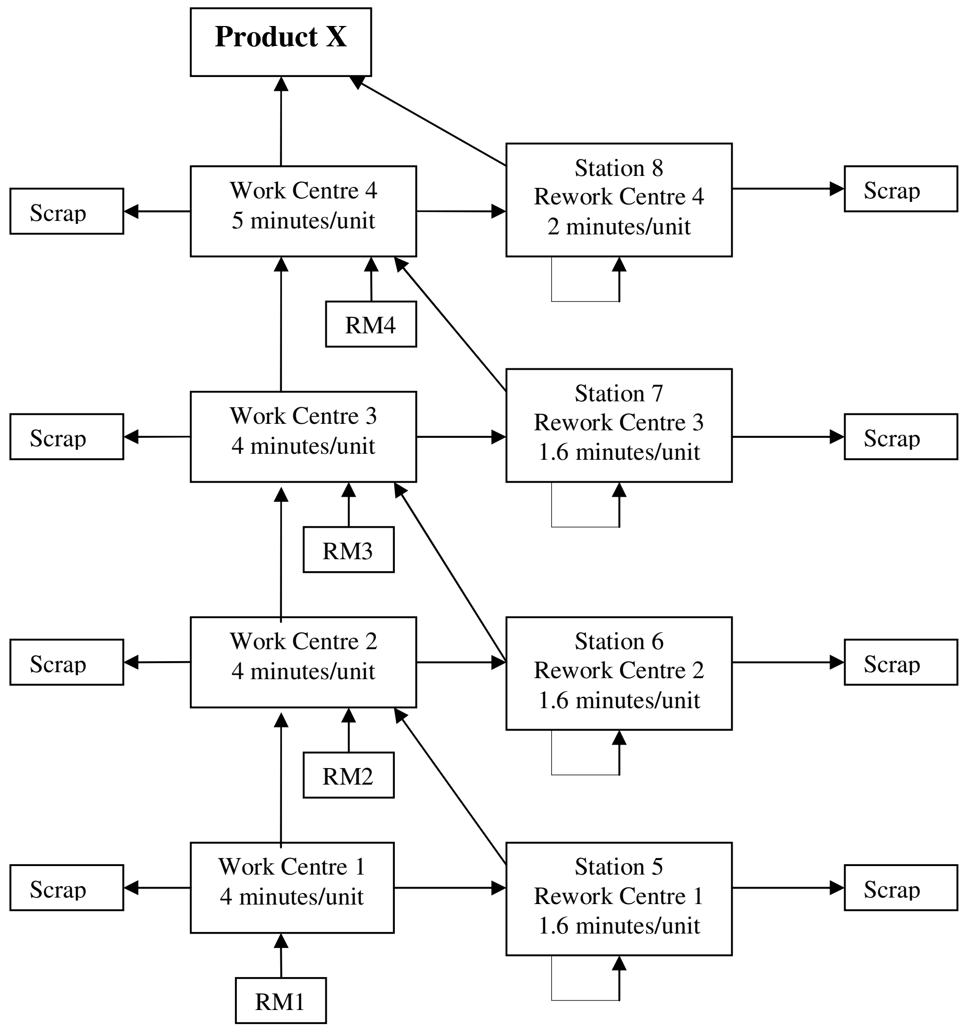


Figure 4.4 Process Flow of Product X in the production system

In addition, selling prices of the products should be determined with respect to the amount of the totally variable costs spent to produce the items. Indeed, selling prices should be computed by adding some rate of profit margin to the totally variable costs of the products. Thus, the totally variable costs are defined in terms of the selling prices of the products.

Parameter values of the sample problem are as follows:

Table 4.1 Selling price and demand values

	PRODUCTS		
	X (1)	Y (2)	Z (3)
Demand (Di)	6400	4800	3200
Selling Price (Spi)	2000	2500	3000

Table 4.2 Upper specification limits (USL) of the products

USL_{ik}	Work center 1	Work center 2	Work center 3	Work center 4
Product X	3.560445	3.356076	2.876521	2.6862429
Product Y	3.452695	3.286486	2.939964	2.7517414
Product Z	3.34637	3.187373	2.960181	2.7197376

Table 4.3 Standard deviation of the processes in terms USL

$SigmaP_{ik}$ (σ_{Pik})	Work center 1	Work center 2	Work center 3	Work center 4
Product X	(1/4)USL	(1/3)USL	(1/6)USL	(1/4)USL
Product Y	(1/3)USL	(1/3.5)USL	(1/4)USL	(1/6)USL
Product Z	(1/2)USL	(1/2.5)USL	(1/5)USL	(1/5)USL

Table 4.4 The deviation of the process mean from the target value in terms of the standard deviation of the processes.

SigmaM	Work center 1	Work center 2	Work center 3	Work center 4
Product X	$0.1\sigma_{P11}$	$0.4\sigma_{P12}$	$0.2\sigma_{P13}$	$0.25\sigma_{P14}$
Product Y	$0.1\sigma_{P21}$	$0.4\sigma_{P22}$	$0.2\sigma_{P23}$	$0.25\sigma_{P24}$
Product Z	$0.1\sigma_{P31}$	$0.4\sigma_{P32}$	$0.2\sigma_{P33}$	$0.25\sigma_{P34}$

Table 4.5 Process capability values of the products at work and rework centers

Cpm_{ik}	Work center 1	Work center 2	Work center 3	Work center 4	Rework center 1	Rework center 2	Rework center 3	Rework center 4
Product X	1.33	0.93	1.96	1.29	1.78	1.33	2.67	1.78
Product Y	1.00	1.08	1.31	1.94	1.33	1.56	1.78	2.67
Product Z	**0.66**	0.77	1.63	1.62	0.89	1.11	2.22	2.22

Table 4.6 Totally variable cost values in terms of selling prices

$TVCP_{ik}$	Work center 1	Work center 2	Work center 3	Work center 4
Product X	$0.08\,SP_1$	$0.1\,SP_1$	$0.18\,SP_1$	$0.125\,SP_1$
Product Y	$0.14\,SP_2$	$0.14\,SP_2$	$0.09\,SP_2$	$0.025\,SP_2$
Product Z	$0.14\,SP_3$	$0.14\,SP_3$	$0.07\,SP_3$	$0.08\,SP_3$

Table 4.7 Loss coefficient (LC) values in terms of selling prices

LC_{ik}	Work center 1	Work center 2	Work center 3	Work center 4
Product X	25	900	1750	48
Product Y	100	325	1100	825
Product Z	35.5	1350	2800	25

Table 4.8 Measurement error (SigmaM) in terms $SigmaP_{ik}$

SigmaM	Work center 1	Work center 2	Work center 3	Work center 4
Product X	$0.08\sigma_{P11}$	$0.06\sigma_{P12}$	$0.05\sigma_{P13}$	$0.1\sigma_{P14}$
Product Y	$0.08\sigma_{P21}$	$0.06\sigma_{P22}$	$0.05\sigma_{P23}$	$0.1\sigma_{P24}$
Product Z	$0.08\sigma_{P31}$	$0.06\sigma_{P32}$	$0.05\sigma_{P33}$	$0.1\sigma_{P34}$

In addition to these values, time and $TVCR_{it}$-totally variable cost needed to reprocess a product- are assumed to be 40% and 50% that of to process the products, respectively ($RET_{it} = 0.4TP_{ik}$ and $TVCR_{it} = 0.5TVCP_{ik}$). Moreover, the assumptions cited in Section 3.1 are also valid in this sample problem.

In addition, the real demand is equal to estimated demand; hence, it is assumed to be well approximated.

The work center whose capacity is not sufficient to produce the product values placed on it, is called as bottleneck station. In Mertoglu (2003) , the work center which is very close to (differing at most 1% form the capacity limit) or at its capacity limit is defined as bottleneck. In this study, we determine the bottleneck station same as in Mertoğlu (2003). Hence, if

- $CAP_k - \sum_{i=1}^{m}(U_{ik}TP_{ik}) \leq (0.01)\, CAP_k$, work center k is identified as bottleneck
- $CAR_t - \sum_{i=1}^{m}(R_{it}RET_{it}RT_{it}) \leq (0.01)\, CAR_t$, rework center t is identified as bottleneck.

For this sample problem the developed LP model is used for 3 improvement periods, each consisting of 4 months.

For this sample problem, improvement rates are defined regarding the capability values of the processes committed at the stations as follows:

- If Cpm<=1.0
  SigmaPx= (1/2) SigmaPx
  Mu-T=0
  Sigma error=0
- If (Cpm>1.0&Cpm<=1.33)
  SigmaPx= (3/4)SigmaPx
  Mu-T=0
  Sigma error=0
- If (Cpm>1.33&Cpm<=2.0)
  SigmaPx= (14/16) SigmaPx
  Mu-T=0
  Sigma error=0
- If Cpm>2.0
  SigmaPx= (15/16) SigmaPx
  Mu-T=0
  Sigma error=0

If process capability value (C_{pm}) is less than 1.0, the standard deviation of the process (SigmaPx) is halved; if C_{pm} is between the values of 1.0 and 1.33, SigmaPx is reduced to the 3/4 of its value; if C_{pm} is between the values of 1.33 and 2.0, SigmaPx is reduced to the 14/16 of its value; if C_{pm} greater than 2.0, SigmaPx is reduced to the 15/16 of its value. For all C_{pm} values, mean of the process is equalized to target value and standard deviation of the measurement error is set to zero, measurement error is wiped.

This sample problem is solved by Throughput-Loss model by using MATLAB. The M.file code (file name: LP_ornek) is given in CD-ROM.

Application of the algorithm on the sample problem

Period 0

No improvement is made yet. T-L model is run with the current values of the period. After the model is terminated optimal product mix is determined,

Product mix: (S_1=0, S_2 =4799.3, S_3 = 0)

Objective function value of this product mix is T-L_0= $0.6282 \cdot 10^6$.

In this case, there is not any bottleneck station because the capacity of the work and rework centers are not fully utilized. The objective function value giving the highest Throughput-Loss value for the estimated values of the next period is chosen from the values below.

If work center 1 is improved; T-L = $1.2578 \cdot 10^6$

If work center 2 is improved; T-L = $3.5736 \cdot 10^6$

If work center 3 is improved; T-L = $1.8704 \cdot 10^6$

If work center 4 is improved; T-L = $0.8234 \cdot 10^6$

If rework center 1 (station 5) is improved; T-L = $0.6285 \cdot 10^6$

If rework center 2 (station 6) is improved; T-L = $0.6284 \cdot 10^6$

If rework center 3 (station 7) is improved; T-L = $0.6283 \cdot 10^6$

If rework center 4 (station 8) is improved; T-L = $0.6282 \cdot 10^6$

Work center (WC) 2 is selected for improvement project. Moreover, it is obviously seen that the objective function values found from the improved values of the rework centers not significantly differ from the T-L_0 value in the 10^6 digest. Because of the good process capabilities of the rework centers, improvement at the of the rework centers does not gain more difference. At the end of this period:

T-L_0(0, 4799.4, 0) = $0.6282 \cdot 10^6$.

Period 1

WC2 has been improved.

After the model is run optimal product mix is determined,

Product mix: (S_1=6400, S_2 = 4799.4, S_3 = 0)

In this case, no bottleneck station occurs again. Objective function value is T-L_1=$3.5736 \cdot 10^6$.

Improvement decision:

If work center 1 is improved; T-L= $5.1239 \cdot 10^6$

If work center 2 is improved; T-L= $4.5464 \cdot 10^6$

If work center 3 is improved; T-L= $6.1096 \cdot 10^6$

If work center 4 is improved; T-L= $3.8293 \cdot 10^6$

If rework center 1 (station 5) is improved; T-L= $3.5738 \cdot 10^6$

If rework center 2 (station 6) is improved; T-L= $3.5736 \cdot 10^6$

If rework center 3 (station 7) is improved; T-L= $3.5736 \cdot 10^6$

If rework center 4 (station 8) is improved; T-L= $3.5736 \cdot 10^6$

Work center 3 is selected for improvement project. At the end of this period:

T-L_1 (6400, 4799.4, 0) =$3.5736 \cdot 10^6$.

Period 2

WC3 has been improved.

Since the work center 3 is loaded very close to its capacity, it is a bottleneck station in the production system.

Product mix: (S_1=6400, S_2 = 4799.4, S_3 = 3168)

Objective function value is T-L_2= $6.1096 \cdot 10^6$.

Improvement decision:

If work center 1 is improved; T-L= $7.7081 \cdot 10^6$

If work center 2 is improved; T-L= $7.1205 \cdot 10^6$

If work center 3 is improved; T-L= $7.2821 \cdot 10^6$

If work center 4 is improved; T-L= $6.3708 \cdot 10^6$

If rework center 1 (station 5) is improved; T-L= $6.1147 \cdot 10^6$

If rework center 2 (station 6) is improved; T-L= $6.1096 \cdot 10^6$

If rework center 3 (station 7) is improved; T-L= $6.1096 \cdot 10^6$

If rework center 4 (station 8) is improved; T-L= $6.1096 \cdot 10^6$

Work center 1 is selected for improvement project. At the end of this period:

T-L_2 (6400, 4799.4, 3168) = $6.1096 \cdot 10^6$.

Period 3

WC1 has been improved.

T-L model is run for product mix determination optimal product mix is

Product mix: (S_1=6400, S_2 = 4800, S_3 = 3200)

In this period, since loaded very close to its capacity, work center 3 is defined as bottleneck.

Objective Function value is T-L_3= $7.7081 \cdot 10^6$

Stop improvement study, since it has been performed for three periods.

Total T-$L_{improvement}$= T-L_1+ T-L_2+ T-L_3

=$17.3913 \cdot 10^6$.

As a result, the determined parameters from Table 5.1 to Table 5.8 all affect the result. Even the bottleneck station is improved in the improvement studies, the work center before the bottleneck work center has a priority over the bottleneck.

4.3 Process Capability Approach

4.3.1 Linear Programming Model

The LP model used for product mix determination is as follows:

$$\text{Max}\left[\sum_{i=1}^{m} (SP_i\ S_i - \sum_{k=1}^{n} U_{ik}\ TVCP_{ik} - \sum_{i=1}^{m}\ \sum_{t=n+1}^{2n}(RT_{it} R_{it} TVCR_{it}\ )\right]$$

$$\text{S.t. } U_{ik} RR_{ik} = R_{i,k+n} \qquad i = 1,\dots,m, \quad k = 1,\dots,n \qquad (1)$$

$$R_{i,t} YRR_{i,t} + U_{ik} YR_{ik} = U_{i,k+1} \qquad i = 1,\dots,m, \quad k = 1,\dots,n-1, \quad t = k+n \qquad (2)$$

$$R_{i,2n} YRR_{i,2n} + U_{in} YR_{in} = S_i \qquad i = 1,\dots,m \qquad (3)$$

$$S_i \le D_i \qquad i = 1,\dots,m \qquad (4)$$

$$\sum_{i=1}^{m} (U_{ik} PT_{ik}) \le CAP_k \qquad k = 1,\dots,n \qquad (5)$$

$$\sum_{i=1}^{m} (R_{it} RET_{it} RT_{it}) \le CAR_t \qquad t = n+1,\dots,2n \qquad (6)$$

$$U_{ik}, S_i, R_{it} \ge 0 \qquad i = 1,\dots,m, \quad k = 1,\dots,n, \quad t = n+1,\dots,2n \qquad (7)$$

4.3.2 Algorithm of the Process Capability Approach

Using process capability ratios is the mostly used and the simplest way of the quality improvement priority determination. Generally, C_p indices are used in practice. However, as C_{pm} is more sensitive to departures of the process mean from the target (Pearn et al., 1992), it is used in this process capability method.

The algorithm for the process capability method is given in Figure 4.5. For the current period, the optimal product mix is determined by solving typical product mix problem regarding the demand, selling price, raw material cost, capacity, yield and rework rates.

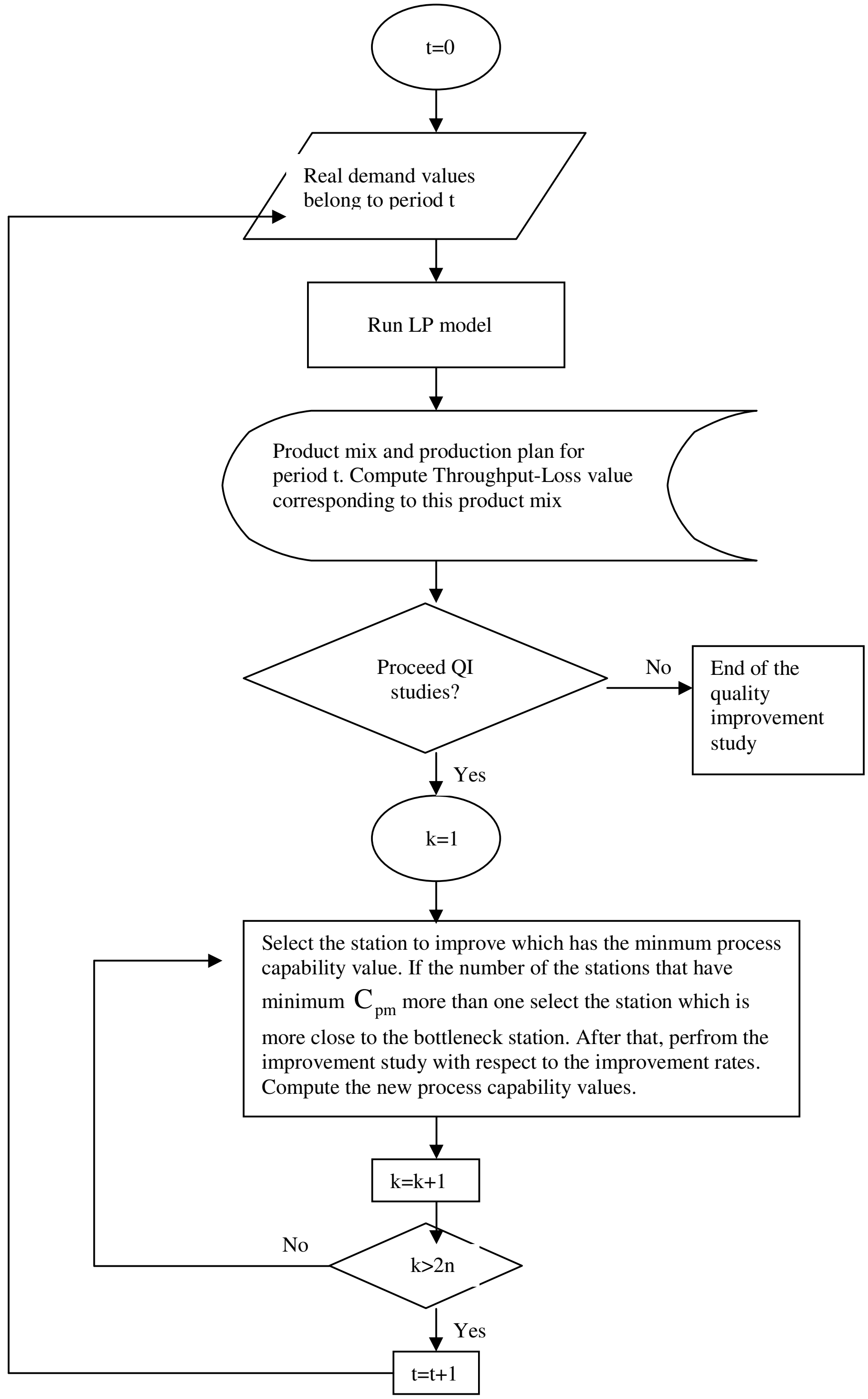


Figure 4.5 Flow of the quality improvement studies with process capability method

Afterwards, the station having the smallest C_{pm} value among the work and rework centers is selected as improvement station. Improvement study is started for the selected station in the beginning and finished at the end of the current period. For the selected station, the new improved C_{pm} value is computed. In the beginning of the succeeding period (t=1), product mix is determined regarding the real demand and improved station's values. For this product mix, T-L value is computed by using objective function formulation in Section 4.2.1. This T-L value is the first improvement period achievement. In addition to the product mix determination, improvement project is selected at the same time among the C_{pm} values. The station that has the minimum capability value is selected as the improvement station and improvement study begins. The algorithm of the improvement studies for the process capability method is performed as the same manner for the succeeding periods.

4.3.3. Application of Process Capability Approach on an Example Problem

In order to compare the T-L model with C_{pm} approach, the example problem defined in Section 5.2.3 is solved with the same values by using the C_{pm} approach. This sample problem is solved by Process capability approach by using MATLAB. The M.file code (file name: LPsade_Cpm_ornek) is given in CD-ROM.

Application of the algorithm on the sample problem

Period 0

No improvement case. LP model in Section 5.3.1 is run with the current values of the period. After the model is terminated optimal product mix is determined,

Product mix: (S_1= 6398.4, S_2 =4799.3, S_3 = 3161.3)

Work center 3 is the bottleneck station. C_{pm} values at this period is given in Table 4.4. Since work center (WC) 1 has the minimum C_{pm} (=0.66), it is selected to improve.

Throughput-Loss value of the product mix determined from typical product mix determination model is T-L_0=$-7.7105 \cdot 10^6$.

Period 1

WC1 has been improved.

After the LP model is run with the improved values, optimal product mix is determined,

Product mix: (S_1= 6398.4, S_2 =4799.9, S_3 = 3193.2)

In this period, work center 3 is the bottleneck station.

Throughput-Loss value is T-L_1= $-6.2155 \cdot 10^6$.

C_{pm} values are determined after the improvement of WC1 as in Table 5.9.

The station with the minimum C_{pm} (=0.77), WC2 is selected for improvement process.

Period 2

WC2 has been improved.

After the LP model is run optimal product mix is determined as

Product mix: (S_1=6400, S_2 =4800, S_3 =3200)

Since its capacity is utilized very close to its limit, work center 3 is the bottleneck station. Througput-Loss value is T-L_2= $5.1239 \cdot 10^6$.

C_{pm} values are determined after the improvement of WC2 as in Table 5.10.

Table 4.9 Process capability values of the products after the first improvement

$C_{pm\ ik}$	Work center 1	Work center 2	Work center 3	Work center 4	Rework center 1	Rework center 2	Rework center 3	Rework center 4
Product X	1.78	0.93	1.96	1.29	1.78	1.33	2.67	1.78
Product Y	2.00	1.08	1.31	1.94	1.33	1.56	1.78	2.67
Product Z	1.33	**0.77**	1.63	1.62	0.89	1.11	2.22	2.22

Table 4.10 Process capability values of the products after the first improvement

$C_{pm\ ik}$	Work center 1	Work center 2	Work center 3	Work center 4	Rework center 1	Rework center 2	Rework center 3	Rework center 4
Product X	1.78	2.00	1.96	1.29	1.78	1.33	2.67	1.78
Product Y	2.00	1.55	1.31	1.94	1.33	1.56	1.78	2.67
Product Z	1.33	1.67	1.63	1.62	**0.89**	1.11	2.22	2.22

Rework Center 1 (station number 5) is selected for improvement project.

Period 3

Rework center 1 has been improved. LP model finds a product mix such as

Product mix: (S_1=6400, S_2 = 4800, S_3 = 3200)

For this period, work center 3 is the bottleneck station that restricts the system from producing more. Troughput-Loss value is T-L_3= $5.1239 \cdot 10^6$.

Stop improvement study, since it has been performed for three periods.

Total T-$L_{improvement}$= T-L_1+ T-L_2+ T-L_3

= $4.0323 \cdot 10^6$.

4.4 Comparison of the Throughput-Loss and Process Capability Approaches

Total T-L values and the stations selected as improvement projects are summarized on Table 4.11

Table 4.11 Results of the methods

Methods	Selected Projects in			Total T-L value
	Period 0	Period 1	Period 2	
Throughput-Loss	2	3	1	$17.3913 \cdot 10^6$
Process Capability (C_{pm})	1	2	5 (Rework center 1)	$4.0323 \cdot 10^6$

According to these results, the T-L method is much better than the process capability method.

In this chapter, by incorporating the inspection error and rework terms into the mathematical model in Köksal (2004), an LP model, T-L model, is developed. Tracing the algorithm in Figure 4.1 performs improvement studies. In order to clarify the use of algorithm, improvement studies are implemented on a sample problem adjusted from the case in Atwater and Chakravorty (1995). The performance of this method is compared with a widely used improvement method, process capability method. However, at this time C_{pm} is used rather than C_p to determine the priorities of the projects. When the studied example is taken into consideration, T-L method gives much better results than process capability method.

In the next chapter, the performance of the T-L method will be studied by using a design of experiment. In addition, at the end of the three periods, the results of the T-L method will be judged against that of the process capability method.

CHAPTER 5

EXPERIMENTAL DESIGNS FOR STUDYING SIGNIFICANT EFFECTS ON THROUGHPUT RESULTS

5.1. Design of Experiments

In this study, up till now, a linear programming (LP) model is developed to support the product mix and quality improvement project selection decisions in a manufacturing environment where rework and inspection error exist. This model is used in a quality improvement algorithm to determine the improvement projects priorities. In addition, this algorithm is explained on a sample problem in Section 4.2.3 and compared with another improvement, process capability, method given in 4.3.3.

In this chapter, the term effects of the T-L model on the Throughput-Loss value are aimed to investigate. If the model were simple or linear, the term effects on the approach could be easily determined. However, T-L model is relatively so complex that the effects cannot be clearly resolved. Thus, to further study the performance of the approach for different conditions, the terms that are thought to be effective is identified; such as deviation from the target, standard deviation of quality characteristic produced in work and rework centers as well as inspection error, loss coefficient and totally variable costs. In this study, it is aimed to determine if there is a term that is more effective than the remainders.

The same production system in the sample problem in Chapter 5 is aimed to examine through a design of experiment. Hence, three products are processed at four work centers. The factors considered significant for this production environment is as follows:

-Standard deviation of the QC values of the products at the work centers ($SigmaPx_{11}$, $SigmaPx_{12}$,…, $SigmaPx_{34}$),

-The deviation of the process mean from the target value for the work (processing) centers ($(\mu_1 - T_1), (\mu_2 - T_2), \ldots, (\mu_4 - T_4)$),

-Standard deviation of the inspection error of the work centers ($SigmaMP_1$, $SigmaMP_2$,…, $SigmaMP_4$),

-Loss coefficient of the QC values of the products processed at work centers (LC_{11},LC_{12},…,LC_{34}),

-Totally variable costs of the products at the work centers ($TVCP_{11}$, $TVCP_{12}$,… $TVCP_{34}$).

In these denotations, first indice expresses the product type (Product X is 1, Product Y is 2 and Product Z is 3) and second indice expresses the work center number. For instance, $SigmaMP_{23}$ denotes the standard deviation of the measurement system for product (Y) type 2 at work center 3.

We tried to investigate as many factors as possible. However, since, we studied the production environment in which 3 products and 4 work (and also 4 rework centers), and inspection error exist, we run into many factors. For this reason, we made the experiment with some starting assumptions and limited the number of the factors to 44 factors defined in the beginning of this page. We assumed that

- Standard deviation of the QC values at the rework center is less than at the work center. ($\mathrm{SigmaRx}_{i,j+n} = 0.75 \cdot \mathrm{SigmaPx}_{ij}$),
- The mean of the reprocessed item is on the target,
- The standard deviation of the inspection error of the rework center is the same that of the work center,
- Loss Coefficients (LCs) of QC values of the reprocessed items are the same that of QC values of the processed items,
- Totally variable cost of the reprocessing operation of the products is half of that of the processing operation ($TVCR_{i,j+n} = 0.5TVCP_{ij}$).

Box-Behnken design, which is an efficient three-level design for fitting second order response surfaces (Myers and Montgomery, 1995), is selected to study the approach's performance. In Box-Behnken designs (BBDs), there is sufficient information for testing lack of fit. It is altitude to central composite designs (CCDs) because it requires less number of runs than CCDs as the number of the factors increases. On the other hand, five levels ($-\alpha$ (axial level), -1, 0, 1, α) can be studied for further analyses in CCD. Nevertheless, the number of the factors that we aimed to study is excessive for CCD, because we cannot design CCD in any statistical package. Therefore, we focused on the BBD and designed its experimentation table in MATLAB7 by using ***bbdesign*** function for 44 factors.

It is known that the small shifts (up to, say, 1.5σ) of the process mean from the target value may not be detected by using Shewart control charts (Montgomery, 2001). Therefore, the deviations up to 1.5σ, is investigated for the difference between process mean and the target value. The levels of the deviations of process mean from the target value for the work center are selected as 0 (the process is at the target), 1σ and 1.5σ.

As stated before, C_{pm} is more sensitive to departures of the process mean and shows better performance than C_p and C_{pk} (Pearn et al., 1992). On the other hand, as mentioned before, C_{pmk} is the most sensitive among the other process capability indices. Since the shifts of the mean from the target more than 1.5σ can be discovered from the Shewart charts, we investigate the shifts up to 1.5σ. When the departure size of the mean from the target is less than 2σ, the difference between the expected values of the C_{pm} and C_{pmk} shows a decrement as the sample size increases. (see Figure 4 and 5, in Pearn et al., 1992). As 100% inspection is made in the production system, C_{pm} is assumed to be sufficient. Thus, we made our decisions on the levels of the process standard deviations based on the C_{pm} values. It is known that the required minimum process capability ratio for a system is 1.33

(Montgomery, 2001). We try to study a range of cases, between a poor (C_{pm} =0.33) and a good (C_{pm} =2) case.

As mentioned in Chapter 5, it is thought that defining standard deviation in terms of the specification limits is not meaningless. By this way, our aim, to scan same values for each processed QC values, is achieved. Thus, we determined standard deviation levels in term of specification limits, a specific rate of the specification limits. We determined the levels of the standard deviation of the QC values processed in work centers as 3σ, 6σ and 12σ.

The inspection error levels of the process and rework are determined regarding the gauge capability measurement rate, precision-to-tolerance (P/T) ratio. The processes are thought to have a good measurement system if their P/T ratio is less than or equal to 10%. If the measurement system of a process has a P/T ratio the value of which is between 10% and 30%, the measurement system is said to be a marginally acceptable system. However, if the P-T ratio value of the system is larger than 30%, it is thought to be an unacceptable system (Montgomery, 2001). In this experimental design study, we aim to research a wide range of P/T values. After we consult to Tümer Arıtürk, we have learned that in industrial practice, the gage capabilities of the measurement systems have so large values that the P/T ratios may be up to a value of 200 % (SPAC Six Sigma Consulting Company, personal communication, 2006). The range that covers the values for the P/T ratio from 5% to 200% is scaned. We determined the inspection error levels regarding this extensive range as 0.1σ, 0.25σ and 1σ which are corresponding to the ranges of 5%-20%, 12.5%-50% and 50%-200% for P/T ratio values, respectively. Here, σ is the standard deviation of the quality characteristic, X_p, for the product processed at work center.

Totally variable cost (TVCP) due to process levels of the products are established in terms of selling prices of the products; a constant rate of the selling prices. TVCP levels of product i for any workstation are identified as $0.12SP_i$, $0.15SP_i$

and $0.18SP_i$. Here SP_i is the selling price value of the i^{th} product. The high and low levels of TVCP are determined after a set of calculations is performed on the approach. The high level is the upper bound for TVCP for which the production system stops manufacturing when all system inputs (all factors) are set at their medium levels. The low level is the lower bound for TVCP for which the production system starts manufacturing again when all system inputs are set to their medium levels. The medium level is selected as midpoint of the high and low levels of TVCP. With these levels, during experimentation, profit margins, the rate of the difference between throughput and totally variable costs (total cost due to process and rework operations) to the totally variable cost of the products, are aimed not to exceed 60% on the average.

Loss coefficient values are determined as 10, 150 and 400. Loss coefficients are assumed not to be correlated with raw material costs and selling price of the products. Thus, the QC of the product, which requires expensive raw materials at a work center, might have a low loss coefficient or vice versa. Hence, loss coefficients are chosen independently from the other factor levels.

The factors and their levels are presented in Table 5.1.

Table 5.1 Factors and their levels used in the experimental design.

Levels	$SigmaPx_{ik}$	$(\mu - T)_k$	$SigmaM_{ik}$	LC_{ik}	$TVCP_{ik}$
1	USL(1/6)	0	0.1σ	10	0.12SPi
2	USL(1/3)	1σ	0.25σ	150	0.15SPi
3	USL(1/1.5)	1.5σ	1σ	400	0.18SPi

Box-Behnken experimental design is performed for T-L model and process capability approach by using MATLAB. This M.file codes (file names : LPsade1 and LPsade_Cpm) are run by using another M.file codes (file names :tasarim4 and tasarim5), respectively. T-L and process capability approach are run for 3796 different cases by using Box-Behnken design. By using M.file codes, bottleneck station is searched for all 3796 different cases as well. Afterwards, the two approaches are compared with respect to their T-L values gained from the three improvement periods. T-L approach deviates maximum 230,3% and on the average 99% from the process capability results. Hence, T-L approach has an enormous altitude over the process capability method. In these cases, work center 3 is found as the bottleneck station through the experiment, with respect to the definition determined in Chapter 4.

5.2. Analysis of Variance (ANOVA) of Experimental Data

The data is gathered using Box-Behnken design and the probability values found from Markov Chain. The data is analyzed and an ANOVA is performed. ANOVA is used to analyze the model constructed for the response variable total Throughput-Loss values for three period times. The model is:

$$Y = \mu_T + \sum \tau_i + \sum \sum \theta_{ij} + \varepsilon$$

where,

μ_T : overall mean,

τ_i : main effect of factor i,

θ_{ij} : interaction effect of factor i and factor j,

ε : error term of the model,

Y : the throughput observation.

ANOVA is used to investigate effects of the independent variables and two-way interactions of the factors and to determine which have significant effect on the response variable.

The following hypothesis on the main and the two-way interaction effects of the factors are tested:

H_0 : There is no main effect (τ_i =0)

H_1 : The effect of factor i is significant ($\tau_i \neq 0$)

H_0' : There is no interaction effect between factor i and factor j (θ_{ij} =0)

H_1' : The interaction effect between factors i and j is significant ($\theta_{ij} \neq 0$)

The two assumptions should be satisfied to interpret the results of ANOVA:

-The random error is normally distributed with mean 0 and a constant variance,

-All pairs of error are independent.

Different graphical tools are used to check the validity of these assumptions (see Figure 5.1, 5.2 and 5.3). The normality plot of the residuals is given in Figure 5.2. In the normality plot, an outlier is obviously seen. The row corresponding to this outlier value is seeked. However, this row does not consist of extreme values of the factors. In this row, SigmaPx24 is at its 1 level- hence, USL_{24}/1.5- and Mu-T4 is at its -1 level-hence, mean of the product 4 is on the target. Therefore, we continue the analysis without eliminating any row of the BBD.

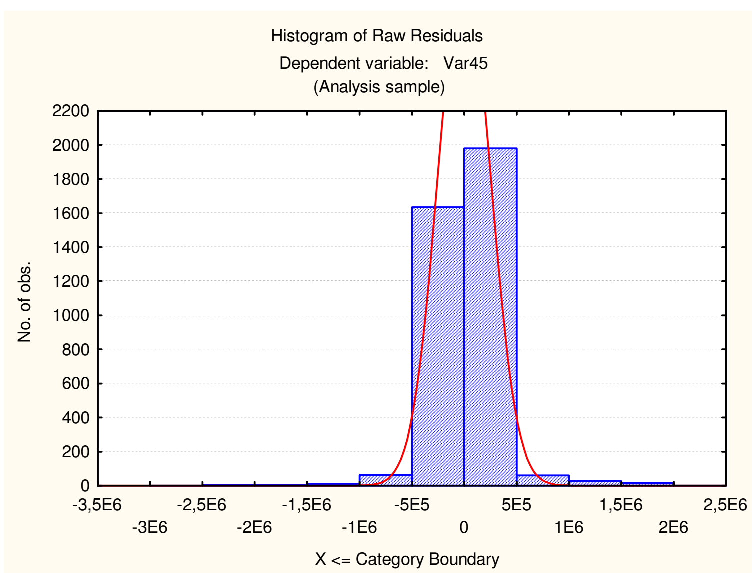


Figure 5.1 Histogram of the residuals

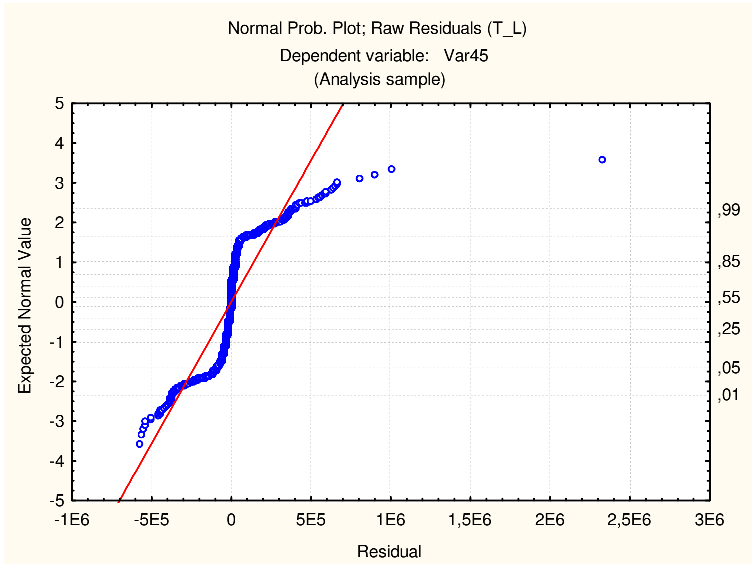


Figure 5.2 Normal Probability plot of the residuals.

Because of the peakness of the residuals' histogram (in Figure 5.1) and the S shape of the normal probability plot (in Figure 5.2), residuals seem not to be normally distributing. Moreover, plot of predicted versus raw residual values in Figure 5.3 does not seem to have a constant variance. Hence, a transformation is needed. Traditional transformations that are designed to achieve constant residual variances, the logarithmic and square root transformations are tested firstly. The residual probability plot of the log transformation in Figure 5.4 seems better. However, constant variance is not attained in the plot of predicted versus residual.

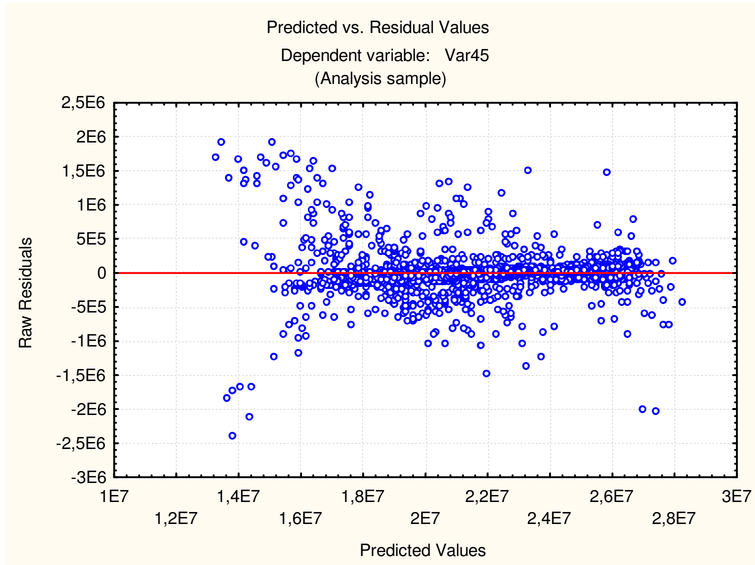


Figure 5.3 The plot of the predicted versus residuals

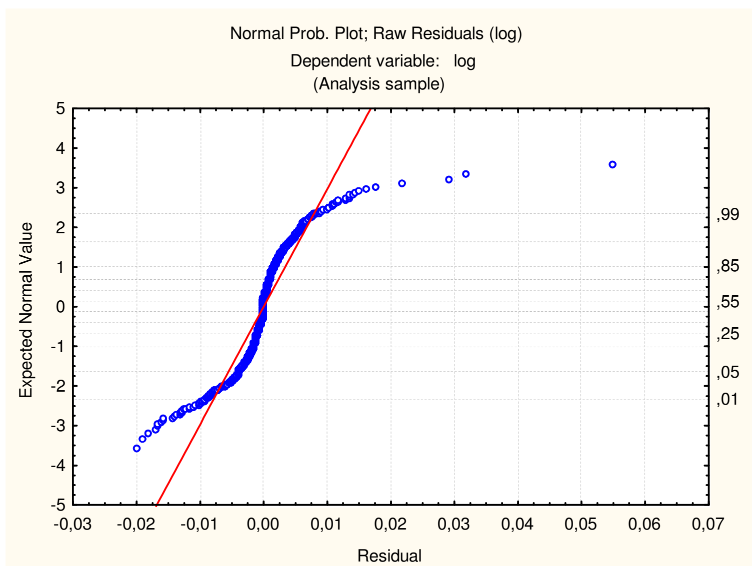


Figure 5.4 The residual probability plot of the log transformed data

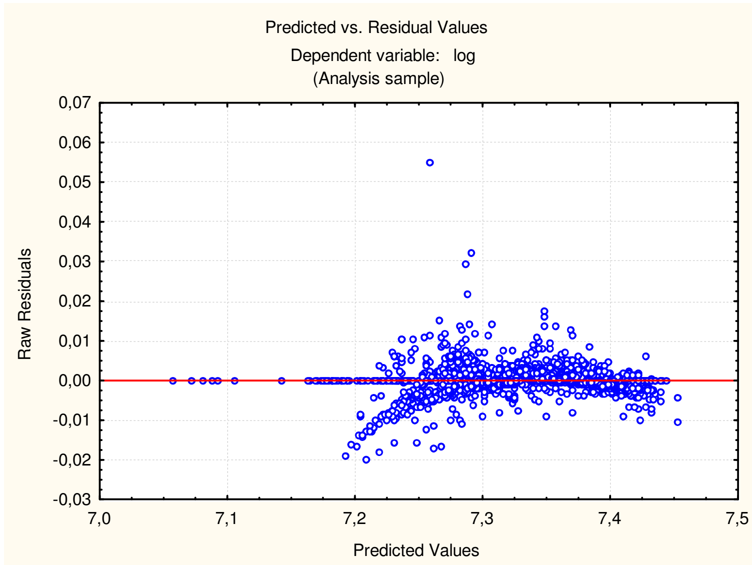


Figure 5.5 The plot of the predicted versus residuals of the log transformed data

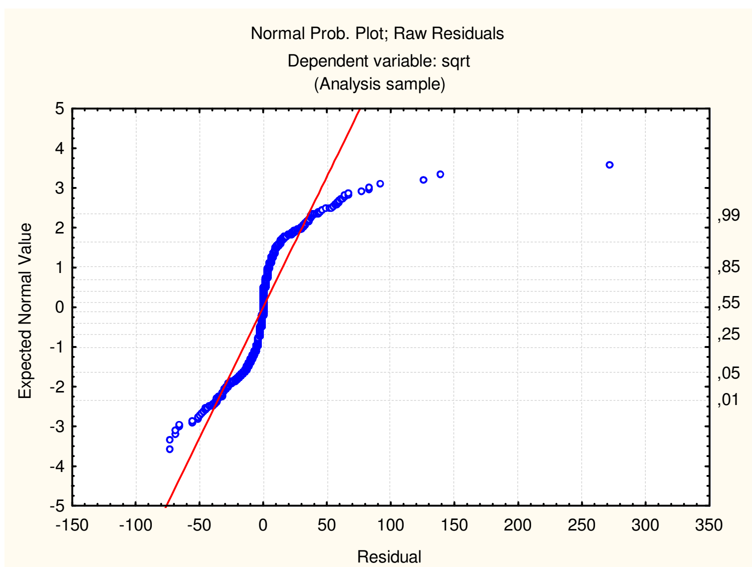


Figure 5.6 The residual probability plot of the square root transformed data

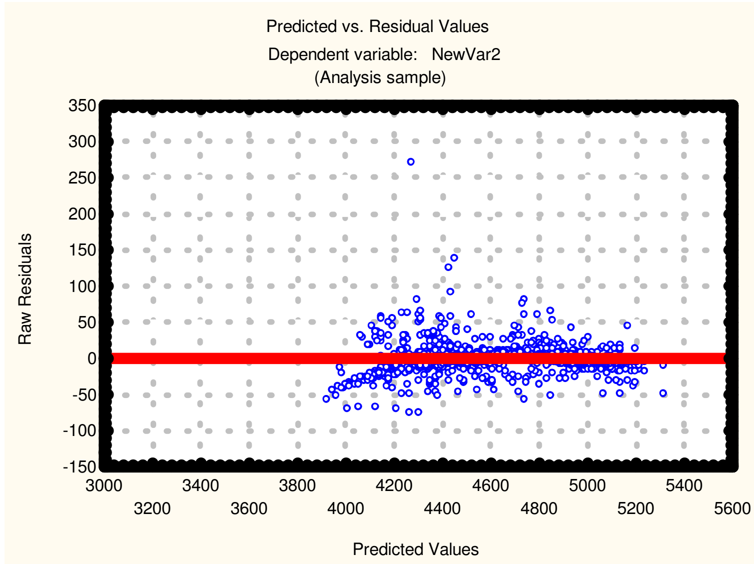


Figure 5.7 The plot of the predicted versus residuals of the square root transformed data

After that, square root transformation is investigated. The residual plot of the residuals for the squared root transformed data seems better than the raw data as well. However, constant variance is again not achieved.

Lastly, the Box-Cox transformation is executed. After the Box-Cox transformation is performed on the data for the λ value of 1.52, the plots in Figure 5.8 and 5.9 is obtained. In Box-Cox transformation we cannot get a better result than log and square root transformations.

Although log, square root and Box-Cox transformations are tested on the data a significant change in the structure of the residuals and improvement in the assumptions could not be achieved.

Design of experiment is constructed to test the main and interaction effects of the factor on the T-L approach. Nonetheless, the ANOVA model does not sufficiently express the T-L approach. Transformations of the data does not result a development at the assumptions. In predicted versus residuals figures, it seems that quadratic terms are needed in the model. The complexity, extreme nonlinearity, of the T-L model may hinder the model to express the approach well.

As stated before, our aim is to determine the factors, which are more effective on the model. Since we have an LP model and do not want to construct a new ANOVA model, further study on the model is not performed. We try to get a general idea about the factors that are more important than the rest of the factors on the approach and not to accurately fit a model by using ANOVA.

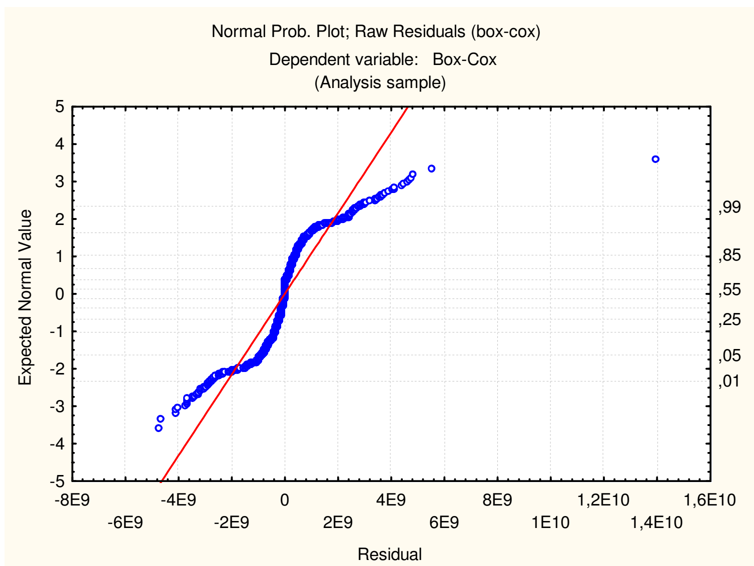


Figure 5.8 The residual probability plot after the Box-Cox transformation

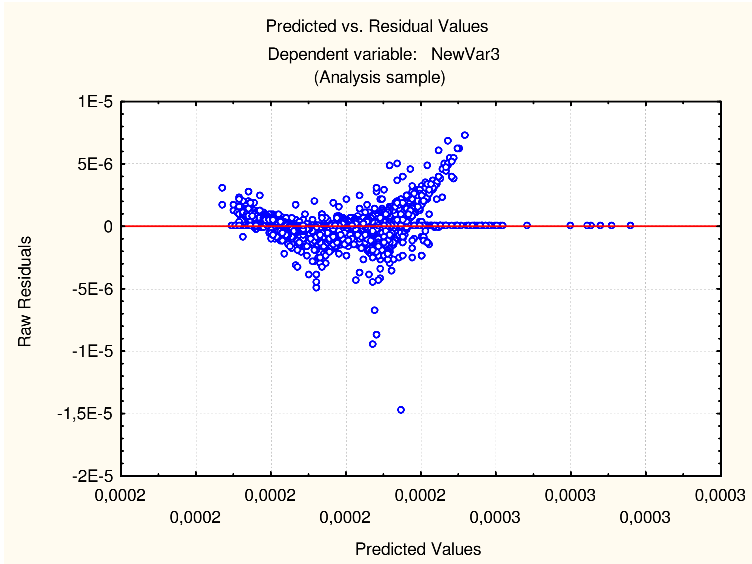


Figure 5.9 The plot of the predicted versus residuals after the Box-Cox transformation

The number of factors is so large that all interaction effects cannot be investigated in the ANOVA study. Therefore, two-way interactions are grouped and each group is separately analyzed with the main effects. Afterwards, the two-way interactions, which are statistically significant, are incorporated and analyzed with main effects. The result of ANOVA is given in Appendix B. The main effects are summarized in Table 5.2.

Even the results of the ANOVA are not very reliable, according to Table 5.2, we can say regarding the related p values of the factors (for the 0.05 alfa level) that all factors significantly affect the model except the standard deviation of product 2 at work center 3, and the standard deviation of the measurement error of the first and second processes. From this result, it is thought that measurement error is not significantly effective on the factors for the work centers prior to the bottleneck center. Hence, measurement error is more effective for the bottleneck or the succeeding stations.

The most of the two-way interactions between standard deviations (SigmaPx*SigmaPx) is statistically significant. We reject the hypotheses H_0' since p=0 for the mostly two-way interactions between standard deviation and the difference between the mean and the target value (SigmaPx*Mu-T), between the standard deviation of the process and loss coefficient (SigmaPx*LC) and between the standard deviation of the process and totally variable cost due to process (SigmaPx*TVCP) given in Appendix G. These two-way interaction effects are significant.

The two-way interaction effect between the mean-target deviation of the first and second process is significant ($\mu - T1 * \mu - T2$). For the stations, before the bottleneck station, mean and target deviations and their interactions should be taken into consideration.

Moreover, the two-way interactions between the deviation of the mean and target and loss coefficient ($\mu\text{-}T * \text{LC}$) and the two-way interactions between loss coefficients are found significant (LC*LC).

In addition, the interactions between the measurement error and the deviation of the mean from the target for the third and fourth work center have significant effect on the model. As indicated before, work center three and four are the bottleneck station and the station following the bottleneck, respectively. Atwater and Chakravorty (1995) have denoted the impact of the constraint (the bottleneck and the succeeding station) on their examined manufacturing system. Although the ANOVA results are not reliable because of the violation of the residual assumptions, it can be said that the measurement error and the deviation of the process mean from the target value are more effective on the bottleneck and the next station than on the other stations. Therefore, during the QI studies, these factors should be investigated carefully and taken into consideration for the bottleneck and the next operation.

Table 5.2 ANOVA results of the main effects

	SS	Degr. of	MS	F	p
Intercept	1,285517E+12	1	1,285517E+12	94,377	0,000000
SigmaPx11	2,265078E+11	2	1,132539E+11	8,315	0,000250
SigmaPx12	2,855393E+12	2	1,427696E+12	104,815	0,000000
SigmaPx13	1,403398E+11	2	7,016992E+10	5,152	0,005839
SigmaPx14	1,499646E+11	2	7,498230E+10	5,505	0,004106
SigmaPx21	3,626124E+12	2	1,813062E+12	133,107	0,000000
SigmaPx22	5,323259E+11	2	2,661630E+11	19,541	0,000000
SigmaPx23	7,455213E+10	2	3,727606E+10	2,737	0,064940
SigmaPx24	2,271443E+12	2	1,135721E+12	83,380	0,000000
SigmaPx31	8,272044E+12	2	4,136022E+12	303,649	0,000000
SigmaPx32	9,996915E+10	2	4,998458E+10	3,670	0,025593
SigmaPx33	5,125054E+11	2	2,562527E+11	18,813	0,000000
SigmaPx34	4,940156E+11	2	2,470078E+11	18,134	0,000000
Mu-T1	1,309558E+12	2	6,547792E+11	48,071	0,000000
Mu-T2	2,251315E+12	2	1,125657E+12	82,641	0,000000
Mu-T3	4,698232E+12	2	2,349116E+12	172,462	0,000000
Mu-T4	5,504955E+12	2	2,752477E+12	202,075	0,000000
SigmaM1	0,000000E-01	2	0,000000E-01	0,000	1,000000
SigmaM2	2,310940E+10	2	1,155470E+10	0,848	0,428240
SigmaM3	3,920817E+12	2	1,960409E+12	143,925	0,000000
SigmaM4	5,670483E+12	2	2,835241E+12	208,151	0,000000
LC11	4,822510E+12	2	2,411255E+12	177,024	0,000000
LC12	5,661577E+11	2	2,830788E+11	20,782	0,000000
LC13	5,693877E+11	2	2,846939E+11	20,901	0,000000
LC14	8,166739E+11	2	4,083369E+11	29,978	0,000000
LC21	1,705436E+13	2	8,527178E+12	626,030	0,000000
LC22	4,397334E+11	2	2,198667E+11	16,142	0,000000
LC23	1,592515E+12	2	7,962575E+11	58,458	0,000000
LC24	3,987818E+11	2	1,993909E+11	14,638	0,000000
LC31	7,660599E+12	2	3,830300E+12	281,204	0,000000
LC32	1,937312E+12	2	9,686559E+11	71,115	0,000000
LC33	3,590334E+12	2	1,795167E+12	131,794	0,000000
LC34	2,015419E+12	2	1,007709E+12	73,982	0,000000
TVCP11	2,209122E+12	2	1,104561E+12	81,092	0,000000
TVCP12	2,224325E+12	2	1,112163E+12	81,650	0,000000
TVCP13	2,251545E+12	2	1,125773E+12	82,649	0,000000
TVCP14	2,254444E+12	2	1,127222E+12	82,756	0,000000

TVCP21	1,939258E+14	2	9,696291E+13	7118,608	0,000000
TVCP22	1,954356E+14	2	9,771778E+13	7174,028	0,000000
TVCP23	1,971173E+14	2	9,855866E+13	7235,762	0,000000
TVCP24	1,975381E+14	2	9,876907E+13	7251,209	0,000000
TVCP31	1,270388E+14	2	6,351941E+13	4663,327	0,000000
TVCP32	1,280918E+14	2	6,404591E+13	4701,981	0,000000
TVCP33	1,291391E+14	2	6,456953E+13	4740,423	0,000000
TVCP34	1,292715E+14	2	6,463576E+13	4745,285	0,000000

5.3. Discussion

In this study, the factors affecting the performance of the T-L approach used to determine product mix and QI project priorities are investigated. Since the residuals assumptions are not satisfied, we cannot say that the constructed ANOVA model express the throughput data well. Although the ANOVA results are not very reliable, according to the results, except the inspection error for work and rework centers 1 and 2, all factors have significant effect on the model. Inspection error of the constraint resources is more effective than that of the other resources. Accordingly, we reach to the same conclusion of as Atwater and Chakravorty (1995) that the constraint resources have great impact on the determination of the QI project priorities and product mix as well.

When we do not incorporate the quality loss criterion into the model, we do not achieve as high Throughput-Loss value as in the T-L mathematical model approach. In order to clarify the altitude of the T-L model with respect to a widely used QI approach, process capability approach, a sample problem is studied in Section 4.2.3 and Section 4.3.3. According to the results of these two approaches derived from this sample problem, T-L model is much better than the process capability approach. By using Box-Behnken design, the treatments of these two models are examined. In accordance with the 3796 different production conditions, T-L model presents a better performance than the process capability approach with the maximum 230.3% and on the average 99% deviation from the process capability results.

In addition, while using process capability method in QI projects, net profit of the improvement study cannot be determined clearly because the quality loss is not taken into consideration. However, with T-L model, net profit is obviously seen from the objective function.

On the other hand, it should not be forgotten that this model is constructed under some assumptions. For the production systems constructed under different assumptions this model may not result in good solutions. Hence, the model may be revised for the different manufacturing settings. For instance, the quality characteristics (QCs) processed at the work centers are assumed to be normally distributing. If QCs have different distributions the probability and expected values computed by using probability density and cumulative function properties of the normal distribution should be calculated in terms of these probability distribution functions. The yield and rework rates of the work and rework centers should be recalculated. In addition, if the QCs have a different distribution rather than normal distribution, the variance of the accepted items used in the expected loss ($\overline{L}_{ik}$) computation should be recalculated as well. Hence, this causes a change at the expected loss value.

Additionally, the QCs are assumed to have a nominal-the-best type of quality loss function. For the QCs that have a different quality loss function, such as larger-the-better, smaller-the-better or asymmetric loss function, the expected quality loss value ($\overline{L}_{ik}$) should be recomputed for this type of loss functions.

All products are assumed to be processed at each work centers. If all products do not pass through all the work centers the constraints from (2) to (6) should be adjusted to this new production environment system. Moreover, scheduling is excluded from the area of the investigation of this study. Set up times of the work centers for products are ignored as well. To incorporate these methods into the model, it should be revised and especially constraint (5) and (6) should be reformulated.

All items processed at the work centers or reprocessed at the rework centers are assumed to be inspected one by one. Hence, 100% inspection is performed at the stations. If acceptance sampling is used rather than 100% inspection, variance of the accepted items should be recomputed. The term, affecting from the truncated variance of the accepted items, is the expected quality loss ($\overline{L}_{ik}$) value of the items sent to customer.

In the examined production system, rework operations are performed at separate stations. If the products are reprocessed at the same work center, the scheduling of all these products should also be taken into account.

A disadvantage of the model is that determining the quality loss coefficient value is really difficult in practice. However, it can be estimated by using the specification limits and the loss at these limits. This loss consists of all the losses such as repair and replacement of the product. It also includes loss to consumer due to the lack of the product during repair operations and transportation cost (Phadke, 1989). Therefore, the determination of the loss coefficient is not a piece of cake.

In this chapter, the effectiveness of the terms that are thought as significant are aimed investigate. The factors are determined and an experimental design is constructed by using these factors. Box-Behnken matrix is and 3796 different cases are studied. All 44 factors, are investigated for their three different levels. By using Box-Behnken design, the treatments of the T-L model and a process capability approach are compared. As a result, T-L model presents a better performance than the process capability approach.

ANOVA is performed on the total T-L values, summation of the three improvement periods. Even the results are not reliable; the outcome of the bottleneck and the succeeding stations having great impact on the QI project selections may be reached according to the ANOVA results.

CHAPTER 6

CONCLUSION AND FURTHER STUDIES

In this study, the effect of inspection error on the product mix and quality projects selection in a manufacturing environment where rework also exists is examined. It is assumed that the products (items) for which rework is necessary are reprocessed at a separate work center and 100% inspection is performed for the products both after rework and processing operations. It is assumed that only one quality characteristic (QC) is processed at a work center and the processing units are independent of each other. Thus, the QC processed at a work center is not affected from the operations at the succeeding work centers.

The QC processed at work centers are assumed to be normally distributed with mean μ_p and variance σ_p^2. In addition, the distribution of the QC of items reprocessed at rework centers is also assumed to be normal with mean μ_r and variance σ_r^2. It is assumed that the items sent to rework centers are reprocessed with a more sophisticated operation. For this reason, it is assumed that the variance of the QC reprocessed at rework center is taken smaller than that of the processed at work center ($\sigma_r^2 < \sigma_p^2$). In addition, the rework operations are assumed to be producing at target ($\mu_r = T$). Besides, QCs are assumed to have nominal-the-best type of quality characteristic.

Another assumption of the examined production system is that the measurement gauge is calibrated. Hence, there is not accuracy problem for the measurement system. In addition, the processes are assumed to be under statistical control. Otherwise, the process capability values are not reliable since the processes are not stable, hence variances are not stable. Process capability indices are only meaningful for the controllable processes.

In Taşeli (2004) the effect of the inspection error on the quality loss value was studied for the production environment constructed under the same assumptions. The formulas used to calculate expected quality loss value ($\overline{L}_{ik}$), which are given in Appendix A, are taken from Taşeli (2004).

For the m products n work centers production systems a mathematical model is developed by using the T-L model in Köksal (2004). The linear programming (LP) model in Köksal (2004) is a tool used to support the product mix and quality improvement project selection decisions. However, this model was constructed for the production environment where rework does not exist and under the assumption of no inspection error in the system. These assumptions are not realistic. Because highly advanced measurement systems may contain errors as well. In addition, in practice, when a product characteristic value does not conform specification limits it does not scrapped. If rework is possible this item is sent to rework and recovered. Therefore, rework and inspection error terms are incorporated to the LP model.

Markov chain approach is used to compute yield and rework rates. The improvement algorithm using the revised LP model is explained on a sample problem and compared with in practice, widely used method, process capability in Section 4.2.3 and 4.3.3. As a result, T-L model presents a better performance than the process capability approach. (See Section 4.4 and Chapter 5).

A design of experiment is performed for the factors that are decomposed from the model. Box-Behnken design is used and three levels are studied for each factor. The residual assumptions that should be satisfied to interpret the ANOVA results are not completely satisfied. Although the results are not reliable, we conclude that all factors have significant effects on the model except the measurement error of the first and second work centers. Measurement error term of bottleneck and the succeeding stations is more effective than that of the other stations. In addition to constraint resources, standard deviation of the processes, the deviation of the mean from the target, measurement error (especially for the bottleneck and the next station), loss coefficient and totally variable costs have significant effect on the model. These factors should not be ignored during the QI studies.

For the production systems constructed under different assumptions this model may not run well. Hence, the model may need a revision for the different manufacturing settings. The possible revisions in the model are explained in Section 5.3.

For further studies, different distributions of the QC may be investigated. In addition inspection error may also be non-normally distributed. This aspect can be taken into account as well. Different quality loss functions for the QC may also be studied. On the other hand, the QCs processed at work centers are assumed to be not affected from the operations at the succeeding work centers. The production environment where the work centers are affecting each other can be analyzed. Another possible research area may be investigating the effect of the measurement system accuracy. In this study, measurement gauge is assumed to be calibrated. However, the system may not be accurate, and the effect of this criterion on the QI priorities determination may also be studied.

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APPENDIX A

MIXTURE DISTRIBUTION OF THE ACCEPTED ITEMS FOR THE REWORK AND INSPECTION ERROR CASE

Taşeli (2004) computed the expected quality loss values for the production environment in which rework and inspection error exist. Here, this formulations is summarized.

The quality characteristic (QC) produced in the processing center, X_p, is assumed to be normally distributed with mean μ_p and variance σ_p^2 ($X_p \sim N(\mu_p, \sigma_p^2)$).
The QC of the reworked item, X_r, is assumed to have a normal distribution with parameters μ_r and σ_r^2 ($X_r \sim N(\mu_r, \sigma_r^2)$).

Since the examined system is under the measurement error effect, the real process QC value, X_p, is observed as Y_p with E_p amount of deviation, where E_p is normally distributed with mean 0 and variance ε_p^2 ($E_p \sim N(0, \varepsilon_p^2)$). Similarly, the real QC value of the reworked item, X_r, is observed as Y_r with E_r amount of deviation. Here, $\mathrm{E_r}$ is assumed to have a normal distribution with mean 0 and variance ε_r^2 ($E_r \sim N(0, \varepsilon_r^2)$).

The joint distribution of the actual and observed process QC values can be denoted as $l_{Xp,Yp}(x, y)$. In the same way as in the processing unit, the joint distribution of the actual and observed quality characteristic can be defined as $h_{Xr,Yr}(x, y)$.

The resulting distribution of the QC of the accepted items coming from both process and rework centers is a mixture of these two truncated distributions mixed at proportions r and t, where r is the rate of the probability the observed QC values are within the specification limits to the probability that the observed QC values are within the scrap limits and t=1-r. r is calculated as follows:

$$r = \frac{\int_{-\infty}^{\infty}\int_{LSL}^{USL} l_{Xp,Yp}(x,y)dydx}{\int_{-\infty}^{\infty}\int_{LLs}^{ULs} l_{Xp,Yp}(x,y)dydx} = \frac{\int_{LSL}^{USL} m_{Yp}(y)dy}{\int_{LLs}^{ULs} m_{Yp}(y)dy}$$

where m(y) is the marginal distribution of Y.

Mixture distribution of the accepted QC, Xa, value is,

$$h_{Xa}(x) = r\frac{\int_{LSL}^{USL} l_{Xp,Yp}(x,y)dy}{\int_{-\infty}^{\infty}\int_{LSL}^{USL} l_{Xp,Yp}(x,y)dydx} + t\frac{\int_{LSL}^{USL} h_{Xr,Yr}(x,y)dy}{\int_{-\infty}^{\infty}\int_{LSL}^{USL} h_{Xr,Yr}(x,y)dydx}$$

$$= \frac{\int_{LSL}^{USL} m_{Yp}(y)dy}{\int_{LLs}^{ULs} m_{Yp}(y)dy} \cdot \frac{\int_{LSL}^{USL} l_{Xp,Yp}(x,y)dy}{\int_{LSL}^{USL} m_{Yp}(y)dy} + t\frac{\int_{LSL}^{USL} h_{Xr,Yr}(x,y)dy}{\int_{LSL}^{USL} n_{Yr}(y)dy}$$

$$= \frac{1}{M'}\int_{LSL}^{USL} l_{Xp,Yp}(x,y)dy + \frac{r}{M''}\int_{LSL}^{USL} h_{Xr,Yr}(x,y)dy$$

where,

$$M' = \int_{LLs}^{ULs} m_{Yp}(y)dy \quad \text{and} \quad M'' = \int_{LSL}^{USL} n_{Xr,Yr}(x,y)dy$$

The first and second moments of this distribution are:

$$E[Xa] = \frac{1}{M'}\{\mu_p[F(\frac{USL-\mu_p}{\sqrt{\sigma_p^2+\varepsilon_p^2}}) - F(\frac{LSL-\mu_p}{\sqrt{\sigma_p^2+\varepsilon_p^2}})]$$

$$+\frac{\sigma_p^2}{\sqrt{\sigma_p^2+\varepsilon_p^2}}[\phi(\frac{LSL-\mu_p}{\sqrt{\sigma_p^2+\varepsilon_p^2}})-\phi(\frac{USL-\mu_p}{\sqrt{\sigma_p^2+\varepsilon_p^2}})]\}$$

$$+\frac{q}{M''}\{\mu_r[F(\frac{USL-\mu_r}{\sqrt{\sigma_r^2+\varepsilon_r^2}})-F(\frac{LSL-\mu_r}{\sqrt{\sigma_r^2+\varepsilon_r^2}})]$$

$$+\frac{\sigma_r^2}{\sqrt{\sigma_r^2+\varepsilon_r^2}}[\phi(\frac{LSL-\mu_r}{\sqrt{\sigma_r^2+\varepsilon_r^2}})-\phi(\frac{USL-\mu_r}{\sqrt{\sigma_r^2+\varepsilon_r^2}})]\}$$

and

$$E[X_a^2]=\frac{1}{M'}\{(\mu_p^2+\sigma_p^2)[F(\frac{USL-\mu_p}{\sqrt{\sigma_p^2+\varepsilon_p^2}})-F(\frac{LSL-\mu_p}{\sqrt{\sigma_p^2+\varepsilon_p^2}})]$$

$$-\frac{2\mu_p\sigma_p^2}{\sqrt{\sigma_p^2+\varepsilon_p^2}}[\phi(\frac{LSL-\mu_p}{\sqrt{\sigma_p^2+\varepsilon_p^2}})-\phi(\frac{USL-\mu_p}{\sqrt{\sigma_p^2+\varepsilon_p^2}})]$$

$$+\frac{\sigma_p^4}{\sigma_p^2+\varepsilon_p^2}[(\frac{LSL-\mu_p}{\sqrt{\sigma_p^2+\varepsilon_p^2}})\phi(\frac{LSL-\mu_p}{\sqrt{\sigma_p^2+\varepsilon_p^2}})-(\frac{USL-\mu_p}{\sqrt{\sigma_p^2+\varepsilon_p^2}})\phi(\frac{USL-\mu_p}{\sqrt{\sigma_p^2+\varepsilon_p^2}})]\}$$

$$+\frac{q}{M''}\{(\mu_r^2+\sigma_r^2)[F(\frac{USL-\mu_r}{\sqrt{\sigma_r^2+\varepsilon_r^2}})-F(\frac{LSL-\mu_r}{\sqrt{\sigma_r^2+\varepsilon_r^2}})]$$

$$-\frac{2\mu_r\sigma_r^2}{\sqrt{\sigma_r^2+\varepsilon_r^2}}[\phi(\frac{LSL-\mu_r}{\sqrt{\sigma_r^2+\varepsilon_r^2}})-\phi(\frac{USL-\mu_r}{\sqrt{\sigma_r^2+\varepsilon_r^2}})]$$

$$+\frac{\sigma_r^4}{\sigma_r^2+\varepsilon_r^2}[(\frac{LSL-\mu_r}{\sqrt{\sigma_r^2+\varepsilon_r^2}})\phi(\frac{LSL-\mu_r}{\sqrt{\sigma_r^2+\varepsilon_r^2}})-(\frac{USL-\mu_r}{\sqrt{\sigma_r^2+\varepsilon_r^2}})\phi(\frac{USL-\mu_r}{\sqrt{\sigma_r^2+\varepsilon_r^2}})]\}$$

where *F(.)* is the cumulative distribution function and $\phi(.)$ is the probability distribution function of a standard normal random variable. The first and second moment of X_a provide the mean and the variance, respectively.

$$\mu_a=E[X_a] \qquad \text{and} \qquad \sigma_a^2=E[X_a^2]-(E[X_a])^2$$

APPENDIX B

THE RESULTS OF THE ANOVA

	SS	Degr. of	MS	F	p
Intercept	1,335020E+12	1	1,335020E+12	100,388	0,000000
SigmaPx11	2,290470E+11	2	1,145235E+11	8,612	0,000186
SigmaPx12	2,876541E+12	2	1,438270E+12	108,152	0,000000
SigmaPx13	6,885267E+09	2	3,442633E+09	0,259	0,771939
SigmaPx14	1,439485E+11	2	7,197424E+10	5,412	0,004503
SigmaPx21	3,627537E+12	2	1,813768E+12	136,388	0,000000
SigmaPx22	5,327562E+11	2	2,663781E+11	20,030	0,000000
SigmaPx23	7,979606E+10	2	3,989803E+10	3,000	0,049921
SigmaPx24	2,258764E+12	2	1,129382E+12	84,925	0,000000
SigmaPx31	8,272627E+12	2	4,136314E+12	311,033	0,000000
SigmaPx32	9,975323E+10	2	4,987662E+10	3,751	0,023610
SigmaPx33	5,101563E+11	2	2,550781E+11	19,181	0,000000
SigmaPx34	4,817696E+11	2	2,408848E+11	18,113	0,000000
Mu-T1	1,325098E+12	2	6,625492E+11	49,821	0,000000
Mu-T2	2,653385E+12	2	1,326692E+12	99,762	0,000000
Mu-T3	4,714817E+12	2	2,357408E+12	177,267	0,000000
Mu-T4	5,504585E+12	2	2,752292E+12	206,961	0,000000
SigmaM1	0,000000E-01	2	0,000000E-01	0,000	1,000000
SigmaM2	2,302635E+10	2	1,151317E+10	0,866	0,420839
SigmaM3	3,920544E+12	2	1,960272E+12	147,404	0,000000
SigmaM4	5,671011E+12	2	2,835506E+12	213,218	0,000000
LC11	4,824867E+12	2	2,412434E+12	181,405	0,000000
LC12	5,578383E+11	2	2,789192E+11	20,974	0,000000
LC13	5,659772E+11	2	2,829886E+11	21,280	0,000000
LC14	8,047510E+11	2	4,023755E+11	30,257	0,000000
LC21	1,705480E+13	2	8,527399E+12	641,224	0,000000
LC22	4,392080E+11	2	2,196040E+11	16,513	0,000000
LC23	1,589833E+12	2	7,949165E+11	59,774	0,000000
LC24	3,987818E+11	2	1,993909E+11	14,638	0,000000
LC31	7,660599E+12	2	3,830300E+12	281,204	0,000000
LC32	1,937312E+12	2	9,686559E+11	71,115	0,000000
LC33	3,590334E+12	2	1,795167E+12	131,794	0,000000
LC34	2,015419E+12	2	1,007709E+12	73,982	0,000000

TVCP11	2,209122E+12	2	1,104561E+12	81,092	0,000000
TVCP12	2,224325E+12	2	1,112163E+12	81,650	0,000000
TVCP13	2,251545E+12	2	1,125773E+12	82,649	0,000000
TVCP14	2,254444E+12	2	1,127222E+12	82,756	0,000000
TVCP21	1,939258E+14	2	9,696291E+13	7118,608	0,000000
TVCP22	1,954356E+14	2	9,771778E+13	7174,028	0,000000
TVCP23	1,971173E+14	2	9,855866E+13	7235,762	0,000000
TVCP24	1,975381E+14	2	9,876907E+13	7251,209	0,000000
TVCP31	1,270388E+14	2	6,351941E+13	4663,327	0,000000
TVCP32	1,280918E+14	2	6,404591E+13	4701,981	0,000000
TVCP33	1,291391E+14	2	6,456953E+13	4740,423	0,000000
TVCP34	1,292715E+14	2	6,463576E+13	4745,285	0,000000
SigmaPx11*SigmaPx12	8,360528E+11	4	2,090132E+11	15,345	0,000000
SigmaPx11*SigmaPx13	3,354238E+12	4	8,385594E+11	61,563	0,000000
SigmaPx11*SigmaPx14	3,577755E+12	4	8,944386E+11	65,666	0,000000
SigmaPx12*SigmaPx13	3,744140E+12	4	9,360351E+11	68,720	0,000000
SigmaPx12*SigmaPx14	3,547506E+12	4	8,868766E+11	65,111	0,000000
SigmaPx12*SigmaPx22	1,060791E+12	4	2,651978E+11	19,470	0,000000
SigmaPx12*SigmaPx23	2,542272E+12	4	6,355680E+11	46,661	0,000000
SigmaPx12*SigmaPx24	2,774874E+12	4	6,937185E+11	50,930	0,000000
SigmaPx12*SigmaPx33	1,968662E+12	4	4,921654E+11	36,133	0,000000
SigmaPx12*SigmaPx34	1,624365E+12	4	4,060913E+11	29,814	0,000000
SigmaPx13*SigmaPx14	3,967880E+12	4	9,919701E+11	72,826	0,000000
SigmaPx13*SigmaPx23	3,861941E+12	4	9,654851E+11	70,882	0,000000
SigmaPx13*SigmaPx24	3,195339E+12	4	7,988348E+11	58,647	0,000000
SigmaPx13*SigmaPx33	2,365935E+12	4	5,914837E+11	43,424	0,000000
SigmaPx13*SigmaPx34	2,444377E+12	4	6,110942E+11	44,864	0,000000
SigmaPx14*SigmaPx23	2,414374E+12	4	6,035935E+11	44,313	0,000000
SigmaPx14*SigmaPx24	4,526933E+12	4	1,131733E+12	83,087	0,000000
SigmaPx14*SigmaPx33	3,781268E+12	4	9,453169E+11	69,401	0,000000
SigmaPx14*SigmaPx34	2,624814E+12	4	6,562036E+11	48,176	0,000000
SigmaPx21*SigmaPx23	9,379798E+11	4	2,344949E+11	17,216	0,000000
SigmaPx21*SigmaPx24	1,101858E+12	4	2,754646E+11	20,223	0,000000
SigmaPx22*SigmaPx23	3,968161E+12	4	9,920402E+11	72,831	0,000000
SigmaPx22*SigmaPx24	4,320802E+12	4	1,080200E+12	79,304	0,000000
SigmaPx23*SigmaPx24	3,732586E+12	4	9,331465E+11	68,508	0,000000
SigmaPx23*SigmaPx33	2,394505E+12	4	5,986262E+11	43,949	0,000000
SigmaPx23*SigmaPx34	2,300204E+12	4	5,750510E+11	42,218	0,000000
SigmaPx24*SigmaPx33	5,332539E+12	4	1,333135E+12	97,873	0,000000
SigmaPx24*SigmaPx34	2,412381E+12	4	6,030953E+11	44,277	0,000000
SigmaPx32*SigmaPx33	3,114548E+12	4	7,786370E+11	57,164	0,000000

SigmaPx32*SigmaPx34	2,664497E+12	4	6,661243E+11	48,904	0,000000
SigmaPx33*SigmaPx34	2,312267E+12	4	5,780667E+11	42,439	0,000000
SigmaPx11*Mu-T1	2,854984E+12	4	7,137460E+11	52,400	0,000000
SigmaPx12*Mu-T1	5,062355E+12	4	1,265589E+12	92,914	0,000000
SigmaPx12*Mu-T2	7,116264E+12	4	1,779066E+12	130,612	0,000000
SigmaPx13*Mu-T3	1,212925E+12	4	3,032312E+11	22,262	0,000000
SigmaPx14*Mu-T1	5,550660E+11	4	1,387665E+11	10,188	0,000000
SigmaPx14*Mu-T2	1,381184E+12	4	3,452961E+11	25,350	0,000000
SigmaPx13*Mu-T1	5,409951E+11	4	1,352488E+11	9,929	0,000000
SigmaPx14*Mu-T4	2,535449E+12	4	6,338624E+11	46,536	0,000000
SigmaPx21*Mu-T1	1,190173E+12	4	2,975432E+11	21,844	0,000000
SigmaPx22*Mu-T1	1,435557E+12	4	3,588892E+11	26,348	0,000000
SigmaPx23*Mu-T1	8,924648E+11	4	2,231162E+11	16,380	0,000000
SigmaPx23*Mu-T2	1,337100E+12	4	3,342750E+11	24,541	0,000000
SigmaPx23*Mu-T3	2,931212E+12	4	7,328029E+11	53,799	0,000000
SigmaPx32*Mu-T1	1,816208E+12	4	4,540519E+11	33,335	0,000000
SigmaPx33*Mu-T2	1,363968E+12	4	3,409921E+11	25,034	0,000000
SigmaPx33*Mu-T1	9,257863E+11	4	2,314466E+11	16,992	0,000000
SigmaPx33*Mu-T3	3,622900E+12	4	9,057250E+11	66,495	0,000000
SigmaPx34*Mu-T1	5,135543E+11	4	1,283886E+11	9,426	0,000000
SigmaPx34*Mu-T2	9,531981E+11	4	2,382995E+11	17,495	0,000000
SigmaPx34*Mu-T4	5,033867E+12	4	1,258467E+12	92,391	0,000000
SigmaPx11*LC11	9,155803E+12	4	2,288951E+12	168,045	0,000000
SigmaPx11*LC12	8,622515E+11	4	2,155629E+11	15,826	0,000000
SigmaPx11*LC13	8,600999E+11	4	2,150250E+11	15,786	0,000000
SigmaPx11*LC14	8,932493E+11	4	2,233123E+11	16,395	0,000000
SigmaPx12*LC11	3,670264E+11	4	9,175661E+10	6,736	0,000021
SigmaPx12*LC12	2,893498E+12	4	7,233745E+11	53,107	0,000000
SigmaPx12*LC13	1,835899E+12	4	4,589748E+11	33,696	0,000000
SigmaPx12*LC14	3,733209E+12	4	9,333023E+11	68,519	0,000000
SigmaPx12*LC23	4,477098E+11	4	1,119274E+11	8,217	0,000001
SigmaPx12*LC24	1,049288E+12	4	2,623220E+11	19,259	0,000000
SigmaPx13*LC11	1,310426E+12	4	3,276064E+11	24,051	0,000000
SigmaPx13*LC12	2,788988E+12	4	6,972471E+11	51,189	0,000000
SigmaPx13*LC13	7,216568E+12	4	1,804142E+12	132,453	0,000000
SigmaPx13*LC14	3,312732E+12	4	8,281830E+11	60,802	0,000000
SigmaPx13*LC23	7,116372E+11	4	1,779093E+11	13,061	0,000000
SigmaPx13*LC24	7,183834E+11	4	1,795958E+11	13,185	0,000000
SigmaPx14*LC11	1,556432E+12	4	3,891081E+11	28,567	0,000000
SigmaPx14*LC12	3,332319E+12	4	8,330797E+11	61,161	0,000000
SigmaPx14*LC13	3,056250E+12	4	7,640624E+11	56,094	0,000000

SigmaPx14*LC14	4,672473E+12	4	1,168118E+12	85,758	0,000000
SigmaPx14*LC23	8,410147E+11	4	2,102537E+11	15,436	0,000000
SigmaPx14*LC24	4,764104E+11	4	1,191026E+11	8,744	0,000001
SigmaPx14*LC33	3,589114E+11	4	8,972785E+10	6,587	0,000028
SigmaPx21*LC21	1,039732E+13	4	2,599330E+12	190,832	0,000000
SigmaPx22*LC22	4,166159E+12	4	1,041540E+12	76,465	0,000000
SigmaPx22*LC23	1,575844E+12	4	3,939610E+11	28,923	0,000000
SigmaPx22*LC24	3,639123E+12	4	9,097808E+11	66,792	0,000000
SigmaPx23*LC13	1,104989E+12	4	2,762474E+11	20,281	0,000000
SigmaPx23*LC14	1,101589E+12	4	2,753973E+11	20,219	0,000000
SigmaPx23*LC22	2,098944E+12	4	5,247360E+11	38,524	0,000000
SigmaPx23*LC23	6,185582E+12	4	1,546396E+12	113,530	0,000000
SigmaPx23*LC24	3,357232E+12	4	8,393079E+11	61,618	0,000000
SigmaPx24*LC13	1,548766E+12	4	3,871916E+11	28,426	0,000000
SigmaPx24*LC14	5,469159E+11	4	1,367290E+11	10,038	0,000000
SigmaPx24*LC22	2,489657E+12	4	6,224144E+11	45,695	0,000000
SigmaPx24*LC23	1,478673E+12	4	3,696683E+11	27,139	0,000000
SigmaPx24*LC24	5,059926E+12	4	1,264981E+12	92,870	0,000000
SigmaPx24*LC33	4,557705E+11	4	1,139426E+11	8,365	0,000001
SigmaPx31*LC31	6,074728E+12	4	1,518682E+12	111,495	0,000000
SigmaPx32*LC32	2,668025E+12	4	6,670062E+11	48,969	0,000000
SigmaPx32*LC33	5,016004E+11	4	1,254001E+11	9,206	0,000000
SigmaPx32*LC34	1,557019E+12	4	3,892549E+11	28,577	0,000000
SigmaPx33*LC13	4,501351E+11	4	1,125338E+11	8,262	0,000001
SigmaPx33*LC14	6,612619E+11	4	1,653155E+11	12,137	0,000000
SigmaPx33*LC24	3,748992E+11	4	9,372480E+10	6,881	0,000016
SigmaPx33*LC32	8,343552E+11	4	2,085888E+11	15,314	0,000000
SigmaPx33*LC33	3,261245E+12	4	8,153112E+11	59,857	0,000000
SigmaPx33*LC34	1,389583E+12	4	3,473958E+11	25,504	0,000000
SigmaPx34*LC13	1,291579E+12	4	3,228947E+11	23,706	0,000000
SigmaPx34*LC14	3,645923E+11	4	9,114808E+10	6,692	0,000023
SigmaPx34*LC23	8,472120E+11	4	2,118030E+11	15,550	0,000000
SigmaPx34*LC32	5,982666E+11	4	1,495666E+11	10,981	0,000000
SigmaPx34*LC34	2,455835E+12	4	6,139587E+11	45,074	0,000000
SigmaPx13*TVCP11	7,984397E+11	4	1,996099E+11	14,655	0,000000
SigmaPx13*TVCP12	8,037609E+11	4	2,009402E+11	14,752	0,000000
SigmaPx13*TVCP13	7,873002E+11	4	1,968250E+11	14,450	0,000000
SigmaPx13*TVCP14	8,098621E+11	4	2,024655E+11	14,864	0,000000
SigmaPx14*TVCP11	9,654704E+11	4	2,413676E+11	17,720	0,000000
SigmaPx14*TVCP12	9,772681E+11	4	2,443170E+11	17,937	0,000000
SigmaPx14*TVCP13	9,715420E+11	4	2,428855E+11	17,832	0,000000

SigmaPx14*TVCP14	9,470358E+11	4	2,367589E+11	17,382	0,000000
Mu-T1*Mu-T2	6,279997E+12	4	1,569999E+12	115,263	0,000000
Mu-T3*SigmaM3	1,148253E+12	4	2,870633E+11	21,075	0,000000
Mu-T4*SigmaM4	1,187619E+12	4	2,969047E+11	21,797	0,000000
Mu-T1*LC11	3,982910E+12	4	9,957276E+11	73,102	0,000000
Mu-T1*LC12	9,906197E+11	4	2,476549E+11	18,182	0,000000
Mu-T1*LC14	6,072938E+11	4	1,518234E+11	11,146	0,000000
Mu-T1*LC21	2,002839E+12	4	5,007097E+11	36,760	0,000000
Mu-T1*LC22	1,359586E+12	4	3,398966E+11	24,954	0,000000
Mu-T1*LC24	8,618728E+11	4	2,154682E+11	15,819	0,000000
Mu-T1*LC31	7,044817E+11	4	1,761204E+11	12,930	0,000000
Mu-T2*LC12	1,941159E+12	4	4,852896E+11	35,628	0,000000
Mu-T2*LC14	1,419629E+12	4	3,549073E+11	26,056	0,000000
Mu-T2*LC24	1,556746E+12	4	3,891865E+11	28,572	0,000000
Mu-T2*LC34	6,444114E+11	4	1,611029E+11	11,827	0,000000
LC12*LC13	1,815641E+12	4	4,539102E+11	33,324	0,000000
LC12*LC14	3,070790E+12	4	7,676975E+11	56,361	0,000000
LC13*LC24	4,705282E+11	4	1,176321E+11	8,636	0,000001
LC14*LC23	6,445310E+11	4	1,611327E+11	11,830	0,000000
LC22*LC24	1,985463E+12	4	4,963657E+11	36,441	0,000000
Error	4,320597E+13	3172	1,362105E+10		