INTERFERENCE ALIGNMENT ON MULTIGROUP MULTI WAY RELAY CHANNELS

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ABSTRACT

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Interference is a major problem for reliable multiuser communications. There are several ways to manage interference which are applied according to the power of interference compared to the power of desired signal. If interference is powerful than the desired signal then interference signal is decoded. Otherwise if it is much weaker than the desired signal it may be considered as noise. If signal powers of the desired signal and the interference are comparable then orthogonalizing all the signals from all the users can be a solution. By orthogonalizing user signals, capacity is shared in between users.

Interference alignment is a relatively new technique used for interference cancellation in multiuser networks. It is based on aligning unwanted signals and desired signals into distinct and orthogonal subspaces. Theoretically, in a K user interference channel, every user can use half of the sum capacity by applying interference alignment.

In this work, we consider interference alignment on multi group multi way relaying channels. Each node in each group is interested in the data streams transmitted by other nodes in the same group. Data streams of the other groups are considered as interference. We develop an interference alignment solution for this scenario, by aligning M nodes in each group into the same subspace at the relay. Therefore the number of antennas needed at the relay are reduced by an order of M. The price paid for this is the increase in the number of time slots needed for the communication between nodes.

Keywords: Interference Alignment, Multiway Relay Channel, Multi Group Multi Way Relaying

ÖZ

ÇOK GRUPLU ÇOK YÖNLÜ RÖLE KANALLARINDA GİRİŞİM HİZALAMA

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Girişim çok kullanıcılı haberleşme sistemlerinde güvenilir haberleşme için majör bir sorundur. Girişimi yönetmek için, istenen sinyal ve girişim sinyalinin gücüne göre uygulanabilecek birkaç yöntem vardır. Eğer girişim sinyalinin gücü, istenen sinyalin gücünden daha yüksek ise, girişim sinyali çözülür. Yada girişim sinyalinin gücü istenen sinyalin gücüne göre çok düşükse gürültü olarak kabul edilebilir. İstenen sinyalin gücü ile girişim sinyalinin gücü birbirine yakın ise tüm kullanıcıların sinyalleri ortogonalize edilmesi bir çözüm olabilir. Kullanıcıların ortogonalizasyonu kapasitenin kullanıcılar arasında paylaşılmasına yol açacaktır.

Girişim Hizalama yöntemi, girişim yok etme için kullanılan nispeten yeni bir yöntemdir. Girişimin ve istenen sinyallerin birbirinden ayrı ve ortogonal alt uzaylara hizalanmasına dayanır. Teorik olarak, girişim hizalanma yönteminin uygulanmasıyla K kullanıcılı girişim kanallarında, her kullanıcı toplam kapasitenin yarısını kullanabilir.

Bu çalışmada çok gruplu çok yönlü röle kanallarında girişim hizalama yöntemi ele alınmıştır. Her grupta yer alan her kullanıcı, kendi gurubundaki diğer tüm kullanıcıların verisiyle ilgilenir. Diğer gruplardaki kullanıcıların verileri girişim olarak değerlendirilir. Her gruptaki M kullanıcının sinyalini rölede aynı alt uzaya

hizalayarak, bu senaryo için bir girişim hizalama yöntemi geliştirildi. Bu şekilde rölede ihtiyaç duyulan anten sayısı M oranında azaltıldı, ancak buna karşılık kullanıcılar arasındaki tüm haberleşmenin sağlanması için gerekli toplam zaman dilimi sayısı arttı.

Anahtar Kelimeler: Girişim Hizalama, Çok Yönlü Röle, Çok Gruplu Çok Yönlü Röle Kanalı

To my family..

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LIST OF ABBREVIATIONS

BC : Broadcast

CDMA: Code Division Multiple Access

DoF : Degrees of Freedom

FDMA: Frequency Division Multiple Access

GSA : Group Signal Alignment

GCA : Group Channel Alignment

IA : Interference Alignment

MC : Multicast

MAC : Multiple Access

SNR : Signal to Noise Ratio

TDMA: Time Division Multiple Access

CHAPTER 1

INTRODUCTION

1.1. Preliminaries

There are two main capacity limiting factor of a single user point to point communication system to decrease its capacity. One of them is signal to noise ratio (SNR), the other one is bandwidth. If these two parameters are determined, then capacity of the related channel is specified by the channel capacity theorem of Shannon.

Capacity of multiuser communications systems are also limited by SNR and bandwidth like point to point systems, however interference also plays an important role in limiting capacity of multiuser systems. Number of mobile devices requiring internet connection increase day by day. These devices can connect internet through local area connections or cellular networks. In order to support more users in a dense environment, cellular cells split into smaller cells. In return, intercell interference increases and throughput of the cell reduces due to intercell interference. Local area networks suffer from interference as the number of access points increase which use same frequency [4].

Traditionally, interference is handled based on interference power compared to desired signal power. This can be summarized as follows [1]:

- Treat as Noise: If the power of interference signal is weak compared to desired signal then interference can be considered as noise and signal detection and decoding can be interpreted under this assumption. Although the power of interference is weak, it is still not noise in nature, and therefore this leads to suboptimal solution.

- Decode: If the power of interference signal is strong compared to desired signal, decoding of interference signal may be preferred. This way rate of desired signal may increase at the cost of reducing other users rate. Moreover it is very complicated when number of users is more than 2.
- Orthogonalization: When the power of interference and desired signal are compared to each other, then orthogonalization of the users channel access is preferred generally. This way every user can communicate interference free, however by orthogonalizing, total sum capacity is shared in between users and as the number of users increase data rate per user will decrease accordingly.

Time Division Multiple Access (TDMA) and Frequency Division Multiple Access (FDMA) can be considered as most common orthogonalizing schemes. While TDMA relies on letting each user transmit distinct time slots, FDMA relies on letting each user at a distinct frequency. Spread spectrum systems such as Code Division Multiple Access (CDMA) is also another technique used to eliminate interference, in which interference is averaged out [5].

Although less efficient compared to orthogonalization, medium access protocol is also used to cope with interference. No matter which scheme is used, total sum capacity of channel is shared in between users [5].

Interference Alignment is a relatively new technique which aims to align interference into one of the signal dimensions which is most of the time antennas, but can also be time slots or frequency blocks. By aligning interference into one of the signal dimensions, desired signals can be resolved using rest of the signal dimensions which are interference free [3].

1.2. Motivation

Multiway relay channels are defined as multiple groups of nodes and multiple nodes in each group, where nodes in a group willing to share its data with other nodes in the same group under the assistance of a relay [6]. Many practical systems can be modeled as multiway relay channel. Social networking scenario where a group can be considered as a friend group willing to share its information with other friends in the same group can be an example. A satellite communication system can be another example. Local information of one node could be wanted to be streamed other nodes in the same group via satellite and since a single satellite would serve a large area, it also would need to serve more than one group [6].

Connecting the nodes in these examples without a relay may not be practical most of the time because of large area coverage requirement. Moreover, relays will be useful in case of power limited nodes and against channel variations.

In multiway relay channels, data stream of nodes which are in a different group are considered as interference for the receiving node. Therefore interference cancellation schemes need to be incorporated into the system, enabling interference free reliable communication.

Relaying schemes for multiway relay systems which have multiple (L) groups and multiple (K) nodes in each group are designed in [10] and [11] where interference alignment is not applied. In [10], a non-regenerative relaying scheme which aims sum rate maximization by making use of several beamforming methods such as zero forcing and minimum mean square error beamforming for broadcasting is presented with $R \geq LK$ antennas needed at the relay. In [11], a half-duplex relaying scheme is presented where network coding is applied for interference cancellation with $R \geq LKd - d$ antennas needed at the relay. Recently interference alignment algorithms are designed for interference cancellation purposes in multiway relay systems for generic scenarios such as L groups, K nodes in each group and each node streams “d” data streams to other nodes. In [8] an upper bound for DoF for L groups and K users in each group is found. In [14] a MIMO Y Channel in which signal space alignment and network coding used together to cancel self-interference at receiving nodes. In [13], an interference alignment algorithm for multi group multiway relay channel for any number of groups but only 2 users in each group is presented. This can also be considered as a K-user interference channel in which node pairs communicate with each other not directly but with assistance of a relay. In [7], an interference alignment scheme for a Multigroup Multiway Relay Channel for any number of groups and any number of nodes is presented where nodes send their information to relay in a single MAC phase, and broadcast the data in several broadcast phases. Each node needs to know CSI of the nodes in the same group. In [7] by aligning $K * d$ data streams into $(K - 1) * d$ dimensional subspace at relay, number of relay antennas is reduced by an amount of $L * d$ compared to algorithms which spatially decodes nodes. A multicast interference alignment algorithm for multigroup multiway relay channels is presented in [9]. Transmission scheme in [9] consists of multiple MAC and multiple MC (multicast) phases. At MAC phase, only a portion of the nodes inside a group transmits. In order to avoid self-interference at the nodes, MC phases are designed such that one of the nodes data is not streamed at a specific MC phase. To achieve this, relay spatially decodes data of nodes at MAC phase. Relay processing matrix consists of receive zero forcing for MAC phase and transmit beamforming based on an algorithm which finds the eigenvectors corresponding to minimum eigenvalues of the interference subspace at the nodes. In [9], every node needs to know global CSI. In [17], an interference alignment based solution is given for the communication of K pair of nodes where there are multiple relays and some of the nodes may be connected to more than one of the relays. In [18], K pair of nodes communicates through a relay, where signal spaces of the pairs are aligned at the relay only partially by utilizing the extra antennas at the relay. In [19], an interference alignment solution and an iterative minimum mean square error (MMSE) based solution, and a comparison of these methods for the relay aided communication of K pair of nodes for multiple relay case is presented. In [20], a similar problem to the one given in [17] is investigated. There are multiple groups and each have multiple nodes, and a whole group is connected to more than a single relay, compared to the multi pair system given in [17]. An interference alignment solution is presented to enable the nodes in the same group to exchange their data stream through the relays.

1.3. Outline of the Thesis

In section 2, we present K-user interference alignment scheme which shows asymptotically every user can have half of the total capacity as in [1] and [2]. In section 3, we present interference alignment scheme on multigroup multiway relay channels described in [7] and present some extensions on system parameters of the system. In section 4, we propose an interference alignment scheme based on aligning some fraction of nodes (subgroup) in the same group into same subspace at the relay.

CHAPTER 2

INTERFERENCE ALIGNMENT

2.1. Interference Channel

Interference channel is the abstract system model in which multiple transmitters aim to send its data to its pair receiver and all the receivers in the channel hear not only its pairs' signals but also other transmitters' signals. Therefore, each receiver is exposed to other transmitters' signals which are actually interference, since each receiving node is willing to communicate only with its partner.

2.2. Degrees of Freedom

Degrees of Freedom (DoF) is a fundamental concept while calculating capacity of wireless channels theoretically. It is very commonly used in interference alignment solutions, therefore we present a mathematical explanation of DoF here as given in [2].

A rate tuple of $(R_1, R_2, \ldots, R_m)$ is considered as achievable, if for long enough codewords, m independent messages $W_1, W_2, \ldots, W_m$ can be decoded with almost zero probability of error. And the capacity region is the closure of $(R_1, R_2, \ldots, R_m)$. Like SISO capacity, capacity region for wireless networks is dependent on noise power and received signal power. Therefore, capacity region is dependent on signal power, channel coefficient from transmitter to receiver and noise power. DoF is a tool defined to understand capacity under the assumption that power is infinite while other parameters unchanged.

If total transmit power is P then the DoF is defined as follows:

$$\eta = \lim_{P \to \infty} \frac{C(P)}{log(P)} \qquad (2.1)$$

This equation can also be written as follows:

$$C(P) = \eta\ log(P) + o\big(log(P)\big) \tag{2.2}$$

Here $o\big(log(P)\big)$ is some function f such that

$$\lim_{\rho \to \infty} \frac{f(\rho)}{log(\rho)} = 0 \tag{2.3}$$

For a SISO channel, when the power is infinite, capacity is also infinite. However, for interference channel, although power is infinite, capacity still will be limited due to interference. DoF is the metric which indicates the interference free signal dimensions in the system, and therefore it is an indicator of the capacity as it is seen from above equations.

2.3. Interference Alignment for 3-User Interference Channel

The main idea of interference alignment is aligning all the interference signals for a receiver into one of the signal dimensions and desired signals into another signal dimension. Since all the transmitters and receivers have only single antenna, symbol extensions is used in order to provide signal dimension for interference and desired signals to be aligned into distinct subspaces.

For 3 – user interference channel, $3n + 1$ degrees of freedom can be obtained by $2n + 1$ of symbol extension for the channel. Therefore for each symbol $\frac{3n+1}{2n+1}$ degrees of freedom can be obtained. Apparently as n increases, degrees of freedom per symbol will approach $3/2$.

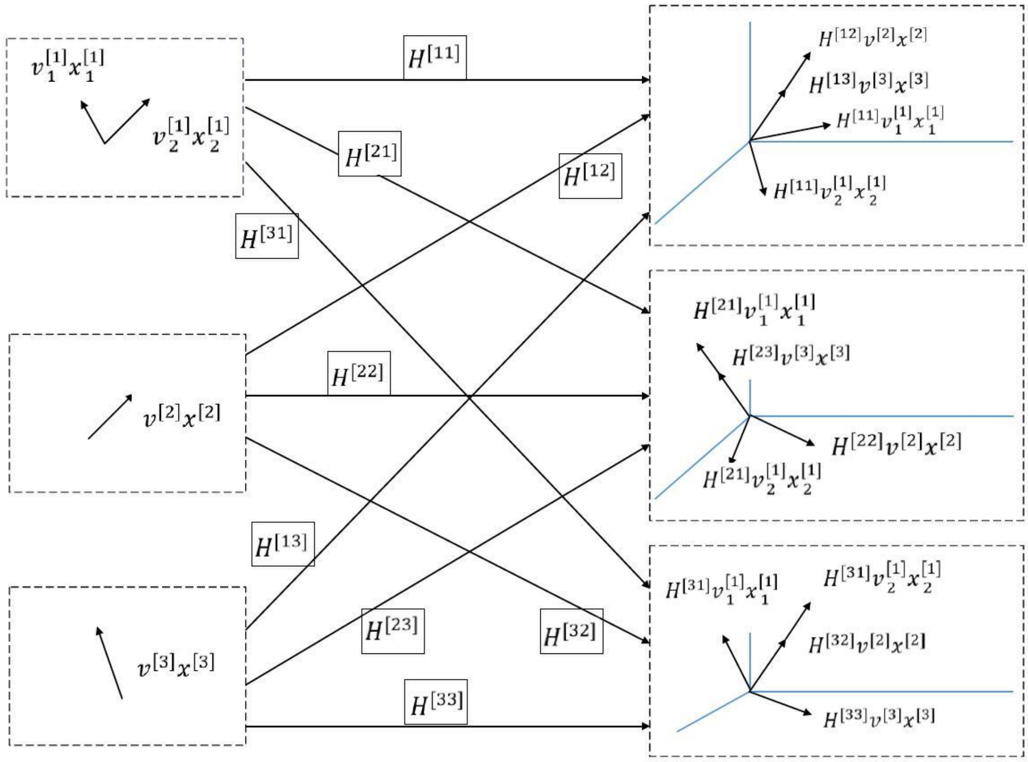

Figure 2.1. Interference Alignment for 3-User Interference Channel

In Figure 2.1 interference alignment scheme for 3-user interference channel with 1 symbol extension can be seen. Letting $n = 1$ we will have $2n + 1 = 3$ symbol extensions for the channel and therefore $4/3$ degrees of freedom will be obtained. User 1 transmit same data along two independent beams which are obtained using the beamforming vectors $\boldsymbol{v}_{\boldsymbol{1}}^{[1]}$ and $\boldsymbol{v}_{\boldsymbol{2}}^{[1]}$ and user 2 and 3 transmit their data along a single beam which are obtained using the beamforming vectors $\boldsymbol{v}^{[2]}$ and $\boldsymbol{v}^{[3]}$. Hence user 1 has 2 DoF and users 2&3 has 1 DoF. Based on this, interference alignment solution is configured as follows:

- Let $\boldsymbol{v}^{[2]} = [1\ 1\ 1]^T$
- At receiver 1, interference from user 2 and 3 needs to be aligned. This requires the following condition to be satisfied:

$$\boldsymbol{H}^{[12]}\boldsymbol{v}^{[2]} = \boldsymbol{H}^{[13]}\boldsymbol{v}^{[3]} \tag{2.4}$$

and therefore,

$$\boldsymbol{v}^{[3]} = \boldsymbol{H}^{[13]^{-1}} \boldsymbol{H}^{[12]} \boldsymbol{v}^{[2]} \tag{2.5}$$

- Similarly at receiver 2, interference from user 1 and user 3 needs to be aligned. Since user 1 transmits along two independent beamforming vectors, only one of them is determined here by aligning with user 3. Alignment condition is as follows:

$$\boldsymbol{H}^{[23]} \boldsymbol{v}^{[3]} = \boldsymbol{H}^{[21]} \boldsymbol{v}_{\mathbf{1}}^{[1]} \tag{2.6}$$

and therefore,

$$\boldsymbol{v}_{\mathbf{1}}^{[1]} = \boldsymbol{H}^{[21]^{-1}} \boldsymbol{H}^{[23]} \boldsymbol{H}^{[13]^{-1}} \boldsymbol{H}^{[12]} \boldsymbol{v}^{[2]} \tag{2.7}$$

- Likewise at receiver 3, interference from user 1 and user 2 needs to be aligned. This time, user 2 will be aligned with 2nd beam of user 1. So the alignment condition is as follows:

$$\boldsymbol{H}^{[32]} \boldsymbol{v}^{[2]} = \boldsymbol{H}^{[31]} \boldsymbol{v}_{\mathbf{2}}^{[1]} \tag{2.8}$$

and therefore,

$$\boldsymbol{v}_{\mathbf{2}}^{[1]} = \boldsymbol{H}^{[31]^{-1}} \boldsymbol{H}^{[32]} \boldsymbol{v}^{[2]} \tag{2.9}$$

2.4. Asymptotic Interference Alignment for K-User Interference Channel

K user interference channel consist of K transmitters and K receivers. Each receiver is only interested in its pairs' signal and all transmitters and receivers have only one antenna. Signal received by receiver $k \in \{1,2,..,K\}$ at time slot $t \in N$ is given as follows:

$$\begin{aligned} Y^{[k]}(t) = H^{[k1]}(t)X^{[1]}(t) + H^{[k2]}(t)X^{[2]}(t) + \cdots \\ + H^{[kK]}(t)X^{[K]}(t) \end{aligned} \tag{2.10}$$

where $X^{[k]}(t)$ is k^{th} transmitters' message signal to be transmitted, $H^{[kj]}(t)$ is the channel coefficient from transmitter j to receiver k at time slot t and $Z^{[k]}(t)$ is white Gaussian noise input to receiver k at time slot t. It is assumed channel coefficients are i.i.d and drawn from a continuous distribution.

As the number of users increase, number of constraints at each receiver to be satisfied increases also. This problem is overcome in [1] asymptotically, by constructing a finite cardinality set which is almost invariant to an arbitrarily large number of linear transformations. We go through the solution as given in [1] and [2].

Suppose we have N linear transformations as $\boldsymbol{T}_1, \boldsymbol{T}_2, \dots, \boldsymbol{T}_N$ which will apply on set $\boldsymbol{V}$ elementwise. So for the set $\boldsymbol{V}$ output of applying linear transformation $\boldsymbol{T}_i$ is as follows:

$$\boldsymbol{T}_i\boldsymbol{V} = \left\{\boldsymbol{T}_i v_1, \boldsymbol{T}_i v_2, \dots, \boldsymbol{T}_i v_{\tau|\boldsymbol{V}|}\right\} \tag{2.11}$$

Another assumption made about linear transformations $\boldsymbol{T}_i$ is that they are commutative, i.e.

$$\boldsymbol{T}_i\boldsymbol{T}_j v_k = \boldsymbol{T}_j\boldsymbol{T}_i v_k \tag{2.12}$$

Main purpose was to construct a set invariant to arbitrarily large number of transformations. This means union of sets which are obtained by application of $\boldsymbol{T}_i$ on set $\boldsymbol{V}$ and set $\boldsymbol{V}$ itself will not have a greater cardinality then set $\boldsymbol{V}$, almost. Mathematically this situation can be expressed as follows:

$$\frac{|I|}{|V|} \to 1 \tag{2.13}$$

where

$$I \triangleq \boldsymbol{V} \cup \boldsymbol{T}_1\boldsymbol{V} \cup \boldsymbol{T}_2\boldsymbol{V} \cup \dots \cup \boldsymbol{T}_N\boldsymbol{V} \tag{2.14}$$

which means applying transformations $\boldsymbol{T}_i$ on set $\boldsymbol{V}$ will align with set $\boldsymbol{V}$ again approximately.

In [1] an iterative scheme is presented to construct set $\boldsymbol{V}$ which will satisfy above conditions. A block diagram of this iterative algorithm is shown in Figure 2.2.

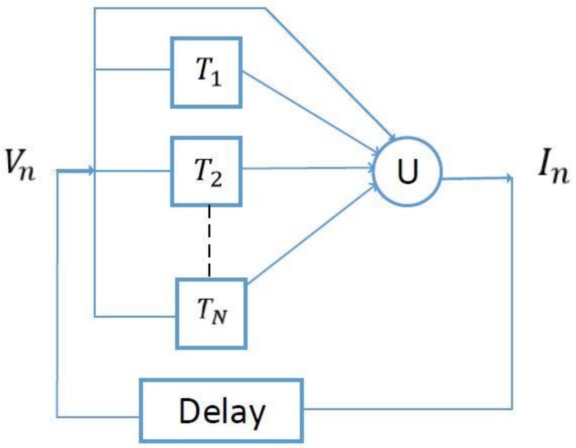

Figure 2.2. Iterative construction of set **V**

Starting with $\boldsymbol{V_1} = 1$ and after first iteration we will have

$$I_1 = \{\mathbf{1}, \boldsymbol{T}_1\mathbf{1}, \boldsymbol{T}_2\mathbf{1}, \dots, \boldsymbol{T}_N\mathbf{1}\} \tag{2.15}$$

Remember that $\frac{|I|}{|V|} \to 1$ needs to be satisfied. At this point $|I| = N + 1$ while $|V| = 1$. Since at each iteration, $\boldsymbol{V_n}$ will get previous version of I_n at 2nd iteration $\boldsymbol{V_2}$ will be set to I_1. So at second iteration we will have the following sets,

$$\boldsymbol{V_2} = \{\mathbf{1}, \boldsymbol{T}_1\mathbf{1}, \boldsymbol{T}_2\mathbf{1}, \dots, \boldsymbol{T}_N\mathbf{1}\} \tag{2.16}$$

$$I_2 = \left\{\mathbf{1}, \dots, \boldsymbol{T}_i\mathbf{1}, \dots, \boldsymbol{T}_i\boldsymbol{T}_j\mathbf{1}, \dots, \boldsymbol{T}_i^2\mathbf{1}\right\} \tag{2.17}$$

But again the condition $\frac{|I|}{|V|} \to 1$ is not satisfied since $|I| = (N + 2) * (N + 1)/2$ and $|V| = N$. At step n, $\boldsymbol{V_n}$ and I_n can be expressed in a more generic way as follows,

$$\boldsymbol{V_n} = \{\, (\boldsymbol{T}_1)^{\alpha_1}(\boldsymbol{T_2})^{\alpha_2} \dots (\boldsymbol{T}_n)^{\alpha_n}\mathbf{1}, \tag{2.18a}$$

$$s.t. \sum_{i=1}^{N} \alpha_i \le n - 1, \tag{2.18b}$$

$$\boldsymbol{\alpha_1}, \dots, \boldsymbol{\alpha_n} \in Z_+ \,\} \tag{2.18.c}$$

and

$$I_n = \{\, (\boldsymbol{T}_1)^{\alpha_1}(\boldsymbol{T_2})^{\alpha_2} \dots (\boldsymbol{T}_n)^{\alpha_n}\mathbf{1}, \tag{2.19a}$$

$$s.t. \sum_{i=1}^{N} \alpha_i \leq n, \tag{2.19b}$$

$$\boldsymbol{\alpha_1}, \dots, \boldsymbol{\alpha_n} \in Z_+ \} \tag{2.19c}$$

All combinations of column vectors satisfying $\sum_{i=1}^{N} \alpha_i \leq n - 1$ constraint will yield combination of $N + n - 1$ taken by N, and so cardinality of $\boldsymbol{V_n}$ is as follows:

$$|\boldsymbol{V_n}| = \binom{N + n - 1}{N} \tag{2.20}$$

and likewise for I_n,

$$|I_n| = \binom{N + n}{N} \tag{2.21}$$

By this result we see that required condition for interference alignment to be successful is satisfied,

$$\frac{|I|}{|V|} = \frac{n + N}{n} \rightarrow 1 \; as \; n \rightarrow \infty \tag{2.22}$$

This result says that, using this scheme one can construct a set which is invariant to an arbitrarily large number of commutative linear transformations.

In a K-user interference channel, every user wants to decode only one of the K transmitting users' data, therefore rest of the K-1 user signals are interference for that node. Considering this for each receiving node there will be K*(K-1) interference channels to be aligned into the same subspace throughout the system. An illustration of interference channels can be seen in Figure 2.3. Aligning N = K*(K-1) channels into same subspace is solved by the above algorithm, so interference is aligned asymptotically. Receiving node can only decode desired signal if this aligned interference subspace is independent of desired signal subspace. Since interference alignment achieved, the only requirement needs to be shown is interference and signal subspaces are distinct.

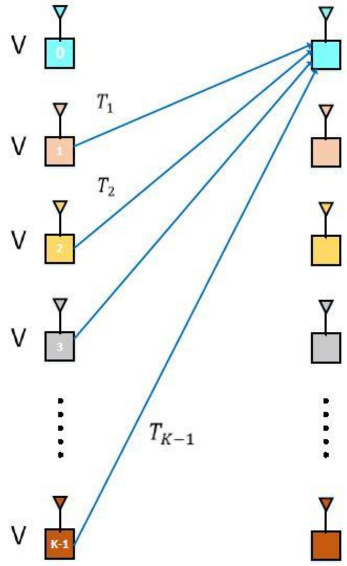

Figure 2.3. Interference which is input to Receiver 0

At this point it can be seen that each user precode their signals into the signal space $\boldsymbol{V}$ which also the space interference are aligned. So how will the desired signals can be separable from interference signals? Since $\boldsymbol{H}^{\boldsymbol{kk}}$ transformations are not used for constructing I (which satisfies $\approx I$) $\boldsymbol{H}^{\boldsymbol{kk}} * \boldsymbol{V}$ spans a distinct space at the nodes since $\boldsymbol{H}^{\boldsymbol{kk}}$ is a generic transformation. As long as total space at the nodes is large enough to contain both the interference and signal space, they will not overlap, hence signal and interference will be separable at the nodes. So the total space size available at the nodes needs to satisfy following condition:

$$S = |\boldsymbol{V}| + |I| \qquad (2.23)$$

If the equation (2.23) is satisfied then,

$$span\left(\boldsymbol{H}^{\boldsymbol{kk}} * \boldsymbol{V}\right) \cap span(\boldsymbol{V}) = \{0\} \qquad (2.24)$$

$$\frac{\left|\boldsymbol{H}^{\boldsymbol{kk}} * \boldsymbol{V}\right|}{S} = \frac{|\boldsymbol{V}|}{|\boldsymbol{V}| + |I|} \rightarrow \frac{1}{2} \qquad as\; n \rightarrow \infty \qquad (2.24)$$

So as we see, signal space size for every receiving node is half of the total signal space size which means as claimed every user can have half of the total capacity.

CHAPTER 3

INTERFERENCE ALIGNMENT ON MULTIGROUP MULTIWAY RELAY CHANNELS

3.1. Introduction

In a multigroup multiway relay channel, there are L groups in the system and there are K nodes in each group. In the assistance of a relay, each node tries to send its data stream to all the other nodes in its own group. Data streams of other groups' nodes are not aimed to be decoded and they are considered as intergroup interference.

A relay assisted interference alignment scheme is suggested as a solution to this problem in [7]. We go through the complete solution given in [7] and present some considerations about constraints of system parameters given in the reference. Next, we extend the given solution for asymmetric number of nodes across the groups and propose a method to exploit this situation to improve the capacity of the nodes in the group with less number of nodes compared to other groups. Finally, we analyze the behavior of the given algorithm when the nodes have rank deficient channel matrices and propose a method to enable the system keep working under this condition. Moreover, we present the sum rate performance of the method proposed for rank deficient channel matrix case.

3.2. System Model

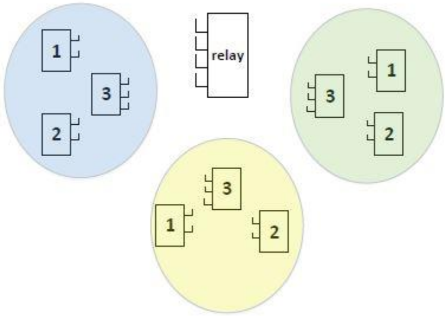

Figure 3.1. Multigroup Multiway Relay Channel Model Example – 3 Groups and 3 Nodes

We summarize the main parameters and assumptions of the system given in [7] as follows. There are L groups, and in each group there are K nodes and each node wants to share d data streams with the nodes in the same group using the assistance of a relay which has R antennas. Each node has N_{lk} antennas. Global Channel State Information is assumed to be known by every node and relay. It is assumed that every node can cancel self interference. There is no direct link between nodes. Since every node can cancel self interference, signals in a group can be mapped on a $(K-1)d$ dimensional subspace at the relay. Therefore, $L(K-1)d$ dimensional subspace is needed at the relay for L groups. So, the relay needs to have

$$R = L(K-1)d \tag{3.1}$$

antennas.

Communication between nodes takes places in one MAC (Multiple Access) phase and $(K-1)$ BC (Broadcast) phases. It is assumed that Global CSI will not change during MAC and BC phases. In the MAC phase, every node sends its signal to relay. In the broadcast phases, relay broadcasts linearly processed received signals. In $(K-1)$ BC phases every node will have $(K-1)$ linearly independent equations and since every node can cancel self interference, each node will be able to solve $(K-1)$ linear equations to obtain $(K-1)d$ dimensional data streams.

$\boldsymbol{d}_{lk}$, denotes the data stream vector to be transmitted to other nodes for node k in group l. $\boldsymbol{V}_{lk}$ denotes the transmit filter matrix. $\boldsymbol{U}_{lk}^{H}$ denotes first stage receive filter matrix. $\boldsymbol{H}_{lk}^{m}$ denotes channel matrix between node and relay at MAC phase and it has a dimension of RxN_{lk}. $\boldsymbol{G}^{p}$ denotes the relay processing matrix applied on the received signal vector at MAC phase, to be transmitted at each BC phase. $\boldsymbol{H}_{lk}^{b}$ denotes channel matrix between relay and node at BC phase and it has a dimension of $N_{lk}xR$. $\boldsymbol{n}_{r}$ denotes the i.i.d. complex Gaussian white noise input to the relay at MAC phase.

Received signal vector after MAC phase at the relay can be expressed as follows:

$$r = \sum_{l=1}^{L} \sum_{k=1}^{K} \boldsymbol{H}_{lk}^{m} \boldsymbol{V}_{lk} \boldsymbol{d}_{lk} + \boldsymbol{n}_{r} \tag{3.2}$$

The received signal vector after p^th BC phase at node (l', k') can be expressed as follows:

$$\begin{aligned} \boldsymbol{y}_{l'k'}^{p} = \boldsymbol{H}_{l'k'}^{b} \boldsymbol{G}^{p} \left(\sum_{\substack{k=1 \\ k \neq k'}}^{K} \boldsymbol{H}_{l'k}^{m} \boldsymbol{V}_{l'k} \boldsymbol{d}_{l'k} + \boldsymbol{H}_{l'k'}^{m} \boldsymbol{V}_{l'k'} \boldsymbol{d}_{l'k'} \right) \\ + \boldsymbol{H}_{l'k'}^{b} \boldsymbol{G}^{p} \sum_{\substack{l=1 \\ l \neq l'}}^{L} \sum_{k=1}^{K} \boldsymbol{H}_{lk}^{m} \boldsymbol{V}_{lk} \boldsymbol{d}_{lk} + \widetilde{\boldsymbol{n}}_{l'k'} \end{aligned} \tag{3.3}$$

In the above equation,

- $\sum_{\substack{k=1 \\ k \neq k'}}^{K} \boldsymbol{H}_{l'k}^{m} \boldsymbol{V}_{l'k} \boldsymbol{d}_{l'k}$ expression is the useful signal for the node (l', k') and it contains data stream of other nodes in the same group.
- $\boldsymbol{H}_{l'k'}^{m} \boldsymbol{V}_{l'k'} \boldsymbol{d}_{l'k'}$ expression is the self interference.
- $\boldsymbol{H}_{l'k'}^{b} \boldsymbol{G}^{p} \sum_{\substack{l=1 \\ l \neq l'}}^{L} \sum_{k=1}^{K} \boldsymbol{H}_{lk}^{m} \boldsymbol{V}_{lk} \boldsymbol{d}_{lk}$ expression is the intergroup interference. Since each node is interested in only the data of the nodes which are in the same group, signals of the nodes in other groups are called intergroup interference.
- $\widetilde{\boldsymbol{n}}_{l'k'}$ denotes the effective noise input to node (l', k'). It is the sum of processed and transmitted relay noise at MAC phase and noise which is directly input to node. It can be expressed as follows:

$$\widetilde{\boldsymbol{n}}_{l'k'} = \boldsymbol{H}^{b}_{l'k'}\boldsymbol{G}^{p}\boldsymbol{n}_{r} + \boldsymbol{n}_{l'k'} \qquad (3.4)$$

Here $\boldsymbol{n}_{l'k'}$ follows CN(0,$\sigma^2_{l'k'}$).

Relay processing matrix and transmit filers are designed as at each node all the useful signals align in a d dimensional subspace and all the intergroup interference are aligned in a $N_{l'k'} - d$ dimensional subspace. This way interference alignment is achieved and hence interference and useful signal are in disjoint subspaces.

First stage receive filter is designed to cancel intergroup interference during each BC phase and its output can be expressed as follows:

$$\begin{aligned} s^{p}_{l'k'} = \boldsymbol{U}^{H}_{l'k'}\boldsymbol{H}^{b}_{l'k'}\boldsymbol{G}^{p} \sum_{\substack{k=1 \\ k\neq k'}}^{K} \boldsymbol{H}^{m}_{l'k}\boldsymbol{V}_{l'k}\boldsymbol{d}_{l'k} \\ + \boldsymbol{U}^{H}_{l'k'}\boldsymbol{H}^{b}_{l'k'}\boldsymbol{G}^{p} \sum_{\substack{l=1 \\ l\neq l'}}^{L}\sum_{k=1}^{K} \boldsymbol{H}^{m}_{lk}\boldsymbol{V}_{lk}\boldsymbol{d}_{lk} + \boldsymbol{U}^{H}_{l'k'}\widetilde{\boldsymbol{n}}_{l'k'} \end{aligned} \qquad (3.5)$$

At Eq.(3.5) one can see intergroup interference is nullified. However, useful signal is still in the equation. If we write these two cases we will have the required conditions to achieve interference alignment at each BC phase at each node. For $\boldsymbol{U}^{H}_{l'k'}$ to nullify intergroup interference following equations needs to be satisfied:

$$\boldsymbol{U}^{H}_{l'k'}\boldsymbol{H}^{b}_{l'k'}\boldsymbol{G}^{p}\boldsymbol{H}^{m}_{lk}\boldsymbol{V}_{lk} = \boldsymbol{0} \; for\ all\ l \neq l' \qquad (3.6)$$

To achieve interference alignment, after applying second stage filter $\boldsymbol{U}^{H}_{l'k'}$ at each node, useful signal needs to lie in a d dimensional subspace, hence, following equation needs to be satisfied:

$$\begin{aligned} rank\left(\boldsymbol{U}^{H}_{l'k'}\boldsymbol{H}^{b}_{l'k'}\boldsymbol{G}^{p}\boldsymbol{H}^{m}_{l'k}\boldsymbol{V}_{l'k}\boldsymbol{d}_{l'k}\right) \; for\ k,k' = 1,..,K \\ and\ l,l' = 1,..,L \end{aligned} \qquad (3.7)$$

Since there are $(K-1)d$ data streams to be resolved at each node, and at each BC phase signal space dimension is only d, $(K-1)$ linearly processed signals received at each BC phase needs to be concatenated and using second stage receive filter,

$(K-1)$ data streams can be resolved. Therefore, estimates of data streams can be expressed as follows:

$$\widehat{\boldsymbol{d}_{lk}} = \boldsymbol{Q}_{l'k'}^{H}\left[\boldsymbol{s}_{l'k'}^{1T} \quad \dots \quad \boldsymbol{s}_{l'k'}^{(K-1)T}\right]^{T} \tag{3.8}$$

In the next section, interference alignment algorithm is described in detail. Design of first and second stage receive filter matrix, transmit filter matrix and relay processing matrix are described.

3.3. Interference Alignment Algorithm

3.3.1. Group Signal Alignment (GSA)

By GSA it is aimed to make Kd data streams to be in $(K-1)d$ dimensional subspace at relay. Although there are Kd data streams broadcasted by the relay for each group, since every node can cancel self interference at each BC phase, $(K-1)d$ dimensional subspace for each group at the relay is adequete.

To achieve GSA, following equation is used to design transmit filter matrices:

$$[\boldsymbol{H}_{l1}^{m} \quad \boldsymbol{H}_{l2}^{m} \quad \dots \quad \boldsymbol{H}_{lK}^{m}]\begin{bmatrix}\boldsymbol{V}_{l1}\\ \boldsymbol{V}_{l2}\\ \vdots \\ \boldsymbol{V}_{lK}\end{bmatrix} = \boldsymbol{0} \tag{3.9}$$

Since every node needs to be in a d dimensional subspace at the relay, $\boldsymbol{V}_{lk}$ needs to be of rank d for all k. To solve the above equation with this constraint, null space dimension of $\boldsymbol{H}_{l}^{m} = [\boldsymbol{H}_{l1}^{m} \quad \boldsymbol{H}_{l2}^{m} \quad \dots \quad \boldsymbol{H}_{lK}^{m}]$ needs to be of d dimensional. To satisfy this condition, following inequality needs to be satisfied:

$$\sum_{k=1}^{K} N_{lk} \geq R + d \qquad l = 1..L \tag{3.10}$$

This can be justified as follows: since $\boldsymbol{H}_{l}^{m}$ is $Rx\sum_{k=1}^{K} N_{lk}$ dimensional and full rank, $\sum_{k=1}^{K} N_{lk}$ needs to larger than $R + d$ for $\boldsymbol{H}_{l}^{m}$ has a d dimensional null space.

3.3.2. Group Channel Alignment (GCA)

GCA problem is very similar to GSA problem. Compared to GCA, this time effective channel from nodes to relay which is expressed as $\boldsymbol{U}_{l'k'}^{H}\boldsymbol{H}_{l'k'}^{b}$ needs to span a d dimensional subspace at the relay and moreover all the effective channels from the relay to all the nodes in a group needs to span a $(K-1)d$ dimensional subspace at the relay. Under these considerations, first stage receive matrix $\boldsymbol{U}_{l'k'}^{H}$ can be designed by solving following equation:

$$\begin{bmatrix}\boldsymbol{U}_{l'1}^{H} & \boldsymbol{U}_{l'2}^{H} & \dots & \boldsymbol{U}_{l'K}^{H}\end{bmatrix}\begin{bmatrix}\boldsymbol{H}_{l'1}^{b} \\ \boldsymbol{H}_{l'2}^{b} \\ \vdots \\ \boldsymbol{H}_{l'K}^{b}\end{bmatrix} = \boldsymbol{0} \tag{3.11}$$

Just like GSA, $\boldsymbol{U}_{l'k'}^{H}\boldsymbol{H}_{l'k'}^{b}$ needs to span a d dimensional subspace at the relay, and to satisfy this requirement once again following equation needs to be satisfied:

$$\sum_{k=1}^{K} N_{lk} \geq R + d \qquad l = 1..L \tag{3.12}$$

3.3.3. Transceive Zero Forcing

After GSA and GCA there are $L(K-1)$ data streams and effective channels. By transmit and receive zero forcing at the relay, $L(K-1)$ data streams will be transmitted to nodes. $\boldsymbol{G^p}$ matrix is the multiplication of three matrices which are receive zero forcing matrix $\boldsymbol{G}_{rx}^{H}$, random block diagonal matrix $\boldsymbol{P_p}$ in order to setup $K-1$ linear combinations at the nodes and transmit zero forcing matrix $\boldsymbol{G}_{tx}$:

$$\boldsymbol{G^p} = \boldsymbol{G}_{tx}\boldsymbol{P_p}\boldsymbol{G}_{rx}^{H} \tag{3.13}$$

Receive zero forcing matrix is the inverse of the basis of subspaces spanned by all the nodes from all the groups. However since, Kd data strems from a group l spans $(K-1)d$ dimensional subspace at relay, when computing receive zero forcing only $(K-1)$ nodes from group l is taken into account.

So, let columns of $\widetilde{\boldsymbol{H}_l^m}$ denote basis for for the $(K-1)d$ dimensional subspace that all of the nodes in group l span at relay in the MAC phase. Then $\widetilde{\boldsymbol{H}_l^m}$ can be described as:

$$\widetilde{\boldsymbol{H}_l^m} = \begin{bmatrix} \boldsymbol{H}_{l1}^m \boldsymbol{V}_{l1} & \boldsymbol{H}_{l2}^m \boldsymbol{V}_{l2} & \dots & \boldsymbol{H}_{l(K-1)}^m \boldsymbol{V}_{l(K-1)} \end{bmatrix} \tag{3.14}$$

Therefore, receive zero matrix at the relay is expressed as follows:

$$\boldsymbol{G}_{rx}^H = \begin{bmatrix} \widetilde{\boldsymbol{H}_1^m} & \widetilde{\boldsymbol{H}_2^m} & \dots & \widetilde{\boldsymbol{H}_L^m} \end{bmatrix}^{-1} \tag{3.15}$$

Similarly let the rows of $\widetilde{\boldsymbol{H}_{l'}^b}$ denote basis for the $(K-1)d$ dimensional subspace spanned by K nodes in group l in the BC phase. Then transmit zero forcing matrix is expressed as follows:

$$\boldsymbol{G}_{tx} = \left[\left(\widetilde{\boldsymbol{H}_1^b}\right)^T \quad \left(\widetilde{\boldsymbol{H}_2^b}\right)^T \quad \dots \quad \left(\widetilde{\boldsymbol{H}_L^b}\right)^T \right]^{(-1)T} \tag{3.16}$$

After transmit zero forcing at BC phase, nodes receive broadcast signals and apply first stage receive filter and eliminate intergroup interference. After first stage receive filter $\boldsymbol{U}_{lj}^H$ applied at node, we will have d linear combinations of $(K-1)d$ streams (useful signals). Since d linear combinations will not be enough to decode $(K-1)d$ data streams, we need to broadcast $K-1$ times each time processing with $\boldsymbol{G}^{\boldsymbol{p}}$ the MAC signal captured by the relay. For this purpose $\mathbf{P_p}$ matrix is changed at each broadcast phase as a random block diagonal matrix.

3.3.4. Group Signal Separation

After $K-1$ BC phases each node has $K-1$ linearly independent equations of d data streams of $K-1$ nodes. By inverting MAC channel, relay processing and BC channel, hence the effective channel from each TX node to specified RX node, and for each BC phases, each node can decode $(K-1)d$ data streams. The effective channel can be expressed as follows:

$$\boldsymbol{H}_{l'k'k}^{(p)eff} = \boldsymbol{U}_{l'k'}^H \boldsymbol{H}_{l'k'}^b \boldsymbol{G}^{\boldsymbol{p}} \boldsymbol{H}_{l'k}^m \boldsymbol{V}_{l'k} \tag{3.17}$$

And all the effective channels from $K-1$ nodes to the receiving node in group l' can be expressed as follows:

$$\boldsymbol{H}_{l'k'}^{(p)eff} = \left[\boldsymbol{H}_{l'k'1}^{(p)eff} \quad \dots \quad \boldsymbol{H}_{l'k'j}^{(p)eff} \quad \dots \quad \boldsymbol{H}_{l'k'K}^{(p)eff}\right] \tag{3.18}$$

And finally effective channel in all BC phases can be expressed as follows:

$$\boldsymbol{H}_{l'k'}^{eff} = \left[\left(\boldsymbol{H}_{l'k'}^{(1)eff}\right)^T \quad \left(\boldsymbol{H}_{l'k'}^{(2)eff}\right)^T \quad \dots \quad \left(\boldsymbol{H}_{l'k'}^{(K-1)eff}\right)^T\right] \tag{3.19}$$

As stated before, by inverting effective channel we can decode $(K-1)d$ data streams from K nodes. Therefore second stage receive filter is expressed as:

$$\boldsymbol{Q}_{l'k'}^{\boldsymbol{H}} = \left(\boldsymbol{H}_{l'k'}^{eff}\right)^{-\mathbf{1}} \tag{3.20}$$

3.4. Sum Rate Performance

3.4.1. Sum Rate Calculation

For MIMO systems where interference is input to receiver in addition to noise, achievable data rate in between transmitting node i and receiving node j is given as follows[12]:

$$R = \log_2|\boldsymbol{I} + \boldsymbol{SINR}| \tag{3.21}$$

where SINR is defined as signal covariance matrix multiplied by inverse of sum of noise covariance matrix and interference covariance matrix.

Sum rate calculation is done for every transmit receive node pairs throughout the whole communication. Signal covariance matrix of the transmitting node, interference(namely intergroup interference) and noise covariance matrix at receiving node is calculated. For each pair, achievable data rate of the link is calculated based on the following formula:

$$R_{ij}^{l} = \frac{1}{T}\log_2\left(\left|\boldsymbol{I} + \boldsymbol{A_j}\left(\boldsymbol{B_j} + \boldsymbol{D_j}\right)^{-\mathbf{1}}\right|\right) \tag{3.22}$$

where,

R_{ij}^{l}: Achievable rate between transmitter node (l,i) and receiver node (l,j)

$\boldsymbol{A_j}$: Signal covariance matrix at node (l, j) due to node (l, i)

$\boldsymbol{B_j}$: Interference covariance matrix at node (l, j) due to intergroup interference

$\boldsymbol{D_j}$: Noise covariance matrix

T : Number of time slots that all the communication take place

Noise free signal received by node (l', k') at pth BC phase is given as follows by simply discarding noise and intergroup interference terms in (3.5):

$$\boldsymbol{x}_{l'k'}^{p} = \boldsymbol{U}_{l'k'}^{H} \boldsymbol{H}_{l'k'}^{b} \boldsymbol{G_p} \boldsymbol{H}_{l'k}^{m} \boldsymbol{V}_{l'k} \boldsymbol{d}_{l'k} \tag{3.23}$$

So concatenated noise and interference free signal vector for transmitting nodes of subgroup vector over $(K-1)$ BC phases will be as follows:

$$\boldsymbol{X}_{l'k'} = \begin{bmatrix} \boldsymbol{x}_{l'k'}^{1} \\ \boldsymbol{x}_{l'k'}^{2} \\ \vdots \\ \boldsymbol{x}_{l'k'}^{(K-1)} \end{bmatrix} \tag{3.24}$$

As explained before, second stage receive filter matrix $\boldsymbol{Q}_{l'k'}^{\boldsymbol{H}}$ is applied to signal vector and therefore signal covariance matrix can be expressed as follows:

$$\boldsymbol{A}_{l'k'} = E\left\{\boldsymbol{Q}_{l'k'}^{\boldsymbol{H}} \boldsymbol{X}_{l'k'} \left(\boldsymbol{Q}_{l'k'}^{\boldsymbol{H}} \boldsymbol{X}_{l'k'}\right)^{\boldsymbol{H}}\right\} \tag{3.25}$$

Likewise, interference only signal input to node (l', k') at pth BC phase can be expressed as follows:

$$\boldsymbol{xi}_{l'k'}^{p} = \boldsymbol{U}_{l'k'}^{H} \boldsymbol{H}_{l'k'}^{b} \boldsymbol{G_p} \sum_{\substack{l=1 \\ l \neq l'}}^{L} \boldsymbol{H}_{l'k}^{m} \boldsymbol{V}_{l'k} \boldsymbol{d}_{l'k} \tag{3.26}$$

Concatenated signal vector over $(K-1)$ MAC phases for intergroup interference will be as follows:

$$\boldsymbol{XI}_{l'k'} = \begin{bmatrix} \boldsymbol{xi}^1_{l'k'} \\ \boldsymbol{xi}^2_{l'k'} \\ \vdots \\ \boldsymbol{xi}^{(K-1)}_{l'k'} \end{bmatrix} \tag{3.27}$$

And, interference covariance matrix can be calculated as follows:

$$\boldsymbol{B}_{l'k'} = E\left\{\boldsymbol{Q}^{\boldsymbol{H}}_{l'k'}\boldsymbol{XI}_{l'k'}\left(\boldsymbol{Q}^{\boldsymbol{H}}_{l'k'}\boldsymbol{XI}_{l'k'}\right)^{\boldsymbol{H}}\right\} \tag{3.28}$$

Concatenated noise vector and noise covariance matrix can be calculated based on noise term in (4.23):

$$\widetilde{\boldsymbol{xn}}^{\boldsymbol{p}}_{l'k'} = \boldsymbol{U}^H_{l'k'}\left(\boldsymbol{H}^b_{l'k'}\boldsymbol{G}_{\boldsymbol{p}}\boldsymbol{n}_r + \boldsymbol{n}^{\boldsymbol{p}}_{l'k'}\right) \tag{3.29}$$

$$\boldsymbol{XN}_{l'k'} = \begin{bmatrix} \widetilde{\boldsymbol{xn}}^1_{l'k'} \\ \widetilde{\boldsymbol{xn}}^2_{l'k'} \\ \vdots \\ \widetilde{\boldsymbol{xn}}^{(K-1)}_{l'k'} \end{bmatrix} \tag{3.30}$$

$$\boldsymbol{D}_{l'k'} = E\left\{\boldsymbol{Q}^{\boldsymbol{H}}_{l'k'}\boldsymbol{XN}_{l'k'}\left(\boldsymbol{Q}^{\boldsymbol{H}}_{l'k'}\boldsymbol{XN}_{l'k'}\right)^{\boldsymbol{H}}\right\} \tag{3.31}$$

Sum rate will be different for each link due to channel variations. There will be K-1 links from each node to rest of the group. Since MAC phase is common for all receiving nodes data rate for the transmitting node has to be the minimum of all the links originating from that transmitting node. And sum rate of the group will be sum of all rates from all nodes to the other nodes in the same group, and sum rate of the whole system will be sum of group sum rates. This can be expressed as follows:

$$R^l_i = (K-1) * \min_j R^l_{ij} \tag{3.32}$$

$$SR = \sum_{l=1}^{L}\sum_{i=1}^{K} R^l_i \tag{3.33}$$

3.4.2. Performance Results

We reproduce sum rate graphics given in [7] for the following scenarios:

Table 3.1. *Scenario Parameters for Sum Rate Simulation*

Scenario	L	K	R	P_r	d	N_{lk}
A	3	3	6	9	1	[2 2 3]
B	3	4	9	12	1	[2 2 3 3]

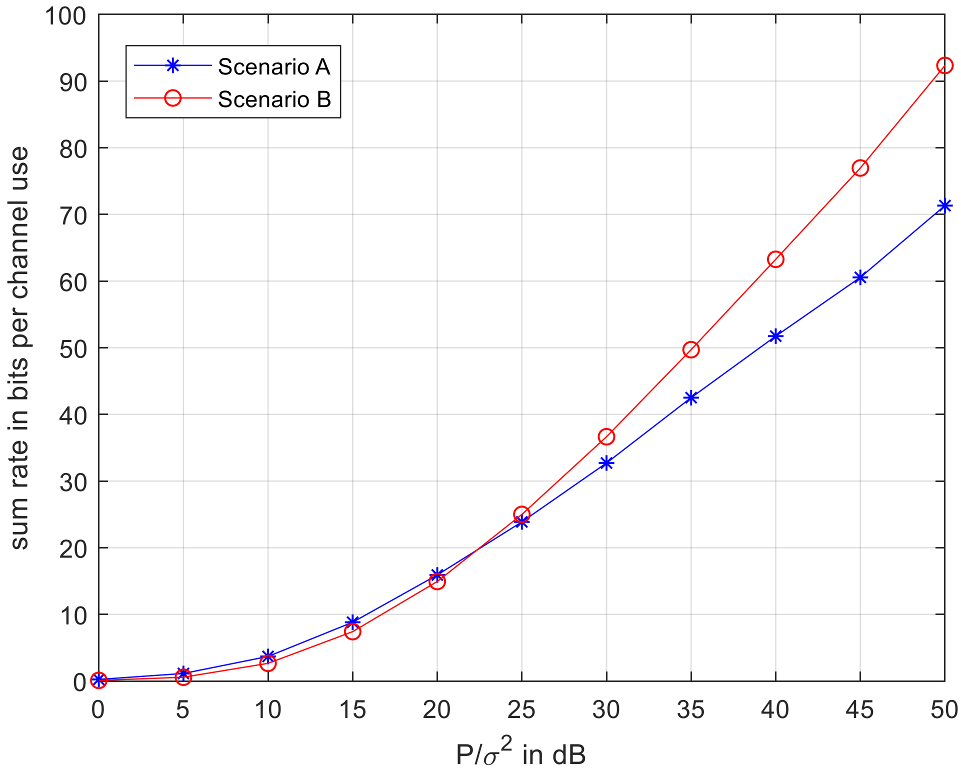

Figure 3.2. Sum Rate Graphics for Scenario A and B

3.5. Some Considerations on System Parameters

3.5.1. Number of Antennas of the Nodes

Number of antennas of the nodes are constrained by group signal alignment and group channel alignment feasibility conditions and the constraint expression is as follows:

$$\sum_{k=1}^{K} N_{lk} \geq R + d \quad l = 1..L \tag{3.33}$$

Namely the expression above states that total number of node antennas in a group must be at least larger or equal to number of relay antennas plus number of data streams.

Number of node antennas in different groups are not constrained nor mentioned in the related work. However in all the scenarios in the given work, number of antennas are symmetric in different groups. However, as long as the constraints discussed above are satisfied, number of node antennas in different groups can be completely different.

3.5.2. Number of Nodes in Different Groups

In the related work it is clearly stated that there are L groups with K nodes in each group. However if we change the relay antenna constraint according to this new situation, interference algorithm can be used when number of nodes in different groups are not equal.

In order to find out constraint on number of relay antennas for this case we can follow a similar way in the related work. In the interference algorithm, relay does not need to spatially separate all the nodes of all groups, instead if each group lies in $K_l d - d$ dimensional subspace at the relay, then at each node after cancelling self interference in $K_l - 1$ BC phases all $(K_l - 1)d$ data streams can be decoded. Therefore we need to constrain number of relay antennas as follows:

$$R = \sum_{l=1}^{L} (K_l - 1)\, d \tag{3.34}$$

Another problem to be solved in case of different number of nodes is number of BC phases. Since we want data to be interchanged in-between nodes, number of BC phases is determined based on group with maximum number of nodes. In some groups there will be BC phases more than needed, and these extra BC phases can be utilized. We will consider this case in section 3.5.4.

3.5.3. Extra Antennas at Nodes

In this scenario we have investigated when nodes have more than required minimum number of antennas by interference alignment algorithm. We used scenario A given in the related work, and changed number of antennas of the nodes. For scenario A the parameters are given as below:

Table 3.2. *Parameters for Scenario A and Scneario A_V*

Scenario	L	K	R	P_R	N_{lk}	P_{lk}
A	3	3	6	9	[2 2 3]	[1 1 1]
A_v	3	3	6	9	[3 3 3]	[3/2 3/2 1]

We tested four variants of scenario A, in which we changed N_{lk} = 3; N_{lk} = 4; N_{lk} = 5 and N_{lk} = 6 for all nodes. In Figure 3.3 we can see the results:

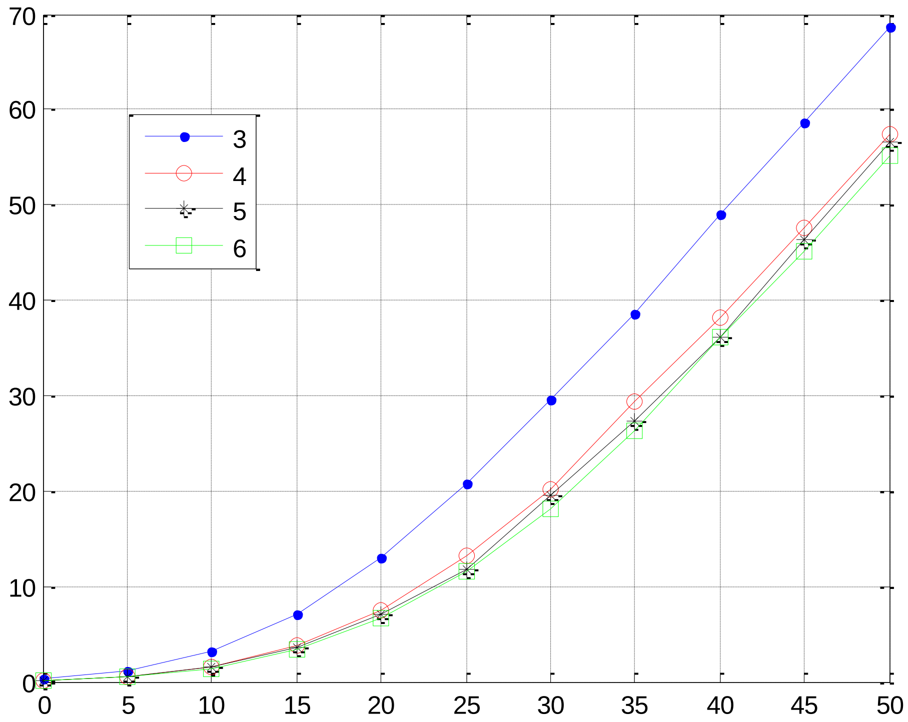

Figure 3.3. Performance results when Nlk =3,4,5,6 for scenario A

We can see that sum rate decreases as the number of antennas increase. We explain this fact in the following discussion. Since we are investigating only the number of antenna nodes we kept d-number of data streams constant in scenario A and therefore, having a larger null space dimension of $\boldsymbol{H}_l^m$ wouldn't be utilized. Since null space dimension of $\boldsymbol{H}_l^m$ is larger than needed, we can increase d too, but in that case we need to increase R also, and even we need to increase number of antennas of the nodes more then we planned. Moreover, changing parameters this way is nothing but another scenario for the given formulation and interference alignment algorithm.

Without changing IA algorithm, adding extra antennas to nodes more than needed is not only useless but also decrease the sum rate because there is a power constraint for the node, and since we distribute power on extra antennas which are not needed, we have a lower SNR at the relay receive and node receive eventually. In return, sum rate decreases, since each link capacity will decrease due to decrease in SNR. To test our hypothesis that decrease in SNR is due to decrease in power per antenna, we increased power of nodes with extra antennas by the same factor that number of antennas increase. That is to say for the case N_{lk} = 3 for all nodes, we increased power of nodes which have originally 2 antennas by a factor of 3/2. Parameters and power compensation factors for Scenario A can be seen in Table 3.1

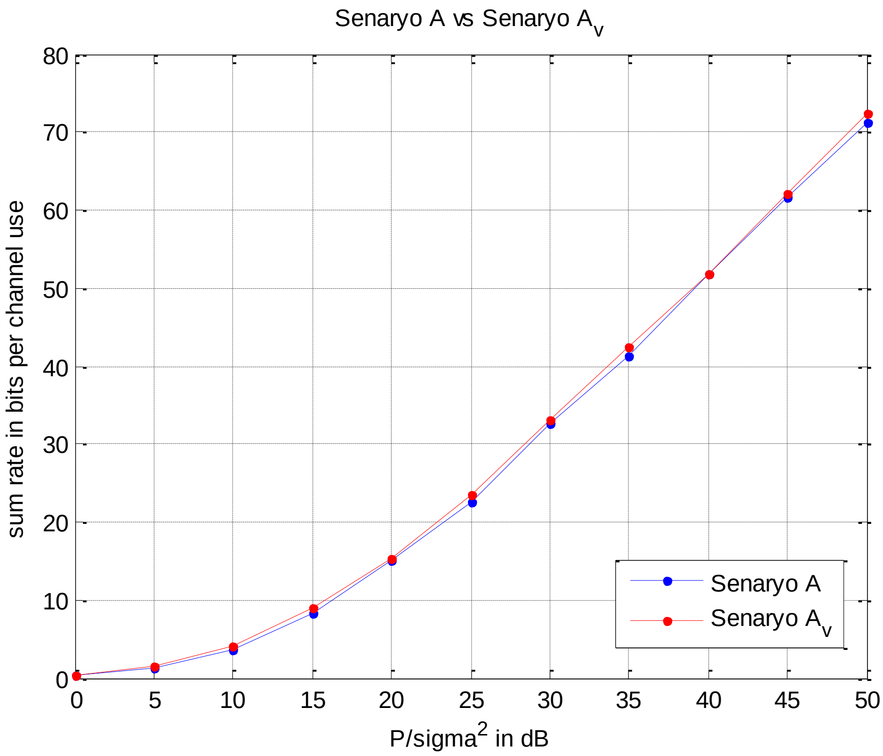

Figure 3.4. Comparison of Scenario A and Scenario Av

We can see that with power compensation, the two scenarios give the same sum rate. This verifies the reason of sum rate decrease is due to power reduction per antenna, and adding extra antennas does not have any role in this interference alignment solution unless these extra nodes are used in a different way.

3.5.4. Groups with Unequal Number of Nodes

In section 3.5.2 it was mentioned that interference alignment algorithm given in [7] can be modified to support different number of nodes for different nodes. However, since necessary number of BC phases is $(K - 1)$ and whenever K is dependent on l

then for the group which has less nodes compared to other nodes will receive BC phase tranmissions from relay more than it needs to decode data streams of the nodes in the same group.

These “extra” BC phases for the group with less number of nodes can be utilized to increase sum rate for the specified group. The main idea is, since BC transmissions are linearly processed MAC phase receive signal and additive noise to node at each BC phase is uncorrelated, we can add up extra BC phases at the receiving node and improve SNR. We change scenario A given before as in the following table:

Table 3.3. *Modified Scenario A for Groups with Unequal Number of Nodes*

Scenario	L	K	R	P_R	N_{l1}, N_{l2}	N_{l3}	PN_{lk}
A_U	3	[3 3 2]	6	9	[2 2 3]	[3 4]	[1 1 1]

Since, sum rate will only improve for the group with less nodes, we focus on sum rate of two nodes in third group. In the following figure we see sum rate for node (3,1) compared when extra BC phases are utilized and not utilized.

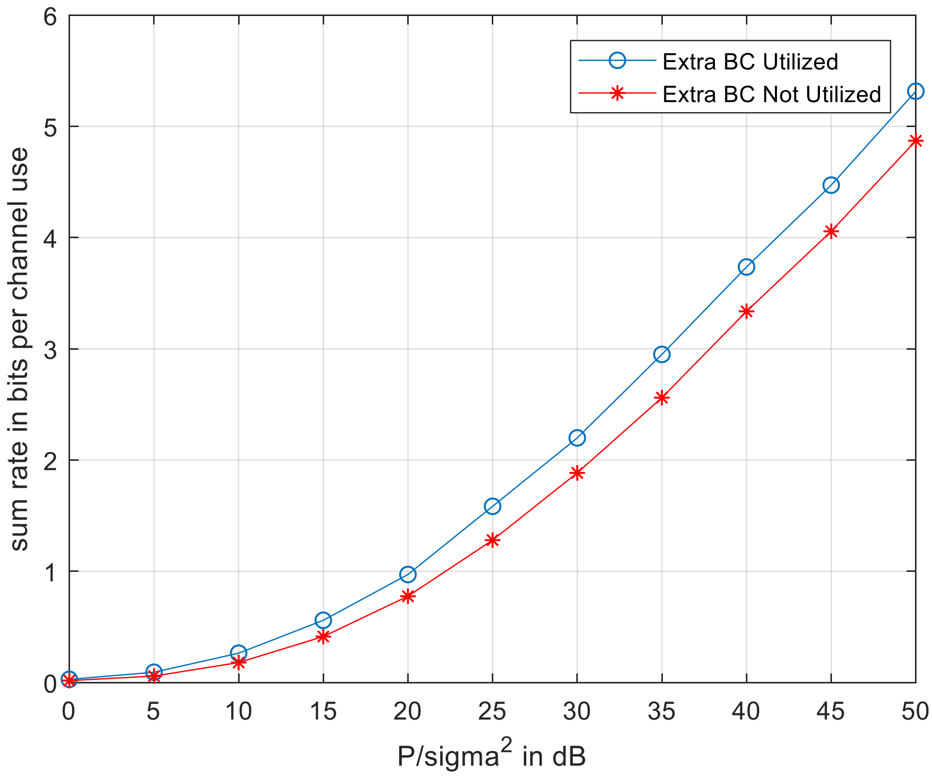

Figure 3.5. Utilizing Extra BC Phases for group with less nodes

3.5.5. Rank Deficient Channel Matrix Considerations

It is assumed that channel matrix of nodes are drawn from an i.i.d. and continuous distribution which guarantees full column rank channel matrices. However, due to several reasons such as poor scattering, too few paths and faulty antenna design which leads to insufficient antennas spacing, channel matrices may be rank deficient [16]. Under this condition we want to know behavior of the interference alignment algorithm given in [7].

When MAC phase channel matrix of one of the nodes is rank deficient, there will be problems about calculating transmit beamforming matrix since transmit beamforming matrix of the nodes are designed as null space of MAC phase channel matrix of the group. Transmit beamforming matrix design formulation is given at (3.9) as follows:

$$[\boldsymbol{H}_{l1}^{m} \quad \boldsymbol{H}_{l2}^{m} \quad \ldots \quad \boldsymbol{H}_{lK}^{m}]\begin{bmatrix}\boldsymbol{V}_{l1}\\ \boldsymbol{V}_{l2}\\ \vdots\\ \boldsymbol{V}_{lK}\end{bmatrix} = \boldsymbol{0}$$

And as mentioned in 3.3.1, null space of $\boldsymbol{H}_{l}^{m} = [\boldsymbol{H}_{l1}^{m} \quad \boldsymbol{H}_{l2}^{m} \quad \ldots \quad \boldsymbol{H}_{lK}^{m}]$ needs to be "d" dimensional and to achieve this, following condition needs to be satisfied:

$$\sum_{k=1}^{K} N_{lk} \geq R + d \qquad l = 1..L$$

Apparently, if channel matrix of one of the nodes is rank deficient, although (3.10) is satisfied, null space of $\boldsymbol{H}_{l}^{m}$ will not have a vector whose elements are all nonzero. Therefore, some of the nodes will not be able to make transmission at all and hence MAC phase communication between nodes and relay will not be possible.

Same applies to broadcast channels and first stage receive matrices too.

In order to solve this problem, diagonal loading concept can be utilized for calculating beamforming matrices. By applying diagonal loading to channel matrix of rank deficient node, null space of $\boldsymbol{H}_{l}^{m}$ will consist of vectors with nonzero elements all. This way, all the nodes will be able make transmission. However, channel itself will be still rank deficient for the specified node. In this section we will examine capacity of individual nodes, where in the same group one of the nodes have a rank deficient channel matrix.

Let MAC phase channel matrix of node (l', k') is rank deficient. Then, while calculating transmit beamforming matrices instead of original channel matrix we can use diagonal loaded MAC phase channel matrix for node (l', k')

$$\boldsymbol{H}_{l'k'}^{m,DL} = \boldsymbol{H}_{l'k'}^{m} + \alpha\boldsymbol{X} \tag{3.35}$$

where $\boldsymbol{X}$ is a matrix same size of $\boldsymbol{H}_{l'k'}^{m}$ with diagonal elements are "1" and rest is "0" as follows:

$$\boldsymbol{X} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{3.35}$$

This way $\boldsymbol{H}_{l'k'}^{m,DL}$ is guaranteed to be full column rank. The parameter α is defined as diagonal loading coefficient and how to determine the α parameter is an optimization problem. Increasing the parameter α too much will dominate the null space solution and hence the beamforming matrices, whereas decreasing it too much will result in almost rank deficient channel matrix again and produce suboptimal results again. However, we didn't study optimizing the α parameter in this work.

By using $\boldsymbol{H}_{l'k'}^{m,DL}$ at transmit beamforming matrix design at the nodes and receive zero forcing calculations at the relay, nodes that have a full column rank MAC phase channel matrix will be able to send its data stream to relay at MAC phase despite the nodes with a rank deficient MAC phase channel matrix, since null space of $\boldsymbol{H}_{l}^{m}$ will have vectors all nonzero.

Diagonal loading can be used for transmit beamforming design and receive zero forcing at the relay. However channel itself will still be rank deficient, and nodes with rank deficient channel matrix, will not be aligned its own subspace at the relay, since receive zero forcing matrix is not designed using original channel matrix which is rank deficient. Therefore, although diagonal loading is used for transmit beamforming and receive zero forcing designs, nodes with rank deficient channel matrices will lose some of their signal power.

For a scenario of L = 3, K = 4 and d = 1, we first present sum rate of nodes $(l,k) = (2,1)$ and $(l,k) = (2,4)$ without rank deficient channel matrix situation. Sum rate of these individual nodes can be seen in Figure 3.6.

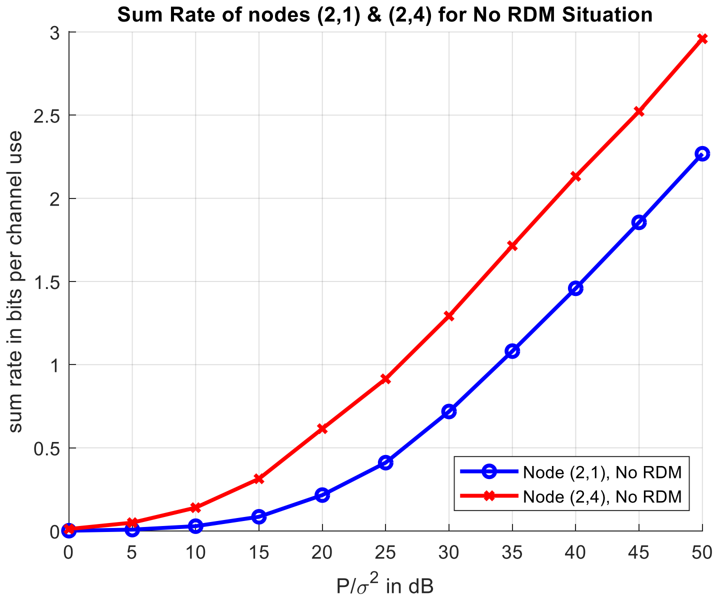

Figure 3.6. Sum Rate of Individual Nodes for No RDM Situation

Next, using singular value decomposition we modify channel matrix of node $(l, k) = (2,1)$ and we zeroize one of the eigenvalues of the channel matrix. No modification is applied to node $(l, k) = (2,4)$. No method used to compensate rank deficient channel matrix effects. Sum rate simulations under these conditions can be seen in Figure 3.7. As it is seen on Figure 3.7 not only node (2,1) but also node (2,4) is not able to communicate at all. This was something expected since the algorithm will return transmit beamforming matrices which equal to $\mathbf{0}$.

Next, we apply diagonal loading in order to cope with rank deficient channel matrix as described above. After applying diagonal loading, node (2,4) achieves sum rate as before. However node (2,1) still far behind compared to no rank deficient channel matrix case. This is again something expected since diagonal loading can only be applied at transmit beamforming design at the nodes and receive zero forcing design at the relay, but the channel itself is still rank deficient and hence zero forcing at the relay for node (2,1) is not quite successful.

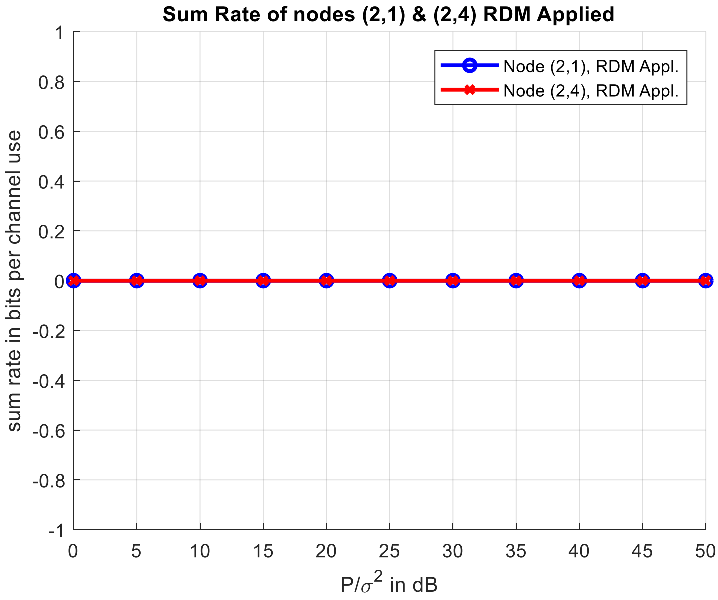

Figure 3.7. Sum Rate of Individual Nodes for RDM Situation

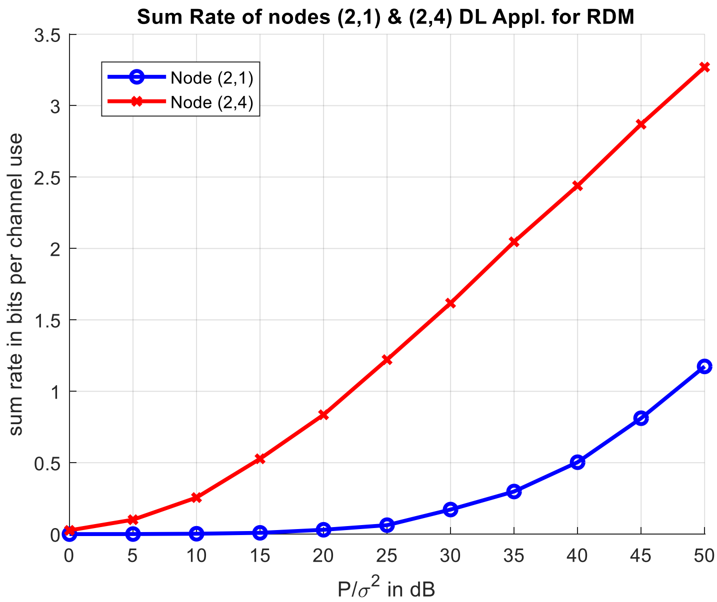

Figure 3.8. Sum Rate of Individual Nodes for Diagonal Loading Applied

CHAPTER 4

SUBGROUP BASED INTERFERENCE ALIGNMENT ON MULTIGROUP MULTIWAY RELAY CHANNELS

4.1. Introduction

In the given interference alignment scheme number of relay antennas is very close to the number of total nodes in the whole system (for d = 1), by just adding extra L antennas to the relay all the nodes can be spatially separated at the relay, hence relay could decode data streams of all nodes and process accordingly.

With this motivation, we propose another method to reduce number of relay antennas without disturbing interference alignment at the nodes.

To achieve this goal, nodes need to lie in a smaller subspace at the relay or two or more nodes need to share same subspace at the relay. If two or more nodes need to share same subspace at the relay, a method need to be devised to decode these two nodes data stream at the receiving nodes since they will add up at the relay.

4.2. Number of Relay Antennas

By designing transmit filters accordingly, we can make M nodes align into the same subspace at the relay. Therefore, relay will need to spatially separate only half of the nodes, and we will need $\frac{LKd}{M}$ dimensional space at the relay. So number of relay antennas can be expressed as follows:

$$R = \frac{LKd}{M} \tag{4.1}$$

4.3. Signal and Channel Alignment of Any Two Nodes

To reduce the number of antennas at the relay, we need to design transmit beamforming matrices of the nodes in the same group such that M of the nodes span the same d dimensional subspace at the relay and this can be expressed as follows:

$$span(\boldsymbol{H}_{li}^{m}\boldsymbol{V}_{li}) = span(\boldsymbol{H}_{li+1}^{m}\boldsymbol{V}_{li+1}) = \cdots = span(\boldsymbol{H}_{li+M-1}^{m}\boldsymbol{V}_{li+M-1}) \quad (4.2a)$$

$$i = (s-1) * M \quad (4.2b)$$

$$s = 1,2,..,K/M \quad (4.2c)$$

where

$$i: Node\ index\ in\ subgroups$$

$$s: Subgroup\ index$$

We define these M nodes which span same subspace at the relay as a subgroup.

In order to satisfy this condition, we need to find a d dimensional intersection between subspaces that node i can span relay and node j can span at relay. To make this feasible, the matrix defined below needs to have d dependent columns for any i & j pair [13]:

$$\boldsymbol{H}_{lij}^{m} = [\boldsymbol{H}_{li}^{m}\boldsymbol{H}_{lj}^{m}] \quad (4.3)$$

Nodes in the same subgroup needs to align not only in MAC phases but also in BC phases. So, we need to design receive filter of each node in the same subgroup such that they will align at the relay into the same subspace in BC phases. This can be achieved similar to MAC phase problem and can be expressed as follows:

$$span\big((\boldsymbol{U}_{li}^{H}\boldsymbol{H}_{li}^{b})^{H}\big) = span\big((\boldsymbol{U}_{lj}^{H}\boldsymbol{H}_{lj}^{b})^{H}\big) \quad (4.4)$$

Again for feasibility,

$$\boldsymbol{H}_{lij}^{b\ H} = \left[\boldsymbol{H}_{li}^{b\ H}\ \boldsymbol{H}_{lj}^{b\ H}\right] \quad (4.5)$$

needs to have d columns dependent to each other for any i & j pair.

4.4. Design of Transmit Beamforming Matrix

We want to design transmit beamforming matrix of the nodes in the same subgroup such that they span the same subspace at the relay. If M > 2 then any 2 node and all of the nodes in the same subgroup needs to span same subspace at the relay. For the pair to span same subspace following equation needs to hold for any i&j pair:

$$\boldsymbol{H}_{li}^{m}\boldsymbol{V}_{li} = \boldsymbol{H}_{lj}^{m}\boldsymbol{V}_{lj} \tag{4.6}$$

which will lead to following equation:

$$\begin{bmatrix} \boldsymbol{H}_{l1}^{m} & -\boldsymbol{H}_{l2}^{m} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{H}_{l2}^{m} & -\boldsymbol{H}_{l3}^{m} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{H}_{l3}^{m} & -\boldsymbol{H}_{l4}^{m} & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{H}_{l(M-1)}^{m} & -\boldsymbol{H}_{lM}^{m} \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_{l1} \\ \boldsymbol{V}_{l2} \\ \vdots \\ \vdots \\ \vdots \\ \boldsymbol{V}_{l(M)} \end{bmatrix} = \boldsymbol{0} \tag{4.7}$$

By satisfying (4.7) we assure that any pair satisfies (4.6).

Since $\boldsymbol{H}_{lm}^{m}$ is $((M-1)*R)\ x\ (M*N)$ (N: Average number of antennas of the nodes) dimensional, and $\boldsymbol{H}_{lm}^{m}$ needs to have d dependent columns to make the nodes in the same subgroup span same subspace at the relay, we can infer that

$$M * N \geq ((M-1)*R) + d \tag{4.8}$$

and so

$$N \geq \frac{(M-1)*R}{M} + \frac{d}{M} \tag{4.9}$$

Since we have

$$R = \frac{LKd}{M} \tag{4.10}$$

then

$$N \geq \frac{(M-1)LKd + Md}{M^2} \tag{4.11}$$

is the necessary average number of antennas at the nodes for the existence of interference alignment solution.

If the above constraints are satisfied then transmit beamforming matrices for the subgroup can be found as follows:

$$\begin{bmatrix} \boldsymbol{V}_{l1} \\ \boldsymbol{V}_{l2} \\ \vdots \\ \vdots \\ \vdots \\ \boldsymbol{V}_{l(M)} \end{bmatrix} = null\left(\begin{bmatrix} \boldsymbol{H}_{l1}^{m} & -\boldsymbol{H}_{l2}^{m} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{H}_{l2}^{m} & -\boldsymbol{H}_{l3}^{m} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{H}_{l3}^{m} & -\boldsymbol{H}_{l4}^{m} & \dots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \boldsymbol{H}_{l(M-1)}^{m} & -\boldsymbol{H}_{lM}^{m} \end{bmatrix}\right) \tag{4.12}$$

4.5. Design of Receive Filter Matrix

Receive filter matrix can be designed similar to transmit beamforming matrix. To pair same nodes into the same channel, following equation needs to be satisfied:

$$\begin{bmatrix} \boldsymbol{U}_{l1}^{H} & \boldsymbol{U}_{l2}^{H} & \dots & \dots & \dots & \boldsymbol{U}_{l(M)}^{H} \end{bmatrix} \begin{bmatrix} \boldsymbol{H}_{l1}^{m} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ -\boldsymbol{H}_{l2}^{m} & \boldsymbol{H}_{l2}^{m} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & -\boldsymbol{H}_{l3}^{m} & \boldsymbol{H}_{l3}^{m} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\boldsymbol{H}_{l4}^{m} & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \boldsymbol{H}_{l(M-1)}^{m} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & -\boldsymbol{H}_{lM}^{m} \end{bmatrix} = \mathbf{0} \tag{4.13}$$

and so,

$$\begin{bmatrix} \boldsymbol{U}_{l1}^{H} & \boldsymbol{U}_{l2}^{H} & \dots & \dots & \dots & \boldsymbol{U}_{l(M)}^{H} \end{bmatrix} = \left(null\left(\begin{bmatrix} \boldsymbol{H}_{l1}^{m} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ -\boldsymbol{H}_{l2}^{m} & \boldsymbol{H}_{l2}^{m} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & -\boldsymbol{H}_{l3}^{m} & \boldsymbol{H}_{l3}^{m} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\boldsymbol{H}_{l4}^{m} & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \boldsymbol{H}_{l(M-1)}^{m} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & -\boldsymbol{H}_{lM}^{m} \end{bmatrix}^{H} \right) \right)^{H} \tag{4.14}$$

Examining (4.14) we can reach (4.9) which constraints the average number of antennas each node in a subgroup needs to have. Therefore we can conclude that average number of antennas each node needs to have is same for both MAC and BC phases to make interference alignment feasible.

Subgroup signal and channel alignment given in (4.12) and (4.14) needs to be done for each subgroup in every group. Then we will have $\frac{LKd}{M}$ available spatial slots for LKd data streams.

One important advantage of the proposed system is hidden in (4.12) and (4.14). As it is seen nodes in a subgroup does not need the channel knowledge of the nodes in other subgroups although all of them in the same group. For example, let K = 4 and M = 2, then for the proposed system every node will only need to know its own channel state information and the channel state information of the other node in its subgroup. However in the reference scheme given in [7], every node needs to know not only its own channel state information but also channel state information of other three nodes in the same group.

4.6. Transceive Zero Forcing

We need receive zero forcing and transmit zero forcing matrices at relay in order to separate pairs in a group and also separate groups after MAC phase and before BC phases. By designing P matrix appropriately we can transmit data streams to nodes.

4.6.1. Receive Zero Forcing Matrix

Receive zero forcing matrix is the inverse of the basis of subspaces spanned by all the nodes from all the groups. However, since Md data streams from a subgroup spans d dimensional subspace at relay, while computing receive zero forcing only one of the nodes from each subgroup is taken into account.

Let columns of $\widetilde{\boldsymbol{H}_l^m}$ denote basis for the $(K-1)d$ dimensional subspace that all of the nodes in group l span at relay in the MAC phase. Then $\widetilde{\boldsymbol{H}_l^m}$ can be described as:

$$\widetilde{\boldsymbol{H}_l^m} = \begin{bmatrix}\boldsymbol{H}_{l1}^m\boldsymbol{V}_{l1} & \boldsymbol{H}_{l(1+M)}^m\boldsymbol{V}_{l3} \cdots \boldsymbol{H}_{l(K-2M+1)}^m\boldsymbol{V}_{l(K-2M+1)} & \boldsymbol{H}_{l(K-M+1)}^m\boldsymbol{V}_{l(K-M+1)}\end{bmatrix} \tag{4.15}$$

As we can see, since subsequent nodes are designed as pair they span same subspace at the relay and only one of them is taken into account in order to obtain $\widetilde{\boldsymbol{H}_l^m}$. And concatenating all $\widetilde{\boldsymbol{H}_l^m}$ matrices of all groups and taking inverse we get receive zero forcing matrix:

$$\boldsymbol{G}_{\boldsymbol{rx}}^H = \begin{bmatrix}\widetilde{\boldsymbol{H}_1^m} & \widetilde{\boldsymbol{H}_2^m} & \dots & \widetilde{\boldsymbol{H}_L^m}\end{bmatrix}^{-\mathbf{1}} \tag{4.16}$$

4.6.2. Transmit Zero Forcing Matrix

In order to calculate transmit zero forcing matrix, we need to obtain the matrix whose rows are a basis for effective channels spanning relay.

$$\widetilde{\boldsymbol{H}_l^b} = \begin{bmatrix}\boldsymbol{U}_{l1}^H\boldsymbol{H}_{l1}^b \\ \boldsymbol{U}_{l(1+M)}^H\boldsymbol{H}_{l(1+M)}^b \\ \vdots \\ \boldsymbol{U}_{l(K-2M+1)}^H\boldsymbol{H}_{l(K-2M+1)}^b \\ \boldsymbol{U}_{l(K-M+1)}^H\boldsymbol{H}_{l(K-M+1)}^b\end{bmatrix} \tag{4.17}$$

If we concatenate $\widetilde{\boldsymbol{H}_l^b}$ matrices for all groups and then take inverse we will get transmit zero forcing matrix:

$$\boldsymbol{G}_{tx} = \left[\left(\widetilde{\boldsymbol{H}_1^b}\right)^T \quad \left(\widetilde{\boldsymbol{H}_2^b}\right)^T \quad \dots \quad \left(\widetilde{\boldsymbol{H}_L^b}\right)^T\right]^{(-1)T} \tag{4.18}$$

4.7. Design of P Matrix

Data received in each MAC phase can only be transmitted in K/M BC phase since we have only LKd/M antennas at the relay. Therefore at each BC phase we broadcast MAC phase data of M nodes to all the nodes in each L groups, namely subgroups in order. These M nodes are the nodes that span same subspace at the relay. We use P

matrix to broadcast MAC phase data of different subgroups to all the nodes in each group. P matrix for $L = 2, d = 2$ and $K = 4$ is given as follows:

$$\boldsymbol{P_1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{4.19}$$

$$\boldsymbol{P_2} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.20}$$

$\boldsymbol{P_1}$ is for BC phase 1 for all MAC phases and $\boldsymbol{P_2}$ is for BC phase 2 for all MAC phases. $\boldsymbol{P_1}$ transmits data of nodes 1&2 to all the nodes in the same group (L=2), and $\boldsymbol{P_2}$ transmits data of nodes 3&4 to all the nodes in the same group. After each MAC phase, K/M BC phases is done. This way data of each MAC phase is transmitted to all nodes in each group.

Combining receive zero forcing matrix, P matrix and transmit zero forcing matrix, we will have relay processing matrix:

$$\boldsymbol{G_p} = \boldsymbol{G}_{tx}\boldsymbol{P_p}\boldsymbol{G}_{rx}^{H}$$

By using P matrices we broadcast different subgroups' MAC phase data to all nodes in the same group. As mentioned before we still need to resolve data streams of nodes that are in the same subgroup. This is done by transmitting data in M MAC phases and pre encoding them before transmit beamforming matrix.

4.8. Designing Pre-Beamforming Encoding Matrix

Since nodes in the same subgroup span the same subspace at the relay, we need to find a way to separate these M nodes' data streams. For this, we use a pre-encoding scheme. Before transmit matrix, we multiply data vector with dxd pre encoding matrix $\boldsymbol{Z_{lk}^{m}}$ which is given for d = 2 as follows:

$$\boldsymbol{Z_{lk}^{m}} = \begin{bmatrix} Z_{lk11}^{m} & Z_{lk12}^{m} \\ Z_{lk21}^{m} & Z_{lk22}^{m} \end{bmatrix} \tag{4.21}$$

Here, m denotes MAC phase, l denotes group, and k denotes node k in group l. Members of $\boldsymbol{Z_{lk}^{m}}$ should be such that, after M MAC phases broadcasted to receiving nodes, data streams of nodes that span same subspace at the relay can be decoded and this can only be achieved if the following condition is satisfied:

$$\boldsymbol{Z} = \begin{bmatrix} \boldsymbol{Z_{l1}^{1}} & \boldsymbol{Z_{l2}^{1}} \\ \boldsymbol{Z_{l1}^{2}} & \boldsymbol{Z_{l2}^{2}} \end{bmatrix} \text{ and rank}(\mathbf{Z}) = 4 \text{ i.e. } \mathbf{Z} \text{ is full rank} \tag{4.22}$$

For subgroup w which contains M nodes, a more general statement for node $\boldsymbol{Z}$ can be given as follows:

$$\boldsymbol{Z_w} = \begin{bmatrix} \boldsymbol{Z_{l(1+\gamma)}^{1}} & \boldsymbol{Z_{l(2+\gamma)}^{1}} & \cdots & \boldsymbol{Z_{l(M+\gamma)}^{1}} \\ \boldsymbol{Z_{l(1+\gamma)}^{2}} & \boldsymbol{Z_{l(2+\gamma)}^{2}} & \cdots & \boldsymbol{Z_{l(M+\gamma)}^{2}} \\ \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{Z_{l(1+\gamma)}^{M}} & \boldsymbol{Z_{l(2+\gamma)}^{M}} & \cdots & \boldsymbol{Z_{l(M+\gamma)}^{M}} \end{bmatrix} \tag{4.23}$$

where $\gamma = \lfloor (w-1)/M \rfloor * M$

and rank($\boldsymbol{Z_w}$) = M*d i.e. $\boldsymbol{Z_w}$ is full rank

$\boldsymbol{Z_w}$ matrix is constructed by concatenating $\boldsymbol{Z_{lk}^{m}}$ matrices horizontally keeping MAC phase constant and changing nodes in the subgroup, and vertically keeping nodes constant and changing MAC phase. When constructing $\boldsymbol{Z_w}$ matrix we use the nodes that span the same subspace at the relay. This is nothing but a system of linear equations. For this system to have a unique solution matrix $\boldsymbol{Z_w}$ needs to be full rank as stated in (4.23). Another point is that, since different subgroups span distinct subspaces at the relay, same precoding matrices can be used in all subgroups.

The constraint in (4.23) can be achieved by random elements for each $\boldsymbol{Z}_{\boldsymbol{lk}}^{\boldsymbol{m}}$. However any other matrix can be selected for $\boldsymbol{Z}_{\boldsymbol{lk}}^{\boldsymbol{m}}$ which will satisfy (4.23). In this study we used Gaussian distributed random elements for $\boldsymbol{Z}_{\boldsymbol{lk}}^{\boldsymbol{m}}$. For other choices performance may change in positive or negative direction or this matrix can be designed to achieve another purpose.

4.9. Resolving Data Streams of Nodes in the Same Subspace

Transmission sequence of the whole system can be configured as M times MAC phase, followed by K/M BC phases for each MAC phase. For example where M = 2 and K = 4, transmission sequence will be as follows:

MAC1, MAC2, BC11, BC12, BC21, BC22

Here, first index of BC denotes MAC phase number and second index denotes BC phase number of the related MAC phase. To be more clear BCQP(for BC12, Q = 1 & P = 2) denotes P^{th} BC phase of Q^{th} MAC phase.

Construction of the $\boldsymbol{Z}$ matrix was described in the previous part. It defines a system of linear equations, which is constructed by pre-encoding matrices applied before transmit beamforming matrix at each node at each MAC phase. Therefore we can consider $\boldsymbol{Z}^{-\boldsymbol{1}}$ as decoding matrix.

At node (l', k') received signal at p^{th} BC phase of m^{th} MAC phase can be expressed as follows:

$$\boldsymbol{s}_{l'k'}^{mp} = \boldsymbol{U}_{l'k'}^{H} \boldsymbol{H}_{l'k'}^{b} \boldsymbol{G}_{\boldsymbol{p}} \sum_{l=1}^{L} \sum_{k=1}^{K} \boldsymbol{H}_{lk}^{m} \boldsymbol{V}_{lk} \boldsymbol{Z}_{\boldsymbol{lk}}^{\boldsymbol{m}} \boldsymbol{d}_{lk} + \boldsymbol{U}_{l'k'}^{H} \widetilde{\boldsymbol{n}}_{l'k'}^{\boldsymbol{mp}} \qquad (4.23)$$

For $l \neq l'$, terms under summation will be intergroup interference and after receive filter matrix $\boldsymbol{U}_{l'k'}^{H}$ they will be cancelled out. Summation for $l = l'$ is the signal of nodes in the same group of the receiving node. $\boldsymbol{Z}_{\boldsymbol{lk}}^{\boldsymbol{m}}$ is the pre-encoding matrix for m^{th} MAC phase. $\widetilde{\boldsymbol{n}}_{l'k'}^{\boldsymbol{mp}}$ denotes the effective noise input to node (l', k'). It is the sum of processed and transmitted relay noise at mth MAC phase and noise which is directly input to node at pth BC phase. It can be expressed as follows:

$$\widetilde{\boldsymbol{n}}_{l'k'}^{\boldsymbol{mp}} = \boldsymbol{H}_{l'k'}^{b}\boldsymbol{G}^{p}\boldsymbol{n}_{r}^{\boldsymbol{m}} + \boldsymbol{n}_{l'k'}^{\boldsymbol{p}} \tag{4.24}$$

Here $\boldsymbol{n}_{l'k'}^{\boldsymbol{p}}$ follows CN(0,$\sigma_{l'k'}^{2}$) and $\boldsymbol{n}_{r}^{\boldsymbol{m}}$ follows CN(0,σ_{r}^{2}).

By design of P matrix, at every BC phase, signals of one of the subgroups in each group is transmitted by the relay. Therefore, at each receiving node, $\boldsymbol{s}_{l'k'}^{pq}$ is concatenated over M MAC phases for each BC phase. We can give an example to be more clear. Let L = 2, K = 4, M = 2 and d = 2. Then at node (1,1), received signal at MAC1, BC11 and MAC2, BC21 will be concatenated to decode data of nodes in subgroup 1. Likewise, received signal at MAC1, BC12 and MAC2, BC22 will be concatenated. Therefore, at each receiving node, there will be concatenated signal vector for each subgroup in the same group and there will be a decoding matrix which is $\boldsymbol{Z}^{-\boldsymbol{1}}$, again for each subgroup. However, decoding matrix may be same for all subgroups since as stated before, they span distinct subspaces at the relay.

For the given example scenario in the previous paragraph, at node (l', k') for first subgroup, $\boldsymbol{S}$ vector which is concatenated signal vector for first subgroup will be formed as follows:

$$\boldsymbol{S}_{l'k'}^{1} = \begin{bmatrix} \boldsymbol{s}_{l'k'}^{11} \\ \boldsymbol{s}_{l'k'}^{21} \end{bmatrix} \tag{4.25}$$

For subgroup w which contains M nodes, a more general statement for node (l', k') can be given as follows:

$$\boldsymbol{S}_{l'k'}^{w} = \begin{bmatrix} \boldsymbol{s}_{l'k'}^{1p} \\ \boldsymbol{s}_{l'k'}^{2p} \\ \vdots \\ \boldsymbol{s}_{l'k'}^{Mp} \end{bmatrix} \tag{4.25}$$

where $p = \lfloor (w-1)/M \rfloor + 1$

Then decoding of transmitted symbols for subgroup w is simply the solution of this set of linear equations, and can be expressed as follows:

$$\hat{\boldsymbol{d}} = \boldsymbol{Z}_w^{-1}\boldsymbol{S}_{l'k'}^{w} \tag{4.26}$$

4.10. Rank Deficient Channel Matrix Considerations

Theoretical considerations for rank deficient channel matrix case are given in 3.5.5. Considerations and solutions for rank deficient channel matrix case given in 3.5.5 applies to proposed interference alignment algorithm too. Therefore we directly present simulation results.

We choose nodes (2,1) and (2,2) since they lie in the same subgroup. First we present sum rate for the case with no rank deficient matrix in Figure 4.1.

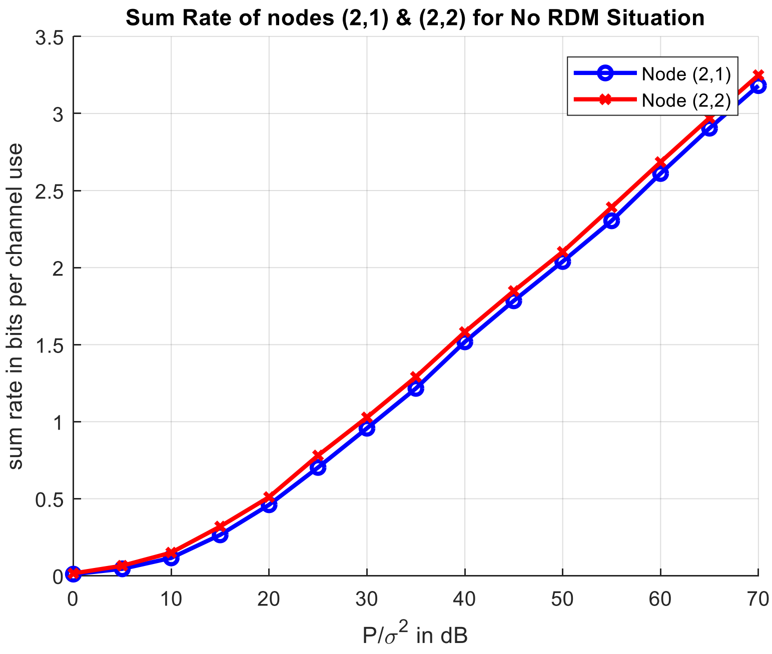

Figure 4.1. Sum Rate of Individual Nodes for No RDM Situation

Next, we use a rank deficient matrix for node (2,1) and do not apply diagonal loading. We see the sum rate results in Figure 4.2. Since (2,1) has a rank deficient matrix, transmit beamforming matrices are **0**.

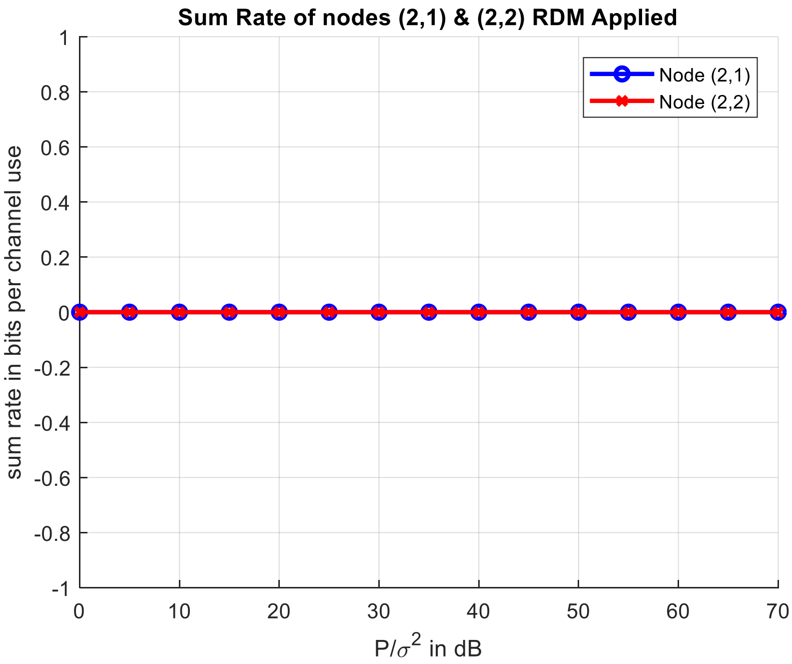

Figure 4.2. Sum Rate of Individual Nodes for RDM Situation

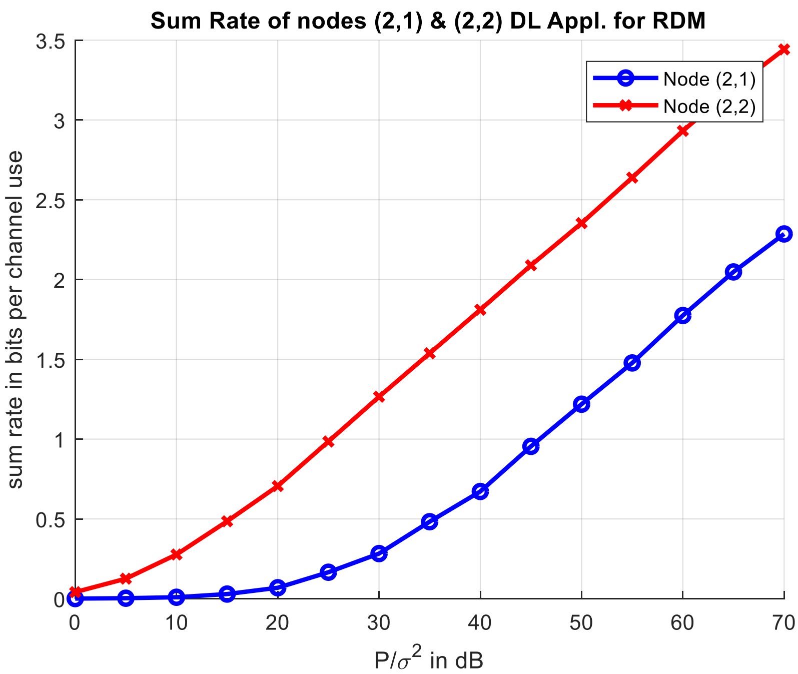

Figure 4.3. Sum Rate of Individual Nodes for Diagonal Loading Applied

Finally, we apply diagonal loading in order to cope with rank deficient matrix of node (2,1). In Figure 4.3 it can be seen that node (2,2) has a sum rate as if there is no rank deficiency problem in the system. However, node (2,1) loose from the sum rate, since channel matrix itself is still rank deficient although diagonal loading applied at transmit beamforming and relay receive zero forcing design.

4.11. Performance Evaluation and Comparison

First we compare necessary relay and node antenna number formulation, channel state information knowledge need at the nodes and number of MAC and BC phases needed for the whole communication. We also present system parameters for which only maximum 3 antennas at the nodes is needed for the proposed system. In addition we present how many antennas are needed at the relay and nodes thoroughly and make comments on these numbers. In addition, we present a 3 dimensional plot of antenna numbers of the nodes and the relay vs number of groups (L) and subgroup size (M); and vs number of nodes in a group (K) and subgroup size (M) for a visual comparison of antenna numbers of the proposed and the reference system.

Next we compare sum rate performances of the proposed system and reference system for same main system parameters such as number of groups, number of nodes and number of data streams in 4.11.3.1. In this comparison, we do not care about how much power is dissipated for the whole communication to take place in two systems, therefore total power usage may be different. We also do not care about total number of antennas in the two system. However, sum rate results are per time slot for both of the systems. We examine the response of the subgroup size of the proposed algorithm by changing it.

Finally, we make a fair comparison of the sum rate performance of the two systems, in terms of total number of antennas needed, total power used by the nodes and the relay and total time slots in section 4.11.3.2. Since, capacity of MIMO systems is dependent on number of antennas and power, results of this fair comparison is very valuable in terms of the absolute performance of the two systems.

4.11.1. Main System Parameters Comparison

For both of the systems, number of antennas at the relay and nodes are formulated in equations (3.1), (3.10), (4.1), (4.9) respectively. We summarize these parameters in the following table:

Table 4.1. *Comparisan of main system paramaters*

Parameter	*Ref[7]*	*Subgroup Based IA*
Relay Antennas	$R = L(K-1)d$	$R = \frac{LKd}{M}$
Node Antennas	$N_{lk} \geq \frac{L(K-1)d + d}{K}$	$N \geq \frac{\left((M-1) * R\right) + d}{M}$
MAC Phase	1	M
BC Phase	$K-1$	$M * \frac{K}{M}$

In the proposed algorithm, as we reduce number of relay antennas nearly in the order of M by aligning M nodes into same subspace at the relay, number of node antennas increase depending on the value of M. For M = K it can be seen that number of node antennas are same for both systems and also number of relay antennas is only L*d. However in return we loose from total time slots that all the communication take place. Another advantage of the proposed system is we can control number of antennas of the relay and nodes by changing parameter M.

In order to gain more insight into how the parameters such as L, K and M change the number of relay and node antennas we plot R and N as a surface depending on K and M while keeping L constant and also depending on L and M while keeping K constant as in Figure 4.4 and Figure 4.5. Although M is not a parameter for the reference algorithm, in order to plot comparable figures we kept parameter M on the y-axis but as it is seen in the figures reference algorithms' antenna numbers don't change on M.

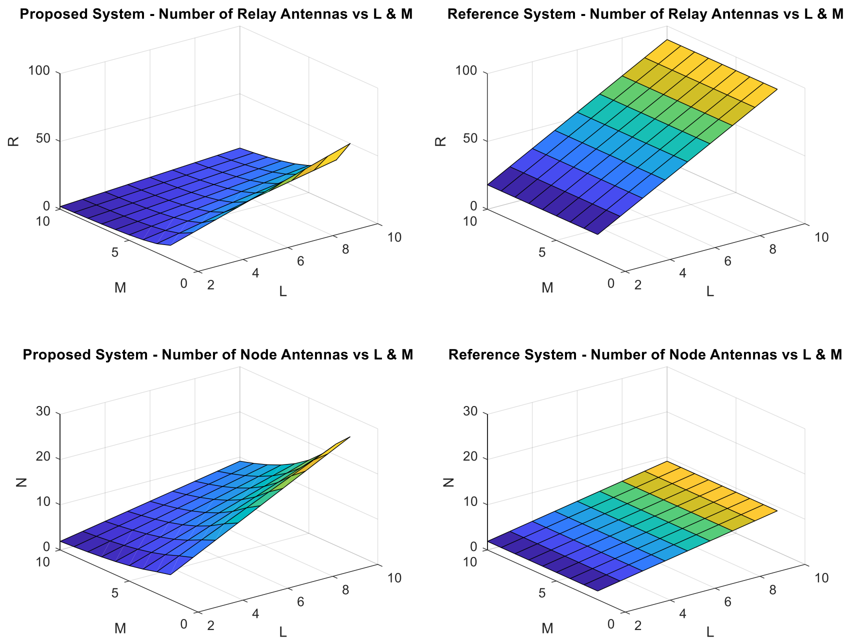

Figure 4.4. Comparison plot of relay and node antenna numbers vs L & M

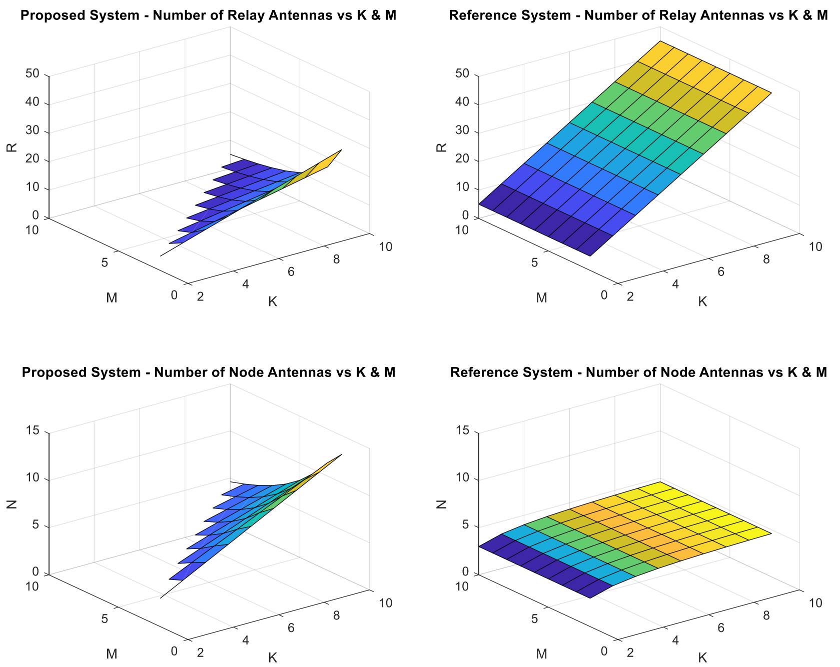

Figure 4.5. Comparison plot of relay and node antenna numbers vs K & M

In Figure 4.4 K = 10 is set and kept constant. Examining Figure 4.4 we see for the reference algorithm R is linearly increases as L increase. However, for the proposed algorithm by controlling M parameter, R value can be reduced as the order of M and this can be clearly seen from the figure. Number of node antennas track the same line for the proposed algorithm and reference algorithm if M = K for the proposed algorithm. For cases M < K proposed algorithm needs more antennas for the nodes.

In Figure 4.5 L = 5 is set and kept constant. Examining Figure 4.5 we see similar results for R. Note that the points where M > K doesn't exist on the plot since M can't be larger than K. Number of node antennas N converges to L*d for the reference algorithm as K increases. However, for the proposed algorithm we need more antennas at the node unless M = K. If M = K then proposed algorithm almost track the same line with the reference algorithm for the number of node antennas, N.

Another important measure which is valuable to compare is computational complexity. In both of the algorithms, mainly matrix inversion and null space calculation is used. For matrix inversion computational complexity is given as $O(n) = n^3$ for an $n \times n$ dimensional matrix. Therefore, since the number of relay antennas will increase as the number of nodes increase for the reference algorithm, computational complexity will also increase in the order R^3 since $\boldsymbol{G}_{rx}^{\boldsymbol{H}}$ and $\boldsymbol{G}_{tx}$ are calculated by calculating $R \times R$ matrix inverses. However, for the proposed algorithm, we can have a lower computational complexity by increasing M. The number of relay antennas for the proposed algorithm are given as $R_1 = L * (K/M) * d$ and for the reference algorithm $R_{org} = L * (K-1) * d$. Therefore, computational complexity of transmit & receive zero beamforming matrix calculation at the relay for the reference algorithm will be $\left(\frac{M(K-1)}{K}\right)^3$ much higher compared to proposed algorithm, depending on the value of K and M. Null space is generally calculated using singular value decomposition. Although there are different algorithms with different accuracy and computational complexity, they are all dependent on size of the matrix. So, again for the proposed algorithm computational complexity of the solution will be reduced compared to reference algorithm, depending on the subgroup size, M.

Less number of node antennas is a very favorable property for today's antenna designs packed into a small form factor like a mobile phone. As the number of antennas increase, they may be correlated and channel matrix may be rank deficient contrary to assumptions made. Therefore we checked for which system parameters, number of node antennas is less than or equal to 3 and tabulated as in Table 4.2.

We can see that for a broad range of parameters, 3 or less node antenna requirement can be satisfied. However, as the number of groups gets higher than 5, neither of the systems can work with 3 or less antennas.

With the proposed system, number of relay antennas decrease and number of node antennas increase. Although higher number of node antennas can be seen as a disadvantage, this fact makes number of node and relay antennas closer to each other. For some scenarios close number of relay and node antennas can be favorable since any node can serve as a relay also. Based on this, in a network any node which doesn't need to transceive data for a period of time can serve as a relay and nodes which are in idle state can be evaluated for highest sum rate. The idle node which can serve as a relay with highest sum rate can be chosen.

In the reference algorithm given in [7], every node needs to know Global CSI. However in the proposed subgroup based interference algorithm, every node needs CSI local to its own subgroup. For M = 2, every node needs to know CSI of only one of the nodes in the same group no matter how large is K. This is a favorable property, since estimating CSI is complex and causes using some of the total sum rate available for this purpose.

Table 4.2. *System Parameters for N* ≤ 3

L	K	d	M	R_1	N_1	R_{org}	N_{org}
2	2	1	2	2	1,50	2	1,50
2	2	2	2	4	3,00	4	3,00
2	3	1	3	2	1,67	4	1,67
2	4	1	2	4	2,50	6	1,75
2	4	1	4	2	1,75	6	1,75
2	5	1	5	2	1,80	8	1,80
2	6	1	3	4	3,00	10	1,83
2	6	1	6	2	1,83	10	1,83
2	7	1	7	2	1,86	12	1,86
2	8	1	8	2	1,88	14	1,88
2	9	1	9	2	1,89	16	1,89
2	10	1	10	2	1,90	18	1,90
3	2	1	2	3	2,00	3	2,00
3	3	1	3	3	2,33	6	2,33
3	4	1	4	3	2,50	9	2,50
3	5	1	5	3	2,60	12	2,60
3	6	1	6	3	2,67	15	2,67
3	7	1	7	3	2,71	18	2,71
3	8	1	8	3	2,75	21	2,75
3	9	1	9	3	2,78	24	2,78
3	10	1	10	3	2,80	27	2,80
4	2	1	2	4	2,50	4	2,50
4	3	1	3	4	3,00	8	3,00
5	2	1	2	5	3,00	5	3,00

4.11.2. Sum Rate Calculation

For MIMO systems where interference is input to receiver in addition to noise, achievable data rate in between transmitting node i and receiving node j is given as follows:

$$R = \log_2|\boldsymbol{I} + \boldsymbol{SINR}| \quad (4.27)$$

where SINR is defined as signal covariance matrix multiplied by inverse of sum of noise covariance matrix and interference covariance matrix.

Sum rate calculation is done for every transmit receive node pairs throughout the whole communication. Signal covariance matrix of the transmitting node, interference (namely intergroup interference) and noise covariance matrix at receiving node is calculated. For each pair, achievable data rate of the link is calculated based on the following formula:

$$R_{ij}^{l} = \frac{1}{T}\log_2\left(\left|\boldsymbol{I} + \boldsymbol{A_j} * \left(\boldsymbol{B_j} + \boldsymbol{D_j}\right)^{-\boldsymbol{1}}\right|\right) \quad (4.28)$$

where,

R_{ij}^{l}: Achievable rate between transmitter node (l, i) and receiver node (l, j)

$\boldsymbol{A_j}$: Signal covariance matrix at node (l, j) due to node (l, i)

$\boldsymbol{B_j}$: Interference covariance matrix at node (l, j) due to intergroup interference

$\boldsymbol{D_j}$: Noise covariance matrix

T : Number of time slots that all the communication take place

Noise free signal received by node (l', k') at mth MAC phase and pth BC phase is given as follows by simply discarding noise and intergroup interference terms in (4.23):

$$\boldsymbol{x}_{l'k'}^{mp} = \boldsymbol{U}_{l'k'}^{H}\boldsymbol{H}_{l'k'}^{b}\boldsymbol{G_p}\boldsymbol{H}_{l'k}^{m}\boldsymbol{V}_{l'k}\boldsymbol{Z}_{l'k}^{m}\boldsymbol{d}_{l'k} \quad (4.29)$$

So concatenated noise and interference free signal vector for transmitting nodes of subgroup vector over M MAC phases will be as follows:

$$\boldsymbol{X}_{l'k'}^{w} = \begin{bmatrix} \boldsymbol{x}_{l'k'}^{1p} \\ \boldsymbol{x}_{l'k'}^{2p} \\ \vdots \\ \boldsymbol{x}_{l'k'}^{Mp} \end{bmatrix} \quad (4.30)$$

As explained before, decoding matrix $\boldsymbol{Z_w^{-1}}$ is applied to signal vector and therefore signal covariance matrix can be expressed as follows:

$$\boldsymbol{A}_{l'k'} = E\left\{\boldsymbol{Z_w^{-1}}\boldsymbol{X}^{w}_{l'k'}\left(\boldsymbol{Z_w^{-1}}\boldsymbol{X}^{w}_{l'k'}\right)^{\boldsymbol{H}}\right\} \tag{4.31}$$

Likewise, interference only signal input to node (l', k') at mth MAC phase and pth BC can be expressed as follows:

$$\boldsymbol{xi}^{mp}_{l'k'} = \boldsymbol{U}^{H}_{l'k'}\boldsymbol{H}^{b}_{l'k'}\boldsymbol{G_p}\sum_{\substack{l=1\\ l\neq l'}}^{L}\boldsymbol{H}^{m}_{l'k}\boldsymbol{V}_{l'k}\boldsymbol{Z}^{m}_{l'k}\boldsymbol{d}_{l'k} \tag{4.32}$$

Concatenated signal vector over M MAC phases for intergroup interference will be as follows:

$$\boldsymbol{XI}^{w}_{l'k'} = \begin{bmatrix}\boldsymbol{xi}^{1p}_{l'k'}\\ \boldsymbol{xi}^{2p}_{l'k'}\\ \vdots\\ \boldsymbol{xi}^{Mp}_{l'k'}\end{bmatrix} \tag{4.33}$$

And, interference covariance matrix can be calculated as follows:

$$\boldsymbol{B}_{l'k'} = E\left\{\boldsymbol{Z_w^{-1}}\boldsymbol{XI}^{w}_{l'k'}\left(\boldsymbol{Z_w^{-1}}\boldsymbol{XI}^{w}_{l'k'}\right)^{\boldsymbol{H}}\right\} \tag{4.34}$$

Concatenated noise vector and noise covariance matrix can be calculated based on noise term in (4.23):

$$\widetilde{\boldsymbol{xn}}^{\boldsymbol{mp}}_{l'k'} = \boldsymbol{U}^{H}_{l'k'}\left(\boldsymbol{H}^{b}_{l'k'}\boldsymbol{G}^{p}\boldsymbol{n}^{\boldsymbol{m}}_{r} + \boldsymbol{n}^{\boldsymbol{p}}_{l'k'}\right) \tag{4.35}$$

$$\boldsymbol{XN}^{w}_{l'k'} = \begin{bmatrix}\widetilde{\boldsymbol{xn}}^{1p}_{l'k'}\\ \widetilde{\boldsymbol{xn}}^{2p}_{l'k'}\\ \vdots\\ \widetilde{\boldsymbol{xn}}^{Mp}_{l'k'}\end{bmatrix} \tag{4.36}$$

$$\boldsymbol{D}_{l'k'} = E\left\{\boldsymbol{Z}_{\boldsymbol{w}}^{-\boldsymbol{1}}\boldsymbol{X}\boldsymbol{N}_{l'k'}^{w}\left(\boldsymbol{Z}_{\boldsymbol{w}}^{-\boldsymbol{1}}\boldsymbol{X}\boldsymbol{N}_{l'k'}^{w}\right)^{\boldsymbol{H}}\right\} \tag{4.37}$$

Sum rate will be different for each link due to channel variations. There will be K-1 links from each node to rest of the group. Since MAC phase is common for all receiving nodes data rate for the transmitting node has to be the minimum of all the links originating from that transmitting node. And sum rate of the group will be sum of all rates from all nodes to the other nodes in the same group, and sum rate of the whole system will be sum of group sum rates. This can be expressed as follows:

$$R_i^l = (K-1) * \min_j R_{ij}^l \tag{4.38}$$

$$SR = \sum_{l=1}^{L}\sum_{i=1}^{K} R_i^l \tag{4.39}$$

4.11.3. Performance Results

4.11.3.1. Sum Rate Comparison for the Same Number of Groups, Nodes and Data Streams

We compare the sum rate performance of the proposed system and reference system for several example scenarios. In these scenarios, all the main system parameters such as number of groups – L, number of nodes in each group – K and number of data streams – d are same for both reference algorithm and proposed algorithm. However, number of antennas at the relay and at the nodes may differ. Number of total time slots may also differ, although sum rate given in the graphics are for per time slot.

Sum rate graphics are obtained with 500 iterations Monte Carlo simulations. Rayleigh flat fading channel model is used for generating channel matrices. Total relay power set to $P_r = 9P$, power of each node is set to $P_n = P$ and kept same for both of the scenarios. The power unit P used to express the power of relay and nodes in an abstract way. Sum rate results are calculated and plotted for different logarithmic scale SNR values and SNR is defined as P/σ^2 where σ^2 is the noise variance input to the nodes and the relay; and P is the abstract power measure used to express relay and node total transmit power. SNR in logarithmic scale which is used in sum rate graphics is given as:

$$SNR_{dB} = 10 * \log_{10}(P/\sigma^2) \tag{4.40}$$

SNR_{dB} defined above is not receive SNR at the nodes or at the relay, since it would not be easy to determine a constant SNR at the relay because of different channel coefficients every node have at MAC phase. Therefore, SNR_{dB} is rather a reference which relates transmit power of the relay and nodes with noise power input to the relay and nodes.

We present scenario parameters and necessary number of relay & node antenna numbers for both of the systems in the following table:

Table 4.3. *Summary of Scenarios – Main System Parameters are Same*

Scenario	L	K	d	M	R_{org}	N_{org}	R_1	N_1
SCN1	2	4	1	2	6	1,75	4	2,5
SCN2	2	6	1	2	10	1,83	6	3,5
SCN3	2	6	1	3	10	1,83	4	3
SCN4	2	6	1	6	10	1,83	2	1,83
SCN5	2	8	1	2	14	1,87	8	4,5
SCN6	2	8	1	4	14	1,87	4	3,25
SCN7	2	8	1	8	14	1,87	2	1,87
SCN8	3	4	1	2	9	2,5	6	3,5
SCN9	3	4	1	4	9	2,5	3	2,5
SCN10	3	6	1	2	15	2,66	9	5
SCN11	3	6	1	3	15	2,66	6	4,33
SCN12	3	6	1	6	15	2,66	3	2,66
SCN13	3	8	1	2	21	2,75	12	6,5
SCN14	3	8	1	4	21	2,75	6	4,75

R_{org} and N_{org} denotes number of relay and node antennas for original scenario given in [7] and R_1 and N_1 denotes number of relay and node antennas for proposed system.

Node antenna numbers are given as average number, and for original scenario, total number of node antennas needs to be satisfied whereas for the proposed system, total number of antennas for the subgroup needs to be satisfied. For example, for SCN1, N_{org} = 1,75, which means total number of antennas in a group needs to be 7, in return number of antennas for the nodes in order can be [2 2 2 1]. Likewise for the proposed system, N_1 = 2,5, which means total number of antennas in a subgroup needs to be 5 (since M = 2 for SCN1), and number of antennas for the nodes in order can be [3 2].

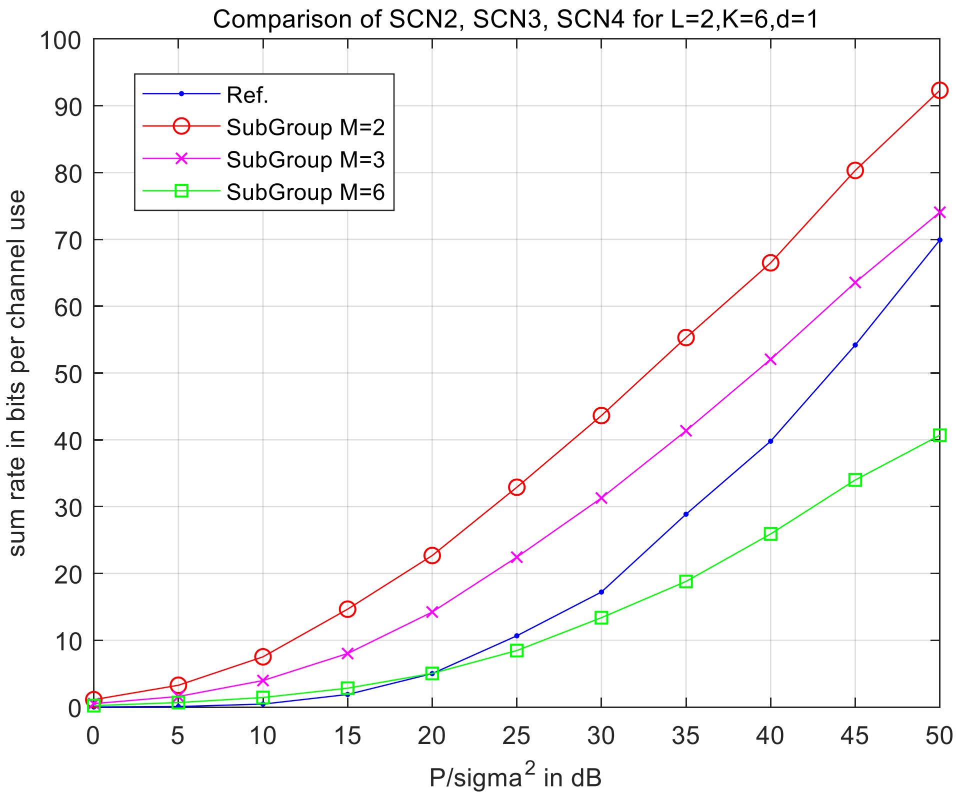

Figure 4.6. Comparison of SCN2, SCN3, SCN4 for L=2,K=6,d=1

In Figure 4.6 we compare original system with the proposed subgroup based interference alignment algorithm for SCN2, SCN3 and SCN4. From the figure it can be seen that as M increase sum rate decrease for the proposed system. So we sacrifice from sum rate as the subgroup size increase which decreases number of relay antennas.

Still, for M = 6 we need R_1 = 2 antennas compared to R_{org} = 10 antennas for the reference IA algorithm and sum rate for the proposed system is slightly higher than or equal to the reference algorithm for P/sigma$^2 \leq$ 20 dB. As we decrease subgroup size (M), we obtain a higher sum rate for the proposed system, but we need to keep in mind that we need more antennas at the nodes for the proposed scenario.

Next we compare sum rate results for the other scenarios given in Table 4.1. Results are similar in the mean of subgroup based algorithms' reaction to increase in M.

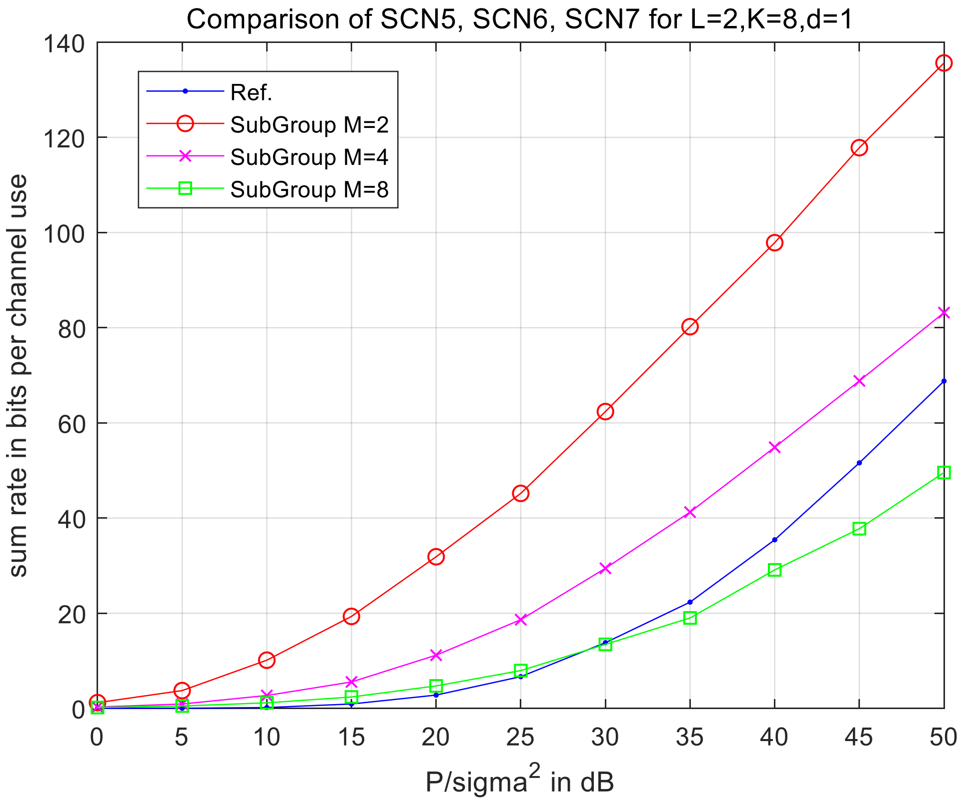

Figure 4.7. Comparison of SCN5, SCN6, SCN7 for L=2,K=8,d=1

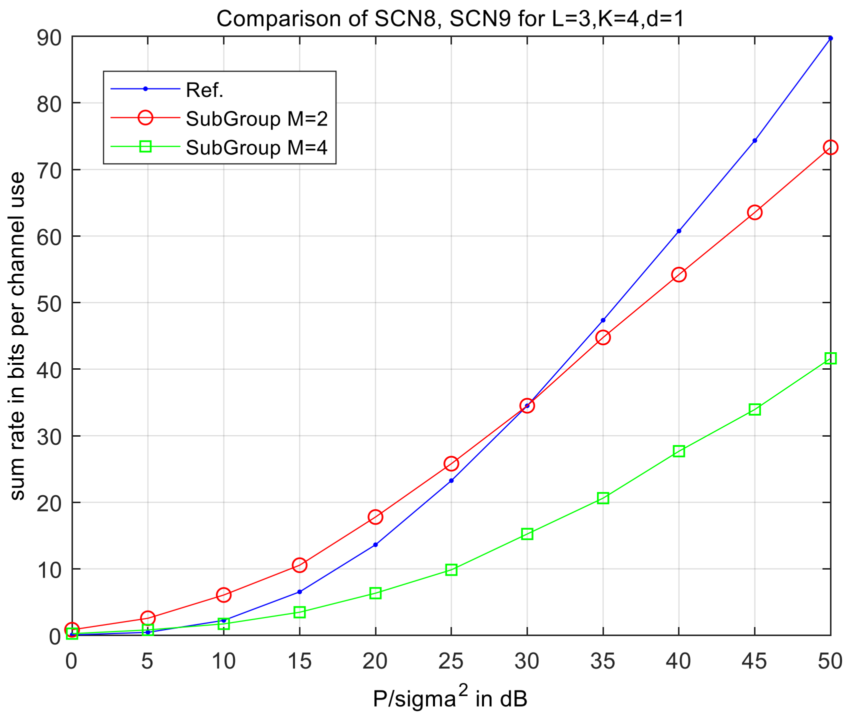

Figure 4.8. Comparison of SCN8, SCN9 for L=3,K=4,d=1

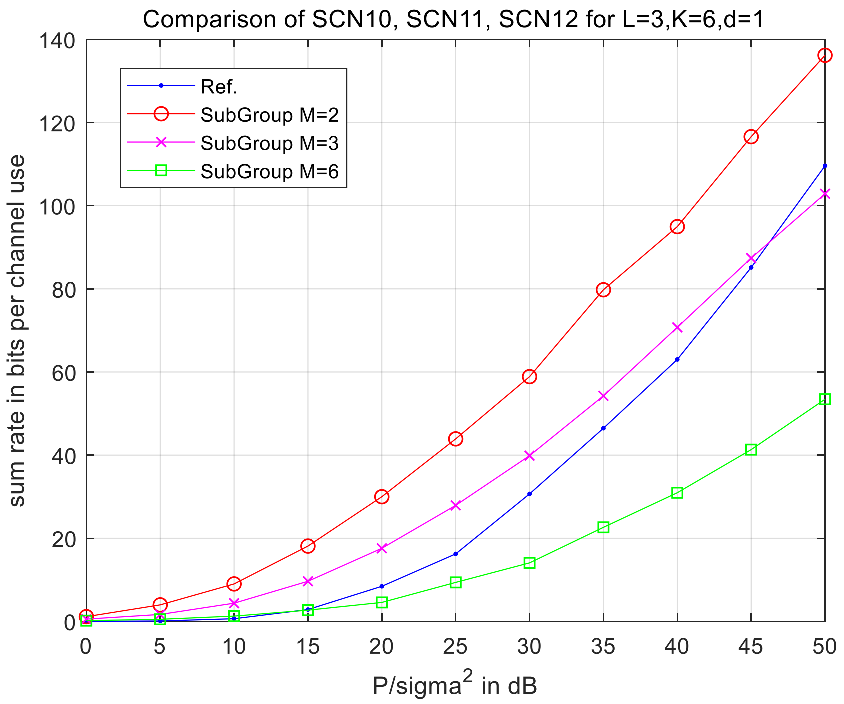

Figure 4.9. Comparison of SCN10, SCN11, SCN12 for L=3,K=6,d=1

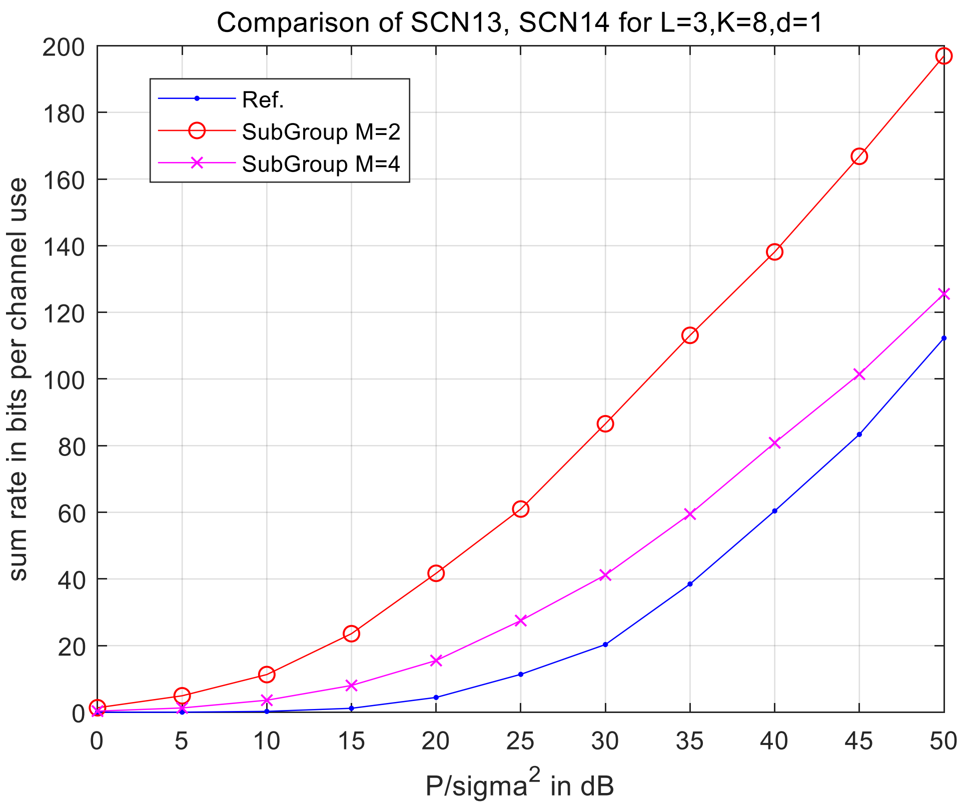

Figure 4.10. Comparison of SCN13, SCN14 for L=2,K=8,d=1

In all of these scenarios, we just kept number of groups, nodes and data streams same for the proposed algorithm and reference algorithm. But since algorithms are different, required number of antennas at the relay and at the nodes are not same for proposed and reference algorithm. Also, number of time slots required for node to node communication through the relay is different for proposed and reference algorithm. Therefore, although the main parameters are same, total used resources such as number of antennas, total used power and total time slots are different and it can be thought that comparison above may not be fair enough.

In the next section we will compare the two algorithms in a more fair perspective.

4.11.3.2. A Fair Comparison Scheme - Sum Rate Comparison for the Same Number of Total Antennas and Total Power Used

In order to make a fair comparison, we searched scenarios in which total number of antennas are same or in %1 proximity to find more scenarios. For these scenarios by changing the relay power appropriately, total power used by the system made equal for the scenarios in each of the algorithms. Therefore, we made total power used by the system equal in each of the algorithms and made total number of antennas in %1 proximity. Note that since we aimed to equalize total number of antennas and total power used, the scenarios we compare the proposed and the reference algorithm may have different number of groups or nodes as opposed to the scenarios we used in the previous section.

We listed the scenarios by changing one of the main system parameters (L, K or d) parameters and keeping rest of the parameters same. This way we can inspect the tendency of the sum rate behavior of the algorithms in respect to changing parameter.

Fair Comparison Scenarios – Increasing Number of Nodes per Group

Table 4.4. *Summary of scenarios for Fair Comparison, L = 2*

Scenario	L_{org}	L_1	K_{org}	K_1	M	R_{org}	R_1	N_{org}	N_1	T_{org}	T_1	P^{rly}_{org}	P^{rly}_1	N^{tot}_{org}	N^{tot}_1	P^{tot}_{org}	P^{tot}_1
SCNF1	2	2	4	5	5	6	2	1,75	1,8	4	10	80	39,6	20	20	248	248
SCNF2	2	2	6	8	8	10	2	1,83	1,88	6	16	80	35,5	32	32	412	412
SCNF3	2	2	8	11	11	14	2	1,88	1,91	8	22	80	30,36	44	44	576	576
SCNF4	2	2	10	14	14	18	2	1,9	1,93	10	28	80	24,86	56	56	740	740
SCNF5	2	2	12	17	17	22	2	1,92	1,94	12	34	120	45,06	68	68	1344	1344
SCNF6	2	2	14	20	20	26	2	1,93	1,95	14	40	120	39,4	80	80	1588	1588

In Table 4.4 we see the scenarios with equal total power and total number of antennas for both the reference algorithm and the proposed algorithm, where subscript “1” means proposed algorithm, whereas subscript “org” means reference algorithm. Number of node antennas are given as average, averaged over the group for the reference algorithm and averaged over subgroup for the proposed algorithm. Total power and total number of antennas for both of the systems can be seen. Number of data streams, “d” is not listed in the table, and it is used as d = 1 for all scenarios and for both of the algorithms.

In the scenarios, L and d doesn’t change at all. Number of nodes in each group, K starts from [K_{org} K_1] = [4 5] and increases up to [14 20] at SCNF6. We see from the sum rate plots that for larger number of nodes proposed algorithm performs better compared to reference algorithm, and as the number of nodes gets smaller reference algorithm performs better. If we examine scenarios, we see that $M = K$ for all scenarios. For the proposed algorithm if $K = M$, total time slots needed is $2 * K$ whereas for the reference algorithm total time slots needed is K. So as we increase K, time slots rate of two algorithms doesn’t change, but as we increase K, in the proposed algorithm we obtain a higher power advantage for BC phases, since we have less relay antennas compared to reference algorithm. This will increase the power per antenna at the relay and hence the sum rate.

We see that, in all of the scenarios, reference algorithm starts to perform better after an P/σ^2 threshold. The P/σ^2 threshold where the reference algorithm performs better depends on the K value. As the K value increase, reference algorithm could only perform better after higher values of the P/σ^2 parameter where the power advantage of the proposed algorithm becomes invaluable due to high SNR.

In the following, we present sum rate plots for the scenarios given in Table 4.4 and also in Table 4.5

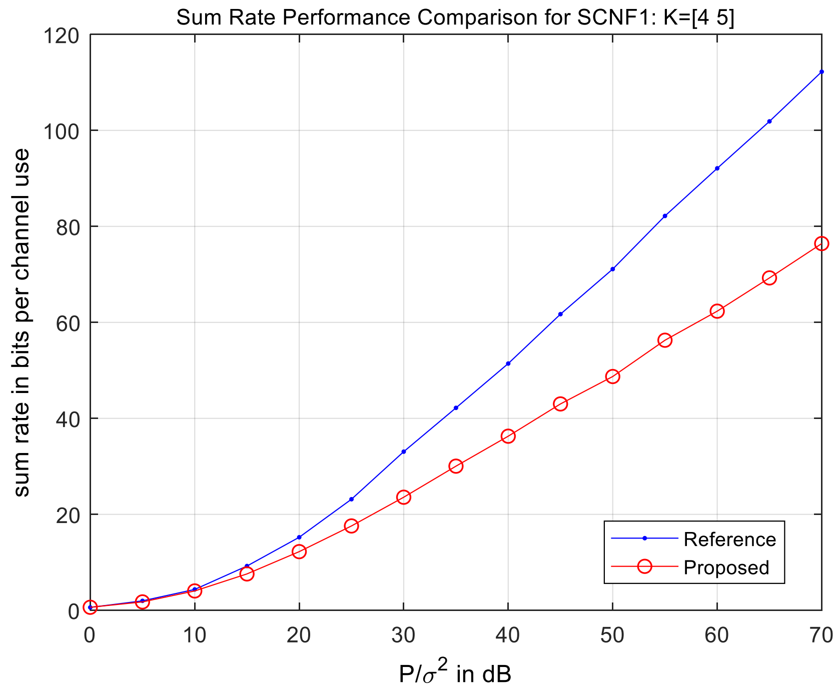

Figure 4.11. Sum Rate Comparison for SCNF1

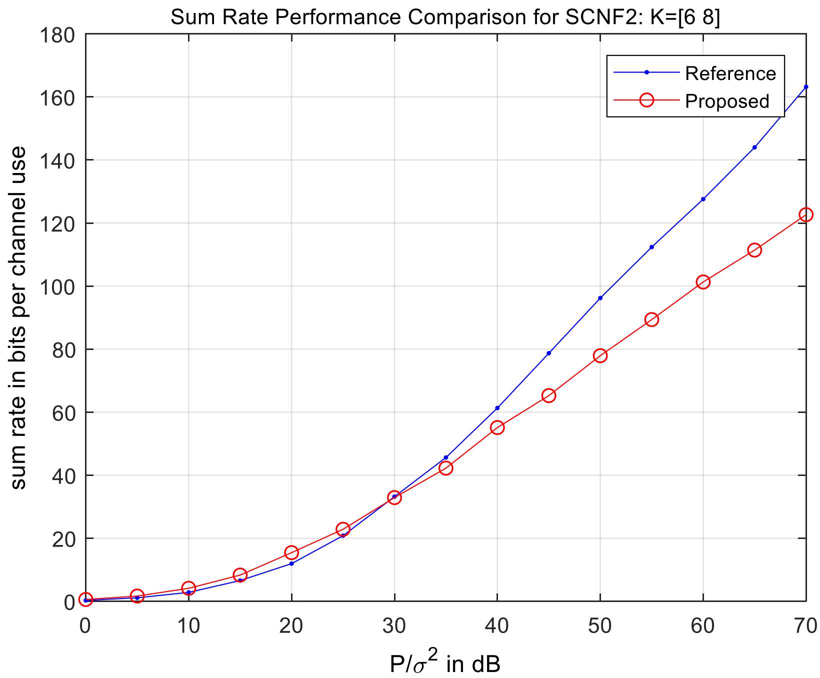

Figure 4.12. Sum Rate Comparison for SCNF2

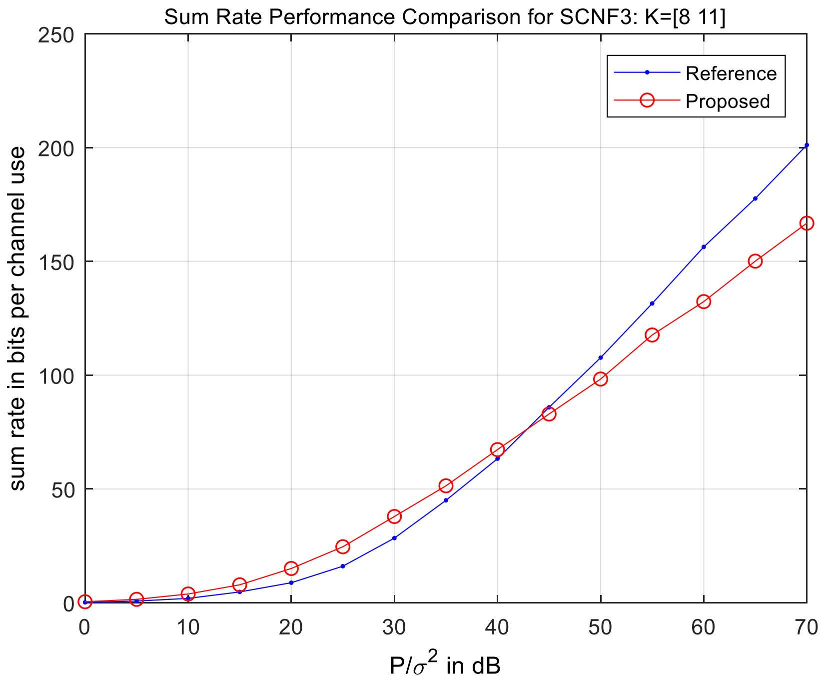

Figure 4.13. Sum Rate Comparison for SCNF3

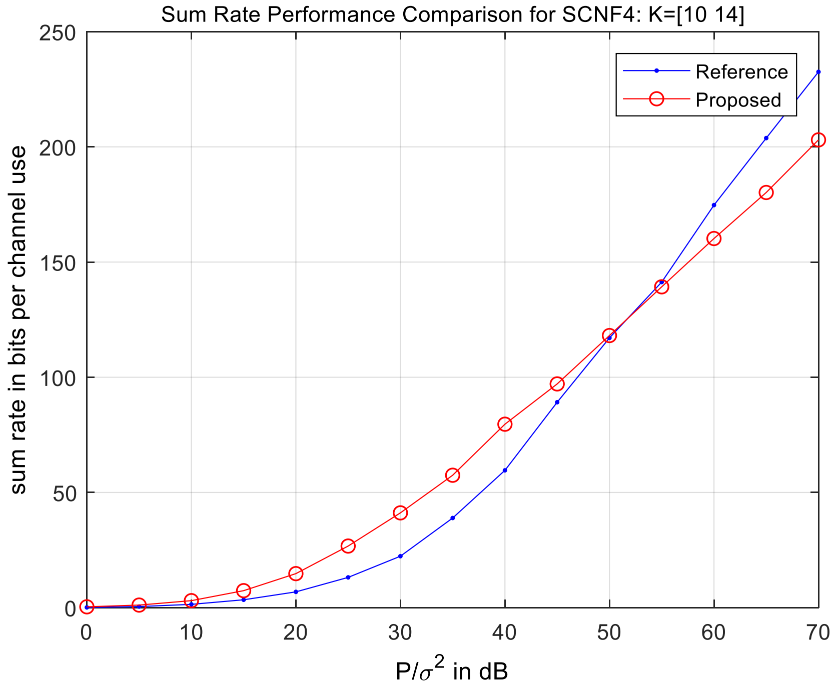

Figure 4.14. Sum Rate Comparison for SCNF4

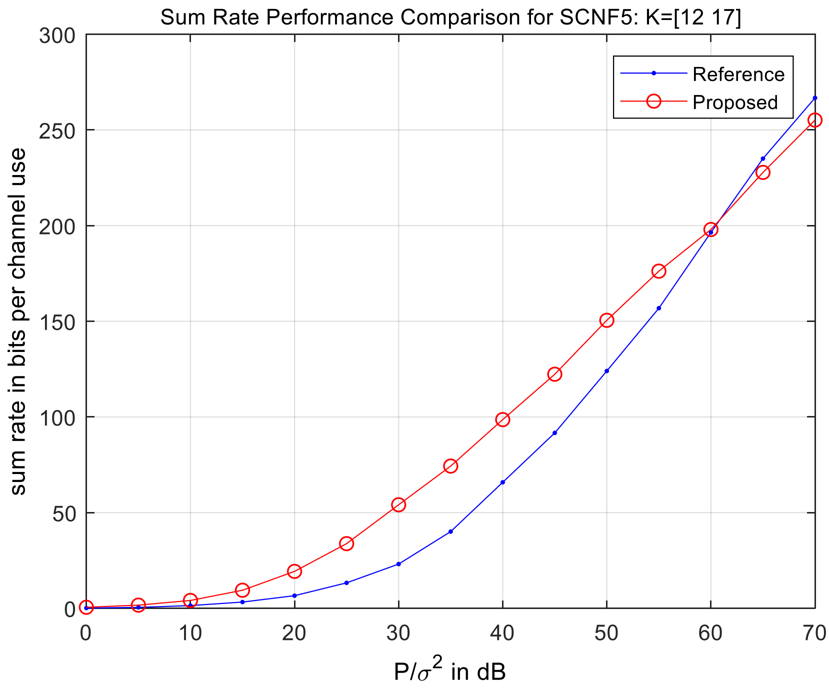

Figure 4.15. Sum Rate Comparison for SCNF5

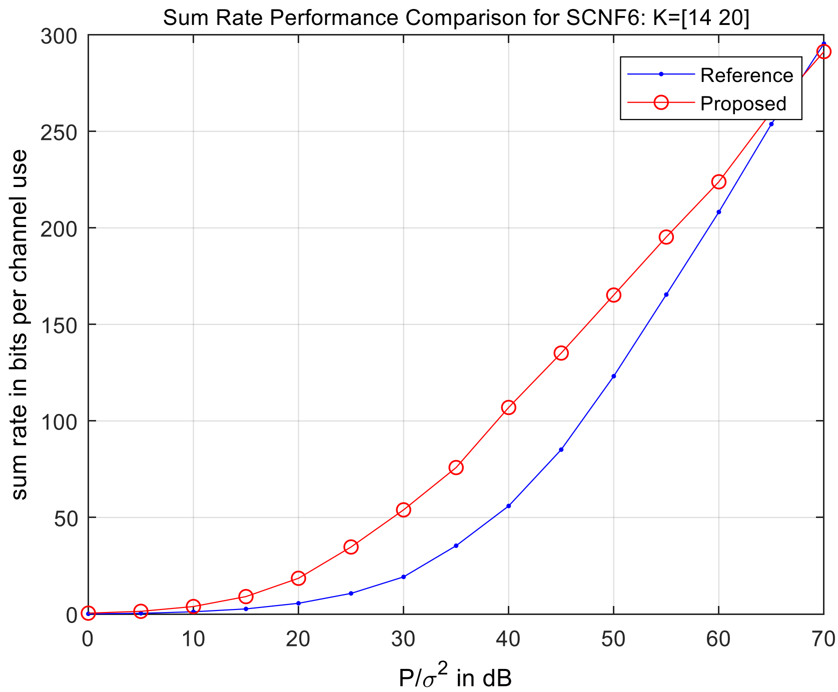

Figure 4.16. Sum Rate Comparison for SCNF6

Likewise, we listed the scenarios for L = 3, d = 1 and for changing values of K in Table 4.5. Sum rate simulation results are very similar with the scenarios where L = 2. As we increase number of nodes per group, again proposed algorithm performs better up to an SNR (P/σ^2) threshold.

Table 4.5. *Summary of scenarios for Fair Comparison, L = 3*

Scenario	L_{org}	L_1	K_{org}	K_1	M	R_{org}	R_1	N_{org}	N_1	T_{org}	T_1	P^{rly}_{org}	P^{rly}_1	N^{tot}_{org}	N^{tot}_1	P^{tot}_{org}	P^{tot}_1
SCNF7	3	3	5	6	6	12	3	2,6	2,67	5	12	80	37,83	51	51	335	335
SCNF8	3	3	8	10	10	21	3	2,75	2,8	8	20	80	28,4	87	87	584	584
SCNF9	3	3	11	14	14	30	3	2,82	2,86	11	28	120	46,07	123	123	1233	1233
SCNF10	3	3	14	18	18	39	3	2,86	2,89	14	36	120	35	159	159	1602	1602

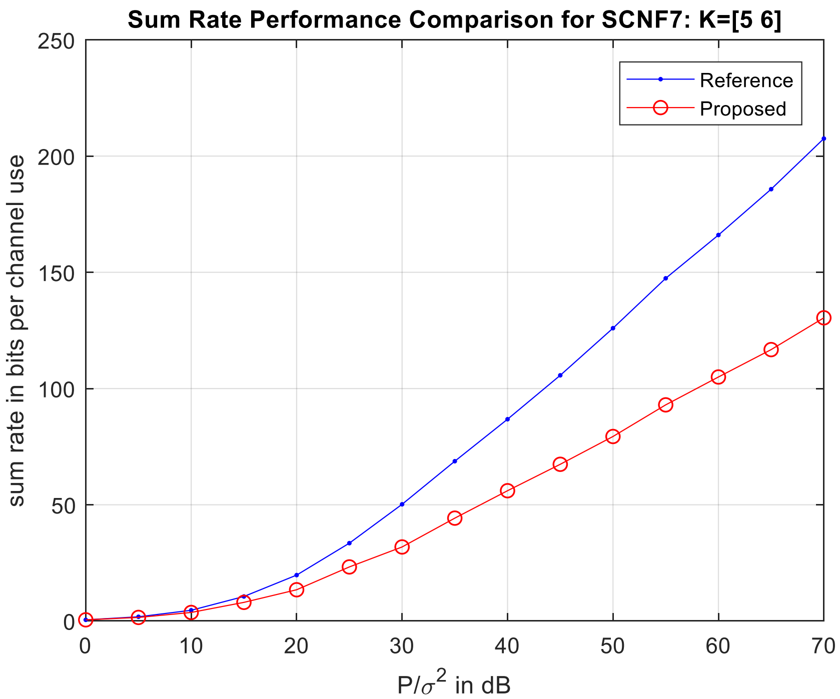

Figure 4.17. Sum Rate Comparison for SCNF7

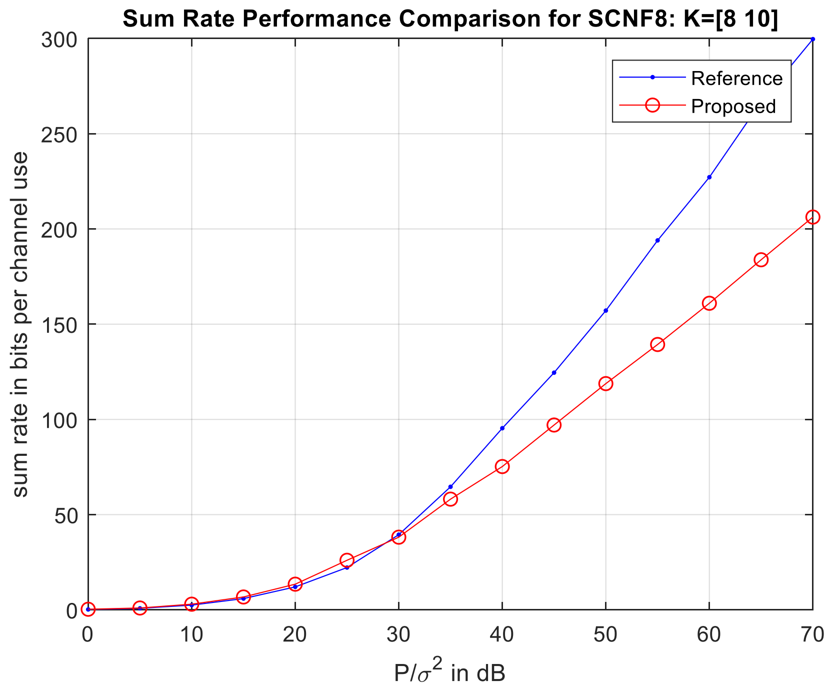

Figure 4.18. Sum Rate Comparison for SCNF8

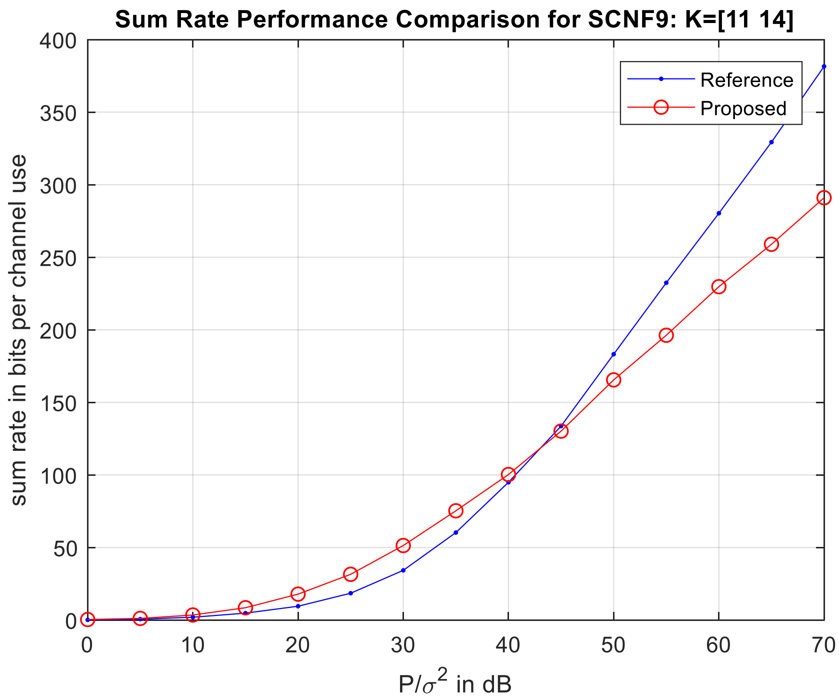

Figure 4.19. Sum Rate Comparison for SCNF9

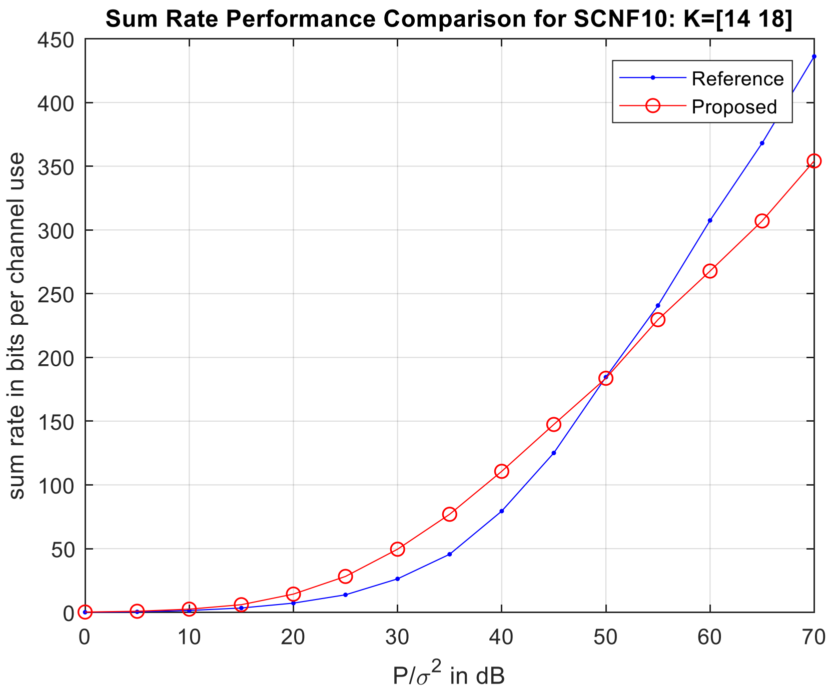

Figure 4.20. Sum Rate Comparison for SCNF10

Fair Comparison Scenarios – Increasing Number of Groups

Next we compare two algorithms when K and d are constant but L is increasing. This way we want to see how the sum rate difference between algorithms change as we increase L. Scenarios for increasing number of groups are summarized in Table 4.6, Table 4.7, Table 4.8 and Table 4.9. In each of the tables, different number of nodes per group are used. Sum rate plots of the scenarios given in the relevant tables, are given after the tables.

We see from the sum rate plots of scenarios (from SCNF11 to SCNF21) with increasing number of groups, relative sum rate performance of the two algorithms doesn't change considerably as the number of groups (L) increase. This is something expected since the number of nodes per group doesn't change in between scenarios and so, subgroup size (M) cannot be increased. Therefore, there is no change in the power advantage at the relay that is offered by the proposed algorithm. Changing the number of groups doesn't change total time slots for both of the algorithms either. So we can conclude that, parameters which effect the sum rate doesn't change by changing L. In all the fair scenarios we could find for increasing L case, the number of nodes per group was not high enough to show that the proposed algorithm performs better. For a scenario with large number of nodes per group, proposed algorithm will still perform better compared to reference algorithm. In the scenarios SCNF20 and SCNF21 we can see the highest number of nodes per group for the increasing L scenarios. And as it is seen clearly, although L is as high as 14 for the reference algorithm and 13 for the proposed algorithm, proposed algorithm performs slightly better up to P/σ^2 is equal to 30 dB.

For $K_{org} = 4$ and $K_1 = 4$, we present sum rate comparison of the two algorithm for increasing L values as seen on Table 4.6.*Table 4.7*

Table 4.6. *Summary of Scenarios for Fair Comparison, K = 4*

Scenario	L_{org}	L_1	K_{org}	K_1	M	R_{org}	R_1	N_{org}	N_1	T_{org}	T_1	P^{rly}_{org}	P^{rly}_1	N^{tot}_{org}	N^{tot}_1	P^{tot}_{org}	P^{tot}_1
SCNF11	8	7	4	4	2	24	14	6,25	7,5	4	6	185,33	133	224	224	588	588
SCNF12	15	13	4	4	2	45	26	11,5	13,5	4	6	344	247	735	728	1092	1092
SCNF13	16	14	4	4	2	48	28	12,25	14,5	4	6	370,67	266	832	840	1176	1176

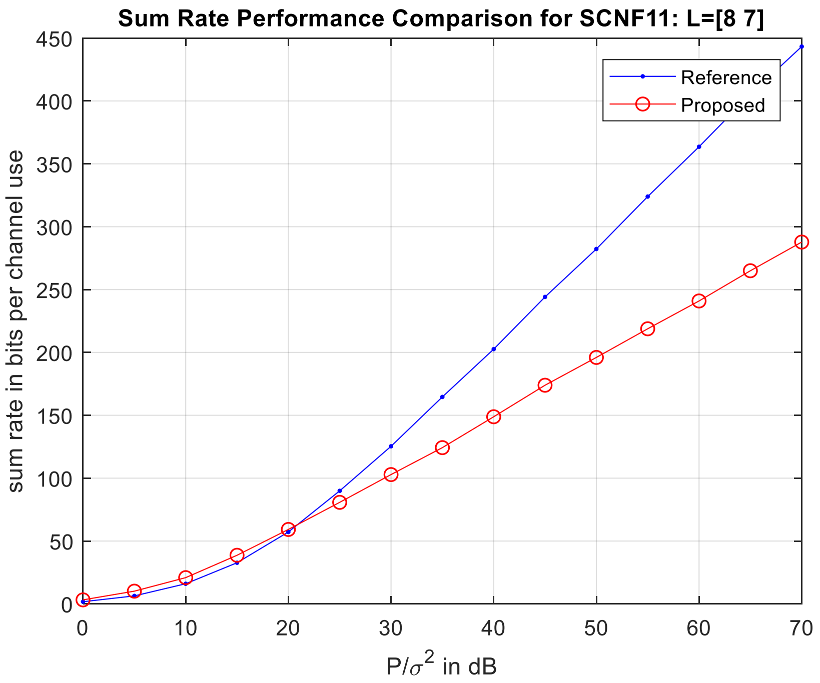

Figure 4.21. Sum Rate Comparison for SCNF11

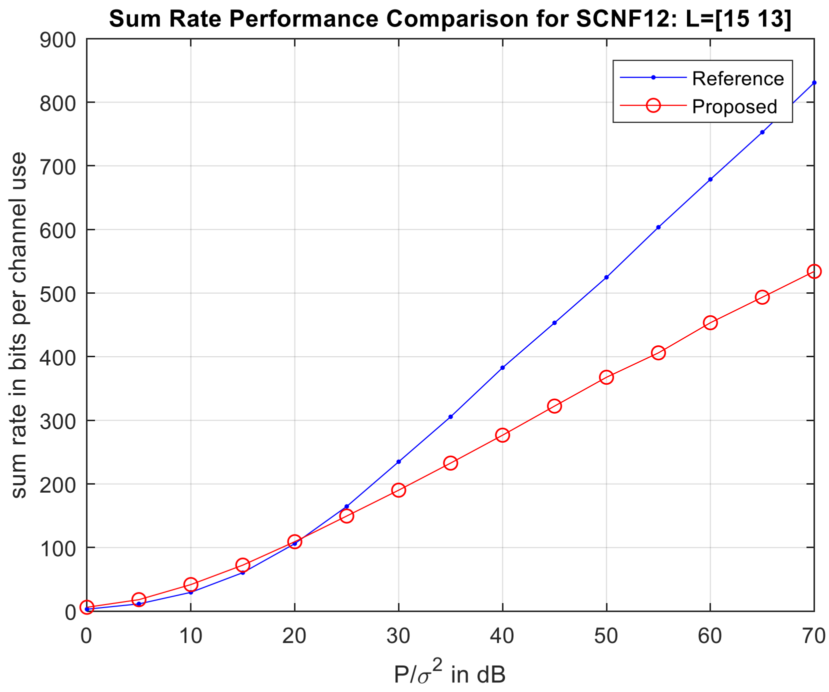

Figure 4.22. Sum Rate Comparison for SCNF12

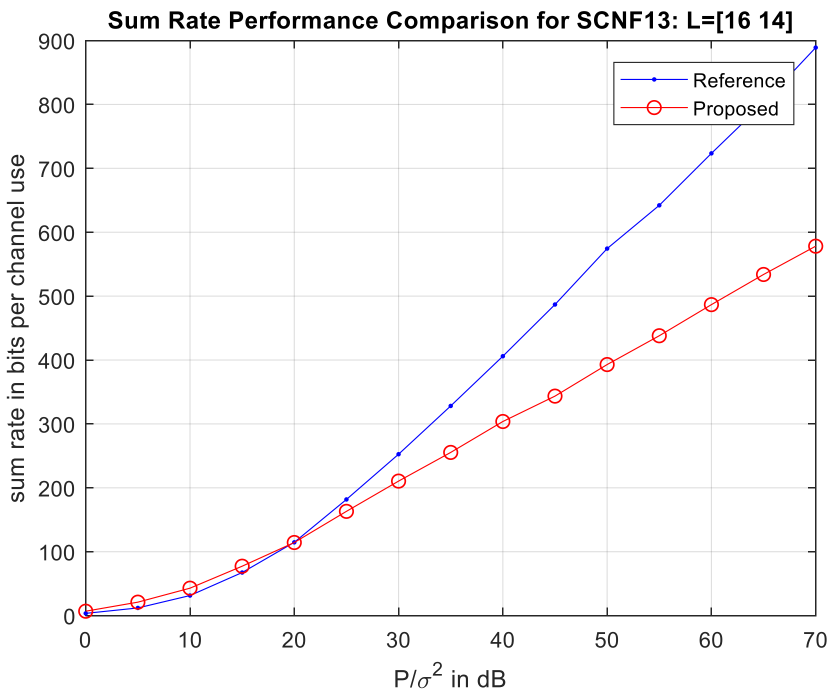

Figure 4.23. Sum Rate Comparison for SCNF13

Again, for $K_{org} = 4$ and $K_1 = 5$, we present sum rate comparison of the two algorithm for increasing L values as seen on Table 4.7.

Table 4.7. *Summary of Scenarios for Fair Comparison, K_{org} = 4, K_1 = 5*

Scenario	L_{org}	L_1	K_{org}	K_1	M	R_{org}	R_1	N_{org}	N_1	T_{org}	T_1	$P^{rly}{}_{org}$	$P^{rly}{}_1$	$N^{tot}{}_{org}$	$N^{tot}{}_1$	$P^{tot}{}_{org}$	$P^{tot}{}_1$
SCNF14	2	2	4	5	5	6	2	1,75	1,8	4	10	45,667	19	20	20	145	145
SCNF15	10	9	4	5	5	30	9	7,75	7,4	4	10	204,16	85,5	340	342	653	653
SCNF16	17	15	4	5	5	51	15	13	12,2	4	10	339,83	142,5	935	930	1087	1087

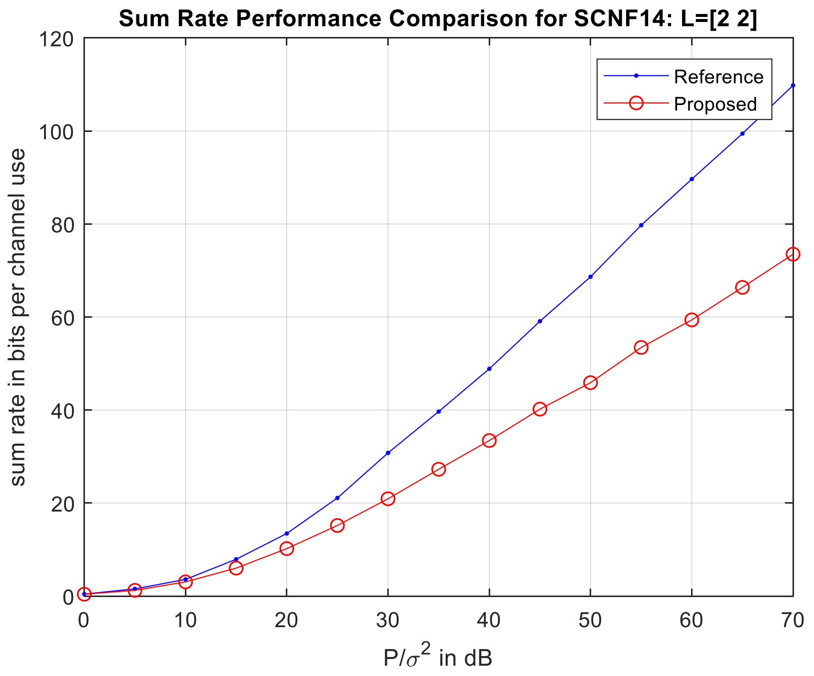

Figure 4.24. Sum Rate Comparison for SCNF14

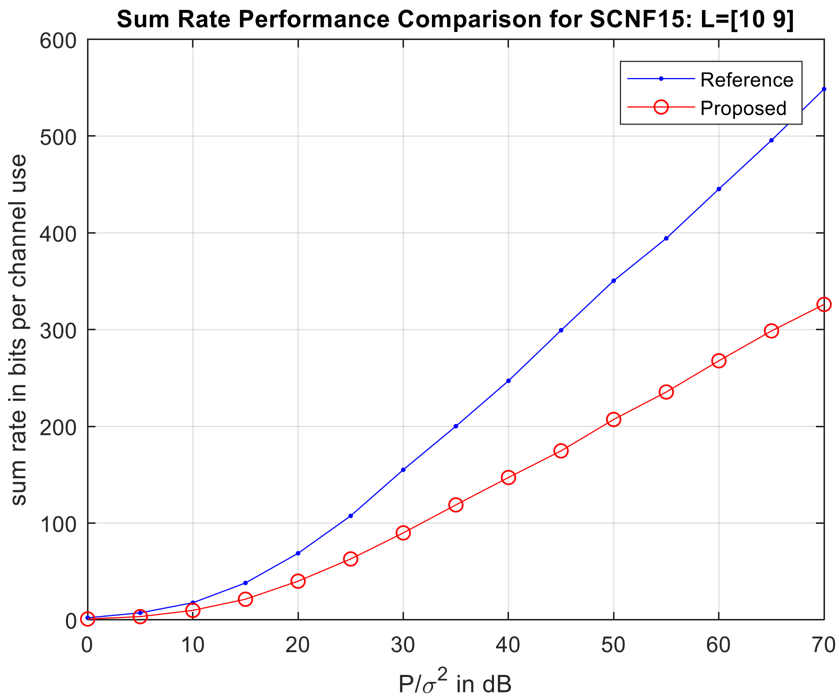

Figure 4.25. Sum Rate Comparison for SCNF15

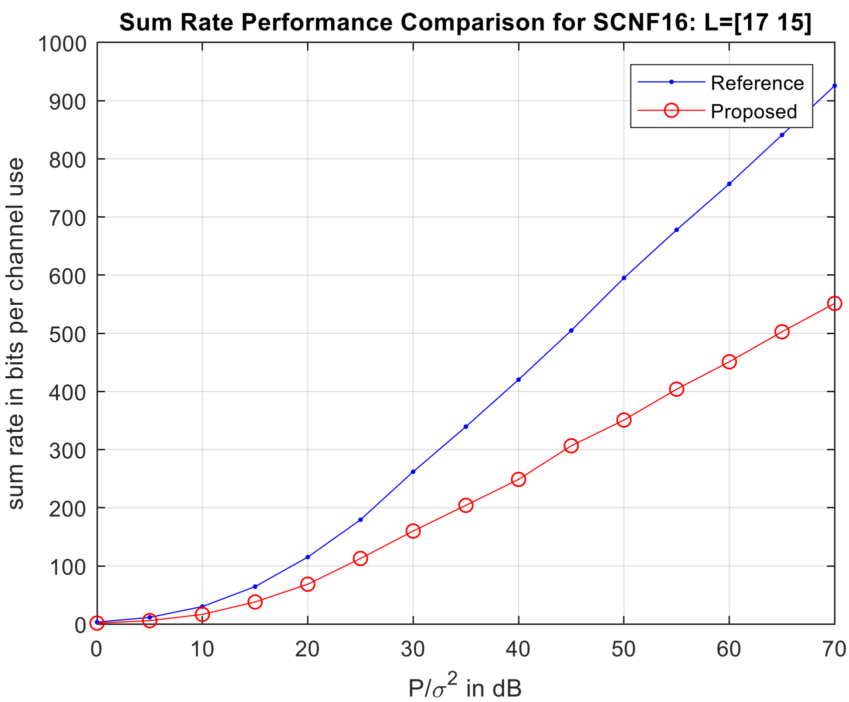

Figure 4.26. Sum Rate Comparison for SCNF16

Here we also present sum rate comparison of the algorithm for increasing L values, for a constant but relatively larger K values compared to previous ones, namely for $K_{org} = 8$ and $K_1 = 10$.

Table 4.8. *Summary of Scenarios for Fair Comparison, K_{org} = 8, K_1 = 10*

Scenario	L_{org}	L_1	K_{org}	K_1	M	R_{org}	R_1	N_{org}	N_1	T_{org}	T_1	P^{rly}_{org}	P^{rly}_1	N^{tot}_{org}	N^{tot}_1	P^{tot}_{org}	P^{tot}_1
SCNF17	3	3	8	10	10	21	3	2,75	2,8	8	20	80,14	28,5	87	87	585	585
SCNF18	12	11	8	10	10	84	11	10,625	10	8	20	292,71	104,5	1104	1111	2145	2145
SCNF19	20	18	8	10	10	140	18	17,625	16,3	8	20	478,57	171	2960	2952	3510	3510

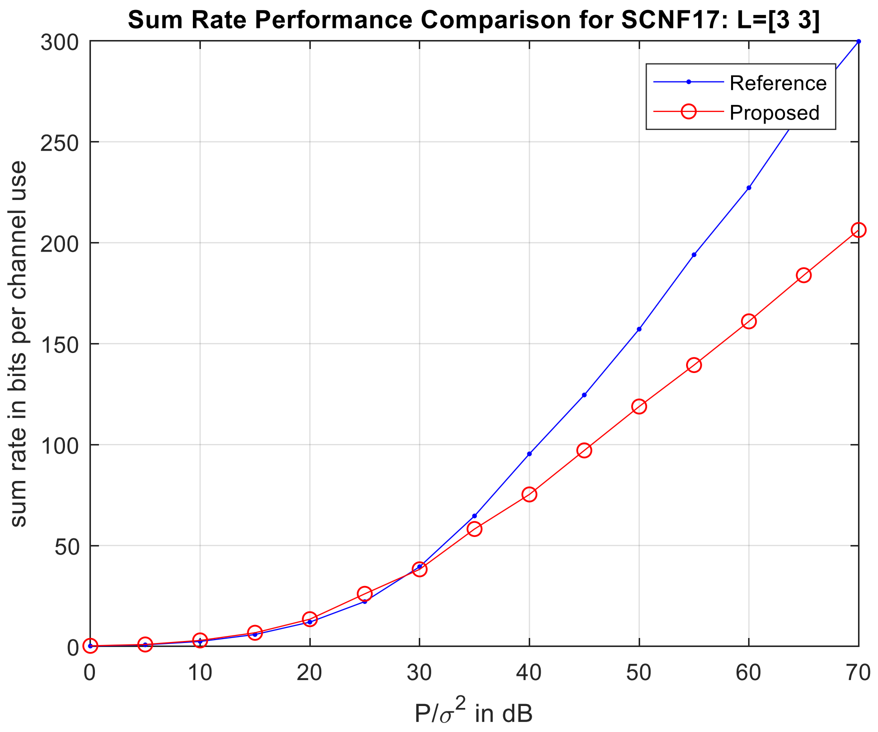

Figure 4.27. Sum Rate Comparison for SCNF17

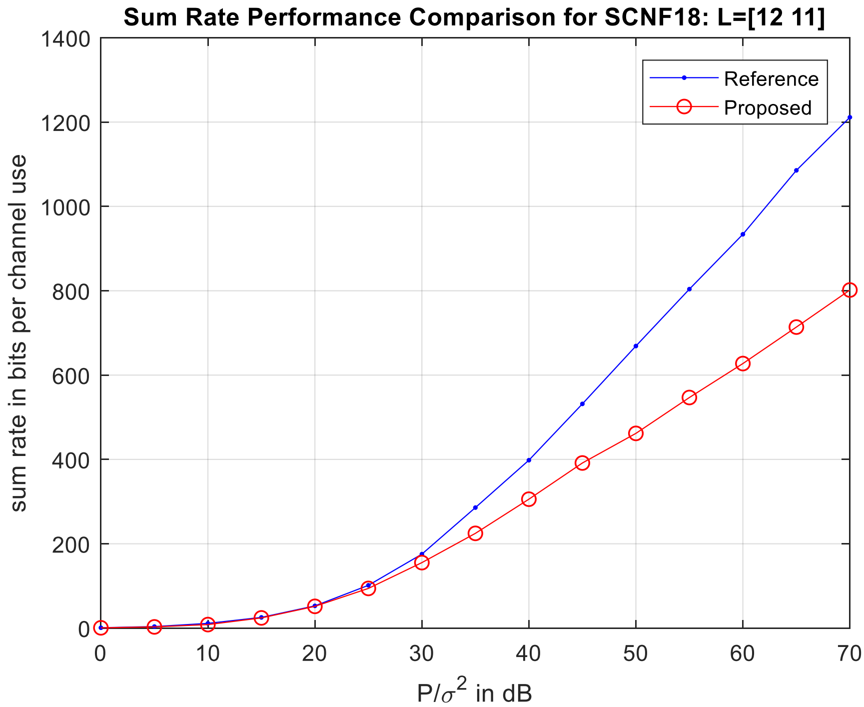

Figure 4.28. Sum Rate Comparison for SCNF18

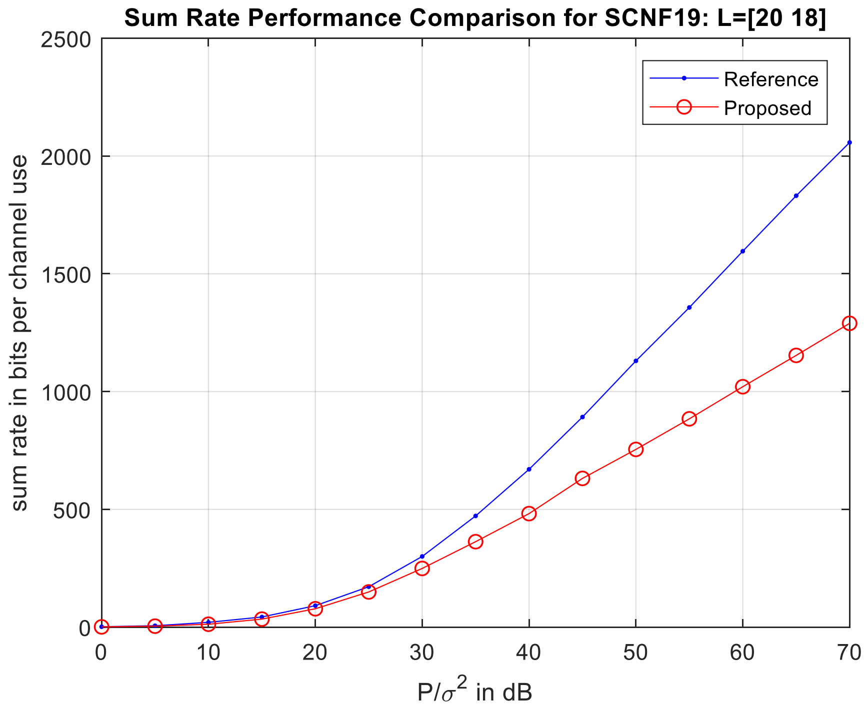

Figure 4.29. Sum Rate Comparison for SCNF19

Finally, we present sum rate comparison of the two algorithm for the largest constant K values can be found with fair parameters to be compared, namely for $K_{org} = 14$ and $K_1 = 13$.

Table 4.9. *Summary of Scenarios for Fair Comparison, K_{org} = 10, K_1 = 12*

Scenario	L_{org}	L_1	K_{org}	K_1	M	R_{org}	R_1	N_{org}	N_1	T_{org}	T_1	P^{rly}_{org}	P^{rly}_1	N^{tot}_{org}	N^{tot}_1	P^{tot}_{org}	P^{tot}_1
SCNF20	4	4	10	12	12	36	4	3,7	3,75	10	24	110,22	38	184	184	1032	1032
SCNF21	14	13	10	12	12	126	13	12,7	12	10	24	357,11	123,5	1904	1885	3354	3354

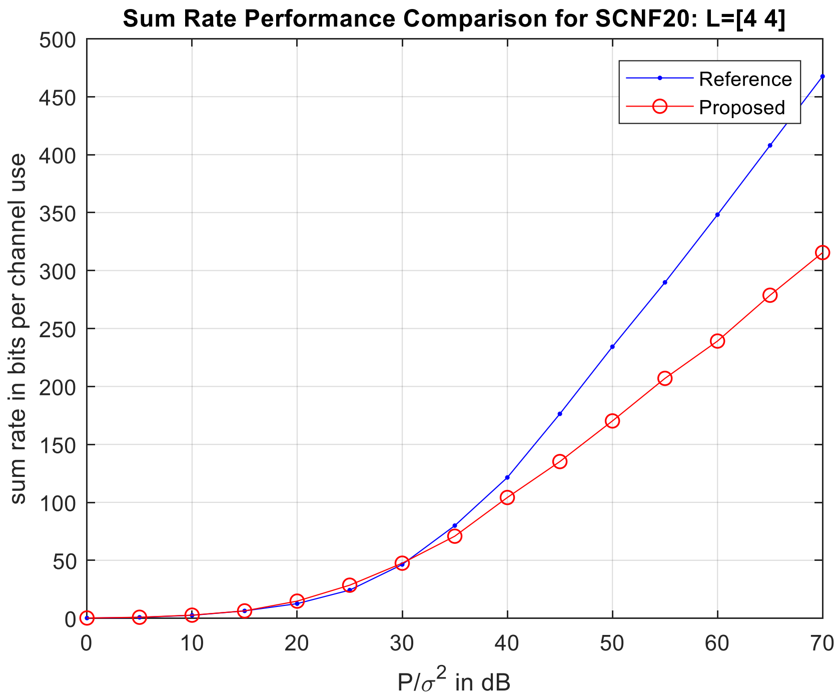

Figure 4.30. Sum Rate Comparison for SCNF20

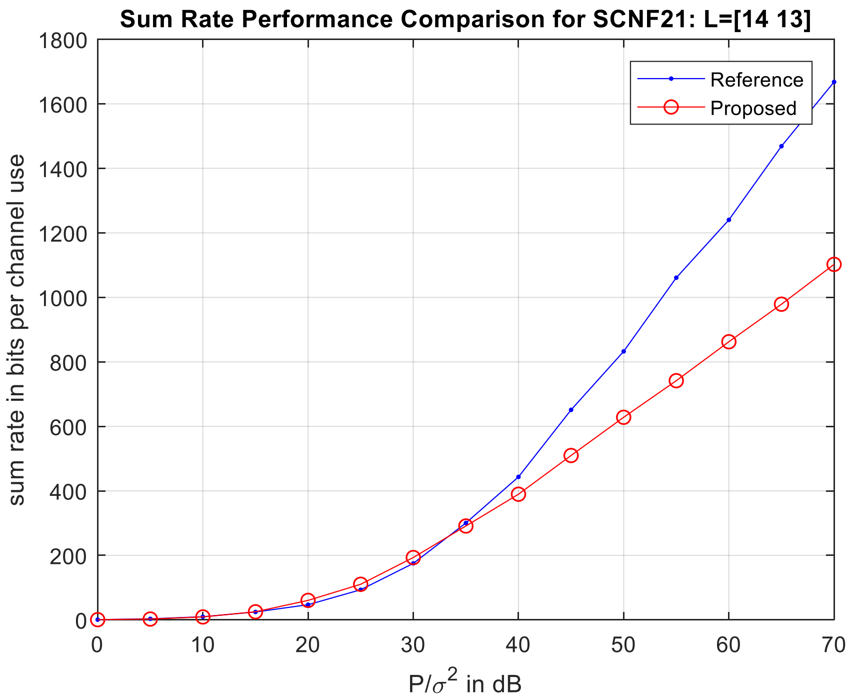

Figure 4.31. Sum Rate Comparison for SCNF21

Fair Comparison Scenarios – Increasing Number of Data Streams

We also searched fair parameter scenarios with changing d and constant L and K. This way we examined the effect of number of data streams – “d” on the sum rate performance of the proposed and reference algorithms. Fair comparison scenarios with increasing number of data streams can be seen in Table 4.10 and Table 4.11.

As it is seen from the sum rate plots of the scenarios from SCNF22 to SCNF29, changing number of data streams “d”, does not cause one of the algorithms perform better. It behaves like a coefficient to the sum rates for both of the algorithms. This is something expected since “d” parameter mainly scales number of antennas for both of the algorithms. We can’t change number of nodes, and therefore subgroup size. So the power advantage of the proposed system at the relay remain same although we increase number of data streams.

Table 4.10. *Summary of Scenarios for Fair Comparison for increasing d, K =[4 5]*

Scenario	L_{org}	L_1	K_{org}	K_1	M	d_{org}	d_1	R_{org}	R_1	N_{org}	N_1	T_{org}	T_1	$P^{rly}{}_{org}$	$P^{rly}{}_1$	$N^{tot}{}_{org}$	$N^{tot}{}_1$	$P^{tot}{}_{org}$	$P^{tot}{}_1$
SCNF22	2	2	4	5	5	1	1	6	2	1,75	1,8	4	10	80	39,6	20	20	248	248
SCNF23	2	2	4	5	5	2	2	12	4	3,5	3,6	4	10	80	39,6	40	40	248	248
SCNF24	2	2	4	5	5	3	3	18	6	5,25	5,4	4	10	80	39,6	60	60	248	248
SCNF25	2	2	4	5	5	4	4	24	8	7	7,2	4	10	80	39,6	80	80	248	248

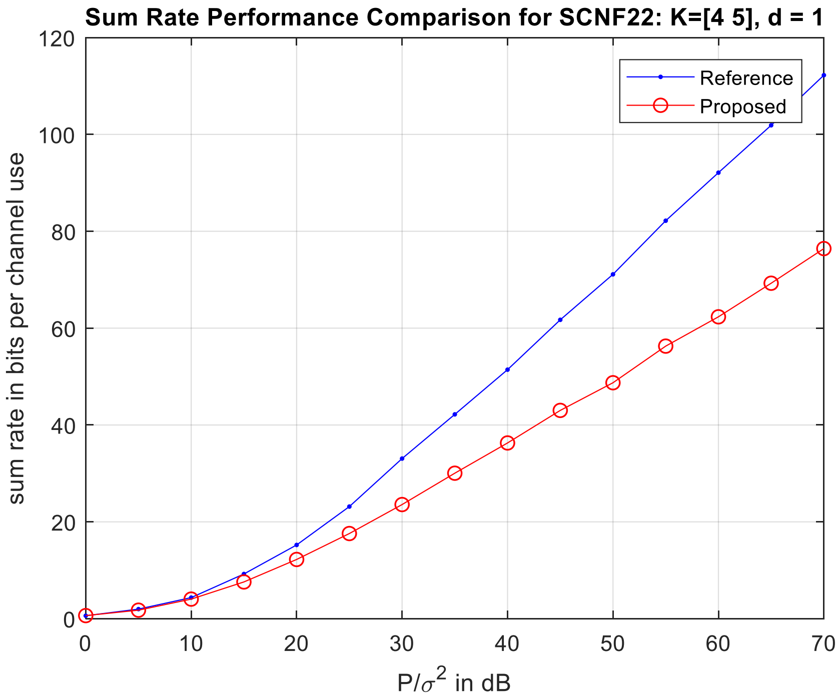

Figure 4.32. Sum Rate Comparison for SCNF22

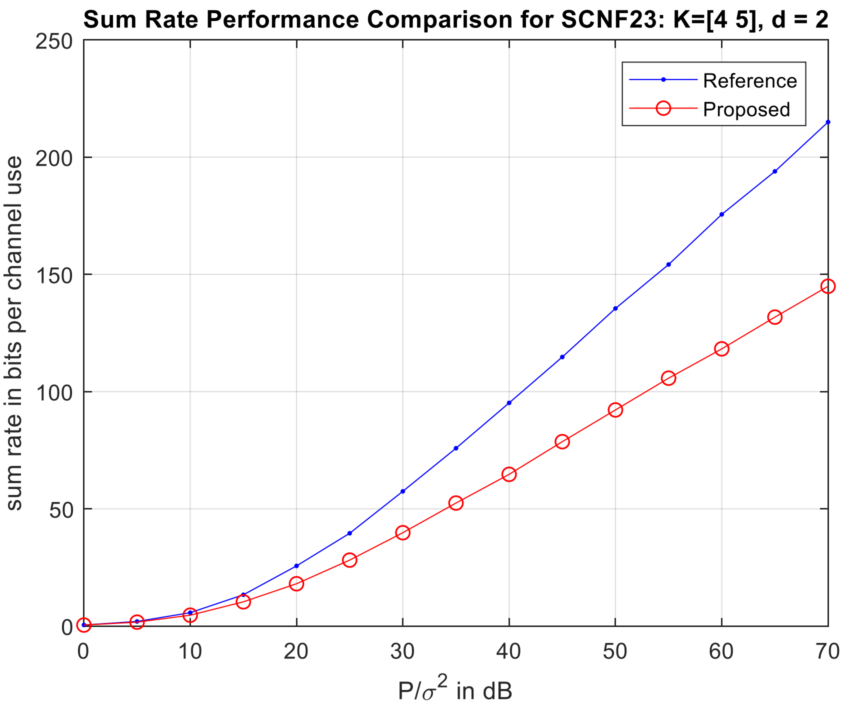

Figure 4.33. Sum Rate Comparison for SCNF23

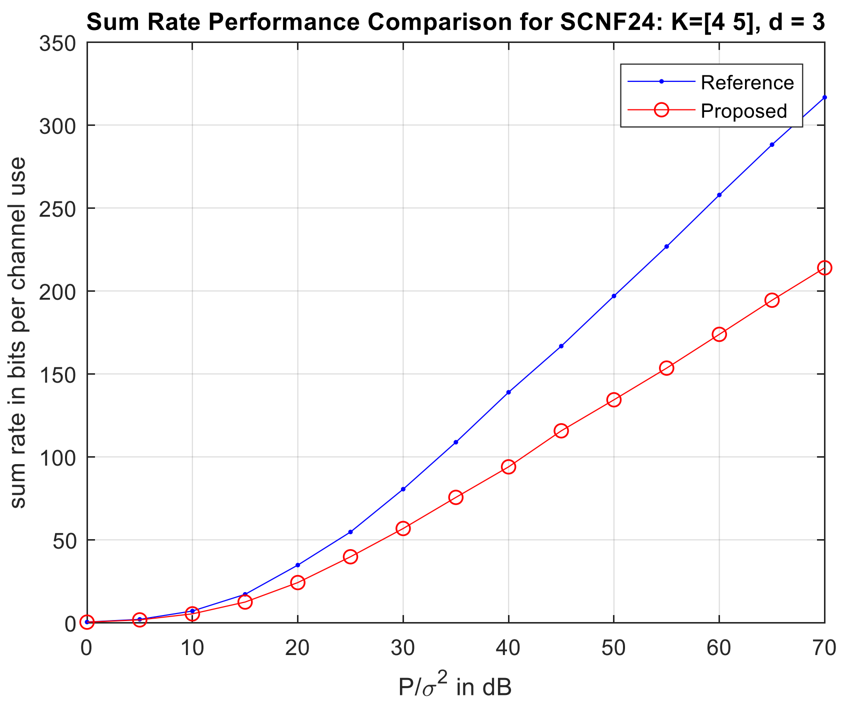

Figure 4.34. Sum Rate Comparison for SCNF24

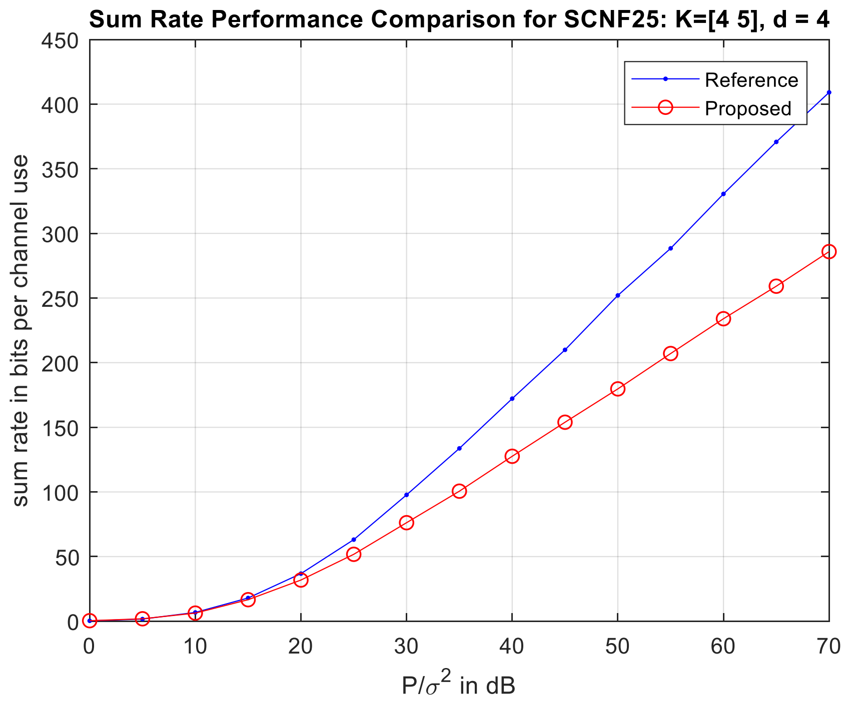

Figure 4.35. Sum Rate Comparison for SCNF25

Table 4.11. *Summary of Scenarios for Fair Comparison for increasing d, K =[6 8]*

Scenario	L_{org}	L_1	K_{org}	K_1	M	d_{org}	d_1	R_{org}	R_1	N_{org}	N_1	T_{org}	T_1	$P^{rly}{}_{org}$	$P^{rly}{}_1$	$N^{tot}{}_{org}$	$N^{tot}{}_1$	$P^{tot}{}_{org}$	$P^{tot}{}_1$
SCNF26	2	2	6	8	8	1	1	10	2	1,83	1,88	6	16	80	35,5	32	32	412	412
SCNF27	2	2	6	8	8	2	2	20	4	3,67	3,75	6	16	80	35,5	64	64	412	412
SCNF28	2	2	6	8	8	3	3	30	6	5,5	5,63	6	16	80	35,5	96	96	412	412
SCNF29	2	2	6	8	8	4	4	40	8	7,33	7,5	6	16	80	35,5	128	128	412	412

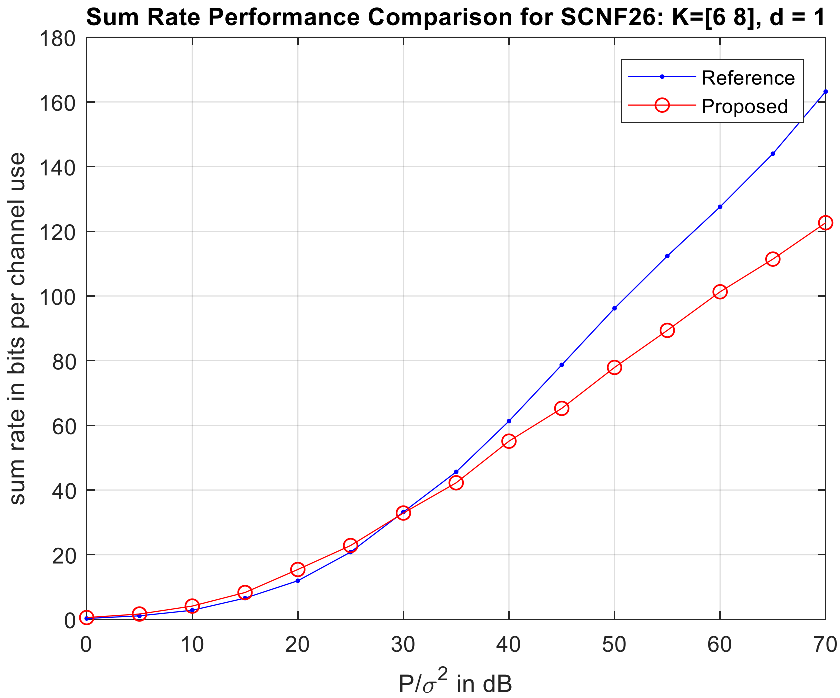

Figure 4.36. Sum Rate Comparison for SCNF26

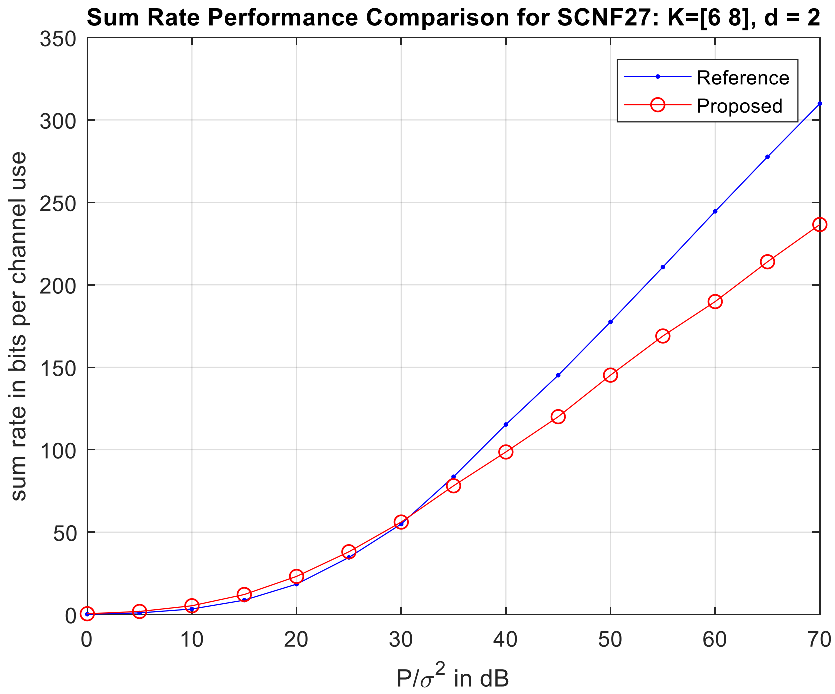

Figure 4.37. Sum Rate Comparison for SCNF27

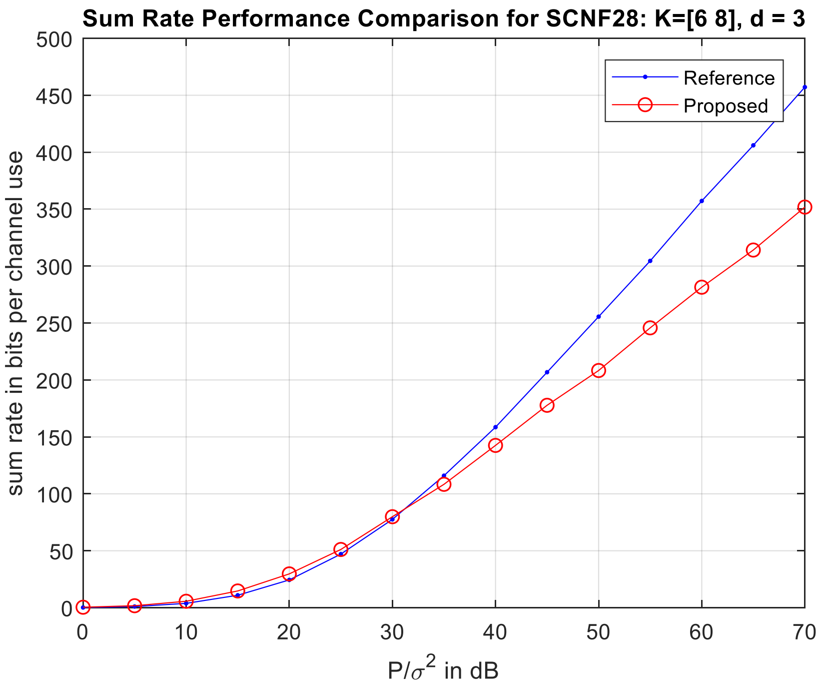

Figure 4.38. Sum Rate Comparison for SCNF28

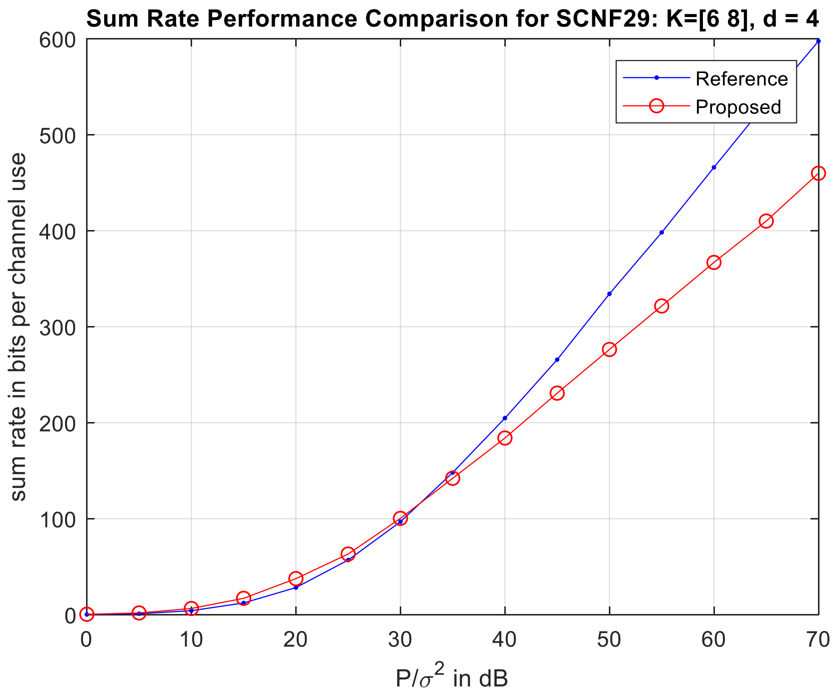

Figure 4.39. Sum Rate Comparison for SCNF29

CHAPTER 5

CONCLUSION

In this work we studied Interference Alignment on multigroup multiway relay channels and proposed a new transmission scheme for multigroup multiway relay channels.

In Chapter 2, we described Interference Alignment algorithm developed for K-user interference channels given in [1]. Method for aligning arbitrary number of interferers into the same subspace at each receiver is described. In a K-user interference channel, how each user can use half of the total channel capacity asymptotically is explained based on this interference alignment algorithm.

In Chapter 3, we described multigroup multiway relay channel system model and also an interference alignment algorithm for multigroup multiway relay channels given in [7]. We simulated the described system and presented sum rate performance of the given algorithm. Moreover, we investigated several aspects of this algorithm. We reformulated the number of relay antennas under the assumption that groups may have different number of nodes. For this case, we also proposed a method which will improve sum rate by utilizing extra broadcast phases for the groups with less number of nodes compared to the other groups in the same system. Finally, we investigated the case for rank deficient channel matrix and proposed a method based on diagonal loading. We have done several simulations for the cases of channel matrix, namely no rank deficiency and rank deficiency case respectively. In case of rank deficiency, the results of the proposed compensation method are presented.

In Chapter 4, we proposed a new transmission scheme based on interference alignment for multigroup multiway relay channels. In this method, we divided groups into subgroups, and aligned all the nodes in a subgroup into the same subspace at the relay. This way, we provided the flexibility to tradeoff between MAC phase slots and the number of relay antennas. We compared the reference algorithm described in Chapter 3 with the proposed algorithm in several aspects. First, main system parameters such as the number of relay and node antennas are compared based on the formulations. Moreover, we plotted the number of relay antennas (R) and number of node antennas (N) parameters versus the number of nodes per group (K) and the subgroup size (M) parameters and then versus the number of groups (L) and the subgroup size (M) parameters as a surface plot. We compared the sum rates of the proposed and the reference algorithm for specific scenarios by changing M values. Then, in order to have a fair comparison between the proposed and the reference system, total power and total number of antennas are used. Hence for the same total number of antennas, total power is set equal for both of the algorithms. Under this case, sum rate is found for different SNR values. It is shown that the proposed method has better sum rate when the number of groups is low and the users in each group is larger.

As a future work to our thesis, a power allocation scheme in-between nodes or between nodes and relay can be investigated for both of the algorithms. A water filling scheme can be utilized to maximize sum rate. Total power budget can be used in an optimum way in between nodes and relay.

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