AN INVESTIGATION ON SEVENTH GRADE STUDENTS' USE OF BAR MODEL METHOD IN SOLVING ALGEBRAIC WORD PROBLEMS

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This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

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ABSTRACT

AN INVESTIGATION ON SEVENTH GRADE STUDENTS' USE OF BAR MODEL METHOD IN SOLVING ALGEBRAIC WORD PROBLEMS

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The purpose of this research is understanding the use of bar model method used in Singapore math curriculum while solving algebra word problems in 7th grades and students' reasons for the solution method preferences in solving algebraic word problems. The data were collected from 10 seventh grade students from a public middle school in Sincan, Ankara, in the spring semester of 2018-2019. In this single case study, an initial assessment was applied to 42 seventh grade students for selecting the participants. Students' errors were analyzed and 10 students were selected. A three-hour instruction about solving algebraic word problems with bar model method was provided these students. After the instruction, a clinical interview with each student was carried out. During these interviews, students solved the algebraic word problems with any method that they want and shared their thoughts about using bar model method.

The results indicated that bar model is an effective method for remedying seventh grade students' errors in solving algebraic word problems. Particularly, it resulted with a positive role on seven of 10 students' performances. This study revealed that bar model method is useful for visualizing the problem content. Although some of the students experienced difficulties in solving problems, nine of the 10 students found this method easier and more interesting. They also preferred the bar model method instead of the algebraic equation method. Thus, this study suggests providing students with more oppourtunities to use the bar model method in algebra problems at various difficulty levels.

Keywords: Bar Model Method, Singapore Math, Algebraic Word Problems, Algebraic Equation, Middle School Students

ÖZ

YEDİNCİ SINIF ÖĞRENCİLERİNİN CEBİR PROBLEMLERİNİN ÇÖZÜMÜNDE BAR MODEL YÖNTEMİNİ KULLANIMININ İNCELENMESİ

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Bu araştırmanın amacı, Singapur matematik öğretim programında yaygın olarak kullanılan bar model yönteminin yedinci sınıf öğrencilerinin cebir problemlerinin çözümündeki kullanımını anlamaktır. Bu çalışma aynı zamanda yedinci sınıf öğrencilerinin denklem problemlerini çözerken kullandıkları metodu tercih etme sebeplerini öğrenmeyi de amaçlamaktadır. Araştırma verileri 2018-2019 eğitim-öğretim yılının bahar döneminde, Sincan, Ankara'da yer alan bir devlet ortaokulundaki 10 yedinci sınıf öğrencisinden toplanmıştır. Tekli durum araştırma deseni kullanılan bu çalışmada, katılımcıları seçmek için 42 yedinci sınıf öğrencisine ön değerlendirme testi uygulanmıştır. Bu testte yapılan hata türleri analiz edilmiş ve çalışma için 10 öğrenci seçilmiştir. Seçilen öğrencilere üç ders saati süren, denklem problemlerini bar model yöntemiyle çözmeyi öğreten bir eğitim verilmiştir. Bu eğitimin ardından, öğrencilerle klinik görüşmeler yapılarak, verilen denklem problemlerini istedikleri yöntemle çözmeleri istenmiş ve bar model yöntemi hakkındaki görüşleri alınmıştır.

Çalışmanın sonuçları yedinci sınıf öğrencilerinin denklem problemleri çözerken kullandıkları bar model yönteminin yaptıkları hataları azaltmakta etkili olduğunu göstermektedir. Bar model yöntemi 10 katılımcının özellikle yedisinin performansı üzerinde olumlu bir role sahiptir. Bu çalışma aynı zamanda bar model yönteminin öğrencilerin problem içeriğini görselleştirmelerinde faydalı olduğunu göstermiştir. Öğrenciler bu yöntemle bir takım zorluklar yaşasalar da, 10 öğrenciden dokuzu yöntemi sevdiğini ve ilgi çekici bulduğunu, bu yöntemi denklem kurma yöntemine tercih ettiğini belirtmiştir. Dolayısıyla çalışmanın verileri, öğrencilere farklı zorluk seviyelerinde problemlerle bar model yöntemini kullanma şansı verilmesi önerilmektedir.

Anahtar Kelimeler: Bar Model Yöntemi, Singapur Matematiği, Denklem Problemleri, Denklem, Ortaokul Öğrencileri

To My Husband and My Family

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LIST OF ABBREVIATIONS

NCTM	National Council of Teachers of Mathematics
MoNE	Ministry of National Education
TIMSS	Trends in International Mathematics and Science Study
PISA	Programme for International Student Assessment
AE	Algebraic Equation
BM	Bar Model
PS	Problem Statement
CPDD	Curriculum Planning & Development Division
CPA	Concrete-Pictorial-Abstract

CHAPTER 1

INTRODUCTION

Algebra is an important area for mathematics education because as Lacampagne (1995) states, “Algebra is the language of mathematics. It opens doors to more advanced mathematical topics for those who master basic algebraic concepts. It closes doors to college and to technology-based careers for those who do not” (p. 237). Similarly, NCTM (2000) states that students should start learning algebra from the beginning of elementary school. They can start by learning Early Algebra and then continue learning algebra until high school because algebra education is important for both university life and work life.

In Turkey, instructional objectives related to algebra are first observed in the 6^{th} grade curriculum. Accordingly, 6^{th} grade students are expected to find n^{th} term in patterns and make sense of algebraic expressions. Moreover in 7^{th} grade, students should do addition and subtraction with algebraic expressions, understand the meaning of equality, and solve one unknown equation and algebra word problems (MoNE, 2018). Evidently, solving algebraic word problems is one of the topics that students are expected to learn in 7^{th} grade.

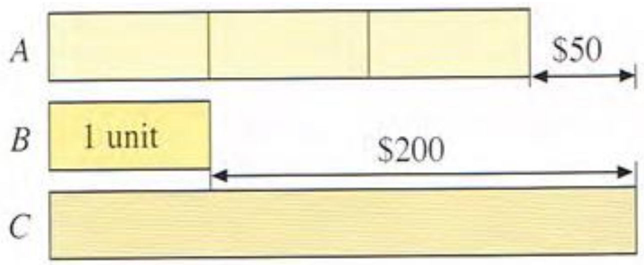
Cummins (1991) states that word problems are significant in mathematics education. Word problems should not be regarded merely as solving equations that are readily given. Instead, words or visuals as givens in the problem are expected to be explained with symbols, letters, and numbers before the problems are solved. A word problem can be defined as a verbal explanation of the problem context in which numerical data are given to be used for some mathematical operations to find the answer to the problem (Verschaffel, Greer, & De Corte, 2000). Bednarz and Janvier (1996) stated that algebra can be regarded as a new and a more effective method for solving word problem, when compared to arithmetic ways of solving problems. Solving word problems by using algebraic methods, like writing an equation, constitutues a common method for various types of problems. That's why learning and solving algebraic word problems are important. Similarly, algebra enables one to understand the relationships among variables in word problems and to convert them into algebraic equations by using numbers and symbols. Therefore, solving word problems and producing different solutions to them are possible with algebra (Özarslan, 2010). Besides, Özarslan (2010) states that transforming algebraic word problems into equations and solving them are important for transition from arithmetic to algebra.

Word problems are significant just like other areas of mathematics. However, solving these problems by using algebra is a serious obstacle for students in their school life (Lawrance, 2007). Despite the importance of algebraic methods in solving word problems, learning and teaching how to solve word problems is difficult (Stacey & MacGregor, 2000). Its difficulty can be attributed to the nature of algebra. Algebra is abstract for some students because it involves variables, symbols and letters (Kieran & Chalouh, 1993). Moreover, in word problems, students are expected to write algebraic equations based on the problem statement and solve the equation; however, this process can be difficult for students because they need to use and manipulate the algebraic symbols (Stacey & MacGregor, 2000). Some studies in the related literature shed light on the reason why students find algebraic word problems difficult (Adu, Assuah & Asideu-Addo, 2015; Jupri & Drivers, 2016; Kayani & Ilyas, 2014; Ladele, 2013). To illustrate, one study reported that students experienced difficulties while solving algebra word problems because they (1) could not know the meaning of symbols and letters, (2) could not understand the problem context, (3) could not write an algebraic equation appropriate for the problem content, and (4) could not solve the equation correctly (Newman, 1983b as cited in Ladele, 2013). TIMSS scores of students in Turkey is explicitly observed to be lower than the international average in the domain of algebra (Bütüner and Güler, 2017). Hence, it is obvious that students in Turkey experience difficulties in algebra. Kayani and Ilyas (2014) emphasized the importance of using different methods to overcome these difficulties and the abstract nature of algebra.

To facilitate the process of solving algebra word problems, using different teaching methods can be helpful. Some teaching strategies are using manipulatives and representations to solve problems which help students in their transition from abstract to concrete. One of the strategies is using representations. NCTM (2000) states that representation is an important way to make problem context more concrete for pupils because they can utilize representations to organize their thoughts. From 6 to 8^{th} grades, students can use this strategy to show, explain or broaden a mathematical concept, and this helps them to solve word problems. Representations could be visual. One of them is diagrams, which can be defined as "displays [of] information in a spatial layout" (Diezmann & English, 2001, p.77). Diezmann and English (2001) emphasized the advantages of diagrams, which are they (1) explain the problem context, (2) simplify complicated problems, and (3) transition from abstract to concrete. This method also helps students to visualize the givens in the problem. Kho (1987) states that visualization is a tool that students can utilize to understand the nature of the problem. If they understand the problem, their probability of solving the problem increases. This diagram method is also used in the mathematics curriculum in Singapore, where it is called the Bar Model method.

Koleza (2015) explains that rectangular bars are used for numbers instead of using algebraic symbols like letters to symbolize unknowns in a problem, in the 'model method', which is also called graphical heuristic. Its name can vary from country to country. For example, the Japanese use the term 'tape diagrams' (Hino, 2019), while Americans use the term 'strip diagrams' (Beckmann, 2004) and Singaporeans use 'bar model' (Clark, 2017). In the present study, the term 'Bar Model Method' will be used. While solving algebra word problems with the bar model method, students should draw rectangular bars to demonstrate the givens in the problem context (Kaur, 2019). However, these bars or models do not represent real objects in the problem; they represent relationships among variables (Ng & Lee, 2009).

For example, Hong, Mei and Lim (2009) explained in their book how the bar model method should be used to improve students' knowledge of basic mathematics concepts and skill in solving word problems. They wrote problems, showed how to solve them by using both the bar model and the algebraic equation. One of the algebra problems was as follows: "A has 3 times as much money as B. B has $200 less than C. C has $50 more than A. Find the total amount of money that A, B and C have" (Hong, Mei and Lim, 2009, p. 56). The problem included three unknowns, and the relationship between the quantities was given in a context. Figure 1.1 depicts the solution of the problem with the bar model method.



2 units = $200 – $50 = $150
1 unit = $150 ÷ 2 = $75

A's money = 3 units = 3 × $75 = $225
B's money = $75
C's money = $225 + $50 = $275

Total amount of money = $225 + $75 + $275 = $575

Figure 1.1 Solution of the problem with the bar model method (Hong, Mei & Lim, 2009, p. 56)

To draw the correct model, B should be represented with a rectangular bar. Afterwards, A and C should be drawn in relation to B as staten in the problem context. Since the problem sentence states that A has 3 times as much money as B, three bars should be drawn for A. Moreover, C should be drawn 200 units more than B because it is stated that B has $200 less than C. Finally, C has $50 more than A, so the difference in units between A and C should be shown as 50. Comprehending the problem and drawing the correct model constitute the first step of the bar model method. The second step in solving the problem with the bar model necessitates using the correct operations and finding the value of one bar. The sequence of the operations leads the solution has been shown in Figure 1.1. This problem could also be solved with the algebraic equation model because the bar model method helps students to not only solve the problem but also write a suitable algebraic equation for the problem context. If students symbolize a bar with x, they can write other quantities according to the number of rectangular bars. As can be observed in Figure 1.2, based on the drawings of the bars in the model, B can be written as x, A can be written as 3x and C can be written as x + 200.

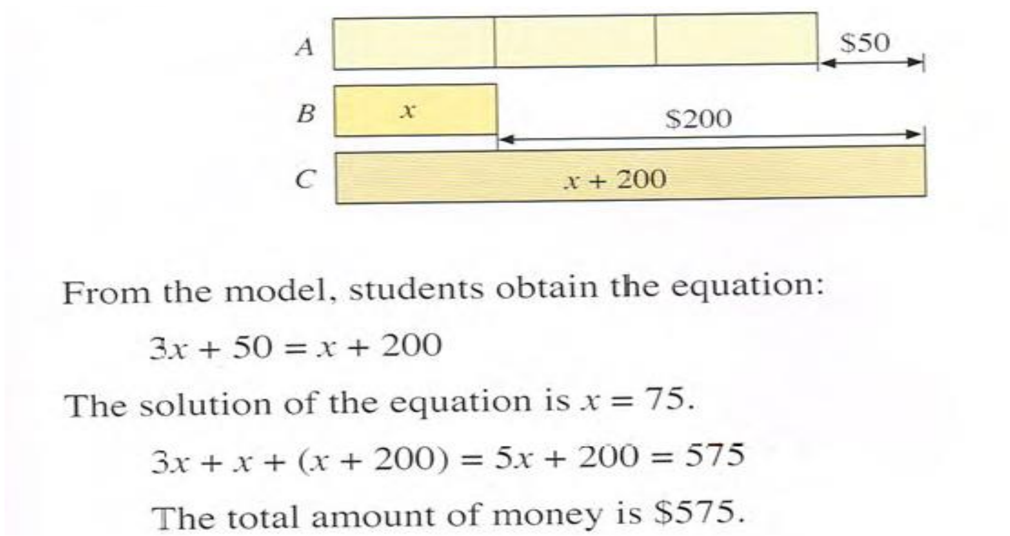


Figure 1.2 Algebraic equation of the problem (Hong, Mei & Lim, 2009, p. 57)

As can be seen in the Figure 1.2, 3x + 50 is equal to x + 200 because they are in alignment. This indicates that the length of the models are the same. Therefore, the equation will be 3x + 50 = x + 200. If this equation is solved, then it can be shown that x is 75. Since the problem asks for the total amount of money, all the quantities should be added.

Since 1980s, the bar model method has been the core visualization methods used in Singapore mathematics curriculum. The scores of students' mathematics exam in Singapore are observed to be very high when compared to those in other countries

(Bütüner & Güler, 2017) which directed some researchers conducting studies to gain insight into the effect of the bar model method (Hoven & Garelick, 2007; Mahoney, 2012; Ng & Lee, 2005; Ng & Lee, 2009; Waight, 2006). These studies mostly showed that the bar model method can support students' mathematical thinking and transition from abstract to concrete. For example, Mahoney (2012) found that using the bar model method affects students' problem solving skills positively. However, there is limited research that shows the impact of the bar model method on solving word problems, particularly algebraic word problems. Moreover, in the accessible literature no study is encountered on the effects of this method in Turkey. Thus, investigating the effects of the bar model method to solve word problems, particularly algebra word problems, is important.

The bar model method is regarded as a bridge between the problem statement and the algebraic equation. Hong and his friends (2009) have maintained that this method helps to write an equation according to the problem. In time, students may not feel the need for the bar model method, and they may solve the problem with an equation. However, some of them may want to draw a model because the bar model method facilitates visualization of different problem statements and abstract quantities in the problem (Kho, 1987).

The bar model method helps students to visualize word problems, but it also enables them to decide which operations to do. Students generally look for clues, like keywords such as more or less, to solve a word problem; however, they can easily understand which operations are appropriate and useful through visual models (Beckmann, 2004). Considering the importance of visualization through bar model, I conjecture that seventh grade students' commonly made errors in algebraic word poblems may decrease with the help of bar model method. Therefore this study provided students a three-hour long instruction on the bar model method and investigated seventh grade students' use of bar model method as they solve algebraic word problems.

1.1.Purpose of the Study and Research Questions

The purpose of this research is understanding the use of bar model method used in Singapore math curriculum while solving algebra word problems in 7th grades and students' reasons for the solution method preferences (i.e., bar model or algebraic equation) in solving algebraic word problems. Specifically, the following research questions are addressed to understand the use of the bar model method while solving algebra word problems in 7th grade:

1. What are the error types that 7th grade students make while solving algebraic word problems?
2. To what extend does the bar model method help 7th grade students remedy the errors that they made while solving algebraic word problems?
3. What are the 7th grade students' reasons for the solution method preferences (i.e., bar model or algebraic equation) in solving algebraic word problems?

1.2. Significance of the Study

There are some studies in Turkey which investigated the strategies and methods that students use while solving algebra word problems (Bal, 2017; Kabael, 2016). Particularly, one study was conducted to compare the mathematics curriculums of Singapore, Turkey and South Korea (Kul & Aksu, 2016). In addition, the Singapore Education System was investigated in a study conducted by Turkish researchers (Levent & Yazıcı, 2014). However, in Turkey, there is a lack of studies that shed light on the effect of using the bar model method on algebra word problems. On the other hand, studies on using the bar model method do exist in other countries. For example, Tagle, Belecina and Ocampo (2016) conducted a study which explores the effect of the pictorial method on 3rd grade students' algebraic thinking in Philippines. They investigated the bar model's effects in topics of decimals and fractions. Ng and Lee (2009) investigated the perception of teachers and students in Singapore regarding the use of the bar model method, in which a test including algebra word problems was used as a data collection instrument. One other study conducted abroad was

Mahoney's study in New Hampshire. He (2012) conducted a study to understand the efficacy of the bar model method in arithmetic word problems. Although some studies exist to understand the bar model method's effectiveness in certain mathematics topics, these are not sufficient as they did not specifically investigate the effects of bar model method while solving algebra word problems. Thus, it is believed that the current study will contribute to the literature.

As mentioned earlier, students have difficulties in solving algebra word problems and, thus, make mistakes. In a study by Adu, Assuah and Asideu-Addo (2010), it is found that most students made mistakes while solving algebra problems. In fact, while they could solve arithmetic word problems, they could not solve problems with algebraic equations. To overcome this obstacle, using different methods could be helpful. The bar model is one of the different methods, and it is believed that this method can be a tool to facilitate transition from concrete to abstract. Indeed, it will be more beneficial if technology-aided teaching methods are used because the use of technology is becoming pervasive day by day and eases students' understanding. Visuality is an important in education and technology provides visuality to students (Zimmermann & Cunningham, 1991). In this respect, the bar model can be used since it can be easily adapted to technology environments. For example, there is a website to use the bar model as a technological manipulative (Thinking Blocks, n.d.).

Mathematical visualization can be defined as "the process of forming images (mentally, or with pencil and paper, or with the aid of technology) and using such images effectively for mathematical discovery and understanding" (Zimmerman & Cunningham, 1991, p. 3). This visualization is very important for mathematics education because Beckmann (2004) states that it helps students to choose the appropriate operation in problem solving. Besides, Temel, Mersin and Dündar (2015) found that visualization is a significant strategy not only for problem solving and but also for learning abstract topics. Since the bar model method relies on visualization, it is believed that investigating the bar model's efficacy is important to reveal whether it is effective in solving algebra word problems and overcoming students' mistakes.

Solving one unknown equation problems has an important place in the middle schools Mathematics Teaching Program in Turkey, especially for 7th grade (MoNE, 2018) and is the participants of the the present study were in grade 7. If students do not learn to write appropriate equations and, hence, cannot solve algebra word problems, they will have difficulty understanding other algebra topics in the following year that is 8th grade. The problem statements in 8th grade necessitate students to especially understand the algebra topics of 7th grade in order to reach the objectives of solving two unknown equations and setting up the two unknown equations. In addition, research studies have proven that students who learn algebra in middle school are more successful in exams and have a better understanding of advanced algebra in later years (Wang & Goldschmidt, 2003). Therefore, using the bar model method, which is an alternative way to solve algebra word problems for 7th grade students, and observing its effects may increase students' both exam scores and their performance in mathematics in the following years.

With this study it is aimed to understand whether using the bar model method in mathematics lessons decreases the errors that students make while solving word problems and students' views about using this method. As a result, this study is viewed as important not only for students, but also for teachers and curriculum developers.

1.3. Definitions of Important Terms

Algebra: Algebra is a field of mathematics that shows general number relationships and includes topics, such as polynomial and equations, and it is a tool that not only represents quantities and numbers with letter symbols, but also make calculations with these symbols (Kieran, 1992).

Word Problem: "Verbal descriptions of problem situations wherein one or more questions are raised the answer to which can be obtained by the application of mathematical operations to numerical data available in the problem statement" (Verschaffel, Greer, & De Corte, 2000, p. 9).

Algebra Word Problem: In this study, the term 'algebra word problem' is used to mean solving word problems with algebra (Usiskin, 1988). Besides, Akkan (2019) states that algebra word problems contain unknown values from the beginning to the end of the solution process. This means that the unknown value is used for operations in algebra word problems.

Unknown: Kieran (1981) defines 'unknown' as letters that represent any or all elements of a given set.

Variable: "A variable is a literal number that may have two or more values during a particular discussion" (Hart, 1951b as cited in Usiskin, 1999, p. 8).

Bar Model: The bar model is a visual method that contains rectangular bars, which are used for numbers instead of using algebraic symbols like letters to symbolize the unknown (Koleza, 2015)

CHAPTER 2

LITERATURE REVIEW

The purpose of the present study was to gain insight into use of the bar model method, which is used in the Singapore mathematics curriculum, plays in the way seventh grade students solve algebraic word problems. Thus, this chapter presents the related literature on algebraic word problems, students' errors in algebraic word problems and students' success rates in the exams on algebra in Turkey. It also includes a section on the bar model method and related works.

2.1. Algebra and Algebra Word Problems

Algebra is one of the important fields in mathematics education. It is very important for both school and work life. In schools, students encounter algebra topics in mathematics problems involving simplified algebraic expressions, equations with one unknown, equations with two unknowns, series, etc. (MoNE, 2018). Algebraic reasoning is important not only for mathematics lessons but also for other lessons, such as physics and chemistry. In this regard, Westbrook (1998) conducted a study to reveal the relationship between algebra and physics courses and the effect of this integration on conceptual understanding of students. A total of 100 ninth grade students were selected as participants in this study, which required the students in a physics class and an algebra-physics integrated class called SAM9 to produce concept maps for some selected topics which were later examined by the researcher. For the algebra and physics integrated class, density and slope were the selected topics. At the end of the research, Westbrook observed that the maps of the SAM9 students had better connections and procedural linkages than students in the physics class. Moreover, the SAM9 students were found to have gained a better understanding of both the physics topics (i.e., density) and algebra topics (i.e., graphing, slope and equations).

In another study, Potgieter, Harding and Engelbrecth (2007) stated that some chemistry topics need an important mathematical foundation and, thus, such topics are found difficult by students. They compared two groups of students. Students in the first group solved problems in an instrument related to the Nernst equation in electrochemistry, which requires algebraic skills, and students in the second group solved problems in an instrument related to algebra topics. The difficulty level of the questions in the instruments were equivalent. The study revealed that students' performance was not at a satisfactory level in both instruments. Students had an inadequate level of competency in not only chemistry problems but also algebra questions. Based on this finding, the research concluded that the reason underlying inadequate chemistry performance could be attributed to students' inadequate mathematical basics. It can be concluded from this study that students who do not have algebraic thinking may experience difficulties in learning physics and chemistry topics. Besides understanding lessons at school, algebra also has an important place in other study areas. As an example, the National Council of Teacher of Mathematics (NCTM) (2000) points to this issue as follows: “Distribution and communication networks, laws of physics, population models, and statistical results can all be represented in the symbolic language of algebra” (p. 37).

According to Usiskin (1995), it is important to teach and learn algebra mainly for three reasons, namely (1) to use it in jobs such as programming, (2) to make financial decisions in daily life, and (3) to acquire knowledge in other disciplines such as chemistry, physics, and business. He also stated that algebra is necessary for making generalizations because it is used to describe patterns and it provides general rules for all mathematic topics. Moreover, as stated by Usiskin (1995), algebra is a language used to solve problems which involve age, work, motion or coins in everyday situations.

Word problems have a significant role in mathematics education and, therefore, it is important to understand this role, especially to investigate algebra and algebraic word problems. According to the NCTM (2000), those students who internalize mathematics can use it in their daily and work life. To reach this vision, word problems is a key to help students use mathematical knowledge in their real life (Chang, 2010).

In addition, solving word problems has an important place in the school mathematics teaching program in Turkey. In seventh grade level, solving word problems that involve equations with one unknown is one of the objectives in algebra (MoNE, 2018). However, the methods utilized to teach problem solving strategies and to develop problem-solving skills in algebra are not sufficient because students still have difficulties while solving equation word problems (Chang, 2010). In this vein, Chang (2010) conducted research with 61 high school students to determine whether or not students can realize the structures of algebraic word problems and the challenges they encounter while solving these word problems. The researcher focused primarily on algebra because algebra is the topic that (1) is important in mathematics education, (2) students often have difficulty with, and (3) is rarely investigated in terms of transfer of learning. Thus, it is important to consider word problems involving equation with one unknown in investigating the conceptual knowledge development as an outcome of teaching algebra. To gain a better understanding of the use of algebra word problems, this section will continue with a section on the difficulties that students experienced in solving algebra word problems.

2.1.1. Students' Common Errors and Difficulties in Solving Algebraic Word Problems

One of the major topics that students often have difficulty with is solution process algebra word problems. In order to develop solution strategies for students to overcome these challenges and to provide suggestions to teachers, it is initially essential to understand these misconceptions and errors. The literature includes studies investigating students' misconceptions and errors as they solve linear equation problems in algebra.

To illustrate, in her study with 124 low achieved 6th grade students, Newman categorized students' errors in problem solving using algebra as (Newman, 1983b as cited in Ladele, 2013): (1) reading recognition, (2) comprehension, (3) transformation, (4) process skills, and (5) encoding.

The first category, reading recognition, suggests that if students do not recognize words and symbols in the problem, their solution process will be slower and even wrong. The second category of difficulty is related to comprehension, it is a crucial step which enables the problem solver to move through the other steps because if the question is not understood, then it cannot be solved accurately. Students need to paraphrase and restate the problem using their own words to successfully complete this category. The third category of error appears in the transformation of the problem situation to the mathematical relations. While solving a mathematics problem, students have to write what they understand from the problem statement by using numbers and math symbols. For algebraic word problems, students should write correct equations and use correct letters and numbers to solve the problem. The fourth category of difficulty, process skills, is related to students' correct application of the mathematical operations. The last difficulty category is encoding and it involves students' reasoning of the answer of the problem. After solving the problem, students need to write the answer in an acceptable form, which means that students should use correct symbols, words or table to write the answer.

According to Newman, students mostly make reading, comprehension and transformation errors. The other two errors seemed to be rare but still maintain their importance for students to solve algebraic word problems. Thus, transformation and process skills errors have been considered more descriptively in this present study as *setting up the equation incorrectly* and *operational mistakes in solving the equation* errors.

Aligning with these five errors and difficulty categories Newman identified five teps for analyzing and solving algebraic word problems (see Figure 2.1).

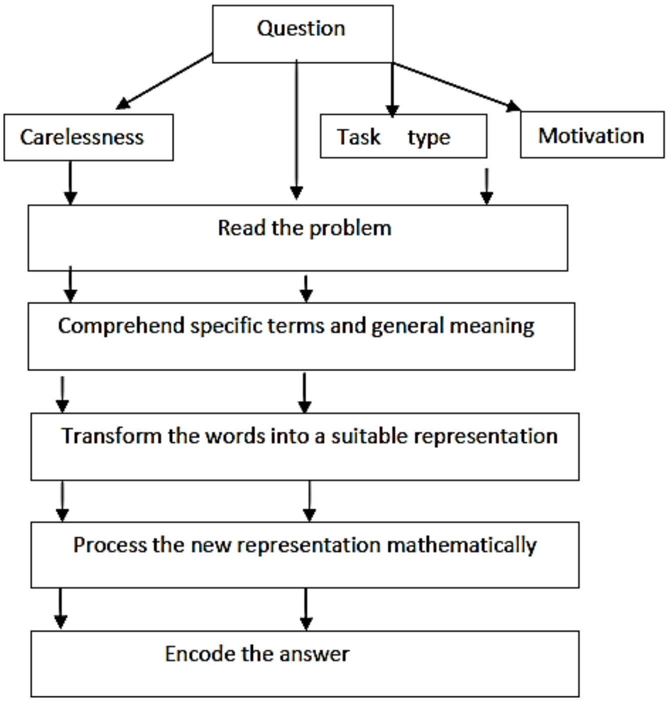


Figure 2.1 Newman's Error Analysis (Newman, 1983b, p. 2 as cited in Ladele, 2013, p. 23)

As can be observed in Figure 2.1, in addition to the five main steps of analysis, there are some error types that can occur in any stage while solving a problem. These errors are carelessness and motivation errors according to Newman (Ladele, 2013). Jha (2012) defined the error of carelessness as the error that causes students to solve the problem incorrectly in their first try and correctly in their second try. Another explanation of the error of carelessness is that a student knows how to solve the problem correctly; however, he/she makes a mistake in any stage of the solution bu later he/she gives the right answer to the same problem. This type of error is called carelessness error (Clements, 2004). Motivation error occurs when the student cannot reach the correct answer because he or she may not want to solve the problem or think that the problem is not worth solving even though he or she has sufficient skills to solve the problem (Clements, 1980).

According to researchers, algebraic word problems are one of the most difficult topics in the field of mathematics (Carpraro & Joffrion, 2006). Therefore, to understand students' difficulties while solving algebraic word problems, Jupri and Drijvers (2016) conducted research with 51 Indonesian students. Jupri and Drijvers (2016) found that students' main difficulty is transformation, which is formulating a mathematical model according to the context of the problem. They stated that this error is related to lack of mathematization, which is defined as

> the activity of organizing and studying any kind of reality with mathematical means, that is, translating a realistic problem into the symbolic mathematical world, and vice versa, as well as reorganizing and (re)constructing within the world of mathematics (Jupri & Drijvers, 2016, p. 2483).

Similarly, Kayani and Ilyas (2014) conducted research about understanding the difficulties students experience in solving word problems in algebra. They studied with 90 students from Pakistan, who were in either 7, 8, 9 or 10th grade and who had different mathematics ability levels. All students, who participated in this study, initially took a 25-item test and subsequently 45 students were selected for interviews. The results revealed four error types, namely transforming or transitioning words into algebraic language, arithmetic operations, using parenthesis, and selection appropriate methods–arithmetic or algebraic–to solve the problem. One of the most common errors is transitioning/transforming error, and 56.8% of the students in the mentioned study had made this error. This shows that many of the students in Pakistan could not translate words in problem sentences into algebraic symbols. The study also revealed that 9 and 10th grade students preferred to use algebraic methods instead of arithmetic methods because their teachers made more use of algebra methods during teaching. According to researchers, possible reasons of these errors might be students' lack of ability in basic arithmetic operations, students' background, the cognitive approach used in teaching algebra, the classroom environment, students' lack of interest, and teachers' teaching methods. Kayani and Ilyas (2014) suggested that using different methods is important for teaching and learning how to solve word problems in algebra. Thus, I considered *using parenthesis incorrectly in writing the equation* as one of the error types in the present study.

Based on the statistics reported by the West Africa Examination Council in 2007, 2011 and 2012, Adu, Assuah, and Asideu-Addo (2015) stated that most students made mistakes at comprehension and transition stages while they were solving word problems. Although some students solved the problems in a reasonable way, most of them could not transform the words into an equation. To understand students' mistakes while solving linear equation word problems, Adu et al. (2015) conducted research with 130 senior high school students in the Central Region of Ghana. The study required these students to solve 10 linear equation problems. Newman's error levels were utilized to analyze students' answers. The study did not reach different conclusions from those reported in the other studies mentioned above. The results showed that 75% of the students had made comprehension errors, 86% made transformation errors and 84% made processing errors. In other words, it was revealed that African students also experienced difficulty in making sense of the word problem (i.e., comprehension), transforming the verbal descriptions of relations to mathematical equations (i.e., transformation), and applying appropriate procedures correctly (i.e., process skills). In this sense, this study also showed that students often have difficulties in solving linear equation word problems. Moreover, Adu (2013 as cited in Adu, Assuah, & Asideu-Addo, 2015) found that although students have an adequate level of arithmetic skills and might be able to solve arithmetic word problems, they might not have the competency to solve algebraic problems or apply basic computation rules to algebra. This is another indication of students' experiencing difficulties in making sense of algebraic relations.

In addition, Egodawatte (2011) studied high school students' misconceptions in algebra in his dissertation. Although this study was conducted with high school students, some word problems, misconceptions and errors used in the study were related with the middle school curriculum. The study revealed three misconceptions and errors, which are reversal error, guessing without reasoning, and incorrect understanding of proportional relationships between variables in problem solving. These errors mostly originated from inaccurate transformation of written or oral language into mathematical symbols. For example, reversal error occurred when students wrote the algebraic expressions according to the word order in the problem. For instance, if the problem states “subtract 3x from 5”, students could write this as “3x – 5”. This error often occurred when the dividend or subtrahend was mentioned in the problem and those were needed to be expressed in algebraic terms. Secondly, when the problem requires setting up an equation to solve a difficult word problem, students often have tendency to guess the answer without engaging in any kind of reasoning. Although they could do mental operations, the researcher stated that the error resulted from guessing. Thirdly, students had difficulty in understanding the relationship between two or three variables that were given in the context of the problem because the students could not understand the relationship between the variables and, thus, wrote an algebraic equation incorrectly. As a result, most students had difficulties while solving algebraic word problems and most of them (71%) guessed without engaging in any reasoning and without using algebraic methods (Egowadatte, 2011). Hence, these two types of errors – guessing without reasoning which is used as *blank guessing error* and incorrect understanding of the relationship between variables which is used as *identifying the unknown incorrectly* – were chosen as types of errors that considered in the present study.

In another study, Ng and Lee (2009) conducted a meta-analysis and synthesized the challenges experienced in solving word problems reported in several studies in this field. As mentioned in this meta-analysis, Küchemann (1981) stated that knowing symbolic words and letters were two of the important aspects in solving algebraic word problems. In another study, Stacey and MacGregor (2000) wrote that transforming words into mathematical symbols and writing an equation appeared as a major obstacle for students. Yet another study by Bednarz and Janvier (1996) found that comprehending relationships between variables in the problem and understanding the context of the problem are two of the most critical cognitive actions, which led to errors in solving algebraic word problems.

In addition to the difficulties and errors pointed out by aforementioned researchers, Booth and Koedinger (2008) indicated some misconceptions of algebra that are particularly related to students' problem solving skills. In this study, 49 high school students learned how to solve equations by using the Cognitive Tutor curriculum, a self-paced intelligent tutor system. In another study, Koedinger and colleagues highlighted this process by indicating that:

> [s]tudents engage in investigations of real world problem situations and use modern algebraic tools (spreadsheets, graphers, and symbolic calculators) to express covariance relationships, to solve problems and to communicate results in intelligent tutor system (Koedinger, Anderson, Hadley, & Mark, 1997, p. 30).

Pre-test and post-tests that include questions measuring the problem solving skills were applied to students to understand their ways of thinking during the problem solving process. In these tests, Booth and Koedinger (2008) especially focused on the equal sign and the negative sign in their measurements to assess students' conceptual knowledge, and they used eight items to measure students' problem solving ability to assess their procedural knowledge. As for the findings, first of all, when students had some misunderstanding regarding the equal and the negative signs, they solved the equations incorreclty. For this reason, they suggested that teachers' initial goal should be to teach the meaning of the equal and negative signs. Moreover, they reported that students who had conceptual knowledge about the topic could apply the right strategies and solve the problems correctly. Hence, knowing basic algebraic rules and the meaning of algebraic terms were important just as the other essentials like transformation or comprehension skills in algebra problem solving.

In summary, the studies investigating the mistakes made by students while solving algebra word problems have been outlined in this section. These studies reported that students experienced difficulties and made mistakes while solving algebra word problems. These mistakes and errors were categorized in different ways by different researchers. First of all, Newman's error categories are as follows: (1) reading recognition, (2) comprehension, (3) transformation, (4) process skills, and (5) encoding. Newman also mentioned two additional error types, which are carelessness and lack of motivation (Newman, 1983b as cited in Ladele, 2013). Secondly, Kayani and Ilyas (2014) found four error types, which are more detailed than Newman's categories. These errors are transforming, arithmetic operations, using parenthesis and selection of appropriate methods for solving the problem. Thirdly, Egodawatte (2011) mentioned three error types that emerged in algebra word problems: reversal error, guessing without reasoning and incorrect understanding of proportional relationship in problem solving. Lastly, Booth and Koedinger (2008) emphasized that insufficient knowledge of basic algebraic rules and the meaning of algebraic terms may cause errors while solving algebra word problems. As seen, these categorization of students' error types have similarities and differences. Considering that I selected five of them for the currect study. In this respect, the literature highly contributed to the design the current study. Apart from studies conducted abroad and mentioned bove, there are other studies that investigated the difficulties experienced by students in Turkey while solving algebra word problems. Thus, this section will continue with an overview of the studies conducted in Turkey.

2.1.2. Research Studies in Turkey

The present section dwells on various studies conducted in Turkey on the difficulties and errors students experience while solving algebra word problems.

Kabael and Akın (2016) conducted a study with 7th grade students to investigate students' problem-solving strategies while solving algebra word problems. Nine students participated in this qualitative study, and clinical interviews were used for collecting data. During the interviews, students were asked to solve one algebra word problem. The problem could be solved using both arithmetic and algebraic methods. At the end of the study, it was found that seven students preferred to use arithmetic methods and two students preferred to use algebraic methods to solve the given problem. This result showed that 7th grade students focused more on using arithmetic methods instead of algebraic methods. In addition, three students tried to solve the problem by setting up an equation; however, they could not write an appropriate equation. According to the researcher, since students used meaningless symbols for the unknowns, they could not write an algebraic equation. Moreover, one student could write correct algebraic equation; however, he/she could not solve this equation because of non-sense making symbolic representation.

In another study, Bal and Karacaoğlu (2017) conducted a study with the purpose of understanding 6, 7 and 8th grade students' algebraic word problem strategies and their errors. 1017 students were chosen for this study in Adana. It was found that students mostly made logic errors while solving algebra word problems. When students experienced difficulty in reading the problem, they could not understand the problem and could not recognize the givens asked in the problem. Similarly, Didiş and Erbaş (2012) aimed to investigate 10th grade students' success in algebra word problems and the factors that affect their success. 217 students had participated in this study, during which a test was applied, and 16 students were chosen for the interviewes. This study showed that students' problem-solving skills in algebra was low. Moreover, the difficulties students experienced while solving algebraic word problems could be accounted for as follows: (1) students could not understand the problem situation and (2) students could not comment on and produce an idea for the problem situation. In addition, the difficulties experienced by students could stem from lack of experience in problem solving.

In brief, studies conducted in Turkey showed that students had difficulties and made mistakes while solving algebra word problems. To gain a better insight into Turkish students' success in the field of algebra, students' performance in international exams, such as Programme for International Student Assessment (PISA) and Trends in International Mathematics and Science Study (TIMSS), could be closely examined. These international exams can enable educators to examine and compare students' success in algebra. PISA aims to understand to what extent test takers apply their mathematical knowledge to their daily life. On the other hand, TIMSS assesses 4th and 8th grade students' math and science knowledge and skills (IEA, 2011).

TIMSS was first applied in 1995. However, Turkey's first participation in this exam was in 1999. 6928 Turkish students participated in TIMSS 2011 (Yücel, Karadağ, & Turan, 2013). The exam involved algebra questions related to patterns, algebraic expressions, equations, inequalities and functions questions (TIMSS, 2009). In Turkey, students could solve questions asking for patterns and basic algebraic computations; however, their success was relatively low in substituting the value of an unknown into an equation or inequality (Kılıç, Aslan-Tutak, & Ertaş, 2014). In another study, Bütüner and Güler (2017) analyzed the changes in 8th grade students' algebra achievements. Turkish 8th grade students' scores were 429 in 1999, 432 in 2007, 452 in 2011, and 458 in 2015, when the grand average is 500. Bütüner and Güler (2017) found that although Turkish students showed positive progress in TIMSS exams, they were below the international average in all the exams. Table 2.1 below displays the TIMSS scores of girls and boys in algebra across the years.

Table 2.1

TIMSS Scores of Girls and Boys in Terms of Content Areas (Bütüner & Güler, 2017)

Content Area	**Countries**	**1999**		**2007**		**2011**		**2015**	
		Boy	**Girl**	**Boy**	**Girl**	**Boy**	**Girl**	**Boy**	**Girl**
Algebra	Taiwan	585	588	622	613	636	621	617	610
	Korean	585	585	596	596	617	616	616	608
	Singapore	578	574	589	569	622	607	630	615
	H. Kong	570	568	573	558	586	579	593	593
	Japan	568	571	560	559	568	572	601	590
	Turkey	*442*	*426*	*447*	*434*	*464*	*446*	*469*	*450*
	Botswana	-	-	404	383	415	399	410	389
	Jordan	446	433	461	436	451	413	438	397
	Morocco	350	354	-	-	360	353	380	366
	South Africa	290	296	-	-	367	356	400	387
	Arabia	-	-	350	338	412	388	398	384
	International mean	489	485	457	444	476	464	489	478

As can be observed in this table, countries such as Singapore, Korea and Taiwan were in first places in all four administrations of the test, and their average scores in algebra were significantly higher than those of students in Turkey (Bütüner & Güler, 2017).

Although the scores of Turkish students have increased every year, their average scores were below the international mean in all administrations. This indicates that mathematics educators need to lay more emphasis on algebra, seek new ways of teaching algebra conceptually and meaningfully. This also signifies that algebra is one of the important topics that mathematics education researchers may focus on in more depth, especially in Turkey.

As mentioned above, it is remarkable that students' level of success in algebra is higher in some countries, such as Singapore. Since the success level of Turkish students in algebra field is low, different teaching methods employed in these countries should be investigated. Therefore, it is important to understand Singapore's teaching methods and practices. That's why this section will continue with the Singapore bar model, which is particularly used in teaching algebra and which is the method that the current study was built on.

2.2. Singapore Mathematics and the Bar Model Method

One of the main characteristics of Singapore Mathematics is learning mathematics using diagram or model drawing, known as the Bar Model Method since 1983 (Ng & Lee, 2009). Using this method, primary school students learn mathematics through visual means supported in concrete ways, which ultimately aims to develop conceptual knowledge. In Singapore, teachers first use concrete materials while they are teaching mathematics. Next, before students learn the topic in abstract ways like using letters for unknowns in algebraic word problems or other mathematics topics, teachers use the pictorial method. Hong, Mei and Lim (2009) describe pictures or diagrams as an important bridge from concrete to abstract learning. This bridge is highly important to understand advanced level algebra in later grades such as high school and university. Although students should make the transition from the pictorial method to abstract and formal methods of solving algebraic problems at the end, some of them may continue to use the Bar Model method in later years (Looi and Lim, 2009). Moreover, according to Kho (1987) students should understand the context of the problem for better use of problem-solving skills, which may be possible when they use visual means.

Kho (1987) stated that this method can be used in most of the mathematics topics like word problems, whole numbers, fractions and ratios, not just algebra word problems. In fact, students start to learn using the bar model method to solve problems in 3th grade. They use the bar model method for very simple problems at first. When they are in 4 and 5th grade, they can solve more difficult and multistep problems with this method. Finally, in 6th grade, they learn how to solve highly difficult problems and then pass on to algebra (Hoven & Garelick, 2007). Ng and Lee (2009) stated that basic problem structures which are learned by Singaporean students in elementary school are part-whole, comparison and multiplication-division. Students learn to solve these types of problems by using the bar model method. To understand the bar model better, an example of a problem-solving procedure will be given for each problem structure. The first problem structure is the part-whole model, which involves the operations of addition and subtraction. In the part-whole model, Madani (2018) states that (1) there is one part and then another part, and these should be added to find the whole and (2) there is a whole and a part and they should be subtracted to find the other part as can be seen in Figure 2.2.

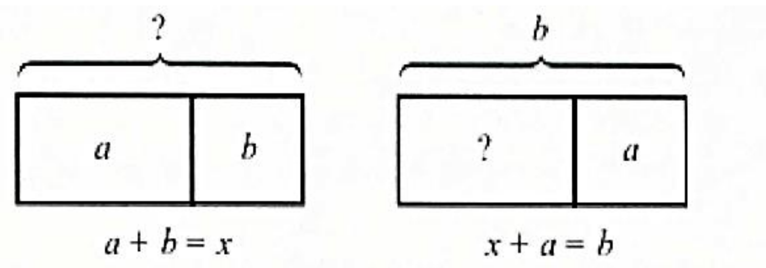


Figure 2.2 Part-whole models: Arithmetic model (on the left) and algebraic model (on the right) (Ng & Lee, 2009, p. 286)

Figure 2.2 shows that part-whole models can be both arithmetic and algebraic. When the whole is unknown, it is arithmetic model. When one of the parts is the unknown, it is algebraic model. The following example can be given for this model: “I have 12 stamps altogether and 5 of them are from Canada. How many are from other countries?” (Ciobanu, 2015, p. 17). For this problem, a rectangular bar should be drawn to represent Canada whose value is 5 and another rectangular bar to represent the other countries, which are not specified. The total value is 12 stamps, so the bar model can be drawn as seen in Figure 2.3.

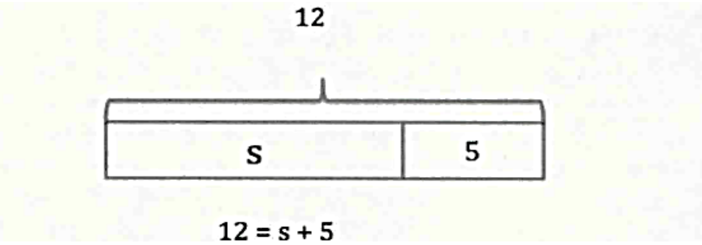


Figure 2.3 Part-whole algebraic model of representation (Ciobanu, 2015, p. 17)

As can be seen in Figure 2.3, the algebraic equation is 12 = s + 5. To find the value of the unknown, 5 should be subtracted from 12. Therefore, the answer is: 12 – 5 = 7

The second problem structure is the comparison model. It is used for comparing two or more unknowns and showing the relationship between these unknowns. The bars' lengths are different from each other and this indicated that the value of the unknowns are different. Thus, the difference between the length of the bars represents the difference between the quantities. Figure 2.4 below shows the comparison model for both an arithmetic problem and an algebraic problem.

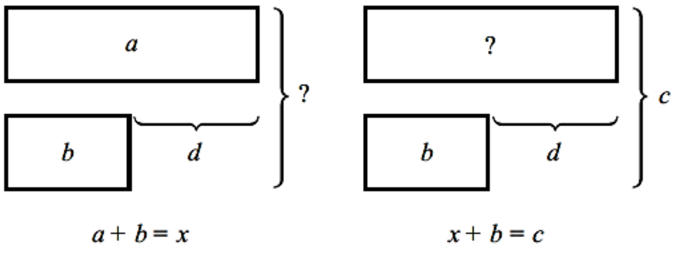


Figure 2.4 Comparison Models: Arithmetic model (on the left) and algebraic model (on the right) (Ng & Lee, 2009, p. 287)

Figure 2.4 shows that the comparison model can be used for algebraic and arithmetic problems like part-whole model. When the model shows the difference between unkowns, it is called the comparison model. To gain a better understanding of this model, the following problem could be given as an example: "145 girls took part in a coloring competition. 34 more boys than girls took part. How many boys took part in the competition?" (Puteh, Tajudin, Adnan, & Aziz, 2017, p. 58). To draw a bar model for this problem, a rectangular bar which represents the girls should be drawn. The length of this rectangular bar is 145 since the number of the girls is given as 145. Then another rectangular bar should be drawn for the number of the boys. The length of this rectangular bar is 34 units bigger than the other bar. In this model, the number of girls and boys will be compared. To find the bigger number, students should add 145 and 34 as can be seen in Figure 2.5.

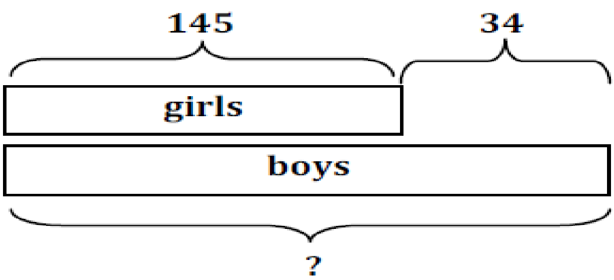


Figure 2.5 Representation of the comparison model (Puteh, Tajudin, Adnan, & Aziz, 2017, p. 58)

According to Figure 2.5, the number of boys is 179 since 145 and 34 should be added. The third problem structure is the multiplication and division model. Ciobanu (2015) states that this model involves problems involving the multiplicative relationship between unknowns. Thus, these unknowns are given multiples of each other as seen in Figure 2.6.

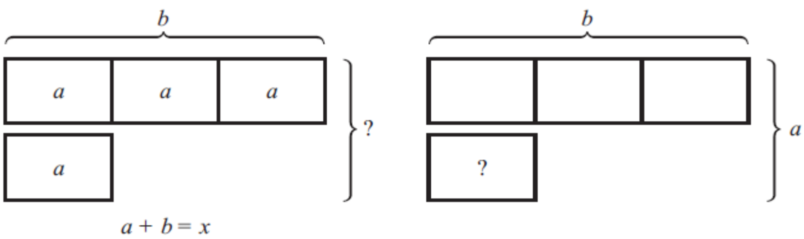


Figure 2.6 The Multiplication and division model for an arithmetic word problem (on the left) and an algebraic word problem (on the right) (Ng & Lee, 2009, p. 289)

As can be observed in Figure 2.6, the multiplication and division model could be both arithmetic and algebraic. Moreover, the lengths of the bars are equal, and one quantity is the multiple of the other variable in the model. The following example illustrates this kind of a model (Hoven & Garelick, 2007, p. 30): “A grocer has 42 apples. 2/7 of them are red, and the rest are green. How many of them are green?” The multiplication and division model may also involve fractions just as in this problem. To solve the problem, the bar model can be drawn as shown in Figure 2.7. Since 2/7 of them are red, there should be two bars for red apples and seven bars for the total number of apples. Therefore, the green apples should be represented with five bars. Moreover, the lengths of all bars should be equal to each other.

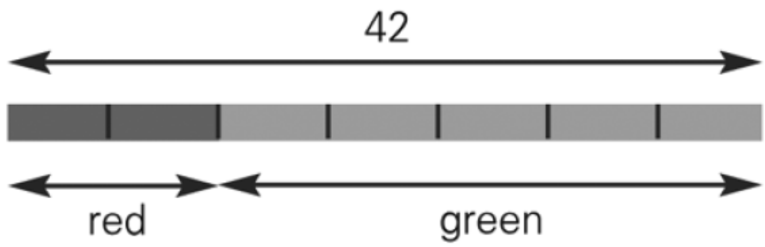


*Figure 2.*7 Representation of multiplication and division model (Hoven & Garelick (2007, p. 30)

There are seven units in the model as can be seen in Figure 2.7. To find one unit’s value, 42 should be divided into 7. Because the division of 42 by 7 equals 6, the value of one unit is 6. The number of green apples could be found with the multiplication of 5 by 6 because there are 5 units representing them. Therefore, the answer is 5 x 6 = 30.

As stated by Mahoney (2012), the history of the bar model method in Singapore goes back to the early 80s when this method was used with the purpose to improve students’ weak word problem-solving skills. Since then, the bar model method has been used in all schools in Singapore. In this method, students read the problem and draw bars (i.e., rectangles) based on the context of the problem. The bar model method involves not only drawing the bars representing the relationship between quantities given in a problem but also arithmetic operations, such as multiplication and subtraction, to reach the answer. However, representing quantities with rectangular bars may still be abstract for students, so they may need to use concrete objects or real pictures of the objects like cars or apples at the beginning in earlier grades (Cai, Ng & Moyer, 2011). Therefore, Singapore Mathematics suggests the Concrete-Pictorial-Abstract approach that starts with concrete manipulatives and results in abstract thinking for which the pictorial method functions as a bridge.

Cai, Ng and Moyer (2011) examined the curriculum of China and Singapore, focusing on the development of students' algebraic thinking in early grades. They examined the Singapore Math Curriculum according to goal specifications, content, and teaching process in algebra. As mentioned above, algebraic topics like simplifying algebraic expressions or using variables are the topics of the 6th grade curriculum. On the other hand, equations and other algebraic structures such as solving algebraic word problems are part of the 7th grade and later years' mathematics curriculum in Singapore. However, in earlier grades, students can improve algebraic thinking skills to make a transition from arithmetic to algebra because they utilize the bar model method to deeply understand part-whole relations between quantities, identify the algebraic relations, and generalize them as algebraic expressions. Hence, solving word problems both arithmetically and algebraically has an important place in the Singapore mathematics curriculum (Curriculum Planning & Development Division [CPDD] 1999, 2000).

According to studies, there is more than one advantage of using the bar model in mathematics education. One of the advantages of the bar model method is that students can make sense of the context of the problem better, meaning that students can learn mathematics topics through seeing and doing (Thiyagu, 2010). Another advantage of the bar model is that it helps students to focus on how to represent the problem instead of just solving it (Cai, Ng & Moyer, 2011). In this sense, the Singapore bar model improves algebraic thinking by facilitating how to determine the unknown and relationship between quantities, which is one of the ways of improving algebraic thinking (Kieran, 2004). Moreover, the bar model method were especially found useful for facilitating the transition from arithmetic to algebra (Hoven & Garelick, 2007).

Students start with basic problems in 3rd grade and solve these problems by using the bar model. They continue to use the model in later grades to solve more difficult and complex problems. Thus, students could easily make a transition to algebra and represent problems symbolically.

2.2.1. Studies Investigating the Bar Model Method

In this section, some international studies investigating the efficacy of the bar model method are summarized.

Before investigating the effects of the bar model method, one should gain deeper insight into Singapore's mathematics curriculum. In this sense, the American Institutes for Research compared the mathematics curricula in the United States and Singapore (Ginsburg, Leinwand, Anstrom, & Pollock, 2005). They investigated the results of TIMSS and realized that the students in Singapore were more successful than those in the United States. They examined the major differences in mathematics assessment, teacher competencies, and mathematics textbooks between the U.S. and Singapore to understand the possible reasons underlying the difference in achievement between the students in the U.S. and those in Singapore. The researchers found that the mathematics textbooks used in Singapore reflected the problem-based approach, the assessments aimed to measure reasoning skills, and their teachers were highly qualified. Moreover, low mathematics ability students had a better chance of understanding and learning the subject than those in the U.S. because alternative teaching methods were provided, and they received help from expert teachers in Singapore. On the other hand, the mathematics education in the U.S. was more traditional. Because definitions and formulas, instead of reasoning, are focused on in their mathematics books, the questions and problems for assessment are not challenging. Furthermore, Ginsburg and his friends (2005) said about teachers in U.S.: "Too many U.S. teachers lack sound mathematics preparation. At-risk students often receive special assistance from a teacher's aide who lacks a college degree" (p. ix). This indicates inadequates of teachers in U.S. compared to Singapore. Another difference is related to the students. The students in The U.S. solve mathematics problems more mechanically, by solely using the mathematics procedures (Ginsburg, Leinwand, Anstrom, & Pollock, 2005). This study revealed that the bar model method used in Singapore enabled students to think deeply and conceptually, which resulted in becoming more successful in international assessments.

Since Singapore students were successful in international exams, Mahoney (2012) conducted a study on the effects of the Singapore bar model method while students were solving word problems including complex ones. In this experimental single-case design study, four students solved word problems throughout eight sessions in three different phases, namely, baseline, intervention, and maintenance. The baseline phase is where students solve some word problems, while the intervention phase is where students take instruction on the bar model method and finally, the maintenance phase is where students solve some word problems using the bar model method. In the baseline phase, the students who did not have any idea about the bar model was required to solve problems involving 10 items. Five of them were multiplicative comparison problems and the other five were fraction problems. The students could not solve the word problems and, thus, could not reach the correct answer in this phase. When the intervention phase began, the students learned how to use the bar model method to solve these kinds of problems in several sessions. After the intervention phase, each student solved the same word problems twice for assessment purposes - once after one week and once after three weeks. The researcher called this phase maintenance. At the end of the study, the researcher found that the students' performances improved after they started to use the bar model method. Students chose correct operations to solve the problem using the bar model method. Drawing a model helps students to see the relation between quantities. Moreover, this study showed that the bar model method can be effective for students who have never learned the method at younger ages.

This international reputation drew the attention of researchers in the U.S., and Waight (2006) investigated the effects of Singapore Mathematics in a regional school district in Massachusetts. At the beginning, students' mathematics grades were very low.

Students' failure rate was 46%, which was a source of disappointment for the school. The school administration started to reflect on ways to solve this dissapointing result and decided to use Singapore Mathematics from 5th grade to 8th grade. Initially, the school started with only six classrooms implementing the Singapore math curriculum. As students became more and more successful, they increased the number of classrooms implementing the Singapore bar model method, and they reached 130 classrooms using this method. Another result of this study was students' enrollment rate in algebra classes. While 25% of the 8th grade students were enrolled in algebra classes at the beginning, all 8th grade students became enrolled at the end of the study. Moreover, the enrollment rate of 9th grade students in algebra classes increased from 25% to 45%. Thus, the researcher pointed out that Singapore Mathematics influenced students' algebra success because the enrollment rate in algebra classes increased.

Another study conducted by Tagle, Belecina and Ocampo (2016), based their research on Bruner's Theory and aimed to develop algebraic thinking among 3rd grade students through pictorial models. His Concrete-Pictorial-Abstract (CPA) approach also supports the Singapore Mathematics curriculum. Concrete methods use manipulatives or concrete objects; pictorial methods are based on drawings, charts or graphs that are drawn by students according to the context of the problem; abstract methods involve using numbers and letters. As mentioned by Hong, Mei and Lim (2006), pictorial methods were considered to function as a bridge between concrete and abstract learning. One of the pictorial methods is the bar model method used in the Singapore mathematics curriculum. Tagle, Belecina and Ocampo (2016) studied with 3rd grade students to understand the effect of the pictorial method for their algebraic thinking levels in their study in the Philippines. They used the pre-test and post-test research design with 28 students. They applied a pre- and post-test to understand students' algebraic thinking levels by using pictorial methods in the lessons, in which the topic were decimals and fractions. At the end of the study, they concluded that pictorial models issuing rectangular bars for each unit in the problems could be a bridge for deciding the abstract operations. They also stated that the pictorial method could help conceptual understanding in addition to procedural knowledge. Thus, this study showed that the bar model method had a powerful effect on developing students' algebraic thinking.

In addition, Swee Fong Ng and Kerry Lee (2009) conducted two studies with 14 teachers and 151 students to understand their perception of the bar model method. In this study, students solved a 10-item math test by using only the bar model method. At the end of the study, they found that this method provided average and high ability students with the opportunity to develop their problem-solving skills. They stated that students, who applied the bar model method partially, needed more practice with the bar model. This study also showed that children could perceive the bar model as a method needed to be memorized and that could be applied to every single problem. However, students need to understand how to draw bar models based on the problem and how bar model drawings could vary across different problems (Ng & Lee, 2009).

It is interesting that 6^{th} grade students in Singapore can solve more complex problems, which are actually at the level of 8^{th} grade. Hoven and Garelick (2007) reported another school in New Jersey using Singapore's bar model method in order to increase students' mathematics achievement. Although the bar model method could lead to positive results, there were some challenges that teachers encountered while teaching with this model because this way of teaching mathematics was slower but addressed the topic more profoundly when compared to the previous math program in New Jersey schools. The school principal expressed that "Singapore's approach is very teacher driven, much slower paced, and goes into much more depth. Teachers aren't used to that" (Hoven & Garelick, 2007, p. 30). On the other hand, the researchers observed that when both students and teachers got used to this method, students learned the essential skills faster. For example, the seeds for multiplication were planted in the first grade because students could develop multiplicative thinking by using bars.

The studies mentioned above indicated that using the bar model method is necessary to develop students' problem solving abilities. Conversely, Clarke (2017) found that students in England do not need an instruction this method to solve word problems.

She conducted a study to understand four 6th grade students' problem-solving strategies and necessity for a new method. Students who participated to this study did not know anything about the bar model method. At the end of the study, two students drew some shapes appropriate for the problem. This supported that using diagrams might be beneficial for problem solving. However, these students used diagrams for different purposes. For example, while one of the students used diagrams to understand the problem, another student used them to check the solutions. According to the researcher, drawing and using diagrams correctly showed that an instruction for bar model method may not be necessary since students were equipped with sufficient strategies and they could invent them on their own. Hoven and Garelick (2007) also stated that the bar model method provides a consistent solution for the problems, which means students are sure what they are going to draw. That's why the researcher stated that using a consistent method, such as the bar model method, could be beneficial for increasing students' levels of achievement.

To sum up, the studies that investigated the efficacy of the bar model method were examined in this section. The strengths of the Singapore mathematics curriculum was explored before investigating the effects of the bar model method. In Ginsburg and his colleagues' study (2005), in which the mathematics curriculum of Singapore and that of U.S. were compared, the findings revealed that Singapore mathematics textbooks reflected a problem-based approach, the assessments aimed to measure reasoning skills, and their teachers were highly qualified. This showed that the bar model method, which is one of the major component of the Singapore mathematics curriculum, is worth exploring. That's why some studies were conducted to understand the effects of the bar model method. Tagle, Belecina and Ocampo (2016) found that the pictorial method, which supports the bar model method, could be a bridge for deciding on the abstract operations and helps conceptual understanding. Another researcher, Mahoney (2012), compared the rate of problems that students solved correctly before and after using bar model method. He found that the bar model method had a powerful effect on solving word problems. In addition, Waight (2006) investigated the effects of the Singapore mathematics curriculum at a school in the U.S. This study revealed that using Singapore mathematics curriculum and the bar model method increased students' success in algebra. Furthermore, Ng and Lee (2009) stated that the bar model method is an important method for increasing the problem solving ability of students; it is nevertheless not an easy method and they should do more practices to master it. Similarly, Hoven and Garelick (2007) stated that the pace of teaching when the bar model method is used could be slower; it needs much practice for fast learning. Understanding the positive sides of the bar model method has been the motivation for my study. However, the number of studies which examined the effectiveness of using the bar model method in the accessible literature in Turkey is limited.Thus, current study was aimed to investigate whether or not the bar model method could help the students to write an appropriate algebraic equation and reduce their errors.

CHAPTER 3

METHODOLOGY

The purpose of this research is to gain an in-depth understanding the use of the bar model method, which is used in the Singapore mathematics curriculum, plays in 7th grade students' ways of solving algebraic word problems. Therefore, the following research question was addressed in this study:

1. What are the error types that 7th grade students make while solving algebraic word problems?
2. To what extend does the bar model method help 7th grade students remedy the errors that they made while solving algebraic word problems?
3. What are the 7th grade students' reasons for the solution method preferences (i.e., bar model or algebraic equation) in solving algebraic word problems?

In this chapter, the research design of the study, the participants, the data collection and analysis procedures, the role of the researcher, and the trustworthiness and credibility of the research explained in detail.

3.1. Research Design

This study was designed as a single case study investigating a case, which in this study was purposefully selected 7th grade students who made six error types while solving word problems. These error types were mentioned in Chapter II and also explained in detailed as describing the case participants. Case study is a research approach which makes use of a variety of data sources to aid in the explanation of a phenomenon. Thus, this approach enables the explanation of a phenomenon through a variety of perspectives rather than through one perspective. Using a variety of perspectives to approach the phenomenon helps to understand and reveal more facets of the phenomenon (Baxter & Jack, 2008). The case study was defined by Creswell (2007) as a research methodology, in which the researcher investigates case/s in depth by means of various data sources. Moreover, Miles and Huberman (1994) defined the case as “a phenomenon of some sort occurring in a bounded context. The case is, “in effect, your unit of analysis” (p. 25).

Of the research methodologies, case study was found to be appropriate to carry out the present research study because seventh grade students in a public middle school had to solve during the 2018-2019 academic year who mades the six error types in solving algebraic word problems forms the subject of this study, which functions as the case. There are four types of case studies: (1) single-case holistic design, (2) single-case embedded design, (3) multiple-case holistic design, and (4) multiple-case embedded design (Yin, 2009). When a study investigates one case, its design is referred to as the single-case design. On the other hand, when a study investigates more than one case, its design is called the multiple-case design. Moreover, embedded design comprises more than one analysis unit of the study, which can be seen in Figure 3.1 (Yin, 2009).

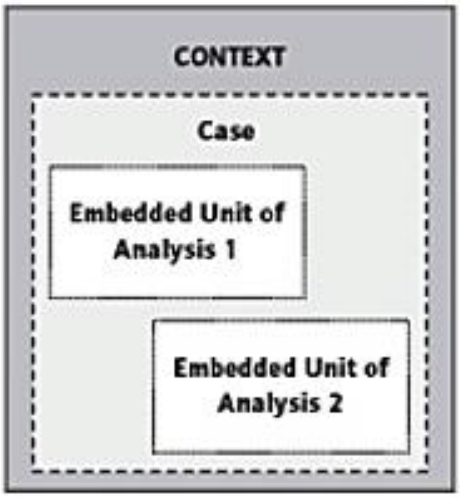


Figure 3.1 Single-case embedded research design (Yin, 2009, p. 46)

As can be observed in Figure 3.1, the embedded design involves more than one unit of analysis. Yin (1994) states that *case* can be defined with unit of analysis, which could be groups, organizations or countries. As a result, the single-case embedded design was employed as the research design of the present study, in which the units of analysis are problem sets involving algebraic word problems. There were three problem sets, which means there were three units of analysis in the study.

In summary, the researcher conducted a study by utilizing the single-case embedded design to make an in-depth exploration of students' solutions involving the bar model method for algebraic word problems after the necessary permissions from the students and their parents were received in accordance with the ethics committee regulations and Ministry of Education (See Appendix A and Appendix B). The case participants are described in detail in the following section.

3.2. Participants

The case participants involved ten 7th grade students (five girls and five boys) in a public middle school in Sincan, Ankara. These participants were selected among 42 students based on their solutions to problems on the initial assessment instrument involving 10-word problems (see Appendix C for initial assessment questions). By using direct instruction during regular class hours, the teacher had taught students how to solve one-unknown equations, how to write an algebraic equation for a word problem, and how to find unknowns in the problem. The objectives of the topics were stated by the Ministry of Education as:

1. to understand the conservation of equality principle,
2. to recognize first order equations with one unknown and write first order equations with one unknown suitable for real-life situations,
3. to solve first order equations with one unknown, and
4. to solve algebraic word problems that require writing first order equations with one unknown (MoNE, 2018).

These objectives were addressed through a teacher-directed method, which is direct instruction. The teacher presented to the class the essential information and taught how to solve questions and problems related with the topic. Then she wrote on the board some different questions and word problems, which the students tried to solve on their own. The teacher checked each of their answers and corrected their mistakes. Based on these mistakes, the teacher explained some parts of the topic again to ensure that the students learned what they had not understood. Before then, each student solved questions on the board. Subsequently the teacher, who was also the researcher, gave the initial assessment to 42 students.

The similar problems asked in the initial assessment were, in fact, solved in class while teaching the topic. Students' mistakes and error types were examined, and each error that was made by the students was categorized. One or two students for each error type were chosen for the study. There were six error types and reasons that resulted in a wrong answer in solving algebraic word problems in the initial assessment: (1) Blank guessing, (2) Identifying the unknown incorrectly, (3) Setting up the equation incorrectly, (4) Using parenthesis incorrectly in writing the equation, (5) Operational mistakes in solving the equation, and (6) Finding the incorrect unknown (which was, in fact, not the answer of the question). Although these errors were mentioned in the previous chapter, a brief explanation of each is as follows:

1. *Blank Guessing*: Students' guessing the answers and operations without any reasoning (Egodawatte, 2011).
2. *Identifying the Unknown Incorrectly*: Students' not being able to identify the unknown variable and its relation with other quantities (Egodawatte, 2011).
3. *Setting Up the Equation Incorrectly*: Students' inability to transfer words into algebraic equations in spite of being able to identify variables and use x correctly (Newman, 1983b as cited in Ladele, 2013).
4. *Using Parenthesis Incorrectly in Writing the Equation*: Students' incorrect usage of the parenthesis while transferring words into algebraic equations. Although this mistake is related to the third error, the researcher categorized it separately because there were many students who had made particularly

the parenthesis error. They generally do not use the parenthesis when required or they use the parenthesis unnecessarily (Kayani & Ilyas, 2014).

5. *Operational Mistakes in Solving the Equation*: Students' inability to solve equations accurately by making some operational errors although the equation is correct (Newman, 1983b as cited in Ladele, 2013).
6. *Finding the Incorrect Unknown as an Answer*: Students' inability to find the correct answer which is not the x but its multiplicative or additive relation with the unknown x. This error type was not existent in the accessible literature; however, the researcher realized that there were many students making particularly this error.

The number of errors made by the students for each error type is presented in Table 3.1. After analyzing students' errors, one or two students were chosen for each error type to identify the case participants. First, Zeynep and Umut were selected for blank guessing. Second, identifying the unknown correctly was one of the biggest challenges in the 4th problem, and Sinem was chosen because she had written x-2, x-4 and x-6 instead of x+1, x+2 and x+3 although she could write x to represent the first number of the four numbers. Ali was also chosen because he could not identify the unknowns correctly in the 1st, 3rd, 5th and 6th problems. Third, there were 18 students who could not write the equation correctly in the 5th problem. Merve was chosen for this error because she wrote the correct unknowns for the short and long sides of rectangle in the question. But, she could not write the correct equation. Moreover, Ece was chosen for this error type because she had not written any equation for the 4th problem even though she had used correct algebraic expressions. Then, there were 32 students who had not used the parenthesis when it was necessary in the first problem, and Melike and Melik were chosen from these 32 students. Next, Mustafa was one of the students who had made an operation mistake while solving the equation in the 4th problem and, therefore, was selected as one of the case participants. Finally, 13 students had found x correctly, while they could not find what the problem asked for in the 6th problem (i.e., finding x as 10 but not finding the number of boys which was $2x – 7$), and Emre was chosen for this error type (See Table 3.2).

Table 3.1

Number of Students' Errors in Each Problem in the Initial Assessment

	Blank guessing	**Identifying the unknown incorrectly**	**Setting up the equation incorrectly**	**Using parenthesis incorrectly in writing the equation**	**Operational mistakes in solving the equation**	**Finding the unknown as an answer**
1) Beş katının 1 eksiği, 1 fazlasının 4 katına eşit olan sayı kaçtır?	0	1	2	32	0	0
2) Bir sayının 2 katının 4 fazlası 26 ise, bu sayı kaçtır?	0	0	2	3	2	0
3) Bir otelde iki ve üç yataklı toplam 35 oda vardır. Bu oteldeki toplam yatak sayısı 85 olduğuna göre, oteldeki üç yataklı oda sayısı kaçtır?	8	14	4	0	2	8
4) Ardışık olan dört sayının toplamı 74 ise, bu sayılardan en büyüğü kaçtır?	12	4	5	0	4	6
5) Bir dikdörtgenin uzun kenarı, kısa kenarının 2 katından 3 fazladır. Dikdörtgenin çevresi 54 m ise, alanı kaç metrekaredir?	6	6	18	1	1	5
6) Bir sınıftaki erkek öğrencilerin sayısı, kız öğrencilerin sayısının 2 katının 7 eksiği kadardır. Bu sınıfın mevcudu 23 ise, sınıftaki erkek öğrenci sayısı	2	10	2	0	0	13
7) Biri diğerinden 5 yaş büyük olan iki kardeşin 6 yıl sonraki yaşları toplamı 31 ise; küçük olan kardeşin şimdiki yaşı kaçtır?	11	12	5	0	3	0

Table 3.1 (continued)

Number of Students' Errors in Each Problem in the Initial Assessment

8) Esma'nın dedesinin yaşı, Esma'nın yaşının 5 katından 5 fazlası kadardır. Esma ile dedesinin yaşları toplamı 77 ise, Esma kaç yaşındadır?	8	4	1	3	4	0
9) Tarık, her gün bir önceki günden 10 sayfa fazla kitap okuyarak 360 sayfalık bir kitabı 5 günde bitirmiştir. Buna göre, son gün kaç sayfa kitap okumuştur?	17	7	1	0	4	5
10) Üç sayıdan birincisi ikincisinin 3 katına, üçüncüsü birincinin 2 fazlasına eşittir. Bu üç sayının toplamı 37'dir. Buna göre, birinci sayı kaçtır?	23	5	2	8	1	0
Total	87	63	42	47	21	37

Table 3.2

Purposeful Sampling of the Case Participants Based on Error Types

Case Participants	Blank guessing	Identifying the unknown incorrectly	Setting up the equation incorrectly	Using parenthesis incorrectly in writing the equation	Operational mistakes in solving the equation	Finding the unknown as an answer
Zeynep	X					
Ece			X			
Melike				X		
Mustafa					X	
Ali		X				
Sinem		X				
Emre						X
Umut	X					
Merve			X			
Melik				X		
Total	2	2	2	2	1	1

Thus, the purposeful sampling method was used to choose these ten students based on their error types. These tens students were chosen not only because they made these errors while solving algebra word problems but also because they had better communication skills, were open to learning, had a higher level of motivation than the other students who had made the same errors.

Case participants were selected from a public middle school in Ankara. Half of the students were attending the school in the mornings and the other half were doing so in the afternoon. There were approximately 1500 students in the school. Most of the students belonged to low and middle socioeconomic status families. Ten of the students participating in this study were 13 years old on average. Their level of success in mathematics, according to the first semester mathematics exam scores, ranged between 50 and 90 percent. Although not all the students participating in this study were very successful in the mathematics lesson, they could express their ideas well in class discussions and were enthusiastic to learn mathematics. The characteristics of the participants, namely their age, classes, average math scores in the first semester and their socio-economic status based on the teacher's observations, are presented in Table 3.3 below.

Table 3.3

Case Participants' Characteristics

Student	Age	First Semester Math Score (%)	Observed SES
Zeynep	13	68	Middle
Ece	12	82	Middle
Melike	13	55	Low
Mustafa	13	69	Middle
Ali	12	63	Middle
Sinem	13	84	Middle
Emre	13	90	Middle
Umut	12	80	High
Merve	13	80	Low
Melik	12	72	High

3.3. Data Collection Procedures

According to Creswell (2009), for gaining an in-depth understanding of the situation in qualitative research, it is beneficial to make use of multiple data collection tools like interviews, observations and documents. Therefore, in the present qualitative study, multiple data collection tools, specifically interview and students' written work, were utilized.

To conduct the present study, the researcher designed a three-class hour instruction to teach students how to use the bar model in solving algebraic word problems. The content of this instruction is described in detail below. After the instruction, the researcher carried out clinical interviews with students, asked them to solve the word problems by using any method that they wanted, and probed their thinking during the semi-structured interviews. Video and audio recordings were made during both the instruction and the interviews.

3.3.1 Instruction

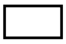
It took three hours for the instruction to be completed. The lessons were held after school hours on Monday, Wednesday and Friday on February, 16th, 18th, and 20th, 2019, respectively because the researcher wanted a free day between the lessons. During the lessons, the researcher taught the students how to solve one unknown equation word problems, which were written by the researcher and examined by an expert in mathematics education (see the problems in Appendix D). These problems were determined based on the seventh grade math curriculum and textbooks. They were similar problems given in the initial assessment that was used to choose the case participants. During the instruction, the researcher initially demonstrated to the students how to solve an algebraic problem using a bar model and then asked the students to solve a similar problem on their own. While the students were solving the problem, they could ask for peer support or the teacher's support. After solving the problems using the bar model method, the researcher taught how to write an appropriate equation. The problems that were solved during the instruction were ordered from easy to more complex ones and grouped into three types, each of which was taught on a separate day of the instruction.

3.3.1.1 Day 1: On the first day, the bar model method was introduced to the students. They learned how to show an unknown with a rectangular bar, the concepts of "more" and "less," and how to represent these concepts in the bar model. Moreover, they learned how to find the numeric value that each rectangular bar represents through arithmetic operations. The problems solved in the first day were as follows:

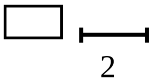
1. Three times a number is 120. So what is this number?
2. If 12 more than 4 times of a number is 132, what is this number?
3. The number which is 8 less than 4 times is 112. So what is this number?
4. Three times the sum of 2 and a number is 36. So what is this number?

To illustrate the bar model solution, a sample bar model solution for the fourth problem is given below.

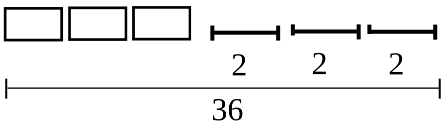
Solution of problem 4 based on the bar model method: In this problem, ‘a number’ is the unknown, so it should be represented with a rectangular bar.

A number: 

Then, the sum of 2 and a number to consider the ordering of the operations in the problem should be drawn. Addition could be drawn using a 2 unit-long line.

The sum of 2 and a number: 

To draw three times of this sum, three bars and three 2 unit-long lines should be drawn. Moreover, the numerical value of whole model is equal to 36.

Three times the sum of 2 and a number: 

According to the model, these 2 unit-long lines should be subtracted respectively:

$$36 - 2 = 34$$

$$34 - 2 = 32$$

$$32 - 2 = 30$$

The numerical value 30 is equal to a value of three bars because three bars were left when 2 unit-long lines were repeatedly subtracted. To find the value of one bar, 30 should be divided into 3, which results in the value of 10 ($30 \div 3 = 10$).

In addition, if x is given for each rectangular bar to represent the unknown, the equation will be $3x + 6 = 36$ because there are three rectangular bars and 6 unit-long lines in total.

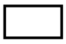
3.3.1.2 Day 2: On the second day, the students learned how to show the unknowns with rectangular bars when there were more than two related quantities in the problem.

They also learned what they had to do if the sum of quantities were given. Moreover, they learned how they could solve problems involving equalities. The problems solved on the second day are given below:

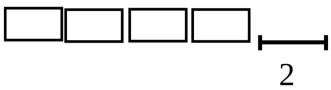
5. The sum of three consecutive numbers is 213. So what is the biggest number?
6. Sema finished her assignment which included 300 questions in 3 days. If she had solved 15 more questions each day, how many questions would she have solved on the second day?
7. 2 more than 4 times of a number and 5 more than 3 times of a number are equal to each other. What is this number?
8. Merve paid 121 TL for 3 skirts and 4 shirts. If one skirt is 10 TL more than the price of one shirt, how much does one shirt cost?

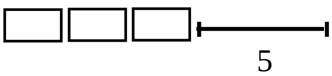
A sample bar model solution for problem 7 is given below.

Solution of problem 7 based on the bar model method: In this problem, 'a number' is the unknown, so it should be represented with a rectangular bar.

A number: 

To represent 2 more than 4 times of a number, initially 4 rectangular bars and then a 2 unit-long line should be drawn. Similarly, initially 3 rectangular bars and then a 5 unit-long line should be drawn to represent 5 more than 3 times of a number in the problem. In addition, the two models should be aligned in length to indicate that two models represent equal quantities.

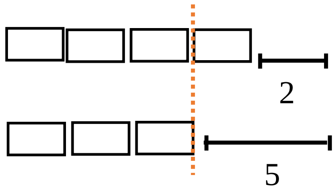
2 more than 4 times of a number: 

5 more than 3 times of a number: 

In equality problems, rectangular bars whose lengths are equal to each other should be removed to see the remaining bars to compare quantities. As can be seen below, when first three rectangular bars are removed, the lengths of one rectangular bar and 2 unit-long line are equal to a 5 unit-long line.

2 more than 4 times of a number:

5 more than 3 times of a number:



Since the lengths of one rectangular bar and a 2 unit-long line are equal to the length of a 5 unit-long line, 2 should be subtracted from 5, which means that 5 – 2 = 3 is the length of the one rectangular bar representing the value of a number.

Moreover, if x is given for each rectangular bar to represent the unknown, algebraic expressions should be 4x + 2 for the first model and 3x + 5 for the second model. To indicate equality, the algebraic equation should be 4x + 2 = 3x + 5.

3.3.1.3 Day 3: On the last day, students learned what they could do if the difference of unknown quantities were given or there were more than two related quantities in the problem. Moreover, the students learned how to solve ‘leg problems’, which, as observed in the initial assessment, was one area students experienced difficulties. The problems solved on the last day of the instruction were as follows:

9. The difference of Ali and his father’s ages is 36. If the father’s age is three times Ali’s age, how old is Ali?
10. The difference of Ali and his father’s ages is 36. If the father’s age is 12 less than 3 times Ali’s age, how old is Ali?
11. The sum of Ahmet, Mehmet and Ali’s ages is 30. Ahmet’s age is one less than Ali’s age and Mehmet’s age is seven more than Ali’s age. So how old is Mehmet?
12. The total number of legs of rabbits and turkeys are 76 in a hencoop. If there are 22 animals in this hencoop, how many turkeys are there?

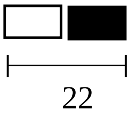
A sample bar model solution for problem 12 is given below.

Solution of problem 12 based on the bar model method: In this problem, ‘number of rabbits’ and ‘number of turkeys’ are the unknowns. Therefore, these will be represented with different colored rectangular bars as shown below.

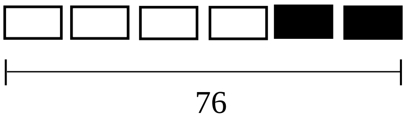
Number of rabbits: 

Number of turkeys: 

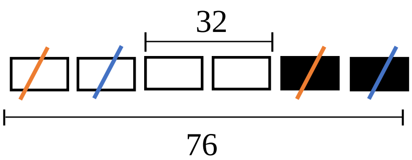
First of all, the total number of animals in the hencoop, which is 22, should be drawn.

Total number of animals: 

Then, the total number of animals’ legs, which is 76, should be drawn. Since rabbits have four legs and turkeys have two legs, there should be four white rectangular bars and two black rectangular bars to represent the legs of the animals.

Total number of legs: 

After drawing the bar model, the focus should be on the value of one white bar and that of the black bar, which is equivalent to 22. In the total number of legs model, one white bar and black bar could both be subtracted since their values are known. This operation needs to be made twice because two white and black bars would be matched.



When the value of one white and the value of one black bar are subtracted from the total number of legs twice, two white rectangular bars are left, which is equal to 32. The operations are as follows:

$$76 - 22 = 54$$

$$54 - 22 = 32$$

Since the value of two white rectangular bars are equivalent to 32, it should be divided into two to find the value of one bar, which results in the value of 16 ($32 \div 2 = 16$). Thus, there are 16 rabbits in the hencoop. Since there are 22 animals in total in the hencoop, the other 6 animals are turkeys: 22 – 16 = 6.

The bar model solutions of all the problems solved during the three-day instruction were provided in Appendix D.

3.3.2. Clinical Interviews

After the instruction, the researcher carried out clinical interviews with each of the 10 students and asked them 10 problems which were similar to the ones in the initial assessment and in the instruction. The set of problems asked during the clinical interviews are provided in Appendix E. While the students were solving these problems, the researcher asked them to use any method they wished to use. They could choose the bar model or directly write an algebraic equation. During the interview, the researcher asked students to explain their thoughts and their methods for each of the problems. The questions that the researcher asked the students to probe their thinking were as follows:

- Can you explain to me what you did?
- Why did you do it in this way?
- Which method did you prefer to use in this problem? Why?
- Which method do you like in general, the algebra method or the bar model method? Why?

- (For the cases applicable) Why do you think that writing an equation is difficult for you?
- Would you like to learn other mathematics topics using the bar model method?

3.4. Data Analysis

In the present study, qualitative data analysis methods, namely content analysis and coding, were used. Content analysis is a way to understand and comprehend human behaviors in indirect ways. It enables the researcher to obtain information, to put it in order and understand qualitative data (Creswell, 2011). According to Elo and Kyngäs (2008) content analysis helps rectify written and verbal words into categories. Moreover, in qualitative research, the researcher collects data using interviews and observations and analyzes these data by means of coding. Coding is defined as obtaining organized data from raw data to comprehend and analyze them more easily (Creswell, 2011).

In brief, ten students participated in the study, they received instruction for three lesson hours, at the end of which researcher interviewed them individually. The content of the lessons was selected and organized by the researcher. The audio and video recordings, which were made of these lessons, were transcribed after each day of the lesson. Moreover, the audio and video recordings of the interviews were also transcribed selectively by focusing on different aspects such as difficulties experienced by the students, the methods chosen by the students, and the research questions.

3.5. Role of the Researcher

Since Johnson (1997) stated that the researcher's opinion, ideas and perspective can affect the results in qualitative research. Creswell (2009) also underlined the significance of the researcher's role in qualitative research, and stated that a researcher should be transparent, give information about her/his past experiences and his/her relationship with the participants. Thus, being transparent and decreasing bias were among my essential aims as a researcher.

I, the researcher of this study, work as a mathematics teacher in the school where the study was conducted. The participants had been my students for two years. There was a camera on the rear side of the classroom and students' faces were not facing to the camera during the video recordings. I also explained to the students that the video recordings and audio tapes would not be shared with anybody and that their real names would not be used in the study. Therefore, the impact of the camera on the students was minimized as much as possible so that the students could act naturally during the instruction and interviews. I explained the purpose of the study and gave information about Singapore mathematics and the bar model method to the participants at the beginning of the research. I also explained that students' participation in instruction, their answers and solutions would not affect their school grades. In addition, in order to reduce bias, I did not express my opinion whatsoever about using the bar model method in mathematics lessons during data collection and analysis procedures. The entire research process was video recorded and audio taped.

As a researcher, I chose participants based on their mistakes to constitute the case to be focused on in the current study. I observed the participants' behaviors, reactions and responses and the difficulties they experienced in using the bar model method during the instruction. Since I was also their teacher, they felt relaxed to ask questions and they had a good communication with me. I also conducted clinical interviews with the participants. In these interviews, I did not make any comments about their solutions and the methods they used in order to maintain my neutral position. When they tried to solve the problem or when they needed my approval, I did not provide any direction to avoid affecting the results of the study. As a teacher, I taught how to solve algebraic word problems by using both algebraic equations and the bar model method. In all the lessons, I answered students' questions, gave them the chance to solve the problems on their own or with their peers and showed them explicitly how to solve the problem to foster their ways of understanding. As a result, I taught them how to use both models by using the same teaching method. This explanation of my role as a researcher and teacher in this study is provided to ensure the validity of the study because researcher's being aware of their own roles and holding a reflective research journal on this issue helps to ensure credibility (Lincoln & Guba, 1985).

3.6. Trustworthiness and Credibility

To understand the quality of a study, the reliability and validity of the research, which is closely related to data collection and analysis, should be looked into (Merriam, 1998). In qualitative research, four concerns, namely credibility, confirmability, transferability and dependability, are listed to ensure reliability and validity (Guba, 1981; Lincoln & Guba, 1985).

The first concern is credibility, which is internal validity in qualitative research. This is used to find out whether the study measures what it actually intends to measure (Shenton, 2004). In the present study, peer examination, triangulation and longitudinal engagement were used to ensure the internal validity of the study (Merriam, 1998). The first strategy was peer examination. In the current study, the researcher studied with a field expert in analyzing the results. They partially did the coding of the transcripts of the audio tapes and video recordings of clinical interviews together. The second strategy was triangulation, which is using different kinds of methods to collect data and different researchers analyzing the same data (Shenton, 2004). In this study, the researcher used different data sources, such as video and audio recordings, interviews and observations to arrive at more detailed and valid results. As also stated above, the codes were triangulated by another researcher's codes. The third strategy was longitudinal engagement, which can be defined as building a trustworthy relationship between the researcher and the participants (Lincoln & Guba, 1985). Since the researcher had been the teacher of the participants for two years, the students acted naturally, felt relaxed and held a sense of trust to the researcher. All these strategies were used to increase the validity of the data.

The second concern is confirmability, which is used to decrease the researcher's bias (Trochim, 2006) Using triangulation could help ensure confirmability (Shenton, 2004). Therefore, in the present study, the researcher's role was explained in the previous section and triangulation was employed.

The third concern is transferability, which is external validity in a qualitative study (Shenton 2004). According to Merriam (1998), it seeks to answer the question of whether the results of the research can be generalized. Although purpose of a qualitative study is not the generalization of the results, transferability could be established by giving detailed explanation about the study and conducting the study with sufficient data. In the present study, the way the case participants were chosen, the data collection tools and the data analysis process were explained in detailed in the previous sections. In addition, 42 students were given an initial assessment test and the researcher interviewed 10 participants. Therefore, giving detailed explanation about the study and conducting the study with sufficient data could help other researchers to transfer the findings of the study.

The fourth concern is dependability, which is reliability in a qualitative study. It refers to finding similar results if the study is replicated in the same context, with similar participants and methods (Shenton, 2004). A detailed explanation of the research process is important for the study to be found trustworthy by other researchers who want to conduct similar studies. It is important for other researchers under which conditions and through which data collection and analysis procedures the current results were reached. Thus, a detailed explanation about how case participants were chosen, the data collection tools and the data analysis process were explained in detail in this chapter. The dates of the lessons and the clinical interviews were written in researcher's journal. Moreover, this journal included the characteristics of the participants, observation notes from instructions and detailed solutions of the problems based on the bar model method. In addition, triangulation is an important strategy for dependability (Merriam, 1998). As mentioned above, triangulation was used by resorting to different data sources and more than one researcher in data analysis. Thus, dependability of the study has been established.

CHAPTER 4

FINDINGS

The present study investigates how the bar model emphasized in the Singapore mathematics curriculum remedy 7th grade students' errors as they solve algebraic word problems in mathematics. To accomplish this purpose, students were asked ten algebra problems and asked to solve them using the methods they preferred. They were also probed by the researcher about their solutions and particularly about the bar model method. These problems were separated into three sets: (1) problems involving quantitative relations but not presented in contextual situations, (2) problems involving quantitative relationships between consecutive numbers, and (3) problems in contextual situations involving two unknown quantities, one of which could be described by the other. Before sharing students' work during clinical interviews, their mistakes in the initial assessment was briefly shared. At the end of this chapter, students' mistakes while using the bar model method was also presented to articulate the sources and/or reasons of their mistakes in algebraic word problems after learning the bar model method.

4.1. Students' Errors in the Initial Assessment

Case participants of the present study were selected among 42 students based on their solutions to problems on the initial assessment instrument involving 10-word problems. Table 4.1 presents the 10 case participants and the error types they demonstrated in the initial assessment questions. In the same table, the number of problems that the students did not give any response to are also shown.

Table 4.1

Case Participants' Errors in Algebraic Word Problems

Students	Blank guessing	Identifying the unknown incorrectly	Setting up the equation incorrectly	Using parenthesis incorrectly in writing the equation	Operational mistakes in solving the equation	Finding the unknown as an answer	No response
Zeynep	2	0	1	0	0	3	4
Ece	0	2	3	0	0	4	1
Melike	3	1	2	0	1	1	1
Mustafa	0	2	3	1	1	1	0
Ali	2	2	3	1	0	1	4
Sinem	0	2	5	2	3	1	0
Emre	1	3	0	1	1	3	0
Umut	2	2	3	1	0	0	3
Merve	0	1	3	1	0	3	4
Melik	0	4	3	1	1	0	3

0 1 2 3 4 5-6

Table 4.1 was also presented as a heat map showing the distribution of errors that each student made. As this table shows, Zeynep mostly made the error of finding the unknown as an answer, while Ece mostly made the error of finding the unknown as an answer. As for Melike, she mostly made the error of blank guessing. Mustafa mostly made the error of setting up the equation incorrectly while Ali mostly made the error of setting up the equation incorrectly. As for Sinem, she mostly made the error of setting up the equation incorrectly. Emre mostly made the errors of identifying the unknown incorrectly and finding the unknown as an answer. Umut mostly made the error of setting up the equation incorrectly, while Merve mostly made the errors of setting up the equation incorrectly and finding the unknown as an answer. Melik mostly made the error of identifying the unknown incorrectly. In addition, Zeynep, Ali, Umut, Merve and Melik had not give any response to three or four of the problems.

4.2. Students' Performances in Solving Algebraic Word Problems

4.2.1. Problem Set 1: Decontextualized Problems Involving Quantitative Relations

This problem set includes four questions, namely P1, P2, P3, and P6. These problems involved one unknown and its quantitative relations described by words, such as 'more than', 'less than', 'equal to' and 'addition.'

The first problem asks: *"If 15 less than 4 times of a number is 35, what is this number?"* [Bir sayının 4 katınınn 15 eksiği 35 ise, bu sayı kaçtır?] In this problem, seven students reached the correct answer by means of the bar model method, and three students solved the problem by writing an algebraic equation. Students' problem-solving processes are explained in detail below.Emre, Melik, Sinem, Melike, Merve, and Umut directly used the bar model method and solved the problem without making any mistakes. They drew rectangular bars entailing correct relationships and set the arithmetic operations accurately. To illustrate, Merve's solution in which the bar model method was used is displayed in Figure 4.1.

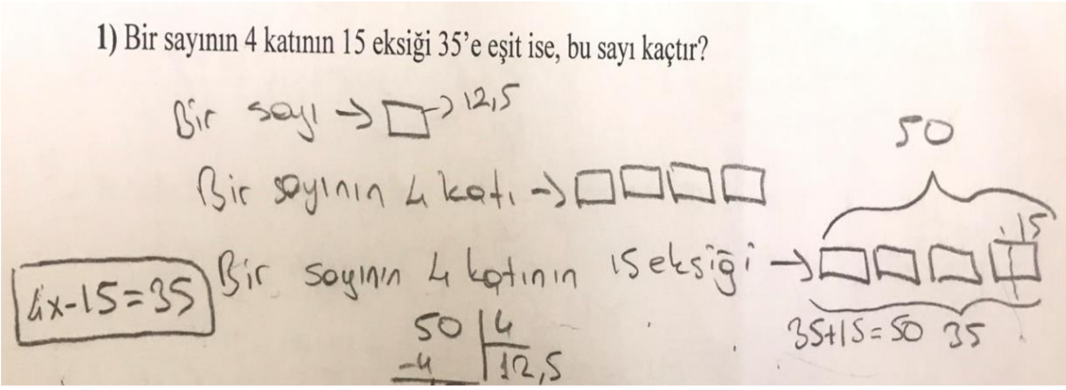


Figure 4.1 Merve's solution to P1

She first represented a number with a rectangular bar. Then she drew four bars to show four times of a number. Then, she split 15 units of the bar because the problem said '15 less than.' When she completed the bar model, she added 15 and 35 because she wanted to complete the bar. Since there were four bars in total, she divided 50 by four and found the answer to be 12.5.

Moreover, the researcher asked the students to write an equation based on the problem and provide an explanation about how they produced the equation. Melike, Sinem, Merve and Umut initially solved the problem by using the bar model. After they used the bar model to solve the problem, they wrote an algebraic equation by considering the words in the problem. For instance, Merve wrote the equation 4x-15=35 as presented in Figure 4.1 for P1. The following dialogue indicates Merve's thoughts about the way she solved the problem.

> **Researcher:** How did you write the equation? Did you look at the bars that you drew or at what was worded in the problem? [Bu denklemi nasıl kurdun? Çizdiğin şekillere mi baktın, yoksa problem cümlesinde söylenenlere mi?]
>
> **Merve:** I wrote the equation according to the statements in the problem. I wrote 4x because it says 4 times of a number. I also wrote -15 because it says 15 less than the number. [Problemdeki kelimelere bakarak denklemi kurdum. 4x yazdım çünkü problemde bir sayının 4 katı diyor. Ayrıca 15 eksiği dediği için de -15 yazdım.]

Researcher: If you can solve the problem by writing an equation, why did you solve it by using the bar model method? [Problemi denklem kurarak çözebiliyorsan, neden bar model yöntemini kullandın?]

Merve: Actually, I can solve it by using either method; however, I preferred the bar model method because that is what you taught us. [Aslında iki yöntemle de çözebiliyorum fakat bar modeli kullanmayı tercih ettim. Çünkü siz bize bu şekilde öğrettiniz.]

This dialogue shows that although students were told that they could use any method they wanted, some students like Merve above thought that they should use the bar model since they received instruction on this method.

On the other hand, two students, Ece and Mustafa, wrote an algebraic equation directly instead of using the bar model method and solved the equation correctly. They said: "Writing an equation is easier than drawing rectangles in this problem."

Another student, Ali, solved the problem as follows:

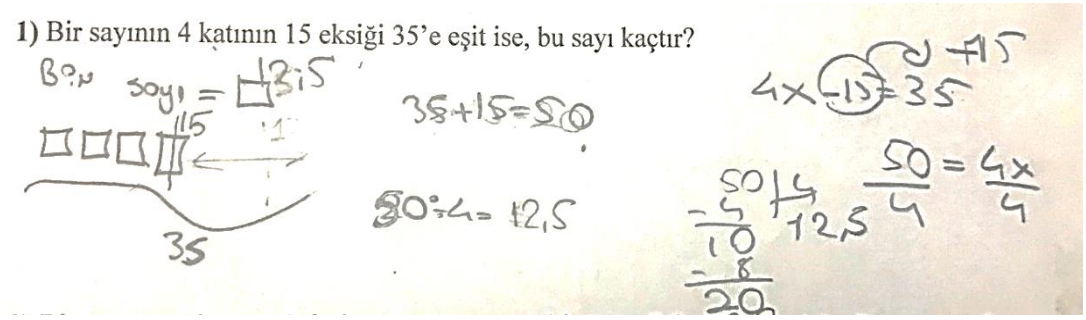


Figure 4.2 Ali's solution to P1

Although Ali could draw bar model correctly, he first subtracted 15 from 35. The following conversational exchange between the researcher and Ali reflects his reasoning:

Researcher: Why did you subtract 15 from 35? [Neden 35'ten 15'i çıkardın?]

Ali: Because the problem said 'less than' so I subtracted 15. [Çünkü problemde 'azdır' diyor, bu yüzden de 15'i çıkardım.]

Researcher: Can you write an equation for this problem? [Bu problem için bir denklem kurabilir misin?]

Ali: Yes, I can write 4x – 15 = 35 [Evet, kurabilirim. 4x – 15 =35]

Researcher: How did you write this equation? Did you read the problem statement or did you look at the bars that you had drawn? [Bu denklemi nasıl yazdın? Problem cümlesini mi okudun, yoksa çizdiğin şekillere mi baktın?]

Ali: I looked at the bars. There are four bars and I gave each bar x so there is 4x. Also, I took out 15 units from one bar, so I subtracted it. Can I solve the equation? [Şekillere baktım. 4 tane kutucuk var ve her birine x verdim; bu yüzden 4x oluyor. Ayrıca bir kutucuktan 15 birim kesmiştim, bu yüzden çıkarma yaptım. Denklemi çözebilir miyim?]

Researcher: Yes, of course. What did you find? [Evet, tabii ki. Ne buldun?]

Ali: I found that x to be 12.5. [x'in 12.5 olduğunu buldum.]

Researcher: Your answers in each solution [*the answer is 5 in the bar model solution, and the answer is 12.5 in the algebraic solution*] are different from each other. What do you think about that? [Bulduğun cevaplar birbirinden farklı. Bu konuda ne düşünüyorsun?]

Ali: I think I made a mistake while subtracting 15 from 35. I think I should have added 15 and 35. [Bence 35'ten 15'i çıkarırken bir hata yaptım. Sanki 15 ile 35'i toplamalıydım.]

Researcher: Why? [Neden?]

Ali: Because I had removed 15 units from the bar, so I should have completed the bar and added 15. [Çünkü ben bir kutucuktan 15 birimlik kısmı kesmiştim. Bu yüzden o kutucuğu bulmak için önce geri tamamlamalıydım, yani 15'i eklemem gerekiyordu.]

Thus, Ali solved the question incorrectly by using the bar model but then reached the right solution by using algebraic equation. However, when he was asked to compare the two solutions, he could detect his mistake and justify that he should have added 15, not by looking at the equation but by using the bar model. That is, he did not say that he should have moved 15 to the other side of the equation with a plus sign (i.e., by resorting to rote memorization of the procedure); instead, he stated that if he added

15, he would complete the fourth bar and so the four bars would be equal to 50 (i.e., 35+15).

Similarly, Zeynep first used the bar model method, subtracted 15 from 35, and divided it by 3 instead of 4 because there were 3 bars left and the fourth bar was incomplete. When she realized that the answer was a repeating decimal when she divided it by 3, she gave up and decided to solve it by using the algebraic method. Zeynep made an operational mistake in this problem. Figure 4.3 below shows Zeynep's both bar model and algebraic solutions to P1.

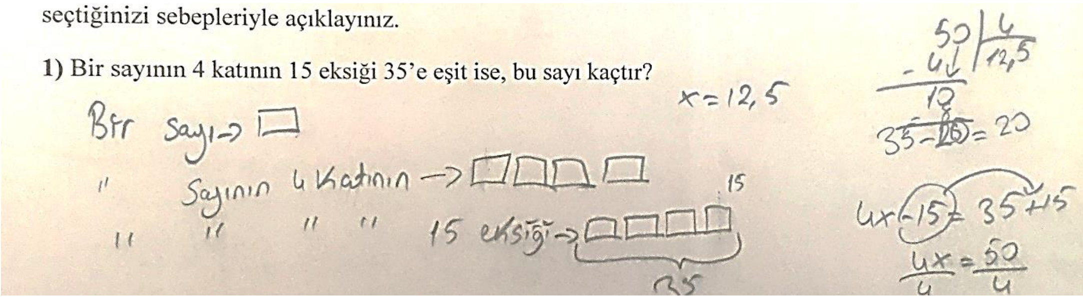


Figure 4.3 Zeynep's solution to P1

In contrast to Ali's case mentioned previously, Zeynep could not detect her mistake in the bar model but wrote the equation and solved it correctly. Both Zeynep and Ali's experiences also showed that knowing both methods gave students the opportunity to check their answers by comparing both solutions. No matter which method they chose, knowing both the algebraic solution and the bar model solution provided them with the opportunity to make comparisons between the two methods of solution.

Thus, in P1, the first preference of eight students regarding the method of solution was the bar model although two students used this method incorrectly. While one student could find his mistake in the bar model, the other student could not find her mistake, but reached the correct answer by using the algebraic equation method. On the other hand, two students solved the problem by directly resorting to the algebraic equation method. Besides, three students used the bar model method while writing an algebraic equation for this problem.

The second problem in this set, P2, also involved two quantitative relations that were equal to each other. Specifically, P2 states: *"3 more than 5 times of a number and 7 more than 4 times of the number are equal to each other. So, what is this number?"* [Bir sayının 5 katının 3 fazlası ile 4 katının 7 fazlası eşittir. Buna göre, bu sayı kaçtır?] In this problem, eight students solved the problem correctly by using the bar model method, while two students reached the correct answer by using the algebraic equation method.

To solve this problem, Emre, Melik, Mustafa, Ali, Merve, Umut, and Melike drew a bar model correctly and reached the correct answer. They explained what they did and why they used the bar model without any hesitation. They showed the equality with rectangular bars and displayed the alignment of the bars properly. Figure 4.4 below shows Melike's solution involving both the bar model and the algebraic equation.

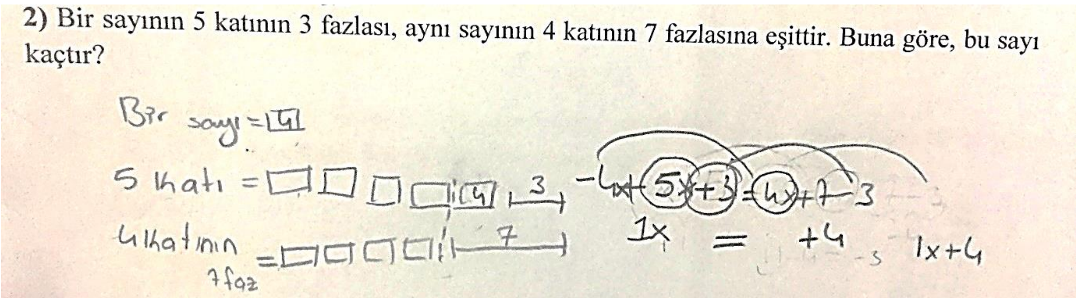


Figure 4.4 Melike's solution to P2

Melike first represented the unknown number with a rectangular bar. Afterwards, she drew 5 bars and a 3 unit-long line in one row and 4 bars and a 7 unit-long line in the second row, and aligned the end points of the two bar models, indicating that the lengths of these two bar models were the same. Then, she drew a dotted line showing that one bar and 3 units are equal to 7 units. From this arithmetic comparison, she found that one bar equals 4 units and wrote 4 inside the fifth bar and the one in the first row. Melike also provided a rational explanation as presented in the following dialogue between the researcher and Melike.

Researcher: How do you know that [*5 bars and*] this 7 unit-long line ends at the point that aligns with [*the four bars and 3 unit-long line*]? [Bu 7 birimlik çizginin diğer şekille aynı hizada biteceğini nereden biliyorsun?]

Melike: Because the problem said that they are equal. So I drew a 7 unit-long line until the end of the 3 unit-long line, so they are in alignment with each other. [Çünkü problemde onların eşit olduğunu söylemiş. Bu yüzden de 7 birimlik çizgiyi, 3 birimlik çizgiyle aynı hizada olacak şekilde çizdim.]

Thus, Melike could rationally explain how she drew the bar model. Afterwards, she wrote the algebraic equation. She represented one bar with x. Since five bars and a 3 unit-long line are equal to four bars and a 7 unit-long line, she wrote the equation as 5x + 3 = 4x + 7. She solved it successfully.

Although two of the students, Sinem and Zeynep, preferred using bar model method, they faced some difficulties. First of all, Sinem drew five rectangular bars correctly. However, she drew a 9 unit-long line instead of a 3 unit-long line in the first row because of carelessness as can be seen in Figure 4.5. She drew four rectangular bars and a 7 unit-long line correctly in the second row. She realized that the length of the one bar and a 9 unit-long line should be equal to a 7 unit-long line. So she subtracted 7 from 9 and found the answer to be 2.

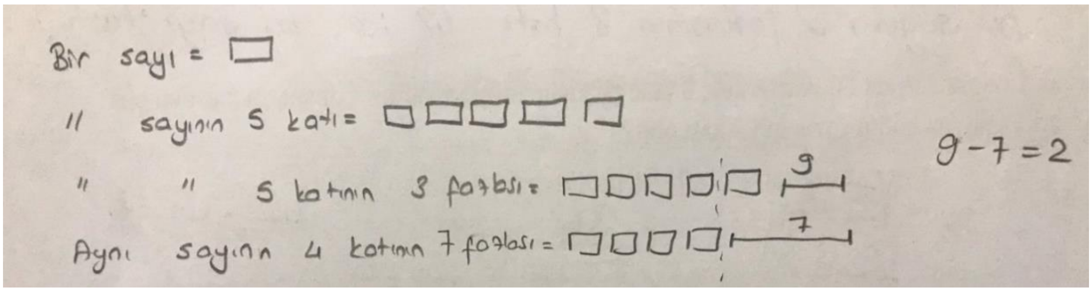


Figure 4.5 Sinem's initial solution to P2

When she wrote an equation based on the word problem and solved it, she found the unknown number to be 4. The following dialogue depicts Sinem's reasoning that was probed by the researcher.

Researcher: Your answers [*the answer is 2 in bar model solution, and the answer is 4 in algebraic solution*] are different from each other? Which answer is correct? [Cevapların birbirinden farklı. Hangi cevabın doğru?]

Sinem: Hmm, I think that the answer is 4 because when I solved the equation, I found that x is 4. [Hmm, sanırım cevap 4 olacak çünkü denklemi çözdüğümde 4 buldum.]

Researcher: So do you trust the equation more than the bar model? [O zaman, sen denkleme bar model yönteminden daha çok güveniyorsun?]

Sinem: Yes, I believe that my equation is true. I must have made some mistakes in the bar model. [Evet, denklemimin doğru olduğuna inanıyorum. Çizdiğimde modelde bir hata yapmış olmalıyım.]

Researcher: Okay. Can you check your solution? [Pekala, cevabını kontrol edebilir misin?]

Sinem: Okay… Oh, I saw my mistake. I wrote nine but I should have written three. I do not know why I wrote that. [Tamam… Aa, hatamı gördüm. Dokuz yazmışım ama üç yazmam gerekiyordu. Neden bu şekilde yazdım bilmiyorum.]

As can be observed in this dialogue, Sinem relied more on the algebraic method than she did on the bar model method. When Sinem checked her solution, she found her mistake immediately. She erased the 9 unit-long line and redrew a 3 unit-long line instead. She already knew that she should have subtracted 3 from 7 because the length of one bar and a 3 unit-long line is equal to a 7 unit-long line as can be seen in Figure 4.6.

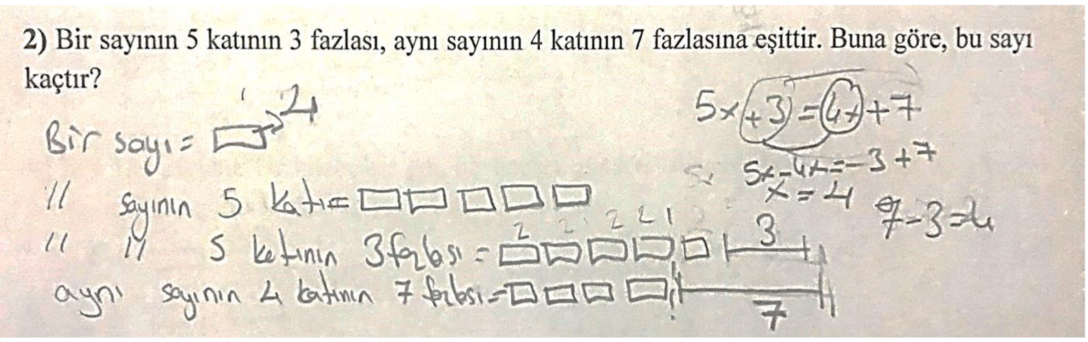


Figure 4.6 Sinem’s revised solution to P2

In brief, Sinem could have solved the problem by using the bar model method but she made a carelessness error. After she checked her answer by using the algebraic equation method, she realized her mistake and corrected it.

Secondly, Zeynep drew bars and lines correctly; however, she could not find the value of a bar by looking at the whole model. She could not do any operation with these bars and lines. The researcher reminded her that she could solve the problem by writing an algebraic equation. Therefore, Zeynep wrote an appropriate equation and solved it without any difficulty as can be seen in Figure 4.7.

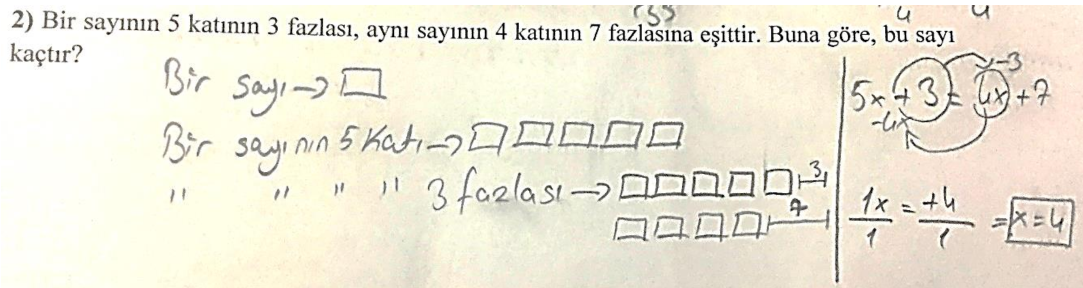


Figure 4.7 Zeynep's solution to P2

Figure 4.7 shows that the student could transform words into the bar model; however, she experienced difficulties in the operations. She made operational errors again in this problem. She could not solve it by using the bar model. However, she wrote the algebraic equation and solved it easily.

On the other hand, Ece solved the problem by directly resorting to writing an equation and did not use the bar model method. When the participants tried to write an equation for the problem, four of the participants looked at their bar model and six of them wrote the equation according to the word problem. These six students did not need to look at the bars that they had drawn; they could already write an equation by reading the problem statement.

Thus, in P2, all the students were found to have used the bar model directly except for one student. This student, Zeynep, could not solve the problem by using this method. Although she solved the problem correctly by using the algebraic equation method, she could not understand how to accurately solve the problem by using the bar model method. Another student, Sinem, made a carelessness error; however, she used the algebraic equation method, realized her mistake and corrected it.

The third problem, P3, involved the addition of two quantitative relationships. It was stated as follows: *"The sum of 1 more than 2 times of a number and 5 less than 3 times of the number is 51. What is this number?"* [Bir sayının 2 katının 1 fazlası ile 3 katının 5 eksiğinin toplamı 51'dir. Buna göre, bu sayı kaçtır?]

In this problem, six students accurately solved the problem by using the bar model method. Two students reached the correct answer by using the algebraic equation method. However, two students could not solve the problem with either of the methods. Besides, five students benefitted from bar model method while writing an algebraic equation in this problem. Students' problem-solving processes are explained in detail below.

First of all, Sinem, Melike, Emre, Umut and Ali solved the problem by using the bar model and they did not experience any difficulties. While Sinem, Emre and Melike wrote an equation with the help of bar model method, Ali could not write the equation at all.

Although Merve and Melik drew bars and lines correctly, they could not arrive at the right answer. First of all, Merve drew a rectangular bar to represent a number. She drew two bars and a 1 unit-long line in the first row. She drew three bars and she split 5 units of the third bar in the second row. Since their total value was equal to 51 and she wanted to complete the third bar, she added 51 and 5. However, she could not continue to solve the problem. The type of error she made was operational. On the other hand, she could write the correct equation and found the unknown value as illustrated in Figure 4.8.

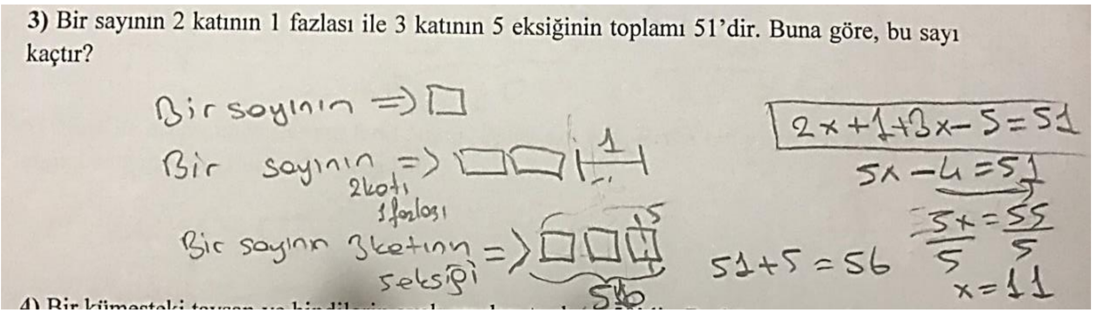


Figure 4.8 Merve's solution to P3

In brief, it was the algebraic method that Merve used to arrive at the correct answer although she had drawn the bar model correctly.

On the other hand, even though Melik had drawn the bar model correctly, he had some confusions. To illustrate, he added 51 and 5, but he did not subtract 1 from 56, as depicted in Figure 4.9.

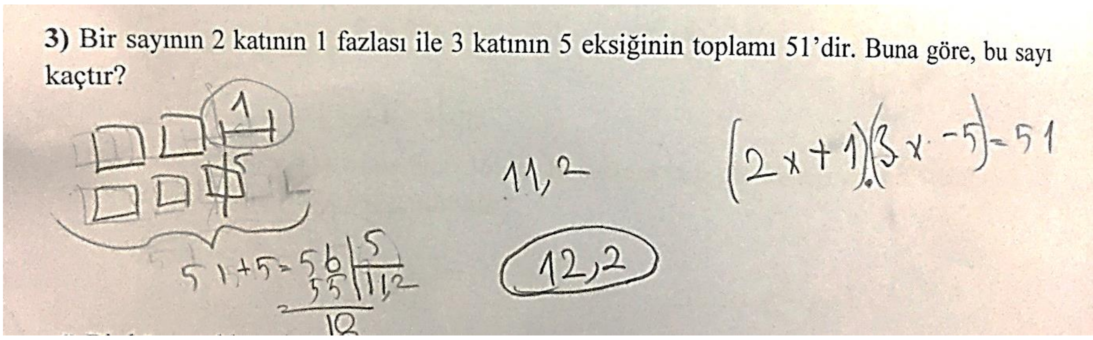


Figure 4.9 Melik's solution to P3

He should have removed the 1 unit-long line; however, he divided 56 into 5 bars directly and added 1 without any reasonable explanation. Moreover, the type of error which he did in this problem was operational. He could neither write the correct algebraic equation. He looked at the bar model method to write the equation. He wrote 2x + 1 to represent the first row and he wrote 3x – 5 to represent the second row. However, he did not put an addition sign between these algebraic expressions; he placed a multiplication sign without any rational reason.

In brief, Melik could not arrive at the correct answer with either of the two methods.

Another student, Mustafa, wanted to solve the problem by writing an algebraic equation first; however, he could not write the equation. For this reason, he continued with the bar model method and could solve it without any difficulty. Moreover, he could correctly write the algebraic equation later by the looking at his bar model. Ece also wanted to solve the problem by just writing an algebraic equation. Different from Mustafa, she could write the correct equation without using the bar model method and found the correct response.

Zeynep made some mistakes like in the previous problem although she drew the correct bar model based on the problem statement. After she drew the correct model, she subtracted 5 from 51 instead of adding them, so she found 46 as an answer. Subsequently, she added 1 and 46, but she was supposed to subtract 1 from the result. Therefore, she could not reach the right answer by using the bar model because she made an operational error, again. Moreover, when she tried to write an algebraic equation to check her solution, she did not use 'addition'. She just wrote two algebraic expressions, such as 2x + 1 and 3x -1 as equal to one another. As seen in Figure 4.10, Zeynep could not reach the correct answer with either of the two methods.

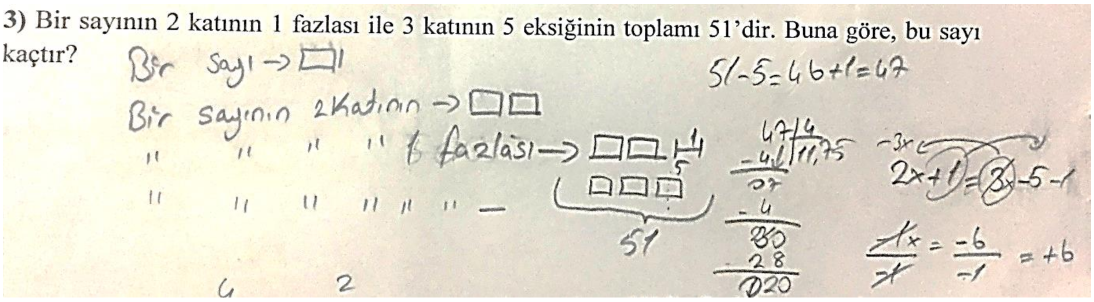


Figure 4.10 Zeynep's solution to P3

Thus, to solve P3, eight students' first preference was to use the bar model method; however, two of them, Melik and Zeynep, could not arrive at the correct answer. Moreover, one student reached the correct answer by using the algebraic equation after she made an operational error with the bar model method. On the other hand, two students' first preference was to use the algebraic equation method. While Ece could solve the problem by using this method, Mustafa could not solve it and continued with the bar model successfully. Besides, while four students benefitted from the bar model method to write the equation, three students could not write the equation at all.

The fourth problem in this set, P6, also involved a quantitative relationship, but different from the ones in P1, P2 and P3; multiple of a quantitative relation is presented in P6 and stated as follows: *"Three times the sum of 2 and a number is 42. So what is this number?"* [Bir sayının 2 fazlasının 3 katı 42'dir. Buna göre, bu sayı kaçtır?]

By using the bar model method in this problem, five students solved the problem correctly, while three students solved it incorrectly because they had made transforming and operational errors. Moreover, one student reached the correct answer by using the algebraic equation method. Besides, nine students benefitted from the bar model method while writing an algebraic equation in this problem even though some of them had written a wrong algebraic equation. The students' problem-solving processes are explained in detail below.

This problem requires the participants to be careful about the sequence of the operations. They should first show the sum of 2 and the unknown with the bar model or algebraic expressions. Afterwards, they had to multiply them with three. If they did this by writing an equation, they needed to place the expression x+2 within parentheses. But if they did that by drawing bar models, they should draw both three rectangular bars and three lines to show the multiples of them.

The students named Mustafa, Sinem, Merve and Umut solved the problem by using the bar model method successfully in their first attempts. For example, as can be seen in Figure 4.11, Merve represented the unknown ('a number') with one rectangular bar. Then, she drew one bar and a 2 unit-long line to show the sum of two and the unknown. After that she drew three bars and three 2 unit-long lines.

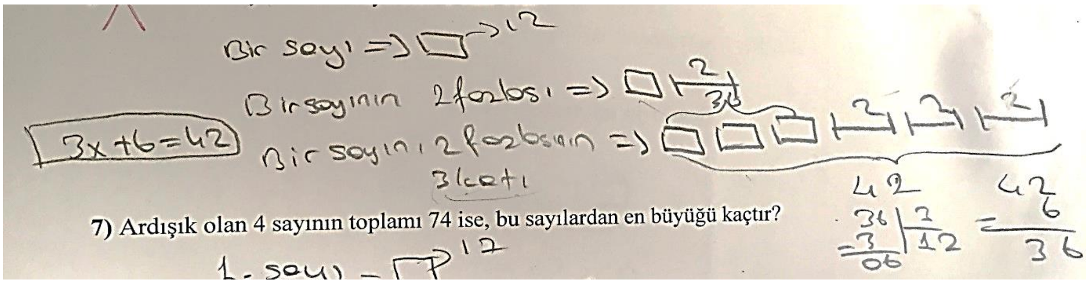


Figure 4.11 Merve’s solution to P6

The following dialogue between Merve and the researcher shows Merve’s explanation about why she drew the bar and the 2 unit-long line three times.

> **Researcher:** I see that you drew three bars and three 2 unit-long lines. Why did you do it like that? [3 tane kutucuk ve 3 tane 2 birimlik çizgi çizdiğini görüyorum. Neden bu şekilde yaptın?]
>
> **Merve:** Because I should draw three pieces from each bar and 2 units lines because the problem said ‘three times the sum of 2 and a number’, so I first drew the sum of 2 and a number. [Çünkü her kutucuktan ve 2 birimlik çizgiden üç adet çizmeliyim. Çünkü problemde bir sayının 2 fazlasının 3 katı diyor, bu yüzden de önce bir sayı ile 2’nin toplamını gösterdim.]
>
> **Researcher:** What does a bar symbolize for us? [Burada bir kutucuk bize neyi sembolize ediyor?]
>
> **Merve:** I do not know the value of a number, so I reprsented the number with a bar. [Bir sayının değerini bilmiyorum, bu yüzden bir sayıya bir kutucuk verdim.]

This dialogue shows that Merve drew three bars and 2 unit-long lines because she wanted to multiply the sum of two and the unknown number by three. Afterwards, she subtracted three times of a 2 unit-long line from 42. Since there are three rectangular bars, she divided 36 by three and found the answer to be 12. Moreover, she wrote the algebraic equation by looking at the bar model. She used x for a bar. Since there were three bars and three 2 unit-long lines, she wrote 3x + 6 and equated it to 42 as follows: 3x + 6 = 42.

While Emre was solving the problem, he faced some difficulties and made errors; however, he found the correct answer at the end. First of all, he represented a number with a rectangular bar and a 1 unit-long line without any explanation. When the researcher asked for his reason, he could not answer and he said: “I guess that’s how we did it in class”. Afterwards, he drew three bars and a 2 unit-long line because the problem statement was follows: “three times the sum of two and a number”. Then, he equated the model to 42 as can be seen in Figure 4.12.

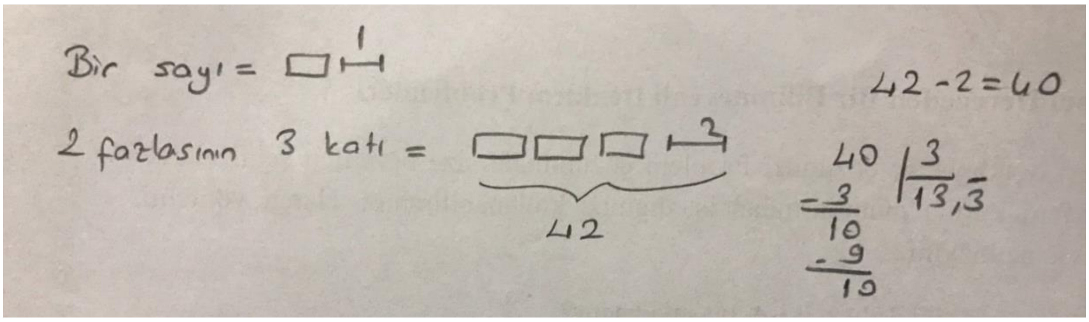


Figure 4.12 Emre’s initial solution to P6

It can be seen that Emre subtracted 2 from 42 and found 40. Subsequently, he divided it by three because there were three bars. When he found the answer to be a repeating decimal number, he decided to reread the problem and think once more. Even though he had made errors in the order of the operations, he instantly realized it and said: “There should be three 2 unit-long lines because the problem said three times the addition of 2 and a number”. Therefore, he drew three bars and three 2 unit-long lines and found the value of the bar correctly, as can be seen in Figure 4.13. Moreover, he could write the algebraic equation based on the bar model as 3x + 6 = 42 at the end.

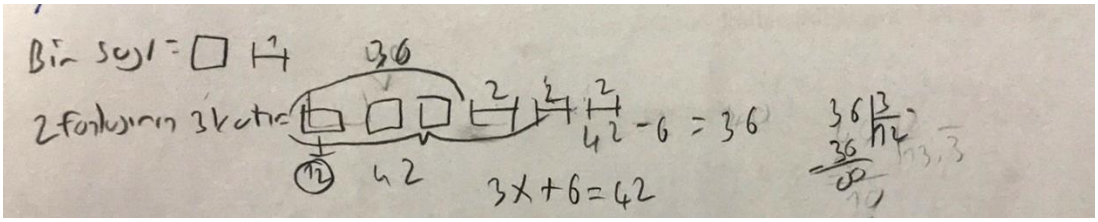


Figure 4.13 Emre’s revised solution to P6

Thus, Emre realized that he had made a mistake in the ordering of the operations and corrected it.

Another student, Melik, could not draw an appropriate bar model based on the problem statement and made a transforming error. Even though he was careful about the sequence of the words, he first drew a rectangular bar and a 2 unit-long line. Then, he added three more bars to indicate the expression, 'three times'. Then he subtracted 2 from 42 and found 40. When he divided 40 by four, he found one bar to be 10 and he wrote the equation as 4x + 2 = 42 based on the bar model. Figure 4.14 shows Melik's solution to P6.

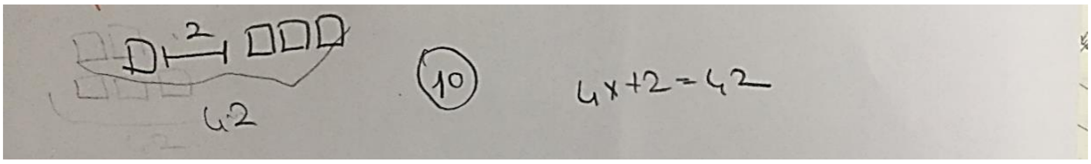


Figure 4.14 Melik's solution to P6

Thus, Melik made a transforming error and drew the bar model wrongly. Since the bar model was incorrect, he could not find the answer. But he wrote correct algebraic equation, which was correct.

Ece drew the bar model correctly; however, she could not solve the problem by using the model as she could not find the value of a rectangular bar. She initially tried to divide 42 by 3, but she said that she could not remember how to proceed with the solution process. So, the researcher reminded her that she could write an algebraic equation instead of solving the problem by means of the bar model method. Afterwards, she wrote the equation easily and did not look at the bar model while writing it. While she was writing the equation, she placed the parentheses correctly and wrote 3.(x + 2) = 42. Then, she solved the equation. An excerpt from the dialogue between the researcher and the student is presented below.

> **Researcher:** Which operation did you do first while you were solving the equation? [Denklemi çözerken önce hangi işlemi yaptım?]

Ece: Uhm… First, I subtracted six from 42 and found 36. [Hmm… Öncelikle 42'den 6'yı çıkardım ve 36 buldum.]

Researcher: Okay. Can you now look at the bar model you drew? Can you show me where we can see six [in your model]? [Pekala. Şimdi çizdiğin modele bakar mısın? Nerede 6 gördüğümüzü bana gösterebilir misin?]

Ece: The value of these 2 unit-longs lines is 6. [Bütün bu 2 birimlik çizgilerin değeri 6 yapar.]

Researcher: If you subtract these six units from 42 as you did in the equation, do you think it will be true? [Denklemde yaptığın gibi, 42'den bu 6 birimlik kısmı çıkarırsan sence doğru olur mu?]

Ece: Ooh… I was going to subtract it, yes, I remember that. If I subtract six from 42, I will find 36. [Aa… Evet, bunu çıkaracaktım, şimdi hatırladım. Eğer 42'den 6'yı çıkarırsam 36 bulacağım.]

Researcher: Okay. Can you show where the value of 36 is in the model? [Tamam. Modelde nerenin değerinin 36 olduğunu gösterebilir misin?]

Ece: I removed these 2 unit-long lines, so three rectangular bars were left in the model. The value of these bars is 36. [Bu 2 birimlik çizgileri çıkardım, bu yüzden modelde geriye sadece 3 tane kutucuk kaldı. Bu kutucukların değeri 36 olur.]

Researcher: So what will do you now? [Peki şimdi ne yapacaksın?]

Ece: I will divide 36 by 3 to find the value of the unknown number. [Bilinmeyen sayının değerini bulmak için 36'yı 3'e böleceğim.]

This dialogue shows that Ece made sense of the operations in the bar model with the help of the algebraic equation method and researcher's prompts. Therefore, Ece found the correct answer with the help of the equation and could solve the bar model as can be seen in Figure 4.15.

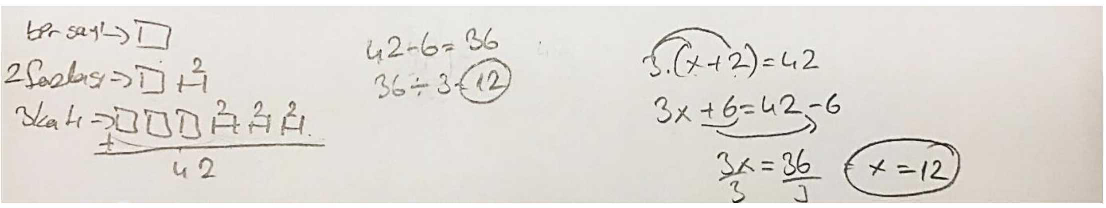


Figure 4.15 Ece's solution to P6

Figure 4.15 shows Ece's solution. Although she could draw the bar model, she made an operational error and could not find the correct answer at first. Then she could find the value of a rectangular bar by using the bar model when she solved the problem by writing an algebraic equation.

Another student, Ali, solved the problem incorrectly because he could not use the bar model correctly. He referred to a number with a rectangular bar; however, he drew only one 2 unit-long line and three bars. In other words, he drew exactly what he read and did not pay attention to the order of the operations. Ali's solution can be viewed in Figure 4.6.

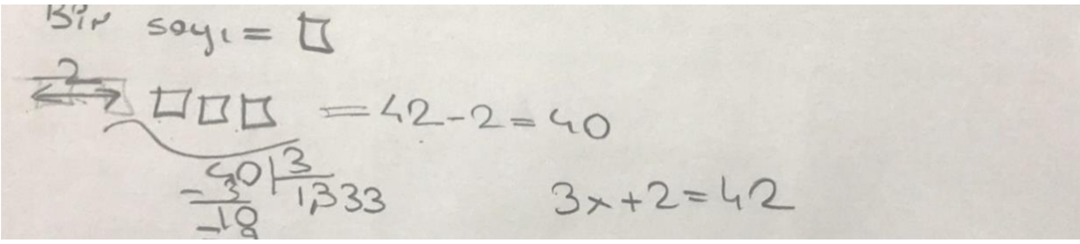


Figure 4.16 Ali's solution to P6

Afterwards, he subtracted 2 from 42 to remove the 2 unit-long line, and he divided 40 by three because there were three bars. Moreover, he wrote the equation by looking at the bar model as follows: 3x + 2 = 42 which was also wrong.

In brief, Ali made the ordering of operation error while trying to solve the problem by means of the bar model method. Since he drew the bar model incorrectly, he could not find the answer.

Melike also made mistakes while drawing the bar model based on the problem statement. Even though she correctly showed the sum of the unknown and 2 with a rectangular bar and a 2 unit-long line, she could not draw multiples of the expression, 'three times'. She just drew one 2 unit-long line and three bars, so she just multiplied the unknown number by three. She also made a mistake while solving and trying to find the value of the unknown because she said that 42 is equal to the sum of the second and third rows in the bar model although she was supposed to look at the bar model only in the third row. Therefore, she found that value of the unknown number to be 9.5.

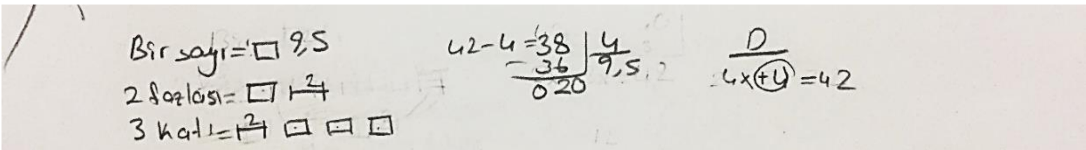


Figure 4.17 Melike’s solution to P6

As a result, Melike made both ordering of operation and operational errors. Moreover, she wrote the algebraic equation incorrectly because she based it on the bar model, but the model was incorrect.

Another student, Zeynep, made the operational error as she did in the previous problems. More specifically, she drew the bar model correctly, but she could not accurately choose which operations to do. She added 6 to 42 instead of subtracting it. Afterwards, she divided the result by three, which is the correct operation. She found the answer to be 16. When she wrote the equation and solved it, she found a different answer because she made use of the bar model while she was writing the equation, and she solved the equation without any mistake. However, she could not decide which answer was true and she did not understand what her mistake was in the solution. She decided to leave the problem with two results, as can be seen in Figure 4.18.

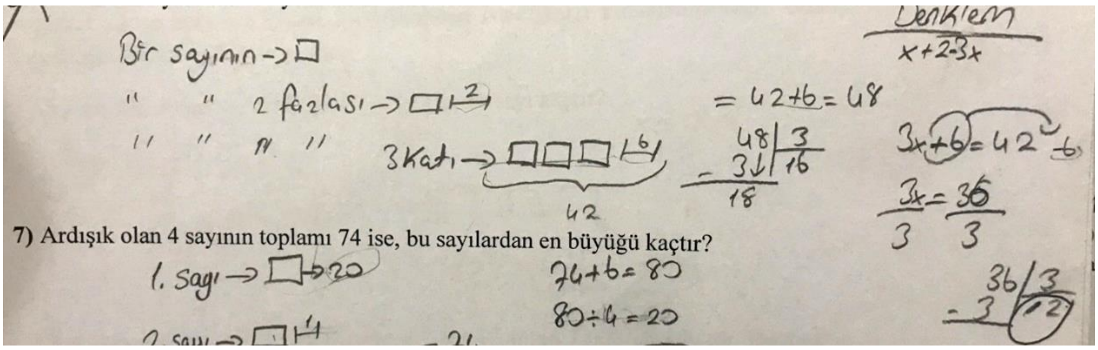


Figure 4.18 Zeynep’s solution to P6

Thus, Figure 4.18 illustrates that Zeynep could draw the correct bar model; however, she made operational errors. Moreover, she could write the algebraic expression thanks to the bar model. However, she could not understand her error and could not decide which answer was true.

To sum up, all the students preferred using the bar model method first. While five students used the bar model correctly, four of them could not solve the problems by means of the bar model method and made some errors, which were transforming and operational errors. In fact, these four students wrote the algebraic equation inaccurately too because they based it on the model that they had drawn. On the other hand, when one student did not solve the problem by using the bar model, she decided to solve it via the algebraic equation method and found correct answer. Nine students resorted to the bar model method to write an algebraic equation.

4.2.2. Problem Set 2: Problems Involving Quantitative Relationships between Consecutive Numbers

This problem set includes three questions, namely P7, P8 and P10. In these problem types, there is more than one unknown. The consecutive relationship between these unknowns were given in the problem statement. This problem set requires students to decide how to represent the first unknown with the bar model or algebraic expression and the other unknowns connected to the first one. Since the consecutive numbers were given in an addition context, they need to add either the bar model of each of the consecutive number or the algebraic equation representing each of the consecutive number. If they use the algebraic equation method, they should write an equation. If they use the bar model method, they should add all the bars and find the value of one rectangular bar.

The first question in this problem set, P7, was as follows: *"If the sum of four consecutive numbers is 74, then what is the biggest number?"* [Dört ardışık sayının toplamı 74 ise, bu sayıların en büyüğü kaçtır?] In this problem, six students solved the problem correctly by using the bar model method. However, four students solved the problem inaccurately by using the bar model method because they made blank guessing, transforming and operational errors. Besides, nine students made use of the bar model method while writing an algebraic equation. Some of them wrote wrong algebraic equations because their bar models were erroneous. Students' problem-solving processes are explained in detail below.

This problem necessitates the students to consider what a consecutive number is and to be careful about how to show all the unknowns by means of the bar model. The students are expected to draw one rectangular bar for the smallest number or write x to represent it. Afterwards, they need to draw the other unknowns, so they are expected to draw one bar and a 1 unit-long line, two 1 unit-long lines, and three 1 unit-long lines, respectively, or write x + 1, x + 2 and x + 3 to symbolize the other unknown numbers.

All the students tried to solve this problem by resorting to the bar model method. Melik, Mustafa, Sinem, Ece, Umut and Merve drew the bar model, showed each unknown via the bar model, summed up the whole model correctly and found the value of a rectangular bar by doing the operations correctly, except for Mustafa. Mustafa had followed all the steps correctly, but he made a careless error while doing the division operation. Since he divided 68 by four and found an incorrect answer, which was 14.5, so he could not reach the value of the biggest number. Figure 4.19 shows Mustafa's solution to P7.

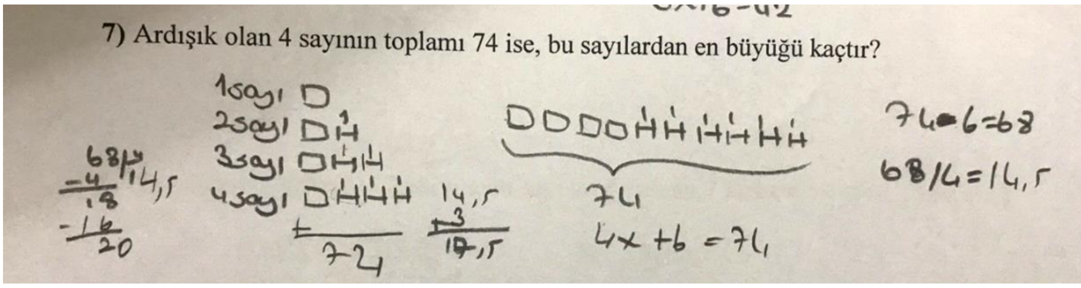


Figure 4.19 Mustafa's solution to P7

As can be observed in the Figure 4.19, Mustafa represented the unknown with a rectangular bar. Since consecutive number is one more than the previous number, he drew one rectangular bar and 1 unit-long line for the second number. He also drew one bar and two 1 unit-long lines for the third number and one bar and three 1 unit-long lines for the fourth number. He added all the numbers, which was equivalent to four bars and six 1 unit-long lines. Since their total value was 74, he first subtracted the values of the lines; that is, he subtracted 6 from 74 and found 68. Then, he divided it by four and found 14.5 instead of 17. When he wrote the algebraic equation in the end, he used the bar model method.

All the students, except for Ece, wrote the algebraic equation based on the bar model. They counted all the rectangular bars and wrote 4x to represent them. They also added the lengths of all the lines and wrote ‘+6’. Therefore, they wrote the equation as 4x + 6 = 74, which was correct. However, Ece did not look at the bar model while writing the equation. First, she solved the problem by using the bar model method correctly. Subsequently, she wrote all the variables separately, such as x for the first number, and x + 1 for the second number. However, she continued writing the equation until x + 4, which means she wrote one additional unknown number: x + x + 1 + x + 2 + x + 3 + x + 4 = 74. When she counted the number of times she had written x, she realized that she had written too many of them because there should be four *x*. So she deleted the last expression, x + 4, and wrote the equation as x + x + 1 + x + 2 + x + 3 = 74. Figure 4.20 shows Ece’s solution process to P7.

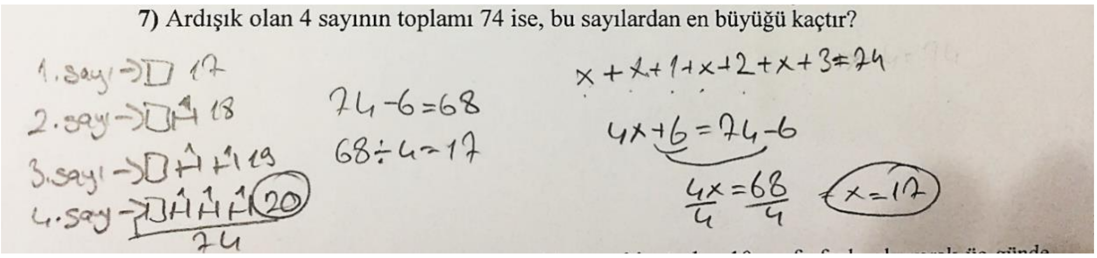


Figure 4.20 Ece’s solution to P7

Most of the students solved this problem correctly. However, some of them, such as Emre and Melike, made some errors. They drew one rectangular bar for the smallest number, and they explained that consecutive numbers increase one by one. However, they drew two rectangular bars for the second number, three rectangular bars for the third number and four rectangular bars for the fourth number; that is increasing the number by one bar at a time instead of adding 1 unit-long lines (see Emre's solution in Figure 4.21). Because the meaning of two bars is two times the first number, they should have drawn a 1 unit-long line for the second number. Therefore, they had drawn ten bars in total, so they divided 74 by 10 and found the value of a bar to be 7.4. Moreover, both of them wrote the equation as 10x = 74 by considering the bar model.

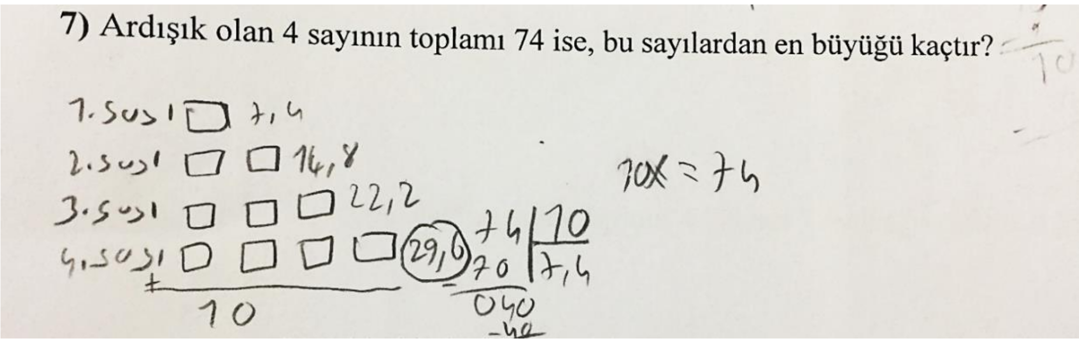


Figure 4.21 Emre's solution to P7

Figure 4.21 shows Emre's solution, which is the same as Melike's solution in this problem. They both made a transforming error.

Secondly, Ali made more than one error. Since he could not make sense with the problem, so he tried to do some operations without any rational reason. He divided 74 by 1, 2 and 3, respectively. Thus, he made the error of blank guessing in this problem. After the researcher asked him how he had represented the smallest number by using the bar model, he drew one bar and a 1 unit-long line for the first number. He also added a 1 unit-long line more for each of the consecutive numbers (see Figure 4.22). However, he did not subtract the lengths of the lines; he just divided 74 by four because there were four bars in total. Therefore, he found the value of the smallest number to be 18.5. He also wrote the equation as 4x = 74, which illustrates that he totally ignored the lines that he had drawn.

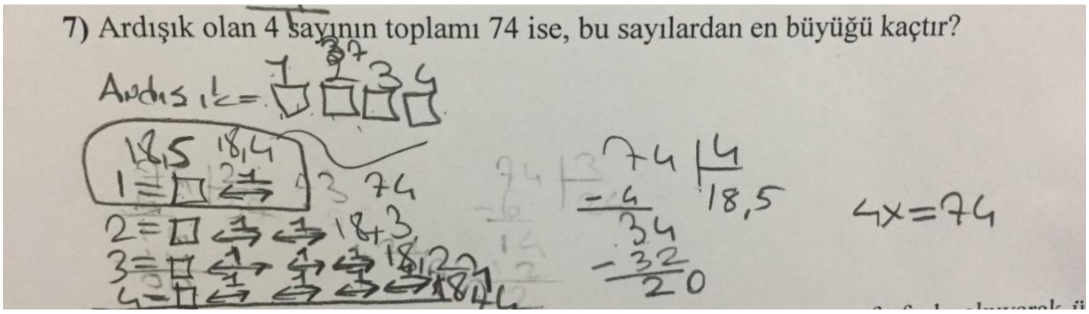


Figure 4.22 Ali’s solution to P7

As a result, Ali made both blank guessing and operational errors while using the bar model method.

Thirdly, Zeynep could draw the correct model and show all the unknowns by using the bar model, but she could not choose the correct operations to do. She again added all the lengths of the lines to 74 instead of subtracting them. After dividing the result by four, she found the value of a rectangular bar to be 20. When she wrote the equation based on the model and then solved it, she found that x was 17, different from the other answer. Although the answer she found in the algebraic equation, which she could solve easily, was correct, she still needed the bar model method to set up this equation.

In brief, in P7, students’ first preference to solve the problem was the bar model but four students used this method incorrectly. Nor could these four students set up correct algebraic equations, so they could not find the correct answer.

The second problem in this set, P8, also involved the consecutive relationship, but different from the P7, the consecutive relationships in P8 were contextualized. Specifically, P8 says the following: *“Elif finished reading her 180-page book in three days by reading 10 pages more than she read the previous day, so how many pages did she read on the first day?”* [Elif 180 sayfalık bir kitabı, her gün bir önceki günden 10 sayfa fazla okuyarak üç günde bitiriyor. Buna göre, ilk gün kaç sayfa kitap okumuştur?] In this problem, nine students solved the problem correctly by means of the bar model method. However, one student solved the problem incorrectly via the bar model method because he made a transforming error. Besides, all the students benefitted from bar model method while writing an algebraic equation in this problem even though one of them wrote the wrong algebraic equation because his bar model was wrong. Students’ problem-solving processes are explained in detail below.

This problem is similar to the previous problem, P7; hence, it should be solved like the former one. Students should draw one rectangular bar or represent page number of the first day with ‘x’. Then, they should draw or write the number of pages read on the other days, which increases 10 per day. When they show the problem context with the bar model or algebraic expressions, they should add up all the components up and equate it with 180. Finally, they should find the number of pages that Elif read on the first day.

All the students, except for Ali, drew the correct model, did the correct operations and found the correct answer in this problem. All of them used the bar model method and wrote the equation by looking at the bar model. They provided a rational explanation of what they had done. For example, as illustrated in Figure 4.23, Mustafa arrived at the correct answer to P8 by using the bar model method.

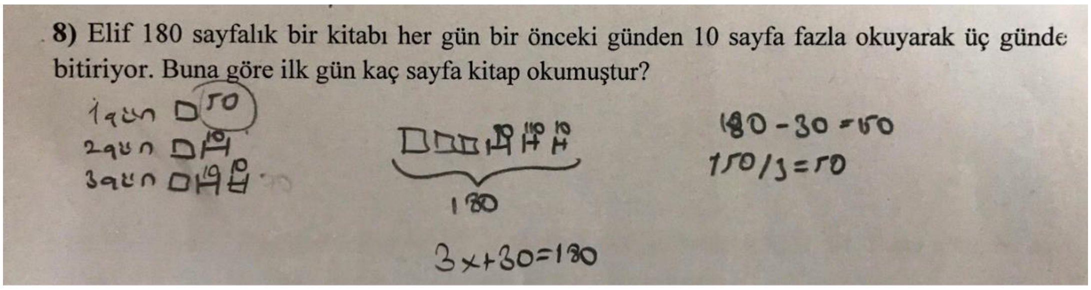


Figure 4.23 Mustafa’s solution to P8

He drew one rectangular bar for the first day, one bar and 10 unit-long line for the second day and one bar and two 10 unit-long line for the third day. He added up all the bars and so drew three bars and three 10 unit-long lines. He equated these bars with

180 because Elif had read 180 pages in total. He subtracted 30 from 180 because the total length of the lines was 30. Since there were three rectangular bars, he divided 150 by three and found the value of one bar. Moreover, he used the bar model to write the algebraic equation. He wrote 3x because there were three rectangular bars and he wrote +30 because the total value of the lengths of the lines was 30. Therefore, he wrote the algebraic equation as 3x + 30 = 180.

Zeynep did not make the same mistake as she had done so in the previous problems, P7 and P6. She added the lengths of the lines instead of subtracting them in problems 1, 2, 3, 6 and 7. However, she decided to subtract 30 more pages from 180 after she drew the bar model correctly (Figure 4.24).

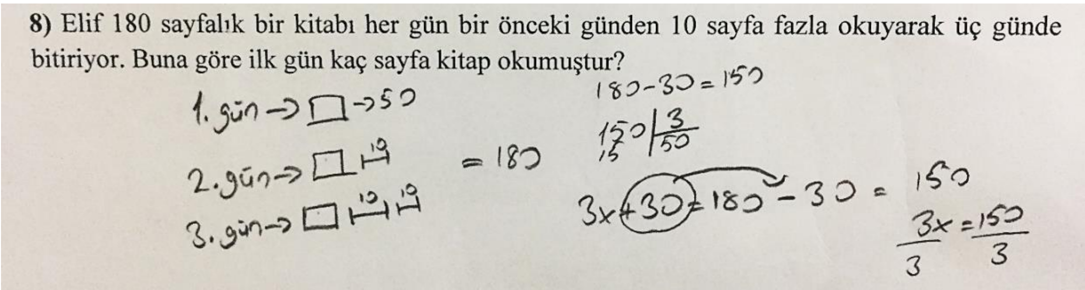


Figure 4.24 Zeynep’s solution to P8

She also justified her answer with an equation after solving it via the bar model method. Following is an excerpt from a dialogue that took place between the researcher and Zeynep after Zeynep had solved the problem:

> **Researcher:** Zeynep, you added the lengths of the lines in the previous problems; however, now you have subtracted these lengths of these lines. Do you have any reason for doing this? [Zeynep, sen önceki problemlerde bu çizgilerin uzunluklarını eklemiştin. Ama şimdi bu uzunlukları çıkardın. Bu şekilde yapmanın bir açıklaması var mı?]
>
> **Zeynep:** I do not know actually. When I added the lengths of the lines, I arrived at an answer that was different from the equations that I had written. However, I wanted to try subtraction in this problem and the answer was exactly the same with the equation. I think I should have used subtraction in the previous problems too. [Aslında tam bilmiyorum. Çizgilerin uzunluklarını eklediğimde, kendi yazdığım denklemlerden farklı sonuçlar elde ettim. Ama bu problemde

çıkarmayı denemek istedim ve gerçekten de denklemle aynı cevabı buldum. Sanırım önceki problemlerde de çıkarma yapmam gerekiyordu.]

Researcher: How do you know that the equation you wrote was correct? Or maybe, you solved the equation wrongly? [Denklemde bulduğun cevabın doğru olduğunu nasıl biliyorsun? Belki de denklemi yanlış çözmüşsündür?]

Zeynep: No, I am sure that I solved the equations correctly. I do not know why, but I was always good at solving equations. Also, I wrote the equations by considering the shapes that I had drawn, so they must be correct. [Hayır, denklemi doğru çözdüğümden eminim. Sebebini bilmiyorum ama denklem çözmekte her zaman iyiydim. Ayrıca denklemleri çizdiğim şekillere bakarak kurdum. Bu yüzden doğru olmalılar.]

This dialogue shows that Zeynep was sure that her equation was correct because she could draw bar models correctly. She could also solve algebraic equations without any problem. Therefore, knowing both methods helped her to find the correct answer. Also, she realized her misconception while solving P8.

On the other hand, Ali was the only student who solved the problem incorrectly. Actually, he found the number of pages read on the first day correctly; however, he made some mistakes while drawing the model. Since he drew the model wrongly, he also wrote the equation wrongly at the end of the problem. He only drew a 10 unit-long line for the first day, as can be seen in Figure 4.25.

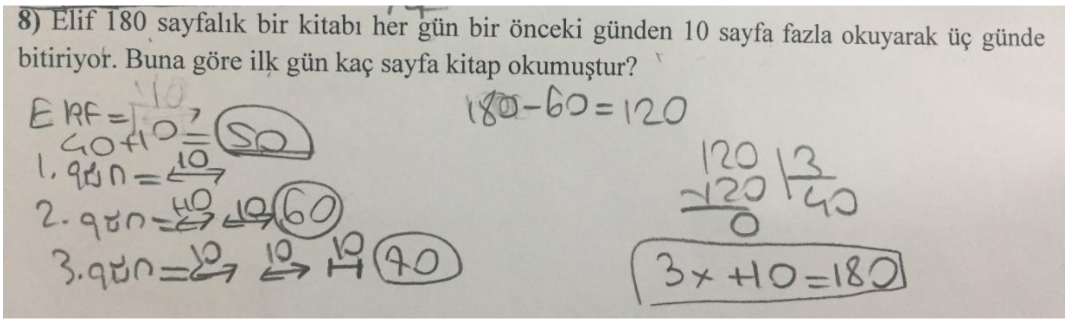


Figure 4.25 Ali’s solution to P8

He did not draw any rectangular bar or any symbol for the unknown quantity. Then, he drew two 10 unit-long lines for the second day and three 10 unit-long lines for the third day. He showed the increasing number of pages each day; however, he needed to use rectangular bars for the unknowns. After the drawing, he added all the lengths of the lines, which equaled 60. He subtracted 60 from 180, the total page of the book. Since there are three days in the problem, he divided the result by three and found 40. He added 40 to 10 because there were a 10 unit-long line for the first day, so he found that Elif read 50 pages on the first day. Moreover, he wrote the equation as 3x + 10 = 180, which was also wrong.

As seen in Figure 4.25, Ali made a transforming error because he could not draw the correct bar model.

In summary, in P8, all students preferred using the bar model method. While nine students solved the problem correctly, one student could not use the bar model correctly because he did not draw any rectangular bar for the unknown value. Besides, one student realized her mistake in the former problems and found the right answer but she did not correct her mistake in the previous problems.

The third problem in this set, P10, is given in a context about ages and involves the additive relationship. Although this problem does not particularly involve consecutive relationship between quantities, the numerical values used in the problem were selected as two consecutive numbers. Specifically, P10 says the following: *"The sum of Harun, Zafer and Ömer's ages is 65. If Harun is 4 years younger than Zafer, and Ömer is 3 years older than Zafer, then how old is Ömer?"* [Harun, Zafer ve Ömer'in yaşları toplamı 65'tir. Harun Zafer'den 4 yaş küçük ve Ömer Zafer'den 3 yaş büyük olduğuna göre, Ömer kaç yaşındadır?] This problem was solved correctly by ten of the students by using the bar model method. Moreover, eight students made use of the bar model method while writing an algebraic equation in this problem. However, one student could not write the correct equation. The students' problem-solving processes are explained in detail below.

The context of this problem is similar to that of P7 and P8 because there are more than two unknowns in this problem as well. There are three people and their ages are unknown. This problem necessitates the students to show one of the unknowns with a rectangular bar or a letter like x. Afterwards, they should show the other quantities based on the problem statement by using the same unknown. When they sum up all the unknowns, the result should be equal 65.

All the students found the correct answer in this problem by using the bar model method even though some of them had some confusions. The first student, Emre, could not decide for which variable to draw a rectangular bar. Emre said, "I think I should draw a rectangular bar for Zafer's age because the problem did not give any information about his age." Afterwards, he successfully solved the problem and found the age of Ömer, as can be seen in Figure 4.26 below.

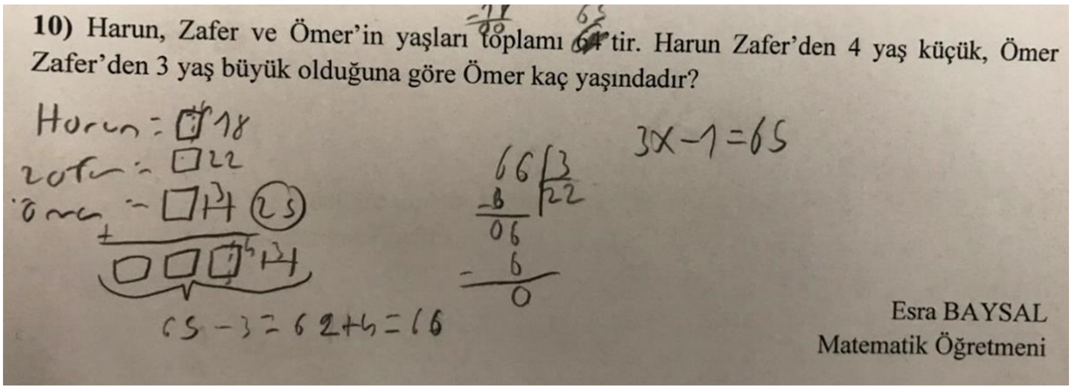


Figure 4.26 Emre's solution to P10

In Figure 4.26, after Emre decided to draw one rectangular bar for Zafer's age, he drew one bar and a 3 unit-long line for Ömer's age because Ömer was claimed to be three years older than Zafer. He also drew one bar and removed 4 units from this bar because Harun was stated to be four years younger than Zafer. Then, he added the whole model, so he drew three bars, 3 unit-long line and removed 4 units from the third bar and equated them to 65. He first subtracted the 3 unit-long line from 65 and found 62. Secondly, he completed the third bar, so he added 62 to 4 and found 66. Since there were three bars, he divided 66 by three and found that Ömer's age was 22. He also wrote the ages of Harun and Ömer as 18 and 25, respectively. Moreover, he set up the equation based on the bar model. He wrote 3x because there were three bars. He said that it was not necessary to write -4 and +3 separately; as their total value is -1, he wrote 3x – 1 and equated it to 65 as follows: 3x – 1 = 65.

Another student, Umut, he drew the model accurately, but made one mistake while solving the problem. The mistake was that although he subtracted the 3 unit-long line from 65 without any difficulty, he did not complete the third bar with 4 units. The subsequent steps followed by student while solving this problem is reflected in dialogue below.

> **Researcher:** You removed 3 units, okay, but why didn't you complete the third bar with four units? [3 birimi çıkardın. Pekala. Ama neden üçüncü kutucuğu 4 birimle tamamlamadın?]
>
> **Umut:** I will add these four units at the end of the solution. [Bu 4 birimi, problemin sonunda ekleyeceğim.]
>
> **Researcher:** Okay! Now what will you do? [Tamam! Peki şimdi ne yapacaksın?]
>
> **Umut:** I will divide 62 by three because there is a total of three rectangular bars. [62'yi 3 böleceğim çünkü toplamda 3 tane kutucuk var.]
>
> **Researcher:** Okay! Could you please tell me your answer? [Peki. Cevabı ne bulduğunu söyler misin?]
>
> **Umut:** Hmm. There is a problem. 62 cannot be divided by three evenly. I think, I made a mistake somewhere in the solution. [Hmm… Bir problem var. 62, 3'e kalansız bölünmüyor. Sanırım bir yerde hata yaptım.]
>
> **Researcher:** Which part do you think you made a mistake in? [Sence nerede hata yaptın?]
>
> **Umut:** I think I should add four units before the division operation. If I add four to 62, I will find 66. I had to divide this result, which is 66, by three. [Bence 4 birimi bölme işleminden önce eklemeliydim. 62 ile 4'ü toplarsam, 66 bulurum. Bu cevabı, yani 66'yı 3 bölmem gerekiyor.]
>
> **Researcher:** Okay. Then what is your final answer? [Tamam. O zaman son cevabın nedir?]
>
> **Umut:** The value of a rectangular bar is 22. [Bir kutucuğun değeri 22'dir.]

Researcher: Now you found the value of a rectangular bar. Then tell me, whose age does the rectangular bar represent? [Sen şimdi bir kutucuğun değerini buldun. Peki bana söyler misin, bu bir kutucuk kimin yaşını temsil ediyor?]

Umut: Zafer's age is represented with one rectangle. Because of that I found Zafer's age. [Zafer'in yaşı bir kutucukla temsil ediliyor. Bu yüzden Zafer'in yaşını buldum.]

Researcher: Whose age does the problem ask for? [Problemde kimin yaşı sorulmuş?]

Umut: Hmm. To solve this problem, I should find Ömer's age. I think so because Ömer has a line length of 3 units more than the rectangular bar; I should add 3 to 22 to find Ömer's age. Therefore, Ömer's age is 25. [Hmm. Ömer'in yaşını bulmalıyım. Sanırım Ömer bir kutucuktan 3 birimlik fazla çizgiye sahip olduğu için, 22 ile 3'ü toplayıp Ömer'in yaşını bulabilirim. Sonuç olarak Ömer 25 yaşındadır.]

In this dialogue, it can be seen that Umut realized his mistake when the result resulted in a repeating decimal number. Moreover, he could realize which unknown he had found.

The students, named Ece and Ali, drew the model correctly, but they made some mistakes while writing the equation. First of all, Ece said that she represented Zafer's age with x. But when she tried to indicate Harun's age, which is defined in the problem as four years younger than Zafer, she only added -4 to the equation. So, she did not use an unknown number. Afterwards, she used x+3 for Ömer's age. To sum up, her equation was $x - 4 + x + 3 = 65$ and she only took into consideration the problem statement, not the bar model. After she checked what she had written, she realized there was a mistake in her equation because while there were three people in the problem, she had only used two unknowns. After realizing her mistake, she added one more x to the equation. To summarize, without considering what she had drawn, Ece correctly set the equation as $x + x - 4 + x + 3 = 65$. On the other hand, Ali only considered the "four years younger" and "three years older" statements and placed -4 and +3 into his equation. So, he wrote the equation as follows: $-4 + 3 = 65$. When researcher saw this equation, she asked Ali: "Won't you use any letter for the unknown?” and “Doesn’t your equation have at least one unknown letter?” After the researcher’s questions, Ali only added one x for Ömer’s age. He did not use any unknowns for Zafer and Harun’s ages. His final equation was as follows: x – 4 + 3 = 65. Ali’s process of arriving at his algebraic equation can be seen in Figure 4.27 below:

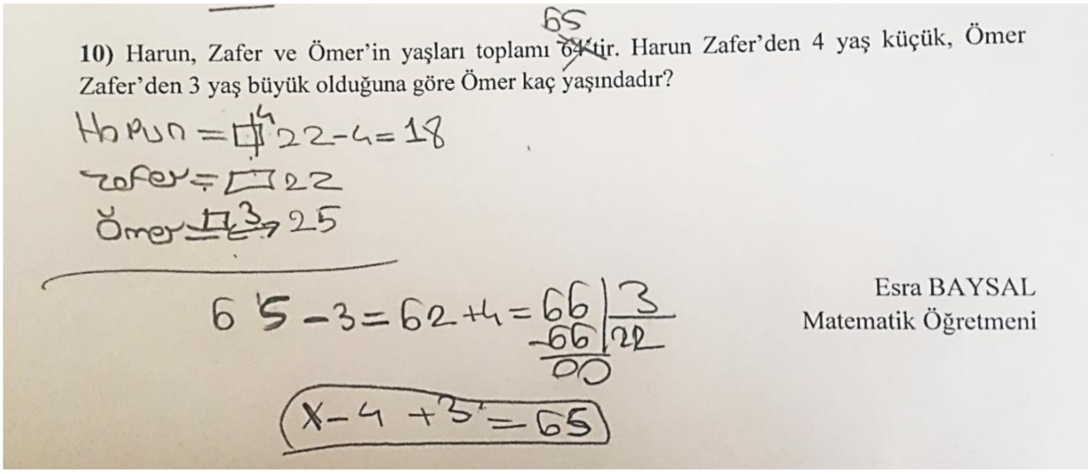


Figure 4.27 Ali’s solution to P10

To sum up, students’ first preference was to use the bar model method for P10. When the researcher warned three students to reconsider their solution process, all the students found the correct answer by using the bar model method. However, one student could not write the appropriate algebraic equation even though s/he had considered the model and solved the problem correctly by using the model.

4.2.3. Problem Set 3: Contextualized Problems with Two Unknown Quantities, One of Which Could be Described by the Other One

The researcher analyzed P1, P2, P3, P6 (problem set 1) and P7, P8, P10 (problem set 2) as two separate sets because the problems in each sets have common points. The remaining problems, P4, P5 and P9, were analyzed separately because these problems, require high-level thinking skills since they were given in contexts. Moreover, the Problems Set 3 include two unknown quantities, one of which could be described by the other one. These problems necessitate the students to first show the unknowns in the problem with the bar model. Subsequently, they should transfer the givens in the problem sentence to the bar model. However, the transfer process in these types of problems is different from that in the other problems regarding operation involved. To illustrate, the students showed the addition of quantities in the model in the previous problem sets, but now they should be able to show the difference of the quantities in P9. Specifically, P9 says the following: *"The difference between Berke's age and his father's age is 36. His father's age is 12 less than 4 times Berke's age. How old is Berke?"* [Berke ile babasının yaşları farkı 36'dır. Babasının yaşı, Berke'nin yaşının 4 katından 12 eksik olduğuna göre, Berke kaç yaşındadır?] This problem was solved correctly by three of the students with the bar model method. These three students benefitted from the bar model method while writing an algebraic equation in this problem. However, seven of the students could not solve the problem by using either of the methods. They made transforming and operational errors. Students' problem-solving processes are explained in detail below. P9 was one of the most challenging problems for the students because the problem requires them to show the difference between the ages of Berke and Berke's father. In the other problem sets, the students showed the addition of the unknowns, and they did not experience many difficulty. However, the students experienced difficulty in showing the difference between the quantities with the bar model. Even some of students, namely Emre, Merve and Umut, made some mistakes and were not completely sure about the steps they followed to solve the problem, they finally found the correct result (16) indicating Berke's age with the bar model method. The first student, Emre, drew the bar model accurately and found Berke's age as Figure 4.28 illustrates.

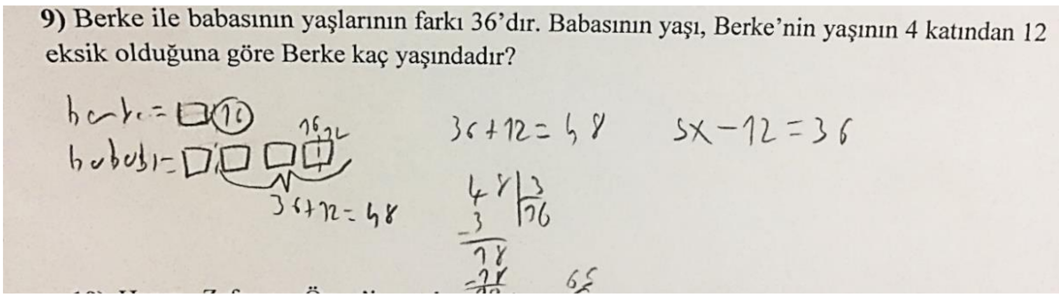


Figure 4.28 Emre's solution to P9

He drew one rectangular bar for Berke’s age, and four rectangular bars for Berke’s father age. He then removed 12 units from the fourth bar to indicate the statement ‘12 less than four times’ in the problem. To show the difference between the ages, he drew a dotted line near the first bars. The difference between the ages indicated with the bar model was placed on the other side of the dotted line. So, he equated this part to 36. He completed the fourth bar, which means he added 36 and 12 and found 48. Since the difference between the ages was represented with three rectangular bars, he divided 48 by three and found 16. Therefore, Berke’s age was found to be 16. For writing the equation, he looked at his all his drawings. Because of there was a total of five rectangular bars in the model, he said that his equation should be 5x – 12 = 36. The student showed the difference in the model correctly, but he wrote addition of all bars in the equation.

The second student, Merve, found the differences between Berke’s age and Berke’s father age by subtracting the rectangular bars representing their ages from each other. The result of Merve’s subtraction was three rectangular bars, and 12 units were removed from the third bar. She did the correct operations and found Berke’s age. She set her equation as 3x -12 = 36, which was also correct. Figure 4.29 below shows Merve’s solution process and her algebraic equation to P9.

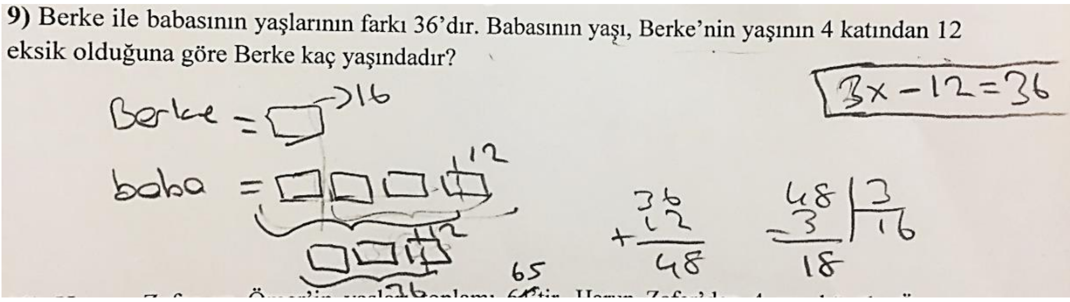


Figure 4.29 Merve’s solution to P9

The third student, Umut, drew a dotted line too and found the difference between the ages correctly, but he could not provide a reasonable response to the researcher, who asked him to explain why he chose to follow the steps he did in his solution to the problem. Umut only said that he remembered the steps from the instruction. He found

Berke's age to be 16, which is correct. He wrote his equation from his drawings as follows: 3x – 12 = 36.

Seven of the students could not find the correct answer in this problem. The common mistake was forgetting to show the difference with the bar model. Because of this mistake, most of the students added all the bars and equated them to 36. Melik, Melike, Sinem and Mustafa added all the bars in their drawings. For this reason, after adding 36 and 12, they divided the sum by 5 and found the value of a rectangular bar, i.e., Berke's age, to be 9.6. These seven students said that they were aware that the problem stated not the sum of the ages but the differences between the ages. But they said that they did not know how to show this data in the model. Because of this reason, except for Mustafa, they wrote their equation as, 5x – 12 = 36. Mustafa set his equation as 4x – 12 = 36 based on the bars representing Berke's father. The solution of another of the student, Sinem, who similarly used addition instead of subtraction is shown in Figure 4.30 below.

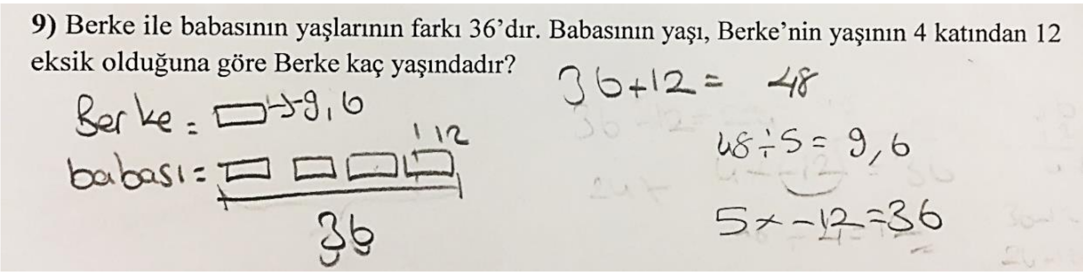


Figure 4.30 Sinem's solution to P9

In brief, Sinem and the other students mentioned above solved the problem by adding the ages. But the problem statement included the difference between the ages, so they made a transforming error.

Ece drew Berke's age and Berke's father age with the bar model, but she also did not know how to show the differences between Berke's age and Berke's father's age. Moreover, she did not know which part of the model is equal to 36. Since the problem stated 'less than 12', she told the researcher that she added 36 and 12 and found 48.

She did not complete the remaining part of the problem. At this point, the researcher asked Ece, “Can you solve this problem by using equations?” Upon the researcher’s question, Ece wrote 4x – 12 = 36 because the problem stated that Berke’s father’s was 12 less than 4 times Berke’s age, but she gave up because she was not sure if the equation was correct or not. Thus, the error she made was also a transforming error.

Ali drew the model correctly as Ece did. But he did not show the difference between Berke and his father’s ages with the model. Neither did he explain which part of the model is equal to 36. Without giving any reasonable explanation, Ali showed the model which indicated that Berke’s father’s age was equal to 36 (see Figure 4.31).

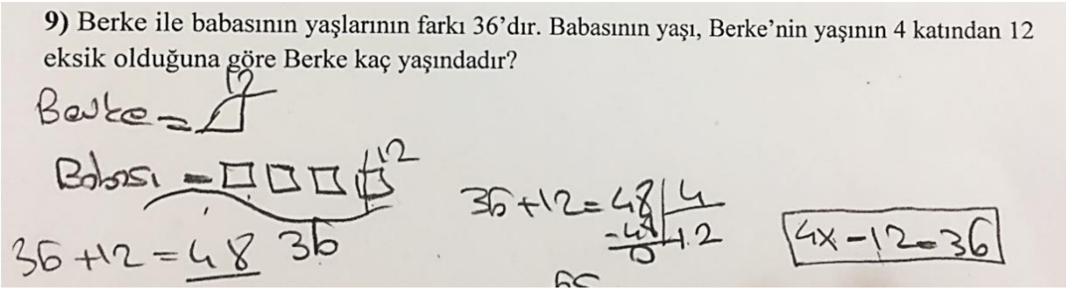


Figure 4.31 Ali’s solution to P9

After that, he tried to find the value of the rectangular bar. Then, he added 36 and 12, and divided the result by four. As a result of these operations, he found Berke’s age to be 12. The researcher asked Ali, “Could you please find Berke’s father age?” Ali responded by saying that the difference between the father’s age and Berke’s age was 36, so Berke’s father’s age should be 48.

At the beginning of her solution, Zeynep wanted to solve the problem without drawing models. But when she tried to write an equation, she only used Berke’s father’s age and wrote 4x – 12 = 36 as her equation without using Berke’s age. When she solved her equation, she found 12 as the value of x, which is also Berke’s age. The researcher wanted Zeynep to solve the problem by drawing a model to justify her answer. Zeynep drew the bar model for Berke’s age and Berke’s father’s age correctly and she equated all the bar models to 36, as shown in Figure 4.32.

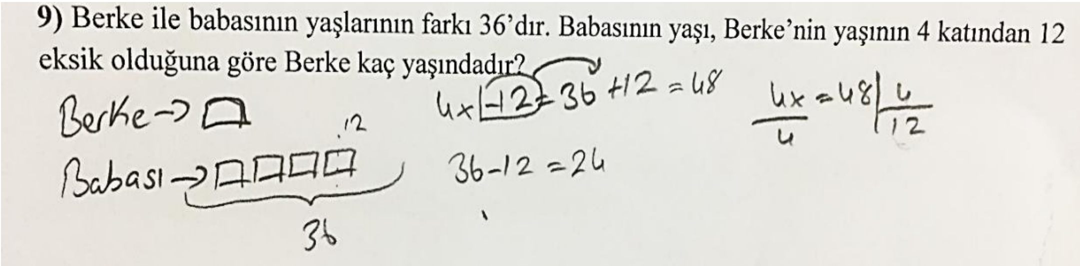


Figure 4.32 Zeynep's solution to P9

Afterwards, she said she needed to subtract 12 from 36. It was wrong because she needs to add 12 to 36 because the problem stated that "His father's age is 12 less than 4 times Berke's age". Zeynep made same mistakes in the other problems too. Even though she could find her mistake in P8, she made the same mistake in P9 as well. The student said that she was not sure about what to do next and she could not solve the rest of the problem, so she gave up solving the problem. She thought that she solved the equation correctly. So, Zeynep made both a transforming and an operational error in this problem because she could not show the difference with the bar model, which is a kind of transforming error, and subtracted 12 from 36, which is a type of operational error.

Thus, nine of the students' first preference was to use the bar model method; however, there were only three students at all who solved the problem correctly. The other students could not find the correct answer. One student wanted to solve the problem by using the algebraic equation method; however, she could not set up the correct equation. She also tried to solve the problem with the bar model method, but again she could not solve it.

The second problem in this set, P4, also involved three unknowns which could be described by the other ones. Specifically, P4 states: *"The number of the legs of rabbits and turkeys in a hencoop is 50. If there are 16 animals in this coop, how many turkeys are there?"* [Bir kümesteki tavşan ve hindilerin ayak sayıları toplamı 50'dir. Bu kümeste toplam 16 tane hayvan olduğuna göre, bunlardan kaç tanesi hindidir?] In this problem, three students solved the problem correctly with the bar model method. These three students could not set up an algebraic equation for the problem. Moreover, five students could not solve the problem by means of either of the methods. They made transforming and operational errors. Two students left the problem blank. Students' problem-solving processes are explained in detail below.

This problem was one of the problems that were difficult for the students and most of them could not find the right answer in the initial assessment, which was applied in order to choose case participants (see Table 3.1). While solving this problem, they should determine the number of turkeys and rabbits in the hencoop by using rectangular bars. They learned that they could use two different bars for turkey and rabbit during the instruction. They could also use different colored bars like white and black for the two unknowns. Afterwards, they should show the total number of animals and the sum of the legs with these bars. To illustrate, Umut was one of the students who solved the problem correctly. His answer is shown in Figure 4.33 below.

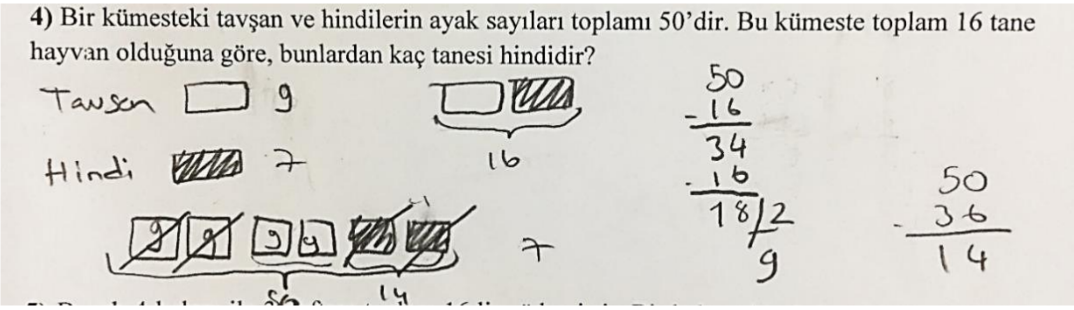


Figure 4.33 Umut's solution to P4

He drew a white rectangular bar for the rabbits and a black rectangular bar for the turkeys. He said that the sum of a white and a black bar is equal to 16 because there are 16 animals in the hencoop. Since turkeys have two legs and rabbits have four legs, there should be four white bars to represent rabbits' legs and two black bars for the turkeys' legs. This model is equivalent to 50 because there are 50 legs in total. At the end of the drawing, he removed two white and black bars from the model where the legs were shown. Since the total value of a black and white bar was 16, he subtracted two corresponding bars, which means he subtracted 16 from 50 twice. After this step, there are two white bars left, which are in total equivalent to 18. So the value of one white bar, which indicates rabbits, is found to be 9 when 18 is divided by two: $18 \div 2 = 9$. He found the number of turkeys to be seven because he subtracted nine from 16. As seen in Figure 4.33, Umut solved the problem with the bar model method; however, he did not write the algebraic equation for this problem.

Umut, Merve and Mustafa were the students who solved the problem only with the bar model method. They drew the correct models and found the number of turkeys in the hencoop. However, there were a few minor calculation mistakes in the solutions of two students. To begin with, Mustafa made a mistake in his subtraction, and due to this mistake, he found the answer wrongly. However, even though he found the wrong answer, his solutions steps and his bar models were correct. Secondly, Merve did all steps correctly. After he found 18, he did not divide 18 by two. At this point, the researcher asked her how many white bars there were left. Upon this question, Merve noticed there were two white bars and she told the researcher she should divide 18 by two. After the division operation, she found the value of one bar to be nine, which represented the number of rabbits in the hencoop. Then she could find the number of turkeys, which turned out to be seven.

In this problem, seven students either did not find the correct answer or did not solve the problem. First of all, Emre drew four white rectangular bars to indicate the rabbits and two black rectangular bars for the turkeys. Actually, Emre drew bar models to represent the legs of animals but he did not state that these bars are equal to 50. He said that he could not solve this problem. Figure 4.34 shows Emre's solution to P4.

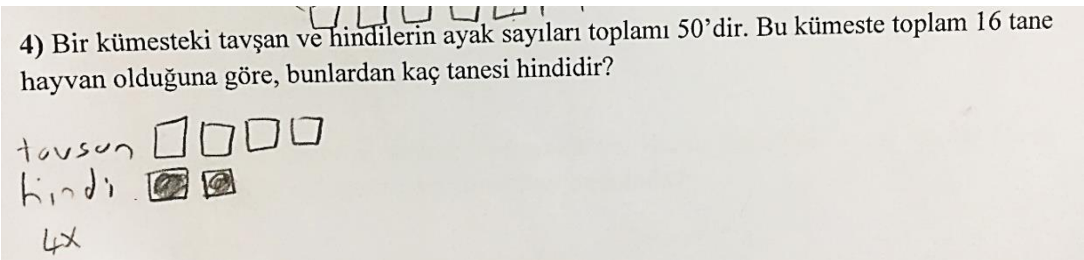


Figure 4.34 Emre's solution to P4

Melike and Sinem drew the bars correctly as Emre did (see Figure 4.34). However, they said that they could not solve this problem. Therefore, they did not continue to solve the problem.

Another student, Ece, accurately drew rectangular bars to represent the number of rabbits and turkeys. She also drew bars for the legs of animals too, and she stated that these bars were equivalent to 50. Even though she indicated the number of animals with a bar model, she could not show the relationship between the bars, which represented the numbers of animals, and 16. Due to this, she gave up solving this problem. Figure 4.35 shows Ece's bar model. She made a transforming error in this problem because she could not draw the bar model correctly.

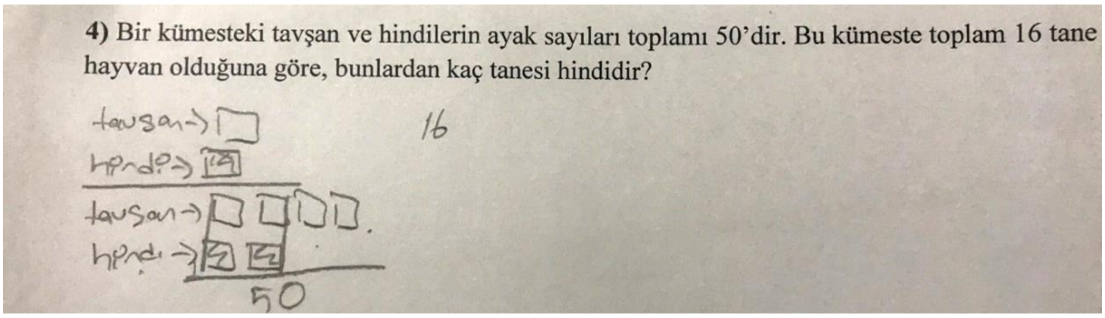


Figure 4.35 Ece's solution to P4

Melik drew bars accurately for both the number of animals and the number of their legs. He also correctly showed that the value of these bars were 16 and 50. He subtracted one white and black bars together, which values are 16 (see Figure 4.36).

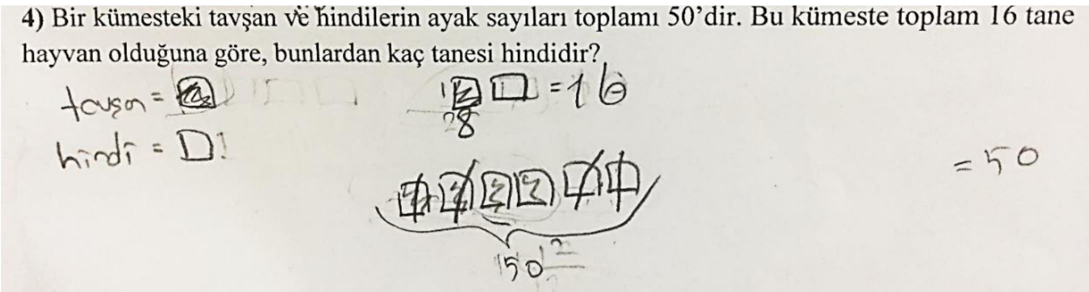


Figure 4.36 Melik's solution to P4

Melik merely relied on rote memorization to solve this problem, so he did not try to find the relationship between the bars. The student did not process any further in this problem. The researcher asked why subtraction was needed and how it was done. The student answered this question by saying, “I do not know, I only remember that we did it this way to solve the problem, but I could not remember the steps that followed to solve this problem.” Subsequently, the student gave up and did not make any comment about the following steps of the solution. As a result, he could draw the correct model, but he could not solve it. So, his error type was operational.

Students, Zeynep and Ali, said that they could not solve this problem. After this statement, they left this problem blank. All of the students, even the ones who solved this problem, could not write the equation to this problem. They all said that they did not remember how to write the equation of this problem.

Thus, the students’ first preference was to use the bar model method, but only three of the students could find the correct answer. Although five of the students tried to solve the problem and made some progress, the other two students did not do anything for the solution of the problem. Moreover, none of the students could write the correct equation for the problem.

The third problem in this set, P5, included two unknown quantities, which are also described by the other ones in a contextual situation. Specifically, P5 was stated as follows: *“Burak paid 16 liras in total for 4 pencils and 3 notebooks. If a pencil costs 50 Kr more than does a notebook, then how many lira is one notebook?”* [Burak 4 kalem ile 3 deftere toplam 16 lira ödemiştir. Bir kalem bir defterden 50 kuruş fazla olduğuna göre, bir defter kaç liradır?] This problem was solved correctly with the bar model method by five students. Two students could set up an algebraic equation for the problem based on the bar model. On the other hand, five students could not solve the problem with either of the methods. They made transforming, blank guessing, identifying unknown incorrectly and operational errors. Students’ problem-solving processes are explained in detail below.

This problem requires the students to use higher level thinking skills because they should show the value of one notebook's and pencil's price, the number of items and their total prices by using the bar model method. First of all, the students should show that a pencil's price is 50 Kr more than a notebook's price by using rectangular bars. Secondly, they should show that there are four pencils and three notebooks. Lastly, they should show that Burak paid 16 liras for the pencils and notebooks in total.

Five students reached the correct answer in this problem; however, some of them had some mistakes. Umut and Sinem solved the entire problem correctly with the bar model method and they wrote the equation by considering the model. Umut wrote the equation as 7x + 2 = 16 and Sinem wrote same equation as 4x + 2 + 3x = 16. To illustrate, Figure 4.37 shows Umut's solution to this problem.

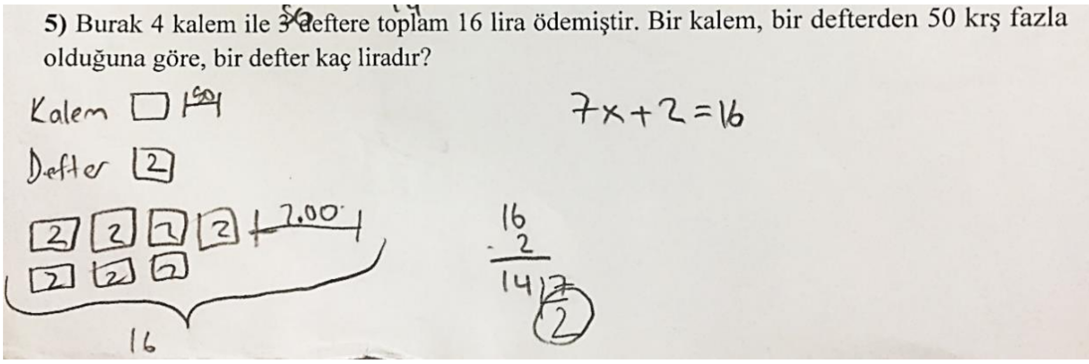


Figure 4.37 Umut's solution to P5

He drew one rectangular bar for the notebook and one bar and a 50 unit-long line for the pencil. Afterwards, he drew four bars and a 2 unit-long line because Burak bought four pencils, so Umut multiplied pencils by four. He also drew three bars because Burak bought three notebooks. Then he equated the entire model to 16 liras. He subtracted two liras from 16 and found 14. Since there were seven bars in total, he divided 14 by seven and found the value of a bar to be 2. Therefore, the price of one notebook was 2 liras. Moreover, he wrote the algebraic equation based on the bar model. There were seven bars and a 2 unit-long line, so he wrote 7x + 2 = 16.

In addition, Melik solved the problem with the bar model correctly; however, he could not write the equation. He initially wrote the equation as 4x + 200 = 3x. When he realized that he had not used 16, he changed the equation and tried a new one. He wrote 4.(x + 50) for the pencils' price and 3x for the notebooks' price. However, he placed a multiplication sign between these algebraic expressions instead of an addition sign. So he wrote the equation as 4.(x + 50) . 3x = 16. He also should have written 0.50 liras instead of 50 Kr.

Another student, Mustafa, showed different unknowns in his bar model, which means he used rectangular bars in different colors. He drew three black bars for the notebooks and four white bars for the pencils. Although he could not show that the price of a pencil was 50 Kr more than the price of a notebook with bar model, he said that the pencils' price were two liras more than the price of the notebooks because there were four pencils (see Figure 4.38).

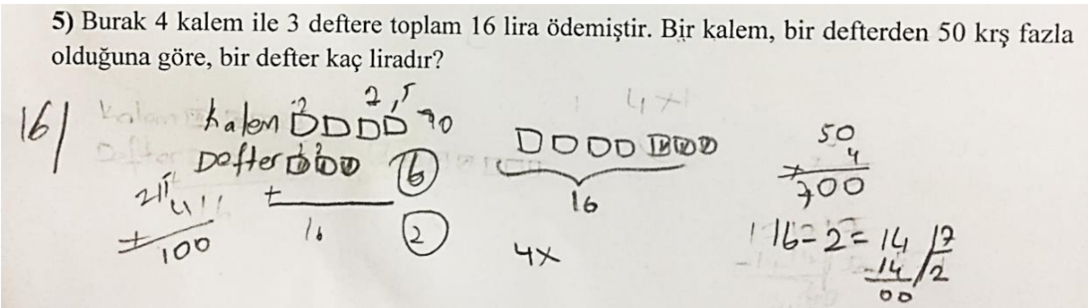


Figure 4.38 Mustafa' solution to P5

Therefore, he subtracted two liras from 16 liras and found 14. Seven bars were left, so he divided 14 by seven and found 2. In summary, although Mustafa represented different unknowns, which were not equal to each other, he divided 14 by seven. He also made identifying the unknown incorrectly and transforming errors in this problem; however, he could find the correct answer. Similar to Mustafa, Emre showed different unknowns separately in his model and solved the problem just like Mustafa did. So, Emre made a transforming error because he could not draw the bar model correctly. He also made the error of identifying the unknown incorrectly because he could not show all the unknowns with the bar model. Nor could they write the equation.

Five students tried to solve the problem with the bar model; however, they faced some difficulties. First of all, like Mustafa and Emre, Merve used two different bars representing different unknowns. She drew a black rectangular bar to represent a notebook's price. She also drew a white rectangular bar and a 50 unit-long line for a pencil's price. When she wanted to show four pencils and three notebooks, she drew four white bars and three black bars; however, she did not multiply the 50 unit-long line by four. Figure 4.39 shows Merve's solution.

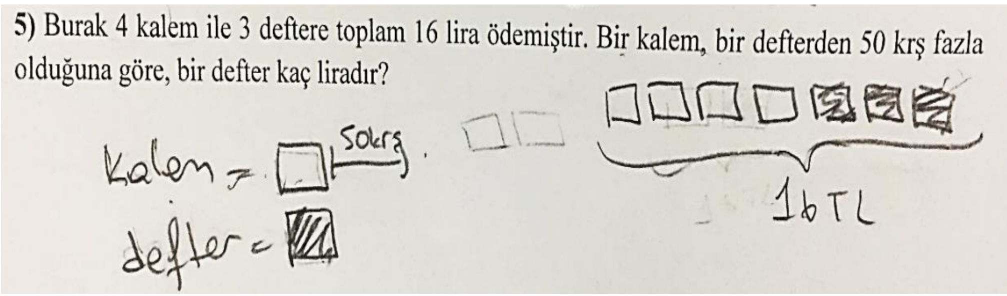


Figure 4.39 Merve's solution to P5

Afterwards, she equated these bars to 16. From this point onwards, she could not continue and could not find the correct answer. She could not correct the bar model, so she made a transforming error. Other student Ali decided to draw a bar model based on the word order in the problem. Since the problem stated that the price of four pencils and three notebooks was 16, he drew four white and three black bars and equated them to 16 (see Figure 4.40).

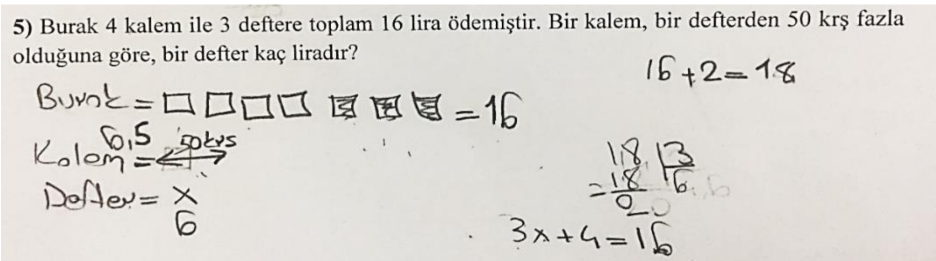


Figure 4.40 Ali's solution to P5

However, he experienced difficulties in showing the second sentence. He drew only a 50 unit-long line to indicate the pencil's price and did not use any unknown. He could decide neither what he should draw to indicate the notebook's price nor for what he would use 'x'. He mixed the bar model method and the equation method with each other. When he tried to solve the problem, he said that the pencils' price were two liras more than the price of the notebooks because there were four pencils. However, he decided to add 16 and 2 and found 18 without any reasonable explanation. Since there were three notebooks and the problem asked for the price of a notebook, he divided 18 by three and found that answer to be six. He did not use the model he had drawn. Besides, he wrote the equation as 3x + 4 = 16. As there were three notebooks, he wrote 3x to represent them. Also, he wrote +4 because there were four pencils and he could not decide which unknown he should use for the number of pencils. As a result, Ali made the errors of blank guessing, identifying the unknown incorrectly and transforming in this problem.

Although Ece drew three bars to represent the notebooks' price and four bars to indicate the pencils' price, she drew just one 50 unit-long line in the first row while multiplying the price of pencils by four. Then, she equated the whole model to 16; however, she could not make sense that she needed to subtract the lengths of the lines. Therefore, she could not continue and, thus, could not find the correct answer. She could not write the equation for this problem. She also made transforming, identifying the unknown correctly and operational errors. Figure 4.41 displays Ece's solution.

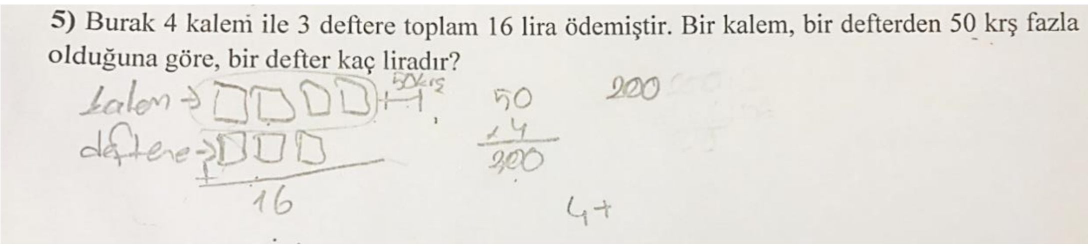


Figure 4.41 Ece's solution to P5

Like Ece, Melike and Zeynep drew three bars to represent the notebooks' price and four rectangular bars to indicate the pencils' price, but they did not continue to solve the problem. They said that they had forgotten how to solve this type of a problem. Therefore, they also made the errors of identifying the unknown correctly and transforming. Hence, in the last problem of the Problem Set 3, all students tried to solve this problem with the bar model; however, only five of them could find the correct answer. The other five students could not use the bar model method appropriately and they could not draw the correct model based on the problem statement. On the other hand, only two students could write the algebraic equation for the problem.

In summary, the researcher grouped the students' errors according to the error types that were used in choosing the participants. Accordingly, the students' error types were as follows: (1) blank guessing, (2) identifying the unknown incorrectly, (3) setting up the equation incorrectly, (4) using the parentheses incorrectly, (5) operational mistakes, (6) finding an incorrect unknown. It is reported by some studies that these error types are more frequently made in the algebraic equation method (Egodawatte, 2011; Kayani & Ilyas, 2014; Newman, 1983b as cited in Ladele, 2013). The researcher categorized the students' errors in the bar model method also based on this classification. The error of identifying the unknown incorrectly emerged when the students could not show each unknown by using an appropriate bar model. The error of setting up the equation incorrectly emerged when students showed each variable but they could continue to draw what was stated in the problem by using the bar model, which is also called the transforming error. The incorrect parentheses error was adapted to the bar model method as ordering of operations. If students need to first add a number and an unknown and then multiply it with a number, they need to show the addition first, and then show the multiplication with the bar model method. Although the students may be able to draw the whole model correctly, they may not be able to find the correct answer because they can do wrong operations. This is called the operational error. Finally, the error of finding an incorrect unknown emerged when the students did not find the wanted unknown. Table 4.2 summarizes the types of participants' errors made in the bar model method, which was explained in detailed above with sample students' work.

Table 4.2

Case Participants' Errors in the Bar Model Method

Students	Blank guessing	Identifying the unknown incorrectly	Setting up the equation incorrectly	Using parenthesis incorrectly in writing the equation	Operational mistakes in solving the equation	Finding the unknown as an answer	No response
Zeynep	0	0	2	1	6	0	1
Ece	0	0	2	1	1	0	0
Melike	0	1	4	1	1	0	0
Mustafa	0	0	1	0	0	0	0
Ali	2	1	3	1	1	0	1
Sinem	0	0	2	0	0	0	0
Emre	0	0	2	0	0	0	0
Umut	0	0	0	0	0	0	0
Merve	0	0	1	0	1	0	0
Melik	0	0	2	0	2	0	0

0	1	2	3	4	5-6

As seen in Table 4.2, no student made finding the incorrect unknown errors and there was only one student who did blank guessing error. On the other hand, most frequent errors are operational and transforming. Moreover, only one student, Umut, did not make any error. The number of errors decreased when compared to the number of errors in the initial assessment, which is also explained in conclusion chapter. These findings indicated that use of bar model method remedied students' challenges as helping them reduce the number of errors and error types.

4.3. The Use of the Bar Model Method across Problem Sets

The 10 problems asked during the clinical interviews were divided into three groups for the analysis. Problem set 1 (P1, P2, P3, and P6) involved quantitative relations but they were not presented in contextual situations. Problem set 2 (P7, P8, and P10) involved quantitative relationships among consecutive numbers. Finally, problems in problem set 3 (P4, P5, and P9) were presented in contextual situations involving two unknown quantities, one of which could be described by the other. In this section, students' responses to each problem set will be presented in terms of the method they preferred to use and the method they used successfully.

The solution paths that students followed in each problem set were mentioned in Chapter IV in detailed, exemplifying with sample student work. In total, 13 different paths that led the students to the answer are listed as follows:

1a) First bar model method was used successfully and then the algebraic equation was written based on the bar model.

1b) First bar model method was used successfully and then the algebraic equation was written based on the problem statement.

2a) First bar model method was used successfully, and then the algebraic equation was written based on the bar model but incorrectly.

2b) First bar model method was used successfully, and then the algebraic equation was written based on the problem statement but incorrectly.

2c) Bar model method was preferred and drawn correctly but no algebraic equation was developed.

3a) First bar model method was preferred but could not be used correctly; and then the correct algebraic equation was written based on the bar model.

3b) First bar model method was preferred but could not be used correctly; and then the correct algebraic equation was written based on the problem statement.

4a) First bar model method was preferred but could not be used correctly; and then the algebraic equation was written based on bar model but incorrectly.

4b) First bar model method was preferred but could not be used correctly and then the algebraic equation was written based on the problem statement but incorrectly.

4c) Bar model method was preferred but drawn incorrectly and no algebraic equation was developed.

5) Algebraic equation method was preferred and written correctly.

6) Algebraic equation method was preferred and written based on problem statement but incorrectly; and then correct bar model method was developed.

7) Algebraic equation method was preferred and written based on problem statement but incorrectly; and then bar model method was developed but incorrectly.

Table 4.3 below presents these solution paths the percentages of students who followed a particular solution path in each problem set.

Table 4.3

*S*tudents' Solution Paths According to Problem Sets

Solution Path	**Problem Set 1**	**Problem Set 2**	**Problem Set 3**
1a) Correct BM → Correct AE based on BM	37.5%	73%	13%
1b) Correct BM → Correct AE based on PS	22.5%	6.6%	
2a) Correct BM → Incorrect AE based on BM	2.5%		6.6%
2b) Correct BM → Incorrect AE based on PS		3.3%	
2c) Correct BM → No AE at all			16.6%
3a) Incorrect BM → Correct AE based on BM	2.5%	3.3%	
3b) Incorrect BM → Correct AE based on PS	7.5%		
4a) Incorrect BM → Incorrect AE based on BM	10%	10%	16.6%
4b) Incorrect BM → Incorrect AE based on PS	5%	3.3%	6.6%
4c) Incorrect BM → No AE at all			33.3%
5) Correct Algebraic Equation	10%		
6) Incorrect AE based on PS → Correct BM	2.5%		
7) Incorrect AE based on PS → Incorrect BM			3.3%

BM: Bar Model

AE: Algebraic Equation

PS: Problem Statement

→: followed by

As seen in Table 4.3, most of the students preferred initially using the bar model to solve the problems. When problem set 1 is examined, it can be seen that the majority of the students (65%) reached the correct result with the bar model method (1a, 1b, 2a, 6). On the other hand, 17.5% of the students reached the correct answer with the algebraic equation method (3b, 5). In addition 42.5% of the students used the bar model or 40% of them used the problem statement to write appropriate equations. This indicates that the percentages of the methods preferred to write an equation were almost equal.

In problem set 2, the majority of the students (82.9%) used the bar model method to find the correct answer (1a, 1b, 2a). On the other hand, most students (76.3%) benefited from the models when writing an equation for the problem (1a, 3a, 4a). In addition, 10% of students tried to write equations by using the bar model, but they could not write the correct equation (4a).

Finally, in problem set 3, only 36.2% of the students were able to reach the correct answer by means of the bar model method (1a, 2a, 2c), and 59.8% did not reach the correct answer with either of the methods (4a, 4b, 4c, 7). In addition, although 13% of the students were successful in writing equations using the bar model (1a), the remaining students either looked at the model and wrote incorrect equation or could not set up an equation in any way. As a result, the bar model method, which is used in the Singapore mathematics curriculum, proved to be a useful method for the questions in problem set 1 and problem set 2, but might not be so effective for the problem set 3, which involves questions that require a higher level of thinking skills.

4.4. Students' Solution Method Prefrences in Solving Algebraic Word Problems

Another research question which the present study addressed was about the students' solution method preferences in solving algebraic word problems. To understand students' reasons and opinions, following the problem-solving session in the clinical interview, students were asked which method they found easier, which method they liked the most, and which method they would prefer to use to solve such problems. In this section, 7th grade students' solution method preferences and their opinions about the bar model method are presented.

Nine students found the bar model easier, mostly because they found the algebraic method confusing and conducive to errors in solving problems. For example, Emre stated:

> I think that solving problems with the bar model is easier because we can engage in reasoning. I am always confused when I try to set up an equation because I have trouble deciding to which variable to assign x. [Bence şekillerle

> problem çözmek daha kolay çünkü mantık yürütebiliyorum. Denklem kurmaya çalıştığımda sürekli kafam karışıyor. Çünkü hangi değişkene x vereceğim konusunda zorlanıyorum.]

Another student, Melik, also stated that he found it most difficult to determine the unknown. The following dialogue illustrates what he thinks about the bar model method:

> **Researcher:** Which method is easier for you? [Sana göre hangi metod daha kolay?]
>
> **Melik:** The bar model method is easier for me since when I try to solve [*the problem*] with the equation method, I get confused and make errors. [Bar model metodu daha kolay geliyor çünkü denklemle çözmeye çalıştığımda kafam karışıyor ve hata yapıyorum.]
>
> **Researcher:** Why do you get confused while you are writing an equation? [Neden denklem yazarken kafan karışıyor?]
>
> **Melik:** Because I can't decide to what [*variable*] I should assign x to. I get confused. Moreover, I get confused about what to do when there are such terms as 'more than' or 'less than' in the problem [*statement*]. That's why I find this method easier than writing an equation. [Çünkü kime x vereceğime karar veremiyorum, kafam karışıyor. Ayrıca problemde 'az' ya da 'çok' gibi kavramlar geçtiğinde ne yapacağımı da karıştırıyorum. Bu yüzden bu yöntem denklem yazmaktan daha kolay geliyor.]
>
> **Researcher:** Which method did you like more? [Hangi yöntemi daha çok sevdin?]
>
> **Melik:** The bar model method. [Bar model yöntemi.]
>
> **Researcher:** Would you want to learn other topics in mathematics with this method? [Diğer matematik konularını da bu yöntemle öğrenmek ister miydin?]
>
> **Melik:** Yes I would like to. [Evet, isterdim.]

Evidently, as Melik indicated, for him the main difficulty is setting up the equation by determining what variable to assign x to, determining the unknowns and deciding which operations the equation involves. He found that the bar model did not entail this difficulty and, therefore, he liked the bar model more. Similarly, Mustafa also pointed to a similar challenge but particularly when the problem involved more than one unknown: “I cannot decide for which unknown I should write x when there is more than one unknown in a problem.” Another student, Ali, thinks that the bar model method is easier for him. He explains the reason underlying his opinion as follows: “The Bar model method is easier for me. I experience difficulties in both writing and solving an equation. [Bar model yöntemi benim daha için daha kolay. Denklemi hem yazarken hem de çözerken zorlanıyorum.]” He was one of the students who found writing and solving algebraic equations complicated.

On the other hand, Sinem stated that her choice of method changed according to the type of problem. Depending on the context of the problem, she could prefer either the bar model method or the algebraic equation method. She explained her view by adding an example: “For example, I can directly write the equation in the fifth problem. Also, the first three problems were easier with an equation. [Örneğin, beşinci problemde direk denklem kurabilirim. Ayrıca ilk üç problem denklemle daha kolaydı.]” So, if the problem is one of the types particularly in the Problem Set 1, which is easy for Sinem, she prefers writing an algebraic equation. Similar to Sinem’s opinion, Ece stated that some types of problems are more suitable for writing algebraic equations. She directly used the algebraic equation method in the first three problems in which quantitative relationship was not given in a problem context. The following dialogue explains what she thinks about using the algebraic equation method:

> **Researcher:** Why did you prefer writing an equation in the first three problems? [Neden ilk üç problemde denklem yazmayı tercih ettin?]
>
> **Ece:** Writing an equation is easier and better in these types of problems. [Bu tarz problemlerde denklem kurmak daha iyi ve kolay geliyor.]
>
> **Researcher:** So why did you draw the bar model in the other problems? [O zaman neden diğer problemlerde şekil çizdin?]
>
> **Ece:** Because it was the first method that came to my mind. Solving other problems with the bar model method is easier I think. I can write an equation better if I draw a bar model. [Çünkü aklıma ilk olarak bu yöntem geldi. Bence diğer problemleri bar model yöntemiyle çözmek daha kolay. Şekilleri çizebilirsem denklemi daha iyi yazıyorum.]

In brief, she thinks that the bar model method helps her in writing an equation in problems involving quantitative relationships between consecutive numbers and in contextualized problems with two unknown quantities, one of which could be ddescribed by the other one. That's why she used the bar model method in the other seven problems. Umut also stated that the bar model was easier for him. The researcher asked him whether he had practiced bar model on his own before the clinical interviews but after the instruction because he solved the problems easily with the bar model method. Moreover, he had difficulties solving problems with the algebraic equation method in the regular classroom. The following dialogue presents his ideas about the bar model method:

> **Researcher:** Which method is easier for you? [Hangi yöntem senin için daha kolay?]
>
> **Umut:** I understood how to solve problems better with the bar model method. It is easier for me. [Problemleri bar model yöntemiyle çözmeyi daha iyi anladım. Benim için bu daha kolay.]
>
> **Researcher:** Umut, did you study for these questions? [Umut, sen bu sorular için çalıştın mı?]
>
> **Umut**: I did not want to study the equation writing problems because I hadn't quite understood the topic, but I understood [it] better now and I studied it with pleasure. [Denklem kurma problemlerine çalışmak istemiyordum çünkü konuyu pek anlamamıştım. Ama şimdi çok iyi anladım ve isteyerek ve zevk alarak çalıştım.]

Evidently, Umut studies mathematics with more pleasure if the topic is easier and easier to understand for him. Different from Umut, Zeynep made a lot of mistakes and could not find the correct answer in many of the problems. Even though she stated that she liked the bar model method during the instruction, she faced challenges in solving the algebraic word problems with bar model during clinical interviews. She could not proceed with the operations although she drew correct bar models. The following dialogue explains what she thinks about her challenges:

> **Researcher:** Which method was easier for you? [Sence hangi yöntem daha kolaydı?]

> **Zeynep:** I used to like the bar model method and I could solve problems in the lessons. I understood the bar model method very well. However, I changed my mind; it is difficult for me to solve problems with this method. I do not remember how to do it. [Bar model yöntemini sevmiştim, derslerde de problemleri bu yöntemle çözebiliyordum. Bar model yöntemini çok iyi anlamıştım. Ama fikrimi değiştirdim, bu yöntemle çözmek benim için zor. Nasıl çözeceğimi hatırlayamıyorum.]
>
> **Researcher:** Why do you think it turned out this way? [Sence neden böyle oldu?]
>
> **Zeynep:** I do not know. I am tired now because school was very tiring today. Also, two weeks passed have passed since the lessons so I may have forgotten [*how to use*] the bar model method. [Bilmiyorum. Şu anda yorgunum çünkü okul bugün çok yorucuydu. Ayrıca derslerin üzerinden iki hafta geçti, bu yüzden bu yöntemi unutmuş olabilirim.]
>
> **Researcher:** So you did not study [*bar model*] before today's meeting, did you? [Bugünkü görüşmeden önce çalışmadın o zaman, değil mi?]
>
> **Zeynep:** No… I could not study. [Hayır... Çalışamadım.]

Although Zeynep made good progress during the instructions, she could not show this progress as she was solving algebraic word problems during the clinical interviews and she made lots of mistakes in solving problems. As can be seen, she stated that she could not practice on her own after the instruction and she was tired on the interview day. These challenges affected students' opinion and preferences about the bar model method in a negative way. Similar to Zeynep, Melike made some mistakes and faced some challenges during the interview. The following dialogue explains her opinion about the bar model method and the mistakes she made.

> **Researcher:** Which method is easier for you? [Hangi yöntem senin için daha kolay?]
>
> **Melike:** Drawing shapes and solving [*problems*] with models is easier for me. [Şekil çizmek ve modellerle çözmek daha kolay geliyor bana.]
>
> **Researcher:** But I think that you experienced some difficulties while you were solving [*problems*] with the bar model method. [Ama sanki bar model yöntemiyle çözerken bir takım zorluklar yaşadın.]

Melike: Yes, I faced difficulties while trying to solve some hard and complicated problems. But still, drawing shapes is easier and easier to understand. [Evet, bazı zor ve karışık problemleri çözerken zorlandım. Ama yine de şekil çizmek daha kolay ve anlaşılır.]

Reseacher: Why did you face difficulties today? Why were you confused? [Peki sence bugün neden zorlandın? Neden kafan karıştı?]

Melike: I think because I did not continue solving problems after the lessons. Moreover, I did not study for today's questions. [Bence problemleri dersten sonra tekrar çözmediğim için böyle oldu. Ayrıca bugünkü sorular için de öncen çalışmadım.]

Researcher: So… If you had studied, could you have solved the problems more easily? [Yani önce çalışsaydın, daha mı kolay çözebilirdin problemleri?]

Melike: Yes, because we solved all of the problems in the lesson and I could solve them. But I think that I forgot some of the things. [Evet, çünkü problemlerin hepsini derste çözmüştük ve ben hepsini çözebilmiştim. Ama sanırım bazı şeyleri unuttum.]

In brief, in Melike's opinion, the bar model method was easier for her and she could solve the problems during the lesson; however, she was confused in the interview. She thinks that if she had studied before, she could have made fewer mistakes. All students, except for Ece, stated they liked the bar model method more. Ece said,

> The equation method is more enjoyable. I would not prefer using the bar model method in other mathematics topics. [Denklem yöntemi daha eğlenceli. Diğer matematik konularında bar model yöntemini tercih etmem.]

On the other hand, Ali stated that he wanted to learn the bar model method in more detail because he liked this method. Moreover, the following dialogue indicates Umut's views toward bar model method.

Researcher: Which method did you like more? [Hangi yöntemi daha çok sevdin?]

Umut: I liked this method more than I did writing an equation. Actually, the bar model method is more tedious because we need to draw shapes. However, it is still better than setting up equations. [Bu yöntemi denklem yazmaktan daha

> çok sevdim. Aslında bar model biraz uğraştırıyor çünkü şekiller çizmemiz gerekiyor. Ama yine de denklem kurmaktan daha iyi bence.]

Evidently, Umut likes the bar model method even though drawing shapes takes more time. Another student, Melike, indicated that she liked drawing pictures in her daily life and that the bar model method was related to drawing, so she liked the bar model method too. Similarly, Merve and Sinem said that they liked the bar model method more than writing algebraic equations. They even wanted to learn about other mathematics topic with this method. For example, Merve stated,

> I like the bar model method more than writing an equation. It would be good if we learned other topics with this method. [Bar model yöntemini, denklem yazmaktan daha çok sevdim. Diğer konuları da bu yöntemle öğrenseydik güzel olurdu.]

The students were also asked which method they preferred using to solve such problems. Moreover, they were asked whether they wanted to learn other mathematics topics with this method or not. The following dialogue is about Emre's ideas regarding which method he prefers using to solve one-unknown problems:

> **Researcher:** Which method are you going to use from now on? For example, will you use the bar model method too in other questions? [Bundan sonra hangi yöntemi kullanacaksın? Mesela, bar model yöntemini diğer sorularda da kullanır mısın?]
>
> **Emre:** It depends on the type of problem. I can use it. Solving problems is easier with the bar model I think. However, I can write the equation in easy problems; for example, I could have solved the second problem with the equation method. [Problem tipine bağlı. Kullanabilirim de. Bence bar modelle problemleri çözmek daha kolay. Ama denklemi de kolay problemlerde kullabilirim, mesela ikinci problem için denklem kurabilirdim.]

Evidently, Emre prefers the algebraic equation method in less challenging problems, which involve one unknown and quantitative relations described with expressions like 'more than', 'less than', 'equal to' and 'addition' even though he likes the bar model method more and he thinks it is easier than writing an equation. On the other hand, Mustafa prefers to use the bar model method in such problems because he said that writing algebraic equations is complicating for him. In addition, Umut, Zeynep and Melike stated that they would like to learn other mathematics topics with this method, so they prefer this method to the algebraic equation method. To illustrate, Zeynep said,

> It will be easier if we learn other topics like percentages with the bar model method. [Diğer konuları, mesele Yüzdeler konusunu, bar model yöntemiyle öğrenseydik benim için daha kolay olabilirdi.]

As another example to support this idea, Melike said,

> For example, I have difficulties in the topic of proportion. Maybe I can learn proportion and percentages better with this method. [Mesela, Orantı konusunda zorlanıyorum. Belki Orantı ve Yüzdeler'i bu yöntemle daha iyi öğrenebilirim.]

In brief, some students see the bar model method as an alternative for the topics that were challenging for them. In general, they liked but they had few experience with the bar model, only during the three-hour long instruction. Still, they found it helpful especially in the problems that they found difficult. For the problems in which quantitative relations were simple and could be expressed in algebraic equations when the order of the operations stated in the problem was followed, they preferred the algebraic expression. However, when the problem is given in context and involve relatively complex quantitative relationship, they preferred bar model method because it is visual-based method.

CHAPTER 5

DISCUSSION AND CONCLUSION

The purpose of the present research is to gain an in-depth understanding the use of the bar model method, which is used in the Singapore mathematics curriculum, plays in 7^{th} grade students' ways of solving algebraic word problems. This study also aimed to reveal reasons of students' solution method preferences while solving algebraic word problems. For these purposes, the students took three lesson hours of instruction on the bar model method. One-to-one clinical interviews were then conducted with each student. During these interviews, the students were asked 10 algebraic word problems, which they could solve with any method they preferred (i.e., the algebraic equation or the bar model method). Students' answers to these problems and the questions posed by the researcher are explained in detail in Chapter IV. In this chapter, the conclusions reached based on the findings presented in the previous chapter are summarized and discussed. In addition, implications for educational practices, limitations of the study and recommendations for further studies are addressed in this chapter.

5.1. Discussion and Conclusion

Students' answers and views are discussed in two parts: (1) the role of the bar model method in students' errors and (2) students' solution method preferences. In the first part, the error types that the students made during the initial assessment test and the types of errors made during the clinical interviews are compared. In addition, the benefits and challenging aspects of the bar model method are presented. In the second part, a discussion on students' opinions about the bar model method, and reasons for their solution method preferences. The results are also compared with previous studies in the accessible literature.

5.1.1. Role of the Bar Model Method in Remediation of the Students' Errors

In the initial assessment test which was used to select the participants, students solved 10 algebraic word problems. Their errors were categorized based on the studies in the literature (Egodawatte, 2011; Kayani & Ilyas, 2014; Newman, 1983b as cited in Ladele, 2013). These error types are as follows: (1) blank guessing, (2) identifying the unknown incorrectly, (3) setting up the equation incorrectly, (4) using the parenthesis incorrectly in writing the equation, (5) operational mistakes in solving the equation, and (6) finding the incorrect unknown as an answer.

Clinical interviews were conducted with the students after the instruction on how to use the bar model method in solving algebraic word problems. In these interviews, the students solved 10 problems with the method they preferred. Most students were found to prefer the bar model method in most of the problems. While doing so, the types of errors that students made most frequently were blank guessing, identifying the unknown incorrectly, transforming, ordering of operation, operational mistakes and finding the incorrect unknown. The tables containing the number of errors that students made according to the error types were presented in Chapter IV (See Table 4.1 and Table 4.2). Figure 5.1 below displays these tables side by side for a comparison of the number of errors.

Table 4.1

Case Participants' Errors in Algebraic Word Problems

Students	Blank guessing	Identifying the unknown incorrectly	Setting up the equation incorrectly	Using parenthesis incorrectly in writing the equation	Operational mistakes in solving the equation	Finding the unknown as an answer	No response
Zeynep	2	0	1	0	0	3	4
Ece	0	2	3	0	0	4	1
Melike	3	1	2	0	1	1	1
Mustafa	0	2	3	1	1	1	0
Ali	2	2	3	1	0	1	4
Sinem	0	2	5	2	3	1	0
Emre	1	3	0	1	1	3	0
Umut	2	2	3	1	0	0	3
Merve	0	1	3	1	0	3	4
Melik	0	4	3	1	1	0	3

0 1 2 3 4 5-6

Table 4.2

Case Participants' Errors in the Bar Model Method

Students	Blank guessing	Identifying the unknown incorrectly	Setting up the equation incorrectly	Using parenthesis incorrectly in writing the equation	Operational mistakes in solving the equation	Finding the unknown as an answer	No response
Zeynep	0	0	2	1	6	0	1
Ece	0	0	2	1	1	0	0
Melike	0	1	4	1	1	0	0
Mustafa	0	0	1	0	0	0	0
Ali	2	1	3	1	1	0	1
Sinem	0	0	2	0	0	0	0
Emre	0	0	2	0	0	0	0
Umut	0	0	0	0	0	0	0
Merve	0	0	1	0	1	0	0
Melik	0	0	2	0	2	0	0

0 1 2 3 4 5-6

Figure 5.1 Students' error types in both initial assesment and clinical interview

A heat map is used to adjust the colors as the numerical value increases in the tables. Accordingly, the difference between the first and the second table can be observed. The number of errors that students made when using the bar model method was less than the number of errors they made when using the algebraic equation method. This is an indication that bar model method is an effective method for seventh grade students in solving algebraic word problems. This conclusion is compatible with that reported in Mahoney's study. Mahoney (2012) found that students' performance improved after they started to use the bar model method.

More specifically, the numerical values in the tables show that the number of errors of all students, except for one student (Zeynep), decreased. In other words, 90% of the participants improved their performances in solving algebraic word problems with the help of the bar model method. While most students (Umut 100%, Mustafa 88%, Sinem 85%, Merve 83%, Emre 77%, Melik 67%, Ece 60%, Ali 38%, and Melike 22%) reduced their errors, there was no change in the number of errors made by Zeynep. Thus, it can be deduced that the bar model method did not have a major effect on the problem-solving skills of the three students whose number of errors either remained the same or decreased by 50% or lower but it had a positive role on the other seven students.

This study also revealed the benefits and challenging aspects of the bar model method. First of all, the positive aspect of the bar model method is a significant reduction in not only students' errors of blank guessing, identifying the unknown, finding the incorrect unknown errors but also the number of questions they did not answer. Only one student made a blank guessing error during the clinical interviews. This shows that while four students, Zeynep, Melike, Emre and Umut, had made the blank guessing error in the initial assessment test, in the clinical interview this type of error reduced to zero. Besides, Cai and his friends (2011) underlined that the bar model helps students to focus on how to represent the problem instead of just solving it. Consistent with this, the findings of the study indicate that the students did not want to just solve the problems, but also wanted to comprehend and represent the problem with the bar model method. Hence, they did not make guesses that lacked reasonable explanations.

Moreover, Kieran (2004) stated that the Singapore bar model improves algebraic thinking by facilitating how to determine the unknown and relationship between quantities, which is one of the ways of improving algebraic thinking. Similarly, the present study revealed that students were able to identify the relationship between unknowns in the problem because the bar model is based on visualization and it is more concrete for them. Since they represented all the unknowns in the problem with rectangular bars, they were able to understand the relationship among the unknowns. Therefore, they made fewer errors of identifying the unknown incorrectly. For example, while Melik made this error four times and Emre three times in the initial assessment, they reduced it to zero during the clinical interviews. In addition, they could more easily understand which unknown they found thanks to the visual models; thus, they could find the unknown that was asked in the problem. During the clinical interviews, no student made the error of finding the incorrect unknown although eight students made this error in the initial assessment.

Besides, one of the most observed difficulties in the initial assessment test was the students' inability to write equations for the given problems. This shows that the students frequently made the transforming error. It was found that six of the 10 students reduced their rate of making this type of an error. Thus, it can be deduced that the bar model method can facilitate the transformation of the problem content by using visuals symbols and numbers. In summary, the bar model method is useful for concretizing and visualizing the problem content.

Secondly, the challenging aspects of the bar model method have been revealed by means of this study. Considering the errors students made in the present study, it can be claimed that students have difficulty in drawing models for complex problems although they also have difficulty in writing algebraic equations for these problems. Students participating in the present study experienced difficulties in drawing models for some of the problems, which required higher level of thinking skills. They may have lacked sufficient practice. Hence, they made transforming errors. For example, the problems that state the difference between unknowns instead of addition or leg problems lead to transforming errors. In addition, one of the students, Zeynep, made six operational mistakes. Although Zeynep drew the appropriate model correctly, she couldn't find the value of a bar because she wasn't sure which operations that she should use. This shows that it can be challenging for some students to find the value of a bar in the bar model method. She also said that solving the equation is easier than finding the value of a bar in the bar model. The researcher observed that one of the underlying reasons of this might be students' tendency to memorize the operations instead of engaging in conceptual thinking (Ng & Lee, 2009). For example, while Zeynep could find the value of a bar using the concept of "length," she tried to remember the order of the operations in the instruction, but she failed to do so.

Similarly, Melike could not draw appropriate model for the ninth problem in which the difference between the unknowns was given. Then, Melike tried to remember the procedure which she followed during the instruction rather than considering subtracting the bars in the model. The researcher realized that one of the reasons underlying the difficulties some students faced was not reviewing what they had learned in the instruction. This might even be related to not having sufficient practice with the bar model; that is indicating three-hour instruction might not be sufficient. For example, Melike stated that she could solve the problems easily in the lesson during the instruction, but then she found the bar model method difficult because she didn't review it at home. In addition to these challenges, the researcher realized that students could not draw the bar model proportionally while they drew correct bar model. For example, after they draw a 1 unit-long line, they should draw twice as long it for drawing a 2 unit-long line. That might be the reason of some students' incorrect solution with the bar model. This problem can overcome with longitinual engagement with the bar model method. When students are more experienced with use of the bar model method, they can draw the bar model more proportionally. Moreover, this problem can overcome with technological tools (Thinking Blocks, n.d.). In summary, it can be challenging for students to draw a suitable model for the problem, represent the givens in a complex problem by drawing rectangular bars, and do the arithmetic operations to find the value of a rectangular bar.

In addition, although students initially preferred the bar model to solve the problems in problem set 1, it was revealed that they did not need this method while writing equations because, as stated by Sinem and Ece during the interviews, problem set 1 included easier problems, and it was found that there was no need to use the bar model for these problems. However, students preferred this method because they liked it more. Conversely, the bar model was found to be effective in solving problems in problem set 2 and in writing the appropriate equation because the problems in problem set 2 involved more than one unknown. In fact, students stated in the interviews that they had difficulties in writing equations for these problems. In brief, it is possible to say that the bar model method is more effective in problems where the sum of more than one unknown is given. Finally, it can be said that students should be provided with more practice in using the bar model method in more complex problems like in problem set 3. Seven students could not use the bar model method to solve the leg problem (who also could not solve the problem with algebraic equation), which they also experienced difficulty in solving in the initial assessment. In addition, the ninth problem, which involved the difference between quantities, was the most difficult question for the students, and thus seven students could not reach the correct answer with the bar model method (who also could not solve the problem with algebraic equation). These results show that students made errors in problems they find different and difficult. This is an indication that this method cannot be said to be completely effective and that students need more practice in using this method (Ng & Lee, 2009).

5.1.2. Students' Solution Method Preferences

The students shared their reasons for the choice of solution method (i.e., bar model or algebraic equation). As stated in Chapter IV (See Table 4.3), most of the students preferred initially using the bar model to solve the problems. Reasons of that explained by the students. First of all, nine students found using the bar model method easier to solve algebraic word problems. As the main reason for this, they stated that it is easier to determine what variable to assign x to, to determine the unknowns, and to decide which operations the equation involves in the bar model method. They also stated that it was easier to draw a model in cases where it was difficult to write an equation; that is, if the problem statement did not reveal the order of unknowns and operations that would be placed in the equation. Sinem said that it is faster to set up equations in easy problems, but that she would still use the bar model for other problems. The reason students preferred this method and found it easier is that it is based on visualization. Ece, on the other hand, said that it was easier to set up the equation because the algebraic equation method was faster. However, Ece stated that the drawing model helped her to write an equation. Therefore, one of the positive aspects of the bar model is that it facilitates the writing an equation.

Secondly, the researcher also asked the students whether or not they liked using the bar model method. All the students, other than Ece, stated that they liked this method more than the algebraic equation method. Umut said that the bar model is more enjoyable and easier to understand, while Melike stated that she liked this method more because she liked drawing. Moreover, Zeynep said that she liked the bar model during the instructions, but she changed her mind because she found it difficult as she experienced some difficulties. Similarly, all students, except Ece, stated that not only did they want to learn this method more in this topic but they also wanted to learn other mathematics topics in which they could use this method. This shows that the approach of the students towards the bar model is positive. Umut said that the bar model was more challenging because they it required them to draw some models and it took a long time to learn and solve, but after he learned the bar model, he came to believe that it was better than the algebraic equation method. This finding is similar to that reported in Hoven and Garelick’s study (2007). The researchers stated that teaching mathematics with the bar model method was slower, but when students got used to this method, they could learn the basic skills faster. In summary, although some of the students experienced difficulties in solving problems and made mistakes, nine of the 10 stated that they found this method easier and more interesting, they liked it more and they preferred the bar model method instead of the algebraic equation method.

5.2. Limitations of the Study and Recommendations for Further Studies

One of the limitations of this study is that the students' instruction on the bar model method was limited to three-lesson hours. On the other hand the bar model is a method that needs practice and is difficult to learn. In Singapore, students learn mathematic topics with the bar model method as of 3rd grade, and they get high scores in the international exams (Hoven & Garelick, 2007). Therefore, further studies could be conducted with students learning the bar model method for a longer period of time. Teaching this method in more detail and with different types of problems may increase the positive impact of the bar model, as suggested by the students during the interviews. Similarly, as reported in previous studies, when the bar model method is learned throughout the whole year, problem-solving and algebraic thinking skills of students could increase (Waight, 2006). One of the limitations of the current study is that the person who taught the bar method in the study, that is the researcher, had not received any training in teaching the bar method. It is recommended that students should be taught this method by someone who has become more specialized and trained in the method. Although Mahoney (2012) found that the bar model method can be effective even for students who never learned the method at younger ages, the difficulties some students experience may be attributed to the fact that they had not learned this method before in a longer period of time, and this is one of the limitations of the study.

The results of the study showed that students perceive the bar model method as an alternative way to better understand the other mathematics topics they have difficulties with. The bar model method could be used for not only algebraic word problems but also fractions, multiplication, percentages or proportions (Kho, 1987). Although this study is limited with seventh grade algebraic word problems, future studies can be conducted with different mathematics topics and different grade levels.

The participants of the present study were selected by means of purposeful sampling method. From among the seventh-grade students in a public school in Ankara, 10 students who were open to communication, motivated and willing to learn were selected. One or two students who had made each type of error were chosen. Further studies might be conducted with more students in different characteristics at different grade levels and showing various academic success, which may increase the transferability.

5.3. Implications for Educational Practices

The conclusion that can be drawn based on the findings of the present study is that the bar model method is an effective method in solving algebraic word problems for seventh grade students. Although the bar model method decreases the number of errors that students make while solving the algebraic word problems, it was observed in the present study that while using the bar model method, students made mistakes and had difficulties in solving complex problems, (i.e., problems in problem set 3). The bar model method is used by students in Singapore as of early ages and students learn basic mathematics topics with this method (Hoven & Garelick, 2007). In Turkey, students learn some mathematics topics with visual teaching methods. For example, students use 'area models' while learning fractions (MoNE, 2018). Although these methods could be basis of the bar model, they were not used on teaching all mathematics topics and they have not specifically features of the bar model method. Taking Singapore as an example, students in Turkey should learn basic mathematics topics with the bar model method as of first grade in elementary school and this could reduce the number of students' mistakes and develop students' quantitative reasoning. For this reason, this method should be used integrated into the mathematics curriculum of every grade level starting from 1st grade.

The result of the present study also provides mathematics teachers, curriculum developers, textbook writers, and teacher educators with basic information about which errors students make while solving algebraic word problems, how bar model method helps seventh grade students in solving algebraic problems and which errors it helps students to avoid. In addition, this study provides information about the effective and challenging aspects of the bar model method. Mathematics teachers, curriculum developers, textbook writers and teacher educators can use this knowledge to prepare an effective educational environment and functional materials for solving algebraic word problems in seventh grade. Teacher educators should also teach the bar model method to pre-service teachers. In addition, it is suggested for teacher educators to design professional development for teachers that would train them on how to teach this method and prepare appropriate lesson plans.

The present study also reveals that the bar model method helps seventh grade students to write appropriate equations for the problem. When preparing a lesson plan, teachers may teach students how to write an appropriate equation for the problem by using the bar model method. In addition, this study has shown that knowing both methods gives students on chance to become aware of their mistakes, correct them and check their answers. Therefore, it is suggested for teachers to prepare lesson plans in which they can use the bar model method first and then the algebraic equation method.

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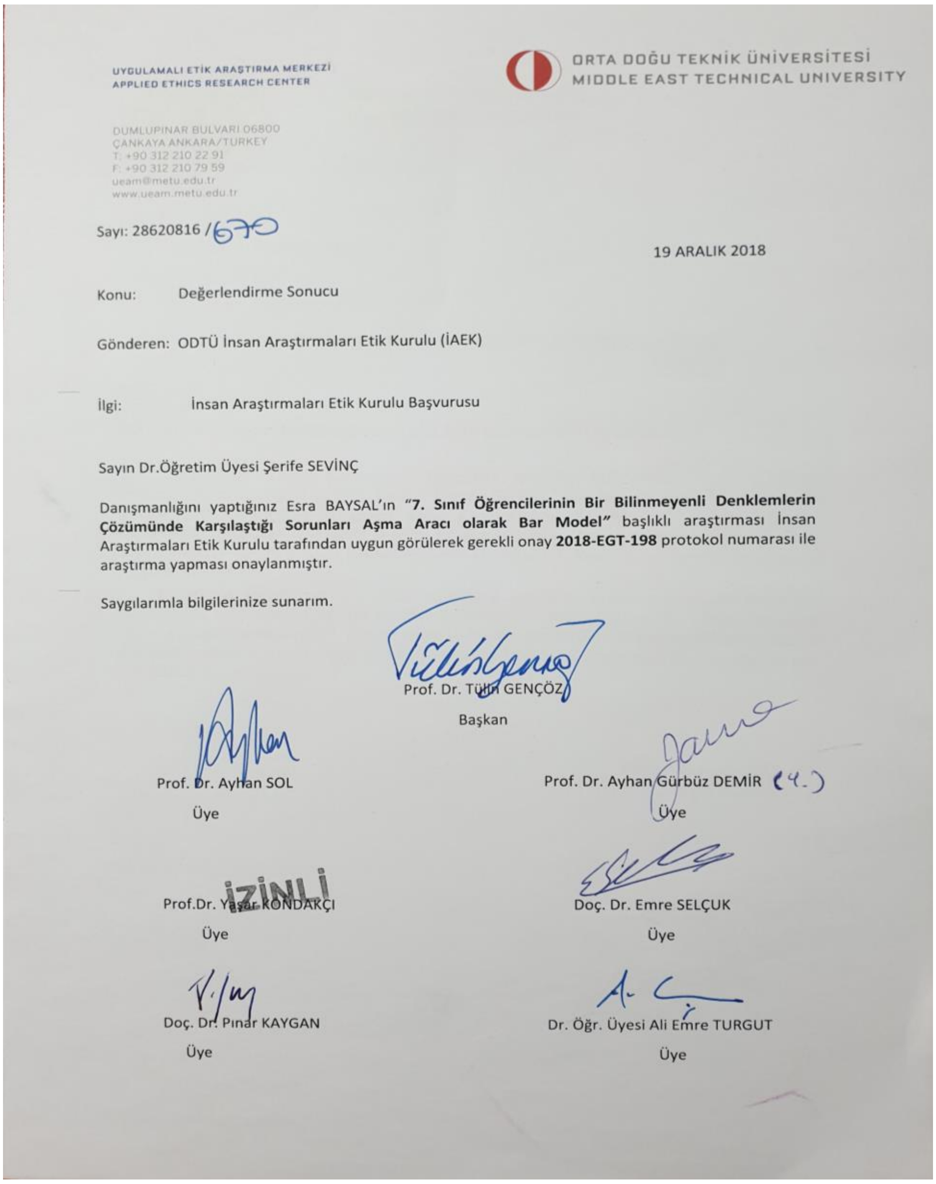
APPENDICES

A. PERMISSION OBTAINED FROM METU APPLIED ETHICS RESEARCH CENTER



UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

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F: +90 312 210 79 59
ueam@metu.edu.tr
www.ueam.metu.edu.tr

Sayı: 28620816 / 670

19 ARALIK 2018

Konu: Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi: İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Dr.Öğretim Üyesi Şerife SEVİNÇ

Danışmanlığını yaptığınız Esra BAYSAL'ın **"7. Sınıf Öğrencilerinin Bir Bilinmeyenli Denklemlerin Çözümünde Karşılaştığı Sorunları Aşma Aracı olarak Bar Model"** başlıklı araştırması İnsan Araştırmaları Etik Kurulu tarafından uygun görülerek gerekli onay **2018-EGT-198** protokol numarası ile araştırma yapması onaylanmıştır.

Saygılarımla bilgilerinize sunarım.

Prof. Dr. Tülin GENÇÖZ
Başkan

Prof. Dr. Ayhan SOL
Üye

Prof. Dr. Ayhan Gürbüz DEMİR (4.)
Üye

Prof.Dr. Yaşar KONDAKÇI
Üye

İZİNLİ

Doç. Dr. Emre SELÇUK
Üye

Doç. Dr. Pınar KAYGAN
Üye

Dr. Öğr. Üyesi Ali Emre TURGUT
Üye

B. PERMISSION OBTAINED FROM MINISTRY OF EDUCATION

T.C.
SİNCAN KAYMAKAMLIĞI
İlçe Milli Eğitim Müdürlüğü

Sayı : 72648821-605.99-E.6245977 26.03.2019
Konu : Araştırma İzni

AHMET ANDİÇEN ORTAOKULU MÜDÜRLÜĞÜNE
SİNCAN

İlgi : İl Milli Eğitim Müdürlüğünün 22.03.2019 tarih ve 5999182 sayılı yazısı.

Orta Doğu Teknik Üniversitesi lköğretim Anabilim Dalı yüksek lisans programı öğrencisi Esra BAYSAL'ın **"7. Sınıf Öğrencilerinin Bir Bilinmeyenli Denklemlerin Çözümünde Karşılaştığı Sorunları Aşma Aracı Olarak Bar Model"** konulu çalışması kapsamında uygulama izin talebi İl Milli Eğitim Araştırma Komisyonunca incelenmiş olup, ekli formların İlçemize bağlı Ahmet Andiçen Ortaokulu 7. Sınıf öğrencilerine uygulanması uygun görülmüştür.

Uygulama formlarının (12 sayfa) uygulama yapılacak sayıda araştırmacı tarafından çoğaltılarak, araştırmanın ilgi (a) genelge çerçevesinde, Müdürlüğümüzün sorumluluğunda, eğitim-öğretimi aksatmayacak şekilde, okul ve kurum yöneticileri de uygun gördüğü takdirde gönüllülük esasına göre yazımız ekinde gönderilen uygulama araçlarının uygulanmasına izin verilmesini rica ederim.

Ahmet Gürsel AVCI
İlçe Milli Eğitim Müdür V.

Ek :1 Adet İlgi Yazı ve Eki

İlçe Milli Eğitim Müdürlüğü Sincan/ANKARA
Elektronik Ağ:sincan06_strateji1@meb.gov.tr

Ramazan DARÇIN
Tel: (0 312) 269 54 46-47/130

Bu evrak güvenli elektronik imza ile imzalanmıştır. https://evraksorgu.meb.gov.tr adresinden c654-4c0a-3c2e-aa08-bf60 kodu ile teyit edilebilir.

C. INITIAL ASSESMENT QUESTIONS

1) Beş katının 1 eksiği, 1 fazlasının 4 katına eşit olan sayı kaçtır?

2) Bir sayının 2 katının 4 fazlası 26 ise, bu sayı kaçtır?

3) Bir otelde iki ve üç yataklı toplam 35 oda vardır. Bu oteldeki toplam yatak sayısı 85 olduğuna göre, oteldeki üç yataklı oda sayısı kaçtır?

4) Ardışık olan dört sayının toplamı 74 ise, bu sayılardan en büyüğü kaçtır?

5) Bir dikdörtgenin uzun kenarı, kısa kenarının 2 katından 3 fazladır. Dikdörtgenin çevresi 54 m ise, alanı kaç metrekaredir?

6) Bir sınıftaki erkek öğrencilerin sayısı, kız öğrencilerin sayısının 2 katının 7 eksiği kadardır. Bu sınıfın mevcudu 23 ise, sınıftaki erkek öğrenci sayısı kaçtır?

7) Biri diğerinden 5 yaş büyük olan iki kardeşin 6 yıl sonraki yaşları toplamı 31 ise; küçük olan kardeşin şimdiki yaşı kaçtır?

8) Esma'nın dedesinin yaşı, Esma'nın yaşının 5 katından 5 fazlası kadardır. Esma ile dedesinin yaşları toplamı 77 ise, Esma kaç yaşındadır?

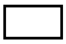
9) Tarık, her gün bir önceki günden 10 sayfa fazla kitap okuyarak 360 sayfalık bir kitabı 5 günde bitirmiştir. Buna göre, son gün kaç sayfa kitap okumuştur?

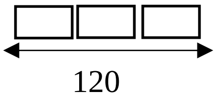
10) Üç sayıdan birincisi ikincisinin 3 katına, üçüncüsü birincinin 2 fazlasına eşittir. Bu üç sayının toplamı 37'dir. Buna göre, birinci sayı kaçtır?

D. PROBLEMS SOLVED DURING INSTRUCTIONS AND THEIR SOLUTIONS WITH BAR MODEL METHOD

Birinci gün

1) Bir sayının 3 katı 120’dir. Buna göre bu sayı kaçtır?

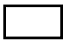
Bir sayı: 

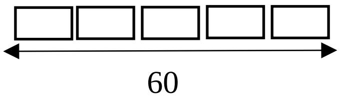
Bir sayının üç katı: 

Bir sayının 3 katı şekil ile gösterilip 120’ye eşit olduğu ifade edilir. Buna göre, bir kutunun değerini bulmak için; 120 : 3 = 40 işlemi yapılır.

<u>Denklemi:</u> Her bir kutu x olursa; 3x = 120

2) Bir sayının 5 katı 60 ise bu sayı kaçtır?

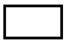
Bir sayı: 

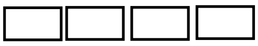
Bir sayının üç katı: 

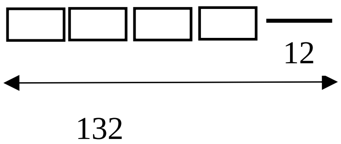
Bir sayının 5 katı şekil ile gösterilip 60’a eşit olduğu ifade edilir. Buna göre, bir kutunun değerini bulmak için;
60 : 5 = 12 işlemi yapılır.

<u>Denklemi:</u> Her bir kutu x olursa; 5x = 60

3) Bir sayının 4 katının 12 fazlası 132 ise bu sayı kaçtır?

Bir sayı: 

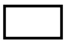
Bir sayının dört katı: 

Bir sayının dört katının 12 fazlası: 

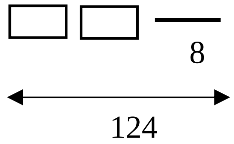
Buna göre, bir kutuyu bulmak için önce 12 birim fazlalık atılır. 132 – 12 = 120 dört kutunun değeridir. Bir kutuyu bulmak için 120 : 4 = 30 işlemi yapılır.

<u>Denklemi:</u> Son şekilde her bir kutu x olursa; 4x + 12 = 132

4) Bir sayının 2 katının 8 fazlası 124 ise bu sayı kaçtır?

Bir sayı: 

Bir sayının iki katı: 

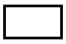
Bir sayının iki katının 8 fazlası: 

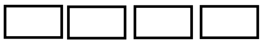
Buna göre, bir kutuyu bulmak için önce 8 birim fazlalık atılır.
124 – 8 = 116 iki kutunun değeridir.
Bir kutuyu bulmak için 116 : 2 = 58 işlemi yapılır.

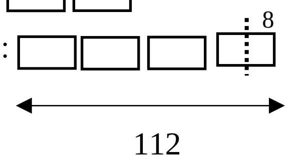
<u>Denklemi:</u> Son şekilde her bir kutu x olursa;

$2x + 8 = 124$

5) Bir sayının dört katının 8 eksiği 112 ise bu sayı kaçtır?

Bir sayı: 

Bir sayının dört katı: 

Bir sayının dört katının 8 eksiği: 

Buna göre, bir kutuyu bulmak için önce 8 birim eksiltilen kısım geri eklenir.
112 + 8 = 120 dört kutunun değeridir. Bir kutuyu bulmak için 120 : 4 = 30 işlemi yapılır.

<u>Denklemi:</u> Son şekilde her bir kutu x olursa;

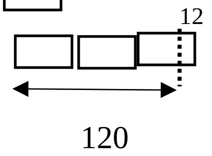
$4x – 8 = 112$

6) Bir sayının 3 katının 12 eksiği 120 ise bu sayı kaçtır?

Bir sayı:

Bir sayının üç katı:

Bir sayının üç katının 12 eksiği:



Buna göre, bir kutuyu bulmak için önce 12 birim eksiltilen kısım geri eklenir. 120 + 12 = 132 üç kutunun değeridir. Bir kutuyu bulmak için 132 : 3 = 44 işlemi yapılır.

<u>Denklemi:</u> Son şekilde her bir kutu x olursa;

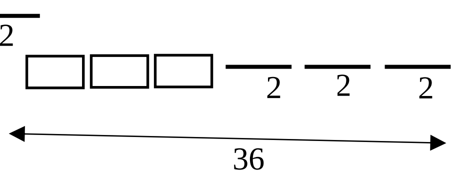
$3x - 12 = 120$

7) Bir sayının 2 fazlasının 3 katı 36 ise bu sayı kaçtır?

Bir sayı:

Bir sayının 2 fazlası:

Bir sayının 2 fazlasının 3

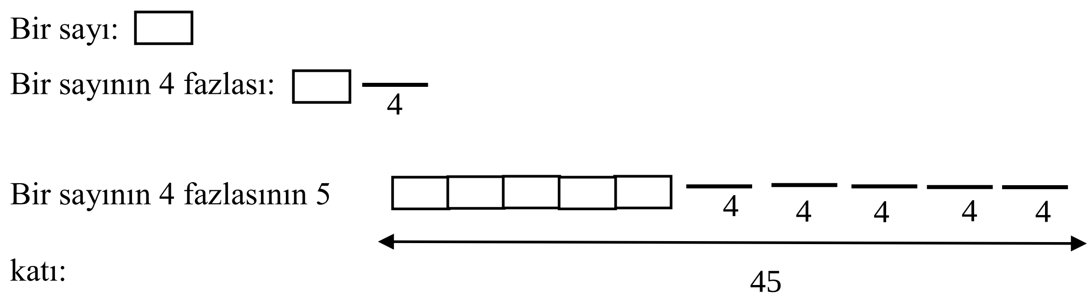


Buna göre, bir kutuyu bulmak için önce 2 birimlik fazlalıklar sırasıyla çıkarılır. 36 – 2 = 34 34 – 2 = 32 32 – 2 = 30 üç kutunun değeridir. Bir kutuyu bulmak için 30 : 3 = 10 işlemi yapılır.

<u>Denklemi:</u> Son şekilde her bir kutu x olursa;

$3x + 6 = 36$

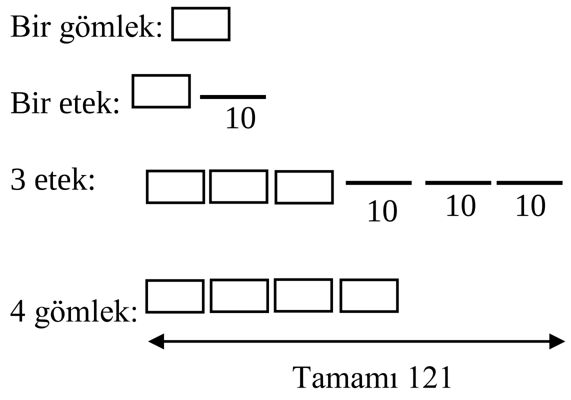
8) Bir sayının 4 fazlasının 5 katı 45 ise bu sayı kaçtır?



Buna göre, bir kutuyu bulmak için önce 4 birimlik fazlalıklar sırasıyla çıkarılır. Burada toplam 20 birimlik fazlalık olduğu için doğrudan 20 de çıkarılabilir.
45 – 20 = 25 beş kutunun değeridir.
Bir kutuyu bulmak için 25 : 5 = 5 işlemi yapılır.

Denklemi: Son şekilde her bir kutu x olursa; $5x + 20 = 45$

9) Esra 3 etek ve 4 gömleğe 121 TL ödemiştir. Bir etek bir gömlekten 10 TL fazla ise, bir gömlek kaç liradır?

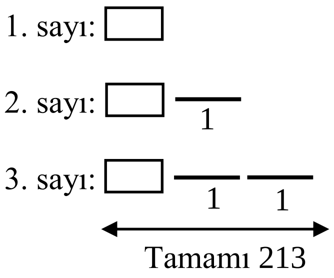


Önce bir gömlek bir kutu ile ve bir etek de bir kutu ve 10 birim fazlalıkla gösterilir.
Ardından 3 etek ve 4 gömlek kutularla ifade edilir.
Buna göre, bir kutuyu bulmak için önce 10 birimlik fazlalıklar sırasıyla çıkarılır. Yani 30 birim çıkarılır.
121 – 30 = 91 toplam 7 kutunun değeridir.
Bir kutuyu bulmak için 91 : 7 = 13 işlemi yapılır. Bir gömlek 13 liradır.

<u>Denklemi:</u> Son şekilde her bir kutu x olursa; 3x + 30 + 4x = 121 olduğu görülür.

İkinci gün

1) Ardışık 3 sayının toplamı 213 ise, en büyük sayı kaçtır?



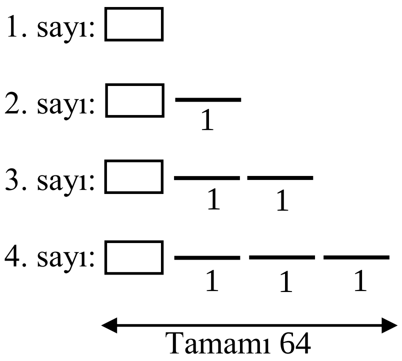
Ardışık sayıların birer birer arttığı hatırlatıldıktan sonra şekil çizilir. Birinci sayıya bir kutu verildikten sonra diğer sayıların da birer birimlik çubuklarla fazlalığı gösterilir.

Buna göre, bir kutuyu bulmak için önce 1 birimlik fazlalıklar sırasıyla çıkarılır. Yani 3 birim çıkarılır.
213 – 3 =210 toplam 3 kutunun değeridir.
Bir kutuyu bulmak için 210 : 3 = 70 işlemi yapılır.

En küçük sayı 70 ise diğer sayılar sırasıyla 71 ve 72'dir.
En büyük sayı 72.

<u>Denklemi:</u> Son şekilde her bir kutu x olursa; x + x + 1 + x + 2 = 213

2) Ardışık 4 sayının toplamı 64 ise bu sayıların en büyüğü kaçtır?



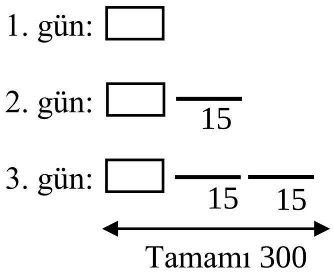
Ardışık sayıların birer birer arttığı hatırlatıldıktan sonra şekil çizilir. Birinci sayıya bir kutu verildikten sonra diğer sayıların da birer birimlik çubuklarla fazlalığı gösterilir.

Buna göre, bir kutuyu bulmak için önce 1 birimlik fazlalıklar sırasıyla çıkarılır. Yani 6 birim çıkarılır.
64 – 6 = 58 toplam 4 kutunun değeridir.
Bir kutuyu bulmak için 58 : 4 = 14,5 işlemi yapılır.

En küçük sayı 14,5 ise diğer sayılar sırasıyla 15,5-16,5 ve 17,5'tir. En büyük sayı 17,5.

<u>Denklemi:</u> Son şekilde her bir kutu x olursa; x + x + 1 + x + 2 + x + 3 = 64

3) Ayşe 300 soruluk ödevinin her gün bir önceki günden 15 soru fazla çözerek 3 günde bitiriyor. Buna göre, Ayşe 2. gün kaç soru çözmüştür?



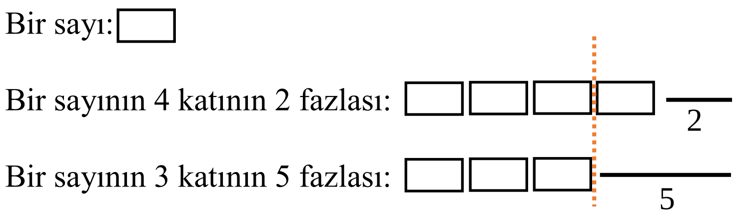
Ayşe'nin birinci gün çözdüğü soru sayısına bir kutu verilip diğer günler de 15'er birim fazlalıkla sırasıyla gösterilir.

Buna göre, bir kutuyu bulmak için önce 15 birimlik fazlalıklar sırasıyla çıkarılır. Yani 45 birim çıkarılır.
300 – 45 = 255 toplam 3 kutunun değeridir.
Bir kutuyu bulmak için 255 : 3 = 85 işlemi yapılır.

İlk gün 85 soru çözmüştür. Buna göre 2. gün 85 + 15 = 100 soru çözmüştür.

<u>Denklemi:</u> Son şekilde her bir kutu x olursa; $x + x + 15 + x + 30 = 300$

4) Bir sayının 4 katının 2 fazlası aynı sayının 3 katının 5 fazlasına eşittir. Bu sayı kaçtır?

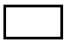


Problemde verilenler sırasıyla kutuyla gösterilir ve eşitlik gösterdiği için şekillerin aynı hizada bitmesine dikkat edilir. İlk üç kutu birbirine eşit olduğu için onlar hizalanır ve kalan kısımdan bir kutu bulunmaya çalışılır.

Buna göre bir kutu ve 2 birimlik çizgi ile 5 birimlik çizgi birbirlerine eşit oldukları için, 5 – 2 = 3 işlemiyle bir kutu bulunur.

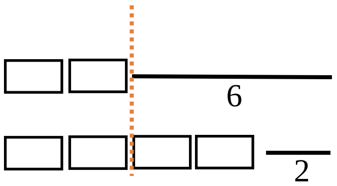
<u>Denklemi:</u> Son şekilde her bir kutu x olursa; $4x + 2 = 3x + 5$

5) Bir sayının 2 katının 6 fazlası aynı sayının 4 katının 2 fazlasına eşittir. Bu sayı kaçtır?

Bir sayı: 

Bir sayının 2 katının 6 fazlası:

Bir sayının 4 katının 2 fazlası:

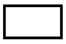


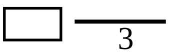
Problemde verilenler sırasıyla kutuyla gösterilir ve eşitlik gösterdiği için şekillerin aynı hizada bitmesine dikkat edilir. İlk iki kutu birbirine eşit olduğu için onlar hizalanır ve kalan kısımdan bir kutu bulunmaya çalışılır.

Buna göre iki kutu ve 2 birimlik çizgi ile 6 birimlik çizgi birbirlerine eşit oldukları için, 6 – 2 = 4 işlemiyle iki kutunun değeri bulunur. Bir kutuyu bulmak için 4 : 2 = 2 işlemi yapılır.

Denklemi: Son şekilde her bir kutu x olursa; $2x + 6 = 4x + 2$

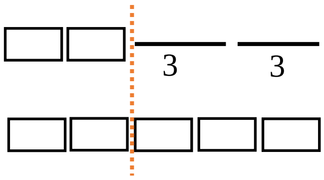
6) Bir sayının 3 fazlasının 2 katı, aynı sayının 5 katına eşittir. Bu sayı kaçtır?

Bir sayı: 

Bir sayının 3 fazlası: 

Bir sayının 3 fazlasının 2 katı:

Bir sayının 5 katı:



Eşitlik gösterdiği için şekillerin aynı hizada bitmesine dikkat edilir. İlk iki kutu birbirine eşit olduğu için onlar hizalanır ve kalan kısımdan bir kutu bulunmaya çalışılır.

Buna göre üç kutu ile 3 birimlik iki çizgi birbirlerine eşit oldukları için, 6 : 3 = 2 işlemiyle bir kutunun değeri bulunur.

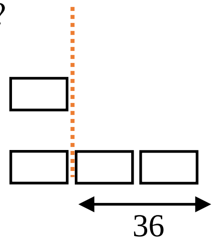
Denklemi: Son şekilde her bir kutu x olursa; $2x + 6 = 5x$

Üçüncü gün

1) Ali ile babasının yaşları farkı 36’dır. Babasının yaşı Ali’nin yaşının 3 katı ise, Ali kaç yaşındadır?

Ali’nin yaşı:

Babasının yaşı:



Ali’nin yaşına bir kutu verilip babasının yaşı da 3 katına göre gösterilir.

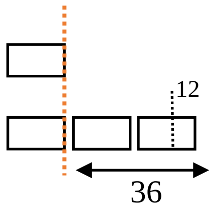
Yaşlarının farkı 36 olduğu için şekil üzerinde fark, kutuların hizalanmasıyla ve aynı kutuların çıkarılmasıyla bulunur. Böylece babasının yaşının Ali’nin yaşından farkı 2 kutu ile gösterilir. Buna göre 2 kutu 36 ise, bir kutuyu bulmak için 36 : 2 = 18 işlemi yapılır. Ali’nin yaşı 18’dir.

<u>Denklemi:</u> Son şekilde her bir kutu x olursa; 3x – x = 36

2) Ali ile babasının yaşları farkı 36’dır. Babasının yaşı Ali’nin yaşının 3 katından 12 eksik ise, Ali kaç yaşındadır?

Ali’nin yaşı:

Babasının yaşı:



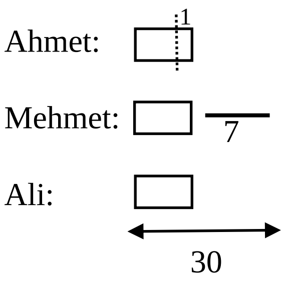
Ali’nin yaşına bir kutu verilip babasının yaşı da 3 katına göre gösterilir. 12 eksiklik ise kutunun kesilmesiyle gösterilir.

Yaşlarının farkı 36 olduğu için şekil üzerinde fark, yine kutuların hizalanmasıyla ve aynı kutuların çıkarılmasıyla bulunur. Böylece babasının yaşının Ali’nin yaşından farkı 2 kutu ve 12 eksiklik ile gösterilir.

Buna göre önce 12 eksiklik geri tamamlanır, yani 36 + 12 = 48 olur. 2 kutu 48 ise, bir kutuyu bulmak için 48 : 2 = 24 işlemi yapılır. Ali’nin yaşı 24’dür.

<u>Denklemi:</u> Son şekilde her bir kutu x olursa; 3x – 12 – x = 36

3) Ahmet, Mehmet ve Ali'nin yaşları toplamı 30'dur. Ahmet Ali'den 1 yaş küçük ve Mehmet Ali'den 7 yaş büyükse, Mehmet kaç yaşındadır?



Ali'nin yaşına bir kutu verildikten sonra, problemdeki diğer kişilerin yaşları da problemdeki bilgilere göre kutuyla gösterilir. Üçünün yaşları toplamı 30'dur.

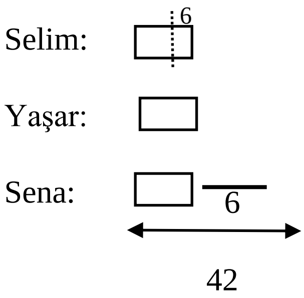
Öncelikle şekildeki 7 birim fazlalık çıkartılır ve sonra 1 birim eksiklik tekrar eklenir.

30 – 7 = 23 ve 23 + 1 = 24. Böylece üç kutunun değeri bulunmuş olur. Bir kutunun değerini bulmak için 24 : 3 = 8 işlemi yapılır. Ali'nin yaşı 8'dir.

Mehmet'in yaşını bulmak için 8 + 7 = 15 işlemi yapılır.

Denklemi: Son şekilde her bir kutu x olursa; $x - 1 + x + 7 + x = 30$

4) Selim, Yaşar ve Sena'nın yaşları toplamı 42'dir. Sena Yaşar'dan 6 yaş büyük ve Selim Yaşar'dan 6 yaş küçükse, Selim kaç yaşındadır?



Yaşar'ın yaşına bir kutu verildikten sonra, problemdeki diğer kişilerin yaşları da problemdeki bilgilere göre kutuyla gösterilir. Üçünün yaşları toplamı 42'dir.

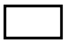
Öncelikle şekildeki 6 birim fazlalık çıkartılır ve sonra 6 birim eksiklik tekrar eklenir.

42 – 6 = 36 ve 36 + 6 = 42. Böylece üç kutunun değeri bulunmuş olur. Bir kutunun değerini bulmak için 42 : 3 = 14 işlemi yapılır. Yaşar'ın yaşı 14'tür.

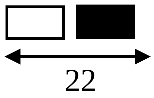
Selim'in yaşını bulmak için 14 – 6 = 8 işlemi yapılır.

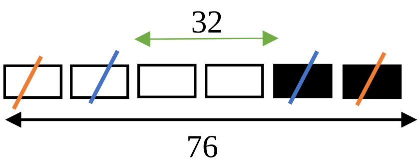
Denklemi: Son şekilde her bir kutu x olursa; $x - 6 + x + x + 6 = 30$

5) Bir kümesteki tavuk ve hindilerin ayaklarının sayısının toplamı 76'dır. Bu kümeste toplam 22 hayvan olduğuna göre, kaç tane hindi vardır?

Tavuk sayısı: 

Hindi sayısı: 

Toplam hayvan sayısı: 

Toplam ayak sayısı: 

Kümesteki toplam hayvan ve toplam ayak sayısı kutu ile gösterilir. Ardından bir boş kutu ile bir dolu kutunun toplamı 22 yaptığı ifade edilir.

İkinci şekilde, değeri bilindiği için bir dolu ve bir boş kutu eşleştirilerek şekilden çıkarılır. Toplam iki tane eş çıkarıldığı için işlemleri şu şekilde olur:

76 – 22 = 54
54 – 22 = 32

Geriye kalan iki boş kutunun değeri 32 ise, bir kutuyu bulmak için 32 : 2 = 16 işlemi yapılır. Boş kutu tavukları gösterdiği için 16 tane tavuk vardır.

22 – 16 = 6 tane de hindi vardır.

Denklemi: Hindi sayısına(dolu kutu) x verilirse, tavuk sayısına(boş kutu) 22 – x verilmelidir ilk şekle bakarak.

Ardından son şekle bakarak; 4.(22-x) + 2x = 76 denir.

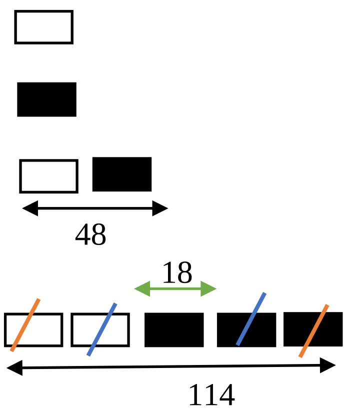
6) Bir otelde 2 yataklı ve 3 yataklı toplam 48 oda vardır. Bu oteldeki toplam yatak sayısı 114 ise, 2 yataklı kaç tane oda vardır?

2 yataklı oda sayısı:

3 yataklı oda sayısı:

Toplam oda sayısı:

Toplam yatak sayısı:



Oteldeki toplam oda ve toplam yatak sayısı kutu ile gösterilir. Ardından bir boş kutu ile bir dolu kutunun toplamı 48 yaptığı ifade edilir.

İkinci şekilde, değeri bilindiği için bir dolu ve bir boş kutu eşleştirilerek şekilden çıkarılır. Toplam iki tane eş çıkarıldığı için işlemleri şu şekilde olur:

114 – 48 = 66
66 – 48 = 18

Geriye kalan bir dolu kutunun değeri 18 ise, 18 tane 3 yataklı oda vardır. Buna göre,

48 – 18 = 30 tane de 2 yataklı oda vardır.

Denklemi: 2 yataklı sayısına(boş kutu) x verilirse, 3 yataklı oda sayısına(dolu kutu) 48 – x verilmelidir ilk şekle bakarak.

Ardından son şekle bakarak; 2x + 3.(48-x) = 114 denir.

E. CLINICAL INTERVIEW PROBLEMS

Birinci Dereceden Bir Bilinmeyenli Denklem Problemleri

Aşağıdaki problemleri açıklayarak çözünüz. Problem çözümünde size öğretilen şekil çizme yöntemi ya da denklem kurma metotlarından istediğinizi kullanabilirsiniz. Hangi yöntemi seçtiğinizi sebepleriyle açıklayınız.

1) Bir sayının 4 katının 15 eksiği 35'e eşit ise, bu sayı kaçtır?

2) Bir sayının 5 katının 3 fazlası, aynı sayının 4 katının 7 fazlasına eşittir. Buna göre, bu sayı kaçtır?

3) Bir sayının 2 katının 1 fazlası ile 3 katının 5 eksiğinin toplamı 51'dir. Buna göre, bu sayı kaçtır?

4) Bir kümesteki tavşan ve hindilerin ayak sayıları toplamı 50'dir. Bu kümeste toplam 16 tane hayvan olduğuna göre, bunlardan kaç tanesi hindidir?

5) Burak 4 kalem ile 3 deftere toplam 16 lira ödemiştir. Bir kalem, bir defterden 50 krş fazla olduğuna göre, bir defter kaç liradır?

6) Bir sayının 2 fazlasının 3 katı 42'dir. Buna göre, bu sayı kaçtır?

7) Ardışık olan 4 sayının toplamı 74 ise, bu sayılardan en büyüğü kaçtır?

8) Elif 180 sayfalık bir kitabı her gün bir önceki günden 10 sayfa fazla okuyarak üç günde bitiriyor. Buna göre ilk gün kaç sayfa kitap okumuştur?

9) Berke ile babasının yaşlarının farkı 36'dır. Babasının yaşı, Berke'nin yaşının 4 katından 12 eksik olduğuna göre Berke kaç yaşındadır?

10) Harun, Zafer ve Ömer'in yaşları toplamı 64'tir. Harun Zafer'den 4 yaş küçük, Ömer Zafer'den 3 yaş büyük olduğuna göre Ömer kaç yaşındadır?

Esra BAYSAL
Matematik Öğretmeni

F. VOLUNTARY PARTICIPATION FORM

Bu araştırma, İlköğretim Matematik ve Fen Eğitimi Programı yüksek lisans öğrencisi Esra Baysal tarafından Assist. Prof. Dr. Şerife Sevinç danışmanlığında yürütülen bir çalışmadır. Bu form sizi araştırma koşulları hakkında bilgilendirmek için hazırlanmıştır.

Çalışmanın Amacı Nedir? Araştırmanın amacı 7. sınıf öğrencilerinin denklem problemlerinin çözümünde Bar Model yönteminin etkisini ortaya çıkarmaktır. Araştırmaya katılmayı kabul ederseniz, sizden beklenen, okuldan sonra size verilecek olan eğitime katılmanız ve ardından size sorulan problemleri açıklayarak çözmektir. Bu çalışmaya katılım ortalama olarak 4 ders saati sürecektir.

Bize Nasıl Yardımcı Olmanızı İsteyeceğiz? Sizlere problem çözümünde yeni bir yöntem öğreteceğiz ve bu eğitime katılmanızı isteyeceğiz. Ardından size verilen soruları açıklayarak çözmenizi isteyeceğiz.

Sizden Topladığımız Bilgileri Nasıl Kullanacağız? Araştırmaya katılımınız tamamen gönüllülük temelinde olmalıdır. Çalışmada sizden kimlik veya kurum belirleyici hiçbir bilgi istenmemektedir. Cevaplarınız tamamıyla gizli tutulacak, sadece araştırmacılar tarafından değerlendirilecektir.

Katılımınızla ilgili bilmeniz gerekenler: Bu çalışma genel olarak kişisel rahatsızlık içerecek soruları içermemektedir. Ancak katılım sırasında sorulardan ya da herhangi başka bir nedenden ötürü kendinizi rahatsız hissederseniz cevaplama işini yarıda bırakıp çıkmakta serbestsiniz. Böyle bir durumda çalışmayı uygulayan kişiye, çalışmadan çıkmak istediğinizi söylemek yeterli olacaktır.

Araştırmayla ilgili daha fazla bilgi almak isterseniz: Bu çalışmaya katıldığınız için şimdiden teşekkür ederiz. Araştırma hakkında daha fazla bilgi almak için İlköğretim Matematik ve Fen Eğitimi yüksek lisans öğrencisi Esra Baysal (E-posta: esra.gedikli@metu.edu.tr) ile iletişim kurabilirsiniz.

Yukarıdaki bilgileri okudum ve bu çalışmaya tamamen gönüllü olarak katılıyorum.

(Formu doldurup imzaladıktan sonra uygulayıcıya geri veriniz).

İsim Soyad Tarih İmza

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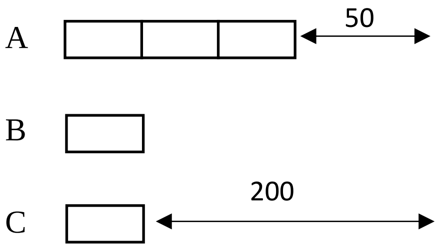
G. TURKISH SUMMARY/TÜRKÇE ÖZET

Giriş ve Alan Yazını

Cebir, matematik öğretiminin kesintisiz olarak devam etmesi için önemli bir faktördür. Çünkü Lacampagne (1995) cebirin matematiğin dili olduğunu ve üst düzey matematik konularını öğrenmek isteyen herkesin temel cebir bilgilerini öğrenmesi gerektiğini dile getirmiştir. Cebirdeki önemli konulardan biri de öğrencilerin yedinci sınıfta öğrenmeleri gereken denklem problemleridir. Bednarz ve Janvier (1996) cebirin, aritmetik yöntemler yerine problem çözmek için yeni ve güçlü bir yöntem olduğunu belirtmişlerdir. Aynı zamanda bir problemi cebirsel yöntemlerle çözmek, yani o probleme uygun bir denklem yazmak aynı problem tiplerinin çözümü için genel bir yöntem sunar. Bu sebeple, cebir problemlerinin cebir alanı içerisinde önemli bir yere sahip olduğu söylenebilir. Fakat cebir problemlerini çözmek öğrenciler için ciddi bir sorun olmuştur (Lawrance, 2007). Bu durumun sebebi cebirin doğasıyla açıklanabilir. Çünkü cebir sayılar, harfler, semboller ve değişkenler içerdiği için öğrencilere göre daha soyut gelmektedir (Kieran & Chalouh, 1993). Öğrencilerin denklem problemlerini çözerken karşılaştıkları hataları ve sebeplerini bulmak için çeşitli araştırmalar yapılmıştır (Adu, Assuah & Asideu-Addo, 2015; Jupri & Drivers, 2016; Kayani & Ilyas, 2014; Ladele, 2013). Bu araştırmaların sonucunda öğrencilerin sembollerin ve harflerin anlamını bilmedikleri, problemin içeriğini anlamadıkları, problem cümlesine uygun denklem kuramadıkları ve bu denklemleri doğru çözemedikleri saptanmıştır (Newman, 1983b aktaran Ladele, 2013). Ayrıca Türkiye'deki öğrencilerin de cebir ve denklem problemleri alanında zorlandıkları TIMSS puanlarından anlaşılabilir çünkü öğrencilerin TIMSS puanları her zaman ortalama puanın altında kalmıştır (Bütüner & Güler, 2017).

Öğrencilerin denklem problemleri çözerken yaptıkları hataların üstesinden gelebilmek için Kayani ve Ilyas (2014) farklı yöntemler denenmesini önermişlerdir. Bu yöntemlerden biri de Singapur matematik müfredatında kullanılan bar model yöntemidir. Koleza (2016) bar model yöntemini bilinmeyeni göstermek için harfler gibi cebirsel semboller yerine dikdörtgen kutular kullanılması olarak tanımlamıştır. Bar model yöntemini anlamak için bir problemin çözümüne bakılabilir. Hong, Mei ve Lim (2009) kitaplarında hem bar model yöntemini hem de denklem kurma yöntemini problem çözerken nasıl kullanılacağını göstermişlerdir. Problemlerden bir tanesi şu şekildedir: "A, B'nin üç katı kadar paraya sahiptir. B'nin parası, C'nin parasından 200 $ daha azdır. C'nin parası ise A'dan 50 $ daha fazladır. A, B ve C'nin sahip oldukları toplam parayı hesaplayınız." Problemi bar model yöntemiyle çözmek için önce A, B ve C'nin sahip oldukları para miktarı Şekil 1'deki gibi dikdörtgen kutularla gösterilmelidir.



Şekil 1 Problemlerin bar model yöntemiyle çözümü

En az paraya sahip olan B olduğu için, B bir birimlik dikdörtgen kutuyla gösterilmiştir. C, B'den 200 $ fazla olduğu için C'ye B ile aynı uzunlukta bir dikdörtgen kutu ve 200 birim uzunluğunda bir çizgi çizilir. A B'nin üç katı olduğu için, aynı uzunlukta üç dikdörtgen kutu A için çizilir. Son olarak, C A'dan 50 $ fazla olduğu için C ile A arasındaki fark 50 birimlik çizgi çizilerek gösterilir. Ardından problemi çözmek için bir kutunun değeri bulunmaya çalışılır. A ile C'nin aynı uzunluktaki ilk kutuları çıkarıldığında, kalan iki kutu ve 50 birimlik çizgi ile 200 birimlik çizginin eşit olduğu görülür. 200'den 50 çıkarıldığında iki kutunun değerinin 150 olduğu görülür. Yani bir kutunun değerini bulmak için 150 ikiye bölünmelidir. $(150 \div 2 = 75)$. B'nin 75 $'a sahip olduğu bulunduktan sonra diğerleri de bulunur ve toplam para hesaplanabilir. Bar model yöntemi sadece denklem problemlerini çözmeye değil, aynı zamanda probleme uygun bir denklem kurmaya da yardımcı olur. Bir kutu x ile gösterilirse, diğer bilinmeyenler de x'e bağlı olarak yazılabilir ve uygun denklem oluşturulabilir.

Bar model yöntemini araştırmak birçok açıdan önemlidir. İlk olarak, öğrencilerin problemleri denklem kurmaya çalışarak çözmekte zorlandıkları ve hata yaptıklarından daha önce bahsedilmişti. Probleme uygun denklem kurmak ve cebirsel problemleri çözmek için bar model etkili bir yöntem olabilir. İkinci olarak, bar model yöntemi teknolojiye uyarlanabilir ve kullanışlı bir metottur. Teknolojinin günden güne ilerlediğini, öğrencilere görsellik açısından avantaj sağladığı ve öğrencilerin matematik konularını anlamasında yardımcı olduğu düşünülürse, bar modelin teknolojiye uyarlanabilir olması önemli bir avantajdır. Örneğin bar model yöntemini teknolojik bir materyal olarak kullanan bir web sitesi vardır (Thinking Blocks, n.d.). Üçüncü olarak, daha önce yapılan çalışmalarda görselleştirmenin matematik eğitimi için önemli olduğu çünkü öğrencilerin problem çözerken uygun işlemleri seçmesinde yardımcı olduğu bulunmuştur (Beckmann, 2004). Bar model yöntemi de görselleştirmeye dayalı bir teknik olduğu için, bu yöntemin denklem problemlerini çözmede ve öğrenci hatalarının giderilmesinde önemli bir rol oynayacağı düşünülmektedir. Son olarak, yedinci sınıf cebir alanındaki denklem problemleri konusuna alternatif bir yöntem olan bar modeli başarılı olduğu takdirde öğrencilerin sınav başarısını arttıracağına ve ileriki yıllardaki matematik performanslarını yükselteceğine inanılmaktadır.

Singapur matematik müfredatında kullanılan bar model yönteminin etkileri, Singapurlu öğrencilerin TIMSS gibi uluslararası sınavlardan başarılı olmasıyla bazı çalışmalar yapılarak incelenmiştir. (Hoven & Garelick, 2007; Mahoney, 2012; Ng & Lee, 2005; Ng & Lee, 2009; Waight, 2006). Bu çalışmalar çoğunlukla bar model yönteminin somutlaştırmaya yardımcı olduğunu ve öğrencilerin matematiksel düşünme becerisini arttırdığını desteklemiştir. Fakat bar model yönteminin özellikle denklem problemlerinin çözümündeki etkisini araştıran çok az sayıda araştırma vardır. Ayrıca Türkiye'deki ulaşılabilir alan yazında bu yöntemin etkisini araştıran bir çalışmaya rastlanmamıştır. Bu bağlamda, bu çalışmanın amacı bar model yönteminin yedinci sınıf öğrencilerinin denklem problemi çözerken kullanımını ve öğrencilerin bu problemleri çözerken kullandıkları yöntemin tercih sebeplerini anlamaktır. Bar model yönteminin denklem problemleri çözme üzerine olan etkisi klinik görüşmeler aracılığıyla ölçülmüştür. Sonuç olarak aşağıdaki araştırma soruları bu çalışma için kararlaştırılırmıştır:

1) Yedinci sınıf öğrencilerinin denklem problemleri çözerken yaptıkları hata türleri nelerdir?
2) Singapur matematik müfredatına dayalı bar model yöntemi yedinci sınıf öğrencilerinin denklem problemlerini çözerken yaptıkları hataları aşmakta ne derece yardımcı olur?
3) Yedinci sınıf öğrencilerinin denklem problemlerini çözerken kullandıkları yöntemi (denklem kurma ya da bar modeli) tercih etmelerinin sebepleri nelerdir?

İlgili alan yazınında, ilk olarak öğrencilerin denklem problemleri çözerken sıklıkla yaptıkları hatalarla ilgili çalışmalara bakılabilir. Bunlardan ilki Newman'ın 124 tane altıncı sınıf öğrencisiyle yaptığı ve onların denklem problemleri çözerken yaptıkları hataları kategorilere ayırdığı çalışmasıdır (Newman, 1983b aktaran Ladele, 2013). Beş kategori sırasıyla: (1) okuma, (2) yorumlama, (3) dönüşüm, (4) süreç, ve (5) kodlama. Bu hataları kısaca özetlemek gerekirse; okuma hatası öğrencinin problem cümlesindeki kelimeleri ve sembolleri tanıyamaması ve problemi okumakta zorluk çekmesi olarak tanımlanabilir. İkinci hata yorumlama hatası ise, öğrencinin problemi kendi cümleleriyle tekrar edememesi ve özetleyememesidir. Üçüncü hata dönüşüm, öğrencinin problemde verilenleri matematiksel sembolleri ve sayıları kullanarak yazamamaktır. Denklem problemlerinde bu hata probleme uygun bir denklem kuramamak anlamına gelir. Dördüncü hata ise problem çözüm sürecinde yapılan hatalardır, yani öğrencinin kurduğu denklemi çözememesi ya da işlem hatası yapmasıdır. Son kategori, kodlama hatası öğrencinin problemi çözdükten sonra cevabı doğru sembolleri ve kelimeleri kullanarak yazamaması olarak tanımlanabilir. Bu hatalardan dönüşüm ve süreç hataları, bu araştırmada kullanılmıştır.

Kayani ve Ilyas (2014) 7, 8, 9 ve 10. sınıf öğrencileriyle yaptığı çalışmada dört tane hata çeşidi tespit etmiştir. Bunlar kelimeleri cebirsel dile dönüştürme, aritmetik işlemler, parantez kullanımı ve problemi çözmek için uygun metodu seçme hataları olarak kategorilere ayrılabilir. Bu araştırma parantezin yanlış kullanımı bir hata çeşidi olarak kullanılmıştır. Diğer bir araştırmacı Egodawatte (2011) lise öğrencilerinin problem çözerken yaptıkları hataları incelemiştir. Bu hatalardan biri herhangi bir mantıksal açıklaması olmadan problemi çözmeye çalışmak ya da cevabı işlem yapmadan tahmin etmektir. Diğer bir hata ise, problemin birden fazla bilinmeyen içerdiği durumlarda öğrencinin bu bilinmeyenler arasındaki ilişkiyi anlayamaması ve buna bağlı olarak da denklemi yazamamasıdır. İki hata türü de bu çalışma için kullanılmıştır.

Türkiye'deki öğrencilerin de denklem problemleri çözerken yaptıkları hatalar incelenmiştir. Kabael ve Akın (2016) çalışmalarında öğrencilerin problem çözerken aritmetik yöntemleri, cebirsel yöntemlere tercih ettiğini bulmuşlardır. Denklem kurmayı tercih eden öğrenciler de anlamsız semboller kullandıkları için denklem kuramamışlar ya da denklemi çözememişlerdir. Aynı şekilde Didiş ve Erbaş (2012) 10. sınıf öğrencilerinin problem çözerken, problemde verilenleri anlamadıklarını, yorum yapamadıklarını ve çözüme ulaşmak için herhangi bir fikir üretemediklerini bulmuşlardır. Ayrıca uluslararası sınavlardan biri olan Trends in International Mathematics and Science Study (TIMSS) sınavında Türk öğrencilerinin sınav puanları yıllar içerisinde yükselse de hep ortalamanın altında kalmıştır (Bütüner ve Güler, 2017). Bu durum matematik eğitimcilerinin cebir alanına daha çok eğilmelerini, cebiri öğretmek için daha anlamlı yeni yöntemler geliştirmeleri gerektiğini göstermektedir.

TIMSS gibi uluslararası sınavlarda ön plana çıkan ülkelerden biri de Singapur'dur. Bu yüzden Singapur'da kullanılan öğretim tekniklerini incelemek önemlidir. Bu yöntemlerden biri de bar model yöntemidir. Yukarıda da açıklandığı üzere bar model yöntemi, problemi görselleştiren, bilinmeyenin bir dikdörtgen şeklindeki kutuyla gösterildiği bir yöntemdir. Bu yöntem Mahoney (2012)'nin çalışmasında belirttiği gibi 80li yılların başında Singapurlu öğrencilerin zayıf problem çözme becerisini geliştirmek için kullanılmaya başlanmıştır. Singapur'da cebir konuları altıncı sınıfta öğretilmeye başlanmaktadır. Fakat daha öncesinde de öğrencilerin cebirsel düşünme becerilerini geliştirmek ve aritmetikten cebire geçişi kolaylaştırmak için bar model yöntemini kullanmaktadırlar.

Bar model yönteminin etkisini araştırmak için birçok çalışma yapılmıştır. Örneğin, Mahoney (2012) dört öğrenciyle bir çalışma yapmıştır. Bu çalışmada öğrenciler önce 10 tane problemi çözmeye çalışmışlar, ardından bar model yöntemini nasıl kullanacaklarına dair bir takım dersler almışlar ve aynı problemleri bar model yöntemiyle çözmeye çalışmışlardır. Çalışmanın sonucunda araştırmacı, bar modelin etkisiyle öğrencilerin performanslarının arttığını, doğru işlemleri seçebildiklerini ve değişkenler arasındaki ilişkiyi daha kolay gördüklerini bulmuştur. Bir diğer araştırmacı Waight (2006) ise Massachusetts'te bir okulda, öğrencilerin matematik notlarının düşük olması sebebiyle altı sınıfta Singapur matematik müfredatını uygulamaya başlamıştır. Bu sınıflardaki öğrencilerin başarısı arttıkça, müfredatın uygulandığı sınıf sayısı da arttırılmış ve en sonunda 130 sınıf bu yöntemi kullanmaya başlamıştır. Bu artış Singapur matematiğinin başarısını göstermektedir. Ng ve Lee (2009) ise bar model yönteminin öğrencilerin problem çözme becerilerini arttırmak için önemli bir faktör olduğunu belirtmiş fakat kolay bir yöntem olmadığını ve öğrencilerin bu yöntemde uzmanlaşması için çok pratik yapılması gerektiğini savunmuştur. Benzer şekilde, Hoven ve Garelick (2007) bar model yöntemi kullanıldığında öğretimin hızını yavaşlatabileceğini ve bu yöntemin bolca pratik gerektirdiğini söylemişlerdir.

Yöntem

Durum deseni Creswell (2007) tarafından bir durumun ya da durumların detaylı bir şekilde ve birden fazla kaynakla araştırıldığı bir araştırma yöntemi olarak tanımlanmıştır. Bu çalışmada da yedinci sınıf öğrencilerinin denklem problemlerini bar model metoduyla çözmelerini birden fazla veri toplama aracı kullanılarak araştırıldığı ve tek bir durum incelendiği için tek durum deseni kullanılmıştır.

Çalışmanın katılımcıları Sincan, Ankara'da bulunan bir devlet ortaokulundan beş kız beş erkek öğrenci olacak şekilde yedinci sınıflardan seçilmiştedir. Yedinci sınıflardan rastgele seçilen 42 öğrenci 10 denklem problemi içeren bir ilk değerlendirme testine tabi tutulmuşlardır. Bu testteki problemler öğretmenin derste çözdüğü problemlerin benzeri olacak şekilde yazılmıştır. Testin sonucunda, öğrencilerin hataları kategorilere ayrılmış ve bu kategorilerden bir ya da iki öğrenci çalışmanın katılımcısı olarak seçilmiştir. Bu hata kategorileri şunlardır: (1) boş tahmin, (2) bilinmeyeni yanlış bir şekilde ifade etmek, (3) denklemi yanlış kurmak, (4) denklem yazarken parantezi yanlış kullanmak, (5) denklemi çözerken işlem hatası yapmak ve (6) yanlış bilinmeyi bulmak. Bu hatalardan ilk beş tanesi alan yazında daha önce bulunan hatalar iken, altıncı hata araştırmacının fark ettiği fakat alan yazında bulunmayan bir hatadır. Bu hataları yapan öğrencilerden, diğerlerine göre motivasyonu yüksek, öğrenmeye istekli, iletişime açık olanlardan toplamda 10 kişi seçilmiştir. Katılımcıların yaş ortalaması 13'tür. Öğrencilerin ilk eğitim-öğretim dönemindeki matematik başarıları 100 üzerinden 50 ile 90 arasındadır. Öğrencilerin eğitim aldığı okul, Ankara'da bir devlet ortaokuludur. Yaklaşık 1500 öğrencinin olduğu bu okulda, öğrencilerin ailelerinin çoğu orta ya da düşük sosyoekonomik düzeydedir.

Creswell (2009) nitel bir araştırmada durumunu daha detaylı bir şekilde inceleyebilmek için birden fazla veri toplama aracı kullanması gerektiğini ifade etmiştir. Nitel bir araştırma olan bu çalışmada gözlem ve görüşme araçları kullanılmıştır. Araştırmacı, katılımcılara üç ders saati boyunca denklem problemlerini bar model yöntemiyle nasıl çözeceklerini öğretmiştir. Bu eğitimin ardından, araştırmacı katılımcılarla klinik görüşmeler yapmıştır, onlardan 10 tane denklem problemini istedikleri yöntemle çözmelerini istemiş ve bar model yöntemi hakkındaki düşüncelerini araştırmıştır. Bu görüşmeler ve ders esnasında hem ses hem de kamera kaydı alınmıştır.

Üç ders saati boyunca devam eden eğitim, birer gün arayla olacak şekilde okul sonraları yapılmıştır. Derslerde yedinci sınıf müfredatındaki denklem problemlerinin (bu problemler ilk değerlendirme testindeki problemlerle benzerdir) bar model yöntemiyle nasıl çözüldüğü öğretilmiştir. Araştırmacı, öğrencilerin aynı zamanda öğretmeni olduğu için, derste kullandığı öğretmen merkezli eğitimi bu eğitimde de kullanmıştır. Her problem tipi için bar model yönteminin nasıl kullanıldığını göstermiş, ardından öğrencilerin tek başına çözmeleri için benzer bir problemi tahtaya yazmıştır. Öğrenciler problemi çözmeye çalışırken hem arkadaşlarından hem de öğretmenlerinden destek almışlardır. Ayrıca öğretmen problemi bar model yöntemiyle çözdükten sonra bar model yöntemiyle problemle uygun denklemin nasıl yazıldığını da göstermiştir. Problemler kolaydan zora olmak üzere üç gruba ayrılmıştır.

1. Gün: İlk gün, öğrencilere bar model yöntemi tanıtılmıştır. Bir problemde bilinmeyi dikdörtgen kutularla nasıl göstereceklerini, problemde ‘az’ ya da ‘çok’ ifadeleri geçtiğinde nasıl model çizeceklerini öğrenmişlerdir. Ayrıca bir dikdörtgen kutunun değerinin nasıl bulunacağını, işlem sıralarını da ilk gün öğrenmişlerdir.

2. Gün: İkinci gün, öğrenciler problemde birden fazla bilinmeyen olduğunda bilinmeyenleri nasıl dikdörtgen kutularla göstereceklerini öğrenmişlerdir. Ayrıca problemdeki sayıların toplamı verildiğinde ya da bir eşitlik verildiğinde ne yapmaları gerektiğini de öğrenmişlerdir.

3. Gün: Üçüncü günde, öğrenciler biraz daha karmaşık problemleri bar model yöntemiyle nasıl çözeceklerini öğrenmişlerdir. Örneğin, sayıların toplamı yerine farkı verildiğinde ya da ikiden fazla bilinmeyen olduğunda nasıl model çizeceklerini öğrenmişlerdir. Ayrıca öğrencilerin ilk değerlendirme testinde en çok hata yaptıkları soru tipi olan ‘bacak problemleri’ de bu günde öğrenilmiştir.

Eğitimin ardından, öğrencilerle birebir yapılan klinik görüşmeler sırasında araştırmacının hazırladığı açık uçlu 10 problemi istedikleri yöntemle çözmeleri istenmiştir. Öğrenciler bar model yöntemini kullanmakta ya da probleme uygun bir denklem kurmakta serbestlerdi. Ayrıca problemlerin çözümü esnasında, katılımcılar araştırmacının bir takım sorularına cevap vermişlerdir. Örneğin; “Ne yaptığını açıklar mısın?”, “Bu problemde hangi yöntemi tercih ettin? Neden?”, “Hangi yöntemi daha çok sevdin?” ya da “Bar model yöntemini diğer matematik konularında da kullanmak ister miydin?”.

Bulgular

Veriler analiz edilirken, araştırmacı öncelikle üç ders saatinin her birinin ve klinik görüşmelerin ardından kamera ve ses kayıtlarını transkript etmiştir. Öğrencilerin eğitim boyunca verdikleri tepkiler araştırmacı tarafından gözlemlenmiştir. Ardından katılımcıların klinik görüşmeler esnasında verdikleri cevaplar kodlanmış, yaptıkları hatalar gruplara ayrılmış ve bar model hakkındaki düşünceleri de incelenmiştir.

Çalışmanın sonuçlarını analiz etmek için, klinik görüşmeler esnasında sorulan problemler üç gruba ayrılmıştır: (1) nicel ilişkileri içeren ancak bağlamsal durumlarda sunulmayan problemler, (2) ardışık sayılar arasında nicel ilişki içeren problemler ve (3) birinin diğeri tarafından tanımlandığı iki bilinmeyen içeren bağlamsal durumdaki problemler.

Birinci problem grubunda, problem 1, problem 2, problem 3 ve problem 6 vardır. Birinci problem: "Bir sayının 4 katınınn 15 eksiği 35 ise, bu sayı kaçtır?" Bu problemde, yedi öğrenci doğru cevaba bar model yöntemini kullanarak ulaşmışken üç öğrenci problemi denklem kurma yöntemiyle çözmüştür. Ayrıca, sekiz öğrencinin ilk tercihi bar model yöntemini kullanmaktı ve üç öğrenci probleme uygun denklem yazarken bar modelinden faydalanmıştır. İkinci problem: "Bir sayının 5 katının 3 fazlası ile 4 katının 7 fazlası eşittir. Buna göre, bu sayı kaçtır?" Bu problemde, sekiz öğrenci bar model yöntemiyle ve iki öğrenci de denklem kurma yöntemiyle doğru cevaba ulaşmıştır. Öğrencilerden biri, probleme uygun modeli çizebilmiş fakat bir kutuyu bulmak için gerekli işlemleri doğru yapamamıştır ve ardından denklem kurarak doğru cevabı bulmuştur. Üçüncü problem: "Bir sayının 2 katının 1 fazlası ile 3 katının 5 eksiğinin toplamı 51'dir. Buna göre, bu sayı kaçtır?" Bu problemde, altı öğrenci bar model yöntemiyle ve iki öğrenci de denklem kurma yöntemiyle doğru cevaba ulaşmıştır. Diğer iki öğrenci ise hiçbir yöntemle soruyu çözememiştir. Ayrıca beş öğrenci denklemi çizdikleri modele bakarak yazmışlardır. Altıncı problem: "Bir sayının 2 fazlasının 3 katı 42'dir. Buna göre, bu sayı kaçtır?" Beş öğrenci problemi bar model yöntemiyle ve iki öğrenci de denklem kurma yöntemiyle doğru çözmüşlerdir.

Üç öğrenci ise hiçbir yöntemle problemi çözememiştir. Ayrıca, bazı öğrenciler yanlış yazsalar da denklemi kurarken bar modelden yararlanmayı tercih etmişlerdir.

İkinci problem grubunda, problem 7, problem 8 ve problem 10 vardır. Problem 7: "Dört ardışık sayının toplamı 74 ise, bu sayıların en büyüğü kaçtır?" Bu problemde, altı öğrenci bar model yöntemiyle doğru cevaba ulaşmıştır. Dört öğrenci ise bu yöntemi kullanırken hata yaptıkları için doğru cevaba ulaşamamıştır. Öğrencilerin dokuzu denklem yazarken çizdikleri modelden yardım almayı tercih etmiştir. Problem 8: "Elif 180 sayfalık bir kitabı, her gün bir önceki günden 10 sayfa fazla okuyarak üç günde bitiriyor. Buna göre, ilk gün kaç sayfa kitap okumuştur?" Bu problem dokuz öğrenci bar model yöntemini kullanarak doğru çözmüştür fakat bir öğrenci modeli yanlış çizdiği için doğru sonuca ulaşamamıştır. Bütün öğrenciler denklem yazarken bar model yönteminden yararlanmak istemişlerdir. Son olarak, problem 10: "Harun, Zafer ve Ömer'in yaşları toplamı 65'tir. Harun Zafer'den 4 yaş küçük ve Ömer Zafer'den 3 yaş büyük olduğuna göre, Ömer kaç yaşındadır?" Bu problem, tüm öğrencilerin bar model yöntemiyle doğru çözdüğü tek problemdir. Ayrıca sekiz öğrenci denklem yazarken bu yöntemden faydalanmıştır.

Üçüncü problem grubunda problem 4, problem 5 ve problem 9 vardır. Bu problemler yüksek seviye düşünme becerileri gerektirdiği ve diğer problemlere göre daha karmaşık olduğu için öğrenciler tarafından çözülmekte zorlanılmıştır. Problem 4: "Berke ile babasının yaşları farkı 36'dır. Babasının yaşı, Berke'nin yaşının 4 katından 12 eksik olduğuna göre, Berke kaç yaşındadır?" Bu problem üç öğrenci tarafından bar model yöntemiyle doğru çözülmüştür. Üçü de denklem kurarken çizdikleri modelden yararlanmışlardır. Fakat yedi öğrenci hiçbir yöntemle problem çözememiştir. Bu problem, diğerlerinden farklı olarak, bilinmeyenlerin farkını vermiştir ve öğrenciler model üzerinde bilinmeyenlerin farkını göstermekte zorlanmışlardır. Problem 5: "Bir kümesteki tavşan ve hindilerin ayak sayıları toplamı 50'dir. Bu kümeste toplam 16 tane hayvan olduğuna göre, bunlardan kaç tanesi hindidir?" Benzer şekilde, bu problem de üç öğrenci tarafından bar model yöntemiyle doğru çözülmüştür. Fakat bu öğrenciler problem uygun bir denklem yazamamışlardır. Geri kalan öğrenciler bar model yöntemiyle bir takım hatalar yapmışlar ya da problemi boş bırakmışlardır. Problem 9: "Burak 4 kalem ile 3 deftere toplam 16 lira ödemiştir. Bir kalem bir defterden 50 kuruş fazla olduğuna göre, bir defter kaç liradır?" Bu problem beş öğrenci tarafından bar model yöntemiyle doğru çözülmüştür. Bu beş öğrencinin ikisi modelden yardım alarak denklem yazabilmişlerdir. Fakat kalan beş öğrenci hiçbir yöntemle doğru çözememişler ve hatalar yapmışlardır.

Tablo 1'e bakıldığında, klinik görüşmeler esnasında hiçbir öğrencinin yanlış bilinmeyeni bulma hatası yapmadığı görülmüştür. Benzer şekilde, sadece bir öğrencinin boş tahmin hatası yaptığı görülmektedir. En çok yapılan hatalar ise işlemsel ve dönüşüm hataları. Sadece bir öğrenci, Umut, problemleri bar model yöntemiyle çözerken hiç hata yapmamıştır.

Tablo 1

Katılımcıların Bar Model Yöntemi Yaptıkları Hatalar

	Blank guessing	Identifying the unknown incorrectly	Setting up the equation wrongy	Using parenthesis incorrectly in writing the equation	Operational mistakes in solving the equation	Finding the unknown as an answer	No response
Zeynep	0	0	2	1	6	0	1
Ece	0	0	2	1	1	0	0
Melike	0	1	4	1	1	0	0
Mustafa	0	0	1	0	0	0	0
Ali	2	1	3	1	1	0	1
Sinem	0	0	2	0	0	0	0
Emre	0	0	2	0	0	0	0
Umut	0	0	0	0	0	0	0
Merve	0	0	1	0	1	0	0
Melik	0	0	2	0	2	0	0

0	1	2	3	4	5-6

Özetle, yedinci sınıf öğrencileri denklem problemlerini çözerken 13 farklı yol izlemişlerdir. Bu yollara bakıldığında öğrencilerin çoğunluğunun problemleri çözerken ilk olarak bar model yöntemini tercih ettikleri görülebilir. Birinci problem grubunda, öğrencilerin çoğunluğu (%65) doğru cevaba bar model yöntemiyle ulaşmışlardır. Fakat bu problem grubunda, öğrencilerin denklem yazarken yararlandıkları yöntem tercihi eşit orandadır. Yani denklem yazarken, problem cümlesine bakan öğrencilerle bar modelden yararlanan öğrencilerin sayısı neredeyse eşittir. İkinci problem grubunda, öğrencilerin büyük bir çoğunluğu (%82,9) bar model yöntemini kullanarak problemi doğru çözmüştür. Ayrıca öğrencilerin çoğu denklem yazarken bar model yönteminden faydalanmıştır. Son olarak üçüncü problem grubunda, öğrencilerin sadece %36,2'si bar model yöntemiyle doğru cevaba ulaşmıştır ve kalan öğrenciler hiçbir yöntemle problemleri doğru çözememiştir. Sonuç olarak, bar model yöntemi birinci ve ikinci problem grubundaki sorular için kullanışlı bir yöntem olsa da üçüncü gruptaki problemler için çok etkili olmayabilir.

Sonuç ve Tartışma

Öğrencilerin cevapları iki bölüm halinde tartışılmıştır: (1) bar model yönteminin öğrencilerin hataları üzerindeki rolü ve (2) öğrencilerin çözüm yöntemi tercihleri. Birinci bölümde, öğrencilerin ilk değerlendirme testindeki, denklem kurma yöntemiyle yaptıkları hata sayıları ile klinik görüşmeler esnasıdna bar model yöntemiyle yaptıkları hata sayıları karşılaştırılmıştır. Şekil 2'de öğrencilerin ilk değerlendirme testindeki ve klinik görüşmelerdeki yaptıkları hataları gösteren tablolar karşılaştırılmıştır.

Table 4.1

Case Participants' Errors in Algebraic Word Problems

Students	Blank guessing	Identifying the unknown incorrectly	Setting up the equation incorrectly	Using parenthesis incorrectly in writing the equation	Operational mistakes in solving the equation	Finding the unknown as an answer	No response
Zeynep	2	0	1	0	0	3	4
Ece	0	2	3	0	0	4	1
Melike	3	1	2	0	1	1	1
Mustafa	0	2	3	1	1	1	0
Ali	2	2	3	1	0	1	4
Sinem	0	2	5	2	3	1	0
Emre	1	3	0	1	1	3	0
Umut	2	2	3	1	0	0	3
Merve	0	1	3	1	0	3	4
Melik	0	4	3	1	1	0	3

0 1 2 3 4 5-6

Table 4.2

Case Participants' Errors in the Bar Model Method

Students	Blank guessing	Identifying the unknown incorrectly	Setting up the equation incorrectly	Using parenthesis incorrectly in writing the equation	Operational mistakes in solving the equation	Finding the unknown as an answer	No response
Zeynep	0	0	2	1	6	0	1
Ece	0	0	2	1	1	0	0
Melike	0	1	4	1	1	0	0
Mustafa	0	0	1	0	0	0	0
Ali	2	1	3	1	1	0	1
Sinem	0	0	2	0	0	0	0
Emre	0	0	2	0	0	0	0
Umut	0	0	0	0	0	0	0
Merve	0	0	1	0	1	0	0
Melik	0	0	2	0	2	0	0

0 1 2 3 4 5-6

Şekil 2 Öğrencilerin ilk değerlendirmede ve klinik görüşmelerde yaptıkları hata türleri

Bu tablolara bakıldığında, bar modelde yapılan hata sayısının denklem kurma yöntemiyle yapılan hata sayısından az olduğu görülmektedir. Bu da bar model yönteminin yedinci sınıf öğrencilerinin denklem problemleri çözerken etkili bir yöntem olduğunu göstermektedir. Ayrıca çalışmaya katılan 10 öğrenciden dokuzu bar modeli yöntemiyle birlikte yaptıkları hata sayısını azaltmıştır. Bu sonuç Mahoney'in çalışmasıyla uyumludur çünkü Mahoney (2012) öğrencilerin bar model yöntemini kullanmaya başladıktan sonra performansının arttığını bulmuştur.

Cai ve arkadaşları (2011) bar model yönteminin öğrencilere problemi çözmeye odaklanmak yerine onu sunmaya odaklanmalarına yardım ettiğini bulmuşlardır. Bu çalışmanın sonuçlarıyla uyumlu bir şekilde, öğrenciler sadece problemi çözmeye değil, aynı zamanda problemi yorumlamaya ve sunmaya çalıştıklarını bulunmuştur. Problemi çözmek için boş tahminlerde bulunmamışlardır. Ayrıca, Kieran (2004) bar model yönteminin problemdeki bilinmeyelerin arasında ilişkisi ifade etmede yardımcı olduğunu belirtmiştir. Benzer şekilde, bu çalışma bar model yöntemiyle öğrencilerin bilinmeyenler arasındaki ilişkiyi gösterebildiklerini ve tüm bilinmeyenleri dikdörtgen kutular aracılığıyla gösterebildiklerini bulmuştur çünkü bu yöntem görselleştirmeye dayalıdır ve öğrenciler için daha somuttur.

Bu çalışmayla birlikte bar model yönteminin bir takım zorlukları da ortaya çıkmıştır. Örneğin, öğrenciler karmaşık problemler için model çizmekte zorlanmışlar. Ayrıca bazı öğrencilere doğru modeli çizdikten sonra, bir kutunun değerini bulmak için gerekli işlemleri yapmak zor gelmiştir. Bu durumun altında yatan sebeplerden birinin öğrencilerin kavramsal düşünme yapmak yerine işlemleri ezberlemeye çalışmaları olduğu düşünülmektedir (Ng & Lee, 2009). Diğer bir yandan, öğrencilerin bu hatayı yapmalarının sebeplerinden birinin ise modeli doğru çizmelerine rağmen orantısız bir şekilde çizmeleri olabilir. Örneğin, bir birimlik çizgiyi çizdikten sonra iki birimlik çizgiyi, diğerinin iki katı olacak şekilde çizmemişlerdir. Bu sorunlar öğrencilerin bar model yöntemiyle uzun sürekli pratik yapmasıyla ya da teknolojik materyaller kullanmasıyla çözülebilir. Bu zorluklar ve öğrencilerin üçüncü problem grubundaki soruları doğru çözemeleri, bu yöntemin tamamen etkili olmadığını ve öğrencilerin daha çok pratik yapması gerektiğini göstermektedir (Ng & Lee, 2009).

Öğrencilerin büyük bir çoğunluğu problemleri çözerken bar model yöntemini kullanmayı tercih etmiştir. Dokuz öğrenci, bu yöntemi daha kolay bulmuşlardır. Öğrenciler bar model yöntemiyle hangi bilinmeyene x vereceklerine ve denklemde hangi işlemleri kullanacaklarına daha kolay karar verdiklerini ifade etmişlerdir. Diğer bir yandan, bir öğrenci ise denklem yazmanın daha hızlı olduğunu ve bu yüzden bazı problemlerde bu yöntemi tercih ettiğini belirtmiştir. Fakat bazı problemlerde ise denklem yazmasına yardımcı olduğunu söylemiştir. Ayrıca öğrenciler bar model yöntemini daha çok sevdiklerini, daha eğlenceli ve ilginç bulduklarını söylemişlerdir. Son olarak ise, öğrencilerden biri bu yöntemi kulanmanın zaman aldığını çünkü çizim yapmanın zorlayıcı olduğunu ama alıştıktan sonra denklem kurma yönteminden daha iyi olduğunu belirtmiştir. Bu bulgu, Hoven ve Garelick'in çalışması ile uyumludur. Hoven ve Garelick (2007) bar model yöntemiyle öğretimin yavaş olduğunu fakat öğrenciler bu yönteme alıştığında temel becerileri daha hızlı öğrendiklerini belirtmişlerdir.

Öneriler

Bu çalışmada öğrenciler bar model yöntemiyle ilgili sadece üç ders saatlik bir eğitim almışlardır. İleride yapılacak çalışmalarda, öğrenciler bu yöntemle ilgili daha uzun, detaylı ve farklı problem tiplerini de içeren eğitim verilebilir. Bu çalışmada, araştırmacı, yani eğitimi veren kişi, daha önce bar model yönteminin öğretimi hakkında herhangi bir eğitim almamış ve uzmanlaşmamıştır. Bu yüzden ileride yapılacak çalışmalarda, öğrenciler bu yöntemde uzmanlaşmış kişilerden eğitim alabilirler. Bar model yöntemi sadece denklem problemlerinde değil, aynı zamanda kesirler, çarpma, yüzdeler ya da orantı konularında da kullanılabilir (Kho, 1987). Bu çalışma yedinci sınıf denklem problemleri konusuyla sınırlı olsa da, ileriki çalışmalar farklı sınıf seviyeleri ve farklı matematik konularıyla yapılabilir.

Bu çalışmanın sonuçları göstermiştir ki bar model yöntemi yedinci sınıf öğrencileri üzerinde, denklem problemleri çözerken olumlu bir etkiye sahiptir. Bu sebeple, bu yöntem Türkiye'de matematik müfredatına entegre edilebilir ve ilkokuldan itibaren tüm matematik konularında kullanılabilir. Bu çalışmanın sonuçları, matematik öğretmenlerine, müfredat geliştiricilerine, ders kitabı yazarlarına ve öğretmen eğitimcilerine bar model yönteminin yedinci sınıf denklem problemleri konusu üzerindeki etkisine dair temel bilgiler vermektedir. Bu bilgileri kullanarak, etkili eğitim ortamları ve materyaller oluşturabilirler. Öğretmen eğitimciler, bar model yöntemi aday öğretmenlere öğretebilirler. Ayrıca, öğretmenler ders planı hazırlarken, denklem problemlerinin çözümünde önce bar model yöntemini ardından denklem kurma yöntemini kullanabilirler çünkü iki yöntemi de bilmek öğrencie hatalarını görme ve onları düzeltme şansı vermektedir.

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