EXPLORING EXTRA DIMENSIONS THROUGH RARE PROCESSES

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ABSTRACT

EXPLORING EXTRA DIMENSIONS THROUGH RARE PROCESSES

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We study the single top quark production and decay mechanisms at one-loop level occurring via flavor-changing neutral currents in the Minimal Universal Extra Dimensions (MUED) model. We make a complete study of the model in the Feynman gauge. Especially, we focus on extracting the complete list of 3- and 4-point interactions and determining the mixings among the gauge eigenstates to form the so-called mass eigenstates. Once we have the complete model analyzed, it is implemented via the LanHEP package and transferred to specialized programs such as FeynArts, FormCalc, and LoopTools for automatic calculations. Finally, we explore the MUED predictions for rare top processes in detail and interpret our results.

Keywords: extra dimensions, rare processes, top quark, physics beyond the Standard Model

ÖZ

NADİR SÜREÇLER ARACILIĞI İLE EKSTRA BOYUTLARIN ARAŞTIRILMASI

Şimşek, Kağan
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Tek ilmek seviyesinde, çeşni değiştiren yüksüz akımlar aracılığıyla gerçekleşen tek üst kuark üretim ve bozunum mekanizmalarını Minimal Evrensel Ekstra Boyutlar (MEEB) modelinde çalışıyoruz. Modelin eksiksiz çalışmasını Feynman ayarında yapıyoruz. Özellikle, 3 ve 4 nokta etkileşimlerinin tam listesinin çıkarılmasına ve kütle özdurumlarının oluşturulması için ayar özdurumlarındaki karışımların belirlenmesine odaklanıyoruz. Bütün modelin analizinden sonra, modeli LanHEP paketiyle uygulayıp FeynArts, FormCalc ve LoopTools gibi otomatik hesaplar için özelleşmiş programlara aktarıyoruz. Son olarak, nadir üst kuark süreçleri icin MEEB öngörülerini detaylıca keşfedip sonuçlarımızı yorumluyoruz.

Anahtar Kelimeler: ekstra boyutlar, nadir süreçler, üst kuark, Standart Model ötesi fizik

To my immortal beloved

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It would be unfair to my parents if I underrate their contributions. Whether I was in need of food, a place to stay, or a car to drive around, my parents were always by my side. Last but not least, I cannot miss the chance to thank my brother, Melih, for he taught me how to *let go* and focus on what really matters to me. Without his lectures at any time of the day, even miles apart, I could hardly organize myself so as to be able to finish my thesis.

TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

LIST OF ABBREVIATIONS

BSM	Beyond the Standard Model
CKM	Cabibbo-Kobayashi-Maskawa
FCNC	Flavor-changing neutral current
KG	Klein-Gordon
KK	Kaluza-Klein
LHC	Large Hadron Collider
LKP	Lightest Kaluza-Klein particle
MUED	Minimal Universal Extra Dimensions
NMUED	Non-minimal Universal Extra Dimensions
SM	Standard Model
UED	Universal Extra Dimensions
VEV	Vacuum expectation value

CHAPTER 1

INTRODUCTION

1.1 The Standard Model

The Standard Model (SM) of particle physics is a well-established theory that explains electromagnetic, strong, and weak interactions in the nature. With the discovery of the long-sought Higgs boson back in 2012, now the full particle set of the SM seems complete [1, 2, 3, 4].

Although the SM very successfully describes practically all existing experimental data, it still suffers from major complications: the hierarchy problem [5], the flavor problem [6], the origin of CP violation [7, 8], to name a few. The hierarchy problem is concerned with the splitting of the fermion masses and with the question why the gravitational force is 10^{34} times weaker than the weak force. In the SM, a dark matter candidate is absent. During the last few years, there have appeared experimental data on the lepton non-universality [9], which cannot be explained within the SM.

All these facts indicate that the SM is not the final theory in particle physics. To address some of these problems, many models have been proposed. On one hand, for example, the string theory has grown to be an independent attempt for a better understanding of the nature; on the other hand, Supersymmetry [10, 11], Two-Higgs-Doublet model [12, 13], and models with extra dimensions of different configurations have appeared as distinct extensions of the SM [14, 15, 16, 17].

1.2 Universal Extra Dimensions

The Universal Extra Dimensions (UED) model, which was proposed by Appelquist *et al.* in 2001 [17], describes a universe with $4 + N$ dimensions where the N extra dimensions are flat and compactified. In the case $N = 1$, the extra dimension is compactified on a circle, and for $N = 2$, it becomes a torus. In this work, we consider the model with one extra dimension, $N = 1$.

The universe in the $5D$ UED model is assumed to be cylindrical. Inside the cylinder extends the usual $4D$ spacetime, and along the tangential direction on the surface lies the extra dimension. The term *bulk* is defined as the complete $5D$ spacetime, whereas a *brane* is specified to be a $4D$ plane at a certain value of the extra dimension.

The UED model is called *universal* simply because all the fields (matter and mediators) are allowed to live in the bulk.

The existing experimental data indicates that the main parameter of the UED, namely the size of the extra dimension, R, lies in the region [18]

$$0.5 \text{ TeV} < R^{-1} < 1 \text{ TeV}. \tag{1.1}$$

At the present time, with the Large Hadron Collider (LHC) exploring the energy region 13 TeV, the extra dimension can be probed and hence the predictions of this model can be checked at the LHC.

In the UED, each SM field has a set of associate particles coming from the extra dimensions, which are called the Kaluza-Klein (KK) partners. Unlike in the Supersymmetry, the SM field and its KK partners have the same spin. A $5D$ field can be decomposed into a sum of the SM mode and its KK partners, where the partner fields are modulated by certain mode functions. This is the so-called KK decomposition.

The $5D$ Lagrangian density (or simply Lagrangian) is integrated over the extra dimension to get the physical states that depend on the usual 4-position vector. This integration is necessary to obtain a field theory whose structure is familiar to us. The mode functions depending only on the extra dimension determine the selection rules for possible interactions after the integration.

The UED is an effective field theory due to the fact that the gauge couplings are dimensionful — indeed, they depend on the size of the extra dimension. When this is the case, the theory at hand becomes non-renormalizable, and thus we need to introduce an appropriate cut-off energy [19, 20].

There are two versions of the UED: minimal (MUED) and non-minimal (NMUED). In the non-minimal case [21, 22, 23, 24, 25, 26], there exist brane-localized terms in the Lagrangian, which are assumed to be absent at the cut-off scale in the minimal version of the model. Due to these brane localizations, we lose certain symmetries in NMUED. Nevertheless, in both versions, there remains a conserved quantum number: the KK parity. The conservation of the KK parity is a major prediction of the model, that is, the lightest KK particle (LKP), which is the *first* KK partner of the photon in the MUED model, is stable, and hence serves as a fine candidate for dark matter.

In this thesis, we focus our attention to the MUED model. After a detailed description of the model and deriving the Feynman rules in the Feynman gauge, we next apply this theory to the analysis of the rare top quark processes occuring by flavor-changing neutral currents (FCNCs).

1.3 Rare top quark processes

The most dominant channel for the top quark decay is $t \to bW$ which occurs at the tree level. The decay width for this process is at the order of 1 GeV. The top quark can also decay into other up-like quarks (practically, u or c) by emitting a neutral gauge boson (a gluon, a Z boson, or a photon) or a Higgs particle. Due to the flavor conservation, these processes cannot take place at tree level unlike the $t \to bW$ case. Assuming the other quark in the final state is c, we need the $t \to c$ transition. In the SM, as well as in the UED, the $t \to c$ transition takes place at one loop level, since direct $t \to cX$ ($X = \gamma, g, Z, h$) vertices are forbidden in the SM (Figure 1.1).

Figure 1.1: The loop that appears in the rare top quark processes occurring by FCNCs in the Feynman gauge.

In Figure 1.1, the d^i ($i = 1, 2, 3$) denote all the down-like quarks (d, s, and b), $\phi^{\pm}$ is the Goldstone scalar associated with the W boson, and both or either of the top quark and the charm quark belongs to an external line. This is the only possible loop for the rare top quark processes occurring by FCNCs in the SM. In writing Feynman amplitudes, two Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, V_{ti} and V_{ci} ($i = d, s, b$), are involved. From the unitarity of the CKM matrix, the amplitude for the Feynman diagrams that contain a loop as in Figure 1.1 will nearly vanish, when summed over the down-like quark generations[1]. This is the reason for the above-mentioned processes to be rare. When we say *rare*, we really mean the following. A straightforward SM calculation shows that the decay widths of these processes are less than 10^{-10} GeV. A similar observation holds true for scattering cross sections of the rare top-production processes.

Intriguingly, there exist only upper bounds for the decay widths of the rare top decay processes given above [27], which are quite large (Table 1.1).

Table 1.1: Bounds on the decay widths of the rare top decay processes.

Process	Branching ratio ($\Gamma/\Gamma_{\text{total}}$)
$t \to c\gamma$	$< 1.7 \times 10^{-3}$
$t \to cZ$	$< 0.24 \times 10^{-3}$
$t \to ch$	$< 1.6 \times 10^{-3}$

Our motivation to study the MUED model is to observe whether extra dimensions can

[1] The exact cancellation would happen if the masses of the quarks in the loop were taken to be degenerate.

account for the large difference between the SM computations and the experimental bounds on the decay widths of the rare top quark decays occurring by FCNCs. Similarly, rare single-top production processes will be studied to see if the MUED model can enhance the SM cross sections.

1.4 Outline of the thesis

This thesis is outlined as follows. In Chapter 2, we analyze the MUED model and, at the end, obtain the complete particle spectrum. After the detailed discussion of the model, we apply it to the following rare top quark processes:

$$t \to c\gamma, \tag{1.2}$$

$$t \to cg, \tag{1.3}$$

$$t \to ch, \tag{1.4}$$

$$t \to cZ, \tag{1.5}$$

$$t \to cgg, \tag{1.6}$$

and

$$gg \to t\bar{c}, \tag{1.7}$$

$$cg \to t\gamma, \tag{1.8}$$

$$cg \to tg, \tag{1.9}$$

$$cg \to th, \tag{1.10}$$

$$cg \to tZ. \tag{1.11}$$

The numerical results for the decay widths and the cross sections are presented in Chapter 3. Chapter 4 contains our conclusion. In Appendix A, complete Feynman diagrams for all the processes are given. The relevant Feynman rules are listed in Appendix B. We wrote an original Mathematica code before moving on to the numerical analysis. By using this code, we were able to quickly compare the vertex factors of the interactions of fermions with scalars and vectors to the ones in the literature. This code is displayed in Appendix C. Our own LanHEP code for the complete MUED model in the Feynman gauge is attached in Appendix D. Let us note that Lan-

HEP is a program designated to extract the complete set of Feynman rules from the Lagrangian-level input.

CHAPTER 2

MINIMAL UNIVERSAL EXTRA DIMENSIONS

In this chapter, we present the formal mathematical construction of the Minimal Universal Extra Dimensions (MUED) model. We start with the convention that we choose for the metric, covariant derivative, and Higgs doublet. Next, we construct the $5D$ cylindrical universe and promote the SM Lagrangian to this universe. As a toy model for the theoretical analysis, we consider the case of a free massive scalar field in $5D$, from which we show the fundamental step in assigning the correct boundary condition to each field in relation with the reflection symmetry introduced to remove the redundant degrees of freedom in the theory, and find out the mass quantization condition. By making analogies for the cases of the vectors and fermions, we obtain the Kaluza-Klein (KK) expansions of all fields and derive the selection rules for the available vertices. Finally, we move on to the extraction of the physical states in the mass basis, and summarize the theoretical results in order to prepare a LanHEP code for the numerical part.

2.1 Conventions

Our metric convention is *mostly minus*:

$$g_{\mu\nu} = (+, -, -, -). \tag{2.1}$$

For the covariant derivative, we take the sign of all the gauge couplings to be plus:

$$\mathscr{D}_\mu = \partial_\mu + ig_s\vec{T}_s \cdot \vec{G}_\mu + ig_w\vec{T}_w \cdot \vec{W}_\mu + ig_y T_y B_\mu \tag{2.2}$$

where

$$\vec{T}_s = \frac{1}{2}\vec{\lambda}, \quad \vec{T}_w = \frac{1}{2}\vec{\tau}, \quad T_y = \frac{1}{2}Y. \tag{2.3}$$

Here, the λ^a ($a = 1, 2, \ldots, 8$) are the Gell-Mann matrices, the τ^i ($i = 1, 2, 3$) are the Pauli matrices, and Y is the hypercharge. The relation among the electric charge, the third component of the weak isospin, and the hypercharge is given as usual by

$$Q = T_w^3 + T_y = \frac{1}{2}\left(\tau^3 + Y\right). \tag{2.4}$$

The $SU(2)$ Higgs doublet is taken to be

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} i(\phi_1 - i\phi_2) \\ h + v + i\phi_3 \end{pmatrix} = \begin{pmatrix} i\phi^+ \\ \frac{1}{\sqrt{2}}(h + v + i\phi_3) \end{pmatrix} \tag{2.5}$$

where $v = 246$ GeV is the Higgs vacuum expectation value (VEV). It should be remarked that there is a factor of i in the CP-odd scalar ϕ^+[1].

2.2 Construction of the $5D$ universe

To begin with, the $4D$ Lorentz indices (denoted by lowercase Greek letters) are promoted to the $5D$ ones (denoted by uppercase Latin letters):

$$\mu = 0, 1, 2, 3 \to M = \mu, 5 = 0, 1, 2, 3, 5. \tag{2.6}$$

With this, the position vector becomes

$$x^\mu \to x^M = (x^\mu, x^5) = (x^\mu, y) \tag{2.7}$$

and as for the derivative, we have

$$\partial_\mu \to \partial_M = (\partial_\mu, \partial_5) = \left(\partial_\mu, \frac{\partial}{\partial y}\right). \tag{2.8}$$

In the meantime, the Minkowski metric becomes

$$g_{\mu\nu} \to g_{MN} = (+, -, -, -, -). \tag{2.9}$$

Scalar and fermionic fields become functions of the usual 4-position, x for short, and the extra dimension, y:

$$\phi(x) \to \phi(x, y), \tag{2.10}$$

$$\psi(x) \to \psi(x, y). \tag{2.11}$$

[1] The reader displeased with this convention can always let $\phi^\pm \to \mp i\phi^\pm$ in what follows. This convention has been chosen on the sole ground that the derivation of the physical states and the coding in LanHEP will be facilitated.

Vector fields are also functions of x and y now; in addition, they obtain a new component:

$$V^{\mu}(x) \to V^{M}(x,y) = (V^{\mu}(x,y), V^{5}(x,y)). \tag{2.12}$$

V^5 is a new degree of freedom, which we treat as a new scalar field in the theory.

We take the extra dimension to be defined on an interval of length $2\pi R$, and then compactify it on a circle (Figure 2.1).

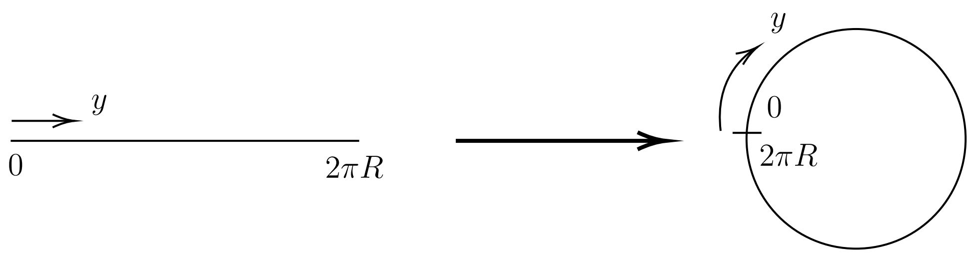

Figure 2.1: Compactification of the extra dimension on a circle of radius R.

This creates a $5D$ cylindrical universe. The usual $4D$ spacetime fills the inside of the cylinder, and the 5th dimension lies on the circle defined on the surface (Figure 2.2).

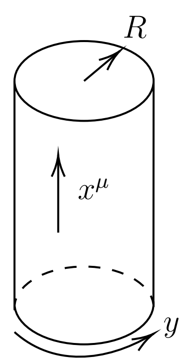

Figure 2.2: The $5D$ cylindrical universe.

2.3 Complete $5D$ Lagrangian of MUED

Embedding the SM Lagrangian into five dimensions is straightforward. We write all the sectors,

$$\mathscr{L} = \mathscr{L}_{\text{gauge}} + \mathscr{L}_{\text{gauge-fixing}} + \mathscr{L}_{\text{higgs}} + \mathscr{L}_{\text{fermion}} + \mathscr{L}_{\text{yukawa}} + \mathscr{L}_{\text{ghost}} \tag{2.13}$$

and the fields are allowed to extend to the extra dimension, as well. This allows us to write

$$\mathscr{L}_{\text{gauge}} = \sum_{a=1}^{8} -\frac{1}{4}(G^a_{MN})^2 + \sum_{i=1}^{3} -\frac{1}{4}(W^i_{MN})^2 - \frac{1}{4}(B_{MN})^2, \tag{2.14}$$

$$\mathscr{L}_{\text{higgs}} = |\mathscr{D}_M H|^2 + \mu_5^2 |H|^2 - \lambda_5 |H|^4 , \tag{2.15}$$

$$\mathscr{L}_{\text{fermion}} = \sum_{f=Q,U,D} \bar{f} i \Gamma^M \mathscr{D}_M f, \tag{2.16}$$

$$\mathscr{L}_{\text{yukawa}} = - y_{u5} \bar{Q} U \tilde{H} - y_{d5} \bar{Q} D H + \text{h.c.} \tag{2.17}$$

and the gauge-fixing terms in the Feynman-'t Hooft gauge will be determined after we obtain the mass eigenstates. The ghosts (or the Faddeev-Popov particles) are irrelevant at the moment. Here, the $5D$ field strength tensors are given by

$$G^a_{MN} = \partial_M G^a_N - \partial_N G^a_M - g_{s5} f^{abc} G^b_M G^c_N, \tag{2.18}$$

$$W^i_{MN} = \partial_M W^i_N - \partial_N W^i_M - g_{w5} \epsilon^{ijk} W^j_M W^k_N, \tag{2.19}$$

$$B_{MN} = \partial_M B_N - \partial_N B_M. \tag{2.20}$$

The $5D$ covariant derivative is taken to be

$$\mathscr{D}_M = \partial_M + i g_{s5} \vec{T}_s \cdot \vec{G}_M + i g_{w5} \vec{T}_w \cdot \vec{W}_M + i g_{y5} T_y B_M \tag{2.21}$$

where g_{s5}, g_{w5}, and g_{y5} denote the $5D$ couplings of the gauge bosons of the strong interactions, weak interactions, and the hypercharge, respectively. The $5D$ $SU(2)$ Higgs doublet is given as

$$H = \begin{pmatrix} i\phi^+ \\ \frac{1}{\sqrt{2}}(h + v_5 + i\phi_3) \end{pmatrix} \tag{2.22}$$

where v_5 is the $5D$ Higgs VEV.

We choose the following convention for the $5D$ Dirac matrices:

$$\Gamma^M = (\gamma^\mu, i\gamma_5) \tag{2.23}$$

which obey the usual anticommutation rule:

$$\{\Gamma^M, \Gamma^N\} = 2g^{MN}. \tag{2.24}$$

The fermionic states Q, U, and D refer to the left-handed $SU(2)$ quark doublet, the right-handed up-type singlet, and the right-handed down-type singlet, respectively. In this work, the leptonic sector is irrelevant to us and it is not going to be explored further.

2.4 Bulk equation of motion for a scalar

Consider the case of a free scalar field, ϕ, of mass m_ϕ. The $5D$ Klein-Gordon (KG) Lagrangian reads

$$\mathscr{L} = \frac{1}{2}(\partial_M \phi)^2 - \frac{1}{2} m_\phi^2 \phi^2. \tag{2.25}$$

The $5D$ Euler-Lagrange equation of motion becomes

$$\partial_M \frac{\partial \mathscr{L}}{\partial(\partial_M \phi)} - \frac{\partial \mathscr{L}}{\partial \phi} = 0. \tag{2.26}$$

Taking the necessary derivatives, we get

$$(⬠ + m_\phi^2)\phi = 0 \tag{2.27}$$

where we define a new operator

$$⬠ := \partial_M \partial^M = \partial_\mu \partial^\mu + \partial_5 \partial^5 = \Box - \partial_5^2 \tag{2.28}$$

It is essential to remark that all the explicit 5$^{\text{th}}$ components are covariant, unless written as contravariant.

Since the $5D$ Lagrangian will be integrated over the extra dimension, it is beneficial to perform a separation of variables between the usual $4D$ coordinates and the extra dimension, y:

$$\phi(x, y) = \sum_{n=0}^{\infty} \phi_n(x) f_n(y). \tag{2.29}$$

Here, the $f_n(y)$ are the so-called mode functions and the $\phi_n(x)$ are the scalar fields. It is assumed that the scalar fields satisfy the $4D$ KG equation:

$$\Box \phi_n = -M_n^2 \phi_n \tag{2.30}$$

where M_n is the physical mass of the state ϕ_n. If we substitute the $5D$ scalar in Equation (2.29) into the equation of motion (2.27), we get

$$\sum_{n=0}^{\infty} -\phi_n \left[f_n'' + (M_n^2 - m_\phi^2) f_n \right] = 0. \tag{2.31}$$

Let

$$m_n^2 := M_n^2 - m_\phi^2. \tag{2.32}$$

We assume that $m_n^2 > 0$ for all positive integers n[2]. Thus, the mode functions that solve Equation (2.31) are given by

$$f_n(y) = A_n \sin m_n y + B_n \cos m_n y \tag{2.33}$$

for all n. With this, the $5D$ scalar field becomes

$$\phi(x, y) = \sum_{n=0}^{\infty} \phi_n (A_n \sin m_n y + B_n \cos m_n y). \tag{2.34}$$

In this summation, the zeroth term, $\phi_0(x)$, will denote the SM mode of the $5D$ scalar field. The rest of the components are simply the KK partners of the SM scalar ϕ_0. However, there is a caveat here: If ϕ is the 5$^{\text{th}}$ component of a vector field, then the cosine tower is extra since there will be a zero mode, which is absent in SM. Therefore, there are redundant degrees of freedom here, which we need to remove systematically.

2.5 The Z_2 symmetry

To eliminate the undesired degrees of freedom, we introduce the Z_2 symmetry on the extra dimension. The reflection symmetry is imposed on the circle such that the components of the fields that lie on the lower part of the circle cannot independently exist from those that lie on the upper part. This is called the S^1/Z_2 orbifolding [28]. We effectively lose the half of the domain at the benefit of eliminating superfluous degrees of freedom (Figure 2.3).

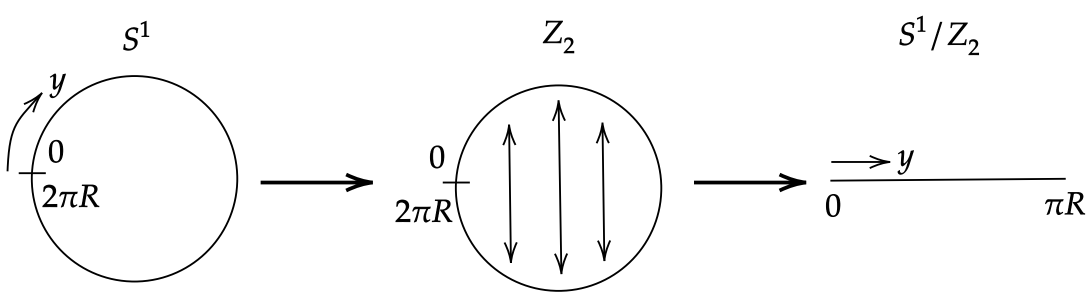

Figure 2.3: The Z_2 symmetry imposed on the circle.

[2] Actually, the hyperbolic solutions, governed by the condition $m_n^2 < 0$, are naturally eliminated for the mass quantization yields the trivial solution, *ergo* the KK partners are decoupled from the theory.

This is equivalent to saying that the fields should be either even or odd under the transformation $y \to -y$:

$$\phi(-y) = \pm\phi(-y). \tag{2.35}$$

One way to achieve this on mathematical grounds is to impose either the Dirichlet boundary condition,

$$\phi\Big|_{y=0,\pi R} = 0 \tag{2.36}$$

or the Neumann one,

$$\partial_5\phi\Big|_{y=0,\pi R} = 0. \tag{2.37}$$

We choose to implement Dirichlet condition to the *new* fields (such as the 5$^{\text{th}}$ component of vectors and the right-handed components of left-handed spinors) so that they will not receive the zeroth mode, hence the SM spectrum will not be spoiled. We define a *Dirichlet* (*Neumann*) *field* to be one that satisfies the Dirichlet (Neumann) condition at the boundaries.

2.6 Mass quantization

After we correctly assign the type of the field (*Dirichlet* or *Neumann*), one boundary condition removes the undesired degrees of freedom and the other determines the mass quantization.

Suppose we have a Dirichlet field. The $5D$ scalar field ϕ given by

$$\phi(x,y) = \sum_{n=0}^{\infty} \phi_n(x)(A_n \sin m_n y + B_n \cos m_n y) \tag{2.38}$$

will vanish at $y = 0$:

$$\phi(x,0) = \sum_{n=0}^{\infty} \phi_n(x) B_n = 0. \tag{2.39}$$

This leaves us with

$$B_n = 0 \quad \forall n \in \mathbb{N}. \tag{2.40}$$

At the other boundary, we have

$$\phi(x,\pi R) = \sum_{n=0}^{\infty} \phi_n(x) A_n \sin m_n \pi R. \tag{2.41}$$

Now the mode function should vanish:

$$\sin m_n \pi R = 0 \tag{2.42}$$

so that

$$m_n \pi R = n\pi, \quad n \in \mathbb{N}^+ \tag{2.43}$$

where we ignore the case $n = 0$ to avoid the trivial solution. Therefore, the mass is quantized as integer multiples of $1/R$:

$$m_n = \frac{n}{R}. \tag{2.44}$$

The $5D$ scalar field ϕ is thus given by

$$\phi(x, y) = \sum_{n=1}^{\infty} A_n \phi_n(x) \sin \frac{ny}{R}. \tag{2.45}$$

To fix the normalization constants A_n, we choose to canonically normalize the kinetic term. Starting from the $5D$ action, we have

$$\begin{aligned} S &\supset \int d^5x \, \frac{1}{2}(\partial_M \phi)^2 \\ &\supset \int d^4x \int_0^{\pi R} dy \, \frac{1}{2} \left(\sum_{n=1}^{\infty} \partial_\mu \phi_n A_n \sin \frac{ny}{R} \right) \left(\sum_{m=1}^{\infty} \partial^\mu \phi_m A_m \sin \frac{my}{R} \right) \\ &\supset \int d^4x \int_0^{\pi R} dy \, \frac{1}{2} \left(\sum_{n,m=1}^{\infty} A_n A_m \sin \frac{ny}{R} \sin \frac{my}{R} \right) \partial_\mu \phi_n \partial^\mu \phi_m \\ &\supset \int d^4x \, \frac{1}{2} \sum_{n=1}^{\infty} (\partial_\mu \phi_n)^2. \end{aligned} \tag{2.46}$$

The normalization constants should satisfy

$$A_n A_m \int_0^{\pi R} dy \, \sin \frac{ny}{R} \sin \frac{my}{R} = \delta_{nm}. \tag{2.47}$$

Since the integral in Equation (2.47) gives $\delta_{nm} \pi R/2$, we obtain

$$A_n = \sqrt{\frac{2}{\pi R}}. \tag{2.48}$$

Therefore, the expansion for a Dirichlet scalar field turns out to be

$$\phi(x, y) = \sum_{n=1}^{\infty} \sqrt{\frac{2}{\pi R}} \phi_n(x) \sin \frac{ny}{R}. \tag{2.49}$$

This expansion is also known as the KK decomposition (or KK tower) for a Dirichlet scalar field. Accordingly, the summation index, n, is called the KK number.

Now suppose we have a Neumann field. The y derivative of the scalar field given in Equation (2.29) vanishes at the point $y = 0$:

$$\begin{aligned}\partial_5\phi\Big|_{y=0} &= \sum_{n=0}^{\infty} \phi_n(x) m_n (A_n \cos m_n y - B_n \sin m_n y)\Big|_{y=0} \\ &= \sum_{n=0}^{\infty} m_n \phi_n(x) A_n \\ &= 0. \end{aligned} \tag{2.50}$$

This produces

$$A_n = 0 \quad \forall n \in \mathbb{N}. \tag{2.51}$$

The vanishing derivative at $y = \pi R$ gives the same mass quantization as for the Dirichlet scalar field:

$$\partial_5\phi\Big|_{y=\pi R} = \sum_{n=0}^{\infty} -m_n \phi_n B_n \sin m_n \pi R = 0 \tag{2.52}$$

so that

$$m_n \pi R = n\pi, \quad n \in \mathbb{N} \tag{2.53}$$

and hence

$$m_n = \frac{n}{R}. \tag{2.54}$$

The KK tower before the normalization now reads

$$\phi(x,y) = \sum_{n=0}^{\infty} B_n \phi_n(x) \cos\frac{ny}{R} = B_0\phi_0(x) + \sum_{n=1}^{\infty} B_n \phi_n(x) \cos\frac{ny}{R}. \tag{2.55}$$

Again, we fix the normalization constants B_n such that the physical states ϕ_n have the canonically normalized kinetic terms:

$$\begin{aligned}
S &\supset \int d^5x \, \frac{1}{2}(\partial_M \phi)^2 \\
&\supset \int d^4x \int_0^{\pi R} dy \, \frac{1}{2} \left(B_0 \partial_\mu \phi_0 + \sum_{n=1}^{\infty} B_n \partial_\mu \phi_n \cos\frac{ny}{R} \right) \\
&\qquad\qquad\qquad \times \left(B_0 \partial_\mu \phi_0 + \sum_{m=1}^{\infty} B_m \partial_\mu \phi_m \cos\frac{my}{R} \right) \\
&\supset \int d^4x \int_0^{\pi R} dy \, \frac{1}{2} \Big[B_0^2 (\partial_\mu \phi_0)^2 + \sum_{n,m=1}^{\infty} B_n B_m \cos\frac{ny}{R} \cos\frac{my}{R} \partial_\mu \phi_n \partial^\mu \phi_m \\
&\qquad\qquad\qquad + \sum_{n=1}^{\infty} B_0 B_n \partial_\mu \phi_0 \partial^\mu \phi_n \cos\frac{ny}{R} \Big] \\
&\supset \int d^4x \, \left[\frac{1}{2}(\partial_\mu \phi_0)^2 + \sum_{n=1}^{\infty} \frac{1}{2}(\partial_\mu \phi_n)^2 \right].
\end{aligned} \tag{2.56}$$

For the SM mode, we get

$$B_0^2 \int_0^{\pi R} dy = 1 \tag{2.57}$$

and hence

$$B_0 = \frac{1}{\sqrt{\pi R}}. \tag{2.58}$$

For the KK modes, we have

$$B_n B_m \int_0^{\pi R} dy \cos\frac{ny}{R} \cos\frac{my}{R} = \delta_{nm}. \tag{2.59}$$

Since the integral in Equation (2.59) yields $\delta_{nm}\pi R/2$, we obtain

$$B_n = \sqrt{\frac{2}{\pi R}}. \tag{2.60}$$

Therefore, the KK decomposition for a Neumann scalar is given by

$$\phi(x,y) = \frac{1}{\sqrt{\pi R}}\phi_0(x) + \sum_{n=1}^{\infty} \sqrt{\frac{2}{\pi R}}\phi_n(x) \cos\frac{ny}{R}. \tag{2.61}$$

2.7 KK decomposition for the vector and fermion fields

2.7.1 The vector case

A similar discussion as to scalar fields holds for vector ones. It is crucial to note that there is no new vector field coming directly from the extra dimension — all the vectors are Neumann fields.

Consider a massive vector field, V^μ, of mass m_V in the Feynman gauge. The $5D$ massive Proca Lagrangian reads

$$\mathscr{L} = -\frac{1}{4}(F_{MN})^2 + \frac{1}{2}m_V^2 V_M^2 - \frac{1}{2}(\partial_M V^M)^2. \tag{2.62}$$

By using the antisymmetric property of the field strength tensor, we have

$$(F_{MN})^2 = (F_{\mu\nu})^2 - 2(F_{\mu 5})^2 \supset F_{\mu\nu}^2 - 2(\partial_5 V_\mu)^2. \tag{2.63}$$

Meanwhile, the mass term contains

$$V_M^2 = V_\mu^2 - V_5^2 \tag{2.64}$$

and the gauge-fixing term includes

$$(\partial_M V^M)^2 = (\partial_\mu V^\mu - \partial_5 V_5)^2 \supset (\partial_\mu V^\mu)^2. \tag{2.65}$$

Now we can safely focus on the vector field V^μ. The free Lagrangian reads

$$\mathscr{L} \supset -\frac{1}{4}(F_{\mu\nu})^2 + \frac{1}{2}(\partial_5 V_\mu)^2 + \frac{1}{2}m^2 V_\mu^2 - \frac{1}{2}(\partial_\mu V^\mu)^2. \tag{2.66}$$

The $5D$ Euler-Lagrange equation of motion for a vector field is given by

$$\partial_M \frac{\partial \mathscr{L}}{\partial(\partial_M V_\alpha)} - \frac{\partial \mathscr{L}}{\partial V_\alpha} = 0, \quad \alpha = 0, 1, 2, 3 \tag{2.67}$$

or, more explicitly,

$$\partial_\beta \frac{\partial \mathscr{L}}{\partial(\partial_\beta V_\alpha)} + \partial_5 \frac{\partial \mathscr{L}}{\partial(\partial_5 V_\alpha)} - \frac{\partial \mathscr{L}}{\partial V_\alpha} = 0. \tag{2.68}$$

Taking the considered derivatives, we obtain

$$-\partial_\beta F^{\beta\alpha} - \partial_\beta g^{\beta\alpha} \partial \cdot V + \partial_5^2 V^\alpha - m^2 V^\alpha = 0 \tag{2.69}$$

or

$$-\Box V^\alpha + \partial^\alpha \partial \cdot V - \partial^\alpha \partial \cdot V + \partial_5^2 V^\alpha - m^2 V^\alpha = 0 \tag{2.70}$$

and hence

$$(\pentagon + m^2) V^\alpha = 0. \tag{2.71}$$

This is the same equation of motion as for the scalar field ϕ; *ergo*, it is not surprising that the same KK decomposition,

$$V^\alpha(x, y) = \frac{1}{\sqrt{\pi R}} V_0^\alpha(x) + \sum_{n=1}^{\infty} \sqrt{\frac{2}{\pi R}} V_n^\alpha(x) \cos\frac{ny}{R}. \tag{2.72}$$

will hold true.

2.7.2 The fermion case

Fermions have a different story for the boundary conditions. To understand this better, we need to re-interpret the reflection symmetry. By changing the origin of the extra dimension, we can re-express the reflection symmetry as if we make the transformation $y \to -y$.

$5D$ fields under the Z_2 symmetry should be either even or odd:

$$\phi(-y) = \pm\phi(y). \tag{2.73}$$

However, for the fermions, we have a chirality issue[3]. This is reflected by considering the following condition [29, 30]:

$$\psi(-y) = \pm\gamma_5\psi(y) \tag{2.74}$$

where we assign the plus (minus) sign to the right- (left-)handed spinor. This is equivalent to imposing the Dirichlet or Neumann boundary conditions at $y = 0$ and $y = \pi R$. Now, all the fields are periodic on the extra dimension, so we can write the fermions in towers of sines and cosines, as well, with the same mass quantization.

Here comes the tricky part. By allowing all possible operators, we may write

$$\psi_R(x,y) = \sum_{n=0}^{\infty}\Big[P_R(A_n \sin m_n y + B_n \cos m_n y) \\ + P_L(C_n \sin m_n y + D_n \cos m_n y)\Big]\psi_{Rn}, \tag{2.75}$$

$$\psi_L(x,y) = \sum_{n=0}^{\infty}\Big[P_R(A'_n \sin m_n y + B'_n \cos m_n y) \\ + P_L(C'_n \sin m_n y + D'_n \cos m_n y)\Big]\psi_{Ln} \tag{2.76}$$

where

$$P_{R/L} := \frac{1 \pm \gamma_5}{2} \tag{2.77}$$

are the right/left-handed projection operators. We see that each left- or right-handed spinor is allowed to have left- and right-handed partners. However, to successfully

[3] The kinetic term of a fermion field contains a single derivative, unlike a boson field, and hence we may not directly impose the aforementioned boundary conditions. Nevertheless, there appears a term $\partial_5(\bar{\psi}\gamma_5\psi)$ in the $5D$ Lagrangian, and this is the most suitable candidate term to impose a reflection condition on.

extract the chiral SM modes, we need to remove at least one of the cosine towers modulated by the projection operators. That is, the left- (right-)handed cosine tower for the right- (left-)handed spinor is a redundant degree of freedom. Now,

$$\begin{aligned}\gamma_5\psi_R(x,y) = \sum_{n=0}^{\infty} \Big[& P_R(A_n \sin m_n y + B_n \cos m_n y) \\ & - P_L(C_n \sin m_n y + D_n \cos m_n y)\Big]\psi_{Rn}.\end{aligned} \tag{2.78}$$

In the meantime,

$$\begin{aligned}\psi_R(x,-y) = \sum_{n=0}^{\infty} \Big[& P_R(-A_n \sin m_n y + B_n \cos m_n y) \\ & + P_L(-C_n \sin m_n y + D_n \cos m_n y)\Big]\psi_{Rn}.\end{aligned} \tag{2.79}$$

Comparing Equations (2.78) and (2.79), we obtain

$$A_n = -A_n, \quad B_n = B_n, \quad -C_n = -C_n, \quad -D_n = D_n \tag{2.80}$$

and this leaves us with

$$A_n = D_n = 0 \quad \forall n \in \mathbb{N}. \tag{2.81}$$

Thus, for the non-normalized right-handed spinor, we get

$$\psi_R(x,y) = \sum_{n=0}^{\infty} \left(B_n P_R \cos m_n y + C_n P_L \sin m_n y\right)\psi_{Rn}. \tag{2.82}$$

After the normalization, this becomes

$$\psi_R(x,y) = \frac{1}{\sqrt{\pi R}} P_R \psi_{R0}(x) + \sum_{n=1}^{\infty} \sqrt{\frac{2}{\pi R}} \left(P_R \cos\frac{ny}{R} + P_L \sin\frac{ny}{R}\right)\psi_{Rn}. \tag{2.83}$$

Similarly for the left-handed spinor, we have

$$\begin{aligned}\gamma_5\psi_L(x,y) = \sum_{n=0}^{\infty} \Big[& P_R(A'_n \sin m_n y + B'_n \cos m_n y) \\ & - P_L(C'_n \sin m_n y + D'_n \cos m_n y)\Big]\psi_{Ln}\end{aligned} \tag{2.84}$$

and

$$\begin{aligned}\psi_L(x,-y) = \sum_{n=0}^{\infty} \Big[& P_R(-A'_n \sin m_n y + B'_n \cos m_n y) \\ & + P_L(-C'_n \sin m_n y + D'_n \cos m_n y)\Big]\psi_{Ln}.\end{aligned} \tag{2.85}$$

Imposing the condition

$$\psi_L(x,-y) = -\gamma_5 \psi_L(x,y), \tag{2.86}$$

we obtain

$$-A'_n = -A'_n, \quad -B'_n = B'_n, \quad C'_n = -C'_n, \quad D'_n = D'_n. \tag{2.87}$$

This gives

$$B'_n = C'_n = 0 \quad \forall n \in \mathbb{N}. \tag{2.88}$$

Hence, the non-normalized KK decomposition for the left-handed spinor reads

$$\psi_L(x,y) = \sum_{n=0}^{\infty} \left(A'_n P_R \sin m_n y + D'_n P_L \cos m_n y\right) \psi_{Ln}. \tag{2.89}$$

After the normalization, this becomes

$$\psi_L(x,y) = \frac{1}{\sqrt{\pi R}} P_L \psi_{L0}(x) + \sum_{n=1}^{\infty} \sqrt{\frac{2}{\pi R}} \left(P_L \cos \frac{ny}{R} + P_R \sin \frac{ny}{R}\right) \psi_{Ln}. \tag{2.90}$$

2.8 Integration limits

After the S^1/Z_2 orbifolding, the domain of the extra dimension becomes $y \in [0, \pi R]$. However, we can still make use of the complete (symmetric) domain.

In the Lagrangian, at the quadratic level, one Neumann field always meets a Neumann field (similarly for a Dirichlet field). For fermions, the left-handed component always goes to another left-handed component. Consider the mass term for a fermion:

$$\begin{aligned}
\mathscr{L} \supset & -m\bar{\psi}_L \psi_R + \text{h.c.} \\
\supset & -m \left[\frac{1}{\sqrt{\pi R}} \bar{\psi}_{L0} + \sum_{n\geq 1} \sqrt{\frac{2}{\pi R}} \bar{\psi}_{Ln} \left(P_R \cos \frac{ny}{R} + P_L \sin \frac{ny}{R}\right)\right] \\
& \times \left[\frac{1}{\sqrt{\pi R}} \psi_{R0} + \sum_{m\geq 1} \sqrt{\frac{2}{\pi R}} \left(P_R \cos \frac{my}{R} + P_L \sin \frac{my}{R}\right) \psi_{Rm}\right] \\
\supset & -m \sum_{nm\geq 1} \frac{2}{\pi R} \left(\bar{\psi}_{Ln} P_L \psi_{Rm} \cos \frac{ny}{R} \cos \frac{my}{R} + \bar{\psi}_{Ln} P_R \psi_{Rn} \sin \frac{ny}{R} \sin \frac{my}{R}\right) \\
& + \text{h.c.}
\end{aligned} \tag{2.91}$$

Therefore, we still need the orthogonality of sines and cosines over the half interval. In order to make this more powerful, we can simply switch to the complete domain, but taking the half of the integral[4]:

$$\int_0^{\pi R} dy \to \frac{1}{2}\int_{-\pi R}^{\pi R} dy. \tag{2.92}$$

In essence, this transformation is possible due to the Z_2 symmetry imposed on the fields, with the fields being completely symmetric or antisymmetric with respect to the transformation $y \to -y$.

2.9 Selection rules

As one can show, using the complete domain has its merits. It is easier to deal with the integration of various products of sines and cosines.

This brings us to the selection rules. In the $5D$ Lagrangian, all the fields are expanded in series of sines and cosines. After integrating out the extra dimension, we get effective selection rules for possible vertices. All the rules (from 2- to 4- point interactions)

[4] The factor $1/2$ is, of course, necessary not to overcount.

are present according to the result of the following integrals:

$$I_{nm} = \frac{1}{2}\int_{-\pi R}^{\pi R} dy\ \cos\frac{ny}{R}\cos\frac{my}{R}, \tag{2.93}$$

$$I'_{nm} = \frac{1}{2}\int_{-\pi R}^{\pi R} dy\ \cos\frac{ny}{R}\sin\frac{my}{R}, \tag{2.94}$$

$$I''_{nm} = \frac{1}{2}\int_{-\pi R}^{\pi R} dy\ \sin\frac{ny}{R}\sin\frac{my}{R}, \tag{2.95}$$

$$I_{nmk} = \frac{1}{2}\int_{-\pi R}^{\pi R} dy\ \cos\frac{ny}{R}\cos\frac{my}{R}\cos\frac{ky}{R}, \tag{2.96}$$

$$I'_{nmk} = \frac{1}{2}\int_{-\pi R}^{\pi R} dy\ \cos\frac{ny}{R}\cos\frac{my}{R}\sin\frac{ky}{R}, \tag{2.97}$$

$$I''_{nmk} = \frac{1}{2}\int_{-\pi R}^{\pi R} dy\ \cos\frac{ny}{R}\sin\frac{my}{R}\sin\frac{ky}{R}, \tag{2.98}$$

$$I'''_{nmk} = \frac{1}{2}\int_{-\pi R}^{\pi R} dy\ \sin\frac{ny}{R}\sin\frac{my}{R}\sin\frac{ky}{R}, \tag{2.99}$$

$$I_{nmk\ell} = \frac{1}{2}\int_{-\pi R}^{\pi R} dy\ \cos\frac{ny}{R}\cos\frac{my}{R}\cos\frac{ky}{R}\cos\frac{\ell y}{R}, \tag{2.100}$$

$$I'_{nmk\ell} = \frac{1}{2}\int_{-\pi R}^{\pi R} dy\ \cos\frac{ny}{R}\cos\frac{my}{R}\cos\frac{ky}{R}\sin\frac{\ell y}{R}, \tag{2.101}$$

$$I''_{nmk\ell} = \frac{1}{2}\int_{-\pi R}^{\pi R} dy\ \cos\frac{ny}{R}\cos\frac{my}{R}\sin\frac{ky}{R}\sin\frac{\ell y}{R}, \tag{2.102}$$

$$I'''_{nmk\ell} = \frac{1}{2}\int_{-\pi R}^{\pi R} dy\ \cos\frac{ny}{R}\sin\frac{my}{R}\sin\frac{ky}{R}\sin\frac{\ell y}{R}, \tag{2.103}$$

$$I''''_{nmk\ell} = \frac{1}{2}\int_{-\pi R}^{\pi R} dy\ \sin\frac{ny}{R}\sin\frac{my}{R}\sin\frac{ky}{R}\sin\frac{\ell y}{R}. \tag{2.104}$$

Here, the number of indices denotes the number of legs at a vertex, and the number of primes indicates the number of sine factors in the integral. All the integrals with an odd number of primes will vanish due to the symmetry:

$$I'_{nm} = 0, \tag{2.105}$$

$$I'_{nmk} = 0, \tag{2.106}$$

$$I'''_{nmk} = 0, \tag{2.107}$$

$$I'_{nmk\ell} = 0, \tag{2.108}$$

$$I'''_{nmk\ell} = 0. \tag{2.109}$$

The nontrivial integrals exist under certain rules applied on the KK numbers involved:

$$I_{nm} = \frac{1}{2}\pi R\delta(m-n), \tag{2.110}$$

$$I''_{nm} = \frac{1}{2}\pi R\delta(m-n), \tag{2.111}$$

$$I_{nmk} = \frac{1}{4}\pi R[\delta(k-m-n)+\delta(k+m-n)+\delta(k-m+n)], \tag{2.112}$$

$$I''_{nmk} = \frac{1}{4}\pi R[\delta(k-m-n)-\delta(k+m-n)+\delta(k-m+n)], \tag{2.113}$$

$$\begin{aligned} I_{nmk\ell} = \frac{1}{8}\pi R[&\delta(k-\ell-m-n)+\delta(k+\ell-m-n)+\delta(k-\ell+m-n) \\ &+\delta(k+\ell+m-n)+\delta(k-\ell-m+n)+\delta(k+\ell-m+n) \\ &+\delta(k-\ell+m+n)], \end{aligned} \tag{2.114}$$

$$\begin{aligned} I''_{nmk\ell} = \frac{1}{8}\pi R[&\delta(k-\ell-m-n)-\delta(k+\ell-m-n)+\delta(k-\ell+m-n) \\ &-\delta(k+\ell+m-n)+\delta(k-\ell-m+n)-\delta(k+\ell-m+n) \\ &+\delta(k-\ell+m+n)], \end{aligned} \tag{2.115}$$

$$\begin{aligned} I''''_{nmk\ell} = -\frac{1}{8}\pi R[&\delta(k-\ell-m-n)-\delta(k+\ell-m-n)-\delta(k-\ell+m-n) \\ &+\delta(k+\ell+m-n)-\delta(k-\ell-m+n)+\delta(k+\ell-m+n) \\ &+\delta(k-\ell+m+n)]. \end{aligned} \tag{2.116}$$

The integrals (2.110) and (2.111) vanish unless $n-m=0$, which implies that the 2-point vertices, namely the kinetic or mass terms or the vector-scalar mixings, respect the KK number conservation (Figure 2.4).

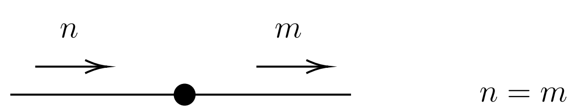

Figure 2.4: KK numbers at a 2-point vertex, which regularly represents a mass term or a derivative (vector-scalar) mixing, are conserved.

Similarly, the integrals (2.112) and (2.113) give zero unless $n \pm m \pm k = 0$ (except for the *all plus* combination). Consequently, 3-point interactions exist if the sum of the KK numbers of the incoming legs is equal to that of the outgoing lines (Figure 2.5).

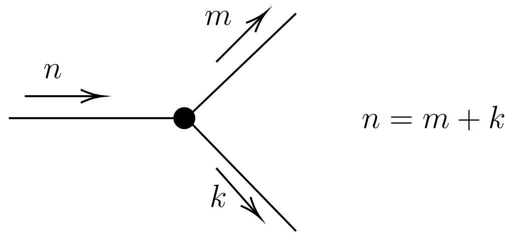

Figure 2.5: The sum of KK numbers incoming to a 3-point interaction is equal to the sum of KK numbers outgoing from the vertex.

Finally, the integrals (2.114) to (2.116) are nontrivial if $n \pm m \pm k \pm \ell = 0$ (except for the *all plus* case). The situation is pictured in Figure 2.6.

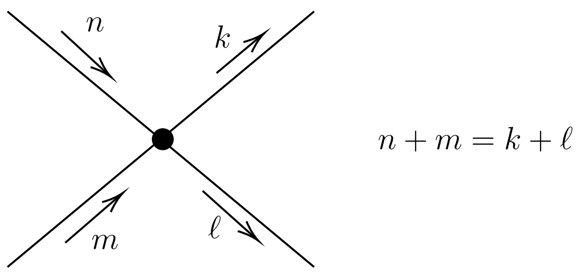

Figure 2.6: The sum of KK numbers incoming to a 4-point interaction is equal to the sum of KK numbers outgoing from the vertex.

These diagrams denote the conservation of KK number,

$$\sum_{\text{in}} n = \sum_{\text{out}} n \tag{2.117}$$

which is a result of the conservation of the 5$^{\text{th}}$ component of the momentum. We quantize the momentum on a circle, so it can be written in integer multiples of some constant (indeed $1/R$); since there are no boundary-localized terms, there is a translational symmetry, *ergo* the 5$^{\text{th}}$ component of the momentum is conserved — just like the usual Feynman rules: We write for any vertex some interaction coupling times $\delta^4(\sum_{\text{in}} k^\mu - \sum_{\text{out}} k^\mu)$, and following this spirit, we write selection rules by using Dirac delta functions over the domain of integers.

2.10 KK parity

In MUED, there is an accidental symmetry. Let τ denote the translation along the extra dimension:

$$\tau : y \to y + \pi R. \tag{2.118}$$

Under this operation, all the mode functions receive a global phase of $(-1)^n$:

$$\sin \frac{ny}{R} \to (-1)^n \sin \frac{ny}{R}, \tag{2.119}$$

$$\cos \frac{ny}{R} \to (-1)^n \cos \frac{ny}{R}. \tag{2.120}$$

Here, $\lambda_n := (-1)^n$ is the eigenvalue of the translation operator and called the KK parity of the n^{th} KK excitation.

If we expand the fields in the $5D$ Lagrangian in KK towers, and if we impose the condition that the $5D$ Lagrangian should be invariant under τ, we see that, in addition to the conservation of the KK number, the KK parity is conserved (Figure 2.7).

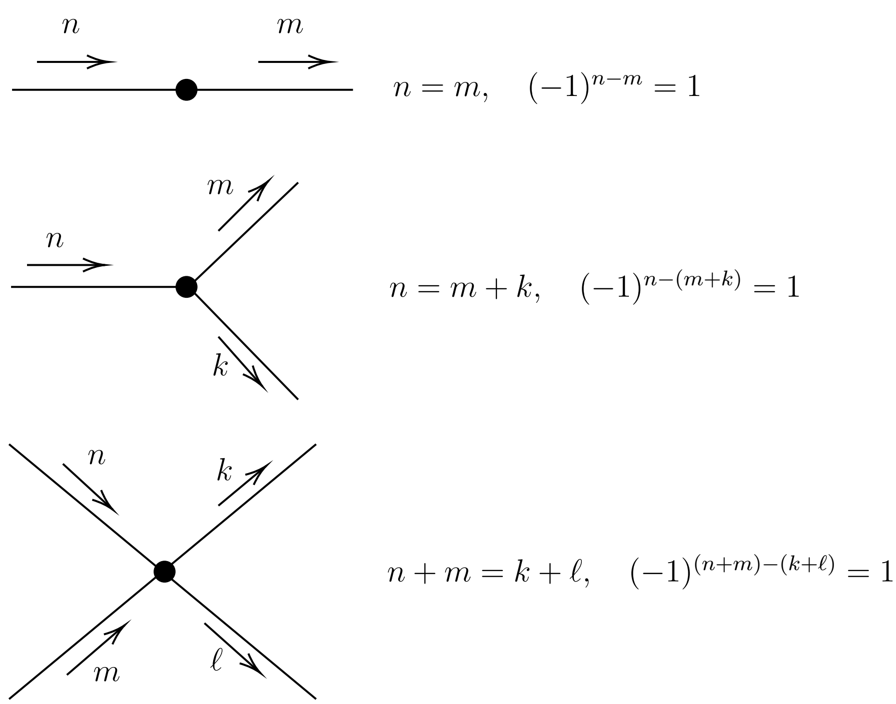

Figure 2.7: The conservation of KK parity is in natural agreement with the conservation of KK number.

Note that the conservation of the KK parity does not bring anything new on top of the conservation of KK number — but it is simply consistent with it. In NMUED, due to boundary-localized terms, we lose the conservation of the KK number, yet the KK parity remains a good quantum number in a disguise.

2.11 MUED as an effective field theory

In a field theory, if the couplings depend on energy, then the theory is simply non-renormalizable and we need to introduce a cut-off scale, Λ. In MUED, the relation between the cut-off energy, the size of the extra dimension, and the maximum KK number to include is given by

$$\Lambda R = n_{\text{max}}. \tag{2.121}$$

From a perturbative point of view and a study of vacuum stability on the mass of the Higgs scalar [31, 32], we can fix

$$n_{\text{max}} = 6. \tag{2.122}$$

This leaves MUED with only one free parameter, namely the size of the extra dimension, R.

Consider the fermion sector, interacting with a $U(1)$ gauge boson:

$$\begin{aligned}\mathscr{L}_{\text{fermion}} &= \sum_f \bar{f} i \Gamma^M \mathscr{D}_M f \\ &\supset \bar{\psi}_L i\gamma^\mu (\partial_\mu + i g_5 A_\mu)\psi_L \\ &\supset \left(\frac{1}{\sqrt{\pi R}}\bar{\psi}_{L0}\right) i\gamma^\mu \left(i g_5 \frac{1}{\sqrt{\pi R}} A_{0\mu}\right)\left(\frac{1}{\sqrt{\pi R}}\psi_{L0}\right).\end{aligned} \tag{2.123}$$

When integrated over the extra dimension, Equation (2.123) gives

$$\begin{aligned}\frac{1}{2}\int_{-\pi R}^{\pi R} dy\, \mathscr{L}_{\text{fermion}} \supset& \frac{1}{\sqrt{\pi R}} g_5 \bar{\psi}_{L0}\gamma^\mu A_{0\mu}\psi_{L0} \\ \supset& - g\bar{\psi}_{L0}\gamma^\mu A_{0\mu}\psi_{L0}.\end{aligned} \tag{2.124}$$

Matching the $5D$ gauge coupling to its $4D$ counterpart, we see

$$g = \frac{g_5}{\sqrt{\pi R}} \tag{2.125}$$

and *ergo* conclude that the $5D$ gauge couplings are dimensionful, hence the theory is non-renormalizable.

Now let us find the relation between v and v_5. Consider the Higgs kinetic term in a $U(1)$-only field theory. The complex scalar field is given by

$$H = \frac{1}{\sqrt{2}}(h + v_5 + i\phi_3). \tag{2.126}$$

For the consistency of the MUED model, only the SM mode of the $5D$ Higgs field should receive a VEV. Thus, it would be more appropriate if we had written Equation (2.126) as

$$H = \frac{1}{\sqrt{2}}(h + i\phi_3) \tag{2.127}$$

where

$$h = \left(\frac{1}{\sqrt{\pi R}}h_0 + v_5\right) + \sum_{n\geq 1}\sqrt{\frac{2}{\pi R}}h_n \cos\frac{ny}{R}. \tag{2.128}$$

Nevertheless, as long as the $5D$ Higgs VEV is not expressed in a sort of KK tower, the notation in Equation (2.126) should work.

Now, the kinetic term of the complex scalar H reads

$$\begin{aligned}
\mathscr{L}_{\text{higgs}} &\supset |\mathscr{D}_M H|^2 \\
&\supset |\mathscr{D}_\mu H|^2 - |\mathscr{D}_5 H|^2 \\
&\supset \left|\left(\partial_\mu + \frac{ig_5}{2}A_\mu\right)\frac{h + v_5 + i\phi_3}{\sqrt{2}}\right|^2 \\
&\supset \frac{1}{2}\left(\frac{g_5 v_5}{2}\right)^2 A_\mu^2 \\
&\supset \frac{1}{2}\left(\frac{g_5 v_5}{2}\right)^2 \left(\frac{1}{\sqrt{\pi R}}A_{0\mu}\right)^2.
\end{aligned} \tag{2.129}$$

If we integrate over the extra dimension, we obtain

$$\frac{1}{2}\int_{-\pi R}^{\pi R} dy\, \mathscr{L}_{\text{higgs}} \supset \frac{1}{2}\left(\frac{g_5 v_5}{2}\right)^2 A_{0\mu}^2. \tag{2.130}$$

If we now match the $4D$ mass of $A_{0\mu}$ with the apparent $5D$ mass (i.e. the mass consisting of the $5D$ constants), we get

$$\frac{g_5 v_5}{2} = \frac{gv}{2} \tag{2.131}$$

and hence

$$v = \frac{v_5}{g/g_5} = \frac{v_5}{1/\sqrt{\pi R}} = v_5\sqrt{\pi R}. \tag{2.132}$$

From Equation (2.131), we also see that

$$g_5 v_5 = gv. \tag{2.133}$$

This allows us to directly write the “apparent $5D$ masses" simply as their $4D$ counterparts — indeed, this is what we did while studying the equations of motion of the scalar and vector fields earlier.

Now let us relate the $5D$ Higgs coefficients, μ_5 and λ_5, to their $4D$ counterparts.

$$\begin{aligned}\mathscr{L}_{\text{higgs}} &\supset \mu_5^2 \left|H\right|^2 - \lambda_5 \left|H\right|^4 \\ &\supset \mu_5^2 \left|\frac{1}{\sqrt{2}}(h + v_5 + i\phi_3)\right|^2 - \lambda_5 \left|\frac{1}{\sqrt{2}}(h + v_5 + i\phi_3)\right|^4 .\end{aligned} \tag{2.134}$$

It should be noted that, for consistency purposes, we take the $5D$ Higgs VEV to be given by the same relation in terms of μ_5 and λ_5 as the $4D$ one:

$$v_5 = \frac{\mu_5}{\sqrt{\lambda_5}}. \tag{2.135}$$

Let us extract the Higgs mass term from the $5D$ Lagrangian,

$$\begin{aligned}\mathscr{L}_{\text{higgs}} &\supset -\mu_5^2 h^2 \\ &\supset -\mu_5^2 \left(\frac{1}{\sqrt{\pi R}} h_0\right)^2\end{aligned} \tag{2.136}$$

and integrate it over the extra dimension:

$$\frac{1}{2}\int_{-\pi R}^{\pi R} dy\, \mathscr{L}_{\text{higgs}} \supset -\mu_5 h_0^2. \tag{2.137}$$

Matching this with the usual SM mass of the Higgs field, we see that

$$\mu = \mu_5. \tag{2.138}$$

This gives us

$$\lambda_5 = \frac{\mu_5^2}{v_5^2} = \frac{\mu^2}{(v\sqrt{\pi R})^2} = \frac{\mu^2}{v^2}\frac{1}{\pi R} = \frac{\lambda}{\pi R}. \tag{2.139}$$

Finally, let us match the $5D$ Yukawa couplings, y_{i5} (where $i = 1, 2$ denotes the upper and lower components of the $SU(2)$ doublets, respectively), to its SM counterpart.

Consider the Yukawa sector:

$$\begin{aligned}
\mathscr{L}_{\text{yukawa}} &\supset -y_{15}\bar{\psi}_L\psi_{1R}\tilde{H} - y_{25}\bar{\psi}_L\psi_{2R}H + \text{h.c.} \\
&\supset -y_{15}\begin{pmatrix}\bar{\psi}_{1L} & \bar{\psi}_{2L}\end{pmatrix}\psi_{1R}\begin{pmatrix}v_5/\sqrt{2} \\ 0\end{pmatrix} + (1\to 2) + \text{h.c.} \\
&\supset -\bar{\psi}_{1L}\frac{y_{15}v_5}{\sqrt{2}}\psi_{1R} + (1\to 2) + \text{h.c.} \\
&\supset -\left(\frac{1}{\sqrt{\pi R}}\bar{\psi}_{1L0}\right)\frac{y_{15}v_5}{\sqrt{2}}\left(\frac{1}{\sqrt{\pi R}}\psi_{1R0}\right) + (1\to 2) + \text{h.c.}
\end{aligned} \tag{2.140}$$

If we integrate Equation (2.140) over the extra dimension, we obtain

$$\begin{aligned}
\frac{1}{2}\int_{-\pi R}^{\pi R} dy\, \mathscr{L}_{\text{yukawa}} &\supset -\bar{\psi}_{1L0}\frac{y_5 v_5}{\sqrt{2}}\psi_{1R0} + (1\to 2) + \text{h.c.} \\
&\supset -\bar{\psi}_{10}\frac{y_5 v_5}{\sqrt{2}}P_R\psi_{10} - \bar{\psi}_{10}\frac{y_5 v_5}{\sqrt{2}}P_L\psi_{10} + (1\to 2) \\
&\supset -\bar{\psi}_{10}\frac{y_5 v_5}{\sqrt{2}}\psi_{10} + (1\to 2).
\end{aligned} \tag{2.141}$$

Now, $y_5 v_5/\sqrt{2}$ should give the usual (non-diagonal) mass matrix, $yv/\sqrt{2}$, so

$$y_5 v_5 = yv \tag{2.142}$$

and thus

$$y = \frac{y_5}{\sqrt{\pi R}} \tag{2.143}$$

just like any gauge couplings. So we can see that the combinations $g_5 v_5$ and $y_5 v_5$ can always be written as gv and yv, respectively, even before we integrate the $5D$ Lagrangian over the extra dimension.

2.12 Final remarks on the notation

To facilitate the notation, we define

$$c_n := \sqrt{\frac{2}{\pi R}}\cos\frac{ny}{R}, \tag{2.144}$$

$$s_n := \sqrt{\frac{2}{\pi R}}\sin\frac{ny}{R} \tag{2.145}$$

to denote the mode functions, and

$$m_n := \frac{n}{R} \tag{2.146}$$

to represent the *universal* mass term. Meantime, let

$$b := \frac{1}{\sqrt{\pi R}} \tag{2.147}$$

which is simply the scale factor for the SM modes. The Einstein summation convention is employed throughout this work. For instance, for the $5D$ Higgs field, we write

$$\begin{aligned} h(x,y) &= \frac{1}{\sqrt{\pi R}} h_0(x) + \sum_{n=1}^{6} \sqrt{\frac{2}{\pi R}} h_n(x) \cos \frac{ny}{R} \\ &= bh_0 + h_n c_n. \end{aligned} \tag{2.148}$$

When we take the y derivative, we include the mass term coming from the argument of the sine or cosine in this summation, as well:

$$\begin{aligned} \partial_5 h(x,y) =& \sum_{n=1}^{6} -\sqrt{\frac{2}{\pi R}} h_n(x) \frac{n}{R} \sin \frac{ny}{R} \\ =& - m_n h_n s_n. \end{aligned} \tag{2.149}$$

We will treat any field without a KK number as a $5D$ field, which depends on both the 4-position and the extra dimension, and fields that carry a KK number as the $4D$ ones, depending only on the 4-position. We will suppress the coordinate dependence of the fields and the mode functions. The $5D$ Lagrangian will be integrated over the extra dimension, y, and the integrated Lagrangian will be denoted by $\xrightarrow{\frac{1}{2}\int_{-\pi R}^{\pi R} dy} \cdots \supset$. For the states in the mass basis and the diagonalized mass matrices, we append an asterisk to the subscript.

2.13 KK decomposition of the fields

Below are the KK decomposition of all the fields in the MUED model in the notation summarized in Section 2.12.

2.13.1 The scalar fields

The KK towers for the usual SM fields, promoted to the $5D$, are as follows:

$$h = bh_0 + h_n c_n, \tag{2.150}$$

$$\phi^{\pm} = b\phi_0^{\pm} + \phi_n^{\pm} c_n, \tag{2.151}$$

$$\phi_3 = b\phi_{30} + \phi_{3n} c_n. \tag{2.152}$$

The KK towers for the 5$^{\text{th}}$ components of the vector fields are as follows:

$$G_5 = G_{5n} s_n, \tag{2.153}$$

$$W_5^1 = W_{5n}^1 s_n, \tag{2.154}$$

$$W_5^2 = W_{5n}^2 s_n, \tag{2.155}$$

$$W_5^3 = W_{5n}^3 s_n, \tag{2.156}$$

$$B_5 = B_{5n} s_n. \tag{2.157}$$

2.13.2 The fermion fields

The KK decomposition of the fermions are given by

$$\psi_L^i = b\psi_{L0}^i + (P_L c_n + P_R s_n)\, \psi_{Ln}^i, \tag{2.158}$$

$$\psi_R^i = b\psi_{R0}^i + (P_R c_n + P_L s_n)\, \psi_{Rn}^i \tag{2.159}$$

where $i = 1, 2$ denotes the upper and lower components of the left-handed $SU(2)$ doublets, respectively, ψ^1 represents the up-like quarks (u, c, and t), and ψ^2 stands for the down-like quarks (d, s, and b). ψ_L^1 and ψ_L^2 together form the left-handed $SU(2)$ doublet, Q. The ψ_R^i are simply the right-handed singlet states.

2.13.3 The vector fields

The KK expansions of the SM vectors are as follows:

$$G^{\mu} = bG_0^{\mu} + G_n^{\mu} c_n, \tag{2.160}$$

$$W^{1\mu} = bW_0^{1\mu} + W_n^{1\mu} c_n, \tag{2.161}$$

$$W^{2\mu} = bW_0^{2\mu} + W_n^{2\mu} c_n, \tag{2.162}$$

$$W^{3\mu} = bW_0^{3\mu} + W_n^{3\mu} c_n, \tag{2.163}$$

$$B^{\mu} = bB_0^{\mu} + B_n^{\mu} c_n. \tag{2.164}$$

2.14 Physical states in MUED

Having armed with the theoretical background and a facilitated notation, we are going to derive the mass eigenstates of the complete particle spectrum of the MUED model in this section. First, we consider the vector and scalar fields, charged and neutral sectors one by one. Then, we focus on the quarks. The gauge-fixing Lagrangian is obtained in the Feynman gauge. The gluon, leptons, and ghosts are treated separately at the end.

2.14.1 The vector and scalar fields

We start with the bosonic sector. In the charged sector, we have the vectors $W^{1\mu}$ and $W^{2\mu}$ and the scalars ϕ_1, ϕ_2, W_5^1, and W_5^2. Due to the existence of two new degrees of freedom, namely W_5^1 and W_5^2, we expect to have a new physical charged scalar, living in the KK tower, besides a charged Goldstone boson. In the neutral sector, there exist the vectors $W^{3\mu}$ and B^{μ} and the scalars ϕ_3, W_5^3, and B_5. In this case, ϕ_3 combined with the two new degrees of freedom, namely W_5^3 and B_5, will contribute to a new physical neutral scalar and the Goldstone bosons associated with the KK partners of the photon and the Z boson.

2.14.1.1 The charged sector

Consider the gauge and Higgs sectors:

$$\mathscr{L} = \mathscr{L}_{\text{gauge}} + \mathscr{L}_{\text{higgs}}. \tag{2.165}$$

We can focus on the $5D$ Proca terms of the first and the second gauge vectors of the weak interactions:

$$\mathscr{L}_{\text{gauge}} = -\frac{1}{4}(W^1_{MN})^2 - \frac{1}{4}(W^2_{MN})^2. \tag{2.166}$$

For a generic non-abelian $5D$ vector V_M, the field strength tensor is given by

$$V^p_{MN} = \partial_M V^p_N - \partial_N V^p_M - g_5 \sigma^{prs} V^r_M V^s_N \tag{2.167}$$

where g_5 is the $5D$ coupling of the interaction that V^μ mediates, and the σ^{prs} are the structure constants. The term proportional to the coupling constant is already quadratic in the fields, thus it will not contribute to the diagonalization of the states. Now, let us take the square of the field strength tensor in Equation (2.167):

$$\begin{aligned}
(V_{MN})^2 =& (V_{\mu\nu})^2 - 2(V_{\mu 5})^2 \\
&\supset -2\left[(\partial_\mu V_5)^2 + (\partial_5 V_\mu)^2 - 2\partial_\mu V_5 \partial_5 V^\mu\right] \\
&\supset -2(\partial_\mu V_5)^2 - 2(\partial_5 V_\mu)^2 + 4\partial_\mu V_5 \partial_5 V^\mu.
\end{aligned} \tag{2.168}$$

Here, the second term will contribute to the mass terms of V^μ by adding the universal mass m_n once the field is expanded, and the last term will be resolved in the gauge-fixing Lagrangian. If we use Equation (2.168) in (2.166), then we get

$$\mathscr{L}_{\text{gauge}} \supset \frac{1}{2}(\partial_5 W^1_\mu)^2 - \partial_\mu W^1_5 \partial_5 W^{1\mu} + \frac{1}{2}(\partial_5 W^2_\mu)^2 - \partial_\mu W^2_5 \partial_5 W^{2\mu}. \tag{2.169}$$

Let

$$W^\pm = \frac{W^1 \mp i W^2}{\sqrt{2}} \tag{2.170}$$

for both the vector and 5$^{\text{th}}$ components. Then,

$$W^1 = \frac{W^+ + W^-}{\sqrt{2}}, \quad W^2 = \frac{-W^+ + W^-}{\sqrt{2} i}. \tag{2.171}$$

Using these in Equation (2.169), we get

$$\begin{aligned}
\mathscr{L}_{\text{gauge}} &\supset \partial_5 W^+_\mu \partial_5 W^{-\mu} - \partial_\mu W^+_5 \partial_5 W^{-\mu} - \partial_\mu W^-_5 \partial_5 W^{+\mu} \\
&\xrightarrow{\frac{1}{2}\int_{-\pi R}^{\pi R} dy} \cdots \supset m_n^2 W^+_{n\mu} W^{-\mu}_n + m_n W^+_n \partial_\mu W^-_{5n} + m_n W^-_n \partial_\mu W^+_{5n}.
\end{aligned} \tag{2.172}$$

The first term in Equation (2.172) is the part of the mass term for the KK W bosons, and the rest is just the derivative mixings to be resolved by the gauge-fixing terms.

Next, consider the Higgs sector:

$$\begin{aligned}\mathscr{L}_{\text{higgs}} &= |\mathscr{D}_M H|^2 - \mathscr{U}(H) \\ &\supset |\mathscr{D}_\mu H|^2 - |\mathscr{D}_5 H|^2 .\end{aligned} \tag{2.173}$$

The covariant derivative is given by

$$\begin{aligned}\mathscr{D} &= \partial + i g_{w5} \vec{T}_w \cdot \vec{W} + i g_{y5} T_y B \\ &= \begin{pmatrix} \partial + \frac{i g_{w5}}{2} W^3 + \frac{i g_{y5}}{2} B & \frac{i g_{w5}}{2}(W^1 - i W^2) \\ \frac{i g_{w5}}{2}(W^1 + i W^2) & \partial - \frac{i g_{w5}}{2} W^3 + \frac{i g_{y5}}{2} B \end{pmatrix} \\ &= \begin{pmatrix} \partial + \frac{i g_{w5}}{2} W^3 + \frac{i g_{y5}}{2} B & \frac{i g_{w5}}{\sqrt{2}} W^+ \\ \frac{i g_{w5}}{\sqrt{2}} W^- & \partial - \frac{i g_{w5}}{2} W^3 + \frac{i g_{y5}}{2} B \end{pmatrix}\end{aligned} \tag{2.174}$$

for both the vector and 5$^{\text{th}}$ components. The hypercharge eigenvalue of the Higgs doublet is $Y = 1$, hence $T_y = \frac{1}{2}$. The Higgs doublet is

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} i(\phi_1 - i\phi_2) \\ h + v_5 + i\phi_3 \end{pmatrix} = \begin{pmatrix} i\phi^+ \\ \frac{1}{\sqrt{2}}(h + v_5 + i\phi_3) \end{pmatrix} . \tag{2.175}$$

Since we only need possible mass terms of the charged particles, we can ignore all the terms from the neutral sector, and the terms already quadratic in fields. This leaves us with

$$\mathscr{D} H \supset \begin{pmatrix} i\partial\phi^+ + \frac{i g_{w5} v_5}{2} W^+ \\ 0 \end{pmatrix} \tag{2.176}$$

where

$$\frac{g_{w5} v_5}{2} = \frac{g_w v}{2} = m_W . \tag{2.177}$$

(See the discussion following Equation (2.143).) Taking the modulus, (2.176) becomes

$$\begin{aligned}|\mathscr{D} H|^2 &\supset (\partial\phi^+ + m_W W^+)(\partial\phi^- + m_W W^-) \\ &\supset \partial\phi^+ \partial\phi^- + m_W^2 W^+ W^- + m_W W^+ \partial\phi^- + m_W W^- \partial\phi^+ .\end{aligned} \tag{2.178}$$

By inserting Equation (2.178) into (2.173) for both the vector and 5$^{\text{th}}$ components, and integrating over the extra dimension, we obtain

$$
\begin{aligned}
\mathscr{L}_{\text{higgs}} \xrightarrow{\frac{1}{2}\int_{-\pi R}^{\pi R} dy} \cdots \supset\ & m_W^2 W_0^{+\mu} W_{0\mu}^- + m_W^2 W_n^{+\mu} W_{n\mu}^- \\
& + m_W W_0^{+\mu} \partial_\mu \phi_0^- + m_W W_n^{+\mu} \partial_\mu \phi_n^- \\
& + m_W W_0^{-\mu} \partial_\mu \phi_0^+ + m_W W_n^{-\mu} \partial_\mu \phi_n^+ \\
& - m_n^2 \phi_n^+ \phi_n^- - m_W^2 W_{5n}^+ W_{5n}^- + m_n m_W W_{5n}^+ \phi_n^- \\
& + m_n m_W W_{5n}^- \phi_n^+ .
\end{aligned}
\tag{2.179}
$$

Totally, from the gauge sector and the Higgs kinetic term, we have

$$
\begin{aligned}
\mathscr{L} \xrightarrow{\frac{1}{2}\int_{-\pi R}^{\pi R} dy} \cdots \supset\ & \{m_W^2 W_0^{+\mu} W_{0\mu}^- + m_{W_n}^2 W_n^{+\mu} W_{n\mu}^-\} \\
& - \{m_n^2 \phi_n^+ \phi_n^- + m_W^2 W_{5n}^+ W_{5n}^- - m_n m_W \phi_n^+ W_{5n}^- \\
& \quad - m_n m_W \phi_n^- W_{5n}^+\} \\
& + \{W_0^{+\mu} \partial_\mu m_W \phi_0^- + W_n^{+\mu} \partial_\mu (m_W \phi_n^- + m_n W_{5n}^-) \\
& \quad + W_0^{-\mu} \partial_\mu m_W \phi_0^+ + W_n^{-\mu} \partial_\mu (m_W \phi_n^+ + m_n W_{5n}^+)\}
\end{aligned}
\tag{2.180}
$$

where we have defined

$$
m_{W_n} = \sqrt{m_W^2 + m_n^2}. \tag{2.181}
$$

The vector fields come out orthogonal, so we only need to diagonalize the scalar fields. The second and the third lines in Equation (2.180) can be written as

$$
\mathscr{L} \supset -S_n^{+\dagger} M_{S_n^+}^2 S_n^+ \tag{2.182}
$$

with the charged scalars collected as a column matrix

$$
S_n^+ = \begin{pmatrix} \phi_n^+ \\ W_{5n}^+ \end{pmatrix}, \quad S_n^{+\dagger} = \begin{pmatrix} \phi_n^- & W_{5n}^- \end{pmatrix} \tag{2.183}
$$

and the mass matrix is given by

$$
M_{S_n^+}^2 = \begin{pmatrix} m_n^2 & -m_n m_W \\ -m_n m_W & m_W^2 \end{pmatrix}. \tag{2.184}
$$

Assume there exists a unitary matrix $U_{S_n^+}$ that diagonalizes the mass matrix:

$$
\begin{aligned}
S_n^{+\dagger} M_{S_n^+}^2 S_n^+ &= S_n^{+\dagger} U_{S_n^+} U_{S_n^+}^\dagger M_{S_n^+}^2 U_{S_n^+} U_{S_n^+}^\dagger S_n^+ \\
&= S_{n*}^{+\dagger} M_{S_n^+ *}^2 S_{n*}^+ .
\end{aligned}
\tag{2.185}
$$

The diagonalized mass matrix has the eigenvalues 0 and $m_{W_n}^2$,

$$M_{S_n^+*}^2 = \begin{pmatrix} 0 & \\ & m_{W_n}^2 \end{pmatrix} \tag{2.186}$$

so that there is one charged Goldstone boson, $G_n^\pm$, associated with the KK W boson. The mass goes to the new physical state, $a_n^\pm$, which is called the charged Higgs boson. We may collect the Goldstone scalar and the charged Higgs in the column vector

$$S_{n*}^\pm = \begin{pmatrix} G_n^\pm \\ a_n^\pm \end{pmatrix}. \tag{2.187}$$

The normalized eigenvectors, $\vec{x}_1$ and $\vec{x}_2$, of the unitary rotation matrix $U_{S_n^+}$ are given by

$$\vec{x}_1 = \frac{1}{m_{W_n}} \begin{pmatrix} m_W \\ m_n \end{pmatrix}, \quad \vec{x}_2 = \frac{1}{m_{W_n}} \begin{pmatrix} -m_n \\ m_W \end{pmatrix}. \tag{2.188}$$

Hence the matrix itself is constructed to be

$$U_{S_n^+} = \frac{1}{m_{W_n}} \begin{pmatrix} m_W & -m_n \\ m_n & m_W \end{pmatrix}. \tag{2.189}$$

From Equation (2.185), we have

$$U_{S_n^+} S_n^+ = S_{n*}^+ \tag{2.190}$$

or, taking the inverse,

$$S_n^+ = U_{S_n^+} S_{n*}^+. \tag{2.191}$$

Therefore, the gauge eigenstates can be written in terms of the mass eigenstates as

$$\phi_n^\pm = \frac{1}{m_{W_n}} (m_W G_n^\pm - m_n a_n^\pm), \tag{2.192}$$

$$W_{5n}^\pm = \frac{1}{m_{W_n}} (m_n G_n^\pm + m_W a_n^\pm). \tag{2.193}$$

The derivative mixing term does not contain the charged Higgs boson:

$$m_W \phi_n^\pm + m_n W_{5n}^\pm = m_{W_n} G_n^\pm. \tag{2.194}$$

We are now in a position to suggest the gauge-fixing Lagrangian. The derivative-mixing terms in the integrated Lagrangian can be collected as

$$\begin{aligned} \mathscr{L}_{\text{gauge-fixing}} \xrightarrow{\frac{1}{2}\int_{-\pi R}^{\pi R} dy} \cdots \supset\ & \partial_\mu W_0^{+\mu} m_W \phi_0^- + \partial_\mu W_n^{+\mu} m_{W_n} G_n^- \\ & + \partial_\mu W_0^{-\mu} m_W \phi_0^+ + \partial_\mu W_n^{-\mu} m_{W_n} G_n^+. \end{aligned} \tag{2.195}$$

Let us try the following:

$$\begin{aligned}\mathscr{L}_{\text{gauge-fixing}} &\supset -\left|\partial_M W^{+M} - m_W\phi^+\right|^2 \\ &\supset -\left|\partial_\mu W^{+\mu} - \partial_5 W_5^+ - m_W\phi^+\right|^2 \\ &\supset -[(\partial_\mu W_0^{+\mu} - m_W\phi_0^+) + c_n(\partial_\mu W_n^{+\mu} - m_n W_{5n}^+ - m_W\phi_n^+] \\ &\quad\times [(\partial_\mu W_0^{-\mu} - m_W\phi_0^-) + c_n(\partial_\mu W_n^{-\mu} - m_n W_{5n}^- - m_W\phi_n^-)].\end{aligned} \tag{2.196}$$

In the light of Equation (2.194), this works.

Now let us briefly summarize this section. The charged vectors $W^\pm$ and their 5$^{\text{th}}$ components are given by the same prescription as in the SM:

$$W^\pm = \frac{W^1 \mp iW^2}{\sqrt{2}}. \tag{2.197}$$

The charged 5$^{\text{th}}$ components mix with the charged scalars to produce the Goldstone bosons associated with the KK W boson and the charged Higgs particle:

$$G_n^\pm = \frac{1}{m_{W_n}}(m_W\phi_n^\pm + m_n W_{5n}^\pm), \tag{2.198}$$

$$a_n^\pm = \frac{1}{m_{W_n}}(-m_n\phi_n^\pm + m_W W_{5n}^\pm). \tag{2.199}$$

The charged Higgs field has mass

$$m_{a_n^\pm} = m_{W_n} = \sqrt{m_W^2 + m_n^2}. \tag{2.200}$$

The gauge-fixing Lagrangian is constructed to be

$$\mathscr{L}_{\text{gauge-fixing}} \supset -\left|\partial_M W^{+M} - m_W\phi^+\right|^2. \tag{2.201}$$

2.14.1.2 The neutral sector

Again, we start with the gauge sector and the Higgs kinetic term:

$$\mathscr{L} = \mathscr{L}_{\text{gauge}} + \mathscr{L}_{\text{higgs}}. \tag{2.202}$$

In the gauge sector, this time we focus on the curvature tensors of the neutral gauge bosons, W^3 and B:

$$\mathscr{L}_{\text{gauge}} = -\frac{1}{4}(W_{MN}^3)^2 - \frac{1}{4}(B_{MN})^2. \tag{2.203}$$

W^3 belongs to a non-abelian vector, so in its field strength tensor, there exists a term proportional to the $5D$ weak coupling, g_{w5}:

$$V^i_{MN} = \partial_M V^i_N - \partial_N V^i_M - g_{w5}\epsilon^{ijk} V^j_M V^k_N. \tag{2.204}$$

Since the terms proportional to the gauge coupling are already quadratic in the fields, we ignore them in the diagonalization. Since the mediator of the hypercharge is an abelian vector, its strength tensor does not contain such a term. Hence, for both W^3 and B, we proceed without the last term in Equation (2.204).

$$\begin{aligned}(V_{MN})^2 &= (V_{\mu\nu})^2 - 2(V_{\mu 5})^2 \\ &\supset -2[(\partial_\mu V_5)^2 + (\partial_5 V_\mu)^2 - 2\partial_\mu V_5 \partial_5 V^\mu] \\ &\supset -2(\partial_\mu V_5)^2 - 2(\partial_5 V_\mu)^2 + 4\partial_\mu V_5 \partial_5 V^\mu. \end{aligned} \tag{2.205}$$

The first term in Equation (2.205) is the kinetic term for the 5th component of the vector field, the second term will contribute to the mass term after KK decomposition, and the last term will be resolved in the gauge-fixing sector.

Substituting Equation (2.205) in (2.203), and integrating over the extra dimension, we obtain

$$\mathscr{L}_{\text{gauge}} \xrightarrow{\frac{1}{2}\int_{-\pi R}^{\pi R} dy} \cdots \supset \frac{1}{2} m_n^2 (W^3_{n\mu})^2 + m_n W^{3\mu}_n \partial_\mu W^3_{5n} + \frac{1}{2} m_n^2 B^2_{n\mu} + m_n B^\mu_n \partial_\mu B_{5n}. \tag{2.206}$$

Next, we consider the kinetic term of the $5D$ Higgs field. To facilitate the computation, we may consider the $U(1) \otimes U(1)$ complex scalar, taking advantage of the fact that W^3 can be treated as another $U(1)$ gauge boson with the generator T^3, the third component of the weak isospin.

$$\begin{aligned}\mathscr{L}_{\text{higgs}} &\supset |\mathscr{D}_M H|^2 \\ &\supset |\mathscr{D}_\mu H|^2 - |\mathscr{D}_5 H|^2 . \end{aligned} \tag{2.207}$$

For both the vector and 5th components, the covariant derivative can be expressed as

$$\mathscr{D} = \partial + i g_{w5} T^3_w W^3 + i g_{y5} T_y B \tag{2.208}$$

where $T^3_w = -1/2$ and $T_y = 1/2$ for the complex scalar field, H, which is given by

$$H = \frac{1}{\sqrt{2}}(h + v_5 + i\phi_3). \tag{2.209}$$

Applying the covariant derivative on the complex scalar field, we obtain

$$\begin{aligned}\mathscr{D}H &= \left(\partial - \frac{ig_{w5}}{2}W^3 + \frac{ig_{y5}}{2}B\right)\frac{h + v_5 + i\phi_3}{\sqrt{2}} \\ &\supset \frac{1}{\sqrt{2}}\left(i\partial\phi_3 - \frac{ig_{w5}v_5}{2}W^3 + \frac{ig_{y5}v_5}{2}B\right) \\ &\supset \frac{i}{\sqrt{2}}\left(\partial\phi_3 - m_W W^3 + m_B B\right)\end{aligned} \tag{2.210}$$

where we have defined the apparent mass term for the mediator of the hypercharge as

$$m_B := \frac{g_{y5}v_5}{2} = \frac{g_y v}{2} \tag{2.211}$$

and the apparent mass of $W_0^{3\mu}$ is defined to be

$$m_W := \frac{g_{w5}v_5}{2} = \frac{g_w v}{2} \tag{2.212}$$

such that $m_B^2 + m_W^2 = m_Z^2$. The absolute square of Equation (2.210) gives

$$\begin{aligned}|\mathscr{D}H|^2 \supset& \frac{1}{2}[(\partial\phi_3)^2 + m_W^2(W^3)^2 + m_B^2 B^2 \\ &+ 2(-m_W W^3\partial\phi_3 + m_B B\partial\phi_3 - m_W m_B W^3 B)] \\ \supset& \frac{1}{2}(\partial\phi_3)^2 + \frac{1}{2}m_W^2(W^3)^2 + \frac{1}{2}m_B^2 B^2 - m_W W^3\partial\phi_3 \\ &+ m_B B\partial\phi_3 - m_W m_B W^3 B.\end{aligned} \tag{2.213}$$

If we substitute Equation (2.213) into (2.207) and integrate over the extra dimension, we obtain

$$\begin{aligned}\mathscr{L}_{\text{higgs}} \xrightarrow{\frac{1}{2}\int_{-\pi R}^{\pi R} dy} \cdots \supset& \frac{1}{2}m_W^2(W_0^3)^2 + \frac{1}{2}m_B^2(B_0)^2 - m_W m_B W_0^{3\mu}B_{0\mu} \\ &- m_W W_0^{3\mu}\partial_\mu\phi_{30} + m_B B_0^\mu\partial_\mu\phi_{30} \\ &\frac{1}{2}m_W^2(W_n^3)^2 + \frac{1}{2}m_B^2(B_n)^2 - m_W m_B W_n^{3\mu}B_{n\mu} \\ &- m_W W_n^{3\mu}\partial_\mu\phi_{3n} + m_B B_n^\mu\partial_\mu\phi_{3n} \\ &- \Big\{\frac{1}{2}m_n^2(\phi_{3n})^2 + \frac{1}{2}m_W^2(W_{5n}^3)^2 + \frac{1}{2}m_B^2 B_{5n}^2 \\ &\quad + m_n m_W W_{5n}^3\phi_{3n} - m_n m_B\phi_{3n}B_{5n} - m_B m_W B_{5n}W_{5n}^3\Big\}\end{aligned} \tag{2.214}$$

where the apparent KK masses of the fields B_n^μ and $W_n^{3\mu}$ are defined as

$$m_{B_n} := \sqrt{m_B^2 + m_n^2} \tag{2.215}$$

and

$$m_{W_n} := \sqrt{m_W^2 + m_n^2}. \tag{2.216}$$

The integrated gauge and Higgs sectors combine to give

$$\begin{aligned}
\mathscr{L} \xrightarrow{\frac{1}{2}\int_{-\pi R}^{\pi R} dy} \cdots \supset \Big\{ & \frac{1}{2} m_W^2 (W_0^3)^2 + \frac{1}{2} m_B^2 - m_W m_B W_0^{3\mu} B_{0\mu} \\
& - m_W W_0^{3\mu} \partial_\mu \phi_{30} + m_B B_0^\mu \partial_\mu \phi_{30} \Big\} \\
& + \Big\{ \frac{1}{2} m_{W_n}^2 (W_n^3)^2 + \frac{1}{2} m_{B_n}^2 B_n^2 - m_W m_B W_n^{3\mu} B_{n\mu} \\
& + W_n^{3\mu} \partial_\mu (m_W \phi_{3n} - m_n W_{5n}^3) \\
& + B_n^\mu \partial_\mu (m_B \phi_{3n} + m_n B_{5n}) \Big\} \\
& - \Big\{ \frac{1}{2} m_n^2 (\phi_{3n})^2 + \frac{1}{2} m_W^2 (W_{5n}^3)^2 + \frac{1}{2} m_B^2 B_{5n}^2 \\
& + m_n m_W W_{5n}^3 \phi_{3n} - m_n m_B B_{5n} \phi_{3n} - m_W m_B W_{5n}^3 B_{5n} \Big\}.
\end{aligned} \tag{2.217}$$

In order to set our convention for the Weinberg mixing angle, we first focus on the SM modes of the vectors.

$$\begin{aligned}
\mathscr{L} \xrightarrow{\frac{1}{2}\int_{-\pi R}^{\pi R} dy} \cdots & \supset \frac{1}{2} m_B^2 B_0^2 + \frac{1}{2} m_W^2 (W_0^3)^2 - m_B m_W B_0^\mu W_{0\mu}^3 \\
& \supset \frac{1}{2} \begin{pmatrix} B_0^\mu & W_0^{3\mu} \end{pmatrix} \begin{pmatrix} m_B^2 & -m_B m_W \\ -m_B m_W & m_W^2 \end{pmatrix} \begin{pmatrix} B_{0\mu} \\ W_{0\mu}^3 \end{pmatrix} \\
& \supset \frac{1}{2} V_0^{\mu T} M_{V_0}^2 V_{0\mu}
\end{aligned} \tag{2.218}$$

where we have defined the column matrix

$$V_0^\mu = \begin{pmatrix} B_0^\mu \\ W_0^{3\mu} \end{pmatrix} \tag{2.219}$$

and the mass matrix

$$M_{V_0}^2 = \begin{pmatrix} m_B^2 & -m_B m_W \\ -m_B m_W & m_W^2 \end{pmatrix}. \tag{2.220}$$

Suppose there exists an orthogonal matrix U_{V_0} that diagonalizes the mixing matrix $M_{V_0}^2$:

$$\begin{aligned}
\mathscr{L} \xrightarrow{\frac{1}{2}\int_{-\pi R}^{\pi R} dy} \cdots & \supset \frac{1}{2} V_0^{\mu T} U_{V_0} U_{V_0}^T M_{V_0}^2 U_{V_0} U_{V_0}^T V_{0\mu} \\
& \supset \frac{1}{2} V_{0*}^{\mu T} M_{V_{0*}}^2 V_{0*\mu}.
\end{aligned} \tag{2.221}$$

The eigenvalues of the mass matrix is nothing but the masses of the photon and the Z boson:

$$M^2_{V_0*} = \begin{pmatrix} 0 & \\ & m_Z^2 \end{pmatrix}. \tag{2.222}$$

The normalized eigenvectors, $\vec{y}_1$ and $\vec{y}_2$, of U_{V_0} are found to be

$$\vec{y}_1 = \frac{1}{m_Z}\begin{pmatrix} m_W \\ m_B \end{pmatrix}, \quad \vec{y}_2 = \frac{1}{m_Z}\begin{pmatrix} -m_B \\ m_W \end{pmatrix} \tag{2.223}$$

so that

$$U_{V_0} = \frac{1}{m_Z}\begin{pmatrix} m_W & -m_B \\ m_B & m_W \end{pmatrix} = \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \tag{2.224}$$

where s_W (c_W) is the (co)sine of the Weinberg mixing angle:

$$c_W = \frac{m_W}{m_Z}, \quad s_W = \frac{m_B}{m_Z}. \tag{2.225}$$

The physical states are given by the rotation

$$V_0^\mu = U_{V_0} V_{0*}^\mu \tag{2.226}$$

or, more explicitly,

$$B_0^\mu = c_W A_0^\mu - s_W Z_0^\mu, \tag{2.227}$$

$$W_0^{3\mu} = s_W A_0^\mu + c_W Z_0^\mu. \tag{2.228}$$

Next, we move on to the KK modes of the vectors. The mixing terms can be collected in the form of matrices as in

$$\begin{aligned} \mathscr{L} \xrightarrow{\frac{1}{2}\int_{-\pi R}^{\pi R} dy} \cdots \supset & \frac{1}{2} m_{B_n}^2 B_n^2 + \frac{1}{2} m_{W_n}^2 (W_n^3)^2 - m_B m_W B_n^\mu W_{n\mu}^3 \\ \supset & \frac{1}{2}\begin{pmatrix} B_n^\mu & W_n^{3\mu} \end{pmatrix}\begin{pmatrix} m_{B_n}^2 & -m_B m_W \\ -m_B m_W & m_{W_n}^2 \end{pmatrix}\begin{pmatrix} B_{n\mu} \\ W_{n\mu}^3 \end{pmatrix} \\ \supset & \frac{1}{2} V_n^{\mu T} M_{V_n}^2 V_{n\mu} \end{aligned} \tag{2.229}$$

Assume there exists an orthogonal matrix U_{V_n} that diagonalizes the mass matrix:

$$\begin{aligned} \mathscr{L} \xrightarrow{\frac{1}{2}\int_{-\pi R}^{\pi R} dy} \cdots \supset & \frac{1}{2} V_n^{\mu T} U_{V_n} U_{V_n}^\dagger M_{V_n}^2 U_{V_n} U_{V_n}^\dagger V_{n\mu} \\ \supset & \frac{1}{2} V_{n*}^{\mu T} M_{V_n *}^2 V_{n*\mu}. \end{aligned} \tag{2.230}$$

The eigenvalues of the mixing matrix determines the masses of the KK photon and the KK Z boson:

$$M^2_{V_n*} = \begin{pmatrix} m^2_{A_n} & \\ & m^2_{Z_n} \end{pmatrix} \tag{2.231}$$

where

$$m_{A_n} = m_n \tag{2.232}$$

and

$$m_{Z_n} = \sqrt{m^2_Z + m^2_n}. \tag{2.233}$$

The eigenvectors, $\vec{z}_1$ and $\vec{z}_2$, of U_{V_n} are found to be

$$\vec{z}_1 = \frac{1}{m_Z} \begin{pmatrix} m_W \\ m_B \end{pmatrix}, \quad \vec{z}_2 = \frac{1}{m_Z} \begin{pmatrix} -m_B \\ m_W \end{pmatrix} \tag{2.234}$$

so that

$$U_{V_n} = \frac{1}{m_Z} \begin{pmatrix} m_W & -m_B \\ m_B & m_W \end{pmatrix} \tag{2.235}$$

which is the same rotation matrix as for the SM modes. Consequently, the Weinberg angle is independent of the KK number. Now, the rotation between the gauge and the mass eigenstates are given by

$$V^\mu_n = U_{V_n} V^\mu_{n*} \tag{2.236}$$

or, more explicitly,

$$B^\mu_n = c_W A^\mu_n - s_W Z^\mu_n, \tag{2.237}$$

$$W^{3\mu}_n = s_W A^\mu_n + c_W Z^\mu_n. \tag{2.238}$$

Next, we study the neutral scalars. The mass terms can be collected in the following arrays:

$$
\begin{aligned}
\mathscr{L} \xrightarrow{\frac{1}{2}\int_{-\pi R}^{\pi R} dy} \cdots \supset & -\Big\{\frac{1}{2}m_n^2(\phi_{3n})^2 + \frac{1}{2}m_W^2(W_{5n}^3)^2 + \frac{1}{2}m_B^2 B_{5n}^2 \\
& + m_n m_W W_{5n}^3 \phi_{3n} - m_n m_B B_{5n}\phi_{3n} - m_W m_B W_{5n}^3 B_{5n}\Big\} \\
\supset & -\frac{1}{2}\begin{pmatrix}\phi_{3n} & B_{5n} & W_{5n}^3\end{pmatrix}\begin{pmatrix} m_n^2 & -m_n m_B & m_n m_W \\ -m_n m_B & m_B^2 & -m_B m_W \\ m_n m_W & -m_B m_W & m_W^2\end{pmatrix} \\
& \cdot \begin{pmatrix}\phi_{3n} \\ B_{5n} \\ W_{5n}^3\end{pmatrix} \\
\supset & -\frac{1}{2} S_n^T M_{S_n}^2 S_n.
\end{aligned}
\tag{2.239}
$$

Assume there exists an orthogonal matrix U_{S_n} that diagonalizes the mass matrix:

$$
\begin{aligned}
\mathscr{L} \xrightarrow{\frac{1}{2}\int_{-\pi R}^{\pi R} dy} \cdots \supset & -\frac{1}{2} S_n^T U_{S_n} U_{S_n}^T M_{S_n}^2 U_{S_n} U_{S_n}^T S_n \\
\supset & -\frac{1}{2} S_{n*}^T M_{S_{n*}}^2 S_{n*}.
\end{aligned}
\tag{2.240}
$$

The eigenvalues of the mixing matrix are 0, 0 and $m_{Z_n}^2$, implying that there appear two Goldstone scalars associated with the KK photon and the KK Z boson, as expected:

$$
M_{S_n}^2 = \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_{Z_n}^2 \end{pmatrix}. \tag{2.241}
$$

The nontrivial eigenvalue indicates the appearance of a new scalar, a_n, with mass

$$
m_{a_n} = m_{Z_n}. \tag{2.242}
$$

Since the eigenvalue 0 is doubly degenerate, we need to perform a Gram-Schmidt orthogonalization procedure. Starting with the eigenvalue equation

$$
\left| M_{S_n}^2 - \lambda_i \right| \vec{q}_i = 0 \tag{2.243}
$$

let us study the case $\lambda_{(1,2)} = 0$:

$$\vec{q}_{(1,2)} = \begin{pmatrix} q_{(1,2)1} \\ q_{(1,2)2} \\ q_{(1,2)3} \end{pmatrix}$$
$$= \begin{pmatrix} q_{(1,2)1} \\ q_{(1,2)2} \\ -\frac{m_n}{m_W} q_{(1,2)1} + \frac{m_B}{m_W} q_{(1,2)2} \end{pmatrix}$$
$$= q_{(1,2)1} \begin{pmatrix} 1 \\ 0 \\ -\frac{m_n}{m_W} \end{pmatrix} + q_{(1,2)2} \begin{pmatrix} 0 \\ 1 \\ \frac{m_B}{m_W} \end{pmatrix} . \tag{2.244}$$

Thus, we might as well take the non-orthogonal eigenvectors, $\vec{q}_1$ and $\vec{q}_2$, to be

$$\vec{q}_1 = \begin{pmatrix} m_W \\ 0 \\ -m_n \end{pmatrix} , \quad \vec{q}_2 = \begin{pmatrix} 0 \\ m_W \\ m_B \end{pmatrix} . \tag{2.245}$$

For the last eigenvalue, $\lambda_3 = m_{Z_n}^2$, we obtain

$$\vec{q}_3 = \begin{pmatrix} m_n \\ -m_B \\ m_W \end{pmatrix} . \tag{2.246}$$

We can switch to the orthogonal set by using the relation

$$\vec{p}_k = \left(1 - \sum_{j=1}^{k-1} \hat{p}_j \hat{p}_j \cdot \right) \vec{q}_k \tag{2.247}$$

where the $\vec{p}_k$ are the mutually perpendicular vectors, with respective unit vectors $\hat{p}_k$. For $k = 1$, we get

$$\vec{p}_1 = \vec{q}_1 \tag{2.248}$$

and hence

$$\hat{p}_1 = \frac{1}{m_{W_n}} \begin{pmatrix} m_W \\ 0 \\ -m_n \end{pmatrix} . \tag{2.249}$$

For $k = 2$, we obtain

$$\vec{p}_2 = \vec{q}_2 - \vec{q}_2 \cdot \hat{p}_1 \hat{p}_1 \tag{2.250}$$

so the normalized vector is given by

$$\hat{p}_2 = \begin{pmatrix} m_B m_n/(m_{W_n} m_{Z_n}) \\ m_{W_n}/m_{Z_n} \\ m_B m_W/(m_{W_n} m_{Z_n}) \end{pmatrix}. \tag{2.251}$$

Since λ_3 is not degenerate, we may directly take

$$\vec{p}_3 = \vec{q}_3 \tag{2.252}$$

and normalize it to unity:

$$\hat{p}_3 = \frac{1}{m_{Z_n}} \begin{pmatrix} m_n \\ -m_B \\ m_W \end{pmatrix}. \tag{2.253}$$

Hence, the rotation matrix U_{S_n} is constructed to be

$$U_{S_n} = \begin{pmatrix} m_W/m_{W_n} & m_B m_n/(m_{W_n} m_{Z_n}) & m_n/m_{Z_n} \\ 0 & m_{W_n}/m_{Z_n} & -m_B/m_{Z_n} \\ -m_n/m_{W_n} & m_B m_W/(m_{W_n} m_{Z_n}) & m_W/m_{Z_n} \end{pmatrix}. \tag{2.254}$$

The rotation between the gauge and the mass eigenstates is given by

$$S_n = U_{S_n} S_{n*} \tag{2.255}$$

or, more explicitly,

$$\phi_{3n} = \frac{m_W}{m_{W_n}} G_{1n} + \frac{m_B m_n}{m_{W_n} m_{Z_n}} G_{2n} + \frac{m_n}{m_{Z_n}} a_n, \tag{2.256}$$

$$B_{5n} = \frac{m_{W_n}}{m_{Z_n}} G_{2n} - \frac{m_B}{m_{Z_n}} a_n, \tag{2.257}$$

$$W^3_{5n} = -\frac{m_n}{m_{W_n}} G_{1n} + \frac{m_B m_W}{m_{W_n} m_{Z_n}} G_{2n} + \frac{m_W}{m_{Z_n}} a_n. \tag{2.258}$$

Finally, let us collect the derivative-mixing terms. For the SM modes, we have

$$\mathscr{L} \xrightarrow{\frac{1}{2}\int_{-\pi R}^{\pi R} dy} \cdots \supset -W_0^{3\mu} \partial_\mu m_W \phi_{30} + B_0^\mu \partial_\mu m_B \phi_{30} \tag{2.259}$$

so the integrated gauge-fixing Lagrangian should contain such terms as

$$\begin{aligned} \mathscr{L}_{\text{gauge-fixing}} \xrightarrow{\frac{1}{2}\int_{-\pi R}^{\pi R} dy} \cdots &\supset -\partial_\mu W_0^{3\mu} m_W \phi_{30} + \partial_\mu B_0^\mu m_B \phi_{30} \\ &\supset -\frac{1}{2}(\partial_\mu W_0^{3\mu} + m_W \phi_{30})^2 - \frac{1}{2}(\partial_\mu B_0^\mu - m_B \phi_{30})^2. \end{aligned} \tag{2.260}$$

The sign of the mass term differs, depending on the sign conventions. We set it by writing the derivative-mixing terms in the mass eigenstates:

$$-m_W W_0^{3\mu}\partial_\mu\phi_{30} + m_B B_0^\mu\partial_\mu\phi_{30} = -Z_0^\mu\partial_\mu m_Z\phi_{30}. \tag{2.261}$$

This helps us determine the $4D$ gauge-fixing terms correctly:

$$\mathscr{L}_{\rm gauge-fixing} \xrightarrow{\frac{1}{2}\int_{-\pi R}^{\pi R} dy} \cdots \supset -\frac{1}{2}(\partial_\mu Z_0^\mu + m_Z\phi_{30})^2 - \frac{1}{2}(\partial_\mu A_0^\mu)^2. \tag{2.262}$$

Let us repeat the procedure for the KK modes:

$$\begin{aligned}\mathscr{L} \xrightarrow{\frac{1}{2}\int_{-\pi R}^{\pi R} dy} \cdots \supset & W_n^{3\mu}\partial_\mu(-m_W\phi_{3n} + m_n W_{5n}^3) + B_n^\mu\partial_\mu(m_B\phi_{3n} + m_n B_{5n}) \\ \supset & -Z_n^\mu\partial_\mu m_{Z_n} G_{Zn} + A_n^\mu\partial_\mu m_{A_n} G_{An} \\ \supset & A_n^\mu\partial_\mu(c_{1n}G_{1n} + c_{2n}G_{2n}) + Z_n^\mu\partial_\mu(c_{3n}G_{1n} + c_{4n}G_{2n})\end{aligned} \tag{2.263}$$

where the sign of the KK Z boson in the second line is to comply with its SM mode. Here, we have defined

$$c_{1n} = -\frac{m_B m_n^2}{m_{W_n} m_Z}, \tag{2.264}$$

$$c_{2n} = \frac{m_n m_W m_{Z_n}}{m_{W_n} m_Z}, \tag{2.265}$$

$$c_{3n} = -\frac{m_W m_{Z_n}^2}{m_{W_n} m_Z}, \tag{2.266}$$

$$c_{4n} = -\frac{m_B m_n m_{Z_n}}{m_{W_n} m_Z}. \tag{2.267}$$

In order to derive the Goldstone bosons, we need to solve the following system:

$$\begin{pmatrix} c_{1n} & c_{2n} \\ c_{3n} & c_{4n} \end{pmatrix} \begin{pmatrix} G_{1n} \\ G_{2n} \end{pmatrix} = \begin{pmatrix} m_{A_n} G_{An} \\ -m_{Z_n} G_{Zn} \end{pmatrix}. \tag{2.268}$$

The solutions are found to be

$$G_{1n} = -\frac{m_B m_n}{m_Z m_{W_n}} G_{An} + \frac{m_W m_{Z_n}}{m_{W_n} m_Z} G_{Zn}, \tag{2.269}$$

$$G_{2n} = \frac{m_W m_{Z_n}}{m_{W_n} m_Z} G_{An} + \frac{m_B m_n}{m_Z m_{W_n}} G_{Zn}. \tag{2.270}$$

Now we are in a position to propose the closed form of the $5D$ gauge-fixing Lagrangian. Let us try the following:

$$\begin{aligned}\mathscr{L} &\supset -\frac{1}{2}(\partial_M W^{3M} + m_W\phi_3)^2 - \frac{1}{2}(\partial_M B^M - m_B\phi_3)^2 \\ &\supset -\frac{1}{2}(\partial_\mu W^{3\mu} - \partial_5 W_5^3 + m_W\phi_3)^2 - \frac{1}{2}(\partial_\mu B^\mu - \partial_5 B_5 - m_B\phi_3)^2 \\ \xrightarrow{\frac{1}{2}\int_{-\pi R}^{\pi R} dy} \cdots &\supset -\frac{1}{2}(\partial_\mu W_0^{3\mu} + m_W\phi_{30})^2 - \frac{1}{2}(\partial_\mu B_0^\mu - m_B\phi_{30})^2 \\ &\quad -\frac{1}{2}(\partial_\mu W_n^{3\mu} - m_n W_{5n}^3 + m_W\phi_{3n})^2 \\ &\quad -\frac{1}{2}(\partial_\mu B_n^\mu - m_n B_{5n} - m_B\phi_{3n})^2. \end{aligned} \tag{2.271}$$

It fits. Let us check the SM modes for the signs:

$$(\partial_\mu W_0^{3\mu} + m_W\phi_{30})^2 + (\partial_\mu B_0^\mu - m_B\phi_{30})^2 = (\partial_\mu A_0^\mu)^2 + (\partial_\mu Z_0^\mu + m_Z\phi_{30})^2. \tag{2.272}$$

which is also fine.

We finalize this section by summarizing the mass eigenstates of the neutral vectors and scalars: B^μ and $W^{3\mu}$ mix to produce the photon and the Z boson with the identical mixing angle for both the SM and the KK modes, hence we might suppress the indices for convenience:

$$B^\mu = c_W A^\mu - s_W Z^\mu, \tag{2.273}$$

$$W^{3\mu} = s_W A^\mu + c_W Z^\mu. \tag{2.274}$$

where s_W (c_W) denotes the (co)sine of the Weinberg mixing angle,

$$c_W = \frac{m_W}{m_Z}. \tag{2.275}$$

In the SM sector, ϕ_{30} remains to be the only neutral Goldstone scalar (associated with the SM Z boson); however, in the KK tower, due to the existence of two new scalar fields, B_5 and W_5^3, we get two Goldstone bosons (associated with the KK photon and the KK Z boson) and a new degree of freedom, a_n, which is called the neutral scalar in the tower:

$$\phi_{3n} = \frac{m_W}{m_{W_n}} G_{1n} + \frac{m_B m_n}{m_{W_n} m_{Z_n}} G_{2n} + \frac{m_n}{m_{Z_n}} a_n, \tag{2.276}$$

$$B_{5n} = \frac{m_{W_n}}{m_{Z_n}} G_{2n} - \frac{m_B}{m_{Z_n}} a_n, \tag{2.277}$$

$$W_{5n}^3 = -\frac{m_n}{m_{W_n}} G_{1n} + \frac{m_B m_W}{m_{W_n} m_{Z_n}} G_{2n} + \frac{m_W}{m_{Z_n}} a_n. \tag{2.278}$$

The neutral scalar a_n has the same mass as the KK Z boson:

$$m_{a_n} = m_{Z_n}. \tag{2.279}$$

The compact form of the $5D$ gauge-fixing Lagrangian reads

$$\mathscr{L}_{\text{gauge}-\text{fixing}} \supset -\frac{1}{2}(\partial_M W^{3M} + m_W\phi_3)^2 - \frac{1}{2}(\partial_M B^M - m_B\phi_3)^2. \tag{2.280}$$

2.14.2 The fermion fields

In the derivation of the physical fermion states in the mass eigenbasis, we need the Dirac and Yukawa sectors.

$$\mathscr{L} = \mathscr{L}_{\text{fermion}} + \mathscr{L}_{\text{yukawa}}. \tag{2.281}$$

The $5D$ Dirac Lagrangian reads

$$\mathscr{L}_{\text{fermion}} = \sum_{f=Q,U,D} \bar{f} i\Gamma^M \mathscr{D}_M f \tag{2.282}$$

where Q denotes the left-handed $SU(2)$ quark doublet, U is the right-handed up-like singlet, and D represents the right-handed down-like singlet. The $5D$ Dirac matrices are taken to be

$$\Gamma^M = (\gamma^\mu, i\gamma_5). \tag{2.283}$$

In our earlier analysis, we observed that the KK mass terms derive from the terms containing the 5$^{\text{th}}$ component of the derivative:

$$\begin{aligned}
\mathscr{L}_{\text{fermion}} &\supset \sum_f \bar{f} i(i\gamma_5)\partial_5 f \\
&\supset -\bar{f}\gamma_5\partial_5 f \\
&\supset -\bar{\psi}_L\gamma_5\partial_5\psi_L - \bar{\psi}_R\gamma_5\partial_5\psi_R
\end{aligned} \tag{2.284}$$

where ψ_L denotes either component of Q and ψ_R is either of U and D. The KK decomposition for the fermion fields read

$$\psi_L = b\psi_{L0} + P_L\psi_{Ln}c_n + P_R\psi_{Ln}s_n, \tag{2.285}$$

$$\psi_R = b\psi_{R0} + P_R\psi_{Rn}c_n + P_L\psi_{Rn}s_n \tag{2.286}$$

with the adjoints

$$\bar{\psi}_L = b\bar{\psi}_{L0} + \bar{\psi}_{Ln} P_R c_n + \bar{\psi}_{Ln} P_L s_n, \tag{2.287}$$

$$\bar{\psi}_R = b\bar{\psi}_{R0} + \bar{\psi}_{Rn} P_L c_n + \bar{\psi}_{Rn} P_R s_n. \tag{2.288}$$

With Equations (2.285)–(2.288) substituted, the Dirac Lagrangian becomes

$$\begin{aligned}
\mathscr{L}_{\text{fermion}} \supset &- \left(\bar{\psi}_{Ln} P_R c_n + \bar{\psi}_{Ln} P_L s_n\right) \gamma_5 \left(-m_m P_L \psi_{Lm} s_m + m_m P_R \psi_{Lm} c_m\right) \\
&- \left(\bar{\psi}_{Rn} P_L c_n + \bar{\psi}_{Rn} P_R s_n\right) \gamma_5 \left(-m_m P_R \psi_{Rm} s_m + m_m P_L \psi_{Rm} c_m\right) \\
\xrightarrow{\frac{1}{2}\int_{-\pi R}^{\pi R} dy} \cdots \supset &- \left(m_n \bar{\psi}_{Ln} P_R \psi_{Ln} + m_n \bar{\psi}_{Ln} P_L \psi_{Ln}\right) \\
&- \left(-m_n \bar{\psi}_{Rn} P_L \psi_{Rn} - m_n \bar{\psi}_{Rn} P_R \psi_{Rn}\right) \\
\supset &- m_n \bar{\psi}_{Ln} \psi_{Ln} + m_n \bar{\psi}_{Rn} \psi_{Rn}.
\end{aligned} \tag{2.289}$$

The chiral modes do not mix in the KK tower; however, the right-handed mode appears with the wrong sign. This can be resolved by including γ_5 in the rotation matrix between the gauge eigenbasis and the mass one.

Meantime, the $5D$ Yukawa sector reads

$$\mathscr{L}_{\text{yukawa}} = - y_{15} \bar{\psi}_L \psi_{1R} \tilde{H} - y_{25} \bar{\psi}_L \psi_{2R} H + \text{h.c.} \tag{2.290}$$

where the subscripts 1 and 2 refer to that component of the left-handed doublet, h.c. stands for the Hermitian conjugate, H is the $SU(2)$ Higgs doublet, and

$$\tilde{H} := i\tau^2 H. \tag{2.291}$$

The only term that we need from the Higgs doublet is the Higgs VEV, so we may safely ignore the rest of the fields:

$$\begin{aligned}
\mathscr{L}_{\text{yukawa}} \supset &- y_{15} \bar{\psi}_L \psi_{1R} \begin{pmatrix} v_5/\sqrt{2} \\ 0 \end{pmatrix} - y_{25} \bar{\psi}_L \psi_{2R} \begin{pmatrix} 0 \\ v_5/\sqrt{2} \end{pmatrix} + \text{h.c.} \\
\supset &- \bar{\psi}_{1L} m_{\psi_1} \psi_{1R} - \bar{\psi}_{2L} m_{\psi_2} \psi_{2R} + \text{h.c.} \\
\supset &- \bar{\psi}_L m_\psi \psi_R + \text{h.c.} \\
\supset &- (b\bar{\psi}_{L0} + \bar{\psi}_{Ln} P_R c_n + \bar{\psi}_{Ln} P_L s_n) m_\psi \\
&\quad \times (b\psi_{R0} + P_R \psi_{Rm} c_m + P_L \psi_{Rm} s_m) \\
\xrightarrow{\frac{1}{2}\int_{-\pi R}^{\pi R} dy} \cdots \supset &- m_\psi \bar{\psi}_{L0} \psi_{R0} - m_\psi \bar{\psi}_{Ln} P_R \psi_{Rn} - m_\psi \bar{\psi}_{Ln} P_L \psi_{Rn} + \text{h.c.} \\
\supset &- m_\psi \bar{\psi}_0 \psi_0 - m_\psi \bar{\psi}_{Ln} \psi_{Rn} - m_\psi \bar{\psi}_{Rn} \psi_{Ln}
\end{aligned} \tag{2.292}$$

where we have defined

$$m_{\psi^i} = \frac{y_{i5} v_5}{2} = \frac{y_i v}{2}. \tag{2.293}$$

Combining the integrated Lagrangians in Equations (2.289) and (2.292), we obtain

$$\begin{aligned}\mathscr{L} \xrightarrow{\frac{1}{2}\int_{-\pi R}^{\pi R} dy} \cdots \supset & - m_\psi \bar{\psi}_0 \psi_0 \\ & - \left\{ m_n \bar{\psi}_{Ln} \psi_{Ln} - m_n \bar{\psi}_{Rn} \psi_{Rn} + m_\psi \bar{\psi}_{Ln} \psi_{Rn} + m_\psi \bar{\psi}_{Rn} \psi_{Ln} \right\}.\end{aligned} \tag{2.294}$$

We choose the following rotation for the physical states:

$$\psi_{Rn} = -\gamma_5 \cos \varphi_{\psi n} \psi_{1n} + \sin \varphi_{\psi n} \psi_{2n}, \tag{2.295}$$

$$\psi_{Ln} = \gamma_5 \sin \varphi_{\psi n} \psi_{1n} + \cos \varphi_{\psi n} \psi_{2n} \tag{2.296}$$

with the adjoints

$$\bar{\psi}_{Rn} = \cos \varphi_{\psi n} \bar{\psi}_{1n} \gamma_5 + \sin \varphi_{\psi n} \bar{\psi}_{2n}, \tag{2.297}$$

$$\bar{\psi}_{Ln} = - \sin \varphi_{\psi n} \bar{\psi}_{1n} \gamma_5 + \cos \varphi_{\psi n} \bar{\psi}_{2n}. \tag{2.298}$$

Inserting Equations (2.295)–(2.298) into (2.294), the relevant terms in the total Lagrangian become

$$\begin{aligned}\mathscr{L} \xrightarrow{\frac{1}{2}\int_{-\pi R}^{\pi R} dy} \cdots \supset & (m_n \cos 2\varphi_{\psi n} + m_\psi \sin 2\varphi_{\psi n}) \bar{\psi}_{1n} \psi_{1n} \\ & + (m_n \cos 2\varphi_{\psi n} + m_\psi \sin 2\varphi_{\psi n}) \bar{\psi}_{2n} \psi_{2n} \\ & + \left\{ (m_\psi \cos 2\varphi_{\psi n} - m_n \sin 2\varphi_{\psi n}) \bar{\psi}_{1n} \gamma_5 \psi_{2n} + \text{h.c.} \right\}.\end{aligned} \tag{2.299}$$

In Equation (2.299), we want to eliminate the mixing between ψ_{1n} and ψ_{2n}:

$$m_\psi \cos 2\varphi_{\psi n} - m_n \sin 2\varphi_{\psi n} = 0. \tag{2.300}$$

This fixes the rotation angle $\varphi_{\psi n}$:

$$\varphi_{\psi n} = \frac{1}{2} \tan^{-1} \frac{m_\psi}{m_n}. \tag{2.301}$$

It should be noted that the physical KK fermions are of the same mass:

$$m_n \cos 2\varphi_{\psi n} + m_\psi \sin 2\varphi_{\psi n} = m_{\psi_n} \tag{2.302}$$

where

$$m_{\psi_n} = \sqrt{m_\psi^2 + m_n^2}. \tag{2.303}$$

2.14.3 Gluons, leptons, and ghosts

Even though we have obtained the physical states of the charged and neutral boson, and of the quarks, there remain the gluons, the leptons, and the Faddeev-Popov particles.

Since the gluons do not interact with the Higgs field in either the SM or the MUED, the SM modes of the gluons will remain massless. However, the KK gluon receives a mass term coming solely from the 5$^{\text{th}}$ derivative in the gauge sector:

$$\mathscr{L} \supset -\frac{1}{4}(G^a_{MN})^2 \supset \frac{1}{2}(\partial_5 G^a_\mu)^2 \xrightarrow{\frac{1}{2}\int_{-\pi R}^{\pi R} dy} \cdots \supset \frac{1}{2}m_n^2(G^a_{n\mu})^2 \tag{2.304}$$

where there exists an implicit summation over the color index $a = 1, 2, \ldots, 8$. By the same token, the 5$^{\text{th}}$ component of the gluon does not mix with any other state. This makes it easier to determine the gauge-fixing Lagrangian:

$$\mathscr{L}_{\text{gauge-fixing}} \supset -\frac{1}{2}(\partial_M G^{aM})^2 = -\frac{1}{2}(\partial_\mu G^{a\mu} - \partial_5 G^a_5)^2. \tag{2.305}$$

Next, we have the leptons. Since the neutrinos in the SM do not possess the right-handed component, it is much more straightforward to deal the leptons. We can expand the electron-like leptons in the same tower as for the down-like quarks. For the left-handed spinors, we have

$$e_L = b e_{L0} + (P_L c_n + P_R s_n) e_{Ln}, \tag{2.306}$$

$$\mu_L = b \mu_{L0} + (P_L c_n + P_R s_n) \mu_{Ln}, \tag{2.307}$$

$$\tau_L = b \tau_{L0} + (P_L c_n + P_R s_n) \tau_{Ln} \tag{2.308}$$

whereas the right-handed spinors are given by

$$e_R = b e_{R0} + (P_R c_n + P_L s_n) e_{Rn}, \tag{2.309}$$

$$\mu_R = b \mu_{R0} + (P_R c_n + P_L s_n) \mu_{Rn}, \tag{2.310}$$

$$\tau_R = b \tau_{R0} + (P_R c_n + P_L s_n) \tau_{Rn}. \tag{2.311}$$

For the neutrinos, we only have the left-handed towers:

$$\nu_{eL} = b \nu_{eL0} + (P_L c_n + P_R s_n) \nu_{eLn}, \tag{2.312}$$

$$\nu_{\mu L} = b \nu_{\mu L0} + (P_L c_n + P_R s_n) \nu_{\mu Ln}, \tag{2.313}$$

$$\nu_{\tau L} = b \nu_{\tau L0} + (P_L c_n + P_R s_n) \nu_{\tau Ln}. \tag{2.314}$$

The physical states for the electron-like leptons are given by the same relation as in Equations (2.295) and (2.296). The KK modes of the electron-like leptons also have the mass

$$m_{\psi_n} = \sqrt{m_\psi^2 + m_n^2}. \tag{2.315}$$

The states given Equations (2.312)–(2.314) are the physical states for the neutrinos. The KK neutrinos have the mass

$$m_{\psi_n} = m_n. \tag{2.316}$$

Finally, we need to include the ghost sector. The KK modes of the ghosts are irrelevant to this work; nevertheless, we will need the SM modes of the gluon ghosts, ω_g. To this end, we will borrow the usual Faddeev-Popov sector:

$$\mathscr{L}_{\text{ghost}} \xrightarrow{\frac{1}{2}\int_{-\pi R}^{\pi R} dy} \cdots \supset i g_s f^{abc} \left(\partial_\mu \bar{\omega}_g^a\right) G^{b\mu} \omega_g^c \tag{2.317}$$

where the f^{abc} are the structure constants for the $SU(3)$ group.

2.15 Summary

We have derived the complete particle spectrum for the MUED model in the Feynman gauge. The masses and the rotations from the gauge basis to the mass one are summarized in Tables 2.1–2.3.

Table 2.1: The spectrum of vector fields in MUED.

Mass eigenstate	Mass	Gauge eigenstate(s)
G_0^μ	0	G_0^μ
$W_0^{\pm\mu}$	m_W	$W_0^{\pm\mu} = \frac{1}{\sqrt{2}}(W_0^{1\mu} \mp i W_0^{2\mu})$
Z_0^μ	m_Z	$Z_0^\mu = c_W W_0^{3\mu} - s_W B_0^\mu$
A_0^μ	0	$A_0^\mu = s_W W_0^{3\mu} + c_W B_0^\mu$
G_n^μ	m_n	G_n^μ
$W_n^{\pm\mu}$	m_{W_n}	$W_n^{\pm\mu} = \frac{1}{\sqrt{2}}(W_n^{1\mu} \mp i W_n^{2\mu})$
Z_n^μ	m_{Z_n}	$Z_n^\mu = c_W W_n^{3\mu} - s_W B_n^\mu$
A_n^μ	m_n	$A_n^\mu = s_W W_n^{3\mu} + c_W B_n^\mu$

In Table (2.1), s_W (c_W) is the (co)sine of the Weinberg mixing angle, and the KK masses are given by the formulae

$$m_{W_n} = \sqrt{m_W^2 + m_n^2}, \quad m_{Z_n} = \sqrt{m_Z^2 + m_n^2}. \tag{2.318}$$

Table 2.2: The spectrum of scalar fields in MUED.

Mass eigenstate	Mass	Gauge eigenstate(s)
h_0	m_h	h_0
$\phi_0^\pm$	m_W	$\phi_0^\pm = \frac{1}{\sqrt{2}}(\phi_{10} \mp i\phi_{20})$
ϕ_{30}	m_Z	ϕ_{30}
h_n	m_{h_n}	h_n
a_n	m_{Z_n}	$a_n = \frac{1}{m_{Z_n}}(m_n\phi_{3n} - m_B B_{5n} + m_W W_{5n}^3)$
$a_n^\pm$	m_{W_n}	$a_n^\pm = \frac{1}{\sqrt{2}m_{W_n}}\left[-m_n(\phi_{1n} \mp i\phi_{2n}) + m_W(W_{5n}^1 \mp iW_{5n}^2)\right]$
G_{5n}	m_n	G_{5n}
$G_n^\pm$	m_{W_n}	$G_n^\pm = \frac{1}{\sqrt{2}m_{W_n}}\left[m_W(\phi_{1n} \mp i\phi_{2n}) + m_n(W_{5n}^1 \mp iW_{5n}^2)\right]$
G_{Zn}	m_{Z_n}	$G_{Zn} = \frac{1}{m_Z m_{Z_n}}(m_Z^2\phi_{3n} + m_n m_B B_{5n} - m_n m_W W_{5n}^3)$
G_{An}	m_n	$G_{An} = \frac{1}{m_Z}(m_W B_{5n} + m_B W_{5n}^3)$

In Table (2.2), the parameter m_B is nothing but the apparent mass of the B_0 boson,

$$m_B = \frac{g_y v}{2} = m_Z s_W \tag{2.319}$$

and the Higgs KK mass is defined as

$$m_{h_n} = \sqrt{m_h^2 + m_n^2}. \tag{2.320}$$

In the KK tower, we have four new degrees of freedom, namely B_{5n}, W_{5n}^3, and $W_{5n}^\pm$. Combined with the KK modes of the $4D$ Goldstone scalars $\phi^\pm$ and ϕ_3, they produce G_{An}, G_{Zn}, $G_n^\pm$, a_n, and $a_n^\pm$. There appear three new fields, living only in the KK tower: the neutral scalar, a_n, and the charged Higgs boson, $a_n^\pm$.

Table 2.3: The spectrum of fermions in MUED.

Mass eigenstate	Mass	Gauge eigenstate(s)
e_0^i	m_{e^i}	e_0^i
ν_0^i	0	ν_0^i
u_0^i	m_{u^i}	u_0^i
d_0^i	m_{d^i}	d_0^i
e_n^{1i}	$m_{e_n^i}$	$e_n^{1i} = -\gamma_5(\cos\varphi_{e^in} e_{Rn}^i - \sin\varphi_{e^in} e_{Ln}^i)$
e_n^{2i}	$m_{e_n^i}$	$e_n^{2i} = \sin\varphi_{e^in} e_{Rn}^i + \cos\varphi_{e^in} e_{Ln}^i$
ν_n^i	m_n	$\nu_n^i = \nu_{Ln}^i$
u_n^{1i}	$m_{u_n^i}$	$u_n^{1i} = -\gamma_5(\cos\varphi_{u^in} u_{Rn}^i - \sin\varphi_{u^in} u_{Ln}^i)$
u_n^{2i}	$m_{u_n^i}$	$u_n^{2i} = \sin\varphi_{u^in} u_{Rn}^i + \cos\varphi_{u^in} u_{Ln}^i$
d_n^{1i}	$m_{d_n^i}$	$d_n^{1i} = -\gamma_5(\cos\varphi_{d^in} d_{Rn}^i - \sin\varphi_{d^in} d_{Ln}^i)$
d_n^{2i}	$m_{d_n^i}$	$d_n^{2i} = \sin\varphi_{d^in} d_{Rn}^i + \cos\varphi_{d^in} d_{Ln}^i$

In Table 2.3, $i = 1, 2, 3$ is the generation index, and the mixing angle $\varphi_{\psi n}$ is given by

$$\varphi_{\psi n} = \frac{1}{2}\tan^{-1}\frac{m_\psi}{m_n} \tag{2.321}$$

where m_ψ is the SM mass of the corresponding fermion. ψ_n^1 and ψ_n^2 are the new degrees of freedom, deriving from the right-handed component of the left-handed KK spinor and the left-handed component of the right-handed KK spinor.

CHAPTER 3

APPLICATIONS OF MUED TO RARE TOP PHYSICS

Once we complete the theoretical considerations of the MUED model, with all the states given in the mass basis and with their rotation angles from the gauge eigenstates to the mass ones, we can apply the model to any desired high-energy physics phenomenology. In this chapter, we present our results of the MUED contributions to rare top quark processes which occur by flavor-changing neutral currents (FCNCs).

Rare top quark processes play a crucial role for new physics searches. Experimentally, we only have upper bounds for the decay widths of the rare top quark decays occurring via FCNCs, which are dramatically larger than the corresponding SM calculations. Since no well-defined measurement of the considered decay widths are yet to be performed, we can hope for a new physics signal in order to account for huge orders of gaps between the SM computations and experimental bounds.

This chapter starts with the list of the processes of our interest, namely the rare top quark decays and single top quark production channels. Next, we discuss the special processes that contains two or more external non-abelian vector fields of the same kind. Then, we show the generic diagrams for the processes considered. Afterwards, our remarks on the numerical analysis are given. Finally, the MUED contributions to the decay widths and cross sections of the processes taken into account are presented.

3.1 Rare top quark processes

In order to perform numerical analysis, one needs the Feynman rules, extracted from the complete Lagrangian in a systematical fashion. To this end, LanHEP [33, 34] appears to be the best solution available at the moment. It is crucial to note that there exists a LanHEP code for the MUED model written by Belyaev *et al.* [29]. Intriguingly enough, the Feynman rules produced by this code seem to contain suspicious vertex factors, especially for the interactions of the quarks with the charged bosons, concerning the CKM matrix elements. Therefore, in accordance with the physical states summarized in Tables 2.1–2.3, we prepared our own LanHEP code for the MUED model, which can be found in Appendix D. By piping the systematically extracted Feynman rules from LanHEP to FeynArts [35] and FormCalc [36, 37][1], we applied the model to the following rare top quark processes that occur by FCNCs:

$$t \to c\gamma, \tag{3.1}$$

$$t \to cg, \tag{3.2}$$

$$t \to ch, \tag{3.3}$$

$$t \to cZ, \tag{3.4}$$

$$t \to cgg, \tag{3.5}$$

and

$$gg \to t\bar{c}, \tag{3.6}$$

$$cg \to t\gamma, \tag{3.7}$$

$$cg \to tg, \tag{3.8}$$

$$cg \to th, \tag{3.9}$$

$$cg \to tZ. \tag{3.10}$$

The full set of Feynman diagrams for the processes (3.1)–(3.10) are presented in Appendix A. The vertex factors relevant to these processes are contained in Appendix B.

[1] LanHEP, FeynArts, and FormCalc are package programs which are used to get the matrix element of a process starting from the Lagrangian of a model.

3.1.1 Processes with two or more non-abelian vectors

Consider the processes

$$t \to cgg, \tag{3.11}$$

$$gg \to t\bar{c}, \tag{3.12}$$

$$cg \to tg \tag{3.13}$$

where we have two external gluons. In the computation of the probability amplitude, the Feynman diagrams that gives the transition amplitude are brought together with those that produce the complex conjugate of the matrix element. Since we have more than one non-abelian vectors of the same kind, namely the gluon, there appear gluon loops due to 3-point gluon self-couplings. In order to conserve the unitarity of the theory [38], we need to include the ghost fields associated with the gluons (Figure 3.1).

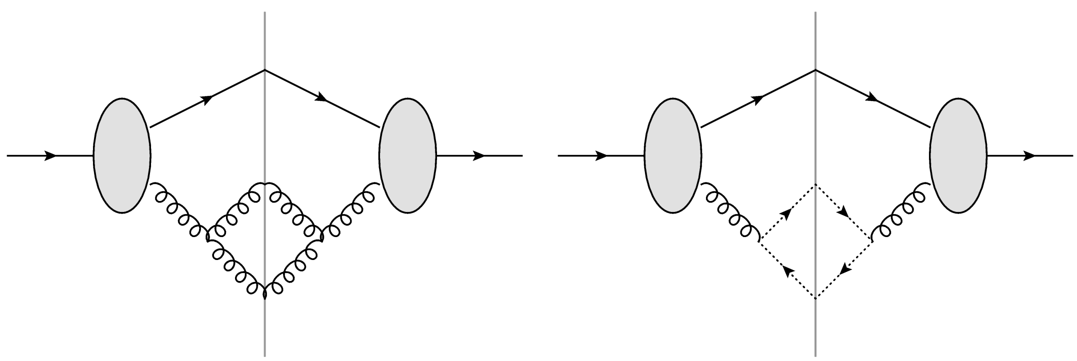

Figure 3.1: Gluon loops in the matrix element squared for the process $t \to cgg$.

This implies that we also have to consider the processes where the gluon is replaced with the gluon ghost:

$$t \to c\bar{\omega}_g\omega_g, \tag{3.14}$$

$$\bar{\omega}_g\omega_g \to t\bar{c}, \tag{3.15}$$

$$c\omega_g \to t\omega_g. \tag{3.16}$$

The ghost fields do not interact with the fermions, thus they will appear only in place of the triple gluon vertices (Figure 3.2).

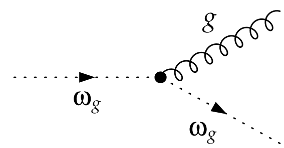

Figure 3.2: Gluon ghosts replaced with two gluons in the triple gluon vertex.

Since the external gluons are SM fields, it is sufficient for us to consider in the MUED model only the SM gluon ghosts due to the conservation of KK number.

3.2 Generic diagrams

Generic Feynman diagrams for the rare top processes are presented below. We classify the diagrams into four: (1) the two-body decays of the top quark given in (3.1)–(3.4), (2) the three-body decay of the top quark given in (3.5), (3) the single top quark production processes via gluon-gluon scattering given in (3.6), and (4) the single top quark production processes via charm-gluon scattering given in (3.7)–(3.10).

To facilitate the notation in the generic diagrams below, we denote the charged bosons in the loops (the W boson, the Goldstone scalar associated with the W boson, and the charged Higgs scalar) by a red line and the outgoing neutral bosons (the photon, the gluon, the Higgs scalar, and the Z boson) by a blue line. Naturally, the quark flavor changes whenever a quark current meets a red line. It is crucial to note that there cannot exist diagrams where the external bosons undergo a self-energy loop; otherwise, the fermion flavor will be conserved.

3.2.1 Two-body top decays

All the possible types of Feynman diagrams for the two-body top decay processes include self-energy and vertex-correction diagrams (Figure 3.3).

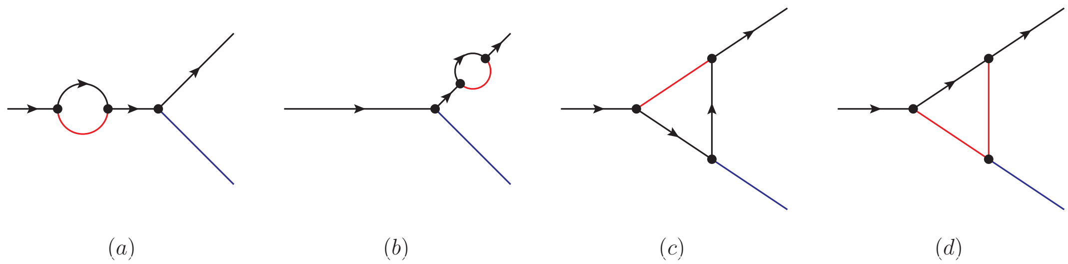

Figure 3.3: Generic self-energy and vertex-correction diagrams for the processes $t \to cX$ ($X = \gamma, g, h, Z$).

It should be noted that the diagram (d) in Figure 3.3 is not available for the process $t \to cg$ since the gluon does not interact with the charged bosons.

3.2.2 Three-body top decay

The three-body top decay process $t \to cgg$ includes self-energy and vertex-correction diagrams (Figure 3.4).

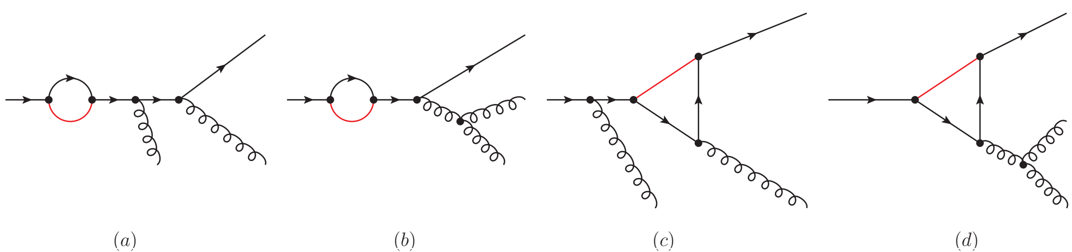

Figure 3.4: Generic self-energy and vertex-correction diagrams for the process $t \to cgg$.

In Figure 3.4, the self-energy diagrams (a) and (b) are to be repeated for all the fermion lines, whether they are virtual or external; moreover, the first gluon can also come out from the charm quark in (a) and (c), and from the down-like quark in (c). It is important to note that we also consider the diagrams with the gluon ghosts. They can be obtained from (b) and (d) in Figure 3.4 by replacing the two outgoing gluons with the gluon ghosts.

3.2.3 Gluon-gluon scattering

For the gluon-gluon scattering, there exist self-energy, vertex-correction, and box diagrams in all of the S, T, and U channels (Figure 3.5).

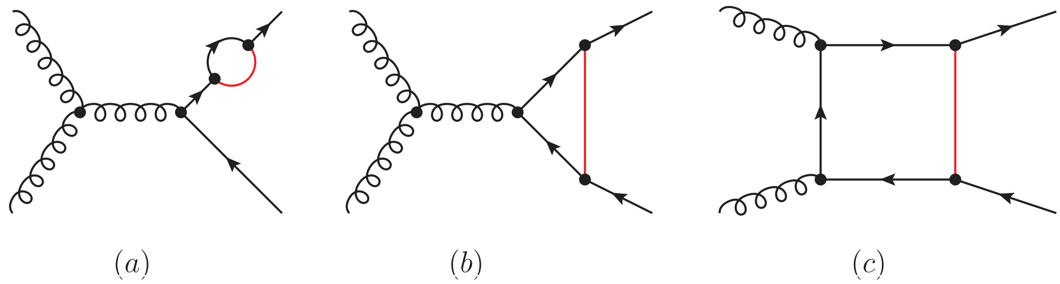

Figure 3.5: Generic self-energy, vertex-correction, and box diagrams for the process $gg \to t\bar{c}$.

The self-energy diagram (a) in Figure 3.5 should be repeated not only for the charm line, but also for the T and U channels. The vertex diagram (b) exists also in the T and U channels. There is a box diagram (c) in the U channel, as well. The S-channel diagrams (a) and (b) contain 3-point gluon self couplings, *ergo* these diagrams should be taken into account with the incoming gluons replaced with their ghost fields.

3.2.4 Charm-gluon scattering

Generic Feynman diagrams for the single top quark production processes via charm quark-gluon scattering contain self-energy, vertex-correction, and box diagrams (Figure 3.6).

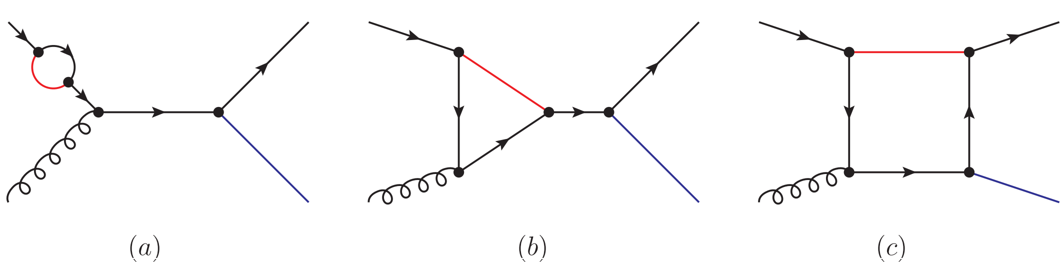

Figure 3.6: Generic self-energy, vertex-correction, and box diagrams for the processes $cg \to tX$ ($X = \gamma, g, h, Z$).

The self-energy diagram (a) in Figure 3.6 is meant to be repeated for all the fermion lines, whether virtual or external, in the S and U channels. It should be noted that when the external state other than the top quark, namely the one with the blue line, is a gluon, there exist 3-point gluon self couplings in the U channel, *ergo* we have to take into account the gluon ghosts in place of the two gluons, both incoming and outgoing.

3.3 Numerical analysis

In the numerical computations of the decay widths and cross sections, we used FormCalc, which prepares the necessary Fortran codes, and then LoopTools to carry out multidimensional integrals of the phase space.

In the process $t \to cgg$, assuming $i, j = 1, 2, 3$ denote the outgoing particles, we imposed cuts on the total energy of each outgoing particle,

$$E_i > 15 \text{ GeV} \tag{3.17}$$

and on the angle between particles i and j,

$$\alpha_{ij} > 15^\circ \tag{3.18}$$

for all i and j.

The scattering events (3.6)–(3.10) were analyzed at the center-of-mass energy

$$\sqrt{s} = 14 \text{ TeV}. \tag{3.19}$$

As for the parton distribution function, we chose CTEQ6L1 [39] from the package LHAPDF [40, 41].

The numerical values of the parameters we used in our analysis are shown in Table 3.1. The fine-structure constants of QED and QCD, and the quark masses are taken at the scale of the mass of the top quark.

Table 3.1: Parameters used in the numerical analysis.

Parameter	Description	Value
g_e	Electric charge	0.313329
g_s	Strong coupling constant	1.21978
$\sin\theta_W$	Sine of Weinberg mixing angle	0.471813
$\sin\theta_{12}$	Sine of θ_{12} in CKM matrix	0.22506
$\sin\theta_{23}$	Sine of θ_{23} in CKM matrix	0.0410788
$\sin\theta_{13}$	Sine of θ_{13} in CKM matrix	0.00357472
m_Z	Mass of Z boson	91.1876 GeV
m_h	Mass of Higgs boson	125.0 GeV
m_d	Mass of down quark	0.0047 GeV
m_c	Mass of charm quark	1.28 GeV
m_s	Mass of strange quark	0.096 GeV
m_t	Mass of top quark	175.0 GeV
m_b	Mass of bottom quark	4.18 GeV

3.4 Results

In this section, we present the decay widths and cross sections of the processes given in (3.1)–(3.10). The results are obtained as a function of the maximum KK number to include, $n_{\max}$, and the inverse radius of the extra dimension, R^{-1}.

3.4.1 Rare top decay processes

All the MUED contributions to the rare top decay processes (3.1)–(3.5) display a similar dependence on the inverse radius of the extra dimension, R^{-1} (Figures 3.7–3.11). From the figures, we make the following observations:

- The MUED contributions asymptotically approach the corresponding SM values as the inverse radius of the extra dimension increases.

- The MUED contributions increase as $n_{\max}$ increases.

- For the values of $n_{\max} = 4$ and $n_{\max} = 6$, the MUED contributions almost coincide.

- The order of the highest relative MUED contributions ranges from 10^{-4} to 10^{-1}, with the highest one in $t \to cg$ and the lowest in $t \to cgg$ at $R^{-1} = 0.2$ TeV.

- Between the experimental ranges of the size of the extra dimension, namely 0.5 TeV $< R^{-1} < 1.0$ TeV, the MUED contributions become nearly negligible.

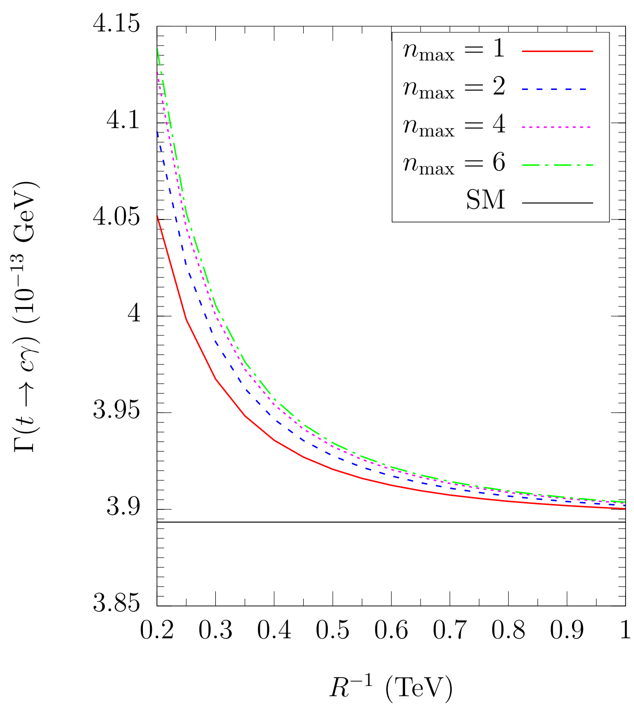

Figure 3.7: MUED contributions to the process $t \to c\gamma$ as a function of the maximum KK number, $n_{\max}$, and the inverse radius of the extra dimension, R.

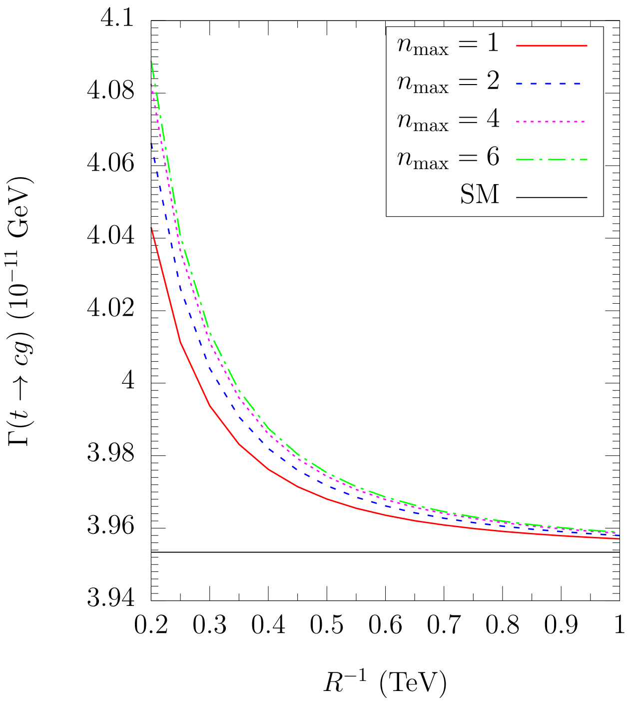

Figure 3.8: MUED contributions to the process $t \to cg$ as a function of the maximum KK number, $n_{\rm max}$, and the inverse radius of the extra dimension, R.

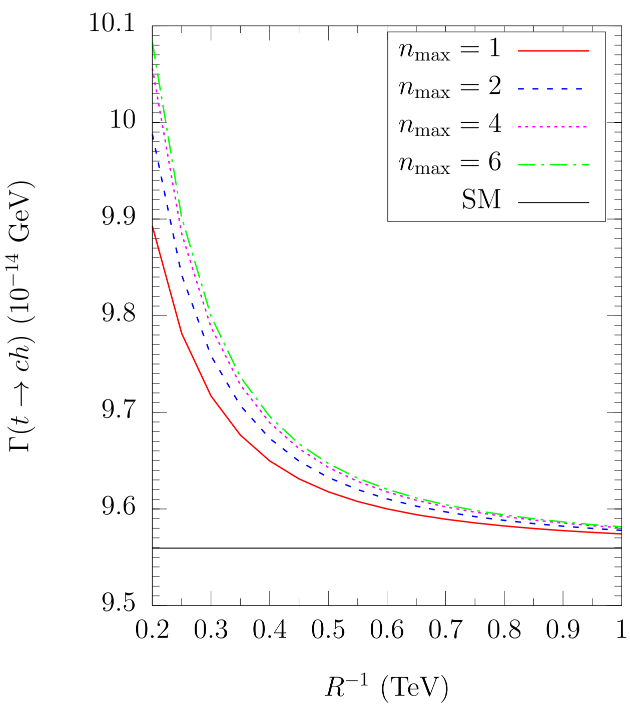

Figure 3.9: MUED contributions to the process $t \to ch$ as a function of the maximum KK number, $n_{\rm max}$, and the inverse radius of the extra dimension, R.

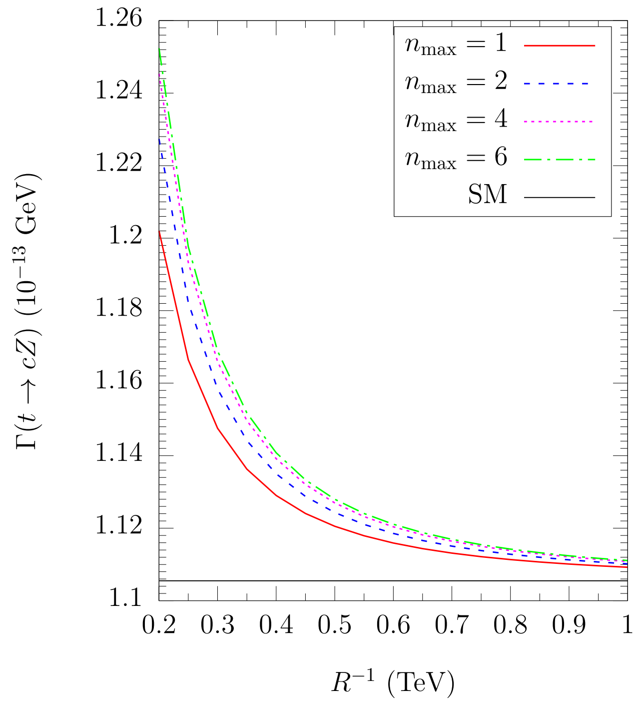

Figure 3.10: MUED contributions to the process $t \to cZ$ as a function of the maximum KK number, $n_{\max}$, and the inverse radius of the extra dimension, R.

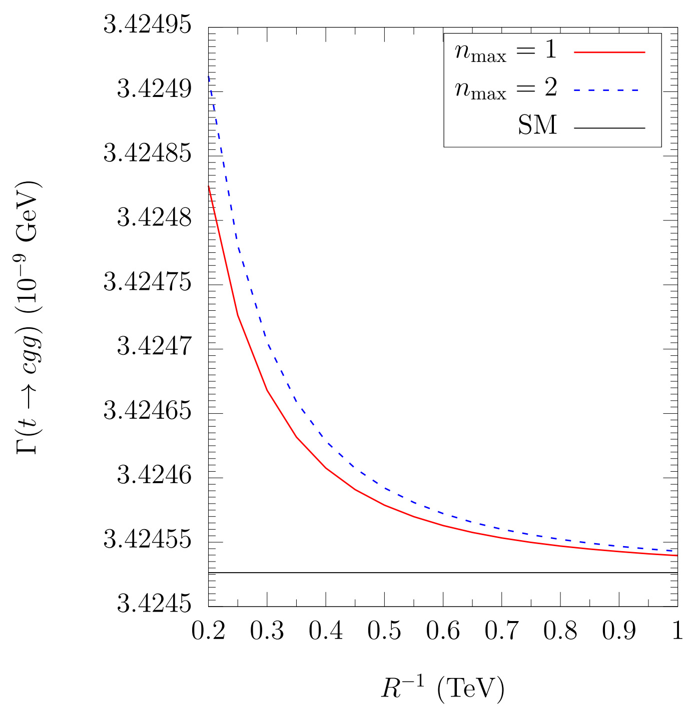

Figure 3.11: MUED contributions to the process $t \to cgg$ as a function of the maximum KK number, $n_{\max}$, and the inverse radius of the extra dimension, R.

3.4.2 Rare single-top production channels

The contributions from the MUED model for the single-top-quark production channels (3.6)–(3.10) display a similar dependence on the inverse radius, R^{-1} (Figures 3.12–3.16), with a notable exception for the process $cg \to th$ (Figure 3.15). From the figures, the following observations can be made:

- The MUED contributions asymptotically approach the corresponding SM values.
- The moduli of the MUED contributions increase with increasing $n_{\rm max}$.
- The MUED contributions nearly coincide for the values of $n_{\rm max} = 4$ and $n_{\rm max} = 6$.
- The order of the highest relative contributions ranges from 10^{-3} to 10^{-2}, with the highest one in $cg \to tg$ at $R^{-1} = 0.2$ TeV.
- For the physically acceptable range of the size of the extra dimension, that is, 0.5 TeV $< R^{-1} < 1.0$ TeV, the MUED contributions can be considered almost negligible.
- For the process $cg \to th$, the MUED contributions are negative for $R^{-1} \gtrsim 0.3$ TeV.

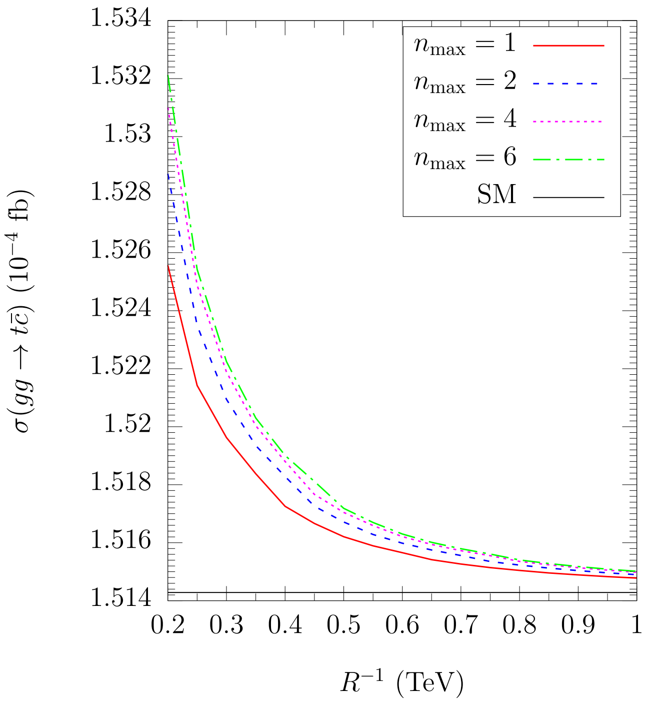

Figure 3.12: MUED contributions to the process $gg \to t\bar{c}$ as a function of the maximum KK number, $n_{\max}$, and the inverse radius of the extra dimension, R.

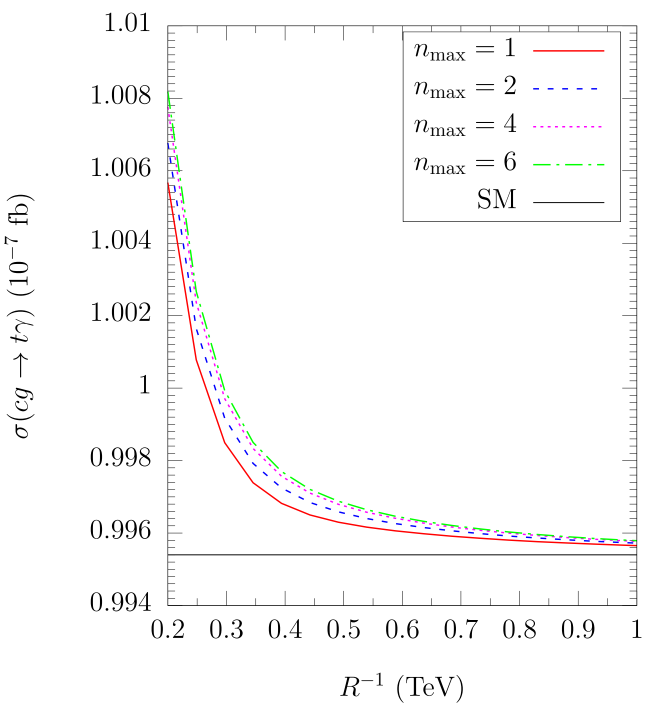

Figure 3.13: MUED contributions to the process $cg \to t\gamma$ as a function of the maximum KK number $n_{\max}$, and the inverse radius of the extra dimension, R.

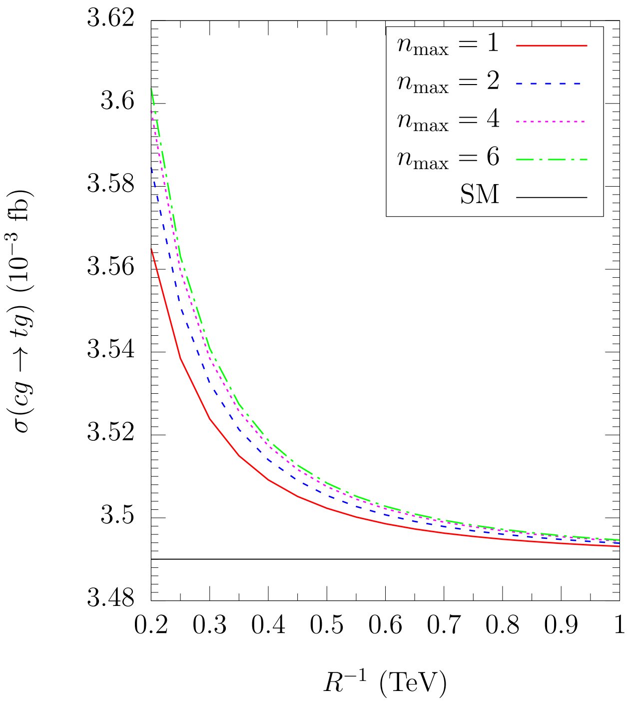

Figure 3.14: MUED contributions to the process $cg \to tg$ as a function of the maximum KK number, n_{max}, and the inverse radius of the extra dimension, R.

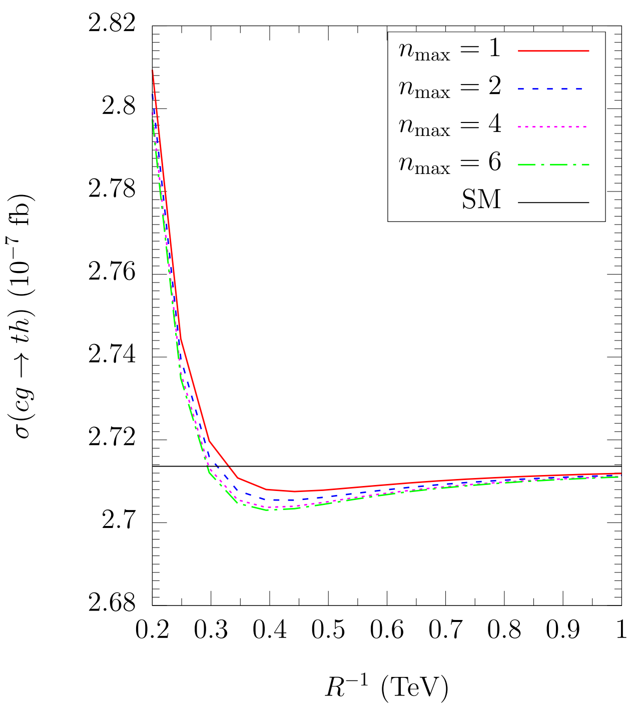

Figure 3.15: MUED contributions to the process $cg \to th$ as a function of the maximum KK number, n_{max}, and the inverse radius of the extra dimension, R.

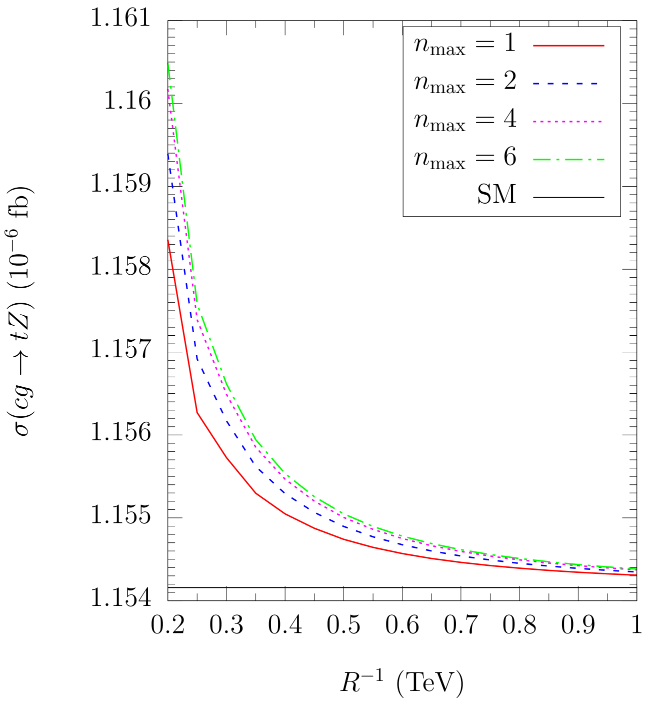

Figure 3.16: MUED contributions to the process $cg \to tZ$ as a function of the maximum KK number, n_{max}, and the inverse radius of the extra dimension, R.

3.5 Summary

The MUED contributions to all the decay and production processes at the maximum KK number $n_{\mathrm{max}} = 6$ are displayed in Figures 3.17 and 3.18, respectively, and the results are summarized in Tables 3.2 and 3.3. The enhancements to the decay widths and cross sections decay with the decreasing R.

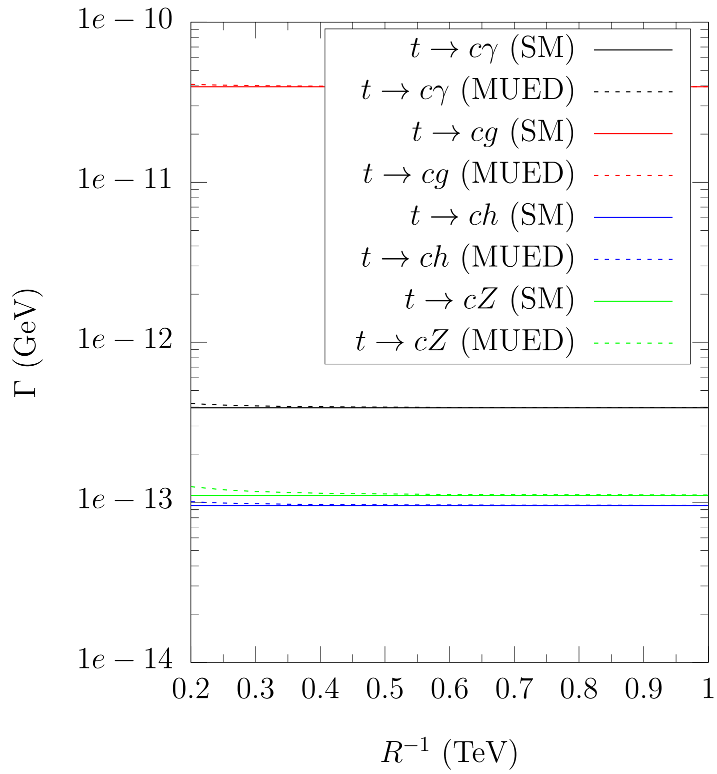

Figure 3.17: MUED contributions to the rare top decay processes at the maximum KK number $n_{\max} = 6$ as a function of the inverse of the size of the extra dimension, R.

Table 3.2: The SM values, Γ, of the decay widths of the rare top decay processes and the contributions from MUED calculations, $\Delta\Gamma$, are shown. The contributions are taken at $R = (1\text{ TeV})^{-1}$ and $n_{\max} = 6$. The observational limits are expressed as the corresponding branching ratios with respect to the main decay mode, $t \to bW$.

Process	Observational limit	Γ (GeV)	$\Delta\Gamma$ (GeV)
$t \to c\gamma$	$< 1.7 \times 10^{-3}$	0.389×10^{-12}	0.103×10^{-14}
$t \to cg$		0.395×10^{-10}	0.549×10^{-13}
$t \to ch$	$< 1.6 \times 10^{-3}$	0.956×10^{-13}	0.220×10^{-15}
$t \to cZ$	$< 0.24 \times 10^{-3}$	0.110×10^{-12}	0.560×10^{-15}

The contributions asymptotically approach the corresponding SM values. This is an expected behavior. There are two ways to see this. Firstly, as the size of the extra dimension decreases, there will be smaller and smaller room for the fields to *leak into*. Doing so, the particles will lose a degree of freedom. When the extra dimension gets too small, there will be no contributions after all. Secondly, the larger the inverse radius of the extra dimension grows, the heavier the KK states become. Recalling that all the KK partners have mass of the form

$$M_n = \sqrt{m_{\text{SM}} + \left(\frac{n}{R}\right)^2} \tag{3.20}$$

where m_{SM} is the corresponding SM mass, we see that at some point the partners will become so heavy that they will decouple from the theory. This, in turn, will diminish the contributions from the extra dimension.

In this sense, the MUED model, having only one free parameter, is extremely predictable.

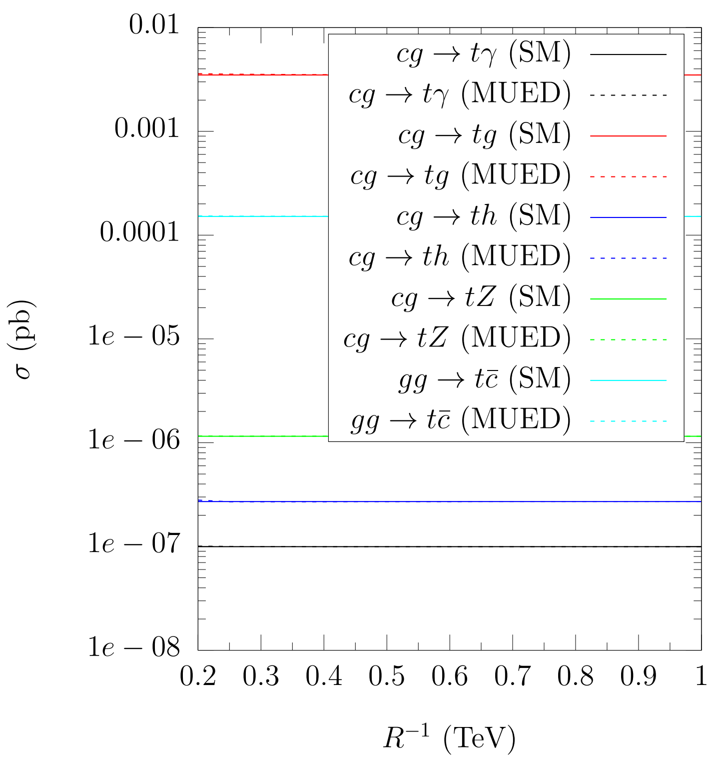

Figure 3.18: MUED contributions to the rare single-top production processes at the maximum KK number $n_{\text{max}} = 6$ as a function of the inverse of the size of the extra dimension, R.

Table 3.3: The SM values, σ, of the cross sections of the rare single top quark production channels and the contributions from MUED calculations, $\Delta\sigma$, are shown. The contributions are taken at $R = (1 \text{ TeV})^{-1}$, $n_{\text{max}} = 6$, and at the center-of-mass energy $\sqrt{s} = 14$ TeV.

Process	σ (pb)	$\Delta\sigma$ (pb)
$gg \to t\bar{c}$	0.151×10^{-6}	0.730×10^{-10}
$cg \to t\gamma$	0.995×10^{-10}	0.354×10^{-13}
$cg \to tg$	0.349×10^{-5}	0.460×10^{-8}
$cg \to th$	0.271×10^{-9}	-0.231×10^{-12}
$cg \to tZ$	0.115×10^{-8}	0.225×10^{-12}

CHAPTER 4

CONCLUSION

In this work, we analyzed the model of Minimal Universal Extra Dimensions in the Feynman-'t Hooft gauge. Firstly, we obtained the mass spectrum of the complete set of particles. Later, we aimed to apply this model to the rare top quark processes occurring by flavor-changing neutral currents.

For performing the numerical analysis, we needed the vertex factors, which were supposed to be systematically extracted from the complete Lagrangian. The most straightforward way to proceed was to construct a LanHEP code. It should be noted that a LanHEP code of the model exists in the current literature by Belyaev *et al.* [29]. However, this code yielded suspicious vertex factors involving the interactions of the quarks with the charged bosons, regarding the CKM matrix elements. In order to systematically check this code for all the interactions that we will need, we prepared an original Mathematica code. Having guaranteed that the LanHEP code at hand was suspicious, we prepared our LanHEP code.

We channeled the Feynman rules extracted from the LanHEP code to FeynArts. By using FormCalc and LoopTools, we obtained the decay widths and the cross sections of the following rare top processes:

$$t \to c\gamma, \tag{4.1}$$

$$t \to cg, \tag{4.2}$$

$$t \to ch, \tag{4.3}$$

$$t \to cZ, \tag{4.4}$$

$$t \to cgg \tag{4.5}$$

and

$$gg \to t\bar{c}, \tag{4.6}$$

$$cg \to t\gamma, \tag{4.7}$$

$$cg \to tg, \tag{4.8}$$

$$cg \to th, \tag{4.9}$$

$$cg \to tZ. \tag{4.10}$$

The contributions to these processes coming from the MUED model are in order 10^{-1} at $R = (200\ \mathrm{GeV})^{-1}$ and decay rapidly to nearly 10^{-3} at $R = (1\ \mathrm{TeV})^{-1}$ for $n_{\mathrm{max}} = 6$ (see Tables 3.2 and 3.3). That is, the contributions deriving from the MUED model to the processes (4.1–4.10) are tiny. This goes on to say that the MUED model cannot be the final extension to the SM. It has only one free parameter, namely the size of the extra dimension, and thus the model is extremely predictable. Therefore, one should not stop at this minimal extension of the SM in an extra-dimensional scenario, and move on with the next-to-minimal (or non-minimal) version of the UED model. In the NMUED model, due to an extended parameter space, it might be possible to obtain relative contributions larger than order unity.

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APPENDIX A

FEYNMAN DIAGRAMS

In this chapter, we present the generic Feynman diagrams for the rare top quark processes given in (3.1)–(3.10). The particle spectrum is summarized in Table A.1. It should be noted that, in the MUED diagrams, there exists an implicit summation over the KK index n from 1 to $n_{\max}$.

Table A.1: The symbols of the particles that appear in the Feynman diagrams.

Label	Definition
$\phi_0^\pm$	SM mode of Goldstone scalar associated with W boson
$G_n^\pm$	KK mode of Golstone scalar associated with W boson
$a_n^\pm$	Charged Higgs boson
d_0^i	SM mode of down-like quarks (d, s, and b)
$d_n^{1/2i}$	KK mode of down-like quarks (d, s, and b) of type $1/2$
c_0	SM mode of charm quark
t_0	SM mode of top quark
g_0	SM mode of gluon
$W_0^\pm$	SM mode of W boson
$W_n^\pm$	SM mode of W boson
Z_0	SM mode of Z boson
A_0	SM mode of photon
ω_{c0}	SM mode of gluon ghost

A.1 Feynman diagrams at SM level

A.1.1 $t \to c\gamma$

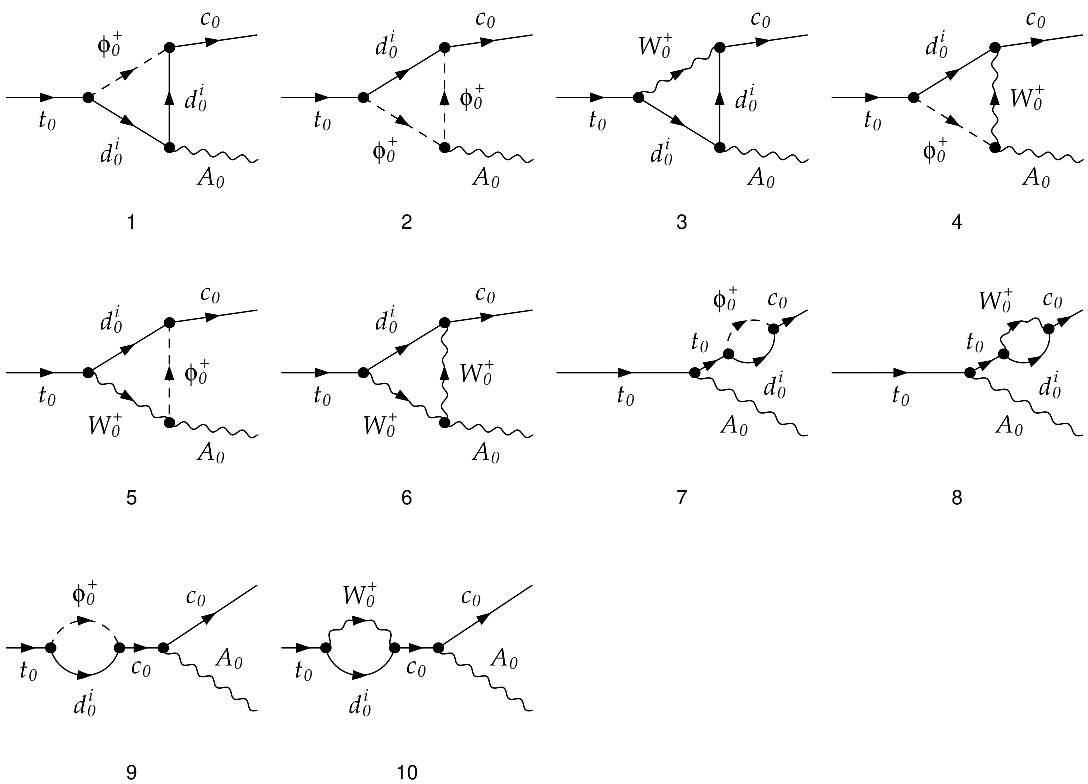


Figure A.1: SM diagrams contributing to the process $t \to c\gamma$.

A.1.2 $t \to cg$

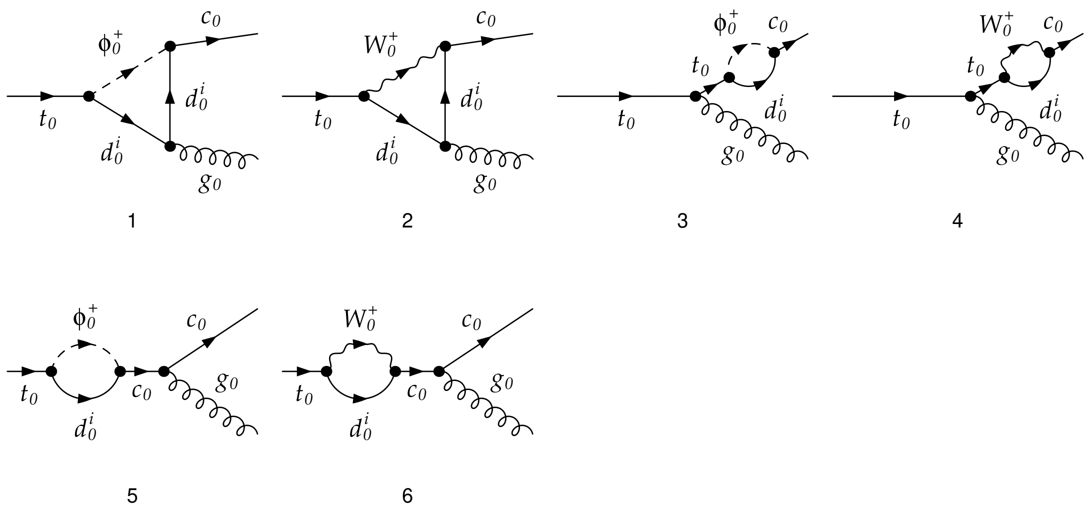


Figure A.2: SM diagrams contributing to the process $t \to cg$.

A.1.3 $t \to ch$

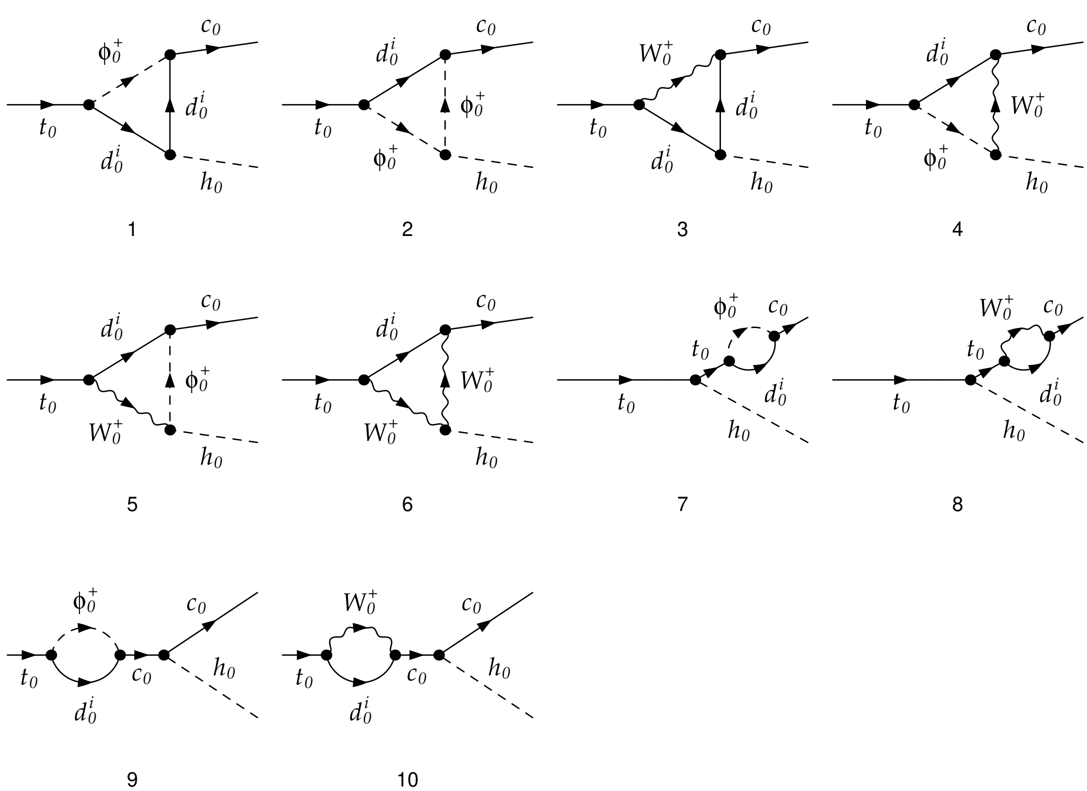


Figure A.3: SM diagrams contributing to the process $t \to ch$.

A.1.4 $t \to cZ$

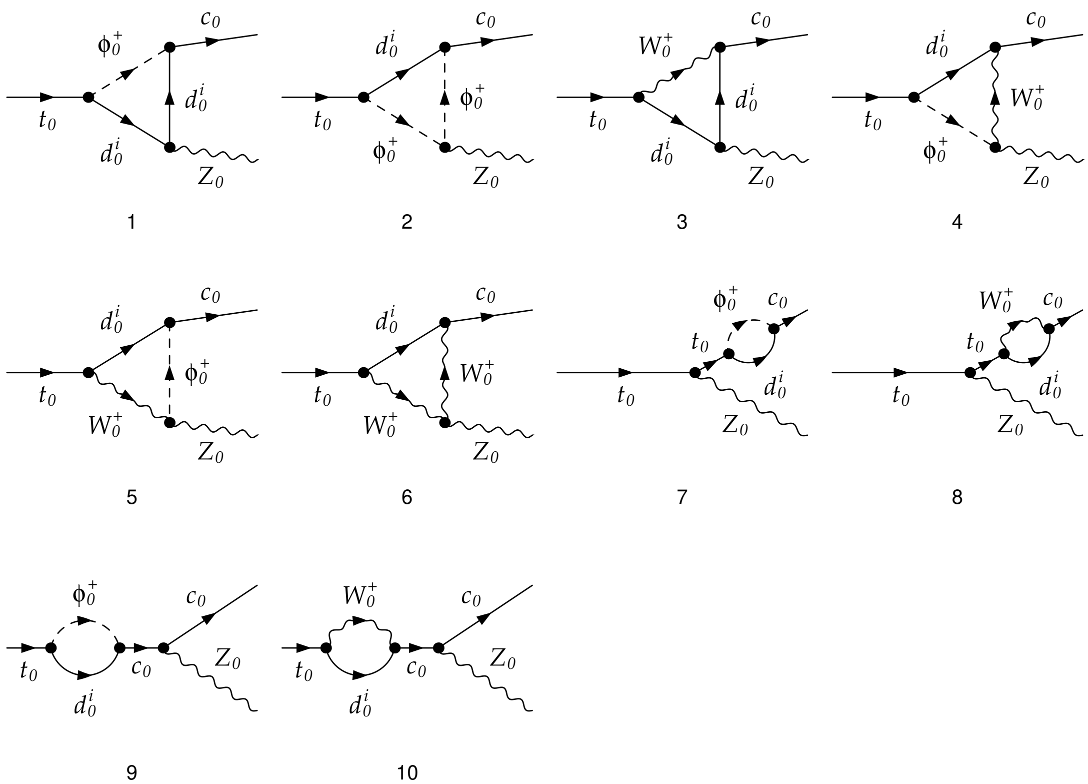


Figure A.4: SM diagrams contributing to the process $t \to cZ$.

A.1.5 $t \to cgg$

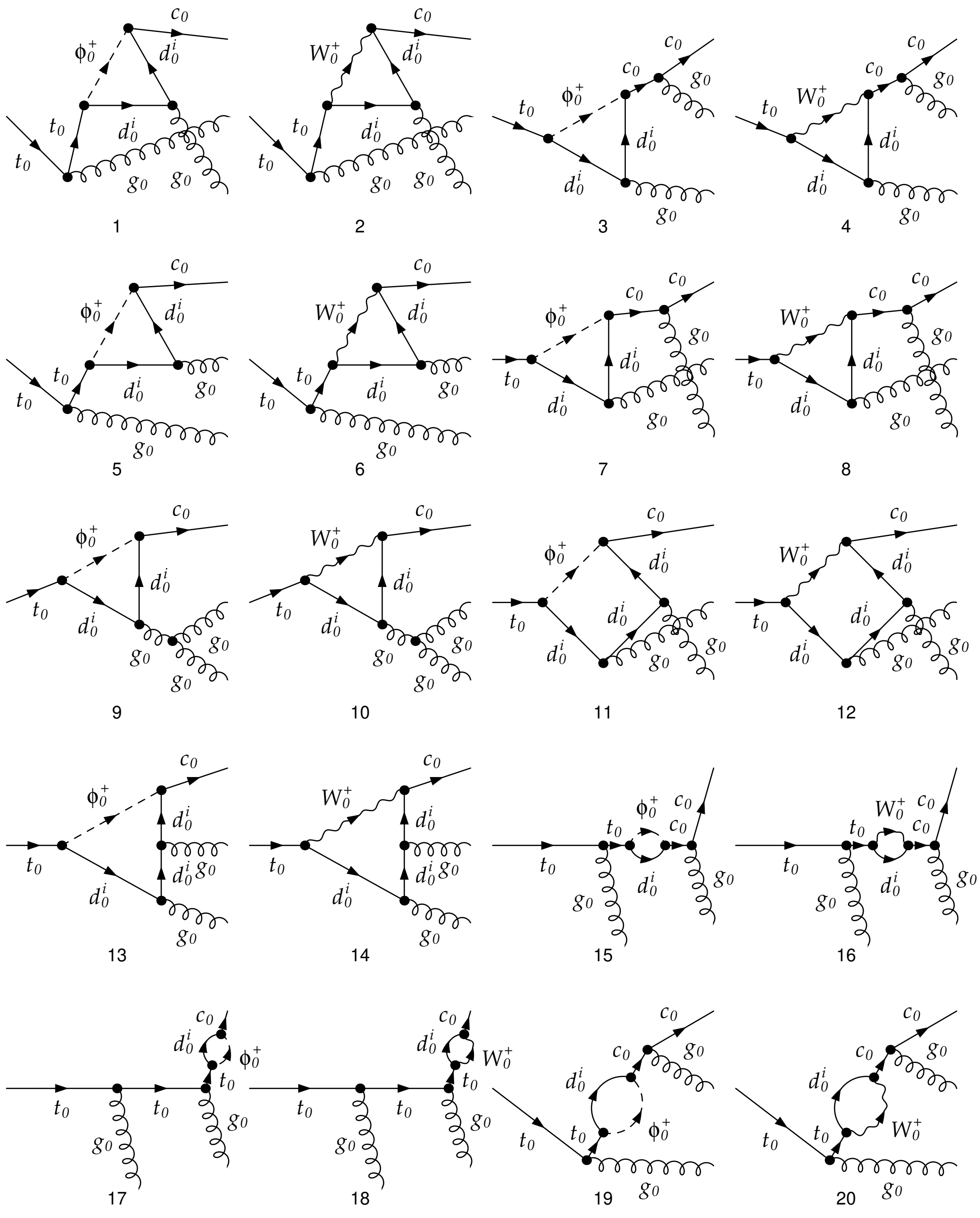


Figure A.5: SM diagrams contributing to the process $t \to cgg$.

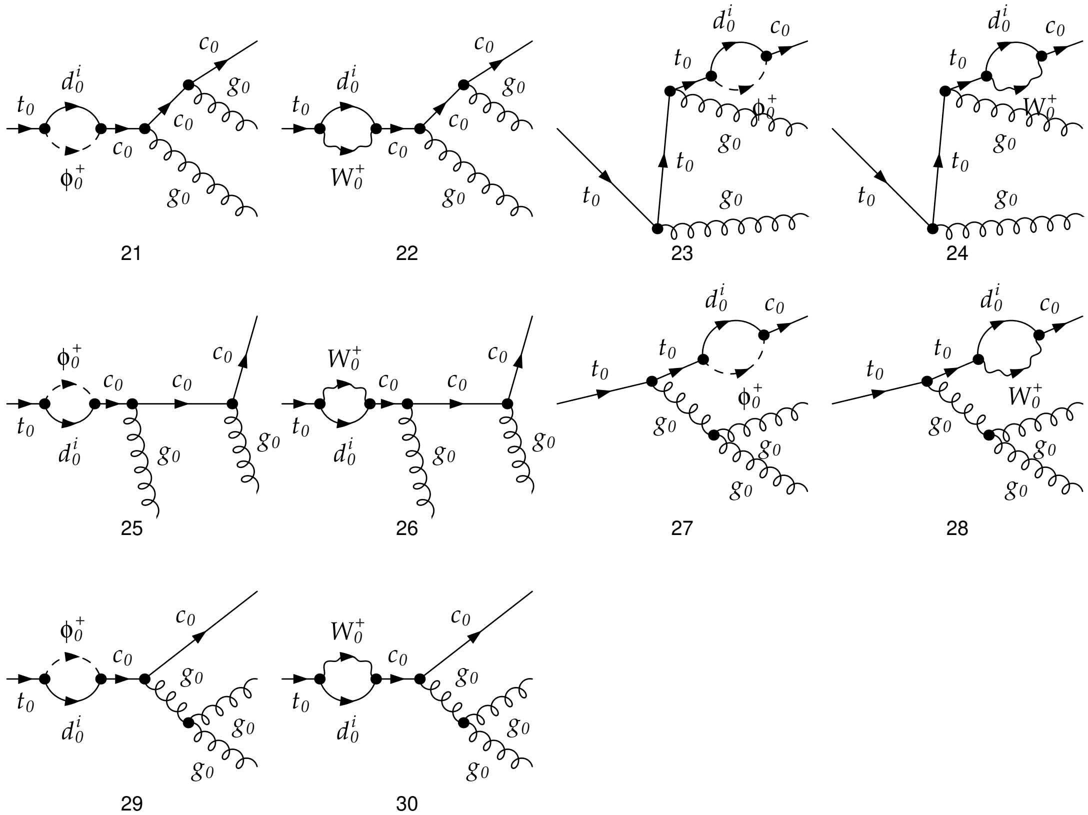


Figure A.6: SM diagrams contributing to the process $t \to cgg$.

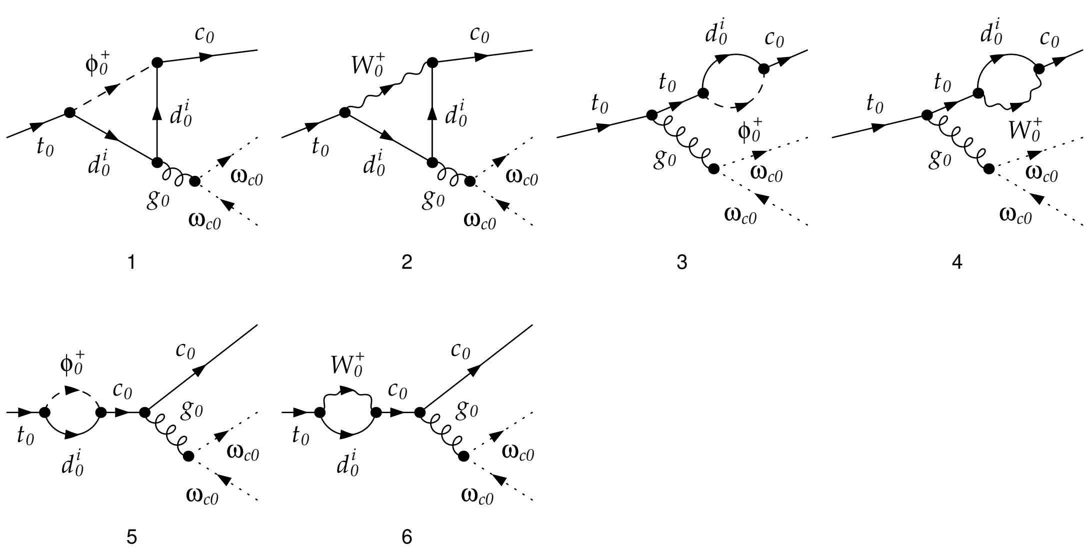


Figure A.7: SM diagrams contributing to the process $t \to cgg$.

A.1.6 $gg \to t\bar{c}$

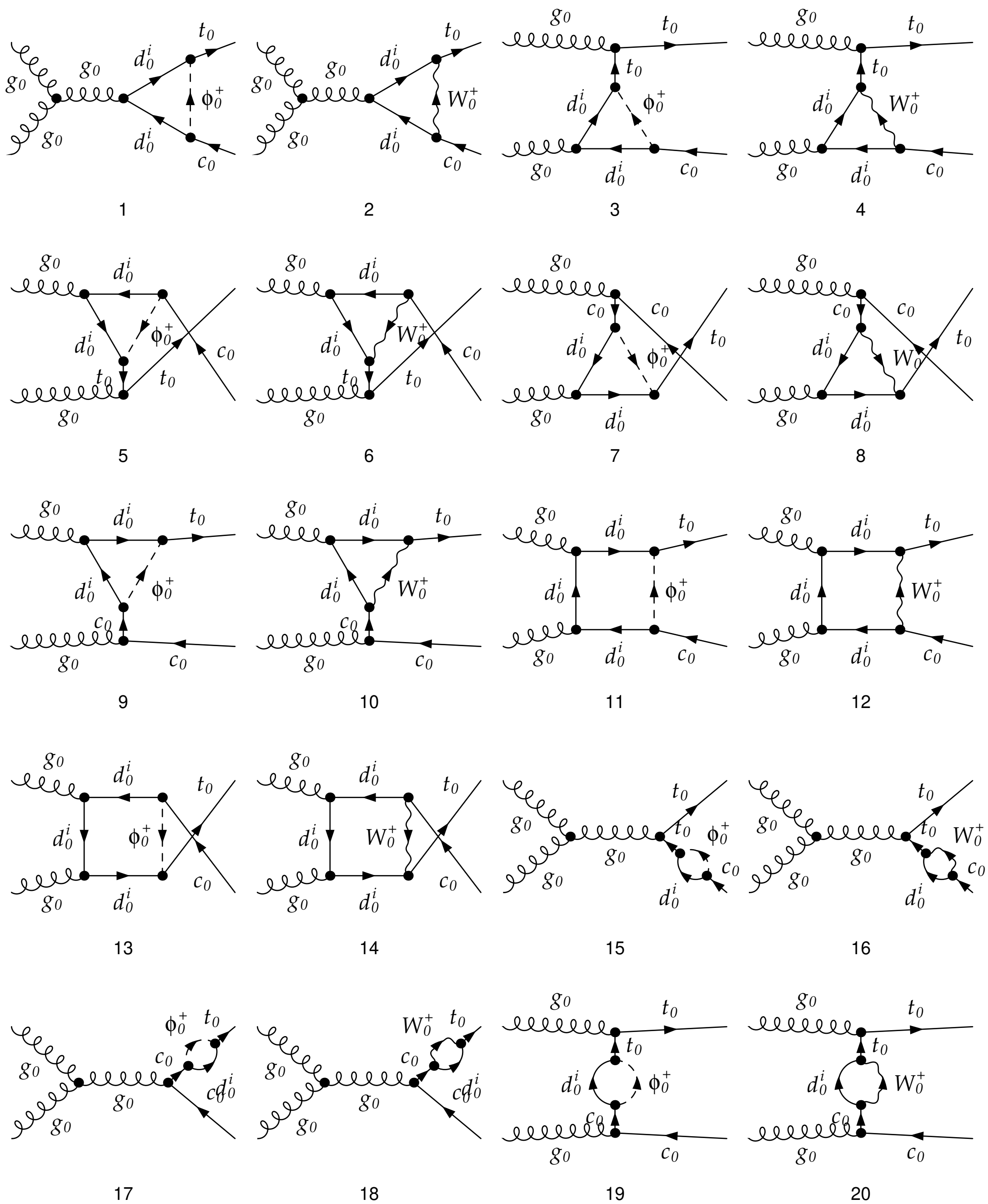


Figure A.8: SM diagrams contributing to the process $gg \to t\bar{c}$.

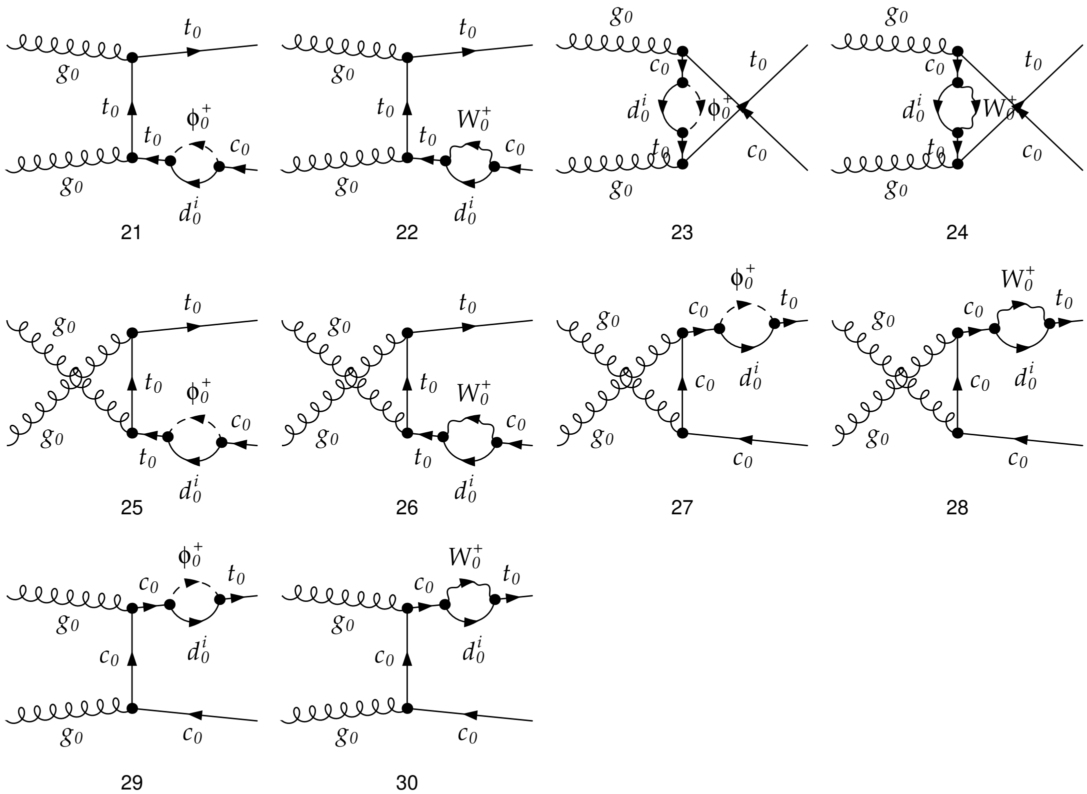


Figure A.9: SM diagrams contributing to the process $gg \to t\bar{c}$.

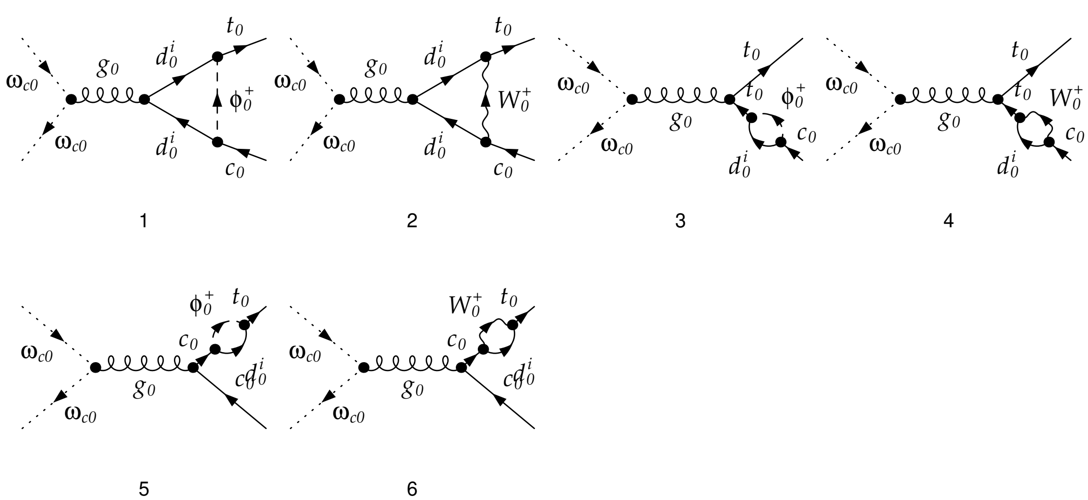


Figure A.10: SM diagrams contributing to the process $gg \to t\bar{c}$.

A.1.7 $cg \to t\gamma$

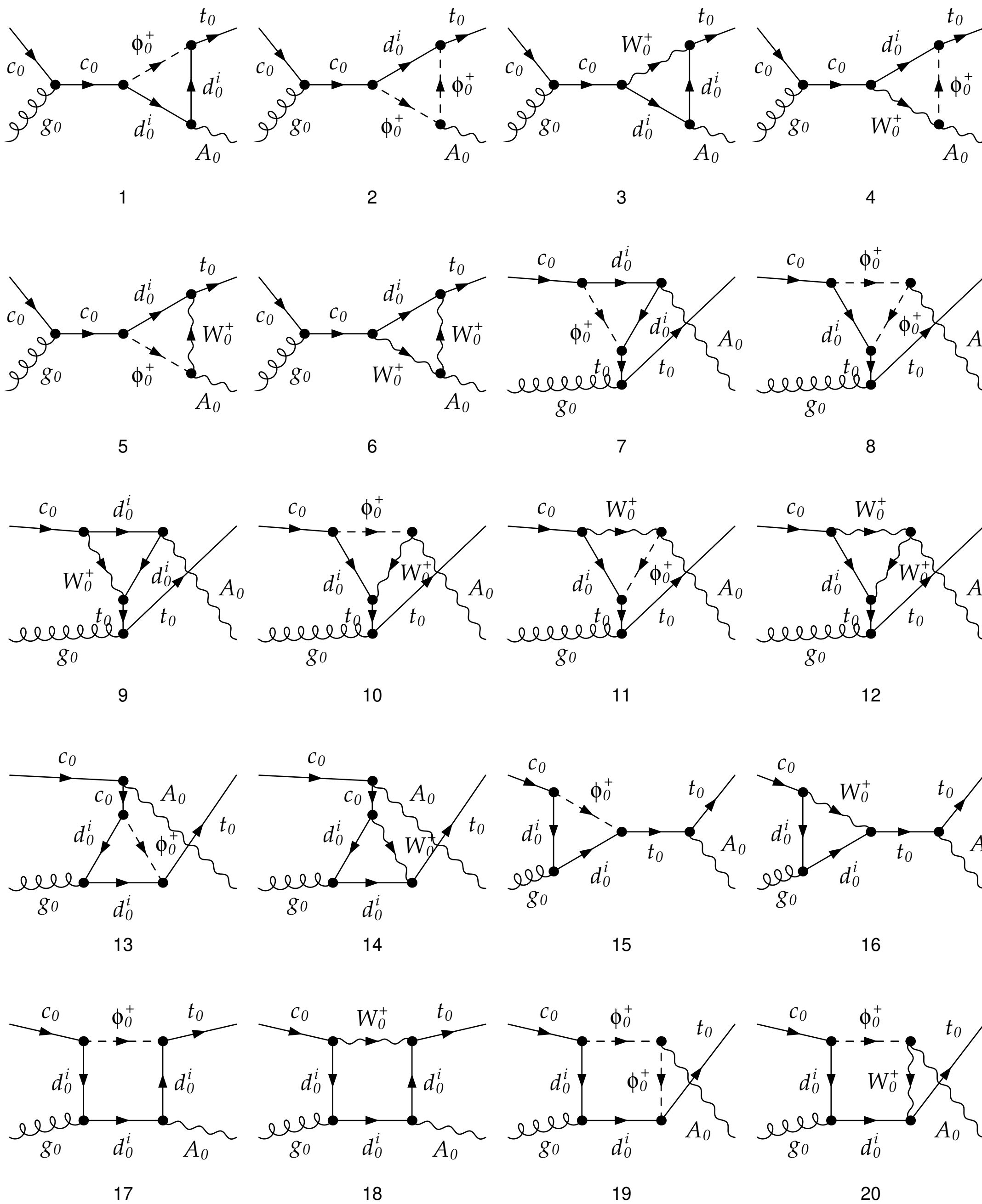


Figure A.11: SM diagrams contributing to the process $cg \to t\gamma$.

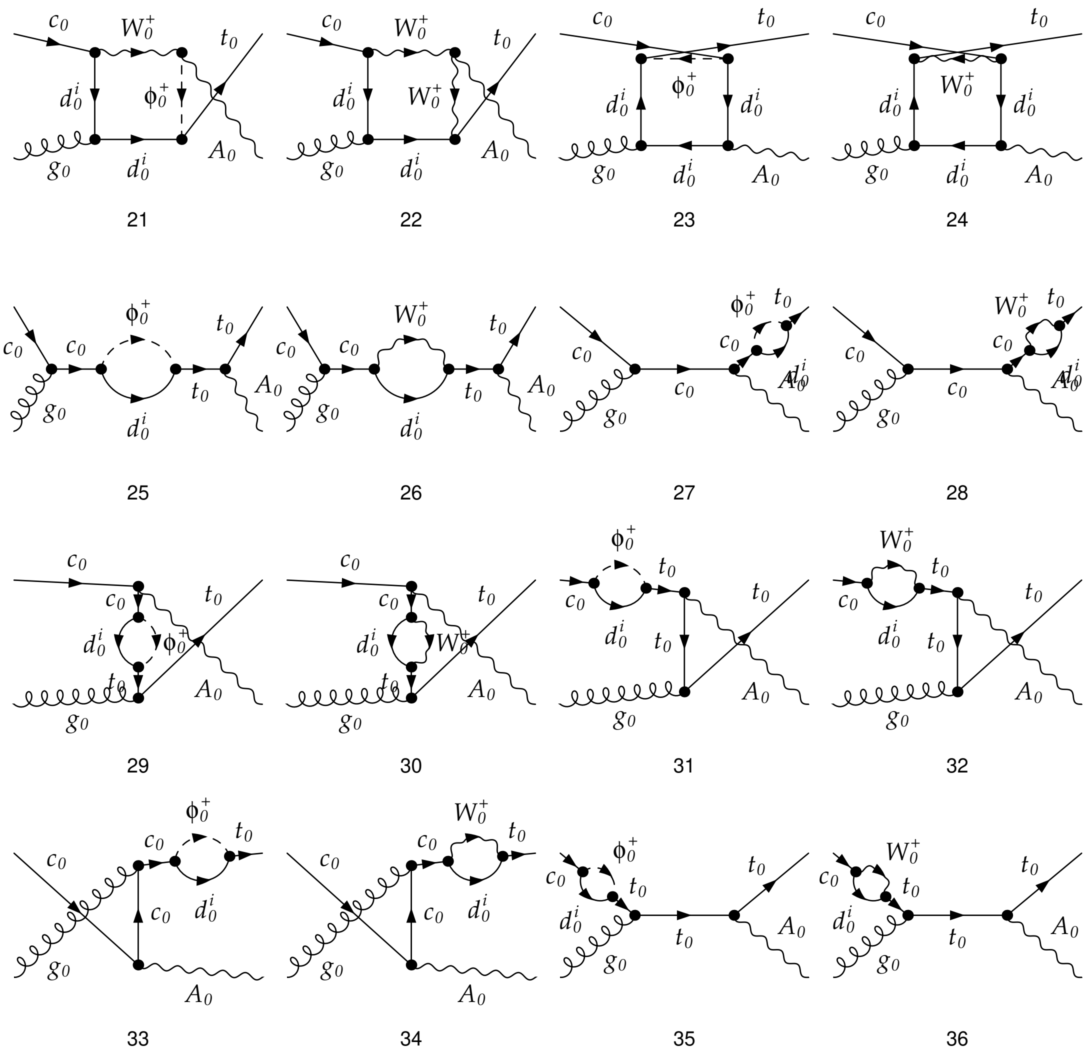


Figure A.12: SM diagrams contributing to the process $cg \to t\gamma$.

A.1.8 $cg \to tg$

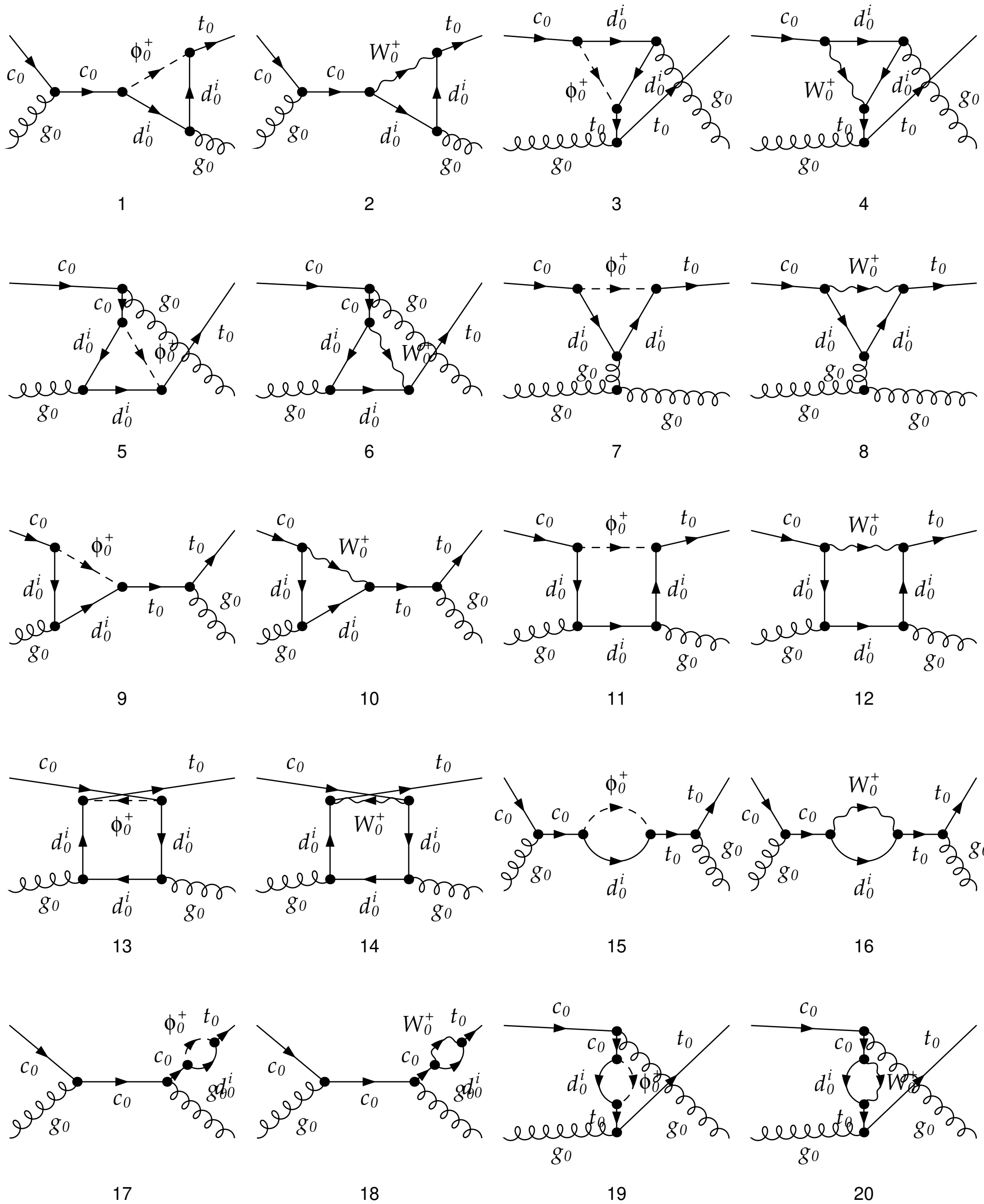


Figure A.13: SM diagrams contributing to the process $cg \to tg$.

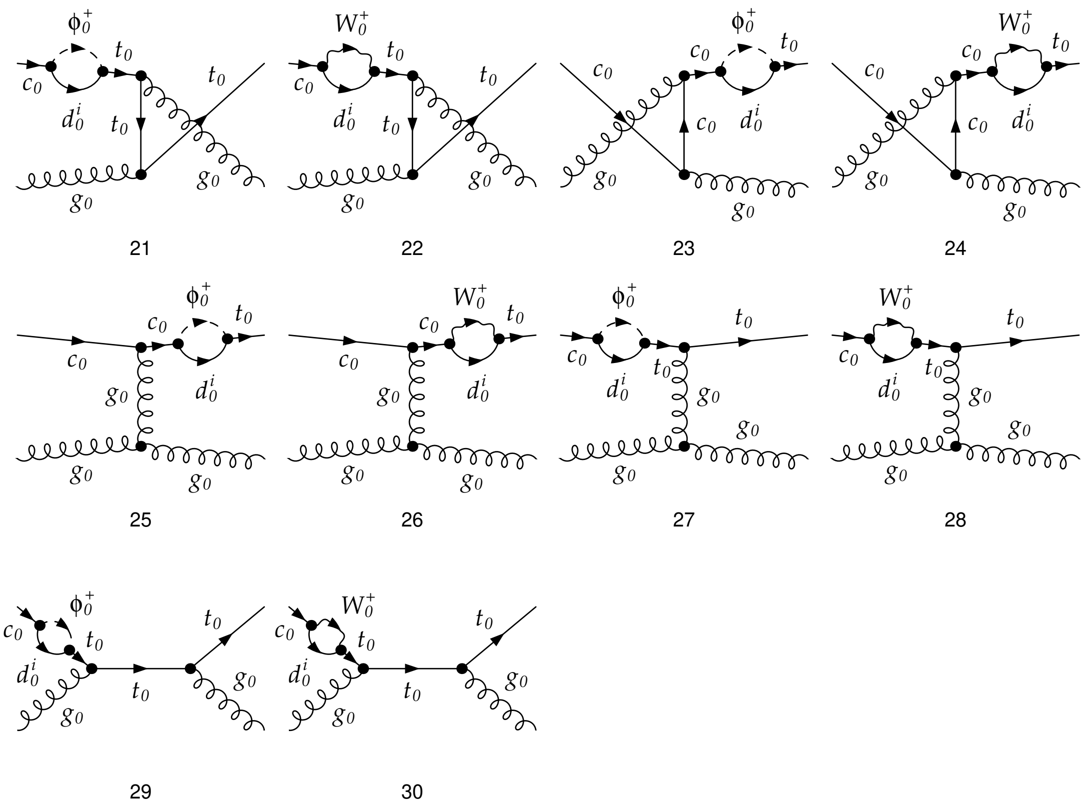


Figure A.14: SM diagrams contributing to the process $cg \to tg$.

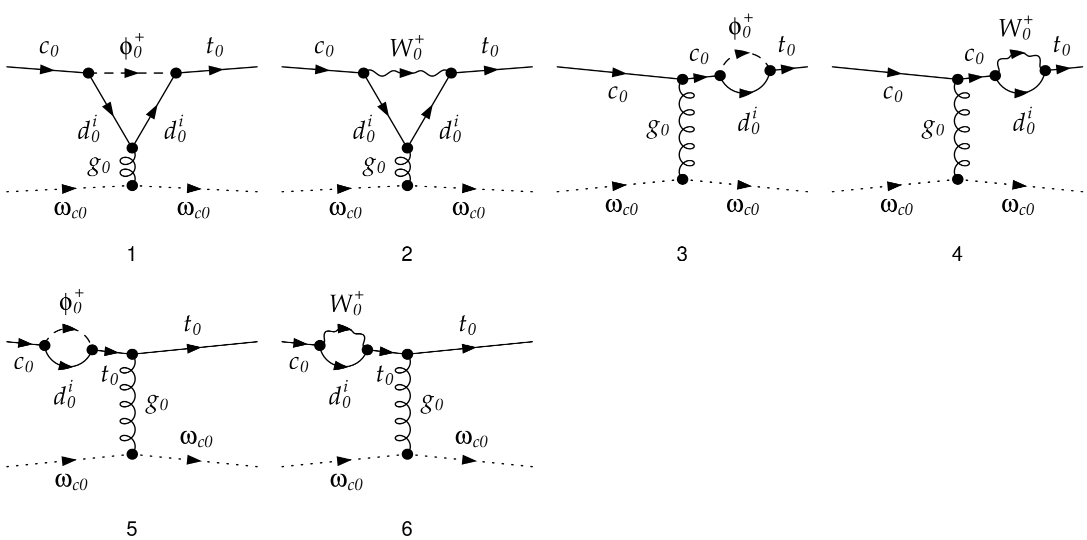


Figure A.15: SM diagrams contributing to the process $cg \to tg$.

A.1.9 $cg \to th$

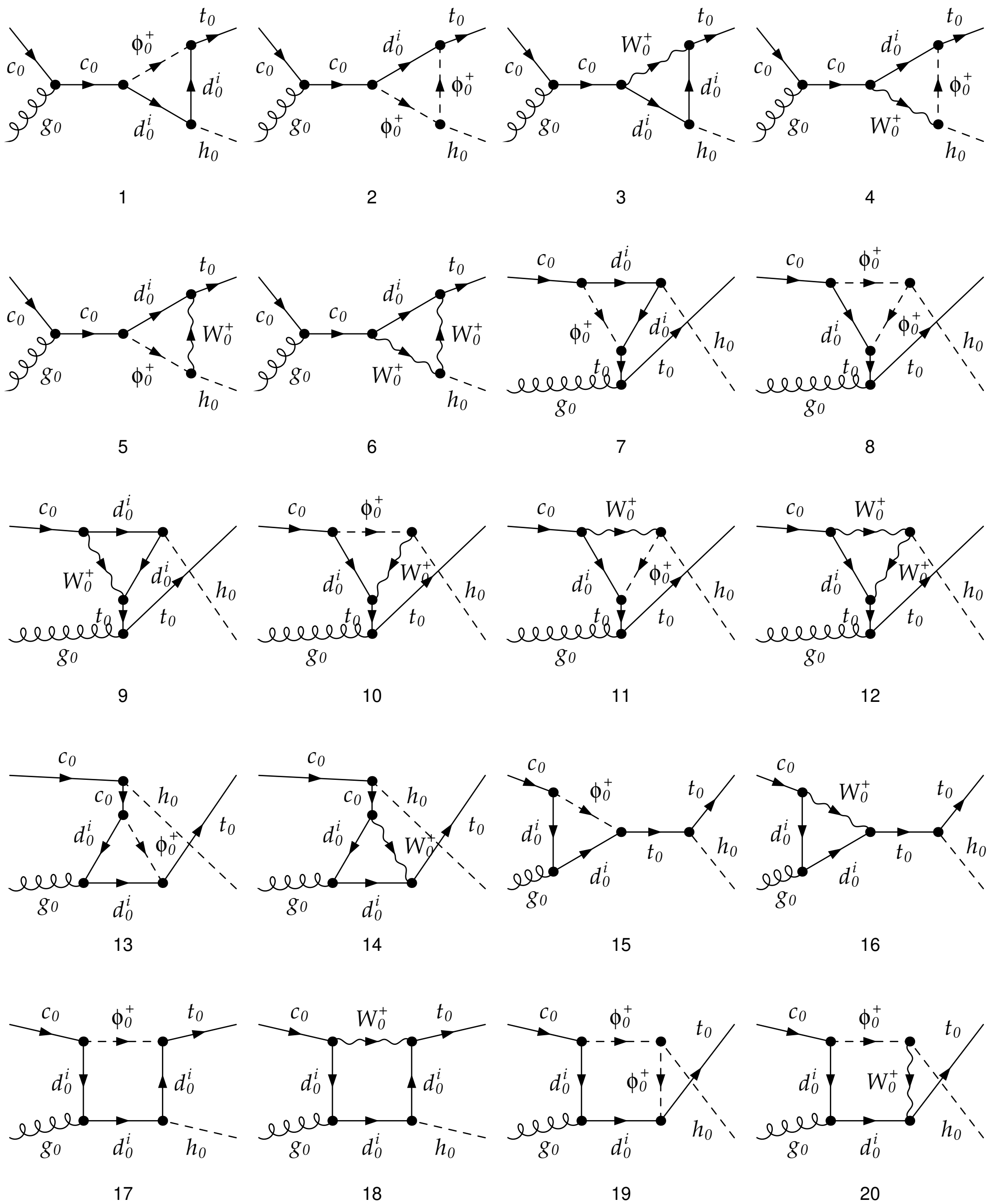


Figure A.16: SM diagrams contributing to the process $cg \to th$.

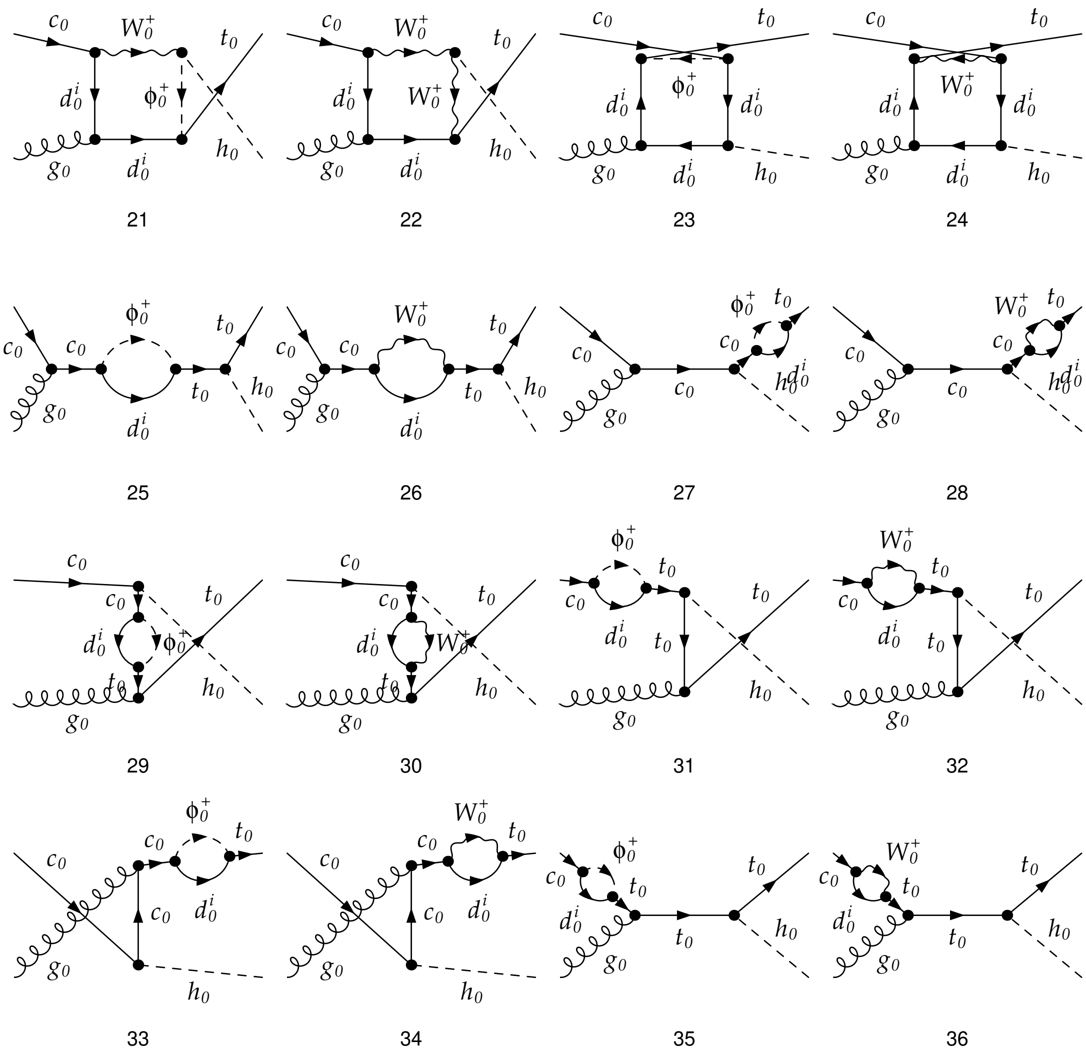


Figure A.17: SM diagrams contributing to the process $cg \to th$.

A.1.10 $cg \to tZ$

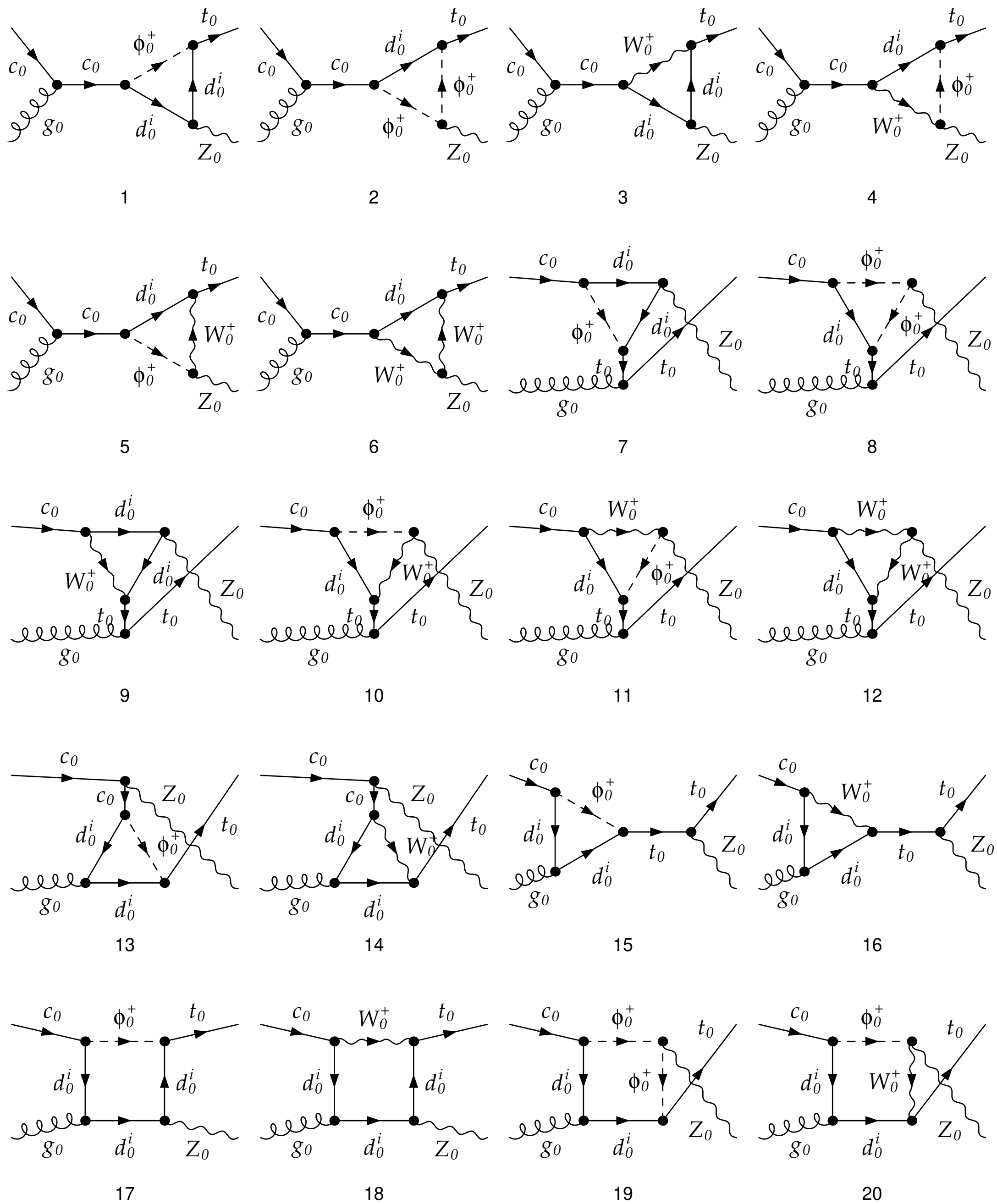


Figure A.18: SM diagrams contributing to the process $cg \to tZ$.

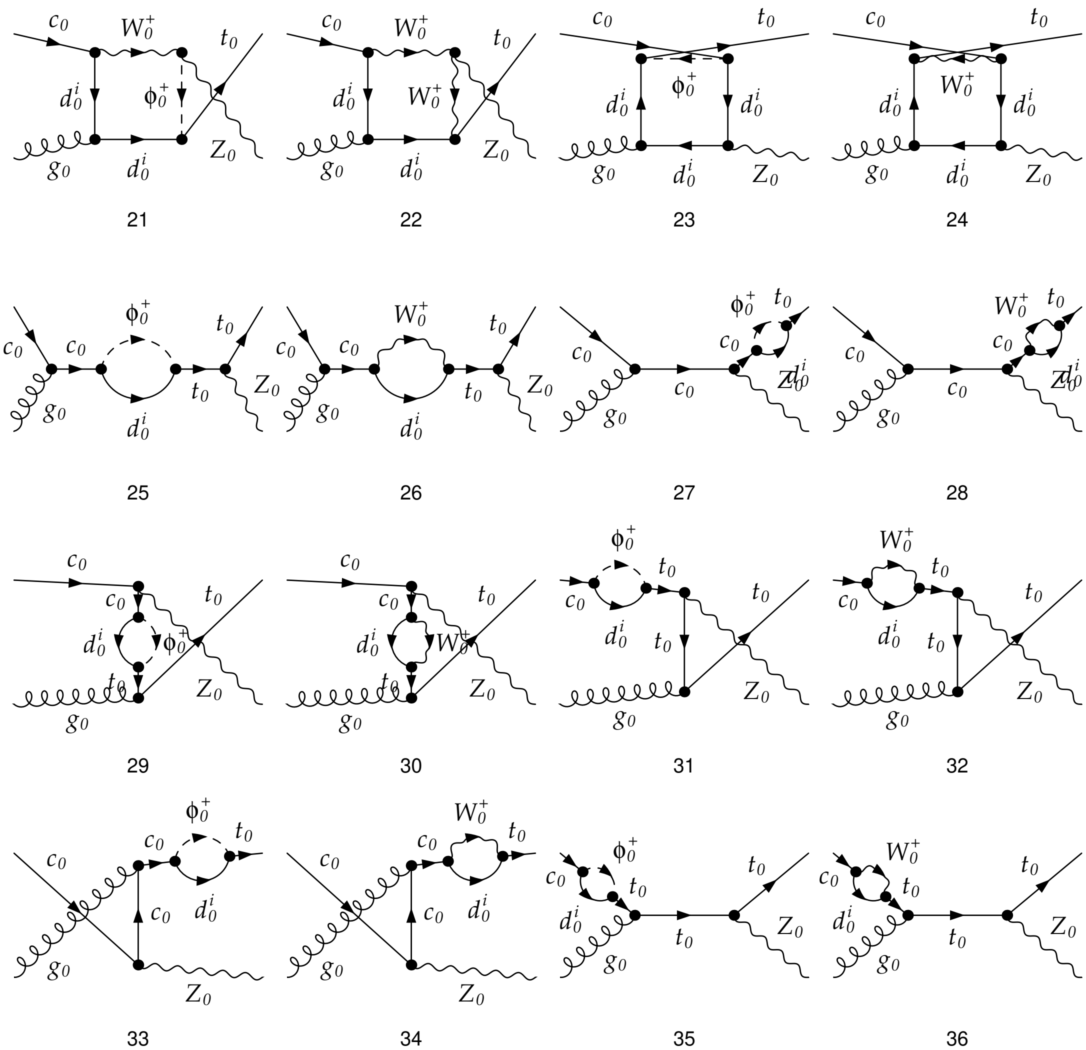


Figure A.19: SM diagrams contributing to the process $cg \to tZ$.

A.2 Feynman diagrams at MUED level

A.2.1 $t \to c\gamma$

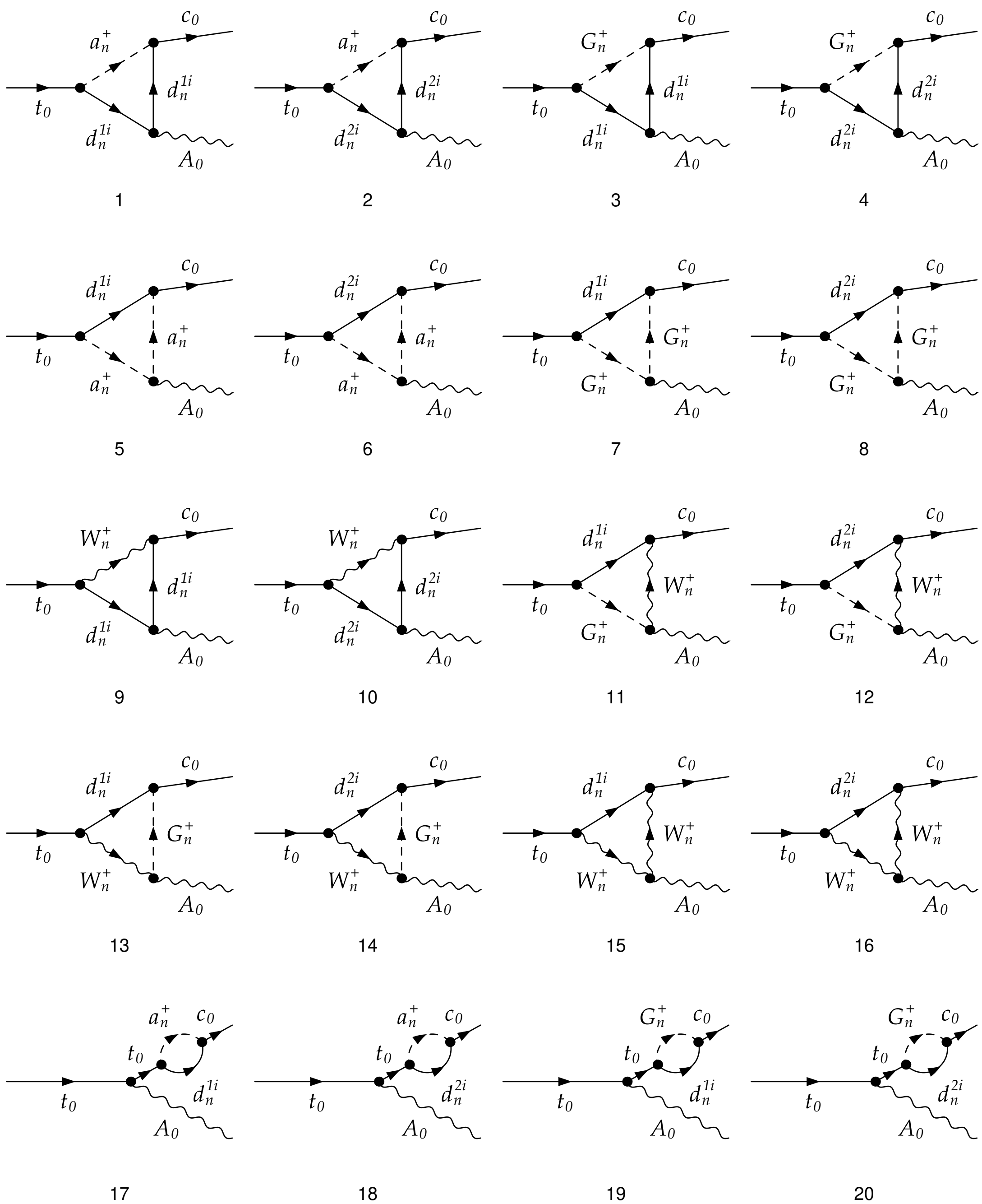


Figure A.20: MUED diagrams contributing to the process $t \to c\gamma$.

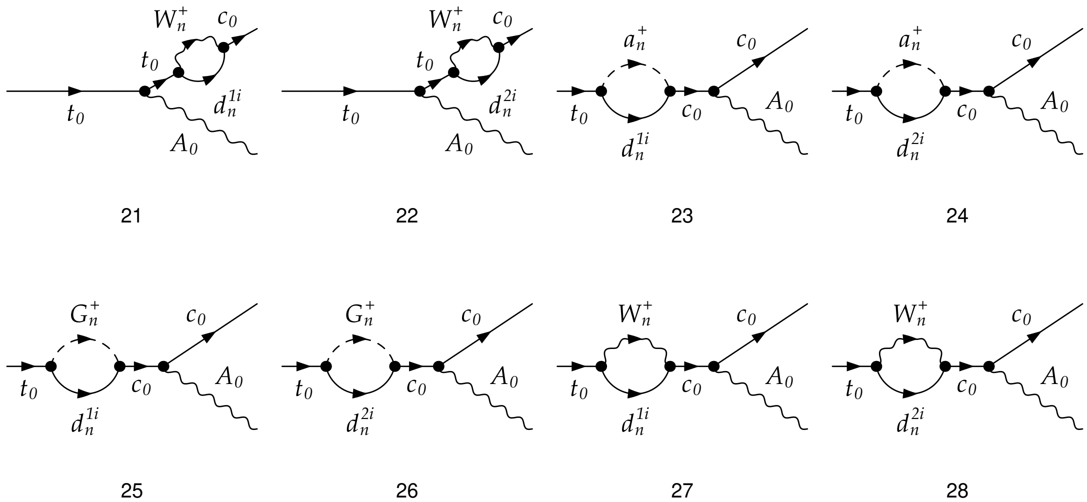


Figure A.21: MUED diagrams contributing to the process $t \to c\gamma$.

A.2.2 $t \to cg$

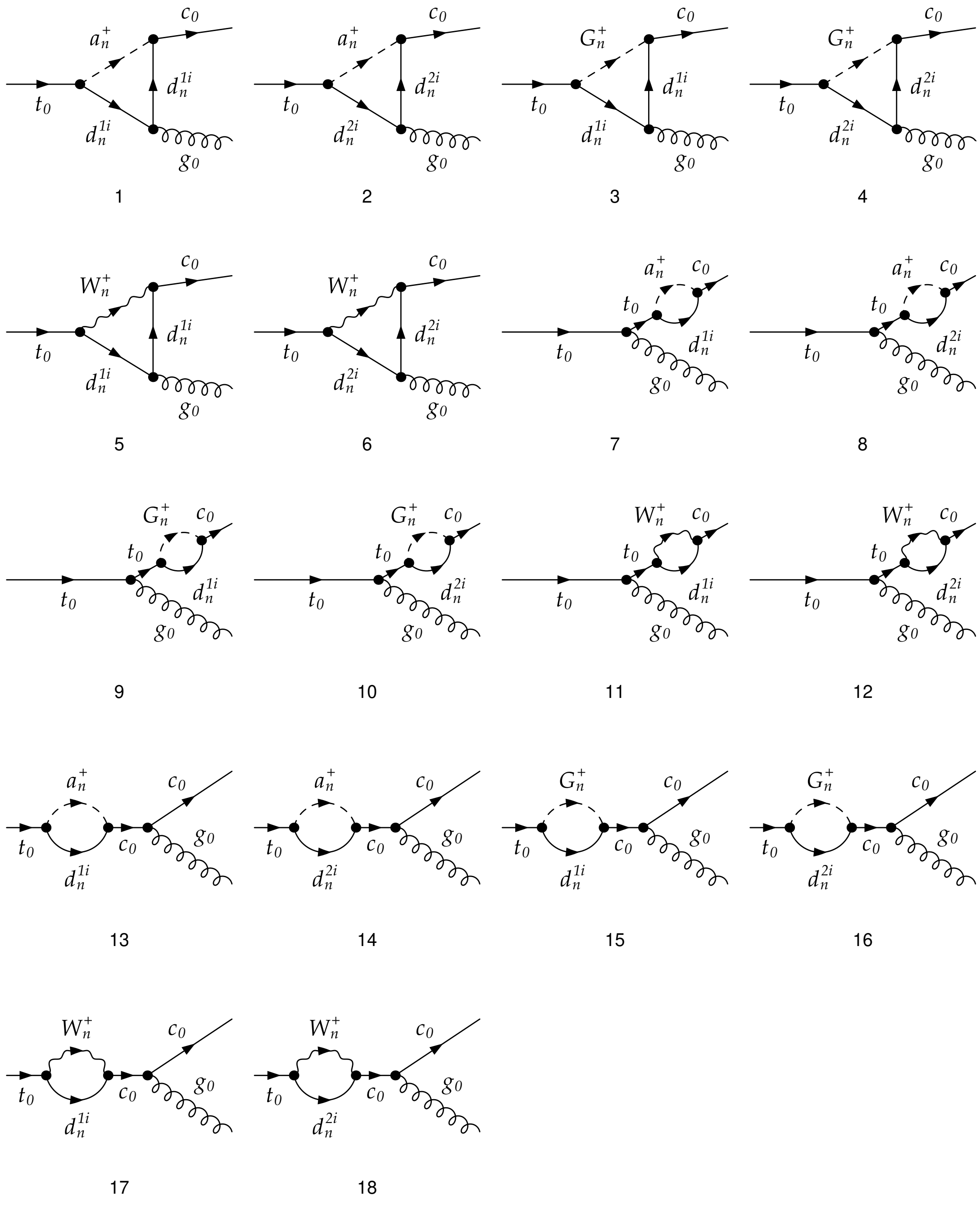


Figure A.22: MUED diagrams contributing to the process $t \to cg$.

A.2.3 $t \to ch$

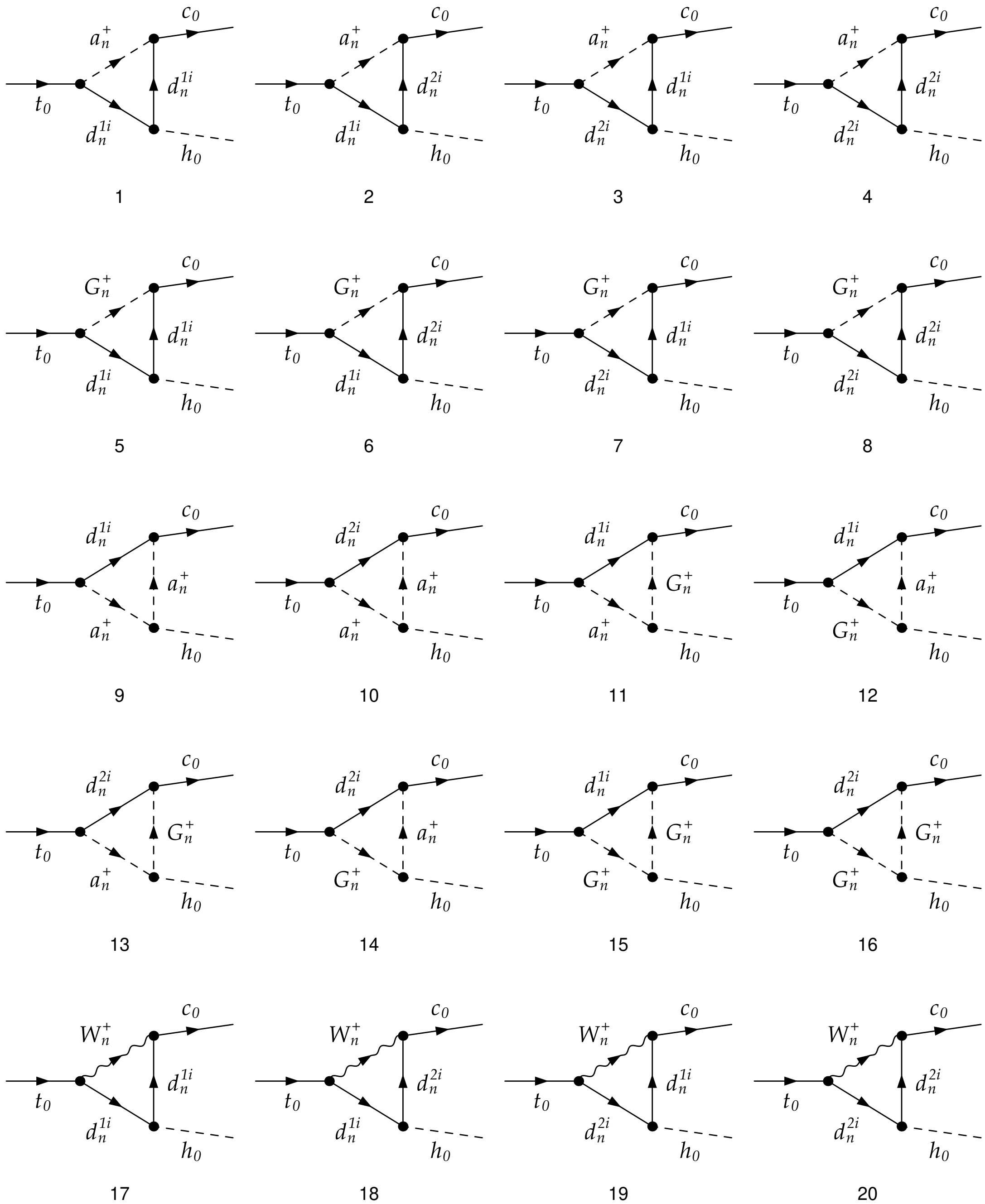


Figure A.23: MUED diagrams contributing to the process $t \to ch$.

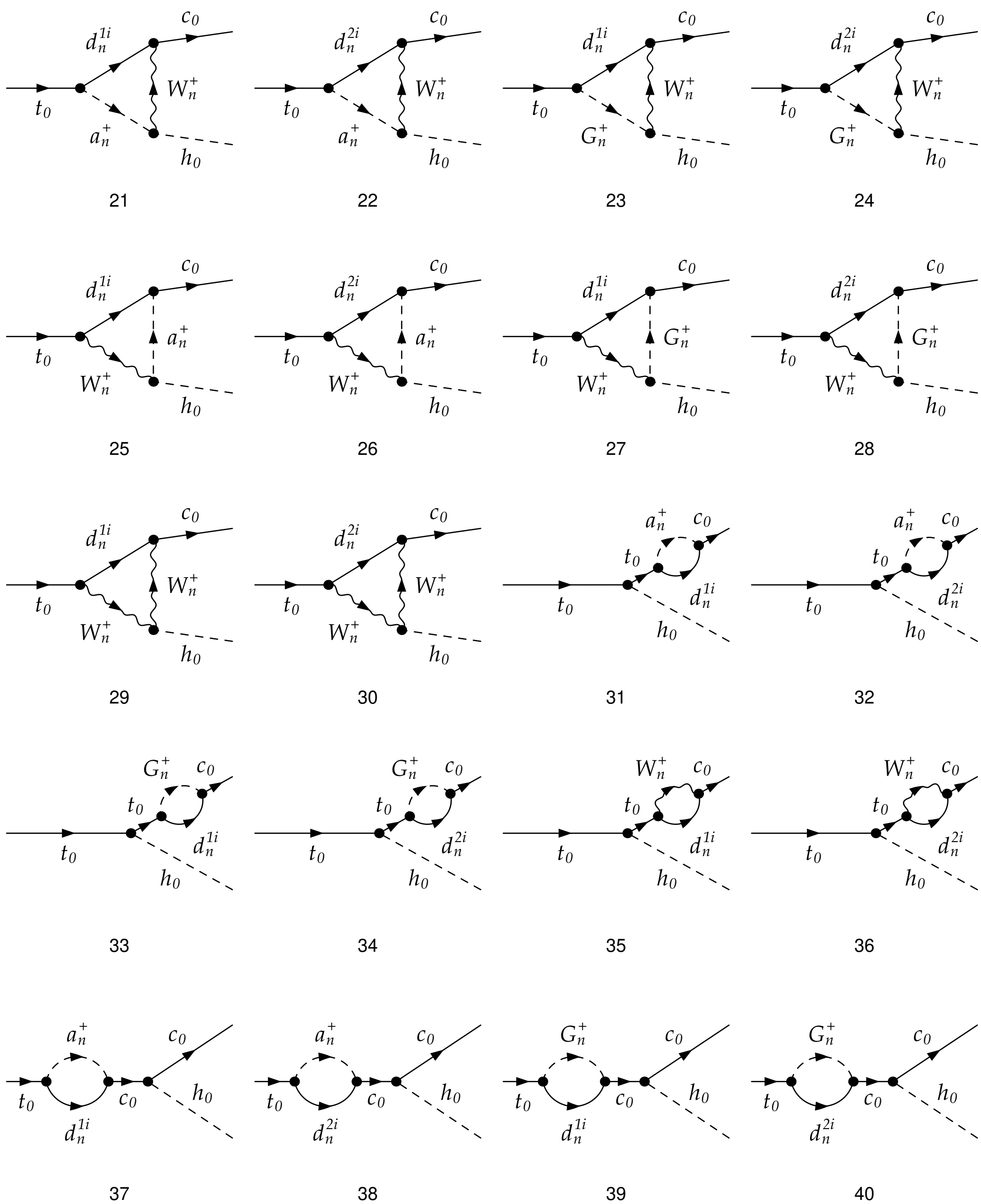
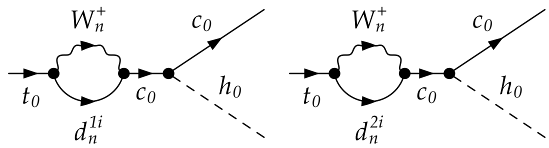


Figure A.24: MUED diagrams contributing to the process $t \to ch$.



41 42

Figure A.25: MUED diagrams contributing to the process $t \to ch$.

A.2.4 $t \to cZ$

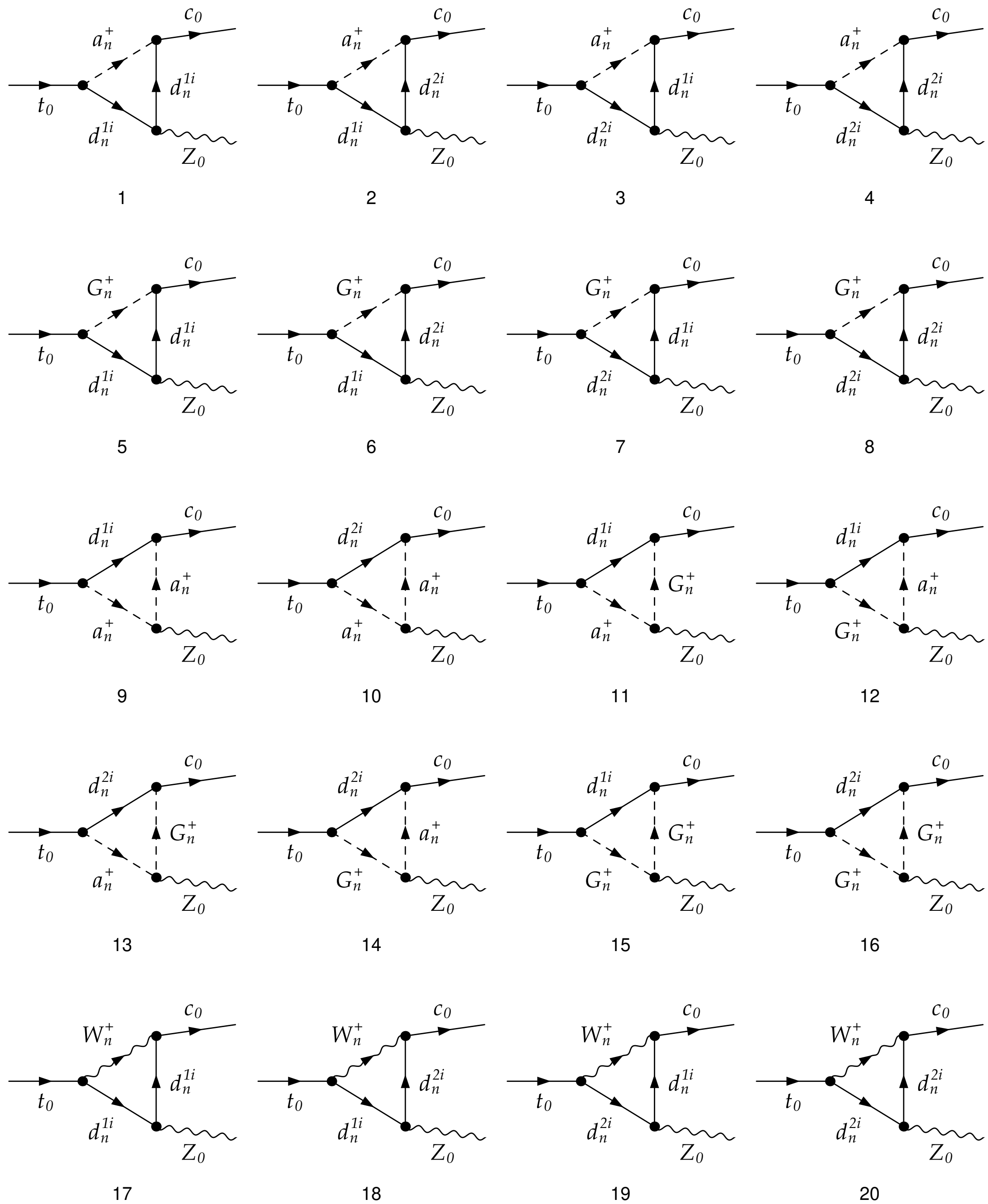


Figure A.26: MUED diagrams contributing to the process $t \to cZ$.

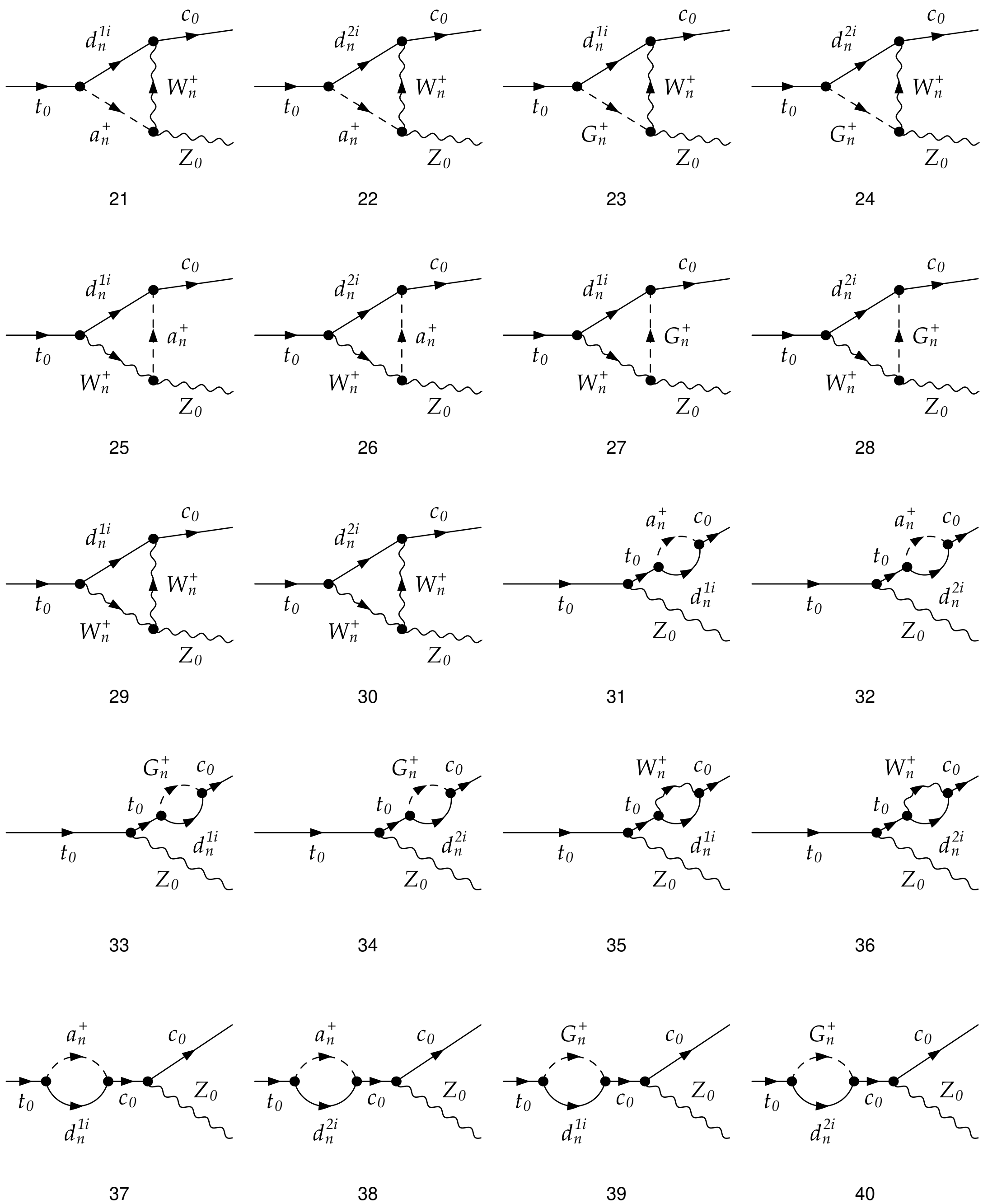


Figure A.27: MUED diagrams contributing to the process $t \to cZ$.

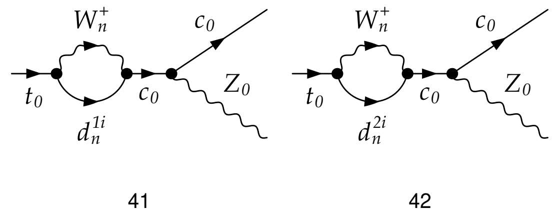


Figure A.28: MUED diagrams contributing to the process $t \to cZ$.

A.2.5 $t \to cgg$

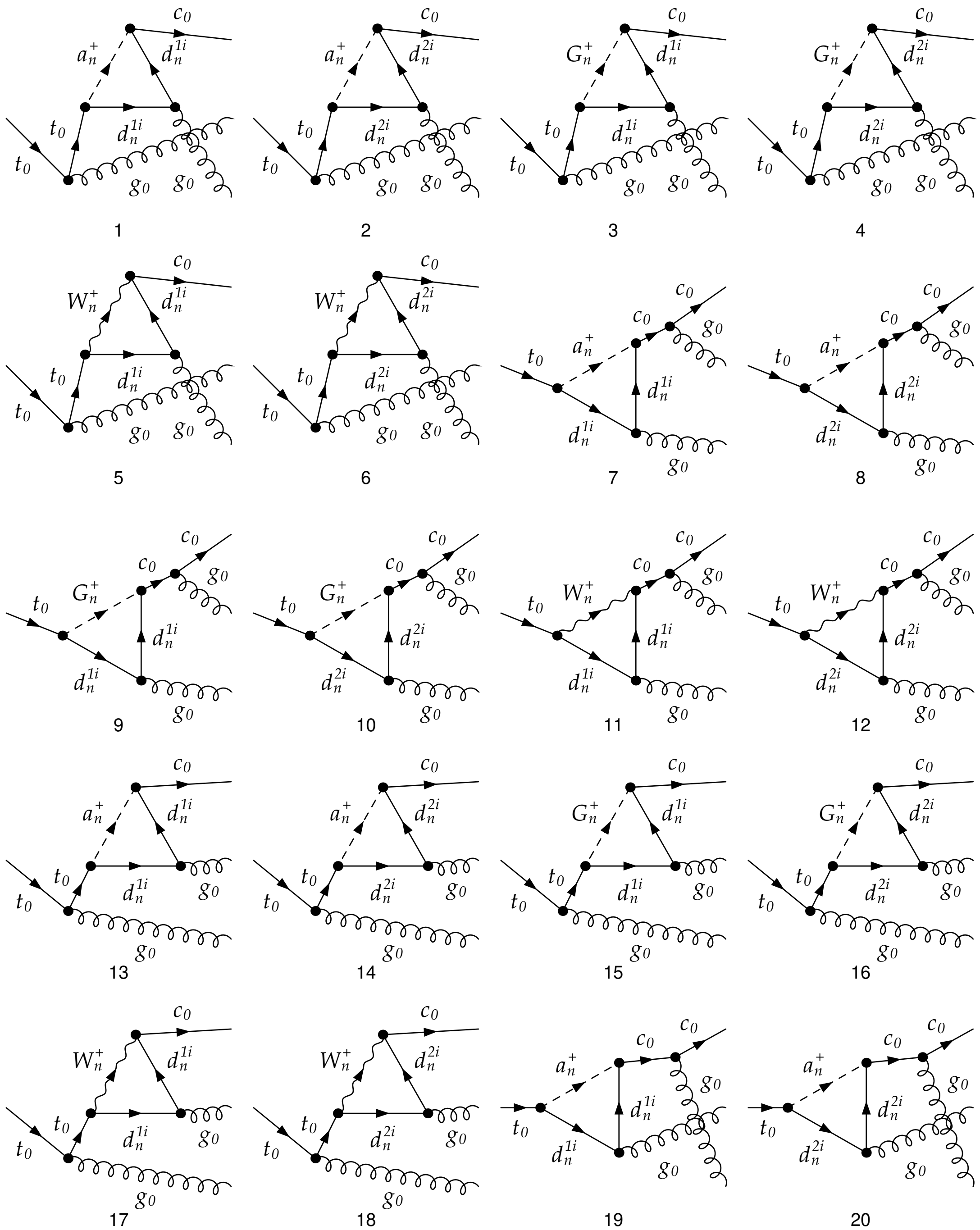


Figure A.29: MUED diagrams contributing to the process $t \to cgg$.

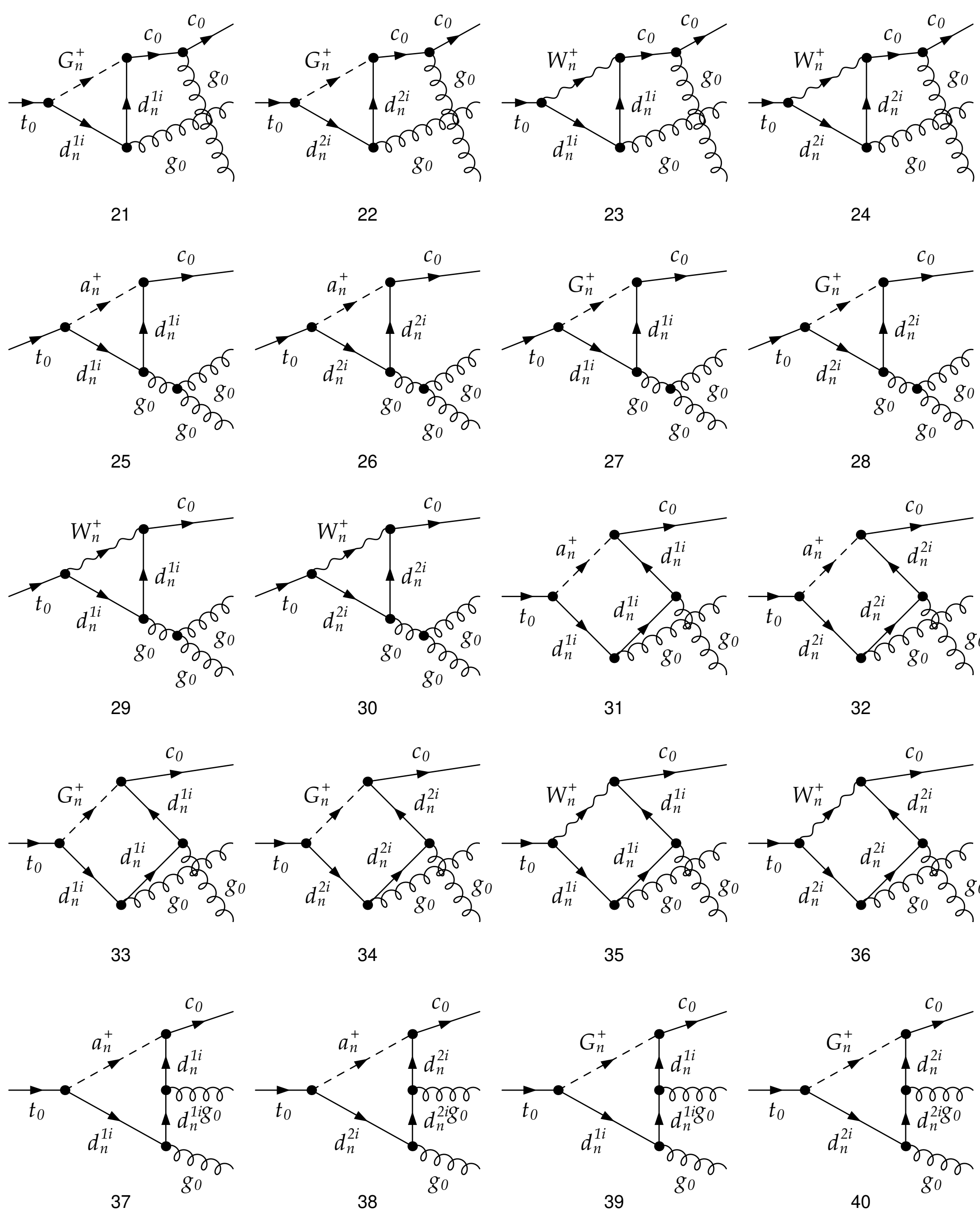


Figure A.30: MUED diagrams contributing to the process $t \to cgg$.

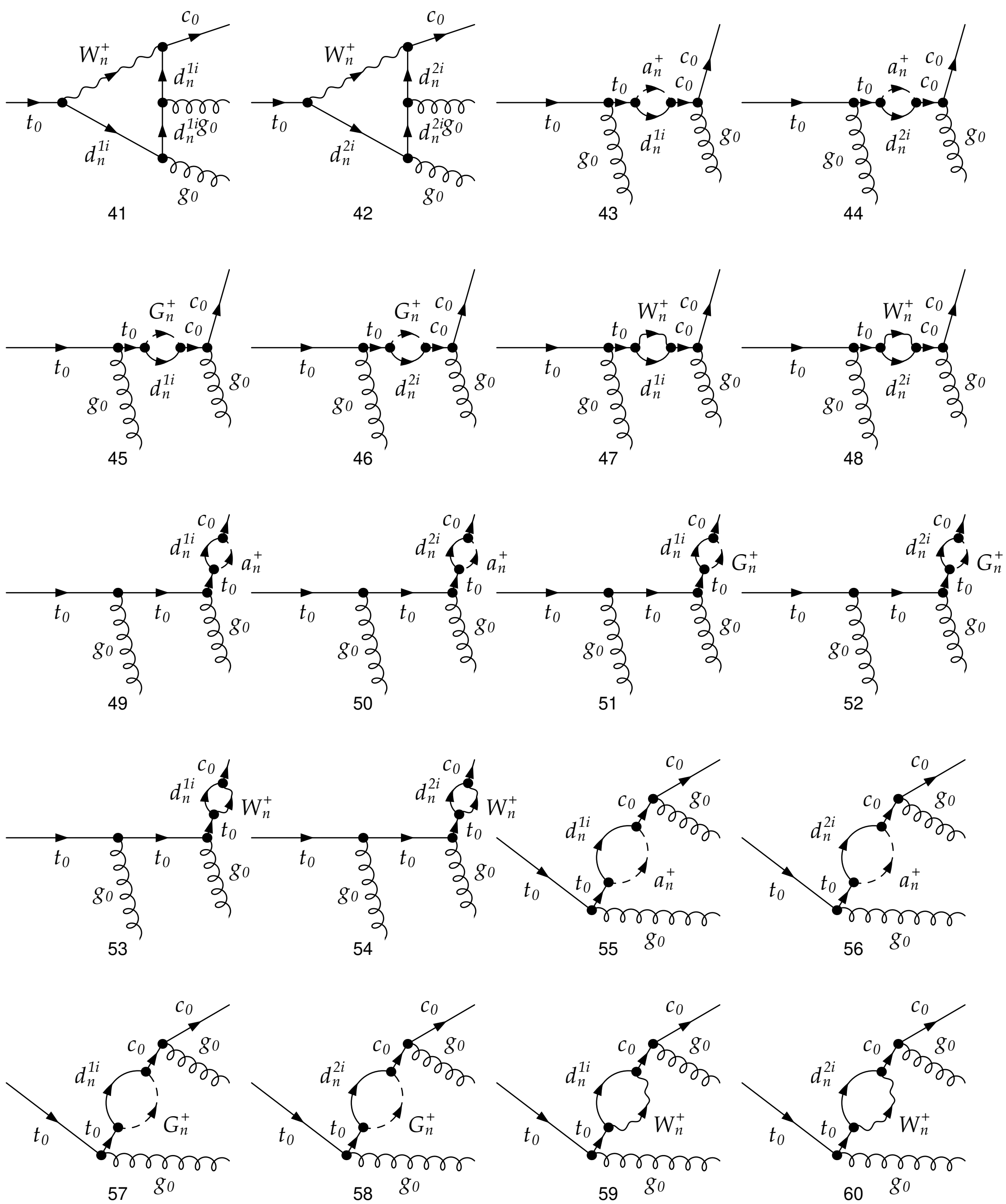


Figure A.31: MUED diagrams contributing to the process $t \to cgg$.

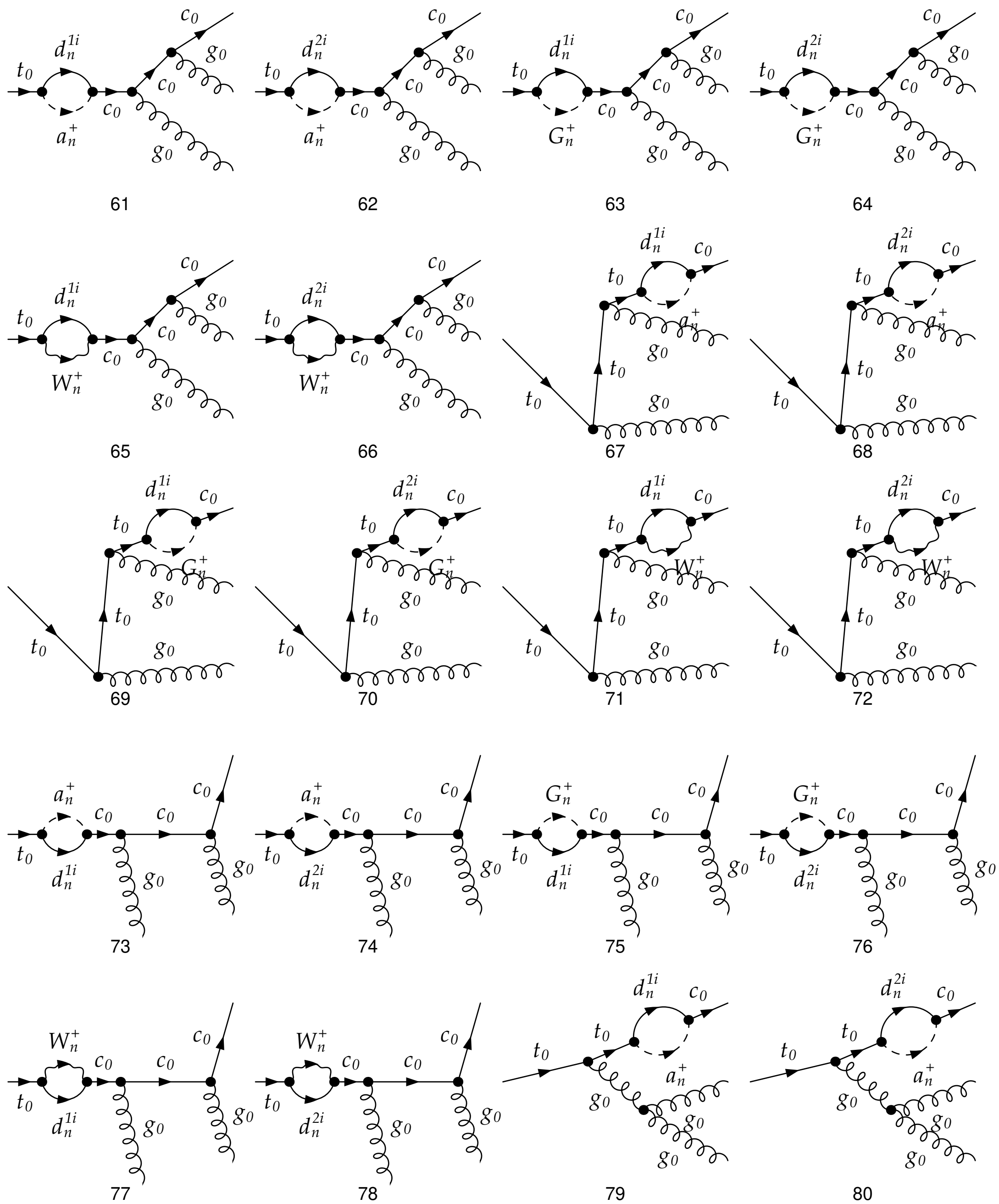


Figure A.32: MUED diagrams contributing to the process $t \to cgg$.

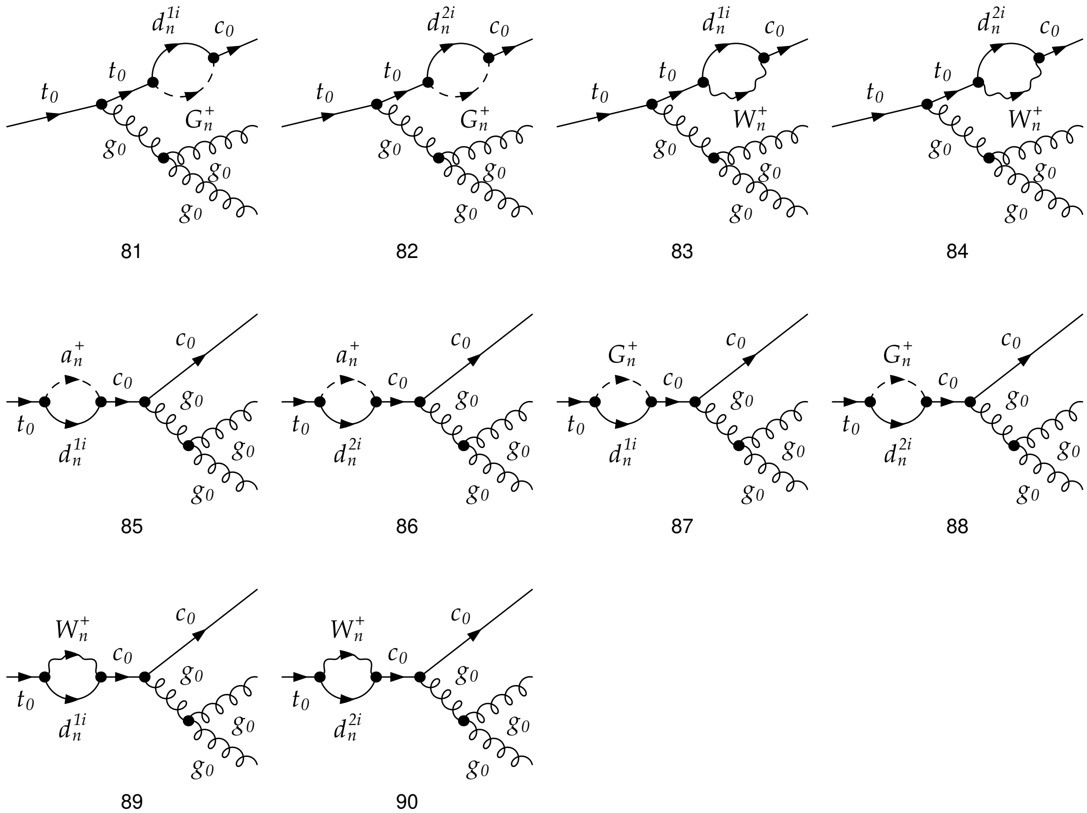


Figure A.33: MUED diagrams contributing to the process $t \to cgg$.

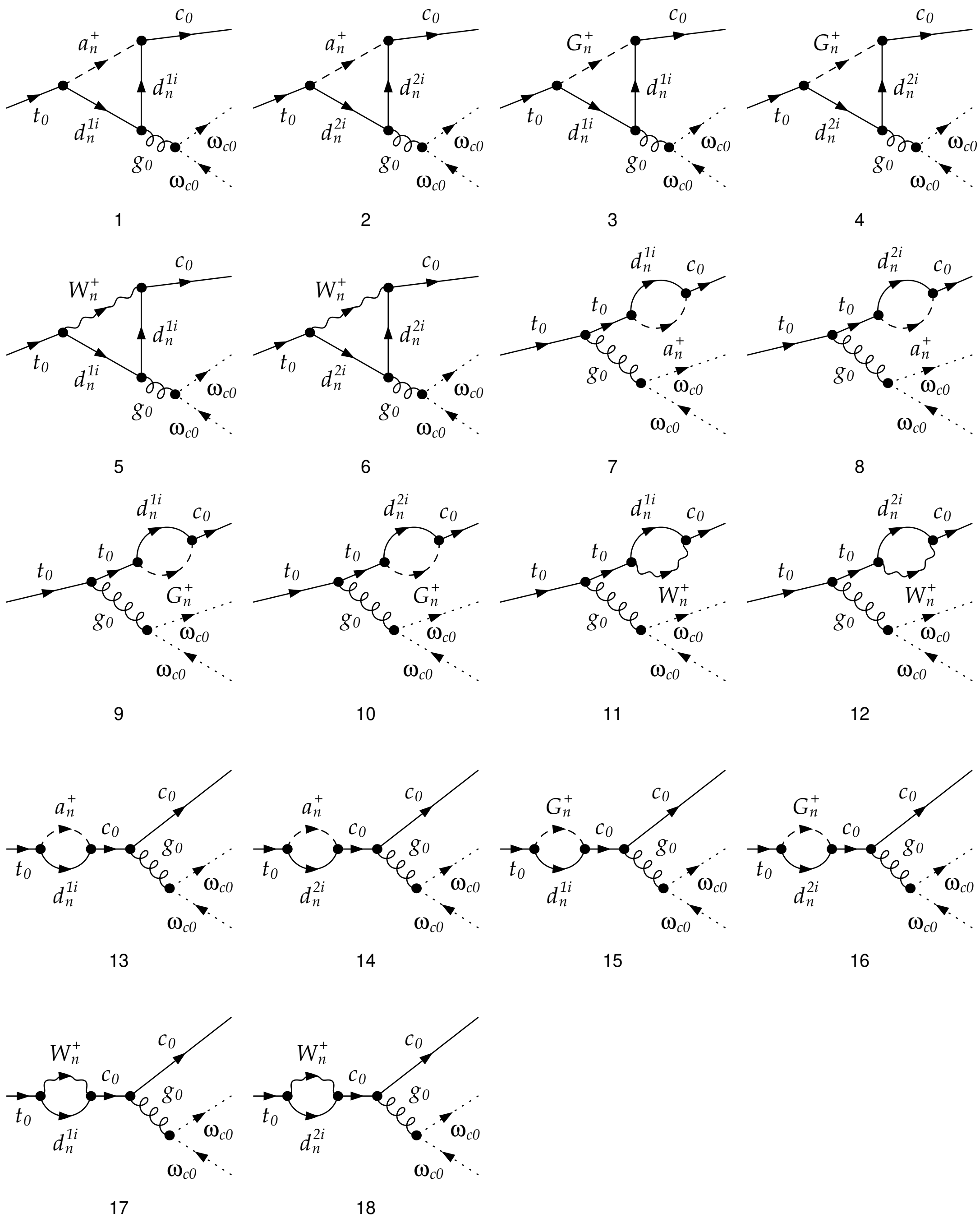


Figure A.34: MUED diagrams contributing to the process $t \to cgg$.

A.2.6 $gg \to t\bar{c}$

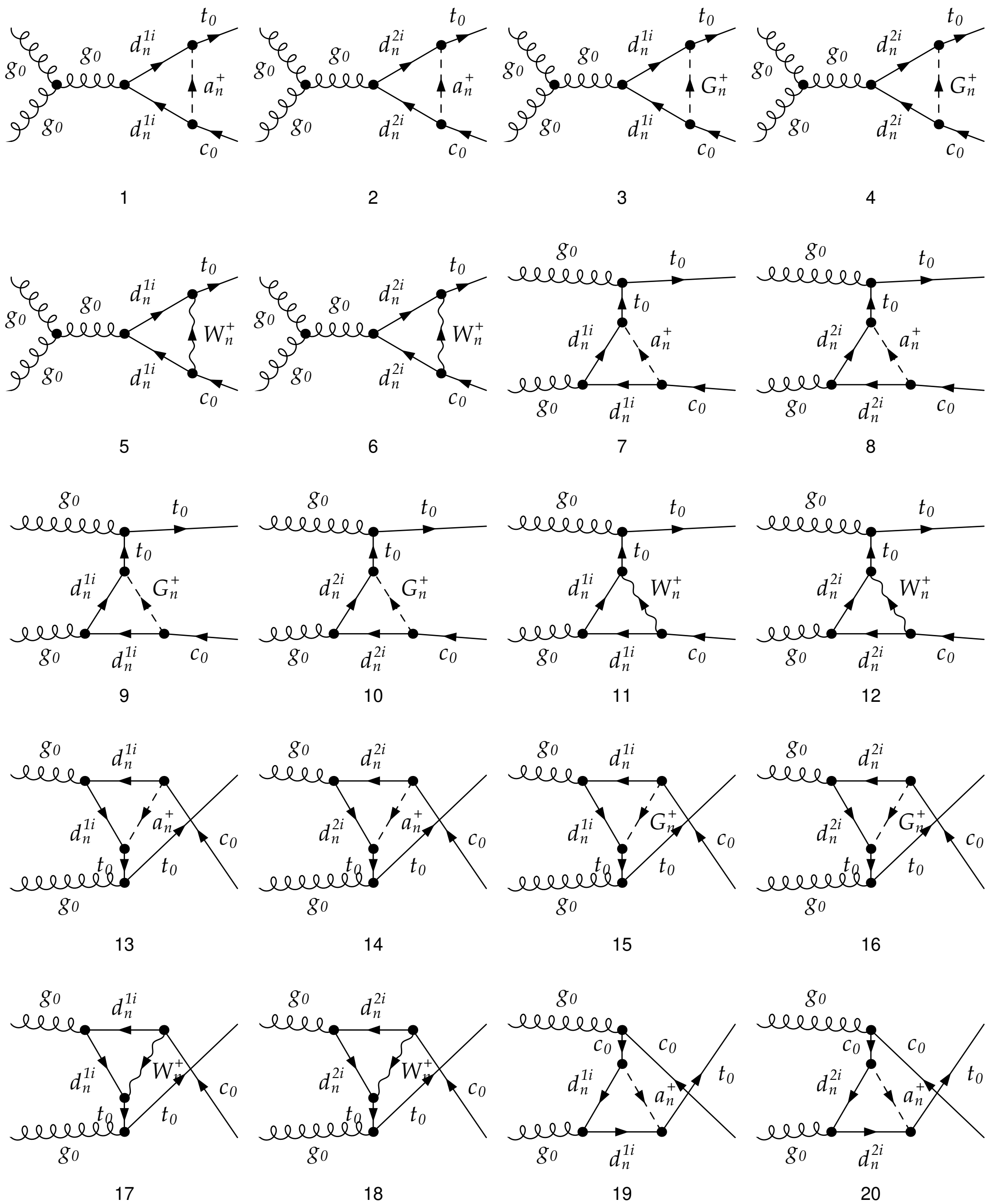


Figure A.35: MUED diagrams contributing to the process $gg \to t\bar{c}$.

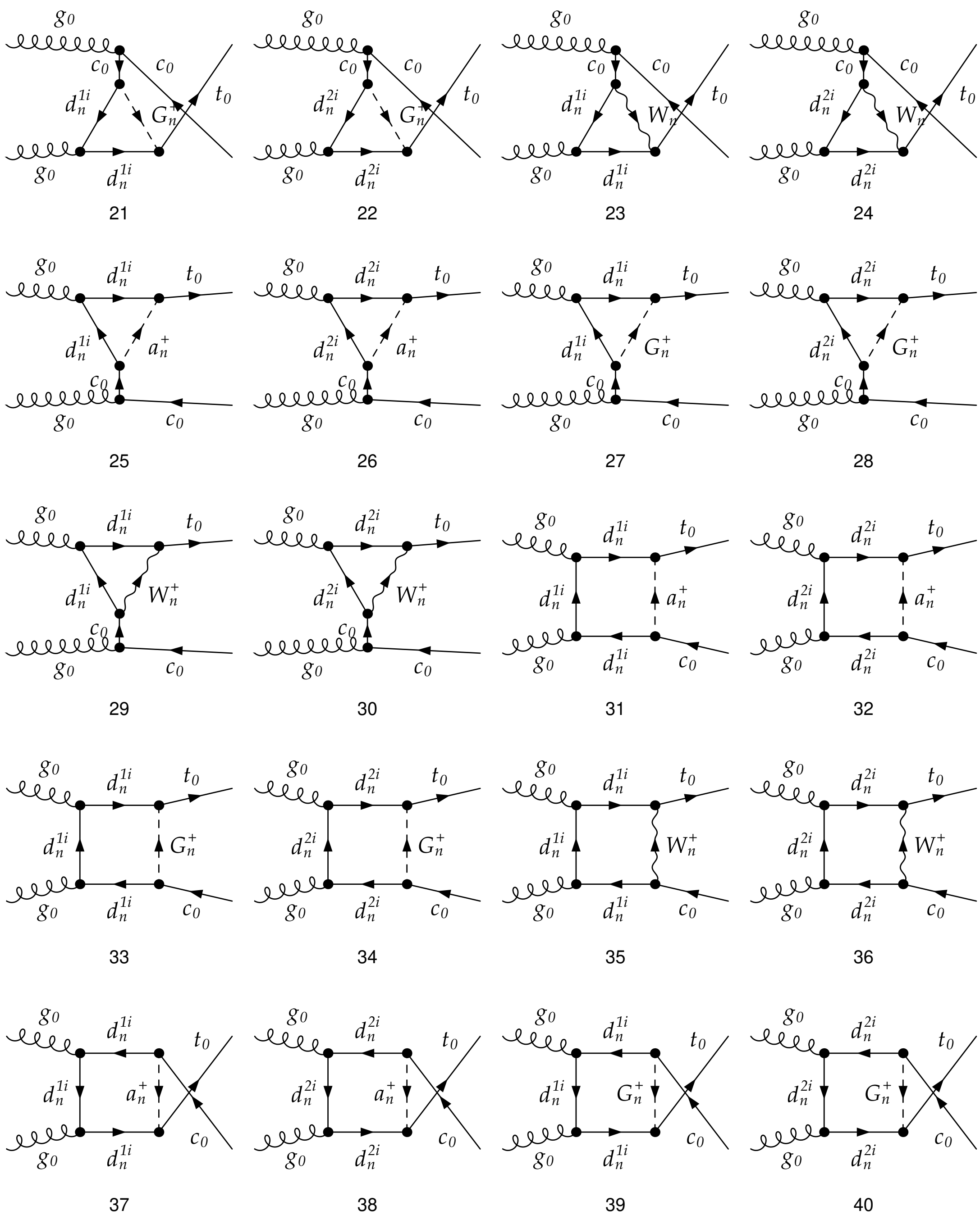


Figure A.36: MUED diagrams contributing to the process $gg \to t\bar{c}$.

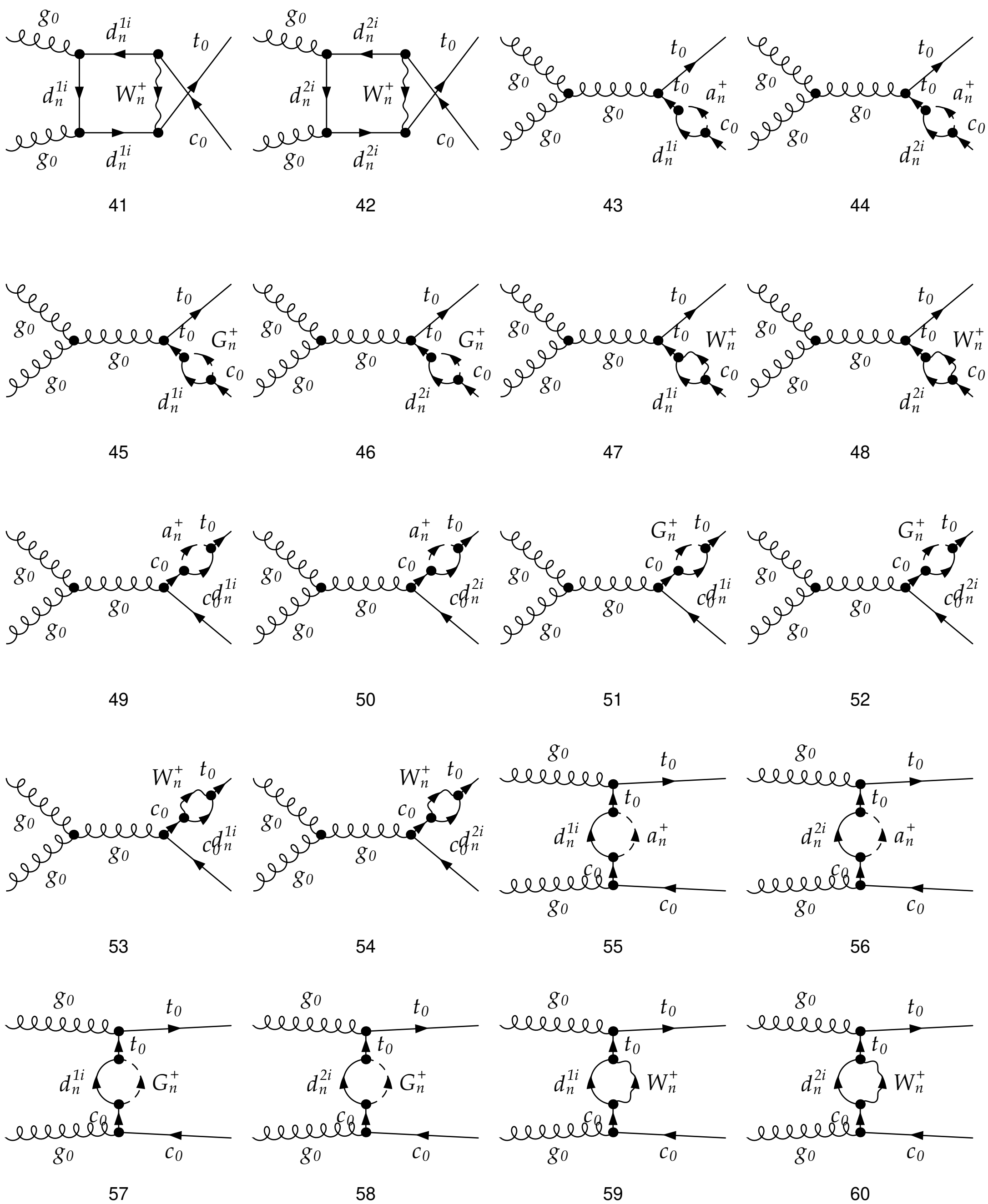


Figure A.37: MUED diagrams contributing to the process $gg \to t\bar{c}$.

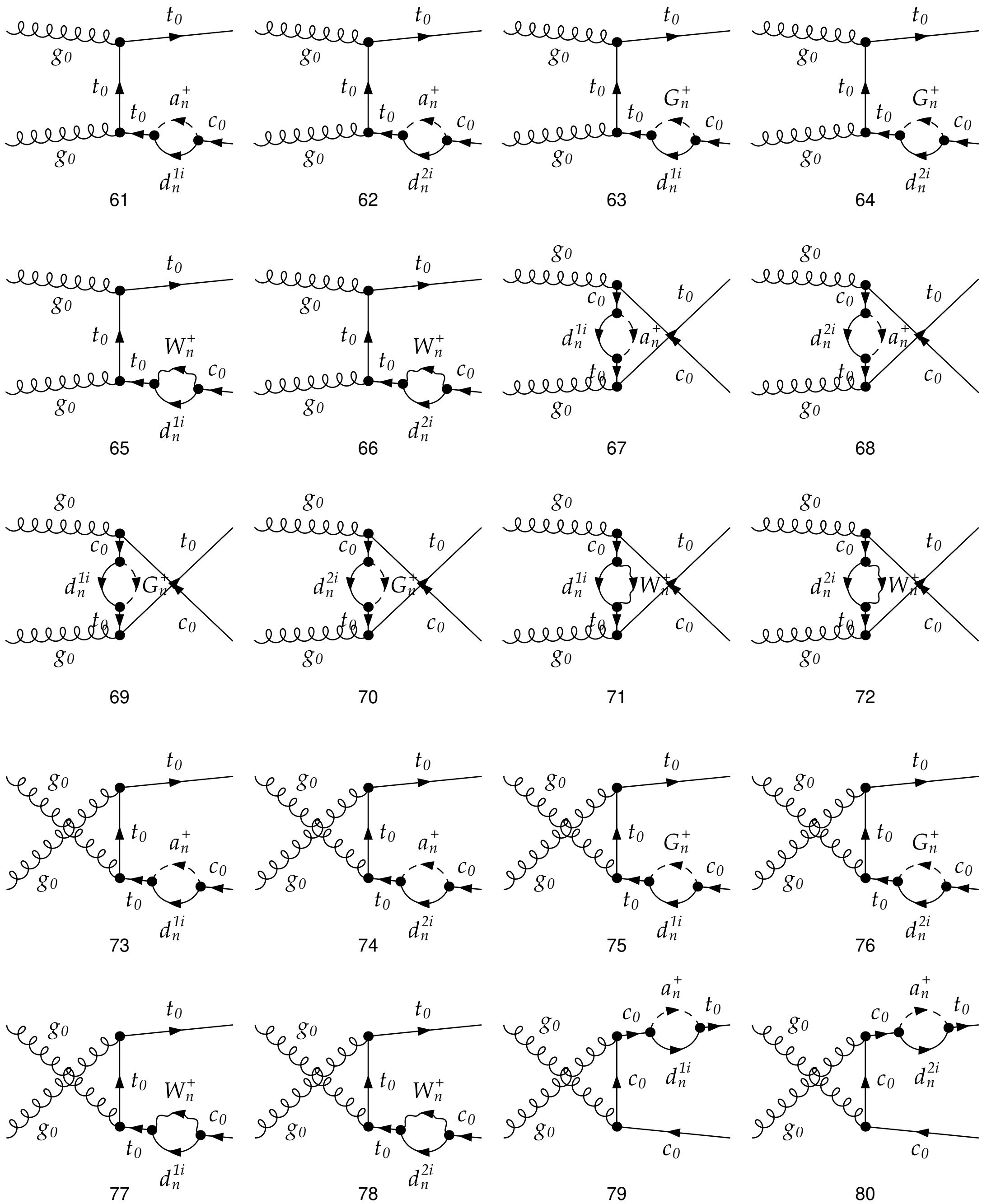


Figure A.38: MUED diagrams contributing to the process $gg \to t\bar{c}$.

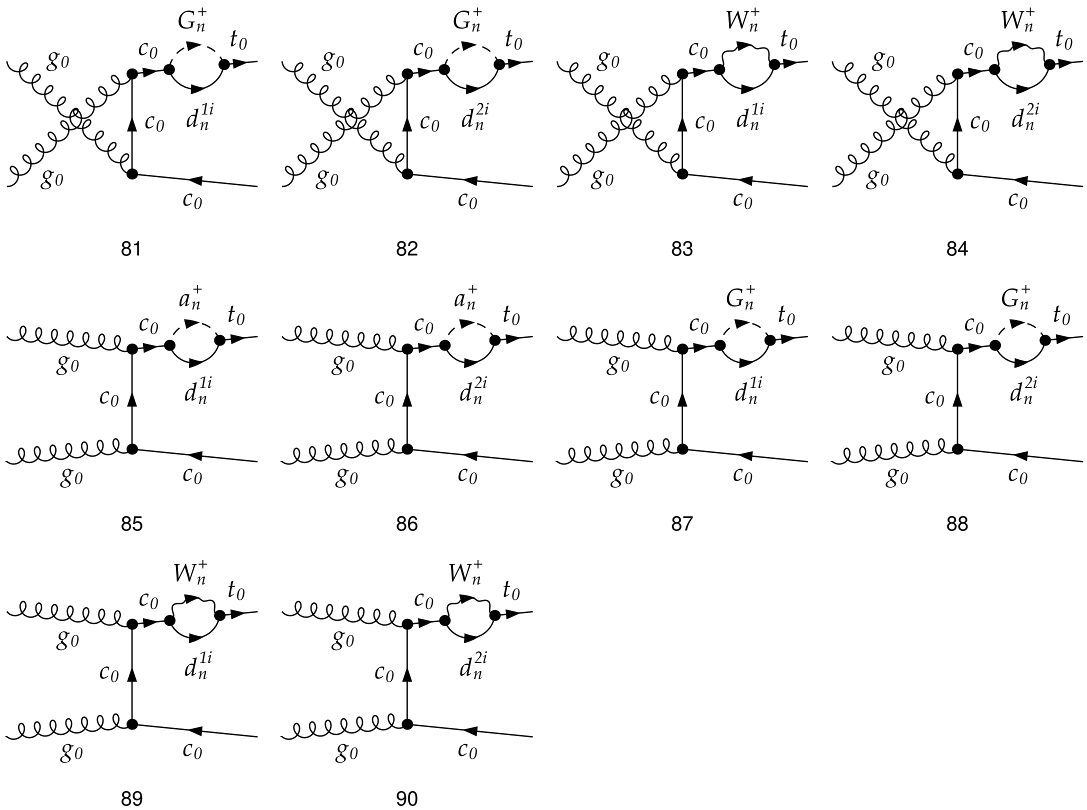


Figure A.39: MUED diagrams contributing to the process $gg \to t\bar{c}$.

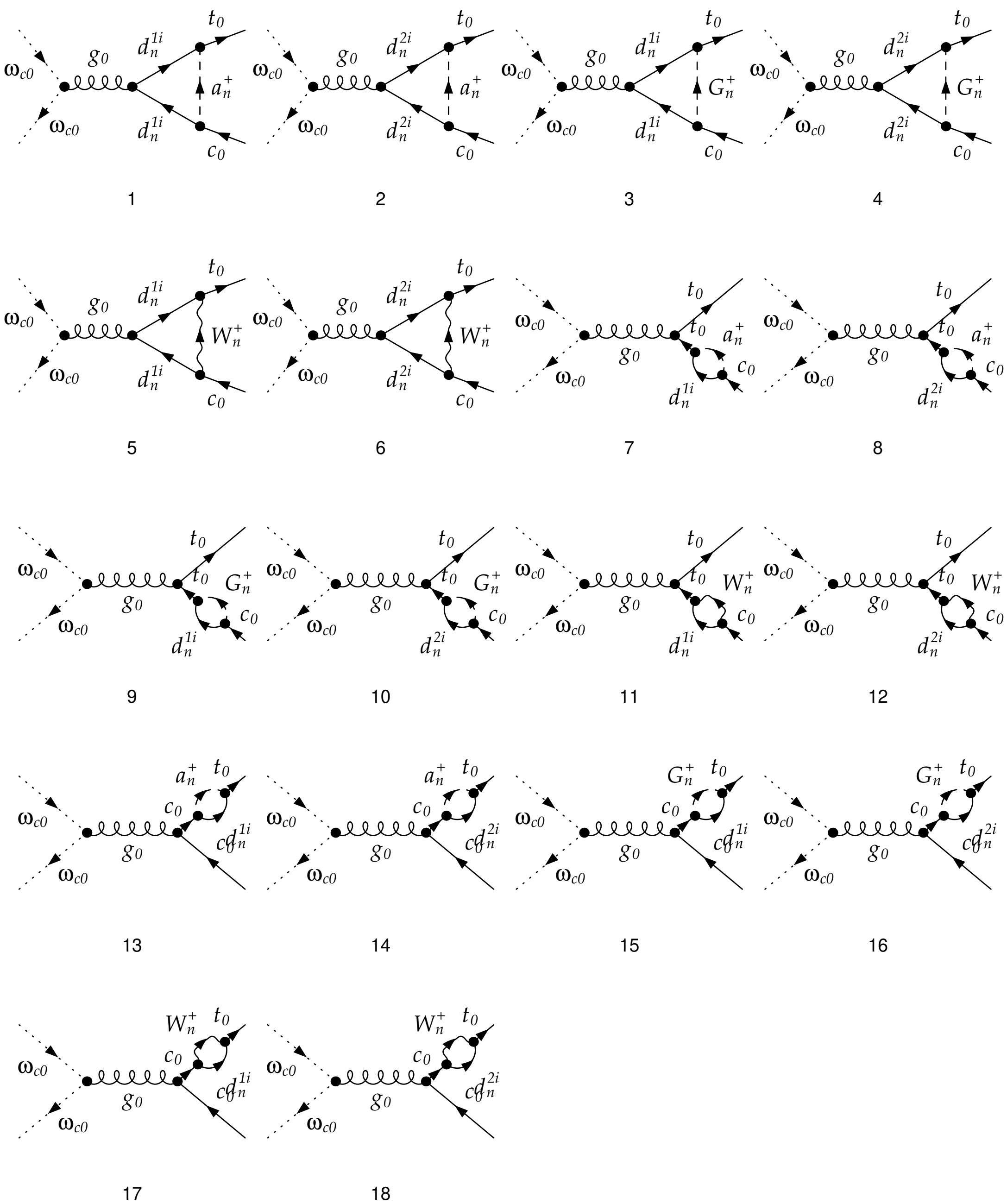


Figure A.40: MUED diagrams contributing to the process $gg \to t\bar{c}$.

A.2.7 $cg \to t\gamma$

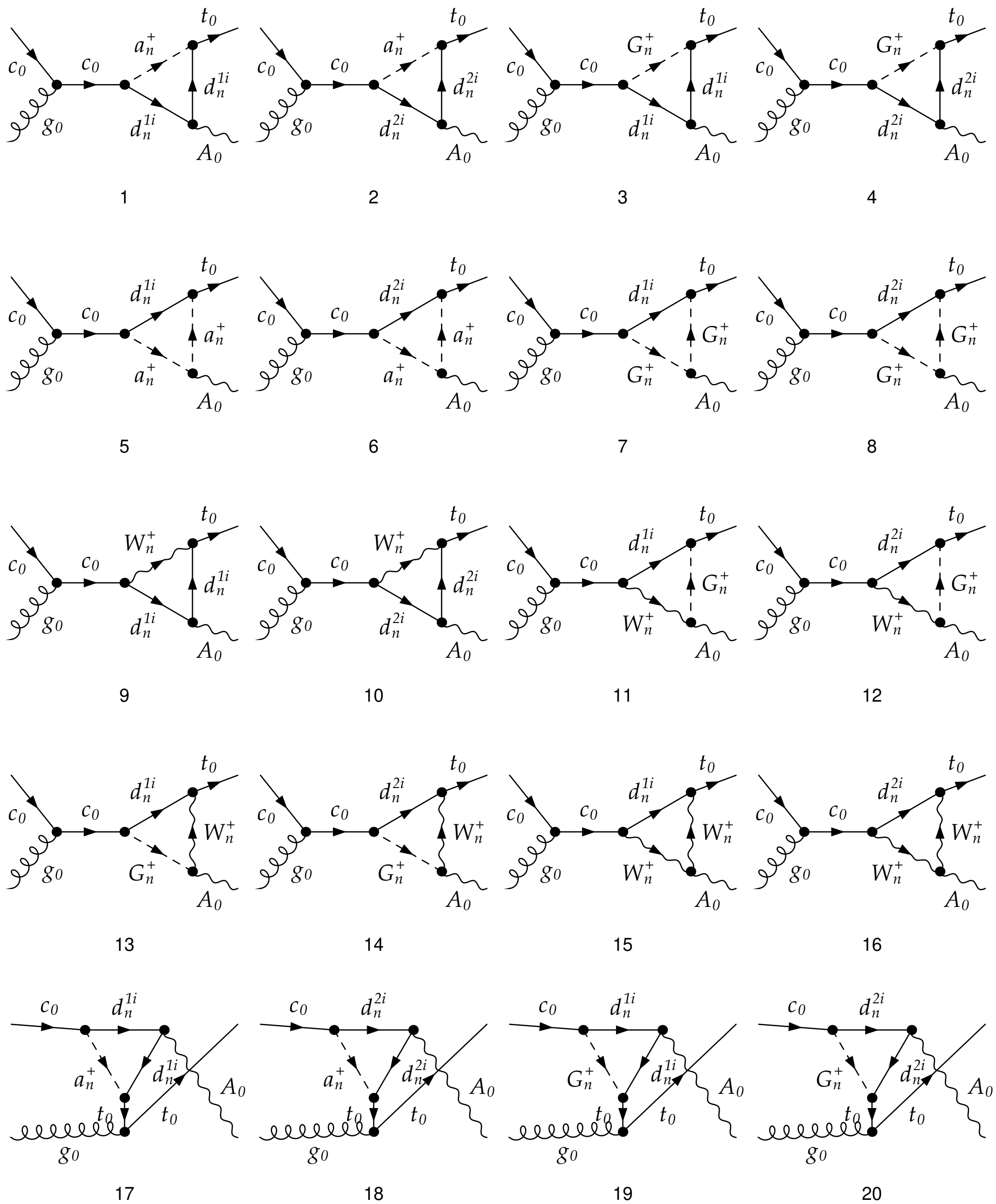


Figure A.41: MUED diagrams contributing to the process $cg \to t\gamma$.

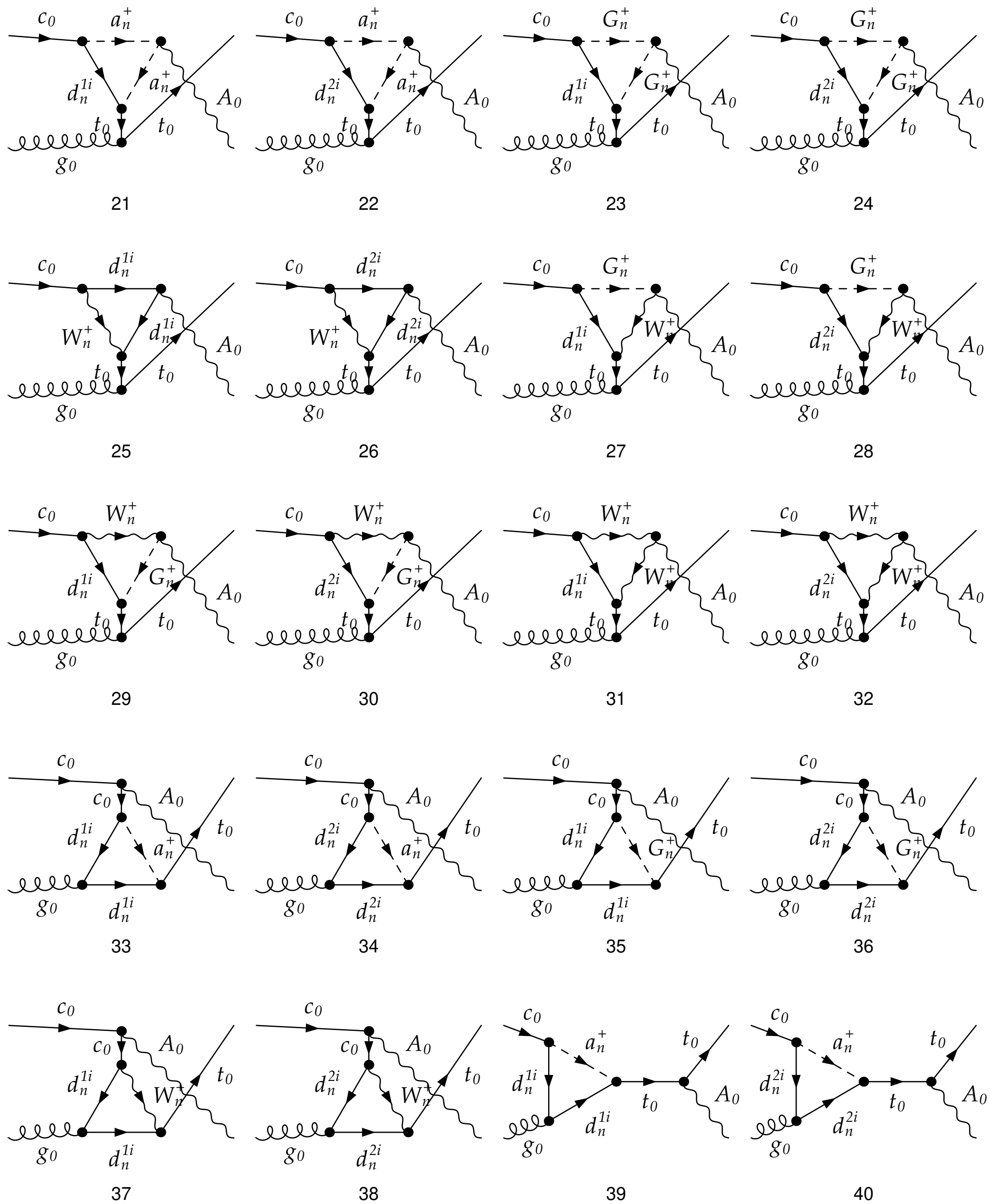


Figure A.42: MUED diagrams contributing to the process $cg \to t\gamma$.

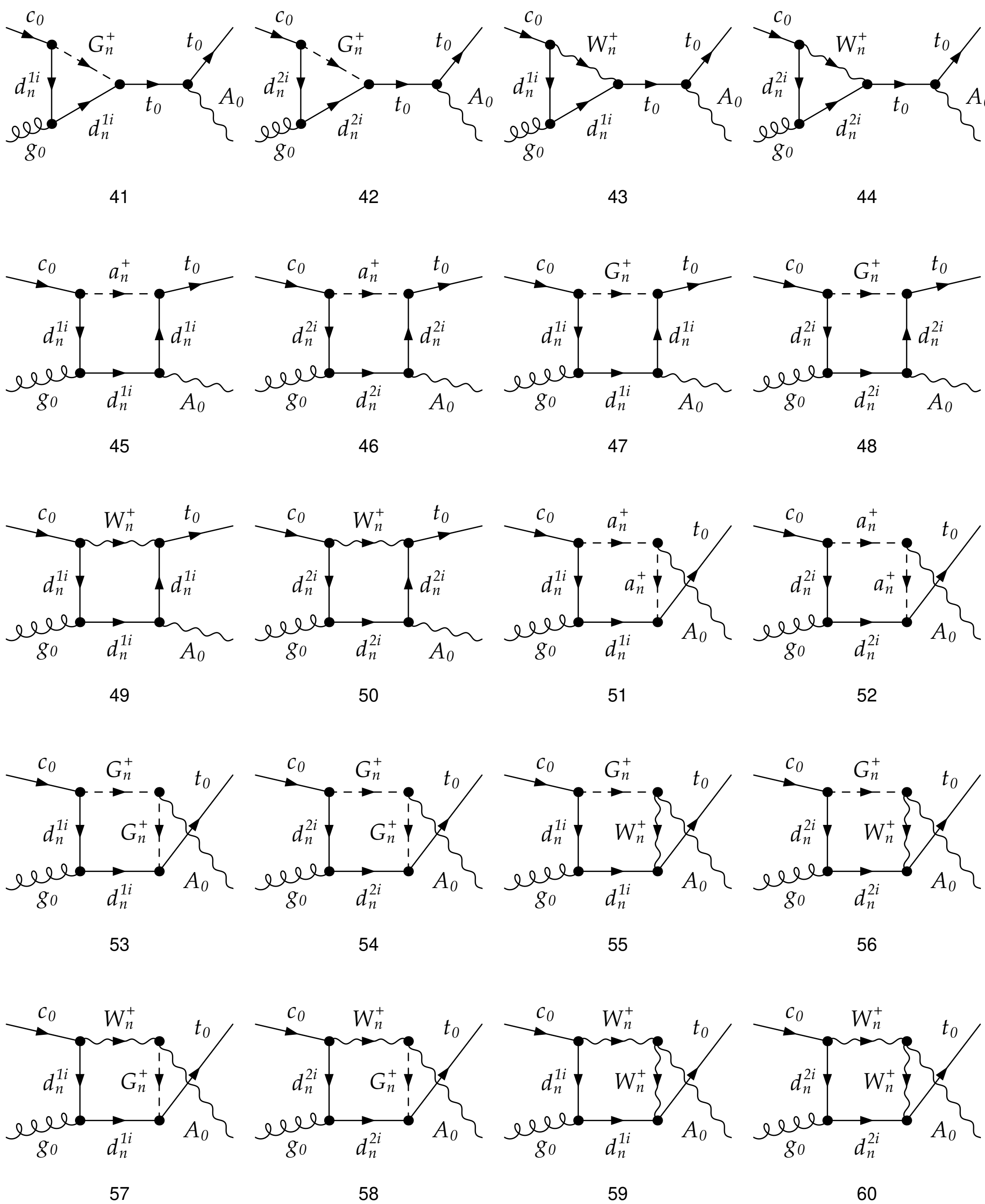


Figure A.43: MUED diagrams contributing to the process $cg \to t\gamma$.

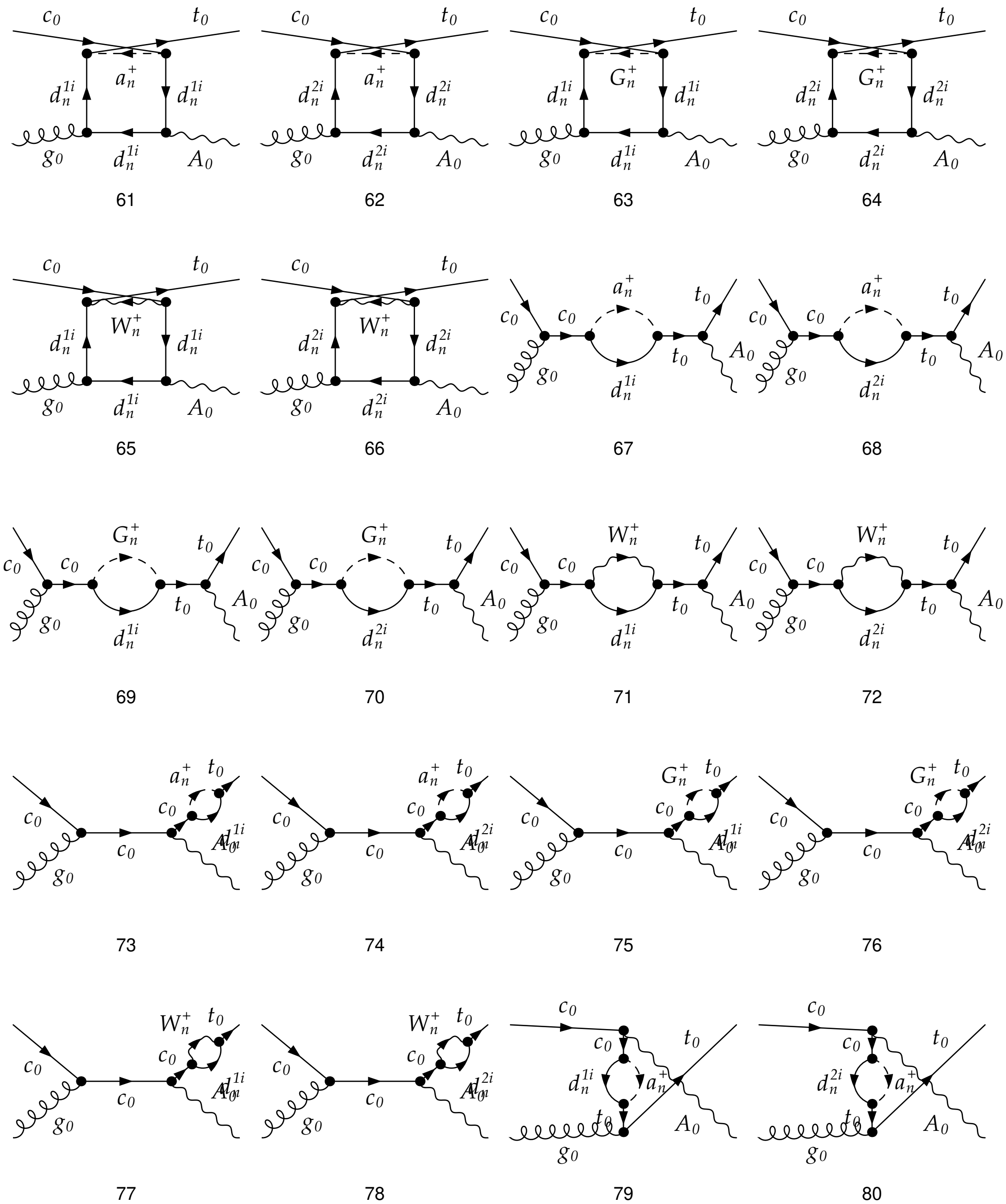


Figure A.44: MUED diagrams contributing to the process $cg \to t\gamma$.

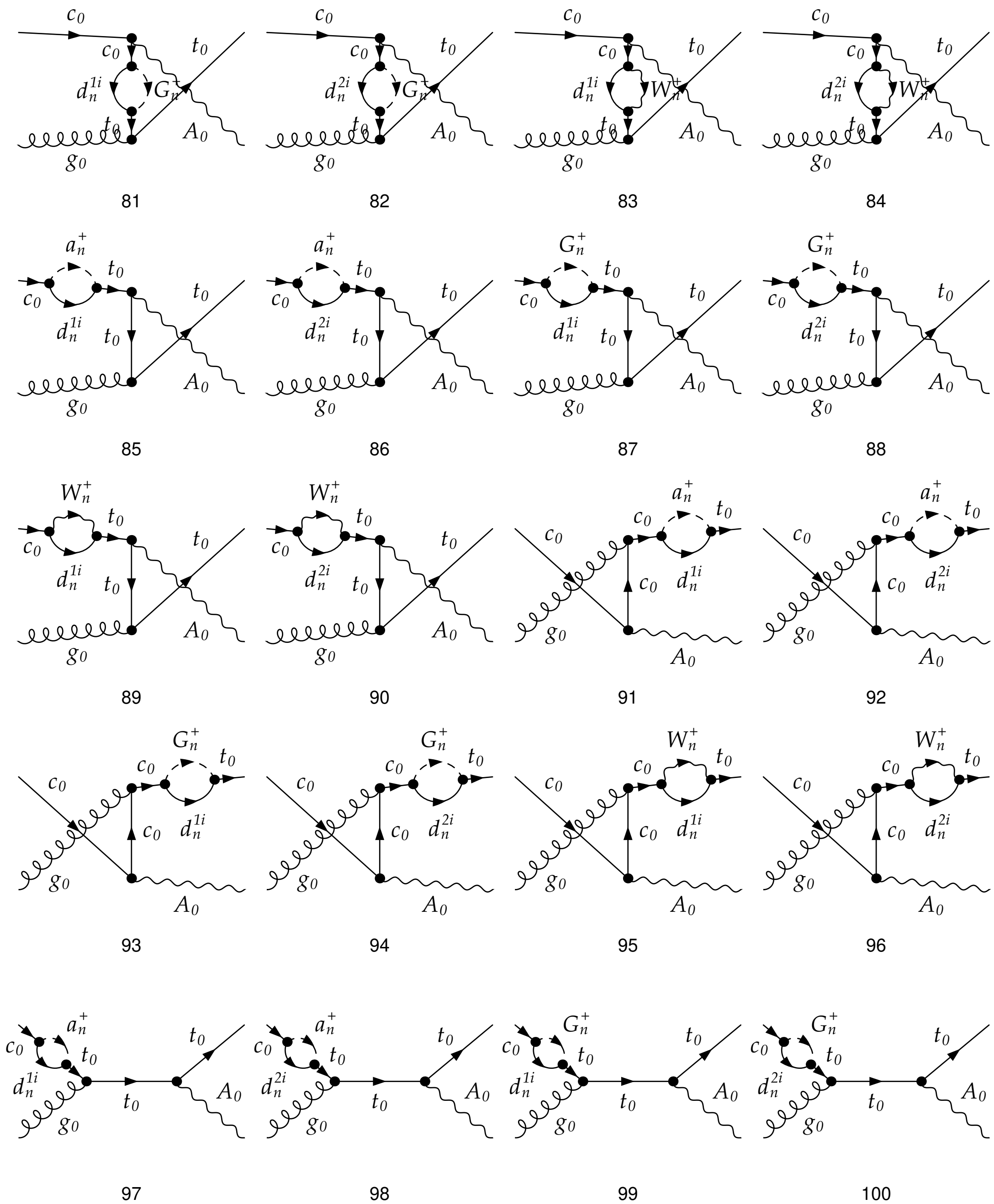
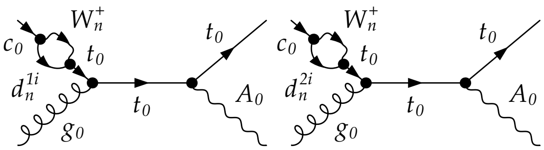


Figure A.45: MUED diagrams contributing to the process $cg \to t\gamma$.



101 102

Figure A.46: MUED diagrams contributing to the process $cg \to t\gamma$.

A.2.8 $cg \to tg$

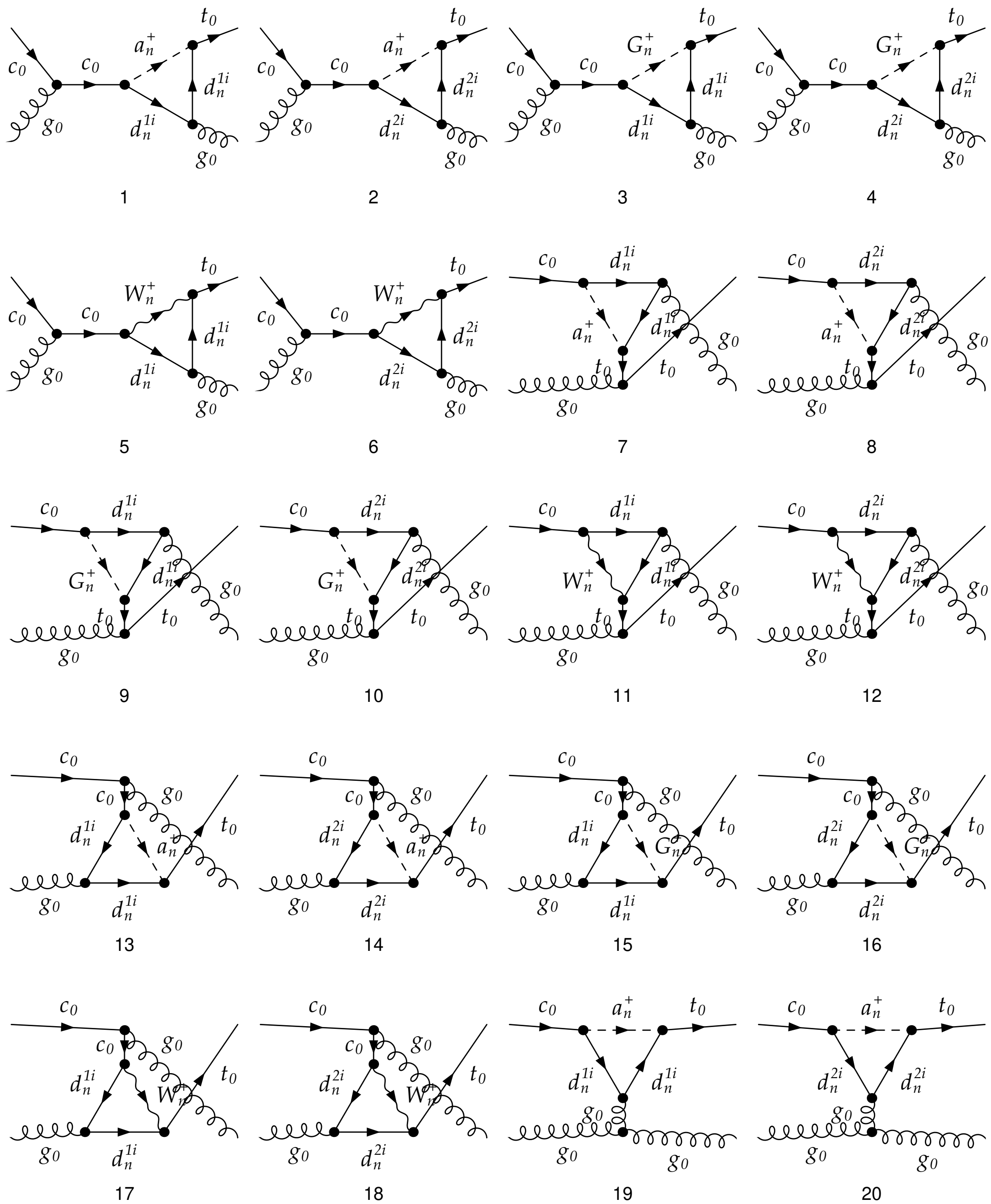


Figure A.47: MUED diagrams contributing to the process $cg \to tg$.

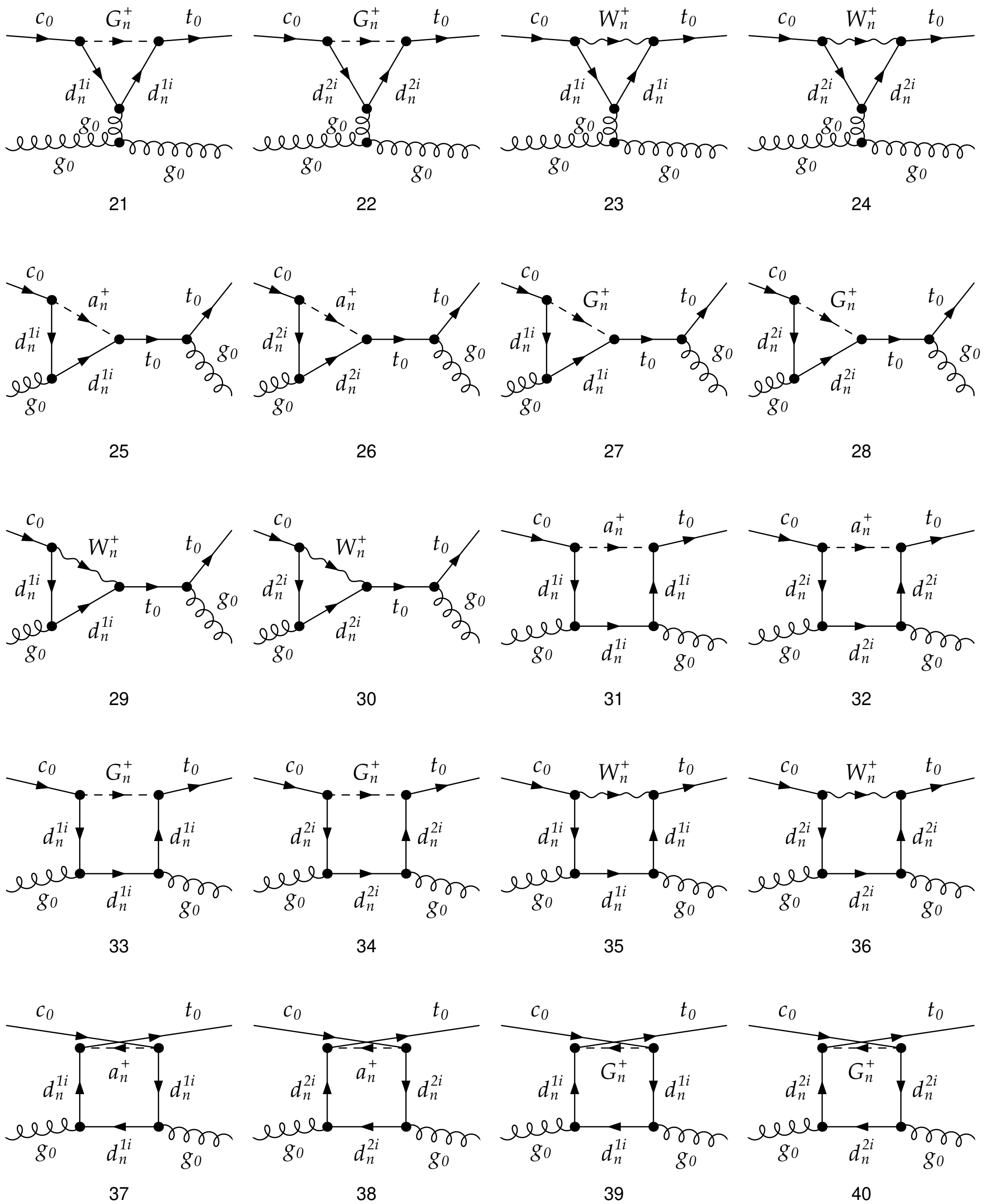


Figure A.48: MUED diagrams contributing to the process $cg \to tg$.

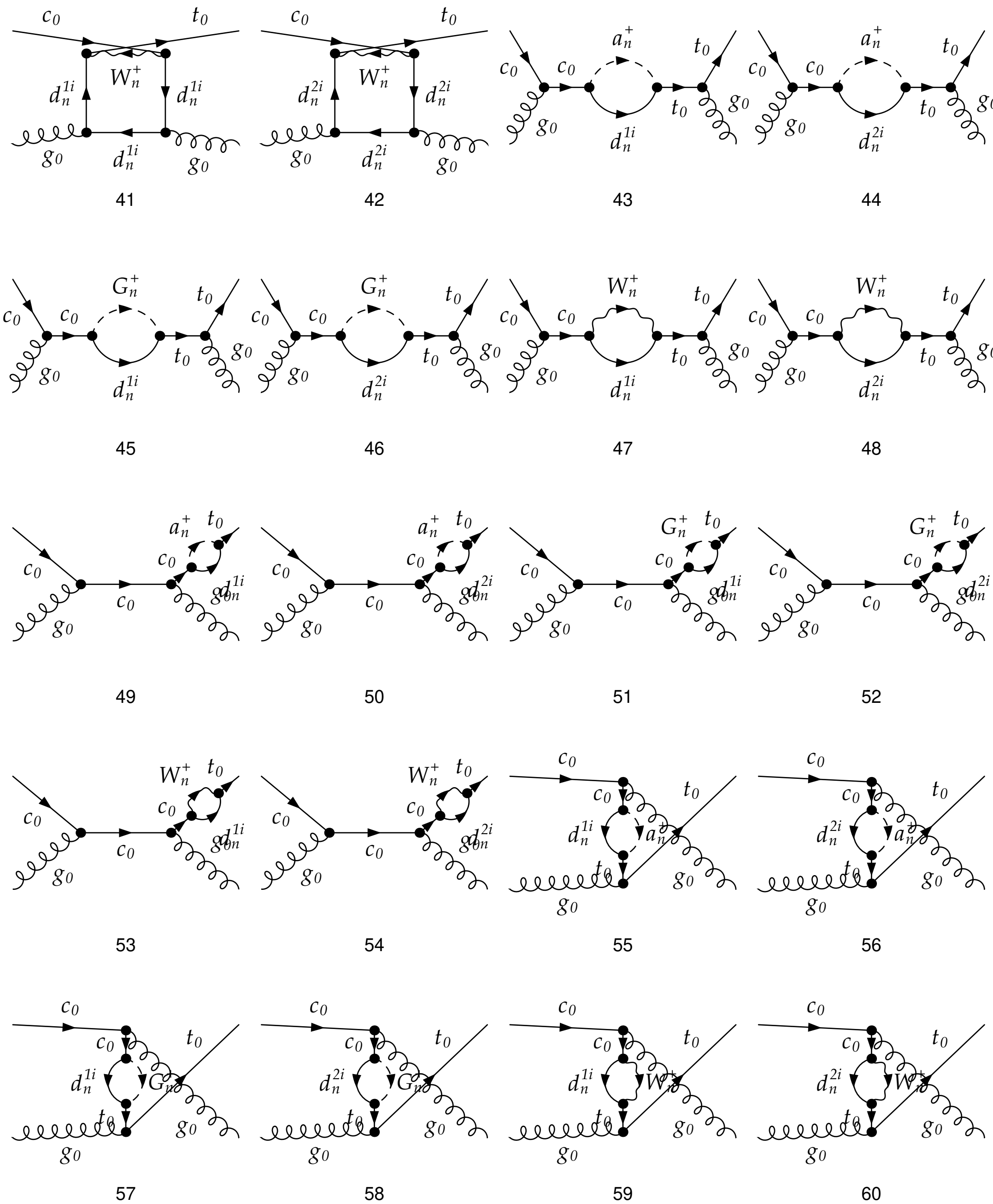


Figure A.49: MUED diagrams contributing to the process $cg \to tg$.

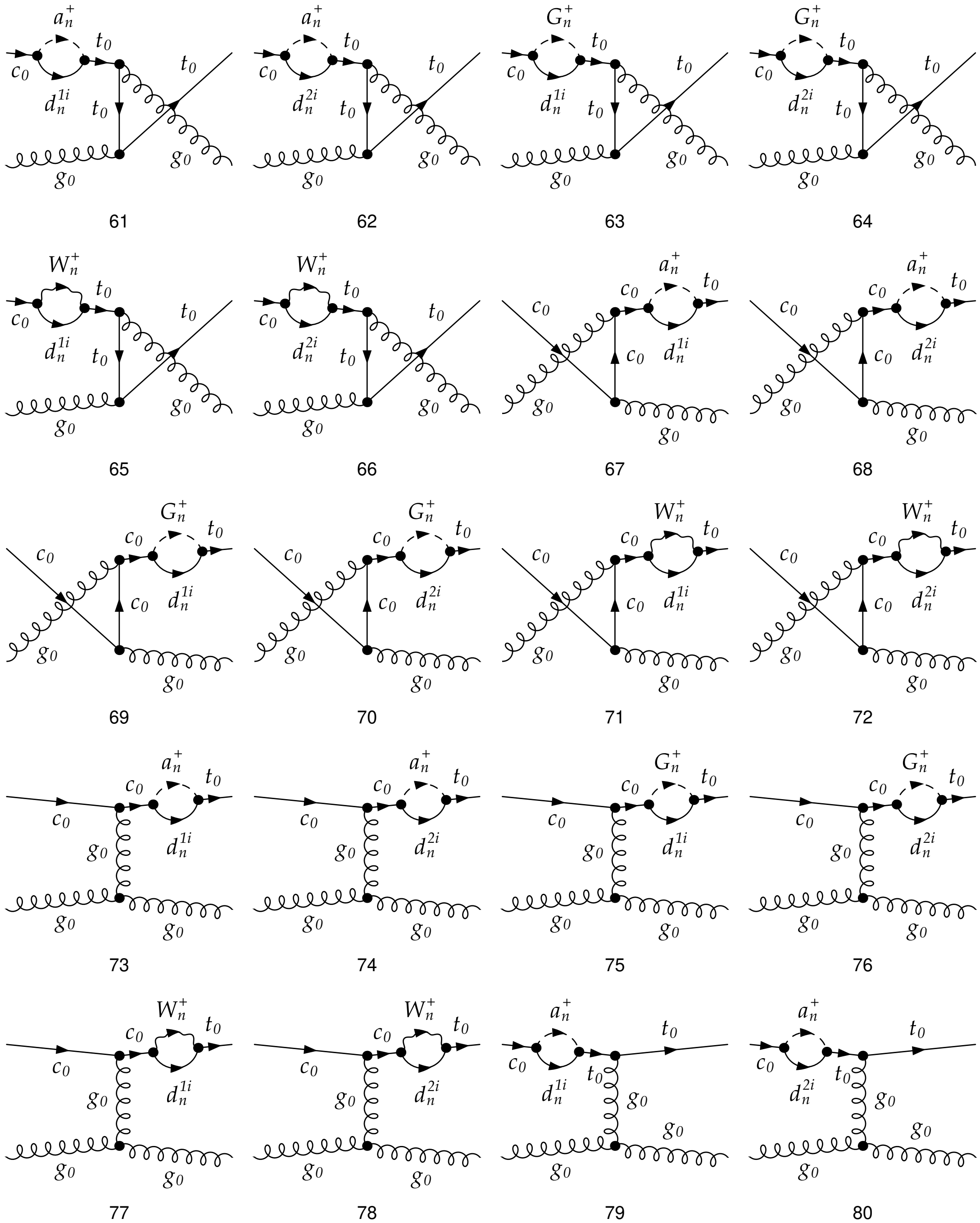


Figure A.50: MUED diagrams contributing to the process $cg \to tg$.

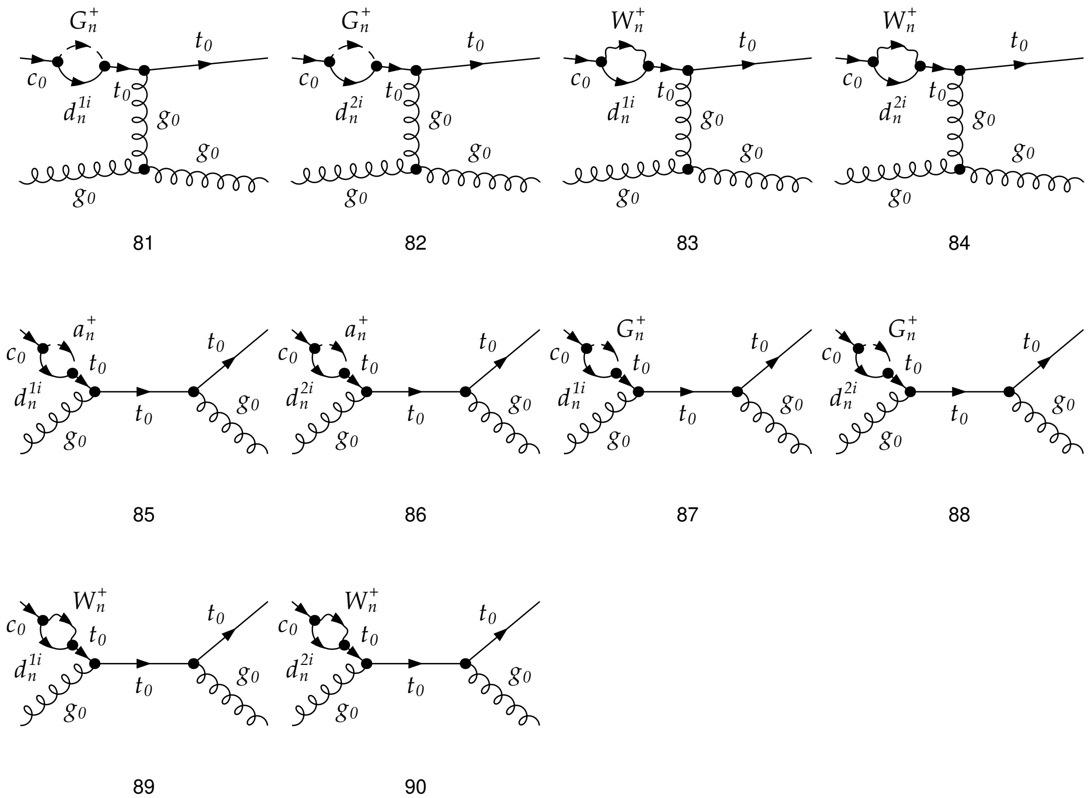


Figure A.51: MUED diagrams contributing to the process $cg \to tg$.

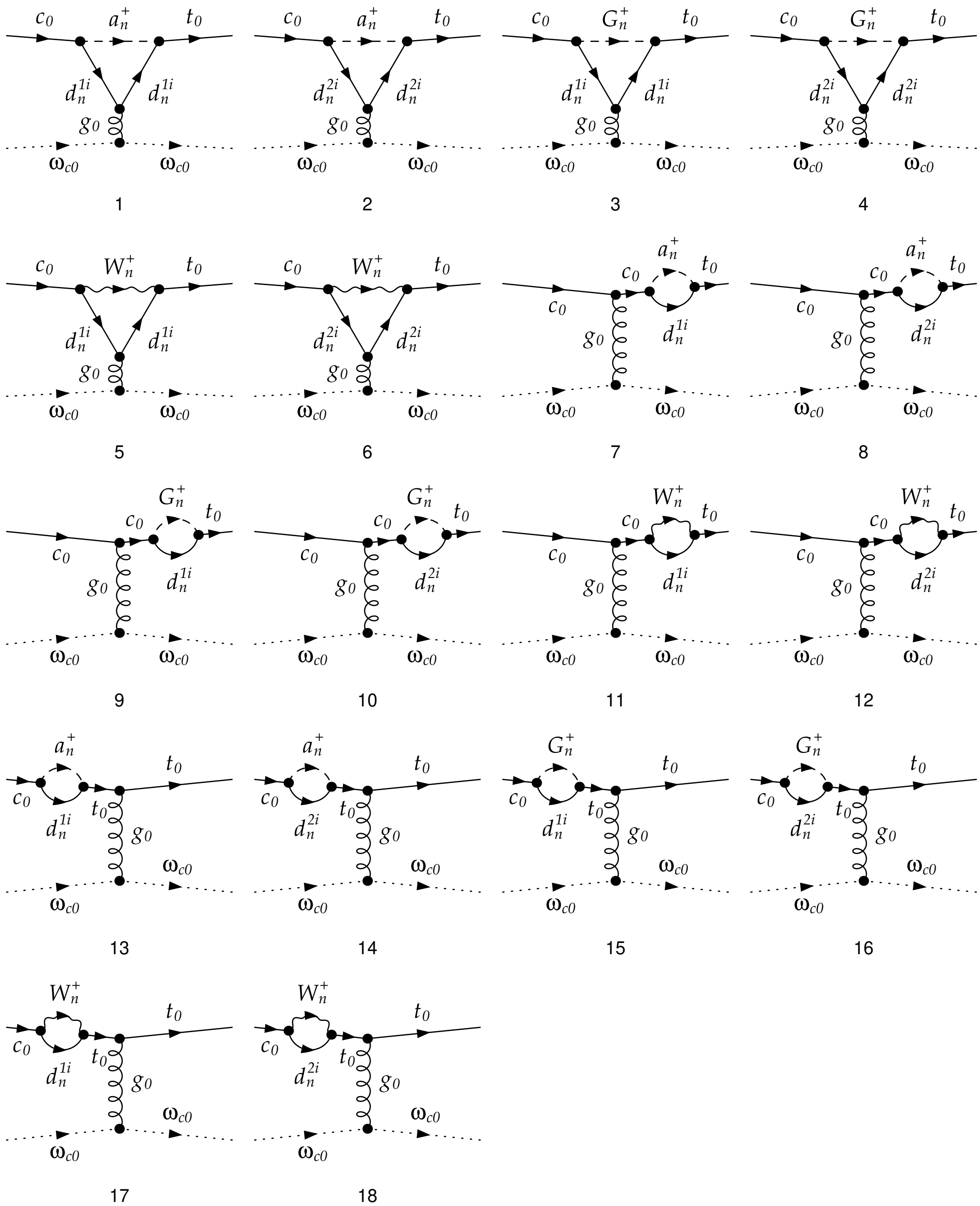


Figure A.52: MUED diagrams contributing to the process $cg \to tg$.

A.2.9 $cg \to th$

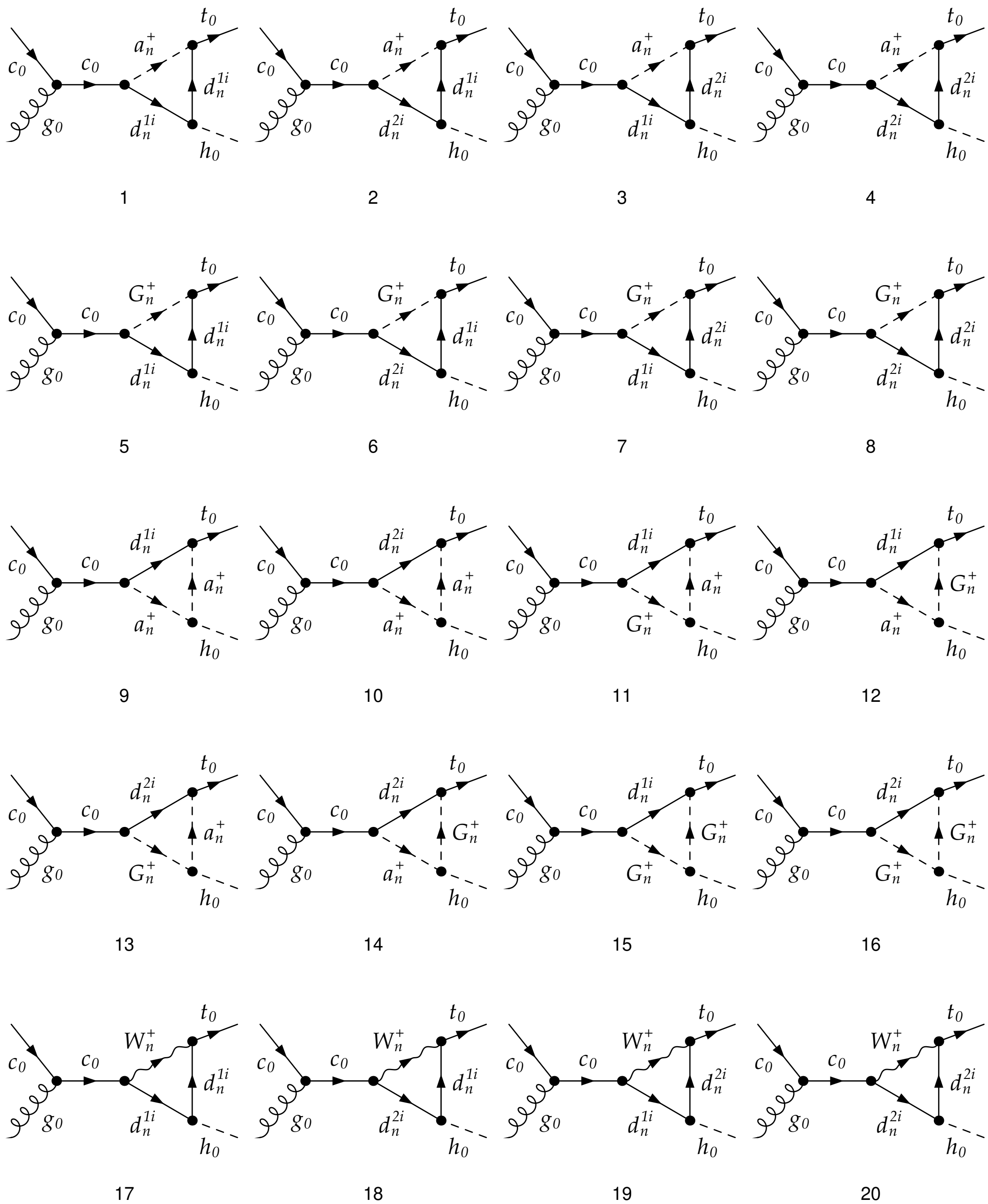


Figure A.53: MUED diagrams contributing to the process $cg \to th$.

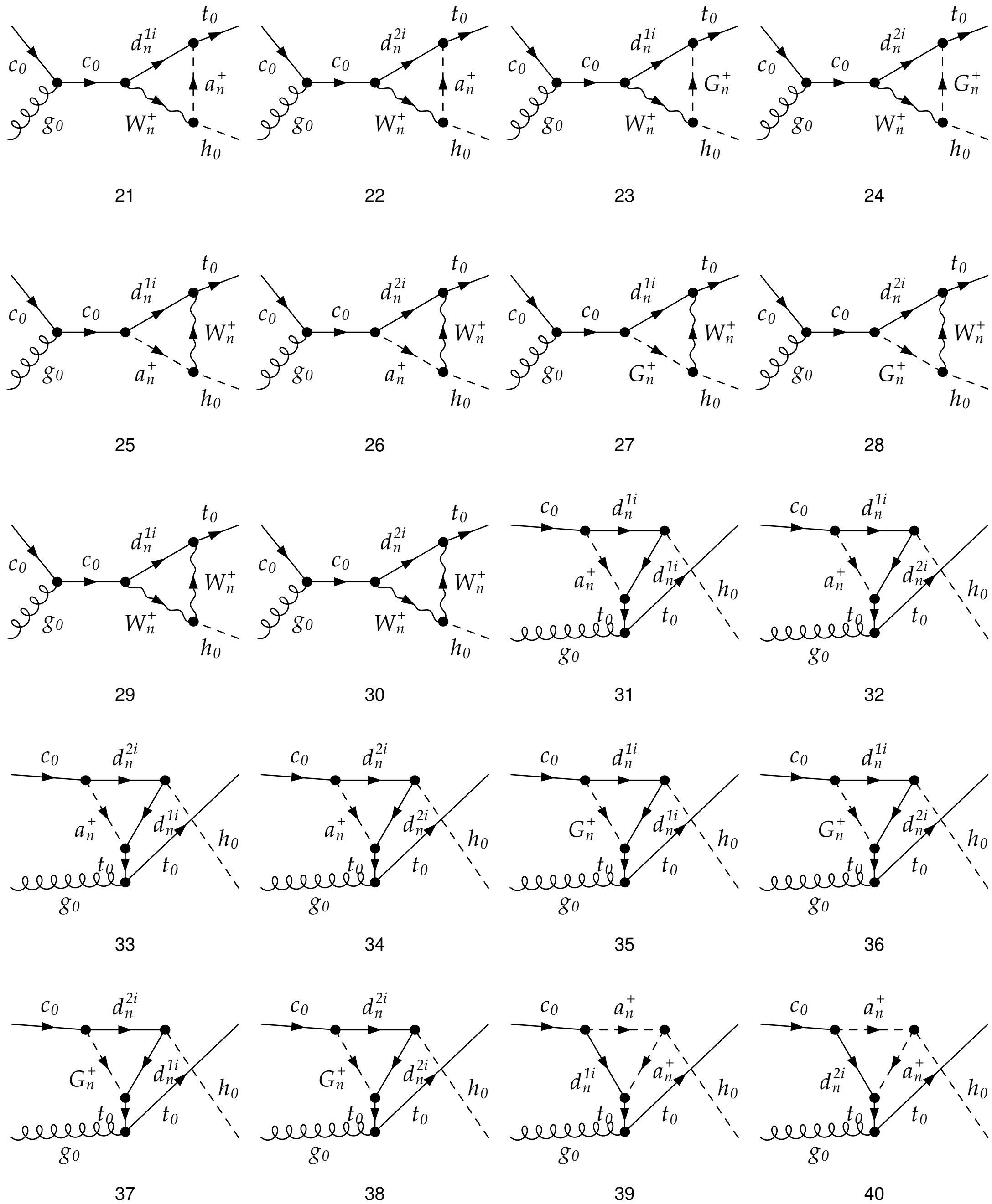


Figure A.54: MUED diagrams contributing to the process $cg \to th$.

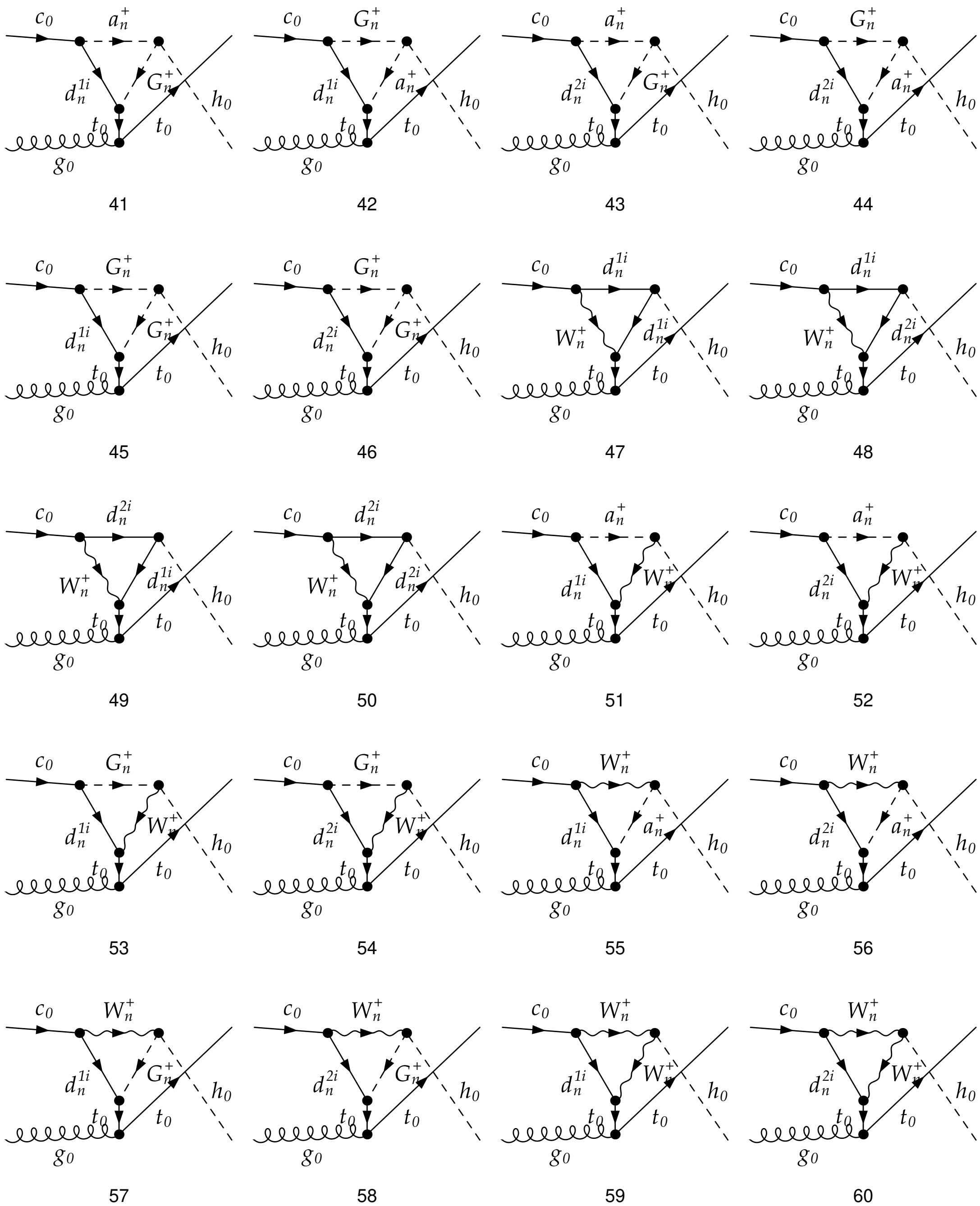


Figure A.55: MUED diagrams contributing to the process $cg \to th$.

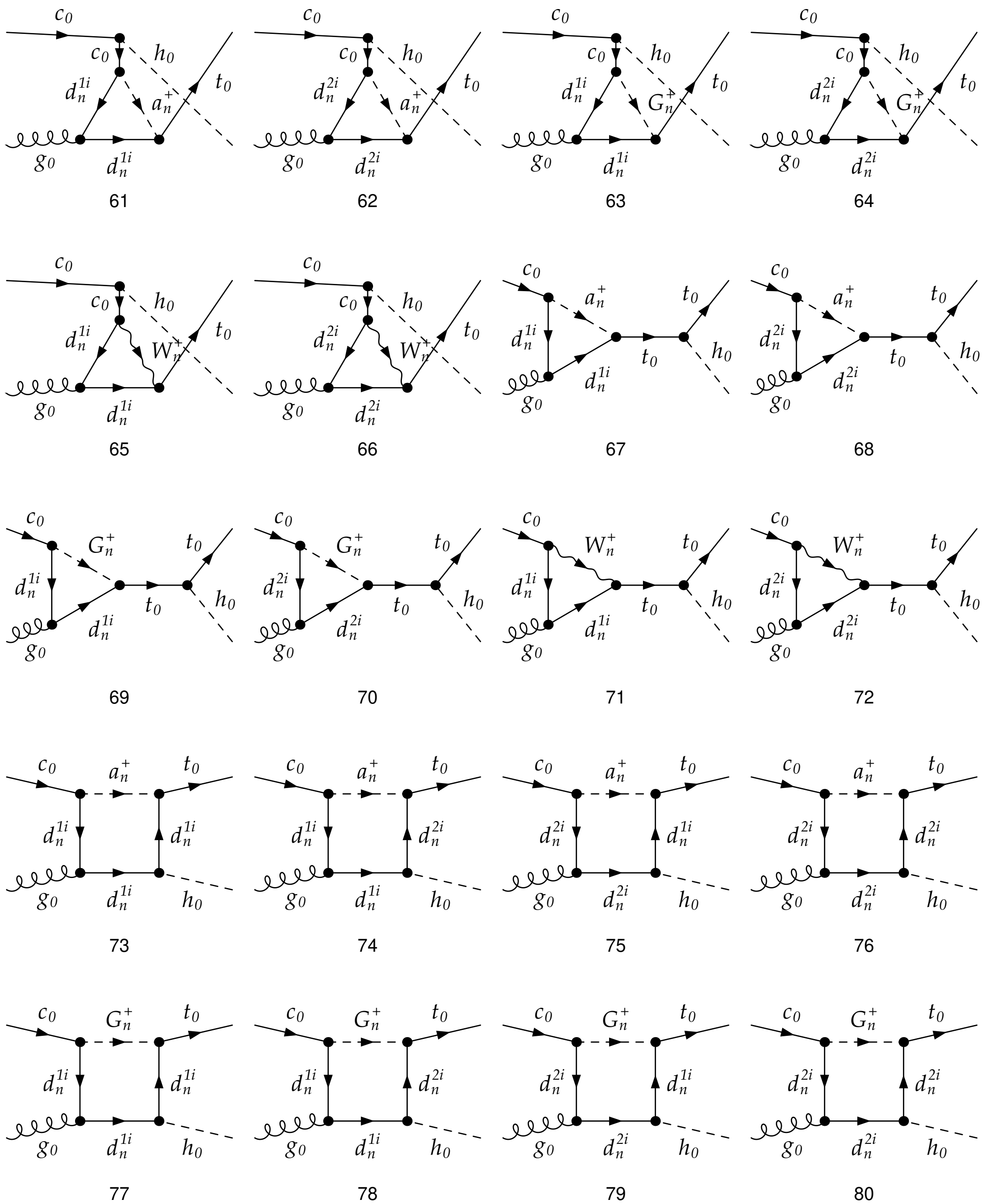


Figure A.56: MUED diagrams contributing to the process $cg \to th$.

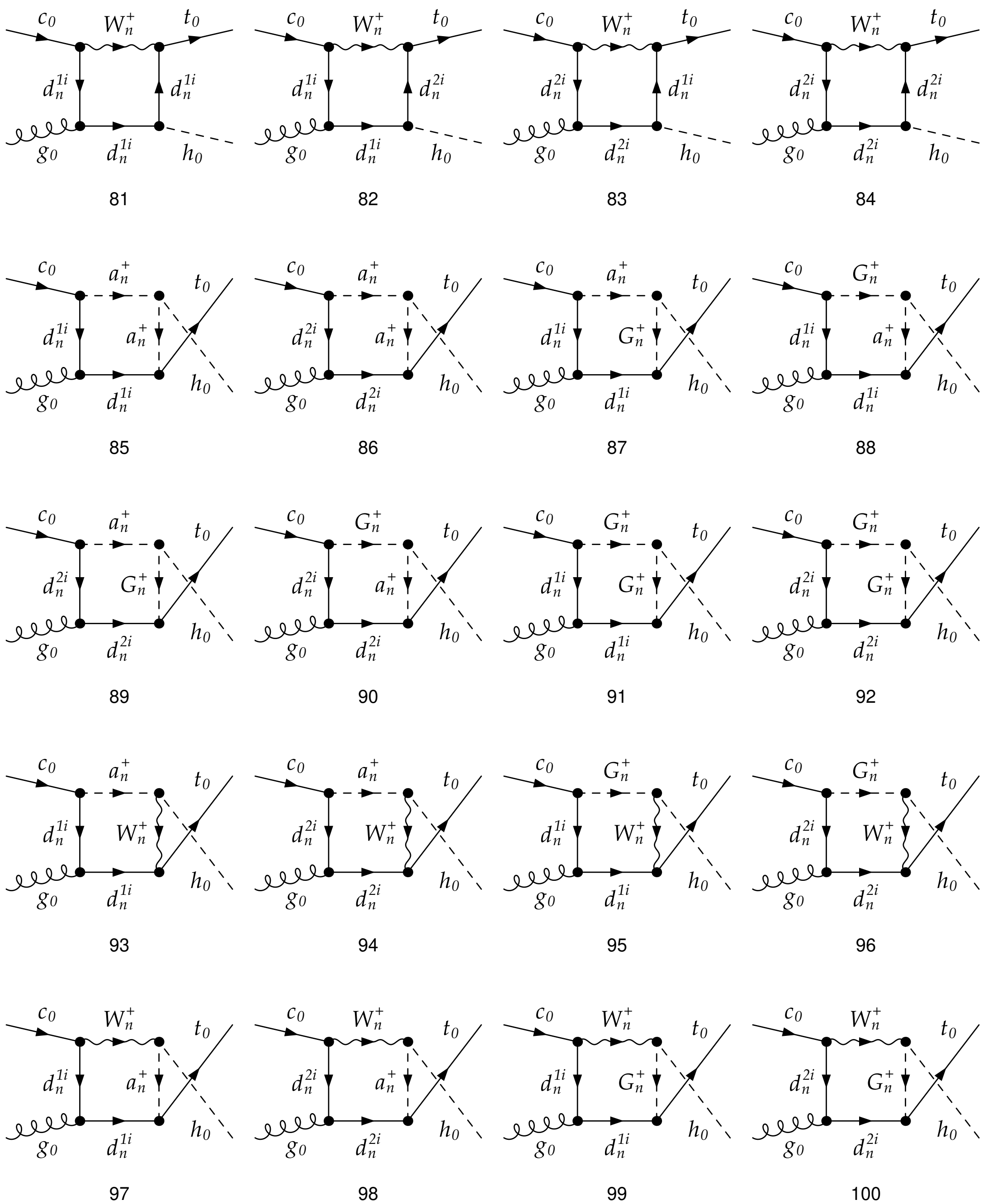


Figure A.57: MUED diagrams contributing to the process $cg \to th$.

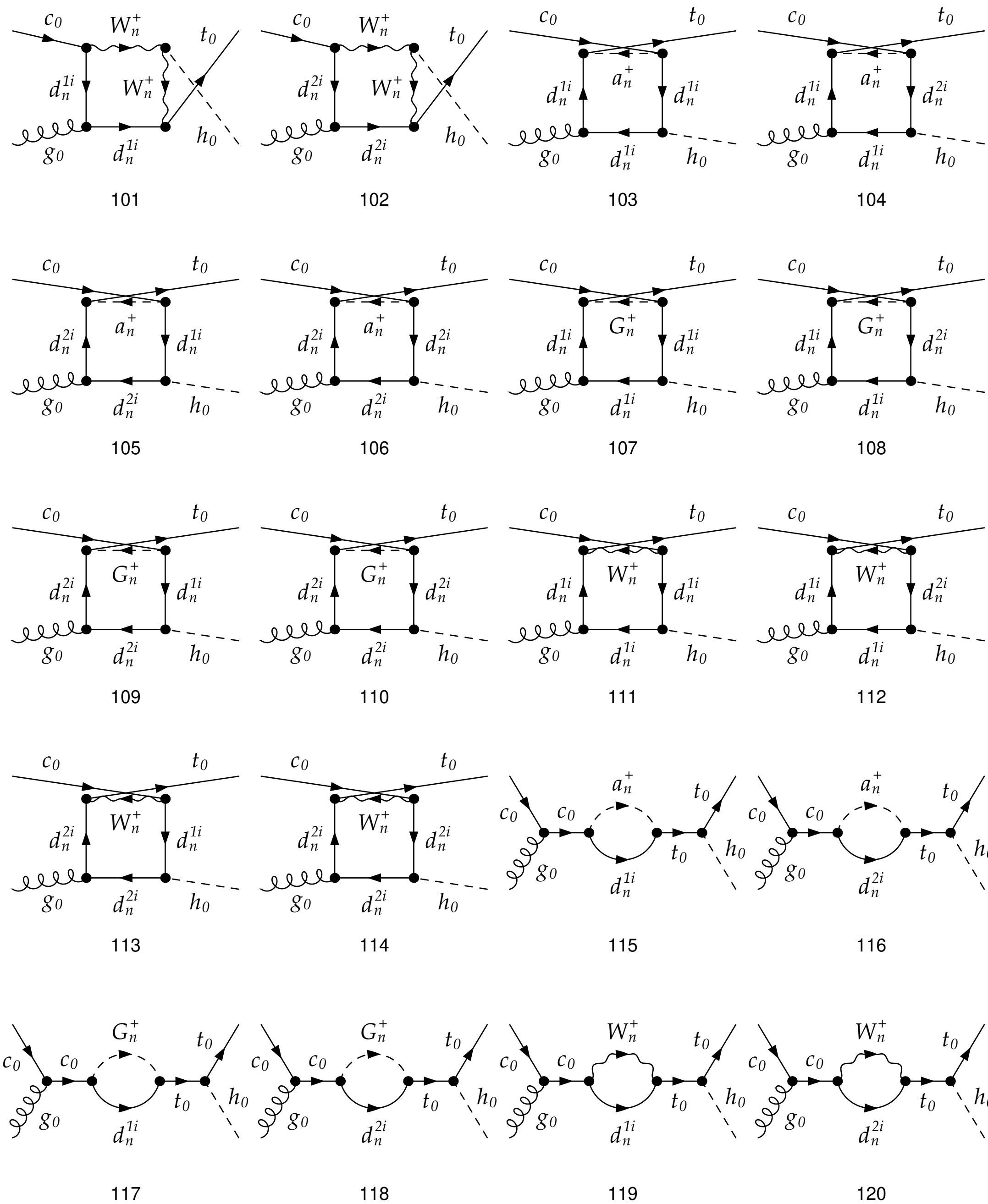


Figure A.58: MUED diagrams contributing to the process $cg \to th$.

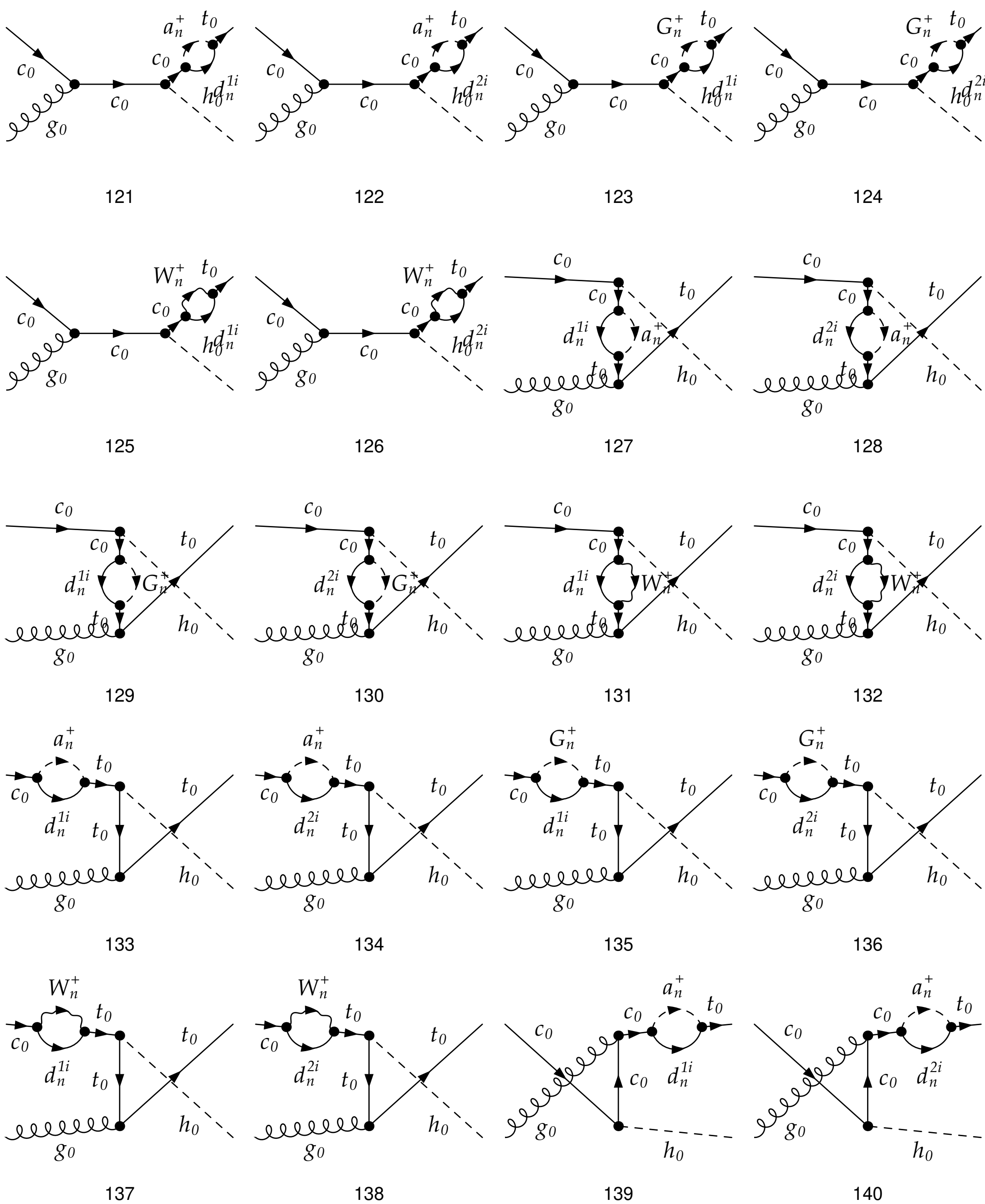


Figure A.59: MUED diagrams contributing to the process $cg \to th$.

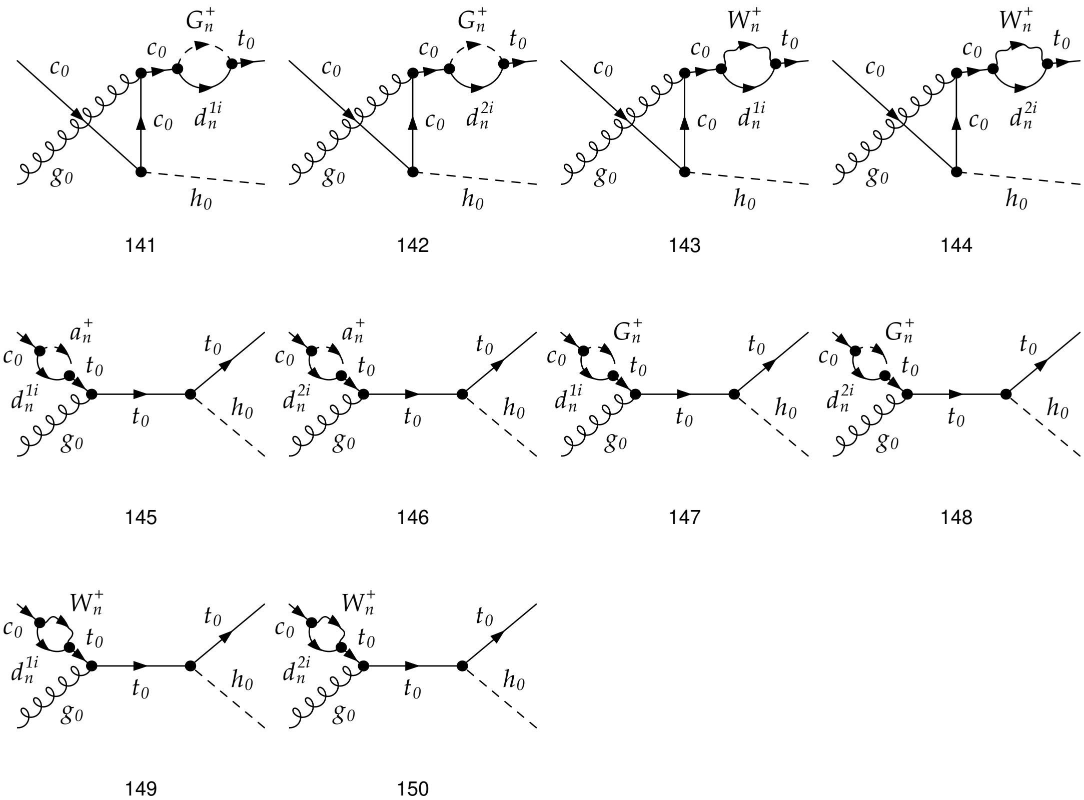


Figure A.60: MUED diagrams contributing to the process $cg \to th$.

A.2.10 $cg \to tZ$

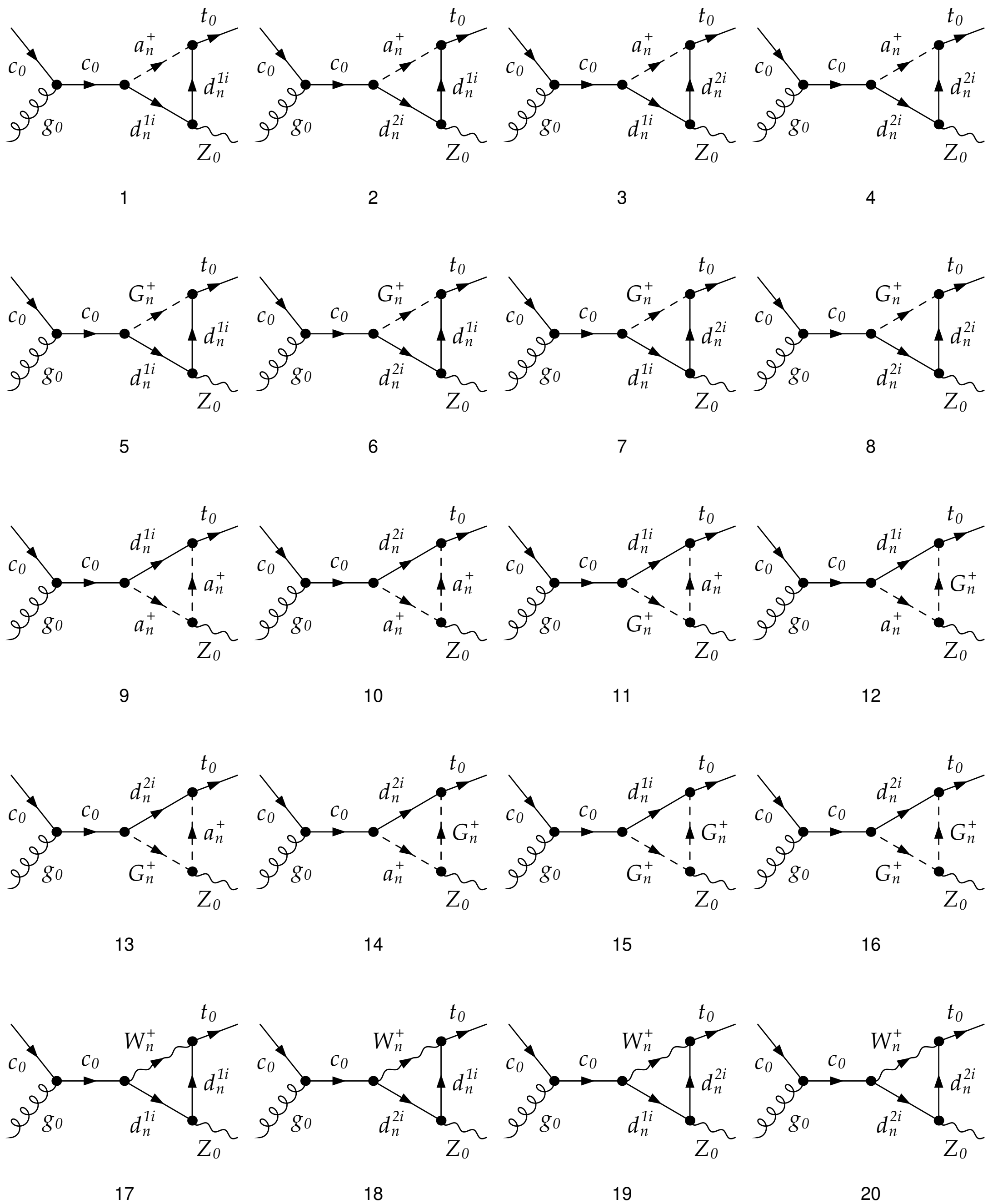


Figure A.61: MUED diagrams contributing to the process $cg \to tZ$.

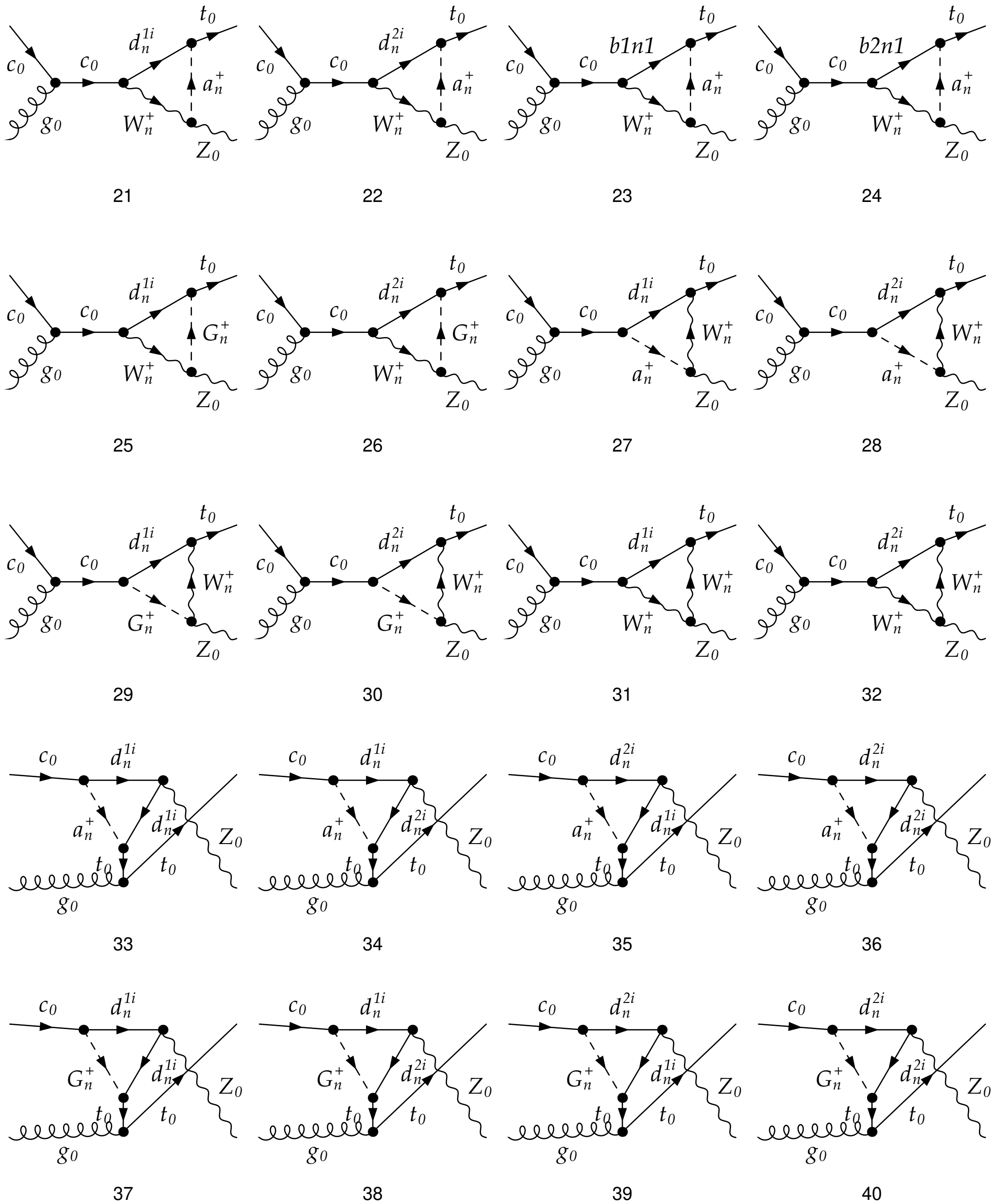


Figure A.62: MUED diagrams contributing to the process $cg \to tZ$.

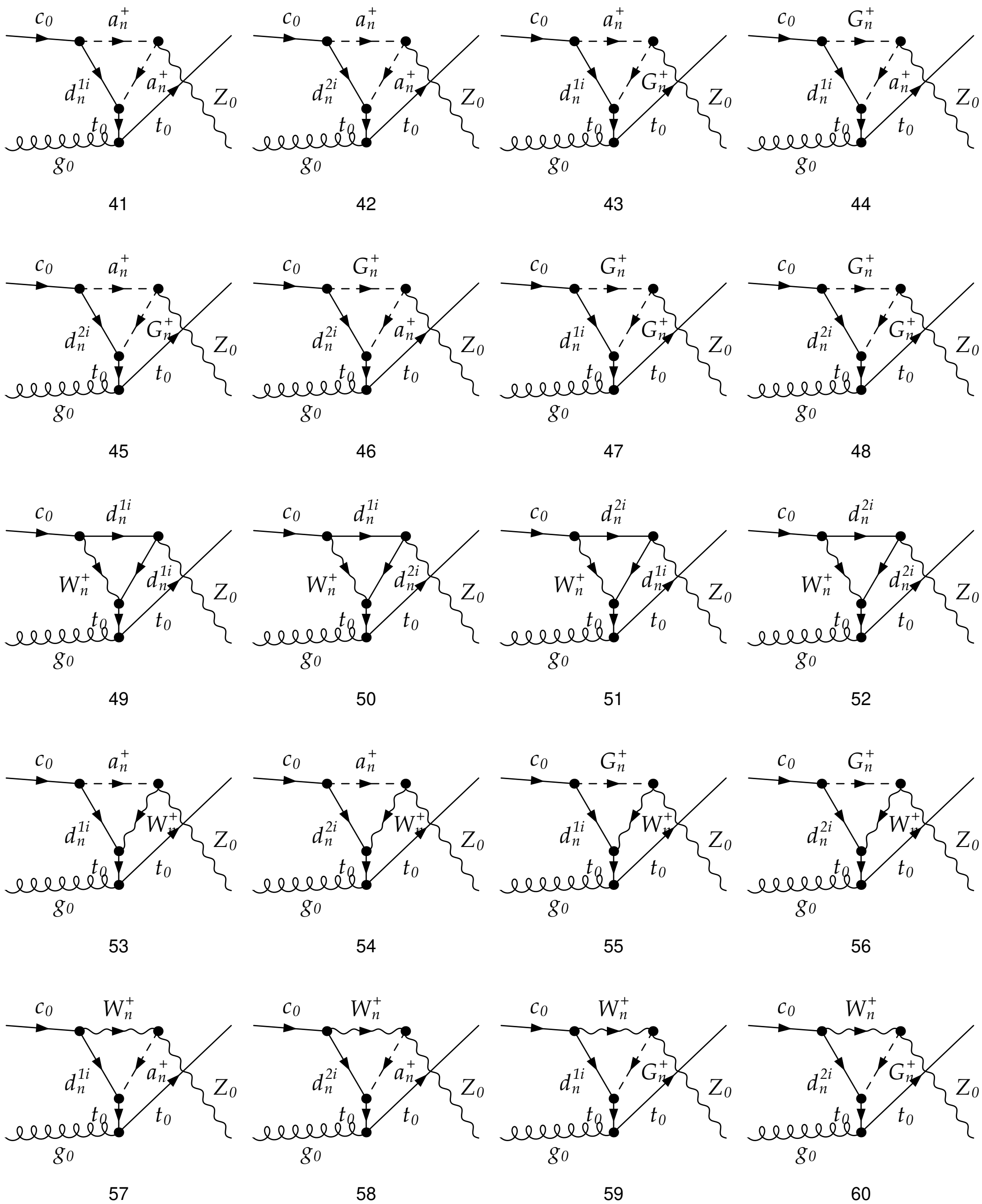


Figure A.63: MUED diagrams contributing to the process $cg \to tZ$.

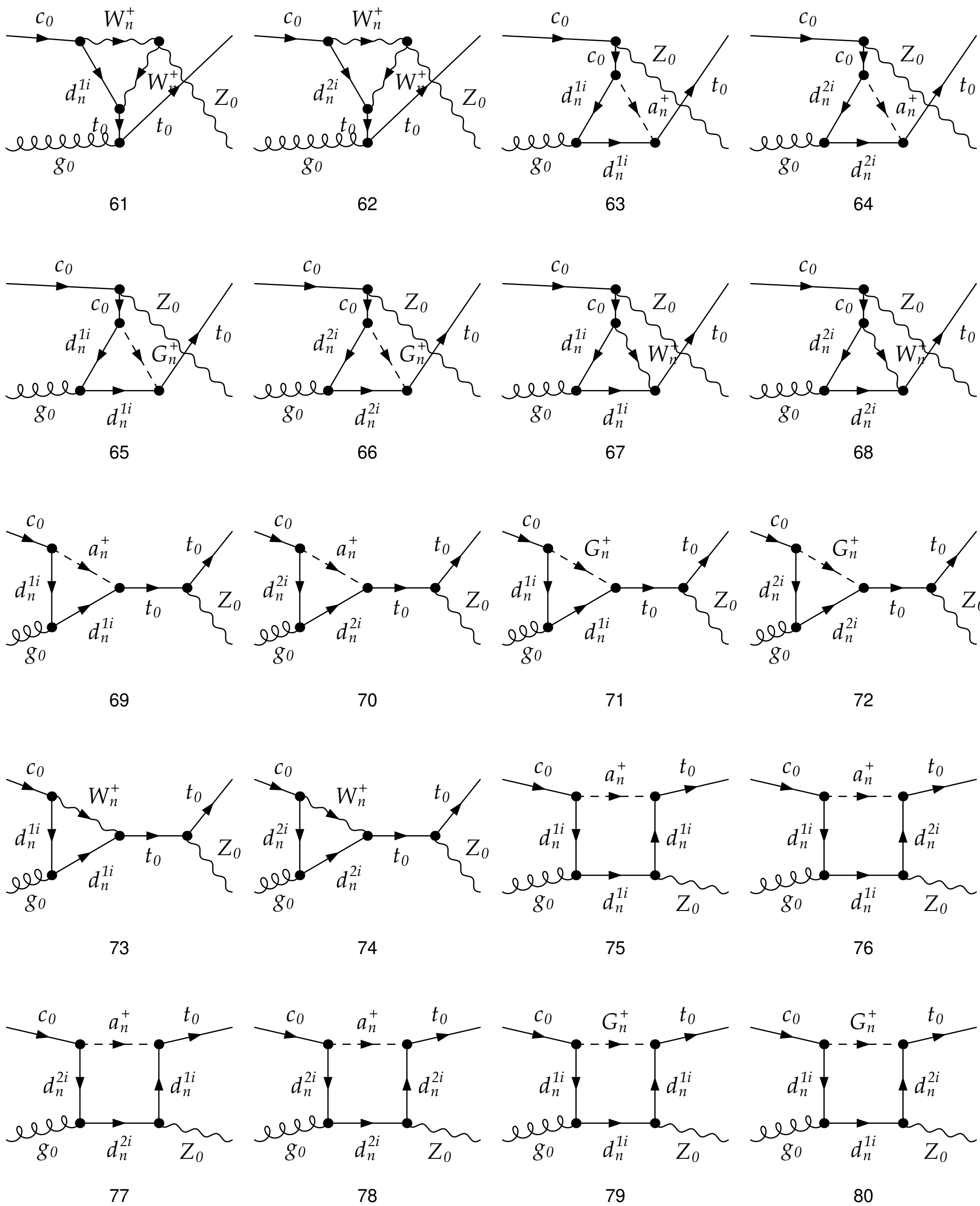


Figure A.64: MUED diagrams contributing to the process $cg \to tZ$.

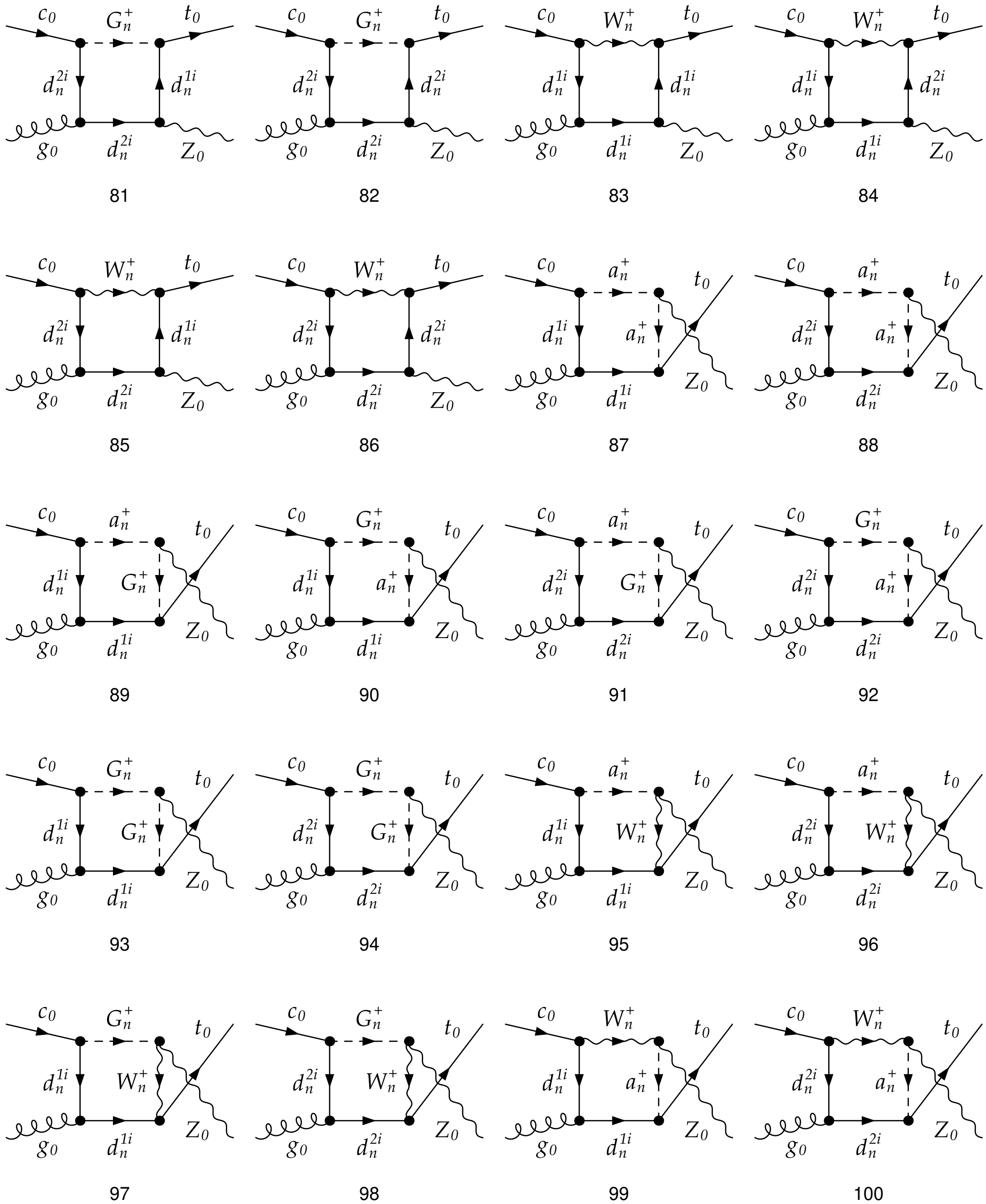


Figure A.65: MUED diagrams contributing to the process $cg \to tZ$.

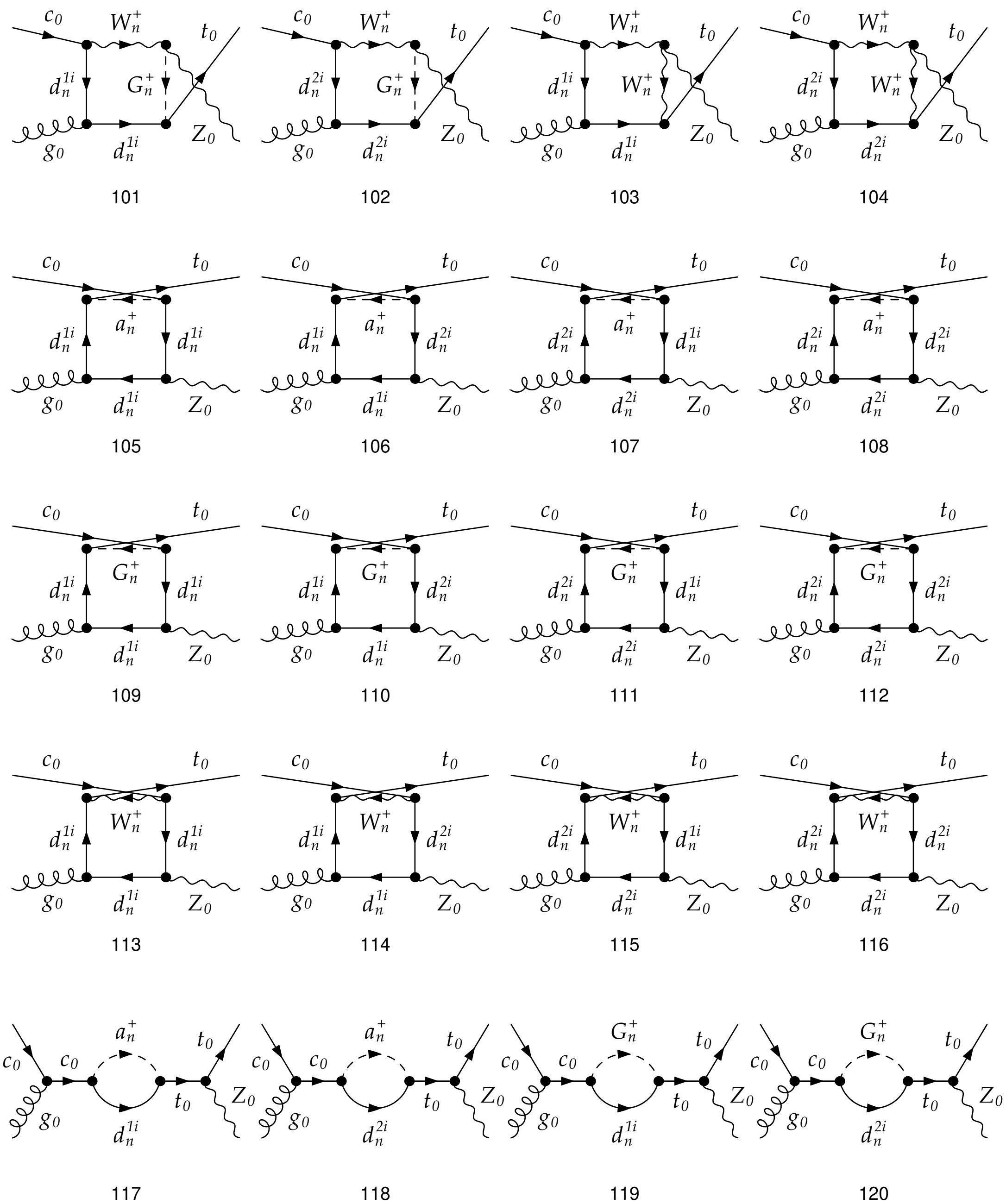


Figure A.66: MUED diagrams contributing to the process $cg \to tZ$.

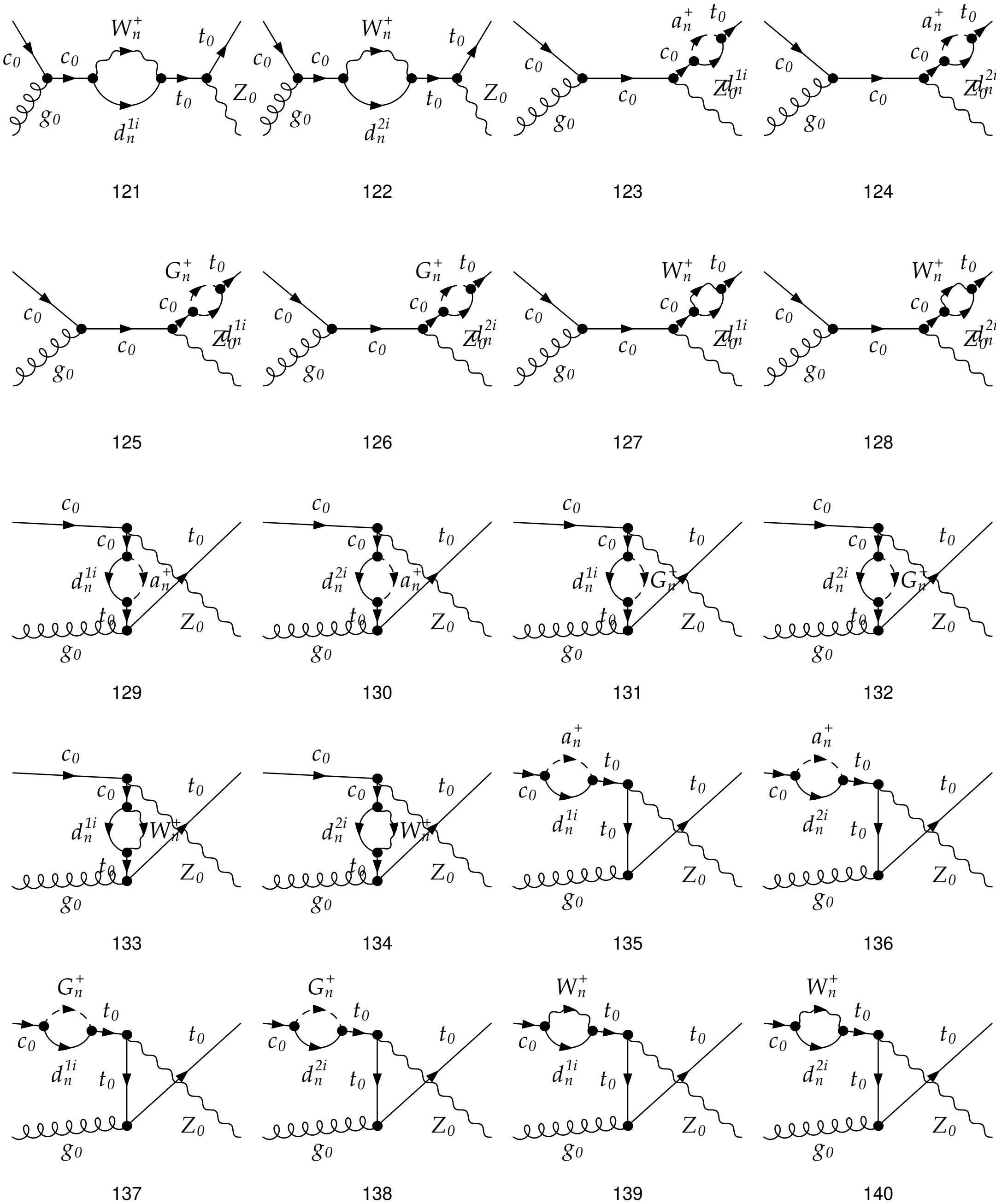


Figure A.67: MUED diagrams contributing to the process $cg \to tZ$.

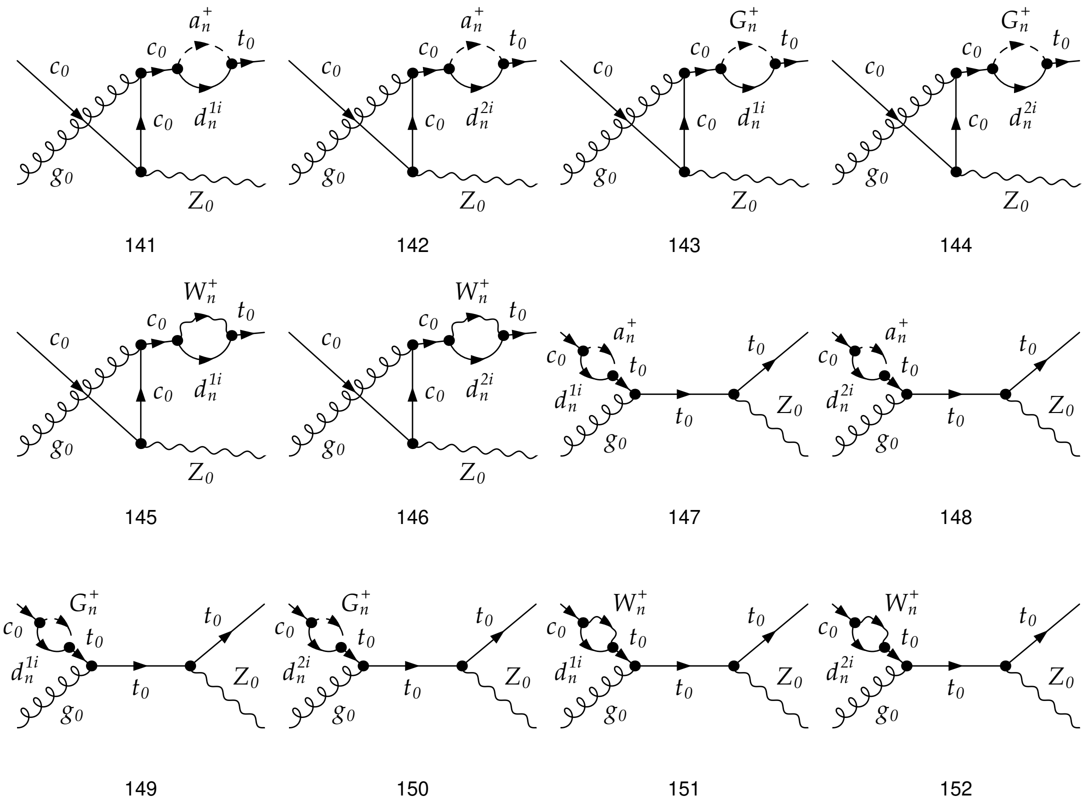


Figure A.68: MUED diagrams contributing to the process $cg \to tZ$.

APPENDIX B

VERTEX FACTORS IN MUED

In this chapter, the relevant vertex factors for the rare top quark processes given in Equations (3.1)–(3.10) are presented. There are no 4-point interactions required. The KK numbers of the fields in the 3-point vertices are considered to be of the form 000 or $nn0$. At one loop, the vertices of the form nmk with all the indices different from 0 are absent due to the conservation of KK number.

In Section B.1, the vertex factors for the interactions of the fermions with the scalars (FFS) and the vectors (FFV). Section B.2 contains the bosonic interactions. We treat gluon self interactions and ghosts separately in Section B.3.

The output has been produced by using LanHEP. It should be noted that the vertex factors should be multiplied by $-i$.

B.1 FFS and FFV interactions

Fields in the vertex	Feynman rules
$\bar{c}_{0ap} \quad c_{0bq} \quad \gamma_{0\mu}$	$-\frac{2}{3}e\delta_{pq}\gamma^{\mu}_{ac}\delta_{cb}$
$\bar{c}_{0ap} \quad c_{0bq} \quad g_{0\mu r}$	$-g_s\lambda^{r}_{pq}\gamma^{\mu}_{ac}\delta_{cb}$
$\bar{c}_{0ap} \quad c_{0bq} \quad Z_{0\mu}$	$-\frac{1}{6}\frac{e}{c_W s_W}\delta_{pq}\gamma^{\mu}_{ac}\big((3-4{s_W}^2)\frac{(1-\gamma^5)_{cb}}{2}-4{s_W}^2\frac{(1+\gamma^5)_{cb}}{2}\big)$
$\bar{c}_{0ap} \quad c_{0bq} \quad h_0$	$-\frac{1}{2}\frac{m_c e}{m_W s_W}\delta_{pq}\delta_{ab}$
$\bar{c}_{0ap} \quad d^{i}_{0bq} \quad W^{+}_{0\ \mu}$	$-\frac{1}{2}\frac{\sqrt{2}V_{ci}e}{s_W}\delta_{pq}\gamma^{\mu}_{ac}\frac{(1-\gamma^5)_{cb}}{2}$
$\bar{c}_{0ap} \quad d^{i}_{0bq} \quad \phi^{+}_{0}$	$-\frac{1}{2}\frac{i\sqrt{2}V_{ci}e}{m_W s_W}\delta_{pq}\big(m_{d^i}\frac{(1+\gamma^5)_{ab}}{2}-m_c\frac{(1-\gamma^5)_{ab}}{2}\big)$

$\bar{d}^i_{0ap}$	c_{0bq}	$W^-_{0\ \mu}$	$-\frac{1}{2}\frac{\sqrt{2}V_{ci}e}{s_W}\delta_{pq}\gamma^\mu_{ac}\frac{(1-\gamma^5)_{cb}}{2}$
$\bar{d}^i_{0ap}$	c_{0bq}	ϕ^-_0	$\frac{1}{2}\frac{i\sqrt{2}V_{ci}e}{m_W s_W}\delta_{pq}\big(m_{d^i}\frac{(1-\gamma^5)_{ab}}{2} - m_c\frac{(1+\gamma^5)_{ab}}{2}\big)$
$\bar{d}^i_{0ap}$	d^i_{0bq}	$\gamma_{0\mu}$	$\frac{1}{3}e\delta_{pq}\gamma^\mu_{ac}\delta_{cb}$
$\bar{d}^i_{0ap}$	d^i_{0bq}	$g_{0\mu r}$	$-g_s\lambda^r_{pq}\gamma^\mu_{ac}\delta_{cb}$
$\bar{d}^i_{0ap}$	d^i_{0bq}	$Z_{0\mu}$	$\frac{1}{6}\frac{e}{c_W s_W}\delta_{pq}\gamma^\mu_{ac}\big((3-2{s_W}^2)\frac{(1-\gamma^5)_{cb}}{2} - 2{s_W}^2\frac{(1+\gamma^5)_{cb}}{2}\big)$
$\bar{d}^i_{0ap}$	d^i_{0bq}	h_0	$-\frac{1}{2}\frac{m_{d^i}e}{m_W s_W}\delta_{pq}\delta_{ab}$
$\bar{d}^i_{0ap}$	t_{0bq}	$W^-_{0\ \mu}$	$-\frac{1}{2}\frac{\sqrt{2}V_{ti}e}{s_W}\delta_{pq}\gamma^\mu_{ac}\frac{(1-\gamma^5)_{cb}}{2}$
$\bar{d}^i_{0ap}$	t_{0bq}	ϕ^-_0	$\frac{1}{2}\frac{i\sqrt{2}V_{ti}e}{m_W s_W}\delta_{pq}\big(m_{d^i}\frac{(1-\gamma^5)_{ab}}{2} - m_t\frac{(1+\gamma^5)_{ab}}{2}\big)$
$\bar{t}_{0ap}$	d^i_{0bq}	$W^+_{0\ \mu}$	$-\frac{1}{2}\frac{\sqrt{2}V_{ti}e}{s_W}\delta_{pq}\gamma^\mu_{ac}\frac{(1-\gamma^5)_{cb}}{2}$
$\bar{t}_{0ap}$	d^i_{0bq}	ϕ^+_0	$-\frac{1}{2}\frac{i\sqrt{2}V_{ti}e}{m_W s_W}\delta_{pq}\big(m_{d^i}\frac{(1+\gamma^5)_{ab}}{2} - m_t\frac{(1-\gamma^5)_{ab}}{2}\big)$
$\bar{t}_{0ap}$	t_{0bq}	$\gamma_{0\mu}$	$-\frac{2}{3}e\delta_{pq}\gamma^\mu_{ac}\delta_{cb}$
$\bar{t}_{0ap}$	t_{0bq}	$g_{0\mu r}$	$-g_s\lambda^r_{pq}\gamma^\mu_{ac}\delta_{cb}$
$\bar{t}_{0ap}$	t_{0bq}	$Z_{0\mu}$	$-\frac{1}{6}\frac{e}{c_W s_W}\delta_{pq}\gamma^\mu_{ac}\big((3-4{s_W}^2)\frac{(1-\gamma^5)_{cb}}{2} - 4{s_W}^2\frac{(1+\gamma^5)_{cb}}{2}\big)$
$\bar{t}_{0ap}$	t_{0bq}	h_0	$-\frac{1}{2}\frac{m_t e}{m_W s_W}\delta_{pq}\delta_{ab}$
$\bar{c}_{0ap}$	${d_n^{1i}}_{bq}$	$W^+_{n\ \mu}$	$\frac{1}{2}\frac{\sqrt{2}V_{ci}e\sin(\varphi_{dn})}{s_W}\delta_{pq}\gamma^\mu_{ac}\frac{(1-\gamma^5)_{cb}}{2}$
$\bar{c}_{0ap}$	${d_n^{1i}}_{bq}$	G^+_n	$-\frac{1}{2}\frac{i\sqrt{2}V_{ci}e}{m_{W_n}ns_W m_W}\delta_{pq}\big(m_n m_W\sin(\varphi_{dn})\frac{(1+\gamma^5)_{ab}}{2}$
			$-m_Z m_{d^i}\cos(\varphi_{dn})c_W n\frac{(1+\gamma^5)_{ab}}{2}$
			$+m_Z m_c c_W n\sin(\varphi_{dn})\frac{(1-\gamma^5)_{ab}}{2}\big)$
$\bar{c}_{0ap}$	${d_n^{1i}}_{bq}$	a^+_n	$-\frac{1}{2}\frac{i\sqrt{2}V_{ci}e}{m_{W_n}s_W m_W n}\delta_{pq}\big(m_W m_Z c_W n\sin(\varphi_{dn})\frac{(1+\gamma^5)_{ab}}{2}$
			$+m_n m_{d^i}\cos(\varphi_{dn})\frac{(1+\gamma^5)_{ab}}{2} - m_n m_c\sin(\varphi_{dn})\frac{(1-\gamma^5)_{ab}}{2}\big)$
$\bar{c}_{0ap}$	${d_n^{2i}}_{bq}$	$W^+_{n\ \mu}$	$-\frac{1}{2}\frac{\sqrt{2}V_{ci}\cos(\varphi_{dn})e}{s_W}\delta_{pq}\gamma^\mu_{ac}\frac{(1-\gamma^5)_{cb}}{2}$
$\bar{c}_{0ap}$	${d_n^{2i}}_{bq}$	G^+_n	$-\frac{1}{2}\frac{i\sqrt{2}V_{ci}e}{m_{W_n}ns_W m_W}\delta_{pq}\big(m_n m_W\cos(\varphi_{dn})\frac{(1+\gamma^5)_{ab}}{2}$
			$+m_Z m_{d^i}c_W n\sin(\varphi_{dn})\frac{(1+\gamma^5)_{ab}}{2}$

			$-m_Z m_c \cos(\varphi_{dn}) c_W n \frac{(1-\gamma^5)_{ab}}{2}\big)$
$\bar{c}_{0ap}$	$d^{2i}_{n\,bq}$	a^+_n	$-\frac{1}{2}\frac{i\sqrt{2}V_{ci}e}{m_{W_n} s_W m_W n}\delta_{pq}\big(m_W m_Z \cos(\varphi_{dn}) c_W n \frac{(1+\gamma^5)_{ab}}{2}$
			$-m_n m_{d^i} \sin(\varphi_{dn}) \frac{(1+\gamma^5)_{ab}}{2} + m_n m_c \cos(\varphi_{dn}) \frac{(1-\gamma^5)_{ab}}{2}\big)$
$\bar{d}^{1i}_{n\,ap}$	c_{0bq}	$W^-_{n\,\mu}$	$\frac{1}{2}\frac{\sqrt{2}V_{ci}e\sin(\varphi_{dn})}{s_W}\delta_{pq}\gamma^\mu_{ac}\frac{(1-\gamma^5)_{cb}}{2}$
$\bar{d}^{1i}_{n\,ap}$	c_{0bq}	G^-_n	$\frac{1}{2}\frac{i\sqrt{2}V_{ci}e}{m_{W_n} n s_W m_W}\delta_{pq}\big(m_n m_W \sin(\varphi_{dn}) \frac{(1-\gamma^5)_{ab}}{2}$
			$-m_Z m_{d^i} \cos(\varphi_{dn}) c_W n \frac{(1-\gamma^5)_{ab}}{2}$
			$+m_Z m_c c_W n \sin(\varphi_{dn}) \frac{(1+\gamma^5)_{ab}}{2}\big)$
$\bar{d}^{1i}_{n\,ap}$	c_{0bq}	a^-_n	$\frac{1}{2}\frac{i\sqrt{2}V_{ci}e}{m_{W_n} s_W m_W n}\delta_{pq}\big(m_W m_Z c_W n \sin(\varphi_{dn}) \frac{(1-\gamma^5)_{ab}}{2}$
			$+m_n m_{d^i} \cos(\varphi_{dn}) \frac{(1-\gamma^5)_{ab}}{2} - m_n m_c \sin(\varphi_{dn}) \frac{(1+\gamma^5)_{ab}}{2}\big)$
$\bar{d}^{1i}_{n\,ap}$	$d^{1i}_{n\,bq}$	$\gamma_{0\mu}$	$\frac{1}{3}e\delta_{pq}\gamma^\mu_{ac}\delta_{cb}$
$\bar{d}^{1i}_{n\,ap}$	$d^{1i}_{n\,bq}$	$g_{0\mu r}$	$-g_s\lambda^r_{pq}\gamma^\mu_{ac}\delta_{cb}$
$\bar{d}^{1i}_{n\,ap}$	$d^{1i}_{n\,bq}$	$Z_{0\mu}$	$\frac{1}{6}\frac{e}{c_W s_W}\delta_{pq}\gamma^\mu_{ac}\big(3\sin(\varphi_{dn})^2\delta_{cb} - 2{s_W}^2\delta_{cb}\big)$
$\bar{d}^{1i}_{n\,ap}$	$d^{1i}_{n\,bq}$	h_0	$-\frac{m_{d^i}\cos(\varphi_{dn})e\sin(\varphi_{dn})}{m_W s_W}\delta_{pq}\delta_{ab}$
$\bar{d}^{1i}_{n\,ap}$	$d^{2i}_{n\,bq}$	$Z_{0\mu}$	$\frac{1}{2}\frac{\cos(\varphi_{dn})e\sin(\varphi_{dn})}{c_W s_W}\delta_{pq}\gamma^\mu_{ac}\gamma^5_{cb}$
$\bar{d}^{1i}_{n\,ap}$	$d^{2i}_{n\,bq}$	h_0	$-\frac{1}{2}\frac{(1-2\sin(\varphi_{dn})^2)m_{d^i}e}{m_W s_W}\delta_{pq}\gamma^5_{ab}$
$\bar{d}^{1i}_{n\,ap}$	t_{0bq}	$W^-_{n\,\mu}$	$\frac{1}{2}\frac{\sqrt{2}V_{ti}e\sin(\varphi_{dn})}{s_W}\delta_{pq}\gamma^\mu_{ac}\frac{(1-\gamma^5)_{cb}}{2}$
$\bar{d}^{1i}_{n\,ap}$	t_{0bq}	G^-_n	$\frac{1}{2}\frac{i\sqrt{2}V_{ti}e}{m_{W_n} n s_W m_W}\delta_{pq}\big(m_n m_W \sin(\varphi_{dn}) \frac{(1-\gamma^5)_{ab}}{2}$
			$-m_Z m_{d^i} \cos(\varphi_{dn}) c_W n \frac{(1-\gamma^5)_{ab}}{2}$
			$+m_Z m_t c_W n \sin(\varphi_{dn}) \frac{(1+\gamma^5)_{ab}}{2}\big)$
$\bar{d}^{1i}_{n\,ap}$	t_{0bq}	a^-_n	$\frac{1}{2}\frac{i\sqrt{2}V_{ti}e}{m_{W_n} s_W m_W n}\delta_{pq}\big(m_W m_Z c_W n \sin(\varphi_{dn}) \frac{(1-\gamma^5)_{ab}}{2}$
			$+m_n m_{d^i} \cos(\varphi_{dn}) \frac{(1-\gamma^5)_{ab}}{2} - m_n m_t \sin(\varphi_{dn}) \frac{(1+\gamma^5)_{ab}}{2}\big)$
$\bar{d}^{2i}_{n\,ap}$	c_{0bq}	$W^-_{n\,\mu}$	$-\frac{1}{2}\frac{\sqrt{2}V_{ci}\cos(\varphi_{dn})e}{s_W}\delta_{pq}\gamma^\mu_{ac}\frac{(1-\gamma^5)_{cb}}{2}$
$\bar{d}^{2i}_{n\,ap}$	c_{0bq}	G^-_n	$\frac{1}{2}\frac{i\sqrt{2}V_{ci}e}{m_{W_n} n s_W m_W}\delta_{pq}\big(m_n m_W \cos(\varphi_{dn}) \frac{(1-\gamma^5)_{ab}}{2}$

			$+m_Z m_{d^i} c_W n \sin(\varphi_{dn}) \frac{(1-\gamma^5)_{ab}}{2}$
			$-m_Z m_c \cos(\varphi_{dn}) c_W n \frac{(1+\gamma^5)_{ab}}{2}\big)$
$\bar{d}^{2i}_{n\,ap}$	c_{0bq}	a^-_n	$\frac{1}{2}\frac{i\sqrt{2}V_{ci}e}{m_{W_n} s_W m_W n}\delta_{pq}\big(m_W m_Z \cos(\varphi_{dn}) c_W n \frac{(1-\gamma^5)_{ab}}{2}$
			$-m_n m_{d^i} \sin(\varphi_{dn}) \frac{(1-\gamma^5)_{ab}}{2} + m_n m_c \cos(\varphi_{dn}) \frac{(1+\gamma^5)_{ab}}{2}\big)$
$\bar{d}^{2i}_{n\,ap}$	$d^{1i}_{n\,bq}$	$Z_{0\mu}$	$\frac{1}{2}\frac{\cos(\varphi_{dn}) e \sin(\varphi_{dn})}{c_W s_W}\delta_{pq}\gamma^\mu_{ac}\gamma^5_{cb}$
$\bar{d}^{2i}_{n\,ap}$	$d^{1i}_{n\,bq}$	h_0	$\frac{1}{2}\frac{(1-2\sin(\varphi_{dn})^2) m_{d^i} e}{m_W s_W}\delta_{pq}\gamma^5_{ab}$
$\bar{d}^{2i}_{n\,ap}$	$d^{2i}_{n\,bq}$	$\gamma_{0\mu}$	$\frac{1}{3} e \delta_{pq}\gamma^\mu_{ac}\delta_{cb}$
$\bar{d}^{2i}_{n\,ap}$	$d^{2i}_{n\,bq}$	$g_{0\mu r}$	$-g_s \lambda^r_{pq}\gamma^\mu_{ac}\delta_{cb}$
$\bar{d}^{2i}_{n\,ap}$	$d^{2i}_{n\,bq}$	$Z_{0\mu}$	$\frac{1}{6}\frac{e}{c_W s_W}\delta_{pq}\gamma^\mu_{ac}\big(3\cos(\varphi_{dn})^2\delta_{cb} - 2{s_W}^2\delta_{cb}\big)$
$\bar{d}^{2i}_{n\,ap}$	$d^{2i}_{n\,bq}$	h_0	$-\frac{m_{d^i}\cos(\varphi_{dn}) e \sin(\varphi_{dn})}{m_W s_W}\delta_{pq}\delta_{ab}$
$\bar{d}^{2i}_{n\,ap}$	t_{0bq}	$W^-_{n\;\mu}$	$-\frac{1}{2}\frac{\sqrt{2}V_{ti}\cos(\varphi_{dn}) e}{s_W}\delta_{pq}\gamma^\mu_{ac}\frac{(1-\gamma^5)_{cb}}{2}$
$\bar{d}^{2i}_{n\,ap}$	t_{0bq}	G^-_n	$\frac{1}{2}\frac{i\sqrt{2}V_{ti}e}{m_{W_n} n s_W m_W}\delta_{pq}\big(m_n m_W \cos(\varphi_{dn}) \frac{(1-\gamma^5)_{ab}}{2}$
			$+m_Z m_{d^i} c_W n \sin(\varphi_{dn}) \frac{(1-\gamma^5)_{ab}}{2}$
			$-m_Z m_t \cos(\varphi_{dn}) c_W n \frac{(1+\gamma^5)_{ab}}{2}\big)$
$\bar{d}^{2i}_{n\,ap}$	t_{0bq}	a^-_n	$\frac{1}{2}\frac{i\sqrt{2}V_{ti}e}{m_{W_n} s_W m_W n}\delta_{pq}\big(m_W m_Z \cos(\varphi_{dn}) c_W n \frac{(1-\gamma^5)_{ab}}{2}$
			$-m_n m_{d^i} \sin(\varphi_{dn}) \frac{(1-\gamma^5)_{ab}}{2} + m_n m_t \cos(\varphi_{dn}) \frac{(1+\gamma^5)_{ab}}{2}\big)$
$\bar{t}_{0ap}$	$d^{1i}_{n\,bq}$	$W^+_{n\;\mu}$	$\frac{1}{2}\frac{\sqrt{2}V_{ti} e \sin(\varphi_{dn})}{s_W}\delta_{pq}\gamma^\mu_{ac}\frac{(1-\gamma^5)_{cb}}{2}$
$\bar{t}_{0ap}$	$d^{1i}_{n\,bq}$	G^+_n	$-\frac{1}{2}\frac{i\sqrt{2}V_{ti}e}{m_{W_n} n s_W m_W}\delta_{pq}\big(m_n m_W \sin(\varphi_{dn}) \frac{(1+\gamma^5)_{ab}}{2}$
			$-m_Z m_{d^i} \cos(\varphi_{dn}) c_W n \frac{(1+\gamma^5)_{ab}}{2}$
			$+m_Z m_t c_W n \sin(\varphi_{dn}) \frac{(1-\gamma^5)_{ab}}{2}\big)$
$\bar{t}_{0ap}$	$d^{1i}_{n\,bq}$	a^+_n	$-\frac{1}{2}\frac{i\sqrt{2}V_{ti}e}{m_{W_n} s_W m_W n}\delta_{pq}\big(m_W m_Z c_W n \sin(\varphi_{dn}) \frac{(1+\gamma^5)_{ab}}{2}$
			$+m_n m_{d^i} \cos(\varphi_{dn}) \frac{(1+\gamma^5)_{ab}}{2} - m_n m_t \sin(\varphi_{dn}) \frac{(1-\gamma^5)_{ab}}{2}\big)$
$\bar{t}_{0ap}$	$d^{2i}_{n\,bq}$	$W^+_{n\;\mu}$	$-\frac{1}{2}\frac{\sqrt{2}V_{ti}\cos(\varphi_{dn}) e}{s_W}\delta_{pq}\gamma^\mu_{ac}\frac{(1-\gamma^5)_{cb}}{2}$

$\bar{t}_{0ap}$ $d^{2i}_{n\,bq}$ G^+_n	$-\frac{1}{2}\frac{i\sqrt{2}V_{ti}e}{m_{W_n}ns_Wm_W}\delta_{pq}\big(m_nm_W\cos(\varphi_{dn})\frac{(1+\gamma^5)_{ab}}{2}$ $+m_Zm_{d^i}c_Wn\sin(\varphi_{dn})\frac{(1+\gamma^5)_{ab}}{2}$ $-m_Zm_t\cos(\varphi_{dn})c_Wn\frac{(1-\gamma^5)_{ab}}{2}\big)$
$\bar{t}_{0ap}$ $d^{2i}_{n\,bq}$ a^+_n	$-\frac{1}{2}\frac{i\sqrt{2}V_{ti}e}{m_{W_n}s_Wm_Wn}\delta_{pq}\big(m_Wm_Z\cos(\varphi_{dn})c_Wn\frac{(1+\gamma^5)_{ab}}{2}$ $-m_nm_{d^i}\sin(\varphi_{dn})\frac{(1+\gamma^5)_{ab}}{2}+m_nm_t\cos(\varphi_{dn})\frac{(1-\gamma^5)_{ab}}{2}\big)$

B.2 Bosonic interactions

Fields in the vertex	Feynman rules
$\gamma_{0\mu}$ $W^-_{0\ \nu}$ $W^+_{0\ \rho}$	$-e\big(p^\rho_1g^{\mu\nu}-p^\rho_2g^{\mu\nu}-p^\nu_1g^{\mu\rho}+p^\nu_3g^{\mu\rho}+p^\mu_2g^{\nu\rho}-p^\mu_3g^{\nu\rho}\big)$
$\gamma_{0\mu}$ $W^-_{0\ \nu}$ ϕ^+_0	$im_Zc_Weg^{\mu\nu}$
$\gamma_{0\mu}$ ϕ^-_0 $W^+_{0\ \nu}$	$-im_Zc_Weg^{\mu\nu}$
$\gamma_{0\mu}$ ϕ^-_0 ϕ^+_0	$e\big(p^\mu_2-p^\mu_3\big)$
$W^-_{0\ \mu}$ $W^+_{0\ \nu}$ $Z_{0\rho}$	$-\frac{c_We}{s_W}\big(p^\nu_3g^{\mu\rho}-p^\nu_1g^{\mu\rho}-p^\mu_3g^{\nu\rho}+p^\mu_2g^{\nu\rho}+p^\rho_1g^{\mu\nu}-p^\rho_2g^{\mu\nu}\big)$
$W^-_{0\ \mu}$ $W^+_{0\ \nu}$ h_0	$\frac{m_Zc_We}{s_W}g^{\mu\nu}$
$W^-_{0\ \mu}$ ϕ^+_0 $Z_{0\nu}$	$-im_Zes_Wg^{\mu\nu}$
$W^-_{0\ \mu}$ ϕ^+_0 h_0	$-\frac{1}{2}\frac{ie}{s_W}\big(p^\mu_2-p^\mu_3\big)$
ϕ^-_0 $W^+_{0\ \mu}$ $Z_{0\nu}$	$im_Zes_Wg^{\mu\nu}$
ϕ^-_0 $W^+_{0\ \mu}$ h_0	$-\frac{1}{2}\frac{ie}{s_W}\big(p^\mu_1-p^\mu_3\big)$
ϕ^-_0 ϕ^+_0 $Z_{0\mu}$	$\frac{1}{2}\frac{(1-2{s_W}^2)e}{c_Ws_W}\big(p^\mu_1-p^\mu_2\big)$
ϕ^-_0 ϕ^+_0 h_0	$-4\frac{m_Zc_W\lambda s_W}{e}$
$Z_{0\mu}$ $Z_{0\nu}$ h_0	$\frac{m_Ze}{c_Ws_W}g^{\mu\nu}$
$W^-_{n\ \mu}$ $W^+_{n\ \nu}$ $Z_{0\rho}$	$-\frac{c_We}{s_W}\big(p^\nu_3g^{\mu\rho}-p^\nu_1g^{\mu\rho}-p^\mu_3g^{\nu\rho}+p^\mu_2g^{\nu\rho}+p^\rho_1g^{\mu\nu}-p^\rho_2g^{\mu\nu}\big)$
$W^-_{n\ \mu}$ $W^+_{n\ \nu}$ h_0	$\frac{m_Zc_We}{s_W}g^{\mu\nu}$
$W^-_{n\ \mu}$ G^+_n $Z_{0\nu}$	$-\frac{ic_We}{m_{W_n}s_Wn^2}g^{\mu\nu}\big({m_Z}^2n^2{s_W}^2-{m_n}^2\big)$

$W^-_{n\ \mu}\quad G^+_n\quad h_0$	$-\frac{1}{2}\frac{im_Z c_W e}{m_{W_n} s_W}\big(p_2^\mu - p_3^\mu\big)$
$W^-_{n\ \mu}\quad Z_{0\nu}\quad a^+_n$	$\frac{im_n m_Z e}{m_{W_n} n s_W} g^{\mu\nu}$
$W^-_{n\ \mu}\quad a^+_n\quad h_0$	$\frac{1}{2}\frac{im_n e}{m_{W_n} n s_W}\big(p_2^\mu - p_3^\mu\big)$
$G^-_n\quad W^+_{n\ \mu}\quad Z_{0\nu}$	$\frac{ic_W e}{m_{W_n} s_W n^2} g^{\mu\nu}\big(m_Z{}^2 n^2 s_W{}^2 - m_n{}^2\big)$
$G^-_n\quad W^+_{n\ \mu}\quad h_0$	$-\frac{1}{2}\frac{im_Z c_W e}{m_{W_n} s_W}\big(p_1^\mu - p_3^\mu\big)$
$G^-_n\quad G^+_n\quad Z_{0\mu}$	$\frac{1}{2}\frac{c_W e}{m_{W_n}{}^2 s_W n^2}\big((1-2s_W{}^2) m_Z{}^2 n^2 p_1^\mu - (1-2s_W{}^2) m_Z{}^2 n^2 p_2^\mu +$ $2m_n{}^2 p_1^\mu - 2m_n{}^2 p_2^\mu\big)$
$G^-_n\quad G^+_n\quad h_0$	$-4\frac{m_Z{}^3 c_W{}^3 \lambda s_W}{m_{W_n}{}^2 e}$
$G^-_n\quad Z_{0\mu}\quad a^+_n$	$\frac{1}{2}\frac{m_n m_Z e}{m_{W_n}{}^2 n s_W}\big(p_1^\mu - p_3^\mu\big)$
$G^-_n\quad a^+_n\quad h_0$	$-\frac{1}{2}\frac{m_n}{m_{W_n}{}^2 e n^3 s_W}\big(m_n{}^2 e^2$ $+m_Z{}^2 c_W{}^2 e^2 n^2 - 8 m_Z{}^2 c_W{}^2 \lambda n^2 s_W{}^2\big)$
$W^+_{n\ \mu}\quad Z_{0\nu}\quad a^-_n$	$-\frac{im_n m_Z e}{m_{W_n} n s_W} g^{\mu\nu}$
$W^+_{n\ \mu}\quad a^-_n\quad h_0$	$\frac{1}{2}\frac{im_n e}{m_{W_n} n s_W}\big(p_2^\mu - p_3^\mu\big)$
$G^+_n\quad Z_{0\mu}\quad a^-_n$	$\frac{1}{2}\frac{m_n m_Z e}{m_{W_n}{}^2 n s_W}\big(p_3^\mu - p_1^\mu\big)$
$G^+_n\quad a^-_n\quad h_0$	$-\frac{1}{2}\frac{m_n}{m_{W_n}{}^2 e n^3 s_W}\big(m_n{}^2 e^2$ $+m_Z{}^2 c_W{}^2 e^2 n^2 - 8 m_Z{}^2 c_W{}^2 \lambda n^2 s_W{}^2\big)$
$\gamma_{0\mu}\quad W^-_{n\ \nu}\quad W^+_{n\ \rho}$	$-e\big(p_1^\rho g^{\mu\nu} - p_2^\rho g^{\mu\nu} - p_1^\nu g^{\mu\rho} + p_3^\nu g^{\mu\rho} + p_2^\mu g^{\nu\rho} - p_3^\mu g^{\nu\rho}\big)$
$\gamma_{0\mu}\quad W^-_{n\ \nu}\quad G^+_n$	$\frac{ie}{m_{W_n} n^2} g^{\mu\nu}\big(m_Z{}^2 c_W{}^2 n^2 + m_n{}^2\big)$
$\gamma_{0\mu}\quad G^-_n\quad W^+_{n\ \nu}$	$-\frac{ie}{m_{W_n} n^2} g^{\mu\nu}\big(m_Z{}^2 c_W{}^2 n^2 + m_n{}^2\big)$
$\gamma_{0\mu}\quad G^-_n\quad G^+_n$	$\frac{e}{m_{W_n}{}^2 n^2}\big(m_Z{}^2 c_W{}^2 n^2 p_2^\mu - m_Z{}^2 c_W{}^2 n^2 p_3^\mu + m_n{}^2 p_2^\mu - m_n{}^2 p_3^\mu\big)$
$\gamma_{0\mu}\quad a^-_n\quad a^+_n$	$\frac{e}{m_{W_n}{}^2 n^2}\big(m_n{}^2 p_2^\mu - m_n{}^2 p_3^\mu + m_Z{}^2 c_W{}^2 n^2 p_2^\mu - m_Z{}^2 c_W{}^2 n^2 p_3^\mu\big)$
$Z_{0\mu}\quad a^-_n\quad a^+_n$	$\frac{1}{2}\frac{e}{m_{W_n}{}^2 c_W n^2 s_W}\big((1-2s_W{}^2) m_n{}^2 p_2^\mu - (1-2s_W{}^2) m_n{}^2 p_3^\mu +$ $2m_Z{}^2 c_W{}^4 n^2 p_2^\mu - 2m_Z{}^2 c_W{}^4 n^2 p_3^\mu\big)$
$a^-_n\quad a^+_n\quad h_0$	$-\frac{m_Z c_W}{m_{W_n}{}^2 e n^2 s_W}\big(m_n{}^2 e^2 + m_Z{}^2 c_W{}^2 e^2 n^2 + 4 m_n{}^2 \lambda s_W{}^2\big)$

B.3 Gluon and ghost vertices

Fields in the vertex	Feynman rules
$g_{0\mu p}\quad g_{0\nu q}\quad g_{0\rho r}$	$ig_s f_{pqr}\big(p_3^\nu g^{\mu\rho} - p_3^\mu g^{\nu\rho} + p_1^\rho g^{\mu\nu} - p_1^\nu g^{\mu\rho} - p_2^\rho g^{\mu\nu} + p_2^\mu g^{\nu\rho}\big)$
$\bar{\omega}_{g0p}\quad \omega_{g0q}\quad g_{0\mu r}$	$-g_s p_1^\mu f_{pqr}$

APPENDIX C

MATHEMATICA CODE

In the literature, there exists a LanHEP code for the MUED model by Belyaev *et al.* [29]. However, this model yields suspicious vertex factors for the interactions of the quarks with the charged bosons, regarding the CKM matrix elements.

We prepared an original Mathematica code to cross-check the vertex factors of the interactions of the fermions with the scalars and vectors at the KK level $nn0$ before moving on to the numerical analysis. Below is presented our Mathematica code. It should be noted that, for convenience, the subscripts are not written by using the `Subscript` command of Mathematica, but as if we were coding in an `.nb` file by using the graphical tools of the software.

```
Clear["Global`*"];

(*

*** here is the convention ***

FFS/nn0 AND FFV/nn0 INTERACTIONS OF QUARKS IN OUR MUED

Gamma5 and PL/PR are always to the right of GammaMu.

SPECTRUM

up: SM mode of up-like Dirac quark
down: SM mode of down-like Dirac quark
up1: KK mode of 1st up-like Dirac quark
up2: KK mode of 2nd up-like Dirac quark
```

```
down1: KK mode of 1st down-like Dirac quark
down2: KK mode of 2nd down-like Dirac quark

Wp: SM mode of W^+ boson
Wm: SM mode of W^- boson
Z: SM mode of Z boson
A: SM mode of photon
g: SM mode of gluon

Wpn: KK mode of W^+ boson
Wmn: KK mode of W^- boson
Pn: KK mode of P boson
Vn: KK mode of V boson
gn: KK mode of gluon

higgs: SM mode of Higgs
Wpf: SM mode of Goldstone of W^+ boson
Wmf: SM mode of Goldstone of W^- boson
Zf: SM mode of Goldstone of Z boson

Wpnf: KK mode of Goldstone of W^+ boson
Wmnf: KK mode of Goldstone of W^- boson
Pnf: KK mode of Goldstone of P boson
Vnf: KK mode of Goldstone of V boson
gnf: KK mode of Goldstone of gluon

hn:KK mode of Higgs
apn and amn: charged scalars in the tower
an: neutral scalar in the tower

*)

(* --- *** --- *)

(* integrator *)

c_n_ := Sqrt[2/(Pi R)] Cos[n (y/R)];
```

```
sn_ := Sqrt[2/(Pi R)] Sin[n (y/R)];

int[a_] := Pi R a /; FreeQ[a, y]
int[a_ b_] := a int[b] /; FreeQ[a, y]
int[a_ + b_] := int[a] + int[b]
int[Cos[n_ y/R]] := 0
int[Sin[n_ y/R]] := 0
int[Cos[n_ y/R] Cos[m_ y/R]] := deltan m Pi R/2
int[Cos[n_ y/R] Sin[m_ y/R]] := 0
int[Sin[n_ y/R] Sin[m_ y/R]] := deltan m Pi R/2
(* for the purposes of studying nn0: *)
int[Cos[n_ y/R] Cos[m_ y/R] Cos[k_ y/R]] := 0
int[Cos[n_ y/R] Cos[m_ y/R] Sin[k_ y/R]] := 0
int[Cos[n_ y/R] Sin[m_ y/R] Sin[k_ y/R]] := 0
int[Sin[n_ y/R] Sin[m_ y/R] Sin[k_ y/R]] := 0
int[Cos[n_ y/R] Cos[m_ y/R] Cos[k_ y/R] Cos[l_ y/R]] := 0
int[Cos[n_ y/R] Cos[m_ y/R] Cos[k_ y/R] Sin[l_ y/R]] := 0
int[Cos[n_ y/R] Cos[m_ y/R] Sin[k_ y/R] Sin[l_ y/R]] := 0
int[Cos[n_ y/R] Sin[m_ y/R] Sin[k_ y/R] Sin[l_ y/R]] := 0
int[Sin[n_ y/R] Sin[m_ y/R] Sin[k_ y/R] Sin[l_ y/R]] := 0

(* refiner *)

ref[a_] := a
ref[a_ b_] := a ref[b]
ref[a_ + b_] := ref[a] + ref[b]
ref[a_ Ak_ Bl_ deltak_ l_] := a An Bn

(* simplifier *)

sim = {
    LambdaL1 LambdaL1co -> 1,
    LambdaR1 LambdaR1co -> 1,
    LambdaL2 LambdaL2co -> 1,
    LambdaR2 LambdaR2co -> 1,
    LambdaL1co LambdaL2 -> CKM,
    LambdaL1 LambdaL2co -> CKMco,
```

```
    LambdaL1co y1 LambdaR1 -> m10 Sqrt[2]/v,
    LambdaL2co y2 LambdaR2 -> m20 Sqrt[2]/v,
    LambdaR1co y1co LambdaL1 -> m10 Sqrt[2]/v,
    LambdaR2co y2co LambdaL2 -> m20 Sqrt[2]/v,
    LambdaL2co y1 LambdaR1 -> CKMco m10 Sqrt[2]/v,
    LambdaR2co y2co LambdaL1 -> CKMco m20 Sqrt[2]/v,
    LambdaR1co y1co LambdaL2 -> CKM m10 Sqrt[2]/v,
    LambdaL1co y2 LambdaR2 -> CKM m20 Sqrt[2]/v
};

(* collector *)

colVec = {
    Q10Bar gamma Q10 -> upBar gamma PL up,
    Q10Bar gamma Q20 -> upBar gamma PL down,
    Q10Bar gamma q10 -> 0,
    Q10Bar gamma q20 -> 0,
    Q10Bar gamma Q1KKL_n -> upBar gamma PL UpKK_n,
    Q10Bar gamma Q1KKR_n -> 0,
    Q10Bar gamma Q2KKL_n -> upBar gamma PL DownKK_n,
    Q10Bar gamma Q2KKR_n -> 0,
    Q10Bar gamma q1KKL_n -> upBar gamma PL upKK_n,
    Q10Bar gamma q1KKR_n -> 0,
    Q10Bar gamma q2KKL_n -> upBar gamma PL downKK_n,
    Q10Bar gamma q2KKR_n -> 0,

    Q20Bar gamma Q10 -> downBar gamma PL up,
    Q20Bar gamma Q20 -> downBar gamma PL down,
    Q20Bar gamma q10 -> 0,
    Q20Bar gamma q20 -> 0,
    Q20Bar gamma Q1KKL_n -> downBar gamma PL UpKK_n,
    Q20Bar gamma Q1KKR_n -> 0,
    Q20Bar gamma Q2KKL_n -> downBar gamma PL DownKK_n,
    Q20Bar gamma Q2KKR_n -> 0,
    Q20Bar gamma q1KKL_n -> downBar gamma PL upKK_n,
    Q20Bar gamma q1KKR_n -> 0,
    Q20Bar gamma q2KKL_n -> downBar gamma PL downKK_n,
```

Q20Bar gamma q2KKR$_n$ -> 0,

q10Bar gamma Q10 -> 0,

q10Bar gamma Q20 -> 0,

q10Bar gamma q10 -> upBar gamma PR up,

q10Bar gamma q20 -> upBar gamma PR down,

q10Bar gamma Q1KKL$_n$ -> 0,

q10Bar gamma Q1KKR$_n$ -> upBar gamma PR UpKK$_n$,

q10Bar gamma Q2KKL$_n$ -> 0,

q10Bar gamma Q2KKR$_n$ -> upBar gamma PR DownKK$_n$,

q10Bar gamma q1KKL$_n$ -> 0,

q10Bar gamma q1KKR$_n$ -> upBar gamma PR upKK$_n$,

q10Bar gamma q2KKL$_n$ -> 0,

q10Bar gamma q2KKR$_n$ -> upBar gamma PR downKK$_n$,

q20Bar gamma Q10 -> 0,

q20Bar gamma Q20 -> 0,

q20Bar gamma q10 -> downBar gamma PR up,

q20Bar gamma q20 -> downBar gamma PR down,

q20Bar gamma Q1KKL$_n$ -> 0,

q20Bar gamma Q1KKR$_n$ -> downBar gamma PR UpKK$_n$,

q20Bar gamma Q2KKL$_n$ -> 0,

q20Bar gamma Q2KKR$_n$ -> downBar gamma PR DownKK$_n$,

q20Bar gamma q1KKL$_n$ -> 0,

q20Bar gamma q1KKR$_n$ -> downBar gamma PR upKK$_n$,

q20Bar gamma q2KKL$_n$ -> 0,

q20Bar gamma q2KKR$_n$ -> downBar gamma PR downKK$_n$,

Q1KKLBar$_n$ gamma Q10 -> UpKKBar$_n$ gamma PL up,

Q1KKLBar$_n$ gamma Q20 -> UpKKBar$_n$ gamma PL down,

Q1KKLBar$_n$ gamma q10 -> 0,

Q1KKLBar$_n$ gamma q20 -> 0,

Q1KKLBar$_n$ gamma Q1KKL$_n$ -> UpKKBar$_n$ gamma PL UpKK$_n$,

Q1KKLBar$_n$ gamma Q1KKR$_n$ -> 0,

Q1KKLBar$_n$ gamma Q2KKL$_n$ -> UpKKBar$_n$ gamma PL DownKK$_n$,

Q1KKLBar$_n$ gamma Q2KKR$_n$ -> 0,

Q1KKLBar$_n$ gamma q1KKL$_n$ -> UpKKBar$_n$ gamma PL upKK$_n$,

```
Q1KKLBarn gamma q1KKRn -> 0,
Q1KKLBarn gamma q2KKLn -> UpKKBarn gamma PL downKKn,
Q1KKLBarn gamma q2KKRn -> 0,

Q2KKLBarn gamma Q10 -> DownKKBarn gamma PL up,
Q2KKLBarn gamma Q20 -> DownKKBarn gamma PL down,
Q2KKLBarn gamma q10 -> 0,
Q2KKLBarn gamma q20 -> 0,
Q2KKLBarn gamma Q1KKLn -> DownKKBarn gamma PL UpKKn,
Q2KKLBarn gamma Q1KKRn -> 0,
Q2KKLBarn gamma Q2KKLn -> DownKKBarn gamma PL DownKKn,
Q2KKLBarn gamma Q2KKRn -> 0,
Q2KKLBarn gamma q1KKLn -> DownKKBarn gamma PL upKKn,
Q2KKLBarn gamma q1KKRn -> 0,
Q2KKLBarn gamma q2KKLn -> DownKKBarn gamma PL downKKn,
Q2KKLBarn gamma q2KKRn -> 0,

q1KKLBarn gamma Q10 -> upKKBarn gamma PL up,
q1KKLBarn gamma Q20 -> upKKBarn gamma PL down,
q1KKLBarn gamma q10 -> 0,
q1KKLBarn gamma q20 -> 0,
q1KKLBarn gamma Q1KKLn -> upKKBarn gamma PL UpKKn,
q1KKLBarn gamma Q1KKRn -> 0,
q1KKLBarn gamma Q2KKLn -> upKKBarn gamma PL DownKKn,
q1KKLBarn gamma Q2KKRn -> 0,
q1KKLBarn gamma q1KKLn -> upKKBarn gamma PL upKKn,
q1KKLBarn gamma q1KKRn -> 0,
q1KKLBarn gamma q2KKLn -> upKKBarn gamma PL downKKn,
q1KKLBarn gamma q2KKRn -> 0,

q2KKLBarn gamma Q10 -> downKKBarn gamma PL up,
q2KKLBarn gamma Q20 -> downKKBarn gamma PL down,
q2KKLBarn gamma q10 -> 0,
q2KKLBarn gamma q20 -> 0,
q2KKLBarn gamma Q1KKLn -> downKKBarn gamma PL UpKKn,
q2KKLBarn gamma Q1KKRn -> 0,
q2KKLBarn gamma Q2KKLn -> downKKBarn gamma PL DownKKn,
```

q2KKLBar$_n$ gamma Q2KKR$_n$ -> 0,
q2KKLBar$_n$ gamma q1KKL$_n$ -> downKKBar$_n$ gamma PL upKK$_n$,
q2KKLBar$_n$ gamma q1KKR$_n$ -> 0,
q2KKLBar$_n$ gamma q2KKL$_n$ -> downKKBar$_n$ gamma PL downKK$_n$,
q2KKLBar$_n$ gamma q2KKR$_n$ -> 0,

Q1KKRBar$_n$ gamma Q10 -> 0,
Q1KKRBar$_n$ gamma Q20 -> 0,
Q1KKRBar$_n$ gamma q10 -> UpKKBar$_n$ gamma PR up,
Q1KKRBar$_n$ gamma q20 -> UpKKBar$_n$ gamma PR down,
Q1KKRBar$_n$ gamma Q1KKL$_n$ -> 0,
Q1KKRBar$_n$ gamma Q1KKR$_n$ -> UpKKBar$_n$ gamma PR UpKK$_n$,
Q1KKRBar$_n$ gamma Q2KKL$_n$ -> 0,
Q1KKRBar$_n$ gamma Q2KKR$_n$ -> UpKKBar$_n$ gamma PR DownKK$_n$,
Q1KKRBar$_n$ gamma q1KKL$_n$ -> 0,
Q1KKRBar$_n$ gamma q1KKR$_n$ -> UpKKBar$_n$ gamma PR upKK$_n$,
Q1KKRBar$_n$ gamma q2KKL$_n$ -> 0,
Q1KKRBar$_n$ gamma q2KKR$_n$ -> UpKKBar$_n$ gamma PR downKK$_n$,

Q2KKRBar$_n$ gamma Q10 -> 0,
Q2KKRBar$_n$ gamma Q20 -> 0,
Q2KKRBar$_n$ gamma q10 -> DownKKBar$_n$ gamma PR up,
Q2KKRBar$_n$ gamma q20 -> DownKKBar$_n$ gamma PR down,
Q2KKRBar$_n$ gamma Q1KKL$_n$ -> 0,
Q2KKRBar$_n$ gamma Q1KKR$_n$ -> DownKKBar$_n$ gamma PR UpKK$_n$,
Q2KKRBar$_n$ gamma Q2KKL$_n$ -> 0,
Q2KKRBar$_n$ gamma Q2KKR$_n$ -> DownKKBar$_n$ gamma PR DownKK$_n$,
Q2KKRBar$_n$ gamma q1KKL$_n$ -> 0,
Q2KKRBar$_n$ gamma q1KKR$_n$ -> DownKKBar$_n$ gamma PR upKK$_n$,
Q2KKRBar$_n$ gamma q2KKL$_n$ -> 0,
Q2KKRBar$_n$ gamma q2KKR$_n$ -> DownKKBar$_n$ gamma PR downKK$_n$,

q1KKRBar$_n$ gamma Q10 -> 0,
q1KKRBar$_n$ gamma Q20 -> 0,
q1KKRBar$_n$ gamma q10 -> upKKBar$_n$ gamma PR up,
q1KKRBar$_n$ gamma q20 -> upKKBar$_n$ gamma PR down,
q1KKRBar$_n$ gamma Q1KKL$_n$ -> 0,

```
    q1KKRBar_n gamma Q1KKR_n -> upKKBar_n gamma PR UpKK_n,
    q1KKRBar_n gamma Q2KKL_n -> 0,
    q1KKRBar_n gamma Q2KKR_n -> upKKBar_n gamma PR DownKK_n,
    q1KKRBar_n gamma q1KKL_n -> 0,
    q1KKRBar_n gamma q1KKR_n -> upKKBar_n gamma PR upKK_n,
    q1KKRBar_n gamma q2KKL_n -> 0,
    q1KKRBar_n gamma q2KKR_n -> upKKBar_n gamma PR downKK_n,

    q2KKRBar_n gamma Q10 -> 0,
    q2KKRBar_n gamma Q20 -> 0,
    q2KKRBar_n gamma q10 -> downKKBar_n gamma PR up,
    q2KKRBar_n gamma q20 -> downKKBar_n gamma PR down,
    q2KKRBar_n gamma Q1KKL_n -> 0,
    q2KKRBar_n gamma Q1KKR_n -> downKKBar_n gamma PR UpKK_n,
    q2KKRBar_n gamma Q2KKL_n -> 0,
    q2KKRBar_n gamma Q2KKR_n -> downKKBar_n gamma PR DownKK_n,
    q2KKRBar_n gamma q1KKL_n -> 0,
    q2KKRBar_n gamma q1KKR_n -> downKKBar_n gamma PR upKK_n,
    q2KKRBar_n gamma q2KKL_n -> 0,
    q2KKRBar_n gamma q2KKR_n -> downKKBar_n gamma PR downKK_n
};

colScaGen[a_ + b_] := colScaGen[a] + colScaGen[b]
colScaGen[a_] := If[FreeQ[a, gamma5], a /. colSca, a /. colSca5]

colSca = {
    Q10Bar Q10 -> 0,
    Q10Bar Q20 -> 0,
    Q10Bar q10 -> upBar PR up,
    Q10Bar q20 -> upBar PR down,
    Q10Bar Q1KKL_n -> 0,
    Q10Bar Q1KKR_n -> upBar PR UpKK_n,
    Q10Bar Q2KKL_n -> 0,
    Q10Bar Q2KKR_n -> upBar PR DownKK_n,
    Q10Bar q1KKL_n -> 0,
    Q10Bar q1KKR_n -> upBar PR upKK_n,
    Q10Bar q2KKL_n -> 0,
```

```
Q10Bar q2KKR_n -> upBar PR downKK_n,

Q20Bar Q10 -> 0,
Q20Bar Q20 -> 0,
Q20Bar q10 -> downBar PR up,
Q20Bar q20 -> downBar PR down,
Q20Bar Q1KKL_n -> 0,
Q20Bar Q1KKR_n -> downBar PR UpKK_n,
Q20Bar Q2KKL_n -> 0,
Q20Bar Q2KKR_n -> downBar PR DownKK_n,
Q20Bar q1KKL_n -> 0,
Q20Bar q1KKR_n -> downBar PR upKK_n,
Q20Bar q2KKL_n -> 0,
Q20Bar q2KKR_n -> downBar PR downKK_n,

q10Bar Q10 -> upBar PL up,
q10Bar Q20 -> upBar PL down,
q10Bar q10 -> 0,
q10Bar q20 -> 0,
q10Bar Q1KKL_n -> upBar PL UpKK_n,
q10Bar Q1KKR_n -> 0,
q10Bar Q2KKL_n -> upBar PL DownKK_n,
q10Bar Q2KKR_n -> 0,
q10Bar q1KKL_n -> upBar PL upKK_n,
q10Bar q1KKR_n -> 0,
q10Bar q2KKL_n -> upBar PL downKK_n,
q10Bar q2KKR_n -> 0,

q20Bar Q10 -> downBar PL up,
q20Bar Q20 -> downBar PL down,
q20Bar q10 -> 0,
q20Bar q20 -> 0,
q20Bar Q1KKL_n -> downBar PL UpKK_n,
q20Bar Q1KKR_n -> 0,
q20Bar Q2KKL_n -> downBar PL DownKK_n,
q20Bar Q2KKR_n -> 0,
q20Bar q1KKL_n -> downBar PL upKK_n,
```

```
q20Bar q1KKR_n -> 0,
q20Bar q2KKL_n -> downBar PL downKK_n,
q20Bar q2KKR_n -> 0,

Q1KKLBar_n Q10 -> 0,
Q1KKLBar_n Q20 -> 0,
Q1KKLBar_n q10 -> UpKKBar_n PR up,
Q1KKLBar_n q20 -> UpKKBar_n PR down,
Q1KKLBar_n Q1KKL_n -> 0,
Q1KKLBar_n Q1KKR_n -> UpKKBar_n PR UpKK_n,
Q1KKLBar_n Q2KKL_n -> 0,
Q1KKLBar_n Q2KKR_n -> UpKKBar_n PR DownKK_n,
Q1KKLBar_n q1KKL_n -> 0,
Q1KKLBar_n q1KKR_n -> UpKKBar_n PR upKK_n,
Q1KKLBar_n q2KKL_n -> 0,
Q1KKLBar_n q2KKR_n -> UpKKBar_n PR downKK_n,

Q2KKLBar_n Q10 -> 0,
Q2KKLBar_n Q20 -> 0,
Q2KKLBar_n q10 -> DownKKBar_n PR up,
Q2KKLBar_n q20 -> DownKKBar_n PR down,
Q2KKLBar_n Q1KKL_n -> 0,
Q2KKLBar_n Q1KKR_n -> DownKKBar_n PR UpKK_n,
Q2KKLBar_n Q2KKL_n -> 0,
Q2KKLBar_n Q2KKR_n -> DownKKBar_n PR DownKK_n,
Q2KKLBar_n q1KKL_n -> 0,
Q2KKLBar_n q1KKR_n -> DownKKBar_n PR upKK_n,
Q2KKLBar_n q2KKL_n -> 0,
Q2KKLBar_n q2KKR_n -> DownKKBar_n PR downKK_n,

q1KKLBar_n Q10 -> 0,
q1KKLBar_n Q20 -> 0,
q1KKLBar_n q10 -> upKKBar_n PR up,
q1KKLBar_n q20 -> upKKBar_n PR down,
q1KKLBar_n Q1KKL_n -> 0,
q1KKLBar_n Q1KKR_n -> upKKBar_n PR UpKK_n,
q1KKLBar_n Q2KKL_n -> 0,
```

q1KKLBar$_n$ Q2KKR$_n$ -> upKKBar$_n$ PR DownKK$_n$,
q1KKLBar$_n$ q1KKL$_n$ -> 0,
q1KKLBar$_n$ q1KKR$_n$ -> upKKBar$_n$ PR upKK$_n$,
q1KKLBar$_n$ q2KKL$_n$ -> 0,
q1KKLBar$_n$ q2KKR$_n$ -> upKKBar$_n$ PR downKK$_n$,

q2KKLBar$_n$ Q10 -> 0,
q2KKLBar$_n$ Q20 -> 0,
q2KKLBar$_n$ q10 -> downKKBar$_n$ PR up,
q2KKLBar$_n$ q20 -> downKKBar$_n$ PR down,
q2KKLBar$_n$ Q1KKL$_n$ -> 0,
q2KKLBar$_n$ Q1KKR$_n$ -> downKKBar$_n$ PR UpKK$_n$,
q2KKLBar$_n$ Q2KKL$_n$ -> 0,
q2KKLBar$_n$ Q2KKR$_n$ -> downKKBar$_n$ PR DownKK$_n$,
q2KKLBar$_n$ q1KKL$_n$ -> 0,
q2KKLBar$_n$ q1KKR$_n$ -> downKKBar$_n$ PR upKK$_n$,
q2KKLBar$_n$ q2KKL$_n$ -> 0,
q2KKLBar$_n$ q2KKR$_n$ -> downKKBar$_n$ PR downKK$_n$,

Q1KKRBar$_n$ Q10 -> UpKKBar$_n$ PL up,
Q1KKRBar$_n$ Q20 -> UpKKBar$_n$ PL down,
Q1KKRBar$_n$ q10 -> 0,
Q1KKRBar$_n$ q20 -> 0,
Q1KKRBar$_n$ Q1KKL$_n$ -> UpKKBar$_n$ PL UpKK$_n$,
Q1KKRBar$_n$ Q1KKR$_n$ -> 0,
Q1KKRBar$_n$ Q2KKL$_n$ -> UpKKBar$_n$ PL DownKK$_n$,
Q1KKRBar$_n$ Q2KKR$_n$ -> 0,
Q1KKRBar$_n$ q1KKL$_n$ -> UpKKBar$_n$ PL upKK$_n$,
Q1KKRBar$_n$ q1KKR$_n$ -> 0,
Q1KKRBar$_n$ q2KKL$_n$ -> UpKKBar$_n$ PL downKK$_n$,
Q1KKRBar$_n$ q2KKR$_n$ -> 0,

Q2KKRBar$_n$ Q10 -> DownKKBar$_n$ PL up,
Q2KKRBar$_n$ Q20 -> DownKKBar$_n$ PL down,
Q2KKRBar$_n$ q10 -> 0,
Q2KKRBar$_n$ q20 -> 0,
Q2KKRBar$_n$ Q1KKL$_n$ -> DownKKBar$_n$ PL UpKK$_n$,

```
    Q2KKRBar_n Q1KKR_n -> 0,
    Q2KKRBar_n Q2KKL_n -> DownKKBar_n PL DownKK_n,
    Q2KKRBar_n Q2KKR_n -> 0,
    Q2KKRBar_n q1KKL_n -> DownKKBar_n PL upKK_n,
    Q2KKRBar_n q1KKR_n -> 0,
    Q2KKRBar_n q2KKL_n -> DownKKBar_n PL downKK_n,
    Q2KKRBar_n q2KKR_n -> 0,

    q1KKRBar_n Q10 -> upKKBar_n PL up,
    q1KKRBar_n Q20 -> upKKBar_n PL down,
    q1KKRBar_n q10 -> 0,
    q1KKRBar_n q20 -> 0,
    q1KKRBar_n Q1KKL_n -> upKKBar_n PL UpKK_n,
    q1KKRBar_n Q1KKR_n -> 0,
    q1KKRBar_n Q2KKL_n -> upKKBar_n PL DownKK_n,
    q1KKRBar_n Q2KKR_n -> 0,
    q1KKRBar_n q1KKL_n -> upKKBar_n PL upKK_n,
    q1KKRBar_n q1KKR_n -> 0,
    q1KKRBar_n q2KKL_n -> upKKBar_n PL downKK_n,
    q1KKRBar_n q2KKR_n -> 0,

    q2KKRBar_n Q10 -> downKKBar_n PL up,
    q2KKRBar_n Q20 -> downKKBar_n PL down,
    q2KKRBar_n q10 -> 0,
    q2KKRBar_n q20 -> 0,
    q2KKRBar_n Q1KKL_n -> downKKBar_n PL UpKK_n,
    q2KKRBar_n Q1KKR_n -> 0,
    q2KKRBar_n Q2KKL_n -> downKKBar_n PL DownKK_n,
    q2KKRBar_n Q2KKR_n -> 0,
    q2KKRBar_n q1KKL_n -> downKKBar_n PL upKK_n,
    q2KKRBar_n q1KKR_n -> 0,
    q2KKRBar_n q2KKL_n -> downKKBar_n PL downKK_n,
    q2KKRBar_n q2KKR_n -> 0
};

colSca5 = {
    Q10Bar gamma5 Q10 -> 0,
```

```
Q10Bar gamma5 Q20 -> 0,
Q10Bar gamma5 q10 -> upBar PR up,
Q10Bar gamma5 q20 -> upBar PR down,
Q10Bar gamma5 Q1KKL_n -> 0,
Q10Bar gamma5 Q1KKR_n -> upBar PR UpKK_n,
Q10Bar gamma5 Q2KKL_n -> 0,
Q10Bar gamma5 Q2KKR_n -> upBar PR DownKK_n,
Q10Bar gamma5 q1KKL_n -> 0,
Q10Bar gamma5 q1KKR_n -> upBar PR upKK_n,
Q10Bar gamma5 q2KKL_n -> 0,
Q10Bar gamma5 q2KKR_n -> upBar PR downKK_n,

Q20Bar gamma5 Q10 -> 0,
Q20Bar gamma5 Q20 -> 0,
Q20Bar gamma5 q10 -> downBar PR up,
Q20Bar gamma5 q20 -> downBar PR down,
Q20Bar gamma5 Q1KKL_n -> 0,
Q20Bar gamma5 Q1KKR_n -> downBar PR UpKK_n,
Q20Bar gamma5 Q2KKL_n -> 0,
Q20Bar gamma5 Q2KKR_n -> downBar PR DownKK_n,
Q20Bar gamma5 q1KKL_n -> 0,
Q20Bar gamma5 q1KKR_n -> downBar PR upKK_n,
Q20Bar gamma5 q2KKL_n -> 0,
Q20Bar gamma5 q2KKR_n -> downBar PR downKK_n,

q10Bar gamma5 Q10 -> -upBar PL up,
q10Bar gamma5 Q20 -> -upBar PL down,
q10Bar gamma5 q10 -> 0,
q10Bar gamma5 q20 -> 0,
q10Bar gamma5 Q1KKL_n -> -upBar PL UpKK_n,
q10Bar gamma5 Q1KKR_n -> 0,
q10Bar gamma5 Q2KKL_n -> -upBar PL DownKK_n,
q10Bar gamma5 Q2KKR_n -> 0,
q10Bar gamma5 q1KKL_n -> -upBar PL upKK_n,
q10Bar gamma5 q1KKR_n -> 0,
q10Bar gamma5 q2KKL_n -> -upBar PL downKK_n,
q10Bar gamma5 q2KKR_n -> 0,
```

q20Bar gamma5 Q10 -> -downBar PL up,
q20Bar gamma5 Q20 -> -downBar PL down,
q20Bar gamma5 q10 -> 0,
q20Bar gamma5 q20 -> 0,
q20Bar gamma5 $Q1KKL_n$ -> -downBar PL $UpKK_n$,
q20Bar gamma5 $Q1KKR_n$ -> 0,
q20Bar gamma5 $Q2KKL_n$ -> -downBar PL $DownKK_n$,
q20Bar gamma5 $Q2KKR_n$ -> 0,
q20Bar gamma5 $q1KKL_n$ -> -downBar PL $upKK_n$,
q20Bar gamma5 $q1KKR_n$ -> 0,
q20Bar gamma5 $q2KKL_n$ -> -downBar PL $downKK_n$,
q20Bar gamma5 $q2KKR_n$ -> 0,

$Q1KKLBar_n$ gamma5 Q10 -> 0,
$Q1KKLBar_n$ gamma5 Q20 -> 0,
$Q1KKLBar_n$ gamma5 q10 -> $UpKKBar_n$ PR up,
$Q1KKLBar_n$ gamma5 q20 -> $UpKKBar_n$ PR down,
$Q1KKLBar_n$ gamma5 $Q1KKL_n$ -> 0,
$Q1KKLBar_n$ gamma5 $Q1KKR_n$ -> $UpKKBar_n$ PR $UpKK_n$,
$Q1KKLBar_n$ gamma5 $Q2KKL_n$ -> 0,
$Q1KKLBar_n$ gamma5 $Q2KKR_n$ -> $UpKKBar_n$ PR $DownKK_n$,
$Q1KKLBar_n$ gamma5 $q1KKL_n$ -> 0,
$Q1KKLBar_n$ gamma5 $q1KKR_n$ -> $UpKKBar_n$ PR $upKK_n$,
$Q1KKLBar_n$ gamma5 $q2KKL_n$ -> 0,
$Q1KKLBar_n$ gamma5 $q2KKR_n$ -> $UpKKBar_n$ PR $downKK_n$,

$Q2KKLBar_n$ gamma5 Q10 -> 0,
$Q2KKLBar_n$ gamma5 Q20 -> 0,
$Q2KKLBar_n$ gamma5 q10 -> $DownKKBar_n$ PR up,
$Q2KKLBar_n$ gamma5 q20 -> $DownKKBar_n$ PR down,
$Q2KKLBar_n$ gamma5 $Q1KKL_n$ -> 0,
$Q2KKLBar_n$ gamma5 $Q1KKR_n$ -> $DownKKBar_n$ PR $UpKK_n$,
$Q2KKLBar_n$ gamma5 $Q2KKL_n$ -> 0,
$Q2KKLBar_n$ gamma5 $Q2KKR_n$ -> $DownKKBar_n$ PR $DownKK_n$,
$Q2KKLBar_n$ gamma5 $q1KKL_n$ -> 0,
$Q2KKLBar_n$ gamma5 $q1KKR_n$ -> $DownKKBar_n$ PR $upKK_n$,

```
Q2KKLBar_n gamma5 q2KKL_n -> 0,
Q2KKLBar_n gamma5 q2KKR_n -> DownKKBar_n PR downKK_n,

q1KKLBar_n gamma5 Q10 -> 0,
q1KKLBar_n gamma5 Q20 -> 0,
q1KKLBar_n gamma5 q10 -> upKKBar_n PR up,
q1KKLBar_n gamma5 q20 -> upKKBar_n PR down,
q1KKLBar_n gamma5 Q1KKL_n -> 0,
q1KKLBar_n gamma5 Q1KKR_n -> upKKBar_n PR UpKK_n,
q1KKLBar_n gamma5 Q2KKL_n -> 0,
q1KKLBar_n gamma5 Q2KKR_n -> upKKBar_n PR DownKK_n,
q1KKLBar_n gamma5 q1KKL_n -> 0,
q1KKLBar_n gamma5 q1KKR_n -> upKKBar_n PR upKK_n,
q1KKLBar_n gamma5 q2KKL_n -> 0,
q1KKLBar_n gamma5 q2KKR_n -> upKKBar_n PR downKK_n,

q2KKLBar_n gamma5 Q10 -> 0,
q2KKLBar_n gamma5 Q20 -> 0,
q2KKLBar_n gamma5 q10 -> downKKBar_n PR up,
q2KKLBar_n gamma5 q20 -> downKKBar_n PR down,
q2KKLBar_n gamma5 Q1KKL_n -> 0,
q2KKLBar_n gamma5 Q1KKR_n -> downKKBar_n PR UpKK_n,
q2KKLBar_n gamma5 Q2KKL_n -> 0,
q2KKLBar_n gamma5 Q2KKR_n -> downKKBar_n PR DownKK_n,
q2KKLBar_n gamma5 q1KKL_n -> 0,
q2KKLBar_n gamma5 q1KKR_n -> downKKBar_n PR upKK_n,
q2KKLBar_n gamma5 q2KKL_n -> 0,
q2KKLBar_n gamma5 q2KKR_n -> downKKBar_n PR downKK_n,

Q1KKRBar_n gamma5 Q10 -> -UpKKBar_n PL up,
Q1KKRBar_n gamma5 Q20 -> -UpKKBar_n PL down,
Q1KKRBar_n gamma5 q10 -> 0,
Q1KKRBar_n gamma5 q20 -> 0,
Q1KKRBar_n gamma5 Q1KKL_n -> -UpKKBar_n PL UpKK_n,
Q1KKRBar_n gamma5 Q1KKR_n -> 0,
Q1KKRBar_n gamma5 Q2KKL_n -> -UpKKBar_n PL DownKK_n,
Q1KKRBar_n gamma5 Q2KKR_n -> 0,
```

```
Q1KKRBar_n gamma5 q1KKL_n -> -UpKKBar_n PL upKK_n,
Q1KKRBar_n gamma5 q1KKR_n -> 0,
Q1KKRBar_n gamma5 q2KKL_n -> -UpKKBar_n PL downKK_n,
Q1KKRBar_n gamma5 q2KKR_n -> 0,

Q2KKRBar_n gamma5 Q10 -> -DownKKBar_n PL up,
Q2KKRBar_n gamma5 Q20 -> -DownKKBar_n PL down,
Q2KKRBar_n gamma5 q10 -> 0,
Q2KKRBar_n gamma5 q20 -> 0,
Q2KKRBar_n gamma5 Q1KKL_n -> -DownKKBar_n PL UpKK_n,
Q2KKRBar_n gamma5 Q1KKR_n -> 0,
Q2KKRBar_n gamma5 Q2KKL_n -> -DownKKBar_n PL DownKK_n,
Q2KKRBar_n gamma5 Q2KKR_n -> 0,
Q2KKRBar_n gamma5 q1KKL_n -> -DownKKBar_n PL upKK_n,
Q2KKRBar_n gamma5 q1KKR_n -> 0,
Q2KKRBar_n gamma5 q2KKL_n -> -DownKKBar_n PL downKK_n,
Q2KKRBar_n gamma5 q2KKR_n -> 0,

q1KKRBar_n gamma5 Q10 -> -upKKBar_n PL up,
q1KKRBar_n gamma5 Q20 -> -upKKBar_n PL down,
q1KKRBar_n gamma5 q10 -> 0,
q1KKRBar_n gamma5 q20 -> 0,
q1KKRBar_n gamma5 Q1KKL_n -> -upKKBar_n PL UpKK_n,
q1KKRBar_n gamma5 Q1KKR_n -> 0,
q1KKRBar_n gamma5 Q2KKL_n -> -upKKBar_n PL DownKK_n,
q1KKRBar_n gamma5 Q2KKR_n -> 0,
q1KKRBar_n gamma5 q1KKL_n -> -upKKBar_n PL upKK_n,
q1KKRBar_n gamma5 q1KKR_n -> 0,
q1KKRBar_n gamma5 q2KKL_n -> -upKKBar_n PL downKK_n,
q1KKRBar_n gamma5 q2KKR_n -> 0,

q2KKRBar_n gamma5 Q10 -> -downKKBar_n PL up,
q2KKRBar_n gamma5 Q20 -> -downKKBar_n PL down,
q2KKRBar_n gamma5 q10 -> 0,
q2KKRBar_n gamma5 q20 -> 0,
q2KKRBar_n gamma5 Q1KKL_n -> -downKKBar_n PL UpKK_n,
q2KKRBar_n gamma5 Q1KKR_n -> 0,
```

```
    q2KKRBar_n gamma5 Q2KKL_n -> -downKKBar_n PL DownKK_n,
    q2KKRBar_n gamma5 Q2KKR_n -> 0,
    q2KKRBar_n gamma5 q1KKL_n -> -downKKBar_n PL upKK_n,
    q2KKRBar_n gamma5 q1KKR_n -> 0,
    q2KKRBar_n gamma5 q2KKL_n -> -downKKBar_n PL downKK_n,
    q2KKRBar_n gamma5 q2KKR_n -> 0
};

(* ultimator *)

ult = {
    upKK_n -> -up1 CU gamma5 + up2 SU,
    UpKK_n -> up1 SU gamma5 + up2 CU,
    upKKBar_n -> up1Bar CU gamma5co + up2Bar SU,
    UpKKBar_n -> -up1Bar SU gamma5co + up2Bar CU,

    downKK_n -> -down1 CD gamma5 + down2 SD,
    DownKK_n -> down1 SD gamma5 + down2 CD,
    downKKBar_n -> down1Bar CD gamma5co + down2Bar SD,
    DownKKBar_n -> -down1Bar SD gamma5co + down2Bar CD,

    Wp0 -> Wp,
    Wm0 -> Wm,
    Z0 -> Z,
    A0 -> A,
    G0 -> g,

    WpKK_n -> Wpn,
    WmKK_n -> Wmn,
    PKK_n -> Pn,
    VKK_n -> Vn,
    GKK_n -> gn,

    h0 -> higgs,
    phip0 -> Wpf,
    phim0 -> Wmf,
    chi0 -> Zf,
```

```
    hKK_n -> hn,
    GpKK_n -> Wpnf,
    GmKK_n -> Wmnf,
    apKK_n -> apn,
    amKK_n -> amn,
    aKK_n -> an,
    GPKK_n -> Pnf,
    GVKK_n -> Vnf,
    G5KK_n -> gnf
};

ultSca0 = {
    gamma5co PR gamma5 -> PR,
    gamma5co PL gamma5 -> PL
};

ultSca1 = {
    gamma5co PR -> PR,
    gamma5co PL -> -PL
};

ultSca2 = {
    gamma5 PR -> PR,
    gamma5 PL -> -PL
};

ultSca3 = {
    gamma5co gamma5 -> 1,
    PL^2 -> PL,
    PR^2 -> PR
};

ultVec1 = {
    gamma5co gamma PR gamma5 -> -gamma PR,
    gamma5co gamma PL gamma5 -> -gamma PL
};
```

```
ultVec2 = {
    gamma5co gamma PR -> -gamma PR,
    gamma5co gamma PL -> gamma PL
};

ultVec3 = {
    gamma5co gamma gamma5 -> -gamma
};

ultVec4 = {
    gamma5co gamma -> -gamma gamma5
};

ultVec5 = {
    gamma5^2 -> 1
};

(* field expansions *)

(* quarks *)

Q1[n_] := (1/Sqrt[Pi R]) Q10 LambdaL1 + Q1KKL_n c_n LambdaL1 +
    Q1KKR_n s_n LambdaL1;
Q2[n_] := (1/Sqrt[Pi R]) Q20 LambdaL2 + Q2KKL_n c_n LambdaL2 +
    Q2KKR_n s_n LambdaL2;
q1[n_] := (1/Sqrt[Pi R]) q10 LambdaR1 + q1KKR_n c_n LambdaR1 +
    q1KKL_n s_n LambdaR1;
q2[n_] := (1/Sqrt[Pi R]) q20 LambdaR2 + q2KKR_n c_n LambdaR2 +
    q2KKL_n s_n LambdaR2;
Q1Bar[n_] := (1/Sqrt[Pi R]) Q10Bar LambdaL1co +
    Q1KKLBar_n c_n LambdaL1co + Q1KKRBar_n s_n LambdaL1co;
Q2Bar[n_] := (1/Sqrt[Pi R]) Q20Bar LambdaL2co +
    Q2KKLBar_n c_n LambdaL2co + Q2KKRBar_n s_n LambdaL2co;
q1Bar[n_] := (1/Sqrt[Pi R]) q10Bar LambdaR1co +
    q1KKRBar_n c_n LambdaR1co + q1KKLBar_n s_n LambdaR1co;
q2Bar[n_] := (1/Sqrt[Pi R]) q20Bar LambdaR2co +
```

```
    q2KKRBar_n c_n LambdaR2co + q2KKLBar_n s_n LambdaR2co;

QDoublet[n_] := {Q1[n], Q2[n]};
QDoubletBar[n_] := {Q1Bar[n], Q2Bar[n]};

(* gauge bosons *)

W1[n_] := (1/Sqrt[Pi R]) (Alpha1 Wp0 + Alpha2 Wm0) +
    (Alpha9 WpKK_n + Alpha10 WmKK_n) c_n;
W2[n_] := (1/Sqrt[Pi R]) (Alpha3 Wp0 + Alpha4 Wm0) +
    (Alpha11 WpKK_n + Alpha12 WmKK_n) c_n;
W3[n_] := (1/Sqrt[Pi R]) (Alpha5 Z0 + Alpha6 A0) +
    (Alpha13 PKK_n + Alpha14 VKK_n) c_n;
B[n_] := (1/Sqrt[Pi R]) (Alpha7 Z0 + Alpha8 A0) +
    (Alpha15 PKK_n + Alpha16 VKK_n) c_n;
G[n_] := (1/Sqrt[Pi R]) G0 + GKK_n c_n;

W15[n_] := (Alpha17 GpKK_n + Alpha18 apKK_n + Alpha19 GmKK_n +
    Alpha20 amKK_n) (s_n/Sqrt[ZW]);
W25[n_] := (Alpha21 GpKK_n + Alpha22 apKK_n + Alpha23 GmKK_n +
    Alpha24 amKK_n) (s_n/Sqrt[ZW]);
W35[n_] := (Alpha25 GPKK_n + Alpha26 GVKK_n + Alpha27 aKK_n)
    (s_n/Sqrt[ZW]);
B5[n_] := (Alpha28 GPKK_n + Alpha29 GVKK_n + Alpha30 aKK_n)
    (s_n/Sqrt[ZB]);
G5[n_] := G5KK_n (s_n/Sqrt[ZG]);

(* higgs components *)

h[n_] := (1/Sqrt[Pi R]) h0 + hKK_n c_n;
phip[n_] := (1/Sqrt[Pi R]) phip0 +
    (Alpha31 GpKK_n + Alpha32 apKK_n) c_n;
phim[n_] := (1/Sqrt[Pi R]) phim0 +
    (Alpha33 GmKK_n + Alpha34 amKK_n) c_n;
chi[n_] := (1/Sqrt[Pi R]) chi0 +
    (Alpha35 GPKK_n + Alpha36 GVKK_n + Alpha37 aKK_n) c_n;
```

```
H[n_] := {I phip[n], (h[n] + v5 + I chi[n])/Sqrt[2]};
HTilde[n_] := {(h[n] + v5 - I chi[n])/Sqrt[2], I phim[n]};
Hco[n_] := {-I phim[n], (h[n] + v5 - I chi[n])/Sqrt[2]};
HTildeco[n_] := {(h[n] + v5 + I chi[n])/Sqrt[2], -I phip[n]};

(* quantum numbers *)

YQDoublet = 1/3;
YQSinglet1 = 4/3;
YQSinglet2 = (-2)/3;
YLDoublet = -1;
YLSinglet1 = 0;
YLSinglet2 = -2;

(* pauli matrices *)

tau1 = {{0, 1}, {1, 0}};
tau2 = {{0, -I}, {I, 0}};
tau3 = {{1, 0}, {0, -1}};
unit2 = {{1, 0}, {0, 1}};

(* couplings and vev *)

gy5 = gy Sqrt[Pi R];
gw5 = gw Sqrt[Pi R];
gc5 = gc Sqrt[Pi R];
v5 = v/Sqrt[Pi R];
v = 2 mW0/gw;
y15 = y1 Sqrt[Pi R];
y25 = y2 Sqrt[Pi R];
y15co = y1co Sqrt[Pi R];
y25co = y2co Sqrt[Pi R];

(* some other parameters *)

If[Z1 == 1, (
    ZW = 1;
```

```
    ZB = 1;
    ZG = 1;
    ZQ = 1;
    Zu = 1;
    Zd = 1
    ),
    Clear[ZW, ZB, ZG, ZQ, Zu, Zd]];

m10 = mu0;
m20 = md0;
Zq1 = Zu;
Zq2 = Zd;

(* covariant derivative minus the del-mu -- only the interactions *)

CovarQDoublet[n_] := I gc5 lambda G[n] unit2 +
    I gw5/2 (tau1 W1[n] + tau2 W2[n] + tau3 W3[n]) +
    I gy5/2 YQDoublet B[n] unit2;
Covar5QDoublet[n_] := (I gc5 lambda G5[n] unit2 +
    I gw5/2 (tau1 W15[n] + tau2 W25[n] + tau3 W35[n]) +
    I gy5/2 YQDoublet B5[n] unit2) ZQ;
CovarQSinglet1[n_] := I gc5 lambda G[n] + I gy5/2 YQSinglet1 B[n];
Covar5QSinglet1[n_] := (I gc5 lambda G5[n] +
    I gy5/2 YQSinglet1 B5[n]) Zq1;
CovarQSinglet2[n_] := I gc5 lambda G[n] + I gy5/2 YQSinglet2 B[n];
Covar5QSinglet2[n_] := (I gc5 lambda G5[n] +
    I gy5/2 YQSinglet2 B5[n]) Zq2;

(* fermion sector *)

LagQ = I gamma QDoubletBar[n].CovarQDoublet[m].QDoublet[k] -
    gamma5 QDoubletBar[n].Covar5QDoublet[m].QDoublet[k] +
    I gamma q1Bar[n] CovarQSinglet1[m] q1[k] -
    gamma5 q1Bar[n] Covar5QSinglet1[m] q1[k] +
    I gamma q2Bar[n] CovarQSinglet2[m] q2[k] -
    gamma5 q2Bar[n] Covar5QSinglet2[m] q2[k] // Expand;
LagQEff = int[LagQ] // Expand;
```

```
LagQRef = ref[LagQEff] // Expand;

(* yukawa sector *)

LagY = -y15 q1[n] QDoubletBar[m].HTilde[k] -
    y15co q1Bar[n] HTildeco[k].QDoublet[m] -
    y25 q2[n] QDoubletBar[m].H[k] -
    y25co q2Bar[n] Hco[k].QDoublet[m] // Expand;
LagYEff = int[LagY] // Expand;
LagYRef = ref[LagYEff] // Expand;
LagFFSFFV = LagQRef + LagYRef /. sim;

(* Alpha factors *)

Alpha1 = 1/Sqrt[2];
Alpha2 = 1/Sqrt[2];
Alpha3 = I/Sqrt[2];
Alpha4 = -I/Sqrt[2];
Alpha5 = CW;
Alpha6 = SW;
Alpha7 = -SW;
Alpha8 = CW;
Alpha9 = 1/Sqrt[2];
Alpha10 = 1/Sqrt[2];
Alpha11 = I/Sqrt[2];
Alpha12 = -I/Sqrt[2];
Alpha13 = SV;
Alpha14 = CV;
Alpha15 = CV;
Alpha16 = -SV;
Alpha17 = omega1 omega5;
Alpha18 = omega2 omega5;
Alpha19 = omega3 omega6;
Alpha20 = omega4 omega6;
Alpha21 = omega1 omega7;
Alpha22 = omega2 omega7;
Alpha23 = omega3 omega8;
```

```
Alpha24 = omega4 omega8;
Alpha25 = gamma31;
Alpha26 = gamma32;
Alpha27 = gamma33;
Alpha28 = gamma21;
Alpha29 = gamma22;
Alpha30 = gamma23;
Alpha31 = mW0/mWn;
Alpha32 = -(mn/(mWn/Sqrt[ZW]));
Alpha33 = (mW0/Sqrt[ZW])/(mWn/Sqrt[ZW]);
Alpha34 = -(mn/(mWn/Sqrt[ZW]));
Alpha35 = gamma11;
Alpha36 = gamma12;
Alpha37 = gamma13;

$Assumptions = {
    mW0 > 0,
    ZW > 0,
    EE > 0,
    CW > 0,
    SW > 0,
    ZB > 0,
    R > 0
};

beta1 = 1/Sqrt[2];
beta2 = 1/Sqrt[2];
beta3 = I/Sqrt[2];
beta4 = -I/Sqrt[2];
beta5 = mn/(mWn/Sqrt[ZW]);
beta6 = (mW0/Sqrt[ZW])/(mWn/Sqrt[ZW]);
beta7 = mn/(mWn/Sqrt[ZW]);
beta8 = (mW0/Sqrt[ZW])/(mWn/Sqrt[ZW]);

gamma1 = mn/Sqrt[(gy v/(2 Sqrt[ZB]))^2 + (gw v/(2 Sqrt[ZW]))^2];
gamma2 = gw/Sqrt[ZW] CV + gy/Sqrt[ZB] SV;
gamma3 = gy CV - gw SV;
```

```
omega1 = Sqrt[ZW] mn/mWn;
omega2 = mW0/mWn;
omega3 = Sqrt[ZW] mn/mWn;
omega4 = mW0/mWn;
omega5 = 1/Sqrt[2];
omega6 = 1/Sqrt[2];
omega7 = I/Sqrt[2];
omega8 = -I/Sqrt[2];

gamma11 = C12 C13;
gamma12 = -C13 S12;
gamma13 = S13;
gamma21 = C23 S12 + C12 S13 S23;
gamma22 = C12 C23 - S12 S13 S23;
gamma23 = -C13 S23;
gamma31 = -C12 C23 S13 + S12 S23;
gamma32 = C23 S12 S13 + C12 S23;
gamma33 = C13 C23;

rule1 = {
    Cos[Theta0V] -> gw/Sqrt[gy^2 + gw^2],
    Sin[Theta0V] -> gy/Sqrt[gw^2 + gy^2],
    Cos[2 ThetaV] -> ((gw^2 - gy^2) v^2/4 +
        (ZW - ZB) mn^2)/Sqrt[((gw^2 - gy^2) v^2/4 +
        (ZW - ZB) mn^2)^2 + (gy gw v^2/2)^2],
    Sin[2 ThetaV] -> (gy gw v^2/2)/Sqrt[((gw^2 - gy^2) v^2/4 +
        (ZW - ZB) mn^2)^2 + (gy gw v^2/2)^2],
    Cos[ThetaS01] -> gw/Sqrt[ZW]/Sqrt[(gy/Sqrt[ZB])^2 +
        (gw/Sqrt[ZW])^2],
    Sin[ThetaS01] -> gy/Sqrt[ZB]/Sqrt[(gy/Sqrt[ZB])^2 +
        (gw/Sqrt[ZW])^2],
    Cos [ThetaS02] -> Sqrt[(gy/(2 Sqrt[ZB]))^2 +
        (gw/(2 Sqrt[ZW]))^2]/Sqrt[(gy/(2 Sqrt[ZB]))^2 +
        (gw/(2 Sqrt[ZW]))^2 + mn^2],
    Sin[ThetaS02] -> -(mn/Sqrt[(gy/(2 Sqrt[ZB]))^2 +
        (gw/(2 Sqrt[ZW]))^2 + mn^2]),
```

```
    Sin[ThetaG] -> gamma1 gamma2/Sqrt[(gamma1 gamma2)^2 + gamma3^2],
    Cos[ThetaG] -> gamma3/Sqrt[(gamma1 gamma2)^2 + gamma3^2]
};

LagFFV = Coefficient[LagFFSFFV, gamma] gamma // Expand;
LagFFS = LagFFSFFV - LagFFV // Expand;

LagFFS0 = colScaGen[LagFFS] // Expand;
LagFFS1 = LagFFS0 /. ult // Expand;
LagFFS12 = LagFFS1 /. ultSca0 // Expand;
LagFFS2 = LagFFS1 /. ultSca1 // Expand;
LagFFS3 = LagFFS2 /. ultSca2 // Expand;
LagFFSFinal = LagFFS3 /. ultSca3 // Expand;

LagFFV0 = LagFFV /. colVec // Expand;
LagFFV1 = LagFFV0 /. ult // Expand;
LagFFV2 = LagFFV1 /. ultVec1 // Expand;
LagFFV3 = LagFFV1 /. ultVec2 // Expand;
LagFFV4 = LagFFV3 /. ultVec3 // Expand;
LagFFV5 = LagFFV4 /. ultVec4 // Expand;
LagFFVFinal = LagFFV5 /. ultVec5 // Expand;

sca[MainField_, FermionBar_, Fermion_] :=
    I Coefficient[LagFFSFinal, MainField
    ToExpression[ToString[FermionBar] <> "Bar"] Fermion]
vec[MainField_, FermionBar_, Fermion_] :=
    I Coefficient[LagFFVFinal, MainField
    ToExpression[ToString[FermionBar] <> "Bar"] Fermion]

(* usage *)

vertex[MainField_, FermionBar_, Fermion_] :=
    If[MemberQ[{Wp, Wm, Z, A, g, Wpn, Wmn, Pn, Vn, gn}, MainField],
    vec[MainField, FermionBar, Fermion],
    sca[MainField, FermionBar, Fermion]] /. rule2 // Simplify
```

APPENDIX D

LANHEP CODE

Once we completed the theoretical analysis of the MUED model in the Feynman gauge, concerning the mass spectrum of the model and the compact form of the various sectors of the complete Lagrangian, we prepared our own LanHEP code for the numerical analysis. Unline the code written by Belyaev *et al.* [29], ours is able to produce the interactions for the maximum KK numbers $n_{\max} = 1, 2, 4, 6$. For the technical reasons (the run time, insufficient RAM, and low CPU), the leptons and the ghosts other than those of the gluons and the KK partners of the ghosts are excluded.

Our own LanHEP code is presented below.

```
model 'mued'/1001.

alias
    nmax=6.

keys
    maxkk=6.

do_if maxkk==1.
let
    cos1=cos(1),
    cos2=0,
    cos3=0,
    cos4=0,
    cos5=0,
    cos6=0,
    sin1=sin(1),
```

```
    sin2=0,
    sin3=0,
    sin4=0,
    sin5=0,
    sin6=0.

do_else_if maxkk==2.
let
    cos1=cos(1),
    cos2=cos(2),
    cos3=0,
    cos4=0,
    cos5=0,
    cos6=0,
    sin1=sin(1),
    sin2=sin(2),
    sin3=0,
    sin4=0,
    sin5=0,
    sin6=0.

do_else_if maxkk==4.
let
    cos1=cos(1),
    cos2=cos(2),
    cos3=cos(3),
    cos4=cos(4),
    cos5=0,
    cos6=0,
    sin1=sin(1),
    sin2=sin(2),
    sin3=sin(3),
    sin4=sin(4),
    sin5=0,
    sin6=0.

do_else_if maxkk==6.
```

```
let
    cos1=cos(1),
    cos2=cos(2),
    cos3=cos(3),
    cos4=cos(4),
    cos5=cos(5),
    cos6=cos(6),
    sin1=sin(1),
    sin2=sin(2),
    sin3=sin(3),
    sin4=sin(4),
    sin5=sin(5),
    sin6=sin(6).

end_if.

option ReduceGamma5=0.

/* parameters */
/**************/

/* free parameters */

parameter
    alphae=1/128,
    pi=acos(-1),
    ge=sqrt(4*pi*alphae),
    gc=1.21978,
    sw=0.471813,
    CKMs12=0.22506,
    CKMs23=0.0410788,
    CKMs13=0.00357472,
    MZ0=91.1876,
    Mh0=125.

/* derived parameters */
```

```
parameter
    cw=sqrt(1-sw**2),
    MW0=MZ0*cw,
    vv=2*(MZ0*cw)/(ge/sw),
    muH=Mh0/Sqrt2,
    lamH=((ge/sw)*Mh0/8/(MZ0*cw))**2.

/* masses */

parameter
    MBB0=MZ0*sw.

parameter
    Mu0=0.0022,
    Md0=0.0047,
    Mc0=1.28,
    Ms0=0.096,
    Mt0=175,
    Mb0=4.18.

_x=[h,Z,W,u,d,c,s,t,b] in parameter
    M_x02=M_x0**2.

parameter
    invR=500,
    R=1/invR.

_n=1-nmax in parameter
    Mhn_n=sqrt(Mh0**2+(_n/R)**2),
    Ma0n_n=sqrt(MZ0**2+(_n/R)**2),
    Macn_n=sqrt(MW0**2+(_n/R)**2),
    Mu1n_n=sqrt(Mu0**2+(_n/R)**2),
    Mu2n_n=sqrt(Mu0**2+(_n/R)**2),
    Md1n_n=sqrt(Md0**2+(_n/R)**2),
    Md2n_n=sqrt(Md0**2+(_n/R)**2),
    Mc1n_n=sqrt(Mc0**2+(_n/R)**2),
    Mc2n_n=sqrt(Mc0**2+(_n/R)**2),
```

```
    Ms1n_n=sqrt(Ms0**2+(_n/R)**2),
    Ms2n_n=sqrt(Ms0**2+(_n/R)**2),
    Mt1n_n=sqrt(Mt0**2+(_n/R)**2),
    Mt2n_n=sqrt(Mt0**2+(_n/R)**2),
    Mb1n_n=sqrt(Mb0**2+(_n/R)**2),
    Mb2n_n=sqrt(Mb0**2+(_n/R)**2),
    MGn_n=(_n/R),
    MAn_n=(_n/R),
    MZn_n=sqrt(MZ0**2+(_n/R)**2),
    MWn_n=sqrt(MW0**2+(_n/R)**2).

_n=1-nmax, _x=[h,a0,ac,u1,u2,d1,d2,c1,c2,s1,s2,t1,t2,b1,b2,G,A,Z,W]
in parameter
    M_xn_n2=M_xn_n**2.

_n=1-nmax in parameter
    MBBn_n=sqrt((MZ0*sw)**2+(_n/R)**2).

/* projection operators */

let
    PL=(1-gamma5)/2,
    PR=(1+gamma5)/2.

/* ckm matrix */

parameter
    CKMc12=sqrt(1-CKMs12**2),
    CKMc23=sqrt(1-CKMs23**2),
    CKMc13=sqrt(1-CKMs13**2).

parameter
    Vud=CKMc12*CKMc13,
    Vus=CKMs12*CKMc13,
    Vub=CKMs13,
    Vcd=(-CKMs12*CKMc23-CKMc12*CKMs23*CKMs13),
    Vcs=(CKMc12*CKMc23-CKMs12*CKMs23*CKMs13),
```

```
    Vcb=CKMs23*CKMc13,
    Vtd=(CKMs12*CKMs23-CKMc12*CKMc23*CKMs13),
    Vts=(-CKMc12*CKMs23-CKMs12*CKMc23*CKMs13),
    Vtb=CKMc23*CKMc13.

OrthMatrix({{Vud,Vus,Vub},{Vcd,Vcs,Vcb},{Vtd,Vts,Vtb}}).

/* fields */
/**********/

/* sm */

scalar
    h0/h0:(Higgs, mass Mh0).

spinor
    u0/U0:(up, color c3, mass Mu0),
    d0/D0:(down, color c3, mass Md0),
    c0/C0:(charm, color c3, mass Mc0),
    s0/S0:(strange, color c3, mass Ms0),
    t0/T0:(top, color c3, mass Mt0),
    b0/B0:(bottom, color c3, mass Mb0).

vector
    G0/G0:(gluon, color c8, gauge),
    A0/A0:(photon, gauge),
    Z0/Z0:(Z, mass MZ0, gauge),
    Wp0/Wm0:(W, mass MW0, gauge).

/* kk */

_n=1-nmax in scalar
    hn_n/hn_n:(Higgs_n, mass Mhn_n),
    a0n_n/a0n_n:(neutral_n, mass Ma0n_n),
    apn_n/amn_n:(charged_n, mass Macn_n).

_n=1-nmax in spinor
```

```
    u1n_n/U1n_n:(up1_n, color c3, mass Mu1n_n),
    u2n_n/U2n_n:(up2_n, color c3, mass Mu2n_n),
    d1n_n/D1n_n:(down1_n, color c3, mass Md1n_n),
    d2n_n/D2n_n:(down2_n, color c3, mass Md2n_n),
    c1n_n/C1n_n:(charm1_n, color c3, mass Mc1n_n),
    c2n_n/C2n_n:(charm2_n, color c3, mass Mc2n_n),
    s1n_n/S1n_n:(strange1_n, color c3, mass Ms1n_n),
    s2n_n/S2n_n:(strange2_n, color c3, mass Ms2n_n),
    t1n_n/T1n_n:(top1_n, color c3, mass Mt1n_n),
    t2n_n/T2n_n:(top2_n, color c3, mass Mt2n_n),
    b1n_n/B1n_n:(bottom1_n, color c3, mass Mb1n_n),
    b2n_n/B2n_n:(bottom2_n, color c3, mass Mb2n_n).

_n=1-nmax in vector
    Gn_n/Gn_n:(gluon_n, color c8, mass MGn_n, gauge),
    An_n/An_n:(photon_n, mass MAn_n, gauge),
    Zn_n/Zn_n:(Z_n, mass MZn_n, gauge),
    Wpn_n/Wmn_n:(W_n, mass MWn_n, gauge).

/* physical fields */
/*****************/

/* scalars */

let
    phip0='Wp0.f',
    phim0='Wm0.f',
    phi30='Z0.f'.

_n=1-nmax in let
    G5n_n='Gn_n.f',
    GAn_n='An_n.f',
    GZn_n='Zn_n.f',
    Gpn_n='Wpn_n.f',
    Gmn_n='Wmn_n.f'.

_n=1-nmax in let
```

```
    Wp5n_n=((_n/R)*Gpn_n+(((ge/sw)*vv/2)/cw)*cw*apn_n)/MWn_n,
    Wm5n_n=((_n/R)*Gmn_n+(((ge/sw)*vv/2)/cw)*cw*amn_n)/MWn_n,
    phipn_n=((((ge/sw)*vv/2)/cw)*cw*Gpn_n-(_n/R)*apn_n)/MWn_n,
    phimn_n=((((ge/sw)*vv/2)/cw)*cw*Gmn_n-(_n/R)*amn_n)/MWn_n,
    W15n_n=(Wp5n_n+Wm5n_n)/Sqrt2,
    W25n_n=(-Wp5n_n+Wm5n_n)/Sqrt2/i.

_n=1-nmax in let
    G1n_n=-MBB0*(_n/R)/(((ge/sw)*vv/2)/cw)/MWn_n*GAn_n
        -(((ge/sw)*vv/2)/cw)*cw*MZn_n/(((ge/sw)*vv/2)
        /cw)/MWn_n*GZn_n,
    G2n_n=(((ge/sw)*vv/2)/cw)*cw*MZn_n/(((ge/sw)*vv/2)
        /cw)/MWn_n*GAn_n-MBB0*(_n/R)/(((ge/sw)*vv/2)/cw)
        /MWn_n*GZn_n,
    W35n_n=-(_n/R)/MWn_n*G1n_n+MBB0*(((ge/sw)*vv/2)/cw)
        *cw/MWn_n/MZn_n*G2n_n+(((ge/sw)*vv/2)/cw)*cw
        /MZn_n*a0n_n,
    BB5n_n=MWn_n/MZn_n*G2n_n-MBB0/MZn_n*a0n_n,
    phi3n_n=(((ge/sw)*vv/2)/cw)*cw/MWn_n*G1n_n+MBB0
        *(_n/R)/MWn_n/MZn_n*G2n_n+(_n/R)/MZn_n*a0n_n.

_x=[h,phip,phim,phi3] in let
    _x=(_x0)*cos(0)+Sqrt2*((_xn1)*cos1+(_xn2)*cos2
        +(_xn3)*cos3+(_xn4)*cos4+(_xn5)*cos5+(_xn6)
        *cos6).

_x=[W1,W2,W3,Wp,Wm,BB,G] in let
    _x5=Sqrt2*((_x5n1)*sin1+(_x5n2)*sin2+(_x5n3)*sin3
        +(_x5n4)*sin4+(_x5n5)*sin5+(_x5n6)*sin6).

let
    WW5={W15,W25,W35}.

/* spinors */

_n=1-nmax, _x=[u,d,c,s,t,b] in parameter
    f_x_n=atan(M_x0/(_n/R))/2,
```

```
    cf_x_n=cos(f_x_n),
    sf_x_n=sin(f_x_n).

_n=1-nmax, _x=[u,d,c,s,t,b] in angle
    sin=sf_x_n,
    cos=cf_x_n.

_n=1-nmax, _x=[u,d,c,s,t,b] in let
    _xRn_n=-gamma5*cf_x_n*(_x1n_n)+sf_x_n*(_x2n_n),
    _xLn_n=gamma5*sf_x_n*(_x1n_n)+cf_x_n*(_x2n_n).

_x=[u,d,c,s,t,b] in let
    _xLL=PL*((_xLn1)*cos1+(_xLn2)*cos2+(_xLn3)*cos3
        +(_xLn4)*cos4+(_xLn5)*cos5+(_xLn6)*cos6),
    _xLR=PR*((_xLn1)*sin1+(_xLn2)*sin2+(_xLn3)*sin3
        +(_xLn4)*sin4+(_xLn5)*sin5+(_xLn6)*sin6),
    _xRR=PR*((_xRn1)*cos1+(_xRn2)*cos2+(_xRn3)*cos3
        +(_xRn4)*cos4+(_xRn5)*cos5+(_xRn6)*cos6),
    _xRL=PL*((_xRn1)*sin1+(_xRn2)*sin2+(_xRn3)*sin3
        +(_xRn4)*sin4+(_xRn5)*sin5+(_xRn6)*sin6).

_x=[u,d,c,s,t,b] in let
    _xL=PL*(_x0)*cos(0)+Sqrt2*((_xLL)+(_xLR)),
    _xR=PR*(_x0)*cos(0)+Sqrt2*((_xRR)+(_xRL)).

/* vectors */

let
    W10=(Wp0+Wm0)/Sqrt2,
    W20=(-Wp0+Wm0)/Sqrt2/i,
    W30=sw*A0+cw*Z0,
    BB0=cw*A0-sw*Z0.

_n=1-nmax in let
    W1n_n=(Wpn_n+Wmn_n)/Sqrt2,
    W2n_n=(-Wpn_n+Wmn_n)/Sqrt2/i,
    W3n_n=sw*An_n+cw*Zn_n,
```

```
    BBn_n=cw*An_n-sw*Zn_n.

_x=[G,W1,W2,W3,BB,Wp,Wm] in let
    _x=(_x0)*cos(0)+Sqrt2*((_xn1)*cos1+(_xn2)*cos2
        +(_xn3)*cos3+(_xn4)*cos4+(_xn5)*cos5+(_xn6)
        *cos6).

let
    WW={W1,W2,W3}.

/* lagrangian */
/**************/

/* higgs sector */

let
    phi={i*phip,(h+vv+i*phi3)/Sqrt2},
    Phi={-i*phim,(h+vv-i*phi3)/Sqrt2},
    phi2=Phi*phi.

let
    Dphi={i*deriv*phip+(i*(ge/sw)/2*W3+i*(ge/cw)/2*BB)
        *(i*phip)+i*(ge/sw)/Sqrt2*Wp*(h+vv+i*phi3)
        /Sqrt2,i*(ge/sw)/Sqrt2*Wm*(i*phip)+deriv*h/Sqrt2
        +i*deriv*phi3/Sqrt2+(-i*(ge/sw)/2*W3+i*(ge/cw)
        /2*BB)*(h+vv+i*phi3)/Sqrt2},
    D5phi={i*deriv5/R*phip+(i*(ge/sw)/2*W35+i*(ge/cw)/2
        *BB5)*(i*phip)+i*(ge/sw)/Sqrt2*Wp5*(h+vv+i*phi3)
        /Sqrt2,i*(ge/sw)/Sqrt2*Wm5*(i*phip)+deriv5/R*h
        /Sqrt2+i*deriv5/R*phi3/Sqrt2+(-i*(ge/sw)/2*W35
        +i*(ge/cw)/2*BB5)*(h+vv+i*phi3)/Sqrt2},
    DPhi={-i*deriv*phim+(-i*(ge/sw)/2*W3-i*(ge/cw)/2*BB)
        *(-i*phim)-i*(ge/sw)/Sqrt2*Wm*(h+vv-i*phi3)
        /Sqrt2,-i*(ge/sw)/Sqrt2*Wp*(-i*phim)+deriv*h
        /Sqrt2-i*deriv*phi3/Sqrt2+(i*(ge/sw)/2*W3
        -i*(ge/cw)/2*BB)*(h+vv-i*phi3)/Sqrt2},
    D5Phi={-i*deriv5/R*phim+(-i*(ge/sw)/2*W35-i*(ge/cw)
```

```
        /2*BB5)*(-i*phim)-i*(ge/sw)/Sqrt2*Wm5*(h+vv
        -i*phi3)/Sqrt2,-i*(ge/sw)/Sqrt2*Wp5*(-i*phim)
        +deriv5/R*h/Sqrt2-i*deriv5/R*phi3/Sqrt2
        +(i*(ge/sw)/2*W35-i*(ge/cw)/2*BB5)*(h+vv-i*phi3)
        /Sqrt2}.

lterm
    DPhi*Dphi-D5Phi*D5phi+muH**2*phi2-lamH*phi2**2.

/* gauge sector */

let
    FB^mu^nu=deriv^mu*BB^nu-deriv^nu*BB^mu,
    FB5^mu=deriv^mu*BB5-deriv5/R*BB^mu,
    FW^mu^nu^a=deriv^mu*WW^nu^a-deriv^nu*WW^mu^a-(ge/sw)
        *eps^a^b^c*WW^mu^b*WW^nu^c,
    FW5^mu^a=deriv^mu*WW5^a-deriv5/R*WW^mu^a-(ge/sw)
        *eps^a^b^c*WW^mu^b*WW5^c,
    FG^mu^nu^a=deriv^mu*G^nu^a-deriv^nu*G^mu^a-gc
        *f_SU3^a^b^c*G^mu^b*G^nu^c,
    FG5^mu^a=deriv^mu*G5^a-deriv5/R*G^mu^a-gc
        *f_SU3^a^b^c*G^mu^b*G5^c.

lterm
    -1/4*FG**2+1/2*FG5**2-1/4*FW**2+1/2*FW5**2-1/4*FB**2
        +1/2*FB5**2.

lterm
    -1/2*(deriv*G-deriv5/R*G5)**2-(deriv*Wp-deriv5/R*Wp5
        -(((ge/sw)*vv/2)/cw)*cw*phip)*(deriv*Wm-deriv5/R
        *Wm5-(((ge/sw)*vv/2)/cw)*cw*phim)-1/2*(deriv*W3
        -deriv5/R*W35+(((ge/sw)*vv/2)/cw)*cw*phi3)**2
        -1/2*(deriv*BB-deriv5/R*BB5-MBB0*phi3)**2.

/* ghost sector */

let
```

```
    ghG='G0.c'*cos(0),
    GhG='G0.C'*cos(0).

lterm
    i*gc*f_SU3*deriv*GhG*G*ghG.

/* fermion sector */

_x=[L,R] in let
    da_x=Vud*d_x+Vus*s_x+Vub*b_x,
    sa_x=Vcd*d_x+Vcs*s_x+Vcb*b_x,
    ba_x=Vtd*d_x+Vts*s_x+Vtb*b_x.

lterm % kinetic
    anti(psiL)*i*gamma*deriv*psiL+anti(psiR)*i*gamma
        *deriv*psiR
        where
            psiL=uL, psiR=uR;
            psiL=dL, psiR=dR;
            psiL=cL, psiR=cR;
            psiL=sL, psiR=sR;
            psiL=tL, psiR=tR;
            psiL=bL, psiR=bR.

lterm % kinetic-5
    -anti(psiL)*gamma5*deriv5/R*psiL-anti(psiR)*gamma5
        *deriv5/R*psiR
        where
            psiL=uL, psiR=uR;
            psiL=dL, psiR=dR;
            psiL=cL, psiR=cR;
            psiL=sL, psiR=sR;
            psiL=tL, psiR=tR;
            psiL=bL, psiR=bR.

lterm % u(1)
    anti(psiL)*i*gamma*i*(ge/cw)/2*YL*BB*psiL+anti(psiR)
```

```
        *i*gamma*i*(ge/cw)/2*YR*BB*psiR
        where
            psiL=uL, YL=1/3, psiR=uR, YR=4/3;
            psiL=cL, YL=1/3, psiR=cR, YR=4/3;
            psiL=tL, YL=1/3, psiR=tR, YR=4/3;
            psiL=dL, YL=1/3, psiR=dR, YR=-2/3;
            psiL=sL, YL=1/3, psiR=sR, YR=-2/3;
            psiL=bL, YL=1/3, psiR=bR, YR=-2/3.

lterm % u(1)-5
    -anti(psiL)*gamma5*i*(ge/cw)/2*YL*BB5*psiL
        -anti(psiR)*gamma5*i*(ge/cw)/2*YR*BB5*psiR
        where
            psiL=uL, YL=1/3, psiR=uR, YR=4/3;
            psiL=cL, YL=1/3, psiR=cR, YR=4/3;
            psiL=tL, YL=1/3, psiR=tR, YR=4/3;
            psiL=dL, YL=1/3, psiR=dR, YR=-2/3;
            psiL=sL, YL=1/3, psiR=sR, YR=-2/3;
            psiL=bL, YL=1/3, psiR=bR, YR=-2/3.

lterm % neutral component of su(2)
    anti(psi1L)*i*gamma*i*(ge/sw)/2*W3*psi1L-anti(psi2L)
        *i*gamma*i*(ge/sw)/2*W3*psi2L
        where
            psi1L=uL, psi2L=dL;
            psi1L=cL, psi2L=sL;
            psi1L=tL, psi2L=bL.

lterm % neutral component of su(2)-5
    -anti(psi1L)*gamma5*i*(ge/sw)/2*W35*psi1L
        +anti(psi2L)*gamma5*i*(ge/sw)/2*W35*psi2L
        where
            psi1L=uL, psi2L=dL;
            psi1L=cL, psi2L=sL;
            psi1L=tL, psi2L=bL.

lterm % charged component of su(2)
```

```
    anti(psi1L)*i*gamma*i*(ge/sw)/Sqrt2*Wp*psi2L
        +anti(psi2L)*i*gamma*i*(ge/sw)/Sqrt2*Wm*psi1L
        where
            psi1L=uL, psi2L=daL;
            psi1L=cL, psi2L=saL;
            psi1L=tL, psi2L=baL.

lterm % charged component of su(2)-5
    -anti(psi1L)*gamma5*i*(ge/sw)/Sqrt2*Wp5*psi2L
        -anti(psi2L)*gamma5*i*(ge/sw)/Sqrt2*Wm5*psi1L
        where
            psi1L=uL, psi2L=daL;
            psi1L=cL, psi2L=saL;
            psi1L=tL, psi2L=baL.

lterm % su(3)
    anti(psiL)*i*gamma*i*gc*lambda*G*psiL+anti(psiR)*i
        *gamma*i*gc*lambda*G*psiR
        where
            psiL=uL, psiR=uR;
            psiL=dL, psiR=dR;
            psiL=cL, psiR=cR;
            psiL=sL, psiR=sR;
            psiL=tL, psiR=tR;
            psiL=bL, psiR=bR.

lterm % su(3)-5
    -anti(psiL)*gamma5*i*gc*lambda*G5*psiL-anti(psiR)
        *gamma5*i*gc*lambda*G5*psiR
        where
            psiL=uL, psiR=uR;
            psiL=dL, psiR=dR;
            psiL=cL, psiR=cR;
            psiL=sL, psiR=sR;
            psiL=tL, psiR=tR;
            psiL=bL, psiR=bR.
```

```
/* yukawa sector */

let
    q1a={uL,daL},
    q2a={cL,saL},
    q3a={tL,baL}.

let
    H={i*phip,(h+vv+i*phi3)/Sqrt2},
    Hcc={-i*phim,(h+vv-i*phi3)/Sqrt2}.

lterm
    -M2/MW0/Sqrt2*(ge/sw)*(anti(pl)*pr*H+anti(pr)*pl
        *Hcc)
        where
            M2=Vud*Md0, pl=q1a, pr=dR;
            M2=Vus*Ms0, pl=q1a, pr=sR;
            M2=Vub*Mb0, pl=q1a, pr=bR;
            M2=Vcd*Md0, pl=q2a, pr=dR;
            M2=Vcs*Ms0, pl=q2a, pr=sR;
            M2=Vcb*Mb0, pl=q2a, pr=bR;
            M2=Vtd*Md0, pl=q3a, pr=dR;
            M2=Vts*Ms0, pl=q3a, pr=sR;
            M2=Vtb*Mb0, pl=q3a, pr=bR.

lterm
    -M1/MW0/Sqrt2*(ge/sw)*(anti(pl)*i*tau2*pr*Hcc
        +anti(pr)*i*pl*tau2*H )
        where
            M1=Mu0, pl=q1a, pr=uR;
            M1=Mc0, pl=q2a, pr=cR;
            M1=Mt0, pl=q3a, pr=tR.

/* conclusion */
/*************/

SetAngle(1-cw**2=sw**2).
```

```
SetAngle(1-CKMc12**2=CKMs12**2).
SetAngle(1-CKMc13**2=CKMs13**2).
SetAngle(1-CKMc23**2=CKMs23**2).

SetAngle(1-cfu1**2=sfu1**2).
SetAngle(1-cfu2**2=sfu2**2).
SetAngle(1-cfu3**2=sfu3**2).
SetAngle(1-cfu4**2=sfu4**2).
SetAngle(1-cfu5**2=sfu5**2).
SetAngle(1-cfu6**2=sfu6**2).

SetAngle(1-cfd1**2=sfd1**2).
SetAngle(1-cfd2**2=sfd2**2).
SetAngle(1-cfd3**2=sfd3**2).
SetAngle(1-cfd4**2=sfd4**2).
SetAngle(1-cfd5**2=sfd5**2).
SetAngle(1-cfd6**2=sfd6**2).

SetAngle(1-cfc1**2=sfc1**2).
SetAngle(1-cfc2**2=sfc2**2).
SetAngle(1-cfc3**2=sfc3**2).
SetAngle(1-cfc4**2=sfc4**2).
SetAngle(1-cfc5**2=sfc5**2).
SetAngle(1-cfc6**2=sfc6**2).

SetAngle(1-cfs1**2=sfs1**2).
SetAngle(1-cfs2**2=sfs2**2).
SetAngle(1-cfs3**2=sfs3**2).
SetAngle(1-cfs4**2=sfs4**2).
SetAngle(1-cfs5**2=sfs5**2).
SetAngle(1-cfs6**2=sfs6**2).

SetAngle(1-cft1**2=sft1**2).
SetAngle(1-cft2**2=sft2**2).
SetAngle(1-cft3**2=sft3**2).
SetAngle(1-cft4**2=sft4**2).
```

```
SetAngle(1-cft5**2=sft5**2).
SetAngle(1-cft6**2=sft6**2).

SetAngle(1-cfb1**2=sfb1**2).
SetAngle(1-cfb2**2=sfb2**2).
SetAngle(1-cfb3**2=sfb3**2).
SetAngle(1-cfb4**2=sfb4**2).
SetAngle(1-cfb5**2=sfb5**2).
SetAngle(1-cfb6**2=sfb6**2).

CheckHerm.
```