MODELING ADVANCED FUND TRANSFER PRICING WITH AN APPLICATION OF HULL-WHITE INTEREST-RATE TREE IN TURKISH BANKING SECTOR

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ABSTRACT

MODELING ADVANCED FUND TRANSFER PRICING
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The Financial Crisis in 2008 has revealed the need for a more advanced management of liquidity risk in financial institutions. This thesis aims to introduce and implement an advanced Fund Transfer Pricing (FTP) model into banking industries of the developing countries. The methodology of the FTP model, constructed in this research, measures the cost of a product's cash-flows by splitting them into a deterministic and a stochastic component. The cost of the deterministic part is assessed as an equivalent of the credit-default premium of an institution, whereas the cost of the stochastic component is modeled by a Brownian Motion. Moreover, in order to forecast the future outlook of FTP rates, a simulation of benchmark Interest Rates with an application of Hull-White model has been performed. The information provided by the expected cost of funding could be a guide to the management of a financial institution. The cost of Basel III liquidity metrics have also been applied into the model, which is one of the main contributions of this thesis to the field of Financial Mathematics. This thesis ends with a conclusion and a preview to future investigations and applications.

Keywords : Fund Transfer Pricing, Hull-White, Interest-Rate Models, Funding Cost, Liquidity

ÖZ

TÜRK BANKACILIK SEKTÖRÜNDE
HULL-WHITE FAİZ ORANI ÖRGÜSÜNÜ UYGULAYARAK
GELİŞMİŞ FON TRANSFER FİYATLAMA MODELLEMESİ

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2008'deki Finansal Kriz, finansal kuruluşlarda likidite riskinin daha gelişmiş yönetimine duyulan ihtiyacı ortaya koymuştur. Bu tez, gelişmiş bir Fon Transfer Fiyatlandırması (FTF) modelinin gelişmekte olan ülkelerin bankacılık sektörlerine tanıtılmasını ve uygulanmasını amaçlamaktadır. Bu araştırmada oluşturulan FTF modelinin metodolojisi, bir ürünün nakit akışlarının maliyetini, deterministik ve stokastik bileşenlere bölerek ölçmektedir. Deterministik kısmın maliyeti, bir kurumun kredi temerrüt primi karşılığı olarak değerlendirilirken, stokastik kısmın maliyeti Brown Devinimi ile modellenmiştir. Ayrıca, gelecekteki FTF oranlarının görünümünü tahmin edebilmek için, Hull-White modeli uygulanarak benchmark faiz oranlarının simülasyonu gerçekleştirilmiştir. Beklenen finansman maliyeti ile sağlanan bilgiler, bir finansal kurumun yönetimine rehberlik edebilir. Basel III likidite ölçümlerinin maliyetinin modele uygulanması da bu tezin Finansal Matematik alanına temel katkılarından biridir. Tez gelecek araştırmalara ve uygulamalara yönelik bir sonuç ve önizleme ile sona ermektedir.

Anahtar Kelimeler: Fon Transfer Fiyatlaması, Hull-White, Faiz Oranı Modelleri, Fonlama Maliyeti, Likidite

To My Family

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LIST OF ABBREVIATIONS

ALM	Asset Liability Management
ASF	Available Stable Funding
FTP	Fund Transfer Pricing
GPL	General Purpose Loan
HQLA	High Quality Liquid Assets
i.i.d.	Independent and Identically Distributed
LCR	Liquidity Coverage Ratio
LIBOR	London Interbank Offered Rate
NII	Net Interest Income
NIM	Net Interest Margin
NSFR	Net Stable Funding Ratio
$\mathbb{R}$	Set of Real Numbers
RSF	Required Stable Funding
st. dev.	Standard Deviation
TNCO	Total Net Cash Outflows

CHAPTER 1

INTRODUCTION

The fundamental function of financial institutions is borrowing funds and lending loans. Institutions pay for funds and gain from loans, and these operations in finance literature are denoted as funding cost and assets return, respectively. The margin between return from assets and cost of funding is the profit of the financial institutions. The larger the margin, the higher the profit. However, this estimation is too broad for specifying the profitability of the institutions. In order to find the ideal margin, one has to set up a one-to-one map of assets with funds. In other words, each asset with specific maturity, risk and costs should be financed by funds having similar characteristics. In real practice, it is impossible to find the exact matching and this was one of the main reasons for the financial crisis in 2008.

Although the crisis in 2008 was the chain reaction of several failures, the mismanagement of risk and liquidity severely worsened the situation. However, banks did not declare insolvency when problem assets were identified, but when a bank run occurred [31]. No matter how strong deposits base banks had, the harshness of run on deposits put financial institutions out of business. The reason for liquidity shortage was threefold. First, demand for asset-backed securities dropped. Second, investors were reluctant to lend money against mortgage backed security as collaterals. Third, mark-to-market assessment of securities caused a huge amount of losses which influenced the funding structure [41]. It is important to note that during the crisis, interbank lending - the source of short-term funding - almost evaporated.

The aftermath of Liquidity Financial Crisis in 2008 has revealed the need for a modification of Fund Transfer Pricing (FTP) methodology [9]. FTP plays a key role in liquidity risk pricing. It is a system created to assess the cost of funds by taking liquidity, interest rate and currency risks associated with lending and taking activities into account [9]. In other words, FTP is an internal pricing system, that encompasses all risks during pricing each balance sheet item, based on its supply or usage [37]. Assets and Liabilities Management (ALM) department or the treasury of banks, which are the central risk management hubs, are generally responsible for setting the FTP rates of all businesses, such as corporate, retail and small and medium-sized enterprises.

Nowadays, banking regulators are closely monitoring the development of internal FTP models and their applications. Germany is the first country to bring up the issue as mandatory. In the U.S., the Federal Reserve, the Federal Deposits Insurance Corporation, and the Office of the Comptroller of the Currency issued the guidance on FTP for large-sized (based on some criteria) banks in March 2016 [18]. The guidance includes general standards of modeling and reporting that are obligatory in implementation of the process. Overall, regulators and banks in developed countries are strengthening the?r financial institutions against shocks, by integrating quantitative FTP models.

Developing countries are also working on applying internal models in their own banking industries. In Turkey, *Banking Regulation and Supervision Agency* (BRSA) has not released any guidance directly related with the standards and application of FTP modeling, yet. However, most of the Turkish banks are using internal methodologies in FTP estimations. Those applications in Turkish banks are generally based on simple models. Unfortunately, simple models have several drawbacks, which were roughly criticized as being one of the main weaknesses of liquidity management during the financial crisis [13].

In literature, there have been some researches on FTP models which have provided an inspiration for this thesis. Schmaltz's work, *A Quantitative Liquidity Model for Banks* [41], was the cornerstone in establishing the model for the cash-flow of products. In his book, Schmaltz investigated all types of cash-flow models and informed the reader with their pros and cons. According to Schmaltz, the optimal model for the cash-flow consisted of three components, namely, the *deterministic*, the *stochastic* and the *jump*. By combination of those components, the cash flow of any product could be modeled. Another research, titled as *Implementation of a Funds Transfer Pricing model with stochastic interest rates* by Danielsson [13], introduced the implementation of short-term interest-rate models into FTP. In [13], Danielsson applied the Monte Carlo Simulation on short-term interest-rate models in order to forecast the total cost of funding in different horizons. Moreover, he analyzed the effect of regulatory measures on transfer-price rates in his work. A more detailed analysis of the impact of the Net Stable Funding Ratio on FTP was pointed out in Jorgensen's master thesis, *Funds Transfer Pricing under Basel III New Requirements, New Implications* [24]. In his thesis, Jorgensen revealed that the implementation of regulatory measures would result in the increase of the FTP rates. The NSFR impact in [24] was observed on different types of assets and liability items, based on their characteristics, such as type, maturity and optionality. Materials that covered the importance of FTP in banking sector and that explained the general concept of FTP include Dermine [14], Dimitriu [16], Levey [29, 30] and Wyle & Tsaig [49].

During our research we put a strong emphasis on pricing the true cost of funding which is the main indicator of profitability in financial institutions. Thus, one of the objectives of this thesis is to provide a guiding methodology of ALM departments in fair pricing of funds. Also, it aims to introduce a stochastic process in finance through the implementation of an advanced FTP model, heavily used in developed countries, on Turkish banks. The model can be used on the product basis. Each balance sheet item, depending on its usage or supply, is charged with a specific FTP rate. The cost of risks related to each item of assets or liabilities is included in its FTP rate, so that in case of failure, the exposure would be covered by the FTP buffer. In addition, the model enables banks to increase *Net Interest Margin* (NIM) by optimizing both interest income and expenses. It separates the cash flow into two parts: *deterministic* and *stochastic*. The deterministic part of cash flow is predictable and has no varying cost, whereas the stochastic part is volatile and incurs cover costs for possible outflows. Unlike simple methods, the advanced method is trying to capture the real cost of the stochastic component, which generally constitutes a small part of the total cash flow.

Furthermore, the advanced FTP model in conjunction with interest rate models can be actively used in the projection of business plans. Precise modeling of an interest rate, which serves as a risk-free rate in advanced FTP methods, will render the management of banks to forecast the true cost of funding. Consequently, banks can take optimal decisions that will provide a higher profit and a lower risk exposure. A short-term interest rate model used in this thesis is constructed by the *Hull-White stochastic interest-rate model* [43]. Adding the liquidity cost to the acquired risk-free yield curve will result in the total cost of funding, applicable to all sorts of analyses.

In addition to the primary objectives that have been emphasized above, one of the main contribution of this thesis is the implementation of Basel III regulatory measures on to the FTP rates. The model of the *Net Stable Funding Ratio's* (NSFR) effect on FTP rates has first been introduced in this work. Due to lack of studies in the literature about the impact of NSFR metric on FTP, the model in this thesis hopefully will emphasize further researches in the area.

The thesis is organized as follows. Chapter 2 begins with the introduction of financial and mathematical concepts that are applied in the models. Then, the framework of the cash flow model is designed. It ends with an analysis of the stochastic interest model for simulating future risk free rates. Chapter 3 covers the establishment of the FTP model. First, it compares the simple methods in terms of advantages and disadvantages. Then, the advanced FTP model is constructed in two parts, namely, the deterministic one and the stochastic one. The stochastic part is modeled by a Brownian motion, whereas the deterministic one is stated as a linear estimation with zero variance. Before applying the model on a product from Turkish banks, the effect of regulatory limits is included. In regulatory part the NSFR impact, one of the main contributions of the thesis, is modeled. Chapter 4, is the application part, where the estimation of the total cost of funding is computed by simulating the benchmark-interest rate curves. Finally, Chapter 5 summarizes the strengths and weaknesses of the model and its application in real circumstances and proposes the pathway for consequent researches. The findings of the NSFR impact on the FTP rates are also clearly mentioned in the last chapter. The appendices of the thesis includes the proof of Vasicek's model's explicit solutions and the derivation of aggregate risk exposure.

CHAPTER 2

PRELIMINARIES

This chapter will introduce both the financial and mathematical tools applied in our FTP model. In the financial tools part, liquidity, the funding items of banks and the regulatory measures are described. It is important to note that the term *liquidity* in this work refers to the flow concept, not to a stock concept. The part on mathematical tools explained the basic principles of probability theory and the stochastic process. Next, we construct the framework of the model, consisting of the *cash flow* and the *funding capacity* of the institution. The prototype of our liquidity model is designed in the last part of this section.

2.1 Financial Tools

In order to understand the general concept of the thesis, it is vital to be familiar with the financial tools, since they describe the behavior of the financial institutions in their operations. This subsection illustrates an overview of banking activities. The part on liquidity shows how banks arrange their resources, whereas the part on funding demonstrates the sources of the funds. The last part of the subsection - regulatory measures - describes the rules of the bank management.

2.1.1 Liquidity

In everyday life the term "liquidity" is mentioned regularly in different sectors of finance. Although they are "interdependent", the meanings differ depending on the area of their use. For this reason, it is important to define the context of the liquidity. In finance literature there are three facets of liquidity - such as *central bank, market* and *funding liquidity* [35]. In this thesis, the funding liquidity is the major concept subject to our analysis; however, for the broad understanding of the issue, other facets of liquidity are also defined.

Central bank liquidity (the basis of national liquidity) is the ability of a central bank to provide liquidity for the economy through financial institutions. It is the supply

Assets	Weight in Total	Liabilities	Weight in Total
Gold and FC	12%	Currency Issued	23%
FX Securities	53%	RR	45%
OMO	21%	Deposits	7%
Other	14%	Other	18%
		Equity and Reserves	6%

Figure 2.1: Balance Sheet - The Central Bank of the Republic of Turkey.

of a monetary base[1] to the financial system on the behalf of central banks' target strategy via different ways of operation [6]. In Turkey, the Central Bank conducts *open-market operations* (OMO) in order to increase or decrease the money supply in the economy. When we analyze the balance sheet of the Central Bank of the Republic of Turkey (CBRT) (see Figure 2.1) in closer details, we can observe that the liquidity supplied to the system is placed under the item *Lending Related to Monetary Policy Operations* (FX Securities and OMO). The lending activities of CBRT is funded heavily by deposits from the banking sector (including the reserve requirements) and from the public.

Market liquidity, as a general concept, is the easiness in trading of financial assets without significant changes in prices [35]. Market liquidity is a vital indicator of market efficiency. Markets, where asset prices are traded with lower deviation from mark-to-market[2] prices, are more efficient in terms of liquidity. The deviation in asset prices during trading in finance is called *liquidity premium* or *hair-cut*. An estimation of liquidity premium depends on several variables such as asset size, time of liquidation, market behavior, etc. Among the specialists, there is no consensus about the method of measuring liquidity premium or hair-cut, but they all accept that the liquidity premium measures the confidence of a market on price of assets [25].

The most important type of liquidity in this work, is the ***funding liquidity***. There are different definitions of funding liquidity, but the most widely used one is that the *funding liquidity* is the capacity of an institution to meet its obligations. At this point, it is noteworthy to state that the word "capacity" indicates only the capacity to fund from external sources, i.e., it does not include the ability to liquidate assets; otherwise, funding liquidity would have the same meaning as market liquidity. External funding encompasses funding from the money market, issuing bonds and loans borrowed from other financial institutions, which also includes the renewal of current contracts. The difference between the extension and acquisition of contracts will be examined in assigning transfer prices to a product.

[1] The sum of currency in circulation and reserve balances.
[2] Mark-to-market is an act of accounting to measure the fair value of financial assets.

After defining the liquidity concepts, now it is time to briefly analyze the risks of liquidity. In brief, *the liquidity risk* is the probability of being illiquid. The higher the probability of not being liquid, the higher the liquidity risk. Since the risks of each liquidity facets are in line with the general definition, it would be adequate to expand only the funding liquidity risk. The funding liquidity risk is realized when a financial institution fails to meets its liabilities [25]. In the banking sector, among risks the funding liquidity risk is treated as the most important solvency indicator of a bank. As it was the case in recent financial crises, banks' inability to meet their obligations during the bank runs resulted in bankruptcy. It is not obvious how to measure the risks of funding liquidity. The problem with quantifying funding liquidity risk is the lack of generally accepted methods of computation such as *Value at Risk* (VaR), *Conditional Value-at-Risk* (CVaR), *Robustified Conditional Value-at-Risk* (RCVaR) or *Loss Given Default* (LGD) applied on credit risks [13, 47].

2.1.2 Funding

Funding plays a crucial role in the liquidity management. The characteristics (*maturity, amount, counterparty,* etc.) of funding sources are the main determinants for cash flow of assets and liabilities of banks. By analyzing those properties, banks construct their own liquidity models. There are two major sources of funding: *Deposits* and *Wholesale Funding*. The wholesale funding also includes money market funding, bond issuance and borrowings from other financial institutions.

The main funding source of commercial banks is given by *deposits*. More than half of the total assets (about 55%) in Turkish banking system are funded by deposits [38]. Deposits in finance literature are shortly defined as the placement of funds to another party for safekeeping. In the balance sheet of banks, deposits are specified as a liability item because banks borrow deposits with the obligation to pay back. Based on the term and the counterparty, different types of deposits exist. Time and demand deposits have varying terms and optionality. In the contracts of time deposits, the maturity and the interest payments are settled beforehand, and the depositor cannot withdraw the money without penalties before the maturity expires[3], whereas demand deposits have no specified maturity and can be withdrawn at any time [7]. In general, demand deposits do not earn interest. Moreover, depending on the source, deposits can be classified as retail, commercial and bank deposits. Some of the developing countries protect depositors up to a certain amount in the case of the insolvency of banks. In Turkey, *Saving Deposit Insurance Fund* (TMSF) protects retail deposits up to TL 100 thousand or up to an equivalent amount of foreign currency deposits or precious metals [38].

Wholesale funding is the second largest source of funding in Turkish banking system, it composes nearly 30% of total liabilities [38]. Repo funding, the interbank market and issued debt securities are the main subsets of wholesale funding. To begin with, the word repo stands for "Repurchase Agreement". *Repo funding* is an agreement in

[3] Depending on the amount, the money in the saving or time accounts can be withdrawn within several days, as specified in the agreement, but banks do not pay interest accruals of withdrawals.

which one party sells securities to another, and simultaneously agrees to repurchase them (or sometimes similar securities) in the future [17]. Another subset of wholesale funding is the *interbank market*, where the banks short in liquidity[4] borrow from other banks with excess liquidity at specified interest rates [39]. The rates are set by large liquidity provider banks, and used as a benchmark such as *London Interbank Offered Rate* (LIBOR) [21]. Finally, *issued debt securities* are the instruments such as bonds, debenture or promissory notes issued with an obligation to repay on a certain date at a specific rate. In Turkey, the share of issued debt securities composes only 4% of the total liabilities [39].

Term and *collateral* are the two main characteristics of the wholesale funding. Funding with less than or equal to 1 year to maturity is defined as *short-term*, whereas the one with higher than 1 year to maturity is named as *long-term*. In terms of collateral, the wholesale funding can be classified as *secured* and *unsecured*. In secured funding, a borrower has to post collateral, while in unsecured funding, funds are obtained without any collateral. *Repo* is an example of secured funding, where securities under repo agreement are used as a collateral. In contrast, in interbank markets, transactions are generally conducted without posting collaterals (except for some rare cases).

2.1.3 Regulatory Measures

Any financial crisis has always had global contingent effects. In order to minimise negative outcomes, there has been a need for a standardised approach of management. In 1974, *Basel Committee* - formerly called as the Committee of Banking Regulations and Supervisory Practices, was founded by the governors of Central Banks of ten countries. Since then, the number of the member countries has increased to 28, including Turkey.[5] The regulations are not binding, rather they function as policy recommendations for the development of financial stability. The committee has released a series of regulatory standards of which the most notable publications are *Basel I*, *Basel II* and *Basel III*.

Basel I: “The Basel Capital Accord” was released in 1988 and it has set minimum ratio for Capital Adequacy. In 1999, the Basel I was replaced by Basel II: “The New Capital Framework”. According to this framework, three main principles were established: “expansion of minimum capital requirements set in 1988 accord, supervisory review of a capital adequacy and internal assessment process, and effective use of disclosure as a lever to strengthen market discipline and encourage sound banking practices” [3].

After Lehman Brother’s collapse in 2008, it was obvious that the standards of Basel II were not enough to cover the severe stresses. Therefore, in December 2010, the committee set out the final version of Basel III: “International framework for liquidity risk measurement, standards and monitoring” and “A global regulatory framework for more resilient banks and banking systems”. Unlike the previous two publications, Basel III also treats liquidity risk management standards. Within the guidance of the

[4] Banks facing shortage in their liquidity portfolio.
[5] For more details visit https://www.bis.org/bcbs/history.htm.

liquidity risk, the two key metrics, the *Liquidity Coverage Ratio* (LCR) and the *Net Stable Funding Ratio* (NSFR), were released.

According to Bank for International Settlements (BIS), LCR measures a bank's short-term capacity to withstand *liquidity risks*. LCR has two components, *High Quality Liquid Assets* (HQLA) and *Total Net Cash Outflows* (TNCO). A formula for the calculation of LCR is given as follows:[6]

$$LCR = \frac{\text{Stock of } HQLA}{TNCO \text{ over the next 30 Calendar Days}} \geqslant 100\%.$$

HQLA based on the liquidity condition is divided into two main subcategories, namely, *level 1* and *level 2* assets. Level 1 assets are the most liquid assets such as cash and central bank reserves and should consist of at least 60% of the stock of HQLA. Level 2 assets are less liquid and cannot cover more than 40% of total HQLA. Examples for level 2 assets are bonds rated AA- or higher, and qualifying common equity shares. The difference between the two subcategories is that the level 2 assets are subject to hair-cut:[7]

TNCO over the next 30 Calendar Days = Total Expected *cash outflows*
- min{total expected *cash inflows*, 75% of total expected *cash outflows*}.

Total expected cash outflows are calculated by multiplying outstanding balances of related liabilities and off-balance sheet items, and their expected run-off rates. On the other hand, total cash inflows are calculated by multiplying contractual receivables by expected flow in rates.

On the other hand, the main objective of NSFR metric in BIS working paper is defined as the requirement for banks to preserve a stable funding with regard to their assets and off-balance sheet items [5]. The calculation of NSFR ratio is executed by the following formula:

$$\frac{ASF}{RSF} \geqslant 100, \tag{2.1}$$

where

ASF: *Available Stable Funding*,
RSF: *Required Stable Funding*.

ASF is measured by applying specified factors according to stability characteristics of an institution's funding resources. More stable funding resources receive higher the ASF factors.[8] On the other hand, RSF calculation is based on the liquidity

[6] The minimum requirement was set up in an increasing order as follows: In 2015: 60%; in 2016: 70%; in 2017: 80%; in 2018: 90%; in 2019: 100%.

[7] The detailed table of assets categories and related weight factor can be found in "Basel III: The Liquidity Coverage Ratio and liquidity risk monitoring" document [4].

[8] The detailed factors of each balance sheet item can be analyzed in the NSFR document [5].

characteristics of assets and its factors have a negative relationship with liquidity properties of the assets. The bank with a better liquidity profile of assets has a lower amount of RSF than the bank with a weaker liquidity profile. It is important to note that the NSFR metric will come into force at the beginning of 2018.

2.2 Mathematical Tools

In this section, the mathematical tools used in the model are introduced. It begins with the introduction of basic fundamentals of probability theory and then defines a Brownian Motion and its properties.

Definition 2.1. ***(Sigma algebra)*** A collection $\mathcal{F}$ of subsets of Ω (sample space or event) is called a *σ-algebra* if it satisfies the following conditions:

a) $\emptyset \in \mathcal{F}$;

b) if $A_1, A_2, A_3, \ldots, \in \mathcal{F}$, then $\bigcup_{i=1}^{\infty} A_i \in \mathcal{F}$;

c) if $A \in \mathcal{F}$, then $A^C \in \mathcal{F}$. [9]

Remark. σ-algebras are closed under the operation of taking countable intersections.

Definition 2.2. ***(Probability measure and space)*** A *probability measure* $\mathbb{P}$ on $(\sigma, \mathcal{F})$ is a function $\mathbb{P} : \mathcal{F} \longrightarrow [0, 1]$ satisfying:

a) $\mathbb{P}(\emptyset) = 0, \mathbb{P}(\Omega) = 1$;

b) if $A_1, A_2, A_3, \ldots$, is a collection of disjoint members of $\mathcal{F}$, such that $A_i \bigcap A_j = \emptyset$ for all pairs i, j satisfying $i \neq j$, then

$$\mathbb{P}(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} \mathbb{P}(A_i).$$

The triple $(\Omega, \mathcal{F}, \mathbb{P})$, comprising a set Ω, a σ-algebra $\mathcal{F}$ of subsets of Ω, and a probability measure $\mathbb{P}$ on $(\Omega, \mathcal{F})$ is called a *probability space* [42].

[9] Recall from set theory, the superscript C refers to *complement* and $A^C = U \setminus A$.

Definition 2.3. ***(Stochastic process)*** A continuous *stochastic process* in a space E endowed with a σ-algebra ξ is a family $(X_t)_{t\geqslant 0}$ of random variables from a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ into (E, ξ).
The measurable space (E, ξ) is referred as the state space. For each $\omega \in \Omega$, the mapping $X(\omega) : t \mapsto X_t(\omega)$ is called the path of the process for the event ω [36].

Definition 2.4. ***(Filtration)*** A *filtration* on $(\Omega, \mathcal{F}, \mathbb{P})$ is an increasing family $(\mathcal{F}_t)_{t\geqslant 0}$ of σ-algebras of $\mathcal{F}$ such that $\mathcal{F}_s \subseteq \mathcal{F}_t$ for all $0 \leqslant s \leqslant t$. In other words, information increases over time.

Remark. It is said that a process $(X_t)_{t\geqslant 0}$ is *adopted* to $(\mathcal{F}_t)_{t\geqslant 0}$ if, for any t, X_t is $\mathcal{F}_t$ - measurable. ($\mathcal{F}_t$ represents the information available at time t.)

Definition 2.5. ***(Martingales)*** Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space, let T be a fixed positive number, and let $(\mathcal{F}_t)_{t\geqslant 0}$ be a filtration of sub-σ-algebras of $\mathcal{F}$. An adopted stochastic process $(M_t)_{t\geqslant 0}$ is called:

(i) *Martingale* if

$$\mathbb{E}[M_t|\mathcal{F}_s] = M_s \text{ for all } 0 \leqslant s \leqslant t \leqslant T,$$

(ii) *Submartingale* if

$$\mathbb{E}[M_t|\mathcal{F}_s] \geqslant M_s \text{ for all } 0 \leqslant s \leqslant t \leqslant T,$$

(iii) *Supermartingale* if

$$\mathbb{E}[M_t\mathcal{F}_s] \leqslant M_s \text{ for all } 0 \leqslant s \leqslant t \leqslant T.$$

Definition 2.6. ***(Brownian motion)*** On a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, a real-valued stochastic process $(W_t)_{t\geqslant 0}$ is called a *Brownian Motion* if for all finite sequences $0 = t_0 < t_1 < \ldots < t_m$, the increments:

$$W_{t_1} = W_{t_1} - W_{t_0}, W_{t_2} - W_{t_1}, \ldots, W_{t_m} - W_{t_{m-1}},$$

are independent, and each of these increments is normal distributed [27] with

$$\begin{aligned} \mathbb{E}[W(t_{i+1}) - W(t_i)] &= 0, \\ Var[W(t_{i+1}) - W(t_i)] &= t_{i+1} - t_i, \qquad \forall\, i = 1, 2, 3, \ldots \end{aligned}$$

Definition 2.7. ***(Filtration for Brownian motion)*** According to Shreve, *Filtration for Brownian motion* is defined as follows: "Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space on which is defined a Brownian motion $(W_t)_{t \geqslant 0}$. A *filtration for the Brownian motion* is a collection of σ-algebras $(\mathcal{F}_t)_{t \geqslant 0}$, satisfying:

(i) ***(Information accumulates)*** For $0 \leqslant s \leqslant t$, every set in $\mathcal{F}_s$ is also in $\mathcal{F}_t$. In other words, there is at least as much information available at the later time $\mathcal{F}_t$ as there is at the earlier time $\mathcal{F}_s$.

(ii) ***(Adaptivity)*** For each $t \geqslant 0$, the Brownian motion W_t at time t is $\mathcal{F}_t$-measurable. In other words, the information available at time t is sufficient to evaluate the Brownian motion W_t at that time.

(iii) ***(Independence of future increments)*** For $0 \leqslant t < u$, the increment $W_u - W_t$ is independent of $\mathcal{F}_t$. In other words, any increment of the Brownian motion after time t is independent of the information available at time t" [43].

Theorem 2.1. *Brownian motion is a martingale.*

Proof. A detailed proof of the theorem can be seen in Shreve [43]. □

2.3 Model Frameworks

A model framework by definition is not the model itself, but rather the variables of the model. In this section, we will define two major components of the liquidity model, namely, *cash flow* and *funding capacity*. Cash flow is the flow perspective of the overall balance sheet items. The funding capacity of a financial institution is its ability to fund the cash outflow.

2.3.1 Cash Flow (CF)

In order to understand cash flow, let us assume that a bank which has only loans in assets and it funds those assets with deposits and equity. The composition of the bank's balance sheet at a point of time (t) is the stock position of the bank (see Figure 2.2). The change in those stocks over a period of time is the cash flow, which can be seen in Figure 2.3.

Since each item of the balance sheet has different maturities, the cash flow varies over time. Incoming cash flows are the *inflows* to a bank, whereas outgoing cash flows are the *outflows* from a bank. The netted amount of in and out flows is the stock position of the balance sheet. The *maturity ladder* of the cash flows consists of the net positions at different time maturities. From Figure 2.4 we can observe inflows to the balance sheet in 12, 36 and 60 months of time periods, while in 6, 24 and 48 months there are outflows from the balance.

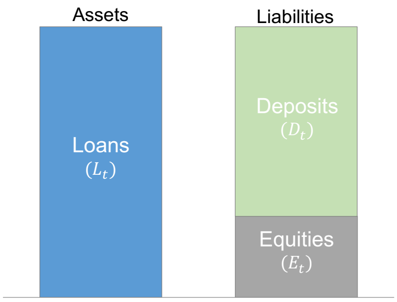

Figure 2.2: Balance Sheet - Stock [41].

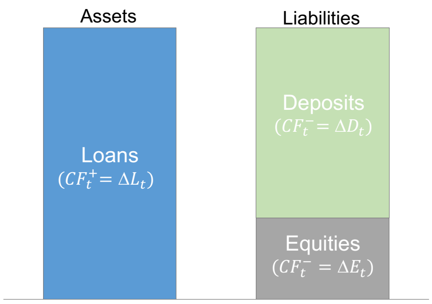

Figure 2.3: Balance Sheet - Cash Flow [41].

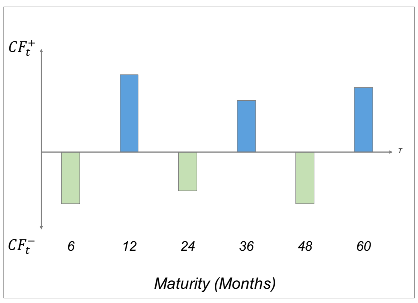

Figure 2.4: Cash-Flow Maturity Ladder [41].

2.3.2 Funding Capacity (FC)

The bank in the previous example will most probably face a liquidity risk due to its distinct product maturity structure. In its assets, the bank has only loans with an average maturity of 3 years, which are generally fixed-term and can not be recalled, except for extreme cases[10]. On the other hand, deposits at the liabilities side are mainly of short-term (3 months maturity) and have the optionality to be withdrawn depending on the desire of the customer. The optionality of the product-cash flows represents the stochastic process. The main reason that banks utilize the products with optionality, is the lower cost of funding. In real circumstances, in order to manage the risk, banks also have cash-equivalent assets, securities and money market receivables in their assets, which can easily be converted into cash to fulfil obligations to depositors in daily operations. When liquid assets are not sufficient to fund outflows, banks borrow from external resources. The ability to fund all outflows is called as *funding capacity*. Banks' main goal in liquidity management is to meet depositors' demand: otherwise, they have to be bailed-out. In mathematical terms, the following equation should hold [41]:

$$FC_t^+ \geqslant CF_t^-, \quad \forall\, t = 0, 1, \ldots, n. \tag{2.2}$$

In summary, cash flow and funding capacity are the key variables to model the liquidity of the banks. In other words, implementation of these two variables is sufficient to construct the basis of a bank's liquidity structure.

2.4 Liquidity-Model Framework

In our *Model Frameworks* section, we have determined the key variables of the liquidity model. Now, it is time to build the model. First, we will model the cash flow of a product and then look at the whole balance sheet's cash flow. In conclusion, the formula of the funding capacity for the total risk exposure will be derived.

2.4.1 Cash-Flow Model

In literature, there are several different cash flow models. The basis of all researches lies on the two aspects of the customer behavior: the *planned* one and the *unplanned* one. Therefore, generally, cash-flow modeling consists of the two parts: a *deterministic* one and a *stochastic* one. The deterministic part of the model refers to liquidity, while the stochastic part is stated as liquidity risk. First we will analyze the modeling of the deterministic section, and then we will look through the stochastic approach of liquidity-risk modeling.

[10] Technically, banks can recall the facility according to credit agreements; however, in the market, recalling is treated as a signal of bankruptcy.

The model of the cash flow with two components as follows:

$$CF^i_{t_k} = \mu^i_{t_k} \cdot \Delta t + \sigma^i \Delta W^i_{t_k}. \tag{2.3}$$

The equation above is also known as *Brownian Motion with drift*[11] [42]. The deterministic part with drift is modeled as:

$$\mu^i_{t_k} \cdot \Delta t,$$
$$\mu^i_{t_k} \in \mathbb{R},$$

where t_k is the *discrete time scale*, defined on $t = 0, \ldots, t_k, \ldots, t_K$, $\Delta t_k = \Delta t$ is the equidistant time scale (here it refers to one day unless otherwise stated), and $\mu^i_{t_k}$ is the expected cash flow at time t_k of the product i. The deterministic component has the following properties:

$$\mathrm{E}[\mu^i_{t_k} \cdot \Delta t] = \mu^i_{t_k} \cdot \Delta t, \tag{2.4}$$
$$\mathrm{Var}[\mu^i_{t_k} \cdot \Delta t] = 0. \tag{2.5}$$

The stochastic component of the equation is given by:

$$\sigma^i \Delta W^i_{t_k},$$
$$\sigma^i \geq 0,$$

where

$$\mathrm{E}[\sigma^i \Delta W^i_{t_k}] = 0, \tag{2.6}$$
$$\mathrm{Var}[\sigma^i W^i_{t_k}] = (\sigma^i)^2 \cdot \Delta t. \tag{2.7}$$

Here, σ^i is constant and measures the sensitivity of a product i to the liquidity shocks, and $(W^i_{t_k})$ is a discrete-time Brownian Motion defined by the probability space $(\Omega, \mathcal{F}, \mathbb{P})$. From the introduction of a Brownian Motion in Definition 2.6 we recall that the change between time t and s $(s < t)$ is normal distributed with zero mean and variance $t - s = \tau$; i.e., $\Delta W_\tau \sim N(0, \tau)$.

Before passing to the next step, it is important to note that the *general process* of any product can be modeled as [41]:

$$CF^i_{t_k} = \mu^i_{t_k} \cdot \Delta t + \sigma^i \Delta W^i_{t_k} + s^i \Delta J^i_{t_k},$$

where $s^i \Delta J^i_{t_k}$ is a jump component which occurs during loss of confidence (bank run) situations. If the jump size is random and independent of the number of jumps, the process is a Compound Poisson Process [25]. Since it is out of scope of this thesis, in our application we ignore the jump component.

[11] The term *Brownian Motion* is used based on Schmaltz's definition in his book, *A Quantitative Liquidity Model for Banks* [41].

The dependence structure of our Brownian component needs further assumptions. In our case, we apply a factor approach and assume that the Brownian component has two factors: the *product-specific* ($\sigma^{i,p} \cdot \Delta W^{i,p}_{t_k}$) and the *systematic* ($\sigma^{i,m} \cdot \Delta W^{i,p}_{t_k}$).[12] Factors are assumed to be independent from each other; furthermore, in order to ease our calculation, we assume that products are independent, too.[13]

The model of our Brownian Motion component:

$$\begin{aligned}
\sigma^i \Delta W^i_{t_k} &= \sigma^{i,p} \Delta W^{i,p}_{t_k} + \sigma^{i,m} \Delta W^m_{t_k}, && (2.8)\\
\rho(\Delta W^{i,p}_{t_k}, \Delta W^m_{t_k}) &= 0, \quad \forall i = 1, \dots, n, && (2.9)\\
\rho(\Delta W^{i,p}_{t_k}, \Delta W^{j,p}_{t_k}) &= 0, \quad \forall i = 1, \dots, n, \quad j = 1, \dots, n, \quad i \neq j, && (2.10)
\end{aligned}$$

where

$\sigma^{i,p}$: sensitivity of product i to $\Delta W^{i,p}_{t_k}$,

$\Delta W^{i,p}_{t_k}$: product-specific liquidity shock,

$\sigma^{i,m}$: sensitivity of product i to the $\Delta W^m_{t_k}$,

$\Delta W^m_{t_k}$: systematic liquidity shock.

The final cash flow of a product:

$$CF^i_{t_k} = \mu^i_{t_k} \cdot \Delta t + \sigma^{i,p} \Delta W^{i,p}_{t_k} + \sigma^{i,m} \Delta W^m_{t_k}. \qquad (2.11)$$

After establishing the product-specific cash flow model, the next step is to *aggregate* the cash flows of all products. The aggregation of deterministic parts is obtained by simple summation, while the stochastic parts require adjustments of factors.

The total cash flow of n products can be represented in the compact martix-vector form:

$$\begin{pmatrix} CF^1_{t_k} \\ CF^2_{t_k} \\ \vdots \\ CF^n_{t_k} \end{pmatrix} = \begin{pmatrix} \mu^1_{t_k} \\ \mu^2_{t_k} \\ \vdots \\ \mu^n_{t_k} \end{pmatrix} \cdot \Delta t + \begin{pmatrix} \sigma^1 \Delta W^1_{t_k} \\ \sigma^2 \Delta W^2_{t_k} \\ \vdots \\ \sigma^n \Delta W^n_{t_k} \end{pmatrix}$$

$$= \begin{pmatrix} \mu^1_{t_k} \\ \mu^2_{t_k} \\ \vdots \\ \mu^n_{t_k} \end{pmatrix} \cdot \Delta t + \begin{pmatrix} \sigma^{1,p} & 0 & \dots & 0 \\ 0 & \sigma^{2,p} & \dots & 0 \\ \vdots & \dots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma^{n,p} \end{pmatrix} \begin{pmatrix} \Delta W^{1,p}_{t_k} \\ \Delta W^{2,p}_{t_k} \\ \vdots \\ \Delta W^{n,p}_{t_k} \end{pmatrix} + \begin{pmatrix} \sigma^{1,m} \\ \sigma^{2,m} \\ \vdots \\ \sigma^{n,m} \end{pmatrix} \Delta W^m_{t_k}.$$

---

[12] The superscripts *p* and *m* in factors stand for *product* and *market*, respectively.

[13] In real circumstances, there exists a significant relationship among products, and a relationship between market and products. This assumption can be investigated and modified in further researches.

Hence, we get the aggregated cash-flow equation:

$$\begin{aligned} CF_{t_k}^A &= (\sum_{i=1}^{n} \mu_{t_k}^i) \cdot \Delta t + (\sum_{i=1}^{n} \sigma^{i,p} \Delta W_{t_k}^{i_p}) + (\sum_{i=1}^{n} \sigma^{i,m}) \cdot W_{t_k}^m \\ &= \mu_{t_k}^A \cdot \Delta t + \sigma^A \Delta W_{t_k}^A, \end{aligned}$$

where

$$\begin{aligned} \mu_{t_k}^A &= \sum_{i=1}^{n} \mu_{t_k}^i, \\ \sigma^A &= \sqrt{\sum_{i=1}^{n} (\sigma^{i,p})^2 + \big[\sum_{i=1}^{n} \sigma^{i,m}\big]^2}. \end{aligned}$$

2.4.2 Funding-Capacity Model (FC)

Based on the definition of the Funding Capacity, in this subsection we will derive an equation to measure the capacity of the liquidity buffer that covers the aggregate exposure σ^A in a confidence level p. Figure 2.5 illustrates the density function $f_{\sigma^A \Delta W_{t_k}^A}$ of an aggregated Brownian Motion. From the setup, the required funding capacity to withstand the exposure σ^A during Δt under the confidence level p is:

$$P(\sigma^A \Delta W_{t_k}^A \leqslant -FC(\sigma^A)) = 1 - p. \tag{2.12}$$

From Definition 2.6 of a Brownian motion, we know that the increments are normal distributed, which results in:

$$\Phi\Big(\frac{\Delta W_{t_k}^A}{\sqrt{\Delta t}} \leqslant \frac{-FC(\sigma^A)}{\sigma^A \sqrt{\Delta t}}\Big) = 1 - p.$$

By solving the equation, we get:

$$FC(\sigma^A) = -\sqrt{\Delta t} \cdot \Phi^{-1}(1-p) \cdot \sigma^A. \tag{2.13}$$

Equation (2.13) states that σ^A units of Brownian standard deviation have to be supported with a funding capacity of $-\sqrt{\Delta t} \cdot \Phi^{-1}(1-p) \cdot \sigma^A$ at the confidence level of p. It is important to note that there is a linear relationship between risk exposure and funding capacity.

2.5 Interest-Rate Models

In this section, we will describe the short-term interest-rate models that are widely accepted in practice, and select the most suitable one for this study. However, before

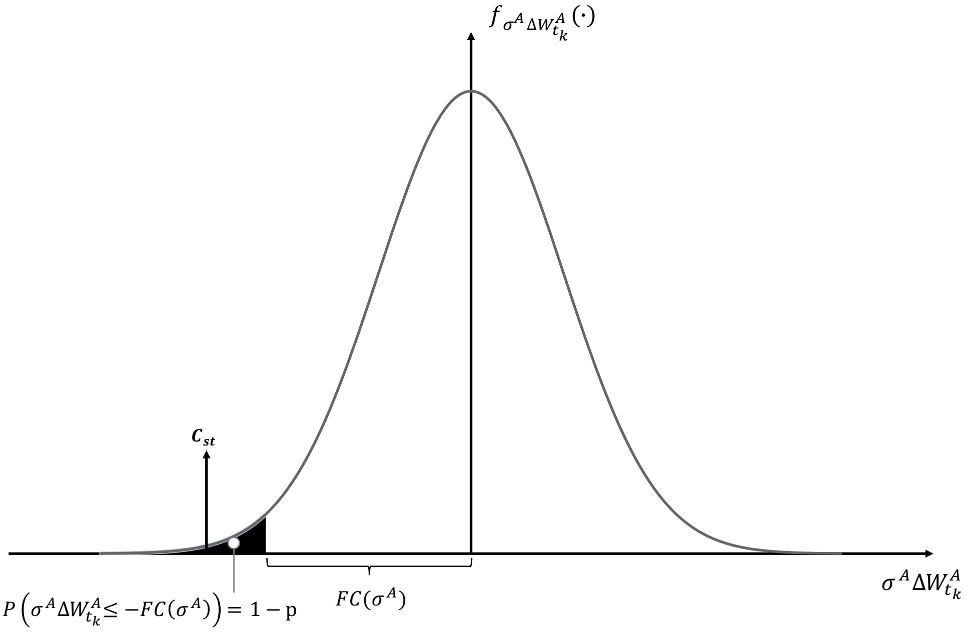

Figure 2.5: Required Funding Capacity [41].

analyzing the models, it is important to discuss the basic properties of interest rates, in particular, the term structure of interest rate.

Mishkin states that "A plot of the yields on bonds with differing terms to maturity but the same risk, liquidity, and tax considerations is called, a *yield curve*, and it describes the term structure of interest rates" [34]. The shape of the yield curves demonstrates the relationship between short - and long-term interest rates. When yield curves slope upward, the long-term interest rates are above the short-term rates; when yield curves are tending downward, the opposite is true; when yield curves are flat, this means that short- and long-term interest rates are the same. Moreover, historical studies have revealed the following facts [34]:

1. The interest rates of bonds with varying maturities act together in time.
2. In case of low short-term interest rates, yield curves tend to have an upward slope; in case of high short-term rates, yield curves are likely to have a downwards slope. (In mathematical contexts, this is called as a *mean reverting process*.)
3. Yield curves are generally upward-sloped.

In the literature, there are several theories that describe the behavior of interest rates. One of the earliest theories are *Expectation Theory* and *Segmented Markets Theory*. Although they lack to explain all three facts simultaneously, they have provided fundamental ideas of their behaviors. By a combination of those fundamental ideas, the *Liquidity-Premium Theory* has been created. The liquidity premium theory explains all three factual behaviors of interest rates on bonds. The main assumptions of the theory is that bonds of different maturities are substitutes and long-term bonds bear higher interest risk than short-term bonds. Based on those assumptions, the liquidity premium

Table 2.1: Yield curves and market expectations according to liquidity-premium theory [34].

Shape of the yield-curve slope	Market expectation of short-term interest rates
1. Steep	Interest rates are expected to rise.
2. Moderate Steep	Interest rates are expected to stay the same.
3. Flat	Interest rates are expected to fall moderately.
4. Inverted	Interest rates expected to fall sharply.

theory is formulated as:

$$r_{nt} = \frac{r_t + r^e_{t+1} + r^e_{t+2} + \ldots + r^e_{t+(n-1)}}{n} + l_{nt}, \quad n \geqslant 1, \tag{2.14}$$

where l_{nt} is the liquidity premium for the n-period interest rate at time t, which is always positive and rises as time to maturity increases [34].

Furthermore, one of the most important features of liquidity premium theory is that by using the slope of the yield curve, one can identify the market expectation of future short-term interest rates. Table 2.1 gives a summary of future short-term interest rate implications.

The *forward yield curve* is the curve of the expected interest rates at future time with different maturities. It can be derived from the current yield curve with the following logic: In an Open-Market Economy, investors have equal return from investing for the whole period, or investing up to some time and reinvesting it to the end of whole period. The equation for the equal returns can be written as:

$$(1 + i_{t+n})^n = (1 + i_{t+k})^k \cdot (1 + i_{t+k,\, t+n})^{n-k}, \qquad k < n. \tag{2.15}$$

where

$$\begin{aligned} n &= \text{length of time period } [0,\ n], \\ k &= \text{length of time period } [0,\ k], \\ n-k &= \text{length of time period } [k,\ n], \\ i_{t+n} &= \text{return for the whole period}, \\ i_{t+k} &= \text{return for period } k, \\ i_{t+k,\, t+n} &= \text{return from period } k \text{ up to period } n. \end{aligned}$$

In this work, forward-yield curves constitute base rates in FTP estimations. By solving Equation (2.15), we get the forward interest rate at time $t + k$ maturing at $t + n$ as follows:

$$i_{t+k,\, t+n} = \left(\frac{(1 + i_{t+n})^n}{(1 + i_{t+k})^k} \right)^{-(n-k)} - 1. \tag{2.16}$$

In short-term interest-rate modeling, the real challenge lies in the shape of the forward-yield curve. The reason is that it requires modeling of the stochastic time evolution of the entire curve. There exists a number of studies on modeling short-term rates, which are classified into two categories: *one-factor models*, since there is only one stochastic variable [11, 20], and *multi-factor models* with multiple stochastic parameters. Due to complicated methodology, multiple-factor models are not discussed in this thesis.

The spot rate $r(t)$ is used as the state variable by one-factor short-term models. The general differential equation of one-factor models has the following form:

$$dr(t) = A(r(t), t)dt + B(r(t), t)dW(t), \tag{2.17}$$

where A and B are drift and diffusion coefficients, respectively. Choice of varying coefficients results in different spot rate dynamics. In the remaining part of this subsection, we will compare and select the model that can best fit the yield curves.

First, we will observe the *Vasicek's* model which was the first to capture a mean reverting process [48]. The *mean-reversion process* implies that in the long-run the short-term interest rates approach to the mean. This process was initially investigated in physics as an *Ornstein-Uhlenbeck* stochastic process. The process is given by the following stochastic differential equation:

$$dr(t) = (\alpha - \beta r(t))dt + \sigma dW(t), \tag{2.18}$$

where α, β, and σ are positive constants. The solution to differential Equation (2.18) can be determined in closed form as:

$$r(t) = e^{-\beta t} r(0) + \frac{\alpha}{\beta}(1 - e^{-\beta t}) + \sigma e^{-\beta t} \int_0^t e^{\beta s} dW(s). \tag{2.19}$$

Proof. For a detailed proof of the equation please refer to Appendix A. □

The explicit solution is a main feature of Vasicek model, by the help of which it is easy to determine all properties of the model. However, Vasicek model has also several disadvantages. In particular, the model drawbacks are that the model does not fit the entire forward yield-curve and it provides negative outcomes.

The next model was developed by John C. Cox, Jonathan E. Ingersoll and Stephen A. Ross as an extension of the Vasicek model [12]. The *Cox-Ingersoll-Ross (CIR)* model represents short-term interest as

$$dr(t) = (\alpha - \beta r(t))dt + \sigma\sqrt{r(t)}dW(t), \tag{2.20}$$

where α, β, σ are positive integers. A principal advantage of the CIR model is that the short-term interest rate can not become negative. If $r(t)$ approaches to zero, on the right side of Equation (2.20) only $\alpha d(t)$ remains and $dr(t)$ goes to a positive value [28]. However, the CIR model, compared to Vasicek, is not a Gaussian process and does not have a closed-form solution, which makes it difficult to analyze [10].

Another extension of Vasicek's model is called *Hull-White (H&W)* model. The extension of this model includes a *time-varying mean reversion* $\alpha(t)$ principal[14]

[14] Compared to the one in Vasicek's model, the mean-reversion process in Hull-White model is time-dependent.

process [22]. This principal allows to fit forward yield curve by steering the volatility of short-term interest rates. The model is the same as Vasicek's model, except that α is time-dependent:[15]

$$dr(t) = (\alpha(t) - \beta r(t))dt + \sigma dW(t), \tag{2.21}$$

with

$$\alpha(t) = \frac{\partial f(0,t)}{\partial t} + af(0,t) + \frac{\sigma^2}{2a}(1 - e^{-2at}). \tag{2.22}$$

The function $f(0,t)$ is the forward rate function at time 0 with maturity t, whereas the first expression on the right-hand side of Equation (2.22) is the partial derivative of the function with respect to time. Since Hull-White model is a Gaussian process and has an explicit solution, it is easy to analyze the model properties as in Vasicek's model. The only weakness of H&W model is the positive probability of negative short-term interest rates.

To sum up, among those three models represented above we choose the Hull-White model. The reason for such a choice is twofold. First, it fits better a forward yield curve than Vasicek's model. Also, unlike CIR model, it is a Gaussian process and has an explicit solution, which is an important implication for the simulation. The drawback of H&W model is in fact arguable. In the recent crisis, some central banks charged negative interests. Let us mention that the Bank of Japan (BoJ) is still applying a negative interest rate policy, in order to fight with Japan's deflationary economy.

[15] In later versions of Hull-White model, α, β, and σ are deterministic functions of time, which is not covered in this work. For more details please refer to [8].

CHAPTER 3

FUND TRANSFER PRICING MODEL

In this section, we will briefly summarize the mechanics and objectives of FTP and then move on to the analysis of the two types of FTP models, namely, *Pooled Average* and *Matched Maturity*. Finally, we will construct the advanced FTP model in question, which includes the model of the NSFR effect (main contribution of our thesis).

In its simplest form, FTP is a process where the central unit (ALM) of a bank collects funds and redistributes them among the business units, acting as an *internal bank* [45]. In case of excesses or shortages of funds, the central unit invests on or borrows from the money market. During the collection and redistribution processes, ALM pays and charges the FTP rates, which results in the decomposition of *net interest margin* (NIM) of a bank. In addition, ALM also manages liquidity position of a bank, by investing excess reserves into securities. The optimization process of the security portfolios is conducted by applying *Conditional Value at Risk* (CVaR) approaches [47].

An illustrative example of the flows between ALM, business units and customers is shown in Figure 3.1. Within a bank, businesses differ by their functioning performances. Some businesses are heavily collecting funds, while others are good at lending. Deposit business collects funds from savers and pays interest. On the other hand, loan business issues loans to borrowers and gets interest. As a result, one side has a surplus of funds, whereas the other one is in shortage. At this point, the central unit of a bank with FTP tool comes into play. ALM of a bank borrows money from the deposit business by paying the deposit FTP rate and lends it to the lending business at a loan FTP rate. Accordingly, there are three units benefiting from the whole transaction. The first unit is deposit business, paying deposit rate to customer and lending at deposit FTP rate. The second one is ALM, borrowing at deposit FTP rate and lending at loan FTP rate. Finally, lending business issues loans at loan rate. The funding of these loans is obtained at loan FTP rate. For the bank, it is a zero-sum gain[1], and the total NIM is the difference between loan and deposit rate.

This is better to understand by a numerical example. Assume that deposit business in Figure 3.1 pays 2% deposit rate to savers and receives 3% deposit FTP rate from ALM; thus, deposit business generates 1% NIM. A bank's lending business unit, on the other hand, borrows at 4% loan FTP rate and charges the customer 5% loan rate, which also

[1] In game theory, when the amount of one person's gain is the same as another's loss, the net change is zero and this is called as zero-sum game.

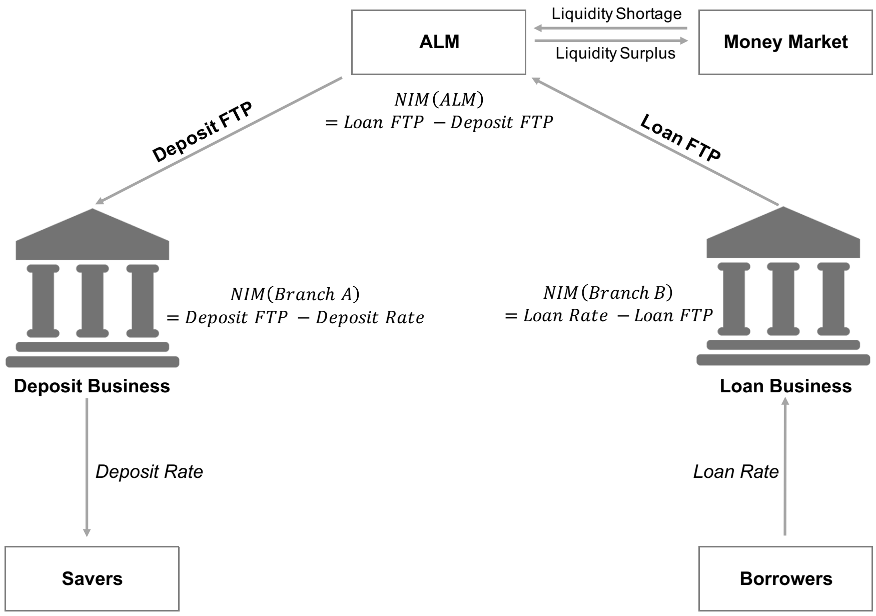

Figure 3.1: Mechanic of FTP within a typical bank [9].

operates with 1% NIM. The NIM of ALM unit is the difference between 4% loan FTP rate and 3% deposit FTP rate. Overall NIM of a bank is the sum of business units' and central unit's NIM, which is equal to 3%. Please note that a bank's NIM can also be calculated by deducting deposit rate (2%) from loan rate (5%).

The decomposition of NIM is one of the most important objectives of the FTP process, which helps a bank to analyze the true performance of business units [29]. In the literature, there are several specifications of FTP process.[2] In general, FTP enables banks to *increase profitability*, to *transfer risks to the central unit* and to *guide management in taking decisions* [16, 29, 30].

3.1 Pooled Average

In practice, there are two most widely used methodologies for fund transfer pricing. These methods are *pooled averages* and *matched maturity* FTP approaches. In this section, we will have a closer look into pooled or weighted average method, which allows for the easiest application.

Pooled average approach is divided into two subcategories: the *single pool* and the *multiple pool* ones. As the name suggests, in the single pool, there is only one rate

[2] For the details of all FTP specifications refer to Dimitriu [16].

obtained to price all balances, whereas in the multiple pool, the assets and liabilities of a bank are grouped into different pools and are priced accordingly.

The analogy in the single-pool approach is simple: the weight of each item in a pool is multiplied by its yield (assets) or cost (liabilities) to form weighted averages. Then, the average weighted yields and costs are assigned as an FTP rate. Under the multiple-pool approach, business units create pools according to some criteria and a set of dimensions: such as type, term, origination, or other fund attributes [49].

Although the pooled-average approach is preferred due to its simplicity, it has several disadvantages. Firstly, the pools created on the level of business units do not take into account the shortfalls and excesses of funding resources, which causes maturity risk. Secondly, ALM does not exactly know the maturity profiles of business pools and operate on an average basis, herewith creating interest risk. Moreover, a pooled average reflects the average rate of existing portfolios, but it does not cover historical rates, which may mislead the management of a bank.

3.2 Matched Maturity

Matched-Maturity method, also called as *Co-Terminus*, is an extended approach of the multiple-pool [49]. Unlike multiple-pool, this approach includes the cash-flow characteristics of the contract and assigns the transfer price accordingly. Tumasyan notes that under matched maturity method, "rates charged for the use of funds and rates credited for providing funds are based on matching the rates on the cost of funding curves" [45]. Matched-maturity approach is one of the most recognized approaches in the financial sector. However, the financial crisis in 2008 showed that this approach could not cover severe liquidity risks, as a result of which the banking regulators of developed countries required further enhancements in FTP methodologies. For instance, the Fed guidance on FTP suggests that matched maturity methodologies would likely be the minimum acceptable practice going forward [40].

In order to understand the matched-maturity process more clearly, let us refer to an example. Suppose a bank which has only two transactions at the same day - a three year mortgage loan at 15% and a six-month saving deposits at 12%. Based on the FTP curve in Figure 3.2, the bank charges 14% FTP rate for loan business. 1% spread between FTP rate and customer rate includes the credit risk in the forms of the expected loss and mark-up[3] of loan business. On the other hand, six-month deposits get 13% from the FTP curve. For sourcing funds in the form of saving deposits, incurring lower cost than wholesale funding, a deposit-gathering business is rewarded with a 1% margin. Finally, the central unit earns 1% NIM for covering interest rate risk. In this case, it is the mismatch in maturity - lending long, borrowing short. In our example, the spread is positive due to the upward sloping FTP curve; however, things may change when the shape of the curve changes.

Compared to pooled approaches, matched-maturity approach better assigns the

[3] Net rate of return after excluding all costs from sale price.

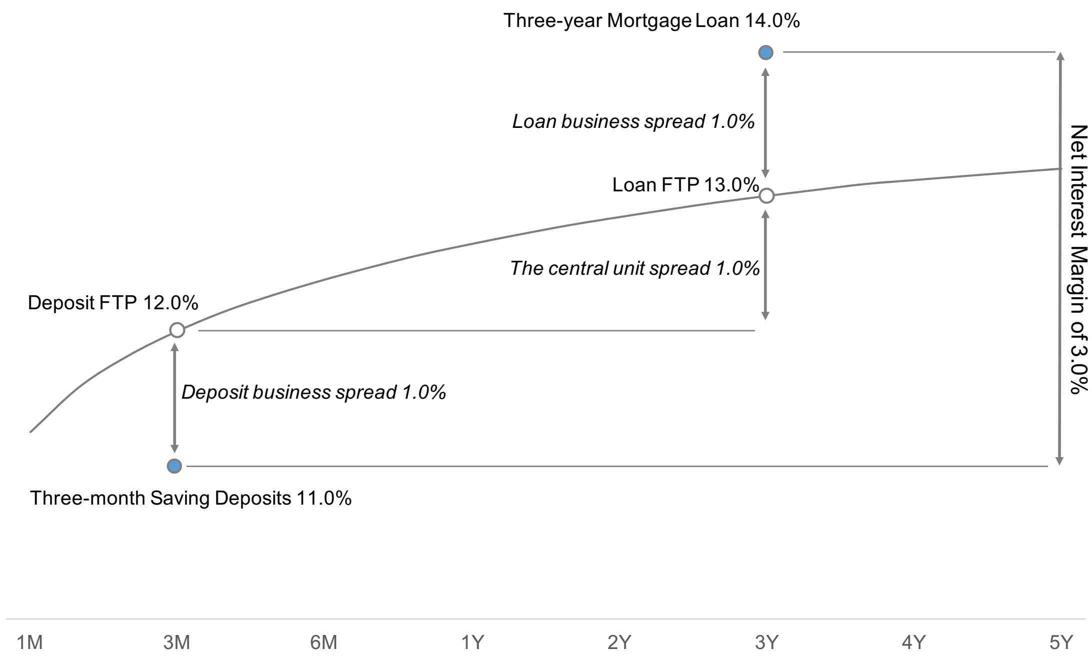

Figure 3.2: Matched-Maturity approach of FTP [30].

profitability margin of each business and of the central unit, which is a core implication in assessing performance and making decisions. However, matched-maturity lacks the ability to catch behavioral aspects of products. For instance, the optionality in saving deposits allows depositors to withdraw deposits before they mature, so that there is no fixed time. From the asset perspective, early repayment of loans causes similar uncertainty. Then the question is, which FTP rate should you charge? To answer this question, one should build a *behavioral model* of customers. The behavior of customers then would be the key component of pricing. To sum up, matched-maturity should be modified in order to reflect the accurate FTP rates.

3.3 Advanced FTP model

This section will introduce a more realistic FTP approach, which embeds optionality and regulatory measures. The cornerstone of the model is based on Christian Schmaltz's book, *A Quantitative Liquidity Model for Banks* [41]. The main idea of the model, which has also been referred to in Section 2.3, is that the cash flow is divided into a *deterministic* and a *stochastic* part. The transfer price of liquidity indicates the deterministic part of cash flow, and the liquidity risk indicates the stochastic part [19]. The summation of these two transfer prices, calculated separately, generates the base FTP rate of a specific product. For further modification of Schmaltz's model, this thesis suggests that the regulatory measures should be included in transfer pricing. The reason for this modification is that the regulatory measures after the big financial crisis are excercising a significant impact on a financial institution's balance sheet. Therefore, reflecting those effects on FTP plays a crucial role in making decisions

[26, 46].

This section is organised as follows: In the first part, we will derive a transfer price model for the deterministic part. Then, the modeling of the stochastic part will be described. In the subsequent part, we will model the effects of regulatory measures and embed them into the transfer pricing. Finally, an illustrative example of an advanced FTP model will be demonstrated.

3.3.1 Deterministic Fund Transfer Pricing

From our "Cash Flow" Subsection 2.4.1, we recall that the deterministic component of cash flow - $\mu^i_{t_k}$. In the literature, there is a consensus on modeling the transfer price of the deterministic component. The function of the deterministic transfer price is defined as follows:

$$TP^D(\mu_{t_k}) := (r(0,t_k) - r_{rf}(0,t_k)) \cdot \mu_{t_k} \cdot \Delta t, \tag{3.1}$$

where

$$\begin{aligned} r(0,t_k) &: \text{funding curve,} \\ r_{rf}(0,t_k) &: \text{risk-free curve.} \end{aligned}$$

In Equation (3.1), the difference between funding curve and risk-free curve is defined as funding spread, which represents funding cost of a bank above the reference rate, and it is scaled by the expected life of the loan.

The only controversial question in the deterministic function is about which market curve should be used as a risk-free curve. In practice, risk-free instruments do not exist. However, banks prefer Triple-A bonds or interest-rate swaps as proxies for risk-free instrument. Since a bond market is not as liquid as swap markets, in practice, many banks choose interest-rate swaps as a risk-free curve. In this thesis, we will use the Turkish-Lira swap curve as a base curve.[4]

The formula in Equation (3.1) indicates the transfer price of a product with only one deterministic cash flow μ at time t_k. For the product i with n deterministic cash flows $\mu^i_{t_0}, \ldots, \mu^i_{t_n}$, the transfer price is the sum of all cash flows:

$$TP^D(\mu^i_{t_0}, \ldots, \mu^i_{t_k}) = \sum_{j=0}^{n} (r(0,t_j) - r_{rf}(0,t_j))\Delta t \cdot \mu^i_{t_j} \cdot (t_j - t_0). \tag{3.2}$$

[4] For short-term rates with less than 1 year to maturity, Turkish Lira and USD cross-currency implied yields - derived from the covered interest-rate parity theorem - are used. For long-term rates Turkish Lira and USD swap rates are applied.

3.3.2 Stochastic Fund Transfer Pricing

In the previous sections, we have defined the Brownian component of cash flow as:

$$\sigma^i \Delta W^i_{t_k} = \sigma^{i,p} \Delta W^{i,p}_{t_k} + \sigma^{i,m} \Delta W^m_{t_k}.$$

which is the sum of the product-specific and the systematic component.

The aggregate exposure of a Brownian component has been derived in Section 2.4.1 as:

$$\sigma^A = \sqrt{\sum_{i=1}^{n} (\sigma^{i,p})^2 + \Big[\sum_{i=1}^{n} \sigma^{i,m}\Big]^2}.$$

The funding capacity to back the aggregate exposure σ^A at a significance level p is stated in Equation (2.13):

$$FC(\sigma^A) = -\sqrt{\Delta t} \cdot \Phi^{-1}(1-p) \cdot \sigma^A.$$

In order to simplify the notation in the above equation, we will denote it as follows:

$$FC(\sigma^A) = FC(1) \cdot \sigma^A,$$

where

$$FC(1) = -\sqrt{\Delta t} \cdot \Phi^{-1}(1-p).$$

3.3.2.1 Transfer Price of Aggregate Exposure

Since the amount of funding capacity is held to back aggregate exposure carries cost, the next step is to determine the cost implied by required funding capacity *FC*. Recall from funding Section 2.1.2 that in terms of collateral the funding is divided into two categories, namely, *secured* (l) and *unsecured* $(1-l)$:

$$FC(\sigma^A) = l \cdot FC(\sigma^A) + (1-l) FC(\sigma^A). \tag{3.3}$$

The function for required cost of funding capacity is defined as follows:

$$\begin{aligned} TP^B(\sigma^A) &= C^R(l \cdot FC(1) \cdot \sigma^A) + C^U((1-l) \cdot FC(1) \cdot \sigma^A) \\ &= C^R(l \cdot FC(1) \cdot \sigma^A) + 0 \\ &= C^R(-l \cdot \sqrt{\Delta t} \cdot \Phi^{-1}(1-p)) \cdot \sigma^A, \end{aligned} \tag{3.4}$$

addressing

C^R : cost function of secured funds,

C^U : cost function of unsecured funds.

The cost function of secured funding costs can be expressed in linear form as:[5]

$$C^R(\sigma^A) = (-l \cdot \sqrt{\Delta t} \cdot \Phi^{-1}(1-p)) \cdot \sigma^A \cdot \Delta Yield. \tag{3.5}$$

At this point, it is important to note that the assumption which unsecured funding incurs zero cost did not hold during the 2008 financial crisis. When Lehman Brothers filed for bankruptcy, the LIBOR rates in interbank lending jumped by more than 360 basis points [21].

3.3.2.2 Transfer Price of an Individual Product

After determining the required funding capacity to cover aggregate stochastic exposure, the next part is disaggregation of funding capacity into products. The independence among products, and between product-specific and market risk, is also valid in this part. Since the independence assumption is too simplistic, to enhance the disaggregation approach, one has to account for diversification effects [41].[6] According to the diversification, the sum of required funding capacity for individual products exceeds the aggregate funding:

$$\sum_{i=1}^{d} FC(\sigma^{p,i}, \sigma^{m,i}) \geqslant FC(\sigma^A). \tag{3.6}$$

The objective here is to adjust individual risks $(\sigma^{i,p}, \sigma^{i,m})$ such that:

$$FC(\sigma^A) = \sum_{i=1}^{d} FC(\sigma^{i,p,adj}) + \sum_{i=1}^{d} FC(\sigma^{i,m,adj}) \leqslant \sum_{i=1}^{d} FC(\sigma^{i,p}) + \sum_{i=1}^{d} FC(\sigma^{i,m}). \tag{3.7}$$

By solving Equation (3.7) we get:

$$\begin{aligned} FC(\sigma^A) &= \sum_{i=1}^{d} FC(\sigma^{i,p,adj}) + \sum_{i=1}^{d} FC(\sigma^{i,m,adj}) \\ &\Leftrightarrow \\ FC(1) \cdot \sigma^A &= FC(1) \cdot \sum_{i=1}^{d} \sigma^{i,p,adj} + FC(1) \cdot \sum_{i=1}^{d} \sigma^{m,i,adj}, \\ \sigma^A &:= \sum_{i=1}^{d} \sigma^{i,p,adj} + \sum_{i=1}^{d} \sigma^{i,m,adj}. \end{aligned} \tag{3.8}$$

[5] The difference in yield indicates the risk premium of the institution, and expresses itself as the difference between the cost of unsecured funding and benchmark yields.

[6] The *Diversification Effect* is measured as the relation of the required funding capacity under 0 correlation to the funding capacity under perfect correlation.

Equation (3.8) states that the allocation of aggregate exposure σ^A is equivalent to the allocation of funding capacity. There are several approaches for the computation of individual risks. However, in this thesis we will use *Adjustment Approach*, which is also additive [41]. The reason for such a choice is twofold: the first one it suits well for a small number of factors (in our case, there are two factors), and the second reason is that it does not require further assumptions.

In this work, we need to calculate two adjustment factors: one measures the effect of *diversification* among products, and the second assesses the effect between product-specific and systematic factors. Let us begin with an estimation of the *adjustment factor* among products.

By embedding Equation (3.8) into the function of Funding Capacity, we obtain:

$$FC(\sigma^A) = -\sqrt{\Delta t} \cdot \Phi^{-1}(1-p) \cdot \sqrt{\sum_{i=1}^{d} (\sigma^{i,p})^2 + \Big[\sum_{i=1}^{d} \sigma^{i,m}\Big]^2}. \tag{3.9}$$

At this point, it is vital to note that lower case superscripts *p* and *m* are used to distinguish products-specific and systematic Brownian risks on product level, whereas upper case superscripts *P* and *M* are referred to the risks across all products. Here, we will derive the equations for two extreme cases. In the first case, where only a product-specific Brownian exposure exists, it holds:

$$\begin{aligned} FC(\sigma^P) &= -\sqrt{\Delta t} \cdot \Phi^{-1}(1-p) \cdot \sqrt{\sum_{i=1}^{d} (\sigma^{i,p})^2 + 0} \\ &= -\sqrt{\Delta t} \cdot \Phi^{-1}(1-p) \cdot \sigma^P. \end{aligned} \tag{3.10}$$

In the second case, only systematic Brownian exposure exists; then, we note

$$\begin{aligned} FC(\sigma^M) &= -\sqrt{\Delta t} \cdot \Phi^{-1}(1-p) \cdot \sqrt{0 + \Big[\sum_{i=1}^{d} \sigma^{i,m}\Big]^2} \\ &= -\sqrt{\Delta t} \cdot \Phi^{-1}(1-p) \cdot \sigma^M. \end{aligned} \tag{3.11}$$

Under perfect correlation, the required funding capacities are additive:

$$FC(\sigma^P + \sigma^M) = FC(\sigma^P) + FC(\sigma^M). \tag{3.12}$$

Now we will derive the formula for the diversification factor κ - kappa, which measures the diversification effect between product-specific and systematic factors. Recall that the formula for the adjustment factor is "actual funding capacity" divided by "funding capacity under perfect correlation":

$$\begin{aligned}\kappa &= \frac{FC(\sigma^A)}{FC(\sigma^P)+FC(\sigma^M)}\\ &= \frac{\sigma^A \cdot FC(1)}{\sigma^P \cdot FC(1)+\sigma^M \cdot FC(1)}\\ &= \frac{\sigma^A}{\sigma^P+\sigma^M}.\end{aligned} \tag{3.13}$$

The adjustments for product-specific (σ^P) and systematic (σ^M) Brownian risks are stated as:

$$\begin{aligned}\sigma^{P,adj} &= \kappa \cdot \sigma^P,\\ \sigma^{M,adj} &= \kappa \cdot \sigma^M.\end{aligned}$$

Having found these adjustments, we can show that the sum of the factors would be equal to the total aggregate exposure:

$$\begin{aligned}FC(\sigma^{P,adj})+FC(\sigma^{M,adj}) &= FC(\kappa \cdot \sigma^P)+FC(\kappa \cdot \sigma^M)\\ &= FC(\frac{\sigma^A}{\sigma^P+\sigma^M} \cdot \sigma^P)+FC(\frac{\sigma^A}{\sigma^P+\sigma^M} \cdot \sigma^M)\\ &= FC(\sigma^A) \cdot (\frac{\sigma^P}{\sigma^P+\sigma^M}+\frac{\sigma^M}{\sigma^P+\sigma^M})\\ &= FC(\sigma^A).\end{aligned}$$

After dividing the funding capacity into factors, the next step is to allocate the funding capacity into individual products (i). The allocation of the systemic factor into individual products is derived in the following form:

$$\begin{aligned}FC(\sigma^{M,adj}) &= FC(\kappa \cdot \sigma^M)\\ &= FC(\kappa \cdot \sum_{i=1}^{d} \sigma^{i,m})\\ &= FC(\sum_{i=1}^{d} \kappa \cdot \sigma^{i,m})\\ &= FC(\sum_{i=1}^{d} \sigma^{i,m,adj}).\end{aligned}$$

From the last two equalities the subsequent equation is derived:

$$\sigma^{i,m,adj} = \kappa \cdot \sigma^{i,m}. \tag{3.14}$$

For the product-specific factor the corresponding formula results as:

$$\begin{aligned} FC(\sigma^{P,adj}) &= FC(\kappa \cdot \sigma^{P}) \\ &= FC\Big(\kappa \cdot \sqrt{\sum_{i=1}^{d} (\sigma^{i,p})^2}\Big) \\ &\leqslant FC(\kappa \cdot \sum_{i=1}^{d} \sigma^{i,p}) \qquad \text{(Triangle Inequality)}. \end{aligned} \tag{3.15}$$

The above inequality reveals that there should be further adjustments incorporating diversification among individual products. The second adjustment factor is defined as:

$$\begin{aligned} \kappa^p : &= \frac{\sigma^P}{\sum_{i=1}^{d} \sigma^{i,p}} \\ &\Leftrightarrow \\ \sigma^P &= \kappa^p \cdot \sum_{i=1}^{d} \sigma^{i,p}. \end{aligned} \tag{3.16}$$

By a combination of Equations (3.15) and (3.16) we derive:

$$\begin{aligned} FC(\sigma^{P,adj}) &= FC(\kappa \cdot \sigma^{P}) \\ &= FC(\kappa \cdot \kappa^p \cdot \sum_{i=1}^{d} \sigma^{i,p}) \\ &= FC(\sum_{i=1}^{d} \kappa \cdot \kappa^p \cdot \sigma^{i,p}) \\ &= \sum_{i=1}^{d} FC(\kappa \cdot \kappa^p \cdot \sigma^{i,p}) \\ &= \sum_{i=1}^{d} FC(\sigma^{i,p,adj}). \end{aligned}$$

From the above the product-specific adjusted exposure is obtained as:

$$\sigma^{i,p,adj} = \kappa \cdot \kappa^p \cdot \sigma^{i,p}, \qquad \forall\, i = 1, 2, \ldots, d. \tag{3.17}$$

Incorporating all factors, we receive the final equation of aggregate exposure:

$$\begin{aligned}
FC(\sigma^A) &= FC(\sigma^{P,adj}) + FC(\sigma^{M,adj}) \\
&= FC(\sum_{i=1}^{d} \sigma^{i,p,adj}) + FC(\sum_{i=1}^{d} \sigma^{i,m,adj}) \\
&= FC(\sum_{i=1}^{d} \kappa \cdot \kappa^p \cdot \sigma^{i,p}) + FC(\sum_{i=1}^{d} \kappa \cdot \sigma^{i,m}) \\
&= \sum_{i=1}^{d} FC(1) \cdot \kappa \cdot (\kappa^p \cdot \sigma^{i,p} + \sigma^{i,m}). \qquad (3.18)
\end{aligned}$$

Therefore, the required funding capacity of product i, with product-specific σ^p and market σ^m risks, is:

$$FC(\sigma^{i,p}, \sigma^{i,m}) = -\sqrt{\Delta t} \cdot \Phi^{-1}(1-p) \cdot \kappa \cdot (\kappa^p \cdot \sigma^{i,p} + \sigma^{i,m}). \qquad (3.19)$$

After deriving the required funding capacity for product i, it is trivial to show that the transfer price of a product i with Brownian exposures, $\sigma^{i,m}$ and $\sigma^{i,p}$ at a confidence level p, equals:

$$\begin{aligned}
TP_i^B(\sigma^{i,p}, \sigma^{i,m}) &= c_R(\sigma^{i,p}, \sigma^{i,m}, l) \qquad (3.20) \\
&= -l \cdot \sqrt{\Delta t} \cdot \Phi^{-1}(1-p) \cdot \kappa \cdot (\kappa^p \cdot \sigma^{i,p} + \sigma^{i,m}) \cdot \Delta Yield.
\end{aligned}$$

3.3.2.3 Optionality

In Equation (3.20), the transfer price of a product is derived for one time lag Δt, where it is assumed that there is no exercise during this period. However, as we have mentioned in our *Funding section*, in real circumstances, indeed there is a frequent numbers of exercises [33]. Therefore, in this subsection, we extend the assumption and denote the full maturity as $T = n \cdot \Delta t$; inserting what into Equation (3.20) shows:

$$\begin{aligned}
TP_i^B(\sigma^{i,p}, \sigma^{i,m}, n \cdot \Delta t) &= c_R(\sigma^{i,p}, \sigma^{i,m}, l) \\
&= -l \cdot \sqrt{n \cdot \Delta t} \cdot \Phi^{-1}(1-p) \cdot \kappa \\
&\quad \cdot (\kappa^p \cdot \sigma^{i,p} + \sigma^{i,m}) \cdot \Delta Yield.
\end{aligned}$$

The whole maturity time T (e.g., 1 year or 365 days), can also be denoted in terms of the average length of repayment period (e.g., 30.4 days) and the number of total repayments (e.g., 12 months), such that:

$$T = n_1 \cdot n_2,$$

where

n_1 : length of time period without exercise (e.g., repayment period of loans),
n_2 : number of exercises (e.g., number of total repayments until the maturity),

and

$$\begin{aligned} TP_i^B(\sigma^{i,p}, \sigma^{i,m}, T, n_2) &= TP(\sigma^{i,p}, \sigma^{i,m}, n_1) \cdot n_2 \\ &= \sqrt{n_1} \cdot TP(\sigma^{i,p}, \sigma^{i,m}) \cdot n_2 \\ &= \sqrt{\frac{T}{n_2}} \cdot TP(\sigma^{i,p}, \sigma^{i,m}) \cdot n_2 \\ &= \sqrt{T \cdot n_2} \cdot TP(\sigma^{i,p}, \sigma^{i,m}). \end{aligned} \tag{3.21}$$

The last expression shows that the Brownian Transfer price grows linearly with respect to the square roots of maturity and the number of exercises [14]. From Equation (3.21), it can be concluded that as the number of exercises rises, the transfer price of a stochastic component also increases. To sum up, the final stochastic transfer price of a product i is represented as:

$$TP_i^B(\sigma^{i,p}, \sigma^{i,m}, T, n_2) = -l \cdot \sqrt{T \cdot n_2 \cdot \Delta t} \cdot \Phi^{-1}(1-p) \cdot \kappa \cdot (\kappa^p \cdot \sigma^{i,p} + \sigma^{i,m}) \cdot \Delta Yield. \tag{3.22}$$

3.3.3 Regulatory Impact

The incorporation of regulatory impact on FTP is a new topic in the area of Financial Mathematics [26]. Up until now, there have been a couple of works only, which are treating the internal cost of regulations. This is due to the fact that new regulatory measures like LCR have come into force recently, whereas the NSFR has not legally been enforced yet. Although not all regulatory measures are strictly implemented, bypassing their impact on determination of FTP will lead to an underpricing the fair value of a product. For the completeness of an advanced FTP methodology, in this thesis, the LCR and the NSFR impacts on the FTP are treated separately.

Thus, this subsection is divided into two parts: first, the impact of existing LCR applications will be analyzed, and then the possible effects of the NSFR application on the FTP rates will be stated. Before moving on to an analysis of the LCR impact on the FTP, let us recall the formula of the LCR from Subection 2.1.3:

$$LCR = \frac{\text{Stock of } \textit{HQLA}}{\textit{TNCO} \text{ over the next 30 Calendar Days}}.$$

The idea in applying LCR by regulators is to force banks to hold a certain amount of liquid assets against their short-term exposure. Holding liquid assets causes an opportunity cost which needs to be covered. The methodology of implementing the LCR impact on the FTP is based on reflecting those opportunity costs on the FTP rates. For the sake of simplicity, we will illustrate the methodology by the following example: Assume that a bank issues a mortgage loan and funds it from cash.[7] Actually, this action can be replicated as an investment of cash to the mortgage-backed securities (MBS). From the above transaction, the HQLA of a bank will decrease, since the class of liquid assets has been downgraded from Level 1 to Level 2B. The fraction of a decrease in the stock of the HQLA is subject to the related haircut (ϕ) among asset levels. In our example, the HQLA stock will decrease by 25% times the notional amount of an investment (N) due to the haircut in MBS. Herewith, the new value of LCR after investment will look like:

$$LCR_{new} = \frac{\text{Stock of } \textit{HQLA} - 0.25 \cdot N}{\textit{TNCO} \text{ over the next 30 Calendar Days}}.$$

Assuming that the initial ratio is equal to the legal required limit for the LCR, the bank needs to fulfill the deficit amount, in our case it is 25% of the notional amount (N), of the HQLA. Likewise, we assume that our bank can acquire unsecured funds and can invest them into Level 1 assets. From the profit and loss (P and L) perspective, this action incurs loss for the bank, since the funding cost (f_c) is higher than the return of Level 1 assets (r_A). Otherwise, there will be an arbitrage opportunity. The maturity of this borrowing has to cover the lifetime of the issued mortgage loan (T). However, as the monthly repayments of the mortgage loan principal are paid, the amount of the borrowing should also be decreased[8], otherwise extra cost for the excess liquidity buffer would occur. In order to calculate the regulatory impact more accurately, in our thesis we would calculate the cost of the fundings' cash-flow. As a result, the *effective cost* of funding can be measured as follows:

$$f_c^{eff} = \sum_{j=0}^{n} \mu_{t_j} \cdot N \cdot (f_c - r_A) \cdot \phi, \tag{3.23}$$

where

$$\begin{aligned} \mu_{t_0} &= 0, \\ \mu_{t_j} &= \text{principal repayments}, \forall\, j = 1, 2, \ldots, n. \end{aligned}$$

Until this point, we have assumed that the initial ratio is equal to the required limit. However, in real circumstances, this is not the case and, generally, banks with a strong liquidity basis have higher ratios. This saves the banks which have an excess HQLA stock from borrowing, and also allows them to use excess liquid assets. As a result, the effective cost of funding will be lower, allowing banks to have a flexibility in pricing

[7] Here, it is important to note that funding directly from cash or by borrowing does not affect the final outcome, since funding from cash and then replacing the amount by borrowing has the same outcome as direct borrowing.

[8] In essence, the same outcome can also be reached by investing an extra reserve amount from repayments.

the FTP rates. The relationship between the proportion of current available HQLA and effective cost of funding (at specific time t), reflected on the FTP rates, can be expressed in the following modified form:

$$f_c^{eff} = \sum_{j=0}^{n} \mu_{t_j} \cdot N \cdot (f_c - r_A) \cdot \phi \cdot \theta_t, \tag{3.24}$$

where

$$\theta_t = \frac{\text{Stock of HQLA}_t}{\text{Total available balance of HQLA}_t}. \tag{3.25}$$

The same logic is used in measuring the NSFR effect on FTP rates. The effective cost of funding, which is needed to accomplish regulatory NSFR ratio, is calculated and embedded in the FTP rate. In order to illustrate this in closer details, let us assume the previous example in the calculation of the LCR effect: The bank issues mortgage loan with a notional amount of N units, and the NSFR of a bank is equal to the regulatory limit of 100%. Recalling from Subsection 2.1.3, the initial NSFR is calculated as:

$$NSFR = \frac{ASF}{RSF}, \tag{3.26}$$

where

ASF: *Available Stable Funding*,
RSF: *Required Stable Funding*.

After the issuance of mortgage loan funded by cash, only RSF will increase by the amount of $\psi \cdot N$, since ASF will change as a result of a move in liability items. Therefore, the bank's new value of NSFR looks as follows:

$$NSFR_{new} = \frac{ASF}{RSF + \psi \cdot N}. \tag{3.27}$$

In order to encompass the decrease in the ratio, the bank needs $\psi \cdot N$ amount of funding, in our example, it is 65% of notional amount (N). The effective cost of this funding can be demonstrated as in the LCR part:

$$f_c^{eff} = \sum_{j=0}^{n} \mu_{t_j} \cdot N \cdot (f_c - r_A) \cdot \psi. \tag{3.28}$$

From the above regulatory effects, one can observe that unique borrowing, covering both of the funding needs, can be established. Thus, all of the requirements are accomplished by one transaction. In other words, to eliminate any double-counting effect of regulatory cost, one should consolidate the LCR and the NSFR costs. Since these costs are differing by their factors ϕ in the LCR and ψ in the NSFR, the maximum of those factors would be enough to cover both of the requirements.[9] The formula for calculating the effective cost of the consolidated regulatory is:

$$f_c^{eff} = \sum_{j=0}^{n} \mu_{t_j} \cdot N \cdot (f_c - r_A) \cdot \max\{\psi, \phi\}. \tag{3.29}$$

The last equation approves our hypothesis that the implementation of regulatory measures has resulted in an increase of FTP rates, which is also shown in Jorgansen's work [24]. The final transfer cost of regulatory measures is expressed in the following form:

$$TP_i^R(\psi, \phi) = \sum_{j=0}^{n} \mu_{t_j} \cdot (f_c - r_A) \cdot \max\{\psi, \phi\}. \tag{3.30}$$

[9] One of the ratios will be on its regulatory limit level, whereas the other one will be higher than the required limit.

CHAPTER 4

APPLICATION

This chapter consists of two sections. In the first section, we will illustrate a numerical application of the product from Turkish financial market. In this example, the liquidity cost and the benchmark curve derived separately and then summed to form the overall funding cost of the product. In the next section, the short-term interest rates are simulated by applying one-factor Hull-White interest-rate model.

4.1 Numerical Example

In this part, the numerical calculation of the methodology will be illustrated. However, before moving on to the calculations, let us briefly summarize the whole FTP model. The total transfer price of a product i is the summation of the *deterministic*, the *stochastic* and *regulatory* components, and the entire formula is represented as:

$$\begin{aligned} TP_i^T(\mu_{t_k}, \sigma^{i,p}, \sigma^{i,m}, T, n_2, \psi, \phi) = {} & TP_i^D(\mu_{t_k}) \\ & + TP_i^B(\sigma^{i,p}, \sigma^{i,m}, T, n_2) \\ & + TP_i^R(\psi, \phi). \end{aligned} \tag{4.1}$$

By replacing the TP_i^D, TP_i^B and TP_i^R with their components, we obtain:

$$\begin{aligned} TP_i^T = {} & \sum_{j=0}^{n} (r(0,t_j) - r_{rf}(0,t_j))\Delta t \cdot \mu_{t_j}^i \cdot (t_j - t_0) \\ & + (-l \cdot \sqrt{T \cdot n_2 \cdot \Delta t} \cdot \Phi^{-1}(1-p) \cdot \kappa \cdot (\kappa^p \cdot \sigma^{i,p} + \sigma^{i,m}) \cdot \Delta Yield) \\ & + \sum_{j=0}^{n} \mu_{t_j} \cdot (f_c - r_A) \cdot \max\{\psi, \phi\}. \end{aligned} \tag{4.2}$$

In our example, we assume that TL 36 000 General Purpose Loan (GPL) is issued for 3 years with monthly principal repayments of TL 1 000. Due to the lack of data, the variables - ($\sigma^{i,p}$) and ($\sigma^{i,m}$) including their adjustment factors which are namely (κ) and (κ^p) - are used as given. However, the readers deeply interested in the calculation methodologies and related researches for Turkish banking sector can refer to [1, 2, 23] and [44]. The average funding spread is assumed to be constant and the approximate calculation[1] of which is equal to 90 BPS per year. The 40% of the required Brownian funding capacity is assumed to be secured by reserves. The Brownian exposure is calculated at 99% confidence level, and the cost of holding reserves against Brownian exposure ($\Delta Yield$ per st. dev.) is equal to the funding spread. Since in Turkish banking sector, a product such as GPL backed-security does not exist, it is not possible to include GPL into the HQLA. Therefore, in our example, the haircut (ϕ) for the LCR is equal to 1. Assuming that the required amount for accomplishing the regulatory limits funded via retail deposits with the maturity of 18 months (deposits are rolled during the life-period of GPL), the NSFR factor (ψ) is 65%. The last variable that should also be stated is the stock proportion of the HQLA in total available HQLA at time t (θ_t), which is 80%. After defining all of the variables, we will put them into components and compute them one by one.

The deterministic component of a GPL is equal to:

$$
\begin{aligned}
(r(0,t_j) - r_{rf}(0,t_j) &= 90 \text{ (BPS)}, \\
(t_j - t_0) &= \frac{365 \text{ (Days)}}{12} \cdot j, \\
n &= 36 && \text{(number of total repayments)}, \\
\mu_{t_0} &= -36\,000 \text{ (TL)}, \\
\mu_{t_j} &= \frac{1\,000}{36\,000} = \frac{1}{36} && \forall t_j, \quad \text{where } j = 1, 2, \ldots, T, \\
\Delta t &= 1 && \text{(daily basis):}
\end{aligned}
$$

$$
TP_i^D(\frac{1}{36}) = \sum_{j=0}^{36} \frac{90BPS}{365Days} \cdot 1 \cdot \frac{1}{36} \cdot \frac{365Days}{12} \cdot j = 138.75 \text{ (BPS)}.
$$

[1] The average of the difference between cost of deposits curve and swap curve, plus average cost of required reserves.

The result of stochastic component is equal to:

$$
\begin{aligned}
l &= 0.4, \\
\sigma^{i,p} &= 0.2, \\
\sigma^{i,m} &= 0.15, \\
T &= 3 \text{ (years)}, \\
n_2 &= 36, \\
\Phi^{-1}(1-0.99) &= -2.3263, \\
\kappa &= 0.7, \\
\kappa^p &= 0.25, \\
\frac{\Delta Yield}{st.\ dev.} &= 90 \text{ (BPS)}:
\end{aligned}
$$

$$
\begin{aligned}
TP_i^B(0.2,\ 0.15,\ 5,\ 36) = &- 0.4 \cdot \sqrt{3 \cdot 365 \cdot 36 \cdot 1} \cdot (-2.3263) \cdot 0.7 \\
&\cdot (0.25 \cdot 0.2 + 0.15) \cdot \frac{90}{365} = 6.38 \text{ (BPS)}.
\end{aligned}
$$

The effect of regulatory measures is equal to:

$$
\begin{aligned}
\theta_t &= 0.8, \\
\phi &= 1, \\
\psi &= 0.65 :
\end{aligned}
$$

$$
TP_i^R(1,\ 0.65) = \sum_{j=0}^{36} \frac{1}{36} \cdot j \cdot \frac{60}{12}\, BPS \cdot 0.8 \cdot \max\{1,\ 0.65\} = 74 \text{ (BPS)}.
$$

According to the results, one can observe that the highest cost in our example incurs the deterministic part, whereas the stochastic part has the lowest part. The reason for such outcome is that our product has mainly a deterministic cash-flow and a lower volatility. Actually, GPLs are generally short-term loans and the amount of each loan is restricted according to the banks' internal policies[2]. Therefore consumers are less willingly to repay the remaining instalments before the maturity, compared to the long-term loans like mortgage. However, if we analyze products such as mortgage loans, demand deposits and saving deposits, the composition of transfer price will vary, since they have higher volatility which has a positive linear relationship with

[2] In Turkey, on average it is about 50 thousand Turkish Lira.

stochastic component of the transfer price. Another important point is that the effect of regulatory measures has a significant weight in total liquidity cost. This result strongly supports our hypothesis that regulatory measures will directly influence the total cost of funding, which will be calculated next.

The total transfer price of a 3 years GPL is equal to:

$$TP_i^T(\frac{1}{36},\ 0.2,\ 0.15,\ 5,\ 36,\ 1,\ 0.65) = 138.75 + 6.38 + 74 \approx 219\ BPS.$$

The 219 BPS is only the total cost of liquidity transfer for one unit of GPL with the maturity of 3 years (73 BPS per year). The total cost of funding actually is the sum of risk-free (swap) curve and the liquidity cost. The calculation methodology of the swap curve (SC) in this thesis is conducted by replicating the cash-flow of GPL funding as stated below [13]:

$$SC_j = \sum_{i=1}^{n} r_{rf}(0, t_{j,i}) \cdot \mu_{t_{j,i}} \cdot (t_{j,i} - t_0), \qquad \forall\, j = 1, 2, \ldots, T, \tag{4.3}$$

where

$$r_{rf}(0, t_{j,i}) = \text{swap rate with maturity } j \text{ at time } i.$$

Based on the above methodology the total funding cost of GPL is calculated and summarized in Table 4.1. According to this table, the total cost of funding for GPL is equal to 2497[3] BPS. Assuming that GPL has *10% (1000 BPS) annual yield*, $1000 \cdot 3 = 3000$ BPS for 3 years. The net profit of a bank will be $3000 - 2497 = 503$ BPS; in our example it is $36000 \cdot 0.0503 \approx 1811$ TL.

4.2 Simulation

In this section, we will simulate the short-term interest rates. Concerning future yields, the market expectations are mainly indicated by the forward yield curve [13, 15]. On the basis of these expectations, the Hull-White model is applied for the simulation of future possible outcomes of short-term interest rates. The data subject to the simulation is the 1 month Turkish Lira and USD implied rates, thus Turkish-Lira swap yield serves as an initial yield curve for the input of the model. Based on the Bloomberg data the swap yield curve of Turkish Lira is shown in Figure 4.1. The shape of the yield curve is inverted; therefore, by referring to Section 2.5 one can infer that the short-term interest rates are expected to fall.

The data are simulated in MATLAB tool[4], by employing the following methodology. At the beginning, the data for TL interest swaps are interpolated (piecewise cubic

[3] The sum of funding costs for GPL maturing in the first, second and third years with 472, 835 and 1 190 BPS, respectively.

[4] The detailed MATLAB code of the simulation is presented in Appendix C.

Table 4.1: Summary: Total Cost of Funding.

Maturity	**1 year**	**2 years**	**3 years**
Swap Rates (%)	11.96 %	11.44 %	11.17 %
	Replicating the cash-flow by applying Equation (4.3):		
SC (BPS)	399	762	1 117
		+	
TP (BPS)	73	73	73
		=	
Funding Cost (BPS)	472	835	1 190

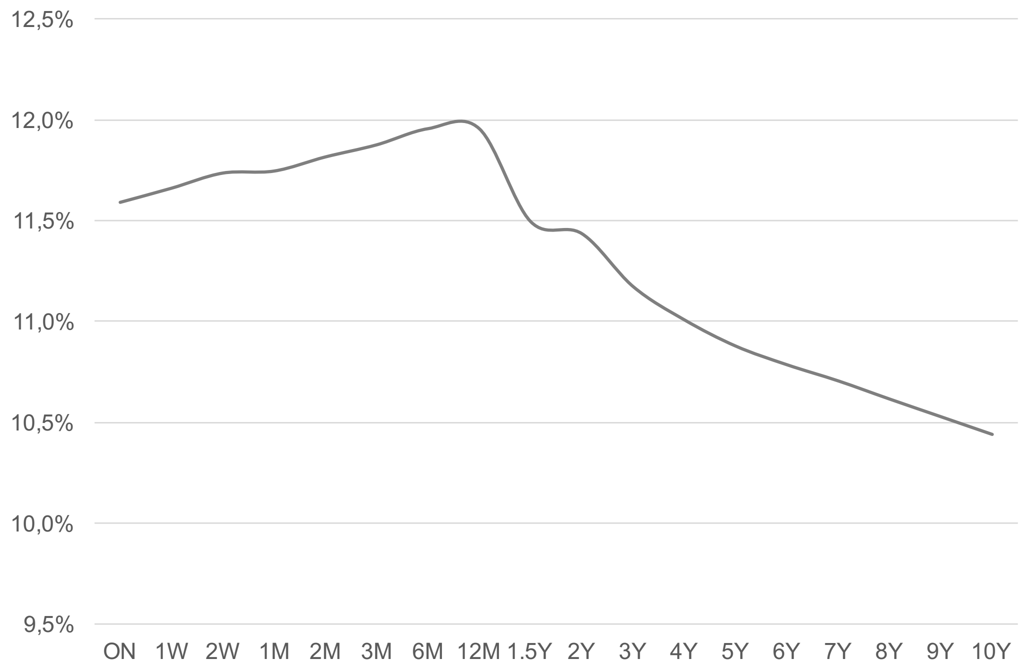

Figure 4.1: Turkish Lira-Swap Yield.

Table 4.2: Simulation Summary: Short-term interest rates.

Time Horizon	**Mean**	**CI: Lower**	**CI: Upper**
1 year	12.12 %	11.95 %	12.30 %
2 year	11.83 %	11.66 %	12.01 %
3 year	11.64 %	11.46 %	11.81 %

Hermite interpolation) with daily (Δt) increments. Then, by running the bootstrap algorithm for the interpolated data, the forward-yield curve are constructed from the time intervals equal to Δt. The parameter estimation of Hull-White model is not a trivial process[5] and out of the scope of this thesis, thus, the values β and σ are used from existing researches in the literature [8, 32].

A summary of our simulation is presented in Table 4.2. The simulation results show that the short term interest rate are declining in a two and three year horizons, which supports the market expectation[6]. The forecasted benchmark rates can be derived by applying the simulated mean into Equation (4.3). Assuming no change in liquidity, one can easily estimate the forecasted funding cost, by arithmetically summing the benchmark rate and the liquidity cost.

[5] Based on data and by data-mining tools such as MARS, GAM and Levenberg-Marquardt methods, and by optimization-supported methods like CMARS, RCMARS, CGPLM and RCPLM, one can identify stochastic and economic dynamics and identify unknown parameters in stochastic differential equations, as a future alternative to more classical SDEs.

[6] The detailed results of the simulation are represented in Appendix D.

CHAPTER 5

CONCLUSION AND OUTLOOK

In our thesis, we have modeled an advanced FTP with the implementation of the stochastic interest rates and the impact of new Basel III regulatory measures. The model incorporates two major parts, namely, liquidity cost and benchmark interest rates. The liquidity cost is divided into the deterministic, the stochastic and the regulatory components, the last of which forms the main contribution of this thesis. The objective of the thesis has been to introduce in Turkey the development and the application of an advanced FTP system, which has already been applied in more developed countries. FTP is an essential tool for internal banks to regulate the bank management. Effectively estimated FTP curves lead to a successive management of costs, risks and future decisions.

In this research, the implied and the cross-currency swap rates, which are used as benchmark rates, formed the base of the FTP curve. The modeling of the stochastic interest rates is conducted by applying one-factor Hull-White model. Goodness of fit and Gaussian characteristics is the prime reason for such kind of a choice. The simulation of the Hull-White provides us with the future possible outcomes of the short-term interest rates. Incorporating future base curve with liquidity cost will help the management of a bank to make their strategic decisions for future periods.

In the financial markets, liquidity is the major concept. The Central bank, market and funding liquidity are three different types of liquidity. In our thesis, the funding liquidity is the main facet subject to the analysis. In its simplest form, the funding liquidity is the capability of a financial institution's assets to meet its obligations. Implementing the measurements of liquidity cost plays a crucial role in FTP modeling. During the 2008 financial crisis, the evaporation of liquidity in the market was the breaking point of financial institutions' bankruptcy. Therefore, the FTP model of this research takes into account the cost of both the deterministic and the stochastic cash-flows of all products into account. The ability to fund those cash-flows is determined by the funding capacity, which is another determinant of our model. Ultimately, cash-flow and funding capacity have been the two fundamental frameworks of an advanced FTP model derived in this research.

One of the significant scientific contributions of this thesis is the implementation of the regulatory measures into the FTP model. Since the NSFR metric has not entered into force yet, there is a lack of research in literature. This thesis initiates the modeling of the NSFR effect into FTP rates and approves its hypothesis that the NSFR metric will result in FTP increase.

The application of real data from Turkish financial market leads us to the following results. First, depending on the characteristics of a product, the cost of liquidity differs. If it is stable - can be determined beforehand - cash flow, the deterministic part consists the major part of total liquidity cost; otherwise, the stochastic cost of liquidity may prevail. The regulatory effect has a significant impact on FTP rates. In the application of the model, it has been revealed that the FTP rate would be 74 BPS lower, providing that the regulatory measures were not taken into consideration.

The application of an advanced model has also its limitations. First, it is a cumbersome task to predict short-term interest rates because it has numerous variables that should be predicted. Macro indicators such as GDP, money supply, current accounts etc., are among indicators that have a significant influence on the evolution of interest rates. Prediction of all those variables could certainly have a positive impact on the estimation of interest rates, but it is difficult to be realistic.

Insufficiency and high cost of data-generating tools is another limitation of the model. An advanced model requires long historical data to predict actual risk factors. However, due to the lack of data, a derivation of factors is not possible. Moreover, calculation of those factors can only be executed by powerful generating tools which have the capacity to operate with a huge amount of data. Those tools demand high qualified users or further education, which in essence, means an extra cost of development for the institution.

Finally, the effect of regulatory measures are based on simple assumptions that need further investigations. After the NSFR entrance into force, new products in the market may emerge, as a result of which, a different impact on the FTP rates may occur.

Despite the limitations, this thesis will hopefully be a guide to further researches on the modeling the effect of regulatory measures on FTP in the literature and to applications in the financial industry.

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APPENDIX A

Proof of Vasicek's Explicit Solution

Recall that the explicit solution is:

$$r(t) = e^{-\beta t}r(0) + \frac{\alpha}{\beta}(1 - e^{-\beta t}) + \sigma e^{-\beta t}\int_0^t e^{\beta s}dW(s). \tag{A.1}$$

In order to proof a claim, that the equation above is an explicit solution of Vasicek's model, we will compute the differential of the right-hand side of (A.1). Here, we use the Itô-Doeblin formula:

$$f(t,x) = e^{-\beta t}r(0) + \frac{\alpha}{\beta}(1 - e^{-\beta t}) + \sigma e^{-\beta t}x,$$

where

$$X(t) = \int_0^t e^{\beta s}dW(s).$$

Derivatives and partial derivatives of the functions $f(t,x)$ and $X(t)$ are:

$$\begin{aligned}
f_t(t,x) &= -\beta e^{-\beta t}r(0) + \alpha e^{-\beta t} - \beta\sigma e^{-\beta t}x \\
&= \alpha - \beta(e^{-\beta t}r(0) + \frac{\alpha}{\beta}(1 - e^{-\beta t}) + \sigma e^{-\beta t}x) \\
&= \alpha - \beta f(t,x), \\
f_x(t,x) &= \sigma e^{-\beta t}, \\
f_{xx}(t,x) &= 0, \\
dX(t) &= e^{\beta t}dW(t), \\
dX(t)dX(t) &= e^{2\beta t}dt.
\end{aligned}$$

The Itô-Doeblin states that

$$\begin{aligned}
&df(t,X(t)) \\
&= f_t(t,X(t))dt + f_x(t,X(t))dX(t) + \frac{1}{2}f_{xx}(t,X(t))dX(t)dX(t) \\
&= (\alpha - \beta f(t,X(t)))dt + \sigma dW(t).
\end{aligned}$$

The last equality is the Vasicek's model, so that we completed the proof.

APPENDIX B

Derivation of Aggregate-Risk Exposure

The Brownian cash flow component of a product i is the sum of product-specific and systemic factors:

$$\sigma^i \Delta W^i_{t_k} = \sigma^{i,p} \Delta W^{i,p}_{t_k} + \sigma^{i,m} \Delta W^m_{t_k}. \tag{B.1}$$

Aggregated exposure is calculated by taking the summation of both sides of the above equation:

$$\sum_{i=1}^{d} \sigma^i \Delta W^i_{t_k} = \sum_{i=1}^{d} \sigma^{i,p} \Delta W^{i,p}_{t_k} + \sum_{i=1}^{d} \sigma^{i,m} \Delta W^m_{t_k}. \tag{B.2}$$

By applying the variance of the aggregate exposure in (B.2) we get:

$$\begin{aligned}
Var(\sigma^A \sum_{i=1}^{d} \Delta W^i_{t_k}) &= Var(\sum_{i=1}^{d} \sigma^{i,p} \Delta W^{i,p}_{t_k} + \sum_{i=1}^{d} \sigma^{i,m} \Delta W^m_{t_k}), \\
(\sigma^A)^2 Var(\sum_{i=1}^{d} \Delta W^i_{t_k}) &= Var(\sum_{i=1}^{d} \sigma^{i,p} \Delta W^{i,p}_{t_k} + \Delta W^m_{t_k} \sum_{i=1}^{d} \sigma^{i,m}), \\
(\sigma^A)^2 \Delta(t) &= Var(\sum_{i=1}^{d} \sigma^{i,p} \Delta W^{i,p}_{t_k}) + Var(\Delta W^m_{t_k} \sum_{i=1}^{d} \sigma^{i,m}) \\
&\quad +2Cov(\sum_{i=1}^{d} \sigma^{i,p} \Delta W^{i,p}_{t_k}; \Delta W^m_{t_k} \sum_{i=1}^{d} \sigma^{i,m}).
\end{aligned}$$

Recall from the properties of the Brownian motion, the variance of the increments ΔW is equal to the difference in time Δt. From the assumption that the systematic factor is independent from product-specific factor, the covariance of the factors in the last expression is equal to 0. In order to simplify the expressions, let us solve the first two expressions on the right-hand side, one by one:

$$
\begin{aligned}
1^{\text{st}} \text{ term} \implies Var(\sum_{i=1}^{d} \sigma^{i,p} \Delta W_{t_k}^{i,p}) &= \sum_{i=1}^{d} Var(\sigma^{i,p} \Delta W_{t_k}^{i,p}) \\
&\quad +2 \sum_{1 \leqslant i < j \leqslant d}^{d} Cov(\sigma^{i,p} W_{t_k}^{i,p}; \sigma^{j,p} W_{t_k}^{j,p}) \\
&= \sum_{i=1}^{d} \left(\sigma^{i,p}\right)^2 \cdot \Delta t,
\end{aligned}
$$

$$
\begin{aligned}
2^{\text{nd}} \text{ term} \implies Var(\Delta W_{t_k}^{m} \sum_{i=1}^{d} \sigma^{i,m}) &= E(\Delta_{t_k}^{m}) \cdot E(\sum_{i=1}^{d} \sigma^{i,m})^2 \\
&\quad -[E(\Delta W_{t_k})]^2 \cdot [E(\sum_{i=1}^{d} \sigma^{i,m})]^2 \\
&= \Delta t \cdot \Big[\sum_{i=1}^{d} (\sigma^{i,m})\Big]^2.
\end{aligned}
$$

By combining the above the expressions, we get the final aggregate exposure as follows:

$$
\left(\sigma^{A}\right)^2 \cdot \Delta t = \sum_{i=1}^{d} (\sigma^{i,p})^2 \cdot \Delta t + \Big[\sum_{i=1}^{d} (\sigma^{i,m})\Big]^2 \cdot \Delta t,
$$

hence,

$$
\sigma^{A} = \sqrt{\sum_{i=1}^{d} (\sigma^{i,p})^2 + \Big[\sum_{i=1}^{d} (\sigma^{i,m})\Big]^2}. \tag{B.3}
$$

The results implies that the aggregate volatility (σ^{A^2}) is the linear sum of the product-specific and the market volatility under the assumption that the market and products, and products within themselves are independent.

APPENDIX C

Matlab Code

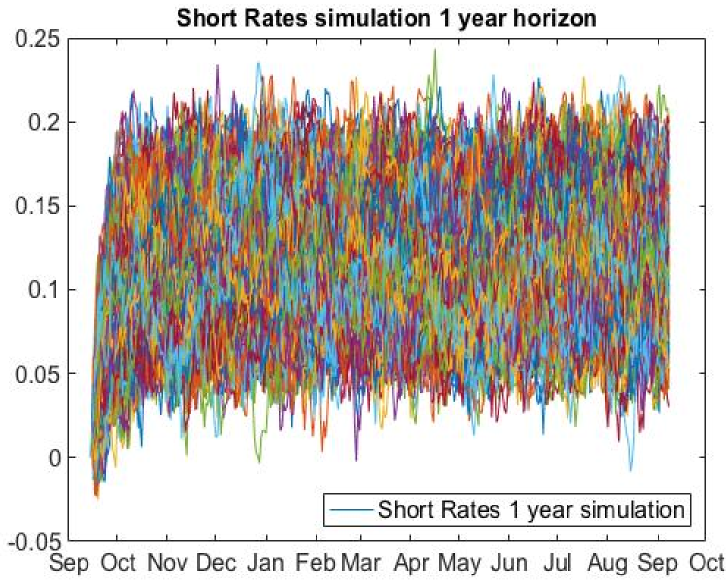
```
clear all, close all, clc

   section 1 getting data swap rates observed on day 15.08.2017

Settle1 = datenum( 15 Aug 2017 );  Day of observation of zerorates
CurveTimes1 = [1/360 7/360 14/360 30/360 61/360 92/360
    0.5 1 1.5 2 3 4 5 6 7 8 9 10];  year amounts in term structure
CurveDates1 = daysadd(Settle1,360 CurveTimes1,1);
ZeroRates1 = xlsread( begzoddata.xlsx ,  G3:G20 ) ;

irdcconst1 = IRDataCurve( Forward ,Settle1,CurveDates1,ZeroRates1,
     InterpMethod , constant );  interpolation of zero curve
irdcpchip1 = IRDataCurve( Forward ,Settle1,CurveDates1,ZeroRates1,
     InterpMethod , pchip );   using const and pchip algorithms

   plotting swap/zero and forward curves for 15.08.2017
figure(1)
PlottingDates1 = daysadd(Settle1,30:1:36010,2);
plot(PlottingDates1, getForwardRates(irdcconst1, PlottingDates1) , b )
hold on
plot(PlottingDates1, getForwardRates(irdcpchip1, PlottingDates1) , r )
plot(PlottingDates1, getZeroRates(irdcconst1, PlottingDates1) , g )
plot(PlottingDates1, getZeroRates(irdcpchip1, PlottingDates1) , yellow )
legend(  Constant Forward Rates , PCHIP Forward Rates ,
         Constant Zero Rates ,...
         PCHIP Zero Rates  , location , SouthEast )
title( Interpolation methods for IRDataCurve objects 15.08.2017)
datetick

   Simulation using algorithm from the thesis
zero1 = getZeroRates(irdcconst1,PlottingDates1);
forwards1 = getForwardRates(irdcpchip1, PlottingDates1);
 forwards2 = getForwardRates(irdcpchip2, PlottingDates2);

alpha = 0.10;    assumed, taken from term project
sigma = 0.0121;   assumed, taken from term project

  to replicate the results use the seed for randn
seed = 13;
randn( state ,seed);

T = 9;  1 ,  3,  5,  9 years horizons are
dt = 1/360;   diffusion term increments
```

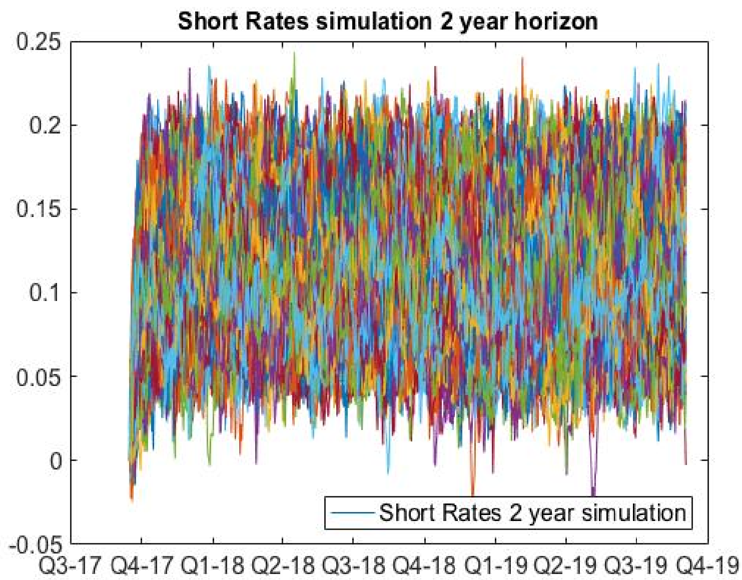
```
daysSim = T/dt;  number of simulated days =
SimPaths = 1000;  simulation trajectories   paths
t = [1:daysSim ] dt;   time vector in day increments for theta
dF = forwards1(2:daysSim + 1)   forwards1(1:daysSim);

Theta = dF(1:daysSim) + alpha forwards1(1:daysSim) + (sigma 2 / (2 alpha)
        (1   exp ( 2 alpha t))) ;
r = zeros(SimPaths,daysSim);
rmean = zeros(daysSim, 1);

 dW = sqrt(dt) randn(SimPaths, daysSim);   changes in Wiener process
dW = randn(SimPaths, daysSim);
r(:,1) = zeros(SimPaths,1);
dW(:,1) = zeros(SimPaths,1);
rmean(1) = 0;

for i = 2:daysSim
     dr(:,i) = (Theta(i)   alpha r(:,i)) dt + sigma dW(:,i);
     r(:,i) = r(:,i 1) + dr(:,i);   Adding the change to r
    r(:,i) = Theta(i) + (1   alpha) r(:,i 1) + sigma dW(:,i 1);
    rmean(i) = mean(r(:,i));
end

   horizon 1 year
oneyear = dt 1;
r1y = r(1:1000, 1:oneyear);
figure(3)  short rates
 PlottingDates1y = daysadd(Settle1,180:1:3601,2);
plot(PlottingDates1(1:oneyear), r1y )
legend( Short Rates 1 year simulation ,  location , SouthEast )
title( Short Rates simulation 1 year horizon )
datetick

   horizon 2 year
twoyear = dt 1 2;
r2y = r(1:1000, 1:twoyear);
figure(4)  short rates
 PlottingDates3y = daysadd(Settle1,180:1:3603,2);
plot(PlottingDates1(1:twoyear), r2y )
legend( Short Rates 2 year simulation ,  location , SouthEast )
title( Short Rates simulation 2 year horizon )
datetick


   horizon 3 year
threeyear = dt 1 3;
r3y = r(1:1000, 1:threeyear);
figure(5)  short rates
 PlottingDates3y = daysadd(Settle1,180:1:3603,2);
plot(PlottingDates1(1:threeyear), r3y )
legend( Short Rates 3 year simulation ,  location , SouthEast )
title( Short Rates simulation 3 year horizon )
datetick

   figures for normality check
figure(6)  Theta
plot(PlottingDates1(1:daysSim), Theta,  m )
legend( Time Varying Theta ,  location , SouthEast )
```

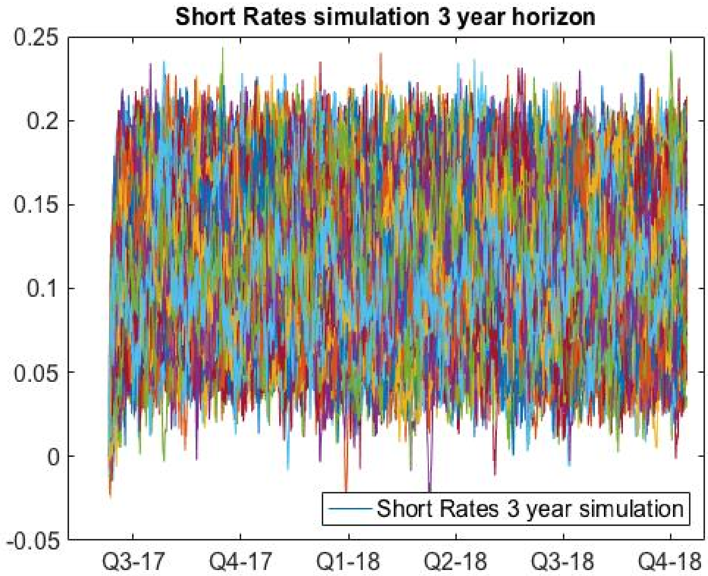
```
title ( Theta )
datetick

figure(7)  histogram short rates
hist(r(:,oneyear), 30)
title ( Histogram 1 year horizon short rate )

figure(8)
hist(r(:,twoyear), 40)
title ( Histogram 2 year horizon short rate )

figure(9)
hist(r(:,threeyear), 40)
title ( Histogram 3 year horizon short rate )


   Statistics of the short rates 1, 2, 3 years
  for simplicity use (approxsigma = variance of the sample)
[muhat1,sigmahat1,muci1,sigmaci1] = normfit(r(:,oneyear));
[muhat2,sigmahat2,muci2,sigmaci2] = normfit(r(:,twoyear));
[muhat3,sigmahat3,muci3,sigmaci3] = normfit(r(:,threeyear));

max1 = max(r1y); max1 = max(max1);
max2 = max(r2y); max2 = max(max2);
max3 = max(r3y); max3 = max(max3);

min1 = min(r1y); min1 = min(min1);
min2 = min(r2y); min2 = min(min2);
min3 = min(r3y); min3 = min(min3);

   Display the Results

disp ( Results from Simulation 1000 trajectories )
fprintf ( 1 year horizon Means of the r   short rate 3.4f. n , muhat1)
fprintf ( 2 years horizon Means of the r   short rate 3.4f. n , muhat2)
fprintf ( 3 years horizon Means of the r   short rate 3.4f. n , muhat3)
```

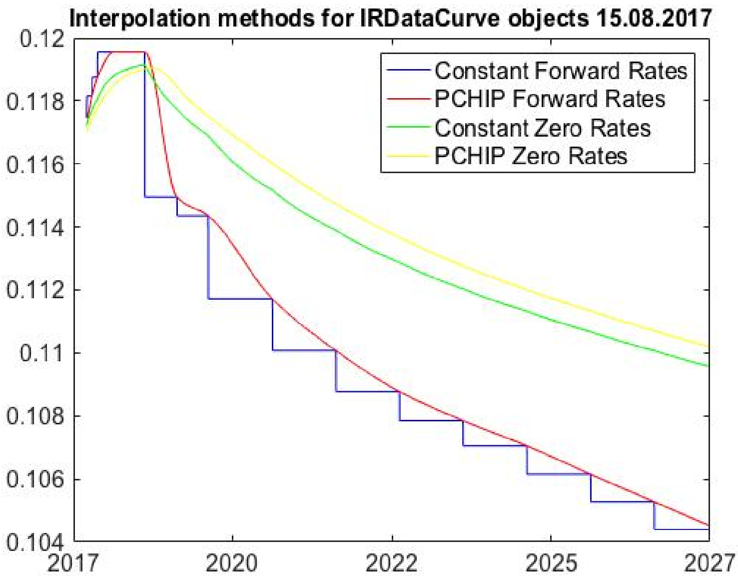
APPENDIX D

Simulation Results






Short Rates simulation 3 year horizon
0.25
0.2
0.15
0.1
0.05
0
-0.05
Short Rates 3 year simulation
Q3-17
Q4-17
Q1-18
Q2-18
Q3-18
Q4-18


Interpolation methods for IRDataCurve objects 15.08.2017
0.12
0.118
0.116
0.114
0.112
0.11
0.108
0.106
0.104
Constant Forward Rates
PCHIP Forward Rates
Constant Zero Rates
PCHIP Zero Rates
2017
2020
2022
2025
2027