INVESTIGATING PRESERVICE MIDDLE SCHOOL MATHEMATICS TEACHERS' NOTICING OF STUDENTS' MATHEMATICAL THINKING IN THE CONTEXT OF LESSON STUDY

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I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

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ABSTRACT

INVESTIGATING PRESERVICE MIDDLE SCHOOL MATHEMATICS TEACHERS' NOTICING OF STUDENTS' MATHEMATICAL THINKING IN THE CONTEXT OF LESSON STUDY

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The purpose of the current study was to examine preservice middle school mathematics teachers' noticing of students' mathematical thinking in the context of lesson study professional development model and determine whether any changes took place in their noticing skills. Qualitative research methodology, in particular, case study was used in order to elaborate what and how preservice teachers notice during lesson study phases which are planning, teaching, reflecting, re-teaching and re-reflecting. With this purpose, lesson study process which included four lesson cycles regarding the subjects of perimeter, area, surface area and prism was conducted in a period of two months. The research took place during the 2014-2015 spring semester. Noticing skills of two senior preservice middle school mathematics teachers were addressed in this study. In data collection process, each phase of the lesson study was recorded with a video camera and the transcripts of video recordings, interviews, observations, lesson plans and field notes were used as data collection tools.

An existing theoretical framework for learning to notice students' mathematical thinking identified by van Es was applied for analyzing the data. Data analysis revealed that there was a gradual development in both preservice middle school mathematics teachers' noticing of students' mathematical thinking throughout the lesson study process. Furthermore, it was thought that five features of the lesson study which were collaboration, investigation on mathematical subject, observation and reflecting, focusing on specific mathematical domain and the cooperating teacher had an impact on the shift in preservice teachers' noticing of students' mathematical thinking.

Keywords: Noticing, Teacher noticing, Lesson study, Students' mathematical thinking, Preservice middle school mathematics teachers.

ÖZ

ORTAOKUL MATEMATİK ÖĞRETMENİ ADAYLARININ FARK ETME BECERİLERİNİN DERS İMECESİ KAPSAMINDA İNCELENMESİ

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Bu çalışmanın amacı ortaokul matematik öğretmeni adaylarının öğrencilerin matematiksel düşünmelerini fark etme becerilerini ders imecesi mesleki gelişim modeli kapsamında incelemek ve bu becerilerde değişimin meydana gelip gelmediğini belirlemektir. Öğretmen adaylarının ders imecesinin planlama, öğretim, yansıtma, tekrar öğretim ve tekrar yansıtma aşamalarında ne fark ettiklerini ve nasıl fark ettiklerini detaylandırmak amacıyla nitel araştırma yöntemlerinden durum çalışması kullanılmıştır. Bu amaçla, çevre, alan, yüzey alanı ve prizma konularına yönelik dört döngüden oluşan ve iki ay süren ders imecesi uygulaması yürütülmüştür. Araştırma 2014-2015 bahar döneminde gerçekleşmiştir. Bu çalışmada iki son sınıf ortaokul matematik öğretmeni adayının fark etme becerileri ele alınmıştır. Veri toplama sürecinde, ders imecesinin her bir aşaması video ile kayıt altına alınmış ve video kayıtların transkriptleri, görüşmeler, gözlemler, ders planları ve alan notları veri toplama aracı olarak kullanılmıştır.

Öğretmen adaylarının öğrencilerin matematiksel düşünme şekillerini fark etme becerilerinin incelenmesinde van Es tarafından tanımlanmış mevcut bir teorik çerçeveden faydalanılmıştır. Veri analizi, her iki ortaokul matematik öğretmen adayının fark etme becerilerinin ders imecesi sürecinde kademeli olarak geliştiğini ortaya koymaktadır. Bunun yanı sıra, öğretmen adaylarının fark etme becerilerindeki değişimde ders imecesi mesleki gelişim modelinin işbirliği yapma, matematiksel konuyu araştırma, gözlemleme ve yansıtma, spesifik matematiksel alana odaklanma ve uygulama öğretmenini dahil etme özelliklerinin etkili olduğu düşünülmektedir.

Anahtar Kelimeler: Fark etme, Öğretmenin fark etmesi, Ders imecesi, Öğrencilerin matematiksel düşünmeleri, Ortaokul matematik öğretmeni adayları.

To My Parents,

Gülsen & Osman Güner

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LS	Lesson Study
PD	Professional Development
MoNE	Ministry of National Education
NCTM	National Council of Teachers of Mathematics

CHAPTER 1

1. INTRODUCTION

In recent years, the concept of noticing has gained importance in terms of effective teaching and learning and the development of this skill is emphasized insistently. It is one of the critical skills that teachers need to have because teaching is a complex activity and it is not likely that teachers see and react to every moment in classroom. Thus, noticing is accepted one of the essential components of teaching expertise (Mason, 2002). According to Liu (2014), “Noticing is a part of everyday life, indicating the act of observing or recognizing something” (p. 1). However, some researchers consider noticing as a concept that teachers focused on (Sherin, Russ & Colestock, 2011; Star, Lynch & Perova, 2011); other researchers address it as the process of paying attention to instructional issues and making sense of these events (Goldsmith & Seago, 2011; Jacobs, Lamb, Philipp & Schappelle, 2011; Kazemi, Elliot, Mumme, Carroll, Lesseig & Kelly-Petersen, 2011; van Es, 2011). Besides, some others add another component in a way that includes making decisions on the improvement of instruction (Jacobs, Lamb & Philipp, 2010; van Es, 2011). Although there have been different views on noticing, researchers defend that this skill is important in terms of development of instruction and student achievement.

Noticing is an activity that includes structuring what is seen (Gibson, 1979). Hence, in spite of focusing on same situations, the ways of sense-making and the points that are noticed can be different from each other (Jacobs, Lamb & Philipp, 2010). Furthermore, the identification of what is noticed by teachers cannot be the indicator of meaningful noticing alone, how it is noticed is important as well (Berliner, 2001; Davis, 2006; Mason, 2002; Star & Strickland, 2008; van Es, 2011; van Es & Sherin, 2002). Noticing includes two main aspects of which are “(a) identifying what is important in a teaching situation and (b) drawing on one's knowledge of teaching and learning to reason about the situation” (van Es & Sherin, 2006, p. 125). This study is based on noticing definition of van Es and Sherin (2006) rather than the other definitions.

Teacher noticing which has become the focus of professional development research and teacher training programs recently (Sherin et al., 2011) is seen as a necessary element of instruction in order to be successful in mathematics education (Choy, 2013). The reason behind significant attention to teacher noticing is that it provides the development of teachers' necessary skills to struggle with the complexity of teaching in accordance with reform efforts (Baş, 2013). Teacher noticing is what teachers see and how they interpret events and details that come up in classroom interactions (Jacobs, Lamb & Philipp, 2010; Mason, 2002) and it is important in terms of the development of instruction (Schoenfeld, 2011). Teacher noticing does not only include attending to noteworthy events in classroom interactions but also making sense of and reasoning about what is attended to by using existing and relevant knowledge (van Es & Sherin, 2002). Noticing has a critical role in providing teachers with learning from their own teaching for it enables them to analyze their practices (Mason, 2002; Sherin et al., 2011). However, it can be difficult on behalf of teachers to pay attention to significant aspects of a lesson, see the relevant points and offer appropriate responses (Star et al., 2011; Star & Strickland, 2007; Vondrová & Žalská, 2013). These important events might be students' unexpected responses (Yang & Ricks, 2012), details about students' understanding (Goodell, 2006) and interactions which are different from what was planned before (Fernandez, Cannon & Chokshi, 2003). It is important to consider these critical events while planning and teaching lessons and also reflecting on them in order to maintain development of instructional practices (Fernandez et al., 2003; Goodell, 2006; Yang & Ricks, 2012) as it yields understanding on how students learn mathematics and how mathematics teaching occurs (Scherer & Steinbring, 2007). Thus, the ability of attending to noteworthy events, linking them with instructional principles and interpreting them in terms of students' thinking is the feature of noticing expertise (van Es, 2011; Yang & Ricks, 2012). In this regard, the examination of teachers' noticing skills is critical while teachers plan, teach and evaluate the lesson since it is expected that good teachers must have the ability to notice (Van Es & Sherin, 2010).

Teachers' noticing is emphasized in teacher education (Sherin & Han, 2004; Sherin & van Es, 2009; Star & Strickland, 2008; van Es & Sherin, 2002, 2008) because it is important for deeper and meaningful students' learning (Hand 2012; Sherin et al.2011; Star and Strickland 2008; van Es 2011) and can develop in time (Hand, 2012; Jacobs et al.2010; Star and Strickland 2008; van Es2011; van Es & Sherin, 2002, 2008). At this point, when novice and expert teachers' noticing skills are compared, it is seen that novice teachers generally do not focus on important events in classroom interactions, instead superficial moments draw their attention (Carter, Cushing, Sabers, Stein & Berliner, 1988; Star & Strickland, 2008; Star, Lynch & Perova, 2011). While expert teachers consider students' understanding and learning details for organizing their instructional practices in a flexible way, novice teachers adhere to their lesson plans rigidly (Berliner, 2001). Likewise, the properties of preservice teachers' noticing skills as novice show that they have tendency to focus on teachers' behaviors and students' actions rather than students' thinking (Levin, Hammer & Coffey, 2009) and have judgmental stance without evidence to support their claims (Davis, 2006). Accordingly, there is need for developing preservice teachers' noticing skills and it should be at the center of teacher preparation programs (Hiebert, Morris, Berk & Jansen, 2007).

Preservice teachers encounter complexities that obstruct learning to teach (Huling-Austin, 1992) and they learn by doing at the beginning of work (Flores, 2006). Hence, training preservice teachers well before becoming teachers is important to assure that they are less influenced by the challenges in the profession, they feel ready for teaching and increase learning to teach (Osmanoğlu, 2010). On the other hand, van Es and Sherin (2002) remark the gap in terms of noticing in teacher education and indicate it as,

> …current programs of teacher education often do not focus on helping teachers learn to interpret classroom interactions. Instead, they focus on helping teachers learn to act, often providing them with instruction concerning new pedagogical techniques and new activities that they can use (p. 572).

Preservice teachers have some opportunities to practice what they have learned during teacher training programs, observe teachers in their classrooms and come close to real instruction setting through the courses such as teaching practice and school experience but their effects are limited and the opportunities given are not enough for their well preparedness (Clift & Brady, 2005). Therefore, teacher education programs should train preservice teachers in a way that they can overcome the challenges by making effective instructional decisions, can acquire what is needed in real teaching and learning practice in accordance with reform efforts and be able to develop their instruction (Osmanoğlu, 2010). At this point, van Es and Sherin (2008) suggest that preservice teachers' ability to attend to significant aspects of classroom events and make sense of them, namely, their noticing skills should be developed with appropriate support in order to preserve improvement in both instruction and student learning. Star and Strickland (2008) also emphasize that preservice teachers' ability to observe is important in terms of making sense of the teaching and learning practice, thus, it should be enhanced. Moreover, researchers claim that development of noticing skills provides development of observation skills so that field observations in the courses of teaching practice and school experience might become more effective and comprehensive. Teacher education programs should give opportunities for teachers to attend to, make sense of and use the sense-making for taking decisions on instruction (van Es & Sherin, 2008). On the other hand, they must be allowed to analyze, discuss on, reason about and reflect on the lessons in real classroom settings in order to improve their ability to make effective decisions in accordance with what they have learned about the relationship between theory and practice and complexities in classrooms (Shulman, 1992). Consequently, it shows the importance of support that ensures this kind of environment.

Most of the studies related to teacher noticing argue that there is need for research on what preservice teachers also notice and how they interpret the events that they have observed in teaching and learning interactions (Hand 2012; Sherin et al., 2011; Star & Strickland 2008; van Es 2011). One of the fundamental skills that preservice teachers have to gain is noticing (Mason 2002; Sherin et al., 2011). However, novice and preservice teachers have more difficulty in focusing on students compared to experienced teachers and their attention reflects rather unrelated events (Carter et al., 1988). When the importance of observation in terms of teacher education programs is considered, not being able to notice significant events influences their professional development in a negative manner because the aim of teacher training is to equip teachers with learning from their own observation (Star, et al., 2011).

On the other hand, mathematics education reform movements highlight that teachers should listen to students' ideas carefully and use them to design the flow of instruction (Ball & Cohen, 1999; National Council of Teachers of Mathematics [NCTM], 2000). It requires attending to their thinking and making sense of ideas that arise during teaching (Rodgers, 2002). Teachers' noticing of student thinking is what lies under effective teaching and high student achievement (Carpenter, Fennema, Peterson, Chiang & Loef, 1989; van Es, 2011). Hence, further learning about teachers' noticing of students' thinking is important for increasing the effectiveness of mathematics teaching and learning. On behalf of teachers' noticing, a range of issues can draw their attention during classroom situations, however, especially the events related to students' mathematical thinking are important in mathematics education (van Es & Sherin, 2010). NCTM (1991) indicates that teachers should pay attention to students' ideas, mathematical approach and thinking in order to increase their mathematical power. Reform movements also emphasize the importance of focusing on the details of students' strategies and ideas that emerge during classroom interactions and their confusions, mistakes and misconceptions related to the subject (Schleppenbach, Flevares, Sims & Perry, 2007). Considering how students think mathematically in instruction is important because it enhances students' mathematical success (Carpenter, Fennema & Franke, 1996). Teachers should listen to what students say (Rodgers, 2002), observe what they do and endeavor to understand how they think (Ball, Lubienski & Mewborn, 2001). Teachers recognize their own students better by attending to how students think and use the knowledge to improve their teaching so that both teachers' and students' learning can take place (Borko, Jacobs, Eiteljrg & Pittman, 2008). Turkish educational system also requires teachers to constitute a classroom environment that reveals students' thinking of querying them appropriately and allows them to discuss for learning about their strategies. They should evaluate the events and tasks throughout the lesson and use this knowledge to improve instruction (Ministry of National Education [MoNE], 2013). Namely, teachers are expected to focus on important events in classroom interactions, understand their meanings and take instructional decisions based on the interpretations of what is noticed as simultaneous events rise in a classroom (Erdik, 2014). Most of the studies also emphasize that teachers need to consider students' possible thinking ways while planning and teaching the lesson (Carpenter et al., 1996; Carpenter, Fennema, Franke, Levi & Empson, 1999; Franke, Carpenter, Levi & Fennema, 2001). However, several studies argue that teachers do not generally endeavor to focus on and make sense of students' mathematical thinking (Kazemi & Franke, 2004). Research shows that teachers can learn from their own students' thinking with the help of professional development efforts (Burns, 2005; Chamberlin, 2005; Fernandez et al., 2003; Franke et al., 2001; Jacobs, Franke, Carpenter, Linda, & Battey, 2007). Thus, teachers need professional development support for developing their noticing of students' mathematical thinking (Jacobs, Lamb & Philipp, 2010).

Studies reveal that teachers can learn and develop themselves in terms of attending to and analyzing the ways of students' thinking through professional development models (Burns, 2005; Chamberlin, 2005; Fernandez et al., 2003; Franke et al., 2001; Jacobs et al., 2007). In order to provide better teaching and student understanding, teachers need to pay attention to students' responses, ideas and reasoning, make sense of how they think and consider these points while planning, teaching and reflecting on the lesson (Goldsmith & Seago, 2013; Jacobs et al., 2010; Schifter, 2001). It is believed that learning more about teachers' noticing of students' thinking allows to improve mathematics instruction (Liu, 2014). Thereupon, noticing of students' mathematical thinking is important (Hiebert, Morris & Glass 2003). Sherin and van Es (2002) advocate that preservice teachers' noticing skills should be given importance in the context of teacher training programs and they should be administered with the opportunities to develop this skill. For this development to take place the environment, which enables us to see the background of teaching, recognize the deficient and failing aspects in the process and develop awareness based on collaboration, is critical (Yang & Ricks, 2012).

It is seen in literature that there are several promising methods to improve teachers' ability to notice students' mathematical thinking. These are (a) using research-based knowledge (Carpenter et al., 1989; Fennema et al., 1996), (b) examining students' written work (Crespo, 2000; Kazemi & Franke, 2004), (c) analyzing videotaped lessons (Masingila & Doerr, 2002; Sherin & Han, 2004), (d) discussions on narrative cases of mathematics instruction (Stein, Hughes, Engle & Smith, 2003). However, although these methods help teachers to gain insight about students' strategies, they are not convenient for reflecting student thinking and developing the implementation (Choy, 2015). Besides, improvement of teachers in terms of attending to and analyzing students' thinking with these kind of practices is not enough (Chamberlin, 2005; Hiebert et al., 2007). In addition, there are some concerns about the use of video-taped lessons or creating video clubs such as differences between teachers' reactions to videos and real teaching, limited noticing depending on out of real context, teachers' tendency to evaluate and effect of researcher perspective in videos (Sherin, Russ & Colestock, 2011). On the other hand, another promising approach in developing teachers' noticing of students' mathematical thinking is the use of professional development models (Schifter, 1998; Stein, Hughe & Smith, 2006).

Effective professional development requires to focus on students' thinking about relevant content (Kazemi & Hubbard, 2008) and Sowder (2007) indicates that professional development based on attending to students' thinking provides opportunity for teachers to design effective instruction and increase student achievement. Noticing includes not only teachers' focus on classroom interactions but also teachers' reasoning, reflections and decisions considering what they attended to (McDuffie, Foote et. al., 2014). One of the approaches that gains importance in teacher education and professional development is the implementation of lesson study that helps teachers to learn focusing on, planning, observing, analyzing, making sense of, discussing about and reflecting on teaching practice in a real classroom setting. Thus, there is a continuous focus and sense making process related to students' thinking and learning in lesson study (Hurd & Licciardo-Musso, 2005). According to Mason (2002) noticing takes place with the combination of one's own experiences, the others' experiences, observations and theories and the power of this combination enables professional development. The collaborative structure of lesson study based on sharing ideas and experiences about the goals of the subject, students' misconceptions and mathematical thinking, and building instruction by considering them assures teachers' professional development (Chokshi & Fernandez, 2004) and might aid to increase their ability to notice students' mathematical thinking.

Lesson study is a model which is based on collaboration among teachers and aims to obtain professional development of teachers, in-service and preservice teachers. It consists of a cycle which includes three main phases: planning, teaching and reflecting. In planning, teachers previously endeavor to predict how students might think mathematically, prepare a lesson plan from the perspective of students and evaluate the content of the lesson that they prepared within collaborative work. Sharing ideas considering these points helps teachers to notice students' possible ways of mathematical thinking and interpret them as well as see each other's mathematical thinking (Lewis, Friedkin, Baker & Perry, 2011). In teaching, whereas one of the teachers from lesson study group teaches the lesson, the others observe the teaching process focusing on many aspects of the lesson (Murata, 2011). Paying attention to students' mathematical thinking in this process is critical in terms of determining how lesson plan facilitates students' learning or makes it more difficult (Yoshida, 2005). In reflecting phase, all teachers share the strong and weak aspects of the lesson, indicate students' thinking and misconceptions that they encountered and make suggestions on how to improve the lesson (Lewis et al., 2011). As in this model, attending to students' strategies, misconceptions, ideas and mathematical understanding; interpreting these important points and considering them while teaching; and making instructional decisions together and enriching the lesson based on this knowledge provides professional development for teachers, improves students' learning and increases the quality of instruction (Borko et al., 2008). Here, the actions of attending to, interpreting and making decisions correspond to the noticing skill (Mason, 2002; Sherin, Jacobs & Philipp, 2011). Noticing is not just a professional skill that only teachers but also preservice teachers need to maintain (Mason 2002; Sherin, Jacobs & Philipp, 2011) because teachers and preservice teachers who do not have enough experience have difficulty in focusing on students, their thinking and important instructional events, making sense of what they noticed and shaping the lesson (1988; Carter et al., 1988). Therefore, most of the researches emphasize the need for investigating preservice teachers' noticing skills (Hand 2012; Sherin, Jacobs & Philipp, 2011; Star & Strickland 2008; van Es 2011). On the other hand, it is suggested that preservice teachers should be provided with opportunities for developing this skill and they should be supported with professional development in order to improve their ability to notice (Jacobs et al., 2010) as the ability of noticing is difficult to have directly (Ball & Cohen, 1999; Sowder, 2007) and professional effort may help to develop it (Goldsmith & Seago, 2011; Kazemi & Franke, 2004; Sherin & Han, 2004; Sherin & van Es, 2009; van Es, 2011; van Es & Sherin, 2008). At this point, lesson study is a model that promotes teachers for attending to students' thinking, hands over opportunities for them to understand students’ strategies and enables them to design the instruction in accordance with this information, that is to say, it may develop their noticing of students’ thinking. Thus, it was decided to issue a professional development environment for preservice teachers in which they might develop their noticing abilities with the opportunities of planning, teaching, analyzing, discussing, reasoning and reflecting. In this study, that was conducted with preservice teachers, lesson study offered a context for analyzing their noticing of students’ mathematical thinking in accordance with the framework and designing the implementation procedure.

1.1 The Purpose of the Study and Research Questions

The purpose of this study was to examine preservice middle school mathematics teachers’ noticing of students’ mathematical thinking in the context of lesson study and determine whether it was effective in the development of this skill. In other words, the aim of this study was to reveal what preservice teachers focused on and how preservice teachers made sense of their attention when they plan, teach and reflect on their lessons, namely, investigate what they noticed and how they noticed it during all lesson study cycles. Besides, it was aimed to examine whether there were changes in the levels of preservice middle school mathematics teachers’ noticing of students’ mathematical thinking during participation in the professional development model. In light of these facts, this study addresses the following research questions:

1. To what extent and how do preservice middle school mathematics teachers notice students’ mathematical thinking in the context of lesson study professional development model?

2. How does lesson study contribute to preservice middle school mathematics teachers’ noticing of students’ mathematical thinking?

1.2 Significance of the Study

Teachers need to consider, listen, evaluate and respond to students’ ideas in order for student-centered instruction to take place (Ball & Cohen, 1999; NCTM, 2000). One way to support this kind of instruction is by helping preservice teachers change their view of classroom instruction from student’s eyes based on his own learning, to teacher’s perspective based on student learning (Stockero, 2014). The approach from “viewing classroom as a student to as a teacher” refers to the concept of teacher noticing (Sherin, Jacobs & Philipp, 2011). School-based field experiences which are a crucial part of teacher education programs are likely to enable this transition (Ishler & Kay, 1981). However, preservice teachers are not successful in focusing meaningfully on and making sense of interactions that came about in classrooms during field experiences (Masingila & Doerr, 2002) and attention to mathematical content and students' understanding is little because novices' ability to notice is not as advanced as expert teachers' (Berliner, 2001). Noticing is not a skill that teachers have or know automatically (Jacobs et al., 2010) rather it is a learnable skill (Jacobs et al., 2010; Sherin, Jacobs & Philipp, 2011) and it can be developed (Schack, Fisher, Thomas, Eisenhardt, Tassel & Yoder, 2013). Teachers need to learn focusing on the significant events that can contribute to students' learning in the complexity of classroom and using students' thinking for better teaching, so that teachers' ability to notice can be developed (Stockero, 2014).

Recent curriculum in Turkey requires teachers to generate a dynamic classroom environment that helps to reveal students' thinking, promotes them to share their ideas and clarify their solution strategies, helps them to encounter real life problems and supports group work (MoNE, 2013). Teachers should attend to important events, evaluate the tasks during the lesson and improve instruction considering their interpretations on these issues (MoNE, 2013). Specifically, they are expected to use their skills of noticing appropriately to guide classroom interactions. Teacher noticing is a critical subject that needs to be focused on in terms of teaching profession and therefore, making investigation on it would be meaningful.

In terms of teachers' noticing in classroom, their attention might be on many different events but specific issues towards students' mathematical thinking are seen more noteworthy in mathematics education (van Es & Sherin, 2010). Mathematics reform movements require teachers to focus on issues reflecting students' thinking such as students' misconceptions, difficulties, mistakes and so on (Schleppenbach et al., 2007). Likewise, the importance of attending to students' thinking in the related content is emphasized (Kazemi & Hubbard, 2008). On the other hand, according to Sowder (2007) professional development efforts based on students' thinking equip teachers with learning on how to create an effective teaching environment for better student learning and achievement. Teachers learn from their teaching when they focus on their students' thinking, hence, this situation maintains improvement for both teachers and students (Borko et al.2008). Besides, ongoing professional development support based on collaboration with preservice teachers and teacher educators and discussion on practices together might increase the impacts of field experiences for more effective learning (Masingila & Doerr, 2002) and improvement of noticing. Although attention to teacher noticing has increased in recent years, knowledge about teachers' ability to notice and the effective ways for developing it are limited (van Es & Sherin, 2010), thus, it is seen that further research on this issue is needed.

Most researches on mathematics education argue that focusing on students' thinking provides changes in instructional practices of teachers (Carpenter, et al., 1989; Franke et al., 2001; Kazemi & Franke, 2004; Steinberg, Empson & Carpenter, 2004). Thereby, instructional practices that reveal and build on students' thinking increase students' mathematical understanding and learning (Hiebert & Wearne, 1993). It is seen that teachers' instructional approach influences the tasks that they use, classroom interactions and student learning (Stein, Smith, Henningsen & Silver, 2000). That is why, teachers' use of instructional practices based on students' thinking is important. There is need for learning more about whether teachers focus on students' thinking and how they understand and make use of them because there are little details about these points in literature (Rhodes, 2007). Consequently, preservice teachers' noticing was addressed by using students' mathematical thinking as a starting point in this study.

The need for attention to students' prior knowledge, thinking and existing comprehension about mathematical concepts and spreading an effort to understand and make sense of them is a significant professional competence in terms of teachers (NCTM, 2000). In Turkey, definitions of the competence are made by Ministry of National Education General Directorate of Teacher Training and Education. The qualifications of knowledge, skills and attitudes that teachers must have are indicated under two headings as "Teaching Profession General Competence" and "Domain Specific Competence". It is stated that teachers should know their students and consider their skills and developmental levels in teachers' general competence MoNE, 2008a) and elementary mathematics teachers should plan and organize teaching of mathematics in accordance with students' mathematical skills, development and thinking in the context of their domain specific qualifications (MoNE, 2008b). The results of research including mathematics teachers' support for their professional development show that teachers who focus on students' mathematical thinking are better in creating student-centered settings as needed at the present time (Carpenter et al., 1989; Fennema, Carpenter et. al., 1996; Franke et al., 2001). However, it is seen that teachers and preservice teachers have difficulty in attending to student thinking, revealing and making sense of it (Crespo, 2000; Steinberg, Empson &Carpenter, 2004; Kazemi & Franke, 2004; Wallack & Even, 2005), especially, their noticing skill is low. It is not easy since students' ideas are changeable and they might use unusual mathematical strategies and notations to reflect their changing way of thinking (Ball, 2001; Schifter, 2001). In the study conducted by Enochs and Junk (2004), it was found that most teachers knew some nonstandard strategies that students produced but did not know some of them in multi-stage operations in mathematics. Baş, Erbaş and Çetinkaya (2011) investigated three mathematics teachers' abilities to anticipate ninth grade students' solution strategies and thinking in algebra. They argued that teachers were not successful in guessing students' mathematical approaches in algebra and making sense of the algebraic thinking behind their strategies. Similarly, in the study of Wallack and Even (2005) one teacher focused on two students' reactions and responses during the process of problem solving and endeavored to interpret and make sense of them. The results showed that what students said and did were not consistent with how the teacher understood and that the teacher had difficulty in making sense of students' mathematical thinking. Most research focuses on the details of what teachers attend to as a characteristic of noticing expertise in general (Choy, 2014). In consequence, this study will contribute to our understanding to what extent preservice teachers attend to and make sense of students' mathematical thinking.

Most of the studies showed that retrospective efforts or analysis of classroom videos (Barnhart & van Es, 2015; Jacobs, Lamb, Philipp &Schapelle, 2011; Kazemi et al., 2011; Sherin, Russ, Colestock, 2011; Stockero, 2014; van Es, 2011), classroom artefacts (Goldsmith & Seago, 2011) and lesson analysis (Santagata, 2011) were used in professional development in order to investigate teachers' noticing. However, these methods are not sufficient in terms of revealing teachers' noticing, especially to clarify how they make sense of the events and make instructional decisions (Sherin, Russ, et al., 2011). The process of analyzing and making sense of significant aspects of instruction in noticing while reflecting about the lesson is important but the effects of these interpretations on planning and teaching of the lessons need to be focused to make it clear (Choy, 2015). On the other hand, the studies mostly examined teacher noticing that happened during or after teaching and there is need for more research in this sense (Choy, 2014). Therefore, the lack of studies which include all phases: planning, teaching and reflecting after implementation, particularly noticing in whole process of lesson drew attention. Hence, it is believed that this study may contribute to literature elaborating what preservice teachers focus on and how preservice teachers make sense of their attention when they plan, teach and reflect on their lessons in a sequence, namely, presenting preservice teachers' noticing within lesson study context in a way that includes all phases.

One of the reasons behind teachers' poor noticing ability is the lack of interest in developing to notice (Choy, 2015). Although there has been research on teacher noticing in recent years, little is known about the development of teachers' ability to notice and how to support their ability (van Es & Sherin, 2010). Mason (2002) emphasizes the importance of improving the ability to notice and offers two ways for it which are providing a setting for a good preparation and use of experience (Mason, 2002). Many researches on teacher noticing advocate studying how preservice teachers examine interactions in the process of teaching and learning (Hand, 2012; Sherin, Jacobs & Philipp, 2011; Star & Strickland, 2008; van Es, 2011) and most of the studies show that participation in a group work based on collaboration increases teachers' attention to students' thinking, reasoning about their strategies and ideas and making sense of classroom interactions (Sherin & van Es, 2009; Sherin & Han, 2004; Jacobs et al., 2010; Borko et al., 2008). Lesson study professional development model based on collaboration is one of the contexts that enables teachers a) to prepare a plan considering students' cognitive processes, misconceptions, learning challenges and possible responses and b) to teach the lesson that they had planned together and observe teaching in the direction of their attention and c) to reflect what they noticed by making sense of them to make instructional decisions and suggestions for improving teaching and learning. Thereupon, this study contributes to the literature as it provides a professional development environment for preservice teachers in which they might develop noticing abilities with the opportunities of planning, teaching, observing, discussing, reasoning and reflecting. It is expected that the results might give clues about how lesson study professional development model influences the development of preservice teachers' noticing skills.

Hughes (2006) reviewed the studies on students' thinking and the ways that were used to focus on it in literature and revealed that in order to help teachers and preservice teachers to focus on students' thinking, some studies used research-based knowledge on students' cognition (Carpenter, et al., 1989; Fennema, et al., 1996; Swaford, Jones &Thornton, 1997) and some of them analyzed students' written work (Crespo, 2000; Kazemi & Franke, 2004). Besides, there were studies which examined videotaped lessons (Masingila & Doerr, 2002; Sherin & Han, 2004) and narrative cases of instruction (Barnett, 1991; Stein et al, 2003) to focus on students' solution approaches and make sense of them. Furthermore, some research used the professional development model to anticipate students' strategies and ways of thinking (Fernandez et. al., 2003; Hughes, 2006; Sisofo, 2010; Wagner, 2003). Most of these studies that examined the improvement of focusing on students' thinking were conducted with teachers and the number of researches on preservice teachers were limited. It shows the need for further studies with preservice teachers related with learning to attend to students' mathematical thinking and incorporating it into their practice during field experience. Hence, it was thought that carrying out the study with preservice middle school mathematics teachers can be useful in terms of literature.

Two main issues arise in the context of examining preservice teachers' ability to notice students' mathematical thinking which are the identification of noticing in specific mathematics domain and the development of this skill (Levin et al. 2009). Teacher noticing offers significant opportunity for examining teachers' engagement with students' thinking (Jacobs, Lamb et al., 2010; Sherin & van Es, 2009; van Es & Sherin, 2002). Most of the studies related to teacher noticing tackle mathematical thinking in general (Walkoe, 2013) without focusing on a particular mathematical domain (van Es & Sherin, 2002, Borko et al., 2008; Goldsmith & Seago, 2011), except a few researches such as the investigation of teacher noticing in whole-number operations (Jacobs et al., 2010), proportional reasoning (Fernandez, Llinares & Valls, 2011), early algebra (Schifter, 2011) and algebraic thinking (Walkoe, 2013). So there is need for further examining this skill in specific mathematics domains. In this respect, it was preferred to address noticing as a specific mathematics domain in this study. For that purpose, it was focused on perimeter, area, surface area and prism in the domain of geometry and measurement. It is important to learn what and how teachers notice in geometry and measurement which is one of the crucial mathematics domains in school curriculum (Yeo, 2008).

"Geometry is one of the best opportunities that exists to learn how to mathematize reality" (Freudenthal, 1973, p. 407). It helps students gain different perspectives (Duval, 1998). "Measurement is an enterprise that spans both mathematics and science yet has its roots in everyday experience" (Lehrer, 2003 as cited in Kellogg, 2010). It enables us to link geometry with real numbers (Clements & Stephan, 2003; Lehrer, 2003). Geometry and measurement include various applications and visualizations that is why the teaching and learning of this branch is difficult (Royal Society, 2001). Being able to make geometric measurements is based on accumulation of various knowledge and abilities and development of this skill is not easy (Clements & Stephan, 2003). For this reason, students have difficulty in understanding and using geometric and measurement concepts and making connections between them (Martin, 2009).

Geometric concepts such as perimeter, area, surface area and prism have taken important roles in mathematics curricula because of their practical properties and usage in daily lives. However, students are not able to comprehend the concepts of perimeter, area, surface area and volume (Martin &Strutchens, 2000) and prisms deeply (Battista & Clements, 1996). According to the results of research such as TIMMS and NAEP and most studies, students have more difficulties and less understanding in perimeter and area concepts than most of the other subjects in the curriculum (Chappell & Thompson, 1999; Thamson & Preston, 2004). Students mostly confuse the concepts of perimeter, area and surface area and they cannot understand them well (Cavanagh, 2007; Clement & Stephan, 2004; D'Amore & Fandino Pinilla, 2006; Danielson, 2005; Martin & Strutchens, 2000). Moreover, students also have difficulty in achieving the objectives related to three dimensional geometry (Katona 2008, Kösa 2011). The challenge in interpreting the static images of three dimensional solids is seen as the main reason behind this situation (Accascina & Rogora, 2006). Students have various mistakes, difficulties and misconceptions regarding these concepts (Cavanagh, 2007; Danielson, 2005; Emekli, 2001; Zacharos, 2006; Tan-Şişman & Aksu, 2009) due to inabilities in understanding the meaning of them completely (Zacharos, 2006).

In spite of the emphasis on geometry and measurement concepts, many studies address to the lack of students' understanding of them (Barret & Clement, 2003; Kenney & Kouba, 1997; Kouba, Brown, Carpenter, Lindquist, Silver & Swafford, 1988). Research also mentions teachers' and preservice teachers' misunderstanding related to these concepts and their insufficient understanding of students' thinking regarding them (Menon, 1998; Reinke, 1997; Simon & Blume, 1994; Tierney, Boyd & Davis, 1990). The results of studies reveal that preservice teachers also have confusion about perimeter and area such as finding area instead of perimeter and using the length x width formula for irregular shapes (Reinke, 1997; Tierney, Boyd &Davis, 1986), difficulties in identifying prism concept and thinking in three dimensions (VanHiele, 1986; Kılıç, 2003; Gökbulut, 2010; Paksu et al., 2012) and pedagogical strengths and weaknesses related to these concepts (Fuller, 1996). Besides, they do not have a clear understanding of their formulas and adequate knowledge regarding them (Simon & Blume, 1994). The preservice teachers' lack of knowledge in this topic is likely to affect their future students' understanding of these concepts (Baturo & Nason,1996) and learning of mathematics negatively (Ball & McDiarmid, 1989). From this point of view, in this study preservice middle school mathematics teachers' noticing skills of students' mathematical thinking related to the subject of perimeter, area, surface area and prism in the learning domain of geometry and measurement and how their noticing skills could be developed through lesson study was investigated.

When preservice teachers go in business as teachers, they encounter a scene which they do not expect because of the complex structure of teaching practice and feel unprepared for teaching (Hebert & Worthy, 2001). This negative feeling especially increases if they are not trained well to handle the complex and difficult situations in real classroom settings and if they do not have extensive experience to transfer their theoretical knowledge to practice (Black & Halliwell, 2000). Being able to make use of what is learned from formal education is only one of the challenges that preservice teachers face. In addition to this, they have difficulty in effective teaching of subject matter, managing the classroom, recognizing students' perspectives, interpreting their learning, anticipating the possible difficulties students have and learning policies too (Hebert & Worthy, 2001). The reasons behind these challenges in the beginning of teaching profession might be deficiencies in teacher education, unfamiliar and difficult responsibilities and adaptation to a new culture (Hebert & Worthy, 2001). At this point, the implementation of lesson study in the course of teaching practice in this study can be useful for preservice teachers to gain experience in order to overcome the difficulties that they might encounter at the beginning of their careers and learn how to transfer theory to practice better. In other words, although it is not the main focus of this study, this research might help with the preparation of preservice teachers for teaching and supply enlightening knowledge about integration of lesson study into the courses such as school experience and teacher practice in teacher training programs.

On the other hand, it draws attention that numbers of research on teacher noticing are scarce in Turkey (Baş, 2013; Erdik, 2014; Osmanoğlu, Işıksal & Koç, 2012; Osmanoğlu, 2010) and even though lesson study has been gaining importance in recent years, still there are limited number of studies related to it in Turkey (Baki, 2012; Baki, Erkan & Demir, 2012; Boran & Tarım, 2016; Budak, Budak, Bozkurt & Kaygın 2011; Bütün, 2012; Eraslan, 2008; Erbilgin, 2013). Therefore, it was necessary to carry out studies on noticing skills of teachers and preservice teachers and the implementation of lesson study in Turkey. It is expected to contribute to the national literature with this study.

In sum, noticing is one of the essential skills that both teachers and preservice teachers need to gain (Mason 2002; Sherin, Jacobs & Philipp, 2011). Current teacher education programs do not pay attention to what and how preservice teachers notice, yet this concept should be an important component of these programs in order to develop it (Hiebert, Morris, Berk & Jansen, 2007). Preservice teachers have difficulty in focusing on important interactions and making sense of them. That is why, most studies indicate the need for research regarding preservice teachers' noticing (Hand 2012; Sherin, Jacobs & Philipp, 2011; Star & Strickland 2008; van Es 2011). Other than that, they should be given various opportunities to improve their noticing skills through professional support (Jacobs, Lamb et al., 2010). Sowder (2007) defends that professional development support based on students' thinking provides a good quality of instruction and high student achievement. In addition, there is need for investigation regarding teacher noticing during planning, teaching and reflecting as most of the studies in that field include noticing during or after teaching (Choy, 2014). It is important to examine and develop future teachers' ability to notice students' mathematical thinking during teacher education programs and lesson study might help to reach this aim. In our study, this environment was maintained by lesson study professional development model which included the opportunities of planning, teaching, analyzing, discussing, reasoning and reflecting so that it might develop preservice teachers' noticing skills.

Moreover, noticing is an important subject that should be investigated, though knowledge about teacher noticing and effective ways to improve it are limited (van Es & Sherin, 2010). Besides, although it is important to use teachers' instructional practices based on students' thinking, there is little research on noticing of students' thinking for better instruction (Rhodes, 2007). At that point, the purpose of this study was to examine preservice middle school mathematics teachers' noticing of students' mathematical thinking in the context of lesson study professional development model and determine whether professional development support was effective in the improvement of this skill. In addition, most of the research on noticing was based on teachers rather than preservice teachers and there are limited studies related to noticing and lesson study in Turkey. On the other hand, the number of investigations of noticing in specific mathematics domain was inadequate. With all of these taken into consideration, it was aimed to reveal what preservice teachers focused on and how preservice teachers made sense of their attention while planning, teaching and reflecting mathematics lessons related to geometry and measurement domain; perimeter, area, surface area and prism concepts in particular.

1.3 Definition of Important Terms

Noticing means looking with professional vision that includes seeing and understanding events (Goodwin, 1994) and it is an action which occurs frequently and in various contexts (Jacobs et al., 2011; Santagata, 2011; van Es, 2011). The skill of noticing includes three main dimensions: "(a) identifying what is important in a teaching situation; (b) using what one knows about the context to reason about a situation; and (c) making connections between specific events and broader principles of teaching and learning" (van Es & Sherin, 2002, p.573).

Teacher noticing is seen as expertise in focusing on and making sense of important aspects of instructional practices (Goodwin, 1994; Sherin & van Es, 2009). Sherin and van Es (2009) indicate teachers noticing as two subskills: (a) identifying what is important in a teaching situation and (b) drawing on one's knowledge of teaching and learning to reason about the situation (van Es & Sherin, 2006, p. 215).

Lesson study is "a method of professional development that encourages teachers to reflect on their teaching practice through a cyclical process of collaborative lesson planning, lesson observation, and examination of student learning" (Lenski & Caskey, 2010, p. 44). The literature generally identifies the stages of the lesson study cycle as planning the lesson plan, implementing the lesson plan, discussing on the implemented lesson, revising the lesson plan, teaching the revised lesson plan and sharing reflections in collaboration (Lewis & Hurd, 2011; Yoshida, 1999; Fernandez, 2002).

CHAPTER 2

2. LITERATURE REVIEW

2.1 Noticing and Teacher Noticing

Noticing means looking with a professional vision that includes seeing and understanding events (Goodwin, 1994). Noticing is an action which occurs frequently and in various contexts (Jacobs et al., 2011; Santagata, 2011; van Es, 2011). Noticing is one of the significant components of mathematics instruction and definition of it varies. Mason (2002) describes it as a discipline and the heart of mathematical practices. Noticing consists of practices which are "reflecting systematically; recognizing choices and alternatives; preparing and noticing possibilities; and validating with others" (Mason, 2002, p. 95) for providing awareness for teachers (Mason, 2002). It is a promising skill in terms of making the process of teaching clear. While van Es and Sherin (2010) identify noticing as focusing on significant aspects and details of specific events in a classroom and making sense of what is focused on with the help of prior knowledge; according to Star and Strickland (2007), noticing substantially includes the former component which refers to attending to important points in a classroom. Jacobs, Lamb and Philipp (2010) expand the definition of it and identify noticing as attending to, making sense of and deciding how to respond. In recent years, teachers' noticing has gained importance in mathematics teacher education to develop teachers' instructional skills (Goldsmith & Seago, 2011; Jacobs et al., 2010; Sherin & van Es, 2009; Star & Strickland, 2007; van Es, 2011). It has become an issue of research not only for in-service teachers but also for professional development of preservice teachers (Star et al., 2011; Star & Strickland, 2008). Noticing helps teachers to increase their knowledge about teaching and learning by reducing the complexity of classroom interactions (Sherin, Jacobs & Philipp, 2011). Teachers can learn to evaluate their own implementations from their practice with the opportunity of noticing (Sherin, Jacobs & Philipp, 2011) and advance the effectiveness of instruction (Schoenfeld, 2011). "Mathematics education reform calls on teachers to base their instruction on the lesson as it unfolds in the classroom, paying particular attention to the ideas that students raise" (NCTM, 2000). Teachers' noticing skills of students' thinking, specifically, attending to and understanding students' thinking and integrating it into instruction are improvable and important in terms of increasing students' achievement. Thus, these skills should be supported through professional development experiences (Goldsmith & Seago, 2011; Jacobs et al., 2010; Sherin & Han, 2004; Sherin & van Es, 2009; Star & Strickland, 2007; van Es, 2011; van Es & Sherin, 2008).

Since noticing is an essential part of mathematics teaching, teacher noticing in mathematics teacher education has gained importance to support teachers to handle with complex events that rise in classroom in a way that enhances the students' achievement (Goldsmith & Seago, 2011; Jacobs et al., 2010; Sherin & van Es, 2009; Star & Strickland, 2007; van Es, 2011). Teacher noticing is seen as an expertise in focusing on and making sense of important aspects of instructional practices (Goodwin, 1994; Sherin & van Es, 2009). In order to increase teaching expertise of teachers, it is thought that the improvement of their noticing should be provided with teacher education (van Es & Sherin, 2002; Sherin, Jacobs & Philipp, 2011). According to van Es and Sherin (2002) three significant dimensions of noticing are as the following:

> (a) identifying what is important or noteworthy about a classroom situation; (b) making connections between the specifics of classroom interactions and the broader principles of teaching and learning they represent; and (c) using what one knows about the context to reason about classroom interactions (p. 573).

First dimension includes attending to noteworthy teaching situations. Due to the complex structure of teaching, teachers should focus on students' reactions in relation to both discourse and action, their ways of thinking on the related topic, what kind of strategies and representations to use for better teaching, how to make students active while learning and so on. However, considering all these aspects simultaneously is not possible, that is why teachers must be able to identify noteworthy events to focus on and respond during lesson. This is important as reform movements require deciding how to shape the instruction throughout implementation (van Es & Sherin, 2002). Second dimension includes interpreting specific events based on relevant instructional principles. Expert teachers are able to address specific practice issues associating them with extensive principles of teaching and learning (Peterson & Cameaux, 1987) whereas novice teachers' abilities cannot go beyond describing events simply (van Es & Sherin, 2002). Third dimension includes benefiting from existing knowledge (e.g. subject matter, students' thinking, context) to interpret the situations as experience and knowledge in a particular domain facilitate the understanding of classroom interactions (Chi, Glaser & Farr, 1988).

What is understood from the same situation changes from person to person because existing experience, knowledge and beliefs impact what is seen (Olson, Roese & Zanna, 1996). Thus, teachers' approaches about what is important to focus on change depending on these factors (Schoenfeld, 1998). It shows that "how individuals analyze what they notice is as important as what they notice" (van Es & Sherin, 2002, p. 575). In accordance with this perspective, Sherin and van Es (2009) indicate teachers' noticing as two subskills: (a) selective attention which means what teachers focus on and (b) knowledge-based reasoning which refers to how teachers interpret what they focus. Selective attention is related to "how the teacher decides where to pay attention at a given moment" (p. 22). Teachers must be able to choose where to direct their attention in the complex structure of classrooms where several events take place at the same time. Therefore, teachers should concentrate on listening to students' ideas carefully in order to gain the ability to determine the significant features of the classroom in terms of lesson's goals (Bell & Cowie, 2001). Knowledge-based reasoning includes "the ways in which a teacher reasons about what is noticed based on his or her knowledge and understanding" (p.22). Teachers make sense of a specific event using their knowledge of subject matter, pedagogy, curriculum or students' thinking. In other words, selective attention is the act of identifying noteworthy events amongst the others and knowledge-based reasoning is the process which includes making sense of and interpreting those events (Walkoe, 2013). There is a strong interaction between these two components. That is, what teachers notice impacts how teachers interpret the events vice versa teachers' sense-making shape the direction of their focus in situations (Sherin & van Es, 2009). Researchers have made many researches on noticing in mathematics education (Sherin & Han, 2004; Sherin & van Es, 2009; van Es & Sherin, 2002, 2006, 2008) and it was argued that teacher noticing is improvable through changes in what they focus such as a shift from teachers' behaviors to students' ideas and in how they reason like the movement from describing simply to elaborating events and having an evaluative approach to an interpretive stance (Sherin & Han, 2004; van Es & Sherin, 2008). Research also emphasizes the importance of attending to students' ideas and mathematical thinking to increase student learning (Ball, Lubienski & Mewborn, 2001; Franke, et al., 2001), but it is difficult for teachers to make sense of student thinking (Heaton, 2000).

Jacobs et. al. (2010) defines noticing of students' mathematical thinking with three components which are "(1) attending to children's strategies (2) interpreting children's understanding and (3) deciding how to respond on the basis of children understandings" (p. 169). The first component refers to the degree of teachers' attention to the important details related to students' mathematical ideas and approach. The more details teachers know about students' mathematical strategies, the more they comprehend students' understanding (Carpenter et al., 1999; Lester, 2007). The second component corresponds to how teachers make sense of students' mathematical thinking that they focused on and how they interpret students' understanding embedded in their strategies. The third component includes teachers' reasoning that they employed to make instructional decisions. It refers to what extent teachers can reflect the things they learned from students' mathematical thinking and understanding. The subskills of attending to students' strategies and interpreting their understanding are included in van Es and Sherin's (2002) identification about teacher noticing-selective attention and knowledge based reasoning however, the focus here is mostly on students' mathematical thinking. Although there are different characterizations towards noticing, the essential subskills are focused on noteworthy instructional events and interpreting these points through existing knowledge (Sherin, Jacobs & Philipp, 2011).

Noticing is a concept that is examined by many researchers (Goldsmith & Seago, 2011; Jacobs et al., 2011; Kazemi et al., 2011; Sherin, Russ, Colestock, 2011; Star et al., 2011; van Es, 2011). Some of them address noticing as focusing on only the events that draw attention (Sherin, Russ, et al., 2011; Star et al., 2011) whereas some of them identify noticing with two actions which are attending to and making sense of instructional moments (Goldsmith & Seago, 2011; Jacobs, Lamb, et al., 2011; Kazemi et al., 2011; van Es, 2011). Besides, noticing can be enhanced in a way that holds another component such as making instructional decisions as well as focusing on and making sense of (Jacobs, Lamb, et al., 2011). There might be some differences between interpretations of what noticing includes but attending refers to being able to see and concentrate on noteworthy events in classroom interactions for most researchers (Barnhart & van Es, 2015; Erickson, 2011; Star et al., 2011). In terms of making sense, van Es (2011) interests in the degree of details and analyses in teachers' interpretations of students' mathematical thinking and the power of offering evidence from observations to support their ideas on mathematics instruction. Her framework copes with what and how teachers notice by tackling the subjects and issues that they focused on and to what extent they elaborate and make their comments explicit (van Es, 2011). Jacobs, Lamb et al. (2011) extend making sense of instructional details and use it in order to decide how to respond in the flow of a lesson. It is defended that making instructional decisions is also important because it is related to the other components which are attending to and making sense of students' mathematical thinking. Teachers decide how to respond for better instruction on the basis of knowledge that they obtain by focusing on and interpreting (Jacobs, Lamb, et al., 2011). Although there are little differences in conceptualization of noticing, researchers have agreement on the importance of noticing in mathematics education (Goldsmith & Seago, 2011; Jacobs, Lamb, et al., 2011, Schifter, 2011; van Es, 2011). According to Mason (2002) noticing occurs with internal and external impulses independently of one's decision to do so, ergo, Choy (2015) proposes to investigate the ways for supporting teachers' noticing to improve.

To speak of what teachers notice, they can notice a wide range of issues in the classroom because their noticing changes in the direction of knowledge, beliefs (Kazemi et al., 2011; Schifter, 2011) and philosophy of education (Erickson, 2011; Schoenfeld, 2011) that they have, accordingly, they might mislead the students to engage in the issues which are less relevant with mathematics instruction (Erickson, 2011). Noticing the important mathematical aspects of instruction is a challenging action for teachers, especially for novice teachers (Star et al., 2011; Star & Strickland, 2008; Vondrová & Žalská, 2013). Similarly, Jacobs et al (2010) argue that most preservice teachers have difficulty in focusing on students' thinking and Santagata (2011) indicates that the challenge in noticing noteworthy events and specific details continues while teachers reflect on the process of teaching and learning. On the other hand, recognizing what significant aspects of instruction are and learning to pay attention to them is substantial for expertise in noticing (Miller, 2011). Thus, it is needed to take the direction of teachers' attention into account due to the vast information that is revealed in a real classroom (Choy, 2015).

In relation to how teachers notice, Jacobs et al. (2010) evaluate teachers' noticing expertise with some criteria. Giving specific mathematical details, making pedagogical comments based on students' thinking, supporting ideas about students' thinking with details gained from classroom observation, making instructional decisions by considering students' thinking and understanding, focusing on individual students and preparing tasks with regard to students' thinking are accepted as indicators which show that teachers begin to gain noticing expertise in how to notice. When the criteria are taken into consideration, it is seen that all of them point to a shift from general approach to specific and elaborative stance. In most studies teachers' noticing expertise is connected with focusing on important features of instruction and the degree of details teachers notice (Goldsmith & Seago, 2011; Jacobs et al., 2010; Jacobs, Lamb, et al., 2011; Stockero, 2014; van Es, 2011). Since how teachers notice is crucial as well as what teachers notice, it is also essential to examine teachers' process of making sense of what they notice, the ability to clarify what they observe and the power of predicting how students learn (Berliner, 2001; Davis, 2006; Mason, 2002; Star & Strickland, 2008; van Es, 2011; van Es & Sherin, 2002).

In noticing, being able to use existing knowledge about the situation to interpret what is focused on and linking practice to theory are important (Jacobs et al., 2011; van Es, 2011; Yang & Ricks, 2012) because benefiting from instructional principles related to the context enables teachers to foresee the students' ways of learning determine strategies and make suggestions on how to improve their learning (van Es, 2011). At this point, van Es (2011) offers a framework for learning to notice students' mathematical thinking. It mentions both what teachers notice and how teachers notice and reflects a developmental trajectory in noticing that includes four levels which are baseline (level 1), mixed (level 2), focused (level 3) and extended (level 4). Some other researchers give importance to the integration of three skills for noticing including attending, interpreting and responding (Barnhart & van Es, 2015; Jacobs et al., 2010; Star & Strickland, 2008; van Es & Sherin, 2002, 2008). Jacobs et al. (2010) explain the relationship between these skills as deciding how to respond to students' thinking is possible through analyzing and making sense of their thinking and sense-making develops with the attention to the details of students' thinking. Barnhart and van Es (2015) emphasize the complexity of these skills and the difficulty in making connections between them.

Noticing of students' mathematical thinking includes attending to noteworthy mathematical ideas of students and interpreting their thinking to make decisions for teaching mathematics (Fernandez, Llinares & Valls, 2013). Being able to distinguish important mathematical points in students' thinking and interpreting them is critical in order to provide teaching for understanding (Fernandez et al., 2013) and make effective instructional decisions (Chamberlin, 2005). Teachers should know how students learn mathematics to improve their mathematical understanding (Schifter,2001; Steinberg, Empson & Carpenter, 2004). It depends on listening to the students and learning from them (Crespo, 2000) and developing teachers' ability to interpret students' mathematical thinking (Eisenhart, Fisher, Schack, Tassel & Thomas, 2011). Research on the development of mathematics teachers emphasize the importance of developing preservice teachers' ability to notice for mathematics teaching (Jacobs et al., 2010; Mason, 2002; van Es & Sherin, 2002). Therefore, it is crucial to examine how preservice teachers attend to and make sense of students' mathematical thinking in different mathematical domains (Hines & McMahon, 2005).

Teachers' expertise in noticing of students' thinking can be improved through experiences in teaching and professional development (Goldsmith & Seago, 2011; Kazemi &

Franke, 2004; Sherin & Han, 2004; Sherin & van Es, 2009; van Es, 2011; van Es & Sherin, 2008) and they need this support to acquire the ability to notice because it is not one of their innate skills (Jacos et. al., 2010). Learning is a natural need for teachers as their teaching ability depends on what they have learned (Darling-Hammond & Sykes, 1999) and thus learning of their profession must be supported and opportunities for learning must be continuous (Jacos et. al., 2010).

Van Es and Sherin made many investigations related to teacher noticing to understand how it occurs in mathematics (van Es & Sherin, 2002, 2006, 2008, 2009). They used videos from mathematics lessons to improve teachers' ability to notice and defended that videotaped lessons encouraged the development of teachers' noticing skills. In addition to van Es and Sherin, some other researchers also used mostly video cases in teacher education (Lampert & Ball, 1998; Osmanoğlu, 2010; Osmanoğlu et al., 2012; Star & Stickland, 2008). In other words, the use of video was conducted by many researchers in the examination of teacher noticing. In the study of van Es and Sherin (2002), they worked on how teachers learn noticing and interpreting interactions that occurred in a classroom. In particular, they investigated how a software tool, VAST, might affect preservice mathematics and science teachers' ability to analyze their own classrooms. They created a trajectory of the developmental levels for noticing and used a framework that they named as learning to notice framework. The research took place in three sessions, preservice teachers were introduced with VAST and used it to gain experience, they analyzed a video reflecting classroom setting with the help of VAST and they discussed on how they integrated this tool into the analysis of their own teaching. It was found that VAST had some effects on the analysis of their own practices and they began to notice a number of noteworthy events in their classroom interactions. They were also able to interpret these specific events that occurred in the classroom videos with respect to instructional principles, in other words, they could make connections between significant events and broader ideas of teaching and learning. Besides, teachers' comments on students' thinking were more interpretive and elaborative after the use of VAST. These properties are the indicators of the improvement in their skill of noticing (van Es & Sherin, 2002).

Sherin and Han (2004) conducted a research with four middle school mathematics teachers to investigate the learning of teachers who participated in one of the professional development video club meetings that took place for a year. In these meetings, they focused on how teachers learned attending to specific classroom interactions and the ways of interpreting them. The findings showed that there were changes in teachers' focus and the analysis of classroom videos over time. Teachers began to attend to and interpret students' strategies and ideas rather than teachers' actions. The characteristics of their comments shifted from the description of students' ideas to a detailed analysis of their thinking. It was argued that teachers learned to focus on specific issues in classroom interactions and how to analyze and make sense of these points through a particular approach, shortly, teachers' noticing advanced (Sherin & Han, 2004).

In the study of Star and Strickland (2007), the focus was on what teachers do not attend as well as what they attend, especially, what is noticed. They used videos of mathematics classrooms from TIMMS and aimed to improve preservice teachers' noticing skills. It was explored whether preservice teachers' ability to attend to significant aspects of classroom interactions developed during the method course. Some changes were made in the content of the course supporting various activities and pre-post assessments were applied. It was found that although preservice teachers' ability to notice details in specific events that took place in classroom was poor, it improved over time throughout the method course and researchers gathered their observations under four categories as classroom environment, tasks, mathematical content, and communication. The findings showed that the ability to notice can be evolved with support.

The aim of van Es and Sherin's study (2008) was to examine what changed in teachers' thinking and noticing through the video club meetings designed to provide a setting for the development of teachers' learning to notice and reason about students' mathematical thinking. They compared teachers' ideas before and after participating in these meetings and determined three developmental paths that teachers exhibited as they learned to notice students' mathematical thinking in the video-based professional development environment: Direct, Cyclical and incremental paths. The research was conducted with the participation of seven fourth and fifth grade elementary teachers in these meetings ten times in a course year. In these meetings, they provided video clips from their own classrooms, which were analyzed and discussed together. Learning to notice framework was used to analyze the data. They named some categories to identify what teachers noticed and how they interpreted their attention: actor, topic, stance and specificity. The results argued that some shifts occurred in these dimensions, to say a few, teachers began to focus further on analyzing and interpreting students' mathematical thinking and think about what they noticed in different ways and also their noticing skills improved. Teachers' views revealed that they think the video club assisted their understanding the importance of revealing students' ideas and forming strategies to understand how they think. In addition to that, how teachers' learning to notice in the context of video-based professional development had effects both on their practice in classroom and students' learning was proved. On the other hand, researchers identified three developmental paths. According to their findings, teachers' approach shifted from a broad perspective to a narrower one on all noticing dimensions in the direct path. Teachers focused on students and their mathematical thinking, referred to specific events and gained interpretive stance. The cyclic path included changing perspectives between broad and narrow perspectives at times. In the incremental path teachers started with a broad perspective and adopted a narrow approach later but only on a few noticing dimensions not on all of them. It was concluded that direct and incremental paths were followed by expert teachers whereas the cyclic path was preferred by novice teachers.

Similarly, van Es and Sherin (2010) investigated the influence of video clubs on teachers' thinking and practice in different contexts. Video club was designed to help teachers learn to notice students' mathematical thinking and see different implementations from the others' classrooms. They watched videotaped lessons and discussed on them during meetings. It was found that teachers began to notice and interpret students' mathematical thinking from the first to the last meetings and attended to more specific events in videos. Teachers also indicated that they recognized the significance of attending to students' thinking, paying attention to students' ideas during teaching and learning about school's mathematics curriculum. Besides, the results revealed that three main changes arose in teachers' instruction in time. Teachers tried to make time for attending to and revealing students' thinking, make their thinking explicit and elaborate on it other than adopting a stance as a learner to consider students' ideas in the context of classroom for learning while teaching. This study shows that teachers' instruction and practice change in accordance with mathematics reform efforts with the help of professional development but researchers emphasize the need for more research (van Es and Sherin, 2009).

Sherin and van Es (2009) continued their investigations on teachers' learning to notice through video club meetings. Researchers searched the influence of video club on teachers' professional vision, particularly, their ability to notice and interpret the important points in a lesson. The aim of the video clubs in the study was to assist the development of teachers' learning to notice in a specific area, in other words, to promote teachers to attend and reason about students' mathematical thinking. Four middle school mathematics teachers participated in one video club for a total of seven meetings whereas seven elementary school teachers participated in another video club for a total of ten meetings. It was argued that video-based professional development setting improved teachers' professional vision and their instruction. In other words, it was claimed that the development of teachers' noticing skills affected their teaching practices. The results showed that, in both video clubs, teachers' noticing of students' mathematical thinking increased. Teachers were not good at attending to students' ideas at the beginning but later they began to focus on and reason about their ideas and approach.

Alsawaie and Alghazo (2010) explored what the effects of video lesson analysis method were on prospective teachers' analysis skills of mathematics teaching. Learning to notice framework was used to evaluate what changed in terms of their noticing skills in both experimental and control groups in time. The results argued that noticing ability improved and noticed points were exceeding in the experimental group compared to the control group. Prospective teachers who participated in the experimental group were better at focusing on important events, making sense of them and making connection with broader educational principles.

In the research based on the use of videotaped lessons or creating video clubs, there are some concerns about these methodologies. When teachers are asked to discuss other teachers' video clips showing their teaching process, teachers' reactions to videos might not be the same as in their real teaching (Sherin, Russ, Colestock, 2011). For example, in their study Levin, Hammer and Coffey (2009) found that there were differences between what a preservice teacher noticed in the discussions during a method class and what and how she noticed during her teaching. According to Sherin, Russ, et al. (2011) when we ask teachers to explain what they noticed and thought about using the videos of their own instruction in video clubs retrospectively, we cannot obtain correct information about their in-the-moment experiences due to being out of "the demands of the classroom" (p. 82). In other words, it limits the teachers' noticing ability to post-event videos. Besides, teachers are likely to exhibit an evaluative approach to one another's performance rather than learning from videos (Thomas, Wineburg, Grossman, Oddmund & Woolworth, 1998). Another concern is that it is hard to understand what and how teachers really notice by relying on videos prepared by researchers as videos reflect their own perspectives. These approaches do not allow for examining teacher noticing that occurs as the implementation of a lesson and teachers analyze videos outside the context (Liu, 2014).

Aside from video based implementations and the issues related to what is noticed, how it is noticed and the development of noticing, Miller and Zhou (2007) investigated teacher noticing in different cultures by comparing teachers in U.S. and Chinese elementary schools and found that U.S. teachers were in a tendency to focus on teacher pedagogy whereas Chinese teachers address to mathematical aspects of the events. Many researches on noticing show that each teacher's construction way of what they see is impacted by their experiences, educational approach, cultural structure and etc. (Miller & Zhou 2007; Star & Strickland, 2007). Individuals might not perceive things in the same way when they attend to the same thing because they reconstruct what they see (Gibson, 1979). These kinds of factors shape the ways teachers notice and thus, gaining the ability to notice is critical for improving profession expertise (Jacobs et al., 2010). Choy (2013) explained what made noticing more productive and described the notation of productive mathematical noticing. He conducted the study with seven mathematics teachers who participated in a lesson study team at a primary school. Lesson study sessions and the lessons were videotaped, the data obtained was analyzed to show how the characterization of teachers' mathematical noticing into more productive or less productive was made and illustrated these concepts through rich instances. It was argued that teachers who had productive mathematical noticing skills were good at reaching viable decisions for their teaching whereas teachers who had less productive mathematical noticing could not help their students to increase their mathematical understanding. Furthermore, it was found that collaboration between teachers increased the productivity of teachers' mathematical noticing.

A few of the studies related to teacher noticing were conducted in the context of professional development programs. In the study of Choy (2015), the aim was to investigate what made noticing more productive in terms of the development of students' reasoning in the context of lesson study professional development model and how to describe the characteristics of this concept. The FOCUS framework was created to analyze teachers' noticing and identify their noticing as productive or less productive. This framework emphasized two important components for providing productive noticing, which are explicit focus and reasoning. According to the results, it was indicated that the FOCUS framework assissted to promote teachers' noticing expertise and it can be used to examine teacher noticing by other researchers.

In national literature, there were few studies related to the concept of noticing. Osmanoğlu (2010) investigated what changed prospective elementary mathematics teachers' noticing through watching video cases and discussing on them. She focused on to what extent their ability to notice changed with the participation in video cases based on teacher education in terms of the roles of teacher and students. Preservice teachers watched six video cases from real mathematics classrooms and discussed on them based on their reflective notes in an online forum. The results showed that prospective teachers' noticing skills developed on behalf of teacher and student roles during the video-case based online discussions.

Baş (2013) investigated the development in teachers' noticing of students' mathematical thinking through a professional development program based on the principles of Models and Modeling Perspective. It was carried out with four secondary mathematics teachers throughout seven periods of one-month each. Each month was comprised of an introductory meeting, modeling task implementation and a follow-up meeting. It was found that teachers' noticing of students' mathematical thinking progressed with the support of professional development.

In the study conducted by Erdik (2014) what mathematics teachers with different years of experience notice and the differences or similarities between teachers' noticing with respect to teaching experience were assessed. Fifteen mathematics teachers participated in the research and three groups consisted of inexperienced, less experienced and experienced teachers. Actor, topic, stance and specificity categories were used to analyze what teachers noticed and there were differences between teachers' noticing in relation with actor, topic and stance categories and how they interpreted what they noticed, especially between experienced and inexperienced teachers.

With these studies taken into consideration, it is seen that most of them were conducted in the context of video clubs through videos from lessons and they investigated what teachers notice, how teachers notice and how to develop teacher noticing (Alsawaie & Alghazo, 2010; Osmanoğlu, 2010; Osmanoğlu et al., 2012; Sherin & Han, 2004; Sherin & van Es, 2009; Star & Stickland, 2008; van Es & Sherin, 2002, 2006, 2008, 2009). A number of them were related to the factors that influence teacher noticing (Miller & Zhou 2007; Star & Strickland, 2007), characterization of noticing (Choy, 2013; 2015) or comparison of teacher noticing in terms of teaching experience years (Erdik, 2014). Besides, few of them were carried out in a professional development context different from video cases (Baş, 2013; Choy, 2015). Most of them were conducted with teachers (Baş, 2013, Choy, 2013, 2015; Erdik, 2014; Miler & Zhou, 2007; Star & Strickland, 2007; Sherin & van Es,2009; van Es & Sherin, 2008, 2010), whereas few of them were carried out with preservice teachers (Alsawaie & Alghazo, 2010; Osmanoğlu, 2010; Osmanoğlu et al., 2012; van Es & Sherin, 2002).

On the other hand, it drew attention that numbers of research on teacher noticing were scarce in Turkey (Baş, 2013; Erdik, 2014; Osmanoğlu et al., 2012; Osmanoğlu, 2010). Besides, some research related to teachers' noticing has addressed how teachers focused on students' thinking, how they make sense of them and how their instructional practices are affected by their attention and interpretations (Jacobs et al., 2010; Sherin & van Es, 2009; van Es & Sherin, 2008). Some other research has focused on the relationship between teachers' attention to students' ideas and their knowledge about students' thinking (Franke et al., 2001; Franke & Khazemi, 2001) and has linked the ability to notice with students' achievement (Carpenter, et al., 1989; Fennema et al., 1996; Jacobs, Franke, Carpenter, Linda &Battey, 2007).

Sherin and van Es (2009) indicate that being able to focus on the details of students' ideas and strategies facilitates to make sense of their thinking so that teachers integrate what they focused on and how they interpreted it into their instructional decisions and shape their classroom practices. This trajectory includes the shift from focusing on teachers and less relevant aspects of the lesson and evaluating the behaviors of students and teachers, to attending to students' thinking, making sense of them and using this information in order to improve instruction (Jacobs et al., 2010). Therefore, it points out that the teachers' ability to notice students' mathematical thinking is an essential part of teaching expertise. Teaching mathematics is not an easy task, thus, teachers should be provided with opportunities for developing both their knowledge of teaching and the ability to notice students' mathematical thinking and reasoning about how to improve their instruction through effective professional development (Coddington, 2014). At this point, this study aimed to investigate preservice teachers' noticing of students' mathematical thinking in the context of a professional development model because there is need for research on preservice teachers' noticing of students' mathematical thinking through effective professional development in Turkey.

2.1.1 Expert and novice teachers

In literature, some studies focused on the differences between expert and novice teachers in terms of what and how they notice. Erickson et al. (1986) investigated how expert and novice teachers observed and made sense of daily classroom interactions whereas some researchers argued how expert teachers discussed on videos that they watched considering teaching principles and analyzed classroom interactions with elaborative stance (Carter et al., 1988; Peterson & Comeaux, 1987). Borko and Livingston (1989) compared the characteristics of novice and expert teachers and found that novice teachers are less able to anticipate students' ideas, reactions or problems and possible situations that might arise in classroom interactions. Expert teachers focused mainly on significant points such as students' understanding and learning rather than less relevant issues such as classroom management whereas novice teachers attended to their own teaching (Borko & Livingston, 1989). Likewise, Hogan, Rabinowitz, and Craven (2003) defended that the focus of expert teachers was on how students learn and understand the relevant concepts not the issues. As opposed to expert teachers, novice teachers attend to evaluating their own performance as teachers.

Some studies emphasized that expert teachers were better at making sense of and recalling the events than novice teachers while watching videos from classroom settings Peterson & Comeaux, 1987; Sabers, Cushing & Berliner, 1991). Sabers, Cushing and Berliner (1991) defended that "experts are able to monitor, understand, and interpret events in more detail and with more insight than either novices or advanced beginners" (p. 71). They found that expert teachers were able to elaborate what they attended to, interpret classroom events and use evaluative judgments instead of descriptive statements. In terms of novice teachers, their descriptions were superficial and not including much details about what they observed. They generally focused on less significant events such as classroom setting and did not make sense of the classroom events (Carter et al., 1988). Similarly, Sabers, Cushing and Berliner (1991) indicated that novice teachers' comments were less detailed and they attended to teachers rather than students as discovered in another research (Sherin & Han, 2004; van Es & Sherin, 2008). According to Krull, Oras and Sisask (2007) expert teachers were more interpretive ad reflective in their expressions, they noticed more classroom events and provided more details about what they noticed than novices.

Jacobs, et al. (2010) also explored the differences in teachers' noticing and found that novice teachers' ability to notice is low. In addition to that, novice teachers were not in tendency to focus on students' strategies and ideas and interpret their thinking, yet both expert and novice teachers had difficulty in deciding how to respond to students thinking when components of noticing were considered. Similarly, Miller (2011) indicated that expert teachers have better understanding and interpretation of classroom teaching and learning. Erdik (2014) also concluded that there were statistically significant differences between the experienced teachers and novice teachers. She also found that there were qualitative differences and similarities between them in terms of what and how they noticed. On the other hand, Crespo (2000) advocated that novice teachers mostly focused on whether students' ideas were correct or not instead of interpreting them.

In the study of Huang and Li (2012), differences between ten expert and ten novice teachers' noticing were investigated. The findings showed that the expert teachers focused primarily on students' mathematical thinking and knowledge. They paid attention to improve students' higher order thinking and knowledge and reveal how they think promoting their participation. What expert teachers noticed about students' mathematical thinking were broader. Besides, their attitudes toward lesson were better and they were more willing than novices. In sum, it was found that there were differences between novice and expert teachers in terms of developing students' mathematical knowledge, mathematical thinking and ability to use them, using necessary instruction tools and determining appropriate teaching methods.

The success of the schools and the achievement of the students are shaped by teachers and their abilities (Wong, 2004). Teachers' quality affects students' performance and leads to students' varied achievement (Wong, 2003). This means that higher quality in teacher abilities, brings more successful students and a more refined education with it (Wong, 2003). Attitudes, educational viewpoints and professional norms of teachers are constituted in the first year of their occupation (Moir & Gless, 2001). Hence, guidance before starting the profession is highly important. Teaching does not have the same amount of challenge for novice and expert teachers and learning to teach is not an easy action (Arditi, 2013). It requires time and experience. The term of novice teachers means the teachers who do not have the abilities of experts (Borko & Livingston, 1989) and it is generally used to represent preservice teachers in this study. Expert teachers are better in stemming the tide of the problems encountered (Darling-Hammond, 1995) during the implementations in contrast with preservice teachers. Other than that, Meyer's study (2004) argues that expert teachers are capable of transferring their prior knowledge into practice so that they are able to focus on students and begin to attain flexibility in their experiences. On the other hand, novice teachers mostly tend to focus on the content of the lesson ruling out the students, have difficulty in noticing the important events in their classroom and in interpreting them correctly (Arditi, 2013). They also try to teach how they learned when they were students (Cooney, 1999). Teachers who have not gained experience and have not gotten the necessary guidance, struggle to reach success in schools and to increase students' achievement (Darling-Hammond, 1995).

Leinhardt (1993) conducted a study to investigate the differences between expert and novice teachers, particularly in planning and teaching. The result of this research shows that expert teachers are able to estimate students' difficulties related to the subject and consider their thinking and understanding in assessment. Inversely, novice teachers are not capable of attending to student thinking and stating what they did. In terms of implementation of the lesson, expert teachers usually take students' answers into consideration while shaping the lesson by adhering to its aim, whereas novice teachers are not able to focus on students' thinking to direct the lesson and consider its goals while deciding on the flow of the lesson. In the study of Livingston and Borko (1990), two preservice secondary mathematics teachers and their practice teachers' performance in schools were examined. The research stated that experienced teachers were capable of noticing key points, extending the content, integrating appropriate materials and estimating students' confusions. However, preservice teachers could not consider students' difficulties while responding them and could not reflect on most of the key concepts in the lesson. The reasons behind preservice teachers' failure were explained with their limited knowledge on students' learning, difficulties, misconceptions and reactions and also limited abilities in presenting different ways. There are also other studies which reached similar results (Zimmerlin & Nelson, 2000).

According to Alvine, Judson, Schein and Yoshida (2004) expert teachers are familiar with what kind of answers and reactions students might give and where they usually make mistakes in each lesson. That is why they know how to react for preventing these mistakes and providing effective instruction. However, novice teachers do not have enough experience to get knowledge about expected student responses, mistakes and difficulties, hence, it is not easy for them to manage the lessons considering these important points. In other words, they do not know how to cope with students in the face of their unexpected responses or questions and they fail to remove their common mistakes. Besides, Fuller (1996) compared novice and experienced elementary teachers' pedagogical knowledge and pedagogical content knowledge and she found that experienced teachers have more conceptual understanding than novice teachers but both groups believed that a good teacher tells students what to do. In Baki's study (2012), it was concluded that novice preservice teachers cannot make a proper time management and have difficulty in the organization of the lesson. According to Baki, they are not able to give students the necessary time for working on the questions and they lose a great deal of time and in the end there remains no time to complete the lesson. These studies reveal the general differences between expert and preservice teachers.

Although education standards are updated and reform movements have been included in the curriculums, most teachers have maintained their traditional approach in their lessons. Therefore, most of the preservice teachers have been impressed by their teachers' ways of teaching and their approach based on a range of rules and procedures in lessons (Doerger, 2003). According to Hiebert et. al. (1997), most novice teachers got used to learning by listening to their teachers passively and doing what was required by the teachers in contrast to the new structure of education which includes reflective participation, construction of knowledge and helping students to learn. It can cause inconsistency between novice teachers' habits of instruction and expectations in the directions of innovations in education (Carter, 2004). Brock and Grady (1997) indicate the properties of novice teachers as it follows; that they cannot focus on specific contents and try to attend to whatever works, have negative attitudes and cannot show flexibility unless necessary support is provided. Due to these facts, it is offered for professional development of preservice teachers in teaching profession to begin as soon as possible (Brock & Grady, 1997; Lambert, Collay, Dietz, Kent & Richert, 1996) in order to keep step with reform movements (Carter, 2004). Accordingly, it is suggested that the quality of teachers and the effectiveness of education can be increased with the aid of professional development programs during teacher training and at every point in their career (Darling-Hammond, 1995). Winstead-Fry (2007) defends that the adoption of effective professional development programs leading novice teachers to overcome the difficulties related to implementation, classroom management, students' learning and curriculum provide accumulation for teachers.

In Japan, new teachers are trained and supervised by expert teachers for a period of one year after they complete teacher training program (Stigler, Fernandez, Yoshida, 1996) and they are asked to join a professional development program (Stigler & Hiebert, 1999). On the other hand, in Turkey, graduated preservice teachers have their own classrooms right away and gain experience by finding their style over time. They participate in service training seminars for only two weeks, however, their contributions are inefficient for improving their teaching skills and increasing students' achievement because of the limited time and the lack of scope (Seferoğlu, 2001). The most important way to increase the effectiveness of education is firstly to secure the development of teachers in many forms (Hill, Rowan & Ball, 2005). Mathematics teaching courses are not adequate to create an environment which allows the transfer of theoretical knowledge into practice (Bransford, Brown & Cocking, 2000). At this point, school experience and teaching practice lessons are essential to constitute a link between theory and practice. Preservice teachers should improve themselves and gain more experience through professional development models such as lesson study in courses before starting teaching in their own classrooms due to the fact that particularly preservice teachers need guidance for doing better in their qualification, knowledge and skills (Korthagen, Loughran & Russell, 2006).

With the help of lesson study, novice teachers learn to anticipate student thinking by sharing ideas with expert teachers and peers and through investigating various resources (Alvine, Judson, Schein & Yoshida, 2004). At this point, this stage can be made easier for novice teachers and preservice teachers with getting experienced teachers' opinions and collaboration among the group members in professional development models (Alvine et al., 2004; O'Connor, 2010). It allows group members to reflect their ideas, learn from each other, enrich their knowledge and it facilitates taking a correct step towards experience (Chokshi & Fernandez, 2005). Lesson study has been conducted and investigated by many researchers (Choski & Fernandez, 2004; Fernandez, 2005; Lewis, 2000) and many studies advise that teaching settings which include discussion and production of the solution and reflection should be formed with preservice teachers (Putnam & Borko, 2000; Fernández, 2010).

With all these studies related to expert and novice teacher considered, it is seen that novice teachers' ability to notice is low and needs to be developed. It is difficult for them to focus on significant events in classroom interactions and important details, especially students' thinking, and making sense of what was seen to improve teaching and learning. Thereupon, preservice teachers should be provided with opportunities for developing their noticing skills since they are novice teachers as well. There is also need for studies related to preservice teachers' noticing of students' mathematical thinking because it is a critical issue for the growth of both teachers and students. Hence, it was aimed to investigate what and how preservice middle school mathematics teachers notice as they plan, teach and reflect the lessons through professional development support, reveal their abilities to notice students' mathematical thinking and examine the contribution of lesson study on their noticing skills' development if it was existing.

2.1.2 Teachers' noticing of students' mathematical thinking

In order to provide effective mathematics instruction teachers must have knowledge including some components. Shulman (1986) identifies knowledge that teachers need to have as (a) subject matter content knowledge, (b) pedagogical content knowledge and (c) curricular knowledge. Subject matter knowledge includes knowing domain-specific concepts, truths, rules and principles and understanding the structures composing the subject matter. Pedagogical content knowledge means knowing how to teach the subject matter. It includes knowing strategies to maintain effective student learning and understanding in teaching specific subjects, and having knowledge about students' misconceptions, existing knowledge, difficulties related to topics and ways to remove those barriers that prevent students' learning.

Curricular knowledge includes knowing relevant instructional materials and tools of subject, alternative curriculum materials and the use of them in teaching. Besides, it is the knowledge of relationship among the subject, relevant topics and class level (Shulman, 1986). Grossman (1990) expands on the scope of pedagogical content knowledge and indicates four significant components of it as (1) knowledge of goals of teaching a subject, (2) knowledge of students (e.g., students' thinking, understanding, existing knowledge, misconceptions and difficulties), (3) knowledge of curriculum, curricular material and resources and (4) knowledge of instructional strategies and representations for teaching a subject.

Knowledge of students is one of the important components for teaching mathematics effectively. Ball, Thames and Phelps (2008) emphasize that teachers must anticipate how students think, where their confusions occur and what draws their attention. They must pay attention to expected students' reactions and what might be difficult or easy for them when determining the tasks for teaching. They must also listen to students' ideas and interpret their thinking carefully. They need to have knowledge about students' mathematical understanding and thinking to be successful in these tasks. Similarly, NCTM (2001) states that focusing on students' thinking and ideas and designing the lessons based on it, is critical in terms of the quality of teaching, and making sense of their mathematical approach allows teachers to create settings that render better students' learning of mathematics. Several studies highlight that teachers' knowledge about and abilities to anticipate, interpret and use students' thinking in their practices help them to boost their students' achievement (Carpenter, Fennema & Franke,1996; Jacobs et al., 2010; Sowder, 2007). It aids in deciding how to respond to students and shape teaching on behalf of teachers appropriately, that is to say, it impacts instructional decisions (Nathan & Koedinger, 2000) and improves students' conceptual understanding (Chamberlin, 2002). However, teachers are neither good at predicting students' reactions, difficulties, misconceptions and what they can do (Goldsmith & Seago, 2011; Nathan & Koedinger, 2000; Kazemi & Franke, 2004), nor in tendency to attend to, anticipate and make sense of students' thinking (Chamberlin, 2002; Goldsmith & Seago, 2011; Jacobs et al., 2010; Kazemi & Franke, 2004; Wallach & Even, 2005) and incorporate it into the lesson and use it for making instructional decisions (Baş, 2013).

Many researches on noticing of students' mathematical thinking show that preservice and practicing teachers' ability to focus on and make sense of students' thinking and distinguish noteworthy events is poor (Sherin & Han, 2004; Sherin & van Es, 2009; van Es, 2011; van Es & Sherin, 2008). In these researches consisting of video-based professional development programs context, it was detected that teachers had tendency to focus on issues such as teachers' instructional approach and behaviors, classroom environment and classroom management instead of students' mathematical thinking, ideas and important aspects of their reasoning. Teachers' ability to notice and understand students' mathematical thinking can be reinforced with professional development programs including investigation, sense making and consideration of students' thinking (Carpenter, Fennema, & Franke, 1996; Sherin, Jacobs & Philipp, 2011; Sowder, 2007).

The concept of mathematical thinking corresponds to acting, working and exhibiting habits as having the mind of a mathematician (Henningsen & Stein, 1997). Researchers advocate that the development of students' mathematical thinking is substantial in terms of mathematics education and success (Ball & Bass, 2003). Focusing on mathematical thinking is important for helping students gain ability to learn mathematics for themselves (Isoda & Katagiri, 2012). Because one essential aim of teaching mathematics is to develop students' mathematical thinking, students should be directed to feel the necessity of this important component of mathematics from preschool to high school (NCTM, 2000). Ball and Bass (2003) indicate that learning to think mathematically is not enough, students should be allowed to elicit their thinking so that teachers can make sense of it.

Teachers should determine mathematical focus and preserve it to develop mathematical concepts and processes. It necessitates maintaining students' engagement with mathematical thinking and ideas rather than just asking them to make calculations (Anthony & Walshaw, 2009). One way for this is to learn students' possible misconceptions and thinking and examine how to incorporate them to lesson tasks in order for students to learn by making mistakes (Anthony & Walshaw, 2009). At this point, collaborative work might be more useful in terms of preparing better tasks for a lesson. Furthermore, revealing students' mathematical thinking is important for evaluating students' learning (Lesh, Hoover, Hole, Kelly, & Post, 2000), thus, teachers should promote students to share their ideas and strategies (Doerr, 2006) and ask questions instead of telling them right away (Mason & Johnston-Wilder, 2006). Listening to students' thinking is also as important as revealing it. Teachers should listen to what students say in a classroom setting carefully (Carpenter, Fennema & Franke, 1996; Davis, 1997; Mason & Johnston-Wilder, 2006), try to see mathematical concepts in their statements and actions and evaluate their mathematical approach with respect to accuracy (Schifter, 2001) in order to channel their attention to the important aspects of a lesson (Mason & Johnston-Wilder, 2006). On the other hand, it is critical to have the ability to make instructional decisions and know how to respond to students' thinking for better teaching but it is difficult for teachers (Herbal-Eisenmann & Breyfogle, 2005; Mason & Johnston-Wilder, 2006). According to Hiebert et. al. (2007), learning from practice requires the evaluation of teaching with the perspective based on student learning and teachers should have four skills which are (a) identifying the goals of instruction, (b) focusing on student thinking and learning, (c) making sense of what they focused in terms of teaching strategies and (d) analyzing student learning for improving. These skills are enclosed in inquiry stance (Hiebert et. al., 2007) and based on students' mathematical thinking (Hiebert et. al., 2007; Hiebert, Morris, Glass, 2003).

Teachers should focus on students' thinking embedded in their statements, works and classroom discourses and make sense of them from the perspective of mathematics instruction and broader educational principles before, during and after lesson to remove the difficulties encountered in the process and provide effective instruction (Goldsmith & Seago, 2013; Jacobs et al., 2010; Schifter, 2001). The actions of attending to, making sense of and making decisions correspond to the concept of noticing (Mason, 2002; Sherin, Jacobs & Philipp, 2011). Expert teachers are able to know where to focus on and make connection between what they see and what they know so that they can make effective decisions on improving instruction in the midst of a complex classroom environment (Berliner, 2001). This skill demands them to be active and abide by the purpose of the lesson on the contrary of being passive and acting spontaneously (Erickson, 2011; Miller, 2011; Sherin, Jacobs & Philipp, 2011) and it also enables them to benefit from existing knowledge and experience to shape the flow of a lesson (Schön, 1991).

In recent years, mathematics educators have comprehended the importance of students' mathematical thinking towards specific subject matter and teachers' ability to reveal students' thinking and considering it in lessons (Grouws, 1992; Lester, 2007; NCTM, 2000) as building on how students think provides effective instruction (Schifter, 1998; Sowder, 2007) and it increases students' achievement (Carpenter et al., 1989; Fennema et al., 1996; Jacobs et al., 2007). The research shows that teachers can learn how to benefit from students' thinking for better instruction with professional development support and maintain this experience even though they have no more support (Franke et al., 2001). In order to build better teaching and learning instruction on students' thinking, teachers must be able to focus on students' strategies, interpret what and how they understand and use this knowledge to make decisions about instruction (Jacobs et. al., 2010). Jacobs, Lamb and Philipp (2010) call this ability which is significant in both teachers' and students' development, as teacher noticing of students' mathematical thinking.

In literature, some studies focused on teachers' noticing of students' mathematical thinking and how to develop this skill but they lacked in number (Kazemi & Franke, 2004; Liu, 2014, Jacobs et al., 2010; Walkoe, 2013). In the study of Jacobs et. al. (2010), it was aimed to investigate noticing of students' mathematical thinking with 131 prospective and practicing teachers through cross-sectional research. Two written measures were used as data collection tools, to assess the three component skills of attending, interpreting and deciding how to respond. It was found that prospective teachers had difficulty in doing what was crucial in these noticing subskills- attending, interpreting and deciding how to respond- of students' mathematical thinking and most of them were not successful in engaging with students' thinking. In terms of the first component, prospective teachers and teachers were not good at attending to children's strategies. This skill includes focusing on significant events within classroom interactions, knowing important mathematical points and selecting them from students' various explanations. Attending to students' strategies might be difficult for teachers since dealing with many things simultaneously is not an easy action. To speak of the second subskill, practicing teachers were better than prospective teachers in interpreting students' understanding. Prospective teachers could not focus on and make sense of how students understand, they further referred to the issues such as the use of tools, alternative ways for solving problems and teachers' actions. It might be derived from the lack of focusing on students' thinking and mathematical knowledge. Deciding how to respond, the third component, consists of selecting the next problem and identifying the next step considering students' mathematical development. Shaping the instruction based on students' understanding is important but it is a complex skill on behalf of teachers. Professional developers should give importance to the improvement of expertise in attending to students' thinking, interpreting it and benefiting from it for instruction (Jacobs et. al., 2010).

Similar to Jacobs et al., Kazemi and Franke (2004) also conducted a research using students' work. The aim was to explore whether the examination of students' work provides development of teachers' understanding and making sense of students' mathematical thinking in a setting based on collaboration with colleagues. In this study, ten teachers met once a month, shared students' work with each other and teachers elaborated on the students' mathematical strategies they noticed while analyzing their works. The results showed that teachers' knowledge about students' mathematical thinking increased in this process and they began to attend more to students' thinking, reveal how they think and understand students' learning.

Walkoe (2013) carried out a study related to noticing in specific mathematics domains and investigated teachers' noticing of students' algebraic thinking. The study was conducted in a video club context as in the research of van Es and Sherin mentioned before. Seven preservice teachers watched videos from algebra classes and discussed on what they noticed during eight weeks. A framework for noticing student algebraic thinking in the video club was created to shape discussions. The results showed that video clubs based on the use of algebraic thinking framework developed teachers' noticing of moments related to students' algebraic thinking deeply. Teachers who were in the video club group interpreted algebraic thinking more in detail compared with the non-video club group.

Liu (2014) investigated how teachers' in-the-moment noticing of students' mathematical thinking occurs while teaching a unit from a reform-based mathematics curriculum. Differently from other studies, it was explored to what extent and how teachers notice in the midst of instruction, not out of context or after the fact as in video clubs or analysis of written works. In the study, the researcher used a new technology which allows two teachers to capture their noticing during their real teaching. The data was obtained from initial interviews, researcher-generated and teacher-generated videotaping and follow-up interviews. It was found that teachers focused on a range of issues as well as students' thinking, and to what extent they noticed was different from each other. Besides, it was argued that teachers' knowledge, beliefs and aims affected their noticing of student thinking.

Research on teachers' ability to notice students' mathematical thinking suggests that their abilities should be developed with support, especially for novice teachers. Besides, further studies are needed with regard to teachers' noticing of students' mathematical thinking because effective mathematics teaching and student achievement are related to being able to notice students' mathematical thinking and shape the flow of instruction considering it appropriately (Jacobs et al., 2010). Likewise, reform-based mathematics education emphasizes engagement with students to understand their thinking and build understanding of mathematics (Maccini & Gagnon, 2002). Teachers' noticing skills of students' mathematical thinking takes place by engaging with students in real classroom settings through professional development which allows focusing on the ways of students' thinking and discussing on them (Jacobs et al., 2010). In other words, teachers become well equipped when they work with students directly, examine their work, collaborate and share ideas with colleagues and instructors about how students think and encounter activities that enable to understand students' thinking better (Jacobs et al., 2010; Sherin, Jacobs & Philipp, 2011). Hence, these kind of opportunities should be provided for preservice teachers to improve their noticing expertise (Jacobs et al., 2010). The knowledge of students' thinking facilitates to evaluate what students understand and learn (Jacobs & Phillip, 2010) but having this knowledge is not easy, it takes time and requires experience (Jacobs et al., 2010). Therefore, teacher educators should teach preservice teachers what and how to notice and improve their skills of noticing of students' mathematical thinking for the reason that preservice teachers' ability to notice students' thinking is poor and they are in need of support and guidance (Sherin, Jacobs & Philipp, 2011).

One of the methods for attending to and interpreting students' thinking is using classroom artefacts (Chamberlin, 2005; Goldsmith & Seago, 2011; van Es & Sherin, 2008) such as students' written work (Ball & Cohen, 1999; Goldsmith & Seago, 2011, 2013; Hiebert et. al., 2003), video clips (Miller & Zhou, 2007; van Es & Sherin, 2008), and students' discussion transcripts (Davis, 1997). These artefacts enable teachers to gain insight on different strategies but they are not convenient for reflecting student thinking and might not obtain sufficient development of practice (Choy, 2015). Besides, knowledge about how teachers' improvement can be maintained in terms of attending to and analyzing students' thinking with these kind of practices might not be enough (Chamberlin, 2005; Hiebert et al., 2007). It is suggested that teachers should have the opportunity to reflect and learn from their own practice for mathematics instruction in accordance with reform movements. In order for that to happen, teachers need to focus on students' thinking and learn about their knowledge and ideas. They must be able to both evaluate their practice by investigating what works and what does not and practice what is learned in order to analyze, refine and improve instruction (Ball & Cohen, 1999). Likewise, the results of many studies indicate that teachers should analyze their own teaching considering students' mathematical thinking (Carpenter et. al., 1996; Carpenter et al., 1999; Franke et. al., 2001) and should participate in implementations including reflection such as lesson study to reflect on instruction (Kullberg, Runesson & Martensson, 2013) since reflection makes teachers' practices more effective (Fernandez et. al., 2003).

Sowder (2007) points that professional development programs should give insight into students' mathematical thinking and learning. Professional development programs including attention to students' thinking aim to train and improve teachers in professional sense through promoting them to investigate, interpret and benefit from students' thinking obtained from lessons, observations, students' works or videos of teaching. Another goal in these programs is to reduce teachers' resistance to change and prompt them for changing their instructional approach to elicit better teaching and learning (Baş, 2013) because it is defended that teachers must promote their knowledge and beliefs based on students' thinking (Carpenter et al., 1989; Nathan & Koedinger, 2000). However, Baş (2013) indicates that how professional development programs support teachers' noticing of students' mathematical thinking is not clear. Similarly, Mason (2002) emphasizes the need for examining teachers' noticing of students' thinking in order to provide improvement of their knowledge and instructional practices. In consequence, it was explored how preservice teachers were supported for focusing on and making sense of students' mathematical thinking in the context of lesson study professional development model to improve instruction. In other words, it was aimed to investigate what and how preservice teachers noticed during planning, teaching and reflecting.

2.1.3 Attending to students' mathematical thinking through lesson study

In a recent research, it was indicated that teachers' knowledge of students' thinking helps to enhance their learning (Hiebert et al., 1997; Ma, 1999; NCTM, 2000; Shulman, 1986). In terms of providing effective instruction based on standards and reform ideas, focusing on content, students' current knowledge and how to incorporate it into lesson (Darling-Hammond et al, 2008), considering students' thinking (Lampert, 2001) and how to improve them is significant (NRC, 2001). Thus, knowledge of content, pedagogy and students' thinking have an important role in creating high quality instruction.

Researches on mathematics education emphasize that teachers improve students' learning, specifically, instruction through focusing on students' thinking (Carpenter et al., 1989; Fennema et al., 1996; Franke et al., 1998). However, teachers are not in tendency to consider students' thinking about the topic while planning the lesson, rather they attend to less significant aspects like motivation, participation, and interest (Brown, 1988). Sisofo (2010) advocates that if teachers gain or develop the ability to focus on students' thinking and integrating them into their lesson design, teachers' success in mathematics teaching increases. Expert teachers are good at aiding students to understand a new topic because they know how to respond to students and use their knowledge of student thinking and subject matter effectively (Hiebert et al., 1997; Ma, 1999). It is believed that preservice teachers might not be able to plan, teach or revise the lesson by attending to students' thinking as much as expert teachers can do, however, they can gain this perspective by experiencing beforehand (Sisofo, 2010). That is why, it is suggested that preservice teachers should be supported to learn using students' thinking in their lessons while they are attending teacher education programs.

Sisofo (2010) indicates that there are three conjectures about preservice teachers' learning based on review of literature and explains them as follows; according to the first conjecture, preservice teachers' learning is based on their prior knowledge as everyone else's. Therefore, it is emphasized that learning which includes gaining new skills, getting necessary knowledge to be taught and impacting beliefs positively must be generated for preservice teachers (Ball & Cohen, 1999). The second conjecture mentions that preservice teachers learn through working on teaching tasks (Darling-Hammond & Sykes, 1999; Putnam & Borko, 2000). Their learning increases when the tasks particularly include attending to students' thinking and mathematical content (Darling-Hammond & Sykes, 1999). The third conjecture indicates that preservice teachers' learning should be based on collaboration among colleagues. Socio-cognitive theory of learning which creates a community of practice environment that provides opportunity for all preservice teachers to share, reflect, discuss and learn about complex instructional issues is supported by many researchers (Ball & Cohen, 1999; Putnam & Borko, 2000). Lesson study is a professional development model which combines these three conjectures for preservice teachers and students' thinking (Sisofo, 2010). Lesson study promotes teachers to consider students' thinking while making decisions about the lesson during the whole process before and after teaching (Fernandez, 2005; Perry & Lewis, 2009; Taylor et al., 2005). Teachers in U.S. and Japan also indicate that lesson study administers teachers to develop their teaching because it prompts them to focus on students' thinking and decide how to manage the flow of lesson using this knowledge (Research for Better Schools, 2002). Thereupon, Sisofo (2010) emphasizes that lesson study can be used to improve preservice teachers' learning and the skills of teaching based on students' thinking in teacher training programs.

In order to be effective in teaching mathematics, teachers must have knowledge of mathematical content, knowledge of student learning and thinking about mathematics (Ball et al., 2001; Fennema & Franke, 1992; Shulman, 1986), need to know about students' current mathematical knowledge, how they learn new mathematics topics, what their mathematical approach, solution strategies, misconceptions towards particular mathematical subjects might be (Hughes, 2006). Schifter (2001) indicates that teachers should consider some important points in their practice which are (1) students' expressions and actions reflecting mathematics (2) whether students' thinking is valid or not in terms of mathematical content (3) mistakes in students' mathematical approach and (4) conceptual issues that students engage in. Teachers need to focus on how students think mathematically, have the ability to make sense of their ways of thinking and based on their thinking they must decide what to do for effective instruction (Hughes, 2006) which refers to teachers' noticing skills (Jacobs et al., 2010). However, most teachers do not have knowledge of students' thinking in specific mathematical content (Ball et al., 2001; Ma, 1999) and although they know how students think, they have difficulty in incorporating it into practice (Ball, 2001; Ball et al., 2001; Lampert, 2001; Schifter, 2001; Schoenfeld, 1998). Expert teachers are able to benefit from students' thinking in their teaching (Lampert, 2001; Leinhardt, 1993; Schoenfeld, 1998) whereas it is not likely for novice teachers Heaton, 2000; Schoenfeld, 1998; Zimmerlin & Nelson, 2000). Therefore, preservice teachers need to be supported through professional education for gaining an instructional approach based on attention to students' mathematical thinking (Hughes, 2006).

Lesson study is one of the professional development models which requires to focus on students' thinking to improve their learning and understanding of content and teaching of the lesson (Hughes, 2006). Lesson study includes the main steps of planning, teaching and reflecting which are all based on students' mathematical thinking. Teachers anticipate students' solution strategies, thoughts and responses that might appear when they encounter questions or new knowledge and they endeavor to ask questions which students could reflect their thinking on and allow them to get rid of their misconceptions during lesson study (Stigler & Hiebert, 1999). Attending to students' thinking while planning and teaching is significant for the improvement of instruction and students' success (Lampert, 2001; Lewis & Tsuchida, 1998; Stigler & Hiebert, 1999). At this point, the results of many researches argue that lesson study which helps focus on student thinking develops science and mathematics education, teachers' ability to teach and achievement of students (Lewis & Tsuchida, 1998; Watanabe, 2002; Yoshida, 1999).

Mathematical thinking entails the use of mathematical techniques, concepts and connections in problem solving (Henderson, Fritz, et al., 2002). Middle school mathematics curriculum indicates that the process of discovery, finding logical relationships and expressing them with mathematical terms underlies mathematical thinking and identifies mathematical thinking as presenting concrete relationships with abstract notations and making generalization (MoNE, 2005). For facilitating students to learn mathematics effectively, teachers must give importance to students' mathematical thinking and integrate it into their teaching practice (Hughes, 2006; MoNE, 2005; Stacey, 2006). According to research towards teachers' focus on students' thinking, the ways to develop students' mathematical thinking are: understanding the concepts that students will construct during lesson (Fernandez & Yoshida, 2004; Lampert, 2001; Leinhardt & Steele, 2005; Masingila & Doerr, 2002; Schifter, 1998), requiring various strategies from students to solve problems (Fennema et al., 1996; Stein et al., 2006), anticipating students' responses and considering their misconceptions (Fernandez & Yoshida, 2004; Masingila & Doerr, 2002; Sherin & Han, 2004; Hughes, 2006), asking questions to students in order to make sense of and promote their own mathematical thinking (Fennema et al., 1996; Fernandez & Yoshida, 2004; Kazemi & Franke, 2004; Masingila & Doerr, 2002). Lesson study includes all these suggestions in the process of planning the lesson, teaching it and reflecting on and refining the lesson. It is argued that teachers who are enrolled in lesson study practice begin to address students' mathematical thinking by determining the goals of the lesson, linking previous knowledge with the relevant topic to be taught, anticipating possible students' responses and promoting to reveal students' strategies in the lesson that they have planned with other colleagues (Fernandez et al., 2003).

Planning and teaching are interrelated with each other powerfully and plans considering students' thinking in-depth have an important role in forming the flow and content of teaching (Huges, 2006). In the study of Fernandez and Yoshida (2004), one Japanese teacher emphasized the benefits of preparing detailed lesson plans as focusing on students' strategies and deciding how to respond to them enables teachers to form effective lesson plans and decrease the anxiety due to teachers' feeling more confident for teaching. It encourages making sense of students' understanding on behalf of teachers during teaching and making instructional decisions to lead students to consider their reactions in order to accomplish the objectives of a lesson. Japanese teachers suggest that while planning the lessons, teachers should have the students' perspective which means seeing the lessons through their eyes (Fernandez & Yoshida, 2004). According to Leinhardt and Steele (2005), focusing on students thinking and using it as planning, teaching and reflecting on the lesson amends teachers to create standards-based instruction and environment including rich mathematical discussions that allow for effective teaching and learning. Schoenfeld (1998) claims that the content of the lesson and way of designing it differ depending on teachers and contexts and it influences teaching practice in the classroom substantially. Hence, there is a significant relationship between planning and teaching and also reflection to improve the lesson is shaped by the data obtained from these processes. The steps of lesson study allow for constitution of a setting which promotes teachers to focus on students' thinking and make use of it to improve both teaching and students' learning so that they can enrich their instructional skills. Many studies show that there is difference between expert and novice teachers (preservice teachers) in terms of being able to use these kind of skills. That is why, it is thought that integrating lesson study professional development model into their teacher education programs might provide them with better preparation and implementation, lets them learn how to improve their lessons and develop their noticing skill.

Fernandez, Cannon and Chokshi (2003) investigated whether lesson study impacts teachers' focus on students' thinking by conducting a research with sixteen American teachers and twelve Japanese teachers that have the role of a coach in the lesson study process. According to the results, although American teachers usually focused on different issues related to the lesson instead of students' thinking, they began to head towards how students think mathematically with the guidance of Japanese teachers and considered this knowledge while planning the lesson. Accordingly, Fernandez et. al. (2003) emphasized that lesson study can be effective in assisting teachers to attend to students' mathematical thinking but they also need the help of an expert to catch the important points. In the study of Wagner (2003), the aim was to research eighteen preservice teachers' views about the critical factors affecting the design of the lesson before participation in lesson study and how they perceived important factors with participation in lesson study. In addition, the impact of lesson study on preservice teachers' focus on students' thinking was examined to make use of it in planning and revision. The results of the study showed that preservice teachers signified many factors that were relevant to students' thinking and other aspects of the lesson but their comments were procedural before participating in the lesson study process. The focus shifted to students' learning rather than teachers' behaviors during the lesson study. Preservice teachers began to indicate anticipation to and use of students' thinking in planning of a lesson as important factors with participation in lesson study whereas they did not express focusing on students' solution strategies before lesson study. It was argued that lesson study directed preservice teachers' attention to pedagogical content knowledge but they were not always successful in using this knowledge to improve their understanding of the students (Wagner, 2003).

Perry and Lewis (2009) carried out a case study investigating what happened with the use of lesson study in the same district for more than four years (Spring 2000-Fall 2004) and through data from interviews he discussed teachers' views about changes in their implementations with participation in the lesson study. The changes were designing tasks that lead students to think and reveal their thinking, working on these tasks for better understanding, promoting students to share various solution strategies by providing opportunities for rich mathematical discourse, focusing on expected students' thinking, thus shaping the lesson considering it and discussing on observations and students' thinking collaboratively. Taylor et. al. (2005) worked with one lesson study group consisting of four elementary mathematics teachers and one teacher educator. Teachers stated the benefits of lesson study as in the following: coming together with colleagues regularly to share ideas, improving practice with the feedback from the others and gaining perspective to attend to students and learning rather than teacher and teaching. Therefore, they endeavored to focus on students' solution strategies, listen to their responses and direct the lesson in accordance with students' thinking. Besides, in the study of Fernandez (2005) conducted with elementary mathematics teachers in a lesson study group, the aim was to investigate the benefits of lesson study on teachers' learning of mathematics. It was argued that lesson study enabled teachers to design the lesson collaboratively discussing on the tasks to ask, when and how to use them, possible students' thinking at different levels of age, students' misconceptions and ways to correct them. The results of these studies show that lesson study supports attention to students' thinking to make decisions for better instruction, provides opportunity for reflective analysis of the process and improves teachers' content and pedagogical knowledge (Riales, 2011). Thus, it can be effective in improving teachers' ability to notice students' mathematical thinking.

Researchers do some studies and build professional development programs to support teachers and preservice teachers in attending to students' thinking and understanding them. One of them is the project of Teaching to the Big Ideas- TBI which took four years. Thirty-six teachers who enrolled in this project analyzed students' mathematical understanding, where misconceptions occurred in their own classrooms and discussed these points coming together. Based on the data obtained from the observation on two teachers during their lessons related to fractions, Schifter (1998) stated that the process developed teachers' ability to focus on students' reactions, listen to their responses and make sense of them. In the study of Kazemi and Franke (2004), primary school teachers asked the same mathematical problems in their lessons and attended to students' answers and solution methods. Later, teachers met to share what kind of responses were given by the students and how students' strategies were and they discussed on the similarities and differences in the lessons in terms of students' mathematical thinking. It was seen that at the beginning of the process, teachers' approach on students' answers were evaluative in terms of success rather than interpretive comments towards understanding them. However, it was argued that teachers could focus on students' thinking and comprehend their ways of solution in times. Kazemi and Franke (2004) indicate that

“Focusing on students' mathematical thinking remains a powerful mechanism for bringing pedagogy, mathematics, and student understanding together. As teachers struggle to make sense of their students' thinking and engage in practical inquiry, they elaborate on how problems are posed, questions are asked, interactions proceed, mathematical goals are accomplished, and learning develops” (p. 108). Steinberg et. al. (2004) focused on one teacher who participated in a professional development program, Cognitively Guided Instruction-CGI, and analyzed the interaction between the teacher and students, mathematical discourses in the classroom and how the teacher engaged in students’ thinking. It was found that the teacher was asking students to explain their solutions but she was in tendency to listen to their answers without inquiry and was not comfortable when she encountered different strategies initially. Although her knowledge of students’ thinking was poor, she began to create richer mathematical discourses considering students’ various approaches later in the period. It was seen that the teacher learned to reveal students’ thinking and reflect this knowledge upon his instructional practice.

Similarly, in national studies, Türnüklü and Yeşildere (2007) conducted a research with forty-five preservice primary mathematics teachers to reveal their knowledge about students’ mathematical thinking in specific content area including fractions, decimals and integers. Four problems were given for preservice teachers to examine, make sense and estimate students’ misconceptions, mistakes, reasoning and suggest ways to remove their incorrect approaches. The result of the study showed that preservice teachers had difficulty in determining the students’ misconceptions and mistakes, finding the reasons behind their difficulties, understanding their ways of thinking, evaluating students’ mathematical knowledge because of their insufficient knowledge and proposing solutions for misconceptions and difficulties. In the study of Akkoç and Yeşildere (2010), development of two preservice teachers’ pedagogical content knowledge in the subject of number patterns was investigated through micro teaching activities in the context of school practicum course. In this process, they observed the mentors’ lessons on numbers pattern, analyzed these lessons in terms of how teachers referred to students’ misconceptions and difficulties, what they used as strategies and how they evaluated students’ mathematical learning and discussed on these issues. Later, they planned and taught the micro-teaching lessons with modelling mentors. It was found that observations and discussions on lessons in the context of school practicum course provided the development of preservice teachers’ pedagogical content knowledge about students’ understanding, difficulties and misconceptions. They began to anticipate students’ thinking and address them in their lessons. Özdemir and Atalay (2016) conducted a study with twenty preservice primary teachers to examine their ability in revealing and making sense of students' mathematical thinking through requiring them to interview with the students regarding fractions and write a report on it. It was argued that preservice teachers were not successful in revealing students' thinking by asking appropriate questions and interpreting their strategies. They were not able to focus on students' thinking or responses and they could not inquire the reasons behind what students did and said in order to understand their mathematical thinking. Besides, preservice teachers' interpretations about students' mathematical thinking were general and superficial and they focused on the correctness of their answers rather than the reasons behind their reasoning. Likewise, the studies in literature support that teachers are in tendency to evaluate students' responses in terms of accuracy (Crespo, 2000; Kazemi & Franke, 2004) and both teachers and preservice teachers have difficulty in interpreting and making sense of their mathematical thinking (Empson & Junk, 2004; Türnüklü & Yeşildere, 2007; Wallack & Even, 2005). The researchers suggested that preservice teachers should gain more experiences and have more settings to think and discuss on their experiences in order to gain these kinds of skills, hence, in-service and preservice training should be structured in a manner to support this development.

During the recent years, teachers' emphasis on attention, comprehension and elaboration on students' thinking and constructing their teaching practice in the direction of this knowledge has accelerated in mathematics education (Ball & Cohen, 1999; Sowder, 2007). NCTM (1991) indicates that mathematics teachers should reveal and give importance to students' thinking and ideas to improve their mathematical power because designing lessons based upon students' mathematical approach increases their achievement (Baş, 2013). However, teachers are not in tendency to focus on and makes sense of students' thinking (Kazemi & Franke, 2004). It is difficult for them since students use various ways for expressing their solutions and ideas and teachers might not be used to them (Ball, 1997). Struggling with many students' strategies and ideas might be overwhelming for teachers (Ball, 2001). In addition, the concern of reflecting what the curriculum requires (Ball, 1997) and teachers' knowledge of mathematics, beliefs and experiences might hinder their effort for focusing on students' thinking (Goldsmith & Seago, 2011). It is also advocated that one of these reasons is teachers' habits of evaluating and interpreting the issues about students quickly and superficially (Kazemi & Franke, 2004; Rodger, 2002; Sherin & van Es, 2009; van Es, 2011; van Es & Sherin, 2008). Furthermore, it is not easy for teachers to isolate their perspective completely which as a result makes it difficult for them to understand what students really say, how they think and see from their point of view (Wallach & Even, 2005).

There is a demand of professional development support for improving teachers' ability to focus on, make sense of and build their teaching practice on students' mathematical thinking (Jacobs et al., 2010) considering the ability to notice does not exist in teachers' nature (Ball & Cohen, 1999; Sowder, 2007) and only professional effort can develop it (Goldsmith & Seago, 2011; Kazemi & Franke, 2004; Sherin & Han, 2004; Sherin & van Es, 2009; van Es, 2011; van Es & Sherin, 2008).

Some ways, such as analyzing video clips of teaching in a real classroom, examining students' written work and discussing teachers' notes of observation, are recommended to increase teachers' attention to students' thinking and enable them to make sense of it (Ball, 1997; Ball & Cohen, 1999; Chamberlin, 2002, 2005; Goldsmith & Seago, 2011; Kazemi & Franke, 2004; van Es, 2011; van Es & Sherin, 2008). Although these kinds of activities are important, they are not enough on their own. One professional development model that promotes teachers' attending to students' thinking, carries opportunities for understanding students' strategies and allows to design the instruction in accordance with this information, which refers to noticing, by combining those kind of suggested activities is lesson study. Consequently, lesson study professional development model was used in this study to investigate preservice teachers' noticing of students' mathematical thinking.

2.2 Lesson Study as an Effective Professional Development Model

After the results that showed good mathematics achievement of Japanese students from the exams such as TIMMS and PISA drew attention, the reasons behind this success were an object of interest and the focus on Japanese teachers' instructional strategies of teaching and learning increased (House & Telese, 2008). The change in their instructional approach from "teaching as telling" to "teaching for understanding" laid the foundation of the success (Lewis & Tsuchida, 1998). In this approach, an education system which ensures meaningful advances in instruction at the right time is used (Burgess, 2006). Lesson study which is a professional development model adopted in this perspective is seen as the most important factor that impacted the achievement in Japan. Because, in other words, it is the reform research model in which teachers endeavor to acquire effectiveness of their teaching practice and learning experiences that are reflected to students (Takahashi & Yoshida, 2004; Yoshida, 1999). Wang-Iverson and Yoshida (2005) indicated that "Japanese lesson study is the core process of professional learning that Japanese teachers use to continually improve the quality of the educational experiences they provide to their students" (p. 3). In general terms, Lesson study is a school based cycle which includes teachers' working collaboratively, collection of data related to student learning (Takahashi & Yoshida 2004; Yoshida, 1999), determination of educational long-term goals for students, teacher-led instruction to achieve these goals in the classroom, collective observation and discussion on the lessons and adopting inquiry-based teaching in practice (Fernandez, 2002; Lewis & Tsuchida, 1998; Stigler & Hiebert,1999). Lesson study supports ongoing development in instruction (Stigler & Hiebert, 1999) through connecting learning of teachers and students and focusing on how to improve daily teaching practices in a way to provide change in student achievement in a positive manner (Takahashi & Yoshida 2004; Yoshida, 1999). The strong side of it is holding based on both a teacher-led and student-focused structure (Lewis, 2002; Stigler, & Hiebert, 1999; Yoshida, 1999).

In literature, there are a range of definitions and types of expressions regarding what lesson study is. Although definitions of lesson study vary somewhat across studies, meaning of them refer to common essential components and features of lesson study in general. Stigler and Hiebert (1999) define lesson study as a problem solving process that maintains improvement in teaching for an extended period of time. Lesson study is seen as the catalyst for change with the shift to the perspective of teaching for understanding (Lewis & Tsuchida, 1997; Lewis & Tsuchida, 1998; Lewis, 2000; Lewis, Perry & Murata, 2006) and it is the broad innovation that allows for practice-based professional development (Stigler & Hiebert, 1999). Boss (2002) indicates that lesson study is a professional development process that builds teachers' instructional skills and connections across grade levels and content area. It is also defined as a school-based professional development model including collaborative efforts of teachers to improve teaching and learning in their classrooms (Stigler & Hiebert, 1999). Furthermore, lesson study is a process that enables teachers to analyze effectiveness of their own teaching in order to reach the aimed learning goals (Fernandez 2002; Fernandez et al., 2003; Lewis, 2000; Lewis & Tsuchida, 1998) and it is a practice that improves students' learning through enriched instruction (Fernandez &Yoshida, 2004; Lewis 2002; Stigler & Hiebert, 1999). In general manner, lesson study is a complex, iterative and Japanese culture originated professional development implementation (Stepanek, 2001; Stigler & Hiebert, 1999) that brings educators together to improve their own practice systematically in collaboration with colleagues (Stepanek, 2001). Lesson study "is a method of professional development that encourages teachers to reflect on their teaching practice through a cyclical process of collaborative lesson planning, lesson observation, and examination of student learning" (Lenski & Caskey, 2010, p. 44). According to Lewis (2006) "Lesson study is not a single, uniform practice, any more than teaching is and takes diverse forms" (p.6). Those variations in definitions and implementations of lesson study may cause misconceptions about its meaning and content. Wang-Iverson and Yoshida (2005, p. 5), elaborate what lesson study is and is not in order to prevent misinterpretations as the following:

Lesson study is:

1. Teacher-led ongoing professional learning
2. Conducted with a common overarching goal
3. Focused on subject content in the context of student thinking
4. Informed by outside expertise through the knowledge of others

Lesson study is not:

1. Teacher training
2. About creating a perfect lesson
3. Done in isolation
4. Doing just one lesson study cycle

Lesson study is a collaborative process in which, respectively, 3-6 teachers come together and form lesson study group (Fernandez, 2002), identify instructional goals and make investigation on it, prepare a detailed lesson plan, one of the teachers from lesson study group teaches the lesson considering a plan in a real classroom while other teachers in a group observe the lesson collecting information on student thinking and performance (Harle, 2008); all of them discuss on what they observed and reflect their ideas about the lessons subsequently, evaluate what worked or what did not work for the students and revise the lesson in the direction of teachers' suggestions. If it is needed, another teacher implements the revised lesson in a different classroom as observation is made by the other teachers. Later, the group members come together to discuss on the lesson again and put the lesson into its final form. A report is written to share the results and the lesson plans with the school (Stigler & Hiebert, 1999; Yoshida, 1999).

Lesson study draws interest because it is different from ineffective and typical professional development models with its essential characteristics. Yoshida (1999) outlines the characteristics of lesson study as the followings:

1) Lesson study is conducted in a real classroom environment that enables teachers to observe both teaching and learning. It presents a collaborative setting in which they evaluate what they discussed throughout the whole process including the preparation

of plan, implementation, observation and reflection so that they can determine the needs for good instruction.

2) Lesson study is a student-centered approach, wherefore, it allows teachers to observe students during the lesson in a classroom and discuss the implementation.
3) Lesson study is a teacher-lead activity in which instructional changes and curriculum development help teachers with their guidance.

According to Lewis (2002), lesson study exhibits four essential features which are (a) determination of long-term goals by teachers concurringly, (b) identification of subject-specific goals, (c) attention to student learning and the development of it (d) observation of the lessons by teachers. There are also several studies which indicate similar certain features of lesson study (Takahashi & Yoshida, 2004; Lewis, 2000; Lewis & Hurd, 2011).

Lesson study necessitates a long span of time since it includes planning, implementation, observation, discussion and evaluation collectively during the process. It is a sustained professional development based on ongoing support of educators, instructors and colleagues. Ongoing supports and feedbacks are critical to maintain success in learning and lead teachers to use necessary knowledge of content and instructional strategies and their skills in new experiences (Harle, 2008). Lesson study necessitates teachers' working collaboratively in a small group during all the phases of it. It is recommended that teachers who form lesson study group should be from the same school, department or grade level to provide high quality learning (Garet et al., 2001). Collaboration promotes productive discussion and sharing ideas in a way that this communication setting impacts change in the teaching practice of teachers (Garet et al., 2001). In lesson study, teachers' discussion on their observation based on collecting data about how students think and what they do and reflection on instructional practices facilitate the evaluation of the poor and powerful aspects of the lesson and its revision (Fernandez & Yoshida, 2004). West-Olatunji, Behar-Horenstein, and Rant (2008) define lesson study as "a form of reflective teaching that uses collaborative dialogue to engage teachers in a collective assessment of their classroom practices" (p. 97). Collaborative efforts and reflective context in lesson study increase experiences in both teaching and learning (Harle, 2008).

Lesson study is a subject specific model and particular content and objectives are focused on. It is defended that focus on content knowledge improves instructional practices and increases students' success (Garet et al., 2001), predominantly, it contributes to the process of teaching and learning. Teachers' subject matter knowledge may not be sufficient enough in practice to evaluate student understanding (Borko & Putnam, 1996). Lesson study also involves asking meaningful questions which can reveal student thinking and make suggestions towards the needs of student learning. As teachers are planning the lesson, they endeavor to integrate student perspectives, anticipate student thinking, design activities and ask questions which empower students to be active and involve in interaction with teacher and peers (Lewis, 2004). One of the essential features of lesson study is enabling teachers to observe all students' learning during the lesson in the classroom. It distinguishes lesson study from the other professional development models and programs. Observations based on student thinking, learning and reactions shape the reflection phase and permit to evaluate and make necessary changes (Harle, 2008). Because lesson study incorporates sustained and collaborative structure and inquiry-based and subject-specific learning, it is seen as a process which also improves teachers' knowledge (Garet et al., 2001). Besides, many studies argue that lesson study has a direct influence on teachers' teaching practice and students' achievement (Chokshi & Fernandez, 2004; Lewis, 2002; Takahashi et al., 2005).

Japanese teachers have detailed knowledge of how students think and understand and benefit from this information in planning and teaching the lesson (Lewis & Tsuchida, 1998; Stigler & Hiebert, 1999; Yoshida, 1999). The shift to student centered approach in Japan is named as lesson study (Lewis & Tsuchida, 1998). In general, lesson study is a professional development model that involves planning, teaching and reflecting. It begins through planning based on collaboration, rich effort and a significant amount of time. In this phase, teachers take students` thinking into consideration while determining the content. After a lesson plan is completed, one member from a small group of teachers teaches the lesson and the rest of the group observe the implementation. Reflection process occurs after the lesson and the teacher who implemented the plan gives a feedback firstly and then the other teachers share their ideas. Optionally, another teacher from the group implements the revised lesson plan and the lesson plan is put into its final form (Hughes, 2006). The literature generally identifies the stages of the lesson study cycle as planning the lesson plan collaboratively, implementing the lesson plan, discussing on the implemented lesson, revising the lesson plan, teaching the revised lesson plan and sharing reflections (Lewis & Hurd, 2011; Yoshida, 1999; Fernandez, 2002).

Lesson study cycle lets teachers focus on achieving the goals of a specific lesson reviewing content area and supports the organization of the lesson considering students` misconceptions, difficulties and needs. At this process, teachers are expected to use various sources and to determine appropriate instructional strategies, activities and materials in terms of students' needs. When teachers pay attention to applying these suggestions, their subject area contents naturally develop and effectiveness of the planned lessons increase in time (Harle, 2008). Lesson study helps to understand the meaning of instructional goals in the context of teaching practice, affects the perceptions of teachers on teaching and learning and supports both teachers` and students` learning with the advantage of working collaboratively (Takahashi & Yoshida, 2004).

Lesson study guides teachers to recognize how to take daily lessons into account and learn from them and it facilitates the learning of how to evaluate their practice (Fernandez et al., 2003). It is a model which is based on investigation, planning, teaching, observation, discussion and reflection. On the other hand, it is also defined as "a form of reflective teaching that uses collaborative dialogue to engage teachers in a collective assessment of their classroom practices" (West-Olatunji, Behar-Horenstein & Rant, 2008, p. 97). The purpose of it is to improve knowledge, belief, skills and effectiveness of teachers, to create a collaborative professional environment and to form a teaching-learning circulation (Lewis, Perry, & Hurd, 2009). It is important since it enables to plan collaboratively, evaluate actual classroom lessons and make suggestions related to the plans of the lessons and the style of teaching and learning for making them better (Lewis, Perry, Hurd & O"Connell, 2006). Lesson study is a cycle in which teachers learn to enrich their educational experiences working with the other members of a group and reflect these experiences to their students (Wang-Iverson and Yoshida, 2005). It helps to increase the quality of education for students while it supports teachers' professional development. It is a learning and teaching process which includes an environment based on designing materials, focusing on students' thinking, analyzing student work, opinions and implications of teachers and working collaboratively as a whole (O'Connor, 2010). Another focus of it is on how students think and learn as well as teachers' development, therefore, it is a model that establishes connection among teacher content knowledge, teaching practices and development of students' learning (O'Connor, 2010).

In general terms, lesson study is a professional development process that teachers work cooperatively in actual classroom environments based on the observation of each other's lesson, discussion and evaluation of the lessons and then revision of them to present powerful instruction (Stepanek, 2001). In addition to teachers, lesson study can also be carried out by preservice teachers in their practice school with the assistance of university teachers and cooperating teachers (Fernandez, 2002). The stages of lesson study include (a) determining the research theme, (b) planning the research lesson (c) teaching the lesson and observation (d) discussion and evaluation on the lesson (e) revising the lesson (f) re-teaching the revised lesson (g) discussion and evaluation on the revised lesson (Fernandez, 2002; Lewis, 2002; Stigler & Hiebert, 1999; Takahashi &Yoshida, 2004; Yoshida, 1999).

According to Fernandez and Yoshida (2004) there are three necessary stages and one optional stage for conducting lesson study in accomplishment. The first stage includes that one member teaches the lesson while the others in the group observe. In the second stage, discussion and evaluation of whether the objectives were accomplished considering the collected data on learning, thinking and behaviors of the students occurs. The third stage involves the revision of the lesson based on analysis and evaluation of teaching process. The last and optional stage is that if the group feels the necessity to improve the lesson, the revised lesson is re-taught and evaluated, later, the last form of the lesson comes into shape. In Japanese teacher training programs, preservice teachers are assisted to learn that important concepts such as the content of topic, expected students' answers and misconceptions, teachers' roles, evaluation criteria are to be considered while planning. Thus, they recognize how to prepare good lesson plans associating these important concepts (Shimizu, 1999). The improvement of teachers' quality should be consistently provided from the very first year of their careers till the end of their professional careers (Moir & Gless, 2001). Lesson study is a model which assures the professional growth of teachers during their instruction process (Lewis, 2002; Stigler & Hiebert, 1999; Takahashi &Yoshida, 2004). Lesson study has the properties of effective professional development. It bases on collaboration, takes student learning and thinking in the center, has an ongoing and teacher-led structure which makes both teachers and students active in the process (Takahashi & Yoshida, 2004).

Stigler and Hiebert (1999) describe the process of lesson study cycle in eight steps (1) defining the problem, (2) planning the lesson, (3) teaching the lesson, (4) reflecting and evaluating the lesson, (5) revising the lesson, (6) teaching the revised lesson, (7) reflecting and evaluating, and (8) sharing results. According to Lewis (2002) typical four steps of lesson study are (a) focus on the lesson study (b) plan the research lesson (c) teach and discuss the research lesson (d) reflect and re-teach. There is consistency in terms of description of lesson study process by some studies in literature (Fernandez, 2002; Fernandez & Yoshida, 2004; Lewis & Hurd, 2011; Yoshida, 1999) and they describe the lesson study cycle as consisting of six phases (1) planning the lesson collaboratively, (2) seeing the lesson plan in action, (3) discussing the lesson plan, (4) revising the lesson plan (5) teaching the revised lesson and (6) shared reflections. It is seen that although a few steps seem to be different and lesson study process is modified differently in some researches, phases of lesson study include each of them in general and have the same content to a large extent. Brief descriptions of main steps of lesson study cycle are indicated below:

Research and Preparation: The members of lesson study determine a unit/ topic that they want to work on by agreeing upon some instructional issues, gather information about this topic and identify content and learning objectives on which they focus discussing collaboratively for several hours a week (Lewis, 2002; Yoshida, 1999). Then, they endeavor to decide on the goals of the lesson and share ideas on what they aim to reach at this process. They make a comprehensive investigation about the topic that they wanted to focus on. They examine the data related to student learning and understanding, research on subject matter and curriculum, discuss on students' difficulties and learning weaknesses which may hinder learning the subject, teaching strategies, and other instructional issues (Lewis, 2002). Teachers utilized from several sources such as textbooks, teacher guidebooks, curriculum and standards in setting the goals (Hurd & Licciardo-Musso, 2005; Lewis, Perry, Hurd, & O'Connell, 2006). They reflect their expectations and ideas about how to improve learning (Takahashi & Yoshida 2004; Yoshida, 1999). During this phase, teachers' subject matter and understanding of the relationship between topic and grade levels became deeper (Hartman, 2004; Stigler & Hiebert, 1999).

Planning the Lesson: Teachers in the lesson study group plan a detailed research lesson collaboratively using the information they obtained from several sources and utilizing each other's experiences (Lewis, 2002). During this phase, teachers share their knowledge, ideas, concerns and suggestions about how to design the lesson for best teaching and learning (Fernandez & Yoshida, 2004). They try to anticipate student thinking, consider student learning including students' misconceptions, difficulties and mistakes, adopt student perspective and estimate their possible motivation and behaviors (Fernandez & Chokshi, 2002; Takahashi & Yoshida, 2004; Takahashi et al., 2005). Besides, teachers consider several teaching and learning materials such as textbooks, manipulative materials, curricula and teaching methods. Hence, by paying attention these critical points for good instruction, teachers get the opportunity to reinforce, improve and correct the deficiencies about their pedagogical content knowledge, mathematical understanding and knowledge of student thinking (Takahashi et al, 2005).

There is the format of a four-column lesson plan adapted from a Japanese lesson study (see Figure 1) which guides for planning the lesson (Lewis, 2002; Matthews, Hlas, & Finken, 2009). Teachers can design their lesson plan writing on this four column table. First column includes learning activities that teacher aimed to do and the questions that they want to inquire. Second column reflects expected student reactions or responses which are likely to be heard and seen when they are engaged in the process. Third column includes responses likely to be given by teachers when they encountered student reactions and key points that teachers should consider. Lastly, fourth column presents how teachers make evaluation of the practice. In brief, a four column lesson plan based on the determination of the lesson content, anticipation of student response, decision making about teacher reactions and evaluation of student learning (Mathews et al., 2009). For effective lesson study, preparation of such a comprehensive and detailed lesson plan is regarded valuable (Fernandez & Yoshida, 2004). To tell the benefits of this format, it can boost preservice teachers to be better in querying and estimating student thinking so that they may adopt a more student-focused approach in their instruction (Hiebert et al., 2007; Mathews et. al., 2009; Sims & Walsh, 2008). Before teaching, making research on student thinking and learning, deciding how to respond to students step by step considering their possible reactions and questions help preservice teachers feel more comfortable and confident during implementation (Sims & Walsh, 2008). Furthermore, it prevents the shift of focus and creates a setting allowing for a more efficient and detailed discussion during planning phase. It also enables to assume students' viewpoints towards the lesson (Hiebert et al., 2007; Lewis, 2002; Matthews et al., 2009; Sims & Walsh, 2008).

Steps of the Lesson: Learning Activities and Key Questions	Expected Student Reactions or Responses	Teacher's Response to Student Reactions/Things to Remember	Goals and Method(s) of Evaluation

Figure 1 Four column lesson plan format

Teaching the Lesson: One of the teachers from lesson study group implements the lesson plan in the classroom while the other teachers observe the lesson taking notes about several aspects such as student learning, behaviors, responses and thinking (Fernandez & Yoshida, 2004; Lewis, 2002,) in order to give feedback on the reflection phase (Yoshida, 1999). They used the collected data to evaluate the effectiveness of their lesson in terms of student understanding and sometimes different teachers and educators are invited to the lesson (Hartman, 2004; Stigler & Hiebert, 1999). The teachers observing the lesson may copy the lesson plan to take notes on it (Lewis, 2006) and come to the classroom with the lesson plan in hand so that it helps teachers to focus on what was aimed in the lesson directly (Fernandez & Yoshida, 2004). Since teachers are familiar with the lesson plan beforehand and have it in hand during the implementation, they can attend to more essential points. However, independent from the use of lesson plan, teachers try to gather information about what students do and say (McQuitty, 2011).

Discussion on the Lesson: Lesson study teachers come together soon after the implementation of the lesson to discuss the teaching practice and reflect their ideas. Here, the emphasize is on "soon after" in order to keep the memory of teachers fresh because it is likely to forget what was observed as time goes on. For example, if research lesson is implemented in the morning, the group can meet in the afternoon (Yoshida, 1999). Discussion begins with the reflections of the teacher who taught the lesson and he/she elaborates what went well or did not and evaluates the success of the lesson. The other teachers share their observations with the colleagues based on their notes from the lesson and make suggestions with respect to improve instruction (Fernandez & Yoshika, 2004; Stigler & Hiebert, 1999). Most lesson study groups use established discussion protocols to discuss and reflect on the lesson more systematically in Japan (Lewis, 2002). There are typical questions which are asked in this phase: "What went well or did not? Should we refine and re-teach the lesson in another classroom? What should be changed?" (Fernandez & Yoshida, 2004; Lewis, 2002; Takahashi et al., 2005). According to McQuitty (2011) as the lesson is planned through the participation of all teachers, the research lesson belongs to the whole group instead of only one teacher who implemented it. Thus, it enables to share the responsibility for the lesson plan and prevents personal criticism of the teacher who taught the lesson (Stigler & Hiebert, 1999). On one hand teachers reflect their ideas on the lesson, on the other hand they decide how to revise the lesson plan with the benefit of their discussion. Another teacher in the group is determined to teach the revised lesson and the lesson study cycle repeats following this fundamental structure (McQuitty, 2011).

This phase provides an effective setting in which teachers share their reactions, feelings and opinions about research lesson and whether the predetermined goals are reached (Elipane, 2012). According to Watanabe (2002) post-lesson discussion is obviously critical and it must be conducted in the way that all members of the lesson study can reflect on the lesson. Discussion helps to understand how teachers make sense of their own observations on the research lesson and gain different perspectives. Differences in the perspectives also yield details on what was observed and eases learning from each other (Elipane, 2012). Besides, it gives an opportunity to the teacher who conducted the lesson for seeing his practice through the eyes of colleagues (Hartman, 2004; Stigler & Hiebert, 1999). On the other hand, discussion is a critical component of lesson study which positively impacts in-service and preservice teachers (Choksi & Fernandez, 2004).

Revising the Lesson (Optional): Teachers revise the lesson based on their observations, suggestions and reflections on the lesson in discussion phase collaboratively. The new lesson plan includes the changes that teachers decided to make for redesigning the lesson (Fernandez & Yoshida, 2004). They might change the wording of the problem, redesign an activity and add or take out questions considering feedbacks of the group and students (Lewis, 2002; Harle, 2008). Misunderstandings and difficulties of the students are the essential concepts that are considered during revision (Eraslan, 2008). Some groups prefer to stop working on the lesson but some maintain to revise and re-teach the lesson (Fernandez & Yoshida, 2004).

Re-teaching the Lesson (Optional): The revised lesson is taught by another teacher from the lesson study group in a different classroom (Stigler & Hiebert, 1999). The reason for changing the teachers is providing them with opportunities for teaching in front of both students and colleagues (Fernandez & Yoshida, 2004). The other teachers observe the revised lesson and take notes again. It enables teachers to evaluate the effectiveness of the changes that they made (Fernandez & Yoshida, 2004). It is possible to revise, reteach and reassess the lesson as many times as teachers believe that it is satisfactory, however, only one more repetition of the cycle is preferred if needed since plentitude of the revisions is likely to decrease the effectiveness of discussions and curriculum progress in time (Chokshi & Fernandez, 2004). Concordantly, it is seen that most of the lesson study practices are revised and retaught in their lessons one time (Fernandez & Yoshida, 2004; Lewis, 2002).

Discussion on the Revised Lesson (Optional): Teachers come together soon after the revised lesson to discuss on what they observed and reflect their ideas and suggestions for the improvement of the lesson (Fernandez & Yoshida, 2004; Yoshida, 1999).

In sum, the first and second steps of lesson study include determining the focus of the lesson, research on the topic, gathering information about the subject content and preparing a detailed lesson plan. In the third step, teaching of the lesson by one teacher, observation of the implementation by group members and taking notes about teaching take place. The fourth step includes discussion on the lesson and reflection of ideas and suggestions to improve the instruction. These steps are essential for the practice of lesson study and the other steps which contain revising, re-teaching and reassessment of the lesson are conducted optionally. The figures shows the process of lesson study cycle (Lewis, Perry & Murata, 2002).

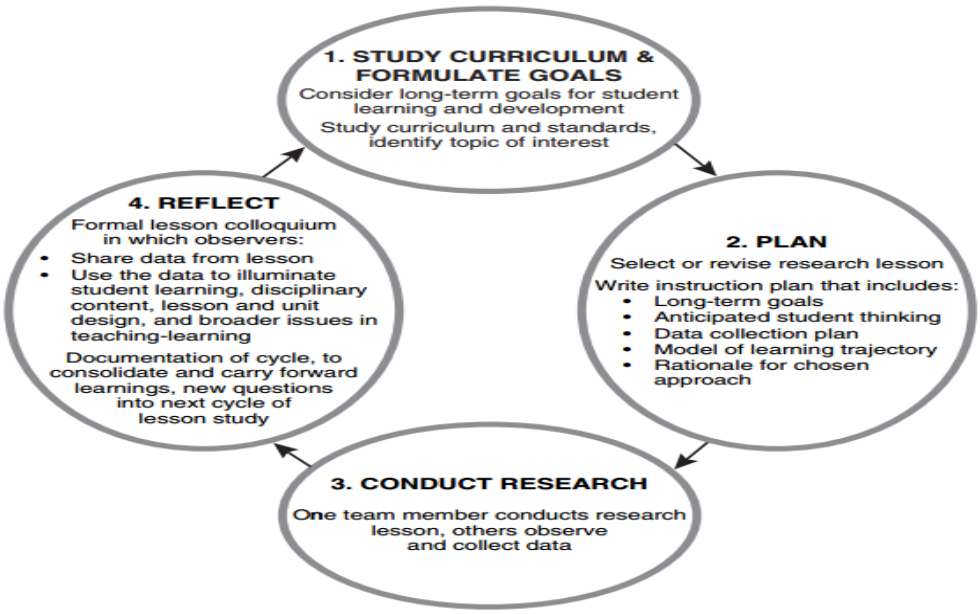

Figure 2 Lesson study cycle

According to Fernandez and Yoshida (2004) lesson study can be conducted in a different context and modified in accordance with conditions. For example, preservice teachers participate in lesson study during field experience courses such as school experience and teaching practice in teacher education programs. They prepare a lesson plan with the guidance of the university teacher and cooperating teacher from the school. They teach a lesson in the classroom of this school and a university teacher, cooperating teacher and other preservice teachers in the lesson study group observe the lesson. In a similar way, first year teachers are involved in lesson study with a mentor teacher. They plan the lesson collaboratively and other teachers forming the lesson study group participate in teaching of the lesson and observe the implementation. Furthermore, it is also widespread in the area of in-service training. On the other hand, lesson study has a perspective that inclusion of outside experts has an important role in strengthening teachers' knowledge and improving instruction. Thus, in Japan and U.S., outside experts (knowledgeable others or advisors) who have knowledge of content, pedagogy and curriculum in the relevant area (Chokshi & Fernandez, 2005; Fernandez, 2002) are often involved in the practice of lesson study to share their ideas, experiences and give feedbacks without dictating (Chokshi & Fernandez, 2004; Fernandez, 2002; Lewis, Perry, Hurd et al., 2006). According to Elipane (2012) "Indeed, the practice of Lesson Study provides a coherent infrastructure for teacher education (pre-service and in-service), which has implications on continuous, cumulative and recursive learning, wherein the teachers themselves play a big part/role in their education".

It is a process which aims to change teacher's knowledge, belief and experiences by providing learning through real practice based on investigation of the subject, planning, implementing and reflecting collaboratively (Lewis, Perry, & Hurd, 2009). Lesson study enables teachers to present their own knowledge on content and pedagogy and ideas about students' thinking and learning as well as listening to the other teachers' ideas on these issues in a manner to develop new knowledge (Chokshi & Fernandez, 2005; Lewis, Perry, & Hurd, 2009). Besides, Fernandez (2005) states that teachers should know how students think, what kind of difficulties and conceptions they have and which strategies are appropriate to overcome the difficulties in order to enhance students' thinking and understanding. Lesson study promotes utilizing observations, productive discussions, student-focused planning and reflections on classroom practice to improve teaching and learning (Fernandez & Yoshida, 2004; Lewis, 2002). The use of lesson study model could meet the need to maintain effective professional development for teachers (Lewis, Perry, & Hurd, 2004).

2.2.1 Benefits of lesson study

Lesson study might be perceived as the preparation of a lesson plan in detail at the beginning, however, this limited approach is not correct because lesson study is primarily based more upon learning the practice of teaching than planning the lesson (Fernandez & Chokshi, 2002; Lewis, 2000). Comprehensive planning gives teachers opportunities for learning the use of new and present instructional strategies better (Fernandez & Chokshi, 2002; Stigler & Fliebert, 1999) and allocates "high-fidelity context on which teachers can build their content knowledge" (Lewis, 2000, p. 17). On the other hand, collaborative nature of it allows for sharing different ideas and promotes teachers to discuss on instructional issues (Lewis, 2000). It also accommodates a setting in which teachers are able to work on the practice and focus on the topic without hurrying up so that the flexible structure decelerates the process (Campbell, 2003). Lesson study draws its strength from anticipation of students' thinking, understanding and reactions (Fernandez et al., 2003; Lewis, 2000). Therefore, "the vision to see children" (Lewis, 2000, p. 14) and the perspective to focus on students rather than the teacher are bestowed upon teachers (Lewis, 2000). Lesson study incorporates theory and practice by giving teachers the role of researchers and using the classrooms as a laboratory (Campbell, 2003). Thus, teachers make sense of theoretical knowledge about education practicing them in the classroom setting (Takahashi & Yoshida, 2004).

Lesson study provides effective professional development with consideration of teachers as adult learners, collaboration, reflection, problem based learning in the classroom and focus on content area (Loose, 2014). In order to improve teaching, promote teacher learning and increase the quality of instruction, the best place is classroom (Stigler & Hiebert, 1999). Webster-Wright (2009) also emphasize that professional learning is accompanied by practice so that classroom setting where teachers routinely encounter difficulties regarding differences in students, materials, time and environmental conditions (Prince, 2016) has an important role in it. At this point, lesson study supports learning in the classroom environment that teachers are used to be in and teach as it occurs (Loose, 2014; Stigler & Hiebert, 1999). It takes shape in the classroom with real students and in real time (Prince, 2016). Takahashi et al. (2013) observed six lesson study groups in public schools in their study and found that lesson study gave teachers the chance to experience what reform required in practice. In addition, Yoshida (2013) argued that the observation step of lesson study which includes gathering data about student thinking and learning during the implementation of the lesson in a real classroom enables more productive discussions on how to make lesson better and enhances the experience in professional development.

Lesson study process necessitates analyzing relevant textbooks and curriculum in depth to determine what students already know, what they need to learn and how fundamental concepts should be taught (Yoshida, 1999; Lewis, 2004) so that teachers become more knowledgeable about the content of subjects, curriculum and the relationship between them. It facilitates teachers to distinguish the important information and concepts, awaken to their own deficiencies and incorrect approaches so that they remove them and make their lessons more effective (Lewis, 2002). Since lesson study is based on a detailed and long term investigation, planning and discussion, the quality of classroom practices advances and planning skills of teachers develop in time (Stigler & Hiebert, 1999). In their research with six teachers, Lewis et al. (2009) revealed that "lesson study enables teachers to strengthen professional community, and to build the norms and tools needed for instructional improvement" (p. 286). Suh and Parker (2010) underline the important aspect of lesson study as collaboration and in their study they found that gathering of teachers' various abilities, experiences and ideas created efficacy that is shared and collegial in lesson study. This environment based on cooperating lets teachers construct and reconstitute knowledge and make sense of how to use it in order to overcome difficulties (Prince, 2016).

Lesson study gives opportunities for teachers to share what they learned from recourses during investigation and their ideas regularly (Lewis, Perry & Hurd, 2004) so that generates an effective learning environment among colleagues (Lewis, 2002) and this collaboration prevents the lack of knowledge that comes with teacher isolation. Teachers become more powerful in their teaching and learning through learning communities based on collaboration in lesson study rather than they do and gain alone (Cossey & Tucher, 2005). Because logical reasoning progresses with differences among colleagues in this shared experiences (Perret-Clermont, 1980), disagreements are the key elements to enrich collaborative efforts (Prince, 2016). At this point, due to teachers' reflecting on practice and making suggestions about the points that need improvement, the reflection phase of lesson study is essential for revealing the conflicts in teachers' ideas so as to provide the development of instruction (Stigler & Hiebert, 1999). Lesson study enables teachers to indicate their critical reflections about practice (Fernandez & Chokshi, 2002) and student learning (Lewis, 2002) deeply through observations for better learning and teaching. The study which includes the participation of preservice teachers in lesson study process of Ricks (2011) showed that reflections posed opportunities for preservice teachers to refine their ideas and to work together more effectively. According to Takahashi et al. (2013) teachers notice what differences and similarities exist in their own practice in comparison with the others' instructional approach with the help of rich reflective experiences, learn from each other and enrich their teaching.

Lewis (2000) expresses three explicit benefits of lesson study as (1) improvement in teachers' subject matter knowledge and teaching skills, (2) support for meaningful collaboration and personal development and (3) in-depth focus on student thinking and sense-making of student learning. In lesson study, teachers experience educational ideas, analyze and evaluate practice in a real classroom so that improvement in their knowledge of content, pedagogy and student learning takes place. Since teachers come together to work on common instructional goals in this process, they acquire new perspectives and knowledge from each other (Freidman, 2005). In the study of Fernandez (2003), it was investigated whether lesson study develops teachers' content knowledge of mathematics studying with thirty mathematics teachers who enrolled in a lesson study process from 1999 to 2001. She indicated that lesson study provided an opportunity for teachers to develop their content and pedagogical knowledge and deepened their comprehension about content area. It was also found that lesson study helped teachers to anticipate student thinking and decide how to respond in the face of unexpected student reactions and events while teaching. To speak of the second benefit, Fernandez, Cannon and Chokshi (2003) put sixteen American and Japanese teachers together in the context of lesson study in their research. It was concluded that even though some steps in the lesson study were difficult for American teachers, the lesson study generated a setting based on effective collaboration with colleagues and supported teachers' individual learning through collaborative efforts. The result of the study conducted by Gilmore and Hawkins (2003) with the participation of fifteen teachers from four elementary and middle schools remarked on teachers' realizing that lesson study was not just planning a lesson and focusing on content, the main goal was to maintain the improvement of instruction. Moreover, it was indicated that lesson study removed the impacts of teacher isolation that weaken the effectiveness of the lesson, with the opportunity of collaboration and learning from each other so that teachers felt more comfortable in practice. The third significant benefit of lesson study is to understand student thinking and learning deeply. Planning phase includes focusing on students' responses, anticipating their thinking and incorporating their perspectives into the lesson for better instruction, thus, lesson study leads teachers to increase students' contributions within the lesson (Fernandez, Chokshi & Cannon, 2003).

Fernandez and Yoshida (2004) indicate various benefits of lesson study in terms of teachers as the following:

(a) Lesson study develops teachers' content knowledge and strengthens their comprehension of essential concepts by aiding them to examine and discuss what they will teach in detail.
(b) It improves teachers' anticipation about students' thinking and understanding the content so that increases their knowledge of teaching.
(c) It enables teachers to determine purposeful goals to reach in terms of both teaching and student learning.
(d) It impacts teaching attitudes of teachers in a positive manner.

(e) It provides opportunity for teachers to evaluate the common plan and their own practice realistically.

In the study of Byrum, Jarrell and Munoz (2002), teachers from 25 high schools appreciated the collaborative structure of lesson study, emphasized their willingness to implement it and drew attention to the cost and the long period of time as negative factors for its implementation. Lewis (2002) conducted interviews with the participants of lesson study in Japan and indicated the key opportunities of lesson study for teachers to improve instruction. According to Lewis's report, lesson study empowers teachers to focus deeply on the goals in terms of content and students, seek for best lessons, enrich knowledge about subject matter and instruction, share responsibility equally and gain the perspective to understand students. Besides, the research of Fernandez, Cannon and Chokshi (2003) regarding lesson study with Japanese and American teachers revealed that teachers gained three lenses which were research lens, curriculum developer lens and student lens during the process. They hypothesized about some ideas, checked them by gathering data and endeavored to make generalizations with the research lens. They determined the sequence of content that they need to teach and learned about what students already know based on the curriculum developer lens. They endeavored to understand how students think and prepared lesson plans using the student lens. Besides, there are other researches on the benefits of lesson study by Lewis and some researchers (Lewis, Perry, & Hurd, 2009; Lewis, Perry, & Murata, 2006). According to the results of their studies which are conducted with lesson study groups, lesson study directly affects and is affected by teacher knowledge, teacher community and teaching-learning resources so that an increase in these areas indirectly improves classroom practice. Lesson study enriches teachers' subject matter knowledge, pedagogical knowledge and knowledge of student thinking and supports teachers to become more aware of the relationships between educational goals for student learning and instruction. It also promotes professional community among teachers increasing their motivation, their abilities of teaching, their collective responsibility feelings and power of determination shared educational long-term goals for student learning and a good quality instruction (Lewis et al., 2009). In other words, lesson study influences teachers' way of thinking and instructional approach through rich collaborative efforts (Hurd & Licciardo-Musso, 2005). Furthermore, lesson study assures the preparation of more effective lesson plans and use of more purposeful materials by utilizing various teaching-learning resources so that it strengthens teaching and learning of students and professional development of teachers (Lewis et al., 2009; Lewis, Perry, & Murata, 2006).

Although there are several difficulties or differences depending on many factors such as culture, conditions which are encountered in the implementation of lesson study, there are important benefits of this profession development model too. In sum, lesson study enables teachers to learn and experience new instructional practices (Byrum et al., 2002), notice what they already know and how their own instructional approaches are (Byrum et al., 2002; Hurd & Licciardo-Musso, 2005), improve their subject matter knowledge (Hurd & Licciardo-Musso, 2005) and pedagogical content knowledge (Fernandez, 2005), enhance self-confidence and integrate instructional goals, reform ideas and standards into classroom practice (Lewis, 2002). Benefits of lesson study include supporting collaboration and reducing teacher isolation, developing observation and reflection, increasing teacher understanding of subject, focusing on students' learning and thinking.

2.2.2 Misconceptions and challenges related to lesson study

There are some misconceptions or lack of knowledge that cause challenges in understanding the meaning of lesson study (Burgess, 2006). Lewis (2002) emphasizes misconceptions about lesson study reviewing the studies in literature and indicates what they are as (1) Lesson study is lesson planning, (2) Lesson study means writing lessons from scratch, (3) Lesson study means writing a rigid "script", (4) Lesson study is writing the "perfect" lesson to be spread to others, (5) The research lesson is a demonstration lesson or expert lesson and (6) Lesson study is a basic research (Burgess, 2006, p. 72). Fujii (2014) conducted a research observing four research lessons in Uganda and Malawi and examined misconceptions about lesson study. According to the results of the research, the arising questions based on these misconceptions were indicated as (a) Is lesson study a workshop? (b) Must the lesson plan be followed rigidly? (c) Is structured problem-solving just solving a task? (d) Is the focus of consideration on teaching or the teacher? (e) Should a research lesson always be re-taught? It was argued that teachers may perceive lesson study as a workshop because the form of workshops or seminars shows similarities with some features of this model. Teachers make incorrect interpretations about the implementation of lesson plan and feel that they have to replicate what was written in the lesson plan instead of modifying it based on the nature of the class. Structured problem solving lessons in lesson study include highlighting key concepts and important points by teachers, student-centered approach and assisting students to gain mathematical thinking ways through not only finding the answers to the questions but also solving problems. However, the important point here can be missed and it can be misunderstood as giving the task and its being solved by students and thinking all concepts are important. The focus of lesson study can be misunderstood as well and teachers attend to the teacher who taught the lesson instead of teaching. On the other hand, lesson study offers re-teaching of the lesson if necessary but this step is optional. Consequently, teachers should prepare lesson plans and conduct research lessons with the thought that reaches the best result (Fujii, 2014).

In spite of having several benefits there are many challenges in implementing lesson study, and concerns about its adaptation to different education systems because of the diversity in cultures (Harle, 2009). Time is one of the challenges in terms of teachers as lesson study takes a long time (Lenski & Caskey, 2009; Lewis, 2002; Stewart & Brendefur, 2005) for planning, teaching, reflecting and evaluating the lesson collaboratively and teachers might not be willing to spend their own free time. Stewart and Brendefur (2005) conducted a research with one lesson study group and argued that teacher learning is affected by teachers' depth efforts for providing the quality of a lesson rather than the number of lesson plans. It points to the other factor, trust among teachers for cooperating, that impacts lesson study. Teachers evaluate the effectiveness of the lesson and refine it based on the data they gathered during observations of the practice, hence, trust is a powerful feeling which could reveal the potential of teachers and the process of lesson study (Lyding, 2012). Formation of trust and a sense of comfort among teachers, not to be disturbed by the existence of colleagues during their observations of implementation in the classroom is essential but also difficult to retain (Harle, 2009). Most teachers do not feel comfortable when their colleagues observe them teaching within the classroom (Lenski & Caskey, 2009; Stewart & Brendefur, 2005) since teachers are likely to perceive the observation of the lesson as an evaluation of themselves (Harle, 2009) though the aim is gathering information to improve instruction.

Moreover, teachers are familiar with being in isolation and making decisions on lesson alone so that observation and reflection by peers makes them uneasy and causes anxiety (Appel, Leong, Mangan, Mitchell & Stepanik, 2007; Caskey & Lenski, 2010; Stigler & Hiebert, 1999). Therefore, teachers should be well informed that observation includes monitoring student learning and evaluating effectiveness of the lesson rather than criticizing what teachers have done so that the positive relationship among teachers increases and their anxiety decreases.

> Working with a majority of teachers who have been teaching at the same school for quite a number of years is no easy task. Neither is it easy to encourage teachers to adopt different approaches to teaching and learning nor

to encourage their involvement in collaborative decision-making when they have not been used to it. (Bezzina & Testa, 2005, p. 146-147).

To adopt the change, teachers need to feel that it is meaningful and they want to be comfortable with it because it is difficult to give up their current approach (O'Connor, 2010). Fowler (2009) explains five educational barriers that make the change difficult as (1) not understanding the change, (2) being unfamiliar with the use of new pedagogy (3) lack of materials, (4) inconsistency between culture and setting of the school and a new policy (5) insufficient motivation. In addition, according to Thornton, Shepperson, and Canavero (2007) "significant institutional change requires high levels of communication, coordination, time, money, and continuous organizational reassessment and realignment" (p. 48). Teachers are not generally able to notice the power of lesson study either since prejudice towards change makes its implementation difficult. They often perceive lesson study as a threat which limits their autonomy (Byrum et al., 2002) and a process which is binding them for a long time (Stigler & Hiebert, 1999) instead of being a model which supports their professional development.

When lesson study process which includes the steps of investigating student understanding and knowledge, anticipating their answers, gathering data about student learning during observation, reflecting about lesson to improve it and refining the lesson based on discussion among teachers (Appel et al., 2007; Stigler & Hiebert, 1999) is considered, the whole process might continue for weeks or months due to the comprehensive structure of lesson study. Accordingly, it seems that teachers' concerns about time are valid at first glance (Lawlor, 2012). Time to observe practice in a classroom during a school day is vital for teachers (McQuitty, 2011) and through school norms, opportunities to work in the context of lesson study are provided during a school day in Japan (Fernandez & Yoshida, 2004). At this point, the concerns about allocation of time and support to integrate the lesson study in schools arise (Appel et al., 2007).

In lesson study, reflections that are truthful and contributing to the development of instruction are critical during discussions instead of insignificant opinions or ideas targeting personal critics (Appel et al., 2007; Wang-Iverson & Yoshida, 2005). Thus, inexpedient discussions diminish the impacts of lesson study. It is seen that taking a long time, sense of losing autonomy, lack of time and support in schools, teachers' habit of isolation and not creating a setting that includes purposeful discourses might cause challenges in the implementations of lesson study appropriately. It is defended that these challenges will sustain to prevent teachers from practicing lesson study for providing instructional development until this model is internalized and becomes prevalent (Appel et al., 2007; Caskey & Lenski, 2010).

According to Burney (2004) for professional development, willingness in experiencing new methods, strategies and ideas, adopting change, being open to make mistakes and learning from them with the other people is crucial. It helps to build up new knowledge and more quality in instruction. Thereby, unwillingness of teachers might create barriers for them in using lesson study and benefiting from its positive effects.

The structure of lesson study that is based on collaboration and the need for inquiry stance also cause challenges in implementing this model (McQuitty, 2011). Lesson study necessitates collaborative work during the whole process and taking responsibility coequally in order to create an effective teaching and learning process, however, teachers are accustomed to conducting lessons with no or little assistance from other teachers (Easley, 2000). Although numerous teachers believe in the significance of collaboration within lesson study (Byrum et al., 2002; Hurd & Licciardo-Musso, 2005), shared planning and teaching in front of the others; adaptation to collaborative efforts might not be easy for teachers (Fernandez, 2002). Besides, it is not always likely to set a common time for every member of the lesson study team because their availabilities for meeting might not coincide, in other words, time commitment might be a problem (Byrum et al., 2002; Wilms & Zell, 2002). On the other hand, one of the essential characteristics of lesson study is having an inquiry stance (Fernandez, 2002). Fernandez (2002) studied with teachers who enrolled in a lesson study and found that teachers had difficulties in conducting lessons that could reveal students' thinking, understanding and answers, interpreting what they observed and extending the results of it. Lack of inquiry stance poses challenges in utilizing all the potential of lesson study and cuts back the effectiveness of lesson study practices (Fernandez, 2002). Moreover, lack of teachers' knowledge or misconceptions about lesson study process are considered as challenges for its implementation. According to some research (Byrum et al., 2002), it is indicated that teachers cannot conduct lesson study effectively due to the lack of their knowledge about it. In sum, time, trust, teacher motivation and willingness, support, isolation, collaboration, autonomy, culture, discourse, allocation of time in schools, inquiry stance, lack of teachers' knowledge are considered as challenges in conducting lesson study.

Chokshi and Fernandez (2004) identify the challenges under three categories as Launching Lesson Study, Understanding Lesson Study and Deepening and Sustaining Lesson Study. Launching lesson study challenges entails that the practitioners do not believe in the adaptation of this model into their lesson as it comes from a different culture, they think that it takes a long time so it is difficult to make time for it and according to them working together during the lesson study process does not seem possible because the teaching styles of all teachers are different. In addition, teachers are not going to feel comfortable due to sharing their class with the other members and observation by the others may make them nervous and shy (Choski & Fernandez, 2004). Challenges to understanding lesson study refer to misconceptions about the model leading to have incomplete or incorrect knowledge related to lesson study. Lesson study is perceived as forming a lesson which is quite a change from what we know before, preparing a lesson plan and just focusing on only one lesson to change it into the best form. Besides, the practitioners think that until lesson study is made regular and continuous for many lessons, conducting it is not going to be very useful. These misconceptions lead them to stay away from conducting lesson study (Choski & Fernandez, 2004). Deepening and sustaining lesson study challenges category involves that making research related topic is not an easy action for anyone and the practitioners think that they have to do this action since the research is an essential part of lesson study thereby, they find conducting a long-term research difficult. In addition, lesson study requires to work, discuss and share with the other members regularly, therefore, it is difficult to keep members together and provide rich dialogs. Although Choski and Fernandez (2004) state these assumptions and concerns on behalf of United States teachers, it seems that their interpretations are obviously valid and common for many teachers in different countries too, especially in Turkey.

2.2.3 Professional development

Ball and Cohen (1999) describe professional development, PD, as a pedagogy that is based on knowledge of how to learn. It is a process in which teachers are active learners, build knowledge through instructional practices and learn from each other. It includes inquiring into practices and student learning and discussing ideas with the others and thereby it supplies improvement of individual capacities due to exposing different interpretations (Ball & Cohen, 1999). Professional development is also stated as a process which includes designing activities not only to increase knowledge, skills and attitudes of teachers but also to impress students' learning in a positive manner (Sparks, 2006). Kennedy (1998) describes professional development in a way that focusing on students' learning helps teachers link to subject matter content, teaching and learning and hence, improves instruction. Teachers` quality and their teaching practices are considerably important in terms of student learning and reform standards (Garet, Porter, Desimone, Birman & Yoon, 2001). Many researchers, experts and policymakers advocate that professional development allows teachers to improve their quality and instruction if conducted effectively (Grove, 2011). Guskey and Yoon (2009) also indicated that "no improvement effort has ever succeeded in the absence of thoughtfully planned and well-implemented professional development" (p. 497). Professional development enables teachers to improve instruction depending on its effective implementation (Spark & Hirsh, 2000), thus, there is a connection between professional development practices and quality instruction. NCTM (2000) suggest that professional development should focus primarily on helping teachers in teaching based on reform-oriented strategies and in constructing knowledge of content, pedagogy and student thinking. According to Wong (2003) the strongest factor which supports training of good quality teachers is professional development models.

There have been many forms of PD from past to present. Some of them had little effect in enhancing the quality of instruction or were not successful in considering some factors such as individual differences, or the complexity of the job (Fullan & Hargreaves,1996). The characteristics of PD in the past are mostly evaluated as ineffective (Guskey & Sparks, 2002) and ineffective PD has been described as "discrete experiences that fail to provide ongoing support and continual feedback to attain long-term, systemic improvements" (Glazer & Hannafin, 2005, p.179). Educators criticized many professional development efforts because of discontinuity and being ineffective in making difference in teacher practice and student learning (Loucks-Horsley, Hewson, Love& Stiles, 2003). Some reasons of this situation are explained with the lack of opportunities in schools for implementation owing to the great amount of time and money needed and existing cultural differences (Richardson, 2000). In addition, isolation and lack of collaboration, little opportunity for reflection on practice and student learning, limited time and lack of learning from each other were seen as the other reasons (Elmore, 2000; Hiebert, Gallimore, & Stigler, 2002). Therefore, the need for more effective professional development has been felt. Although there are various models which have different and similar aspects, a range of research has identified the characteristics and key features of effective professional development (Grove, 2011; Prince, 2016).

According to a publication of *Designing Professional Development for Teachers of Science and Mathematics*, there are five principles of effective professional development (Loucks-Horsley, Hewson, Love, Dyasi, Friel, Mumme, Sneider, & Worth, 1998). These principles are:

1. Professional development experiences must have students and their learning at the core- and that means all students.

2. Excellent mathematics teachers have a very special and unique kind of knowledge that must be developed through their professional development learning experiences.
3. Principles that guide the improvement of student learning should also guide professional learning for teachers and other educators.
4. The content of professional learning must come from both research and practice.
5. Professional development must align with and support system-based changes that promote student learning. (p.3)

Garet, Porter, Desimone, Birman and Yoon (2001) indicate three important features of professional development which influence teachers' knowledge, skills and practices positively. Professional development must focus on content knowledge and provide teachers with an environment which includes their active participation in quality planning, implementation and discussion. Besides, it should attend to how activities are perceived by teachers as a part of their learning. According to Darling-Hammond and McLaughlin (2011) effective PD must include teachers' teaching, inquiry, observation, evaluation and reflection; it must be based on collaboration and sharing knowledge with colleagues; it must be related to the issues of teaching and student learning and it must be supported with coaching and modelling. In effective professional development, participants have both learner and teacher roles and they encounter the uncertainties of both roles but they learn to struggle with these difficulties (Darling-Hammond & McLaughlin, 2011).

Loucks-Horsley (2003) states the qualities of effective PD as: (a) PD should base on effective teaching and learning which is well-defined (b) PD should enable teachers to improve knowledge of content and pedagogy and analyze their practices (c) PD should include investigation and teachers are considered as adult learners (d) PD should give teachers the opportunity to work with colleagues and to share responsibility to improve instruction (e) PD should assign teachers with leadership roles (f) PD should enable to connect with major goals of reform movements and education systems (g) PD should base on student learning and include ongoing evaluation and improvement.

Similarly, features of high quality PD are defined as collaboration among participants, attending to the content of subject matter, consideration of students' learning and providing opportunities for active participation (Desimone, 2009). An effective professional development program should be parallel with common goals and model teaching strategies and involve practice and continuous feedback (Desimone, 2009). In general, it includes small groups of teachers, implementation in a classroom setting (Darling-Hammond, 2004), reflection on practice (Garet et al., 2001; Cohen & Hill, 2000), the real needs and interests of school staff and ongoing long term learning (Merriam, Cafferella, & Baumgartner, 2007). Teachers' subject matter knowledge, pedagogy and knowledge of student thinking are important components of effective PD and high quality instruction (Carpenter et al., 1996). Besides, consideration of students' possible misconceptions and current knowledge provides development of instruction as well as teacher learning (Carpenter et al., 1996). Professional development that comprises of understanding students' thinking is important for the improvement in teachers' teaching so that student achievement rises (Carpenter et al., 1996; Desimone, 2009). According to Kennedy (1999) professional development that focuses on knowledge of subject matter, student thinking and student learning is more likely to be effective.

In summary, effective PD includes investigation, examination, reflection and refinement of practice collaboratively during the process (Ball & Cohen, 1999; Sims & Walsh, 2008). The components of it are inquiry, collaboration, sharing knowledge, peer observation and leadership of teacher (Kelleher, 2003). Effective professional development enables continuous and supported opportunities for teachers to collaborate on instruction in classroom setting, learn from their own classrooms, try to meet the needs related to specific content and attend to student thinking (Harle, 2008). Teachers should analyze their own practices and share their reflective ideas to increase the quality of instruction (Byrum, Jarrell, & Munoz, 2002) and learn from teaching (Sims & Walsh, 2008), thereby, PD allocates a setting which has positive effects on teacher learning (Grove, 2011).

Effective professional development is important to be successful in education systems for the reason that teachers' role and their teaching are key in student learning and keeping up with the reform movement (Garet et al., 2001). The NCTM Professional Standards for Teaching Mathematics (1991) indicate that one important element influencing a teacher's professional development is reflection on learning and teaching both alone and with colleagues. Effective professional development provides an interactive setting for teachers to reflect on their own practices (Kilpatrick & Silver, 2000). Teachers are more likely to go through the difficulties arising from reform movements through working with colleagues (Kilpatrick & Silver, 2000). Teacher learning and new understandings arise when teacher's knowledge and experience is combined with the others (Lambert et al., 1996). Researchers defend that teachers should be handed the opportunities for effective professional development (Heibert, Gallimore, & Stigler, 2002; Stigler & Heibert, 1999) because it is the key to improve instruction and consequently improve student achievement (Garet et al., 2001; Grove, 2011). In general, teachers` experiences take place without collaboration and focusing on student learning, with little inquiry in teaching practice and lack of learning from each other and in an insufficient amount of time (Elmore, 2000; Hiebert, Gallimore, & Stigler, 2002). Effective professional development presents rich learning experiences for teachers so that it brings new perceptions (Desjean-Perrotta & Buehler, 2000). Stigler and Hiebert (1999) highlight that successful instruction depends on a good quality teacher, hence teachers need the support of sustainable and effective professional development for providing improvement in instruction (Ball & Cohen, 1999, Gordon, 2004) and student learning (Stigler & Hiebert, 1999; Yamnitzky, 2010). Many educators and researchers emphasized the importance of providing teachers an environment where they can learn how to teach better through training. It shows the substantial role of effective professional development and teacher training programs in terms of teaching profession (Harle, 2008).

One of the models which contains most of the characteristics of effective professional development within itself is lesson study (Garet et al., 2001). Doig and Groves (2011) indicate "…teachers learn best by doing and building their own understandings rather than being told" (p. 78). Lesson study is this kind of professional development approach which influences teacher training and student learning in a positive manner since it allows teachers to teach and observe a lesson, evaluate student learning and make interpretations about the lesson (Doig & Groves, 2011). In this respect, lesson study professional development model was used to help preservice teachers gain various and rich field experiences and develop their skills of noticing as well as the other profession skills in the present study. In other words, preservice middle school mathematics teachers' levels of noticing students' mathematical thinking were examined in the context of lesson study.

2.2.4 Challenges in preservice teacher education

The goal and focus of education shifted from basic reading, writing and computing skills to helping students gain knowledge and learn strategies for logical reasoning on history, science, technology, social studies, mathematics and art (Bransford, Brown, & Cocking, 2000). In accordance with changes in reform movements, teacher education attends to develop teachers' instructional practice with the help of schools (Hartman, 2004) and professional development models have come into prominence. It is defended in many countries that a good education and high quality training should be maintained to make teachers more effective in the classroom (Yoshida, 1999).

According to Frykholm (1999) preservice and novice teachers are in tendency to use ways of teaching which they are familiar from their own learning process based on what their teachers used. Although preservice teachers become a bit more active and have the opportunity of gaining field experience during training through the courses such as teacher practice and school experience, their background of teaching styles based on mostly what they have learned from their teachers previously restricts the effectiveness of instruction (Artzt & Curcio, 2003). Hammerness et al (2005) describes three essential problems that are encountered in teacher education programs:

1) Students are accustomed to observing their teachers for many years in the learning process, however, this habit can cause various misconceptions that are difficult to change depending on teachers' deficiencies or mistakes.
2) Because the process includes both understanding what was intended and doing the necessary things concurrently, it might be difficult in terms of preservice teachers. Thus, a teacher educator should enable preservice teachers to do teaching practice and reflect on the process of learning and teaching during their teacher training programs.
3) Considering students' possible responses, different needs and unexpected reactions is likely to be too complex for preservice teachers. That is why, they need help for learning how to think systematically about students, deciding how to response and gaining habits of reflection on practice.

Hammerness et al (2005) also emphasize that teacher education programs form the first step for teaching profession and they are not completely enough to improve preservice teachers' knowledge, skills, beliefs and attitudes which are necessary for effective teaching since they need ongoing support for lifelong learning. According to Baki (2008) no matter how well a curriculum and substructure is possessed, it is not likely to have success in the education system without an increase in teachers' quality because change in education depends on teachers to a large extent (Fullan, 1993). On the other hand, inefficacy of teachers in education is noticed in related studies and this situation reveals the importance of teacher education and professional development (Kranier, 2011).

One of the important challenges in teacher education is training effective mathematics teachers (Hiebert et al., 2007). In general, preservice teachers do not comprehend the complex structure of teaching (Grossman, Compton, Ingra, Ronfeldt, Shahan & Williamson, 2009), so they do not feel the need for professional support (Ball & Cohen, 1999; Kennedy, 1999). However, it is crucial to give preservice teachers an ongoing professional learning setting

(Chassels & Melville, 2009; Ganesh & Matteson, 2010; Hiebert et al., 2007). It is difficult for preservice teacher to prepare plans and conduct lessons, specially, in the first years (Carrier, 2011). Hence, teacher education programs should provide environments which include guidance of experts, practice of planning and teaching in real but less stressful circumstances and feedback (Fernandez, 2005; Ganesh & Matteson, 2010; Grossman et al., 2009). However, the courses in teacher training programs are not good at reflecting real classroom conditions (Grossman et al., 2009) and they are mostly based on introduction of theory rather than practice actively (Fernandez, 2005). Due to the fact that preservice teachers are not able to link what they learned in courses with field experience (Lampert & Ball, 1999), allowing them to transform their theoretical knowledge to practice with collaboration of experts and peers has an important role in the improvement of preservice teachers (Chassels & Melville, 2009; Morris et al., 2009). In other words, preservice teacher education should include not only classroom practice but also collaboration and inquiry (Wagner, 2003).

Hughes (2006) indicates the important points that need to be considered in mathematics teacher education to prepare more effective teachers as: (1) preservice teachers' beliefs about teaching, learning and students, (2) putting theory into practice and showing how to integrate theoretical knowledge into real teaching, (3) providing opportunities for predominantly focusing on students' thinking, (4) supporting field experience. In addition, many studies include suggestions about preservice teachers' learning. It is offered that opportunities should be presented for preservice teachers not only involving to use their previous knowledge and habits but also gain new skills and perspectives so that their beliefs and knowledge change in a positive manner (Ball & Cohen, 1999). Preservice teachers should be encountered with activities which reveal students' thinking and mathematical content (Darling-Hammond & Sykes, 1999) so that they can learn from teaching practice (Darling-Hammond & Sykes, 1999; Putnam & Borko, 2000). Besides, an environment based on collaboration should be provided so that preservice teachers can learn with and from each other (Ball & Cohen, 1999; Putnam & Borko, 2000).

In contrast with expert teachers' knowledge, preservice teachers have limited knowledge and experience, but as long as they are well-supported in their training at university, the quality of their instruction will increase in the first years of their career (Baki, 2012). Instruction of a subject is not an easy cognitive activity because it embodies organization and integration of different knowledge and skills (Leinhardt & Greeno, 1986). Therefore, it is considerably valuable for preservice teachers to practice based on planning, teaching and evaluating in real classrooms to handle this situation (Darling-Hammond, 2005). It shows the importance of courses which enable preservice teachers to use their own knowledge and put it into practice and overcome the complex structure of classroom practices such as teaching practice and school experience as well as method courses (Baki, 2012; Yoshida & Jackson; 2011). These kinds of courses give opportunities to implement knowledge and skills learned from the lessons towards profession of a teacher with the help of school and university teachers in real school settings and so that it contributes to individual and professional development (Katrancı, 2008).

Cochran, DeRuiter and King (1993) indicate that as teachers are aware of students' thinking and understanding, the effectiveness of their instruction increases. However, Baki (2012) found in her research that preservice teachers were not able to determine the levels of students, estimate their possible difficulties and identify related subjects to form conceptual structure in teaching. It shows preservice teachers need support for increasing their experience and knowledge. Borko and Putnam (1996) reviewed several studies and indicated that preservice teachers are not open to change with the help of instruction in education programs. The results of the studies show that field experience setting and the support from university and cooperating teachers should be provided for preservice teachers in order to teach in accordance with the goals of the teacher education program. Kagan (1992) reports that most of the preservice teachers think that theory and practice are different from each other and the accordance between the content of the lessons in university and the lessons in real classroom is low. Kagan also concludes that novice teachers respectively need the knowledge of classroom management, teaching and integrating both of them to shift their own attention to students' thinking and learning. As it is understood, preservice teachers have difficulties in taking necessary actions such as planning, teaching, evaluating considering students' thinking and need support for improvement of their own knowledge and skills to become effective teachers. Professional development has made its importance apparent at this point.

On the other hand, the courses of teaching practice and school experience based on taking an active role are one of the key components and the most helpful part (Koerner, 1992) of teacher training programs. According to Graham (2006) cooperating teachers have a critical role in the success of preservice teachers gaining field experience but in general, the needs of this role are not perceived correctly and preservice teachers usually cannot receive support that they need. They view these courses as a procedure rather than a guidance for learning from experience. However, Graham suggests the constitution of discussion and a sharing environment which is led by cooperating teachers and based on collaboration for education of preservice teachers. Frykholm (1998) highlights that when preservice teachers and novice teachers discuss about several aspects of practice such as the learner, pedagogy, mathematics and reform and reflect upon teaching process, their preparation will be more effective. Similarly, Curcio and Artzt (2005) recommend "careful lesson planning, anticipation of student misconceptions and constructive reflection on a lesson after instruction" to both novice and expert teachers (p. 604). It is seen that many researchers have emphasized the components of effective professional development such as collaboration, reflection, sharing knowledge and inquiry in teacher training.

NCTM (1999) states that "teachers take responsibility for their own growth and development" (p.168) to have positive effects on students' achievement. Because teaching is a cultural based activity which is resistant to change (Stigler & Hiebert, 1999), teachers have a critical role in the educational reform (Hiebert & Stigler, 2000). Although Paker (2008) underlines the importance the cooperating teachers' and instructors' feedbacks, he also indicates that preservice teachers get insufficient feedbacks and support from them related to the phases of planning, teaching and reflecting. Practice on teaching and reflecting upon this process are key to providing improvement of the skills which are necessary to be an effective teacher (Loughran, 2002). When the studies are examined, it is seen that they possess collaborative work of teachers and preservice teachers, planning the process of teaching and learning together, observation of each other's lessons and reflection on implementations in terms of positive and negative aspects (Fernandez & Yoshida, 2004; Fernandez, 2005). In this respect, professional development supports drawing attention to the contribution on preservice teachers' improvement (DarlingHammond & McLaughlin, 1995).

Stigler and Hiebert (1999) indicate that if the courses in teacher training enable preservice teachers to plan collaboratively and implement the lesson plans and evaluate the lessons, the culture of teaching will change in a positive manner. Wilson, Floden and Ferrini-Mundy (2001) emphasize the importance of teaching experience by stating that both expert and novice teachers believe teaching experience is the key to the success of teacher training and becoming effective teachers. However, the importance of it is ruled out in general. In teacher education programs, preservice teachers should be assisted to improve their knowledge while teaching and experiencing beyond university (Darling-Hammond & Hammerness, 2005). One model which has the essential features of effective professional development model is lesson study. According to Stigler and Hiebert (1999) lesson study professional development model should be integrated into teacher education programs, method courses and the lessons on teaching profession.

Implementation of lesson study with preservice teachers helps advancing in teacher education programs (Hiebert, et al., 2007). Lesson study enables teachers to improve their instructional skills by planning a lesson with collaboration among peers, providing a real setting for teaching, sharing ideas about the lesson and refining it together (Takahashi & Yoshida, 2004). The meaning of lesson study for preservice teachers is more than practicing the steps of this model. It creates professional learning communities for them, helps them fill the deficiencies in their knowledge about subject matter and pedagogy, makes effective reflection and evaluation on the process by gaining inquiry stance (Chassels & Melville, 2009; Chokshi & Fernandez, 2005). On the other hand, lesson study effects the roles of participants and lets preservice teachers undertake the teacher role in this process (Berliner, 1994). Besides, each of them have individual strengths and weakness, the collaborative nature of lesson study aids to reduce the weaknesses and add a new approach on strengths through learning from each other (Mostofo, 2013). Researches on lesson study which are conducted with preservice teachers show that there are challenges as well as benefits of the model but it can be very effective in their development by modifying it depending on the conditions (Carrier, 2011; Chassels & Melville, 2009; Fernandez, 2005; Ganesh & Matteson, 2010; Sims & Walsh, 2008). Many researchers promote the use of lesson study in preservice teacher education programs in order to provide novice teachers with a setting which enriches their perspectives about teaching and learning through purposeful sharing (Chassels & Melville, 2009).

Stigler and Hiebert (1999) also advise incorporation of lesson study into preservice training programs. The studies related to lesson study generally focus on describing the benefits of lesson study (Chokshi & Fernandez, 2004; Fernandez & Yoshida, 2004; Lewis, 2002; Lewis & Tsuchida, 1998; Wagner, 2003; Watanabe, 2002) and they mostly attend to the implementation of lesson study (Chokshi & Fernandez, 2005). Besides, most of these studies were conducted with elementary teachers, few of them were related to preservice teachers (Hughes, 2006; Gurl, 2009). Although attention toward the use of lesson study was on teachers in numerous studies in literature, it is recommended that lesson study can be practiced with preservice teachers effectively (Carrier, 2011; Chassels & Melville, 2009Fernandez, 2005; Ganesh & Matteson, 2010; Sims & Walsh, 2008). In this respect, there is need for research related to preservice teachers in the context of lesson study.

2.2.5 Effects of professional development on mathematics teachers

Current mathematics reform movements predominantly aim to improve mathematical content knowledge of teachers, methods of mathematics teaching and success of students in mathematics and draw educators' and teachers' attention to focus on these issues (Harle, 2008) Whole reform initiatives argued the key role of professional development in the aimed changes' coming true (NCTM, 2000). It was also emphasized that mathematics teachers in all levels (The Mathematical Education of Teacher, 2001), thus, teacher training programs (National Commission on Mathematics and Science Teaching for the 21st Century [NCMST], 2000) should be supported permanently with high quality professional development to increase achievement in mathematics. Preservice teacher education programs have an important role in shaping the future teachers' beliefs and providing them experiences of practice (Thomas & Pedersen, 2003).

Good mathematics instruction in accordance with reform movements helps for the improvement of student achievement (Ross, McDougall & Hogaboam-Gray, 2002; Hiebert, 1999) because old approaches in mathematics teaching do not meet the needs anymore (Hiebert, 1999). Perry (2000) defends that teachers must empower students to be able to determine appropriate methods when needed and comprehend why they work or not instead of explaining the problem solutions directly for providing success in mathematical understanding in the context of reform expectations. Yet, explanation of the solution step by step is seen as the indicator of good instruction and so that teachers endeavor not to cause confusion in the name of students with their old perspectives (Cooney et al., 1998). The important point to be attended is not telling students what was intended and instead, allowing them to structure the knowledge under the guidance of the teacher (Arditi, 2013). However, practicing the changes in mathematics education is not easy (Ross et al., 2002). The reasons behind these challenges are teachers' beliefs (Battista, 1994), knowledge (Ball, Bass & Hill ,2004), collaborations (Stein & Brown, 1997) and structure of school (Hall & Hord, 2006). When these factors are examined, teachers' insufficiency in mathematical understanding and knowledge about pedagogy and teaching strategies makes it difficult to consider reform practices and decreases effectiveness of teachers (Hill, Rowan, & Ball, 2005). Teachers need to have more and rich mathematical knowledge to make sense of what students said and did and knowledge about how to teach mathematics for leading students to correct mathematical inferences and also need to use them in a flexible manner in the implementation of reforms in mathematics instruction (Ross et al., 2002; Smith, 1996). If teachers' beliefs are not accordant with reforms of mathematic teaching, it is difficult to adopt needed changes into practices (Tepylo, 2008). When the effects of inconsistent beliefs and poor knowledge on mathematics reforms are considered, it is thought that these challenges can be reduced with professional development based on collaboration (Hill, 2004).

According to NCTM (1991) preservice teachers and in-service mathematics teachers should learn to consider the nature of mathematics, the ways of teaching and students' learning of mathematics which are necessary for being effective in instruction. One of the important challenges that is encountered in teacher education is preparation of effective mathematics teachers (Hiebert et al., 2007) and presenting effective professional development opportunities for them (Loucks-Horsley et al., 2003). The results of the studies related to effectiveness of mathematics teachers, teacher education, professional development and students' learning advocate attending to the content of mathematics, using instructional strategies, being supported by effective professional development and collaboration with other teachers (Darling-Hammond, 2004). Being able to determine mathematical tasks in accordance with learning objectives, provide an appropriate setting, engage students in discourse and make sense of students' learning and thinking depends primarily on how teachers comprehend mathematics (NCTM, 2006). At this point, professional development is valuable in assisting teachers to improve both their own knowledge and skills and students' mathematical understanding (Harle, 2008).

The interest of mathematics reformers and educators in effective professional development has continued to maintain improvement of mathematics instruction. They endeavor to find effective ways for providing good quality training in teacher education programs because good training brings good learning and teaching mathematics with it (Ball, 2002). Teachers who are well-supported and have knowledge of mathematics and skills of teaching have positive impact on students' learning of mathematics and qualified teachers are the most important factor for student achievement (Flores, 2007). According to the research of Cwikla (2002) which included the evaluations of the relationship between teachers and students' mathematics performance, and students' achievement in mathematics was impressed by effective teachers in a positive manner whereas poor teachers had negative effects on students' mathematics performance. Cohen and Hill (2000) examined changes in the practice with self-reports of 1000 fourth grade teachers who enrolled in professional development process based on mathematics curriculum. According to the reports, teachers indicated that an increase in student's mathematics performance occurred and their content knowledge skills improved since the process helped focus on content. Consequently, the quality of teachers is an essential component in shaping students' achievement (Coble & Piscatelli, 2002). Similarly, the result of several studies outline that success of students is directly related to teacher education and their knowledge, ability and experience (Darling-Hammond & Ball, 1998). Furthermore, the investigations in reference with the relationship between student achievement and teacher education have been carried out and the focus has been on the concept of professional development in which mathematics teachers enrolled. The results of these studies show that the increase in students' mathematics achievement links to effective professional development (Yamnitzky, 2010).

Carpenter and Franke (1998) conducted their research with 22 mathematics and science teachers and concluded that the feature of taking a long time of professional development had positive effects on teachers in terms of understanding students' thinking and works. In Ball and Cohen's (1999) research based on the implementation of professional development studies with mathematics teachers, it was found that the process which included collaboration, inquiry, classroom based learning, and analyzing of student thinking affected the practice of teacher and student learning. Moreover, teachers' knowledge increased with making sense of student thinking and considering it in practice and they learned more information from the other teachers through collaboration. In line with the review of 93 studies, Kennedy (1999) indicated that professional development based on subject matter knowledge and knowledge of student thinking was more effective on students. In the study of Carpenter et al., (1989), 20 first grade teachers participated in professional development which aimed to improve teachers' anticipation about students' mathematical thinking on a specific mathematics content throughout four weeks. It was argued that depending on teachers good understanding of how students think, they improved mathematics instruction integrating this knowledge into their decisions and hereby, students' mathematics performance increased.

The review of literature reveals the strong relationship between good mathematics instruction, well-supported teachers, student achievement and effective professional development (Cohen & Ball, 1999; DuFour, DuFour, Eaker, & Many, 2006). Among professional development models, lesson study draws attention with its structure based on active participation, student thinking and ongoing support in mathematics (Harle, 2008). It is indicated in literature that teachers who participated in lesson study begin to focus more on their practices to supply students with the best learning and their understanding of mathematics increases (Fernandez & Chokshi, 2002; Lewis, 2002; Watanabe, 2002). However, the background of these positive effects based deeply on the investigation of students' learning, how to integrate students' thinking into practice and how to form collaboration that provides development of their knowledge in mathematics and pedagogy (Ball & Bass, 2001). Lesson study is a professional development model that helps providing high quality instruction and training successful students in all content areas (Lewis, 2004). Educators generally link lesson study with effective mathematics and science instruction (Lewis, 2002; Yoshida, 1999). Many studies have been conducted on lesson study as professional development model focused on mathematics and science curriculum (Fernandez et al., 2003; Heibert & Stigler, 2000; Murata & Takahanshi, 2002) and investigated the effects of it in mathematics teaching (Lewis, 2002; Doig & Groves, 2011). In order to benefit from lesson study in mathematics education in real terms, teachers must anticipate students' mathematical thinking, understand how they learn mathematics, collaborate and reflect necessary changes in practice (Fernandez & Cannon, 2005; Fernandez, Cannon, & Chokshi, 2003; Lewis, 2000; Stepanek, 2001; Stigler & Hiebert, 1999; Watanabe, 2002).

2.2.6 Implementation of lesson study in Turkey

Lesson study has been gaining importance in recent years but there are limited number of studies related to lesson study in Turkey (Baki, 2012; Baki, Erkan & Demir, 2012; Boran & Tarım, 2016; Budak, Budak, Bozkurt & Kaygın 2011; Bütün, 2012; Eraslan, 2008; Erbilgin, 2013). Baki (2012) conducted a lesson study with preservice teachers in order to improve instruction in teacher training program and examine the effect of lesson study implementation in Turkey. She found that lesson study had positive effects on the professional development of preservice teachers and provided improvement on their mathematics knowledge for teaching. The researcher suggests that the development of preservice teachers' mathematics knowledge for teaching should be maintained through some lessons in the context of teacher training program.

Baki, Erkan and Demir (2012) implemented a lesson study model with the students in a 6^{th} grade classroom in order to form an effective lesson plan which aims to increase the students' knowledge and investigate whether lesson study serves the purpose or not. They concluded that the discourse among the students increased so that their learning became more substantial during lesson study process. It was also indicated that the lesson plan that had been prepared by several mathematics teachers was quite beneficial and the students got through to the expected learning level with this study. They recommend that teachers should investigate about mathematical topics, plan the lessons based on this investigation and share information coming together as in lesson study. Budak, Budak, Bozkurt, Kaygın (2011) found that lesson study increased preservice teachers' knowledge of concept teaching and removing students' misconceptions and helped to use materials more actively. Erbilgin (2013) investigated preservice teachers' perception about lesson study by using an assessment form consisting of 14 questions. Lesson study groups including three preservice teachers each were created and they conducted the lesson study process in the context of mathematics teaching course II. It has been reached that they generally have positive opinions on the implementation of this model and accepted the contribution of it on their professional development. These findings show that lesson study model can be used in our country as differently from its origin, Japan.

Eraslan (2008) evaluates the feasibility of effective professional development strategies of lesson study such as continued collaboration, observation of each other and learning about the students' thinking and reacting ways (Darling-Hammound & McLaughlin, 1995) in Turkey. He states that teachers in Turkey are not able to observe each other since they are not willing to share their private classroom environment with the others. On the other hand, teacher education programs integrated observation into the lessons of school experience and teaching practice but it can be said that preservice teachers do not completely achieve the goals of these lessons owing to the lack of help provided by the cooperating teacher and lecturer. In addition to observation, lesson study enables teachers to evaluate their own and the others' teaching skills and strategies (Stepanek, 2001) so that they can learn new approaches from each other working as a group. However, Eraslan (2008) indicates that there is no opportunity for creating a comprehensive setting like that in Turkey.

Eraslan (2008) looks for an answer to the question of "Can lesson study work in Turkey as it works in Japan?" According to him, some factors such as teachers' cultural roles, curriculum, opportunity for flexible scheduling and university teacher may affect the implementation of lesson study in Turkey. Viewpoints and roles of teachers about teaching and learning are influenced by the culture of their society (Stigler & Hiebert, 1999). Japanese teachers pay attention to teach mathematics by showing the relationship among the mathematical concepts, facts and procedures (Stigler and Hiebert, 1999), they try to show tolerance to students in case of spending too much time to solve problems and making mistakes and create a platform to query the students' solution ways and to make students understand key mathematical points in problem solving (Shimizu, 1999). However, Turkish teachers usually focus on algorithms and rules while teaching mathematics, ask familiar problems based on mostly operational skills and solve problems explaining the steps without questioning

(Berberoğlu, Çelebi, Özdemir, Uysal & Yayan, 2003). Therefore, cultural differences in teachers' roles may cause difficulty in integration of lesson study into our education system.

Lesson study necessitates long and extra time to plan, observe and discuss the lesson. In Turkey, teachers' programs are appropriate to make time for the implementation of this model. However, Eraslan (2008) also expresses that in order to give motivation for participation, teachers should be supported by the government at the beginning. Since teachers in Turkey use the same curriculum, the lessons suggested after lesson study process can be used by the other Turkish teachers in accordance with grade levels of lessons. Another positive factor is that experts in their fields have important roles in lesson study process, Turkish university teachers can play a part in conducting lesson study model successfully. Eraslan (2008) emphasizes that if teachers in Turkey show interest, spend necessary time and energy, lesson study is going to be beneficial in terms of the development of both students' learning and teachers' profession. In addition, he suggests that the effectiveness of lesson study's integration into school experience and teaching practice lessons should be investigated in the context of teacher training programs with participation of preservice teachers, university teacher and cooperating teacher.

In the study of Bütün (2012), lesson study professional development model was integrated into the course of teaching practice in elementary mathematics education program of a university in Turkey. The research was conducted with the participation of twenty-six preservice teachers and problems encountered in the implementation of lesson study were investigated. He addressed the challenges separately in three topics which are the main steps of lesson study, planning, teaching and reflecting. The results of the research showed that preservice teachers had the greatest difficulty in anticipating students' challenges and misconceptions and offering solutions for them in planning phase. Fennema and Franke (1992) state that having knowledge about student thinking and understanding their difficulties related to topic and incorporating this knowledge into lesson depends on professional experience and is an important indicator of professional competency. It was also difficult for preservice teachers to generate the content, design the activities and determine their duration as they do not have enough knowledge about the students that they will teach the lesson and their potential in the classroom (Bütün, 2012). It is believed that lack of knowledge related to students in practice is one of the challenges in the implementation of lesson study (Chassels & Melville, 2009). Another problem encountered during planning was maintaining consistency between the sequence in teachers' instruction programs and topics in lesson study in practice schools. According to the results related to the teaching phase, the perspective of cooperating teachers in practice school and habits of teaching prevented conducting the research lessons as planned (Bütün, 2012). It is thought that this problem arises because of teachers' and schools' not understanding the function and implementation process of lesson study model. In addition, the other problems faced in teaching phase were lack of experience in classroom management, insufficient prior knowledge and physical conditions (Bütün, 2012). The results of the reflecting phase showed that preservice teachers had difficulty in evaluating the effectiveness of their lessons. Furthermore, they indicated that teachers' absence to the discussion meeting in order to reflect their ideas or lack of teachers' feedback posed challenges in recognizing the students and evaluating the lesson correctly (Bütün, 2012). Literature also argues that teachers have problems in participating the meetings before and after lessons due to the reasons stemmed from workload and not finding available time (Chassels & Melville, 2009; Cajkler & Wood, 2015).

Boran and Tarım (2016) investigated six secondary mathematics teachers' views about lesson study in Turkey. It was found that all teachers thought that lesson study had considerably positive effects on their professional development. Teachers also indicated that they learned from various experiences, gained different perspectives and utilized from collaboration (Stigler & Hiebert, 1999). Negative views of teachers about lesson study were the existence of camera for recording, difficulty of coming together and time. On the other hand, teachers who participated in the research defended that lesson study is practicable in Turkey.

Lesson study has a major role in the Japanese education system, particularly, for elementary and middle grade science and mathematics education since it develops teachers' learning, teaching practice and student achievement (Lewis & Tsuchida, 1998; Watanabe, 2002; Yoshida, 1999). Adaptation of lesson study in Turkey is suggested (Baki, 2012; Eraslan, 2008, Erbilgin, 2013) in order to improve the professional and personal development of teachers' and preservice teachers', instruction and students` learning. Erbilgin (2013) emphasizes that making research towards the implementation of lesson study can contribute to the development of education system in our country.

In traditional preservice teacher education, they have little opportunity to gain experience in heightening their own understanding of the concepts in a content (Coddington, 2014). Teachers' experiences in mathematics is the same owing to being in the same education system and it includes memorization and computation without deeper understanding of concepts (Ball, 1990; Ball, 1997). In addition to limited knowledge, preservice teachers do not feel adequate in mathematics and this causes anxiety as well. Ergo, their experience, knowledge and feeling have influence on their practices (Ball, 1997; Drew, 2011). Researchers concerning professional development in mathematics indicated that teachers' knowledge and skills should be developed continuously and opportunities for reasoning about and making sense of essential concepts should be provided (Mewborn, 2003). Thus, many studies have endeavored to develop and strengthen teachers' noticing and reveal students' thinking by promoting them (Blanton & Kaput, 2005; Franke, Carpenter et al., 1998; Franke, et al., 2007; Jacobs et al., 2010; Sherin, & van Es, 2009; van Es & Sherin, 2006). Several professional development programs were designed to improve teachers' learning from students' thinking (Blanton & Kaput, 2005; Franke et al., 2009; Jacobs et al., 2007). Attending to students' thinking in these programs helped them to learn from students, gain inquiry stance, make instructional decisions and get high student achievement (Franke et al., 1998). These studies argue that focusing on students' thinking is critical in terms of teaching and learning and provides teachers' professional growth and change (Blanton & Kaput, 2005; Franke et al., 2009; Jacobs et al., 2007). Therefore, there is need for examining what teachers' attend to and how they reason about students' thinking, in particular, their noticing skills.

2.3 Geometry and Measurement

According to NCTM (2000), being more interested in geometry and measurement in schools is important. Geometry, particularly in elementary schools, is indicated as "…has everything to do with measuring" (van den Heuvel-Panhuizen, 2005, p. 7). Measurement is defined relating to geometry and it is stated as "Measuring is a process by which a number is assigned to an attribute of an object or event" (Reys, Lindquist, Lambdin, Smith, & Suydam, 2003, p. 322). In other words, measurement provides connection between geometry and real numbers (Clements & Stephan, 2003; Lehrer, 2003). Although geometry and measurement compose significant domain in mathematics, teaching of it in middle grades is based on formulaic structure that students memorize formulas, rules, properties and definitions of geometric concepts without conceptual understanding (Fuys, Geddes, & Tischler, 1988; NCTM, 2000; Steele, 2006). There is a strong overlap between geometry and measurement (NCTM, 2000).

According to NCTM (2000), students need to progress from informal deduction based on empirical effort to formal deduction; from using the examples of shapes to applying their properties and relationships between them; from finding the perimeter, area, surface area and volume to generating patterns and generalizations. In the recent Turkish middle school mathematics curriculum, geometry and measurement were combined under a common learning area (MoNE, 2013). According to this mathematics curriculum, the learning domain of geometry and measurement aims to train students who are able to identify, draw and express the features of basic geometric concepts such as straight line, ray, polygons, prism, angle and height. Furthermore, one of the aims is to ensure that students are able to calculate perimeter and area of polygons and surface area of three dimensional solids such as prism, cylinder and pyramid (MoNE, 2013). In literature, many studies in this domain indicate that students' success is low because students have difficulty in learning even basic geometric concepts (Dane, 2008; NCTM, 2000). Besides, the results of international comparisons and assessments such as Trends in International Mathematics and Science Study [TIMSS]- 2007, Programme for International Student Assessment [PISA]- 2006, PISA- 2009 (EARGED, 2001) and national investigations (Bekdemir & Işık, 2007; Olkun & Aydoğdu, 2003) show that Turkish students have low achievement in these exams. It is believed that one of the reasons behind students' failure in learning geometric concepts and low achievement in this domain is teachers' approach (Olkun & Aydoğdu, 2003) as there is a tendency based on memorization of the essential properties, formulas and rules and representation of inadequate samples in these lessons (Fujita & Jones, 2007). On the other hand, most researches reveal that preservice teachers also have difficulties and misconceptions related to geometric concepts (Bozkurt & Koç, 2012; Koç & Bozkurt, 2011; Linchevski, Vinner, & Karsenty, 1992). Therefore, geometry and measurement is a learning domain which is needed to work on.

The concepts of perimeter, area, surface area and prism have taken important roles in mathematics curricula because of their practical properties and usage in daily lives. Hence, the students are expected to attain basic skills and conceptual knowledge about them (Kenney & Kouba, 1997; Martin & Strutchens, 2000; Ferrer, et al., 2001). The concept of area can be explained as "the amount of surface of a region" and perimeter "the distance around the region" (Dickson, 1989, p. 79). In other words, perimeter represents the total length around a figure, and area refers to the measurement of two dimensional spaces covered by a figure. These concepts provide a basis for learning or engaging in the other geometric concepts. For instance, the knowledge of perimeter helps to find the length of the unknown edge of a figure and the knowledge of area helps to understand and calculate the surface area of prisms. It also shows how geometric concepts are closely related to each other.

On the other hand, prism is a three dimensional solid which is frequently encountered as concrete objects in daily life. It is one of the most basic and essential elements of geometry and the most fundamental component of space geometry (Dane, et al., 2012). The concept of prism can be defined as “a polyhedron with an n-sided polygonal base, a translated copy of it to another plane, and n other faces joining corresponding sides of the two bases” (Marchis, 2012, p. 35). In other words, if the prismatic region is defined as the surface consisting of the pieces of a plane whose intersection points are parallel to each other and “the prismatic region bounded by two parallel bases” is called prism (Yemen-Karpuzcu & Işıksal-Bostan, 2013). All these concepts are both related to geometry and measurement, namely, based on geometric measurement. In school mathematics, geometric measurement is described as “the assignment of a numerical value to an attribute of an object” (NCTM, 2000, p. 44), such as length, area, surface area or volume of an object. Geometric measurement is a complex skill which combines the knowledge of concepts and the abilities to use them that develop slowly over years (Clements & Stephan, 2003; Committee of Inquiry into the Teaching of Mathematics in Schools & Cockcroft, 1982; Lehrer, 2003).

Students should be given various opportunities to understand the meaning of geometric concepts, to recognize the properties and relations regarding them and to learn how to measure geometric figures and objects that they encounter in their daily life (Freudenthal, 1973; van den Heuvel-Panhuizen, 2005). Students have difficulty in understanding and using geometry and measurement concepts (Martin, 2009) so it is a challenge for them to combine and link them together. Moreover, students are not able to comprehend the concepts of perimeter, area, surface area and volume (Martin &Strutchens, 2000) and prisms deeply (Battista & Clements, 1996). The reasons of not possessing a strong understanding of these geometric concepts are seen as teaching of the formulas by isolating them from each other and related geometric concepts; exposing the relationships and differences among perimeter, area, surface area, volume and prisms in an insufficient way and emphasizing further geometric aspects of the concepts rather than the measurement structures of them (Lehrer, 2003).

2.3.1 Students’ difficulties and misconceptions in perimeter, area, surface area and prism

Assessments of TIMSS, NAEP and MCAS show that students have difficulty in understanding and putting the concepts of geometry and measurements into practice (Barret& Clement, 2003; Martin 2009). They do not understand the meaning of perimeter, area, surface area and volume (Martin & Strutchens, 2000). Other than that, they are not able to understand the difference between these concepts clearly and recognize their nature based on measurement (Lehrer, 2003). For example, students do not realize that multiplication of two dimensions corresponds to the concept of area (Outhred & Mitchelmore, 2000) that is why they do not perceive the result as measurement (Lehrer, 2003).

Students often confuse perimeter and area with each other (Cavanagh, 2007; Clement & Stephan, 2004; D'Amore & Fandino Pinilla, 2006; Danielson, 2005; Kouba et al., 1988; Steele, 2006). It is likely that students apply wrong operations, for example, multiplication of edges instead of adding (Kenney & Kouba, 1997), or they use inappropriate measurement units, such as linear units versus square units (Chappell & Thompson, 1999; Steele, 2006) and they find area instead of perimeter or vice versa (Kouba, et al., 1988; Kenney & Kouba, 1997). Similar results have been reported with the other studies, they also indicate that students have crucial misconceptions related to perimeter, area, surface area and volume and demonstrate difficulty in determining measurement units to represent these concepts (Chappell & Thompson, 1999; Fuys et al., 1998; Martin & Strutchens, 2000). Students may also think that all rectangles of a given area need to have the same perimeter or all rectangles of a given perimeter need to have the same area (D'Amore & Fandino Pinilla, 2006). They have a misconception that the rectangle with the bigger perimeter also has a bigger area than the others (D'Amore & Fandino Pinilla, 2006). The reason of these confusions is related to the lack of conceptual understanding of these concepts (Kouba et al., 1988). According to Danielson (2005), they stemmed from giving definitions and formulas of these concepts without clear understanding of their meanings and relationships. On the other hand, students have difficulty in understanding conservation of area (Kouba et al., 1988), calculating areas of irregular figures (Lindquist & Kouba, 1989) and comparing areas of geometric shapes (Cavanagh, 2007). One of the misperceptions of both students and teachers may be that as the perimeter of a shape increases, area of it also increases or vice versa (Reinke, 1997; Ma, 1999). Noticing of the changeable relationship between area and perimeter is one of the most difficult issues to understand (Ferrer et al., 2001).

Students also have difficulty in achieving the objectives related to three dimensional geometry (Katona 2008, Kösa 2011). The challenge in interpreting the static images of three dimensional solids is seen as the main reason behind this situation (Accascina & Rogora, 2006). Transferring a three dimensional solid into two dimensional paper may cause optical illusions thus, confusions since it is a difficult action (Baki, Kösa, Karakuş, 2008). Besides, the inabilities in teaching two dimensional geometry in terms of supplying students with rich experiences directs students to memorize the rules, concepts and formulas (Özen, 2009). In literature, there are studies arguing that students and preservice teachers had difficulties in defining geometric concepts and misconceptions regarding them (Işıksal, Koç, & Osmanoglu, 2010; Koç & Bozkurt, 2011; Zembat, 2007). It is not easy for students to indicate examples which are not geometric figures; draw the nets of a rectangular prism; understand the concepts of perimeter, area, surface area and volume which are related to measurement; reach generalizations through the properties of geometric concepts (Martin & Strutchens, 2000). Various studies argue that students have difficulty in determining the number of cubes in a rectangular prism composed of unit cubes (Battista & Clements, 1996; Ben-Chaim, Lappan, & Houang, 1985; Hirstein, 1981). The relationships and features of two dimensional figures are based on the measurement of sides, perimeter and area and students are more comfortable with them in comparison to three dimensional figures. While engaging in three dimensional figures, students have to know and use three linear dimensions, physical appearances and isometric representations of three dimensional solids and measurement of surface area and volume (Steele, 2006). However, Steele indicates that coordinating these relationships in three dimensional figures is more complex for students and various misconceptions related to this subject arise in their minds. Specifically, some studies concluded that preservice teachers demonstrated difficulties while identifying the concept of prism (Altaylı, Konyalıoğlu, Hızarcı, & Kaplan, 2014; Bozkurt & Koç, 2012). However, definitions are obviously important to render students' best understanding of a mathematical concept (Edwards & Ward, 2008). Moreover, it is challenging for preservice teachers to think in three dimensions (VanHiele, 1986; Gökbulut, 2010; Paksu et al., 2012).

Although it is important for students to acquire comprehension about these concepts, their understanding is insufficient (Kenney & Kouba, 1997; Kouba, Brown, Carpenter, Lindquist, Silver, & Swafford, 1988) and their learning might happen by memorizing the formula. Due to the fact that the formula of each concept is presented without interrelating, it may cause difficulties in geometry (Martin, 2009). For example, the students may describe the area as the product of length and width instead of discovering the idea of covering surface (Dickson, 1989). The research of Mulcahy (2007) confirmed this view since it was found that students tend to solve the problems about these concepts by applying their formulas instead of understanding their meaning. Based on this view, Sherman and Randolph (2003) designed a study in which twenty-seven 4th grade students participated to examine their knowledge about area and perimeter. They found that the students could apply the formulas of area and perimeter without knowing either the reason of use or meaning of the formulas.

Linear measurement which provides a basis for measuring perimeter is frequently applied and often misused by early elementary students (Lehrer, 2003). Regarding linear measurement, Bragg and Outhred (2000) conducted a study with 120 students who are between 6-10 years old. They investigated students' performance while engaging in the length measurement tasks. It was advocated that although most of the students could make measurements by using a ruler and use informal units, they could not link linear units with a formal scale. It was also found that mistakes related to alignment arose while students were measuring with a ruler. It was thought that the reason behind this kind of a mistake might be starting from one (Ellis & Siegler, 1995). Bragg and Outhred (2000) emphasized that "…if students do not understand how scales are constructed, they will not have the basic knowledge to relate measurement of length and number lines, nor have the foundation to develop area, volume and other higher order mathematical applications" (p. 118). Martin (2007) also stated that the lack in teaching of linear measurement was likely to prevent learning of fundamental concepts and ideas regarding measurement. Besides, Lehrer et al. (1998) explored the development of 37 students' (6-8 ages) understanding related to length measurement. They discovered that most students used linear units related to perimeter for area measurement.

Similarly, many researches revealed that students applied inappropriate measurement units (Chappell & Thompson, 1999; Curry, Mitchelmore & Outhred, 2006; Steele, 2006). Curry et al. (2006) explored the development of students' understanding in perimeter, area and volume concurrently. It was argued that students did not recognize the importance of using appropriate units for these concepts. It might be an indicator of their unawareness of what they found and measured. It is suggested that students should understand basic measurement principles such as the use of appropriate units, the relationship between the unit and measurement and interactions in units (Martin, 2009).

The results of some studies addressing the measurement aspects of geometric concepts proclaimed that students have difficulties in measuring the area of figures (Cavanagh, 2007; Danielson, 2005; Zacharos, 2006). They applied incorrect formulas to find the area such as "area= height+ base" and "area= total lengths of sides of a figure" (Zacharos, 2006) because they do not have a strong understanding of the meanings of either geometric concepts or their formulas. Zacharos (2006) concluded that students had a tendency to use area formula "area = base × height" rather than finding the area by dividing the rectangle into units or covering it with unit squares in a similar way. However, students generally focused on the formula as a simple way to find the area instead of its meaning (Özdoğan, 2010). Likewise, Olkun and

Aydoğdu (2003) indicated students' challenges in calculating area by benefitting from the relationship among triangle, rectangle and parallelogram. At this point, their suggestion was not to teach geometric figures separately from each other.

Moreover, calculation of the area of irregular figures is difficult for students (Cavanagh, 2007; D'Amore & Fandino Pinilla, 2006). Students generally encounter regular figures and make calculations on them in their lessons. Therefore, when they see these kinds of figures they endeavor to find the perimeter instead of area because they get used to calculating the area of regular figures and using area formula. However, these figures are not appropriate for applying area formula directly, therefore students prefer not to use this formula and find perimeter and they prefer to use area formula and calculate it incorrectly (Cavanagh, 2007; Zacharos, 2006). Another concept that should be given importance in measurement instruction is conservation of area, however, it is neglected in general (Clement & Stephan, 2004). Thus, students have difficulty in understanding that when the same parts forming a shape are reconstructed in a way that will create a new shape, the area does not change. Therefore, it is suggested to provide opportunities for students to cut the shapes into pieces and reconstruct different shapes from the same parts and to realize that area remains the same.

Winarti, Amin, Lukito and Gallen (2012) investigated whether letting students' exploring the relationship between perimeter and area in early phases of learning process would support their understanding of perimeter and area concepts. Third grade students and mathematics teachers of one elementary school were participants of this research. In this study, Design research and three basic principles in The Realistic Mathematics approach were used in order to develop learning of perimeter and area. A pre-test was used to learn students' knowledge about perimeter and area whereas a pro-test was used to determine the progress in their knowledge about these concepts. It was concluded that although the class activities in teaching experiment helped some students in understanding perimeter and area and the reason of multiplying length and width to find area, some students still had difficulties in understanding these concepts and the differences between them.

Machaba (2016) examined thirty 10^{th} grade students' understanding and explanation of the concepts of area and perimeter. Their problem solving strategies about area and perimeter problems, their misconceptions, and the reasons of these misconceptions were also investigated. In order to collect data about them, a written test and clinical interviews were conducted with the students. It was found that the students did not have sufficient conceptual understanding of area and they could not appropriately explain a perimeter. Besides, they represented misconceptions about the relationship between the concepts of area and perimeter. They explained the reason of these situations based on students' inadequate prior knowledge about area and perimeter. In accordance with the findings of the study, the researcher provided some suggestions.

In addition to perimeter and area, various studies exposed students' challenges regarding the concepts of surface area, volume and prisms. Hirstein (1981) indicated that students have problems regarding rectangular prisms. The result of his study revealed that students tend to consider the unit cubes that they could see in the rectangular prism or the faces of a rectangular prism which they could count the unit cubes. He linked this difficulty with students' confusions related to surface area and volume. Similarly, Ban-Chaim et al. (1985) investigated 5^{th}-8^{th} grade students' strategies and difficulties while counting the unit cubes in a rectangular prism. Four mathematical approaches were found as follows; students counted the number of visible faces; they counted the number of visible faces and doubled it; they counted the number of visible cubes or they counted the number of visible cubes and doubled it. These findings showed that students perceived three dimensional solids as two dimensional figures or they did not understand three-dimensionality in prisms and they were not aware of the hidden parts in them either. There was a consistency between the results obtained from the studies of Hirstein (1981) and Ben-Chaim et al. (1985).

Battista and Clements (1996) researched on 3^{th} and 5^{th} grade students' understanding regarding the volume of three dimensional solids. The result of the study advocated that it was very challenging for the students to enumerate the number of unit cubes in a rectangular prism. It was thought that the reason of this challenge might be based on inappropriate or insufficient mental construction of a rectangular prism. In the study of Battista and Clements (1998), 5^{th} grade students were asked to find the surface area and volume of rectangular prisms consisting of unit cubes. Although students used formulas to find surface area and volume, none of them were capable of establishing the connections between the formula and a spatial structuring with unit cubes. He indicated that:

> Students who spatially structure an array into columns or layers generally calculate the total by skip counting or multiplying by the number of cubes in a column or layer. Alternatively, many students structure an array as an unrelated set of rectangular-prism faces. They determine the number of cubes visible on all or some of these faces, usually counting cubes along the prism's edge more than once. (p. 405)

It was advocated that after students gained the ability to coordinate representations of rectangular prism, the volume of it should be teaught otherwise problems might arise. With continued studies on volume, surface area, and spatial sense, Battista (2002) tried to facilitate students' layer conception understanding of a rectangular prism's volume with the help of a web-based tablet that he generated. This tablet enabled students to add the cubes one by one, in rows, columns or layers into an empty rectangular prism. It also allowed them to open and close the side of a rectangular prism so that students could see the relationships between three dimensions, layers and volume. It was advocated that teaching the volume of a rectangular prism in this way was more effective than the formula as this system provided the development of numeration strategies and coordination of the images of figures.

Martin (2007) examined the effectiveness of the investigation of perimeter, area, surface area and volume concurrently with 4th grade students during their performance in which they tried to recognize the properties regarding these concepts and understand the relationships among the properties of different dimensions. The results of this exploratory study revealed an increase in students' performance and understanding in terms of area, surface area and volume and use of appropriate measurement units. Similarly, Martin (2009) tried to attain evidence that 4th grade students could understand the concepts of perimeter, area, surface area and volume and the relationships among them through teaching concurrently instead of teaching in sequence. The effects of non-traditional manipulatives on students' understanding of these four concepts were investigated. The findings proved that the factors that affected students' success positively in this study were manipulative materials; the tasks and exploratory activities based on concurrent teaching of perimeter, area, surface area and volume; correct mathematical language and cooperating learning environment.

In Turkey, various studies were designed to research students' understanding of perimeter and area concepts. Emekli (2001) conducted a study with the aim of examining 7th and 8th grade learners' misconceptions on these concepts. It was observed that the students confused the concepts of area and perimeter, had difficulty in using their formulas in the problems. The findings of Dağlı's study (2010) were parallel to the findings of this study. Dağlı (2010) focused on the misconceptions of 5th grade learners about area and perimeter in the study. 262 fifth grade students participated in the study. According to the findings of the study, students had difficulty in understanding these concepts and they had the biggest challenge in understanding the concept of area. The confusion of these concepts was observed. Other than that, the students had difficulty in computing area and perimeter, and finding the side lengths of the shapes in spite of knowing their area and perimeter. Tan-Şişman and Aksu (2009) designed a study with 134 seventh grade students. It was observed that the participants could not successfully use the area and perimeter formulas although being aware of their differences and understanding these views. Hence, they had difficulty in understanding the situations in which the area of the shapes was preserved and the perimeter of them changed.

In the studies of Olkun (1999), elementary students' success and difficulties in finding the number of unit cubes in a rectangular prism were explored. The results of these studies asserted that students had difficulty in understanding the structure of a rectangular prism and physical materials of prisms helped to understand them. Olkun (2001) also investigated how students conceptualized the volume of a rectangular prism through various drawings. It was found that students understood quicker with concrete prisms rather than drawings; they perceived big prisms more complicated and they also had difficulty in envisaging the prisms' systematic structure based on row, column and layers with unit cubes.

Moreover, Zembat (2007) focused on an instructional design about the construction of volume formula a rectangular prism. He conducted an action research on 22 seventh graders of an art school who did not know this concept. He examined the way that the students constructed this formula in their minds. He also stated the students' developmental process of constructing this formula. He found that filling up the base layer and adding up a number of layers until the box is filled are students' mental activities observed in the instructional design. Based on the findings, he explained that the students acquired the ideas for Cavalieri's principle by reflecting themselves while engaging in the activities. Cantürk Günhan and Özen (2010) examined the students' viewpoints about learning prisms by drama method. In this study, the students participated in drama lessons designed based on the concept of prisms. These lessons were also organized about the area and volume of prisms. They explained that the students' creativeness was funny and beneficial for learning prisms.

In relation to prisms, surface area and volume concepts of students, Dane et al. (2012) conducted a study regarding rectangular prism with primary and secondary school students who were at 5^{th}, 8^{th} and 11^{th} grades. They investigated how students perceived rectangular prism by using four different types of models. Three of them were frequently encountered models whereas one of them was less frequently encountered. It was found that as the grade increased, the success in naming the models correctly increased. In other words, students used the basic concepts related to rectangular prisms such as edges, corner, surface, and surface area more consistently, and established relationships between them and the grade. Besides, it was observed that students named frequently encountered models more appropriately than the ones infrequently encountered. The answers of students also displayed that they confused two dimensional figures such as square and rectangle, three dimensional solids such as rectangular prism and cube; concepts such as side-edge, side-surface, and angle-corner while naming and making comments related to them. Tekin-Sitrava and Işıksal-Bostan (2014) focused on 8^{th} grade students' performance, solution strategies, difficulties and their reasoning about finding the volume of a rectangular prism in the activities. This qualitative study was conducted with middle school students. It was discovered that although students represented medium level of performance in the questions, they formed a limited number of strategies about the solution of problems. On the other hand, they stated that unfortunately there were students who could not find the volume of a rectangular prism and had difficulties in problems.

The literature shows that there are existing difficulties in understanding perimeter, area, surface area and prisms for different grade levels of learners. Some studies explain the reason of this situation as misconceptions and lacking conceptual understanding (Ma, 1999). The previous research examined the misconceptions in the measurements of area and perimeter by focusing on students' studying problems and teachers' describing and defining them (Menon, 1998; Reinke, 1997; Simon & Blume, 1994). Moreover, research shows that students' misconceptions are held by prospective and in-service teachers too (Kellogg, 2010). The students do not successfully make separation among the concepts of area and perimeter based on different reasons (Kouba et al., 1988). For example, they can apply wrong algorithms for computations and pay attention to inaccurate units of measure (Chappell & Thompson, 1999; Kenney & Kouba, 1997; Kouba, et al., 1988). Kouba et al. (1988) explain that the main reason of this case is not having sufficient level of conceptual understanding about these concepts. Furthermore, even students who have high achievement find challenge in understanding the relationships between formulas and perimeter, area, surface area and volume (Boaler & Humphreys, 2005). Based on the review of many researches' results, Steele (2006) indicates that both students and teachers have difficulty in establishing connections among the abstract properties of geometric figures, concrete measurement of dimensions and calculation of surface area and volume of prisms through cube buildings.

2.3.2 Preservice teachers' knowledge of perimeter, area, surface area and prism

Many studies also concluded that teachers and preservice elementary teachers had misunderstanding related to the concepts of area and perimeter (Menon, 1998; Reinke, 1997; Simon & Blume, 1994; Steele, 2006; Tierney, Boyd & Davis, 1990). Besides, they pointed out the inadequacy of preservice teachers' reasoning about how students might think in the subjects of perimeter and area. It can be said that preservice teachers had deficient knowledge, understanding and reasoning regarding perimeter and area (Kellogg, 2010). Preservice teachers also confused area, surface area and volume and relied on the formulas of measurement concepts without conceptual understanding (Enochs & Gabel, 1984; Latt, 2007). They have some misconceptions and lack of knowledge regarding perimeter, area, surface area and prisms (Livy, Muir & Maher, 2012). These gaps in their understanding may cause problems in the future because teachers' knowledge effects teaching and student learning (Ball & McDiarmid, 1989). Similarly, Kellogg (2010) advocates that preservice teachers will reflect their such misconceptions to their future students and their students will also have them. In general, preservice teachers' experiences in measuring show similarities with elementary and middle school students'.

It has been observed that preservice teachers tend to confuse the concepts of area and perimeter. Reinke (1997) conducted a study focusing on that case. 76 preservice elementary teachers are asked to describe the ways of finding the perimeter and area of shaded parts of some given figures. In their written reports, it was observed that 22% of the participants explained how they would find the area even though they were wanted to find the perimeter of the shapes. Besides, it was found that they made explanations for area more successfully than finding the perimeter of them. The participants explained the area of a circle including 360 degrees based on the semicircle including 180 degrees. In that explanation, they used the word "including" instead of "covering" due to their lacking knowledge. Furthermore, Tierney et al. (1986) conducted a similar research in a mathematics course with preservice teachers. Throughout this course, the preservice teachers' misconceptions about area and perimeter were examined. This investigation was performed by collecting data through their journal writings. It was observed that preservice teachers tended to think that findings area was equal to a number. In this view, a preservice teacher counted the pegs around a figure in order to compute its area since s/he tried to find a reasonable number. Another participant explained that finding the area was not reasonable since it could not be measured directly by a tool. In that research, it was observed that they confused what to count to find area and perimeter since they might not have sufficient understanding about the meanings and units of area and perimeter. In some other research, it was realised that preservice teachers had challenges with irregular shapes (Maher & Beattys, 1986). Moreover, Tierney et al. (1990) argued that preservice teachers endeavored to use the length x width formula in order to calculate the area of irregular shapes.

Most of them even do not understand why the length and width should be multiplied to find the area of a shape (Simon & Blume, 1994).

Baturo and Nason (1996) conducted a study with 16 preservice teachers regarding their content knowledge and pedagogical content knowledge in the subject of area. They were clinically interviewed while they worked on a set of eight tasks that were developed for the study. The results of the study revealed that preservice teachers' subject matter knowledge was limited to provide students with meaningful understanding of relevant mathematical concepts in the subject. Researchers also made some inferences about preservice teachers such as they would demonstrate difficulties in using multiple and appropriate concrete representations while introducing area concepts, their responses to students' questions would be cognitively superficial and they would have challenges in linking area to the other subjects in the curriculum. Furthermore, Fuller (1996) compared 26 preservice elementary teachers' and twenty-eight experienced elementary teachers' pedagogical content knowledge regarding perimeter and area. It was concluded that only one of the experienced teachers could give the correct procedural and conceptual answer whereas most of the preservice teachers offered answers which lacked in terms of mathematical content and pedagogy.

Similarly, Menon (1998) examined preservice teachers' understanding of perimeter and area. Fifty-four postgraduate (elementary school) preservice teachers participated in the study. Four tasks about perimeter and area were conducted with the participants. They were also required to prepare a problem to be asked to students about the understanding of perimeter. Based on the findings of the study, it was seen that preservice teachers tended to have procedural knowledge about the concepts rather than a conceptual and relational understanding of them. Fuys et al., (1998) investigated 8 preservice teachers' and 5 practicing teachers' knowledge in geometry and measurement topics. They concluded that preservice teachers and teachers exhibited similar misconceptions with students. They had difficulty in explaining the features to define geometric figures with the concept of area and confused linear and square units.

Furthermore, Henry and Soyibo (2006) designed a study to find out whether preservice teachers' performance on perimeter, area and area-perimeter was satisfactory; the differences among their performance on each concept were significant in terms of their gender, cohort, age and mathematical abilities; and there were significant correlations among independent variables and their performance on each concept. The study was conducted with 200 preservice teachers. Data collection tools were an 18-item multiple-choice test and four structured questions on perimeter, area and area-perimeter. The findings revealed that preservice teachers' performance on perimeter, area and area-perimeter was barely satisfactory; there were significant differences in their performance in favour of students with high mathematical abilities whereas there were no significant differences in their performance in terms of other independent variables; and the relationship between their performance on each concept and their mathematical abilities was positively significant but weak.

In the study of Yew, Zamri and Lian (2011), they analyzed preservice secondary school mathematics teachers' subject matter knowledge of calculating perimeter and area of a composite figure. The participants of this case study research were 8 preservice teachers who were in the 4th year of their teacher education programs. The data was obtained from their responses on a particular task related to the subject through clinical interviews. It was argued that most of the preservice secondary school mathematics teachers' procedural knowledge of calculating perimeter and area of composite figure was enough and they could understand measurement convention (e.g. "perimeter-linear units; area-square units). It was also found that they did not check the correctness of their responses. Moreover, Livy, Muir and Maher (2012) investigated preservice teachers' understanding of area and perimeter. They reported primary pre-service teachers' understanding and misconceptions of area and perimeter by focusing on their mathematical knowledge for teaching mathematics. 17 preservice teachers who were in the final year of their teacher education programs took part in the study. It was observed that most pre-service teachers had sufficient procedural understanding of area and perimeter, tended to represent similar misconceptions and had an insufficient level of mathematical knowledge of area and perimeter.

Preservice teachers' knowledge of surface area, volume and prism concepts were focused on as well as perimeter and area. Enochs and Gabel (1984) investigated 128 preservice teachers' misconceptions related to the measurement of surface area and volume through six tasks. In the task related to finding the volume of a rectangular prism, although %77 of preservice teachers indicated that volume was found through multiplying length, width and height, only %44 of them felt certain that the formula of base area times height could be used to find the volume of a rectangular prism. This high contrast of percentages in different formulas of volume was unexpected. It was also concluded that preservice teachers confused surface area and volume and used formulas inappropriately. This result was linked to relying on the use of formulas rather than understanding conceptual meanings of surface area and volume (Enochs & Gabel, 1984).

Vistro (1991) investigated preservice elementary teachers' conceptual and procedural knowledge of perimeter, area, surface area and volume. The responses of 65 preservice teachers to a written test including problems related to these four measurement concepts and the data obtained from interviews were analyzed. The researcher tried to uncover preservice teachers' basic geometric concepts, their procedures in measuring these concepts and the connection between their conceptual and procedural knowledge. This study argued that preservice elementary teachers knew basic geometric concepts related to perimeter and procedures to measure it; they also had knowledge about the concepts regarding area but had a tendency to rely on the formula of A=LxW while finding area; they know the basic concepts of volume but are used to applying the formula of V=LxWxH for rectangular prisms; and their knowledge about surface area is low and they confuse the concepts of area and surface area with each other. In general term, the result of the study showed that preservice teachers had various misconceptions related to geometry and measurement. Similarly, Latt (2007) tried to explore preservice teachers' understanding of perimeter, area, volume and the relationships between them. The results of the study demonstrated that preservice teachers' understanding of these four concepts, except for area, was limited. They had a tendency to apply formulas based on rote learning in order to calculate them.

One of the important concepts in the domain of geometry and measurement which is related to perimeter, area, surface area and prism is dimension. Although it is a fundamental idea in mathematics, teachers do not generally feel the need to explain this concept, as a result students are not able to understand its meaning fully (Skordoulis, Vitsas, Dafermos & Koleza, 2009). In general terms, this concept is defined as the extension of an object in any direction and mathematically, it is expressed as each of the length, width and height from three directions which is considered in measuring of straight lines, surfaces or objects (Peker & Karakuş, 2013). In literature, there are some researches on this concept. Ebersbach (2009) designed a study in which the participants were 19 kindergarten students, 20 first grade students, 17 third grade students and 17 adults. They were expected to estimate the volume of rectangular prisms in different sizes through unit cubes. The researcher analyzed which features they used to define the three dimensions while engaging in this action. The results of the study proved that there was no difference between the features the participants used to define three dimensions and they applied the concepts of length, width and height equally. Skordoulis et al. (2009) endeavored to examine the role of the Cartesian x-y plan while preservice teachers were determining the value of the dimensions of an object. It was discovered that Cartesian plane was effective in determining the dimensions of objects. It was also disclosed that preservice teachers had difficulty in understanding and finding the number of dimensions of a figure or object.

In Turkey, preservice teachers have confusions and difficulties similar to students too. Şimşek (2011) investigated preservice mathematics teachers' pedagogical content knowledge related to the subjects of perimeter and area in terms of students' potential difficulties. 5 senior preservice teachers participated in this case study. The findings revealed that preservice teachers had inadequate knowledge about students' difficulties related to perimeter and area. Furthermore, it was found that they also faced the same difficulties that students had. Besides, preservice teachers lacked knowledge and skills to determine and eliminate students' difficulties in related subjects. Similarly, Yenilmez and Koza-Çiftçi (2014) tried to identify pre-service high school teachers' knowledge of perimeter and area in their study. They conducted a case study with 36 pre-service teachers. They detected that most pre-service teachers did not have sufficient knowledge of these concepts. Şimşek and Boz (2015) conducted a study with 85 preservice primary mathematics teachers. The researchers explored preservice teachers' knowledge of length measurement. They examined whether preservice teachers were able to determine students' different mathematical approaches related to counting the points and lines on a ruler instead of units which was one of the common misconceptions students had. It was concluded that preservice teachers' conceptual understanding of length measurement was inadequate.

National studies demonstrated that preservice teachers had difficulties and lack of knowledge in regard to prisms as well as students. Bozkurt and Koç (2012) explored 158 first year preservice elementary mathematics teachers' knowledge of prism. It was argued to be challenging for preservice teachers to define the concept of prism and they could not use sufficient mathematical language in general. It was also ascertained that preservice mathematics teachers were not able to indicate the crucial features in their definitions of prism. Çetin et al. (2012) investigated preservice primary school teachers' perceptions of rectangular prism by using four different models, three of which were frequently encountered models whereas one of them was less frequently encountered. Researchers interviewed with 113 preservice teachers on these models. The findings showed that most of them named the three models that they were familiar appropriately and explained the features and relationships of relevant geometric concepts. However, their success was low in naming the model that they encountered rarely, which was the skeleton of a rectangular prism. Therefore, the use of skeleton model was suggested for providing contribution to a better construction of such concepts such as prism and pyramid.

In the study of Gökbulut and Ubuz (2013), preservice primary teachers' knowledge on prism was searched. 4 preservice teachers' definitions and examples were analyzed based on some criteria. It was argued that preservice teachers had inadequate knowledge regarding prism and so it was difficult for them to define this concept. Besides, it was concluded that preservice teachers could not use mathematical language well either, they did not know the necessary principles for the definition of prism and their undergraduate degree did not have an effect on the ability to identify and exemplify the prism. At this point, the lack of providing appropriate definitions and examples in mathematics content lessons was emphasized and accordingly, making the lessons richer in terms of these concepts was suggested.

Similarly, Gökkurt and Soylu (2016) designed a study with 6 preservice middle school teachers in order to examine their mathematical content knowledge regarding prisms. The researchers considered some dimensions in teachers' content knowledge of prism which were identifying the concept of prism, drawing open and close images of prisms, recognizing the prisms which were in open and close forms and explaining the logical justifications underlying surface area and volume formulas of prisms. According to the findings, it was obtained that teachers had problems in identifying the concept of prism, determining the basic elements and recognizing different surface developments of cube. It was found that teachers drew similar prism types independent from the years of their teaching experiences and they were good at drawing prisms. Moreover, the findings showed that some of the teachers used incorrect concepts in their instructional explanations. For example, the concept of ceiling was used instead of base or upper base. It was parallel to the findings of Bozkurt and Koç (2012). Besides, it was found that teachers could draw only two different surface developments of prisms and they had difficulty in identifying open and close forms of prisms.

In literature, it was seen that most of the studies were related to three dimensional solids including rectangular prism, cube, cone, pyramid and cylinder instead of focusing on a particular geometric shape. The results proved that preservice teachers had difficulty with geometric shapes. When the findings regarding prisms were considered, it was seen that preservice teachers had misconceptions; they made mistakes in their definitions and they did not know all the necessary properties (Altaylı et al., 2014; Bozkurt & Koç, 2012; Çetin et al. 2012; Gökbulut & Ubuz, 2013; Gökkurt and Soylu, 2016).

Differently from the existing studies, Toptaş (2014) explored preservice primary teachers' explanations regarding the term of "edge" which was one of the basic concepts in prism like the other three dimensional solids. 195 preservice teachers participated in this study and they were required to explain the edge term at three levels and determine three appropriate questions. The researcher analyzed their explanations in terms of the mathematical thinking development phases. According to the findings, three different categories emerged which were a) some of the preservice teachers could both explain the edge term and ask three appropriate questions b) some of them could explain the term, however, they could not ask appropriate questions and c) some could neither explain the term nor ask appropriate questions according to mathematical thinking developmental phases.

In Turkey, some researchers also made investigations on the concept of dimension which was a fundamental concept for geometry and measurement. Ural (2011) conducted a study with 2nd, 3rd and 4th grade preservice elementary mathematics teachers and postgraduate students to learn how they determined the number of dimensions of a figure. In the study, it was discovered that preservice mathematics teachers had misconceptions regarding geometric figures and objects and they demonstrated difficulty in understanding and recognizing dimensions as one, two or three. In addition, while determining the dimensions, they applied the features such as area-volume, number of axis, length-width-height and plane-space position. Ural (2014) also made a similar investigation with 15 mathematics teachers and aimed to determine which criteria teachers used to decide on the dimensions of a geometric figure and how the consistency between criteria and decision was. The findings showed that half of the teachers used criteria; the common criteria were area-volume and length-width-height; and there was inconsistency between the criteria they used and the decisions about dimension. It was thought that the reason of these results stemmed from the lack of their knowledge on geometric concepts. Likewise, Paksu, Musan, İymen and Pakmak (2012) tried to determine 46 preservice primary teachers' concept images related to dimension. According to the findings, it was advocated that the knowledge of preservice teachers on dimension was inadequate and their concept images were not correct. Besides, they considered various criteria such as the number of vertexes, edges, diagonal and visible faces while determining the number of dimension.

When the confusions, difficulties and mistakes of both students and preservice teachers regarding perimeter, area, surface area and prism concepts were considered, the importance of planning and teaching based on students' mathematical thinking and investigating preservice teachers' noticing of these points arose. One of the topics in teacher education that was paid more attention is student thinking (Kellogg, 2010). Teachers should know how students think mathematically (e.g. the subjects of perimeter and area), what kind of difficulties and misconceptions they may have regarding the subject and what the possible reasons behind these obstacles are (Lehrer, 2003; Simon & Blume, 1994). Therefore, preservice teachers with this equipment focus on students' thinking so that future mathematical instruction and student learning may become more effective (Ball et al., 2001). From this point of view, in this study, preservice middle school mathematics teachers' noticing skills of students' mathematical thinking related to the subject of perimeter, area, surface area and prism in the learning domain of geometry and measurement were investigated. The literature related to perimeter, area, surface area and prism and previous studies about students', teachers' and preservice teachers' understanding of these concepts helped to determine specific mathematics domain that included a large variety of students' mathematical thinking for the current study.

2.4 Summary

One of the professional skills that both teachers and preservice mathematics teachers should gain to be a successful mathematics teacher is noticing (Mason 2002). Noticing of students' thinking provides changes in teachers' instructional practices (Franke, Carpenter, Levi & Fennema, 2001; Kazemi & Franke, 2004; Steinberg, Empson & Carpenter, 2004). Instructional practices which promote and build on students' thinking increase mathematics teaching and learning (Hiebert & Wearne, 1993). In other words, teachers' ability to notice students' mathematical thinking is important for increasing students' achievement (Carpenter, Fennema, Peterson, Chiang & Loef, 1989; van Es, 2011). It can be difficult even for teachers to attend to noteworthy aspects of the lesson, making sense of what they observed and offer appropriate responses (Star, Lynch & Perova, 2011; Star & Strickland, 2007; Vondrová & Žalská, 2013). Therefore, there is need for developing preservice teachers' noticing skills by centering on teacher preparation programs (Hiebert, Morris, Berk & Jansen, 2007).

Studies reveal that teachers can learn and develop their noticing skills through professional development models (Burns, 2005; Chamberlin, 2005; Fernandez, Cannon, & Chokshi, 2003; Franke, Carpenter, Levi, & Fennema, 2001; Jacobs, Franke, Carpenter, Linda, & Battey, 2007). Lesson study is a model which aims to maintain professional development of teachers, in-service and preservice teachers. It consists of a cycle which includes three main phases: planning, teaching and reflecting. In planning, teachers previously endeavor to predict how students might think mathematically, prepare a lesson plan from the perspective of students and evaluate the content of the lesson that they prepared within collaborative work. Sharing ideas enables teachers to notice students' possible ways of mathematical thinking and interpret them as well as see each other's mathematical approach (Lewis, Friedkin, Baker & Perry, 2011). The collaborative structure of lesson study based on exchanging ideas about the goals of the lesson, students' mathematical thinking and building instruction by considering them provides professional development for teachers (Chokshi & Fernandez, 2004). Research also points to teachers' and preservice teachers' misunderstanding related to the subjects of perimeter and area which were included in geometry and measurement domain and their insufficient understanding of students' thinking regarding these concepts (Menon, 1998; Reinke, 1997; Simon & Blume, 1994; Tierney, Boyd & Davis, 1990). It is important to learn what and how teachers notice in geometry and measurement which is one of the crucial mathematics domains in school curriculum (Yeo, 2008).

To sum up, providing opportunities for preservice teachers to participate in the lesson study process might help to increase their ability to notice students' mathematical thinking and improve them professionally. In the light of the previous studies on noticing, lesson study and the subjects of perimeter, area and prism, this study aimed to investigate preservice middle school mathematics teachers' noticing skills of students' mathematical thinking related to the subjects of perimeter, area, surface area and prism in the learning domain of geometry and measurement and how their noticing skills could be developed through lesson study. The present study also aimed to contribute to the limited literature on preservice teachers' noticing skills and the use of lesson study in teacher education in Turkey.

CHAPTER 3

3. METHODOLOGY

The purpose of this study was to investigate preservice middle school mathematics teachers' abilities to notice students' mathematical thinking in the context of lesson study professional development model. In this study, qualitative research methodology, in particular, case study was used in order to elaborate what and how preservice teachers notice during the phases of planning, teaching and reflecting and present an in-depth picture of the process. Research design, participants, researcher's role, research procedures, data collection and data analysis processes and trustworthiness of the study are presented in this chapter.

3.1 Research Design

Research design can be chosen in regard to the types of the research questions, the role of the researcher and the focus of the study (Yin, 2003). In this study, it was aimed to investigate and examine preservice teachers' noticing skills during the participation in professional development model and qualitative research approach was used. Qualitative research method enables researchers to gain a viewpoint about people in their social and cultural settings (Stake, 1995). Besides, it helps to get more detailed information about participants and to determine important points by interpreting the data (Walker, 1985). Thus, this approach is appropriate for investigating and elaborating what and how preservice teachers notice throughout planning, teaching and reflecting phases in lesson study.

> Qualitative research is a situated activity that locates the observer in the world. It consists of a set of interpretive, material practices that make the world visible. These practices transform the world. They turn the world into a series of representations including field notes, interviews, conversations, photographs, recordings, and memos to the self. At this level, qualitative research involves an interpretive, naturalistic approach to the world. This means that qualitative researchers study things in their natural settings, attempting to make sense of, or interpret, phenomena in terms of the meanings people bring to them (Denzin & Lincoln, 2005, p. 3).

Qualitative research can be defined as a process that entails revealing the events and perceptions in a kind of realistic and holistic way through data collection methods such as observation, interview and document analysis (Creswell, 2007). It aims to explain the relationships among a set of results considering obtained data, investigate and understand social facts in their own context (Glaser, 1978). It involves collecting and analyzing a variety of data that uncover the routine and problematic situations and identify their meanings for individuals. There is an effort for better understanding of the issue and human experience through interpretive practices and making it more visible (Denzin & Lincoln, 2005).

According to Creswell (2007) qualitative research begins with assumptions and research questions towards social or human problems and necessitates to examine individuals or groups through a theoretical lens or a worldview. Researchers use qualitative approach based on inquiry, data collection that takes place in a natural setting and data analysis that is inductive and includes creating patterns or themes in order to research the problem. The results comprise of participants' views, the reflexivity of the researcher and thick and detailed description and interpretation of the problem. Besides, there are several common features of qualitative research: (a) natural setting, (b) researcher as key instrument, (c) multiple sources of data, (d) inductive data analysis, (e) participants' meaning, (f) emergent design, (g) theoretical lens, (h) interpretive inquiry and (i) holistic account. If we address qualitative research methodology in the direction of these characteristics, researchers collect data by themselves through examining and observing the events in a natural context where participants encounter the problem or experience the issue. The objective of this study was to determine the meaning that preservice teachers gave to their experiences in lesson study process. Accordingly, lesson study process was implemented within the course of teacher practice so that preservice teachers had a chance to maintain the study in the natural settings which they were familiar with. Furthermore, a prevalent framework was used in order to interpret preservice teachers' noticing skills and their experiences during lesson study in a more effective way. In qualitative research, multiple data sources such as interviews, written documents and observations are used and researchers organize the data to create patterns, categories and themes through inductive approach. The aim is to learn and reflect how participants ascribe a meaning to the problem or issue, not researchers' meaning. Additionally, qualitative research process is flexible and some changes may occur in time. Researchers interpret what they see and hear and adopt interpretive inquiry into the research. Moreover, it is endeavored to draw the big picture and identify details of the problem or issue (Creswell, 2007). Similarly, interviews, preservice teachers' lesson plans, observations, field notes and video recordings were utilized to collect data in the present study. It was intended to obtain rich data about the topic that was investigated to present more detailed and realistic information. Thus, it was centered on preservice teachers' noticing of students' mathematical thinking and changes on it for four lesson study cycles and during approximately two months to provide an in-depth picture of the whole process.

Qualitative research focuses on what experiences individuals had and how they interpreted them. Merriam (1998) stated "qualitative researchers are interested in studying the meaning people have constructed, that is, how they make sense of their world and the experience they have in the world" (p. 6). The nature of qualitative research is based on endeavoring to obtain in-depth information about human experience or the events through detailed examination of human behaviors and sense-making the reasons behind these behaviors (Trochim & Donnelly, 2008). They emphasize its interpretive and naturalistic structure. Furthermore, the aim of it is not to generalize the results to broader situations directly, rather it is to have detailed information about experiences and understand them. In this study, two cases were addressed in details to present the nature of preservice teachers' noticing skills and changes on these skills over time rather than making generalizations about the results.

On one hand, Creswell (2007) indicates when it is appropriate to use qualitative research in a detailed way. Qualitative research is conducted when there is need for investigation of a problem and detailed understanding of the issue. It is used to promote individuals to share their views and to make and report research in a more flexible way. Moreover, it is conducted because of the desire to understand the natural settings in which participants experience the problem or issue. On the other hand, researchers use qualitative research to support quantitative results, elaborate and develop theories and when "quantitative measures and statistical analyses do not fit the problem" (p. 40). However, carrying out qualitative research is not easy. It takes a long time, includes the use of many resources and requires a strong commitment. In sum, "a qualitative approach is appropriate to use in order to study a research problem when the problem needs to be explored; when a complex, detailed understanding is needed; when the researcher wants to write in a literary, flexible style; and when the researcher seeks to understand the context or settings of participants" (Creswell, 2007, p. 51).

In the context of this study, the goal was to investigate preservice teachers' experiences and behaviors in terms of noticing within the process of professional development which includes practicing in real classroom settings and working with colleagues. In other words, it was sought to learn about the details of what and how they noticed, understand their approach and views and evaluate the development in their abilities to notice. Hence, there was need for implementation of qualitative research to gain detailed understanding of preservice teachers' noticing skills and experiences in a natural context, examine how they constructed meanings, learn their perspectives and present the big picture of the whole process. At this point, this current study was based on qualitative research methodology, in particular, case study approach was used since it was suitable for the purpose of the research.

3.2 Case Study

Case study is a qualitative approach in which the researcher investigates a case or cases over time and in detail through a rich data collection involving multiple resources of information such as observation, interviews, audiovisual material, documents and presents the description of case or cases (Creswell, 2007). Even the most distinct feature of it is focusing on a current event, case, situation, individual, several individuals or groups and analyzing in detail (Stake 1995). In other words, it is "an in-depth description and analysis of a bounded system" (Merriam, 2009, p. 40). Researchers reveal in-depth information about the issue and its context through this elaborative analysis (Putney, 2010). Whereas Stake (1995) states that case study is the preference of what to study, other researchers describe it as a strategy of inquiry, a methodology or a comprehensive strategy for research (Denzin & Lincoln, 2005; Merriam, 1998; Yin, 2003). Merriam (1998) states that case study is "an intensive, holistic description and analysis of a single instance, phenomenon or social unit" (p. 21). According to Creswell (2007) the main definition of case study integrates investigating and understanding an issue using a case (a bounded system) or cases (multiple bounded systems) as specific illustrations. In this study, the case was two senior preservice middle school mathematics teachers' noticing of students' mathematical thinking in the context of lesson study professional development model. The aim was to investigate what and how preservice teachers notice in detail, understand how they ascribed meaning to the experiences and practices within the context and examine the development of their abilities to notice during the process in the present study.

Stake (1995) explains the procedure of conducting a case study and indicates that it begins with the decision on whether case study approach fits the research problem. Researchers make a detailed description of the case or cases which might be an individual, several individuals or an event. What kind of purposive sampling to use while selecting the case is decided. Extensive data collection is made based on multiple sources such as observation, interview and documents. Detailed description of the case is reflected through holistic or embedded analysis of the data (Yin, 2003). Researcher might address to more specific issues to elaborate the complex structure of the case as well as this description. Finally, the meaning of the case is reported with an interpretive stance (Stake, 1995). However, there are still some challenges in conducting the case study. Researchers must describe the case or cases by setting the boundaries in detail and indicate a rationale for selecting the case (Creswell, 2007).

Case study is a useful and effective method for gaining knowledge about the actions and reflections in the process and practicing educational programs (Stake, 1995). Snow and Anderson (1991) refer to case studies as

> relative holistic analyses of systems of actions that are bounded socially, spatially, and temporally; they are multi-perspectival, and polyphonic; they tend toward triangulation; they allow for the observation of behavior over time and thus facilitate the processual analysis of social life; and they have an open-ended, emergent quality (p. 152).

Case study is preferred to enhance knowledge about the case or cases rather than generalize the results (Stake, 1995) and it is used commonly for studying complex and specific situations in their naturalistic contexts in order to reveal and present detailed and meaningful understanding of them (Yin, 2003). Case study is appropriate for presenting in-depth characteristics of identifiable cases (Stake, 1995) and gaining insights into practices. Yin (2003) advocates that case study deals with contemporary events in real settings through an empirical inquiry and searches for answers to the questions in the form of how and why. Besides, it is used more for learning about the issue and the meaning of it and focusing on the process rather than the outcomes (Merriam, 1998). In this study, preservice teachers consistently engaged in the actions of planning, teaching and reflecting based on collaboration with colleagues in a real classroom or university settings. Moreover, it was a utilized framework for learning to notice students' mathematical thinking to understand preservice teachers' skills, behaviors, views and the meaning that they gave to their experiences in the lesson study process. These points refer to the study of a specific issue in its natural context in order to disclose detailed and meaningful understanding of it, learn the meaning of it and identify in-depth characteristics of events which are the key features of case study.

Researchers generally conduct a case study for three different purposes: "to produce detailed descriptions of a phenomenon, to develop possible explanations of it, or to evaluate the phenomenon (Gall, Gall & Borg, 2007). Three types of case studies are defined in literature as intrinsic case study, instrumental case study and multiple case study. In an intrinsic case study, the aim is to understand specific individual(s) or situation better, describe the properties of a case in detail and learn more about the phenomenon by studying it in depth. It necessitates to understand a single case with all its parts (Fraenkel, Wallen & Hyun, 2011). In an instrumental case study, the aim is to find results that support the phenomenon rather than reveal the properties of a particular case, gain insight into a phenomenon or refine a theory (Yin, 2003). Here, the case is used as a tool for reaching more global goals beyond a particular case (Fraenkel et al., 2011), it is of secondary interest and enables supportive understanding (Stake, 1995). In multiple case study, the aim is to investigate differences within and between cases and replicate the procedure in cases (Yin, 2003). It entails to study more than one case simultaneously as a part of the whole research (Fraenkel et al., 2011). Case study is an appropriate approach for examining and identifying various abilities and experiences of the participants during their engagement in lesson study professional development since it provides an in-depth understanding of the process (Rock, 2005). Therefore, case study, in particular, intrinsic case study was used to present detailed descriptions of preservice teachers' noticing skills and a full picture of the whole process in this study.

3.3 Participants

In this study, four senior preservice middle school mathematics teachers who enrolled in the program of elementary mathematics education at a university in northern Turkey participated in a lesson study process from the beginning to the end of the study. One of them was a female student and three of them were male students. The real names of preservice teachers were not used in this study, instead pseudonyms were given as Semih, İnci, Mehmet and Hasan. However, in this study, one female student (İnci) and one male student (Semih) were chosen in order to represent both genders and explore their skills and experiences in greater depth as the intensity of the lesson study process was extensive. Criterion sampling, which is one of the purposive sampling techniques, is employed in order to determine the students to participate in lesson study group. It includes the selection of cases or individuals that satisfy an important criterion (Gall et al., 2007). In other words, the fundamental feature in this technique is working on all cases which carry a range of the predetermined criteria. In this research, the selection of participations was made by taking three criteria into consideration (1) having completed the most part of the courses related to teaching profession, learning-teaching methods and techniques and having prepared themselves for starting a career, thus, being a senior student (2) being new to lesson study and (3) being a volunteer for joining the lesson study process.

These three concepts were important in the selection of participants: first reason, if the preservice teachers did not take the courses associated with teaching profession, their prior knowledge would not be sufficient enough to follow the lesson study process, they would have difficulties while planning the lessons, implementing the plans and evaluating the process. The background obtained from these courses supports carrying out the lesson study professional development model accomplishedly and forms a basis for revealing their noticing skills. Second reason, the aim of this study was to examine preservice teachers` abilities to notice students' mathematical thinking as they participated in the lesson study, therefore, being unfamiliar with this model was important in terms of the accurate interpretation of the data and arousing participants' interest in the study. Third reason, lesson study is a model which is based on collaborative working, teaching, discussing and sharing ideas. Thus, if participants are not willing to perform the requirements in the lesson study, appropriate results may not be obtained. Before the study, preservice teachers were informed about the process of the research and lesson study professional development model. Participants were selected from the willing preservice teachers who wanted to incorporate into this kind of environment and develop themselves in the professional field.

On the other hand, this study required practice in a real classroom setting with preservice teachers and the support of the cooperating teacher in the school. Accordingly, this study was conducted in the context of a teaching practice course in order to provide an appropriate environment and also gain insight on the integration of the professional development model into the course. At this point, one middle school was randomly selected from the school list of the teaching practice course in teacher education program of a university and it was visited. The procedure of the research was expressed and permission to conduct the study was granted. All mathematics teachers were willing to help with the process of practice and one of them was assigned as the cooperating teacher of the preservice teachers by considering that s/he has a course with 5th grades in accordance with specific mathematics subject of the research. She was requested to allow for teaching in her lessons, share her ideas about teaching of mathematics subject, observe the lessons that were implemented by preservice teachers and contribute to the process with her feedbacks.

3.3.1 Description of the research setting and participants

Setting

The middle school in which teaching and reflecting phases of lesson study were conducted with the help of cooperating teacher is one of the public schools situated in the northern part of Turkey. The number of students is around 880, the number of classrooms is 28 and the number of students in each class is between 20 and 30. The students who enrolled in the school have a range of achievement levels from low to high. 5th and 6th grade students start the lesson at 13.00 p.m. and finish at 18.40 p.m. whereas 7th and 8th grades start the lesson at 07.10 a.m. and finish at 12.50 p.m. The school schedule consisted of seven class periods. These periods were 40 minutes long and there were 10-minute breaks between classes. The implementations of lesson plans that were prepared by preservice teachers were made in cooperating teacher's classes which were 5/F and 5/G. There were 23 students in 5/F and 24 students in 5/G. There was no smartboard or computer in the classrooms but they had projectors.

The cooperating teacher, Ezgi, had twelve years of teaching experience. She did not have any experience in lesson study but she was willing to help the students as much as she could. She had courses with only the 5th and 6th grade students during the period of the research. In order to gather data about the cooperating teacher and help the students become accustomed to the preservice teachers and the researcher, one of each lessons in 5/F and 5/G, which are the cooperating teacher's classes, were observed before starting the implementations. In general, she teaches the lesson by stating the important information relevant with the subjects in the beginning, asking questions and requesting the students to solve them and following the order and the tasks in the textbook without benefitting from technology and materials very often. She has a teacher-centered instructional style but uses questioning as a supportive tool. Therefore, the students are used to listening to what the teacher tells and then solve the relevant questions to reinforce the subjects.

The semester during which preservice teachers participated in this study was their last semester of a four-year program. Preservice middle school mathematics teachers selected were all from the same teacher education program and the same class of a university. It was important to choose preservice teachers from the same university because it was thought that participants who knew each other would adopt lesson study and arrange a common time for meetings and implementations in an easier way. These participants took same courses from the same faculty members together at university. It was also considered that a similar background and akin experiences of preservice teachers would facilitate to interpret the data.

Case 1: İnci

İnci, a 22 year old female, graduated from an Anatolian High School which was classified as a qualified school in Turkey. GPA of İnci was 3.15 and she completed the courses of mathematics teaching methods I and II by taking BB grades, geometry by taking CC grade and school experience with AA grade. She did not work in a private tutoring institution but she offered private mathematics lessons to 3 students during university years. She was willing to become a mathematics teacher at the earliest and getting ready for KPSS (Public Personnel Selection Examination) exam to start working in a middle school. She was also interested in lesson study and wanted to develop herself by participating in this process. Her hesitation was more related to time as she had to follow the courses at university, participate in each phase of this study, which would take a long time, and go to the KPSS preparation courses for passing the examination in scholarly writing. She did not take any training connected with teaching profession and mathematics instruction before, except from the courses at university. She was talkative and good at expressing her thoughts comfortably. She was good with the other members in the lesson study group. While determining İnci as a case, it was considered that İnci had good characteristics and willingness in terms of obtaining rich data and she was the only female student in this study. The desire for representing both a female and a male student from the lesson study group influenced the selection of cases.

Case 2: Semih

Semih, a 23 year old male, graduated from an Anatolian High School. Semih's GPA was 2.87 and he completed the courses of mathematics teaching method I by taking CB grade and teaching method II by taking BB grade, geometry with CB grade and school experience with AA grade. He had private teaching experiences. He had worked in a private tutoring institution for a semester and he also offered private mathematics lessons to 10 students during university years. He wanted to be a teacher but he was not as eager as İnci. He was not studying hard to get ready for KPSS exam. His primary goal was to complete his courses at university successfully. In addition to becoming a mathematics teacher, he was thinking of enrolling in a graduate program and trying his chance to be an academician. Thus, he was not determined about what he wanted. He did not take any training related to teaching profession and mathematics instruction before, except for the courses at university. He was eager to participate in the lesson study process and felt comfortable with this model due to his previous experiences with various students. He was a humorous and responsible person. He was willing to carry out the extra duties as well as what was required. He was good with the other members in the lesson study group and open to work collaboratively. He had a talkative and moderate personality. He was also good at expressing his thoughts comfortably like İnci. While determining Semih as a case, it was considered that Semih was more willing to participate in this process and his GPA was between the other two male students. Therefore, it was thought that selection of Semih might provide better representation.

3.4 Procedure of the Research

Lesson study is a professional development which helps to learn about how students comprehend best by improving lessons through collaboration with colleagues (Adams, 2013). The purpose of this study was to investigate preservice middle school mathematics teachers' noticing of students' mathematical thinking during lesson study phases. In other words, the focus was on revealing what and how preservice teachers notice, what they attend to and how they make sense of their attention throughout planning, teaching, reflecting, re-teaching and re-reflecting on the lessons. Their noticing skills exposed during the lesson study are likely to affect their learning from practice (Choy, 2015), thus this study might elaborate the role of this skill in the context of lesson study. In a present research, lesson study was used to maintain an environment in order to disclose preservice teachers' noticing through anticipating students' thinking and responses, considering alternative mathematical approaches to respond to students' reasoning, reflecting systematically, discussing on the ideas and deciding what to do with the colleagues (Mason, 2002).

3.4.1 Pilot study

Before conducting the main study, a pilot study was conducted with four preservice middle school mathematics teachers, three males and one female, in order to see the degree of feasibility of the research and determine the deficiencies and the points that should be changed. Participants were senior students who were enrolled in a course of teaching practice in an elementary mathematics education program at a university. Their real names were not used and they were called as Metin, Hakan, Sedat and Feride. Preservice teachers were informed about the requirements of the research process and the first meeting took place in the university's seminar hall for postgraduate students. In this meeting, lesson study model was introduced to them through a power point presentation that elaborates the cyclical structure and steps of the lesson study process because they need to understand lesson study cycle before using it. Examples of videos towards the implementation of lesson study and lesson plans from literature were showed to them as well in order to provide better understanding of the professional development model.

One of the videos was about subtracting a two-digit number from a three-digit number. In this video, one Japanese teacher aimed to present a subtraction problem in which the numbers were hidden and only the difference was known. The teacher expected the students to fill in the blanks, to find the number combinations and to notice certain rules hidden in the answer of the problem (The first rule was that minuend and subtrahend increased by one and the second rule was that there was the same number equations as the value of the difference. For example, there were 3 ways of subtracting when the difference was 3). In the video, the teacher took advices of the other teachers on how to proceed the lesson. After planning the lesson, the teacher conducted the lesson in his classroom while seven people, who were the teachers and faculty members, observed this lesson. In reflection, the teacher reflected his analysis about the students at first and the other members proposed alternative ways to present the problem and gave advices for improving the lesson. This video included three phases that were planning, teaching and reflecting respectively. The video lasted twelve minutes and the preservice teachers watched it from beginning to end.

The second video was regarding an implementation with kindergarten students. In this video, seven teachers discussed on what they would do and how they would conduct the lesson. There were various books in front of them and they benefitted from these sources while planning. In teaching, one of the teachers read a story and asked some questions about it to the students. In this phase, the other group members observed the lesson and took notes. In the reflecting phase, they tried to analyze students’ responses related to the story and talked about how to help students to understand the bigger picture of the story. They shared their opinions to develop the lesson. This video was fourteen minutes long but the preservice teachers did not watch all of it. The phases of lesson study were briefly shown. These videos have been used mostly to introduce lesson study, concrete the procedure of it, help preservice teachers to understand the content of lesson study phases and give them an idea of what to do. During this meeting the subject matter, which is equalizing denominator of fractions in 5th grades, to design a lesson was determined since it is a topic that students have difficulty in understanding. It took approximately 40 minutes.

At the end of this introductory meeting, preservice teachers were given the task of making an investigation about teaching of fractions, in particular, equalizing denominator of fractions. Sources such as mathematics textbooks, mathematics teacher guidebooks and some mathematics education books were supplied. Preservice teachers were required to learn what students know and do not know about this subject and what misconceptions and difficulties they might have, to take notes on what kind of activities and questions should be asked and to think about the sequence of them. After the examination of the sources, preservice teachers went to their practice school to receive suggestions from their cooperating teacher, Can, about the topic so as to get help while planning the lesson. He did not want to be included in the whole planning process due to his works and long duration of planning. He made suggestions related to teaching of equalizing dominator of fraction, emphasized the important points and answered the questions of preservice teachers. This conversation took about twenty minutes and preservice teachers took notes about what he said in this process to consider while planning the lesson.

In the planning phase, preservice teachers met in the university seminar hall to share their ideas and suggestions based on their investigations and experiences. They endeavored to determine the activities and questions that would be included in the lesson plan, discussed on how students think mathematically, made suggestions on how to respond to them and concluded which materials to use. They used the four column lesson plan tool and tried to prepare the lesson plan by filling in these columns. It took approximately seventy minutes but it could not be finished. Therefore, preservice teachers met for the second time to complete the lesson plan. They continued planning together until they completed it and it took almost three hours. After planning, interviews towards the planning phase were conducted with each preservice teacher separately and they prepared the materials which they decided to use while teaching the lesson.

In the teaching phase, one of the preservice teachers who volunteered to teach was selected. Hakan conducted the lesson in front of an instructor at the university before teaching in a real classroom and took some advices to consider. Later, this preservice teacher taught the lesson in a real classroom, 5/B at the school whereas the other preservice teachers, cooperating teacher and the researcher observed the lesson without interacting with the students in the classroom. Preservice teachers took notes on the copy of the lesson plan about the lesson to discuss after implementation together. Cooperating teacher was requested to take notes while observing the lesson to give feedback. The researcher also took notes on the copy of the lesson plan. Teaching occurred in two lesson periods, namely, eighty minutes.

In the reflecting phase, cooperating teacher gave feedback on the lesson and made some suggestions briefly and went to his classroom because he had a lesson. Then, preservice teachers shared their ideas about the effectiveness of the lesson design on the day that the lesson was taught and discussed on the teaching phase in accordance with discussion protocol. They evaluated what worked well or did not and what should be changed. They emphasized unexpected responses and made suggestions to revise the lesson plan considering the points that they dwelled on. The lesson was revised in the direction of this information. After reflecting on the lesson, interviews towards the teaching phase were conducted with each preservice teacher separately. Preservice teachers gathered to watch the video of the lesson together by taking notes. After watching it, they discussed on the lesson and shared what they noticed. Another preservice teacher, Sedat, from the lesson study group taught the revised lesson in another classroom 5/E while the other preservice teachers and the researcher were observing and taking notes on the lesson plans. The cooperating teacher did not participate in the observation indicating that he had work. Similarly, they discussed on the revised lesson again and tried to improve the lesson design collaboratively. After reflecting on the revised lesson, interviews towards the re-teaching phase were conducted with each preservice teacher separately. Preservice teachers got together to watch the video of the revised lesson while taking notes. After watching it, they discussed on the lesson and shared what they noticed. All the phases of the lesson study and interviews were recorded with a camera to examine preservice teachers’ noticing of students’ mathematical thinking.

Table 1 Timeline of the Pilot Study

Date	Activity	Description of Activity	Duration	Meeting Place
Introduction of Lesson study & Preparation				
11.02.2015	Introductory Meeting and Determination of Subject	Preservice teachers watch a power point slide about the nature of lesson study and examples of lesson study videos, examine examples of lesson plans and determine a teaching goal.	40 minutes	University Seminar Hall
17.02.2015	Meeting with Cooperating Teacher	Preservice teachers get suggestions from the cooperating teacher about teaching after investigating and gathering information on the subject matter.	20 minutes	School Conference Room
Planning				
19.02.2015	Planning the Lesson	Preservice teachers begin to plan the lesson by considering many sources and suggestions from the cooperating teacher to meet the goal	70 minutes	University Seminar Hall
20.02.2015	Maintaining Planning the Lesson	Preservice teachers continue to work on planning until the lesson plan is complete	180 minutes	University Seminar Hall
23.02.2015	Interviews and Preparation of Materials	Interviews towards planning were conducted with each preservice teacher individually. Preservice teachers prepared the necessary materials for the lesson together.	20-30 minutes	University Seminar Hall
Teaching				
25.02.2015	Lesson 1 at University	One preservice teacher, Hakan, teaches the lesson and an instructor observes him. The instructor shares ideas and makes some suggestions about the teacher pedagogy and implementation.	120 minutes	University Seminar Hall
26.02.2015	Lesson 1 at School and Observation	Preservice teacher, Hakan, teaches the planned lesson in a real classroom 5/B. Other preservice teachers, the cooperating teacher and researcher observe the lesson and take notes.	80 minutes	Cooperating Teacher’s Classroom
Reflecting				
26.02.2015	Post-lesson discussion and revision of the lesson	The cooperating teacher shares his opinions, makes some suggestions briefly and leaves the group. Preservice teachers discuss on the lesson, make recommendations about the changes needed for improving the lesson plan and they revise the lesson.	15 + 65 minutes	School Conference Room

Table 1 (continued)

27.02.2015	Interviews	Interviews towards teaching were conducted with each preservice teacher individually.	20-30 minutes	University Seminar Hall
02.03.2015	Watching Video and Discussion	Preservice teachers watched the video of the lesson 1, took notes and discussed on the lesson.	80 + 30 minutes	University Seminar Hall
Re-teaching				
03.03.2015	Lesson 2 at University	One preservice teacher, Sedat, teaches the revised lesson while an instructor, whom he gets suggestions from, is observing him	120 minutes	University Seminar Hall
04.03.2015	Lesson 2 at School and observation	Preservice teacher, Sedat, teaches the revised lesson in a classroom, 5/E. Other preservice teachers, the cooperating teacher and researcher observe the lesson and take notes.	80 minutes	Cooperating Teacher's Classroom
Re-reflecting				
04.03.2015	Post-lesson discussion and revision of the revised lesson	Preservice teachers discuss on the lesson, make recommendations about the changes needed for improving the lesson plan and they revise the lesson.	30 minutes	School Conference Room
05.03.2015	Interviews	Interviews towards re-teaching were conducted with each preservice teacher individually.	20-30 minutes	University Seminar Hall
06.03.2015	Watching Video and Discussion	Preservice teachers watched the video of the lesson 2, took notes and discussed on the lesson.	80 + 30 minutes	University Seminar Hall

After one lesson study cycle related to equalizing denominator of fractions was conducted in the pilot study, some modifications seemed to be needed in this study. The researcher gained more experience from the pilot study on how to conduct this study and received some recommendations from her advisor. At first, it was seen that the number of preservice teachers in the lesson study group was quite adequate for the implementation of the lesson study cycles in the main study. Arranging a common time for all preservice teachers to gather, teach and discuss was difficult because of the courses that they need to participate at university and the course which they need to attend in order to prepare themselves for KPSS exam to become a teacher. Therefore, it would be more difficult organizing the research schedule for more preservice teachers. On the other hand, it was observed that preservice teachers learned from each other and completed each other's deficiencies so that more and different experiences provided a richer environment. Thus, it was realized that fewer preservice teachers would decrease the effectiveness of the lesson study process. It was decided to conduct the main study with four preservice teachers in this direction so that the uncertainty about the number of participants in lesson study group would diminish.

With the pilot study, it was seen that only one lesson study cycle required plenty of time and great effort. However, preservice teachers were very busy because of their courses in teacher training program and preparation for KPSS exam. Besides, it was realized that if the process had a longer time period, they would be bored and become unwilling to keep on participating. It would be ineffective asking them to prepare, implement and evaluate more lesson plans. On the other hand, the purpose of this study was to reveal preservice teachers' noticing skills and their development, if any existed, in detail. Hence, fewer number of lesson study cycles would not be enough in terms of evaluating the research process appropriately. At this point, it was decided to conduct four lesson study cycles and enable each preservice teacher to implement one lesson and one revised lesson.

It was also understood that cooperating teachers were unwilling to spend too much time in this process due to the intensity of their course schedules and difficulties of arranging appropriate time for meetings. They offered to share their opinions and make suggestions to improve the lesson plans and implementations briefly. They accepted to observe the teaching phases to give feedbacks on the lessons. They also indicated that they would be open to help if preservice teachers asked for it. Therefore, it was arranged to meet the cooperating teacher before planning the lesson and after teaching the lesson in order to utilize her experiences and take recommendations about teaching of mathematics subject in the main study.

In the pilot study, an instructor who was a member of department of mathematics education at a university was requested to observe preservice teachers' practice of teaching before conducting the implementation in a real classroom and to give some advices in order to improve the lesson. It was noticed that conducting the lesson and the revised lesson in front of the instructor at university before teaching in practice school took a long time. Preservice teachers offered to practice teaching by themselves instead of coming to university for one more day. Furthermore, it was seen that the instructor made suggestions related to teacher pedagogy, terminology, and classroom management rather than the content of the lesson plan and she was also unwilling to observe and give feedbacks for longer. It was realized that taking this kind of help was difficult for both preservice teachers and the instructor. That is why, they resolved not to make practice of teaching the lessons at university before the real implementation.

Pilot study showed that a long period of time elapsed from the first step -planning of the lesson to the last step- reflecting on the revised lesson. In other words, there would be one or two weeks between planning of one lesson and another, owing to the completion of one lesson study cycle consisting of several steps. However, in schools, when teaching of one subject starts, it continues without having a break, namely, different subjects are not taught until one subject is completed. In the main study, the aim was to teach all the objectives of perimeter, area, surface area and prism in the domain of geometry and measurement. Therefore, the lessons must be implemented one after another not to hinder the flow of the curriculum and mathematics course at school. At this point, it was preferred to prepare all the lesson plans until the teaching time of the subject is due so as not to cause confusion in students and block the natural setting of the mathematics course. On the other hand, this situation may have caused preservice teachers to forget some of the important points that they had talked about during planning and to miss them out while teaching the lesson. Thus, preservice teachers were wanted to review and practice teaching of the lesson plans by themselves especially before the implementations.

In the pilot study, preservice teachers were wanted to watch the videos of lessons after reflecting phases by considering some studies in literature. They were required to discuss on the lesson and elaborate what they noticed based on the notes that they took. However, it was realized that preservice teachers were bored while they were watching the videos and took few notes. When asked to share their opinions about the lesson and what they noticed, it was seen that they could say nothing or very few things different from the data obtained from post-lesson discussions and interviews. Besides, preservice teachers emphasized that they observed the lesson carefully and still remembered what they observed and indicated that they did not need to watch the lesson videos again. It was realized that they found this step unnecessary and time-consuming and they were unwilling to participate in. Consequently, it was decided to remove the step of watching videos, considering the attitudes of preservice teachers and the similar comments on the videos in order not to reduce the effectiveness of the research.

The analysis of the pilot study also helped to gain insight about the appropriateness of predetermined coding categories for investigating preservice teachers' abilities to notice students' thinking. The data obtained from the pilot study could be assigned to the coding categories and the need for modification of categories was not felt. Some interview questions were refined in the light of the results of the analysis. The pilot study also showed that the lesson study provided a good context for learning about preservice teachers' noticing and encouraged to maintain the research by expanding the process in the main study.

3.4.2 Main study

Lesson study process, as in the pilot study, was conducted with four senior preservice middle school teachers, three males and one female, who enrolled in elementary mathematics education program at a public university in the north of Turkey. Pseudonyms for all preservice teachers were used which were İnci, Semih, Mehmet and Hasan. In this study, the data of İnci and Semih was analyzed and presented because of the intensity of the process. They were different from the participants in the pilot study yet from the same practice school. All four preservice teachers were in the same program and had similar experiences. In this program, they are required to pass the courses of mathematics such as analysis, geometry, statistics and possibility, and analytic geometry, mathematics education and general education such as physics, history, and technology. All of them had already passed mathematics courses, most of the mathematics education courses and teaching methods courses. These courses intend to develop the mathematical proficiency, the pedagogical competencies and the dispositions of preservice teachers. Senior preservice teachers were purposively selected as they had completed most of the courses and they were close to becoming teachers. During this study, they were in their last semester and they were taking teaching practice course which aims to enrich their field experience and prepare them for teaching profession through real settings. In this course, they are asked to prepare lesson plans, implement these plans in a real classroom at a practice school with the help of a cooperating teacher and instructor and prepare a teaching portfolio. It also includes the evaluation of preservice teachers' performance and feedbacks from the cooperating teacher and instructor to improve their teaching skills. The present study occurred in the context of a teaching practice course during 2014-2015 spring semester. Similar to the pilot study, data sources such as interviews, observations, field notes and video recordings of each phase in lesson study cycles were used in this qualitative research.

In this study, the researcher worked in coordination with the cooperating teacher, Ezgi, from the practice school where the cooperating teacher of the pilot study worked too. She was a mathematics teacher in her twelfth year teaching at the time of the study. This teacher allowed each preservice teacher to teach the lesson on the subject of perimeter, area, surface area and prism in her classroom twice for this study. The researcher briefed the cooperating teacher on the research process and lesson study model. The cooperating teacher's role was to share opinions and make suggestions about teaching of the subject before planning phase, observe and take notes during teaching phase and give feedbacks and make recommendations about the lesson to improve it in discussion step. She was also very helpful in organizing teaching dates in her classrooms. Duration of her comments before planning and after teaching changed between 10 and 20 minutes.

Introduction of Lesson study and Preparation

As in the pilot study, first meeting with preservice teachers took place in university's seminar hall and they were informed about the requirements of the research process in detail. In this meeting, lesson study model was introduced to them through a power point presentation that elaborates the cyclical structure and steps of the lesson study process as they need to understand lesson study cycle before using it. Examples of videos towards the implementation of lesson study and lesson plans from literature were showed to them as well to maintain better understanding of the professional development model. They were asked to complete four lesson study cycles on one specific mathematics domain in 5^{th} grade. It was opted to focus on perimeter, area, surface area and prism in the domain of geometry and measurement because students mostly confuse the concepts of perimeter, area and surface area and they cannot understand them well (Martin & Strutchens, 2000). It took approximately 45 minutes.

At the end of this introductory meeting, preservice teachers were given the task of making an investigation about teaching the subjects of perimeter, area, surface area and prism. Sources such as mathematics textbooks, mathematics teacher guidebooks, curriculum and some mathematics education books were supplied. Preservice teachers were desired to learn what students know and do not know about this subject, what their misconceptions, possible responses and difficulties might be and how these confusions and difficulties might be removed. At this point, anticipating students' thinking and responses both provide to develop teachers' mathematical knowledge for teaching (Lewis et al., 2011) and to examine their noticing of students' mathematical thinking. They were also wanted to take notes on what kind of activities and questions should be asked and think about the sequence of them and materials. They examined the objectives of the subject of perimeter, area, surface area and prism and learned what they need to enable students to obtain in detail with the help of middle school mathematics curriculum. They decided to shape the content of lesson study cycles in the direction of the objectives in the relevant unit. The assignment of the objectives was made in terms of lesson study cycles as the following.

Table 2 The Objectives of the Subjects of Perimeter, Area, Surface Area and Prism in Curriculum

Lesson Study Cycles	**Objectives**
Cycle 1	Students will be able to: "Calculate perimeters of polygons; create different figures which have the given perimeters."
Cycle 2	"Calculate the area of a rectangle; use square centimeter and square meter." "Estimate a given area using square centimeter and square meter units."
Cycle 3	"Create different rectangles with the given area." "Solve problems that require to calculate the area of a rectangle."
Cycle 4	"Recognize a rectangular prism and identify its essential features." "Draw surface developments of a rectangular prism and decide whether different surface developments belong to the rectangular prism or not." "Calculate the surface area of a rectangular prism."

After examination of the sources, preservice teachers went to their practice school to receive the cooperating teacher's suggestions about the topic in order to get help while planning the lesson. She emphasized where students mostly have difficulty and which points should be paid attention to, made suggestions about teaching of the perimeter, area, surface area and prism and answered the questions of preservice teachers. This conversation took about twenty minutes and preservice teachers took notes about what she said in this process to consider while planning the lesson.

Planning the Lesson

These phases included preparation of lesson plans that would address teaching objectives of the subject. As it was mentioned while explaining the implications of the pilot study, it was agreed to prepare all four lesson plans until teaching time of the unit came at the practice school of the main study. Therefore, preservice teachers came together many times to complete the lesson plans. They met in the university seminar hall to share their ideas and suggestions based on their investigations and experiences. While working on planning a lesson, preservice teachers collaboratively endeavored to form detailed lesson plans, which one of the preservice teachers would use to teach the lesson in a real classroom whereas the others would use to follow the lesson and take notes on during observation. In this process, they used the four column lesson plan tool (see Appendix A) and tried to prepare the lesson plans by filling these columns. They also utilized mathematic textbooks, teacher guide books and various mathematics education books. They attempted to determine the activities and questions that would be included in the lesson plan, discussed on how students think mathematically and how teachers react, made suggestions about how teacher should respond to students and determined the materials to use as planning required it in the lesson study. While planning, preservice teachers were expected to explain why they offered an activity or question to be in the lesson plan so as to understand how they linked students' thinking with teaching activities and the objectives of the subject.

At the end of the planning phase, four completed lesson plans towards the objectives in each lesson study cycle were obtained respectively. In this direction, the first lesson plan was related to teaching of perimeter and the second lesson plan was on area and use of units. The third lesson plan mainly included a combination of perimeter and area and served as a repetition. In the fourth lesson plan, it was aimed to introduce rectangle prism, teach its properties and calculation of surface area. Sometimes, one meeting was not enough to prepare the lesson plan and it took longer. Therefore, preservice teachers met for the second time to complete the lesson plans. They completed each of the first and third lesson plan in one meeting but they prepared each of the second and fourth plan in two meetings. After planning, semi-structured interviews towards the planning phase were conducted with each preservice teacher individually (see Appendix B.1) and they prepared the materials which they arranged to use as teaching the lesson. These four lesson plans based on collaborative work were also used as a data source to see whether lesson study helped preservice teachers learn how to focus on and make sense of students' mathematical thinking when planning, teaching and revising the lessons, namely, to evaluate their noticing skills.

Teaching the Lesson

Doig and Groves (2011) indicate that, "teaching the research lesson forms the core of Japanese Lesson Study, providing both the opportunity to test the lesson plan in the classroom and opportunity for observation and reflection" (p.81). In this phase, the preservice teacher had the responsibility of being the main teacher that day s/he taught the class. As it was mentioned in the pilot study, each preservice teacher made two implementations one of which was the lesson and the other revised lesson so that they included two different cycles of teaching phases. Before beginning to teach the lessons, preservice teachers determined which lesson plan would be implemented by whom. Information about preservice teachers who were the main teachers of cycles and classrooms that teaching occurred were as shown below.

Table 3 Summary of Lesson Study Cycles at Middle School

	LS Cycle 1	**LS Cycle 2**	**LS Cycle 3**	**LS Cycle 4**
Lesson	Lesson 1	Lesson 3	Lesson 5	Lesson 7
Preservice Teacher	Hasan – 5/F	Semih – 5/F	İnci – 5/F	Mehmet – 5/F
Revised Lesson	Lesson 2	Lesson 4	Lesson 6	Lesson 8
Preservice Teacher	İnci – 5/G	Mehmet – 5/G	Hasan – 5/G	Semih – 5/G

In each lesson study cycle, one of the preservice teachers conducted the lesson in a real classroom at the practice school according to the schedule that they had determined previously whereas the other preservice teachers, cooperating teacher and the researcher observed the lesson without interacting with the students in the classroom. Preservice teachers took notes on the copy of lesson plans related to the lesson to discuss together after implementation. In this process, they were not asked to pay particular attention to any issues in order to learn what they noticed themselves and how they made sense without leading. Besides, it was emphasized that while they were observing the teaching of the lesson, the purpose was not to evaluate their peers instead the goal was to improve the lesson prepared as a team. The cooperating teacher was requested to observe the lesson and take notes to give feedback. The researcher also took notes on the copy of the lesson plans to use as a data source. The same procedure was maintained in each teaching phase of four lesson study cycle. Except for the last lesson, each teaching occurred in two lesson periods, particularly, eighty minutes. Preservice teachers decided to increase the duration of teaching the last lesson from two lesson periods to three during their revision of it because they could not complete teaching.

Reflecting on the Lesson

In each reflecting phase, preservice teachers, the cooperating teacher and researcher met in the school conference room soon after the lesson on the day that the lesson was taught.

The cooperating teacher was given the first opportunity to speak and share her opinions. She gave feedback on the lesson, made some suggestions briefly and left the group. It took ten or fifteen minutes in general. Later, the main teacher of the lesson, who is the preservice teacher that taught the lesson, explained her impressions of what was successful or not in the lesson by making self-reflection and offered some recommendations. After this, other preservice teachers added their comments, suggestions and questions based on their detailed notes from the lesson. In this process, collegial dialogue was encouraged in order to reflect on the lesson collaboratively. The researcher facilitated the post-lesson discussion with questions discussion protocol (see Appendix C) such as what went well or did not and what should have been done differently. Preservice teachers shared their ideas about the effectiveness of the lesson design, discussed on the points that needed refinement based on their observation notes and made suggestions to revise the lesson plan. After reflecting on the lesson, interviews towards teaching phase were conducted with each preservice teacher individually to elaborate their noticing of students' thinking with questions and understand what they learned about students' thinking (see Appendix B.2). It was also aimed to learn their thoughts, perceptions and reactions about lesson study as a professional development model as well as the change in their noticing thanks to interviews. The same process was followed in each reflecting phase of four lesson study cycle. Reflecting among preservice teachers varied between 30 to 80 minutes.

Re-teaching the Lesson

In lesson study, re-teaching is conducted by another preservice teacher from the lesson study group in another classroom. In this study, since determination of main teachers of lessons was made considering this principle, preservice teachers taught the revised lessons based on this schedule while other preservice teachers, the cooperating teacher and researcher were observing and taking notes. The same procedure in teaching phase was continued in each re-teaching phase.

Re-reflecting on the Lesson

In each re-reflecting phase, preservice teachers discussed on the revised lessons and tried to improve the lesson design collaboratively. The same procedure in reflecting phase was continued in each re-reflecting phase. After reflecting on the revised lesson, interviews towards the re-teaching phase were conducted with each preservice teacher individually. Whole phases of lesson study and interviews were recorded with a camera to examine preservice teachers' noticing of students' mathematical thinking. The researcher was present in each phase of the lesson study, listened to the discourse of preservice teachers, watched their interactions, took notes, observed the lessons and recorded the whole process in order to understand how preservice teachers were making sense of their new experience, evaluating their noticing skills and elaborating the nature of the setting.

Table 4 Timeline of the Main Study

Date	Activity	Duration
Introduction of Lesson study & Preparation		
30.03.2015	Introductory Meeting and Determination of Subject	40 minutes
02.04.2015	Meeting with the Cooperating Teacher	20 minutes
	Lesson Study Cycle 1	
Planning		
03.04.2015	Planning the Lesson	190 minutes
06.04.2015	Interviews and Preparation of Materials	20-30 minutes
Teaching		
27.04.2015	Lesson 1 at School and Observation	80 minutes
Reflecting		
27.04.2015	Post-lesson discussion and revision of the lesson	15 + 40 minutes
28.04.2015	Interviews	20-30 minutes
Re-teaching		
29.04.2015	Lesson 2 at School and observation	80 minutes
Re-reflecting		
29.04.2015	Post-lesson discussion and revision of the revised lesson	10 + 30 minutes
30.04.2015	Interviews	20-30 minutes
	Lesson Study Cycle 2	
Planning		
08.04.2105	Planning the Lesson	95 minutes
09.04.2015	Maintaining Planning the Lesson	90 minutes
10.04.2015	Interviews and Preparation of Materials	20-30 minutes
Teaching		
06.05.2015	Lesson 3 at School and Observation	80 minutes
Reflecting		
06.05.2015	Post-lesson discussion and revision of the lesson	10 + 80 minutes
07.05.2015	Interviews	20-30 minutes
Re-teaching		

Table 4 (continued)

08.05.2015	Lesson 4 at School and observation	80 minutes
Re-reflecting		
08.05.2015	Post-lesson discussion and revision of the revised lesson	10 + 50 minutes
08.05.2015	Interviews	20-30minutes
	Lesson Study Cycle 3	
Planning		
13.04.2015	Planning the Lesson	160 minutes
14.04.2015	Interviews and Preparation of Materials	20-30 minutes
Teaching		
11.05.2015	Lesson 5 at School and Observation	80 minutes
Reflecting		
11.05.2015	Post-lesson discussion and revision of the lesson	10 + 45 minutes
12.05.2015	Interviews	20-30 minutes
Re-teaching		
13.05.2015	Lesson 6 at School and observation	80 minutes
Re-reflecting		
13.05.2015	Post-lesson discussion and revision of the revised lesson	10 + 65 minutes
14.05.2015	Interviews	30-35 minutes
	Lesson Study Cycle 4	
Planning		
16.04.2015	Planning the Lesson	85 minutes
17.04.2015	Maintaining Planning the Lesson	135 minutes
20.04.2015	Interviews and Preparation of Materials	20-30 minutes
Teaching		
18.05.2015	Lesson 7 at School and Observation	80 minutes
Reflecting		
18.05.2015	Post-lesson discussion and revision of the lesson	10 + 80 minutes
19.05.2015	Interviews	30-45 minutes
Re-teaching		
20.05.2015	Lesson 8 at School and observation	120 minutes
Re-reflecting		
20.05.2015	Post-lesson discussion and revision of the revised lesson	10 + 75 minutes
21.05.2015	Interviews	30-45 minutes

In the process of the main study, until the steps of one lesson study cycle were completed, it was not proceeded to the teaching phases of another cycle. In other words, for example, lesson 1 was conducted in the teaching phase and post lesson discussion and revision of this lesson took place in the reflecting phase. It was followed by lesson 2, namely, the revision of the first lesson plan. Later, preservice teachers discussed on this revised lesson in the re-reflecting phase. After completion of lesson study cycle 1, the teaching process of the second lesson plan began and similar steps occurred for cycle 2. The rest of the lesson study cycle continued in this pattern and allowed teachers to work collaboratively, share their ideas and learn to improve their plans and teaching by learning from each other.

3.4.3 The context of the study

In the process of lesson study, preservice teachers met the cooperating teacher first to learn her ideas and suggestions about teaching of the subjects before planning the lesson. After preservice teachers made investigations about perimeter, area, surface area and prism, they often gathered to prepare the lesson plans towards the objectives of the mathematical topic. In this process, they used the four column lesson plan tool so that they discussed on the content of the lesson, students' possible thinking, teacher's reactions and materials. They endeavored to determine the activities and questions that would be included in the lesson through utilizing various sources. After lesson plans were completed, teaching of them by preservice teachers occurred in the real classrooms on specified days in the middle school as indicated before. Immediately after teaching, reflecting on the lesson happened in the school conference room. Cooperating teacher observed each lesson and shared her ideas and suggestions related to the lesson. Preservice teachers also expressed their opinions and discussed on how to improve the lesson. A similar procedure was maintained for each lesson study cycle.

Lesson Study Cycle 1

In the first planning phase, the aim was to teach the concept of perimeter and the objective of the first cycle was "Calculate perimeters of polygons; create different figures which have the given perimeters". Preservice teachers settled to focus on the relationship between measurement and perimeter because the calculation of perimeter was based on the action of measuring and included geometry and measurement domain. They thought that students should realize the meaning of perimeter regarding measurement. They decided to show a picture in the form of a rectangle. They agreed to ask students to find its perimeter by rounding the picture with the rope and measuring the length of the rope with a ruler and also by measuring each edge of the picture separately with a ruler and summing these values. In this way, they meant to show the equality of both results and make students feel the measurement aspect of perimeter. They also expected students to comprehend that perimeter was the sum of all edge lengths rather than memorizing the formula of it.

After the first activity, preservice teachers resolved to reinforce the meaning of perimeter and make students reach to the conclusion that perimeter is the sum of all existing edges. They decided to give figures which were a triangle, a quadrilateral and a pentagon. Later, they expected students to complete a table which required to write in each edge length of a figure and perimeter of it and to do this operation for each figure. They thought that students would write three edge lengths, find the perimeter by summing all of them and see that perimeter of a triangle was the sum of three edges. Students would sum four edges of a quadrilateral and realize that its perimeter was the sum of four edges. After repeating it for a pentagon, they would see that the perimeter of a pentagon was the sum of five edges. In this way, preservice teachers aspired to show the relationship between the number of edges and perimeter and make students understand that perimeter was the sum of all edges of the figure.

Preservice teachers talked about preparing an exercise paper which included four visual questions about summing the given edges to find the perimeter or finding one edge from the given perimeter in order to reinforce the subject. Preservice teachers also discussed on students' general tendency to memorize the formulas rather than internalizing the meaning of them. Therefore, they wanted to show how perimeter formula was obtained instead of writing it directly. They thought that students should recognize the meaning of 2x(a+b), 2a+2b and 4a. They also paid attention to the importance of different representations and discussed on showing the equality of a+a+b+b, 2x(a+b) and 2a+2b. Here, they aimed to concretize the perimeter formulas by visualizing. At this point, they decided to prepare two pieces of isometric paper material in a way that they could stick on boards in the classrooms. They planned to draw a rectangle in the form of axb on one isometric paper and then move the edges' end to end to the side of another isometric paper. At first, they settled to move the edges as a,a,b,b respectively to show the formula of 2a+2b and then move them as a,b,a,b respectively to give the formula of 2x(a+b). After giving the perimeter formula of a rectangle, they agreed to draw a square in the form of axa and then move the edges to the side as a,a,a,a to show that the perimeter formula of a square is 4a. After, they drew a rectangle figure in the form of 3x6 cm and asked its perimeter and they also gave a verbal question including one edge length of a square to find its perimeter in the lesson plan. In sum, they decided to move the edges end to end to the side and show how formulas formed.

The other objective included to create different figures which had the same perimeter. Preservice teachers talked about how to help students feel that there might be different figures with the same perimeter. They determined to draw four different rectangles each of which had a perimeter equivalent to 10 cm but the edges were different without writing their values. These figures were in the forms of 1x9, 2x8, 3x7 and 5x5 and some of them were in a horizontal position whereas some were in a vertical position. They chose to get students' estimations about the perimeter of these figures with the reasons first and then want them to calculate perimeters after writing the value of edge lengths on the figures. In this way, they strived to make students notice that perimeters of all figures were equal and different figures had the same perimeter.

After giving the meaning of perimeter; showing the relationship between perimeter and measurement; introducing the perimeter formulas of a rectangle and square; and making students feel different figures which had the same perimeter, preservice teachers resolved that continuing with questions would be appropriate. They looked for various questions from mathematics textbooks. They paid attention to ask the questions which both would and would not include figures because the cooperating teacher had suggested them to let the students draw sometimes. They also tried to combine perimeter and different kinds of concepts and asked the questions which included a rectangle and square together or a rectangle and triangle. After reinforcing the concept of perimeter with questions, preservice teachers paid attention to pedagogical issues and concluded to finish the lesson with repetition of what students learned in the lesson. The revised lesson plan 1 was presented in Appendix D.1.

Teaching of the lesson was conducted by Hasan in 5/F classroom. After the reflecting phase, preservice teachers decided to make the questions in the exercise paper more difficult and changed some of them in the direction of each other's and cooperating teacher's comments. In the first lesson plan, they used "l" and "s" letters for representing respectively long edge and short edge while introducing the perimeter formula of a rectangle. However, the cooperating teacher expressed that this use caused confusion for students and suggested to use the letters of "a" and "b" which students were familiar with. Therefore, preservice teachers changed the letters and used "a" for the short edge and "b" for the long edge. Re-teaching of the revised lesson was conducted by İnci in 5/G classroom. After the re-reflecting phase, they did not make any changes in the lesson plan.

Lesson Study Cycle 2

In the second planning phase, the aim was to teach the concept of area and the objectives of the second cycle were “Calculate the area of a rectangle; use square centimeter and square meter” and “Estimate a given area using square centimeter and square meter units”. Preservice teachers agreed to start the lesson by repeating the previous lesson related to perimeter before proceeding to the area concept. Preservice teachers planned to prepare two pieces of plotting papers as a material to use because area means covering the interior region. They intended to utilize this material for covering with squares and calculating area by counting them. The cooperating teacher had also suggested to emphasize that perimeter was the whole outer length of a shape and the meaning of area was the interior region. From this viewpoint, they chose to show these concepts through visualizing. Preservice teachers opted to create a rectangle with a band on plotting paper and to show that the band surrounding the rectangle was the perimeter of it. Then, they thought to cover the interior region of this figure with a piece of cardboard and show that it was the area of the rectangle to explain the difference.

Later, preservice teachers agreed to draw a figure on plotting paper and wanted students to cover this figure with unit squares. They thought that students would find the number of unit squares by counting in the interior region of the figure and then understand how to calculate area. In other words, they would realize that the area of a figure was equivalent to the sum of unit squares within the interior region. Preservice teachers also decided to prepare an exercise paper which included six figures that were divided into unit squares to reinforce the calculation of area by counting the unit squares.

The objective of this cycle included the use of area measurement units, m^2 and cm^2. Preservice teachers talked about how to make students understand the need for using standard measurement units at first. They settled to draw two same rectangles side by side on a plotting paper and ask students to cover one of them with a square in the form of 1x1 and the other with a square in the form of 2x2. They thought that it would facilitate students’ realization that different measurement units gave different results and this situation necessitated the use of standard measurement units to obtain the same results. From here, they aspired to give area units m^2 and cm^2. They concluded to do a similar activity for length measurement and wanted students to measure the long edge length of their desks with their hand spans. From here, they aimed to repeat length units, m and cm.

Preservice teachers did not want to give the area formulas of a rectangle and square straightway. At this point, they resolved to use the relationship between the unit squares in rows and columns. They agreed to draw a rectangle in the form of 3x6. Later, they aimed to query the number of unit squares in each row and the repetition number of the row throughout the column. They though that students would realize that the number in the row was multiplied with the number on the column to find the area. They also believed students would see that the result which they obtained by counting the squares one by one was equivalent to the multiplication of the long and short edges of a rectangle. However, they thought that only one example would not be enough to reach this inference, therefore they considered giving another rectangle in the form of 4x5 and do the same thing for it too. After working on these two rectangles, they decided to support students to reach the area formula as short edge times long edge and axb. After introducing the area formula of a rectangle, they agreed to ask two questions one of which was visual and the other was verbal and also one of them was towards finding the area using the formula directly and the other included a combination of perimeter and area. Later, preservice teachers agreed to follow the same procedure for giving the area formula of a square and determined two squares in the form of 6x6 and 4x4 in order to enable students to reach the formula. Similarly, they asked two questions one of which was towards finding the area with one given edge and the other included to find the area with perimeter in order to allow students to use the area formula.

The objective of this cycle involved estimating the area as well as calculating it. Preservice teachers also wanted to combine perimeter and area, and determined an activity in accordance with their goals. This activity included first to estimate and later calculate the perimeter and area of three given figures. At first, the top of the figures would be covered with different colorful cardboard pieces and students would be asked to compare the perimeter of the figures and then the area of the figures. After their estimations about perimeter and area were written on the board, the colorful cardboard pieces would be removed and students would calculate both perimeter and area. Therefore, by calculating they would see that the perimeter of the two figures were equal whereas the area of both were different. It would benefit them to compare their estimations and results and reinforce the concepts of perimeter and area.

After giving the meaning of area; showing the relationship between measurement and area; teaching to find the area covering with unit squares and counting them; introducing the area formula of a rectangle and square; and making students estimate the perimeter and area, preservice teachers agreed to maintain the lesson with questions. They endeavored to ask different kinds of questions including; covering a rectangle with a square, finding the area based on the addition of a rectangle and square areas or subtraction of the area of a square from a rectangle and combining the perimeter and area. They also decided to finish the lesson with a short repetition. The revised lesson plan 2 was presented in Appendix D.2.

Teaching of lesson was conducted by Semih in 5/F classroom. After the reflecting phase, preservice teachers negotiated to change all the figures in the exercise paper. They made them more difficult and added half squares as well as unit squares as the cooperating teacher indicated they were too easy and suggested to give half squares in the form of triangles to aid students turn two triangles into one square. She also offered to talk about the relationship between measurement units and daily life. Therefore, preservice teachers considered asking students where these measurement units were used in daily life and added this question to the lesson plan. Besides, preservice teachers pointed to the difficulty of covering the rectangles and squares while giving the area formulas because their size was so big that this operation took a lot of time. They also stated that there was no need for second figures since students could tell the area formula at the first figure. Thus, preservice teachers made the sizes smaller such as the rectangle 3x4 and the square 3x3 and removed the second figures while introducing area formulas. Re-teaching of the revised lesson was conducted by Mehmet in 5/G classroom. After the re-reflecting phase, they concluded to remove only one figure from the exercise paper considering it included half of a rectangle in the form of triangle not a square, so students had difficulty and could not understand how to solve it.

Lesson study Cycle 3

In the third planning phase, the aim was to teach conservation of area and reinforce the concepts of perimeter and area, and the objectives of the third cycle were “Create different rectangles with the given area” and “Solve problems that require to calculate the area of a rectangle”. Preservice teachers determined to start the lesson with the repetition of previous subjects. They focused on the concept of conservation of area by looking at the objective of creating different rectangles with the same area. They resolved to prepare a rectangle from a cardboard and stick it in horizontal, vertical and diagonal positions respectively on the board. They expected students to notice that a rectangle could be in different prototypes but it was still the same rectangle and the edges of a rectangle did not change with rotation so the numerical value of the area did not change.

After determination of the first activity, preservice teachers agreed to reinforce the conservation of area and the constitution of different figures with the same area. They aimed to show this by creating one geometric figure from another with the same area. They decided to create a triangle from a rectangle using the same pieces, a rectangle from a square and a trapezium from a parallelogram in order to make students realize that area did not change because the pieces in both figures were the same. In the first activity, they showed that the area of the same figure did not change and they utilized numerical justifications; here differently from the first one, without needing to use a mathematical formula, they chose to show there was no change of the area in different figures.

Preservice teachers decided to draw a figure which would include 16 unit squares on a plotting paper. At first, they wanted to ask students to cover it with unit squares by counting one by one and say the area of it. Later, they opted to ask a student to create a rectangle from this figure by using the same unit squares and removing them from the plotting paper next, and then ask another student to create a square from this rectangle by using the same unit squares and next removing them from the plotting paper again. Therefore, they concentrated on making students create different figures with the same area by covering them with the same amount of unit squares and also understand that different figures might have the same area. Besides, it was settled to ask students to calculate the area of a rectangle and square by using the formula. Thus, it was aimed to emphasize that area could be found both by counting the unit squares in the interior region and by the formula, and accordingly another objective was to make them find that the area did not change, by showing the relationship between the area of a rectangle and a square.

Preservice teachers also intended to reinforce the concepts of perimeter and area by using them together. They determined a question which included four rectangles in the form of 1x12, 3x4, 4x3 and 2x6 and required to write the short edge, long edge, perimeter and the area of each figure. Here with this question, preservice teachers aimed to make students realize that different figures which had the same area did not need to have the same perimeter.

After teaching the conservation of area, preservice teachers endeavored to determine different kinds of questions. For example, questions included creating different rectangles which had the same area, integrating knowledge on prior subjects such as fraction and rounding, covering a rectangle with squares, combining perimeter and area. Moreover, preservice teachers negotiated to finish the lesson with repetition as in the other plans. The revised lesson plan 3 was presented in Appendix D.3.

Teaching of the lesson was conducted by İnci in 5/F classroom. After the reflecting phase, preservice teachers decided to remove one question about rounding the edges of a rectangle because the cooperating teacher indicated that students would not compare their estimations with the real results as they did not know multiplication in decimals. Re-teaching of the revised lesson was conducted by Hasan in 5/G classroom. After the re-reflecting phase, both preservice teachers and the cooperating teacher emphasized the loss of time in one question related to creating different rectangles with the same area in two implementations. At this point, the cooperating teacher suggested to allow one of the students to draw one rectangle instead of waiting for too long. Thereby, it could lead the other students to understand what was required. In this direction, preservice teachers agreed to draw one of the rectangles by themselves at first in order to guide the students and then call them. They thought that if they continued to use 28 as area, there would be only two rectangles left to find after the teacher drew one. Therefore, they also concluded to change the area into 24 to increase the number of rectangles.

Lesson Study Cycle 4

In the fourth planning phase, the aim was to teach the features of rectangular prisms, open and close images of them and their surface area and the objectives of the fourth cycle were "Recognize a rectangular prism and identify its essential features", "Draw surface developments of a rectangular prism and decide whether different surface developments belong to the rectangular prism or not" and "Calculate the surface area of a rectangular prism". They resolved to start the lesson with the repetition of the previous lesson. They settled to use various computer programs in which they would show the elements of a prism, and open and close forms of them. They also agreed to prepare concrete materials to increase visualization. They wanted to emphasize the relationship between dimensions and the concepts that students had learned such as one dimension, length and perimeter; two dimensions, length and width and area; three dimensions, length, width and height and prism. They also talked about showing the constitution of a prism with the addition of height. They preferred to draw a point and then create a line segment. Here, they wanted to query why it was used to proceed to the relationship of length and perimeter. Later, they agreed to create a rectangle by adding width on length to proceed from two dimensions to area. Lastly, they wanted to create a rectangular prism by adding height on length and width to draw attention to three dimensions of a prism. They meant to show the formation of a rectangular prism step by step instead of indicating that it was a three dimensional figure which included length, width and height. Preservice teachers talked about showing how a prism formed by opening and closing it with the help of a computer program.

Preservice teachers agreed to continue by using concrete models of a rectangular prism, tetragonal prism and cube as the cooperating teacher indicated to teach these three types of rectangular prisms. They determined to introduce the essential features of each prism by showing and counting on these models respectively. They aimed to show edge, vertex, base, face, lateral face, height and find the number of face, edge and vertex for each prism by counting on the concrete models. They chose to write these features and draw open and close images of them on the board side by side. They aimed to make students find what was asked and to realize the differences and similarities between them by comprehending instead of giving the information straight away. They also concluded to ask prisms' places of use in daily life to concretize these concepts

Preservice teachers thought that open and close images of all prism types needed to be given with the help of both computer programs and concrete materials. In order to show that one prism might have different open images and each image might not turn into a prism, they chose to present different open images of prisms and show whether they would close with the help of both computer programs and concrete materials. Here, they attempted to prevent or reduce students' incorrect inferences by helping them feel what was intended rather than expecting students to give correct answers regarding formation of all prisms directly.

The objective of this cycle also included calculating the surface area of rectangular prisms. Preservice teachers did not want to give the surface area formula of each prism right away. Instead, they agreed to teach that the meaning of surface was equivalent to the sum of areas of all faces. They resolved to find the area of each face and write them into the relevant region so that students would see that the surface area was the sum of all faces' areas when they saw the open image of a prism. They decided to open the concrete model of a rectangular prism, calculate each face on it and then find the surface area by adding all area values, and make the same operations for a tetragonal prism and cube respectively.

After introducing the features of each prism, showing the similarities and differences between them, representing open and close images of prisms on both concrete models and computer programs, allowing students to understand the formation of a prism and teaching the calculation of the surface area of each prism, preservice teachers wanted to determine some questions related to calculating the surface area of a rectangular and tetragonal prism. They preferred to ask one of them by giving the figure and the other verbally. They also asked a question about the surface area of a cube by combining perimeter in the question. They paid attention to ask different questions. The revised lesson plan 4 was presented in Appendix D.4.

Teaching of the lesson was conducted by Mehmet in 5/F classroom. After the reflecting phase, preservice teachers settled to make no changes in the lesson plan but increase the lesson hours from two to three hours to implement the lesson plan completely in the direction of their own ideas and the cooperating teacher's suggestion. Re-teaching of the revised lesson was conducted by Semih in 5/G classroom. After the re-reflecting phase, preservice teachers decided to increase the lesson hours by one more hour, namely, four hours in total due to the intensity of this lesson plan and they did not make any other changes.

3.5 Researcher Role

The researcher had the roles as a facilitator in following the flow of the lesson study phases and a researcher in collecting data. The researcher paid attention to emphasize that her main role was as an observer in this process. Before beginning the study, the researcher informed the participants about the requirements of the research process, gave detailed information about lesson study through written and video examples and guided preservice teachers to reach educational resources for their investigation of mathematical content. As a facilitator, the researcher introduced the concept of lesson study, helped them to follow the steps in the lesson study process and supplied them with a four column lesson plan tool that is mostly used by other lesson study groups in literature (Lewis, 2002). In planning phase, the researcher endeavored to provide not wandering away from the topic through statements like "What kind of a plan will you prepare?", "What will you write next?" and "How will you continue the lesson?". In teaching phase, researcher observed the implementation of the lesson plan in the classroom and took field notes about the lesson. The students in the classroom knew the role of the researcher and the researcher did not interact with the students. In reflecting phase, researcher had a role of a facilitator and directed preservice teachers to share their ideas about teaching, what went well or did not and discussed on what to change for the improvement of the lesson. The researcher took mainly the role of an observer as a participant and had a "highly observational" stance (Guest et al., 2013, p. 89). Therefore, the researcher did not make any contributions to how preservice teachers designed the lesson plans, did not limit their observations during the teaching process and did not make any suggestions in the reflecting phase. In brief, the researcher was present within all lesson study professional development phases and her presence was as unobtrusive as possible.

The researcher observed and video-recorded the whole process without making any evaluation about preservice teachers or what they did. The researcher organized appropriate dates in the direction of preservice teachers' convenience for planning meetings, teaching, reflecting meetings and interviews in order to ensure the flow of the research. With the help of these various roles, the researcher gained insight into what and how preservice teachers notice, had better understanding of the research context, mathematical content, preservice teachers' approach, experience and anticipation of students' thinking and collected a set of data such as lesson plans, observation notes which were supportive for the interpretation of data.

3.6 Data Collection

Common sources of case studies to understand and describe the phenomenon are interviews, observations and documents emerged during the process (Merriam, 1998). Using multiple methods to collect data provides a clearer view, better understanding, richer and truer interpretation of the study (Yin, 2003).

Data collection took two months and included four lesson study cycles which consist of preparation of four lesson plans, four teaching and four re-teaching in two 5^{th} grade classrooms and eight reflecting processes on these lessons in total. Each lesson study cycle involved eight data collection steps.

Table 5 Summary of the Data Collection Process

Cycle 1	**Cycle 2**	**Cycle 3**	**Cycle 4**
Videotaping of planning 1	Videotaping of planning 2	Videotaping of planning 3	Videotaping of planning 4
Interview after planning	Interview after planning	Interview after planning	Interview after planning
Videotaping and observation of teaching	Videotaping and observation of teaching	Videotaping and observation of teaching	Videotaping and observation of teaching
Videotaping of reflecting on the lesson	Videotaping of reflecting on the lesson	Videotaping of reflecting on the lesson	Videotaping of reflecting on the lesson
Interview after teaching	Interview after teaching	Interview after teaching	Interview after teaching
Videotaping and observation of re-teaching	Videotaping and observation of re-teaching	Videotaping and observation of re-teaching	Videotaping and observation of re-teaching
Videotaping of reflecting on the revised lesson	Videotaping of reflecting on the revised lesson	Videotaping of reflecting on the revised lesson	Videotaping of reflecting on the revised lesson
Interview after re-teaching	Interview after re-teaching	Interview after re-teaching	Interview after re-teaching

3.6.1 Interviews

Interview is a powerful tool which assists to understand how people make sense of their world and experiences (Merriam, 2002). Interview helps to obtain more information about individuals' past actions, experiences, feelings and thoughts (Rapley, 2004). Kvale and Brinkman (2009) indicate that "the qualitative research interview attempts to understand the world from the subjects' point of view, to unfold meaning of their experiences, to uncover their lived world prior to scientific explanations" (p.1). In other words, interviews help to get rich information about the subject being studied and enables triangulation of information from other sources (Denzin & Lincoln, 2005; Lincoln & Guba, 1985). In this study, semi-structured interviews were used to elaborate what and how preservice teachers noticed during the lesson study, learn how participants have constructed the meaning of their lesson study experiences and increase the validity of the study. In other words, it was applied to illustrate preservice teachers' understandings, feelings and thoughts on the process including the lesson plans that were prepared collaboratively, experiences of teaching, discussions based on their own implementations, learning and observations. Semi-structured interviews with each preservice teacher were conducted after both planning and teaching phases with a set of prepared questions. Interview questions were developed based on the relevant literature on professional development and noticing. Interviews lasted approximately between 30 and 60 minutes. Each of the interviews was recorded with a video camera and fully transcribed.

Interviews centered on preservice teachers' perceptions of effectiveness of a lesson plan and the success of a lesson, students' unexpected responses related to mathematics subject and their noticing within each lesson study session. The aim was to understand preservice teachers' opinions associated with lesson study, knowledge about teaching, how they use it in lesson study process and what they learned. In particular, the researcher's focus was on what preservice teachers noticed about students' mathematical thinking and how they made sense of it as making decisions about the lessons. By asking questions during the interview parts after planning and teaching, the goal was to promote preservice teachers to elaborate their own reasoning, knowledge of mathematics teaching and their own understanding and observation of students' thinking. Besides, the interviews tried to expose preservice teachers' expectations about students' ways of thinking and solution approaches after planning and what was different from their expectations after teaching. It was also aimed to understand whether changes in their noticing occurred in more detail and whether lesson study contributed to their professional development or not. For the purpose of expanding on preservice teachers' noticing throughout the lesson study process, interviews were conducted individually. Interviews enabled to understand what they attended to while planning, teaching and discussing and how they interpreted the moments that they captured. Transcripts of video recordings were used to demonstrate preservice teachers' noticing but they could not fully reflect their thinking. Therefore, another purpose of the interviews was to reduce these deficiencies and obtain richer information about their noticing. Interviews helped to increase the depth of understanding of preservice teachers' thinking and ideas. They allowed to reveal preservice teachers' impressions on the lessons, interpretations of their experiences and ideas on students' mathematical approach. They were also used to verify what preservice teachers believed that they noticed about students' mathematical thinking during the lesson study.

3.6.2 Observations

Observation is a valuable method which enables to observe what happens directly, to gain new viewpoints and to understand the event in-depth (Merriam, 2002). Marshall and Rossman (1989) identify observation as "the systematic description of events, behaviors, and artifacts in the social setting chosen for study" (p.79). Observations are important considering they serve to examine the cases in their natural settings, understand the awareness degree of the participants in the process and help to interpret the things that participants are not willing to talk about (Patton, 2002). In this study, the data obtained from observations was used to describe the settings that lesson study process took place, what type of activities, behaviors and feelings were displayed, what experiences participants had and how participants gave meaning to their own observations, discussions and works based on collaboration in the settings. In order to understand better and interpret to what extent and how preservice teachers noticed students' mathematical thinking during instruction, whole implementations of the lessons were observed by the researcher as well as lesson study group members. Observations supplied a wide range of data in a very short time. The role of the researcher during the teaching phase was as a non-participant observer and she took field notes on teaching and classroom interactions since it is recommended for effective observation (Merriam, 1988). These notes contribute to a rich description of settings, activities, experiences, participants during the study and writing notes without delay facilitates remembering what happened during the process (Lofland & Lofland, 1984). Field notes include researcher's writings about what was observed and thought in the observation settings (Bogdan & Biklen, 1992). For example, field notes included information about students, their behaviors, classroom environment and what teacher did in this study. Besides, field notes were kept for each professional development phase to elaborate the whole picture of what was happening in lesson study cycles. Field notes were useful for recalling the important points and supporting the other data. They empowered the researcher to make sense of preservice teachers' comments during the lesson study process while analyzing the data.

3.6.3 Video recordings

Video recording is helpful in terms of capturing both planned and unplanned situations (McKenney & Reeves, 2012). It can be utilized in examining teachers' noticing (Jacobs, Lamb, et al., 2011; Kazemi et al., 2011; Miller, 2011; Seidel et al., 2011; Star et al., 2011; van Es & Sherin, 2008) and in collecting data during the lesson study process (Lewis et al., 2009). Video recording provides to capture the authenticity of the classroom and presents details of the teaching and learning process (Baecher, Rorimer, Smith, 2012). Videos enable to record potentially useful interactions and behaviors which may be overlooked, to trace the learning of the participants during the lesson study cycle (Lewis, Perry & Murata, 2006) and to review the same data again and again for correct interpretations (Latvala, Vuokila-Oikkonen & Janhonen, 2000). In this study, a video camera was used to record preservice teachers' planning discourse; capture preservice teachers' teaching and the interaction between preservice teacher who conducted the lesson and students during the teaching phase; and record their ideas and reflections on the lessons in order to study what and how preservice teachers notice. In other words, video recordings were used in order to capture all of the lesson study process. Recordings of preservice teachers' discourses during planning the lesson collaboratively, teaching the lesson by interacting with the students and reflecting their ideas and suggestions for improvement of the lesson is important to gain insight into their noticing and also to triangulate the data. Each record of the lesson study phases supplied data to see what was happening and how the lesson study process seemed to be influential in developing preservice teachers' abilities to notice, plan, teach and revise the lesson based on students' mathematical thinking.

3.6.4 Documents

In addition to interviews, observations and videos, data were collected from relevant documents in order to support and enhance the findings. According to Merriam (2009) the data from documents deliver descriptive information for better understanding of the situation, verifying the hypotheses and supporting analysis. The documents in this research were lesson plans that were constituted by preservice teachers, researcher notes on the meetings and lessons, what seemed like important and general summary of what was done in the same day the meeting occurred or the next day at the latest and the transcripts of video recordings. Lesson plans allow to learn more about preservice teachers' lesson planning and the skills of attending to, making sense of and considering students' mathematical thinking as planning, teaching and revising the lesson.

3.7 Data Analysis

Research design and analysis methods vary depending on different purposes of research (Knafl & Howard, 1984). In data analysis, the purpose is to organize the data, reveal the meaning of it and present realistic conclusions (Polit & Beck, 2006). All qualitative research engages in making interpretations about situations and conditions but these interpretations change in accordance with the details and the level of abstraction based on analysis method (Patton, 2002). In data analysis, the flexibility of adopting inductive or deductive approaches or combination of them changes depending on the aim of the research (Cho & Lee, 2014; Elo & Kyngäs, 2008). These two approaches differentiate based on how codes or categories are developed (Cho & Lee, 2014). An inductive approach is used when prior knowledge related to the phenomenon that has been studied is limited or there are no previous studies about it and hence, codes, categories or themes are directly generated by the researcher from the data (Elo & Kyngäs, 2008; Hsieh & Shannon, 2005). It is particularly appropriate to develop a theory rather than describe a theory or verify a previous theory (Zhang & Wildemuth, 2009). A deductive approach is used when the aim is to test a previous theory in a different situation or compare categories at different periods (Hsieh & Shannon, 2005; Elo & Kyngäs, 2008) and therefore, existing codes or categories from prior relevant theory, research, or literature are used (Cavanagh, 1997). The use of coding categories from previous studies provides to support and compare the findings with the relevant research (Zhang & Wildemuth, 2009).

In this study, deductive approach was used to analyze the data in order to display preservice teachers' ability to notice students' mathematical thinking and to present whether there were changes in their skills over time with participation in the lesson study process. In other words, it was aimed to analyze preservice teachers' noticing of students' mathematical thinking in planning, teaching and reflecting phases of lesson study by using appropriate coding categories and then classify their skills in terms of noticing levels systematically to reveal the development in their skills and to identify the meanings of the data obtained. Inductive category development includes the following steps and the figures show the process (Cho & Lee, 2014).

a) the research question,

b) the determination of category and levels of abstraction,

c) the development of inductive categories from material,

d) the revision of categories,

e) the final working through text, and

f) the interpretation of results.

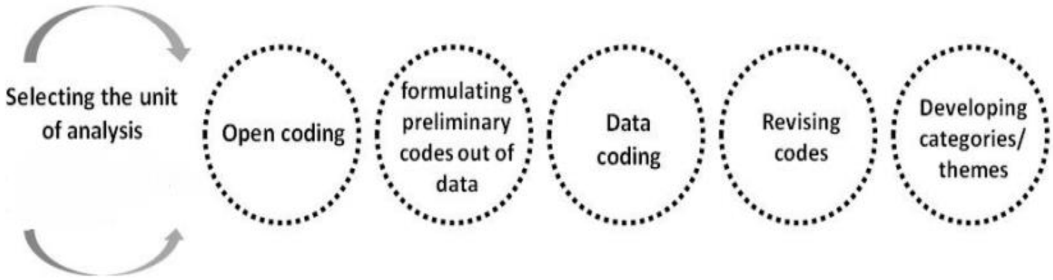

Figure 3 Procedure for an inductive approach

Second and third steps differentiate in deductive category development as

b) theoretical-based definitions of categories, and

c) theoretical-based formulation of coding rules (as cited in Cho & Lee, 2014, p. 10).

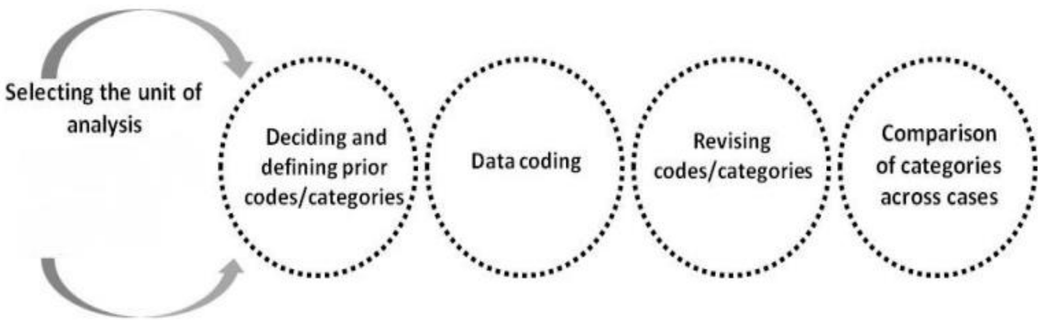

Figure 4 Procedure for a deductive approach

In the present study, relevant literature on teacher noticing was examined in detail to decide on an appropriate method for analyzing the data in accordance with the purpose of the study. Although there were different types of conceptualization of noticing notion in the studies, they generally referred to the same dimensions with minor differences. While determining the commonly used and central coding categories, it was seen that van Es's (2011) framework for learning to notice student mathematical thinking included most of the dimensions and sub-dimensions in the other studies. In this direction, it was preferred to use the existing theoretical framework and predetermined coding categories to analyze preservice teachers' noticing skills as deductive approach was adopted

3.7.1 Coding categories

In this study, data was composed of interviews, observations, written documents and video recordings of whole lesson study phases. Video recordings were fully transcribed and transcripts were read by the researcher in detail to examine and analyze the data better because it is recommended that the researcher should engage in data for obtaining the sense of whole through reading more than once (Polit & Beck, 2003). Units of analysis were selected which included whole transcripts of lesson study process, interviews, observations, field notes and lesson plans. Literature considering teacher noticing was investigated and revised in order to choose the data analysis method. The researcher determined relevant coding categories which are used commonly and examined how they were used in analyses of the data. The data were coded with the help of *framework for learning to notice student mathematical thinking* (van Es, 2011).

In this framework, noticing is separated into two central categories as "What Teachers Notice" and "How Teachers Notice" in order to understand the nature of noticing and there are four dimensions to analyze the data: *Actor, Topic, Stance* and *Specificity.* First two dimensions, namely, actor and topic refer to what teachers notice whereas the last two dimensions, stance and specificity refer to how teachers notice. *Actor* is the person that participants focus on in their comments (student, teacher, curriculum developers, self, other). *Topic* refers to what participants focus on and the issues they identify (classroom management, classroom environment, climate, mathematical thinking, pedagogy, other). van Es and Sherin (2006) describe these concepts by stressing *mathematical thinking* refers to the expressions about mathematical ideas, strategies and understandings. Pedagogy refers to teaching techniques and strategies. *Classroom management* addresses the statements related to the mechanism in the classroom and the strategies that the teachers use to handle the classroom events. *Climate* addresses the classroom's social environment. *Classroom environment* refers to the physical setting of the classroom and demographics of the students and teachers (Star & Strickland, 2008).

Stance is related to the analytic approach of participants in analyzing the practice (describe, evaluate, interpret). Here, *describe* refers to the statements participants restate about the events that occurred in the setting; *evaluate* refers to the judgmental statements that reflect participants' comments on what was good or bad, and suggestions about what should have been done differently; *interpret* refers to the statements that participants make inferences about their observations and the reasons behind the events and try to explain their meanings. *Specificity* concerns the depth of analysis that participants use to discuss the events they notice (general, specific). *General* refers to the statements that include few details about participants' thinking and broad generalizations and reflects the whole class. *Specific* refers to the statements in which participants provide evidence to support and elaborate their thinking and reflect particular individuals or issues (van Es & Sherin, 2010; van Es, 2011). Van Es and Sherin (2010) also described another dimension, *Evidence*, which examines the source of evidence in their research. However, it was not taken into consideration in this study since it is not regarded with the purpose of the research. Table 6. shows the dimensions which were used in the analysis of preservice teachers' noticing skills in this study.

Table 6 Coding Categories for Analysis of Teacher Noticing

Dimensions of Noticing	**What Teachers Notice**		**How Teachers Notice**	
Sub-Dimensions	Actor	Topic	Stance	Specificity
Description of Dimension	Who is identified	What is discussed	How the event is analyzed	How the level of detail is provided
Categories in Dimension	Student Teacher Curriculum developers Self Other	Mathematical thinking Teacher pedagogy Classroom management Classroom environment Climate Other	Describe Evaluate Interpret	General Specific

In the present study, the transcripts of four lesson study cycle processes and interviews were coded by the researcher with the help of coding categories to reveal what and how preservice teachers notice during lesson study process. In order to ensure the reliability of the coding procedure, a faculty member in mathematics education was requested to code the data as well. A randomly selected transcript of planning, teaching, reflecting and re-reflecting phases from four lesson study cycles were read and coded independently by the faculty member. Both the researcher and the faculty member analyzed the data by considering the coding categories and theoretical framework. The coding of the researcher and the faculty member were compared and the inter-rater reliability was found about 83%. The difference between the two coders was discussed so as to increase the percentage of the agreement and a consensus was reached on the codes. When they thought differently from each other, both of them examined the studies using the framework for learning to notice student thinking (van Es, 2011) and the sample sentences for coding categories in literature together, discussed on the samples and the transcripts and tried to determine the characteristics of which categories were dominant in the quotations.

A developmental trajectory from baseline to extended level which illustrates the growth in learning to notice in time was generated by van Es (2011) through these coding categories. With this framework for learning to notice student mathematical thinking, the aim was to display teachers' development in attending to noteworthy issues in students' mathematical thinking and making sense of their observations based on evidence. In this study, the levels of preservice teachers' noticing were categorized through the framework of van Es (2011) in order to project their noticing skills and understand whether there were emergent developmental shifts.

3.7.2 Framework for learning to notice student mathematical thinking

Van Es (2011) identified a framework for learning to notice student thinking which promotes to examine a trajectory of development in two dimensions, what teachers notice and how teachers notice, with four levels of noticing: Level 1 (Baseline), Level 2 (Mixed), Level 3 (Focused) and Level 4 (Extended). In this study, this framework was used to investigate preservice teachers' abilities to notice student mathematical thinking, determine their levels of noticing and reveal the development in their noticing skills, if it existed, in the context of lesson study professional development model. The content of noticing levels and developmental shifts from Level 1 to Level 4 with respect to two dimensions of noticing (van Es, 2011) were described below.

Level 1 (Baseline): In terms of what to notice, participants attend to a range of issues such as teacher pedagogy, behaviors of students, the class's learning, participation and classroom climate. They are in tendency to refer to the students as a whole class and are concerned generally with themselves. Regarding how to notice, participants offer general impressions and indicate their observations by oversimplifying the complex structure of classroom interactions. Besides, their comments are descriptive and evaluative in nature and they do not supply evidence to support their claims or they offer few details. At this stage, participants make comments like "*They all wanted to volunteer*" and "*The class is engaged*".

Level 2 (Mixed): As for what to notice, participants attend primarily to teacher pedagogy but they begin to focus on students' mathematical thinking as well. The shift in attending from the class as a whole to particular students occurs. As for how to notice, participants maintain the general impressionistic stance but they also start to point out noteworthy events on one hand. Moreover, they continue to make evaluative comments on their observations, however, an effort for making sense of the events begins as well. Although they refer to some particular students or events in order to support their analysis, they are not consistent in extending their explanations or elaborating their observations. At this level, participants offer comments such as "*I like how he borrowed*" and "*They do not get it*".

Level 3 (Focused): In relation to what to notice, participants begin to focus primarily on particular students and their mathematical thinking. The shift in the focus from a range of issues to students' mathematical approach is the essential feature that distinguishes this level from Level 1 and Level 2. They stop centering on themselves and seeing the class as a whole. Regarding how to notice, participants reason on what they observed and endeavor to provide justifications about why the events happened. They address particular noteworthy events and offer evidence from these events in order to make inferences about students' mathematical thinking and understanding. Their comments are interpretive in nature and give details to support their claims. They attempt to elaborate their observation with various interpretations and explanations. For instance, participants make an explanation like "*She was using two different approaches, estimation and the traditional algorithm, to solve the problem*".

Level 4 (Extended): Participants continue to examine how students think mathematically by considering their comments and explanations in detail. They try to make sense of what they observed and provide details from their observations to support their analysis. Differently from the other levels, participants consider the relationship between students' ways of thinking and teacher pedagogy while interpreting what they focused on. Participants associate a specific student's thinking way to a particular instructional principle, offer alternative pedagogical solutions and make connections between their ideas, what they observed and the broader teaching and learning principles. In other words, they discuss on and interpret the interactions that they noticed using different ways by linking them with the essential features of teaching such as assessment and language. For example, participants make a comment like "*So maybe we need to really rethink our assessment of students*". Table 7 shows the levels of noticing which were used in the analysis of preservice teachers' noticing skills in this study (van Es, 2011).

Table 7 Framework for Learning to Notice Student Mathematical Thinking

	What Teachers Notice	**How Teachers Notice**
Level 1 (Baseline)	Attend to whole class environment, behavior, and learning and to teacher pedagogy.	Form general impressions of what occurred. Provide descriptive and evaluative comments. Provide little or no evidence to support analysis.
Level 2 (Mixed)	Primarily attend to teacher pedagogy. Begin to attend to particular students' mathematical thinking and behaviors.	Form general impressions and highlight noteworthy events. Provide primarily evaluative with some interpretive comments. Begin to refer to specific events and interactions as evidence.
Level 3 (Focused)	Attend to particular students' mathematical thinking.	Highlight noteworthy events. Provide interpretive comments. Refer to specific events and interactions as evidence. Elaborate on events and interactions.
Level 4 (Extended)	Attend to the relationship between particular students' mathematical thinking and between teaching strategies and student mathematical thinking.	Highlight noteworthy events. Provide interpretive comments. Refer to specific events and interactions as evidence. Elaborate on events and interactions. Make connections between events and principles of teaching and learning. On the basis of interpretations, propose alternative pedagogical solutions.

The data obtained from four lesson study cycles were analyzed with the help of coding categories: *actor*, *topic*, *stance* and *specificity* and noticing levels: *Level 1 (Baseline)*, *Level 2 (Mixed)*, *Level 3 (Focused)* and *Level 4 (Extended).* Selecting unit of analysis is an important initial step that refers to decide which data will be analyzed depending on the research questions (Cho & Lee, 2014). According to De Wever, Schellens, Valcke, and Van Keer (2006) the unit of analysis might be a sentence, a paragraph or a complete message in a research. With respect to the unit of analysis, in the present study, the transcripts of planning, teaching, reflecting, re-teaching and re-reflecting phases for each lesson study cycle and preservice teachers' answers to interview questions were examined. The paragraphs from the data were selected as the unit of analysis and coded. The researcher formed a table including what and how both preservice teachers noticed in each of the lesson study phases and interviews. After the data were coded, the percentage of each level was calculated. Preservice teachers' noticing levels were determined for all phases in lesson study and for each lesson study cycle. Determination of noticing levels took place by considering the characteristics of which level was predominant and had the highest percentage in each phase. For instance, a preservice teacher who had the distribution of noticing characteristics at Level 1 for 60%, at Level 2 for 30% and at Level 3 for 10% was classified as Level 1 (see Tables 8 and 9). Assigning a level enabled to interpret preservice teachers' abilities to notice students' mathematical thinking, developmental shifts in their noticing and how lesson study may have influenced their noticing of student thinking.

3.8 Trustworthiness

The concepts of reliability and validity are the most important elements which show the qualification of the study (Daymon & Holloway, 2003). Lincoln and Guba (1985) name the validity and reliability of the qualitative studies as trustworthiness and a state that four criterions for trustworthiness need to occur. These are credibility, transferability, dependability, and confirmability and they respectively refer to internal validity, external validity/generalizability, reliability and objectivity.

Credibility is related to the confidence in the accuracy of the findings obtained from the subjects, informants and the context in the study (Lincoln & Guba, 1985). In qualitative studies, it generally bases on the experiences of individuals as they are lived and interpreted by the informants (Krefting, 1990). According to Lincoln and Guba (1985) peer examination which includes checking the research process and presenting findings by experienced colleagues is important for validating the researcher's honesty. When colleagues check the categories and present negative situations, credibility of the study increases. Besides, in order to ensure credibility, Merriam (1998) suggests to use triangulation, member checks, peer examination, long-term observation and Shenton (2004) differently advices a prolong engagement, negative case analysis, thick description and examination of the other studies. Therefore, an expert in mathematics education was requested to examine the results of the research process and the findings were reconstructed by negotiating together in this study.

Other than that, interviews, observations, video recording, field notes and lesson plans are used as multiple data sources for data triangulation. Thick description of the study was presented as well as peer examination and the findings of previous studies were assessed in order to increase the credibility of the research.

Transferability includes generalizability of the findings and results to similar cases and events (Lincoln & Guba, 1985). Shenton (2004) emphasizes that "…it is the responsibility of the investigator to ensure that sufficient contextual information about the fieldwork sites is provided to enable the reader to make such a transfer" (p. 69). In order to assure transferability, the researcher needs to present how the study was conducted and give data in detail (Lincoln & Guba, 1985). In the study, the researcher focused on presenting a thick description of the nature of the lesson study process for each cycle stage by stage. In addition, the researcher attempted to clarify the content and procedure of the research, selection of participants, the process and methods of data collection and the timeline of the study in detail.

Dependability is related to whether the results of the research are dependable and whether there is consistency between the results and data (Merriam 1998). The consistency of a study can be affirmed by comprehensively clarifying each step of the research (Yin, 2003). Similarly, Shenton (2004) emphasizes that the research design, implementation procedure of the research, the ways of data collection and the followed steps in the study should be explained in detail to prove dependability in a qualitative study. He also suggests to use multiple data collection methods, explain the data collection process and define the formation or use of coding categories because providing credibility also supports the dependability. In this study, the research endeavored to describe how the process was conducted, how data were collected and how categories were used in a way that the other researchers understand and repeat the study. In order to assure dependability of coding through ensuring the credibility, a randomly selected transcript of planning, teaching, reflecting and re-reflecting phases from four lesson study cycles were read and coded independently by the researcher and the expert. Differences between the two coders were discussed to increase the percentage of the agreement and a consensus was reached on the codes. On the other hand, higher reliability can be maintained through code lists that were generated deductively rather than inductively (Catanzaro, 1988). Deductive approach and predetermined coding categories based on an existing theoretical framework were also utilized in this study.

Confirmability is related to researcher objectivity in interpretations of the results. Each researcher has their own perspective and presents the same case differently from each other.

It refers to “the extent to which the characteristics of the data, as posited by the researcher, can be confirmed by others who read or review the research results” (Bradley, 1993, p.437). In order to ensure confirmability, it is suggested to utilize multiple types of data sources rather than one data collection method (Herr & Anderson, 2005), explain the decisions and provide a detailed description of the process (Shenton, 2004). Another method for confirmability is to form the chain of evidence which means presenting the report about the content and problems of the study. The important point is that the report must include a sufficient number of interviews and observation quotations (Yin, 2003). In this study, triangulation and rich description of content were utilized and the analyses were supported by quoting from the transcripts of interviews and video recordings in order to ensure confirmability.

CHAPTER 4

4. FINDINGS

In the previous chapter, the methodology for evaluating two preservice teachers' noticing of students' mathematical thinking in the context of lesson study professional development model was described in details. In this chapter, what and how preservice teachers notice and how the development of their noticing in the context of lesson study process is will be presented. Multiple sources were used to collect data:

- Individual interviews of preservice teachers after planning, teaching and re-teaching
- Video recordings for each phase of lesson study
- Researcher's and preservice teachers' lesson observation notes
- Lesson plans that preservice teachers prepared

4.1 Case of İnci

4.1.1 Lesson study cycle 1

4.1.1.1 What and how İnci noticed in planning phase

First lesson plan was related to teaching of perimeter and the objective of the first cycle was "Calculate perimeters of polygons; create different figures which have the given perimeters". Before planning the lesson preservice teachers went to the practice school to get help and recommendations from the cooperating teacher about the subject. They listened to her ideas, took notes and asked the questions they had in their minds. The cooperating teacher generally indicated that students confuse the concepts of perimeter and area, thus firstly they should help them to understand that perimeter is the sum of all edge lengths, and then calculate the perimeter of different figures, use materials to concretize and ask questions to reinforce. All preservice teachers met to prepare the first lesson plan at the university seminar hall bringing the various sources such as mathematics textbooks, teacher's guide book, the curriculum and different mathematics education books. The following are presentations of examples based mainly on İnci's expressions from the planning meeting of lesson study cycle 1.

Preservice teachers discussed on how to start the lesson for a while. They shared their suggestions and referred to different points. Semih emphasized that they needed to focus on the meaning of perimeter in order to teach the calculation of it whereas İnci suggested to start teaching the subject with a problem. Hasan drew other preservice teachers' attention to the concept of measurement because perimeter, area and surface area were included in the learning domain of geometry and measurement and he believed that students had to feel it. On the other hand, Mehmet suggested to give a material and ask the perimeter of it. How to start teaching perimeter according to their comments are as it follows:

> **İnci:** According to me, we can start asking a problem such as "there is a field and the farmer wants to enclose it with a picket fence. How much fence is needed?". There is a similar example in the book as well. Here, it does not start the subject with the perimeter of a rectangle instead it gives numeric values and asks for calculation.
> **…**
> **Hasan:** According to me, students should know the meaning of measurement because they will engage in it consistently
> **İnci:** Yes, we can start like that. We can ask students "What is measurement?" and "What comes into your mind when you hear the concept of perimeter?"
> **…**
> **Mehmet**: We can prepare a material and require students to measure the perimeter using a ruler or their hands.
> **…**
> **İnci:** The cooperating teacher also suggested us to use materials, I think it can be good. Maybe, we can give a scenario or play a game with the students but I am not sure how we can do that.

In this excerpt, preservice teachers made their suggestions about the introduction of the lesson. İnci focused on an example in a textbook and shared the content of it with the other members in the group. She offered to start with a numerical problem rather than informing students about the subject directly. Here, she expected students to calculate the perimeter without showing how to find it at first. She did not elaborate why she suggested it and how her idea was related to the overall mathematical goal of the lesson. Her suggestion was based on what one of the books included rather than associating her idea to students' mathematical thinking or teaching principles. According to her, it could be asked because there was a similar example in the book. Therefore, her approach was very broad and superficial in nature.

Besides, she attended to cooperating teacher's suggestions in her notes and she supported the use of materials considering her ideas. She did not offer details of the cooperating teacher's suggestions and provided no evidence to support why she agreed with her idea. She only evaluated the use of materials as "good". She also offered to play a game related to the subject with the students but it got rejected by the others because of the physical conditions of the classrooms and limited time of lessons. However, she did not mention what kind of a game it was, the content of it and how it served the aim or students' mathematical learning so these points were not clear. She only made a suggestion probably based on knowledge about teaching but she did not give details to support her approach because she was not sure about it as well. In terms of what İnci noticed, her focus was on several issues such as textbook, the cooperating teacher and her own pedagogical approach rather than students' mathematical thinking. She did not make any connection between what she attended to and students' thinking and learning. Regarding how she noticed, her comments were unspecific and she emphasized generic teaching and learning approaches such as solving a problem, using a material and playing games without dilating upon. She offered descriptive and evaluative expressions in nature. For example, she restated the problem and evaluated use of material (e.g. "It can be good"). Furthermore, she provided little or no details to promote her ideas. Because of attending to mathematically insignificant aspects with a general approach, her noticing in this dialog had prevalent properties of Level 1.

Preservice teachers had difficulty in deciding how to start the lesson. There were various ideas and reaching a consensus took time. They maintained to work on selecting an activity to draw students' attention. Mehmet mentioned the materials that were normally used in this subject such as isometric paper and plotting paper. He offered to prepare and use them to calculate the perimeter of different figures. İnci agreed to use these two materials but did not accept to use them as Mehmet stated. The following extract shows the dialog between them:

> **Mehmet:** There are some materials such as isometric paper and plotting paper. If we use these two main materials instead of plenty other materials, I think it will be more effective. We can prepare unit squares and ask students to form different figures on the material. For example, we will give students 10 unit squares, students will constitute a figure like "d" and find the perimeter of it. Another student will form a rectangle and calculate its perimeter. Then, they will realize that both have the same perimeter.
> **İnci:** But is not it more towards teaching the subject of area? We are trying to introduce perimeter now. We will use the same things as in area, students will cover them by using unit squares. Therefore, we will fall into repetition in this situation.

In the quotation above, while they were talking about introducing the concept of perimeter, Mehmet indicated an idea in respect to the equalities of perimeters of different figures. In his suggestion, preservice teachers did not notice that when students used 10 unit squares and created different figures, area would be the same but the perimeter might not be. However, Mehmet indicated that figures would have the same perimeter. İnci also could not capture her incorrect mathematical approach and did not consider students' mathematical thinking. She focused rather on the similarities of activities and the place of materials' use. Her approach was general and evaluative in nature. She did not agree with Mehmet's suggestion because she occasionally encountered this kind of an example in teaching the subject of area. Her comments included few details and she did not elaborate how students might think. Hence, her noticing exhibited the features of Level 1.

İnci shared a mathematical approach that was included in a book and suggested to round the edges of a shape with a rope. She aimed to make the students measure the length of the rope that is used to round all the edges of the shape so that they would find its perimeter. The others liked this idea and focused on how to use it in the lesson. They discussed in what kind of context this activity should be given. The conversation on this issue is as noted below:

> **İnci:** For example, I noticed in one book that a rope was used to measure the perimeter. They rounded all edges with the rope, and then cut it after all edges were included. Later, they measured the whole rope that was used with a ruler and found the perimeter of the shape. I think, it is a good activity and we can use it to show perimeter. It is also an activity of measurement. In another book, there is a picture and someone wants to frame it, quite similar to that activity. It asks whether the perimeter or area should be found.
> **Mehmet:** It is good but it can be asked after the area is taught. This query can even be useful to reduce their confusions about perimeter and area.
> **İnci:** Ok, but I liked this context, we can use it.
> **Semih:** Yes, we can show a picture and want them to round it with a rope and measure its perimeter with a ruler.
> **İnci:** According to me, we should do this so that the students can learn that perimeter is based on measurement.

In this excerpt, İnci attended to how the teacher should show students the connection between perimeter and measurement. She explained an activity in a book which is related to this point in order to get the others' opinions. She considered student' mathematical thinking and understanding (e.g. "Students can learn that perimeter is based on measurement"). Regarding how to notice, she indicated how it was related to the purpose of the lesson, however, she did not elaborate how the students would make this connection and how they would think mathematically in this process. Would the students understand that perimeter is the sum of all edges, surrounding of the figure or something else? These questions were not detailed. Therefore, she was inconsistent in elaborating and giving details to make her opinions clear. Her comments included mainly a description of the activity and her evaluation about the use of it (e.g. "It is a good activity" and "I liked this context"). She also made interpretive comments on the students' learning of the relationship between perimeter and measurement but it was rather general and she did not provide evidence to support her claims. Thus, this mixed approach showed that her noticing had the characteristics of Level 2.

Preservice teachers continued to talk about how to organize the activity. Hasan suggested to ask students their predictions about the perimeter of the picture, write them on the board and find the nearest answer before measuring. In addition to this, Semih offered to ask the students how to find how much rope was needed. İnci wondered how to react if the students said that they needed to know the length of the edges instead of measuring around the picture. She also asked what to do if the students did not understand that they found perimeter with this activity. She usually queried possible students' responses asking other preservice teachers and how to respond students in this situations. İnci's questions showed that she focused on teacher pedagogy and students' mathematical thinking. She wondered how teachers should conduct the lesson if unexpected responses were given by indicating an alternative mathematical approach. However, she stated her ideas with general expressions without detailing about why they might think as she indicated. Her comments were descriptive and evaluative in nature. Although she considered the students' thinking and identified a few of them, she did not provide enough elaboration regarding her claims. Thus, her noticing here was considered as Level 2.

While preservice teachers were endeavoring to decide how to maintain the lesson, they talked on many issues such as asking the students where perimeter was used in daily life, requiring students to create different figures in a certain amount, giving known figures such as triangle, rectangle, parallelogram and thrombus or not and asking problems. Later, they decided to give some figures on a paper and want students to calculate the perimeter of them. The following is a presentation of the dialog between the preservice teachers:

İnci: In one source, there are some different geometric figures and their perimeter is asked.
Mehmet: Yes, it is very good, such a paper can be given. We can draw figures on isometric paper and ask them to write the value of each perimeter counting the unit squares along the edges.
İnci: I think, we should use isometric papers later, we can write the numeric value of the edges on them directly. Therefore, the students will find perimeter by summing all edges.
Hasan: Isn't using isometric paper much better than using plain paper?
İnci: In the first activity, we aimed to teach students that perimeter is the sum of all edge lengths of a figure. According to me, before counting units, students should sum the edges directly and we should use isometric paper later.
Semih: Will we leave the values of some edges blank?
Hasan: I think so, there should be blanks.
…

İnci: Do the students know the equality of opposing parts? Can they find the unknown edges using the known values?
…

In this quotation, İnci focused on aiding the students to engage in finding perimeter by summing the edges of different figures. She tried to choose what and how to ask in terms of the purpose of the lesson and students. She wanted to reinforce that perimeter was the sum of all edges because they aimed to express this point in a previous example. However, her comments were not detailed enough to reflect her intention. After the other members' questions, she made more explanations about her suggestion. She also attended to the use of material and she explained her idea about using isometric paper later in terms of the flow of the lesson. Besides, she considered what the students had already known and their abilities to solve the questions. However, she did not provide details about how the students might think mathematically or what their possible mistakes might be in these kinds of questions. Her noticing was considered as Level 2 because she focused both on teaching and the students' learning issues. Besides, her comments were general and she was inconsistent in elaborating her ideas.

During the planning process, İnci asked general questions about the implementation of their decisions on the lesson such as "Will we want students to write everything?", "How will we make them write this?", "Will we ask them to draw figures?" and indicated broadscale suggestions such as "We should not give exercise sheets one after another" and "Group work can be applied with 4-5 students". Here, she focused on several issues such as the implementation of the lesson plan, teacher pedagogy and student learning rather than students' mathematical thinking. At these points, she offered overall expressions instead of providing details about her ideas and used descriptive and evaluative comments in nature. Her noticing here mostly exhibited the properties of Level 1.

In the direction of the first objective preservice teachers wanted to create different polygons which had the same perimeter with the students. They first decided to use isometric paper and require students to draw on it. They looked for questions towards this aim in the books. İnci suggested giving some figures and asking what they look like. She aimed to repeat some known geometric figures such as a trapezium, rhombus, parallelogram, triangle and rectangle because the students had learned their features before. Semih hesitated about whether the students knew or did not know these figure but İnci was sure. They discussed on giving these figures and using isometric paper for a while. İnci emphasized that the important point was to require the students to draw different figures which had the same perimeters. İnci's some comments in this conversation are as in the following:

> **İnci:** We can want students to draw a figure with a perimeter of, for example, 20 cm. It does not a matter if they create a pentagon or another figure.
> **Semih:** It sounds complicated. We should indicate what we expect carefully. I think, the key point is that the perimeter must be 20 cm. In other words, the students should find the numbers whose sum is 20
> **İnci:** I think, we lost too much time here because we will still design how to tell that perimeter of a square is 4a and perimeter of a rectangle is 2a+2b. Furthermore, we should ask problems that include finding perimeter from edges vice versa finding edges from perimeter.

In terms of what İnci noticed, İnci focused on the content of a question, repetition of some mathematical concepts, time and next concepts that would be taught. She did not consider students' mathematical thinking while making suggestions about the question. She did not provide details about how students' mathematical approach might be in this example or what they would learn with it. The reason for offering this question was not clear, "Would students realize that different figures might have the same perimeter?" and "How would they learn?". She evaluated the question in terms of the purpose of the lesson and described what was expected from students without elaborating. Her comments were rather unspecific and judgmental (e.g. "Students should find the numbers whose sum is 20"). On the other hand, she attended to time and next steps so she evaluated the time spent as too much. She made suggestions about what kinds of problems should be asked but she did not give justification about her ideas. She offered them probably based on previous knowledge and experiences. However, her approach was not detailed and it was mostly based on general teaching and learning issues. Thus, her noticing predominantly had the prevalent properties of Level 1.

One point that preservice teachers focused while teaching the concept of perimeter was the students' learning perimeter formulas of a rectangle and square comprehensively. In the activity related to the perimeter of a rectangle and square, Mehmet suggested forming a straight line using cardboard with a width of 5 cm in order to represent long and short edges of a rectangle and lengths would change depending on their decisions. Later, he offered to move them to the isometric paper on the board end to end as width, length, width and length, namely a,b,a,b to show that perimeter formula is 2.(a+b). Preservice teachers liked this idea and agreed on giving the formulas like that. However, Hasan and İnci indicated the need for changing the material. Their comments on how to make the practice of this activity better are as below:

Hasan: There is a problem about the cardboard. Some parts of edges on ends will be extra while creating a rectangle. We can use some rope.
İnci: Rope is very thin so they may not see it.
Semih: They have width and they will overlap on ends when you create a rectangle first. It is big and it will not be a correct presentation.
…

They decided to follow the same way for giving the perimeter formula of a square and they talked on how to represent one edge of the figure. Their conversation continued as below:

İnci: We will do the same thing again. How many edges are there? Four. We can do it like that in a way that it is 4a from a+a+a+a.
…
Hasan: We can say "e" for the edge. "e" might represent the edge better.
Semih: I guess, they understand.
İnci: Ok. We can decide depending on the class. If they understand it, we use "a", if they do not, it remains as "e". After the formula, we can ask questions which require calculating the perimeter of a square vice versa finding one edge from the perimeter divided by four.
Semih: In addition, we should ask questions that we can use a rectangle and square together. The cooperating teacher also said that we should place one within the other. We can combine them like that.

In this excerpt, İnci focused on the material that was offered for telling the formula of a rectangle and square and evaluated the suitability of it in a broad manner (e.g. "The rope is very thin so they may not see it"). In Mehmet's suggestion, the width of cardboard which would be used to represent short and long edges of a rectangle would not enable creating a correct rectangle due to overlapping. However, she did not consider this incorrectness in terms of students' mathematical thinking, instead she addressed to the structure of materials. She did not elaborate how students' possible mathematical approach might be or what kind of confusions might arise in this situation. In addition, she again made suggestions about how to maintain the lesson and what kind of questions should be added on the lesson plan. Yet, she did not provide evidence to support her ideas and did not associate her suggestions to the students' learning and thinking. "Why was it important to ask questions that included both perimeter and area?", "How did she think they would contribute to students?" or "How would the questions serve the purpose of the lesson?". She did not elaborate these points sufficiently. Her approach was quite general and evaluative. Therefore, her comments reflected the evidence of Level 1.

During the interview that followed the end of planning, she indicated what they aimed in this process. She explained what the expected students' responses, difficulties and misconceptions were and how they considered students' mathematical thinking while planning as in the following:

> **İnci:** We thought that we should help students to comprehend the meaning of perimeter. Therefore, we allowed for measurement of perimeter with a rope and addition of the edges of figures end to end by moving them on isometric paper in order to concretize the concept. We used materials to reduce misconceptions. For example, we prepared exercise sheets including some figures and we wrote numerical values on the edges but we left some edges blank for the students to find themselves. I thought that they might not find them and sum only the numbers that were written. They may have difficulty in understanding the formula of perimeter but I do not think so. In particular, the relationship between 2(a+b) and 2a+2b may be complicated in my opinion because it is regarding further algebraic expressions. I think that they will comprehend perimeter since we followed a way from simple to complex.

In this explanation, according to İnci, students might have difficulty in understanding the meaning of perimeter and formula of perimeter. She indicated that they endeavored to concretize through some materials and activities to reduce these difficulties. She also stated that they followed a way from simple to complex referring a teaching principle. Besides, she addressed a possible mistake that the students might make. She indicated that the students might not consider the edges whose number were not written on while summing all the edges to find perimeter. Thus, she emphasized that they paid attention to ask these kind of questions to correct if the students made mistakes. Here, it is seen that she has some knowledge about students' difficulties and expected mistakes and tried to consider them while planning. However, when it was compared with her other attentions, the percentage of focus on students' mathematical thinking was smaller.

İnci indicated what she largely focused on as planning and what she noticed in this process. The following is presentation of her noticing through her comments:

> **İnci:** My primary focus was on how to teach the objective. In other words, I aimed to show the calculation of perimeter and meaning of it in the best way. Besides, I paid attention to provide a basis for the objectives of the next subject because they were related with each other. I noticed that I did not know students' some misconceptions regarding perimeter. For example, one of the members said that students counted points instead of distance while finding the length of an edge. It seemed different to me at that time and I thought about why they counted points. Moving edges did not come to my mind while telling perimeter formula of a rectangle and square. This approach attracted my attention when my friend explained it. For example, there was a problem combining a rectangle and square. We would find one edge of a square from the perimeter of a rectangle and then calculate the perimeter of the square from its edge. I think it might be difficult for the students but it was suitable for their levels and a good example.

İnci's comments showed that her focus was on teaching the lesson in accordance with the objective and by associating to the next mathematical issues. She indicated that she did not know students' some misconceptions and difficulties and had deficiencies about students' mathematical thinking. She emphasized that she noticed these points in the process and learned from the other members while sharing ideas with each other. She also stressed the importance of planning collaboratively and contribution of it to her noticing.

In terms of what İnci noticed during planning, her focus was on a range of issues such as teacher pedagogy, textbook, cooperating teacher, selection of examples, materials, similarities of activities offered, purpose of the lesson, time, repetition and implementation of lesson. In addition, she attended to how students' think mathematically in some situations and endeavored to consider it as designing the lesson. However, when the whole planning process was evaluated, her attention was on teaching and learning issues rather than students' mathematical thinking. Regarding how İnci noticed, her comments were very common in nature. She mostly used descriptive and evaluative expressions. She did not provide details to support her ideas and did not explain how students might think. When İnci's all comments in the planning phase were considered, although her comments varied between Level 1 and Level 2, her noticing predominantly had the features of Level 1.

4.1.1.2 What and how İnci notice in reflecting phase

Teaching of the first lesson plan was made by Hasan in 5/F classroom. The first lesson plan included the objective of "Being able to calculate perimeters of polygons; create different figures which have the given perimeters". Due to the aim was to focus on case of İnci here, teaching of Hasan was not presented. While Hasan was implementing the lesson plan, the other preservice teachers, cooperating teacher and researcher observed this lesson and took notes. After teaching was completed, the reflecting meeting occurred immediately at the school conference room. In the reflecting phase, preservice teachers were asked to reflect their thoughts about the lesson including; what worked well, what did not work, what the difficulties were, what they observed about the students' actions and thinking and also what should be changed about the lesson to improve it and what the rationales for the changes were.

This phase began with the comments of the cooperating teacher briefly and she reflected her thoughts on the lesson. The cooperating teacher first talked about one student who sometimes prevented the implementation of the lesson. She indicated that he was a problematic student and suggested not allowing him to block the lesson too much. She emphasized that when the students were recognized well, such situations would diminish. She said that the first activity was good but it took a long time so it should be reduced. She stated that while students were summing the edges to find perimeter in one question, they confused which edges they wrote or not. Thus, they missed some lengths to include in calculation or they added twice. At this point, she suggested determining a point on one corner, wanting students to write the edge lengths respectively until they reach this point in order not to be confused while calculating. Furthermore, she offered to draw geometric figures on isometric paper when students were required to compare perimeters of these figures. She thought that students could count the edges and they could notice that perimeters of all figures were the same but they made anticipated comments. She indicated that materials and the content of the lesson plan were appropriate in general. She also suggested asking more difficult questions towards the end of the lesson. On the other hand, in order to help with the students' confusion, she offered to use a cross instead of a point for showing multiplication and the letters of a,b instead of the letters of l,s for representing long and short edges because students got used to seeing these representations.

In accordance with discussion protocol, İnci shared her opinions about the lesson, evaluated the effectiveness of the lesson plan and made some suggestions to improve it. The following dialogs show what and how İnci noticed during the teaching phase.

> **İnci:** The first activity was implemented wrongly and they had difficulty in measuring the edges and pieces of the rope. Therefore, he lost time for cutting and the activity did not serve its purpose. It was not easy to manage the classroom either. Sometimes, the students made noise to come to the board or answer the question. However, although they were willing to participate the lesson, their all answers were not correct. On the other hand, they were more comfortable by marking each line between two points in order to find perimeter through counting one by one. There were individual differences among the students.

In terms of what to notice, İnci's focus was on a wide variety of issues such as implementation, time, teacher pedagogy, students' behaviors and classroom management. She mentioned how the general classroom setting was and what the students did (e.g. "The students made noise to come to the board or answer the question"). She did not attend to particular students and their mathematical thinking. She referred the practical way that students preferred to count edges. Regarding how to notice, she reflected her general impressions with a superficial approach. Her comments were mostly descriptive and evaluative in nature (e.g. ". It was not easy to manage the classroom either"). She did not elaborate what she observed. For example, she made evaluation of the students without providing details about her assessment (e.g. “There are individual differences among the students”). Therefore, her noticing exhibited characteristics of Level 1.

> **İnci:** In the first activity including measurement of a picture with a rope, Hasan did not wait for the students’ responses long enough. He should have made students more active and spoken to them, however, he quickly said sentences like “we found perimeter here” and “we summed all edges”. He should have given more opportunities for the students to think and receive these inferences…Besides, Hasan did not usually use units. In the question which included comparing the perimeter of different rectangles, one student said that it was a square for a rectangle. Hasan stated that a square was a particular type of rectangle but his explanation was irrelevant. He might have said that it was not a square because the lengths of all edges were not equal… Students had difficulty in finding the lengths of edges. They could say that opposing edges were the same in a rectangle but they missed out finding and adding the unknown edges to find perimeter. They substantially summed the known edges as calculating the perimeter.

In terms of what to notice, İnci focused primarily on teacher’s pedagogy, students’ mistakes and difficulties (e.g. “Hasan did not wait for the students’ responses long enough” and “They generally summed the known edges as calculating the perimeter”). She attended to what he should have done differently. Besides, she referred to one student’s response, however, she focused on the teacher’s approach here rather than the student’s mathematical thinking. Regarding how to notice, İnci offered what she observed in a broad scale. For example, she did not elaborate on which points students had difficulties in, why they could not find the lengths of the edges or why they did not consider the blank edges. She predominantly made descriptive and evaluative comments. She described what the students said and what the teacher did and evaluated teachers’ teaching approach (e.g. “He lost time here, I think” and “They could say that opposing edges were the same in a rectangle”). She gave advices on what should have been done differently. She endeavored to provide justification about some events but these comments were not sufficient enough. She sometimes made references to support her analyses but she chiefly maintained a broad approach. Therefore, she was inconsistent in offering details. Her noticing in this quotation was considered as evidence of Level 2.

> **İnci:** Hasan should not have used isometric paper for the questions in the exercise sheet while solving them. He made a mistake and lost time here, I think… There was a problem which included finding perimeter of a figure, calculating the fence for rounding it three times and finding the amount of money for enclosing with fence. Hasan lost too much time here therefore he could not ask the next questions. He indicated how to find money verbally. It would have been better if the students were wanted to write the problems and their solutions in their notebooks. Some questions should have been more difficult… The students could not draw the figure on the isometric paper in their notebooks. It made me surprised because isometric paper and their notebook were similar and I expected them to have no difficulty here.

In this excerpt, İnci attended to the mistakes of the teacher and students, time, pedagogical issues and teaching. She focused on mathematically insignificant aspects of the lesson and the whole class instead of particular students by referring them *as "they"* and "*students".* She made some suggestions about the implementation and the content of lesson but she did it in general manners. For example, why she offered not to indicate problems verbally or why the questions should have been more difficult were not clear. She presented few or no details and justifications about her ideas. Her comments included the description of what she observed or evaluation of the teacher and students' approach (e.g. "It made me surprised" and "He indicated how to find money verbally"). She also made judgmental comments with no evidence (e.g. "I expected them to have no difficulty here"). Due to the fact that she oversimplified the complexity of the classroom, her noticing here had prevalent properties of Level 1.

> **İnci:** In contrast to what the cooperating teacher said, I also thought that not using isometric paper in the question aiming to show that different figures might have the same perimeter would have been better. If Hasan had drawn these figures on isometric paper, the students would have counted the units and seen that they all had the same perimeter. They would have focused to only calculate the perimeters and would not have captured the important point. However, when they learned that the perimeters of all figures were equal after their own estimations, they were surprised and it was more effective. Thus, I think the teacher should not have used isometric paper here.
> **Hasan:** Yes, I agree.
> **İnci:** I also noticed that the students counted the points instead of the distance to find the lengths of edges on isometric paper.

In terms of what to notice, İnci attended mostly to the implementation of the lesson, teacher pedagogy and students' mathematical thinking. However, she did not address particular students and continued referring to the class as a whole with words such as "*they*", "*students*" (e.g. "Students would have counted the units and seen that they all had the same perimeter"). Regarding how to notice, İnci made mostly evaluative comments with an interpretive stance. For example, she emphasized that students could have made estimations about the perimeters instead of counting the units on isometric paper so that it would have increased the effectiveness of activity. Although she endeavored to explain the students' mathematical strategies, she did it in a general way (e.g. "They would have only focus on calculating the perimeters"). She tried to provide evidence from her observation to support her analysis but she offered few details about students' mathematical thinking. This mixed approach was the characteristic of Level 2 with the focus on teaching and student learning issues, evaluative approach and insufficient elaboration.

After the reflecting phase, individual interviews were made with the preservice teachers in order to obtain richer information about their noticing and reveal the missing points that preservice teachers might have forgotten to mention. İnci's comments differently from what she shared in the reflecting phase were as below.

> **İnci:** Hasan implemented the first activity based on measurement of perimeter with the rope incorrectly. He could not show the length of the whole rope was equal to the sum of all edges. He cut the rope in accordance with the lengths of edges and wanted the students to measure them separately. Then, the students measured the edges on the picture again. They actually did the same thing. However, the rope had to be whole and it had to show the perimeter. He also lost too much time there. Moreover, one student consistently tried to block the lesson but the other students were obviously willing. Materials also drew their attention.

In this extract, İnci's initial focus was on the teacher and teacher's approach in the lesson. She referred to the implementation mistake of the teacher and what he did step by step. She also addressed one student's behavior and the class enthusiasm rather than students' mathematical thinking. Her comments were descriptive and evaluative (e.g. "He also lost too much time there"). She did not provide evidence to support her analysis and had a general impressionistic stance. She did not present details about how the mistake in the implementation might influence students' mathematical thinking, how students' reactions were or whether they understood the aim of the activity. She only addressed this situation in terms of its correctness and she did not associate it to students' learning. Due to the fact that the focus was on the issues such as behavior, participation, teaching, teacher pedagogy, her approach was general and her elaboration was little. Thus, her comments in this quotation exhibited the evidence of Level 1.

In terms of what İnci noticed during the reflecting phase, she focused on teacher pedagogy, the students' and teacher's behaviors, classroom enthusiasm, students' responses, implementation of lesson and students' mathematical thinking. In other words, she attended to a range of issues, in particular, less mathematical aspects of the lesson. Regarding how to notice, she did not make references to details to draw inferences about students' mathematical thinking. She offered general impressions about what she observed. She mainly described and evaluated what she noticed using little or no interpretive stance. Hence, although she made comments which were considered as Level 1 and Level 2, her noticing in the reflecting phase had further the features of Level 1.

4.1.1.3 What and how İnci notice in re-teaching phase

The findings of this episode obtained from the implementation of the first lesson plan included the objective of "Being able to calculate perimeters of polygons; create different figures which have the given perimeters". The revision of the lesson taught by İnci in 5/G classroom. The findings from İnci's lesson were presented and her noticing during the teaching phase was revealed.

During the planning phase, the preservice teachers discussed on how to start the lesson and they aimed to make students understand that perimeter was an activity of measurement. Therefore, they decided to ask students to round a picture in the form of a rectangle with a rope and measure the length of the whole rope with ruler. Later, they thought to require the students to measure the long and short edges separately and sum all edges. Therefore, they aimed for the students to see that the results of both ways were equal and to feel the measurement aspect of perimeter rather than formula. The dialog below describes how İnci conducted the activity and what she did to help the students develop the sense of meaning of perimeter:

> **İnci:** I have a picture and I want to frame it. However, I have to find how much rope is needed. Can you help me? According to you, how many cms of rope is needed approximately?
> **S1:** One edge is 10 cm.
> **İnci:** How much rope in total?
> **S1:** 70
> **İnci:** 70? Another.
> **S2:** 50
> **S3:** 60
> **İnci:** 60 what? Cm. Is there anyone who has another prediction?
> **S4:** 36
> **İnci:** Another. (She writes numbers on the board).
> **S5:** 80 cm
> **İnci:** Now, what should I do in order to find how much rope is needed?
> **S6:** We will measure here (Student shows the length) and then here (Student shows the width) and multiply both of them.
> **İnci:** Can someone help me? (One student comes to the board). Does everybody see?
> **Students:** Yes.
> **İnci:** Round the picture with the rope. I will cut the rope when you round it completely. Hold one end of the rope and I will hold the other. Measure the length of the rope.

Figure 5 The activity regarding measurement of perimeter

S7: 158.
İnci: Write it on the board. We could not open and hold the rope accurately. Now, can another student help me to measure the edges of the picture separately? (Another student comes). Measure the edges. How long?
S8: 30.
İnci: Let's write it here. Measure the other edge.
S8: 50.
İnci: Write it. Find the other edges.

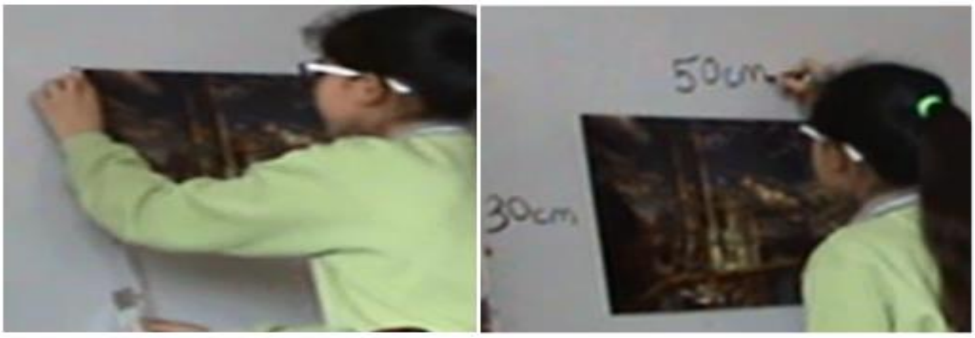

Figure 6 Second part of the activity performed by another student

S8: Is not the other edge 30?
İnci: Ok. Here is 30.
S8: There is no need to measure the other long edge. It is 50.
İnci: Ok. Sum all of them. How much is the total?
S8: 160.
İnci: It is closer to the value that we found at first, isn't it?
S9: 2 more.
İnci: It resulted from the structure of the rope and holding it wrongly. This result had to be 160 instead of 158. If you think both results as 160, they are the same, aren't they? What did we calculate here?
S9: Area of it.

İnci: Area. Another answer?
S10: Surroundings.
İnci: Another.
S11: Perimeter.
İnci: Yes, we found the perimeter of it. Your predictions about the perimeter of the picture are not close to the real result, are they?
S12: Yes, we made smaller predictions.

İnci started the lesson trying to draw students' attention with a question. She expected the students to estimate the rope was needed for the whole picture, namely, perimeter of it at first. After she learned students' predictions, she asked them how to find the amount of rope needed. One student expressed how to find area, as multiplication of length by width, instead of perimeter. However, İnci ignored the wrong answer and focused on doing what was in her mind. Here, student S6 confused area and perimeter with each other but İnci preferred not to attend this gap. She did not respond the student in a way that could reveal why he thought mathematically in this way. She could not know what to say to the students and tried to lead them to perimeter. She indicated what to do when the answers that she wanted were not given. She helped the students while rounding the picture with the rope and measuring it. However, an error occurred here because of not rounding or holding the rope accurately. Indeed, the perimeter of the picture was 160 cm so the length of the rope had to be 160 cm. However, it was found as 158 cm because of the error in measurement. When the result was found differently from what was expected, İnci made an explanation about this situation in order not to cause confusion in students (e.g. "We could not open and hold the rope accurately"). In the second part of this activity, İnci wanted another student to measure the edges with a ruler separately. Here, the student saw that opposing edges were equal and she wrote them directly without measuring again. This student summed all edges and found 160 cm. The important point was to show students that the measurement around a figure was the same with the sum of all edges, namely, perimeter. In other words, they aimed to make students understand the relationship between perimeter and measurement. However, İnci wanted the students to ignore the error and think that they were equal because these values did not come equal in this activity as planned. Therefore, the activity could not reach its purpose completely. When İnci asked the students what was found, one student said area again. İnci did not ask why he gave this answer and focused on the other students. When she got the correct answers, she compared the initial predictions of the students with the real measurement.

It can be argued that İnci's noticing in this dialog was Level 1 because she was not able to attend to students' mathematical thinking and promote them to elaborate their mathematical approach. She did not listen to the students long enough and could not respond them in accordance with their answers. Although she noticed some wrong answers, she did not try to learn the reasons of them. Therefore, she missed the opportunity to correct this confusion. For example, she heard that one student indicated the concept of area instead of perimeter, however, she did not query why he thought that area was found. She reacted to the wrong answer by asking the question such as “Another answer”. Her approach was quite common and she did not support the students to elaborate their reasoning. She indicated what should be done without allowing them to understand the purpose. She mainly focused on what she had in her mind instead of students’ mathematical thinking and understanding. Thus, her noticing can be characterized as Level 1.

After the initial activity, İnci continued the lesson with the next activity which aimed to reinforce comprehension of the meaning of perimeter and show the relationship between number of edges and perimeter. In other words, they wanted to make students feel that perimeter was the sum of all edge lengths in a figure. For example, there are three edges in a triangle, its perimeter equals to the sum of three edges. Similarly, perimeter is the sum of four edges for a quadrilateral. They aimed to show that perimeter was found by summing all of them depending on how many edges there are. In the planning phase, they decided to draw one triangle, rectangle and pentagon on the board and want students to complete a table which included writing edge lengths and finding the perimeter of each figure. The following dialog describes the interaction between İnci and the students:

> **İnci:** (She draws the figures and a table on the board). Let’s start. Who wants to complete the table for a triangle (One student comes). How many edges are there in the first figure? Write the length of the first edge.

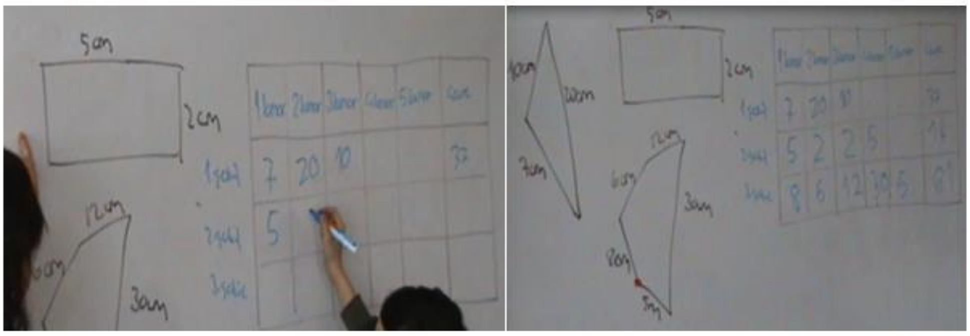

Figure 7 The question regarding the meaning of perimeter

S13: 7.
İnci: Ok. The second edge?
S13: 20.
İnci: The third edge?
S13: 10.
İnci: What did you find in total?
S13: 37.
İnci: Ok. Write it under under the topic of perimeter in the table. (Another student comes). What is the first edge for the rectangle?
S14: 5
İnci: The second?
S14: 2
İnci: The others?
S14: 2 and 5.
İnci: Ok. What did you find in total?
S14: 14.
İnci: Write it in the table. (Another student comes). Start here. What are the edges?
S15: 8, 6, 12, 30, 5.
İnci: What is that in total?
S15: 61.
İnci: Ok. Did you write it in your notebooks? According to you, is there a relationship between the perimeter of the figures and their edges?
S16: I think, there is.
İnci: What is it?
S16: One edge is 10 cm, the other is 7 cm and the other edge is 20 cm. The edges help us find the perimeter.
İnci: How do they help? What do we do with these edges?
S16: we sum them.
İnci: Yes, we find perimeter by summing all edges, don't we? Let's write the definition of perimeter… What did we do to find the perimeter of a triangle?
Students: We summed 3 edges.
İnci: What did we do if the figure had 4 edges?
Students: We summed 4 edges.
İnci: And 5 edges?
Students: We summed 5 edges.
İnci: If it had 10 edges, what would we do?
Students: We would sum 10 edges.
İnci: Yes, it is good. We would sum all the edges. What can we say about perimeter and edges?
Students: No matter how many edges there are, we sum all of them.

İnci drew a triangle, rectangle and pentagon and wanted the students to write each edge of the figures and perimeters of them. The students could complete the table easily. She leaded the students to give the answers which were needed for completing the table and solving the question instead of telling them directly. She attended to what they said and went step by step in the direction of their answers. For the triangle, student S13 wrote three edges that he saw and found the perimeter by adding in mind. For the rectangle, student S14 wrote edges utilizing the knowledge of equality of opposing edges and summed all of them. İnci guided the students with overall questions like “What is the next edge?” and “What did you find in total?”. For pentagon, İnci paid attention to put a starting point and want the students to write the edges respectively in order not to miss any. In the first lesson implementation, the students confused which edge they wrote or not as a result they found the perimeter wrong. Hence, the cooperating teacher suggested drawing a starting point in order to facilitate the students’ following the edges. It was seen that İnci considered this suggestion in teaching. Student S15 wrote edges and added them on the board.

After perimeter was found for each figure, İnci tried to draw students’ attention to the relationship between edges and perimeter. However, student S16 did not give the answer corresponding to the question and gave a general answer. He probably could not understand what İnci intended to ask. İnci wanted the student to elaborate his expressions asking questions such as “How do they help?” and “What do we do with the edges?”. Student S16 could indicate that they summed all edges as in İnci’s guidance and she accepted it as an indicator of understanding. However, it cannot be sufficient enough to think that they captured the main point here. Later, most probably, she noticed this deficiency and decided to make students feel it in terms of the number of edges. She asked what happened when edge numbers change as 3, 4, 5 or 10 step by step. After this query, the students reached what preservice teachers aimed in planning and they could indicate that no matter how many edges there are, they added them to find perimeter.

İnci mostly maintained the flow that she had in her mind and guided the students to get the answers that she wanted to hear. She focused on getting the students to say expressions such as “sum of all edges” or “how many edges there are”. Although she endeavored to attend the answers of the students and responded them in accordance with their answers, she was inconsistent in revealing their mathematical thinking. She sometimes wanted the students to give more details about their mathematical approach through questions but she sometimes did not query their reasoning. İnci listened to the students and attended to their answers in both an evaluative and interpretive way. She evaluated whether the students’ answers and explanations were correct and enough. Besides, she tried to interpret the students’ thinking to figure out what they understood. She asked basic questions step by step to sustain a clear solution process and had a general approach. She could not focus on individual responses while questioning the relationship between perimeter and number of edges. Therefore, she could not notice what the mistakes or confusions were and decided on the students' understanding considering the whole class. Hence, her noticing exhibited the features of Level 2.

After this activity, İnci gave an exercise paper, which they changed after the first implementation and added a few difficult questions in the direction of the cooperating teacher's suggestion, to all the students. In the planning phase, the preservice teachers thought that they should reinforce perimeter by asking questions and allowing students to work on them after telling what perimeter was and how it was found. Thus, they prepared an exercise paper and decided to wait for the students to engage in solving questions. The following dialog describes the interactions in the classroom:

İnci: Let's solve the question respectively on the board. Who wants to come for the first question? (One student comes). What will we do to find perimeter?

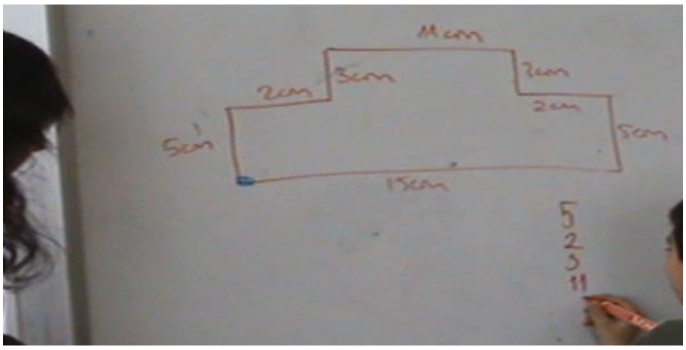

Figure 8 First question related to perimeter in the exercise paper

S17: We will sum all of them.
İnci: Start from here (She marks a starting point) and write the numbers.
S17: 46.
İnci: Did everyone find it as 46?
Students: Yessss.
İnci: Come for the next question. (Another student comes).

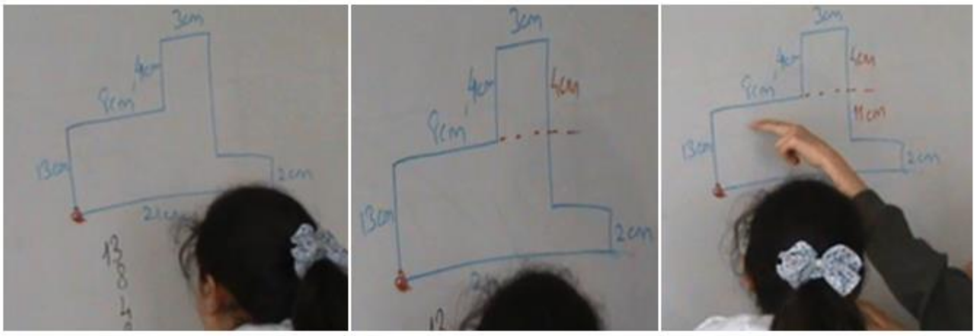

Figure 9 Second question related to perimeter in the exercise paper

S18: (She writes all the numbers that she saw and adds them but she does not find the unknown edges).
Students: These unknown edges should be found. She did not add them.
İnci: Allow your friend to solve…How can you find this edge? (She cannot give an answer). Did everyone find the unknown edges?
Students: Yes.
İnci: What should I do to find this edge? If I draw a line here, does the opposite edge become 4 as well?
S18: Yes.
İnci: How can we find the remaining part of this edge? (Another student gives an answer).
S19: We will subtract 2 from 13.
İnci: What is it?
S19: 11. We will sum 11 and 4. The unknown edge becomes 15. (It is the unknown edge in the vertical).
İnci: Ok. Good. How can we find the other unknown edge?
S20: We will sum these two edges (He points 8 and 3 in top edges) and subtract from 21.
İnci: What does it become?
S20: 8 plus 4 plus 3..
İnci: There must not be 4.
S20: Ok. Sorry.The total is 11 from 8 plus 3. Now, If we subtract 11 from 21, this unknown edge becomes 10. (It is the unknown edge in the horizontal).
İnci: Ok. What is the perimeter of the figure now?
Students: 76.
İnci: Did everyone find it as 76?
Students: Yes.

In the first example, the student found the answer easily. Here, İnci paid attention to draw a starting point for the student not to skip any edge and ask him what he did. In the next question, there were unknown edges in the vertical and horizontal. Student S18 did not consider these edges while calculating the perimeter of the figure. She only summed the numbers that she saw on the edges. The other students in the classroom noticed her mistake and indicated the need for finding the unknown edges firstly. İnci waited for S18 to find the edges but she could not give any answers. At this point, İnci preferred to give a clue about the solution path and showed how to find a part of the edge. She expected the student to find the other part of the same edge in a similar way but she could not continue with the solution. İnci asked the other students from the class, one of the students S19 saw that the opposite of this part was 13 so he offered what should be done correctly. When İnci got the correct answer, she did not ask why he followed a way like this and focused on the other unknown edge. Similarly, another student told what he did for finding this edge but he did not elaborate why he made these operations. Students described what they did mathematically rather than why they followed these steps. After all the edges were found, İnci wanted the students to calculate the perimeter and the questions were solved with the help of a few students.

İnci's noticing here was considered as Level 1 because she was not able to focus on the students' mathematical thinking and promote them to elaborate the reasons behind their mathematical approach. She mostly evaluated whether the students' statements were correct mathematically. İnci did not ask the students why they followed this kind of strategy and did not try to learn that there were students who thought differently. She missed the opportunity to determine mistakes or confusions if there existed any. İnci asked the questions towards description such as "What is it?" and "What is the perimeter?" rather than questions towards interpretation about why they did it like this and how they decided to solve in this way. İnci followed a way which enabled to reach the solution of the question immediately and considered the correct answers. Her approach was very broad and she did not support the students to elaborate their reasoning. She mainly focused on what she had in her mind instead of students' mathematical thinking and understanding. Thus, her noticing can be characterized as Level 1.

İnci: Who wants to come for the next question?
S21: I could not solve this question.
İnci: Ok. Come and solve together.
S21: (Student writes 4 on some unknown edges. Then, he subtracts 8 from 21 and writes 13 on the top edge).

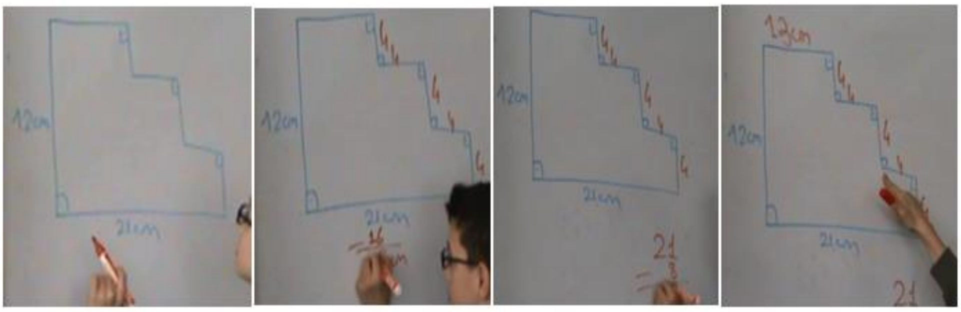

Figure 10 Third question related to the perimeter in the exercise paper

İnci: Why did you equalize these edges to 4?
S21: Because there are right angles.
İnci: When we see right angles, do we write the same numbers?... Ok. All of these edges cannot be the same. Thus, we cannot write 4. If I move this edge to the side, is it the same? (The second edge in the vertical position on the right side and red arrow shows it).
Students: Yes.
İnci: If I move the other edge in a similar way, what is the total of all these edges? (She refers to the red dotted line).
Student: 12 cm.
İnci: Now, we will find the sum of unknown edges in the horizontal similarly. If I move these two edges up without changing, what is the length of this dotted line? (She refers to the yellow dotted line).
Students: 21 cm.

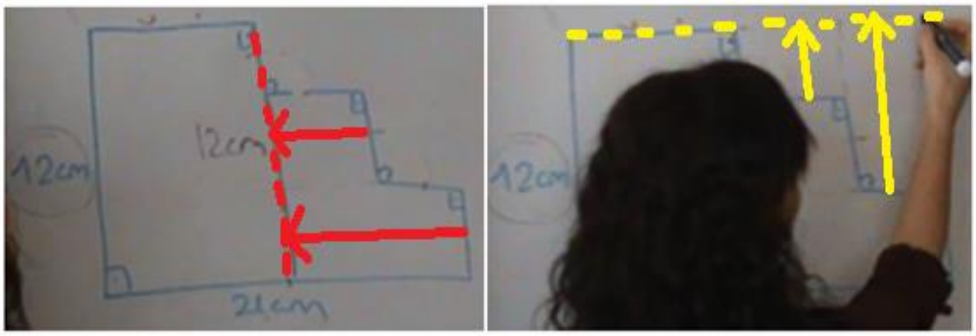

Figure 11 Fourth question related to perimeter in the exercise paper

İnci: Ok. The sum of unknown edges in the horizontal became 21. What is the perimeter of this figure now?
Students: 66.
İnci: Did you understand how we found the sum of unknown edges?
Students: Yes.
İnci: Do you want me to repeat.

Students: No.

This question seemed different to the students and they had difficulty in solving it. When student S21 came to solve the question, his misconception about the concepts of right angles and equality arose. Although there was no equality sign, he considered all three pieces in vertical on the right side as equal. Reasoning in a similar way in the previous question, the opposing edge was 12 cm and he probably divided it by 3 because he thought the pieces were equal. Therefore, he found each edge as 4. Later, although there was no information or sign about equality, he considered the edges in the horizontal as equal to these edges and wrote 4 on them as well. It was noticed that it was possible in terms of the students to focus on only the image without considering the necessary conditions. He was likely to perceive these edges as equal due to optical illusion. His approach was not based on mathematical logic. İnci queried why he wrote 4 on each edge and learned the reason was the right angle. However, she did not respond the student in a way to reveal his mathematical thinking more deeply. She did not promote him to elaborate his approach with questions such as "Why did you write 4 when you saw the right angle?", "What is the meaning and function of the right angle?" or "When do we write the same numbers?". After only one answer, she preferred to explain and solve the question. Although she encountered the mistake and confusion while controlling the student's answer, she did not consider that other students might have similar or different kinds of problems and did not endeavor to learn others' mathematical thinking on these points. Instead of listening to the students, she focused on explaining how to solve the problem step by step through general questions such as "what is the sum of all these edges" and "what is the length of this dotted line?".

İnci's noticing in this dialog was accepted as Level 2 because she was inconsistent in promoting students to elaborate their mathematical thinking. She queried the reason behind students' mathematical strategy at first but she did not maintain this approach later. She focused on the solution of the question accurately rather than how students thought mathematically. She did not try to learn the other students' mathematical approach or reveal their existing mistakes or confusion. She exhibited an evaluative approach rather than an interpretive stance. She focused mostly on the correctness of the responses rather than revealing and interpreting their mathematical thinking. She tried to listen to the students but she mostly tried to present what was in her mind. Because of this mixed approach, her noticing had the prevalent features of Level 2.

İnci: Ok. From the students who did not raise hands. Come for the next question. Solve the question explaining how you thought.
S22: The shaded region is a square. Perimeter of it is 8 cm. When I divide 8 by 4, I find one edge as 2 cm. All these edges become 2. I will sum all of them. (He draws a small line on the edges by counting how many there are).

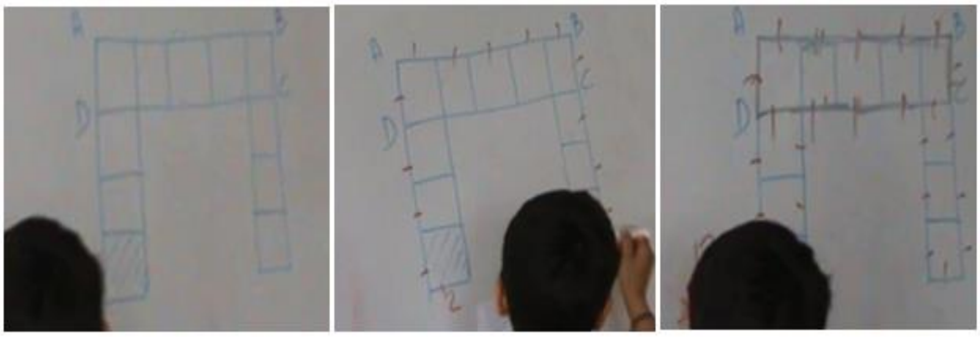

Figure 12 Fifth question related to perimeter in the exercise paper

İnci: What does the question require from us?
S22: Perimeter.
İnci: Which perimeter?
S22: Perimeter of ABCD rectangle.
İnci: Count there. I am drawing a line around to make it explicit. The perimeter of it is asked only.
S22: From 12 times 2, 24.
İnci: Ok. Did everyone understand?... Ok.

In this question, İnci wanted student S22 to solve the question explaining how he thought mathematically. Student S22 could find the length of one edge but he tried to calculate the perimeter of the whole figure, which was not required. He described what he did step by step accurately but İnci did not query why he followed these steps and she did not want him to elaborate his thinking. She did not ask why he divided 8 by 4 or why he multiplied 12 by 2. Student S22 probably thought that the perimeter of the figure was asked as in the previous questions without paying attention to the question. İnci noticed his mistake and leaded him to notice what was asked so that he could find the correct answer.

İnci listened to the student and allowed him to explain his mathematical strategy instead of indicating what to do. She also preferred not to tell him to find perimeter of ABCD and led him to notice what was asked. She wanted the student to explain how he solved the question but she did not try to make sense of his thinking by asking the reasons behind his steps. She listened to the students' comments and attended to their answers in an evaluative way rather than an interpretive stance. Although she tried to focus on what the students said, her responses revealing the students' thinking and her guidance to enable them to give details about their mathematical approach were limited. This mixed approach exhibited the characteristics of Level 2.

İnci: Who wants to come? (One student comes). How do we find perimeter?
S23: By adding.
İnci: Ok. Add the given edges. The perimeter was given. What should we do to find the unknown edges?

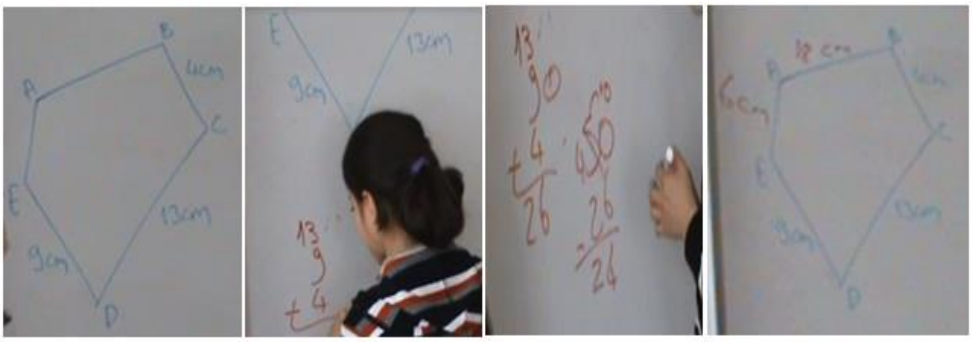

Figure 13 Sixth question related to perimeter in the exercise paper

S23: We need to subtract the known edges from perimeter. Sum of the given edges are 26 from 9 plus 13 plus 4. When we subtract 26 from 50, the unknown edges come 24.
İnci: Ok. The question indicates that length of AB is 3 times the length of AE. Then, how can we find these edges?
S23: (The student cannot answer).
Students: We will multiply 24 by 3.
Students: We will divide 24 by 4.
İnci: Ok. Divide by 4. What is AE?
S23: 6 cm. The other is 18 cm.

It was aimed to reinforce the concept of perimeter through differentiating the question type. The students were expected to find edge lengths utilizing perimeter and the relationship between these edges. İnci asked some questions to get the answers that she wanted to hear such as "How do we find perimeter" and "What should we do to find the unknown edges?". The student S23 could find the sum of two edges but she could not find them separately using the relationship. At this point, İnci did not endeavor to learn what she would say and think mathematically. İnci focused on the answers of the class rather than promoting the student to reveal her approach. However, some students gave wrong answers and offered to multiply 24 by 3. İnci ignored this answer and she did not respond these students in a way that could elaborate why they thought like that. When she heard the correct answer, she indicated what student S23 should do and the problem was solved without explaining the mathematical strategy.

İnci's noticing here was accepted as Level 1 because she was not able to attend to students' reasoning and encourage them to elaborate their mathematical approach. She did not listen to the students well enough and could not respond them in accordance with their answers. She did not react to the wrong answers or make sense of students' mathematical thinking. Her approach was so general and descriptive. She restated the question and described what to do. She also evaluated the comments in terms of their correctness. Therefore, her noticing predominantly was in Level 1.

After solving the questions in the exercise paper, İnci maintained the lesson with an activity which aimed to show the logic behind perimeter formula of both a rectangle and square. In the planning phase, the preservice teachers thought that the students should make sense of what the meaning of 2x(a+b), 2a+2b and 4a was. They did not want to give the formula directly and aimed to concretize the process by visualizing. They believed that if the students understood how to find perimeter without memorizing formulas, they could internalize this concept and confusion about perimeter and area would decrease. In this direction, they decided to move the edges end to end to the side and show how formulas were formed. The following dialog describes the interaction between İnci and students:

> **İnci:** This is isometric paper. You can consider it as your notebooks. Each distance between two points is the same. I drew a rectangle and named the length as "a" and the width as "b". I am moving the length to the side and then I put the other length at the end of it. Later, I moved short edges and added end to end. It is respectively a, a, b, b. What are we doing while finding perimeter?
> **Students:** Summing all edges.
> **İnci:** Ok. What is perimeter?
> **Students:** a+a+b+b.
> **İnci:** There are 2a and 2b, aren't there? What can we say for the perimeter of a rectangle?
> **Students:** 2a+2b.
> **İnci:** Ok. Now, I am moving one long edge and then one short edge respectively. Thus, it becomes a, b, a, b. Here, there are two pieces of a+ b, aren't there? Can we say 2x(a+b) for perimeter?
> **Students:** Yes.

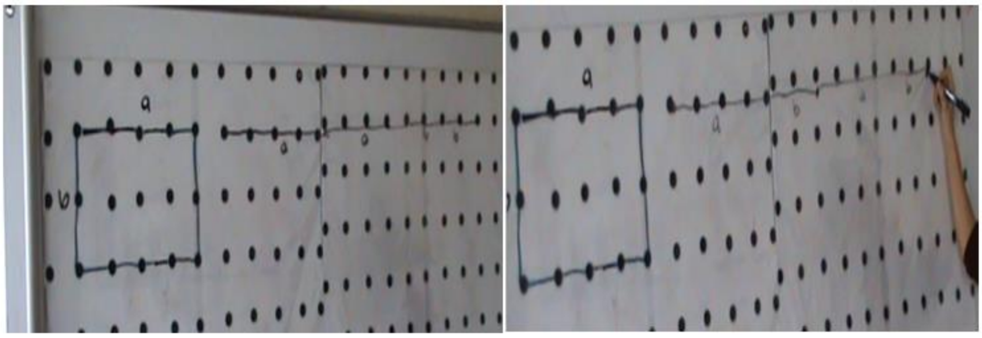

Figure 14 The activity to teach perimeter formula of a rectangle and square

İnci: Ok. I drew a square and named one edge as “a”. Now, I am moving the edges to the side and adding them end to end. How many “a”s are there?
Students: 4.
İnci: What can we do to find perimeter?
Students: Multiply one edge by 4.
İnci: What is the perimeter of a square?
Students: 4a.

It is seen that İnci started the lesson with an introduction of the material and described the function of it giving the example of notebook to facilitate understanding. She tried to guide the students to give the answers which were needed for the lesson. However, her comments were directing the students to say the same things with her rather than making them reason on the logic of the activity (e.g. “Here, there are two pieces of a+ b, aren’t there? Can we say 2x(a+b) for perimeter?”). She attended to what the students said and went step by step in the direction of their answers. She wanted to show formula presentations of a rectangle from simple to more complex a+a+b+b, 2a+2b and 2x(a+b). She followed the same process for the formula of a square. In planning, preservice teachers aimed to help the students reach the formulas by themselves through these activities. They also wanted to show the equality of rectangle formulas and make students feel the measurement aspect of perimeter. However, she allowed the students to state the formula rarely and she indicated them by herself in most cases. She did not focus on the purposes of the activity completely. Her approach was general and she interacted with the whole class. She interpreted the students’ answers as indicators of their understanding of what she intended. She felt that the activity went well and achieved the goal.

İnci’s noticing during this activity was classified as Level 1 because although she endeavored to attend the students’ reactions, she did not respond the students in a way that could reveal their mathematical thinking and she did not query the reasons behind the students’ answers. She focused on the whole class rather than specific students. Although she listened to the ideas of the students, she did not try to make sense of their mathematical approach. She mostly conserved the flow that she had in her mind and guided the students to get the answers that she wanted to hear. She maintained a pattern of questioning which provided to help students notice what was intended through a general approach instead of getting more details about their thinking and learning. She had a descriptive approach and restated only what was on the board. Hence, her noticing mostly exhibited the properties of Level 1.

İnci asked questions that required students' using formulas to solve them. Later, İnci continued the lesson with a question which was based on estimation of perimeters of different figures. One of the aims in this lesson was to show students that there might be different figures which had the same perimeter. In the planning phase, preservice teachers decided to draw different rectangles whose edge lengths were different from each other without writing their values. They agreed to learn what the students would say about their perimeters at first. After writing the values of the edges, they expected the students to calculate the perimeter of each figure and notice all of them were equal. They decided not to give the numbers at the beginning to increase the effect of it on the students. The following dialog describes the interaction between İnci and the students:

İnci: According to you, the perimeter of which figure is bigger than the others?

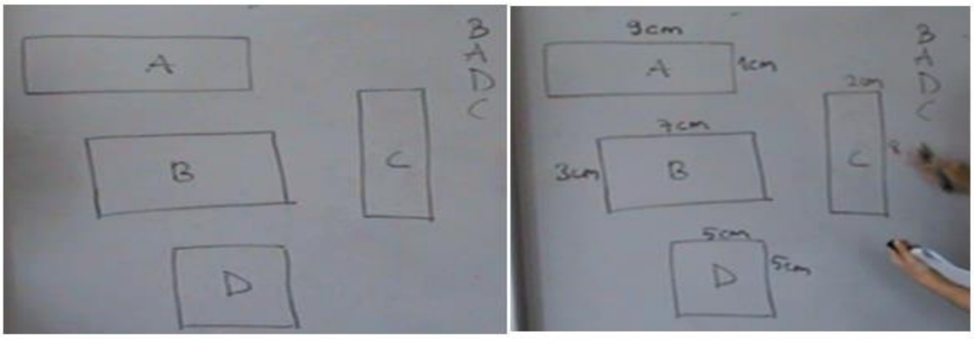

Figure 15 The question regarding different figures with the same perimeter

Students: A.
Students: B.
Students: C.
Students: D.
İnci: Why did you say B?
S24: Because it is thicker.
İnci: Why did you say A?
S25: It is longer.
İnci: Why did you say C?
S26: It is in the end.

İnci: I am writing the values of edges. Calculate each of their perimeter.
Students: All of them are equal.
İnci: Yes. All are equal. Then, different figures can have the same perimeter, cannot they?
Students: Yes.
İnci: How did you find perimeters?
Students: By summing.
İnci: For A?
S27: I summed long and short edge and then multiplied by 2.
İnci: For B?
Students: Alike.
İnci: For D?
S28: Multiplying one edge by 4.
İnci: What can we say for perimeters?
Students: 20
Students: Equal.
İnci: Are these figures the same?
Students: No.
İnci: But perimeters are the same.
Students: Yes.

The above dialog showed that this activity was implemented suitably in accordance with the purpose and created the expected effect. While students were estimating the perimeters of the figures, they gave various answers by focusing on only the images of the figures without considering the edge lengths.. The preservice teachers thought that there might be students who could notice the deficiency of the concept of edge length in calculation of perimeter by giving answers such as "We need to know the edge lengths to find perimeter", however, there were no students who made comments towards these points. When the students were asked to explain the reasons behind their answers, it was also seen that they gave expected answers such as "It is longer" and "because it is thicker". It was argued that the preservice teachers made correct predictions about the students' mathematical thinking. After all edges were written, students noticed that perimeter of each figure was equal to the others by calculating numerically and they were surprised. They understood that their previous answers were not correct (e.g. "All of them are equal"). İnci noticed that the students comprehended the relationship between perimeter and edge lengths. She wanted the students to tell how they found perimeters and what they noticed again and again. She endeavored to show them that different figures could have the same perimeter.

İnci's noticing in this activity was accepted as Level 3 because she did not only listen to the students' answers about perimeter of the figures, but also questioned them in a way that helped to reveal their mathematical thinking. She guided them to give the answers which were needed for the lesson instead of telling them directly. When she heard the incorrect answers, she wanted more details about the reasons behind them. She focused on specific students and their mathematical thinking rather than the whole class. She promoted the students to make more explanations about their reasoning through questions such as “Why did you say A?”, “How did you find perimeters?” and “What can we say for perimeters?”. She attended to the students’ ideas and tried to make them catch that different figures could have the same perimeter and feel that interpretation about perimeter without considering edge lengths might not be correct. She listened to the students’ comments and attended to their answers in an interpretive manner because she tried to make sense of their mathematical approach to comprehend what the students understood. She accepted their correct answers as indicators of their understanding. However, she did not accept them immediately, she continued to query for each figure. Because she focused on the students’ mathematical thinking and guided them to give details about their mathematical approach and she had further an interpretive approach, her noticing was considered as Level 3.

İnci went on asking questions to reinforce what students learn about perimeter. The next question included three step. The interaction in the classroom was as following:

İnci: If the distance between two points is 1 m, how much fence was needed to enclose a field similar to this figure for one time?

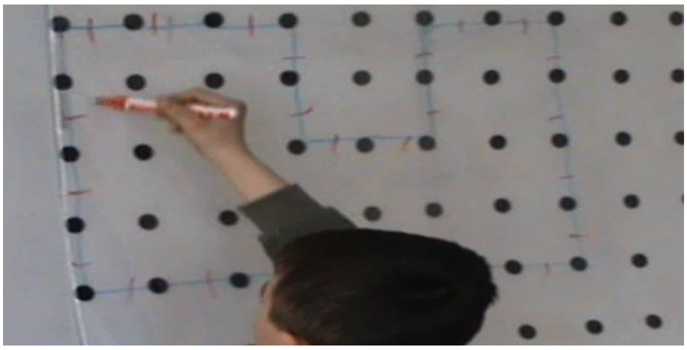

Figure 16 The question related to calculation of perimeter

S29: 26. (He counts distances).
İnci: Ok. How much fence was needed to enclose the field for three times?
S30: We enclosed it once. Therefore, we need to round it twice more.
İnci: If we enclose it for three times, which number should be used to multiply?
S30: 2.
Students: 3.
İnci: If we enclose it for three times, we need to multiply by three.
Students: 78.
İnci: If the cost of 1 m of fence is 12 TL, when we enclose the field four times how much money is needed? (One student comes). Solve the question.

S31: At first we found perimeter as 26. I multiply it by 4. It is 104. Then, I multiply 104 by 12 and it is 1248.

In this example, İnci paid attention to call three different students on the board. Perimeter was not asked directly as in the previous problems but students were expected to understand what was asked in the different contexts. In the first step, student S29 could find the perimeter of the figure drawing a line between each two points and counting these lines one by one. The whole class could understand that they needed to find perimeter and solve it easily. In the second step, whereas perimeter was needed to be multiplied by 3 in accordance with the question asked, student S30 thought that perimeter was found once in the first step and there was need for two more times in order to find the perimeter for three times. She should have multiplied perimeter by 3 directly or she should have multiplied perimeter by 2 and added it on the first result. However, she only multiplied perimeter by 2. Her expression and solution was not consistent. Here, there might be two situations. She might have thought correctly but found an incorrect solution or she might have reasoned incorrectly but expressed it correctly. İnci's question to make the student's mathematical thinking clear was not effective to understand her mathematical approach because student S30 might have misunderstood İnci's question or might not have expressed her ideas completely. İnci could have given more opportunities for the student to elaborate on her thinking. However, she preferred to indicate what to do and wanted to learn the answer from the whole class. For the last step, İnci wanted student S31 to explain his mathematical strategy and listened to his solution step by step but she did not try to learn why the student made these mathematical operations. When she saw his solution was correct, she did not need to query his mathematical approach.

İnci did not attend to students' ideas and respond them in a way that could promote them to reveal their thinking. She did not query the reasons behind their strategies. She could not give the students opportunities to present details about their mathematical approaches. She ignored the wrong answer and continued solving the question focusing on the correct response. She tried to correct the mistake indicating what to do directly rather than promoting the student to reason on it. She mostly followed the flow that she had in her mind and guided the students to get the correct solution. Her approach was general and evaluative. For example, she evaluated the explanation of student S31 as good and decided not to say more on it. Thus, her noticing here mainly exhibited the properties of Level 1.

İnci asked another question which combined perimeter of a square, triangle and trapezium. In this question, students were expected to find one edge of the square by dividing its perimeter (48 cm) by four. Then, they needed to find the unknown edge of a triangle by subtracting other two edges from its perimeter (36 cm). After all the edges were found, they were expected to calculate the perimeter of trapezium by summing all the edges. The following dialog shows the interaction between İnci and students:

İnci: Let's listen to your friend.
S32: I added 48 and 9.
İnci: Why?
S32: To find the result.
İnci: What is the perimeter of the square?
S32: 48 cm.
İnci: What can we do to find one edge of it?
S32: Divide by 4. It is 12 cm.
İnci: What is the perimeter of the triangle?
S32: 36.
İnci: We sum 12 and 9 and subtract from 36 to find the other edge of the triangle.
S32: 15 cm.
İnci: Now, what should we do to find the perimeter of the trapezium?
S32: Add the edges.
İnci: Which edges?
S32: Four edges, 12, 12, 15 and 21.

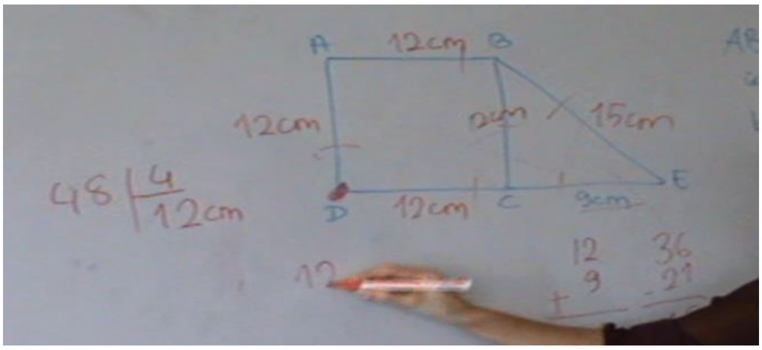

Figure 17 The question including combination of different figures

Here, the student who came to the board had difficulty in solving this question. At first, the student S32 offered to sum the perimeter of the square, 48cm, and one edge of the triangle, 9 cm, to find the perimeter of the figure. However, he missed that one edge of the square was on the inside of the figure and could not be used to calculate the perimeter of the trapezium. Besides, there was one unknown edge of the tringle that needed to be found in order to calculate the perimeter of trapezium. When İnci noticed these points, she preferred to solve the problem by herself through general leading questions such as "What is the perimeter of the square?" and "What can we do to find one edge of it?". The aim of these questions was to show clearly what she did step by step rather than query the student's mathematical thinking.

She allowed student S32 to make mathematical operations but did not promote him to reason on the question. She did not listen to the students long enough and could not respond them in accordance with their answers.

İnci did not present opportunities for the students to discuss on the important mathematical points and could not lead them to make more explanations. She mostly maintained the flow that she had in her mind and guided the students to get the answers that she wanted to hear. She asked superficial questions step by step to provide a clear solution of the problem and had a general approach. She restated the visual problem verbally and used descriptive comments. Therefore, her noticing was considered as Level 1.

When wall the dialogs during teaching were considered, İnci's noticing predominantly exhibited the characteristic of Level 1. Her initial focus was on making what she intended in parallel with the lesson plan rather than students' mathematical thinking. She sometimes did not listen to the students and indicated what to do directly or she sometimes listened to the students but did not try to make sense of their mathematical approach. She chiefly considered the students as a whole and ignored the wrong answers. She could not promote the students in a way that could reveal the details of their thinking. Her approach was quite general and descriptive. She mostly focused on the correctness of students' responses and exhibited an evaluative approach as well. Although some classroom interactions were evaluated as Level 2 and Level 3, when what and how she noticed was considered as a whole, her noticing was further in Level 1.

4.1.1.4 What and how İnci notice in re-reflecting phase

After the lesson plan was implemented by Hasan in 5/F classroom in the teaching phase and preservice teachers discussed about what should be changed or paid attention for the next implementation, re-teaching of first lesson plan was made in another classroom 5/G by İnci. While İnci was implementing the lesson plan, the other preservice teachers, cooperating teacher and researcher observed this lesson and took notes. In the discussion phase which was conducted just after re-teaching, the cooperating teacher and all preservice teachers were wanted to reflect their thoughts about the lesson including; what worked well, what did not work, what the difficulties were, what they observed about the students' actions and thinking and also what should be changed about the lesson to improve it and what the rationales for the changes were.

This phase began with the comments of the cooperating teacher briefly and she reflected her thoughts on the lesson. She indicated that questions' level of difficulty was good this time and they were appropriate in terms of class' mental level. She emphasized the importance of variety in the type of questions for appealing to each student and not bothering them. She stated that students again confused perimeter and area with each other and comprehension of area was more difficult for them. When one preservice teacher asked whether the lesson served the purpose of the objective, the cooperating teacher indicated the efficiency of the lesson plan and did not offer any changes. She also gave an example from her observation to support her claim. She emphasized that some students, who were not interested in the lesson and did not raise hands in general, were willing to participate in the lesson and come to the board for solving the questions. She expressed that she liked isometric paper material because it was similar to students' notebooks and helped students to draw figures more easily. She also indicated that it was more permanent visually. In this lesson, a cross was used instead of a point in order to show multiplication and the letters of "a, b" instead of the letters of "l, s" to represent long and short edges in the direction of the cooperating teacher's previous suggestions. She emphasized that paying attention to these points reduced the students' confusions because they expect to see the subjects the way they had encountered instead of in different representations. She evaluated this lesson as sufficient and appropriate and did not suggest any changes in terms of either the lesson plan or teaching.

In accordance with discussion protocol, İnci shared her opinions about the lesson that she taught, evaluated the effectiveness of the lesson plan and made some suggestions to improve it. The following dialogs show what and how İnci noticed during the re-teaching phase as the teacher of the lesson.

> **İnci:** I tried to stick to the plan and not to skip the important points. I could not complete a few questions as I lost time while solving the questions in the exercise sheet. I thought that I gave sufficient time while waiting for the students to solve the questions but nobody raised hands for some questions. Thus, I had to wait for them. Revision of the lesson plan was good and the students could understand the subject… There were some questions they could not solve or they made mistakes. I noticed the wrong answers but I could not decide whether I should correct them immediately or later. I also had difficulty in reducing the students to silence.

In terms of what to notice, her initial focus was on her own teaching approach and pedagogy (e.g. "I tried to stick to the plan" and "I lost time while solving the questions"). She also attended the classroom management, the class's behavior and learning (e.g. "I also had difficulty in reducing the students to silence"). Regarding how to notice, İnci offered general impressions. For example, she indicated that the students could understand the subject, however, her comment was superficial. She did not make reference to how she made this inference. She mostly described what she could do or not with short sentences and used evaluative comments as well (e.g. “I could not complete a few questions” and “Revision of the lesson plan was good”). She also addressed the difficulties that she encountered as teaching. She did not focus on specific students and their mathematical thinking. She referred to the students as a whole and did not provide details to support her claims. Because she focused on a range of issues related to less mathematical aspects of the lesson and maintained a general impressionistic approach toward her analysis, her noticing here was considered as evidence of Level 1.

> **İnci:** Right angle caused confusion in one question and the student considered three unknown edges as equal and divided by three. When I asked why they were equal, he explained the reason of equality with right angles. They had difficulty here. Besides, they could find the unknown edges in the vertical but I think they could not understand how to find the unknown edges in the horizontal. In addition, they had difficulty in the question where one edge was three times the other. Most of them offered to divide 24 by 3, a few students could say to divide 24 by 4 correctly. On the other hand, I thought how I should apportion 24 as 6 and 18, how I should express it. One student said it and I repeated.

In this excerpt, İnci focused on students’ mathematical thinking and understanding. She also attended her own teaching approach addressing the difficulty that she encountered. She first referred to a specific student’s response and tried to make sense of how he thought mathematically. She elaborated why the student might have given such an answer by addressing his explanation (e.g. “He explained the reason of equality with right angles). Although she focused on an individual student, she continued to refer to the whole class (e.g. “They had difficulty here”). She addressed the whole class’s mathematical understanding (e.g. “They could find the unknown edges in the vertical but I think they could not understand how to find the unknown edges in the horizontal”). However, her comments were very broad. Why she thought like this or why the students offered to divide by 3 instead of 4 were not elaborated. She did not give justifications about students’ mathematical strategies. She preferred to identify the noteworthy events but did not provide details to promote her claims. Therefore, she was inconsistent in elaborating her ideas. She sometimes offered little or no evidence to develop her analysis. Although she began to make some interpretations about students’ mathematical thinking, she did it in general ways and maintained an evaluative approach (e.g. “Most of them offered to divide 24 by 3, a few students could say to divide 24 by 4 correctly”). Because of the mixed approach, her noticing was accepted as evidence of Level 2.

> **İnci:** Material drew attention of the students and facilitated their learning. I also noticed that students in 5th grade were attentive to the image of the figures. For

> example, while drawing a rectangle, if it was similar to a square, they asked whether it was a square. Therefore, we must be careful to draw the figures correctly because they focus on this point. In general, we tried to explain that it was not a square because all edges were not equal when students did not understand. However, the figure must be drawn more specifically. Students should understand long edge is bigger than short edge when they look at it. For example, even though we wrote 2 and 3 on the edges, they perceived it as a square without considering the numerical values and inequality of edges because the lengths of edges look alike in view. Hence, according to me, realistic drawing is more persuasive.

In this quotation, İnci primarily focused on teacher pedagogy, students' mathematical thinking and understanding. She referred to the noteworthy events from her observation to explain how students think and how they seemed to interpret some concepts. She endeavored to elaborate students' incorrect mathematical approach toward the images of the figures with examples (e.g. "… they perceive it as a square without considering the numerical values and inequality of edges because the lengths of edges look alike in view"). She made some suggestions about what should be done (e.g. "we must be careful to draw figures correctly"). She focused on students' mathematical approach but continued to refer the students as a whole. On the other hand, she attended the students' behaviors and learning but in overall manners (e.g. "Material drew attention of the students and facilitated their learning"). How materials helped the students was not detailed and she did not make reference to the examples from her observation for supporting her claims. Her comments were evaluative with an interpretive stance. She tried to make evaluation through interpreting some events in terms of students' mathematical approach. For example, she assessed realistic drawing as more persuasive because students were in tendency to focus on images. In some comments, her elaboration was limited and unspecific. Thus, her noticing here was further accepted as Level 2.

> **İnci:** After the first implementation of this lesson plan, the cooperating teacher said that the letters of l and s confused the students. Therefore, we used the letters of a and b while giving the formulas as usual. However, I think there is no difference between the use of a,b and l,s. On the contrary, l and s was more meaningful because "l" was used to represent the long edge and "s" was used to represent the short edge in line with their first letter. Students got used to a and b and when they encountered with different representations such as l and s, they could not generalize and transfer knowledge. They focused on specific rather than general issues. The cooperating teacher's another suggestion was to put a starting point on the figure to prevent missing an edge or calculating twice while finding the perimeter. As she said, it worked well and reduced the operation mistakes.

In this excerpt, İnci focused predominantly on teaching and student learning issues. She maintained to address pedagogical issues and indicated her ideas about the use of letters for representation of edges with an interpretive stance. She offered simple impressions about mathematical approach of the students rather than particular students' mathematical thinking or making sense of them (e.g. "they could not generalize and transfer knowledge"). She tried to provide justifications about her claims but her approach was further general. She explained the reasons through tendency of the students without providing details (e.g. "They focus on specific rather than the general issue"). She mostly made evaluative and interpretive comments in nature. She attempted to elaborate her claims but not give enough details to support her analysis. Therefore, her noticing here had exhibited the prevalent properties of Level 2.

After the reflecting phase, individual interviews were conducted with the preservice teachers to reveal the missing points and detail their noticing. What İnci indicated differently from the reflecting phase was as in the following.

> **İnci:** As is in the first implementation, I lost time while measuring the perimeter of the picture in the first activity and I could not obtain the necessary results to show the equality of measurements similar to Hasan… In one question, it was asked to find necessary fence to enclose the given figure for three times. In the first step, the perimeter of the figure was found and the student offered to multiply perimeter by 2 instead of 3 in the second step. There was this kind of misconception. Besides, they confused perimeter and area. When I asked what we found after measurement of the edges, the answer of area was given. Similarly, when I asked how we found perimeter, one of the students said "through multiplication of edges", having area in his mind… In general, I attended what students said and whether they gave expected answers. I paid attention to implement what we planned correctly and without skipping any points.

In this excerpt, İnci's focus was on the implementation of the lesson, teaching and students' answers. She addressed her mistake in an activity but did not elaborate how it affected students' mathematical thinking or whether it caused any confusions. She only described the situation in a general manner without details. She also referred to some students' incorrect responses, however, she did not endeavor to make sense of students' mathematical thinking. She provided little or no evidence about her observations to strengthen her claims. She offered overall impressions about what occurred. Her comments were predominantly descriptive and evaluative. Due to the fact that she attended to teacher pedagogy and students' responses largely yet she did not reason on their strategies or elaborate how students might think mathematically, her noticing here was considered as evidence of Level 2.

When İnci's all comments in the re-reflecting phase after re-teaching were considered, her noticing had mainly the features of Level 2. In terms of what to notice, she mostly focused on her own pedagogy, students' mathematical thinking and understanding. She mentioned particular students and notable mathematical aspects of the lesson. However, she also continued referring to the whole class. Regarding how to notice, although she mainly evaluated what she observed, she also adopted an interpretive stance and tried to make sense of students' responses. However, she did it in broad manners. She was inconsistent in elaborating and providing details about the events to support her claims. Although she made comments which were accepted as Level 1 and Level 2, her noticing was further in Level 2.

4.1.2 Lesson study cycle 2

4.1.2.1 What and how İnci notice in planning phase

The second lesson plan was related to teaching of area and the objectives of the second cycle were "Being able to calculate the area of a rectangle; use square centimeter and square meter" and "Estimate a given area using square centimeter and square meter units". Before planning the lesson, the preservice teachers went to the practice school to get help and recommendations about the subject from the cooperating teacher. They listened to her ideas, took notes and asked the questions coming to their minds. The cooperating teacher indicated that area measurement units should be introduced and materials should be used. She also suggested explaining the relationship between perimeter and length measurement units and area and area measurement units in order to reduce the use of inappropriate units because making this connection is difficult for students. She emphasized the confusion of students about perimeter and area and she recommended presenting the difference between them well. She also signified that we should give area of a rectangle and square only at this grade. All preservice teachers met to prepare the first lesson plan at the university seminar hall bringing the various sources such as mathematics textbooks, teacher's guide book, curriculum and different mathematics education books. The following are presentations of examples based on mainly İnci's expressions from the planning meeting of lesson study cycle 2.

This lesson plan was towards teaching of area and use of measurement units. The preservice teachers thought about how to start the lesson and they tried to decide whether area or measurement units should be given at first. The interaction between the preservice teachers is as in the following:

> **Semih:** In the previous lesson, we taught perimeter and we used unt as a measurement unit generally but students know length measurement units already.
> **İnci:** But they do not use them occasionally. We should remind them and emphasize paying attention to using appropriate units. How will we start the lesson?
> **Mehmet:** At first, we can give unit conversion.
> **Semih:** We can calculate the area of a rectangle.
> **İnci:** I think we can repeat the previous lesson shortly asking what they learned before.
> **…**

Hasan: We can multiply units.
Mehmet: Meter times meter, square meter and then we can write mxm=m^2
Semih: Can't we give it directly as in the book? Do we need to show this?
İnci: Students write in their notebooks without understanding. For example, 6th grades do not use units. Thus, these students will not pay attention either. Yet, I am not sure that 5th grade students know exponential numbers. 6th grades know them but they confuse a lot yet 5th grades may not have this knowledge.

As for what to notice, İnci focused primarily on pedagogical issues and students' mathematical understanding (e.g. "I think we can repeat the previous lesson shortly" and "Students write in their notebooks without understanding"). She also addressed general habits of the students while doing mathematics. She indicated her hesitation about students' mathematical knowledge. Therefore, it showed the deficiency in the preservice teachers' knowledge and how it affected their planning. Regarding how to notice, although she identified the crucial events, she did it in a general way. Her comments were judgmental and evaluative (e.g. "They do not use them ocacsionally" and "These students will not pay attention either"). She endeavored to explain her suggestions but she was limited in elaborating her ideas. She did not focus on students' mathematical thinking instead considered their approach in a general way. Thus, her noticing was considered as evidence of Level 2.

The preservice teachers decided to start with repetition and they continued talking about how to maintain the lesson. İnci suggested teaching area through covering the interior region of the figure by benefitting from the books. Their conversation shaped on this suggestion is as it follows:

İnci: I think we can ask the students to cover the inside of a figure in order to help them understand what area is. The cooperating teacher said that the students confused perimeter and area. It is likely that they can find perimeter instead of area or vice versa. Thus, they must realize that area is the part which is covered, namely, the interior region so that they can comprehend the difference between area and perimeter better. They will understand the meaning of area. There is something similar to what I indicated in the book. It requires coloring the inside of a rectangle and finding the number of unit squares. Then, it asks the relationship between the number of unit squares and edge lengths of a rectangle. Students can notice how edge lengths were related to area. It can be good. We can proceed to area of a rectangle from covering.
Mehmet: Yes, hereby, we can show that area is the interior region but we need material.

In this excerpt, İnci's main focus was on students' mathematical thinking. She attended to their confusion and tried to explain what to do in order to reduce mistakes with the reasons. She elaborated what kind of mistakes might arise because of this confusion (e.g. "they can find perimeter instead of area or vice versa"). She provided details about how her suggestions were related to the overall goals of the lesson and what it would sustain in terms of students' mathematical understanding (e.g. "… they can comprehend the difference between area and perimeter better"). She referred to the noteworthy events and made reference to evidence about her ideas. She endeavored tried to make sense of students' mathematical thinking (e.g. "Students can notice how edge lengths are related to area"). Hence, her noticing exhibited further the prevalent properties of Level 3.

After preservice teachers agreed on İnci's suggestion, they talked about how to present this activity. Later, İnci drew the others' attention to introduce and use measurement units again. İnci insisted on making students understand why unt^2, cm^2 and m^2 are used in area. The interaction between preservice teachers is as the following:

> **İnci:** We say that mxm= m^2 or untxunt= unt^2 but students do not know that. They only use these units but they do not recognize why cm becomes cm^2 in area. We can show that when we multiply meter by meter, how many meters there are, 2, so it is square meter, m^2, in order to ensure better learning.
> **Mehmet:** It may be abstract, according to me.
> **İnci:** I think we should show them from where square meter comes. If we say these are length measurements and these are area measurements as you said, it will be too easy for us and abstract for the students indeed. We will make no explanation of them in this case and want them to learn by heart in a way. I think for example, instead of 3x5 as finding area, we can write 3unt x 5unt at first and indicate that we write how many units there are on it, so it becomes unt^2.
> **Semih:** Yes, it can be done for m and cm, consequently the transition to these units becomes easier.
> **İnci:** Yes, when we write cm x cm, how many cm are there? 2. Thus, we will show that it will be written on it and it becomes cm^2.
> **Semih:** A problem arises here. Students have not learnt the area of a rectangle yet.
> **İnci:** Therefore, I suggest that you teach the area of a rectangle at the beginning and then we can give measurement units.

In this quotation, İnci focused on the implementation of the lesson and students' mathematical knowledge. She attended to pedagogical issues such as providing justification for better learning, preventing rote learning, abstractness and the order of the concepts. She indicated what students do and know in general (e.g. "They only use these units but they do not know why cm becomes cm^2 in area"). She described her suggestions but she did not elaborate in terms of students' mathematical thinking. Although she made evaluative comments, she partly adopted an interpretive stance (e.g. "It will be too easy for us and abstract for the students indeed"). She considered students' mathematical approach but she further focused on how to give units. In other words, she mostly attended to teaching and learning issues with a broad approach. Hence, her noticing was accepted as Level 2.

The preservice teachers aimed to enable the students to obtain the formula of a rectangle and square's area by themselves instead of giving the formula directly. They focused on what kind of activities or questions should be given. The conversation between them is as it follows:

> **İnci:** For example, we can draw a rectangle in the form of 3x6 and students will count the unit squares and find the area as 18 at first. Then, we will ask them "How many unit squares are there in the first row?" 6. In the second one, 6. In the third one, 6. Later, students find the area by multiplying 3 by 6. Thus, students will realize that they found area as 18 by counting one by one and multiplying the edges as well. We will query the relationship between area and edge lengths of a figure here so that they can understand how to find area. However, is only one example enough according to you?
> **Mehmet:** No, we can give another example like a rectangle 4x5 and they can do the same thing. Later, we can give formula as axb.
> **İnci:** After, the students learn axb, we should show that untxunt=unt^2 in these examples.
> **Hasan:** Later, we can do the same for teaching the area of a square.
> **Mehmet:** We should prepare an exercise work sheet including different figures.
> **İnci:** In books, there are many figures on plotting paper and it is asked to calculate area by counting the unit squares. Will we draw half squares in the form of a triangle? Can students complete two half squares to a whole unit square? Can they think of it?
> **Hasan:** There are these kinds of questions in the book.
> **Mehmet:** No, they cannot. We should not ask.

In terms of what to notice, İnci primarily focused on students' mathematical thinking and her comments were based on it. Since she aimed to help students to reach the formula of rectangle area with the activity that she offered, she also paid attention to whether the students could make generalizations or not. She referred to students' mathematical understanding and thinking (e.g. "Students will realize that they found area as 18 by counting one by one and multiplying edges as well"). Her comments on whether the students would be able to think of obtaining one whole square from two triangles showed that she considered students' mathematical thinking while shaping the lesson plan. Regarding how to notice, she mainly made interpretive comments and provided details to draw inferences about students' understanding and to support her claims. She endeavored to make sense of how students would reach the area formulas of a rectangle and square. She elaborated how her suggestions were related to the overall goals of the lesson. She focused on mathematical aspects of the lesson, in particular, students' thinking and understanding rather than the issues such as classroom management, participant etc. She also had an interpretive stance. Her noticing had further the features of Level 3.

While the preservice teachers were talking about the activity which was related to the use of standard measurement units, Semih also offered to present another similar example using hand span. The conversation between İnci and Semih on this issue is as in the following.

> **Semih:** We can also want students to measure the length of the desk with their own hand span. There will be many different results. It will be more basic and visual for students.
> **İnci:** But, we have already used materials and done a similar thing and it is also related to measuring length rather than area.
> **Semih:** Yes, we did it for area but not for length measurement. It takes only one or two minutes.
> **İnci:** Yes, it does not take much time but I do not think that it is necessary. Besides, there may be students who find the same answers.
> **Semih:** In any case, there will be students who find different answers because they have different hand spans.

In this quotation, İnci evaluated the suggestion of another member in the group in terms of the relevance with the subject and the convenience in teaching. She thought that it was further related to length measurement units but the subject was area and same answers might cause confusions for students. She also added having performed a similar activity previously as another reason of why she believed the suggestion was unnecessary. Her approach was rather common and evaluative in nature. She did not focus on students' thinking or reason on their mathematical approach. She further paid attention to the implementation of the lesson. Thus, her noticing was considered as Level 1.

During the planning phase, İnci made comments considering students' mathematical approach such as "Do students know the concept of exponential numbers?", "Can they think of it?" and "We should show why we use square centimeter or square meter in area while using centimeter or meter in perimeter". Besides, she also made comments on how to conduct the lesson for the students to internalize the objectives well "It can be abstract in this way", "We should change the order of the activities to attain better connection between the concepts" and "We should not use many materials it might cause confusion as well". Here, she focused primarily on teaching and learning issues. She paid attention to how teacher pedagogy should be and considered students' mathematical understanding. At these points, she sometimes provided details to support her ideas and gave justifications about her suggestions but she still offered general expressions. She had an evaluative approach with an interpretive stance in nature. Her noticing mostly was considered as evidence of Level 2 in some dialogs.

After the preservice teachers completed giving the related concepts, they tried to determine the questions to reinforce area. They discussed on which questions can be asked or not. The interaction between them is as in the following.

> **İnci:** What do you think about this problem which includes finding the area of a square with a perimeter of 40 cm? It involves combination of perimeter and area so it can be good.
> **Mehmet:** Yes, I agree.
> …
> **İnci:** We can also give a rectangle and want them to subtract a square from it because I noticed that 6^{th} grade students had difficulty in solving this kind of a question. They will calculate the area of a rectangle and square separately and then subtract the area of the square.
> **Semih:** We can show it on cardboard. It can be better for visualization.
> **İnci:** I think we do not need that because we will use materials sufficiently. We should not distract students' attention.
> **Semih:** We may not draw correctly.
> **…**
> **İnci:** We can finish summarizing the lesson through questions such as "What is area?", "How do we find area of a rectangle and square?" and "What is the difference between perimeter and area?".

In terms of what to notice, İnci focused on teaching and students' mathematical understanding. She referred to students' confusions or difficulties and associated the questions with them. She also attended to pedagogical issues such as repetition, use of materials and attention. Regarding how to notice, she was inconsistent in providing details to promote her claims. She sometimes made justification about her comments (e.g. "I think we do not need because we will use materials sufficiently"). However, she sometimes did not elaborate her ideas, for example, what kind of difficulties students had was not clear. Her comments were predominantly evaluative rather than interpretive. She did not focus on students' mathematical thinking or try to make sense of them. This mixed approach was characteristic of Level 2.

During the interview that followed the end of planning, she indicated what they aimed in this process. She explained what expected students' responses, difficulties and misconceptions were and how they considered students' mathematical thinking while planning as in the following:

> **İnci:** Students use measurement units without knowing why they use them. Therefore, I think we should show where they come from. In both perimeter and area, the operations are made with edge lengths of the figures so students confuse these concepts with each other… In one activity, we wanted the students to cover the same of two rectangles with different two squares to show the need for use of standard measurement units. Students may have difficulty there. Besides, they may have confusion in proceeding from unt, cm, m to unt^2, cm^2 and m^2. We will try to show it

> in a simple way to facilitate their understanding and to repeat what they learnt. We always endeavored to consider students' mathematical thinking and possible responses while planning the lesson.

In this excerpt, according to İnci students forget to use units or cannot write appropriate units because they do not know why they should use them. Therefore, she stressed the importance of showing the difference between length and area measurement units with reasons. She also addressed the students' confusion about perimeter and area. Thus, she emphasized that they tried to facilitate understanding what these concepts meant. Besides, she thought that covering the same figure with different unit squares might cause confusion and appear different but they aimed to draw attention to the need for use of standard measurement units. They tried to cause a confusion at first and promote the students to reason on different situations presuming that learning would be more effective that way. It is seen that she considered expected students' reactions while planning the lesson.

İnci indicated what she briefly focused on as planning and what she noticed in this process. The following is presentation of her noticing through her comments:

> **İnci:** I focused on giving the meaning of area in the best way. We could say that area is multiplication of edges directly but we preferred using materials to tell concretely and facilitate understanding. We had read what kind of misconceptions and difficulties related to the subject students might have before so that we could estimate more or less. We tried to determine the activities and questions toward reducing their confusions and mistakes. On the other hand, in this process, different ideas were presented by the other members and they effected the lesson plan positively. If I had prepared alone, it would not have been as rich as the current plan.

İnci's comments showed that her focus was on teaching, learning and students' mathematical approach. Her explanations revealed that she paid attention to pedagogical issues such as using materials, making concrete, explaining the meanings of concepts and not rote learning. It is also understood that she tried to shape the lesson plan based on the students' mathematical thinking to the best of her knowledge. On the other hand, she emphasized the contribution of the other group members and the importance of sharing different ideas in improving the lesson plan.

When İnci's all comments during the planning phase were considered, her noticing was accepted as Level 2 because she predominantly focused on pedagogical issues and considered students' mathematical thinking. She generally tried to base her suggestions on students' confusions, difficulties or mistakes. She addressed the noteworthy events rather than less mathematical issues but she did in general ways. Her comments were mostly evaluative with an interpretive stance. She endeavored to provide details about her ideas but she was inconsistent. Thus, although there were comments in Level 1, Level 2 and Level 3, her noticing mostly had the features of Level 2.

4.1.2.2 What and how İnci notice in reflecting phase

Teaching of the second lesson plan was made by Semih in 5/F classroom. The second lesson plan included the objectives of “Being able to calculate the area of a rectangle; use square centimeter and square meter” and “Estimate a given area using square centimeter and square meter units”. Since the aim was to focus on İnci’s case, teaching of Semih was not given here. As Semih was implementing the lesson plan, the other preservice teachers, cooperating teacher and researcher observed this lesson and took notes. After teaching was completed, the reflecting meeting occurred immediately at school conference room. In the reflecting phase, the preservice teachers were asked to reflect their thoughts about the lesson including; what worked well, what did not work, what the difficulties were, what they observed about the students’ actions and thinking and also what should be changed about the lesson to improve it and what the rationales for the changes were.

This phase began with the comments of the cooperating teacher briefly and she reflected her thoughts on the lesson. She indicated that implementation of the lesson and teaching of it was good. She suggested asking questions which included completing half unit squares in the form of a triangle to obtain whole unit squares in the exercise sheet. She stated that students could find area counting whole unit squares one by one directly and offered us to give half unit squares in order to make the questions a little difficult and vary them. She expressed that students had foreknowledge about area measurements such as cm^2 and m^2 but they should be reminded in the entry of the lesson. She also expressed that although students encountered unit squares (unt^2) in the books, this concept confused their minds and they did not perceive it as a measurement unit. Thus, she suggested telling what cm^2 and m^2 meant firstly. She advised us to make connections with daily life and ask the place of their use in life so as to concrete these concepts. On the other hand, the cooperating teacher emphasized that students confused perimeter and area as a result they confused length measurements and area measurements with each other as well. She drew preservice teachers’ attention to show the difference between these units and write the appropriate unit for each measurement at every turn in order to provide better understanding. She indicated that combining perimeter and area with questions was good because it facilitated learning the subject and reduced the confusion between them. She also emphasized that students liked covering with unit squares and it ensured the comprehension of the area concept better. She indicated that the lesson plan was progressing from easy to difficult, the order of the activities and questions were appropriate and the content of it was sufficient.

In accordance with discussion protocol, İnci shared her opinions about the lesson as an observer, evaluated the effectiveness of the lesson plan and made some suggestions to improve it. The following dialogs show what and how İnci noticed during the teaching phase.

> **İnci:** Semih started the lesson with repetition and tried to make the students active in the lesson through questions. It was good. He emphasized that the interior region was area and the length around the figure was perimeter. Students were willing to go to the board and they liked covering the interior region of the figures with unit squares. Materials drew their attention. While solving the questions in the exercise sheet, although the questions would not be drawn on the plotting paper, Semih confused and drew it on plotting paper. Then, he realized that it took time and did not continue drawing the figures on it. He also made a mistake while drawing a figure and thus, a difference occurred between what the students found on the paper and what was found on the board. A few students calculated incorrectly because they made a mistake as counting the unit squares.

İnci focused on a range of issues including teacher pedagogy, student enthusiasm, the behavior of teacher and student, implementation of the lesson and student learning (e.g. “Students were willing to go to the board” and “Semih started the lesson with repetition”). Her focus was predominantly on the class as a whole referring to “students” and “they”. She did not attend to students’ mathematical thinking and did not try to make sense of them. İnci’s comments were so general in nature. She mostly described what she noticed with short expressions and also used evaluative comments (e.g. “He realized that it took time” and “It was good”). She provided few or no details to support her claims. She mostly had an impressionistic stance and attention on various issues. Therefore, her noticing had the prevalent features of Level 1.

> **İnci:** In one question related to covering the same rectangles with two different identical squares, Semih promoted the students to reason on why the numbers of squares to cover the rectangles were different as we had decided in planning. Students gave expected answers such as “one identical square was bigger than the other”, “one square was 1x1 and the other was 4x4”. Here, I noticed that we used a big number for rectangle area because covering with 32 unit squares took time and students started to make noise. We could have used a smaller rectangle. Semih gave an example toward measurement with hand span to reinforce the reason of this difference and show the necessity of using standard measurement units, however, there were students who confused what hand span was and how measurement was done with it. He corrected this confusion.

In terms of what to notice, İnci's initial focus was on teacher pedagogy and students' mathematical approach. She made reference to evidence of the responses and confusions of the students but she continued referring to them as a whole rather than focusing on particular students (e.g. "Students gave expected answers such as "one identical square was bigger than the other", "one square was 1x1 and the other was 4x4"). However, she did that in general ways. Although she attended to student thinking, she did not make justification about it. Why students gave these kinds of responses was not detailed and she only described them. She also made suggestions about what should be done differently based on her observation. Her comments were mainly descriptive and evaluative. Because she focused on teaching and learning issues and maintained a broad approach by giving limited details, her noticing here was considered as evidence of Level 2.

> **İnci:** As planning the lesson, we thought that we should make students discover the formula of area from the relationship between column and row, and we also thought that one example was not enough and asked two questions in order to help students generalize the area formula of a rectangle and square. However, they could understand formula at the first example and Semih decided to skip the second examples for both rectangle and square feeling that it would not be necessary. I think he gave the correct decision at this point. Students gave expected answers and Semih emphasized the appropriate units each time.

In this excerpt, İnci attended to teacher pedagogy and students' mathematical understanding primarily (e.g. "Semih emphasized the appropriate units each time" and "Students could understand formula at the first example"). She did not focus on particular students' mathematical thinking and reason on their strategies. She referred to the class as a whole and offered common impressions. She predominantly used evaluative comments (e.g. "I think he gave the correct decision at this point). She provided few or no details about what she observed. She tried to make justification about the teacher's decision in teaching but she did not elaborate what kind of answers the students gave. She was inconsistent in providing evidence to strengthen her claims. Hence, her noticing further exhibited the properties of Level 2.

> **İnci:** In one question, the students were asked to find the area of a square with one edge of 7 cm. Some students said 28 as the answer because they found perimeter instead of area. They confused area and perimeter with each other. They calculated perimeter more easily but had difficulty in finding area. In particular, when perimeter and area are combined in question, they find it difficult for solving. Besides, students do not use measurement units, they usually write numbers and leave it like that. At this point, Semih occasionally tried to remind using appropriate units.

In terms of what to notice, İnci focused on students' mathematical strategies and teacher pedagogy. She referred to the mathematical understanding of the whole class (e.g. "They calculated perimeter more easily but had difficulty in finding area"). She attended the noteworthy events and endeavored to make sense of students' mathematical thinking (e.g. "Some students said 28 as the answer because they found perimeter instead of area). However, although she had focused on a few students' mathematical approach, she continued referring to the whole class. Regarding how to notice, she further offered overall impressions rather than giving details to support her interpretations. She addressed students' confusions, difficulties and habits but in a general manner. Her comments were mostly descriptive and evaluative in nature although she adopted an interpretive stance on some comments. She was inconsistent in making reference to the evidence from her observation to support her claims. Thus, her noticing further exhibited the prevalent features of Level 2.

> **İnci:** In one activity related to estimating perimeter and area of three different figures, Semih implemented it incorrectly. He should have covered the figures with colored cardboard and he should have asked the students to estimate which perimeter and area was bigger. After estimation, he should have removed the cardboard and he should have helped students realise their mistakes. However, they saw the unit squares in the figures and got confused while finding perimeter. Semih had to write the lengths of edges one by one to facilitate understanding. The students did not estimate area and counted the unit squares directly. Therefore, the activity did not reach its goal.

In this quotation, İnci's initial focus was on teacher pedagogy and implementation of the lesson. She attended to specific events and drew attention to the mistakes of the teacher. She indicated what should had been done differently elaborating the reasons. She addressed the students' difficulty in calculating perimeter because of this mistake. However, her comments were unspecific and she did not give details about students' confusions or what kind of mistakes occurred. She further evaluated the effectiveness of the implementation rather than offering interpretive comments. Because İnci attended to teaching and learning issues, she did not focus on particular students' mathematical thinking and maintained a broad approach, her noticing was considered as Level 2.

> **İnci:** Semih skipped a few questions depending on the situation and I think it was a good decision. I do not know the reason but the students generally get bored in the second part of lessons. They might have got used to the materials or us. Although we warned them to be silent, they continued to make noise. When they get bored especially towards the end of the lesson, no matter how many interesting examples we present, it is difficult to draw their attention. I think we tried to present area in the best way but we should have removed the second examples while giving formula of a rectangle and square's area. We should add questions in the exercise sheet that involve completing half unit squares as the cooperating teacher said. We should emphasize

> the difference between length and area measurement units. We may replace difficult questions because we generally cannot complete them. I think we should put some more difficult problems in the beginning. It may increase attention but I am not sure, it may be worse.

In terms of what to notice, İnci focused on classroom management, students' interest, teacher's approach and the behavior of the students and teacher. She did not attend to specific students and their mathematical thinking rather she focused on the class as a whole. Regarding how to notice, her comments were very mainstream in nature. She described what she noticed without providing details (e.g. "Semih skipped a few questions depending on the situation"). She also offered evaluative comments about the teacher's behavior, teaching or students' participation. (e.g. "No matter how many interesting examples we present, it is difficult to draw their attention). She made some suggestions to improve the lesson in a general way. Due to the fact that she focused a range of issues which were less related to the mathematical aspects of the lesson and had an impressionistic stance, her noticing here was considered as evidence of Level 1.

After the reflecting phase, individual interviews were conducted with the preservice teachers to reveal the missing points and detail their noticing. What İnci indicated differently from the reflecting phase was as following.

> **İnci:** Students confused perimeter and area. One point that drew my attention was that they could say what perimeter of a rectangle or area of square was but they made mistakes while solving questions, in particular, combination of them. We taught both perimeter and area. If we had not taught them, I would think the problem might have derived from the teacher. Thus, I do not understand why they still confuse them. Maybe, they need time to internalize the subject. I wonder whether finding perimeter seems easier, therefore they say it at first without considering what was asked. They generally do not use units and they forget them. They could understand that area was covering and they could find it by counting the unit squares. Although a few students made mistake and found area as one more or less because of counting wrongly, they did not have a difficulty here. They could draw the figures on plotting paper to their notebooks easily… Substantially, I focused on responses of the students, controlled whether expected answers were given and Semih implemented the lesson plan well.

In this excerpt, İnci focused mainly on students and their mathematical approach. She addressed where the students had confusions and difficulties but she did it in general ways. She did not try to provide justification about students' mathematical thinking. She described what students could do or not through expressions without details. She also had an evaluative approach with an interpretive stance (e.g. "Maybe, they need time to internalize the subject"). She did not focus on particular students' mathematical thinking and kept referring to the class as a whole. She presented evidence from her observation to support her claims but she did not make reference to the details. Her comments during the interview showed that her noticing further reflected the properties of Level 2.

When İnci's all comments during the reflecting phase were considered, her noticing was predominantly accepted as Level 2 because she focused primarily on teacher pedagogy and students' mathematical approach. Although she attended to students' mathematical approach, she offered general impressions instead of giving details to strengthen her ideas. She sometimes tried to elaborate her claims but did not make sense of students' mathematical thinking or reason on them. Her approach was mostly descriptive and evaluative in nature. Hence, İnci's noticing in the reflecting phase was further considered as evidence of Level 2.

4.1.2.3 What and how İnci notice in re-reflecting phase

Teaching of revision of the second lesson plan was made by Mehmet in 5/G classroom. As Mehmet was implementing the lesson plan, the other preservice teachers, cooperating teacher and researcher observed this lesson and took notes. After teaching was completed, the reflecting meeting occurred immediately at school conference room. In the re-reflecting phase, preservice teachers were wanted to reflect their thoughts about the lesson including; what worked well, what did not work, what the difficulties were, what they observed about the students' actions and thinking and also what should be changed about the lesson to improve it and what the rationales for the changes were.

This phase began with the comments of the cooperating teacher briefly and she reflected her thoughts on the lesson. She signified that completing half squares made a difference in questions and the connection between daily life and the subject was maintained this time. She stated that the students did not know the concepts of row and column and when they were explained in the beginning of the lesson, students could understand better. It was good that the questions were from easy to difficult and the students could comprehend the solution of the questions both in the exercise sheet and on the board. She also expressed that it was normal encountering the students who confused perimeter and area as well as the students who understood the subject easily. At this point, the cooperating teacher stressesed that the students would learn better in time as they kept solving questions. She pointed out that the students were at a small age group therefore they could forget or confuse what they learned. She emphasized that there was no problem from the beginning to the end in the lesson and many issues such as area, problems, area measurements and use of them were presented well. She indicated that there was no need to add or change anything. After her suggestions were considered and applied in the revision of the second lesson plan, she found the lesson sufficient and did not make any suggestions to improve it.

In accordance with discussion protocol, İnci shared her opinions about the lesson, evaluated the effectiveness of the lesson. The following dialogs show what and how İnci noticed during the re-teaching phase.

> **İnci:** We mainly heard expected answers from the students . However, there were still students who offered to sum the edges of a figure when area was asked. Students could say that the interior region was area. Mehmet taught the difference between perimeter and area well. He tried to call different students and his communication was good. He reminded the measurement units when students forgot to write them. He was interested in the students individually while they were solving the question in the exercise sheet. He asked questions such as "What did you do?" and "What did you find?" and he repeated area. All of them were good. He discoursed on completing half squares and made needed explanations and showed that two half squares in the form of triangles constituted one square by visualizing it through movement when students had difficulty with them. It was good as well, however, he could not have command of the class at times. In the activity which included covering two same rectangles with two different squares, one student said that there was need for 12 squares in the second rectangle because 12 squares were used in the first rectangle. Mehmet emphasized that the square to cover the second rectangle would not be the same. When he asked the reason of the difference in the number of squares that were needed to cover the same rectangles, the students could say that one square was bigger than the other and they did not have the same dimensions.

In this excerpt, İnci focused primarily on teacher pedagogy and evaluated teaching approach (e.g. "Mehmet gave the difference between perimeter and area well" and "He tried to call different students"). She also attended to particular students' mathematical thinking but she continued referring to the whole class and she did that in a broad manner (e.g. "One student said that there was need for 12 squares in the second rectangle because 12 squares were used in the first rectangle"). She further described the response of the student and she did not try to make sense of his mathematical thinking. Considering they will cover the second rectangle with the same number of squares, the students might have thought that they did not need to count the squares again and thus answered as 12 or there might be another reason. However, İnci did not explain possible reasons behind this answer. She mostly addressed pedagogical issues such as communicating, drawing attention, reminding, and visualizing. She highlighted the noteworthy events but offered general impressions. Her comments were predominantly descriptive and evaluative in nature (e.g. "All of them were good"). She endeavored to describe what she observed step by step to strengthen her claims but she was inconsistent in providing detailed interactions. Therefore, her noticing had the prevalent properties of Level 2.

İnci: The students could give examples related to use of area from daily life. Mehmet asked the name of the figures and their properties each time. He tried to remind previous concepts. He explained the meaning of row and column and drew students' attention to the relationship between row, column, lengths of edges and area. He also tried to emphasize the difference between perimeter and area measurements and I think he did well. He also preferred to shade area instead of covering in one question because he thought that it would take time and it was a clever decision. In one question related to finding the number of 2x2 squares needed to cover the square in the form of 8x8, some students divided 8 by 2 and found 4, some of them multiplied 8 by 4 and found perimeter. Mehmet needed to give clues and direct the students to find the answer.

In terms of what to notice, İnci's initial focus was on teacher pedagogy and students' mathematical approach (e.g. "He tried to remind previous concepts" and "Some students divided 8 by 2 and found 4"). She mainly attended to the teacher and described what Mehmet did with general expressions without details. İnci did not focus on students' mathematical thinking or provide justifications of them. Regarding how to notice, she presented the examples from students' responses, however, she did not reason on their mathematical thinking. Why students divided or multiplied 8 by 4 and how they might have thought was not elaborated. She did not make interpretive comments rather she evaluated what she observed (e.g. "it was a clever decision"). She referred to specific moments but she focused mostly on teaching and learning issues and continued with a broad approach. Hence, her noticing exhibited the features of Level 2.

İnci: Mehmet indicated area of a square could be represented with a^2 as well as axa but the students could not understand it well enough. The representation of axa was more comprehensible for them and I guess we should not have discoursed a^2 in that moment. However, for example, one student said that it looked like exponential numbers. It also showed that they had knowledge. The question toward estimating perimeter and area of the given figures was obviously good. I liked this activity but Mehmet did not query the reasons of students' answers sufficiently. He put a point on the corner of the figures while calculating perimeters and it was good. Students did not have difficulty as calculating perimeter. In Semih's class, there were students who missed some edges but it did not happen in the second implementation. Students did not have difficulty in finding area by counting the unit squares either, instead they made mistakes while using area formula. Our deficiencies were toward the implementation rather than the content of the lesson plan. Thus, there is no need to change it, I think.

In this quotation, İnci attended to teacher pedagogy and students' mathematical understanding (e.g. "Mehmet did not query the reasons of students' answers sufficiently" and "Students could not understand it well enough"). She commented about the whole class instead of focusing on particular students' mathematical thinking. She mostly evaluated and described what happened with general statements (e.g. "I liked this activity" and "He put a point on the corner of the figures while calculating perimeters"). Why students had difficulty or not and why she liked this activity were not clear. She was not good at elaborating her ideas. She tried to promote her claims through descriptive comments but she did not make sense of what she observed or interpret them in terms of students' thinking. The main focus was on teaching and learning issues and an impressionistic approach was predominant although she referred to the noteworthy events and interactions. Thus, her mixed approach was characteristic of noticing in Level 2.

After the re-reflecting phase, individual interviews were made with the preservice teachers to elaborate their noticing and learn the missing points if there were any. What and how İnci noticed during the re-teaching phase is as in the following:

> **İnci:** There were many students who found perimeter instead of area. They multiplied edge length by 4 to find the area of a square or they divided area by 4. It might be because four edges of a square are equal. When we asked area and perimeter, they could indicate what area and perimeter was but I guess multiplying by 4 comes easy. They did not try to understand the question or think it over enough. Maybe, the question might have seemed complicated or they might have made a mistake because they wanted to give the answer immediately. They could understand area of a rectangle but they had problem in area of a square. Somehow, I thought that they would understand area of a square more easily than a rectangle since all edges of it are equal. In contrast, students had difficulty in area of a rectangle and they could understand area of a square in Semih's lesson. It showed that there might be difference from student to student. I do not know what we can do actually to overcome this confusion. We aimed to make students reach area formula axa by using a 3x3 square, however, one student overgeneralized or made a mistake and multiplied the edge length of the square by 3 in the following question. Mehmet implemented the lesson plan well. The students were good at using appropriate units in comparison with the other class. They could understand which units they would use in perimeter or area.

İnci's comments during the interview showed that she focused primarily on students' mathematical approach and teacher pedagogy. She made reference to the evidence of students' strategies from the lesson. She tried to make interpretation about the reasons of students' mistakes but she did that in a general manner (e.g. "I guess multiplying by 4 comes easy" and "They did not try to understand the question or think it over enough"). She indicated what she expected and what was different from her assumptions. She compared the students' mathematical approach with the approach of the students in the first implementation of the second lesson plan. She also focused on particular students' strategies but her elaboration about students' thinking was limited. She generally did not disclose details to support her claims. Her comments in this dialog were evaluative with an interpretive stance but general as well. Thus, her noticing was further considered as evidence of Level 2.

İnci indicated what she mainly focused on as planning and what she noticed in this process. The following is presentation of her noticing through her comments:

> **İnci:** I focused on the answers of the students and how successfully Mehmet could implement the lesson plan. I am looking at the class deeper while observing. We got to know the students and they knew us too. We are learning how to react and deal with the students.

İnci's comments related to her focus during the re-teaching phase support that her noticing had the properties of Level 2. She also indicated that this process helped her know the students reciprocally and improve their teaching.

When İnci's all comments during the re-reflecting phase were considered, her noticing was considered as evidence of Level 2 because she focused primarily on teacher pedagogy and students' mathematical approach. She referred to students' confusions, difficulties and mistakes but she was not good at providing justifications about them. Although she attended to students' mathematical approach, she made general comments instead of making reference to the evidences to support her claims. Her elaboration was inconsistent and her approach was mostly descriptive and evaluative. She also adopted an interpretive stance from time to time but is was limited. She did not try to make sense of students' mathematical thinking. Therefore, İnci's noticing in the re-reflecting phase further reflected the properties of Level 2.

4.1.3 Lesson study cycle 3

4.1.3.1 What and how İnci notice in planning phase

The third lesson plan was mostly towards reinforcement of perimeter, area and combination of them and the objectives which the third cycle included were "Being able to create different rectangles with the given area" and "Solve problems that require to calculate the area of a rectangle". Before planning the lesson the preservice teachers went to the practice school to get help and suggestions about the subject from the cooperating teacher. They listened to her ideas, took notes and asked the questions coming to their minds. The cooperating teacher generally indicated that no matter how much repetition or explanation was done, the students kept confusing perimeter and area with each other, thus, problems including combination of them should be asked. She suggested asking questions which involved figures and wanted the students to find perimeter and area, vice versa giving problems which did not include figures and required the students to draw them. All preservice teachers met to prepare the third lesson plan at the university seminar hall bringing the various sources such as mathematics textbooks, teacher's guide book, curriculum and different mathematics education books. The followings are presentations of examples based mainly on İnci's expressions from the planning meeting of lesson study cycle 3.

The preservice teachers discussed on how to start the lesson and they decided to repeat what the students had seen in pervious lessons through questions such as "What was perimeter?" and "What was area of rectangle?". Before presenting that there were different rectangles which had the same area directly, they wanted to make students feel the conservation of area. In other words, they aimed to show when a figure was rotated, relocated or was not exposed to addition and subtraction, area of it remained the same. The conversation among the preservice teachers about this issue is as in the following:

> **İnci:** We had talked that we should give the concept of conservation of area in the previous planning. There were examples in a book… There is an activity here. First, the rectangle is horizontal, later it is turned over to vertical. I think we can start with it because students get used to horizontal prototype of a rectangle and they are likely to think area of it changes when it is rotated to vertical position or perceive it as a different figure. Misconception may emerge here.
> **Hasan:** We should also put the rectangle in a diagonal position. Students can get confused when they see it as well.
> **Semih:** Yes. You are right. Do not we need to ask the students to find the area?
> **İnci:** We may make them both calculate the area of the rectangle and feel that area is the colored region. Therefore, students find the same numerical value for area and see that the interior colored region is the same each time. They will also see that rotation does not change area.
> **…**
> **Semih:** Then, we should ask what we can say about area after each rotation.
> **İnci:** If the students say that area does not change, what will we do?
> **Hasan:** We will emphasize that addition or subtraction was not done on it.
> **İnci:** If the students say that area changes, what and why they may say?
> **Mehmet:** They are likely to say that the figure has changed because according to the students it is not a rectangle anymore.
> **İnci:** They may say that the place of it has changed.
> **Semih:** The figure in the new position may appear bigger.
> **İnci:** Ok. How should we respond for them to understand that the figure is still a rectangle?... We can highlight the edges and numerical calculation. What was the length? 40 cm. What was the width? 20 cm. What was the area? 800 cm^2. For vertical and diagonal positions, we can ask these values and area again and again. Thus, they can say that area does not change.

In this excerpt, İnci focused on introducing the concept of conservation of area and offered an example from a book. She endeavored to explain how her suggestions were related to mathematical goals of the lesson. She also considered students' specific mathematical thinking and mistakes (e.g. "Students get used to horizontal prototype of rectangle … perceive it as different figure"). She reasoned through the students' possible strategies and used details to draw inferences about students' mathematical understanding (e.g. "They also will see that rotation does not change area"). She focused on how students may think mathematically in each situation and also promote the other members to attend to this point. Thus, preservice teachers try to guess students' mathematical approach and provide justification about the reason behind their possible reactions (e.g. "They might say that the place of it has changed"). She also paid attention to how to respond the students to reduce their mistakes by elaborating her ideas. This interaction was important because it could enable the group members to discuss on where the specific confusions arose. İnci focused primarily on how the students think mathematically and considered it as shaping the content of the lesson. She referred to the noteworthy and specific points. She presented details and evidence to strengthen her claims. Therefore, her noticing in this dialog exhibited the prevalent features of Level 3.

The preservice teachers kept discussing how they would conduct this activity. There were different opinions about presenting the concept of conservation of area with the same figure or using different figures for each position that they decided. The interaction between the preservice teachers is given below:

> **İnci:** The area was the red colored region. We can ask whether this region changes.
> **Semih:** We can prepare three rectangles which are the same size and stick them in horizontal, vertical and diagonal positions respectively. We can show the area of each figure was equal by putting one on top of the other. Therefore, we help the students understand that they are the same independent of their positions.
> **İnci:** We should put these figures side by side and it must be the same figure in order to understand that it is the same figure so the area is the same. We must show the edges are 20 cm and 40 cm in each position. If we use different figures, this may cause confusion and it does not serve the purpose. Ok. I found it. We can stick a rectangle in a horizontal position on the board at first, draw around it and write the edge lengths. Then, we can move the same rectangle to the side in a vertical position and draw around it again. We make it for the diagonal position so that the students can notice that the same figure was used, no change was made and edges were the same in each position. Thus, they will realize that rotation and relocation did not change its area, the area was the red region and the edges did not change so the numerical values were equal.

In this quotation, İnci's initial focus was on how the concept should be taught and students' mathematical understanding. She did not agree with Semih's suggestion about teaching and she shared her own opinions by elaborating. She thought that the students should feel the conservation of area on the same figure and understand area did not change with rotation or relocation. She endeavored to explain how her suggestions were related to the overall mathematical goals of the lesson. She believed that different figures would not serve this purpose correctly and might cause confusion. Her comments were further interpretive in nature. She reasoned on students' mathematical approach and made interpretations about their thinking. She tried to make reference to the evidence which supports her comments and provides justification about her ideas (e.g. "They will see that … the edges did not change so numerical values were equal). She gave details about her ideas by considering students' mathematical thinking. Hence, her noticing here was considered as evidence of Level 3.

Preservice teachers talked about how to continue lesson after first activity. They thought that they showed conservation of area on same figure but they should also make students feel this concept on different figures. They also aimed to reinforce the constitution of different figures which had same area. They discussed on an example in the book and how to present it to the students. The following dialog shows the interaction between the preservice teachers:

> **İnci:** We can use another example to reinforce the conservation of area. Here, a triangle was created from a rectangle, a rectangle was created from a square and a parallelogram was created from a trapezium. What do you think?
> **Mehmet:** Students do not know the area of a trapezium.
> **İnci:** We will not ask them to calculate area. We will just ask whether area changes when we move the parts of the figure to the next side and create a different figure with them. For example, they may say that perimeter changes as a result area changes too.
> **Mehmet:** If students say that, we cannot show that area does not change by calculating the area of the figures because they do not know the area of a trapezium and parallelogram.
> **İnci:** We do not need to give the mathematical formula and show numerically. We can emphasize that area was the red colored region and ask whether any changes occurred on it. We can make them feel the equality by visualizing.

Here, the preservice teachers had a disagreement about including an activity in the lesson plan. Mehmet defended that it was not appropriate for the students' level because they did not know finding the area of some geometric figures in this activity. İnci offered to make students feel the equality of interior region without calculating area numerically. İnci focused further on how to teach the subject better and she tried to consider students' mathematical approach in this process. She reasoned on students' possible mathematical thinking (e.g. "They may say that perimeter changes so that area changes"). However, it was limited and her comments were general. She mainly addressed teaching and learning issues (e.g. "We can make them feel the equality by visualizing"). She generally described her suggestions in a broad manner and evaluated the others' ideas. She was inconsistent in providing details to support her comments. Thus, her noticing further had the properties of Level 2.

Preservice teachers offered various opinions concurrently as planning the lesson. While they were discussing on the activity which involved creating a triangle from a rectangle, they talked about finding area by covering and counting with unit squares and creating different figures with the same area. However, they had a disagreement about the order of the activities. The discussion among preservice teachers is as in the following:

> **Mehmet:** We can want the students to cover the interior region of a figure with unit squares so that the students can see that area is not based only on formula but also on covering.
> **İnci:** Yes, we can stick unit squares on the board and want them to create different figures such as L, T. Thus, they can see that each figure has the same area numerically because the same number of unit squares is used. They will understand that different figures have the same area by counting the unit squares.
> **Mehmet:** According to me, first we should give this activity that requires them to calculate area numerically rather than the activity that includes creating a triangle, rectangle and parallelogram.
> **İnci:** But first we should make them perceive the conservation of area without calculation. They need to understand that rotation or moving same parts does not change area. It is easier, we will not ask them to find anything. In the activity that you said, they will move the unit squares to the side and create different figures. Hence, we should give the other activity first, I think. So they can make it themselves.
> **Semih:** Here, the students will see that area does not change. They will create L from T and notice that T has six unit squares and L has six unit squares.
> **İnci:** But here, they will say that area is the red region in the first figure and it is still the red region in the second one as well. They will think that there are same pieces in both figures and realize equality of areas. The same logic. In the first activity, we showed that area was the same in the same figure in terms of both numerical value and interior region. In the second one, we can display on different figures in terms of the interior region without calculation and in the third activity, we can exhibit it on different figures through numerical values. I think, it is step by step from easy to difficult in this way.

In this excerpt, İnci focused primarily on students' mathematical understanding and thinking. She considered students' mathematical approach as evaluating the other members' ideas and making her own suggestions. She referred to possible students' responses to elaborate her ideas (e.g. "They will think that there are same pieces in both figures and realize equality of areas"). She tried to give justification about students' thinking (e.g. "They can see that each figure has the same area numerically because the same number of unit squares is used). She associated her comments with the purpose of the lesson and made reference to the details to support her ideas. She explained why the activity that she mentioned should be given first with the reasons. She made evaluative and interpretive comments. She generally addressed the noteworthy events on the basis of the evidence of students' thinking patterns. Thus, her noticing here was considered as evidence of Level 3.

During the planning phase, İnci also asked expressions regarding students' mathematical approach such as "How may students think here?", "What kinds of answers can the students give?", "Do the students know the concept of rounding off" and "We cannot ask this kind of a question because students have not seen permutation yet". She made many suggestions in terms of both serving the purpose of the lesson and ensuring students' understanding better. She generally offered opinions on what can be done to reduce the mistakes and endeavored to elaborate her opinions with reasons. Her comments were mainly interpretive and included details to support her comments. She predominantly referred to the students' mathematical thinking as shaping the lesson plan. Therefore, İnci's noticing exhibited the properties of Level 3 in some dialogs.

During the interview that followed the end of planning, İnci indicated what they aimed in this process. She explained what expected students' responses, difficulties and misconceptions were and how they considered students' mathematical thinking while planning as in the following:

> **İnci:** We will relocate the cardboard and for example, create a triangle from a rectangle. Area will not change but students can get confused and think that area changed due to relocation or turning into a different figure. We will try to minimize misconceptions with concrete materials. Students are likely to say that area is bigger considering only the image but each area is equal. Students may think that area changes because perimeter changes, misconceptions may arise here. In other words, they may associate area with perimeter. We tried to give representations which they do not encounter a lot in order to make them reason and reveal their misconceptions. Therefore, we will have a chance to reduce them.

In this excerpt, according to İnci, the students were likely to have confusion and give wrong answers when the figures relocated or changed although they had the same area. She thought that when they saw different prototypes of the same figure which they did not encounter very often, it might cause confusion as well. She indicated the reason of the misconceptions as optical illusion and not considering the meaning of numerical values. She addressed that students did not reason on what they learned in general. It is seen that the preservice teachers had some knowledge about students' confusions and difficulties and they especially tried to reveal them in order to correct them in more effective ways. The comments showed that they shaped the lesson plan based on students' mathematical thinking.

İnci expressed what she chiefly focused on as planning and what she noticed in this process. The following is presentation of her noticing through her comments:

> **İnci:** We mostly focused on teaching with materials in order to enable the students to learn the new concepts better. We paid attention to determine the questions including relocation and creating different figures because the students should understand their effects. We also attended to find different activities and questions to present possible alternatives during the lesson. We focused on expected students' confusions, mistakes and how they might think mathematically. We tried to constitute the content of the

> lesson plan based on this knowledge. I also noticed that some suggestions were not appropriate for students' level. This was our third lesson plan. I think we have gained experience in the process so it was completed in a shorter time in comparison with the first two lesson plans.

İnci's comments showed that her focus was on the lesson considering pedagogical issues and students' mathematical thinking. She indicated that she paid attention to using materials, presenting rich content and students' mental levels. Her expressions supported that the students' mathematical approach was predominantly considered during planning the lesson. She also emphasized that this process increased their experiences and affected their abilities to prepare the lesson plan in a positive way.

When İnci's comments during the whole planning process were considered, her noticing was considered as evidence of level 3 because she predominantly focused on students' mathematical thinking and made suggestions in accordance with the objectives of the lesson by considering how students think mathematically. She generally referred to possible students' responses and promoted the other members of the group to focus on these points. She elaborated her ideas and explained the reasons behind her comments to support her claims. She used details to interpret students' understanding. Her focus was on more mathematical aspects of the lesson rather than the issues such as classroom management, student interest or time. Hence, her noticing in the planning phase had the prevalent features of Level 3.

4.1.3.2 What and how İnci notice in teaching phase

The findings of this episode obtained from the implementation of the third lesson plan included the objectives of "Being able to create different rectangles with the given area" and "Solve problems that require calculating the area of a rectangle". The first implementation of the third lesson was conducted by İnci in 5/F classroom and the revision lesson taught by Hasan in 5/G classroom. The findings from İnci's lesson were presented and her noticing during the teaching phase was revealed.

İnci began repeating the previous lesson and tried to listen to what the students had learned before moving on to the new objective. During the planning phase, the preservice teachers discoursed on remembering the necessary concepts in order to ensure them not to forget and conduct the lesson better. İnci helped the students to repeat how to find the area of a rectangle and square and she used rectangle formula to switch to the concept of the conservation of area. Although the objective of this lesson did not include this concept directly, the preservice teachers thought that it was associated with creating different rectangles which had the same area and discussed the importance of making the students feel it. They thought that the students were familiar with the rectangles in a horizontal position rather than in vertical and diagonal positions because they did not usually encounter these prototypes. From this point of view, they decided to rotate the same rectangle, as a result they aimed both to show area did not change and reveal whether different prototypes of rectangles would cause confusion or not. İnci implemented the first activity which served this purpose, and answers indicating a change in area were given by the students as anticipated by the preservice teachers. The dialog below describes how İnci conducted the activity and what she did to help the students sense the conservation of area:

İnci: What did we see in the previous lesson?
Students: Measurement of area.
İnci: Which figures' area did we learn?
S1: Square
İnci: What was the area of a square?
S1: Multiplication of one edge with another edge.
İnci: All edges of a square are equal, right? If we say "a" for one edge, what will happen?
S2: a times a.
İnci: Yes. The area of a square is a times a (axa) in this case, isn't it? Which other shape's area did we see?
S3: Rectangle.
İnci: How do we find the area of a rectangle?
S3: We multiply width by length.
İnci: Very good. What was this figure? (She attaches a rectangle on the board in a horizontal position).

Figure 18 The figure of a rectangle in a horizontal position

Students: A rectangle.
İnci: Who wants to calculate the area of this rectangle? (She calls a student to the board). You draw in your notebooks as well.
S4: We multiply width and length, so it is 800 from 20 times 40.
İnci: What is its unit?
S4: Square centimeter (cm^2).

İnci started the lesson with a quick recap of what they did in the previous lesson. She guided the students to give the answers which were needed for the lesson instead of telling them directly. She attended to what they said and went step by step in the direction of their answers. When one student expressed the area of the square like multiplication of one edge with another edge, she also wanted him to indicate it with a formula and allow him to indicate axa, rather than just saying it. She did not make any explanations about area because the students did not give wrong answers in repetition and she found their answers sufficient enough. After İnci attached a rectangle, student S4 found the area of the figure explaining his solution without difficulty, however, he did not pay attention to the unit. İnci noticed the missing point and asked the student to write appropriate the unit. He did not have a problem writing the unit after İnci's warning. It showed that the students usually forget to use the units as preservice teachers had assumed in the previous phases. İnci maintained the activity by rotating the rectangle from the horizontal position to vertical position.

İnci: If I attach this figure like that, does the area of it change? (She attaches a rectangle on the board in a vertical position)
Students: Yes.
İnci: Why does it change?
S4: The edges changed, so the area changed too.
Students: No.
İnci: Why does not it change?
S5: Because the edges are equal.
İnci: Then, what can we say about area of this figure?
Students: 800 cm^2 again.
İnci: If I attach this figure like that, does the area of it change? (She attaches a rectangle on the board in a diagonal position)

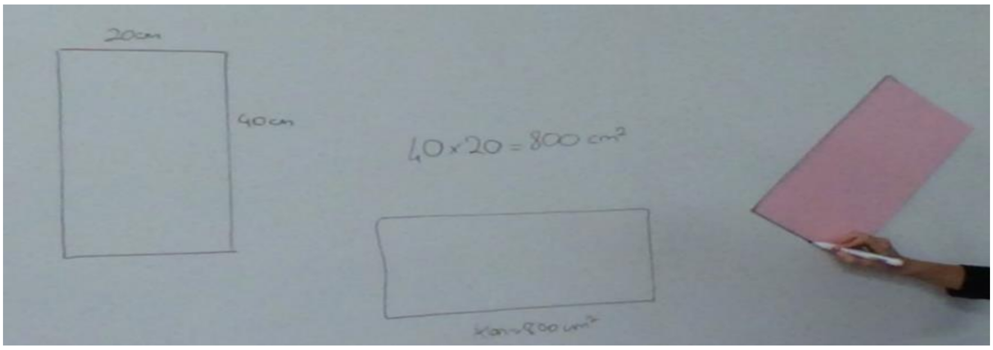

Figure 19 The demonstration of conservation of area by rotating the same figure

Students: No

İnci: Why does not it change?
S6: The edges are still the same.
İnci: I did nothing to the edges. Then, can I say that the area of this figure is equal to 800 cm^2?

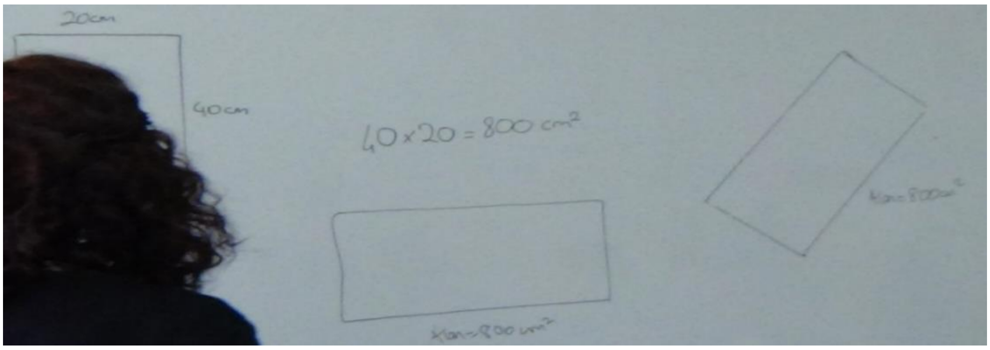

Figure 20 The numerical representation of the area equality by rotating

Students: Yes.
İnci: In that case, when I rotate the figure, does the area of the figure change as long as the edges remain the same?
Students: No.

As decided in planning, İnci moved and rotated the same rectangle and queried whether its area changed or not. They thought that there might be students who would say both "yes" and "no" for the change of area. Similarly, students gave both answers. İnci asked the reason of their thinking, as they had agreed on during the planning phase to decide whether the students really understood what she intended or not (e.g. "Why does not it change?" and "Why does it change?"). The students' answer "The edges changed so the area changed" showed that some of them perceived it as a new figure with rotation. The students' answer of "Because the edges are equal" and "The edges are still the same" indicated that some of them perceived no change and felt the conservation of area. She wanted to show area did not change proving with numbers to reinforce this realization. Therefore, she drew the attention to the first measurement of area and queried whether the area still was 800 cm^2 in the new position. The students could deduce that it was again 800 cm^2 from multiplication of 20 cm by 40 cm. After İnci rotated the rectangle in the diagonal position, the students again explained there was no change in area due to the uniformity of the edges. İnci tried to probe how the students' thinking was and she attempted to ask the students to give justification for their answers in each position. She felt that the activity went well and achieved the goal. She endeavored to make the students notice by themselves that the edges of a rectangle did not change through movement and rotation of the same rectangle and the rectangle could be in a different prototype but it was still the same rectangle, therefore, the area does not change. She interpreted the students' answers as indicators of their understanding of what she had intended.

It can be argued that İnci's noticing in this dialog was in Level 3 because she was able to attend to students' reasoning and promote them to explain the reasons behind their answers. She listened to the students' ideas and responded them in accordance with their answers rather than expressing what was intended or making them see the implicit knowledge in the activity directly. She was able to clarify what the students meant with the uniformity of the edges using area formula for each position, thus, she also focused on giving the key points on one hand. On the other hand, she listened to the students' comments and attended to their answers in an interpretive way. She tried to make sense of students' thinking to find out whether the students understood or what they understood through an interpretive stance (Davis & Renert, 2014). Because she focused on the students' mathematical thinking and explored their reasoning, she responded the students in a way that could reveal their thinking, guided them to give details about their mathematical approach and she had further an interpretive approach, her noticing exhibited the characteristics of Level 3.

After the initial activity, İnci went on with the next activity which aimed to reinforce comprehension of the conservation of area and the constitution of different figures which have the same area. In the planning phase, the preservice teachers thought that they showed the area did not change on the same figure and utilized numerical justifications in the first activity. Differently from the first one, they decided to show there was no change in area through different figures and without using the mathematical formula here. They wanted the students to sense what they had intended through basic visualization. The following dialog describes the interaction between İnci and the students:

> **İnci:** What is this figure? (She attaches a rectangle which contains two triangles on the board).
> **Students:** A rectangle.

Figure 21 The figure of a rectangle

İnci: What can we say about the area of this figure?
S7: Nothing.
İnci: Can we say nothing? Hmm (She asks another student who raised her hand). What can I say?
S8: This edge is equal to that one (She shows the opposite long edges). This one is equal to that edge as well (She shows the opposite short edges).
İnci: What can I say for its area?
S8: I need to multiply this edge by that one (She shows one long and one short edge).
İnci: Can I say that whole pink region is the area of the figure?
Students: Yes.
İnci: If I change the figure like this, what is this now? (She moves two small triangle pieces of the rectangle to the side and creates a big triangle).

Figure 22 The demonstration of conservation of area in the first example

Students: A triangle.
İnci: Did any change occur in the area of the figure?
S9: No, it did not.
İnci: Why not?
S10: Yes, it did.
İnci: Why did it occur?
S10: Teacher, there are four edges here (He points to the rectangle), there are three edges here (He points to the triangle).
İnci: Alright. What was the area of a rectangle? It was the interior region, right? Namely, I had used these pieces and formed a rectangle at first. Did I ever change the size of the figure? Did I ever cut it, make it smaller or add something on these pieces?
Students: No.
İnci: Then, did its area change?
Students: No.

It was seen that İnci conducted the lesson querying even the most basic concepts such as the name of the figures and change in area. After she attached each figure on the board, she checked whether the students knew the geometric figures or not and their answers showed that they had learned them. She wrote the name of each figure under it. She tried to learn their general ideas about area of the figures and here, she expected the students to indicate area as the interior region or the colored part without using formula and not in terms of numbers. However, when she asked the area of a rectangle, student S7 gave an unexpected answer that we could say nothing about area. This answer was obviously important because he may have thought that there were no values on the edges, therefore, it was difficult to make a comment about what it was. If he could have reasoned in terms of numerical values and had given this answer, it would have showed that his mathematical approach was good. However, İnci did not expect this answer, hence, she did not attend to what the student meant and did not endeavor to reveal the mathematical thinking of S7. It was difficult to understand what he was referring to without clarifying his response of "nothing". She went on allowing another student to answer but this time student S8 explained one common property of a rectangle and indicated that the opposite edges were equal to each other. It showed that the students were in tendency to share what they know without considering what really was pointed. When İnci could not get the response in her mind, she emphasized that she was asking the ideas related to area. After İnci insisted on area, S8 addressed the area formula of a rectangle since he did not understand what the teacher had expected. At this point, it can be said that her question statement was not clear enough for the students. Because they did not understand what İnci had intended, they could not give the expected answers. In this case, İnci endeavored to direct their attention to the "colored, interior part" by pointing to the pink region or yellow cardboard.

She further guided the students to reason in a way that they would notice what she had meant using directed questions such as "Can I say that whole pink region is the area of the figure?". Maybe, it was not difficult to say that area means the interior region, however, İnci's instructions were not enough to reveal this knowledge. When İnci queried whether area changed or not after creating a triangle from the rectangle, there was a conflict about it. Some students indicated no change, whereas some defended a change in the area. Although the preservice teachers had guessed both reactions, they did not expect that difference in the number of the figures' edges would cause confusion. She did not focus on the student S10's answer of "there are four edges here, there are three edges here" and did not give the opportunity to elaborate her thinking because İnci could not know how to respond to this expression. She tried to explain that area did not change associating it to the interior region and emphasizing that there was no increase or decrease on it. She continued to the activity by showing the conservation of area between a square and rectangle and also a trapezium and parallelogram.

İnci: What is this figure? (She attaches a square which includes two rectangles on the board).

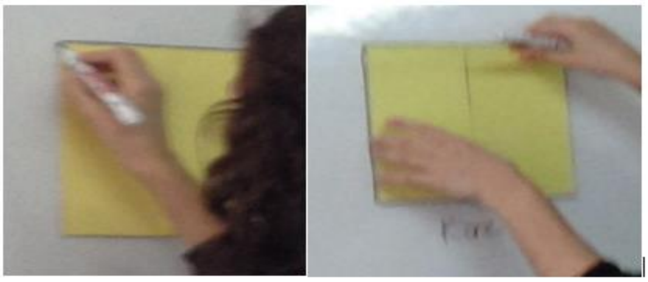

Figure 23 The figure of a square

Students: A square.
İnci: What can I say about the area of this figure?
S11: All edges are equal to each other.
İnci: Can I say that whole yellow region is the area of this figure?
Students: Yes.
İnci: If I moved these pieces to the next side and attached them like this, what happens? (She moves two rectangle pieces of the square to the next side and creates a rectangle).

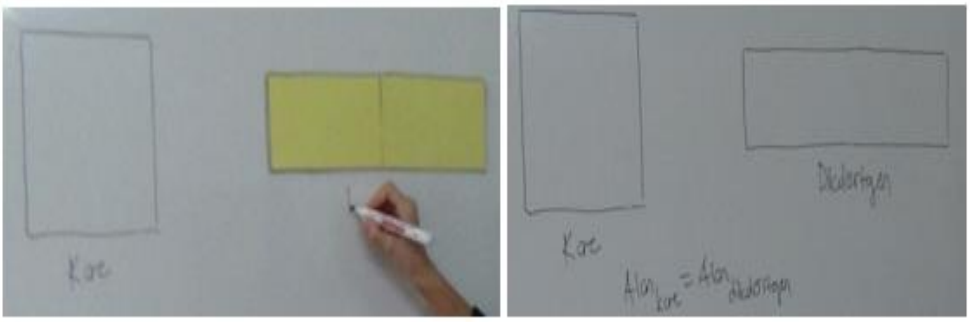

Figure 24 The demonstration of conservation of area in the second example

Students: A rectangle.
İnci: Does the area change?
Students: No.
Students: Yes.
İnci: Does the area of the figure change?
Students: No:
İnci: Why doesn't it change?
S12: Because you did not make any change on it, you did not make it smaller or larger.

İnci: Did I cut something from or add on these yellow pieces of cardboard.
Students: No:
İnci: I just moved and placed them differently. Does the area change?
Students: No.
İnci: Ok. What is the name of this figure? (She attaches a trapezium on the board).

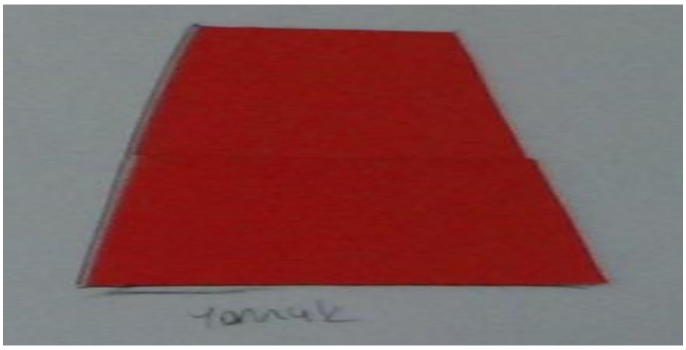

Figure 25 The figure of a trapezium

Students: A trapezium.
İnci: What can I say for the area of it?
S13: It is equal to the red region of the cardboard.
İnci: Yes, the area of the figure is its interior region right? What happens, if I place this figure that way? (She moves the pieces to the side and creates a parallelogram).

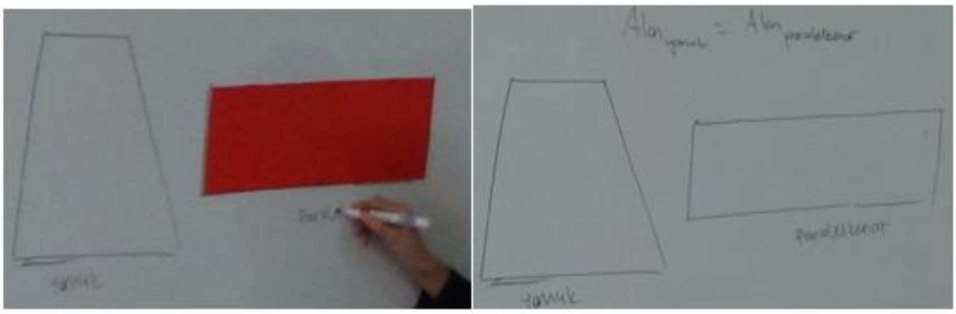

Figure 26 The demonstration of conservation of area in the third example

Students: A parallelogram.
İnci: Is there any change in the area of the figure?
Students: No.
İnci: Why didn't it change?
S14: You did not make it smaller or bigger.
S15: You made no change on the cardboard.
İnci: Yes. What I did, I separated the whole into pieces and changed their places. Then, can I say that when I move or rotate the figure, its area does not change?

Students: Yes.

Similarly, İnci attached a square on the board and asked the name of the figure and what can be said about it. Student S11 described one general property of a square about equality of all the edges instead of commenting on area. İnci herself indicated whole yellow region was area again. It is seen that İnci and the students were referring to different aspects of the same figure and it may be the indicator of lack of understanding. There were still students who said that area changed when a rectangle was created from the square. After the answer of "yes" for change, İnci asked again what happened to area but her this question further was like the signal which gave clue about incorrectness of the answer. Therefore, the students who said "yes" changed the answer as "no" without much thinking so as to say what the teacher had expected. Besides, the other students explained why area did not change with similar statements as İnci had used in the previous example. At this point, it cannot be said that the students noticed the conservation of area completely, however, İnci accepted it as an indicator of understanding and did not investigate reasoning behind the answer of "yes". She maintained the activity and attached a trapezium on the board. The students could explain that area was the whole red region and area did not change when a parallelogram was created from the trapezium because the cardboard pieces remained the same, no reduction or extension occurred. At the end of the activity, İnci summarized the key ideas, emphasized that the movement or rotation of the same pieces did not alter the area and made them feel the concept of conservation of area.

İnci's noticing during this activity was classified as Level 2 because although she endeavored to attend the students' reactions, her responses revealing their thinking were limited. She sometimes queried the reasons behind the students' answers. Additionally, she did not pay attention to their explanations. Although she listened to the ideas of the students, she did not attempt to understand what they were thinking. She mostly conserved the flow that she had in her mind and guided the students to get the answers that she wanted to hear. When the intended answers were not given, she ignored the wrong answer or she emphasized the key points instead of helping the students to catch them. She could have provided more opportunities for the students to elaborate their reasoning but when she heard the correct answers, she thought that they were enough. She also asked leading questions, however, she could have asked the questions which would assure more explanation about their knowledge. On the other hand, she listened to the students' comments and attended to their answers in an evaluative way rather than an interpretive stance. She focused on whether the correct answers were given rather than interpreting mathematical thinking. She maintained a pattern of questioning which enabled helping the students notice what was intended through a general approach instead of getting more details about students' thinking and learning. Although she tried to focus on what the students said, her responses to reveal the students' thinking and her guidance to give details about their mathematical approach were limited and she had further an evaluative approach. This mixed approach exhibited the characteristics of Level 2.

After the second activity, İnci continued with the activity which aimed to give the definition of the conservation of area. In the planning phase, the preservice teachers thought that they should show the students that area could be found covering with unit squares as well as using formula and the number of them would give area. Therefore, they aimed to make the students count the number of unit squares in a figure, then help them create different figures covering with the same number of unit squares. The students were supposed to notice that area remained the same even though the figure changed, and different figures could have the same area. The following dialog describes the interaction between İnci and the students:

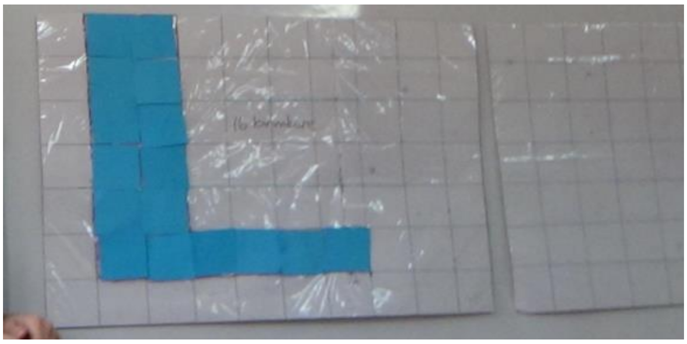

Figure 27 The activity to create different figures with unit squares

İnci: Who wants to find the area of this figure?
S16: 16.
İnci: 16, what?
S16: 16 unt^2.
İnci: Ok. Who wants to come and create a rectangle from these unit squares? Area will be 16 unt^2. (S17 goes to the board and starts to do what was asked). What is your friend doing?
Students: He is forming a rectangle.

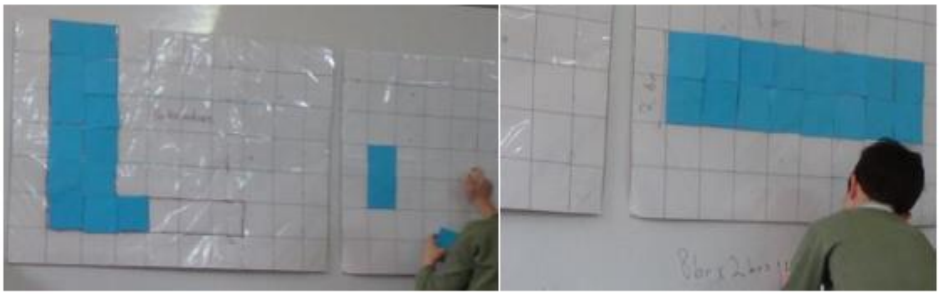

Figure 28 The example of creating a rectangle by covering with unit squares

İnci: (After the student created a rectangle, İnci asks questions to the class). What is the width?
Student: 2.
İnci: What is the length?
Students: 8.
İnci: How do we find area?
S17: Multiplying 2 by 8. 2 unt times 8 unt equals to 16 unt^2.

It was seen that İnci conducted the lesson step by step and without causing any confusion. She constituted a figure placing 16 small colored unit squares randomly. Here, she expected the students to find its area by counting the unit squares inside and feel that the number of unit squares in the interior region symbolize area. It was not difficult for the students to find the number and they counted the unit squares one by one. Later, student S17 came to the board and created a rectangle in the form of 2x8 moving unit squares to the next plotting paper. When İnci asked area, it was noticed that student S17 used the formula to find it and she could not say 16 unt^2 directly catching that the number of unit squares was the same in both figures. Here, the student forgot to write the name of the appropriate unit but İnci reminded it and asked the name of the new figure to emphasize again. Later, she maintained the activity.

İnci: Who wants to come and create a square from these unit squares? (S18 goes to board and starts to do what was requested). What is the length of one edge?

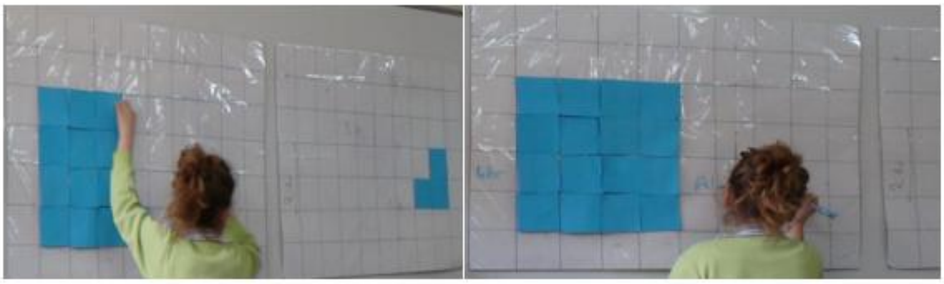

Figure 29 The example of creating a square by covering with unit squares

S18: 4 unt.
İnci: What is the area of this square?
S18: 16 unt^2.
İnci: How did you find it?
S18: I just moved them.
İnci: What do you mean?
S18: There were 16 unit squares here (She points to the rectangle). So, it is again 16 unt^2 in the square.
İnci: Yes, you are right. How else can we find this result?
S19: From the formula, 4 times 4, 16 unt^2.
İnci: What did your friend create shortly before?
Students: A rectangle.
İnci: What was the area of that rectangle?
Students: 16.
İnci: What did your friend create now?
Students: A square.
İnci: What is its area?
Students: 16.
İnci: What can we say about area?
Students: Area did not change.
İnci: Can we create different figures which have the same area?
Students: Yes.

After formation of a rectangle, student S18 created a square using the unit squares in the rectangle. İnci queried how he found it and tried to reveal how he thought mathematically. It was noticed that S18 did not need to calculate area using the formula again and realized the same number of unit squares were used, so he answered as 16 unt^2 directly. She saw that the student felt the conservation of area. Yet, İnci also drew attention to another way to emphasize area by both counting the unit squares in the interior region and using the formula. She tried to show the relationship between the area of a rectangle and square and allowed them to reach the conclusion that area did not change. By questioning the students, she emphasized that the figures were a rectangle and square, namely, they were different but their areas were the same. She gave the definition of the conservation of area after she thought that the students noticed the key point. She followed the flow as they had decided in the planning phase and did not have any problem about implementation of this activity.

İnci's noticing in this activity was accepted as Level 3 because she did not only listen to the students' answers about area of the figures, but also queried them in a way that helped to reveal their mathematical thinking. When she heard an unclear answer or wanted more details about the reasons behind the students' answers, she promoted them to make more explanation about their reasoning through questions such as "What do you mean?" and "How did you find it?". She attended to the students' ideas and tried to make the students catch that different figures can have the same area and supported them to feel the conservation of area rather than indicating what was intended. She listened to the students' comments and attended to their answers in an interpretive way because she endeavored to make sense of their mathematical approach to comprehend what students understood. She accepted their correct answers as indicators of their understanding. However, she did not accept them immediately, she continued to query for each figure. Because she focused on the students' mathematical thinking and probed their reasoning, she responded the students in a way that could reveal their thinking, guided them to provide details about their mathematical approach and she had further an interpretive approach, her noticing was in Level 3.

After the third activity, İnci wrote a question which involved calculating both the perimeter and area of the figures. With this question, the preservice teachers aimed to make a general repetition of these concepts, reinforce that different figures with the same area could be drawn and stress that different figures with the same area did not need to have the same perimeter. The following dialog describes the interaction between İnci and the students:

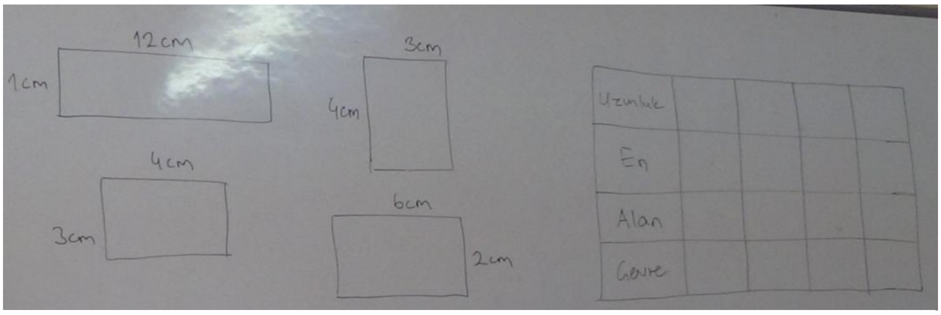

Figure 30 The question to show the relationship between area and perimeter

İnci: Let's write this in your notebooks and solve it on the board. Who wants to come for the first figure? (One student comes). What is the length of this figure?
S19: 12 cm.

İnci: What is the width?
S19: 1cm.
İnci: What is the area?
S19: 12 times 1, 12 cm^2. Now, we will find the perimeter. If this edge is 12, the opposite one is 12. If here is 1, here is 1 as well and perimeter is 2 times 12 plus 2 times 1, 26.
İnci: What is its unit? It is cm^2 for the area and cm for the perimeter, do not forget it. Who wants to come for the second figure? (One student comes). First, complete the table.
S20: Length is 4 cm, width is 3 cm, area is 12 cm^2 and perimeter is 14 cm.
İnci: How did you find that?
S20: From 4 cm times 3cm, area is 12 cm^2. When I sum all the edges, 3 plus 4 plus 3 plus 4 equals 14 cm.
İnci: Yes. As your friends did, we can sum all edges separately or we can multiply equal edges with two and then sum them in order to find perimeter. Who wants to come for the third figure?
S21: Length is 3cm, width is 4 cm, area is 12 cm^2 and perimeter is 14 cm.
İnci: For the last figure? (There are some students who are indicating that all areas are the same).
S22: Length is 6 cm, width is 2 cm, area is 12 cm^2 and perimeter is 16 cm.

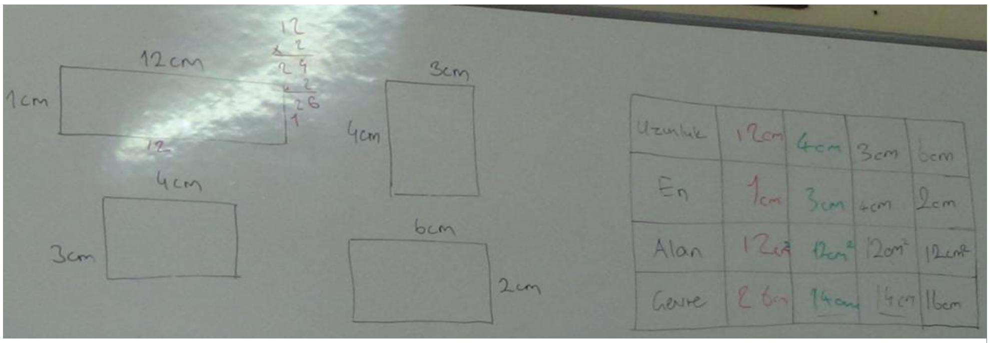

Figure 31 The completed table by the students

İnci: What can we say about the areas of these figures?
Students: Areas of all figures are equal.
İnci: What can we say about the perimeter of these figures?
Students: Perimeters of all figures are different.
İnci: All of them?
Student: Except the 2 of them.
İnci: What can we say about the perimeter of these figures which have the same area?
Students: It can be different or the same.

İnci continued with a question to repeat the subjects they had learnt until that moment and to summarize them through a table. She leaded the students to give the answers which were needed for completing the table and solving the question instead of telling them directly.

She attended to what the students said and went step by step in the direction of their answers. İnci paid attention to call different students to the board so as not to bother the other students and to determine better whether there were mistakes or confusions as they had presumed in the planning phase. She used summative statements about different ways of finding perimeter pointing to the solution ways of the students. Moreover, when İnci drew the figures and wrote the length of the edges on them, some students were deceived by the image. They perceived the second figure as a square and asked whether it was a square or not. Before the teacher's comment, the other students responded their friends and said that the edges were different therefore it could not be a square. Later, İnci emphasized that all edges had to be equal in order to be a square. It was noticed that it was possible for the students to focus only on the image without considering numbers . Here, the students did not have difficulty finding the area and perimeter of the figures and completed the table easily. However, although some of them noticed all areas were equal, some of them did not catch the key point until the whole table was completed.

When İnci heard the answers of "all were the same", she did not focus on this moment, did not query what the meaning was and continued with the solutions of the students who were at the board. Similarly, there were two rectangles which had the same edges but one of them was in a vertical position (4 cm x 3 cm) and the other was in a horizontal position (3 cm x 4 cm). The students did not notice that they were the same, thus, they calculated area and perimeter again. They could not say that the measurement of perimeter and area were the same as in the previous figure directly, because both figures included the same numbers. At this point, İnci could not enable the students to see this point and did not respond the students in a way to reveal their mathematical thinking. Although she made activities about the conservation of area and emphasized rotation did not change area from the beginning of the lesson, she could not transfer the previous knowledge here. She did not provide opportunities for the students to make connection with what they had learned before and she could not enhance students' reasoning as they had discoursed in the planning phase. Therefore, it was seen that this question could not elicit what they intended because it was easy for the students to calculate area and perimeter for these figures, the important point was to notice the hidden relationship. Besides, there was no evidence that students realized the reason why perimeter of these two figures were equal. When the students said that perimeter of the figures with the same area could be the same or different, İnci could not query students' mathematical thinking about the equality of these two figures' area and perimeter. She could not draw students' attention to the equality of the two figures' edge lengths so the students might not have established the relationship between the sameness of area and perimeter in them. She mainly focused on the accuracy of the students' responses and evaluated their mathematical understanding considering these.

It can be argued that İnci's noticing in this dialog could be considered as Level 2 because although she endeavored to attend the answers of the students and responded them in accordance with their answers, she was not good at revealing their mathematical thinking. She sometimes wanted the students to present more details about their mathematical approach through questions but she sometimes did not query their reasoning. She did not give students the opportunity to discuss on the important mathematical points and could not lead them to make more explanations. She mostly maintained the flow that she had in her mind and guided the students to get the answers that she wanted to hear. When she heard the correct answers, she thought that they were enough. Her attention to the students' comments had an evaluative approach rather than an interpretive stance. She focused mostly on the correctness of their responses rather than interpreting mathematical thinking. She asked general questions step by step to provide a clear solution of the problem and had a general approach. Although she tried to get the students' answers and listened to what they said, her guidance to elaborate their mathematical approach was limited. In addition, she could not present opportunities for the students to elicit their mathematical reasoning. Hence, her noticing exhibited the features of Level 2.

Later, İnci went on with a question which asked how many different rectangles with areas of 28 unt^2 could be created. The preservice teachers wanted to ask a different kind of question so as to reveal more mathematical thinking. The following dialog describes the interaction between İnci and the students:

> **İnci:** How many different rectangles can we create using 28 unit squares? Draw the figures in your notebook. Namely, what is the area in the question?
> **Students:** 28 unt^2.
> **İnci:** In other words, how many different rectangles with areas of 28 unt^2 can be drawn? (The students try to find the answer for quite a while).
> **S23:** 3.
> **S24:** 4. (She checks the answer and notices that the student found a wrong number pair such as 9x3. She guides the student asking the multiplication of which numbers is 28).
> **S25:** 6. (When she looks at the student's notebook, she sees that the student wrote 3 correct number pairs and then, he changed the place of width and length for each figure and found 6 as the answer).

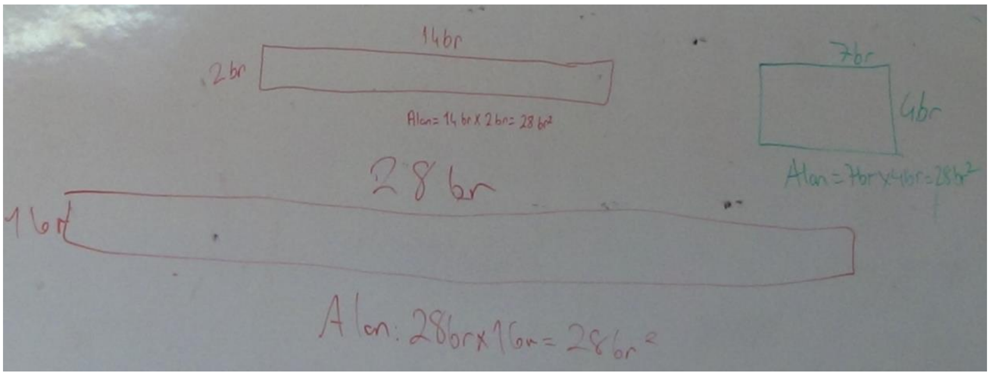

Figure 32 The question related to drawing different rectangles with the same area

> **İnci:** Let's solve it on the board. (She goes on walking around the class and controls what kind of figures students draw). Ok. Come for one of them (She points to one student). You for another and you for the next.

Here, the main aim was to let students create different rectangles with the same area in accordance with the objective of the lesson and think of the possibilities. It was a good question for querying the students' reasoning, however, İnci could not provide opportunities to reveal students' mathematical thinking. She controlled the answers walking around the class and she tried to correct the mistakes of the students individually but she did not consider calling them to the board or did not make an explanation to the whole class about them. She did not respond to the students who gave wrong answers in a way that could elicit how they thought mathematically, she checked what they did in their notebooks instead. Here, she noticed that some students who said the answer as 6 found 3 correct number pairs but later, they found 6 by changing the place of width and length.

When she saw a wrong number pair, she tried to guide the student to see his mistake but she did not make any explanation for changing the place of length and width. Although she emphasized that rotation did not change area and it was the same figure from the beginning of the lesson, it was seen that there were still students who got confused at this point. However, she did not clarify it or did not guide them to think on why the answer was 3 in order to overcome this confusion. She preferred to call three students, who found the correct answer, to the board. After they drew their figures and wrote the number pairs, it remarked that students pay attention to draw the figures in a way that could represent and show the difference between their real dimensions. Thus, it is seen that the students usually pay attention to the image and teachers should consider this point because it may be misleading for them. Although the question was solved, it was unclear whether they noticed the difference between the answer of 3 and 6 and where they made a mistake. İnci followed a way which could not disturb the flow of the lesson and she could not reveal students' reasoning as they decided in the planning phase.

It can be argued that İnci's noticing here was in Level 1 because she was not able to attend to students' reasoning and encourage them to elaborate their mathematical approach. She did not listen to the students long enough and could not respond them in accordance with their answers. Although she noticed some wrong answers, she could not aid the students to make explanations about how they thought. She also interfered with another student when she noticed a wrong number pair for a rectangle but her guidance was very limited. She reacted to the wrong answer by restating the question through keywords such as "multiplication of which numbers". On the other hand, although she encountered some mistakes and confusions while controlling some student answers, she did not consider that the other students may have the same problems and did not endeavor to learn the others' mathematical thinking on these points. She preferred to correct the students individually or not to interfere with some of them, and thus she could not reflect on what she noticed to the whole class. Therefore, she missed the opportunity to determine whether there were the same problems and to lead the whole class to reach a correct approach. She could not elicit the students' mathematical thinking and did not allow them give details about their solution strategies. İnci followed a way which made it possible to reach the solution of the question immediately and considered the correct answers. She mainly focused on what she had in her mind instead of students' mathematical thinking and understanding. Thus, her noticing here was considered as evidence of Level 1.

In the planning phase, the preservice teachers wanted to ask problems including different concepts such as "proportion", and "times" to reinforce the subject. Thus, İnci asked this kind of a question towards the end of the lesson and let the students work on it as they had decided. The following dialog describes the interaction between İnci and the students:

> **İnci:** Let's write another question. The length of a rectangle is 28 cm and the width is 4 cm. If the length decreases in the proportion of 1/2 and the width increases 3 times, what kind of a change occurs in the area of this rectangle? (She gives students some time to work on the problem).
> **S26:** 168.
> **S27:** 2.
> **İnci:** What is 168? Shall we solve it on the board?
> **S28:** It increases $24cm^2$.

İnci: Who wants to come? (She chooses one of them, the student starts to solve it right away). Draw a new rectangle as well as the initial shape. Tell us what you did step by step.
S29: I divided 28 by 2. The new length became 14 cm. Then, I multiplied 4 by 3. The new width became 12 cm.
İnci: What was the area of the initial rectangle?
S29: It was 112 cm^2 from 28 times 4. The new rectangle is 168 cm^2 from 14 times 12. Hence, it increases 56 cm^2.
İnci: Did everyone find the answer as 56 cm^2.
Students: Yes.

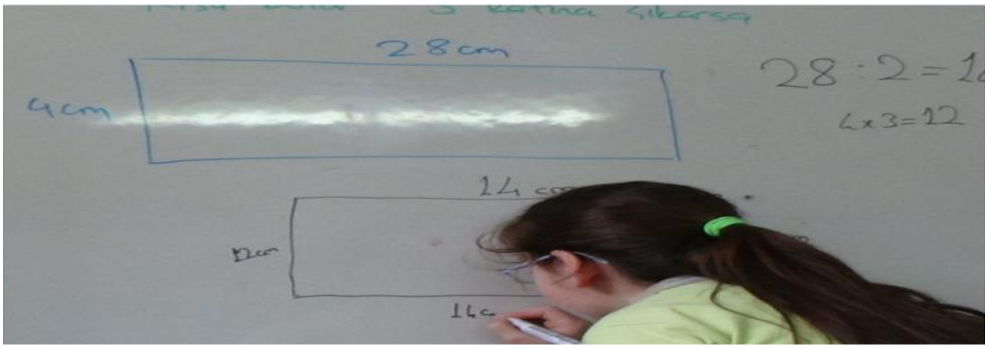

Figure 33 The question regarding calculation of the change in area

İnci wanted students to make some mathematical change on the edges and determine the difference between the initial and new rectangle in terms of area. Most of the students did not have difficulty in solving this problem, unlike the preservice teachers' anticipation. There were different answers such as 168, 24 and 56. However, she ignored the wrong answers and did not respond them in a way that could reveal their mathematical thinking. For example, the student who found the answer as 2 had calculated the new length as 14 and the new width as 12 correctly but she had found the difference between the edges (14-12=2) instead of the difference between areas as indicated in the problem. However, İnci could not notice the student's mistake because she did not query how the student reasoned and what kind of a path the student followed. She missed the opportunity to realize what the student understood from the problem and to correct the mistake. Presumably, 168 was the area of the new rectangle and student S26 forgot to find the difference or could not understand the meaning of change but İnci could not consider them. On the other hand, after she determined a student who found the correct answer and called him to the board, she wanted the students to give more details about her solution and listened to the student's mathematical approach. However, she was in tendency not to wander from the main flow even though she heard incorrect responses. She attended to the intended answer and mathematical approach behind it but she did not focus on the reasons behind the wrong answers. When student S29 explained her response step by step and clearly, İnci did not need to repeat the solution as she had checked whether there were different answers. Her question was superficial because some students had given different answers at the beginning. Yet, İnci liked the solution and explanation of student S29 and there was no hesitation or objection from the other students. She interpreted this situation as an indicator of their understanding and proceeded to the next question.

İnci's noticing in this dialog can be classified as Level 2 because her attending to students' ideas and her responses promoting the students to reveal their thinking were limited. It is seen that she did not focus on the wrong answers and did not try to learn how the students found 168 or 24, but she paid attention to request more explanation about the student's solution strategy from her at the board. İnci did not indicate what was intended or did not make the students see the implicit knowledge directly and she listened to what the student at the board said. Although the wrong answers might have been a signal of a possible gap in students' understanding of area or the concepts of proportion and times, İnci did not consider them. She could not provide opportunities for these students to present details about their mathematical approach and hence, she did not do anything to correct these mistakes. She exhibited an evaluative approach rather than an interpretive stance. For example, she evaluated the explanation of student S29 as good and decided not to say more on it. Her response and leading was broadscale. Thus, her noticing here further exhibited the properties of Level 2.

İnci continued the flow of the lesson and asked a question which involves placing squares in a rectangle. The preservice teachers wanted to make the question a little bit more difficult and reveal different solution strategies through it. The following dialog describes the interaction between İnci and the students:

> **İnci:** Another question. How many squares with an edge of 3 cm can be drawn in a rectangle whose length is 24 cm and width is 12 cm without leaving any blank? (She allowed them to solve it in their notebooks first).
> **S30:** 24.
> **İnci:** Did everyone find 24?
> **S31:** 32.
> **İnci:** Is there anyone who found 32? Who wants to solve it? Ok. You come. How did you solve the question?
> **S31:** In order to find the area of the rectangle, we multiply 24 by 12, it is 288 cm^2. Then, we multiply 3 by 3 because all edges a square are equal and area is 9 cm^2. We divide 288 by 9 and find the answer as 32.
> **İnci:** Draw the figure. How many 3s are there in 24?
> **S31:** 8.

İnci: Then, I can break it into 8 pieces here? (She pointed the length which is 24 cm). How many 3s are there in 12?
S31: 4.
İnci: Ok. All pieces are 3cm?
S31: Yes.
İnci: There are 8 pieces here (She pointed the length) and there are 4 pieces here (She pointed the width). It is 32 from 4 times 8. It is the same as what you found, right?
Students: Yes.

İnci asked the question and waited for the students to find the answer for a while. Most of the students could solve the problem and obtained 32, however, there were different answers as well. İnci asked a question "Did everyone find 24" but her statement was a little misleading on behalf of the students because the answer was not 24. She did not pay attention her discourse and did not research the reasons behind the wrong answer. She did not ask the student S30 how he solved the problem and how he thought mathematically. The mistake of S30 here was that although he found the area of the rectangle as 288 cm^2 multiplying 12 by 24, he calculated the perimeter of the square, which is 12, instead of area. Hence, he divided the area of the rectangle by the perimeter of the square and found the answer incorrectly as 24. However, İnci could not notice the mistake because she ignored the answer and called another student, who gave the correct answer, to solve the question at the board. Doing this she missed the opportunity to determine where S30 made a mistake or had a confusion, as a result she missed the chance to correct it.

She allowed students S31 to explain his solution and listened to his mathematical approach step by step. However, she did not respond to him in a way that could reveal his mathematical thinking. Why he multiplied 24 by 12 and why he divided 288 by 9 were not made clear. She did not query his reasoning or did not try to make sense of it and accepted the correct answer as an indication of understanding. However, giving the correct answer does not show that it was really understood. The students may have memorized these kinds of questions but not internalized them. Hence, trying to implicit their mathematical thinking is important. İnci did not control whether the other students realized the solution or did not try to learn alternative paths used by the other students. After student S31 explained his solution, she attempted to visualize the solution to make it more comprehensible for the class and presented it in a different way. Although she asked questions in a way that the students would understand what was in her mind, she was deficient in elaborating why she did it in this way.

İnci's noticing was classified as Level 2 because she mainly had a mixed approach. She wanted one student to make an explanation about the solution and cared for listening to ideas but she ignored the wrong answer of another student and did not try to learn how he thought mathematically. Hence, she missed the opportunity to determine what kind of a mistake was made and learn the reasons behind the answer. On the other hand, she listened to the student's solution strategy and tried to learn her mathematical approach but she did not respond in a way that could reveal the details of her mathematical thinking sufficiently and so she could not make sense of her mathematical understanding. Therefore, her approach was further evaluative in terms of accuracy of the answer, she was not able to interpret the student's reasoning. In addition, she focused on what was in her mind as well and she tried to indicate her solution approach instead of eliciting their thinking. Therefore, her noticing in this dialog mostly exhibited the properties of Level 2.

When the whole process of teaching was considered, İnci's noticing predominantly exhibited the features of Level 2. Her focus was on the students' mathematical thinking and understanding. She listened to their ideas and responses instead of indicating what she had in her mind. Although she endeavored to attend to the students' reactions, her responses that could reveal their thinking were limited. She queried the reasons behind their answers but she did not pay attention to their explanations. She focused on both particular students and the whole class and paid attention to follow the lesson plan correctly. Her approach was evaluative with an interpretive stance in nature. She evaluated the correctness of the students' responses. She was inconsistent in revealing the details of their mathematical thinking and responding them in such a way. Therefore, she sometimes missed the chance to correct the students' mistakes or did it in basic ways. İnci's noticing during the teaching phase was further in Level 2.

4.1.3.3 What and how İnci notice in reflecting phase

Teaching of the third lesson plan was made by İnci in 5/F classroom. The third lesson plan included the objectives of "Being able to create different rectangles with the given area" and "Solve problems that require to calculate the area of a rectangle". While İnci was implementing the lesson plan, the other preservice teachers, cooperating teacher and researcher observed this lesson and took notes. After teaching was completed, the reflecting meeting occurred immediately at the school conference room. In the reflecting phase, preservice teachers were wanted to reflect their thoughts about the lesson including; what worked well, what did not work, what the difficulties were, what they observed about the students' actions and thinking and also what should be changed about the lesson to improve it and what the rationales for the changes were.

This phase began with the cooperating teacher's brief comments and she reflected her thoughts on the lesson. The cooperating teacher indicated that the activities were enough and appropriate in terms of the objectives of the lesson, and understanding of the students was good, thus, there was no need to change anything in the content of the lesson. She emphasized that seeing perimeter and area together consistently was important in terms of understanding them better and this lesson made it possible. She stated that making students learn the meaning of area with unit squares took time but enabled concretization. She indicated that she liked incorporating different mathematical concepts such as "fractions", and "times" into problems because it helped to evaluate their previous learning. However, she suggested removing a question towards estimation because the students did not know multiplication of decimals and they would not be able to compare them by finding the real results. She also offered to solve more questions if any time was left. She evaluated the content of the lesson as appropriate and enough in terms of enabling the students to reach the objectives. She indicated that the activities and questions were successful to provide better understanding through visualization and concretization and to reinforce the subject.

In accordance with discussion protocol, İnci shared her opinions about the lesson that she taught, evaluated the effectiveness of the lesson plan and made some suggestions to improve it. The following expression shows what and how İnci noticed during the teaching phase.

> **İnci:** I started the lesson through reminding what we had taught in the previous lessons. I noticed that one student said "4a" for the area of a square. Indeed, he said the area formula of a square at first and the equality of all edges of a square. However, when I tried to emphasize what he said, he might have supposed that I was asking something else. I expected the answer of "a times a", but I think I forced him for "4a" unwittingly. Later, as we had guessed, when I rotated the rectangle in a vertical position, the answers of both "area changed" or "area did not change" were given in the first activity which was related to changing the position of the same rectangle. The students might have seen that it had a bigger area as a lateral, therefore, they might have thought that its area became bigger. One student said that the edges changed so the area changed. Because the length of the rectangle in the first position became the width in the second position and vice versa here, he perceived the figures as different rectangles. Therefore, some of them got confused and they said that area changed. However, when I drew their attention on the rectangle's edges' remaining the same in the activity, I saw that they understood what we wanted them to gain.

When the whole statement was considered, İnci's noticing was accepted to be in Level 3 since she focused on the students' responses and tried to make sense of how they think mathematically. She referred to specific moments and the students, and identified the noteworthy events (e.g. "I noticed that one student said "4a" for the area of a square" and "the answers of both “area changed” or “area did not change” were given in the first activity). She explained why the student might have given such an answer. Further, she used details to make inferences about their mathematical understanding referring to their answers. She endeavored to reason on students’ mathematical thinking and used mostly interpretive comments such as “The length of the rectangle in the first position became width in the second position … some of them got confused and they said that area changed”. She noticed the confusions of the students in some activities, elaborated how she tried to diminish their incorrect approaches and also emphasized that expected answers were given by the students. Focusing on particular students’ approaches, referring to the remarkable actions, explaining students’ thinking and providing evidence to support her comments show a shift from Level 2 to Level 3, therefore, in this expression her noticing mainly had the features of Level 3.

> **İnci:** I gave the definition of conservation of area. Later, I called some students to the board and wanted them to form different figures with unit squares. The students were good at creating a rectangle whose area was 16 unt^2 by sticking unit squares on the board and also creating a square from this rectangle using the same unit squares. They could make this transition and did not have a problem. In another activity, there was a table to be completed. It involved writing the perimeter, area and the length of edges. The students stated that area was the same for all figures, it was good and they also understood that perimeter could be the same or different for the figures which have the same area.

She focused primarily on teaching and students’ mathematical understanding. She had a tendency to describe what she did step by step while teaching the lesson in a general manner (e.g. “I gave the definition of conversation of area” and “I called some students to the board”). She also made evaluative comments but she provided little or no evidence about her assessment in terms of students’ mathematical thinking. She reflected common features of the lesson and mentioned some specific moments but she continued referring to the whole class. She indicated that the activity served the purpose of the lesson because it made the students feel that there could be different figures which had the same area. The statement of “The students stated that area was the same for all figures” showed that she accepted it as evidence of her claim. Her comments were less related to students’ mathematical thinking and unspecific in nature. Her elaboration was inconsistent and insufficient. Her noticing was considered as evidence of Level 2 with respect to common features of this level.

> **İnci:** In the question requiring to create different rectangles using 28 unit squares, the students gave the answers of 1 and 28, 2 and 14, 4 and 7, therefore, 3 different rectangles were formed as it should be. However, there were students who said that 6 different rectangles could be formed. I examined these students’ figures in their notebooks, looked at what they wrote and noticed that they changed the place of width

> and length and therefore they found the answer as 6. However, most of the students found 3 rectangles in general. Nevertheless, they were aware that there were 3 pairs but they were not sure about whether they should accept it as 3 or 6. Therefore, different answers were given but we cannot say that what they made is completely wrong.

In terms of what to notice, İnci focused on particular students' answers and explained why they gave answers like that. She referred to the important events and indicated the students' common mistakes and she described what they did. She also endeavored to make sense of their mathematical thinking and made references to evidence of their solution to give details and inferences. Her comments had an interpretive stance in nature. She attempted to present details to support her thoughts. Hence, her noticing here was considered as Level 3.

> **İnci:** … there was no difficulty in the next question. They could find the new length of the rectangle quite easily after it decreased in the ratio of $\frac{1}{2}$, they had calculation errors only. Later, in the question related to covering the rectangle with squares, the students found the answer as 24 but I could not understand how they got it.
> **Mehmet:** I looked at one of the students' notebook. He found the area of the rectangle correctly, then he found the perimeter of the square and divided the area of the rectangle by the perimeter of the square. Most of the students solved it like this, I think. A few answers as 32 were given by some students as it should be, however, the students who found 24 were in majority. According to me, they did not understand the problem and did not realize that it included covering and thus the area was asked, or they might have confused the concepts of area and perimeter again.
> **İnci:** Yes, you are probably right. I lost too much time in the question including formation of different rectangles with areas of 28 unt^2.
> **Semih:** Yes, you lost 15 minutes. You should not wait for the students so much.
> **İnci:** I wanted to ask a question related to daily life which required finding the area of each room in a house, but I had no time so I had to skip a few questions.

Her focus was on the process of teaching and she conveyed the setting of teaching directly. She handled a question and evaluated students' general approach on it. She also focused on their solution but did not make reference to the details of what kind of errors they made. Her comments were so overall and descriptive. Furthermore, she did not make sense of what the students tried to do while solving one of the questions (e. g. "the students found the answer as 24 but I could not understand how they got it"). She did not try to analyze and interpret students' mathematical thinking because she did not understand what they did. Later, her focus shifted from the students as a whole to her own teaching. She focused on how she used time in the lesson and which questions she missed, different from the lesson plan, namely, she addressed less mathematical aspects of the lesson. On the whole, she focused on a range of issues from "time of the lesson" to "on-task behaviors of the students" and presented a brief description of a part of the lesson. She provided little or no evidence to support her comments. Therefore, this quote reflected her noticing had further evidence of Level 1. On the other hand, Mehmet's comment drew attention. While İnci indicated that she could not understand how the students solved the problem and found 24 as the teacher of this lesson, Mehmet expressed a point that he noticed about it during observation. He described what the student did step by step to give details for drawing inferences and endeavored to make sense of how mathematical thinking of the student was. He reasoned out the student's solution and made interpretive comments. Here, it was understood that the preservice teacher who conducted the teaching phase might not have noticed every noteworthy event due to many factors such as following the lesson plan, maintaining teaching and classroom management whereas the preservice teachers who observed the lesson had many chances to focus and notice many issues. In addition, one learned from the others' noticing as a result they completed each other's deficiencies while discussing.

After the reflecting phase, individual interviews were conducted with the preservice teachers to reveal missing points and detail their noticing. What İnci indicated differently from the reflecting phase was as in the following.

> **İnci:** When I asked whether area changed or not after rotating and relocating the figures, some students indicated that it changed. Indeed, it was an expected answer for us but I asked the reason so as to understand their thinking. They gave answers like "the figure became bigger", "the edges changed" or "it turned into another figure". When we created a triangle from a rectangle an interesting answer was given "one figure had four edges while the other had three edges". We showed finding area of a rectangle and square until then. They knew that a rectangle and square had four edges, the opposing edges were equal in a rectangle and all edges were equal in a square and area was found by multiplying two edges. However, a triangle consisted of three edges and they did not know how to calculate the area of a triangle. He might have had difficulty in deciding which edges to multiply or he might have concluded that the edge became longer in the horizontal and accordingly its area increased.

İnci's focus was on her own teaching and particular students' mathematical thinking. She noticed the students' mistakes and she indicated the students' answers like "the figure became bigger" or "the edges changed" directly, particularly failing responses, which she noticed in order to explain their approach. Regarding how to notice, she endeavored to reason on the students' thinking strategies and referred to specific events from her observation using details to draw inferences. Her comments were mainly interpretive and had specificity to support her claims. Her noticing was remarkable because it reflected a variety of mathematical thinking of the students and how the students might have confused these approaches. She also emphasized that they encountered the reactions that they expected while planning the lesson.

Because she focused on specific students' thinking and important mathematical points and elaborated her analyses supporting through examples from the lesson, her noticing was considered as evidence of Level 3.

> **İnci:** One student said that the rectangle transformed into square. He realized that we created one figure from another figure without making any change, therefore, the area did not change. I think, he explained it well. Later, I could use this approach and I stated that a parallelogram formed from a trapezium. In addition, one student solved a question and wrote the measurement of area into the figure, it was good. When I created different figures I emphasized that the interior region reflected the area. Thus, this situation showed the student's mathematical understanding was good because if she had not understood the relationship between the interior region and area, she could not have written the answer into the figure. Besides, in the question involving creation of different rectangles using 28 unit squares, one student's answer was 4 different from the others. I paid attention to it because mostly the answers were given as 3 or 6 in the class. I controlled how he solved the question looking at his notebook. I saw that he found a wrong number pair in addition to 1x28, 2x14 and 4x7. He might not have checked by making inverse operation whether 28 could be obtained with multiplication of these number pairs. He probably said 4 since he did not notice one of them was incorrect because of hurrying up to give the answer.

In this excerpt, it was understood that İnci's focus was predominantly on specific students and their mathematical thinking, misconceptions and mistakes (e.g. "one student wrote square unit instead of length" and "he found a wrong number pair in addition to 1x28, 2x14 and 4x7"). She highlighted the notable events describing what the students did in terms of mathematical aspects to support her claims. She made connection between their mathematical thinking and what were taught and she also evaluated the effect of the lesson (e.g. "if she had not understood the relationship between the interior region and area, she could not have written the answer into the figure"). Her comments were mostly interpretive and detailed in nature (e.g. "He might not have checked by making inverse operation"). She attempted to interpret how the students might have thought. She referred to the students' solutions and mistakes and presented evidence from what she noticed to explain their thinking and to support her claims. She also indicated common reasons for the students' mistakes as their hasty behaviors. Thus, her noticing mainly exhibited the features of Level 3.

> **İnci:** I focused on whether the students understood the subject or not and their answers because I would maintain to shape the lesson according to their feedbacks. I endeavored to emphasize the important points that we discoursed on while planning the lesson. I controlled whether the students could solve the problems, where and why they made mistakes. I focused on how I could lower to their levels and how I could teach the lesson in the simplest terms.

İnci summarized what she generally focused on during the lesson in the end. She indicated that she paid attention to the students' mathematical understanding and thinking, their responses and mistakes. Her comments proved her noticing here to be in Level 3.

When İnci's all comments during the post-teaching phase were considered, her noticing predominantly exhibited the prevalent properties of Level 3. In terms of what was noticed, even though İnci sometimes referred to issues such as classroom management, duration of the lesson, the students' behaviors, she focused mostly on her own pedagogy and further mathematical thinking and understanding of the students. She usually paid attention to what specific students said and did rather than referring to the class as a whole and highlighted the noteworthy events and interactions in terms of mathematical aspects of the lesson. Regarding how she noticed, İnci chiefly indicated her ideas on students' possible thinking ways and tried to reason on how the students thought mathematically making references to the evidence from the lesson. She used details to draw inferences about the students' understanding and elaborated students' approaches and confusions. She also endeavored to make sense of students' mathematical thinking. She made descriptive comments to present details of the lesson and evaluative comments to indicate her approach. However, her comments to provide justification about what she noticed were further interpretive in nature. Therefore, although her some comments exhibited the features of noticing in Level, Level 2 and Level 3, her noticing was further considered as evidence of Level 3 during the reflecting phase.

4.1.3.4 What and how İnci notice in re-reflecting phase

After the third lesson plan was implemented by İnci in the teaching phase and the preservice teachers discussed on what should be changed or paid attention for the next implementation, re-teaching of the third lesson plan was made in another classroom, 5/G, by Hasan. Since the aim was to focus on case of İnci here, Hasan's teaching was not presented. While Hasan was implementing the lesson plan, the other preservice teachers, cooperating teacher and researcher observed this lesson and took notes. In the re-reflecting phase which was conducted just after re-teaching, the cooperating teacher and all preservice teachers were requested to reflect their thoughts about the lesson including; what worked well, what did not work, what the difficulties were, what they observed about the students' actions and thinking and also what should be changed about the lesson to improve it and what the rationales for the changes were.

This phase began with the comments of the cooperating teacher briefly and she reflected her thoughts on the lesson. The cooperating teacher indicated that the students who normally did not attend to the lesson were active in this lesson. She also stated that the subject was understood because the students participated in the lesson and could solve the questions. She expressed that repetition of the concepts of perimeter, area and their units constantly was good. On the other hand, she pointed to the loss time in one question related to creating different rectangles which consisted of 28 small unit squares. She suggested allowing one of the students, who had found the answer, to draw one rectangle on the board instead of waiting for too long. Thereby, it could lead the other students to understand what was required and they could continue to find the other rectangles. She also emphasized that the students did not think the multiplications of which numbers were 28, they counted small unit squares and it was necessary to make the students think on it. She indicated the lesson plan was good enough and did not offer any change in terms of the content and implementation.

In accordance with discussion protocol, İnci shared her opinions about the lesson that she observed, evaluated the effectiveness of the lesson plan and made some suggestions to improve it. The following statements show what and how İnci noticed during the re-teaching phase.

> **İnci:** Hasan made a repetition at the beginning of the lesson, it was good. When he gave a rectangle in the horizontal position, he required the students to find area and he wrote the value inside it. He did this to emphasize because area corresponded to the interior region. He rotated the rectangle in the vertical position and asked what happened in the new situation. The students said it was the same, but what remained the same or what they indicated were not clear. They might have addressed the figure not area. Hasan should have emphasized what happened to the area because answers were given like "the measurements were the same" or "the figure was the same" differently from what we had expected. They noticed that the edges did not change but they did not associate it to area. Hasan also should have stated that area remained the same because the edge lengths did not change. In the second activity, Hasan attached the rectangle on the board, one student immediately said that it was composed of two triangles. One student perceived the figure as an equilateral triangle although there were no properties supporting this idea and queried whether it was or not. I noticed that the students were in tendency to label something without considering the necessary criterions. When Hasan asked the class to make comments about the areas of the figures comparing the old and new figures, none of them said that area changed, they noticed area remained the same and explained their thinking with the figures' being composed of the same pieces.

In this excerpt, İnci predominantly attended to teacher's pedagogy, students' mathematical answers and reactions (e.g. "Hasan made a repetition at the beginning of the lesson" and "One student perceived it as an equilateral triangle"). Although she had focused on a few individual students and their mathematical thinking, she continued to address the class as a whole saying “they” and “the students” (e.g. “I noticed that the students were in tendency to qualify something without considering the necessary criterions”). She mostly made evaluative comments which had an interpretive stance and although she indicated preliminary interpretations of students’ mathematical approach, she did it overall (e.g. “They noticed that the edges did not change but they did not associate it to area”). She indicated her ideas about the behaviors of the teacher and the implementation of the lesson. She also gave advices on what should have been done then and what should be done later. She endeavored to provide justification about some events but these comments were not sufficient enough. She sometimes made reference to the evidence to support her analyses but usually, she maintained a broad and evaluative approach. Therefore, she was inconsistent in offering details. Her noticing in this quotation was considered as evidence of Level 2.

> **İnci:** Some students confused perimeter and area again and found perimeter, not area. Students were mostly unaware of this situation because the important thing for them was to get the answer. Thus, they did not focus on whether area or perimeter was asked, instead they wanted to give the answer quickly no matter if it was correct or not. There were a few students who forgot to use the units or wrote incorrect units. Hasan reminded them not to forget units in each time and tried to emphasize the relationship between units. It was good. I think there is a connection between use of inappropriate units and the confusion of perimeter and area. If the students had understood this relationship, they would not have made mistakes while using them. I think they will internalize the subjects in time by solving questions.

In this excerpt, İnci’s main focus was on students’ mathematical approaches and teacher pedagogy. She noticed that the confusion of perimeter and area still continued although it was the third lesson and the students repeated them in each lesson. She thought that the students needed time to internalize the subject. She endeavored to explain the reason of calculating perimeter instead of area but she did in a brief manner. She associated it to students’ tendency to find answer without paying attention what was asked and to focus on the numbers instead of providing details of their mathematical strategies. Similarly, she evaluated students’ understanding but she did not elaborate her assessment (e.g. “If the students had understood this relationship, they would not make mistakes as using them”). Her comments were evaluative with interpretive stance and she offered broadscale impressions about the whole class. Although she referred to the noteworthy events, her elaboration on the basis of the evidence from her observation was limited. Therefore, her noticing here was considered as evidence of Level 2.

> **İnci:** When Hasan wanted the students to create a rectangle through 16 small unit squares, one of the student said that 6 times 2. The mean of 16 unit squares was area of the rectangle but this student did not consider it. He thought of perimeter rather than area and calculated perimeter as 6 plus 2 is 8 and 8 times 2 was equal to 16. Therefore, he gave an answer as a rectangle in the form of 6x2 and made mistake. He confused perimeter and area and did not notice what was asked to find. In the question related to creating different rectangles of which areas were 28 unt^2, the students found incorrect answers. The student who said 7 as answer found 3 correct rectangles, changed the place of width and length and found as 6 rectangles, then she calculated perimeter as well as area and found 7 rectangles. It showed that some students continued to confuse both concepts, yet, the number of them were a few.

In terms of what was notice, İnci primarily focused on particular students and their mathematical thinking (e.g. “The mean of 16 unit squares was area of the rectangle but this student did not consider it”). She predominantly attended students’ solution strategies. Regarding how to notice, she emphasized mathematically important points and drew attention to the remarkable events. She initially attempted to reason on what she observed. She explained how the students might have thought as solving the question and gave justifications about their answers (e.g. “He thought of perimeter rather than area and calculated perimeter as 6 plus 2 eight and eight times 2 was equal to 16”). She provided details from her observation to support her analysis about students’ mathematical thinking. She stressed that the students’ mistakes resulted from the confusion of the concepts of perimeter and area. She made mainly interpretive comments as well as some evaluative comments with an interpretive stance (e.g. “It showed that some students continued to confuse both concepts” and “I liked his approach because the students immediately understood what he intended”). Therefore, her noticing exhibited prevalent features of Level 3.

After the reflecting phase, individual interviews were conducted with the preservice teachers to reveal missing points and detail their noticing. What İnci indicated differently from the reflecting phase is as in the following:

> **İnci:** Hasan implemented the lesson plan well, he could not solve only one or two questions, like me. Both of us spent a lot of time in the question related to creating different rectangles with 28 unt^2. In the question which includes completing a table, Hasan confused the concept of width and length, it was probable that he had an absent state at times. The important point was that the students noticed his mistake and corrected it immediately. It showed that they knew these concepts well. Besides, Hasan leaded the students well even if some incorrect answers were given by the students. I tried not to miss the answers and reactions of the students. I can say that the implementation of the lesson was good. I focused on how the reactions, answers and confusions of the students were rather than the implementation of the teacher.

In this quotation, İnci focused on a range of issues including implementation of the lesson, teacher pedagogy, time, mistakes of the teacher and students' learning (e.g. "Hasan implemented the lesson plan well" and "They knew these concepts well"). She offered her general impressions without providing details from her observations to support her analyses. Her comments were descriptive and evaluative in nature. She referred to the whole class rather than attending to particular students' mathematical thinking or she did not interpret what she observed in terms of students' mathematical approach. Thus, her noticing here was further in Level 1.

When all comments of İnci in the re-reflecting phase were considered, her noticing predominantly had the features of Level 2. On the whole, she focused on the students' mathematical thinking as well as the issues of teaching and teacher's pedagogy. She further referred to the whole class although she attended to particular students and endeavored to make sense of their mathematical approach. She made quotations from the students' answers and described what had occurred in the lesson. She evaluated teaching and student learning with an interpretive stance and made some suggestions. She was inconsistent in providing details about her claims. Although she tried to reason through the students' strategies, she made basic comments without details. There were some comments reflecting the properties of Level 1, Level 2 and Level 3 but her noticing in the re-reflecting phase was further considered as Level 2.

4.1.4 Lesson study cycle 4

4.1.4.1 What and how İnci notice in planning phase

The fourth lesson plan was mostly towards prisms, open and close forms of them and surface area. The objectives which the fourth cycle included were "Being able to recognize a rectangular prism and identify its essential features", "Draw surface developments of a rectangular prism and decide whether different surface developments belong to the rectangular prism or not" and "Calculate the surface area of a rectangular prism" Before planning the lesson the preservice teachers went to the practice school to get help and suggestions about the subject from the cooperating teacher. They listened to her ideas, took notes and asked the questions coming to their minds. The cooperating teacher mainly indicated that the students had difficulty in drawing the prisms, so open images and close forms of prisms should be drawn together by the students step by step. She suggested us to show what would happen when prism was opened vice versa what kind of a prism would constitute when an open figure was closed. She stated that students could not understand how to proceed to three dimensions, thus, relationship between dimensions and concepts such as perimeter, area and prism should be given well and students should feel this process. She advised us to use computer programs and concrete materials to show three dimensions better. All preservice teachers met to prepare the fourth lesson plan at the university seminar hall bringing the various sources such as mathematics textbooks, teacher's guide book, curriculum and different mathematics education books. Below are presentations of examples based mainly on İnci's expressions from the planning meeting of lesson study cycle 4.

While the preservice teachers were discussing on the lesson plan, İnci focused on some points such as naming prisms, differences among prism types, students' specific confusions and perceptions. She drew the others' attention on these issues and explained her opinions. Other preservice teachers agreed with İnci's interpretations and they talked about what should be done. Interaction among the preservice teachers is as in the following:

> **İnci:** I noticed that the book focused on how to name the prism. We should show that we name a prism according to its base. Some of the students may try to name a prism considering different faces. On the other hand, teaching how to name a cube may be confusing because it is an exception to the rule. For example, in cube, one student may think that each face is a square so it must be a tetragonal prism according to its base but it has a special name. We need to show the differences among all three types of prisms and their naming. Students may not understand the difference between them so it is an important issue for me.
> **Mehmet:** We can give rectangular and tetragonal prisms and promote the students to compare what is different so that they will see that the bases are different and there is a relationship between the figures on the bases and names of the prisms. Then, we can do the same for a tetragonal prism and cube with more explanation.

In this excerpt, İnci's main focus was on students' mathematical thinking. She referred to students' specific mathematical approach related to naming prisms (e.g. "For example, in cube, one student may think that each face is a square so it must be a tetragonal prism according to its base"). She also made justifications about why students might think as she indicated. For example, she associated the incorrect naming and confusion in cube to its being an exception to the rule. She endeavored to reason on students' strategies and making sense of their thinking. She provided details about her claims and exhibited an interpretive perspective. She referred to the noteworthy events and how specific confusions might arise. Therefore, her noticing had the prevalent properties of Level 3.

The preservice teachers talked about the relationship among prism types and how students were likely to perceive them. İnci shared her ideas and suggestions about this issue. The following are her comments:

İnci: One of the important points is that we name a figure as a prism because lateral faces are a rectangle. Thus, students may use this knowledge for naming a prism and instead of naming a figure according to its base, some of the students are likely to consider the object as a rectangular prism since lateral faces are rectangles.
Semih: Yes, there will be students who interpret it like that.
İnci: On the other hand, the base of a tetragonal prism is a square and all faces of a cube are squares, therefore students may not understand how these objects are also rectangular prisms.
Mehmet: They will probably not be able to make a connection between them if we do not help them.
İnci: Some students may think that a tetragonal prism and cube are not the same types of objects with rectangular prisms because of not perceiving that square is a kind of rectangle. We should make them notice this inference by querying so that they can accept both a tetragonal prism and cube are also rectangular prisms, namely, a special type of a rectangle prism.

In terms of what was noticed, İnci centrally focused on students' mathematical thinking and perceptions. She referred to students' possible thinking ways and how to consider them in teaching (e.g. "Instead of naming according to its base, the students are likely to consider the object as a rectangular prism since lateral faces are rectangles). Regarding how noticing was, she pointed the important issues in terms of highly mathematical aspects of the lesson and she did that in elaborative ways. She offered justifications about her ideas and suggestions in detail. For example, she explained the reason behind the incorrect perception about a tetragonal prism and cube as the lack of knowledge about the relationship between a rectangle and square. She endeavored to make sense of students' thinking and reasoned on how their mathematical approach might have been. She made reference to the details about her comments to promote her claims. Hence, her noticing exhibited the prevalent features of Level 3.

The preservice teachers thought that students had difficulty in understanding and visualizing the prisms in their minds because they were three dimensional. Thus, they felt that there was need for concretization with models and linking to daily life in order to maintain better understanding. The dialog on these issues is presented as below:

İnci: We can start the lesson with repetition of the previous subjects, as usual. Then, we will introduce a rectangular prism and show its elements such as edges, vertexes, face, height, and base firstly.
Semih: Ok. We can show them on the model and then draw them on the board by writing the names.
İnci: Yes. We can also associate the rectangular prism model to the classroom. For example, after we show edge on the concrete model, we will want a student to show the edge in the classroom and we will do it for all the other elements.
Hasan: I think this association is very good.

İnci: Thus, we will also have presented an example from daily life for better understanding.

In this quotation, İnci primarily focused on pedagogical issues and students' mathematical understanding. She referred to the concepts such as repetition, daily life and concretization. Her comments were generally on teaching and student learning issues (e.g. "we will also have presented an example from daily life for better understanding"). She mentioned the important events but her approach was quite broad. She offered descriptive and evaluative comments in nature (e.g. "We will want a student to show the edge in the classroom and we will do it for all other elements). She provided few details about her ideas and did not relate to students' thinking. Thus, her noticing further had the characteristics of Level 2.

During the planning phase, preservice teachers made many suggestions to improve the lesson plan. İnci tried to think on behalf of students and suggested using different colors for some faces to show equality and help student understanding. Interaction among the preservice teachers is as in the following:

İnci: We talked about there being a difference on the bases and lateral faces in terms of the figures. I think we should use different colors to point these differences because students may have difficulty in understanding the concepts of base and lateral face and which faces were equal to each other on the prism. Coloring in this way may reduce confusion.
Semih: Yes, we cover bases with cardboard in the same color and lateral faces have the same color but in a different color from the base.
Hasan: But students may confuse which face is the base or ceiling.
İnci: Both of them are bases since it is the same when the figure is rotated. We do not use the concept of ceiling. It actually causes confusion.
Hasan: How will we mention about it?
İnci: Lower base and upper base, if we need to, but both are bases.
Hasan: Ok.

In terms of what to notice, İnci focused predominantly on students' mathematical understanding and pedagogical issues. She referred to students' difficulties and confusions but she did it in a general way (e.g. "Students may have difficulty in understanding the concepts of base and lateral face and which faces were equal to each other on the prism"). Regarding how to notice, although she made comments on the noteworthy points, her approach was broad. She was inconsistent in providing details about her claims. For example, she did not explain what kind of confusion might arise whereas she mentioned where students might have difficulty. However, she did not elaborate her comments sufficiently. Her comments were evaluative and also interpretive in nature (e.g. "Coloring in this way may reduce confusion"). She did not provide justification about some of her interpretations. For example, why she thought that using different names for faces on the bases might cause confusion was not detailed. On the other hand, she warned her friend in terms of a teaching issue and tried to complete the lack in his knowledge. She mostly attended to teaching and learning with a general approach. Thus, her noticing here was considered as evidence of Level 2.

As planning the lesson, preservice teachers tried to pay attention to some issues in terms of not only providing better understanding but also not causing confusion. İnci indicated some of her concerns about the decisions related to teaching of the subject and the other preservice teachers also shared their own ideas. Interaction among the preservice teachers is as in the following:

> **İnci:** We discoursed that bases would be in the same color, lateral faces would have the same color differently from the base. It came into my head that difference in colors might cause a confusion while finding the surface area. Some of the students may think that bases have a different color so they will not be included in calculation.
> **Semih:** Surface area includes the sum of all faces. All of them are faces although they have different colors or names. The colors will help to understand which faces are equal to each other. Therefore, I do not think that the students will have a confusion like you mention.
> **Mehmet:** I am not sure. It may cause confusion.
> **Semih:** But we will explain why we use different colors and what they represent. I think using the same colors will make understanding difficult, to the contrary.
> **İnci:** On the other hand, there may be some students who think that when a tetragonal prism is rotated, the rectangle will become a base instead of the square and the object will turn into a rectangle prism. We should emphasize that rotation will not change the object.

In this excerpt, İnci's main focus was on students' mathematical thinking and their specific confusions. İnci referred to how some students' possible reasoning might be in an elaborative way (e.g. "Some of the students may think that bases have a different color so they will not be included in calculation"). She tried to make sense of their thinking and explain why these kinds of confusions might arise. For example, she linked the mistake in naming of prism with the confusion deriving from rotation and she also associated confusion in surface area to different colors. She gave details about her claims with a predominantly interpretive stance. Hence, her noticing exhibited common properties of Level 3.

The preservice teachers always made suggestions and shared their ideas to decide what to include in the lesson plan together. Sometimes, they had difficulty in reaching a consensus due to their different approaches but they shaped the content by discussing together.

> **İnci:** We should pay attention to draw realistically because we noticed in the previous lessons that students might perceive the figures wrongly based on the images.

Semih: Yes. We should draw in a way that students could feel the difference between the figures.
Mehmet: We can match the open figures and prisms as Hasan said.
İnci: It is already difficult to decide whether an open figure will form a prism for students and you expect the students to match them too. I think they cannot do it.
Mehmet: Why do not they understand? We will give different examples for all types of prisms. They should be able to, I think.
…
İnci: Should we give the definitions of some terms such as edge, vertex and prism in order to reinforce understanding?
Hasan: We should. Students can use them for repetition later.

In terms of what to notice, İnci focused primarily on the issues of teaching and learning. She considered students' mathematical understanding and made her suggestions based on it. She referred to students' common mathematical approaches which she encountered in her previous experience (e.g. "We should pay attention to drawing realistically because in the previous lessons we noticed that students might perceive the figures wrongly based on the images). Regarding how to notice, she indicated the important points but her approach was general and inconsistent. Her elaboration for some of her comments was quite little whereas she tried to provide details for some of her ideas. She had an evaluative and interpretive stance (e.g. "I think they cannot do it). Therefore, her noticing was further considered as evidence of Level 2.

While preparing the lesson plan, the preservice teachers sometimes discussed on some issues that they had talked about before, in order to improve it in the direction of their own concerns related to students' thinking and understanding. İnci focused on how to introduce the prism concept and indicated some points that she wanted to add.

İnci: We said that students needed to realize that prisms formed through gaining height in the third dimension. Semih had suggested to progress from two dimensions to three dimensions to teach the meaning or prism but one dimension is missing in this case.
Semih: But it is enough to show how to obtain a prism.
İnci: Yes, but we should give them respectively for better understanding. The cooperating teacher also had suggested to associate dimensions to the concepts of perimeter, area and prisms. Students may not realize that perimeter includes one dimension which is length and area depends on two dimensions which are length and width. Because of this lack in substructure, students may not think that prism is obtained with the addition of height on these dimensions. Thus, some of them may perceive prism as a separate concept.
Mehmet: We can show a prism and point to the length, width and height on it.
İnci: I think we should do something more comprehensive. We can use a computer program and draw a point at first. Later, we will form a straight line by extending the point and emphasize that it represents the length and one dimension. We will query what is found with length and students will make connection among perimeter, length and one dimension.

> **Hasan:** Then, we will add width on length by drawing and students will realize that there are two dimensions and area is found by using both of them.
> **İnci:** Yes. Later, we will add height to the rectangle and obtain a prism. Students will see how a prism forms and it is a three dimensional object. Thus, we will progress step by step from one dimension to three dimensions and from perimeter to prism by showing the relationships. Therefore, students will understand the meaning of prism better.
> **Mehmet:** I think it is a good way.

In terms of what to notice, İnci centrally focused on students' mathematical understanding and possible reasoning. She referred to how students might think in the process of learning the relationships between dimensions and some mathematical concepts (e.g. "Students may not think that prism is obtained with the addition of height on these dimensions" "Some of them may perceive prism as a separate concept"). She also endeavored to provide justification about students' mathematical approach. For example, she related the difficulty in understanding the relationships to the lack in substructure. She explained how her suggestions were related to the goals of the lesson and how they would contribute to the students' learning in an interpretive way. She presented details about her ideas associated to students' reasoning. She referred to the noteworthy events by elaborating. Thus, her noticing exhibited further the features of Level 3.

During the interview that followed the end of planning, İnci indicated what they aimed in this process. She explained what expected students' responses, difficulties and misconceptions were and how they considered students' mathematical thinking while planning as in the following:

> **İnci:** We will show open and close forms of the prisms and give both appropriate and inappropriate examples to make students understand formation of prisms. They know prisms as a concept but they will see them in such detail for the first time so we will build a substructure for their subsequent learning. Three dimensional objects are complex for students and their abstract thinking is not developed yet so we will try to concrete them as far as possible through materials, computer programs and associating to daily life. I am concerned that students may not realize that one prism may have different open figures and different prisms may have the same type of open image. We endeavored to consider students' difficulties and how to make their learning better.

İnci indicated that she further expected difficulty related to opening and closing of prisms because it was based more on abstract thinking. At this point, it was understood that they endeavored to increase visualization and concretization to facilitate learning. Her comments showed that they paid attention to shape the lesson plan in the direction of the important points.

İnci indicated what she focused on as a whole while planning and what she noticed in this process. The following is a presentation of her noticing through her comments:

> **İnci:** By and large, we focused on what students could and could not understand and what kind of confusions and difficulties might arise. We discussed on how to introduce the subject of prism and show its meaning. We really talked about this issue very much. I noticed that I did not know some things and we tried to complete our own deficiencies. I can say that we are better in planning and thus, discussing together influences this process positively.

İnci's comments revealed that their focus point was the students' mathematical thinking and understanding while planning the lesson and how to teach the subject in the best way for students. She indicated that they did not avoid spending too much time in discussion of an issue even if it was a simple matter. She also emphasized the contribution of lesson study process.

When İnci's comments during the whole planning process were considered, her noticing further exhibited the features of level 3 because she predominantly focused on students' mathematical thinking and elaborated the relationship among her suggestions, students reasoning and overall goals of the lesson. She generally referred to possible student strategies and promoted the other members of group to focus on these points. She offered details about her ideas and provided justifications about her comments to support her claims. Her approach was mostly interpretive and she tried to make inferences about students' understanding. Hence, her noticing in planning was considered as evidence of Level 3.

4.1.4.2 What and how İnci notice in reflecting phase

Teaching of the fourth lesson plan was made by Mehmet in 5/F classroom. The fourth lesson plan included the objectives of "Being able to recognize a rectangular prism and identify its essential features", "Draw surface developments of a rectangular prism and decide whether different surface developments belong to the rectangular prism or not" and "Calculate the surface area of a rectangular prism". Since the aim was to focus on case of İnci, Mehmet's teaching was not presented. While Mehmet was implementing the lesson plan, the other preservice teachers, cooperating teacher and researcher observed this lesson and took notes. After teaching was completed, the reflecting meeting occurred immediately at the school conference room. In the reflecting phase, the preservice teachers were requested to reflect their thoughts about the lesson including; what worked well, what did not work, what the difficulties were, what they observed about the students' actions and thinking and also what should be changed about the lesson to improve it and what the rationales for the changes were.

This phase began with the comments of the cooperating teacher briefly and she reflected her thoughts on the lesson. She indicated that visual representations and concrete models enabled students' better understanding of the elements and properties of prisms. She stated that when the prisms were drawn on the board, they became two dimensional and accordingly, students had difficulty in perceiving the back side of them. At this point, she expressed that she liked the contribution of visualization with various computer programs and concrete materials that the preservice teachers used. Owing to the fact that the preservice teachers could give only the surface area of a rectangular prism, the cooperating teacher suggested increasing the lesson hours in order to complete the surface area of a tetragonal prism and cube and solve more questions or decrease some activities related to opening and closing the prisms with computer programs. She explained why she offered them to reduce activities based on computer programs saying that students learned better by doing themselves on the models yet she emphasized the importance of both. She also indicated that the other necessary concepts, except the surface area of a tetragonal prism and cube, were taught in a good way and the content was efficient enough to achieve the objectives.

In accordance with discussion protocol, İnci shared her opinions about the lesson that she observed, evaluated effectiveness of the lesson plan and made some suggestions to improve it. The following dialogs show what and how İnci noticed during the teaching phase.

> **İnci:** The elements of prism were understood by the students. Students did not confuse them and they could notice that the number of edges was 12, the number of vertices 8 and the number of faces 6 for each rectangle prism, tetragonal prism and cube without the teacher's indication. After they counted the components of a rectangular prism, they could say the number of components for a tetragonal prism and cube without need for counting again. Being able to make this inference quickly was good. Besides, one student likened the classroom to a rectangular prism, linking with daily life by himself. It was good that Mehmet promoted the students to find the elements of a prism in the classroom but he named the floor as base and the upper base as ceiling. However, both bottom and top were accepted as base in a prism. Although we talked about it in planning, he forgot it. I thought that his explanation might cause confusion. We should pay attention to use appropriate terms. We also need to present explicit figures since students got confused when the prisms looked like each other and had difficulty in determining the type of prisms.

In terms of what to notice, İnci primarily focused on students' mathematical understanding and teacher pedagogy. She addressed to students' responses and confusions to make inference about their understanding (e.g. "The elements of prism were understood by the students" and "Students got confused when the prisms looked like each other"). Her attention was on the whole class rather than specific students. Regarding how to notice, she did not try to make sense of students' thinking, rather she described the important interactions in the classroom and evaluated these events (e.g. "Being able to make this inference quickly was good"). She endeavored to elaborate students' learning and teacher's approach, her comments were limited in terms of details. She offered general expressions with a broad perspective in nature (e.g. "We should pay attention to use appropriate terms"). Thus, her comments here were considered as evidence of Level 2.

> **İnci:** After Mehmet queried what was different in a cube, students noticed that all faces were square. One student indicated that all of them were base. He probably wanted to refer to naming according to base and stress that it did not matter which face was accepted as base. He did not make an incorrect comment there. He probably though that each faces were square and equal to each other so that when cube was rotated, one face might be both a lateral face and base. Another student made an explanation for tetragonal prism like that it was combination of a rectangular prism and cube, its bases were square and lateral faces were rectangle. His comments showed that he noticed the differences and similarities among these three prisms. His interpretation was different from the others' approach.

In this quotation, İnci's main focus was on particular students and their mathematical thinking. She referred to these students' responses and tried to reason on how they thought mathematically. For example, she associated the student's comment about base to noticing equality of all faces. She identified the remarkable events to explain students' thinking and provided details about them to strengthen her analyses. Her comments had an interpretive stance in nature (e.g. "His comments showed that he noticed the differences and similarities among these three prisms"). She tried to explain why she thought that these specific approaches were different. Therefore, it was accepted that her noticing further had the properties of Level 3.

> **İnci:** In the activity related to determining an open figure would form a prism when it was closed, one student counted the faces on the image and he thought that it would form a prism because there were 6 faces, based on the number of faces in prisms that they had learned but actually it could not close. He made a lacking inference because it might or might not form a prism if it had 6 faces. Similarly, another student answered that we had drawn this open figure in the notebook, it had to be a rectangular prism but it was not even similar to what was in their notebooks. Even if it was the same, I understood that she did not notice that there might be various open images which would constitute a prism. I saw that she was in tendency to specialize and oversimplify the complex structure of the subject. The key point that the student needed to notice here was that one prism might have more than one open image. Thus, we especially used computer programs and concrete materials with this aim.

In terms of what was noticed, İnci focused centrally on specific students and the details of their mathematical thinking by considering their explanations. She referred to these students' reactions and tried to reason on mathematical approach that they used to solve the questions. For example, she made an interpretation about the student's incorrect inference related to forming a prism with 6 faces. According to İnci, this student endeavored to find the short path for deciding whether an open figure would close by making a generalization about the number of faces in a prism. Her comments were also illuminating since they revealed the students' tendency to oversimplify and where and how this specific approach might arise. Her comments were interpretive in nature. She also made reference to evidences from her observation to support her claims. Thus, her noticing exhibited the prevalent features of Level 3.

After the reflecting phase, individual interviews were conducted with the preservice teachers to reveal missing points and detail their noticing. What İnci indicated differently from the reflecting phase was as in the following.

> **İnci:** When Mehmet drew the square on the base of the prism, one student perceived it as a parallelogram because its side view was drawn to make them feel three dimensions. That student focused on the image of the figure and he could not think of it as a whole. Another student could make a deduction that each square was a rectangle at the same time so that a tetragonal prism was a kind of a rectangular prism as well. We aimed to emphasize this point to teach rectangular prism types but we did not assume that the students could explain it. His responses were unexpected for us.

In this excerpt, İnci's focus was on particular students and their mathematical approaches. She indicated how the students were thinking about some mathematical concepts and how they perceived them. She reasoned through one student's strategy whose explanations she had used. For example, she interpreted the student's explanation saying that he could understand that square was a kind of a rectangle and he made deduction about prism types based on this knowledge. She offered some citable events related to students' thinking and understanding and provided details from the lesson to support her analyses. Her comments were mostly interpretive and toward making sense of what she observed. Hence, her noticing had further characteristics of Level 3.

> **İnci:** Mehmet could only tell the surface area of a rectangular prism. He could not proceed to tetragonal prism and surface area of a cube because his time was finished. However, the students could indicate how to find area of rectangles and squares in a rectangular prism and used measurement units correctly. On the other hand, if they had thought over that areas of all faces were summed to find the surface area of a rectangular prism, it would be possible for them to understand how to find surface areas of a tetragonal prism and cube in the same sense. I think they should be able to make this inference from what Mehmet taught…The students chiefly had difficulty while drawing images of prisms and Mehmet explained it verbally to them. His directions were good. I thought that the students would have difficulty in

understanding prisms because they are three dimensional however their reactions were better than I had expected.

In terms of what to notice, İnci attended primarily to teacher pedagogy, students' mathematical approaches and difficulties. She indicated what the teacher could tell or could not complete and his approach in teaching. She also referred to students' learning and difficulties but she did it in general ways (e.g. "The students could indicate how to find area of rectangles and squares" and "The students generally had difficulty while drawing images of prisms"). She focused the class as whole rather than individual students. Regarding how to notice, İnci endeavored to interpret possible students' mathematical thinking and elaborated what kind of deduction she expected from the students. According to İnci, students should be able to find the surface of a tetragonal prism and cube based on the knowledge related to surface area of rectangular prism. Although she presented details about her ideas, she also offered overall impressions. Thus, she was inconsistent in making reference to the evidence in order to support her claims. Her comments were elaborative and she adopted an interpretive stance partly (e.g. "His directions were good"). Therefore, İnci's noticing was considered as evidence of Level 2.

When İnci's all comments in the reflecting phase were considered, her noticing further had the characteristics of Level 3. She mainly focused on the students' mathematical thinking, their responses and teacher's pedagogy. Although she made comments about the whole class, she mostly attended to specific students. İnci endeavored to make sense of students' mathematical thinking and explain how they might think mathematically. She highlighted the noteworthy events and interactions with an interpretive stance. Although she used evaluative comments, her approach was rather interpretive. She attempted to elaborate her claims by offering evidence from what she observed in general. Although some of her comments reflected the properties of Level 2 and Level 3, her noticing in reflecting was further considered as Level 3.

4.1.4.3 What and how İnci notice in re-reflecting phase

After the fourth lesson plan was implemented by Mehmet in classroom 5/F during the teaching phase and the preservice teachers discussed on what should be changed or paid attention to for the upcoming implementation, re-teaching of the fourth lesson plan was made in another classroom 5/G by Semih. As the aim was to focus on İnci's case here, teaching of Semih was not presented. While Semih was implementing the lesson plan, the other preservice teachers, cooperating teacher and researcher observed this lesson and took notes. In the re-

reflecting phase which was conducted just after re-teaching, the cooperating teacher and all preservice teachers were required to reflect their thoughts about the lesson including; what worked well, what did not work, what the difficulties were, what they observed about the students' actions and thinking and also what should be changed about the lesson to improve it and what the rationales for the changes were.

This phase began with the cooperating teacher's brief comments and she reflected her thoughts on the lesson. The cooperating teacher indicated that the lesson was good on a broadscale, the only problem was related to drawing of the prisms. She expressed that both Semih and the students had difficulty in drawing the figures so they started to make noise unavoidably. She remarked that Semih's drawing caused some confusion regarding the type of prism. She suggested to draw as well as possible because the students paid attention to the image of the figures. She emphasized that the students could understand the elements and properties of prisms, open and close situations of them, formation of prisms, naming of them and calculation of surface area. She also stated that lesson was implemented in detail through computer programs, concrete materials and visualizing. She explained some of the students' problems with three dimensional figures. She indicated the content of the lesson was good and sufficient as it was.

In accordance with discussion protocol, İnci shared her opinions about the lesson that she observed, evaluated the effectiveness of the lesson plan and made some suggestions to improve it. The following statements show what and how İnci noticed during the re-teaching phase.

> **İnci: S**tudents could understand how to obtain a prism by establishing the connection among the concepts of length, width and height and three dimension. Semih emphasized that the figure on the base helped to name the prism. Thereupon, the students reasoned that if the base was a square, it would be a tetragonal prism in this case, all by themselves. Their effort to make inference was good. On the other hand, when Semih asked the number of edges, vertexes and surfaces, the students gave various answers such as 4, 5, 8 in contrast to the other class. In Mehmet's class, the students could indicate these numbers correctly right away, but these students gave incorrect answers and needed to count on the concrete models. Then, the students could notice that all of the elements were the same, namely, there were 8 vertexes, 12 edges and 6 surfaces for each rectangular prism, tetragonal prism and cube.

In terms of what to notice, İnci focused primarily on what the teacher did and students' mathematical approach. She pointed to the mathematical understanding of the whole class rather than attending to specific students (e.g. "Students could understand how to obtain a prism"). She also described what the teacher did and said in order to make comments about the students' approach. She identified the noteworthy moments but she offered basic impressions without providing details. For example, although she made reference to evidence of students' responses to support her claims, her explanation about how they understood prism, naming of prisms and elements of them were not elaborated sufficiently. Therefore, she was inconsistent in giving details to strengthen her analyses. Her comments were evaluative but an interpretive stance was adopted partly (e.g. "Their effort to make inference was good"). She also compared students' performances in two different classes based on her observations and evaluated what was different. Because İnci attended to the mathematical approach of the whole class with a broad perspective, her noticing was further considered as evidence of Level 2.

> **İnci:** In the activity related to opening and closing of the prism, one student could notice that there were 5 surfaces so one surface was missing and it would not form a prism. Another student also saw that one surface was extra because there were 7 surfaces so it would not be a prism. They could give these answers without thinking for long because they made a deduction based on the prism types which had 6 surfaces. On the other hand, they could have reached an overgeneralization that each figure with 6 surfaces would constitute a prism but we gave examples which could prevent this misconception and Semih directed the students on this point well. In Mehmet's class, there were no answers regarding extra or missing surfaces. One student perceived the square on the base of the tetragonal prism as a parallelogram. Another student responded him that "it was three dimensional and it seemed like a parallelogram because we looked at it from the side view. She tried to correct the misperception of another student herself.

In this excerpt, İnci's central focus was on particular students and their mathematical thinking. She addressed their responses and made sense of how they thought mathematically. She reasoned through the strategies that these students used to answer the questions related to constitution of prism. She used details from her observation to explain the reasons behind students' answers. She also referred to their possible overgeneralizations and how this specific approach might arise (e.g. "They could have reached an overgeneralization that each figure with 6 surfaces would constitute a prism"). She highlighted the important events and her comments were interpretive in nature. She provided details from what she observed and endeavored to make inference about students' mathematical thinking to promote her claims. Thus, her noticing had the prevalent features of Level 3.

> **İnci:** When Semih indicated that cube was a special prism, one student asked the reason for its being special. They could not understand this statement and needed more explanation. Normally, we named a prism according to the figure on the base. Thus, the students might have thought that it was a tetragonal prism because the base was a square, however, lateral faces were not rectangles. I think if Semih had said that all faces were square in a cube and that was what made it special, his explanation would have been better in terms of understanding and showing the differences.

Semih: I tried to show them.
İnci: Yes, you did it well. For example, one student offered to multiply one face's area by 6 in order to find the surface area of a cube because he thought that there were 6 faces on a cube. However, the key point that he missed here was that each face was a square and equal to each other. The area of one face would not be multiplied by 6 if all the faces in a cube were not the same. This student might not have considered this issue. Semih emphasized the reason for multiplication by 6 and the area of one square and how to obtain the formula of a cube's surface area by making reference to the equality of faces. In one question related to calculating the surface area of a prism. One student found one lateral surface by multiplying width by length, namely, he found 56 from7 times 8. She continued by multiplying 56 by 4 because there were four equal lateral surfaces. Later, she found 44 for one face's area of base. I think she made an operation mistake there and found 44 instead of 49 from 7 times 7 unwittingly. However, her mathematical approach was correct.

In terms of what was noticed, İnci focused primarily on the details of students' mathematical thinking and understanding (e.g. "Students might have thought that it was a tetragonal prism because the base was a square, however, lateral faces were not rectangles"). She also attended to the teacher's approach and made some suggestions about it with reasons and by considering students' understanding. She referred to particular students and moments instead of the whole class. Regarding how she noticed, she endeavored to reason on her observation and make sense of the students' mathematical thinking. For example, she associated one student's offer, including multiplication by 6, with the number of faces and addressed the missing point in his consideration. It was important because İnci revealed a specific approach that the student might have had. She provided details about the students' responses, mistakes and strategies that they used in order to support her claims. She mostly mentioned specific events and interactions with an interpretive stance (e.g. "The key point that he missed here was that each face was a square and equal to each other"). Hence, her noticing further exhibited properties of Level 3.

After the re-reflecting phase, individual interviews were conducted with the preservice teachers to reveal missing points and detail their noticing. What İnci indicated differently from the reflecting phase was as in the following.

İnci: For the most part, students could understand how to name a prism, recognize elements and properties of them, learn how to obtain a prism, identify various open and close prisms and see how to find their surface area. Semih associated the subject to daily life at the end of the lesson so it affected students' understanding negatively. For better learning, they needed this concretization in the beginning. Because of time loss, he could not solve some questions and gave an exercise sheet as homework. In the revision of the first implementation of this lesson, we decided to increase lesson hours from 2 to 3 but I saw that the content was very rich, so according to me we needed to increase it to 4. In the first two lessons, we can introduce types of rectangular

> prisms, teach the elements and properties of them, draw open and close images of them and show constitution of a prism. In the last two lessons, we can tell the concept of surface area for each prism, how to calculate them and reinforce the subject by solving various questions. I think implementation of this lesson plan can be more efficient in this way.

In this excerpt, İnci focused on various issues such as teacher pedagogy, students' understanding, time and implementation of the lesson. She made some suggestions about teaching by explaining the reasons. She pointed to the whole class's mathematical understanding instead of focusing on particular students and making sense of their thinking. Her comments were so unspecific without details. She also had a descriptive and evaluative approach in nature (e.g. "I think implementation of this lesson plan can be more efficient in this way). She provided little or no evidence from her observation to support her claims. For example, her comments about students' understanding or her assessment that "For better learning, they needed this concretization in the beginning." were not clear. Because she attended to a range of points with a general impressionistic stance, her noticing here was accepted as evidence of Level 1.

When all comments of İnci in the re-reflecting phase were considered, her noticing mostly exhibited the features of Level 3. She predominantly focused on the students' mathematical thinking, their solution strategies as well as the issues of teaching and teacher's pedagogy. She sometimes referred to the whole class but she further attended to particular students. İnci endeavored to make sense of students' mathematical thinking and reason through their strategies. She addressed the students' responses and how interactions occurred during the lesson in order to support her analyses. Although she offered impressions about teaching and learning issues, she linked them to students' thinking and understanding. Although she made evaluation about the lesson, her comments were mostly interpretive. She attempted to present details about what she observed. Although some of her comments reflected the properties of Level 1 and Level 2, her noticing in the re-reflecting was further considered as Level 3.

4.1.2 Summary of findings from case 1 - İnci

During the lesson study process, four lesson study cycles were conducted. Each cycle included phases of planning, teaching, reflecting, re-teaching and re-reflecting respectively. Preservice teachers worked collaboratively in planning, reflecting and re-reflecting phases whereas they implemented the lesson individually in teaching and re-teaching phases while the other group members were observing.

In the first lesson study cycle, İnci's noticing was at Level 1 in planning, reflecting and re-teaching phases whereas it was at Level 2 in the re-reflecting phase. Teaching of the first lesson plan was conducted by another preservice teacher so information towards this phase was not presented. During planning and reflecting phases, in terms of what to notice, the prevalent indicators of Level 1 were that İnci focused on a range of issues such as teacher pedagogy, materials, similarities of activities, time, implementation and students' behaviors. In other words, she attended to the issues which were mathematically less significant. For example, İnci focused on whether students should write everything or draw each figure. Regarding how to notice, she mostly made descriptive and evaluative comments. Besides, her approach was so general and did not present details about her ideas to support them. For example, she made some assessments about students but they were not clear in terms of the basis of the evidence. In re-teaching, regarding what to notice, the main characteristics of Level 1 were that İnci's initial focus was on doing what she had intended in parallel to the lesson plan rather than students' mathematical thinking. She mostly considered the students as a whole and ignored the wrong answers. For example, she reacted to a wrong answer by asking the question "Another answer?". In terms of how to notice, she could not promote the students in a way that could reveal the details of their thinking. She did not listen to the students long enough and indicated what they should do straight away. Her comments were quite descriptive and evaluative in nature since she focused on the correctness of the students' responses. For example, İnci asked questions towards description such as "What is it?" and "What is the perimeter?" rather than questions towards interpretation. During the re-reflecting phase, in terms of what to notice, the robust evidences of Level 2 were that she mostly focused on teacher pedagogy and students' mathematical thinking rather than less mathematical aspects of the lesson. Although she mentioned particular students, she continued addressing the whole class. Regarding how to notice, she evaluated what she observed at large but she also adopted an interpretive stance. She was inconsistent in elaborating and providing details about the events to support her claims.

In the second lesson study cycle, İnci's noticing was at Level 2 in planning, reflecting and re-reflecting phases. Because teaching and re-teaching of the second lesson plan were conducted by different preservice teachers, information about these phases were not indicated. In terms of what to notice during planning, reflecting and re-reflecting phases, the dominant indicators of İnci's noticing being in Level 2 were that she predominantly focused on teacher pedagogy and students' mathematical thinking. She addressed the noteworthy issues such as students' confusions, difficulties or mistakes but she did it in general ways and she was not good at making justifications about them. For example, she focused primarily on teaching and students' learning issues through comments such as "Do students know the concept of exponential numbers?" and "It can be abstract in this way". Although she tried to consider students' thinking, her approach was broad. Regarding how to notice, although she attended to students' mathematical approach, she offered overall impressions instead of giving details to strengthen her ideas. Her comments were mostly evaluative with an interpretive stance but this stance was limited. She was inconsistent in elaborating her ideas. She sometimes made reference to the evidence to support her ideas but at times she offered basic comments without details. For example, although she described the answers of the students, she generally did not elaborate possible reasons behind these answers.

In the third lesson study cycle, İnci's noticing was at Level 3 in planning and reflecting phases whereas it was at Level 2 in teaching and re-reflecting phases. Re-teaching of the third lesson plan was conducted by a different preservice teacher, thus, information about it was not given. During planning and reflecting, in terms of what to notice, the predominant characteristics of Level 3 were that she predominantly focused on students' mathematical thinking and made suggestions in accordance with the objectives of the lesson by considering how students think mathematically. She attended to particular students' responses rather than the whole class. Regarding how to notice, she endeavored to make sense of students' mathematical thinking. She used details to draw inferences about the students' understanding and elaborated on students' approaches and confusions. Her comments were predominantly interpretive. In teaching, regarding what to notice, the main indicators of Level 2 were that İnci's focus was on students' mathematical thinking and what she had in her mind. She listened to the students' responses chiefly instead of indicating what she intended directly but she sometimes preferred a telling mode to maintain the flow of the lesson. She focused on both particular students and the whole class. Regarding how to notice, she was inconsistent in revealing the details of students' mathematical thinking and responding them in the right way. Her approach was evaluative with an interpretive stance in nature. She often missed the opportunities to correct students' mistakes or did it in general ways. During the re-reflecting phase, in terms of what to notice, the main indicators of Level 2 were that İnci further referred to the whole class although she attended to particular students. She focused on both teacher pedagogy and student thinking. She indicated broadscale impressions about teaching and learning issues. Regarding how to notice, her comments were mostly descriptive and evaluative in nature but she also adopted an interpretive stance. Her elaboration about what she observed was limited.

In the fourth lesson study cycle, İnci' noticing was at Level 3 during all planning, reflecting and re-reflecting phases. Teaching and re-teaching of the fourth lesson plan were conducted by different preservice teachers so information about them was not presented. During planning, reflecting and re-reflecting phases, in terms of what to notice, the prevalent features of İnci's Level 3 noticing were that she predominantly focused on students' mathematical thinking and signaled to particular students' mathematical strategies. Regarding how to notice, she made justifications about her comments and elaborated on the relationship among her comments, students' reasoning and overall goals of the lesson. Her approach was mainly interpretive. She endeavored to make sense of students' mathematical thinking and provide details about her ideas and observations. The following table presents İnci's noticing during lesson study cycles.

Table 8 İnci's Noticing Process in Lesson Study

	Planning	**Teaching**	**Reflecting**	**Re-teaching**	**Re-Reflecting**
Cycle 1	Level 1 *[63% L1, 37% L2]*	-	Level 1 *[60% L1, 40% L2]*	Level 1 *[60% L1, 30% L2, 10% L3]*	Level 2 *[20% L1, 80% L2]*
Cycle 2	Level 2 *[14% L1, 57% L2, 29% L3]*	-	Level 2 *[29% L1, 71% L2]*	-	Level 2 *[100% L2]*
Cycle 3	Level 3 *[20% L2, 80% L3]*	Level 2 *[14% L1, 57% L2, 29% L3]*	Level 3 *[17% L1, 17% L2, 66% L3]*	-	Level 2 *[25% L1, 50% L2, 25% L3]*
Cycle 4	Level 3 *[43% L2, 57% L3]*	-	Level 3 *[40% L2, 60% L3]*	-	Level 3 *[25% L1, 25% L2, 50% L3]*

Note. L1= Level 1; L2= Level 2; L3= Level 3

It is seen from the table that İnci's noticing varied between Level 1, Level 2 and Level 3. Her noticing progressed from Level 1 to Level 3 during planning phases. Besides, her noticing increased from Level 1 to Level 3 in reflecting whereas it progressed from Level 2 to Level 3 in the re-reflecting phase. It is also seen that her noticing was at Level 1 in her first implementation and it changed as Level 2 in her second implementation. İnci's noticing was low in early cycles but a shift began to take place as lesson study cycles continued. In general terms, there was a linear increase in her noticing during the lesson study process.

4.2 Case of Semih

4.2.1 Lesson study cycle 1

4.2.1.1 What and how Semih notice in planning phase

The first lesson plan was related to teaching perimeter and the objective of the first cycle was "Being able to calculate perimeters of polygons; create different figures which have the given perimeters". As mentioned in case 1, preservice teachers met to prepare the first lesson plan at the university seminar hall bringing the various sources after they listened to the ideas and suggestions of the cooperating teacher. The following are presentations of examples based mainly on Semih's expressions from the planning meeting of lesson study cycle 1.

At the beginning of the planning process, preservice teachers discussed on how to start the lesson. Preservice teachers offered their suggestions. Some of them were refused or some of them were improved. A conversation among preservice teachers is as the following:

> **Semih:** We should teach students the meaning of the perimeter concept at first. Instead of starting teaching immediately with the figure, the point that should be focused is what perimeter is. In one book, it is also offered to start from the length of line segments.
> **Mehmet:** Will we give definition? What is a triangle? What is quadrilateral?
> **Semih:** No, students know figures. We will just make them feel that triangle is the sum of three edges and quadrilateral is the sum of four edges.
> **…**
> **Semih:** We should introduce the subject with something that is easy but remarkable.
> **İnci:** We can ask a problem at first.
> **Hasan:** Perimeter is a measurement activity. Students should know the definition of measurement. We can ask what comes into their head when they hear the concepts of measurement and perimeter.
> **Semih:** Hi, students. What is measurement? What is perimeter? It does not sound good at the beginning. We need to present the subject more strikingly. For example, we can prepare a small material.

In this excerpt, Semih primarily focused on teaching and pedagogical issues rather than students' mathematical thinking. He addressed what should be done and shared his suggestions. However, he did not elaborate his ideas. Why the meaning of perimeter was important or how the material would serve teaching were not clear. He offered general expressions. He gave importance to the points such as making an interesting entry, drawing students' attention, using a particular material. His comments were mostly descriptive and evaluative (e.g. "In one book, it is also offered to start from the length of line segments" and "It does not sound good at the beginning"). He provided little or no evidence to support his suggestions and associate them with students' mathematical approach. Thus, his noticing here was accepted as Level 1.

Preservice teachers continued to talk about the content of the plan. It was taking time to reach a consensus. There were different ideas. The discourse among preservice teachers is as it follows:

> **Hasan:** We can give figures without writing any numbers and ask which of them is bigger.
> **Semih:** We need to make students comprehend the concept of perimeter first.
> **Mehmet:** What do you think?
> **Semih:** We can find a basic problem. We will not teach perimeter directly but make students feel it. For example, students can find how much fence is needed to enclose a field.
> **…**
> **Semih:** How many questions will we ask in the exercise sheet?
> **İnci:** 6 is enough I think.
> **Semih:** What about 4? We can make the last question difficult as well.
> **İnci:** Let's ask 5.
> **Semih:** Ok. Then, 5.

In terms of what to notice, Semih attended to the order of the content and the number of the questions. He did not address the students' mathematical thinking or their confusions. Here, they focused what to do and their own ideas. Regarding how to notice, his comments were quite general in nature. He did not make any reference to the details to support what he said. He probably thought that students should learn perimeter well at first for being able to compare the figure but he did not provide justification about his idea. His comments were predominantly descriptive. Since the students were not the focus of this dialog and details and interpretation were not provided, his noticing had the prevalent properties of Level 1.

While planning the lesson, İnci offered to measure around a picture in order to show perimeter. The other members agreed on this suggestion by thinking that it would provide to understand that perimeter was an activity based on measurement and it would draw students' attention. They talked about how to improve it. The conversation about this issue is as in the following:

> **…**
> **Semih:** Here, we will ask students to round the picture with a rope and measure perimeter with a ruler. However, they will not notice that perimeter is the sum of edge lengths. Should we cut the rope in accordance with the edges and show the sum of them or not?
> **İnci:** I think the rope should remain as a whole.
> **Semih:** Students should also find the perimeter by summing the edges end to end to understand this concept.

İnci: We can stick the picture on the board and write the edge lengths on it.
Semih: If we write directly, it does not make sense. Students should measure the edges too.
Mehmet: It is good enough. Hence they will find the length of the whole rope and the sum of edges and they will see that both are equal.

In this excerpt, Semih's primary focus was on students' mathematical understanding. He noticed the point which was missed in another member's suggestion and tried to improve it. According to him, measuring only around the figure was not enough therefore the students needed to see the total of it. He thought that making this connection would provide to understand the meaning of perimeter. He provided justification about his ideas and referred to noteworthy points. However, his elaboration about students' mathematical thinking was limited in some parts. His comments were evaluative and interpretive in nature. He generally focused on teaching and learning issues but maintained a broad approach. Hence, his noticing further reflected the features of Level 2.

After preservice teachers agreed on İnci's suggestion related to measuring the perimeter of a picture, they talked about how to include estimation of perimeter in this activity. Semih made a suggestion referring to the concept of prize in education. Exchanges among preservice teachers are given below:

Semih: We will ask students' estimations about perimeter at first, right?
Mehmet: Yes, they will estimate and then they will measure with a ruler.
Semih: Will we write estimations on the board?
Hasan: Yes, we should use the board.
Semih: We can give candy to the students who came to the board and all students after the lesson is finished as a prize.
İnci: We cannot give candy for everything. We should determine for which activity or question it will be given.
Hasan: This age group is small. It can be a problem and distract their attention.
Semih: Ok. We can make students clap the student who has the best estimation about perimeter, then.

In this quotation, Semih focused predominantly on the implementation of the lesson. He referred to less mathematical aspects of the lesson (e.g. "Will we write estimations on the board?"). Likewise he made comments related to pedagogical issues and offered to give a reinforcer to the students. His comments were also inconsistent, he suggested to give candy both to the students who came to the board and then offered to give everyone as well. The problems in his suggestions were noticed by the others. Semih had a general approach and did not provide details to support his ideas. His comments were descriptive in nature. His noticing exhibited the common features of Level 1.

Preservice teachers talked about the materials which they decided to use in the lesson. The conversation below presents their comments on this issue:

> **Semih:** Students will wrap the rope around the picture, and then measure the length of the whole rope. Will they use a tape or ruler? Which one is better?
> **Mehmet:** They cannot measure with a ruler because the picture will be big.
> **Semih:** How big will it be?
> **Mehmet:** I thought as 50x70 cm.
> **Semih:** It is too big. We do not need such a big thing. I think 30x50 is enough.
> **İnci:** Yes, I agree.
> **Semih:** Will we call one student for both measuring the rope and edges or more?
> **İnci:** One can measure the rope, the other can measure the edges and add them.
> **Semih:** It is better to activate as many students as possible.

In this excerpt, Semih's questions showed that his focus was further on the materials and the points related to implementation (e.g. "Will they use a tape or ruler?" and "How big will it be?"). He addressed pedagogical issues by mentioning the active participation of students too. He offered general expressions without details. His comments were descriptive and evaluative in nature. He did not focus on students' mathematical thinking or consider their thinking while offering ideas in this dialog. He mostly shared the questions in his mind with the others in order to learn their opinions and shape teaching. Thus, his noticing was considered as Level 1.

In the direction of the first objective preservice teachers wanted to create different polygons which had the same perimeter with the students. They discussed on giving geometric figures such as trapezium, rhombus, parallelogram, triangle and rectangle which were known by the students and using isometric paper to draw on it for a while. The comments in this conversation are as the following:

> **İnci:** We can ask students to draw a figure with a perimeter of 20 cm for example. It does not matter whether a student creates a pentagon or another figure.
> **Semih:** It sounds complicated. We should indicate what we expect carefully. I think, the key point is that the perimeter must be 20 cm. In other words, students should find the numbers whose sum give 20.
> **Hasan:** Yes, you are right.
> **Semih:** We can give isometric paper to the students for drawing on them. We say that the distance between two points is 1br and require them to draw figures.
> **İnci:** We say that for example, "draw a parallelogram with a perimeter of 20 unt" by indicating which figure we want.
> **Hasan:** Something preoccupied my mind. While drawing a parallelogram, two edges are sloping. They will be a radical expression. Can students perceive it?
> **Semih:** We will not want them to find the value there. They will draw the figures visually and write the numbers on them.
> **İnci:** I do not know how we can do that. We can draw these figures and give the values instead of asking students to do these.

Semih: We are unable to settle this matter. In this case, we should limit the figures with rectangle and square or we should not give isometric paper, it can be confusing or we may cause misconceptions in future.

In terms of what to notice, Semih focused on teaching and students' mathematical thinking. He indicated what should be done such as speaking clearly and using the material. After one member's warning about students' mathematical perception, he focused on this issue. At this point, he offered his ideas based on students' mathematical thinking. He tried to interpret his ideas but his comments were inconsistent in terms of providing details and general from time to time. He had an evaluative and interpretive approach (e.g. "It can be confusing or we may cause misconceptions in future"). He interpreted the possible situations in terms of students' learning and evaluated which way would be better for designing this question. His mixed approach reflected the characteristic of Level 2.

After preservice teachers determined how to give the concept of perimeter and prepared an exercise sheet to reinforce it, they thought on what else to tell the students. They researched the alternatives and necessary points to include in the plan. Semih offered to show that perimeter was the total of all edges of a figure regardless of which figure it was. The other members liked his idea and interaction related to this issue is presented below:

Semih: I think students need to comprehend that three edge lengths are summed to find the perimeter of a triangle, four edge lengths are summed for quadrilateral, namely, as how many edges there are, that many edges are summed. Therefore, they will see it and understand perimeter better. We can make students reach a generalization like that.
Mehmet: That would be great. I had also noticed it while researching the subject.
İnci: Ok. We draw one triangle, one rectangle and one pentagon on the board.
Semih: We can draw a summarizing table as in the book too. They will write the edge lengths and the perimeter of each figure on it in order to notice what we had intended more easily.
Hasan: What should we ask to lead the students?
Semih: What is the relation between edge lengths and perimeter?
Mehmet: According to me, we should emphasize the number of edges.
İnci: I agree. What is the relationship between perimeter and the number of edges?
Semih: In addition to that, what did we do for finding the perimeter of each figure? What can we say in general?

In terms of what to notice, Semih focused predominantly on students' mathematical thinking and further mathematical aspects of the lesson. He referred to noteworthy events and tried to make sense of students' mathematical understanding. Regarding how to notice, he indicated how students would think and reach generalization in detail. He provided justifications about his suggestion and associated it with students' thinking. He described what should be done step by step to strengthen his ideas. His comments mostly were interpretive and elaborative in nature. Thus, his noticing here was considered as evidence of Level 3.

Preservice teachers tried to determine what kind of problems should be asked to enable students to make practice and reinforce their understanding through different kinds of questions. They made their suggestions and tried to enrich the content. Semih's ideas about this issue is as follows:

> **Semih:** We can ask a question related to comparison and students try to find the figure with a bigger perimeter.
> **Mehmet:** Do we need to ask comparison? Our aim is to calculate perimeter.
> **Semih:** But we can derive questions in such a way that includes comparison. However, asking later may be better.
> **…**
> **Semih:** One edge length of a known figure can be asked by giving its perimeter. We want them to find an edge using perimeter.
> **Hasan:** Yes, we should ask that.
> **Semih:** Here, there is a problem. Perimeter of a pentagon is 108 cm, one edge is not given and it asks this edge. It is better giving it on the figure. Besides the cooperating teacher said that we should give a number and ask them to draw the figure or vice versa give the figure and ask to calculate. Besides, we can make it a bit more difficult. We cannot give two edges and give a relationship between them like that one is three times the other and want students to find the length of these edges.
> **…**
> **Hasan: S**tudents have a misconception related to finding perimeter. They count the points instead of the distance between two points. For example, it is likely that students say 5br by counting points but it is 4 unt. We can ask an example related to it.
> **Semih:** Unless we reinforce understanding of perimeter and give area formulas, we should not ask this kind of question. We may give it towards the end of the lesson but not now.

In this excerpt, Semih focused on the content of the plan and shared his suggestions ideas about the questions. However, he did not explain why he offered them. He generally described the context of them without interpretation. His comments were general in nature. Whereas one member referred an issue related to the students' misconception, Semih evaluated it in terms of the suitability for the order of the concepts (e.g. "We may give it towards the end of the lesson but not now"). He did not focus on students' mathematical thinking. He made descriptive comments and did not make reference to the evidences to support his ideas. Thus, his noticing was accepted as Level 1.

During planning the lesson, Semih also focused on less mathematical aspect of the lesson. He made comments such as "Will we make students write all of these?", "If we use geometry board, students will not see elastic bands on it or they will break", "We should express what we want well", "We should not give students isometric papers to draw figures, it will be difficult to follow them". He made suggestions about the implementation of the lesson. He did not tell his ideas and offered general comments. He mostly referred to the issues such as teaching, material etc. Hence, Semih's noticing reflected the features of Level 1 in some dialogs.

Preservice teachers aimed to teach the perimeter formula of rectangle and square by visualizing rather than writing the formula directly. They wanted to concrete the formula for students and provide better understanding. At this point, Semih offered to give it on a numeric example. For example, he explained his idea liked that students would compute 10+10+5+5 and they would see two of 10 and two of 5. He advocated that formula of perimeter in the form of 2a+2b could be given from this point of view. On the other hand, Mehmet defended to show it through material rather than teaching it based on a numerical question. After discussion on the details, he offered to draw a rectangle in the form of axb on an isometric paper which was stuck on the board and moved the edges to the next isometric paper as adding a and a in vertical position, b and b in horizontal position in order to show perimeter formula. The interaction among preservice teachers about this issue is presented below:

> **Semih:** You said that we would add the edges in vertical and horizontal positions. I think it can be a problem.
> **Mehmet:** I wanted to be like that particularly because students see that two edges are vertical and the other two edges in horizontal. I wanted to add them in a manner that the students could see them on the figure.
> **Semih:** According to me, we should add all of the edges end to end, being in horizontal or vertical position is not a matter, but we should not sum them partially. I think it does not serve the purpose in this way because we said that perimeter was the sum of all edges. However, here it was like half, not completed. It may cause confusion.
> **Mehmet:** Yes, you are right. I thought that students would see the equality of the edges easily.
> **Semih:** At the beginning of the lesson, we will add the edges end to end and draw as a line segment. If we make it here like this, it will support what we had said before as well.

In terms of what to notice, Semih focused on the suitability of an activity in terms of students' mathematical understanding. He stated that the activity would not serve the purpose of the lesson because it would be represented differently than what was indicated in the meaning of perimeter. He attended the difference here and reasoned on this situation considering students. Regarding how to notice, he tried to provide details to support his ideas. However, he sometimes did not clarify his claims. For example, what kind of confusions may arise were not explained and general comments were made. His comments were mainly evaluative and interpretive (e.g. "I think it can be a problem" and "it will support what we said as well”). Students’ possible thinking was not elaborated and a broad approach was maintained. Thus, his noticing in this dialog was considered as evidence of Level 2.

During the interview that followed the end of planning, he indicated what they had aimed in this process. He explained what expected students’ responses, difficulties and misconceptions were and how they considered students’ mathematical thinking while planning as the following:

> **Semih:** Our aim was to gain the meaning of the concept of perimeter and how to calculate it. Students will measure perimeter with ruler, estimate it and feel its meaning. We discussed on the difficulties that students might have. I do not think that students have difficulty in measuring. We will try to formulize perimeter of a rectangle and square with materials. Understanding how we reach the formulas may be difficult. We will try to facilitate it giving several examples and repeating.

In this excerpt, according to Semih students might have difficulty in understanding how the perimeter formulas are obtained. He did not think that students would have a lot of confusions on this objective. He indicated that they endeavored to overcome the difficulties or confusions through examples and repetition. His comments showed that he tried to consider students’ thinking but it is seen that he did not convey his claims well enough and used general expressions as in the planning phase.

Semih indicated what he generally focused on as planning and what he noticed in this process. The following is the presentation of his noticing through his comments:

> **Semih:** While planning, the content must be appropriate in terms of students’ mental level. We researched from many sources not only one book. Maybe, we can miss an important point or give inappropriate example, thus, benefitting from the books important. I also focused on the order of what we would teach. There are some concepts without giving their foreknowledge, we cannot teach. Therefore, we tried to pay attention to learning what students knew or not and teach the concepts which constituted the substructure of the subject. For example, we discussed about the flow of the lesson with the other members. Different ideas arouse. Working collaboratively was good. I noticed that if I missed something or said wrong, they completed the deficiencies and vice versa. It helped us to think further as students.

Semih’s comments showed that his focus was on students’ mental level, teaching order of the concepts and teaching the lesson in a good way. He emphasized the benefit of searching from different sources in shaping the lesson plan. It is seen that preservice teachers needed this kind of support to complete their deficiencies. In addition, he highlighted the importance of collaborative work and this process in terms of an environment including rich ideas, learning from each other and thinking like the students.

In terms of what Semih noticed during planning, his focus was on a range of issues such as teacher pedagogy, students' understanding the order of the content, students' behaviors, selection of questions, materials, the content of the lesson and implementation. In addition, she attended to students' mathematical thinking and tried to consider it in some cases. However, when the whole planning process was evaluated, his attention was on less mathematical aspects of the lesson rather than students' mathematical thinking. Regarding how Semih noticed, he had such a general approach. His comments were mostly descriptive and evaluative although he sometimes adopted interpretive stance. He did not make reference to the evidences to support his ideas and offered general expressions. He also did not try to make sense of students' mathematical thinking too much. When whole comments of Semih in planning phase were considered, although his comments varied in Level 1, Level 2 and Level 3, his noticing predominantly had the features of Level 1.

4.2.1.2 What and how Semih notice in reflecting phase

Teaching of the first lesson plan was made by Hasan in 5/F classroom. The first lesson plan included the objective of "Being able to calculate perimeters of polygons; create different figures which have the given perimeters". Since the aim was to focus on Semih's case here, teaching of Hasan was not presented. While Hasan was implementing the lesson plan, the other preservice teachers, cooperating teacher and researcher observed this lesson and took notes. After teaching was completed, reflecting meeting occurred immediately at school conference room. In the reflecting phase, preservice teachers were wanted to reflect their thoughts about the lesson including; what worked well, what did not work, what difficulties there were, what they observed about the students' actions and thinking and likewise what should be changed about the lesson to improve it and what the rationales for the changes were.

In accordance with discussion protocol, Semih shared his opinions about the lesson that he observed, evaluated the effectiveness of the lesson plan and made some suggestions to improve it. The following dialogs show what and how Semih noticed during teaching phase:

> **Semih:** I think that we should not use a red pen on the isometric paper on the board because it could not be recognized from the back of the class. It is better using a blue or black pen. Hasan should not have cut the rope according to the edges. Our aim was to show that the length of the whole rope was equal to the sum of all edges but he measured and combined the edges twice in this case. He had stuck isometric paper on the board earlier. He should have used it after solving the question in the exercise sheet. Students tried to draw the figures but they could not. It took time. On the other hand, students were willing to participate in the lesson. They tried to share what they knew. We paid attention to use materials and I noticed that students liked them and they increased students' attention to the lesson.

In terms of what to notice, Semih focused on a range of issues including appropriateness of the color of the pen, mistakes of the teacher in the implementation of the lesson, participation, students' behaviors and time. He did not attend to students' mathematical thinking rather he addressed less mathematical aspects of the lesson (e.g. "Students were willing to participate the lesson" and "He stuck isometric paper on the board earlier"). Regarding how to notice, he reflected his general impressions and did not elaborate his claims. His comments were descriptive and evaluative in nature (e.g." It is better using a blue or black pen"). He did not make reference to the details to support his analyses. For example, how students might think mathematically after the mistake in the implementation were not detailed. His noticing here exhibited the prevalent properties of Level 1.

> **Semih:** In the activity regarding the relationship between the number of edges and the perimeter of a figure, Hasan queried students' answers with questions such as "What would we do, if we had 10 edges, 100 edges and 1000 edges?" in order to reinforce that perimeter was the sum of all edges. It was good. I also noticed something. Would not we move the edges of a rectangle adding side by side?
> **Mehmet:** Yes, but he stuck the second isometric paper in a vertical position instead of horizontal. Thus, he could not move all edges end to end because there was no space.
> **Semih:** Therefore he moved them in the form of L and it did not serve the purpose. One student said that the square was a special type of a rectangle and I liked his statement. On the other hand, one student could not solve the question while on the board. Hasan called another student instantly. He could have helped this student because he might have felt bad when the other student came.

In this quotation, Semih's primary focus was on teacher pedagogy and what teacher did. He referred to teacher's mistake in the implementation. He tried to relate his observation to the mathematical goals of the lesson. However, his comments were descriptive and general in nature (e.g. "Therefore he moved them in the form of L and it did not serve the purpose"). He attended to some particular students too but did it in a general way. He did not endeavor to make sense of their mathematical thinking rather he focused on what they did and how they behaved. He made evaluative comments too (e.g. "I like his statement" and "He might have felt bad when the other student came"). He was inconsistent in providing details to support his claims. For example, he attempted to explain why he appreciated teacher's approach but he did not explain why he liked one student's response. Since he attended mainly to teaching and learning issues with a broad approach and few details, his noticing was considered as evidence of Level 2.

> **Semih:** When Hasan stuck the three figures on the board, he asked which one was bigger. He should have expressed it like "the perimeter of which figure is bigger?".

> Saying clearly what was indicated is important. After estimating the perimeter of these figures, students noticed that all of their perimeters were equal by calculating numerically. Their comments such as "All perimeters were the same" and "You asked an obfuscatory question" showed their realization and astonishment. Expected students' answers have been received, to name a few, they said "the edge here was longer, thus, perimeter was bigger" or "all the edges of this figure were the same thus its perimeter was bigger". This question enabled to produce different students' reactions accordingly I think it was all right. In some questions, Hasan wanted students to draw figures but they had difficulty in it. It might have been better if he had drawn them himself. Besides, I think the questions were simple for students' mental level in general. We can change some of them with more difficult questions.

In terms of what to notice, Semih focused mostly on teacher pedagogy and students' mathematical thinking. He referred to specific students' statements to strengthen his claims and he accepted them as the indicator of students' mathematical understanding. He referred to students' answers to support his claims. However, he did not make sense of their thinking or provide justification about why they gave such answers. He pointed to the noteworthy events but he maintained a general approach. He was inconsistent in providing details about his analysis to support his claims. For example, he evaluated one question as good because it helped to get different reactions from students but he did not make reference to the evidence from his observation for why he thought that the questions were simple in general. He made some suggestions about the lesson and teacher. His comments were evaluative but he also adopted an interpretive stance as well (e.g. "I think the questions were simple for students' mental level in general"). Thus, his noticing had the features of Level 2.

After the reflecting phase, individual interviews were made with preservice teachers in order to obtain richer information about their noticing and reveal missing points that preservice teachers might have forgotten to mention. Semih's comments, differently from what he had shared in the reflecting phase, are as below:

> **Semih:** Students knew that perimeter was found by summing the edge lengths and the perimeter of a rectangle and square. However, they confused perimeter and area. When we asked, some students gave area as an answer instead of perimeter. Students had difficulty while measuring the edges. Petty errors arouse because they could not hold the ruler correctly. Hasan made some mistakes as implementing the lesson. He taught it in general terms but he missed some details and important points. Hasan sometimes had problems due to not knowing the students and their cognitive levels. When I heard the answers which we expected, I noticed that we substantially could estimate and consider students' thinking correctly while planning… I tried to focus on students' answers because students gave different answers and we began to learn how students thought. I paid attention to the implementation of the lesson, what he could teach or not. Besides, students' behaviors were important for me. While discussing on the lesson, it was difficult to say what was wrong or deficient because the member who taught the lesson might feel that we criticized him. Thus, all of us were attentive to

> our expressions because we had the same responsibility. I also noticed that I missed some important points or did not think they were important but the other members indicated them or completed my deficiencies.

In this excerpt, Semih attended to what teacher could do or not and students' mathematical approach. He referred to the students as a whole not individually (e.g. They confused perimeter and area"). He offered general impressions about the teacher and class instead of providing details to promote his claims. For example, he did not make reference to the evidence of what kind of problems the teacher had and what the expected answers were. He mostly made evaluative comments with an interpretive stance (e.g. "Petty errors arouse because they could not hold the ruler correctly"). Besides, Semih evaluated their own planning and saw the expected answers as the indicator of their doing something correctly. Because he did not focus on particular students' mathematical thinking and had a broad perspective, his explanations were considered as evidence of noticing in Level 2. On the other hand, Semih indicated his focus was on students' answers, teaching and the behaviors of students and teacher. Likewise, his own comments supported that his noticing was in Level 2. It is also seen that Semih stated his concern about causing wrong perceptions like criticism while discussing and their attention for sharing responsibility.

When Semih's all comments during the reflecting phase are considered, his noticing had predominant properties of Level 2. In terms of what to notice, he predominantly focused on teacher pedagogy and students' mathematical approach. He sometimes attended to particular students and their mathematical answers, however, he continued to refer to the class as a whole. Regarding how to notice, he highlighted noteworthy events but did it in general ways. He sometimes gave details about his observation to support his claims but he sometimes provided little or no evidence to elaborate his ideas. His comments were descriptive and mostly evaluative. He adopted an interpretive approach too even though just a little. Although some of his comments were considered as Level 1, his noticing mostly exhibited the features of Level 2.

4.2.1.3 What and how Semih noticed in re-reflecting phase

Whereas teaching of first lesson plan was conducted by Hasan in classroom 5/F, re-teaching of the first lesson plan was made in another classroom 5/G by İnci. While İnci was implementing the lesson plan, the other preservice teachers, cooperating teacher and researcher observed this lesson and took notes. In the re-reflecting phase which was conducted just after re-teaching, the cooperating teacher and all preservice teachers were asked to reflect their thoughts about the lesson including; what worked well, what did not work, what difficulties there were, what they observed about the students' actions and thinking and what should be changed about the lesson to improve it and what the rationales for the changes were.

In accordance with discussion protocol, Semih shared his opinions about the lesson, evaluated effectiveness of the lesson plan and made some suggestions to improve it. The following dialogs show what and how Semih noticed during the re-teaching phase as an observer of the lesson:

> **Semih:** I sometimes saw 3 or 4 students who were on the board to answer the questions at the same time. İnci could not intervene in this situation therefore the classroom became complicated.
> **İnci:** I did not call them but they came with their own will.
> **Semih:** Yes, you did not call them. Although you wanted them to respond in their desk, they came anyway. Besides, she paid attention to put a starting point considering the cooperating teacher's feedback, it was good. I think the questions were good in general. However, some of them should have been given towards the end of the lesson. The order of them may be changed. İnci's attempt to call different students was well. According to me, she was hasty in some questions and intervened early. It might have been better to wait for a while. However, she probably considered the time.

In terms of what to notice, Semih primarily focused on the overall classroom environment, teacher pedagogy, behaviors of students and content of the lesson (e.g. "She was hasty in some questions and intervened early and "The order of them may be changed"). He attended to a range of issues and the whole class rather than particular students' mathematical thinking. Regarding how to notice, Semih described what he observed through a general impressionistic stance. In addition, his comments were evaluative (e.g. "İnci's attempt to call different students was well"). He did not make any interpretation about the noteworthy events. He did not provide details from the lesson to support his analysis. For example, how he made assessments about questions or why he made suggestions about the order of them were unclear. Due to the fact that Semih attended to a wide variety of issues with a general approach and did not explain his claims on the basis of evidence from his observations, his noticing here had common properties of Level 1.

> **Semih:** In the first activity, perimeter was found as 157 with the rope whereas it was 160 with the ruler. There was a measurement error again. I think we should scale the length of the rope and cut it beforehand. Then, students should make measurement with it because they are not able to hold the rope correctly. Besides, students' estimations about the perimeter of this figure were not successful. The students in another class had made closer estimations. In the question related to generalization, after İnci asked how to find the perimeter of a triangle, quadrilateral and pentagon, she also queried the relationship between the number of edges and perimeter for the figures with ten edges and twenty edges as Hasan had done in the previous lesson. It was good. She paid attention to emphasize the important points. In the question

including unknown edges, she leaded the students to find them at first because some students did not consider them while finding perimeter.

In terms of what to notice, Semih focused mainly students' mathematical approach and teacher pedagogy. He addressed to an implementation mistake and made suggestions on what should be done considering it on behalf of students. However, it was related to students' abilities of measurement rather than their thinking. He referred to the students as a whole and made a comparison between the students in both classes. He reflected an evaluative approach but he also had an interpretive stance (e.g. "She leaded the students to find them at first because some students did not consider them while finding perimeter"). He was inconsistent in giving details to support his claims. He made an assessment about the teacher's behavior without elaborating, whereas he tried to explain his suggestions with the reasons. Although he highlighted specific moments and important events such as the mistake of the teacher and students, his comments were very general. Therefore, his noticing was considered as evidence of Level 2.

Semih: Students had a tendency to say the answer quickly without writing in their notebooks. The important point for them was to say something after a quick effort for solving the question, they did not mind whether it was right or wrong. I noticed that some students wanted special attention and expected to be given permission to speak constantly instead of the other students. There were numerous students that is why it was not possible to do so. Accordingly, when they were not allowed to speak, they gave up on following the lesson.
İnci: I tried to call different students as far as possible.
Mehmet: There were other students who wanted to give answers too yet they did not take offence at the teacher when they were not called. We cannot always let the same students answer.
Semih: I am speaking in general. We should try to call students to be active in the lesson. It must be neither more nor little. However, the class tried to listen and follow the lesson without getting bored in general.

In this quotation, Semih's dominant focus was on nonmathematical aspects of the lesson. He attended to the issues such as students' behaviors, participation and classroom enthusiasm. He offered very general impressions and he did not provide evidence to strengthen his claims (e.g. "Students had a tendency to say the answer quickly" and "The class tried to listen and follow the lesson without getting bored in general"). He made simple suggestions related to the implementation rather than associating his comments to students' thinking. (e.g. "We should try to call students to be active in the lesson"). His comments were mostly descriptive and evaluative in nature. For example, he described students' habits during solving the questions and evaluated the participation of the class as good. Thus, his noticing was mainly considered as evidence of Level 1.

> **Semih:** One question was difficult for the students thus no one could solve it. One student conceived three edges as equal because they all had right angles but I do not know how he came to that conclusion. Thus, İnci needed to solve it. While there was a discourse on the perimeter of a square, one student said that one edge would be multiplied with four because all edges were equal. I think it was a good explanation. In the question related to estimating the perimeter of three different figures, some students said that "A was bigger because it was thicker", "C was bigger because it was longer". However, when İnci wrote the numerical values of the edges, the students calculated the perimeter and noticed immediately that they were all equal. Here, İnci asked why they said A, B or C each time. Her guidance was good. One student tried to find the perimeter of a whole figure but a different part was being asked. İnci drew student's attention to this point since he missed it.

In this excerpt, Semih's primary focus was on teacher pedagogy and students' mathematical thinking. Although he focused on particular students' mathematical approach, he did not make sense of their thinking (e.g. "One student conceived three edges as equal because they all had right angles but I do not know how he came to that conclusion"). He referred to specific student answers to strengthen his analyses but his comments were rather general and without justification or a detailed description of his observation. He also mentioned teacher's pedagogical approach which was to query students' thinking and guide them. He mostly made evaluative comments (e.g. "I think it was a good explanation" and "Her guidance was good"). He was inconsistent in providing details about the interactions in the classroom. Since he focused further on teaching and learning issues with a general perspective, his noticing exhibited the properties of Level 2.

> **Semih:** Communication of İnci with the students was well. However, disruptive noises occurred sometimes. Students were willing to come to the board. Besides I took notes on adding a prize question because students had asked whether there would be any. I think their teacher asks them such questions in general.
> **Mehmet:** But we do not know their teacher's opinion for this kind of a question.
> **Semih:** For example, there was one question which nobody could solve. We can give a prize for it and allow some more time for solving it… İnci told some parts quickly. She could have waited a bit more for them to write it down. In addition, while calculating the perimeter, she put a starting point as considering the cooperating teacher's suggestion after the first implementation. It turned out to be a good idea. Her classroom management, tone of voice and control on the lesson plan was really good. According to me, we do not need to add on or remove anything from the lesson plan.

In terms of what to notice, Semih focused on a range of issues such as classroom management, participation, student enthusiasm, teacher pedagogy and the behavioral issues (e.g. "Students were willing to come to the board" and "İnci told some parts quickly"). His focus was on the class as a whole and his comments were quite general. He described what he observed with superficial statements without details and interpretations. He did not focus on students' mathematical approach or did not try to make sense of their thinking. He mostly attended to less mathematical aspects of the lesson. His claims were not clear. For example, he did not state how putting a starting point was related to the mathematical goal of the lesson. He offered evaluative comments in nature. He provided little or no evidence to support his ideas instead he offered general impressions. Hence, his comments predominantly had the features of Level 1.

After the re-reflecting phase, individual interviews were made with preservice teachers in order to obtain richer information about their noticing and reveal the missing points that preservice teachers might have forgotten to mention. Semih's comments differently from what he shared in the re-reflecting phase were as below:

> **Semih:** Using materials facilitated both our teaching and students' learning. They provided visualization and active participation of students. However, some mistakes occurred in measurements because students could not use the ruler properly. On the other hand, İnci tried not to make same mistakes in the first implementation but she sometimes could not prevent this situation. She had to skip some questions because time was not enough. However, she implemented the lesson plan well in general terms.

In this quotation, it is seen that Semih's focus was on teacher's approach and pedagogical issues such as visualization, active participation and benefit of material. He attended to points which were mathematically insignificant. His comments were piece by piece and quite general. For example, what kind of mistakes he referred to or how he made assessment about teacher's implementation was not clear. He made predominantly descriptive and evaluative comments with a superficial approach. He offered few or no details about his observations to promote his claims. Thus, his noticing exhibited common features of Level 1.

When Semih's all comments in the re-reflecting phase were considered, his noticing mainly had the prevalent features of Level 1. He attended to a range of issues which addressed less mathematical aspects of the lesson rather than students' mathematical thinking. He also focused on the whole class instead of particular students. He generally did not provide details or offered little evidence to support his ideas. He had a predominantly general impressionistic stance. He described what he observed without elaborating and evaluated the events in a general manner. Although some of his comments were considered as Level 1 and Level 2, his noticing in the re-reflecting phase had the properties of Level 1 mostly.

4.2.2 Lesson study cycle 2

4.2.2.1 What and how Semih notice in planning phase

The second lesson plan was related to teaching of area and the objectives of the second cycle were “Being able to calculate the area of a rectangle; use square centimeter and square meter” and “Estimate a given area using square centimeter and square meter units”. Before planning the lesson preservice teachers went to the practice school to get help and recommendations about the subject from the cooperating teacher. They listened to her ideas, took notes and asked the questions that came to their minds as mentioned before. The following are presentations of examples based mainly on Semih’s expressions from the planning meeting of lesson study cycle 2.

While planning the lesson, preservice teachers agreed on showing the meaning of area through covering with unit squares in order to make students understand the difference between perimeter and area better. They talked about how to present this meaning. The following presents the conversation among preservice teachers:

> **Semih:** We can prepare a plotting paper using a piece of white cardboard. We draw unit squares on it and then cover it with transparent gelatin in order to write and erase on it easily. Later, we prepare colorful unit squares. Students cover inside the figures which we draw and find the area by counting them one by one. For example, one student counts as 18 so that s/he can understand area is 18 unit squares.
> **Hasan:** We can cut a box of sugar cubes and place the cubes of sugar in it.
> **Semih:** But what you mentioned is related to three dimensional figures and the concept of volume not area. For example, we can outline the edges of a rectangle with some colorful tape on a plotting paper, and prepare another rectangle piece of cardboard in a different color for the interior region, and then we stick that rectangle piece on the plotting paper at the board. Here, the tape represents perimeter and cardboard the area. Therefore, we can show the difference between perimeter and area clearly in this way.
> **Mehmet:** Yes, it is possible.
> **İnci:** You said we will stick a whole rectangle inside the taped rectangle but how will its area be calculated?
>
> **Semih:** No, there will be no numeric calculation. We just try to give the concept of area and the meaning of it. We expect students to understand that the edges around a figure form the perimeter and the interior region is area.
>
> **Mehmet:** Thus, we will emphasize the difference between them.

In terms of what to notice, Semih’s focus was especially on students’ mathematical understanding and teaching of the subject. He concentrated on teaching the meaning of area and the difference between area and perimeter because of students’ confusion regarding these points. He tried to elaborate how his suggestions about implementation were related to the overall goals of the lesson. He focused on the content of activities rather than students' mathematical thinking. On the other hand, he noticed that another member's statement was not related to the objective of this lesson and he explained the reason for it. Regarding how to notice, he provided details about what should be done step by step but his comments related to students were quite general with fewer details (e.g. "Student counts as 18 in order to understand area is 18 unit squares"). He referred to the noteworthy events but his approach was mostly descriptive and evaluative. He was inconsistent in providing details about his ideas. His noticing had further properties of Level 2.

Preservice teachers maintained conversations on what to include in the lesson plan during planning. Semih indicated one activity which caught his attention and he offered to do it. The interaction among preservice teachers was as the following:

> **Semih:** There is an activity here and I do not know what you think but I liked it. There are different figures which were separated into unit squares but they are covered with colorful cardboards at first. Students are required to estimate which figure has a bigger perimeter and area. After the cardboards are removed, students see whether their answers were correct by counting the edges and unit squares. This lesson plan includes the objective based on estimation that is why it can be appropriate.
> **İnci:** I think it is good. We can use it.
> **Semih:** We should pay attention to ask the reasons behind their answers here. Why they say A, B or C is important in order to learn how they think and to correct them. What can they say?
> **İnci:** They say that it seems bigger.
> **Hasan:** Its edge is longer.
> **Semih:** They may think that it is more corrugated thus it is bigger.
> **Mehmet:** According to me, the perimeter results will make them more surprised. It seems difficult.

In this excerpt, Semih predominantly focused on the issues of teaching and students' mathematical thinking. He explained how the activity which he noticed was related to the goal of the lesson. He tried to channel the other group members' attention to students' mathematical thinking and they reasoned on specific student responses by focusing on that issue. Semih considered students' strategies while designing the activity for lesson plan. He provided details about his ideas to strengthen his suggestion. He addressed specific points and tried to make sense of students' thinking. He used interpretive comments in general. He provided justifications about his ideas (e.g. "Why they say A, B or C is important in order to learn how they think and correct them"). Therefore, his noticing was further considered as evidence of Level 3.

In addition to the meaning of area, measurement units and area formulas, preservice teachers discussed on whether to give unit conversion and its necessity. There were some hesitations and different ideas about this issue. Interaction among preservice teachers on it is presented as below:

Semih: For example, one square in the form of 1mx1m is prepared from a carton and the area of it is found as 1 m^2. Later, one edge of this square is measured in terms of dm and area is found as 100 dm^2. Thus, it aids to show unit conservation and relationship between dm^2 and m^2 concretely. Can we use it?
İnci: Unit conservation may be difficult for students. According to me, we should not try to teach it. It is not our main goal directly.
Semih: Ok. Then, we do the activity which includes measuring the area of a figure with $cm^{2.}$ Later, we query which measurement units are used to measure the area of their school garden and we emphasize that meter which is bigger than centimeter as a measurement unit can be used for it. We indicate their usage and differences verbally.
Hasan: Will we give the unit conservation table in the shape of steps?
Semih: We can state that we use cm and mm for small figures and dm and m for bigger figures. Later, we can indicate that we use cm^2 and mm^2 for small areas and dm^2 and m^2 for bigger areas.
İnci: You can also give that 1m = 100 cm here.
Semih: No, there is no need to mention unit conservation. Knowing only measurement units and their order is enough for students.
Mehmet: I agree.

In this quotation, Semih focused mainly on teaching and learning issues. He concentrated on teaching the relationship between area measurement units. He offered an activity for it but he described it and asked the other ideas about the use of it. However, he provided few details about why he especially offered it or how it will canalize students to think. He referred to pedagogical issues such as linking with examples from daily life, concreting, and explaining place of use too but he did not elaborate his ideas. His approach was rather general. He made mostly descriptive and evaluative comments (e.g. "Knowing only measurement units and their order is enough for students"). He did not focus on students' mathematical thinking or did not associate his comments with their understanding. He considered the objective rather than the students but in addition he focused on the mathematical aspects of the lesson. Thus, his comments here were accepted as the indicator of Level 2.

Preservice teachers discussed on how to teach the measurement units and why they are used. They did not want to say the area measurement units right away and looked for more effective ways. They paid attention to associate units with the concept of area too. Semih offered an activity from a book to make students notice the need for measurement units. The dialogs below show the interaction among the group members on this issue:

Semih: I also liked another activity in a book. Normally, students do not know how to compare two different measurement units. Here, for example, there are two same rectangles and one of them is covered with big squares whereas the other one is covered with small unit squares.
Mehmet: These rectangles have the same area.
Semih: Yes, they do. Is it included in the scope of the next objective in third plan?
İnci: I think it is here. Why do you think that it should be in the next lesson plan?
Semih: I agree. It is further related to the measurement units. It shows that if different measurement units are used, the results will be different therefore we can show the need for standard measurement units and we can proceed to m^2 and cm^2 from this point.
Mehmet: Ok. We can use them here.
Semih: Here, the area of rectangles is the same but the needed unit squares will be different. According to you, how do students think or what can they say?
Hasan: They can say that the size of unit squares is different hence they are different.
Mehmet: 1x1 square is used for one rectangle whereas 2x2 square is used for the other.
Semih: One is bigger that is why fewer unit squares are needed. The other is smaller consequently more unit squares are needed.
İnci: All we said has the same meaning. Similar answers are given I think.
Semih: Ok. We will emphasize this point if they do not notice. Thus, they will realize that different results are obtained with different units and why certain measurement units are used.

In terms of what to notice, Semih's main focus was on students and their mathematical thinking. He reasoned on how they think mathematically and he promoted the other members to focus on it too. He referred to possible students' answers (e.g. "One is bigger that is why fewer unit squares are needed" and "The other is smaller consequently more unit squares are needed"). He explained how his suggestion was related to the goal of the lesson. In general, he provided details about his ideas to draw about student understanding. He addressed the noteworthy events and his comments were interpretive in nature (e.g. "… therefore we can show the need for standard measurement units…"). As a consequence, his noticing had the features of Level 3.

During the planning phase, Semih made comments considering students' mathematical approach and pedagogical issues "Students need to reach generalization of area formulas", "How can students give answers?", "We should pay attention to simplifying and visualizing" and "It may be difficult for students". He made his suggestion considering the students and the goals of the lesson. However, he did it in a broad manner and provided justification about his ideas occasionally. He had an evaluative approach with an interpretive stance in nature. His noticing mostly was considered as evidence of Level 2 in some dialogs.

Preservice teachers talked about what kind of problems should be asked to the students. They made various suggestions and discussed on them. The conservation among preservice teachers is as the following:

> **Semih:** We talked about students' difficulties and misconceptions in the questions which were based on the relationship between edges. We can ask what happens to the area of a rectangle when one edge is multiplied with 3 and the other edges are divided with 2.
> **Mehmet:** Yes, it can be asked.
> **Semih:** Besides we can give the area and one edge of a rectangle and want its perimeter. We can combine them to reduce confusion and it can be good.
> **İnci:** We can also use a rectangle and square together.
> **…**
> **Semih:** We can finish the lesson by repeating. We can ask questions such as "What is meter?", "Where do we use it in real life?", "What is area?", "Where do we use it?" and Why do we need meter and area?

In this excerpt, Semih focused on pedagogical issues and students' mathematical approach. He addressed general teaching and learning topics such as repetition, real life, and place of use. He considered students' difficulties or confusions while offering questions to ask. However, he did it in general ways. He did not elaborate what the difficulties or misconceptions of students were. He described the questions in his mind and evaluated them in terms of helping students' mathematical understanding (e.g. "We can combine them to reduce confusion and it can be good"). He tried to provide justification about his suggestions but his comments were not detailed and linking to students' mathematical thinking was weak. Thus, his comments were considered as evidence of noticing in Level 2.

During the interview that followed the end of planning, he indicated what they had aimed in this process. He explained what expected students' responses, difficulties and misconceptions were and how they considered students' mathematical thinking while planning as the following:

> **Semih:** Our aim was to support students to calculate and estimate area and use measurement units of cm^2 and m^2. We tried to give the difference between area and perimeter. I think students will feel that area is covering and it equals to the sum of the unit squares in it as well. Similarly we tried to combine perimeter and area and repeat what we taught in order to show the difference between them. Students may think one area is bigger than the other due to optical illusion. We selected our examples considering these kinds of points and paid attention to use different figures. For example, I offered to measure the length of the desk with my hand span. Everybody has a different hand span thus different results will be obtained although the same thing is measured. Some students may say that "my hand is big" or "my hand span is small". We will emphasize the need of standard measurement units because of this differences. We will show unt and unt^2 relationship and how to obtain unt^2. Then, we

> will give cm^2 and m^2. Students may be shy because we do not spend time with them too much, thus, we will constantly ask whether they understood or they had a problem.

In this quotation, according to Semih, students are likely to confuse the concept of perimeter and area with each other. Therefore, he emphasized showing the difference well. He explained how the group decided to respond the students so as to reduce their confusions and mistakes. At this point, he referred to covering, calculating the unit squares in the interior region, combining perimeter and area, using different figures and repetition. He mentioned how students may think mathematically and he emphasized that they tried to consider the students while planning.

Semih indicated what he generally focused on while planning and what he noticed in this process. The following is the presentation of his noticing through his comments:

> **Semih:** We know that students have some misconceptions and confusions in this subject. I focused on how we can simplify the concepts to make students understand better. In education, we need to follow an order from easy to difficult. We paid attention to this point and we even made changes in the order of some activities or questions in the lesson plan. Geometry is a difficult mathematical domain and I had difficulty in designing the lesson too. Besides, the age group is younger therefore it was complicating as well. On the other hand, while teaching the lesson, there will be the other members, the cooperating teacher and you. It makes me a little excited but it will let us see the mistakes better.

Semih's comments showed that his focus was on teaching and learning issues with a broad perspective. He referred to pedagogical issues such as the order from easy to difficult, the need of age group and lowering in students' mental level. He also addressed his own difficulties and lack of knowledge. On the other hand, he emphasized one negative and one positive aspect of lesson study, according to him. He indicated that the participation of the other members and teacher into the lesson causes excitement whereas their feedback provides rich assessment.

When Semih's all comments during the planning phase were considered, his noticing was accepted as Level 2 because he mostly focused on pedagogical issues and considered students' mathematical thinking and understanding. He endeavored to associate his comments with the overall goals of the lesson. He addressed the noteworthy events rather than less mathematical issues but he did it in general ways. His comments were predominantly evaluative with an interpretive stance. He was inconsistent in making reference to the details to support his ideas. Thus, although there were comments in Level 2 and Level 3, his noticing predominantly had the features of Level 2.

4.2.2.2 What and how Semih notice in teaching phase

The findings of this episode obtained from the implementation of the second lesson plan included the objectives of "Being able to calculate the area of a rectangle; use square centimeter and square meter" and "Estimate a given area using square centimeter and square meter units". Teaching of the second lesson plan was conducted by Semih in 5/F classroom. The findings from Semih's lesson were presented and his noticing during teaching phase was revealed.

Semih started the lesson with the repetition of the previous subject and tried to remind students what they had learned about perimeter before proceeding to the next subject. He controlled whether the students had problem or confusion to interfere in this deficiency. After he was sure that there was no need to discourse on it, he made an entry to the concept of area. During the planning phase, preservice teachers focused on giving the difference between perimeter and area and the meaning of them. Therefore, they wanted to show that through visualizing. The next dialog shows how Semih conducted the activity and how the interaction occurred in this process:

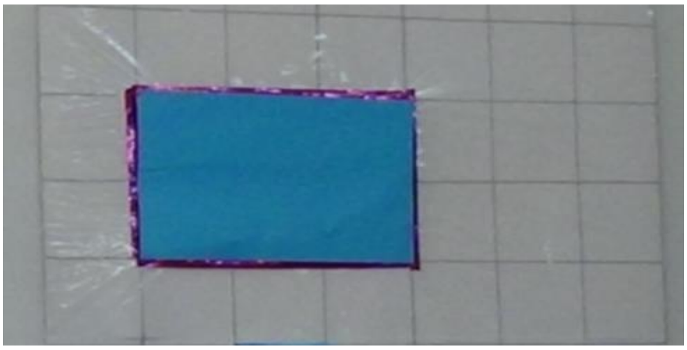

Figure 34 The demonstration of the difference between perimeter and area

Semih: What am I drawing here?
Students: A rectangle.
Semih: What does the length of this tape give us?
S1: Perimeter.
Semih: I want one of you to stick this cardboard on the figure (One student comes). What was the purple tape? It was the perimeter of a rectangle. What is the blue cardboard inside the rectangle?
S2: Area.
Semih: Yes, the edges around the figure form the perimeter and the interior region is area.

Later, Semih continued the lesson through covering a figure with unit squares for the students to find the number of unit squares in the interior region of the figure and understand how to find area. Here, preservice teachers aimed to show that area can be found by counting the number of unit squares as well as using the formula. They also wanted to give the meaning of area in this way rather than rote learning of it. The dialog among Semih and students is as the following:

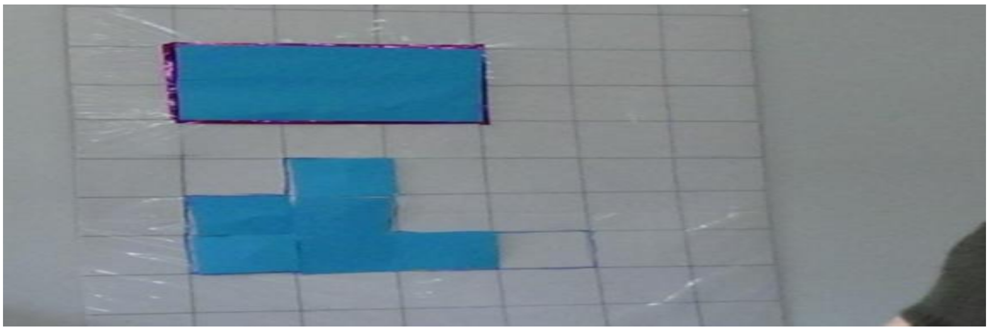

Figure 35 The example of calculation of area with unit squares

Semih: Now, the squares that you saw on the board are equal to each other, namely, their edges and interior regions are the same. I want help from one of you (One student comes). I will give these colorful unit squares and you will stick them on the material, ok?
S3: Yes, teacher.
Semih: Look at the board. Your friend placed the figures. What did we say for them? Unit square. Here, how many unit squares are there in this figure?
S3: 7.
Semih: Then, what is the area of the figure? 7 unit squares, right?
Students: Yes.
Semih: There is no unaccountable point I think.

Semih created a rectangle using a tape and wanted to draw students' attention to the difference between perimeter and area through materials. After Semih drew a rectangle, he asked the name of the figure and the meaning of the length of the tape. He wanted to repeat what they had learnt before. Students could give the answers as preservice teachers had foreseen in planning. Since he heard the correct response, he did not focus on this point to obtain more detailed information. After the inside of the figure was covered with cardboard, students could say its area, however, Semih did not want them to elaborate their ideas. He considered that correct answers were enough and accepted them as an indicator of students' understanding. He did not even query the difference between perimeter and area to see if there were any misconceptions or confusions, instead he preferred to indicate them himself. Although the purpose of this activity was to focus on this point in order to reduce confusion related to these two concepts, Semih further focused on conducting the activity.

Later, he paid attention to introduce the materials and gave information about their properties. He expected students to find the area of the figure by covering it with unit squares. The main aim here was to show that the number of unit squares in the interior region of the figure was equal to the area of it, by concretizing. Students could find it easily but he did not make the necessary emphasis here. He did not allow the students to convey their thinking and their understanding. He rather attended to whether the result was found correctly. In the planning phase, preservice teachers decided to reinforce area through counting the unit squares in the figures. They prepared an exercise sheet which included these kinds of questions. After Semih showed an example related to covering, he gave students an exercise sheet to find the area of figures. He waited for the students for a while and then started to solve the questions. He asked the answers verbally. When he heard that some students found one more or less, he wanted the students to count the unit squares again. After students found the correct answer, he indicated that they made a mistake while counting. It appeared that Semih noted specifically what kind of mistakes students made in regard to finding the area but his response was limited in revealing their thinking clearly.

It can be argued that Semih's noticing in this dialog was Level 1 because he was not able to attend to students' mathematical thinking and promote them to elaborate their mathematical approach. He did not listen to the students efficiently and thus could not respond them in a way to see whether they had different reasoning patterns. He focused on the correctness of the responses, however, giving correct answers does not necessarily show students' understanding. Thus, his approach was very broadscale and evaluative in nature. He asked general questions to get these answers and he made deductive comments instead of students. He further indicated what he had in his mind and focused on conducting the activity without permitting students to understand the purpose. Thus, his noticing can be accepted as Level 1.

During the planning phase, preservice teachers talked about how to show the need for use of standard measurement units and then to proceed with cm^2 and m^2. They decided to ask students to cover two same rectangles with different unit squares. They thought that students would find the number of the necessary squares for the same figures differently as a result there would be confusion. They believed that students would realize that different measurement units gave different results. How Semih conducted this activity and interaction in the classroom is presented below:

Semih: You see plotting paper on the board. Think it as your notebook. Your notebook is also plotting. I will draw a figure and you will draw it accordingly. 4 unt rightward and 8 unt downward and then draw the same figure next to it. If there is anybody who does not understand, I can help.

Students: No. We will draw the same rectangles.

Figure 36 The example of covering the rectangle with 1x1 squares

Semih: Ok. I need someone (One student comes). Cover the first figure with these squares. All of them are equal. How many squares did you use to cover the rectangle?

S4: 32.

Figure 37 The example of covering the rectangle with 2x2 squares

Semih: Ok. Thank you. I need someone else (One students comes). Cover the second figure with thees squares. How many squares did you need to cover the rectangle?

S5: 8.

Figure 38 The demonstration of the same rectangles with different size squares

Semih: Both rectangles were the same but we found the number of squares for covering them different from each other. According to you, why?
S6: One square is bigger than the other.
S7: In the first rectangle, we covered one by one. In the second rectangle we covered four each.
Semih: What do you mean with one and four?
S7: First square's area is one and the area of the second square is four.
Semih: How do you know that the area of the second figure is four?
S7: There are four unit squares under it.
Semih: Good. Another answer?
S8: The size of squares was different.
Semih: Yes, we used different unit squares and we obtained different results. Is there any point that you did not understand?
Students: No, teacher.
Semih: Ok. There are desks in front of you. They are the same. I want you to measure the long edge of this desk with your hand span. What was hand span? When you open your hand, it is the distance between this finger and that finger, right? (He shows on his hand).
Students: Yes.
Semih: What did you find?
S9: 7.
S10: 7,5.
S11: 7.
S12: 6.
Semih: Although the length of your desk was the same, why did you find different results?
S13: Because lengths of our hand spans are different.
S14: Someone's hand is big, someone's hand is small.
Semih: Good. In order not to cause this kind of confusion, there are standard measurement units which are accepted by everyone. For area, some of these measurement units that we use are m^2 and cm^2.

It is seen that Semih endeavored to explain the purpose of the materials and facilitate understanding by associating with students' notebook. He guided the students with comments such as "4 unt rightward and 8 unt downward" too while drawing the figure in order to prevent drawing on the notebooks correctly. Students did not have difficulty in covering the rectangles with squares and finding the answers. After Semih queried the reason behind the difference between the number of unit squares for two same rectangles, students gave the answers that preservice teachers had expected while planning the lesson. Semih focused on particular students' responses and tried to make sense of how they thought mathematically. For example, student S7 noticed the area of squares but Semih did not find her answer enough and wanted to learn more about her comments (e.g. "How do you know that the area of the second figure is four?). Similarly, he tried to learn whether there were different thinking patterns with questions such as "Another answer?". He paid attention to check whether there were any points which could not be understood by students. He promoted the students to explain their ideas and allowed them to reason on the question. After he believed that there would be no different answers, he maintained the activity. At this point, he again paid attention to remind the needed concepts not to cause a mistake and explained how to measure with a hand span by visualizing. He asked why different answers were found to learn more about their thinking. After he was convinced that their answers were enough, he accepted their responses as indicator of their mathematical understanding but he continued to query until getting the necessary explanations. He conducted the activity step by step in accordance with its goal.

Semih's noticing in this activity was accepted as Level 3 because he did not only listen to the students' answers about area of the figures, but also queried them in a way that could reveal their mathematical thinking. He guided students to give the answers which were needed for the lesson instead of telling them directly. He focused on specific students and their mathematical thinking rather than the whole class. He encouraged the students to provide more details about their reasoning through questions such as "According to you, why?" and "What do you mean with one and four?". He attended to students' ideas and supported them to understand that when different measurement units are used, the obtained results would be different. He listened to the students' comments and attended to their answers in an interpretive manner because he endeavored to make sense of their mathematical approach to comprehend what students understood. He accepted their correct answers as indicators of their understanding. However, he did not accept immediately, he continued to query until different answers were not given. Because he focused on the students' mathematical thinking and guided them to provide details about their mathematical approach and he had an interpretive approach, his noticing was considered as evidence of Level 3.

As planning the lesson, preservice teachers did not want to give area formulas of rectangle and square directly, instead they aimed to support students to obtain the formulas by themselves. Thus, they decided to make students feel the area formula from the relationship between row and column by querying the number of unit squares in each row and repetition number of the row throughout the column. The following dialog describes what kind of interaction occurred between Semih and the students:

Semih: Draw the figure with me in your notebooks. What did we say for this square?

Students: Unit square.
Semih: Remember this plotting paper on the board was the same as in your notebook. Now, draw the figure. Is there anyone who did not come to the board? You come.

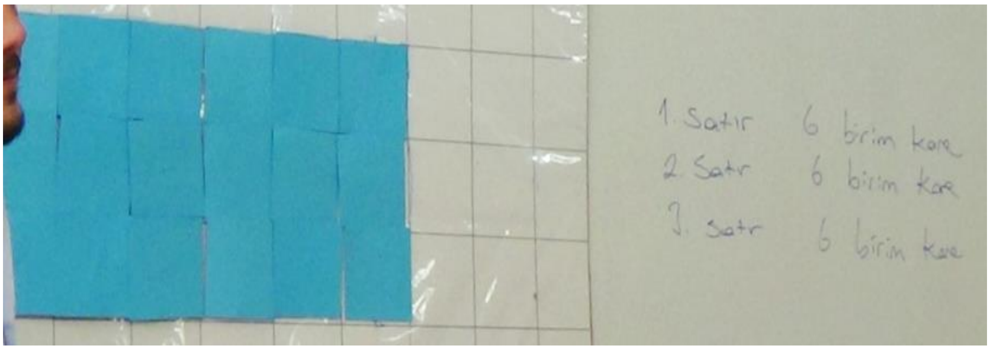

Figure 39 The activity for introducing the area formula of a rectangle and square

Semih: How do we name the horizontal line? Row or length. What is the name of the vertical line? Column or width, right?
Students: Yes.
Semih: While your friend is sticking the unit squares, you will also shade the same squares in your notebooks. Follow him. Cover the first row. How many unit squares are there in the first row?
S15: (He sticks the unit squares by counting) 6.
Semih: How many unit squares are there in the second row?
Students: 6.
Semih: How many unit squares are there in the third row?
Students: 6.
Semih: What is the area of this rectangle?
Students: 18 unit squares.
Semih: How many rows are there?
Students: 3.
Semih: How many columns are there?
Students: 6.
Semih: What kind of relationship is there between area and edge lengths?
S15: We multiplied 3 and 6.
Semih: In other words, how did we find area? (He asks the class).

S16: By multiplying edges.
Semih: Which edges?
Students: Long and short edges.

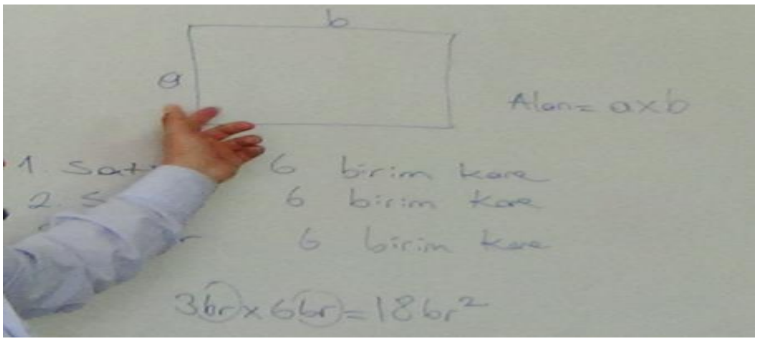

Figure 40 The example of calculation of rectangle area

Semih: Here, we multiply 3 unit and 6 unit and then it comes 18 unit squares. We can write this expression like that 3br x 6br = 18unt^2. There is one unt next to 3 and there is another unt next to 6. When I multiply them, I write in the form of unt^2 because there are two unt and I write 2 on unt. Did you understand?
Students: Yes.
Semih: While finding the area of a rectangle, what did we do? We multiplied the short edge and long edge or we multiplied the width and length. If we say "a" for the short edge and "b" for the long edge, how do we write area formula in general?
Students: axb.

Semih started the activity with recalling related concepts such as row, column, width and length. He encouraged the students to see the relationship between the number of unit squares in a row and column. He wanted students to reach the conclusion that when the number of unit squares in one row was repeated throughout the column, the area- the unit squares which were needed to cover the interior region of the rectangle- was found. He asked general questions in order to lead the students to this point step by step. He helped the students to give the needed answers for generalization instead of indicating what he intended. In this direction, students gave necessary answers and they could say how to find the area of a rectangle. Semih' comments regarding how to proceed from unt to unt^2 were quite unspecific. He told what was in his mind directly and did not promote students to rationalize their thinking on this issue. He accepted students' expression "yes" as an indicator of understanding without listening to any comments.

Semih mostly maintained the flow that he had in his mind and guided the students to give the answers that he wanted to hear. He focused on getting the students to say expressions such as "By multiplying edges" or "axb". Although he endeavored to attend the students' reactions and responded them in accordance with their answers, he was inconsistent in promoting students to elaborate their mathematical thinking. He wanted students to provide more details about their mathematical approach through questions but he did in a general way. He did not query their reasoning sufficiently. Semih listened to the students and attended to their answers in an evaluative manner. He evaluated whether students' answers and explanations were correct and enough. He asked basic questions step by step to provide a clear solution process and had a general approach rather than interpreting their thinking. He could focus on individual responses minimally while querying the relationship between the unit squares in a row and column. Therefore, he could not notice whether there were mistakes or confusions and he decided on students' understanding taking the whole class into consideration. Because of this mixed approach, his noticing had prevalent features of Level 2.

After giving area formula of a rectangle, Semih continued with two questions to reinforce this concept. The following dialog represents the interaction between Semih and students:

> **Semih:** What is the area of a rectangle with a width of 5 cm and a length of 12 cm?
> **S17:** 34.
> **Semih:** How do we find the area of a rectangle?
> **S18:** 60, 5 times 12 is 60.
> **Semih:** We learned measurement units. There is one cm here and another here. How many cm are there? Two. Thus, what is the unit for the area here? Cm^2. What is the area?
> **S18:** 60 cm^2.
> **Semih:** Another question, what is the area of a rectangle with a perimeter of 54 cm and a width of 12 cm? (He waits for a while). Ok. Let's solve it. You come.
> **S19:** Perimeter is 54 cm and width is 12 cm. If one short edge is 12 cm, the opposite edge is 12 cm as well. Thus, I will multiply 12 with 2, it equals 24 cm.
> **Semih:** What did you find?
> **S19:** I found the sum of short edges. I will subtract 24 from 54 because the perimeter is 54 cm. It equals 30 cm.
> **Semih:** What is 30 cm?
> **S19:** The sum of long edges. If I divide 30 with 2, I find one long edge which is 15 cm.
> **Semih:** How do we find the area of a rectangle?
> **S19:** I multiply 15 and 12. Area equals 180 cm^2.
> **Semih:** Did everyone find it like that?
> **Students:** Yes.

The first question was towards using area formula directly. Thus, most of the students could find the answer easily. One student gave a wrong answer but Semih probably interpreted that student S17 found perimeter instead of area. Therefore, he tried to lead the students to focus on area with a question like "How do we find the area of rectangle?". However, he did not focus on this answer by asking the whole class and his response could not provide to reveal explicitly what S17 was thinking. Semih focused further on the correctness of the answer. It is seen that the students forgot to use measurement units in general. Semih reminded them to use the appropriate unit by emphasizing why this unit had to be written. However, in the second question, Semih's approach was more elaborative as well as evaluative. He tried to learn the details of student's mathematical strategy and make sense of it. His questions were towards understanding what the student did while solving the question (e.g. "What did you find?" and "What is 30 cm?"). He checked whether the student knew what she found in each step in order to draw inference about her understanding.

Semih's noticing in this dialog was considered as Level 2 because he was inconsistent in promoting students to elaborate their mathematical thinking. He endeavored to understand what one student did mathematically in detail whereas he did not focus on another student's wrong answer. He further exhibited an evaluative approach and he adopted an interpretive stance too. He attended to the correctness of the responses but he listened to the students and allowed them to explain their mathematical strategy instead of indicating what to do. This mixed approach exhibited the characteristics of Level 2.

Semih maintained the lesson with the activity related to area formula of square in a similar way that he used for rectangle. After students reached the formula axa for square, he asked two questions to make students understand it better. The conversation between Semih and students is as below:

Semih: Find the area of a square with a perimeter of 40 cm. (He calls one student) Why did you divide 40 by 4?
S20: Because it has four edges.
Semih: Only, because of having four edges?
S20: In order to find one edge.
Semih: We divide by 4 because square's all four edges are equal to each other. What did you find for one edge?
S20: 10 cm.
Semih: How do we find area?
S20: I will multiply 10 by 10, the area is 100.
S21: So we found the interior region of it as the area.
Semih: Yes. In other words, we found the interior region of a square. From cmxcm, area measurement unit comes cm^2, right? Write it next to 100. Do not forget units.
S20: 100 cm^2.
Semih: I will ask a question verbally. Answer me. One edge of a wall in the shape of a square is 7 m. What is the area of it?
S22: 28.
Semih: What do we do while finding area? What did you find?
S23: He found perimeter.

Semih: There is a square and one edge of it is 7 m. What is the area of it?
S24: We multiply 7 by 4.
Semih: But we said that if we multiply 7 by 4, we find perimeter not area.
S25: 14.
Semih: Ok. Let's do it by drawing instead of talking.

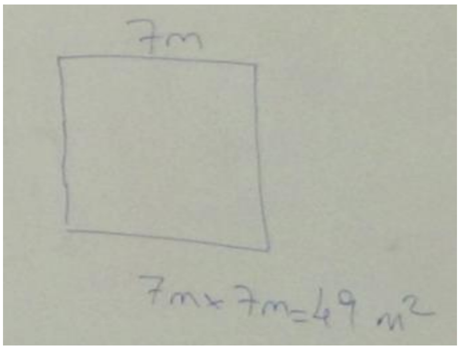

Figure 41 The example of calculation of square area

Semih: All edges are equal and 7 m, right?
Students: Yes.
Semih: What is area formula of a square?
S26: axa. Thus, from there, 7 times 7, 49.

Semih endeavored to learn the details of student S20's mathematical strategy and wanted to understand whether the student knew what he found as doing mathematical operations. When Semih asked the student why he made division, the student's responses of "Because it has four edges" and "In order to find one edge" did not correspond to Semih's question. The student did not make incorrect comments but he could not indicate the important point related to the equality of all edges, namely, the main reason for this operation. His responses showed that the student did not understand what Semih intended. Semih endeavored to direct his attention to equality of edges by emphasizing like "Only, because of having four edges?" but the student could not give the answer in a way that Semih expected. It seemed that Semih and the student were referring to different aspects of the same issue and it might be a signal of lack in mathematical understanding. At this point, Semih preferred to explain the reason of this mathematical operation instead of asking the other students and learning their mathematical thinking. There might have been students who gave correct or different answers, hence Semih missed the opportunity to elaborate the students' mathematical approach. Similarly, using appropriate measurement unit while indicating the result was forgotten. Semih paid attention to and reminded this point again as the preservice teachers had agreed upon in the planning phase. Later, he asked a basic question towards using area formula of a square.

Preservice teachers had thought that students would solve this question easily and they had decided to ask it verbally in the planning phase. However, the students confused perimeter and area with each other and they found the perimeter instead of area here. Semih attempted to make students realize what they found with leading questions. Although some students understood that they made a mistake, some of them maintained to give answers referring to perimeter. Semih had difficulty in guiding the students and restated his comments in different ways (e.g. “What did you find?”, “We find perimeter not area” and “What is the area of it?”). Student S25 gave the answer as 14 but Semih ignored this wrong answer and did not ask the student how he found this answer. After his questions did not help the students find the area of the square, he needed to draw a figure and emphasize its area formula for the students to find the correct answer.

Although he endeavored to attend the answers of the students and responded them in accordance with their answers, he was inconsistent in revealing their mathematical thinking. He wanted students to give more details about their mathematical approach through questions. Yet, he often did not query their reasoning. Semih listened to the students and attended to their answers in an evaluative and interpretive manner. He evaluated whether students’ answers and explanations were correct and enough. Besides, he tried to interpret students’ thinking to figure out what students understood. He asked basic questions step by step to provide a clear solution process and had a general approach. He queried the reason behind student’s mathematical strategy partly but he could not maintain this approach continuously. Semih’s attention to students’ ideas and his responses to promote the students’ mathematical thinking revelations were limited. He tried to learn how students’ mathematical approaches were, however, he could not question students’ strategies sufficiently. Thus, his noticing here was considered as evidence of Level 2.

During the planning phase, preservice teachers focused on an activity which included first estimating and later calculating the perimeter and area of the given figures. It was aimed to show students their mistakes, if any existed, by enabling them to compare estimations and real results. Preservice teachers thought that they would combine perimeter and area in this manner and it might help to reduce the students’ confusions about these concepts. Normally, the teacher should have covered the figures with colorful cardboard before estimation and then he should have asked the students to make a calculation by removing them. However, Semih forgot to cover the figures and the students saw the unit squares on them. Thus, he conducted the activity with some deficiencies. The interaction in the classroom is as the following:

Semih: I am sticking three figures on the board. According to you, the perimeter of which figure is bigger?
S27: All of them are the same.
S28: B.
Semih: Why did you say B?
S28: Because there are more squares on it. Here, there are 8 but the others have less.
Semih: Is there anyone who has a different opinion?
S29: Figure C.
Semih: How can we find the edge lengths?
S29: By multiplying.
Semih: We find area by multiplying, but here are no lengths. These figures consist of unit squares, right?
Students: Yes.
Semih: Let's look at how many units there are around the figures. Did not you understand? There are unit squares inside of these figures. Now, we will count the units around them. How many are there here? 2. Here? 3. We were summing edge lengths while finding perimeter. Ok. I will write them on clearly (He refers to the red numbers). 2+2+3+3=10 unt for the first figure. Perimeter of it is 10 unt. Who wants to answer for figure B?

Figure 42 The question related to calculating perimeter and area

S30: (He counts each unit by marking on it) perimeter of figure B is 14 unt.
Semih: Come for figure C.
S31: 13 unt.

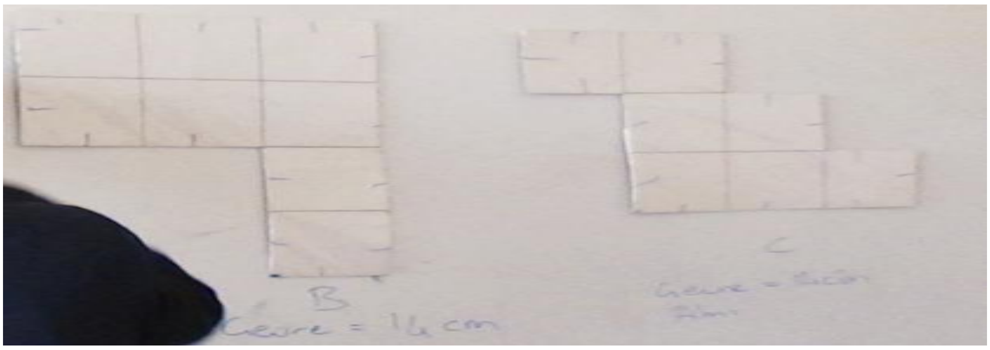

Figure 43 The example of students' solution regarding perimeter

Semih: Is it 13? Let's count together. 1, 2, 3, ….14, right? Namely, perimeter of B and C is the same and bigger than the perimeter of A. Did you understand perimeter?
Students: Yes.
Semih: What can we say about the area of these figures? How do we find area?
S32: By multiplying.

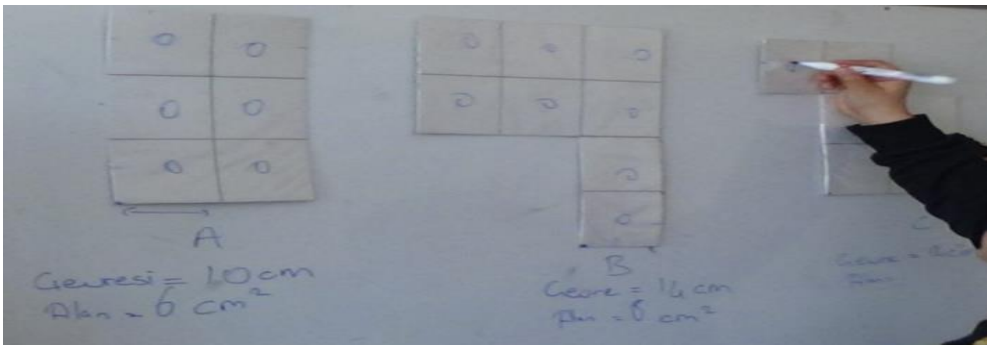

Figure 44 The example of students' solution regarding area

Semih: They consist of unit squares therefore we can find area by counting them too, can't we? Which area is bigger?
S33: B.
Semih: Why did you think as B?
S33: A has 6 unit squares, B has 8 and C has 7 unit squares.
Semih: Is there any other answer?
S34: I thought similarly.
Semih: Which area is bigger?
Students: B.
Semih: We counted unit squares to find area and summed the edge lengths to find perimeter.

When Semih heard students' different estimations about the perimeters of the figures, he asked some of them the reason behind their answers. However, he did not focus on all of them and did not promote students to explain their mathematical thinking. For example, he asked why the student said "B" but he did not want an explanation about why the student said "all of them are the same" or "C". On the other hand, while Semih was referring to edge lengths, one student addressed area. His respond indicated that he did not understand what Semih indicated. Semih focused on conducting the activity and preferred to tell what to do instead of allowing the students reason on it. When he noticed a mistake he guided the student to recount the edges again. He asked how to find area and the student referred to the formula but Semih indicated its meaning based on counting instead of learning the other students' ideas. It was also seen that the students marked around the edges with small lines and the inside of the unit squares with small dots in order to understand where they counted. However, he did not query the meaning of these marks for the students.

It can be argued that Semih' noticing in this dialog was Level 2 because he was inconsistent in revealing students' mathematical thinking and elaborating their strategies. Semih mostly maintained the flow that he had in his mind and guided the students to get the answers that he looked for. Since he made a mistake while conducting the activity, it could not serve the purpose of the lesson completely and might cause incorrect answers. Although he tried to learn more details about some students' answers, he did not pay attention to some responses. His approach towards students' reactions was evaluative and interpretive. He sometimes could not decide how to respond the students and told what he had in his mind directly rather than promoting students to reason on it. Therefore, his noticing further had the features of Level 2.

Preservice teachers wanted to enrich the content of the problems by combining perimeter and area or rectangle and square in order to reinforce area and perimeter. They also wanted to ask the change of area with addition or subtraction. Semih asked students one of these kind of questions and the following shows the dialog between them:

Semih: What is this figure?
Students: A rectangle.
Semih: Its length is 12 cm and is 10 cm. How do we find the area of a rectangle?
S35: We multiply length by width. From 12 times 10, area is 120 cm^2.

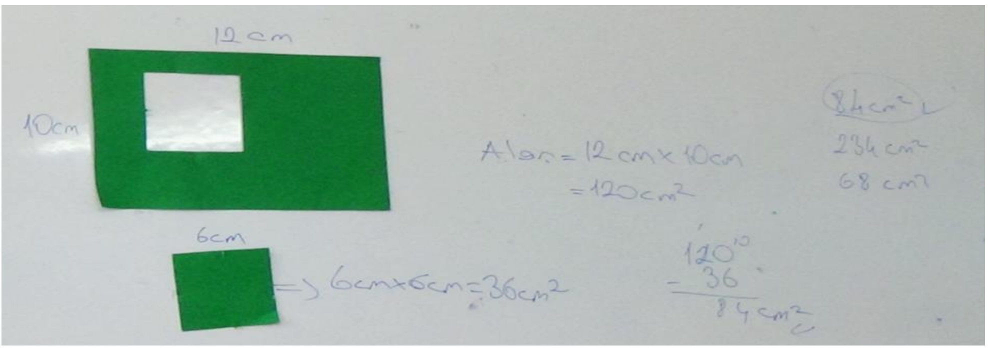

Figure 45 The example of the students' solution of area problem

Semih: When I subtracted this square from it, did the area change?
Students: Yes.
Semih: One edge of the square is 6 cm. When it is removed from the rectangle, what is the area in the new situation? What did you find?

S36: 84 cm^2.
S37: 234 cm^2.
S38: 68 cm^2.
Semih: Look at the solution. What was the area of the rectangle at first?
Students: 120 cm^2.
Semih: How do we find the area of a square?
Students: Multiplying one edge by the other.
Semih: Then, from 6 times 6, the area of the square is 36 cm^2. I will remove this part from the rectangle. How can I find the remaining area?
Students: Subtracting 36 from 120.
Semih: From 120-36, it is found as 84 cm^2. The students who found 84 found the correct solution.

In this dialog, it is seen that Semih predominantly focused on the correct solution of the question. Previously, the students could find the area of a rectangle but when they were asked to find the remaining area after subtraction of a square, there were different answers. However, Semih ignored the wrong answers and maintained the solution by himself in a general manner. He did not attend to students' mathematical thinking He told what he intended straight away instead of allowing students to explain their strategies.

Semih's noticing during this activity was classified as Level 1 because although he endeavored to attend to the students' reactions, he did not respond the students in a way that could reveal their mathematical thinking and he did not query the reasons behind the students' answers. He focused on the whole class rather than specific students. Although he listened to the ideas of the students, he did not try to make sense of their mathematical approach. He mostly maintained the flow that he had in mind and guided the students to get the answers that he wanted to hear. He maintained a pattern of questioning which helped students notice what was intended through a general approach instead of getting more details about their thinking and learning. He had a descriptive approach and restated only what was on the board. Hence, his noticing mostly exhibited the properties of Level 1.

When the whole process of teaching was considered, Semih's noticing mainly had the characteristic of Level 2. His focus was on students' mathematical thinking and understanding. He mostly listened the students' ideas and responses instead of indicating what he had in his mind. However, in the cases which the students could not understand what Semih meant, he needed to tell them what to do directly. Although he strived to attend to the students' answers and promote them to elaborate their mathematical approach, his responses to reveal their thinking were inconsistent. He sometimes inquired the reasons behind the students' answers but at times he did not pay attention to their explanations. He focused on both particular students and the whole class. His approach was evaluative with an interpretive stance in nature.

He evaluated the correctness of students' responses. Semih's noticing during the teaching phase was usually in Level 2.

4.2.2.3 What and how Semih notice in reflecting phase

Teaching of the second lesson plan was made by Semih in 5/F classroom. The second lesson plan had the objectives of "Being able to calculate the area of a rectangle; use square centimeter and square meter" and "Estimate a given area using square centimeter and square meter units". While Semih was implementing the lesson plan, the other preservice teachers, cooperating teacher and researcher observed this lesson and took notes. After teaching was completed, the reflecting meeting occurred immediately at the school conference room. In the reflecting phase, preservice teachers were required to reflect their thoughts about the lesson including; what worked well, what did not work, what difficulties there were, what they observed about the students' actions and thinking and also what should be changed about the lesson to improve it and what the rationales for the changes were.

In accordance with discussion protocol, Semih shared his opinions about the lesson that he taught, evaluated the effectiveness of the lesson plan and made some suggestions to improve it. The following dialogs show what and how Semih noticed during teaching phase:

> **Semih:** While solving the questions in the exercise sheet, I drew two of them on plotting paper and wanted a student to cover them with unit squares but it would have been better without covering or just by counting the squares. It caused a loss of time.
> **Mehmet:** But then, there was no covering here, you did. Students should have counted.
> **Semih:** Would not I want to cover? Ok. I forgot it… Students were good in general but they were shy about talking and got bored sometimes. I tried to do my best in order to make them engage in the questions and activities. Therefore, I thought that this lesson served its purpose. Students liked covering with unit squares and materials drew their attention but they took time too. Maybe, we can reduce the size of rectangles and the squares for covering them. It was difficult to cover an area with 36 and 32 unit squares. We can reduce the numbers and ask them to shade some figures instead of covering them in order to find area. This way, loss of time may decrease.

In this excerpt, Semih focused on a range of issues including his own practice, time, students' behaviors, implementation and interest of the classroom (e.g. "Students liked covering with unit squares" and "I tried to do my best in order to make them engage in the questions and activities"). He also made suggestions about the content of the lesson considering time loss instead of the issues such as students' mathematical thinking and understanding in order to improve the lesson plan. Regarding how to notice, Semih described what he noticed in a general way without details. Besides he offered judgmental and evaluative comments (e.g. "Students were good in general" and "I thought that the lesson served its purpose"). However, what he meant with these kinds of statements were not clear. He provided little or no details about what he observed to support his claims. He reflected his general impressions on various issues rather than students' mathematical thinking by referring the students as a whole. Therefore, his noticing here was considered as evidence of Level 1.

> **Semih:** At the beginning of the lesson, students could say what perimeter of rectangle and square were while repeating the previous subject. I liked their answers. In the first activity, I tried to present that perimeter was the sum of edges around the figure and area was the interior region of it. At the end of the lesson, students could indicate this difference well. Thus, I thought that they could understand these concepts better. There were students who found the answers as one more or less in the questions in the exercise sheet but I think they understood finding area by counting the unit squares in general. When I asked a simple question including use of area formula of a square, students found perimeter instead of area. They confused both concepts generally.

In this quotation, Semih's main focus was on students' responses and their mathematical approach. Semih indicated what students could do or not during the lesson. He also referred to their mathematical understanding and confusions (e.g. "They could understand these concepts better" and "They confused both concepts generally"). However, he did it in a broad manner with no elaboration and superficial descriptive statements. He addressed the students' mistakes too but he did not try to make sense of their mathematical thinking. He did not focus on particular students and referred to the class as a whole. His comments were judgmental and evaluative (e.g. "I think they understood finding area by counting the unit squares in general") but how he made this judgement was unclear. He offered general impressions and provided no or little evidence from his observation to support his analyses. Thus, his noticing had prevalent features of Level 2.

> **Semih:** In the activity which included covering same two rectangles with different sized squares, students explained the difference between the needed number of squares with a statements like "one square is bigger than the other" and "their size is not the same" as we had expected. They could notice what we had intended. Then, I proceeded to standard measurement units from this point and taught them how to obtain unt^2 but the students had difficulty in measurement units. On the other hand, they could obtain area formula of a rectangle and square on the first figures. Thus, I did not need to repeat the same thing for the second figures. Besides, I forgot to cover the figures before students estimated their perimeter and area. Thus, this activity did not reach the aim and students confused how to find perimeter and area at first. Students could understand what we wanted to teach in general.

In terms of what to notice, Semih focused on his own implementation and students' mathematical approach. In addition he addressed to some difficulties and confusions the students had in an overall way (e.g. "Students had difficulty in measurement units" and "Students confused how to find perimeter and area at first"). He referred to the whole class instead of particular students. He did not focus on students' mathematical thinking and reason on their strategies. Regarding how to notice, although Semih mentioned noteworthy events, his approach was rather general. He described these events without providing details and did not make interpretation about the reason of students' mathematical approach. He mostly used descriptive and evaluative comments in nature. He was inconsistent in elaborating his claims. For example, he made reference to the evidence of students' responses but he did not provide details about what kind of difficulty students had. Hence, his noticing exhibited the features of Level 2.

After the reflecting phase, individual interviews were made with preservice teachers in order to obtain richer information about their noticing and reveal the missing points that preservice teachers might have forgotten to mention. Semih's comments differently from what he shared in the reflecting phase are as below:

> **Semih:** Students did not have many misconceptions or didn't give too many different answers. Students gave answers that we expected in terms of being both correct and incorrect. In estimation question, some of them counted the number of unit squares in the figures instead of the edges around them in order to find the perimeter. Actually, students knew the conceptual meaning of these concepts but they confused their formulas and had difficulty while making operation. Besides, one student made division by four. He did a correct operation but his explanation was deficient. The reason for the operation was not just having four edges, but the important point was the equality of all the edges in a square. I noticed that there was need to support their mathematical understanding by questioning what and why they did. Students also forgot to use measurement units or they could not use appropriate units. In order to reduce confusion about perimeter, area and units and provide better comprehension of these concepts, we can give examples from daily life as the cooperating teacher suggested. Moreover, I think that internalization of the subject will occur in time and with more practice.

In terms of what to notice, Semih's primary focus was on students' mathematical understanding, teaching and learning. He addressed to the general difficulty and confusion related to perimeter and area (e.g. "Students knew the conceptual meaning of these concepts but they confused their formulas and had difficulty while making the operation"). He emphasized pedagogical issues such as the need for querying students' ideas and integration of daily life examples for better mathematical understanding. He highlighted the noteworthy events but offered his general impressions without making reference to the specific events. He indicated that expected answers were given by students yet the points of what these answers were and how they were related to the goal of the lesson were not clear. He did not provide enough details about his claims. Although he sometimes focused on particular students, he continued to refer to the students as a whole. He did not attend to how students thought mathematically rather he described how their mathematical approach was and made judgmental comments without details. Thus, his noticing here was considered as evidence of Level 2.

> **Semih:** The cooperating teacher gave good feedbacks and we will consider them in the revision of the lesson. For example, we discussed on the question related to completing half squares to a whole square to find the area by counting but we did not ask it because of the hesitation about its suitability for the students. After the cooperating teacher's feedback, we decided to include these kinds of questions to improve the lesson plan. On the other hand, I noticed that while observing the lesson, I was more comfortable but while teaching, I had to follow the lesson plan in general terms, ask questions and engage the students. Thus, losing attention was normal due to having many responsibilities at the same time. I lost it too and I forgot to cover the figures with colorful cardboard and I should not have used plotting paper while solving some questions but I did and lost time. Students were generally willing to participate in the lesson but they sometimes got bored. I tried to motivate them in such cases. I endeavored to call different students to the board, pay attention to the students' answers and respond them in a correct way in order not to cause confusion.

In terms of what to notice, Semih focused on various issues such as feedbacks, his own teaching, time, students' enthusiasm and behaviours (e.g. "Students were generally willing to participate in the lesson" and "I did and lost time"). He indicated the cooperating teacher's contributions to improve the lesson plan and decrease his own lack of knowledge. Likewise he compared his responsibilities while observing and implementing the lesson and described the difference between them. Besides he referred to pedagogical issues such as calling different students, motivation and attention. Regarding how to notice, he had a general impressionistic stance. He described what he noticed with few or no details. He did not focus on students' mathematical thinking or make sense of them. He was not good at elaborating his analyses and made general assessments. Thus, his noticing had further the properties of Level 1.

When Semih's all comments during the reflecting phase were considered, his noticing mostly exhibited features of Level 2. In terms of what to notice, he predominantly focused on pedagogy, students' mathematical approach, teaching and learning issues. He sometimes paid attention to particular students and their mathematical answers, however, he continued to refer to the class as a whole. Regarding how to notice, he highlighted the noteworthy events yet in general manners. He sometimes gave details supporting his claims about his observation but he also continued to show little or no evidence to unfold his ideas. His comments were descriptive and mostly evaluative. He offered judgmental statements without making reference to the evidence supporting his claims. Although some of his comments were considered as Level 1, his noticing mostly exhibited the features of Level 2.

4.2.2.4 What and how Semih notice in re-reflecting phase

After the second lesson plan was implemented by Semih in 5/F during the teaching phase and preservice teachers discussed what should be changed about it or what should be paid attention for the next implementation, the re-teaching of the second lesson plan was conducted in another classroom 5/G by Mehmet. While Mehmet was implementing the lesson plan, the other preservice teachers, cooperating teacher and researcher observed this lesson and took notes. In the discussion phase which was conducted just after re-teaching, the cooperating teacher and all preservice teachers were wanted to reflect their thoughts about the lesson including; what worked well, what did not work, what difficulties there were, what they observed about the students' actions and thinking and also what should be changed about the lesson to improve it and what the rationales for the changes were.

In accordance with discussion protocol, Semih shared his opinions about the lesson, evaluated the effectiveness of the lesson plan and made some suggestions to improve it. The following dialogs show what and how Semih noticed during the re-teaching phase.

> **Semih:** Mehmet tried to motivate the students to answer his questions and he called different students. It was good. In the questions on the exercise sheet, students could calculate area by counting the units squares easily. First, they counted the number of all the squares, later they counted half squares. They could think two halves as a whole and summed all of them at the end. Students applied everything step by step and I liked their strategies. The last question seemed difficult for the students and they found different answers like 17, 19. However, it was completely our mistake, I noticed that we asked a question which was not appropriate for their levels. Mehmet always checked whether there were points that the students could not understand and dealt with the students by walking around the class. After Mehmet showed how to obtain unt^2, he proceeded to cm^2 and m^2. Then, students could make generalizations such as km^2, hm^2, dm^2 for area measurement units. Seeing that was surprising.

In terms of what to notice, Semih's main focus was on teacher pedagogy and students' mathematical thinking. He addressed the students' strategies and understanding providing details from his observation (e.g. "Students could calculate area by counting the units squares easily" and "They could think two halves as a whole and summed all of them at the end"). Besides he referred to what the teacher did and his pedagogical approach such as motivation, promoting different students and dealing with students. Regarding how to notice, he tried to make sense of students' mathematical thinking but he also continued to offer general impressions. For example, he reasoned on how students followed a step by step approach mathematically for half squares but he did not provide interpretation about students' mistakes or difficulties. Therefore, he was inconsistent in giving details from his observation to support his analyses. He mostly made evaluative and interpretive comments (e.g. "Seeing that was surprising" and "Students could make generalizations such as km^2, hm^2, dm^2 for area measurement units"). He evaluated their performance while planning too and criticized one question for not being suitable for students' mental levels. Although he focused on the noteworthy events, his approach was further general. In addition he continued to refer to the class as a whole instead of particular students. Thus, his noticing here was considered as evidence of Level 2.

> **Semih:** When Mehmet asked how to find area of a square, some students said that ax4 or by multiplying one edge by four, namely, perimeter instead of area. Besides, there was a student who gave the answer of multiplying one edge by two. I think that it was expected to hear ax4 because they confused perimeter and area and they wanted to make an operation with 4 because all edges of a square were equal. However, why did they say ax2? I do not know.
> **Mehmet:** They might have confused it with a^2.
> **İnci:** We multiply two edges. It might be the reason.
> **Semih:** I could not make sense of it. The reason of this answer should have been queried. Mehmet endeavored to show area of a square by emphasizing that square was a particular type of a rectangle and area would be found using the same path meaning area would be axa. It was good but I think students could not understand. When Mehmet also showed axa in the form of a^2, students had difficulty, although we showed the same thing with measurement units. Before giving these concepts, we can remind that 2x2 is 2^2 and square of 2 is 4, 3x3 is 3^2 and square of 3 is 9 and 5x5 is 5^2 and square of 5 is 25 in order provide better understanding. If I remember it correctly, one student used the statement "as exponential numbers" undoubtedly they knew this concept and we can use it to recall.

In this excerpt, Semih focused on students' mathematical thinking and teaching approach of teacher. He referred to students' answers to support what he indicated and he tried to make sense of how they thought mathematically. However, he could reason on some answers but he could not interpret how the students' mathematical approach was in some cases (e.g. "I could not make sense of it"). Correspondingly he indicated the mathematical points that the students could not understand and had difficulty, however, he did not rationalize them and offered his general impressions. Although he focused minimally on particular students, he maintained to address the whole class. He mostly attended to teaching and learning issues and made suggestions by explaining the reason for them. His comments were predominantly evaluative and interpretive (e.g. "I think that it was expected to hear ax4 because they confused perimeter and area" and "It was good but I think students could not understand"). Although he highlighted the important moments, he was inconsistent in making reference to the details supporting his claims. Therefore, his noticing further had the features of Level 2.

> **Semih:** In the question which included to cover a square in the form of 8x8 with a smaller square in the form of 2x2, one student found how many squares would be in one edge by dividing 8 by 2 and found it as 4. He did the same thing for the other edge and he found there would be 4 squares on this edge. He separated the edges into small squares by drawing vertical and horizontal lines. Therefore, he visualized how many small squares would fit inside a big square in this way. Later, he multiplied 4 by 4 and found answers as 16 correctly. He tried to find a practical path instead of drawing at every turn. He understood and emphasized it would be the same result if the squares were counted. Everybody cannot have this mathematical approach, it was good. When bigger numbers are given, he can also do it without drawing the figure.

In terms of what to notice, Semih focused mainly on a particular student and his mathematical thinking. He explained his strategy step by step using details from the lesson to draw inference about his mathematical approach that he used to solve the question. Regarding how to notice, he tried to make sense of student's thinking and what he appeared to understand. Semih elaborated on what he observed making reference to the evidence promoting his claims. His comments were predominantly interpretive (e.g. "He tried to find a practical path instead of drawing at every turn"). Because Semih focused on specific events and particular students' mathematical thinking and provided details about them with an interpretive stance, his noticing was considered as evidence of Level 3.

After the re-reflecting phase, individual interviews were made with preservice teachers in order to obtain richer information about their noticing and reveal the missing points that preservice teachers might have forgotten to mention. Semih's comments differently from what he had shared in the re-reflecting phase were as below:

> **Semih:** I had difficulty in teaching the area of a rectangle in the first implementation of this lesson plan but Mehmet further had difficulty in teaching the area of a square in the revision of it. Students generally answered with perimeter, as 4xa, instead of area although Mehmet tried to show their mistake. In perimeter, they were summing all the edges but they counted the unit squares or multiplied the edges in area. Thus, they began to confuse them because it was difficult for them to decide whether they needed to make addition or multiplication. They had some problems in understanding measurement units too but they will comprehend better in time by solving questions. In the first lesson, we did not give examples from daily life related to the use of these units but in this lesson, it helped understanding and students could give various examples that they encountered in their lives. For example, one student said that we used m^2 for calculating bigger area and his approach was good.

In this quotation, Semih's primary focus was on teaching and learning issues. He addressed students' incorrect answers, confusions and difficulties (e.g. "Students generally answered with perimeter, as 4xa, instead of area" and "They also had some problems in understanding measurement units"). He endeavored to make sense of students' confusions about perimeter and area. He reasoned on why they might have indicated perimeter instead of area and explained his ideas about this issue. On the other hand, he did not explain the encountered problems related to measurement units in detail and he pointed them in a general way. Although he attended to important events and interactions, he mainly had a broad perspective. He made both evaluative and interpretive comments in nature (e.g. "His approach was good" and "They began to confuse them because it was difficult for them to decide whether they needed to make addition or multiplication"). In general, he continued to refer to the students as a whole rather than focusing on particular students and there was inconsistency in his comments. Thus, his noticing reflected prevalent properties of Level 2.

> **Semih:** I focused on what kinds of answers were given by the students and whether they gave responses that we had expected. For example, in a question related to estimation of both perimeter and area of the given figures, one student could give the correct answer for their area. They were different figures and I did not expect to hear a correct estimation. On the other hand, one student found the perimeter of a square by multiplying 7 by 4 as 28 instead of calculating the area multiplying 7 by 7 as 49. However, it did not make me surprised because we expected this confusion. I paid attention to the use of measurement units and generalization of area formulas. I also attended to whether the teacher of the lesson could teach what we aimed and used the materials correctly. For example, while learning estimations of students, Mehmet did not query the reason of students' answers. However, how students thought mathematically was important to correct their mistakes or misconceptions. There were some deficiencies in implementation. I compared the difference between the first implementation and revision of the lesson.

In this excerpt, it is seen that Semih's focus was on students' mathematical approach and teacher pedagogy. He explained what he attended by making reference to the evidence from his observation to support his claims. He addressed students' answers and implementation of teacher. He mostly provided descriptive and evaluative comments and his approach was very broad. His own comments supported that his noticing was further in Level 2.

When Semih's all comments during the re-reflecting phase were considered, his noticing mainly had characteristic of Level 2. In terms of what to notice, he focused primarily on teacher pedagogy and students' mathematical thinking. He attended to particular students and their mathematical strategies but he further considered the class as a whole. Regarding how to notice, he highlighted the noteworthy events but his approach was often general whereas he sometimes exhibited an interpretive perspective. He endeavored to provide details about his observation to support his claims but he provided little or no evidence to elaborate his ideas. His comments were descriptive, evaluative and interpretive but evaluative stance was predominant. Correspondingly he tried to make sense of students' mathematical thinking and reasoned on their strategies. However, this approach was limited. Although there were comments which had features of Level 2 and Level 3, his noticing mostly exhibited the features of Level 2.

4.2.3 Lesson study cycle 3

4.2.3.1 What and how Semih notice in planning phase

The third lesson plan was mostly toward the reinforcement of perimeter, area and combination of them and the objectives which the third cycle included were "Being able to create different rectangles with the given area" and "Solve problems that require to calculate the area of a rectangle". All preservice teachers met to prepare the third lesson plan at the university seminar hall bringing the various sources such as mathematics textbooks, teacher guide book, curriculum and different mathematics education books. The followings are presentations of examples based mainly on Semih's expressions from the planning meeting of lesson study cycle 3.

During the planning phase, preservice teachers made various suggestions regarding both implementation and content of the lesson. They shared ideas and tried to decide which one of them they will do or not. The following is Semih's some comments as planning:

> **Semih:** We should start with repetition to check whether students remember the previous concepts with questions such as "What is area?" and "How do we calculate the area of a rectangle and square?". Later we need to determine effective activities which serve the purpose of the lesson.
> **…**
> **Semih:** We should call different students and control whether there were students who had different ideas in each time. Do we need to give the definition of area?
> **İnci:** We gave perimeter. I think it is better to make students write in their notebooks.
> **…**
> **Semih:** We should ask questions related to daily life such as calculating the area of a room or carpet. Students are likely to encounter these problems in their lives accordingly the subject becomes more meaningful to them.

In this excerpt, Semih focused on a range of issues, in particular, he referred to pedagogical issues such as repetition, promoting different ideas and association with daily life. His comments had further the characteristics of suggestion in order to learn the others' ideas (e.g. "We should ask questions related to daily life" and "Do we need to give the definition of area?"). His approach was rather general and toward the whole class. He made comments which were descriptive and evaluative in nature (e.g. “The subject becomes more meaningful to them”). He provided few or no details to promote his ideas. Why he made these kinds of suggestions was not rationalized and how they were related to the goals of the lesson was not clear. Thus, his noticing had prevalent properties of Level 1.

Preservice teachers discussed how to show creating different figures with the same area measurements and also repeat area conservation. At first, they thought of creating a rectangle from another figure having the same amount of unit squares and then doing the same thing for a square but Semih offered to combine them in order to provide transition between quadrilaterals. The conversation about this issue is as in the following:

> **Semih:** We stated that we would give a figure consisting of 6 unit squares in the form of L and then ask students to create a rectangle with them. In a similar way, we would give a figure consisting of 4 unit squares in the form of T and tell students to create a square with them. It seems scattered. I think we should combine them together. We can determine a number which is suitable for making both a rectangle and square.
> **İnci:** How many unit squares will we use?
> **Semih:** For example, it can be 16. We will stick them, for example, as L on plotting paper. First, a student will find the area of this figure is 16 unit squares by counting. Later, the student will create a rectangle in the form of 2x8 or 1x16 as he wishes by covering with unit squares and moving them to the plotting paper on the side. Then, he will create a square in the form of 4x4 by moving them to the next plotting paper. Therefore, they will obtain different quadrilaterals which have the same area in parallel with the objective of the lesson.

In this quotation, Semih’s primary focus was on teaching and learning issues (e.g. “We can determine a number which is suitable for making both rectangle and square”). He addressed his suggestion considering students’ mathematical approach. In addition to that he endeavored to explain how it is related to the goal of the lesson but in general manner without deepening his comments (e.g. “They will obtain different quadrilaterals which have the same area in parallel with the objective of the lesson). He described what he had in his mind through elaborating step by step. However, he was inconsistent in elaborating his ideas. For example, he offered to make a change in the implementation but he did not provide enough details about the reason for it. Because of his mixed approach, his noticing here was considered as evidence of Level 2.

During the planning phase, preservice teachers tried to consider students’ various confusions and difficulties and constitute the content based on this kind of knowledge. Semih focused on a possible misconceptions of students and suggested to show that perimeters of the figures did not need to be the same when area measurements of them were equal. The other preservice teachers agreed on this idea. The interaction among the group about this issue is as below:

> **Semih:** I think we should integrate perimeter into some questions to reduce students' confusion. Students should notice that different figures with equal area measurements may have the same perimeter or not because they are in tendency to make overgeneralization. They may think that if areas are equal, perimeters also need to be the same. Therefore, we need to prevent these kinds of misconceptions if they exist. I suggest showing the relationship between perimeter and area in the figures which have the same area.
> **Hasan:** We will give different figures wherefore they can calculate perimeter as well as area.
> **Semih:** It should contain only rectangles because they do not know area of each figure but they will calculate both area and perimeter. For example, the figures can be rectangles in the form of 1x32, 2x16 and 4x8, area of each figure is 32. Then, we can use a table as in the book. Students write lengths of edges, perimeter and area for each figure to see the differences and similarities easily.
> **Mehmet:** 32 is big for covering.
> **Semih:** It can be 3x4, 2x6 and 12x1 as well. Students find perimeter and area. Then, we will query the relationship between area measurements of figures. They will see that each area is equal. Later, we will query the relationship between perimeter measurements of figures. They will realize that perimeter measurements do not need to be the same when area measurements are equal.

In terms of what to notice, Semih's focus was mainly on students and their mathematical thinking. He addressed their possible misconceptions and mathematical approach (e.g. "They are in tendency to make overgeneralization). Besides, he considered students' prior knowledge while making suggestions (e.g. "It should contain only rectangles because they do not know area of each figure"). Regarding how to notice, he endeavored to make sense of students' mathematical thinking and reason on their possible strategies. He elaborated how his suggestions were related to the overall goals of the lesson and provided details to support his ideas. His comments were interpretive in nature (e.g. "They may think that if areas are equal, perimeters also need to be the same"). This conversation was important because it helped to discuss on a specific confusion and how to remove it though detailed comments. Thus, his noticing exhibited further the features of Level 3.

Preservice teachers talked about what kinds of problems should be asked to reinforce the subject of area. They tried to add different types of questions to vary the content too. The dialog among preservice teachers is as in the following.

> **Semih:** The objective of the lesson includes solving area problems as well. We can combine square and rectangle and tell students to cover a particular area so as to reinforce what we taught. For example, we can ask how many squares with an edge of 2 cm we need to cover a rectangle in the form of 24x12 cm.

Hasan: Students cannot solve it.
Semih: But they learned area of a rectangle and square and how to find area by covering. We did many examples through materials and by concretizing. I think we need to ask this kind of question.
Mehmet: Students may not understand that it is related to area and instead calculate perimeter. They may find areas but they may not decide how to continue to the solution. We need to reveal their mistakes.
İnci: Yes, it should be asked.
Mehmet: We can use decimals to indicate the edges of one rectangle and want students to find its area.
Semih: Do students know the concept of rounding off?
Hasan: Yes, they do.
Semih: I am not sure about it. If they know, we can ask. We can also want them to estimate first and then to calculate. The edges can be 5,9 and 10,1, they are easier for rounding off. They consider 5,9 as 6 and 10,1 as 10 and find 60 from there approximately.

In terms of what to notice, Semih focused on teaching, student learning and students' mathematical approach. He paid attention to consider students' prior knowledge while planning the lesson (e.g. "Do students know the concept of rounding off?"). He addressed pedagogical issues such as reinforcement, use of materials, concretization. He endeavored to explain the reason of his suggestion by associating it to what students had learned before rather than considering students' mathematical thinking or confusions. Thus, he had a very generic approach. Regarding how to notice, he did not try to make sense of students' mathematical thinking. He mostly used descriptive and evaluative comments and he was inconsistent in elaborating his ideas. For example, he tried to provide justification about the covering problem but why he offered estimation for the other question was not detailed sufficiently. Therefore, his noticing was considered as evidence of Level 2.

Semih paid attention to reveal students' possible mistakes and how to teach the objective in a different way. He offered a question toward this issue and his explanation about it is as below.

Semih: The objective includes creating different rectangles which have an indicated area. Thus, we can tell students to make a rectangle, for example, with an area of area 28 unt^2 and ask them how many different rectangles may be obtained. Students will try to draw rectangles. Maybe, they will draw the same figures through changing width and length by forgetting that rotation does not change area hence mistakes might arise. Here, students need to find the appropriate number pairs which give 28 and they may find incorrect number pairs. I think it can help to reveal if confusion or difficulty exist. They will create different figures which have the same area, use area formula of rectangle and repeat what we taught from the beginning of the lesson with this question.

In this excerpt, Semih' main focus was on students and their mathematical thinking. He addressed the objective of the lesson and associated it with students' possible mistakes and confusions. He tried to make sense of their mathematical strategies (e.g. "They will draw the same figures through changing width and length" and "They may find incorrect number pairs"). He explained how his suggestion was related to the goals of the lesson. He predominantly used interpretive comments He referred to noteworthy events and provided details about his ideas to make inferences of the students' mathematical approach. Hence, his noticing had further characteristics of Level 3.

During the planning phase, Semih used expressions considering students' mathematical approach such as "If students' wrong answer or misconception is revealed, what would we do?", "Students might think quite differently from each other" and "Do students know this concept?". He made many suggestions considering both goals of the lesson and students' mathematical thinking and understanding. He tried to elaborate his comments with reasons to support his ideas but they were limited occasionally. His comments were both evaluative and interpretive. He referred to students' mathematical approach, teaching and learning issues in general. Thus, his noticing had the characteristic of Level 2 in some dialogs.

During the interview that followed the end of planning, Semih indicated what they aimed in this process. He explained what expected students' responses, difficulties and misconceptions were and how they considered students' mathematical thinking while planning as the following:

> **Semih:** We thought that students got used to only one prototype of rectangle and when we change it in a way to be in vertical and diagonal positions, their misconceptions could be revealed. They might think that the area of a rectangle changed when it was rotated. Similarly, creating different figures with the same pieces might cause confusion for the students. We aimed to remove difficulties through both visualizing and calculating numerically. Thus, we tried to determine the activities and questions considering students' confusions and mistakes in general. Students might overgeneralize that one area measurement belonged to only one figure. Therefore, we endeavored to show that there might be different figures which have the same area.

In this excerpt, according to Semih students may have some confusions and misconceptions related to the subject. It is understood from his comments that they discussed on these points as a group and considered the students' mathematical thinking while planning the lesson. Similarly, he mentioned how to respond the students in order to help them to understand the subject.

Semih indicated what he generally focused on as planning and what he noticed in this process. The following is presentation of his noticing through his comments:

> **Semih:** I noticed that, our expectations were high or low at the beginning but now, we are better in preparing a suitable content for the lesson according to students' levels. Determining the questions and the order of the activities became easier for us. We gained speed in preparing lesson plans and learned to lower ourselves to the students' levels. We paid attention to check students' prior learning because they were likely to forget what they had learned. We tried to progress from easy to difficult, gave problems related to daily life, visualized what we taught with materials in order to provide permanence and promoted active participation of the students for better learning.

He evaluated the progression of themselves in planning the lesson. His comments showed that he thought they were better in thinking like the students and designing the lesson considering this point. He emphasized the positive effect of this process on his and his group friends' performance in planning. Besides it is understood that they paid attention to various pedagogical issues and utilized this knowledge to shape the plan.

When Semih's all comments during the planning phase were observed, his noticing was considered as evidence of Level 2 because he mostly focused on pedagogical issues and students' mathematical thinking and understanding. He endeavored to provide justifications about his comments by linking them with the goals of the lesson but his explanations were not enough in some ways. Although he referred to the important points, he maintained a general approach too. His comments were predominantly evaluative with an interpretive stance. He was inconsistent in making reference to the details to support his ideas. Thus, although there were comments in Level 1, Level 2 and Level 3, his noticing further exhibited properties of Level 2.

4.2.3.2 What and how Semih notice in reflecting phase

Teaching of the third lesson plan was made by İnci in 5/F classroom. The third lesson plan included the objective of "Being able to create different rectangles with the given area" and "Solve problems that require to calculate the area of a rectangle". Since the aim was to focus on Semih's case here, teaching of İnci was not presented. While İnci was implementing the lesson plan, the other preservice teachers, cooperating teacher and researcher observed this lesson and took notes. After teaching was completed, the reflecting meeting occurred immediately at school conference room. In the reflecting phase, preservice teachers were wanted to reflect their thoughts about the lesson including; what worked well, what did not work, what the difficulties were, what they observed about the students' actions and thinking and also what should be changed about the lesson to improve it and what the rationales for the changes were.

In accordance with discussion protocol, Semih shared his opinions about the lesson, evaluated the effectiveness of the lesson plan and made some suggestions to improve it. The following dialogs show what and how Semih noticed during the teaching phase.

> **Semih:** When İnci obtained a triangle from a rectangle with colorful cardboard on the board, one student indicated that area measurements of these figures were not equal. He explained the reason of his answer like that one figure had four edges whereas the other had three edges. He tried to establish a relationship between the number of edges and area. It was a different kind of answer. He might have thought that one figure had a bigger area because it had more edges. Similarly, one student said that the rectangle was obtained from the square therefore the area of these two figures was the same. His answer showed he could notice that both figures had the same pieces, thus, area did not change. Another student explained his ideas about the area of the figures through referring to the colorful pieces of cardboard. It showed that he made sense of the meaning of area as the interior region.

In terms of what was noticed, Semih predominantly focused on particular students and their mathematical thinking. He referred to students' answers and mathematical approach to strengthen his analyses (e.g. "One student said that the rectangle was obtained from the square therefore the area of both figures was the same"). Regarding how he noticed, Semih highlighted the noteworthy events through an interpretive stance. He reasoned on students' possible mathematical strategies and tried to make sense of how they thought mathematically (e.g. "He might have thought that one figure had a bigger area because it had more edges"). He provided details about what he observed to support his ideas and make interferences about the events. He made comments on how specific confusions arose and how students comprehended some mathematical concepts. Therefore, his noticing exhibited prevalent features of Level 3.

> **Semih:** İnci gave different rectangles and students found the perimeter and area measurements of them. Students indicated that the perimeter of the figures which had the same area could be the same, different or the powers of each other. It made no sense why he used the expression of "powers". I think she comprehended 26 as a power of 12 but she confused 26 with 24 if she thought like that. Likewise, in the question related to finding different rectangles with the area measurements of 28 unt^2, some of the students made expected mistakes. For example, one student found one incorrect number pair as 3x9. The result of it was 27 but the student did not need to control whether it was equal to 28. Moreover, in one question about finding the difference between the new and old area measurements by changing the edges, one student found new edges by dividing one of them by two and multiplying the other by three. Later, he found the difference between the edges instead of area measurements.

> He probably missed out what was asked. İnci tried to direct him by asking the multiplication of which numbers is 28.

In this excerpt, Semih's primary focus was on specific students and their mathematical thinking. He reasoned through the strategies which students used to solve the questions. He addressed what kind of mathematical strategies students used by making reference to the evidence from his observation. He tried to make sense of their mathematical thinking and explain why students might have applied these strategies (e.g. "I think she comprehended 26 as a power of 12 but she confused 26 with 24 if she thought like that"). His comments were mostly interpretive and addressed specific moments from the lesson. He focused on the specifics of students' mathematical thinking and provided insight into their approach (e.g. "He found the difference between edges instead of area measurements. He probably missed out what was asked"). He rationalized what he observed to promote his interpretations. Thus, his noticing had the characteristics of Level 3.

> **Semih:** Students confused perimeter and area measurements with each other. For example, they wrote unt instead of unt^2 or vice versa. İnci paid attention to remind them to use the appropriate unit measurements when students forgot to. She also clarified some important points through visualizing. She always tried to query the reasons behind the students' answers and whether there were different answers. She asked the name of each figure and made repetition occasionally. Her teaching approach was well.

In terms of what to notice, Semih mainly focused on teacher pedagogy and students' mathematical approach. He addressed students' confusion and how the teacher intervened in this situation. He mentioned how the teacher behaved in the lesson and what kind of a pattern she followed as teaching. He endeavored to elaborate the teacher's approach whereas he did not provide details about students' mathematical thinking. He made reference to no evidence about his comment "Students confused perimeter and area measurements with each other" in order to strengthen his analysis. He attended to pedagogical issues such as visualizing, querying, repetition. He referred to some noteworthy events but his approach was quite broad. He offered his general impressions and evaluated what he observed (e.g. "Her teaching approach was well"). He considered the students as a whole and attended to teaching and learning issues in general. His comments included few details supporting his claims. Therefore, his noticing was considered as evidence of Level 2.

After the reflecting phase, individual interviews were made with preservice teachers in order to obtain richer information about their noticing and reveal the missing points that preservice teachers might have forgotten to mention. Semih's comments differently from what he shared in the reflecting phase were as below.

> **Semih:** When we rotated the rectangle, students might have thought that the width and length were changed that is why the area changed but İnci tried to eliminate this perception through numerical calculation. A similar perception arose in another question towards the end of the lesson. Students mostly found the number of rectangles as 3 or 6. One student explained his ideas saying there were 3 rectangles but if we changed the width and length, there would be 6 rectangles. His answers showed the confusion about rotation. However, another student responded that when the figure was rotated, nothing changed correspondingly it had to be 3.

In this quotation, Semih focused mostly on students and their mathematical thinking. He referred to specific students' mathematical responses and how a confusion might influence the responses (e.g. "When the figure was rotated, nothing changed correspondingly it had to be 3"). He examined the details of students' approach to draw inferences about their understanding. He drew attention to the student's hesitation about his own answer and associated it with the lack of understanding. He tried to explain the reasons behind different answers through addressing students' own explanations. His comments were predominantly interpretive and elaborative (e.g. "When we rotated the rectangle students might have thought that the width and length were changed so the area changed"). He endeavored to make sense of what he observed and made reference to the evidences from the lesson to promote his ideas. Hence, his noticing further reflected the properties of Level 3.

> **Semih:** While drawing the figures, we cannot make an exact sized copy of them because it is difficult to draw correctly on the board. When students liken one figure to a square or the edges seem equal, they are likely to perceive the figure as a square although it is not. For example, one student asked whether one figure was a square when he saw it on the board. I noticed that it did not matter if the edge lengths were written or not. If this student had attended to the numbers on the edges, he could have understood that the figure was a rectangle but his primary focus was on the image of the figure. Thus, he thought that it was a square. İnci endeavored to draw their attention to the inequality of the edges through questions to show that it was not a square.

In terms of what was noticed, Semih focused on students and their mathematical thinking centrally. He referred to particular students' mathematical approaches and where specific mistakes arose. He attended to the noteworthy events to explain students' thinking. Regarding how he noticed, Semih attempted to make sense of students' mathematical strategies and explained the reason for the incorrect perception with the focus being only on the image of the figure and the lack of attention to numerical values on the basis of the evidence from his observation. He provided details to strengthen his analysis and made mostly interpretive comments about students' mathematical thinking (e.g. "If this student had attended to the numbers on the edges, he could have understood that the figure was a rectangle”). His comments were important because it revealed how a student might have confusion in distinguishing the figures from each other. Therefore, his noticing predominantly reflected the features of Level 3.

> **Semih:** There were students who had different levels and accordingly they gave various responses. I focused on their answers and whether expected or different kinds of answers were given. There was less confusion about perimeter and area in this lesson. They began to understand these concepts better, they made only a few mistakes. I also attended to whether students could understand the subject and how the teacher taught the lesson. I noticed that her interest in the students and feedbacks were good. She listened to the students’ responses and tried to correct their mistakes by concretizing with materials.

In this excerpt, Semih focused on students’ mathematical understanding and teacher pedagogy rather than less mathematical aspects of the lesson. He did not attend to particular students and their mathematical thinking. He referred to the mathematical understanding of the whole class by comparing with the previous lessons (e.g. “There was less confusion about perimeter and area in this lesson”). He also paid attention to the teacher’s approach and pedagogical issues. His comments were general and evaluative in nature. For example, he made an assessment about students’ confusion “They began to understand these concepts better, they made only a few mistakes”, however, he provided few details about this assessment. He offered his general impressions about the students and teacher. Thus, his noticing further had the properties of Level 2.

When Semih’s all comments in the reflecting phase were considered, his noticing predominantly exhibited the features of Level 3. He especially focused on particular students’ mathematical thinking and their responses. He further attended to particular students’ mathematical approaches instead of considering the whole class. He reasoned on the solution strategies of the students and attempted to make sense of their mathematical thinking. He made a quotation from the students’ answers and described what had occurred in the lesson to make reference to the evidence in order to strengthen his claims. He made evaluative comments but his approach was mainly interpretive. He examined the important events and elaborated on what he had observed. Although there were comments that had the properties of Level 2 and Level 3, his noticing in the reflecting phase was further considered as Level 3.

4.2.3.3 What and how Semih notice in re-reflecting phase

After the third lesson plan was implemented by İnci in 5/F during the teaching phase and preservice teachers discussed about what should be changed or paid attention for the next implementation, the re-teaching of the third lesson plan was made in another classroom 5/G by Hasan. Because the aim was to focus on Semih's case here, Hasan's teaching was not presented. While Hasan was implementing the lesson plan, the other preservice teachers, cooperating teacher and researcher observed this lesson and took notes. In the re-reflecting phase which was conducted just after re-teaching, the cooperating teacher and all the preservice teachers were asked to reflect their thoughts about the lesson including; what worked well, what did not work, what the difficulties were, what they observed about the students' actions and thinking and also what should be changed about the lesson to improve it and what the rationales for the changes were.

In accordance with discussion protocol, Semih shared his opinions about the lesson that he observed, evaluated the effectiveness of the lesson plan and made some suggestions to improve it. The following dialogs show what and how Semih noticed during the re-teaching phase.

> **Semih:** In this classroom, when Hasan rotated the rectangle, the students did not give the answer that the area changed in contrast to the other class. They could notice that the figure did not change. While creating a figure from another figure, students explained that the area of the figures were equal through statements such as "Both figures have the same pieces consequently the area measurements of them are equal" and "We did not make any addition or subtraction so the area remained the same". Students could not understand which and how edges would be changed in one question. Hasan should have helped them to understand by drawing the figure. Students could solve the question which included covering a rectangle with squares well. I noticed that the students understood what we had intended in this lesson.

In terms of what to notice, Semih focused primarily on students' mathematical approach and teacher pedagogy. He mentioned what students could understand or not in general ways without giving details (e.g. "They could notice that the figure did not change" and "Students could not understand which and how edges would be changed in one question"). He compared the students' performances in both classrooms based on their responses through a statement like "The students did not give the answer that the area changed in contrast to the other class". He also referred to the teacher's approach and made a suggestion related to it in terms of students' understanding. Regarding how to notice, although he made reference to the evidence of students' responses, he was inconsistent in elaborating the details of their mathematical thinking and he did not try to make sense of them. For example, what kinds of strategies students followed and how he noticed their understanding, these points were not detailed. He identified the important events but he offered general impressions. He made evaluative comments with an interpretive stance (e.g. “I noticed that the students understood what we had intended in this lesson”). Because he focused mostly on teaching and learning issues with a broad perspective, his noticing was considered as evidence of Level 2.

> **Semih:** In the question related to covering a rectangle with a square and finding the necessary number of squares, one student tried to cover with squares in the form of 1x1 instead of 3x3. Thus, he claimed that there would be more squares. Hasan showed that he used different squares and that is why he found a different result. In addition, one student added up only one long and one short edge to find the perimeter. We emphasized that while calculating perimeter they should sum all the edges that they saw. The student might sum only the edges which he saw the numerical values on. He added 12 and 1 and found the perimeter as 13 instead of 26. Hasan encouraged the student to find the unknown edges and then she could find the perimeter. Here, the student might have thought of area because two edges were multiplied in area or she might have forgotten to multiple by 2 as in perimeter. On the other hand I think the student might have confused the formulas of area and perimeter with each other.

In terms of what to notice, Semih focused primarily on particular students and their mathematical thinking (e.g. “One student summed only one long and one short edge to find the perimeter”). He attended to the details of these students’ strategies and explanations. He addressed their responses and solutions step by step. Regarding how to notice, he endeavored to reason through what he observed. He highlighted the noteworthy events and made various explanations about the possible reasons of the strategies that specific students had used to solve the questions. He tried to make sense of their thinking and made reference to the evidence from his observation to support his claims (e.g. “The student might sum only the edges which he saw the numerical values on”). He elaborated what kind of and how specific confusions might arise and he mostly made interpretive comments. Hence, his noticing had the prevalent properties of Level 3.

> **Semih:** Students were expected to create a rectangle by using 16 unit squares. One student used 15 unit squares and created a rectangle in the form of 3x5 and 1 square remained. He could not create a correct rectangle. He moved the unit squares randomly without considering the multiplication of which numbers would be 16 or the multiplication of which edges would give that area. He thought that when he moved all of them he would obtain 16 unit squares but he did not focus on obtaining a rectangle or maybe he overlooked the emphasis was on a rectangle. Thus, he made a mistake.

In terms of what was noticed, Semih’s main attention was on a particular student and his mathematical thinking rather than the other issues. He examined the details of that student’s mathematical strategy by considering what he did and said. Regarding how he noticed, Semih provided evidence from the lesson about what the student did step by step in order to strengthen his claims. He aimed to make sense of the student's thinking and reasoned on his strategies that he used in the activity (e.g. "He removed the unit squares randomly without considering the multiplication of which number would give 16"). He addressed the noteworthy moments and explained how and where specific confusions were revealed. His comments predominantly were interpretive in nature. Hence, his noticing exhibited the prevalent features of Level 3.

After the re-reflecting phase, individual interviews were made with preservice teachers in order to obtain richer information about their noticing and reveal the missing points that preservice teachers might have forgotten to mention. Semih's comments differently from what he shared in the re-reflecting phase were as below.

> **Semih:** There was less participation in this class but the answers of the students were better in comparison with the other class. Besides, I think the feedbacks of the cooperating teacher were helpful because we had disagreement about what to do the question which included to create different rectangles with the area measurement of 28 unt^2 or some different points. We considered her ideas while making decisions about the lesson plan. Moreover, concretization with materials provided to increase the students' motivation and facilitate their understanding. Students did not have much difficulty in this lesson. Confusions or mistakes were less than we had expected. I noticed that they did not confuse perimeter and area a lot anymore and they could solve the questions for most of the time.

In terms of what to notice, Semih focused on a range of issues such as the participation of the students, the contribution of the cooperating teacher, materials, interest and student learning. He did not attend to particular students' mathematical thinking instead he paid attention to less mathematical aspects of the lesson. He referred to the class as a whole and offered general impressions (e.g. "I noticed that they did not confuse perimeter and area a lot anymore"). His comments were judgmental and evaluative in nature (e.g. "concretization with materials provided to increase the students' motivation and facilitate their understanding"). However, he did not provide evidence to support his claims. For example, he did not elaborate on his assessment about two classes. He mainly had a general impressionistic stance. In other words, he told what he saw and described the events without making sense. Thus, his noticing further exhibited the characteristics of Level 1.

When Semih's all comments during the re-reflecting phase were considered, his noticing exhibited features of Level 3 predominantly. In terms of what to notice, he focused predominantly on particular students' mathematical thinking although he attended to teacher pedagogy and referred to the whole class partially. He addressed the students' responses and mathematical strategies. Regarding how to notice, he mainly highlighted the important moments and interactions through an interpretive stance. He tried to demonstrate what he observed on the basis of evidence from the lesson in order to support his analyses. Similarly he tried to make sense of students' mathematical thinking and reasoned on their strategies. Although there were comments which had features of Level 1, Level 2 and Level 3, his noticing mostly had the characteristic of Level 3.

4.2.4 Lesson study cycle 4

4.2.4.1 What and how Semih notice in planning phase

The fourth lesson plan was mostly on prisms, open and close forms of them and surface area and the objectives of the fourth cycle were "Being able to recognize a rectangular prism and identify its essential features", "Draw surface developments of a rectangular prism and decide whether different surface developments belong to the rectangular prism or not" and "Calculate the surface area of a rectangular prism". All preservice teachers met to prepare the third lesson plan at the university seminar hall bringing the various sources such as mathematics textbooks, teacher's guide book, curriculum and different mathematics education books. The following are presentations of examples based mainly on Semih's expressions from the planning meeting of lesson study cycle 4.

During the planning phase, preservice teachers made various suggestions regarding both the implementation and content of the lesson. They shared ideas and tried to decide which suggestions should be considered. Below are some of Semih's comments as planning:

> **Semih:** There are plastic cubes. After we show a cube, we can add another next to it and it turns into a tetragonal prism. Later, we can add another cube next to them and the object turns into a rectangular prism. Thus, students obtain a tetragonal prism from a cube and a rectangle prism from a tetragonal prism. They will see three types of prisms in the same place so that it may help them understand the differences and similarities among them better.
> **İnci:** We will make concrete models from cardboards so as for me there is no need for this activity again.
> **Mehmet:** I do not think that there will be enough time to do that. This lesson will be more intensive.

In terms of what to notice, Semih focused predominantly on teaching and student learning issues. He made some suggestions regarding the implementation by trying to indicate how it was related to the goals of the lesson and students' mathematical understanding (e.g. ". They will see three types of prisms in the same place so that it may help them understand the differences and similarities among them better"). However, his approach was so general without elaboration. He did not attend to students' thinking or make sense of it. Regarding how to notice, his comments were descriptive and evaluative. He described his suggestions and evaluated how their effects might be. He tried to explain his ideas to convince the others but provided few details in his comments. Therefore, his noticing is further considered as evidence of Level 2.

Preservice teachers talked about the relationship between open and close forms of prisms. They discussed which point was more important, what they should expect from the students and suggestions about how to teach some concepts. The following dialog presents the interaction among preservice teachers:

> **Semih:** We expressed that we would show prisms and then present the open figures of them. Now, we should give the open figure and want students to form prisms. We should support them to think bipartitely to understand the subject in all aspects.
> **İnci:** Will we do it for one example or for more than one different open images?
> **Semih:** At first, for each type of prisms and then for different open figures.
> **Hasan:** I insist on giving, for example, 4 open figures and 4 prisms and requiring students to match them.
> **Semih:** But we are talking about how to give the relationship between open and close forms at first.
> **Hasan:** They will ultimately see and form the prisms by themselves.
> **Semih:** Before teaching, how will they know which open images will correspond to which prism? Abstract thinking is difficult for them. They may not visualize how to close and make connections between open and close forms.
> **Mehmet:** Even we are struggling. It is unfair to expect students to reason in an advanced stage.
> **Semih:** So we need to ask more simple questions related to the formation of a prism. Rather than asking which prism will form, we can ask if a prism will be constituted when we close the open form of it, and they could answer saying true or false.
> **Mehmet:** It is sensible.

In this excerpt, Semih's primary focus was on students' mathematical understanding. He also referred to pedagogical issues by drawing the others' attention to supporting bipartite thinking and considering students' mental levels. He indicated what students could do and what might be difficult for them and considered these points while evaluating the others' ideas (e.g. "Abstract thinking is difficult for them"). Although he identified the noteworthy events, he made general comments (e.g. "We should support them to think bipartitely to understand the subject in all aspects"). He was inconsistent in giving details about his ideas. The depth of the details that he provided was changeable as including more details or little elaboration. His comments were both evaluative and interpretive in nature. He evaluated what kind of a path should be followed by making interpretations about students' understanding. Because he mostly attended to teaching and students' learning issues with a broad perspective, his noticing had the characteristics of Level 2.

While planning the lesson, Semih indicated students' possible reasoning about the subject and shared his ideas with the other preservice teachers. Semih's expressions are as below:

> **Semih:** Students may think that one prism only has one open figure by overgeneralizing. They may not notice that there can be different open figures of prisms. Thus, we need to show it visually and emphasize this point. I think we can utilize computer programs because there are various alternatives.
> **Mehmet:** Yes, they are very good.
> **Semih:** Our aim is also to help students decide whether an open figure will form a prism. Students may think that each open figure will constitute a prism so we need to present inappropriate examples to remove this incorrect approach. They should reason on what kinds of open figures close. One program requires choosing the answer as true or false considering formation of prism and then it closes the open figure and shows the answer. I think we can use it. We ask students their ideas first and then show how the figure closes.

In terms of what was noticed, Semih centrally focused on students' mathematical thinking and their specific approaches (e.g. "Students may think that one prism only has one open figure by overgeneralizing" and "Students may think that each open figure will constitute a prism"). Regarding how he noticed, he made suggestions considering students' thinking and providing justifications about his ideas. He tried to make sense of students' thinking through predominantly interpretive stance. In other words, he tried to think about the possible reasons behind students' responses and reactions. Besides he made reference to the details about the relationship among his comments, students and the goals of the lessons to strengthen his claims. He addressed where and how specific confusions might arise and what should be done for this issue so his comments were important (e.g. "Students may think that each open figure will constitute a prism so we need to present inappropriate examples to remove this incorrect approach"). Hence, his noticing exhibited the prevalent properties of Level 3.

Preservice teachers tried to determine how the content of the plan would be and which questions should be asked by explaining their ideas. They talked about the use of computer programs to support learning. The conversations about this issue is as the following:

> **Hasan:** After students closed the open figures of different types of prisms and saw the examples which did not form prisms by trying themselves with the cartons concretely, we can use computer programs.
> **İnci:** Ok but we need to determine the open figures on the program.
> **Semih:** Choose the open figures that are different from what we had used before.

Mehmet: Pay attention that they are close to the number of appropriate and inappropriate examples for formation of the prism.
Semih: For example, there are 5 faces on this figure. Students may think that one face is missing if they associate the number of faces on prims that we will show.
Hasan: Can they think like that?
Semih: If not all, some of them can reason in this way. Similarly, we can use figures which have more faces. After a few examples, students can make this inference. According to me, these kinds of figures are more important to facilitate understanding.

In this quotation, Semih's main focus was on students' mathematical thinking and their possible strategies. He referred to how they might reason while deciding the formation of prisms (e.g. "Students may think that one face is missing if they associate the number of faces on prisms that we will show"). He indicated what can be expected from the students in terms of understanding and deducing. He provided justifications about his claims. For example, he offered to give examples of open figures which had one more or less face because he thought that students could make inference about formation of prisms based on the numbers of faces. He also tried to make sense of students' thinking. He had an interpretive stance and pointed to the important issues. Thus, his noticing had the characteristics of Level 3.

During the planning phase, Semih gave importance to students' understanding the meaning of prism. He explained his suggestions about how to proceed to the third dimension in a way that students could notice the constitution of a prism. The following conversation presents mainly Semih's comments on this issue:

Semih: Before teaching the prism types, we need to make students understand the concept of prism and how to obtain it.
İnci: We may query the figures and dimensions on it.
Semih: Yes. You are right. We should give the concept of dimension. Students learned the dimensions of length and width until now. The difference in prisms is height so becoming a three dimensional object. Students may not think that there is a relationship between dimensions and prisms and that is why they are called three dimensional objects.
Mehmet: We can compare some things with one dimension and two dimensions.
Semih: What did students see before? The concept of area which is has two dimensions. We can prepare a material that will be a rectangle or square. Students will look at the figure and show two dimensions at first. Later, we will uphold the upper base and height will start to increase so that the students will be able to realize that height becomes the third dimension and understand how a prism develops. Thus, they make a connection between dimensions and prism and comprehend the meaning of it.
Hasan: It sounds very sensible.

In terms of what to notice, Semih focused predominantly on students' mathematical thinking and understanding. He indicated that students might not think there was a relationship between dimensions and prism and they might not realize why prisms were called as three dimensional objects because of this lack. Thus, he gave his suggestions supporting students to reason on this issue. Regarding how to notice, he provided justifications about his ideas and made detailed explanations on how they were related to the goal of the lesson. He pointed the noteworthy events and used interpretive comments. He reasoned on students' mathematical approach by elaborating. Hence, his noticing was further considered as evidence of Level 3.

Preservice teachers talked about how to teach the concept of surface area and calculation of it for each prism type. In addition to that they discussed giving a general formula or not and making the students feel the short path for calculation. The ideas and suggestions of preservice teachers are presented as below:

> **…**
> **Semih:** Students can calculate the area of each face with area formula and then we can write the results on each face. Later, we will open the prism and students will see by themselves that surface area is the sum of these areas.
> **İnci:** Students find surface area by adding each area one by one in this way.
> **Mehmet:** Yes, it will be more effective like that.
> **Semih:** Ok. We can make them see how to find it shortly too. For example, the opposite faces are equal on a rectangular prism. Students should be able to think that different areas are added and then it is multiplied by 2. Similarly, students may not make inference like that surface area of tetragonal prism is 2 times base area and 4 times lateral face area whereas cube's surface area equals to 6 times area of one square.
> **Mehmet:** We should not give formulas.
> **Semih:** It will be difficult for them adding one by one each time and they may miss or write some faces twice. Thus, we promote them to make inference about the same faces by emphasizing verbally. For example, consider the base is 16 and lateral face is 20 in a tetragonal prism. After finding the surface area adding one by one, we can direct students to notice that it can be written like 2 times 16 and 4 times 20 through a numerical sample.
> **İnci:** I agree. It will be better.
> **Semih:** There are three types of prisms and they have different number of bases and lateral faces. If we try to give surface areas of them with a general formula, it will probably cause confusion and students will make mistakes because of remembering wrongly. However, if they feel the meaning of surface area as I said, they will not need to use a formula.

In this excerpt, Semih focused centrally on students' mathematical thinking, their possible difficulties and confusions that might arise (e.g. "Students may not make inference like that surface area of tetragonal prism is 2 times base area and 4 times lateral face area"). He endeavored to reason through the students' mathematical strategies and interpret them step by step. He also provided justifications about his suggestions. For example, he did not offer to give a general formula so as not to cause confusions and mistakes due to focusing on the formula and remembering incorrectly. On the other hand, he suggested to make students feel short paths in order to remove the possibility of missing or adding more faces. He provided details to support his ideas and make inference about students' mathematical understanding. His noticing further reflected the properties of Level 3.

During the interview that followed the end of planning, Semih indicated what they aimed in this process. He explained what expected students' responses, difficulties and misconceptions were and how they considered students' mathematical thinking while planning as the following:

> **Semih:** We will try to teach the meaning of the prism, how to obtain prisms, the elements and properties of them, opening and closing them and surface area. I think students will mostly have difficulty in understanding the important points related to open and close forms of prisms and formation of them because it requires visualizing in their minds. We will associate it with daily life, use concrete materials and computer programs to facilitate students' understanding. We paid attention to benefit from visualizations and illustrative directions. We will try to reinforce the subject through similar examples considering students' thinking.

In this excerpt, according to Semih, students may have some difficulties regarding the subject. It is understood that they tried to consider students' thinking and understanding during planning and paid attention to pedagogical issues such as association with daily life, using materials and concretization.

Semih indicated what he generally focused on while planning and what he noticed in this process. The following is the presentation of his noticing through his comments:

> **Semih:** I can say that we are getting faster and better in planning day by day. In this subject, I felt that we had a lack of knowledge too so the source book helped us quite a lot in shaping the lesson plan. I noticed that observation of the lesson by us and the cooperating teacher was really important. Especially, feedback of the cooperating teacher helped us to interpret students' understanding, the effectiveness of our implementation and improve the lesson plan.

He evaluated the progression of themselves in planning the lesson. He also referred to the lack of their own knowledge and the contribution of sources books, observations and feedbacks of the cooperating teacher at this point. His comments showed that he found the lesson study process useful.

When Semih's all comments in the planning phase were considered, his noticing predominantly exhibited the features of Level 3. He predominantly focused on students' mathematical thinking and their specific confusions and difficulties rather than less mathematical aspects of the lesson. Likewise he considered pedagogical issues while planning but students' general mathematical approach was more in the foreground. He reasoned on possible strategies of the students and attempted to make sense of their mathematical thinking.

He mostly provided justification about his ideas and explained how his suggestions were related to the overall goals of the lesson. He made evaluative comments but his approach was mainly interpretive. He pointed to noteworthy events and elaborated on his claims. Although there were comments that had the properties of Level 2 and Level 3, his noticing in the planning phase was further considered as Level 3.

4.2.4.2 What and how Semih notice in reflecting phase

Teaching of the fourth lesson plan was made by Mehmet in 5/F classroom. The fourth lesson plan included the objectives of "Being able to recognize a rectangular prism and identify its essential features", "Draw surface developments of a rectangular prism and decide whether different surface developments belong to the rectangular prism or not" and "Calculate the surface area of a rectangular prism". While Mehmet was implementing the lesson plan, the other preservice teachers, cooperating teacher and researcher observed this lesson and took notes. After the teaching was completed, the reflecting meeting occurred immediately at school conference room. In the reflecting phase, the preservice teachers were wanted to reflect their thoughts about the lesson including; what worked well, what did not work, what the difficulties were, what they observed about the students' actions and thinking and also what should be changed about the lesson to improve it and what the rationales for the changes were.

In accordance with discussion protocol, Semih shared his opinions about the lesson that he observed, evaluated the effectiveness of the lesson plan and made some suggestions to improve it. The following dialogs show what and how Semih noticed during the teaching phase.

> **Semih:** He queried the similarities and differences among the types of prisms utilizing concrete models. Thus, students did not have difficulty in understanding these properties. When students see the prisms on the paper, they cannot calculate the surface area because they cannot perceive that there are faces behind a prism. When we showed with concrete models and computer programs, they could see the faces at the back of the prisms. In the questions related to estimating whether an open figure would form a prism, students gave answers quickly and without thinking too much. However, in open figures including 6 surfaces, they need to envisage and we cannot expect that from 5^{th} grade students right away. Students tried to classify prisms according to the open figures but it was not possible because there might be plenty of different open figures. They noticed that there would be different images of open prisms and each open figure would not need to constitute a prism.

In terms of what to notice, Semih focused primarily on teacher pedagogy and students' mathematical approach. He referred to students' general perceptions related to prism and difficulties in visualizing it in their minds (e.g. "They cannot perceive that there are faces behind a prism”). Equally,he mentioned pedagogical issues such as the use of various materials, suitability for students’ level and teacher guidance. Regarding how to notice, Semih highlighted noteworthy events but in general ways. He was inconsistent in providing details about the interactions to support his analyses. His comments about what students understood were not elaborated by making reference to the evidences from his observation. How he made these kinds of assessments about students were not clear. His comments were evaluative with an interpretive stance in nature (e.g. “in open figures including 6 surfaces, they need to envisage and we cannot expect that from 5th grade students right away”). Due to the fact that he did not focus on particular students and make sense of their mathematical thinking and he also had a broad perspective, his noticing here was considered as evidence of Level 2.

> **Semih:** One student’s answer drew my attention. He did not prefer to count the edges one by one on the model, instead he tried to find them in a shorter path. He explained his strategy that there were 4 edges in the front, there were 4 behind likewise, there were 2 on the right side because he had counted 2 of them before and there were 2 on the left side similarly. Then, he found the numbers of edges as 12 from 4+ 4+ 2+ 2. In other words, the student calculated the number on one side and then the number on the opposite of this side. Later, he considered non-common edges and found all the edges. It was unexpected for us because we thought that the students would need to count one by one. Besides, one student asked whether it would not be a cube when the open image was closed. Here, the student tried to generalize what kind of a prism would be obtained as well as whether a prism would form when the open figure was closed. He could not think that a different prism might have the same type of open images and one prism might have more than one open images.

In this quotation, Semih’s main focus was on particular students and their mathematical thinking. He reasoned through students’ mathematical strategies and endeavored to make sense of them (e.g. “Student calculated the number on one side and then the number on the opposite of this side. Later, he considered non-common edges and found all the edges”). He referred to the students’ answers to strengthen his claims. He mostly used interpretive comments in nature. For example, he explained the reason of the student’s approach with tendency of generalization. He mentioned the important moments and elaborated his ideas through offering evidences from the lesson. Thus, his noticing exhibited the prevalent features of Level 3.

> **Semih:** In the questions on opening and closing the prism, one student realized in his mind that an open figure would not form a prism by visualizing. He offered to cut one face and then add it to where he showed. He tried to benefit from the open figures which he knew that they would close and constitute a prism. However, he did not consider that there might be different open figures which did not form prisms so he suggested to render the figures in a way that could enable to obtain prisms. I thought that a tetragonal prism might be perceived as a rectangular prism too when it was

> rotated horizontally. The students might think that the base was now rectangular so it turned into a rectangular prism from a tetragonal prism. However, they did not have this kind of confusion. They did not pay attention or understood well.

In terms of what was noticed, Semih attended centrally to specific students and their mathematical thinking. He examined the details of students' mathematical thinking and referred to a specific confusion that might arise. Regarding how he noticed, he reasoned on what kind of strategies the students used and might use. He offered comments about the noteworthy events and provided details to support claims (e.g. "He did not consider that there might be different open figures which did not form prisms"). He tried to make sense of how the students thought mathematically and he made interpretations about it (e.g. "I also thought that a tetragonal prism might be perceived as a rectangular prism when it was rotated horizontally"). He made reference to the evidences from what he observed. Therefore, his noticing had the characteristics of Level 3.

After the reflecting phase, individual interviews were made with preservice teachers in order to obtain richer information about their noticing and reveal the missing points that preservice teachers might have forgotten to mention. Semih's comments differently from what he shared in the reflecting phase were as below:

> **Semih:** This lesson had the highest participation and the students' interest was good. Their answers were good and different. Mehmet could not complete the surface area of a tetragonal prism and cube and ask some questions. The content of the plan was appropriate but intensive. Thus, we will try to be faster in the revision of the lesson. We decided to increase the lesson hours from 2 to 3 as a group but we made no change. Students liked the computer programs and concrete materials. It caused a little noise but increased their attention. Associating the subject to daily life was important and students could give examples. I saw that using materials and making students active were useful in terms of understanding better.

In terms of what to notice, Semih's focus was primarily on the whole classroom environment, students' behaviors, teacher pedagogy, participation and time. In other words, he referred to a range of issues and the whole class rather than particular students' mathematical thinking. Regarding how to notice, he described what happened in the lesson with general statements without details (e.g. "Mehmet could not complete the surface area of a tetragonal prism and cube and ask some questions" and "It caused a little noise but increased their attention"). His comments were also evaluative (e.g. "Students liked the computer programs and concrete materials"). He oversimplified the complexity of the classroom and provided few or no details to support his analyses. Hence, his noticing had the prevalent features of Level 1.

When all of Semih's comments in the reflecting phase were considered, his noticing predominantly had the characteristics of Level 3. He mainly focused on particular students' mathematical thinking, strategies and teacher's pedagogy. He sometimes attended to the students as a whole class but he further attended to particular students' mathematical approach. He reasoned on what he observed and aimed to make sense of students' mathematical thinking. He made reference to the evidences of students' answers and provided details about the noteworthy events and interactions in the classroom in order to support his claims. He made descriptive and evaluative comments but his comments were mostly interpretive. Although there were comments that had the properties of Level 1, Level 2 and Level 3, his noticing in the reflecting phase was further considered as Level 3.

4.2.4.3 What and how Semih notice in re-teaching phase

The findings of this episode obtained from the implementation of the fourth lesson plan included the objectives of "Being able to recognize a rectangular prism and identify its essential features", "Draw surface developments of a rectangular prism and decide whether different surface developments belong to the rectangular prism or not" and "Calculate the surface area of a rectangular prism". The fourth lesson plan was implemented by Mehmet in 5/F during the teaching phase and preservice teachers discussed on it about what should be changed or paid attention to for the next implementation, re-teaching of the fourth lesson plan was made in another classroom 5/G by Semih. The findings from Semih's lesson were presented and his noticing during the teaching phase was revealed.

In the planning phase, preservice teachers thought that they needed to associate dimensions to what they referred. They decided to show the relationship among one dimension, length and perimeter; two dimensions, length and width and area; three dimensions, length, width and height and prism. Thus, they aimed to reinforce the previous subjects and make students understand how to obtain a prism. They utilized a computer program and decided to draw a point, then create a line segment to represent length and one dimension, later add width on length to represent area and two dimensions and then add height on length and width to show prism and three dimensions. The following presents the interactions between Semih and the students:

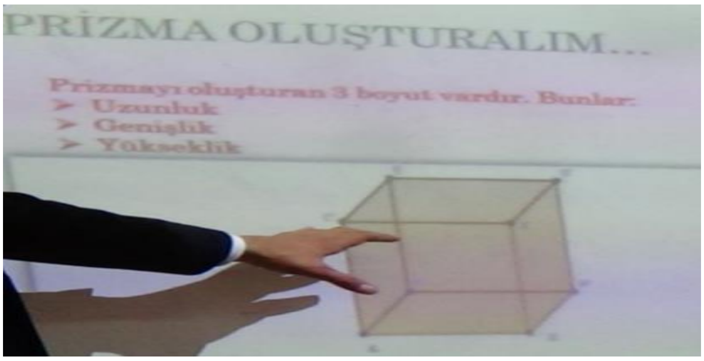

Figure 46 The demonstration of prism formation with a computer program

Semih: What is there?
Students: Point.
Semih: What did I create by moving the point?
S1: A line segment.
Semih: What do we find using line segments? Perimeter or area?
Students: Perimeter.
Semih: How do we calculate perimeter?
S2: By summing all the edges around the shape.
Semih: Yes, namely, by adding lengths of all line segments. Thus, what is the feature that we used here?
S3: Length.
Semih: Good. We can say that there is only one feature, namely, one dimension, cannot we?
Students: Yes.
Semih: What did I create by moving this line?
S4: Parallelogram.
Semih: It seems a parallelogram because we look at side view of the figure but it is a rectangle.
Semih: How many features are there?
S5: Two. Length and width.
Semih: Yes, I drew width as well as length and I created a geometric figure which had two dimensions, right?
Students: Yes.
Semih: What was the interior region of a figure?
S6: Area.
Semih: What happened now? What I added on length and width? (He points height).
Students: Height.
Semih: How many dimensions are there on this object?
S7: Three dimensions which are length, width and height.
Semih: Yes. We call the three dimensional objects as prism.

Semih started the lesson by going from a point to a prism with the help of a computer program. He tried to encourage the students to see the relationship between the numbers of dimensions and some mathematical concepts such as perimeter, area and prism. He conducted the lesson step by step through general questions toward getting conceptual answers such as line segment, perimeter and length rather than obtaining deeper information about reasoning of the students. He also made some directions instead of allowing students to think and answer for some questions (e.g. "What do we find using line segments? Perimeter or area?). He generally followed what he had in his mind. When a student perceived the base of a prism as a parallelogram instead of rectangular, he tried to make an explanation about looking from different angles but he did not consider it was enough to remove the hesitation of the student. He was mainly in tendency to focus on teaching what he had intended. He expected the students to confirm what he said instead of allowing them to think. Thus, evidence showed that the students realized the aimed relationship was limited. Students might not connect the number of dimensions to relevant concepts. Because of Semih's general approach, making inference about students' understanding was difficult.

It can be argued that Semih's noticing here was in Level 1because he did not try to listen how students were thinking instead he followed a discourse to get the answers and key words that he wanted from the students. He asked general questions towards description and confirmation such as "What is there?" and "I created a geometric figure which had two dimensions, right?" rather than questions towards interpretation. His questioning was more evaluative and correct answer-oriented. He did not promote the students in a way that could support and reveal their mathematical thinking rather he tried to lead the students notice the implicit knowledge in his questions. Semih focused on giving the relationship between three dimensions and prism so the mode of telling the subject dominated this part of the lesson. Thus he missed the opportunity to determine mistakes or confusions if there existed any. He referred to the class as a whole and exhibited a general approach. He mainly focused on what he had in his mind instead of students mathematical thinking and understanding. Thus, his noticing had the characteristics of Level 1.

Preservice teachers talked about showing formation of a prism by opening and closing with the help of a computer program. They also wanted to rotate a prism in order to facilitate understanding and realizing the back sides of the object because students had difficulty in visualizing it in their minds. Explanations of Semih related to this issue is as follows:

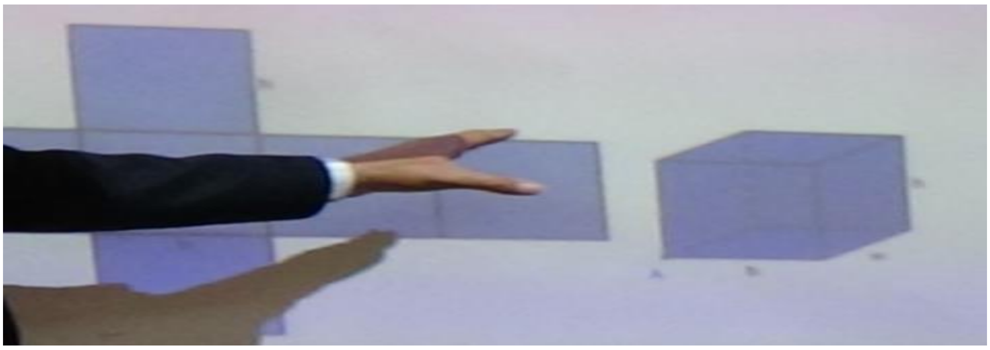

Figure 47 The figure of devlelopments of a prism with the computer program

Semih: You will see the formation of a prism here by opening and closing it... There was a prism at first and it turned into an open figure. Then, we saw that prism obtained by folding this open figure again. There were three dimensions of the object which were length, width and height as we talked before. You see them here as well.

After visualization through the computer program, Semih continued to the lesson with concrete models and introduced the elements of prisms for better understanding. He drew the image of the prism on the board and wrote the elements of it to be permanent.

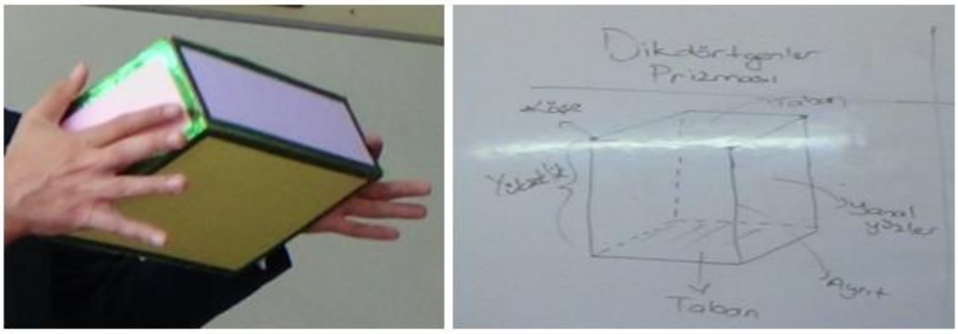

Figure 48 The figure of identification of a rectangular prism’s features

Semih: How do we name the intersection points of the edges?
Students: Vertex.
Semih: Ok. What do we say for each figure in yellow, pink or purple?
S8: Square.
S9: Rectangle.
S10: Face.
Semih: Yes, it is called “face”. What is the name of these lines that I pointed with my finger?
Students: Edge.

Semih: When I put the prism in a way that the yellow region will be at the bottom, how can we call this face?
S11: Underside.
S12: Base.
Semih: Yes, it is base. There is another yellow region across of this face. How can we call it?
S13: Base again.
Semih: Why did you say base?
S13: When we rotate the prism and put the upper yellow region to the bottom, it will be the new base in the same sense.
Semih: What is the figure on lateral faces?
Students: Rectangle.
Semih: What is the figure on the base?
Students: Rectangle.
S14: Each face is a rectangle.
Semih: Thus, we can name this prism as rectangular prism, right?
Students: Yes.
S15: If it was a square?
Semih: I will show it later.

Semih introduced the elements of the prism on the concrete model separately and expected students to indicate their names. He tried to learn what students knew about the subject instead of saying the names of the elements. Some students gave different answers because of Semih's questions but they were not totally wrong. For example, Semih referred to the yellow figure as "element", one student perceived what he meant as a geometric figure and he said rectangle instead of face. Here, it was seen that what Semih had intended and what the student understood was different from each other. Thus, there was need for more clear statements. He not only asked general questions which could reveal knowledge of the students related to the subject but also queried reasoning of particular students about the base concept. The students could make inference about the concept of the base. Semih tried to emphasize how prism was named but his approach was general.

Semih's noticing in this dialog can be considered as Level 2 because he was inconsistent in promoting students to elaborate their mathematical thinking. Although he focused on particular students' mathematical reasoning and tried to make sense of it, he maintained to consider the responses of the whole class. He further asked general and descriptive questions requiring students to give answers related to concepts and terms instead of questioning their mathematical approach. He evaluated the answers in terms of correctness but he also adopted an interpretive stance. He sometimes listened to the students but he mostly endeavored to tell what he intended. His approach was limited in terms of making inference about students' mathematical understanding. Because of this mixed approach, his noticing had prevalent features of Level 2.

During the planning phase, preservice teachers discoursed on teaching properties of prism with models concretely so that the students could find the number of elements through counting by themselves. They aimed to make students find what was asked by comprehending instead of giving information directly and forcing them for rote learning. The conservation between Semih and the students is as below:

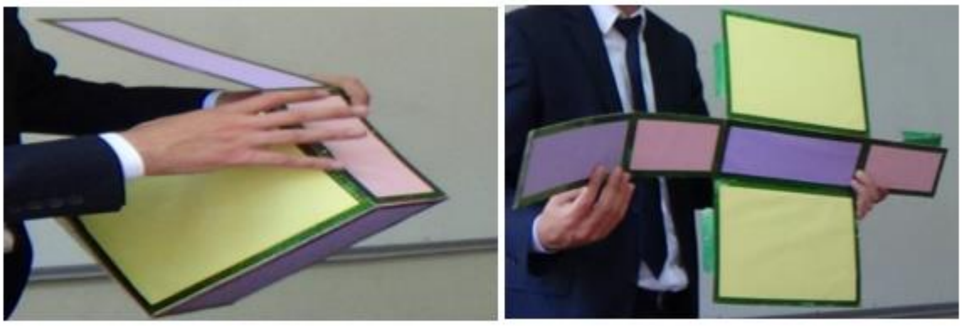

Figure 49 The demonstration of surface development of a rectangular prism

Semih: We have just seen the concepts of vertex, edge and face. Now look at the prism model. How many faces are there on this object?
Students: 6.
Semih: Explain how you found it.
S16: There are 4 faces on lateral and 2 on bases so there 6 six faces on a prism.
Semih: How many vertexes are there on this prism?
S17: 12.
S18: 8.
S19: 6.
S20: 7.
S21: 11.
Semih: Who wants to find the number by counting?
S22: One, two, three, four and on the other part, five, six, seven and eight.
Semih: Yes, you see that there are 8 vertexes. How many edges are there?
S22: (He counts the edges) 12.
Semih: In this case, there are 6 faces, 8 vertexes and 12 edges on the prism, right?
Students: Yes.

Semih continued to the lesson by asking the numbers of elements. He showed all faces of prism by opening it so that students counted and found the number of faces easily. Later, they gave various answers for the number of vertexes. They might have made mistakes while counting them and they might have missed or considered more than one vertices or they might have counted the wrong elements instead of them. However, Semih did not query how the students obtained these numbers. He preferred to show how to find the answer on the model concretely. The students could notice the number better after counting one by one. On the other hand, Semih asked a different kind of question and wanted the students to reason on whether height was an edge.

Semih: Can we say that height is also an edge?
S23: Yes.
S24: No.
Semih: Why did you say no?
S24: I do not know.
Semih: These all lines are edges, right?
S24: Yes.
Semih: When you consider the lines between two bases, what did we say for it?
S24: Height.
Semih: You just said that this was an edge and you now said that it was height.
S24: So height is an edge as well.

Students' responses to this question showed that they were unsure about it although it was easy to notice. When Semih heard the wrong answer, he tried to learn the student's reasoning for this idea but the student could not explain his responses. It showed that the students might have given answers without thinking too much. Thus, Semih tried to make students notice the relationship between them by leading the students with questions instead of indicating the answer directly. His approach was sometimes general but it was also towards obtaining information about students' thinking.

Semih's attention to the students' ideas and his responses in a way that could reveal students' mathematical thinking were limited. He could not provide the students with opportunities to present details about their mathematical approaches. He preferred not to focus on the wrong answers instead he tried to correct them showing on the models. Thus, he missed the opportunity to determine where the lack of knowledge or confusion existed. His approach was general and evaluative mostly but he also attempted to adopt an interpretive stance. He focused on what he had intended but in addition he directed the students to make inference about the subject instead of telling the necessary information straight away. For example, he did not say that height was an edge and leaded the student to reach the correct answer. Although he focused on specific students, he mainly considered the class as a whole. His noticing reflected the prevalent features of Level 2.

In this lesson plan, the aim was to teach three types of prisms which are a rectangular prism, tetragonal prism and cube. Therefore, the preservice teachers decided to focus on each prism and introduce their elements and properties separately. They aimed to support the students to notice similarities and differences among prism types and reinforce the essential concepts. One of the interactions regarding this issue in the classroom is as in the following:

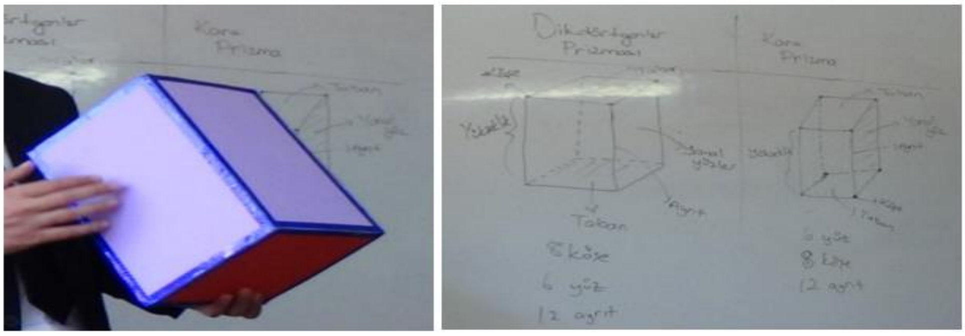

Figure 50 The figure of identification of a tetragonal prism's features

Semih: Now, look at this figure.
S25: Tetragonal prism.
Semih: Why did you call it as a tetragonal prism?
S25: Because it looks like a square.
Semih: Which side looks like a square?
S25: The base is a square.
Semih: Yes, it is a tetragonal prism because the base is a square. In this case, how do we name the prisms?
Students: According to the base.
Semih: Ok. Let's repeat the properties of this prism. How many vertexes are there?
S26: 4.
S27: 8.
Semih: Ok. We count together (He points to the vertexes with his finger on concrete prism model). One, two, …, eight. How many faces are there?
S28: 4.
S29: 6.
Semih: Look at, there are 6 faces (He shows the faces). How many edges are there? Come and show.
S30: 8.
Semih: You counted the vertices. Where were the edges?
S30: Ok. 12.

Here, Semih wanted to question the elements of a tetragonal prism and the properties of it through a concrete model by visualizing. When he showed the prism without indicating its type, the student S25 named the object as a tetragonal prism. Semih queried the reasoning of this student with a question such as "Why did you call it as a tetragonal prism?". He tried to be sure about student's mathematical understanding by obtaining more information.

However, when Semih asked the numbers of prism elements, there were different answers and Semih did not try to learn how the students found these numbers. For example, student S28 might have indicated the number of lateral faces as 4 instead of all faces due to confusion. However, Semih missed the opportunity to understand the mistake and correct it. In order to show the answers, he needed to count the elements on the model one by one. After he identified the features of a tetragonal prism, he continued by drawing its figure on the board.

Semih: Now, I am drawing a tetragonal prism and writing its elements and properties on the board. Then, you will write it in your notebooks.
S31: Teacher, is it a square?
S32: Are you drawing a parallelogram?
Semih: It is a square. Why did it look like a parallelogram while drawing a prism?
S33: Because it is three dimensional.
S34: Because we look at the side view of the object.
Semih: Yes, you are right. What are the numbers of elements?
S35: 6 faces, 8 vertexes and 12 edges.
S36: All of them are the same.
Semih: What are the geometric figures on the base?
Students: Square.
Semih: What are the geometric figures on lateral faces?
Students: Rectangle.
S37: All faces were rectangle in a rectangular prism.

Semih noticed student S32 perceived the geometric figure wrongly and he tried to learn the other students' understanding and he started a discussion to listen to the others' approaches and promoted the students to share their thinking ways. It was seen that some students understood the reason for perceiving the square as a different geometric figure and associated this situation with its three dimensional model and looking from different angles. At this point, Semih did not need more explanation because the students tried to help their own friend. On the other hand, Student S36 indicated that "All of them were the same" while Semih was questioning the numbers of prism elements. However, Semih did not query what the student meant with this expression and maintained the flow of the lesson. The student probably noticed the relationship between the number of both rectangular prism and tetragonal prism elements and emphasized the sameness of them. However, Semih could not reveal the student's mathematical thinking since he did not focus on this answer. It is seen that Semih tried to listen to students' responses but there was inconsistency in his approach.

Semih's noticing in this dialog was classified as Level 2 because he sometimes was able to attend to students' thinking and encourage them to give details about the reasons behind their answers but he sometimes did not focus on students' responses and did not try to obtain more explanations about their mathematical approaches. He listened to the students' ideas and responded them in accordance with their answers rather than indicating what was intended or making them see the implicit knowledge in the activity directly. However, when he could not get the correct answers he directed the students with general questions. On the other hand, he exhibited both an evaluative and interpretive stance while listening to the students' comments and attending to their answers. He endeavored to make sense of students' thinking but in an unspecific way in some moments. Hence, his noticing exhibited the characteristics of Level 2.

After Semih discoursed on the properties of a rectangular prism and tetragonal prism, he followed the same procedure for the cube. He showed a concrete model of it and queried the numbers of the elements and characteristics of the object. The dialog between Semih and the students is as in the following:

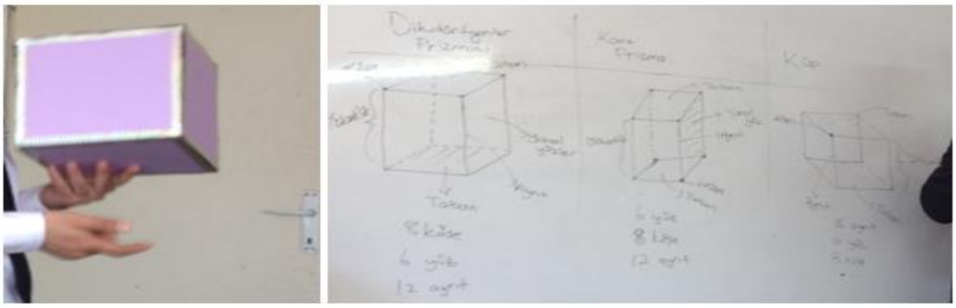

Figure 51 The figure of identification of a cube's features with a concrete material

Semih: Look at this object.
S38: Square.
S39: Cube.
Semih: Which geometric figures does it include?
Students: Square.
Semih: What is the base?
Students: Square.
Semih: What are the figures on lateral faces?
Students: Square.
S40: All faces are square.
Semih: What is the name of this figure?
S41: Tetragonal prism.
S42: Cube.
Semih: Why did you call it as a tetragonal prism?
S41: We named the previous prism as a tetragonal prism because its base was a square. This object's base was a square also.
Semih: How did we describe tetragonal prism?
S41: The bases were square and the lateral faces were rectangle.

Semih: What is different here from a tetragonal prism?
S41: All faces are square in this object.
Semih: Yes, bases are square and lateral faces are square here. Thus, how can we call this object?
S42: Cube.
Semih: Ok. Let's look at the numbers of vertexes, faces and edges.
S43: 12 edges, 6 faces and 8 vertexes as in the previous prisms.
Semih: Why did you say that?
S43: Because rectangular prism and tetragonal prism had 12 edges, 6 faces and 8 vertexes and cube is similar to them.
Semih: Do you agree with your friend?
Students: Yes.

When Semih showed the model of a cube, student S38 indicated the geometric figure on it whereas student S29 expressed the name of it before listening to the question. Their answers showed that they had some knowledge about the issue. Students noticed the properties related to the faces of the cube (e.g. "All faces are square"). However, there was hesitation about the name of the prism. Student S41 named the cube as a tetragonal prism. She considered the properties of the previous prisms and how their naming was made according to the geometric figure on the base. She thought that the base was square so it would be a tetragonal prism, however, she missed the important point related to lateral faces. She transferred her previous knowledge to the new situation incompetently. Semih noticed what the student did not consider and directed her attention to the difference between tetragonal prism and this object by questioning the figures on the bases and lateral faces instead of indicating what he intended directly. He allowed students to reason on the question and he was willing to listen to the student's thinking. On the other hand, student S43 could express the number of the elements without needing to count again through making inference from the previous prism. Semih queried the reason behind this answer. The student thought that cube was similar to a rectangular prism and tetragonal prism and there was only a small difference among them due to the geometric figures on the bases and lateral faces, consequently she indicated that a cube had 12 edges, 6 faces and 8 vertices. Then, Semih finished the lesson summarizing the properties through general questions.

Semih's noticing in this conversation was classified as Level 3 because he endeavored to attend to students' reactions and responded them in a way that he could reveal their thinking. He promoted particular students to reason on some issues and wanted more explanation about the reasons behind their comments. He mainly listened to the students' ideas and endeavored to make sense of their thinking. When students gave unexpected answers, he did not ignore them. In this case, he did not prefer to indicate the correct answers directly instead he helped them to catch their mistakes and correct them by asking various questions. He attended to students' answers with an interpretive stance to make inference about their mathematical understanding. His noticing further exhibited features of Level 3.

During the planning phase, preservice teachers thought that the students needed to see both close and open images of all three types of prisms separately. They also agreed that using only concrete materials could not be enough to show different sights of them clearly because of limited rotation. Thus, they decided to use a computer program in order to open, close and rotate all prisms for better visualizing and learning. The following comments reflect Semih's explanation on this issue.

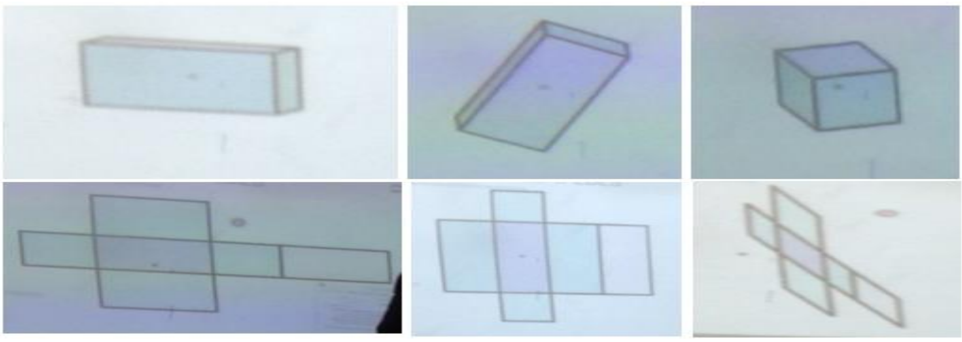

Figure 52 The demonstration of surface developments of rectangular prisms

> **Semih:** Now, we see different sights of prisms as both open and close. Remember when I was drawing a tetragonal prism, you perceived the square at the bottom as a parallelogram because of the side view, namely, looking from a different angle. You also see them from different perspectives now.

Preservice teachers thought that the students might make an overgeneralization like there was only one open image of a prism, one kind of open image had to belong to only one prism or all open images could form a prism. In order to show that one prism might have different open images and each image might not turn into a prism, they decided to present different open images of prisms and show whether they would close with the help of both computer programs and concrete materials. They preferred to use both computer programs and concrete materials because they believed their benefits would be different and increase the richness of the content. Here, their aim was to make students feel these key points and prevent possible misperceptions rather than expecting them to offer correct estimations regarding formation of all prisms. The following conversation presents interactions between Semih and the students:

Figure 53 The example of different surface developments of rectangular prisms

Semih: Look at the open figures. They are being folded and forming a prism, right?
Students: Yes.
Semih: I will choose different open images from the next side and you will see how they are closing and how a prism is obtained. Be aware that the open figures are different and form a prism.
…
Semih: Now, we will continue on concrete materials. According to you, can a prism be obtained from this cardboard by folding it?
Students: No.
Students: Yes, a prism forms.
Semih: Who wants to come to the board? You come. (He points to a student).
S47: One face was missing. The top of the prism remained open.
Semih: Does a prism form in this case?
S47: No. If there was one more face to close the top, we could obtain it.
Semih: You come for the next one. Will a prism be constituted?
Students: No.
Students: Yes.
Semih: Think a little bit. You answer without thinking.
S48: Yes, I obtained a prism from this cardboard.
Semih: Ok. You come. What do you think?
Students: Forms.
Students: Does not form.
S49: No.
Semih: Why?
S49: Because one face remained as extra.
Semih: What did you notice in this activity?
S49: Some open figures formed a prism whereas some of them did not.
Semih: Yes. Each open image does not form a prism, right?
Students: Yes.

In the process which included showing the open and close forms of all types of prisms and rotating them, Semih referred to students' incorrect perceptions about the geometric figure on the base. Using the computer program, he endeavored to clarify that the figures could be perceived differently depending on angles of view by rotating the prisms and showing the forms of them which students were familiar and unfamiliar with. Later, he wanted to show the students how to obtain a prism by folding the faces and there might be different open images which could constitute a prism or not. He called different students to solve the questions and promoted the students to explain the thinking behind their answers in detail with the questions such as "Does a prism form in this case?" and "Why?". He maintained the activity with the help of technology and provided students with the opportunity to practice on formation of prisms.

Semih: Now, you will estimate whether the open figure will form a prism or not in the computer program. Think first and then answer. What do you think for this figure?
S50: It dooes not form.
Semih: Why not?
S50: Because there are 5 faces and one face will remain open.
Semih: Good. For the next question, what do you say?
Students: It dooes not form.
Students: It forms.
Semih: Ok. Let's see. It does not form a prism. Look at the next question.
S51: There are 7 faces. One will be more so it does not.
Semih: How did you understand there will be one face more?
S51: There were 6 faces on the other prisms.
Semih: Yes. How many faces do rectangular prism, tetragonal prism and cube have?
Students: 6.
Semih: Thus, some of your friends decided whether a prism will form or not by counting the number of faces. When they found less or more than 6 such as 5 and 7, they said the answer directly. Did you understand?
Students: Yes.

Similarly, he tried to learn the details of students' thinking through such questions "How did you understand there will be one face more?" and "why". Although students S48, S49, S50 and S51 gave correct answers, Semih tried to learn more about their reasoning and control the possible gaps in their understanding. Student S48 and S49 could notice that the open figures would not form prisms because of missing and extra faces after folding. Similarly, S49 and S50 could make correct comments related to the formation of prisms by making inference about the number of faces before folding the faces. Semih was willing to listen to the students' responses by querying their mathematical approaches.

Semih's noticing here can be accepted as Level 3 because he did not only listen to the students' answers about formation of the prisms, but also queried them in a way that helped to reveal their mathematical thinking. He wanted more details about the reasons behind the students' answers and he promoted them to make more explanations about their reasoning through questions such as "What did you notice in this activity?" and "How did you understand there will be one face more?". He attended to the students' ideas and tried to make the students catch that each open figure might not turn into a prism and prism might have different open images rather than indicating what was intended. He listened to the students' comments and attended to their answers in an interpretive manner because he tried to make sense of their mathematical thinking to learn about what the students understood. Thus, his noticing mostly had the properties of Level 3.

One of the objectives of the lesson was to tell how to calculate the surface area of all three types of prisms. The preservice teachers thought that teaching the meaning of surface area and calculation of it step by step on concrete models by opening all faces would be more effective than giving their formulas directly. Thus, they decided to find the area of each face and write them in the relevant region for the students to see that surface area was sum of all faces' areas when they saw the open image of the prism. They also thought that the students might reach general formulas of them in this way.

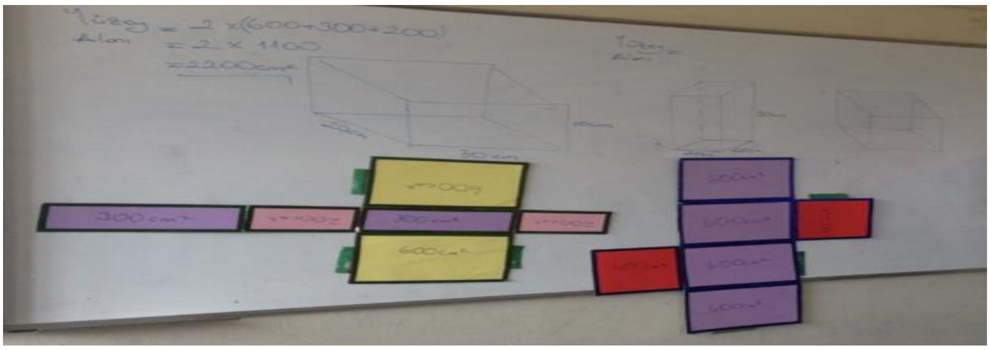

Figure 54 The demonstration of the surface area in rectangular prism types

Semih: What is this object?
Students: A rectangular prism.
Semih: What is the name of this yellow figure?
Students: Rectangle.
Semih: If the length is 30 cm and the width is 20 cm, what is the area of it?
Students: 600 cm^2.
Semih: Let's look at the purple region. The length is 30 cm and the width is 10 cm. What is its area?

Students: 300 cm^2.
Semih: Where is the same of this rectangle?
Students: Behind it.
Semih: So what will we do?
S52: Write the same area into the rectangle behind it.
Semih: Ok. I wrote it. What did we find in the yellow region? 600 cm^2. What will we do?
S53: Write 600 in the other yellow region.
Semih: Ok. There is another rectangle which is pink. The length is 20 cm and the width is 10 cm. What is the area?
Students: 200 cm^2.
Semih: Where is the same of it?
S54: Across this face.
Semih: Yes. When I open this rectangular prism, the sum of all faces' areas will give the surface area of it. Let's sum all areas. 600 + 300 + 200. Do we have the same result again?
Students: Yes. We need to multiply them by 2.
Semih: Yes. What will we find in this case?
Students: 2200 cm^2.
Semih: What did we do while finding the surface area of a rectangle prism?
S55: We found each area and summed all of them.
Semih: Yes. Another way?
S56: We found the areas and multiplied by 2 instead of adding separately.

Semih questioned what the object was and how to find area to repeat the previous learning for helping to transfer it here and make an introduction to the subject. He tried to make following the solution and learning easier therefore he asked the questions step by step. The students could find the area of rectangles that Semih referred respectively. He repeated the same kind of questioning pattern in a manner to help students give the answers and say the keywords that he wanted to hear. He asked generic questions to show the solution of the question rather than learning more about students' thinking. His questions were further evaluative and towards getting close answers. Although the students were able to give correct answers to these questions, they were not enough on their own to reveal whether the students understood what was intended. On the other hand, he also attended to include students in the process instead of telling what he had in his head directly. For example, he directed the students to consider geometric figures which had the same area, doing that he aimed to show there was no need for calculation again. He promoted students to notice some important points by themselves such as multiplication by 2 instead of summing all edges. However, his approach was limited in aiding students to reason and make inferences.

Semih mostly maintained the flow that he had in his mind but he guided the students to notice the important points and the implicit knowledge by themselves rather than telling them directly. He listened to the students' answers and conducted the solution of the question including them. Because he did not hear a wrong answer, he accepted this situation as the indicator of students' understanding. Thus, he did not need to query students' reasoning and to make sense of it. Although he endeavored to attend the students' reactions and responded them in accordance with their answers, his approach was limited in promoting students to elaborate their mathematical thinking. Semih focused more on getting students to indicate the necessary terms and calculations than promoting a discourse to reveal their thinking. Besides, his questioning was more evaluative and towards obtaining correct answers. He asked basic questions step by step to provide a clear solution process and had a general approach rather than an interpretive stance. Therefore, he could not notice what mistakes and confusions were and he determined students' understanding considering the whole class. Because of this mixed approach, his noticing had prevalent features of Level 2.

After rectangular prism, Semih continued to the lesson with the surface area of a tetragonal prism in a similar way. The conversation between Semih and the students is presented as below:

Semih: What is the name of this object?
Students: Tetragonal prism.
Semih: What was the figure on the base of this prism?
Students: Square.
Semih: One edge of this square is 20 cm. What is the area of it?
S57: 400 cm^2.
Semih: Where is the same of it?
S57: Across. The red region again.
Semih: What are the figures on lateral faces?
Students: Rectangle.
Semih: The width is 20 cm and the length is 30 cm. What is the area?
S58: 500.
S59: No. 600 cm^2.
Semih: How do we find the area?
S58: By adding.
Students: By multiplying length and width.
Semih: What is the area?
S59: 600 cm^2 from 20 times 30.
Semih: What will we do?
S60: We will write 600 into each rectangle.
Semih: Yes. You are right. What is the surface area of this tetragonal prism?
S60: We will sum all of them so it is 3200 cm^2.
Semih: What is the red square?
Students: Base.
Semih: How many bases are there?
Students: 2.
Semih: Thus, what can we do?
S61: We can multiply 2 by 400.
Semih: Yes. Similarly, how can we write the areas of rectangles?

S61: We multiply 4 by 600.
Semih: In this case, 800 from 2x400 and 2400 from 4x600. The surface area is 3200 cm^2 from the sum of them. How do we find the surface area of a tetragonal prism?
S62: Finding each area.

Semih: If we want to indicate it in a general form, we can say that the surface area of a tetragonal prism is equal to the total of 2 times base area and 4 times lateral face area. Did you understand?

Students: Yes.

Semih stated the surface area of a tetragonal prism by asking basic questions toward repetition again. When he asked the area of the geometric figures on the bases and lateral faces individually, he noticed that student S58 made mistake. He did not try to learn how this student obtained the answer but he also endeavored to lead the students with a question on the other hand (e.g. "How do we find area?). Student S58 indicated the concept of perimeter instead of area indicating "by adding". If she had calculated by adding, she should have found the answer as 100 but she found 500. However, Semih did not focus on this issue and continued to the lesson considering the correct answer. He preferred not to tell what was intended directly and expected the students to notice some points. His responses revealing students' thinking to make inference about their understanding were inconsistent. For example, student S60 offered writing 600 into each rectangle but Semih did not query the reason for his comment. He did not try to understand whether this student noticed the equality of lateral faces and accepted it directly as the correct answer without supporting the student to make more explanations about his comments. On the other hand, while Semih was telling how to calculate the surface area of a tetragonal prism, one student's question showed his hesitation about the geometric figure. The following addressed this conservation.

S63: Teacher, is the figure with the area of 600 cm^2 a rectangle or square?
Semih: One edge is 20 cm and the other is 30 cm. What do you think?
S63: A rectangle?
Semih: What can we say about the properties of a square?
Students: All edges are equal.
Semih: Are edges in this figure equal to each other?
S63: No, they are different. Ok, it is a rectangle.

While talking about the figures on bases and lateral faces, student S63 confused rectangle and square with each other. Semih tried to reveal his knowledge and thinking to make him notice what he missed and help him answer the question by himself. When Semih drew attention to the equality of edges in a square, the student could see that this figure was not a square. He also leaded the students to reach a general formula of surface area with some questions instead of indicating it directly.

Semih's noticing in this conservation was accepted as Level 2 because he listened to the students' answers and attempted to query them in a manner that could help to reveal their mathematical thinking but his approach was inconsistent. He guided the students to catch the important terms and make the calculation which were needed for the lesson instead of telling them directly. However, he also asked general questions which were towards learning students' knowledge about the concepts and calculation rather than promoting them to interpret mathematical issues. When he heard incorrect answers, he did not ignore them but could not sufficiently question the details about the reasons behind them from time to time. He focused on both the whole class and specific students. He listened to the students' comments and attended to their answers in both an evaluative and interpretive way. For example, he evaluated the students' area calculations in terms of correctness whereas he approached the confusion of the student related to square and rectangle in an interpretive stance. He attended the students' responses and tried to complete the missing parts by leading the students through a series of questions instead of telling what he had in his mind. Thus, his noticing was considered as Level 2.

After tetragonal prism, Semih proceeded to the surface area of a cube in a similar questioning pattern. The following dialog presents the interaction between Semih and the students.

Semih: What is this object?
Students: A cube.
Semih: What are all faces of a cube?
Students: Square.
Semih: One edge of it is 20 cm. What is the area of one square?
Students: 400 cm^2.
S64: Is the answer 2400?
Semih: What will we do to find the surface area of a cube?
S65: We will sum the area of each face.
Semih: Area of one square is 400 cm^2, how many 400 cm^2 are there?
Students: 6.
Semih: What can we say?
S65: 2400 cm^2.
Semih: Ok. What did you notice while finding the surface area of a cube?
S66: In the previous prism, we multiplied by 2 but here we multiplied by 6.
Semih: Why?
S66: Because there were 6 faces.
Semih: Is the only reason that there are 6 faces? There were 6 faces on a tetragonal prism too.
S66: All faces are square.

Semig: What do you mean?
S66: Their areas are equal so we multiplied by 6.
Semih: Yes, it is ok now. The reason is not only having 6 faces but also the equality of these faces. There were 6 faces in the other prisms but we did not multiply by 6. We paid attention to which faces were the same.

In this excerpt, it was understood that student could find the surface area of a cube easily. Student S64 actually gave the correct answer but Semih maintained his questioning in order not to break the flow of teaching for enabling the other students' understanding. He endeavored to promote students to reason and reach a general formula of a cube's surface area through questions such as "What did you notice while finding the surface area of a cube?". He caught an important point in student S66's comments which might reveal the gap in his understanding. Thus, he attempted to learn more about the thinking behind that student's answers. He asked interpretive questions like "Is the only reason that there are 6 faces?" and "What do you mean?" to query how he thought mathematically. He also tried to make the student notice the lack in his approach instead of telling what he had in his head or accepting the student's answer with an evaluative stance. Semih thought that the student might have made an overgeneralization about multiplication by 6 without considering the equality of faces, namely, the needed conditions. Therefore, he preferred to check whether there was a situation like that and to prevent constitution of this kind of misconception if it had existed previously.

It can be argued that Semih's noticing was in Level 3 because he was able to attend to students' thinking and promote them to justify their answers. He listened to the students' ideas and responded them in accordance with their answers rather than indicating what was intended or making them see the implicit knowledge directly. Through an interpretive stance, he tried to make sense of students' thinking to find out whether the students understood or what they understood. He noticed that there might be missing points in a particular student's answer and he preferred to support him to provide details about his approach in order to remove the misconception or overgeneralization tendency of the student if any existed, even though the student gave a correct answer. Hence, his noticing exhibited the characteristics of Level 3.

In the planning phase, preservice teachers decided to reinforce the concept of surface area through asking questions after calculation of it was taught for each prism. Therefore, they determined some questions and decided to allow students to solve them. The following dialog includes the solution process of one of those questions.

Semih: Calculate the surface area of a classroom which has a length and width of 7 cm and height of 8 cm and is shaped like a prism.
S67: 56.
S68: 44.
S69: 49.
Semih: Who wants to come? (One student comes to the board). If the length and width is 7, what is the figure on the base?
Students: A square?
Semih: What do we do to find the surface area of a tetragonal prism?
S69: We look at the base, 49 from 7 times 7.
Semih: Yes, you found the base. What will you do to find the area of one lateral face? In this case, short edge is 7 cm and long edge is 8 cm for this rectangle, right?
S69: Yes. 7 times 8 is 56. If there are two, 115.
Semih: Let me help you. We found the area of one base as 49. How many bases are there? Two. Then, I will multiply 49 by 2.
Students: 96.
Students: 98.
Semih: 98 m^2. What did your friend find by multiplying 7 by 8?
Students: 56.
Semih: Namely, he found the area of one lateral face as 56.

This question asked the students to calculate the surface area of a tetragonal prism. However, they made operation mistakes while multiplying the edges. Semih tried to lead the students by emphasizing the shapes on the bases and lateral faces. Since they still could not find the correct values, he preferred to show them what to do by asking general questions such as “How many bases area there?” and” What did your friend find by multiplying 7 by 8?”. While he was telling the solution, one student asked a question. This conversation was presented below.

S70: Why did he multiply 7 by 8?
Semih: This is a tetragonal prism. Lateral face is a rectangle. Long edge of it is 8 because it is height as well. Short edge is 7 because it is also the same with one edge length of the square on the base. In order to find the area of this rectangle, we multiplied 7 by 8.
S70: Ok.
Semih: How many lateral faces are there?
Students: 4.
Semih: What do we need to do?
S71: Multiply 4 by 56. It is 224.
Semih: What is the surface area of this prism?
S72: 1204.
S73: 322.
S74: 324.
Semih: We will sum the area of two bases and four lateral faces. 98 plus 224, it is equal to 322 m^2.

In this excerpt, students were asked to calculate the surface area of a tetragonal prism but they had difficulty in finding the area of both bases and lateral faces. It was seen that they multiplied the numbers wrongly. Due to the fact that there was very little time for the bell to ring, Semih tried to solve the question quickly in order to complete the subject. Thus, he generally told what he intended rather than querying students' reasoning. Students gave various and incorrect answers but he ignored them and preferred to focus on the correct responses. Wrong answers were likely to express the gaps in their understanding and Semih preferred to follow a telling mode. Students had difficulty in visualizing the shape of the prism in their mind and understanding the meaning of it so they could not solve it. It was likely that most of the students could not comprehend the object referring to a tetragonal prism in the problem by considering the edge lengths. If Semih had drawn the tetragonal prism and written the edge lengths on the board, there might have been more students who would understand and solve the problem. However, he made explanations verbally and directed the students to give the answers which he wanted to hear. Wrong answers of the students probably addressed to the lack of understanding, incorrect thinking or operation mistakes but Semih did not want the students to elaborate the reasons behind these responses rather he focused on the correctness of them.

Semih's noticing during this interaction was classified as Level 1 because although he tried to attend to the students' reactions, he did not respond the students in a way that could reveal their mathematical thinking and he did not query the reasons behind their answers. He focused on the whole class rather than specific students. Students gave wrong answers but Semih ignored them, he focused on the correct responses instead. Thus, he did not catch the gaps in their mathematical understanding. Although he listened to the ideas of the students, he did not attempt to make sense of their mathematical approach. He mostly maintained the flow that he had in his mind and guided the students to get the answers that he looked for. He maintained a pattern of questioning which helped the students notice what was intended through a general approach instead of getting more details about their thinking and learning. Hence, his noticing mostly had the properties of Level 1.

When the whole process of teaching was considered, Semih's noticing predominantly exhibited the features of Level 2. His focus was on mathematical thinking and understanding of both the whole class and particular students. He listened to the students' ideas and responses instead of indicating what he had in his mind directly but he also preferred to apply the mode of telling from time to time. He both endeavored to promote the students to reason on important points through interpretive questions and asked general questions towards getting the necessary terms and calculations from them. Thus, his responses to reveal the students' thinking in order to make inference about it was inconsistent. On the one hand, he was willing to listen to the students' answers so that he could vary his explanations and directions, on the other hand, he was in tendency to maintain the flow that he had intended. Therefore, his noticing in the re-teaching was considered as Level 2.

4.2.4.4 What and how Semih notice in re-reflecting phase

After the fourth lesson plan was implemented by Mehmet in 5/F during the teaching phase and preservice teachers discussed on it about what should be changed or paid attention for next implementation, re-teaching of the fourth lesson plan was made in another classroom 5/G by Semih. While Semih was implementing the lesson plan, the other preservice teachers, the cooperating teacher and researcher observed this lesson and took notes. In the re-reflecting phase which was conducted just after re-teaching, the cooperating teacher and all preservice teachers were wanted to reflect their thoughts about the lesson including; what worked well, what did not work, what the difficulties were, what they observed about the students' actions and thinking and also what should be changed about the lesson to improve it and what the rationales for the changes were.

In accordance with discussion protocol, Semih shared his opinions about the lesson that he taught, evaluated the effectiveness of the lesson plan and made some suggestions to improve it. The following dialogs show what and how Semih noticed during the re-teaching phase.

> **Semih:** When I repeated the previous subjects, it was good that the students could indicate what perimeter, area and conservation of area was. I introduced the elements of prism such as edge, vertex and surface with concrete models. When I asked the number of them, students gave various answers such as 5, 7 and 8. Thus, I counted these elements on a rectangular prism to express how to find them and the students could find them for a tetragonal prism and cube easily. Besides, the students did not have difficulty in indicating which geometric figures were on sides and bases of prisms. On the other hand, I tried to show different opening and closing representations of prisms. However, the students said their answers randomly and directly. They were in tendency to indicate that the figure could form a prism or not without thinking on it.

In terms of what to notice, Semih focused primarily on his own pedagogy as the teacher of the lesson and students' mathematical approach. He referred to the students' responses and what they could do or not. He offered comments about the tendency of the whole class instead of attending particular students' mathematical thinking (e.g. "They were in tendency to indicate that the figure could form a prism or not without thinking on it”). He also mentioned pedagogical issues such as using concrete models and different representations to facilitate understanding. Regarding how to notice, he drew attention to important interactions but he offered general impressions. He provided few details to support his claims and he did not try to make sense of students’ thinking. For example, why students gave incorrect answers for the numbers of elements of prisms or what kind of answers the students indicated regarding the constitution of prisms after closing. His comments were descriptive and evaluative in nature (e.g. “It was good that the students could indicate what perimeter, area and conservation of area was”). Because he attended teaching and learning issues with a general stance, his noticing here was considered as evidence of Level 2.

> **Semih:** While we calculated the surface area of a prism, the students asked whether the area of one face was found because we found the area of only one figure until now. This kind of question made me surprised and I tried to teach that the surface area of a prism was the sum of all area measurements. I had as much difficulty in drawing the prism on the board as the students. I could not explain them how to draw it easily. While I was trying to draw a cube, I drew it as a tetragonal prism by mistake. The students perceived the sides of the figure as a rectangle and they did not accept it as a cube. I caused a confusion accidentally. They paid attention to the image of the figures. I think they could not have made this kind of an objection if they had not understood. In a rectangular prism, I emphasized that the prism was named according to the figure on the base. Thus, the students could recognize a tetragonal prism themselves with this knowledge.

In this excerpt, as the teacher of the lesson Semih focused mostly on his own pedagogy and the students’ mathematical thinking. He referred to their confusions and difficulties highlighting the important moments (e.g. “. The students perceived the sides of the figure as a rectangle and they did not accept it as a cube). He also indicated the deficiencies that he saw in himself. He tried to offer interpretations about the students’ thinking and the reasons behind their mathematical approaches but in unspecific ways. He associated their suggestion to find the area of one face to old habits and linked their confusions with his own mistake. He was inconsistent in elaborating his claims. He made reference to the students’ answers as evidence. Additionally, he offered general impressions and judgements. For example, he did not give details about how he made assessment on the recognition of naming a prism. His comments were evaluative with an interpretive stance (e.g. “I think they could not have made this kind of an objection if they had not understood”). Therefore, his noticing had the prevalent properties of Level 2.

> **Semih:** I lost time at the beginning of the lesson because the projector did not work, we tried to solve this problem. Later, there was no curtain in the classroom so the

> reflection on the board could not be seen well. Besides, I laid emphasis on rectangular prism more than needed. However, I could not proceed quickly because the students said that they could not write or draw fast enough. In addition to the computer environment, we planned to show whether prisms would be constituted from open figures with concrete materials. Students came to the board to understand how the open figure would close but the cardboard pieces were so thin that the students had difficulty while folding them. I think we needed to use thicker cardboard. They lost their attention sometimes and I had difficulty in managing the classroom.

In terms of what was noticed, Semih's focus was on a range of issues including the implementation of the lesson, time, materials, students' interest, classroom environment and management. He attended to the physical conditions of the classroom and described what kind of negative events arose. He referred to on-task behaviors of the whole class and teaching issues (e.g. "I could not proceed quickly because the students said that they could not write or draw fast enough"). As for how he noticed, his comments were quite general and descriptive. He provided few or no details to support his claims with a broad impressionistic stance. In general, his attention was on less mathematical aspects of the lesson rather than the students' mathematical thinking, accordingly he did not try to make sense of their strategies. Hence, his comments exhibited the evidence of noticing in Level 1.

> **Semih:** When I asked how many surfaces a tetragonal prism had, one student answered as 4. I think she confused lateral faces and did not consider lower and upper bases as faces that is why she found 4 instead of 6. There was a table which included writing the names of the geometric figures on the lateral face and base of each prism. However, the students did not understand what I asked and some of them gave answers such as 8, 12. I think my direction was deficient there. Thus, the students probably tried to find the number of edges or vertices on the base as 8. Besides, in the exercises which necessitated considering whether the open figure would form a prism, one student directly noticed that there were 7 faces so one face would be extra and a prism would not be obtained. Similarly, another student saw that there were 5 faces so one face would be missing and it would not be a prism. These students could reason on and find a general path about the formation of prisms by looking at 6 faces of a rectangular prism, tetragonal prism and cube.

In this quotation, Semih mainly focused on the students and their mathematical thinking. He referred to specific students' mathematical approach and tried to make sense of their strategies (e.g. "One student directly noticed that there were 7 faces so one face would be extra and a prism would not be obtained"). He also provided details about the students' reasoning with interpretations in order to draw inferences about their understanding. For example, he thought that the students tried to make generalizations by looking at the number of faces on each prism that they had learned. He examined specific events through making reference to the evidences from his observation with an interpretive stance in order to strengthen his analyses. Thus, his noticing had the properties of Level 3.

After the reflecting phase, individual interviews were made with the preservice teachers in order to obtain richer information about their noticing and reveal the missing points that preservice teachers might have overlooked. Semih's comments differently from what he shared in the reflecting phase were as below:

> **Semih:** The students endeavored to find the easy way to decide whether the open figures would be a prism or not. They focused on the number of the faces and eliminated the figures which had more or less than 6 faces. They could reach this inference by themselves. The important point here was that the students needed to understand each open figure with 6 faces would not form a prism. We gave appropriate and inappropriate examples to make students notice this difference. I think they could understand what we intended because we showed them with computer programs and used concrete materials.

In terms of what to notice, Semih focused primarily on students' mathematical understanding and teacher's approach. He talked about pedagogical issues such as presenting appropriate and inappropriate examples and using materials. He referred to the mathematical understanding of the whole class but he in a broadscale (e.g. "I think they could understand what we intended"). Regarding how to notice, he did not make reference to the evidences from the lesson to support his interpretations, instead he offered general impressions with statements not including details. On the other hand, he tried to refer the students' responses (e.g. "They focused on the number of the faces and eliminated the figures which had more or less than 6 faces"). However, he was inconsistent in elaborating his claims and had a broad perspective. He used evaluative and interpretive comments in nature. Therefore, his noticing further reflected the characteristics of Level 2.

> **Semih:** While calculating the surface area of a tetragonal prism, one student said 500 for the area of a rectangle with a lateral face of prism instead of 600 from 20 times 30. I asked her how she found 500 and how to calculate the area of rectangle. She said by summing but if she had done that, she would not have obtained 500. She confused the concepts or made an operation mistake. Besides, while teaching the surface area of a cube, the student noticed the need for multiplying by 6 but he might have overgeneralized that one area would be found and it would be multiplied by 6 for each prism because they had 6 faces as well. However, the figures on a prism might not be the same. I tried to complete the lack in this approach by drawing attention to the equality of each figure in a cube.

In this quotation, Semih' main focus was on particular students and the details of their mathematical thinking by considering their explanations. He paid attention to students' strategies to solve the questions. His comments were noteworthy because he addressed how and where a student might make incorrect generalizations about some concepts (e.g. "The student noticed the need for multiplying by 6 but he might have overgeneralized that one area would be found and it would be multiplied by 6 for each prism"). He provided details regarding the students' responses and interactions in the classroom in order to support his claims and make inference about students' understanding. He attempted to make sense of these students' mathematical thinking and made interpretive comments about what he observed (e.g. "She said by summing but if she had done that, she would not have obtained 500"). Hence, his noticing was further considered as evidence of Level 3.

When Semih's all comments during the re-reflecting phase were considered, his noticing exhibited predominantly characteristics of Level 2. In terms of what to notice, he focused primarily on teacher pedagogy and students' mathematical thinking. Although he attended to specific students' mathematical approach, he further considered the students as a whole. Regarding how to notice, he referred to the noteworthy events with a broad perspective. He tried to reason through what he observed and offer evidence to support his claims but his elaboration was mostly inconsistent. He also tried to make sense of students' mathematical thinking and reasoned on their strategies. However, this approach was limited. Although there were comments which had features of Level 1, Level 2 and Level 3, his noticing mostly had the features of Level 2.

4.2.2 Summary of findings from case 2 - Semih

In the first lesson study cycle, Semih's noticing was at Level 1 in the planning and re-reflecting phases whereas it was at Level 2 in the reflecting phase. Teaching and re-teaching of the first lesson plan were conducted by different preservice teachers, thus, information towards these phases was not indicated. During the planning and re-reflecting phases, in terms of what to notice, the prevalent indicators of Level 1 were that Semih's focus was on a range of issues including teacher pedagogy, students' understanding, the order of the content, students' behaviors, selection of questions, materials, the content of the lesson and implementation. His attention was on less mathematical aspects of the lesson rather than the students' mathematical thinking. For example, he attended to the structure of materials, classroom environment or speaking clearly. Regarding how to notice, he described his ideas through general expressions without making reference to the evidence in order to support what he said. He also offered evaluative comments from time to time. For example, he indicated that he could not understand how the student thought mathematically or what he did as an indicator of Level 1. In the reflecting phases, regarding what to notice, he predominantly focused on teacher pedagogy and students' mathematical approach as further referring to the class as a whole. Regarding how to notice, he highlighted the noteworthy events but in general ways. His comments were mostly evaluative and he sometimes adopted an interpretive approach even a little. He was inconsistent in providing details about what he observed to promote his claims.

In the second lesson study cycle, Semih' noticing was at Level 2 during the whole planning, teaching, reflecting and re-reflecting phases. Re-teaching of the second lesson plan was conducted by another preservice teacher so information of it was not presented. During planning, reflecting and re-reflecting phases, in terms of what to notice, the dominant indicators of Level 2 were that Semih mostly focused on pedagogical issues and considered students' mathematical thinking and understanding. He generally did not focus on particular students' mathematical thinking and had a broad perspective. Regarding how to notice, his comments were predominantly evaluative with an interpretive stance. He provided limited justification and elaboration about his comments. In the teaching phase, regarding what to notice, the robust characteristics of Level 2 were that Semih focused on students' answers and understanding and also what he had in his mind. He tried to listen the students' ideas and allowed them to explain their ideas. However, in the cases which the students could not understand what he referred to, he needed to tell them what to do directly. As for what to notice, although he endeavored to attend to the students' answers and promote them to clarify their mathematical approach, his responses revealing their thinking were inconsistent. He sometimes queried the reasons behind the students' answers but he did not sometimes pay attention to their explanations.

In the third lesson study cycle, Semih's noticing was at Level 2 in the planning phase, whereas it was at Level 3 in reflecting and re-reflecting phases. Because teaching and re-teaching of the third lesson plan were conducted by different preservice teachers, information about them was not given. During the planning, in terms of what to notice, the main indicators of Level 2 were that Semih focused primarily on teacher pedagogy and students' mathematical thinking. He sometimes attended to particular students and their mathematical strategies but he further referred to the students as a whole. Regarding how to notice, his comments were descriptive, evaluative and interpretive from time to time but an evaluative stance was predominant. Although he referred to the important points, he still maintained a general approach. Reasoning on students' strategies and elaboration of his ideas were limited. In reflecting and re-reflecting phases, regarding what to notice, the common characteristics of

Level 3 were that Semih predominantly focused on particular students' mathematical thinking and their responses. He reasoned on the solution strategies of the students and attempted to make sense of their mathematical thinking. In terms of how to notice, his approach was mainly interpretive. He made reference to the evidences of students' answers and provided details about the noteworthy events and interactions in the classroom in order to support his claims. Correspondingly he explained where and how specific confusions or mistakes of the students might arise.

In the fourth lesson study cycle, Semih' noticing was at Level 3 in the planning and reflecting phases whereas it was at Level 2 in the re-teaching and re-reflecting phase. Since teaching of the fourth lesson plan was conducted by another preservice teacher, information about it was not indicated. During the planning and reflecting phases, in terms of what to notice, the common indicators of Level 3 were that Semih predominantly focused on particular students' mathematical thinking, their specific confusions and difficulties rather than less mathematical aspects of the lesson. Regarding how to notice, he mostly provided justifications about his ideas and explained how his suggestions were related to the overall goals of the lesson. He reasoned on what he observed and provided details to make inference about the students' mathematical understanding. His comments were mainly interpretive in nature. In the re-teaching phase, regarding what to notice, the prevalent features of Level 2 were that Semih's focus was on mathematical thinking and understanding of both the whole class and particular students. He listened the students' ideas and responses instead of indicating what he had in his mind directly but he also preferred to apply the mode of telling from time to time. Regarding how to notice, he endeavored to promote the students to reason on the important points through interpretive questions but he also asked general questions towards getting the necessary terms and calculations from them. Thus, his responses revealing the students' thinking in order to make inference about their understanding was inconsistent. During the re-reflecting phase, in terms of what to notice, the common characteristics of Level 2 were that Semih focused primarily on teacher pedagogy and students' mathematical thinking. Although he attended to specific students' mathematical approaches, furthermore he considered the students as a whole. Regarding how to notice, he referred to the noteworthy events with a broad perspective and a rather evaluative approach. He endeavored to reason through what he observed and offer evidence to support his claims but his elaboration was mostly inconsistent. The following table presents Semih's noticing during lesson study cycles.

Table 9 Semih's Noticing Process in Lesson Study

	Planning	Teaching	Reflecting	Re-teaching	Re-Reflecting
Cycle 1	Level 1 *[60% L1, 30% L2, 10% L3]*	-	Level 2 *[25% L1, 75% L2]*	-	Level 1 *[67% L1, 33%L2]*
Cycle 2	Level 2 *[67% L2, 33% L3]*	Level 2 *[29% L1, 57% L2, 14% L3]*	Level 2 *[40% L1, 60% L2]*	-	Level 2 *[80% L2, 20% L3]*
Cycle 3	Level 2 *[17% L1, 50% L2, 33% L3]*	-	Level 3 *[33% L2, 67% L3]*	-	Level 3 *[25% L1, 25% L2, 50% L3]*
Cycle 4	Level 3 *[33% L2, 67% L3]*	-	Level 3 *[25% L1, 25% L2, 50% L3]*	Level 2 *[20% L1, 50% L2, 30% L3]*	Level 2 *[17% L1, 50% L2, 33% L3]*

Note. L1= Level 1; L2= Level 2; L3= Level 3

It is seen from the table that Semih's noticing varied between Level 1, Level 2 and Level 3. His noticing progressed from Level 1 to Level 3 during planning phases whereas his noticing increased from Level 2 to Level 3 in the reflecting phase. It is also seen his noticing increased from Level 1 to Level 3 and then back to Level 2 during the re-reflecting phase. On the other hand, the results showed that his noticing was at Level 2 during both teaching and re-teaching phases. It can be said that Semih's noticing was low at the beginning of the study but a gradual progress occurred towards the end of the lesson study cycles.

CHAPTER 5

5. DISCUSSION AND CONCLUSION

The purpose of this study was to examine preservice middle school mathematics teachers' noticing of students' mathematical thinking in the context of lesson study and determine whether professional development support was effective in the improvement of this skill. In other words, the aim of this study was to reveal what preservice teachers focused on and how preservice teachers made sense of their attention while planning, teaching and reflecting mathematics lessons, and namely, investigate what they noticed and how they noticed during whole lesson study cycles. Besides, it was aimed to examine the development of preservice middle school mathematics teachers' noticing of students' mathematical thinking during participation in this process. In the light of these foci, this study sought to answer the following research questions:

1. To what extent and how do preservice middle school mathematics teachers notice students' mathematical thinking in the context of lesson study professional development model?

2. How does lesson study contribute to preservice middle school mathematics teachers' noticing of students' mathematical thinking?

5.1 Preservice Middle School Mathematics Teachers' Noticing of Students' Thinking

The findings pointed out that both İnci's and Semih's ability to notice students' mathematical thinking was low when whole lesson study process was considered. In particular, it predominantly exhibited the features of Level 2. They focused on various issues such as behaviors of the teacher, students' interest, classroom management, teacher pedagogy, student learning, classroom environment, materials rather than focusing on specifics of students' mathematical thinking directly. Although their focus was on a range of points, they attended mostly to both teacher's pedagogical strategies and students' mathematical approach. Similarly, Jacobs et al. (2010) indicated that prospective teachers preferred to discuss on what they generally knew about teaching and learning mathematics such as use of tools or multiple ways for solving a problem. İnci and Semih focused on some particular students but they maintained to address the class as a whole. Their comments were descriptive and evaluative in nature yet they also adopted an interpretive stance partially. They had a tendency to offer their general ideas and impressions instead of providing details to support their claims. Albeit they identified noteworthy events, there was inconsistency in elaborating these issues throughout the lesson study process. This study revealed that noticing was not an easy action for preservice teachers. Likewise, many of the studies related to noticing argued that noticing can be challenging for novice teachers (Erickson, 2011; Star et al., 2011; Star & Strickland, 2008; van Es & Sherin, 2002), particularly, preservice teachers.

Data analysis revealed that noticing of İnci and Semih varied between Level 1 and Level 2 early on in this study (in the first and second lesson study cycles). Instead of focusing on the details of students' mathematical thinking, they attended to less mathematical aspects of the lesson which was one prevalent indicator of an inadequacy in Level 3 and their observations did not further reflect interrelated events (Carter et al., 1988). Similarly, the results of some researches revealed that preservice teachers were likely to focus on different instructional details which were relevant or irrelevant (Borko et al., 2008; Erickson, 2011; Sherin, Russ, et al., 2011; Star et al., 2011; Star & Strickland, 2008). Erickson (2011) explained the reason of this situation with the simplicity of attending to more common aspects of the lesson rather than the specifics of students' mathematical thinking. When the data obtained from early lesson study cycles were considered, it was seen that both İnci and Semih described what they observed in a general way without addressing the details of the events and evaluated them without making sense of the meanings. In other words, their comments were mostly descriptive and evaluative in nature (Chamberlin, 2005; Goldsmith & Seago, 2011; Kazemi & Franke, 2004; Sherin & Han, 2004; Sherin & van Es, 2009; van Es, 2011; van Es & Sherin, 2008). They were in tendency to restate the students' answers but they did not try to reason through the strategies that students had used. Windschitl (2004) also advocated that preservice teachers struggled to support their ideas. Additionally, the studies in literature supported that teachers were in tendency to evaluate students' responses in terms of accuracy (Crespo, 2000; Kazemi & Franke, 2004) and both teachers and preservice teachers had difficulty in interpreting and making sense of their mathematical thinking (Empson & Junk, 2004; Türnüklü & Yeşildere, 2007; Wallack & Even, 2005). The findings of relevant studies, in parallel to the findings of this study, argued that instead of paying attention to students' thinking, participants paid attention to a range of issues with a broad perspective at the beginning of the study (Goldsmith & Seago, 2011; Jacobs, Lamb, & Philipp, 2010; Kazemi & Franke, 2004; McDuffie et al., 2014; Sherin & Han, 2004; Sherin & van Es, 2009; Star & Strickland, 2007; van Es, 2011; van Es & Sherin, 2008).

On the other hand, the findings showed that both İnci's and Semih's noticing varied between Level 2 and Level 3 later on in this study (in the third and fourth lesson study cycles). These participants began to focus more on particular students' mathematical thinking and make sense of them. They endeavored to provide justifications about what they observed and details to make deductions about students' understanding. They began to address the details of important student responses by trying to give them meaning. Their comments turned mostly into interpretive ones in nature. The findings revealed that preservice teachers' noticing about students' mathematical thinking developed gradually throughout the process of lesson study. It can be argued that noticing of students' mathematical thinking is an ability that can be developed through professional development models. There are many studies in literature which supported this finding and emphasized the importance of this kind of support (Goldsmith & Seago, 2011; Jacobs, Lamb, & Philipp, 2010; Lewis, Friedkin, Baker &Perry, 2011; Sherin & Han, 2004; Sherin & van Es, 2009; Star & Strickland, 2007; van Es, 2011; van Es & Sherin, 2008). In parallel to the findings of this study, the results of some studies confirm that professional development experience increased the ability to notice students' mathematical thinking substantially (Ball, 1997; Ball & Cohen, 1999; Chamberlin, 2002, 2005; Goldsmith & Seago, 2011; Kazemi & Franke, 2004; Jacobs, Lamb, & Philipp, 2010; Jacobs, Lamb, Philipp, & Schappelle, 2011; van Es, 2011; van Es & Sherin, 2008). Correspondingly, investigations argue that teachers can learn a lot from thinking of the students in their own classes through professional development (Breyfogle & Herbal-Eisenmann, 2004; Burns, 2005; Chamberlin, 2005; Fernandez, Cannon, & Chokshi, 2003; Franke, Carpenter, Levi, & Fennema, 2001; Jacobs, Franke, Carpenter, Linda, & Battey, 2007).

Noticing includes two dimensions which are what to notice and how to notice. In terms of what to notice, focus of both İnci and Semih was on a range of issues from classroom management to participation of students but they generally paid attention to teacher pedagogy and partially students' mathematical approach in the early cycles of the lesson study. Later, their attention began to shift particularly on students' mathematical thinking and strategies rather than paying attention to insignificant aspects of the lesson. Their focuses were varied in Level 1, Level 2 and Level 3 during the lesson study, some shifts occurred from Level 1 to Level 2 and then Level 2 to Level 3 towards the end of the study. They began to make suggestions by considering possible student approaches, attend to the specifics of student thinking and referred to particular student responses. For example, İnci's and Semih's focus was on the issues such as teacher pedagogy, the behaviors of students' and the teacher, classroom enthusiasm, classroom management, students' responses, implementation of the lesson and students' mathematical thinking. However, the order, relation and intensity of them were variable. Although there was a gradual increase in both İnci's and Semih's noticing, the results showed that İnci began to focus more specifically on students' mathematical thinking compared to Semih. The changes in what Semih noticed were not as perceptible as İnci's. More than half of İnci's noticing in lesson study phases varied between Level 1 and Level 3 whereas more than half of Semih's noticing was at Level 2. In other words, it can be said that İnci focused on less important mathematical issues at the beginning of the study but she attended more to students' mathematical thinking lately in comparison with Semih. The predominance of Level 2 in Semih's noticing showed that he primarily focused on teacher pedagogy while attending to some details of students' strategies and adopted mostly this approach throughout the process. Since Semih had more teaching experience than İnci, his background might have been influential on his attention to teacher and students. Despite the fact that both preservice teachers attended to different points, their main focus was on teaching and student learning issues. They referred to the whole class while planning, teaching and reflecting.

As the cycles progressed, İnci and Semih began to pay attention to particular students' responses which they considered as significant and their mathematical thinking. Their focus shifted from irrelevant and mathematically insignificant issues to more essential aspects of the lesson. Tendency of İnci and Semih was akin to each other and showed a similar progress in terms of what they noticed. The difference between them was more in the teaching phase of the lesson study. İnci had a tendency to ignore the wrong answers and follow what she had in her mind during teaching. She generally considered students as a whole and focused chiefly on the correctness of students' responses earlier. Semih endeavored to focus on students' thinking as well as what he had intended and responded them in accordance with their answers in teaching. He focused on both particular students and the whole class. Researchers (Bardsley, 2006; Wickstrom, 2014) stated that teachers in professional development began to focus more on students' mathematical thinking and the details of their strategies over time instead of focusing on the correctness of the responses or unimportant issues.

The analysis of data also revealed that there were changes in how İnci and Semih noticed similar to the shift in what they noticed. In the early cycles of the lesson study, regarding how to notice, both İnci and Semih shared their ideas and offered their impressions in a broad scale. They indicated their personal judgements without elaborating sufficiently.

They referred to students' responses through direct quotations but did it in general ways. They did not try to interpret the reasons behind students' thinking, instead their comments were descriptive and evaluative in nature. They provided little or no details to support their claims. As opposed to low reasoning at the onset of study, some gradual progress from Level 1 to Level 3 took place over time in terms of how they noticed. They began to make sense of students' mathematical thinking more carefully with a dominant interpretive stance. They also started to make justifications about their suggestions and to give more details to support their ideas. For example, İnci offered such overall comments and sometimes, the reasons behind her suggestions or claims were not clear. She mainly shared her ideas based on the knowledge from the courses at university rather than explaining the meaning of her comments or linking them to students' mathematical thinking. Correspondingly, Semih was inconsistent in providing details to strengthen what he indicated. He occasionally endeavored to give justifications about his ideas yet he sometimes did not elaborate on his comments or observations. Both of them exhibited descriptive and evaluative approaches. They described the responses of students without interpretation in the early phases.

Although there was a similar shift about how İnci and Semih noticed, there were differences between them in terms of in which phases and to what extent these changes occurred. İnci showed more noticeable development about how she noticed. In other words, her approach was so general and did not provide details about her ideas to support them at the beginning but she tried to make sense of students' mathematical thinking further and use details to draw inferences about the students' understanding with an interpretive stance lately compared to Semih. Albeit there were some changes in how Semih noticed, this shift was not as great as İnci's. Semih began to reason on students' mathematical strategies with an interpretive stance yet he predominantly maintained inconsistency in his explanations and approach. Similar findings were obtained from İnci and Semih regarding how they noticed in planning and reflecting phases. However, in the teaching phase, İnci could not promote the students in a way that could reveal the details of their thinking earlier. Her approach was general and descriptive. She also evaluated the students' responses in terms of correctness. Semih endeavored to promote students to explain their answers to draw inference about their mathematical understanding. Even so, at times he was inconsistent in revealing students' mathematical thinking. Much as he continued to evaluate students' responses, his approach adopted an interpretive stance. In time, both İnci and Semih began to question the reasons behind students' strategies more often so that their approach resulted in interpretive responses at the least. Previous studies in mathematics teacher programs also reported development in noticing, from descriptive and evaluative approach to a more analytic and interpretive stance (Crespo, 2000; van Es & Sherin, 2002).

There was a shift from Level 1 to Level 3 but noticing of İnci and Semih was predominantly at Level 2 when overall lesson study cycles were considered. When the development of İnci and Semih was compared, it was observed that there were more changes in İnci's noticing of students' mathematical thinking. The evidence showed that İnci's noticing occurred at Level 2 and Level 3, which was very close to each other. In other words, İnci began to progress from focusing on teacher pedagogy and whole class mathematical approach to the details of particular students' mathematical thinking more successfully than Semih. Furthermore, İnci's comments included more details and justifications in comparison to Semih's responses. İnci's academic success was higher than Semih, accordingly this situation may have resulted from this difference. What these preservice teachers attended to and how they made sense changed gradually over time in a positive manner. Although their noticing levels did not change occasionally or went back among phases, all findings indicated an improvement in their noticing skills. Similarly, Osmanoğlu (2010) found that prospective teachers started to notice new aspects of classroom interactions and became more competent in what to notice and focus on over time during video-case-based discussions. This finding of the study can be considered as an evidence that preservice teachers' skills of noticing students' mathematical thinking can be developed through professional development models. Several studies in literature also support what has been claimed in this study (Fernández, Llinares, &Valls, 2011; Rodgers, 2002; Star, Lynch, & Perova, 2011, Star & Strickland, 2007; Walkoe, 2013).

In brief, some changes about what preservice teachers focused on and how they made sense of their own attention took place over time. There were similarities and differences between their noticing because although the same events are considered, they are likely to be different from what and how they were noticed (Jacobs et al., 2012). Liu (2014) also found that teachers focused on a range of issues as well as students' thinking and to what extent they noticed student thinking was respective. The findings of this study can be accepted as a supporting evidence for the gradual development in preservice teachers' noticing of students' mathematical thinking during the process of lesson study though there were variations in their noticing skills within and among cycles. Likewise, van Es and Sherin (2008) asserted that in-service teachers' noticing skills developed and they focused more on and made sense of students' mathematical thinking at the end of the video club sessions. Fernandez, Llinares and

Valls (2012) investigated the development in preservice teachers' noticing of students' mathematical thinking in an on-line context. The findings of the study showed that preservice teachers demonstrated difficulties in focusing on and interpreting students' mathematical thinking in a specific mathematics subject (addictive and multiplicative thinking) and they described mathematically insignificant aspects of the events. It was argued that some preservice teachers developed their noticing of students' mathematical thinking through interacting with each other in on-line discussions. In parallel to the findings of the present study, Baş (2013) stated that teachers' ability to notice student thinking developed with the support of a professional development model. Osmanoğlu (2010) also indicated there were significant changes in prospective teachers' ability to notice classroom interactions and mathematical content at the end of the study. In Coddington's study (2014) with teachers, it was seen that there were notable shifts in teachers' levels of noticing of students' mathematical thinking in professional development. Its results revealed that teachers went ahead from offering general comments about student thinking to indicating noteworthy events based on students' specific mathematical thinking and giving justifications about students' responses with evidence. It showed the importance of professional development for changes in noticing of students' thinking (Carpenter et al., 1999; Fennema et al., 1996; Jacobs et al., 2010).

Flake (2014) investigated the development of teachers' mathematical knowledge in teaching and their ability to notice a child's mathematical thinking over the course of a semester, in which they were involved in a mathematics methods course and a field experience in an elementary classroom. In contrast with the findings of the present study, it was found that there was no statistically significant change in their ability to notice the students' mathematical thinking. It was advocated that the course of a semester based only on working in an elementary classroom was not enough in terms of time and opportunity for preservice teachers to develop their noticing skills of students' thinking and they mostly utilized their prior knowledge to interpret students' responses (Sherin, 2011). At this point, it can be said that noticing is neither a skill which teachers have naturally or experience without professional support nor does it develop by itself (Sherin, Jacobs & Philipp, 2011). It is an important matter that is needed to focus on in order to present instruction of good quality (Choy, 2013) and provide the development of both teachers and preservice teachers (Hand 2012; Mason 2008, 2011; Santagata et al., 2007; Sherin et al., 2011; Star & Strickland, 2008; van Es 2011). Thus, there is need for appropriate environment and professional support to enable the improvement of preservice teachers' noticing skills. On the other hand, it was also realized that none of İnci's and Semih's noticing skills exhibited the features of Level 4 in the current study. It showed that these preservice teachers were not able to make connections between students' mathematical thinking and broad educational principles.

5.1.1 Preservice middle school mathematics teachers' noticing in planning phase

In planning phase, preservice teachers endeavored to prepare a common lesson plan including learning activities and questions, expected students' responses and determined teachers' responses. When noticing of İnci and Semih was considered during planning phases, they focused on a wide range of issues such as content, objectives of the lesson, time, richness in the activities and questions, use of materials, technological support, pedagogy and students' mathematical thinking and understanding. Their noticing varied among Level 1, Level 2 and Level 3 while planning the lesson. Both preservice teachers' noticing skills were low and at Level 1 initially in the planning phase. Diversity in their attention was high and their comments were general. They did not try to interpret ideas considering how students might think mathematically. Their main focus was to form a content in accordance with the purposes of the lesson by determining activities and questions. In the study of Star et al. (2011), it was detected that preservice teachers had challenges in focusing on the important mathematical details of the lesson tasks and mathematical content. Similarly, Fernandez, Llinares, and Valls (2012) argued that prospective secondary mathematics teachers were not able to link students' mathematical strategies to the problems. However, as the lesson study cycles continued, preservice teachers began to focus further on the concepts regarding student learning and pedagogy. Likewise, noticing skills of İnci and Semih progressed from Level 1 to Level 2 in the second lesson planning phase. They referred to what students' need to learn and what should be given in which way by considering pedagogical issues. They also tried to share what they knew about students' difficulties and mistakes and consider these points during planning. Nevertheless, their ability to prepare a plan and take instructional decisions that promoted students' thinking was limited.

Other than that, their approach was nonspecific and based mostly on theoretical knowledge that they had learned in undergraduate courses. They described what they knew about the subject and students' learning in relevant subjects and evaluated appropriateness of the activities or questions. Still, the relationship between what they knew about students and what they suggested for implementation of the lesson was not strong. They were inconsistent in supplying details about their claims. Thus, they offered general ideas as well as descriptive and evaluative comments with an attempt to elaborate. There was a difference between İnci's and Semih's noticing in the third lesson planning phase. İnci's noticing increased from Level

2 to Level 3 whereas Semih's noticing remained at Level 2. İnci began to offer comments considering students' possible mathematical thinking and explaining the details of their strategies. She provided justifications to support her ideas and associated her comments with the purpose of the lesson. However, Semih continued to focus on both teacher pedagogy as well as students' thinking. He did not refer to students' possible mathematical approach as İnci did. He was also inconsistent in elaborating his suggestions with evidence and made limited interpretive comments. The difference in their academic achievement may be the reason of İnci's progression to a higher level before Semih did. Late in, İnci's noticing remained at Level 3 and Semih's noticing progressed from Level 2 to Level 3 in the fourth planning phase. Level 4 necessitates establishing connection between students' mathematical thinking and broader instructional principles. Due to the fact that this ability is difficult and based on rich teaching experiences, both preservice teachers might not proceed to this level. They further began to focus on students' mathematical thinking and reason on their possible strategies. They endeavored to consider students' expected mistakes, difficulties and confusions while determining what should be included in the content of the plan in accordance with the objectives of the lesson. They promoted the group members to attend to what kind of answers students might give and how they should respond them in order to diminish their confusions and difficulties and provide opportunities for students to articulate their reasoning. They began to offer more details about their ideas and gave justifications about their comments. They were more detailed and specific in identifying students' mathematical thinking. They tried to make sense of specific students' thinking and link their suggestions with students' reasoning through an interpretive stance.

Zembal- Saul, Star and Krajicik (1999) indicated that one of the important struggles of preservice teachers was not being able to estimate students' prior knowledge on the subject. Karal-Eyüboğlu (2011) also found that preservice teachers were inadequate in estimating what students already knew and that is why in planning and teaching, they included information and concepts that the students had not learned before. In this study, both İnci and Semih used some comments like "Do students know this concept?", "Should we teach it?", "Is it difficult for them?", "Can they think like that? and "Did they learn it before?". These were the indicators that they were not sure about some of their knowledge related to students' prior knowledge but they tried to consider students' mathematical thinking while planning the lesson. Thus, it can be said that lesson study environment helped preservice teachers to learn from each other about their lack of students' prior knowledge and to notice students' mathematical thinking.

Planning is one crucial part of lesson study in which participants have the opportunity to strengthen and increase their mathematical understanding and knowledge on student thinking (Takahashi et al., 2005). Focusing on students' mathematical thinking during planning a lesson is important in terms of improving instruction and students' achievement (Lampert, 2001; Lewis & Tsuchida, 1998; Stigler & Hiebert, 1999). Hughes (2006) explored how preservice secondary mathematics teachers' attention to students' mathematical thinking in the process of lesson planning improved over time in a course at university. The results of the study indicated that preservice teachers improved their ability to focus on students' mathematical thinking significantly when planning a lesson during the course of teaching lab. Hughes (2006) also stated that there was a relationship between preservice teachers' attention to students' mathematical thinking and high-level tasks which they prepared towards students' thinking and reasoning mathematically. High-level tasks require to anticipate student thinking and determine appropriate responses as planning (Henningsen & Stein, 1997). In other words, their increasing attention to students' mathematical thinking helped to determine the content of the lesson plan in a way that could reveal students' thinking and develop conceptual understanding (Lampert, 2001; Lewis & Tsuchida, 1998; Stigler & Hiebert, 1999). The results of the study supported the findings which were obtained from the present study in terms of development of preservice teachers' focus on students' mathematical thinking over time in planning.

In contrast to the finding from the late cycles of this study, Mapolelo (1999) advocated that preservice teachers were not generally good at preparing a lesson plan according to students' difficulties and errors and generating a lesson content which assists to reveal students' thinking. In this study, it was seen that both İnci and Semih began to pay attention to plan the lesson considering students' mathematical thinking over time. Likewise, in the study of Mostofo (2013) which investigated the influence of lesson study on instructional planning of preservice secondary mathematics teachers, it was found that preservice teachers' ability to estimate students' misconceptions and responses were improved and lesson study influenced their planning positively. The other researches on lesson study were also congruent with these results (Hiebert et al., 2007; Mathews et. al., 2009; Sims & Walsh, 2008).

During the lesson study, İnci and Semih gained better understanding of the subjects as they made investigations about them and it affected their planning. The increase in their noticing levels and their own comments supported this result as well. İnci's comments such as "There is something in the book similar to what I indicated" and "Beforehand we had read what kind of misconceptions and difficulties related to the subject students might have so that we could estimate more or less" showed that she utilized the sources during planning and she also found investigation related to mathematical subjects useful. Correspondingly, Semih made comments like that "In one book, it is also offered to start from the length of line segments" and "We researched from many sources not only one book. Maybe, we can miss an important point or give an inappropriate example, thus, benefitting from the books is essential". These expressions revealed that Semih also thought that reading about mathematical subjects contributed to his knowledge about students' thinking and understanding and facilitated planning. Both İnci's and Semih's approach showed that they made suggestions or indicated their ideas based on what they learned from the investigations that they had made from various books before starting planning. Paying attention to the sources of students' confusions and making sense of their difficulties about the subject is crucial in giving instructional decisions and designing the tasks throughout lesson study cycles (Choy, 2015).

In planning, İnci shared her general ideas based on her previous knowledge on teaching and learning, suggestions of the cooperating teacher or the content of some books. However, with the increasing consideration of students' mathematical thinking, she endeavored to reason about students' possible responses, confusions and difficulties, and the ways to learn them. For instance, she began to ask the other group members how students might think and how to respond them when a task is suggested to be included in the lesson plan. Likewise, while Semih was offering basic comments related to the implementation of the lesson at the beginning, he began to consider the details of students' mathematical strategies and their reasons. Therefore, he tried to suggest activities or questions which would guide students to explain their thinking later. For instance, he began to emphasize the reasons behind students' answers in order to correct their failing mathematical approach and to offer some tasks in this direction. When these preservice teachers canalized their focus on mathematically significant aspects of the lesson and details of students' thinking throughout planning, the lesson plan became more supportive in revealing students' thinking and responding them in accordance with their answers during the teaching phase of the lesson cycle. In other words, what preservice teachers focused on and how they made sense of them influenced their perspectives in teaching and reflecting phases. Similarly, Choy (2015) found that teachers' attention during planning enabled them to respond to students' thinking during the lesson. Harle (2009) indicated that the development in making sense of students' thinking was a major factor which promoted teachers to make changes in their teaching. Some studies emphasized that it was necessary to apprehend the difficulties and mistakes of students and take measures considering these details for good mathematics instruction (Fennema & Franke, 1992; Shulman, 1986; Park & Oliver, 2008). At this point, it can be said that lesson study setting enriched the lesson plans, allowed to consider students' mathematical thinking and impacted the design of the lesson positively. Also in his study, Wagner (2003) ascertained that during the lesson study the focus of preservice teachers shifted to students' learning rather than the behaviors of teachers. Lesson study changed their perception about the worthiness of students' mathematical thinking and preservice teachers began to anticipate and use students' thinking while planning the lesson and to see focusing on students' solution strategies in teaching as important factors.

Likewise, some research advocated that professional development programs which were based on the principle of investigation and interpretation of students' thinking enabled the improvement of noticing by gaining different perspectives (Carpenter, Fennema, & Franke, 1996; Franke, Carpenter et al., 2001; Sherin et al., 2011; Sowder, 2007) and increased the ability to notice students' mathematical thinking (Ball, 1997; Ball & Cohen, 1999; Chamberlin, 2002, 2005; Goldsmith & Seago, 2011; Kazemi & Franke, 2004; van Es, 2011; van Es & Sherin, 2008).

5.1.2 Preservice middle school mathematics teachers' noticing in teaching and re-teaching phase

In teaching and re-teaching phases, İnci's and Semih's noticing varied between Level 1 and Level 2. When all of the teaching processes were considered, their noticing of students' mathematical thinking was low. İnci's noticing was at Level 1 in the first implementation and it increased to Level 2 in the second implementation whereas Semih's noticing was at Level 2 in both two implementations. For their own first implementations, the reason behind Semih's higher noticing in comparison with İnci might be that Semih had teaching experience in a private tutoring institution for a semester and gave private lessons to many students differently from İnci. Furthermore, İnci made her first implementation of the lesson in the first cycle whereas Semih made his first implementation in the second cycle. Until his teaching, Semih had the chance to observe two lesson implementations. These may have given him ideas about what he should pay attention to. On the other hand, both İnci and Semih made mistakes in some of the activities during the lesson or implemented some steps incompletely. Therefore, they faced some challenges in achieving the purposes of the lesson. It is likely that such situations arise from the inexperience of preservice teachers and their noticing is not as good as experienced teachers. Similarly, some studies argued that inexperienced teachers and preservice teachers had difficulty in shaping the lesson by focusing on students and noteworthy events (Carter et al., 1988). Vondrová and Žalská (2013) also argued that preservice teachers had difficulty in noticing mathematical details. The reason behind the low noticing of İnci and Semih while teaching in comparison with the planning and reflecting phases may be that preservice teachers worked collaboratively during these phases but they taught the lessons individually. Thus, a change in noticing through individual attempts may take longer without a rich interacting environment of collaboration. At this point, it can be said that the deficiencies in preservice teachers' individual noticing decrease and their noticing enriches when they discourse together and share their ideas.

Other than that, İnci and Semih needed to engage in a lot of things at the same time such as following a lesson plan, interacting with students, conducting activities and maintaining classroom management during teaching. These actions may be difficult for them owing to their lack of experience. Although Semih had more teaching experience than İnci, both preservice teachers conducted limited implementations in front of the whole class. Teaching is a challenging activity and noticing students' thinking during the implementation is difficult even for teachers. Therefore, expecting a quick development of their ability to notice students' thinking during teaching cannot be realistic. Moreover, there are so many interactions which happen at any given issue and time in class that they may prevent preservice teachers from focusing on students' thinking (Pilitsis, 2014). Also, their attention is divided into many diverse points while engaging in these actions. In other words, it is needed to pay attention to many things in the complex structure of a classroom with various distractions (Jacobs et al., 2010). Pilitsis (2014) indicates that development of preservice teachers' noticing while teaching is difficult "due to the cognitive load involved in attending to the messy contexts of real classroom interactions". In the present study, Semih's comments also showed that he had difficulty in teaching due to many actions that he needed to follow (e.g. "I noticed that while observing the lesson, I was more comfortable but while teaching, I had to follow the lesson plan in general terms, ask questions and engage with the students. Thus, losing attention was normal because of having many responsibilities at the same time. I lost my attention too"). Therefore, the reason for preservice teachers' low noticing in teaching phases may be these obstacles.

At Level 1, preservice teacher, İnci, was in tendency to ignore the wrong answers and and she did not respond or promote students in a fashion that could reveal their mathematical thinking. Her main focus was on maintaining what was intended in the lesson plan rather than listening to the students. Classroom discourse was further based on descriptive and evaluative comments in nature. At Level 2, focus of İnci and Semih was on both students' mathematical thinking and what they had in their minds. They sometimes listened to the students' answers yet they sometimes rarely indicated what they intended straightway. Their attention was on particular students as well as the whole class. They were inconsistent in revealing the details of students' mathematical thinking and responding them in such a way. Their approach was mostly evaluative but they also attempted to interpret students' understanding. They sometimes missed the opportunities to correct students' mistakes or did it in a broadscale. Similarly, the study of Mapelelo (1999) which took place with the participation of mathematics preservice teachers revealed that they could not consider students' answers adequately or give enough feedbacks and clues. Haydar, Vatuk and Angulo (2009) found that novice teachers had a negative attitude in responding to students' wrong answers and making dialogs with students in a manner that reveals their thinking. It was also observed that they were not willing to allow the students to correct their own mistakes and they were in tendency to indicate the correct answer directly. In the study of Wagner (2003) the results showed that preservice teachers focused on students' thinking except they did not respond to it. They attended to the errors and endeavored to make sense of them, however, they did not provide opportunities for students to make their errors clear and reveal their thinking. Instead, they tried to understand the errors by themselves. It was argued that preservice teachers identified common students' errors but they did not integrate this knowledge into teaching in a way that could promote students' thinking.

A shift in İnci's noticing occurred during the teaching phase and it increased from Level 1 to Level 2. In planning, preservice teachers tried to prepare lesson plans in the best way they could. İnci participated actively in planning, shared her ideas and listened to the others' opinions. She also observed the lessons and reflected her ideas on them with the others. Interactions in this process, exchanges of ideas about students' mathematical thinking and preparation for the lesson beforehand might be the explanations of this shift. Moreover, her comments such as "In this process, different ideas were presented by the other members and they effected the lesson plan positively. If I had prepared it alone, it would not have been as rich as the current plan" and "I am looking at the class deeper while observing. We are learning how to react and deal with the students" supported that the steps in the lesson study might have effects on the development of İnci's teaching. Similarly, teachers in U.S. and Japan also indicate that lesson study helps teachers to develop their teaching because it prompts them to focus on students' thinking and decide how to manage the lesson using this knowledge (North West Teacher, 2001, 2003; Research for Better Schools, 2002). Mostofo (2013) spotted in his research that preservice teachers' focus of their teaching shifted away from themselves to the students through the lesson study. Carter (2004) investigated the effects of lesson study on novice teachers' practice. Accordingly, a shift in their teaching in terms of eliciting more student thinking in the classroom was detected through lesson study. However, the findings of the present study also revealed that Semih's noticing remained in the same level during the teaching phase although inci's noticing developed. The extent to which teachers notice student thinking during instruction might vary (Sherin et al., 2008). The reason of it might be the difference between the subjects that İnci and Semih taught. İnci taught perimeter, conservation of area and repeated perimeter and area by using some concrete materials in her lessons.

On the other hand, Semih told the concepts of area, surface area and rectangular prism in his lessons. He intensely used both concrete materials and computer programs to facilitate the understanding of prisms. Students have more difficulty in prisms rather than the concepts of perimeter and area since prisms are three dimensional. Therefore, the difficulty and the intensity of the subject might have influence on Semih' noticing. Moreover, it is thought that this result might have stemmed from the limitation of the present study. This study entailed only two implementation opportunities for each preservice teacher that is why it is a fairly short time to expect change. It is thought that if preservice teachers are provided with more opportunities for teaching, their noticing skills will increase as the shifts in their noticing were more explicit during the planning and reflecting phases which had more intensity. The findings from teaching phases can be interpreted like that the reflection of change in preservice teachers' noticing on their teaching experiences might take time. Considering even teachers have difficulty in incorporating students' thinking into the practice (Ball, 2001; Ball, Lubienski & Mewborn, 2001; Lampert, 2001; Schifter, 2001; Schoenfeld, 1998; Sherin, 2002), it is normal to take time for preservice teachers too.

On the other hand, it can be argued that noticing of İnci and Semih in planning and reflecting phases have influence on their perspectives in teaching. For example, İnci's noticing in planning and reflecting phases was at Level 1 and her noticing exhibited the features of Level 1 in the teaching phase of lesson cycle 1. As lesson cycles progressed, her noticing increased to Level 2 and Level 3 and her noticing had the properties of Level 2 in the teaching phase of the lesson cycle 3. There was a consistency or close relationship between İnci's noticing levels in both planning and teaching. She attended to the issues which were mathematically less significant and she mainly made descriptive and evaluative comments in the first planning phase. Also, her main focus was on doing what she had intended rather than students' mathematical thinking and she evaluated students' responses in terms of correctness without promoting explanation for their thinking in teaching of the first cycle. Furthermore, she predominantly focused on students' possible mathematical thinking and she tried to make sense of students' strategies with an interpretive stance in the third lesson planning. Similarly, she attended to students' thinking and listened to their responses. However, in teaching of the third cycle, she sometimes maintained what she had intended with mostly evaluative approach. Although her noticing levels were not the same for planning and teaching in this cycle, they were related to each other.

Likewise, Semih's noticing in planning and reflecting phases varied predominantly between Level 2 and Level 3 and his noticing in teaching phases reflected the characteristics of Level 2. Semih basically focused on pedagogical issues and tried to consider students' mathematical thinking and understanding. He also provided limited justification and elaboration about his comments in the second planning. In parallel, he focused on what he had in his mind and also the students' answers. His responses that could reveal students' thinking were inconsistent and he sometimes preferred to tell them what to do directly. Moreover, Semih predominantly focused on particular students' mathematical thinking, their specific confusions and difficulties with elaborative approach in an interpretive stance in the fourth lesson planning rather than less mathematical aspects of the lesson. Similarly, he focused on particular students' mathematical thinking and listened to their responses to make inferences about their understanding. Nonetheless, he sometimes preferred to apply the mode of telling from time to time and asked general questions which were not sufficient enough to reveal students' thinking. His noticing in planning and teaching during the two cycles was the same or close to each other. Besides, İnci's comment that "It was an expected answer for us but I asked the reason so as to understand their thinking" or Semih's comments such as "When I heard the answers which we expected, I noticed that we substantially could estimate and consider students' thinking correctly while planning" and "it did not make me surprised because we expected this confusion. I paid attention to the use of measurement units and generalization of area formulas" showed that their noticing in planning allowed to shape their lessons. Likewise, Choy (2015) advocated that what and how teachers noticed during planning affected how they responded to the students in class. Besides, focusing on and listening to students' reasoning enables them to interact with students by promoting their mathematical thinking (Franke et al., 2009; Isoda & Katagiri, 2012; Smith & Stein, 2011).

In planning, İnci had difficulty in deciding whether a mathematical concept, relation or representation would be appropriate for students' cognitive levels. She also had lack of knowledge about how mathematical subjects were related. The findings showed that she began to make up her deficiencies with the help of preparation for the subjects and sharing with her group friends. Her comments in planning and reflecting phases such as "If the students say that area does not change, what will we do?", "We tried to give representations which students do not encounter too much in order to make them reason and reveal their misconceptions" and "I also noticed that some suggestions were not appropriate for students' level", and her approach to students during teaching, for example "Why doesn't it change?" and "What do you mean?" pointed that she also gained a tendency to take decisions by considering students' thinking and her tendency in planning influenced her teaching.

In planning, Semih attended more to the implementation of the lesson, the order of the content and pedagogical issues such as drawing attention, reinforcement and materials. He had difficulty in lowering to the students' levels through thinking like them because he gave private mathematics lessons to students who were upper than 5^{th} grade. His statements such as "Geometry is a difficult mathematical domain and I also had difficulty designing the lesson" and "age group is younger so it was complicating" also supported that planning the lesson considering students' mathematical strategies was not easy for him. However, his comments in planning and reflecting phases such as "According to you, how do students think?" or "what can they say?" and "I noticed that there was need for supporting students' mathematical understanding by querying what and why they did that" and also his responses to the students in teaching saying "Why did you say it like that?" and "What do you think?" showed that he began to consider students' thinking over time throughout the lesson study. Thus, it is thought that as preservice teachers' noticing of students' mathematical thinking during planning increases, their noticing of significant mathematical details in classroom interactions during teaching will change positively. Planning phase includes focusing on students' responses, anticipating their thinking and incorporating their perspectives into the lesson for better instruction, hence, lesson study leads teachers to increase the contributions of students within the lesson (Fernandez, Chokshi & Cannon, 2003).

On the other hand, Yeşildere and Akkoç (2010) examined preservice teachers' microteachings and the activities that they had prepared related to different representations and rules of patterns. It was found that although the preservice teachers had prepared a good lesson in accordance with the instructional schedule, they could not use it during the teaching process. Similar findings were obtained in the study of Baki (2012) and it was revealed that preservice teachers had some problems in implementing the activities that they had determined in planning appropriately. Mostofo (2013) indicated that much as planning of the lesson was based on collaboration, its teaching was done on an individual basis. Accordingly, he defended that some minor adjustments on the lesson plan or various implementations of the same lesson were carried out by preservice teachers during teaching in lesson study. At this point, Stylianides and Stylianides (2007) indicated that although mathematical activities had an important role in students' learning, the activity which was selected by teachers might not warrant good teaching. In this study, it was also observed that İnci and Semih could not implement some activities exactly as they had aimed or in accordance with their purposes. They had struggles at some points and made mistakes though they had discoursed on them deeply in the planning phase. For example, minor errors occurred while measuring the length of the rope to find the perimeter of a rectangle in İnci's first teaching. Correspondingly, Semih forgot to cover the figures on with colorful cardboard pieces, so the activity did not reach its aim in terms of providing estimation and students had some confusions related to perimeter and area. Besides, in an activity related to area, he should not have wanted students to cover the figure with unit squares but he forgot it that is why he lost time there. Thus, this situation reveals their need for more opportunities in teaching experience through teacher education programs.

5.1.3 Preservice middle school mathematics teachers' noticing in reflecting and re-reflecting phase

The findings revealed that İnci's and Semih's noticing in reflecting and re-reflecting phases varied between Level 1 and Level 2 early on in this study (in the first and second lesson study cycles), in particular, their noticing skills were predominantly in Level 2. Preservice teachers' focus was on a wide variety of issues such as classroom management, teacher pedagogy, behavior of students and the teacher, classroom environment and students' understanding. They generally attended to students' answers but they did not try to make sense of them. They described what they observed and evaluated the teacher's behaviors in terms of suitability or students' responses in terms of correctness. Their focus was on the whole class rather than particular students. They were inconsistent in providing details about their claims. Besides, their comments were mostly general. Although both İnci and Semih demonstrated similar noticing skills, what they focused on and how they made sense of them was different at some points. Albeit İnci focused on a range of issues, she tended to evaluate the performance of the other group members and her own implementations based on their consistency with the lesson plan. Thus she focused on the teacher mostly and reflected what the teacher could do or could not do during teaching. According to Osmanoğlu (2010), the reason of preservice teachers' focus on teacher roles might be that they start to make assumptions on the role of a teacher and see themselves as teachers rather than students. İnci's comments supported that she showed regard to the lesson plan (e.g. "I paid attention to implement what we had planned correctly and without skipping any points"). She made comments related to teachers' classroom management, use of time, expressions and behaviors. She usually described the events without the attempt to sense-making. She also made some suggestions toward the content of the lesson such as asking more difficult problems and using small numbers. On the other hand, Semih attended to students' answers and behaviors more than İnci did. Although he referred to what students said or did, he chiefly evaluated their answers in terms of correctness. His comments supported that he paid attention to students in a broad manner (e.g. "I focused on what kind of answers were given by the students and whether they gave the responses that we had expected"). He also focused on the issues such as materials, teacher pedagogy, students' willingness and made some suggestions related to implementation like calling different students, making them active and giving prizes to reinforce them.

The findings showed that noticing of İnci and Semih varied between Level 2 and Level 3 later on in this study (in the third and fourth lesson study cycles), in particular, their noticing skills were predominantly in Level 3. They not only endeavored to focus on the details of students' responses but also paid attention to establish a connection between their responses and the subject towards the later phases of reflecting. They tried to understand whether the lesson plan content served the purpose of the lesson in terms of students' confusions and difficulties by making an association between what they observed and their knowledge about the subject. They noticed some gaps in students' understanding and how these gaps would cause confusion as the mathematical subjects progressed. This approach enabled these preservice teachers to gain insight into students' thinking about the subject. Except for the third re-reflecting phase, İnci's noticing was at Level 3 in reflecting and re-reflecting phases of the third and fourth cycles. Interestingly, although İnci's primary focus was on the teacher and implementation of the lesson in the early cycles, she began to pay less attention to what teacher did or not. Rather, she attended to particular students and what kind of mathematical strategies they used. Her own comment that "I focused on what the reactions, answers and confusions of the students were instead of the implementation of the teacher" also supported the finding related to the shift in her noticing. İnci also addressed students' thinking deeply in most of her comments and made interpretations about the reasons of their solution ways. She used some comments as below:

> "I focused on whether the students understood the subject or not and their answers so that I could maintain to shape the lesson according to their feedbacks. I endeavored to emphasize the important points that we had discoursed on while planning the lesson. I controlled whether the students could solve the problems, where and why they made mistakes"

Her reflection comments showed that she began to attend to students' responses and make sense of their mistakes. It was also understood that her noticing of students' thinking enabled her to shape her lesson. Similarly, Semih generally focused on students' confusions, mistakes and difficulties and he checked whether expected answers were given by the students or whether there were unexpected answers. Except the last re-reflecting phase, his noticing was at Level 3 in reflecting and re-reflecting phases of the third and fourth cycles. He sometimes attended to teacher pedagogy but his focus was mostly on students. He also tried to give justifications about students' challenges and to make sense of their mathematical understanding. He referred to the details of their approach by representing evidence from his own observations. His comments like "One student's answer drew my attention" and "That student might sum only the edges which he saw the numerical values on" supported the findings about his noticing. Both İnci's and Semih's noticing skills were at Level 3 in the reflecting phases of the third and fourth cycles. However, İnci's noticing increased from level 2 to level 3 whereas Semih's noticing went back from Level 3 to Level 2 in re-reflecting phases of the last two cycles. As indicated before, Semih taught rectangular prism and surface area in his second implementation and the re-reflecting phase, in which he went back to Level 2, included discussions on this lesson. Since he taught a complicated and content-intensive subject, his low noticing in the teaching phase might take its effect in the re-reflecting phase in a similar way.

When teachers interpret students' mistakes, they can take measures to remove the mistakes that they observed (Choy, 2015). In this study, both İnci and Semih endeavored to make suggestions related to students' confusions and mistakes to prevent misunderstanding. Preservice teachers' observations during the teaching phase helped to present evidence and justifications about their suggestions in reflecting. They tried to make comments based on the students' responses and reactions that they noticed. Paying attention to consider and promote students' thinking in classroom interactions influences teachers' approach and the goals of the lesson positively (Franke et al., 2007; Jacobs et al., 2010). In this study, it was observed that both preservice teachers' focus and reasoning about students' knowledge and cognitive difficulties had an effect on their approach in planning, teaching and reflecting. Similarly, according to the findings of Choy's (2015) study, teachers' noticing of students' thinking enabled them to prepare tasks which targeted students' confusions and the obstacles in planning, to respond to students more effectively during teaching and to discourse more mathematical aspects of classroom interactions in reflecting. Wagner (2003) indicated that although preservice teachers had difficulty in incorporating students' thinking into instruction, their comments in reflecting revealed that they were attending to students' thinking. It was also realized that preservice teachers' focus on student thinking was crucial to develop their reflective inquiry. The support of the others and reflecting were seen as important factors for transferring learning into practice (Bardsley, 2006) and for professional development during teacher preparation (Cochran, et al, 2009; Hiebert, Morris, Berk, & Jansen, 2007). Similarly, it was advocated that the repetition and reflection process was crucial in noticing students' thinking in detail (Wickstrom, 2014). The results of many researches argued that lesson study empowered development of teachers' reflections related to students' thinking (Byrum et al., 2002; Fernandez et al., 2003; Grove, 2011; Sims & Walsh, 2009). On the other hand, it can be said that lesson study professional development model provided a context for İnci and Semih to anticipate students' thinking while designing the content of the lesson plan, and helped them learn from their own and friends' teaching by observing and enabled to improve the lesson by making more connection between students' responses, the goals of the lesson and reflections of the group members.

In the reflecting phase, it was found that preservice teachers could influence and change each other's thinking through sharing their ideas and discussing together (e.g. giving up one preservice teacher's suggestion and adopting the other's idea). Besides, it was observed that what preservice teachers focused on and how they made sense of what they focused might be the same or different from each other. Thus, the environment in which each preservice teacher discussed on their opinions and suggestions in the reflecting phase contributed to preservice teachers' noticing, especially at times they could not notice some aspects, through providing them to learn from the others. For example, İnci suggested not to use isometric paper in a question which aimed to show that different figures might have the same perimeter although the cooperating teacher offered to use it after the first implementation. İnci thought that if the figures were drawn on isometric paper, students would count the units and see that they all had the same perimeter directly. She believed that the students would only focus on calculation and overlook the key point. Her comments persuaded the other members in the group and they decided not to use isometric paper to make the question more effective. Furthermore, İnci indicated that she could not understand one student's mathematical strategy while teaching the lesson in the reflecting phase. Another preservice teacher from the group explained what he observed and the reasons behind the student's solution. Therefore, this interaction helped İnci to learn the details of this student's mathematical thinking. Likewise, in the re-reflecting phase, Semih indicated that he could not make sense of why one student said a particular kind of mathematical expression while observing the lesson. The other group members were not sure about it either but all of them reasoned on the student's possible mathematical thinking together and talked about how to remove this mistake. Besides, Semih highlighted the importance of reflecting on his behalf with such a statement "I also noticed that I missed some important details or did not think they were important yet the other members indicated them or completed my deficiencies". Semih's comment also showed that he learned what to focus on and how to make sense of the events through interacting with the other members. It can be said that reflecting and collaborating had an impact on his noticing. Likewise, other studies emphasized the importance of focusing on and making sense of teachers' comments in reflecting (Barnhart & van Es, 2015; Berliner, 2001). At this point, it can be said that especially the collaborative structure of lesson study makes significant contributions both to prepare a better lesson content and to increase noticing. Various studies in literature stress that such professional development models have positive effects on noticing skills (Franke, Carpenter vd., 2001; Lewis vd., 2011) and there is need to create such environments (Runesson & Martensson, 2013; Yang & Ricks, 2012).

At the beginning of the study it was expected to be challenging for preservice teachers to focus on mathematically significant issues and make sense of them. It was also assumed for preservice teachers to notice most of the details and the meanings of students' mathematical thinking at the end of the study. As presumed, the results of this study revealed that preservice teachers began to focus on students' responses which were worthy of consideration and mathematically significant. They also endeavored to make sense of students' mathematical thinking, reason on their strategies and make justifications about their comments throughout the lesson study process.

5.2 The Important Features of Lesson Study that Influence Noticing

Data analysis reveals that professional development support with lesson study can provide some shifts in preservice teachers' noticing of students' mathematical thinking positively. The scantiness of attention to and sense-making of students' thinking at the beginning of the study confirmed that preservice teachers needed to learn how to use this kind of thinking when planning, teaching and revising the lesson. Similarly, Sisofo (2010) emphasizes this need for preservice teachers because there is a relationship between mathematics instruction of good quality and consideration of students' thinking. The data of this study showed that İnci and Semih began to focus on students' mathematical thinking more often toward the end of the lesson study process. Furthermore, the levels of the details and the depth of the analyses that they used in their explanations, justifications and suggestions increased. Correspondingly, in the study of Corcoran (2008) including lesson study process with preservice teachers, it was seen that preservice teachers' self-confidence and sense-making about how students think mathematically increased. In parallel, Wagner (2003) conducted lesson study implementations and found that preservice teachers began to draw each other's attention more to students' thinking in this process. Harle (2008) indicated that teachers valued the incorporation of students' thinking in planning, teaching and reflecting of mathematics lessons. Similarly, "Participants' beliefs and knowledge about teaching and learning shifted toward a perspective grounded in children's mathematical thinking" (Carpenter et al., 1989, p. 119).

The focus of lesson study is to anticipate and build on students' mathematical thinking and to enable preservice teachers to use students' thinking as the basis for instruction (Wagner, 2003). "We suspect that making student thinking visible affects the types of exercises and activities teachers incorporate in the lesson. It is challenging to design ways to make student thinking visible that are also pedagogically purposeful" (Murata & Takahashi, 2002, p. 1881). Lesson study creates a common area in which preservice teachers share and talk about students, their strengths and weaknesses, and which misconceptions they may have as learning a specific mathematical topic in order to prepare a lesson towards learning needs of the students (Lewis, 2002; Wagner, 2003). Teaching and observing the same lesson provides to acquire information on and gain insights into students' mathematical thinking processes (Lewis, 2002). Furthermore, Watanabe (2001) defends that one possible greatest contribution to the success of lesson study is made through anticipation of students' thinking. "Teachers learn best by studying, doing and reflecting; by collaborating with other teachers; by closely looking at students and their work; and by sharing what they see" (Darling-Hammond, 2003, p. 278).

The cases of the current study enabled preservice teachers to acquire new viewpoints on students' mathematical thinking as they participated in lesson study process through interacting with each other consistently. They had a chance to observe the other members' instructions based on the lesson plan that they all had prepared together, to see different classroom interactions and to discuss them together. They adopted lesson study with collaborative working and exchange of the ideas so that they motivated each other to achieve the goals in the process. Through discourse on critical aspects of how to teach, noteworthy events in classroom instructions and the patterns to improve the lesson; considering and reasoning on students' mathematical thinking, preservice teachers might learn how to create an effective teaching environment for better student learning and achievement (Borko et al.2008). They might also expand their experience about the complexity of real classroom instruction (Butler et al., 2006) through interpreting the events that occurred in a classroom setting (Lundeberg et al., 1999; Shulman J., 1992). Therefore, preservice teachers are expected to develop their noticing skills and attain new and various perspectives depending on the richness of opportunities in lesson study.

Lesson study draws its strength from anticipation of students' thinking, understanding and reactions (Fernandez et al., 2003; Lewis, 2000). Therefore, it gives teachers "the vision to see children" (Lewis, 2000, p. 14) and the perspective to focus on students rather than the teacher (Lewis, 2000). Fernandez (2003) also found that lesson study helped teachers to focus on students' thinking, interpret them and decide how to respond in the face of unexpected student reactions and events while teaching. One reason for the positive shift in preservice teachers' noticing of students' mathematical thinking is quite likely this strong perspective in lesson study. Moreover, various factors may have a role in the changes in İnci's and Semih's noticing of students' mathematical thinking. According to the data analysis carried out within this study, these preservice teachers' discourses referred particularly to five features of lesson study which were collaboration, investigation on mathematical subject, observation and reflecting, specific mathematical domain and cooperating teacher.

5.2.1 Collaboration

Preservice teachers worked collaboratively at almost every step of the lesson study. After investigation of mathematical concepts, they endeavored to plan the lesson by sharing their ideas and learning from each other. They interfered in each other's opinions, improved the suggestions and offered various ideas. Although they taught the lesson individually, the other preservice teachers who observed the lesson reflected what they noticed and transferred the contribution of group work to the next phase. Preservice teachers made various comments about what they knew regarding students' confusions, difficulties and mistakes and they anticipated students' mathematical thinking in the lesson study process. Furthermore, they listened to each other's comments related to students' thinking carefully, discussed on them, learned what and how the other group members think, gained different perspectives and completed the lack of knowledge. The mutual exchange of their ideas influenced their noticing in a positive way. It also provided a more effective process and supported their professional development (Chamberlin, 2002, 2005) through the support to focus on and make sense of students' thinking (Baş, 2013).

In this study, both İnci and Semih indicated that if they had planned the lessons on their own, they would not be as good as they are now. They emphasized the importance of working together and the contributions of the other group members to them at every turn. Therefore, it is thought that collaborative work affected preservice teachers' noticing of students' mathematical thinking positively. The findings of other studies also supported the benefits of collaboration to the development of teachers' noticing of students' mathematical thinking (Baş, 2013; Chamberlin, 2004, 2005; Kazemi & Franke, 2004; Osmanoğlu, 2010). An environment based on cooperating enables teachers to construct and reconstitute knowledge and make sense of how to use it in order to overcome difficulties (Prince, 2016). In this study, preservice teachers constructed their knowledge of students' thinking related to perimeter, area, surface area and prism with the help of sharing what they had learnt from recourses during investigation and learning from each other's ideas regularly (Lewis, Perry & Hurd, 2004). In literature, several studies emphasized the importance of interacting with each other in promoting teachers to notice more about the events in the classroom and analyze them (Sowder, 2007; van Es & Sherin, 2002, 2008). For example, İnci indicated her lack of knowledge about the subject and stressed that she learned it from other group members and she found working together effective (e.g. "I noticed that I did not know some things and we tried to complete our own deficiencies. I can say that we are better in planning, and discussing together influences this process positively"). Similarly, she pointed to the contribution of collaboration for gaining different perspectives with such a comment "Moving edges did not come to my mind while telling the perimeter formula of a rectangle and square. This approach attracted my attention when my friend explained it". Likewise, Semih underlined that collaborative work helped him to understand students' thinking and their viewpoints (e.g. "Different ideas arose. Working collaboratively was good. I noticed that if I missed something or said something wrong, they completed the deficiencies and vice versa. It helped us to think further as if we were students"). According to the findings of the present study, both İnci and Semih began to focus more on student thinking and mathematically significant aspects of the instructions and make sense of these components. They had an opportunity to see and interpret these issues with the help of different perspectives which were enhanced thanks to the interaction and collaboration among themselves (Yadav et al., 2007). They became more open to alternative ideas and enhanced their confidence in their lessons with collaborative working (Carrier, 2011; Chassels & Melville, 2009; Ganesh & Matteson, 2010; Mostofo, 2013).

Collaboration promotes productive discussion and exchange of ideas (Thompson & Zeuli, 1999). The shared experiences as a group provides important opportunities to support preservice teachers' learning (Putnam & Borko, 2000). This collaboration prevents the lack of knowledge that comes with teacher isolation and compared to working alone, teachers become more competent (Cossey & Tucher, 2005). Thus, preservice teachers endeavored to use this knowledge while, planning, teaching and reflecting the lesson, in particular, towards the end of the lesson study process. West-Olatunji, Behar-Horenstein, and Rant (2008) define lesson study as "a form of reflective teaching that uses collaborative dialogue to engage teachers in a collective assessment of their classroom practices" (p. 97). It can be said that collaborative efforts and reflective context in lesson study increased preservice teachers' noticing of students' thinking.

5.2.2 Investigation on mathematical subject

In this study, preservice teachers gathered information about mathematical subjects of perimeter, area, surface area and prism, and they made a comprehensive investigation on the content and learning objectives of it. They examined student learning and thinking related to the subject, investigated misconceptions, difficulties and mistakes of the students and researched on teaching strategies and other instructional issues. They utilized from several sources such as student textbooks, teacher guidebooks, curriculum and standards, additional education books in order to determine what students have already learnt, what they need to learn and how the fundamental concepts should be taught as indicated in literature (Yoshida, 1999; Lewis, 2002; 2004; Lewis, Perry, Hurd, & O'Connell, 2006). Therefore, they became more knowledgeable about the content of the subjects, curriculum, students' mathematical thinking and the relationship between them. They tried to anticipate student thinking, adopt student perspective and estimate their possible motivation, responses and behaviors (Fernandez & Chokshi, 2002; Lewis, 2004b; Takahashi & Yoshida, 2004; Takahashi et al., 2005). Therefore, investigation on the mathematical subject was considered as another important feature of lesson study that influenced preservice teachers' ability to notice students' mathematical thinking.

In the present study, İnci emphasized that through making investigation related to the subjects that they would teach, she began to notice the relationship between the concepts better, estimate students' possible mathematical thinking and determine the activities and questions to reduce students' confusions, difficulties and mistakes. For instance, she indicated that students might perceive the same figure as separate one when it is rotated or they might think that its area changed. She offered to ask this kind of a question to reveal students' misconception, if they had any, in order to remove it. Likewise, Semih pointed that students did not know how to compare two different measurement units and they did not understand the need for standard units based on his readings on the subjects. He suggested to cover two same rectangles with different squares to complete students' gaps and they discussed on how students think mathematically in this activity. Semih's own comment that "I felt that we also had a lack of knowledge, so source books helped us quite a lot in shaping the lesson plan" also supported him to gain insight about students' mathematical thinking and to consider these elements while planning. In this study, preservice teachers often indicated the positive effect of investigation on mathematical subjects. Yoshida and Jackson (2011) also emphasized that the investigation in the planning phase had an essential role in taking good instructional decisions and developing teachers' knowledge of students. In Osmanoğlu's study (2010), she also found that with the help of interacting with each other through discussions and making use of various educational sources such as books and curriculum, preservice teachers began to focus on and talk more about the issues related to student thinking and learning such as motivating students to think and reason, and letting students build their own knowledge; conducting student-centered lessons; student understanding and learning, and how teachers can facilitate it ; being able to understand student questions and what they say; having students explain and defend their answers; ensuring student understanding. In other words, she referred to the shift in preservice teachers' noticing of student' mathematical thinking over time and the influence of in-depth investigation about the concepts.

Research on prevalent subjects provided opportunities for preservice teachers to attend to important mathematical concepts such as student thinking, confusion and struggles, anticipate expected student responses, analyze the relevant information and interpret the readings in terms of students' mathematical thinking. Literature also includes parallel expressions to the findings of this study. It was argued that lesson study facilitated to focus on achieving the goals of a specific lesson by investigating the content area thoroughly and it supported to organize the lesson considering students` misconceptions, difficulties and needs (Harle, 2008). Furthermore, some studies advocated that it prevented the shift of focus and provided to create a student-focused setting allowing for more efficient and detailed discussion on students' mathematical thinking on the subjects. Therefore, this process enforced to get students' viewpoints towards the subject, namely, facilitated to notice students' specific mathematical thinking (Hiebert et al., 2007; Lewis, 2002; Mathews et al., 2009; Sims & Walsh, 2008).

On the other hand, preservice teachers used a lesson plan format which was applied in lesson study studies to guide and facilitate planning. It allowed them to consider learning activities and questions; students' possible responses and reactions; teachers' responses to students and how to make evaluation of practice. Therefore, this comprehensive and detailed lesson plan format might have directed İnci and Semih to focus on students' mathematical thinking and make sense of them to give instructional decisions while determining the content of the lesson. In other words, it might have impacted these preservice teachers' noticing of students' mathematical thinking. Writing detailed lesson plans by using this kind of a lesson plan format might influence the development of preservice teachers' abilities in planning (Mostofo, 2013) and teaching because preservice teachers were contemplating students' responses previously (Sims & Walsh, 2008). Similarly, some studies indicated that this lesson plan format could help preservice teachers to be better in querying and estimating student thinking so that they might adopt a more student-focused approach in their instruction (Hiebert et al., 2007; Mathews et. al., 2009; Sims & Walsh, 2008).

5.2.3 Observation and reflecting

Preservice teachers were able to observe concrete mathematical thinking and reactions of the students directly and take notes about what they observed during the implementations of the lessons. Later, they had the opportunity to discuss on what worked and what did not through their observations with the group and also think about how to improve the lesson based on these shared opinions. In the planning phase, preservice teachers made further comments with the help of their previous knowledge, readings and intuitions. However, it can be said that observing the lesson and reflecting on it provided more concrete and clear noticing of students' specific mathematical thinking. In this study, for example, İnci noticed that while observing the lesson, another group member asked good questions to support students' understanding of perimeter in addition to what they had planned before. In the reflecting phase, preservice teachers expressed their appreciation and İnci decided to use the same questions in the revision lesson to teach better. Besides, İnci emphasized the positive effects of observation on her teaching with such a statement “I am looking at the class in a more thorough manner while observing. We are learning how to react and deal with the students”. In a similar way, Semih noticed that students made overgeneralizations about the nets of prisms while observing another group member’s implementation. In the reflecting phase, he drew the others’ attention to students’ thinking and he decided to make extra explanations about these points to prevent students’ confusions in teaching of the revision lesson. Moreover, preservice teachers were not sure about students’ foreknowledge regarding some mathematical concepts. Semih’s comment that “One student used “as exponential numbers” statement so they knew this concept and we can use it for recalling” revealed that observing students’ concrete mathematical thinking facilitated to complete his lack of knowledge. It was also observed that observation of the lesson helped both İnci and Semih to understand whether their approach as a group was appropriate in terms of students’ cognitive levels. In this respect, İnci indicated that “I also noticed that some suggestions were not appropriate for students’ level” whereas Semih pointed that “Our expectations were high or low at the beginning but now, we are better at preparing a suitable content for the lesson according to students’ levels”. Furthermore, İnci and Semih emphasized the importance of exchanging ideas through interacting together in the reflecting phase for both improving the lesson and learning from each other. Preservice teachers’ similar comments were the indicators that they viewed these steps significant in assisting their noticing. Because teachers come together to work on common instructional goals in this process, they acquire new perspectives and knowledge from each other (Freidman, 2005). Therefore, the chance to observe specific student thinking and classroom interactions, and reflect on these components might have increased preservice teachers’ ability to notice students’ mathematical thinking.

Similarly, Jacobs et al (2010) indicate that teachers’ noticing skills of students’ mathematical thinking occurs by engaging with students in real classroom settings through focusing on the patterns of students’ thinking and discussing on them. Observations based on student thinking, learning and reactions shape the reflection phase and assure to evaluate and make necessary changes (Harle, 2008). In parallel to the findings of the present study, Sisofo (2010) found that having access to observe student thinking helped preservice teachers to use it while explaining their ideas with reasons for revision of the lessons. In lesson study, teachers’ discussion on their observation based on collecting data about how students think and what they do, and reflecting on instructional practices facilitates to evaluate the poor and powerful aspects of the lesson and revise it (Fernandez & Yoshida, 2004; Lewis, 2004). Yoshida (2013) argues that the observation step of lesson study which includes gathering data about student thinking and learning during lesson implementation in a real classroom provides more productive discussions on how to make lesson better and enhances the experience of professional development.

Philpp, Ambrose, Lamb, Sowder, Schappelle and Sowder (2007) conducted an experimental research on the changes in the beliefs of preservice teachers' mathematics teaching and learning. The researchers formed three groups: one group learned about students' mathematical thinking by watching them on videos or working with them directly; one group visited elementary school classrooms and the other one was the control group. It was seen that the most sophisticated beliefs developed in the group which preservice teachers studied students' mathematical thinking during mathematics instructions. Reflection among preservice teachers was considered as the reason of this development in this study since reflecting process helped to comprehend students' mathematical thinking better with the help of the focus on how to keep interactions with students mathematically and sharing ideas as a group. The results of many studies suggested teachers to observe and analyze their teaching by considering students' mathematical thinking (Carpenter et. al., 1996; Carpenter, Fennema, Franke, Levi, & Empson, 1999; Franke et. al., 2001; Franke et. al., 2007). It was also emphasized that preservice teachers should reflect their own implementations and share their ideas with the others in the group (Loughran, 2002; Darling- Hammond, 2006; Borko ve Putmon, 1996).

5.2.4 Cooperating teacher

In this study, cooperating teacher could only offer her ideas and suggestions before planning; observe each lesson and give feedbacks; and share her ideas about the implementation and lesson content in order to improve it after each instruction due to the lack of time in her work program. Although there was a limitation in the cooperating teacher's participation into the lesson study process, preservice teachers' comments pointed highly to her contribution. In the present study, İnci's comments showed that she gave importance to the suggestions of the cooperating teacher and she tried to consider them while planning and teaching the lesson. For example, after she observed the first implementation, the cooperating teacher suggested to add a starting point on the geometric figure in order to prevent students' mistakes such as missing an edge or calculating twice while finding the perimeter. İnci tried to use a starting point while teaching the revision lesson and she indicated that "it worked well and reduced the operation mistakes". Moreover, İnci, focused on students' confusion about perimeter and area and how to reduce it because the cooperating teacher remarked students' mistakes and difficulties based on this deficiency. It was understood that İnci found cooperating teachers' comments useful and paid attention to them. On the other hand, İnci was not sure about whether students could complete two half triangles into a whole square and Semih also had hesitation about its suitability for students as well as the other group members. However, after the first implementation, the cooperating teacher offered to add turning half triangles into a whole square on the lesson plan in order to make the questions in the exercise paper more difficult. Feedback of the cooperating teacher encouraged both İnci and Semih to learn more about students' thinking. Furthermore, for instance, since the cooperating teacher highlighted that students had difficulty in understanding the relationship between dimensions and the concepts of perimeter, area and prism, Semih focused on how students might think while engaging in these concepts and how to remove students' gaps. Moreover, Semih highlighted the contribution of the cooperating teacher with such a statement "Especially, the cooperating teacher's feedback helped us to interpret students' understanding, the effectiveness of our implementation and improve the lesson plan". All these comments showed that they gave importance to the feedbacks and suggestions of the cooperating teacher and considered them while revising the lesson. It can be argued that she helped the professional development of preservice teachers. Therefore, it is thought that it is highly probable that her comments served as good guidance for developing their knowledge and noticing of students' mathematical understanding and thinking.

Similarly, Graham (2006) indicates that cooperating teachers have a critical role in the success of preservice teachers' gaining field experience. Paker (2008) underlines that preservice teachers get insufficient feedbacks and support from cooperating teachers related to the phases of planning, teaching and reflecting. Accordingly, in our study, the cooperating teacher's support was limited due to her busy schedule. In the experimental study of Evertson and Smithey (2000) which was conducted with preservice teachers, an experimental group worked with experienced teachers in the course of teaching practice whereas the control group maintained the course traditionally. At the end of the study, it was clear that preservice teachers in the experimental group were more successful in the issues such as preparing effective lesson plans and arranging the time. In the study of Hughes (2006), preservice teachers indicated that the cooperating teacher directed their attention to students' mathematical thinking and the planning process. Likewise, Gurl (2009) conducted a lesson study with preservice secondary mathematics teachers and cooperating teachers and indicated that the lessons of preservice teachers were strongly influenced by the cooperating teachers. Its findings argued that there was some evidence of the cooperating teacher's impacts the on preservice teachers' focus on students' mathematical thinking while planning. Burke (2015) also indicated that preservice teachers were able to focus on noteworthy events, analyze and interpret these events and make sense of teaching with the support and guidance of an instructional coach. Similarly, many researches emphasized the important and influential role of the cooperating teacher in teaching practices of pre-service teachers (Mossgrove, 2006).

5.2.5 Specific mathematical domain

Preservice teachers do not often have opportunities to interact with students in the class because the kinds of courses taken at university which enable them to learn about students' thinking on different mathematical topics are limited (Sisofo, 2010). Therefore, this situation may cause preservice teachers' deficiency in anticipating students' thinking. It is highly probable that preservice teachers' knowledge about students' mathematical thinking is based on their own experiences, their interactions with students in courses taken at university, their readings about students' thinking (Sisofo, 2010) and what they had learnt when they were students. In this study, it was observed that considering a specific mathematical domain provided an opportunity for İnci and Semih's deep learning about students' mathematical thinking related to the subject area and complete their lack of knowledge with the help of a detailed investigation before planning. Thus, they endeavored to pay attention to how to use students' thinking while planning, teaching and reflecting on the lesson. It was thought that considering one particular mathematical domain and specific subjects related to each other during lesson study prevented losing attention and promoted the preservice teachers to concentrate on one point so that it gave them more opportunities to notice what and how students think.

In the current study, İnci focused on the relationship between measurement and perimeter, area and surface area. She paid attention to show how they can be measured concretely. Since the nature of these concepts based on measuring, she insisted on helping students to learn the meaning of these concepts firstly. For example, she suggested to measure the outer side of a figure with a rope and find its whole length, and then measure the edges separately and find the sum of them. In this way, she aimed to show that the length around a figure was equivalent to the sum of the edges in order to make students feel the meaning of perimeter. She also offered to ask students to cover the inside of a figure in order to enable them to realize that area is the part which is covered, namely, the interior region. Accordingly, she aimed to show these concepts' meaning based on measurement and also help students to comprehend the difference between area and perimeter better. Besides, İnci thought that students may have confusion in passing from br, cm, m to br^2, cm^2 and m^2, and they may use inappropriate units. She emphasized to show the relationship between perimeter and linear units and also area and square units in a simple manner to facilitate students' understanding. Moreover, İnci's comment that "I paid attention to provide a basis for the objectives of the next subject because they were related to each other" showed that she benefitted from the relevant objectives.

Similarly, since Semih knew that students confused perimeter and area with each other, he focused on teaching the difference between them in the best way to prevent their mistakes. He thought that students' lack of understanding on perimeter would cause difficulties and incorrect calculations (e.g. finding the area instead of the perimeter or vice versa). He suggested to show that perimeter was the outer side of a figure whereas area was the interior region of this figure through visualizing and using materials in order to concrete their meaning for better understanding. In addition, in order to reduce students' confusions about perimeter and area, Semih offered to combine them by giving perimeter and expecting students to find one edge and area or vice versa. Likewise, Semih focused on the relationship between area and surface area since surface area is based on knowledge regarding the area of the rectangle and square in the types of rectangular prisms. He thought that students might confuse the concepts of area and surface area with each other. He suggested to calculate the area of each face in a prism by using the appropriate area formula and write the values on the faces. He aimed to show that the meaning of surface area was the sum of area measurements on all faces by opening a prism. In this study, while designing the lessons according to the objectives, both İnci and Semih made suggestions using a similar approach which was based on measurement. Their approaches showed that they tried to use their knowledge related to perimeter as teaching area and transfer the knowledge of perimeter and area into prism and surface area. They could notice the relationship between the subjects and how students' lack in one of them affected the other since they considered only this topic and investigated what to do deeply. Therefore, they had more chance to focus on and make sense of students' mathematical thinking about these concepts as well as getting detailed information about the subjects.

On the contrary, Sisofo (2010) accepted the focus on a particular topic as a limitation of the lesson study and advocated giving a chance for preservice teachers to learn students' thinking on various mathematical topics. In Baki’s study (2012) which involved the implementation of lesson study with preservice teachers, the researcher wanted each preservice teacher in the experimental group to prepare a lesson plan towards only one objective which was unrelated and different from each other. She discovered that preservice teachers in both experimental and control groups were not successful in preparing tasks in terms of students’ mathematical difficulties related to subject. The reason of this deficiency in contribution of lesson study was explained with not working subject dependent, namely, working by considering the objectives which did not complete each other. Therefore, considering a specific mathematical domain and its related objectives might have increased the success of preservice teachers’ focusing on and making sense of students’ mathematical thinking about perimeter, area, surface area and prism in whole lesson study process in this study. Baki (2012) also suggested to conduct lesson study implementations depending on specific mathematical topics to increase the effectiveness of the process. Likewise, Fernandez et al., (2012) dealt with one particular mathematics subject in their study regarding the development of noticing of students’ mathematical thinking. It was argued that the specific mathematics subject influenced the change of preservice teachers’ noticing skills positively. Walkoe (2013) also suggested to go beyond from considering noticing of mathematics thinking in a general way, to particular mathematics domains.

If we summarize these important features of lesson study that influenced preservice teachers’ noticing of students’ mathematical thinking, the benefits of lesson study included supporting collaboration and reducing teacher isolation, developing observation and reflection, increasing teacher’s understanding of mathematical subjects, providing guidance of the cooperating teacher and focusing on students’ learning and thinking. In order to maintain development of instruction, studies in literature highlighted the importance of focusing on students’ thinking, elaborating the reasons behind their possible strategies and discussing on them by sharing with each other (Carpenter et al., 1999; Lester, 2007). It is argued that teachers who participated in lesson study process begin to refer to students’ mathematical thinking by associating relevant knowledge about the topic to the goals of the lesson, anticipating possible students’ responses and supporting students in a way that reveals their mathematical approach during teaching the lesson that the group members had planned together (Fernandez, Cannon & Chokshi, 2003). Providing opportunities for preservice teachers during their teacher education process through investigating, working collaboratively, learning from their own teaching, observing the other’s teaching, reflecting, building theory to practice and making them ready for the complex structure of a real classroom environment in lesson study might enhance the effectiveness of teacher education on preservice teachers' noticing skills.

5.3 Implications of the Study

There was a positive shift in preservice teachers' noticing of students' mathematical thinking with participation in the lesson study process because it provided many opportunities for preservice teachers to carry their theoretical knowledge into practice. Furthermore, traditional teacher training is not enough in terms of teaching profession and making them ready for the complexity of classrooms (Shulman, J., 1992). Thus, when it comes to teacher education benefitting from professional development models such as lesson study is important as it supplies an effective environment for development of teacher profession. Teacher education programs should also enable preservice teachers to notice, interpret and make pedagogical decisions (Osmanoğlu, 2010). Accordingly, it can be suggested that engaging in lesson study should be employed in most of the periods of teacher education programs.

When the effect of collaboration in this study was considered, practices in respect to the development of noticing skills can be conducted in the context of professional development programs like lesson study by throwing experienced teachers and inexperienced preservice teachers together. Similarly, these kind of practices can be integrated into the courses for professional development of preservice teachers such as school experience and teaching practice in order to both systematize these courses more and support students to get more efficiency from them. In this direction, it is suggested to teach the scope, aim and phases of lesson study model to teachers and preservice teachers so that preservice teachers can be empowered to start their profession as more qualified teachers with enriched experiences. At this point, it can be offered to initiate a course which provides opportunities for preservice teachers to learn lesson study deeply and to practice it intensely.

Osmanoğlu (2010) indicates that in addition to the experiences obtained from the courses such as teaching practice and school experience, preservice teachers can improve their noticing skills with the help of mathematics videos from real classrooms. She also suggested to use such video cases and discuss on them in teacher education in order to develop preservice teachers' noticing skills and pedagogical content knowledge. Thereupon, various video-based cases in real mathematics classrooms can be constructed from lesson study implementations to use in other courses for several purposes at university in order to increase preservice teachers' profession. Consequently, it is offered to use lesson study model and construct video-based cases from its implementations in order to create source for another undergraduate course which aims to make preservice teachers see the realities of classroom settings and teaching (Butler et al., 2006; Lundeberg et al., 1999; Shulman J., 1992); learn new knowledge; improve their noticing skills and gain new viewpoints (Barnett & Tyson, 1999).

In literature, several studies offer to supply preservice teachers with knowledge about students' mathematical thinking during teacher education programs. According to Philpp et al., (2007), one way for it is to infuse students' thinking into mathematics content courses so that they will be ready for building on students' thinking as future teachers by learning more information about this issue and experimenting with students' mathematical thinking. Similarly, Baki (2012) suggested to enrich the content of courses in education faculties which entailed to determine students' misconceptions, difficulties and mistakes over mathematical subjects. She emphasized that though lesson study was effective in this development, it was not enough on its own. Mapelelo (1999) also advocated to talk about students' misunderstandings in university settings since they were related to students' understanding and learning of mathematics directly. Likewise, it was observed that preservice teachers' knowledge and ability to notice of students' mathematical thinking were important to be able to prepare effective lesson plans, to teach the lessons in a fashion that will consider and reveal their thinking and to reflect critical ideas to improve mathematics instructions in this study. Therefore, it is thought that the courses which aim to teach students' possible mathematical thinking patterns, misconceptions, challenges and mistakes concerning various mathematics topics should be given or increased and many opportunities which show how to use students' mathematical thinking through experiencing should be provided in teacher education programs of universities.

5.4 Limitations and Recommendations

In this study, it was focused only on preservice teachers' noticing in the context of lesson study because qualitative research took a long time. However, the topic of students' noticing should also be investigated since both teacher and student noticing are likely to be related to each other. It is thought that students' noticing in classroom interaction will help to determine the effects of what was designed by teachers on students' reasoning (Lobato, Hohensee, & Rhodehamel, 2013). Therefore, research on this relationship can be useful in future studies. Seeking answers for the questions "how does teacher noticing have an effect on students' noticing in classroom interaction?" vice versa "how does student noticing affect teacher's noticing of student thinking?" may be crucial to clarify how to develop this skill.

On the other hand, this research was conducted with two preservice teachers who were in their senior year of teacher education program so the number of people in the study group was too small to make generalizations on other contexts. This limitation was stemmed from the wide structure of lesson study and multiplicity of lesson cycles. It can be repeated with more participants through considering only main stages of lesson study- planning, teaching and reflecting- instead of adding re-teaching and re-reflecting and for fewer lesson cycles. More research is needed in order to refer the generalizability of this study. This study is also limited to researcher's own teaching and learning approach. Although various data sources such as video recording, observations and interviews were used for triangulation, it is likely to be researcher's bias even a little. The nature of noticing and development of this skill in other professional development contexts as well as lesson study can be explored in future studies in order to illuminate how different settings influence noticing. Furthermore, similar studies that are conducted through lesson study may be helpful in finding out possible effects of lesson study on teacher education. Therefore, replication of this study can be conducted with preservice teachers from separate departments or universities. Besides, the findings showed that some factors such as collaboration, observation and reflecting supported preservice teachers' ability to notice. The factors which influence noticing of students' thinking may be another point that should be investigated.

In this study, the participation of the cooperating teacher was limited due to her work program. Thus, her contribution to the lesson study process was not as much as it was supposed to be. She observed each lesson and participated in reflecting phases following teaching. Nevertheless, after she shared her ideas and made suggestions related to lesson, she had to leave the group due to her next lesson. Similarly, she indicated her opinions, answered preservice teachers' questions and made suggestions to help them while planning the lesson but she could not be present in all planning processes because planning took a long time. Thus, this study can be replicated with participation of one or more cooperating teachers on the condition that they are fully involved in each phase.

Moreover, this study is limited to a specific mathematical domain of geometry and measurement and the subjects of perimeter, area, surface area and prism. It can be conducted by employing distinct mathematics domains and subjects in order to understand whether and how teacher noticing is influenced by the change in them. Besides, it gave preservice teachers an opportunity to learn about and notice students' thinking related to these subjects deeply. However, it might be accepted as a limitation on the other hand as preservice teachers could not go beyond noticing of students' thinking for these mathematical subjects. More studies can be conducted for various subjects in more mathematical domains. Thus, the effectiveness of one particular mathematical domain or various mathematical topics in different domains on noticing can be further investigated.

In this study, possible effects of lesson study on preservice teachers' noticing of student thinking was explored. How their knowledge, beliefs and goals or other factors may influence their noticing can be examined too. Moreover, experimental research can be done to explore the effect of these kinds of factors on noticing of student thinking. On the other hand, some studies highlight that teachers' knowledge and abilities to anticipate, interpret and use students' thinking in their practices help them to increase their students' achievement (Carpenter, Fennema & Franke,1996; Jacobs, Lamb & Philipp, 2010; Sowder, 2007). Even so, there are few researches on the relationship between teachers' noticing of student thinking and students' mathematical achievement. Thus, future studies regarding this issue can be conducted.

Furthermore, the transcripts of the video recordings in each lesson study phase, interviews, and observation notes were used to examine and make meaning of preservice teachers' ability to notice students' mathematical thinking and their development in this study. Interpretation of the data may present more limited information than the original content because it included the researcher's inferences. Therefore, noticing of participants can be reported with the support of technology which provides access to teachers' noticing of student thinking in-the-moment and hands teachers the control to select moments for discussion while they were teaching. At this point, teachers' or preservice teachers' noticing can be reported through applying this kind of technology and its effects for professional development opportunities can be investigated.

5.5 Implications for my Future Career

Conducting this research helped me to shape my future plans. I will try to investigate how to integrate lesson study into undergraduate courses. I will teach what it is and how to practice it in order to educate teachers when I become a faculty member. I want to provide opportunities for preservice teachers to work collaboratively, learn how to prepare efficient lesson plans, observe and analyze real classroom settings and enrich their knowledge by discussing together before starting the profession. I will use video-based cases from lesson study implementations to improve noticing skills of preservice teachers and knowledge about the realities of classroom settings. I will endeavor to find different ways for developing preservice teachers' ability to notice. I will also research other issues such as pedagogical content knowledge and mathematical understanding when I conduct lesson study in my courses. I am also thinking on how to include teachers in the lesson study process in more effective ways. I will also investigate the feasibility of lesson study with teachers in Turkey. I hope my research helps teacher educators raise more qualified teachers.

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APPENDICES

APPENDIX A: FOUR COLUMN LESSON PLAN TOOL

Steps of the lesson: learning activities and key questions	Student activities and expected reactions/ responses	Teacher's response to student reactions/ Things to remember	Method(s) of evaluation

APPENDIX B: INTERVIEW QUESTIONS

B.1 Pre-Teaching Interview Questions

1. What were the goals of the lesson according to the plan that you prepared?
2. What kind of a relation was there between the lesson plan that you had prepared and the students’ existing knowledge?
3. What were the expected students’ responses and reactions during teaching according to the lesson plan? How did you take these into consideration while preparing your lesson plan?
4. How did preparing a lesson plan affect your noticing of what and how students think?
5. What were the aspects that you focused on and got your attention while preparing the lesson plan?
6. What were the positive and negative factors that affected preparing your lesson plan?
7. Can you elaborate on your planning process?

B.2 Post-Teaching Interview Questions

1. What were students’ responses and reactions during teaching the lesson?
2. What kind of a relation was there between the expected and observed students’responses and reactions?
3. How did observing the lesson affect your noticing of students’ thinking regarding the subject?
4. What were the aspects that you focused on and got your attention while observing the lesson?
5. What were the the positive and negative factors that affected teaching the lesson?
6. How did participating in the reflecting phase affect your noticing of students’ thinking?
7. What were the aspects that you focused on and got your attention in the reflecting phase?
8. What were the the positive and negative factors that affected the reflecting phase after the lesson?

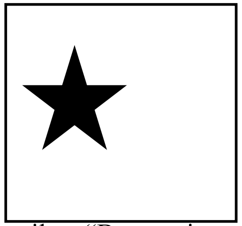
APPENDIX C: DISCUSSION PROTOCOL

Reflecting on the Lesson:

1. All participants are given time to review their observation and notes about the lesson. They are requested to think on what worked, what did not work, what difficulties there were, what changes might be made.
2. The cooperating teacher briefly indicates his/her ideas and makes suggestions to improve the lesson.
3. First the teacher of the lesson describes the goals of the lesson briefly and then reflects his/her thoughts about the lesson, including:
    - What worked well
    - What did not work
    - What difficulties there were
    - What differences there were between the lesson and the lesson plan
    - What should be changed about the lesson to improve it and what the rationales for the changes are
4. After sharing of the teacher who conducted the lesson, the other lesson study group members comment on the lesson based on their observation data from the lesson, including:
    - What worked well
    - What did not work
    - What difficulties there were
    - What they observed about the students` reactions and thoughts during the lesson
    - What differences there were between the lesson and lesson plan
    - What should be changed about the lesson to improve it and what the rationales for the changes are

APPENDIX D: THE REVISED LESSON PLANS

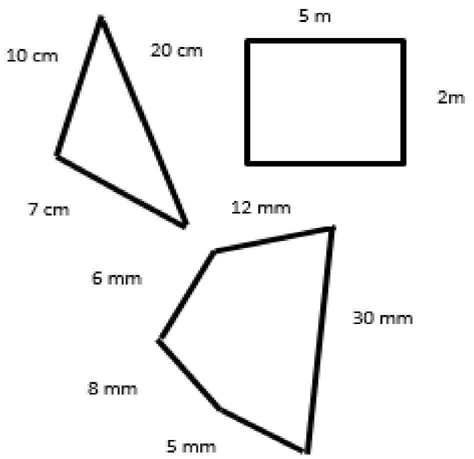
D.1 The Revised Lesson Plan 1

Dersin Adımları: öğrenme aktiviteleri ve anahtar sorular	**Öğrenci aktiviteleri/ beklenen öğrenci tepkileri ve yanıtları**	**Öğrenci tepkilerine karşı öğretmen yanıtları/ göz önünde bulundurulması gerekenler**	**Amaçlar ve Değerlendirme Yöntemleri**
Etkinlik 1:  – Öğrencilere "Bu resmin etrafını şeritle süslemek istiyorum. Etrafına ne kadar şerit gerekir?" şeklindeki bir soruyla derse başlanır. – Tahminler tahtaya yazılır. – Sonrasında "Sizce bunu nasıl hesaplayabiliriz?" diye sorulur. – Bir öğrenciye şerit verilir ve öğrenciden resmin etrafına şeridi çekmesi istenir. – Daha sonra başka bir öğrenciden şeridi açıp uzunluğunu cetvelle ölçmesi istenir. – Öncesinde aldığımız tahminler arasında cevaba en yakın tahmini söyleyen öğrenci alkışlanır. – Sonrasında başka bir öğrenci tahtaya alınır, resmin kenarlarını tek tek ölçmesi ve ölçtüğü uzunlukları toplaması istenir. İlk	– Öğrenci 100,125,130 gibi tahminlerde bulunur. – Ölçeriz. – İple etrafından geçeriz.		– Çevrenin kenarların toplamı olduğunu hissetmeleri sağlanır.

bulduğumuz toplamla ikinci toplamın aynı olduğunu öğrencilerin fark etmeleri sağlanır.

– Öğrencilere “Biz şimdi neyi hesaplamış olduk?” sorusu yöneltilir.

Etkinlik 2:



1. kenar	2. kenar	3. kenar	4. kenar	5. kenar	T

– Öğrenciler tek tek tahtaya kaldırılarak tablo doldurulur.

– Resmin etrafını
– Resmin kenarını
– Resmin dışını
– Resmin çevresini

– Kenarları topladık.

– 3 kenarın uzunluklarını topladık.

– 4 kenarın uzunluklarını topladık.

– 5 kenarın uzunluklarını topladık.

– Evet.

– Çevreyi hesaplamak için ne yaptık?

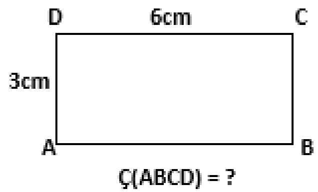
– Üçgenin çevresini bulmak için ne yaptık?

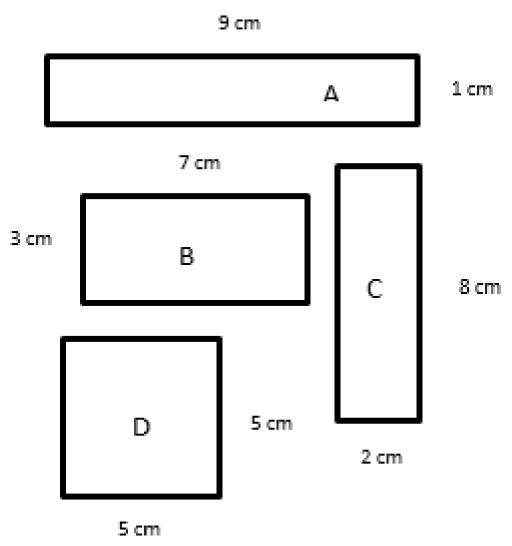
– Dörtgenin çevresini bulmak için ne yaptık?

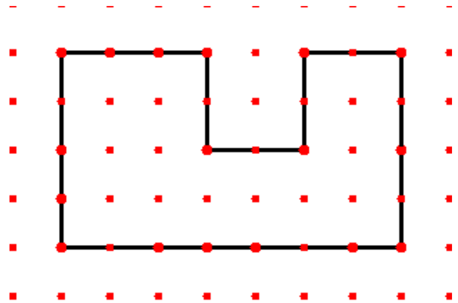
– Beşgenin çevresini bulmak için ne yaptık?

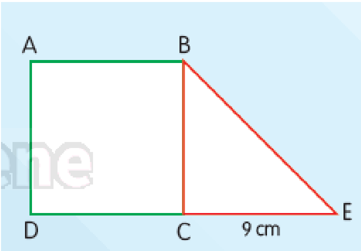
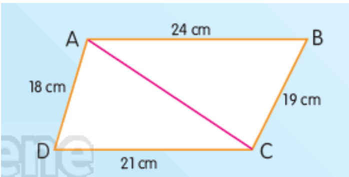
– “Çokgenimiz kaç kenarlıysa o kenarları topluyoruz.” Diyebilir miyiz?

– Çevrenin kenarların toplamı olduğu vurgulanır.
– Çevrenin tanımı deftere yazdırılır.
– Bir çokgenin çevre uzunluğu o çokgenin bütün kenar uzunluklarının toplamına eşittir.

– Öğrencilere “Şekillerin çevre uzunlukları ile kenar uzunlukları arasında nasıl bir bağıntı vardır?” sorusu sorulur. **Etkinlik 3:** – Öğrencilere çevre uzunluklarını hesaplamayla ilgili alıştırma kağıdı dağıtılır. – Öğrencilerden bu soruları cevaplandırmaları istenir. **Etkinlik 4:** *Dikdörtgenin Çevre Formülüne Ulaşma* – Tahtaya iki tane büyütülmüş izometrik kağıt asılır. – İzometrik kağıtlardan birine bir tane dikdörtgen çizilir. Diğerine bu çizilen dikdörtgenin kenarları önce uzun kenarlar ardından kısa kenarlar uç uca eklenecek şekilde taşınır. – Buradan 2(uzunkenar)+2(kısakenar) toplamının çevre olduğu vurgulanır. – Ardından silinip dikdörtgenin kenarları (uzunkenar)+(kısakenar), (uzunkenar)+(kısakenar)şeklinde taşınır ve çevrenin 2.(uzunkenar+kısakenar) olduğu gösterilir. – Ç = 2a+ 2b = 2(a + b) **SORU 1:**  			– Öğrenci, etkinlik kağıdında verilmeyen kenarları fark edip bulmaya çalışacak mı? – Öğretmen, dikdörtgenin çevresinin 2a+2b=2(a+b) olduğunu vurgular

Karenin Çevre Formülüne Ulaşma – Büyütülmüş izometrik kağıtların birine kare çizilir. Diğerine ise çizilen karenin kenarları art arda eklenerek Ç=(kenar)+(kenar)+(kenar)+(kenar) olduğu gösterilir. Buradan Ç= 4a formülü verilier. – Öğrencilere kaç kenarı topladığımız sorulur. **SORU 2:** Bir kenarı 7 cm olan karenin çevresi nedir? **Etkinlik 5:** İzometrik kağıda şu şekiller çizilir.  1. Sizce hangisinin çevresi daha büyük olabilir?	– 4 tane kenar – Evet – 28 – 49 – Cevaplar öğrencilerden alınır. – A daha büyüktür. – B daha büyüktür. – C daha büyüktür. – D daha büyüktür.	– Karenin çevresi için 4.(kenar) diyebilir miyiz? – Neden A dedin? – Neden B dedin? – Neden C dedin? – Neden D dedin?	– Karenin çevresinin 4a olduğu vurgulanır.

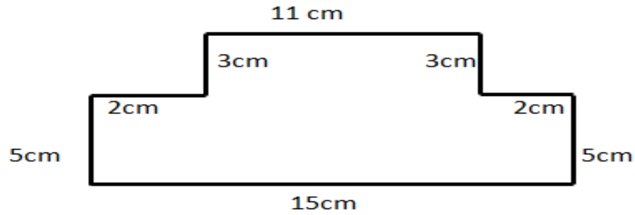
– Sonrasında öğrenciler kaldırılır, şekillerin çevrelerini bulmaları istenir. 2. Bu çokgenlerin çevreleri için ne dersiniz? 3. Bu çokgenler birbirinin aynısı mıdır?	– Daha büyük sayılar var. – Kenarları daha uzun – Evet – Hayır – Aynısıdır çünkü çevreleri eşittir. – Aynısı değildir çünkü kenarlar farklıdır.	– Neden? – Neden?	– Kenarları farklı olan çokgenlerin çevrelerinin aynı olabileceği gösterilir.
SORU 3:  Şekli çizilir. 1. İki noktanın arasındaki uzaklığı 1 metre kabul edersek, Ali bey bu tarlanın etrafını bir kez dikenli tel ile çevirirse ne kadar tel gerekir? 2. Ali bey, bu tarlanın etrafını 3 kez dikenli tel ile çevirmek isterse ne kadar tele ihtiyacı olur? 3. Telin metresi 12 TL olduğuna göre tarlanın etrafı 4 kez dikenli tel ile çevrildiğinde Ali Bey ne kadar ücret öder?	– Kenarları topluyorduk – 3 ile çarparım. – 4 ile çarparım sonra 12 ile	– Öğrencinin yapamaması halinde “Çevreyi bulmak için ne yapıyorduk?” şeklinde yönlendirici bir soru sorulur. – “Kenarları sayalım” – “3 kez tel çekiyorsam ne yaparım?”	

– Soruları öğrencilere sorulur ve defterlerine çözmeleri istenir. – Sonrasında öğrenciler tahtaya kaldırılarak sorular cevaplanır. **SORU 4:** Çevresi 30 cm olan bir dikdörtgenin uzun kenarı 8 cm'dir. Bir kenarı, dikdörtgenin kısa kenarının uzunluğuna eşit olan karenin çevresi kaç cm'dir? – Öğrencilerden önce defterlerine çözmeleri istenir. Daha sonra bir öğrenci tahtada çözmesi için tahtaya kaldırılır. **SORU 5:**  Yukarıdaki yamuk ABCD karesi ile BCE üçgeninden oluşmuştur. Karenin çevresi 48 cmve üçgenin çevresi 36 cm'dir. ICEI=9 cm ise ABED yamuğunun çevresi kaç cm'dir? **SORU 6:** 	- 2 uzun, 2 kısa	– Öğrencinin yapamaması halinde tahtaya başka bir dikdörtgen çizilir. – "Kaç tane kısa kenarımız, kaç tane uzun kenarımız var?" – "2 tane uzun kenarımızın uzunluklarının birbirine eşit olduğunu biliyoruz. O halde uzun kenarlarımızın toplamını çevreden çıkarırız ve kalan uzunluğu 2 tane kısa kenarımıza eşit olarak paylaştırırız."	– Problemler anlaşıldı mı? – Çevre kavramı oluştu mu? – Öğretmen görselleştirmeye dikkat edebilir.

Yukarıdaki şekilde kenar uzunlukları verilen ACD üçgeninin çevresi 73 cm’dir. Buna göre ABC üçgeninin çevresi kaç cm’dir? **Dersin Tekrarı:** 1. Bugün ne öğrendik? 2. Çevre neydi? 3. Dikdörtgenin çevresi neydi? 4. Karenin çevresi neydi? 5. Aynı çevrede farklı şekiller oluşturabilir miyiz?			

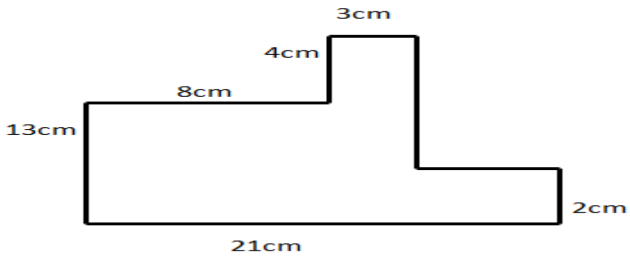
ALIŞTIRMALAR

1.



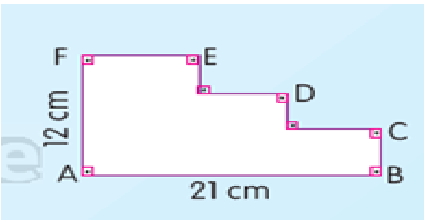
Şeklin çevresi nedir?

2.



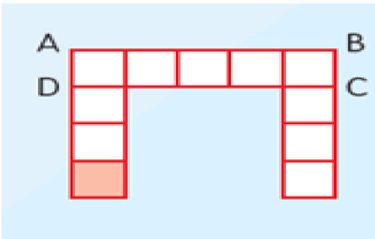
Şeklin çevresini bulunuz.

3.



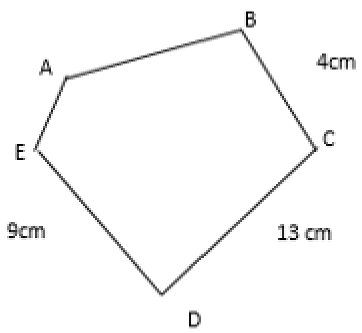
Yandaki şeklin çevresini bulunuz.

4.



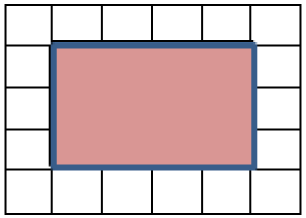
Yandaki şekil eş karelerden oluşmuştur. Taralı bölgenin çevre uzunluğu 8 cm ise ABCD dikdörtgeninin çevresi kaç cm'dir?

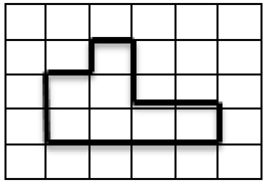
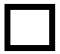
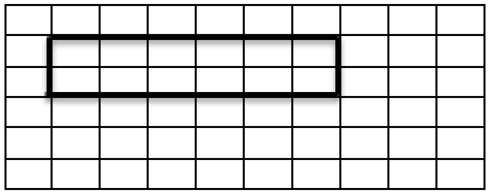
5.

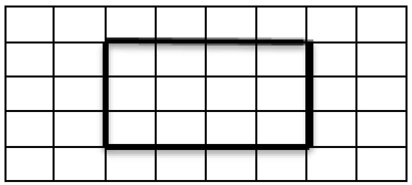


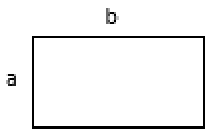
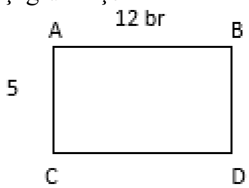
Çevresi 50 cm olan yandaki şeklin AB kenarı AE kenarının 3 katı olduğuna göre AB kenarının uzunluğu kaç cm'dir?

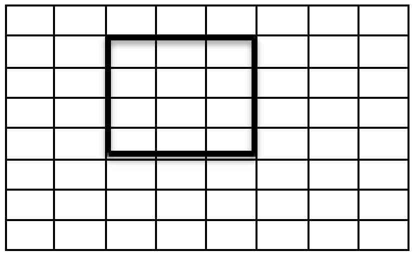
D.2 The Revised Lesson Plan 2

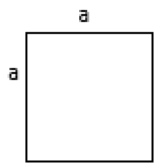
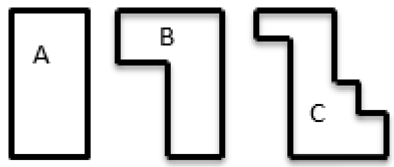
Dersin Adımları: öğrenme aktiviteleri ve anahtar sorular	Öğrenci aktiviteleri/ beklenen öğrenci tepkileri ve yanıtları	Öğrenci tepkilerine karşı öğretmen yanıtları/ göz önünde bulundurulması gerekenler	Amaçlar ve Değerlendirme Yöntemleri
Hatırlatma: – Dersin başlangıcında öğrencilere "Bir önceki dersimizde neler görmüştük?" sorusu sorularak bir önceki dersin kısa bir tekrarı yapılarak derse başlanır.	– Çevre hesaplama – Dikdörtgenin çevresi – Ç=2x(a+b) – uzun kenar ile kısa kenarı toplayıp iki ile çarparak.	– Dikdörtgenin çevresini nasıl buluyorduk?	
Etkinlik 1:  – Tahtaya yapıştırılan kareli kağıt üzerine yukarıdaki şekilde bant yardımıyla bir dikdörtgen çizilir. – Bir öğrenci tahtaya kaldırılır ve öğrenciden önceden dikdörtgenin iç bölgesini kaplayacak şekilde hazırlanmış olan renkli kartonu yapıştırması istenir. – Bu şekildeki şeridin uzunluğu bize neyi verir?	– Karenin çevresi – 4xa – Bir kenarını dört ile çarparak. – Şeklin çevresini verir	– Karenin çevresini nasıl buluyorduk?	

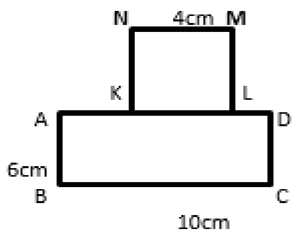
– Şeridin içinde kalan alanın bize alanı verdiği ifade edilir. **Etkinlik 2:**   – Şeklindeki birim kareler hazırlanarak öğrenciden şekli kaplayıp alanını bulması istenir. – Kaç birim kare var? **Alıştırma Çözümü** – Daha sonra öğrencilere alıştırma kağıdı verilir. Çözmeleri için belli bir süre verildikten sonra tahtada çözmeleri istenir. **Etkinlik 3:**  – Tahtada yukarıdaki şeklin aynısından iki farklı kareli kağıda çizilir. – Bir öğrenci tahtaya kaldırılarak ilk şekli $1br^2$lik özdeş karelerle kaplayıp sonucu şeklin altına yazması istenir.	– 7 tane – 7 birim kare oldu.	– Şeklin alanı ne olmuş oldu? – Bir yeri ölçmek için farklı birimler kullanırsak farklı sonuçlar elde etmiş oluruz. Bu sebeple alan ölçmede aynı sonuçlar elde edebilmek amacıyla standart ölçü birimlerini kullanmamız gerekiyor. Bunun için bizler de alan ölçerken standard ölçü birimleri olan cm^2 ve m^2 gibi birimler kullanıyoruz.	– Alanın tanımı kavratılır. – Alan ve çevrenin farklı kavramlar olduğu vurgulanır. – Birim karenin ne demek olduğu vurgulanır – Sıranın bir kenarını uzunluğu öğrencilere karışlarını kullanarak ölçtürülür. Farklı sonuçlar çıktığı için uzunluk ölçerken herkes tarafından geçerli kabul edilen ölçü birimleri olan cm ve m kullanıldığı belirtilir.

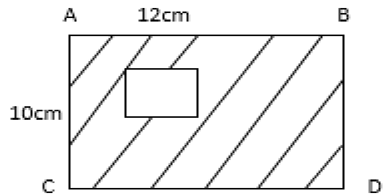
– Bir başka öğrenci de tahtaya kaldırılarak bu sefer diğer şekli $4br^2$lik özdeş karelerle kaplaması ve çıkan sonucu şeklin altına yazması istenir. – Dikdörtgensel bölge aynı olduğu halde birim kare sayıları neden farklıdır? – *Günlük hayatta cm^2 ve m^2' yi nerelerde kullanıyoruz örnek verebilir misiniz? **Etkinlik 4:** – 3x4 bir dikdörtgen çizilir. Öğrenci tahtaya kaldırılır ve özdeş birim karelerle şekli sayarak kaplaması, dikdörtgenin altına alanını yazması istenir.  **3 tane 4 birim kare bulunduran satır** – 1.satırda kaç tane birim kare var? – -Şeklin altına "3 tane 4 birim kare bulunduran satır" ifadesi yazılarak öğrencilerin görmeleri sağlanır.	– Ölçmek için kullandığımız şekiller farklı. – 4 tane – 4 tane – 4 tane – Evet – Hayır	– 2.satırda kaç tane birim kare var? – 3.satırda kaç tane birim kare var? – O halde üzerinde 4 birim kare bulunan 3 tane satır var, diyebilir miyiz? – Hayır cevabı gelmesi durumunda ise satır ve sütun kavramları vurgulanarak öğrenci yönlendirilir. – Alanı 12 birim karedir.	– Çevre açısından da aynı olduğu vurgulanır. – Günlük hayatta cm^2 ve m^2 nin kullanıldığı yerler açıklanır. – Satırın sütun kadar ya da sütunun satır kadar yinelenmesinin alanı verdiği vurgulanır.

– Alan ile kenar uzunlukları arasında bir ilişki var mıdır? – Buradan hareketle dikdörtgenin alanının, (kısakenar)x(uzunkenar)=(en)x(boy) olduğu gösterilir.  – kısa kenar: a – uzun kenar: b dersek – Alan= axb olur. **SORU 1:** – Aşağıdaki şeklin alanını hesaplayalım.  – 5x12=60 br^2 – (br)x(br)=(br^2) – (5br)x(12br)=(60br^2) – (cm)x(cm)=(cm^2) – (m)x(m)=(m^2) – Şeklin çevresi kaç br'dir?	– Kenar uzunlukları çarpılır. – Kısa kenar ile uzun kenarı çarparız. – 34 br	– İstenen cevap gelmezse "3 kere 4", "3 defa 4" şeklinde yönlendirici ifadeler kullanılır.	– Alan=en x boy= axb olduğu vurgulanır. – Alanın biriminin br^2, cm^2 ve m^2 olduğu vurgulanır. – Alanda neden br^2, cm^2, m^2 kullanıldığı, çevrede neden br, cm, ve m kullanıldığı ve bunlar arasındaki

– Çevre → uzunluk → br, m, cm – Alan → uzunluk → genişlik br², cm² ve m² **SORU 2:** Çevresi 54 cm kısa kenarı 12 cm olan dikdörtgenin alanı kaç cm^2'dir? **Etkinlik 5:** – Şekilleri çizilir. İki öğrenci tahtaya kaldırılır ve özdeş birim karelerle şekilleri sayarak kaplamaları, şekillerin altlarına alanlarını yazmaları istenir.  **3 tane birim kare olan 3 satır** – 1.satırda kaç tane birim kare var? – -Şeklin altına "<u>3 tane birim kare olan 3 satır</u>" şeklinde yazılarak öğrencilerin, bir önceki örnekte de yapmış olduğumuz gibi, karenin alanını ile kenar uzunlukları arasındaki ilişkiyi görmeleri sağlanır. – Alan ile kenar uzunlukları arasında bir ilişki var mıdır?	-180 cm^2 – 3 tane – 3 tane – 3 tane – Evet – Kenar uzunlukları çarpılır. – Bir kenarını diğer kenar ile çarparız.	– 2.satırda kaç tane birim kare var? – 3.satırda kaç tane birim kare var? – -O halde "üzerinde 3 birim kare bulunan 3 satır var" diyebilir miyiz? – -Alanı 9 birim karedir.	geçişin nasıl yapıldığı hissettirilir. – -Problem anlaşıldı mı? – Öğrenciler çözüme ulaşabildiler mi? – Soruların sonunda birimlerin mutlaka yazılması gerektiği vurgulanır.

– -Buradan hareketle karenin alanının, (kenaruzunluğu)x(kenar uzunluğu) oluğu gösterilir.  – Alan=a x a olur.			– Karenin alanının (kenar)x(kenar) olduğu vurgulanır. – Karenin alanının axa olduğu vurgulanır.
SORU 3: Bir kenarı 7m olan kare şeklindeki duvarın alanı nedir? (Sözel olarak sorulur.)	– $49m^2$		– Problemler anlaşıldı mı?
SORU 4: Çevresi 40 cm olan kare şeklindeki bir çerçevenin alanını bulunuz. – Öğrencilere çözmeleri için süre verildikten sonra bir öğrenci tahtaya kaldırılarak çözmesi istenir.	– $100m^2$		– Öğrenci çevreden kenarı bulup alanı hesaplayabildi mi? – Öğrenci çevre ile alan arasında bağlantı kurabildi mi? – Soruların sonunda birimlerin mutlaka yazılması gerektiği vurgulanır.
Etkinlik 6:  – Üzeri renkli fon kartonlarla kaplı şekiller tahtaya yapıştırılır. Öncelikle öğrencilerden şekillerin çevre ve alanlarına yönelik tahminler alınır. – Sizce A, B, C şekillerinden hangisinin çevresi daha büyük?”	– A – B	– Neden A? – Neden B?	

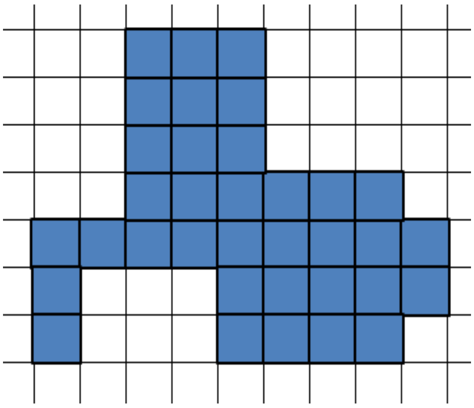
	– C – Daha kıvrımlı – Daha düz	– Neden C?	
– Sizce A, B ve C şekillerinden hangisinin alanı daha büyük?” – Öğrencilerden alınan cevaplar tahtaya yazılır. – Kareler sayılarak çözüme ulaşılır. – En yakın cevabı veren öğrenci sınıf tarafından alkışlanır. –	– A – B – C – Daha çok yer kaplıyor – Daha az yer tutuyor	– Neden A? – Neden B? – Neden C?	
SORU 5: Bir kenarı 8m olan kare şeklindeki bir yer, bir kenarı 2m olan kare fayanslarla döşenecektir. Bu iş için kaç tane fayans gerekir? **Alternatif:**  KLMN kare, ABCD dikdörtgen olmak üzere yukarıdaki şeklin alanını bulunuz.	– 76 cm^2		– Öğrenci şeklin alanının kare ve dikdörtgenin alanının toplamına eşit olduğunu fark edebildi mi?

SORU 6:  ABCD dikdörtgen, içteki şekil ise bir kenarı 6cm olan bir kare olduğuna taralı alanı bulunuz. **SORU 7:** Alanı 81 cm2 olan karenin bir kenarının uzunluğu, bir dikdörtgenin uzun kenarının uzunluğuna eşittir. Dikdörtgenin uzun kenarı kısa kenarının 3 katına eşit olduğuna göre dikdörtgenin çevresi kaç cm'dir? **SORU 8:** Alanı 48 cm^2 olan bir dikdörtgenin uzun kenarı 12 cm'dir. Bu dikdörtgenin içine çizilen 3 eş kareden birinin alanı kaç santimetre kare olur? **Tekrar:** – Öğrencilere genel bir tekrar yaptırılır. – Alan neydi? – Dikdörtgenin alanı neydi? – Karenin alanı neydi? – Çevre ile alan arasındaki fark nedir?	– 84 cm^2 – 24 cm – 16 cm^2		– Öğrenci alanın eksildiğinin farkına vardı mı? – Öğrenci alandan çevre hesaplamaya geçebildi mi? – Ders anlaşıldı mı?

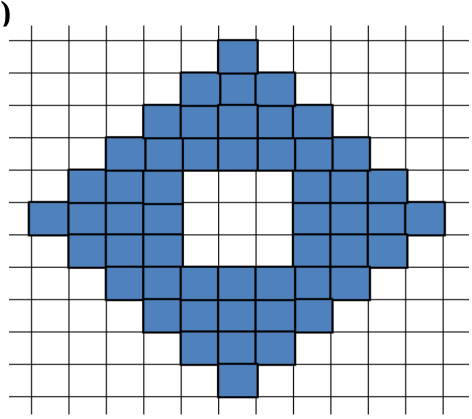
ALIŞTIRMALAR

Aşağıdaki şekillerin alanlarının kaç birim kare olduğunu bulunuz.

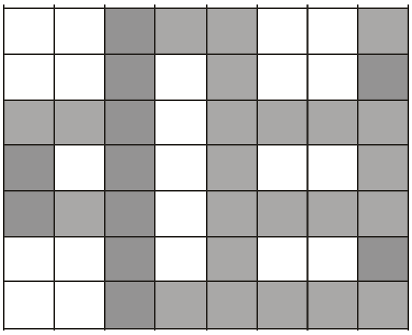
1)



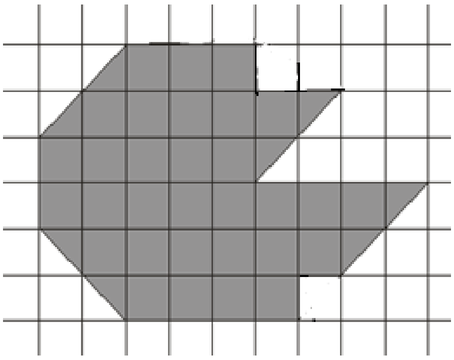
2)



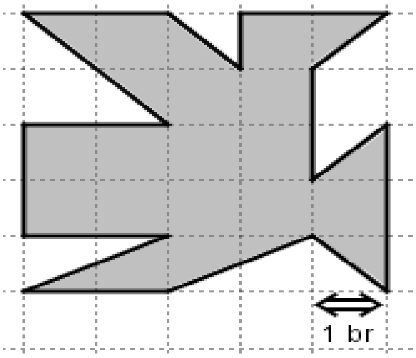
3)



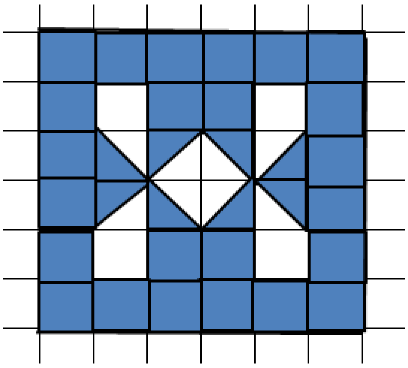
4)



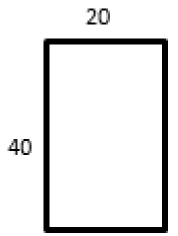
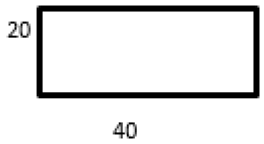
5)

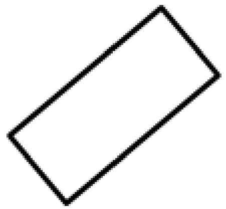


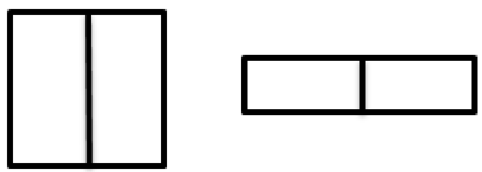
6)

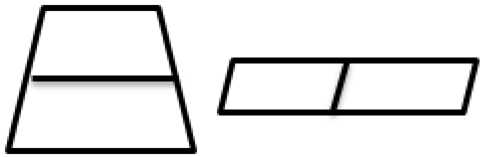
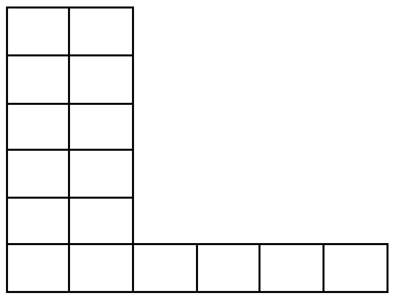


D.3 The Revised Lesson Plan 3

Dersin Adımları: öğrenme aktiviteleri ve anahtar sorular	Öğrenci aktiviteleri/ beklenen öğrenci tepkileri ve yanıtları	Öğrenci tepkilerine karşı öğretmen yanıtları/ göz önünde bulundurulması gerekenler	Amaçlar ve değerlendirme yöntemleri
Hatırlatma: – Dersin başında öğrencilere "Bir önceki dersimizde neler görmüştük?" sorusu sorulur. **Etkinlik 1:**  – Şekli tahtaya yapıştırılır ve bir öğrenciden alanını hesaplaması istenir. (tahtaya çizilir) 	– Alan bulma – Dikdörtgenin alanı – Karenin alanı – Uzun kenar x Kısa kenar – Kenar x Kenar	– Dikdörtgenin alanını nasıl buluyorduk? – Karenin alanını nasıl buluyorduk?	– Geçen dersin hatırlatması yapılır.
– Şekil yukarıdaki hale getirilir ve öğrenciye "şu anki şeklimizin alanı nedir?" diye sorulur. (tahtaya çizilir) – Daha sonra şeklimiz aşağıdaki gibi yapıştırılır.	– Alan değişmez, aynı olur. – Alan değişir. – Çünkü şeklin üzerinde bir değişiklik yapmadık.	– Neden? – Neden? – Arkadaşınızdan farklı düşünen var mı?	

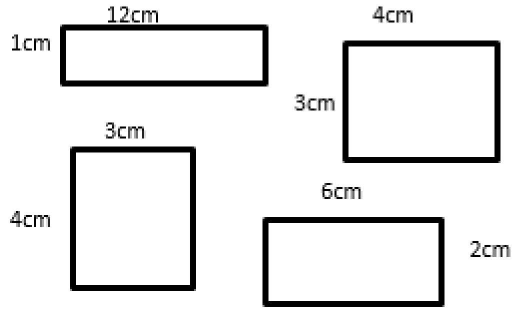
 – Şimdiki şeklimizin alanı nedir?	– Şekil değişti. – Şekil büyüdü. – Alan aynıdır, değişme olmaz. – Alan değişir. – Şekil değişmedi – Şekil artık dikdörtgen değil – Şekil değişti – Şekil büyüdü – 20cm – 40cm – 20cm – 40cm – Hayır – Evet – Hayır	– Neden? – Neden? – Yatay şekil için “kısa kenarımız ne kadar?” – “uzun kenarımız ne kadar?” – Eğik şekil için kısa kenarımız ne kadar?” – “uzun kenarımız ne kadar?” – Şekillerin kenarları değişti mi? – O halde alanı kenarları çarparak bulduğumuz için, kenarlar da değişmediğinden alan da değişmedi, diyebilir miyiz? – Hayır cevabı veren öğrencilere dikdörtgenin alanı hatırlatılarak yönlendirilir.	– Dikdörtgende kenar değişmediği sürece alanın sabit kalacağı gösterilmeye çalışılır. – Şekilleri döndürdüğümüzde alanların değişmediği anlaşıldı mı?

Etkinlik 2:  – Şekilleri tahtaya yapıştırılır. – Bu şeklimiz nedir? – Şekillerin isimleri altlarına yazılır. – Taralı alanların büyüklükleri arasında fark var mıdır?	– Dikdörtgen ve üçgen – Evet – Hayır – Kenar değişti – Şekil değişti – Biri dikdörtgen biri üçgen – Taralı alan değişmedi – Karton aynı kaldı – Renk aynı	– Neden? – Neden? – Taralı alanın bir parçasını alıp yan tarafa ekledik. Taralı alanın tamamında bir değişiklik olmadığı için alan değişmemiştir.	– Bütünün bir parçasını alıp şekli değiştirdiğimizde bütünün boyutunda bir değişme olmadığından alanda da değişme olmaz. – Alanın korunumu anlaşıldı mı? – Öğrenci aynı alana sahip farklı şekiller olabileceğini sezdi mi?
 – Bu şekillerimiz nedir? – Şekillerin altına isimleri yazılır. – Taralı alanların büyüklükleri arasında fark var mıdır?	– Kare ve dikdörtgen – Evet – Hayır – Şekil değişti – Kenarlar değişti	– Neden? – Neden? – Taralı alanın bir parçasını alıp yan tarafa	– Bütünün bir parçasını alıp şekli değiştirdiğimizde bütünün boyutunda bir değişme olmadığından alanda da değişme olmaz.

 – Bu şekillerimiz nedir? – Şekillerin altına isimleri yazılır. – Taralı alanların büyüklükleri arasında fark var mıdır?” **Etkinlik 3:**  – Şekli verilir. Öğrenciden şeklin kaç birim kareden oluştuğunu bulması ve şeklin altına yazması istenir.	– Dikdörtgen daha uzun – Karton aynı kaldı – Renkler aynı – Yamuk ve paralelkenar – Evet – Hayır – Şekil değişti – Kenarlar değişti – Dikdörtgen daha uzun – Karton aynı kaldı – Renkler aynı	ekledik. Taralı alanın tamamında bir değişiklik olmadığı için alan değişmemiştir. – Neden? – Neden? – Taralı alanın bir parçasını alıp yan tarafa ekledik. Taralı alanın tamamında bir değişiklik olmadığı için alan değişmemiştir.	– Alanın korunumu anlaşıldı mı? – Öğrenci aynı alana sahip farklı şekiller olabileceğini sezdi mi?

– Sınıftan başka bir öğrenci seçilir ve bu birim karelerden dikdörtgen oluşturması ve alanını matematiksel olarak ifade etmesi istenir. – 2x 8 = 16 birim kare – Bir başka öğrenci seçilerek tahtadaki dikdörtgenin birim karelerinin yerlerini değiştirerek kare oluşturması istenir. – Şeklimiz kaç birim kareydi? – -Alan korunumunun tanımı verilir. – -Bir şekil küçük parçalara ayrılıp, bu parçaların yerleri değiştirilerek düzenlenip yeni şekil oluşturulduğunda alanda bir değişikliğin olmamasına alan korunumu denir. **Etkinlik 4:** – Sonrasında bir öğrenciye 10 birim kare verilir ve tahtada farklı şekiller oluşturması istenir. Her defasında alanın değişmediği ifade edilir.	– 16 birim kare – Kenar x Kenar – 4 birim	– Karenin alanı neydi? – Alanı 16 br^2 olan karenin kenarı kaç birim olmalı? – Cevap gelmezse “Hangi sayıyı kendisiyle çarparsam 16 eder?” şeklinde öğrenci yönlendirilir. – Önceki örneklerimizi bu tanımla ilişkilendirebilir miyiz? – Alan korunumu o örneklerde var mıdır?	– Birim karelerin yerini değiştirerek aynı alana sahip farklı şekillerin oluşturulabileceğini görür. – Alan korunumu tanımı yapılır.

Etkinlik 5:



Uzunluk				
Genişlik				
Alan				
Çevre				

- Öğrencilerden şekillerin çevrelerini ve alanlarını bularak tabloyu doldurmaları istenir.
- Şekillerin arasında nasıl bir ilişki var

SORU 1:

- 28 birim kare kullanarak kaç farklı dikdörtgen oluşturabiliriz? Şekilleri defterlerinize çiziniz.
- Öğrencilerden gelen cevaplar tahtaya çizilir.

SORU 2:

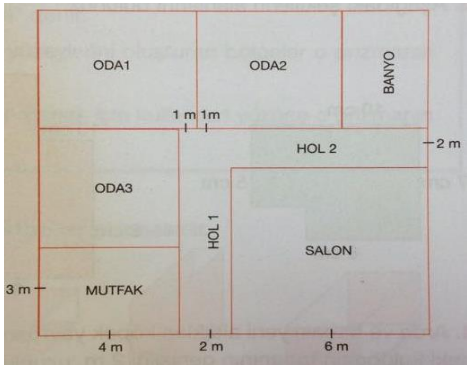
- Bir dikdörtgenin uzun kenarı 28 cm kısa kenarı 4 cm'dir. Bu dikdörtgenin uzun kenarı $\frac{1}{2}$ oranında azaltılır kısa kenarı 3 katına

- Alanlar aynı
- Çevreler aynı da olabilir, farklı da olabilir.

- 56

- Çevreleri arasında nasıl bir ilişki var?
- Demek ki alanları aynı olan şekillerin çevreleri farklı da olabilir, aynı da olabilir.

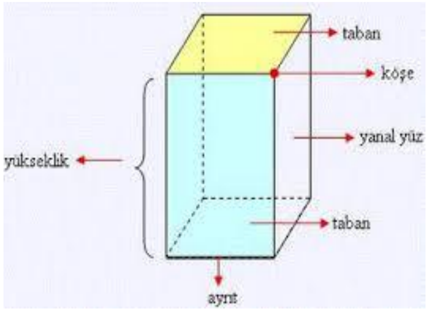
- Problem anlaşıldı mı?
- Öğrenciler çözüme ulaşabildiler mi?

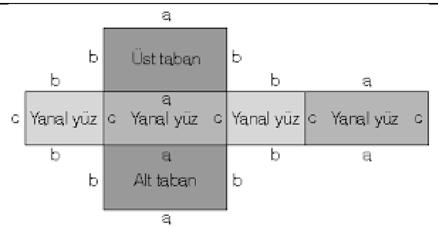
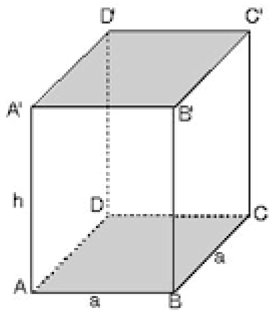
çıkartılırsa dikdörtgenin alanında nasıl değişim olur? **SORU 3:** – Uzun kenarı 24cm, kısa kenarı 12cm olan bir dikdörtgenin içine hiç boşluk kalmayacak şekilde bir kenarı 3cm olan karelerden kaç tane çizebiliriz? **SORU 4:** – Boyu eninin 4 katı olan bir dikdörtgenin çevresi 60 cm olduğuna göre bu dikdörtgenin alanı kaç cm^2dir? – Sınıftan bir öğrenci seçilerek tahtada çözüme ulaşması istenir. **SORU 5:**  – Evimiz dikdörtgen şeklindedir ve planı yukarıdaki gibidir. SALON' umuzun alanı 42 m^2, ODA 1, ODA 2 ve BANYO'nun oluşturduğu dikdörtgenin alanı ise 60 m^2'dir. Verilenlere göre evimizin alanı kaç m^2'dir? **SORU 6:** Doğan, kirlenen halılarını yıkatmak için Halı Yıkamacıya verecektir. Halıların boyutları 5m ve 2 m, 4	– 32 – 144 cm^2 – 168 m^2		– Problemler anlaşıldı mı? – Öğrenciler çözüme ulaşabildiler mi?

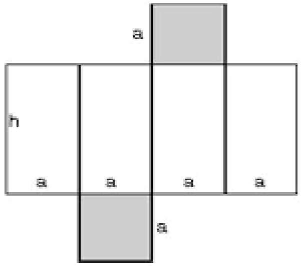
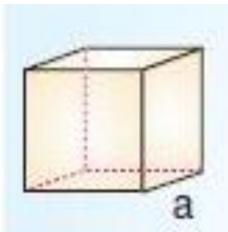
ve 6m, 3 ve 5m dir. Halı yıkama ücreti m^2 başına 2tl olduğuna göre – Doğan ne kadar ücret öder? – Doğan'ın 100lirası varsa ne kadar parası kalır? **Tekrar:** – Bugün ne öğrendik? – Alanları aynı olan farklı şekiller oluşturabilir miyiz? – Alanları aynı olan şekillerin çevreleri için ne söyleyebiliriz?	– 98 – 2		

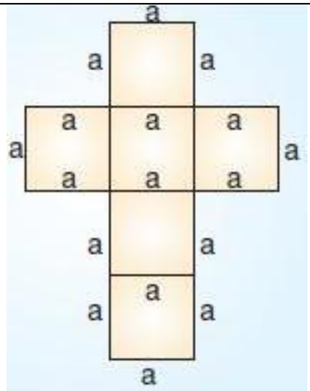
D.4 The Revised Lesson Plan 4

Dersin Adımları: öğrenme aktiviteleri ve anahtar sorular	Öğrenci aktiviteleri/ beklenen öğrenci tepkileri ve yanıtları	Öğrenci tepkilerine karşı öğretmen yanıtları/ göz önünde bulundurulması gerekenler	Amaçlar ve değerlendirme yöntemleri
Hatırlatma: – "Bir önceki dersimizde neler görmüştük hatırlayalım." – Alanları eşit farklı şekiller oluşturabilir miyiz?	– Dikdörtgenin ve karenin alanını – Alan birimlerini – Evet – Birim karelerin ya da kartonun yerini değiştirmiştik. – Olabilir de olmayabilir de.	– Nasıl oluşturuyorduk şekillerimizi? – Şekillerin çevrelerinde bir değişiklik oluyor muydu?	– Bir önceki dersin tekrarı yapılarak neler öğrenilmiş olduğu hatırlatılır.
Etkinlik 1: – Bilgisayar programı yardımıyla önce bir nokta çizilir ardından bir uzunluk çizilir burada tek bir özellik bakımından inceleyerek çevre bulduğumuz, ardından genişlik çizilir burada iki özellik bakımından inceleyerek alan hesapladığımız açıklanır. Daha sonra yükseklik çizilerek şeklin 3 boyut kazandığı ve taban alanına yükseklik kazandırılarak oluşan bu şeklin "Prizma" olarak adlandırıldığı açıklanır. ***Dikdörtgenler Prizmanın Tanıtılması*** – Açılır kapanır olarak hazırlanan dikdörtgenler prizması somut modeli üzerinde öğrencilere prizmanın elemanları tanıtılır.	– Nokta – Doğru parçası – Çizgi – Uzunluk – Kare – Dikdörtgen – Uzunluk ve genişlik	– Bu nedir? – Noktayı uzattım bir çizgi çizdim. Buna ne deriz? – Bu çizgiyi şu şekilde genişlettim ne oldu? – Hangi özellikler var burada?	– Prizmanın tanımı deftere yazdırılır. – Dikdörtgende en ve boy olduğundan 2 boyut, prizmada en, boy ve yükseklik olduğundan 3 boyutlu olduğu vurgulanır.

 – Materyal üzerinde yukarıdaki gibi isimlendirmeler yapılarak sınıf içinde dolaşılıp öğrencilerin modeli yakından görmeleri sağlanır. – Somut model üzerinde tanıtılan elemanların günlük hayatla ilişkisini kurmak için sınıf ortamını dikdörtgenler prizması olarak düşünmeleri öğrencilerden istenerek sınıf içinde elemanlar tekrar vurgulanır. – Öğrencilere prizma modeliyle ilgili sorular sorulur. – Bu dikdörtgenler prizmasının kaç yüzü vardır? – Kaç ayrıtı vardır? – Kaç köşesi vardır? – Bu prizmanın tabanlarındaki geometrik şekil nedir? – Bu prizmanın yanal yüzlerindeki geometrik şekil nedir? – Model açık hale getirilir. Prizmanın elemanları tekrar edilir.	– Üç boyutlu – Prizma – Yükseklik – Uzunluk, genişlik ve yüksekliği var – 3 tane özellik var – 6 yüz – 12 ayrıt – 8 köşe – Dikdörtgen – Dikdörtgen	– Şekli biraz daha değiştiriyorum. Ne oldu? – Hangi özelliği ekledim? – Bu şekillere neden üç boyutlu deniyor sizce? – Tabanı ve yanal yüzleri dikdörtgen olan prizmaya <u>dikdörtgenler prizması</u> denir.	– Modelin dikdörtgenler prizması olduğu vurgulanır. – Prizmanın elemanlarının anlaşılması sağlanır. – Dikdörtgenler prizmasının özellikleri tanıtılır. – Tabanlarında yüz olduğu, yükseliğin de ayrıt olduğu vurgulanır.

 Dikdörtgenler prizmasının açılmış hâli – Dikdörtgenler prizmasının açık ve kapalı halleri, elemanlar üzerinde gösterilecek şekilde tahtaya çizilir ve öğrencilerden bunları deftere çizmeleri istenir. ***Kare Prizmanın Tanıtılması*** – Somut kare prizma modeli üzerinde prizma elemanları tanıtılır.	– 4 – 2 – Tabanlar – Karşılıklı yüzler	– Kaç tane yanal yüz var? – Kaç tane taban var? – Eşit olan yüzeler var mı?	– Taban ve yan yüzlerindeki geometrik şekillerin ne olduğu vurgulanır.
 – Sınıf içinde dolaştırılarak öğrencilerin görmeleri sağlanır. – Bu cismin tabanları hangi geometrik şekildir? – Bu cismin yan yüzleri hangi geometrik şekildir? – Somut materyal açılarak kare prizmanın açınımı öğrencilere gösterilir.	– Kare – Dikdörtgen	– Tabanı kare olduğu için, tabanı kare yanal yüzleri dikdörtgen olan prizmaya kare prizma denir.	– Kare prizmayı oluşturan geometrik şekiller ve özellikleri vurgulanır.

 – Kare prizmanın özellikleri anlatılır. Tabanlar kare, yanal yüzler eşit gibi.	– 4 – 2 – Tabanlar – Karşılıklı yüzler	– Kaç tane yanal yüz var? – Kaç tane taban var? – Eşit olan yüzeler var mı?	
Küpün Tanıtılması – Somut küp modeli üzerinde prizma elemanları tanıtılır.  – Bu prizmanın taban yüzleri hangi geometrik şekilden oluşmaktadır? – Bu cismin yan yüzleri hangi geometrik şekildir.	– Kare – Kare	– Kare de bir dikdörtgendir. Bütün yüzleri kare olan prizmaya özel olarak küp denir.	
– Somut materyal açılarak küpün açınımı öğrencilere gösterilir.			– Küpün tüm yüzlerinin kare ve kenarlarının eşit olduğu gösterilir.



- Küpün özellikleri anlatılır. Tabanlar kare, yanal yüzeyler kare, bütün ayrıtlar eşit, bütün yanal yüzler eşit gibi.
- Dikdörtgenler prizması, kare prizma ve küp arasındaki farkı taban ve yan yüzler açısından net görmelerini sağlayacak tabloyu öğrencilerden doldurmaları istenir.

	Yan Yüzler	Taban
Dikdörtgenler Prizması	Dikdörtgen	Dikdörtgen
Kare Prizma	Dikdörtgen	Kare
Küp	Kare	Kare

Etkinlik 2:

- Bilgisayar programı yardımıyla prizmaların döndürülerek açılıp kapanışları farklı yönlerden gösterilir.

- 4
- 2
- Hepsi yan yüz ya da hepsi taban
- Hepsi kare
- Hepsi eşit

- Dikdörtgenler prizması hep dikdörtgen
- Kare prizma hem dikdörgen hem de kareden oluşuyor
- Küp hep kare
- Bütün hepsi dikdörtgenler prizması

- Kaç tane yanal yüz var?
- Kaç tane taban var?
- Eşit olan yüzeler var mı?

- Üç prizma çeşiti arasındaki benzerlik ve farklılıklar neler?

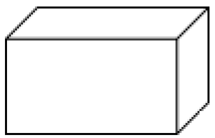
- İsminin özel bir durum olduğu açıklanır.

- Prizmalar arasındaki farklar vurgulanır ve pekişmesi sağlanır.

- Prizmaların açınım ve kapanışlarının kavranması sağlanır.

- Önceki somut modellerdeki açık haller ile bu etkinlikte gösterilen açık prizma modellerinin farklı olduğunu öğrencilerin anlaması sağlanır.

Etkinlik 3: – Dikdörtgenler prizması, kare prizma ve küpün ilk gösterimden farklı şekillerdeki açınımları öğrencilere gösterilir. Hangi açınımın hangi prizmaya ait olduğunu tahmin etmeleri istenir. Sırasıyla öğrenciler kaldırılarak şekilleri kapatmaları ve hangi prizmanın oluştuğunu sınıfa göstermeleri istenir. – Kapandığında prizma oluşturmayan açınımlar verilerek öğrencilerden hangi şekil oluşacağını tahmin etmeleri istenir. Öğrenciler tahtaya kaldırılarak açınımların kapandığında prizma oluşturmadığını sınıfa göstermeleri istenir. **Etkinlik 4:** – Prizma açınımlarının kapanabilen ve kapanamayan çeşitli örnekleri bilgisayar programı yardımıyla gösterilir. **Etkinlik 5:** ***Prizmaların Yüzey Alanlarının Hesaplanması*** – Kapalı şekildeki dikdörtgenler prizması için sırasıyla kenar uzunlukları ölçülerek üzerine yazılır. Öğrencilerden her bir yüzün alanını hesaplayarak üzerine yazmaları istenir. Daha sonra kapalı model açılır ve prizmanın yüzey alanın, tüm yüzey alanlarının toplamı olduğu gösterilerek yüzey alanları hesaplanır. – Aynı süreç kare prizma ve küpün yüzey alanlarını hesaplama için de yapılır ve öğrencilerin kare prizma için 2xtaban alanı + 4xyanal yüz, küp için 6xyüz genel sonucuna ulaşmaları sağlanır.	– axb – axa – Dikdörtgen – Dikdörtgen ve kare – 4 dikdörtgen, 2 kare – Kare – 6 eşit kare	– Dikdörtgenin alanını nasıl buluyorduk? – Karenin alanını nasıl buluyorduk? – Dikdörtgenler prizmasi hangi alanlardan oluşacak? – Kare prizma hangi alanlardan oluşacak? – Küpte hangi alanları bulecağız?	– Farklı açınımların olabileceği ve her açınımın kapanıp prizma oluşturmayabileceği vurgulanır. – Prizmaların yüzlerinin ne olduğu, kare ve dikdörtgenin alanın hesaplanışı öğrencilere sorularak hatırlamaları sağlanır. – – Prizmaların kapalı ve açık halleri tahtaya çizilir. Kapalı halinde ayrıt uzunlukları, açık halinde de şekilleri içine alan ölçüleri yazılır. – Çocuğunda yüzey alanının bütün yüzeylerin toplamı olduğunu görmesi sağlanır. – -Kare prizma ve küp için yüzey alanı formülleri öğrencilerle birlikte oluşturularak

Soru 1:  – Yukarıdaki dikdörtgenler prizmasının yüzey alanını bulunuz. – Bir öğrenciden tahtada soruyu çözmesi istenir. **Soru 2:** – Uzunluğu ve eni 7 m yüksekliği 8 m olan prizma şeklindeki bir sınıfın yüzey alanını hesaplayınız. **Soru 3:** – Bir kenar uzunluğu 2 cm olan bir zarın yüzey alanı kaç cm^2'dir?	– $2(10x8)+2(10x6)+2(6x8)=376$ cm^2 – $2(7x7)+4(7x8)=322$ m^2 – $6x(2x2)=24$ cm^2		tahtaya ve deftere yazılır. – Problemler anlaşıldı mı? – Öğrenci şeklin kare prizma olduğunu fark edebildi mi?

APPENDIX E: VITA

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EDUCATION

Degree	Institution	Year of Graduation
PHD	Middle East Technical University, Elementary Education	2017
MS	Marmara University, Mathematics Education	2012
BS	İstanbul University, Mathematics Education	2010

WORK EXPERIENCE

Year	Place	Enrollment
2015- Present	İstanbul University, Faculty of Education	Research Assistant
2013-2015	Ondokuz Mayıs University, Faculty of Education	Research Assistant
2012-2013	Marmara University, Faculty of Education	Research Assistant
2011-2012	Ondokuz Mayıs University, Faculty of Education	Research Assistant

FOREIGN LANGUAGES

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2. Şengül, S. & **Güner, P.** (2012). *Proof Schemes Used by Mathematics Teachers Candidates.* 1st International Eurasian Conference on Mathematical Sciences and Applications, Pristina, Kosovo, 3-7 September.

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APPENDIX F: TURKISH SUMMARY/TÜRKÇE ÖZET

ORTAOKUL MATEMATİK ÖĞRETMENİ ADAYLARININ FARK ETME BECERİLERİNİN DERS İMECESİ MESLEKİ GELİŞİM MODELİ KAPSAMINDA İNCELENMESİ

Giriş

Son dönemde önem kazanan ve üzerinde durulan kavramlardan birisi fark etme becerisidir. Fark etme, günlük yaşamda bir şeyi gözlemleme, anlamlandırma ve ayırt etme gibi anlamlara gelmektedir (Miller, 2011). Öğretmenin fark etmesi ise sınıf içi etkileşimlerde ne üzerinde durduğu ve gerçekleşen olayları nasıl yorumladığıdır (Jacobs, Lamb & Philipp, 2010; Mason, 2002). Neyin önemli olduğunu tanımlayabilmeyi ve bu durumlar üzerinde akıl yürütüp, gözlemlenenleri anlamlandırmayı kapsamaktadır (Van Es & Sherin, 2002). Fark etme görülenin yeniden yapılandırıldığı bir aktivite olduğundan (Gibson, 1979), aynı olay gözlemlenmesine rağmen odaklanılan noktalar ve çıkartılan anlamlar birbirinden farklı olabilmektedir (Jacobs, Lamb & Philipp, 2010). Bunun yanı sıra, neyin fark edildiğinin belirgin bir şekilde tanımlanması, fark etmenin anlamlı olduğunu göstermek için yeterli değildir. Nasıl fark edildiği, odaklanılan olayların nasıl anlamlandırıldığı ve bu doğrultuda nasıl kararlar alındığı da üzerinde durulması gereken boyutlardır (Berliner, 2001; Davis, 2006; Mason, 2002; Seidel & Stürmer, 2014; Star & Strickland, 2008; van Es, 2011; van Es & Sherin, 2002). Fark etmenin iki temel yönü bulunmaktadır: Bunlar, "a) bir öğretim durumunda neyin önemli olduğunu tanımlama ve b) durum hakkında akıl yürütmek için öğretme ve öğrenme bilgisinden faydalanma" (van Es & Sherin, 2006, p. 125) şeklindedir. Önemli olaylara odaklanma, bunları öğretim ilkeleri ile ilişkilendirme ve öğrencilerin düşünme şekilleri açısından yorumlama fark etme becerisinin göstergedir (van Es, 2011; Yang & Ricks, 2012). İyi bir öğretmenin bu beceriye sahip olması beklenmektedir (Van Es & Sherin, 2010).

Öğretmen adayları mesleğe başladıklarında çeşitli zorluklarla karşılaşmakta (Huling-Austin, 1992) ve pek çok şeyi yaparak öğrenmektedir (Flores, 2006). Diğer bir yandan, van Es and Sherin (2002) öğretmen eğitimdeki eksiklikleri vurgulayarak, programların öğretmen adaylarına sınıf etkileşimlerde neye odaklanmaları ve bunları nasıl yorumlamaları gerektiğini öğretme hususunda yardımcı olmadığını dile getirmektedir. Öğretmen adayları okul deneyimi ve öğretmenlik uygulaması gibi derslerle öğrendiklerini uygulamaya aktarma, öğretmenleri sınıflarında gözlemleme ve gerçek öğretim ortamında bulunma gibi bazı fırsatlar elde etmektedir. Fakat bu fırsatlar sınırlıdır ve öğretmen adaylarını mesleğe hazırlamak için yeterli değildir (Clift & Brady, 2005). Öğretmen adaylarının başarılı bir öğretmen olmak için kazanması gereken mesleki becerilerden birisi fark etmedir (Mason 2002; Sherin et al. 2010). Bu noktada, van Es and Sherin (2008) öğretmen adaylarının sınıf ortamında gerçekleşen olayların önemli yönlerine dikkat edebilme ve bunları anlamlandırabilme becerilerinin, yani, fark etme becerilerinin gerekli destek sağlanarak geliştirmesi gerektiğini savunmaktadır.

Diğer bir taraftan, matematik eğitimi reform hareketlerine göre öğretmenler öğrencilerin düşüncelerini dikkatli bir şekilde dinlemeli ve bunu öğretimin akışını düzenlerken kullanmalıdır (Arvold, Tumer, & Cooney, 1996; Ball & Cohen, 1999; Hiebert et al., 1996: NCTM, 2000). Bu ise ders sırasında öğrencilerin nasıl düşündüklerine odaklanmayı ve onları anlamlandırmayı gerektirmektedir (Rodgers, 2002). Öğretmenin öğrencilerin düşünme şekillerini fark etme becerisi etkili öğretimin ve yüksek öğrenci başarısının altında yatan nedenlerden birisi olarak görülmektedir (Carpenter, Fennema, Peterson, Chiang & Loef, 1989; van Es, 2011). Sınıf ortamında pek çok konu öğretmenin dikkatini çekebilir fakat öğrencilerin matematiksel düşünmelerinin, hatalarının ve yanılgılarının ön plana çıktığı durumlar matematik eğitimi açısından önemlidir (van Es & Sherin, 2010). Öğrencilerin matematiksel düşünmelerini fark etme becerilerindeki artışın matematik eğitiminin gelişimini sağladığı düşünülmektedir (Liu, 2014). Fakat pek çok araştırma öğretmenlerin öğrencilerinin matematiksel olarak nasıl düşündüklerine odaklanmadıklarını ve stratejilerinin arkasında yatan nedenleri anlamaya çalışmadıklarını ortaya koymaktadır (Kazemi & Franke, 2004; Koellner-Clark & Lesh, 2003). Ayrıca, çalışmalar öğretmenlerin öğrencilerinin düşünme şekillerini fark etme becerilerinin mesleki gelişim modelleri yardımıyla gelişebileceğini de vurgulamaktadır (Kazemi & Franke, 2004; Koellner-Clark & Lesh, 2003).

Öğretmen eğitiminde ve mesleki gelişim çalışmalarında önem kazanan yaklaşımlardan birisi ders imecesi (lesson study) modelidir. Ders imecesi mesleki gelişim modeli öğretmenlere gerekli noktalara dikkat etme, planlama yapma, dersi gözlemleme ve analiz etme, olayları yorumlama, derse ilişkin düşüncelerini yansıtıp bunlar üzerinde tartışma fırsatları sunulan ve bu becerilerinin gelişmesini destekleyen bir modeldir. Dolayısıyla, ders imecesinde öğrencilerin düşünme ve anlama şekilleri üzerine devamlı bir odaklanma ve anlamlandırma süreci mevcuttur (Hurd & Licciardo-Musso, 2005). Mason (2002)'a göre fark etme kişinin kendi tecrübelerinin, başkalarının tecrübelerinin, gözlemlerinin ve teorilerin birleşmesiyle gerçekleşmekte ve bu birleşim mesleki gelişime olanak tanımaktadır.

Ders imecesi öğretmen ve öğretmen adaylarının mesleki anlamda gelişimlerini sağlamayı hedefleyen işbirliğine dayalı bir modeldir. Bu model ders imecesi grubunu oluşturan bireylerin bir araya gelerek yürüttükleri planlama, öğretim ve yansıtma aşamalarını kapsayan bir döngüden oluşmaktadır (Baki, 2012; Lewis, 2002; Lewis & Tsuchida, 1998). İhtiyaca bağlı olarak öğretim ve yansıtma aşamaları ders revize edilerek tekrar edilmektedir. Ders imecesinin uygulanma şekilleri kültürel farklılıklara bağlı olarak çeşitlilik gösterse de modelin temel aşamaları ve süreç öğeleri değişmemektedir (Murata, 2011). *Planlama aşamasında,* katılımcılar öncelikle konunun öğretimine, öğrencilerin konuyla ilgili anlamalarına, düşünme stratejilerine ve öğrenme sırasında yaşadığı zorluklara, öğretim programının içeriğine ve öğretme stratejilerine yönelik çeşitli kaynaklardan araştırmalar yapmaktadır (Lewis, 2002). Araştırarak öğrendiklerinden, kendi deneyimlerinden ve gözlemlerinden faydalanarak fikirlerini paylaşmakta ve dersin içeriğini oluşturmaya çalışmaktadır (Fernandez & Yoshika, 2004). Grup üyeleri bu süreçte konuyla ilgili matematiksel etkinlikler ve sorular belirlemekte, bunların farklı çözüm yolları üzerine tartışmakta, öğrencilerin bunlara verebilecekleri olası yanıtları düşünmekte ve kendilerinin öğrencilerin yanıtları karşısında nasıl dönüt verebilecekleri üzerine konuşmaktadır (Murata, 2011). Üyelerin bu noktalara dikkat ederek fikirlerini birbirleriyle paylaşmaları ise kendilerinin matematiksel olarak nasıl düşündüklerini görmelerinin yanı sıra öğrencilerin de olası matematiksel düşünme şekillerini fark etmelerini ve anlamlandırmalarını kolaylaştırmaktadır (Lewis, Friedkin, Baker & Perry, 2011).

Öğretim aşamasında, hazırlanan ders planı grup üyelerinden birisi tarafından sınıf ortamında uygulanırken, gruptaki diğer üyeler dersi gözlemlemekte ve detaylı notlar almaktadırlar (Lewis, 2002; Fernandez & Yoshika, 2004). Yürütülen dersin hangi açılardan öğrencilerin öğrenmesini kolaylaştırdığını ya da zorlaştırdığını belirlemek içinse gruptaki bireylerin öğrencilerin nasıl düşündüklerine dikkat etmeleri oldukça önemlidir (Yoshida, 2005). *Yansıtma aşaması,* dersin öğretiminden sonra grup üyelerinin bir araya gelmesiyle gerçekleşmektedir. Dersin öğretimini yapan kişi yürüttüğü derse ilişkin kendi düşüncelerini ve paylaşmak istediği noktaları diğerlerine aktarmakta, diğer grup üyeleri de sürece ilişkin tuttukları notlara dayanarak fikirlerini bildirmekte ve dersi nasıl geliştirecekleri üzerine önerilerde bulunmaktadırlar. Daha sonra, yapılan değerlendirmeye dayanarak uygun görülen değişiklikler yapılmakta ve ders planı revize edilmektedir (Fernandez & Yoshika, 2004). İsteğe bağlı olarak, revize edilen ders planı diğer bir grup üyesi tarafından başka bir sınıfta uygulanmaktadır. Aynı sürecin gerçekleşmesiyle, ders sonunda dersin öğretimine yönelik fikirler yansıtılmakta ve ders planına son şekli verilerek ders imecesi döngüsü tamamlanmaktadır (Lewis, 2002). Kısacası, ders imecesi öğrencilerin öğrenmelerinin göz önünde bulundurularak grup üyeleri tarafından planlamanın yapıldığı, dersin yürütüldüğü, gözlemlerin yapıldığı ve derse ilişkin düşüncelerin grupça paylaşıldığı okul tabanlı bir mesleki gelişim modelidir (Lewis, Perry, Friedkin & Roth, 2012).

Ders planlarının oluşturulabilmesi ve bunların hedeflendiği gibi derse aktarılabilmesi kolay değildir (Leinhardt & Greeno, 1986; Resnick, 1987) ve öğretmen adayları açısından, sahip oldukları bilgi, tecrübe gibi pek çok nedenden dolayı bunları yapabilmek daha da zorlu bir süreci kapsamaktadır. Bu modelde olduğu olduğu gibi, öğretmenin öğrencilerinin düşünme stratejilerine ve konuyu nasıl anladıklarına dikkat etmesi, öğretim sırasında bu noktaları göz önünde bulundurması ve dersi buradan yaptığı çıkarımlara dayanarak zenginleştirmesi hem öğretmenin hem de öğrencinin gelişimini sağlamaktadır (Borko, Jacobs, Eiteljrg & Pittman, 2008). Burada önemi vurgulanan dikkat etme, anlamlandırma ve karar verme eylemleri ise araştırmacılar tarafından fark etme olarak adlandırılan beceriye karşılık gelmektedir (Mason, 2002; Sherin, Jacobs & Philipp, 2011). Bu beceri sadece öğretmenlerin değil aynı zamanda öğretmen adaylarının da kazanması gereken temel mesleki becerilerden birisidir (Mason 2002; Sherin et al, 2011). Çünkü yeterli tecrübeye sahip olmayan öğretmen ve öğretmen adayları öğrenciye ve önemli öğretimsel durumlara odaklanarak dersi şekillendirmekte zorlanmaktadır (Berliner, Stein, Sabers, Clarridge, Cushing, & Pinnegar, 1988; Carter, Cushing, Sabers, Stein, & Berliner, 1988). Dolayısıyla, pek çok araştırma öğretmen adaylarının da öğrenme ve öğretme etkileşimlerinde neler fark ettiği ve bunları nasıl yorumladığı konusunda çalışılması gerektiğini ortaya koymaktadır (Hand 2012; Mason 2008, 2011; Santagata, Zannoni & Stigler; 2007; Sherin et al., 2011; Star & Strickland 2008; van Es 2011). Sherin ve van Es (2005) öğretmen adaylarına fark etme becerilerini geliştirebilecekleri ortamların sunulması gerektiğini savunmaktadır. Bu noktada, ders imecesi öğrencilerin matematiksel düşünme biçimlerine odaklanma, onların kullandıkları stratejileri anlama ve bu doğrultuda dersi şekillendirme fırsatı verdiği için bu modelin öğretmen adaylarının fark etme becerilerin geliştirebileceği düşünülmektedir. Dolayısıyla, bu çalışmada ders imecesi modeli ortaokul matematik öğretmen adaylarının öğrencilerin matematiksel düşünmelerini fark etme becerilerini incelemede ve araştırma sürecinin tasarlanmasında bir bağlam sağlamıştır.

Diğer taraftan, "Geometri gerçeği nasıl matematikleştireceğini öğrenmeyi sağlayan en iyi fırsatlardan birisidir" (Freudenthal, 1973, p. 407) ve farklı bakış açıları kazanmaya yardım eder (Duval, 1998). "Ölçme hem matematiği hem de bilimi kapsayan ancak gündelik tectübeye dayanan bir girişimdir" (Lehrer, 2003 as cited in Kellogg, 2010) ve geometri ile sayılar arasındaki bağın kurulmasını sağlar (Clements & Stephan, 2003; Lehrer, 2003). Geometri ve ölçme pek çok uygulamayı ve gösterimi barındırdığı için öğrenmesi ve öğretmesi zor bir alandır (Royal Society, 2001). Çünkü geometrik ölçümler yapabilme çeşitli bilgi ve becerilerin birikimine dayanan ve gelişimi kolay olmayan bir beceridir (Clements & Stephan, 2003; Lehrer, Jaslow, & Curtis, 2003). Bu nedenle, öğrenciler geometrik ve ölçmeyle ilgili kavramları anlamakta ve kullanmakta, ayrıca bunlar arasında ilişkiler kurmakta zorluk yaşamaktadır (Martin, 2009). Benzer şekilde, ölçme ile geometrik kavramları ilişkilendirmek ve bu bilgiyi problem çözme sürecine transfer edebilmek öğrenciler açısından kolay değildir (Cavanagh, 2008; Danielson, 2005; Emekli, 2001; Kordaki & Potari, 1998; Zacharos, 2005; 2006; Tan Şişman & Aksu, 2009; Zembat, 2009). Literatürde pekçok çalışma en temel kavramları öğrenmede bile zorluk yaşadıklarından dolayı öğrencilerin bu alanda başarısının düşük olduğunu ortaya koymaktadır (Akuysal, 2007; Çetin & Dane, 2004; Dane, 2008; Mullis et al., 2000; NCTM, 2000; Prescott, Mitchelmore, & White, 2002). Öğretmenin yaklaşımı (Olkun & Aydoğdu, 2003); kavramların anlamlarını içselleştirmek yerine öğrencilerin kavramları, formülleri, kuralları ezberleme eğilimi ve yeterli olmayan örnek gösterimleri öğrencilerin bu kavramları anlamakta yaşadıkları zorlukların ve bu alandaki düşük başarısının nedenleri olarak görülmektedir (Fujita & Jones, 2007; Yılmaz, Turgut & Kabakçı, 2008). Öte yandan, pekçok araştırma öğretmen adaylarının da geometric kavramlarla zorluklara ve kavram yanılgılarına sahip olduklarını göstermektedir (Alkış-Küçükaydın & Gökbulut, 2013; Bozkurt & Koç, 2012; Gutierrez & Jaime, 1999; Koç & Bozkurt, 2011; Linchevski, Vinner, & Karsenty, 1992; Özsoy & Kemankaşlı, 2004; Tunç & Durmuş, 2012). Dolayısıyla, geometri ve ölçme üzerinde durulması gereken öğrenme alanlarından birisidir.

Çevre, alan, yüzey alanı ve prizma kavramları günlük hayattaki pratik özellikleri ve kullanımlarından dolayı matematik müfredatında önemli bir yere sahiptir. Dolayısıyla, öğrencilerin bu kavramlarla ilgili temel becerilere ve kavramsal bilgilere sahip olması beklenmektedir Kenney & Kouba, 1997; Martin & Strutchens, 2000; Ferrer, et al., 2001). Fakat öğrenciler çevre, alan, yüzey alanı (Martin &Strutchens, 2000) ve prizma (Battista & Clements, 1996) kavramlarını tam olarak anlayamamaktadır. Bu durumun nedenleri, formüllerin birbirlerinden ve ilişkili olduğu geometric kavramlardan izole edilerek öğretilmesi; bu kavramlar arasındaki ilişkilerin ve farklılıkların yeterince gösterilememesi; ve bu kavramların geometrik yönlerinin ölçmeye dayanan yapılarından daha çok vurgulanması şeklinde açıklanmaktadır (Battista, 1999; Clements & Battista, 1992; Lehrer, 2003; Lehrer et al., 2003).

Çalışmalar öğrencilerin yanı sıra öğretmen ve öğretmen adaylarının da çevre ve alan kavramlarıyla ilgili yanlış anlamalarının ve öğrencilerin bu kavramlara yönelik sahip oldukları matematiksel düşünmelerini anlamada eksiklerinin olduğunu ortaya koymaktadır (Menon, 1998; Reinke, 1997; Simon & Blume, 1994a; Tierney, Boyd & Davis, 1990; Woodward & Byrd, 1983). Ayrıca, öğretmen adaylarının çevre, alan, yüzey alanı ve hacim kavramlarını birbirine karıştırdığı ve kavramsal anlama olmaksızın ölçme kavramlarının formüllerine güvendikleri görülmektedir (Enochs & Gabel, 1984; Vistro, 1991; Latt, 2007; Sáiz, 2003). Bunun yanı sıra, öğretmen adayları çevre, alan, yüzey alanı ve prizmalarla ilgili bilgi eksikliklerine ve bazı kavram yanılgılarına sahiptir (Livy, Muir & Maher, 2012). Öğretmen adaylarının bu konulardaki eksikliklerinin kendi öğretimini, öğrencilerin ilgili konulardaki matematiksel anlamalarını ve öğrenmelerini olumsuz etkilemesi muhtemeldir (Ball & McDiarmid, 1989; Baturo & Nason1996). Bu noktadan hareketle, bu çalışmada ortaokul matematik öğretmen adaylarının öğrencilerin matematiksel düşünmelerini fark etme becerileri geometri ve ölçme alanında yer alan çevre, alan, yüzey alanı ve prizma konuları kapsamında ele alınmıştır.

İlgili literatür göz önünde bulundurulduğunda, bu çalışmada ortaokul matematik öğretmen adaylarının öğrencilerin matematiksel düşünmelerini fark etme becerilerini ders imecesi mesleki gelişim modeli kapsamında incelemek ve bu modelin öğretmen adaylarının fark etme becerilerinin gelişiminde etkili olup olmadığını belirlemek amaçlanmıştır. Başka bir deyişle, bu çalışmanın amacı ortaokul matematik öğretmen adaylarının dersi planlarken, öğretirken ve derse ilişkin düşüncelerini yansıtırken neye odaklandıklarını ve odaklandıkları olayları nasıl anlamlandırdıklarını ortaya koymaktır. Ayrıca, öğretmen adaylarının öğrencilerin matematiksel düşünme şekillerini fark etme becerilerinde ders imecesi uygulamasına katılımları sürecinde değişikliğin meydana gelip gelmediğini araştırmaktır. Bu doğrultuda çalışmanın araştırma soruları aşağıdaki gibidir:

1. Ortaokul matematik öğretmen adayları öğrencilerin matematiksel düşünme şekillerini ders imecesi mesleki gelişim modeli kapsamında ne derece ve nasıl fark eder?
2. Ders imecesi ortaokul öğretmen adaylarının öğrencilerin matematiksel düşünme şekillerini fark etme becerisine nasıl katkıda bulunur?

Araştırmanın Önemi

Matematik eğitiminde pek çok araştırma öğrencilerin düşünme şekillerine odaklanmanın ve bunları anlamlandırmanın öğretmenlerin öğretim uygulamalarını değiştirdiğini ortaya koymaktadır (Campbell & White, 1997; Carpenter, Fennema, Peterson, Chiang & Loef, 1989; Franke, Carpenter, Levi & Fennema, 2001; Kazemi & Franke, 2004; Lin, 2006; Lubinski & Jaberg, 1997; Mewborn, 2006; Steinberg, Empson & Carpenter, 2004; Vacc & Bright, 1999). Öğrencilerin nasıl düşündükleri üzerine inşa edilen ve onların düşünme şekillerini ortaya çıkarmayı hedefleyen öğretim uygulamalarına bağlı olarak da öğrencilerin matematiksel anlamalarını ve öğrenmelerini arttırdığı sonucuna ulaşılmıştır (Fennema, Franke, Carpenter & Carey, 1993; Hiebert & Wearne, 1993). Literatürde öğretmenlerin öğrencilerin düşünme şekillerine odaklanıp odaklanmadıkları ve bunları nasıl anlamlandırdıkları hakkında detaylı bilgi yer almadığından bu konuların araştırılmasına ihtiyaç vardır. Fark etme öğretmenlerin yanı sıra öğretmen adaylarının da kazanması gereken bir beceri olduğundan bu çalışmanın ortaokul matematik öğretmen adaylarının fark etme becerilerini ortaya koyarak alana katkı yapacağı düşünülmektedir.

Son dönemlerde fark etme becerilerine gösterilen ilgi artmış olmasına rağmen öğretmenlerin fark etme becerileri ve bu beceriyi geliştirme yolları hakkında sahip olunan bilgi sınırlıdır (van Es & Sherin, 2009), dolayısıyla, bu konuyla ilgili daha fazla araştırma yapılmasına ihtiyaç vardır. Bunun yanı sıra, yapılan araştırmaların çoğu fark etme becerisinin göstergesi olarak öğretmenlerin neye odaklandıklarını araştırmıştır (Choy, 2014). Bu araştırmada ise neye odaklanıldığının yanı sıra odaklanılan noktaların nasıl anlamlandırıldığı üzerinde de durulmuştur. Dolayısıyla bu çalışmanın öğretmen adaylarının öğrencilerin matematiksel düşünme biçimlerine ne derece odaklandıklarını ve bunları nasıl anlamlandırdıklarını ortaya koymada faydalı olacağı düşünülmektedir.

Araştırmaların birçoğu fark etme becerilerini incelemek için geriye dönük ya da olay sonrasını yansıtan durumları ele almıştır. Diğer bir yandan, çalışmalar daha çok dersin öğretimi sırasında ya da sonrasında gerçekleşen fark etme becerilerini incelediğinden bu anlamda daha çok çalışmanın yapılmasına ihtiyaç vardır (Choy, 2015). Bu nedenle planlama, öğretim ve öğretim sonrası yansıtma aşamalarının hepsini kapsayan yani bütün bir ders imecesi sürecini yansıtan çalışmaların eksikliği dikkat çekmektedir. Bu çalışma ortaokul matematik öğretmen adaylarının dersi planlarken, öğretim yaparken ve ders üzerine düşüncelerini yansıtırken neye odaklandıklarını ve bunları nasıl anlamlandırdıklarını, yani, bu üç aşamayı barındıran ders imecesi sürecindeki fark etme becerilerini detaylı bir şekilde ortaya koyarak alana katkı sağlayabilir.

Öğretmen adayları öğretmen olarak mesleğe başladıklarında, sınıf ortamının kompleks yapısından dolayı beklemedikleri bir sahneyle karşılaşmakta (Veenman, 1984) ve kendilerini öğretmek için hazır hissetmemektedirler (Hebert & Worthy, 2001). Öğrendiklerini kullanmada yaşadıkları zoluklara ek olarak, öğretmen adayları dersi etkili öğretme, sınıf yönetimi, öğrencilerin bakış açısını kazanma ve onların nasıl düşündüklerini öngörme gibi konularda da problemler yaşamaktadır (Brock & Grady, 1996; Davies & Ferguson, 1997; Hebert & Worthy, 2001). Asıl amaç olmamasına rağmen, bu çalışma öğretmen adaylarının öğretmenlik mesleğine hazırlanmalarına yardım edebilir ve ders imecesi mesleki gelişim modelinin okul deneyimi ve öğretmenlik uygulaması gibi öğretmen yetiştirme programlarına entegre edilmesi hakkında aydınlatıcı bilgi sağlayabilir.

Az sayıda çalışma dışında, tam sayılarda işlerler (Jacobs et al., 2010), orantısal düşünme (Fernandez, Llinares & Valls, 2011) ve cebirsel düşünme (Walkoe, 2013), fark etme becerisiyle ilgili çalışmaların büyük bir çoğunluğu matematiksel düşünmeyi belirli bir alana odaklanmadan genel bir şekilde ele almaktadır (van Es & Sherin, 2002, Borko, Jacobs, Eiteljorg, & Pittman, 2008; Goldsmith & Seago, 2011). Bu nedenle fark etme becerilerinin spesifik matematik öğrenme alanlarında daha fazla araştırılmasına ihtiyaç vardır. Bu amaçla, bu çalışmada geometri ve ölçme öğrenme alanında yer 5. sınıf alan çevre, alan, yüzey alanı ve prizma konularına odaklanılmıştır. Bunun yanı sıra, literatür incelendiğinde fark etme becerilerine yönelik çalışmaların ağırlıklı olarak öğretmenler ile yapıldığı ve öğretmen adaylarıyla yapılan çalışmaların sınırlı olduğu görülmüştür (Hughes, 2006). Bu doğrultuda, araştırmanın ortaokul matematik öğretmen adaylarıyla yürütülmesine karar verilmiştir

Diğer bir yandan, fark etme becerilerine yönelik Türkiye'de yapılan çalışmaların azlığı dikkat çekmektedir (Baş, 2013; Erdik, 2014; Işıksal et al., 2012; Osmanoğlu, 2010). Benzer şekilde, ders imecesi mesleki gelişim modeli son dönemlerde önem kazanmasına rağmen, Türkiye'de bu konuya yönelik yürütülen çalışmaların sayı sınırlıdır (Baki, 2012; Baki, Erkan & Demir, 2012; Boran & Tarım, 2012; Budak, Budak, Bozkurt & Kaygın 2011; Bütün, 2012; Eraslan, 2008; Erbilgin, 2013). Dolayısıyla, Türkiye'de öğretmen ve öğretmen adaylarının fark etme becerilerine ve ders imecesi mesleki gelişim modelinin uygulanmasına yönelik çalışmaların yapılmasına ihtiyaç vardır. Bu çalışmanın ulusal literature katkıda bulunması beklenmektedir.

Yöntem

Bu çalışmada, ortaokul matematik öğretmen adaylarının fark etme becerilerini ve bu becerilerdeki değişimi detaylandırarak sürecin geniş kapsamlı bir resmini sunmak için nitel araştırma desenlerinden biri olan durum çalışması kullanılmıştır. Durum çalışması araştırmacının bir olayı ya da olayları zaman içerisinde ve farklı kaynaklardan zengin veri elde ederek detaylı bir şekilde araştırdığı bir yöntemdir (Creswell, 2007). Bu araştırmada, durum çalışması ders imecesi mesleki gelişim modelinin planlama, öğretim, yansıtma, tekrar öğretim ve tekrar yansıtma aşamalarında öğretmen adaylarının ne ve nasıl fark ettiklerini detaylı bir şekil de araştırmak; bu kapamda deneyimlerine ve uygulamalarına yükledikleri anlamları öğrenmek; ve süreç içerisinde fark etme becerilerinde meydana gelen değişimleri incelemek amacıyla tercih edilmiştir. Sonuçları genellemekten ziyade, iki öğretmen adayına odaklanılarak fark etme becerilerini ve zaman içerisinde meydana gelebilecek değişimleri daha detaylı bir şekilde sunmak hedeflenmiştir. Öğretmen adayları işbirliğine dayanan planlama, öğretim ve yansıtma gibi eylemleri alışkın oldukları üniversite ya da gerçek sınıf ortamlarında gerçekleştirmiştir.

Katılımcılar

Bu çalışmada, bir kız üç erkek olmak üzere dört son sınıf ortaokul matematik öğretmen adayı ders imecesi uygulama sürecinin tamamında yer almıştır. Fakat bu sürecin kapsamının yoğun olmasından dolayı hem her iki cinsiyeti temsil etmek hem de deneyimlerini ve becerilerini daha detaylı incelmek amacıyla içlerinden bir kız (İnci) bir erkek (Semih) olmak üzere iki öğretmen adayı ele alınmıştır. Her bir öğretmen adayına takma isimler verilmiş ve diğer iki öğretmen adayı için Mehmet ve Hasan isimleri kullanılmıştır. Çalışmanın katılımcılarını belirlemek için amaçlı örnekleme tekniklerinden ölçüt örnekleme kullanılmıştır. Bu teknik önceden belirlenen önemli ölçütleri karşılayan olay ya da kişilerin seçimini kapsamaktadır (Gall et al., 2007). Bu çalışmada, katılımcıların belirlenmesinde üç ölçüt göz önünde bulundurulmuştur: (1) öğretmenlik mesleğiyle ilgili dersleri büyük bir kısmını tamamlamış, öğretim yöntem ve tekniklerini öğrenmiş ve kendini öğretmenlik mesleğine hazırlamış, yani, son sınıf öğrencisi olmak (2) ders imecesi kavramına yeni olma ve (3) sürece katılmaya gönüllü olma. Araştırma belirlenen öğretmen adaylarıyla 2014-2015 bahar döneminde gerçekleştirilmiştir.

Bu çalışma, öğretmen adaylarıyla gerçek sınıf ortamlarında uygulama yapmayı ve bu süreçte okuldaki uygulama öğretmeninden destek almayı gerektirmektedir. Dolayısıyla, bu çalışma uygun ortamı sağlamak ve ayrıca ders imecesi mesleki gelişim modelinin öğretmenlik uygulaması dersine dahil edilmesi hakkında fikir sahibi olmak amacıyla, öğretmenlik uygulaması kapsamında yürütülmüştür. Bu ders kapsamında üniversite tarafından oluşturulan ortaokul listesinden bir okul rast gele seçilmiştir. Bu sürece yardımcı olacak uygulama öğretmeni araştırmada ele alınan matematik konuları doğrultusunda 5. Sınıflarlar dersinin bulunmasına ve gönüllü olmasına göre belirlenmiştir. Uygulama öğretmeni Ezgi bu süreçte, planlamadan önce öğretmen adaylarına konuların öğretimiyle ilgili fikirler vermiş, onlara derslerin öğretimini yapmaları için izin vermiş, her bir dersi gözlemlemiş ve derse ilişkin görüşlerini ve önerilerini paylaşmıştır.

Verilerin Toplanması

Araştırmanın verileri 8 hafta süren ve 5. sınıf çevre, alan, yüzey alanı ve prizma konularının öğretimine yönelik gerçekleşen dört ders imecesi döngüsünün uygulamalarından elde edilmiştir. Öğretmen adayları döngülerde yer alan kazanımlara yönelik nasıl planlar hazırlayacaklarını ve derslerin öğretimini nasıl yapacaklarını belirlemeye çalışmıştır. Planlama aşamasında, öğretmen adayları öncelikle konuya yönelik detaylı araştırmalar yapmış ve dersin ilgili hocasından öğretime ilişkin fikir ve önerilerini almıştır. Ardından bir araya gelerek ders planını oluşturmaya çalışmış ve gerekli materyalleri hazırlamıştır. Uygulama aşamasında, belirlenen bir öğretmen adayı dersin öğretimini yaparken ders, diğer grup üyeleri, uygulama öğretmeni ve araştırmacı tarafından gözlemlenmiş ve bu süreçte çeşitli notlar alınmıştır. Dersin öğretimi uygulama öğretmeninin 5. sınıflarından seçilen şubelerde yapılmıştır. Ders planının uygulanmasının hemen ardından, öğretmen adayları, uygulama öğretmeni ve araştırmacı bir araya gelerek yansıtma aşamasını gerçekleştirmiştir. Bu süreçte uygulama öğretmeni kendi düşüncelerini paylaştıktan sonra gruptan ayrılmaktadır. Bu aşamada, başta dersin öğretimini yapan öğretmen adayı olmak üzere bütün öğretmen adayları derse ilişkin görüşlerini paylaşmakta, dersi değerlendirmekte ve önerilerini dile getirmektedir. Her bir döngü benzer şekilde ders imecesi aşamalarına uygun olarak yürütülmüştür. Araştırmacı, süreç içinde gözlemci rolünü üstlenmiş, uygulamaya dahil olmamış, veri toplamış ve uygulamanın aksamadan yürütülmesinde kolaylaştırıcı rol oynamıştır. Ders imecesi döngüsünün tüm aşamaları video ile kayıt altına alınmış ve videolar transkript edilmiştir. Araştırmanın verileri öğretmen adaylarıyla dersin öğretiminden önce ve sonra gerçekleşen görüşmeler, tüm süreç boyunca yapılan gözlemler ve tutulan alan notları, video transkriptleri ve öğretmen adaylarının oluşturduğu ders planları olmak üzere birçok kaynaktan faydalanılarak toplanmıştır. Her bir ders imecesi döngüsünün kapsadığı kazanımlar aşağıdaki tabloda yer almaktadır.

Tablo 1 Ders İmecesi Döngüleri ve İlgili Kazanımlar

Ders İmecesi Döngüleri	**Kazanımlar**
Döngü 1	"Çokgenlerin çevre uzunluklarını hesaplar; verilen bir çevre uzunluğuna sahip farklı şekiller oluşturur"
Döngü 2	"Dikdörtgenin alanını hesaplar; santimetrekare ve metrekareyi kullanır." "Belirlenen bir alanı santimetrekare ve metrekare birimleriyle tahmin eder"
Döngü 3	"Verilen bir alana sahip farklı dikdörtgenler oluşturur." "Dikdörtgenin alanını hesaplamayı gerektiren problemleri çözer."
Döngü 4	"Dikdörtgenler prizmasını tanır ve temel özelliklerini belirler." "Dikdörtgenler prizmasının yüzey açınımlarını çizer ve verilen farklı açınımların dikdörtgenler prizmasına ait olup olmadığına karar verir." "Dikdörtgenler prizmasının yüzey alanını hesaplar."

Dört döngüden oluşan ders imecesi sürecinde, dört ders planı hazırlanmış, dört öğretim ve dört tekrar öğretim yapılmış ve dört yansıtma ve dört tekrar yansıtma aşaması gerçekleşmiştir. Her bir döngü sekiz veri toplama adımı içermektedir. Veri toplama sürecini gösteren tablo aşağıdaki gibidir.

Tablo 2 Veri Toplama Süreci

Döngü 1	Döngü 2	Döngü 3	Döngü 4
Ders Planı 1'in video ile çekimi	Ders Planı 2'in video ile çekimi	Ders Planı 3'ün video ile çekimi	Ders Planı 4'ün video ile çekimi
Planlama sonrası görüşme	Planlama sonrası görüşme	Planlama sonrası görüşme	Planlama sonrası görüşme
Öğretimin gözlemlenmesi ve video ile çelimi	Öğretimin gözlemlenmesi ve video ile çelimi	Öğretimin gözlemlenmesi ve video ile çelimi	Öğretimin gözlemlenmesi ve video ile çelimi
Ders üzerine yansıtma aşamasının video ile çekimi	Ders üzerine yansıtma aşamasının video ile çekimi	Ders üzerine yansıtma aşamasının video ile çekimi	Ders üzerine yansıtma aşamasının video ile çekimi
Öğretim Sonrası görüşme	Öğretim Sonrası görüşme	Öğretim Sonrası görüşme	Öğretim Sonrası görüşme
Tekrar öğretimin gözlemlenmesi ve video ile çelimi	Tekrar öğretimin gözlemlenmesi ve video ile çelimi	Tekrar öğretimin gözlemlenmesi ve video ile çelimi	Tekrar öğretimin gözlemlenmesi ve video ile çelimi
Revise ders üzerine yansıtma aşamasının video ile çekimi	Revise ders üzerine yansıtma aşamasının video ile çekimi	Revise ders üzerine yansıtma aşamasının video ile çekimi	Revise ders üzerine yansıtma aşamasının video ile çekimi
Tekrar öğretim sonrası görüşme	Tekrar öğretim sonrası görüşme	Tekrar öğretim sonrası görüşme	Tekrar öğretim sonrası görüşme

Verilerin Analizi

Öğretmen adaylarının, ders imecesi mesleki gelişim modeli kapsamında öğrencilerin matematiksel düşünmelerini fark etme becerilerini incelemek ve bu becerilerde değişimin meydana gelip gelmediğini belirlemek amacıyla içerik analizi tekniği kullanılmıştır. İçerik analizi, verilerin tema ve örüntüleri tanımlaya ya da kodlamaya dayanan sistematik sınıflandırma süreci yoluyla yorumlanmasını sağlayan bir tekniktir (Hsieh & Shannon, 2005). Burada amaç, verileri temsil eden uygun kavramlar bulabilmek ve bunun için verileri indirgemektir (Patton, 2014).

Öğretmen adaylarının öğrencilerin matematiksel düşünmelerini fark etme becerilerini yorumlamak için ise Van Es (2011) tarafından geliştirilen dört düzeyden oluşan bir teorik çerçeveden faydalanılmıştır. Van Es fark etmeyi, öğretmenler ne fark eder ve öğretmenler nasıl fark eder şeklinde iki temel kategoriye ayırmakta ve bu iki kategoriyi de dört boyutta değerlendirmektedir. İlk kategori, yani öğretmenlerin neyi fark ettiği, Özne (Actor) ve Konu (Topic) boyutlarını içermektedir. Özne, öğrenci, öğretmen, kendisi, başkaları gibi kime odaklanıldığını belirten boyuttur. Konu, matematiksel düşünme, pedagojik stratejiler, sınıf yönetimi, ortam gibi hangi konunun tanımlandığını yansıtmaktadır. İkinci kategori, yani öğretmenlerin nasıl fark ettiği, Tutum (Stance) ve Belirginlik (Specificity) boyutlarını kapsamaktadır. Tutum, öğretmenin fark ettiklerini yorumlamadaki analitik yaklaşımıdır.

Grubun öğretme ve öğrenmeye yönelik sorgulamasının verimli olup olmadığını, gözlemlediklerini yorumlayışını ve değerlendirişini ele almaktadır. Bu tanımlama, değerlendirme ve yorumlama olarak üçe ayrılmaktadır. Tanımlama, gerçekleşen olayların yeniden dile getirilmesidir. Değerlendirme, öğretmenin neyin iyi ve kötü olduğunu belirtmesini ya da farklı yapılması yönünde verdiği yargıları içermektedir. Yorumlama ise öğretmenin gözlemlerinden çıkarım yaptığı, olayın neden öyle gerçekleştiğini açıklamaya çalıştığı cümleleri ifade etmektedir. Belirginlik, öğretmenin açıklamalarındaki detayın seviyesini yansıtmaktadır. Öğretmenin düşüncelerini yansıtırken genel izlenimlerinden mi bahsettiğine yoksa gerekçeleriyle ve detaylı bir şekilde mi ifade ettiğine odaklanmaktadır. Tablo 3 fark etme becerilerinin analizinde kullanılan boyutları göstermektedir.

Tablo 3 Fark Etme Becerilerinin Analizinde Kullanılan Boyutlar

Ne Fark Etti		**Nasıl Fark Etti**	
Özne (Actor)	Konu (Topic)	Tutum (Stance)	Belirginlik (Specificity)
Kim tanımlandı?	Ne tartışıldı?	Analitik yaklaşım nasıldı?	Detay düzeyi nasıldı?
Öğrenci, Öğretmen, Kendisi, Başkaları vb.	Matematiksel Düşünme, Öğretmenin Pedagojisi, Öğrencilerin Öğrenmesi, Öğretim Sınıf Ortamı Sınıf Yönetimi vb.	Tanımlayıcı Değerlendirici Yorumlayıcı	Genel Detaylı

Her iki kategoride (Ne Fark Etti-Nasıl Fark Etti) bu dört boyut ele alındığında ise fark etme becerisinde zaman içerisindeki gelişimi ortaya koyabilecek dört düzeyden oluşan bir teorik çerçeve ortaya çıkmaktadır: Düzey 1 (Baseline), Düzey 2 (Mixed), Düzey 3 (Focused) ve Düzey 4 (Extended). Bu çalışmada öğretmen adaylarının fark etme becerilerini yorumlamak için kullanılan kategoriler ve bunların kapsamları (Van Es, 2011) aşağıdaki gibidir:

Düzey 1 (Baseline): Katılımcı, fark edilenler açısından, bütün sınıfın davranışları, öğrencinin öğrenmesi, sınıf ortamı ve öğretmenin pedagojisi gibi bir dizi konuya odaklanır. Öğrencilerden bir bütün, bir sınıf olarak bahsetme eğilimindedir. Nasıl fark etti boyutu bazında, genel izlenimlerinden bahseder ve gözlemlediklerini basite indirgeyerek aktarır. Bunun yanı sıra, katılımcının açıklamaları daha çok tanımlayıcı ve değerlendirici nitelikte olup yaptığı açıklamaları destekleyici kanıt sunmaz ya da çok az detay belirtir.

Düzey 2 (Mixed): Katılımcı, ne fark etti boyutunda, öncelikli olarak öğretmenin pedagojisine dikkat eder fakat öğrencilerin matematiksel düşünmelerine de odaklanmaya başlar. Bütün sınıfı ele almaktan belirli öğrencilere de dikkat etmeye doğru bir eğilim başlar. Nasıl fark ettiği kapsamında, genel izlenimlerinden bahsetmeye devam eder fakat önemli olayları da tanımlar. Ayrıca, gözlemlediklerini değerlendirmeye devam ederken aynı zamanda bunları anlamlandırmaya da çalışır. Açıklamalarını desteklemek için belirli öğrencilerden ve anlardan bahsetmesine rağmen yorumlarını genişletmekte ve gözlemlediklerini detaylandırmakta tutarsızdır.

Düzey 3 (Focused): Katılımcı, fark edilenler açısından, ağırlıklı olarak belirli öğrencilere ve onların matematiksel düşünmelerine odaklanmaya başlar. Odağın başka noktalardan artık öğrencilerin matematiksel yaklaşımlarına kayması bu kategoriyi ilk iki düzeyden ayıran belirgin bir özelliktir. Nasıl fark etti boyutunda, katılımcı gözlemledikleri üzerine akıl yürütür ve bunların nedenlerini gerekçelendirmeye çalışır. Önemli durumları ele alır, öğrencilerin matematiksel düşünmeleri ve anlamalarıyla ilgili çıkarım yapabilmek için buradaki detayları kullanır. Yaptıkları açıklamalar yorumlayıcı nitelikte olup bu açıklamaları detaylandırmaya çalışır.

Düzey 4 (Extended): Katılımcı, öğrencilerin matematiksel olarak nasıl düşündüklerini onların çeşitli yorumlarını ve açıklamalarını göz önünde bulundurarak detaylı olarak incelemeye devam eder. Ne gözlemlendiği üzerine akıl yürütür ve düşüncelerini desteklemek için gözlemlerinden detaylar sunar. Diğer düzeylerden farklı olarak, katılımcı burada gözlemlediklerini yorumlarken öğrencinin düşünme şekliyle öğretmenin pedagojisi arasındaki ilişkiyi göz önünde bulundurur. Belirli bir öğrenci düşünme şeklini spesifik bir öğretim yaklaşımıyla ilişkilendirir, alternatif öğretim yaklaşımları önerebilir ve gözlemledikleriyle geniş öğretme ve öğrenme prensipleri arasında bağlantı kurmaya çalışır.

Ders imecesinin planlama, öğretim, yansıtma, tekrar öğretim ve tekrar yansıtma aşamalarından elde edilen veriler, yukarıda ifade edilen (özne, konu, tutum ve belirginlik) boyutlar ve kategoriler (Düzey 1, Düzey 2, Düzey3 ve Düzey 4) göz önünde bulundurularak analiz edilmiştir. Öğretmen adaylarının fark etme becerilerinin hangi düzeyde olduğu her bir ders imecesi aşaması için tespit edilmiş ve gelişimsel süreç ortaya konulmaya çalışılmıştır. Araştırma verileri iki araştırmacı tarafından birbirinden bağımsız olarak incelenmiş ve Van Es (2011) tarafından oluşturulan kategoriler dikkate alınarak yorumlanmıştır. Araştırmanın geçerlik ve güvenirliğini arttırmak amacıyla görüşme, gözlem, doküman ve video kayıt gibi farklı veri toplama araçları kullanılmıştır. Çalışmanın içeriği, katılımcıları, işlenişi, veri toplama aşamaları ayrıntılı ve zengin açıklamalar kullanılarak sunulmuştur. Ayrıca, sonuçlar araştırma verilerinden detaylı bilgiler yansıtılarak desteklenmiştir.

Bulgular

Ders imecesi uygulama sürecinde dört ders döngüsü yürütülmüş ve her bir döngüde planlama, öğretim, yansıtma, tekrar öğretim ve tekrar yansıtma aşamaları gerçekleştirilmiştir. Öğretmen adayları planlama, yansıtma ve tekrar yansıtma aşamalarında iş birliğine dayalı olarak çalışırken dersin öğretimini ve tekrar öğretimini diğer grup üyeleri kendilerini gözlemlerken bireysel olarak yürütmüşlerdir. Bu süreçte, her bir öğretmen adayı dört planlama aşamasına katılmış, iki öğretim ve altı gözlem yapmış ve ayrıca toplamda sekiz yansıtma aşamasında yer almıştır. Bu çalışmada iki ortaokul matematik öğretmen adayından, İnci ve Semih, elde edilen verilerin bulguları detaylı bir şekilde ele alınmıştır.

İnci

Tablo 4 İnci'nin Ders İmecesi Uygulamasında Fark etme Süreci

	Planlama	Öğretim	Yansıtma	Tekrar Öğretim	Tekrar Yansıtma
Döngü 1	Düzey 1	-	Düzey 1	Düzey1	Düzey2
Döngü 2	Düzey 2	-	Düzey 2	-	Düzey2
Döngü 3	Düzey 3	Düzey 2	Düzey 3	-	Düzey2
Döngü 4	Düzey 3	-	Düzey3	-	Düzey3

Tabloda görüldüğü gibi, ders imecesi sürecinde İnci'nin fark etme becerileri Düzey 1, Düzey 2 ve Düzey 3 arasında çeşitlilik göstermiştir. Planlama aşamasında fark etme becerileri Düzey 1' den Düzey 3'e doğru kademeli olarak artmıştır. Yansıtma aşamasında bu becerileri Düzey 1'den Düzey 3'e yükselirken, tekrar yansıtma aşamasında Düzey 2'den Düzey 3'e doğru ilerlemiştir. Bunu yanı sıra, ilk ders uygulamasında fark etme becerileri Düzey 1'de iken ikinci uygulamasında Düzey 2 olarak değişmiştir. Bu süreçte, İnci'nin fark etme becerileri ilk döngülerde düşük bulunmuş fakat döngüler ilerledikçe olumlu yönde bir değişimin gerçekleştiği görülmüştür. Genel anlamda, ders imecesi uygulamasına katılım sürecinde İnci'nin fark etme becerilerinde kademeli bir artış olduğu sonucuna ulaşılmıştır. Diğer bir deyişle, öğretmen adayı matematiksel olarak önemli olmayan çeşitli konulara odaklanmaktan öğrencilerin spesifik matematiksel düşünmelerine dikkat etmeye doğru bir eğilim sergilemiştir. Bunun yanı sıra, detay içermeyen genel kullanmak yerine düşüncelerini desteklemek amacıyla yorumlarını gerekçelendirmek ve öğrencilerin nasıl düşündüğünü anlamlandırmak için çaba harcamaya başlamıştır. Bu süreçte, tanımlayıcı ve değerlendirici bir yaklaşımdan yorumlayıcı bir bakış açısına doğru bir geçiş gerçekleşmiştir. Öğretim aşamasında ise başta öğrencilerin yanıtlarını çok fazla dinlememe, yanlış cevaplarını göz ardı etme ve zihninde tasarladığı şekilde dersi anlatma eğiliminde olduğu görülmüştür. İkinci uygulamada ise öğrencilere matematiksel düşünme şekillerini açıklamaları için fırsatlar vermeye ve stratejilerini açıklamaları için destekleyici sorular sormaya çalışmasına rağmen bir yandan da hedeflediği şekilde dersi işlemeye dikkat etmiş, istediği yanıtları alamayınca hemen kendisi anlatmış ve ara ara yanlış cevapları görmezden gelmiştir. Öğretim aşamasında az da olsa İnci'nin fark etme becerilerinde gelişim olduğu sonucuna ulaşılmıştır.

Semih

Table 5 Semih'in Ders İmecesi Uygulamasında Fark etme Süreci

	Planlama	**Öğretim**	**Yansıtma**	**Tekrar Öğretim**	**Tekrar Yansıtma**
Döngü 1	Düzey1	-	Düzey2	-	Düzey1
Döngü 2	Düzey2	Düzey2	Düzey2	-	Düzey2
Döngü 3	Düzey2	-	Düzey3	-	Düzey3
Döngü 4	Düzey3	-	Düzey3	Düzey2	Düzey2

Tabloda görüldüğü üzere, Semih'in fark etme becerileri ders imecesi aşamalarında Düzey 1, Düzey 2 ve Düzey 3 özelliklerini sergilemiştir. Planlama aşamasında fark etme becerileri Düzey 1'den Düzey 2'ye yükselirken yansıtma aşamasında Düzey 2'den Düzey 3'e doğru bir değişim olmuştur. Diğer taraftan, tekrar yansıtma aşamasında Semih'in fark etme becerilerinin Düzey 1'den Düzey 3'e yükseldiği fakat son aşamada Düzey 2'ye geri düştüğü sonucuna ulaşılmıştır. Bunun yanı sıra, bulgular Semih'in fark etme becerilerin her iki öğretim uygulamasında da Düzey 2'de olduğunu göstermiştir. Genel olarak, Semih'in fark etme becerilerinin uygulamanın başlarında düşük olduğu zaman içerisinde yavaş yavaş gelişim gösterdiği söylenebilir. Bir başka deyişle, ders imecesi sürecinin başlarında Semih öğrencilerin matematiksel düşünmelerinden ziyade öğretmenin pedagojisi, öğretmen ve öğrenci davranışları, öğrencilerin isteklilikleri, materyal, ders içeriği ve sınıf ortamı gibi pek çok farklı konuya odaklanmıştır. Fakat zaman içerisinde Semih'in öğrencilerin matematiksel olarak nasıl düşündüklerine daha fazla odaklanmaya başladığı sonucuna ulaşılmıştır. Bu süreçte, başlarda ifadelerinde tutarsızlıklar söz konusudur. Bazen yaptığı yorumları gerekçelendirmeye çalışsa da bunu ya genel bir dil ile yaptığı ya da nedenlerini detaylandırarak açıklamadığı sonucuna ulaşılmıştır. Süreç ilerledikçe, Semih'in öğrencilerin matematiksel düşünmelerinin arkasında yatan nedenlerin neler olabileceğine dair yorumlayıcı bir bakış açısıyla akıl yürütmeye çalıştığı görülmüştür. Öğretim aşamasında ise Semih hem spesifik öğrenci yanıtlarına odaklanmış hem de sınıfı bir bütün olarak görmeye devam etmiştir. Öğrencilerin matematiksel düşünmelerini ortaya çıkarmada yaklaşımı tutarsızdır. Öğrencilerden bazen nasıl düşündüklerini açıklamasını istemesine rağmen bazen sadece duyma istediği kavramı söylemelerini ya da beklediği hesaplamaları yapmalarını sağlamak için genel sorular sormuştur. Her zaman öğrencileri dinlememiş ve doğrudan anlatmayı tercih ettiği kısımlar da olmuştur. Bu aşamada genel olarak hem öğrenciyi göz önünde bulundurduğu hem de kendine odaklandığı karma bir yaklaşım sergilemiştir. Öğretim sırasında Semih'in fark etme becerilerinde değişim gözlemlenmemiştir.

Tartışma ve Sonuç

Bu çalışmadan elde edilen bulgular öğretmen adayları İnci ve Semih'in fark etme becerilerinin düşük olduğunu ve ağırlıklı olarak Düzey 2'nin özelliklerini sergilediğini göstermektedir. Ne fark ettikleri bazında, öğretmen adayları doğrudan öğrencilerin matematiksel düşünmelerine odaklanmaktan ziyade öğretmenin davranışları, öğrencinin katılımı, sınıf yönetimi, öğretmenin pedagojisi, öğrencinin öğrenmesi, materyal kullanımı ve sınıf ortamı gibi pek çok farklı konuya odaklanmıştır. Üzerinde durdukları noktalar çeşitlik gösterse de, çalışmanın genelinde bu öğretmen adaylarının öğretmenin pedagojik stratejilerine ve öğrencilerin matematiksel yaklaşımlarına dikkat ettikleri, yani, Düzey 2'nin özelliklerini yansıttıkları sonucuna ulaşılmıştır. Benzer şekilde Jacobs et al. (2010) öğretmen adaylarının daha çok materyal kullanımı, problemin farklı çözüm yoları içermesi gibi genel olarak matematik öğretimi ve öğrenimine yönelik neler bildikleri hakkında tartıştıklarını dile getirmiştir. Nasıl fark ettikleri boyutunda, İnci ve Semih bazı spesifik öğrencilere ve onların matematiksel düşünmelerine odaklanmıştır fakat genel olarak sınıfı bir bütün olarak görme eğilimindedirler. Yorumları genel olarak tanımlayıcı ve değerlendirici olmasına rağmen yorumlayıcı bakış açısını da benimsemişlerdir. Yorumlarını destekleyici ifadeler kullanmak yerine genel izlenimlerini ve düşüncelerini yansıtma eğilimi göstermişlerdir. Önemli olayları ele almalarına rağmen bu konuları detaylandırmakta yeterince iyi olmadıkları sonucuna ulaşılmıştır.

İnci ve Semih'in fark etme becerileri ders imecesi sürecinin başlarında (1. ve 2. Döngü) Düzey 1 ve Düzey 2'nin özelliklerini göstermiştir. Bu süreçte öğretmen adayları öğrencilerin matematiksel düşünme biçimlerinin detaylarına odaklanmak yerine dersin matematiksel olarak çok önemli olmayan boyutlarına odaklanmışlardır (Borko et al., 2008; Erickson, 2011; Sherin & Star, 2011; Sherin, Russ, et al., 2011; Star et al., 2011; Star & Strickland, 2008). Düşüncelerini ya da gözlemlediklerini çok fazla detaylandırmadan basit ifadelerle açıklamaya ve anlamlandırmadan yüzeysel olarak değerlendirmeler yapmaya çalıştıkları sonucuna ulaşılmıştır. Diğer bir deyişle, bu döngülerde yorumları genel olarak tanımlayıcı ve değerlendrici niteliktedir Chamberlin, 2005; Goldsmith & Seago, 2011; Kazemi & Franke, 2004; Sherin & Han, 2004; Sherin & van Es, 2009; van Es, 2011; van Es & Sherin, 2008). Benzer şekilde, diğer araştırmalar da çalışmaların başlarında katılımcıların öğrencilerin düşünme biçimlerinden ziyade bir dizi konuyu geniş bir bakış açısıyla ele aldıklarını göstermektedir (Goldsmith & Seago, 2011; Jacobs, Lamb, & Philipp, 2010; Kazemi & Franke, 2004; McDuffie et al., 2014; Sherin & Han, 2004; Sherin & van Es, 2009; Star & Strickland, 2007; van Es, 2011; van Es & Sherin, 2008).

İnci ve Semih'in fark etme becerilerinin ders imecesi sürecinin sonlarına doğru (3. ve 4. Döngü) Düzey 2 ve Düzey 3'ün özelliklerini taşıdığı sonucuna ulaşılmıştır. Bu katılımcılar daha fazla öğrencilerin matematiksel düşünmelerine odaklanmaya ve onları anlamlandırmaya çalışmışlardır. Fark ettiklerini gerekçelendirmeye ve öğrencilerin matematiksel anlamalarına yönelik çıkarım yapmaya yönelik çaba harcamaya başladıkları görülmüştür. Öğrencilerin spesifik yanıtlarından örnekler vermeye ve nasıl düşünmüş olabileceklerini ve nedenlerini yorumlamaya çalışmışlardır. Bu doğrultuda, öğretmen adaylarının öğrencilerin matematiksel düşünmelerini fark etme becerilerinde ders imecesi sürecinde yavaş yavaş ilerleme olduğu ve bu becerilerin mesleki gelişim modelleri yardımıyla geliştirilebileceği söylenebilir (Goldsmith & Seago, 2011; Jacobs, Lamb, & Philipp, 2010; Lewis, Friedkin, Baker &Perry, 2011; Sherin & Han, 2004; Sherin & van Es, 2009; Star & Strickland, 2007; van Es, 2011; van Es & Sherin, 2008).

Planlama aşamasında her iki öğretmen adayının fark etme becerileri de Düzey 1'den Düzey 3' doğru bir ilerleme olduğu sonucuna ulaşılmıştır. Öğretmen adayları süreç içerisinde öğrencilerin matematiksel düşünme biçimlerine odaklanmaya ve olası stratejileri üzerinde akıl yürütmeye başlamışlardır. Öğrencilerden beklenen kavram yanılgılarını, hataları ve zorlukları göz önünde bulundurma ve diğer grup üyelerini bu konular üzerinde düşünmeye teşvik etme eğilimi kazandıkları gözlemlenmiştir. Daha detaylı yorumlar yapmaya ve ifadelerini destekleciyi gerekçeler sunmaya başladıkları sonucuna ulaşılmıştır. Benzer şekilde Hughes (2006) matematik öğretmen adaylarının öğrencilerin matematiksel düşünmelerini fark etme becerilerinin planlama aşaması boyunca geliştiğini ve Mostofo (2013) ders imecesi modelinin matematik öğretmen adaylarının öğrencilerin yanılgılarını ve yanıtları fark etme becerilerini geliştirdiğini ortaya koymuştur. Ders imecesi üzerine yapılan diğer çalışmaların da bu sonuçları destekler nitelikte olduğu görülmüştür (Hiebert et al., 2007; Mathews et. al., 2009; Sims & Walsh, 2008).

Öğretim aşamasında, İnci'nin fark etme becerilerinin ilk ders öğretiminde Düzey 1 seviyesinde olduğu ikinci uygulamasın da ise Düzey 2'ye yükseldiği sonucuna ulaşılmıştır. Semih'in fark etme becerilerinin ise her iki öğretim uygulamasında Düzey 2'nin özelliklerini sergilediği görülmüştür. Semih'in daha yüksek fark etme becerileri göstermesinde İnci'ye kıyasla daha fazla öğretim tecrübesine sahip olmasının etkili olabileceği düşünülmektedir. Bu süreçte İnci'nin Düzey 1'deki fark etme becerileri öğrencilerin yanlış cevaplarına odaklanmama, bu cevapların nedenlerini sorgulamama, öğrencilere matematiksel düşünmelerini açıklamaları için fırsat vermeme, zihninde öğretmeyi tasarladığı konulara odaklanma, öğrencileri çok fazla dinlemeyip doğrudan anlatma ve değerlendirici bir yaklaşıma sahip olma özelliklerini göstermiştir. İnci ve Semih'in Düzey 2'deki fark etme becerileri ise benzerlik göstermektedir. Bu süreçte, öğretmen adayları hem öğretmeyi düşündükleri konuya hem de öğrencilerin matematiksel düşünmelerine odaklanmışlardır. Bazen öğrencilerin yanıtlarını dinlemiş bazen ise kendi zihinlerinde mevcut olan şeyi doğrudan anlatmayı tercih etmişlerdir. Spesifik öğrenci cevaplarına odaklanmaya başlamalarına rağmen çoğunlukla sınıfa bir bütün olarak yaklaşmışlardır. Öğretmen adaylarının öğrencilerin matematiksel düşünme biçimlerini ortaya çıkarmada ve bu doğrultuda onlara cevap vermede tutarsız oldukları sonucuna ulaşılmıştır. Genel olarak, değerlendirmeci bir yaklaşım sergilemelerine rağmen öğrencilerin cevaplarını yorumlamak için çaba harcamaya başladıkları da görülmüştür. Bazen öğrencilerin yanlış cevaplarını düzeltme fırsatını kaçırdıkları ya da bunu çok genel şekilde yaptıkları sonucuna ulaşılmıştır.

Öğretim aşaması genel olarak değerlendirildiğinde her iki öğretmen adayının da fark etme becerilerinin düşük olduğu sonucuna ulaşılmıştır (Vondrová & Žalská, 2013). Planlama ve yansıtma aşamalarına kıyasla bu beceride daha az gelişim gözlemlenmesinin sebebi öğretmen adaylarının öğretimi diğer aşamalardaki gibi işbirliği içinde değil bireysel yürütmesi, sadece iki öğretim uygulaması yapma fırsatının sunulması ve sınıf ortamının karmaşık yapısında pek çok eylemi aynı anda yürütme tecrübelerinin az olması olabilir. Bunun yanı sıra, İnci ve Semih'in öğretim sırasında çeşitli hatalar yaptıkları ya da bazı aktiviteleri amacına uygun olarak yürütemedikleri sonucuna ulaşılmıştır. Öğretmen adayların tecrübelerindeki eksikliklerin bu durumların nedeni olabileceği düşünülmektedir (Berliner et al., 1988; Carter et al., 1988). Literatür de yer alan bazı çalışmaların sonuçları da öğretmen adaylarının planladıklarını uygulamaya aktarmada problemler yaşadığını ortaya koymaktadır (Baki, 2012; Mostofo, 2013; Stylianides & Stylianides, 2007; Yeşildere and Akkoç, 2010). Ayrıca, her iki öğretmen adayının planlama aşamasındaki fark etme becerilerinin onların öğretim aşamasında neye odaklandıklarını, olayları nasıl anlamlandırdıklarını ve öğrencilere verdikleri cevapları etkilediği düşünülmektedir (Choy, 2015; Fernandez, Chokshi & Cannon, 2003). Benzer şekilde, diğer çalışmalar da öğrencilerin düşünmelerine odaklanmanın öğretim sırasında onların stratejilerini ortaya çıkaracak şekilde etkileşim ortamı oluşturmayı etkilediğini savunmaktadır (Brodie, 2010d; Franke et al., 2009; Isoda & Katagiri, 2012; Smith & Stein, 2011).

Yansıtma aşamasında, İnci ve Semih'in ders imecesinin başlangıçtaki döngülerinde (1. ve 2. Döngü) fark etme becerilerinin Düzey 1 ve Düzey 2 arasında çeşitlilik gösterdiği fakat ağırlıklı olarak Düzey 2'de olduğu sonucuna ulaşılmıştır. Öğretmen adaylarının ifadeleri onların dersi gözlemlerken genellikle öğretmene ve öğrencilerin davranışlarına odaklandıklarını, öğrencilerin cevaplarına dikkat ettiklerini fakat bunları çok fazla anlamlandırmaya çalışmadıklarını göstermektedir. Öğretmen adayları öğrencilerin verdiği cevapları ya da eylemleri tanımlamakta, doğruluk ya da yanlışlığı bakımından değerlendirmekte fakat bunların nedenlerini yorumlamaya çalışmamıştır. Öğretmenin uygulamasında eğitimsel anlamda beğendiği noktaları vurgulamış, yanlış yapılan noktaları dile getirmiş ve nasıl düzeltilmesi gerektiği hakkında fikirler bildirmiştir. Öğretmen adaylarının özlemlerine ilişkin genel izlenimlerini dile getirme ve bunu yaparken ifadelerini çok fazla detaylandırmama eğiliminde oldukları sonucuna ulaşılmıştır. Fakat ders imecesi döngüleri ilerledikçe (3. ve 4. Döngü) fark etme becerilerinin Düzey 2 ve Düzey 3 arasında çeşitlilik göstermeye başladığı görülmüştür. İnci ve Semih'in derse ilişkin düşüncelerini yansıtırken öğrencilerin matematiksel düşünmelerine daha fazla odaklanmaya, nedenleri hakkında akıl yürütmeye, gözlemledikleri olaylardan çeşitli örnekler sunarak düşüncelerini desteklemeye ve öğrencilerin matematiksel yaklaşımlarını anlamlandırmaya başladıkları sonucuna ulaşılmıştır. Çeşitli çalışmalar da ders imecesi modelinin öğretmenin öğrencilerin matematiksel düşünmelerine yönelik yansıtıcı ifadelerini geliştirdiğini ortaya koymaktadır (Byrum et al., 2002; Fernandez et al., 2003; Grove, 2011; Sims & Walsh, 2009; Wilms & Zell, 2002).

Bu çalışmadan elde edilen veriler, ders imecesi mesleki gelişim modeli ile öğretmen adaylarının öğrencilerin matematiksel düşünmelerini fark etme becerilerinin geliştirilebileceğini ortaya koymaktadır. Öğretmen adaylarının çalışmanın başlarındaki düşük fark etme becerileri planlama, öğretim ve yansıtma gibi aşamalarda bu becerilerin geliştirilmesi gerektiğini göstermektedir. Ders imecesi gücünü, öğrencilerin matematiksel düşünmelerini, anlamalarını ve yanıtlarını ön görme çabasından almaktadır (Fernandez et al., 2003; Lewis, 2000). Bu çalışmada İnci ve Semih'in fark etme becerilerinde pozitif değişimin gerçekleşmesinde ders imecesinin bu güçlü bakış açısının etkili olması muhtemeldir. Bunun yanı sıra, çeşitli pek çok faktör bunda rol oynamış olabilir. Öğretmen adaylarının süreç içinde yaptığı yorumlar baz alındığında, ders imecesi mesleki gelişim modelinin, özellikle, iş birliği içinde çalışma; matematiksel konuya yönelik araştırma yapma; gözlemleme ve yansıtma; spesifik matematiksel alana odaklanma ve uygulama öğretmeninden destek alma ile ilgili özelliklerinin bu gelişimde etkili olduğu düşünülmektedir.

Öğretmen adayları öğretim hariç neredeyse ders imecesinin bütün aşamalarında iş birliği içerisinde çalışmıştır. Bu süreçte, öğrencilerin düşünmelerine yönelik bildiklerini paylaşmış, birbirlerinin düşüncelerini öğrenmiş, farklı bakış açıkları kazanmış ve birbirlerinin eksiklerini tamamlamışlardır. Dolayısıyla, işbirliği içinde fikir alışverişinin onların öğrencilerin matematiksel düşünmelerini fark etme becerilerini olumlu etkilediği düşünülmektedir. Çalışmaların sonuçları da bu bulgularla paralellik göstermektedir (Baş, 2013; Chamberlin, 2004, 2005; Kazemi & Franke, 2004; Koellner-Clark & Lesh, 2003; Osmanoğlu, 2010; Zawojewski, Chamberlin, Hjalmarson, & Lewis, 2008). İnci ve Semih de iş birliği içinde çalışmalarının hem kendilerine katkı sağladığını hem de süreci olumlu etkilediğini dile getirmiştir.

Bu çalışmada öğretmen adayları geometri ve ölçme alanında yer alan çevre, alan, yüzey alanı ve prizma konularına yönelik kapsamlı araştırma yapmıştır. Çeşitli kaynaklardan faydalanarak öğrencilerin bu konulardaki yanılgılarını, zorluklarını ve hatalarını incelemişlerdir. Dolayısıyla, bu bilgileri kullanarak öğrencilerin olası düşünme şekillerini tahmin etmeye, onların bakış açılarını kazanmaya ve bu doğrultuda ders planı hazırlamaya dikkat etmişlerdir (Fernandez & Chokshi, 2002; Lewis, 2004b; Takahashi & Yoshida, 2004; Takahashi et al., 2005). Bu durumun öğretmen adaylarının fark etme becerilerini etkilediği düşünülmektedir. Yoshida and Jackson (2011) planlama aşamasında yapılan araştırmanın öğretmen adaylarının öğretimsel kararlarını etkilediğini dile getirmiştir. Osmanoğlu (2010) da öğrencilerin matematiksel düşünmelerine odaklanmayı ve onları anlamlandırmayı sağladığını, yani, fark etme becerilerini etkilediğini belirtmiştir. Her iki öğretmen adayı çeşitli kaynaklardan araştırma yapmanın konularla ilgili eksik bilgilerini tamamlamalarına yardımcı olduğunu vurgulamıştır.

Öğretmen adayları gerçek sınıf ortamlarında somut olarak öğrencilerin matematiksel düşünme biçimlerini gözlemleme ve bunlar üzerine grupça tartışarak fikirlerini paylaşma, yeni bakış açıları kazanma ve birbirlerinden öğrenme fırsatı elde etmiştir. Freidman (2005) da bu sürecin yeni bilgiler edinmek ve diğerlerinin nasıl düşündüğünü anlamak adına faydalı olduğunu savunmaktadır. Jacobs ve ark. (2010) öğretmenlerin öğrencilerin matematiksel düşünmelerini fark etme becerilerinin gerçek sınıf ortamında onlarla etkileşim halindeyken gerçekleştiğini belirtirken, Harle (2008) öğrencilere yönelik gözlemlerin yansıtma aşamasını şekillendirdiğini ve dersin gelişimini sağladığını vurgulamıştır. Öğretmen adayları gözlemlerken öğrendiklerini ve yansıtırken farklı perspektifler kazandıklarını dile getirmiştir.

Bu çalışmada, uygulama öğretmenin katılımı sınırlı olmasına rağmen hem İnci hem de Semih uygulama öğretmeninin fikirlerinin ve tavsiyelerinin kendileri açısıdan faydalı olduğunu pek çok kez vurgulamıştır Bunun yanı sıra, öğretmen adaylarının uygulama öğretmeninin dönütlerine önem verdikleri ve uygulamalarında onun tavsiyelerini göz önünde bulundurarak hareket ettikleri görülmüştür. Benzer şekilde literatürde yer alan birçok çalışma öğretmen adayları açısından uygulama öğretmeninin önemini ve etkili rolünü vurgulamaktadır (Burke, 2015; Borko & Mayfield, 1995; Graham, 2006; Gurl, 2009; Hughes, 2006; Mossgrove, 2006; Paker, 2008; Vacc & Bright, 1999).

Öğretmen adayları geometri ve ölçme alanında belirlenen konularla ilgili detaylı araştırma yaparak öğrencilerin nasıl düşünebileceklerini anlamaya ve kendi bilgi eksikliklerini kapatmaya çalışmışlardır. İnci ve Semih'in konular arasında ilişki kurmaya, bir konudaki eksikliğin diğerini nasıl etkileyeceğine dikkat etmeye ve öğrencilerin olası yanılgılarını, zorluklarını ve hatalarını bu süreçte göz önünde bulundurmaya çalıştıkları sonucuna ulaşılmıştır. Dolayısıyla, öğrencilerin bu konularla ilgili nasıl düşündüklerine daha fazla odaklanma ve anlamlandırma fırsatı elde ettikleri düşünülmektedir. Benzer şekilde, Fernandez ve ark. (2012) spesifik matematik konularına odaklanmanın öğretmen adaylarının fark etme becerilerini olumlu etkilediğini savunmuştur. Walkoe (2013) ise fark etme becerilerini genel bir yaklaşım yerine belirli matematiksel alanlarda ele almak gerektiğini dile getirmiştir. Baki (2012) araştırmasında öğretmen adaylarından ilişkili olmayan, birbirinden bağımsız kazanımlar üzerinde çalışmasını istemiş ve bu durumun olumsuz etkisinin olduğu sonucuna ulaşmıştır. Bu doğrultuda, konu bağımlı ve birbirini tamamlayan kazanımlarla çalışmaların yürütülmesini tavsiye etmiştir.

APPENDIX G: TEZ FOTOKOPİSİ İZİN FORMU

ENSTİTÜ

Fen Bilimleri Enstitüsü

Sosyal Bilimler Enstitüsü

Uygulamalı Matematik Enstitüsü

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YAZARIN

Soyadı : GÜNER
Adı : PINAR
Bölümü : İLKÖĞRETİM

TEZİN ADI (İngilizce) : INVESTIGATING PRESERVICE MIDDLE SCHOOL MATHEMATICS TEACHERS' NOTICING OF STUDENTS' MATHEMATICAL THINKING IN THE CONTEXT OF LESSON STUDY

TEZİN TÜRÜ : Yüksek Lisans Doktora

1. Tezimin tamamından kaynak gösterilmek şartıyla fotokopi alınabilir.
2. Tezimin içindekiler sayfası, özet, indeks sayfalarından ve/veya bir bölümünden kaynak gösterilmek şartıyla fotokopi alınabilir.
3. Tezimden bir bir (1) yıl süreyle fotokopi alınamaz.

TEZİN KÜTÜPHANEYE TESLİM TARİHİ: