PRE-SERVICE MIDDLE SCHOOL MATHEMATICS TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE REGARDING THE AREA OF TRIANGLES

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF SOCIAL SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

ASLI BİLİK

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
THE DEPARTMENT OF ELEMENTARY EDUCATION

SEPTEMBER 2016

Approval of the Graduate School of Social Sciences

Prof. Dr. Tülin GENÇÖZ
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

Prof. Dr. Özgül YILMAZ TÜZÜN
Head of Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

Assoc. Prof. Dr. Mine IŞIKSAL-BOSTAN
Supervisor

Examining Committee Members

Assoc. Prof. Dr. Erdinç ÇAKIROĞLU (METU, ELE)

Assoc. Prof. Dr. Mine IŞIKSAL-BOSTAN (METU, ELE)

Assist. Prof. Dr. Işıl İŞLER (METU, ELE)

Assist. Prof. Dr. Reyhan TEKİN-SİTRAVA (KU, ELE)

Assist. Prof. Dr. Mesture KAYHAN ALTAY (HU., ELE)

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name : Aslı BİLİK

Signature :

ABSTRACT

PRE-SERVICE MIDDLE SCHOOL MATHEMATICS TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE REGARDING THE AREA OF TRIANGLES

Bilik, Aslı

M. S., Department of Elementary Education

Supervisor: Assoc. Prof. Dr. Mine Işıksal-Bostan

September 2016, 181 pages

The purpose of this study was to investigate pre-service middle school mathematics teachers' pedagogical content knowledge on the concept of the area of triangles. In this respect, pre-service teachers' pedagogical content knowledge with respect to students' possible misconceptions/ difficulties regarding the area of triangles and the strategies that pre-service teachers employed to overcome these misconceptions/difficulties were examined. To conduct an in-depth investigation regarding the purpose of the study, qualitative research methodology was utilized. Two pre-service middle school teachers were selected through purposive sampling. Data was gathered through semi-structured pre-interviews and post-interviews, classroom observations, and field notes.

The findings revealed that pre-service middle school teachers were able to provide a variety of possible misconceptions/difficulties that students may have regarding the area of triangles. In this regard, findings of the research showed that

most of the specified misconceptions/difficulties were related to the concept of height, which is a prerequisite prior knowledge for the concept of area. In addition, pre-service teachers generally employed discussion strategy during their practice teaching to overcome the misconceptions/difficulties regarding the area of triangles held by sixth grade students.

Keywords: Mathematics Education, Pre-service Teachers, Pre-service Mathematics Teachers, Pedagogical Content Knowledge, Area of Triangles

ÖZ

ORTAOKUL MATEMATİK ÖĞRETMEN ADAYLARININ ÜÇGENİN ALANI KONUSUNA İLİŞKİN PEDAGOJİK ALAN BİLGİLERİ ÜZERİNE BİR ÇALIŞMA

Bilik, Aslı

Yüksek Lisans, İlköğretim Bölümü

Tez Yöneticisi: Doç. Dr. Mine Işıksal Bostan

Eylül 2016, 181 sayfa

Bu çalışmanın amacı, ortaokul matematik öğretmen adaylarının üçgenin alanı konusu ile ilgili pedagojik alan bilgilerini incelemektir. Bu bağlamda, öğretmen adaylarının üçgenin alanı konusu ile ilgili olası öğrenci zorluklarını/kavram yanılgılarını belirlemeye yönelik bilgileri ve bunları gidermeye yönelik ders esnasında kullandıkları öğretimsel stratejiler incelenmiştir. Çalışmanın amacı doğrultusunda derinlemesine bir araştırma yapabilmek için, nitel araştırma yöntemi kullanılmıştır. Çalışmanın örneklemini oluşturan iki ortaokul matematik öğretmen adayı ise amaçlı örneklem yöntemi ile seçilmiştir. Veri yarı yapılandırılmış görüşmeler, ders gözlemleri ve doküman analizleri kullanılarak toplanmıştır.

Bulgular öğretmen adaylarının, öğrencilerin üçgenin alanı konusunda sahip olabilecekleri çeşitli zorluklar/kavram yanılgıları belirttiklerini göstermiştir. Bu bağlamda, öğretmen adayları tarafından belirlenen kavram yanılgılarının geneline bakıldığında, bunların büyük çoğunluğunu yükseklik kavramına ilişkin

zorluklar/kavram yanılgılarının oluşturduğu görülmüştür. Buna ek olarak, öğretmen adaylarının öğrencilerin ders esnasında ortaya çıkardıkları üçgenin alanı ile ilgili zorluklarını/kavram yanılgılarını ortadan kaldırmak için genellikle tartışma yöntemini kullanmayı tercih ettikleri görülmüştür.

Anahtar Kelimeler: Matematik Eğitimi, Öğretmen Adayları, Matematik Öğretmen Adayları, Alan Eğitimi Bilgisi, Üçgenin Alanı

To my parents,

Ali BİLİK and Nesrin BİLİK

ACKNOWLEDGEMENTS

First of all, I would like to present my deepest gratitude to my thesis advisor, Assoc. Prof. Dr. Mine IŞIKSAL-BOSTAN. This study would not have been implemented without your critical feedbacks and your valuable advices. Even for the midnight you give me feedback. I always feel your believe in me for being successful even I lose my self reliance and you always have supported and encouraged me to study. Thank you very much for everything.

I am thankful to my committee members, Assoc. Prof. Dr. Erdinç ÇAKIROĞLU, Assist. Prof. Dr. Işıl İŞLER, Assist. Prof. Dr. Reyhan Tekin-Sitrava, and Assist. Prof. Dr. Mesture KAYHAN ALTAY for their valuable feedback and encouragement.

I would like to thank my family which is so patient in my graduate learning. My father, Ali BİLİK and my mother Nesrin BİLİK always listen my problems and encouraged me whenever I needed them. They were always with me even though we were apart. My sisters Hatice BİLİK and Eda BİLİK and my brother, Osman BİLİK were always believed in me for being successful in the graduate period. They were always ready helping me in my problems. Especially you Eda, you are always with me and my study partner during my sleepless night.

Last, but by no means least, thanks to my fiancé Onur DURMAZ, who was always ready to offer emotional support and encouragement. He is always patient even when I intolerable. I feel very fortunate to have a partner in life as you.

I also want to thank to my friend, Çağla ADIGÜZEL. Your help, suggestions and support at my worse time helped me a lot. Thanks for being such a good friend.

I would like to thank my homemates, Dilanur TAŞKIN and Naime KAYA and also Neşe ŞERBETCİ for their help, support, encouragement. You always endure my stressfull situations without any complaint. Thank you also for providing me a seperate room for my study, for the cookies and for the tea at nights.

I would also like to express my thanks to Ayşenur PEKER and Sinem KESKİN for your close friendship, support and encouragement. You were patient during this path, even I did not allocate much time for you. I feel so lucky to have friends like you.

Semanur KANDİL, thank you for your support and patient when answering my endless questions. I also thank to the lovely participants who contributed to this study by spending their time and honestly responding instruments.

My final thanks are for TUBITAK (The Scientific and Technological Research Council of Turkey) for financial support to me during M. S. The support makes most of things easier for me.

TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

CHAPTER I

INTRODUCTION

All students need to learn and understand mathematics, and student achievement can be improved by effective teaching of mathematics (Brophy, 1986; Troisi, 1983; National Council of Teachers of Mathematics [NCTM], 2000). However, effective teaching of mathematics resembles a puzzle which is complex and needs to be solved (Hill, Schilling, & Ball, 2004). It is obvious that teachers have a crucial role in students' understanding of mathematics (Hiebert et al., 1997). It leads to questions about what teachers need to know for effective teaching (NCTM, 2001). In this respect, teachers' knowledge of mathematics has become the most critical factor for effective teaching (Tirosh, 2000).

Since teachers have an important role in education (NCTM, 2001), teachers' knowledge become a critical factor to understand requirements of a sophisticated teacher. In this respect, a great number of studies have been conducted to clarify what an effective teachers' required to know and the components of teachers' knowledge. Primarily, Shulman (1987) identified seven categories of teacher knowledge necessary for developing students' understanding, namely (1) subject matter content knowledge (SMCK), (2) pedagogical content knowledge, (3) curriculum knowledge, (4) general pedagogical knowledge, (5) knowledge of learners and their characteristics, (6) knowledge of educational context, (7) knowledge of educational ends, purposes, and values, and their philosophical and historical grounds. The first three of the seven categories are related to content-specific knowledge; the remaining four categories are general knowledge (Rowland, Turner, Thwaites, & Huckstep, 2009). The first category, subject matter content knowledge (SMCK), was defined by Shulman (1986) as "*the amount and organization of knowledge per se in the mind of the teacher*" (p. 9). In another definition, it is asserted that SMCK includes both substantive and syntactic knowledge. The substantive knowledge knows the facts, concepts and processes of mathematics. On the other hand, the syntactic knowledge knows how to prove and disprove an idea (Shulman, 1986). The second category, PCK, is the combination of content knowledge and pedagogical knowledge. It was defined by Shulman as "the particular form of content knowledge that embodies the aspects of content most germane to its teachability" (2004, p. 203). In this respect, PCK includes knowledge of students' prior knowledge, (mis)conceptions and difficulties regarding a concept, the strategies used to overcome those misconceptions/difficulties. Also, it includes the knowledge required to teach the concept in a comprehensible way. In short, all required strategies, methodologies and explanations to make the students comprehend the concept can be inherent in pedagogical content knowledge. The last category, CK, is about knowing how the topics across the school continuum are organized and knowing how the resources should be used (Hill et al., 2004; Shulman, 1986).

Following Shulman’s categorization, the categories of teachers' knowledge have been recategorized or represented differently by many other researchers (Ball, Thames, & Phelps, 2008; Grossman, 1990; Rowland et al., 2009). Grossman categorized teacher knowledge into four components: (a) general pedagogical knowledge, (b) subject matter knowledge, (c) the pedagogical content knowledge, and (d) knowledge of the context. She systemized Shulman's categorization and added a new component to it. Furthermore, in this model, curriculum knowledge was included as one of the subcategories of PCK (as cited in Fernandez, 2014).

Hill, Rowan and Ball (2005) define mathematical knowledge for teaching, which refers to the mathematical knowledge that is used to teach mathematics. Ball (1988) stated that making someone else learn mathematics naturally requires mathematical knowledge. By expanding the categories proposed by Shulman's for content knowledge, mathematical knowledge for teaching was divided into four domains (Ball et al., 2008): Common content knowledge (CCK), specialized content knowledge (SCK), Knowledge of content and student (KCS), and Knowledge of content and teaching (KCT). The first two, CCK and SCK, were placed under subject matter knowledge. The other two, KCS and KCT, were considered to be components of pedagogical content knowledge. In this categorization, SCK differs from Shulman's categorization and is defined as knowing the representation of mathematical ideas, novice solution methods of problems, and explanation of rules and procedures (Hill, Ball, & Schilling, 2008). On the other hand, CCK is explained as being able to solve a mathematical problem in a correct way. In addition, KCS is related to knowledge of students, so that, a teacher needs to know students' difficulties and thoughts about a topic. Lastly, KCT is related to necessary knowledge required to organize a lesson (Ball et al., 2008).

Rowland and his colleagues (2009) offer a new breath to assess mathematical content knowledge in the action of teaching. This new framework is called 'Knowledge Quartet' since student teachers' content knowledge was categorized into four dimensions (Rowland et al., 2009). While Ball (1990) categorized the different types of mathematics teachers' knowledge in her framework, in knowledge quartet the importance is on the situations in lesson plans and in the teaching environment in which mathematics content knowledge can be observed. Different from the other studies, Rowland and his colleagues (2009) aim to examine how different types of teacher knowledge affects an ongoing teaching environment. To be able to investigate the influence of interrelated types of knowledge on teaching, observation of mathematics lessons plays an important role in this framework. In addition, it helps to enhance teaching by means of critical reflection and the provision of feedback on teaching (Rowland et al., 2009).

Throughout many years, researchers have investigated teachers' knowledge in relation to effective teaching (Ball, 1990; Ball et al., 2008; Carlsen, 1999; Grossman, 1990; Rowland, Huckstep, & Thwaites, 2005; Shulman, 1986; 1987). In many of these studies, researchers interpreted that subject matter knowledge and pedagogical content knowledge are important aspects of teacher' knowledge for teaching mathematics effectively (Hill et al., 2004; Savas, 2011). However, even if teachers know the subject to teach well, they may have some instructional and pedagogical concerns (Ball et al., 2008). In this respect, there is a growing body of evidence supporting that pedagogical content knowledge is the most important element in the domain of teacher knowledge for effective teaching (An, Kulm, & Wu, 2004; Park & Oliver, 2008).

1.1 Statement of the Problem

To teach mathematics well, a profound understanding of mathematical knowledge is not sufficient (Ball et al, 2008; Turnuklu & Yesildere, 2007). It also requires having an in-depth understanding of pedagogical content knowledge, which necessitates knowing how a topic can be easily understood, what students' prior knowledge and misconceptions are, which teaching strategies can be employed to overcome these misconceptions, and which examples, explanations and demonstrations are effective for the enhancement of students' comprehension of the topic (Shulman, 1986). In this regard, studies should be conducted to investigate teachers' pedagogical content knowledge.

Principles and Standards for School Mathematics offers five content standards, namely number and operations, data analysis and probability, algebra, geometry and measurement, which students need to learn from prekindergarten to K-12. Those of measurement are one of the important content in school curriculum since its application can be commonly found in real life settings (Cavanagh, 2008; NCTM, 2014). The literature on students' learning of measurement reported that students experience problems in understanding the concept of measurement. They just memorize formulas and try to solve the questions without understanding their meaning (Tan- Sisman & Aksu, 2009). Moreover, it was concluded in a number of studies that teachers' knowledge affects students' learning outcomes (Hatisaru, 2013; Lenhart, 2010).

Students' prior knowledge significantly affects their learning (Hewson & Hewson, 1983). When teachers organize their instruction on students' prior knowledge, their achievement will be affected positively (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). Accordingly, teachers' pedagogical content knowledge of students' conceptions and misconceptions become an important factor and it significantly affects their teaching (Carpenter, Fennema, Petersen, & Carey, 1988). Based on the results of previous studies, teacher candidates have difficulty in determining students' misconceptions on specific topics (Isik, Ocal, & Kar, 2013;

Turnuklu & Yesildere, 2007). Therefore, to teach effectively, teachers need to know students' conceptions and misconceptions about a specific topic. Moreover, they need to know the source of these misconceptions and how to overcome them. However, studies show that teacher candidates have insufficient knowledge about overcoming students' misconceptions (Yemen-Karpuzcu, Isiksal-Bostan, & Ayan, 2013). In this respect, studies should be conducted to investigate teachers' knowledge of students' misconceptions/difficulties of students on specific subjects and their knowledge of strategies to overcome these misconceptions/difficulties.

Thus, the aim of the present study is to fulfill a gap in the related literature to some extent by investigating middle school pre-service mathematics teachers' pedagogical content knowledge on a specific measurement subject. In this respect, the concept of area was chosen as the focus of the study since students have a superficial level of understanding regarding the concept of area and they have some misconceptions/difficulties regarding this topic (Cavanagh, 2008; Tan-Sisman & Aksu, 2009; Orhan, 2013). Since teachers have a significant impact on promoting students' learning and eliminating their existing misconceptions, studies investigating teachers' pedagogical content knowledge have also been conducted on the concept of area (Baturo & Nason, 1996; Simsek, 2011; Yeo, 2008; Yew, Zamri, & Yian, 2010). However, there is a limited number of studies relative to teachers' knowledge on the area of the triangle (Gutierrez & Jaime, 1999). Therefore, the content of the study has been narrowed down to the area of the triangle. In this respect, this study intended to contribute to the literature by investigating pre-service middle school mathematics teachers' knowledge of students' possible misconceptions/difficulties regarding the area of triangle and their knowledge of the teaching strategies used to overcome these misconceptions.

In line with this aim, the present study seeks to answer the following research question:

1. What is the nature of pre-service middle school mathematics teachers' pedagogical content knowledge regarding the area of triangles?
    a. What do pre-service middle school mathematics teachers know about misconceptions/difficulties held by 6^{th} grade students related to the area of triangles?

b. What kinds of instructional strategies do pre-service middle school mathematics teachers employ to overcome the misconceptions/difficulties held by 6^{th} grade students related to the area of triangles during practice teaching?

1.2 Definition of Important Terms

Pre-service middle school mathematics teachers

Pre-service middle school mathematics teachers are senior students in elementary mathematics education. They are enrolled in a four-year teacher education program in Turkey and have not begun teaching in a regular class yet.

Pedagogical Content Knowledge

Pedagogical content knowledge is defined by Shulman (2004) as "*the particular form of content knowledge that embodies the aspects of content most germane to its teachability*" (p. 203).

In this study, pedagogical knowledge refers to pre-service middle school teachers' knowledge of misconceptions/difficulties held by sixth grade students regarding the area of triangles and their knowledge of the instructional strategies employed to overcome the misconceptions/difficulties of the students during teaching.

Area is the amount of space in an enclosed figure (Yeo, 2008).

Misconception is a student conception that causes an error regularly (Smith, Andrea, diSessa & Roschelle, 1993).

Difficulty is students' mental obstacles in the process of perceiving, comprehending and attributing meaning to a concept (Bayazıt, 2008).

In this study, the term misconception was used for sixth grade students' misconstruction or incomplete knowledge and insufficient prior knowledge regarding position of height, the relationship between altitude and elements of triangle, the relationship between area and other concepts and the formula of triangle's area and insufficient prior knowledge pertaining to the area of triangles. On the other hand, the term difficulty is used for sixth grade students' mental obstacles which were encountered while constructing a height to a corresponding base or constructing a perpendicular from a point to a provided line segment. Additionally, obstacles in understanding the meaning underlying the area concept and understanding where the formula of triangles' area is derived from and how to use that solve a problem can be also included in the difficulty term.

The terms difficulty and misconception were not examined separately instead the term misconception/difficulty was used through this study.

Instructional Strategy

In this study instructional strategy term was used for the methodologies and techniques that pre-service teachers employed to overcome students'misconception or difficulty regarding area of triangles.

1.3 Significance of the Study

Subsequent to the statement of Shulman (1986) on pedagogical content knowledge, there has been an increasing amount of research on it. However, the studies that have been conducted demonstrated that teachers' levels of PCK are not sufficient (Carpenter et al., 1988; Turnuklu & Yesildere, 2007). Novice teachers do not have sufficient knowledge to organize their lesson according to students' prior knowledge and to assess learning strategies (Carpenter et al., 1988). In addition, teachers are unable to identify students' errors in the solutions they provide for the problems. Even if they can identify the errors, they cannot determine the reasons underlying those errors (Esen & Cakiroglu, 2012; Turnuklu & Yesildere, 2007). Therefore, more studies should be conducted to assess teachers' PCK and their results should be used to improve teachers' PCK.

For over 25 years, pedagogical content knowledge of teachers became the main concern of many studies. So far, several studies have been conducted to investigate pre-service and in-service teachers' pedagogical content knowledge on several mathematics topics (An, Kulm, & Wu, 2004; Baker & Chick, 2006; Basturk & Donmez, 2011; Bukova-Guzel, 2010; Burgess, 2006; Carpenter et al., 1988; Fuller, 1996; Gökkurt, Sahin, Soylu, & Soylu, 2013; Isik, Ocal, & Kar, 2013; Isiksal & Cakiroglu, 2011; Shin, 2011). A diverse collection of subject areas in mathematics have been focused on in these research studies including teachers' knowledge on statistics (Burgess, 2006), probability (Shin, 2011), fractions (An, Kulm, & Wu, 2004; Gökkurt, Sahin, Soylu, & Soylu, 2013; Isik, Ocal, & Kar, 2013; Isiksal, 2006; Isiksal & Cakiroglu, 2011; Turnuklu & Yeşildere, 2007), integers (Kubar, 2012), functions (Haciomeroglu, 2005, Hatisaru, 2013; Karahasan, 2010), trigonometry (Fi, 2003), and limit and continuity (Dönmez, 2009). In contrast to an extensive number of research studies on mathematics topics, there are a limited number of studies investigating teachers' pedagogical content knowledge on geometry and measurement topics. So far, teachers' knowledge on geometric shapes (Gökbulut, 2010; Gökkurt, 2014), solid objects (Bukova-Guzel, 2010), quadrilaterals (Aslantutak, 2009; Ozcakir, 2013), decomposing and recomposing one-dimensional and two-dimensional figures (Lenhart, 2010), volume of 3D solids (Tekin-Sitrava, 2014), measurement specifically on the concepts of length, area and volume (Esen, 2013), area formulae (Yew, Zamri, & Yian, 2010), and area and perimeter (Simsek, 2011; Yeo, 2008) have been examined. According to the results of the accessible literature, it was noticed that that teachers' pedagogical knowledge regarding measurement was not sufficient. In order to provide information to teacher educators, it is significant to conduct studies investigating teachers' knowledge on measurement topics.

Due to its significance, throughout the literature there is an extensive number of studies on specific measurement topics. These studies indicate that a vast majority of students have a superficial level of understanding the concept of the area. A weak understanding of the concept of the area and rote memorization of the formulas lead to the emergence of misconceptions (Cavanagh, 2008; Gokdal, 2004;Huang & Witz, 2013; Kamii & Kysh, 2006; Orhan, 2013; Outhred & Mitchelmore, 2000; Tan-Sisman & Aksu, 2009; Zacharos, 2006). In this respect, teachers have an important role in making the concept of the area more meaningful to students and in overcoming their misconceptions by means of appropriate teaching strategies. However, limited number of studies have been conducted on the concept of the area (Baturo & Nason, 1996; Simsek, 2011; Yeo, 2008; Yew, Zamri, & Yian, 2010). The main focus of those studies was teachers' knowledge of students’ misconceptions regarding the confusion of the area and perimeter, the relationship between the area and perimeter (Simsek, 2011), teaching strategies used in teaching the area and perimeter (Yeo, 2008), subject matter knowledge of teachers (Baturo & Nason, 1996) and knowledge on the area formulae (Yew, Zamri, & Yian, 2010). When the accessible literature was reviewed, only one study which is conducted by Simsek (2011) was found to have examined the pedagogical content knowledge of pre-service mathematics teachers regarding students' difficulties related to the topics of the perimeter and area. Although it seems that pre-service teachers' PCK as regards to student difficulties related to the area was investigated, the difference is rooted in the content. Therefore, Simsek investigated students' difficulties regarding the relation between the perimeter and the area concept, also, the area of rectangles and squares were investigated in his study. However, in the present study, the researcher aimed to investigate pre-service teachers' knowledge of students' misconception/difficulty and the instructional strategies employed to overcome those misconceptions/difficulties regarding area of triangles topic, also regarding the heightof triangles. Since these topics have not been addressed before as the focus of any accessible study, it is significant to conduct this study on teachers' knowledge of students' misconceptions/difficulties regarding the area of the concept of the triangle and knowledge of the instructional strategies employed to overcome those misconceptions/difficulties.

Furthermore, researchers examined different PCK components in their studies. Since PCK is a complex construct, hence, instead of investigating all components, concentrating on a small number of components is more meaningful (Bahcivan, 2005). Therefore, in the present study, teachers' knowledge of student difficulties/misconceptions regarding the area of the triangle and their knowledge of the instructional strategies used to eliminate these misconceptions were chosen as the components of PCK to be investigated. In this way, extensive information was aimed to be gathered on the components, which were intended to contribute to PCK literature by providing in-depth information on those components.

When the studies on teachers' PCK were examined, it was found that in most of the studies, the data were gathered through paper pencil tests, discussions, questionnaires or interviews (Baker & Chick, 2006; Fuller, 1996; Isik, Ocal, & Kar, 2013; Turnuklu & Yeşildere, 2007;). Seeking pedagogical content knowledge while observing teaching in a natural class environment makes studies more meaningful and it enables the researcher to understand the link between the knowledge and the teaching in action (Ball, Lubienski, & Mewborn, 2001; Thwaites, Huckstep, & Rowland, 2005). By observing the natural teaching environment, that the researcher can observe how teachers use knowledge base within the classroom environment and how the translation of knowledge into the ongoing learning environment takes place and provides a chance to identify teachers' developmental needs (Hegarty, 2000). For this reason, the data collection procedure employed in the present study was the observation of the ongoing learning environment in the real classroom context. Hence, this study is intended to make a contribution to mathematics teachers so that they can enhance their knowledge of student difficulties and useful strategies to employ in teaching.

As for the participants of the present study, pre-service teachers were chosen since pre-service teachers will become teachers in the future, it is important to investigate their knowledge on certain topics. Thus, policy makers and teacher educators can determine the content of the course, and new courses could be offered according to the results of the studies. In this respect, it is significant to conduct such a study to contribute to the literature.

CHAPTER II

LITERATURE REVIEW

The purpose of the study was to investigate pre-service middle school mathematics teachers' knowledge of misconceptions/difficulties held by sixth grade students regarding the triangle area concept and their knowledge of strategies used to overcome these misconceptions/difficulties. In the following sections, review of the literature was presented in five parts. First of all, the definition of pedagogical content knowledge and different PCK models were reviewed. In order to become familiarized with the framework of the present study, research studies regarding teachers' knowledge on mathematics topics revealed through the knowledge quartet will be reviewed. Subsequently, information regarding area measurement and the definitions of error, misconception and difficulty were provided. In addition, studies related to students' conceptions and misconceptions/difficulties related to the concept of area and studies on mathematics teachers' knowledge on the concept of area were presented. The chapter concluded with a summary of the reviewed literature.

2.1 The Nature of Pedagogical Content Knowledge

In the 1970s the area of interest in research studies was primarily on subject matter. However, in the 1980s interest was geared towards pedagogy. Thus, as of those time studies have been conducted to answer questions regarding how teachers plan lessons, assess student understanding, and arrange time (Shulman, 1986). However, these studies did not focus on the subject matter; what's more, these studies ignored some questions as how teachers decided on how to teach a subject and what to teach, how they questioned students' learning, and by which instructional strategy they overcame students' misconceptions. Shulman and his colleagues referred these gaps as "missing paradigm" in the studies on teaching and they focused on these gaps in their research program entitled "Knowledge Growth in Teaching". Through this program they attempted to gain insight into the way new knowledge is obtained, how old knowledge is organized and the way new knowledge is constructed based on the combination of both old and new knowledge (Shulman, 1986). As a result of the study, Shulman (1986) first introduced the term pedagogical content knowledge as "a particular form of content knowledge that embodies the aspect of content most germane to its teachability" (Shulman, 1986, p. 9). He (1986) categorized the pedagogical content knowledge components as knowledge of strategies used for teaching a specific subject in a comprehensible way and knowledge of learners including students' (pre)conceptions and difficulties regarding a specific subject. Shulman (1986) also stated that students misconceptions and overcoming those misconceptions were important components of the pedagogical content knowledge.

After Shulman's (1986) clarification of PCK, a great degree of value has been attached to PCK, and a large number of studies have been conducted to clarify the components of PCK and some scholars have expanded its definition and (sub) components (Grossman, 1990; Magnusson, Krajcik, & Borko, 1999; Mark, 1990 Shulman, 1986; 1987; Tamir, 1988).

Grossman extended the components of PCK clarified by Shulman and specified four components. The first component, *conceptions of purpose for teaching subject matter,* was specified by Grossman as an overarching component additional to Shulman's components. Teacher's knowledge and beliefs about a specific subject, why a subject is taught at a specific grade level and why it is important to teach students were explained by the first component. The second component, which was also specified by Shulman, was *knowledge of students understanding*. The knowledge of student's prior knowledge, misconceptions and difficulties regarding a specific subject was included in this component. Another component, *curricular knowledge,* includes knowledge of curriculum materials needed for effective teaching of a subject and arrangement of the subjects. Although Shulman conceptualized curricular knowledge within a separate category in teachers' knowledge, Grossman specified it as one of the components of PCK. Despite its different position in clarification of Shulman and Grossman, the clarification of it was the same. Both Shulman and Grossman clarified curricular knowledge including the knowledge of the organization of the topics of a subject area in the same year, and also those in the past and the following years. Moreover, the knowledge of the topics of other subject areas taught at the same time was included in this knowledge. The last component, *knowledge of instructional strategies,* includes knowledge of effective representations, examples, and explanations used to make the subject more meaningful to learners as specified by Shulman. Grossman's clarification of the PCK components is presented in Figure 2.1 (as cited in Jing-Jing, 2014, p. 413).

Pedagogical Content Knowledge		
Conceptions of Purposes for Teaching Subject Matter		
Knowledge of Students' Understanding	Curricular Knowledge	Knowledge of Instructional Strategies

Figure 2.1 Grossman's clarification of the PCK components (as cited in Jing-Jing, 2014, p. 413)

Marks (1990) suggested a new PCK model based on the results of the research that he conducted with eight mathematics teachers to investigate PCK regarding equivalence of fractions in fifth grade. Four main categories of PCK were derived from the analysis of the interviews: Subject matter for instructional purposes, students' understanding of the subject matter, media for instruction in the subject matter, and instructional processes for the subject matter. In this model, subject matter is at the center among the four main components of PCK. In addition, different from Grossman's model, Marks identified a new component, media for instruction. Besides, Marks (1990) identified subcategories under each of the main categories, which are presented below in Figure 2.2.

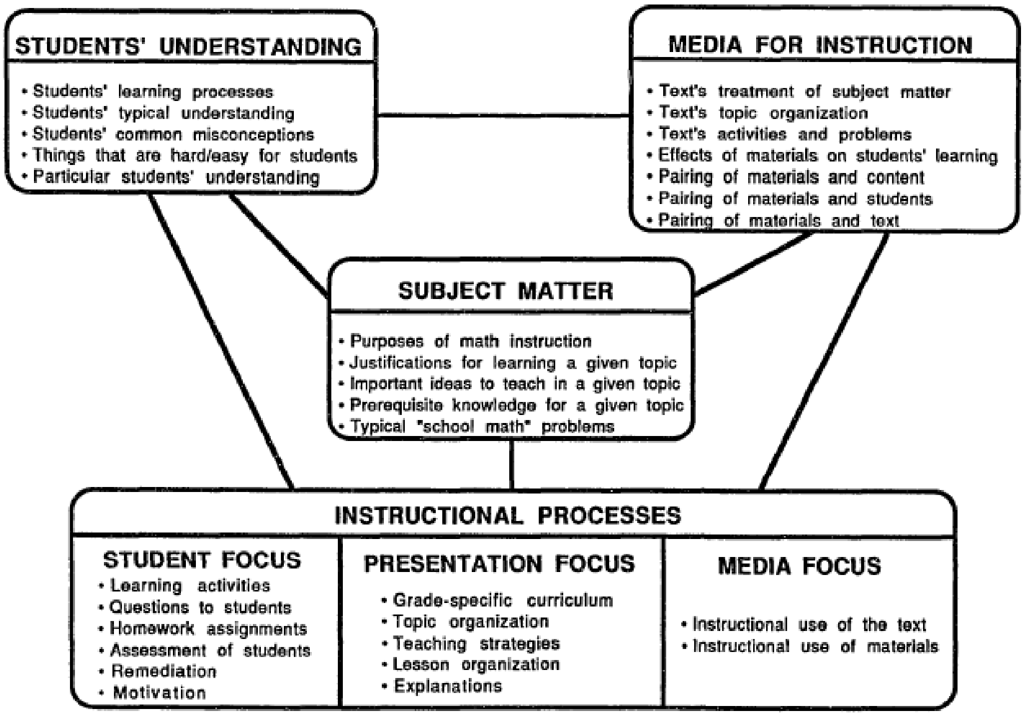


Figure 2.2 Marks' clarification of pedagogical content knowledge in 5th grade equivalence of fractions (Marks, 1990, p. 86)

In the model, Marks added on Shulman's specification by including in the model the knowledge of assessment, which is not included in Grossman's clarification. Contrary to Grossman, he directed the focus of the study purely on teachers' knowledge and excluded teachers' beliefs.

Magnusson, Krajcik and Borko (1999) defined PCK as transformation of subject matter knowledge, pedagogical knowledge, and knowledge of context. There is a reciprocal relationship between these knowledge domains, and PCK stands at the heart of the teacher's knowledge. By expanding Grossman's model (1990), they clarified a PCK model for science teaching, including five components: orientation towards science teaching, knowledge and beliefs about science curriculum, knowledge and beliefs about students' understanding of specific science topics, knowledge and beliefs about assessments in science, and knowledge and beliefs about instructional strategies for teaching science (see Figure 2.3).

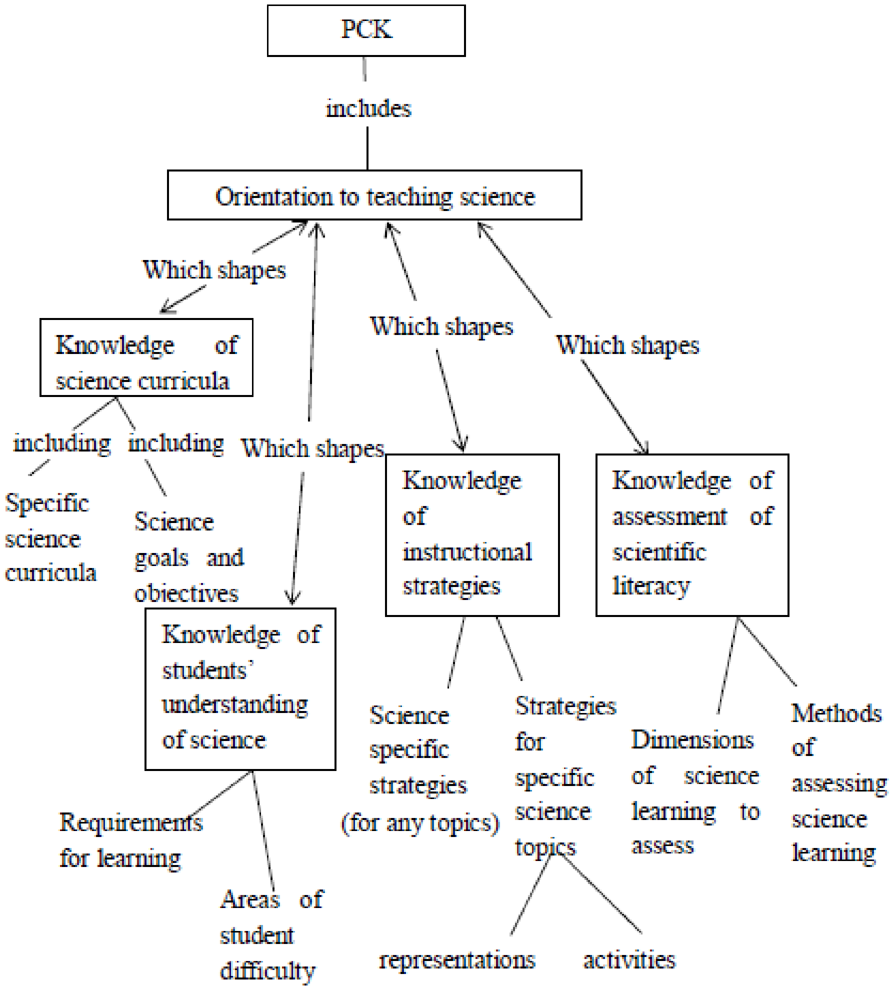


Figure 2.3 Components of PCK for science teaching (Magnusson et al., 1999, p. 99)

In the model, the name of component in Grossman's PCK model, namely 'conceptions of purposes for teaching subject matter', was changed into 'orientation to teaching science'. Although Grossman, in her specification of the component, includes knowledge and beliefs about a specific subject matter, the clarification of Magnusson et al. includes knowledge and beliefs about a specific topic in science teaching. Hence, they narrowed down the subject into science teaching. Additionally, Magnusson and her friends extend the specification of PCK by including knowledge of assessment just as in Marks'.

Another study that was conducted by Park and Oliver (2008) was on that reexamined the components of PCK. Knowledge of three experienced chemistry teachers were examined through multiple sources including classroom observations, interviews, lesson plans, and field notes. The data analysis revealed that PCK consisted of two dimensions, named as understanding and enactment, with six components. These components were located to form a hexagon model with PCK at its center. In this respect, Figure 2.4 diagrams Park and Oliver's model of PCK for science teaching. The five components within the model proposed by Magnusson et al. model were integrated into this model also. These components are orientation to teaching science, knowledge of science curriculum, knowledge of assessment of science learning, knowledge of students' understanding in science, and knowledge of instructional strategies. Similar to the model of Magnusson et al., orientation to teaching science was specified at the top of the model since teachers' beliefs and knowledge about a specific topic affect their decisions about planning, teaching and assessing. In addition to the pre-determined components, Park and Oliver's model offered a new affective component to PCK for science teaching called 'teacher efficacy'. Teacher efficacy is related to teacher beliefs regarding determining effective teaching strategies for achieving some pre-established educational goals (Park & Oliver, 2008).

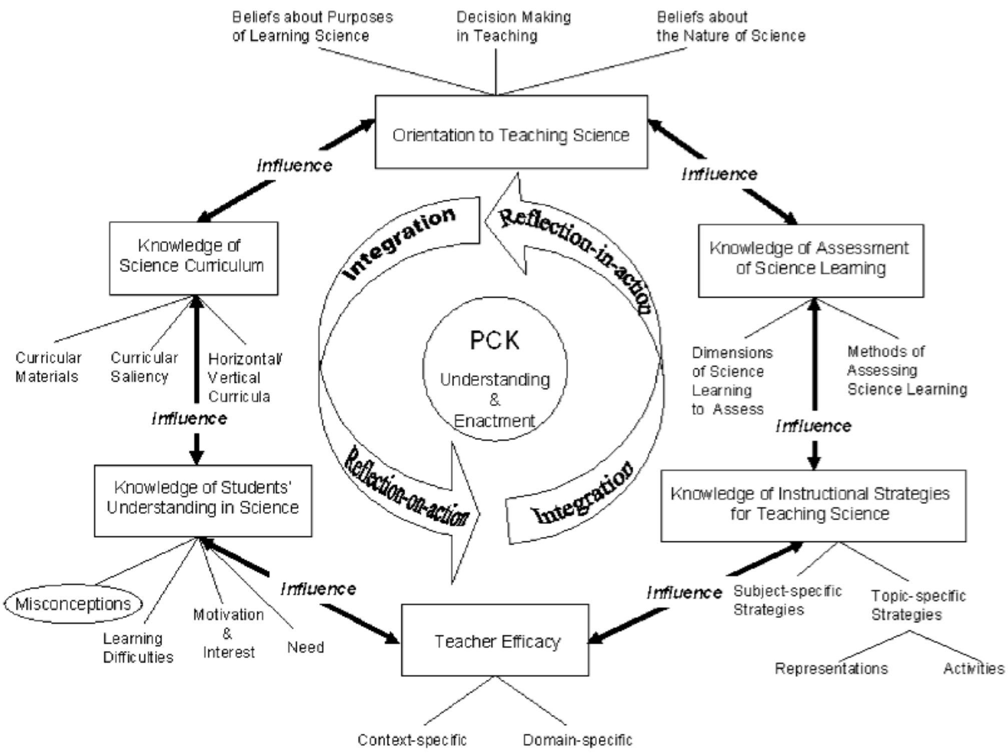


Figure 2.4 Park and Oliver's hexagon model of pedagogical content knowledge for science teaching (2008, p. 279)

Contrary to the teacher knowledge frameworks proposed thus far, Ball, Thames, and Phelps (2008) specifically aim to investigate the nature of teachers' knowledge in mathematics by questioning the knowledge necessary for effective teaching. They defined PCK as a conduit to connect knowledge and practice of teaching and developed a 'practice based theory of mathematical knowledge for teaching'. Within this approach, Ball and her colleagues focus on the term 'mathematical knowledge for teaching (MKT),' which refers to the mathematical knowledge required for teaching mathematics in a comprehensible way (Ball et al., 2008). According to the results of the analysis, they built on Shulman's categories of teacher knowledge. Figure 2.5 presents the dimensions and sub-dimensions of content knowledge in relation with two categories of Shulman (1986), which are subject matter knowledge and pedagogical content knowledge.

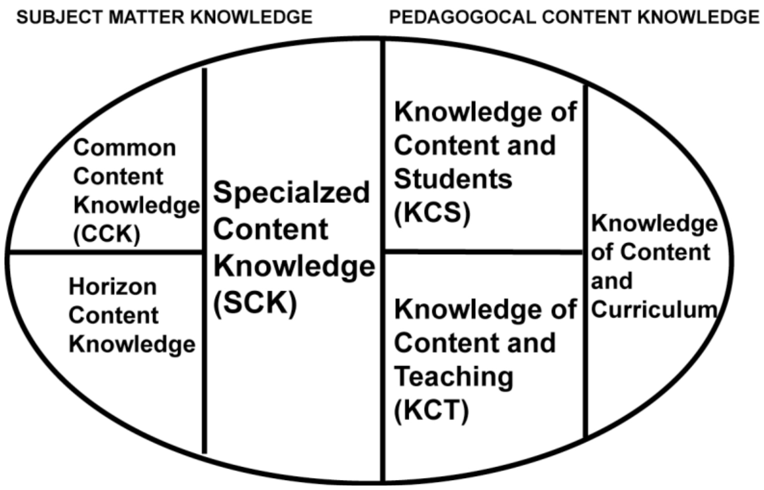


Figure 2.5 Ball et al.'s domains of mathematical knowledge for teaching (Ball et al., 2008, p. 403)

As can be seen in the Figure 5, Ball and her colleagues divide Shulman's subject matter knowledge into three domains: common content knowledge (CCK), specialized content knowledge (SCK), and horizon content knowledge, and his pedagogical content knowledge is subdivided into three domains: knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum.

The common content knowledge (CCK) domain is defined as the mathematical knowledge found in every well-educated person in common, that is, the knowledge required to answer a mathematic questions and this knowledge is not specific to teachers (Ball et al., 2008). To solve a mathematical problem correctly, to use the mathematical terms accurately, and to become aware of a wrong answer can be included in this type of knowledge, and this knowledge is not unique to teachers (Ball et al., 2008). Contrary to CCK, the specialized content knowledge (SCK) domain is unique to teaching and peculiar to teachers. A teacher who has this type of knowledge can use mathematical language properly, can make clear explanations and justifications about a mathematical idea, and can also use appropriate representations to express a mathematical idea effectively (Ball et al., 2008). There is another domain of subject matter knowledge called horizon content knowledge. The knowledge about the connection of the mathematics topics in the curriculum is included in this type of knowledge. Hence, the teacher who has this knowledge can establish a relationship between the topics that students have learned in their preceding years and those that they will learn in the following years (Ball et al., 2008).

In addition to the domains of subject matter knowledge, knowledge of content and students (KCS) is conceptualized as a domain within pedagogical content knowledge. Ball et al. (2008) stated that it is a combination of knowledge of mathematical ideas and knowledge about students. Hence, a teacher with this knowledge can predict what students think, in which areas students have difficulty, whether a task is easy or hard for the students, how they can learn easily, and what can motivate them. Moreover, knowledge of students' conceptions, misconceptions, and difficulties and knowledge of the reason underlying these misconceptions or difficulties can be included in this type of knowledge (Ball et al., 2008). The second subcomponent of pedagogical content knowledge is specialized as knowledge of content and teaching (KCT), which is the combination of knowledge about teaching and knowledge about mathematics and is about the design of the course. This type of knowledge includes the decisions regarding the sequence of the contents, choice of examples, choice of most effective instructional strategy, method, and representations used. Furthermore, it contains instructional decisions regarding when the teacher should provide deeper explanations, or solve new questions (Ball et al., 2008). The last subcategory of pedagogical content knowledge is determined as knowledge of the content and curriculum, which corresponds to Shulman's curricular knowledge category.

As in the theory of Ball et al., that dwells on practice, Rowland, Turner, Thwaites, and Huckstep (2003) aim to develop a practice based framework to investigate teachers' content knowledge by observing mathematics lessons. To this end, researchers at various universities, specifically Cambridge, London, Durham, and York Universities, conducted a collaborative study to examine prospective elementary mathematics teachers' subject knowledge in mathematics (Rowland, Thwaites, & Huckstep, 2003). The importance of observing ongoing teaching was highlighted in this study. For the study, an audit assessment of 16 items was administered to 149 pre-service teachers. Two supervisors of these pre-service teachers examined the relation between the assessment of audit items and the teachers' competency in mathematics teaching; the teachers' subject knowledge was categorized as low, medium and high (Goulding, Rowland, & Barber, 2002). Twelve pre-service teachers from each category of subject knowledge were chosen for observation, and two mathematics lessons of each were observed and videotaped. The grounded approach was used to analyze the data. As a result of the analysis, 18 provisional codes were generated (Rowland et al., 2003). Subsequently, these codes were categorized into four units, which were referred to as *'knowledge quartet'* (Rowland, 2005).

The four dimensions of the knowledge quartet are named as *foundation, transformation, connections, and contingency.* Table 2.1 presents these four dimensions and the related codes under these dimensions.

Table 2.1: Knowledge Quartet Model (Rowland et al., 2009, p. 29)

Dimensions	Codes
Foundation	Adheres to textbook Awareness of purpose Concentration on procedures Identifying errors Overt subject knowledge Theoretical underpinning Use of terminology
Transformation	Choice of examples Choice of representations Demonstration
Connection	Anticipation of complexity Decisions about sequencing Making connections between procedures Making connections between concepts Recognition of conceptual appropriateness
Contingency	Deviation from agenda Responding to children's ideas Use of opportunities

The first dimension of the knowledge quartet is *foundation,* which is related with teachers' knowledge learned in their teacher education. It consists of teachers' knowledge of mathematics content, knowledge of mathematics pedagogy, and beliefs regarding mathematics. Foundation knowledge is essential for the other three dimensions those are transformation, connection, and contingency since all the decisions that we make during teaching is an outcome of this dimension. Shulman's subject matter knowledge and theoretical pedagogical content knowledge coincide with foundation knowledge (Rowland et al., 2009).

The second dimension is *transformation,* which coincides with Shulman's pedagogical content knowledge and can be seen in both the plan and act of teaching. The core question mentioned in this dimension is "*what it means to teach a subject*" (Rowland et al., 2009, p. 30). While teaching a subject, teachers need to represent the content differently to be able to make it comprehensible to students. The presentation of the topic can be in the form of analogies, illustrations, explanations and demonstrations. Moreover, the choice of examples is an important factor in teaching since learning can be realized by the use of examples (Rowland, 2005).

The third dimension of the knowledge quartet is *connection,* which concerns *"the coherence of the planning or teaching across an episode, lesson or series of lessons"* (Rowland et al., 2009, p. 31). It includes teachers' knowledge of the anticipation of complexity as well as sequencing the topics, and establishing a connection between procedures and concepts. In addition to the ordering of the topics of the instruction, teachers also need to order the sequence of the exercises and the questions posed to the students (Rowland & Turner, 2007).

The last dimension is *contingency, which* refers to unexpected events in a learning environment. Even if the teacher plans a lesson completely, some unplanned events can occur. This category questions teachers' responses to these unexpected events or the unplanned questions of the students. The teacher with high contingency knowledge can be able to anticipate students' possible responses and they can be prepared to respond to children's comments (Rowland et al., 2009).

To sum up, after Shulman's pronunciation of the PCK concept, many educational researchers conducted studies to extend and to clarify the teachers' pedagogical content knowledge containing different categories and sub-categories (Ball et al., 2008; Grossman, 1990; Magnusson et al., 1999; Marks, 1990; Park & Oliver, 2008; Rowland et al., 2009; Shulman, 1986, 1987). In addition to studies relevant to the specification of PCK as examined above, there are other specifications also (Carlsen, 1999; Cochran et al., 1993; Fernandez-Balboa & Stiehl, 1995; Loughran et al., 2001; Tamir, 1988). Table 2.2 summarizes different PCK models with different sub-components conceptualized by different researchers. As it can be inferred from Table 2.2, there is no consensus on the components of PCK. However, two components, namely (i) representation and instructional strategies and (ii) students learning and conceptions were specified by all of the scholars after first being clarified by Shulman.

Table 2.2 Different conceptualizations of PCK (Lee & Luft, 2008, p. 1346)

	Knowledge of							
Reference	Subject matter	Representations and instructional strategies	Student learning and conceptions	General pedagogy	Curriculum and media	Context	Purpose	Assessment
Shulman (1987)	a	PCK	PCK	a	a	a	a	b
Tamir (1988)	a	PCK	PCK	a	PCK	b	b	PCK
Grossman (1990)	a	PCK	PCK	a	PCK	a	PCK	b
Marks (1990)	PCK	PCK	PCK	b	PCK	b	b	b
Cochran et al. (1993)	PCKg	b	PCKg	PCKg	b	PCKg	b	b
Fernandez-Balboa and Stiehl (1995)	PCK	PCK	PCK	b	b	PCK	PCK	b
Magnusson et al. (1999)	a	PCK	PCK	a	PCK	a	PCK	PCK
Carlsen (1999)	a	PCK	PCK	a	PCK	a	PCK	b
Loughran et al. (2001)	b	PCK	PCK	b	PCK	b	PCK	PCK

Notes: a, distinct category in the knowledge base for teaching; b, not discussed explicitly; PCK, pedagogical content knowledge; PCKg, pedagogical content knowing.

In addition to the researchers presented in Table 2.2, Ball et al. and Rowland et al. (2009) also conceptualized the knowledge used in mathematics teaching, different from other studies' subject areas. On the other hand, Rowland and his colleagues directed their attention to the observation of an ongoing learning environment. Additionally, a model of mathematical knowledge was established from the data of the pre-service teachers. Therefore, in the present study, knowledge of instructional strategies and knowledge of student learning and conceptions were investigated in the light of the knowledge quartet model proposed by Rowland et al. (2009).

The definition of PCK and the models of PCK were reviewed in this section. In the next section, studies which used knowledge quartet as a framework concerning teachers' PCK are reviewed.

2.2 Studies on Mathematics Teachers' Knowledge with Knowledge Quartet

Since 2003, 'Knowledge Quartet' has been used as a framework in the research studies concerning pre-service and beginning mathematics teachers' mathematical knowledge. While a considerable number of studies on pre-service teachers' mathematical content knowledge have been conducted abroad, the relevant number of studies was quite low in Turkey. The literature review showed that these studies have been conducted to raise awareness in the investigation of ongoing learning environment and exploring teachers' knowledge reflections on the learning environment (Johnson, 2011; Kula, 2011; Kula & Guzel, 2014; Livy, 2010; Petrou, 2009; Rowland, 2005; Rowland et al., 2005; Rowland, Thwaites, & Huckstep, 2003; Rowland, Thwaites, & Jared, 2011a, b; Rowland & Turner, 2007; Turner & Rowland, 2008).

One of the significant studies contributing to the literature of teachers' knowledge by means of the knowledge quartet framework is conducted by Turner (2005, 2007, 2009a, 2009b, 2011). Within the scope of his dissertation, he aimed to investigate beginning teachers' knowledge of mathematics and mathematics pedagogy by looking into their teaching and planning, and he also aimed to examine the usefulness of 'the knowledge quartet' in reflecting and developing the beginning teachers mathematical content knowledge. To this end, he conducted a four-year longitudinal study with twelve trainee teachers. The data analysis of the first year revealed that trainee teachers' explanations of their reasoning underlying their teaching made them realize their improper teaching strategies. The reasons behind these teaching strategies were described as published schemes, the National Numeracy Strategy (NNS), and the teaching style of their mentors. Hence, they indicated that if there were any limitation on teaching, they would teach differently. While reflecting on their teaching, they also realized the inconsistency between their beliefs regarding teaching and their own teaching. Therefore, they looked for ways to make their teaching more compatible with their beliefs with the help of knowledge quartet framework.

In 2007, Turner focused on the choice and use of representations of elementary school teachers. The results showed that the use of representations were inappropriate in teaching since they were not chosen based on the requirements of students' learning outcomes.

In another study, Turner (2009a) aimed to investigate the beginning teachers' ability to respond to students' incorrect answers to questions or incorrect statements during teaching. To this end, the development of those three trainees was followed for four years in the light of knowledge quartet. The results showed that being able to respond to the contingent actions of the students was an important factor for effective teaching, and it was concluded that the items found in the knowledge quartet guide beginning teachers by showing the significant points that they need to pay attention. Hence, it helps to enhance the development of beginning teachers' PCK concerned with knowledge of students' errors/misconceptions.

In 2009b, Turner published a report on the conceptions of one beginning teacher, Kate, who participated in the fourth year of the study with three other trainees. While the analysis of the first year-data showed that Kate's focus was on procedures, in the second year her focus shifted to the number of different representations. In the third year, Kate emphasized conceptual understanding of the students instead of procedures. Additionally, she stated that reflecting on her teaching with the help of knowledge quartet improved her mathematics teaching. In addition, Turner (2011) presented a paper on Kate's mathematical content knowledge revealed through the foundation dimension which is one of knowledge quartet's dimensions. So that, the development of Kate's mathematical content knowledge was followed for four years in the light of knowledge quartet. The results revealed that the reflections and the discussions on practice developed her mathematical content knowledge for teaching.

Similar to Turner, Livy (2010) conducted a four-year longitudinal study investigating the relationship between mathematical content knowledge and pedagogical content knowledge in teaching. In 2010, she reported the findings of one of the seventeen pre-service teachers' practice and mathematical content knowledge in the light of knowledge quartet. The analysis of the data indicated that Lisa did not demonstrate different solutions or different representations to promote the learning of the students because of limited knowledge in teaching strategies. In conclusion, the researcher suggested that pre-service teachers' mathematical content knowledge needs to be developed through practice.

Until this point, all the research studies mentioned were conducted in UK. However, there is also another study in the literature which was conducted by Petrou (2010) in Cyprus. The aim of the study was to examine the relationships between Cypriot pre-service teachers' mathematical content knowledge and their teaching practice with the help of knowledge quartet. As a result of the study, it was found that knowledge quartet was an extensive tool that could be used to analyze the observations of the mathematics lessons in Cyprus. On the other hand, the use of mathematics textbooks was also important to investigate Cypriot classrooms. However, it was not mentioned within the framework. Therefore, knowledge quartet was adapted for the use of mathematics textbooks. For this reason, the researcher recommended that the differences between the developed conditions of the framework and its applied conditions needed to be considered in order to adapt it to the contexts of other countries.

The studies mentioned above investigated pre-service or beginning teachers' mathematical content knowledge in teaching in other countries. In contrast to the extensive number of studies conducted abroad specifically in the UK, there is a limited number of studies conducted in Turkey regarding teachers' knowledge revealed through knowledge quartet (Dogan & Isiksal, 2014; Kula, 2011). In one of those studies, Kula (2011) used knowledge quartet as a tool for the analyses. The aim of her study was to investigate pre-service secondary mathematics teachers' mathematical content knowledge in the act of teaching the concept of limit. The data were gathered from four pre-service teachers by means of observation, interview, and lesson plans. To begin with, the participants prepared a lesson plan for each of the four lessons, and a pre-interview was conducted with each participant before the observation to discuss their plans. Subsequently, a post interview was conducted following the observations. Both the observations and interviews were recorded and transcribed for analysis. The results of the study indicated that pre-service teachers' beliefs affect the preparation and teaching processes of the lessons. It was also found that pre-service teachers' knowledge of students' misconceptions and difficulties was limited. Their inadequate knowledge of students' possible misconceptions caused the emergence of new misconceptions. At this point, teachers realized their inadequacies in the act of teaching and stated that feedback given contributed to their knowledge and they would take into consideration the points mentioned while planning other lessons. Therefore, it is believed that critical reflections of novice teachers' lessons will be helpful to develop their knowledge.

As previously stated, one of the purposes of the present study was to investigate pre-service elementary mathematics teachers' knowledge of students' misconceptions/difficulties regarding the area of the triangle concept. After this section, which focused on studies related to teachers' knowledge revealed through knowledge quartet that is the framework of the study, the next section directs its focus to the concept of area.

2.3 Measurement

Principles and Standards for School Mathematics offers five content standards, namely number and operations, data analysis and probability, algebra, geometry and measurement, which students need to learn from prekindergarten to K-12. Those of measurement are one of the important content in school curriculum since its application can be commonly found in real life settings (Cavanagh, 2008; NCTM, 2014, Ministry of National Education [MoNE], 2014). Bright defined the measurement concept as the comparison between an attribute of an object and chosen unit of measure (as cited in Zembat, 2014).

When the area measurement examined in the Turkish Mathematics Curriculum at third grade, students are expected to understand the meaning of the area concept. At fourth grade, students are expected to distinguish the relation between the perimeter and the area concepts. Moreover, students should be able to realized the act that area of a shape is formed by unit squares. At fifth grade, students should be able to calculate the area of rectangles. At sixth grade, students are expected to explain the meaning of perpendicular line segment and height concept. In addition, students should be able to calculate the area of parallelogram and the area of triangles. At seventh grade, students are expected to calculate the area of trapezium and the area of rhomb, also, they should solve the relevant problems. However, at eight grade, a specific area measurement topic is not involved in the curriculum. Hence, it can be concluded that area measurement took an important place in the curriculum.

2.3.1 Area Measurement

One of the important domains of measurement is the area (Outhred & Mitchelmore, 2000). Baturo and Nason defined area as "the amount of surface that is enclosed with a boundary" (1996, p. 238). To be able to understand the measurement of the area, Stephan and Clements (2003) offered four foundational concepts: *Partitioning, unit iteration, conservation, and structuring an array.* Partitioning can be explained as cutting a two dimensional space into smaller regions to be able to count them mentally. The other concept, unit iteration, is about iterating a specified area unit until completely covering the region without any gaps or overlaps. On the other hand, area conservation means that when a region is cut and rearranged into another shape, its area does not change. The last concept, structuring an array, is related to realizing the underlying structure of the arrays so that there is equal number of units in each row (or column) in a rectangular region. In this regard, Cavanagh (2008) asserted that investigating multiplicative nature of arrays is critical for students' meaningful learning. Not being able to investigate the multiplicative structure underpinning the area formula can cause superficial understanding of the area by students. The understanding of the area was divided into three levels by Zhou (2012) (Table 2.3).

Table 2.3 The Concept Map of the Area Attribute Dimension (Zhou, 2012, p. 7)

Level	Description of Thinking	Indicators
At 1	Students view objects holistically based on their appearances.	• Have some intuitive notions of area as the size of a space, but this understanding is limited to some at surfaces of familiar shapes. • Can make some straightforward visual comparisons of two areas. • Recognize that cutting symmetrical figures in the middle would divide their areas in halves.
At 2	Students abstract the attribute of area from its various geometrical representations.	• Generalize the notion of area beyond surfaces of familiar shapes. • Recognize that the total area is unaltered in certain transformation, such as dividing a shape and rearranging its components. • Use cut-and-paste (actual or imaginary) method to make a given shape into another given shape in order to compare the areas of incongruent figures.
At 3	Students gain better insight into the nature of area attribute and can apply this understanding to solve new problems beyond the comparison of areas.	• Transform an existing shape to a new shape to solve problems about areas. (This *differs from the level 2 performance. At level 2, the shapes of comparisons are given and students are required to find ways to transform one shape into another. At level 3, however, the specific configuration of the target shape, or even the need to reconfigure the shapes, may not be apparent.)* E.g., given a shape with a part of it shaded, students can find the relation between the shaded part and the unshaded part by appropriately dividing and recomposing the figure.

In the first level, students can compare the areas of two shapes visually. However, they cannot consider the fact that different shapes can have the same area. They may also believe that the area measure is directly proportional to the length measure; in other words, students may think that if the area of a shape increases by the same amount of increase of the perimeter of the shape or vice versa. In the next level, students realize that different shapes can have the same area. Moreover, they understand that decomposing a shape and recomposing a new shape does not change its area. In the last level, students recompose new shapes from the parts of other shapes even if the question did not ask them to do so (Zhou, 2012).

In this section, key concepts for understanding the area concept and levels of understanding this concept are presented. In contrast to the importance given to the area measurement in the school curriculum, students' understanding of the concept was at superficial level (Tan-Sisman & Aksu, 2009). Since they could not comprehend the concept in a meaningful way, they experienced some difficulties and held some misconceptions regarding the area concept (Cavanagh, 2008). In the next section, the difficulties the students experienced and the misconceptions they held regarding the area concept will be presented.

2.4 Difficulty and Misconception

The terms 'misconception' and 'mistake' were defined differently in the literature. In one of them, mistake or error is defined as calculation errors made by students, while misconception is defined as "conceptual obstacles making learning difficult" (Biber, Tuna, & Korkmaz, 2013). In another study, misconception was defined as a student conception that causes an error regularly (Smith, Andrea, diSessa, & Roschelle, 1993). Corresponding to Smith and his colleagues' definition, Riccomini (2005) referred to misconception as common errors in that repeated mistakes produce error patterns.

Errors can be seen easily in learners' speech or solutions. However, misconceptions cannot be seen at first glance (Luneta & Makonye, 2010) and arise from students' prior learning (Biber, Tuna, & Korkmaz, 2013; Luneta & Makonye, 2010; Olivier, 1989; Riccomini, 2005; Smith et al., 1993; Turkdogan, Baki, & Cepni, 2009). To unveil the misconceptions, you need to root around the given answer of the learner.

On the other hand, the term difficulty is defined by Bayazıd (2015) as students' obstacles in the process of comprehension, signification, and understanding of a concept. Thus, difficulty is used to define the problems that the students experience in mathematics learning in general. In this respect, the term difficulty includes both errors and misconceptions within itself.

In the present study, misconception and error were not examined separately. Instead, the term misconception/difficulty was used together to refer to refer to students' limited conceptions and insufficient prior knowledge pertaining to the area of triangles.

The source of these mistakes and misconceptions of students and the ways to deal with them should be determined. What's more, those misconceptions need to be overcome before they affect students' learning process of subjects in the future (Biber, Tuna, & Korkmaz, 2013; Simsek, 2011; Turkdogan, Baki, & Cepni, 2009). Therefore, a large body of research has been conducted to investigate students' understanding of the concept of the area. Students' misconceptions regarding measurement were also investigated in these research studies. In the next section, studies on students' conceptions and misconceptions on the concept of area are presented.

2.4.1 Studies Related to Students' Conceptions and Misconceptions/Difficulties Regarding the Area Concept

It is important to give some place for studies on students' conceptions and misconceptions/difficulties in this section since teachers' knowledge of the misconceptions students hold and the difficulties they experience in relation to the area concept was a dimension of this study, and it can provide researchers and in-service teachers with some insight.

In the study of Huand and Witz (2013), children's understanding of area and the area formula of a rectangle were investigated. The sample of the study consisted of 22 fourth-graders from a public elementary school in Taipei, Taiwan. Three treatments concerning area measurement, geometry motions, and both geometry motions and area measurement were administered to the selected three classrooms. Subsequently, a paper-and-pencil test on area measurement and one-to-one interviews were performed. The results of the study indicated that some of the students defined the concept of area as measuring the area. Also, to find the area of a rectangle, they used the formula by rote memorization; they could not explain how the multiplication of base and height works. Moreover, some of the students mixed perimeter and the area concept in that to find the area of a rectangle they summed the length of the sides.

With a similar aim of the study conducted by Huang and Witz (2013), Cavanagh (2008) tried to investigate seventh graders' understanding of the area in Sydney. The participants included 43 students, who first completed a paper-test containing five questions based on the area of rectangles and triangles. Afterwards, 12 of those students were interviewed one to one following two weeks of lessons on area measurement of squares, rectangles and triangles. In the first question, students were asked to define area. Seventy two percent of them defined area as "space inside the shape" while 12% of them did so as "length times width" after the lessons. In another question, the area of a right angled triangle (the sides being given as 3, 4, 5 cm) with 1 cm intervals being marked on its sides was asked. Forty percent of the students tried to draw grids (after covering area measurement in class), while the percentage was 21% before the lessons. Some of the students forgot to divide by 2, and some of the students multiplied the three sides of the triangle. In question four, in which the area of an L-shaped figure with unknown sides was asked, only 23% of the students were able to calculate the area correctly. Many of the students tried to find the perimeter of the shape instead of its area. Finally, in question five that had two parts, students were asked to draw a rectangle with an area of 24 cm^2 and in the second part, students were asked to draw a nonrectangular shape with the same area. However, many of the students drew a shape that had a perimeter of 24 cm for the both of the questions. In the interview part of the study, the students were given three pieces of cardboard; one of them was a 10 cm by 8 cm rectangle, while the other one was a right-angled triangle having sides of 10, 12 and 15.5 cm, and the last shape was a parallelogram with a 10 cm base and an 8 cm perpendicular height. The question asked whether the area of the given triangle and parallelogram could be equal to the area of the given rectangle, and what the reasoning underlying their answer was. By placing the triangle over the rectangle, eight of the 12 students answered that the triangle had a smaller area than the rectangle, while 3 of them claimed the reverse by saying that the sides of the triangle were longer than those of the rectangle. The last student claimed that the two areas could not be compared because the shapes were different. For the parallelogram, all of the 12 students answered that the rectangle had a smaller area than the parallelogram since they compared the slant height of the parallelogram with the perpendicular height of the rectangle.

Another researcher investigating students' understanding, strategies and misconceptions as regards area measurement was Zacharos (2006). The researcher examined the effect of a special teaching course and different use of tools on students' measurement strategies. One hundred and six students participated in the research; 56 of them were in the experimental group and 50 of them were in the control group. In the experimental group emphasis was on conceptual understanding of area measurement; however, in the control group, the emphasis was on the use of formulas rather than conceptual understanding of the concept. After the instruction, a one-to-one interview was conducted with each student. The results of the study revealed that the students used the area=$base \times height$ strategy for each shape regardless of whether the shape was a rectangle or a parallelogram. Moreover, the students used *area=base+height* strategy or *area=total lengths of sides of a figure* to find the area of a geometric shape.

Another study worth mentioning is a master's thesis conducted by Orhan (2013) in Turkey. The purpose of the study was to investigate the common errors of middle school students in the domain of area and perimeter. One hundred and eleven private middle school students (6^{th}, 7^{th} and 8^{th} grade) participated in the study. Procedural and conceptual tests on area and perimeter developed by the researcher were administered. Procedural knowledge tests require using formulas and computations to find the areas and perimeters of given figures, while a conceptual knowledge test requires the test taker to understand the rationale underlying the concepts. When the answers of the students were examined, it was seen that students could calculate the perimeter of the square with ease, but they were unable to calculate one side of the square when its area was given. They generally divided the area by four to find the length of one side. Moreover, the students had some misconceptions while calculating the area of a parallelogram. The first misconception that was identified was that students used the slant height instead of the given perpendicular height as did students in the study of Cavanagh (2008). Another common error that was determined was that the students used the perpendicular height even to calculate the area; they divided the product of the height and the side by two. In addition, when the area of the triangles was asked for, it was observed that students did not divide the multiplication of the height and the length of the side by two. Furthermore when the area of a composite figure was asked, the students tried to calculate the perimeter of the polygon rather than its area.

In another study, Gökdal (2004) investigated misconceptions in relation to proportion of 8^{th} and 11^{th} grade students in the topics of area and volume. To determine the misconceptions, the researcher administered a test with 16 open-ended questions to 562 students. The results of the analysis corresponded with the results of Cavanagh's study in that the students used the concepts of the perimeter and area interchangeably. They also did not know the concepts of base and the corresponding height. Moreover, to find the area of a right angle triangle whose three side lengths were given, some of the students multiplied its perpendicular edges and forgot to divide it by 2, and some of the students multiplied the three sides of the triangle, while some of them multiplied one of the perpendicular sides with the hypotenuse. In addition, some of the students calculated its perimeter rather than its area.

Similar domains of measurement, perimeter and area were studied in a study by Tan-Sisman and Aksu (2009), as in Orhan's study (2013). The researchers examined 7th grade students' understanding of the concepts of the perimeter and the area. Eight open-ended questions, which were posed to 134 seventh grade students, served as the data source of the study. The results of the study revealed that students' understanding of the area and the perimeter were at superficial level. Moreover, students used the perimeter formula to calculate the area of a figure or vice versa. Another study was conducted by Tan-Sisman and Aksu (2012) on three domains of measurement: length, area and volume. In this study, the researchers aimed to examine 6th grade students' procedural and conceptual knowledge and their problem solving skills in relation to these concepts. The Conceptual Knowledge Test, the Procedural Knowledge Test and the Word Problem Test were administered to 445 sixth grade students. The results showed that students' performances in all tests were very low. Therefore, it was concluded that students did not know the meaning of measurement and they had some problems in solving word problems.

Similar to the concepts researched by Tan-Sisman and Aksu (2012), Dagli (2010) conducted a study not only to investigate 5th graders' understanding of perimeter, area and volume, but also to determine their misconceptions regarding these concepts. A 40 item-test was administered to 262 fifth graders. The results of the study coincide with the results of the above studies in terms of confusion between the area and the perimeter concept. In addition, students experienced difficulty mostly in area measurement problems.

To sum up, the conducted studies manifestly show that students have some misconceptions regarding the concept of the area. These were summarized by Esen (2013), as follows: Difficulty in understanding the area as a space inside a figure, difficulty in understanding transitivity and conservation of the area, confusing the area and perimeter, difficulty in understanding that the area and perimeter are directly related in that one determines the other, difficulty in applying the formula to find the area of a rectangle or that of plane figures other than rectangles, difficulty in interpreting the results of the procedure, confounding linear and square units, and difficulty in understanding the inverse relationship between the size of the unit and the number of units (p. 45).

Eliminating the source of the misconceptions requires improving teachers and teacher candidates' knowledge regarding mistakes and misconceptions (Turkdogan, Baki, & Cepni, 2009). However, teacher candidates have difficulty in determining students' misconceptions (Turnuklu & Yesildere, 2007). Moreover, teachers are unable to decide on the appropriate instructional strategies for related misconceptions even if they are able to determine the source of misconceptions in students' mathematics work (Riccomini, 2005).

In this respect, studies have been conducted to investigate pre-service and in-service teachers' pedagogical concept knowledge in terms of students' misconceptions on certain topics. In this part of the literature review, studies on students' conceptions and misconceptions in the area concept have been presented. In the next section, studies on teacher knowledge of the area concept are outlined.

2.4.2 Studies on Mathematics Teachers' Knowledge on the Concept of Area

Misconceptions in relation to the area concept do not merely belong to students. Some teachers or teacher candidates may also hold these misconceptions. In this part of the chapter, studies related to teachers' content knowledge on the area concept and pedagogical content knowledge are reviewed.

One study by Murphy (2012) investigated the relationship between pre-service teachers' subject matter knowledge (SMK) on the area concept and the planned lesson to teach that concept. The results of the analyzed data by four prospective mathematics teachers showed that limited subject matter knowledge on a topic was caused by rule oriented teaching although a sufficient SMK on the topic led the teacher to use inquiry based teaching.

In another study, Baturo and Nason (1996) evaluated freshman elementary mathematics teachers' subject matter knowledge on the area concept. Eight tasks assessing pre-service teachers' SMK on the area concept were applied to 16 first-year pre-service teachers during interviews. The results of the study revealed that prospective teachers' SMK was insufficient. The areas of rectangular shapes would be taught to make calculations based on rote memorization without engaging in meaningful learning since knowing the formulae and the way to calculate was perceived to be more important than understanding the rationale of the underlying rules. Not knowing the relation between a rectangle and a triangle, the students could not understand why it is divided by two when calculating the area of the triangles. Moreover, it was also concluded that pre-service teachers with limited SMK would experience some difficulties while teaching the concept in an effective and meaningful way.

Another study by Yew, Zamri and Lian (2010) yielded similar results to those reported by Baturo and Nason (1996). The researchers conducted a study to investigate pre-service teachers' knowledge of the area formulae. The analyzed data obtained from eight pre-service teachers showed that none of the eight pre-service teachers was able to develop a formula for rectangular areas, although they all remembered the formula for the area of a rectangle. Moreover, only two of the eight participants were able to develop the formula for triangular areas based on the formula of a rectangular area. It can be concluded from the results that the pre-service teachers did not learn the formulae in a meaningful manner; they learnt them via rote memorization without relating the formulae for the area of a rectangle to that of a triangle or the others.

Since teachers' limited knowledge of concept definitions can cause students to experience difficulties and to hold misconceptions regarding that concept, Bozkurt, Koç and Yılmaz (2014) aimed to examine pre-service teachers’ knowledge regarding definitions of the concepts of length, area and volume. In this respect, a form requiring the definitions of these terms and asking for the relationship between the terms was administered to 85 freshman elementary mathematics teachers. The results revealed that the area concept came to prominence as quantification rather than as a concept. In addition, some freshmen defined the area as length multiplied by width. In the light of these results, the researchers inferred that interpretation of the meaning as a formula or a dimension could result in misconceptions and difficulties for students.

With a similar aim to the research of Bozkurt et al. (2014), Ayan, Yemen-Karpuzcu and Isiksal-Bostan (2014) investigated pre-service teachers' subject matter knowledge regarding the concepts of the perimeter, area, surface area and volume and the relationship between these concepts. Two open-ended problems were directed to 22 elementary mathematics teachers. The first problem asked which geometric figure with the same perimeter had the largest area. Sixteen out of 22 participants gave the square as an answer, while the circle or the hexagon was the answer provided by six of the participants. To answer the question, thereof the six pre-service teachers who gave the circle as an answer had formed geometric figures and compared their areas. On the other hand, three of the six pre-service teachers who gave the hexagon as an answer stated that as the number of the sides increased, so did the area. The second problem asked which solid with the same surface area had the largest volume. Only two participants answered the problem correctly by giving sphere as the answer and just one of them answered both of the questions correctly. Accordingly, pre-service mathematics teachers failed to understand the relationships between these concepts; however, they did not have any difficulty in calculating the area and perimeter of a shape or the surface area and volume of a solid. The researchers specified the probable reason underlying this situation by stating that while teaching these concepts more importance was given to operational skills rather than to activities involving the interpretation of relationships between concepts. Thus, they concluded that it is important for prospective teachers to comprehend the concepts in depth since these prospective teachers will teach these concepts in future.

The studies mentioned above were related to teachers' subject matter knowledge on the area concept. In addition to these studies, there are also some research studies examining teachers' pedagogical content knowledge (PCK) in relation to the concept of the area.

Yeo (2008) questioned how beginning mathematics teachers’ PCK on area and perimeter affects grade 4 students in the act of teaching. To this end, five of a beginning teacher’s lessons on area and perimeter were observed and video-taped. The results indicated that the teacher had a sufficient level of PCK since his choice of examples was fruitful for students’ meaningful learning; also he selected activities according to students' needs. Just in one situation he showed lack of PCK in that he could not realize the students' lack of conception in defining area. When students defined the area as length times width, he did not explain that it is a concept rather than a formula.

To prevent students' misconceptions and difficulties, teachers need to anticipate students' misconceptions and difficulties in relation to the area concept and to know the strategies to overcome these misconceptions. Therefore, SMK and PCK of class teachers as regards the area concept were investigated by Dogan and Isiksal (2014). Data were collected from a fourth grade teacher by means of a pre-interview, a post-interview and observation of five lessons. Analysis of the gathered data was carried out by knowledge quartet. The results indicated that the teacher was unaware of the goals and objectives of the lesson, so the teacher was incapable of relating the objectives. Since the teacher had not mentioned the connection between the methods of solution, the students found the area of the rectangles by counting one by one instead of using the formula of length times width.

Another study, which was conducted as a master's thesis in Turkey by Simsek (2011) sought to investigate pedagogical content knowledge of prospective mathematics teachers with respect to the difficulties students experienced in area and perimeter. The participants of the study consisted of five senior students from Cumhuriyet University. Interviews and observations were utilized as data collection tools. Three scenarios about the relationship between the perimeter and area were used in the interview. Subsequent to interviews, pre-service teachers' pre-planned lessons were observed and videotaped. The analysis of the transcribed data revealed that teacher candidates did not have sufficient knowledge of students' misconceptions; even they themselves had these misconceptions. Moreover, they did not have adequate knowledge of educational strategies to eliminate these misconceptions.

2.5 Summary of Literature Review

In the relevant literature, a large body of research has attempted to shed light on the conceptualization of teachers' knowledge. Following the PCK of Shulman (1986), a countable number of researchers conducted studies to extend and to clarify the definition of pedagogical content knowledge and its different categories and sub-categories(Grossman, 1990; Magnusson et al., 1999; Marks, 1990; Park & Oliver, 2008; Shulman, 1986; 1987). Different from the categorizations proposed so far of teachers' knowledge or specifically PCK, Ball et al. (2008) and Rowland et al.(2009) focused on the nature of mathematical content knowledge in the act of teaching. Contrary to the study by Ball and her colleagues, Rowland et al. (2009) gave more importance to the observation of ongoing learning environments rather than the administration of questionnaires.

Due to its significant place in real life settings, measurement is one of the important mathematical content areas in the school curriculum (NCTM, 2014). As it can be seen in related literature, one of the mostly used concepts in research is area measurement. Although measuring area was central to an extensive number of studies, conducted research showed that most of the students had a superficial level of understanding the area concept and their limited learning caused them to hold misconceptions and to experience difficulties (Cavanagh, 2008; Dagli, 2010; Gokdal, 2004; Huang & Witz, 2013; Tan-Sisman & Aksu, 2012; Orhan, 2013; Zacharos, 2006; Zhou, 2012).On the other hand, teachers' knowledge regarding students' difficulties and misconceptions and their knowledge of appropriate instructional strategies were significant for effective teaching and for eliminating students' difficulties and misconceptions (Riccomini, 2005; Turkdogan, Baki, & Cepni, 2009). However, the extensive amount of evidence indicated that teachers and teacher candidates have insufficient knowledge regarding misconceptions and difficulties; in fact, even they, themselves, had these misconceptions (Baturo & Nason, 1996; Yew, Zamri, & Lian, 2010). Moreover, they did not have adequate knowledge of instructional strategies to eliminate these misconceptions (Simsek, 2011). Since teachers' knowledge is a significant factor for students' achievement, more studies should be conducted on teachers' knowledge on specific topics.

Even though measurement is one of the important content areas, reviewed literature revealed that comprehension of the measurement topics, especially that of the area concept was limited (Cavanagh, 2008; Tan-Sisman & Aksu, 2009; Zacharos, 2006). Although teachers' pedagogical knowledge on measurement has a crucial role in student achievement (Hatisaru, 2013; Lenhart, 2010), there is a limited number of research on teachers' knowledge regarding the area concept (Dogan & Isiksal, 2014; Simsek, 2011; Yeo, 2008). Since teachers' pedagogical content knowledge has a significant effect on students' achievement and is important for effective teaching, the purpose of the present study is to investigate pedagogical content knowledge of pre-service teachers in relation to the area concept. In this respect, the foci of the present study was narrowed down to pre-service mathematics teachers' knowledge regarding students' difficulties and misconceptions and the knowledge of strategies to overcome those difficulties and misconceptions regarding the area of triangles.

CHAPTER III

METHODOLOGY

The purpose of the study was to investigate middle school pre-service mathematics teachers' pedagogical content knowledge of the concept of the area of triangles. In this respect, pre-service teachers' knowledge of misconceptions/difficulties that students may have regarding the concept of the area of triangles was investigated. Additionally, pre-service teachers' knowledge of the instructional strategies that were used during practice teaching to overcome misconceptions/difficulties held by students was examined.

In this chapter, the design of the study, the context of the study, the participants of the study, the data collection techniques and the data collection tools, data analysis, trustworthiness and ethical considerations are explained.

3.1 The Design of the Study

To understand the meaning behind a phenomenon and to gain insight into the phenomenon, qualitative research designs are commonly used (Merriam, 1998). Hence, the qualitative research methodology was utilized in the present study in order to conduct an in-depth investigation of pre-service teachers' knowledge of misconceptions/difficulties that may be held by students regarding the area of triangles and their knowledge of strategies that were used to eliminate these misconceptions/difficulties of the students during the practice of teaching.

Merriam (1998) regarded the qualitative research as an umbrella including different types of inquiry and it allows us to understand a phenomenon without changing the natural environment. Similar to the definition made by Merriam (1998), Creswell (1998) defined qualitative research as "an inquiry process of understanding based on distinct methodological traditions of inquiry that explore a social or human problem" (p.15). In other words, qualitative designs provide insight into the meaning of a phenomenon that people have constructed. Moreover, such research studies help researchers to describe in detail the quality of a particular situation (Frankel, Wallen & Hyun, 2012).There are four overlapping characteristics that have been emphasized by different writers to understand the nature of qualitative research. One of the four characteristics of qualitative designs is that the importance is placed on understanding the phenomenon by examining the point of views of participants. Another characteristic is related to the researcher since data collection and data analysis are conducted by the researcher. Another one is that extensive amount of data is gathered from observations, interviews and documents and they form larger categories. Finally, the last characteristic is that the findings provide deep knowledge (Merriam, 2009).

After providing some information regarding the qualitative research design, information on the case study as one of the types of qualitative research design is presented in the following section.

3.1.1 Case Study

The qualitative case study design was employed in this study. Creswell (1998) defined case study as "an exploration of a bounded system or a case (or multiple cases) over time through detailed, in depth data collection involving multiple sources of information rich in context. This bounded system is bounded by time and place, and it is the case being studied a program, an event, an activity, or individuals" (p. 61). Similarly, Merriam (2009) defined case study as "an in-depth description and analysis of a bounded system" (p. 40).

As it can be concluded from the definitions, the most important characteristic of the case study research is the nature of the case. The case can be a student, a teacher, a program, a class, or a school (Creswell, 1998; Frankel, Wallen, & Hyun, 2012; Merriam, 1998). In this respect, the case of the present study was two senior pre-service middle school mathematics teachers enrolled in the middle school mathematics teacher education program in the Middle East Technical University (METU). Since the purpose of the study was to gain in-depth understanding of the pre-service teachers' knowledge of misconceptions/difficulties that may be held by students regarding the area of triangles and their knowledge of strategies that they used to eliminate these misconceptions/difficulties of students in their practice of teaching, the single case study was employed. The pre-service teachers' pedagogical content knowledge served as a holistic unit of analysis in the study. In addition, the context of the study was the middle school mathematics education program. In the next part, detailed information about the context of the study is provided.

3.1.2 Context of the Study

In this part of the study, information about the context of the study, which was the elementary mathematics education program, the courses that were taken to graduate from the university and practice school environment are provided in a detailed manner.

The Elementary Mathematics Education (EME) Program at Middle East Technical University (METU) is a four-year program. The courses included in the program are from the department of Mathematics, Physics, Statistics, Turkish Language, Modern Languages, History, Computer Education and Instructional Technology, Educational Sciences and Elementary Education. The courses of the EME program are provided in Table 3.1 (METU, 2013). Subsequent to this program, pure mathematics and science courses are offered in the first two years. In the following years, pre-service teachers attend educational courses. In their first semester of their third year, pre-service teachers take the Methods of Teaching Mathematics I course. The course content includes the objectives of the mathematics teaching, methods, techniques and materials in mathematics teaching and teaching approaches in school mathematics. In the following semester, the Methods of Teaching Mathematics II course is offered to pre-service teachers. During the course, pre-service teachers learn and discuss effective teaching methods for meaningful learning of the concepts of numbers, algebra, geometry, measurement, probability and data analysis. Moreover, students' misconceptions related to these concepts are analyzed. In their senior year, school experience and practice teaching in elementary education courses are offered to the pre-service teachers. In the first semester they just attend real classroom settings and observe real classroom environments. However, in their final semester before graduation, they both observe and have practice experiences in their cooperating schools.

Table 3.1 Courses in the Elementary Mathematics Education Program at METU

FIRST YEAR	
First Semester	Second Semester
MATH 111 Fundamentals of Mathematics	MATH 112 Discrete Mathematics
MATH 117 Calculus I	MATH 118 Calculus II
MATH 115 Analytic geometry	MATH 116 Basic Algebraic Structures
ENG 101 English for Academic Purposes I	CEIT 100 Computer Application In Education
EDS 200 Introduction to Education	ENG 102 English for Academic Purposes II
IS 100 Introduction to Information Technologies and Applications	
SECOND YEAR	
Third Semester	Fourth Semester
PHYS 181 Basic Physics I	PHYS 182 Basic Physics II
MATH 219 Introduction to Differential Equations	MATH 201 Elementary Geometry
STAT 201 Introduction to Probability& Stat. I	STAT 202Introduction to Probability& Stat. II
ELE 221 Introductional Principles and Methods	ELE 225Measurement and Assessment
EDS 220 Educational Psychology	ENG 211Academic Oral Presentation Skills
Any one of the following set	Any one of the following set
HIST 2201 Principles of Kemal Atatürk I	HIST 2202 Principles of Kemal Atatürk II
HIST 2205 History of the Turkish Revolution I	HIST 2206 History of the Turkish Revolution II
THIRD YEAR	
Fifth Semester	Sixth Semester
MATH 260 Linear Algebra	ELE 310 Community Service
ELE 341 Methods of Teaching Mathematics I	ELE 329 Instructional Technology and Material Development
	ELE 342 Methods of Teaching Mathematics II
	EDS 304 Classroom Management
Any one of the following set	Any one of the following set
TURK 105 Turkish I	TURK 106 Turkish II
TURK 201 Elementary Turkish	TURK 202 Intermediate Turkish
TURK 305 Oral Commination	TURK 306 Written Commination
Elective	Restricted Elective
Elective	
FOURTH YEAR	
Seventh Semester	Eight Semester
ELE 301 Research Methods	ELE 420 Practice Teaching in Elementary Education
ELE 419 School Experience	EDS 416 Turkish Educational System and School Management
ELE 465 Nature of Mathematical Knowledge for Teaching	EDS 424 Guidance
Restricted Elective	Elective
Elective	

In addition to the elementary mathematics education program, practice school environment should be considered under the context of the study.. The study was conductedin a public middle school in Ankara, Turkey. It was a double shift school in which one group of students take education in the morning, and the second group in the afternoon. The sixth graders who took part in the study were in the morning group. The sixth grade students were generally between 10 and 11 years old. Moreover, students were generally from families with high SES.In addition, the classrooms in which participants’ teaching were observed consisted of 32-35 students. The classrooms had green boards andthere were no computers or smart boards in the classes.

3.1.3 Sampling and Participant

In this section, detailed information is given about the case of the study, which was two senior pre-service middle school mathematics teachers.

As for sampling, probability and non-probability sampling are two basic types of sample selection. If the generalizability of the results of the study from the sample to the population is the aim of the study, then probability sampling can be implemented. However, the goal of qualitative studies is not to generalize the results to the population. Since generalizing the findings is not a goal of qualitative studies, non-probability sampling methods are chosen by qualitative researchers rather than probability sampling (Frankel, Wallen & Hyun, 2012; Merriam, 1998). Patton (1990) indicated that selecting *information-rich cases* is the most important factor during participant selection. In this way, the researcher can gain a rich source of in-depth information from small number of cases. Since the sample selected purposefully contributes to the purpose of the research, Patton (1990) called it purposeful sampling.

Since this study aimed to arrive at an in-depth understanding of pre-service teachers' knowledge of misconceptions/difficulties that may be held by students regarding the area of triangles and their knowledge of strategies that were used to eliminate these misconceptions/difficulties of students in the practice of teaching, generalization was not a concern of the present study, and thus, the non-probabilistic sampling method was employed. To be more specific, as one type of non-probabilistic sampling method, the purposive sampling was used in the study.

The participants of the study were selected from METU during the 2014-2015 academic year based on some criteria. One reason why the participants were selected from METU was the high quality of education offered at the university. Therefore, it was assumed that the researcher would be able to get rich data about students' misconceptions/difficulties regarding the concept of the area of triangles, and about instructional strategies which can be used to overcome these misconceptions/difficulties of students. Another reason for the selection of participants from METU was easy access of participants. Since the researcher conducted interviews and observations with the participants, she needed to spend plenty of time with the participants. Hence, easy access of the participant was an important criterion for the researcher and thus for the study. In addition, according to accessible literature, conducted studies revealed that experience was an important factor for the development of teachers' pedagogical content knowledge. Therefore, it is assumed that seniors pedagogical content knowledge was more developed when compared to other undergraduate levels in the elementary mathematics education program at METU since they took pedagogically-rich courses in their third and the fourth years. In this respect, since seniors had completed all the offered courses in their final semester, the participants of the present study were chosen from among seniors. In addition, since the researcher was to observe the participants in a real classroom environment at their internship schools and since the school experience course is offered in the final year of education, the participants needed to be chosen from among the seniors. Therefore, the researcher chose the sample of the study from among the seniors.

There is no right answer to how many samples is suitable for case studies (Merriam, 2009). Patton (1990) indicated that it is more valuable to gain rich information from a smaller number of samples than getting smaller amount of information from large samples. Thus, two volunteers and information-rich seniors, who were in the same internship school, were selected as the participants of the study.

The participants of the present study were two female pre-service middle school mathematics teachers who were in their last semester of the undergraduate program in METU. Pseudonymous names were used and the confidentiality of the collected data was ensured to the participants. Hatice was used as a pseudonym for the first pre-service teacher and Eda for the other pre-service teacher. Both of the participants had graduated from an Anatolian Teacher Training High School. Although Hatice had enrolled in the teacher education program willingly, Eda was unwilling to become a mathematics teacher at the beginning. However, over the years, she had become pleased about becoming a teacher. Thus, during the research study, both of the pre-service teachers were willing to be a mathematics teacher. In addition, they had taken all the required must courses in their undergraduate programs. Moreover, until the last semester, Hatice and Eda had taken five elective courses. In this regard, both Hatice and Eda had taken hands-on activities in mathematics instruction, problems solving in mathematics, and mathematical modeling for teachers as elective courses. Additionally, Hatice had taken the teaching of geometry concepts course and Eda had taken the Instruction to Psychology course. Furthermore, neither had prior teaching experience in a real classroom.

3.2 Data Collection Procedure

The data was collected from the two pre-service middle school mathematics teachers at Middle East Technical University during the spring semester of the 2014-2015 academic year.

In order to ask the pre-determined pre-service teachers about their decisions regarding their participation in the study, the researcher met with the two pre-service teachers and explained the purpose of the study to them. Moreover, the data collection process of the thesis and their responsibilities were clearly explained. In addition, the researcher informed the participants about the classroom observations and the hours during which their teaching could be observed. Additionally, the observation dates were decided on together with the participants. Since the researcher considered using a video camera for the observations and the interviews, the participants were asked whether or not it was a problem for them and, thus, their permission was taken. Confidentiality of the data was also ensured in that it was explained that no one else could have access to the data. The researcher and her supervisor were the only people who could reach the data, and during data collection and analysis the pseudonyms were to be used instead of their real names. Since the pre-service teachers were taking the practice teaching course during the semester when the data were collected, it was also ensured that their grades relevant to the practice teaching course were not to be affected by anything else related to this study. Additionally, they were told that they were not obligatory to participate in the study since voluntariness was an important factor for the study. At the end of the conversation, the pre-service teachers volunteered to participate in the study. The time schedule for the data collection procedure is presented in Table 3.2.

Table 3.2 Time schedule for data collection

Date	**Events**
September 2014- January 2015	Development of data collection tools (observation protocol, interview protocol, and teaching scenarios) Selecting and meeting the participants
January 2015- February 2015	Obtaining permission from the METU Ethical Committee and Ankara Provincial Directorate for National Education
February 2015- March 2015	Pilot interviews and rewriting the questions
March 2015- June 2015	Data collection (lesson plan activity, teaching observation, pre-interview and post-interviews)
June 2015- March 2016	Preparing data for analysis & data analysis

The required permission was obtained from the METU ethical committee to conduct the study and from Ankara Provincial Directorate for National Education to conduct the observation. After receiving the permission of the Ankara Provincial Directorate, the permission of the school administrator and the mathematics teacher who gave her lesson to the trainees were asked.

3.3 Data Collection Tools

In qualitative studies, three main techniques are used to collect data. One of them is the observation of the actual environment to learn what is actually going on. The other form of collecting data is the interview. By means of the interview, the researcher can learn about people's thoughts, their knowledge and the reason behind a behavior. In addition to the observation and interview, documents can be used as a data collection tool (Frankel, Wallen, & Hyun, 2012). Merriam (2009) pointed out that the most common form of data collection in educational studies is interviewing. Also, when the results are gained through different types of data collection tools called triangulation, researcher bias is reduced and hence validity of the study is enhanced (Frankel, Wallen, & Hyun, 2012; Merriam, 2002).

In this respect, to gain a deeper understanding of pre-service teachers' knowledge of misconceptions/difficulties that students may have regarding the concept of the area of triangles and their knowledge of the strategies that were used during practice teaching to overcome misconceptions/difficulties held by students, data were gathered through three types of data collection tools: semi-structured interviews, observation of the ongoing learning environment in a real classroom context and documents (lesson plans, field notes). Each data collection tool is explained in the following sections.

3.3.1 Interviews

Interviews help the researchers to understand the reason behind a situation or a behavior and the thoughts of people (Frankel, Wallen, & Hyun, 2012). The aim of an interview is to gain new and valuable information that fills the missing part in other forms of collected data. In this respect, interviews were conducted in order to gather detailed information about pre-service teachers' knowledge of misconceptions/difficulties that students may have regarding the concept of the area of triangles.

Merriam (2009) classified the interviews into three types according to its degree of structure; highly structured interviews, semi-structured interviews, and unstructured interviews. In highly structured interviews, the interview questions and their orders are predetermined so that the researcher asks the questions as if h/she is reading a survey form. In semi-structured interviews, the researcher has several guiding questions but the questions and their order are flexible and can change according to the answer of the participants. On the other hand, in unstructured interviews there is not any type of pre-determined questions. In this study, the semi-structured interview was utilized. These interviews were conducted as pre-interviews related to lesson planning activity before the teaching observation and as post-interviews conducted after the classroom observations.

After the lesson planning activity, the first interview was conducted with the participants before the observation of their lesson. The purpose of this pre-interview was to clarify their thoughts on their prepared lesson plans. Hence, participants had a chance to explain the rationale underlying the questions they had prepared and integrated into their plans, in other words, why she preferred to ask such a question/problem. Hence a detailed picture of pre-service teachers' knowledge and thoughts about misconceptions/difficulties that students may have regarding the area of triangles were determined more accurately. In this respect, some of the questions posed to the pre-service teachers during the pre-interview are provided in the following paragraph, and the protocol of pre-interview questions is provided in Appendix A.

For instance, to understand whether pre-service teachers take into consideration the misconceptions/difficulties that students may have pertaining to the area of triangles while organizing their lesson, the pre-service teachers were posed the following question: "What did you take into consideration while preparing the lesson plan associated with the altitude of the triangle/ the area of the triangle?" Another example question that was asked in order to gain insight into pre-service teachers' knowledge of strategies was "Which instructional strategies did you consider to use o overcome students' misconceptions/difficulties regarding the altitude of the triangle/ the area of the triangle during your instruction?" It was realized that in her lesson plan, Hatice had written, "observe students in part A to detect their misconceptions about constructing a perpendicular from a given point to a line segment" and she provided students with some misconceptions/difficulties regarding the height concept and the way to answer those misconceptions/difficulties. Regarding these circumstances, to lead her into explaining what her knowledge of students' misconceptions/difficulties regarding the height concept/ the area of triangles was, the following general questions were asked during the pre-interview: "Why did you prefer to include them in your plan?, In what way can it be useful for you as a teacher?" Additionally, regarding this situation, to understand the source of pre-service teachers' knowledge regarding students' misconception/difficulties about the concept of the area of triangles, they were asked the following questions: "Where did you encounter these misconceptions/difficulties? How did they come to your mind?" Moreover, Hatice planned to use an activity on constructing a perpendicular from a given point to a line segment in order to teach the concept of the altitude of triangles. To understand the rationale underlying the preparation of this activity, the pre-service teacher was asked the question that follows: "Why did you prefer to teach the concept of the altitude of triangles by constructing perpendicular line segments from a given point to a line?" Moreover, it was seen in the lesson plan on the altitude of triangles that Hatice grouped the prepared questions in the activity in that she planned to make students draw first the a, b, d questions then the f, g, ı, and finally the e, h, k, j questions. To clarify her thoughts concerning this grouping of the questions, she was asked the following questions: "Is there any reason underlying this grouping? Why did you prefer to group them?" As a last example, pre-service teachers prepared some problems in their lesson plans. To understand their thoughts in-depth and the reason underlying the prepared questions, the pre-service teachers were asked the following: "Why did you prefer to ask such a question to the students?, "What was your purpose in asking them?"

After the observation of the pre-service teachers' lessons, a post-interview was conducted with each participant. The purpose of this interview was to clarify their thoughts, decisions and behaviors in teaching the topic to the class. The researcher gets a chance to ask clarifying questions about the observed lessons by means of the interview. Also the participants get a chance to observe themselves and can explain the reasons behind their behaviors. Hence, the researcher can understand what the pre-service teachers had in their mind while doing anything during their teaching, what they actually did or whether they acted in accordance with their plans by means of the post-interview. In this respect, some of the questions posed to the pre-service teachers during the post-interview are provided in the following paragraph, and the protocol of post-interview questions can be found in Appendix C.

As an example of the post-interview questions, to clarify pre-service teachers' knowledge of the misconception/difficulties that students may have regarding the area of triangles/altitude of triangles, and to understand the difference between their knowledge of students' misconceptions/difficulties regarding the area of triangles, they were posed the following questions: "Which misconceptions/difficulties may the students have regarding the area of triangles/altitude of triangles? Which of these misconceptions/difficulties did you encounter during your teaching practice? Was there any misconception/difficulty that you encountered for the first time during your teaching?" Additionally, to specify in which parts of the lesson they had difficulty in while eliminating the misconceptions/difficulties that emerged during teaching, the following questions were asked: "Did you experience any difficulty during practice teaching? Did you experience any difficulty in responding to students' questions or in eliminating misconceptions/difficulties held by students?" Moreover, to understand whether their thoughts about what they did coincided with what they actually did, and to examine pre-service teacher' knowledge of strategies employed to overcome students' errors, the following question was asked: "Which strategies do you think you employed to eliminate the misconceptions/difficulties that the students had during teaching? As a last example, during her practice teaching, Eda asked the students a problem that necessitated the calculation of a triangle's area. In the problem, a garden looking like a triangle with a base of 25 m and a height of 15 m corresponding to this base were given. Subsequently, some of the students answered the problem as 100, 200, and 400 m^2. To gain insight into the thoughts of the pre-service teacher about the answers provided by the students and to find out her knowledge regarding students' misconceptions/difficulties underlying students' erroneous answers, the following questions were asked: "Did you think about how the students found these answers? What can be the reason behind the erroneous answers of the students? Do you think that the instructional strategy you employed to overcome students' errors was influential for students' understanding? Can you explain your thought?"

During the preparation of the interview protocols, related studies (Isiksal, 2006; Karahasan, 2010; Kubar, 2012; Kula, 2011; Simsek, 2011; Tekin-Striva, 2014) were examined. In the light of examined studies and especially by the help of Kula's study (2011) since her conducted interviews were relevant to the purpose of the interview of this study, the outlines of the interview protocols were prepared.

To understand whether the instrument would serve the purpose of the study, first, the questions were discussed with an expert, who is an instructor in the mathematics education department. The appropriateness of the questions for the purpose and the clarity of the question were discussed with the expert. After the discussion with the expert, two in-service mathematics teachers and two graduated students from the Elementary Mathematics Education Program at METU were discussed with also to gain more insight into the questions that were planned to be asked. The difficulties that the participants could have, the clarity of the questions and the meanings they express were discussed by the participants. Hence the questions were revised according to gathered suggestions.

In the light of these discussions, the questions were narrowed down and elaborated. For instance, found unrelated to the aim of this study, the following question was omitted: "Can the real life problems be effective for generating such a question/instructional strategy/solution/thought?" Instead, more specific questions were added to the pre- interview questions: "Why did you prefer to ask such a problem/question to the students? What was your purpose in asking these questions/problems? Which difficulties/misconceptions may students have regarding the topic? How do you come to know that it is a difficulty/misconception? What did you think about the way to overcome misconceptions/difficulties?" Additionally, more reasoning questions were preferred for the post interview: "What were you thinking when this was happening? What could the student be thinking of when s/he was doing/asking/answering...? Did this instructional strategy/explanation adequate for students' understanding?".

After this the questions were revised, the final versions of the interviews questions were obtained. Then, they were administered to the two pre-service teachers. In this regard, the pre-interviews and the post interviews were conducted with the two pre-service teachers regarding the altitude of triangles and the area of triangles separately. They were carried out face to face with the pre-service teachers one by one. Moreover, each interview lasted about half an hour. At the beginning of each interview, the purpose of the interview was explained to the participants.

3.3.2 Classroom Observation

Qualitative researchers use three techniques to collect data and observation is one of them. Patton (2014) defined the major purpose of the observation as "to see firsthand what is going on rather than simply assume we know. We go into a setting, observe, and describe what we observe" (p. 331). As the purpose defined by Patton (2014), Merriam (1998) emphasized the difference between observations and interviews. According to Merriam, a phenomenon can be observed in its natural setting without requiring any chance. However, in an interview the location is changed and designed for this purpose. In addition, observation provides firsthand data, whereas an interview yields secondhand data. Therefore, to make an in-depth examination of the pre-service teachers' knowledge on students' misconceptions/difficulties regarding the area of triangles, and also, their knowledge of strategies employed to overcome the misconceptions/difficulties that students hold about the area of triangles during teaching were observed. The observation of the teaching practices gave the researcher a chance to determine how pre-service teachers respond to students' thoughts in a real classroom environment. Moreover, it allowed the researcher to observe the participants' PCK regarding the concept of the area of triangles in their ongoing teaching. In addition, whether pre-service teachers deviated from their lesson plans during teaching or in which situations they changed their plans was observed.

The teaching practices of two pre-service mathematics teachers in the area of triangles were observed at their internship schools where they were attending within the scope of practice teaching in their elementary education course. Four lessons of each participant were observed and video-taped with permission.

All of the eight lessons were observed by the researcher and taped by a video camera from the rear end of the classrooms in order not to disturb the students and not to distract their attention. During or right after the classroom observations, field notes were taken about the relevant lesson. An observation protocol was used to guide the researcher while taking notes about the lessons. The observation protocol is presented in Appendix E. The observation protocol includes noteworthy points regarding the purpose of the study. To illustrate, some of the noteworthy points determined in the protocol were as follows: "Students solved a question inaccurately because of a misconception/difficulty and the pre-service teacher did not notice/ noticed the misconception/difficulty". The other point determined in the protocol was that "students demonstrated different misconceptions/difficulties that were not specified by the pre-service teachers before the lesson and pre-service teacher did not notice/ noticed the misconception/difficulty". In addition, as a last point to take into consideration during pre-service teachers' practice teaching, it was observed whether "the pre-service teachers employ a instructional strategy to eliminate students' misconceptions/difficulties".

3.3.3 Lesson Planning

To examine the pre-service middle school mathematics teachers' knowledge of common misconceptions and difficulties held by sixth grade students regarding the area of triangles, possible sources and the strategies used to overcome these misconceptions/ difficulties, the pre-service teachers were asked to prepare a lesson plan according to the objectives. The participants prepared their lesson plans according to the following objectives: '*Students should be able to draw an altitude of any side of a triangle*' and *'students should be able to formulate a instructional strategy to find the area formula of a triangle and solve related problems.* Moreover, they were reminded that the lesson plans should be prepared for at least four 40-minute lessons. Two weeks were given for the preparation of the lesson plans.

The researcher told the pre-service teachers that the lesson plan can include grade level of students, duration of the lesson, prerequisite knowledge and skills, objectives, materials, teaching methods and techniques, teaching processes from introduction to closure, assessment and homework. They were also told that they could add additional information apart from the mentioned titles. In addition, the participants were told that they were free to use any source while preparing their lesson plans. Furthermore, it was reminded that the plan should be prepared in a detailed information rich-manner. Additionally, participants were asked to include the questions to be asked to the students, the problems to be solved during the lesson, the activities, homework etc. and whatever they had planned to use during the lesson. The pre-service teachers prepared lesson plans are provided in Appendix G.

3.4 Data Analysis

Merriam (2009) defined data analysis as "the process of making sense out of data" (p. 175) and she added that it is the most challenging part of a qualitative study. The data analysis comprises three processes, namely consolidating, reducing, and interpreting (Merriam, 1998).

In this study, two pre-service teachers' lesson plans, pre-interviews and post-interviews, and observations of the teaching practices were analyzed to examine the pre-service teachers' knowledge of misconceptions/difficulties that students may have regarding the area of triangles and their knowledge of the strategies that were employed during practice teaching to overcome misconceptions/difficulties held by students. To begin with, all the video-records regarding pre-interviews, post-interviews and classroom observations were transcribed. Then, first elimination of transcribed data carried out according to two research questions, so that; irrelevant data was removed from the transcription. Afterwards, the interrelated data were gathered together according to two research questions separately.

To identify pre-service teachers' knowledge of misconceptions/difficulties that sixth grade students may have regarding area of triangles in the scope of first question, collected transcripts related with first research question were analyzed and a general list of codes were developed. Then, the data were analyzed again and again until the codes of the misconceptions/difficulties saturated for data; in other words, until different misconceptions/difficulties did not emerge. The codes were further titled under themes so that similar codes were categorized under specific themes. After that, the obtained list of codes and themes were discussed with an expert, who was an instructor in the mathematics education department. Considering gathered remarks, the researcher obtained the finalized themes and the codes. In this respect, the knowledge of pre-service teachers on misconceptions/difficulties that students may have regarding the concept of the area of triangles were categorized into three main categories and relevant subcategories. The names of the main categories were determined as misconceptions/difficulties regarding the concept of height, regarding the concept of area and regarding the formula for the area of the triangle.

Same process that was followed to answer the first question was carried out to answer the second question. For the second question, the data collected from the classroom observations were analyzed to investigate pre-service teachers' knowledge of instructional strategies that were carried out during practice teaching to overcome misconceptions/difficulties held by students regarding the area of triangles.

The determined codes and themes were established upon the responses of the participants, the available literature and the researcher’s own experiences with the data.

3.5 Trustworthiness

Validity and reliability are the most important factors of research studies. Collection, analyses, and interpretation of the data and the display of the findings have a significant role in obtaining valid and reliable knowledge (Merriam, 1998; 2009). In quantitative studies, validity refers to "*the appropriateness, meaningfulness, correctness, and usefulness of the inferences a researcher makes,* "while reliability refers to "*the consistency of scores or answers from one administration of an instrument to another from one set of items to another*" (Frankel, Wallen, & Hyun, 2012, p. 147). For qualitative research studies Lincoln and Guba (1985) offered the use of different terminologies instead of internal validity, external validity, reliability and objectivity. Correspondingly, the term credibility is used rather than internal validity, dependability/consistency for reliability, transferability for external validity, and confirmability rather than objectivity (Lincoln & Guba, 1985).

3.5.1 Credibility

Merriam (1998) pointed out that credibility, or internal validity, searches for the answers of the questions; "*How research findings match reality? How congruent are the findings with reality? Do the findings capture what is really there? Are investigators observing or measuring what they think they are measuring?*" (p. 201). Therefore, credibility of the findings is the most important thing for research studies (Creswell, 2012). To increase the credibility of research findings, Merriam (1998) proposed six strategies: triangulation, member checks, long-term observation, peer examination, participatory or collaborative modes of research, and researcher's biases. To ensure credibility, triangulation, member checks, and researcher’s biases were used in this study. Triangulation is the most employed strategy to increase internal validity of qualitative research studies (Merriam, 2009). Creswell (2012) defined triangulation as a process to gather data from different individuals or different data collection methods in qualitative research.

In this study, triangulation was carried out by multiple sources of data, in other words, as Creswell’s terms, different types of data and different methods of data collection. In this regard, lesson plans, semi-structured pre- and post-interviews, and classroom observations were used to triangulate the data.

3.5.2 Dependability/Consistency

In quantitative studies, the term reliability refers to whether the same research findings can be gathered if the study is replicated (Merriam, 2009). However, it cannot be possible in social sciences since human behavior changes from moment to moment. Therefore, for qualitative studies the term dependability or consistency of the results is used rather than reliability (Lincoln & Guba, 1986). Merriam (1998) defined dependability as *"whether the results are consistent with the data collected"* (p. 206).

Merriam (2009) enlisted the strategies to ensure dependability and consistency as *triangulation, peer examination, investigator's position,* and *audit trail.* To enhance the dependability of the results in this study, the researcher triangulated the multiple sources of data. Moreover, the design and the context of the study, the data collection tools, the data collection procedure, and the data analysis processes of the study were explained explicitly in previous sections of the methodology chapter to ensure the dependability of the study. Additionally, a clearly defined position of the research can be used to increase the dependability of the study (Merriam, 1998). Thus, in qualitative studies all the processes of a study as data collection, data analyses, and the interpretation of the data are carried out by the researcher (Merriam, 2009). Therefore, the researcher of the present study undertook her biases and assumptions regarding the research by making explanations pertaining to them. The detailed explanation relevant to the researcher's position is provided in a following section.

3.5.3 Transferability

Merriam (2009) explained external validity as *"the extent to which the findings of one study can be applied to other situations. That is, how generalizable are the results of a research study?"* (p. 223). However, generalizability is not the concern of qualitative studies (Merriam, 2009). Instead of generalizability, Lincoln and Guba (1986) proposed the term *transferability* for qualitative studies. To promote transferability of a study there are some strategies that can be used. One of them is providing rich, comprehensive descriptions of the research context so that one can assess the degree to which it matches with the real context as a close match will ensure the transferability of the findings (Merriam, 1998). In this study, the process of data collection, the context and the sample have been described in detail. Hence, the researchers can conduct similar studies with different contents or in different contexts, thus increasing the transferability of the study.

3.6 Researcher’s Role and Bias

In a qualitative study, the researcher is the only person who collects, analyzes and interprets the data (Merriam, 2009). Since the researcher has the sole control over the data, it can lead to the researcher's bias. Therefore, researcher bias, dispositions, and assumptions need to be undertaken by the researcher throughout the study (Merriam, 2009).

In the process of classroom observations, the researcher was a non-participant observer. In this respect, the researcher observed pre-service teachers' practice of teaching within the scope of a practice teaching course from the rear end of the class. Since the students were used to interns attending their classes, it was assumed that this would help to avoid the manipulation of her effect. For this reason, classes were observed before the real observations so that students could get used to the researcher and to being recorded by a camera. In addition to the students, the pre-service teachers could also be affected. To minimize this effect, the researcher spent time with them. Therefore, they met with the researcher pertaining to permission twice for the pre-interviews. In addition, the participants and the researcher talked via telephone and social network for the preparation of the lesson plans. Additionally, the researcher repeatedly said, "There is no right answer for the questions, just try to understand your thoughts, and explaining your ideas in a detailed manner is the most important thing for the study." Consequently, it is assumed that they were not affected by the researcher.

3.7 Limitations of the Study

There are some limitations of the study. One of the limitations is about the researcher's effect on the natural classroom environment. The researcher’s role and bias were presented in the relevant section of the methodology chapter.

The other limitation concerns the generalizability of the results. Since the study was conducted with two pre-service teachers, the results cannot be generalized to other pre-service teachers. Therefore, the findings of the study are limited to the data gathered from the two pre-service teachers. Furthermore, since the concept of the study was the area of the triangle, the results cannot be generalized to other concepts. Thus, the findings of the study were restricted to the concept of the area of triangles. Moreover, the findings of the study were limited to data sources as pre-post interview, classroom observations and lesson plans.

CHAPTER IV

FINDINGS

The aim of the study was to investigate pre-service middle school mathematics teachers' pedagogical content knowledge on the concept of the area of triangles. In this respect, pre-service teachers' pedagogical content knowledge with respect to students' possible misconceptions/ difficulties regarding the area of triangles and the strategies that pre-service teachers used to overcome these misconceptions/difficulties were examined. In this chapter, the findings of the study are presented under two sections. Each section refers to one of the research questions and also includes subsections. The first section examines pre-service mathematics teachers' knowledge on misconceptions/difficulties of students regarding the area of triangles, and the second section is dedicated to the strategies that pre-service teachers employed to overcome students' misconceptions/difficulties.

For each section, codes were established based upon the responses of the participants, the available literature and the researcher's own experiences with the data. The codes were further titled within the themes according to the analysis of lesson plans, pre- and post-interviews, and classroom observation transcripts. While reporting the findings of the analysis, detailed information is given by providing excerpts from the lesson plans, interviews and observation transcripts.

4.1 Pre-service Teachers' Knowledge on Students' Misconceptions/Difficulties

One of the aims of the present study was to examine the pre-service elementary mathematics teachers' knowledge of common misconceptions and difficulties held by sixth grade students on the area of triangles. To this end, pre-service teachers were asked to prepare a lesson plan based on the objectives that follow: 'S*tudents should be able to draw the altitude of any of the sides of a triangle'* and *'Students should be able to formulate a strategy to find the area formula of a triangle and solve related problems.'* . To examine their knowledge in depth, their lectures were observed. In addition, a pre-interview was conducted to clarify their views regarding their prepared lesson plans, and a post interview was conducted to clarify their thoughts, decisions and behaviors in lecturing the topic to the class. The term misconception/difficulty was used in this study to refer to students' limited conceptions and insufficient prior knowledge pertaining to the area of the triangle**.** In this respect, pre-service teachers' perception of students' mistakes corresponds to pre-service teachers' knowledge of misconception/difficulty held by sixth grade students regarding the area of triangles.

Based on the analysis of the conducted interviews and the observation transcripts, it can be stated that the pre-service teachers provided a variety of possible misconceptions/difficulties that students may have regarding the area of the triangle. The list of the misconception/difficulties specified by the two pre-service teachers including their relevant categories is summarized below in Table 4.1.

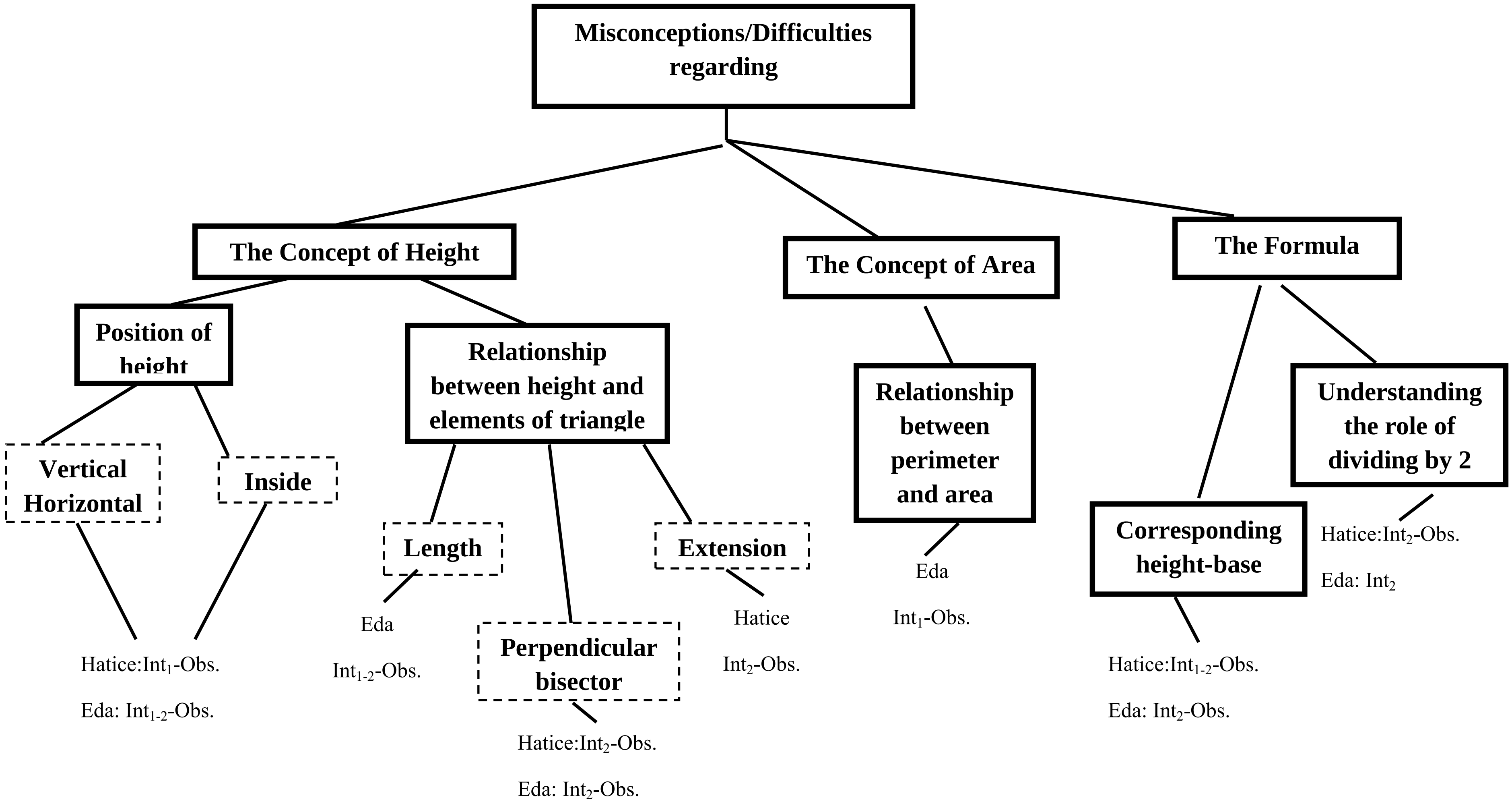


Table 4.1 The summary of misconceptions/difficulties specified by two pre-service teachers (Int$_1$ refers to pre-interview, Int$_2$ refers to post-interview, and Obs refers to classroom observation)

During the analysis of the lesson plans, one thing that specifically attracted attention was that before the researcher asked the teachers a question as to what misconceptions/difficulties could the students have, both of the pre-service teachers gave some of them a place in their lesson plans. Table 4.1 summarizes the misconceptions/difficulties that students might have regarding the concept of the area of triangles as specified by the pre-service teachers. As can be seen in Table 4.1, pre-service teachers' knowledge regarding students' misconceptions/difficulties in the concept of area of triangles could be categorized into three main groups and relevant subcategories. The main categories are named as misconceptions/difficulties *regarding the height, regarding the area and regarding the formula.* In this regard, they were explained in a detailed manner under each section. In the following section, misconceptions/difficulties regarding the concept of height are explained.

4.1.1 Difficulties/Misconceptions Regarding the Concept of Height

When Table 4.1 is examined, it can be seen that most of the misconceptions/difficulties stated by the participants is related to the concept of height, which is an element of the triangle and prerequisite knowledge for the area of triangles.

As stated above, there were three categories which were formed according to the analysis of the pre-service teachers' perceptions regarding students' possible misconceptions/difficulties in the area of triangles. One of these categories, students' misconceptions/difficulties regarding the concept of height, could be further divided into subcategories as misconceptions/difficulties regarding the position of height and those regarding the relationship between the elements of the triangle and height. Under the subcategory of the misconceptions/difficulties regarding the position of height, there were two misconceptions/difficulties specified by two pre-service teachers. To be specific, pre-service teachers stated that students may think that the height should always be either vertical or horizontal. Additionally, they highlighted that students may think that the height should always be inside the triangle. As for, students' misconceptions/difficulties regarding the relationship between elements of the triangle and height, pre-service teachers stated that students may think that height is a perpendicular bisector of the base. Moreover, they underlined that students may have the misconception that the side of a triangle cannot be extended. Finally, they emphasized that students may consider the hypotenuse as the height.

In the following sections, each misconception/difficulty mentioned is examined under the subcategory it belongs to in detail. To make it more meaningful and understandable to the reader, they are supported with excerpts taken from the observation or interview transcripts.

4.1.1.1 Misconception/Difficulties regarding the Position of the Height

In this section, students' possible misconceptions/difficulties as 'height is always vertical or horizontal to the ground' and 'height is always inside the triangle', specified by the pre-service teachers while expressing their knowledge of misconception/difficulties, are examined.

i. Height is Always Vertical or Horizontal

Both of the participants proposed that students may hold a misconception as 'height is always vertical or horizontal to the ground'. To determine whether there is such a thought in students' mind, both of the pre-service teachers prepared some questions in their lesson plans, accordingly. To illustrate, Hatice prepared the following question to ask the students in an activity sheet. The corresponding part taken from her lesson plan is presented below.

Question: Draw the perpendiculars from point F and I to the line segments.

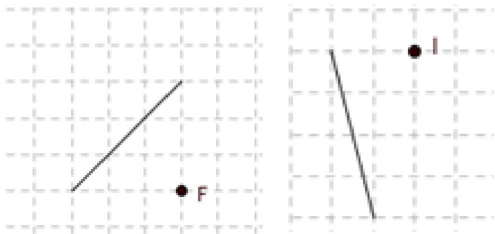


Figure 4.1 Figure of the questions from lesson plan of Hatice

In addition to the questions, the expected answers of the students to the questions and the reason behind the answers were presented in Hatice's lesson plan with possible strategies to deal with each student’s expected answers. To illustrate, In F, students may draw a perpendicular as shown in the below picture.

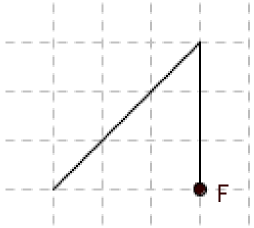


Figure 4.2 Hatice's presentation of possible students' construction

Because they may think that a perpendicular will always be vertical.

In G and I, students may draw the perpendiculars as shown in the pictures below.

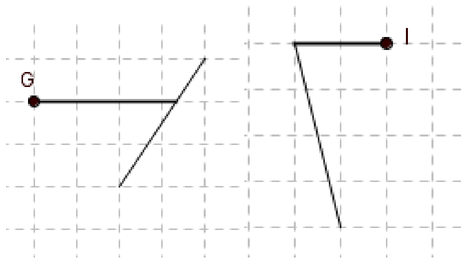


Figure 4.3 Hatice's presentation of possible students' constructions

Because they may think that a perpendicular can be drawn only vertically or horizontally. (Lesson plan of Hatice)

As can be seen from the example, the pre-service teacher not only prepared the questions regarding students' difficulty/misconceptions but also added possible student answers. Hatice further explained the aim of the prepared questions as follows:

> The aim of the above questions is to enable students to understand that altitude is a perpendicular line segment and can be constructed on any base. ... Students think that altitude is only vertical. Thus, they could not understand how they could draw altitudes to all the sides of the triangle, especially which are not vertical. (Lesson plan of Hatice)

In addition to the prepared questions corresponding to students' relevant misconceptions/difficulties and the expected answers of the students to the prepared questions, the pre-service teacher included the way to respond to each mistake in their lesson plans. Although Hatice gave a large place in her lesson plan to students' misconceptions, Eda mentioned the misconceptions/difficulties superficially in her plan just by asking, 'Is it always vertical or horizontal?'

In the following section, pre-service teachers' specifications regarding students' possible thoughts on whether the height should always be inside the triangle is examined.

ii. Height is Always Inside the Triangle

In addition to pre-service teachers' knowledge regarding 'height is always vertical or horizontal to the ground', the misconception that 'height is always inside the triangle,' which was proposed by both of the pre-service teachers, can be considered as an element of teacher knowledge on students' misconceptions/difficulties regarding the area of triangles.

The pre-service teachers indicated that students might believe that height cannot be outside of the triangle; in other words, height should be always inside the triangle. The results of the analysis revealed that both of the pre-service teachers reflected their knowledge of the misconception/difficulty both in their lesson plans and during the pre-interview. To illustrate, Eda prepared a question to ask during the lesson as follows: *'Is the height always inside the triangle?'* The corresponding part of the pre-interview, which was conducted to understand the aim of Eda, is given below.

> Height in the obtuse triangle cannot be distinguished by the students, that is, where the height needs to be drawn, whether inside or outside, or can it be outside? I thought that more emphasis should be placed on this topic to prevent the perception that the height can merely be inside the triangle and cannot be outside of the triangle. (Eda-Int_1)

This statement of Eda can be accepted as an enlightenment concerning the knowledge of misconception/difficulty. On the other hand, Hatice indicated the same misconception/difficulty in her lesson plan with a little difference in that she specified it in relation to line segments as stated in the first misconception/difficulty. In other words, she expressed that the students may have errors since they believe that height should always be on the line segment.

Up to this point, pre-service teachers' perceptions regarding the misconception/difficulties that students may have in relation to the position of height has been presented. In the following section, pre-service teachers' knowledge of students' misconceptions/difficulties regarding the relationship between the height and the elements of the triangle is examined.

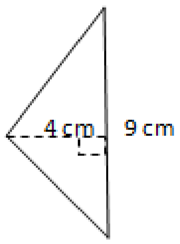
4.1.1.2 Misconception/Difficulties Regarding the Relationship between the Height and the Elements of the Triangle

In the light of data analysis, from the designated misconceptions/difficulties that could be held by students, three of them fell under this title. In this respect, students' possible misconceptions that height is a perpendicular bisector of the base, that the side of a triangle cannot be extended, and that the hypotenuse of a triangle is its height are examined in the sections below in a detailed manner.

i. Height is a Perpendicular Bisector of the Corresponding Base

When the pre-service teachers' classroom observations were analyzed, it was seen that during pre-service teachers' classroom observations, some of the students made some errors in consequence of their thoughts that height is always a perpendicular bisector of the corresponding base, and Eda realized at that moment that students' had such a misconception. To illustrate, an excerpt taken from the observation of the pre-service teacher is provided below. Eda wrote the following question on the board:

Question:



Find the area of the triangle.

Figure 4.4 Figure of the question from lesson plan of Hatice

Solution of the student: The base is 9 cm. When we divided it by two, it turns out to be 4.5 cm and 4.5 cm...

After the student's solution, Eda explained to the class the solution method of their friends:

Eda Teacher: Ayşe divided the triangle into two parts. She wrote 4 cm for the height and 9 cm for the base. Hilal, you divided it into two as 4.5 and 4.5. However, have you been told that the triangle was isosceles or equilateral? Did you divide into two because of the appearance of the triangle?

Student: Yes, because of its appearance.

Eda Teacher: Does the height always divide the base into two? For example, in this question did the height divide the base into two?

The class: Yeess

Eda Teacher: Now, look here. If the triangle in the question were an isosceles triangle or an equilateral triangle, then what Hilal said could be true. However, now we cannot assert that the height divides the base into two pieces as 4.5 and 4.5... (Eda-Obs.)

In the light of the above excerpt taken from Eda's classroom observation, it can be inferred that the pre-service teacher had knowledge related to the relevant misconception. Although in the beginning the pre-service teacher had not mentioned that students might have the misconception that height is always a perpendicular bisector of its corresponding base, she afterward underlined this misconception/difficulty of the students.

ii. A Side of a Triangle Cannot Be Extended

According to the analysis of the classroom observation it was seen that the pre-service teacher noticed students' misconceptions that a line segment cannot be extended, and realized students' lack of knowledge on the extension concept. The dialogue taken from the classroom observation is provided below:

Hatice Teacher asked the following question to the class: Draw a perpendicular line segment from point E to the given line segment. Then, a student drew an extension of the line segment and the perpendicular as shown with an arrow in picture below.

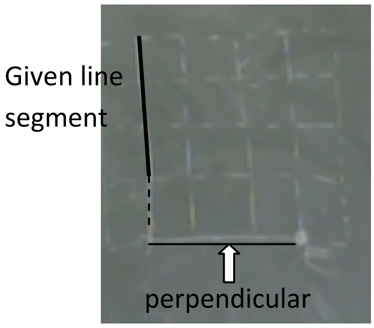


Figure 4.5 An example of students' answer from Hatice's lesson

Student-B: It does not exist here (by implying the drawn extension)

Student-C: Is it a ray? (Referring to the line segment)

Hatice Teacher: A line segment

Student-C: I cannot draw it (implying the extension) if it is a line segment.

Hatice Teacher: Even so, state your idea about the question.

Student-C: I would draw the extension if it were a ray. ... It can be extended since it is a ray.

After a while, some other students could not understand why they extended the line segment. The relevant part of the observation was as follows:

Student-C: The thing that I could not understand was that we learned the line segment did not extend. However, we extended it.

Hatice Teacher: It does not extend. We said the extension of the line segment.

During the conducted post-interviews, Hatice said, "I did not think that they would have difficulties in learning the extension of a line segment". As it can be understood from the statement, although the relevant misconception/difficulty that students may have did not come to her mind before the lesson, she afterwards highlighted that students may think that line segments cannot be extended. In this respect, during the post-interview she said, "... *students lack what can be the extension of a line segment because they don't know of such a concept."* The statement of Hatice can be accepted as evidence of the pre-service teacher’s knowledge of the relevant misconception/difficulty that students may have regarding extension.

iii. Confusion of the Length and Height

The last indicator of the pre-service teachers' knowledge as regards misconceptions/difficulties that students may have in relation to the relationship between the height and the elements of a triangle was students' confusion of the length and the height. To be clearer, the term length can be correlated to the hypotenuse of a right triangle.

One of the pre-service teachers, Eda, identified the following misconception/difficulty that may be held by students while differentiating the length and the height. First of all, in her lesson plan she prepared a question given below to ask the class and she explained the reason underlying it during the pre-interview as "When I thought about the difficulties the students experienced in relation to concept of height, the confusion between height and length came to my mind. That’s why I tried to give the ladder problem". The problem was given below:

> Problem: The painter Rasim wants to paint his house. Since he cannot reach the higher part of the wall, he will use a ladder of 1.5 m length. When he ascends the highest stair of the ladder, what can be his height in relation to the ground? Show it by drawing on the shape.

Figure 4.6 Presentation of the ladder from Eda’s lesson

Afterwards, the pre-service teacher stated, “If students confused the length of the ladder and its height, then I would expect them to make this mistake, that the height of the painter on the last stair of the ladder will be 1.5 m since the length of the ladder is 1.5 m" (Eda-Int_1).

Additionally, she stated, "I think the length of the ladder here refers to the hypotenuse of the triangle, while the height of the painter from the ground refers to the height of the triangle. I think what the students can confuse could derive from these concepts" (Eda-Int_1).

In her statements, the pre-service teacher highlighted that students' can get confused while deciding on the height of a right angled triangle. Thus, the hypotenuse and the vertical height of the right angled triangle can be confused by students.

4.1.2 Difficulties/Misconceptions regarding the Concept of Area

As stated above, the misconceptions/difficulties of students as specified by the pre-service teachers could be divided into three basic categories. The first category, which was the pre-service teachers' knowledge of difficulties/misconceptions that students may have regarding the concept of height, has been examined above. In this section, the second category, which is pre-service teachers' knowledge of misconceptions/difficulties that students may hold in relation to the concept of area, is investigated. Misconceptions/difficulties of students regarding the concept of area refer in this study to students' misconceptions/difficulties between the concept of area and other measurement concepts, such as perimeter or volume.

When the data were analyzed, it was seen that one of the specified misconceptions/difficulties that students may have regarding the concept of the area of triangles fall under the relevant category. In this regard, the pre-service teachers underlined that students' inadequate knowledge of the concepts of perimeter and area lead students to think that there is a direct relationship between the perimeter of the figure and its area. In the following section, pre-service teachers' knowledge of the students' misconceptions/difficulties regarding the relationship between these are examined.

4.1.2.1 Difficulties/Misconceptions regarding the Relationship between Perimeter and Area

Analysis of Eda’s lesson plans revealed that the pre-service teacher prepared an activity regarding students’ perceptions that when the area decreases, the perimeter also decreases. In the activity, she planned to distribute a parallelogram to each student in the class. She further had the students calculate the perimeter of the parallelograms. Then, students were to cut the parallelograms into two equal pieces and calculate the perimeter of the obtained triangles. At the end, Eda stated, "The knowledge that when the area of a shape decreases by half, its perimeter does not decrease by half is dwelled upon. They are told that the area concept is not directly proportional to the perimeter concept" (Lesson plan of Eda). As a result of her statements gathered from the analysis of her lesson plan, it can be said that Eda has the knowledge of students’ misconception/difficulty that there is a directly proportional relationship between the perimeter of a shape and its area.

During the pre-interview, Eda's statements were consistent with her knowledge of the corresponding misconception. She expressed the reason underlying her preparation of the activity with her statements below:

(I prepared such an activity) to prevent students from having such a misconception; to prevent them from thinking that when we divide a parallelogram into two equal triangles, its area is divided into two and its perimeter is also (Eda- Int_1).

Additionally, she stated, I’m actually trying to show them whether the perimeter of a triangle decreases by half when its area is divided by two. Since students have some difficulties regarding these concepts, they need to understand the difference between these concepts (Eda- Int_1).

In the following section, pre-service teachers' knowledge of misconceptions/difficulties held by students regarding the formula for the area of the triangle formula is examined.

4.1.3 Difficulties/Misconceptions regarding the Formula for the Area of a Triangle

So far, pre-service teachers' knowledge of misconceptions/difficulties that students may have regarding the concept of height and the concept of area has been presented. In this section, the last category with respect to misconceptions/difficulties related to the formula is examined. In the light of the analysis, the last category could be further divided into two. In this respect, pre-service teachers underlined that students may have misconceptions/difficulties in understanding the role of dividing by two in the formula and in establishing the height and its corresponding base for the formula. In this regard, pre-service teachers' perceptions regarding the specified misconceptions/difficulties that may hold by students are examined in the following two sections, respectively.

4.1.3.1 Difficulties/Misconceptions Regarding the Comprehension of the Role of '2' in the Formula

In the light of the accessible literature and the pre-service teachers' expressions, it can be stated that pre-service teachers highlight the fact that, while calculating the area of a triangle, students might not understand the role of the number two within the formula; moreover, students may forget to divide the multiplication of the base and the corresponding height by two.

Based on the analysis of the classroom observations, it was observed that during her teaching practice, Hatice realized that some of the students were ignoring the 2 in the formula while calculating the area. During the post interview, when she was asked to state students' possible misconceptions/difficulties regarding the concept of area, she specified that students may forget to divide the product of the base and the corresponding height by two. In addition, she claimed that some students can multiply the base and the height to find the area of a triangle but they can ignore the division of 2 found in the formula. In this respect, during her post interview, the pre-service teacher expressed the following:

> During calculation they think of the parallelogram. I think, they think of the formula as the base multiplied by the height as in the area of the parallelogram. However, they cannot think that we divide the parallelogram into two. So, they forget to divide it by 2 (Hatice- Int_2).

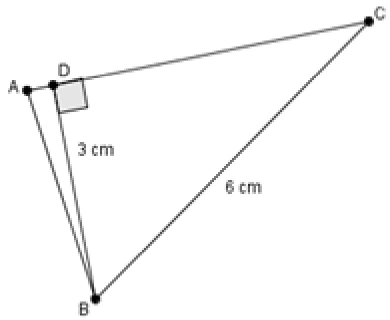
A similar misconception/difficulty was specified by Eda also during her post interview. In addition to the misconception of the students stated above, in Eda's class, one of the students solved an area problem by dividing the product into two twice; in other words, the student found the answer in a proper manner and then she divide the result by two again. During the post interview, the pre-service teacher mentioned that students may have misconceptions/difficulties in understanding the role of 2 in the area formula for the triangle. Then, the pre-service teacher stated the following with regard to the student's' solution:

> Well, I think she got confused. We stated that half of the area of a parallelogram gives the area of the triangle. I think, she got confused and divided the area of the triangle by two again. So, she confused the old information with the new one. (Eda- Int_2).

In this section, pre-service teachers' knowledge of the misconception/difficulty that students may have in understanding the role of two has been examined. The following section dwells on pre-service teachers' knowledge regarding the misconception/difficulty that may be held by students in establishing the height and its corresponding base for the formula.

4.1.3.2 Difficulties/Misconceptions in Establishing the Height and the Corresponding Base

The analysis of the pre-service teachers' lesson plans revealed that one of the pre-service teachers had highlighted that students may have a misconception/difficulty in establishing the height and its corresponding base for the formula. To specify the students with this difficulty/misconception, Hatice prepared and placed a problem into her lesson plan as follows:



Problem: Ahmet found the area of the triangle on the left side as 9 cm^2 but he was not sure of his answer. What do you think about his answer? Explain.

(Lesson plan of Hatice)

Figure 4.7 Figure of the problem from lesson plan of Hatice

She further explained her expected answer for the question during the conducted pre-interview as follows:

> The students would say three multiplied by six divided by two. While finding the area we multiply the base with the corresponding height and then it is divided by two. However, when the students see a height here, they will most probably directly multiply them and divide the result by two without thinking about the corresponding base (Hatice- Int_1).

As can be seen in the above statements of Hatice, it has been underlined in relation with the question that students may not be able to establish the height and its corresponding base to calculate the area of the triangle. On the other hand, although prior to her teaching Eda did not mention that students might have misconceptions/difficulties while establishing the height and its corresponding base, during the post-interview she emphasized that students might not be able to establish the height and its corresponding base while calculating the area of the triangle. The statements taken from the post interview of Eda are provided below as evidence of her knowledge.

> The effect of height (regarding misconception/difficulty) was one area of the triangle concept. Since students experience difficulty in "specifying height and its corresponding base," they also struggle in the concept of area (Eda- Int_2).

Hence, the analysis of the lesson plans, pre-interviews, classroom observation transcripts and post-interviews revealed that both of the pre-service teachers possessed knowledge regarding students' misconception/difficulty in relation to establishing the height and its corresponding base, which was required to calculate the area of a triangle.

To conclude, in this part of the chapter, two pre-service teachers' knowledge of misconceptions/difficulties that students may have regarding the concept of the area of triangles has been examined. According to the participants' statements, the accessible literature and the researcher's experiences with the data, the pre-service teachers' knowledge of misconceptions/difficulties that students may have regarding the concept of the area of triangles were investigated and categorized into three main categories called misconceptions/difficulties regarding the concept of height, regarding the concept of area and regarding the formula for the area of triangles. In the following section, the pre-service teachers' knowledge of the strategies to overcome students' possible misconceptions/difficulties pertaining to the concept of the area of triangles were investigated.

4.2 Pre-service Teachers' Knowledge of the Instructional Strategies to Overcome Students' Misconceptions/Difficulties

In the previous part of the chapter, pre-service teachers' knowledge on elementary students' misconceptions/difficulties regarding the concept of the area of triangles has been examined in accordance with the aim of the study and, while doing so, the first research question was tried to be answered. In the rest of the chapter, the answer of the second research question, that is *"What kind of strategies do pre-service elementary mathematics teachers use to overcome the misconceptions/difficulties held by the 6th grade students related to the area of triangles during practice teaching?"* was investigated.

In this study, the instructional strategies term refers to the methodologies and techniques that pre-service teachers used or planned to use when students showed an indication of a misconception or difficulty during the lesson.

Based on the analysis of the conducted interviews and the observation transcripts, it can be stated that the pre-service teachers used and provided various strategies to overcome possible misconceptions/difficulties of students regarding the area of triangles. In this respect, the strategies used by pre-service teachers during their practice teaching to overcome the misconceptions/difficulties held by the students were discussion, demonstration, didactic approach, cognitive conflict, and direct teaching. The summary of these strategies is provided in Table 4.2. The specified misconceptions/difficulties that students may have regarding the concept of the area of triangles and the strategies used to overcome the corresponding misconceptions/difficulties are presented in the mentioned table. In the following sections, each instructional strategy is examined in detail with corresponding excerpts taken from the classroom observations and the pre-service teachers' statements.

Table 4.2 Summary table of the strategies used by the pre-service teachers to overcome the misconceptions/difficulties of student

Category of misconception/difficulty	Misconception/Difficulty regarding the Area of Triangles	Strategies				
		Direct Teaching	**Discussion**	**Demonstration**	**Didactic approach**	**Cognitive conflict**
Regarding the concept of height	Vertical-horizontal			x		x
	Inside	x	x	x		
	Perpendicular bisector				x	
	Extension	x	x			
	Length		x			
Regarding the concept of area	Distinguish between perimeter and area		x			
Regarding the formula	Forgot to divide by two					x
	Corresponding height base	x	x			x

4.2.1 Discussion

In the light of the pre-service teachers' knowledge regarding strategies used to overcome students' misconceptions and difficulties regarding the concept of the area of triangles, discussion was one of the strategies used by pre-service teachers to eliminate misconceptions/difficulties held by sixth grade students during the lesson. In the class where class discussion was conducted, students' multiple points of view were presented in relation to the problems or ideas; furthermore, students responded to different ideas and reflected on their own ideas.

During classroom observation, Hatice noticed that some of the students thought that a line segment could not be extended, thus realizing students' incomplete knowledge of the extension concept. In addition, the pre-service teacher encountered students who thought that height should always be inside the triangle while working on a question. In this respect, the question in which students' misconceptions/difficulties emerged is presented below.

Question: Draw a perpendicular line segment from point E to the given line segment.

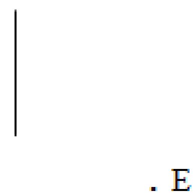


Figure 4.8 Figure of the question from lesson plan of Hatice

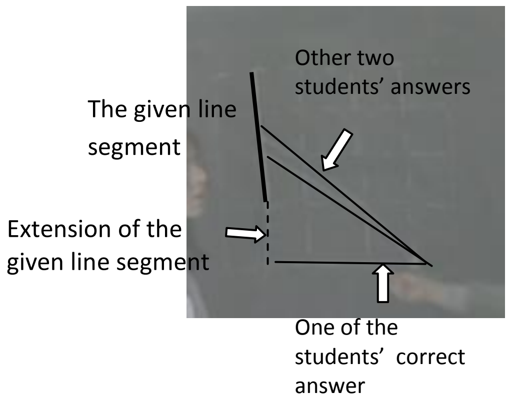


Figure 4.9 An example of students' different answers from Hatice's lesson

After she wrote the question on the board, Hatice asked the class to draw the different answers of the students. In the meantime, some of the students produced their misconception/difficulty that a line segment could not be extended. When the pre-service teacher encountered such a misconception/difficulty, she conducted a class discussion to eliminate it as follows:

Hatice Teacher: Which one of the line segments is the perpendicular one?

Student-A: The ones that have an angle of 90^0 between them.

Hatice Teacher: Is this correct, or this one, drawn by Osman or Onur? How do we decide which one is correct?

Student-A: I mean that we measure whether the angle between the line segments and the base is 90^0.

Hatice Teacher: Come and measure it. You (referring to the class) measure with your protractor on your worksheets. (After a while) What was the angle?

Student-A: 60^0

Hatice Teacher: One of them is 60^0, and the other one is 70^0. Were they the perpendicular line segments?

Students: No. The line segment found at the bottom is the correct one.

At this point although some of the students found the perpendicular line segment, the teacher continued to ask further questions to the students who could not understand how an extension of a line segment could be drawn. The remaining part of the discussion was as follows:

Hatice Teacher: But the line segment extended to this point. How could I draw this part (indicating the line segment)?

Student B: We couldn't.

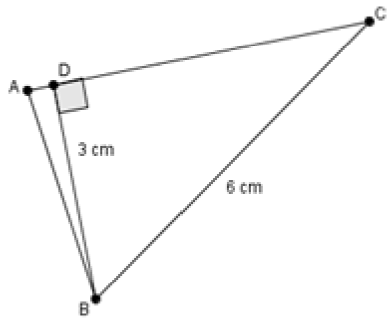
Student C: I can move this line from here to there; it does not matter as the length of it does not change. Therefore, this segment here is not necessary.

Figure 4.10 A figure demonstrating the explanation of student

> Hatice Teacher: What your friends said was important. Listen. She said that "drawing the line segment (extension) is not necessary." Additionally, we thought that if this line segment (the base) was a little longer, it would be called as the extension, and then we could draw as Buğra did since the angle in between would be 90^0. So, I can draw from out of the line segment. It doesn't necessarily have to be drawn on the line segment. (Hatice-Obs.).

The pre-service teacher directed some questions to the class and created a discussion environment to overcome students' misconstruction of the extension of a line segment, which is also the misconception that the height is always inside the triangle; in other words, for this question, the misconception is that the perpendicular is always on the line segment. At the end of the discussion, the students reached the conclusion that drawing an extension was not necessary to draw a perpendicular line segment from a point to a line segment, and that the perpendicular did not necessarily have to be drawn on the provided line segment.

The other example pertaining to students' misconception/difficulty in establishing the height and corresponding base while calculating the area of a triangle was taken from the observation of Hatice. The analysis of the lesson plan revealed that the pre-service teacher prepared a problem to specify whether or not the students were able to determine the height and its corresponding base to calculate the area of the triangle. When the data gathered from classroom observation was analyzed, it was seen that some of the students had some problems in determining the height corresponding to the base as the pre-service teacher had expected prior to the lesson. When she encountered the students' misconception that any height could be multiplied by any base, she tried to eliminate this thought by creating a classroom discussion environment. Although the question was given in the relevant misconception/difficulty section of the chapter, it is presented in this chapter again to make the reading coherent to the reader. Below the problem, an excerpt taken from the observation transcript of Hatice is provided.



Problem: Ahmet found the area of the triangle on the left side as 9 cm^2 but he was not sure of his answer. What do you think about his answer? Explain.

Hatice Teacher: Ahmet calculated the area of the given triangle as 9 cm^2 but he was not sure about his answer. What do you think about the answer of Ahmet?

Students: He answered correctly, teacher.

Student A: 3 cm belongs to [BC], doesn't it?

Hatice Teacher: I don't know. Who wants to talk? Tell us Oğuz.

Student B: It is correct.

Hatice Teacher: Why is it correct? Can you explain it to us?

Student B: We multiply 6 and 3, and it equals 18. We further divide the product by two, and it equals 9.

Student C: The only person who said that it is not correct is me.

Hatice Teacher: Oğuz said that product of 6 and 3 equals 18. Did you think like that since they were a height and a base?

Student B: Yes.

Hatice Teacher: Then, you divided the product by two and got 9 cm^2... Does everyone agree with the explanation of Oğuz?

Majority of the students: Yees.

Hatice Teacher: Is there anyone who does not agree?

Student D: Ahmett.

Hatice Teacher: You and Ahmet don't agree. Arda doesn't also. Why don't you agree, Ahmet? Can you explain it to us?

The student: Teacher, the height was drawn from point B to the base [AC]. Therefore, we need to multiply the given height and the base [AC] and the length of the base was not provided.

Hatice Teacher: Who is listening to Enes? Anıl, can you explain it to us also?

The student: Teacher, I agree with Ahmet because 6 cm belongs to the side [BC]. If the length of [AC] is not 6 cm also, then the answer will not be correct.

Student E: I want to say something. I think, what they said is not correct because the height was drawn here to [AC]. I mean that Ahmet found a height.

Hatice Teacher: What did we write here Ozan? There was written a side of the triangle and the height corresponding to that side of the triangle. We didn't say whichever side; it needs to be the corresponding side of it. In that case, this height corresponds to which side of the triangle?

Students: [AC]

Hatice Teacher: Do I know |AC|, Taha?

Students: No

Hatice Teacher: Then, can I multiply this and this?

Students: Noo.

Hatice Teacher: We say a side and its corresponding height. Where does this belong to? Here, as Arda and Ahmet said. Therefore, I need to find here. So, I cannot multiply these two just because I immediately saw these two.

As it can be seen from the provided excerpt, when students showed their incomplete knowledge regarding establishing the height and its corresponding base, the pre-service teacher guided the students by means of questions. Both the proponents and the opponents of Ahmet's thought had the chance to state their ideas. Eventually, the teacher explained the correct answer.

There was another example on which the discussion method was carried out. In this exceprt which was taken from the classroom observation transcript of Eda, the pre-service teacher aimed to eliminate the students' misconception/difficulty that was related to the confusion between the concepts of length and height referring to a side and the height of a right-angled triangle. At first, the pre-service teacher prepared a problem to specify whether or not the students were able to distinguish between the height and the length concepts. The analysis of the classroom observation data showed that students were not be able to distinguish between the height and the length concepts while working on the problem as the pre-service teacher had expected. When the pre-service teacher observed the students' error emerging from this misconception/difficulty, she preferred to use the discussion method to eliminate their inadequate knowledge regarding the height and the length concepts. The problem asked and the excerpt taken from the observation transcript of Eda is provided below.

Problem: The painter Rasim wants to paint his house. Since he cannot reach the higher part of the wall, he will use a ladder of 1.5m length. When he ascends the highest stair of the ladder, what can be his height with respect to the ground? Show it by drawing on the shape.

Student A: There were ten stairs of the ladder and its height was 1.5 m. Therefore, I divided 1.5 m by 10. So, for each stair he ascends 15 cm.

Eda Teacher: Read the problem again. Does it say the height is 1.5 m.? It's said that the length of the ladder is 1.5 m.

Student A: Yes, therefore 15 cm for the ascent of each stair.

Eda Teacher: Where is the length of the ladder?

Student A: It is here [He drew the height of the end point of the ladder on the board].

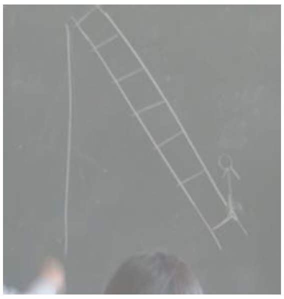


Figure 4.11 An example of student's answer from Eda's lesson

Eda Teacher: This is the given ladder, so its length is here [she showed its length].

Student A: The length is here, and the height is here [He corrected himself].

Eda Teacher: So, when you ascend each stair, can you still say that the length should be divided by ten and that he moves 15 cm forward?

Student B: Teacher, since it was stated that the ladder includes ten stairs, my friend divided it by ten.

Eda Teacher: Well then, what is the height of the painter on the first stair?

Student B: 15 cm

Eda Teacher: But 15 cm is here [She showed the length of the first stair]. Is there any other idea? Do you agree with your friend?

Students: Yees.

Eda Teacher: Haven't we just stated that the height of the endpoint was here? We did not state the length of the ladder.... If the length of the ladder is 1.5 m, then can its height also be 1.5m?

Students: No, it can't.

Eda Teacher: It can't be the same, can it? If it (the length) is 1.5 m, then it (the height) has to be something different from 1.5 m. It is even smaller than the length when we look at them.

Student C: Because it is skewed.

Eda Teacher: Because it is skewed, isn't it? For example, think about the right angled triangle since you know it well. In the right angled triangle, the side which is not one of the perpendiculars is always longer than the others. In that case, the length of the ladder is longer than its height. So, I cannot say both of them are 1.5 m. Now, can you show the heights of the first stair, second stair, third stair... by constructing?

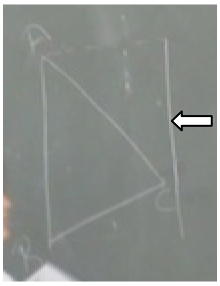
Student A: It will be here for the first stair.

...

Eda Teacher: Do not confuse height and length. ... Height is not the same as the length of the ladder. It is the distance between the end point of the ladder and the ground (Eda-Obs.)

As can be seen, to eliminate, students' misconception/difficulty regarding the confusion of the height and the length concepts, the pre-service teacher preferred to use the discussion method. In this respect, she asked some leading questions to enable the students to find the correct answer of the problem by realizing their misconstruction. However, whatever she asked the students could not reach the correct answer. Therefore, she explained it herself.

Another example in which the pre-service teachers used discussion as a instructional strategy was related to students' misconception/difficulty regarding the position of height. The analysis of the classroom observations showed that the pre-service teacher encountered one of the students' misconception/difficulty in that students thought the height should be always vertical or horizontal to the ground. This example was referred to by Eda as the most difficult to overcome since it was unexpected and she did not experience such an example before. When she asked the height of [BC] to the class, some of the students drew similar line segments as given below. She further tried to eliminate their misconception by asking some questions. The excerpt taken from the corresponding observation of Eda is given below.



A student's construction of height

Figure 4.12 An example of sstudent's answer from Eda's lesson

Eda Teacher: How did you draw it, from which point to which base?

Student A: I drew it from A to base C.

Eda Teacher: Why did you draw it like this?

Student A: At random.

Student B: Can I draw it? My drawing looks like his drawing but the line segment continue up to base C.

Student C: Teacher, can we draw it from A to B?

Eda Teacher: Do you mean that this side ([AB]) of the triangle is the height? It is not a right angled triangle, is it?

Student C: It is skewed.

Eda Teacher: Is the height drawn from point A? Any one else?

Student D: From point A to point B?

Eda Teacher: From point A to base [BC].

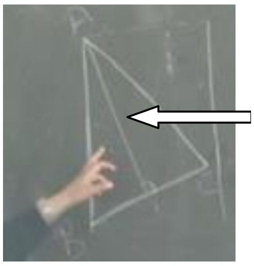


Figure 4.13 An example of student's answer from Eda's lesson

Eda Teacher: Why do you think it is correct? Tell us Ahmet.

Student E: [Shows by moving his hand] Because the triangle is not like that, it is like this. Therefore, the perpendicular line segment will be like this, not like that.

Eda Teacher: Yes, its base is inclined. In the other examples the bases were horizontal to the ground, so we always drew it vertical to the ground. However, the base is inclined now. I think that Ayşe drew the vertical line segment to the ground since she thought that the base was horizontal as before (Eda- Obs.).

To eliminate students' misconception during her practice teaching, first of all she tried to understand the reason underlying their answer by asking some questions to the students. After one of the students gave the correct answer, she asked a student to explain the reason of the inclined drawing of the height. Subsequently, she explained the reason underlying their errors and the correct answer.

As a last example of this instructional strategy, pre-service teachers preferred to use the discussion method to overcome students' misconceptions/difficulties regarding the role of 2 found in the formula for the area of triangles. The example taken from the observation transcript of Hatice is provided. In the example, a problem asked by the pre-service teacher is also provided.

Problem: There is a garden looking like a triangle. The length of one of the bases of this triangle is 25 m and the length of the height corresponding to this base is 16 m. Pepper seedlings were planted to one fourth of the garden and tomato seedlings were planted to the rest of the garden. Find out how many cm^2 were their areas? (Lesson plan of Hatice)

Hatice Teacher: Let's take your answers. Who was able to find the answers? What is your answer?

Student A: 100

Hatice Teacher: Sanem found 100.

Student B: Teacher, which one did she find tomato or pepper?

Student C: No, it is the area.

Student D: I found 100, also.

Student E: Wrong, I think it is 200.

Student F: 200

Hatice Teacher: Anybody else? Arda what did you find?

The Student: I found the area of pepper to be 300 and the tomato as 100.

Hatice Teacher: Well, Sinem you found the area first, and then calculated the others, didn't you? How did you find the area of the triangle?

The student: I multiplied 25 by 16.

Hatice Teacher: Why did you multiply them? [Calling out to the class] She said that she found the area as 400. Did you find the same answer?

Students: Yees.

Hatice Teacher: How do we calculate the area? Why 200?

Student G: No, it is not 200. We will divide it by two.

Hatice Teacher: Why do we divide it by two?

Student G: a*h/2

Hatice Teacher: Well, the formula. ... How do we calculate the area, Ali?

The student: We multiply...

Hatice Teacher: Do I just multiply?

Student H: We divide the product by two also.

Hatice Teacher: So, the base is 25 m, the height is 16 m, divide the product by two, 200 m^2 (Hatice-Obs.).

The pre-service teacher guided the students with her questions. By asking leading questions, she aimed to eliminate their misconception with regard to the problem. Students made some errors emerging from their confusion of the area of the parallelogram and the area of the triangle, in a word; they forgot to divide the product of the base and height by two.

In this section, the discussion as one of the strategies used by the pre-service teachers to overcome misconceptions/difficulties held by students while solving questions regarding the concept of the area of triangles has been examined by providing excerpts from the real classroom teachings. In the following section, demonstration, which was used as another instructional strategy by the pre-service teachers to eliminate students' misconceptions/difficulties regarding the concept of the area of triangles, is presented.

4.2.2 Demonstration

Another instructional strategy which was derived from the knowledge of the pre-service teachers used for eliminating students' misconceptions/difficulties regarding the concept of the area of triangles during practice teaching was demonstration. In the demonstration strategy, first of all the teacher shows the students how to do a task by providing consecutive guidelines. Then, the teacher asks the students to carry out what they have seen on their own.

As an example for this instructional strategy, an excerpt taken from the observation transcripts of Hatice can be used. In this respect, during classroom observation, Hatice asked the students to draw a perpendicular line segment from a point to a line segment. However, the analysis of the observations revealed that Hatice observed that some of the students had a misconception that the height should always be vertical or horizontal to the ground. Then, she tried to eliminate this misconception/difficulty of the students regarding the position of the height by utilizing the demonstration strategy. The relevant dialogue between the students and Hatice is provided below.

Hatice Teacher: Which of the line segments was perpendicular to the given line segment?

A Student: It needs to be perpendicular.

Hatice Teacher: How do I understand whether it is perpendicular or not?

A Student: The angle on the intersection of the line segments should be 90^0.

Hatice Teacher: Let's measure the angle on the intersection. Look at the board. First, I'll demonstrate how to measure, and then you can measure the other intersections. Let's first look for the horizontal line segment. I place the protractor in this manner, so that the original line segment overlaps 0^0. Then, look at the overlapping degree with the horizontal line segment. Is it 90^0?

Figue 4.14 A figure of Hatice's demonstration from the lesson

Students: No.

Hatice Teacher: Then, let's look at the vertical one. Is it 90^0?

Students: No.

Hatice Teacher: Let's look at the last one. Is it 90^0? As you see, it became 90^0, so the height is this one. Now, apply what you observed to your worksheets with your protractors (Hatice-Obs.).

The statements of Hatice functioned as evidence of her knowledge of the demonstration method that she used to overcome students misconception/difficulty regarding the position of height. Moreover, the analysis of the lesson plan of Hatice revealed that she planned to use this instructional strategy when she encountered during instruction such a thought that height should be always vertical or horizontal.

Another example regarding the use of the demonstration strategy was taken from the classroom observation of Eda. The analysis of the classroom observations revealed that during instruction of the pre-service teacher, some of the students had difficulty/misconception regarding the identification of the height of the relevant base for a given base of an obtuse triangle. To eliminate the misconception/difficulty held by the students with regard to specifying the height of the asked base of the triangle, Eda preferred to apply the demonstration strategy by using the corner of a paper instead of a protractor. The relevant part of the observation is provided below.

> Look at the board. Now I will show you how to draw a perpendicular line segment. The corner of this paper is 90^0, right? I have not got a protractor but I need to draw a 90^0, don't I? I need to draw a perpendicular line segment to this. So, I located the sides that intersect at a relevant corner of the paper on the base and the peak point of the height which will be drawn. Such a height emerged. Now, you draw it like that in your notebooks (Eda-Obs.).

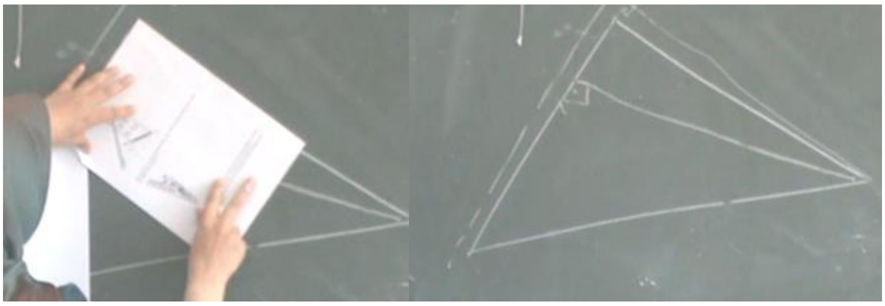


Figure 4.15 A figure of Eda's demonstration from the lesson

In this question, Eda observed, as she expected, that the students had difficulty while drawing the height in an obtuse triangle. They also displayed the misconception/difficulty that the height is always inside the triangle. Therefore, to overcome the difficulty/misconception in question, the pre-service teacher used students’ knowledge that the intersection's angle of the base and the height is 90^0. Then, she demonstrated how to draw a 90^0 angle on the intersection by using the corner of a sheet of paper. Furthermore, she asked the students to draw the height in the same manner as she had done. This situation reflects the knowledge of the pre-service teacher regarding the demonstration method.

After examining the demonstration as the pre-service teachers’ knowledge of instructional strategy, in the following sections their knowledge on cognitive conflict strategy used to overcome students' misconceptions/difficulties regarding the concept of the area of triangles was examined.

4.2.3 Cognitive Conflict

In the light of the knowledge of the two pre-service teachers on strategies, the third instructional strategy used to eliminate students’ misconceptions/difficulties in the area of triangles was cognitive conflict. In this instructional strategy the teacher does not state the misconception/difficulty held by students. Instead, she waits for the students to realize their own error with the help of leading questions. The aim of those leading questions was to provoke conflict in students' thoughts (Swan, 2001). To illustrate, during instruction, Hatice asked the students to draw a perpendicular line segment to a given line segment from point F. Then, the students drew their varying answers on the board. As can be seen in the picture below taken from the board, some answers of the students contained the misconception/difficulty pertaining to their thought that height should always be vertical or horizontal to the ground. In this respect, a scene from the lesson observation of Hatice is provided.

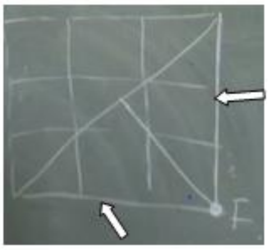


Figure 4.16 An example of students’ different answers from the Hatice’s lesson

Hatice Teacher: Which line segment is the perpendicular one? How can we decide?

A Student: It must be exactly perpendicular.

Hatice Teacher: How can I understand its perpendicularity?

A Student: An angle of 90^0 is formed at the intersection point of the given line segment and the perpendicular line segment.

Hatice Teacher: Your friend said that it must be 90^0. Let's see if it is correct by measuring the angle between the perpendicular and the given line segment (Here, the pre-service teacher refers to measuring the angles from previous examples that are known to be correctly measured). ... For example, I'm placing the protractor on the angle [referring to the first question, A]. What is the angle in between the line segment and the perpendicular?

A Student: 90^0

Hatice Teacher: Yes, it is 90^0, isn't it? Then, let's measure the angle in the examples B, C, D. I placed the protractor on B. What is the angle in between?

A Student: 90^0

Hatice Teacher: Then, let's look at C.

A Student: 90^0

Hatice Teacher: In the same way, let's look at C. What is it?

A Student: 90^0

Hatice Teacher: Then, what can we say about the angle between the perpendicular and the line segment? What is the angle in between?

A Student: 90^0

Hatice Teacher: Then our target while looking for the correct answer will be this. Now, we can first look at the horizontal one (she passed onto question F).

Hatice Teacher: Now, let's measure the angle with the protractor. Is it 90^0 (she refers to the horizontal construction of the students)?

Students: Noo.

Hatice Teacher: So, is it the perpendicular line segment?

Student: No.

Hatice Teacher: Is the other one (referring to the vertical construction of the students) 90^0? Not. Let's look at your final construction.

A Student: 90^0 (Hatice-Obs.)

As can be seen in the excerpt provided above, the teacher first made students remember how they could understand the perpendicularity of a line segment. When the angle between their drawings and the given line segment was measured in the previous examples which were correct, the students observed that the angle was required to be 90^0. Then, to check the accuracy of their answer, the angle that they had constructed was also measured and students who constructed erroneous line segments observed that the required angle was not 90^0 in their constructions. This situation provoked a conflict in students' thoughts. Thus, they noticed that their answers were not correct. As a result, they had a chance to correct their answer.

The above statements of the pre-service teacher revealed the teacher knowledge regarding cognitive conflict strategy in that to overcome this misconception of students, she first asked some questions to provoke a conflict in students' minds. Hence, they were able to realize their own mistakes and correct them.

4.2.4 Didactic Approach

After the examples provided regarding pre-service teachers' knowledge of cognitive conflict strategy, which was preferred to eliminate students' misconceptions/difficulties, there was another instructional strategy used by pre-service teachers to eliminate students' misconceptions/difficulties regarding the concept of the area of triangles. In this respect, didactic approach was examined in relation with classroom observations in the light of pre-service teachers' knowledge of strategies. Didactic approach was specified by Swan (2001) as a way to respond to students' misconceptions/difficulties; in this approach, the students were directly informed of their misconception/difficulty and the reason underlying it by means of a mathematical language. Since the correction of the misconception/difficulty was essential in this approach, they were generally corrected by the teacher.

For instance, while students tried to solve a question during the lesson, a misconception/difficulty regarding students' thought that height is a perpendicular bisector of the corresponding base emerged and Eda realized it. To overcome this misconception/difficulty of the students, she used the didactic approach as illustrated in the excerpt below.

> In the question, there was a triangle with a height of 4 cm and a base of 9 cm.
>
> A student: The height is 4 cm. Since the base is 9 cm, it turns out to be 4.5cm by 4.5 cm when it divided by two...
>
> Eda Teacher: Hilal divided the triangle into two parts. The height is given as 4 cm and the base as 9 cm. You divided it into two as 4.5 by 4.5. However, are we told that the triangle is isosceles or equilateral? Did you divide it into two because of the appearance of the triangle?
>
> A student: Yes, because of its appearance.
>
> Eda Teacher: Does the height always divide the base into two? For example, did it hold for the corresponding question?
>
> A student: Yeess.
>
> Eda Teacher: Now, look here. If the triangle in the question were an isosceles triangle or an equilateral triangle, then what your friends said could be true. However, now we cannot assert that the height divides the base into two pieces as 4.5 and 4.5. Therefore, we should perform the operation directly with the base and the height (Eda-Obs.).

The analysis of the observation transcripts revealed that while eliminating the misconception/difficulty held by the students regarding the position of the height, the pre-service teacher first tried to understand the thoughts of the students underlying the error. After ensuring the misconception/difficulty behind their error, Eda informed the students that the height can divide the corresponding base into two if the types of triangle were isosceles or equilateral based on sides. Otherwise, dividing the base into two equal pieces was an error. Subsequent to informed the students of their misconception, Eda explained the solution.

4.2.5 Direct Teaching

The last instructional strategy used by pre-service teachers to eliminate students' misconceptions/difficulties regarding the concept of the area of triangles was direct teaching. In this instructional strategy, the teacher directly explains the method of solution when a misconception/difficulty emerges. So, students were passive receivers when compared to the other strategies used by the pre-service teachers up to now.

The analysis of the observation transcripts revealed that the pre-service teachers preferred to use the direct teaching method to explain a concept the second time while eliminating students' misconceptions/difficulties. For instance, as previously indicated, Eda asked a question to the students regarding the construction of the height of an obtuse triangle. However, as she had expected, students thought that the height should always be inside the triangle. This misconception/difficulty of the students caused errors while constructing the height of the given triangle. Therefore, to eliminate those errors, she used the demonstration method at first as shown by Figure 4.15. However, some of the students still had a misconception/difficulty in understanding how height could be constructed outside of the triangle. For this reason, she tried to explain that how a height can be outside of an obtuse triangle again by using the direct teaching method. The corresponding part of the observation is presented below:

> A student: Why didn't we draw it inside the triangle?
>
> Eda Teacher: From here? ... I could not obtain the angle of 90^0 from inside the triangle. But, what did we write for the definition of the height? It is a perpendicular line segment drawn from the corner to the

base, right? In this regard, the angle of the perpendicular line segment is 90^0. So, I have to draw it outside the triangle (Eda-Obs.)

After her explanation, she realized that the students were unable to understand her statements. Therefore, she additionally stated,

> Let's show it in a clearer manner to you [She drew a different obtuse triangle which was more inclined]. It is an obtuse triangle, isn't it? Let's draw its height. ... By the way, I'm trying to draw it from this side of the triangle to the base. It can't be this. Neither can it be this...The drawn line segments are becoming more and more upright aren't they? However, as you see it went out of the triangle. There can be a height at outside of the triangle because my base is here and my corner is here. I'm trying to a height from this point towards the base. [At this point, she used a paper to show the perpendicularity as used before] I drew the height by using my sheet of paper by combining the peak point and the base. So, the height is here, outside of the triangle (Eda- Obs.).

Figure 4.17 A figure of Eda's presentation from the lesson

The analysis of the observation revealed that while eliminating the thought of the students such that height should be always inside the triangle, Eda tried to explain why the height of the corresponding base required to be constructed at the outside of the triangle. While making explanations, Eda also constructed what she said on the board. Hence, while Eda was transferring her knowledge, the students were passive learners.

As another example of the direct teaching strategy, the excerpt taken from the observation of Hatice is given. The analysis of the observation transcripts showed that Hatice also used the direct teaching method to overcome students' misconception/difficulty related to their lack of knowledge of extension. While working on a question during practice teaching, the pre-service teacher encountered a misconception/difficulty of students in that they thought that a line segment could not be extended. The question that the pre-service teacher asked was about constructing the height of an obtuse triangle. The pre-service teacher stated that, since she did not expect to experience students' lack of knowledge regarding the extension of a line segment and she was not aware of such a difficulty/misconception before the lesson, she expressed this situation as the most difficult one to overcome. The related excerpt taken from the observation of Hatice is given below:

> A Student: The thing that I cannot understand is that we learned that the line segment cannot extend in any way.
>
> Hatice Teacher: It does not extend. We said the extension of it.
>
> The Student: But we extended it.
>
> Hatice Teacher: We said its extension. If that were the case, we would think. We would think if it were a little bit longer than this line segment. If the line segment were like this, I could draw this height. However, it is (the longer one) different from this (the extension). It is the extension of this line segment. Iı, the extension of something ... Have you ever heard the 'extension' term? Extension of a line segment can be thought like a line, which continues at the level of line segment. We can extend from the beginning or the end of the line segment (Hatice- Obs.).

Like Eda, whose example is provided above, Hatice also used the direct teaching method to eliminate students' misconceptions/difficulties. In this respect, she tried to explain the meaning of the ‘extension’ term.

To sum up, in line with aim of the study, the following two research questions were tried to be answered: What is pre-service elementary mathematics teachers’ knowledge of misconceptions/difficulties held by the 6th grade students related to the area of triangles and what kind of strategies do pre-service elementary mathematics teachers use to overcome the misconceptions/difficulties held by the 6th grade students related to the area of triangles during practice teaching? The summary tables formed as a result of the data analysis were provided for both of the questions separately at the beginning of the relevant sections. In this respect, in light of the analyzed data, the knowledge of the pre-service teachers as regards to misconceptions/difficulties that students may have regarding the concept of the area of triangles were categorized into three main categories and relevant subcategories. The names of the main categories are determined as misconceptions/difficulties (i) regarding the concept of height, (ii) regarding the concept of area and (iii) regarding the formula for the area of triangles. Thus, the main categories and their subcategories were summarized in Table 4.1. On the other hand, the knowledge of the pre-service teachers of the strategies used to overcome misconceptions/difficulties that students held during practice teaching regarding the concept of the area of triangles were examined under five headings as discussion, demonstration, didactic approach, cognitive conflict, and direct teaching.

CHAPTER V

CONCLUSION, DISCUSSION, AND IMPLICATIONS

The purpose of the study was to investigate two pre-service middle school mathematics teachers' knowledge of misconceptions/difficulties held by sixth grade students regarding the concept of the area of triangles and of instructional strategies used to overcome these misconceptions/difficulties. In line with this purpose, conclusions and discussions drawn from research findings in the light of the literature, implications and recommendations for further research studies will be addressed in this chapter.

The conclusion drawn from the findings of the study is discussed under two sections with regard to two research questions. In the first section, the findings regarding the pre-service elementary mathematics teachers' knowledge of common misconceptions and difficulties held by sixth grade students regarding the area of triangles is discussed based on the previous literature. In the second section, findings regarding the pre-service elementary mathematics teachers' knowledge of instructional strategies to overcome misconceptions/difficulties in the area of triangles held by sixth grade students are discussed.

5.1 Pre-service Teachers Knowledge of Students' Misconceptions/Difficulties

In this section, findings of the research pertaining to pre-service teachers' knowledge of students' misconceptions/difficulties regarding the concept of the area of triangles is discussed based on the related literature.

Carpenter et al. (1988) mentioned that teachers' pedagogical content knowledge of students' conceptions and misconceptions is an important factor for effective teaching and this knowledge significantly affects teachers' decisions regarding teaching. The thoughts of the pre-service teachers in the present study seem to be parallel with the statements found in the literature (Carpenter et al., 1988) in that findings of the research showed that pre-service teachers place more emphasis on misconceptions/difficulties of students regarding the area of triangles while organizing their lessons. In this respect, the pre-service teachers stated that being aware about students' possible misconception/difficulties on specific topics can help teachers to answer appropriately and to overcome those misconception/difficulties effectively.

In the light of analysis of the data, it was revealed that pre-service teachers thought that being aware of students' misconceptions/difficulties regarding a concept is necessary for qualified teaching. In this respect, findings of the research revealed that pre-service teachers provided a variety of possible misconceptions/difficulties that students may have regarding the area of triangles, and they were categorized under three headings: misconceptions/difficulties regarding (i) the concept of height, (ii) the concept of area and (iii) the formula. The misconceptions/difficulties of students situated under these three headings were also stated in the literature (Cavanagh, 2008; Gökdal, 2004, Herskowitz, 1989; Moreira & Contente, 1997; Orhan, 2013).

Findings of the research revealed that most of the specified misconceptions/difficulties were related to the concept of height, which is a prerequisite prior knowledge for the concept of area. State differently five misconceptions/difficulties out of eight were relative to the concept of height. This might stem from pre-service teachers' thought that students' prior knowledge significantly affects their learning which was stated by Hewson and Hewson (1983) also. The misconceptions/difficulties regarding the concept of height specified by pre-service teachers in this study were also found in the literature. Students can say the height should be vertical or horizontal to the ground or inside the triangle (Herskowitz, 1989). In this regard, pre-service teachers pointed out that students' misconception/difficulty might stem from the fact that in general students are asked to find the altitudes of an object according to floor or in the provided questions at schools the height of the horizontal base of an object is asked. Therefore, students are not being able to experience different height constructions of different bases. Hence, students developed the thought that height should be always vertical or horizontal. Additionally, pre-service teachers determined another possible misconception/difficulty of students regarding the concept of height as students may think that height should be always inside the triangle. Hershkowitz (1989) mentioned that the reason of the misconception/difficulty regarding the height should be always inside the triangle is stemmed from the fact that the concept image of classification of the triangle by angle. Pertaining to this, concept image is defined as representations and images which evokes when a concept is read or heard (Gutierrez & Jaime, 1999). In this respect, the concept image of the triangle created in students' mind is an acute triangle and it has only internal altitudes (Hershkowitz, 1989). Therefore, the students tend to draw internal line segments inside the triangles. Furthermore, as it was specified by Eda, students may think that the height should be perpendicular bisector of a side (Gutierrez & Jaime, 1999). Then the pre-service teacher explained that this thought of students might be stemmed from the fact that the position of height looks like dividing the base into two equal pieces so students think that height is perpendicular bisector of the base. Moreover, in the literature it was stated that students can confuse the perpendicular height with the slant edge (Cavanagh, 2008; Orhan, 2013). Regarding students this difficulty, Eda specified that students may use the terms height and the length interchangeably since they have deficient knowledge about the definition of the height. In addition to misconceptions/difficulties identified in the literature, there was another one specified by the pre-service teacher in this study. In this respect, the pre-service teacher stated that students can say that the side of a triangle cannot be extended since they lacked knowledge of the concept of extension. The basis of the specification of this misconception/difficulty based on a situation occurred in the lesson. So that, Hatice asked a question to students regarding constructing a height from a point to a given line segment during practice teaching. To construct this height, pre-service teachers required to construct it on the extension of the base. Although one of his friend construct the height correctly on the extension of the base, one of the students stated that it could not be constructed since a line segment cannot be extended.

In addition to pre-service teachers' specification of the misconceptions/difficulties regarding the concept of height, there are also findings regarding the misconceptions/difficulties regarding the concept of area and the formula mentioned by them. These findings were also consistent with the studies in the literature (Cavanagh, 2008; Gökdal, 2004; Gutierrez & Jaime, 1999; Moreira and Contente, 1997; Orhan, 2013). According to the findings of the study, pre-service teachers mentioned that students may not be able to specify the base of a triangle and the corresponding height of this base. In this regard, the students can say that a base and a height are necessary to calculate the area of a triangle without thinking about which base corresponds to the given height. For this reason, they can have difficulty in the area concept. In addition, Moreira and Contente (1997) indicated that students' inadequate knowledge of the perimeter and the area concept cause students to think that there is a direct relationship between the perimeter of the figure and its area. The misconception/difficulty found in the literature was also stated by the pre-service teachers of the present study. In this regard, Eda indicated that students might think that division of a parallelogram into two triangles with equal areas requires the division of its perimeter by two. In other words, the students think that as the area decrease in half, perimeter decrease in half also. Eda also indicated that this misconception/difficulty might stem from students' lack of knowledge regarding the area and perimeter concepts. Since students cannot define the area and perimeter concepts properly, they cannot differentiate these two and confuse with one another. The parallel statements were also concluded by Tan-Sisman (2010) in that sixth grade students' superficial level of understanding regarding the length, area and volume concepts results in mistakes and difficulties about these topics. She might be state this sentence since the students' misconceptions/difficulties root in students' superficial level of understanding of the concepts (Tan-Sisman, 2010). Furthermore, the pre-service teacher might hold this misconception once when she was a student. Therefore, at the present time, she might understand the thoughts of students and where the students have difficulty in fact. In a similar vein, the findings of the study showed that pre-service teachers specified students' lack of understanding the role of 2 found in the area formula of a triangle. In this respect, pre-service teachers stated that during calculation, students remember the area of a parallelogram but they may forget to divide the parallelogram into two equal pieces. Hence, they may ignore the division by two in the formula. This misconception/difficulty was provided in some studies in the relevant literature so that to calculate the area of a triangle, students multiply the base and the height perpendicular to this base, but they forgot to divide this multiplication by two (Cavanagh, 2008; Gökdal, 2004; Orhan, 2013). So that, in these studies researchers also indicated that students ignore ‘2’ found in the formula. As the reason of emergence of this misconception, the pre-service teachers might think that students confuse their new knowledge with old ones as it can be understood from their statements. The educational courses that they took or an experience that they gained from a familiar middle school student might stem their specification of this knowledge. Hence, these experiences provided by a course or a student might help preservice teachers in developing their knowledge on students’ misconceptions/difficulties. Since pre-service teachers' knowledge of students' misconceptions/difficulties regarding the concept of the area of triangles was supported by the previous studies in the literature, it could be inferred that pre-service teachers' knowledge of students' misconceptions/difficulties regarding the concept of the area of triangles was sufficient. In same manner, Tekin-Sitrava (2014) concluded that middle school teachers had sufficient knowledge of students' errors regarding the volume of 3D solids. In this respect, the findings of Tekin-Sitrava (2014) were parallel to the present study in terms of pre-service teachers' knowledge of students' misconception/difficulties.

There might be several factors affecting pre-service teachers’ knowledge of misconceptions/difficulties regarding the area of triangle. One of these factors might be the courses that they took before this study. In this regard, Elementary Mathematics Education Program at METU offers some elementary education courses in which pre-service teachers can be informed regarding the students’ misconceptions/difficulties. In this regard, the courses that pre-service teachers took in their third year were the Methods of Teaching Mathematics I and II. By the help of these courses pre-service teachers prepared lesson plans for mathematics lessons including different teaching methods. In addition, they analyzed students’ misconceptions/difficulties on specific learning area of mathematics. The other course that pre-service teachers took in their first semester of fourth year was the

Nature of Mathematical Knowledge for Teaching. By means of this course, pre-service teachers specified the misconception/difficulties that student may hold and describe the strategies that can be employed to overcome these misconceptions/difficulties. Moreover, they learned to anticipate possible students' misconceptions/difficulties regarding specific mathematics topics and to organize their lesson with regard to those anticipated misconceptions/difficulties (METU, 2013). In this respect, during post interviews Hatice stated that she could be able to specify these students' misconceptions/difficulties in a detailed manner since she made a research in the scope of an elementary education course on students' misconceptions/difficulties while constructing altitude of a line segment from a pre-determined point. In addition, Eda indicated that they examined an article regarding students' misconceptions/difficulties on the height concept in the scope of a course. Therefore, the pre-service teachers' knowledge on misconceptions/difficulties regarding the height concept might stem from the courses which pre-service teachers took before this study. In the same manner, during a discussion which was conducted to talk about students' possible misconceptions and difficulties with respect to a specific topic or during a presentation of their prepared lesson plans or activities, they might have examined students' conceptions and misconceptions regarding the area concept in general or regarding the area of triangles. Moreover, they might have discussed about in which situations students may have difficulty. Hence, they can think that students may have these misconceptions/difficulties and it would be an obstacle for students' meaningful learning, so, they might think that they should consider students' those misconceptions/difficulties while teaching in their future classes. Moreover, they might have prepared lesson plan and activities regarding the area concept, area of triangles or altitude of triangles. After planning they might have discussed on the weaknesses and strengths of this prepared lesson plan/activity also. Therefore, pre-service teachers' knowledge on the misconceptions/difficulties that students might have regarding the altitude of triangles and area of triangles might stem from the instructions received in the scope of the elementary education courses.

Another factor affecting determination of the misconception/difficulty by pre-service teachers might be the experiences of the pre-service teachers that they encounter during tutorials. In this regard, during the pre-interviews, the pre-service teachers stated that while they tutored middle school students, they encountered where students have difficulty in understanding the area concept and solving problems. Moreover, when the students made an error while they were calculating the area of a triangle, the pre-service teachers might have asked some leading questions to understand the rationale behind their methods of solution. Hence, they might have faced with some students' misconceptions regarding the area of triangles behind students' explanations. Thus, the pre-service teachers' knowledge on misconceptions/difficulties regarding the area of triangles might stem from their experiences which gained by means of tutorials that they had before this study.

5.2 Pre-service Teachers' Knowledge of the Instructional Strategies Employed to Overcome Students' Misconceptions/Difficulties

Since teachers' pedagogical content knowledge of misconceptions/difficulties held by students was an important factor for students' meaningful learning (Carpenter et al., 1988), the knowledge of strategies required to overcome those misconceptions/difficulties also becomes an important factor. In this respect, findings regarding the pre-service elementary mathematics teachers' knowledge of strategies to overcome misconceptions/difficulties in the area of triangles held by sixth grade students are discussed in the light of previous studies.

Findings of the present study revealed that pre-service teachers employed various strategies to overcome misconceptions/difficulties regarding the area of triangles held by sixth grade students during their practice teaching. Cognitive conflict, didactic approach, discussion, demonstration, multiple representation, and direct teaching were the strategies used by pre-service teachers in their practice of teaching. Those strategies were also stated in the literature as approaches to respond to student errors. Swan (2001) specified didactic approach as a way to respond to students' mistakes, in that in this approach, the student is directly informed of his/her mistake and the reason of the mistake is stated using a mathematical language. Since the correction of the mistake is essential in this approach, the mistake is generally corrected by the teacher. As another approach, Swan (2001) suggested cognitive conflict to respond to students' mistakes. In this approach, the teacher asks some questions to provoke conflict in students' thoughts. With the help of the questions, it is aimed to make students realize their misconceptions. In a similar vein, Borasi

(1994) mentioned that when a misconception arises in the act of teaching, arranging discussion with the class to eliminate students' errors is essential for meaningful learning.

The implementation of different instructional strategies might stem from the educational courses that the pre-service teachers took before the study. Since the participants of the study were in their last semester of their education, they have taken the pedagogy related courses, such as Methods of Teaching Mathematics and Nature of Mathematical Knowledge for Teaching. The purposes of these courses are to describe the misconceptions of students regarding specific contents, offering strategies to overcome these misconceptions, predicting misconceptions of students and taking them into consideration while planning lessons as stated above section. Moreover, various teaching approaches and techniques for meaningful mathematics learning are included among the purposes of these courses (METU, 2013). Therefore, pre-service teachers' decisions on the strategies employed to overcome students' misconceptions/difficulties during practice teaching might be stem from the courses which were taken before this study. In this respect, the effect of the method course on pre-service teachers' decisions regarding teaching strategies was also encountered in a study by Isiksal (2006).

In addition to pedagogical courses as Methods of Teaching Mathematics and Nature of Mathematical Knowledge for Teaching, School Experience course might have an effect on the pre-service teachers' decisions regarding implementation of the instructional strategies. During the course, pre-service teachers observe real classroom environment and prepare lessons with appropriate instructional strategies considering students with diffent ages and abilities (METU, 2013). Moreover, following to this course the pre-service teacher took the Practice Teaching course. So that, this course was taken by the participants of the study at the same semester with the study. Before the observation of their teaching, they observed the classes approximately one month. Hence, during these observations at the intern schools the pre-service teachers might be impressed from their mentor teachers teacherswhom they observed and worked with. The instructional strategy employed by these observed teachers might be admired by participants of the study so they might usedthese strategies during their teaching.

The findings of the research showed that pre-service teachers gave more emphasis to explanations of the students regarding the reason behind their calculations during their practice teaching. For this reason, they always asked "why" questions to the students after receiving an answer to the question. In this way, they tried to understand the weaknesses of the students. Since the misconceptions/difficulties that students may have can be revealed from the students' response to the question asked by teacher (Aydın & Delice, 2008), it could be inferred that pre-service teachers' way to understand students' misconceptions/difficulties might be regarded as effective. After understanding the difficulty/misconception of the students at that moment, they tried to guide them with questions and conducted discussions with the class to provide a meaningful learning environment. Pre-service decision about conducting a discussion environment might stem from the educational courses that are mentioned above. Since in these courses they were taught that it is important to include students in the lesson and students need to participate actively during teaching. Moreover, the importance of asking questions to students to understand their reasoning behind their answer might have been also stated many times in these courses. Hence, these statements of their instructors might make an impression in their mind.

The other factor affecting their decision about conducting discussion environment might be one of the teachers who stick in their mind in their middle or high school teaching years. So that, the instructional strategy that this teacher used and the pre-service teachers remembered as the most effective for their learning in those years might have an effect on the decisions todays. By means of discussion majority of the class had a chance to express their thoughts regarding the problem asked. Moreover, the students were able to realize their friends' different thoughts. Since the correct answer was provided by their friends and also by the pre-service teachers, students who had a misconception/difficulty could understand where the error was, and could learn in a meaningful manner. In addition, the student who was timid might have observed pre-service teacher's way to respond to wrong answers and might be encouraged to talk about his/her answer and thoughts. In this respect, this instructional strategy which was employed may be regarded as effective to eliminate students' misconceptions/difficulties.

Both of the pre-service teachers prepared lesson plans of four lessons. In the course of organization of the lesson, pre-service teachers took into account the students' possible misconceptions/difficulties regarding the area of triangles as they have expressed during interviews. They also considered the way to eliminate those misconceptions/difficulties. However, during practice teaching, the pre-service teachers encountered some other student's misconceptions/difficulties regarding the area of triangles, which they had not thought about before the lesson. The analysis of the results revealed that both of the pre-service teachers preferred to employ the direct teaching method under these circumstances. For instance, in addition to the misconceptions/difficulties that she herself had determined, Hatice faced another misconception/difficulty regarding students' lack of knowledge in the extension of a line segment. To eliminate students' misconception/difficulty that "a line segment cannot be extended", she preferred to directly explain the meaning of the extension term. However, employing the direct teaching method to overcome the misconceptions/difficulties could not be an effective way for students' meaningful understanding (Zembat, 2008). The reason of implementing direct teaching method when pre-service teachers encountered the misconception/difficulty that they did not identify before the lesson might stem from feeling uncomfortable to eliminate it since they were unplanned how to respond the students' misconceptions/difficulties and unexperienced regarding teaching. Since the pre-service teachers were unprepared about the way to eliminate the misconception/difficulty, it might have led pre-service teachers to feel the least comfortable to eliminate. Therefore, the pre-service teachers might have preferred to employ the direct teaching method. Hence, it might be concluded that pre-service teachers' knowledge of strategies employed to eliminate students' misconceptions/difficulties regarding the area of triangles was limited to the strategies that they had planned to employ for the pre-determined misconceptions/difficulties.

As stated above, the findings revealed that pre-service teachers were not able to employ strategies to eliminate the misconceptions/difficulties that they were not familiar before the lesson. As an example of these misconceptions/difficulties, the construction of a height in inclined figures can be provided. The statement of Eda given below can provide evidence for the difficulty they experienced: "I could not do anything to eliminate it because I encountered it the first time" (Eda, Post-interview). In the light of this statement it can be stated that having difficulty in eliminating students' misconceptions/difficulties might stem from that pre-service teachers' are being unexperienced. After the lesson, to understand the difference between experienced and unexperienced situation regarding the 'construction of a height in inclined figures' she was asked that what she would have done to eliminate it if she were given another chance to eliminate this misconception/difficulty. Although the pre-service teacher had difficulty in overcoming the student's misconception/difficulty during teaching as she had expressed, after the lesson she provided different strategies to overcome. Hence, it can be inferred from this finding that pre-service teachers' knowledge of student misconceptions/difficulties has an effect on their knowledge of strategies in that when a teacher experiences how students can answer or which misconceptions/difficulties they may previously have, s/he could improve students' knowledge easily. The parallel finding regarding the effect of teachers' knowledge of misconception/difficulty on teachers' knowledge of instructional strategy to overcome was also found in the literature (Gökkurt, 2014). As Gökkurt's (2014) study revealed that teachers were able to employ appropriate methods, techniques and strategies to eliminate students' errors if they have the knowledge on students' prior knowledge and on the underlying cause of the error that students made. Hence, it can be stated that if the pre-service teachers were familiar with these misconceptions/difficulties, they could implement different instructional strategies to overcome the students' misconceptions/difficulties regarding area of triangles.

In addition to the findings mentioned so far, there is another one important finding of the present study. In the light of the analysis of the data, it can be concluded that pre-service teachers became aware of some of students' misconceptions/difficulties during their practice teaching although they did not recognize them in their plans or during pre-interviews. In line with this, data revealed the evidence that teaching in the natural class environment leads pre-service teachers to improve their knowledge of misconceptions/difficulties, and accordingly, their knowledge of the ways to overcome those misconceptions/difficulties. During actual teaching, pre-service teachers had a chance to observe how students think, how their solution methods for the questions are, what their reasoning behind their answers is, where they have difficulty in, and so on. Hence, their knowledge might develop in this sense, especially regarding the knowledge on students. Thus, this finding displayed the importance of practice teaching courses for pre-service teachers' development regarding the knowledge on students’ misconceptions/difficulties of specific mathematics topics and the knowledge on the strategies which might be employed to overcome those misconceptions/difficulties by providing actual learning environments.

According to the findings of the research, although pre-service teachers’ actions were mostly consistent with their plans, there was one exception. In this occasion, the pre-service teacher planned an activity to eliminate students’ misconception/difficulty that as the area decreases the perimeter also decreases in the same amount. However, during teaching she did not use the activity; instead, she preferred to ask whether there was a direct relationship between the area and the perimeter of a parallelogram. Then, after one counter example given by a student, she passed onto something else although she had planned a variety number of questions to make the relation between area-perimeter concepts comprehensible to students. Therefore, students may not be able to comprehend the relation between the perimeter and the area concept. The reason of this inconsistency between the planned lesson and the actual one might stem from the fact that during practice teaching, pre-service teacher might think that the class hour would not be enough for completing the required objectives. On the contrary, during practice teaching since the pre-service teacher did not implement her plan regarding the area-perimeter relationship, the remaining part of the lesson lasted shorter than a class hour. Thus, she did not decide on what to do for the remaining period of time, then, she preferred to continue with the lesson of the other day. But, she was not able to carry out an effective lesson for student meaningful learning of the concept. Hence, she might have realized the requirements of preparing and acting through a lesson plan. In this regard, Ozden, Usak, Ulker and Sorgo (2013) concluded in their study that asking pre-service teachers to prepare a lesson plan for a specific topic and for specific age group was able to promote their consideration about students’ prior knowledge and possible difficulties/misconceptions. Moreover, this might also stem from the fact that pre-service teacher might have thought that students' comprehension levels for performing the prepared activity were not at sufficient level, so she might have decided not to employ the activity during practice teaching. Pertaining to the possibility of this thought of the pre-service teacher it might be stated that before the lesson in which perimeter-area relation was mentioned the pre-service teacher attended two more lessons. Hence, she could have an opportunity to evaluate students' pre-conceptions, conceptions, etc. regarding the altitude of a triangle and area of triangle. Thus, she might evaluate the students' required performances and knowledge on prepared activity as insufficient so she might have decide on not to provide it to the students.

5.3 Implications

The purpose of the study was to investigate two pre-service middle school mathematics teachers' pedagogical content knowledge of the misconceptions/difficulties held by sixth grade students regarding the concept of the area of triangles and their knowledge of strategies used to overcome these misconceptions/difficulties.

In the present study, pre-service teachers' provided varied misconceptions/difficulties that students might have regarding the concept of the area of triangles. Additionally, they employed various strategies to overcome these misconceptions/difficulties during practice teaching. In this regard, the study offers implications for policy makers, curriculum developers and teacher educators. One of the implications provided for teachers educators is that while seeking pre-service teachers' knowledge mathematical knowledge development, 'knowledge quartet framework' can be used as a tool to discuss and reflect on pre-service teachers' teaching practice (Rowland, Turner, Thwaites, Huckstep, 2009). In this regard, 'Knowledge Quartet' helps pre-service teachers in deciding what is important to consider and what need to be considered in the context of teacher knowledge. Moreover, knowledge quartet gives the opportunity to understand the link between knowledge and teaching in action by observing how to use knowledge base during teaching, and how to translate knowledge into ongoing learning environment (Rowland et. al., 2009). In this respect, it is thought that the use of knowledge quartet might affect pre-service teachers' development of knowledge when used to reflect on the knowledge after a microteaching activity and after an actual classroom practice. In this sense, in-service teachers may also implement knowledge quartet as a reflection to their teaching. Hence, they can have a chance to evaluate negative and positive ways of their teaching.

In addition to the implications stated related to practice courses and microteaching activities, there is another implication for teacher educators, so that, teacher educators can enhance the content of the method courses or practice teaching courses by providing teaching scenarios to pre-service teachers. Teaching scenarios refer to simulations of real classroom situations, into which students' misconceptions/difficulties can also be integrated. With the help of these scenarios, pre-service teachers can be aware of possible students' misconceptions/difficulties regarding a specific topic. They can also examine students' thoughts and can provide strategies to eliminate those thoughts.

In addition to the implications provided for teacher educators, there are also implications for policy makers and curriculum developers. As a result of the findings of the study, the effect of pedagogical courses was evidently seen in pre-service teachers' planned lessons and also in their practice of teaching. In this regard, the statement of one of the pre-service teachers could be regarded as evidence; in other words, she expressed that the knowledge of students' misconceptions/difficulties that she used while planning her lesson had come from the Methods of Teaching Mathematics course. In this regard, an implication for curriculum developers of pedagogical courses could be that the content of these courses is required to designed in such a manner that more emphasis could be given to students' misconceptions/difficulties regarding specific topics and the strategies and methods may be employed to eliminate those misconceptions/difficulties. Hence, pre-service teachers could create a meaningful learning environment in their future classes. Also, there is another implication for policy makers such that more pedagogically rich courses for specific contents of mathematics education could be included into teacher education programs. Hence, pre-service teachers can learn students' misconceptions and difficulties on specific topics in a detailed manner. In addition, they can learn the way to teach and to eliminate misconceptions/difficulties of students. Hence, pre-service teachers can teach more effectively and create meaningful learning environments during their future practices of teaching.

Another implication for curriculum developers and policy makers could be that practice teaching courses may be made more comprehensive such that it can be offered in additional semesters before the senior year or can require more time to participate in real classrooms, and more chance can be given to pre-service teachers to teach in real classrooms. Moreover, microteaching activity is an effective method to provide pre-service teachers a chance to acquire stronger pedagogic skill for effective teaching (Ananthakrishnan, 1993). Also, it enables obtaining immediate feedback after each teaching practice (Ananthakrishnan, 1993). Hence, pre-service teachers can obtain an opportunity to improve his/her knowledge on teaching and can realize the noteworthy points required to be taken into consideration during teaching. In this respect, Simbo (1989) conducted a study to investigate the effect of microteaching activities on pre-service teachers' performances in their actual practice teaching classroom. As a result of his study, it was concluded that microteaching activities have a statistically significant effect on teaching performances of pre-service teachers in a positive manner. Therefore, microteaching activities could be performed to keep track of pre-service teacher development of mathematical knowledge. Hence, pre-service teachers may be able to realize possible students' misconceptions/difficulties regarding specific topics and can learn effective ways to eliminate these misconceptions/difficulties during microteaching activity or during conducted discussions regarding their microteaching activity.

After providing implications drawn in the light of the findings of the study, recommendations for further studies and limitations of the present study are presented in following sections.

5.4 Recommendations for Further Studies

The aim of the study was to investigate two pre-service middle school mathematics teachers' pedagogical content knowledge of misconceptions/difficulties held by sixth grade students regarding the concept of the area of triangles and their knowledge of strategies used to overcome these misconceptions/difficulties. In line with this aim, recommendations for future research studies are provided in this section.

Measurement is one of the important contents since it widely used in the real life setting (NCTM, 2014). Therefore, the content of this study was determined as measurement, the focus of which was then narrowed down to the area of triangles to conduct an investigation in a depth manner. However, relevant studies showed that students have a superficial level of understanding regarding the concept of area and they have some misconceptions/difficulties regarding this topic since they have learned the formulae of area of some geometric figures in a rote manner (Cavanagh, 2008: Orhan, 2013, Tan-Sisman & Aksu, 2009). Since teachers have a significant impact on promoting students' learning and eliminating their existing misconceptions, more studies investigating teachers' pedagogical content knowledge on misconceptions/difficulties that students may have about area of different figures could also be conducted (Baturo & Nason, 1996; Yeo, 2008; Yew, Zamri & Yian, 2010; Simsek, 2011). Hence, the effective ways to teach these topics can be presented to teachers and teacher educators. Moreover, the other topics of measurement are also important to investigate and the literature requires a more complete picture regarding the measurement learning area. The research conducted to investigate teachers' knowledge on measurement topics help us to understand where students have difficulties or misconceptions, how students' those misconceptions/difficulties can be overcame, which strategies are effective for meaningful learning of students, and so on. For this reason, more research studies could be conducted on other measurement topics like perimeter, volume and the metric unit of measure. Hence, teachers can create meaningful learning environment for students.

Since PCK is a complex construct, researchers suggest that instead of investigating all components at a time, investigating a small number of components in a deep manner is more meaningful (Bahcivan, 2005). In this respect, the focus of this study was limited to two components of PCK, namely the knowledge of misconceptions/difficulties of students regarding the area of triangles and the knowledge of the strategies used to overcome these misconceptions/difficulties during teaching. On the other hand, the effect of teachers' knowledge of students' learning was indicated by the researchers in the literature (Lenhart, 2010; Hatısaru, 2013). However, the relationship between teachers' knowledge and students' learning was not within the scope of this research. Therefore, the relationship between teachers' knowledge and students' learning regarding specific measurement topics could be examined in further studies. Hence, it could be understand that how teachers' knowledge affected the students' learning. Moreover, more studies could be conducted to examine other components of PCK regarding the concept of the area of triangles. Hence, this provides a look from a broader perspective.

Researchers mentioned that teachers' subject matter knowledge has a significant impact on their pedagogical content knowledge (Isiksal, 2006). Therefore, further studies could be conducted to investigate pre-service teachers' subject matter knowledge regarding the concept of the area of triangles in addition to pre-service teachers' pedagogical content knowledge. Hence, the relationship between the pedagogical content knowledge and subject matter knowledge can be seen more clearly.

In addition, the pioneers of PCK indicated that experience is an important factor for the development of PCK (Grossman, 1990; Shulman, 1987) However, the findings of a study by Tekin-Sitrava (2014) did not support this statement. On the contrary, the findings of her study revealed that experienced teachers' knowledge of students' errors regarding the volume of 3D shapes and the strategies to overcome these errors were not sufficient. Thus, further studies could be carried out to investigate in-service teachers' knowledge of students' misconceptions/difficulties regarding the concept of the area of triangles and the strategies to overcome these misconceptions/difficulties.

This study was conducted in one semester of a teacher education program. To investigate the development process of pre-service teachers' and in-service teachers' mathematical knowledge, further longitudinal studies could be conducted. Accordingly, pre-service teachers' improvements in their knowledge regarding a specific topic could be followed starting from their early years in the teacher education program to their experienced years as an in-service teacher. Hence, the whole map of knowledge can be obtained and teacher educators can gain insight into the development process of teachers' knowledge in this way.

REFERENCES

An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teachers in China and the US. *Journal of Mathematics Teacher Education*, *7*(2), 145-172.

Ananthakrishnan, N. (1993). Microteaching as a vehicle of teacher training--its advantages and disadvantages. *Journal of postgraduate medicine*, *39*(3), 142.

Aslan-Tutak, F. (2009). *A study of geometry content knowledge of elementary preservice teachers: The case of quadrilaterals.* (Unpublished doctoral dissertation). University of South Florida, USA.

Ayan, R., Yemen-Karpuzcu, S. ve Işıksal-Bostan, M. (2014). Matematik öğretmen adaylarının alan bilgileri: Sabit çevre değişen alan, sabit yüzey alanı değişen hacim. *11.Ulusal Fen Bilimleri ve Matematik Eğitimi*, (pp. 1070-1071).

Aydın, E. ve Delice, A. (2015). Ölçme ve değerlendirmeye kavram yanılgıları perspektifinden bir bakış. In M. F. Özmantar, E. Bingölbali, H. Akkoç (Eds.), *Matematiksel kavram yanılgıları ve çözüm önerileri* (4th ed.), (pp.393-436). Ankara: Pegem Akademi.

Bahçivan, E. (2012). *Assessment of high school physics teachers pedagogical content knowledge related to the teaching of electricity.*(Unpublished Doctoral Dissertation). Middle East Technical University, Turkey.

Baker, M., & Chick, H. L. (2006). Pedagogical content knowledge for teaching primary mathematics: A case study of two teachers. In *29th Conference of the Mathematics Education Research Group of Australasia* (Vol. 1, pp. 60-67).

Ball, D. L. (1988). Unlearning to teach mathematics. *For the learning of mathematics*, 40-48.

Ball, D. L. (1990). The mathematical understandings that prospective teachers bring to teacher education. *The elementary school journal*, 449-466.

Ball, D. L., Hill, H.H., & Bass, H. (2005). Knowing mathematics for teaching:Who knows mathematics well enough to teach third grade and how do we decide? *American Educator,* 14-46.

Ball, D. L., Lubienski, S. T., & Mewborn, D. S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge.*Handbook of research on teaching*, *4*, 433-456.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407.

Basturk, S., & Dönmez, G. (2011). Examining pre-service teachers‟ pedagogical content knowledge with regard to curriculum knowledge. *International Online Journal of Educational Sciences, 3*(2), 743-775.

Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics, 31*(3), 235-268.

Baxter, J. A., & Lederman N. G. (1999). Assessment and measurement of pedagogical content knowledge. In J. Gess-Newsome & N. G. Lederman (Eds.), *Examining pedagogical content knowledge* (pp. 147-162). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Bayazıt, I. (2015). Fonksiyonlar konusunun öğreniminde karşılaşılan zorluklar ve çözüm önerileri, In M. F. Özmantar, E. Bingölbali, H. Akkoç (Eds.), *Matematiksel kavram yanılgıları ve çözüm önerileri* (4th ed.), (pp.91-119). Ankara: Pegem Akademi.

Biber, Ç., Tuna, A., & Korkmaz, S. (2013). The mistakes and the misconceptions of the eighth grade students on the subject of angles.*European Journal of Science and Mathematics Education Vol*, *1*(2), 50.

Borasi, R. (1994). Capitalizing on errors as" springboards for inquiry": A teaching experiment. *Journal for research in mathematics education*, 166-208.

Bozkurt, A., Koç, Y., & Yılmaz, B. (2014). İlköğretim Matematik öğretmenliği Birinci Sınıf Öğrencilerinin Uzunluk, Alan ve Hacim Kavramlarına Yönelik Yaptıkları Tanımların İncelenmesi, *11.Ulusal Fen Bilimleri ve Matematik Eğitimi*, 1213.

Brophy, J. (1986). Teacher influences on student achievement. *American Psychologist*, *41*(10), 1069-1077.

Bukova-Guzel, E. (2010). An investigation of pre-service mathematics teachers pedagogical content knowledge: Example of solid objects. *Scientific Research and Essays*, *5*(14), 1872-1880.

Burgess, T. A. (2006). A framework for examining teacher knowledge as used in action while teaching statistics. In *Proceedings of the Seventh International Conference on Teaching Statistics*.

Carlsen, W. (1999). Domains of teacher knowledge, In J. Gess-Newsome & N. G. Lederman (Eds.). *Examining pedagogical content knowledge: The construct and its implications for science education* (pp. 95-132). Boston: Kluwer

Carpenter, T. P., Fennema, E., Peterson, P. L., & Carey, D. A. (1988). Teachers' pedagogical content knowledge of students' problem solving in elementary arithmetic. *Journal for research in mathematics education*, 385-401.

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C. P., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal*, *26*(4), 499-531. doi:10.3102/00028312026004499

Cavanagh, M. (2008). Reflections on measurement and geometry. *Reflections*, *33*(1), 55.

Chick, H. L., Baker, M., Pham, T., & Cheng, H. (2006). Aspects of teachers' pedagogical content knowledge for decimals. In J. Novotna, H. Moraova, M. Kraka & N. Stehlikova (Eds.) *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 297-304). Prague: PME.

Cochran,K. F., DeRuiter, J. A., & King, R. A. (1991, April). Pedagogical content knowledge: A tentative model for teacher preparation. Paper presented at the Annual Meeting of the American Educational Research Association, Chicago, IL.

Cochran, K. F., DeRuiter, J. A., & King, R. A. (1993). Pedagogical content knowing: An integrative model for teacher preparation. Journal of Teacher Education, 44, 263-272.

Creswell, J. W. (1998). Qualitative inquiry and research design: Choosing among five traditions. Thousand Oaks, California: Sage Publications.

Creswell, J. W. (2012). *Educational Research: Planning, conducting, and evaluating quantitative and qualitative research* (4th ed.). Pearson Education, Inc.: Boston.

Dagli, H. (2010). *İlköğretim beşinci sınıf öğrencilerinin çevre, alan ve hacim konularına ilişkin kavram yanılgıları.* Unpublished master thesis, Afyon Kocatepe University, Turkey.

Doğan, S. ve Işıksal, M. (2014). Alan ölçme konusuna yönelik matematiksel alan bilgisinin incelenmesi. *11.Ulusal Fen Bilimleri ve Matematik Eğitimi*, 700.

Donmez, G. (2009). *Matematik öğretmen adaylarının limit ve süreklilik kavramlarına iliskin pedagojik alan bilgilerinin değerlendirilmesi.* Unpublished Master Thesis, Marmara university, Turkey.

Esen, Y. (2013). *Development of a test for assessing teachers' mathematical content knowledge for teaching geometric measurement at elementary grade level.* (Unpublished Doctoral Dissertation). Middle East Technical University, Turkey.

Esen, Y. ,& Cakiroglu, E. (2012). Ilköğretim matematik öğretmen adaylarının hacim ölçmede birim kullanmaya yönelik kavrayışları. *MED, 1*, 21-30.

Fernandez, C. (2014). Knowledge base for teaching and pedagogical content knowledge (PCK): Some useful models and implication for teachers' training. *Problems of Education in the 21st Century, 60*(60), 79-100.

Fernandez-Balboa, J. M., & Stiehl, J. (1995). The generic nature of pedagogical content knowledge among college professors. Teaching and Teacher Education, 11(3), 293–306.

Fi, C. D. (2003). Preservice secondary school mathematics teachers' knowledge of trigonometry: Subject matter content knowledge, pedagogical content knowledge and envisioned pedagogy.

Fraenkel, J., Wallen, N., & Hyun, H.H. (2012). *How to design and evaluate research in education* (8th ed.). Boston: McGraw Hill.

Fuller, R. A. (1996). Elementary Teachers' Pedagogical Content Knowledge of Mathematics. Paper presented at t*he Mid-Western Educational Research Association Conference.* Chicago, IL.

Gokbulut, Y. (2010). Sınıf öğretmeni adaylarının geometrik cisimler konusundaki pedagojik alan bilgileri. (Yüksek lisans tezi). Gazi Universitesi, Ankara.

Gokdal, N. (2004). *Ilköğretim 8. sınıf ve ortaöğretim 11. sınıf öğrencilerinin alan ve hacim konularındakı kavram yanılgıları.* Unpublished master thesis, Gazi University.

Gokkurt, B., Sahin, O., Soylu, Y., & Soylu, C. (2013). Examining preservice teachers' pedagogical content knowledge on fractions in terms of students' errors. *International Online Journal of Educational Sciences, 5*(3), 719-735.

Goulding, M., Rowland, T., & Barber, P. (2002). Does it matter? Primary teacher trainees' subject knowledge in mathematics. *British Educational Research Journal*, *28*(5), 689-704.

Graham, K. J., & Fennell, F. S. (2001). Principles and standards for school mathematics and teacher education: Preparing and empowering teachers. *School Science and Mathematics*, *101*(6), 319-327.

Grossman, P. L. (1990). *The Making of a Teacher: Teacher Knowledge and Teacher Education.* New York: Teachers College Press, Columbia University.

Gutiérrez, A., & Jaime, A. (1999). Preservice primary teachers' understanding of the concept of altitude of a triangle. *Journal of Mathematics Teacher Education*, *2*(3), 253-275.

Haciomeroglu, G. (2016). Prospective Secondary Teachers' Subject Matter Knowledge and Pedagogical Content Knowledge of the Concept of Function. (Unpublished doctoral dissertation). University of Florida, USA.

Hatisaru, V. (2013). *Teachers' knowledge of content and students about the function concept and its interrelation with student learning outcomes in vocational high schools.* Unpublished Doctoral dissertation, Middle East Technical University, Turkey.

Hegarty, S. (2000). Teaching as a knowledge-based activity. *Oxford review of education*, *26*(3-4), 451-465. doi: 10.1080/713688541

Hershkowitz, R. (1989). Visualization in Geometry--Two Sides of the Coin.*Focus on learning problems in mathematics*, *11*, 61-76.

Hewson, M. G., & Hewson, P. W. (1983). Effect of instruction using students' prior knowledge and conceptual change strategies on science learning. *Journal of Research in Science Teaching*, *20*(8), 731-743. doi: 10.1002/tea.3660200804

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K. C., Wearne, D., Murray, H., ... & Human, P. (1997). Making sense. *Teaching and learning mathematics with understanding*.

Hill, H. C., Ball, D. B., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers" topic-specific knowledge of students. *Journal for Research in Mathematics Teacher Education*, 39(4), 372-400.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American educational research journal*, *42*(2), 371-406.

Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *The Elementary School Journal*, *105*(1), 11-30.

Huang, H. M. E., & Witz, K. G. (2013). Children's conceptions of area measurement and their strategies for solving area measurement problems. *Journal of Curriculum and Teaching*, *2*(1), 10-26. doi: 10.5430/jct.v2n1p10

Isik, C., Ocal, T., & Kar, T. (2013). Analysis Of Pre-Service Elementary Teachers'pedagogical Content Knowledge In The Context Of Problem Posing.

Isiksal, M. (2006). *A study on pre-service elementary mathematics teachers' subject matter knowledge and pedagogical content knowledge regarding the multiplication and division of fractions.* Unpublished doctoral dissertation, Middle East Technical University, Turkey.

Isiksal, M., & Cakiroglu, E. (2011). The nature of prospective mathematics teachers' pedagogical content knowledge: the case of multiplication of fractions. *Journal of Mathematics Teacher Education*, *14*(3), 213-230.

Isiksal, M., Koç, Y., Bulut, S., & Turhan, T. A. (2007). An analysis of the new elementary mathematics teacher education curriculum in turkey. *The Mathematics Educator, 17* (2), 41–51.

Jing-Jing, H. U. (2014). A critical review of pedagogical content knowledge'components: Nature, principle and trend. *International Journal of Education and Research*, *2*, 411-424.

Johnson, T. L. (2011). *Elementary pre-service teachers' mathematical knowledge for teaching: using situated case studies and educative experiences to examine and improve the development of MKT in teacher education.* Unpublished Doctoral Dissertation, University of North Carolina, USA.

Kamii, C. and Kysh, J. (2006). The difficulty of "length x width": Is a square the unit of measurement? *Journal of Mathematical Behavior*, 25, 105-115.

Karahasan, B. (2010). *Preservice secondary mathematics teachers" pedagogical content knowledge of composite and inverse function.* Unpublished doctoral dissertation, Middle East Technical University, Turkey.

Kartal, T., Ozturk, N., & Ekici, G. (2012). Developing pedagogical content knowledge in preservice science teachers through microteaching lesson study. *Social and Behavioral Sciences, 46,* 2753-2758.

Kinach, B. M. (2002). A cognitive strategy for developing pedagogical content knowledge in the secondary mathematics method course: Toward a model for effective practice. *Teaching and Teaching Education, 18,* 51-71.

Kubar, A.(2012). *Pre-service elementary mathematics teachers' knowledge about definitions of integers and their knowledge about elementary students' possible misconceptions and errors in describing integers.* Unpublished master thesis, Gazi University, Turkey.

Kula, S. (2011). *Examining mathematics preservice teachers' subject matter and pedagogical content knowledge by using knowledge quartet: The case of limit.* (Unpublished master thesis). Dokuz Eylül University, Turkey.

Kula, S., & Bukova- Guzel, E. (2014). Matematik ve matematik öğrenimi bilgisi ışığında dörtlü bigi modelindeki beklenmeyen olaylar bilgisi. *Turkish Journal of Computer and Mathematics Education, 5*(1), 89-107.

Lee, E. and Luft, J. A. (2008). Experienced secondary science teachers' representation of pedagogical content knowledge. *International Journal of Science Education, 30*(10), 1343–1363.

Lenhart, S. T. (2010). *The effect of teacher pedagogical content knowledge and the instruction of middle school geometry.* Unpublished doctoral dissertation, Liberty University, Virginia.

Lincoln, Y. S., & Guba, E. G. (1985). Naturalistic Inquiry. Thousand Oaks, California: Sage Publications.

Livy, S. (2010). A" Knowledge Quartet" Used to Identify a Second-Year Pre-Service Teacher's Primary Mathematical Content Knowledge. *Mathematics Education Research Group of Australasia*.

Luneta, K., & Makonya, P. J. (2010). Learner errors and misconceptions in elementary analysis: A case study of a grade 12 class in South Africa. *Acta Didactica Napocensia, 3*(3), 35-46.

Magnusson, S., Krajcik, J., & Borko, H. (1999). Nature, sources and development of pedagogical content knowledge for science teaching, In J. Gess-Newsome & N. G. Lederman (Eds.). *Examining pedagogical content knowledge: The construct and its implications for science education* (pp. 95-132). Boston: Kluwer.

Makonye, J. P., & Luneta, K. (2014). Mathematical errors in differential calculus tasks in the Senior School Certificate Examinations in South Africa. *Education as Change*, *18*(1), 119-136.

Marks, R. (1990). Pedagogical content knowledge. From a mathematical case to a modified concept. Journal of Teacher Education, 41(3), 3-11.

Mason, J. (1996). *Qualitative researching*. London: Sage Publications Ltd.

Maxedon, S. J. (2003). *Early childhood teachers" content and pedagogical knowledge of geometry.* Unpublished doctoral dissertation, The University of Arizona, USA.

Merriam, S. B. (1998). *Qualitative research and case study applications in education (Rev. ed.).* San Francisco, CA: Jossey-Bass.

Merriam, S. B. (2009). *Qualitative research: A guide to design and implementation*. San Francisco: Jossey-Bass.

Middle East Technical University (2013). General Catalog 2013-2015. METU Printing Studio Press.

Moreira, C. Q., and Contente, M. do R. (1997). The Role of Writing to Foster Pupil's Learning About Area. In E. Pehkonen (Ed.), *Proceedings of the 21st PME International Conference, 3*, 256-263.

Morine-Dershimer, G., & Kent, T. (1999). The complex nature and sources of teachers' pedagogical knowledge, In J. Gess-Newsome & N. G. Lederman (Eds.). *Examining pedagogical content knowledge: The construct and its implications for science education* (pp. 21-50). Boston: Kluwer

Murphy, C. (2012). The role of subject knowledge in primary prospective teachers' approaches to teaching the topic of area. *Journal of Mathematics Teacher Education*, *15*(3), 187-206.

National Council of Teachers of Mathematics, (2000). Principles and Standards for School Mathematics. Reston, VA: NCTM.

National Council of Teachers of Mathematics, (2014). Principles to actions: Ensuring mathematical success for all. Reston, VA: NCTM

Olivier, A. 1989. Handling pupils' misconceptions. *Pythagoras 21,* 9-19.

Orhan, N. (2013). An investigation of private middle school students' common errors in the domain of area and perimeter and the relationship between their geometry self-efficacy beliefs and basic procedural and conceptual knowledge of area and perimeter. (Unpublished master thesis). Middle East Technical University, Turkey.

Outhred, L., & Mitchelmore, M. C. (2000). Young children's intuitive understanding of rectangular area measurement. *Journal for Research in Mathematics Education, 31*, 144-167.

Ozcakır, B. (2013). *The effect of mathematics instruction supported by dynamic geometry activities on seventh grade students' achievement in area of quadrilaterals.* Unpublished Doctoral Dissertation, Middle East Technical University, Turkey.

Ozden, M., Usak, M., Ulker, R., & Sorgo, A. (2013). Effects of lesson preparation methods on prospective primary teachers pedagogical content knowledge. *Journal of Environmental Protection and Ecology 14*(3), 1432-1442.

Ozmantar. M. F. ve Bingölbali, E. (2009). Sınıf öğretmenleri ve matematiksel zorlukları. *Gaziantep Üniversitesi Sosyal Bilimler Dergisi*, *8*(2), 401-427.

Park, S., & Oliver, J. S. (2008). Revisiting the conceptualization of pedagogical content knowledge (PCK): PCK as a conceptual tool to understand teachers as professionals. *Research in Science Education*, *38*(3), 261-284.

Patton, M. (1990). *Qualitative evaluation and research methods* (pp. 169-186). Beverly Hills, CA: Sage

Patton, M. Q. (2014). *Qualitative research and evaluation methods* (4th ed.). Thousand Oaks, CA: Sage Publications, Inc.

Petrou, M. (2009). Adapting the knowledge quartet in the cypriot mathematics classroom. *CERME 6–working group 10*, 2020-2039.

Principles and Standarts for School Mathematics, (2000). Executive summary Yearbook. Reston,VA: National Council of Teachers of Mathematics.

Riccomini, P. J. (2005). Identification and remediation of systematic error patterns in subtraction. *Learning Disability Quarterly*, *28*(3), 233-242.

Rowland, T. (2005, January). The Knowledge Quartet: A tool for developing mathematics teaching. In *Conference of Finnish Mathematics and Science Education Research Association* (p. 11).

Rowland, T. (2009). *Developing primary mathematics teaching: Reflecting on practice with the Knowledge Quartet*. India: Sage.

Rowland, T., Huckstep, P., & Thwaites, A. (2003). The knowledge quartet. *Proceedings of the British Society for Research into Learning Mathematics*, *23*(3), 97-102.

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education*, *8*(3), 255-281.

Rowland, T., Jared, L. & Thwaites, A. (2011a, July). Triggers of contingency in mathematics teaching. In *35th Conference of the International Group for the Psychology of Mathematics Education Developing Mathematical Thinking*.

Rowland, T., Jared, L., & Thwaites, A. (2011b). Secondary mathematics teachers' content knowledge: The case of Heidi. In *Proceedings of the Seventh Congress of the European Society for Research in Mathematics Education* (pp. 2827-2837).

Rowland, T., & Turner, F. (2007). Developing and using the 'knowledge quartet': A framework for the observation of mathematics teaching. *The Mathematics Educator*. *10(*1), 107-124.

Rowland, T., & Turner, F. (2008). How shall we talk about 'subject knowledge' for mathematics teaching. *Proceedings of the British Society for Research into Learning Mathematics*, *28*(2), 91-96.

Savas, M. (2011). *Investigating pre-service science teachers' perceived technological pedagogical content knowledge regarding genetics.* Unpublished Master Thesis, Middle East Technical University, Turkey.

Shin, B. (2011, July). An Analysis Of Teachers'pedagogical Content Knowledge On Probability. In *35th Conference of the International Group for the Psychology of Mathematics Education Developing Mathematical Thinking*.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational researcher*, 4-14.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard educational review*, *57*(1), 1-23.

Shulman, L. S. (2004). Those who understand: Knowledge growth in teaching. In S. M. Wilson (Ed.), *Wisdom of practice: Essays on teaching, learning, and learning to teach* (pp. 189-215). San Francisco: Jossey-Bass.

Simbo, F. K. (1989). The effects of microteaching on student teachers' performance in the actual teaching practice classroom. *Educational Research,31*(3), 195-200.

Simsek, N. (2011). Investigation of pedagogical content knowledge of prospective mathematics teachers related to perimeter and area topics in the context of students' difficulties Unpublished master's thesis, Gazi University, Turkey.

Smith, J. P., diSessa, A. A., & Roschelle, J. (1993). Misconceptions reconceived: a constructivist analysis of knowledge in transition. *The Journal of the Learning Sciences 3*(2), 115–163.

Smith, D., Eggen, M., & Andre, R. S. (1993). A transition. *The Journal Of The Learning Sciences*, *3*(2), 115-163.

Stephan, M., & Clements, D. H. (2003). Linear and area measurement in prekindergarten to grade 2. *Learning and teaching measurement*, 3-16.

Swan, M. (2001). Dealing with Misconceptions in Mathematics. in Gates, P. (Ed.). *Issues in mathematics teaching* (pp. 147-165). Roulledge Palmer: London.

Tamir, P. (1988). Subject matter and related pedagogical knowledge in teacher education. *Teaching and Teacher Education, 4*(2), 99-110.

Tan- Sisman, G. (2010). Sixt grade students' conceptual and procedural knowledge and word problem solving skills in length, area, and volume measurement. Unpublished doctoral dissertation, Middle East Technical University, Turkey.

Tan- Sisman, G., & Aksu, M. (2009). Seventh grade students' success on the topic of area and perimeter. *Elementary Online Education, 8*(1), 243-253

Tan- Sisman, G., & Aksu, M. (2012). Sixth Grade Students' Performance on Length, Area, and Volume Measurement. *Education & Science/Egitim ve Bilim*, *37*(166).

Tekin-Striva, R. (2014). An investigation into middle school mathematics teachers" subject matter knowledge and pedagogical content knowledge regarding the volume of 3D solids. Unpublished doctoral dissertation, Middle East Technical University, Turkey.

Thwaites, A., Huckstep, P., & Rowland, T. (2005). The knowledge quartet: Sonia's reflections. In *Proceedings of the sixth British Congress of Mathematics Education held at the University of Warwick* (pp. 168-175).

Thwaites, A., Jared, L., & Rowland, T. (2011). Analysing secondary mathematics teaching with the Knowledge Quartet. *Research in Mathematics Education*, *13*(2), 227-228.

Tirosh, D. (2000). Enhancing prospective teacher' knowledge of children's conceptions: The case of division of fractions. *Journal for Research in Mathematics Education, 31*(1), 5-25.

Troisi, N. F. (1983). *Effective Teaching and Student Achievement*. Publications, National Association of Secondary School Principals, 1904 Association Drive, Reston, VA 22091.

Turkdogan, A., Baki, A., & Cepni, S. (2009). The Anatomy of Mistakes: Categorizing Students' Mistakes in Mathematics within Learning Theories. *Turkish Journal of Computer and Mathematics Education (TURCOMAT)*, *1*(1).

Turner, F. (2005) "I woundn't do it that way': trainee teachers reaction to observations of their own teaching. Hewitt, D. (Ed.) *Proceedings of the British Society for Research into Learning Mathematics, 25* (3), pp. 87-92.

Turner, F. (2007). Beginning teachers' use of representations. *Proceedings of the British Society for Research into Learning Mathematics, November 2007*, 102-107.

Turner, F. (2009a). Developing the Ability to Respond to the Unexpected. *Informal Proceedings of the British Society for Research into Learning Mathematics. Paper presented in Cambridge*.

Turner, F. (2009b) Kate's conceptions of mathematics teaching: Influences in the first three years. *CERME 6–Working Group 10*, 1861.

Turner, F. (2011, July). Mathematical content knowledge revealed through the foundation dimension of the KQ. In *35th Conference of the International Group for the Psychology of Mathematics Education Developing Mathematical Thinking.*

Turner, F., & Rowland, T. (2008). The knowledge quartet: A means of developing and deepening mathematical knowledge in teaching. In *Mathematical Knowledge in Teaching Seminar Series: Developing and Deepening Mathematical Knowledge in Teaching, Loughborough University, Loughborough. Retrieved on January* (Vol. 30,p. 2013).

Turnuklu, E. B., & Yesildere, S. (2007). The pedagogical content knowledge in mathematics: Preservice primary mathematics teachers' perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers,* 1.

Yemen-Karpuzcu, S., Işıksal-Bostan, M., & Ayan, R. (2013, May). İlköğretim matemtik öğretmen adaylarının tam sayılarda çıkarma işleminde öğrenci hatalarını gidermeye yönelik öğretimsel açıklamaları. In A. Bülbül (Chair), *12. matematik sempozyumu*. Symposium conducted at the meeting of MATDER, Ankara.

Yeo, J. K. K. (2008). Teaching area and perimeter: Mathematics-pedagogical-content knowledge-in-action. Peper presented at *the 31st Annual Conference of the Mathematis Education Reseach Group of ustralasia Incorporated* (MERGA, 2008).

Yin, R. K. (2014). *Case study research: Design and methods*. Sage publications.

Yew, W. T., Zamri, S. N. A. S., & Lian, L. H. (2010). Examining Preservice Teachers' Knowledge of Area Formulae. *Procedia-Social and Behavioral Sciences*, *8*, 198-206.

Zacharos, K. (2006). Prevailing educational practices for area measurement and students' failure in measuring areas. *Journal of Mathematical Behaviour, 25,* 224–239.

Zembat, İ. Ö. (2015). Kavram yanılgısı nedir? In M. F. Özmantar, E. Bingolbali, H. Akkoç (Eds.), *Matematiksel kavram yanılgıları ve çözüm önerileri* (4th ed.). (pp. 1-8). Ankara: Pegem Akademi

Zhou, W. (2012). *Dimensions and levels of students' understanding of area measurement.* (Doctoral Dissertation). Vanderbilt University, USA.

APPENDICES

A. PRE-INTERVIEW PROTOCOL (IN ENGLISH)

I am Aslı BİLİK. I am a graduate student from the Department of Elementary Science and Mathematics Education at METU. The purpose of this study was to investigate pre-service middle school mathematics teachers' pedagogical content knowledge on the concept of the area of triangles. The purpose of this pre-interview was to clarify your thoughts on your prepared lesson plans regarding altitude of triangles/area of triangles. If you feel uncomfortable for any reason during this interview, you have a right to end at that moment. The interview was supposed to last about half an hour. If it is not a problem for you, I want to record this interview with a video camera. Thank you for your participation.

QUESTIONS

- What did you take into consideration while preparing the lesson plan associated with the altitude of the triangle/ the area of the triangle? / Which points do you focus on while planning?
- How did planning the course in that way come to your mind? What were your sources?
- What did you presented in your plan?
- What can be students prior knowledge regarding altitude of the triangle/ the area of the triangle?
- What kind of prerequisite knowledge do you think students need in order to learn altitude of the triangle/ the area of the triangle topics?

If pre-service teacher includes anything in her plan regarding possible students' misconception/difficulty;

- Why did you prefer to include these misconceptions/difficulties that students may have regarding altitude of the triangle/ the area of the triangle in your plan? / In what way can it be useful for you as a teacher?
- Where did you encounter these misconceptions/difficulties? / How did they come to your mind?
- In what way do you think can altitude of the triangle/ the area of the triangle be difficult to teach? / What are the factors making teaching it difficult?
- What are the difficulties related to teaching altitude of the triangle/ the area of the triangle topic?
- Which difficulties/misconceptions may students have while learning altitude of the triangle/ the area of the triangle topics?

Ask following about the prepared questions/activities;

- Why did you prefer to ask such a question to the students? / What was your purpose in asking them?
- Why did you plan to emphasize this knowledge?
- Which error(s) do you think the students will make in answering this question?
- *(if difficulty/misconception stated)* Which misconceptions/difficulties do you think the students may have in answering this question?
    - How do you come to know this difficulty/misconception? / How did this difficulty/misconception come to your mind?
    - What can be the reason behind the students' erroneous answer?
    - Can you explain, what could be the student thinking when s/he solving the question?
    - Which instructional strategies did you consider to use to overcome students' this misconceptions/difficulties during your instruction?

B. PRE-INTERVIEW PROTOCOL (IN TURKISH)

Merhaba, benim adım Aslı Bilik. Orta Doğu Teknik Üniversitesi Eğitim Fakültesi İlköğretim Bölümü, İlköğretim Fen ve Matematik Eğitimi Programında yüksek lisans öğrencisiyim. Bu çalışmanın amacı ilköğretim matematik öğretmen adaylarının üçgenin alanı konusuna ilişkin alan öğretimi bilgilerini incelemektir. Hazırlamış olduğun ders planıyla ilgili bilgi amaçlı bir görüşme yapmak istiyorum. Görüşmeler esnasında herhangi bir nedenden ötürü kendini rahatsız hissedersen görüşmeyi sona erdirmede serbestsin. Mülakat yaklaşık 30 dakika sürecektir. Senin için bir sorun olmuyorsa görüşmeleri video kaydına almak istiyorum. Katılımın için şimdiden çok teşekkür ederim

SORULAR

- Ders planını hazırlarken üçgenin alanı/üçgenin yüksekliği ile ilgili neleri göz önünde bulundurdun? Planı hazırlarken nelere odaklandın?
- Bu şekilde planı hazırlamak nasıl aklına geldi? Hangi kaynaklardan yararlandın?
- Ders planında nelere yer verdin?
- Öğrencilerin üçgenin alanı/üçgenin yüksekliği ile ilgili ön bilgilerinin neler olabileceğini düşünüyorsun?
- Öğrencilerin üçgenin alanı/üçgenin yüksekliği konularını öğrenmek için hangi tür bilgilere ihtiyacı olduğunu düşünüyorsun?

Eğer öğretmen adayı ders planında herhangi bir öğrenci zorluğuna/kavram yanılgısına yer verdiyse;

- Neden öğrencilerin üçgenin alanı ve üçgenin yüksekliği konularındaki sahip olabilecekleri bu zorluklara/kavram yanılgılarına ders planında yer vermek istedin?
- Bir öğretmen olarak ders planında öğrencilerin kavram yanılgılarına/zorluklarına yer vermek, hangi açıdan etkili olacaktır?
- Bu öğrenci zorluklarıyla/kavram yanılgılarıyla nerede karşılaştın? Bu öğrenci zorlukları/kavram yanılgıları nasıl aklına geldi?
- Hangi yönlerden üçgenin yüksekliği/ üçgenin alanı konularını anlatmak zordur? Bu konuları öğretmeyi zorlaştıran faktörler neler?
- Öğrenci üçgenin yüksekliği/üçgenin alanı konusunu öğrenirken hangi zorluklara/kavram yanılgılarına sahip olabilir?

Ders planında yer alan hazırlanmış sorular/aktiviteler için şunları sor;

- Neden böyle bir soru sormayı tercih ettin? Bu soruyu sorma amacın ne?
- Neden bu düşünceyi vurgulamak istedin?
- Bu soruyu çözerken öğrenciler nasıl hatalar yapabilirler?
- *(Eğer zorluk/kavram yanılgısı ifadelerini kullandıysa)* Bu soruyu çözerken öğrenciler nasıl zorluk/kavram yanılgısı gösterebilirler?
    - Bu zorluğu/kavram yanılgısını nereden biliyorsun? Bu zorluk/kavram yanılgısı nasıl aklına geldi?
    - Öğrencinin bu hatalı cevabının altında yatan sebep ne olabilir?
    - Öğrenci bu soruyu cevaplarken ne gibi düşüncelere sahip olabilir?
    - Öğrencilerin bu zorluğunu/kavram yanılgısını ortadan kaldırmak için hangi öğretimsel stratejiyi kullanırsın?

C. POST-INTERVIEW PROTOCOL (IN ENGLISH)

The purpose of this pre-interview was to clarify your thoughts, decisions and behaviors during practice teaching that I had observed. If you feel uncomfortable for any reason during this interview, you have a right to end at that moment. The interview was supposed to last about half an hour. If it is not a problem for you, I want to record this interview with a video camera. Thank you for your participation.

QUESTIONS

- What do you think about the lesson that you teach?
- Is there any difference between the lesson that you planned and the lesson that you taught, can you explain them? If yes, why did these differences occur?
- During pre-interview you said that, you would apply this question/activity. However, you did not. Why? Can you explain.
  - Which factors were affected your decisions on this change? How did they cause to this?
- Did you experience any difficulty during practice teaching?

Regarding the situations occurred during practice teaching;

- While student was answering this question what were you thinking?
- What do you think about how the student found this erroneous answer? What could be the student's thoughts while s/he was saying/solving this?
- What can be the reason behind the erroneous answer of the student?
  - Why the student might have this difficulty/misconception at that point?
- Do you think that the strategy you employed to overcome students' errors was influential for students' understanding? Can you explain your thought.

  - Did you experience any difficulty in responding to students' questions or in eliminating misconceptions/difficulties held by students?
  - What were you thinking while doing …?
  - Why did you employ this strategy? In which way it helps you to eliminate students' misconception/difficulty?

- Now, when you think about the possible students' misconceptions/difficulties, which misconceptions/difficulties may the students have regarding the area of triangles/altitude of triangles?
- Which of these misconceptions/difficulties did you encounter during your teaching practice?
- Was there any misconception/difficulty that you encountered for the first time during your teaching? Can you explain?
  - In what way do you think will the knowledge of this misconception/difficulty affect your teaching on this topic?
  - If you were given another chance to eliminate the students' this misconception/difficulty, will you change your educational strategy employed? What are these changes?

D. POST-INTERVIEW PROTOCOL (IN TURKISH)

Bu çalışmanın amacı ilköğretim matematik öğretmen adaylarının üçgenin alanı konusuna ilişkin alan öğretimi bilgilerini incelemektir. Gözlemlemiş olduğum ders anlatımlarınla ilgili, ders esnasında gerçekleşen durumları ve senin düşüncelerini açıklığa kavuşturmak amacıyla bu görüşmeyi düzenledim. Görüşmeler esnasında herhangi bir nedenden ötürü kendini rahatsız hissedersen görüşmeyi sona erdirmede serbestsin. Mülakat yaklaşık 30 dakika sürecektir. Senin için bir sorun teşkil etmiyorsa görüşmeleri video kaydına almak istiyorum. Katılımın için şimdiden çok teşekkür ederim.

SORULAR

- Anlattığın ders ile ilgili ne düşünüyorsun?
- Planladığın ders ile uyguladığın ders arasında hiç farklılık var mı? Eğer varsa, neden bu farklılıklar ortaya çıktı?
- Yapılan ön görüşmede bu soruyu soracağını/bu aktiviteyi uygulayacağını söylemiştin, ancak sormadın/uygulamadın. Sebebini açıklar mısın?
  - Bu değişimde etkili olan faktörler neler? Bunu yapmana nasıl sebep oldu?
- Ders anlatımı sırasında herhangi bir zorluk yaşadın mı?

Ders esnasında olan olaylarla ilgili;

- Öğrenci bu soruyu cevaplarken bununla ilgili ne düşündün?
- Öğrencinin bu hatalı cevabı nasıl bulduğunu düşünüyorsun? Öğrenci bu soruyu çözerken ne düşünmüş olabilir?
- Öğrencinin bu hatalı cevabının altında yatan sebep ne olabilir?
  - Öğrenci neden bu noktada kavram yanılgısı/zorluğa sahip?

- Öğrencilerin bu kavram yanılgısını/zorluğunu yok etmek için kullandığın yöntem etkili oldu mu sence? Bu konudaki düşüncelerini açıklar mısın?
    - Öğrencilerin bu zorluklarını/kavram yanılgılarını ortadan kaldırmaya çalışırken hiç zorluk yaşadın mı? Açıklar mısın?
- ….. yaparken ne düşünüyordun?
- Neden bu öğretimsel stratejiyi kullandın? Öğrenci zorluğunu/kavram yanılgısını yok ederken nasıl yardım edebilir?
- Şimdi üçgenin alanı/üçgenin yüksekliği konusundaki öğrenci zorluklarını/kavram yanılgılarını düşündüğünde, öğrencilerin hangi zorluklara/kavram yanılgılarına sahip olduğunu düşünüyorsun?
- Bu zorluklardan/kavram yanılgılarından hangisiyle ders esnasında karşılaştın?
- İlk defa ders esnasında karşına çıkan bir öğrenci kavram yanılgısı/zorluğu oldu mu? Açıklar mısın?
    - Yeni tanıdığın bu kavram yanılgısının/zorluğun, hangi anlamda bu konudaki ders anlatımını etkileyeceğini düşünüyorsun?
    - Eğer sana ikinci bir şans verilseydi, öğrencilerin ders esnasında ortaya çıkardığı ve senin yeni tanıdığın bu zorluğu/kavram yanılgısını ortadan kaldırmak için uyguladığın öğretimsel stratejiyi değiştirir miydin? Evetse, bu değişiklikler neler olurdu?

E. OBSERVATION PROTOCOL (IN ENGLISH)

Pre-service Teacher: Date:

Subject/Objective:

The notewothy points to take into consideration;

- Students solved a question inaccurately because of a misconception/difficulty and the pre-service teacher did not notice the misconception/difficulty.

Explain:

- Students solved a question inaccurately because of a misconception/difficulty and the pre-service teacher noticed the misconception/difficulty

Explain:

- Students demonstrated different misconception/difficulty that was not specified by the pre-service teachers before the lesson and pre-service teacher did not notice the misconception/difficulty

Explain:

- Students demonstrated different misconception/difficulty that was not specified by the pre-service teachers before the lesson and pre-service teacher noticed the misconception/difficulty.

Explain:

- Pre-service teachers employ a strategy to eliminate students' misconceptions/difficulties.

Explain:

F. OBSERVATION PROTOCOL (IN TURKISH)

Öğretmen Adayı: Tarih:

Konu/Kazanım:

Dikkat edilmesi gereken önemli noktalar;

- Öğrenciler sahip oldukları bir kavram yanılgısı/zorluktan dolayı soruyu yanlış bir şekilde çözüyor ve öğretmen adayı bu hatayı/kavram yanılgısını fark edemiyor.

Açıkla:

- Öğrenciler sahip oldukları bir kavram yanılgısı/zorluktan dolayı soruyu yanlış bir şekilde çözüyor ve öğretmen adayı bu hatayı/kavram yanılgısını fark ediyor.

Açıkla:

- Öğrenci ders esnasında öğretmen adayının daha önceden belirtmediği bir kavram yanılgısı/zorluk gösteriyor ve öğretmen adayı bu hatayı/kavram yanılgısını fark edemiyor.

Açıkla:

- Öğrenci ders esnasında öğretmen adayının daha önceden belirtmediği bir kavram yanılgısı/zorluk gösteriyor ve öğretmen adayı bu hatayı/kavram yanılgısını fark ediyor.

Açıkla:

- Öğretmen adayı öğrencinin yanılgısını/kavram yanılgısını yok etmeye yönelik bir öğretimsel strateji kullanıyor.

Açıkla:

G. LESSON PLANS OF THE PRE-SERVICE TEACHERS

HATICE'S LESSON PLAN-1

Name: Hatice

Grade Level: 6

Topic: Altitude of triangle

Duration: 2 lesson hour

Prerequisite Knowledge and Skills: Students should be able to

- explain basic properties of a triangle.
- explain what perpendicularity is.

Objectives: Students should be able to

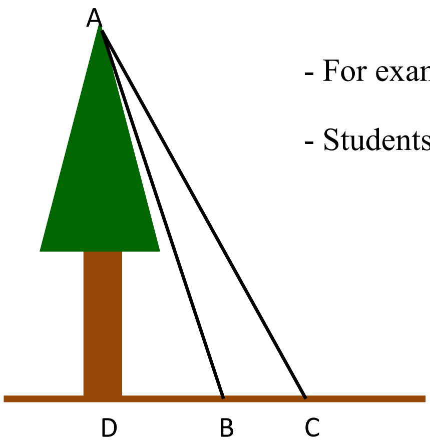
- draw altitude of any side of a triangle.

Materials: Activity sheet, protractor

Teaching Methods: Questioning, Discussion

<u>Starting</u>

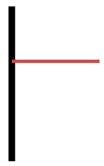
- Ask what comes to students' minds when they hear the word altitude.
- Draw a tree to the board and ask what they understand from altitude of this tree is 3 m. Want students to show which distance is 3 m.



- For example, ask IABI or IACI is 3 m.

- Students can probably say that IADI is 3 m.

- Then, ask altitude changing your base. For example, put your book such that it is vertical to the floor and put your pencil horizontally as in below:



- Then, ask what the altitude of that pencil according to the book?

- Students can say the vertical distance, but emphasize that they should look according to the book.

(The aim of above questions is to provide students feel that altitude is perpendicular line segment to any base. Since in real life it is generally asked to find altitudes of objects according to horizontal base, students can only think that altitude is vertical and then they could not understand how they can draw altitudes to all sides of the triangle which are not vertical in general.)

- Then, ask students whether they have learnt drawing altitude of any geometric shape.

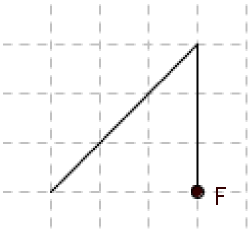
- Say that today they will learn drawing altitudes of another geometric shape.

- Distribute activity sheets to students. While distributing, say that "You will work individually and sometimes there will be discussions. I will say you what you should do."

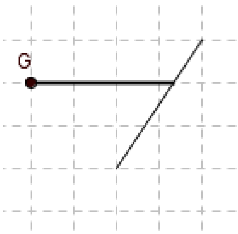
Middle

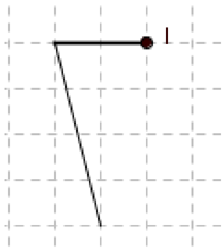
- The important thing to draw altitude of a triangle or any shape is drawing a perpendicular to a line from a point. Therefore, activity starts with drawing perpendicular to line segments from given points.

- Say students that firstly they will do the part A and they have 15 minutes.
- While students are working, observe students' works and determine what their common errors are. Then, according to these, decide the questions which will be asked.
- After 15 min, want from students making errors to show their drawings on the board.
- Students probably will not have difficulty in drawing perpendiculars to line segments in A, B and D. However, they may draw perpendicular wrong from C since the point is under the line segment. If students have difficulty in C, ask does it matter the point is under or above the line segment?
- In F, students can draw perpendicular as shown in the below picture



because they think that perpendicular will always be vertical.
- Then, find what the angles are between line segments and perpendiculars in A, B and D by using protractor. (Expected answer is 90°)
- Say that the angle between line segment and perpendicular should be 90°.
- Ask whether the angle between line segment and perpendicular in F is 90° or not.
- Students can easily realize their mistake since the angles are different.
- Discuss how to draw perpendicular and let them to try some and measure angles for each of them until they reach correct one.
- In G and I, students can draw the perpendiculars as shown in the below pictures

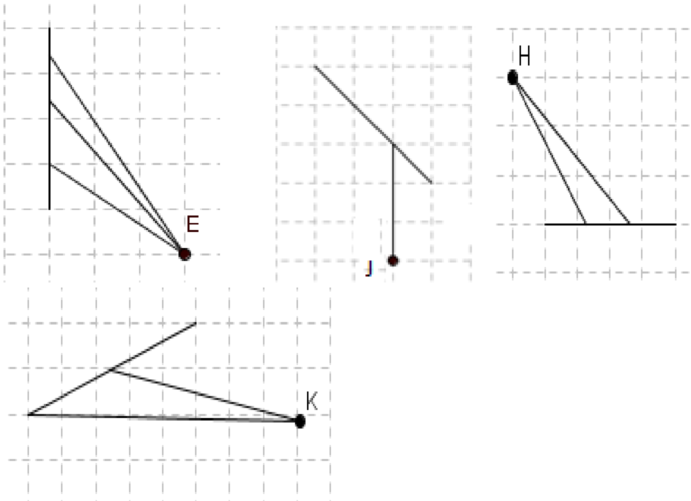




because they may think as in A, B and D that is perpendicular can be drawn only vertical or horizontal.

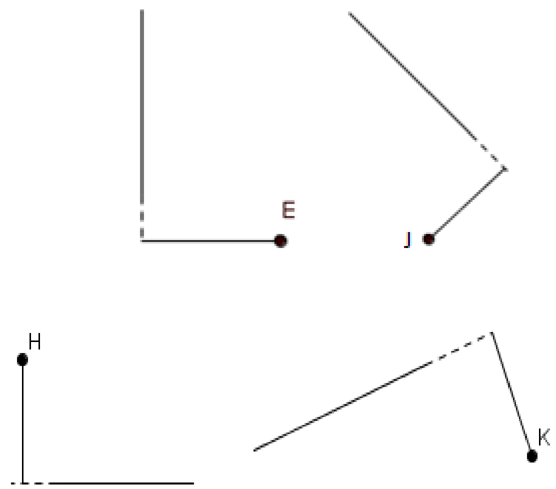
- Ask how they can determine whether their drawings are correct or not.
- If students cannot remember what they should do, say what they did to control F.

- Ask same questions in F for G and I.
- Ask students what they learnt from examples A, C, D, F, G and I. (Expected answers: both line segment and perpendiculars need not to be only vertical or horizontal. The important thing at this point is angle between them should be 90°.)
- Summarize these key points after student say their thinking and want students to write these on their sheets under part A.
- In E, H, K and J, students can draw the perpendiculars as shown in the below pictures



since they think the perpendicular must not be outside of the line segment also in J they can think perpendicular should be vertical.

- Want students to measure the angles between perpendiculars and line segments. Some students may realize the angles are not 90° before saying this statement by remembering the earlier examples.
- Say students to find perpendicular by trying as before, but most of them cannot find because they always try to draw on the line segments.
- Then, ask that if the line segments were a little longer, what their answer would be.
- After they found answer, say that if we cannot draw a perpendicular to a point on the line segment, we can draw extension of the line segment to draw a perpendicular as shown in the picture below and then draw perpendicular.



- Ask students what they have learnt from the examples E, H, K and J. (Expected answer: Perpendicular can also be drawn outside of the line segment.
- Summarize this key point after students' answers and want students to write it under part A.
- Want students to draw correct perpendiculars onto the grid paper in part B. (5 min)
- Want them to combine the point with the end points of the line segment with colorful pencils.
- Ask the following questions:
  1) What is the formed shape? (triangle)
  2) Which element of triangle can perpendicular of the line segment be? (altitude of the triangle)
     - If students cannot say that element is altitude, want students to remember the tree at the beginning of the lesson or pencil and to think what the perpendicular was which is 3 meters long.
  3) Looking your triangles, what can you say about the altitude of a triangle?
  4) Is it always vertical?
  5) Is it always inside the triangle?
     - After students say no, ask where altitude can be (outside or a side of triangle)
     Then, ask when it is outside of the triangle and when it is a side of the triangle. Want students to write answers to their sheets.
  6) Is it possible to draw altitude to all sides of the triangle?
     - After students say yes, want students to remember the book-pencil example at the beginning of the lesson and say that our book can be any side of triangle and we find altitude of pencil according to the book.

- After these questions, ask that “Looking all the things up to now, how do you define the altitude and what are the properties of altitudes in a triangle?” (Expected answer is altitude is perpendicular drawn from a vertex of triangle to the side opposite of that vertex. Therefore, we can draw altitudes to all sides in a triangle and they need not to be vertical. Also, they can be in outside of the triangle or can be a side of the triangle.)

- Students can have difficulty in defining altitude. In this case, show some triangles they formed and ask what they drew from point A, K and G.

- After students tried making definition, summarize their thoughts and want students to write definition and properties on their sheets.

- Then, say that you will work on the questions in part C. (20 min)

- Observe students while they are working. Ask how they drew altitudes.

- After 20 min, first ask students who drew altitudes wrong. Then, give permission to others to correct their friends.
- In part C, students are expected to write; for example, [AC] $\perp$ [CD]. If students always write [AC] is perpendicular to [CD], ash which symbol we use to say that two line or line segments are perpendicular to each other.
- Do this part until there are 5 minutes. If it does not finish, take part C from students to see whether students learnt the topic or not.

End

- Ask students what they learnt from this lesson about altitude of triangle.
- After students’ answers, teacher summarizes all things they learnt.

Assessment

- Observe students in part A to detect their misconceptions about drawing perpendicular to a line from a point.

- Observe them while they are working on part C and showing their answers to the part C on the board to see whether they have learnt drawing altitudes of triangle.

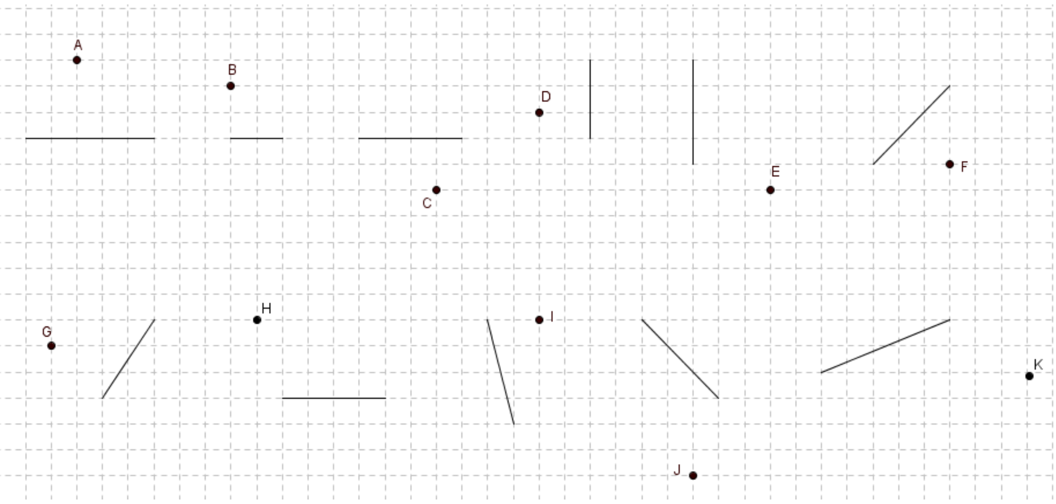
- Use anecdotal notes to record your observations.

- If part C does not finish in the lesson, take them and check students’ answers.

Etkinlik Kağıdı

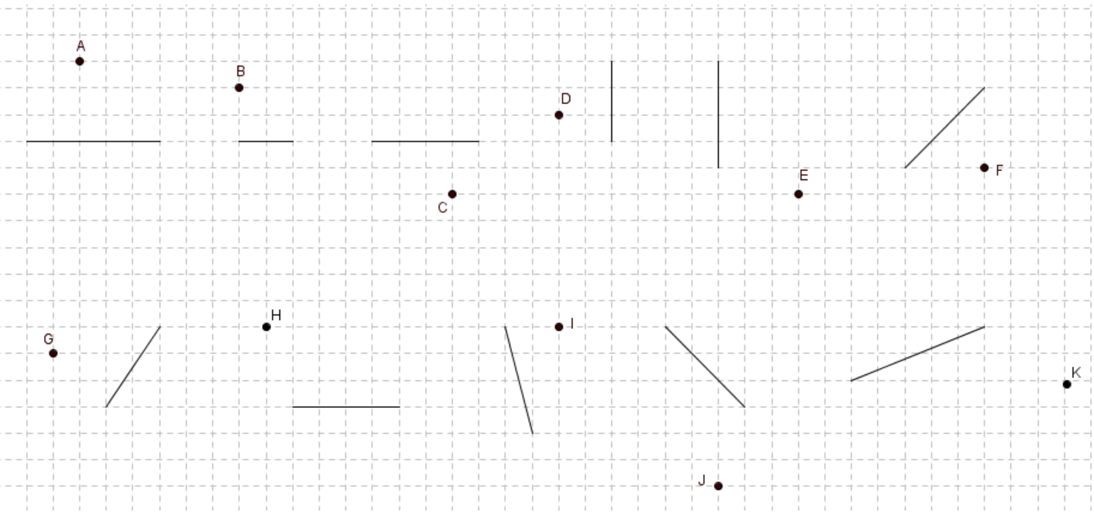
Bölüm A

Aşağıdaki doğru parçalarına verilen noktalardan dikme çizin.



Bölüm B

Öncelikle aşağıdaki doğru parçalarına verilen noktalardan doğru dikmeleri çizin. (A bölümüne göre) Daha sonra renkli kalemlerle verilen noktayı doğru parçasının uç noktaları ile birleştirin ve soruları cevaplayın.



Sorular

1) Doğru parçasına çizilen dikme üçgenin hangi elemanıdır?

……………………………………………………………………………………………

2) Üçgenleri incelediğinizde üçgenin yüksekliği hakkında ne söyleyebilirsiniz?

- Her zaman dik bir doğru parçası mıdır?

……………………………………………………………………………………………

- Her zaman üçgenin içinde midir?

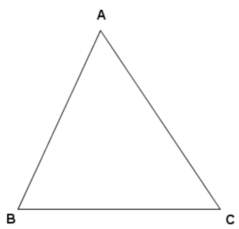
……………………………………………………………………………………………

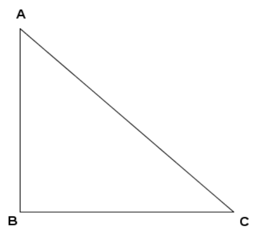
- Üçgenin bütün kenarlarına yükseklik çizilebilir mi?

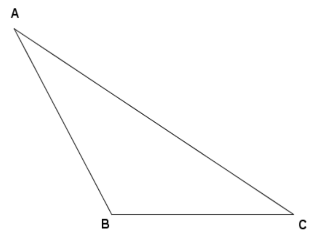
……………………………………………………………………………………………

Bölüm C

Aşağıdaki üçgenlerin bütün yüksekliklerini çizin ve isimlendirin. Yüksekliklerin hangi kenara dik olduğunu belirleyin ve verilen boşluklara yazın.(Gerekli olursa iletkinizi kullanabilirsiniz)

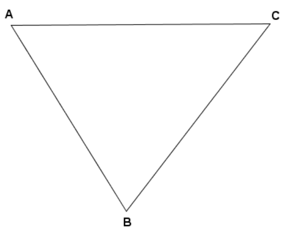


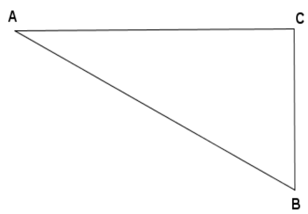


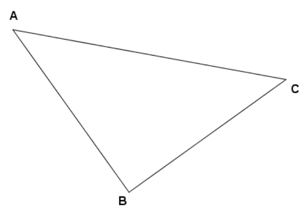


………………… ……………………… …………………

………………… ……………………. …………………







………………… …………………… ………………………

………………… …………………… ………………………

HATICE'S LESSON PLAN-2

Name: Hatice

Grade Level: 6

Topic: Area of triangle

Duration: 2 lesson hour

Prerequisite Knowledge and Skills: Students should be able to

- draw altitude of any side of a parallelogram.
- form area formula of parallelogram and solve problems about it.

Objectives: Students should be able to

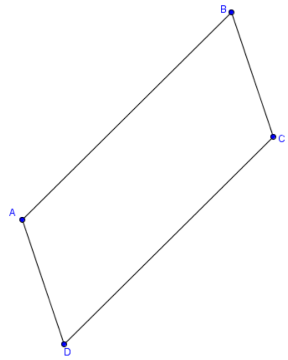
- form area formula of a triangle and solve problems about it.

Materials: Activity sheet, triangles, ruler, protractor, glue

Teaching Methods: Questioning, Discussion

<u>Starting</u>

- Start lesson with asking questions about previous lessons.
- Ask what they have learnt in the last lesson. They should say that they learnt drawing altitudes of triangle.
- Want one of them to come to the board and draw a triangle and altitudes of it.
- If student draws acute triangle, draw a right and an obtuse triangle and ask altitudes of some of the sides. Want some students to show them and ask others whether their friends' answers are correct or not.
- Ask what they have learnt about parallelogram. Students should say that they learnt drawing altitudes of a parallelogram and finding area of it.
- Draw the below parallelogram to the board and want students to draw altitudes of [AD] and [DC], and then say the area of the parallelogram. If students cannot draw, ask what altitude was and say them to remember how they draw altitudes of a triangle.



- Ask where students see triangles in their lives and then show some pictures.

- Say that since triangles are important for us, we will learn some more things about them.

- Say that to learn more things about triangles, they will do an activity.

- Distribute activity sheets and triangles to the students. Each student will get six same triangles.

- While distributing materials, say, "You will work individually during the activity. We will start together and I will say you what you should do."

<u>Middle</u>

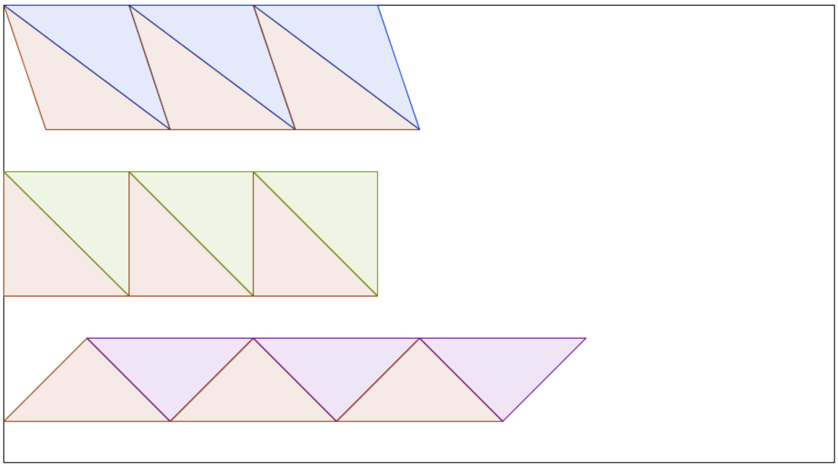
- After distributing materials, start reading the introduction part of the activity. Then, read the first question.

- Ask students what they should do.

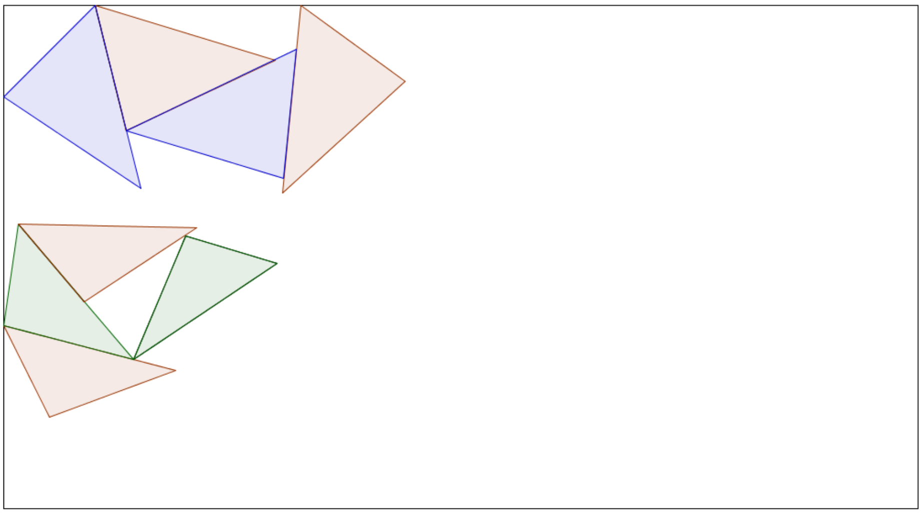
- Getting answer from few students, say, "As your friends said, you are a wall tile master and you will cover the wall in your sheets with the tiles. Tiles are the triangles in your hands. Now, you should think a little about how to arrange your tiles. Then, I will take your opinions and then you will decide how to put the tiles and glue them."

- Observe students while they are thinking. After about 5 minutes, take students' opinions.

- Expect students to arrange tiles in such a way:



- Observe students that arrange tiles in such a way that there are so many gaps between tiles or they did not form a pattern with tiles as in below.



Firstly, ask those students' opinions. Then, ask class what they think about their friends' opinions. If they say that they can make such an arrangement, ask "Are tiles that you see in your homes or in other places arranged like that? Did you see any gap between them?" If they say that, they cannot do like that, give permission to a student who arranged tiles so that two of the triangles formed a parallelogram. Ask why they cannot arrange tiles as their friends said and then want to show his arrangement.

- After all students agree with those students who formed a parallelogram, want others to arrange their tiles in that way and want to glue them to their sheets.

- If students say that there are gaps in sides of the wall, say them that those gaps will be filled last according to your covering.

After students finished the first question, move to the second one.

- Say that "You would also cover the floor, but your tiles finished. How do you determine the number of tiles needed for the floor?" Give students 3 minutes to think. Observe students while they are thinking.

- Expect students to say that if they know the area of a tile, they can find how many tiles are needed.

- Get students' answers after about 3 minutes. After each student's opinion, ask class what they think about that idea. Then, ask whether there are any different ideas.

- If students cannot come up with an idea, ask that "What should you know about these triangle tiles to find the number of tiles?"

- Students can say that we should know how much place those triangles cover. In this case, ask "How can we express the place that triangles cover in another word? Did we learn finding place that some other shapes cover before? If they could not answer, draw a square to the board and ask how much place that square cover?"

- Students can see the parallelogram that two triangles form and can use area of parallelogram to find the number of tiles. If such a case happens, ask how triangle tiles are related with those parallelograms. After they say that two of them make one parallelogram, ask whether they can use also area of triangle tiles to reach the number of tiles.

- After students agree with they need to find the area of tiles, want them to write the answer to their sheets and then want them to glue their activity sheets to their notebooks. Say that we will continue with our notebooks. (First lesson will probably finish here. Say that they will find the area of the triangle in the second lesson.)

- Say that "Now, think how you can find area of your triangle tile. You have a ruler and protractor and you can also get help from the wall that you covered in the first question. You can draw your triangles to your notebooks or you can work with ones that you glued. After 10 minutes, you will share your reasoning and findings with class."

- Expect students to find the area of triangle using area of parallelogram. They know how to find area of parallelogram. They should draw altitudes of it using their protractors and after measuring length of a side and corresponding altitude, they will find area of parallelogram. They should say that since area of parallelogram is equal to two times area of triangle, we should divide area of parallelogram by 2 to get area of the triangle.

- Move around the class while students are trying to find the area of triangle.

- If students cannot see getting help from the area of parallelogram, ask "Do you see a shape in your wall which is familiar to you? Remember the previous lessons." When students see the parallelogram, ask "Can you use that parallelogram to find area of the triangle?"

- If students find area of the triangle, first ask their reasoning. Then, draw some other triangles; for instance, if they found area of acute triangle, draw right and obtuse triangle, to their notebooks and ask how they can find area of those triangles. The aim of this question is to provide students to reach the formula of the area of triangle.

- After 10 minutes, call students who found the area of acute, obtuse and right triangles to the board and explain their reasoning and solutions respectively. Also, want others to write them on their notebooks. Then, ask others whether they agree with them and whether there is different reasoning or not.

- Ask class "Which element of the triangle is altitude of the parallelogram?" showing three triangles on the board. (Expected answer is altitude of the triangle) Then, ask sides that correspond to those altitudes in each triangle.

- Ask that "So, how did you find the area of those triangles?"

- If they could not answer, say that you are finding area of parallelogram, multiplying length of side and altitude that corresponds to that side, and then ask what they can say about the area of triangle.

- Expect students to say that they should multiply length of side and altitude that corresponds to that side and then should divide the result by 2.

- Getting answer from few students, say that to find the area of a triangle, draw a triangle to the board, they should multiply length of side and corresponding altitude and then should divide by 2 as their friends said and write

Area of triangle= $\frac{\text{Length of side x Corresponding altitude}}{2}$ and want students to write it on their notebooks. Also, they will write under this equation to find the area of a triangle multiply length of side and corresponding altitude and then divide by 2.

- If you have time, pass to the problems. If you can, project each problem to the board respectively and want students to write them on their notebooks. If you cannot, read them aloud and want from students to write.

- Give 3 minutes to students to solve the problem.

- After 3 minutes, take students' answers. Firstly, choose student who gave a wrong answer and want to explain his reasoning to detect his mistake. Then, want from a student giving correct answer to explain his solution.

- Continue solving questions until there are 7 minutes to the break. Want students to write the other questions on their notebooks. Those will be their homework.

- If you cannot pass problems, distribute them as homework.

<u>End</u>

- Ask students what they have learnt today and how they learnt it.

- After students, summarize the lesson.

Assessment

- Observe students while they are trying to find area of triangle.

- Observe students while they solve problems to check whether they understood how to find area of triangle.

- Use anecdotal notes to record your observations.

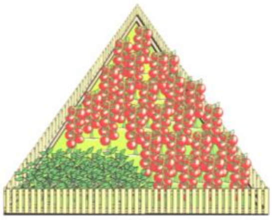
- If you give problems as homework, observe students next lesson whether they could solve the problems or not.

Pictures





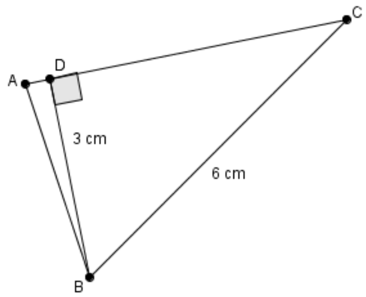

PROBLEMS



1) Üçgen şeklindeki bir bahçenin kenarlarından biri 25 m ve bu kenara ait yükseklik ise 16 m'dir. Bu bahçenin $\frac{1}{4}$ 'üne biber, geri kalan kısmına ise domates fidesi dikilmiştir. Biber ve domates fidesi dikilen bölgelerin kaçar metrekare olduğunu bulunuz.

2) Bir hediye kutusunun tabanı dik üçgen biçimindedir. Bu kutunu taban alanı 54 cm^2 ve dik kenarlardan birinin uzunluğu 9 cm'dir. Kutunun diğer dik kenar uzunluğu kaç cm'dir?

3) Ahmet yanda verilen üçgeninin alanını 9 cm^2 bulmuştur fakat cevabından emin değildir. Ahmet'in cevabı hakkında ne düşünüyorsunuz? Açıklayın.



4) Bir apartman dairesinin balkonunun zemini üçgen şeklindedir. Balkonun zemininin bir kenarının uzunluğu 5 m'dir. Bu balkonun zemini için toplam 5 m^2 karo taşı kullanılmıştır. Buna göre 5 m'lik kenara ait yükseklik kaç m'dir?

ETKİNLİK KAĞIDI

Ne kadar fayans gerekli?

Yeni yapılan bir evin banyosuna fayans ustası olarak size verilen üçgen şeklindeki fayansları döşemeniz istenmektedir. Bütün duvarlar ve yer bu fayanslar ile döşenecektir.

1) Aşağıda döşeme yapılacak olan duvar verilmiştir. Fayansları nasıl döşeyeceğinizi düşünün. Döşeme işleminin nasıl olacağına karar verdikten sonra fayanslarınızı duvara yapıştırın. (Kenarda kalan küçük boşluklar daha sonra döşemelere uygun olarak doldurulacaktır.)

2) Duvarları bitirdikten sonra son olarak yere yapılacak olan döşemeniz kaldı fakat elinizdeki fayans bitti. Yere döşeyeceğiniz fayans miktarını nasıl belirlersiniz? Açıklayın.

EDA'S LESSON PLAN-1

Dersin Adı: Matematik

Sınıf: 6. sınıf

Ünitenin Adı: Geometri ve Ölçme

Konu: Üçgende Yükseklik Çizme

Süre: 2 ders saati

Öğrenci Kazanımları ve Hedef Davranışları:

1) Üçgende bir kenara ait yüksekliği çizer.

2) Geniş açılı üçgenlerdeki yüksekliği inceler

Ünite Kavramları ve Sembolleri:

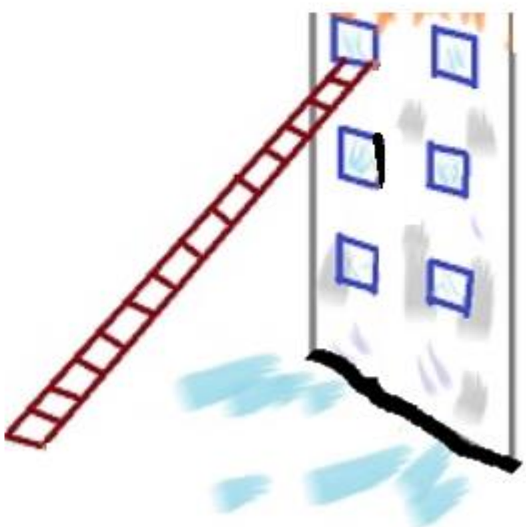
Yükseklik: Bir üçgenin bir köşesinden karşı kenarına indirilen dik doğru parçasına üçgenin o kenara ait yüksekliği denir.

Öğretme-Öğrenme Yöntem ve Teknikleri: Sunuş, Soru-Cevap, Beyin Fırtınası

Kullanılan Araç ve Gereçler: Gönye, Cetvel, Noktalı kağıt.

Öğretme- Öğrenme Etkinlikleri:

Dikkat çekme:



Şekildeki binanın 3. katına bir merdiven dayanmıştır. Merdivenin penceredeki ucunun yerden yüksekliğini çizerek gösteriniz.

Boyacı Rasim usta evinin duvarlarını boyamak istiyor. Yüksek yerlere boyu yetişmediğinden merdiven kullanma ihtiyacı duyan Rasim usta 1,5 m boyundaki merdivenle boyama işlemini bitiriyor. Merdivenin en üst basamağına çıktığında Rasim ustanın her bir basamakta yerden yüksekliğini şekil üzerinde çizerek gösterebilir misiniz?

Pisa (Piza) kulesi İtalya'da bulunmaktadır. Bu kulenin özelliği eğik olmasıdır. Pisa kulesinin yerden yüksekliğini şekil üzerinden gösteriniz.

<u>Derse Geçiş:</u>

Bireysel Öğrenme Etkinlikleri:

- Noktalı kağıtlar dağıtılır.
- Bir noktadan bir kenara dikme nasıl çizildiği hatırlatılır. önce buna dair birkaç örnek yapılır.
- Cetvel yardımıyla değişik üçgenler çizilmesi istenir. (üç çeşit üçgeni de içerecek şekilde)
- Öğretmen de aynı zamanda tahtaya üç çeşit üçgen çizer.
- Tahtada her üçgen üzerinden gönye kullanılarak bir kenara ait yüksekliğin karşı köşeden çizilebileceği gösterilir.
- Aynı işlemi noktalı kağıt üzerinden öğrencilerin de gönye kullanarak yapması beklenir.
- Özellikle geniş açılı üçgenler üzerinde çalışılır.

Güdüleme:

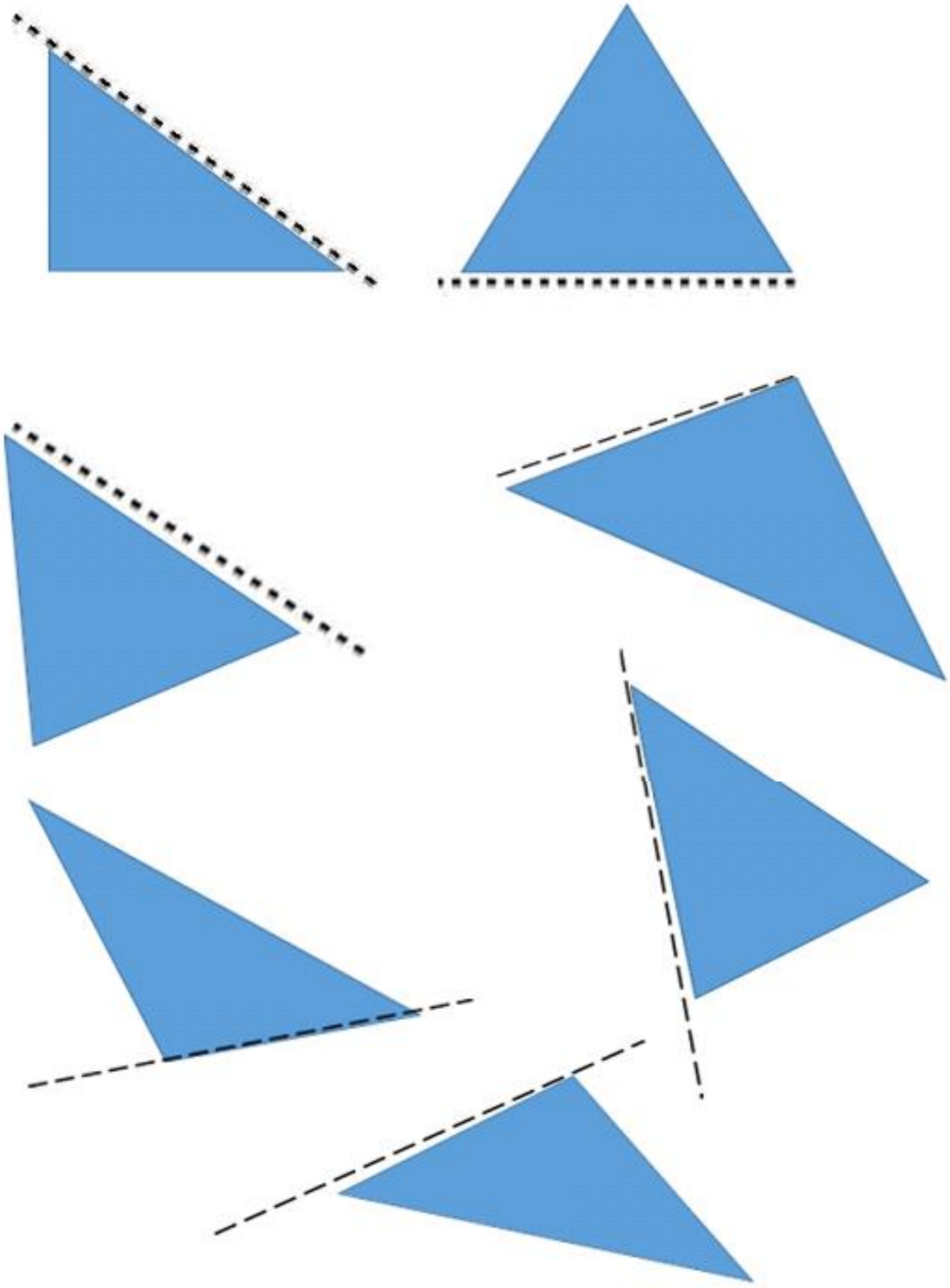
- Çizilen yüksekliklerin aralarındaki farkların tartışılması.
- "Hepsi yatay mı dikey mi yoksa farklı şekillerde mi?"diye sorulur.
- Öğrencilerin kağıtlarından her çeşitten bulunan yükseklikler gösterilir.
- yoksa tahtaya farklı üçgenler çizerek gösterilir.
- "Yükseklik her zaman üçgenin içinde mi olur? "diye sorulur.
- Dışında olan yükseklikler de gösterilir.
- Alıştırma yapmak için hazırlanan etkinlik kağıdının diğer sayfasına geçilir.
- Verilen üçgenler üzerinde yüksekliklerin gösterilmesi istenir.

Özet:

- Öğretmen çevremizdeki yüksekliklere örnek ister.
- bugün genel olarak ne öğrendikleri sorulur.
- Üçgende yüksekliğin tanımından bahsedilir.
- Yüksekliğin özelliklerinin üzerinden geçilir.
- Etkinlik kağıdının dosyalarına yerleştirilmesi istenir.

ETKİNLİK KAĞIDI

Noktalı çizgilerin olduğu kenarı taban kabul ederek o kenara ait yüksekliği çiziniz.



EDA'S LESSON PLAN-2

Dersin Adı: Matematik

Sınıf: 6. sınıf

Ünitenin Adı: Geometri ve Ölçme

Konu: Üçgende Alan Hesaplama

Süre: 2 ders saati

Öğrenci Kazanımları ve Hedef Davranışları:

1)Üçgenin alan bağıntısını oluşturur; ilgili problemleri çözer.

Ünite Kavramları ve Sembolleri:

Alan: Sınırlı kapalı yüzeylerin ölçüsü.

Birim kare: Her bir kenarı bir birim olan kareye birim kare adı verilmektedir.

Üçgenin Alanı: Bir üçgenin alanı, bir kenarın uzunluğu ile bu kenara ait yüksekliğin uzunluğunun çarpımının yarısına eşittir.

Üçgende Alan Formülü: (a x h) / 2

Öğretme-Öğrenme Yöntem ve Teknikleri: Sunuş, Soru-Cevap, Beyin Fırtınası

Kullanılan Araç ve Gereçler: Gönye, Cetvel, Kareli Kağıt, Makas, Dörtgen şeklindeki kartonlar.

Öğretme- Öğrenme Etkinlikleri:

Dikkat Çekme:

Ali'ye okuldaki tasarım dersi için maket bir yelkenli gemi yapmaya karar vermiştir. Şekildeki yelkenliyi yapmak isteyen Ali'nin üzerinde düşündüğü bir nokta vardır o da geminin yelkenleri için ne kadar bez kullanacağını bilememesidir. Geminin direk boyu 10 cm ve yelkenin alt tabanının bağlanacağı diğer yatay direk 3cm olduğuna göre Ali'ye ne kadar bez kullanacağı konusunda nasıl yardımcı olabilirsiniz?

Derse Geçiş:

- Öğrencilerin yukarıdaki soru ile alakalı görüşleri tartışılır.
- Tam sonuca gelmeden tekrar soruya dönmek üzere konu orda bırakılır.
- Kartondan yapılan paralelkenarlar dikdörtgenler ve kareler dağıtılır.
- Ellerindeki bu dörtgenler üzerinde cetvel yardımıyla birer köşegen çizilmesi istenir.
- Daha sonra bu köşegen boyunca şekil kesilir.
- Oluşan iki parçanın hangi şekil olduğu sorulur ve her bir çeşit dörtgenden ne elde ettikleri tartıştırılır.
- Daha önce öğrendikleri bilgilerden yararlanarak (kare, dikdörtgen ve paralelkenarın alanını bulma) üçgenin alanının nasıl bulunacağı hakkında beyin fırtınası yaptırılır.

Güdüleme:

- Üçgenin alanının paralelkenarın alanının yarısı olduğu tartışılarak öğrencilerle birlikte çıkarım yapılması
- Taban ve tabana ait yüksekliği çarpıp ikiye böldüğümüzde alanı bulduğumuz söylenmesi
- Yukarıda yapılan etkinliğe göre üçgenin alanı hakkında genel bir yargıya varılması
- Etrafımızdaki alanını hesaplayabileceğimiz üçgensel şekillerden örnekler verilmesi.
- Üçgenin alan formülünün paralelkenarın formülünden geldiğinden bahsedilmesi.(üçgenin alanının paralel kenarın alanının yarısı olduğundan bahsedilmesi)

- Yukarıdaki soruya tekrar dönülerek öğrenilenlerle sorunun çözülmesi.

Etkinlik - 1

- Ellerindeki parçaları birleştirerek kartonun eski halinin çevresi hesaplatılır.
- Keserek elde ettikleri üçgenlerin çevreleri hesaplatılır.
- Daha sonra da alan yarıya indiğinde çevrenin yarıya inmediği üzerinde durulur. (aralarında doğru orantı olmadığı söylenir)

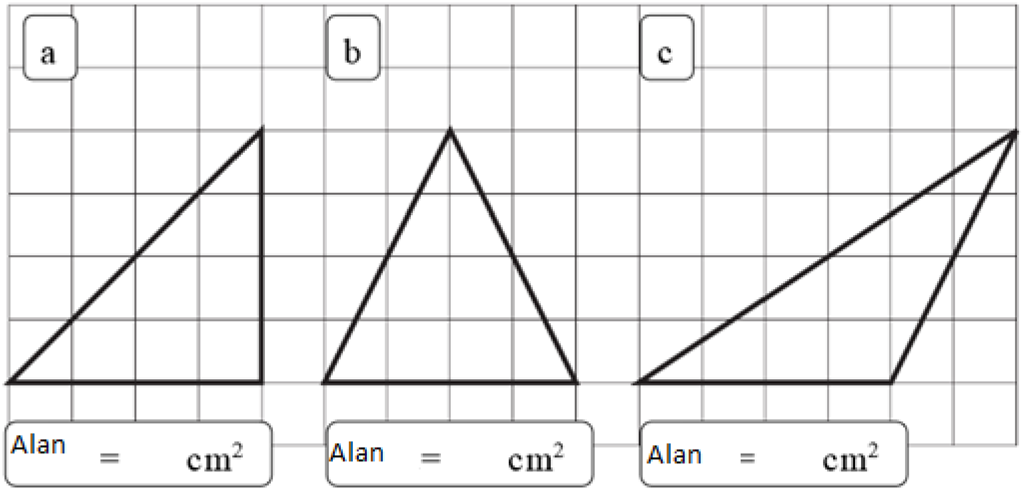
Etkinlik -2

- Keserek elde ettikleri üçgenlerin üzerine tüm yükseklikleri çizmeleri istenir.
- Üçgenin alanını aynı üçgene ait farklı kenar ve o kenara ait yükseklikle hesaplaması istenir ve alanın aynın olup olmadığı sorulur.
- Etkinlik kağıdı dağıtılır. Bireysel çalışmaları istenir.
- Bu etkinlikle beraber öğrencilerin aynı alana sahip farklı üçgenler olduğunu fark etmeleri amaçlanır.
- Etkinliğin birinci sorusunda birim kareleri nasıl sayacakları üzerinde durulur.(dikdörtgene tamamlama gibi.)
- Bu konuda beyin fırtınası yaptırılır. her öğrencini istediği yolu kullanması söylenir.

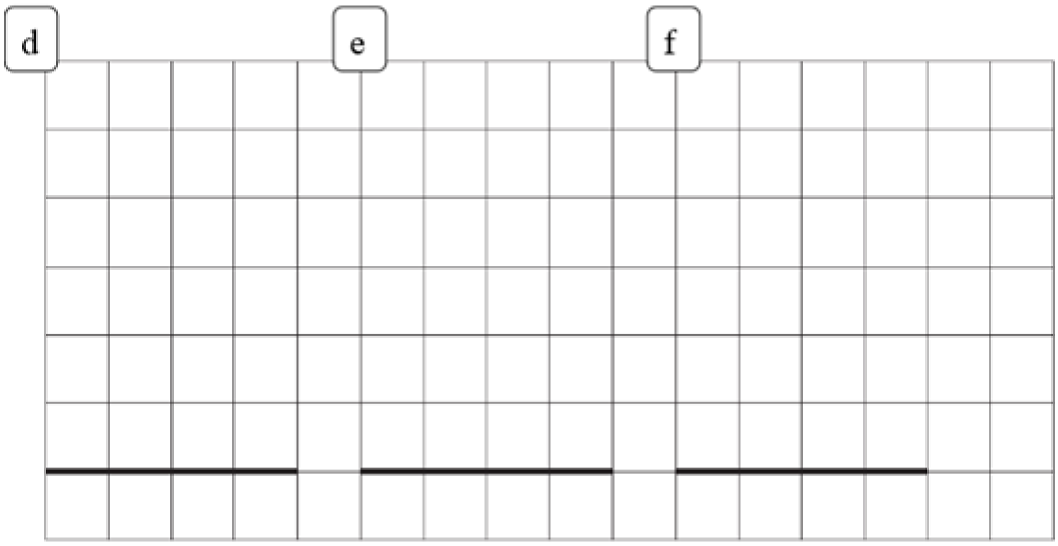
ETKİNLİK KAĞIDI

ÜÇGENDE ALAN HESAPLAMA

1. Kareleri sayarak üçgenlerin alanlarını bulunuz.



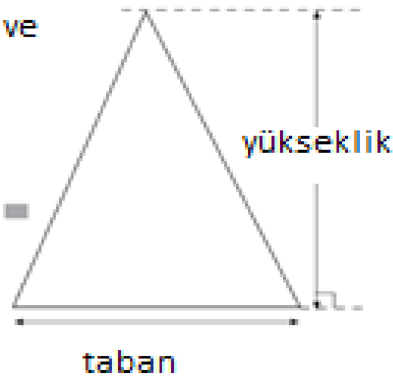
2. Aşağıda verilen doğruları çizilecek üçgenlerin birer kenarı kabul ederek , yukarıdaki üçgenlerle aynı alana sahip üç farklı üçgen çiziniz.



3. Yandaki şekil üçgenin tabanının ve yüksekliğinin nasıl ölçüldüğünü göstermektedir.

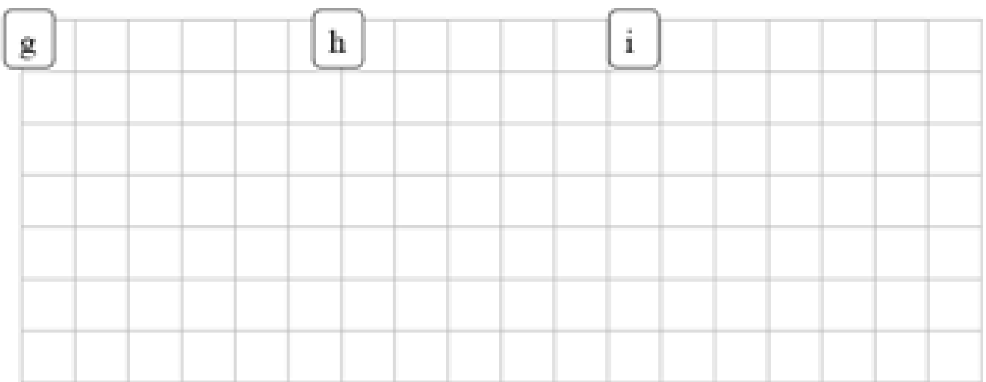
1. ve 2. sorulardaki üçgenlerin yükseklik ve tabanlarını hesaplayınız

Üçgen	Taban	Yükseklik
a		
b		
c		
d		
e		
f		



Sizce üçgenlerin alanları neden aynı?

1. ve 2. sorulardaki üçgenlerle aynı alana sahip farklı yükseklik ve tabanları olan üçgenler oluşturunuz.



Üçgen	Taban	Yükseklik	Alan
g			
h			
i			

H. TURKISH SUMMARY

ORTAOKUL MATEMATİK ÖĞRETMEN ADAYLARININ ÜÇGENİN ALANI KONUSUNA İLİŞKİN PEDAGOJİK ALAN BİLGİLERİ

GİRİŞ

Öğrencilerin ihtiyacı olan şey matematiği anlamak ve öğrenmek olduğundan bu alanda öğrenci başarısı ancak etkili matematik öğretimi ile sağlanabilir (Brophy, 1986; NCTM, 2000; Troisi, 1983). Şu açık bir gerçek ki öğretmenler öğrencilerin matematiği anlamlandırmalarında etkili bir öneme sahiptir (National Council of Teachers of Mathematics [NCTM], 2001). Bu da akıllara öğretmenin etkili bir öğretim için nelere ihtiyacı olduğu sorusunu getirir (NCTM, 2001). Bu bağlamda, etkili matematik öğretimi için gerekli en önemli faktörlerden birisi öğretmen bilgisidir (Tirosh, 2000).

İlgili yayın ve araştırmalar incelendiğinde etkili bir öğretmen olmanın gerekliliklerini açığa çıkarmak isteyen ve öğretmen bilgisinin bileşenlerini belirlemeyi amaçlayan birçok çalışma bulunmaktadır. Shulman'ın (1986; 1987) yapmış olduğu çalışmalar bunların ilki niteliğindedir. Shulman (1987) çalışmasında öğrencilerin bir konuyu kavramalarının sağlanması için yedi tür öğretmen bilgisinin gerekliliğini belirtmiştir. Bunlar; konu alan bilgisi, genel pedagoji bilgisi, müfredat bilgisi, öğrenciler ve onların özellikleri hakkındaki bilgi, eğitim ortamı ve şartları bilgisi, eğitimsel amaçlar, hedefler ve değerler ve bunların felsefi ve tarihsel temelleri bilgisi ile pedagojik alan bilgisidir. Bunlardan pedagojik alan bilgisi ilk defa Shulman (1986) tarafından ilgili alana tanıtılmış ve bir alan öğretmenini alan uzmanından ayıran bilgi olarak ifade edilmiştir. Bu yönüyle pedagojik alan bilgisi bir konuyu en iyi ifade eden gösterimleri, örnekleri ve açıklamaları içermektedir. Diğer bir deyişle bir konuyu anlaşılır kılmakta etkili olan yöntemleri içermektedir. Ayrıca, öğrencilerin belli konuları öğrenirken getirdikleri ön bilgilerinin neler olduğunu, en çok zorlandıkları noktaları ya da kavram yanılgılarına sahip oldukları noktaları, bu kavram yanılgılarının sebeplerini ve gidermek için kullanılabilecek öğretimsel stratejleri, konuyu öğretirken neyin öğrencilerin anlamasını kolaylaştırdığını ya da zorlaştırdığını bilmeyi gerektirmektedir (Shulman, 1986).

Yapılan çalışmalar öğretmen bilgisi ile öğrenci başarısı arasındaki ilişkiyi gözler önüne sermiştir (Ball, 1990; Grossman, 1990; Rowland, Huckstep, & Thwaites, 2005; Shulman, 1986; 1987). Ancak bu çalışmalar etkili bir matematik eğitimi için derin matematik bilgisinin tek başına yeterli olmadığını (Ball, Thames & Phelps, 2008; Turnuklu & Yesildere, 2007), aynı zamanda derin pedagojik alan bilgisinin de etkili öğretim için gerekli olduğunu göstermiştir (Shulman, 1986). Bu bağlamda, yapılan çalışmaların büyük çoğunluğu etkili bir pedagojik alan bilgisinin önemli bir öğretmen bilgisi bileşeni olduğunu destekler niteliktedir (An, Kulm & Wu, 2004; Park & Oliver, 2008).

Matematik eğitiminde bulunan beş öğrenme alanından birisi olan ölçme, hem günlük hayatta sıklıkla karşılaşılan bir kullanım alanına sahip olması hem de diğer matematik alanlarının her biriyle ilişkili olması sebebiyle okul müfredatlarında önemli bir yere sahiptir (Cavanagh, 2008; NCTM, 2014). Günlük hayatta sıklıkla kullanılan bir konu olmasına rağmen ilgili yayın ve araştırmalar incelendiğinde genellikle öğrencilerin ölçme kavramlarını anlamakta zorluk yaşadıkları ve mantığını anlamadan ezberledikleri formüller ile soruları çözmeye çalıştıkları görülmüştür (Tan- Sisman & Aksu, 2009). Bu bağlamda öğretmenlerin, öğrencilerin sahip olduğu kavramları ve bu kavramlara ait yaşayabilecekleri zorlukları/kavram yanılgılarını göz önünde bulundurarak, dersi öğrenci bilgisine göre planlamalarının öğrencilerin başarısını önemli ölçüde artırdığını destekleyen çalışmalar ilgili yayın ve araştırmalarda mevcuttur (Lenhart, 2010; Carpenter, Fennema, Petersen, & Carey, 1988). Ancak, yapılan bazı çalışmalar öğretmen adaylarının ve mesleğe yeni başlamış öğretmenlerin dersi öğrencilerin bilgisine göre planlamakta zorlandıklarını (Carpenter et al., 1988), bunun yanı sıra öğrenci zorluklarını/kavram yanılgılarını tanımakta ve bunların altında yatan sebepleri belirlemekte yetersiz kaldıkları görülmüştür (Esen & Cakiroglu, 2012; Turnuklu & Yesildere, 2007).

Bu bilgiler ışığında, bu çalışma, ilgili yayın ve araştırmalarda bulunan eksikliklerin bir kısmını gidermek amacıyla öğretmen adaylarının öğrencilerin alan kavramına ait kavram yanılgılarını/zorluklarını belirleme bilgileri ve bunları gidermekiçin kullanılabilecek öğretimsel strateji bilgileri bağlamında pedagojik alan bilgilerini incelemeyi amaçlanmıştır. Bu bağlamda, bu çalışmanın odağında bulunan alan kavramı, daha detaylı bilgi elde edilebilmesi açısından üçgenin alanı konusuna sınırlandırılmıştır.

Araştırma Soruları

Bu çalışma kapsamında yanıtlanmaya çalışılan araştırma sorusu ve alt araştırma soruları aşağıda verilmiştir.

1. Ortaokul matematik öğretmen adaylarının üçgenlerin alanına ilişkin pedagojik alan bilgilerinin doğası nedir?
   a. Ortaokul matematik öğretmen adayları altıncı sınıf öğrencilerinin üçgenlerin alanı konusu ile ilgili sahip oldukları kavram yanılgıları/zorlukları ile ilgili ne gibi bilgilere sahiptirler??
   b. Ortaokul matematik öğretmen adaylarının altıncı sınıf öğrencilerinin üçgenlerin alanı konusu ile ilgili kavram yanılgılarını/zorluklarını gidermek için kullandığı öğretimsel stratejiler nelerdir?

YÖNTEM

Çalışma Deseni

Nitel çalışma yöntemleri genellikle doğal bulunduğu ortamı değiştirmeden bir olayın arkasındaki anlamı daha iyi anlamak ve bu durum ile ilgili daha derinlemesine bir öngörü kazanmak amacıyla kullanılır (Merriam, 1998). Nitel çalışma yöntemlerinden biri olan durum çalışması ise araştırılan durumu daha derinlemesine ve çoklu veri toplama yöntemi kullanarak incelenmesine olanak sağlar (Creswell, 1998). Bu sebeple, bu çalışmada 2 ortaokul matematik öğretmen adayının, öğrencilerin üçgenin alanı konusu ile ilgili zorluklarını/kavram yanılgılarını belirlemeye yönelik bilgilerini ve bunları gidermek için kullandıkları öğretimsel strateji bilgilerini derinlemesine incelemek amacıyla durum çalışması yöntemini kullanmak uygun olacaktır. Bu doğrultuda, bu çalışmanın durumunu ODTÜ İlköğretim Matematik Eğitimi Programı'nda kayıtlı, son sınıfta bulunan 2 öğretmen adayı oluşturmaktadır.

Katılımcılar

Bu çalışmanın katılımcılarını, amaçlı örnekleme yöntemi kullanılarak seçilmiş 2 ortaokul matematik öğretmen adayı oluşturmaktadır. Bu katılımcılar, 2014-2015 eğitim-öğretim yılında bazı kriterlere dayanarak ODTÜ'den seçilmişlerdir. Katılımcılara Hatice ve Eda rumuzları verilmiştir. Katılımcıların eğitimlerinin son döneminde olmalarına ve gerekli tüm dersleri tamamlamış olmalarına dikkat edilmiştir. Katılımcılar gönüllülük esasına dayanarak seçilmiştir.

Veri Toplama Araçları

Öğretmen adaylarının öğrencilerin üçgenin alanıyla ilgili sahip olabilecekleri kavram yanılgılarını/zorluklarını belirleme durumlarını ve bu kavram yanılgılarını/zorlukları gidermek amacıyla kullandıkları öğretimsel stratejileri belirlemek ve derinlemesine araştırabilmek amacıyla yarı yapılandırılmış görüşmeler, ders gözlemleri ve doküman analizleri kullanılmıştır.

Yarı-yapılandırılmış Görüşme

Öğretmen adaylarının, öğrencilerin üçgenin alanıyla ilgili sahip olabilecekleri kavram yanılgılarına/zorluklarına ilişkin bilgilerini incelemek amacıyla yarı yapılandırılmış görüşmeler düzenlenmiştir. Bu görüşmeler iki aşamada gerçekleştirilmiştir. Bunlardan ilki, ön-görüşme olarak ders anlatımlarından önce hazırladıkları ders planları ile ilgili daha detaylı bilgi edinmek amacıyla düzenlenmiştir. Ön-görüşmeler aracılığıyla öğretmen adayları hazırladıkları soru/cümle/etkinlik örneklerini daha detaylı açıklama imkânına sahip olmuşlardır. İkinci uygulanan görüşme, son görüşme olarak adlandırılmış ve öğretmen adaylarına staj okullarında anlattıkları dört dersin ardından uygulanmıştır. Son görüşmeler aracılığı ile öğretmen adaylarının ders esnasında yaptıkları açıklamaların ve davranışlarının altında yatan sebeplerin incelenmesi; ayrıca öğretmen adaylarının, öğrencilerin çözüm yollarından ya da açıklamalarından yaptıkları çıkarımlarla ilgili bilgi alınması amaçlanmıştır.

Görüşme formu oluşturulmadan önce pedagojik alan bilgisiyle ilgili çalışmalar ve dokümanlar incelenmiş ve her iki görüşme içinde amaca uygun taslak görüşme formu oluşturulmuştur (Isiksal, 2006; Karahasan, 2010; Kubar, 2012; Kula, 2011; Simsek, 2011; Tekin-Striva, 2014). Daha sonra alanında uzman, matematik eğitimi bölümünden bir akademisyenle soruların amaca uygunluğu ile ilgili bir görüşme yapılmıştır. Uzmanla yapılan bu görüşmeden sonra sorulardan çıkarılabilecek anlamlar ve soruların açıklığı ile ilgili iki matematik öğretmeni ve henüz alanda çalışmaya başlamamış iki matematik öğretmeniyle sorular tartışılmış ve görüşme formlarının son hali oluşturulmuştur.

Görüşmeler, öğretmen adaylarının staj derslerinde üzerinde duracakları kazanımlar doğrultusunda, üçgenin alanı ve üçgenin yüksekliği konularında iki ayrı oturum şeklinde ve her bir öğretmen adayıyla bire bir düzenlenmiştir. Öğretmen adaylarının da izniyle bu görüşmeler video kameraya kaydedilmiştir. Her görüşme yaklaşık olarak yarım saat sürmüştür.

Sınıf Gözlemi

Yarı yapılandırılmış görüşmelerin yanında ders gözlemleri de yapılmış ve öğretmen adaylarının bilgilerinin gerçek sınıf ortamındaki yansımaları incelenmiştir. Bu gözlemler, öğretmen adaylarının, ders esnasında öğrencilerin ortaya çıkarmış oldukları zorlukların/kavram yanılgılarının farkına varıp varmadıklarını incelemek ve öğretmen adaylarının hangi öğretimsel stratejileri kullanarak öğrencilerin bu zorluklarını/kavram yanılgılarını gidermeye çalıştıklarını araştırmak amacıyla yapılmıştır. Bu bağlamda, iki öğretmen adayının, öğretmenlik uygulaması dersi kapsamında gittikleri staj okullarında dörder ders saati gözlemlenmiştir. Ders gözlemi sırasında ders anlatımları, gerekli izinlerle video-kaydına alınmıştır. Ayrıca, araştırmacıya ders esnasında dikkat etmesi gereken önemli noktalar hakkında rehber olması açısından ilgili yayında bulunan çalışmaların da yardımıyla bir gözlem protokolü hazırlanmıştır (Aydın, 2012; Tekin-Sitrava, 2014). Bu protokole uygun olarak, araştırmacı ders esnasında not almıştır.

Ders Planı

Öğretmen adaylarından, öğrencilerin üçgenin alanı konusunda sahip olabilecekleri kavram yanılgıları/zorlukları ile ilgili bilgilerini araştırmak amacıyla öğretmen adaylarına üçgenin alanı ve üçgenin yüksekliği konularında ikişer ders planı hazırlamaları istenmiştir. Öğretmen adaylarının ders planı hazırlama süreçlerinde herhangi bir müdahalede bulunulmamıştır. Planları, anlatacakları derslere uygun olarak toplam dört ders saatini kapsayacak şekilde detaylı olarak hazırlamaları istenmiştir.

Veri Analizi

Bu çalışmada öğretmen adaylarının ders planları, ön-görüşme ve son-görüşmeleri ve ders gözlemleri araştırmanın amacı doğrultusunda analiz edilmiştir. İlk olarak, yarı-yapılandırılmış görüşmelerin video kayıtları ile öğretmen adaylarının dörder saatlik ders anlatım videoları bilgisayar ortamında ses döküm raporlarına dönüştürülmüştür. Daha sonra bu ses dökümlerinden araştırma sorularıyla ilgisi olmadığı düşünülenler ayrıştırılmış ve her bir soru ile ilgili olan dökümler bir araya getirilmiştir. Daha sonra birinci soru için bir araya getirilen ses dökümleri aracılığıyla genel anlamda bir kod listesi oluşturulmuştur. Yeni kod oluşmayıp, kodlar doyuma ulaşıncaya kadar bu işlem tekrar edilip hazırlanan raporların üzerinden geçilmiştir. Ardından bu kodlar onlarla ilgili ana temalar oluşturularak, uygun temaların altında toplanmıştır. Elde edilen kodlar ve temalar alanında uzman bir matematik eğitimi akademisyenine gösterilmiş ve alınan görüşler sonucunda kodlara ve temalara son şekli verilmiştir. Buna bağlı olarak, öğretmen adaylarının üçgenin alanı konusundaki öğrencilerin sahip olabilecekleri kavram yanılgıları/zorluklara ilişkin bilgileri üç ana kategoriye ayrılmıştır; yükseklik kavramına ilişkin, alan kavramına ilişkin ve üçgenin alan formülüne ilişkin zorluklar/kavram yanılgıları. İkinci sorudaki kod ve kategorileri belirlemek için aynı süreç uygulanmıştır. Bunun sonucunda öğretmen adaylarının, ders sürecinde gözlemledikleri öğrenci kavram yanılgılarını/zorluklarını gidermek amacıyla kullandıkları öğretimsel stratejiler beş başlığa ayrılmıştır. Bunlar; tartışma, gösterip yaptırma, didaktik yaklaşım, bilişsel çatışma ve doğrudan anlatımdır.

BULGULAR VE TARTIŞMA

Öğretmen Adaylarının Öğrencilerin Zorluklarına/Kavram Yanılgılarına ilişkin Bilgisi

İlgili yazın ve çalışmalardan bazıları, öğretmenlerin pedagojik alan bilgi bileşenlerinden biri olan öğrenci zorlukları/kavram yanılgıları hakkındaki bilgilerinin etkili öğretim için önemli bir bilgi bileşeni olduğunu ve bu bilginin öğretmenin ders ile ilgili kararlarını önemli ölçüde etkilediği belirtmektedir (Carpenter et al., 1988). Çalışmaya katılan öğretmen adaylarının düşüncelerinin, bu çalışmalarla benzerlik gösterdiği görülmüştür. Bununla ilgili olarak, öğretmen adayları belirttikleri öğrencilerin sahip olabilecekleri kavram yanılgılarının/zorlukların bazılarını öğretmenlik uygulaması kapsamında yaptıkları ders anlatımı sırasında fark edip tanımlarken, bazılarını ise ders planını hazırken belirtmiş hatta planlarında bunlara yönelik sorulara da yer vermiştir. Yapılan görüşmeler sırasında öğretmen adaylarına, bu şekilde öğrencilerin konuya ilişkin kavram yanılgılarını/zorluklarını dikkate alarak dersi planlamalarının sebebi sorulduğunda, "eğer öğretmen yapacağı derse ilişkin öğrencilerin kavram yanılgılarını/zorluklarını düşünerek dersini planlarsa, ders esnasında öğrencilerin bunlardan kaynaklanan hatalarıyla karşılaştığında etkili bir şekilde bu sorunu çözebilir" cevabını vermişlerdir.

Yapılan analizler sonucunda elde edilen bilgiler ışığında, öğretmen adaylarının öğrencilerin üçgenin alanı konusunda sahip olabilecekleri çeşitli zorlukları/kavram yanılgılarını belirttikleri görülmüştür. Bunlar, ilgili yayın ve araştırmalar, katılımcıların cevapları ve araştırmacının veri ile deneyimi ışığında üç ana bölüme ayrılmıştır: yükseklik kavramına ilişkin zorluklar/kavram yanılgıları, alan kavramına ilişkin zorluklar/kavram yanılgıları ve üçgenin alan formülüne ilişkin zorluklar/kavram yanılgıları. Bu bağlamda, belirlenen kavram yanılgılarının geneline bakıldığında, bunların büyük çoğunluğunu yükseklik kavramına ilişkin zorluklar/kavram yanılgılarının oluşturduğu görülmüştür. Öğretmen adaylarının, öğrencilerin ön öğrenmelerinin, öğrenecekleri yeni konu üzerinde önemli bir etkiye sahip olduğunu düşünmeleri bunun sebebi olabilir. Ki bu düşünce alandaki bazı çalışmalar tarafından da desteklenmektedir (Hewson & Hewson, 1983).

Analizlerden elde edilen bilgilere göre öğretmen adayları, öğrencilerin yükseklik kavramına ilişkin sahip olabilecekleri zorluklardan/kavram yanılgılarından biri olarak öğrencilerin, yüksekliğin her zaman yatay veya dikey çizilmesi gerektiğini düşünebileceklerini belirtmiştir. Aynı öğrenci zorluğu/kavram yanılgısı, ilgili alandaki bir çalışmada öğretmen adaylarının belirttikleri şekilde verilmiştir (Herskowitz, 1989). Öğretmen adayları, yapılan görüşmeler sırasında öğrencilerin bu düşüncelerinin altında yatan sebeple ilgili şunu belirtmiştir: "Genelde derslerde verilen örneklerde öğrencilerden yere paralel olan kenara ait yüksekliği çizmesi istenir ya da o yükseklik verilir. Bu yüzden, öğrenciler bu duruma aşina olurlar ve farklı bir kenara yükseklik çizmeleri istendiğinde yere dikey çizmeye çalışırlar". Bu zorluğun/kavram yanılgısının yanında buna benzer olarak öğrencilerin, yüksekliğin her zaman üçgenin içinde olması gerektiğini düşünebileceklerini belirtmişlerdir. Hershkowitz (1989) de çalışmasında bu duruma, öğrencilerin zihinlerinde yer eden kavram görüntülerinin sebep olduğunu vurgulamıştır. Öyle ki, öğrencilerin zihinlerinde canlanan üçgen görüntüsü genellikle dar açılı üçgen şeklindedir ve dar açılı üçgenin tüm yükseklikleri üçgenin içinde bulunmaktadır. Bu sebeple, öğrenciler üçgene ait yüksekliğin içerden çizilmesi gerektiğini düşünmektedir.

Bildirilen diğer bir öğrenci zorluğu/kavram yanılgısı ise diğer yayınlarda da yer alan öğrencilerin yüksekliğin kenar orta dikme olduğunu düşünmeleri ile ilgilidir (Gutierrez & Jaime, 1999). Öğretmen adayları, öğrencilerin böyle düşünmelerinin sebebinin verilen şeklin görünüşünden kaynaklanabileceğini belirtmişlertir. Öyle ki, çizilen yüksekliğin, kenarı ortadan ikiye bölüyormuş gibi görünmesinden dolayı öğrencilerin bu düşünceye kapılabileceklerini ifade etmiştir.

Yükseklikle ilgili bir diğer öğrenci zorluğu/kavram yanılgısı olarak ilgili yayın ve araştırmalarda öğrencilerin yükseklik ile hipotenüsü karıştırmaları verilmiştir (Cavanagh, 2008; Orhan, 2013). Bununla ilgili olarak Eda, öğrencilerin yükseklik ile uzunluk kavramını çoğu zaman karıştırdıklarını belirtmiştir.

Şimdiye kadar öğretmen adayları tarafından belirtilen öğrenci zorlukları/kavram yanılgıları aynı zamanda ilgili yayın ve araştırmalar tarafından da desteklenmiştir. Ancak, bu çalışmada öğretmen adayları, erişilebilen yayınlar içerisinde örneğine rastlanmayan bir öğrenci yanılgısı/zorluğu daha belirtmişlerdir. Öyle ki, ders esnasında Hatice öğrencilere bir doğru parçasının dışına, verilen bir noktadan dik bir doğru parçası çizmeyi gerektiren bir soru sormuştur ve buna cevap olarak bir öğrenci “eğer verilen bu doğru parçası bir ışın olmuş olsaydı, bunu uzatarak bu dikliği çizebilirdim” demiştir. Ders sonrası yapılan görüşmelerde Hatice, öğrencinin bu hatasının aslında uzantı kavramını bilmemesinden kaynaklı olduğunu belirtmiştir.

Öğretmen adayları, belirttikleri yüksekliğe ait kavram yanılgılarının yanında alan kavramına ve üçgenin alanı formülüne ilişkin de çeşitli öğrenci zorlukları/kavram yanılgıları belirtmiştir. Yapılan analizler sonucunda öğretmen adaylarının, öğrencilerin taban ve tabana ait yüksekliği belirlemede zorluk/kavram yanılgısı yaşayabileceklerini belirttikleri görülmüştür. Bununla ilgili olarak öğretmen adayları, öğrencilerin soruda herhangi bir yükseklik ve bir taban uzunluğu gördüklerinde, doğrudan bu iki uzunluğu formüle yerleştirerek bir sonuç elde etmeye çalışabileceklerini ifade etmişlerdir. Öğrencilerin bu zorluluğuna/kavram yanılgısına Gökdal’ın (2004) çalışmasında da rastlanmaktadır.

Bu duruma ek olarak, öğretmen adaylarından Eda, öğrencilerin alan ve çevre kavramını ayırt etmede zorlandıklarını, hatta aralarında doğrudan orantılı bir ilişki bulunduğunu düşündüklerini belirtmiştir. Öğrencilerin bu zorluğunun/kavram yanılgısının sebebi olarak ise Eda, alan ve çevre kavramlarını öğrencilerin tam olarak tanımlayamadıklarını ve bu kavramları ayırt etmede zorlandıklarını belirtmiştir. Ayrıca, öğrencilerin bu kavramları karıştırmalarının temelinde ise ezbere dayalı öğrenmenin bulunabileceğini ifade etmiştir.

Öğretmen adaylarının belirttiği son öğrenci zorluğu/kavram yanılgısı ise öğrencilerin, üçgenin alanı formülünde bulunan ikiye bölme işlemini anlamlandıramamaları olmuştur. Bunun altında yatan sebebin ise öğrencilerin daha önceki öğrenmeleri olabileceği belirtilmiştir. Öyle ki, öğretmen adayları, öğrencilerin üçgenin alanını hesaplarken aslında aynı taban ve yüksekliğe sahip paralelkenarların alanının yarısı kadar olduğunu unuttuklarını ve bu sebeple formülde bulunan ikiyi görmezden geldiklerini belirtmişlerdir. Çalışmada elde edilen bu sonuç ile ilgili alanda bulunan bazı çalışmaların sonuçları paralellik göstermektedir (Cavanagh, 2008; Gökdal, 2004; Orhan, 2013).

Öğretmen adaylarının, öğrenci zorlukları/kavram yanılgıları bilgisini etkileyen çeşitli sebepler olabilir. Bu sebeplerden ilki, öğretmen eğitimi ile ilgili aldıkları pedagojik dersler olabilir. Bu bağlamda, eğitimlerinin 3. yılında Matematik Öğretme Yöntemleri I –II dersleri verilmektedir ve bu ders sürecinde öğretmen adaylarına, öğrencilerin matematik öğrenme alanlarında sahip olabilecekleri çeşitli zorlukları/kavram yanılgılarını inceleme fırsatı sunulmaktadır (ODTÜ, 2013). Bunun yanında, bu dersler kapsamında matematik dersleri için çeşitli öğrenme yöntemlerini içeren ders planları hazırlamaları istenmektedir (ODTÜ, 2013). Ayrıca, öğretmen adaylarına son senelerinde Öğretme için Matematik Eğitimi ersi verilmektedir. Bu ders kapsamında öğretmen adayları, matematik konularındaki öğrenci zorluklarını/kavram yanılgılarını belirleyerek bunların kaynaklarının neler olabileceğini ve hangi yöntemlerle bunları giderebileceklerini tartışmaktadır (ODTÜ, 2013). Bununla ilgili olarak, Hatice, aldığı bir eğitim dersi kapsamında yükseklik ile ilgili bir araştırma yaptığını ve bu araştırma sebebiyle öğrencilerin yükseklikle ilgili zorluklarının/kavram yanılgılarının farkında olduğunu ifade etmiştir. Buna ek olarak, Eda, aldığı bir eğitim dersi kapsamında yükseklik ile ilgili öğrenci kavram yanılgılarının bulunduğu bir makale incelemesi yaptıklarını belirtmiştir. Bu yüzden, öğretmen adaylarının almış oldukları pedagojik eğitim dersleri, hazırladıkları ders planlarında öğrencilerin olası kavram yanılgılarının/zorluklarının üzerinde durmalarının sebebi olabilir.

Öğretmen adaylarının öğrenci zorlukları/kavram yanılgıları bilgisini etkileyen diğer bir faktör verdikleri özel dersler olabilir. Yapılan görüşmeler sırasında öğretmen adayları belirttikleri bazı zorluklar/kavram yanılgıları ile verdikleri özel dersler sırasında karşılaştıklarını belirtmiştir. Öyle ki, öğretmen adayı bu dersler sırasında öğrencinin üçgenin alanını hesaplarken yaptığı bir hatanın altında yatan sebebi bulmak amacıyla bazı sorular sorarak öğrenci zorluğunu/kavram yanılgısını fark etmiş olabilir.

Öğretmen adaylarının, öğrencilerin kavram yanılgılarını/zorluklarını belirlemelerinin son sebebi olarak kendilerinin de eskiden bu zorluklara/kavram yanılgılarına sahip olmaları olabilir. Böylece, öğretmen adayları eskiden kendisinin yaşadığı zorlukları/yanılgılarını düşünerek, öğrencilerin de bunlara sahip olabileceklerini düşünmüş olabilirler.

Öğretmen Adaylarının Öğretimsel Strateji Bilgisi

Ders gözlemlerinden elde edilen bilgiler ışığında öğretmen adaylarının, öğrencilerin üçgenin alanı konusuna ilişkin ortaya çıkardıkları kavram yanılgılarını/zorluklarını gidermek amacıyla kullandıkları öğretimsel stratejiler incelendiğinde beş farklı öğretimsel strateji kullandıkları görülmüştür. Bunlar; tartışma, gösterip yaptırma, didaktik yaklaşım, bilişsel çatışma ve doğrudan anlatımdır. Bu bağlamda, Swan (2001) bilişsel çatışmayı şu şekilde tanımlamıştır: Öğrencinin verdiği hatalı cevabın üzerine öğretmen soracağı sorularla öğrencinin hatasını fark etmesini sağlar. Böylece öğrencinin düşüncelerinde bulunan tutarsızlıkla yüzleştirilerek doğru cevabı elde etmesi sağlanır. Ders gözlemlerinin analizlerinden elde edilen sonuçlara göre Hatice'nin önceden beklediği bir öğrenci zorluğu/kavram yanılgısı için bu yöntemi kullandığı görülmüştür. Bu yöntemi kullanmasının sebebi, öğrencilerin zihinlerinde çatışma yaratmak için soracağı soruları dersten önce hazırlama fırsatı bulmuş olması olabilir.

Bilişsel çatışma yönteminin yanı sıra, öğretmen adaylarının derslerinde karşılaştıkları öğrenci zorluklarını/kavram yanılgılarını ortadan kaldırmak için didaktik yaklaşım kullandığı söylenebilir. Bu yaklaşıma göre, öğretmen önce öğrenciye yaptığı hatayı, nedeniyle birlikte matematiksel bir dille ifade eder. Daha sonra hata öğretmen tarafından düzeltilir (Swan, 2001). Öğretmen adayının bu yöntemi kullanmasının bir nedeni olarak, öğrenci zorluğuyla/kavram yanılgısıyla ders esnasında karşılaştığı için, öğrencinin daha pasif olduğu bu yöntemi kullanmayı tercih ettiği söylenebilir.

Diğer bir öğretimsel strateji olarak öğrencilerin kavram yanılgılarını/zorluklarını düzeltmek amacıyla sınıf tartışmaları düzenlenmiştir. Borasi (1994) bununla ilgili olarak ders esnasında bir kavram yanılgısı görüldüğü anda bunu gidermek için sınıf tartışması düzenlemenin, öğrencilerin anlamlı öğrenmeleri için gerekli olduğunu belirtmiştir. Bu bağlamda, öğretmen adaylarının sadece ders öncesinde belirledikleri öğrenci kavram yanılgıları/zorlukları için sınıf tartışması düzenlemeleri dikkat çeken bir noktadır. Bu yüzden, öğretmen adaylarının bu öğretimsel stratejiyi kullanmalarının sebebi olarak zorluğa/kavram yanılgısına aşina olmaları gösterilebilir.

Öğrencilerin aktif olduğu ve görsel anlamda desteklendiği diğer bir yöntem olan gösterip yaptırma yöntemi, öğretmen adayları tarafından önceden kullanmayı planladıkları öğrenci zorluklarını/yanılgılarını gidermek için uygulanmıştır.

Öğretmen adayları, derslerden önce belirttikleri öğrenci zorluklarının/kavram yanılgılarının dışında, sınıfta beklenmedik bir öğrenci kavram yanılgısı/zorluğu ile karşılaştıkları durumlarda düz anlatım yöntemini kullanmayı tercih etmişlerdir. Öğretmen adayları bu şekilde beklenmedik bir durum ile karşılaştığında, kendilerini güvende hissetmediklerinden doğrudan anlatım yöntemini tercih etmiş olabilir. Bu durumda, öğretmen adaylarının üçgenin alanı konusuna yönelik öğrencilerin sahip oldukları kavram yanılgılarını/zorlukları gidermek için ders esnasında kullandıkları öğretimsel strateji bilgileri, daha önceden uygulamayı planladıklarıyla sınırlıdır.

Yukarıda da belirtildiği gibi öğretmen adayları, belirledikleri öğrenci zorluklarından/kavram yanılgılarından farklı bir öğrenci kavram yanılgısı/zorluğu ile karşılaştıklarında, bunları gidermek için uygun öğretimsel stratejileri kullanamadıkları belirlenmiştir. Bununla ilgili olarak, yapılan son görüşmeler sırasında Eda, "İlk defa böyle bir düşünceyle karşılaştığımdan dolayı kavram yanılgısını/zorluğu gidermek için hiçbir şey yapamadım" ifadelerini kullanmıştır. Öğretmen adayının bu şekilde zorlanmasının sebebi, gerçek sınıf ortamında yaptıkları ders anlatımına karşı deneyimsiz olmaları olabilir. Bu durumla ilgili olarak öğretmen adayına, kendisine yeniden fırsat verilseydi, karşılaştığı öğrenci zorluğunu/kavram yanılgısını gidermek için ne yapacağı sorulduğunda ise çeşitli öğretimsel stratejiler belirterek bunları kullanabileceğini ifade etmiştir. Bu yüzden, öğretmen adaylarının öğrenci yanılgısı/zorluğu bilgisinin, bu zorlukları/yanılgıları gidermek için kullanılabilecek öğretimsel stratejiler bilgisinin üzerinde bir etkisi olduğu söylenebilir. Çalışmanın bu sonucuna paralel olarak, Gökkurt (2014) çalışmasında öğretmen adaylarının, öğrencilerin hatalarının altında yatan sebepleri bildiklerinde bu hataları gidermek için uygun tekniği ve stratejiyi kullanabildikleri sonucunu elde etmiştir. Bu sebeple, öğretmen adaylarının üçgenin alanı konusundaki öğrenci zorlukları/kavram yanılgıları bilgisine sahip olduğu durumlarda, bunları gidermek için uygun öğretimsel stratejileri kullanabildikleri söylenebilir.

ÖNERİLER

Yapılan çalışmada pedagoji temelli derslerin öğretmen bilgisine etkisi açıkça görülmektedir. Bu bağlamda, hem bu derslere giren akademisyenlerin hem de bu derslerin içeriğini oluşturan program geliştiricilerin pedagojik içerikli derslere gerekli önemi vermeleri ve öğretmen adaylarının anlamlı öğrenme ortamları oluşturabilecek, yeterli pedagojik donanıma sahip birer öğretmen olarak mezun olmaları sağlanmalıdır.

Ayrıca gerçek sınıf ortamında bulunmanın, öğretmen adaylarının özellikle öğrenci bağlamında bilgisini olumlu yönde etkilediği görülmüştür. Bu sebeple, öğretmen adaylarına daha fazla gerçek sınıf ortamında bulunma fırsatı sunulmalıdır. Ayrıca, öğretmen adaylarının bilgi gelişimlerini inceleyebilmek amacıyla mikro-öğrenme etkinlikleri yapılmalıdır. Bunun yanında, eğitim derslerinde öğretmen adaylarına gerçek durumlardan uyarlanmış senaryolar verilerek durum değerlendirmesi yapmaları istenebilir. Böylelikle öğretmen adayları daha fazla gerçek hayat örneği deneyimleme fırsatı bulacaklardır.

Bu çalışmanın konusu, daha detaylı veri elde edebilmek amacıyla genel alan kavramından üçgenin alanı konusuna özelleştirilmiştir. Ancak, ilgili alanda yapılan alan kavramı ile ilgili araştırmalar, öğrencilerin alan kavramını kavrayışlarının yüzeysel olduğunu ve formülleri ezberleyerek yaptıkları işlemlerin bazı kavram yanılgılarının doğmasına sebep olduğu sonucunu ortaya koymaktadır (Cavanagh, 2008; Huang & Witz, 2013; Zacharos, 2006). Bu sebeple, öğretmen adaylarının diğer geometrik şekillerin (kare, dikdörtgen, yamuk, paralelkenar…) alanı konusu ile ilgili olası öğrenci zorluklarını/kavram yanılgılarını belirlemeye yönelik bilgileri ve bunları gidermeye yönelik kullandıkları öğretimsel stratejileri derinlemesine incelemek amacıyla da çalışmalar yapılabilir. Öğretmen adaylarının öğrenci zorluk/kavram yanılgı bilgisi ve bunları gidermek için kullanılabilecek öğretimsel strateji bilgisi konularındaki eksikliklerini gidermeye yönelik eğitimler verilmesi, öğretmen adaylarının yeterli bilgi donanımıyla mezun edilmesi açısından önemlidir. Ayrıca, aynı çalışmalar şuan alanda aktif olarak çalışan öğretmenlerle de düzenlenebilir. Böylece, öğretmen adaylarıyla bilgilerinin karşılaştırılması yapılarak eksik olan bilgileri tamamlamaya yönelik çalışmalar yapılabilir. Diğer bir yandan, bu çalışmada pedagojik alan bilgisinin sadece iki bileşeni çalışılmış olup bunlar hakkında derinlemesine bilgi elde edilmeye çalışılmıştır. Bu sebeple, yapılacak olan çalışmalarda pedagojik alan bilgisinin diğer alt boyutlarının da çalışılması önem taşımaktadır. Böylelikle öğretmen bilgisine daha geniş bir perspektiften bakma fırsatı sunulabilir.

I. TEZ FOTOKOPİ İZİN FORMU

ENSTİTÜ

Fen Bilimleri Enstitüsü

Sosyal Bilimler Enstitüsü

Uygulamalı Matematik Enstitüsü

Enformatik Enstitüsü

Deniz Bilimleri Enstitüsü

YAZARIN

Soyadı : BİLİK
Adı : Aslı
Bölümü : İlköğretim Fen ve Matematik Eğitimi

TEZİN ADI (İngilizce) : Pre-Service Middle School Mathematics Teachers' Pedagogical Content Knowledge Regarding The Area of Triangles

TEZİN TÜRÜ : Yüksek Lisans Doktora

1. Tezimin tamamından kaynak gösterilmek şartıyla fotokopi alınabilir.

2. Tezimin içindekiler sayfası, özet, indeks sayfalarından ve/veya bir bölümünden kaynak gösterilmek şartıyla fotokopi alınabilir.

3. Tezimden bir bir (1) yıl süreyle fotokopi alınamaz.

TEZİN KÜTÜPHANEYE TESLİM TARİHİ: