HYDRO INFLOW FORECASTING AND VIRTUAL POWER PLANT PRICING IN THE TURKISH ELECTRICITY MARKET

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ABSTRACT

HYDRO INFLOW FORECASTING AND VIRTUAL POWER PLANT PRICING IN THE TURKISH ELECTRICITY MARKET

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Hydro inflow forecasting with most accurate quantitative models is a very crucial subject for effective hydro optimization, virtual power plant pricing, volume risk management and weather derivatives pricing in the Turkish electricity market. Predicting increase or decrease in hydro inflow, seasonal characteristics of hydrological years such as wet, dry or normal, allow the decision-makers to economically use water for optimal periods, quantify of volume risk and determine effective portfolio management strategies. In this study, we focus on defining and pricing a hydroelectricity power plant as a Virtual Power Plant (VPP). For pricing of this non-standard option, we worked on inflow and price scenarios and optimization model with the possible real world constraints. For the hydro inflow forecasting that will be used in optimization model, we applied Seasonal Autoregressive Integrated Moving Average model with Exogenous Variable (SARIMAX), whereas lagged indexed precipitation data, having the highest correlation with historical inflow data, is included as exogenous variable. In addition to point forecast of hydro inflow, we generated various inflow scenarios by using the distribution of model fit residuals as a stochastic processes for defined VPP. Moreover, we worked on hydro optimization problem where objective function is maximizing the expected value of generation by tracing to generated inflow and price scenarios. Price scenarios are simulated by using the hourly shape of historical Day Ahead Market (DAM) prices. As a result, we could analyze the optimization outputs according to different price and inflow levels. For defined VPP, Volume at Risk measure is expressed to explain the meaning of risky volume for the valuation of VPP. Furthermore, in the last section of the study, by the help of the flexibility of optimization model, by using different constraints, we worked on the constructions, solutions and evaluations of different optimization cases as a significant contribution in academic literature and common practice in current electricity markets.

Keywords : Hydro Inflow Forecasting, Hydro Optimization, Virtual Power Plant Pricing, Energy and Commodity Market, Valuation and Decision in Electricity Market, Volume at Risk

ÖZ

HİDROELEKTRİK SANTRALLER İÇİN AKIM TAHMİNİ VE TÜRKİYE ELEKTRİK PİYASASINDA SANAL HİDROELEKTRİK SANTRAL FİYATLAMASI

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Türkiye elektrik piyasasında, hidroelektrik santrallerine gelen akım tahmininin kesinliği yüksek olacak şekilde yapılması, hidro optimizasyonu, sanal hidroelektrik santrallerinin fiyatlanması, hacim risklerinin yönetimi ve iklim türev araçlarının fiyatlanması için çok önemlidir. Hidroelektrik santrallere gelen akımdaki artış ve azalışı ve hidrolojik döngüde mevsimsel karakteristiğin doğru tahmini sayesinde, portföy yönetimi daha etkin bir şekilde yapılabilir. Bu çalışmada sanal bir hidroelektrik santralinin kısıtları ile tarif edilmesi ve fiyatlanması üzerinde durulmuştur. Bu standart olmayan ürünün fiyatlanması için de, akım ve fiyat senaryoları, ayrıca gerçeğe yakın kısıtlarıyla optimizasyon modeli üzerinde çalışılmıştır. Optimizasyon modelinde kullanılacak akım tahmini için Mevsimsel ARIMA (SARIMAX) Modeline başvurulmuştur. Modelde gecikmeli ve endeksli yağış datası, akım datası ile en yüksek korelasyona sahip olacak şekilde oluşturulmuş ve harici etkileyen olarak kullanılmıştır. Akım tahminine ek olarak, model hatalarının dağılımı ve olasılıklı süreçler kullanılarak çeşitli akım serileri oluşturulmuştur. Ayrıca farklı akım ve fiyat serilerini içeren hidro optimizasyon modeli, amaç fonksiyonu üretimin beklenen değerini maksimize edecek şekilde kurulmuş ve sanal hidroelektrik santralinin fiyatlanmasında kullanılmıştır. Fiyat senaryoları, geçmiş market fiyatlarının saatlik karakteri kullanılarak simüle edilmiştir. Böylece belirlenen farklı akım ve fiyat seviyeleri için optimizasyon çıktıları elde edilmiştir. Ayrıca belirlenen sanal santral için Riske Maruz Hacim kavramı ile sanal santral değerlemesinde riskli hacim ifade edilmiştir. Son olarak da kurulan optimizasyon modelinin de

esnekliği yardımıyla, farklı kısıtlar kullanılarak farklı optimizasyon problemlerinin kurulması, çözülmesi ve değerlendirilmesi üzerinde çalışılmış ve bu sayede hem literatüre hem de elektrik piyasası çalışmalarına katkı sağlanması hedeflenmiştir.

Anahtar Kelimeler: Hidro Akım Tahmini, Hidro Optimizasyonu, Sanal Hidroelektrik Santrallerinin Fiyatlanması, Enerji ve Emtia Piyasası, Elektrik Piyasasında Değerleme ve Karar Verme, Riske Maruz Hacim

To my beloved family

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LIST OF ABBREVIATIONS

BIST	Borsa İstanbul
BPM	Balancing Power Market
BRSA	Banking Regulation and Supervisory Agency
CA	Competition Authority
CMB	Capital Markets Board
DAM	Day Ahead Market
EİE	Electric Power Resources Survey and Development Administration
EMRA	Energy Market Regulatory Authority
EPİAŞ	Turkish Energy Stock Market
EÜAŞ	Turkish Electricity Generation Company
HEPP	Hydro Electricity Power Plant
MAPE	Mean Absolute Percentage Error
MENR	The Ministry of Energy and Natural Resources of Turkey
MEP	Market Exchange Price
MYTM	National Load Dispatch Center
OTC	Over the Counter
PMUM	Market Financial Settlement Center
RoR	Run-of-river
SMP	System Marginal Price
TEİAŞ	Turkish Electricity Transmission Company
TEK	Turkish Electricity Administration Commission
VHPP	Virtual Hydroelectricity Power Plant
VIOP	Borsa İstanbul Futures and Options Market
VPP	Virtual Power Plant

CHAPTER 1

INTRODUCTION

Turkish electricity market has been growing rapidly in terms of both demand side and correspondingly supply side by regarding social and economical development in Turkey. Among the reasons for electricity demand side growth, Gross Domestic Product (GDP), Industrial Production Index (IPI), employment, labor force, export-import volumes can be indicated as main ones. On the other hand, to meet this energy demand having increasing trend, various energy sources have been used in Turkey. The commonly used supply sources for electricity are natural gas and LNG, hydro, import and hard coal, lignite and wind. Moreover, in Turkey, investments for renewable energy sources, i.e., solar, wind, geothermal, hydro, biomass etc., have also been on the rise in recent years. Natural gas and LNG is the biggest energy source in Turkey, but it is imported from foreign countries such as Russia, Iran, Azerbaijan, Algeria and Nigeria. Therefore, hydraulic capacity can be said as the biggest territorial source in Turkish electricity portfolio.

Hydroelectricity power plants (HEPPs) are separated into two main groups which are Run-of-River (RoR) HEPPs and HEPPs with Reservoirs in Turkey. RoR HEPPs are must-run, this means that these power plants are operated if it has inflow being higher than minimum turbining level. The difference between RoRs and reservoirs is that reservoirs can store water and dispatch it whenever the power plant operator wants. Therefore, depending on the time and amount of inflow is minor for reservoirs' electricity generation. However, RoRs are in need of inflow to generate electricity and this implies that seasonality, drought and wetness affect a RoR's generation severely.

In general scheme of trading side, since inflow affects electricity generation in both reservoirs and RoRs, we can say that there is an obvious relation, i.e. negative correlation, between inflow and system price. In similar way, when we consider for generation side, for hydroelectricity producers, the aim is maximizing revenue by the optimization of available source. This means that dispatch of a hydro electricity power plant should be scheduled by regarding the producing energy at the relatively high priced hours [5]. However, inflow and price are uncertain and stochastic drivers in that problem. Therefore, to optimize the hydro portfolio and have optimal scheduling, inflow and price are needed to be predictable [23].

The water has an opportunity cost, since the water streams to the reservoirs at no cost and the variable cost of hydro production is very low, but the amount of water available is limited and uncertain. Marginal cost of water (water value) depends on reservoir volume, production capacity of the system, supply capacity and demand expectations, expected spot market prices and predicted inflow amount to the reservoir [30].

Literature has many works about hydro inflow forecasting by using different methods from various model families. Artificial Neural Network (ANN) and Fuzzy logic have many applications in hydrology: Dawson and Wilby (1998) [11], Abrahart and See (2000) [1], Khadr and Schlenkhoff (2014) [20]. Moreover, Thomas - Fiering Model, firstly published by Thomas and Fiering (1962), is also applied for many case studies in literature. In Chapter 3, to predict monthly inflow to defined Virtual Dam, Seasonal Autoregressive Integrated Moving Average Model with exogenous variable (SARIMAX) is constructed. Actually, seasonal time series models are very common in power markets. Especially, hydrological time series can be modelled by statistical models with regarding the relation between observations. These models are generally successful to capture the hydrological cycles and seasonality for hydro inflow forecasting. Seasonal Arima (SARIMA) Model also belongs to family of probabilistic concepts and mathematical statistics [34]. According to Chow (1964), there is a dependence between successive observations of hydrological data [9]. In the study of Çevik (2002), monthly inflow data of Yeşilırmak River was modelled by Seasonal ARIMA (SARIMA) model and obtained forecasts by means of this method [8]. In this study, as a contribution, indexed and lagged precipitation data was used as an exogenous variable obtained regarding the highest correlation with historical inflow series in an addition to the constructed univariate seasonal model. On the other hand, we applied Monte Carlo Simulation technique by using the parameters of model residuals' distribution and addendum of point forecast and these simulations, we generated inflow scenarios for the defined Virtual Hydro Power Plant. For time series analysis and forecasting, R Programming (https://cran.r-project.org/) was used.

In Chapter 4, we study on hourly Day-Ahead-Market (DAM) prices. In electricity markets, especially in wholesale and retail sides, pricing electricity is very important, but difficult issue, as well. To supply the electricity demand, generation mix is formed by regarding the marginal costs of the different fuel-typed power plants for each hour. Generation mix influences electricity prices and pricesetter is at where the demand is supplied. For electricity producers and traders, forecasting the value of electricity is very essential to make profit. Electricity price simulations is one of the hot topics in the energy markests. Burger et al. (2004)[7] worked on the Spot Market Price Simulation (SMaPS) Model for Germany market price. Moreover, ARMA - GARCH and mean reverting models were applied by Haario and Kauranne (2010) [19]. For monthly price levels, we used historical electricity prices and for hourly shape modeling, we applied the similar technique as inflow scenarios for stochastic behavior of electricity prices with the purification of seasonality and 24 hour periodicity by using SARIMA model constructed by for each month separately [7].

When we consider Virtual Power Plant (VPP) concept in hydro power business, as an addition to physical asset of a hydroelectricity power plant, this virtual HEPP can be represented as a non-standard product or an option. According to Zurborg (2010), physical constraints and execution decisions belong to the physical part, when bidding decisions and settlement are included in financial part of VPP concept [37]. Despite the fact that physical constraints can be included in VPP concept, VPP distinguishes the technical operation from economic dispatch. Therefore, Virtual Power Plant can be used for balancing or hedging portfolio by power traders. VPP option buyers get a right to use capacity within the scope of defined contractual constraints, such as marginal cost, operating hours, amount of capacity etc., without the operational risk [6]. This concept is a brand new concept in Turkish electricity market and VPPs are traded by electricity producers and power traders by means of bilateral agreements or auctions mainly. In Chapter 4, by using the generated inflow and price scenarios, we constructed an optimization model that helps to calculate fair value of defined VPP for given time horizon by using Xpress Optimization Programming - XPress 7.7 - 64 Bit (http://www.fico.com/en/products/fico-xpress-optimization-suite).

In Chapter 5, various optimization cases are obtained by means of the flexibility of constructed optimization model. In this model, reservoir levels (initial, end, minimum, maximum etc.) are assigned as parameters and could be changed. These changes are categorized in cases and studied 4 different situations, namely Case 1 - 4.

This study aims to establish a complete work including the importance of hyraulic capacity of Turkish electricity market, hydro inflow forecasting, optimization, pricing and risk management for a virtual power plant. Hydroelectric power comprises of run-of-river hydro power plants and dammed hydro power plants in Turkey and these cover 35% of total installed capacity as of the end of 2015. Therefore, forecasting inflow with high accuracy is very essential for effective hydro optimization, weather derivatives, virtual power plant pricing and volume risk management.

The main objectives of this study can be listed as follows by summarizing;

• To forecast hydro inflow and generate inflow scenarios for a defined Virtual Power Plant (VPP),

• To simulate electricity prices by using historical hourly shapes,

• To optimize hydraulic capacity of VPP by regarding inflow and price scenarios,

• To calculate the value of VPP as an financial contract,

• To generate various optimization cases by use of the constraints of optimization model,

• To contribute to the literature of hydro inflow modeling, VPP pricing and volume risk management by considering power traders and producers.

CHAPTER 2

Turkish Electricity Market

Turkish energy sector consists of the electricity, natural gas, petroleum and LPG markets. These markets are regulated by Energy Market Regulatory Authority (EMRA). Although, state-owned authority dominates these markets, privatized portion in the markets is increasing year by year. In this chapter, we explain Turkish electricity market in detail.

2.1 History of the Turkish Electricity Market

In 1902, first electricity generation in Turkey occurred by 2 kW-hydroelectricity power plant in Tarsus and first remarkable electricity production was in İstanbul by means of Silahtarağa Thermal Power Plant in 1913. Until 1970, there were some important initiatives such as constructing power plants with miscellaneous dimensions, i.e., Çatalağzı and Tunçbilek thermal power plants, Sarıyar, Kemer, Hirfanlı, Demirköprü HEPPs, and establishing govermental institutions ”Electric Power Resources Survey and Development Administration (EİE)” and ”The Ministry of Energy and Natural Resources of Turkey (MENR)” to regulate and control electricity market and production [26].

In 1970, ”Turkish Electricity Administration Commission (TEK)”, which had caused monopolistic electricity market in Turkey, was established. Generation, transmission and distribution of electricity had been managed by TEK for 23 years. After that, between 1994 and 2003, generation and transmission of electricity was carried out by ”Turkish Electricity Joint Stock Company (TEAŞ)” and distribution has separated and administrated by ”Turkish Electricity Distribution Joint Stock Company (TEDAŞ)”. By the way, in 1987, Karakaya Dam - 1,800 MW and in 1994, Atatürk Dam - 2,400 MW which has still the largest installed capacity in Turkey, were constructed. Hence, the hydroelectricity portfolio started to develop substantially. However, after 1993, because of the increase in the number of natural gas power plants, thermal capacity started to rise.

Furthermore, in 1984, to incite the private sector for building and operating the electricity generation, ”Built Operate and Transfer (BOT) Model” was introduced. In this model, electricity is produced by private investors and sold to the national grid, the state-owned electricity authority, or even to private end users [27].

In 2004, TEAŞ was divided into three subgroups; ”Turkish Electricity Generation Joint Stock Corporation (EÜAŞ)”, ”Turkish Electricity Transmission Joint Stock Corporation (TEİAŞ)” and ”Turkish Electricity Trading Joint Stock Corporation (TETAŞ)”. Moreover, TEDAŞ was separated into 21 distribution regions and as of 2013, share transfer agreements were completed for privatization of them [32]. On the other hand, in the generation side, privatization is proceeded, especially for thermal portfolio of EÜAŞ, summarized in Figure 2.1.

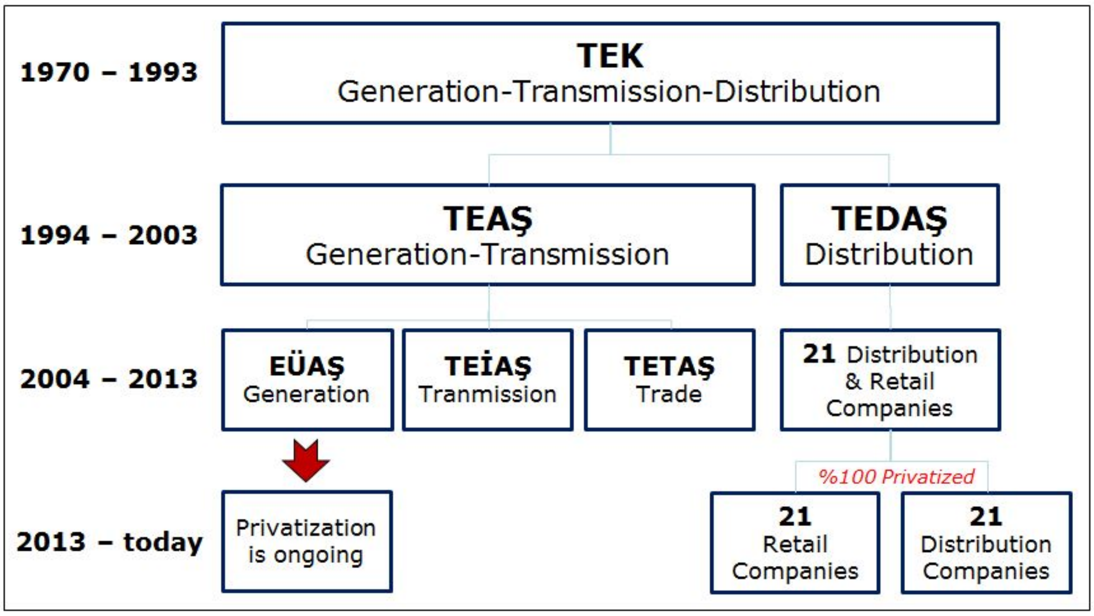


Figure 2.1: History of Electricity Market in Turkey

By regarding recently published ”Balancing and Settlement Regulation” by Energy Market Regulatory Authority (EMRA) (Law no. 6446), Turkish Electricity market can be divided into four sub-market which are derivative market, day-ahead market (DAM), intraday market and balancing power market (BPM) that will be explained in next subsections in detail.

Derivative market is mid- and long-term market (weeks/ months/ years) in which optimization of production and meeting supply needs are essentials. This new market, called Futures and Options Market (VIOP), is operated by Borsa İstanbul (BIST) and regulated by EMRA, Capital Markets Board (CMB) and Competition Authority (CA).

Day-ahead market, which has short-term (D+1) time horizon, has been existing since 2011 and balance between generation and consumption is required. It is operated by EPİAŞ and regulated by EMRA and CA. EPİAŞ is established in 2015 and its shareholder structure has been consisting of 3 groups which are Type A, Type B and Type C. 30% share is Type A and belongs to TEİAŞ, 30% share is Type B and belongs to BIST and lastly, 40% share is Type C and is allocated to market participants and can be transferred among the companies.

The second new development in Turkish electricity market is *intraday market* (very short-term/ hours) as of June 2015. Its market operator and regulators are same as day-ahead market.

Finally, current *balancing power market* is operated by TEİAŞ for the security of system. This is real-time market and regulated by EMRA and CA [14].

According to Installed Capacity 2015 Report of TEİAŞ [33], Turkish electricity generation portfolio, regarding companies, comprises of 'Independent Power Producers (IPPs)', 'State-owned Power Plants (EÜAŞ and power plants under EÜAŞ control)', 'Build Operate and Transfer (BOT) Power Plants', 'Build Operate (BO) Power Plants', 'The Autoproducers', 'The Transferring of Operating Rights (TOR) Power Plants' and 'Unlicensed Power Plants' displayed in Figure 2.2. As seen in the Figure 2.2, the biggest share belongs to IPPs (59%) and increase in that share is expected in next years for the liberalization of the market.

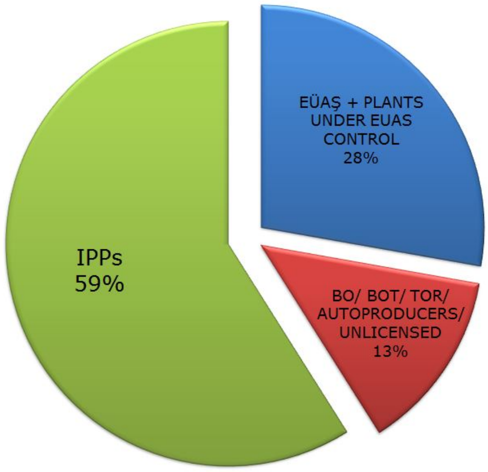


Figure 2.2: Installed Capacity by Companies in Turkey

2.1.1 The Physical Market

Since electricity is a non-storable commodity, for balancing the system, secure physical delivery is very crucial. In Turkey, system operator is National Load Dispatch Center, as a subsidiary of TEİAŞ and also manages ancillary services [14]. To understand the physical market operations better, we will consider electricity market in five parts which are bilateral agreements and contracts on physical delivery, day-ahead market (DAM), balancing power market (BPM), intraday market, settlement and reporting.

Until the new regulation, Market Financial Settlement Center (PMUM) had been responsible for financial contracts and settlement. By means of the establishment of EPİAŞ, all financial and settlement issues have been under EPİAŞ's responsibility. For day-ahead (D+1) operations, bilateral agreements, bid and offers, options, forwards and contracts for physical electricity delivery are logged into the system of EPİAŞ at the day before (D) from 09:30 until 11:30. After offer verification, market clearing and evaluation of objections, final notification of Market Exchange Price (TL/MWh) and transaction volume are issued by EPİAŞ at the one day before at 14:00 [35]. MEP is determined by the intersection of sales and purchase offers for each hour of day-ahead. Merit Order Curve, that is a step-wise function, is occured for each hour via ranking available sources of energy to supply the demand with the object of maximizing social welfare. Figure 2.3 is presented to explain the construction of Merit Order Curve generically. According to offers of market participants, supply curve is occured and where the demand intersects the supply curve, MEP is determined.

Figure 2.3: Market Exchange Price

Balancing power market (BPM) is real time market in that physical delivery is realized and according to imbalance direction in the system (loading or deloading), System Marginal Price (SMP) is determined as shown in Figure 2.4. By comparing the supply and demand amount, loading and de-loading instructions are delivered to market participants for the balance of system and according to the direction, SMP is determined in this real time market.

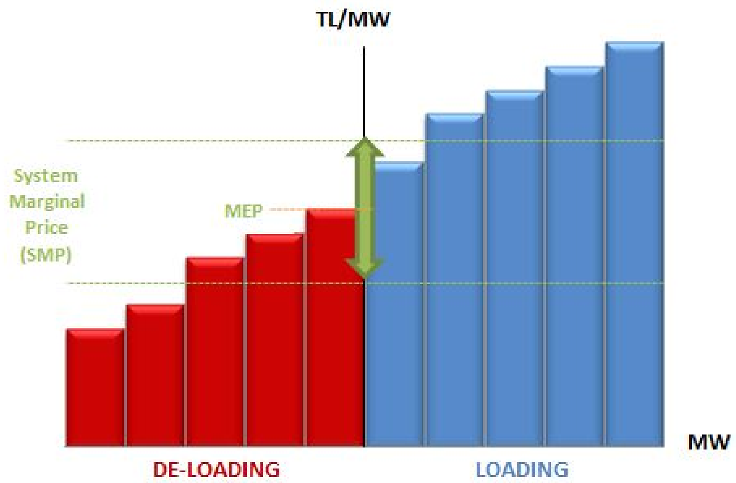


Figure 2.4: System Marginal Price

Intraday market, that has come newly, has real-time operation as well as balancing power market, but also it provides trading opportunities to market participants. Last issue for physical market is the settlement and reporting of DAM activities, BPM activities and imbalances. Settlement of day-ahead market is daily basis, while balancing power market, energy imbalance, ancillary services are settled and reported monthly.

2.1.2 The Financial Market

In financial market, there is no physical electricity delivery and it includes futures and forward contracts as distinct from physical market, even though its short history, financial electricity market is developing rapidly in Turkey [12].

According to Seim [30], the financial market in energy sector is generally used for making profit from volatility in market price, managing risk, hedging price and contributing to liquidity of the market.

In Over-The-Counter (OTC) market, in addition to physical deals, financial contracts are also traded, where dealers are market makers and quote bid and ask prices themselves. As of March 2015, traded volumes in the OTC market are shown in Figure 2.5. Liquidity in OTC market changes month by month and does not have an obvious character.

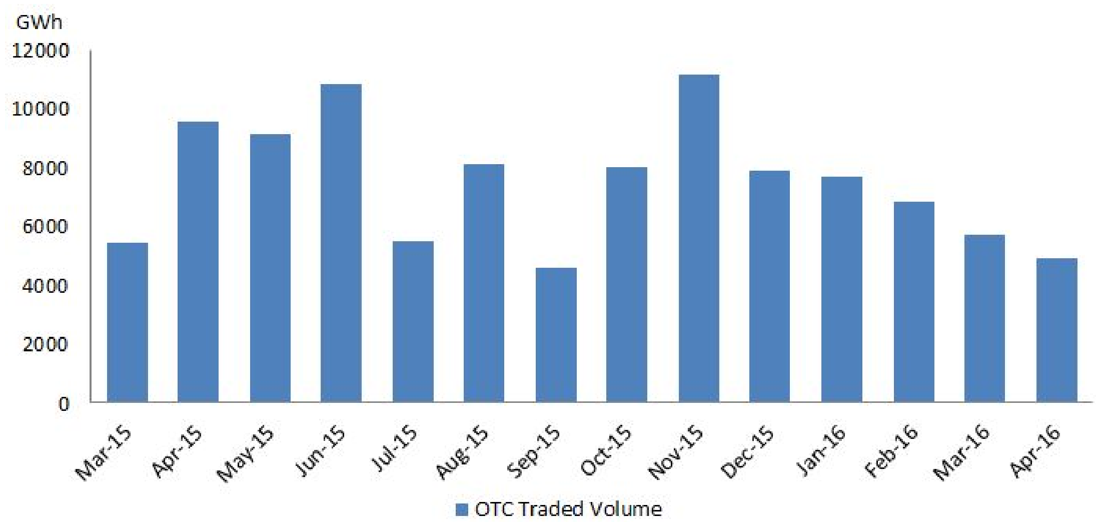


Figure 2.5: OTC Traded Volume

Another platform for financial power contracts in Turkish electricity market is Borsa İstanbul Futures and Options Market (VIOP) where baseload electricity is traded and regulated by Borsa İstanbul (BIST) in the current situation. By comparing these financial enviroments, VIOP is more transparent and developing market. As seen in Figure 2.6, there has been a sharply increasing trend in VIOP traded volume of baseload electricity future contracts. While liquidity in VIOP is increasing because of the transparency and accessibility, traded volume in OTC market decreases, e.g. decrease by approximately half of traded volume is observed for April year over year.

Other contracts traded in VIOP are listed in two main groups; Future Contracts that are single stock future contracts, equity index future contracts, currency future contracts, precious metals (gold) future contracts, commodity future contracts and foreign indices future contracts and Option Contracts that are single stock option contracts, equity index contracts and USD/TRY option contracts.

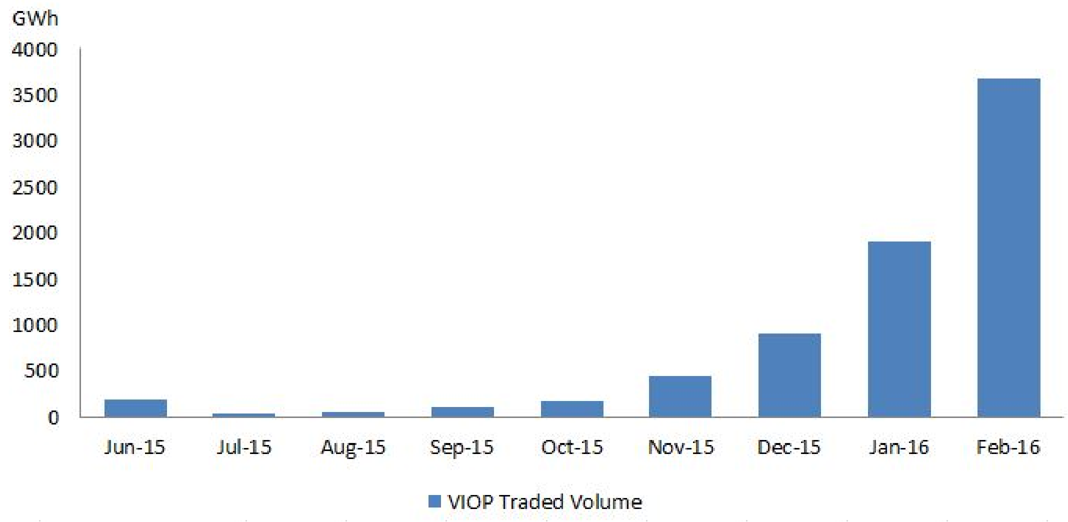


Figure 2.6: VIOP Traded Volume

2.2 Hydro Power in Turkey

In Turkey, hydraulic capacity is the second biggest source for electricity production. The three largest Turkish dams are; 2,405 MW Atatürk Dam with estimated mean annual generation of 6.98 TWh, 1,800 MW Karakaya Dam with approximately 6.68 TWh/year and 1,330 MW Keban Dam with 5.78 TWh annually electricity generation. These dams, that are state run power plants, owned by EUAS and locate at Fırat River basin.

Hyraulic installed capacity - both Run-of-River and Reservoir capacity - comprises 35% of Turkish electricity production portfolio as Figure 2.7 shows, and this equals to approximately 66.6 TWh annual generation in 2015 [33]. Moreover, in Figure 2.8, annual hydro generation is demonstrated with capacity factors that are calculated by dividing annual generation by the annual hydro installed capacity.

Moreover, hydraulic is one of the energy sources that is supported by government. According to Renewable Energy Law (Law No: 6094, 2010), feed in tariffs and incentives for electricity generation via utilization of renewable sources as in Table 2.1, such that power plants commissioned during the period 2005 - 2015 will be able to benefit from guaranteed prices for a period of 10 years.

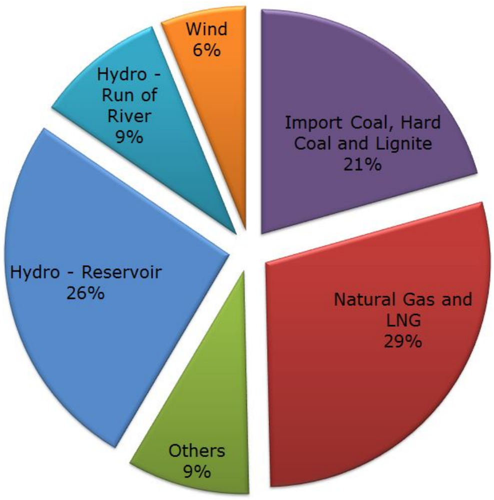


Figure 2.7: Installed Capacity by Fuel Types in Turkey

Table 2.1: Feed in Tariff for Renewable Energy Source Utilization

Type	Feed in Tariff (US¢/ kWh)
Hydro	7.3
Wind	7.3
Geothermal	10.5
Bio-fuel (Incl. solid waste)	13.3
Solar	13.3

2.2.1 Relation between Hydro Inflow and Electricity Prices

As known, electricity prices depend on some fundamental parameters for example, electricity demand, capacity development, climate changes, GDP growth etc. Additionally, hydraulic generation capacity plays an important role for electricity production and there is an uncontrovertible relation between hydro generation and electricity prices. Especially, must-run hydro power plants affect prices crucially because of taking place in the left side of Merit Order Curve. This implies that there is a negative correlation between hydro inflow and electricity prices. When we compare the monthly inflow to dams ($Mio.\ m^3$) (TEİAŞ, 2016) and monthly electricity prices (TL/MWh) (Market Financial Settlement Center, 2016), moves in the opposite directions can be observed easily in the Figure 2.9. Therefore, in the market, ability to forecast hydro inflow is one of the key factor for electricity pricing.

As seen from Table 2.2, 2014 was a dry year in that electricity generation from hyraulic power is 20% lower than 2013 and this is an essential reason increase in electricity prices in 2014.

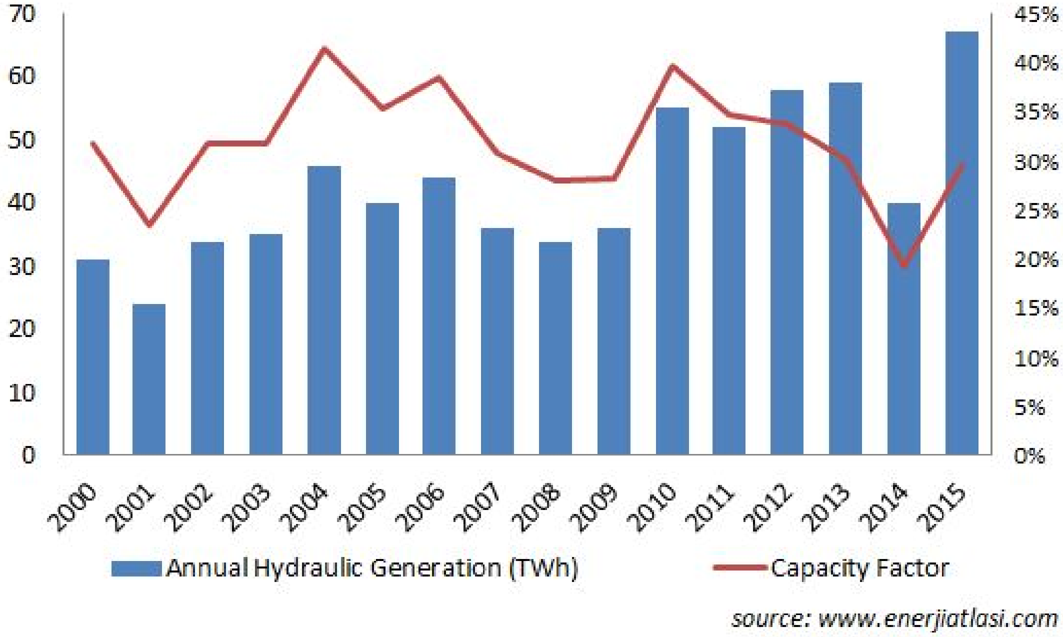


Figure 2.8: Annual Hydraulic Generation

Table 2.2: Annual Total Inflow to Dams vs. Average Electricity MEP

Date	Inflow to Dams (Mio.m^3)	Market Exchange Price (TL/MWh)
2010	76,246	121.1
2011	65,559	125.7
2012	58,346	149.7
2013	51,794	150.0
2014	29,744	164.1
2015	58,947	137.9

2.2.2 Hydro Power Business in Turkey

In Turkey, importance of energy derivatives gets higher with the contributions of the more educated market participants, ease in regulation, institutionalization, recognition of derivatives, comprehension ability in risk management etc. [3].

While hydro power is significantly important in the generation portfolio of Turkey as physical commodity, using hydro power as financial product has been developing day by day. For instance, for electricity producers, optimizing capacity against the spot market is a very important ability in financial area. Moreover, optimizing hydro asset against the forward/future market and forming hedging strategies are other necessities for producers. At the same time, liquidity of trade of hydro capacity as an option has been increasing in Turkish electricity market. This optionality for capacity can be considered as real option and Virtual Power Plant concept.

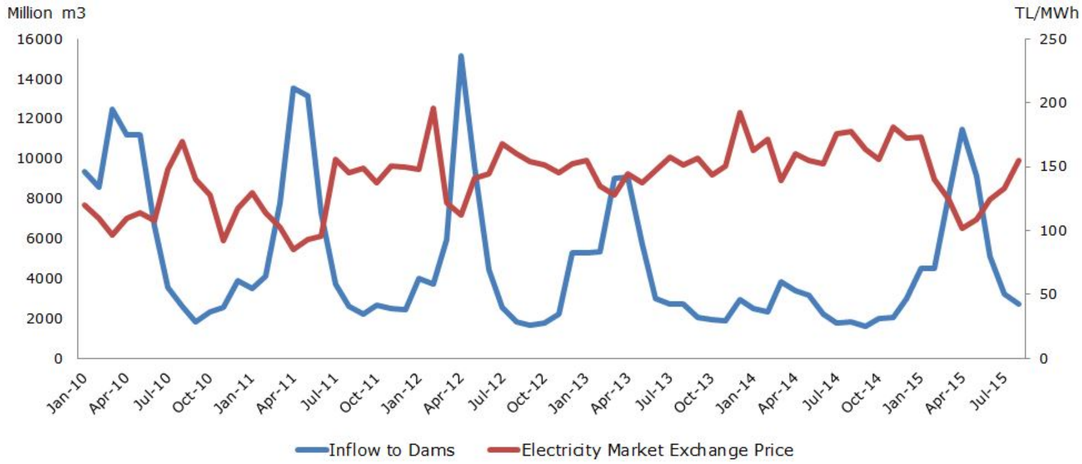


Figure 2.9: Relation between Monthly Hydro Inflow and Electricity Prices

Virtual Power Plant (VPP) is a new concept in Turkish electricity market and this can be analyzed in two subgroups which are financial and physical parts. Financial part of VPP consists of valuation of VPP, bilateral agreements/auctions and settlement. In physical part, physical delivery and real-world constraints are the main subjects. When the VPP buyer has a right to buy electricity by regarding the constraints of agreement, VPP seller is obliged to provide electricity to counter party by generating (if production is possible) or buying from market (if VPP seller is not a producer). Therefore, ability to predict the value of VPP is very important and the target in this thesis is to construct an optimization model giving the fair value of defined virtual hydro power plant with respect to inflow and price scenarios.

2.3 The Flow of the Proposed Methodology

We represent the steps as a flow in general scheme in Figure 2.10 for better understanding of the target of the study. In fortcoming chapters, we explain these steps in detail.

The organization of the thesis is as follows;

In Chapter 2, Turkish electricity market is explained in terms of history, regulation and operation. In Chapter 3, to forecast the inflow to VHPP, SARIMAX Model is constructed. In Chapter 4, we work on hourly Day-Ahead-Market (DAM) prices to generate electricity price scenarios. By using the generated inflow and price scenarios, we constructed an optimization model by using Mixed Integer Linear Programming. In Chapter 5, various optimization cases are obtained by means of the flexibility of constructed optimization model. In this model, by means of the change in reservoir levels (initial, end, minimum, maximum) and operating hours, we obtain 4 different optimization cases. In Case 1, VHPP values were obtained according to different initial and end reservoir levels. In Case 2 and Case 3, VPP values were obtained according to different initial, end and maximum reservoir levels. Lastly, in Case 4, operating hours were constrained and virtual hydro power plant was required to be dispatched only at the peak hours (08:00-19:00/ all days in a week). Therefore, we had a chance to analyze different VHPP values for the same inflow and price levels because of the change in model constraints. To sum up, we obtain a flexible and reliable tool that values defined VHPP.

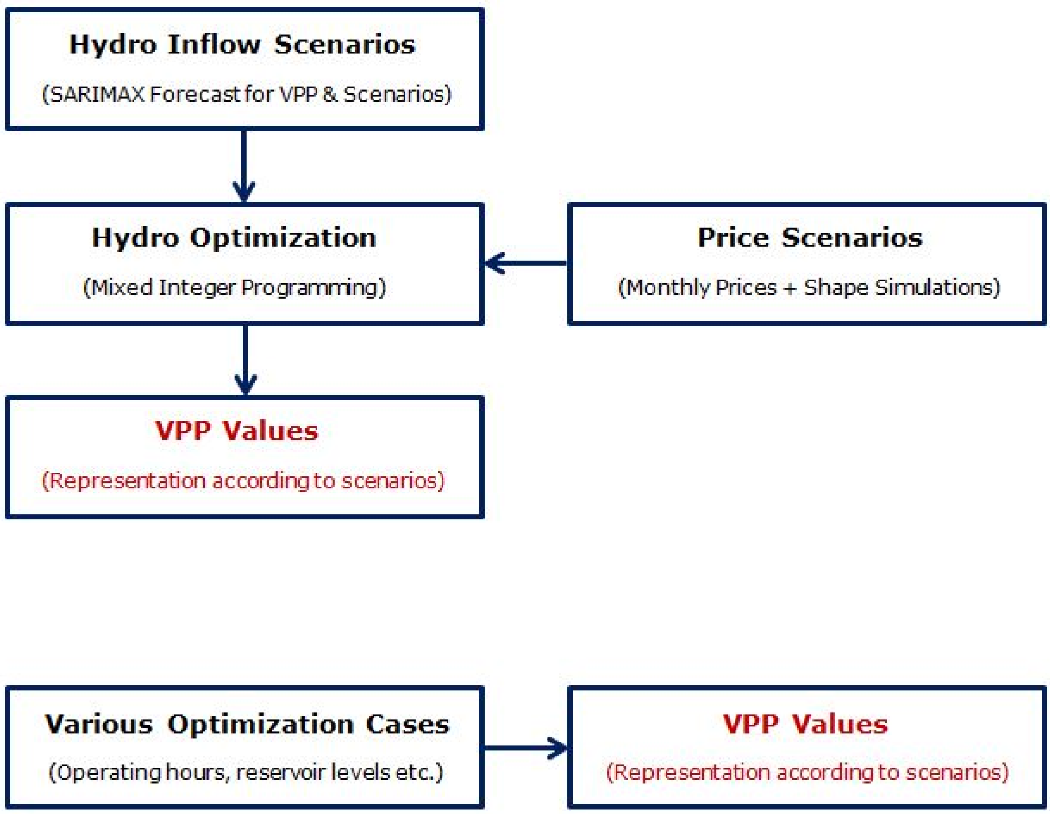


Figure 2.10: The Flowchart of the Proposed Methodology

CHAPTER 3

HYDRO INFLOW FORECASTING

An important factor in hydroelectricity power plant pricing is the hydro inflow behaviour during a year as mentioned earlier. For this reason, the knowledge on the future realizations plays an important role in estimating the value of hydroelectricity power plant. As it can be observed in Figure 3.1, the hydro flow occurences show time and seasonal dependency. To forecast the future values, we fit a stochastic model using time series methods. Road map is determined for hydro inflow modelling according to the inferences from time series tests and analyses.

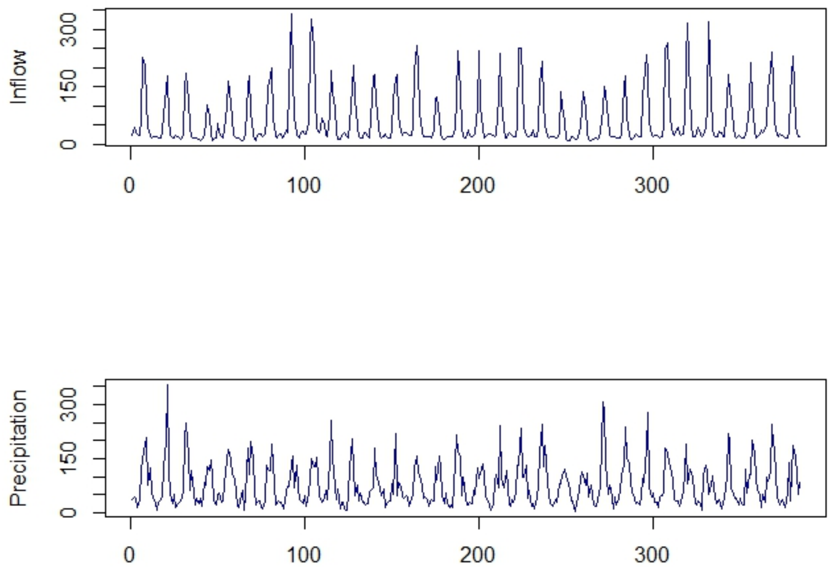


Figure 3.1: Hydro Inflow and Precipitation Series

Monthly average inflow data (m^3/s) for the time period between October 1979 and September 2011, obtained as a result of feasibility study of Arkun Dam and Hydroelectricity Power Plant which is located on Çoruh River in provincial border of Erzurum and Artvin, is taken as the real life data. Arkun Dam, whose construction was started in 2010, belongs to EnerjiSA Generation Company, has been in operation since second quarter of 2014 and it has three 78 MW capacity main turbines and two 5.4 MW capacity environmental turbines, summing up to 244.8 MW. It has 14 km. energy tunnel which is the longest tunnel in EnerjiSA projects and annual generation of Arkun HEPP is approximately 780.1 GWh.

In the model, as an exogenous variable, we used Speedwell precipitation data that consists of various stations with different weights with lag of 5 months. To get this series, we used 52 precipitation stations over Turkey and chose 15 significant stations with the weigths, that are given in Table 3.1, regarding to get highest correlation with inflow series.

Table 3.1: Station Weights for Indexed Precipitation Series

City Name	Station ID	Weight
ALANYA	TURK_17310	0.0249
ANAMUR	TURK_17320	0.0656
BALIKESIR	SYNOP_WMO_17150	0.0571
BINGOL	TURK_17203	0.0039
BODRUM	TURK_17290	0.1293
CANAKKALE	TURK_17112	0.0433
DALAMAN	SYNOP_WMO_17295	0.1939
EDIRNE	TURK_17050	0.0334
FINIKE	TURK_17375	0.1032
GIRESUN	TURK_17034	0.0311
HOPA	TURK_17042	0.0915
INEBOLU	TURK_17024	0.0075
MUGLA	TURK_17292	0.0232
SILIFKE	TURK_17330	0.0459
SINOP	TURK_17026	0.1463

3.1 Data Characteristics

Monthly average inflow of 384 observations and precipitation data obtained from 15 stations are analyzed based on certain characteristics which are required. In Table 3.2, basic descriptive statistics can be found for both inflow and precipitation data.

Table 3.2: Descriptive Statistics

Property	Arkun Inflow Series	Precipitation Series
Mean	62.230	77.910
Standard Deviation	68.071	60.833
Skewness	1.7468	1.3202
Kurtosis	2.3573	1.7131
Shapiro - Wilk	0.7170	0.8737
Jarque - Bera	287.85	160.73

According to descriptive statistics of series given in 3.2, distributions are not normal and Pearson test [28], whose null hypothesis is that the data are sampled from normal distribution, gives that series do not come from normal distribution.

Moreover, histograms of Arkun inflow and precipitation series in Figure 3.2 show also that they are not normally distributed. Because of the log-normal characteristics of the original data and to normalize distributions of series, logarithm of the data will be used in modelling.

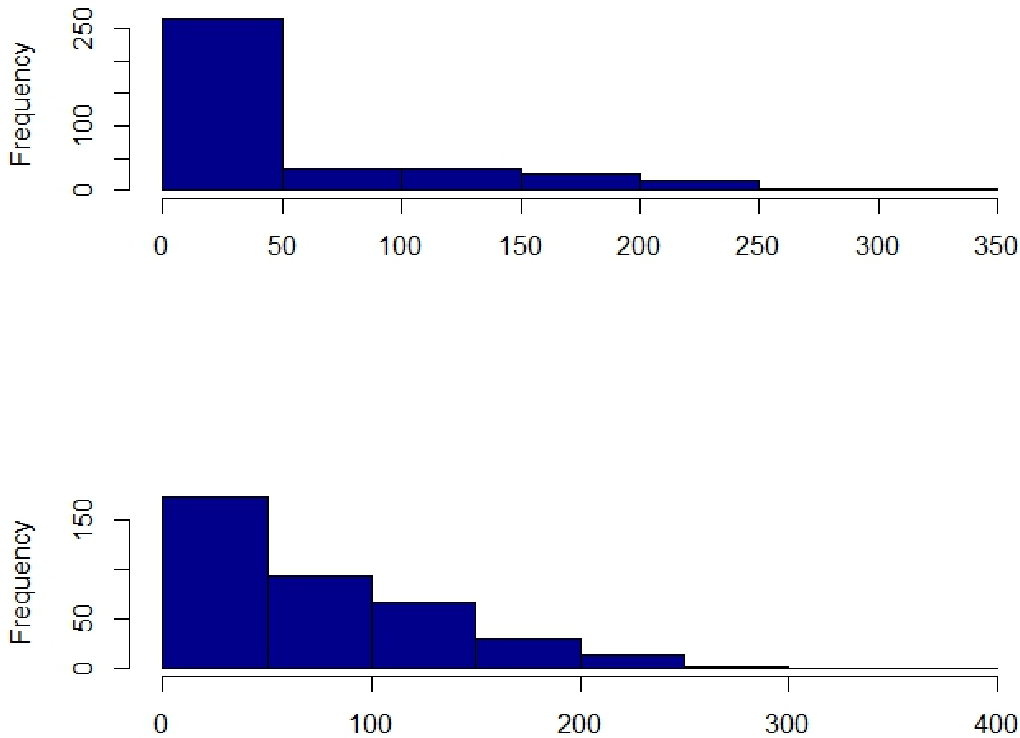


Figure 3.2: Histograms of Inflow and Precipitation Series

Analyzing stationarity of data is very essential for time series modelling. Stationarity means that the probability distribution is the same for all starting values of t. If it is not stationary, transformation of non-stationary data into stationary one is an important step in time series approach (Box and Jenkins, 1976) [24].

For stationarity of main data, as an unit root test, Augmented Dickey Fuller Test (Dickey and Fuller, 1979) is applied and also KPSS Test (Kwiatkowski, Phillips, Schmidt and Schin, 1991) is used as an complement of the test for the presence of unit root. For the time series modelling, data should be stationary and so not include a unit-root.

ADF (Augmented Dickey Fuller) Test [13] refers to

$$\Delta X_t = \alpha + \beta t + \theta X_{t-1} + \varphi_1 \Delta X_{t-1} + ... + \varphi_{p-1} \Delta X_{t-p+1} + \varepsilon_t$$

where p is the lag order of autoregressive process for time series, α is constant, β is the coefficient of the time trend and $\theta = 0$ is the null hypothesis of the ADF test.

$H_0 : \theta = 0$ (i.e. Time series data is not stationary and it should be differenced)

$H_1 : \theta < 0$ (i.e. Time series data is stationary and there is no need to make the data be differenced)

Table 3.3: ADF Test

	Dickey-Fuller	Lag order	p-value
Inflow Series	-13.4285	7	0.01
Precipitation Series	-13.6006	7	0.01

Since p-value is smaller than 0.05, the null hypothesis of the ADF test is rejected for both hydro inflow and precipitation series. This means that inflow and precipitation data do not have unit root, i.e. they are stationary according to ADF test.

Similarly, KPSS test is a commonly used unit root test to check stationary in time series (Kwiatkowski, et al., 1991) [22]. Its null hypothesis is the inverse of ADF test as follows;

$H_0 : \theta = 0$ (i.e. Time series data is stationary and there is no need to make the data be differenced)

$H_1 : \theta < 0$ (i.e. Time series data is not stationary and it should be differenced)

Table 3.4: KPSS Test

	KPSS Level	Truncation Lag Parameter	p-value
Inflow Series	0.051	4	0.1
Precipitation Series	0.044	4	0.1

Since p-value is greater than 0.05, the null hypothesis of the KPSS test is accepted. This means that both hydro inflow and precipitation data do not have a unit root, i.e. they are stationary according to also KPSS test.

As a result of stationarity tests, at the 0.05 level of significance, we can say that Arkun inflow and precipitation data are stationary and we do not need differencing the inflow and precipitation data.

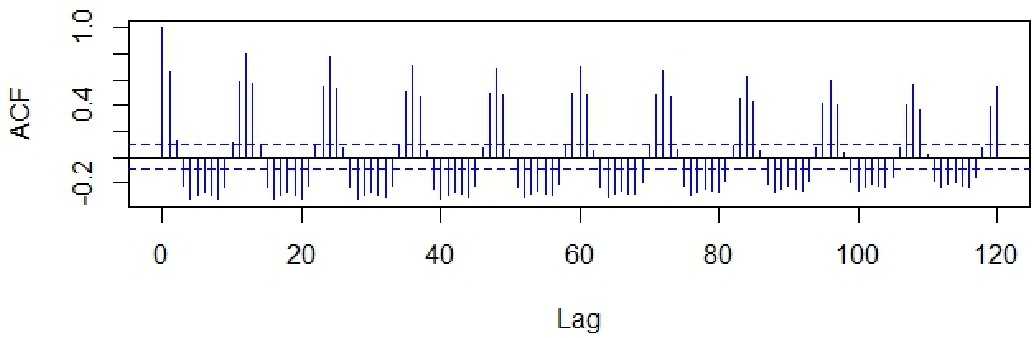
For the time series process $\{X_i, \forall i \in \tau\}$, the autocorrelation function (ACF) is an important indicator of the serial correlation and is represented as

$$\rho(s,t) = \frac{\gamma(s,t)}{\sqrt{\gamma(s,s)\gamma(t,t)}} \tag{3.1}$$

for all $s, t \in \tau$ where the autocovariance function is defined as follows (Shumway, 2011) [31]. Here,

$$\gamma(s,t) = cov(X_s, X_t) = E[(X_s - \mu_s)(X_t - \mu_t)] \tag{3.2}$$

The ACF is used to measure the linear predictability of X_t by regarding just X_s. By taking $s = t+k$, we can say that this is the ACF of lag k of the time series. A 10-years-lag autocorrelation function in Figure 3.3 of original inflow data represents the annual hydrological and seasonal cycle clearly. Moreover, ACF of precipitation series Figure 3.3 also shows the seasonality in data. Therefore, the ACF graph can be interpreted as that in forecasting model, seasonality must be taken into the account.



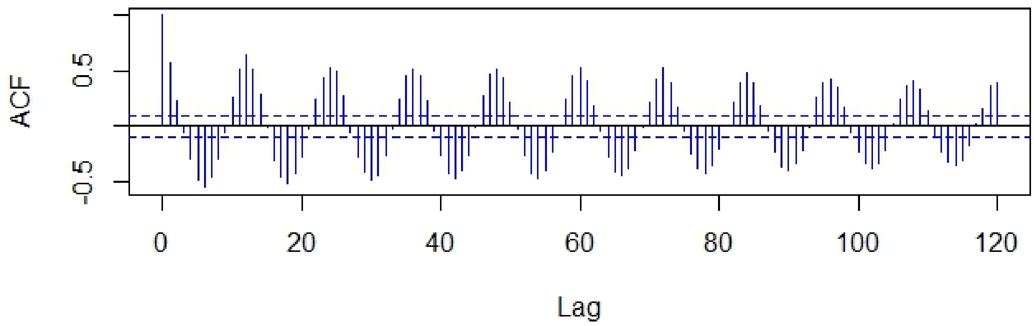


Figure 3.3: Autocorrelation Functions of Inflow and Precipitation Series

The partial ACF (PACF) gives the autocorrelation between X_t and X_{t+k} with the linear dependence of X_t on X_{t+1} through X_{t+k-1} removed [4]. Figure 3.4 shows that both inflow and precipitation series are partially autocorrelated for 12 months - 1 year.

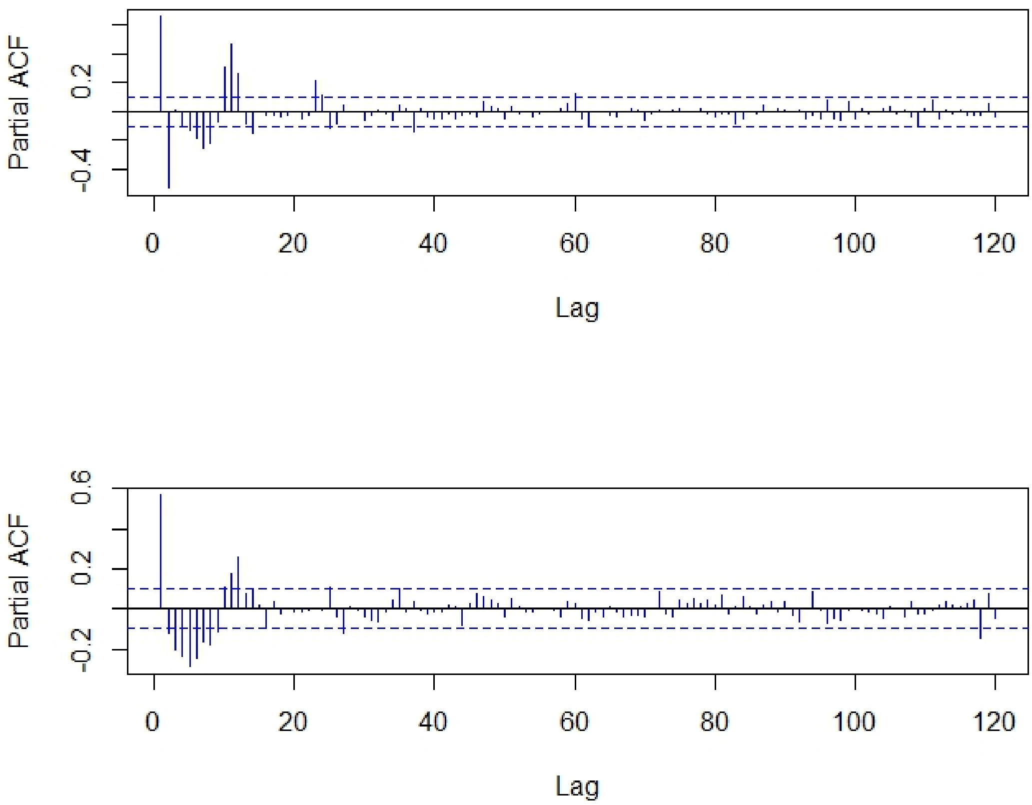


Figure 3.4: Partial Autocorrelation Functions of Inflow and Precipitation Series

3.2 Seasonal ARIMA Model with Exogenous Variable

When seasonal component that repeats every s observations is observed in the series, a seasonal time series model is appropriate for forecasting. Especially, monthly hydro inflow data has seasonal component obviously regarding hydro inflow cycle, especially from beginning of October to end of September.
To capture seasonality component, SARIMA models are incorporated.

Let $x_t \sim$ ARIMA$(p, q, d)x(P, Q, D)_s$, then

$$ARIMA(p, q, d)x(P, Q, D)_s \ ; \ \Phi_P(B^s)\phi(B)\nabla_s^D\nabla^d x_t = \Theta_Q(B^s)\theta(B)\omega_t \quad (3.3)$$

$$\Phi_P(B^s) = 1 - \Phi_1 B^s - \Phi_2 B^{2s} - ... - \Phi_P B^{Ps}$$
$$\Theta_Q(B^s) = 1 + \Theta_1 B^s + \Theta_2 B^{2s} + ... + \Theta_Q B^{Qs}$$

where ω_t is the usual white noise process. The ordinary autoregressive and moving average components are represented by polynomials $\phi(B)$ and $\theta(B)$ of orders p and q respectively, and seasonal autoregressive and moving average components by $\Phi_P(B^s)$ and $\Theta_Q(B^s)$ of orders P and Q, and ordinary and seasonal difference components by $\nabla^d = (1-B)^d$ and $\nabla_s^D = (1-B)^D$ [31].

When we use SARIMA model with exogenous variables $y_{(k,t)}$, the mathematical representation becomes;

$$ARIMA(p,q,d)x(P,Q,D)_s \;\; ; \;\; \Phi_P(B^s)\phi(B)\nabla_s^D\nabla^d z_t = \Theta_Q(B^s)\theta(B)\omega_t \tag{3.4}$$

$$z_t = x_t - \beta_1 y_{(1,t)} - \beta_2 y_{(2,t)} - ... - \beta_b y_{(b,t)}, \tag{3.5}$$

This model is called as SARIMAX, as it contains exogenous variable. In SARIMAX model, z_t is the auto-correlated regression residuals where x_t is the observed output at time t. $y_{(k,t)}$ represents the kth exogenous input variable at time t and b is the number of exogenous input variables. The ordinary autoregressive and moving average components are represented by polynomials $\phi(B)$ and $\theta(B)$ of orders p and q respectively.

3.3 Parameter Estimation and Inflow Scenarios

Based on the ACF - PACF properties plausible models are fitted and the best fitting model is selected based on AIC statistics.

Table A.2 shows that ARIMA$(3,0,0)(4,0,0)_{12}$ with drift is the best one.

Best fit in Figure 3.5, obtained by model ARIMA$(3,0,0)(4,0,0)_{12}$ with drift, is illustrated for the in-sample data. The model has $\sigma^2 = 0.07219$ and log likelihood = -55.41. AIC = 130.82, AICc = 131.41 and BIC = 170.32. In the best model, ordinary and seasonal autoregressive parameters are found as $p = 3$ and $P = 4$ with $s = 12$.

After finding the best model fit for inflow whose parameters are demonstrated in Table 3.5 where exogenous is precipitation series, we analyze if the model is appropriate by performing residual analyses. By regarding the autocorrelation function of residuals in Figure 3.6 and applying Bartels test whose null hypothesis is that the sequence is distributed randomly, we can say that residuals are distributed in random manner [2].

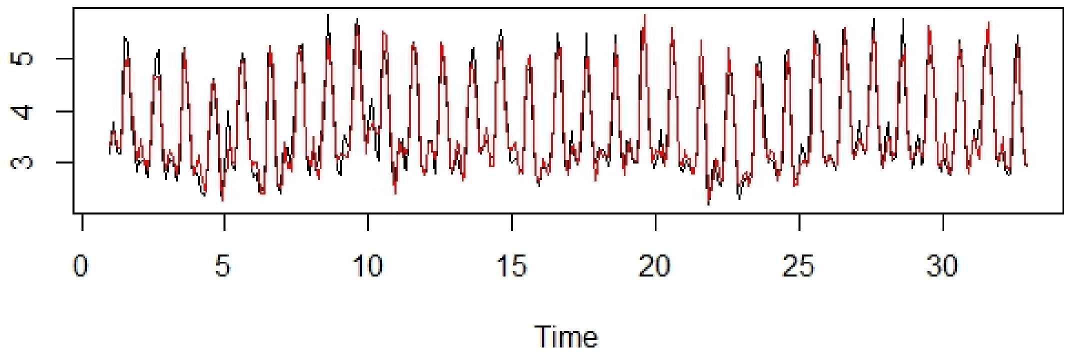


Figure 3.5: Model Fit

Table 3.5: The Parameter Estimations of SARIMAX

	Coefficient	s.e.	P-values
AR1	0.6811	0.0529	0.000000e+00
AR2	-0.0934	0.0628	1.371190e-01
AR3	0.1632	0.0525	1.902464e-03
SAR1	0.1233	0.0528	1.954585e-02
SAR2	0.3243	0.0491	3.841327e-11
SAR3	0.3387	0.0506	2.172418e-11
SAR4	0.1750	0.0546	1.347480e-03
Intercept	3.6600	0.6410	1.132342e-08
Exogenous	0.0137	0.0229	5.490811e-01

The residuals given in Equation (3.6),

$$e_t(h) = Y_{t+h} - \hat{Y}_t(h) \tag{3.6}$$

should be checked for efficiency reasons [34]. Several error measures listed below are calculated and presented in Table 3.6.

Mean Error (ME):

$$\frac{1}{n}\sum_{t=1}^{n} e_t$$

Root Mean Square Error (RMSE):

$$\sqrt{\frac{1}{n}\sum_{t=1}^{n}(\hat{Y}_t - Y_t)^2}$$

Mean Absolute Error (MAE):

$$\frac{1}{n}\sum_{t=1}^{n}|(\hat{Y}_t - Y_t)|$$

Mean Absolute Percentage Error (MAPE):

$$\frac{1}{n}\sum_{t=1}^{n}\frac{|(\hat{Y}_t - Y_t)|}{\hat{Y}_t} \times 100$$

Table 3.6: Training Set Error Measures

Error Measure	Value
ME	0.000304
RMSE	0.2686861
MAE	0.2028392
MPE	-0.508461
MAPE	5.471788

To minimize error between actual and forecasted values, seasonal time series model applies the theoretical basis of minimum mean squared error [21].

To check if the serial correlation disappears, we apply Ljung - Box test to ensure that residuals are independently distributed. The *Q* statistics given in equation 3.7,

$$Q = n(n+2)\sum_{h=1}^{k}\frac{\hat{\rho}_h^2}{n-h} \tag{3.7}$$

where ρ_h is the autocorrelation at lag h, k is the number of tested lags and n is the sample size and whose distribution is Chi-square.

Since $\chi^2 = 8.835$, df = 11, p-value = 0.6371, the null hypothesis of Ljung - Box test is fail to be rejected by significance level of 0.05.

To check the normality of residuals, Shapiro - Wilk normality test (Shapiro and Wilk, 1965) is used. W statistics in equation 3.8 allows us to check whether the sample follows normal distribution.

$$W = \frac{\left(\sum_{i=1}^{n}\alpha_i y_i\right)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2} \tag{3.8}$$

where α_i are coefficients and come from a normal distribution by regarding means, variances and covariances of the order statistics of a sample whose size is equal to n.

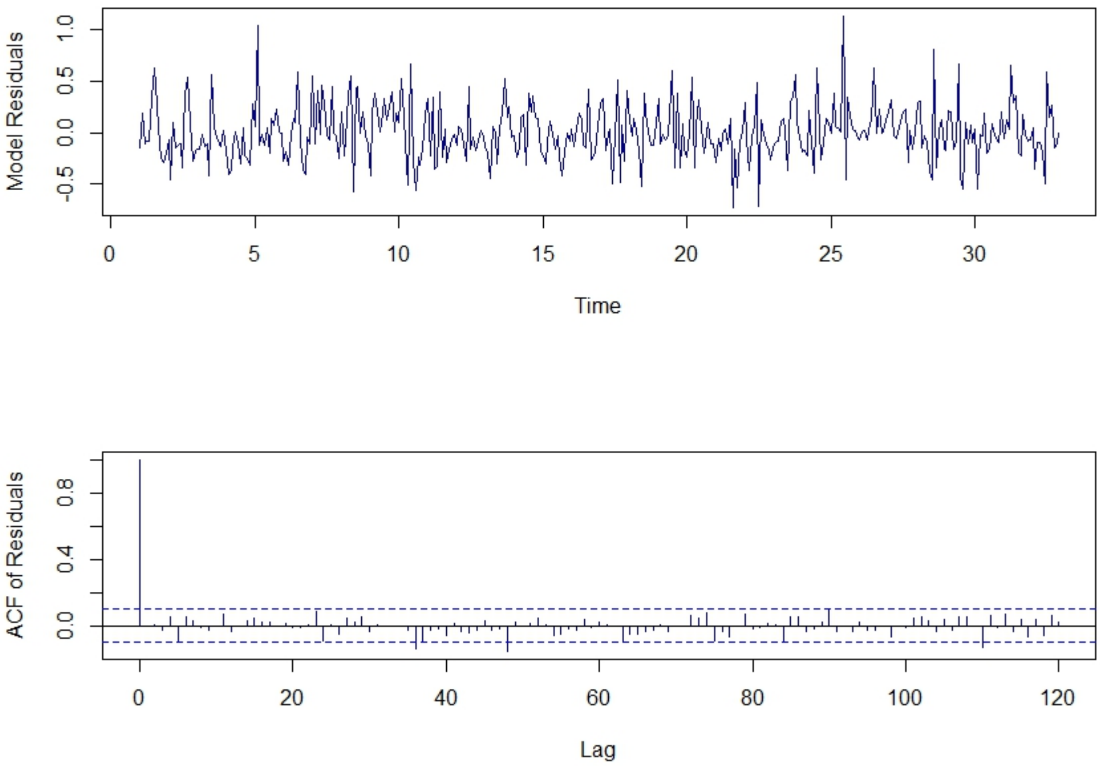


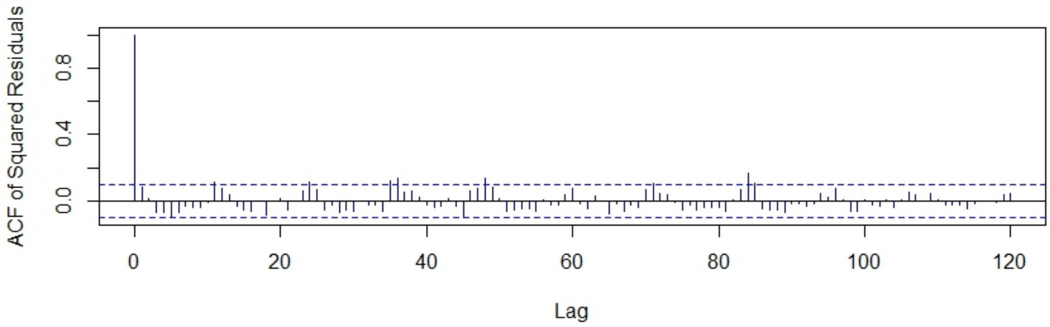
Figure 3.6: Residuals of the Estimated Model

According to Shapiro - Wilk normality test, residuals are not normally distributed and also as seen in Q-Q plots in Figure 3.8 [29]. Furthermore, Pearson test justifies non-normality.

Residuals are accepted as t distributed as the result of analyses as shown in Figure 3.9. By using the parameters given in Table 3.7, we estimate parameters of t ditribution [10].

Table 3.7: Statistics of Residuals

	Residuals	T-Residuals Est.	T-Residuals sd
Mean	0.0003039	-0.0135316	0.01346685
Standard Deviation	0.2690365	0.22650388	0.01550268
df	-	6.61952827	2.49587810
loglik	-	-34.59595	



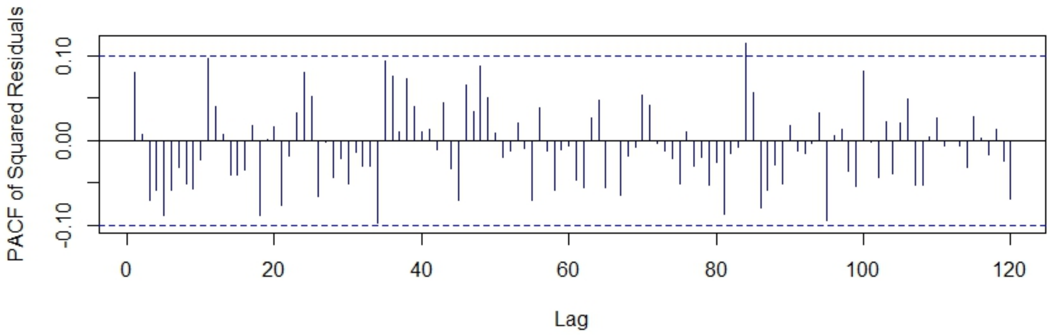


Figure 3.7: ACF and PACF of Squared Residuals of the Estimated Model

To check the presence of the heteroscedasticity, autocorrelation and partial autocorrelation functions of squared residuals, shown in Figure 3.7, are analyzed. Furthermore, we applied Lagrange Multiplier (LM) test for autoregressive conditional heteroscedasticity (ARCH) for model residuals and null hypothesis of test, which is no ARCH effects, is accepted according to p-value test for 0.05 significance level.

By using SARIMAX model for inflow data, we obtain monthly point forecast which is shown in Figure 3.10. On the other hand, randomness of residuals are proven and parameters of t distributed fit are estimated to generate inflow scenarios. Based on fitted model, a simulation analysis is performed the 100 inflow scenarios. It is clear that hydrological pattern is observed in point forecast of 12-months inflow and generated inflow simulations. Among these scenarios shown in Figure 3.11, 15 inflow scenarios, that are ranked in 80% confidence interval of point forecast, have been used in optimization model.

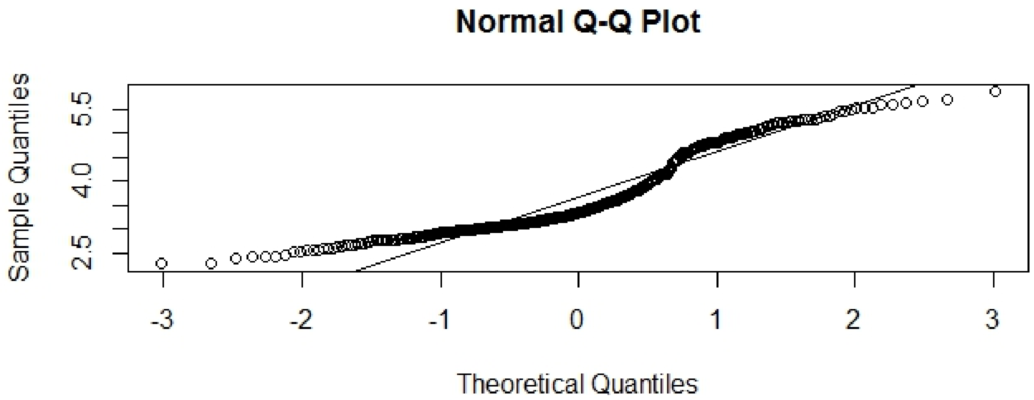


Figure 3.8: Q - Q Plot to Test Normality of Residuals

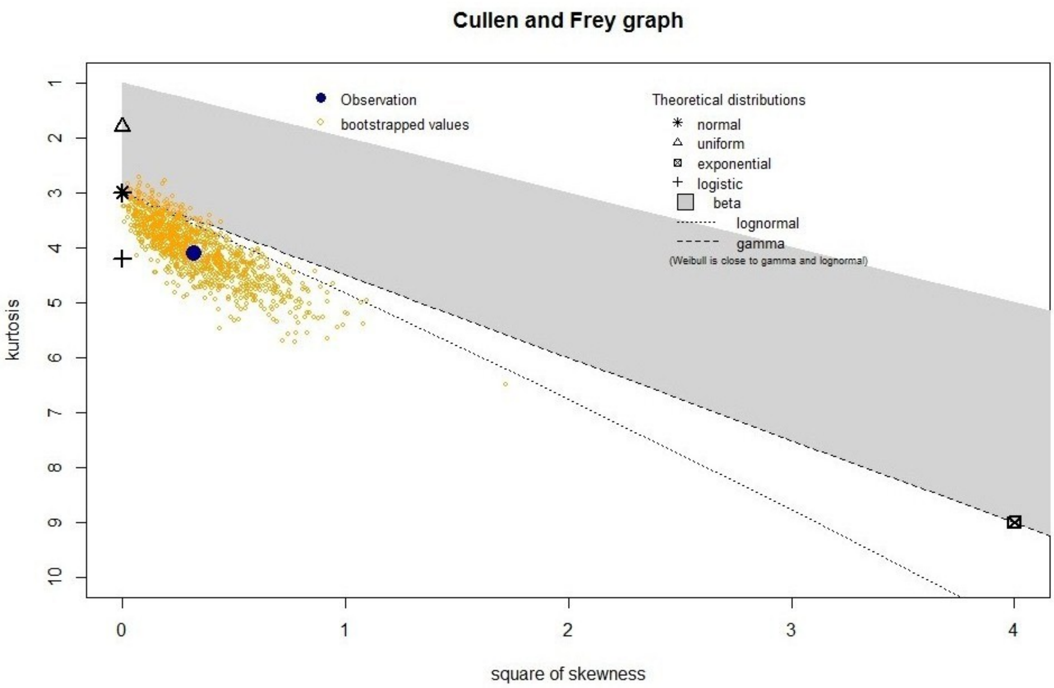


Figure 3.9: Cullen and Frey Graph of Residuals

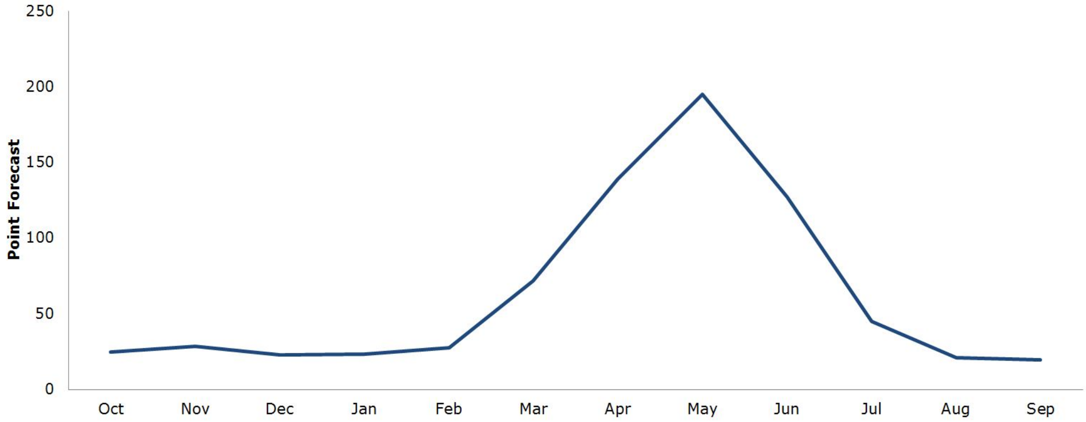


Figure 3.10: Point Forecast by using SARIMAX Model (m^3/s)

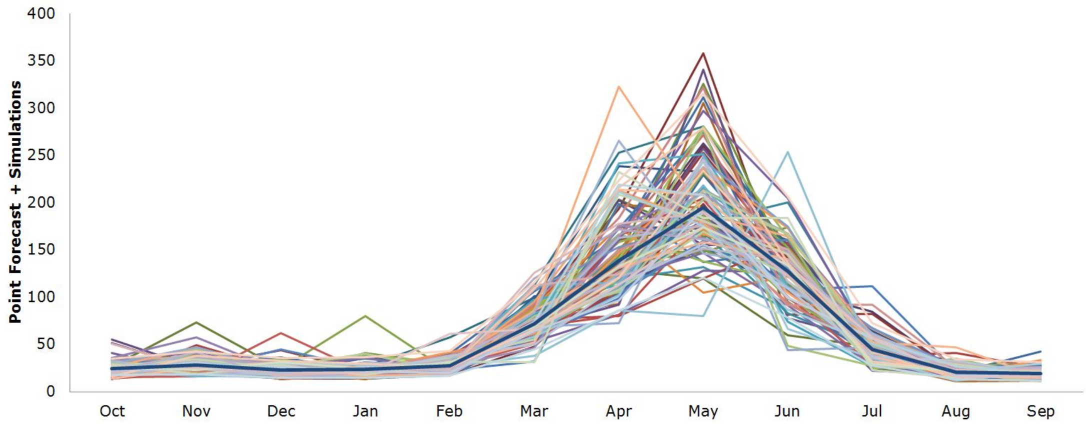


Figure 3.11: Inflow Scenarios (m^3/s)

CHAPTER 4

VIRTUAL POWER PLANT (VPP) PRICING

Virtual Power Plant requires also the determination of price scenarios which constitute the input for hydro optimization. In this section, by using the historical spot electricity market prices, price simulations are obtained.

4.1 Data Definition

For price simulations, historical hourly Day Ahead Market (DAM) prices observed between 2011 and 2015 are used. Among 43,800 observations, 29/02/2012 and national and religious holidays are extracted. For hourly shapes, we separate prices according to months by using the average of five years as shown in Figure 4.2. When we analyze the historical electricity prices, we confront some characteristics which are generally observed in electricity prices such as seasonal patterns, periodicities and spikes [7]. Because of the Natural Gas curtailment for electricity generation in the history, that is insufficiency in the NG supply for the NG power plants, some spikes (e.g., 2000 TL/MWh) are observed in Figure 4.1(a). To eliminate spikes, we use the distribution, mean and standard deviation of the historical data and obtain truncated hourly prices as seen in Figure 4.1(b). Two-sided three sigma deviations from average value of price series are used as truncation limits and truncated price series, which has statistics in Table 4.1, is obtained.

Table 4.1: Descriptive Statistics

	Hourly Prices	Truncated Hourly Prices
Minimum	0.0	0.0
1st Quarter	120.0	120.0
Median	150.0	150.0
Mean	145.5	145.1
3rd Quarter	177.0	177.0
Maximum	2000.0	292.0

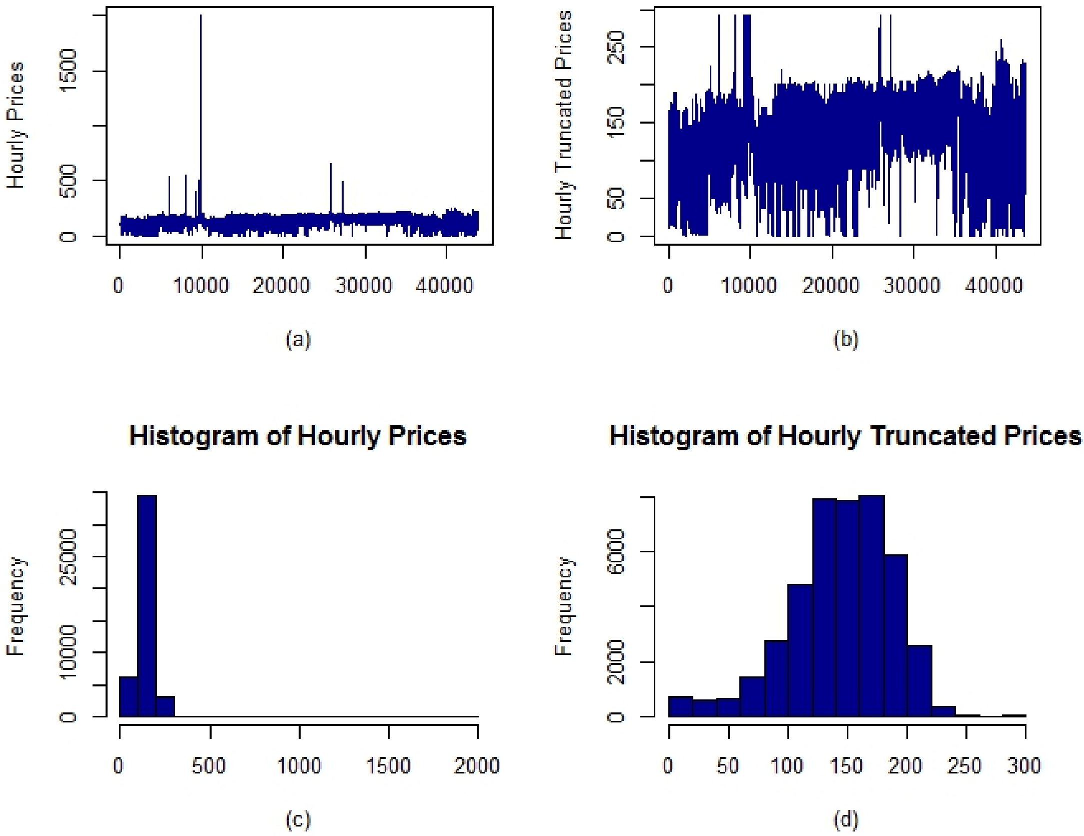


Figure 4.1: Hourly Prices vs. Truncated Hourly Prices

4.2 Price Scenarios

In this section, to generate 100 price scenarios displayed in 4.4, we generate 25 different price scenarios randomly for each month by regarding the monthly price distribution and for each one of these 25 scenarios, we simulated 4 hourly shape curves by using SARIMA model for deseasonalization and elimination of the periodicity. For these models, some important graphs are shown in detail at the end of Chapter and SARIMA model selections for each months are found in Appendix A. For each month, best SARIMA fit is obtained and after deseasonalization, by using the model noises, we combined monthly simulations by adding successively and price series for 8760 hours are generated.

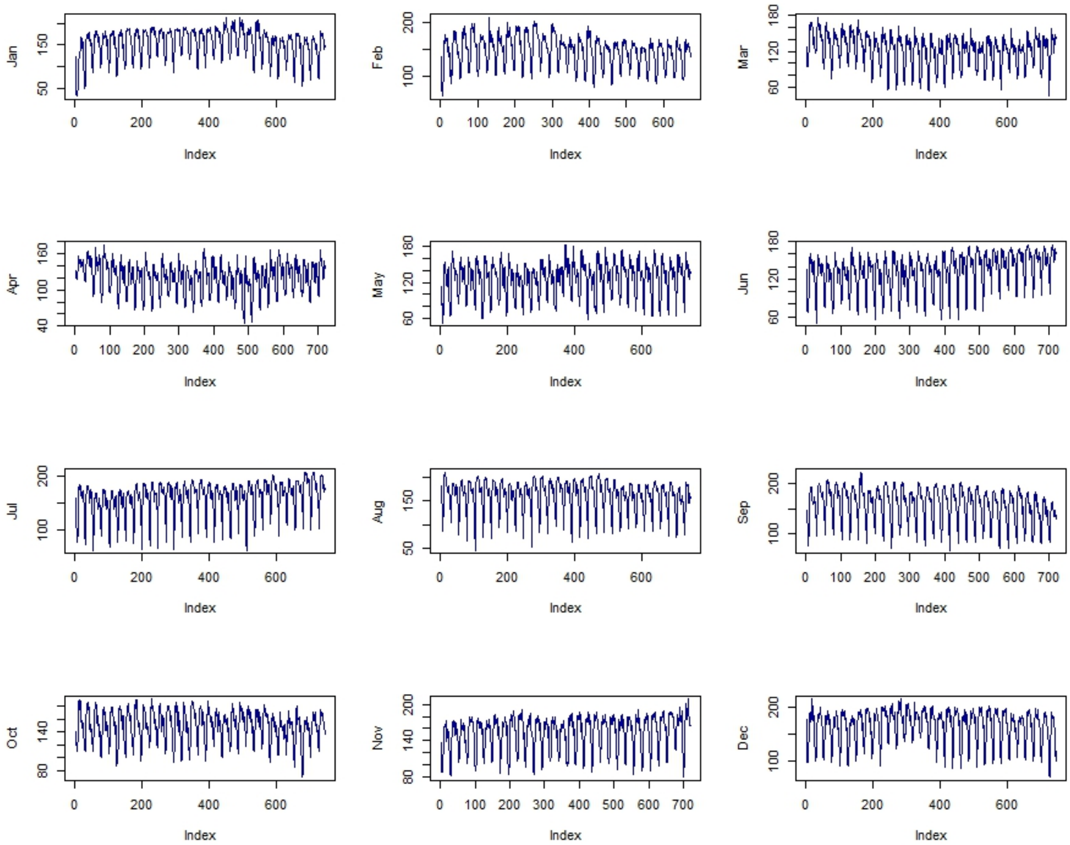


Figure 4.2: Hourly Price Shapes by using the Average of 5 Years (2011-2015)

4.3 Hydro Optimization

In this part, constraints in the optimization problem is defined and expressed for VHPP that will be priced in hourly granularity. For inflow to virtual reservoir, inflow scenarios generated in Chapter 3.3, are utilized. Moreover, price scenarios obtained as mentioned in previous section, will be used in optimization.

Hydro optimization problem is constructed by using volume levels of reservoir V_k, forecasted inflow series to reservoir I_k by means of integrated inflow forecasting model, market price simulations p_k, electricity generation G_k, marginal cost for VPP C_k and spillage of water S_k where $k = 1, ..., T$ denoting the time period. Spillage of water is an undesirable condition for the hydro optimization. Since it is favorable that the spillage should converge to zero, a penalty is used as a constraint [25].

The volume levels for each k is a function of a period ahead volume, inflow, electricity generation and spillage of water as given in Equation (4.1).

$$
\begin{aligned}
V_k &= V_{k-1} + I_k - G_k - S_k \\
V_{k-1} &= V_{k-2} + I_{k-1} - G_{k-1} - S_{k-1} \\
&\;\;. \\
&\;\;. \\
V_T &= V_{T-1} + I_T - G_T - S_T.
\end{aligned} \tag{4.1}
$$

Therefore, the objective function is

$$
\begin{aligned}
max \sum_{i=1}^{T} E[(p_i - C_i).G_i - M.S_i] \qquad (4.2) \\
s.t \quad 0 \leq G_i < G_{max} < G_{cap}, \\
0 \leq V_{min} \leq V_i < V_{max}, \quad i = 1, 2, ..., T,
\end{aligned}
$$

where M is sufficiently large, with the Initial and End Reservoir Levels V_1 and V_T, respectively. Maximum Level V_{max}, Minimum level V_{min}, Maximum generation G_{max}, Installed capacity G_{cap} and time horizon of the option $1, ..., T$ are given in any special contract as specific constraints.

Mixed Integer Linear Programming

Mathematical Programming is applied for the solution of an optimization problem which consists of a function of many parameters and is subject to a set of constraints. Mixed-Integer Linear Programming (MILP) is a member of the mathematical programming family, that restricts the decision variables to be integer during the selection of optimal solution as a process of minimizing or maximizing a linear function. The main steps of MILP can be summarized as identification of decision variables, definition of objective function and determination of constraints [18]. We apply MILP for optimization problem because of the fact that, dispatch problems have mixed linear-integer structure [15].

Hourly hydro optimization for defined VPP is constructed for a hydro year (8760 hours) by regarding the generated inflow and price scenarios under the given constraints by using MILP and Xpress Solver [16]. Because of the hourly bidding in the market, model is constructed discretely and it gives the optimized generation curve by regarding the objective function.

Generated 15 inflow $I_0, ..., I_T$ and 100 price $p_1, ..., p_T$ series for a time span, $T = 8760$ are employed in the optimization model. Price scenarios are ranged according to increasing annual average of series where inflow scenarios are arranged by considering the increasing annual sum. This means that, we expect the highest VPP value at the intersection of the last price scenario and the last inflow scenario among all scenarios.

For a hundred price and one inflow scenario, run time come out approximately 40 minutes. However, parallel runs are possible in Xpress Solver. Therefore, we obtain 1500 VPP values approximately in 2 hours.

4.4 VPP Pricing

MILP is the most flexible method to express real-world constraints in the optimization. Moreover, this approach is suitable to calculate option delta and implement the dynamic delta hedging for VPP [6].

The optimization outputs (Base Case) for 8760 hours according to given inflow and price scenarios for defined VPP with the assumptions as given in Table 4.2; In Base Case, we assume that V_{max} is 70.2 GWh by regarding the maximum stored energy of defined VPP, V_0 and V_T are assumed as equal to half of V_{max} as given in Table 4.2.

Table 4.2: Base Case - Optimization Assumptions

Initial Uptime	1
Minimum Uptime	1
Year End	8760
Days	365
Initial Level (GWh)	35.1
End Level (GWh)	35.1
Maximum Level (GWh)	70.2

For each VPP value, $Z_{i,p}$, that is obtained for inflow scenarios $i = 1, ..., m$ and price scenarios $p = 1, ..., n$, fair value of VPP is calculated as in equation (4.3)

$$VPP = \frac{1}{m.n} \sum_{p=1}^{n} \sum_{i=1}^{m} Z_{i,p}. \tag{4.3}$$

For VPP pricing as a real option, we need to consider extra concepts. Intrinsic and extrinsic values are some of these. According to philosophers, *"Intrinsic value of something means the value that something has "in itself," or "for its own sake," or "as such," or "in its own right." Extrinsic value is value that is not intrinsic"* [36].

Under the light of definition of intrinsic and extrinsic values of a hydro power plant, we interpret the calculations of these values as follows: VPP value for the expected market price curve gives the intrinsic value of VPP. Moreover, by subtracting intrinsic value from fair value of VPP, an extrinsic value for VPP is attained.

According to each price and inflow scenario, we obtained VPP values in TL that are given in Figure 4.5 for Base Case constraints as a surface. Moreover, to demonstrate the sensitivity of VPP values, for the first price scenario, change in VPP values according to inflow scenarios can be observed in Figure 4.6 and for the first inflow scenario, change in VPP values according to price scenarios can be observed in Figure 4.7. The minimum VPP value is gotten at the intersection of the first price and the first inflow scenarios. As mentioned before, price scenarios are sorted ascendingly according to annual price average. Whereas, inflow scenarios are sorted ascendingly according to annual total inflow amount. Therefore, we expect the minimum value at the intersection of first scenarios and the maximum value at the intersection of last scenarios. In Figure 4.5, the inflow scenario axis shows that higher inflow level leads to higher VPP values as expected. On the other side, when we analyze VPP values according to price scenario axis, we conclude that monthly price scenarios are more significant than shape scenarios for the change in value.

4.5 Volume at Risk

For a hydroelectricity power plant, we have both price and volume risks. Price movements affect the value of generated electricity and this must be taken into the account in risk management point of view [17]. We need to determine between which levels electricity prices can oscillate and in a defined confidence interval, to which level the value of VPP can go down.

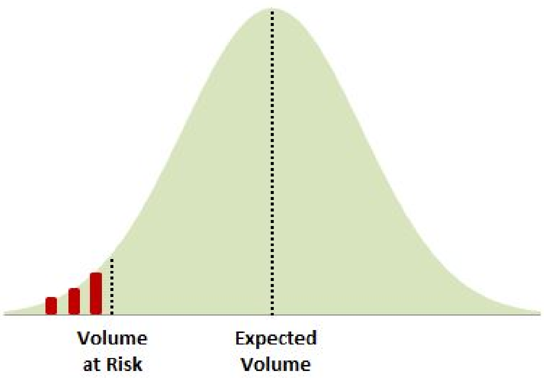


Figure 4.3: Volume at Risk Demonstration

On the other hand, Volume at Risk concept is expressed to define risky volume that are calculated by means of the distribution of VPP values obtained from optimization model. By considering the distribution, in 95% confidence interval, deviation from average value is regarded, e.g., 1.645 is the coefficient of subtracted deviation as seen in Figure 4.3.

By using this methodology, we could calculate Volume at Risk for each optimization case. Inspiring from the philosophy of VaR, we propose the similar measure, called Volume at Risk (VoaR).

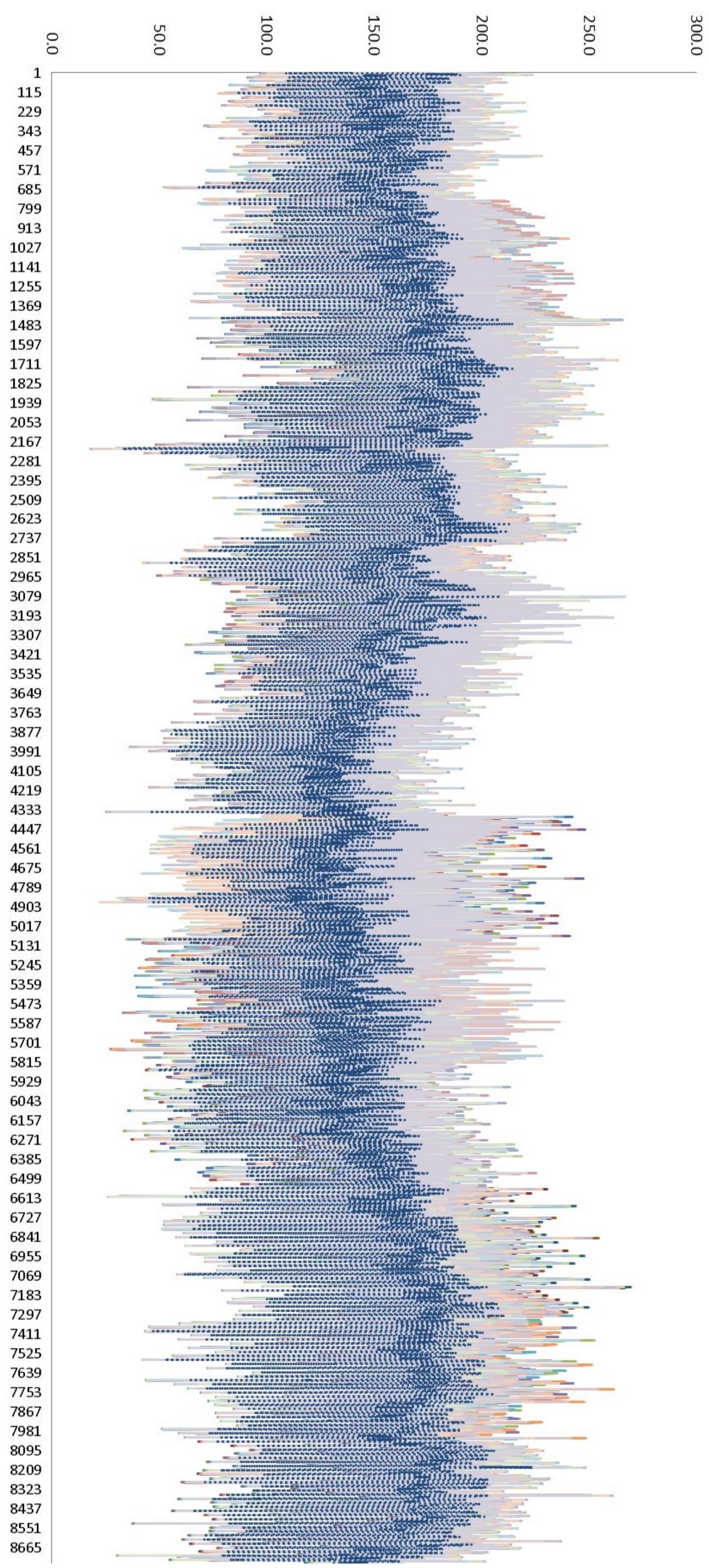


Figure 4.4: Price Scenarios (TL/MWh)

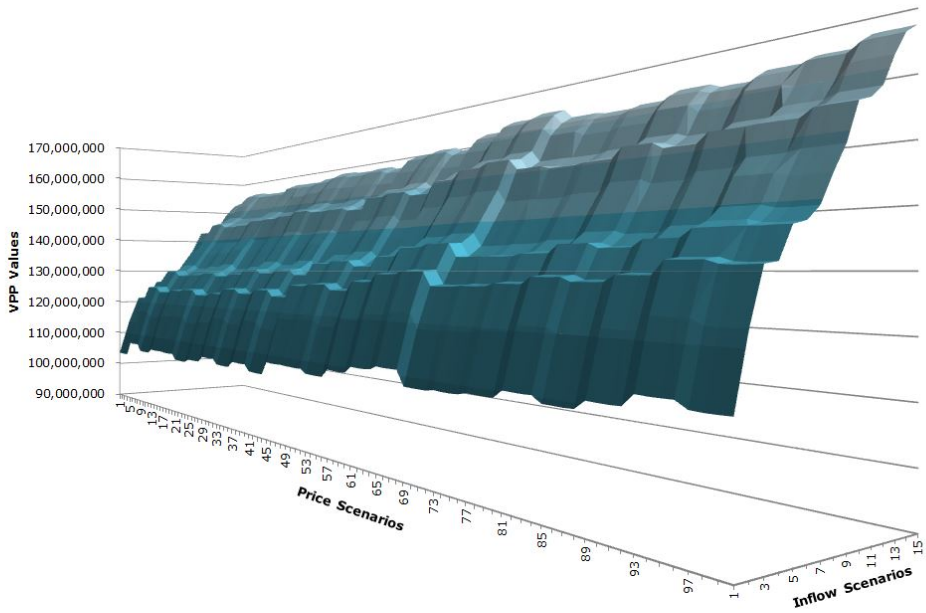


Figure 4.5: Surface of VPP Values for Base Case

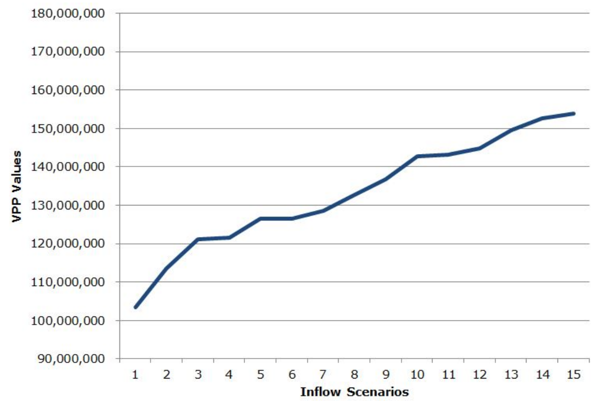


Figure 4.6: VPP Values according to Inflow Scenarios

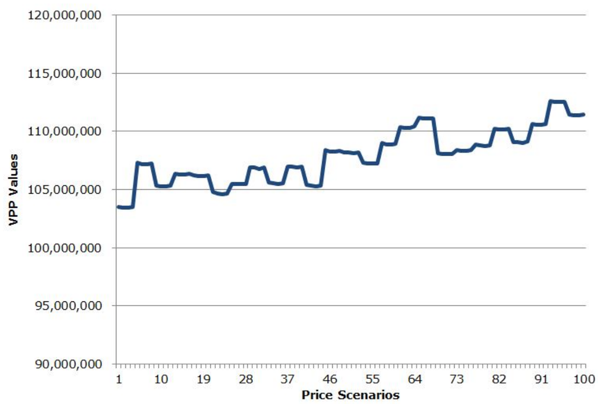


Figure 4.7: VPP Values according to Price Scenarios

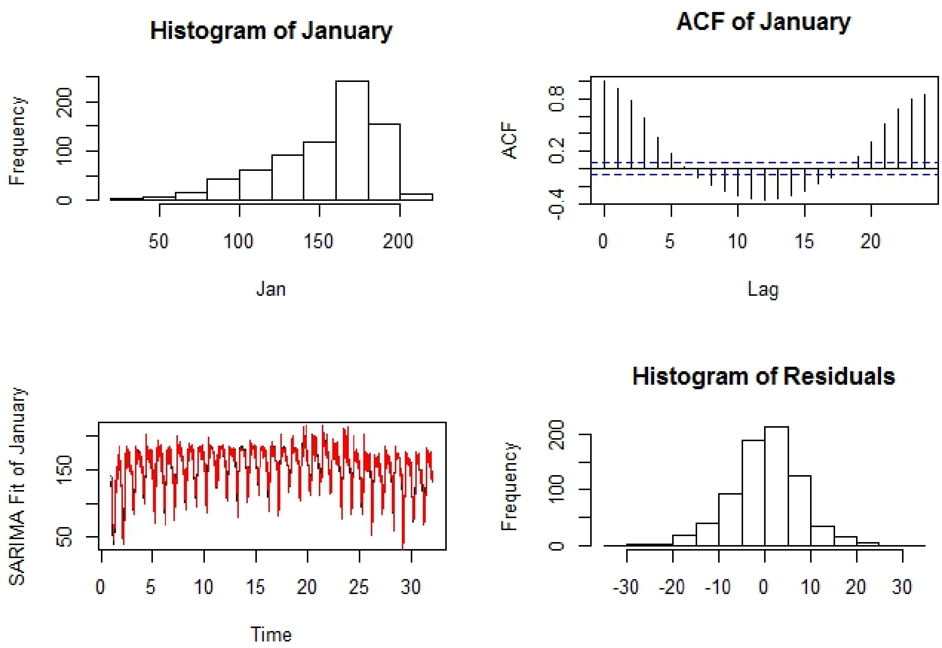


Figure 4.8: Some Important Plots of January Price and Its Modeling

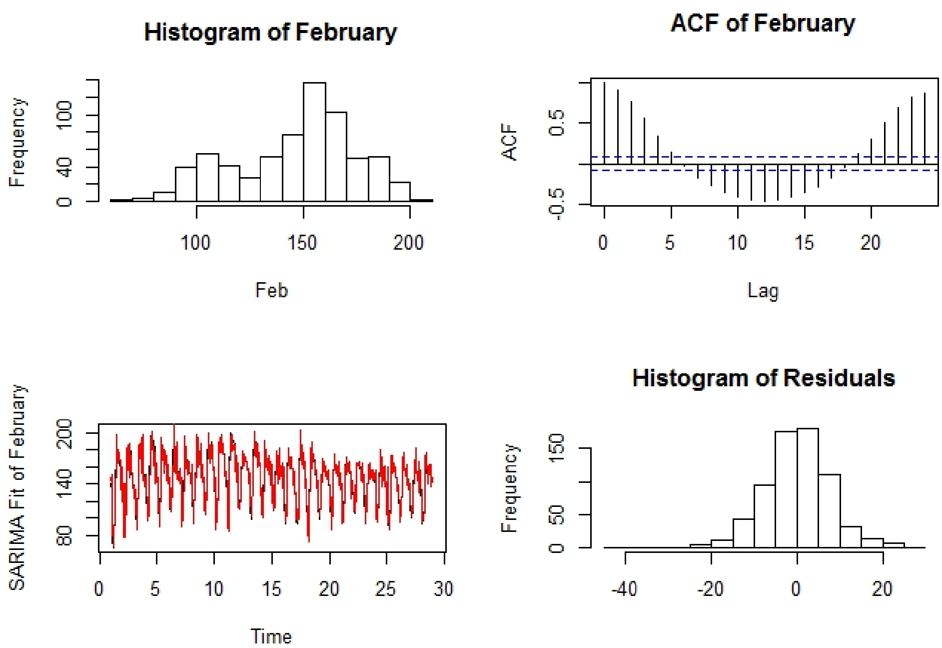


Figure 4.9: Some Important Plots of February Price and Its Modeling

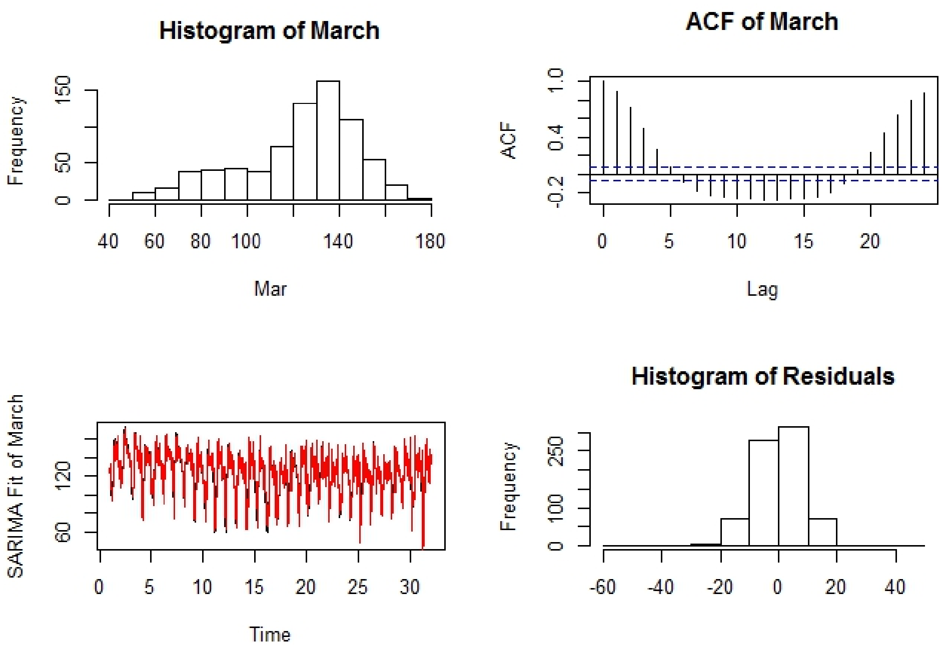


Figure 4.10: Some Important Plots of March Price and Its Modeling

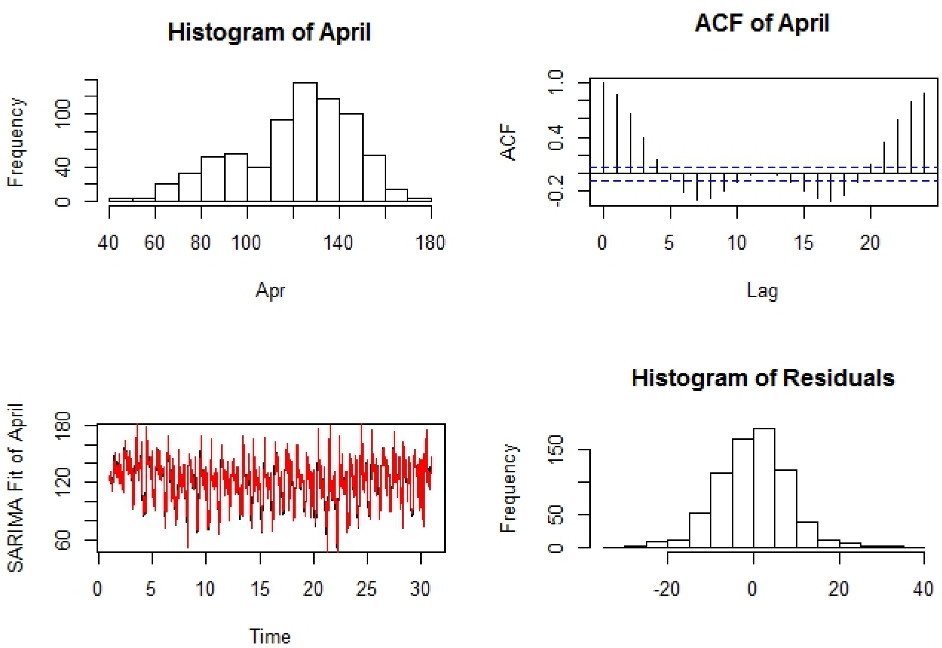


Figure 4.11: Some Important Plots of April Price and Its Modeling

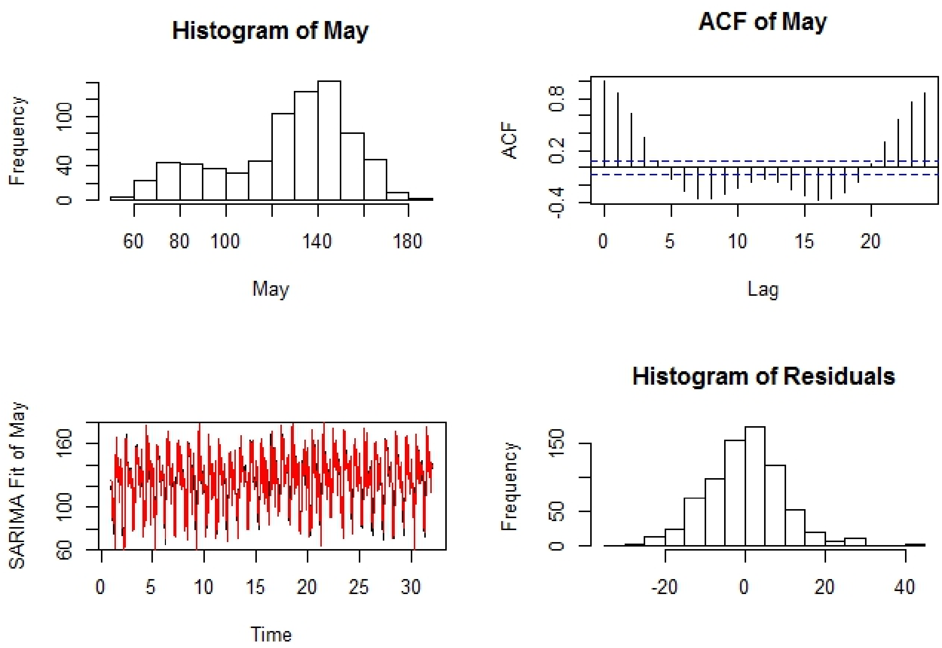


Figure 4.12: Some Important Plots of May Price and Its Modeling

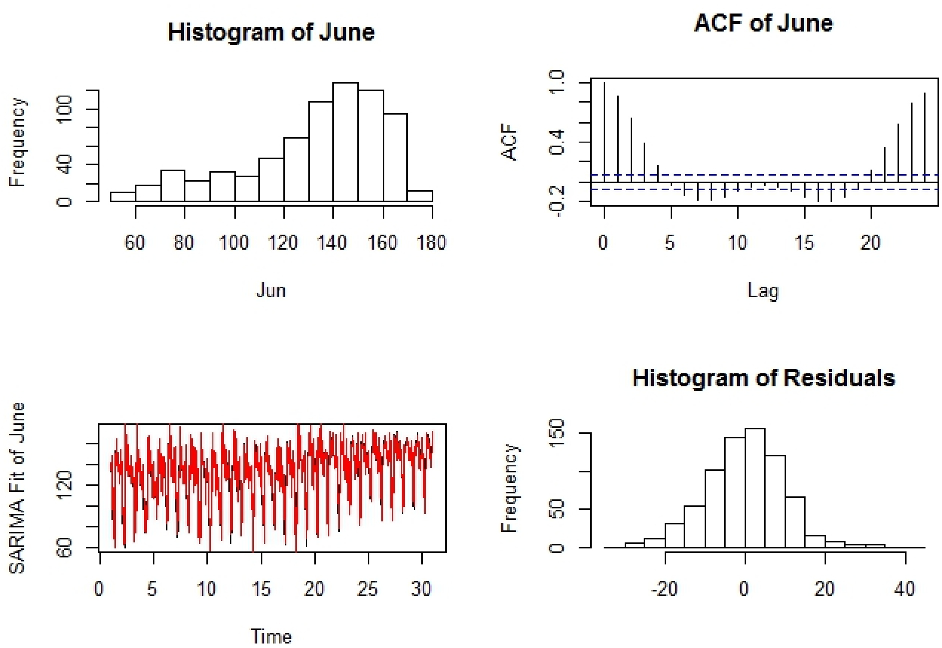


Figure 4.13: Some Important Plots of June Price and Its Modeling

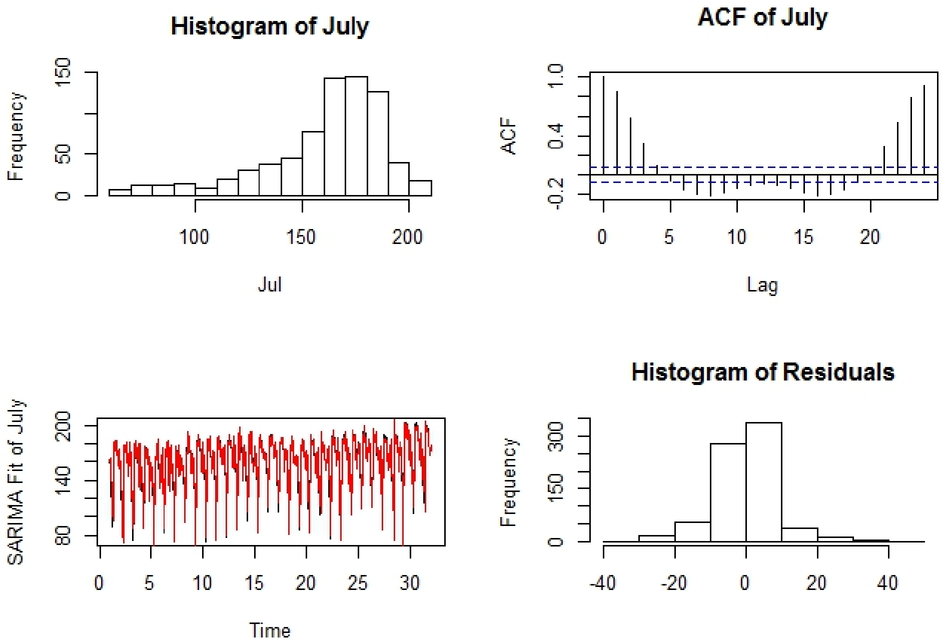


Figure 4.14: Some Important Plots of July Price and Its Modeling

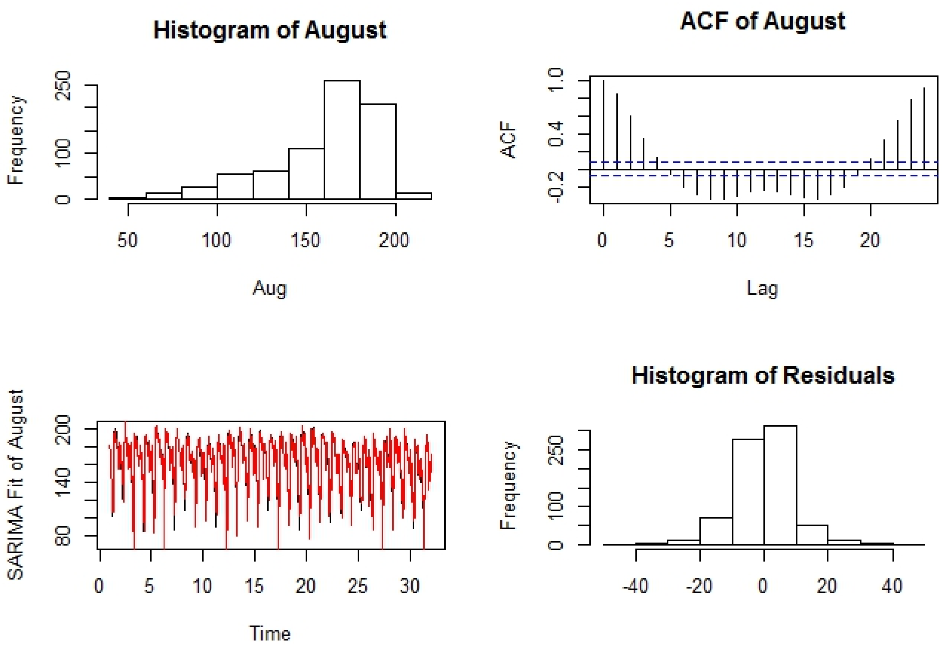


Figure 4.15: Some Important Plots of August Price and Its Modeling

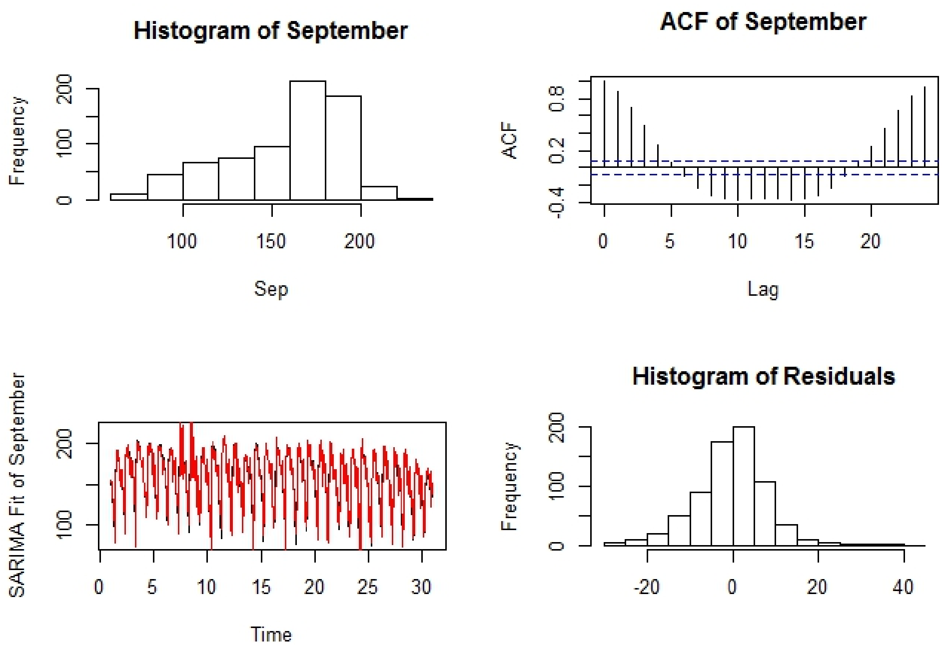


Figure 4.16: Some Important Plots of September Price and Its Modeling

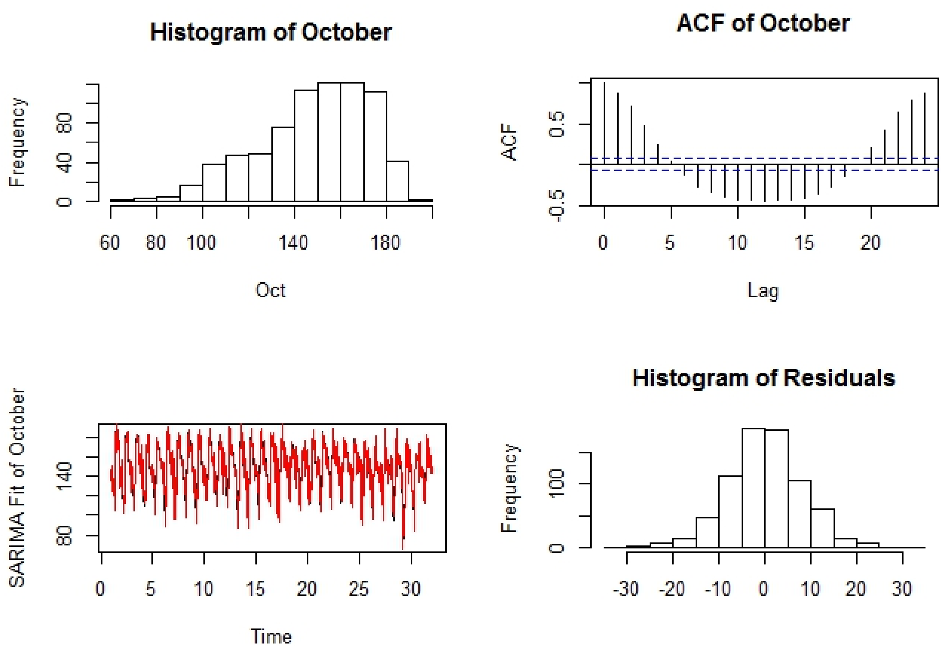


Figure 4.17: Some Important Plots of October Price and Its Modeling

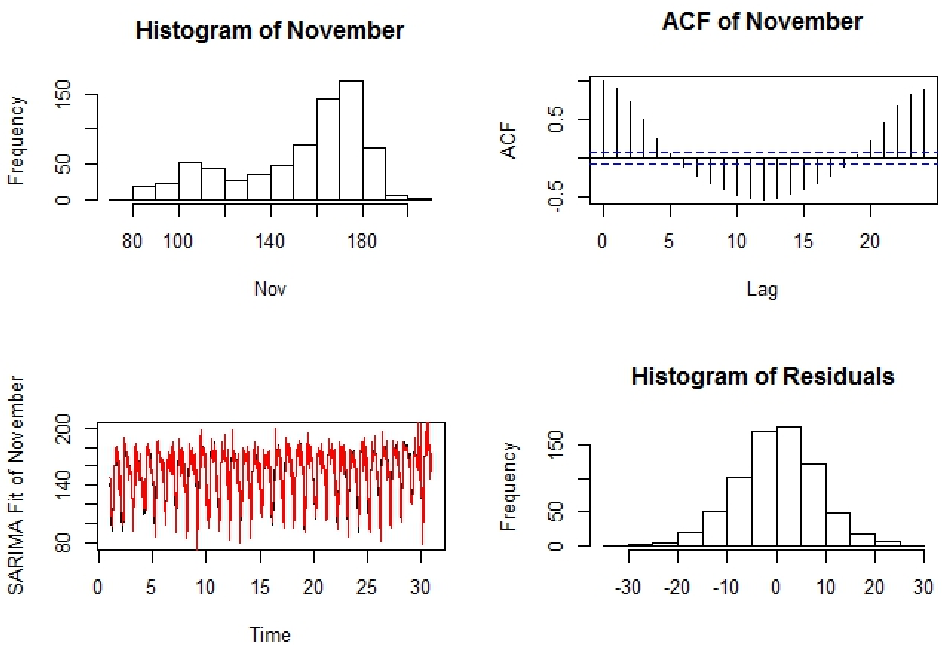


Figure 4.18: Some Important Plots of November Price and Its Modeling

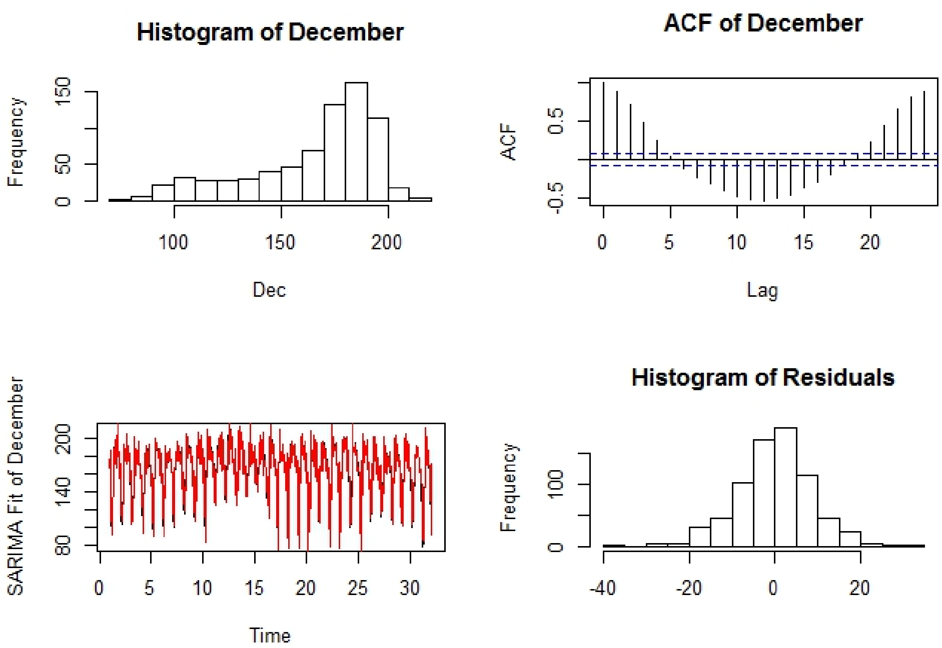


Figure 4.19: Some Important Plots of December Price and Its Modeling

CHAPTER 5

VARIOUS CASES IN HYDRO OPTIMIZATION

As the scenarios are derived and quantified for Base Case, some variations on the parameters are selected and categorized in 4 groups to illustrate the sensitivity of the optimization modelon VPP values. The outline of the cases are presented in Table 5.1.

Table 5.1: Parameter Values for Base and Other Cases

	Base Case	Case 1	Case 2	Case 3	Case 4
Initial Uptime	1	1	1	1	1
Minimum Uptime	1	1	1	1	1
Year End	8760	8760	8760	8760	8760
Days	365	365	365	365	365
Initial Level (GWh)	35.1	0	17.6	0	35.1
End Level (GWh)	35.1	0	17.6	0	35.1
Maximum Level (GWh)	70.2	70.2	35.1	35.1	70.2
Operating Hours	All	All	All	All	Peak

5.1 VPP Values Under Case 1

In Case 1, we assumed that V_{max} is same as Base Case, on the other hand, V_0 and V_T are equal to zero as given in Table 5.1. This implies that VPP is defined as empty at the initial time, so generation is possible by means of the inflow for initial time regardless the price levels.

Optimization results with these assumptions are shown in Figure 5.1. In this case, the aim is to compare the value of the defined hydro virtual power plant in case of that initial and end reservoir levels are zero, with the Base Case results. Since for the initial generation we need to inflow and price scenarios stay the same, we expect that VPP values in Case 1 are lower than Base Case. Thus, Figure 5.1 also shows that VPP values are lower than ones in Figure 4.5.

Moreover, another difference in Case 1 from Base Case, increase in inflow amount cause sharper increases in VPP values. On the other hand, in lower price levels, VPP values are smoother and have similar sight with Base Case, but at the higher price levels, because of the generation decision (holding water for the higher priced hours or generating electricity), transitions between price scenarios are also sharper than Base Case. Again in this case, shape scenarios creates small differences for VPP values.

5.2 VPP Values Under Case 2

In Case 2, we assume that V_{max} is the half of Base Case, and V_0 and V_T are equal to the half of V_{max} as given in Table 5.1.

Optimization results with these assumptions are shown in Figure 5.2. In this case, the aim is to compare the value of the defined hydro virtual power plant in case of that maximum reservoir level is half of the maximum reservoir level in Base Case. Moreover, initial and end reservoir levels in Case 2 are also the half of initial and end reservoir levels in Base Case.

In Case 2, since maximum reservoir level is lower, holding water for higher priced hours is constrained by the half of maximum reservoir level. This means that we expected also lower VPP values than Base Case. By comparing with Case 1, because of the non-zero initial and end levels, smoother surface for VPP values can be seen in Figure 5.2. However, in this case also, at the intersection of higher price and higher inflow scenarios, VPP values are increasing progressively.

Change in constraints leads to change in intrinsic and extrinsic values of VPP, despite the price scenarios are same as in Base Case.

5.3 VPP Values Under Case 3

In Case 3, we assume that V_{max} is the half of Base Case, and V_0 and V_T are equal to zero as given in Table 5.1. This implies that VPP is defined as empty at the initial time, so generation is possible by means of the inflow for initial time regardless the price levels.

Optimization results with these assumptions are shown in Figure 5.3. In this case, the aim is to compare the value of the defined hydro virtual power plant in case of that maximum reservoir level is half of the maximum reservoir level in Base Case. Moreover, initial and end reservoir levels in Case 3 are zero.

Since for the initial generation we need to inflow and price scenarios stay the same, we expected that VPP values in Case 3 are lower than Base Case. Thus, Figure 5.3 also shows that VPP values are lower than ones in Figure 4.5.

Moreover, in Case 3, increase in inflow amount cause sharper increases in VPP values as in Case 1 because of the zero initial and end reservoir levels. On the other hand, in lower price levels, VPP values are smoother and have similar sight with Base Case, but at the higher price levels, because of the generation decision (holding water for the higher priced hours or generating electricity), transions between price scenarios are also sharper than Base Case like Case 1.

5.4 VPP Values Under Case 4

In Case 4, we assume that V_{max}, V_0 and V_T are same as Base Case, as given in Table 5.1. In that case, we assumed that VPP could produce electricity at only peak hours (08:00-19:00/ all days in a week).

Optimization results with these assumptions are shown in Figure 5.4. In this case, we compare the value of the defined hydro virtual power plant in case of that operating hours are constrained. In Case 1, Case 2 and Case 3, constraints are changed for reservoir levels. However, at that case, we assume that VPP can produce electricity at only peak hours (08:00-19:00/ all days in a week).

As seen in Figure 5.4, since operating hours are constrained and at the almost each price scenario, we have similar increasing shape according to inflow scenarios. When we analyze the hourly generation, since in the optimization model, a penalty is imposed for spillage, operation in full capacity is observed frequently. However, not to operate in offpeak hours leads to decrease in VPP value.

At last, as a summary, VPP values for each case are shown in Table 5.2. By using hourly price forward curve or market expectation, intrinsic value can be calculated for VPP. We need to note that if a stock has significantly lower intrinsic value than market value, this means that the value of stock is overestimated. Therefore, according to trade side, i.e option buyer or seller, intrinsic value must be taken into the account.

Table 5.2: VPP Values for Optimization Cases

Mio. TL	Base Case	Case 1	Case 2	Case 3	Case 4
VPP Value	138.9	125.5	130.8	125.1	120.6

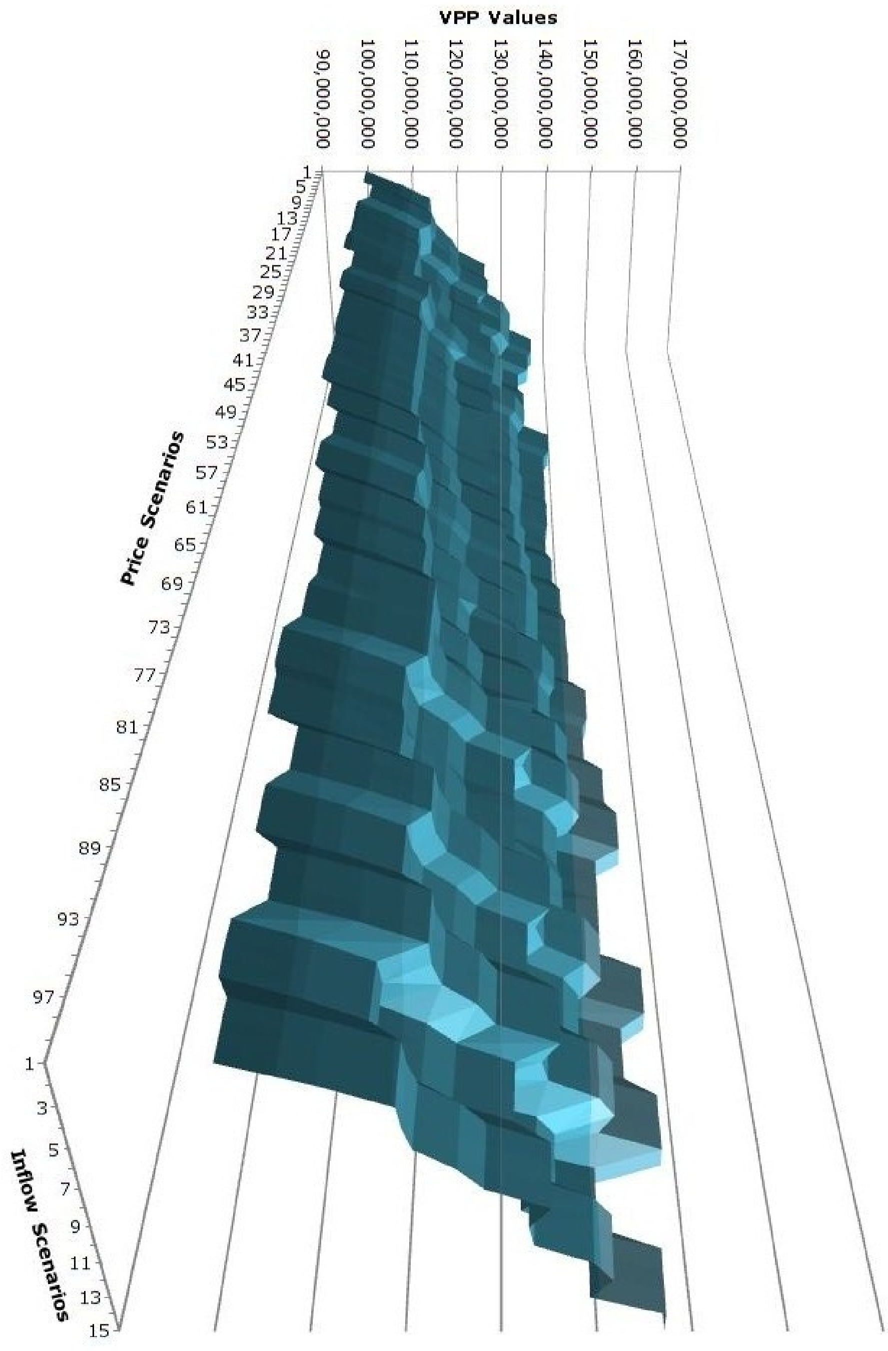


Figure 5.1: Surface of VPP Values for Case 1

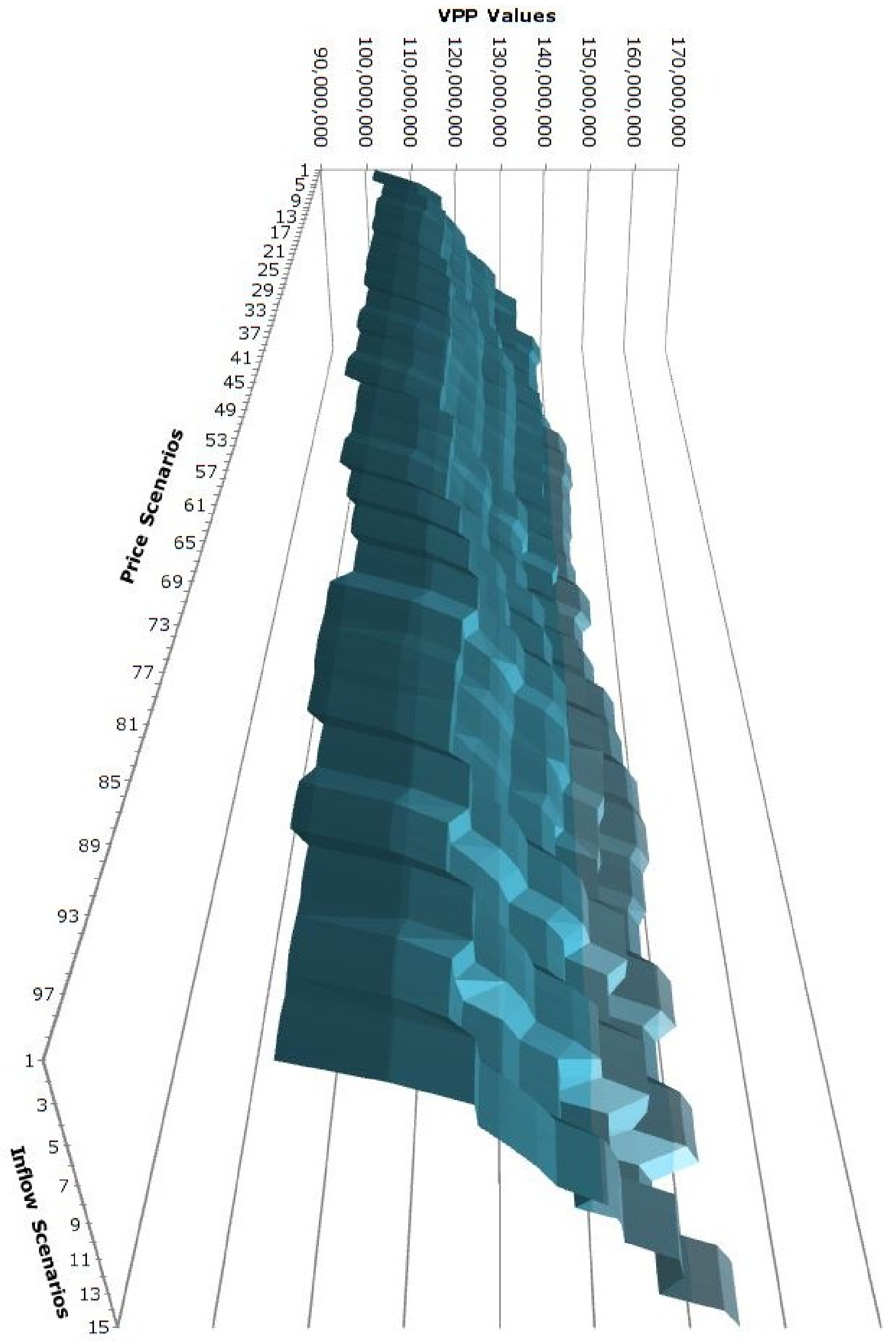


Figure 5.2: Surface of VPP Values for Case 2

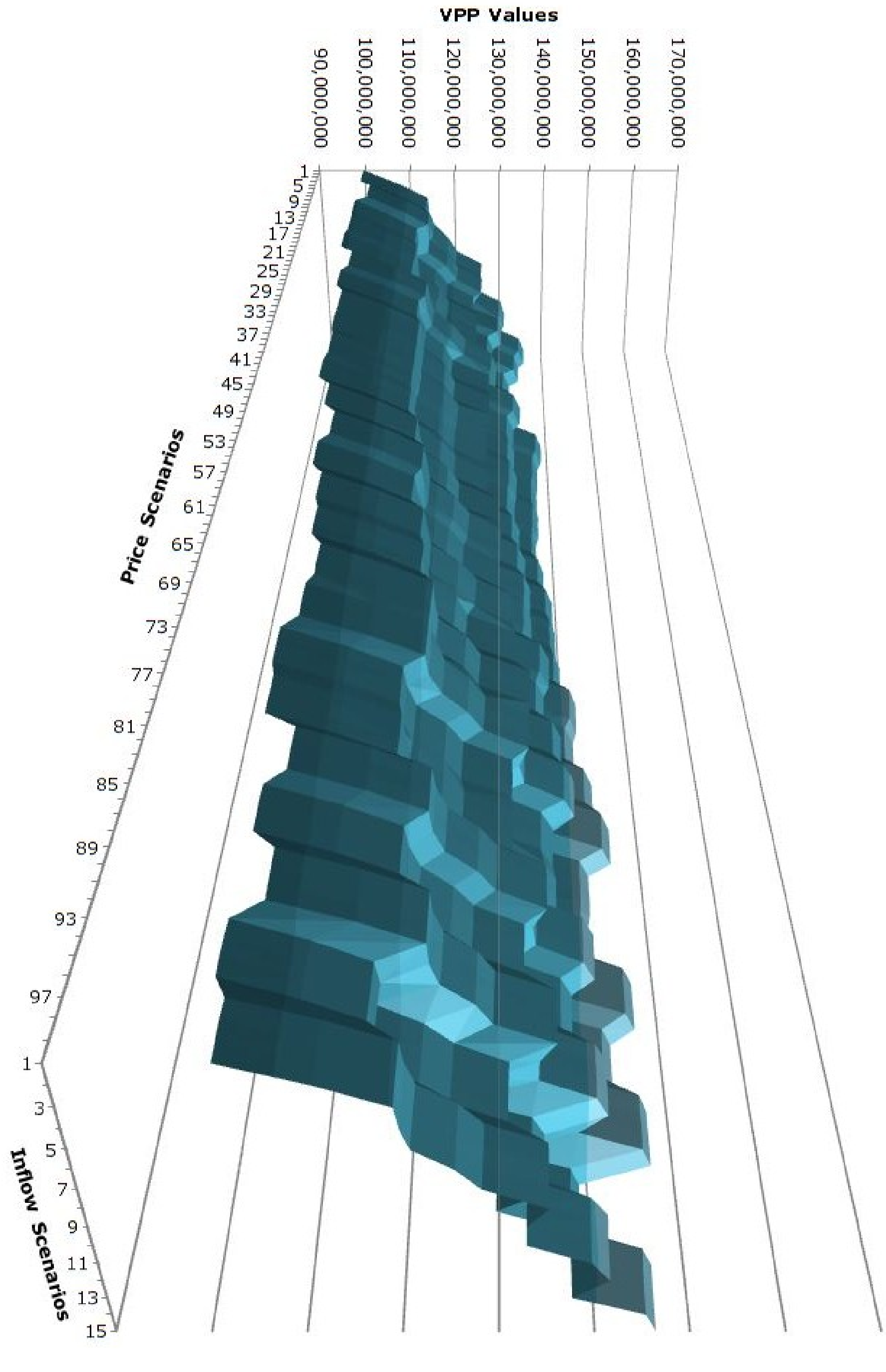


Figure 5.3: Surface of VPP Values for Case 3

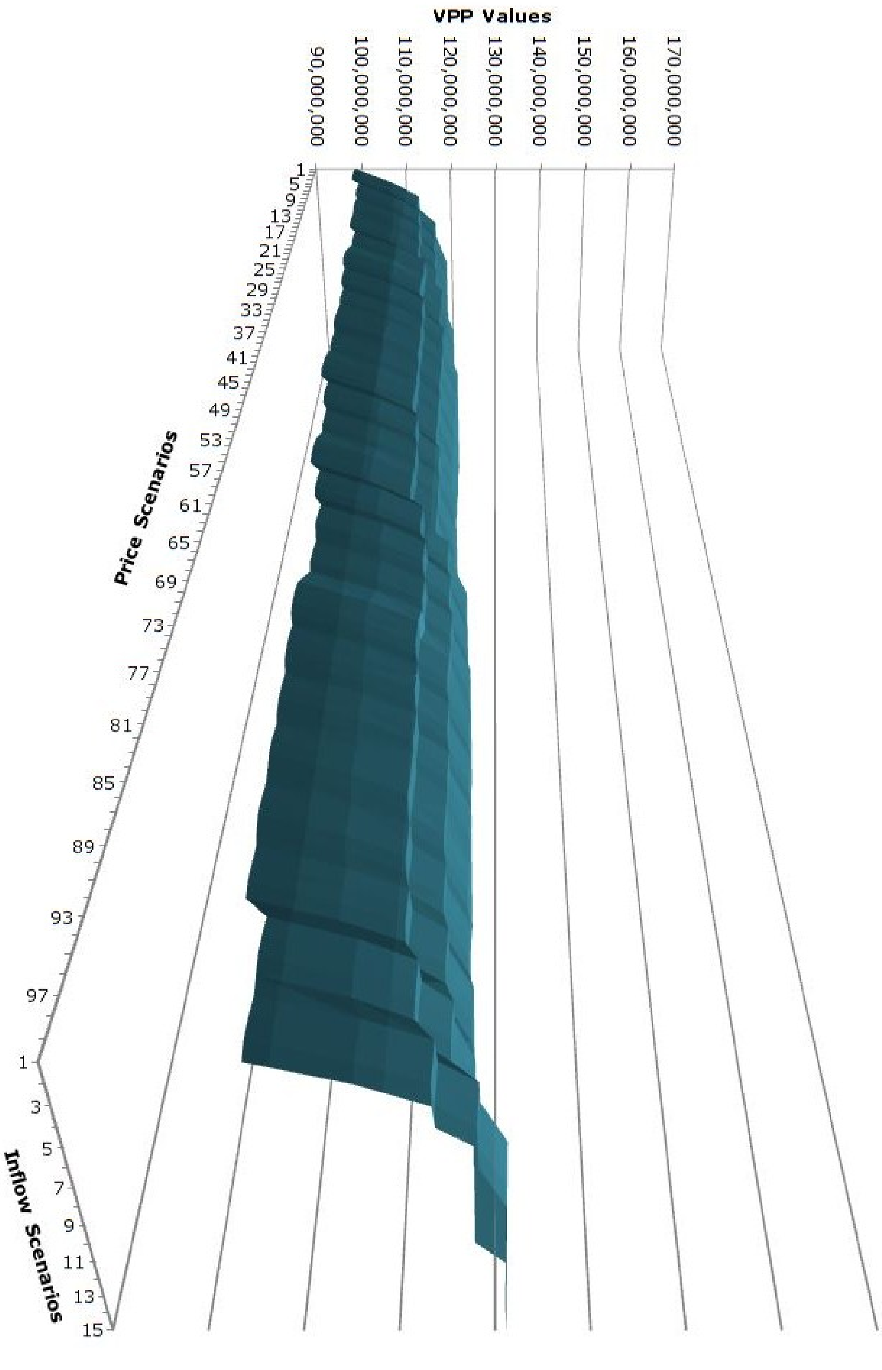


Figure 5.4: Surface of VPP Values for Case 4

CHAPTER 6

CONCLUSION AND COMMENTS

In Turkey, electricity business is growing rapidly by means of the development in the perception of risk management. Especially, energy derivatives become more popular for making profit from volatility in market price, managing risk, hedging price and so on.

In this study, aim is to establish a complete work for Virtual Power Plant (VPP) pricing. For this purpose, we work on forecasting hydro inflow and generate inflow scenarios for a defined VPP, simulating electricity prices by using historical hourly shapes, optimizing hydraulic capacity of VPP by regarding inflow and price scenarios. Moreover, we calculate the value of VPP as an financial contract, generate various optimization cases by use of the constraints of optimization model and aim to make a contribution to the literature of hydro inflow modeling, VPP pricing and volume risk management by considering power traders and producers.

In the consideration of real option, we define a Virtual Hydro Power Plant with the possible real-world constraints such as reservoir levels, generation constraints, costs and so on. First of all, we work on hydro inflow forecasting model to compose inflow series for VPP. For this purpose, we construct a seasonal time series model with precipitation as an exogenous variable. In addition to the point forecasts, obtained from this hydro inflow forecasting model, we generate inflow scenarios to see the VPP values for different inflow levels.

Secondly, to use in the scheduling and dispatching of VPP, we generate electricity price scenarios. In this scenarios, we consider both monthly and hourly day-ahead market prices. For monthly prices, we use historical electricity prices and for hourly shape modeling, we model shapes according to characteristics of each month by regarding the peak/offpeak shape.

After obtaining inflow and price scenarios, we design an hourly optimization model for VPP that dispatchs water according to hourly price scenarios in deference to the defined constraints by means of MILP. The objective function of the optimization problem is maximizing revenue that is calculated as the summation of product of price and generation less marginal cost for the whole time period. As an output of this optimization model, we obtain a VPP value for the intersection of each price and inflow scenario. By defining generic constraints, we set up our Base Case and evaluate optimization results accordingly. Furthermore, we establish Volume at Risk concept that is to imply risky volume levels for the defined Virtual Hydro Power Plant.

By using the flexibility of optimization model, we apply some changes in constraints and we generate various optmization cases. These are;

In Case 1, initial and end reservoir levels of VPP are changed. The difference of Case 1 from Base Case is that initial and end reservoir levels are zero in Case 1. Therefore, for the generation, VPP need to save the water and because of this reason, VPP values are lower in Case 1 than Base Case.

In Case 2, we use half of initial, end and maximum reservoir levels. Hence, we constrain the levels for holding water and we analyze results by comparing with Base Case.

In Case 3, maximum reservoir level of VPP is the half of one in Base Case and initial and end levels are zero. In this case, lower VPP values than previous cases have been expected and outputs are also in this direction.

In Case 4, instead of the change in reservoir levels, we change the operating hours. We assume that VPP could produce electricity at only peak hours (08:00-19:00/ all days in a week). By analyzing the hourly generation, since in the optimization model, a penalty is applied for spillage, operation in full capacity is observed frequently and a relatively smooth surface is obtained. However, since generation in offpeak hours is impossible, VPP values decrease in Case 4.

This thesis presents a complete work step by step on the purpose of valuation for a defined VPP. Ability to define and estimate the value of a VPP is very beneficial for the developing derivative market in Turkey. As a summary, this study establishes all details of VPP concept and contributes to risk management strategies in electricity market. By means of this constructed flexible pricing tool that values defined VHPP, option market making can be easier for power traders.

As further works, there are some aspects that are open to improvement. For example, since inflow data could not have more frequent granularity, we use monthly data for modeling. Moreover, since product-based OTC prices are not public, historical day-ahead electricity price data is used and it is restricted by 5 year. As a result, providing more historical price data may lead to more healty forecasting results. Therefore, we can put more accurate forecasting into the optimization model and get more precise results for VPP value.

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APPENDIX A

Table A.1: SARIMAX Models for Inflow Forecasting

Model	Model Components	AIC
ARIMA(0,0,0)	with drift	793.9386
ARIMA(1,0,0)(1,0,0)[12]	with drift	292.9217
ARIMA(0,0,1)(0,0,1)[12]	with drift	525.7503
ARIMA(1,0,0)	with drift	657.8371
ARIMA(1,0,0)(2,0,0)[12]	with drift	168.2924
ARIMA(1,0,0)(2,0,1)[12]	with drift	∞
ARIMA(1,0,0)(3,0,1)[12]	with drift	∞
ARIMA(0,0,0)(2,0,0)[12]	with drift	358.9759
ARIMA(2,0,0)(2,0,0)[12]	with drift	170.7821
ARIMA(1,0,1)(2,0,0)[12]	with drift	169.6911
ARIMA(2,0,1)(2,0,0)[12]	with drift	170.2777
ARIMA(1,0,0)(2,0,0)[12]	with zero mean	∞
ARIMA(1,0,0)(3,0,0)[12]	with drift	123.6871
ARIMA(1,0,0)(4,0,1)[12]	with drift	118.9908
ARIMA(0,0,0)(4,0,1)[12]	with drift	321.6318
ARIMA(2,0,0)(4,0,1)[12]	with drift	∞
ARIMA(1,0,1)(4,0,1)[12]	with drift	118.5502
ARIMA(2,0,2)(4,0,1)[12]	with drift	∞
ARIMA(1,0,1)(4,0,1)[12]	with zero mean	∞
ARIMA(1,0,1)(3,0,1)[12]	with drift	∞
ARIMA(1,0,1)(5,0,1)[12]	with drift	∞
ARIMA(1,0,1)(4,0,0)[12]	with drift	116.1907
ARIMA(0,0,1)(4,0,0)[12]	with drift	181.9974
ARIMA(2,0,1)(4,0,0)[12]	with drift	106.0071
ARIMA(2,0,0)(4,0,0)[12]	with drift	104.772

Table A.2: SARIMAX Models for Inflow Forecasting Cont'd

Model	Model Components	AIC
ARIMA(3,0,1)(4,0,0)[12]	with drift	93.19252
ARIMA(3,0,1)(4,0,0)[12]	with zero mean	∞
ARIMA(3,0,1)(3,0,0)[12]	with drift	114.7167
ARIMA(3,0,1)(5,0,0)[12]	with drift	∞
ARIMA(3,0,1)(4,0,1)[12]	with drift	∞
ARIMA(3,0,1)(5,0,1)[12]	with drift	∞
ARIMA(4,0,1)(4,0,0)[12]	with drift	∞
ARIMA(3,0,0)(4,0,0)[12]	with drift	91.0268
ARIMA(3,0,0)(4,0,0)[12]	with zero mean	∞
ARIMA(3,0,0)(3,0,0)[12]	with drift	112.5355
ARIMA(3,0,0)(5,0,0)[12]	with drift	∞
ARIMA(3,0,0)(4,0,1)[12]	with drift	∞
ARIMA(3,0,0)(5,0,1)[12]	with drift	∞
ARIMA(4,0,0)(4,0,0)[12]	with drift	94.09194
The Best Model	ARIMA(3,0,0)(4,0,0)[12]	with drift

Table A.3: SARIMA Model for January Price

Model	Model Components	AIC
ARIMA(0,0,0)	with drift	7327.954
ARIMA(1,0,0)(1,0,0)[24]	with drift	∞
ARIMA(0,0,1)(0,0,1)[24]	with drift	6140.271
ARIMA(0,0,0)	with zero mean	9643.21
ARIMA(0,0,1)(1,0,1)[24]	with drift	∞
ARIMA(0,0,1)	with drift	6594.665
ARIMA(0,0,1)(0,0,2)[24]	with drift	5966.887
ARIMA(1,0,1)(0,0,2)[24]	with drift	5304.599
ARIMA(1,0,0)(0,0,2)[24]	with drift	5336.396
ARIMA(1,0,2)(0,0,2)[24]	with drift	5260.943
ARIMA(2,0,3)(0,0,2)[24]	with drift	5213.844
ARIMA(2,0,3)(0,0,2)[24]	with zero mean	5310.162
ARIMA(2,0,3)(1,0,2)[24]	with drift	∞
ARIMA(2,0,3)(0,0,1)[24]	with drift	5363.005
ARIMA(1,0,3)(0,0,2)[24]	with drift	5246.096
ARIMA(3,0,3)(0,0,2)[24]	with drift	5205.624
ARIMA(3,0,2)(0,0,2)[24]	with drift	5225.88
ARIMA(3,0,4)(0,0,2)[24]	with drift	5207.033
ARIMA(2,0,2)(0,0,2)[24]	with drift	5222.589
ARIMA(4,0,4)(0,0,2)[24]	with drift	5214.626
ARIMA(3,0,3)(0,0,2)[24]	with zero mean	∞
ARIMA(3,0,3)(1,0,2)[24]	with drift	∞
ARIMA(3,0,3)(0,0,1)[24]	with drift	5355.114
ARIMA(4,0,3)(0,0,2)[24]	with drift	5219.585
The Best Model	ARIMA(3,0,3)(0,0,2)[24]	with drift

Table A.4: SARIMA Model for February Price

Model	Model Components	AIC
ARIMA(0,0,0)	with drift	6409.167
ARIMA(1,0,0)(1,0,0)[24]	with drift	∞
ARIMA(0,0,1)(0,0,1)[24]	with drift	5393.453
ARIMA(0,0,0)	with zero mean	8640.367
ARIMA(0,0,1)(1,0,1)[24]	with drift	∞
ARIMA(0,0,1)	with drift	5770.78
ARIMA(0,0,1)(0,0,2)[24]	with drift	5180.205
ARIMA(1,0,1)(0,0,2)[24]	with drift	4734.023
ARIMA(1,0,0)(0,0,2)[24]	with drift	4752.314
ARIMA(1,0,2)(0,0,2)[24]	with drift	4717.046
ARIMA(2,0,3)(0,0,2)[24]	with drift	4662.416
ARIMA(2,0,3)(0,0,2)[24]	with zero mean	∞
ARIMA(2,0,3)(1,0,2)[24]	with drift	∞
ARIMA(2,0,3)(0,0,1)[24]	with drift	4795.762
ARIMA(1,0,3)(0,0,2)[24]	with drift	4710.948
ARIMA(3,0,3)(0,0,2)[24]	with drift	4666.535
ARIMA(2,0,2)(0,0,2)[24]	with drift	4660.711
ARIMA(2,0,2)(0,0,2)[24]	with zero mean	∞
ARIMA(2,0,2)(1,0,2)[24]	with drift	∞
ARIMA(2,0,2)(0,0,1)[24]	with drift	4795.821
ARIMA(3,0,2)(0,0,2)[24]	with drift	4664.49
ARIMA(2,0,1)(0,0,2)[24]	with drift	4660.324
ARIMA(2,0,1)(0,0,2)[24]	with zero mean	∞
ARIMA(2,0,1)(1,0,2)[24]	with drift	∞
ARIMA(2,0,1)(0,0,1)[24]	with drift	4793.78
ARIMA(3,0,1)(0,0,2)[24]	with drift	4662.927
ARIMA(2,0,0)(0,0,2)[24]	with drift	4722.913
The Best Model	ARIMA(2,0,1)(0,0,2)[24]	with drift

Table A.5: SARIMA Model for March Price

Model	Model Components	AIC
ARIMA(0,0,0)	with drift	6928.433
ARIMA(1,0,0)(1,0,0)[24]	with drift	∞
ARIMA(0,0,1)(0,0,1)[24]	with drift	5896.383
ARIMA(0,0,0)	with zero mean	9308.433
ARIMA(0,0,1)(1,0,1)[24]	with drift	∞
ARIMA(0,0,1)	with drift	6281.542
ARIMA(0,0,1)(0,0,2)[24]	with drift	5686.824
ARIMA(1,0,1)(0,0,2)[24]	with drift	5323.894
ARIMA(1,0,0)(0,0,2)[24]	with drift	5325.482
ARIMA(1,0,2)(0,0,2)[24]	with drift	5309.578
ARIMA(2,0,3)(0,0,2)[24]	with drift	5285.852
ARIMA(2,0,3)(0,0,2)[24]	with zero mean	∞
ARIMA(2,0,3)(1,0,2)[24]	with drift	∞
ARIMA(2,0,3)(0,0,1)[24]	with drift	5421.676
ARIMA(1,0,3)(0,0,2)[24]	with drift	5304.49
ARIMA(3,0,3)(0,0,2)[24]	with drift	5286.76
ARIMA(2,0,2)(0,0,2)[24]	with drift	5284.907
RIMA(2,0,2)(0,0,2)[24]	with zero mean	∞
ARIMA(2,0,2)(1,0,2)[24]	with drift	∞
ARIMA(2,0,2)(0,0,1)[24]	with drift	5419.647
ARIMA(3,0,2)(0,0,2)[24]	with drift	5282.268
ARIMA(3,0,1)(0,0,2)[24]	with drift	5286.295
ARIMA(2,0,1)(0,0,2)[24]	with drift	5293.32
ARIMA(4,0,3)(0,0,2)[24]	with drift	5286.201
ARIMA(3,0,2)(0,0,2)[24]	with zero mean	5365.975
ARIMA(3,0,2)(1,0,2)[24]	with drift	∞
ARIMA(3,0,2)(0,0,1)[24]	with drift	5421.741
ARIMA(4,0,2)(0,0,2)[24]	with drift	5283.4
The Best Model	ARIMA(3,0,2)(0,0,2)[24]	with drift

Table A.6: SARIMA Model for April Price

Model	Model Components	AIC
ARIMA(0,0,0)	with drift	6684.763
ARIMA(1,0,0)(1,0,0)[24]	with drift	∞
ARIMA(0,0,1)(0,0,1)[24]	with drift	5674.977
ARIMA(0,0,0)	with zero mean	8982.412
ARIMA(0,0,1)(1,0,1)[24]	with drift	∞
ARIMA(0,0,1)	with drift	6058.461
ARIMA(0,0,1)(0,0,2)[24]	with drift	5473.212
ARIMA(1,0,1)(0,0,2)[24]	with drift	5163.951
ARIMA(1,0,0)(0,0,2)[24]	with drift	5174.503
ARIMA(1,0,2)(0,0,2)[24]	with drift	5155.986
ARIMA(2,0,3)(0,0,2)[24]	with drift	5138.051
ARIMA(2,0,3)(0,0,2)[24]	with zero mean	5244.084
ARIMA(2,0,3)(1,0,2)[24]	with drift	∞
ARIMA(2,0,3)(0,0,1)[24]	with drift	5256.256
ARIMA(1,0,3)(0,0,2)[24]	with drift	5152.649
ARIMA(3,0,3)(0,0,2)[24]	with drift	5144.571
ARIMA(2,0,2)(0,0,2)[24]	with drift	5139.457
ARIMA(2,0,4)(0,0,2)[24]	with drift	5151.046
ARIMA(3,0,4)(0,0,2)[24]	with drift	5152.886
The Best Model	ARIMA(2,0,3)(0,0,2)[24]	with drift

Table A.7: SARIMA Model for May Price

Model	Model Components	AIC
ARIMA(0,0,0)	with drift	7064.512
ARIMA(1,0,0)(1,0,0)[24]	with drift	∞
ARIMA(0,0,1)(0,0,1)[24]	with drift	6031.807
ARIMA(0,0,0)	with zero mean	9358.763
ARIMA(0,0,1)(1,0,1)[24]	with drift	∞
ARIMA(0,0,1)	with drift	6420.746
ARIMA(0,0,1)(0,0,2)[24]	with drift	5841.651
ARIMA(1,0,1)(0,0,2)[24]	with drift	5563.952
ARIMA(1,0,0)(0,0,2)[24]	with drift	5577.33
ARIMA(1,0,2)(0,0,2)[24]	with drift	5559.266
ARIMA(2,0,3)(0,0,2)[24]	with drift	5533.413
ARIMA(2,0,3)(0,0,2)[24]	with zero mean	5656.074
ARIMA(2,0,3)(1,0,2)[24]	with drift	∞
ARIMA(2,0,3)(0,0,1)[24]	with drift	5633.306
ARIMA(1,0,3)(0,0,2)[24]	with drift	5545.679
ARIMA(3,0,3)(0,0,2)[24]	with drift	5535.046
ARIMA(2,0,2)(0,0,2)[24]	with drift	5531.716
ARIMA(2,0,2)(0,0,2)[24]	with zero mean	∞
ARIMA(2,0,2)(1,0,2)[24]	with drift	∞
ARIMA(2,0,2)(0,0,1)[24]	with drift	5633.291
ARIMA(3,0,2)(0,0,2)[24]	with drift	5532.616
ARIMA(2,0,1)(0,0,2)[24]	with drift	5529.803
ARIMA(2,0,1)(0,0,2)[24]	with zero mean	5652.327
ARIMA(2,0,1)(1,0,2)[24]	with drift	∞
ARIMA(2,0,1)(0,0,1)[24]	with drift	5631.514
ARIMA(3,0,1)(0,0,2)[24]	with drift	5535.074
ARIMA(2,0,0)(0,0,2)[24]	with drift	5561.217
The Best Model	ARIMA(2,0,1)(0,0,2)[24]	with drift

Table A.8: SARIMA Model for June Price

Model	Model Components	AIC
ARIMA(0,0,0)	with drift	6868.341
ARIMA(1,0,0)(1,0,0)[24]	with drift	∞
ARIMA(0,0,1)(0,0,1)[24]	with drift	5826.201
ARIMA(0,0,0)	with zero mean	9118.759
ARIMA(0,0,1)(1,0,1)[24]	with drift	∞
ARIMA(0,0,1)	with drift	6234.17
ARIMA(0,0,1)(0,0,2)[24]	with drift	5668.459
ARIMA(1,0,1)(0,0,2)[24]	with drift	5368.424
ARIMA(1,0,0)(0,0,2)[24]	with drift	5377.981
ARIMA(1,0,2)(0,0,2)[24]	with drift	5355.826
ARIMA(2,0,3)(0,0,2)[24]	with drift	5355.14
ARIMA(2,0,3)(0,0,2)[24]	with zero mean	∞
ARIMA(2,0,3)(1,0,2)[24]	with drift	∞
ARIMA(2,0,3)(0,0,1)[24]	with drift	5446.536
ARIMA(1,0,3)(0,0,2)[24]	with drift	5350.119
ARIMA(1,0,4)(0,0,2)[24]	with drift	5349.095
ARIMA(0,0,3)(0,0,2)[24]	with drift	5405.281
ARIMA(2,0,5)(0,0,2)[24]	with drift	5350.046
ARIMA(1,0,4)(0,0,2)[24]	with zero mean	∞
ARIMA(1,0,4)(1,0,2)[24]	with drift	∞
ARIMA(1,0,4)(0,0,1)[24]	with drift	5456.04
ARIMA(0,0,4)(0,0,2)[24]	with drift	5365.136
ARIMA(2,0,4)(0,0,2)[24]	with drift	5351.058
ARIMA(1,0,5)(0,0,2)[24]	with drift	5349.862
The Best Model	ARIMA(1,0,4)(0,0,2)[24]	with drift

Table A.9: SARIMA Model for July Price

Model	Model Components	AIC
ARIMA(0,0,0)	with drift	7137.365
ARIMA(1,0,0)(1,0,0)[24]	with drift	∞
ARIMA(0,0,1)(0,0,1)[24]	with drift	6002.292
ARIMA(0,0,0)	with zero mean	9696.508
ARIMA(0,0,1)(1,0,1)[24]	with drift	∞
ARIMA(0,0,1)	with drift	6457.807
ARIMA(0,0,1)(0,0,2)[24]	with drift	5697.53
ARIMA(1,0,1)(0,0,2)[24]	with drift	5433.904
ARIMA(1,0,0)(0,0,2)[24]	with drift	5452.329
ARIMA(1,0,2)(0,0,2)[24]	with drift	5425.668
ARIMA(2,0,3)(0,0,2)[24]	with drift	5409.962
ARIMA(2,0,3)(0,0,2)[24]	with zero mean	5522.858
ARIMA(2,0,3)(1,0,2)[24]	with drift	∞
ARIMA(2,0,3)(0,0,1)[24]	with drift	5646.61
ARIMA(1,0,3)(0,0,2)[24]	with drift	5425.04
ARIMA(3,0,3)(0,0,2)[24]	with drift	5411.283
ARIMA(2,0,2)(0,0,2)[24]	with drift	5408.618
ARIMA(2,0,2)(0,0,2)[24]	with zero mean	5534.351
ARIMA(2,0,2)(1,0,2)[24]	with drift	∞
ARIMA(2,0,2)(0,0,1)[24]	with drift	5645.468
ARIMA(3,0,2)(0,0,2)[24]	with drift	5409.214
ARIMA(2,0,1)(0,0,2)[24]	with drift	5407.639
ARIMA(2,0,1)(0,0,1)[24]	with drift	5649.039
ARIMA(3,0,1)(0,0,2)[24]	with drift	5407.579
ARIMA(3,0,0)(0,0,2)[24]	with drift	5417.503
ARIMA(2,0,0)(0,0,2)[24]	with drift	5425.525
ARIMA(4,0,2)(0,0,2)[24]	with drift	5399.587
ARIMA(4,0,2)(0,0,1)[24]	with drift	5641.698
ARIMA(5,0,2)(0,0,2)[24]	with drift	5416.528
ARIMA(4,0,1)(0,0,2)[24]	with drift	5418.21
ARIMA(4,0,3)(0,0,2)[24]	with drift	5401.351
ARIMA(5,0,3)(0,0,2)[24]	with drift	5414.725
The Best Model	ARIMA(4,0,2)(0,0,2)[24]	with drift

Table A.10: SARIMA Model for August Price

Model	Model Components	AIC
ARIMA(0,0,0)	with drift	7206.034
ARIMA(1,0,0)(1,0,0)[24]	with drift	∞
ARIMA(0,0,1)(0,0,1)[24]	with drift	6043.371
ARIMA(0,0,0)	with zero mean	9707.627
ARIMA(0,0,1)(1,0,1)[24]	with drift	∞
ARIMA(1,0,1)(0,0,2)[24]	with drift	5514.324
ARIMA(1,0,0)(0,0,2)[24]	with drift	5548.674
ARIMA(1,0,2)(0,0,2)[24]	with drift	5515.833
ARIMA(0,0,0)(0,0,2)[24]	with drift	6294.305
ARIMA(2,0,2)(0,0,2)[24]	with drift	5517.104
ARIMA(1,0,1)(0,0,2)[24]	with zero mean	5609.588
ARIMA(1,0,1)(1,0,2)[24]	with drift	∞
ARIMA(1,0,1)(0,0,1)[24]	with drift	5700.895
ARIMA(2,0,1)(0,0,2)[24]	with drift	5499.354
ARIMA(2,0,0)(0,0,2)[24]	with drift	5512.517
ARIMA(3,0,2)(0,0,2)[24]	with drift	5488.858
ARIMA(3,0,2)(0,0,1)[24]	with drift	5666.983
ARIMA(4,0,2)(0,0,2)[24]	with drift	5509.39
ARIMA(3,0,1)(0,0,2)[24]	with drift	5511.276
ARIMA(3,0,3)(0,0,2)[24]	with drift	5489.368
ARIMA(4,0,3)(0,0,2)[24]	with drift	5486.298
ARIMA(4,0,3)(0,0,2)[24]	with zero mean	5603.711
ARIMA(5,0,3)(0,0,2)[24]	with drift	5494.051
ARIMA(4,0,4)(0,0,2)[24]	with drift	5486.919
ARIMA(5,0,4)(0,0,2)[24]	with drift	5483.122
ARIMA(5,0,4)(0,0,1)[24]	with drift	5655.807
ARIMA(6,0,4)(0,0,2)[24]	with drift	5468.479
ARIMA(6,0,3)(0,0,2)[24]	with drift	5470.516
ARIMA(6,0,5)(0,0,2)[24]	with drift	5470.557
ARIMA(6,0,4)(0,0,2)[24]	with zero mean	∞
ARIMA(6,0,4)(1,0,2)[24]	with drift	∞
ARIMA(6,0,4)(0,0,1)[24]	with drift	5652.915
The Best Model	ARIMA(6,0,4)(0,0,2)[24]	with drift

Table A.11: SARIMA Model for September Price

Model	Model Components	AIC
ARIMA(0,0,0)	with drift	7120.868
ARIMA(1,0,0)(1,0,0)[24]	with drift	∞
ARIMA(0,0,1)(0,0,1)[24]	with drift	5964.434
ARIMA(0,0,0)	with zero mean	9368.432
ARIMA(0,0,1)(1,0,1)[24]	with drift	∞
ARIMA(0,0,1)	with drift	6405.732
ARIMA(0,0,1)(0,0,2)[24]	with drift	∞
ARIMA(0,0,1)(1,0,2)[24]	with drift	∞
ARIMA(1,0,1)(0,0,1)[24]	with drift	5498.53
ARIMA(1,0,0)(0,0,1)[24]	with drift	5533.47
ARIMA(1,0,2)(0,0,1)[24]	with drift	5487.282
ARIMA(2,0,3)(0,0,1)[24]	with drift	5445.108
ARIMA(2,0,3)(0,0,1)[24]	with zero mean	5561.638
ARIMA(2,0,3)(1,0,1)[24]	with drift	∞
ARIMA(2,0,3)	with drift	5785.627
ARIMA(2,0,3)(0,0,2)[24]	with drift	5231.569
ARIMA(1,0,3)(0,0,2)[24]	with drift	5257.577
ARIMA(3,0,3)(0,0,2)[24]	with drift	5224.674
ARIMA(3,0,2)(0,0,2)[24]	with drift	5222.774
ARIMA(2,0,1)(0,0,2)[24]	with drift	5228.541
ARIMA(4,0,3)(0,0,2)[24]	with drift	∞
ARIMA(3,0,2)(0,0,2)[24]	with zero mean	5330.951
ARIMA(3,0,2)(1,0,2)[24]	with drift	∞
ARIMA(3,0,2)(0,0,1)[24]	with drift	5437.956
ARIMA(2,0,2)(0,0,2)[24]	with drift	5267.468
ARIMA(4,0,2)(0,0,2)[24]	with drift	5228.969
ARIMA(3,0,1)(0,0,2)[24]	with drift	5228.321
The Best Model	ARIMA(3,0,2)(0,0,2)[24]	with drift

Table A.12: SARIMA Model for October Price

Model	Model Components	AIC
ARIMA(0,0,0)	with drift	6843.816
ARIMA(1,0,0)(1,0,0)[24]	with drift	∞
ARIMA(0,0,1)(0,0,1)[24]	with drift	5844.567
ARIMA(0,0,0)	with zero mean	9580.126
ARIMA(0,0,1)(1,0,1)[24]	with drift	∞
ARIMA(0,0,1)	with drift	6257.675
ARIMA(0,0,1)(0,0,2)[24]	with drift	5678.306
ARIMA(1,0,1)(0,0,2)[24]	with drift	5301.372
ARIMA(1,0,0)(0,0,2)[24]	with drift	5300.341
ARIMA(2,0,1)(0,0,2)[24]	with drift	5302.558
ARIMA(1,0,0)(0,0,2)[24]	with zero mean	∞
ARIMA(1,0,0)(1,0,2)[24]	with drift	∞
ARIMA(1,0,0)(0,0,1)[24]	with drift	5438.516
ARIMA(0,0,0)(0,0,2)[24]	with drift	6108.917
ARIMA(2,0,0)(0,0,2)[24]	with drift	5300.259
ARIMA(3,0,1)(0,0,2)[24]	with drift	5249.348
ARIMA(3,0,1)(0,0,2)[24]	with zero mean	∞
ARIMA(3,0,1)(1,0,2)[24]	with drift	∞
ARIMA(3,0,1)(0,0,1)[24]	with drift	5361.364
ARIMA(4,0,1)(0,0,2)[24]	with drift	5255.851
ARIMA(3,0,0)(0,0,2)[24]	with drift	5273.018
ARIMA(3,0,2)(0,0,2)[24]	with drift	5250.73
ARIMA(4,0,2)(0,0,2)[24]	with drift	5256.638
The Best Model	ARIMA(3,0,1)(0,0,2)[24]	with drift

Table A.13: SARIMA Model for November Price

Model	Model Components	AIC
ARIMA(0,0,0)	with drift	6867.182
ARIMA(1,0,0)(1,0,0)[24]	with drift	∞
ARIMA(0,0,1)(0,0,1)[24]	with drift	5773.258
ARIMA(0,0,0)	with zero mean	9314.657
ARIMA(0,0,1)(1,0,1)[24]	with drift	∞
ARIMA(0,0,1)	with drift	6227.329
ARIMA(0,0,1)(0,0,2)[24]	with drift	5555.897
ARIMA(1,0,1)(0,0,2)[24]	with drift	5188.484
ARIMA(1,0,0)(0,0,2)[24]	with drift	5192.811
ARIMA(1,0,2)(0,0,2)[24]	with drift	5176.913
ARIMA(2,0,3)(0,0,2)[24]	with drift	5117.654
ARIMA(2,0,3)(0,0,2)[24]	with zero mean	5242.269
ARIMA(2,0,3)(1,0,2)[24]	with drift	∞
ARIMA(2,0,3)(0,0,1)[24]	with drift	5221.17
ARIMA(1,0,3)(0,0,2)[24]	with drift	5151.273
ARIMA(3,0,3)(0,0,2)[24]	with drift	5125.477
ARIMA(2,0,2)(0,0,2)[24]	with drift	5116.503
ARIMA(2,0,2)(0,0,2)[24]	with zero mean	5245
ARIMA(2,0,2)(1,0,2)[24]	with drift	∞
ARIMA(2,0,2)(0,0,1)[24]	with drift	5224.773
ARIMA(3,0,2)(0,0,2)[24]	with drift	5123.963
ARIMA(2,0,1)(0,0,2)[24]	with drift	5119.641
The Best Model	ARIMA(2,0,2)(0,0,2)[24]	with drift

Table A.14: SARIMA Model for December Price

Model	Model Components	AIC
ARIMA(0,0,0)	with drift	7180.038
ARIMA(1,0,0)(1,0,0)[24]	with drift	∞
ARIMA(0,0,1)(0,0,1)[24]	with drift	6053.194
ARIMA(0,0,0)	with zero mean	9740.966
ARIMA(0,0,1)(1,0,1)[24]	with drift	∞
ARIMA(0,0,1)	with drift	6508.307
ARIMA(0,0,1)(0,0,2)[24]	with drift	5866.16
ARIMA(1,0,1)(0,0,2)[24]	with drift	5468.26
ARIMA(1,0,0)(0,0,2)[24]	with drift	5480.47
ARIMA(1,0,2)(0,0,2)[24]	with drift	5444.503
ARIMA(2,0,3)(0,0,2)[24]	with drift	5394.981
ARIMA(2,0,3)(0,0,2)[24]	with zero mean	∞
ARIMA(2,0,3)(1,0,2)[24]	with drift	∞
ARIMA(2,0,3)(0,0,1)[24]	with drift	5491.404
ARIMA(1,0,3)(0,0,2)[24]	with drift	5426.154
ARIMA(3,0,3)(0,0,2)[24]	with drift	5402.865
ARIMA(2,0,2)(0,0,2)[24]	with drift	5397.295
ARIMA(2,0,4)(0,0,2)[24]	with drift	5392.938
ARIMA(3,0,5)(0,0,2)[24]	with drift	5398.86
ARIMA(2,0,4)(0,0,2)[24]	with zero mean	∞
ARIMA(2,0,4)(1,0,2)[24]	with drift	∞
ARIMA(2,0,4)(0,0,1)[24]	with drift	5485.864
ARIMA(1,0,4)(0,0,2)[24]	with drift	5418.495
ARIMA(3,0,4)(0,0,2)[24]	with drift	5401.508
ARIMA(2,0,5)(0,0,2)[24]	with drift	5394.703
The Best Model	ARIMA(2,0,4)(0,0,2)[24]	with drift