# MAXIMIZING ENERGY GENERATION OF A CASCADE HYDROPOWER SYSTEM

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 $\mathbf{B}\mathbf{Y}$ 

VEHBİ KARAEREN

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# Approval of the thesis:

## MAXIMIZING ENERGY GENERATION OF A CASCADE HYDROPOWER SYSTEM

submitted by VEHBİ KARAEREN in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering Department, Middle East Technical University by,

Prof. Dr. Gülbin Dural Ünver Dean, Graduate School of <b>Natural and Applied Sciences</b>		
Prof. Dr. Ahmet Cevdet Yalçıner Head of Department, Civil Engineering		
Prof. Dr. A. Burcu Altan-Sakarya Supervisor, Civil Engineering Dept., METU		
Examining Committee Members:		
Prof. Dr. Mustafa Göğüş Civil Engineering Dept., METU		
Prof. Dr. A. Burcu Altan-Sakarya Civil Engineering Dept., METU		
Prof. Dr. İsmail Aydın Civil Engineering Dept., METU		
Assoc. Prof. Dr. Mete Köken Civil Engineering Dept., METU		
Kenan Hayfavi Deputy General Manager, GAMA Enerji A.Ş.		
	DATE:	24.11.2014

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name : Vehbi Karaeren

Signature :

## ABSTRACT

## MAXIMIZING ENERGY GENERATION OF A CASCADE HYDROPOWER SYSTEM

Karaeren, Vehbi

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Electricity has become one of the main pillars of a well-functioning society and hydroelectric power plants (HEPPs) are one of the main providers of energy in Turkey that are also renewable. To exploit the energy production capacity of existing HEPPs in Turkey and efficient usage of water, operational optimization could help immensely. A non-linear constrained optimization algorithm with an extension, namely a heuristic that provides starting points, has been implemented to maximize the energy production of a cascade HEPP system based on monthly inflow data.

A cascade of four dams along the Murat River, which is one of the two main streams of Euphrates River at the Eastern part of Turkey, was studied to test the abilities of the mentioned algorithm. A 40-year average monthly flow data was used as input and results of cascade and single optimization were compared.

Results showed that the developed MATLAB<sup>®</sup> script can converge to an optimum solution that satisfies the constraints. The comparison study made with individual runs of each HEPP showed that a 5% higher energy production was attained with the cascade variant. A drawback was the higher computational time (which was still within comparable limits), which can be further improved by implementing more efficient algorithms.

**Keywords:** Hydropower, Optimization, Maximization, Electricity Generation, Cascade Dams, Multireservoir Systems, MATLAB

#### KASKAT BARAJLARDA ENERJİ ÜRETİMİNİN MAKSİMİZASYONU

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Elektrik, gelişmiş toplumlarda önemli yaşam kaynaklarından biri olmaya başlamış ve Hidroelektrik Enerji Santralları (HESler) da yenilenebilir olmakla birlikte Türkiye'nin enerji tedariğinde ana enerji kaynaklarından biridir. İşletme optimizasyonu, Türkiye'deki mevcut HES'lerin enerji üretim kapasitelerinden daha iyi faydalanılmasına ve suyun verimli kullanılmasına son derece katkı sağlayabilir. Kaskat HES sisteminin enerji üretiminin aylık akım verileri kullanılarak maksimize edilmesi amacıyla, farklı başlangıç noktaları sağlayan bulgusal doğrusal olmayan kısıtlı optimizasyon algoritması kullanılmıştır.

Algoritmadaki yeterlikleri test etmek için, Türkiye'nin doğusunda yer alan Fırat Nehri'nin iki ana kolundan biri olan Murat Nehri üzerindeki dört barajdan oluşan kaskat sistemi üzerinde çalışılmıştır. 40 yıllık ortalama aylık akım verileri girdi olarak kullanılmış olup kaskat ve tek baraj optimizasyon sonuçları karşılaştırılmıştır.

Elde edilen sonuçlar, geliştirilmiş MATLAB kodunun kısıtları sağlayan optimum çözüme yakınsayabildiğini göstermektedir. Karşılaştırma sonucunda kaskat sistem optimizasyonu yapılarak, barajların tek tek optimize edilmesine göre %5 daha fazla enerji üretimi sağlanabildiği görülmektedir. Yüksek işlem süresi (hala karşılaştırılabilir limitler dahilinde olmak üzere) bir dezavantaj olmakla birlikte daha etkili algoritmalar kullanılarak ileride geliştirilebilir.

Anahtar Kelimeler: Hidroelektrik, Optimizasyon, Maksimizasyon, Elektrik Üretimi, Kaskat Barajlar, Çoklu Rezervuar Sistemleri, MATLAB

ÖΖ

TO MY FAMILY...

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# **TABLE OF CONTENTS**

ABSTRA	СТ	V
ÖZ		vi
ACKNOV	WLEDGEMENTS	viii
TABLE C	OF CONTENTS	ix
LIST OF	FIGURES	xi
LIST OF	TABLES	xiv
СНАРТЕ	CRS	
1. INTRO	DDUCTION	1
1.1 Stat	tement of the Problem	1
1.2 Pur	pose and Scope	3
1.3 Ove	erview	4
2. LITER	ATURE REVIEW	5
2.1 Sing	gle Dam Operation	5
2.2 Case	cade Operation	8
3. OPTIM	AIZATION PROCESS	11
3.1 For	mulation of Hydropower Generation	11
3.2 Nun	nerical Formulation of the Problem	13
3.3 Sett	ing the Optimization Algorithm	17
4. CASE	STUDY	19
4.1 The	Selected Cascade	19
4.2 Out	line of the Case Study	27
4.2.1	Optimization with Minimum Flow Data	
4.2.2	Optimization with Average Flow Data	
4.2.3	Optimization with Maximum Flow Data	50

4.3 Con	parison of Results	72
4.3.1	Energy Generation Comparison of Single and Cascade Operation	72
4.3.2	Comparison of Optimization Performance	74
5. CONCI	LUSION	77
5.1 Sum	ımary	77
5.2 Rec	ommendations	77
6. REFEF	RENCES	79
APPEND	IX A	
APPEND	IX B	

# LIST OF FIGURES

#### **FIGURES**

Figure 1-1: Energy resources	2
Figure 3-1: A cascade hydropower system illustration	13
Figure 4-1: Hydropower plants located on Murat River	20
Figure 4-2: The illustration of the studied cascade	20
Figure 4-3: Water elevation vs. volume curve for UK	22
Figure 4-4: Water elevation vs. volume curve for LK	23
Figure 4-5: Water elevation vs. volume curve for B1	24
Figure 4-6: Water elevation vs. volume curve for B2	25
Figure 4-7: Operation head for UK with minimum flow data	
Figure 4-8: Operation head for LK with minimum flow data	
Figure 4-9: Operation head for B1 with minimum flow data	34
Figure 4-10: Operation head for B2 with minimum flow data	34
Figure 4-11: Turbined volume for UK with minimum flow data	35
Figure 4-12: Turbined volume for LK with minimum flow data	35
Figure 4-13: Turbined volume for B1 with minimum flow data	
Figure 4-14: Turbined volume for B2 with minimum flow data	
Figure 4-15: Energy generation of UK with minimum flow data	37
Figure 4-16: Energy generation of LK with minimum flow data	37
Figure 4-17: Energy generation of B1 with minimum flow data	
Figure 4-18: Energy generation of B2 with minimum flow data	
Figure 4-19: Operation head for UK with average flow data	44
Figure 4-20: Operation head for LK with average flow data	44
Figure 4-21: Operation head for B1 with average flow data	45
Figure 4-22: Operation head for B2 with average flow data	45
Figure 4-23: Turbined volume for UK with average flow data	46

Figure 4-24: Turbined volume for LK with average flow data	46
Figure 4-25: Turbined volume for B1 with average flow data	47
Figure 4-26: Turbined volume for B2 with average flow data	47
Figure 4-27: Energy generation of UK with average flow data	48
Figure 4-28: Energy generation of LK with average flow data	48
Figure 4-29: Energy generation of B1 with average flow data	49
Figure 4-30: Energy generation of B2 with average flow data	49
Figure 4-31: Operation head for UK with maximum flow data without spillway	
upper bound	55
Figure 4-32: Operation head for LK with maximum flow data without spillway	
upper bound	55
Figure 4-33: Operation head for B1 with maximum flow data without spillway	
upper bound	56
Figure 4-34: Operation head for B2 with maximum flow data without spillway	
upper bound	56
Figure 4-35: Turbined volume for UK with maximum flow data without spillway	
upper bound	57
Figure 4-36: Turbined volume for LK with maximum flow data without spillway	
upper bound	57
Figure 4-37: Turbined volume for B1 with maximum flow data without spillway	
upper bound	58
Figure 4-38: Turbined volume for B2 with maximum flow data without spillway	
upper bound	58
Figure 4-39: Energy generation for UK with maximum flow data without	
spillway upper bound	59
Figure 4-40: Energy generation for LK with maximum flow data without	
spillway upper bound	59
Figure 4-41: Energy generation for B1 with maximum flow data without spillway	
upper bound	60
Figure 4-42: Energy generation for B2 with maximum flow data without spillway	
upper bound	60

Figure 4-43: Operation head for UK with maximum flow data with spillway
upper bound
Figure 4-44: Operation head for LK with maximum flow data with spillway
upper bound
Figure 4-45: Operation head for B1 with maximum flow data with spillway upper
bound67
Figure 4-46: Operation head for B2 with maximum flow data with spillway upper
bound67
Figure 4-47: Turbined volume for UK with maximum flow data with spillway
upper bound
Figure 4-48: Turbined volume for LK with maximum flow data with spillway
upper bound
Figure 4-49: Turbined volume for B1 with maximum flow data with spillway
upper bound
Figure 4-50: Turbined volume for B2 with maximum flow data with spillway
upper bound
Figure 4-51: Energy generation for UK with maximum flow data with spillway
upper bound
Figure 4-52: Energy generation for LK with maximum flow data with spillway
upper bound
Figure 4-53: Energy generation for B1 with maximum flow data with spillway
upper bound71
Figure 4-54: Energy generation for B2 with maximum flow data with spillway
upper bound
Figure 4-55: Difference in Cascade and Individual Runs Regarding Feedback75

# LIST OF TABLES

# TABLES

<b>Table 4-1:</b> Water elevations and corresponding reservoir volumes for UK
<b>Table 4-2:</b> Water elevations and corresponding reservoir volumes for LK
<b>Table 4-3:</b> Water elevations and corresponding reservoir volumes for B1
<b>Table 4-4:</b> Water elevations and corresponding reservoir volumes for B2
<b>Table 4-5:</b> Main characteristics of the dams in the studied cascade system
Table 4-6: Optimization result of single operation with minimum flow data
for UK
Table 4-7: Optimization result of single operation with minimum flow data
for LK
Table 4-8: Optimization result of single operation with minimum flow data
for B1
Table 4-9: Optimization result of single operation with minimum flow data
for B2
Table 4-10: Optimization result of cascade operation with minimum flow data
for UK
Table 4-11: Optimization result of cascade operation with minimum flow data
for LK
Table 4-12: Optimization result of cascade operation with minimum flow data
for B1
Table 4-13: Optimization result of cascade operation with minimum flow data
for B2
Table 4-14: Optimization result of single operation with average flow data
for UK
Table 4-15: Optimization result of single operation with average flow data
for LK

Table 4-16: Optimization result of single operation with average flow data
for B141
Table 4-17: Optimization result of single operation with average flow data
for B2
Table 4-18: Optimization result of cascade operation with average flow data
for UK
Table 4-19: Optimization result of cascade operation with average flow data
for LK
Table 4-20:         Optimization result of cascade operation with average flow data
for B1
Table 4-21: Optimization result of cascade operation with average flow data
for B2
<b>Table 4-22:</b> Optimization result of single operation with maximum flow data and
without spillway upper bound for UK
<b>Table 4-23:</b> Optimization result of single operation with maximum flow data and
without spillway upper bound for LK
<b>Table 4-24:</b> Optimization result of single operation with maximum flow data and
without spillway upper bound for B1
Table 4-25: Optimization result of single operation with maximum flow data and
without spillway upper bound for B2
Table 4-26: Optimization result of cascade operation with maximum flow data
and without Spillway Upper Bound for UK53
Table 4-27: Optimization result of cascade operation with maximum flow data
and without Spillway Upper Bound for LK
Table 4-28: Optimization result of cascade operation with maximum flow data
and without Spillway Upper Bound for B154
Table 4-29: Optimization result of cascade operation with maximum flow data
and without Spillway Upper Bound for B254
Table 4-30: Optimization result of single operation with maximum flow data and
with spillway upper bound for UK

Table 4-31: Optimization result of single operation with maximum flow data and
with spillway upper bound for LK
Table 4-32: Optimization result of single operation with maximum flow data and
with spillway upper bound for B163
Table 4-33: Optimization result of single operation with maximum flow data and
with spillway upper bound for B263
Table 4-34: Optimization result of cascade operation with maximum flow data
and with spillway upper bound for UK64
Table 4-35: Optimization result of cascade operation with maximum flow data
and with spillway upper bound for LK64
Table 4-36: Optimization result of cascade operation with maximum flow data
and with spillway upper bound for B165
Table 4-37: Optimization result of cascade operation with maximum flow data
and with spillway upper bound for B2
<b>Table 4-38:</b> Optimization results by using minimum inflow data
<b>Table 4-39:</b> Optimization results by using average inflow data
Table 4-40: Optimization results by using maximum inflow data without spillway
upper bound
Table 4-41: Optimization results by using maximum inflow data with defined
spillway upper bound

#### **CHAPTER 1**

#### **INTRODUCTION**

#### 1.1 Statement of the Problem

Electricity has become one of the main pillars of a well-functioning society. Electric power can be generated from different resources that are mainly listed into two groups as renewable and non-renewable as shown in Figure 1-1. Since the non-renewable resources are scarce, the efficient usage of renewable energy resources is becoming much more important nowadays. Although renewable energy resources are known to be replenished naturally within the life period of a human being; by the virtue of the above-mentioned scarcity, today it is necessary to use the renewable resources optimally and reduce non-renewable energy source dependency.

Optimization is a mathematical tool that is used to find the best optimal solution for a specified equation in order to maximize or minimize the result by considering some defined constraints. This defined tool can be useful in many different areas; such as engineering, management, economics, marketing, etc. Due to its wide applicability, these algorithms make the implementation of engineering aspects to any problem possible.

Hydropower is the power generated from moving water through a vertical head difference and as presented in Table 1-1, it is the commonly used resource in Turkey. In this regard, the operational optimization of hydropower plants is significant in order to increase the effective usage of installed power in Turkey and increase the electricity generation all over the world, as well. There are two types of hydropower plants in terms of the storage characteristics; one is without reservoir that is named as run of river (ROR) hydroelectric power plants (HEPP), and the other one is with reservoirs in which construction of a dam body is prerequisite. Since the running

water have to flow through the turbines in ROR type HEPPs, it is not possible to have an operational optimization; however for the ones with the storage, the electricity generation can be managed and duly maximized. The operational optimization of a unique dam certainly increase the energy generation; however by taking into consideration a cascade hydropower generation system, the energy increase is far more than a unique dam optimization.

Since all large hydropower plants located on main streams are cascade, the operational optimization is become more of an issue. The cascade hydropower plants all over the world generally government owned; however there are also some private sector investments. Whatever the effected community is, the rate of return is the main asset for the initial investment decision and during the operation period, an optimized use of water mean a lot. In this regard, instead of a random operation, a scheduled operation will increase the total energy generation during all the year round.



Figure 1-1: Energy resources

As of 30.09.2014	RESOURCES	Installed Power (MW)	Percentage in Total (%)	Plants in Operation
	Coal	14,034.3	20.57	30.0
	Natural Gas + LNG	21,190.7	31.06	232.0
Non- renewable Resources	Others-1 (Fuel-oil + Asphaltite + Naphtha + Diesel Oil)	678.1	0.99	19.0
	Other-2 (Multifuel - Solid + Liquid + Natural Gas)	4,741.7	6.95	51.0
Renewable Resources	Hydropower	23,454.9	34.38	504.0
	Geothermal and Other Renewables	626.1	0.92	63.0
	Wind Energy	3,483.9	5.11	87.0
	Solar Energy	20.3	0.03	73.0
	TOTAL	68,230	100	1,059

**Table 1-1:** Installed power in Turkey according to the resources

#### **1.2** Purpose and Scope

The main objective of this study is to use non-linear constrained optimization algorithm for the maximization of electricity generation for a cascade hydropower system. In this regard, hydraulic modeling of a cascade is investigated. The data of the monthly inflows to the reservoirs are considered and a monthly basis optimization is carried out by using maximum, minimum and average flow data.

In order to maximize power generation in a cascade hydropower system, a non-linear optimization algorithm is implemented. A case study of a cascade with four dams is examined in order to compare the single dam operation with a cascade operation. The scope of this study contains the use of only one non-linear algorithm and thus a comparison of convergence success, result accuracy etc. are not discussed herein.

#### 1.3 Overview

This thesis is divided into five chapters. The current chapter presents the problem statement and clarifies the scope and the purpose of this study. Chapter 2 compiles relevant literature sources in a review format. The optimization process, objective function formulation for power production and its formulation in MATLAB is explained in Chapter 3. The fourth chapter presents a case study with the optimization procedure explained in the previous chapter. Results are presented and advantages as well as disadvantages of the used algorithm are discussed. The final chapter concludes this thesis by summarizing the findings together with the recommendations. The implementation of optimization into MATLAB is defined in Appendix-A.

#### **CHAPTER 2**

#### LITERATURE REVIEW

There are several models and optimization functions that can be combinatorially studied for different claims and purposes by many engineers and scientists. The methodology of using a Nonlinear Programming (NLP) simulation for the purpose of investigating the effects of maximizing the hydropower generation reviewed and understood with respect to several literatures.

#### 2.1 Single Dam Operation

First studies related to single reservoir operations were conducted in 1980s. For example, Yeh (1985) performed a study to examine the state-of-the-art of mathematical models which were enhanced for reservoir operations. In this study linear programming (LP), dynamic programming (DP), NLP and simulation were studied and the essentials of the usage of those models with the aid of computers and the difficulties that scientist and engineers face at the optimization procedure of reservoirs are introduced. Yeh performed this detailed literature reviews over all mathematical optimization models and their characteristics; and stated the advantages and some future work suggestions for each optimization methodology.

In 1990s, Simonovic conducted a study (1992) to represent the gap between practical and theoretical applications of the mathematical models for the reservoir optimizations. Two examples were illustrated for closing the gap. First example was about a simple simulation optimization model for reservoir sizing. Second example was about the advantages of knowledge based technology in the single multipurpose reservoir analysis. Wurbs (1993) defined the aim of his study as to select reservoir-system analysis models successfully and determine which methods are most feasible in different types of decision situations.

In 2000s, Barros et al. (2003) performed a study, where an NLP model was developed for the Brazilian hydropower system which has a capacity of 69,375 MW and includes 75 hydropower plants. The model was solved by using NLP and also the NLP was linearized and solved with LP (linear programming) and SLP (successive linear programming). When a comparison made among those three methods, LP seems good enough to solve the problem but SLP was much faster to reach the desired convergence. However, NLP was claimed as the most precise and suitable programming for real-time operation. It was stated that NLP model satisfied the demand and supply more energy and NLP model was alleged as the most useful method for real-time operation.

The aim of the study of Wang et al. (2004) was to develop a short-term hydro generation optimization model to increase energy production of large-scale hydro systems. This model includes a cycling module and a transition module. The cycling module determined the end-of-study content and discharge delay releases of each reservoir. Then, optimal generation scheduling was obtained by using the transition module. A direct search procedure (DSP) was also shown in this study. It was stated that the DSP can overcome nonlinearity and large number of reservoirs. The engineers of Fujian Electric Power Company Ltd. (FEPCL) are using this model and they are planning the hourly or half hourly generation by using this model.

Barros et al. (2005) conducted a study to compare different objective functions for optimization of complex hydropower systems. Objective function should be carefully specified due to the fact that it directly affects the operation systems. For this purpose, six different functions were analyzed in this study. The functions were; minimizing the stored potential energy's loss, minimizing storage deviations from targets, maximizing production of energy, minimizing spilled energy, minimizing energy complementation, maximizing secondary energy's profit. On the lights of results, minimizing complementation of energy selected as the best objective function for the studied hydropower system; which is focused on minimizing the use of alternative energy resources.

The aim of the study of Cheng et al. (2008) is to represent that Chaos Genetic Algorithm (CGA) is more relevant and effective to obtain optimum solution than the Genetic Algorithm (GA). In this study, premature convergence problem of GA was stated and CGA is used to overcome premature local optimum and increase the convergence speed of genetic algorithm. CGA integrates powerful global searching capability of the GA with that of powerful local searching capability of the Chaos Optimization Algorithm (COA). With respect to their study, they came up with a result that CGA can improve convergence speed and solution accuracy.

The objective of Yoo's study (2009) is to develop a linear objective function as an alternative to a nonlinear function to maximize hydropower energy production. The linear objective function model was analyzed and applied to operation of Yongdam Dam and HEPP. Although, the probable maximum energy generation of the HEPP is 214 GWh/year; it was concluded that this model could generate 184GWh annual energy.

In Sulek's study (2012) a new hybrid optimization method was suggested to solve a hydrothermal coordination problem and the hydro sub-problem is worked out by using hybrid optimization method. This method combined the traditional numerical methods and Genetic Algorithms. The study performed to the Slovak power system. The results express the efficiency of this hybrid method. However, according to the same results, the execution time of hybrid optimization method still not considered as quickly as desired and also this method requires too much hardware equipment.

Lu et al. (2013) performed a study to find an optimization procedure for Zhelin reservoir. The need of this new method is caused by the decrease of the water level and so the power production because of the decreasing rainfalls. For this purpose, three different methods applied; progressive optimization algorithm (POA), particle swarm optimization (PSO) and genetic algorithm (GA). In addition, the minimization of water consumption rate is chosen as the objective function. According to results of

these three methods, it can be concluded that POA is the most feasible method for the investigated case.

#### 2.2 Cascade Operation

First studies related to multi reservoir operations were conducted in the end of 20<sup>th</sup> century. To illustrate, Wardlaw and Sharif (1999) conducted a study to show that accurate and feasible solutions were obtained by using Genetic Algorithm (GA) approach. It was resulted from the study that GA application might be applied to large finite horizon multi reservoir system. Moreover, in this study, the benefits of the GA approach were stated. Providing accurate results over longer time horizons and also providing solutions which were very close to the optimum solutions were some of the benefits of the GA approach.

In 2000s, Labadie (2004) performed a study to obtain maximum efficiency in operation and existing reservoir systems. The aim of this study was to investigate the state-of-the-art in reservoir system optimization in detail with respect to overcome high dimensional, dynamic, nonlinear and stochastic characteristics of reservoirs. It is found out that even though the optimization can be applied to the reservoir operation, the implementation can be only be improved by involving the decision makers in terms of system development.

Hinçal et al. (2010) conducted a study to represent the efficiency of Genetic Algorithm in multi reservoirs' optimization. A computer code in Fortran Programming language was generated for this study. The code was applied to three reservoirs, the Blue Mesa, the Morrow Point and the Crystal Reservoirs in the Coloroda River. The real operational data were compared to the results obtained from this study and it was concluded that using genetic algorithm method was very feasible solution and this method could be used in optimization processes.

The objective of Jothiprakash and Arunkumar's study (2013) was to optimize multireservoir system including multiple hydropower plants to maximize the production of hydropower and meet the irrigation demands by using a NLP model. The NLP model was implemented to Koyna Hydroelectric Project (KHEP) to maximize the generation of hydropower. The results obtained from this study demonstrated that by relaxing the tribunal constraint on releases could increase the generation of hydropower and optimal releases met demand over long period of operation.

In the paper of Zheng et al. (2013), the improved Adaptive Genetic Algorithm (AGA) method and the application of this method in short term joint optimal operation of Qing River cascade hydropower stations were stated. The results obtained from this study presented that the improved Adaptive Genetic Algorithm (AGA) could produce more brilliant solution in the same algebra. Moreover, it could be also understood that the quantity of maximum power generation was not equal to benefit of maximum power generation.

Yang et al. (2013) demonstrated different strategies for the characteristics of cascade reservoirs optimization and for the untimely convergence problem of Genetic Algorithm. First strategy was to generate solution space generation for producing applicable initial population. Second strategy was to find chaos optimization for optimizing initial population. Third strategy was about suggesting new selective, trigonometric operators to sustain population variety. Finally, the fourth strategy was to admit applicable possibility of crossing and mutation to promote the performance of convergence speed.

#### **CHAPTER 3**

## **OPTIMIZATION PROCESS**

#### 3.1 Formulation of Hydropower Generation

As previously defined, hydropower is the power generated from moving water through a vertical head difference. Watermills are the first illustration of the use of hydropower that has a history since ancient ages. Late in 19<sup>th</sup> Century the hydropower was firstly used for generating electricity and the definition of hydroelectricity had emerged for the first time. Hydroelectricity is the electricity generated from hydropower plants. The formulation of the hydroelectricity generation is given in Equation 3-1.

$$P = \rho g H Q \eta \tag{3-1}$$

where

P = Power Generation (Watt)

 $\rho$  = Density of the water (kg/m<sup>3</sup>)

g = Gravitational Acceleration (m/s<sup>2</sup>)

H = Water Head (m)

Q = Discharge through turbines (m<sup>3</sup>/s)

 $\eta$  = Overall efficiency of hydropower plant which can be defined as;

$$\eta = \eta_{\rm H} \cdot \eta_{\rm G} \cdot \eta_{\rm T} \cdot \eta_{\rm Tr} \tag{3-2}$$

where

 $\eta_H$  - Hydraulic efficiency due to head losses

 $\eta_T$  - Turbine efficiency

 $\eta_G$  - Generator efficiency

 $\eta_{Tr}$  - Transformer efficiency

Maximization of Equation (3.1) is used as the objective function of the optimization problem. The assumptions and the basis of the modeling are as listed below:

- The density of the water is taken as 1000 kg/m<sup>3</sup>.
- The gravitational acceleration is taken as  $9.81 \text{ m/s}^2$ .
- Overall efficiency of hydropower plants are taken as 0.9 as constant by ignoring the hydraulic loss variation.
- Monthly basis operation is carried out with monthly basis data.
- Tail water level is stable for each and every studied dam and is taken as a specified value.
- Head is defined as a function of storage. The relation between reservoir volume and head is correlated by regression and the regarding equations are as mentioned in Chapter 4; storage characteristics are defined by the relationship between volume-area-elevation.
- Environmental flow requirements are not considered.
- The modeling is executed by starting at full capacity of each and every reservoir, at maximum water levels, on October which is the start of a water year.
- The maximum probable discharge through turbines is calculated by considering the maximum discharge capacity of the turbines for each and every dam.

- Since the river bed is so large and all the dams have large reservoirs, it is assumed that the water released from an upstream dam will reach to the downstream one without any lag.
- The evaporations from the reservoirs are not considered.
- Since monthly operation is carried out, required release of environmental flow for each hour is not considered.
- In addition to the release from an upstream reservoir to a downstream one, intermediate flows between reservoirs are also considered.

An illustration for a cascade hydropower system is presented in Figure (3-1) in which downstream reservoirs are fed by the release from the upstream reservoirs and additionally first three reservoirs are also fed by an intermediate flow.



Figure 3-1: A cascade hydropower system illustration

#### **3.2** Numerical Formulation of the Problem

The hydropower generation function expressed in Section 3.1 is used to form the objective function which will later on be maximized by using an optimization algorithm. The above illustrated model (Figure 3-1) is used in the formulation. There are specific constraints of this problem, which are given below.

 There exists a minimum and maximum flow that needs to be satisfied in order to ensure that the HEPP is producing energy and these flow rates are determined for each and every dam by considering the minimum and maximum turbine discharge capacities.

- As expressed in Section 3.1 all four dams are assumed to start with maximum storage, thus start with at their maximum water heads. It is forced that after one year of operation, the reservoir level will be the same in order to have periodicity. On this wise, the conservation of mass law is satisfied by making the total inflow equal to the total outflow. (Inflow = Outflow).
- The reservoir storage throughout the year must be within the specified levels. This will ensure that enough water head is available and by defining the maximum storage, the spillage water level is determined.
- The flow through the spillway is taken to be greater or equal to 0.

Maximization of total energy generation is considered as the objective function which is function of both the outflow and spill through each reservoir.

Note that, for the cascade system used, all the outflows from the reservoirs are used for energy production purposes. Hence, there is only outflow to produce energy through turbines or spill through the spillway.

Based on the above given restrictions, the objective function and the constraints for this optimization problem can be formulated as maximization of energy;

$$\max\left(\sum_{j=1}^{M}\sum_{i=1}^{N}E(Q,SP)_{i,j}\right)$$
(3-3)

where

N = 12 (number of months)

M = 4 (number of HEPPs in the related cascade system)

 $Q_{i,j}$  = Turbined Flow (Outflow through turbines from reservoir j at month i) (m<sup>3</sup>)

 $SP_{i,j}$  = Spilled Flow from reservoir j at month i (m<sup>3</sup>)

Note that, for each month the storage is calculated for each reservoir taking the difference of total inflows and total outflows. Total outflow is equal to the summation of discharged and spilled water, which is the release of the dam and will flow through a downstream dam. Total inflow is equal to the summation of the released flow from an upstream reservoir and intermediate flows.

The storage equation is defined in a loop, where the storage at the end of time step "i" is dependent on the storage at the end of time step "i-1", the inflow during time step "i", and the turbined and spilled flow during "i". This is written in the following format.

$$S_{i,j} = S_{i-1,j} + I_{i,j} + (Q_i + SP_i)_{j-1} - (Q_i + SP_i)_j \quad \text{for } i=1,...N; j=1,...M$$
(3-4)  
where

 $S_{i,j}$  = Reservoir Storage Volume of reservoir j at the end of month i

 $I_{i,j}$  = Intermediate flow (Inflow to the reservoir j during month i, apart from the release from an upstream reservoir)

Conservation of mass equation for each reservoir is satisfied. It should be noted that in order to satisfy periodicity, the storage level at the beginning and at the end of each year is forced to be equal to each other. This is enforced by equating the summation of all the inflows to the summation of outflows for each reservoir throughout the year.

$$(Vin)_{j} = (Vout)_{j}$$
 for j=1,...M (3-5)

where

$$(Vin)_{j} = \sum_{i=1}^{N} Q_{i,j-1} + SP_{i,j-1} + I_{i,j}$$
 for i=1,...N; j=1,...M (3-6)  
$$(Vout)_{j} = \sum_{i=1}^{N} Q_{i,j} + SP_{i,j}$$
 for i=1,...N; j=1,...M (3-7)

The following equations give the minimum and maximum limit on the outflow, spill through spillways and storage, respectively.

$$Qmin_{i,j} \le Q_{i,j} \le Qmax_{i,j} \qquad \text{for } i=1,...N; j=1,...M$$
(3-8)

$$SPmin_{i,j} \le SP_{i,j} \le SPmax_{i,j}$$
 for i=1,...N; j=1,...M (3-9)

$$\operatorname{Smin}_{i,j} \le \operatorname{Smax}_{i,j} \qquad \text{for } i=1,...N; j=1,...M$$
(3-10)

Since MATLAB requires matrices in order to perform a robust and computationally efficient calculation, the above given equations need to be reformulated in a matrix format. An additional consideration while writing the problem in MATLAB is the reservoir storage function (continuity equation). Time-dependent nature of this equation renders conventional modeling approaches inadequate, thus a different approach, namely a "black box" type input-output symbolic function has been created to overcome this problem.

A symbolic computation capability of MATLAB has been exploited to calculate the power generation with time dependent reservoir storage. The end result of this "black box" equation requires the following input values in order to construct a proper objective function. The rest is calculated automatically during the optimization.

- Inflow to each reservoir. (V<sub>in</sub>)
- Initial reservoir storage values. (S<sub>0</sub>)

The optimization problem is based on monthly operation (N = 12 months) and there are 4 HEPPs in the cascade, a V<sub>in</sub> vector of 4Nx1 is defined, which will be denoted as  $\underline{V}_{in}$  hereafter. Additionally, 4 scalar S<sub>0</sub> values are defined for each HEPP.

The decision variables ( $Q_{i,j}$  and  $SP_{i,j}$ ) are defined in an 8Nx1 vector format, where the first 4N variables represent the turbined flow and the other 4N represent the spilled flow. This vector is denoted as <u>X</u>.

By using the hydropower generation formula (Equation 3-1) monthly energy generation is defined in Equation 3-9 in MWh.

$$E_{i,j} = \left(9810 \cdot \frac{Q_{i,j}}{3600} \cdot \left(Hop(S_{i,j}) - Htw_j\right) \cdot \eta\right) \cdot 10^{-6} \qquad \begin{array}{c} \text{for } i=1,...N\\ j=1,...M \end{array} (3-11)$$

where

 $Hop(S_{i,j}) = Operational water head which is a function of S_{i,j}$ . (m) Htw = Tail water head which is constant for each dam (m)

Thus, the energy generation can be simplified as a function of discharge and spill  $(Q_{i,j} \text{ and } SP_{i,j})$ .

$$E_{i,j} = f(Q_{i,j}, S_{i,j}) \qquad \text{for } i=1,...N; \ j=1,...M \qquad (3-12)$$

The operation head is a regression equation derived from the relationship between the head and the storage level and is unique for each reservoir. These equations are presented in Section 4.1 for all four reservoirs.

After this, the generated power of each HEPP is summed up to obtain the total annual cascade power generation, as shown in the equation given below.

$$E_{total} = \sum_{j=1}^{M} \sum_{i=1}^{N} E_{i,j}$$
(3-13)

These operations are defined symbolically in MATLAB and by employing the intrinsic "*matlabFunction*" script; an m-file is created that works in a black-box type of manner.

The constraints are defined in another MATLAB script and these are also formulated in matrix format. The derivation of the matrices are given in the Appendix-A

#### 3.3 Setting the Optimization Algorithm

The optimization algorithm used for this problem is the intrinsic MATLAB function, *fmincon*, which is a gradient-based method that is designed to work on problems where the (nonlinear multivariable) objective and constraint functions are both continuous and have continuous first derivatives.

There are four algorithms that can be used with this function, namely interior-point (which is used for this problem), trust-region-reflective, SQP and lastly active-set. It is optionally possible to introduce the Hessian, but it is also possible to run without it. The interior-point algorithm is able to handle large, sparse problems. Bounds are satisfied at all iterations, and it can recover from NaN or Inf results (MATLAB Documentation, 2014). It contains the flexibility of switching between a line search

method that computes steps by factoring the primal-dual equations and a trust region method that uses a conjugate gradient iteration (Waltz et al, 2006).

Most of the default settings of *fmincon* have been kept, the optimization settings that need to be mentioned are as follows.

- Optimization algorithm: Interior-Point
- Maximum number of iterations: 5000 (can be increased if convergence is unattainable)
- Termination tolerance on decision variables: 10<sup>-8</sup>

In addition to *fmincon*, the *GlobalSearch* object was implemented, which contains options that affect how the solver algorithm searches for a global minimum. It was used in an attempt to increase variability in minima search direction, in a sense to create a "pseudo"-metaheuristic search method (Ugray et al, 2007)

The GlobalSearch algorithm performs the following steps:

- Run *fmincon* from initial starting point. If this run converges, *GlobalSearch* records the start point and end point for an initial estimate on the radius of a basin of attraction. Furthermore, *GlobalSearch* records the final objective function value for use in the score function. The score function is the sum of the objective function value at a point and a multiple of the sum of the constraint violations.
- Generate Trial Points. *GlobalSearch* uses the scatter search algorithm to generate a set of trial points. Trial points are potential start points.
- *GlobalSearch* evaluates the score function of a set of trial points. It then takes the point with the best score and runs *fmincon* from that point. Then it initializes the Basins of Attraction.
- If *fmincon* runs with success (converges), it creates a *GlobalOptimSolution*.

#### **CHAPTER 4**

## **CASE STUDY**

#### 4.1 The Selected Cascade

The studied cascade is located on the Murat River, which is one of the two main streams of Euphrates River at the Eastern part of Turkey within the provincial borders of Muş, Bingöl and Elazığ. The cascade is utilizing the hydroelectric potential between Alparslan-II Dam and HEPP, and Keban Dam and HEPP. The river section, at which the cascade is located, is sketched in Figure 4-1. The projects will be executed as a private sector investment under the Turkish Energy Regulations that enables the owner to operate the cascade for 49 years. The usage permission of the data is given by the owner within the scope of this thesis and the related letter is attached as Appendix-B.

There are five projects in the above mentioned cascade and named as below, from upstream to downstream:

- Upper Kaleköy Dam and HEPP (UK)
- Lower Kaleköy Dam and HEPP (LK)
- Gözeler Weirs and HEPPs
- Beyhan-1 Dam and HEPP (B1)
- Beyhan-2 Dam and HEPP (B2)

Since the Gözeler Weirs and HEPPs project is a run of river type and as stated in Chapter 1, it is not possible to make an operational optimization to a ROR type HEPP, the other four large dam projects are considered in the optimization process.



Figure 4-1: Hydropower plants located on Murat River

Links between reservoirs are:

- outflow from UK is inflow to LK
- outflow from LK is inflow to B1
- outflow from B1 is inflow to B2.

There are also intermediate flows to the reservoirs. The links between the reservoirs and the intermediate flows are illustrated in Figure 4-2.



Figure 4-2: The illustration of the studied cascade
By considering the hydraulic regime, all the four large dams have similar design with varying capacities where they include three large turbines and one small turbine for environmental flow purposes. Due to the specified heads and discharges, all the turbines are Francis type. Thus, operational discharge range is limited with respect to turbine supplier's specifications.

According to the hydraulic design of the studied dams, maximum and minimum operation levels of the HEPPs have already been determined and taken as a specific value for each and every dam in the optimization; which determines the maximum and minimum reservoir storage volumes that are obtained by using the head vs. storage relationship. The data given in Table 4-1 to Table 4-4 are used in the calculations.

As described in Chapter 3, after plotting the given data, a regression curve is fitted to the data of head versus storage for each reservoir in order to determine the respective reservoir storage volume for the operational water head. The fitted regression equations are used in optimization. Each HEPP has its own head storage relationship that is plotted with the regression curves and equations are given in Figure 4-3 to Figure 4-6 for Upper Kaleköy, Lower Kaleköy, Beyhan-1, and Beyhan-2 Dams, respectively. The reservoir storage volume of a dam at a time step is calculated by using Equation 3-4, and corresponding water elevation is estimated by using the corresponding regression equations.

The water head used in the calculations of hydroelectricity generation which is given by Equation 3-1, is calculated by subtracting the constant tail water heads of each dam that are given in Table 4-5, from the water elevations that are estimated by using the above mentioned regression equations.

Water Elevation (m)	Total Volume of the Reservoir (m <sup>3</sup> )
1210	406,200,000
1220	525,040,000
1230	665,480,000
1232	696,161,000
1235	783,759,500

 Table 4-1: Water elevations and corresponding reservoir volumes for UK



Figure 4-3: Water elevation vs. volume curve for UK

Water Elevation (m)	Total Volume of the Reservoir (m <sup>3</sup> )
1080	195,650,000
1090	284,540,000
1100	396,930,000
1110	535,250,000

Table 4-2: Water elevations and corresponding reservoir volumes for LK



Figure 4-4: Water elevation vs. volume curve for LK

 Table 4-3: Water elevations and corresponding reservoir volumes for B1

Water Elevation (m)	Total Volume of the Reservoir (m <sup>3</sup> )
970	195,314,000
980	343,174,000
990	647,766,000



Figure 4-5: Water elevation vs. volume curve for B1

Table 4-4: Water elevations and corresponding reservoir volumes for B2

Water Elevation (m)	Total Volume of the Reservoir (m <sup>3</sup> )
900	85,176,000
910	149,030,000



Figure 4-6: Water elevation vs. volume curve for B2

Main characteristics of the HEPPs are as shown on Table 4.5.

MAIN CHARACTERISTICS	UK	LK	B1	B2
Maximum Operation Water Level (m)	1235	1102.5	982	905
Minimum Operation Water Level (m)	1210	1085	977	902
Tail Water Level (m)	1103	1020	905	870
Maximum Head (m)	132	82.5	77	35
Minimum Head (m)	107	65	72	32
Discharge (Large Turbine) (m <sup>3</sup> /s)	182.4	199.1	272.9	272.9
Discharge (Small Turbine) (m <sup>3</sup> /s)	17.7	20.2	50	50
Total Probable Discharge (m <sup>3</sup> /s)	564.9	617.5	868.7	868.7
Total Probable Discharge Volume (m <sup>3</sup> /month)	1,464,220,800	1,600,560,000	2,251,670,400	2,251,670,400
Max. Reservoir Volume (m <sup>3</sup> )	770,547,042	438,584,475	442,588,235	117,113,665
Min. Reservoir Volume (m <sup>3</sup> )	409,504,946	238,812,785	324,941,176	97,956,577

 Table 4-5: Main characteristics of the dams in the studied cascade system

# 4.2 Outline of the Case Study

As stated in Chapter 3, a MATLAB code was written for maximizing the energy generation of a cascade system. The provided 40 years data taken between the years of 1967 – 2006 is used in the calculations. The cascade defined in the previous section is analyzed by considering three different cases; i.e. the optimization is carried out by using minimum, maximum and average flow data. The input for the average flow is calculated by taking average of summation of the data for each month. The input data for minimum and maximum flow to the reservoirs are the ones at the years 2000 and 1988, respectively.

The program is run for both the cascade operation and single dam operation, separately. Calculation time was approximately 24 minutes for the single runs (roughly 1.5 hours for all four dams) and around 4 hours for the cascade case. This implies that runtime for the cascade case is within an order of magnitude in comparison with the single runs and can still be computed with an ordinary office PC. The calculations were made on an Intel<sup>®</sup> Core<sup>™</sup> i7 CPU Q740 machine with 6GBs of RAM and Windows 7 OS. A possible improvement could be made by introducing a more efficient algorithm that has better performance for big numbers of decision variables.

## 4.2.1 Optimization with Minimum Flow Data

The data set, at year of 2000, at which the yearly sum of all monthly flows to the reservoirs results with a minimum value, is used in the calculations. The optimization outputs are given in Tables 4-6 to 4-13. Tables 4-6 to 4-9 summarize the optimization results considering single operation of each reservoir, whereas Tables 4-10 to 4-13 give the results for cascade operation for UK, LK, B1 and B2, respectively. The final results of decision variables, turbined and spilled volumes during each month, are given in tables together with the operational water head at the end of each month, which are calculated by using the corresponding storages.

As it is stated in the previous chapter, the reservoir operations are started with full reservoir volumes, this means that in the beginning of October, head of the reservoirs are at their maximum values that are specified in Table 4-5 and since the conservation of mass law is satisfied, the maximum head values can be also observed in the tables and in the figures at September which shows the result for the end of September that is also the beginning of October. The maximized energy generation at each month is as shown in the tables. The illustrations of the results are plotted in the Figures 4-7 to 4-18; those which Figures 4-7 to 4-10 show the operation head, Figures 4-11 to 4-14 show turbined volumes and Figure 4-15 to 4-18 show energy generation at each month for respective reservoirs.

By running the optimization using the minimum flow data, total volume of flow to the reservoir is not higher than the total discharge capacity of the turbines of each HEPP, thus there is no spill flow in minimum flow runs; which is an expected result since the dams are designed for not to waste any water. The operation head and turbined volume for UK and B2 are almost same without any significant deviation that the optimization results are uniform; in other words UK and B2 have identical energy generation with minimum flow data. Overall comparison will be made in Section 4.3.

UK (Single Run)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	131.79	196,138,043.01	0.00	63,393.15
November	131.79	199,262,152.74	0.00	64,402.88
December	131.79	208,932,041.45	0.00	67,528.25
January	131.79	210,222,619.80	0.00	67,945.38
February	131.79	193,136,429.31	0.00	62,423.00
March	131.79	250,586,127.19	0.00	80,991.14
April	131.79	417,598,456.02	0.00	134,970.65
May	131.79	274,832,115.98	0.00	88,827.60
June	131.79	138,003,069.45	0.00	44,603.53
July	131.79	69,952,504.29	0.00	22,609.12
August	131.79	81,110,875.29	0.00	26,215.59
September	131.79	149,274,311.09	0.00	48,246.47
			Total	772,156.76

 Table 4-6: Optimization result of single operation with minimum flow data for UK

Table 4-7: Optimization result of single operation with minimum flow data for LK

LK (Single Run)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	80.83	224,320,469.44	0.00	44,470.97
November	80.84	207,356,836.97	0.00	41,110.13
December	82.50	203,438,865.80	0.00	41,162.04
January	81.74	239,050,771.27	0.00	47,923.69
February	75.56	300,986,886.37	0.00	55,775.08
March	72.75	394,742,829.24	0.00	70,431.98
April	82.50	596,412,633.45	0.00	120,672.91
May	82.50	361,610,864.99	0.00	73,165.18
June	81.04	183,114,419.84	0.00	36,394.11
July	79.08	98,532,285.63	0.00	19,109.82
August	78.94	90,650,706.98	0.00	17,550.01
September	82.50	113,616,852.65	0.00	22,988.24
			Total	590,754.14

B1 (Single Run)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	77.00	227,743,097.48	0.00	43,007.58
November	76.62	241,305,466.13	0.00	45,343.41
December	76.63	250,096,831.52	0.00	47,004.21
January	77.00	287,953,485.95	0.00	54,377.86
February	75.43	332,209,744.37	0.00	61,456.52
March	76.51	469,648,381.71	0.00	88,122.69
April	77.00	1,223,963,288.76	0.00	231,136.29
May	72.25	645,180,950.32	0.00	114,321.80
June	73.97	151,213,161.19	0.00	27,432.72
July	77.00	85,185,165.82	0.00	16,086.58
August	75.15	79,747,329.27	0.00	14,697.41
September	77.00	121,375,445.11	0.00	22,920.84
			Total	765,907.90

Table 4-8: Optimization result of single operation with minimum flow data for B1

Table 4-9: Optimization result of single operation with minimum flow data for B2

B2 (Single Run)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	35.00	227,743,099.03	0.00	19,548.90
November	35.00	232,346,895.56	0.00	19,944.08
December	35.00	269,841,482.06	0.00	23,162.52
January	35.00	277,167,404.45	0.00	23,791.36
February	35.00	367,240,289.60	0.00	31,522.99
March	35.00	423,037,420.97	0.00	36,312.47
April	35.00	1,235,543,704.41	0.00	106,055.98
May	35.00	559,528,301.55	0.00	48,028.51
June	35.00	210,737,526.31	0.00	18,089.18
July	35.00	111,313,449.70	0.00	9,554.87
August	35.00	80,231,080.53	0.00	6,886.84
September	35.00	120,891,693.45	0.00	10,377.04
			Total	353,274.73

UK (Calculated in Cascade)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	131.79	196,138,043.06	0.00	63,393.31
November	131.79	199,262,152.85	0.00	64,403.05
December	131.79	208,932,041.70	0.00	67,528.43
January	131.79	210,222,619.98	0.00	67,945.55
February	131.79	193,136,429.44	0.00	62,423.17
March	131.79	250,586,127.20	0.00	80,991.35
April	131.79	417,598,456.02	0.00	134,971.00
May	131,79	274,832,115.98	0.00	88,827.83
June	131.79	138,003,069.45	0.00	44,603.64
July	131.79	69,952,504.30	0.00	22,609.18
August	131.79	81,110,875.31	0.00	26,215.65
September	131.79	149,274,311.10	0.00	48,246.59
			Total	772,158.75

 Table 4-10: Optimization result of cascade operation with minimum flow data for UK

 Table 4-11: Optimization result of cascade operation with minimum flow data for LK

LK (Calculated in Cascade)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	82.50	205,311,944.53	0.00	41,541.02
November	82.30	209,690,303.02	0.00	42,323.96
December	82.50	220,113,925.46	0.00	44,535.93
January	80.09	257,967,074.00	0.00	50,667.57
February	79.56	236,439,271.37	0.00	46,131.84
March	82.47	329,470,550.03	0.00	66,635.87
April	79.97	736,196,934.95	0.00	144,387.76
May	80.23	358,696,786.59	0.00	70,574.53
June	82.50	140,481,324.65	0.00	28,423.76
July	76.71	142,266,995.12	0.00	26,764.61
August	82.50	22,945,658.49	0.00	4,642.61
September	82.50	154,253,655.20	0.00	31,210.33
	Anna and Anna	and the second sec	Total	597.839.79

B1 (Calculated in Cascade)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	77.00	227,743,099.03	0.00	43,007.58
November	77.00	232,346,895.63	0.00	43,876.97
December	76.18	269,841,482.14	0.00	50,411.73
January	77.00	277,167,404.52	0.00	52,340.99
February	73.94	367,240,289.68	0.00	66,596.04
March	77.00	423,037,421.04	0.00	79,887.44
April	77.00	1,235,543,704.50	0.00	233,323.16
May	75.89	559,528,301.59	0.00	104,140.00
June	75.08	210,737,526.32	0.00	38,805.41
July	77.00	111,313,449.69	0.00	21,020.71
August	75.13	80,231,080.53	0.00	14,782.52
September	77.00	120,891,693.72	0.00	22,829.49
	And the second		Total	771,022.03

**Table 4-12:** Optimization result of cascade operation with minimum flow data forB1

Table 4-13: Optimization result of cascade operation with minimum flow data	a for
B2	

B2 (Calculated in Cascade)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	35.00	227,743,099.13	0.00	19,548.90
November	35.00	232,346,895.64	0.00	19,944.08
December	35.00	269,841,482.14	0.00	23,162.52
January	35.00	277,167,404.53	0.00	23,791.36
February	35.00	367,240,289.67	0.00	31,522.99
March	35.00	423,037,421.05	0.00	36,312.47
April	35.00	1,235,543,704.48	0.00	106,055.98
May	35.00	559,528,301.59	0.00	48,028.51
June	35.00	210,737,526.34	0.00	18,089.18
July	35.00	111,313,449.75	0.00	9,554.87
August	35.00	80,231,080.58	0.00	6,886.84
September	35.00	120,891,693.50	0.00	10,377.04
		An other states and the second second second second second second second second second second second second se	Total	353,274.73



Figure 4-7: Operation head for UK with minimum flow data



Figure 4-8: Operation head for LK with minimum flow data



Figure 4-9: Operation head for B1 with minimum flow data



Figure 4-10: Operation head for B2 with minimum flow data



Figure 4-11: Turbined volume for UK with minimum flow data



Figure 4-12: Turbined volume for LK with minimum flow data



Figure 4-13: Turbined volume for B1 with minimum flow data



Figure 4-14: Turbined volume for B2 with minimum flow data



Figure 4-15: Energy generation of UK with minimum flow data



Figure 4-16: Energy generation of LK with minimum flow data



Figure 4-17: Energy generation of B1 with minimum flow data



Figure 4-18: Energy generation of B2 with minimum flow data

## 4.2.2 Optimization with Average Flow Data

The average of the 40 years data is used in these calculations. The optimization outputs are given in Table 4-14 to 4-21. Tables 4-14 to 4-17 summarize the optimization results considering single operation of each reservoir, whereas Tables 4-18 to 4-21 give the results for cascade operation for UK, LK, B1 and B2, respectively. The final results of decision variables, turbined and spilled volumes during each month, are given in tables together with the operational water head at the end of each month, which are calculated by using the corresponding storages.

The maximized energy generation at each month is as shown on the tables. The illustrations of the results are plotted in Figures 4-19 to 4-30; those which Figures 4-19 to 4-22 show the operation head, Figures 4-23 to 4-26 show turbined volumes and Figure 4-27 to 4-30 show energy generation at each month for respective reservoirs. The final results of decision variables, turbined and spilled volumes during each month, are given in tables together with the operational water head at the end of each month, which are calculated by using the corresponding storages.

As in the case of optimization with using the minimum flow data, the total volume of flow to reservoirs is not higher than the total discharge capacity of the turbines of each HEPP, hence again there is no spill flow in average flow runs.

UK (Single Run)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	131.79	205,933,739.48	0.00	66,559.18
November	128.28	305,817,234.77	0.00	96,211.39
December	131.15	168,974,299.84	0.00	54,349.68
January	131.79	339,807,343.34	0.00	109,828.04
February	124.79	371,485,905.82	0.00	113,688.67
March	125.56	478,279,071.47	0.00	147,279.62
April	131.79	950,697,336.50	0.00	307,271.83
May	131.18	795,686,863.80	0.00	255,992.68
June	131.79	194,244,902.53	0.00	62,781.27
July	126.85	182,424,240.70	0.00	56,752.05
August	130.75	0.14	0.00	0.00
September	131.79	119,963,506.63	0.00	38,773.02
			Total	1,309,487.42

 Table 4-14: Optimization result of single operation with average flow data for UK

 Table 4-15: Optimization result of single operation with average flow data for LK

LK (Single Run)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	77.93	270,452,719.69	0.00	51,691.04
November	80.03	309,446,642.35	0.00	60,732.62
December	77.45	232,387,691.94	0.00	44,141.40
January	82.50	309,175,601.45	0.00	62,555.89
February	82.26	410,284,520.51	0.00	82,774.59
March	72.40	708,157,740.62	0.00	125,747.50
April	82.50	1,089,663,735.64	0.00	220,473.03
May	82.50	968,068,043.82	0.00	195,870.42
June	77.39	307,525,212.42	0.00	58,370.50
July	78.92	181,185,129.69	0.00	35,070.52
August	76.69	33,566,039.87	0.00	6,313.27
September	82.50	59,169,873.00	0.00	11,971.91
			Total	955,712.70

B1 (Single Run)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	77.00	299,213,280.43	0.00	56,504.18
November	73.25	454,869,817.85	0.00	81,710.69
December	77.00	209,148,373.37	0.00	39,496.10
January	72.00	477,375,831.86	0.00	84,295.03
February	77.00	357,371,653.01	0.00	67,486.96
March	75.05	966,454,703.82	0.00	177,892.82
April	75.80	1,576,754,380.04	0.00	293,125.62
May	77.00	1,292,421,501.36	0.00	244,064.11
June	74.14	476,440,860.13	0.00	86,632.38
July	77.00	134,317,995.38	0.00	25,364.95
August	75.17	80,433,742.20	0.00	14,828.28
September	77.00	22,804,727.53	0.00	4,306.50
			Total	1,175,707.61

Table 4-16: Optimization result of single operation with average flow data for B1

Table 4-17: Optimization result of single operation with average flow data for B2

B2 (Single Run)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	35.00	299,213,280.58	0.00	25,683.72
November	35.00	454,869,817.71	0.00	39,044.89
December	35.00	209,148,373.80	0.00	17,952.77
January	35.00	477,375,831.60	0.00	40,976.75
February	35.00	357,371,653.07	0.00	30,675.89
March	35.00	966,454,703.68	0.00	82,958.06
April	35.00	1,576,754,380.01	0.00	135,344.65
May	35.00	1,292,421,501.37	0.00	110,938.23
June	35.00	476,440,860.33	0.00	40,896.49
July	35.00	134,317,995.73	0.00	11,529.52
August	32.00	99,590,824.40	0.00	7,815.89
September	35.00	3,647,644.73	0.00	313.10
	-		Total	544,129,96

UK (Calculated in Cascade)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	131.78	206,192,009.32	0.00	66,637.78
November	129.89	271,941,592.23	0.00	86,631.37
December	131.76	186,838,395.36	0.00	60,375.49
January	131.75	356,497,186.53	0.00	115,191.69
February	131.74	230,105,777.96	0.00	74,347.55
March	131.75	491,011,431.58	0.00	158,649.72
April	131.72	1,080,086,063.03	0.00	348,924.72
May	114.55	1,059,239,566.90	0.00	297,570.02
June	128.49	4,298,430.00	0.00	1,354.55
July	131.79	824,556.36	0.00	266.50
August	131.31	92,618,333.06	0.00	29,825.69
September	131.79	133,661,102.67	0.00	43,200.18
			Total	1,282,975.26

Table 4-18: Optimization result of cascade operation with average flow data for UK

Table 4-19: Optimization result of cascade operation with average flow data for LK

LK (Calculated in Cascade)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	82.49	218,668,857.24	0.00	44,238.57
November	82.48	299,574,502.28	0.00	60,599.67
December	82.47	220,944,954.86	0.00	44,689.93
January	82.47	383,532,350.12	0.00	77,574.09
February	82.46	266,319,278.80	0.00	53,859.23
March	82.44	608,602,876.63	0.00	123,047.27
April	82.48	1,333,817,315.60	0.00	269,811.61
May	82.50	1,231,406,824.94	0.00	249,152.07
June	82.36	60,875,531.13	0.00	12,296.20
July	82.49	15,542,789.44	0.00	3,144.55
August	82.50	100,672,740.24	0.00	20,368.09
September	82.50	139,124,929.71	0.00	28,149.32
			Total	986,930.63

B1 (Calculated in Cascade)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	76.99	247,581,885.76	0.00	46,750.05
November	76.99	356,833,340.51	0.00	67,373.32
December	76.98	286,159,986.70	0.00	54,025.97
January	76.97	434,334,240.79	0.00	81,989.47
February	76.96	331,201,363.80	0.00	62,515.93
March	76.97	820,983,172.18	0.00	154,973.63
April	76.99	1,838,041,959.81	0.00	347,053.61
May	77.00	1,583,726,382.57	0.00	299,071.13
June	77.00	162,517,825.35	0.00	30,690.20
July	77.00	35,953,064.72	0.00	6,789.37
August	77.00	104,457,509.08	0.00	19,725.94
September	77.00	145,816,135.73	0.00	27,536.28
			Total	1,198,494.90

Table 4-20: Optimization result of cascade operation with average flow data for B1

Table 4-21: Optimization result of cascade operation with average flow data for B2

B2 (Calculated in Cascade)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	34.97	247,795,901.54	0.00	21,249.81
November	34.94	356,976,273.91	0.00	30,593.01
December	34.92	286,295,321.12	0.00	24,520.75
January	34.92	434,345,241.94	0.00	37,199.15
February	34.92	331,178,772.76	0.00	28,366.42
March	34.94	820,857,967.95	0.00	70,348.32
April	34.97	1,837,896,139.66	0.00	157,612.41
May	35.00	1,583,537,321.31	0.00	135,914.35
June	34.99	162,535,917.10	0.00	13,949.26
July	34.99	36,003,743.54	0.00	3,089.24
August	34,99	104,459,212.42	0.00	8,962.86
September	35.00	145,725,053.75	0.00	12,508.67
			Total	544,314.26



Figure 4-19: Operation head for UK with average flow data



Figure 4-20: Operation head for LK with average flow data



Figure 4-21: Operation head for B1 with average flow data



Figure 4-22: Operation head for B2 with average flow data



Figure 4-23: Turbined volume for UK with average flow data



Figure 4-24: Turbined volume for LK with average flow data



Figure 4-25: Turbined volume for B1 with average flow data



Figure 4-26: Turbined volume for B2 with average flow data



Figure 4-27: Energy generation of UK with average flow data



Figure 4-28: Energy generation of LK with average flow data



Figure 4-29: Energy generation of B1 with average flow data



Figure 4-30: Energy generation of B2 with average flow data

#### 4.2.3 Optimization with Maximum Flow Data

The data set of 1988, in which the sum of all flows to reservoirs resulting with the maximum value, is used in the calculations for two cases; i.e. the first case is spilled volume is not constrained and the second one is limited, except at two specific months. The reason why this limitation is needed will be discussed in the following section.

#### 4.2.3.1 Study without Spillway Upper Bound

The optimization outputs are given in Table 4-22 to 4-29. Tables 4-22 to 4-25 summarize the optimization results considering single operation of each reservoir, whereas Tables 4-26 to 4-29 give the results for cascade operation for UK, LK, B1 and B2, respectively. The final results of decision variables, turbined and spilled volumes during each month, are given in tables together with the operational water head at the end of each month, which are calculated by using the corresponding storages.

The maximized energy generation for each month is as shown on the tables. The illustrations of the results are plotted in Figures 4-31 to 4-42; those which Figures 4-31 to 4-34 show the operation head, Figures 4-35 to 4-38 show turbined volumes and Figure 4-39 to 4-42 show energy generation at each month for respective reservoirs.

The results show excessive spills that occur even in dry season, which indicates that the algorithm is not able to come up with an expected energy generation. Most probably, the code is stuck in local minima and cannot converge to a global optimum solution. This result in such an outcome where the cascade case actually generates less energy compared to the single runs, which should not be the case. This was a result to be expected for a gradient-based algorithm, although an effort to overcome local minima has been made by implementing the "*GlobalSearch*" algorithm. A heuristic algorithm that contains intrinsic mutational characteristics should be employed to overcome getting stuck in low slope areas and local minimas.

UK (Single Run)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	125.86	324,041,352.16	176.13	100,022.41
November	131.79	110,592,298.81	0.02	35,744.18
December	131.79	324,883,952.74	16,638.33	105,004.70
January	131.70	533,305,057.06	865,398.04	172,256.73
February	131.79	299,092,043.70	297.10	96,668.58
March	107.15	1,175,604,148.22	574,574.11	308,918.96
April	131.79	1,464,220,799.62	980,512,846.29	473,246.09
May	131.79	1,464,220,799.44	1,245,403,637.58	473,246.09
June	131.79	716,875,237.70	461.28	231,698.94
July	131.24	120,039,735.77	83.07	38,637.78
August	125.26	194,316,737.03	47.57	59,694.37
September	131.79	143,471.62	0.64	46.37
			Total	2,095,185.20

**Table 4-22:** Optimization result of single operation with maximum flow data and without spillway upper bound for UK

**Table 4-23:** Optimization result of single operation with maximum flow data and without spillway upper bound for LK

LK (Single Run)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	82.50	343,701,139.83	1,077,026.56	69,541.48
November	82.50	234,883,511.22	0.54	47,524.27
December	82.50	477,689,780.03	278,138.08	96,651.57
January	82.50	592,216,964.05	1,282,118.04	119,824.00
February	82.50	375,893,176.58	204,560.26	76,054.94
March	82.50	1,360,106,518.25	0.07	275,192.05
April	82.50	1,600,559,999.96	1,215,671,876.88	323,843.31
May	82.50	1,600,559,999.96	1,210,531,429.04	323,843.31
June	82.50	798,821,430.23	31,992.77	161,626.54
July	82.50	167,319,806.95	0.07	33,854.03
August	82.50	204,543,091.45	0.05	41,385.46
September	82.50	8,014,852.10	0.04	1,621.66
			Total	1,570,962.60

B1 (Single Run)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	77.00	385,140,749.05	723,631.41	72,730.94
November	77.00	422,955,386.06	376.26	79,871.95
December	77.00	727,763,822.04	2.24	137,432.74
January	77.00	679,855,593.92	427,609.08	128,385.63
February	77.00	505,555,828.18	81.86	95,470.43
March	74.45	1,745,844,509.96	299.83	318,775.11
April	77.00	2,251,670,399.14	1,319,543,117.31	425,211.07
May	77.00	2,251,670,399.13	1,021,913,978.74	425,211.07
June	77.00	1,022,163,887.66	5.06	193,027.98
July	77.00	271,484,843.33	1.18	51,267.88
August	77.00	219,593,240.58	0.98	41,468.54
September	77.00	10,135,178.15	0.85	1,913.95
3			Total	1,970,767.28

**Table 4-24:** Optimization result of single operation with maximum flow data andwithout spillway upper bound for B1

**Table 4-25:** Optimization result of single operation with maximum flow data andwithout spillway upper bound for B2

B2 (Single Run)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	35.00	385,140,749.53	385,864,380.97	33,059.52
November	35.00	422,955,386.15	59,041,302.12	36,305.43
December	35.00	727,763,821.99	42,476,813.03	62,469.43
January	35.00	679,855,593.90	679,741,803.19	58,357.10
February	35.00	505,555,828.13	6.19	43,395.65
March	35.00	1,745,844,509.76	130,923,919.74	149,858.93
April	35.00	2,251,670,398.87	1,319,543,116.70	193,277.76
May	35.00	2,251,670,398.70	1,021,913,978.15	193,277.76
June	35.00	1,022,163,887.80	5,721,293.52	87,739.99
July	35.00	271,484,842.71	4.10	23,303.58
August	35.00	219,593,240.39	134,693,456.86	18,849.33
September	35.00	10,135,177.06	0.14	869.98
			Total	900,764.46

UK (Calculated in Cascade)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	131.14	197,198,119.01	19,841,417.34	63,422.87
November	130.44	225,064,475.06	25,950,096.73	72,001.31
December	130.92	307,392,033.19	6,150,524.12	98,699.23
January	131.10	508,487,991.93	18,943,204.36	163,495.44
February	131.79	268,235,628.41	15,533,724.53	86,695.57
March	112.79	1,116,114,674.94	8,397.66	308,746.98
April	131.79	1,459,585,941.91	1,045,203,352.64	471,748.08
May	131.26	1,464,220,799.89	1,258,987,179.34	471,369.49
June	131.31	715,641,517.15	0.02	230,468.62
July	131.36	104,058,967.40	730,107.44	33,523.20
August	131.60	68,267,580.14	56,930.33	22,033.55
September	131.79	129,037,130.19	0.26	41,705.68
			Total	2,063,910.02

**Table 4-26:** Optimization result of cascade operation with maximum flow data and without Spillway Upper Bound for UK

**Table 4-27:** Optimization result of cascade operation with maximum flow data and without Spillway Upper Bound for LK

LK (Calculated in Cascade)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	82.05	229,975,457.90	12,968,234.25	46,275.91
November	82.29	364,355,630.60	8,226,840.49	73,529.20
December	79.93	483,176,166.85	10,335,686.63	94,715.36
January	77.79	596,786,744.54	14,438,435.70	113,849.27
February	73.59	389,472,123.61	19,223,440.62	70,290.05
March	82.50	1,198,139,011.70	179,511.54	242,420.96
April	77.01	1,579,604,371.78	1,359,299,027.96	298,353.96
May	82.21	1,574,109,564.24	1,191,295,264.68	317,360.10
June	82.19	797,758,626.68	96,778.95	160,797.60
July	81.52	148,975,147.21	10,741,009.45	29,782.94
August	82.18	69,748,041.00	1,227,313.06	14,057.46
September	82.50	132,388,648.23	866,335.32	26,786.36
			Total	1,488,219.17

B1 (Calculated in Cascade)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	75.37	291,739,025.95	30,745,198.80	53,923.40
November	73.88	570,769,568.14	24,758,469.13	103,423.14
December	73.04	736,168,302.99	16,090,956.92	131,875.37
January	72.73	703,357,359.61	12,739,776.35	125,465.68
February	73.40	514,371,130.74	8,114,622.03	92,594.12
March	77.00	1,439,378,488.07	0.03	271,815.83
April	73.22	2,228,538,772.62	1,614,165,243.56	400,206.86
May	74.12	2,213,391,688.16	993,400,444.33	402,355.88
June	75.00	1,000,419,982.19	3,691.40	184,021.74
July	76.44	229,687,117.99	287,916.87	43,061.41
August	76.58	75,531,834.38	7,236,880.92	14,186.24
September	77.00	119,745,469.26	5,801,001.56	22,613.03
			Total	1,845,542.70

**Table 4-28:** Optimization result of cascade operation with maximum flow data andwithout Spillway Upper Bound for B1

**Table 4-29:** Optimization result of cascade operation with maximum flow data andwithout Spillway Upper Bound for B2

B2 (Calculated in Cascade)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	34.59	294,579,381.78	30,547,005.05	24,987.03
November	35.00	567,459,065.73	25,426,809.67	48,709.27
December	35.00	730,313,789.84	21,945,470.95	62,688.31
January	34.78	698,269,936.90	19,216,313.24	59,565.21
February	32.00	515,458,361.81	24,795,362.73	40,453.17
March	35.00	1,370,277,907.76	49,943,493.46	117,621.23
April	35.00	2,176,745,620.48	1,665,958,395.83	186,846.40
May	35.00	2,210,941,163.93	995,850,968.43	189,781.66
June	35.00	1,000,423,673.81	0.04	85,873.87
July	32.00	232,276,355.80	16,855,765.85	18,229.05
August	32.00	75,241,221.69	7,527,493.53	5,904.93
September	35.00	99,520,503.00	6,868,880.70	8,542.59
			Total	849,202.73



Figure 4-31: Operation head for UK with maximum flow data without spillway upper bound



Figure 4-32: Operation head for LK with maximum flow data without spillway upper bound



Figure 4-33: Operation head for B1 with maximum flow data without spillway upper bound



Figure 4-34: Operation head for B2 with maximum flow data without spillway upper bound


Figure 4-35: Turbined volume for UK with maximum flow data without spillway upper bound



Figure 4-36: Turbined volume for LK with maximum flow data without spillway upper bound



Figure 4-37: Turbined volume for B1 with maximum flow data without spillway upper bound



Figure 4-38: Turbined volume for B2 with maximum flow data without spillway upper bound



Figure 4-39: Energy generation for UK with maximum flow data without spillway upper bound



Figure 4-40: Energy generation for LK with maximum flow data without spillway upper bound



Figure 4-41: Energy generation for B1 with maximum flow data without spillway upper bound



Figure 4-42: Energy generation for B2 with maximum flow data without spillway upper bound

### 4.2.3.2 Study with Defined Spillway Upper Bound

It is observed from the results of the study without a spillway upper bound that the code is not able to converge to a solution without excessive spill discharges during the dry season which is not expected. In this regard, the spillway upper bound is limited to zero for all the months except April and May. This approach is carried out by considering the total inflow volumes to the reservoirs; i.e. April and May are the only two months in which the total inflow to the reservoir is more than the probable maximum discharge through turbines that are given in Table 4-5. The optimization results are tabulated in Tables 4-30 to 4-37.

Tables 4-30 to 4-33 summarize the optimization results considering single operation of each reservoir, whereas Tables 4-34 to 4-37 give the results for cascade operation for UK, LK, B1 and B2, respectively; and the illustration of results can be observed in Figures 4-43 to 4-54; those which Figures 4-43 to 4-46 show the operation head, Figures 4-47 to 4-50 show turbined volumes and Figure 4-51 to 4-54 show energy generation at each month for respective reservoirs.

UK (Single Run)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	131.53	207,002,340.26	0.00	66,776.42
November	131.46	236,280,106.93	0.00	76,176.64
December	131.79	316,251,967.83	0.00	102,214.78
January	131.79	531,905,799.08	0.00	171,915.56
February	117.67	544,221,483.76	0.00	157,055.32
March	109.26	911,564,292.38	0.00	244,268.63
April	128.34	1,286,980,120.58	1,257,869,526.51	405,071.35
May	131.79	1,367,635,855.53	1,263,622,526.15	442,029.19
June	131.79	716,875,694.06	0.00	231,699.09
July	130.91	128,181,521.64	0.00	41,154.96
August	129.70	102,579,282.26	0.00	32,628.77
September	131.79	83,739,277.02	0.00	27,065.10
			Total	1,998,055.80

**Table 4-30:** Optimization result of single operation with maximum flow data andwith spillway upper bound for UK

 Table 4-31: Optimization result of single operation with maximum flow data and with spillway upper bound for LK

LK (Single Run)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	79.14	266,134,370.29	0.00	51,652.00
November	82.50	322,175,928.02	0.00	65,186.26
December	82.50	469,319,294.83	0.00	94,957.96
January	82.50	591,234,426.18	0.00	119,625.20
February	82.50	621,226,879.57	0.00	125,693.61
March	79.81	1,126,236,435.30	0.00	220,433.94
April	82.50	1,600,559,999.96	1,285,043,531.21	323,843.31
May	82.50	1,600,559,999.96	1,132,165,373.71	323,843.31
June	82.50	798,853,418.11	0.00	161,633.01
July	82.50	175,461,509.74	0.00	35,501.35
August	82.50	112,805,589.11	0.00	22,824.10
September	82.50	91,610,657.00	0.00	18,535.70
			Total	1,563,729.73

B1 (Single Run)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	77.00	307,220,578.03	0.00	58,016.30
November	77.00	510,248,182.38	0.00	96,356.54
December	77.00	708,284,348.36	0.00	133,754.19
January	77.00	688,849,385.28	0.00	130,084.04
February	77.00	750,685,065.01	0.00	141,761.24
March	77.00	1,451,995,390.40	0.00	274,198.44
April	72.00	1,401,298,887.52	2,416,912,681.69	247,441.36
May	77.00	2,251,670,399.84	825,900,864.60	425,211.07
June	77.00	1,022,163,886.21	0.00	193,027.98
July	72.00	397,273,601.38	0.00	70,150.57
August	77.00	10,208,679.30	0.00	1,927.83
September	77.00	93,730,992.00	0.00	17,700.39
			Total	1,789,629.96

**Table 4-32:** Optimization result of single operation with maximum flow data and with spillway upper bound for B1

**Table 4-33:** Optimization result of single operation with maximum flow data andwith spillway upper bound for B2

B2 (Single Run)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	35.00	307,220,578.04	0.00	26,371.05
November	35.00	510,248,182.37	0.00	43,798.43
December	35.00	708,284,348.36	0.00	60,797.36
January	35.00	688,849,385.28	0.00	59,129.11
February	35.00	750,685,065.01	0.00	64,436.93
March	35.00	1,451,995,390.39	0.00	124,635.65
April	35.00	2,251,670,399.98	1,566,541,169.23	193,277.76
May	35.00	2,251,670,399.98	825,900,864.46	193,277.76
June	35.00	1,022,163,886.21	0.00	87,739.99
July	35.00	397,273,601.38	0.00	34,100.97
August	35.00	10,208,679.63	0.00	876.29
September	35.00	93,730,991.67	0.00	8,045.63
			Total	896,486.93

UK (Calculated in Cascade)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	131.79	200,338,621.81	0.00	64,750.80
November	131.79	234,295,209.43	0.00	75,725.80
December	131.79	324,900,595.11	0.00	105,010.08
January	131.79	531,905,800.55	0.00	171,915.56
February	131.79	301,357,001.85	0.00	97,400.63
March	107.15	1,176,144,761.14	0.00	309,070.73
April	131.79	1,464,220,793.57	980,546,811.70	473,246.09
May	131.79	1,464,220,793.71	1,245,403,641.72	473,246.09
June	131.79	716,875,693.44	0.00	231,699.09
July	131.79	105,940,094.78	0.00	34,240.56
August	131.79	74,616,627.58	0.00	24,116.60
September	131.79	133,943,347.61	0.00	43,291.40
			Total	2,103,713.42

**Table 4-34:** Optimization result of cascade operation with maximum flow data and with spillway upper bound for UK

**Table 4-35:** Optimization result of cascade operation with maximum flow data and with spillway upper bound for LK

LK (Calculated in Cascade)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	82.50	221,075,264.20	0.00	44,730.43
November	82.50	358,586,424.70	0.00	72,553.24
December	82.50	477,967,923.15	0.00	96,707.85
January	82.50	591,234,427.77	0.00	119,625.20
February	82.50	378,362,397.88	0.00	76,554.54
March	65.01	1,559,772,557.02	0.00	248,671.26
April	82.50	1,600,559,993.67	1,016,005,842.97	323,843.30
May	82.50	1,600,559,993.69	1,210,531,433.19	323,843.30
June	82.50	798,853,418.28	0.00	161,633.01
July	82.50	153,220,082.15	0.00	31,001.21
August	82.50	84,842,931.92	0.00	17,166.38
September	82.50	141,814,722.40	0.00	28,693.55
			Total	1,545,023.27

B1 (Calculated in Cascade)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	77.00	262,161,480.26	0.00	49,507.23
November	77.00	546,658,684.94	0.00	103,232.39
December	77.00	716,932,973.02	0.00	135,387.41
January	77.00	688,849,396.34	0.00	130,084.04
February	77.00	507,820,585.16	0.00	95,898.11
March	72.00	2,003,131,512.19	0.00	353,722.78
April	77.00	2,251,670,393.77	1,062,256,424.12	425,211.06
May	77.00	2,251,670,392.78	1,021,913,981.68	425,211.06
June	77.00	1,022,163,883.98	0.00	193,027.98
July	77.00	257,385,110.99	0.00	48,605.25
August	77.00	99,893,073.64	0.00	18,864.06
September	77.00	143,935,049.13	0.00	27,181.05
		C	Total	2,005,932.43

**Table 4-36:** Optimization result of cascade operation with maximum flow data andwith spillway upper bound for B1

**Table 4-37:** Optimization result of cascade operation with maximum flow data andwith spillway upper bound for B2

B2 (Calculated in Cascade)	Head (m)	Turbined Volume (m <sup>3</sup> )	Spilled Volume (m <sup>3</sup> )	Energy Generation (MWh)
October	35.00	262,161,487.47	0.00	22,503.29
November	35.00	546,658,687.37	0.00	46,923.81
December	35.00	716,932,972.23	0.00	61,539.73
January	35.00	688,849,391.20	0.00	59,129.11
February	35.00	507,820,583.19	0.00	43,590.05
March	32.01	2,022,231,510.35	0.00	158,749.07
April	35.00	2,251,670,393.68	1,043,156,424.36	193,277.76
May	35.00	2,251,670,394.16	1,021,913,985.39	193,277.75
June	35.00	1,022,163,886.27	0.00	87,739.99
July	35.00	257,385,111.81	0.00	22,093.29
August	35.00	99,893,071.85	0.00	8,574.57
September	35.00	143,935,042.67	0.00	12,355.02
			Total	909.753.44



Figure 4-43: Operation head for UK with maximum flow data with spillway upper bound



Figure 4-44: Operation head for LK with maximum flow data with spillway upper bound



Figure 4-45: Operation head for B1 with maximum flow data with spillway upper bound



Figure 4-46: Operation head for B2 with maximum flow data with spillway upper bound



Figure 4-47: Turbined volume for UK with maximum flow data with spillway upper bound



Figure 4-48: Turbined volume for LK with maximum flow data with spillway upper bound



Figure 4-49: Turbined volume for B1 with maximum flow data with spillway upper bound



Figure 4-50: Turbined volume for B2 with maximum flow data with spillway upper bound



Figure 4-51: Energy generation for UK with maximum flow data with spillway upper bound



Figure 4-52: Energy generation for LK with maximum flow data with spillway upper bound



Figure 4-53: Energy generation for B1 with maximum flow data with spillway upper bound



Figure 4-54: Energy generation for B2 with maximum flow data with spillway upper bound

### 4.3 Comparison of Results

# 4.3.1 Energy Generation Comparison of Single and Cascade Operation

As it is expected, the energy generation of the cascade operation is more than the total energy generation of the single operation results for the cases, except the optimization by using maximum inflow data without spillway upper bound; however this case should be considered as a local minima result and by considering the remaining three cases, the increase in the energy generation is raised as the inflow increase.

The optimization results by using minimum inflow data is tabulated in Table 4-38. After taking output data from MATLAB, the benefit is calculated by subtracting the energy generation at single operation from the energy generation at cascade operation. Hereby, the increase percentage is calculated by taking the ratio of benefit over single operation and the increase is only 0.49%, which is an ignorable proportion.

HEPPs	Single Operation (MWh)	Cascade Operation (MWh)	Benefit (MWh)	Percentage (%)
UK	772,157	772,157	0	0.00
LK	590,754	597,840	7,086	1.20
<b>B1</b>	765,908	771,022	5,114	0.67
<b>B</b> 2	353,275	353,275	0	0.00
Total	2,482,094	2,494,293	12,199	0.49

Table 4-38: Optimization results by using minimum inflow data

The results in Table 4-39 show that maximization by using average inflow data is again result with an ignorable increase in energy generation.

HEPPs	Single Operation (MWh)	Cascade Operation (MWh)	Benefit (MWh)	Percentage (%)
UK	1,309,487	1,282,975	-26,512	-2.02
LK	955,713	986,931	31,218	3.27
B1	1,175,708	1,198,495	22,787	1.94
B2	544,130	544,314	184	0.03
Total	3,985,038	4,012,715	27,677	0.69

Table 4-39: Optimization results by using average inflow data

As it is previously expressed, maximum data is used by considering two cases, due to the unfavorable results that are observed for the optimization without spillway upper bound. The optimization results by using maximum inflow data with infinite and defined spillway upper bound are given in Tables 4-40 and 4-41, respectively.

upper bound					
HEPPs	Single Operation (MWh)	Cascade Operation (MWh)	Benefit (MWh)	Percentage (%)	
UK	2,095,185	2,063,910	-31,275	-1.49	
LK	1,570,963	1,488,219	-82,744	-5.27	

1,845,543

6,246,875

849,203

-125,224

-51,561

-290,805

-6.35

-5.72

-4.45

1,970,767

6,537,680

900,764

**B1** 

**B2** 

Total

 Table 4-40: Optimization results by using maximum inflow data without spillway upper bound

Since the upper bound restriction is implemented at the results shown in below table, the energy generation of single operation is less than the generation of without spillway upper bound; in other words, the energy generations of the HEPPs are decreased in single operation when there is a restriction on the decision variables for all individual HEPP. To express explicitly, the total energy generation of 6,537 GWh decreased to 6,248 GWh, by defining spillway upper bound.

HEPPs	Single Operation (MWh)	Cascade Operation (MWh)	Benefit (MWh)	Percentage (%)
UK	1,998,056	2,103,713	105,657	5.29
LK	1,563,730	1,545,023	-18,707	-1.20
<b>B1</b>	1,789,630	2,005,932	216,302	12.09
B2	896,487	909,753	13,266	1.48
Total	6,247,902	6,564,423	316,521	5.07

 Table 4-41: Optimization results by using maximum inflow data with defined spillway upper bound

The advantage of optimization of a cascade system can be observed conveniently from the optimization results by using maximum inflow data with defined spillway upper bound. Although the energy generation of LK decreased, total energy generation of the cascade increased.

Since the total energy generation at single dam operation is decreased by defining spillway upper bound; the comparison of energy generation at single operation without spillway upper bound (6,537 GWh) and total energy generation at cascade operation with spillway upper bound (6,564 GWh) is also carried out; nonetheless, the total energy generation is increased as 0.41%.

# 4.3.2 Comparison of Optimization Performance

A comparison of optimization performance can be made from two different points of views, namely in terms of computational time and convergence rate, and accuracy of calculated minima.

A big advantage of doing a cascade-scale optimization is that every HEPP will have feedback from each other, thus enabling a "global" optimum and by considering the cascade system as a whole, the maximization of total energy generation is handled, without favoring one HEPP over another and considering the energy generation as a total sum. This phenomenon is illustrated in Figure 1 given below. A single HEPP optimization in series may result in inadequate water inflow to an important (potentially high energy production) reservoir and hinder optimum energy generation. This is also observed in this problem where the cascade run yielded a higher energy value as optima.



Figure 4-55: Difference in Cascade and Individual Runs Regarding Feedback

A drawback of performing a cascade optimization may occur depending on the optimization method used, which is computational time. Efficiency of the solver may cause a computational complexity from negligible to a combinatorial increase. The use of *fmincon* with *GlobalSearch* for the cascade problem resulted in a fair increase of computational time in comparison with individual runs. The main reason was due to the more complex search space (96 dimensions instead of 12), and thus a higher number of trial points.

## **CHAPTER 5**

### CONCLUSION

### 5.1 Summary

A 4-HEPP cascade system was considered for a monthly large-scale optimization study that intended to maximize energy generation. A MATLAB script was developed that takes initial reservoir storage and inflow values as inputs and constructs a symbolic black-box script that is later used in an optimizer. The optimization method used for this study is the intrinsic *fmincon* function together with the GlobalSearch object, which results in a heuristic search algorithm.

Results show that without defining any upper bound on spillway flow when maximum flow is considered, the algorithm jam in local minima, which can be expected for a non-linear optimization.

A comparison study made with single dam optimization of each HEPP showed that a better optimum was attained with the cascade variant. Optimization of a cascade system can increase the total produced energy up to 5% by comparing with optimization of single dam operation.

A drawback was the higher computational time (which was still a reasonable duration), which can be further improved by implementing efficient algorithms.

## 5.2 **Recommendations**

All the work done may be reanalyzed by using the daily data, if available; and a carried out daily operation conduce to revenue optimization by considering the electricity prices. The used method can be changed and a heuristic method (Genetic

Algorithm, Evolutionary Strategy, Particle Swarm Optimization etc.) can be used in order to solve the optimization problem. It is well known that the heuristic methods can solve complex problems with lower solution times, if required conditions are met.

Another option to improve the work can be to progressively reanalyze end results. After monthly optimization, the end results can be used to obtain weekly values, which can be used to obtain daily values and so on. This method renders a turbinebased operational optimization feasible with a personal computer since it does not handle every decision variable at the same time.

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## **APPENDIX A**

# DERIVATION OF CONSTRAINT MATRICES FOR MATLAB OPTIMIZER

Mathematical formulation of the objective function is given in equation A-1. The constraints are given between equations A-2 to A-5. The constraints are conservation of mass equation for each reservoir, and minimum and maximum limit on outflow, spill and storage, respectively.

$$\max\left(\sum_{j=1}^{M}\sum_{i=1}^{N}E(Q,SP)_{i,j}\right)$$
(A-1)

subject to

$$\sum_{i=1}^{N} (Q + SP)_{i} = \sum_{i=1}^{N} R_{i}$$
 (A-2)

$$Q_{\min,j} \le Q_j \le Q_{\max,j}$$
for j=1,...M(A-3) $SP_{\min,j} \le SP_j \le SP_{\max,j}$ for j=1,...M(A-4) $S_{\min,j} \le S_j \le S_{\max,j}$ for j=1,...M(A-5)

where

N = 12 (number of months)

M = 4 (number of HEPPs in the related cascade system)

 $Q_{i,j}$  = Turbined Flow (Outflow through turbines from reservoir j at month i)

 $SP_{i,j}$  = Spilled Flow from reservoir j at month i

 $R_i$  = Inflow to a reservoir that is the total inflow considering the outflow and spill from an upper reservoir and the intermediate flow between reservoirs

The intrinsic MATLAB function *fmincon* requires the constraints to be defined in a matrix format. Below-given paragraphs explain how these matrices were defined and sample calculations to help better understand the underlying concepts.

The equality constraints formulate the balance between initial and final reservoir limits. It has the form  $A_{eq} \cdot X = \overline{B}_{eq}$  where matrix  $A_{eq}$  and vector  $\overline{B}_{eq}$  are defined as follows.

$$A_{eq} = \begin{bmatrix} J_{1,2N} & 0_{1,2N} & 0_{1,2N} & 0_{1,2N} \\ -J_{1,2N} & J_{1,2N} & 0_{1,2N} & 0_{1,2N} \\ 0_{1,2N} & -J_{1,2N} & J_{1,2N} & 0_{1,2N} \\ 0_{1,2N} & 0_{1,2N} & -J_{1,2N} & J_{1,2N} \end{bmatrix}_{4x8N} \text{ and } \overline{B}_{eq} = \begin{bmatrix} \sum_{i=1}^{n} I_{i,1} \\ \sum_{i=1}^{n} I_{i,2} \\ \sum_{i=1}^{n} I_{i,3} \\ \sum_{i=1}^{n} I_{i,4} \end{bmatrix}_{4x1}$$
(A-6)

where "J" is a matrix of ones and "0" is a matrix of zeros. Since N is equal to 12, the  $A_{eq}$  matrix has a size of 4x96, vector X has a size of 96x1 and vector  $\overline{B}_{eq}$  has a size of 4x1.

A sample calculation for the first reservoir can be done by taking the first 1x96 submatrix of  $A_{eq}$  and the first element of vector  $\overline{B}_{eq}$ . The J and 0 matrices are shown below in equation A-7.

$$J_{1,2N} = \begin{bmatrix} 1\\1\\1\\1\\1 \end{bmatrix}_{1x24}^{T} 0_{1,2N} = \begin{bmatrix} 0\\0\\0\\0\\0 \end{bmatrix}_{1x24}^{T}$$
(A-7)

Thus, a submatrix consisting of the first row would consist of 24 ones and 72 zeros. The X vector is a column vector of size 96x1 and consists of monthly outflow and spill flow values. The first element of  $\overline{B}_{eq}$  gives the sum of monthly inflows to the first reservoir. The matrix multiplication would result in the following relationship (equation A-8) which needs to be satisfied at all times.

$$1 \cdot Q_{1,1} + 1 \cdot Q_{2,1} + \dots 1 \cdot Q_{12,1} + 1 \cdot SP_{1,1} + 1 \cdot SP_{2,1} + \dots 1 \cdot SP_{12,1} \dots \\ + 0 \cdot Q_{1,2} + 0 \cdot Q_{2,2} + \dots + 0 \cdot SP_{11,4} + 0 \cdot SP_{12,4} \qquad = \sum_{i=1}^{n} I_{i,1} \quad (A-8)$$

The lower- and upper bounds are defined for each decision variable (i.e. turbined and spilled flow) and is also in vector format as shown.

$$\overline{LB}_{8Nx1} = \begin{bmatrix} 0\\0\\\vdots\\0 \end{bmatrix}_{8Nx1} < \overline{X}_{8Nx1} < \overline{UB}_{8Nx1} = \begin{bmatrix} UB_{Q,1}\\UB_{SP,1}\\\vdots\\UB_{Q,4}\\UB_{SP,4} \end{bmatrix}_{8Nx1}$$
(A-9)

....

where  $UB_{Q,1} = 1464220800 \text{ m}^3$ ;  $UB_{Q,2} = 1600560000 \text{ m}^3$ ;  $UB_{Q,3} = 2251670400 \text{ m}^3$ ;  $UB_{Q,4} = 2251670400 \text{ m}^3$  and  $UB_{SP} = \infty \text{ m}^3$ 

The minimum and maximum permitted limit on the storage is formulated through the inequality constraint. It has the form  $A \cdot X \leq \overline{B}$  where matrix A and vector  $\overline{B}$  are defined as given in equations A-10 and A-11.

$$A = \begin{bmatrix} -L(J)_{N} & -L(J)_{N} & 0_{N,N} & 0_{N,N} & 0_{N,N} & 0_{N,N} & 0_{N,N} & 0_{N,N} \\ L(J)_{N} & L(J)_{N} & 0_{N,N} & 0_{N,N} & 0_{N,N} & 0_{N,N} & 0_{N,N} \\ L(J)_{N} & L(J)_{N} & -L(J)_{N} & -L(J)_{N} & 0_{N,N} & 0_{N,N} & 0_{N,N} \\ -L(J)_{N} & -L(J)_{N} & L(J)_{N} & L(J)_{N} & 0_{N,N} & 0_{N,N} & 0_{N,N} \\ 0_{N,N} & 0_{N,N} & L(J)_{N} & L(J)_{N} & -L(J)_{N} & -L(J)_{N} & 0_{NXN} & 0_{NXN} \\ 0_{N,N} & 0_{N,N} & -L(J)_{N} & -L(J)_{N} & L(J)_{N} & 0_{NXN} & 0_{NXN} \\ 0_{N,N} & 0_{N,N} & 0_{N,N} & 0_{N,N} & L(J)_{N} & L(J)_{N} & -L(J)_{N} & -L(J)_{N} \\ 0_{N,N} & 0_{N,N} & 0_{N,N} & 0_{N,N} & -L(J)_{N} & -L(J)_{N} & -L(J)_{N} \\ 0_{N,N} & 0_{N,N} & 0_{N,N} & 0_{N,N} & -L(J)_{N} & -L(J)_{N} & L(J)_{N} \\ \end{bmatrix}_{8N \times 8N}$$
(A-10)

$$\overline{B} = \begin{bmatrix} S_{max} - C_1 \\ S_{min} + C_1 \\ S_{max} - C_2 \\ S_{min} + C_2 \\ S_{max} - C_3 \\ S_{min} + C_3 \\ S_{max} - C_4 \\ S_{min} + C_4 \end{bmatrix}_{8Nx1} \quad \text{where} \quad \overline{C}_{m=1:4} = \begin{bmatrix} S_{0,m} + \sum_{i=1}^{n=1} I_{i,m} \\ \vdots \\ S_{0,m} + \sum_{i=1}^{n=12} I_{i,m} \\ S_{0,m} + \sum_{i=1}^{n=12} I_{i,m} \end{bmatrix}_{Nx1} \quad (A-11)$$

 $L(J)_N$  denotes a lower triangular matrix of ones with a size of 12x12.  $0_{N,N}$  denotes a matrix of zeros of size 12x12. By the use of lower triangular ones matrices, each row of matrix A represents the sum of inflow into and subtraction of outflow from a reservoir for every time step (month) along the cascade, when multiplied with the decision vector  $\overline{X}$ . The rows of vector $\overline{B}$ , on the other hand, denote the maximum and minimum allowable reservoir storage limits at each month.

An example calculation for the first month will help demonstrate the procedure that is being followed. Equation A-12 shows the lower triangular matrix of ones and the zeros matrix. The matrix multiplication of the first row of A and vector X is given in equation A-13.

$$L(J)_{N} = \begin{bmatrix} 1 & 0 & 0 \\ \vdots & \ddots & 0 \\ 1 & \cdots & 1 \end{bmatrix}_{12x12} \quad \text{and} \quad 0_{N,N} = \begin{bmatrix} 0 & 0 & 0 \\ \vdots & \ddots & 0 \\ 0 & \cdots & 0 \end{bmatrix}_{12x12} \quad (A-12)$$

$$\begin{array}{rl} -1 \cdot (1 \cdot Q_{1,1} + 0 \cdot Q_{2,1} + 0 \cdot Q_{3,1} + \dots + 0 \cdot Q_{12,1} + 1 \\ & \cdot SP_{1,1} + 0 \cdot SP_{2,1} + 0 \cdot SP_{3,1} + \dots \\ & + 0 \cdot SP_{12,1} + 0 \cdot Q_{1,2} + 0 \cdot Q_{2,2} \\ & + \dots + 0 \cdot SP_{11,4} + 0 \cdot SP_{12,4}) \end{array}$$

$$\begin{array}{rl} (A-13) \\ Smin + S_{0,1} + I_{1,1} \\ Smin + S_{0,1} + I_{1,1} \end{array}$$

## **APPENDIX B**

## PERMISSION LETTER REGARDING DATA USE

