

THE EFFECT OF USING DYNAMIC GEOMETRY SOFTWARE ON EIGHT  
GRADE STUDENTS' ACHIEVEMENT IN TRANSFORMATION GEOMETRY,  
GEOMETRIC THINKING AND ATTITUDES TOWARD MATHEMATICS AND  
TECHNOLOGY

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## **ABSTRACT**

### **THE EFFECT OF USING DYNAMIC GEOMETRY SOFTWARE ON EIGHT GRADE STUDENTS' ACHIVEMENT IN TRANSFORMATION GEOMETRY, GEOMETRIC THINKING AND ATTITUDES TOWARD MATHEMATICS AND TECHNOLOGY**

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The aim of this study was to investigate the effects of Dynamic Geometry Software-Assisted Instruction on 8<sup>th</sup> grade students' mathematics achievement in transformation geometry (fractals, rotation, reflection, translation), geometric thinking, and attitudes toward mathematics and technology compared to the Regular Instruction. The Static-Group Pretest-Posttest research design was adopted in this weak experimental research study. The study was conducted during the fall semester of the 2012-2013 academic year in a private elementary school in Bilkent district in Ankara\TURKEY. The sample of the study consisted of 34 eight grade students (17 male and 17 female). The study lasted 10 class hours in three weeks. For the treatment, two intact classes were used and each of these classes was chosen as the experimental and the control group randomly. The experimental group students were taught the subject of transformation geometry by the researcher with Dynamic

Geometry Software-Assisted Instruction using GeoGebra while the control group students were taught the same content by the mathematics teacher of the class with the Regular Instruction. In order to gather data, Mathematics Achievement Test (MAT), van Hiele Geometric Thinking Level Test (VHL) and Mathematics and Technology Attitude Scale (MTAS) were administered to the students as measuring instruments. The Quantitative Data Analyses were done by using Independent-samples *t*-test. The results of the study indicated that the Dynamic Geometry Software-Assisted Instruction had a significant effect on students' mathematics achievement in transformation geometry and geometric thinking positively compared to the Regular Instruction. However, the Dynamic Geometry Software-Assisted Instruction had no significant effect on students' attitude towards mathematics and technology.

**Keywords:** Dynamic Geometry Software, GeoGebra, Mathematics Achievement, van Hiele Geometric Thinking Levels, Attitude Towards Mathematics and Technology, Transformation Geometry.

## ÖZ

### DİNAMİK GEOMETRİ YAZILIMI KULLANIMININ SEKİZİNCİ SINIF ÖĞRENCİLERİNİN DÖNÜŞÜM GEOMETRİSİ KONUSUNDAKİ BAŞARISI, GEOMETRİK DÜŞÜNMESİ VE MATEMATİK VE TEKNOLOJİYE YÖNELİK TUTUMLARI ÜZERİNE ETKİSİ

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Bu çalışmanın amacı, Dinamik Geometri Yazılımı Destekli Öğretimin, Geleneksel Öğretim ile karşılaştırıldığında, 8. sınıf öğrencilerinin dönüşüm geometrisi konusundaki matematik başarıları, geometrik düşünmesi ve matematik ve teknolojiye yönelik tutumları üzerine etkisini incelemektir. Çalışmanın araştırma modeli Statik Grup Öntest-Sontest araştırma desenidir. Çalışma, 2012-2013 eğitim-öğretim yılı güz döneminde Ankara ilinin Bilkent ilçesinde özel bir ilköğretim okulunda gerçekleştirilmiş ve 10 ders saati (3 hafta) sürmüştür. Çalışmanın örneklemini bu okulda öğrenim gören, 17'si kontrol 17'si deney grubunda olmak üzere 34 (17 kız ve 17 erkek), 8. sınıf öğrencisi oluşturmaktadır. Çalışmada kullanılmak üzere okulda halihazırda var olan iki adet 8. sınıf seçilmiş ve bu sınıfların her biri deney ve kontrol grubu olarak rastgele seçilmiştir. Uygulama sürecinde deney grubu öğrencilerine dönüşüm geometrisi konusu araştırmacı tarafından GeoGebra kullanılarak Dinamik Geometri Yazılımı Destekli Öğretim ile öğretilmiş, kontrol grubu öğrencilerine ise

aynı konu sınıfın matematik öğretmeni tarafından Geleneksel Öğretim kullanılarak öğretilmiştir. Veri toplama aracı olarak van Hiele Geometrik Düşünme Düzeyi Testi (VHL), Matematik Başarı Testi (MAT) ve Matematik ve Teknoloji'ye Yönelik Tutum Ölçeği (MTAS) kullanılmıştır. Elde edilen verilerin sayısal analizleri SPSS paket programında Bağımsız Örneklem t-testi kullanılarak yapılmıştır. Analizlerin sonuçları, Dinamik Geometri Yazılımı Destekli Öğretimin, Geleneksel Öğretim ile karşılaştırıldığında, 8. sınıf öğrencilerinin dönüşüm geometrisi konusundaki Matematik Başarısı ve Geometrik Düşünme üzerinde istatistiksel olarak anlamlı ve olumlu bir etkiye sahip olduğunu ancak öğrencilerin Matematik ve Teknolojiye Yönelik Tutumları üzerinde istatistiksel olarak anlamlı bir etkiye sahip olmadığını ortaya koymuştur.

Anahtar Kelimeler: Dinamik Geometri Yazılımı, GeoGebra, Matematik Başarısı, van Hiele Geometrik Düşünme Düzeyleri, Matematik ve Teknolojiye Yönelik Tutum, Dönüşüm Geometrisi.

*To My Beloved Family*



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## **LIST OF ABBREVIATIONS**

DGS	Dynamic Geometry Software
CAS	Computer Algebra System
CAI	Computer-Assisted Instruction
DGSI	Dynamic Geometry Software-Assisted Instruction
RI	Regular Instruction
MoNE	Ministry of National Education
NCTM	National Council of Teachers of Mathematics
ICT	Information and Communication Technology
MAT	Mathematics Achievement Test
VHL	van Hiele Geometric Thinking Level Test
MTAS	Mathematics and Technology Attitude Scale
EG	Experimental Group
CG	Control Group
N	Sample size
df	Degree of freedom
Sig.	Significance
$\alpha$	Significance Level



## **CHAPTER 1**

### **INTRODUCTION**

The public education system, which not only helps communities to develop and improve, but also enables individuals to develop, cannot be thought independently of the systems that construct the structures of the society and the technological advances in our epoch where we face new technological changes every day (Yenice, 2003). Besides, as known, the major purpose of public education is to enable individuals to accumulate and acquire knowledge, and to guide individuals by showing how to use this knowledge and in what way. However, in order to accomplish such purposes of education, the methods that are commonly used remain insufficient and there is a need for new teaching methods for the accomplishment of the purposes of public education and permanent learning. To this end, new educational technological tools should be taken advantage of (Uzunboyulu, 1995).

In the information era we live in, it is inevitable that technology affects how we teach and how we learn. As a result of research studies for new approaches to the process of teaching-learning, which have been conducted for many decades, new supportive techniques that enable effective teaching and learning have been developed. One of these techniques is based on the integration of technology into the educational field, indirect use of ICT in classrooms. Many research studies showed that ICT (Information and Communication Technology) is useful as a supportive tool in the teaching and learning environment. In the mathematics classroom, the use of ICT can help students and teachers perform better in calculations, analyses of data, exploration of mathematical ideas and concepts and the association of these ideas and concepts with real life examples, thus resulting in permanent and effective learning in mathematics and higher mathematics achievement (Doktoroğlu, 2013; Saha, Ayub & Tarmizi, 2010; Toker, 2008; Yemen, 2009).

The use of technology in mathematics education not only helps students construct their visual representations of mathematical ideas and concepts, summarize and analyze data, and interpret these data, but also enables students to investigate every area of mathematics, such as geometry, algebra, and statistics (NCTM, 2000). The National Council of Teachers of Mathematics (NCTM) considers technology as one of their six principles for school mathematics and states:

‘Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning.’(p. 11)

Computers are one of the most common tools of technologically-enriched learning environments that make teaching of mathematics more effective, and the number of computer laboratories and the use of computers in the learning environment at schools are increasing day by day (Baki, 2001). Thus, integration of technology into mathematics education, indirectly the application of ICT (Information and Communication Technology), is mainly done by the use of computers in the learning environments. The use of computers provides extensive opportunities for facilitating, supporting and enriching the learning of mathematics in schools. Web-based interactive learning objects, interactive applets, spreadsheets and graphing programs are some types of ICT applications, which are currently being used in mathematics education through computers.

Computer-Assisted Instruction (CAI) is one of the most commonly used and investigated methods of instruction and is implemented with the use of computers in the learning environment (Çelik & Çevik, 2011). Baki (2002) asserted that Computer-Assisted Instruction helps teachers to introduce new contents and materials, teach new subjects, promote new skills and test them, and repeat and remind them when needed. According to Baki (2002), computers can be utilized to teach a subject easily in accordance with the subject’s level of difficulty. He also states that the load, complexity, and level of the details of the topic can be set according to the students’ learning level. Such kinds of assistance that the Computer-Assisted Instruction provides affect and change the learning environment positively. Taking into consideration that effective teaching of mathematics is mainly ensured

by means of effective mathematics teachers, the use of computers in learning environments assumes great importance in enriching the learning environment and enhancing the quality of teaching-learning process.

There are two main types of systems that can effectively support teaching and learning of mathematics: *computer algebra systems* and *dynamic geometry systems*. *Computer algebra systems* (CAS) are used to represent the abstract mathematical concepts such as integers, rational numbers, complex numbers, polynomials, functions and equation systems and solve mostly algebraic problems (Davenport, Siret & Tournier, 1993). Those systems (e.g. Derive, Mathematica, Livemath) help students to improve computational skills, to discover, visualize and practice mathematical concepts. CAS not only facilitates students' learning, but also helps teachers to improve effective teaching materials and enable teachers to establish effective communication between the students and themselves, as well as supporting distance education (Majewski, 1999).

*Dynamic geometry system* focuses on the learning and teaching of Geometry, particularly Euclidean Geometry, and solving the problems with respect to geometry concepts. It also focuses on the relations among points, lines, angles, polygons, circles and other geometrical concepts (Sangwin, 2007). The term "dynamic" refers to manipulating, resizing and dragging the figure to observe the differences. Dynamic mathematics/geometry softwares (e.g. GeoGebra, Cabri, Geometer's Sketchpad, Cindrella) also offer students and teachers useful facilities for using both Computer Algebra Systems and Dynamic Geometry Systems together (Hohenwarter & Lavicza, 2009).

NCTM's (2000) Principles and Standards for School Mathematics also state that Dynamic Geometry Softwares can be used to enhance student learning and continues:

"The effective use of technology in the mathematics classroom depends on the teacher. Technology is not a panacea. As with any teaching tool, it can be used well or poorly. Teachers should use technology to enhance their students' learning opportunities by selecting or creating mathematical tasks that take advantage of what technology can do efficiently and well — graphing, visualizing, and computing." (p.25)

NCTM (2008) also remarks that the use of interactive geometry softwares, computer algebra systems, applets, interactive presentation devices, spreadsheets, and calculators have an important place for permanent and effective learning of mathematics. Besides, NCTM (2008) emphasizes that use of technology in education is essential for teaching and learning of mathematics and, therefore, all schools should have necessary technological substructure and equipment for the active use of educational technologies in mathematics education. NCTM (2000) emphasizes the use of DGS in mathematics classrooms and states the following goals that are aimed to achieve;

- Exploring properties of rectangles and parallelograms using dynamic software.
- Learning about length, perimeter, area, and volume of similar objects using interactive figures.
- Learning about properties of vectors and vector sums using dynamic software.
- Understanding ratios of areas of inscribed figures using interactive diagrams.

The Ministry of National Education (MoNE) of Turkey agrees with the principles and suggestions of NCTM (2012). MoNE (2013) also suggested that the mathematics teachers to utilize technological tools (i.e. dynamic mathematics softwares) in mathematics classrooms to make the mathematics teaching more effective. Parallel with this suggestion, which encourages mathematics teachers to use technology in learning environments, Turkey has recently (in February 2012) initiated the FATİH (Movement of Enhancing Opportunities and Improving Technology) Project which aims at providing students with equal opportunities in education and improving the technological substructure of the schools. The Ministry of Education started to equip all 620,000 schools, including preschools, primary, and secondary institutions, with tablets and LCD smart boards. It is aimed to achieve active use of ICT in every classroom throughout the country and enrich the learning environment till the end of 2013 (MoNE, 2010). In this way, the students will be

provided with technologically enriched learning opportunities which provide them with better understanding of mathematics due to the technological tools used in mathematics education such as computers with dynamic geometry softwares. Kokol-Voljc (2007) stated three main characteristics of DGS as; a dynamic model of paper and pencil with dragging mode, a combined sequence of commands to form a macro, and the motions of geometrical objects which are visualized as a locus.

In dynamic learning environments, when the students drag the points or figures through dynamic tools, they achieve different goals (Arzarello, Micheletti, Olivero, Robutti, Paola, & Gallino, 1998; Hollebrands, Laborde & Strasser, 2006; Rivera, 2005). Students prefer using three types of dynamic movements; wandering dragging, lieu muet dragging, and dragging test (Arzarello et al., 1998). In wandering dragging, students aim at observing the regularities and discovering the results while dragging. In lieu muet dragging, students aim at preserving some regularity in the construction. They drag a point to observe the difference while other variables are constant. The third type, dragging test, means observing changes to test a hypothesis during dragging.

Although there are many advantages of constructions made with DGS, the construction activities done by paper-and-pencil should not be ignored since both dynamic and paper-and-pencil environments make great contributions to students' conceptual development (Kokol-Voljc, 2007). Therefore, in the present study, both paper-and-pencil and GeoGebra as a DGS were used to benefit the advantages of both environments.

One of the most useful and versatile dynamic geometry softwares is GeoGebra which was selected as a dynamic geometry software for the present study. GeoGebra combines Computer Algebra Systems (CAS) and Dynamic Geometry System into one easy-to use system. GeoGebra, created by Hohenwarter in 2001, is a free dynamic geometry, algebra, and calculus software for both teachers and students to make teaching and learning more effective. One of the unique properties of GeoGebra is that it integrates algebra view, graphic view, and spreadsheet view in a single interface (Preiner, 2008). GeoGebra not only provides students with facilities to experiment the mathematical ideas and to associate mathematical concepts with

the real life examples, but also helps students to examine the relation between algebraic and geometrical concepts better.

GeoGebra can also be used in many ways in the teaching and learning of mathematics: for demonstration and visualization as it can provide different representations; as a construction tool since it has the abilities for constructing shapes; for investigation to discover mathematics because it can help to create a suitable atmosphere for learning; and for preparing teaching materials using it as a cooperation, communication and representation tool (Hohenwarter & Fuchs, 2004). One of the most important reasons for GeoGebra to be used are; it has a variety of language options (including Turkish) and has easily accessible online lessons/activities on the official website ([www.geogebra.org](http://www.geogebra.org)), and it is a free open-source software (Bijedic & Hamulic, 2009).

GeoGebra can be defined as an effective and important tool in establishing relationships between geometry and algebra concepts in elementary mathematics since it proved its capability and potential in mathematics education (Hohenwarter & Jones, 2007). The software can be used with students ranging from elementary level to college level, aged from 10 to 18, beginning with simple constructions up to the integration of functions. Students can explore mathematics alone or in groups and the teacher tries to be a guide in the background, giving support when students need help. Students' results of their experiments with GeoGebra constitute the basis for discussions in class so that teachers can have more time to concentrate on fundamental ideas and mathematical reasoning (Schumann, 1992). Researches indicated that GeoGebra has a positive effect on students' mathematics achievement on geometry concepts covered in the primary mathematics curriculum (Bilgici & Selçik, 2011; Doktoroğlu, 2013; İçel, 2011).

In the national mathematics curriculum of Turkey established by MoNE, the subject of transformation geometry is covered in eighth grade mathematics. Klein (1870) stated that transformation geometry is the main subject learnt in geometry (as cited in Junius, 2002). However, the related literature showed that both students and teachers have difficulties in understanding and teaching the subject since the topic is a little more abstract than the other topics of mathematics



(Harper, 2002). To put it differently, different and more effective teaching methods and tools that facilitate the learning of the topic are needed (Boulter, 1992). Therefore, it is important to teach the topic of transformation geometry accurately and effectively in eighth grade mathematics.

The van Hiele Model of Geometric Thinking is a theory which offers a model for explaining and describing students' geometric reasoning (van Hiele, 1986). This theory resulted from the two Dutch mathematics educators' doctoral works, Dina van Hiele-Geldof and Pierre van Hiele, at the University of Utrecht in the Netherlands. Pierre van Hiele formulated the five levels of thinking in geometry and discussed the role of insight in the learning of geometry. The van Hiele theory has been applied to clarify students' difficulties with the higher order cognitive processes, which is essential to success in high school geometry. In this theory, if students are not taught at the proper Hiele level that they are at or ready for it, they will face difficulties and they cannot understand geometry. Since the current National Middle School Mathematics Curriculum of Turkey aims at raising "geometric thinkers", it is important to investigate the effect of dynamic geometry software on students' progress through geometric thinking levels and seek for a correlation between the students' geometric thinking levels and mathematics achievement, which may serve the purposes of the curriculum, based on the results of the present study.

Attitude towards mathematics, which refers to a student's self-reported enjoyment, interest and level of anxiety toward mathematics (Pilli, 2008), plays a crucial role in the learning of mathematics and achievement in mathematics (Arslan, 2008; Peker & Mirasyedioğlu, 2003). Thus, investigating the effectiveness of the instruction using dynamic geometry software, which may establish a positive attitude towards mathematics, is important for students' mathematics learning and achievement.

Considering the above mentioned statements, the present research study will investigate the effects of Dynamic Geometry Software-Assisted Instruction using GeoGebra on 8<sup>th</sup> grade students' Mathematics Achievement in transformation geometry, Geometric Thinking, and Attitudes toward Mathematics and Technology.

## 1.1 Purpose of the study

The aim of this study is to examine the effects of Dynamic Geometry Software-Assisted Instruction using GeoGebra on 8<sup>th</sup> grade students' mathematics achievement in transformation geometry, geometric thinking and attitudes toward mathematics and technology.

## 1.2 Research Questions and Hypotheses

The study aims at investigating the following main and sub-research questions. To examine the research questions of the study, four null hypotheses were stated below.

**Main Research Question:** What is the effect of the Dynamic Geometry Software-Assisted Instruction on 8<sup>th</sup> grade students' Mathematics Achievement in Transformation Geometry, Geometric Thinking, and Attitude towards Mathematics and Technology?

To examine the main research question, three sub-problems were addressed:

**Sub-Problem 1)** What is the effect of the Dynamic Geometry Software-Assisted Instruction on 8<sup>th</sup> grade students' mathematics achievement in transformation geometry (fractals, rotation, reflection, translation)?

**(Sub-Problem 1)  $H_0$ :** There is no statistically significant mean difference between the students taught by the Dynamic Geometry Software-Assisted Instruction and those taught by Regular Instruction with respect to Mathematics Achievement Test (MAT) posttest scores.

**Sub-Problem 2)** What is the effect of the Dynamic Geometry Software-Assisted Instruction on 8<sup>th</sup> grade students' attitudes toward mathematics and technology?

**(Sub-Problem 2)  $H_0$ :** There is no statistically significant mean difference between the students taught by the Dynamic Geometry Software-Assisted Instruction and

those taught by the Regular Instruction with respect to Mathematics and Technology Attitude Scale (MTAS) posttest scores.

**Sub-Problem 3)** What is the effect of the Dynamic Geometry Software-Assisted Instruction on 8<sup>th</sup> grade students' geometric thinking?

**(Sub-Problem 3)  $H_0$ :** There is no statistically significant mean difference between the students taught by the Dynamic Geometry Software-Assisted Instruction and those taught by the Regular Instruction with respect to van Hiele Geometric Thinking Level Test (VHL) posttest scores.

### **1.3 Significance of the study**

Technology integration into mathematics classrooms is important to the field of education, not only because today's society is becoming more and more advanced and reliant upon technology but also because schools are beginning to embrace technology as an essential part of their curricula (Özel, Yetkiner & Capraro, 2008). As a result of research studies for new approaches to the process of teaching-learning by the help of technology use in learning environments, which have been conducted for many decades, new supportive techniques that enable effective teaching and learning have been developed. One of these effective techniques is implemented with the assistance of computers in mathematics classrooms which is the application of ICT as a kind of integration of technology into mathematics education.

Since the early 1980s, there has been a growing interest in computers as a tool to ease students' learning. The importance of using technology effectively and properly as a learning tool has been stressed by many researchers. Therefore, a dynamic mathematics or geometry software that encourages students to explore and express mathematical ideas is becoming a crucial issue (Işıksal & Aşkar, 2005). Moreover, the use of computer in classrooms has been expanding owing to the positive effects of Computer-Assisted Learning in mathematics (Souter, 2001). In addition, since mathematics is abstract in its nature, it is important for students to visualize abstract mathematical concepts in a Dynamic Learning Environment via computers (Jones & Bills, 1998). As the existing teaching methods remained

insufficient, the present study is significant as it investigates the effectiveness of Dynamic Geometry Software as a supportive tool in the teaching of mathematics, thus, contributing to the mathematics teaching in practice.

A study by Aiken (1972) indicated that attitudes also play an essential role in learning mathematics and using computers may lead to more positive attitudes in students. Hence, this study is significant since it can improve not only the practice of Dynamic Geometry Software-Assisted Instruction, but can also establish a positive attitude towards mathematics because it is based on the application of Computer-Assisted Instruction enriched with the use of Dynamic Mathematics Software, GeoGebra.

The related literature documented the positive effects of using dynamic geometry software. However, the effect of the Dynamic Geometry Software-Assisted Instruction is still needed to be investigated. To put it differently, there are not many studies which investigate the effects of DGS on students' geometric thinking and attitude towards mathematics. Thus, this study will contribute to the mathematics education literature.

Another significance of this study arises from the lack of in-depth experimental research studies on transformation geometry and fractals in Dynamic Learning Environment through DGS. The transformational geometry is an important topic in the K-12 mathematics curriculum (Harper, 2002). According to Desmond (1997), Edwards and Zazkis (1993), and Law (1991), both students and pre-service teachers have difficulties in understanding the motions of reflection, rotation, and translation. Besides, it is difficult to teach the subject of transformation geometry effectively for teachers in crowded classes because it requires much work and drawings. Hence, this study is significant since it may provide an insight into the teaching of transformation geometry in a dynamic learning environment for mathematics teachers.

Studies of Fuys, Geddes and Tischler (1988), Senk (1989), Shaughnessy and Burger (1985), and Usiskin (1982) have revealed that students' van Hiele geometric thinking level is a good predictor of the students' achievement in geometry. It is expected from the outcome of the study that the DGS-Assisted Instruction has a

positive effect on students' geometric thinking, hence, an increase in students' mathematics achievement. This may be another significance of the present study.

According to Çakıroğlu, Güven and Akkan (2008), mathematics teachers evaluated themselves as incapable of designing, conducting and evaluating a technology-supported learning environment. The lesson plans, activity sheets and worksheets related to the topic of Transformation Geometry and Fractals, which were prepared to be used during the study in dynamic learning environment through GeoGebra, may be considered as examples for the mathematics teachers so that this study can encourage mathematics teachers who have concerns about the use of technology or Dynamic Geometry Softwares (e.g. GeoGebra, GSP, Cabri) in mathematics classrooms and suggest them ideas about technology use in mathematics classrooms. In this way, it can help students by providing them with permanent and effective learning of mathematics. Therefore, this study can contribute not only to the mathematics education literature, but also to the teacher education, educational practice, curriculum development, educational field, and educational policy making.

This study may also lead to subsequent research studies on new teaching methods or supportive components to the existing teaching methods based on the results to find an answer to the question of "How do we teach mathematics better?" Findings of this study may be also significant in validating the usage of dynamic geometry software while teaching by employing Dynamic Geometry Software-Assisted Instruction.

In the light of the literature review and the lack of the research in the field, this study will be conducted by considering its significance in teaching and learning of mathematics, that is, contribution to mathematics education. Thus, this research study will provide insight into the effects of dynamic geometry environment on students' Mathematics Achievement in Transformation Geometry, Geometric Thinking, and Attitude towards Mathematics and Technology. The findings of the study may shed light on the design of technology-supported learning environment and instructions. Also, the information derived from this study can serve as foundations for development of curricular considerations.

#### **1.4 Definitions of the Important Terms**

***Dynamic Geometry Software*** is defined as a kind of computer software that enables students and teachers to visualize geometric figures and shapes, explore geometric relationships and concepts, make and test conjectures in a dynamic learning environment by manipulating the objects such as dragging, constructing, rotating, translating in order to understand the concepts of geometry (Goldenberg & Couco, 1998). In this study, GeoGebra was used to teach the subject of transformational geometry.

***Dynamic Geometry Software-Assisted Instruction*** is an instruction which is based on the delivery of the activities and tasks using Dynamic Geometry Software. In this learning environment, the teacher gives students instructions about the dynamic activities and tasks after a brief explanation about the topic while students explore the relationships between the concepts and draw conclusions through these activities and tasks. In this study, the experimental group students were taught with Dynamic Geometry Software-Assisted Instruction using the dynamic GeoGebra activities and tasks.

***Regular Instruction*** refers to a teacher-centered, textbook-based teaching approach. Regular Instruction includes teaching through lectures, note-taking, question-answer and exercises. In regular learning environments, the teacher acts as a knowledge transmitter and sometimes asks questions to the students. Rules, definitions, strategies and generalizations related to the topic are given first, and then examples are provided. The students are passive listeners and note-takers in this learning environment (Duartepe, 2004). In this study, the control group students were taught with such kind of Regular Instruction.

***Achievement*** is defined as “something accomplished successfully, especially by means of exertion, skill, practice or perseverance” (Thorndike & Barnhart, 1993, p. 46). In this study, achievement means the total measurement of the scores of mathematics achievement test. In another words, the achievement is what the MAT measures.

**Attitude** is defined as “those beliefs formed from a combination of experiences measured in the domains of mathematics” (Capraro, 2000, p. 8). In this study, attitude means the total measurement of the scores of attitude towards mathematics scale. In short, attitude is what the MTAS measures.

**Attitude towards mathematics and technology** refers to student’s self-reported enjoyment, interest and level of anxiety toward learning mathematics with technology (Pilli, 2008).

**Transformational Geometry** is defined as “a subset of geometry in which students learn to identify and illustrate movement of shapes in two and three dimensions. The three types of movement are slides (translations, as when a figure is moved on a page), flips (reflections, that is, when a figure is turned over in three dimensions), and turns (rotations, when a figure is rotated 90° without being flipped).” (Kirby & Boulter, 1999, p.285). In the present study, transformation geometry includes the patterns such as fractals, and the movement of the figures such as reflection, translation, rotation and combination of these.

**van Hiele Geometric Thinking Levels** are defined as the levels which describe the way that students reason about shapes and other geometric ideas. The van Hiele Theory of Geometric Thinking outlines the hierarchy of these levels through students’ progress as they develop geometric ideas (Usiskin, 1982).

### **1.5 Assumptions of the study**

There are several assumptions in the present study. First of all, it was assumed that all the instruments were administered to the experimental and control groups under the same standard conditions. Moreover, the subjects of the study were assumed to be sincere while responding to the test items. In addition, it was assumed that the students from different classes did not interact and communicate about the items of the post-achievement tests before the administration of these tests. It was also assumed that the differences of the implementers had no effect on the results of the study.

## **1.6 Limitations of the study**

The present study has some limitations. Firstly, subjects were not assigned to the experimental and the control group randomly. Therefore, the study was a weak experimental study. Besides, the results of the study were limited to the population with similar characteristics. Moreover, the results of the study were restricted to the topics of Transformation Geometry and Fractals. Hence, this limited focus restricted the generalization of the results of the study to other contents in mathematics. Furthermore, duration of the treatment was three weeks. This duration was short in gaining evidence regarding the improvement of students' geometric thinking and attitudes toward mathematics and technology. In addition, the results of the study were limited due to the instruments used to measure certain variables. Thus, different results could be obtained if different instruments were used. Finally, the differences between the implementers were limitations in terms of the internal validity of the study.



## **CHAPTER 2**

### **LITERATURE REVIEW**

In this chapter, the literature related to the present study is reviewed. The chapter is split into eight parts. First of all, technological tools used in Mathematics Education are mentioned. Then, the literature concerning the use of Dynamic Geometry Software in Mathematics Education is reviewed. In the following part, the research studies focused on the effects of GeoGebra as a Dynamic Geometry Software on students' mathematics learning are mentioned. Next, the literature regarding the van Hiele Theory and van Hiele Geometric Thinking Levels are given. Afterwards, the literature related with the role of technology use in attitude towards mathematics is emphasized. Later on, the transformation geometry topic as a sub-learning area of geometry and the fractals as a sub-learning area of transformation geometry are reviewed. In the last part, a consistent summary of the literature reviewed in this chapter is drawn.

#### **2.1 Technology Usage in Mathematics Education**

Over the last quarter of a century, advances and novelties in technology have become a very important factor in everyday life. Besides, technological advances and developments brought new perspectives to the process of education and educational mentality. These advances necessitated changes in the qualifications of the triple of individual, information and society. With the emergence of the information society, need of the individuals who use and advance technology also increased. In other words, the skills of critical thinking and creativeness became a standard in modern-day society. Reaching these standards can only be done with qualified and sufficient education. The use of technology in the learning environment not only helps education for maintaining in accordance with the necessities of the era, but also provides individuals with opportunities for growing adequately (Ersoy, 2003).

The power of new technologies as one of the strongest forces in the contemporary growth and evolution of mathematics and math teaching are technology and technological advances which obviously affect how we learn and teach mathematics (Goldenberg, 2000). Moreover, the traditional methods used in classrooms remain insufficient in terms of meeting all the criterion of a quality teaching and learning of mathematics (Alakoç, 2003). It is the common viewpoint of educators that the existing problems related to the teaching cannot be solved by using the traditional teaching methods (Aktüment & Kaçar, 2003). As Usiskin (1982) and Fuys, Geddes, and Tischler (1988) promoted, the role of instruction is crucial in teaching and learning geometry. The more systematically structured the instruction, the more helpful it will be for middle school students to overcome their difficulties and to increase their understanding of geometry. Hence, the common opinion of many researchers, mathematics teachers, and studies focus on the notion that the novelties in mathematics education and technology integration into mathematics education support students' understanding of mathematics, and they suggest the use of technology in mathematics classrooms (Hollebrands, 2003).

Furthermore, the mathematics education researchers have a parallel interest in investigating the effect of technology on learning and teaching mathematics, and the curriculum. Technology tools provide powerful range of visual representations which help teachers to focus students' attention to mathematical concepts and techniques (Zbiek, Heid, Blume & Dick, 2007). Thus, technological tools, such as Computers, Graphic Calculators, Interactive White Boards, Web-Based Applications, Dynamic Mathematics/Geometry Softwares, are started to widely use in mathematics classroom and many studies investigated to determine the effectiveness of technology in mathematics education (Baki, 2001; Borwein & Bailey, 2003; Doğan, 2012; Ersoy, 2003; Hollebrands, 2003; Koehler & Mishler, 2005; Lester, 1996; NCTM, 2000).

Technology use not only plays a crucial role in mathematics education, but also helps mathematics educators to better capture the attention of the students and provide students with better understanding of mathematics and mastering the mathematical concepts (Khouyibaba, 2010). However, the integration of technology

in the learning and teaching of mathematics requires special attention in many respects (Iranzo, 2009). Technology environments allow teachers to adapt their instruction and teaching methods more effectively to meet their students' needs (NCTM, 2008). By integrating educational tools into their everyday teaching practice, teachers can provide creative opportunities for supporting students' learning and fostering the acquisition of mathematical knowledge and skills.

Parallel with the awareness of the increasing importance of new technologies in everyday life, several educational organizations started to develop technology-related standards (Lawless & Pellegrino, 2007), trying to encourage the integration of new technology in learning environments. For instance, the National Council of Teachers of Mathematics (NCTM, 2008), which is the world's largest association of mathematics teachers considered technology as one of their six principles for school mathematics and continues:

'Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.'(p. 11)

Computers are one of the mainly used technologies in learning environments. Increasing load of information, instruction process that is being more complicated day by day, and the purposes and standards of quality and contemporary education mandated the use of computers in education (Baki, Güven & Karataş, 2004). In order to win the race in the road of modernization, almost all countries enhanced their efforts of utilization of computers in all fields, especially in educational field. Computers as the most favorite tools of the 21<sup>st</sup> century affect human life and society. First and foremost, computers bring innovations and radical changes to education systems with bringing to other fields of the countries (Mercan, Filiz, Göçer & Özsoy, 2009). Computers are extremely crucial since they can provide a variety of rich experiences that allow students to be actively involved with mathematics (McCoy, 1996). In mathematics teaching, computers have fostered entirely new fields. As to educational field, they've raised the importance of certain ideas, made some problems and topics more accessible, and provided new ways to represent and handle mathematical information, affording choices about content and pedagogy that

we've never had before (Goldenberg, 2000). Moreover, the computers offer students immediate access to the web, where they can find additional resources and use interactive sites to investigate mathematical concepts.

Over the years, the computers have become vital for business and economy and 'computer literacy' is considered a very important skill in modern-era society. Especially for young people who have grown up having access to computer technology at home, computers have become common tools for communication, text processing, and last but not least, playing games. As in many other fields, computers were started to utilize in educational field through learning environments. On the one hand, successful students can be supported more effectively than ever by nurturing their individual interests and mathematical skills. On the other hand, weaker students can be provided with activities that meet their special needs and help them to overcome their individual difficulties. Thusly, students "may focus more intently on computer tasks" and "may benefit from the constraints imposed by a computer environment" (Preiner, 2008). Moreover, the development and rapid growth of the Internet in combination with its increasing accessibility for the public has opened up a whole new digital world (Ersoy, 2003).

Technological advances which we face in the era we live in and the approach of Computer-Assisted Instruction had effects also on the mathematics teaching in the schools (Akkoç, 2008). The use of computers in classrooms has been expanding, in part, owing to the positive effects of Computer-Assisted Instruction of mathematics (Souter, 2001). Thus, millions of schools around the world started to utilize Computer-Assisted Instruction in the learning environments. There are many studies which indicate the positive effect of Computer-Assisted Instruction on students' mathematics learning (Altın, 2012; Andiç, 2012; Balkan, 2013; Çoban-Gökkaya, 2001; Hangül, 2010; Helvacı, 2010; Tayan, 2011; Tor & Erden, 2004; Sulak, 2002; Şataf, 2010; Şen, 2010)

Computer-Assisted Instruction can be defined as a method of utilization of computers in learning environments which aims at making students' recognize their own deficiency and performance through mutual interaction, control their learning with getting instant feedbacks, and making students more interested in lesson by the

help of graphics, audio, animations and figures. The mathematics teaching that is done by utilizing the cognitive tools based on the computers is defined as Computer-Assisted Mathematics Instruction (Baki, 2002). Computer-Assisted Mathematics Instruction have been started to be important in terms of forming learning environments in the field of mathematics education (İpek & Akkuş-İspir, 2010). Since 1950s, many countries, firstly in Italy, then United States of America, initiated studies for extending the Computer-Assisted instruction by integrating it into their curricula (Mercan, Filiz, Göçer & Özsoy, 2009). The purpose of giving computers place in the learning environments is to grow productive, creative, successful, critical thinker, problem solver and adequate individuals in order to improve certain knowledge, skill and attitude. Thus, all of these goals may be fulfilled by utilizing the computers in the teaching learning process (Aktümen ve Kaçar, 2003).

Ersoy (2003) conducted a study on the use of computers and calculators in teaching and learning mathematics to contribute in developing strategies and developments in mathematics teaching process. The results of his study showed that the students need to understand how to use technology tools in their learning experiences. When integrated properly into the teaching and learning process, computers improve student proficiency in mathematics. Through different software applications, computers reduce the cognitive load of mathematical learning (Kozma, 1987; Liu & Bera, 2005). As a supportive tool, interactive mathematics computer programs such as *Geometer's Sketchpad* (Jackiw, 1995) and virtual modeling and visualization tools also provide students with dynamic multiple representations and support their understanding as they interact with concepts in a variety of ways (Flores, Knaupp, Middleton, & Staley, 2002; Garofalo, Drier, Harper, Timmerman, & Shockey, 2000).

Additionally, students can develop and demonstrate deeper understanding of mathematical concepts and are able to cope with more advanced mathematical contents in technology-enriched learning environments than in 'traditional' teaching environments (NCTM, 2008). Students can benefit in different ways from technology integration into everyday teaching and learning. New learning opportunities are provided in technological environments, potentially engaging

students of different mathematical skills and levels of understanding with mathematical tasks and activities (Hollebrands, 2007). By the help of the visualization of mathematical concepts and exploring mathematics in multimedia environments, students' understanding in a new way can be fostered.

Van Voorst (1999) reported that technology is useful in helping students view mathematics less passively, as a set of procedures, and more actively as reasoning, exploring, solving problems, generating new information, and asking new questions. Furthermore, he claims that technology helps students to visualize certain math concepts better and it also adds a new dimension to the teaching of mathematics. Laborde, Kynigos, Hollebrands and Strasser (2006) summarized technology use in mathematics education as following;

“(...) Research on the use of technology in geometry not only offered a window on students' mathematical conceptions of notions such as angle, quadrilaterals, transformations, but also showed that technology contributes to the construction of other views of these concepts. Research gave evidence of the research and progress in students conceptualization due to geometrical activities (such as construction activities or proof activities) making use of technology with the design of adequate tasks and pedagogical organization. Technology revealed how much the tools shape the mathematical activity and led researchers to revisit the epistemology of geometry” (Laborde et al., 2006, p. 296).

## **2.2 Dynamics Geometry Softwares in Mathematics Education**

In our day, technology progresses rapidly and provides new opportunities for meaningful mathematics education. Also, continuous improvement of the computer technology not only increases the quality and quantity of educational softwares, but also constantly varies the alternatives for educational purposes (MoNe, 2013). Thus, the use of computer softwares in mathematics, especially in geometry, become widespread gradually. Also, As to Turkey, current K-12 mathematics curriculum used in schools supports mathematics instruction done by the assistance of the dynamic mathematics softwares (MoNe, 2013). Moreover, the effectiveness of Dynamic Geometry Software (DGS) in mathematics classroom is a widely researched area (Baki, Kosa & Güven, 2011; Christou, Mousoulides, Pittalis & Pitta-

Pantazi, 2004; Güven, Baki & Çekmez, 2012; Habre, 2009; Pandiscio, 2010; Stols & Kriek, 2011).

Dynamic Geometry Softwares (DGS) are the computer softwares which allow users to construct geometric figures, measure some variables of these figures to determine the properties of them, drag the figures through the screen, make geometric constructions, hypothesize about these constructions and test these hypotheses, and enable users to make generalizations (Baki et al., 2001). Students can learn mathematics easier and more permanent owing to all of these features of DGS.

Dynamic environments allow users to change the appearance of the geometric figures while mathematical relationships on the figure are still preserved (Goldenberg & Couco, 1998). In this environment, the visual figures are enriched with dynamic movements to help students in developing their strategies and improving their mathematical understanding. Visualization is among the one of the most important aspects of geometric thinking (NCTM, 2008); therefore, it has vital importance. The students drag and move the points to observe changes in the relationships on the figures by using the software. In DGS environments, to check conjectures and to construct of conjectures explanations and verification are possible by means of drag mode. There are numerous researches aimed to investigate the facilities of drag mode in Dynamic Geometry Software (e.g., Hölzl 1996, Arzarello et al. 2002; Jones, 1996; Jones, 2000; Sowder& Harel, 1998). Jones (2000) mentioned the facilities of drag mode in DGS as following;

“By operating in this fashion, dynamic geometry environments appear to have the potential to provide students with ‘direct experience’ of geometrical theory and hereby break down what can all too often be an unfortunate separation between geometrical construction and deduction make it possible for students to focus on what varies and what is invariant in a geometric figure and enable students to gain more a meaningful idea of proof and proving” (p.2).

While the students use dragging options of the dynamic environments, they have different goals (Arzarello, Micheletti, Olivero, Robutti, Paola, & Gallino, 1998; Hollebrands, Laborde & Strasser, 2006; Rivera, 2005). The students mostly prefer three types of dynamic movements; *wandering dragging*, *lieu muet dragging*, and *dragging test* (Arzarello et al., 1998). In *wandering dragging*, students' aim is to observe the regularities and exploring interesting results while dragging (Zbiek et al., 2007). In *lieu muet dragging*, the students aim to preserve some regularity in the construction (Zbiek et al., 2007). They drag a point to observe the difference while other variables are invariant. The third type, *dragging test*, means observing changes to test a hypothesis during dragging (Zbiek et al., 2007).

Hollebrands (2003) conducted a study on the use of the Geometer's Sketchpad, a dynamic geometry software as a technological tool, to examine the nature of students' understanding of geometric transformations including reflections, translations, dilations, and rotations. The case study approach and constant comparison method were used with 16 tenth grade students. The students experienced a seven-week instructional period. The data sources were students' worksheets, observations, and interview documents. The researcher analyzed data in-depth and used a research framework to characterize students' understanding of geometric concepts and their methods in interpreting of geometrical representations. Hollebrands (2003) suggested that with the use of technology, students' understanding of transformations were critical for promoting the improvement of deeper understanding of transformations as functions. The study was seen as a first step to see how technology affects students' understanding of geometry.

Moreover, since GeoGebra, an open source dynamic geometry software, provides the opportunity to construct and dynamically visualize geometric figures (Hohenwarter & Fuchs, 2004), Fahlberg-Stojanovska and Trifunov (2010) investigated a study to show how GeoGebra improved students' understanding of construction and geometric proof. They conducted a qualitative exploratory study by using tasks that include construction and proof problems for the relations on the triangles. The results showed that using GeoGebra in these tasks improves the percentage of students that are able to solve the triangle construction and proof



problems (Fahlberg-Stojanovska & Trifunov, 2010). This result is consistent with that of Christou et al (2004) and Pandiscio (2010) in terms of DGS's effectiveness in justification and verification of both geometric and algebraic problems' solutions.

Güven (2002) expressed that according to the findings of many studies, while students regard mathematics as a crowd of formulas that should be memorized in traditional learning environments, their ideas change in DGS environments and in this sense they regard mathematics as a whole of relationships which need to be investigated. Therefore, DGS is a great teaching and learning method that enhances students' skills of understanding mathematical relationships and justifications (Jiang, 2002).

One of the advantages of dynamic geometry software is providing student with observing the different constructions of the same object during the interaction with DGS. In this case, constructions in dynamic geometry differ from drawing with static paper and pencil learning environment. Aarnes and Knutzon (2003) mentioned this facility of DGS as "DGS gives an easier access to this insight than would have been possible by pencil and paper construction, because the point may be moved" (p.3). Owing to this movement, students recognize the various positions of the object rather than its specific-size and position which provide them to make conjectures and generalizations. Researches on Dynamic Geometry Software largely focused on its potential as a conjecturing tool and as a way to investigate what kind of processes occurred during the constructions in geometrical contexts (Arcavi & Haddas, 2000; Goldenberg & Cuoco, 1998; Laborde & Capponi, 1994).

Experimental studies of Hoyles and Sutherland (1989) and Noss (1987) revealed that students come to understand many ideas and processes related to the geometrical concepts through an appropriate invention in a meaningful way. Several researchers dealt with the effects of computer based learning and dynamic geometry software in developing students' understanding in geometry and found that the use of technology, particularly use of dynamic geometry software, is helpful for pupils in terms of developing their understandings of geometrical concepts since interacting with dynamic geometry software can help students explore, conjecture, construct and

explain geometrical relationships (Hativa, 1984; Jones, 2000; Jones, 2001; McCoy, 1991; Marrades, & Guitérrez, 2000; Velo, 2001).

In their study, Balacheff and Kaput (1996) defined the visible part of the geometry activity of the learner as making distinction between the drawings and figures. They pointed out that dynamic geometry environments provide the distinction between drawings and figures. Laborde (1993) made the distinction between drawing and figure in the following way: “drawing refers to the material entity while figure refers to a theoretical object” (p.49).

In another study, Ubuz and Üstün (2004) aimed at investigating student’s development of geometrical concepts through a dynamic learning environment. They preferred to use Geometer’s Sketchpad as a dynamic geometry software. They investigated the students’ understanding of and performance in lines, angles and polygons (triangles, square, rectangle, parallelogram), compared to traditional learning environment with pretest-posttest design. As a result of their study, comparison of the pre-and post-test means of the students indicated that the treatment resulted in marked improvement in their performance in lines, angles, and polygons in the experimental group, who received treatment with GSP. They promoted that Geometer’s Sketchpad enables students to test whether their geometric constructions work in general or whether they have discovered a special case of the original construction and further stated that GSP is used for exploration and guided discovery which enables students to test their conjectures and be more engaged in their learning.

As a Dynamic Geometry System, Geometry Supposer (Schwartz & Yerushalmy, 1984), also provides opportunity for students’ in conjecturing and reasoning. In Geometry Supposer, students chose a figure, such as rectangle and perform measurement operations on it. Several studies related to Geometry Supposer cited evidence that students who use this program performed better than the ones who did not use (Lampert, 1988; Wiske & Houde, 1988; Yerushalmy, Chazan, & Gordon, 1987).

*Cabri-géométrie* (Laborde, 1990) is another dynamic geometry software, in which constructions can be made simply by dragging mode. In Cabri environment,

invariant properties belonging to the shapes retained, whereas its size and position can be changed by dragging action. This property of Cabri provides students to validate their conjectures. Across studies, several findings are consistent on the benefits of the use of Cabri-*géométrie* (Arzarello et al., 1998; Laborde, 2001; Mariotti, 2001).

One of the recent studies related to the effects of using dynamic geometry software is the study of Gawlick (2002). The purpose of the study was to investigate how the step from experimental to regular dynamic geometry software use will probably take place in the classroom. He presented the results of their study concerning differential effects of using dynamic geometry software on students' achievement. As a result of the study, some steps which are necessary in integrating dynamic geometry software to a learning environment were underlined. According to the results of the study, one of the important issue, that should be considered in integrating DGS into classroom is the necessity of change of educational environment accordingly. Gawlick (2002) asserted that, "teachers must be put into a position to develop new teaching sequences, and schools must have the equipment to make dynamic geometry home work and assessment possible" (p.91).

In the study of Jones (2001) which aimed to gain information about interpretations of 12-year old students while using dynamic geometry software. Analysis of the data from the study indicated that the use of DGS can assist students in making progress towards more mathematical explanation. She further mentioned that, especially in the early stages, the dynamic nature of the software influenced the form of explanation of students.

Another study related to the DGS was conducted by Hölzl (1999) which focused on examining the long-term effects of dynamic geometry software use in a classroom setting, where dynamic geometry software was an integral part of the learning environment. The study results indicated that Dynamic Geometry Software possesses significant potential on transformation geometry and the application of dynamic geometry software should only be realized after thorough consideration.

Gillis (2005) conducted another research study to investigate students' conjectures with comparing static and dynamic geometry environments. The data

collected were examined both quantitatively and qualitatively. Qualitative data were collected by means of observations of participant, a survey, participant interviews, and a qualitative analysis of the conjectures which were made by the students in both dynamic and static environments. The results of the study indicated that the students who used dynamic geometry software were found more successful in making relevant conjectures. Furthermore, the correctness of their conjectures was higher when compared to students working in a static geometry environment.

Marrades and Gutiérrez (2000) conducted two case studies which aimed at investigating the ways dynamic geometry software improves students' understanding of the nature of mathematical proof and their proof skills while secondary school students working with Dynamic Geometry Software. The purpose of study was to teach geometric concepts and properties, and to help students to improve their proof skills and conception related to the nature of mathematical proof. After the analyses of the students' to proof problems, they observed the types of justifications produced, and verified the usefulness of learning in dynamic geometry computer environments to improve students' proof skills.

Laborde (2001) presented an analysis of teaching sequences involving dynamic geometry software. Teaching sequences used in the study were developed by teachers over a time span of three years. The result of her study indicated that when dynamic geometry software was a visual representative of the data, it became an essential component to understand the tasks through the teaching process. On the last stage of the study, the technology began to shape the conceptions of the mathematical objects that the students construct. As a result of the study, Laborde (2001) asserted that the integration of computer technology into mathematics classrooms is a long and difficult process.

Mariotti (2000) carried out a long-term teaching experiment with the purpose of clarifying the role of a dynamic geometry software, in the teaching and learning process. The study conducted with the 9<sup>th</sup> and 10<sup>th</sup> grade students of a scientific high school as a part of a coordinated research project. The functioning of specific elements of the software was described and analyzed as instruments used by the teacher in classroom activities. As a result of the study, Mariotti (2000) stated that

the students were greatly facilitated by the use of dynamic software that affords visualization, exploration and the use of problem solving strategies.

In the light of all these studies, the facilities of dynamic geometry software usage in learning environments can be sum up with three main advantages. Firstly, dynamic geometry environments help students to create mental models for thinking about geometric shapes (Jones, 2001; Üstün & Ubuz, 2004; Velo, 2001). Secondly, students do not have to memorize the properties of geometrical shapes since they learn by doing. Thirdly, dynamic geometry softwares allow students to experience the property in action before using it at a more formal level (Laborde, 1995). In the following part, information about GeoGebra which was used in the present study as a dynamic geometry software and the research studies related to this software will be presented.

### **2.2.1 GeoGebra**

GeoGebra, developed by Marcus Hohenwarter and Yves Kreis in 2001, is a free dynamic mathematics software (DMS) developed for teaching and learning of mathematics in elementary school, secondary school and the college level. GeoGebra combines Computer Algebra Systems (CAS) and Dynamic Geometry System into one easy-to use system. That is, it combines the functions of a dynamic geometry software (DGS) with the features of a computer algebra system (CAS). Hence, it provides linking mathematics with algebra, geometry and calculus by including both dynamic geometry and computer algebra tools (Hohenwarter & Preiner, 2007b).

This free dynamic geometry, algebra, and calculus software was developed both for teachers and students to make teaching and learning of mathematics more effective and permanent. GeoGebra can be defined as an effective and important tool in establishing relationship between geometry and algebra concepts in elementary mathematics since it proved its capability and potential in mathematics education (Hohenwarter & Jones, 2007). The software can be used with students ranging from elementary level to college level, aged from 10 to 18, beginning with simple constructions up to the integration of functions. The students can explore mathematics alone or in groups and the teacher tries to be a guide in the background

who gives support when students need help. The students' results of their experiments with GeoGebra constitute the basis for discussions in class so that teachers can have more time to concentrate on fundamental ideas and mathematical reasoning (Schumann, 1992).

GeoGebra is one of the most popular DGS all around world. There are 300,000 visitors from 188 different countries (March, 2008). It is estimated that more than 100,000 teachers already use GeoGebra to construct both static and dynamic mathematics materials for improving their students' learning (Preiner, 2008). The software is freely available at [www.geogebra.org](http://www.geogebra.org). A screenshot from GeoGebra window is presented in Figure 1 below.

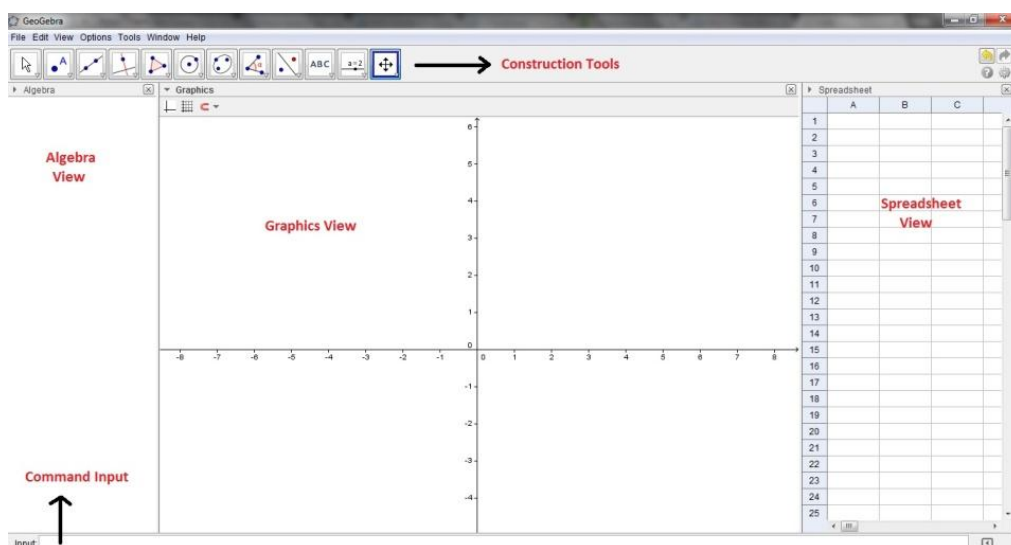


Figure 1. Screenshot from GeoGebra user interface

“GeoGebra is open source software under the GNU General Public License<sup>1</sup> and freely available at [www.geogebra.org](http://www.geogebra.org). Thereby, either an installer file can be downloaded, or GeoGebra can be launched directly from the Internet using GeoGebra *WebStart*. Since the software is based on Java, it is truly platform independent and runs on every operating system. Furthermore, GeoGebra is multilingual not only in its menu, but also in its commands, and was translated by volunteers from all over the world into more than 35 languages” (Preiner, 2008, p. 35). Hohenwarter, Hohenwarter, Kreis and Lavicza (2008) stated the importance of

using open-source softwares as: “Open-source packages do not only offer opportunities for teachers and students to use them both at home and in the classroom without any restriction, but they also provide a means for developing support and user communities reaching across borders. Such collaboration also contributes to the equal access to technological resources and democratization of mathematics learning and teaching” (p.8).

One of the unique properties of GeoGebra is it integrates algebra view, graphic view, and spreadsheet view in a single interface (Preiner, 2008). GeoGebra not only provides students with facilities to experiment the mathematical ideas and to associate mathematical concepts with the real life examples, but also helps students to examine the relation between algebraic and geometrical concepts better. GeoGebra can also be used in many ways in the teaching and learning of mathematics: for demonstration and visualization since it can provide different representations; as a construction tool since it has the abilities for constructing shapes; for investigation to discover mathematics since it can help to create a suitable atmosphere for learning; and for preparing teaching materials using it as a cooperation, communication and representation tool (Hohenwarter & Fuchs, 2004).

GeoGebra can be used not also to visualize mathematical concepts and ideas, but also to create instructional materials. Also, GeoGebra has the potential to encourage the student-centered learning, active student participation, collaborative learning, and discovery learning by experimenting mathematical ideas, theorems and using interactive explorations (Preiner, 2008). GeoGebra enables teachers and students to make strong connections between geometry and algebra (Hohenwarter & Jones, 2007). In other words, GeoGebra supports visualization skills of learners in a computerized dynamic environment (Hacıömeroğlu, 2011) as well as their understanding of algebraic operations and equations. Moreover, like all DGS, GeoGebra also has a dragging tool called as “slider”. Algebraically, it is a variable that has a value for its interval. Graphically, it is a segment that allows the user to change the value of the variable by dragging (Bu & Hacıömeroğlu, 2010).

### 2.2.1.1 Advantages of GeoGebra

GeoGebra is an open source software that includes both dynamic geometry and computer algebra tools (Hohenwarter & Fuchs, 2005). That is, it combines almost all features of DGS and CAS environments into a single software. Diković (2009) stated some of the main advantages of GeoGebra as following;

- GeoGebra is more user-friendly than a graph calculator owing to its easy-to-use user interface multilingual menus, commands and help.
- It promotes guided discovery, cooperative and experimental learning, multiple presentations and students' products in mathematics.
- GeoGebra was created to provide students with better understanding of mathematics. Students can manipulate objects by dragging around the plane of drawing or using sliders to test mathematical ideas and see how these movements affect other variables. In this way, students have the opportunity to solve problems by investigating mathematical relations dynamically.
- Users can personalize their own creations through the adaptation of interface (e.g. font size, language, quality of graphics, color, coordinates, line thickness, line style and other features).
- The algebra input allows the user to construct new objects or to modify the existing ones by the command line. The worksheet files can easily be published as Web pages.
- It encourages teachers to use and assess technology in mathematics classrooms.

In addition to the abovementioned advantages, GeoGebra is open source software under the GNU General Public License<sup>1</sup> and freely available at [www.geogebra.org](http://www.geogebra.org). Thereby, either an installer file can be downloaded, or GeoGebra can be launched directly from the Internet using GeoGebra *WebStart*. Since the software is based on Java, it is truly platform independent and runs on every operating system. Moreover, GeoGebra is multilingual not only in its menu, but also



in its commands. It was translated by volunteers from all over the world into more than 35 languages (Preiner, 2008).

#### **2.2.1.2 Disadvantages of GeoGebra**

As all the Dynamic Geometry Softwares (e.g. Cabri, Geometer's Sketchpad, Autograph) have, GeoGebra has also some deficiencies. Diković (2009) stated the deficiencies of GeoGebra as following;

- Students or other users, who don't have previous programming experience, may have difficulty with entering algebraic commands in the input box. Even though the basic commands are not difficult to find out, they might feel uncomfortable or embarrassed while using it.
- Some teaching approaches such as learning by discovery or experimenting may not be appropriate for many students.
- Future layers that will be added to GeoGebra should include more symbolic features of CAS such as complex applications and 3D extensions.

When we consider the literature review as a whole, although some disadvantages of GeoGebra exist, we can conclude that the use of DGS in learning environment can be used as a useful technological tool that makes the teaching and learning more effective, permanent, easier, and funnier (Dikovic, 2009; Hohenwarter & Fuchs, 2005).

#### **2.2.2 Dynamics Geometry Software and Mathematics Achievement**

In this part, some research studies on DGS use in mathematics teaching and its effect on students' mathematics achievement were reviewed.

One of these studies was conducted a study by Yemen (2009) with 50, 8<sup>th</sup> grade students to investigate the effects of technology-assisted instruction using Dynamic Geometry Software on 8<sup>th</sup> grade students' achievement and attitudes in analytical geometry. The students in both groups, experimental and control groups, were instructed for five weeks time span. The Geometer's Sketchpad was selected as

Dynamic Geometry Software in the study. Results of the study revealed that the students taught by Technology-Assisted Method scored higher, on the average, than the students taught by Traditional Method on an Analytical Geometry Achievement. However, there was no statistically significant difference found between the groups' post-test scores of attitude towards mathematics. She explained the reason why the attitude has not changed depending on the time span of the study since the changes in attitude may require a time span which is longer than five weeks. Consequently, she concluded that the DGS enhanced students' mathematics achievement more than the traditional method did. She also concluded that DGS had no significant effect on students' attitude toward mathematics.

Another quasi-experimental pretest-posttest control group design research study was conducted by Toker (2008) to investigate the effects of using Dynamic Geometry Software (DGS), Geometer's Sketchpad, while teaching by guided discovery compared to paper-and-pencil based guided discovery and traditional teaching method on sixth grade students' van Hiele geometric thinking levels and geometry achievement. The study was conducted in a private school in Çankaya, Ankara, Turkey and it lasted six weeks. The sample of the study consisted 47, 6<sup>th</sup> grade students in the school. In order to gather data, Geometry Achievement Test (GAT) and van Hiele Geometric Thinking Level Test (VHL) were used. The results of the study indicated that there was a significant effect of methods of teaching on means of the collective dependent variables of the sixth grade students' scores on the POSTVHL after controlling their PREVHL scores and there was a significant effect of methods of teaching on means of the collective dependent variables of the sixth grade students scores on the POSTGAT after controlling their PREGAT scores. In other words, guided discovery teaching method using dynamic geometry software (The Geometer's Sketchpad) was significantly more effective on students' van Hiele Geometric Thinking Level and Geometry Achievement than the other methods were.

Similarly, İşıksal (2002) carried out a study with the purpose of investigating the effect of spreadsheet and dynamic geometry software on the mathematics achievement and mathematics self- efficacy of 7<sup>th</sup> grade students. The research was conducted with 64 seventh grade students at a private elementary school. During the

study, experimental groups received Autograph-Based Instruction (ABI) and Spreadsheet Based Instruction (SBI) respectively whereas the control group received Traditionally Based Instruction (TBI). The study lasted three weeks. Mathematics Achievement Test (MAT) and Mathematics Self-Efficacy Scales (MSES) were used as pre and posttests. Results of the study indicated that ABI and TBI groups had significantly greater mean scores than SBI group with respect to mathematics achievement. However, there was no significant mean difference between the ABI and TBI groups with respect to mathematics achievement. Also, ABI group had significantly greater mean scores than TBI group with respect to mathematics self-efficacy. However, there was no significant mean difference between ABI and SBI groups and there was no significant mean difference between SBI and TBI groups with respect to mathematics self-efficacy. Moreover, there was a statistically significant correlation between post-test scores of Mathematics Self-Efficacy and Mathematics Achievement, Mathematics Achievement and Computer Self-Efficacy, Mathematics Self-Efficacy and Computer Self-Efficacy.

Mercan (2012) conducted another quasi-experimental study with 37, seventh grade students, 17 students in experimental group and 20 students in control group, to investigate the effects of dynamic geometry software GeoGebra on students' mathematics achievement in Transformation Geometry and retention levels. The research design of the study was the pretest-posttest control group design. While the experimental group students were taught with GeoGebra-based course for two weeks in accordance with Ministry of National Education curriculum, control groups students were taught with Traditional Instruction. During this 2-week-course, students were provided with GeoGebra construction activities involving active use of GeoGebra. The measurement instruments, Achievement Tests and Retention Tests, were prepared for the particular units by Mercan (2012) and were administered to both groups as pre-test and post-test, before and after the treatment, respectively. Results of the study revealed that GeoGebra was found to affect the achievement and learning of students positively. Moreover, there was a significant difference between the post retention test results in favor of the experimental group.

Parallel with the study results of Yemen (2009), Aydoğan (2007) also conducted a study with the aim of investigating the effects of using a dynamic geometry software The Geometer's Sketchpad together with open-ended explorations on 6<sup>th</sup> grade students' performance in polygons and congruency and similarity of polygons. The study consisted of 134 students in total. While the students in the control group were taught via traditional instruction, the students in the experimental group were taught the same topic by open-ended explorations in a dynamic geometry environment using The Geometer's Sketchpad. Geometry Test (GT) and Computer Attitude Scale (CAS) were used as data collection instruments. All students had taken the GT as pre-test, post-test, and delayed post test. However, CAS was administered only to the experimental group at the end of the instruction. Furthermore, some qualitative data were collected through video-taped classroom observations and interviews with selected students. The results of the study also showed that experimental group achieved significantly better than the control group students. In addition, a statistically significant correlation between CAS and GT was observed. The results of the study indicated that dynamic geometry environment together with open-ended explorations significantly improved students' performances in polygons and congruency and similarity of polygons.

Sarı-Yahşi (2012) performed another research study to compare the effects of The Geometer's Sketchpad and GeoGebra dynamic geometry software programs' on 7<sup>th</sup> grade students' mathematics achievement and retention levels in the topic of Transformation Geometry. The pretest-posttest control group design adopted in the study which was conducted with 72, 7<sup>th</sup> grade students, two experimental groups (48 students) and one control group (24 students). Control group was selected among three 7<sup>th</sup> grade classes randomly. While the subject of transformation geometry were taught the first experimental group using GeoGebra dynamic and taught the second experimental group using The Geometer's Sketchpad, the control group were taught the same subject using Traditional Instruction. The worksheets and classroom activities prepared by Sarı-Yahşi (2012) were used to teach the classes for six weeks. The study results indicated that Computer-Assisted Instruction method using Dynamic Geometry Softwares were significantly more effective than the Traditional

Instruction in terms of mathematics achievement. Also, it was found that the students taught with Computer-Assisted Instruction method using dynamic geometry softwares had higher retention level than the control group students had.

Similar to the study of Sarı-Yahşi (2012), Baharvand (2001) investigated the effects of using Geometer's Sketchpad compared to instruction by teacher-lecture and pencil-and paper activities on performance of students', students' retention level, and students' attitude toward learning geometric concepts. 26 seventh grade students received instruction by teacher-lecture and another seventh grade class with 24 students learned the same concepts using the Geometer's Sketchpad. The results of the study indicated that the students taught with the GSP scored significantly higher on the posttest than the control group.

Pilli (2008) also carried out another experimental study with the purpose of examining the effects of the computer software named as *Frizbi Mathematics 4* on 4<sup>th</sup> grade students' mathematics achievement in the units of multiplication of natural numbers, division of natural numbers, and fractions, retention, attitudes toward mathematics and attitude toward Computer-Assisted Learning. While the control group (26 students) were taught using a lecture-based traditional instruction, the experimental group (29 students) were taught using educational software, namely *Frizbi Mathematics 4*. The groups were compared on achievement of mathematics, retention, and attitude toward mathematics and Computer-Assisted Learning. Scores on achievement tests were collected three times; at the beginning of the study, immediately after the intervention, and 4 months later. Mathematics Attitude Scale and Computer-Assisted Learning Attitude Scale were administrated only two times; at the beginning of the study and immediately after the completion of the study. Results of the study revealed significant difference between the groups on the post achievement tests and attitude scales in favor of experimental group. However, statistically significant differences in favor of treatment group, on the retention tests was attained on the multiplication and division units but not on fractions. The evidence indicates that *Frizbi Mathematics 4* for learning and teaching mathematics at the primary school level in Turkish Republic of Northern Cyprus (TRNC) is an effective tool.

In the following section, some of the research studies related to the GeoGebra use in mathematics teaching as a dynamic mathematics software and its effect on students' mathematics achievement were reviewed in detail.

### **2.2.3 GeoGebra and Mathematics Achievement**

There are many research studies indicating that GeoGebra enhance students' academic achievement. Some of these studies were mentioned in this section.

One of these studies was conducted by Bilgici and Selçik (2011) with 32, 7<sup>th</sup> grade students from two different schools to investigate the effects of GeoGebra in the learning of the Polygons on 7<sup>th</sup> grade students' mathematics achievement. The experimental group (17 students) were taught by Computer-Assisted Instruction using several GeoGebra worksheets prepared, while the control group (15 students) were taught in a Computer-free learning environment for 11 teaching hours in a primary school. The experimental group received instruction of GeoGebra for 2 hours before the treatment is implemented. Results of the study revealed that the difference between the experimental and the control groups after the treatment is statistically significant. This result indicates that Computer-Assisted Instruction utilizing GeoGebra enhanced students' achievement scores more than the Computer-free Instruction did. It was also found that the experimental group students carried out more effective learning with Computer-Assisted Instruction utilizing GeoGebra and retained what they learnt more than they retained after they learned via computer-free instruction. As a result, the researchers concluded that the use of DGS in mathematics education enhanced students' mathematics achievement and retention level more than the traditional method did per se.

In another research, Saha, Ayub and Tarmizi (2010) studied with 53 secondary school students to investigate the effects of GeoGebra on mathematics achievement in the learning of Coordinate Geometry. The sample of the study was assigned into two groups as high visual-spatial ability students (HV) and low visual-spatial ability students (LV) according to the Spatial Visualization Ability Test. Results of the study revealed that there was a significant difference between the control group and GeoGebra group in favor of the GeoGebra group related to the

mean performance scores. The results of study also indicated that there was no significant difference between the high visual-spatial ability (HV) students taught with GeoGebra and the high visual-spatial ability (HV) students taught with Traditional Instruction in terms of the mean posttest performance scores. The results of study also showed that there was no significant difference between the low visual-spatial ability (LV) students taught with GeoGebra and the low visual-spatial ability (LV) students taught with Traditional Instruction in terms of the mean posttest performance scores. This finding showed that LV students who had undergone learning Coordinate Geometry using GeoGebra was significantly better in their achievement rather than students underwent the traditional learning. In other words, the study results showed that the GeoGebra enhanced the LV students' mathematics performance in Coordinate Geometry. Consequently, the results of this study revealed that Computer-Assisted Instruction (using GeoGebra) as a supportive tool to the Traditional Instruction is more effective than Traditional Instruction per se.

Similarly, Furkan, Zengin, and Kutluca (2012) conducted a study to determine the effects of dynamic mathematics software GeoGebra on 10<sup>th</sup> grade students' achievement in trigonometry. The sample of the study consisted of 51, tenth grade students. The experimental group students were undergone to the lessons arranged with the GeoGebra in Computer-Assisted Instruction, while the students in control group were taught with constructivist instruction. The data collected after 5 weeks of the application. The test results indicated that there was a significant difference between the experimental and the control groups' achievement scores in trigonometry. This difference was in favor of the experimental group which had lessons with GeoGebra.

Parallel with the study results of Furkan, Zengin, and Kutluca (2012), İçel (2011) conducted a study to analyze effects of dynamic mathematics software GeoGebra on eight grade students' achievement in the subject of triangles. The sample of the study consisted of 40 (20 in experimental group and 20 in control group), 8<sup>th</sup> grade students. The experimental group students were instructed with the planned activities that were constructed with GeoGebra, while the control group students were taught with traditional method in accordance with the official

curriculum textbook for six class hours, two weeks in total. A pre-test (consisted of 13 questions), a post-test and a recall test were administered to the groups both before and after the treatment to collect data. The post-test and recall test, which consisted of 11 questions, were identical. The recall test was administered to the students one month after the study completed. Results of the study revealed that the experimental group students scored higher on the post-test than the students in the control group. The total recall test results showed that GeoGebra was also effective in enhancing the permanence of the acquired knowledge. The students in the experimental group scored higher on the recall test than the students in the control group.

Zengin (2011) also carried out another experimental quantitative study with 51 students at the high school level to determine the effect of GeoGebra on both achievement and attitude toward mathematics. The researcher designed GeoGebra workshops for the experimental group and used a pretest posttest control group design. Similar to the study of İçel (2011), it was found that GeoGebra has a positive effect on mathematics achievement. However, there was no significant difference between the experimental and control group in terms of their attitudes towards mathematics (Zengin, 2011).

Filiz (2009) conducted a quasi-experimental study with 25 elementary school students (12 in treatment and 13 in control group) to investigate the effect of using GeoGebra and Cabri Geometry II Dynamic Geometry Softwares in a Web-based setting on students' achievement and the development of learning experiences during this process. For this purpose, four objectives of 8<sup>th</sup> grade geometry learning field were selected and a web site including dynamic geometry softwares and worksheets related with the subject were prepared for the students. As a result of the study, a significant difference was found in favor of the treatment group in which web-based materials were used. Moreover, it was found that a more effective learning is experienced by students taught with web based learning materials when compared to students taught with Traditional Instruction. The results of the study also revealed that dynamic geometry softwares improved students' inference and hypothesizing skills.



## **2.3 The van Hiele Theory of Geometric Thinking**

The van Hiele Theory was developed by two Dutch mathematics educators in separate doctoral dissertations at the University of Utrecht in 1957, Pierre Marie van Hiele, and his wife Dina van Hiele-Geldof. The theory has been applied to explain why many students have difficulty with the higher order cognitive processes, particularly proof, required success in high school geometry. The van Hieles theorized that students who have trouble are being taught at a higher van Hiele level than they are at or ready for. The theory outlines the hierarchy of levels through students' progress as they develop geometric ideas. Put it differently, the van Hiele model explains the stages of human geometric reasoning. The theory also offers a remedy: go through the sequence of levels in a specific way (Usiskin, 1982). Van Hiele Levels are sequential and progress from one level to another depends more on the content and method of instruction than on age or biological maturation. A teaching-learning process is necessary to move the student from one level to the next (Duartepe,2004). The theory has three aspects: the existence of levels, properties of the levels, and the movement from one level to the next (van Hiele, 1957).

### **2.3.1 van Hiele Geometric Thinking Levels**

van Hiele states that all students progress in geometrical thinking through five sequential and hierarchical levels named as the levels of Recognition, Analysis, Order, Deduction, and Rigor (van Hiele, 1959; van Hiele, 1986, van Hiele-Geldof, 1984).

#### **2.3.1.1 Level 0 (Recognition)**

At Level 0, students view figures holistically by their appearance. They can learn names of figures and recognize a shape as a whole. For example, squares and rectangles seem to be different. They identify shapes according to the shapes' some physical features, such as "largeness", "pointedness", etc. However, they cannot notice or explain the properties of components. If students are introduced to a certain shape, then they are able to name when they see it again but without giving

explanations concerning properties of its parts. When asked to explain why a particular quadrilateral is a square, a typical response would be, “because it looks like one.”. Students may be able to distinguish one figure from another simply based upon its appearance (Usiskin, 1982).

#### **2.3.1.2 Level 1 (Analysis)**

At Analysis Level, a student can identify properties of figures. For example, the student knows the properties of a square such as; a square has four congruent sides; a square has congruent diagonals; a square has four right angles; the diagonals of a square bisect each other; the diagonals of a square are perpendicular; opposite sides of a square are parallel. They reason about geometric concepts by means of an informal analysis of shapes’ parts and properties. These properties could be realized by a variety of activities such as observation, measuring, cutting, and folding. At this level necessary properties of the figure could be understood. However, each property is perceived as isolated and unrelated, no property implies any other. Therefore, relations between properties and definitions are not understood (Duatepe, 2004).

#### **2.3.1.3 Level 2 (Order)**

Students at Level 2 can logically order figures and relationships, but still does not operate with a mathematical system. That is, simple deduction can be followed, but proof is not understood. Students logically order the properties of concepts, form abstract definitions, and distinguish between the necessity and sufficiency of a set of properties in determining a concept. The relationship between properties can be established, hierarchies can be built and the definitions can be understood, properties of geometric figures are deduced one from others. For example, the student can see that a square is a rectangle; but a rectangle may not be a square. However, the importance of deduction cannot be understood at this level (Usiskin, 1982).

#### **2.3.1.4 Level 3 (Deduction)**

Students at Level 3 understand the significance of deduction and the roles of postulates, theorems, and proof. Proofs can be developed and written with

understanding. Students can construct proofs of theorems, understand the role of axioms and definitions, and the meaning of necessary and sufficient conditions. “Students can reason formally by logically interpreting geometric statements such as axioms, definitions, and theorems” (Battista & Clements, 1992, p.428). For example, students understand the fact that the definition of “quadrilaterals in which all sides and angles are equal” and the definition of “quadrilaterals in which all angles are perpendicular and adjacent sides are equal” could be proved to be equal and both can define a square (Duartepe, 2004).

#### **2.3.1.5 Level 4 (Rigor)**

Students at this level are able to reason Non-Euclidean geometry and explore other axiomatic systems. They understand the necessity for rigor and are able to make abstract deductions. Furthermore, they are able to make connections and see relationships between different axiomatic systems (Usiskin, 1982). Students compare different geometries based on different axioms and study them without concrete models. They can establish consistency of a set of axiom, and equivalence of different sets of axioms, create an axiomatic system for a geometry. Theorems in different axiomatic systems could be established (Duartepe, 2004).

To sum up, the first level of geometric thinking begins with nonverbal thinking. The student at level 0 perceives a figure as a whole shape and does not perceive their parts. He/she might say, "It is a rectangle because it looks like a door". At level 1, properties can be recognized but properties are not logically ordered yet. At level 2, properties are logically ordered; one property precedes or follows from another property. But at this level, the intrinsic meaning of deduction, that is, the role of axioms, definitions, theorems, and their converses are not understood. At level 3 deduction and construction of proof can be understood. Different axiomatic systems can be understood at level 4. This model has been studied and validated by numerous researchers (Burger & Shaughnessy, 1986; Fuys, Geddes & Tischler, 1988; Hoffer, 1981; Mayberry, 1981; Senk, 1983; Senk, 1989; Usiskin, 1982).

### **2.3.2 Research Studies on the van Hiele Theory**

There are various studies which were conducted to discover the implications of the van Hiele Theory for current K-12 geometry curriculum in dynamic learning environments. Many research studies indicated that the van Hiele Geometric Thinking Levels are helpful for describing the development of geometric thinking of students from elementary level to the college level (Burger & Shaughnessy, 1986; Fuys et al., 1988; Han, 1986; Hoffer, 1983; Usiskin, 1982; Wirszup, 1976). Some of these studies were mentioned in this section.

One of the first major studies on the van Hiele Theory was performed by Usiskin (1982, as cited in Fuys, 1985). Usiskin developed a multiple-choice test to measure students' van Hiele Geometric Thinking Levels and this test has been widely used by other researchers. Usiskin (1982) developed this test to find out if the test could predict students' achievement in geometry. He tested 2900, 10<sup>th</sup> graders and looked for a correlation between their van Hiele Geometric Thinking Levels and Geometry Achievement. The study results indicated that there was a moderately strong correlation ( $r=.64$ ) between the subjects' Geometry Achievement and van Hiele Geometric Thinking Level. The study results also revealed that the students were generally at Level 0 or Level 1, hence, most of the students were not ready for high school geometry.

Another experimental study was carried out by Öztürk (2012) with 52, 8<sup>th</sup> grade students to investigate the effects of dynamic mathematics software GeoGebra on students' mathematics achievement and van Hiele geometric thinking levels in teaching of trigonometric ratios and slope. Pretest-posttest control group design was adopted as research design of the study. The students assigned into the groups according to the results of "Achievement Test" and "van Hiele Geometrical Thinking Level Test". While the experimental group (26 students) were instructed with Computer-Assisted Teaching materials using GeoGebra, the control group (26 students) were taught with Traditional Instruction based on Constructivism approach. Retention Level Test was also administered to the students six weeks after the treatment ended. The study results indicated that Computer-Assisted Instruction (CAI) using dynamic mathematics software, GeoGebra, had a significant effect on

students' mathematics achievement in trigonometric ratios and slope compared to the Traditional Instruction. However, the CAI using GeoGebra had no significant effect on students' van Hiele Geometric Thinking Level. The study results also revealed that the experimental group students' Retention Level was significantly higher after six weeks.

Toker (2008) also conducted a quasi-experimental pretest-posttest control group design research study to investigate the effects of using Dynamic Geometry Software (DGS), Geometer's Sketchpad, while teaching by guided discovery compared to paper-and-pencil based guided discovery and traditional teaching method on sixth grade students' van Hiele geometric thinking levels and geometry achievement. The study was conducted in a private school in Çankaya, Ankara, Turkey and it lasted six weeks. The sample of the study consisted 47, 6<sup>th</sup> grade students in the school. In order to gather data, Geometry Achievement Test (GAT) and van Hiele Geometric Thinking Level Test (VHL) were used. The results of the study indicated that there was a significant effect of methods of teaching on means of the collective dependent variables of the sixth grade students' scores on the POSTVHL after controlling their PREVHL scores and there was a significant effect of methods of teaching on means of the collective dependent variables of the sixth grade students scores on the POSTGAT after controlling their PREGAT scores. In other words, guided discovery teaching method using dynamic geometry software (The Geometer's Sketchpad) was significantly more effective on students' van Hiele Geometric Thinking Level and Geometry Achievement than the other methods were.

Similarly, Moyer (2003) conducted a study to examine the effects of using Geometer's Sketchpad (GSP) on the increase in student's achievement and van Hiele Geometric Thinking Levels in geometry instruction. He used a non-equivalent control group design in his study. The subjects were selected from four intact geometry classes. Two teachers had two classes, one of which used GSP throughout the study. The researcher designed content pre-test and two content posttests, one for each chapter of content. The results of his study indicated that the use of GSP did not have a significant effect on the increase in students' van Hiele levels and geometry achievement. He recommended that further research studies should address the

investigation into what teacher skills are necessary in order to use GSP effectively as an instructional tool in mathematics classrooms. He also recommended that a research study concerning the use of GSP should be conducted throughout the whole year instead of studying the chosen chapters.

Parsons, Stack and Breen (1998) conducted a study with 11 eighth-graders to determine if Computer-Assisted Instruction (CAI) could improve students' van Hiele levels, specifically to level 2 (informal deduction, based on a 0-4 numbering scheme). The students were administered van Hiele Geometric Thinking Level Test (Usiskin, 1982, as cited in Fuys, 1985) before and after the treatment. The sample of the study began with about 18% below level 0, 45% at level 0, and 36% at level 1. Result of study showed that there was a significant increase in van Hiele Geometric Thinking Levels from the pretest to posttest. The students were also administered two other pre/post tests: one on standard geometry content and the other on vocabulary. Statistical analysis showed that there was no significant difference between pretest and posttest for either of standard geometry content and vocabulary tests. Parsons, Stack and Breen claimed that CAI had a significant impact on van Hiele levels and helped students get to level 2.

Another quasi-experimental research study was performed at a primary school by Tutak and Birgin (2009) with 38, 4<sup>th</sup> grade students' to investigate the effects of the Computer-Assisted Instruction (CAI) using dynamic geometry software, Cabri, on students' van Hiele geometric thinking level. The research pattern of the study was pretest-posttest control group design. While the experimental group were consisted of 21 students, the control group was consisted of 17 students. Whereas the experimental group students was instructed with CAI using Cabri, the control group students were instructed by Traditional Instruction. In order to collect data, "van Hiele Geometric Thinking Level Test" was administered to both groups as pretest and posttest. The results of the study revealed that the Computer-Assisted Instruction using dynamic geometry software had a significant effect on the students' van Hiele Geometric Thinking Level compared to the Traditional Instruction.

In the research study of July (2001), she documented and described 10<sup>th</sup> grade students' geometric thinking and spatial abilities as they used Geometer's Sketchpad (GSP) to explore, construct, and analyze three-dimensional geometric objects. Then he found out the role that can dynamic geometry software, such as GSP, play in the development of students' geometric thinking as defined by the van Hiele theory. He found there was evidence that students' geometric thinking was improved by the end of the study. The teaching episodes using GSP encouraged level 2 thinking of the van Hiele theory of geometric thinking by helping students to look beyond the visual image and attend to the properties of the image. Via GSP students could resize, tilt, and manipulate solids and when students investigated cross sections of Platonic Solids, they learned that they could not rely on their perception alone. In addition teaching episodes using GSP encouraged level 3 of the van Hiele thinking by aiding students learn about relationships within and between structure of Platonic solids.

Similar to the study of Moyer (2003), Meng and Idris (2012) conducted a study to explore if students' geometric thinking and achievement in solid geometry could be enhanced through phase-based instruction using manipulatives and The Geometer's Sketchpad (GSP) based on the van Hiele theory. The researchers employed a case study research design and purposeful sampling to select eight case study participants from a class of mixed-ability Form One students. The results of the study showed that the teaching intervention could enhance the participants' geometric thinking and achievement in solid geometry.

Besides, Chang, Sung, and Lin (2007) performed another research study to investigate the learning effects of GeoCAL, a multimedia learning software which is based on van Hiele Geometric Thinking Level Theory, on each of the geometric thinking levels and overall geometric thinking. The subjects of the study were 2<sup>nd</sup> elementary school students of an average age of eight who have not previously had formal lessons in geometry. The study results indicated that, with the exception of recognition ability, GeoCAL produced significant learning effects on visual association, description/analysis and abstraction/relation as well as overall geometric thinking.

Idris (2007) carried out a quasi-experimental pretest-posttest control group design research study in a secondary school with 65, Form Two students to investigate the effects of using The Geometer's Sketchpad (GSP) on students' achievement in geometry, the van Hiele Level, and to get students' views on learning geometry with GSP. While the experimental group underwent the lessons using the Geometer's Sketchpad for ten weeks, the control group students were taught by the traditional approach. The van Hiele Geometry Test was administered to determine students' level of geometric thinking according to the van Hiele Theory. The questionnaire and checklist were administered to explore the students' response towards the use of the Geometer's Sketchpad in learning of geometry. The descriptive analysis showed that most of the students agreed that the GSP is a useful tool for learning of geometry. The study results also revealed that the experimental group differed significantly from the control group in terms of geometry achievement and change in van Hiele geometric thinking level after the treatment.

In another research, Clements, Battista, and Sarama (2001) designed a research-based curriculum using the Logo, a graphic oriented educational programming language, to investigate how elementary students learn geometric concepts. The aim of their project was also to assess student learning in this micro world setting, and characterize how Logo facilitates students' learning. Clements, Battista, and Sarama developed a curriculum as Logo Geometry (LG) with the theories of Piaget and van Hiele as the underlying models to inform curricular and assessment decisions. One group of students participated in the LG curriculum (experimental group) while another group did not (control group). The results of their study indicated that the Logo geometry students scored significantly higher than control students on total achievement tests, made double the gains of the control groups. Furthermore, students in the LG classes showed higher gains in describing properties of shapes (level 2 thinking on a 1-5 numbering scheme) than students in the control group. This results supported a premise of the LG curriculum that having students engage in construction of more complex paths (shapes) helps them to transition between level one thinking (visual) to level two thinking (analysis). Moreover, LG students did better than those in the control group in identifying lines



of symmetry for a given figure, and justifying why pairs of figures were congruent. Additionally, the experimental group showed better understanding of slides, flips, and turns than the control group based on a number of motions sorting activities. As a conclusion, the study results showed that there was a significant difference between the experimental group (Logo curriculum) and the control group in terms of total achievement test scores in favor of the experimental group.

In the next part, some of the research studies on technology use in mathematics teaching and its effect on students' attitude towards learning mathematics with technology will be presented.

## **2.4 Technology and Attitude Towards Mathematics**

Attitude towards mathematics is defined as a belief formed from a combination of experiences measured in the domains of mathematics (Capraro, 2000). In other words, attitude towards mathematics refers to a student's self-reported enjoyment, interest and level of anxiety toward mathematics (Pilli, 2008) and plays a crucial role in the learning of mathematics and achievement in mathematics (Arslan, 2008; Peker & Mirasyedioğlu, 2003).

Besides, Ma and Kishor (1997) stated that there is a general belief that students learn more effectively when they are interested in what they learn and that they will achieve better in mathematics if they like mathematics. Thus, investigating the effectiveness of the instruction using dynamic geometry software, which may establish a positive attitude towards mathematics, is important for students' mathematics learning and achievement. In this study, the attitude towards mathematics and technology means students' attitude toward learning mathematics with technology which was measured by Mathematics and Technology Attitude Scale (MTAS). In this part, some research studies related to the use of technology (e.g. CAI, DGS) in mathematics teaching and its effect on students' attitude towards mathematics will be presented.

One of these research studies was conducted by Yousef (1997) to investigate the effects of using the GSP on the high school students' attitudes towards geometry. One of the results of his study indicated that the scores of the pretest and posttest of

the students in the experimental group were significantly different. Another result indicated that there was a significant difference between the control and experimental groups in the gain of the scores from the pretest to the posttest.

Another quasi-experimental pretest-posttest control group design research study was carried out in a secondary school by Idris (2007) with 65, Form Two students to investigate the effects of using The Geometer's Sketchpad (GSP) on students' achievement in geometry, the van Hiele Level, and to students' views on learning geometry with GSP. The questionnaire and checklist were administered to explore the students' response towards the use of the Geometer's Sketchpad in learning of geometry. The descriptive analysis results indicated that the most of the students showed positive reactions toward using this software in learning of geometry and agreed that the GSP is a useful tool for learning of geometry.

Similarly, Baki and Özpınar (2007) performed another research study to investigate the effects of Computer-Based Instruction using Logo on students achievement, retention level, and attitude towards mathematics. Throughout the treatment, a Logo-based instructional material for 6th grade was designed and implemented in a primary school. While 35 students in control group were taught without computer-based activities, 33 students in experimental group were taught with computer-based activities for six lessons. At the end of the study, semi-structured interview was conducted to get the students' views on this application. The study results indicated that the Computer-Based Instruction using Logo affected students' attitude towards mathematics positively more than the traditional instruction affected. Similarly, Sulak and Allahverdi (2002) and Özdemir and Tabuk (2004) also found in their studies that the Computer-Assisted Instruction affected students' attitudes towards mathematics positively.

Besides, Pilli (2008) carried out a study with the purpose of examining the effects of the computer software *Frizbi Mathematics 4* on 4<sup>th</sup> grade students' mathematics achievement, retention, attitudes toward mathematics and attitude toward computer assisted learning. A series of ANOVAs for repeated measures revealed significant difference between the groups on the post achievement tests and attitude scales in favor of experimental group.

Similar to the study of Yousef (1997), Yemen (2009) also conducted a study with 50, 8<sup>th</sup> grade students to investigate the effects of Technology-Assisted Instruction using the Geometer's Sketchpad, on 8<sup>th</sup> grade students' achievement in analytical geometry and attitudes toward mathematics. The study results indicated that the use of DGS had no significant effect on students' attitude towards mathematics.

## 2.5 Transformation Geometry

The subject of transformation geometry as a sub-learning area of geometry consists of the motions of translation, reflection and rotation (Karakuş, 2008; Pleet, 1990). According to Klein (1870), the transformational geometry is the basic subject of learning geometry (as cited in Junius, 2002). Similarly, Boulter (1992) stated that transformational geometry consists of mental, graphical or physical motions of two- or three-dimensional geometrical shapes. These motions can be expressed like: slides (translation), flips (reflections), and turns (rotations) as given in the Figure 2 (Boulter, 1992).

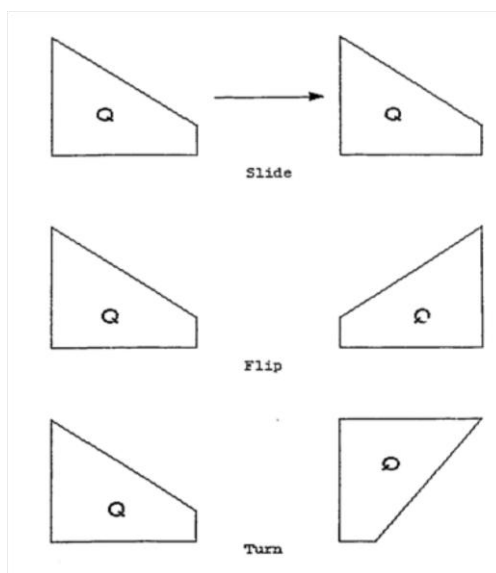


Figure 2. Simple rigid transformation (Boulter, 1992, p.4)

In addition to abovementioned information, Poincaré (1913) remarked that Geometry aims at studying a particular group; and that general group concept preexists potentially in the mind of an individual. He also alleged that mathematical

group is a set which satisfies associatively, identity and inverse and there are mathematical group structures which take place in our minds. The idea of group structure which preexists in our minds may sound strange. However, if you think groups as transformations such as rotations and reflections and if you link them with Poincaré's (1913) notion of motor space and motion of solids as the true source of geometry, it become more understandable (as cited in Junius, 2002).

The features of geometric objects and properties of transformations should not be considered independently from each other (as cited in Bouckaert, 1995). Instead, they should be thought as relating properties with each other in order to provide gradual learning of how to prove. It was also stated that the symmetries or automorphisms can be defined as a concept which is used to establish a connection between the features of objects and the properties of transformations. Moreover, it was described as transforming an object into itself with considering its structure (as cited in Bouckaert, 1995). Transformation geometry, which can be characterized as the study of geometric objects in the plane, links the properties of transformations to the properties of objects. In addition, geometric transformations provide discovering and/or proving characteristics of geometric objects; forming patterns like friezes, rosettes, wallpapers; classifying geometric objects; perceiving the chirality of an object (as cited in Bouckaert, 1995).

The application of transformation geometry can be seen in many areas in the literature. For instance, Pumfrey and Beardon (2002) states that the Art goes in harmony with Mathematics over the centuries. Knuchel (2004) pointed out that, considering the tessellations which were the products of Islamic civilization and brought to Europe by Arab conquests in the thirteenth century, the connection can be seen clearly since they were composed of the rotation, reflection and translation of the objects in a plane such that there are no gaps or overlaps. Pumfrey and Beardon (2002) also claimed that tessellation is a common feature of decorative art and can be frequently encountered around us. Also, the questions of how patterns are made of and how objects move in the space can be clarified with the motions of translation, reflection and rotation. Thus, elementary mathematics curriculum should give greater importance to Transformational Geometry.

Gürbüz (2008) stated that the students should be able to construct patterns using equal polygonal regions and make tessellations with the activities of cutting, folding and sticking papers while they learn the subject of transformation geometry. Thus, students can discover the relationship among the geometric shapes by constructing, drawing, measuring, visualizing, comparing, changing the shapes and classifying them and they develop spatial intuition through this activities (Gürbüz, 2008).

California State Department of Education (1985) mentioned that in order to provide better understanding of the geometric concepts, such as congruence, similarity, parallelism, symmetry and perpendicularity, instruction of geometry should utilize transformations in the plane such as reflections, translations, and rotations (as cited in Pleet, 1990). Similarly, Harper (2002) claimed that transformation geometry topic is an important topic which should be taken part in the K-12 mathematics curriculum. Particularly, for the students between the grades of 9<sup>th</sup> and 12<sup>th</sup>, transformations should be used as a significant tool in solving geometric and non-geometric problems (Harper, 2002).

Knuchel (2004) also stated that the learning of symmetry as a sub-learning area of transformation geometry subject has a crucial role for elementary school students because it provides them with making sense of the facts around them in a different context and creating their own patterns. Moreover, she mentioned that the transformational geometry brings the life and mathematics together in a concrete and meaningful way. It is important for students to comprehend the concepts of geometry and symmetry through the way which makes them think that everything they see around them has a strong foundation in mathematics, even if it is not directly related to it.

Boulter (1992) stated that in order to provide students with conceptual understanding of transformational geometry topic, instructors must create an environment where the motions such as those in Figure 2, can be simulated. Further, he added that various teaching methods should be used while teaching the topic transformational geometry. Put it differently, individual differences among the students should be taken into account while teaching this topic. Instructors should

lead students and provide the relations clearly since it is important in terms of constituting conceptual understanding and reasoning for students. Thus, teaching the topic of transformation geometry with using Dynamic Geometry Software (GeoGebra) can help to achieve the abovementioned goals in the present study.

As mentioned above, the transformational geometry is considerably important and essential sub-learning area of geometry in K-12 mathematics curriculum (Desmond, 1997). Furthermore, both students and pre-service/in-service mathematics teachers have difficulties in understanding the motions of reflection, rotation, and translation (Desmond, 1997; Edwards & Zazkis, 1993; Law, 1991). This is one of the reasons for investigating the instruction of transformation geometry in the present study.

### **2.5.1 Fractals**

Fractal is another attracted and substantial sub-topic of transformational geometry for the students. Mandelbrot (1991) stated the two main roles of fractal geometry as describing the geometry of nature and the geometry of chaos. Mandelbrot (1977) also clarified the origin of the term of fractal as follows (p.1):

I coined the term fractal from the Latin adjective *fractus*. The corresponding Latin verb *frangere* means “to break”: to create irregular fragments. It is therefore sensible- and how appropriate for our needs! - That, in addition to “fragmented” (as in *fraction* or *refraction*). *Fractus* should also mean “irregular”, both meanings have been preserved in fragment (as cited in Miller, 1998).

According to Briggs and Peat (1989), the first fractal samples were encountered over a hundred years ago (as cited in Miller, 1998). These strange shapes, which could not be identified by the traditional Euclidean concepts of shapes, lines and calculus, were constructed by using new recursive or iterative technique (Jones, 1993; Stewart, 1996). Mathematicians panicked at the end of the nineteenth century due to these shapes (Jones, 1993) and avoided from these shapes said Miller (1998). Frame and Manderlbrot (2002) defined these shapes as “monsters shapes” (p.12).

Bovill (1996) asserted that the fractal geometry is the study of geometrical shapes which are never-ending, self similar, meandering cascade when zoomed in. Fraboni and Moller (2008) stated that the fractals are self similar across different scales, that is to say, fractal is a geometric pattern that is repeated at ever smaller scales to produce irregular shapes and surfaces that cannot be presented classical Euclidean Geometry and it is a shape which consists of small copies of itself. This makes the fractals different and more appealing than other Euclidean figures. Comparison of Euclidean Geometry and Fractal Geometry were presented in Table 1 (Pietgen & Saupe, 1988, p.26).

Table 1. Comparison of Euclidean Geometry and Fractal

<b>EUCLIDEAN</b>	<b>FRACTAL</b>
<b>Traditional (&gt;2000yr)</b>	<b>Modern monsters (~10yr)</b>
Based on characteristic size	No specific size or scaling
Suits manmade objects	Appropriate for natural shapes
Described by formula	(Recursive) algorithm

Source: Pietgen & Saupe, 1988, p.26.

As mentioned in the Table 1 above, Euclidean Geometry have existed for more than 2000 years. However, the fractal geometry, which have existed for approximately 10 years, is much more newer than the Euclidean Geometry. Also, euclidean geometry is based on characteristic size, suits manmade objects and can be described by formula while the fractal geometry has not got specific size or scaling. Also, fractal geometry is merely appropriate for natural shapes and can be defined by an algorithm. In addition to the abovementioned expressions, Miller (1998) stated that fractal geometry is a lot richer than Euclidean geometry in point of the lines, shapes, objects in nature, patterns and forms compared to Euclidean geometry. Furthermore, Yazdani (2007) expressed that fractals differ from classical geometry in terms of its beauty and impressiveness. Classical or Euclidean geometry have been working for the development of mathematics, science, and engineering for centuries.

However, as to the ordinary events and shapes surrounding us, it has failed. Complicated rough objects, irregular lines, such as mountains and clouds, could not be explained by classical geometry. That is why fractal geometry is an extremely important topic. For instance, Mandelbrot (1977) mentioned that:

Why is geometry often described as “cold” and “dry”? One reason lies in its inability to describe the shape of a cloud, a mountain, a coastline, or a tree. Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in straight lines (as cited in Miller, 1998).

As opposed to Mandelbrot, Galileo (1975) ignored nature’s true shapes as they are irregular and, hence, inapprehensible. Galileo (1975) stated that (p. 241):

Lines are called regular when, having a fixed and definite description, they are susceptible of definition and of having their properties demonstrated. Thus the spiral is regular, and its definition originates in two uniform motions, one straight and the other circular. So is the ellipse, which originates from the cutting of a cone or a cylinder. Irregular lines are those which have no determinacy whatever, but are indefinite and casual and hence indefinable; no property of such lines can be demonstrated, and in word nothing can be said about them (as cited in Miller, 1998).

As it was stated above, while Galileo states that irregular lines cannot be defined since they do not have any features, Mandelbrot has managed to define them and caused us to understand them. Thanks to Mandelbrot’s invention of fractal geometry, it became possible to explain the complex rough objects, irregular lines, forms, patterns as well as smooth ones such as snowflakes, ferns, coastlines, mountain ranges, tree branches, river-bed patterns, clouds, and so on (Miller, 1998).

Similarly, Kröger (2000) mentioned that, in recent years, the concept of fractal geometry has become famous in natural sciences and it has been used to describe different phenomena as plant growth, the description of turbulence, the shape of mountain, clouds, mixture of liquids, the shape of brain tumors or lungs, models of economy, or the frequency of occurrence of letters and words. Frame and Mandelbrot (2002) stated the importance of fractals as follows (p.12):



.....fractal geometry is rich in open conjectures that are easy to understand, yet represent deep mathematics. First, they did not arise in earlier mathematics, but in the course of practical investigations in diverse natural sciences, some of them are old and well established, others are newly revived, and a few are altogether new. We feel very strongly that those fractal conjectures should not be reserved for the specialists, but should be presented to the class whenever possible.

Yazdani (2007) stated that the objects in nature such as ferns, snowflakes, coastlines, and mountains have formed more complicated geometric figures. He also stated that the fractal geometry is essential in terms of curricular considerations since it inspires the concepts of geometry taught in K-12 and high school mathematics curriculum. According to Yazdani (2007), students should be asked to discover various objects in nature which do not seem to be composed of polygons, lines, circles, or square so that they can realize how mathematics is related with the real life events during the activities in mathematics classrooms.

Frame and Mandelbort (2002) claimed that the fractal geometry needs simulation and visualization more than it needs the proof. Furthermore, the fractals can make mathematics more interesting and fun without breaking the rules of mathematical proofs. Fraboni and Moller (2008) stated in their another study that offering mathematical ideas using fractals may give a new impulse to the classroom environment. They went forward with stating that fractal geometry makes students develop new point of view on their understanding of mathematical concepts and encourages their creativity in problem solving. Students can make sense of some topics such as symmetry, number sequences, ratio and proportion, measurement, and fractions through fractal geometry. In addition, Fraboni and Moller (2008) stated that fractal geometry offers teachers great flexibility since its instruction can be modified according to the level of the students and to the time restrictions.

In the light of the study results that were mentioned above, the topic of fractal is an extremely important sub-learning area in teaching of mathematics. Besides, both students and pre-service/in-service mathematics teachers have difficulties in understanding the fractal geometry since it is a difficult topic to apprehend. These

statements may be considered the main reason for investigating the instruction of fractal geometry in the present study.

## **2.6 Summary of the Literature Review**

Technological tools, such as Graphic Calculators, Interactive White Boards, Computers, Web-Based Applications, Dynamic Mathematics/Geometry Softwares, are started to widely use in mathematics classroom and many studies indicated that the technology use in mathematics education is an effective and essential tool in the learning and teaching of mathematics. The common opinion of many researchers, mathematics teachers, and studies focus on the notion that the technology integration into mathematics education is essential and capable of making the teaching of mathematics more effective. (Baki, 2001; Borwein & Bailey, 2003; Doğan, 2012; Ersoy, 2003; Hollebrands, 2003; Koehler & Mishler, 2005; Lester, 1996; NCTM, 2000).

Computers are one of the mainly used technologies in learning environments. Thus, computer use in mathematics classrooms has been expanding owing to the positive effects of Computer-Assisted Instruction. Many studies existing in the mathematics education literature indicated the positive effects of Computer-Assisted Mathematics Instruction on students' mathematics learning and mathematics achievement (Akkoç, 2008; Aktümen ve Kaçar, 2003; Altın, 2012; Andiç, 2012; Baki, 2002; Baki, Güven & Karataş, 2004; Balkan, 2013; Çoban-Gökkaya, 2001; Hangül, 2010; Helvacı, 2010; İpek & Akkuş-İspir, 2010; Sulak, 2002; Şataf, 2010; Şen, 2010; Tayan, 2011).

Dynamic Geometry Softwares (DGS), as a technological tool in learning environment, offers students useful facilities for using both computer algebra system and a dynamic geometry software and enhance students' understanding of mathematics. (Hohenwarter & Lavicza, 2009). Research studies on DGS focus on the idea that DGS facilitate and support students' understanding of mathematics, help students to visualize abstract mathematical concepts, and test mathematical ideas in a dynamic learning environment (Baki, Kosa & Güven, 2011; Christou, Mousoulides, Pittalis & Pitta-Pantazi, 2004; Fahlberg-Stojanovska & Trifunov, 2010; Gawlick,

2002; Gillis, 2005; Goldenberg & Couco, 1998; Güven, Baki & Çekmez, 2012; Habre, 2009; Hollebrands, 2003; Hölzl, 1999; Jones, 2001; Laborde, 2001; Marrades & Gutiérrez, 2000; Mariotti, 2000; Pandiscio, 2010; Pandiscio, 2010; Stols & Kriek, 2011).

GeoGebra, as a dynamic geometry software, not only provides students with facilities to experiment the mathematical ideas and to associate mathematical concepts with the real life examples, but also helps students to examine the relation between algebraic and geometrical concepts better (Hohenwarter & Jones, 2007). GeoGebra has also the potential to encourage the student-centered learning, active student participation, collaborative learning, and discovery learning by experimenting mathematical ideas, theorems and using interactive explorations (Preiner, 2008). Many studies on the effectiveness of GeoGebra indicated that GeoGebra has the positive effects on students' mathematics learning, mathematics achievement and attitude towards learning mathematics through dynamic mathematics softwares (Ayub & Tarmizi (2010) Bilgici & Selçik, 2011; Furkan, Zengin & Kutluca, 2012; Filiz, 2009;. İçel, 2011; Saha, Zengin, 2011).

The van Hiele Model of Geometric Thinking Levels is helpful for describing the development of students' reasoning in geometry and predicting students' achievement in geometry at the levels ranging from elementary level to the college level (Burger & Shaughnessy, 1986; Fuys et al., 1988; Han, 1986; Hoffer, 1983; Usiskin, 1982; Wirszup, 1976). The results of various research studies which were conducted to discover the implications of the van Hiele Theory for current K-12 geometry curriculum in dynamic learning environments indicated that Dynamic Geometry Softwares (DGS) help students to progress between the geometric thinking levels and increase their geometric thinking levels (Chang, Sung & Lin, 2007; Clements, Battista & Sarama, 2001; July, 2001; Meng & Idris, 2012; Moyer, 2003; Öztürk, 2012; Parsons, Stack & Breen, 1998; Toker, 2008; Tutak & Birgin, 2009; Idris, 2007).

Research studies related to the effects of Dynamic Geometry Software (DGS) use in mathematics education on students' attitude towards learning mathematics with dynamic geometry softwares indicated the positive effects of DGS on students'

attitude towards learning mathematics in a dynamic learning environment (Baki & Özpınar, 2007; Idris, 2007; Pilli, 2008; Yousef, 1997).

The Transformation Geometry is a considerably important and essential sub-learning area of geometry in K-12 mathematics curriculum (Desmond, 1997). California State Department of Education (1985) mentioned that in order to provide better understanding of the geometric concepts, such as congruence, similarity, parallelism, symmetry and perpendicularity, instruction of geometry should utilize transformations in the plane such as reflections, translations, and rotations (as cited in Pleet, 1990). The transformational geometry consists of understanding the mental, graphical or physical motions of two- or three-dimensional geometrical shapes which are extremely important for success in geometry (Boulter, 1992).

Fractal is another attracted and substantial sub-topic of transformational geometry for the students. The fractal geometry is essential in terms of curricular considerations since it inspires the concepts of geometry taught in K-12 and high school mathematics curriculum (Yazdani, 2007). Offering mathematical ideas using fractals may give a new impulse to the classroom environment and the fractal geometry may help students to develop new point of view on their understanding of mathematical concepts and encourages their creativity in problem solving. Students can make sense of some topics such as symmetry, number sequences, ratio and proportion, measurement, and fractions through fractal geometry (Fraboni & Moller, 2008).

In the light of the related studies that were mentioned above, there are few studies which provide insight into the teaching of transformation geometry and fractals in a dynamic learning environment and its effect on students' geometric thinking and achievement. Thus, this study aimed at investigating the effects of dynamic geometry software (GeoGebra) on students' mathematics achievement in transformation geometry, geometric thinking and attitude towards mathematics and technology. Besides, the related literature provided significant information for choosing the appropriate research design, data collection instruments, and the statistical data analysis procedure for the objectives of the study.

## CHAPTER 3

### METHODOLOGY

This chapter presents information about the research design, the sampling procedure, the population and the sample group, the data collection procedures and instruments, the reliability and validity of the instruments, the design of the instruction, the analysis method of the data collected, the teaching and learning materials, treatment, and lastly the internal and external validity issues of the study.

#### 3.1 The Research Design

The aim of this study was to investigate the effects of the Dynamic Geometry Software-Assisted Instruction GeoGebra on 8<sup>th</sup> grade students' mathematics achievement in transformation geometry (fractals, rotation, reflection, translation), Geometric Thinking and Attitude Towards Mathematics and Technology. In the present study, the cause-and-effect relationship was investigated. However, random assignment was not used to form the groups since the two already-existing groups were used to compare. Therefore, the present study was a *weak experimental design study* which has a research design of *the Static-Group Pretest-Posttest design* to test the hypotheses of the study (Fraenkel, Wallen & Hyun 2011). The research design of the study is summarized in Table 2.

Table 2. Research design of the study

Group	Pretest	Treatment	Posttest
EG	MAT, VHL, MTAS	DGSI	MAT, VHL, MTAS
CG	MAT, VHL, MTAS	RI	MAT, VHL, MTAS

**EG:** Experimental Group

**CG:** Control Group

**DGSI:** Dynamic Geometry Software-Assisted Instruction

**RI:** Regular Instruction

**MAT:** Mathematics Achievement Test

**VHL:** van Hiele Geometric Thinking Level Test

**MTAS:** Mathematics and Technology Attitude Scale

### 3.2 Population and Sample

In the present study, convenience sampling method was used. The researcher chose a private elementary school for the implementation since there was a group of students to study with, a mathematics teacher to implement the treatment, and a computer laboratory enabling students to work with computers using DGS available for the study at this school. The students were also available for a basic GeoGebra training by the researcher for one week before the study began. Hence, the sample was conveniently available for the study. The school was also chosen due to its suitable technological infrastructure. There was a computer laboratory which had 25 computers, a projector and a smart board at this school. These technological devices were needed during the study. The school was located in a university campus and had 600 students in total. In this school, there were three already-existing 8<sup>th</sup> grade classrooms which were formed according to the Placement Test of the school. However, two of these classrooms were selected as the sample of the study since the students in these classrooms had similar mathematics achievement levels according to their previous mathematics grades and placement test results. Besides, the pretests were conducted to determine whether the groups were equal in terms of dependent variables of the study. Thus, the present study was conducted with these two 8<sup>th</sup> grade classrooms (8-B and 8-C), with 34 students in total.

Each of these classrooms was chosen as the experimental group and the control group randomly. The classroom of 8-B was chosen as the experimental group and 8-C as the control group in the present study. There were 17 students in both classes. Both classes in which the study was conducted had equal classroom settings and conditions except computers with DGS in the experimental group's learning environment. The number of the subjects in each group is presented in Table 3 below.

Table 3. Distribution of the subjects in terms of the group and gender

Gender	Experimental Group (Dynamic Geometry Software-Assisted Instruction)	Control Group (Regular Instruction)	Total	Percentage
Female	8	9	17	50%
Male	9	8	17	50%
Total	17	17	34	100%

The experimental group (8-B with 17 students) were instructed by Dynamic Geometry Software-Assisted Instruction (supported with GeoGebra activities), and the control group (8-C with 17 students) were instructed by Regular Instruction. The experimental group was instructed by the researcher and the control group was instructed by the mathematics teacher of the classroom.

All 8<sup>th</sup> grade private elementary school students in Çankaya district/Ankara were identified as the target population of the study. This was the population to which the results of the study were generalized. All 8<sup>th</sup> grade students (51 students in three eight grade classes) at the school in which the study was conducted was the accessible population. In other words, the two already existing classes of the 8<sup>th</sup> grade students (8-B and 8-C) at this private elementary school in Bilkent district/Ankara were used as the sample of the present study.

As regards the major characteristics of the sample group comprising the 34, 8<sup>th</sup> grade students, their age ranged between 14 and 15, and they had a high socio-economic status. Since, in their former years, the students had taken various

Informatics Technologies courses, which made them Computer literate, it was assumed that all 8<sup>th</sup> grade students of the school had a minimum required knowledge of computer use, which they used during the treatment.

### **3.3 The Data Collection Instruments**

In the present study, a quantitative research methodology was used. In order to collect data, three instruments were used; Mathematics Achievement Test (MAT), van Hiele Geometric Thinking Level Test (VHL), and Mathematics and Technology Attitude Scale (MTAS). All of these three instruments' pilot studies were conducted by the developers of the instruments, and the reliability and validity issues were addressed. All these issues are discussed below.

#### **3.3.1 The Mathematics Achievement Test (MAT)**

The Mathematics Achievement Test (MAT) was developed by Akay (2011) and was used to address students' achievement in Transformation Geometry (fractals, rotation, reflection, translation) (See Appendix A).

The rationale for the selection of the MAT was that this instrument aims at measuring the variable (Mathematics Achievement of the students) that the researcher intended to measure in the present study. Also, this instrument's content and objectives were appropriate for the study since it included 14 open-ended questions on the topic of Transformation Geometry prepared with the consideration of the objectives (See Appendix B and D) given in the curriculum published by the Ministry of National Education (2009-2010). These questions were related to the objectives that the researcher wanted to investigate in the study. Moreover, this achievement test was used in an experimental study (Akay, 2011), which investigates the effect of a different teaching method (peer instruction method) on students' mathematics achievement in transformation geometry. As the present study also investigated the effects of a different teaching method (DGS-Assisted Instruction using GeoGebra) on students' mathematics achievement, this instrument was appropriate for the study. The scoring of MAT was done according to the rubric



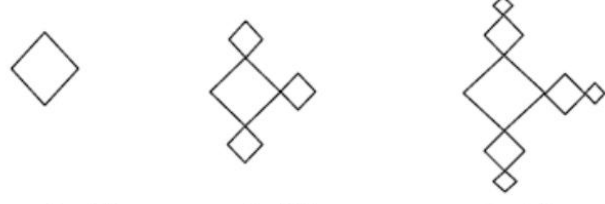
which was prepared by the developer of the instrument (Akay, 2011) (See Appendix C). The possible scores of the MAT ranged between 0 and 78.

This instrument was used as a pretest one week before the beginning of the study to determine whether the students in the experimental and control groups differed from each other in terms of academic achievement. Also, MAT was implemented as a posttest to both groups one week after the intervention was completed.

In addition, the reliability and validity issues of MAT were examined by the developer of the instrument. The inter-rater coefficient was calculated as .98 for the pilot study and the same coefficient was calculated as .99 for the main study. Therefore, it can be said that reliability values of the MAT, for its implementation in both the pilot and main study, were high, which is an indication of reliability. More specifically, in the present study, the reliability value of MAT was calculated as .94. Below is a sample question from MAT and the corresponding objective which is aimed at measuring by means of the the question.

**Objective 1:** Students should be able to construct and draw patterns with line, polygon and circle models and decide which patterns are fractals.

1.



1.şekil 2.şekil 3.şekil 4.şekil

a) Yukarıda ki şekiller, 1.şeklin orantılı olarak küçültülmüş ya da büyütülmüş halleri ile inşa edilmiş, her adımda aynı kural uygulanmış bir örüntü müdür (fraktal)? Cevabınızı açıklayınız.

b) Aynı kural devam etseydi bu örüntüde ki 4.şekil nasıl olurdu yukarıya çiziniz.

c) Çizdiğiniz 4.şekilde kaç eşkenar dörtgen vardır?

Figure 3. Sample question from Mathematics Achievement Test (MAT) aiming at measuring Objective 1

### **3.3.2 van Hiele Geometric Thinking Level Test (VHL)**

The van Hiele Geometric Thinking Level Test (VHL), including 25-multiple choice items developed by Usiskin (1982) and translated into Turkish by Duatepe (2000a), were used to determine students' van Hiele Geometric Thinking Levels (See Appendix E). The reliability and validity issues of the VHL were examined by Duatepe (2000a), and the Cronbach Alpha reliability measures were found as .82, .51, and .70, for the first, second, and third level, respectively. In the present study, the reliability values of the MAT were calculated as .80, .49, and .68 for the first, second, and third level, respectively.

The van Hiele Geometric Thinking Level Test (VHL) was conducted to both groups as a pretest one week before the beginning of the study to determine whether the students in the experimental and control groups differed from each other in terms of geometric thinking. The VHL was also implemented as a posttest to both groups one week after the intervention was completed.

The rationale underlying the selection of VHL was that this instrument has 25 questions which aim at determining the same variable the researcher intended to measure in the study, students' geometric thinking levels. Also, this instrument can measure specific skills, such as ordering the properties of the figures, identifying and comparing the figures, and deduction, which constitute geometric thinking levels of the students.

In van Hiele Geometric Thinking Level Test, there are five levels which are represented by certain items. The first five items represent level 1, the second five items represent level 2, the third five items represent level 3, the fourth five items represent level 4, and the last five items represent level 5. According to van Hiele, primary school mathematics enables students to reach only the third level (van Hiele, 1986). Therefore, only the first 15 questions were considered in this study. The students' geometric thinking was investigated based on the students' scores on the van Hiele Geometric Thinking Level Test which was prepared according to the van Hiele Theory. Each question in the VHL was assessed by giving one for each correct

answer and zero for each incorrect answer. Since the first 15 questions of the test were considered in the study, possible scores of the VHL ranged between 0 and 15.

The questions in the first level are related to identifying triangles, rectangles, squares, and parallelograms. The questions in the second level are about the properties of squares, rectangles, diamonds, rhombuses, isosceles triangles, and radius and tangent of the circle. The questions in the third level are on ordering properties of triangles, simple deduction, comprehending hierarchy among squares, rectangles and parallelograms, and comparing rectangle and parallelograms (Duartepe, 2004). The objective of each question of the VHL is presented in Appendix F.

Below is a sample question from VHL and the corresponding objective, which it aimed to measure:

**Question: 6**

**Representing van Hiele Level: 2**

**Objective:** Comprehend properties of a square

6- PQRS bir karedir.  
Aşağıdakilerden hangi özellik her kare için doğrudur?

a) [PR] ve [RS] eşit uzunluktadır.  
b) [OS] ve [PR] diktir.  
c) [PS] ve [OR] diktir.  
d) [PS] ve [OS] eşit uzunluktadır.  
e) O açısı R açısından daha büyüktür.

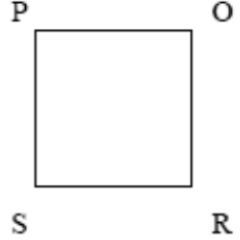


Figure 4. Sample question from van Hiele Geometric Thinking Level Test (VHL) aiming at measuring the properties of square

### 3.3.3 Mathematics and Technology Attitudes Scale (MTAS)

The Mathematics and Technology Attitudes Scale (MTAS), which was developed by Barkatsas et al. (2007) and translated into Turkish by Boyraz (2008), was used to determine students' attitudes toward learning mathematics with technology (See Appendix G). The MTAS was conducted to both groups as a pretest one week before the beginning of the study to determine whether the students in the

experimental and control groups differed from each other in terms of attitude towards learning mathematics with technology. The MTAS was also implemented as a posttest to both groups one week after the intervention was completed. Moreover, this instrument was appropriate for the present study since the MTAS served the purposes and the objectives of the present study with its content.

The Mathematics and Technology Attitudes Scale (MTAS) consists of 20 items and five subscales which are mathematical confidence [MC], confidence with technology [TC], attitude towards learning mathematics with technology [MT], affective engagement [AE] and behavioral engagement [BE]. These subscales were investigated to reveal students' attitudes toward learning mathematics with technology. In this attitude scale, the students were asked to indicate the extent of their agreement with each statement, on a five-point scale from strongly agree to strongly disagree (scored from 5 to 1). A different but similar response set were used for the BE subscale. A five-point system was again used: nearly always, usually, about half of the time, occasionally, hardly ever (scored again from 5 to 1). Since the aim was to measure different attitudinal and behavioral characteristics using the same scale, two different rating systems were used in MTAS.

The reliability issue was addressed by Boyraz (2008) and the internal reliability, as measured by calculating the Cronbach alpha coefficient (Boyraz, 2008), for each section in the test was found to be *Mathematical confidence*: .85; *Attitude towards learning mathematics with technology*: .87; *Confidence with technology*: .78; *Behavioral engagement*: .73 and *Affective engagement*: .66. In the present study, the Cronbach alpha coefficients were calculated for each subscale as *Mathematical confidence*: .81; *Attitude towards learning mathematics with technology*: .82; *Confidence with technology*: .75; *Behavioral engagement*: .72 and *Affective engagement*: .64. Possible scores of the MTAS ranged from 20 to 100. Sample items which represent the subscales of MTAS are presented in Table 4.

Table 4. Sample items representing the five subscales of MTAS

Subscale	Item sample
Mathematical confidence [MC]	I know that I can cope with difficulties in mathematics.
Confidence with technology [TC]	I am good at using computers.
Attitude towards learning Mathematics with technology [MT]	I like using mathematics softwares in learning of mathematics.
Affective engagement [AE]	I can take good grades on mathematics.
Behavioral engagement [BE]	I try to answer to the questions which teacher asks.

### 3.4 Variables

In experiments, the independent variable was the variable that was controlled and manipulated by the researcher, whereas the dependent variable was not manipulated. Instead, the dependent variable was observed or measured for variation as a presumed result of the variation in the independent variable (Fraenkel, Wallen & Hyun 2011). The variables in this study were classified as independent and dependent variables. Classification of those variables are presented in Table 5 below.

Table 5. Classification of the variables of the study

Name	Type of variable	Type of value
Posttest score on Mathematics Achievement Test	Dependent	Continuous
Posttest score on van Hiele Geometric Thinking Level Test	Dependent	Continuous
Posttest score on Mathematics and Technology Attitudes Scale	Dependent	Continuous
Treatment	Independent	Categorical

### **3.4.1 Independent Variable**

The independent variables of the study were the treatments (instruction methods) implemented, and had two categories as Dynamic Geometry Software-Assisted Instruction using GeoGebra and Regular Instruction (without using any dynamic mathematics software). These variables were considered as categorical variables and measured on a nominal scale. Besides, the researcher asked the teacher of the control group not to use any technological tools (graphics calculators, projector etc.) that support or facilitate the learning process. Therefore, independent variables which were instructional methods were controlled.

### **3.4.2 Dependent Variable**

Dependent variables of the study were the students' posttest scores on the Mathematics Achievement Test (as measured by POSTMAT), the van Hiele Geometric Thinking Level Test (as measured by POSTVHL), and the Mathematics and Technology Attitudes Scale (as measured by POSTMTAS).

All of these variables were interval and continuous. The possible minimum and maximum scores ranged from 0 to 78 for the POSTMAT, 20 to 100 for the POSTMTAS, and 0 to 15 for the POSTVHL, respectively.

## **3.5 Procedures**

The aim of this study was to investigate the effects of the Dynamic Geometry Software-Assisted Instruction using GeoGebra compared to Regular Instruction (traditional textbook-based instruction) on the 8<sup>th</sup> grade students' mathematics achievement in transformation geometry (fractals, rotation, reflection, translation), Geometric Thinking and Attitude Towards Mathematics and Technology.

The study was conducted during the fall semester of the 2012-2013 academic year in a private elementary school in Ankara/TURKEY. The time schedule for the lessons and lesson plans were prepared, and the purpose and procedure of the study were explained to the participants before the study began.

For this study, GeoGebra was used as a dynamic geometry software in the experimental group. Both groups were instructed for a time span of three weeks (ten class hours in total for each group) and taught the same content to reach exactly the same objectives in the cognitive domain with different teaching methods. There were five hours of mathematics lessons in each week, and each lesson hour lasted 40 minutes in both groups. The experimental group students learned transformation geometry topics with GeoGebra, whereas the control group students learned the topics in a Regular Instruction Environment (in a computer-free, non-technologically equipped classroom), which was based on a textbook approach using chapters related to transformation geometry from the textbook prepared by the Ministry of National Education for the eighth grade students.

The researcher instructed the experimental group students in the computer laboratory but he was also present in the control group during the treatment to observe the teacher who instructed the students in a regular classroom. The mathematics teacher of the students in the control group took place during the instruction to the experimental group as an observer in order to check and confirm that the researcher as an instructor did not have any bias. The teacher took notes during all class hours. In both groups, the students were only guided in the activities and they constructed their own learning by following the steps in the activities. Also, to familiarize the EG students with the researcher, the researcher was present in the EG for one week prior to treatment and pretests.

Lesson plans, activity sheets and worksheets for each group were prepared based on the textbook which were developed by considering the objectives of the eight grade mathematics suggested by Ministry of National Education. The activities in the textbook were rearranged, prepared and done on GeoGebra in the experimental group, whereas the same activities were done on the blackboard in the control group. Both groups worked on the activity sheets and worksheets by paper-pencil. The same content, examples and questions were used in both groups to reach the same objectives. The only difference between the activities was the use of GeoGebra. Activity sheets were distributed to the students in the middle of each class hour.

Worksheets were given to the students at the end of the class hour for the purpose of assessment.

The prepared lesson plans were checked by a mathematics educator who is a faculty member, and two experienced mathematics teachers to determine whether they were mathematically correct and appropriate for achieving the objectives (i.e. putting the concepts and definitions given in order, recommendations that support the integrity of the lesson, supplementation of activities, and so on). According to their comments and recommendations, all lesson plans were revised to obtain a consistency between the objectives and content of the activities. Experimental group students were trained for the usage of basic tools of GeoGebra for week 1 (four class hours) in a computer laboratory which was technology equipped. They were taught the usage of the essential tools of the software and making the basic construction in GeoGebra using these tools, such as constructing a regular polygon, rotating an object around a point, or reflecting an object. The training session was done one week before administering the pretests.

One week after completing the GeoGebra training in the experimental group, MAT, VHL, and MTAS were administered as pretests to the EG and CG students one week before the treatment began. The same tests were administered to both groups as posttest to examine the effect of the DGS-Assisted Instruction using GeoGebra one week after the treatment session ended. MTAS and VHL were conducted during the first mathematics lesson of the week (in two lesson hours), and MAT in the second lesson of the same week (approximately in two lesson hours). Students were given one lesson hour to complete the MTAS, one lesson hour for the VHL, and two lesson hours for the MAT. All the students in both groups completed the tests on their own.

After the pretests were conducted to the groups, the students were instructed for three weeks. Afterwards, MAT, VHL, and MTAS were administered to the EG and CG students as posttests one week after the end of the intervention. The reason for the one week delay of the implementation of the posttests was the students' inconvenience owing to another subject's exam. There was a time span of four weeks between the implementations of the pretests and posttests.



An outline of the procedure of the study was given in Table 6 below.

Table 6. Outline of the procedure of the study

	Experimental Group	Control Group
Pretests	MAT, VHL, MTAS	MAT, VHL, MTAS
Treatment	Dynamic Geometry Software-Assisted Instruction (DGSI)	Regular Instruction (RI)
Posttests	MAT, VHL, MTAS	MAT, VHL, MTAS

**MAT:** Mathematics Achievement Test

**VHL:** van Hiele Geometric Thinking Level Test

**MTAS:** Mathematics and Technology Attitude Scale

**PREMAT:** Pretest score of Mathematics Achievement Test

**POSTMAT:** Posttest score of Mathematics Achievement Test

**PREVHL:** Pretest score of van Hiele Geometric Thinking Level Test

**POSTVHL:** Posttest score of van Hiele Geometric Thinking Level Test

**PREMTAS:** Pretest score of Mathematics and Technology Attitude Scale

**POSTMTAS:** Posttest score of Mathematics and Technology Attitude Scale

The researcher gave the same homework assignments to both groups after each lesson. These assignments were provided from the textbook. After the treatment session ended, the researcher administered the posttests to all the groups in order to elicit their understandings. The content of the weekly plans, their order and administration of the tests are summarized in Table 7.

Table 7. Content of the weekly plans, their order and administration of the tests

Week	Content of the week	Class hour
1 <sup>st</sup>	Administration of Pretests	3
2 <sup>nd</sup>	Fractals	4
	Translation through a line	
3 <sup>rd</sup>	Reflection through a coordinate axis	4
	Rotation around the origin	
4 <sup>th</sup>	Reflection with translation	2
5 <sup>th</sup>	Administration of Posttests	3

### **3.6 Treatment**

This part includes information about the description of the treatment for the experimental and control groups.

#### **3.6.1 Treatment in the Experimental Group**

The experimental group was instructed transformation geometry (fractals, rotation, reflection, translation) by the researcher by means of the DGS-Assisted Instruction using GeoGebra for three weeks, ten class hours in total. The lessons were held in the computer laboratory of the school which was fully technologically equipped. Each student studies single-handedly with a computer which had the software of GeoGebra. Activities related to the objectives, which the students used during the lessons, were sent to the students' computers in the computer lab before the lessons began. The treatment in the experimental group was based on the activities in GeoGebra. The activity sheets were prepared in a way that the teacher guided the students in order to make them explore their ideas in a dynamic geometry environment. Besides, the students conjectured and explored geometric concepts and ideas using GeoGebra software.

The students were given worksheets in the classroom sessions to ensure as much consistency as possible in the teaching of the unit. Since the experimental group students were trained for four class hours for the basic use of GeoGebra, they had no difficulty in working on activities on GeoGebra.

In the first few minutes of each lesson hour, the content of the lesson was introduced to the students. The students were asked about their expectations from the lesson and students' questions related to the topic were clarified. After the introduction of the topic, brief explanation about the lesson was made by the researcher. The students were asked some intriguing questions and were given some motivating information about everyday life related to the topic. They were asked what they knew about the topic. For instance, firstly, in the lesson in which fractals were studied, students were asked what they remembered about the patterns from their previous classes. Then, they discovered the difference between the fractals and

patterns by doing GeoGebra activities and activity sheets. The detailed lesson plans are given in Appendix J. Screenshots of the abovementioned GeoGebra activities are given in Figure 5 below.

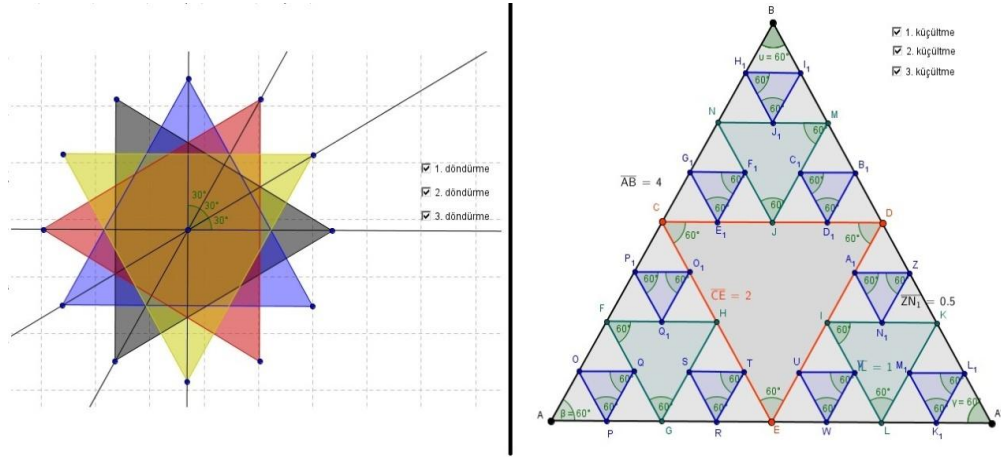


Figure 5. Screenshot from a GeoGebra activity related with the difference between fractal and pattern

In the activities which are given in Figure 5, firstly, the students discovered that the pattern, on the left-side, was constructed by rotating the blue, red, and yellow triangles clockwise around their centers by an angle of  $30^\circ$  with the aid of the explanations given by the researcher and their own manipulations, such as dragging or resizing the object. Subsequently, the students discovered that the fractal, on the right-side, was a pattern constructed with a shape's minimized or enlarged self-similar patterns. Thus, the students realized the difference between the fractal and pattern. Then, their understanding related to the targeted objective was examined through an activity sheet. In the activity sheet, they were asked to determine which shape was a pattern and which were fractals. Exercise samples from the activity sheet are presented in Figure 6.

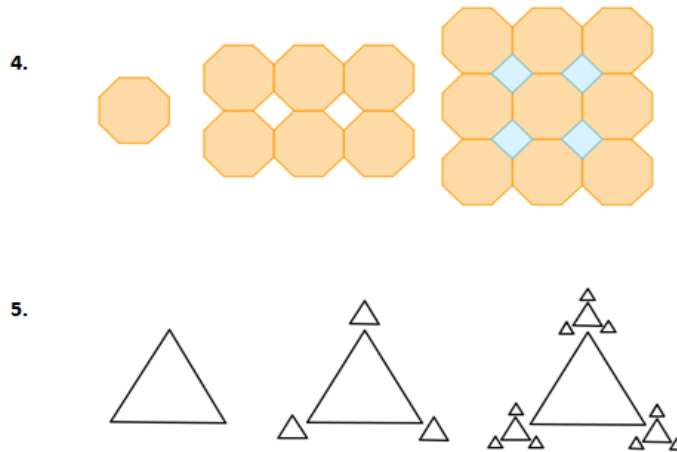


Figure 6. Screenshot from an activity related with the difference between fractal and pattern

The researcher as an instructor also utilized the projector during the treatment in the experimental group (e.g. both for showing real life examples and pictures from the nature to associate the topic with everyday life and introducing the GeoGebra activities to the students ). In this way, the students discovered how mathematics was related with daily life and they figured out the purpose of the activity. After the researcher stressed the important terms and mentioned the key points of the topic with a GeoGebra activity through the projector, the students were given directions to open specific GeoGebra activities from their computers (e.g. open the activity of “kılavuz-etkinlik-1”). The students followed the researcher’s instructions.

After a brief explanation for the activity, the students started to work on a specific GeoGebra activity. Students constructed, dragged and resized the objects which were displayed on the screen dynamically. They observed the results of the movements or manipulations they did. In this learning environment students created their own understanding of transformation geometry. In addition, the students were active participants in the learning process in that they were imagining, communicating, exploring and expressing their ideas. While the students were dealing with the activities, the researcher gave feedback on the students’ errors and guided them about their questions.

The researcher acted as a facilitator to make students develop transformation geometry concepts and guide them to reach targeted goals. Students were free to make observations, ask questions, and make conjectures in the lessons. Afterwards, the researcher distributed the activity sheets and asked the students to read the activities. Then, the students started to work on the activity sheets. The researcher never gave the correct answer to the students directly; he always tried to make students find the correct answers on their own through asking questions.

When each of the activities was completed by all the students, the answers of the questions were discussed in the class. The researcher not only checked all the students' answers and gave feedback related to their answers, but also made the students aware of the correct and incorrect answer by encouraging them. In this way, the researcher had an idea on the students' understanding, misunderstandings and errors. At the end of the lesson, the researcher distributed worksheets to the students as a mini quiz to elicit their understanding of the topic.

Class hours periods were 80 minutes consisting of two block lessons. At the end of each period, the teacher gave homework assignments to the students. Lesson plans, activity sheets, and worksheets used for the experimental group are presented in Appendix J and Appendix K. The design of a lesson hour in the experimental group is summarized in Table 8.

Table 8. The design of a lesson hour in the experimental group

Part of the lesson	Teacher Activity	Student Activity	Duration
Introduction	<p>Introduce the topic</p> <p>Ask intriguing questions and give motivating information about everyday life related to the topic</p>	<p>Express expectation from the lesson</p> <p>Listen to the explanations and key terms/definitions of the topic</p>	5 min.
Development	<p>Give students directions to open and work on the relevant GeoGebra activity</p> <p>Distribute the activity sheets</p>	<p>Work on the GeoGebra activity</p> <p>Fill in the activity sheet</p>	25 min.
Assessment	Distribute the worksheet	Fill in the Worksheet	5 min.
Closure	<p>Review the important parts of the topic</p> <p>Assignment for the next class</p>	Note the homework assignment	5 min.

Classroom environment of the experimental group and the students working on GeoGebra and activity sheet are shown in Figure 7 below.



Figure 7. Views from the experimental group classroom environment

### **3.6.2 Treatment in the Control Group**

The control group was instructed transformation geometry (fractals, rotation, reflection, translation) by the mathematics teacher of the classroom with the Regular Instruction for three weeks, ten class hours in total. The lessons were held in a regular classroom environment. The Regular Instruction was teacher-centered and based on a textbook which involved making use of chapters related to transformation geometry prepared by the Ministry of National Education for the eighth grade students. Instruction in the control group was mostly based on giving explanation, rules and the strategies about the topic which were needed to solve the questions. Moreover, the researcher was present as an observer during the treatment process in the control group.

The teacher's role in the control group was a knowledge transmitter for the students. The concepts were explained and their definitions were given to the students by the teacher and the teacher solved some examples on the blackboard by writing and drawing. Then, the teacher allowed students to take notes. The students in this group were passive participants in the learning process; that is, they were just responsible for listening to the teacher, taking notes and solving the problems the teacher asked.

After the teacher solved a few examples and gave the rules, the students were asked to solve similar questions to the examples. Sometimes, the teacher wrote exercises onto the board and called the students to solve them. These questions were from the students' textbook. The CG students worked on the same activity sheets and worksheets as those of EG students. All the exercises, questions, activity sheets, and the worksheets were the same as the ones in the experimental group. The only difference between the activities was the use of GeoGebra. The lessons were continued by solving the questions in the worksheets. The students in the control group were expected to listen to the teacher, take notes written on the blackboard and solve the exercises. At the end of each class period, the teacher gave the students a homework assignment from their textbook. The homework assignment was also the same as the one given to the students in the experimental group.

The lessons in the control group were held as follows: Information, rules or strategies were explained to solve the exercises, sample exercises were given, similar exercises were solved by the students and a homework assignment was given at the end of the class. The comparison of the instruction processes of the experimental and control groups is given in Table 9.

Table 9. The comparison of the instruction process of the experimental and control groups

Group	Environment	Roles of teacher	Roles of students
Experimental Group	Computer Laboratory	Introduce the topic and the purpose of the GeoGebra activity	Deal with GeoGebra, activity sheets, and worksheets
		Guide the students	
		Monitor the students' work	Discuss their work with the class
		Control the study environment	
Control Group	Regular Classroom Environment	Give information	Take notes
		Present the topic	Listen to the teacher and solve questions similar to those of the teacher
		Solve questions	

In this part, the data collection procedure and the treatment both in the experimental and control groups have been explained. In the following part, data analysis will be dwelled on.

### 3.7 Analysis of Data

Quantitative data analysis was used to analyze the data gathered through the Mathematics Achievement Test (MAT), the van Hiele Geometric Thinking Level Test(VHL), and the Mathematics and Technology Attitude Scale (MTAS). The data



analysis of the study was done using the PASW Statistics 18 program. The data obtained were analyzed using descriptive and inferential statistical analyses.

In descriptive statistics, the mean, median, minimum and maximum test scores, standard deviation, skewness and kurtosis values of the pretest and posttest scores of the dependent variables were computed for both experimental and the control group. Box plots and histograms were also used in order to investigate the general characteristics of the sample. In inferential statistics, in order to investigate the effects of different instructional methods on the 8<sup>th</sup> grade students' Mathematics Achievement in transformation geometry (fractals, rotation, reflection, translation), Geometric Thinking, and Attitude Towards Mathematics and Technology, Independent-samples t-test was conducted as inferential statistical procedure. Before conducting the tests, all assumptions of the tests were checked.

Firstly, Independent-samples t-test was conducted to determine whether the experimental and the control group differ significantly in terms of their mathematics achievement level, geometric thinking, and attitude towards mathematics and technology. Therefore, all pretest scores (PREMAT, PREVHL, and PREMTAS) of the experimental and the control group were compared. Secondly, Independent-samples t-test was conducted again to explore whether there was a statistically significant difference between posttest scores of the control group and posttest scores of the experimental group after the treatment session ended. The hypotheses were tested at the significance level of .05 since it is the mostly used value in educational studies.

### **3.8 Internal Validity**

Internal validity means that observed differences on the dependent variable are directly related to the independent variable, and not due to some other unintended variable (Fraenkel, Wallen & Hyun 2011). Possible threats to the internal validity of the study and how they were minimized or controlled are discussed in this part.

The possible threats to the internal validity alter according to the research design in educational studies. Since the static-group pretest-posttest research design was adopted in the present study, there were some threats to internal validity. Highly

likely threats to internal validity were the threats of subject characteristics, mortality, location, instrumentation, testing, history, implementation and novelty (Fraenkel, Wallen & Hyun 2011).

Although the students were not randomly assigned to the experimental and the control groups, the researcher controlled the subject characteristics threat by having equal groups to compare since two classes which had similar academic achievements were chosen as the sample of study. These classes were chosen based on students' previous mathematics grades and placement test results. The Placement Test was conducted to all the students before they enrolled in the school, and this test also included a mathematics achievement test. Therefore, it was assumed that the groups were similar in terms of the mathematics achievement level and this threat was controlled.

Loss of subjects in a study refers to the mortality (Fraenkel, Wallen & Hyun 2011). In the present study, mortality threat was controlled by assuring that the groups did not differ in numbers lost. Even if the number of lost students in one group was more than another group, this did not distort the results of the study since the subject characteristics threat was controlled by having equal groups to compare. Also, the lost subjects' test scores were replaced with the mean test scores of the students who took the tests. Thus, mortality threat was controlled.

Instrumentation threat was controlled by assigning the same data collector for both groups. The mathematics teacher of the classes conducted the pretests and the posttests to the EG and the CG. Therefore, the data collector characteristics threat to internal validity was prevented. Since the same data collector (the mathematics teacher of the classes) was used for administration of the pretests and the posttests, the data collector bias threat was also controlled. In order to control the instrument decay, the data collection schedule was planned and the scoring procedure was carried out by another mathematics teacher. The students' papers were scored by this mathematics teacher using the given rubric while scoring in order to prevent distortion of the data in such a way as to make certain outcomes (such as support for the hypothesis) more likely. Furthermore, this mathematics teacher scored the tests

without knowing whose answers were being scored. Thus, this threat was also controlled.

Testing threat refers to the fact that a pretest can make students more aware, sensitive, and responsive towards the subsequent treatment (Fraenkel, Wallen & Hyun 2011). However, testing threat was controlled by administering the posttests four weeks after the pre-tests were conducted in order to prevent recalling the questions in the tests. History was not a threat in the present study either since any unexpected events did not occur during the treatment and the administration of the pretests and posttests.

Location threat was also controlled since both classes used the same textbook and had equal classroom settings and conditions (resources, class size, etc.). To state it differently, all the conditions under which the study conducted, except for the primary independent variable (instructional method), were standardized. The only difference between the classes was the presence of the computers with GeoGebra, which was a requirement for the experimental group treatment.

Implementation threat was present in the study since the groups were taught by two different instructors; the researcher and the mathematics teacher of the school. Therefore, instruction in both groups might have been affected by the instructors' individual differences such as teaching skills or other characteristics related to the outcome of the study. However, the mathematics teacher was asked not to give additional verbal explanation or strategy and solve any additional exercise during the lessons. Thus, this threat was also minimized.

For ethical reasons, one week after this study, the topics were covered again under regular mathematics sessions for the control group students. Students were instructed through GeoGebra in computer laboratory. Therefore, all participants in this study had the opportunity to study in a dynamic mathematics software-based learning environment. Hence, novelty threat was controlled.

### 3.9 External Validity

The external validity is the extent to which the result of a study can be generalized. The extent to which the sample represents the population of interest is the population generalizability (Fraenkel, Wallen & Hyun 2011).

The two already-existing classes of 8<sup>th</sup> grade students (8-B and 8-C) at a private elementary school in Bilkent district/Ankara were used as the sample of the study. All the 8<sup>th</sup> grade private elementary school students in Çankaya district/Ankara were identified as the target population of study. This was the population from which the results of the study were generalized. Since there was no other private elementary school nearby Bilkent district, all the 8<sup>th</sup> grade students in the school in which the study was conducted was the accessible population.

Since convenience sampling method was used for selecting the sample of the study, the generalizability of the research results were limited only to the subjects who have similar characteristics with the subjects participated in this study and these results cannot be generalized to a larger population regarding external validity. In other words, the results of this study can be applied to a broader population of samples who have similar characteristics and conditions with the ones in this study (e.g. eight grade private elementary school students nearby Bilkent district).

Ecological generalizability is defined as the degree to which the results of a study can be extended to other settings or conditions (Fraenkel, Wallen & Hyun 2011). The tests were administered in regular classroom settings during the regular lesson hours. There were two classes with approximately 20 students in each class. The conditions of the classrooms were quite similar, and the sitting arrangements and the lighting conditions were equal in both classrooms. Thus, the threats to the ecological validity were controlled.

In this chapter, the design of the study, population and sample, instruments, data collection procedure, data analysis, assumptions, limitations, internal and external validity issues of the study have been explained. In the next chapter, the results of the study will be given.

## CHAPTER 4

### RESULTS

The aim of this quantitative experimental study was to investigate the effects of using Dynamic Geometry Software on 8<sup>th</sup> grade students' mathematics achievement in transformation geometry, geometric thinking, and attitudes toward mathematics and technology.

This chapter presents the descriptive statistics related to Mathematics Achievement Test, van Hiele Geometric Thinking Level Test, and Mathematics and Technology Attitude Scale, and inferential statistics related to the research questions. The study aimed at investigating the following research questions:

**Main Research Problem:** What is the effect of the Dynamic Geometry Software-Assisted Instruction on 8<sup>th</sup> grade students' Mathematics Achievement in Transformation Geometry, Geometric Thinking, and Attitude towards Mathematics and Technology?

To examine the main problem, three sub-problems were addressed:

**SP1)** Is there a significant mean difference between the group taught by the Dynamic Geometry Software-Assisted Instruction and the group taught by Regular Instruction with respect to Mathematics Achievement posttest scores?

**SP2)** Is there a significant mean difference between the group taught by the Dynamic Geometry Software-Assisted Instruction and the group taught by Regular Instruction with respect to van Hiele Geometric Thinking Level posttest scores?

**SP3)** Is there a significant mean difference between the group taught by the Dynamic Geometry Software-Assisted Instruction and the group taught by Regular Instruction with respect to Mathematics and Technology Attitude posttest scores?

#### **4.1 Missing Data Analyses**

There were some missing data in pretests and posttests. In control group, the students with id 8 and id 13 did not take POSTMAT, the student with id 17 did not take PREMAT, and the student with id 7 did not take PREMTAS. In experimental group, the student with id 14 did not take PREVHL and PREMTAS. These students' related test scores were replaced with the mean score of the students who took the tests since the mean score was the appropriate measure of central tendency for continuous variables (Pallant, 2011).

#### **4.2 Analysis of Pretest Scores of the Experimental Group and the Control Group**

Prior to comparison of the experimental and the control group to investigate the effectiveness of the DGS-Assisted Instruction, Independent-samples t-test was conducted firstly to determine whether the groups differ significantly in terms of their mathematics achievement level, geometric thinking, and attitude towards mathematics and technology according to their pretest scores of Mathematics Achievement test, van Hiele Geometric Thinking Level Test, and Mathematics and Technology Attitude Scale.

##### **4.2.1 Assumptions of Independent-Samples t-test**

Before conducting Independent-Samples t-test, assumptions which were discussed in Pallant (2011), were checked. These assumptions were the level of measurement, random sampling, independence of observations, normality, and homogeneity of variance. The assumptions were checked for all pretest and posttest scores of MAT, VHL and MTAS.

#### **4.2.1.1 Level of measurement**

According to this assumption, the dependent variable is measured at the interval or ratio level which requires using a continuous scale instead of discrete categories (Pallant, 2011). In the present study, the dependent variables were the pretest and the posttest scores of the Mathematics Achievement Test, van Hiele Geometric Thinking Level Test, and Mathematics and Technology Attitude Scale which were continuous variables and were measured on ratio scale. Therefore, the assumption of Level of Measurement were verified.

#### **4.2.1.2 Random Sampling**

In the present study, convenience sampling method was adopted. Sample of the study which consisted of the students from two different 8<sup>th</sup> grade classrooms was selected according to their previous mathematics grades. Thus, this assumption was not verified. However, in real-life research, this situation does not cause major problems (Pallant, 2011).

#### **4.2.1.3 Independence of Observations**

To verify this assumption, the researcher observed both groups during the administration of all pretests and posttests. According to the observations, it was concluded that the participants of the study did all tests by themselves and the measurement of a participant was not influenced by another participant. Therefore, the independence of observations assumption was also validated.

#### **4.2.1.4 Normality**

Since the sample size is less than 30, Kolmogorov-Smirnov and Shapiro-Wilk statistic and histograms were used to assess the normality of the distribution of the test scores. Kolmogorov-Smirnov and Shapiro-Wilk statistic table and Histograms related to the all pretest and posttest scores of the experimental group and the control group were given in the Table 10 below.

Table 10. Kolmogorov-Smirnov statistic for pretest and posttest scores of the groups

		Kolmogorov-Smirnov			Shapiro-Wilk		
Group		Statistic	df	Sig.	Statistic	df	Sig.
PREMAT	EG	.167	17	.200	.916	17	.126
	CG	.154	17	.200	.948	17	.419
PREVHL	EG	.196	17	.083	.907	17	.090
	CG	.219	17	.059	.923	17	.169
PREMTAS	EG	.136	17	.200	.922	17	.158
	CG	.183	17	.134	.928	17	.200
POSTMAT	EG	.132	17	.200	.971	17	.835
	CG	.109	17	.200	.973	17	.875
POSTVHL	EG	.187	17	.117	.944	17	.373
	CG	.172	17	.193	.934	17	.255
POSTMTAS	EG	.124	17	.200	.950	17	.460
	CG	.136	17	.200	.953	17	.502

EG: Experimental Group

CG: Control Group

As it is seen from the table above, both Sig. values of Kolmogorov-Smirnov and Shapiro-Wilk statistic for all pretest and posttest scores of the groups are greater than .05 ( $p > .05$ ). Moreover, as it can be seen in the descriptive statistics tables of the scores which were given above, the skewness and kurtosis of the scores' distribution have values between the -1.00 and +1.00. Therefore, it can be concluded that pretest and posttest scores of the experimental group and the control group are normally distributed. Also, the twelve histograms below with normal curves support the normality of the groups' pretest and posttest scores related to the Mathematics Achievement Test, van Hiele Geometric Thinking Level Test, and Mathematics and Technology Attitude Scale.



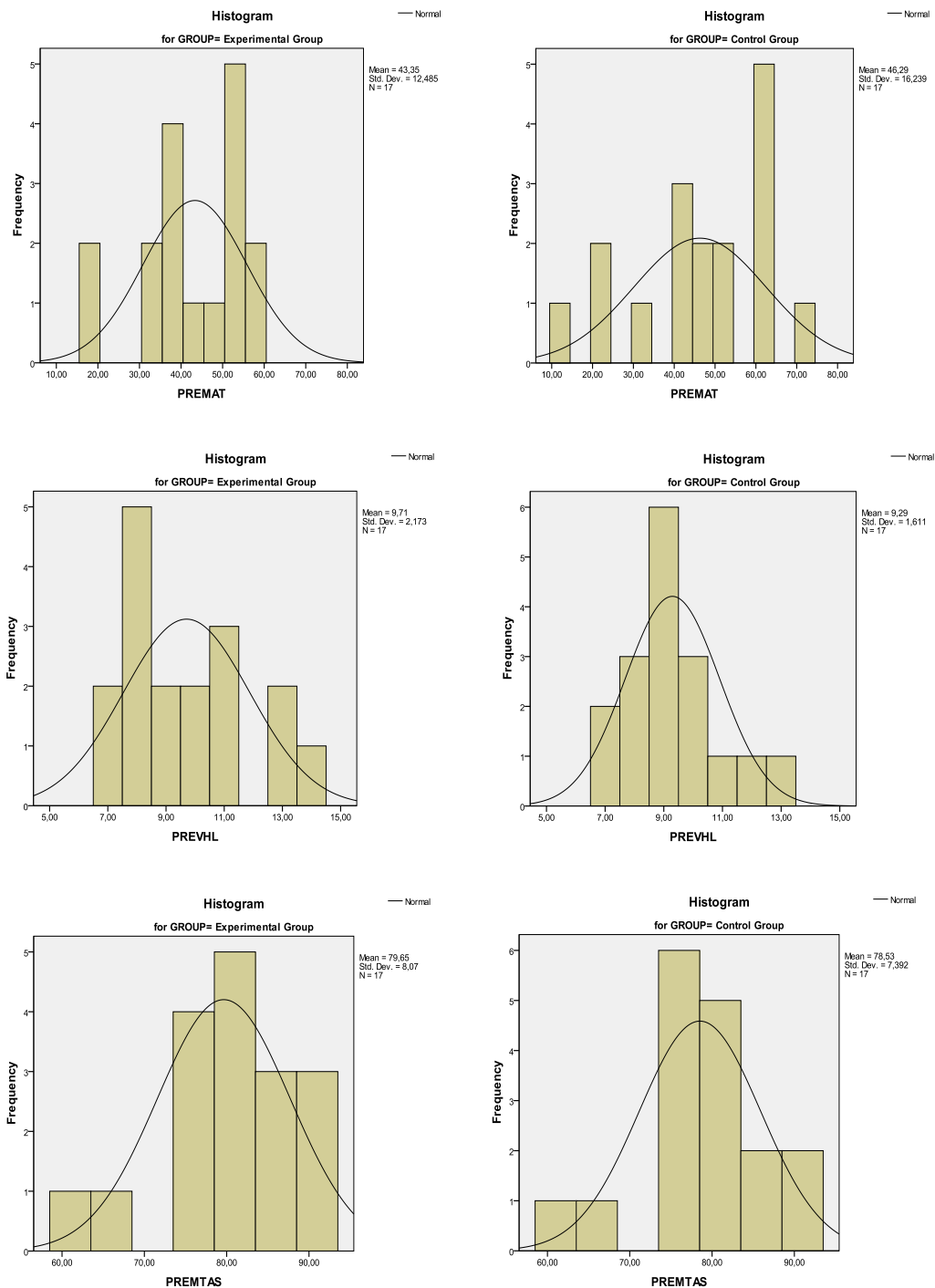


Figure 8. Histograms of the pretest scores for both groups

Histograms with normal curves given above support the normality of both groups' pretest scores related to the Mathematics Achievement Test, van Hiele Geometric Thinking Level Test, and Mathematics and Technology Attitude Scale.

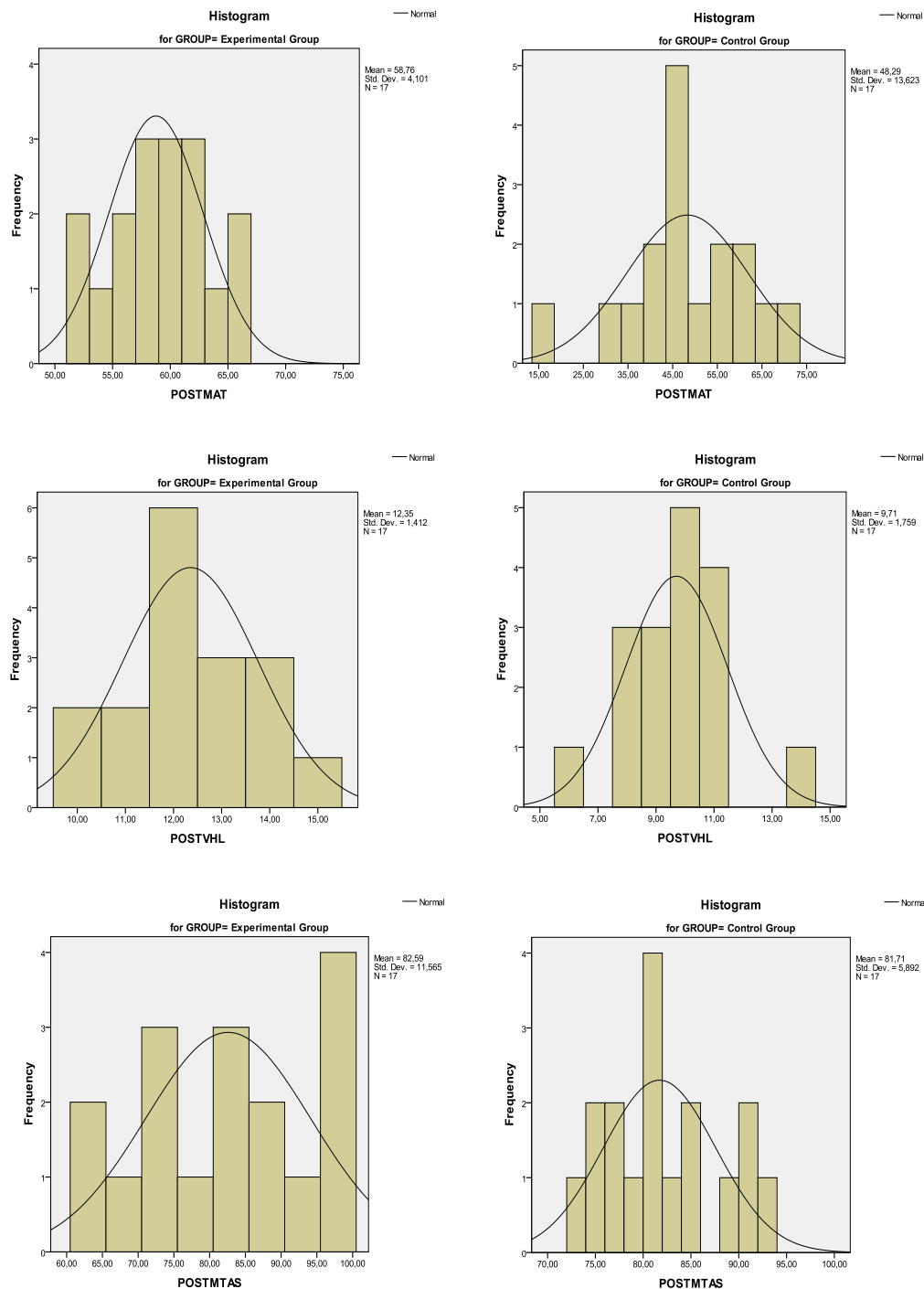


Figure 9. Histograms of the posttest scores for both groups

Histograms with normal curves given above support the normality of both groups' posttest scores related to the Mathematics Achievement Test, van Hiele Geometric Thinking Level Test, and Mathematics and Technology Attitude Scale.

#### 4.2.1.5 Homogeneity of Variance

In order to check this assumption, Independent Samples t-test was performed and Levene's Test for Equality of Variances results were investigated. The hypotheses tested and test results and were given below.

**H<sub>0</sub>:**  $\sigma_1^2 = \sigma_2^2$  (The two populations have equal variances)

**H<sub>1</sub>:**  $\sigma_1^2 \neq \sigma_2^2$  (The two populations do not have equal variances)

Table 11. Levene's Test results for Independent-samples t-test

Levene's Test for Equality of Variances		
	F	Sig.
PREMAT	.82	.369
PREVHL	2.66	.112
PREMTAS	.174	.679

According to Levene's test results, it can be seen that the significance values of PREMAT, PREVHL, and PREMTAS were greater than the value of .05. Since the value of Sig. was greater than the test value ( $\alpha = .05$ ), the null hypothesis was failed to reject and it was concluded that the two populations have equal variances in tests. In other words, homogeneity of variances assumption was verified.

After checking the assumptions, Independent-Samples t-test was conducted firstly to ensure the experimental and the control group do not differ significantly in terms of their mathematics achievement level, geometric thinking, and attitude towards mathematics and technology according to their pretest scores of Mathematics Achievement test, van Hiele Geometric Thinking Level Test, and Mathematics and Technology Attitude Scale. The null hypothesis and the inferential statistics table of the test were given below.

Null Hypothesis: There is no statistically significant difference between the two groups' pretest scores on Mathematics Achievement Test, van Hiele Geometric Thinking Level Test and Mathematics and Technology Attitude Scale.

Table 12. Independent-samples t-test results of the groups' pretest scores on Mathematics Achievement Test, van Hiele Geometric Thinking Level Test and Mathematics and Technology Attitude Scale

t-test for Equality of Means			
	t	df	Sig. (2-tailed)
PREMAT	-.592	32	.558
PREVHL	.628	32	.535
PREMTAS	.421	32	.677

An Independent Samples t-test was conducted to check whether there is a significant difference between the groups' mathematics achievement level, geometric thinking, and attitude towards mathematics and technology before the treatment.

Analysis results revealed that there was no statistically significant difference between the experimental group ( $M=43.35$ ,  $SD=12.48$ ) and the control group ( $M=46.29$ ,  $SD=16.23$ ;  $t(32)= -.59$ ,  $p>.05$ , two-tailed) in terms of mathematics achievement level according to the groups' pretest scores on Mathematics Achievement Test. There was no statistically significant difference between the experimental group ( $M=9.70$ ,  $SD=2.17$ ) and the control group ( $M=9.29$ ,  $SD=1.61$ ;  $t(32)= .628$ ,  $p>.05$ , two-tailed) in terms of geometric thinking according to the groups' pretest scores on van Hiele Geometric Thinking Level Test. Similarly, there was no statistically significant difference between the experimental group ( $M=79.64$ ,  $SD=8.06$ ) and the control group ( $M=78.52$ ,  $SD=7.39$ ;  $t(32)= .421$ ,  $p>.05$ , two-tailed) in terms of attitude towards mathematics and technology according to the groups' pretest scores on Mathematics and Technology Attitude Scale. Therefore, it can be concluded that the groups do not differ significantly in terms of the dependent variables of the study before the treatment process begins.

#### **4.3 The Effect of DGS-Assisted Instruction on Students' Mathematics Achievement**

In this section, descriptive statistics, inferential statistics and the findings related to the analysis of the scores on Mathematics Achievement Test (MAT) were given.

#### 4.3.1 Descriptive Statistics of the Mathematics Achievement Test (MAT)

Descriptive statistics related to the pretest (PREMAT) and posttest (POSTMAT) scores of Mathematics Achievement Test for the experimental group and the control group was presented in Table 10 below. Descriptive Statistics Table was presented below to give information about the mean scores, median, standard deviations, the values of skewness and kurtosis, and the minimum and maximum scores regarding Mathematics Achievement Test for both groups. The Mathematics Achievement Test was evaluated out of 78 point.

Table 13. Descriptive statistics of the groups' pretest and posttest scores on Mathematics Achievement Test

	Groups			
	EG		CG	
	PREMAT	POSTMAT	PREMAT	POSTMAT
N	17	17	17	17
Mean	43.35	58.76	46.29	48.29
Median	45.00	59.00	46.00	48.00
Std. Deviation	12.48	4.10	16.23	13.62
Minimum	18.00	52.00	12.00	16.00
Maximum	59.00	66.00	71.00	72.00
Skewness	-.687	-.049	-.318	-.728
Kurtosis	-.288	-.533	-.576	-.497

Table 10 demonstrates an overall summary of the descriptive statistics obtained from the pretest and posttest scores on Mathematics Achievement Test of experimental and control groups. As it can be seen in the Table 10, both groups' posttest mean scores were higher than the pretest mean scores. Moreover, the mean score of experimental group increased from 43.35 to 58.76 while the mean score of control group increased from 46.29 to 48.29 at the end of the study. In other words,

the increase in Mathematics Achievement Test scores of the experimental group is higher than the increase in Mathematics Achievement Test scores of the control group.

In addition to the numerical descriptive statistics, clustered box plots were also performed in statistical analysis. The clustered box plots of the PREMAT and the POSTMAT for the experimental group and the control group were given in Figure 10 below.

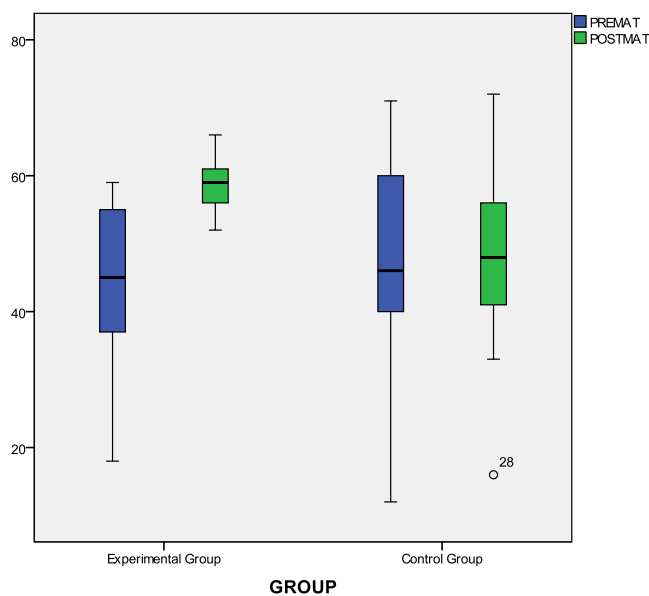


Figure 10. Clustered Box Plot of the PREMAT and POSTMAT for the experimental group and the control group

As seen from the box plot, the box includes mid 50% and each whisker represents upper and lower 25% of the scores (Green, Salkind, & Akey, 2003). Therefore, it can be concluded from the box plot that 75% of the experimental group scored 45.00 or higher on PREMAT and scored 59.00 or higher on POSTMAT. Also, 75% of the control group scored 46.00 or higher on PREMAT and 48.00 or higher POSTMAT. In addition, there was a lower outlier which represents a lower extreme score in POSTMAT of the control group. Moreover, the skewness and kurtosis of the scores' distribution have values between the -1.00 and +1.00 which verifies that the scores are normally distributed.

#### 4.3.2 Inferential Statistics of the Mathematics Achievement Test (MAT)

Independent-samples t-test was conducted to explore whether there was a statistically significant difference between posttest scores of the experimental group and control group in terms of Mathematics Achievement Test after the treatment session ended. The following hypothesis was tested through Independent-samples t-test:

Null Hypothesis 1: There is no statistically significant mean difference between the group taught by the Dynamic Geometry Software-Assisted Instruction and the group taught by Regular Instruction with respect to Mathematics Achievement Test posttest scores.

Table 14. Independent Samples t-test results of the groups' posttest scores on Mathematics Achievement Test

t-test for Equality of Means			
	t	df	Sig. (2-tailed)
POSTMAT	3.03	32	.005

An Independent Samples t-test was conducted to compare the groups' posttest scores on Mathematics Achievement Test. There was a statistically significant difference between the experimental group ( $M=58.76$ ,  $SD=4.10$ ) and the control group ( $M=48.29$ ,  $SD=13.62$ ;  $t(32)= 3.03$ ,  $p<.05$ , two-tailed) in terms of mathematics achievement level according to the groups' post scores on Mathematics Achievement Test. The Eta square statistic (.22) indicated a medium effect size (Cohen, 1988) as practical significance of the treatment.

#### 4.4 The Effect of DGS-Assisted Instruction on Students' Geometric Thinking

In this section, descriptive statistics, inferential statistics and the findings related to the analysis of the scores on van Hiele Geometric Thinking Level Test (VHL) were given.

#### 4.4.1 Descriptive Statistics of the van Hiele Geometric Thinking Level Test (VHL)

Descriptive statistics related to the pretest (PREVHL) and posttest (POSTVHL) scores of van Hiele Geometric Thinking Level Test for the experimental group and the control group was presented in Table 11. The van Hiele Geometric Thinking Level Test was evaluated out of 15 point.

Table 15. Descriptive statistics of the groups' pretest and posttest scores on van Hiele Geometric Thinking Level Test

	Groups			
	EG		CG	
	PREVHL	POSTVHL	PREVHL	POSTVHL
N	17	17	17	17
Mean	9.70	12.35	9.30	9.70
Median	9.00	12.00	9.00	10.00
Std. Deviation	2.17	1.41	1.61	1.75
Minimum	7.00	10.00	7.00	6.00
Maximum	14.00	15.00	13.00	14.00
Skewness	.636	.038	.778	.275
Kurtosis	-.657	-.451	.605	.722

Table 15 demonstrates an overall summary of the descriptive statistics obtained from the pretest and posttest scores on van Hiele Geometric Thinking Level Test of experimental group and control group students. As it can be seen in the Table 11, both groups' posttest mean scores were higher than the pretest mean scores. Besides, the mean score of the experimental group increased from 9.70 to 12.35, while the mean score of the control group increased from 9.30 to 9.70 at the end of the study. Put it differently, the increase in van Hiele Geometric Thinking Level Test



scores of the experimental group is higher than the increase in van Hiele Geometric Thinking Level Test scores of the control group.

In addition to the numerical descriptive statistics, clustered box plots were also performed in statistical analysis. The clustered box plots of the PREVHL and the POSTVHL for the Experimental Group and the Control Group were given in Figure 11 below.

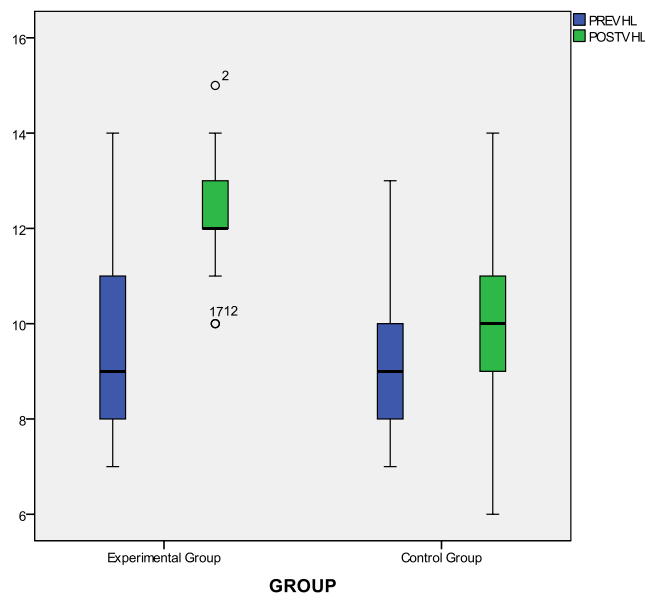


Figure 11. Clustered Box Plot of the PREVHL and POSTVHL for the experimental group and the control group

As seen from the box plot, the box includes mid 50% and each whisker represents upper and lower 25% of the scores (Green, Salkind, & Akey, 2003). Therefore, it can be concluded from the box plot that 75% of the experimental group scored 9.00 or higher on PREVHL and scored 12.00 or higher on POSTVHL. Also, 75% of the control group scored 9.00 or higher on PREVHL and 10.00 or higher on POSTVHL. In addition, there were two lower outliers which represent lower extreme scores and one upper outlier which represents a higher extreme score in POSTVHL of the experimental group. Moreover, the skewness and kurtosis of the scores' distribution have values between the -1.00 and +1.00 which verifies that the scores are normally distributed.

#### 4.4.2 Inferential Statistics of the van Hiele Geometric Thinking Level Test (VHL)

Independent-samples t-test was conducted to explore whether there was a statistically significant difference between posttest scores of the experimental group and control group in terms of van Hiele Geometric Thinking Level Test after the treatment session ended. The following hypothesis was tested through Independent-samples t-test:

Null Hypothesis 2: There is no statistically significant mean difference between the group taught by the Dynamic Geometry Software-Assisted Instruction and the group taught by Regular Instruction with respect to van Hiele Geometric Thinking Level Test posttest scores.

Table 16. Independent-samples t-test results of the groups' posttest scores on, van Hiele Geometric Thinking Level Test

t-test for Equality of Means			
	t	df	Sig. (2-tailed)
POSTVHL	4.83	32	.000

An Independent Samples t-test was conducted to compare the groups' posttest scores on van Hiele Geometric Thinking Level Test. There was a statistically significant difference between the experimental group ( $M=12.35$ ,  $SD=1.41$ ) and the control group ( $M=9.70$ ,  $SD=1.75$ ;  $t(32)=4.83$ ,  $p<.05$ , two-tailed) in terms of geometric thinking according to the groups' post scores on van Hiele Geometric Thinking Level Test. The Eta square statistic (.40) indicated a medium effect size (Cohen, 1988) as practical significance of the treatment.

Besides, the study result also indicated that there was a moderately strong correlation ( $r=.53$ ) (Cohen, 1988) between the students' posttest scores of Mathematics Achievement Test and van Hiele Geometric Thinking Level Test.

#### 4.5 The Effect of DGS-Assisted Instruction on Students' Attitude Towards Mathematics and Technology

In this section, descriptive statistics, inferential statistics and the findings related to the analysis of the scores on Mathematics and Technology Attitude Scale (MTAS) were given.

#### 4.5.1 Descriptive Statistics of the Mathematics and Technology Attitude Scale (MTAS)

Descriptive statistics related to the pretest (PREMTAS) and posttest (POSTMTAS) scores of Mathematics and Technology Attitude Scale for the experimental group and the control group was presented in Table 12. The Mathematics and Technology Attitude Scale was evaluated out of 100 point.

Table 17. Descriptive statistics of the groups' pretest and posttest scores on Mathematics and Technology Attitude Scale

	Groups			
	EG		CG	
	PREMTAS	POSTMTAS	PREMTAS	POSTMTAS
N	17	17	17	17
Mean	79.64	82.58	78.52	81.70
Median	80.00	82.00	79.00	81.00
Std. Deviation	8.06	11.56	7.39	5.89
Minimum	61.00	63.00	61.00	73.00
Maximum	90.00	100.00	90.00	92.00
Skewness	-.875	-.158	-.865	.282
Kurtosis	.672	-.957	.938	-.982

Table 17 demonstrates an overall summary of the descriptive statistics obtained from the pretest and posttest scores on Mathematics and Technology Attitude Scale of the experimental group and the control group students. As it can be seen in the Table 12, both groups' posttest mean score were higher than the pretest mean score. Also, the mean score of experimental group increased from 79.64 to

82.58 while the mean score of control group increased from 78.52 to 81.70 at the end of the study. State differently, the increase in Mathematics and Technology Attitude scores of the control group is higher than the increase in Mathematics and Technology Attitude scores of experimental group.

In addition to the numerical descriptive statistics, clustered box plots were also performed in statistical analysis. The clustered box plots of the PREMTAS and the POSTMTAS for the experimental group and the control group were given in Figure 12 below.

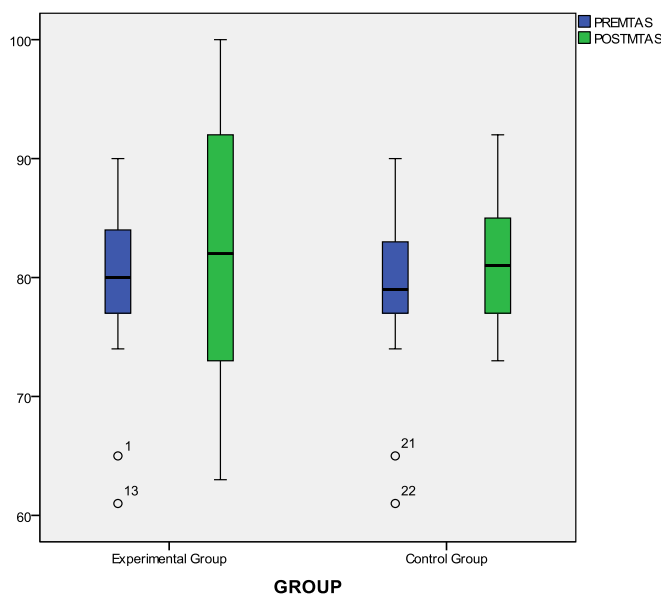


Figure 12. Clustered Box Plot of the PREMTAS and POSTMTAS for the experimental group and the control group

As seen from the box plot, the box includes mid 50% and each whisker represents upper and lower 25% of the scores (Green, Salkind, & Akey, 2003). Therefore, it can be concluded from the box plot that 75% of the experimental group scored 80.00 or higher on PREMTAS and scored 82.00 or higher on POSTMTAS. Also, 75% of the control group scored 79.00 or higher on PREMTAS and 81.00 or higher on POSTMTAS. In addition, there were two lower outliers which represent lower extreme scores in PREMTAS for both groups. Moreover, the skewness and

kurtosis of the scores' distribution have values between the -1.00 and +1.00 which verifies that the scores are normally distributed.

#### 4.5.2 Inferential Statistics of the Mathematics and Technology Attitude Scale (MTAS)

Independent-samples t-test was conducted to explore whether there was a statistically significant difference between posttest scores of the experimental group and control group in terms of Mathematics and Technology Attitude Scale after the treatment session ended. The following hypothesis was tested through Independent-samples t-test:

Null Hypothesis 3: There is no statistically significant mean difference between the group taught by the Dynamic Geometry Software-Assisted Instruction and the group taught by Regular Instruction with respect to Mathematics and Technology Attitude Scale posttest scores.

Table 18. Independent Samples t-test results of the groups' posttest scores on Mathematics and Technology Attitude Scale

t-test for Equality of Means			
	t	df	Sig. (2-tailed)
POSTMTAS	.28	32	.781

An Independent Samples t-test was conducted to compare the groups' posttest scores on Mathematics and Technology Attitude Scale. There was no statistically significant difference between the experimental group ( $M=82.58$ ,  $SD=11.56$ ) and the control group ( $M=81.70$ ,  $SD=5.89$ ;  $t(32)=.28$ ,  $p>.05$ , two-tailed) in terms of attitude towards mathematics and technology according to the groups' post scores on Mathematics and Technology Attitude Scale. The Eta square statistic (.002) indicated a small effect size (Cohen, 1988) as practical significance of the treatment.

## 4.6 Summary of the Results

The descriptive statistics including sample size, mean, standard deviation, minimum and maximum scores, skewness and kurtosis reported the demographics of the sample in Table 13, Table 15 and Table 17.

According to the analysis of the test results, the experimental group students' pretest mean score on mathematics achievement test (PREMAT) was 43.35 ( $SD = 12.48$ ) while the posttest mean score (POSTMAT) was 58.76 ( $SD = 4.10$ ). On the other hand, the control group students' pretest mean score on the same achievement test was 46.29 ( $SD = 16.23$ ) while the posttest mean score was 48.29 ( $SD = 13.62$ ). There was no statistically significant mean difference between the two groups' pretest scores on Mathematics Achievement Test (PREMAT). However, there was a statistically significant mean difference between the two groups' posttest scores on Mathematics Achievement Test (POSTMAT) in favor of the experimental group.

The experimental group students' pretest mean score on van Hiele Geometric Thinking Level Test (PREVHL) was 9.70 ( $SD = 2.17$ ) while the posttest mean score (POSTVHL) was 12.35 ( $SD = 1.41$ ). On the other hand, the control group students' pretest mean score on van Hiele Geometric Thinking Level Test was 9.30 ( $SD = 1.61$ ) while the posttest mean score was 9.70 ( $SD = 1.75$ ). There was no statistically significant mean difference between the two groups' pretest scores on van Hiele Geometric Thinking Level Test (PREVHL). However, there was a statistically significant mean difference between the two groups' posttest scores on van Hiele Geometric Thinking Level Test (PREVHL) in favor of the experimental group.

The experimental group students' pretest mean score on Mathematics and Technology Attitude Scale (PREMTAS) was 79.64 ( $SD = 8.06$ ) while the posttest mean score (POSTMTAS) was 82.58 ( $SD = 11.56$ ). On the other hand, the control group students' pretest mean score on Mathematics and Technology Attitude Scale was 78.52 ( $SD = 7.39$ ) while the posttest mean score was 81.70 ( $SD = 5.89$ ). There was no statistically significant mean difference between the two groups' pretest scores on Mathematics and Technology Attitude Scale (PREMTAS).

In this chapter descriptive statistics and the inferential statistics of the study was explained. In the following chapter, discussions, implications and recommendations related to the study will be given.

## **CHAPTER 5**

### **DISCUSSION, IMPLICATIONS AND RECOMMENDATIONS**

The aim of this study was to investigate the effects of the Dynamic Geometry Software-Assisted Instruction using GeoGebra on 8<sup>th</sup> grade students' mathematics achievement in transformation geometry (fractals, rotation, reflection, translation), geometric thinking and attitudes toward learning mathematics with technology. This chapter consists of the discussion of the study results and implications and recommendations for further research studies.

#### **5.1 Students' Mathematics Achievement**

An Independent-samples t-test was conducted to investigate the effect of the DGS-Assisted Instruction using GeoGebra on experimental group students' scores on Mathematics Achievement Test (MAT). The results of the statistical analyses revealed that there was a statistically significant mean difference between the experimental group taught by the DGS-Assisted Instruction and the control group taught by Regular Instruction with respect to posttest scores of Mathematics Achievement Test (MAT). This result which indicates the positive effect of dynamic geometry software GeoGebra on students' mathematics achievement is consistent with previous research studies in the literature (Bilgici & Selçik, 2011; Filiz, 2009; Furkan, Zengin, & Kutluca, 2012; İçel, 2011; Saha, Ayub & Tarmizi, 2010; Zengin, 2011).

Several reasons may account for the positive effect of the DGS-Assisted Instruction using GeoGebra on students' achievement and geometric thinking. The main reason might be the use of dynamic geometry software which provided students with exciting, interesting and visual way of learning. This learning environment attracted students' attention to the lesson and provided active student participation in



the present study as it was also found in the studies of Boyraz (2008) and Choate (1992).

Visualization helps students to better understand abstract concepts in a more concrete way (Hacıömeroğlu, 2011). Thus, another possible reason that affected experimental group students' mathematics achievement can be the visualization of the mathematical concepts and ideas which might be provided by dynamic geometry software. To put it differently, the dynamic learning environment might have provided students with a visual way of learning the topic of transformation geometry in the present study. The importance of visualization is defined as the main and core component in the teaching and learning of geometry according to the results of previous research studies (Battista, 1994; Bishop, 1989; Gutiérrez, 1996; Harnisch, 2000; Hershkowitz, 1989; Reed, 1996).

As previously stated, dragging is a dynamic movement which allows DGS users to test the hypotheses, observe the regularities and changes and resize the objects (Arzarello et al., 2002). Thus, the features of dynamic geometry softwares, such as dragging and representations of the concepts both graphically and algebraically, may account for the experimental group students' higher achievement in mathematics than the control group. The results obtained in this study are consistent with the results of previous research concerning the effects of dragging feature of dynamic geometry softwares (Arzarello et al. 2002; Jones, 2000; Healy & Hoyles, 2001; Hölzl, 1996; Sträßer, 2001). This reason can be explained by the comparison of the traditional learning environment with static paper-pencil environment, in which students do not have a chance to observe changes, with the dynamic learning environment, which provides students with a rich learning experience by enabling them to realize the specific properties (i.e. the square has four equal side lengths in any size) and changeable characteristics of the shapes such as side length, area and perimeter. While students deal with the static drawings with paper-pencil and these drawings present the figure as in the form of its general case in the static learning environment, dynamic learning environment via dynamic geometry software provides students with construction of a figure dynamically which

enable them to resize or drag the object to observe the changes and make their own generalizations related to the certain shape. During a construction, when a shape is dragged from its corner, it conserves the properties which are related to its constrain. Although the size and its position change, the shape remains the same. This kind of characteristic of dynamic geometry environment enables students to comprehend the shape with its important properties.

It was reported in a study by Hohenwarter et al. (2008) that GeoGebra as a dynamic geometry software helped students to make them better understand the topic with concrete real life examples via visualization in a dynamic learning environment. Furthermore, the students were active participants during the whole class since the lesson prepared required active involvement of students such as making constructions, working on the activities, testing the mathematical ideas and hypotheses. All of these might have been the reason for high mathematics achievement. Moreover, the instant and quick feedback opportunity that students have in a dynamic learning environment may be another reason for the better understanding of the topic and higher achievement since students could instantly see what they did correct or wrong. Also, the instructor's role as a guide rather than a "knowledge transmitter" may be another reason for the experimental group students' higher mathematics achievement in transformation geometry.

Another possible reason underlying the experimental group students' higher mathematics achievement in transformation geometry can be the immediate calculation and transformation opportunity through visualization and dragging that dynamic geometry software provided. By means of these opportunities, the students did not have to memorize some formulas in order to calculate and transform some variables such as change in the coordinate of a point when it was reflected about the x-axis, change in the coordinates of an object when it was rotated around a point by angle or area and perimeter of a shape when it was exposed to a motion of transformation. For instance, students observed that the area or perimeter of a shape remained the same in the reflection of the same shape via visualization and dragging opportunities that DGS provided. In this context, the traditional method in

mathematics teaching is criticized since it compels students to memorize mathematical formulas because of its lack of supportive components such as visualization (Fuys, Geddes, and Tischler, 1988; Mayberry, 1983). To exemplify, traditional instruction merely involves giving students the rules, such as axis of a point turns into its opposite sign and ordinate stays the same when it is reflected about y-axis. Thus, merely memorizing rules without understanding the idea behind them eventually end up with forgetting or confusing the knowledge obtained. Dynamic geometry softwares not only provide understanding of these calculations but also making generalizations. In this study, students resized and dragged the figure, reflected the coordinates of the shape, translated an object by a segment in dynamic environment so that they could immediately observed the changes and make conclusions about certain motions of transformation. Such a property enabled students to make their own conjectures about the motions of reflection, translation, and rotation. This may also account for the better understanding of the topic and higher mathematics achievement of the experimental group students, who underwent the DGS-Assisted Instruction using GeoGebra.

## **5.2 Students' Geometric Thinking**

An Independent-samples t-test was conducted to investigate the effect of the DGS-Assisted Instruction using GeoGebra on experimental group students' scores on van Hiele Geometric Thinking Level Test (VHL). The results of the statistical analyses revealed that there was a statistically significant mean difference between the experimental group taught by the DGS-Assisted Instruction using GeoGebra and the control group taught by Regular Instruction with respect to posttest scores of van Hiele Geometric Thinking Level Test (VHL). In other words, the experimental group students scored significantly higher on van Hiele Geometric Thinking Level Test than the control group. This result which indicates the positive effect of dynamic geometry software on students' geometric thinking level is consistent with related previous studies in the literature (Clements, Battista & Sarama, 2001; Chang, Sung & Lin, 2007; Idris, 2012; July, 2001; Meng & Öztürk, 2012; Moyer, 2003; Parsons,

Stack & Breen, 1998; Tutak & Birgin, 2009; Toker, 2008; Idris, 2007). These experimental studies investigated the same research question as that in the present study focused on examining the change in students' geometric thinking level after the students were instructed with Dynamic Geometry Software. The researchers of the abovementioned studies linked the positive progress in geometric thinking level with the features of dynamic geometry software which helped students to develop their geometric reasoning like it was concluded in the present study.

Several reasons may account for the positive effect of the DGS-Assisted Instruction on students' geometric reasoning level. Achievement in geometry and van Hiele Geometric Thinking Level are moderately strongly correlated ( $r=.64$ ) (Burger 1985; Burger & Shaughnessy, 1986; Geddes et al. 1982; Geddes, Fuys & Tichler, 1985; Mayberry, 1981; Shaughnessy & Burger, 1985; Usiskin, 1982). Thus, the main reason for higher increase in geometric thinking level of experimental group students may be attributed to their higher increase in mathematics achievement due to the use of dynamic geometry software as mentioned above.

The progress in experimental group students' geometric thinking indicated a medium effect size (Cohen, 1988). This effect size can be increased by extending the time span of the treatment process. In this study, the treatment process lasted three weeks and this time span may not be enough to draw exact conclusions about the progress of the students' geometric thinking. This argument is supported by the results of previous studies (Johnson, 2002; van Hiele-Geldof, 1984). In order to investigate the increase in students' geometric thinking and their progress, longer time span is needed (Johnson, 2002; van Hiele-Geldof, 1984). In other words, to observe the long-term effects and larger effect size of the treatment with respect to the students' geometric thinking, a longer time span of treatment process may be needed.

Research studies also revealed that the elementary and middle school students' van Hiele levels of geometric reasoning can be increased by developing systematically planned mathematics instruction (Wirszup, 1976; Fuys, Geddes, & Tischler, 1986). In these studies, similar to the present study, positive progress

between geometric thinking levels was observed due to the consistent and planned mathematics instruction which was in accordance with the students' level of development. Thus, the DGS-Assisted Instruction using GeoGebra can be considered as a planned, effective, and systematically constructed instruction since preliminary preparation was required before the treatment began in the present study (i.e. lesson plans, activities, work sheets, activity sheets). Therefore, this study may provide an example of a systematic and planned mathematics instruction, which was supported and enriched by the dynamic features of GeoGebra.

### **5.3 Students' Attitude towards Mathematics and Technology**

An Independent-samples t-test was conducted to investigate the effect of the DGS-Assisted Instruction using GeoGebra on experimental group students' scores on Mathematics and Technology Attitude Scale (MTAS). The results of the statistical analyses revealed that there was no statistically significant mean difference between the experimental group taught by the DGS-Assisted Instruction using GeoGebra and the control group taught by Regular Instruction with respect to posttest scores of MTAS. In other words, the DGS-Assisted Instruction using GeoGebra had no significant effect on students' attitude towards learning mathematics with technology after the treatment session ended.

Even though the students expressed their thoughts verbally that they liked learning mathematics with technology and found GeoGebra as a useful tool in learning mathematics, the increase in experimental group students' MTAS posttest scores after the treatment was not statistically significant according to the statistical analyses. This result is consistent with the study of Yemen (2009). In her study, she did not find a significant effect of dynamic geometry use on students' attitude towards mathematics either and accounted for this result with the short duration (4 weeks) of the treatment, which was almost the same time span as the present study. As previously mentioned, the treatment duration of the present study lasted three weeks and this time span may not have been sufficient to change 8<sup>th</sup> grade students' attitude towards mathematics since the change in the students' attitude towards

mathematics necessitate a longer time span (Hannula, 2002). In this context, the main reason for the non-significant increase in students' attitude towards learning mathematics with technology might be the eighth grade students' already-formed attitudes toward mathematics after the middle school years. Hence, observing a significant change in students' attitude towards learning mathematics with technology can be possible by conducting long-term studies covering different learning areas at the same grade level (e.g. instructing students different topics by DGS-Assisted Instruction for the entire year).

The other reason for the non-significant change in students' attitude towards learning mathematics with technology might be the students' familiarity to the technological devices, such as computers and tablets. Another possible reason for this result could be due to the attitude scale which was used in the present study. That is, different results related to the students' attitude towards learning mathematics with technology might be obtained if a different instrument was used to measure students' attitude towards mathematics and technology.

However, the result of the present study is inconsistent with some other research studies in the literature which indicates the positive effect of dynamic geometry software use on students' attitude towards mathematics and students' positive reactions to learn mathematics with technology (Baki & Özpınar, 2007; Idris, 2007; Özdemir & Tabuk, 2004; Pilli, 2008; Sulak & Allahverdi, 2002; Yousef, 1997). The possible reason for this inconsistency between the present study and previous studies may be explained by the time span difference of the treatment processes since these experimental research studies lasted longer than the present study lasted. Thus, the time span of those studies might have been sufficient to affect students' attitude towards mathematics. Another possible reason for this inconsistency might be the difference in the grade levels of the students participating in these studies and the present study. The abovementioned studies were conducted with different grade levels and on different subjects from those in the present study.

The following part focuses on implications for teachers, teacher educators, students, curriculum developers and policy makers based on the findings of this research study.

#### **5.4 Implications**

Several implications could be deduced for mathematics teachers, teacher educators, students, curriculum developers, and educational policy makers based on this research study.

The results of this study showed that DGS-Assisted Instruction using GeoGebra in the teaching of transformation geometry had significant effects on students' achievement and geometric thinking. Since mathematics is abstract in its nature, GeoGebra as a dynamic geometry software helped students to make them better understand the transformation geometry and fractals with concrete real life examples via visualization in a dynamic learning environment. Besides, the students were active participants during the whole class since the lesson prepared required active involvement of the students, such as making constructions, working on the activities, testing the mathematical ideas and hypotheses. Thus, mathematics teachers should integrate technological tools into their classrooms and they should know how to use dynamic geometry softwares adequately, effectively and systematically. This study may provide them with an example of this application to make them aware of the positive influence of dynamic geometry softwares on students' understanding of mathematics. In addition, mathematics teachers should be provided with opportunities to develop effective teaching methods with the help of technology integration (i.e. lessons conducted on dynamic geometry softwares). They should be provided with in-service education courses on the integration of technology into mathematics teaching to help them gain the necessary competency for teaching with computers.

Furthermore, mathematics teachers should be aware of different teaching methodologies, which can be applied in mathematics classrooms, and they should pay special attention to the student-centered and technology enriched instruction

methods. These methods can be easily applied and do not require much time and money and they provide conceptual understanding of mathematics. The teachers should also take into account that achievement in geometry and geometric thinking are moderately strongly correlated (Usiskin, 1982), as it was found in the present study and the fact that the use of dynamic geometry software affects students' geometric thinking significantly. Due to this positive correlation, teachers should be aware of the importance of geometric thinking and the fact that it can be increased over time if appropriate materials and teaching methods are used. Considering all the advantages dynamic geometry software provided and the correlation between mathematics achievement and geometric thinking, the mathematics teachers are recommended to use such softwares in their mathematics lessons while they are teaching different subjects through longer time span to provide better understanding and permanent learning and to get better results in mathematics teaching.

As for teacher educators, faculties of education should include various courses to train prospective teachers for adequate and effective use of technological tools in mathematics teaching since such skills were needed and used as the main part of the instruction given to the students by the researcher. As a result of the use of such skills in the instruction process, which were given by the researcher as a mathematics teacher, provided students with higher mathematics achievement and progress through geometric thinking level in the present study. Since the mathematics teachers might not have time to develop their technological skills when they become inservice mathematics teachers, it is important for prospective teachers to experience the use of technology in mathematics teaching when they study at undergraduate level at the faculties of education. Thus, the prospective mathematics teachers should be equipped with necessary practical and theoretical knowledge and they should be competent in integration of technology into mathematics learning environment, such as conducting DGS-Assisted Instruction, Computer-Assisted Instruction, Computer-Based Instruction and smart boards effectively, or integrating other computer technologies into mathematics teaching, before they start to teach at the schools as an in-service mathematics teacher. In addition, the use of these



alternative teaching methods and supportive tools should be encouraged. In this way, teachers can make their mathematics teaching more effective so that students may be provided with a better understanding of mathematics.

According to the results of the study, it was concluded that elementary school students' mathematics achievement increased at the end of the teaching with DGS-Assisted Instruction. As it was mentioned before, one week training process for the basic use of GeoGebra was given to the students prior to the treatment in the present study. This training process was needed since the students were not familiar to the dynamic geometry software and they were not able to use it. Thus, students at elementary level should be provided the opportunity to use dynamic mathematics softwares regularly to gain necessary knowledge and skills to use them appropriately and adequately. In order to remedy this gap, elementary students may take elective courses to enrich and practice their knowledge in a dynamic learning environment. Such an application may be an integral part of mathematics teaching and students may consolidate their learning regularly. In this context, the technological resources (i.e. hardware, software, internet access) and course options of the schools may be refined for K-12 students.

As it was also found in the present study, curriculum developers should pay special attention to the moderate correlation between students' mathematics achievement and geometric thinking in the present study as it was also found in the studies of Burger (1985), Burger and Shaughnessy (1986), Geddes et al (1982), Geddes, Fuys and Tichler (1985), Mayberry (1981), Shaughnessy and Burger (1985), Usiskin (1982). Concordantly, the mathematics curriculum can be refined and designed to raise "geometric thinkers". Curriculum developers should also consider the effectiveness of DGS-Assisted Instruction on the development of geometric reasoning and take into account the results of the present study during the curriculum development process. Moreover, the integration of dynamic mathematics softwares into mathematics curriculum and its importance should be highly emphasized rather than merely remaining as a recommendation as in the Teacher Guide Textbook, which says "Dynamic Geometry Software may be utilized". For instance, curriculum

developers may insert dynamic mathematics-based activities or tasks in the textbook as applications of the topics in a dynamic learning environment. Also, the teachers should be provided with extra time for the use of dynamic mathematics softwares in the teaching of the topics covered in the K-12 mathematics curriculum.

One further implication can be suggested for the mathematics textbooks. The mathematics textbooks for elementary students are in need of concrete activities that help students to improve geometric thinking. These activities should also be applicable in a dynamic learning environment. In other words, the activities based on dynamic mathematics software should be included in the mathematics textbooks for the elementary students.

In the following part, recommendations for further research studies are offered in the light of the results of the present study.

### **5.5 Recommendations for Further Research Studies**

In the present study, the main purpose was to investigate the effect of DGS-Assisted Instruction on 8<sup>th</sup> grade students' mathematics achievement in transformation geometry (fractals, rotation, reflection, translation), geometric thinking and attitude towards mathematics and technology. In this section, some recommendations are suggested for further studies in the light of the findings of the present study.

This study was based on the topic of transformation geometry taught in 8<sup>th</sup> grade mathematics lessons as stated in the National Mathematics Curriculum of Turkey. Hence, the results of the present study cannot be generalized to the other grade levels and other contents of mathematics. It is strongly recommended for further research studies to conduct this instruction method with different grade levels and to cover different learning areas of mathematics. For instance, longitudinal research studies may be conducted in order to examine the long-term effects of DGS-Assisted Instruction on students' mathematics achievement, geometric thinking and attitude towards mathematics and technology. That is to say, the effect of this

instruction method can be investigated with a group of students ranging from the 6<sup>th</sup> to 8<sup>th</sup> grade. In this way, their increase or decrease in mathematics achievement, development of geometric reasoning and change in attitude towards mathematics can be understood in detail.

The treatment process of this study lasted three weeks. In order to gain evidence related to the long-term effects of DGS-Assisted Instruction on students' mathematics achievement, geometric thinking and attitude towards mathematics, further research studies could be conducted through a longer time span of treatment. For instance, in order to observe a significant change in students' attitude towards mathematics, research studies which lasts longer can be carried out. Also, the effects size of the treatment can be increased by conducting long-term studies covering different learning areas at the same grade level.

This study was conducted in a private school, in which the class sizes were too small. Especially in public schools, class sizes are not as small as the ones in this study. All of the students in the experimental group, which received DGS-Assisted Instruction had a chance to use a computer on their own. In crowded classrooms, such kind of setting may not be satisfied. Therefore, similar studies should be conducted with different class sizes in order to determine the effect of class size on achievement of students, their geometric thinking and attitude towards mathematics and technology. Since this study was conducted at a private school, the subjects were from a high socioeconomic status. Thus, further studies can also be conducted at public schools in order to determine the effect of school type and/or socioeconomic issues on achievement of students, geometric thinking and attitude towards mathematics and technology.

Convenience sampling method, which limits the generalizability, was chosen in the present study. Further research studies may be conducted with students chosen randomly from a public or private elementary school. In this way, the researchers may also have a chance to increase the generalizability of their study results to a broader population which has similar characteristics to the sample of their study. In

other words, the present study should be replicated with a larger randomly selected sample.

In this study, the experimental and control groups were taught by two different mathematics teacher, the researcher and the mathematics teacher of the school in which the study was conducted. Thus, the differences between the implementers can be considered as a limitation in terms of the internal validity of the study. Therefore, further experimental studies are recommended to be conducted with teaching carried out by the same implementers of the treatment.

The quantitative research methodology was adopted in the present study. That is, the study was restricted with the analysis of quantitative data. Hence, in order to provide in-depth insight into the effects of dynamic mathematics softwares on students' achievement, geometric thinking level and attitude towards mathematics, qualitative research methodologies, such as observation and interviews, are also recommended to be used.

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## APPENDICES

### APPENDIX A

#### MATHEMATICS ACHIEVEMENT TEST

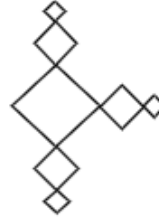
1.



1.şekil



2.şekil



3.şekil

4.şekil

a) Yukarıda ki şekiller, 1.şeklin orantılı olarak küçültülmüş ya da büyütülmüş halleri ile inşa edilmiş, her adımda aynı kural uygulanmış bir örüntü müdür (fraktal)? Cevabınızı açıklayınız.

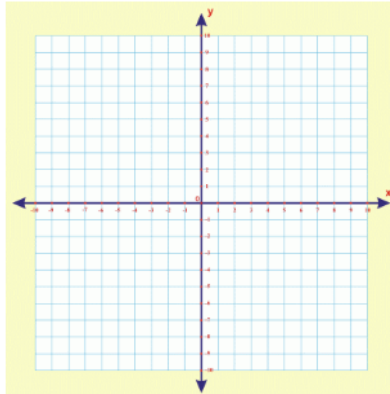
b) Aynı kural devam etseydi bu örüntüde ki 4.şekil nasıl olurdu yukarıya çiziniz.

c) Çizdiğiniz 4.şekilde kaç eşkenar dörtgen vardır?

2. Aykut'un bir köpeği ve bu köpeğinin bir kulübesi vardır.

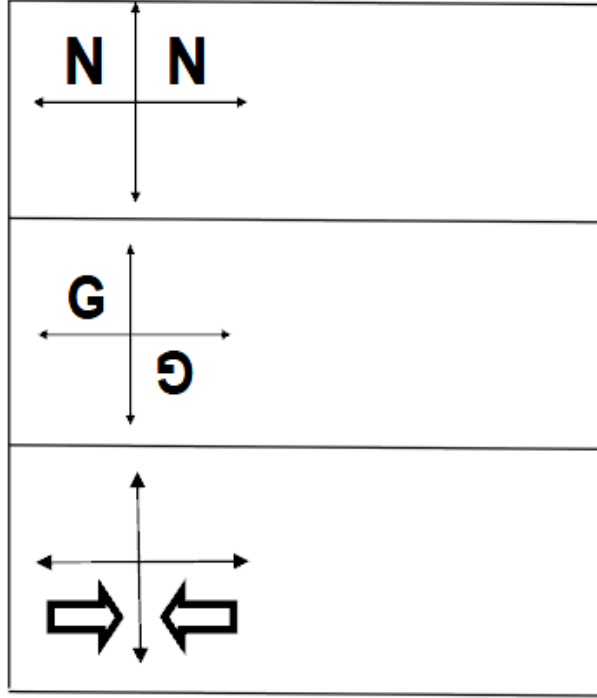
a) Bu kulübenin yerini beğenmeyen Aykut, kulübeyi evin etrafında saat yönünde  $90^\circ$  döndürmek istiyor. Aşağıdaki koordinat düzlemi üzerine köşelerinin koordinatları  $K(3, -3)$ ,  $L(6, -3)$ ,  $M(3, -8)$  ve  $N(6, -8)$  olarak belirlenen kulübeyi ve de dönme hareketi sonrasındaki yerini çiziniz. (evi orijin noktası olarak kabul ediniz)

b) Dönme hareketi sonrasında oluşan yeni kulübenin şeklini y ekseninde yansıtınız ve oluşan şeklin koordinatlarını şeklin köşelerine yazınız.



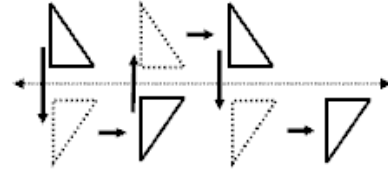


3. Aşağıdaki çizimlerde, şekillere hangi dönüşüm hareketlerinin yaptırıldığını belirleyip şeklin yanına yazınız.



4. Yandaki şekilde yapılmış olan dönüşüm hareketlerini sırasıyla aşağıdaki noktalı yere yazınız.

.....



5.



1.adım



2.adım

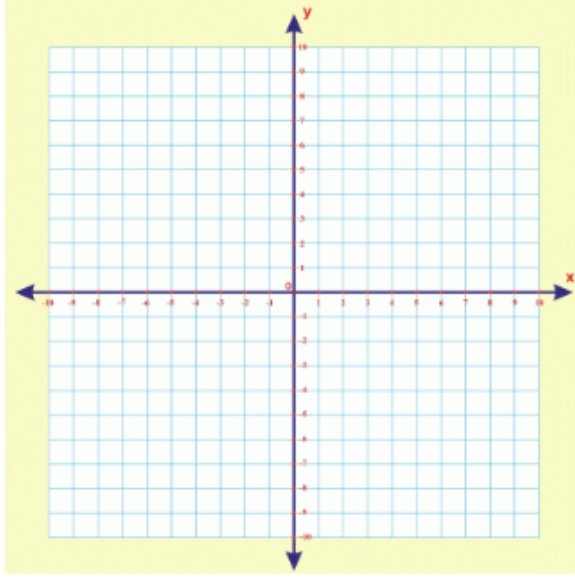
3.adım  
(Fraktal ise)

3.adım  
(Örüntü ise)

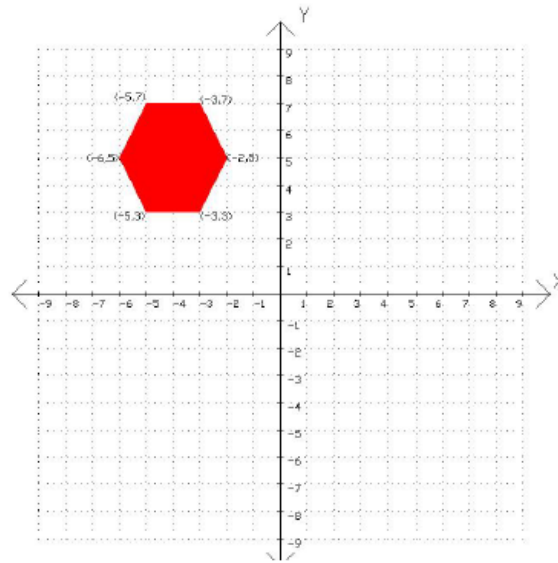
a) Yukarıda ilk 2 adımı verilen örüntünün 1.adımdaki şeklin orantılı olarak küçültülmüş ya da büyütülmüş halleri ile inşa edilmiş, her adımda aynı kural uygulanmış bir örüntü (fraktal) olabilmesi için 3.adım ne olmalıdır? Cevabınızı açıklayın.

b) Yukarıda ilk 2 adımı verilen şeklin 3.adımını siz belirleyiniz ve fraktal olmayan bir örüntü oluşturunuz. Cevabınızı açıklayın.

6. Aşağıdaki koordinat düzlemi Ali'nin evinin banyosunun yukarıdan görüntüsüdür. Banyoda var olan bir kelebeğin bacaklarının koordinatları  $A(1,2)$ ,  $B(6,2)$ ,  $C(6,6)$  ve  $D(1,6)$  şeklindedir. Bu kelebek koordinat düzlemine göre 8birim aşağıya yürürse aynadaki bacaklarının görüntüsünün koordinatları nasıl olur eksen üzerinde gösteriniz. (aynayı x eksenini olarak düşününüz)



7. Koordinatları  $A(-5,7)$ ,  $B(-3,7)$ ,  $C(-2,5)$ ,  $D(-3,3)$ ,  $E(-5,3)$  ve  $F(-6,5)$  şeklinde verilen bir uçurtma, rüzgârın etkisiyle koordinat düzlemi üzerindeki orijin etrafında saat yönünde  $270^\circ$  lik bir dönme hareketi yapıyor. Uçurtmanın koordinat düzlemi üzerindeki yeni görüntüsünü çizin ve koordinatlarını şeklin üzerine yazınız.



8. Aşağıdaki şekiller belli bir kurala göre dizilmiştir. Bu kuralı bulunuz ve 4.adımı bu kuralı göz önünde bulundurarak çiziniz. Cevabınızı açıklayınız.



1.adım



2.adım

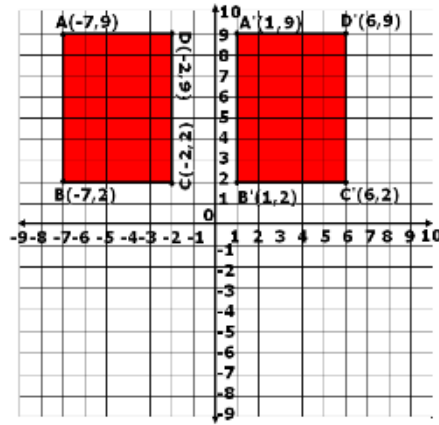


3.adım

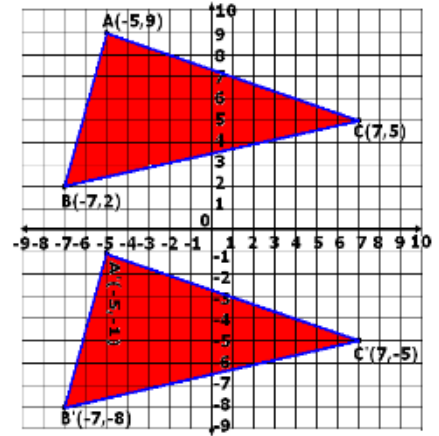
4.adım

9. Aşağıdaki grafiklerde yapılmış olan dönüşüm hareketlerini koordinat eksenlerinin altına yazınız.

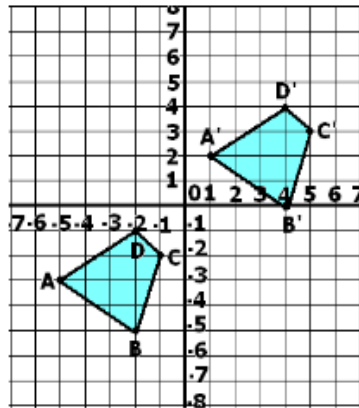
a)



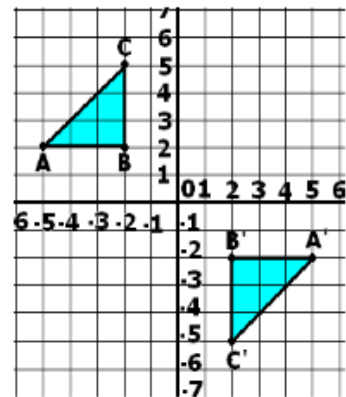
b)



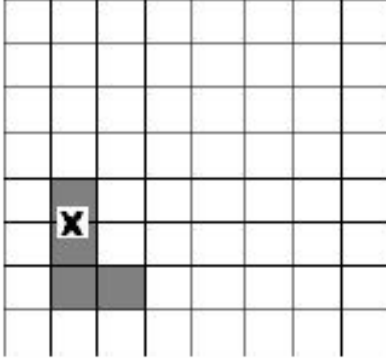
c)



d)



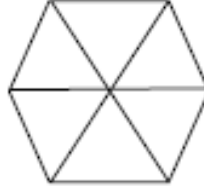
10. Aşağıdaki şekil birim karelerden oluşmuştur. X cismi yukarı yönde 4 birim, sağa doğru 3 birim ötelenirse şeklin son hali nasıl olur çiziniz ve cevabınızı açıklayınız.



11. Aşağıda ici taralı olarak verilmiş altıgenin (1. şekil), saatin tersi yönünde  $60^\circ$  döndürülmüş halini yanındaki 2. şekil üzerinde çiziniz ve cevabınızı açıklayınız.



1. şekil

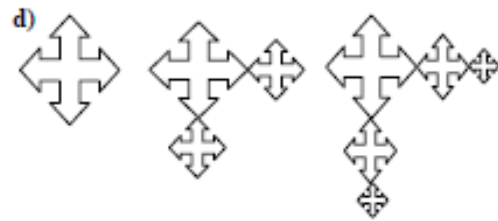
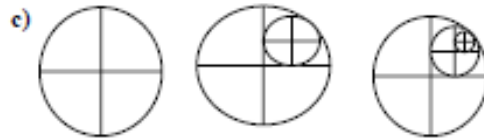
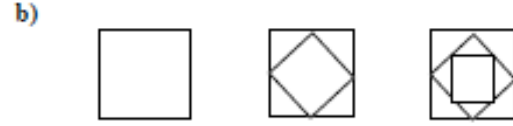


2. şekil

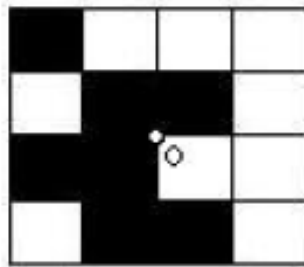
12. Aşağıda verilen sözcüğün aynadaki görüntüsünü çiziniz.



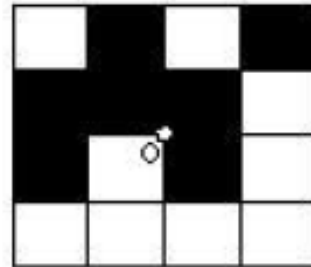
13. Aşağıda ilk 3 adımı verilen şekillerden hangisi veya hangileri fraktal (şeklin orantılı olarak küçültülmüş ya da büyütülmüş halleri ile inşa edilmiş, her adımda aynı kural uygulanmış bir örüntü) hangileri değildir? Cevabınızı şekillerin altına açıklayınız.



14.



Şekil-1



Şekil-2

Yukarıdaki merkezi "O" harfi ile gösterilen koordinat ekseninde, Şekil-1'i kullanarak Şekil-2'yi elde etmek istiyorsak, şekil-1'e nasıl bir dönüşüm hareketi uygulanmalıdır açıklayınız.

## APPENDIX B

### SPECIFICATION TABLE OF MATHEMATICS ACHIEVEMENT TEST

Questions	Objectives						Cognitive Level Steps			
	1	2	3	4	5	6	Comprehend	Application	Analysis	Synthesis
1	x					x	x	x	x	
2			x	x				x		x
3		x	x	x			x		x	
4		x	x		x				x	
5	x							x	x	
6		x	x					x		x
7				x			x			
8				x			x	x	x	
9		x	x	x	x				x	
10		x						x		
11				x				x		
12			x				x			
13	x								x	
14				x			x			

## APPENDIX C

### OPEN-ENDED QUESTIONS' SCORING RUBRIC OF MATHEMATICS ACHIEVEMENT TEST

Scores	Answer Types
0	<ul style="list-style-type: none"> <li>No answer</li> <li>Completely irrelevant or off-topic answer</li> </ul>
1	<ul style="list-style-type: none"> <li>Partial understanding without explanation (e.g. in question 6 it was expected from students both to translate and to reflect the shape. If a student was able to translate the shape but was not able to reflect the shape or vice versa and if s/he was not able to explain the result)</li> <li>Some hints that show the mathematical understanding or mathematical concepts (fractals, rotation, reflection, translation, etc.) familiarity (e.g. similar to the above example, if student was able to translate or rotate or reflect the object correctly even that is not the expected correct result)</li> <li>Minimal understanding of the task</li> <li>Misunderstanding of the question and the correct answer through that misunderstanding without explanation (e.g. in question 6, although it was asked students to translate the shape 8 units, a student translated 10 units or it was asked students to translate the shape 8 units, a student translated 10 units or it was asked students to translate the shape down but s/he translated the shape up or it was asked students to reflect the shape upon the x axis but s/he reflected the shape upon the y axis correctly without explanation.)</li> </ul>
2	<ul style="list-style-type: none"> <li>Correct answer without explanation (e.g. in question 1, the answer was correct but there was no explanation.)</li> <li>Mistake sourced drawing</li> <li>Correct rule application but wrong result (e.g. in question 5, the definition of fractals was correct but the drawing was incorrect or any other explanation was correct but drawing was incorrect or in question 10, although it was asked to translate 4 units, student translated 3 units but s/he explained the result as it was 4 units, i.e. only drawing was incorrect)</li> <li>Limited success resulting in an inconsistent or flawed explanation</li> <li>Correct drawing without explanation (e.g. question 8 and 11, an example is available in Appendix H)</li> </ul>

- 
- |   |   |
|---|---|
|   | <ul style="list-style-type: none"><li>• Insufficiency and lacking in some minor ways of answer or explanation (e.g. in question 5, while defining fractals the main difference between fractals and patterns was not explained, i.e. lack of information or explanation)</li></ul>  |
| 3 | <ul style="list-style-type: none"><li>• Correct answer with sufficient explanation (e.g. in question 5 both the definition of fractals and the drawings were correct, and in question 14 the shape's rotation direction and rotation angle were correct)</li><li>• A response demonstrating full and complete understanding</li></ul> |
-



## **APPENDIX D**

### **OBJECTIVES MEASURED BY MATHEMATICS ACHIEVEMENT TEST**

**Objective 1:** Students should be able to construct and draw patterns with line, polygon and circle models and decide which patterns are fractals.

**Objective 2:** Students should be able to translate a polygon through a coordinate axis or a line and to draw its image after translation.

**Objective 3:** Students should be able to draw a polygon's image after making a reflection through a coordinate axis and translation through any line.

**Objective 4:** Students should be able to explain rotation motion, draw shapes after rotation on a plane and according to the given angle, and draw the image of a polygon under the rotation motion around the origin on a coordinate axis.

**Objective 5:** Students should be able to determine the image of shapes after making translation with reflection and construct it.

**Objective 6:** Students should be able to construct patterns and decide the number of shapes in the patterns.

## APPENDIX E

### VAN HIELE GEOMETRIC THINKING LEVEL TEST

# VAN HIELE GEOMETRİ TESTİ

## YÖNERGE

Bu test 25 sorudan oluşmaktadır. Sizden testteki her soruyu bilmeniz beklenmemektedir.

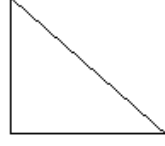
Kitapçığı açtığınızda;

- 1- Bütün soruları dikkatlice okuyun.
- 2- Doğru olduğunu düşündüğünüz seçenek üzerinde düşünün. Her soru için tek bir doğru cevap vardır. Cevap kağıdına doğru olduğunu düşündüğünüz seçeneği işaretleyin.
- 3- Soru kağıdındaki boşlukları çizim yapmak için kullanabilirsiniz.
- 4- İşaretlemiş olduğunuz cevabı değiştirmek isterseniz, ilk işareti tamamen siliniz.
- 5- Bu test için size verilecek süre 35 dakikadır.

### VAN HIELE GEOMETRİ TESTİ

1- Aşağıdakilerden hangisi ya da hangileri karedir?

- a) Yalnız K
- b) Yalnız L
- c) Yalnız M
- d) L ve M
- e) Hepsi karedir.



K

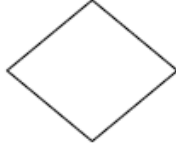


L

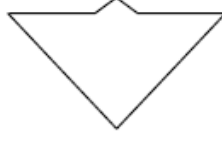


M

2- Aşağıdakilerden hangisi ya da hangileri üçgendir?



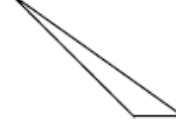
U



V



Y



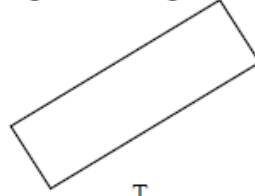
Z

- a) Hiçbiri üçgen değildir.
- b) Yalnız V
- c) Yalnız Y
- d) Y ve Z
- e) V ve Y

3- Aşağıdakilerden hangisi ya da hangileri dikdörtgendir?



S



T



U

- a) Yalnız S
- b) Yalnız T
- c) S ve T
- d) S ve U
- e) Hepsi dikdörtgendir.

4- Aşağıdakilerden hangisi ya da hangileri karedir?



F



G



H



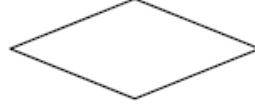
I

- a) Hiçbiri kare değildir.
- b) Yalnız G
- c) F ve G
- d) G ve I
- e) Hepsi karedir.

5- Aşağıdakilerin hangisi ya da hangileri paralelkenardır?



K



L



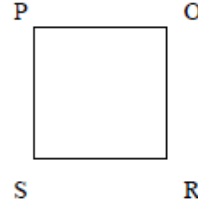
M

- a) Yalnız K
- b) Yalnız L
- c) K ve M
- d) Hiçbiri paralel kenar değildir.
- e) Hepsisi paralel kenardır.

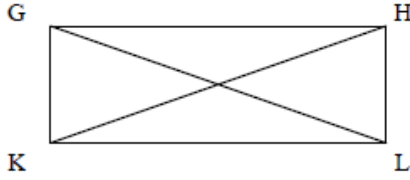
6- PQRS bir karedir.

Aşağıdakilerden hangi özellik her kare için doğrudur?

- a) [PR] ve [RS] eşit uzunluktadır.
- b) [OS] ve [PR] diktir.
- c) [PS] ve [OR] diktir.
- d) [PS] ve [OS] eşit uzunluktadır.
- e) O açısı R açısından daha büyüktür.

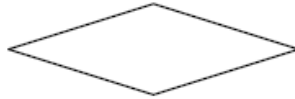


7- Bir GHJK dikdörtgeninde, [GL] ve [HK] köşegendir. Buna göre aşağıdakilerden hangisi her dikdörtgen için doğrudur?



- a) 4 dik açısı vardır.
- b) 4 kenarı vardır.
- c) Köşegenlerinin uzunlukları eşittir.
- d) Karşılıklı kenarların uzunlukları eşittir.
- e) Seçeneklerin hepsi her dikdörtgen için doğrudur.

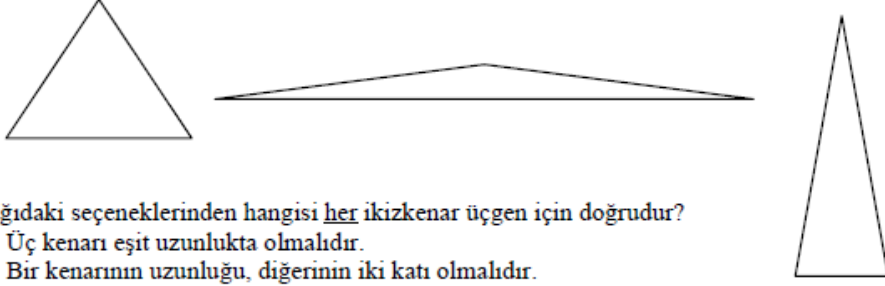
8- Eşkenar dörtgen tüm kenar uzunlukları eşit olan, 4 kenarlı bir şekildir. Aşağıda 3 tane eşkenar dörtgen verilmiştir.



Aşağıdaki seçeneklerinden hangisi her eşkenar için doğru değildir?

- a) İki köşegenin uzunlukları eşittir.
- b) Her köşegen, aynı zamanda açıortaydır.
- c) Köşegenleri birbirine diktir.
- d) Karşılıklı açılarının ölçüsü eşittir.
- e) Seçeneklerin hepsi her eşkenar dörtgen için doğrudur.

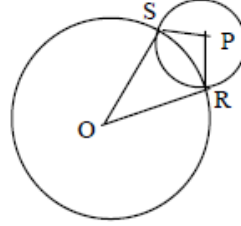
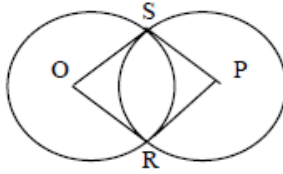
9- İkizkenar üçgen, iki kenarı eşit olan üçgendir. Aşağıda üç ikizkenar üçgen verilmiştir.



Aşağıdaki seçeneklerinden hangisi her ikizkenar üçgen için doğrudur?

- Üç kenarı eşit uzunlukta olmalıdır.
- Bir kenarının uzunluğu, diğerinin iki katı olmalıdır.
- Ölçüsü eşit olan en az iki açısı olmalıdır.
- Üç açısının da ölçüsü eşit olmalıdır.
- Seçeneklerinden hiçbirisi her ikizkenar üçgen için doğru değildir.

10. Merkezleri birbirinin içinde yer almayan ve merkezleri P ve O ile adlandırılmış olan iki çember 4 kenarları PROS şeklini oluşturmak üzere R ve S noktalarında kesişirler. Aşağıda iki örnek verilmiştir.



Aşağıdaki seçeneklerinden hangisi her zaman doğru değildir?

- PROS şeklinin iki kenarı eşit uzunlukta olacaktır.
- PROS şeklinin en az iki açısının ölçüsü eşit olacaktır.
- [PO] ve [RS] dik olacaktır.
- P ve O açılarının ölçüleri eşit olacaktır.
- Yukarıdaki seçeneklerin hepsi doğrudur.

11. Önerme S: ABC üçgeninin üç kenarı eşit uzunluktadır.

Önerme T: ABC üçgeninde, B ve C açılarının ölçüleri eşittir.

Buna göre aşağıdakilerden hangisi doğrudur?

- S ve T önermeleri ikisi de aynı anda doğru olamaz.
- Eğer S doğruysa, T de doğrudur.
- Eğer T doğruysa, S de doğrudur.
- Eğer S yanlışsa, T de yanlıştır.
- Yukarıdaki seçeneklerin hiçbirisi doğru değildir.

12. Önerme 1: F şekli bir dikdörtgendir.

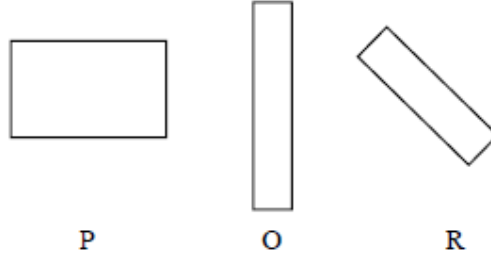
Önerme 2: F şekli bir üçgendir.

Bu iki önermeye göre aşağıdakilerden hangisi doğrudur?

- Eğer 1 doğruysa, 2 de doğrudur.
- Eğer 1 yanlışsa, 2 doğrudur.
- 1 ve 2 aynı anda doğru olamaz.
- 1 ve 2 aynı anda yanlış olamaz.
- Yukarı seçeneklerin hiçbirisi doğru değildir.

13. Aşağıdaki şekillerden hangisi ya da hangileri dikdörtgen olarak adlandırılabilir?

- a) Hepsi
- b) Yalnız O
- c) Yalnız R
- d) P ve O
- e) O ve R



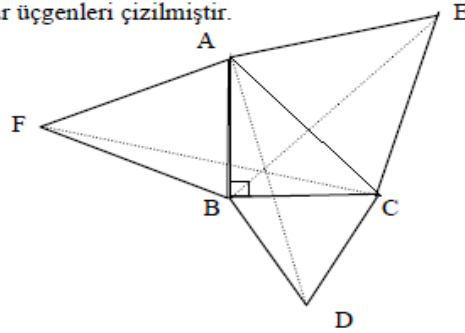
14. Tüm dikdörtgenlerde olup, bazı paralelkenarlarda olmayan özellik nedir?

- a) Karşılıklı kenarları eşittir.
- b) Köşegenler eşittir.
- c) Karşılıklı kenarlar paraleldir.
- d) Karşılıklı açıları eşittir.
- e) Yukarıdaki seçeneklerin hiçbiri doğru değildir.

15- Aşağıdakilerden hangisi doğrudur?

- a) Dikdörtgenlerin tüm özellikleri, tüm kareler için geçerlidir.
- b) Karelerin tüm özellikleri, tüm dikdörtgenler için de geçerlidir.
- c) Dikdörtgenin tüm özellikleri, tüm paralel kenarlar için geçerlidir.
- d) Karelerin tüm özellikleri, tüm paralel kenarlar için geçerlidir.
- e) Yukarıdaki seçeneklerin hiçbiri doğru değildir.

16- Aşağıda bir ABC dik üçgeni verilmiştir. ABC üçgeninin kenarları üzerinde; ACE, ABF ve BCD eşkenar üçgenleri çizilmiştir.



Bu bilgilerden [AD], [BE] ve [CF] ortak bir noktadan geçtikleri kanıtlanabilir. Bu kanıt size neyi ifade eder?

- a) Yalnızca bu üçgen için; [AD], [BE] ve [CF] nin ortak bir noktası olduğundan emin olabiliriz
- b) Sadece bazı dik üçgenlerde; [AD], [BE] ve [CF] nin ortak bir noktası vardır.
- c) Herhangi bir dik üçgende, [AD], [BE] ve [CF]nin ortak bir noktası vardır.
- d) Herhangi bir üçgende, [AD], [BE] ve [CF]nin ortak bir noktası vardır.
- e) Herhangi bir eşkenar üçgende, [AD], [BE] ve [CF]nin ortak bir noktası vardır.

17- Aşağıda bir şeklin üç özelliği verilmiştir.

Özellik D: Köşegenleri eşit uzunluktadır. Özellik S: Bir karedir. Özellik R: Bir dikdörtgendir.

Bu özellikler dikkate alındığında aşağıdakilerden hangisi doğrudur?

- a) D gerektirir S, o da gerektirir R.
- b) D gerektirir R, o da gerektirir S.
- c) R gerektirir D, o da gerektirir S.
- d) R gerektirir S, o da gerektirir D.
- e) S gerektirir R, o da gerektirir D.

18- Aşağıda iki önerme verilmiştir.

I- Eğer bir şekil dikdörtgense, köşegenleri birbirini ortalayarak keser.

II- Eğer bir şeklin köşegenleri birbirini ortalayarak kesiyorsa şekil dikdörtgendir.

Buna göre aşağıdakilerden hangisi doğrudur?

- a) I'in doğru olduğunu kanıtlamak için, II nin doğru olduğunu kanıtlamak yeterlidir.
- b) II'nin doğru olduğunu kanıtlamak için, I in doğru olduğunu kanıtlamak yeterlidir.
- c) II'nin doğru olduğunu kanıtlamak için, köşegenleri birbirini ortalayan bir dikdörtgen bulmak yeterlidir.
- d) II nin yanlış olduğunu kanıtlamak için, köşegenleri birbirini ortalayan dikdörtgen olmayan bir şekil bulmak yeterlidir.
- e) Yukarıdaki seçeneklerin hiçbiri doğru değildir.

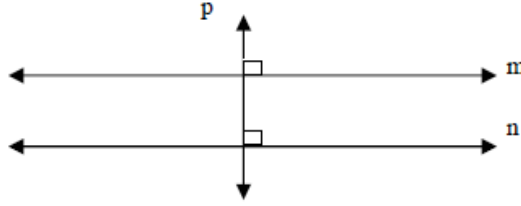
19- Aşağıdaki üç ifadeyi inceleyin.

{1} Aynı doğruya dik olan iki doğru paraleldir.

{2} İki paralel doğrudan birine dik olan doğru, diğerine de diktir.

{3} Eğer iki doğru eş uzaklıktaysa paraleldir.

Aşağıdaki şekilde, m ve p, n ve p doğrularının birbirine dik olduğu verilmiştir. Buna göre yukarıdaki cümlelerden hangisi ya da hangileri m doğrusunun n doğrusuna paralel olmasının nedeni olabilir?



- a) Yalnız {1}
- b) Yalnız {2}
- c) Yalnız {3}
- d) {1} ya da {2}
- e) {2} ya da {3}

20- Aşağıdaki ifadelerden hangisi doğrudur?

Geometride,

- a) Her terim tanımlanabilir ve her doğru önermenin doğru olduğu kanıtlanabilir.
- b) Her terim tanımlanabilir ama bazı önermelerin doğru olduğunu varsaymak gerekir.
- c) Bazı terimler tanımsız kalmalıdır, ama bütün doğru önermelerin doğruluğu kanıtlanabilir.
- d) Bazı terimler tanımsız kalmalıdır ve doğru olduğu varsayılmış bazı önermelere gerek vardır.
- e) Yukarıdaki seçeneklerinden hiçbiri doğru değildir.

21- Bir açıyı üçlemek demek onu üç eşit parçaya bölmek demektir. 1847 yılında, P.L. Wantzel bir açının yalnızca pergeli ve işaretlenmemiş cetvel kullanarak üçlenemeyeceğini kanıtlamıştır. Bu kanıttan nasıl bir sonuca varabilirsiniz?

- a) Açılar yalnızca pergeli ve işaretlenmemiş cetvel kullanarak iki eş parçaya ayrılamazlar.
- b) Açılar yalnızca pergeli ve işaretlenmiş cetvel kullanarak üçlenemezler.
- c) Açılar herhangi bir çizim aracı kullanarak üçlenemezler.
- d) Gelecekte, birinin yalnızca pergeli ve işaretlenmiş cetvel kullanarak açıları üçlemesi mümkün olabilir.
- e) Hiç kimse, açıları yalnızca pergeli ve işaretlenmemiş cetvel kullanarak üçleyecek genel bir yöntem bulamayacaktır.



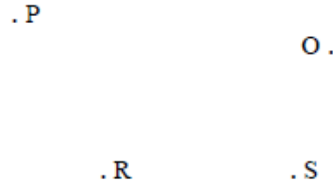
22- Ali adlı bir matematikçinin kendi tanımladığı geometriye göre, aşağıdaki önerme doğrudur.

Bir üçgenin iç açılarının ölçüsü toplamı 180 dereceden azdır.

Buna göre aşağıdakilerden hangisi doğrudur?

- a) Ali üçgenin açılarını ölçerken hata yapmıştır.
- b) Ali mantıksal bir hata yapmıştır.
- c) Ali doğru sözcüğünün anlamını bilmiyordur.
- d) Ali bilinen geometridekilerden farklı varsayımlarla başlamıştır.
- e) Yukarıdaki seçeneklerden hiçbiri doğru değildir.

23- F geometrisinde, her şey alışık olduklarımızdan farklıdır. Burada sadece dört nokta ve 6 doğru vardır. Her doğru iki nokta içerir. Eğer P, O, R ve S nokta ise, {P,O}, {P,R}, {P,S}, {O,R}, {O, S} ve {R, S} doğrulardır.



Kesişme ve paralel terimlerinin F- geometrisindeki kullanımı şöyledir: {P, O} ve {P,R} doğruları P' de kesişirler çünkü P {P, O} ve {P,R} ın ortak noktasıdır. {P, O} ve {R, S} doğruları paraleldir çünkü ortak hiçbir noktaları yoktur.

Buna göre, aşağıdakilerden hangisi doğrudur?

- a) {P, R} ve {O, S} kesişirler.
- b) {P, R} ve {O, S} paraleldir.
- c) {O, R} ve {R,S} paraleldir.
- d) {P, S} ve {O, R} kesişirler.
- e) Yukarıdaki seçeneklerin hiçbiri doğru değildir.

24- İki ayrı geometri kitabı 'dikdörtgen' sözcüğünü iki farklı şekillerde tanımlamıştır. Buna göre aşağıdakilerden hangisi doğrudur?

- a) Kitaplardan birinde hata vardır.
- b) Tanımlardan biri yanlıştır. Dikdörtgen için iki farklı tanım olamaz.
- c) Bir kitapta tanımlanan dikdörtgenin özellikleri diğer kitaptakinden farklı olmalıdır.
- d) Bir kitapta tanımlanan dikdörtgenin özellikleri diğer kitaptakiyle aynı olmalıdır.
- e) Kitaplarda tanımlanan dikdörtgenlerin farklı özellikleri olabilir.

25- Varsayalım aşağıdaki önerme I ve II yi kanıtladınız.

I. Eğer p ise q dir.

II. Eğer s ise q değildir.

Buna göre önerme I ve II den aşağıdakilerden hangisi çıkartılabilir?

- a) Eğer s ise, p değildir.
- b) Eğer p değil ise q değildir.
- c) Eğer p veya q ise s dir.
- d) Eğer p ise s dir.
- e) Eğer s değil ise p dir.



## APPENDIX F

### OBJECTIVES OF EACH TASK FOR THE FIRST 15 ITEMS OF VAN HIELE GEOMETRIC THINKING LEVEL TEST

Question	Level	Objective
1	1	Identifying square
2	1	Identifying square
3	1	Identifying rectangle
4	1	Identifying triangle
5	1	Identifying parallelogram
6	2	Comprehend properties of square
7	2	Comprehend properties of rectangle
8	2	Comprehend properties of diamond
9	2	Comprehend properties of isosceles triangles
10	2	Comprehend properties of radius and tangent of a circle; and comprehend properties of rhombus
11	3	Show simple deduction related to properties of triangle
12	3	Show simple deduction related to rectangle and triangle
13	3	Comprehend hierarchy between square and rectangle
14	3	Compare rectangle and parallelogram
15	3	Comprehend hierarchy between square and rectangle and parallelogram

## APPENDIX G

### MATHEMATICS AND TECHNOLOGY ATTITUDE SCALE

#### MATEMATİK VE TEKNOLOJİYE YÖNELİK TUTUM ÖLÇEĞİ

Bu ölçek bir bilgi testi değildir ve bu nedenle hiçbir sorunun “doğru cevabı” yoktur. Aşağıda yer alan sorularla Geometer-Sketchpad yazılımı ile yapmış olduğunuz dersleriniz hakkındaki fikirleriniz öğrenilmek istenmektedir. Verilen yargı cümlelerini okuyarak kendi düşüncenizi en iyi yansıtan yalnız bir seçeneği işaretleyiniz.

Adı , Soyadı:

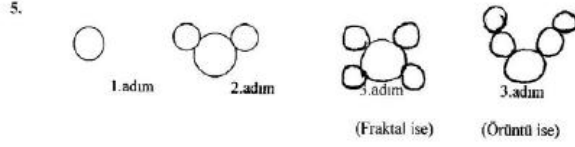
Sınıfı: No: Yaş : Cinsiyet : (E) (K)

	Hemen hemen hiç	Ara sıra	Yaklaşık yarın yarıya	Genellikle	Hemen hemen her zaman
1. Matematikte zor konsantre olurum.					
2. Öğretmenin sorduğu sorulara cevap vermeye çalışırım.					
3. Hata yaptığımda onları düzeltene kadar çalışırım.					
4. Eğer bir problemi çözmeyi başaramazsam, çözmek için başka fikirler denemeye devam ederim.					
5. Bilgisayar kullanmakta başarılıyım.					
6. VCR, VCD, DVD, MP3 ve cep telefonu gibi teknolojik aletleri kullanmakta başarılıyım.					
7. Birçok bilgisayar sorununu çözebilirim.					
8. Okul için gerekli olan herhangi bir bilgisayar programını iyice öğrenebilirim.					
9. Beynim matematiğe iyi çalışır.					

	Kesinlikle katılmıyorum	Katılmıyorum	Emin değilim	Katılıyorum	Kesinlikle Katılıyorum
10. Matematikten iyi notlar alabilirim.					
11. Matematikteki zorluklarla başa çıkabileceğimi biliyorum.					
12. Matematikte kendime güveniyorum					
13. Matematikte yeni şeyler öğrenmeye ilgi duyuyorum.					
14. Matematikte emeğinizin karşılığında ödüllendirilirsiniz.					
15. Matematik öğrenmek eğlencelidir.					
16. Matematik sorularını çözdüğüm zaman bir çeşit memnuniyet hissedirim.					
17. Matematik için bilgisayar yazılımları/programları kullanmayı seviyorum.					
18. Matematikte bilgisayar yazılımları/programları kullanmak, fazladan sarf edilen zaman, emek ve efora değer.					
19. Bilgisayar yazılımları/programları kullanıldığı zaman matematik daha ilginç hale gelebilir.					
20. Bilgisayar yazılımları/programları matematiği daha iyi öğrenmeme yardım edebilir.					

## APPENDIX H

### SAMPLES OF STUDENT ANSWERS

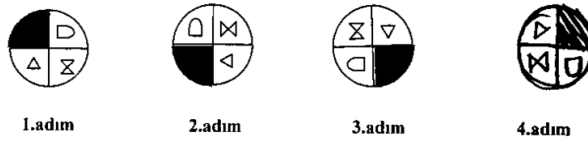


a) Yukarıda ilk 2 adımı verilen örüntünün 1.adımdaki şekli, orantılı olarak küçültülmüş ya da büyütülmüş halleri ile inşa edilmiş, her adımda aynı kural uygulanmış bir örüntü (fraktal) olabilir mi? Cevabınızı açıklayın.

Bunun için ilk önce her kenarına birer tane daire koyulmalı. 3.adımda örüntü olursa iki kenarına fraktal olur 5a dört kenarına koyulmalıdır.

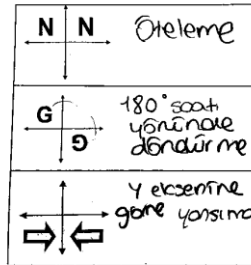
In the student answer related to the question 5 which was given above, the student was given 0 point since both the drawing and the explanation of the question was incorrect.

8. Aşağıdaki şekiller belli bir kurala göre dizilmiştir. Bu kuralı bulunuz ve 4.adımı bu kuralı göz önünde bulundurarak çiziniz. Cevabınızı açıklayınız.



In the student answer related to the question 8 which was given above, the student was given 2 point. Although the drawing was correct, there was no explanation of the question written.

3. Aşağıdaki çizimlerde, şekillere hangi dönüşüm hareketlerinin yapıldığını belirleyip şeklin yanına yazınız.



In the student answer related to the question 3 which was given above, the student was given 3 point since the answer was completely correct.

## **APPENDIX J**

### **LESSON PLANS OF EXPERIMENTAL GROUP**

#### **LESSON PLAN 1**

**Area of Learning:** Geometry

**Sub-area of Learning:** Pattern and Tessellation

**Grade level:** 8<sup>th</sup>

**Objective(s):**

- Students should be able to construct patterns and decide the number of shapes in the patterns
- Students should be able to construct and draw patterns with line, polygon and circle models and decide which patterns are fractals.

**Duration:** 40 + 40 minutes (2 class hours)

**Key Terms:** Pattern, fractal

**Resources / Materials:** Computer with GeoGebra software for each student, projector, pencil, activity sheet, worksheet

**Skills:** Computer usage, Geometrical Thinking, Mathematical Reasoning, Mathematical Correlation

**Prerequisite Knowledge:** Line, polygon, circle, computer-literacy.

**Activities (Description of the procedures):**

#### **I. STARTING**

- Students are introduced the topic of today.
- Students are remembered the concept of pattern and they are asked if they know what pattern means and how a pattern is constructed.
- Students are shown some examples of patterns through projector (examples from the real life and the nature such as honeycombs, carpet models, and decorations composed of the patterns).

- Students are asked to open GeoGebra to work on the pattern activities. Teacher tells students the aim of the activity and what they are supposed to do in the activity.
- The name of the GeoGebra file to open are told (e.g. open the file of “kılavuz-etkinlik 1”) and necessary directions/instructions about the activity are given (e.g. rotate the triangle around the center by angle of  $30^\circ$ ).
- After working on the pattern activities, the students were asked what they realized about the patterns.
- Students are asked to create their own patterns on GeoGebra. Teacher guide students when needed. Then, students’ works are checked.

## **II. MIDDLE**

- Students are asked if they have ever heard about “fractal”.
- After a brief verbal explanation of the concept of fractal, the students are shown fractal examples from the nature such as plants, land forms etc.
- Students are asked to open GeoGebra to work on the fractal activities. Teacher tells students the aim of the activity and what they are supposed to do in this activity.
- The name of the GeoGebra file to open are told and necessary directions/instructions about the activity are given (e.g. open the file of “yaprak-fraktal”).
- After working on the fractal activities in GeoGebra, the students were asked some exploring questions and they were discovered what fractal means and how a fractal is composed of.
- Students are remembered the previous pattern activities and they are asked if there is a difference between the fractal and the pattern. The answer of this question is discussed with the students.
- After working on the fractal activities, the students were asked to create their own fractals on GeoGebra. Teacher guide students when needed. Then, students’ works are checked.

- Activity sheet related to the difference between fractal and pattern is distributed to the students (Appendix G).
- The students are asked to write their answers to the questions given in the activity sheet.

### **III. END**

- The main points and important definitions of the topic are summarized.
- The concept of fractal and the difference between fractal and pattern is clarified.
- Students' questions related to the topic are answered.
- Worksheet as a mini quiz is distributed to the students to elicit information about students' learning and understanding with respect to the topic (Appendix G).
- The students are given some minutes and they are asked to do the exercises given on the worksheet.
- The answers of the questions given on the worksheet were discussed with the students and they are made to realize the right and the wrong answer.
- Students are asked to save their work on GeoGebra in a folder.
- Homework assignment from the textbook is given.
- The students are told the next lesson's topic.

Screenshots from the GeoGebra activities used in this lesson were given below:

### Pattern Activities;

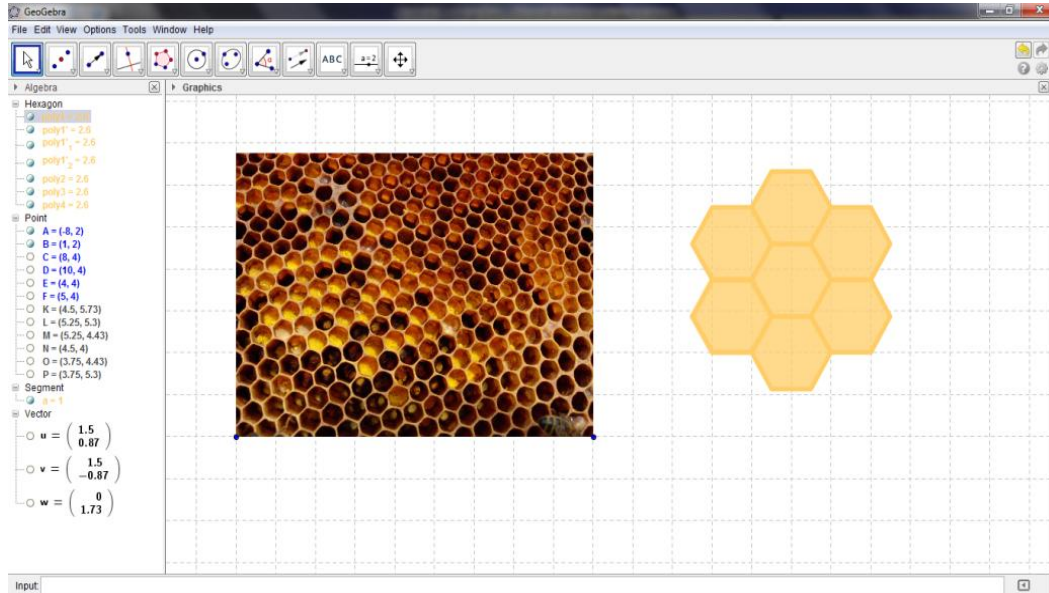


Figure 13. Honeycomb activity

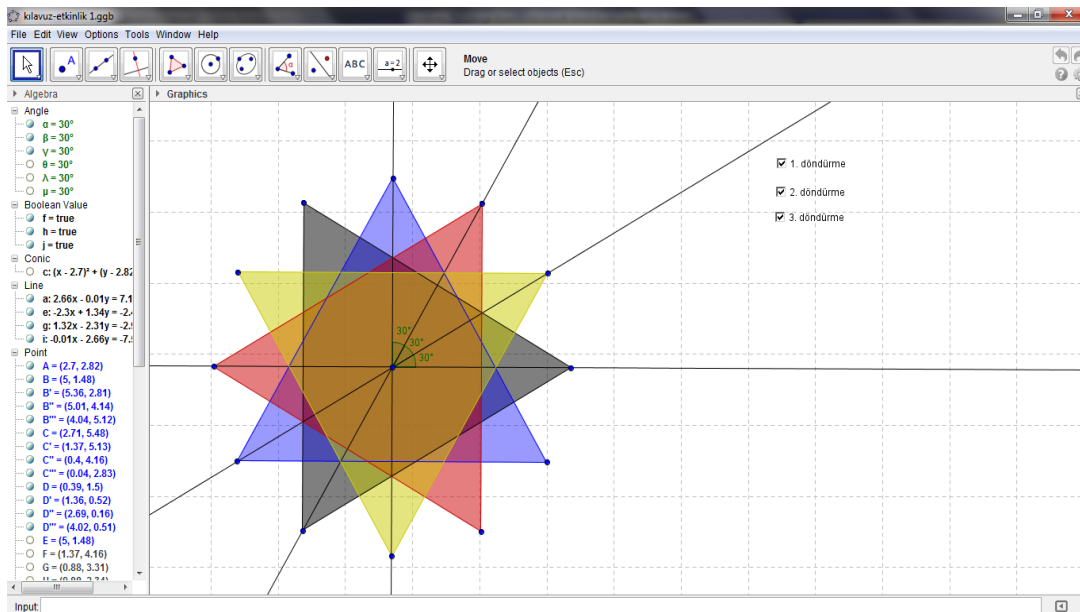


Figure 14. Pattern composed of a rotated triangles activity



## Fractal Activities;

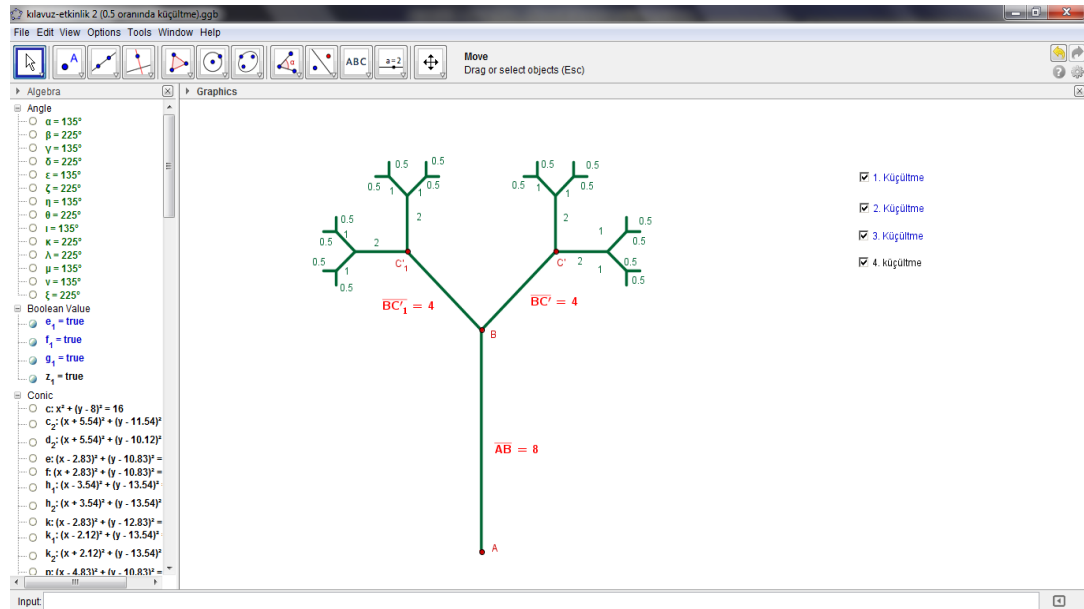


Figure 15. Fractal activity

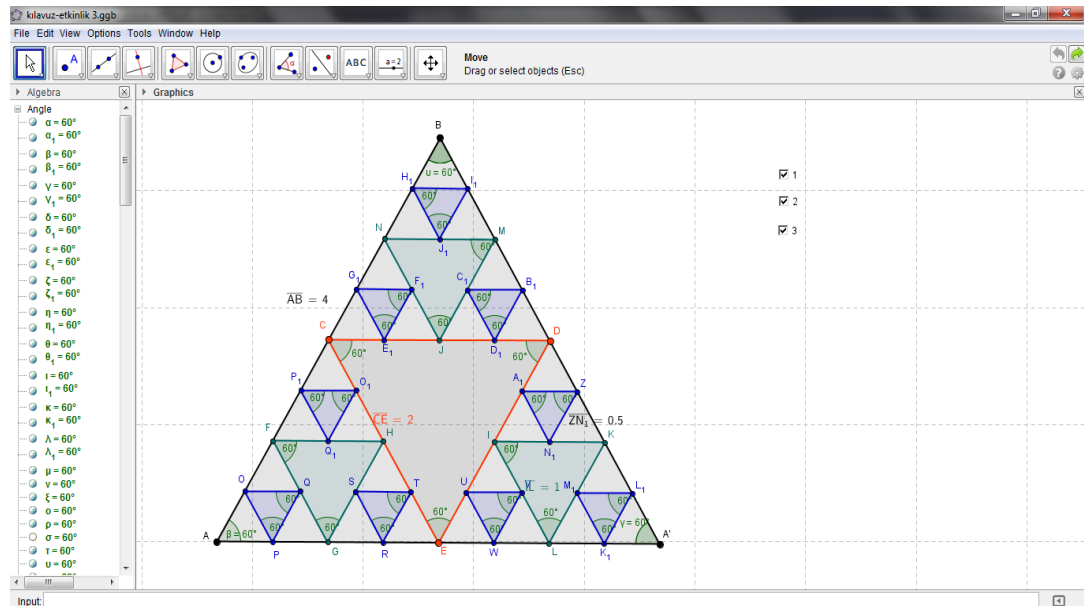


Figure 16. Sierpinski triangle activity

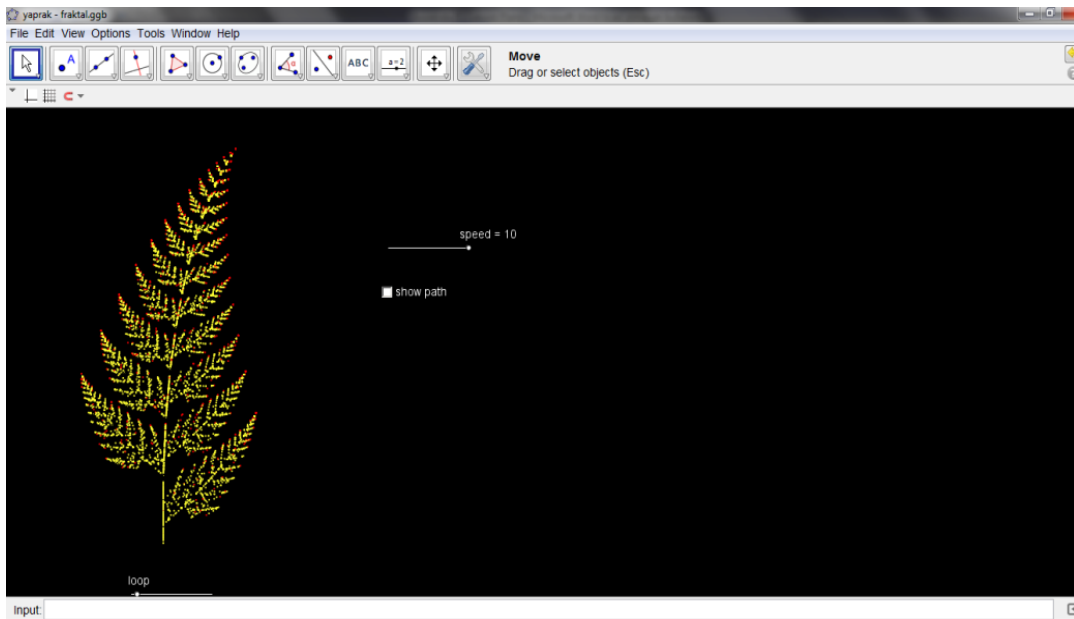


Figure 17. Leaf fractal activity

**Fractal examples from the nature;**

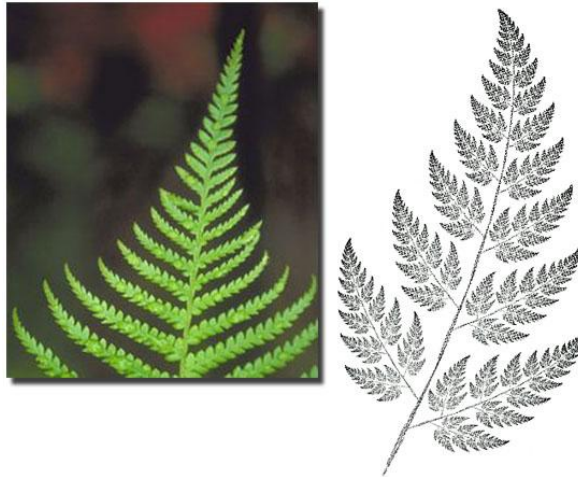


Figure 18. Fractal examples from the nature

## LESSON PLAN 2

**Area of Learning:** Geometry

**Sub-area of Learning:** Transformation Geometry

**Grade level:** 8<sup>th</sup>

**Objective(s):**

- Students should be able to translate a polygon through a coordinate axis or a line and to draw its image after translation.

**Duration:** 40 + 40 minutes (2 class hours)

**Key Terms:** Translation through a coordinate axis or a line

**Resources / Materials:** Computer with GeoGebra software for each student, projector, pencil, activity sheet, worksheet

**Skills:** Computer usage, Geometrical Thinking, Mathematical Reasoning, Mathematical Correlation

**Prerequisite Knowledge:** Line, polygon, coordinate system, computer-literacy.

### Activities (Description of the procedures):

#### I. STARTING

- Students are introduced the transformation geometry and the topic of today.
- Students are asked what transformation means in Mathematics? After the answer of this question is taken from the students, the content of transformation geometry topic (reflection, translation, rotation) is mentioned as a brief information for the beginning of the lesson.
- Students are told the motions of translation, reflection and rotation are explained with transformation geometry in Mathematics. Then, teacher starts to give deeper information about the translation which is the topic of today.
- Students are remembered the motion of translation which they learned in the 6<sup>th</sup> grade. Then, the students are asked how they describe the motion of translation and asked to tell the sorts of translation (e.g. translation to the right, left, up, and down).

- Students are also asked the properties of the motion of translation and they are asked the questions of “what kind of changes occur when an object is translated? , what changes and what stays the same in a translated object?”
- Then, the students are given the real life examples of the motion of translation (e.g. the one skiing, moving car throughout a straight road, people moving in a bank queue etc.).

## **II. MIDDLE**

- After a small discussion related to the topic, the students are distributed the activity sheets related to the translation through a coordinate axis and a line (Appendix G).
- Students are asked to read the activity sheet. Then, they are asked to answer the questions given in the activity sheet while working on the GeoGebra. They are told that they are supposed to examine the explanations with respect to the topic on GeoGebra.
- The students are asked to open GeoGebra to work on the translation through a coordinate axis / a line activity. Teacher tells students the aim of the activity and what they are supposed to do in the activity.
- The name of the GeoGebra file to open are told (e.g. open the file of “kılavuz-etkinlik 6”) and necessary directions/instructions about the activity are given (e.g. teacher says “There are four sliders given in this GeoGebra activity. Two of them (slider e and slider g) represent the translation through x-axis and y-axis, respectively, another two sliders represent the translation through two different lines. You may move the slider “e” by 1 unit and move the slider “g” by 3 unit and see what changes or stays the same in the shape after translation).
- While students work on the GeoGebra activity, they are asked some exploring questions to make them discover and realize the changes in the translated object such as change in the axis or ordinate of a point.

- The students are given some time to deal with the activity by themselves. They translate the object both through a coordinate axis and a line by manipulating the slider. They observe the changes in the coordinates of a point via dynamic text after the shape (all points in an object) is translated. Teacher guide students when needed.
- After working on the GeoGebra activity, the students are asked to make a generalization about the image of a translated object and its new coordinates. Thus, students are made to realize the generalization about the translation through a line and a coordinate axis.
- Then, the students are asked to write their answers to the questions given in the activity sheet.
- Lastly, it is summarized that the translation through a line means the translation of all points in the shape parallelly by a specific direction (through x or y-axis) with a specific unit of translation. Also, the students are discovered that an object does not differ from its translated image in terms of the shape, direction, size or area.

### **III. END**

- The main points and important definitions of the topic are summarized.
- The concept of the motion of translation through a coordinate axis or a line is clarified.
- Students' questions related to the topic are answered.
- Worksheet as a mini quiz is distributed to the students to elicit information about students' learning and understanding with respect to the topic (Appendix G).
- The students are given some minutes and they are asked to do the exercises given on the worksheet.
- The answers of the questions given on the worksheet were discussed with the students and they are made to realize the right and the wrong answer.
- Students are asked to save their work on GeoGebra in a folder.

- Homework assignment from the textbook is given.
- The students are told the next lesson's topic.

Screenshots from the GeoGebra activities used in this lesson were given below:

Translation through a line/coordinate axis activities;

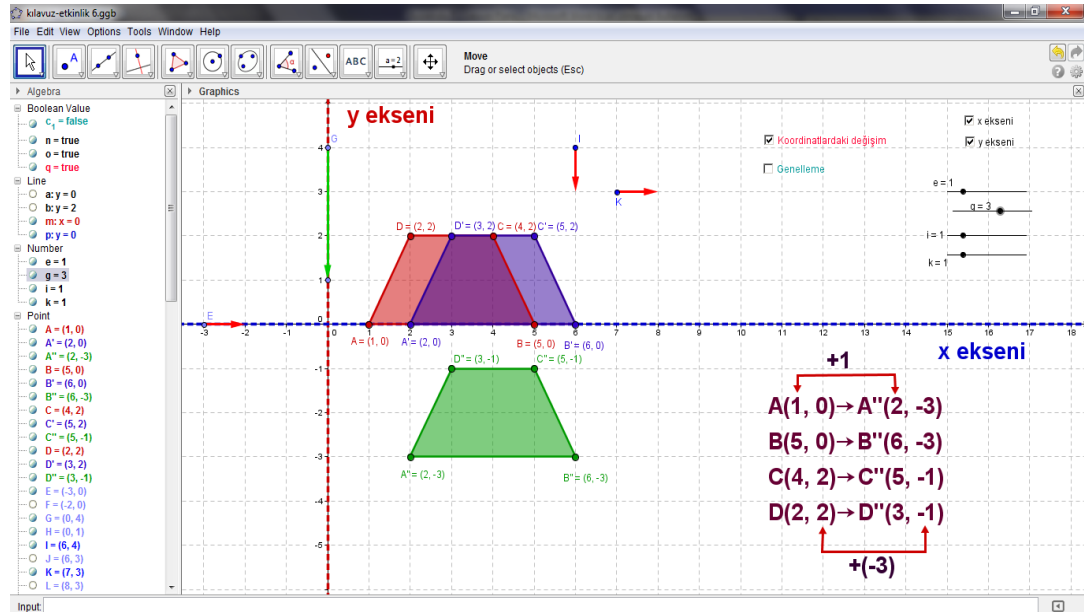


Figure 19. Translation through x-axis and y-axis

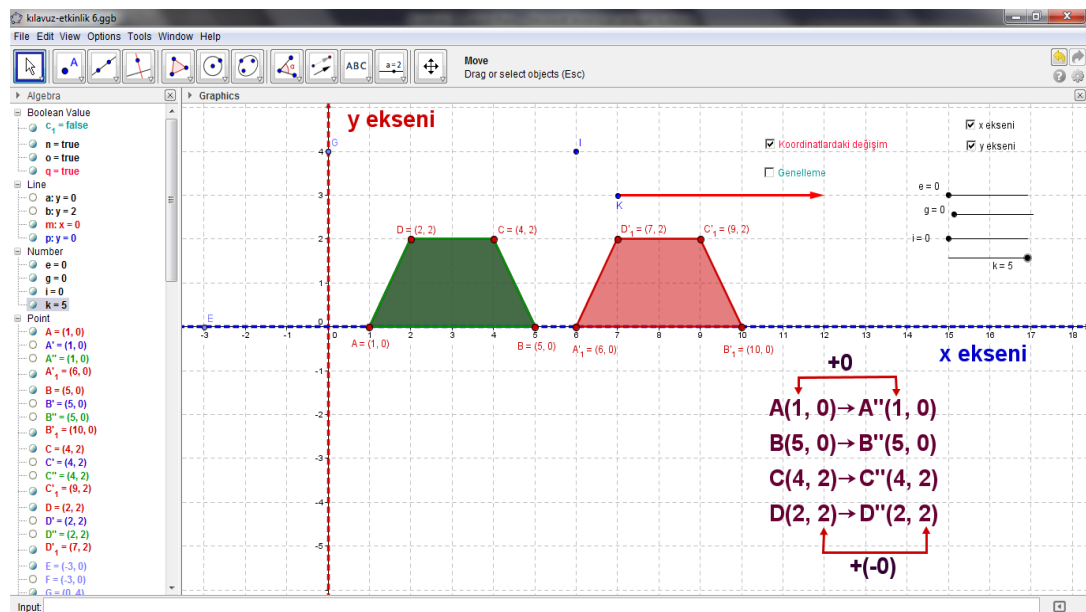


Figure 20. Translation through line K

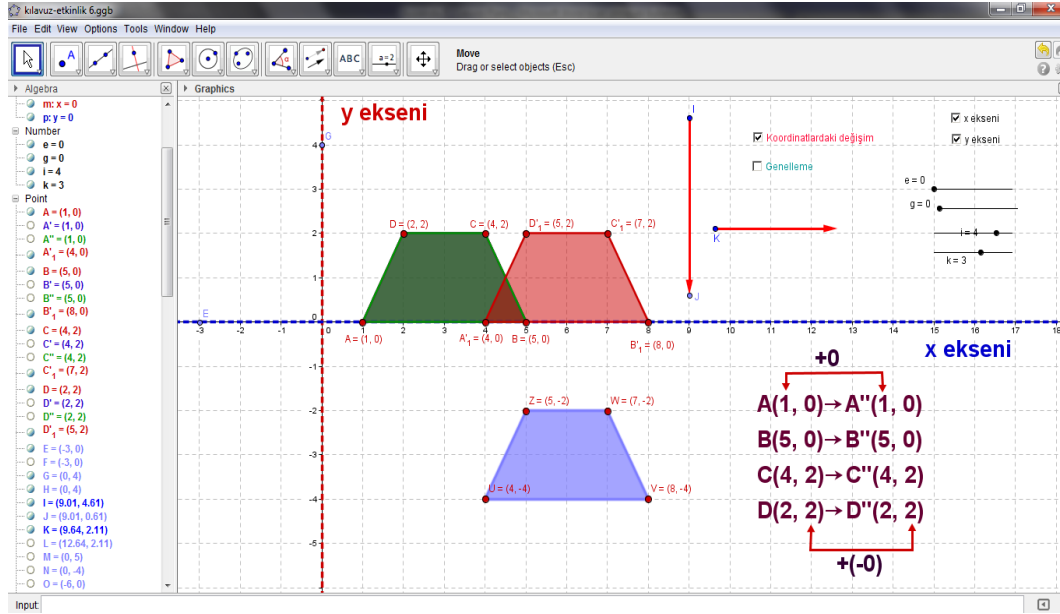


Figure 21. Translation through line K and line J

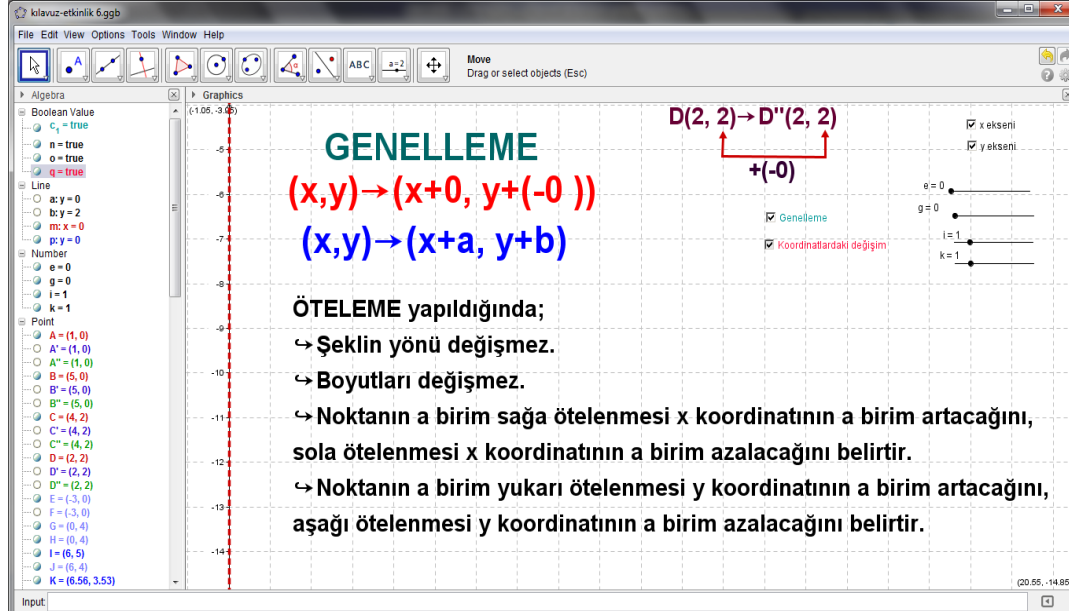


Figure 22. Generalization related to the translation through a coordinate axis / a line



### LESSON PLAN 3

**Area of Learning:** Geometry

**Sub-area of Learning:** Transformation Geometry

**Grade level:** 8<sup>th</sup>

**Objective(s):**

- Students should be able to draw a polygon's image after making a reflection through a coordinate axis.

**Duration:** 40 + 40 minutes (2 class hours)

**Key Terms:** Reflection through a coordinate axis

**Resources / Materials:** Computer with GeoGebra software for each student, projector, pencil, activity sheet, worksheet

**Skills:** Computer usage, Geometrical Thinking, Mathematical Reasoning, Mathematical Correlation

**Prerequisite Knowledge:** Line, polygon, coordinate system, computer-literacy.

#### **Activities (Description of the procedures):**

##### **I. STARTING**

- Students are introduced the topic of today.
- Students are remembered the motion of reflection which they learned in the 7<sup>th</sup> grade. Their knowledge related to the reflection is checked.
- Students are asked how they describe the motion of reflection. They are also asked to tell where they see the concept of reflection in everyday life and how they use the motion of reflection in daily life (e.g. rear view mirror, cheval glass, sliding door and so on)
- Then, the students are given the real life examples of the motion of reflection and shown some pictures from the usage of reflection in real life to make students able to see how mathematics is associated with the real life.
- Students are also asked the properties of the motion of reflection and they are asked the questions of “what kind of changes occur when an object is

reflected? , what changes and what stays the same in the image of a reflected object?”

- The answers of the questions aforementioned are discussed with the students.

## **II. MIDDLE**

- Before the motion of reflection is started to discuss, the concept of symmetry is mentioned.
- The definition of the concept of symmetry is given briefly and students are orientated to discover the fact the reflection is the same transformation as the symmetry about a line or the mirror symmetry.
- Students are orientated to realize that the symmetry axis is the coordinate axes (x-axis and y-axis) in the reflection through a coordinate axis.
- After a small discussion related to the topic, the students are distributed the activity sheets related to the reflection through a coordinate axis (Appendix G).
- Before working on the activity sheets, students’ knowledge related to determining a point in coordinate system is checked. Teacher tells students several ordered pairs and asks students to tell him the ordinate and the axis of the ordered pairs.
- Then, students are asked to read the activity sheet. They are asked to find answer to the questions given in the activity sheet while working on the GeoGebra. They are told that they are supposed to examine the explanations with respect to the topic on GeoGebra.
- The students are asked to open GeoGebra to work on the reflection through a coordinate axis activity. Teacher tells students the aim of the activity and what they are supposed to do in the activity.
- The name of the GeoGebra file to open are told (e.g. open the file of “kılavuz-etkinlik 5 (y-eksenine göre yansıma)”) and necessary directions/instructions about the activity are given (e.g. teacher says “There are three check boxes given in this GeoGebra activity. These check boxes are prepared to reflect the

triangle about y-axis and reveal the changes occurring in the coordinates of the triangle. Firstly, activate the first check box to reflect the triangle about y-axis. Then, activate the second check box to see the change in the coordinates of the reflected triangle. Lastly, move the right-side triangle dynamically by dragging its vertices to change its coordinates and create a new triangle. You can also resize the triangle and observe what kind of changes occurs in its reflection. Lastly, activate the third checkbox to see the generalization about a polygon's reflection about y-axis. You can create your own polygon (e.g. a rhombus) and reflect it about y-axis.

- The same procedure is followed for discussing the reflection about x-axis with working on the GeoGebra activity named as “kılavuz-etkinlik 5 (y-eksenine göre yansıma)”.
- In order to provide better understanding of the reflection, students are told to open another GeoGebra activities named as “Bart-Simpson and Ambulans”.
- While students work on the GeoGebra activities, they are asked some exploring questions to make them discover and realize the changes in the image of reflected triangle such as change in the axis and ordinate of a point or differences between the original shape and its image in terms of size, form, direction etc.
- The students are given some time to deal with the activity by themselves. Firstly, they are discovered that the reflection of a polygon is the reflection of all points in this polygon and the image is the mergence of reflected points. Then, students are orientated to discover the generalization about the change in the coordinates when a polygon is reflected about x-axis or y-axis. Teacher guide students when needed.
- After working on the GeoGebra activity, the students are asked to make a generalization about the properties of the image of a reflected object about coordinate axes and its new coordinates. Thus, students are made to realize the generalization about the reflection of a polygon about a coordinate axis

and the image's properties which change and stay the same as the original shape.

- Then, the students are asked to write their answers to the questions given in the activity sheet.
- It is summarized that the reflection of a polygon is the reflection of all points in this polygon and the image is the mergence of reflected points. Students are remembered the fact that the motion of reflection, the symmetry about a line, and the mirror symmetry are all the same transformations. Also, the students are discovered that an image of a reflected shape does not differ from the original shape in terms of form, size, and area, but differs in terms of direction and place.
- Students are asked to measure the image's (reflected triangle's) area and length of the sides through GeoGebra so that they could see there is no difference between the image and the original shape in terms of area or size. However, there is a difference between the image and the original shape in terms of direction and place.
- Students are discovered that reflection of a point (ordered pair) about x-axis transforms the sign of ordinate into the reverse sign but does not change the axis (  $(x,y) \rightarrow (x,-y)$  ). Also, reflection of a point (ordered pair) about y-axis transforms the sign of axis into the reverse sign but does not change the ordinate (  $(x,y) \rightarrow (-x,y)$  ).

### **III. END**

- The main points and important definitions of the topic are summarized.
- The concept of the motion of reflection through a coordinate axis is clarified.
- Students' questions related to the topic are answered.
- Worksheet as a mini quiz is distributed to the students to elicit information about students' learning and understanding with respect to the topic (Appendix G).

- The students are given some minutes and they are asked to do the exercises given on the worksheet.
- The answers of the questions given on the worksheet were discussed with the students and they are made to realize the right and the wrong answer.
- Students are asked to save their work on GeoGebra in a folder.
- Homework assignment from the textbook is given.
- The students are told the next lesson's topic.

Screenshots from the GeoGebra activities used in this lesson were given below:

Reflection through a coordinate axis activities;

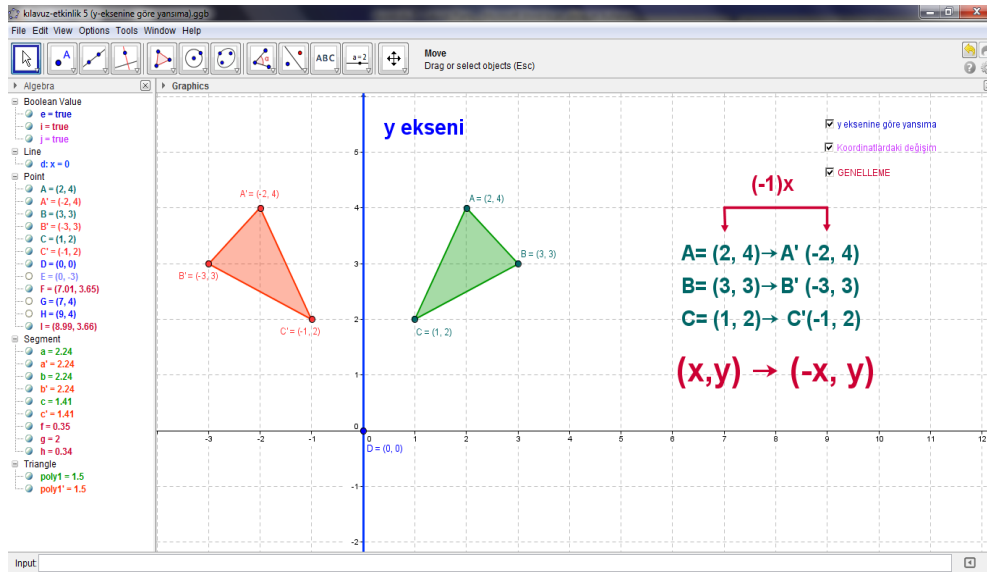


Figure 23. Reflection of a polygon about y-axis activity

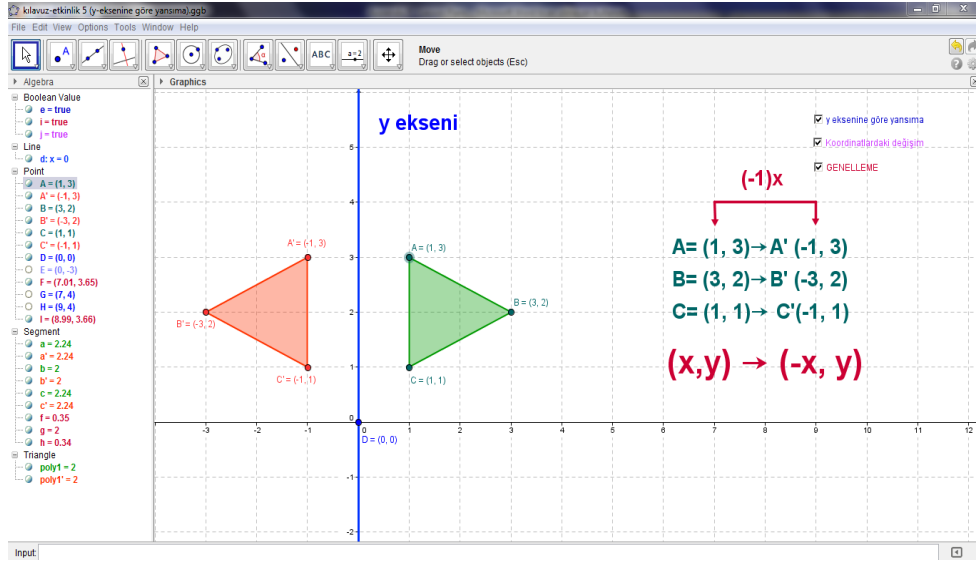


Figure 24. Reflection of a different polygon about y-axis activity

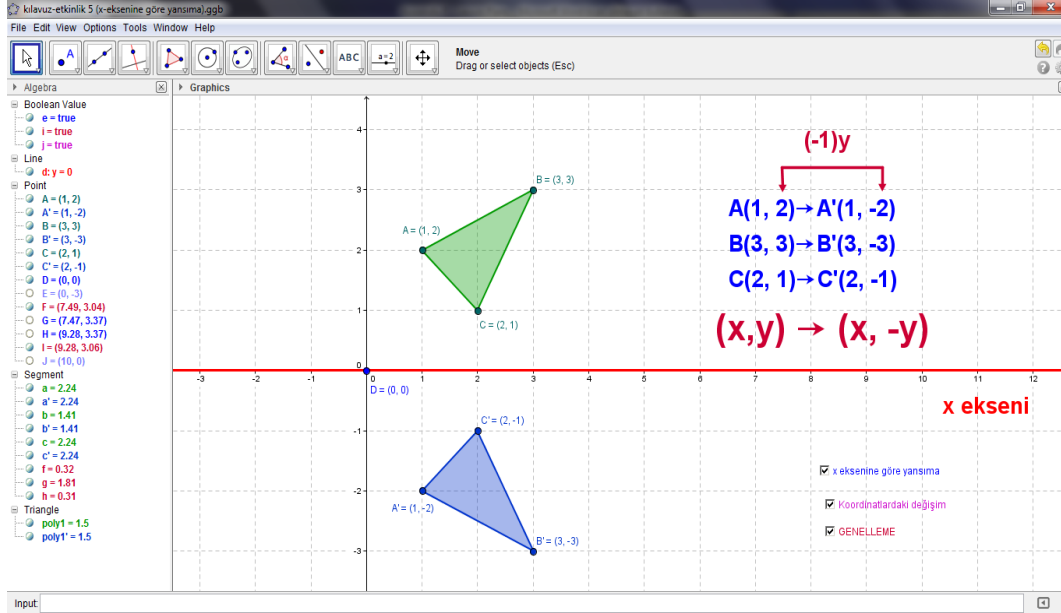


Figure 25. Reflection of a polygon about x-axis

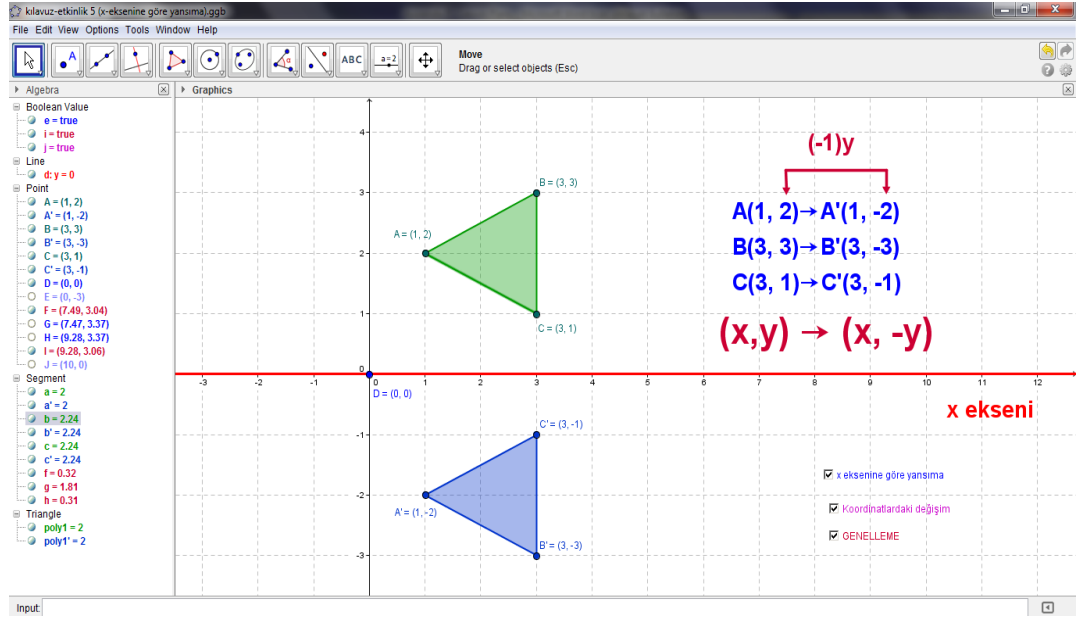


Figure 26. Reflection of a different polygon about x-axis activity

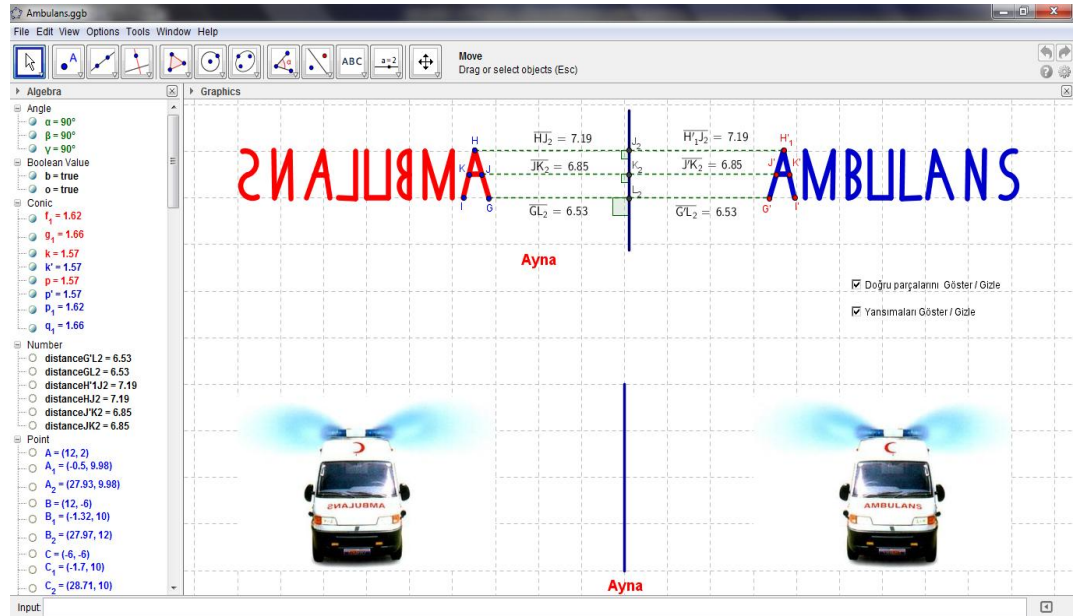


Figure 27. Ambulance activity

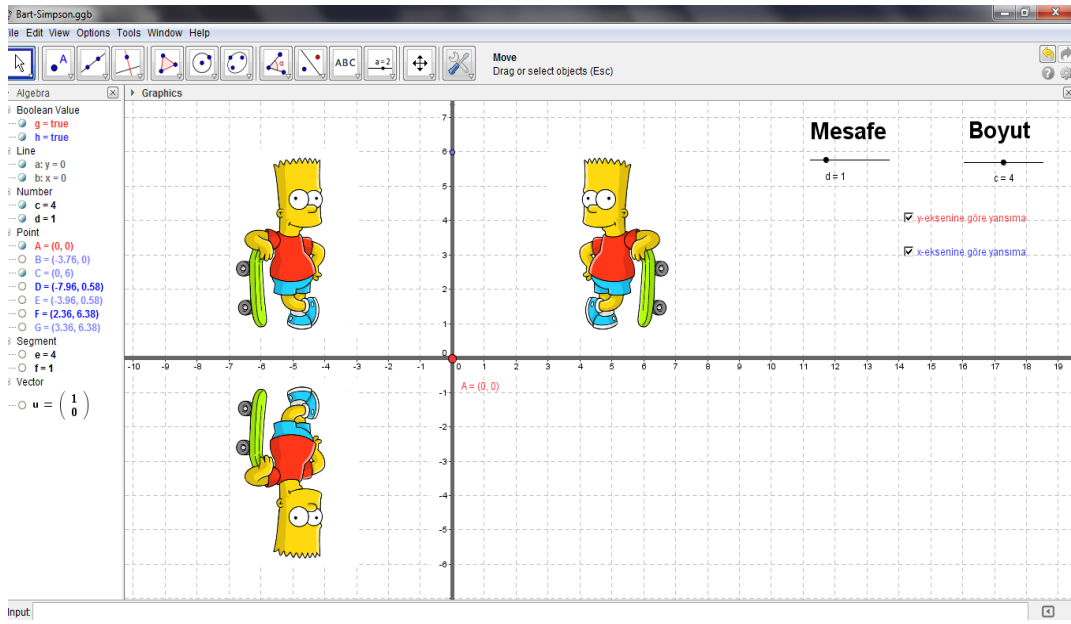


Figure 28. Bart-Simpson activity

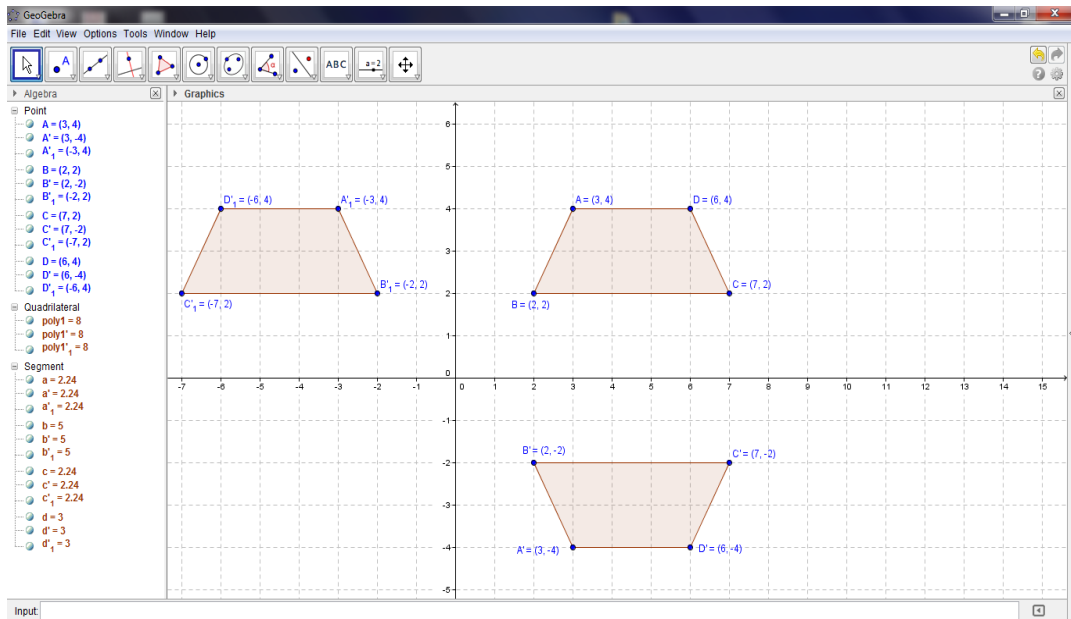


Figure 29. Sample of a student work related to the reflection of a rhombus about x-axis and y-axis



## LESSON PLAN 4

**Area of Learning:** Geometry

**Sub-area of Learning:** Transformation Geometry

**Grade level:** 8<sup>th</sup>

**Objective(s):**

- Students should be able to explain rotation motion, draw shapes after rotation on a plane by given angle, and draw the image of a polygon under the rotation motion around the origin on a coordinate axis.

**Duration:** 40 + 40 minutes (2 class hours)

**Key Terms:** Rotation around the origin

**Resources / Materials:** Computer with GeoGebra software for each student, projector, pencil, activity sheet, worksheet

**Skills:** Computer usage, Geometrical Thinking, Mathematical Reasoning, Mathematical Correlation

**Prerequisite Knowledge:** Line, angle, polygon, coordinate system, computer-literacy.

**Activities (Description of the procedures):**

### I. STARTING

- Students are introduced the topic of today.
- Students are remembered the motion of rotation which they learned in the 7<sup>th</sup> grade. Their knowledge related to the rotation motion is checked.
- Students are asked how they describe the motion of rotation. They are also asked to tell where they see the concept of rotation in everyday life and how they use the motion of rotation in daily life (e.g. clock hand rotating around the clock, the earth rotating around the sun, wheel rotating on a car, opening a door, compact discs. Anything that rotates (spins) around a central point and so on). The terms of “clockwise” and “counter clockwise” are remembered.

- Then, the students are given the real life examples of the motion of rotation and shown some pictures from the usage of rotation in everyday life to make students able to see how mathematics is associated with the real life.
- Students are also asked the properties of the motion of rotation and they are asked the questions of “what kind of changes occur when an object is rotated? , what changes and what stays the same in the image of a rotated object?”
- The answers of the questions aforementioned are discussed with the students and the mathematical definition of the rotation motion is given such as “A rotation is a transformation that is performed by spinning the object around a fixed point known as the center of rotation”.

## **II. MIDDLE**

- Students are asked if they have ever heard about “rotation around the origin”.
- After a brief verbal explanation of the concepts of the rotation around the origin, the centre of rotation, and rotation angle, it is mentioned that the centre of rotation is the point of (0,0) in rotation around the origin.
- Key terms and necessary information related to the topic are given.
- After a small discussion related to the topic, the students are distributed the activity sheets of the rotation around the origin (Appendix G).
- Students are asked to read the activity sheet. Then, they are asked to answer the questions given in the activity sheet while working on the GeoGebra activity. They are told that they are supposed to examine the explanations with respect to the topic on GeoGebra.
- The students are asked to open GeoGebra to work on the rotation around the origin activities. Teacher tells students the aim of the activities and what they are supposed to do in the activities.
- The name of the GeoGebra file to open are told (e.g. open the file of “Rotation about the origin by angle”) and necessary directions/instructions about the activity are given (e.g. teacher says “In this GeoGebra activity, There is a slider which manipulates the angle of rotation and three check

boxes that reveal the changes in the area, coordinates, and the length of the vertices of the triangle being rotated. Adjust the slider alpha for different angles (e.g.  $60^\circ$ ,  $90^\circ$ ,  $180^\circ$  etc.) to rotate the given triangle and see what changes or stays the same in the shape after rotation. Then, you can activate the check boxes to see the changes in the area, coordinates, and the length of the vertices of the rotated triangle.

- The students are asked first to rotate the three points of the triangle around the origin by angle. Then, they are asked to rotate the triangle around the origin by the same angle. In this way, they are discovered the fact that rotation of a shape is the rotation of all points belongs to this shape and the image after rotation is the mergence of these rotated points
- The students are asked to rotate the triangle counter clockwise as well.
- The same procedure is followed for discussing the second question given in the activity sheet (rotation of a hexagon around the origin by  $180^\circ$ ) with working on the GeoGebra activity named as “Rotation about the origin by angle 2”.
- While students work on the GeoGebra activity, they are asked some exploring questions to make them discover and realize the changes in the rotated object such as change in the axis or ordinate of a point belongs to the object.
- The students are given some time to deal with the activity by themselves. Teacher guide students when needed. They rotate the triangle around the origin by different angles by manipulating the slider and move the triangle dynamically by dragging its vertices to change its coordinates and create a new triangle, then rotate it. They can also resize the triangle and observe what kind of changes occurs in its image after rotation and observe the changes in the coordinates of a point via dynamic text after the shape (all points in an object) is rotated.
- Students are asked to measure the image’s (rotated triangle’s) area and length of the sides through GeoGebra so that they could see there is no difference between the image and the original shape in terms of area or size. However,

there is a difference between the image and the original shape in terms of direction and place.

- After working on the GeoGebra activity, the students are asked to make a generalization about the rotated object and its new coordinates. Firstly, they are discovered that the rotation of a polygon, just like in the reflection and translation motions, is the rotation of all points in this polygon and the image is the merge of these rotated points. Then, students are orientated to discover the generalization about the change in the coordinates when a polygon is rotated around the origin by angle in a clockwise or counterclockwise direction.
- Then, the students are asked to write their answers to the questions given in the activity sheet.
- Lastly, the students are mentioned the rotational symmetry. They are told what the rotational symmetry means and how it is determined in a shape. The students are discovered the rotational symmetry angles of some polygons (e.g. square, hexagon etc) through a few examples.

### **III. END**

- The main points and important definitions of the topic are summarized.
- The concept of the rotation around the origin is clarified.
- Students' questions related to the topic are answered.
- Worksheet as a mini quiz is distributed to the students to elicit information about students' learning and understanding with respect to the topic.
- Students are asked to open the file of "WorkSheet 2 - rotation about the origin" (Appendix G). Teacher tells students the aim of the activities and what they are supposed to do in the activities. Also, necessary directions/instructions are given.
- The students are given some time and they are asked to do the exercises given on the worksheet after they work on the dynamic worksheet of GeoGebra.

- The answers of the questions given on the worksheet were discussed with the students and they are made to realize the right and the wrong answer.
- The students are also discovered that the rotation around the origin by  $90^\circ$  clockwise is the same rotation as the rotation around the origin by  $270^\circ$  anticlockwise.
- After the students work on the dynamic worksheet in GeoGebra, they are asked to write their generalization to the worksheet.
- The students' answers were checked and they are discovered the correct rules related to the change of the coordinates of the shape after rotation around the origin by specific angles.
- The students are orientated to draw a conclusion that “every time the shape is rotated through  $90^\circ$  in a clockwise direction, the coordinates change according to the rule of  $(x, y) \rightarrow (y, -x)$ . When the shape is rotated through  $90^\circ$  in an anticlockwise direction, the coordinates change according to the rule of  $(x, y) \rightarrow (-y, x)$ . When the shape is rotated through  $180^\circ$ , the coordinates change according to the rule of  $(x, y) \rightarrow (-x, -y)$ . When the shape is rotated through  $360^\circ$ , its coordinates do not change, stays the same”.
- Students are asked to save their work on GeoGebra in a folder.
- Homework assignment from the textbook is given.
- The students are told the next lesson's topic.

Screenshots from the GeoGebra activities used in this lesson were given below:

### Rotation around the origin activities;

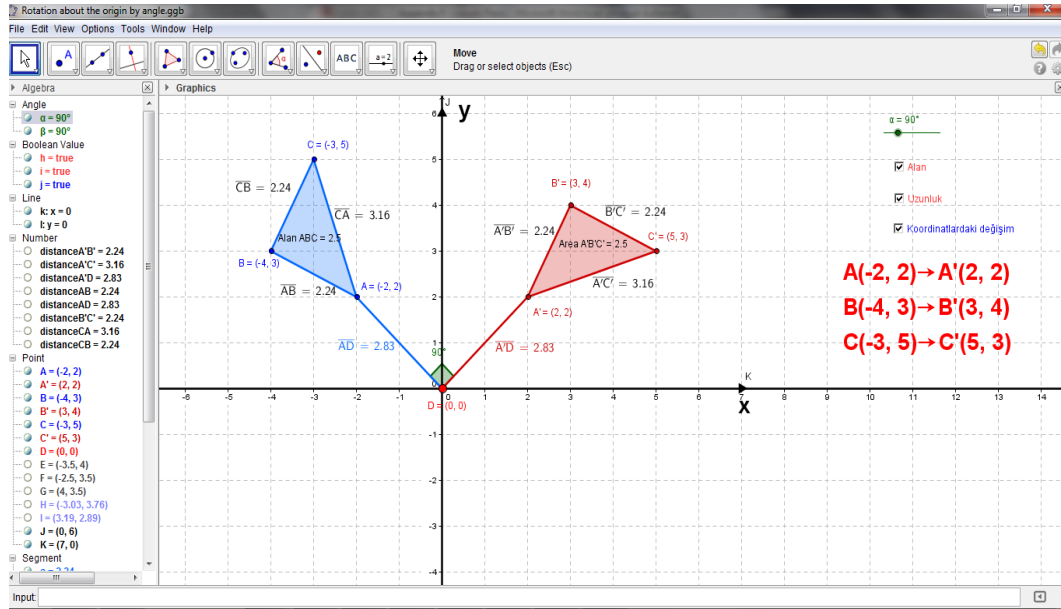


Figure 30. Rotation of a triangle around the origin through 90° clockwise activity

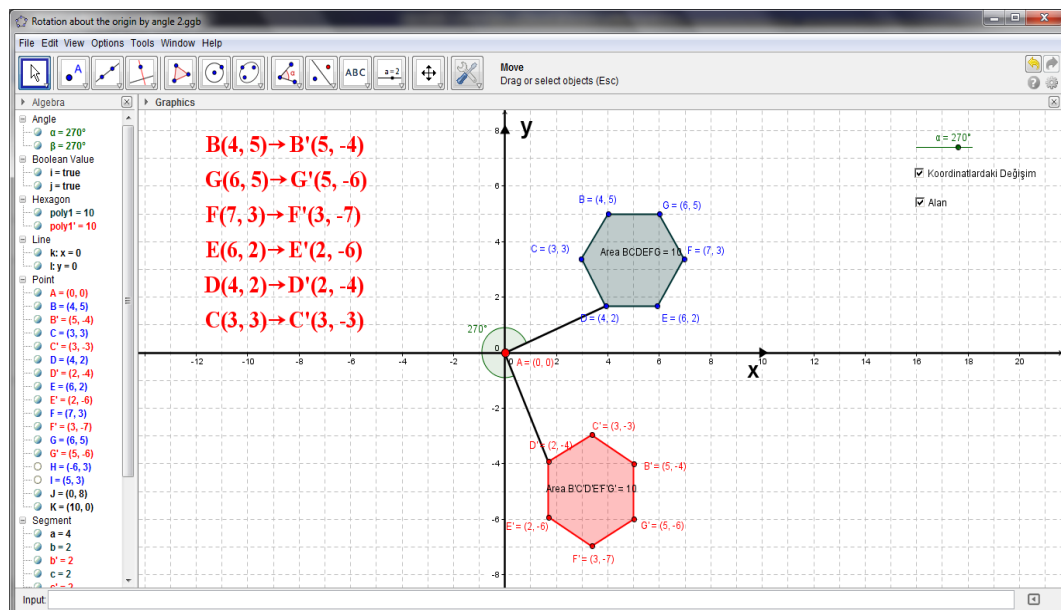


Figure 31. Rotation of a hexagon around the origin through 270° anticlockwise

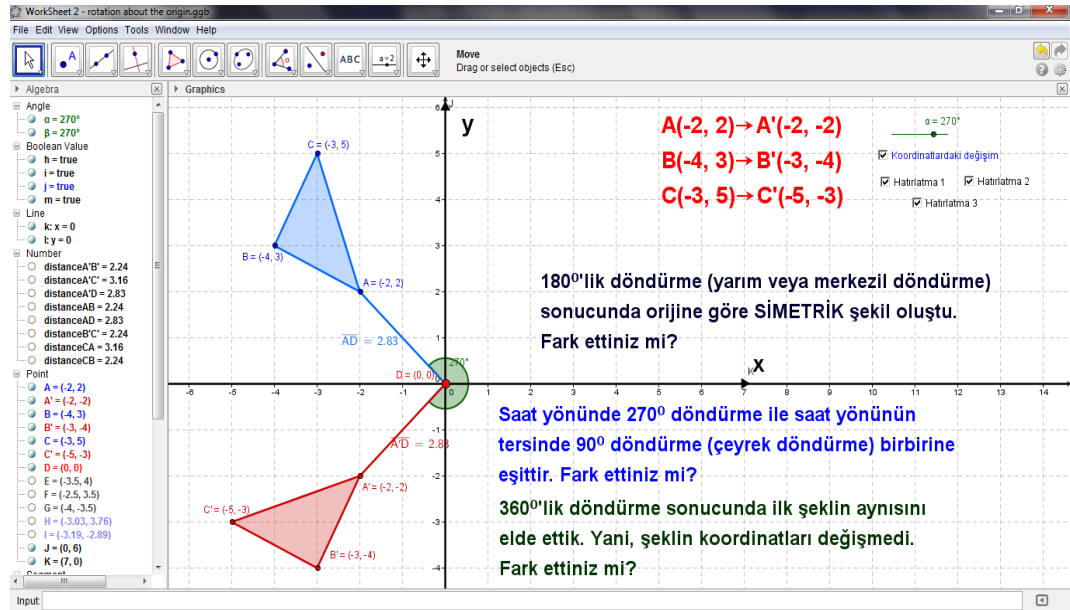


Figure 32. Screenshot from the dynamic worksheet - Rotation of a hexagon around the origin through 270° anticlockwise

### Rotation examples from the everyday life;



Figure 33. Rotation examples from the everyday life

## LESSON PLAN 5

**Area of Learning:** Geometry

**Sub-area of Learning:** Transformation Geometry

**Grade level:** 8<sup>th</sup>

**Objective(s):**

- Students should be able to determine the image of shapes after making translation with reflection and construct it.

**Duration:** 40 + 40 minutes (2 class hours)

**Key Terms:** Reflection with translation

**Resources / Materials:** Computer with GeoGebra software for each student, projector, pencil, activity sheet, worksheet

**Skills:** Computer usage, Geometrical Thinking, Mathematical Reasoning, Mathematical Correlation

**Prerequisite Knowledge:** Translation motion, line, polygon, coordinate system, computer-literacy.

### Activities (Description of the procedures):

#### I. STARTING

- Students are introduced the topic of today.
- Students are remembered the motions of reflection and translation which they learned in the previous lessons. Their knowledge related to the reflection and translation is freshened.
- Then, students are asked how the motion of reflection and translation can be used together and also asked what this transformation is called.
- A brief verbal explanation and necessary information related to the reflection with translation is given.
- Students are asked to tell where they see this transformation in everyday life and how they use it in daily life (e.g. every time we take a step, we do the motion of reflection with translation)



- Then, the students are given the real life examples of the motion of reflection (e.g. tile patterns which comprise of the reflection with translation) to make students able to see how mathematics is associated with the real life.
- The answers of the questions aforementioned are discussed with the students.

## **II. MIDDLE**

- In order to make students realize that every time they take a step, they do the motion of reflection with translation, they are told to open the GeoGebra activity names as “kılavuz-etkinlik 8 (1)” and work on it.
- After a small discussion related to the topic, the students are distributed the activity sheets related to the reflection with translation (Appendix G). Then, students are asked to read the activity sheet. They are asked to find answer to the questions given in the activity sheet while working on the GeoGebra. They are told that they are supposed to examine the explanations with respect to the topic on GeoGebra.
- The students are asked to open GeoGebra to work on the reflection with translation activities. Teacher tells students the aim of the activity and what they are supposed to do in the activity.
- The name of the GeoGebra file to open are told (e.g. open the file of “kılavuz-etkinlik 8 (1)”) and necessary directions/instructions about the activity are given. In this activity, students are orientated to comprehend the motion of the translation after reflection.
- The same procedure is followed for discussing the motion of the reflection after translation with working on the GeoGebra activity named as “kılavuz-etkinlik 8 (1)”.
- After working on the first two activities, students are told to open another activity named as “kılavuz-etkinlik 8 (2 ve 3ün eşitliği)”. By this activity, they are orientated to discover the fact that image of an object which is reflected after it was translated through a line is the same as the image of the same object which is translated after it was reflected through a line.

- In order to provide better understanding of the topic, students are told to open another GeoGebra activity named as “kılavuz-etkinlik 8 (4)”.
- Students are also mentioned that there is no point and no line stay fixed except for the line of reflection in the motion of reflection with translation.
- The students are given some time to deal with the activity by themselves. While students work on the GeoGebra activities, they are asked some exploring questions to make them discover and realize there is no difference between the reflection after translation and the translation after reflection.
- The students are asked to write their answers to the questions given in the activity sheet.

### **III. END**

- The main points and important definitions of the topic are summarized.
- The concept of the reflection with translation is clarified.
- Students’ questions related to the topic are answered.
- Worksheet as a mini quiz is distributed to the students to elicit information about students’ learning and understanding with respect to the topic (Appendix G).
- The students are given some minutes and they are asked to do the exercises given on the worksheet.
- The answers of the questions given on the worksheet were discussed with the students and they are made to realize the right and the wrong answer.
- Students are asked to save their work on GeoGebra in a folder.
- Homework assignment from the textbook is given.
- The students are told the next lesson’s topic.

Screenshots from the GeoGebra activities used in this lesson were given below:

Reflection with translation activities;

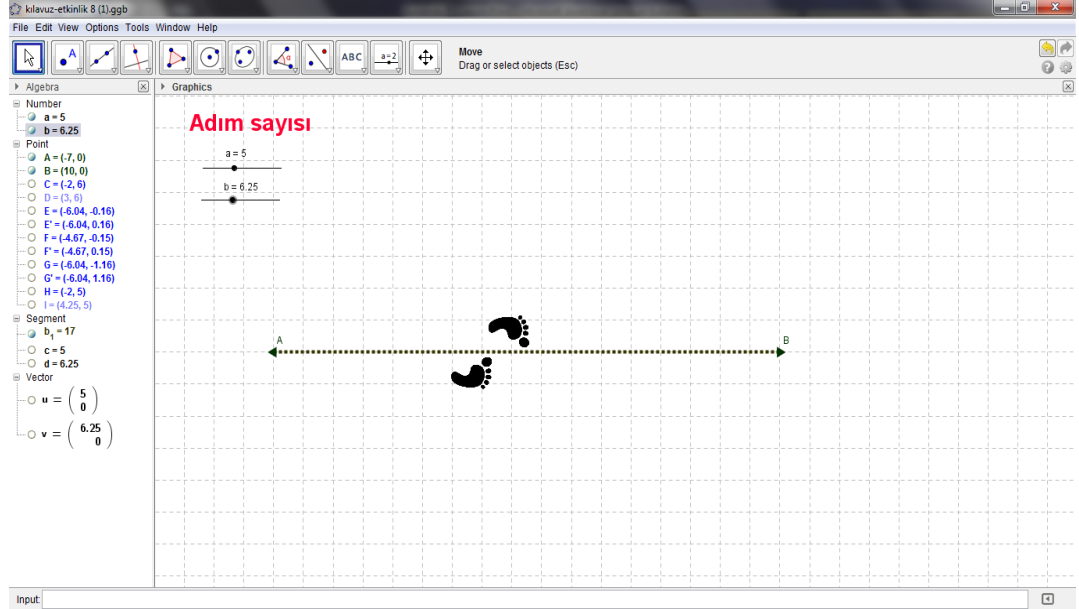


Figure 34. Step activity

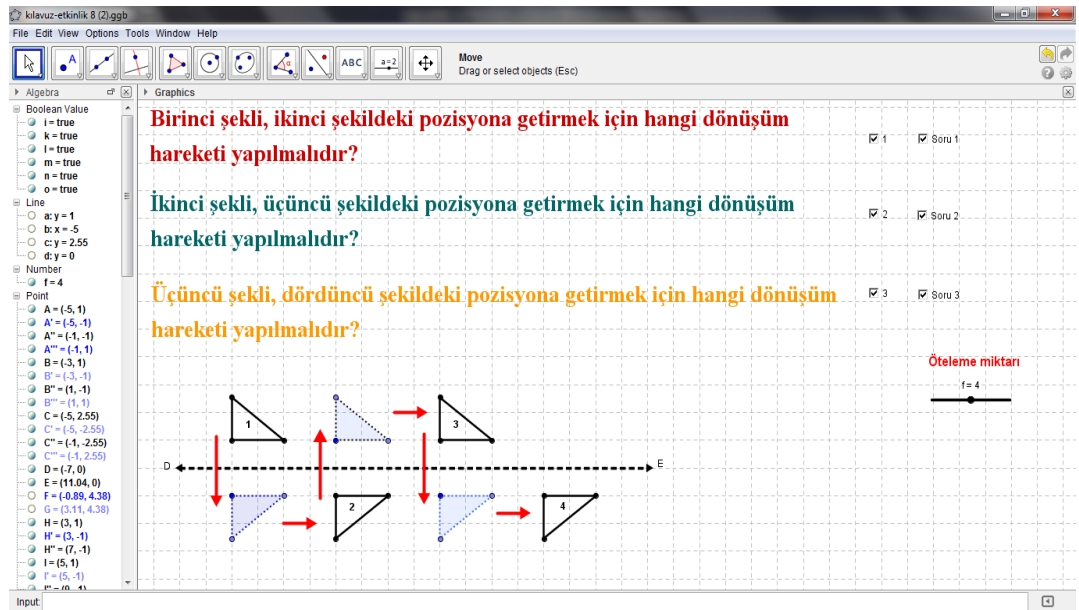


Figure 35. Translation after reflection activity

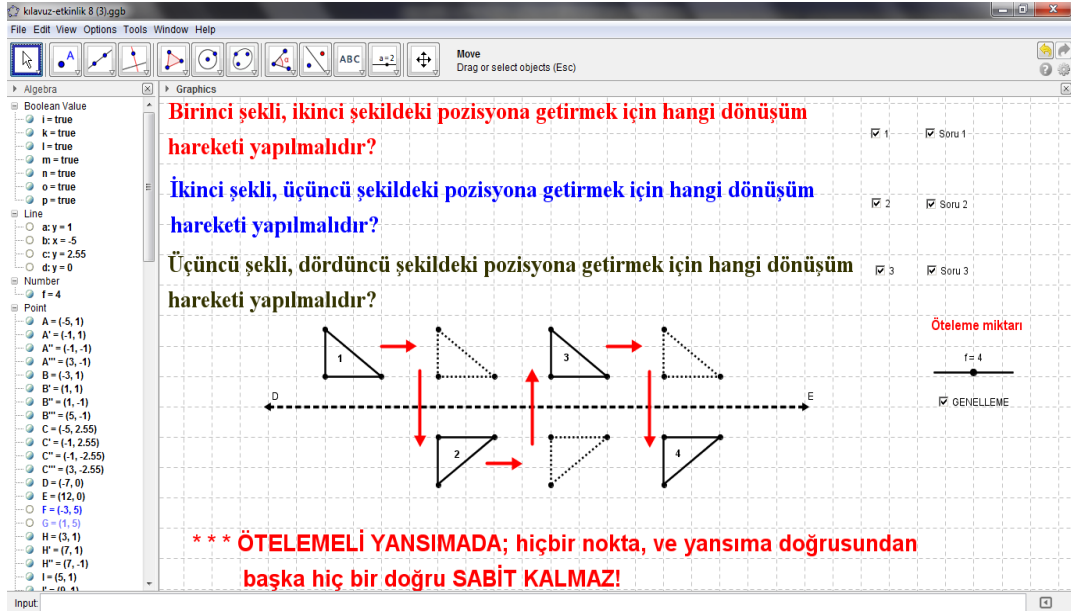


Figure 36. Reflection after translation activity

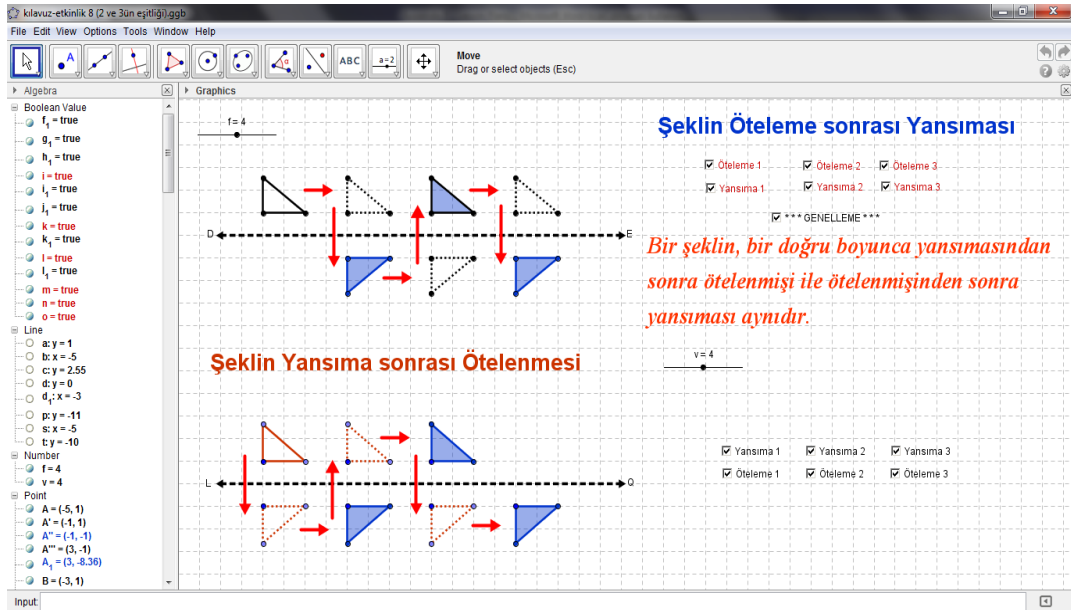


Figure 37. Equality of the translation after reflection and the reflection after translation

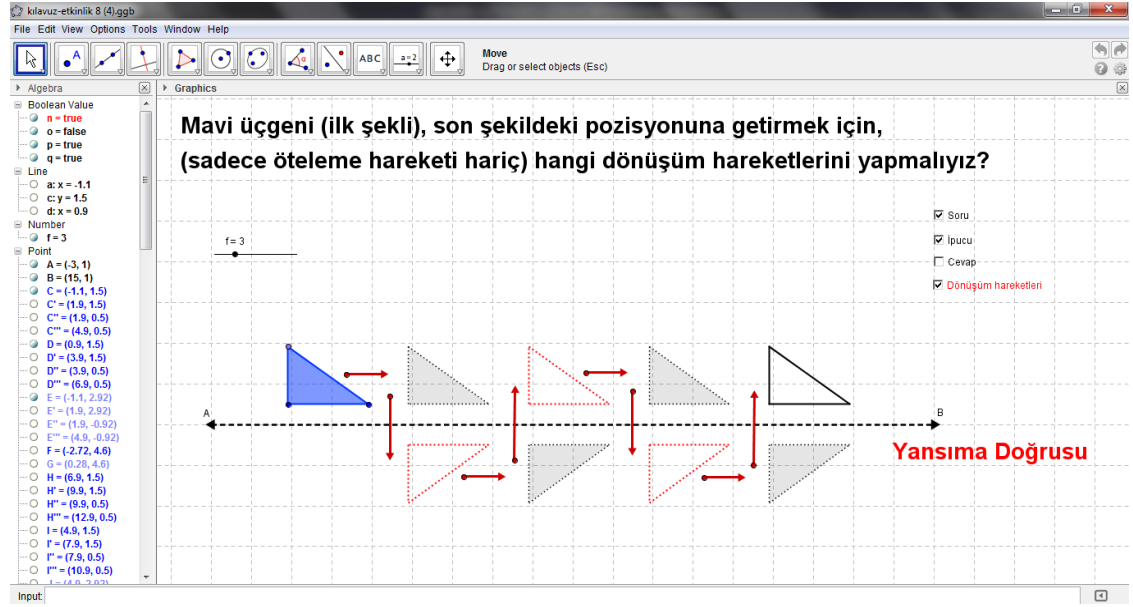


Figure 38. Dynamic GeoGebra question related to the topic

## APPENDIX K

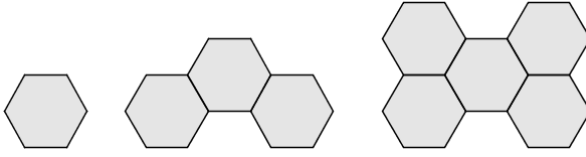
### ACTIVITY SHEETS AND WORKSHEETS

Activity sheets and worksheets used in the topic of Fractal;

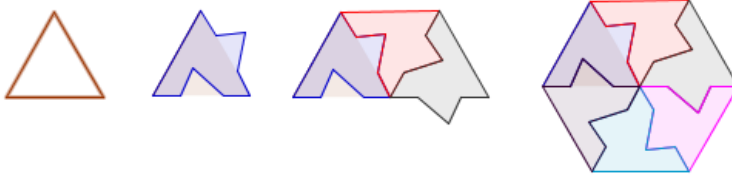
#### FRAKTAL - AKTİVİTE KAĞIDI

Aşağıdaki şekil gruplarından fraktal olanları belirleyiniz.

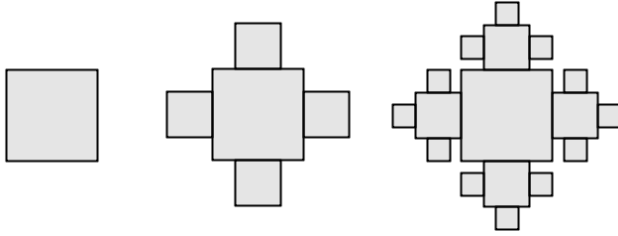
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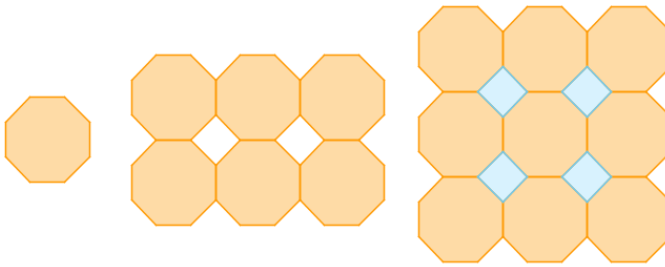
2.



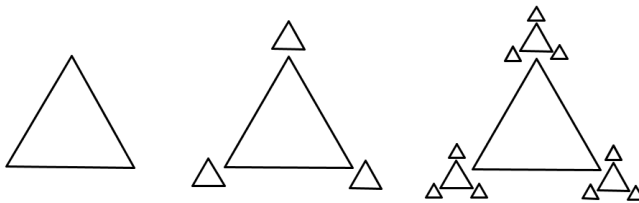
3.



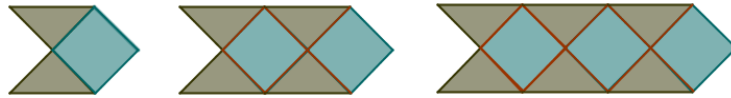
4.



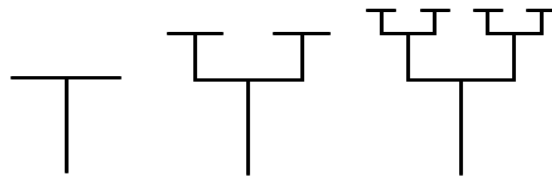
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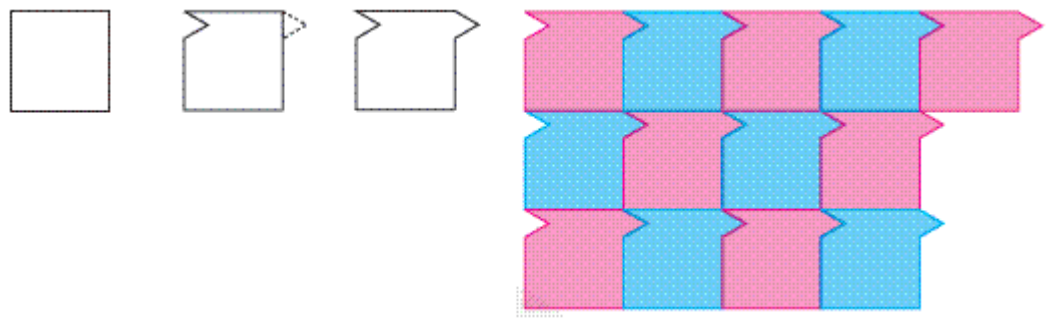
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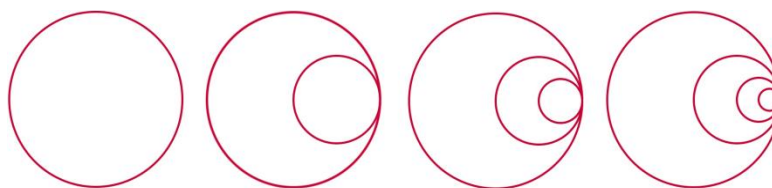
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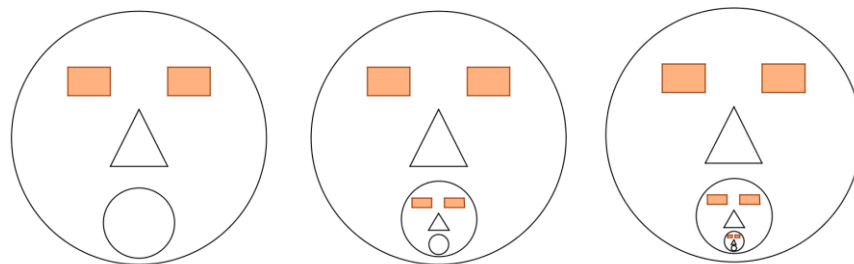
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9.

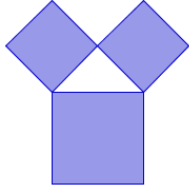


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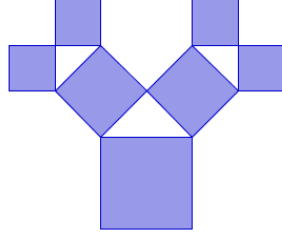


## FRAKTAL - ÇALIŞMA KAĞIDI

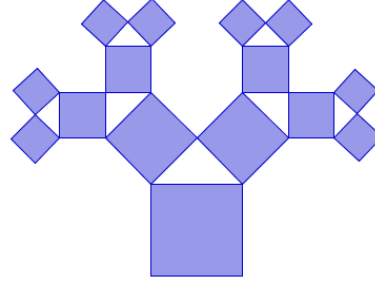
1.



1. adım



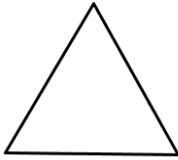
2. adım



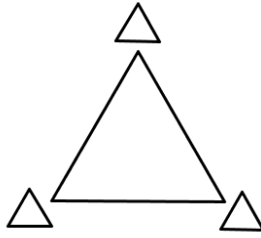
3. adım

- a) Yukarıda ilk üç adımı verilen şekil bir fraktal mıdır? Neden?
- b) Bu örüntünün 4. Adımını çiziniz.
- c) Bu örüntünün 4. Adımında kaç dörtgen bulunur?

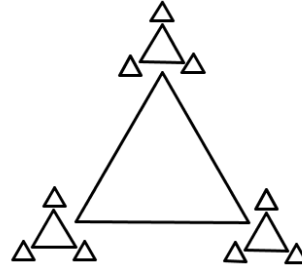
2.



1. adım



2. adım



3. adım

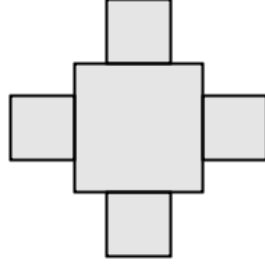
- a) Yukarıda ilk üç adımı verilen şekil bir fraktal mıdır? Neden?
- b) Bu örüntünün 4. Adımını çiziniz.
- c) Bu örüntünün 4. Adımında kaç üçgen bulunur?



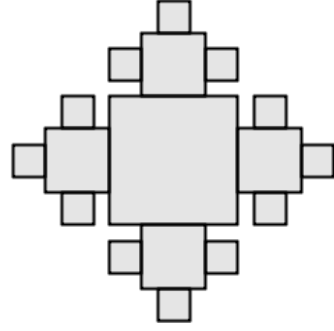
3.



1. adım



2. adım



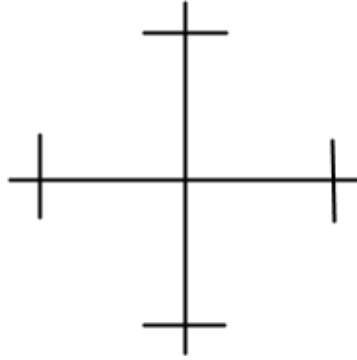
3. adım

- a) Yukarıda ilk üç adımı verilen şekil bir fraktal mıdır? Neden?
- b) Bu örüntünün 4. Adımını çiziniz.
- c) Bu örüntünün 4. Adımında kaç dörtgen bulunur?

4.

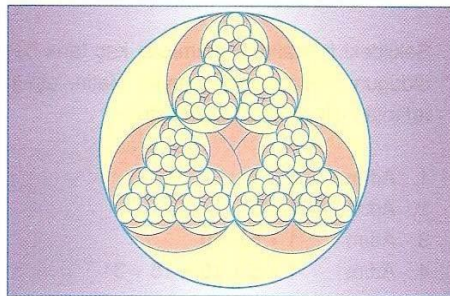


1. adım



2. adım

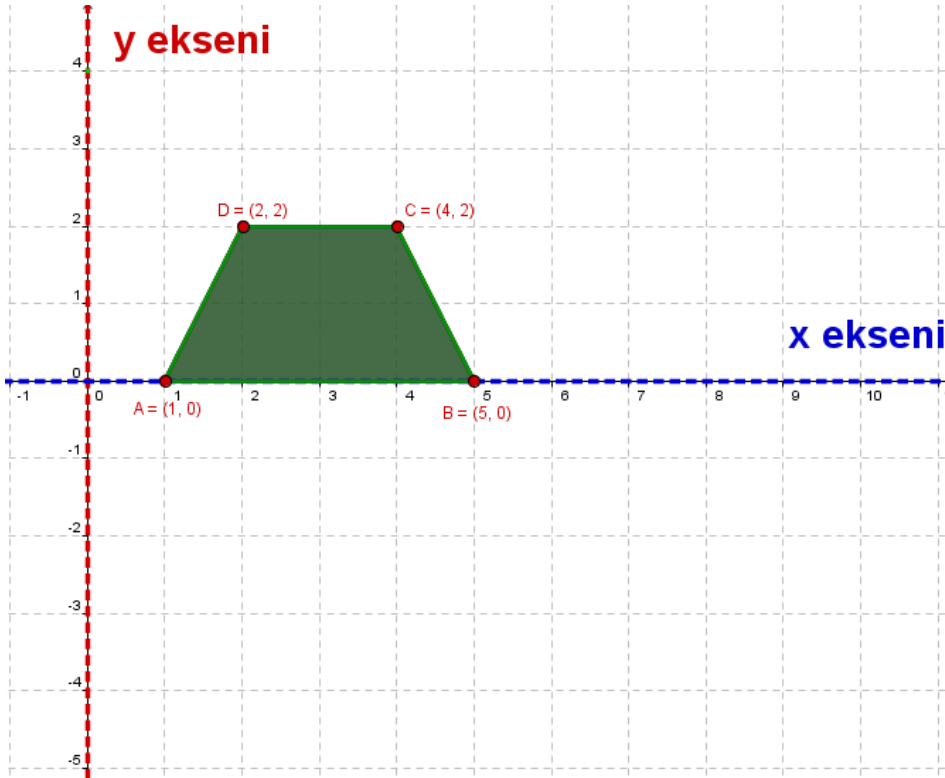
- a) Yukarıda 1. ve 2. Adımları verilen örüntünün fraktal olabilmesi için 3. Adım ne olmalıdır? Çiziniz.
  - b) 3. Adımdaki “+” sayısını hesaplayınız.
5. Bir fraktalın kaçınıcı adımında aşağıdaki şekil meydana gelir?



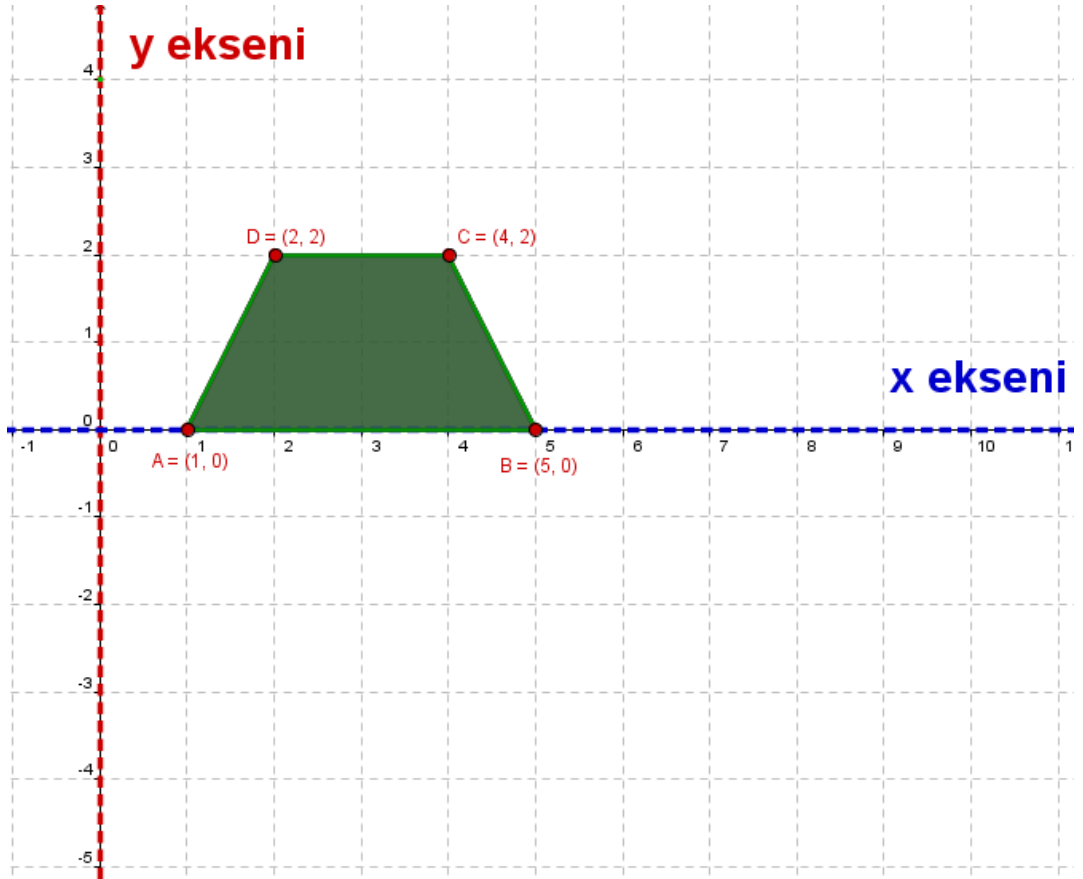
**Activity sheets and worksheets used in the topic of the translation through a coordinate axis;**

**DOĞRU BOYUNCA ÖTELEME - AKTİVİTE KAĞIDI**

1. Aşağıda verilen ABCD yamuğunu x ekseninde 1 birim sağa, y ekseninde 3 birim aşağıya ötelesek;
  - a. ABCD yamuğunun öteleme sonrası geldiği yer (görüntüsü) nasıldır?
  - b. Şekli ötelediğimizde A, B, C, ve D noktalarının koordinatlarında ve genel olarak ABCD yamuğunda nasıl bir değişiklik olur (Alan, kenar uzunlukları vb..)?
  - c. Bir şeklin öteleme hareketi sonrasında oluşan görüntüsü için nasıl bir genelleme yapabilirsiniz?

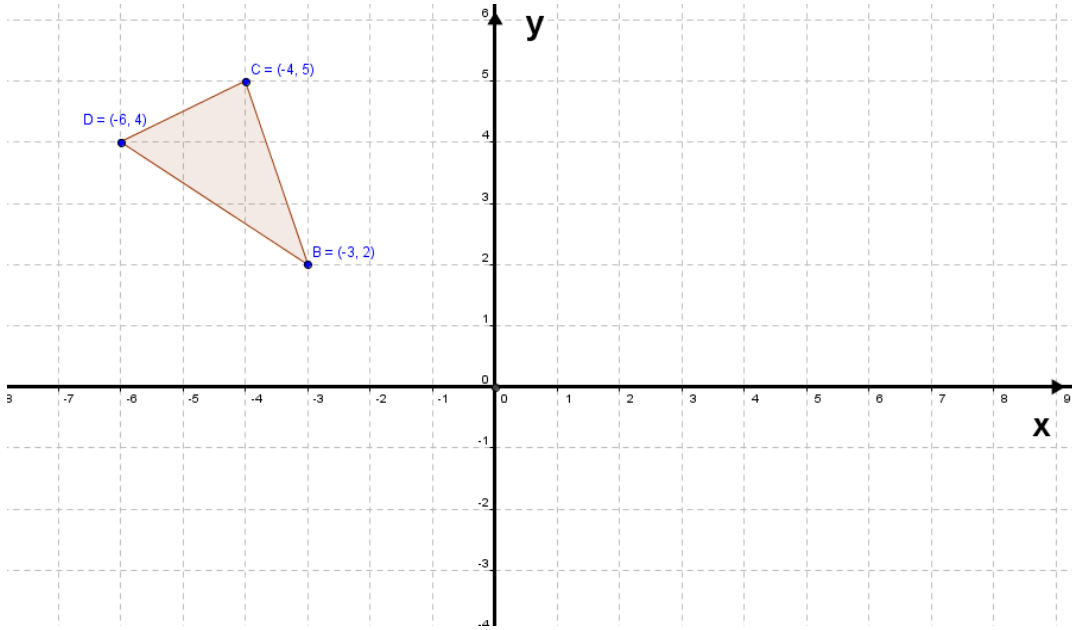


2. Aşağıda verilen ABCD yamuğunu y eksenine paralel 3 birim aşağıya, x eksenine paralel 1 birim sağa ötelesek;
- d. ABCD yamuğunun öteleme sonrası geldiği yer (görüntüsü) nasıldır?
  - e. Şekli ötelediğimizde A, B, C, ve D noktalarının koordinatlarında ve genel olarak ABCD yamuğunda nasıl bir değişiklik olur (Alan, kenar uzunlukları vb..)?
  - f. Bir şeklin öteleme hareketi sonrasında oluşan görüntüsü için nasıl bir genelleme yapabilirsiniz?

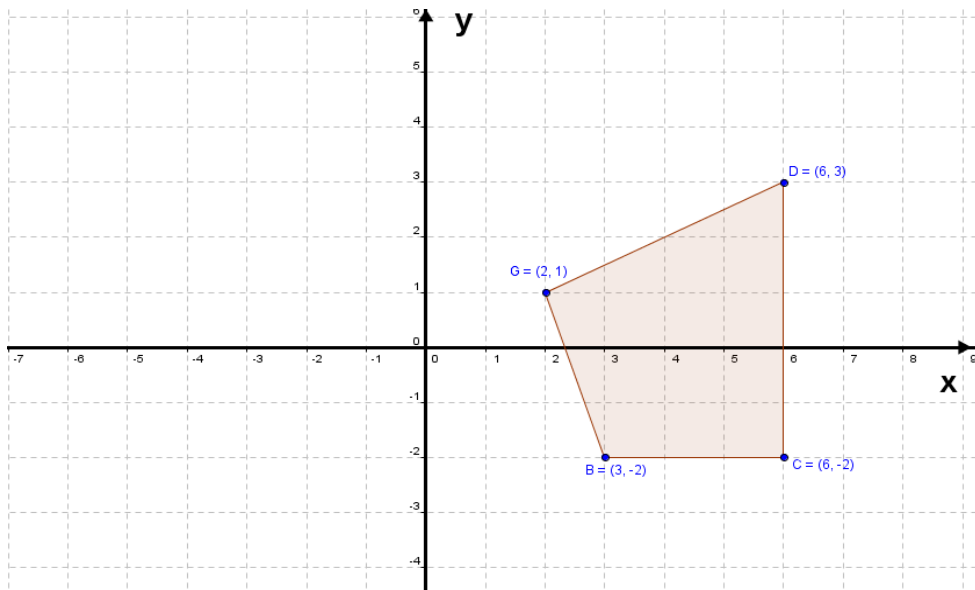


## DOĞRU BOYUNCA ÖTELEME - ÇALIŞMA KAĞIDI

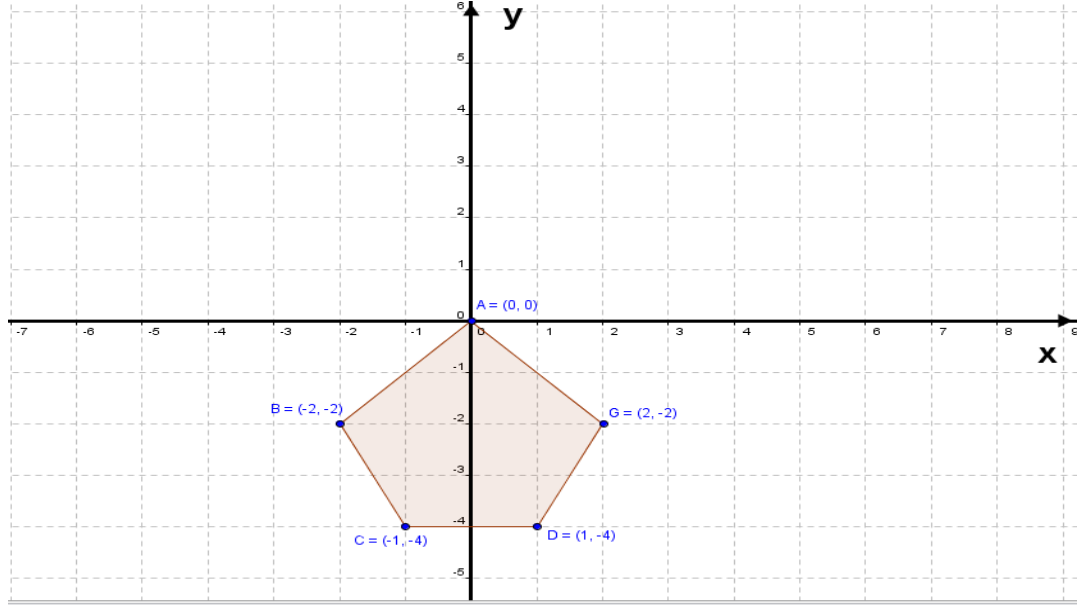
1. Aşağıda verilen DBC üçgeni x ekseninde 7 birim sağa, y ekseninde 6 birim aşağıya ötelenirse görüntüsü nasıl olur?



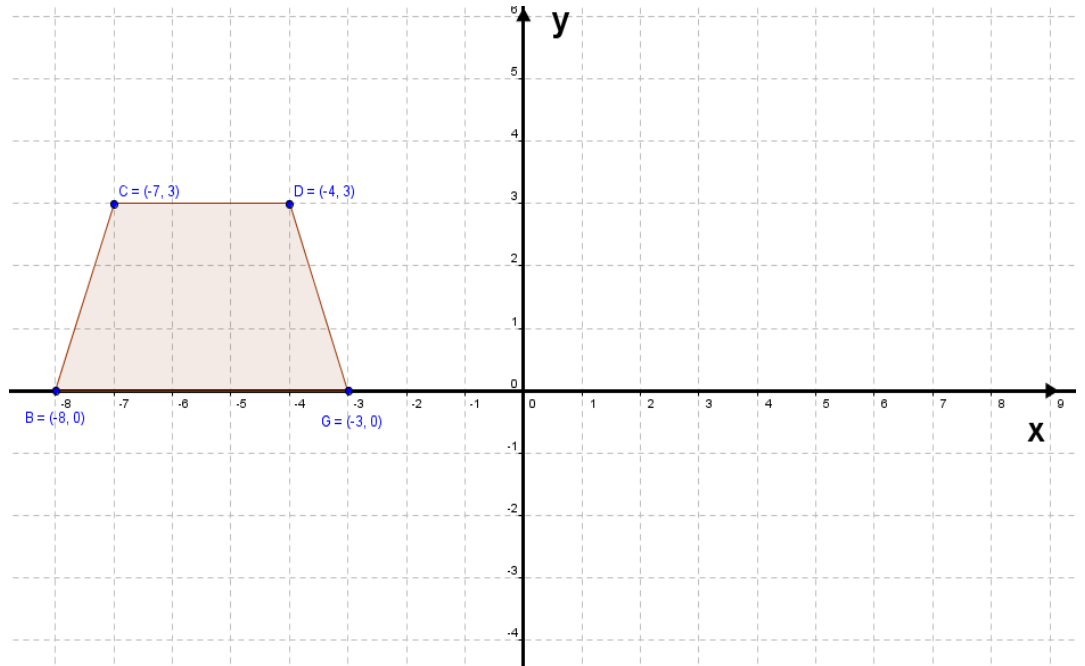
2. Aşağıda verilen DCBG dörtgeni x ekseninde 8 birim sola, y ekseninde 3 birim yukarıya ötelenirse görüntüsü nasıl olur?



3. Aşağıda verilen ADEGF beşgeni x ekseninde 4 birim sağa, y ekseninde 6 birim yukarıya ötelenirse görüntüsü nasıl olur?



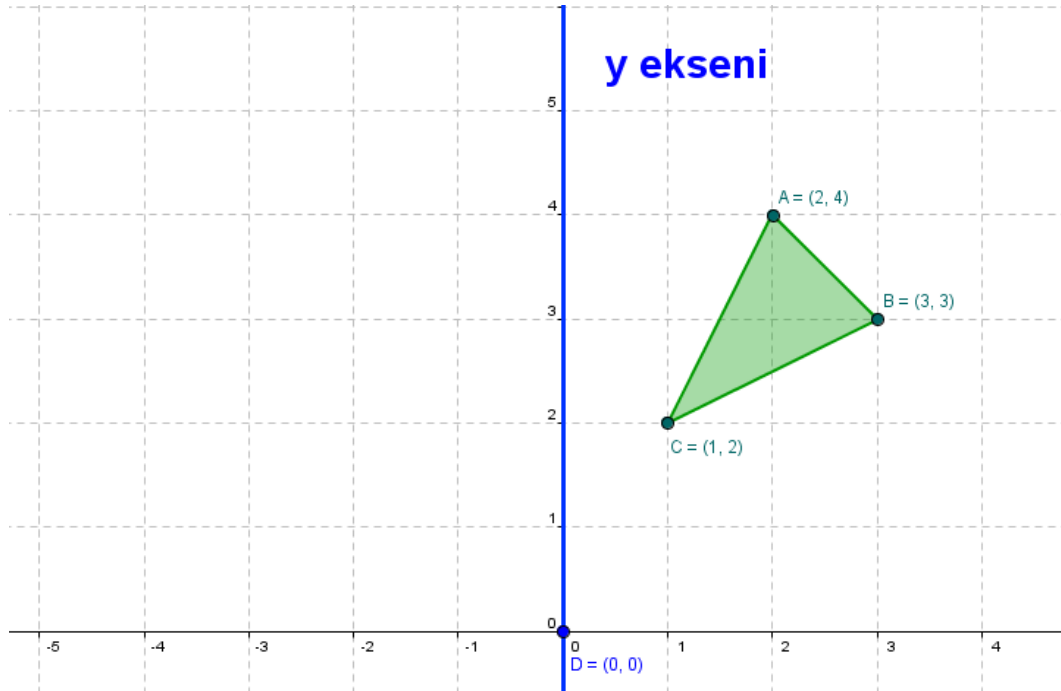
4. Aşağıda verilen CDBG yamuğu x ekseninde 9 birim sağa, y ekseninde 4 birim aşağıya ötelenirse görüntüsü nasıl olur?



**Activity sheets and worksheets used in the topic of Reflection about a coordinate axis;**

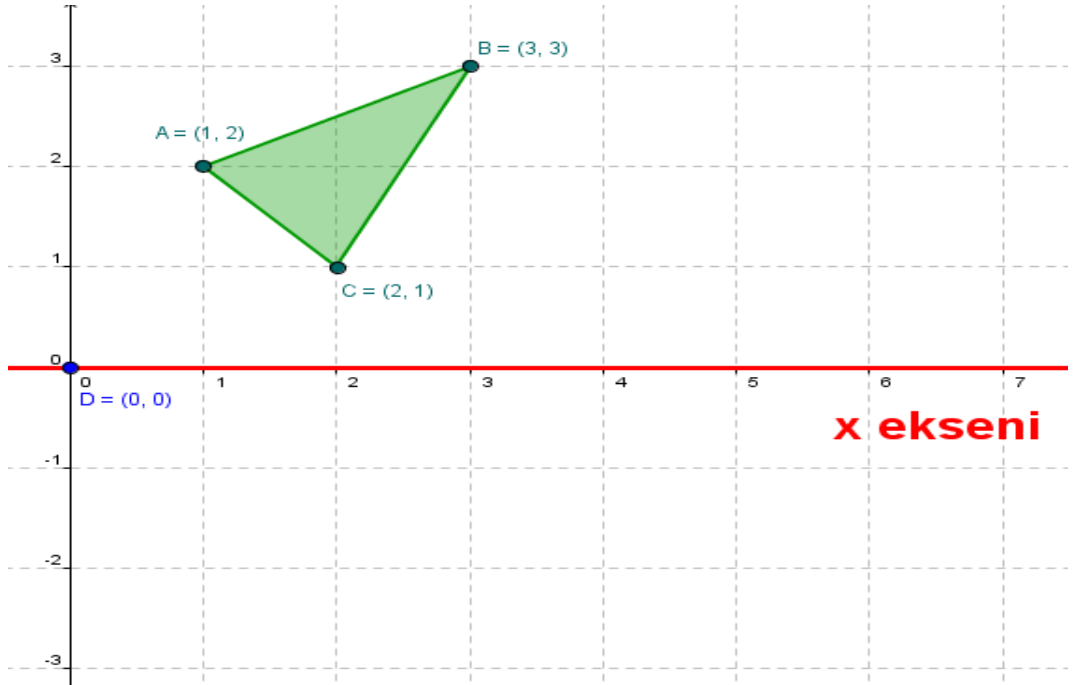
**EKSENLERE GÖRE YANSIMA - AKTİVİTE KAĞIDI**

1. Aşağıda verilen ABC üçgenini y-eksenine göre yansıttığımızda;
  - a. ABC üçgeninin görüntüsü nasıl olur?
  - b. Şekli yansıttığımızda A, B, ve C noktalarının koordinatlarında ve genel olarak ABC üçgeninde nasıl bir değişiklik olur (Alan, kenar uzunlukları vb..)?
  - c. Bir şeklin y-eksenine göre yansımadaki görüntüsü için nasıl bir genelleme yapabilirsiniz?



2. Aşağıda verilen ABC üçgenini x-eksenine göre yansıttığımızda;

- d. ABC üçgeninin görüntüsü nasıl olur?
- e. Şekli yansıttığımızda A, B, ve C noktalarının koordinatlarında ve genel olarak ABC üçgeninde nasıl bir değişiklik olur (Alan, kenar uzunlukları vb..)?
- f. Bir şeklin x-eksenine göre yansımadaki görüntüsü için nasıl bir genelleme yapabilirsiniz?



## EKSENLERE GÖRE YANSIMA - ÇALIŞMA KAĞIDI

GeoGebra aktivite sayfasında verilen üçgenin köşelerini hareket ettirerek aşağıdaki tabloda verilen C, D, ve E koordinatlarına ayarlayınız. Sonra, şekli x ve y eksenlerine göre yansıtmak için kontrol kutularını tıklayınız ve C, D, ve E noktalarının yeni koordinatlarını bulunuz. Daha sonra, bulduğunuz koordinatları aşağıdaki tablodaki boşluklara yazınız. Son olarak, bir çokgenin x-eksenine ya da y-eksenine göre yansımada çokgenin koordinatlarının nasıl değiştiğine dair sizden bir **genelleme** yapmanız beklenmektedir.

Yansıma eksen	Noktalar		
	C(-6,8)	D(-8,4)	E(-2,2)
x-ekseni			
y-ekseni			

Yansıma eksen	Noktalar		
	C(2,6)	D(1,-2)	E(7,2)
x-ekseni			
y-ekseni			

Yansıma eksen	Noktalar		
	C(4,-2)	D(3,-7)	E(8,-5)
x-ekseni			
y-ekseni			

Yansıma eksen	Noktalar		
	C(-6,-1)	D(-7,-6)	E(-2,-4)
x-ekseni			
y-ekseni			



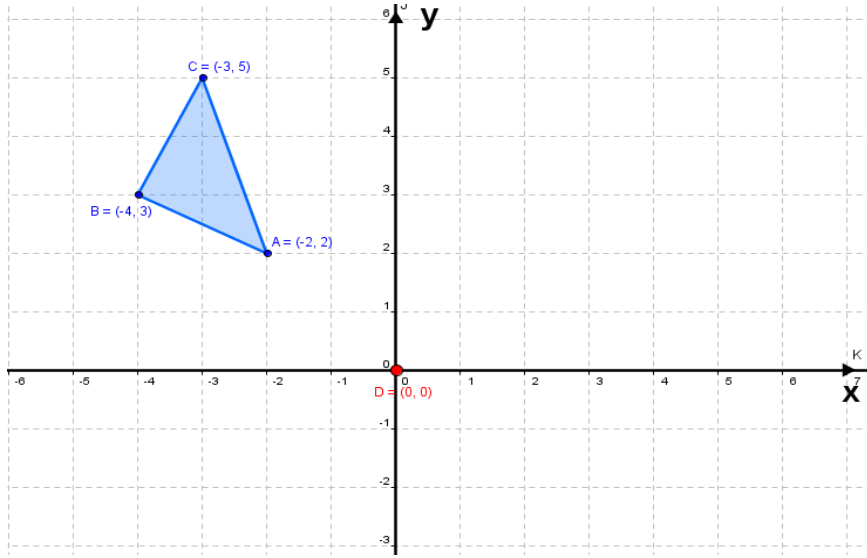
**GENELLEME:** Koordinatlarından birisi (a,b) olan bir şekli;

- x eksenine göre yansıttığımızda (a , b) koordinatı (..... , ..... ) olur.
- y eksenine göre yansıttığımızda (a , b) koordinatı (..... , ..... ) olur.

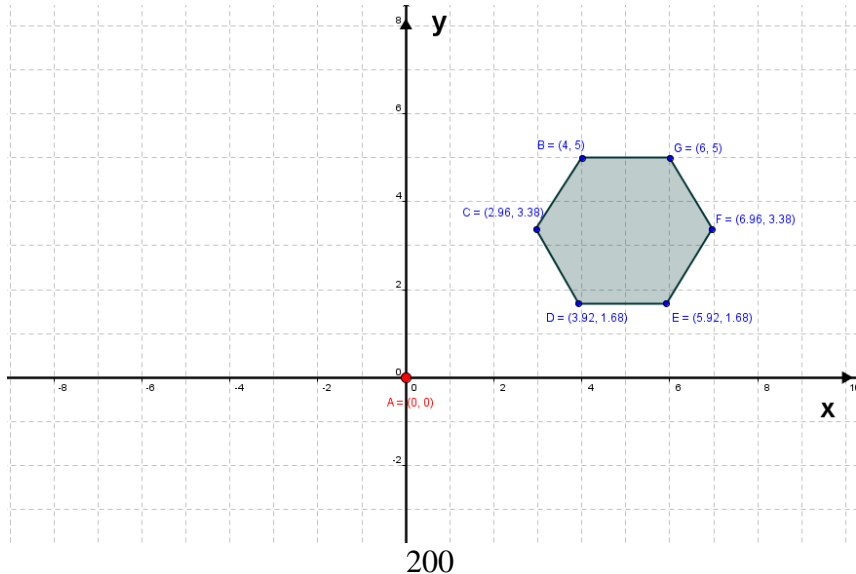
**Activity sheets and worksheets used in the topic of Rotation around the origin;**

**ORİJİN ETRAFINDA DÖNME - AKTİVİTE KAĞIDI**

1. Aşağıda verilen ABC üçgeni orijin etrafında saat yönünde  $90^\circ$  döndürülürse görüntüsü nasıl olur? (**Dosya: Rotation about the origin by angle**)



2. Aşağıda verilen BGFEDC altıgeni orijin etrafında saat yönünün tersi yönde  $180^\circ$  döndürülürse görüntüsü nasıl olur? (**Dosya: Rotation about the origin by angle** 2)



## ORİJİN ETRAFINDA DÖNME - ÇALIŞMA KAĞIDI

GeoGebra aktivite sayfasında (**Dosya: WorkSheet 2 - rotation about the origin**) verilen üçgeni aşağıdaki tabloda istenen açılarda ve yönlerde orijin etrafında döndürünüz. Döndürme sonrasında, tabloda verilen boşluklara üçgenin döndürme hareketi sonrasında elde edilen koordinatlarını yazınız. Son olarak sizden, bir çokgenin orijin etrafında belirli açılarda döndürülmesi sonucu çokgenin koordinatlarının nasıl değiştiğine ilişkin bir **genelleme** yapmanız beklenmektedir.

ABC üçgeninin köşe koordinatları	ABC üçgeninin orijin etrafında <u>saat yönünde 90°</u> döndürüldüğü nde koordinatları	ABC üçgeninin orijin etrafında <u>saat yönünde 180°</u> döndürüldüğü nde koordinatları	ABC üçgeninin orijin etrafında <u>saat yönünde 270°</u> döndürüldüğü nde koordinatları	ABC üçgeninin orijin etrafında <u>saat yönünde 360°</u> döndürüldüğü nde koordinatları
A(-2,2)	A( , )	A( , )	A( , )	A( , )
B(-4,3)	B( , )	B( , )	B( , )	B( , )
C(-3,5)	C( , )	C( , )	C( , )	C( , )

ABC üçgeninin köşe koordinatları	ABC üçgeninin orijin etrafında <u>saat yönünün tersi yönde 90°</u> döndürüldüğü nde koordinatları	ABC üçgeninin orijin etrafında <u>saat yönünün tersi yönde 180°</u> döndürüldüğü nde koordinatları	ABC üçgeninin orijin etrafında <u>saat yönünün tersi yönde 270°</u> döndürüldüğü nde koordinatları	ABC üçgeninin orijin etrafında <u>saat yönünün tersi yönde 360°</u> döndürüldüğü nde koordinatları
A(-2,2)	A( , )	A( , )	A( , )	A( , )
B(-4,3)	B( , )	B( , )	B( , )	B( , )
C(-3,5)	C( , )	C( , )	C( , )	C( , )

## SAAT YÖNÜNDE DÖNME İÇİN GENELLEME:

Koordinatlarından biri (a,b) olan bir şekli, **orijin etrafında saat yönünde;**

**90° döndürdüğümüzde** (a,b) koordinatı (..... ,.....)' e dönüşür.

**180° döndürdüğümüzde** (a,b) koordinatı (..... ,.....)' e dönüşür.

**270° döndürdüğümüzde** (a,b) koordinatı (..... ,.....)' e dönüşür.

**360° döndürdüğümüzde** (a,b) koordinatı (..... ,.....)' e dönüşür.

## SAAT YÖNÜNÜN TERSİ YÖNDE DÖNME İÇİN GENELLEME:

Koordinatlarından biri (a,b) olan bir şekli, **orijin etrafında saat yönünün tersi yönde;**

**90° döndürdüğümüzde** (a,b) koordinatı (..... ,.....)' e dönüşür.

**180° döndürdüğümüzde** (a,b) koordinatı (..... ,.....)' e dönüşür.

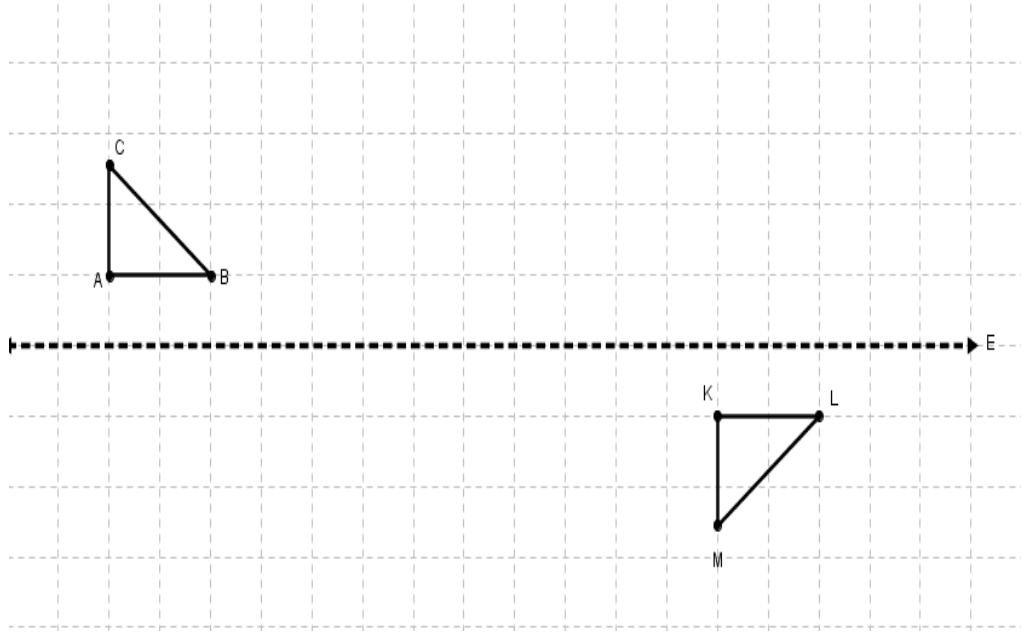
**270° döndürdüğümüzde** (a,b) koordinatı (..... ,.....)' e dönüşür.

**360° döndürdüğümüzde** (a,b) koordinatı (..... ,.....)' e dönüşür.

**Activity sheets and worksheets used in the topic of Reflection with translation;**

**ÖTELEMELİ YANSIMA - AKTİVİTE KAĞIDI**

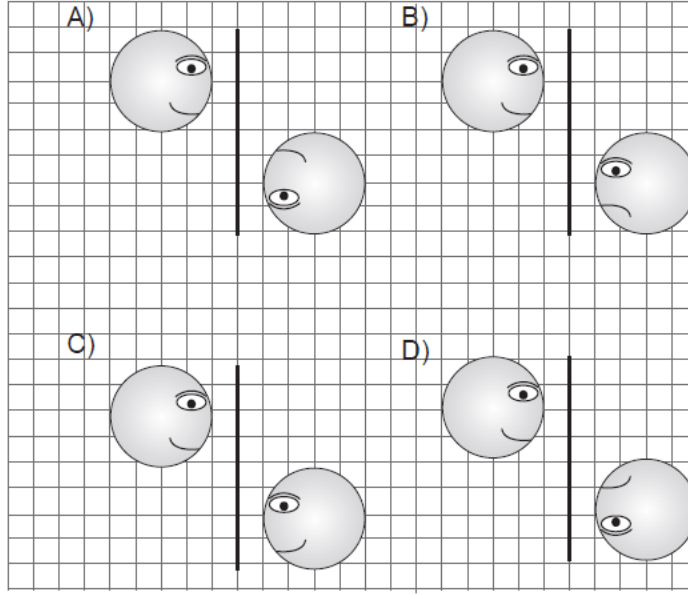
1. Aşağıda verilen ABC üçgenini, KLM üçgeninin pozisyonuna getirmek için hangi dönüşüm hareketleri yapılmalıdır?



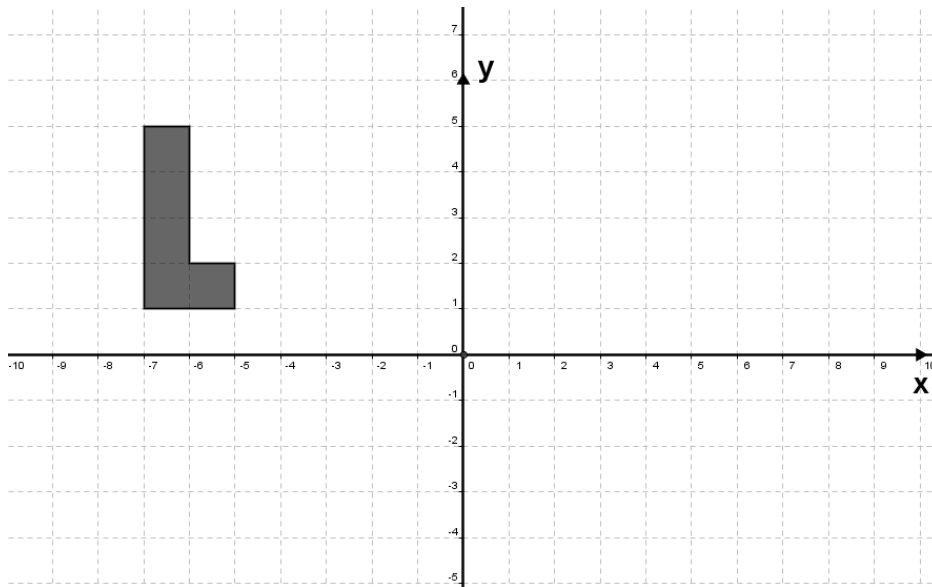
2. Bir şeklin bir doğru boyunca yansımasından sonra ötelenmişi ile ötelenmişinden sonra yansıması aynı mıdır?

## ÖTELEMELİ YANSIMA - ÇALIŞMA KAĞIDI

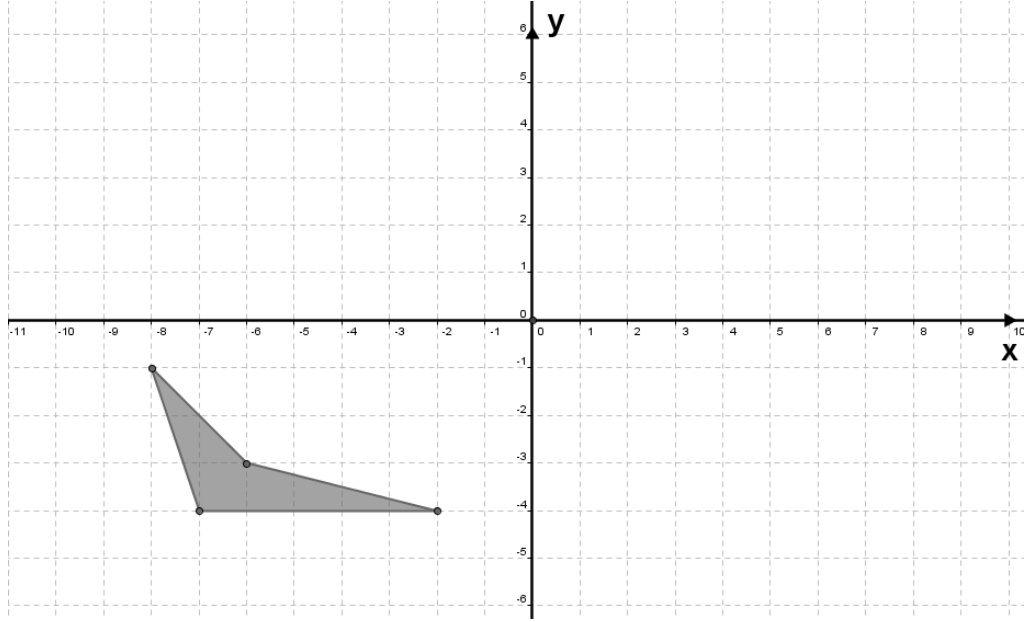
1. Aşağıdakilerin hangisinde verilen şekiller, birbirinin ötelemeli yansımasıdır?



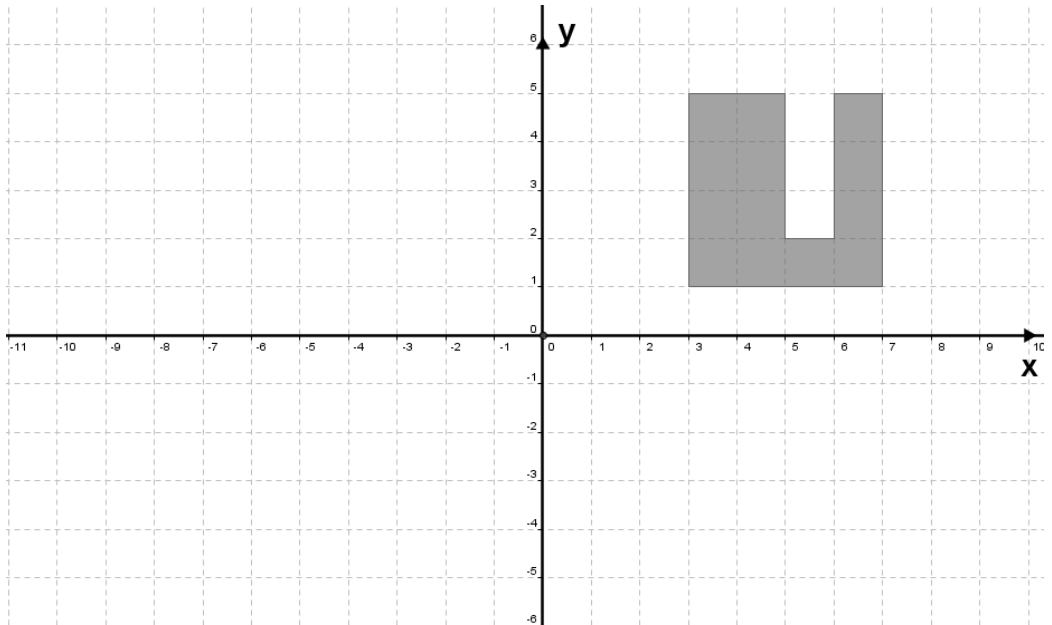
2. Aşağıda verilen şeklin y ekseninde aşağıya doğru 6 birim ötelenmesi sonrasında y eksenine göre yansıması nasıldır?



3. Aşağıda verilen şeklin x eksenine göre yansıması sonrasında x ekseninde sağa doğru 9 birim ötelenmesi sonrasında görüntüsü nasıldır?



4. Aşağıda verilen şeklin y eksenine göre yansıması sonrasında y ekseninde aşağıya doğru 4 birim ötelenmesi sonrasında görüntüsü nasıldır?



## APPENDIX L

### RAW DATA OF THE STUDY

#	Group	PREMAT	PREVHL	PREMTAS	POSTMAT	POSTVHL	POSTMTAS
1	1	57	9	65	62	12	70
2	1	59	7	74	58	15	74
3	1	38	13	84	60	13	92
4	1	46	8	77	54	11	65
5	1	18	7	89	52	12	100
6	1	52	11	88	63	12	96
7	1	45	11	83	55	13	73
8	1	40	11	77	58	14	80
9	1	55	8	90	61	12	82
10	1	53	8	79	66	12	88
11	1	35	10	80	52	13	81
12	1	40	8	74	59	10	63
13	1	55	14	61	65	14	90
14	1	20	NA	NA	59	12	96
15	1	55	13	89	61	14	96
16	1	37	9	80	58	11	73
17	1	32	8	84	56	10	85



18	2	40	7	84	44	8	85
19	2	24	8	80	47	9	80
20	2	31	9	78	63	11	85
21	2	51	9	65	34	10	73
22	2	43	8	61	39	8	81
23	2	71	13	84	72	14	90
24	2	52	10	NA	56	10	74
25	2	45	9	79	NA	9	75
26	2	43	9	90	53	10	76
27	2	60	11	83	62	11	90
28	2	12	7	77	16	6	88
29	2	60	12	82	64	11	79
30	2	61	10	89	NA	11	92
31	2	63	9	77	47	10	83
32	2	61	10	74	54	10	81
33	2	24	9	74	41	8	77
34	2	NA	8	80	33	9	80

**\*Group 1:** Experimental Group (8-B / 9 male and 8 female)

**\*Group 2:** Control Group (8-C / 8 male and 9 female)

**\*NA:** Not attended the test

## APPENDIX M

### ETHICAL PERMISSION OF THE RESEARCH



T.C.  
ANKARA VALİLİĞİ  
Milli Eğitim Müdürlüğü

Sayı : B.08.4.MEM.0.06.20.01-60599/64327  
Konu : Araştırma İzni  
Mustafa Buğra AKGÜL


31/08/2012

ORTA DOĞU TEKNİK ÜNİVERSİTESİNE  
(Öğrenci İşleri Daire Başkanlığı)

İlgi: a) MEB Yenilik ve Eğitim Teknolojileri Genel Müdürlüğünün 2012/13 nolu genelgesi.  
b) Üniversitenizin 10/08/2012 tarih ve 9665 sayılı yazısı.

Üniversiteniz Sosyal Bilimler Enstitüsü yüksek lisans öğrencisi Mustafa Buğra AKGÜL' ün "Dinamik geometri yazılımı kullanımının 8. sınıf öğrencilerinin dönüşüm geometrisi konusundaki akademik başarıları, Van Hiele geometrik düşünme düzeyleri ve matematik ve teknolojiye yönelik tutumları üzerine etkisi" konulu tezi ile ilgili çalışma yapma isteği Müdürlüğümüzce uygun görülmüş ve araştırmanın yapılacağı İlçe Milli Eğitim Müdürlüğüne bilgi verilmiştir.

Mühürlü anketler (15 sayfadan oluşan) ekte gönderilmiş olup, uygulama yapılacak sayıda çoğaltılması ve çalışmanın bitiminde iki örneğinin (CD/disket) Müdürlüğümüz Strateji Geliştirme Bölümüne gönderilmesini rica ederim.

  
İlhan KOÇ  
Müdür a.  
Şube Müdürü

EKLER :  
Anket (15 sayfa)

06.09.12\*014813

## APPENDIX N

### TEZ FOTOKOPİSİ İZİN FORMU

#### ENSTİTÜ

Fen Bilimleri Enstitüsü	<input type="checkbox"/>
Sosyal Bilimler Enstitüsü	<input checked="" type="checkbox"/>
Uygulamalı Matematik Enstitüsü	<input type="checkbox"/>
Enformatik Enstitüsü	<input type="checkbox"/>
Deniz Bilimleri Enstitüsü	<input type="checkbox"/>

#### YAZARIN

Soyadı : Akgül

Adı : Mustafa Buğra

Bölümü : İlköğretim Fen ve Matematik Alanları Eğitimi

**TEZİN ADI** (İngilizce) : The effect of using Dynamic Geometry Software on Eighth Grade Students' Achievement in Transformation Geometry, Geometric Thinking and Attitudes Toward Mathematics and Technology.

**TEZİN TÜRÜ** : Yüksek Lisans ☒ Doktora ☐

1. Tezimin tamamı dünya çapında erişime açılsın ve kaynak gösterilmek şartıyla tezimin bir kısmı veya tamamının fotokopisi alınsın. ☐
2. Tezimin tamamı yalnızca Orta Doğu Teknik Üniversitesi kullanıcılarının erişimine açılsın ☐
3. Tezim bir (1) yıl süreyle erişime kapalı olsun ☒

**TEZİN KÜTÜPHANEYE TESLİM TARİHİ:**