CAUSAL RELATIONS AMONG 12TH GRADE STUDENTS' GEOMETRY KNOWLEDGE, SPATIAL ABILITY, GENDER, AND SCHOOL TYPE

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ABSTRACT

CAUSAL RELATIONS AMONG 12TH GRADE STUDENTS' GEOMETRY KNOWLEDGE, SPATIAL ABILITY, GENDER, AND SCHOOL TYPE

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The purpose of this study is to invetigate the causal relationships among 12th grade students' geometry knowledge regarding prisms and pyramids, spatial ability, gender, and school type. Path analysis was used to test the relationships among knowledge factors (declarative, conditional, and procedural knowledge), spatial ability factors (spatial visualization, mental rotation, and spatial perception ability), gender (female and male), and school type (general high schools and Anatolian high schools). Knowledge factors and spatial ability factors were determined by carrying out confirmatory factor analysis for the Prisms and Pyramids Knowledge Test and Purdue Spatial Visualization Test separately.

Results revealed the bilateral relations among students' declarative, conditional and procedural knowledge; and the bilateral relations among spatial visualization, mental rotation, and spatial perception ability.

When relations among spatial ability factors and knowledge factors were examined, the importance of the students' spatial abilities on geometry performance was exposed explicitly. Spatial visualization and mental rotation ability have positive direct effects on all knowledge factors. Additionally, spatial perception ability have positive direct effect on declarative and procedural knowledge.

On the other hand, school type has positive direct effects on students' geometry knowledge factors and spatial ability factors. These effects exposed the superiority of students in Anatolian high schools in respect of students in general high schools.

Moreover, direct effects of gender on mental rotation ability, spatial perception ability, and declarative knowledge were found. Although, results presented the male superiority in mental rotation and spatial perception abilities, direct effect of gender on declarative knowledge indicate the female advantage.

Keywords: Mathematics Education, Geometry Knowledge, Spatial ability, School Type, Gender, Path analysis

ÖZ

12. SINIF ÖĞRENCİLERİNİN GEOMETRİ BİLGİLERİ, UZAMSAL YETENEKLERİ, CİNSİYETLERİ VE OKUL TÜRLERİ ARASINDAKİ NEDENSEL İLİŞKİ

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Bu çalışmanın amacı 12. sınıf öğrencilerinin prizma ve piramit hakkında geometri bilgileri, uzamsal yetenekleri, cinsiyetleri ve okul türleri arasındaki nedensel ilişkiyi araştırmaktır. Bilgi faktörleri (tanımsal, koşullu ve işlemsel bilgi), uzamsal yetenek factörleri (uzamsal görselleştirme, zihinsel döndürme ve uzamsal algı yeteneği), cinsiyet ve okul türleri (Anadolu lisesi ve genel lise) arasındaki ilişkileri test etmek için path analizi kullanılmıştır. Bilgi faktörleri ve uzamsal yetenek faktörleri Prizma ve Piramit Testi ve Purdue Uzamsal Görselleştirme Testi için yapılan doğrulayıcı factor analizleri ile tanımlanmıştır.

Araştırma sonuçları öğrencilerin tanımsal, koşullu ve işlemsel bilgileri arasında iki yönlü ilişki olduğunu göstermiştir. Benzer şekilde, sonuçlar uzamsal görselleştirme, zihinsel döndürme ve uzamsal algı yetenekleri arasında iki yönlü ilişki olduğunu ortaya koymuştur.

Bilgi faktörleri ve uzamsal yetenek faktörleri arasındaki ilişkiler incelendiğinde, öğrencilerin uzamsal yeteneklerinin üç boyutlu geometri başarımları üzerindeki önemi açığa çıkmaktadır. Uzamsal görselleştirme ve zihinsel döndürme yeteneklerinin tüm bilgi faktörleri üzerinde doğrudan etkisi vardır. Ek olarak, uzamsal algı yeteneği tanımsal bilgi ve işlemsel bilgi üzerine doğrudan etkisi vardır.

Ayrıca, okul türü öğrencilerin tanım bilgisi, koşul bilgisi, işlem bilgisi, uzamsal görselleştirme yeteneği, zihinsel döndürme yeteneği, uzamsal algı yeteneği üzerinde pozitif dogrudan etkiye sahiptir. Bu etkiler Anadolu liselerinde okumakta olan öğrencilerin genel liselerdeki öğrencilere göre üstünlüğünü göstermektedir.

Buna ek olarak, cinsiyetin zihinsel döndürme yeteneği, uzamsal algı yeteneği ve tanımsal bilgi üzerine doğrudan etkisi olduğu bulunmuştur. Sunuçlar zihinsel döndürme ve uzamsal algı yeteneklerinde erkeklerin üstünlüğünü göstermesine rağmen, cinsiyetin tanımsal bilgi üzerindeki doğrudan etkisi kızların üstünlüğünü göstermektedir.

Anahtar Kelimeler: Matematik Eğitimi, Geometri Bilgisi, Uzamsal Yetenek, Okul Türü, Cinsiyet, Path analizi

To my family…

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LIST OF ABBREVIATIONS

PPKT	: Prisms and Pyramids Knowledge Test
G1-G55	: Prisms and Pyramids Knowledge Test Items
DecK	: Declarative Knowledge
ConK	: Conditional Knowledge
ProK	: Procedural Knowledge
PSVT	: Purdue Spatial Visualization Test
P1-P36	: Purdue Spatial Visualization Test Items
SPerA	: Spatial Perception Ability
MRotA	: Mental Rotation Ability
SVisA	: Spatial Visualization Ability
CFA	: Confirmatory Factor Analysis
SEM	: Structural Equation Modeling
3D	: Three-Dimensional
AGFI	: Adjusted Goodness-of-Fit-Index
AIC	: Akaike Information Criterion
CAIC	: Consistent AIC
CFI	: Comparative Fit Index
ECVI	: Expected Cross Validation Index
GFI	: Goodness-of-Fit Index
NFI	: Normed Fit Index
NNFI	: Nonnormed Fit Index
RMSEA	: Root Mean Square Error of Approximation
PGFI	: Parsimony GFI, PNFI: Parsimony NFI
RMR	: Root Mean Square Residual
S-RMR	: Standardized RMR

CHAPTER I

INTRODUCTION

Geometry has an important place in mathematics curriculum. School geometry should enable students to analyze two- and three-dimensional geometric objects, describe spatial relations, apply transformations, and use spatial abilities and geometric modeling to solve problems (MEB, 2010a, 2010b, 2011; NCTM, 2000). In addition to these geometric ideas, it can help students gain insight to understand the nature and the beauty of mathematics, recognize and apply the geometric ideas and relationships into other disciplines such as science, art, architecture, and everyday life (MEB, 2010a, 2010b, 2011; NCTM, 2000). Therefore, geometry knowledge has prominent place not only inside school but also outside. Unfortunately, numerous studies revealed that many students had low achievements and negative attitudes towards geometry. They also encountered difficulties in geometry, developed misconceptions, and failed to go beyond seeing geometric figures (Battista, 2007; Clements & Battista, 1992; Duatepe-Paksu & Ubuz, 2007; Mistretta, 2000; Ubuz, 1999; Usiskin, 1972).

Geometry, as a branch of mathematics, can be considered as a tool for understanding, describing and interacting with the space. So, idealized models of the physical world can be constructed in 'the science of space' (Mammana & Villani, 1998). Usiskin (1987) described four dimensions of geometry to conceptualize it: a) geometry as the study of visualization, drawing, and constructions of figures; b) geometry as the study of real physical world; c) geometry as the vehicle for representing mathematical or other concepts whose origin is not visual or physical; d) geometry as an example of a mathematical system. The first three dimensions emphasized the visual aspects of geometry, as they require the use of spatial reasoning. Geometry helps us understand the physical world by using visual elements such as symbols, points, lines, arrows, curves, angles, two and three-dimensional figures for modeling. Seeing these constituents is not enough to understand given visual stimulus, transforming the visual information according to given rules and making inferences required (Tversky, 2005a, 2005b). What is understood from such visual elements depends on spatial ability and the domain specific knowledge (knowledge of geometry) of the visualizer. To think and operate on the geometric element, visualizer combines his/her knowledge of geometry and spatial ability, and determines what knowledge he/she should notice and how he/she would organize that knowledge (Downs & DeSouza, 2006).

One of the theoretical perspectives on the development of geometric knowledge is cognitive science that attempts to integrate theoretical work from psychology, philosophy, linguistics, and artificial intelligence (Clements & Battista, 1992). Almost all cognitive science models about knowledge deal with types of knowledge and acquisitions of them. The distinction between the knowledge of concepts and procedures plays an important role in understanding knowledge acquisition in mathematics education. Although terminology of knowledge types is not the same, there are overlaps in meanings. For instance, while Piaget (as cited in Hiebert & Lefevre, 1986) distinguishes between conceptual understanding and successful action, Hiebert and Lefevre (1986) distinguish conceptual and procedural knowledge. All among these classifications, one of the knowledge types refers to “knowing that” and another refers to “knowing how” respectively. In detail, knowing how (procedural knowledge) includes rules, algorithms and procedures for solving mathematical task. On the other hand, knowing that (declarative and conceptual knowledge) is “the web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information”

(Hiebert & Lefevre, 1986. p.3). Thus, both the knowledge of facts, generalizations, and the relations among these facts constitutes knowing that. However, establishing relations among facts entails the knowledge of why. Hence, “knowing why” can be separated from knowing that and knowledge can be classified into three as declarative (knowing that), conditional (knowing when and why), and procedural (knowing how) (Alexander & Judy, 1988; Mason & Spence, 1999; Schunk 2000; Smith & Ragan, 2005). Schunk (2000) defined declarative knowledge as knowing facts, generalizations, theories, and hypothesis; procedural knowledge as knowing how to perform cognitive activities and apply concepts, rules and algorithms; and conditional knowledge as knowing when and why to employ forms of declarative and procedural knowledge. To summarize, declarative knowledge forms the basis of actions, procedural knowledge provides actions like applying algorithms, and conditional knowledge helps to explain situation and predict the results of actions (Schunk, 2000).

When the students’ have well-organized knowledge, this organization helps them have access to relevant information easily and to apply it in answering process (Chinnappan, 1998; Prawat, 1989; Rittle-Johnson, 1999; Schoenfeld, 1986). Accordingly, the relations among knowledge types are required for success. Most research dealing with the types of knowledge has exposed that there is a significant positive relationship between knowledge types and that development of one type of knowledge is linked to development of the other (Aydin, 2007; Aydin & Ubuz, 2010; Hiebert & Lefevre, 1986; Mason & Spence, 1999; Rittle-Johnson & Siegler, 1998; Rittle-Johnson, 1999). Thus, performance on tasks is not only based on specialization in procedural knowledge, but also specialization in declarative and conditional knowledge (Schoenfeld, 1986, 1988; Skemp, 1976). Measuring all these knowledge types separately and determining the interrelations among them is important for interpreting students’ knowledge and difficulties.

Educational research studies have been mainly devoted to the identification of factors affecting teaching and learning. For that purpose, many theories and models were developed to explain factors of individual learning (e.g. The Carroll's Model, the Cooley-Leinhardt Model, the Bloom's Model, the Harnischfeger-Wiley Model, the Bennett's Model, the Gagné's Model, the Glaser's Model, the Piaget's Model, the Bruner's Model; the Walberg's Model). All these models highlighted the importance of quality and quantity of instruction on students' learning and understanding. Moreover, learners' gender and ability were widely specified factors that affect learning. For instance, Walberg's model of educational productivity (Walberg, 1981) hypothesizes that student's individual differences such as ability and gender influence outcomes of education.

Recently, with the technological revolution, the popularization of computers and other media tools, and increase in computerized learning environments, researchers have become more conscious about the importance of spatial ability. In general, spatial abilities are concerned with imagination of visual stimuli and mental manipulation of it in two- or three-dimensional space by generation, retention, retrieval, transformation, and representation of visual information (Clements & Battista, 1992; Clements & Sarama, 2007a; Kovac, 1989; Linn & Petersen, 1985; Lohman, 1993). Psychometric studies of spatial ability have shown that spatial ability is not uni-dimensional. Numerous efforts have been made to differentiate spatial ability into at least two or more sub-abilities. Some researchers studied spatial ability under two constructs (Carpenter & Just, 1986; Ekstrom, French, Harman, & Dermen, 1976; French, 1951, as cited in McGee ,1979; McGee, 1979). First construct, named as spatial perception or orientation, is the ability to determine spatial relationships with respect to the orientation of one's body. The second one, visualization or manipulation, is ability to mentally manipulate, rotate, twist, invert or fold-unfold a pictorially presented object (McGee, 1979). Some other researchers studied spatial ability under three constructs by considering mental rotation as a separate ability (Linn & Petersen, 1985; Lohman, 1993; Thurstone, 1938, as cited in Sternberg, 2003). For instance, Linn and Petersen's meticulous meta-analysis research (1985) presented a framework for spatial abilities. They categorized spatial ability processes into three constructs based on solving processes of the tasks with cognitive and psychometric rationales. Spatial perception, similar to previous categorization, is the ability to determine spatial relationships with respect to the orientation of one's body. However, they separated visualization ability into two: mental rotation and spatial visualization. Mental rotation ability refers to the ability of envisioning the rotation of an object rapidly and correctly. On the other hand, visualization ability refers to other complex manipulations of presented object such as folding-unfolding. The studies on spatial abilities generally reported positive correlational relationships among different spatial abilities (Hegarty & Waller, 2004; Hegarty et al., 2006; Karaman & Yontar Toğrol, 2010).

In addition to attempts to define and understand spatial abilities, researchers also search for the affects of spatial abilities on learning and understanding mathematics (Archavi, 2003; Ethington & Wolfle, 1984; Wai, Lubinski, & Benbow, 2009), geometry (Battista, Wheatley & Talsma, 1982; Casey, Nuttall & Pezaris, 2001; Clements & Battista, 1992; Gutiérrez, 1992, 1996; Hannafin, Truxaw, Vermillion, & Liu, 2008; Lean and Clements, 1981; Malara, 1998; Parzysz, 1988; Parzysz, 1991), chemistry (e.g. Bodner & Guay, 1997), geology (Titus & Horsman, 2009), and many other disciplines.

Numerous studies revealed that spatial ability is essential for geometric thought and enhancing students' spatial abilities is one of the roles of geometry education (Battista, 1990, 2007; Casey, et al., 2001; Clements & Battista, 1992; Gutiérrez, 1996; McGee, 1979). Spatial ability is declared as a fundamental element in learning and teaching three-dimensional geometry (Gutiérrez, 1996). Researchers are convinced that students' performance in geometry not only related to their knowledge of geometry but also their spatial ability.

Moreover, the Ministry of National Education (MEB) supports this view by emphasizing the contribution of spatial abilities to the comprehension of geometric/mathematical concepts and theorems, and the development of problem solving and logical thinking skills (MEB, 2011). In the new Secondary School Geometry Curriculum, development of the students' spatial abilities is given importance and the objectives of the program encompass developing students' spatial abilities (MEB, 2011). Likewise, National Council of Teachers of Mathematics (NCTM) emphasizes the relation between geometry knowledge and spatial ability and mentions about the importance of students' spatial abilities in order to study geometry in one, two, and three dimensions in a variety of situations (NCTM, 2000).

> "Through the study of geometry, students will learn about geometric shapes and structures and how to analyze their characteristics. Spatial visualization …is an important aspect of geometric thinking. … Geometric thinking and spatial reasoning offer ways to interpret and describe physical environments and can be important tools in problem solving. … Spatial reasoning is helpful in using maps, planning routes, designing floor plans, and creating art." (NCTM, 2000, p. 41)

> "Students should develop visualization skills … that allow them to turn, shrink, and deform two- and three-dimensional objects. Later, they should become comfortable analyzing and drawing perspective views, counting component parts, and describing attributes that cannot be seen but can be inferred. Students need to learn to physically and mentally change position, orientation, and size of the objects in systematic ways as they develop their understanding about congruence, similarity, and transformations." (NCTM, 2000, p.43)

We live in a three-dimensional (3D) world; and around us there are many 3D geometric shapes. The 3D geometric shapes comprise a fundamental portion of the geometry knowledge students need to have during education. Students encountered with prisms and pyramids concepts from elementary grades. In most of the countries (e.g. Australia, Germany, Turkey), curriculum documents propose that students should develop knowledge of prisms and pyramids during elementary education and expand this knowledge through geometry courses in secondary education. They learn some of the terminology used to describe prisms and pyramids, their mathematical properties, and how to calculate their surface area and volume. However, literature on knowledge of geometry was generally limited to plane geometry and elementary level. For instance, Battista (1990) investigated high school students' mental rotation ability, logical reasoning, problem solving, and geometry achievement on angles, polygons, circles, congruencve, similarity, and coordinate geometry, strategies. He reported gender differences in geometric and spatial thinking, and in relations between students' abilities and geometry performance on plane geometry. Ambrose and Kenehan (2009) conducted an experimental study to understand the improvement on elementary students' thinking on 3D geometry. Thus, literature review showed that research on prisms and pyramids in secondary schools is markedly absent in educational research.

Although aforementioned objectives are same for all secondary grade learners, students in different schools generally do not have equivalent opportunities to reach them. It is wellknown that school environment has a significant effect on academic performance (Higgins et al., 2005). Berberoğlu and Kalender (2005) studied the effect of school type differences on students' performance in Turkey in the PISA 2003 data. They reported that students in science high school, Anatolian high schools, private high school, and Police College are superior to the students in public high schools, vocational high schools, and Anatolian vocational high schools (Berberoğlu & Kalender, 2005). In another study, Berberoğlu (2005) examined the sources of variation in mathematical literacy skills of students. In the analysis, he classified schools as general schools (including public high schools, vocational schools and Anatolian vocational schools) and private high schools (including science high schools, Anatolian high schools, private high schools, and Police College). Results of the school difference analysis showed that private schools are more successful than general high schools and the difference was nearly two standard deviations. He reported that private school students are less anxious on mathematics than general high school students are, and they have more positive and disciplined class atmosphere than general high school students have. In addition, their self- efficacy and self-concept are greater than general high school students' self- efficacy and self-concept. Moreover, private schools offer mathematics extension courses, extracurricular mathematics activities, and mathematics competitions more frequently. On the other hand, in general high schools, teachers have low expectations of students, the student-teacher relations are poor, and students are not being encouraged to achieve their full potential. In a more recent study, Alacacı and Erbaş (2010) investigated the effects of school characteristics on students' mathematics performances in the PISA 2006. Similar to Berberoğlu, results indicated the advantage of Anatolian high schools on general high schools. Results of hierarchical linear modeling analysis revealed that 55% of the variance is attributable to differences between school types. School type was used as a dimension of the between-school variance and its potential relation to between-school variance was declared. Consequently, the type of school appears to be a variable that cannot be neglected in educational studies in Turkey.

Gender difference in students' mathematics and spatial ability performances has long been investigated through narrative and meta-analytic reviews. Maccoby and Jacklin (1974) reported the males' superiority in mathematics during the high school years. Researchers attributed this difference to their greater interest in quantitative area. In addition, they emphasized that the gender difference in mathematics was probably not as great as the difference in spatial ability. Ethington and Wolfle (1984) reported the significant gender difference in mathematics performance and they attributed this difference to the positive attitudes toward mathematics and prior mathematics performance. In addition, Ethington and Wolfle (1984) found that the influence of these variables on performance was stronger for males than for females. Differently, Dees (1982) did not find any gender difference in geometry learning and indicated that females were equally able to learn geometry. Battista (1990) reported the male advantage on geometry achievement and geometry problem solving. More recently, Ai (2002) reported that gender differences in growth in mathematics varied by one's initial status in mathematics.

On the other hand, gender differences in spatial ability in favor of males were frequently reported (Ethington & Wolfle, 1984; Kaufman, 2007; Linn & Petersen, 1985; Voyer, Voyer, & Bryden, 1995), especially for mental rotation ability and spatial visualization ability. The origins of the gender differences in spatial ability tasks have been investigated through different aspects. With the emphasis on gender differences, most of the studies explained the individual differences with genetic, hormonal, neurological, environmental factors or complex interactions among these (Coluccia & Louse, 2004; Kaufman, 2007; Kimura, 1996; Linn & Petersen, 1985; Maccoby & Jacklin, 1974; Mohler, 2008; Newcombe, Bandura, & Taylor, 1983). Aforementioned studies on gender differences revealed the importance of students' gender as a variable in spatial abilities and mathematics performance.

1.1 Purpose of the Study

Having established these facts, it seems necessary to examine the relationships among students' knowledge on prisms and pyramids, spatial ability, gender and school type by using path analysis in structural equation modeling technique. Path analysis enable researcher to measure the direct and indirect effects that one variable has upon another. Moreover, comparison of the magnitude of the directs and indirect effects lead usto identify the causal mechanism (Olobatuyi, 2006; Peyrot, 1996). Consequently, the purpose of the current study is to explore the interrelations among geometry knowledge types (declarative, conditional, and procedural knowledge), interrelations among spatial abilities (spatial visualization, mental rotation, and spatial perception ability), investigate the direct and indirect effects of spatial abilities on geometry knowledge types, and the direct and indirect effects of gender and school type on knowledge types and spatial abilities by using path analysis in structural equation modeling technique.

Main research problem of this study is:

How 12^{th} grade students' geometry knowledge types, spatial abilities, gender, and school type are related?

Thus, this study will seek answers to the following research questions:

- What is the path model that explains the interrelations among students' declarative, conditional and procedural knowledge of geometry?
- What is the path model that explains interrelations among students' spatial visualization, mental rotation, and spatial perception abilities?
- What is the path model that explains the relationship among spatial abilities and knowledge types?
- What is the path model that explain the effect of gender on spatial abilities and knowledge types?
- What is the path model that explain the effect of school on spatial abilities and knowledge types?

1.2 The Hypothesized Model of the Study

Path Model was tested by model trimming approach according to empirical standards. Accordingly, path analyses began with the just-identified model (Figure 1.1) and continued by simplifying it via eliminating paths according to statistical criteria. Chi-square difference test was used to test the statistical significance of the decrement in overall fit (Kline, 2005).

Hypotheses of the model were:

H_o: The linear structural model is not statistically significant.

H_a: The linear structural model is statistically significant.

Hypothesized (saturated model) model which was indicating all possible relations was presented in Figure 1.1. In the saturated model, existence of reciprocal relationships among the variables concerning knowledge types; reciprocal relations among the variables concerning three types of spatial ability was hypothesized. Additionally, in the model, students' school types and gender had an effect on all knowledge types and all spatial abilities. Furthermore, existence of the effect of each spatial ability factor on all knowledge type was hypothesized.

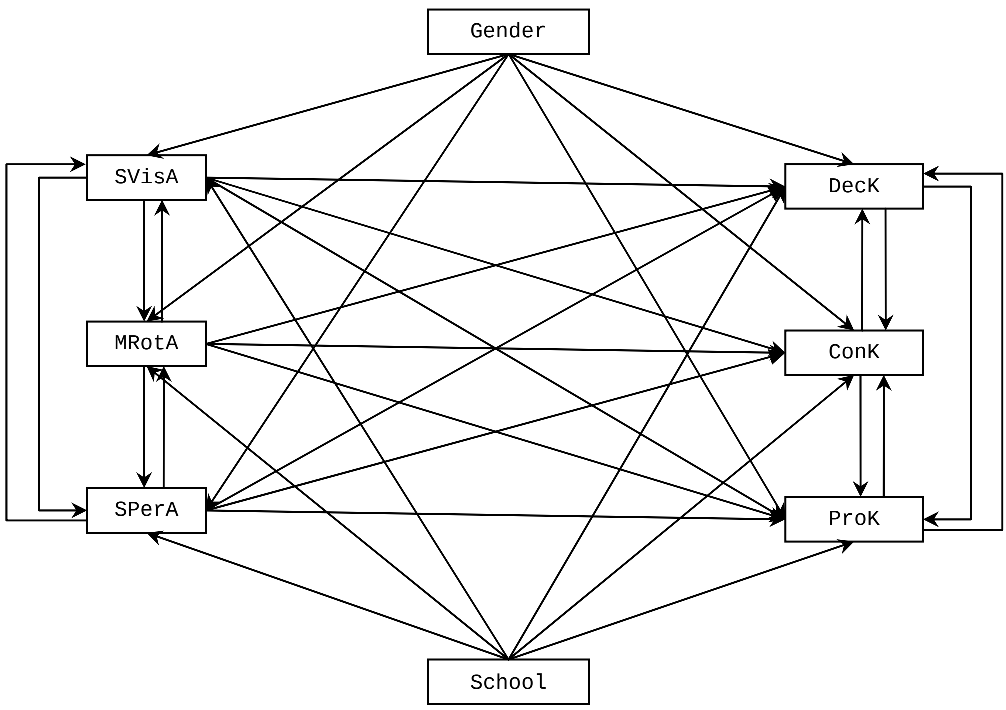


Figure 1.1 Hypothesized model representing the relationships among knowledge types, spatial abilities, gender, and school type

Note: (DecK: declarative knowledge, ConK: conditional knowledge, ProK: procedural knowledge, SVisA: spatial visualization ability, MRotA: mental rotation ability, SPerA: spatial perception ability, School: school type)

1.3 Significance of the Study

Geometry can be considered as the origin of the visualization in mathematics but if we examine the papers and books (Battista, 2007; Gutiérrez, 1996; Zimmerman & Cunningham, 1991), we find many of them focusing on the teaching or learning about plane geometry and only a few focusing on space geometry. Thus, this study hopes to contribute to the literature by focusing on secondary students' knowledge on prisms and pyramids and spatial ability.

Previous research demonstrated the interaction between the knowledge of concepts and knowledge of procedures (Hiebert, 1986). However, these studies disregarded the discrimination of knowledge of concepts as declarative and conditional. In this study, knowledge of geometry was investigated through declarative, conditional, and procedural knowledge. Such distinction provides a way of understanding students' failures and successes. Furthermore, considering knowledge in a framework that distincts knowledge into three types, investigating relationships among them provides deeper understanding about the structure of students' knowledge system.

The findings will provide insight for the relationships among declarative, conditional, and procedural knowledge performances on prisms and pyramids within secondary school. Studies indicated that knowledge of concepts and knowledge of procedures are learned in tandem rather than independently (Aydin, 2007; Rittle-Johnson & Siegler, 1998; Rittle-Johnson et al., 2001). Conceivably, the relations among knowledge types should be investigated. Delineating how the three types of knowledge interact with each other would provide teachers further suggestions about the elements that should be included during geometry teaching.

Spatial ability is an important ability for understanding and operating visual elements (Mohler, 2008; Sternberg, 2003). Some researchers, who investigated the spatial ability and its relations with other variables, generally used a single test score as an indicator of spatial ability (Casey et al, 2001). Others mostly discriminated spatial ability into two constructs (Carpenter & Just, 1986; Hegarty & Waller, 2004; McGee, 1979) and reported correlational relations (Hegarty et al., 2006). Considering spatial ability in a framework that distinguishes spatial ability into three abilities, and investigating relations among them would provide deeper understanding about the structure and nature of spatial abilities.

The investigation of the relations among spatial abilities and geometry performance is not a new subject. However, studies investigating this relationship are generally carried out with the data of nationwide exams, standardized tests, geometry tests that include a few items from main topics (Casey et al, 2001; Wai et al., 2009). The literature shows that most of the studies used a single variable an indicator of performance or spatial ability to investigate the relations among them (Casey et al, 2001). Moreover, literature on knowledge of geometry was mostly limited to elementary school or 2D geometry. There is a lack of research concerning the secondary school students' performance regarding prisms and pyramids in 3D geometry. In the current study, students' performance was measured by a purposely developed test which includes items related to declarative, conditional, and procedural knowledge on prisms and pyramids. Students' spatial abilities are measured by a test that includes items on three constructs namely spatial visualization, mental rotation, and spatial perception abilities. The findings will provide empirical evidence that there are relations among spatial abilities and knowledge of geometry regarding prisms and pyramids.

Moreover, deeper understanding about the structure and nature of spatial abilities and knowledge types, and the relationships among these may aid effective mathematics teaching environments. Thus, results of this study aims to give support to geometry teachers through illustrating variables related to students' knowledge of geometry on prisms and pyramids.

In educational research, the constructs are complex and multidimensional statistical designs are more powerful. Using linear combinations of variables increases the chance of discovering relationships or differences that single variable designs could not determine (Guarino, 2004). Structural equation modeling provides greater flexibility to test the structure coefficients in a causal model. Accordingly, the present study employs an advanced statistical technique (path analysis using Structural Equation Modeling technique) in order to investigate the causal relations among variables in the model. The investigation of the effects of spatial abilities on geometry knowledge types will provide further comprehension about the relations among them. Besides, determination of direct and indirect effects of spatial abilities on knowledge factors will help teachers to design purposive educational environments.

Furthermore, estimation of the group differences on variables is analyzed through the specification of a multiple indicators and multiple causes model (MIMIC model) where factors with effect indicators are regressed on one or more dichotomous cause indicators such as gender and school type (Kline, 2005). Thus, the results of this study will be important for the identification of the factors affecting geometry performance and spatial abilities.

The investigation of the school as a cause indicator of spatial abilities and knowledge types will provide further information about the outcomes of schools that follow different educational policies. The determination of educational outcome differences will help to identify activities that facilitate geometry teaching and learning.

This study will be a step for the investigation of the relationships among knowledge types in relation to spatial abilities considering interrelations among spatial abilities and interrelations among knowledge types. In addition, direct and indirect effects of gender and school differences were not neglected. The relations emerged from this study provide suggestions for teachers, instructional designers and mathematics education researchers. The findings present evidence for the relations among abilities and performances, and understanding these relations will provide insight for the development of comprehensive educational environments. New ideas for the curriculum development can emerge based on the relations asserted in this study.

A better understanding of different cognitive abilities underlying geometry performance will lead to better curriculum standards, teaching activities, and recommendations for geometry teaching, and for educational interventions.

1.4 Variables of the Study

School Type (School): There are various types of high schools in Turkey. General high schools were public schools that accept all students who want to enter these high schools. Anatolian high schools refer to public high schools that admit their students based on Secondary Education Institutions Entrance Exam (OKS) score. These schools provided more lessons in selected foreign language (English, German or French). Tenth grade students in both schools used to choose one of areas: Turkish Language - Mathematics, Science, Social Science, and Foreign Language. Geometry courses are offered Turkish Language – Mathematics and Science areas based on the same curriculum.

Knowledge: Knowledge is defined as interconnected facts and generalizations of organized information (Gagné, Wager, Golas, & Keller, 2005). Students acquire knowledge in three different forms: declarative, conditional, and procedural knowledge.

Declarative Knowledge (DecK): Declarative knowledge refers to “knowing that” that requires recalling, remembering, describing, and listing facts, names and organized information (Alexander & Judy, 1988; Schunk, 2000). Knowledge of concept definitions, recall of facts, formulae, and components of relevant geometric object are the samples of declarative knowledge.

Conditional Knowledge (ConK): Conditional knowledge refers to “knowing why” that involves the knowledge of when and where to access facts or employ particular procedures (Alexander & Judy, 1988; Schunk, 2000). Making connections among concepts, generating explanations, and performing condition-action processes are the samples of conditional knowledge.

Procedural Knowledge (ProK): Procedural knowledge refers to “knowing how” that includes application of rules and principles (Alexander & Judy, 1988; Schunk, 2000). Knowing what to do and how to do, determination of the procedure, recalling the steps of the procedure, applying the steps correctly in correct order, and confirming the results are samples of procedural knowledge.

Spatial Abilities: Spatial abilities refer to abilities that include generation, retention, retrieval, transformation, and representation of visual information (Linn & Petersen, 1985; Lohman, 1993). Spatial ability can be distinguished into three: spatial visualization, mental rotation, and spatial perception ability.

Spatial Visualization Ability (SVisA): It is related to the tasks that involve “complicated, multistep manipulations of spatially presented information” (Linn & Petersen, 1985, p.1484).

Mental Rotation Ability (MRotA): This ability includes the rapid and correct rotation of a visual object mentally (Linn & Petersen, 1985).

Spatial Perception Ability (SPerA): It is another type of spatial ability that is required to determine spatial relationships with respect to the orientation of viewers’ bodies (Linn & Petersen, 1985).

CHAPTER II

LITERATURE REVIEW

This chapter starts with the literature review on knowledge, then followed by literature on spatial ability. Next, the literature about the relationship between geometry performance and spatial ability were explained Then, literature on gender difference was presented. Review continues with the school differences. Finally, summary of the related literature were presented.

2.1 Knowledge

Research in cognitive science has produced intensive literature on knowledge. Knowledge is defined as interconnected facts and generalizations of organized information (Anderson, 2005; Gagné et al., 2005; Schunk, 2000). There is plenty of research that was devoted in defining and discussing issues related to various types of knowledge involved in mathematics learning and teaching. Within the mathematics education literature, various forms of knowledge have been mentioned e.g., instrumental, relational, conceptual, procedural, formal, visual, knowing that, knowing how, knowing why, knowing to, intuitive, analytical, implicit, explicit, tacit, elementary and advanced (Hiebert, 1986; Tirosh, 1999).

Hiebert (1986) provided specific literature on knowledge of mathematics. Hiebert and Lefevre (1986) examined knowledge of mathematics in terms of conceptual and procedural knowledge. They defined conceptual knowledge as "a connected web of knowledge, a network in which the linking relationships are prominent as the discrete pieces of information. Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network" (p.3). Procedural knowledge, as Hiebert and Lefevre (1986) defined, is composed of formal language of mathematics and rules, algorithms or procedures used to solve tasks. From their descriptions, it can be inference that the essence of conceptual knowledge is the cognitive connections between the pieces of information, and the knowledge of formal language in other words symbol representation system of mathematics take part in procedural knowledge. However, small pieces of information can be thought as a different kind of knowledge. Besides, knowledge of symbol system could found place in declarative knowledge because it is a piece of knowledge that can be used to make connections and used in algorithms.

Differently, Anderson (1982, 1985, 1987, 1996, 2005) used the term declarative knowledge to define the knowledge of a set of facts and stated that declarative knowledge mostly came in the form of rules. He defined productions as the units of procedural knowledge that refers to steps in which a problem is solved. The system of productions constituted learners procedural knowledge. Apart from other researchers, he used the term conceptual knowledge for the abstraction of set of facts and experiences to general categorizations of the properties of those facts and experiences.

Alexander and Judy (1988), in their report based on an extensive review of literature, defined domain specific knowledge as declarative, conditional and procedural knowledge. According to Alexander and Judy, declarative knowledge refers to factual information and it is "knowing that"; whereas conditional knowledge involve the knowing when and where to access facts or employ particular procedures. Procedural knowledge is selecting declarative knowledge into functional units and in other words, it is "knowing how". In addition, they mentioned about strategic knowledge as a form of procedural knowledge that refers to the knowledge of strategies which are goal directed procedures that intentionally evoked prior to, during or after the performance of a task.

Similar to Alexander and Judy (1988), Mason and Spence (1999) described three type of knowledge: knowing that, knowing how and knowing why. They discussed some epistemological distinctions among them and stated that these three types of knowledge constitutes the core of the institutionalized education. However, these knowledge types are not enough to develop awareness that enables students to know how to use this knowledge in new situations. Depending of this idea, authors proposed three approaches and offered a new knowledge – knowing- to act – that enables students to act creatively rather than reacting to stimuli with trained behavior. They stated that knowing-to act is an active knowledge which is presented in a moment when it is required, and teaching and testing it is harder than knowing that, knowing how and knowing why. That is why knowing-to was not involved in our study.

From a social cognitive point of view in conjunction with information processing theory, Schunk (2000) mentioned that students acquire knowledge in three different forms. Facts, generalizations, theories, hypotheses and truths about world events constitute students' declarative knowledge; rules, algorithms and the knowledge of how to perform cognitive activities constitute procedural knowledge; and knowledge of "when to employ forms of declarative and procedural knowledge and why it is important to do so" (p.82) constitute conditional knowledge.

Differentiation about knowledge of concepts and knowledge of procedures has been declared by many studies in the field of mathematics; however, studies that differentiated conceptual knowledge as declarative and conditional knowledge are seldom. Detailed descriptions of three different types of knowledge were presented in the following sections.

2.1.1 Declarative Knowledge

As cited in Smith and Ragan (2005), Gagne (1985) distinguished possible learning outcomes and one of them is declarative knowledge. Declarative knowledge is mostly described as "knowing that" that requires recalling, remembering, describing, and listing facts, names and organized information (Alexander & Judy, 1988; Anderson, 2005; Mason & Spence, 1999; Ryle, 1949; Schunk, 2000; Smith & Ragan, 2005). Declarative knowledge mostly came in the form of rules, facts, and hypothesis (Anderson, 1982, 1983a; 1983b; 1985, 1987, 1996, 2005). Knowledge of concept definitions, recall of facts, formulae, and components of relevant geometric object are the samples of declarative knowledge. It is parallel to Bloom's level of recall and understanding and acquisition of it is equated to rote learning. Thus, declarative knowledge can be seen as a low-level learning outcome. Smith and Ragan (2005) stated that declarative knowledge constitutes the basis for students to learn objectives that are more complex. Schunk (2000) stated that declarative knowledge is often processed automatically and "meaningfulness, elaboration, and organization enhance the potential for declarative information to be effectively processed and retrieved." (p.154). Meaningfulness is important for learning declarative knowledge because integrating new knowledge to the existing knowledge helps transforming knowledge in long-term memory. In similar fashion, elaboration promotes storage of knowledge by adding new knowledge to the knowledge being learned in the form of examples or details that serve to link new and old information. Organizing knowledge by breaking it into small parts simplifies the acquisition of knowledge since well-organized knowledge is easier to process. Thus, linking, elaboration and organization are important for declarative knowledge to be efficiently processed and retrieved (Schunk, 2000; Smith & Ragan, 2005). However, retrieval of declarative knowledge is often slow and conscious (Anderson, 2005; Schunk, 2000). The use of declarative knowledge was slow because the learner had to reclaim specific facts and interpret them independently. Even assuming student knows the answer to a question, s/he may have to think again and answer consciously.

2.1.2 Conditional Knowledge

According to Alexander and Judy (1988), conditional knowledge involves the knowing why, when, and where to access facts or employ particular procedures. It is concerned about the relational rules and principles. Schunk (2000) defined conditional knowledge as “knowledge of when to employ forms of declarative and procedural knowledge and why it is important to do so” (p.82). The opinions of Schunk (2000) stressed the role of conditional knowledge for achievement by stating “conditional knowledge helps students select and employ declarative and procedural knowledge to fit task goals.”(p.179). It includes ‘if-then’ or ‘condition-action’ statements that explain the relations among concepts in a particular domain (Aydin, 2007). These relations among concepts can also be described in cause and effect relationship. ‘If’ statement includes the cause or the condition that changed and ‘then’ refers to the effect or action that revealed from the cause. Depending on this idea, knowledge of facts and procedures does not guarantee achievement if the knowledge of when, why and in which condition that knowledge was employed was lacking. Conditional knowledge enables learner to predict what will happen when one condition is changed, to explain why the situation fits, and to select the appropriate algorithms in different circumstances. To predict, explain, or control circumstances, one needs the use of conditional knowledge (Smith & Ragan, 2005).

2.1.3 Procedural Knowledge

As cited in Smith and Ragan (2005), another possible learning outcome that Gagné (1985) mentioned is procedural knowledge (intellectual skills). Procedural knowledge is “knowing how” that includes application of rules and principles (Alexander & Judy, 1988; Anderson, 1996; Hiebert & Lefevre, 1986; Schunk, 2000; Smith & Ragan, 2005). In addition, Alexander and Judy

(1988), use the declarative knowledge to clarify the definition of procedural knowledge. They stated that procedural knowledge is selecting declarative knowledge into functional units. Anderson's (1996) theory of Adaptive Character of Thought (ACT-R) hypothesized that procedural knowledge is stored as a production system which is a network of rules. The common point that aforementioned studies identified is that procedural knowledge includes the application of declarative knowledge that includes the knowledge of rules and procedures. It is parallel to Bloom's levels of application. Clearly, both the knowledge of what to do and how to apply this knowledge were vital for procedural knowledge. Smith and Ragan (2005) presented general information processing analysis for a procedure as follows:

> "1. Determine whether a particular procedure is applicable.
> 2. Recall the steps of the procedure.
> 3. Apply the steps in order, with decision steps is required during the procedure.
> 4. Confirm that the end result is reasonable." (p.87)

Schunk (2000) mentioned that practice is essential to establish procedural knowledge. If students apply and alter procedures to the different forms of content, the transfer of procedural knowledge to long-term memory realized. Retrieval of procedural knowledge is faster than declarative knowledge (Schunk, 2000). Once learner acquires procedural knowledge, they retrieve it quickly and automatically. If students learn how to perform procedures, they do not have to think about steps consciously.

2.1.4 Relations among Declarative, Conditional and Procedural Knowledge

Various researchers discussed the relations among knowledge types. For instance, Alexander and her colleagues (Alexander & Judy, 1988; Alexander, Schallert, & Hare, 1991) explicated the connection among knowledge types. In respect to their ideas, declarative knowledge is factual knowledge that can be used in certain processes or routines (procedural knowledge). Additionally, conditional knowledge is the understanding of where and when to access declarative and procedural knowledge. They emphasized that the acquisition of one type of knowledge does not guarantee the acquisition of another type. Alexander and Judy (1988) hypothesized that declarative, conditional, and procedural knowledge has vital role for the efficient and effective utilization of accurate process; and accurate process contributes to the utilization and acquisition of declarative, conditional, and procedural knowledge. Additionally, they hypothesized that as knowledge in the domain increases, strategic processing is altered. On the other hand, they stated that the nature of the domain and the structure of the task affected the importance of domain specific knowledge.

In the same way, Mason and Spence (1999) explained the relations among knowledge types and suggested that knowledge types are formed sequentially. At first declarative knowledge was formed. Then, connections between facts that constitute conditional knowledge were developed. At last, procedural knowledge provides actions based on declarative and conditional knowledge. They stated that

> "… knowing-that [declarative knowledge] forms the ground, the base energy upon which all else depends and on which actions depend; knowing-why [conditional knowledge] provides an overview and sense of direction that supports connection and link making and assists reconstruction and modification it difficulties arise en route. Knowing-how [procedural knowledge] provides action, thinks to do, changing the situation and transforming it, and providing the various knowings with fresh situations upon which to operate" (p146).

Hiebert and Lefevre (1986) emphasized that two types of knowledge are distinct, but linked in critical and reciprocally beneficial ways. They questioned the differences and similarities between conceptual and procedural knowledge. Hiebert and Lefevre (1986) emphasized that it is difficult to imagine someone possessing conceptual and procedural knowledge as entirely independent systems. In fact, although it is possible to consider procedures without concepts, it is not easy to imagine conceptual knowledge that is not linked with some procedures. According to them, this is due to the fact that procedures translate conceptual knowledge into something observable.

> “Mathematical knowledge includes significant and fundamental relationships between conceptual and procedural knowledge. When the concepts and procedures are not connected, students may generate answers but not fully understand what they are doing. … They stated that meaning is generated as relationships between units of knowledge are recognized or created, so the conceptual knowledge must be learned meaningfully. Procedures, on the other hand, may or may not be learned with meaning. They proposed that procedures that are learned with meaning are procedures that are linked to conceptual knowledge. In similar vein, conceptual knowledge cannot be generated directly by rote learning, in contrast, procedures can be.” (Hiebert & Lefevre, 1986)

Anderson (1982, 1983a; 1983b; 1985, 1987, 1996, 2005) mentioned that people’s knowledge switches from explicit use of declarative knowledge to direct application of procedural knowledge rapidly by proceduralization process. According to Anderson, declarative knowledge and the repeated use of declarative knowledge in procedures (productions) give rise to the acquisition of procedural knowledge.

According to Smith and Ragan (2005), declarative knowledge is strongly tied to other types of knowledge and declarative knowledge is necessary to understand problems in order to solve them. Furthermore, the knowledge of procedures requires a form of declarative knowledge that includes the recall of a list of steps necessary to complete a skill. From this point of view, all procedures have a declarative knowledge component – knowledge of steps and knowing what to do. However, having this knowledge is not enough. The student must integrate the concepts within the procedures. He concluded that the problem solving requires combination of conditional and procedural knowledge with a basis of declarative knowledge.

As to Schunk’s (2000) ideas, declarative and procedural knowledge interacts, and retrieval of both is necessary for learning. Having declarative knowledge is typically a prerequisite for implementing procedures successfully.

Nevertheless, retrieval of it is slower than procedural knowledge. Additionally, procedural knowledge is retrieved often automatically, while retrieval of declarative knowledge is conscious. In line with these ideas, he concluded that, learner may have difficulty in learning because they lack domain specific declarative knowledge in other words they do not understand the prerequisite steps. Moreover, the opinions of Schunk stressed the role of declarative and procedural knowledge for conditional knowledge. However, he added that having declarative and procedural knowledge does not guarantee to perform in conditional knowledge. Well performance depends on both having knowledge of facts and procedures, and when and why to select and employ that knowledge. Furthermore, he stated that conditional knowledge is a form of declarative knowledge because it is the relations of "knowing that".

With respect to these studies, it is important to perceive the interrelations among declarative, conditional, and procedural knowledge to characterize performance. Conclusively, research should be explicitly designed to infer the causal relations among knowledge types, and instruments used to ascertain knowledge should be sensitive to theoretical orientation (Alexander & Judy, 1988; Alexander et al., 1991; Anderson, 2005).

The examination of the interaction among knowledge types is critical to understand their role in mathematical performance (Hiebert & Lefevre, 1986; Rittle-Johnson, Siegler, Alibali, 2001). Most of the empirical research in the field of mathematics education distinguished the knowledge of mathematics as conceptual and procedural knowledge. The association between conceptual and procedural knowledge has long been investigated in various domains such as counting (Cowan, Dowker, Christakis, & Bailey, 1996; Gelman, Meck & Merkin, 1986; Gelman & Meck, 1986) , arithmetic (Baroody & Gannon, 1984; Baroody & Ginsburg, 1986; Byrnes & Wasik, 1991; Cauley, 1988; Cowan & Renton, 1996; Fuson & Briars, 1990; Hiebert & Wearne, 1986, 1996; Knuth, McNeil, & Alibali, 2006; Perry, 1991; Resnick, 1982; Rittle-Johnson &

Alibali, 1999; VanLehn, 1986), number concepts (Sinclair & Sinclair, 1986); fractions (Mack, 1990; Moss &Case, 1999; Rittle-Johnson, 1999), percent (Lembke & Reys, 1994), proportional reasoning (Ahl, Moore & Dixon, 1992; Dixon & Moore, 1996), probability (Renkl, 1997); problem solving (Hiebert &Wearne, 1986; Silver, 1986), calculus (Engelbrecht, Harding, & Potgieger, 2005), and geometry (Aydin, 2007; Huang & Witz, 2011; Pesek & Kirschner, 2000; Schoenfeld, 1986; Webb, 1979). To assess conceptual knowledge students' interpretations relevant to concepts, their evaluation on whether the given statement was true or not, their predictions and explanations for the cause and consequence relations were used. To determine procedural knowledge students' use of formulas, procedures, and algorithms while solving problems were evaluated. All of these studies except for Resnick (1982) indicated the significant relationship between conceptual and procedural knowledge. Resnick's results on multidigit addition and subtraction suggested that procedural and conceptual knowledge were unrelated.

Most of these studies (Baroody & Gannon, 1984; Byrnes & Wasik, 1991; Cauley, 1988; Cowan et al. 1996; Cowan & Renton, 1996; Dixon & Moore, 1996; Engelbrecht et al., 2005; Hiebert & Wearne, 1996; Knuth et al., 2006; Ahl et al., 1992; Resnick, 1982; Rittle-Johnson & Alibali, 1999; Webb, 1979) provide at least correlational support for the idea that conceptual and procedural knowledge are related. However, these studies did not show whether the two types of knowledge influenced one another.

Qualitative studies based on interviews (Lembke & Reys, 1994; Mack, 1990; Moss & Case, 1999; Hiebert & Wearne, 1996) were conducted with elementary school students from 4th grade to sixth grade. Results of these studies were similar. Mack (1990) suggested that sixth grade learners could construct meaningful procedural knowledge by building upon declarative knowledge within their informal knowledge. She added more evidence to that conceptual knowledge should be taught prior to procedural knowledge. Hiebert and Wearne's (1996) longitudinal study with elementary school students included tasks assessing declarative knowledge (understanding task), procedural knowledge (skills) and conditional knowledge (understanding skills). Results revealed the important relation between conceptual and procedural knowledge by pointing that conceptual understanding helps students to invent new procedures, modify old ones to solve new problems, and make sense of procedures. Moreover, understanding tasks and procedures exhibited close relations to understanding of conditional knowledge that requires modification of relevant declarative knowledge. Moss and Case (1999) interviewed with 29 forth grade students in an experimental research. Experimental group received instructional sessions in which concepts and relations among concepts were emphasized; control group received traditional instructions including exercises, rules, and computations. The qualitative analysis yielded similar results with previous research. Students in both groups showed some improvement on problems that required the application of conventional algorithms. However, treatment group got more correct answers than control group did; they also reasoned about non-routine problems and demonstrated deeper understanding. In another study, Lembke and Reys (1994) found that students who successfully solved computational problems also exhibited well performance in conceptual knowledge questions. On the contrary, students with insufficient declarative knowledge and conditional knowledge had difficulties in procedural tasks.

A number of studies on knowledge acquisition suggest that procedural knowledge is based on conceptual knowledge (concept-first theories). According to concept-first theories, students acquire conceptual knowledge, and use their conceptual understanding to constrain application and transfer of procedures (Baroody & Gannon, 1984; Byrnes & Wasik, 1991; Cowan & Renton, 1996; Gelman & Meck, 1986; Siegler & Crowley, 1994; Star et al, 2005). These studies emphasized that conceptual understanding develops before the use of procedures, which embody those concepts. Conceptual knowledge seems to precede related procedural knowledge in the domain of integer addition and subtraction (Byrnes, 1992), fraction addition (Byrnes & Wasik, 1991), proportional reasoning (Dixon & Moore, 1996), single digit addition (Baroody & Gannon, 1984; Cowan & Renton, 1996; Siegler & Crowley, 1994). Hiebert and Wearne (1996) presented evidence that level of conceptual understanding predicts future procedural knowledge. They found that first grade students with greater conceptual knowledge had greater procedural knowledge in third and fourth grade. Experimental studies have shown that instructions that include conceptual rationale for procedures lead to greater procedural knowledge than procedure oriented instruction (Fuson & Briars, 1990; Hiebert & Wearne, 1996; Rittle-Johnson, 1999). The study of Byrnes and Wasik (1991) showed that simple fraction concepts develop prior to use of correct procedure for addition of fractions. However, they stated that conceptual knowledge is necessary but not sufficient for the acquisition of procedural knowledge.

In contrast, some other studies hypothesized that knowledge begins at an implicit procedural level and over time becomes explicit and well understood (procedure-first theories). Rittle-Johnson and Siegler (1998) reviewed that knowledge of the procedure for counting and fraction multiplication precedes understanding of the underlying concepts. Rittle- Johnson and Alibali (1999) indicated that improvement in procedural knowledge can be causally related to improvements in conceptual knowledge. Gelman and her colleagues reported that counting skills precede understanding counting principles. Briars and Siegler (1984) reported similar results that students accurately count before they understand one to one and order irrelevance principles. In a similar vein, Byrnes and Wasik (1991) noted that students, who did not understand the basic fraction concepts, solved fraction multiplication problems. Thus, procedural knowledge tended to precede conceptual knowledge for fraction multiplication. Repeated experiences and trial-error procedures help learner to select appropriate procedures then relevant concept understanding emerges (Gelman et al. 1986).

Overall, previous research suggested that there exist potential relations between conceptual and procedural knowledge of mathematics. However, these studies reported the relations between conceptual and procedural knowledge without distinguishing conceptual knowledge as declarative and conditional knowledge. Review of the studies revealed that the topics studies have been mainly limited to elementary school mathematics and a few studies exist in the area of geometry.

The studies deal with the types of knowledge were rare in the domain of geometry. Most of them were investigated students' knowledge on two-dimensional geometry such as triangles (Aydın, 2007; Aydın & Ubuz, 2010), and polygons (Pesek & Kirshner, 2000).

In a more recent study, Aydın and Ubuz (2010) presented a structural equation modeling study on geometry knowledge of triangles. They distinguished geometry knowledge into three: declarative, conditional, and procedural knowledge. A model that demonstrates the reciprocal relationship among these knowledge types was confirmed. Additionally, they stressed the role of each type of knowledge in knowledge of triangles. The results of the study provided additional evidence that greater knowledge of concepts is associated with greater knowledge of procedures or vice versa.

Accordingly, studies yielded that the relation between conditional and procedural knowledge helps students to have control over their performance, and to carry out meaningful procedures. Moreover, studies provided evidence that learners' failure to explain relations among facts, principles, and procedures affects their performance in procedural tasks. Consequently, having sufficient declarative knowledge is essential to build conditional knowledge, to develop appropriate procedures, and having conditional knowledge helps to adopt declarative knowledge and procedures to unfamiliar situations. The procedural performance satisfies the justification of declarative and procedural knowledge.

2.2 Spatial Ability

Since 1920s, research studies have determined a spatial ability factor in the cognitive tests administered and spatial ability has been considered as an important component in many intelligence models. Since then, many attempts have been made to define the spatial ability, its properties, and dimensions. Several researchers used different terms about this spatial factor: spatial ability, visual reasoning, imagination, spatial thinking, imagery, visualization, mental image, spatial images, spatial visualization ability, visualization ability, spatial imagery and many others… Besides, different researchers emphasized different properties of the spatial ability and defined this spatial factor in a different way. For instance, Kelley (1928, as cited in McGee, 1979) identified a spatial ability factor and described it as the mental manipulation of shapes. Thurstone (1938, as cited in Sternberg, 2003) identified seven primary mental abilities including spatial ability and defined it as the ability entails "visualizing shapes, rotations of objects and how pieces of a puzzle would fit together" (p. 28). More studies that are recent emphasized the cognitive properties of spatial ability and defined it as the ability of generating transforming, retaining, retrieving visual stimulus (Linn & Petersen, 1985; Lohman, 1993). As it can be seen, general agreement is that spatial ability is about the mental manipulation of objects and their parts in two or three dimensional space (Burnett & Lane, 1980; Clements & Battista, 1992; Kovac, 1989; Olkun, 2003).

In addition to the confusion about the definition of spatial ability, there exists a dissension on its dimensions. Although research has clearly shown that spatial ability is not a uni-dimensional concept, the categorization of the factors and their relationship remain unclear. Numerous efforts have been made to differentiate spatial ability into at least two or more sub-abilities. Some researchers distinguished spatial ability into two categories such as spatial visualization and spatial orientation (McGee, 1979); mental rotation and perspective taking (Hegarty & Waller, 2004), orientation and manipulation (Carpenter & Just, 1986). Some others distinguished it into three groups such as spatial relations and orientation, visualization, and kinesthetic imagery (Michael, Guilford, Fruchter & Zimmerman, 1957, as cited in McGee, 1979); visualization, orientation, and relation (Lohman, 1979, as cited in Linn & Petersen, 1985); spatial perception, mental rotation, and spatial visualization (Linn and Petersen, 1985). In a different way, Maier (1996) examined spatial ability in five sub-abilities as spatial perception, visualization, mental rotation, spatial relations, and spatial orientation.

McGee (1979), in his review of spatial ability literature, supported the existence of at least two spatial ability factors: spatial visualization and spatial orientation. According to McGee, spatial visualization ability is “the ability to mentally rotate, manipulate, and twist two- or three- dimensional stimulus object” (p.896). Spatial orientation ability is “the comprehension of the arrangement of elements within a visual stimulus pattern, the aptitude to remain unconfused by changing orientations in which a spatial configuration may be presented and an ability to determine spatial orientation with respect to one’s body” (p.897)

Hegarty and Waller (2004), like (McGee, 1979), distinguish spatial ability into two: mental rotation and perspective taking. Hegarty and Waller discussed the difference between these sub-abilities in terms of mental transformations that their framework is based on. Similar to Thurstone (1950, as cited in McGee, 1979), they conceptualized that perspective taking is related to egocentric transformations (imagining themselves moving) and mental rotation is related to object-based transformations (imagining object is moving with respect to a reference). They studied with undergraduate students and used six different paper-and-pencil tests of spatial abilities (the Card Rotation Test, the Flags test, the Vandenberg Mental Rotations Test, the object Perspective Test, the Money Standardized test of Direction Sense and the Pictures Test). They tested that perspective taking and mental rotation are different sub-abilities of spatial ability. They used structural equation modeling for confirmatory factor analysis. The results of the study revealed that data provides a good fit to two-factor model, which means perspective taking and mental rotation abilities are distinct abilities. In addition, their study supported that the measures of perspective taking and mental rotation abilities are highly correlated.

Linn and Petersen (1985), in their detailed meta-analysis study, defined spatial ability as a “skill in representing, transforming, generating, and recalling symbolic, nonlinguistic information” (p.1482). They focused on spatial ability categories in terms of solving processes on the basis of the cognitive and psychometric rationales. Accordingly, they categorized spatial ability processes into three constructs as spatial perception, mental rotation, and spatial visualization. Spatial visualization tasks involve “complicated, multistep manipulations of spatially presented information” (p.1484) and can be done efficiently using an analytic process. Mental rotation tasks are different from other types of spatial abilities in terms of solving processes and “involve a Gestalt-like analogue process” (p.1484) and can be done efficiently using Gestalt-like mental rotation process analogous to physical rotation of the stimuli. In spatial perception tasks respondents are required to “determine spatial relationships with respect to the orientation of their own bodies” (p.1482) and can be done efficiently using kinesthetic process. Thus, Linn and Petersen’s three-factor model with spatial visualization, mental rotation, and spatial perception constituted a basis on the framework for spatial ability.

Although the factor names and definitions differed in studies, common descriptions were used to define factors. In this study, the framework proposed by Linn and Petersen (1985) was used to define spatial ability and its dimensions.

2.2.1 Spatial Visualization Ability

The spatial visualization ability is related with the tasks that involve "complicated, multistep manipulations of spatially presented information" (p.1484, Linn & Petersen, 1985). Additionally, some spatial visualization ability tasks may involve the processes required for spatial perception and mental rotation. These tasks characteristics promote use of a well-organized analytic strategy that requires a repertoire of strategies for given task and keeping track of multistep procedures to finish the task (Linn & Petersen, 1985). The strategy repertoire for spatial visualization items might include the propensity to rely on gravitational and kinesthetic cues (Linn & Petersen, 1985). Well-known tasks for this ability include surface development and paper folding.

2.2.2 Mental Rotation Ability

Some of the earlier researchers did not differentiate mental rotation from spatial visualization (e.g. Ekstrom et al.1976). However, Linn and Petersen (1985) presented comprehensive rationales for the differentiation of the mental rotation ability.

Mental rotation tasks are distinct tasks that involve "a Gestalt-like analogue processes" (p.1984) and often require "a cognitive process analogous to the physical rotation of an object" (p.1488, Linn & Petersen, 1985). These tasks can be offered in 2D or 3D representations. Furthermore, these tasks may involve the processes required for spatial perception. Purdue Spatial Visualization Test of Rotations (Guay, 1976), Shepard & Metzler Mental Rotation Test (Shepard & Metzler, 1971), Card Rotation Test (French et al, 1963, as cited in McGee, 1979), and Cube Comparison Test (Ekstrom et al., 1976) are widely used to assess this ability.

2.2.3 Spatial Perception Ability

The term spatial perception ability is related with the tasks that respondents are required to "determine spatial relationships with respect to the orientation of their own bodies" (p.1482) and can be done efficiently using kinesthetic process (Linn & Petersen, 1985). Linn and Petersen (1985) mentioned that the performance on spatial perception could be influenced by the "knowledge about physical principles, propensity to rely on gravitational and kinesthetic cues, and propensity to combine test features analytically" (p.1487).

2.2.4 Importance of Spatial Abilities

It is clear that spatial ability has an imperative place in human thought. Many researchers have presented evidences to demonstrate its important role in various fields such as mathematics (Arcavi, 2003; Clements & Sarama, 2007a, 2007b; Dreyfus, 1991; Nemirovsky & Noble, 1997; Nuttall, Casey, & Pezaris, 2005; Wai, Lubinski, & Benbow, 2009), geometry (Battista, 1990, 2007; Battista, et al., 1982; Clements & Sarama, 2007a, 2007b; Gutiérrez, 1996; Malara, 1998; McGee, 1979; Owens &, Outhred, 2006; Parzysz, 1988, 1991; Pittalis & Christou, 2010; Presmeg, 2006), chemistry (Bodner & Guay, 1997), geology (Titus & Horsman, 2009), engineering (Alias, Black, & Gray, 2002; Nemeth, 2007; Olkun, 2003; Onyancha, Onyancha, Derov & Kinsey, 2009; Sorby, 1999, 2009), and art (Haanstra, 1996).

Lohman (1989) emphasized the importance of abilities by stating "understanding abilities means understanding individual differences in learning and development" (p.359). Thus, understanding the structure of spatial ability and its role in learning mathematics specifically in geometry is crucial to understand students' individual differences in learning and development.

Wai et al, (2009) presented the findings of a longitudinal research on spatial ability for science, technology, engineering, and mathematics domains. Participants of their study were drawn from grades 9 to 12 with the sample of

400000 students who were tracked for 11+ years. In their article, influence of spatial ability on science, technology, engineering, and mathematics domains has been supported through the presentation of the findings that link decades of longitudinal research. The results of the study suggest that spatial ability is one of the most important characteristic among youngsters who have educational and occupational accomplishment in science, technology, engineering, and mathematics domains and it plays a crucial role in constituting educational and educational outcomes.

2.2.5 Relations among Types of Spatial Abilities

The studies on spatial abilities mostly used correlational analysis to investigate the relations among them. Studies (Hegarty et al., 2006; Karaman & Yontar Toğrol, 2010) were presented evidence for the significant relationship among the types of spatial abilities. In addition, structural equation modeling studies that measure spatial abilities with different tests allowed the spatial ability factors to be correlated while they are testing their theoretical models (Hegarty & Waller, 2004).

Linn and Petersen (1985) connected different spatial abilities by means of the strategies used to solve spatial tasks. As mentioned before, selection of appropriate strategy and having repertoire of strategies have an effect on performance in spatial visualization ability. Additionally, a strategy to solve a spatial visualization item might require using gravitational and kinesthetic cues that mainly characterize spatial perception performance as well as mental rotation (Linn & Petersen, 1985). Thus, spatial visualization items could be solved using a range of processes associated with spatial perception and mental rotation.

2.3 Learning Geometry

Geometry has an important place in mathematics curriculum. School geometry should enable students to analyze two- and three-dimensional geometric objects, describe spatial relations, apply transformations, and use spatial abilities and geometric modeling to solve problems (MEB, 2010a, 2010b, 2011; NCTM, 2000). In addition to the geometric ideas, geometry can help students to gains insight to understand the nature and the beauty of mathematics, recognize and apply geometric ideas and relationships into other disciplines such as science, art, architecture and everyday life (MEB, 2010a, 2010b, 2011; NCTM, 2000). Therefore, geometry knowledge has prominent place not only inside school but also outside the school.

From a developmentalist point of view, child's geometric thought progresses through stages with the help of social interaction and active engagement with surrounding (Piaget & Inhelder, 1967). In examining consecutive stages, Piaget and Inhelder (1967) made a distinction between spatial perception and spatial imagery and arrived at a theory of geometric intuition that emphasize the importance of difference between perceptual and conceptual space. Sensory-motor activities such as handling objects, turning them over, and moving them about embraces the child's behavior. These activities allow objects physical permanence together with size and shape. When sensory-motor activities are supported by imagination, conceptual space may be said to begin. Consequently, children's different actions (physical or mental) develop understanding of topological (proximity, separation, order, connectedness, enclosure, and continuity), projective, and Euclidean relationships (similarity, reflection, parallelism and distance) correspondingly.

Similarly, Van Hiele (as cited in Usiskin, 1982) proposed another developmentalist theory. The van Hiele model of geometric thinking postulated that development of geometric thinking progresses through five stages. In the recognition level, students' perception is only visual. Next, student analyzes geometric objects in terms of their properties. Subsequently, in the informal deduction level, thinking is more theoretical that allows interrelating previously comprehended properties. In the forth level, the roles of elements of an axiomatic system and deduction are understood. The learner in the last level can explore different geometries and undertake comparison of various deductive systems.

The opinions of Duval (1998) stressed the cognitive complexity of geometry learning. He stated that learning geometry involves three kinds of cognitive processes that can perform separately.

- " visualization processes with regard to space representation for the illustration of a statement, for the heuristic exploration of a complex situation, for a synoptic glance over it, or for a subjective verification.
- construction processes by tools: construction of configurations can work like a model in that the actions on the representative and the observed results are related to the mathematical objects which are represented
- reasoning in the relationship to discursive processes for extension of knowledge, for proof, for explanation" (p.38)

According to Duval (ibid.), these cognitive processes are closely related, and relation is cognitively needed for proficiency in geometry. He emphasized that the coordination between these three kinds of processes can occur after the differentiation between different visualization processes and between different reasoning processes.

Although studies used different terminologies, they all pointed out that geometry learning is a complex and dynamic process and they all asserted the importance of visualization processes in geometrical thinking (Duval, 1998; Fishbein, 1993; Hershkowitz, 1990; Piaget & Inhelder, 1967; Presmeg, 2006).

Despite the lack of research on 3D geometry learning, the conducted studies support the models that emphasize the role of visualization in learning 3D geometry.

2.4 The Relationship Between Geometry Education and Spatial Abilities

In spite of the disagreements on definitions of spatial abilities, many researchers in the field of mathematics education have declared the importance of spatial abilities and visual reasoning (Ambrose & Falkner, 2002; Battista, et al., 1982; Bishop, 1980; Casey et al, 2001; Clements & Sarama, 2007a; Del Grande, 1987; Dreyfus, 1991; Farrell, 1987; Fennema & Sherman, 1977, 1978; Fennema & Tartre, 1985;Gutiérrez, 1996; Herskowitz, 1998; Herskowitz, Parzysz, & van Dormolen, 1996; Lohman, 1989; McGee, 1979; Moses, 1977, Pittalis & Christou, 2010; Presmeg, 2006; Sherman & Fennema, 1977; Zimmerman & Cunningham, 1991).

Farrell (1987) mentioned that spatial ability is one of the student variables of particular interest in geometry learning. She stated that geometry requires the discrimination ability with visual material and this ability (spatial abilities) helps students to locate the hidden or turned shapes in order to solve problems. Del Grande (1987) proposed the existence of reciprocal relationship between spatial abilities and learning geometry concepts. He stated that the spatial abilities should make it possible to design geometry programs and developing mathematical activities will improve learners' spatial performance. Similarly, Hoffer suggested that (as cited in Del Grande, 1987, p.126)

> "It appears that visual perception skills and geometry concepts can be learned simultaneously, since geometry requires that the students recognize figures, their relationships, and their properties. Geometry could easily be taught and included with a visual perception training program so as to improve students' visual perceptions."

As mentioned before, Lohman (1989) emphasized the importance of abilities for learning and development in terms of understanding individual differences. Thus, understanding the structure of spatial ability and its role in learning mathematics specifically in geometry is crucial to understand students' individual differences in learning and development.

Dreyfus (1991) emphasized that spatial reasoning is the intuitive, supportive, global and preliminary stage in the reasoning process and it supports further reasoning. Herskowitz (1998) asserted this intuitive support and gone one-step further by stating that “visual reasoning is the backbone of a rigorous proof” (p.36).

Gutiérrez (1996) discussed the confusion in terminology to be used in the field of spatial ability and offered a model characterizing the activity of visualization in mathematics. He attempted to integrate several elements defined by Presmeg (1986), Bishop (1980), Yakimanskaya (1991) and others. He considers visualization in mathematics as “a kind of reasoning activity based on the use of visual or spatial elements, either mental or physical, performed to solve problems or prove properties.” (p.9) Visualization, according to Gutiérrez (1996), is integrated by four elements:

1. Mental image: Any kind of cognitive representation of a mathematical concept or property by means of visual or spatial elements.
2. External representations: Any kind of verbal or graphical representation of concepts or properties (pictures, drawings, diagrams, etc.) that help to create or transform mental images and to do visual reasoning.
3. Processes of visualization: A mental or physical action where mental images are involved.
4. Visualization abilities: A set of abilities to perform necessary processes with specific mental images for a given problem.

Gutiérrez (1996) stated that geometry could be considered as the origin of the visualization in mathematics, and visualization in a fundamental element in learning and teaching three-dimensional geometry. He reported his model of spatial visualization and the application of the model. In his research, he aimed to analyze the ways students solve the activities and determine the kinds of mental images and visualization abilities they have used. He studied with students aged from seven to 17 years old and asked them to solve activities that rotation of the solids on computer screen from initial position to a target position. Based on the excerpts of two students (one second grader and one eight grader), he mentioned the importance of abilities like mental rotation, perceptual consistency and perception of spatial relationships. He concluded that the origin of the second grader's difficulties was the lack of spatial abilities like mental rotation and perception of spatial relationships. On the other hand, eight-grader easily achieved the activity by foreseeing the results of a series of rotations and she used the ability of perception of spatial positions.

Owens and Outhred (2006) reviewed research on geometry and spatial ability. They stated that the research in geometry and spatial ability was focused on problem solving in geometry, constructions of geometric concepts, the role of spatial ability, and the visual and contextual aspects of conception. However, they pointed out the lack of research on 3D geometry learning. As mentioned before, most of the studies on 3D geometry and visualization were conducted with elementary students (Ambrose & Falkner, 2002; Ambrose & Kenehan, 2009; Pittalis & Christou, 2010) or in- or pre-service teachers (Battista et al., 1982). Some others (Battista, 1990; Kirby & Boulter, 1999) studied with secondary school students; however, their studies were mostly on plane geometry topics (angles, triangles, polygons etc...). Since the context of studies differ, they all revealed the importance of spatial ability on geometry performance.

Pittalis and Christou (2010) conducted a study with 269 students from 5th to 9th grade students to describe and analyze the structure of 3D geometry thinking by identifying four types of reasoning and to examine their relations with spatial ability. Second order measurement model results of 3D Geometry Thinking Test showed that four types of reasoning, which include the representation of 3D objects, spatial structuring, measurement, and conceptualization of mathematical properties could describe 3D geometric thinking. Third order measurement model analysis of Spatial Abilities Test indicated that spatial ability could be explained under three factors of spatial visualization, spatial orientation (similar to spatial perception factor of Linn and Petersen, 1985) and spatial relations (similar to mental rotation factor of Linn and Petersen, 1985). The structural model analysis results showed that students' spatial ability is a strong predictor of four types of reasoning in 3D geometry.

Ambrose and Falkner (2002) conducted a qualitative experimental study with the first and second graders to develop spatial understanding through building polyhedrons. They explored students' spatial thinking and took notes during instruction. At first, they introduced the concept of closed structures. Then, students were asked to build one polyhedron and to write a description of it. Children made a variety of structures and described their models with holistic terms such as house, spaceship, or other familiar objects. Next task includes the building as many different structures as students could using six triangles and two squares to show students that enumerating the shapes in a polyhedron was insufficient to describe the model. Three different models were selected and analyzed by students. Discussions about the constructed shapes revealed that students persisted in seeing models as being fanciful objects and did not notice that the point was the organization of the faces. In addition, it was observed that some students dissected models into smaller three-dimensional parts. Following students interpretations of geometric descriptions, students were asked to reconstruct a polyhedron from a description. Analyses yielded that students' spatial understanding was increased and their later descriptions of polyhedrons include more information about the model than their descriptions at the beginning of the study.

Battista et al. (1982) conducted a study that investigates the importance of spatial visualization for geometry learning in pre-service teachers (N=82). They designed a geometry course to give participants opportunity to participate activities involving spatial components. For instance manipulation of concrete models, paper folding and/or using cutouts were experienced to examine symmetry of polygons and transformational geometry. Based on correlation they concluded that spatial ability can be improved by instruction and it is an important factor in geometry learning.

In another study, Battista (1990) studied the role of spatial visualization ability in learning, problem solving and gender differences in high school geometry with 145 high school geometry students (75 male and 53 female) from a middle-class, Midwestern community. He used Purdue Spatial Visualization Test: Rotations (Guay, 1977) to measure students' spatial visualization ability to mentally visualize rotations of objects in space; Logical Reasoning Test (experimenter constructed) to assess students' ability to draw conclusions in logical syllogistic format; Cooperative Mathematics Tests, geometry Part 1 and Form B to test students' knowledge of geometry on angle relations and measures, parallel lines, triangles, area and perimeter, circles, polygons, the Pythagorean theorem, congruence, similarity, proof and coordinate geometry, and Geometric Problem Solving/Strategies Test (experimenter constructed) to assess students' ability to solve geometric problems and to determine their strategies they used in solving these problems. Correlation analyses and regression analyses of this study indicated that spatial visualization and logical reasoning were important factors in geometry achievement and geometric problem solving for high school students. Moreover, males scored significantly higher than females on spatial ability (mental rotation) and high school geometry.

Kirby and Boulter (1999) investigated the effects of two types of instruction on transformational geometry upon performance and spatial ability. Seventy students from 7th and 8th grade were divided into two instructional groups (traditional approach versus teaching incorporating object manipulation and imagery). They used the Linn and Petersen's (1985) classification for spatial abilities and Hidden Patterns, Card Rotations and Surface Development Tests developed by Ekstrom et al. (1976) to assess spatial perception, mental rotation, and spatial visualization abilities respectively. Pretest scores of spatial ability and geometry performance, gender, and instructional condition were included in the regression analysis. Results revealed that posttest geometry performance was predicted by pretest spatial ability, pretest geometry performance and the interaction of pretest geometry and condition. Subjects who performed better in pretest geometry performed better in experimental group, while those who performed lower in pretest geometry performed better in the control group. However, results indicated that instructional conditions did not make a considerable difference. Follow-up regression analysis showed that posttest spatial ability was predicted by pretest spatial ability, handedness, pretest geometry, and interaction of handedness and pretest spatial ability. That is, spatial ability scores were not related to instructional condition, in both groups students' spatial abilities were improved over the course of the study. This result suggests that instructional program targeted spatial ability could facilitate the development of spatial ability. Moreover, results did not indicate any gender difference in neither spatial ability nor geometry performance.

Ambrose and Kenehan (2009) carried out a qualitative experimental study to understand the development of 3rd grade (8-9 years old) students' thinking in three-dimensional geometry. They conducted lessons in which students built and described polyhedra. Three tasks were given as pre-post assignments. In two of the tasks, students were asked to compare cube and pyramid, and a hexagonal prism and a hexagonal antiprizm. In the last task, they were asked to describe a compound polyhedron with a square on top and a cube on the bottom. The assignments, videotapes, photographs, and students' written work were analyzed through holistic and componential descriptors. Results of the study revealed that students advanced in their geometric reasoning and began to identify, enumerate, and notice relationships between component parts of polyhedra.

Some other rstudies focused on the activities that aims to develop students' visualization and 3D geometry knowledge. For instance, Pohl (1987) presented six activities and 12 exercises on tetrahedron and octahedron. She stated that one of the best ways to learn to visualize three-dimensional objects is to construct models that demonstrate the object. With the help of models, learner can experience many spatial relationships, discover and visualize various properties of three-dimensional space (Pohl, 1987).

2.5 Gender Differences

2.5.1 Gender Differences in Spatial Abilities

There are various explanations and hypotheses concerning the gender differences in spatial ability. Most of them reported the finding that males perform better than females on spatial tasks especially in mental rotation tasks (Battista, 1990; Ben-Chaim, Lappan, & Huang, 1988; Linn & Petersen, 1985; Maccoby & Jacklin, 1974; McGee, 1979; Mohler, 2008; Newcombe et al., 1983; Nuttall, et al., 2005; Halpern, Beninger, & Straight, 2011). Numerous studies aimed at determining the sources of variance in spatial ability. The origins of the gender differences in spatial ability tasks have been investigated through different aspects. With the emphasis on gender differences, most of the studies explained the individual differences with genetic, hormonal, neurological, and environmental factors (Linn & Petersen, 1985; Maccoby & Jacklin, 1974; Newcombe, et al., 1983).

In their comprehensive review, Maccoby and Jacklin (1974) indicated the consistent superiority of males on spatial tasks and this advantage was emerged in early adolescence and maintained in adulthood. One of the attributions to this difference is to biological factors. They presented evidence of a recessive sex-linked gene that contributes an element to high spatial ability in addition to heritability of it. Additionally, researchers mentioned about the effects of hormones on performance in spatial tasks and reported that androgen is an important negative factor in spatial ability. Moreover, some studies reviewed by Maccoby and Jacklin attempted to explain the gender difference in spatial ability with cerebral dominance. However, these studies did not report consistent findings. For example, Kimura and his colleagues (as cited in Maccoby & Jacklin, 1974) point out that cerebral functions relevant to spatial ability tend to be localized in the right hemisphere and found greater localization among males for certain spatial tasks being apparent at age 5. On the other hand, Buffery (as cited in Maccoby & Jacklin, 1974) found that females to be more lateralized on spatial task at ages 3-4.

Linn and Petersen (1985) supported Maccoby and Jacklin's inference and identified that gender difference exists in favor of males in two of three categories of spatial ability (mental rotation and spatial perception). However, this difference is large only for mental rotation and medium for spatial perception. They attributed the gender differences to selection and efficient application of solution strategies. They detected that females consistently select less efficient and less accurate strategies for spatial tasks that involve mental rotation and spatial visualization. They stated that gender differences are detected as soon as mental rotation could be measured. Conversely, they discussed the inconsistent findings on gender differences in spatial visualization and reported the lack of gender difference.

Nemeth (2007) studied the development of spatial ability with engineering students. They reported that male and female students' performances in spatial abilities are different. However, while this difference remained the same with instruction, development of male students was more significant. Thus, results revelaled that the improvement of spatial ability is even higher for male students, which yields that gender differences were getting stronger.

Similarly, Ben-Chaim et al. (1988) and Battista (1990) reported male superiority in their studies. Ben-Chaim et al. (1988) investigated the gender differences in spatial ability in the context of an experimental study that includes three-week visualization instruction and concluded that gender differences in spatial abilities exist in favor of males. Likewise, Battista (1990) conducted a crosssectional study on the role of spatial visualization ability in learning, problem solving and gender differences in high school geometry. He stated that male and female students were significantly differed and reported the superiority of males on mental rotation ability.

On the other hand, Fennema and Sherman studies (Fennema & Sherman, 1977, 1978; Sherman& Fennema, 1977) concluded that there are no gender differences in spatial ability among middle school students. Ertekin and İrioğlu (2012) investigated the gender differences in upper elementary students' mental rotation ability and results revealed that there is no significant gender difference in students' mental rotation ability.

In addition to investigating gender differences, some researchers have attempted to explain the underlying causes of the gender gap. For instance, Linn and Petersen (1985), attributed the lack of gender difference in spatial visualization to the effect of significant gender differences in spatial perception and mental rotation abilities. Furthermore, Linn and Petersen mentioned the effects of biological factors, hormonal changes at puberty, pubertal maturation, genetic factors and X-linked recessive major gene for spatial ability. In a different way, they proposed that these factors interact with sex-typed experiences and sex-role experiences to produce better performance. Besides, they highlighted the role of solution strategies that were applied to tasks (Linn & Petersen, 1985).

Some researchers focused on the role of biological factors (Halpern et al., 2011) and stated that gender differences in favor of males in spatial abilities lead the explanations through the biological factors. Some others emphasized the role of experiences by mentioning about the difference in males and females' experiences across lifespan and interaction between sex-role and sex-type experiences (Baenninger & Newcombe, 1989; Linn & Petersen, 1985;

Newcombe et al., 1983; Tobin-Richards & Petersen, 1981, as cited in Linn & Petersen, 1985).

2.5.2 Gender Difference in Mathematics Performance

There are numerious studies on gender differences in mathematics performance. Most of them reported male superiority, some reported female advantage, however, some other no gender differences. Literature presents inconsistent findings on gender differences in mathematics performance.

Maccoby and Jacklin (1974) reported the males' superiority in mathematics during the high school years. Researchers attributed this difference to their greater interest in quantitative area. In addition, they emphasized that the gender difference in mathematics was probably not as great as the difference in spatial ability. Casey, et al. (2001) conducted a study with eighth grade students to compare spatial ability with mathematics self confidence as mediators of gender differences in mathematics. They used the Vandenberg Mental Rotation test, the Water Level Test, academic self confidence questionnaire, the Mechanical Reasoning subtest of DAT, selected items from the 8^{th} Grade TIMMS items were administered. They decomposed significant gender differences favoring males in path analysis.

Dees (1982) investigated the gender difference in grades seven through 12 from 12 different schools specifically for geometry performance. Similar to, aforementioned studies, she found that males were superior in content knowledge upon entering the geometry courses. Conversely, when the entering scores of students were adjusted, results revealed that girls were equally able to learn geometry. Thus, no gender differences were reported in geometry learning.

Ai (2002) conducted a 3-level longitudinal and multilevel modeling study based on the data collected by Longitudinal Study of American Youth. The results of the separate group analyses suggested that there were large gender differences in initial status and growth rate. The level two analysis of gender indicated that between gender differences were found only in low achievers group. However, no differences were found for high achievers group. For low group, females started higher than boys. Ai (2002) indicated that gender differences in growth in mathematics varied by one's initial status in mathematics. Gender gap in growth rate was not statistically significant.

Some researchers investigated the possible causes of gender differences proposed that gender differences in mathematics arise from the gender differences in spatial abilities (Casey, et al., 2001; Friedman, 1995).

2.6 The Influence of School on Students Academic Performances

Researches have concluded that both the educational environment and the learner have an influence in performance. Research on gender differences in mathematics education has highlighted the roles of factors such as teachers, parents and schools as important determinants of gender differences (Ai, 2002).

Ai (2002) conducted a 3-level longitudinal and multilevel modeling study based on the data collected by Longitudinal Study of American Youth. Results indicated that gender gap in growth in mathematics performance varied across schools. In some schools, girls' average growth was higher, whereas in others boys' average growth rate was higher. For those who started high, there was no gender difference.

School type differences in students performance was analyzed by Berberoğlu (2005) and Berberoğlu and Kalender (2005) in the PISA 2003 data. Berberoğlu and Kalender (2005) studied on the PISA 2003 data of 4855 students at the age of 15. Based on the MANOVA analyses, they indicated that school type has main effect in students' performance. They reported that students in science high school, Anatolian high schools, private high school, and Police College were superior to the students in public high schools, vocational high schools, and Anatolian vocational high schools. In another study, Berberoğlu (2005) studied on the mathematical literacy skills of 4855 students at the age of 15. The findings of the study indicated that the Turkish students were performing lower than the students of other OECD countries were, and performances of students in different schools were different. However, when the data reanalyzed in terms of school differences, the results were distorted. Results of the school difference analysis presented that private schools were more successful than general high schools and the difference was nearly two standard deviation. Private high schools were placed nearly one standard deviation over the international average and general high schools were placed nearly one standard deviation below the international average. Berberoğlu also investigated the possible causes of these differences and reported that these schools were different in terms of student characteristics and educational environment. Private school students were less anxious on mathematics than general high school students were, and they had more positive and disciplined class atmosphere than general high school students had. In addition, their self-efficacy and self-concept were more than general high school students' self-efficacy and self-concept. The contributions of these differences on mathematical literacy were obvious. Moreover, private schools offer mathematics extension courses, extracurricular mathematics activities, and mathematics competitions more frequently. On the other hand, in general high schools, teachers have low expectations of students, the student-teacher relations are poor, and students are not being encouraged to achieve their full potential.

In a more recent study, Alacacı and Erbaş (2010) investigated the effects of school characteristics on students' mathematics performances in Turkey. They analyzed the PISA 2006 data of 4942 fifteen-year-old students from 76 provinces. Results of hierarchical linear modeling analysis indicated the advantage of Anatolian high schools on general high schools and revealed that 55% of the variance was attributable to between-schools. School type was used as a dimension of the between-school variance and its potential relation to between-school variance was declared. Consequently, the type of school appears to be a variable that cannot be neglected in educational studies in Turkey.

2.6.1 High Schools and the place of Prisms and Pyramids in Geometry Curriculum

2.6.1.1 High Schools

In Turkey, pre-school education is not obligatory. Children who are between 6-14 years old go to elementary schools. Eight-year elementary education is obligatory in Turkey. At the end of the elementary school period, most of the students enter the the Secondary Education Institutions Entrance Exam (Orta Öğretim Kurumları Sınavı, OKS). This test is conducted by Ministry of National Education. It involves 25 questions on Turkish Language, 25 questions on mathematics and geometry, 25 questions on science, and 25 questions on social studies. Students between 14-18 years old attend to the high school, which is not obligatory. The students who want to continue their education in high schools are placed to the high schools based on their performance in the OKS and according to their choices. The Anatolian high schools, private high schools and Anatolian teacher high schools, science high schools, social science high schools accept students based on their OKS performance. General high schools accept all students who want to enter these high schools. Consequently, at the end of the elementary education, students are stratified based on the OKS results and they continue their education in high schools in which similar achievement level colleagues exist. Thus, the Turkish Secondary School Education System is organized along achievement-stratified school types.

The most successful students prefer to continue their education in science high schools. The education at these schools is intensively based on mathematics, geometry, physics, chemistry, and biology. The subsequent preferred schools are Anatolian high schools (including teacher Anatolian high schools). These schools offer education in different areas such as science, Turkish Language and Mathematics, social science, and foreign language. These schools carry out similar curriculums. However, students in teacher Anatolian high schools have to take courses about education such as Educational Psychology, Teaching Methods Course, and Introduction to Teaching Profession. The students who want to attend the social science high school have opportunity to choice social science or Turkish Language–mathematics area and their education concentrated on social science courses. Private high schools offer similar courses but students attending to private high schools have to pay a certain fee during schooling years. Thus, private schools have better economical circumstances. Public high schools also give chance to select science, Turkish Language-Mathematics, social science, and foreign language areas; however, opportunities in these schools are more limited. High schools that accept students based on OKS empower extracurricular activities, science or mathematics competitions, and use of laboratories more than public high schools (Berberoğlu, 2005). In addition, in order to be a teacher at schools like Anatolian high schools, science high schools, and social science high schools, teachers have to be successful in Teacher Selection Exam (Fen Liseleri, Sosyal Bilimler Liseleri, Spor Liseleri, Her türdeki Anadolu Liseleri Öğretmenlerinin Seçme Sınavı). Thus, not only the students in Anatolian high schools, science high schools, and social science high schools but also the teachers were selected.

2.6.1.2 The place of Prisms and Pyramids in Geometry Curriculum

Since the participants of this study educated through former secondary school geometry curriculum (MEB, 1992), only the characteristics of them were presented here.

The 3D geometric shapes comprise a fundamental portion of the content knowledge high school students need to have (number of objectives on 3D geometric shapes/ total number of objectives in high school curriculum = 0.23).

Students encounter with prism concepts from 1st grade and pyramid concepts from 5th grade. Curriculum documents propose that students should develop some knowledge of prisms and pyramids during elementary education.

Geometry was taught for 10th, 11th, and 12th grades based on the curriculum developed in 1992. Curriculum objectives were separated through grades 10, 11, and 12. Geometry-I in grade 10 included geometric concepts (point, line, plane, coordinate, and angle) and triangles; Geometry-II in grade 11 included polygons, circles, and geometric places; and Geometry-III in grade 12 included space geometry, right projection, prisms, pyramids, cylinders, cones, and spheres. Time allowed for each geometry course was two hours a week and all the secondary schools followed the same curriculum. Within the secondary school geometry curriculum, there were 44 objectives and 344 target behaviors. Prisms and pyramids have a considerable place with three objectives, 30 behaviors and two objectives, 18 behaviors respectively. Students, who completed the Geometry-III course, were expected to define prisms and pyramids, comprehend their area and volume, and to make applications about their area and volume. The curriculum objectives include mostly acquisition of declarative and procedural knowledge. Conditional knowledge objectives have place only in pyramids subject.

The MEB (2011) recommend that students should develop the abilities to identify prisms and pyramids, recall their properties, construct prisms and pyramids by paper folding, and perform length, area and volume calculations throughout elementary and secondary education. This included attaining proficiency in declarative, conditional, and procedural knowledge about prisms and pyramids by the end of grade 12.

2.7 Summary

Knowledge is defined as interconnected facts and generalizations of organized information (Anderson, 2005; Gagné, et al., 2005; Schunk, 2000). There is a plenty of research on defining and discussing the types of knowledge within mathematics education literature. Hiebert (1986) distinguishes between conceptual and procedural knowledge; Anderson (1982, 1983a; 1983b; 1985, 1987, 1996, 2005) distinguishes between declarative, conceptual, and procedural knowledge; Alexander and Judy (1988) distinguishes between declarative, conditional, procedural, and strategic knowledge; Mason and Spence (1999) distinguishes between knowing that, knowing why, knowing how, and knowing to; Schunk (2000) distinguishes between declarative, conditional, and procedural knowledge. The difficulty of assessing knowing to or strategic knowledge leads us to orient another framework. Thus, the rationale for the distinction of declarative and conditional knowledge leads the selection of framework for interpreting knowledge in terms of essential components. As a result, the present study undertakes the knowledge within its three essential components: declarative knowledge (knowing that), conditional knowledge (knowing where and why), and procedural knowledge (knowing how). Declarative knowledge refers to the knowledge of facts, rules, hypothesis, formulas that requires recalling, remembering, describing, and listing. Conditional knowledge refers to knowledge of when and why to use appropriate declarative and procedural knowledge. It makes possible to predict the results of condition changes and to establish the relations among concepts. Procedural knowledge includes the application of rules and algorithms. Declarative knowledge forms the basis for conditional and procedural knowledge. Conditional knowledge provides a general idea including connections among facts (declarative knowledge). It is also an understanding of where and why to use declarative and procedural knowledge. Procedural knowledge provides actions based on declarative and conditional knowledge. Reasonable organization of knowledge (conditional knowledge) strengthens the understanding of facts (declarative knowledge). Similarly, repeated use of correct procedures gives rise to the consolidation of declarative and conditional knowledge (Anderson, 2005; Aydin, 2007; Aydin & Ubuz, 2010; Mason &

Spence, 1999). The students' knowledge structure has long been an important issue in mathematics education in various domains such as algebra (e.g. Byrnes & Wasik, 1991; Rittle- Johnson & Alibali, 1999; Rittle- Johnson & Siegler, 1998), calculus (e.g. Engelbrect et al., 2005), and geometry (e.g. Aydin, 2007; Aydin & Ubuz, 2010; Pesek & Kirshner, 2000; Webb, 1979). Qualitative methods (Lembke & Reys, 1994, Mack, 1990; Moss & Case, 1999; Hiebert & Wearne, 1996) or experimental methods (Byrnes & Wasik, 1991; Fuson & Briars, 1990; Hiebert & Wearne, 1996; Rittle-Johnson, 1999) were used to investigate the students' knowledge structure and the relations among knowledge types. All of these studies mentioned about the relations between knowledge types; however, most of them did not distinguish declarative knowledge and conditional knowledge.

Review of literature revealed that these studies are limited to elementary school mathematics and the studies focused on geometry have been mainly limited to 2D geometry such as polygons, triangles. Teaching and learning of three-dimensional objects such as prisms and pyramids take place in 12th grade geometry curriculum. In Turkey, University Entrance Exam may be the reason for the dearth of the studies on 3D geometry. Almost all of the 12th graders focused on this exam. Thus, researchers generally may not prefer to study with them in order to control threats.

Spatial ability are concerned with imagination of visual stimuli and mental manipulation of it in two- or three-dimensional space by generation, retention, retrieval, transformation, and representation of visual information (Clements & Battista, 1992; Clements & Sarama, 2007a; Kovac, 1989; Linn & Petersen, 1985; Lohman, 1993). Factor analytic studies presented evidence that spatial ability is not uni-dimensional. The framework proposed by Linn and Petersen (1985) demonstrated that spatial ability can be considered under three constructs: spatial visualization, mental rotation, and spatial perception. Spatial visualization tasks involve "complicated, multistep manipulations of spatially presented information" (p.1484) and can be done efficiently using an analytic process. Mental rotation tasks are distinct from other types of spatial abilities in terms of solving processes and "involve a Gestalt-like analogue process" (p.1484) and can be done efficiently using Gestalt-like mental rotation process analogous to physical rotation of the stimuli. In spatial perception tasks, respondents are required to "determine spatial relationships with respect to the orientation of their own bodies" (p.1482) and can be done efficiently using kinesthetic process. The review of the literature showed that the reports seek for the relations among spatial abilities were used mainly correlational analysis and reported significant relations.

Moreover, reports on relations between spatial abilities and geometry performance were particularly based on correlational or variance analysis. Most of the studies used a single score for spatial ability and geometry performance such as national exam scores, course grades or a total score of a test. When the structure of these variables was considered, analyzing these variables in terms of their separate constructs provide more detailed insight on them. Furthermore, geometry education models were investigated, most of them developed their own definitions for the term connected with spatial abilities, and they integrated this term for geometry education. For instance, Zimmerman and Cunningham (1991), similar to Presmeg (2006), mentioned that their use of the term visualization differs from the common usage in psychology. They defined it as a process of forming mental images and using these images for mathematical discovery. The present study views spatial abilities similar to the common usage in cognitive science as imagination of visual stimuli and mental manipulation of it in space. Thus, this ability was not one of the usual components of the school curriculum. Therefore, spatial ability is informally acquired (Ben-Chaim et al., 1988). Since the frameworks of the present study is not include a developmentalist view, the progress way of students' spatial abilities or 3D geometry knowledge is beyond the scope of the present study. This study seeks for evidence for more detailed relations among spatial abilities and geometry knowledge.

The results of the studies on gender differences were controversial. Most of the studies mentioned the male superiority on spatial abilities and mathematics performance. Moreover, some studies propose that gender differences in mathematics arise from the gender differences in spatial abilities (Friedman, 1995). However, the contrary results exist especially in researches that study with covariates.

The effects of school type differences on students' development have been studied frequently. In Turkey, the advantage of schools, which accept students based on OKS results, was evident for many variables. However, during the review of the literature, studies on spatial ability and 3D geometry were not encountered.

Many studies have looked at the influence of gender, school type, and spatial ability in mathematics education; however, they analyzed them individually instead of taking into account several of these sources at the same time. A few studies existed that have considered the effects of several factors and they mostly used multiple linear regression.

Ma and Kishor (1997) suggested using structural equation modeling in order to explain causal relations. Structural equation modeling analysis has enabled researchers to study the factors simultaneously. Path analysis, as a type of structural equation modeling technique, assesses the contribution of a variable to another in a non-experimental situation and the fundamental difference between path analysis and a regression model is that dependent variable can also appeare as an independent variable in equations (Jöreskog & Sörbom, 1993a).

The review of the literature suggested that there is a need for further studies to explore the causal relations among spatial abilities and students' performance in different knowledge types on 3D geometry. In addition, variables such as gender and school types should be considered to identify the related factors affecting teaching and learning. With respect to these circumstances, the present study investigates the causal relations among spatial abilities, causal relations among knowledge types on 3D geometry, and causal relations among spatial abilities, knowledge types, gender and school type by using Path Analysis using Structural Equation Modeling technique.

CHAPTER III

METHODOLOGY

3.1 Design of the Study

This study was designed as a cross-sectional survey study of association among geometry knowledge on prisms and pyramids, spatial ability, gender and school type.

3.2 Population and Sample

The target population of this study was all 12th grade secondary school students in Eskişehir who enrolled in geometry course in their schools during the 2009-2010 academic year. The accessible population was all 12th grade students who enrolled in geometry course in Anatolian and general high schools during 2009-2010 academic year in two central districts of Eskişehir.

According to January 2010 Secondary Education Statistics, 3633 12th grade students enrolled in Geometry course from 42 secondary schools scattered through 14 districts in Eskişehir. All the Anatolian High Schools and General High Schools located in two central districts participated in the study. There were ten Anatolian and ten general high schools in these two districts and the total number of 12th graders who enrolled in geometry courses in these schools was 2918. Thus, the accessible population for this study was 2918 12th grade students, who enrolled in geometry course in their schools, from 20 different schools in two central districts of Eskişehir during 2009-2010 academic year.

Prior to the study, ethics committee approval (Appendix A) and permission of the Eskişehir National Education Directorate (Appendix B) for the pilot study and main study were obtained.

Pilot studies were carried out on students who has similar properties with population, but not who were constitute a part of the final sample. To conduct pilot study analyses The Purdue Spatial Visualization Test and Prisms and Pyramids Knowledge Test were administered to 1067 and 849 students respectively in 2008-2009 academic year. Detailed information on pilot study sample was presented in following section.

The main study data was collected from students who are different from the previous sample. The Purdue Spatial Visualization Test and Prisms and Pyramids Knowledge Test were administered to 1845 and 1651 students respectively. The instruments of the study were administered to 2257 students in total. Since each instrument was administered in different times, the loss of participants was inevitable. Some of the participants were absent during the first or second collection of data. The number of the participants who took both tests were 1275. Since the method for missing data was listwise deletion, the sample size for this study reduced to 1161. No outliers were detected. As a result, the data obtained from the 1161 (501 male and 660 female) 12th grade secondary students who completed both instruments constituted the sample of the study. Distribution of the students to schools types was given in Table 3.1. The sample size is large enough for path analysis using Structural Equation Modeling. The ages of the sample ranged from 16 to 21 with a mean of 18.10. Since this sample size exceeded the 10% of the target population (40%), the results of the study can be generalized to the population.

Table 3.1 Distribution of students by gender and type of the school attended

	# of schools	male	female	Total
General High School	10 (50%)	188 (36%) (38%)	340 (64%) (52%)	528
Anatolian High School	10 (50%)	313 (49%) (62%)	320 (51%) (48%)	633
Total	20	501	660	1161

3.3 Data Collection Instruments

In this study, Purdue Spatial Visualization Test (PSVT) and Prisms and Pyramids Knowledge Test (PPKT) were used to collect data. At first, PSVT was administered. After the teacher finished the Prisms and Pyramids unit, PPKT was administered. Besides, in each test demographic data such as name, age, gender and school name was gathered to match the tests of students. Each instrument and reliability-validity analysis were presented in the following sections.

3.3.1 Purdue Spatial Visualization Test (PSVT)

3.3.1.1 Description

Purdue Spatial Visualization Test Battery (Appendix C) was developed by Guay (1976). This test includes three sections: (i) Developments, (ii) Rotations and (iii) Views. Developments section consists of 12 questions designed to see how well participants visualize the folding of developments into 3D objects. In this section, an open form of a 3D model was given with a shaded part that shows the bottom surface; and participants were expected to select the folded form of model that fits the given open form among five objects. Thus, Developments section assesses students' spatial visualization ability.

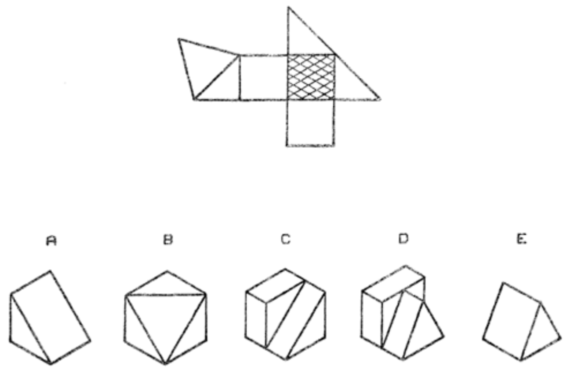


Figure 3.1 Sample item from Developments section of PSVT

Rotations section consists of 12 questions to see how well participants can visualize the rotation of 3D objects. In this section, an example of the rotation is represented and participants are expected to select the rotated form of given model with same rotation type. Therefore, Rotation section assesses students' mental rotation ability.

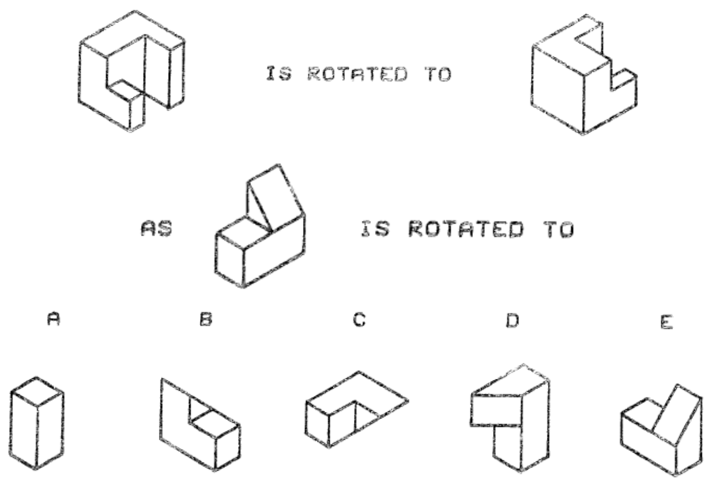


Figure 3.2 Sample item from Rotations section of PSVT

Views section consists of 12 questions designed to see how well participants visualize what 3D objects look like from various viewing positions. In this section, a model in a transparent cube and a black dot that identifies the desired viewing position is given and participants are expected to select the view that looks like the object as seen from the viewing position. Hence, Views section assesses students' spatial perception ability.

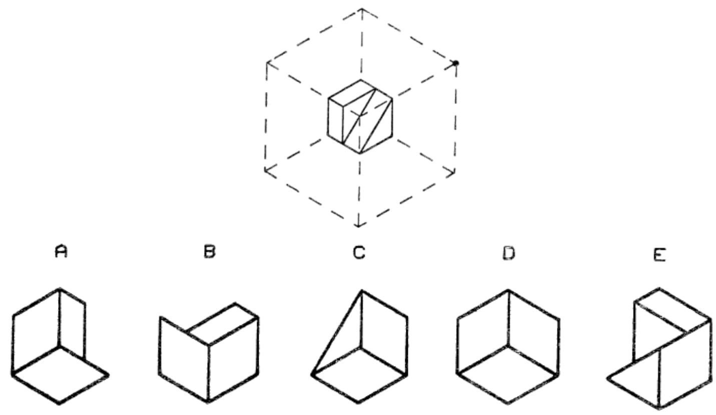


Figure 3.3 Sample item from Views section of PSVT

The test was individually administered to students in class approximately for 40 minutes. The correct items were scored as 1 and incorrect answers were scored as 0. So, the scale scores of a student in each section is the number of items answered correctly. The test directions and sample items can be found in Appendix D.

3.3.1.2 Content Validity of the PSVT

Since the test does not include any wording except the guidelines, the comprehensibility of the test was not an issue. Double translations for directions were done by researcher and a research assistant from the Department of Educational Sciences at the Middle East Technical University. Furthermore, a reseacher from educational sciences and a mechanical engineer, who was an expert on technical drawing and visualization, reconciled that figures were well drawn and the test was related to spatial ability. The structure of the test was not changed; so the format of the instrument could be considered as valid. Additionally, Guay (1976) stated that the test is suitable for use with subjects aged 13 years or older (see Appendix C)

3.3.1.3 Construct Validity of PSVT

PSVT was administered to 1067 students (543 male, 510 female and 14 missing at the end of the fall semester of 2008-2009 academic year. This sample was not constituted a part of the final sample. The ages of the sample ranged from 16 to 20 with a mean of 17.93. The PSVT was given in class and took approximately 40-45 minutes to administer.

The percentages of missing data were between 2.7 % and 12.4 %. In this study, in the view of the fact that participants had enough time to answer all questions, all missing items were coded as 0 to form asymptotic covariance matrix.

The Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy test revealed a value of 0.929 (greater than 0.6) and the Bartlett's Test of Sphericity tests was significant ($\chi^2(630)=6695.759$, $p=.00$, $\alpha=0.05$). These findings indicated that the data was appropriate for the factor analysis.

The confirmatory factor analysis (CFA) model was run with the fixed value of 1.00 for the first item for each set of measurement coefficients (λ_x parameters). Since the PSVT data was binary, polychoric correlations and asymptotic covariance matrices were obtained by PRELIS and saved in files to be read by SIMPLIS for confirmatory factor analysis. Three subsections of the PSVT (spatial visualization, mental rotation and spatial perception ability) were allowed to correlate each other.

CFA, based on the data from 1067 high school students, was performed to provide evidence to the factor structure through LISREL 8.7 for Windows (Jöreskog & Sörbom, 2004) on the three subtests of the PSVT. A three-factor model was hypothesized. As mentioned before, items one to 12 serve as the indicators of the spatial visualization ability (SVisA), 13 to 24 serve as the indicators of the mental rotation ability (MRotA), and 25 to 36 serve as the indicators of the spatial perception ability (SPerA). Thus, the structural equation model was involving three latent variables as SVisA, MRotA and SPerA and 36 observed variables as items of the test. The hypothesized model is presented in Appendix E, where circles represent latent variables (factors) and rectangles represent measured variables (items).

The CFA supported the three-factor model with significant loadings. The final SIMPLIS syntax for the PSVT is given in the Appendix F. There are 1067 participants and 36 observed variables. With 36 observed variables, there are 666 data points. The final model indicates that 75 parameters to be estimated; therefore, the model is over-identified and is tested 591 dfs. The ratio of cases to observed variables is 29.64 and the ratio of cases to estimated parameters is 14.63. These ratios indicate the adequacy of sample size (Tabbachnic & Fidell, 2007).

CFA model of PSVT was evaluated in terms of goodness-of-fit indices. A Satorra-Bentler chi-square of 1027.83 with 591 degrees of freedom at a significance level $p=<0.001$ indicates significant Chi-Square. As known, $\chi 2$ is sensible to large sample size (Byrne, 1998; Schumacker & Lomax, 2004). The value of the Normed Chi-Square (NC) in terms of which $\chi 2/df$ was 1.74, that is less than five times the model degrees of freedom, indicates a good fit to the data (Kelloway, 1998; Schermelleh-Engel, Moosbrugger, & Müller, 2003). Although chi-square was significant, other fit indices supported the hypothesized model and indicated a good fit to the data (Table 3.2). For instance RMSEA= 0.026 (<0.05), GFI=0.98 (>0.95) and the standardized NFI=0.99 (>0.95).

Table 3.2 Goodness-of-fit indices of the model for PSVT

Fit Index	Value
χ2	1027.83
χ2/df	1.74
RMSEA	0.026
CN	699.95
GFI	0.98
AGFI	0.98
PGFI	0.98
RMR	0.05
S-RMR	0.05
NFI	0.99
PNFI	0.93
NNFI	1.00
CFI	1.00
IFI	1.00
RFI	0.99

Note. *AGFI* = Adjusted Goodness-of-Fit-Index, *AIC* = Akaike Information Criterion, *CAIC*=Consistent *AIC*, *CFI* = Comparative Fit Index, *ECVI* = Expected Cross Validation Index, *GFI* = Goodness-of-Fit Index, *NFI* = Normed Fit Index, *NNFI* = Nonnormed Fit Index, *RMSEA* = Root Mean Square Error of Approximation, PGFI= Parsimony GFI, PNFI=Parsimony NFI, *RMR* = Root Mean Square Residual, *SRMR* = Standardized RMR.

The LISREL estimates of parameters in the model in which the coefficients appeared between 0.56 and 1.17 and all t-values were significant at $p<0.05$. For each of the observed variables that represented the latent variables, R^2, λ_x and the measurement error (δ) associated with the observed variable were presented in Table 3.3.

Table 3.3 Standardized solutions, R^2, λ_x, and the measurement error (δ) associated with the observed variables of PSVT

Latent Variables	Observed Variables	Standardized Solutions	λ_x	δ	R^2
Spatial Visualization Ability (SVisA)	P1	0.58	1.00	0.66	0.34
	P2	0.46	0.79	0.79	0.21
	P3	0.49	0.83	0.76	0.24
	P4	0.38	0.65	0.85	0.15
	P5	0.55	0.94	0.70	0.30
	P6	0.62	1.06	0.61	0.39
	P7	0.68	1.16	0.54	0.46
	P8	0.61	1.03	0.63	0.37
	P9	0.67	1.15	0.55	0.45
	P10	0.62	1.05	0.62	0.38
	P11	0.60	1.03	0.64	0.36
	P12	0.34	0.59	0.88	0.12
Mental Rotation Ability (MRotA)	P13	0.64	1.00	0.59	0.41
	P14	0.69	1.07	0.53	0.47
	P15	0.61	0.96	0.63	0.37
	P16	0.60	0.93	0.64	0.36
	P17	0.58	0.91	0.66	0.34
	P18	0.60	0.95	0.63	0.37
	P19	0.50	0.78	0.75	0.25
	P20	0.63	0.98	0.630	0.40
	P21	0.68	1.07	0.53	0.47
	P22	0.46	0.73	0.78	0.22
	P23	0.61	0.95	0.63	0.37
	P24	0.36	0.56	0.87	0.13
Spatial Perception Ability (SPerA)	P25	0.58	1.00	0.66	0.34
	P26	0.59	1.01	0.66	0.34
	P27	0.67	1.16	0.55	0.45
	P28	0.68	1.17	0.54	0.46
	P29	0.68	1.17	0.54	0.46
	P30	0.64	1.09	0.60	0.40
	P31	0.65	1.11	0.58	0.42
	P32	0.53	0.91	0.72	0.28
	P33	0.60	1.02	0.65	0.35
	P34	0.65	1.11	0.58	0.42
	P35	0.61	1.05	0.63	0.37
	P36	0.56	0.96	0.69	0.31

Table 3.4 displays the estimates for the covariances between the latent constructs.

Table 3.4 Covariance matrix of latent constructs of PSVT

	SVisA	MRotA	PerA
SVisA	0.34		
MRotA	0.30	0.41	
SPerA	0.24	0.30	0.34

The summary statistics for fitted residuals for the model yielded the smallest fitted residual as -0.16 and the largest residual as 0.18. Since the fitted residual values were less than two in absolute value (Kelloway, 1998), the fitted residuals for the model indicated a good fit. On the other hand, the summary statistics for standardized residuals for the model yielded the smallest standardized residual as -3.19 and the largest standardized residual as 4.56. The structure of both residuals displayed a similar shape and the steamleaf plots of both residuals were approximately normal which indicated a good fit. The steamleaf plots and summary statistics of residuals are presented in AppendixG.

Evaluation of the model according to the goodness-of-fit indices regarding their criteria showed that there is a good fit between model and the data. Accordingly, specified observed variables indicated the related latent variables of the PSVT and it was confirmed that PSVT could be used to assess the students' spatial visualization, mental rotation and spatial perception ability with Development, Rotation and View subsections.

3.3.1.4 Convergent Validity of the PSVT

Numerous researchers reported the relationship among the sub-abilities of spatial ability. (Hegarty & Waller, 2004) and the relationship between spatial ability and geometry performance (Battista, 1990; Battista et al., 1982; Clements & Battista, 1992; Hannafin et al., 2008). Correlational analysis was employed among sub-abilities of PSVT and geometry grades (Geo) taken in previous semester to provide evidence for convergent validity. The correlations among the subsections of PSVT showed significant relationships with *r*s ranging from 0.547 to 0.621. The subsections of PSVT showed significant correlations with geometry grade taken in previous semester with *r*s ranging from 0.327 to 0.352. Results were presented in Table 3.5

Table 3.5 Correlations among the spatial abilities and geometry grades

		SVisA	MRotA	SPerA
MRotA	Pearson *r*	0.599**		
	p	<0.001		
	N	1067		
SPerA	Pearson *r*	0.547**	0.621**	
	p	<0.001	<0.001	
	N	1067	1067	
Geo	Pearson *r*	0.336**	0.352**	0.327**
	p	<0.001	<0.001	<0.001
	N	781	781	781

**. Correlation is significant at the 0.01 level (2-tailed).
*. Correlation is significant at the 0.05 level (2-tailed).

3.3.1.5 Discriminant Validity of the PSVT

The superiority of the theoretical model among three models was investigated. One of the alternative models was one-factor model that all items loaded on a single factor proposing that factors of the PSVT were not statistically divergent. As one of the frameworks on spatial ability proposes that spatial ability can be categorized into two. Thus, another alternative model was two-factor model that items loaded on two factors as visualization and mental rotation ability. In this two-factor model, P13 to P24 serve as the indicators of mental rotation ability and rest serve as the indicators of visualization ability. Independence model is the null model of the target model with no parameters estimated. The model comparisons are presented in Table 3.6.

Table 3.6 Comparison of goodness-of-fit statistics for tests of discriminant validity of the PSVT

Model	χ^2	df	χ^2/df	$\Delta\chi^2$	Δdf
Target Model	1027.83	591	1.74	-	-
One-factor Model	1575.93	594	2.65	548.1	3
Two-factor Model	1416.07	593	2.38	388.21	2
Independence Model	59301.22	630	94.13	58273.39	39

The comparisons of the target model to the one-factor and two-factor models indicated that target model showed a better fit to the data than the one-factor and two-factor models. The chi-square difference tests supported this superiority with $\Delta\chi^2$ = 548.1 and Δdf=3; and $\Delta\chi^2$ = 388.21 and Δdf=2. This result supported that it is unlikely to take one-factor and two-factor models as a correct alternatives and that PSVT has a multidimensional structure with three factors.

As expected, the independence model (null model) had a poorer fit to the data than the target model. The chi-square difference test supported this superiority with $\Delta\chi^2$ = 58273.39 and Δdf=39.

Accordingly, these comparisons provide evidence that the items of the PSVT discriminate into the three factors which is shown in CFA.

3.3.1.6 Subgroup Validity of the PSVT

Hinkin (1995), in his review of scale development, suggested assessing the groups that would be expected to differ on the measure would provide further evidence of the construct validity. Several studies investigated the gender difference in spatial ability and reports the differences in mental rotation and spatial perception in favor of males (Linn & Petersen, 1985). Conversely, no gender difference was declared on spatial visualization (Linn & Petersen, 1985). So, in this study gender was expected to differentiate students on the factors of PSVT. Thus, independent sample t-tests were carried out by using IBM SPSS Statistics 20. The assumption of normal distribution was checked by looking at the Q-Q plot, the skewness and kurtosis values, and no violations were observed. The equality of variance assumption was checked with the Levene's Test. Equality of variance assumption was violated. Thus, the corresponding t-value was evaluated to test the difference.

The results supported the previous research and revealed that there is a significant mean difference between male and female students in spatial visualization factor ($t(1040.614)=4.045$), mental rotation factor ($t(1001.278)=7.547$) and spatial perception factor ($t(969.326)=6.612$). Male students have significantly higher spatial abilities than females. Although the gender difference on spatial visualization factor was not expected, the gender difference on remaining factors supported the subgroup validity for this instrument.

3.3.1.7 Reliability of the PSVT

Internal-consistency estimates of reliability were examined for test and for each latent variable after latent variables were determined. Reliability analyses were conducted separately for test and for each sections of PSVT by SPSS. The alphas for the PSVT and the scales are reasonable, with the coefficient alphas above 0.70. The alpha reliability coefficients were computed as 0.88 for full test and 0.73 for spatial visualization, 0.76 for mental rotation and 0.78 for spatial perception factors.

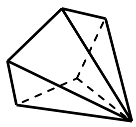
3.3.2 Prisms and Pyramids Knowledge Test (PPKT)

3.3.2.1 Description

The Prisms and Pyramids Knowledge Test (Appendix H), developed by the researcher, is a selected response test including 40 questions measuring students' knowledge on prisms and pyramids. The test includes three sections: (i) What is this?, (ii) True or False?, (iii) Which one is true? First section includes 12 multiple choice items (with three choices) designed to see how well students can identify a prism or a pyramid from the given drawing. Second section includes 11 true-false items designed to see how well students can recall the properties of prisms and pyramids. These two sections with the total number of 23 items were developed to assess students' declarative knowledge on prisms and pyramids. Sample declarative items were presented below.

Declarative Knowledge Question (What is this?)

Examine the drawings of the three dimensional solids given below. Mark the appropriate choice for the classification of the solid or describe the solid.

1.  a) prism b) pyramid c) Other : ……

Declarative Knowledge Question (True or False?)

Examine the expressions given below. Mark **T** for the expressions which are **<u>always correct</u>**, and mark **F** for the expressions which are false.

23 A triangular pyramid with all faces are congruent to an equilateral triangle is called regular tetrahedron. (T) (F)

The last section includes 17 multiple choice questions (with five choice) designed to see how well students can establish relationships among prisms, pyramids and their elements; and perform on calculation procedures about prisms and pyramids. Seven items were designed to assess students' conditional knowledge and ten items were designed to assess students' procedural knowledge. Sample conditional and procedural items were given below.

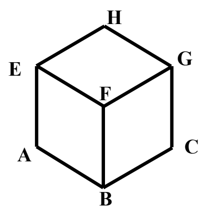
Conditional Knowledge Question (Find the correct answer)

28 – "consider a square prism with a height of 5 units and one side of base length of 4 units. Inside this prism, there exist 60 $unit^3$ water. If a cube with a side of 3 units is placed into this prism and it sank, …"
Which of the following statements completes the expression above accurately.

a) the amount of the water inside the prism increases.
b) 7 $unit^3$ water overflows.
c) the height of the water inside the prism increases 3units.
d) the amount of the water inside the prism becomes 87 $unit^3$.
e) the height of the water inside the prism does not change.

Procedural Knowledge Question (Find the correct answer)

24- The length of the edge of cube in the Figure is 2 cm. What is the minimum length that a spider crosses to reach the point G from point A?



a) 6
b) $2+2\sqrt{2}$
c) $2\sqrt{5}$
d) $2+2\sqrt{3}$
e) 4

The test was individually administered to students in class approximately for 40 minutes. The correct items were scored as 1 and incorrect answers were scored as 0. So, the scale scores of a student in each section is the number of items answered correctly.

3.3.2.2 PPKT Development Procedure and Content Validity of the PPT

The PPKT was developed to measure students' geometrical knowledge on prisms and pyramids. Various test books, a variety of geometry books and standardized tests such as university entrance exam items, TIMSS items and PISA items were used to construct a question pool on solid geometry (e.g. Aichele & Wolfe, 2007; Fogiel, 2004; İşçi, 2006; Püskülcü & Çiftçi, 2008). All questions were categorized according to knowledge type (declarative, conditional and procedural) and examined in terms of required knowledge to solve questions. It is seen that most of the questions in these resources are mostly procedural questions.

At first, constructed-response items were developed based on the Secondary School Geometry Curriculum (MEB, 1992). Constructed-response items had been preferred to assess students' knowledge and computational processes. Besides, constructed-response version of the test gave chance to determine students' difficulties, calculation errors and misconceptions. However, when the sample size was considered, disadvantages of constructed-response items for the evaluation and grading process were determined such as time spent administering, scoring, objectivity problems, and reliability.

The selected-response item format is the most appropriate format for the test developers in effective measurement of cognitive achievement or ability (Downing, 2006; Haladyna, 1997). Item formats such as the multiple-choice item, multiple-choice variants such as matching, true-false, item sets, alternate-choice are common item forms useful to test developers (Downing, 2006; Haladyna, 1997). Downing (2006) and Haladyna (1997) mentioned the advantages of selected-response formats. Selected-response items encourage content validity by allowing representative sampling of the content domain.

They can be efficiently scored with objectivity in scoring. For multiple-choice and essay tests covering the same content, multiple choice will have higher reliability. Piloting and new item tryout is more easily carried out for selected-response items than for construct-response items.

Thus, the researcher decided to use the selected-response format. Multiple formats were used such as true-false, three- and five-choice questions. Three-choice and true-false items were developed to assess declarative knowledge, and multiple-choice items with five-choices were developed to assess students' conditional and procedural knowledge. Mostly, question form was used for procedural items and completion form was used for conditional items. Procedural items were direct questions in which the requested answer was asked in a straight line. On the other hand, particular attention was paid to include condition-action or if-then statements in conditional items. Items, choices and distracters were carefully developed. While developing items, giving clues about the right answer for other items, opinion based items, trick items were avoided. As pictorial representations give clues about the structures of solids, pictorial representations of solids were avoided as much as possible. While developing distracters, students' typical calculation errors and misconceptions were considered.

Finally, a 48-item test was developed. Test included 14 three-choice items on identification of prisms or pyramids; 18 true-false items on definitions, properties, and area-volume formulas, and 16 five-choice items on calculation of area, lateral area, volume, height, lateral height, length of an edge, length of a diagonal, predicting the result when some properties of prism or pyramid changes, and predicting the relationship between prisms and pyramids. This version of the test including 32 declarative knowledge items, eight conditional knowledge items and eight procedural knowledge items was submitted to the supervisor, two research assistants and a high school mathematics teacher. All researchers and teacher were asked for comment on clarity of questions, their face and content validity, appropriateness of choices, the appropriateness of its content to the objectives, the appropriateness of its content to high school geometry curriculum, and mathematical correctness. In addition, one of the research assistants, who studied knowledge types, was asked for comment on the categorization of questions into knowledge types. Then, this draft version was administered to six elementary mathematics pre-service teachers to check the clarity of questions. All of them were administered the same question set however three pre-service teachers solved the construct-response form of the five-choice items to collect alternative responses. These alternative responses provided additional distracters and gave chance to determine possible calculation errors and misconceptions. Two true-false items were determined as contradictory items. One of them was sometimes true and the incorrectness of the other was obvious. Another true-false item was determines as inappropriate since it involves procedural knowledge. Moreover, the lack of items on definitions of prisms and pyramids and on procedural knowledge items regarding pyramids as declared. Taking into account all suggestions, three true-false items were omitted and eight true-false items on definitions of prisms and pyramids were added to declarative knowledge items, and two items on pyramids were added to procedural knowledge items. With last revisions, final version of the geometry test including 55 items (37 items on declarative knowledge, ten items on procedural knowledge, and eight items on conditional knowledge) was developed and the supervisor, research assistants and teacher, checked it.

No more revisions were made on the test and this test was confirmed to be appropriate for 12^{th} grade students and valid to administer. The table of specification and the distribution of the questions among the test items according to knowledge types were given in Table 3.7.

Table 3.7 Table of specification of PPKT and distribution of the questions among the test items according to knowledge types

Question	OBJECTIVES	DecK	ProK	ConK
1 - 14	Identification of a prism or pyramid from the given drawing.	√		
17 - 37	Recall properties of prisms or pyramids	√		
38	Calculating the shortest way between opposite corners of the cube.		√	
39	Predicting what will happen to the total area of a cube, if a small cube is removed from one of the corners.			√
40	Predicting the shape of water in a cube, if cube is moved in different ways.			√
41	Given the circumference of a base and the height of a hexagonal prism, calculate the total area of this prism.		√	
42	Given the increase in length of edges and change the total area of a cube, calculate the length of edges of preceding cube.		√	
43	Predict what will happen if a cube is put into a square prism containing water.			√
44	Given the dimensions of trapezoid prism, calculate the volume of this prism.		√	
45	Predict what will happen if the properties of prism is changed.			√
46	Given the three different lateral areas of a rectangular prism, calculate the length of the diagonal of this rectangular prism.		√	
47	Calculate the ratio between the volume of a square prism and the volume of a triangular pyramid that is formed by cross-section of the preceding prism.		√	
48	Establish relationship between a pyramid and a prism which have equal base areas and heights.			√
49	Given the length of the base edge and the height of a right square pyramid; calculate the lateral area of this pyramid.		√	
50	Justify the relationship between the height and the base area of a square pyramid and the height and the base area of a small pyramid which is formed by a parallel cross-section of the preceding pyramid			√
51	Given the length of the base edge and the height of a right square pyramid, calculating the length of lateral edge of this pyramid.		√	
52	Predict what will happen if the properties of a pyramid is changed.			√
53	Given the length of the base edge and the height of a right square pyramid, calculate the volume of the truncated pyramid which is formed by a parallel cross-section that has the definite distance from the apex		√	
54	Justify the relationship between the height and the volume of a square pyramid and the height and the volume of a small pyramid which is formed by a parallel cross-section of the preceding pyramid			√
55	Given the length of the base edge and the volume of a right triangular pyramid, calculating the height of this pyramid.		√	
	Total	37	10	8

In this last version of the test, some questions were adapted and some were taken as it is. Item 38 is about spider on the wall or ant on the box, Item 39 is about removed piece from a prism, Item 41 is about covering a box, and question 49 is about covering a tent. Additionally, Item 51, 53 and 55 are standard calculation questions. They are all communal questions and can be seen in any book about geometric objects (e. g. Fogiel, 2004; İşçi, 2006; Püskülcü & Çiftçi, 2008). Item 42 and 43 is adapted from Püskülcü and Çiftçi (2008), Item 44 was adapted from Aichele and Wolfe (2007). All adaptations were done with changes in the structure and numbers. Item 53 was taken from Püskülcü and Çiftçi (2008, p.295, question 12). Other Item were developed by the researcher based on her experiences on properties and definitions of prism and pyramids. All questions were appropriate for 12th grade concerning the objectives of the geometry curriculum. The researcher has developed all the choices for these questions by taking into consideration students' typical calculation errors and misconceptions.

The test was administered to ten (number of students) 12th grade students from two different schools in order to determine the time it took to complete, check the clarity of questions, the adequacy of test duration and the difficulty of the questions. One of the students solved questions at his home with no restriction in time. He told that he was not a very successful student in geometry and he was bored through the end of the test and did not solve the last five questions. Other nine solved questions in class with their mathematics teacher with time restriction of 40 minutes class hour. Teacher was informed about the test and asked to write proposed questions but the students did not propose any. As a result this final version of the test was confirmed to be appropriate and valid to administer individually in class approximately for 40 minutes.

This last version of the test was administered to 849 students, who attended Geometry3 course at school and finished prisms and pyramids topic, during 2008-2009 academic year. Test was administered by the researcher or the classroom teacher with the presence of the researcher in class and took approximately 40-45 minutes to administer. Sixty-five cases did not answer at least one section of the test. This might be due to their unwillingness to participate in the study. So, they were excluded in view of the fact that participants had enough time to answer all questions. Eventually, subsequent analyses were conducted with 784 participants (342 male, 339 female and 103 unknown). The ages of the sample ranged from 16 to 21 with a mean of 17.73.

Primarily, item analysis was conducted with the whole test items by using the ITEMAN version 3.5 computer program (1993) to compute and examine the statistical properties of participants' responses to test and to an individual test item. Analyses were conducted with 55 items. Results revealed that G5 and G24 are not appropriate items with negative discrimination index (Table 3.8). After dropping them, the 53 items were analyzed again.

Table 3.8 Item analysis results of Item 6 and Item 28

		Item Statistics			Alternative Statistics					
Seq. No.	Scale-Item	Prop. Correct	Biser.	Point Biser.	Alt.	Prop. Endorsing		Biser.	Point Biser.	Key
5	1-5	0.14	0.11	0.15	A	0.64	0.54	0.73	0.16	?
					B	0.13	0.20	0.03	-0.20	
CHECK THE KEY C was specified, A works better					C	0.14	0.10	0.21	0.15	*
					Other	0.08	0.00	0.00	-0.22	
24	1-24	0.20	-0.06	-0.01	A	0.76	0.70	0.79	0.06	?
CHECK THE KEY					B	0.20	0.24	0.18	-0.01	*
B was specified, A works better					Other	0.04	0.00	0.00	-0.12	

Crocker and Algina (1986) noted that "for the items where guessing is more likely occur [1- 37 in our test] it is desirable to construct items with p values somewhat higher than .50" (p.98). Additionally, Henryssen (1971, as cited in Crocker & Algina, 1986) when the average biserial correlation between item and total test score is in the range .30 to .40, the ideal item difficulty level should be between .40 and .60; but as the average biserial correlation increases above .60, a wider range of item difficulties may be acceptable. In line with these, when the items were examined according to the frequency of missing, correct and incorrect responses, item difficulties, biserial, point biserial scores, and discrimination indices, items G1, G16, G17, G18, G20, G27, G28, G29, G32, G33, G34, G35 and G40 were determined as low or very low discriminating items (Table 3.9).

Table 3.9 Distribution of PPKT items through discrimination indices

	discrimination indices	Items
low	.20 <D≤.30	G1, G17, G20, G28, G29, G32, G33, G34, G40
very low	D≤.20	G16, G18, G27, G35

After dropping abovementioned 15 items (G1, G5, G16, G17, G18, G20, G24, G27, G28, G29, G32, G33, G34, G35, and G40), items were analyzed again. Results of the frequency of missing, correct, and incorrect responses, item difficulties, biserial, point biserial scores, and discrimination indices were presented in Table 3.10.

Consequently, PPKT was confirmed to be appropriate for 12^{th} grade students and valid to administer individually in class approximately for 40 minutes. Since multiple-choice items were used, they can be evaluated as correct and incorrect. An item answered correctly by the participants is scored as 1, and an item answered incorrectly by the participants is scored as 0. The knowledge test is a collection of items that are distributed to three separate subtests in terms of related knowledge type. Thus, each subtest score is determined by summing the item scores or counting the correct answers of the participant in related items and the total test score is computed by summing three subtest scores.

Table 3.10 Missing, correct, and incorrect responses, item difficulties, biserial, point biserial scores, and discrimination indices of PPKT items

	items	missing	% of missing	correct	incorrect	difficulty (*p*)	point biserial	discrimination index
Declarative Knowledge	G2	11	1.4	622	151	0.79	0.40	0.41
	G3	84	10.71	499	201	0.64	0.56	0.47
	G4	16	2.04	668	100	0.85	0.38	0.47
	G6	25	3.19	656	103	0.84	0.44	0.50
	G7	36	4.59	592	156	0.76	0.53	0.54
	G8	13	1.66	698	73	0.89	0.32	0.47
	G9	51	6.51	627	106	0.8	0.43	0.48
	G10	9	1.15	647	128	0.83	0.41	0.48
	G11	101	12.88	462	221	0.59	0.61	0.51
	G12	34	4.34	512	238	0.65	0.54	0.48
	G13	59	7.53	549	176	0.7	0.44	0.41
	G14	21	2.68	529	234	0.67	0.34	0.33
	G15	4	0.51	364	416	0.46	49	0.39
	G19	10	1.28	520	254	0.66	0.54	0.46
	G21	25	3.19	614	145	0.78	0.37	0.34
	G22	24	3.06	573	187	0.73	0.40	0.38
	G23	35	4.46	428	321	0.55	0.71	0.56
	G25	38	4.85	361	385	0.46	0.44	0.57
	G26	53	6.76	480	251	0.61	0.41	0.48
	G30	18	2.3	581	185	0.74	0.48	0.53
	G31	38	4.85	523	223	0.67	0.47	0.56
	G36	29	3.7	615	140	0.78	0.35	0.33
	G37	31	3.95	532	221	0.68	0.43	0.48
Conditional Knowledge	G39	64	8.16	421	299	0.54	0.66	0.77
	G43	99	12.63	368	317	0.47	0.68	0.78
	G45	134	17.09	261	389	0.33	0.70	0.71
	G48	99	12.63	353	332	0.45	0.72	0.84
	G50	133	16.96	310	341	0.4	0.67	0.71
	G52	148	18.88	203	433	0.26	0.64	0.57
	G54	136	17.35	310	338	0.4	0.68	0.76
Procedural Knowledge	G38	29	3.7	432	323	0.55	0.63	0.81
	G41	80	10.2	247	457	0.32	0.50	0.57
	G42	83	10.59	390	311	0.5	0.59	0.77
	G44	68	8.67	392	324	0.5	0.68	0.86
	G46	94	11.99	316	374	0.4	0.69	0.83
	G47	135	17.22	260	389	0.33	0.60	0.70
	G49	131	16.71	218	435	0.28	0.61	0.65
	G51	128	16.33	199	457	0.25	0.58	0.59
	G53	169	21.56	146	469	0.19	0.49	0.42
	G55	126	16.07	221	437	0.28	0.56	0.61

3.3.2.3 Construct Validity of PPKT

Confirmatory factor analysis is a theory thesting- model that enables researcher to test hypothesized factor structure of the data collected via a measurement tool (Stevens, 2002). Researcher searches for a fit between the observed and predetermined model representing the number and the indicators of factors in order to validate the factor structure of the data.

As PPKT had a priori specified theoretical model, confirmatory factor analysis rather than exploratory factor analysis was conducted to test for the multidimensionality of the instrument (Bollen, 1989). LISREL 8.7 for Windows (Jöreskog & Sörbom, 2004) was used to test that PPKT is a multidimensional test composed of three factors namely declarative knowledge, conditional knowledge and procedural knowledge and provide evidence for construct validity (Jöreskog & Sörbom, 1993a). When the observed variables are binary, it is suggested to compute polychoric correlations and asymptotic covariance matrix and to use weighted least squares method of estimation (Byrne, 1998; Jöreskog & Sörbom, 1993a, 1993b; Kline, 2005; Schumacker & Lomax, 2004). Therefore, for the analysis correlation matrix and asymptotic covariance matrix should be computed. Since the PSVT data was binary, polychoric correlations and asymptotic covariance matrices were obtained by PRELIS and saved into files to be read by SIMPLIS for confirmatory factor analysis. The percentages of missing data were between 0.5 % and 21.6 %, and all missing items were coded as 0 to calculate asymptotic covariance matrix. The CFA model was run with the fixed value of 1.00 for the first item for each set of measurement coefficients (λx parameters). Three factors of the PSVT were allowed to correlate with each other.

The Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy test revealed a value 0.913 (> 0.6) and the Bartlett's Test of Sphericity tests was significant

($\chi^2(780)=5042.868$, $p=<0.001$, $\alpha=0.05$). These findings indicated that the data was appropriate for the factor analysis.

The CFA model was run with the fixed value of 1.00 for the first item for each set of measurement coefficients (λ_x parameters). Since the PPKT data was binary, polychoric correlations and asymptotic covariance matrices were obtained by PRELIS and saved into files to be read by SIMPLIS for confirmatory factor analysis. Three subsections of the PPKT (Declarative, Conditional and Procedural Knowledge) were allowed to correlate each other.

A CFA, based on the data from 784 high school students, was performed to provide evidence to the factor structure through LISREL 8.7 for Windows (Jöreskog & Sörbom, 2004) on the three subtests of the PPKT. A three-factor model was hypothesized. The items that serve as the indicators of the Declarative Knowledge (DecK), Conditional Knowledge (ConK) and Procedural Knowledge (ProK) were shown in Table 3.11. Thus, the structural equation model involves three latent variables as Dec, Con and Pro, and 40 observed variables as items of the test.

Table 3.11 The indicators of the subsections of the PPKT

Latent Variable		Item
Declarative Knowledge	DecK	G2, G3, G4, G6, G7, G8, G9, G10, G11, G12, G13, G14, G15, G19, G21, G22, G23, G25, G26, G30, G31, G36, G37
Conditional Knowledge	ConK	G39, G43, G45, G48, G50, G52, G54
Procedural Knowledge	ProK	G38, G41, G42, G44, G46, G47, G49, G51, G53, G55

Additionally, considering the modification indices with highest values and their justification four covariance terms were added to syntax in order to improve the model. The hypothesized model is presented in Appendix I, where circles represent latent variables (factors) and rectangles represent measured variables (items). The modification indices provided by LISREL improve the overall fit indices. The change in improvement in the fit indices can be seen in Table 3.12. Besides, the paths that modification indices indicate can be interpreted substantively and it is usual that the items in the same subtest can be correlated since they measure same ability. Moreover, the reduction in chi-square and estimated loadings were same as what modification indices predicted. All loadings were significant. The final SIMPLIS syntax for the PPKT is given in the Appendix J. There are 784 participants and 40 observed variables. With 40 observed variables, there are 820 data points. The final model indicates that 87 parameters to be estimated; therefore, the model is over-identified and is tested 733 dfs. The ratio of cases to observed variables is 19.6 and the ratio of cases to estimated parameters is 9.01. This ratio is adequate given that the reliability of the subtests of the PPKT.

Table 3.12 Comparison of fit indices

Model	$\chi2$	df	$\chi2$/df	RMSEA	GFI	NFI	S-RMR
1st model	2198.52	737	2.98	0.050	0.95	0.97	0.093
Final model	1509.49	733	2.06	0.037	0.97	0.98	0.079

CFA model of PPKT was evaluated in terms of goodness-of-fit indices. A Satorra-Bentler chi-square of 1509.49 with 733 degrees of freedom at a significance level p=<0.001 indicates significant Chi-Square. As known, $\chi 2$ is sensible to large sample size (Byrne, 1998; Schumacker & Lomax, 2004). The value of the Normed Chi-Square (NC) in terms of which $\chi 2/df$ was 2.06, that is less than five times the model degrees of freedom, indicated a good fit to the data (Kelloway, 1998; Schermelleh-Engel, et al., 2003). Although chi-square was significant, other fit indices supported the hypothesized model and indicated a good fit to the data (see Table 3.13). For instance RMSEA= 0.037 (<0.05), GFI=0.97 (>0.95) and the standardized NFI = 0.98 (>0.95).

Table 3.13 Goodness-of-fit indices of the model for PPKT

Fit Index	Value
$\chi 2$	1509.49
$\chi 2/df$	2.06
CN	422.61
RMSEA	0.037
RMR	0.08
S-RMR	0.08
GFI	0.97
AGFI	0.97
PGFI	0.87
NFI	0.98
NNFI	0.99
PNFI	0.92
CFI	0.99
IFI	0.99
RFI	0.98

Table 3.14 Covariance matrix of latent constructs of PPKT

	DecK	ConK	ProK
DecK	0.29		
ConK	0.37	0.58	
ProK	0.33	0.54	0.51

The diagonally weighted least square estimates appeared between 0.61 and 1.31 and all t-values were significant at $p < .05$. Table 3.14 displays the estimates for the covariances between the latent constructs. Additionally, for each of observed variable that represents the latent variables, standardized solutions, R^2, λ_x, and the measurement error associated with the observed variable (δ) were presented in Table 3.15.

Table 3.15 Standardized solutions, R^2, λ_x, and the measurement error (δ) associated with the observed variables of PPKT

Latent Variables	Observed Variables	Stardardized solutions	λ_x	δ	R^2
DecK	G2	0.54	1.00	0.71	0.29
	G3	0.49	0.91	0.76	0.24
	G4	0.59	1.11	0.65	0.35
	G6	0.64	1.19	0.59	0.41
	G7	0.66	1.23	0.56	0.44
	G8	0.60	1.13	0.63	0.37
	G9	0.58	1.08	0.67	0.33
	G10	0.57	1.07	0.67	0.33
	G11	0.54	1.01	0.71	0.29
	G12	0.58	1.07	0.67	0.33
	G13	0.43	0.81	0.81	0.19
	G14	0.33	0.61	0.89	0.11
	G15	0.41	0.77	0.83	0.17
	G19	0.55	1.02	0.70	0.30
	G21	0.43	0.81	0.81	0.19
	G22	0.49	0.91	0.76	0.24
	G23	0.70	1.31	0.51	0.49
	G25	0.52	0.97	0.73	0.27
	G26	0.50	0.94	0.75	0.25
	G30	0.64	1.19	0.59	0.41
	G31	0.62	1.16	0.61	0.39
	G36	0.46	0.86	0.78	0.22
	G37	0.49	0.92	0.76	0.24
ConK	G39	0.76	1.00	0.42	0.58
	G43	0.74	0.98	0.45	0.55
	G45	0.78	1.02	0.40	0.60
	G48	0.87	1.15	0.24	0.76
	G50	0.63	0.83	0.60	0.40
	G52	0.76	1.00	0.43	0.57
	G54	0.68	0.90	0.53	0.47
ProK	G38	0.71	1.00	0.49	0.51
	G41	0.58	0.82	0.66	0.34
	G42	0.65	0.92	0.57	0.43
	G44	0.78	1.09	0.39	0.61
	G46	0.82	1.16	0.32	0.68
	G47	0.69	0.97	0.52	0.48
	G49	0.71	1.00	0.49	0.51
	G51	0.66	0.93	0.56	0.44
	G53	0.54	0.76	0.70	0.30
	G55	0.62	0.88	0.61	0.39

The summary statistics for fitted residuals for the model yielded the smallest fitted residual as -0.26 and the largest residual as 0.34. Since the fitted residual values were less than 2 in absolute value (Kelloway, 1998), the fitted residuals for the model indicated a good fit. On the other hand, the summary statistics for standardized residuals for the model yielded the smallest standardized residual as -4.22 and the largest standardized residual as 7.02. The structure of both residuals displayed a similar shape and the steamleaf plots of both residuals were approximately normal which indicated a good fit. The steamleaf plots and summary statistics of residuals were presented in Appendix K.

Evaluation of the model according to the goodness-of-fit indices regarding their criteria showed that there is a good fit between model and the data. Accordingly, specified observed variables indicated the related latent variables of the PPKT and it was confirmed that PPKT can be used to assess the students' knowledge on prisms and pyramids with Declarative Knowledge, Conditional Knowledge and Procedural Knowledge factors.

3.3.2.4 Convergent Validity of the PPKT

Numerous researchers reported the relationship among the knowledge types (Aydin, 2007; Aydin & Ubuz, 2010; Hiebert & Lefevre, 1986; Mason & Spence, 1999; Rittle-Johnson & Siegler, 1998; Rittle-Johnson, 1999). Correlational analysis was employed among knowledge types, which were determined by PPKT, and geometry grades taken in previous semester to provide evidence for convergent validity. The correlations among the factors of PPKT showed significant relationships with *r*s ranging from 0.678 to 0.791. The factors of PPKT showed significant correlations with geometry grade taken in previous semester with *r*s ranging from 0.426 to 0.518. Results were presented in Table 3.16.

Table 3.16 Correlations among the knowledge types and geometry grades

		DecK	ConK	ProK
ConK	Pearson Correlation	0.676**		
	Sig. (2-tailed)	<0.001		
	N	784		
ProK	Pearson Correlation	0.668**	0.791**	
	Sig. (2-tailed)	<0.001	<0.001	
	N	784	784	
Geometry	Pearson Correlation	0.502**	0.426**	0.472**
	Sig. (2-tailed)	<0.001	<0.001	<0.001
	N	362	362	362

** Correlation is significant at the 0.01 level (2-tailed).

3.3.2.5 Discriminant Validity of the PPKT

To investigate discriminant validity, the superiority of the theoretical model among three models was investigated. One of the alternative models was one-factor model that all items loaded on a single factor proposing that factors of the PPKT were not statistically divergent. As one of the frameworks proposes that knowledge of mathematics distinguishes into two, other alternative model was two-factor model that items loaded on two factors as conceptual and procedural knowledge. Thus, declarative and conditional items were combined to serve as the indicators of the conceptual knowledge and rest serve as indicators of the procedural knowledge. Independence model is the null model of the target model with no parameters estimated. The model comparisons are presented in Table 3.17.

The comparisons of the target model to the one-factor and two-factor models indicated that target model showed a better fit to the data than the one-factor and two-factor models. The chi-square difference tests supported this superiority with $\Delta\chi^2$ = 131.61 and Δdf=3, and $\Delta\chi^2$ = 99.74 and Δdf=2, which were statistically significant. This result supported that it is unlikely to take one-factor and two-factor models as a correct alternate and that PPKT has a multidimensional structure.

Table 3.17 Goodness-of-fit statistics for tests of discriminant validity of PPKT

Model	χ2	df	χ2/df	Δχ2	Δdf
Target Model	1509.49	733	2.06	-	-
One-factor Model	1641.10	736	2.23	131.61	3
Two-factor Model	1609.23	735	2.19	99.74	2
Independence Model	82387.56	780	105.63	80878.07	67

As expected, the independence model (null model) had a poorer fit to the data than the target model. The chi-square difference test supported this superiority with $\Delta\chi^2$ = 80878.07 and Δdf=67, which was statistically significant.

Accordingly, these comparisons provide evidence that the items of the PPKT discriminate into the three factors which is shown in CFA.

3.3.2.6 Reliability of the PPKT

The alphas for the PPT and the scales are high with the coefficient alphas above or equal to 0.80. The alpha reliability coefficients were computed as 0.93 for full test and 0.82 for Declarative Knowledge, 0.80 for Conditional Knowledge and 0.80 for Procedural Knowledge sections.

3.3.3 Summary

Consequently, validity and reliability analyses of the PSVT revealed that the general model of the PSVT components with three scales is a reasonable representation of the data. PSVT was accepted as an appropriate test to assess

12th grade students' spatial ability in three factors. Also, analyses of the PPKT suggested that the general model of the PPKT components with three scales is a reasonable representation of the data and demonstrate that PPKT is an appropriate instrument to assess 12th grade students' knowledge on Prisms and Pyramids.

Taken together, construct validity, discriminant validity, convergent validity and reliability analyses revealed that all data collection instruments were valid and reliable. In addition, based on these results, six latent variables which were included in the path model were determined.

3.4 Threats to Internal Validity

The possibility that characteristics of the subjects in a study may account for observed relationships is known as subject characteristics threat (Fraenkel & Wallen, 2000). In the present study, the 12th grade subjects who were enrolled in geometry course at school were selected, but most of the characteristics of the subjects could not be controlled in this study. Thus, the subject characteristics could be a threat for the present study.

No matter how carefully the subjects of a study are selected, it is probable to lose some as the study progresses (Fraenkel & Wallen, 2000). This threat is known as mortality. In the present study, multiple data collection instruments were administered at different times and lose of subjects was inevitable. In addition, at the first data collection point the H1N1 influenza epidemic and at the second data collection point university entrance exams affected students' participation. Despite these effects, nearly 32% of the population participated in the study. Thus, administrations of the tests to more than the needed number of subjects helped to surmount this threat. Also, subjects lost were similar to those remaining on pertinent characteristics. Therefore, the mortality could not be a threat for the present study.

Location threat refers to the possibility that results are due to characteristics of the location in which a study is conducted, thereby producing a threat to internal validity (Fraenkel & Wallen, 2000). In this study, instruments were always administered in the actual schools and classes of the students. Since, the study did not include any manipulation and the subjects were in their ordinary environment, the location could not be an essential threat for the study.

Instruments and procedures used in collecting data may also constitute a threat to the internal validity of a study. Changes in instrument over time, characteristics of the data gatherers and data collector bias can create problems. (Fraenkel & Wallen, 2000). Data collector of the study was the researcher with the presence of classroom teacher. Researcher was always presented at school during data gathering to control data collection procedures. No changes were done in the instrument over time. Since the instruments have objective scoring criteria, scoring could not be a threat. Thus, the instrumentation could not be a threat for this study.

Testing is a threat that refers to improved scores on a posttest that are a result of subjects having taken pretest (Fraenkel & Wallen, 2000). The present study is not an intervention study and instruments were used only one time. Hence, the testing could not be a threat for the present study.

The possibility that results are due to an event that is not part of a study, but which may affect performance on variables is known as history threat (Fraenkel & Wallen, 2000). In the present study, subjects were 12^{th} grade students who were generally focused on University Entrance Exams. It was understandable that these exams were the most important thing in their life. Additionally, data was occasionally collected in the same day that subjects have course exams. The researcher tried to control for some of the conditions; however, it was hard to say that history was not a threat for the study.

Changes that occur in subjects as a direct result of the passage of time may affect their performance (Fraenkel & Wallen, 2000). Since the present study was not an intervention study and the instruments were administered only one time, maturation could not be a threat for the present study.

The way in which subjects view a study and their participation in it can create a threat to internal validity (Fraenkel & Wallen, 2000). The aim and the procedure of the study was explained in detail in order to control this threat however attitude of the subjects could be a threat for the study.

The possibility that results are due to variations in the implementations of the treatment in an intervention study is known as implementation threat (Fraenkel & Wallen, 2000). The implementation could not be a threat for the present study.

Lastly, the regression threat may be present whenever change is studied in a group that is extremely low or high in its pre-intervention performance (Fraenkel & Wallen, 1996). Since, there was no intervention in this study; regression could not be a threat for the present study.

3.5 Potentially Confounding Variables

It has been reported that experiences have an effect on persons spatial ability development. For instance, Baenninger and Newcombe (1989) reported a weak but reliable relation between spatial experiences and spatial development. In a more recent study, Tang (2006) reported that experiences on computer, music, and art have a significant effect on students' spatial ability. In addition, gender differences in the activities were reported. Alternatively, socioeconomic status of the students' family and economic condition of schools can be thought as confounding variables. Levine et al. (2005) socioeconomic status may affect the development of spatial abilities. Thus, gender likely determines interest, activities, and experiences; and socioeconomic status likely determines access to preferred activities. In the present study, however, the data was not collected about the students' experiences and socioeconomic status. Thus, they can be thought as confounding variables, neglecting these variables may have an effect on the results of this study.

3.6 Ethical Issues

As known, there are three very important issues that every researcher should address: protecting participants from harm, ensuring confidentiality and the deception of the participants.

The data collection procedure of this project was carried out in students' actual locations that were their own classrooms; the protection of participants from harm was ensured.

Since the data was gathered through three separate sessions, gathering names of students needed for match their data. In order to set confidentiality of the students, schools and classes were labeled with numbers or letters for the whole assessment.

In this study, necessary permissions from ethical committee, National Education Directorate, school administrations, and class teachers were provided. School administrations, teachers, and participants were informed about study and test instructions. Therefore, it could be said that the deception of the students will not be an issue in this study.

Another ethical aspect is about the permissions to use the PSVT. The PSVT test was bought from Educational Testing Service on January 1, 2009. Ordering information can be found in Appendix L.

3.7 Data Collection

Instruments were administered by the researcher or the classroom teacher with the presence of the researcher in class and it takes approximately 40-45 minutes to administer each instrument.

Data collection was conducted in two phases. In the first phase, the instruments were administered to different samples for pilot study and main study. Data for pilot study and confirmatory factor analysis was collected from the schools that are proper between the end of the fall semester and midst of spring semester of 2008-2009 academic year. For the administration of PPKT, researcher paid attention to check the completion of the Prisms and Pyramids Unit. The second phase included the administration of instruments to sample between the end of the fall semester and end of spring semester of 2009-2010 academic year. At first PSVT was administered, approximately one month later PPKT was administered. The time between the administration of PSVT and PPKT changed because of the curriculum schedule difference from school to school.

3.8 Data Analysis

After collection procedures finished, the data entry was done by the researcher. Data were entered directly from the test booklets. Females were coded as '1' and males as '0'. General high school was coded as '0' and Anatolian high schools as '1'. Data check and cleaning phases contained detection of all anomalies and errors. Data check process was carried out by comparison of randomly selected booklets data with computer data. Determined errors were corrected by controlling related booklet.

Then, the data was investigated in terms of descriptive analyses such as missing data, data cleaning and descriptive statistical procedures. The data files were imported from IBM Statistics 20 to PRELIS. The program was run to supply needed steps for model testing. Then, confirmatory factor analyses were conducted for all instruments using LISREL 8.7 with SIMPLIS command language for Windows (Jöreskog & Sörbom, 2004) in order to confirm and determine latent variables of the study. Syntaxes were presented in Appendix F and Appendix J. Additionally; item analyses were conducted for PPKT before CFA in order to check the statistical properties of students' responses to the test and individual items. After that, reliability and validity analyses were carried out using IBM Statistics 20. Then, additional confirmatory factor analyses were conducted separately for PSVT and PPKT to confirm the structure of tests for the main study data. Lastly, path analysis with multiple indicators and multiple causes was used to test connections among variables. Path analysis from structural equation modeling family is a technique for observed variables that test the relationships among three or more variables and it is far more powerful then the most of other associational research techniques (Fraenkel & Wallen, 1996, 2000; Guarino, 2004). Multiple indicators and multiple causes model, in which factors with effect indicators are regressed on one or more dichotomous cause indicators that represent group membership, was one way of estimating group differences on variables (Kline, 2005). Analyses were conducted using LISREL 8.7 with SIMPLIS command language for Windows (Jöreskog & Sörbom, 2004) in order to investigate causal relationships among variables. The data file was imported to PRELIS to produce the necessary files for path analysis. Covariance and asymptotic covariance matrices were produced for the path model testing. In the analysis of the study, the significance level was taken to be 0.05. Analyses were conducted by using the listwise deletion method and Weighted Least Squares estimation method in modeling. The general strategic framework to test path model was model generating by trimming. Model was tested by model trimming according to empirical standards. Although, literature suggests a model that spatial ability had an effect on geometry performance, no detailed prior theory was found about the relationships between the factors of knowledge and spatial abilities. Thus, model-trimming approach can be seen as exploratory analysis. Accordingly, path analyses began with the just-identified model and continued by simplifying it by eliminating paths according to statistical criteria. Chi-square difference test was used to test the statistical significance of the decrement in overall fit (Kline, 2005).

3.8.1 Missing Data

The PPKT and PSVT was coded as (1) for correct and (0) for incorrect items. The total scores of the students for each variable were computed by counting correct items. Since the Weighted Least Square method of estimation was used to test model, the asymptotic covariance matrices were need to be calculated. The asymptotic covariance matrices can only be calculated with listwise deletion method. For CFAs, missing items were coded as (0), thus CFA data does not include missing. However, total scores of the participants were calculated by counting correct items by considering missing data in order to determine the cases that did not have scores for variables. These circumstances lead us to use listwise deletion method for missing data in Path analysis.

3.8.2 Procedures for Effect Size and Sample Size

3.8.2.1 Effect size

The effect size is an indicator of the amount of variability in the dependent variable that can be accounted for by the independent variable (Cohen, Cohen, West, & Aiken, 2003). An effect simply is a measure of the strength of the relationship between variables. A multiple correlation (R), a squared multiple correlation (R^2) and an adjusted squared multiple correlation (R^2_{adj}) are the multiple correlation indices. These indices assess how well the linear combination of predictor variables in the regression analysis predicts the criterion variable. Thus, the effect size is approximately equivalent to the R^2 used in multiple regressions. Cohen, et al. (2003) suggested that effect sizes can be measured in terms of R^2 and gave a reference for effect sizes (small= .01, medium= .09, and large= .25).

The effect sizes in measures of R^2 for the latent variables were given in the Table 3.18.

Table 3.18 The effect sizes in measures of R^2 for the latent variables

Latent variables	R^2
SVisA	0.27
MRotA	0.35
SPerA	0.27
DecK	0.36
ConK	0.54
ProK	0.51

The effect sizes in measures of R^2 for latent variables of this study have effect sizes between 0.27 and 0.54 that indicate large effect size. Included variables explained 0.27 of the variance of spatial visualization ability, 0.35 of the variance of mental rotation ability, 0.27 of the variance of spatial perception ability, 0.36 of the variance of declarative knowledge, 0.54 of the variance of conditional knowledge, 0.51 of the variance of procedural knowledge.

3.8.2.2 Sample size

Structural equation modeling is a large sample technique, and more than 200 cases could be considered as large (Kline 2005, Tabachnick & Fidell, 2007). Since the sample size of the present study was large (N=1161), sample size would not be a problem. In addition, according to the table of minimum sample sizes needed for calculations (MacCallum, Browne, & Sugawara, 1996), the sample size of this study is adequate.

CHAPTER IV

RESULTS

Results of the study were presented in two sections as preliminary analysis and path analysis. Preliminary analysis section includes the descriptive analysis and assumption tests for path analysis. Path Analysis section includes the spatial ability, geometry knowledge, gender, and school type model testing.

4.1 Preliminary Analysis

This section includes confirmatory factor analysis of the main data to provide additional information about the measurement model or data and provide evidence for the factor structure of the administered tests. Additionally, descriptive statistics, correlation matrix of variables were presented to give additional information about sample.

4.1.1 Confirmatory Factor Analysis and Constitution Variables of the Study

The standardized solutions, measurement coefficients, measurement errors, and squared multiple correlations obtained from the CFA was conducted for the main study data, and reliability coefficients for the latent variables and tests were presented in Appendix M and Appendix N respectively. The results confirmed the construct validity of the tests. In addition, the alpha reliability coefficients were computed as 0.843 and 0.863 for PSVT and PPKT respectively. Alpha coefficients for spatial visualization, mental rotation, spatial perception, declarative knowledge, conditional knowledge and procedural knowledge factors were 0.717, 0.672, 0.744, 0.738, 0.703, and

0.720 respectively. These results demonstrated internal consistency of each test. Although, the reliability coefficient for mental rotation section was lower than 0.70, it was acceptable according to Nunnally (1978, as cited in Tang, 2006). All these results revealed the appropriateness of the tests for this study.

Measurement models for PSVT and PPKT were evaluated in terms of goodness-of-fit indices. A Satorra-Bentler $\chi2(591)$= 922.80 and SB$\chi2(732)$= 1962.07 indicates significant Chi-Square for PSVT and PPKT respectively. The values of $\chi2/df$ were 1.56 and 2.68 indicates a good fit (<5) to the data for PSVT and PPKT respectively (Kelloway, 1998; Schermelleh-Engel, et al., 2003). Other fit indices supported the hypothesized models and indicated a good fit to the data (Table 4.1).

Table 4.1 Goodness-of-fit indices of the models for PSVT and PPKT

Fit Index	PSVT	PPKT
$\chi2$	922.80	1962.07
$\chi2/df$	1.56	2.68
CN	848.14	585.14
RMSEA	0.022	0.035
RMR	0.05	0.07
S-RMR	0.05	0.07
GFI	0.98	0.96
AGFI	0.98	0.96
PGFI	0.87	0.86
NFI	0.98	0.97
NNFI	0.99	0.98
PNFI	0.92	0.91
CFI	0.99	0.98
IFI	0.99	0.98
RFI	0.97	0.97

Based on the results of the CFA, variables were formed by the composition of students' responses to related items. Thus, total scores for the variables declarative knowledge, conditional knowledge, procedural knowledge, spatial visualization ability, mental rotation ability, and spatial perception ability were calculated.

4.1.2 *Descriptive Statistics and Assumptions*

Prior to data analysis, data was examined through SPSS program for accuracy of data entry, missing values, distribution attributes and the assumptions. The listwise deletion method was used in the model testing in order to calculate the asymptotic covariance matrix. The descriptive statistics of variables was presented on Table 4.2 and Table 4.3 respectively.

Structural equation modeling is a large sample technique, and more than 200 cases could be considered as large (Kline 2005, Tabachnick & Fidell, 2007). According to the table of minimum sample sizes needed for tests and for power calculations (MacCallum, Browne, & Sugawara, 1996), the sample size of this study is adequate.

The assumption of univariate normality was checked by looking at the Q-Q plot, the skewness and kurtosis values, and no violations were observed. Since the estimation method was weighted least square, the multivariate normality was not an issue. The measured variables were screened for outliers. Cases with standardized scores exceeds 3.29 ($p<0.001$) are considered as potential univariate outliers and the criterion for multivariate outliers is Mahalonobis distance at $p<0.001$ (Tabachnick & Fidell, 2007). Mahalonobis distances were evaluated and outliers were not observed.

Linear relationship among the pairs of measured variables assessed through inspection of matrix of scatterplots, all were oval shaped which indicates the normal distribution and linear relationship (Tabachnick & Fidell, 2007).

Table 4.2 Descriptive statistics of variables and total scores on PSVT

		General High Schools			Anatolian High Schools			Total		
		Male	Female	Total	Male	Female	Total	Male	Female	Total
	Valid N	188	340	528	313	320	633	501	660	1161
Spatial Visualization	Mean	4.92	4.82	4.86	6.14	5.75	5.94	5.68	5.27	5.45
	Std. Dev.	2.81	2.57	2.66	2.94	2.83	2.89	2.95	2.74	2.84
	Median	5	4	4	6	6	6	5	5	5
	Mode	5	4	4	4	4	4	4	4	4
	Maximum	12	12	12	12	12	12	12	12	12
	Minimum	0	0	0	0	0	0	0	0	0
	Range	12	12	12	12	12	12	12	12	12
	Skewness	0.53	0.56	0.55	0.26	0.29	0.28	0.35	0.44	.412
	Kurtosis	-0.44	-0.12	-0.25	-0.70	-0.59	-0.64	-0.65	-0.40	-.510
Mental Rotations	Mean	5.09	4.18	4.50	6.27	5.38	5.82	5.82	4.76	5.22
	Std. Dev.	2.34	2.15	2.26	2.53	2.37	2.49	2.52	2.34	2.47
	Median	5	4	4	6	5	4	6	5	4
	Mode	5	4	4	5	4	6	5	4	4
	Maximum	11	11	11	12	12	12	12	12	12
	Minimum	0	0	0	0	0	0	0	0	0
	Range	11	11	11	12	12	12	12	12	12
	Skewness	0.19	0.62	0.47	0.06	0.26	0.18	0.14	0.45	.325
	Kurtosis	-.07	0.41	0.09	-0.52	-0.44	-0.51	-0.39	-0.17	-.342
Spatial Perception	Mean	4.67	4.01	4.25	6.04	5.09	5.56	5.53	4.53	4.96
	Std. Dev.	2.67	2.34	2.48	3.25	2.90	3.11	3.11	2.68	2.92
	Median	4	4	4	6	5	4	5	4	4
	Mode	3	3	3	5	4	4	3	3	3
	Maximum	12	12	12	12	12	12	12	12	12
	Minimum	0	0	0	0	0	0	0	0	0
	Range	12	12	12	12	12	12	12	12	12
	Skewness	0.63	0.82	0.77	0.21	0.60	0.41	0.40	0.77	.625
	Kurtosis	-0.18	0.55	0.26	-0.92	-0.35	-0.72	-0.73	0.14	-.321
PSVT	Mean	14.68	13.02	13.61	18.44	16.22	17.32	17.03	14.57	15.63
	Std. Dev.	6.16	5.42	5.74	7.23	6.55	6.98	7.08	6.20	6.70
	Median	14	13	13	17	15.50	16	16	13	14
	Mode	15	13	13	15	11	11	15	13	13
	Maximum	31	32	32	36	35	36	36	35	36
	Minimum	2	0	0	4	4	4	2	0	0
	Range	29	32	32	32	31	32	34	35	36
	Skewness	0.65	0.86	0.80	0.43	0.60	0.53	0.54	0.77	.693
	Kurtosis	0.05	1.03	0.60	-0.65	-0.09	-0.39	-0.40	0.40	.012

Table 4.3 Descriptive statistics of variables and total scores on PPKT

		General High Schools			Anatolian High Schools			Total		
		Male	Female	Total	Male	Female	Total	Male	Female	Total
	Valid N	188	340	528	313	320	633	501	660	1161
Declarative Knowledge	Mean	14.69	14.36	14.48	17.89	18.18	18.04	16.69	16.21	16.42
	Std. Dev.	3.81	3.41	3.56	3.43	3.05	3.24	3.90	3.76	3.82
	Median	15	14	14	18	19	18	17	16	17
	Mode	15	13	15	18	20	20	18	18	18
	Maximum	23	22	23	23	23	23	23	23	23
	Minimum	4	5	4	6	7	6	4	5	4
	Range	19	17	19	17	16	17	19	18	19
	Skewness	-0.26	-0.01	-0.11	-0.61	-0.79	-0.70	-0.49	-0.33	-.389
	Kurtosis	-0.36	-0.30	-0.33	-0.04	0.42	0.18	-0.25	-0.56	-.436
Conditional Knowledge	Mean	2.14	1.61	1.80	3.49	3.74	3.62	2.98	2.65	2.79
	Std. Dev.	1.63	1.45	1.54	1.97	2.0	1.99	1.96	2.04	2.01
	Median	2	1	2	3	4	4	3	2	3
	Mode	1	1	1	3	4	4	1	1	1
	Maximum	7	7	7	7	7	7	7	7	7
	Minimum	0	0	0	0	0	0	0	0	0
	Range	7	7	7	7	7	7	7	7	7
	Skewness	0.72	0.88	0.83	0.01	-0.04	-.01	0.30	0.51	.410
	Kurtosis	0.13	0.37	0.31	-0.99	-0.98	-0.99	-0.87	-0.74	-.820
Procedural Knowledge	Mean	2.91	2.22	2.47	4.51	4.72	4.62	3.91	3.43	3.64
	Std. Dev.	1.92	1.81	1.88	2.45	2.46	2.46	2.39	2.49	2.46
	Median	3	2	2	5	5	5	4	3	3
	Mode	1	1	1	5	7	5	2	1	1
	Maximum	9	9	9	10	10	10	10	10	10
	Minimum	0	0	0	0	0	0	0	0	0
	Range	9	9	9	10	10	10	10	10	10
	Skewness	0.68	1.01	0.87	0.14	-0.03	0.05	0.40	0.54	.461
	Kurtosis	0.03	0.88	0.44	-0.91	-0.75	-0.84	-0.72	-0.59	-.663
PPKT	Mean	19.74	18.19	18.74	25.89	26.65	26.27	23.58	22.29	22.85
	Std. Dev.	5.84	5.35	5.58	6.59	6.27	6.43	6.98	7.19	7.12
	Median	19	17	18	26	27	27	23	21	22
	Mode	15	16	16	22	27	27	22	16	16
	Maximum	36	37	37	39	38	39	39	38	39
	Minimum	6	5	5	11	8	8	6	5	5
	Range	30	32	32	28	30	31	33	33	34
	Skewness	0.38	0.69	0.58	-0.14	-0.25	-0.20	0.09	0.26	.179
	Kurtosis	-0.32	0.61	0.16	-0.69	-0.66	-0.68	-0.77	-0.82	-.818

High correlations (>.90) among variables can create multicollinearity. One of the methods for diagnosing multicollinearity is examining the bivariate correlations among variables (Tabachnick & Fidell, 2007; Stevens, 2002). Correlation analysis results do not indicate the existence of multicollinearity (see Table 4.4).

Table 4.4 Correlations among variables

	1	2	3	4	5	6
1 Spatial Visualization	1					
2 Mental Rotations	0.515**	1				
3 Spatial Perception	0.470**	0.502**	1			
4 Declarative Knowledge	0.313**	0.372**	0.307**	1		
5 Conditional Knowledge	0.315**	0.337**	0.290**	0.547**	1	
6 Procedural Knowledge	0.319**	0.344**	0.304**	0.559**	0.721**	1

Note: All p values were <0.001 and N= 1161

** r is significant at the 0.01 level (2-tailed).

4.2 The Prisms and Pyramids Knowledge, and Spatial Ability Model with Gender and School Type

Data analysis began with saturated model and then, the model was trimmed according to empirical standards that specify the paths were deleted or added according to statistical criteria. Subsequently, chi-square difference test was used to test the statistical significance of the decrement in overall fit, as paths were trimmed (Kelloway, 1998; Kline, 2005).

In the saturated model presented in Figure 1.1, it was hypothesized that there would be reciprocal relationships among the variables concerning knowledge types; reciprocal relations among the variables concerning three types of spatial ability. Additionally, students' school types and gender had an effect on all knowledge types and all spatial abilities. Furthermore, there would be relationships from the variables concerning the three types of knowledge to three types of spatial abilities. Since, it was saturated model (see Model 1 in

Table 4.5); it has no degrees of freedom and therefore can never be rejected. However, the path coefficients from gender to conditional knowledge, procedural knowledge and spatial visualization ability, and path coefficients from spatial perception to conditional knowledge were not statistically significant. Consequently, the model respecification was guided by empirical considerations and model was trimmed. Non-significant paths from gender to conditional knowledge, procedural knowledge and spatial visualization ability and from spatial perception to conditional knowledge were deleted step by step (Model 1a).

Following the recommendations of Jöreskog (1993a, 1993b), alternative models were tested to select a model as most appropriate in representing the sample data. Based on the the results of some studies proposing that geometry education improves spatial abilities (Kaufman, Steinbügl, Dünser, & Glück, 2005), we assumed that students' knowledge of prisms and pyramids affect their spatial ability performance. Then, the saturated alternative model (Figure 4.1), was tested (Model 2). Following that, non-significant paths from school to spatial visualization, mental rotation and spatial perception, from gender to spatial visualization, declarative knowledge, conditional knowledge, and procedural knowledge, from procedural knowledge to mental rotation, and from conditional knowledge to spatial perception were deleted step by step. Then, alternative model was tested again (Model 2a). The summary of the goodness-of-fit indices and the model comparison were presented in Table 4.5.

Table 4.5 Goodness-of-fit statistics and comparisons for the spatial ability, geometry knowledge, gender, and school type model

Model	$\chi2$	df	$\chi2$/df	RMSEA	RMR	GFI	AGFI	CFI	AIC	CAIC	ECVI
Model1	0	0									
Model1a	1.66	5	0.33	0.0	0.02	0.99	0.99	1.00	63.66	251.43	0.05
Model2	0	0									
Model2a	11.67	9	1.3	0.01	0.05	0.99	0.99	0.99	65.66	251.43	0.05

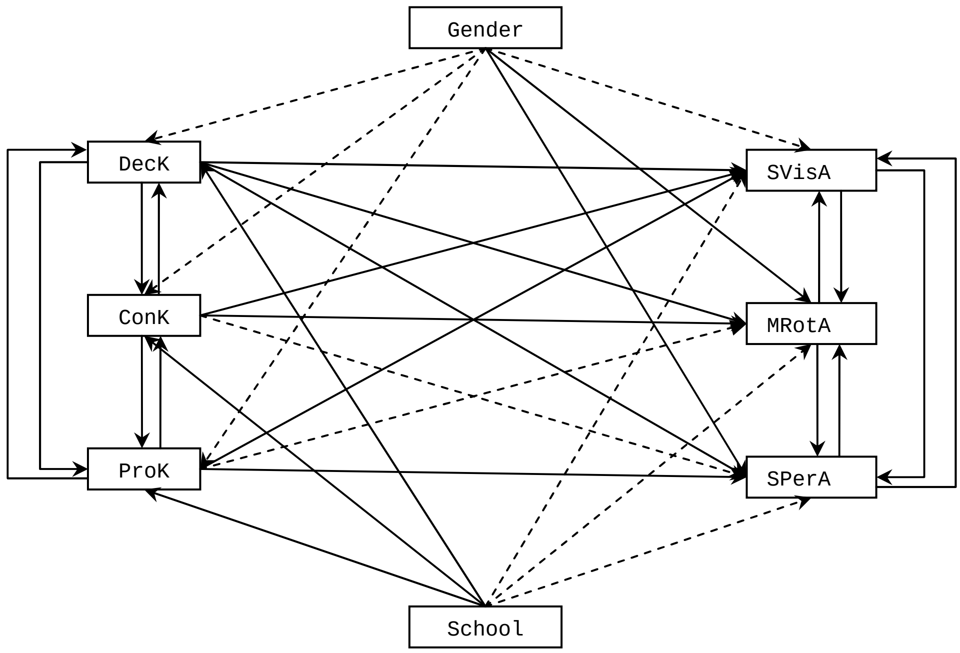


Figure 4.1 Alternative model

Note: (i) Dashed lines indicate non significant paths. (ii) DecK: Declarative Knowledge, ConK: Conditional Knowledge, ProK: Procedural Knowledge, SVisA: Spatial Visualization Ability, MRotA: Mental Rotation Ability, SPerA: Spatial Perception Ability, School: School Type.

The hypothesized model and alternative model were compared according to fit indices. To compare the non-nested alternative models with the same data fit index comparison of Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC) and Expected Cross-Validation Index (ECVI) were suggested in addition to RMSEA, RMR, GFI, AGFI, CFI (Byrne, 1998). As Hu and Bentler (1995) and Byrne (1998) stated smaller values of AIC, CAIC and ECVI represents a better fit of the hypothesized model. CAIC and ECVI indices were equal and AIC index is smaller for target model. Additionally, $\chi2$/df, RMSEA, RMR and CFI indices indicates that the target model is better than alternative model. According to non-nested model comparison criteria, Model1a represents a better fit than Model 2a. As a result, Model 1a presented in Figure 4.2 was considered to be the final path model of the The Prisms and Pyramids Knowledge, and Spatial Ability with Gender and School Type. The syntax of Model 1a was given in the Appendix O.

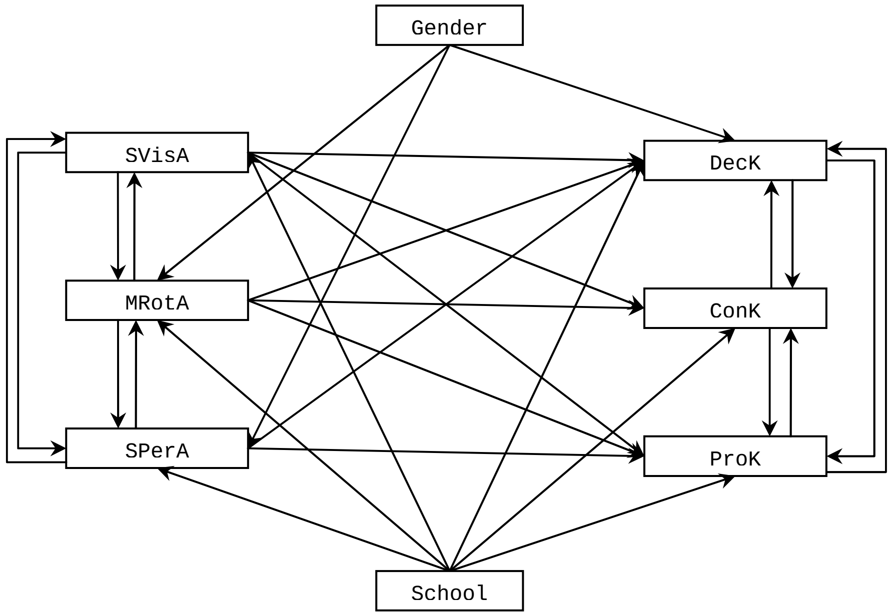


Figure 4.2 The Prisms and Pyramids Knowledge, Spatial Ability, Gender and, School Type Model

Note: (i) Non significant paths were deleted.

The Prisms and Pyramids Knowledge, and Spatial Ability Model with Gender and School Type was evaluated in terms of goodness-of-fit indices. As indicated in Table 4.6, the final model demonstrated a non-significant chi-square value of $\chi^2(5)= 1.66$ with $p= 0.89$. The value of the Normed Chi-Square (NC) in terms of which $\chi2/df$ was 0.33. The GFI and AGFI of the model was 0.99. The RMR and S-RMR values of the model were 0.02 and 0.004, respectively. RMSEA of the model was 0.0. Moreover, RMSEA of the model was demonstrated to be in the 90 percent confidence interval for RMSEA, which was from 0.00 to 0.028. NFI and NNFI of the model were 0.99 and 1.00, respectively. All values indicated a good fit to the data. The values of CFI and IFI were 1.00, and RFI was 0.98. The Expected Cross Validation Index (ECVI) of the model was 0.05 and it is among the 90 percent confidence interval for ECVI which was from 0.05 to 0.06. Furthermore, this value was found to be smaller than the value for saturated model (0.062) and the value for independence model (0.93). The investigation of goodness-of-fit indices of the model regarding their criteria revealed that there is a good fit between the model and the data.

Table 4.6 Goodness-of-fit Indices of the final Model for PPKT and PSVT

Fit Index	Value
$\chi 2$	1.66
$\chi 2$/df	0.33
RMSEA	0.0
RMR	0.02
S-RMR	0.004
GFI	0.99
AGFI	0.99
PGFI	0.11
NFI	0.99
NNFI	1.00
PNFI	0.14
CFI	1.00
IFI	1.00
RFI	0.98

Structural equations are used to describe the linear relationship between dependent variables and a set of causal variables with estimated direct effect coefficients. Each line presented in Figure 4.2 represents a direct effect of one variable on another. The arrow starts from the cause variable and points to the effect. As it can be seen from the regression equations above, statistical estimates of direct effects are path coefficients which are interpreted as regression coefficients in multiple regression. As it can be seen from the structural equations given below, spatial abilities have reciprocal relations.

Additionally, students' in Anatolian high schools perform better on each spatial ability. Moreover, students' performance on mental rotation and spatial perception ability tasks depends on their gender. The squared multiple correlations for spatial visualization, mental rotation, and spatial perception abilities indicated that the predictors explained 27%, 35%, and 27% of the variance respectively.

$$SVisA = 0.20*MRotA + 0.18*SPerA + 0.59* School, R^2= 0.27$$

$$MRotA = 0.20* SVisA + 0.17* SPerA + 0.77* School - 0.69*GENDER, R^2= 0.35$$

$$SPerA = 0.18* SVisA + 0.17*Rot + 0.78* School - 0.64*GENDER, R^2= 0.27$$

The structural equations of declarative knowledge, conditional knowledge and procedural knowledge indicated that knowledge types have reciprocal relations. All knowledge types depends on spatial visualization and mental rotation abilities. Differently, declarative and procedural knowledge depends on special perception ability. Students' school difference had an effect on all knowledge types. Apart from others, conditional knowledge and procedural knowledge does not depend on students' gender. The squared multiple correlations for declarative knowledge, conditional knowledge and procedural knowledge indicated that the predictors explained 36%, 54%, and 51% of the variance respectively.

$$DecK = 0.12* SVisA + 0.26* MRotA + 0.09* SPerA + 0.09* ConK + 0.14* ProK + 2.55* School + 0.38* GENDER, R^2= 0.36$$

$$ConK= 0.06* SVisA + 0.05* MRotA + 0.09* DecK + 0.30* ProK + 0.71* School, R^2= 0.54$$

$$ProK = 0.07* SVisA + 0.06* MRotA + 0.05* SPerA + 0.14* DecK + 0.30* ConK + 0.90* School, , R^2= 0.51$$

In addition to direct effects, path analysis provide information about indirect effects which involve one or more mediator variables (Kline, 2005). These mediator variables in indirect effects are transmit some of the causal effects of prior variables onto subsequent variables. There are many indirect effects that are represented in Figure 4.2. One corresponds to the path Gender→MRotA→DecK, and it reflects that students gender affects their performance in mental rotation tasks, which in turn influences performance in declarative knowledge items. There are many other paths that indicate the effect of gender on declarative knowledge mediated by other variables or combinations of them. With the combination of direct and indirect effects, total effects were determined.

The detailed analyses of estimated coefficients were interpreted in detail based on direct, indirect, and total effects. Direct, indirect and total effects of the path model were shown in Table 4.7.

Table 4.7 Direct, indirect and total effects

		Effects	1	2	3	4	5	6	School Type	Gender
		Direct	0.00	0.20	0.18				0.59	0.00
1	SVisA	Indirect	0.10	0.06	0.06				0.45	-0.34
		Total	0.10	0.26	0.24				1.04	-0.34
		Direct	0.20	0.00	0.17				0.77	-0.69
2	MRotA	Indirect	0.06	0.09	0.06				0.41	-0.22
		Total	0.26	0.09	0.24				1.19	-0.91
		Direct	0.18	0.17	0.00				0.78	-0.64
3	SPerA	Indirect	0.06	0.06	0.09				0.39	-0.22
		Total	0.24	0.24	0.09				1.18	-0.86
		Direct	0.12	0.26	0.09	0.00	0.09	0.14	2.55	0.38
4	DecK	Indirect	0.14	0.12	0.13	0.04	0.06	0.05	1.00	-0.39
		Total	0.27	0.38	0.22	0.04	0.15	0.19	3.55	-0.01*
		Direct	0.06	0.05	0.00	0.09	0.00	0.30	0.71	0.00
5	ConK	Indirect	0.10	0.11	0.09	0.06	0.12	0.06	1.08	-0.11
		Total	0.15	0.16	0.09	0.15	0.12	0.36	1.80	-0.11
		Direct	0.07	0.06	0.05	0.14	0.30	0.00	0.90	0.00
6	ProK	Indirect	0.12	0.14	0.09	0.05	0.06	0.13	1.23	-0.16
		Total	0.19	0.20	0.14	0.19	0.36	0.13	2.13	-0.16

* Indicates non-significant result

The examination of reciprocal relations among knowledge types revealed that declarative knowledge has a bilateral positive direct effect on both conditional knowledge (β=0.09) and procedural knowledge (β=0.14), and procedural knowledge has a bilateral positive direct effect on conditional knowledge (β=0.30). These direct effects were strengthened with indirect effects. The total effects reported in Table 4.7 indicate that the largest total effect was between procedural and conditional knowledge.

Similarly, spatial visualization ability has a bilateral positive direct effect on both mental rotation ability (β=0.20) and spatial perception ability (β=0.18), and mental rotation ability has a bilateral positive direct effect on spatial perception ability (β=0.17). These direct effects were strengthened with indirect effects. The total effects reported in Table 4.7 indicate that the largest total effect was between spatial visualization and mental rotation abilities.

Spatial visualization ability has positive direct effect on declarative knowledge (β=0.12), conditional knowledge (β=0.06), and procedural knowledge (β=0.07). It also has positive indirect effect on declarative knowledge (β=0.14), conditional knowledge (β=0.10), and procedural knowledge (β=0.12). These indirect effects could be mainly attributed to the direct effect of this variable on mental rotation and spatial perception abilities. Mental rotation ability has positive direct effect on declarative knowledge (β=0.26), conditional knowledge (β=0.05), and procedural knowledge (β=0.06). The indirect effects of mental rotatin ability, which could be mainly attributed to the direct effect of this variable on spatial visualization and spatial perception abilities, on declarative knowledge is 0.12, on conditional knowledge is 0.11, and on procedural knowledge is 0.14. Spatial perception ability has positive direct effect on declarative (β=0.09) and procedural knowledge (β=0.05). It has positive indirect effect on declarative knowledge (β=0.13), conditional knowledge (β=0.09), and procedural knowledge (β=0.09). These indirect effects could be mainly attributed to the direct effect of this variable on spatial visualization and mental rotation abilities. As is seen in Figure 4.2 and Table 4.7, there are reciprocal relations among knowledge types. Thus, knowledge factors also behave as mediator variable on each other.

The results presented in Table 4.7 showed that School Type has a positive direct effect on declarative knowledge (Γ=2.55), conditional knowledge (Γ=0.71), procedural knowledge (Γ=0.90), spatial visualization ability (Γ=0.59), mental rotation ability (Γ=0.77), and spatial perception ability (Γ=0.78). School Type also has indirect effect on declarative knowledge (Γ=1.00), conditional knowledge (Γ=1.08), procedural knowledge (Γ=1.23), spatial visualization ability (Γ=0.45), mental rotation ability (Γ=0.41), and spatial perception ability (Γ=0.39). Although school type has an effect on all knowledge types and spatial abilities, it has stronger effects on declarative and procedural knowledge (see Figure 4.2 and Table 4.7). Accordingly, declarative and procedural knowledge can be thought as major mediator variables. The total effects of School Type on declarative knowledge is Γ=3.55, conditional knowledge is Γ=1.80, procedural knowledge is Γ=2.13, spatial visualization ability is Γ=1.04, mental rotation ability is Γ=1.19, and spatial perception ability is Γ=1.18. These direct, indirect and total effects exposed the role of school in geometry learning and spatial ability performance. The interpretation of these results indicates that Anatolian high schools students perform better than general high school students in prisms and pyramids knowledge and spatial abilities. The total effect of school type on declarative knowledge indicates the largest difference exist in declarative knowledge.

As a result of coding, negative effect of gender indicates male advantage and positive effect indicates female advantage. According to Table 4.7, gender has a positive direct effect on declarative knowledge (Γ=0.38). Differently, it has a negative direct effect on mental rotation ability (Γ=-0.69) and spatial perception ability (Γ=-0.64). Thus, direct effects indicate the female superiority in declarative knowledge and male superiority in mental rotation and spatial perception abilities. Gender has negative significant indirect effect on declarative knowledge (Γ=-0.39), conditional knowledge (Γ=-0.11), procedural knowledge (Γ=-0.16), which could be attributed to the direct effect of this variable on mental rotation and spatial perception abilities. However, gender also has negative significant indirect effect on spatial visualization ability (Γ=-0.34), mental rotation ability (Γ=-0.22), and spatial perception ability (Γ=-0.22), which could be attributed to the interrelations among knowledge types . As a result, gender has negative significant total effect on spatial visualization ability (Γ=-0.34), mental rotation ability (Γ=-0.91), spatial perception ability (Γ=-0.86), conditional knowledge (Γ=-0.11), and procedural knowledge (Γ=-0.16). In contrast, gender has a non-significant total effect on declarative knowledge (Γ=-0.01). Examining indirect, direct, and total effects exposed a complex picture. The direct effects of gender on mental rotation and spatial rotation abilities were strengthened with the indirect effects. The direct, indirect, and total effects confirmed the gender difference in mental rotation ability and spatial perception ability in favor of males. The largest gender difference was observed in mental rotation ability within all variables. Although, gender has no direct effects on spatial visualization ability, conditional, and procedural knowledge, with indirect effects male superiority was detected on these variables. The positive significant direct effect and negative significant indirect effect of gender on declarative knowledge take away the effect of gender on declarative knowledge.

These results indicated that spatial visualization, mental rotation, and spatial perception abilities has strong interrelations among each other. Declarative, conditional, and procedural knowledge is affecting each other, although to a lesser extent than in spatial abilities. With the combination of direct and indirect effects of spatial visualization, mental rotation, and spatial perception abilities on knowledge types, it is obvious that spatial visualization, mental rotation, and spatial perception abilities are important variables that have effect on students' geometry knowledge regarding prisms and pyramids. As is seen in Table 4.7, determined indirect effects could be attributed to the spatial ability factors, which has strong bilateral relations. Moreover, when the total effects were examined it is found that, the strongest total effects of spatial abilities were on declarative knowledge.

CHAPTER V

DISCUSSION, CONCLUSION AND IMPLICATIONS

This chapter includes the discussion of the results, conclusion, and educational implications. Additionally, limitations of the study and recommendations for future research studies were presented.

5.1 Summary of Results

The present study investigates the relationship among students' spatial ability, geometry knowledge on prisms and pyramids, school type and gender. The Purdue Spatial Visualization Test (PSVT) developed by Guay (1976) and the Prisms and Pyramids Knowledge Test (PPKT) was used to collect data related to students' spatial visualization ability, mental rotation ability, spatial perception ability, declarative knowledge, conditional knowledge and procedural knowledge.

The results of the path analysis revealed that declarative knowledge have a bilateral positive direct effect on both conditional knowledge and procedural knowledge, and procedural knowledge have a bilateral positive direct effect on conditional knowledge. These direct effects were strengthened with indirect effects.

Similarly, spatial visualization ability has a bilateral positive direct effect on both mental rotation ability and spatial perception ability, and mental rotation ability has a bilateral positive direct effect on spatial perception ability. These direct effects were strengthened with indirect effects.

Spatial visualization ability has a positive direct effect on declarative knowledge, conditional knowledge, and procedural knowledge. It also has a positive indirect effect on declarative knowledge, conditional knowledge, and procedural knowledge. Mental rotation ability has a positive direct effect on declarative knowledge, conditional knowledge, and procedural knowledge. It also has a positive indirect effect on declarative knowledge, conditional knowledge, and procedural knowledge. Spatial perception ability has a positive direct effect on declarative and procedural knowledge. It has a positive indirect effect on declarative knowledge, conditional knowledge, and procedural knowledge. With the combination of direct and indirect effects, total effects were determined. These results indicated that spatial visualization ability, mental rotation ability, and spatial perception ability are important variables that have an effect on 3D geometry knowledge.

Gender has a positive direct effect on declarative knowledge. Conversely, it has a negative direct effect on mental rotation ability and spatial perception ability. Direct effects indicate the female superiority in declarative knowledge and the male superiority in mental rotation and spatial perception abilities. As expected, gender does not have significant direct effect on spatial visualization ability. This supports the findings of Linn and Petersen (1985). Gender also has negative significant indirect effect on declarative knowledge, conditional knowledge, procedural knowledge, spatial visualization ability, mental rotation ability, and spatial perception ability. Consequently, gender has a non-significant total effect on declarative knowledge and negative total effect on conditional knowledge, procedural knowledge, spatial visualization ability, mental rotation ability, and spatial perception ability. For that reason, there is no significant gender difference on declarative knowledge and there is a significant gender difference in favor of males on all other variables.

School Type has a positive direct effect on declarative knowledge, conditional knowledge, procedural knowledge, spatial visualization ability, mental rotation ability, and spatial perception ability. School Type also has indirect effect on declarative knowledge, conditional knowledge, procedural knowledge, spatial visualization ability, mental rotation ability, spatial perception ability. As a result, school type has significant total effects on declarative knowledge, conditional knowledge, procedural knowledge, spatial visualization ability, mental rotation ability, spatial perception ability. These direct, indirect, and total effects exposed the superiority of students in Anatolian high schools in knowledge of prisms and pyramids and spatial ability.

5.2 Discussion

5.2.1 Relations among Knowledge Types

In spite of the different terminologies used, literature on knowledge on mathematics mostly mentions about the distinction of conceptual and procedural knowledge (e.g. Hiebert, 1986; Skemp, 1976). However, confirmatory factor analyses and discriminant validity analysis conducted in this study revelaed that knowledge can be classified into three: declarative, conditional, and procedural knowledge.

Results of this study presented the bilateral relations among students' declarative, conditional and procedural knowledge. These results support the Aydın and Ubuz's (2010) finding, that there are reciprocal relations among these knowledge types. This finding was also in accordance with previous research that emphasized the reciprocal relations among conceptual knowledge and procedural knowledge (Rittle-Johnson & Siegler, 1998; Rittle-Johnson, et al., 2001; Rittle-Johson & Koedinger, 2005). Declarative knowledge forms the base for conditional and procedural knowledge (Schunk, 2000). Declarative knowledge affects the conditional knowledge by improving understanding of concepts and relations between concepts (Aydın & Ubuz, 2010). The relations between concepts cannot be constructed without the knowledge of definitions and facts or vice versa. Similarly, declarative knowledge has a bilateral relation with procedural knowledge that indicates the improvement in knowledge of concepts leads to improvement in selection and application of correct algorithms or vice versa (Aydın & Ubuz, 2010). A student knowing the properties of pyramid and prism could provide an explanation for the relationships between the properties of prism and pyramid that has equal heights and base areas. Such an explanation satisfies the justification of declarative knowledge. This relation encourages the learner in selection and application of correct algorithm. Bilateral relations between conditional knowledge and procedural knowledge demonstrate that establishing relations among facts reinforce the correct use of algorithms or vice versa. Finding and evaluating the correctness of answer and the reasonable explanation for used algorithms necessitates both conditional and procedural knowledge.

Literature indicated that knowledge of concepts and knowledge of procedures are learned in tandem rather than independently (Aydin, 2007; Rittle-Johnson & Siegler, 1998; Rittle-Johnson et al., 2001). The results also support this idea that the knowledge of geometry has a structure that cannot be constructed independently. Knowledge types support each other. Nevertheless, the results of this study can not be used to explain developmental structure of knowledge for instance as a support concept-first or procedure first theories.

5.2.2 Relations among Spatial Abilities

Studies on spatial ability have shown that it is not uni-dimensional. However, confirmatory factor analyses and discriminant validity analysis conducted in this study revelaed that spatial abilities can be classified into three: spatial visualization, mental rotation, and spatial perception ability.

Results revealed the bilateral relations among spatial visualization, mental rotation, and spatial perception. These relations support the findings of previous studies that presented the correlational relations among spatial abilities (Hegarty & Waller, 2004; Hegarty et al., 2006; Karaman & Yontar Toğrol, 2010).

The bilateral relation between spatial visualization ability and mental rotation ability is reasonable. To rotate a visual object mentally, visualize should completely understand the structure of the it. If the structure of the object could not be comprehended, one cannot consider the place of the elements of the object when position of it changed. On the other hand, if one could mentally rotate the object, this rotation helps to understand the structure of the object and consider the properties of unseen parts of it. Similar relations can explain the bilateral relations among spatial visualization and spatial perception abilities. In order to visualize an object with respect to orientation of one's body, the structure of the elements should be appreciated or vice versa. The bilateral relation between spatial perception ability and mental rotation ability is also reasonable. Both abilities are related to visualizing the object from different points. In mental rotation ability, the position of the object is changing; conversely, in spatial perception ability, the position of visualizer is changing. The reason for the relation may be explain via this connection.

5.2.3 Relations among Knowledge Types and Spatial Abilities

Spatial visualization ability is related with the tasks that involve complicated, multistep manipulations of objects. Spatial visualization ability had a positive effect (direct, indirect, and total) on declarative, conditional, and procedural knowledge. This indicates that spatial visualization ability lead learners to make effective progress on declarative knowledge, conditional knowledge, and procedural knowledge questions. For example, consider the sample item asking 'the minimum length that a spider cross' (Item24 in PPKT). In order to find the find the correct answer, one should visualize the net of a cube (Figure 5.1). After this, visualization process helps student to think that all faces of the cube are square. Then, student notice that if all faces of the cube are square then the way of the spider pass through the hypotenuse of a triangle whose three sides are in the ratio $1 : \sqrt{3}: 2$ or the Pythagoras theorem is required to find the distance asked for.

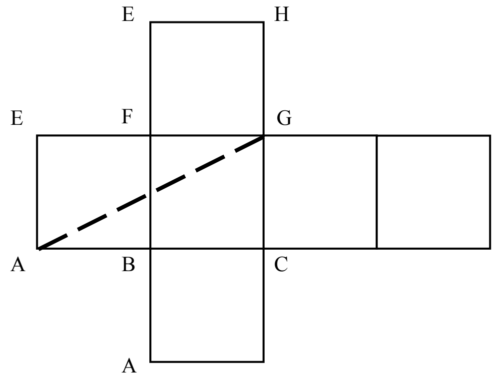


Figure 5.1 Solution diagram for Item G24

Mental rotation ability has a positive effect (direct, indirect, and total) on declarative knowledge, conditional knowledge, and procedural knowledge. Mental rotation of an object is considered as a cognitive process that is analogous to the physical rotation. Similar to spatial visualization ability, results provided evidence that spatial visualization ability leads students to make effective progress on declarative knowledge, conditional knowledge, and procedural knowledge. Rotation of an object facilitates understanding not only the structure of the object but also relations among elements of it. For instance, consider prototypical images and the items in first part of PPKT that includes the identification of a prism or pyramid from the given drawing. Mental rotation ability may help to interpret visual information. Consider the objects in Figure 5.2. Figure 5.2 (a) presents a prototypical image for pentagon pyramid, and Figure 5.2 (b) presents a rotated form of it. To realize that these two illustrations represent the same object, one should use his/her mental rotation ability efficiently. If one does not have effective mental rotation ability, he/she cannot identify the object represented in Figure 5.2 (b) and answer the item asking whether it is a pyramid or prism.

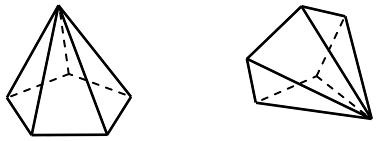


Figure 5.2 Sample declarative knowledge item

(Figure b is used in G1)

The relations between spatial perception ability and knowledge types is not as uncomplicated as other abilities and knowledge types. Spatial perception ability has a positive direct effect on declarative and procedural knowledge. It had positive indirect effect on declarative knowledge, conditional knowledge, and procedural knowledge. Although spatial perception ability has no significant direct effect on conditional knowledge, it has an indirect effect on conditional knowledge. Like all other spatial abilities, spatial perception ability also helps learner to interpret the visual information. When the total effects of the model are examined, significant total effects of spatial perception ability on all knowledge types are observed.

When magnitude of effects were inspected, it is determined that spatial abilities has larger effect on declarative knowledge. Spatial abilities help to judge the visual information. However, it is not enough. In order to understand the given stimulus, transforming the visual information according to facts and making inferences is required (Tversky, 2005a, 2005b). That may be the reason of smaller direct effects of spatial abilities on conditional and procedural knowledge. Since conditional knowledge is about condition-action processes, and procedural knowledge includes application of rules and principles, this result is reasonable. As spatial abilities have largest effects on declarative knowledge and declarative knowledge has an effect on conditional and procedural knowledge, it is also logical to observe larger indirect effects of spatial abilities on conditional and procedural knowledge. Spatial abilities open the way of selecting adequate facts related to the situation and declarative knowledge mediated the effect of spatial abilities on conditional and procedural knowledge.

These results indicated that spatial visualization ability, mental rotation ability, and spatial perception ability are important variables that had an effect on knowledge of prisms and pyramids. Thus, results support the idea that students with high spatial abilities performed better than did students with low spatial abilities on geometry tasks (Hannafin et al, 2008).

5.2.4 Gender Differences

The direct effects support the literature that indicated the male superiority in mental rotation and spatial perception (Coluccia & Louse, 2004; Ethington & Wolfe, 1984; Kaufman, 2007; Linn & Petersen, 1985). Moreover, similar to their findings, larger difference in mental rotation than in spatial perception is found in present study. As has been found by Linn Petersen (ibid), male and female students' performances in spatial visualization tasks did not differ. However, indirect effect of gender on spatial visualization gives an idea about gender difference. The interrelations among the spatial abilities are the source of this effect. For instance, gender has an effect on mental rotation ability and mental rotation ability has an effect on spatial visualization ability. Such relations originate the indirect effects, as a result the total effect of gender on spatial visualization ability is detected. The effect of gender on spatial visualization ability was mediated by mental rotation and spatial perception abilities. This study presents empirical evidence for gender difference in favor of males on spatial abilities. Unfortunately, the data is not sufficient to explain the sources of this difference. Researchers hypothesize several reasons for gender differences in spatial abilities. Sex-linked recessive genes, hormones, neurological factors, child rearing, educational environments, experiences, culture, load of working memory or complex interactions between these could underlie the gender differences (Coluccia & Louse, 2004; Kaufman, 2007; Kimura, 1996; Linn & Petersen, 1985; Maccoby & Jacklin, 1974; Mohler, 2008; Newcombe, et al., 1983). The participants of this study were at the puberty stage, one possible reason for this gender difference could be hormones. The role of the educational environment and culture would play an important role in gender difference too. Kelley, (1988, as cited in Farooq, 2009) claimed that teachers interact more with boys than girls in math and science instruction. They were also found to ask boys more questions and provide boys more response opportunities. Thus, another possible reason for gender difference could be student-teacher interaction in educational environments.

When the direct effects of gender on knowledge types were examined, it is found that gender had positive (which indicates the female advantage) direct effect only on declarative knowledge. Covarying out the spatial abilities eliminated gender difference in conditional and procedural knowledge. This finding supports the findings of previous studies that exposed female superiority (Berberoğlu, 1995; Ubuz, 1999). The possible explanations for these differences were the procedural knowledge focused education. Most of the instructions and tasks that students encounter was procedural. So, both females and males have equal chance to overcome difficulties. Similarly, the lack of instruction based on conditional knowledge leads similar results. On the other hand, declarative knowledge was presented everywhere. Students easily achieve it by reading or memorizing. Female students' learning strategies (Davis & Carr, 2002; Kenney-Benson, Pometrantz, Ryan, & Patrick, 2006) may be one of the reasons of this difference. When the indirect effects were examined, it is found that gender has negative (which indicates the male advantage) indirect effects on all knowledge types. With this indirect effect, the total effect of gender on declarative knowledge becomes non-significant.

According to total effects of gender, results revealed that male students' performance on conditional and procedural knowledge is better than female students' performance. Besides, no gender effect was observed on declarative knowledge. A number of studies reported there is no gender differences in geometry performance (Huntley et al, 1990; Ma, 1995; Park & Norton, 1996). Furthermore, a number of researchers have reported the males are better than females in geometry (Battista, 1990; Maccoby & Jacklin, 1974). In contrast, some others have indicated the opposite (Ubuz, 1999). The result of the present study is supporting the studies that indicate the male dominance in mathematics related fields (Battista, 1990, Maccoby & Jacklin, 1974; Tartre & Fennema, 1995). The results of this study are supported by the results of the Fennema-Sherman studies (Fennema & Sherman, 1977, 1978; Sherman & Fennema, 1977) in terms of gender differences in spatial abilities and geometry performance.

5.2.5 School Type Differences

The direct, indirect, and total effects of School Type indicate the superiority of students in Anatolian high schools in 3D geometry knowledge and spatial abilities. This result supports the studies that declared the advantage of Anatolian high schools (Alacacı & Erbaş 2010; Berberoğlu, 2005; Berberoğlu & Kalender, 2005). There may be several reasons for that advantage. The possible explanations for this difference may be the socioeconomic status of children (Alacacı & Erbaş 2010). Berberoğlu (2005) explains the unequity among school types through students' anxiety, self-efficacy, and self-concept. Moreover, it is known that Anatolian high schools offer mathematics extension courses, extracurricular mathematics activities, and mathematics competitions more frequently than general high schools. These activities may help students not only develop their geometry performance but also their spatial abilities Furthermore, in general high schools, teachers have low expectations of students, the student-teacher relations are poor, and students are not being encouraged to achieve their full potential. On the other hand, opportunities that different types of schools have constitute the quality of school and the quality of instructions in schools. For instance, having a projector, hands on materials, a mathematics laboratory or qualified teachers are sample different elements that form learning environments. One possible reason for school type difference could be such elements. Another possible explanation for this finding is the teacher factor. The effect of teachers experience and teaching is evident. Not only students in Anatolian high schools but also teachers were selected. Selected teachers may be those who are more likely to use more visual oriented methods. Thus, the quality of the instruction may affect the performance of students on prisms and pyramids, and spatial abilities. Students may lack in academic experience with spatial displays. Moreover, they may lack in appropriate skills and strategies for dealing with spatial displays.

5.3 Conclusion

Results of this study revealed the bilateral relations among students declarative, conditional, and procedural knowledge. These interrelations indicate that they are learned in tandem rather than independently. Thus, declarative knowledge forms the basis for conditional and procedural knowledge. Conditional knowledge provides the connections among facts and rules and strengthens the understanding of facts. It also leads the understanding of where and why to use declarative and procedural knowledge. Use of correct procedures gives rise to the consolidation of declarative and conditional knowledge.

Results of this study revealed the bilateral relations among students' spatial visualization, mental rotation, and spatial perception abilities. Conditional, and procedural knowledge. The reciprocal relations among spatial abilities were expected as complete understanding of visual stimuli includes understanding of all parts and the relations between the parts of that object. That is, students, who understand the relations among the parts of a visual stimulus and perform multistep manipulations of spatially presented information, tend to perform better on cognitive processes analogous to the physical rotation of an object and determine the spatial relationships with respect to the orientation of visualizers' body. Furthermore, students, who perform well on cognitive processes analogous to the physical rotation of an object, tend to perform better on multistep manipulations of spatially presented information and determine the spatial relationships with respect to the orientation of visualizers' body. Finally, students, who determine the spatial relationships with respect to the orientation of visualizers' body, are inclined to understand the relations among the parts of a visual stimulus, perform multistep manipulations of spatially presented information, and perform better on cognitive processes analogous to the physical rotation of an object.

The effects of spatial abilities on geometry performance revealed the importance on students' abilities in geometry education. Improvement in students' spatial abilities directly related to improvement in 3D geometry performances. Thus, the attempts to be made to improve students' abilities appear to be crucial.

The direct effects of gender indicate the female advantage on declarative knowledge. However, as a consecuence of the indirect effects, the total effects of gender on knowledge types revealed that no gender difference exists in declarative knowledge and that males perform better than females on conditional and procedural knowledge items. Similarly, the effects of gender on spatial abilities revealed that male students' performances are better than female students' performance.

The significant positive direct and indirect effects of school type on students' performance in declarative, conditional, and procedural knowledge, spatial visualization, mental rotation, and spatial perception indicate the Anatolian high school students' superiority on 3D geometry knowledge and spatial abilities.

5.4 Implications

This study provided support in favor of the fact that knowledge of mathematics distinguishes into three: declarative, conditional, and procedural knowledge. In addition, this study hopes to attract teachers' attention to undertake the responsibility of teaching geometry by emphasizing the bilateral relations among knowledge types. The knowledge distinction leads the alternative ways for instructions in mathematics courses and analysis of the relations among knowledge types can provide different perspectives for teaching and learning geometry. Proficiency in geometry is not only based on specialization in procedural knowledge, but also specialization in declarative and conditional knowledge. It has become clear that neither type of mathematical knowledge should be taught to the exclusion of the other. The acquisition of knowledge of concepts, relations among concepts, and procedures should be considered at the same time in geometry instructions for improving understanding. Teachers could develop discoursive-learning environment that promotes students to progress on their declarative, conditional, and procedural knowledge. Instruction that emphasize the concepts, relations among concepts, and how they relate to steps in a procedure likely lead to increases in declarative, conditional, and procedural knowledge. Accordingly, students will be able to make transformations among concepts and procedures by means of discourse that involves the comprehension of meaning (Smith & Ragan, 2005). For instance, figures should be included in their lessons in order to provide multiple representations of prisms and pyramids. Students should encounter with both typical and protypical examples, and figures of non-examples. Representing prisms and pyramids from different point of views, in other words rotated views, enable students to identify them correctly. Furthermore, definitions of terms related to prisms or pyramids should be clearly presented in relation to figural representation. For instance, after the height of a pyramid was defined, it should be shown on the figures of different pyramids. Moreover, difference between the height of the pyramid and the slant height should be mentioned by indicating figural representations. Thus, emphasizing the differences and relations between concepts is important. Such approach not only lead to develop comprehensive declarative knowledge but also develop students' conditional knowledge. Knowledge of relations also enhance declarative knowledge. A final step that combine declarative, conditional, and procedural knowledge was required. A question asking the amount of wrapping paper needed to cover a box with given the lengths of the base edges and height of pyramid will give student opportunity to scrutinize and use his or her knowledge.

The curriculum designs and instructional methods to teach geometry should include activities that increase students' spatial abilities. There is a consensus that spatial abilities cn be developed by appropriate instructions. Geometry instructors need to take the effect of spatial abilities on the performance in declarative, conditional, and procedural knowledge into account when designing instructional environments. Developing students' spatial abilities would help students to overcome difficulties in understanding visual representations of geometric figures and led to better performance. The determined effects of spatial abilities on knowledge types presented evidence that integration of spatial elements in geometry teaching is crucial. Presented relations might help educators to design targetted training in teaching environments for students. Previously mentioned learning environments that meets students with different representations of prisms and pyramids such as typical and protypical examples, and figures of non-examples may lead students develop their declarative knowledge with the help of mental rotation and spatial perception abilities. Another activity that includes paper folding of a pyramid, may lead students to understand the properties of it. Paper folding activities which is related to spatial visualization ability may help students to discriminate the height of a pyramid from slant height. Moreover, hands on materials such as models of prisms and pyramids not only help to visualize the geometrical concepts but also provides opportunities to work on for example cross-sections and different point of views.

The superiority of students in Anatolian high schools pointed out the existence of different qualifications in different schools. This situation indicates the inequalities in educational opportunities. Accordingly, results revealed the necessity of educational policies that should eliminate the school types and differences between schools by providing equal learning opportunities, conditions, and qualification. Determination of the possible causes of these differences is required. The prevalent use of extracurricular activities and extension courses among all types of schools should provide students equal educational opportunities to develop performance in spatial abilities and geometry knowledge. Activities that support learning by influencing spatial ability may in turn influence the quality of instruction because teachers may perform more effectively in more intellectually responsive classes (Haertel, Walberg & Weintein, 1983).

Finding a standardized test that focus on a specific subject in geometry is not very easy. The PPKT developed in this study might help researchers and educators to determine students' declarative, conditional, and procedural knowledge on Prisms and Pyramids. So, the results of PPKT provide detailed information about students' acquisition on different knowledge types. Thus, the students' difficulties and common mistakes in each knowledge type might be determined without any other test.

When the nationwide assessment instruments were examined, it was seen that the majority of items were procedural. In spite of the relations among declarative, conditional, and procedural knowledge, discrimination of students could not be done merely based on procedural items. Thus, the existence of types of knowledge should be taken into consideration when developing tests.

Mathematics teachers for the most part, have not taken any course on epistemology or spatial ability. We can not expect mathematics teachers to teach geometry considering epistemology together with spatial ability, if they have only been educated on the teaching of mathematics. In order to prepare teachers to use spatial elements correctly in their instructions, in-service and pre-service teacher training programs should include courses about epistemology and spatial abilities. Such courses could help in- and pre-service teachers to develop their spatial abilities and knowledge on epistemology. These acquirements might lead teachers to prepare geometry lessons targeting both developing and utilizing students' spatial abilities.

5.5 Limitations

This study was conducted with the 12th grade students who were trained based on previous mathematics and geometry curriculum in elementary and secondary education respectively. Thus, the results of this study cannot be generalized to students who were trained after the improvement efforts in curriculums.

This study tried to explain the causal relations among knowledge types by using path model. The model supported the studies that show the iterative relations among knowledge types. Since the framework of this study is not developmentalist, this study cannot contribute to the literature that discusses the developmental procedure of knowledge. The results of this study cannot be interpreted as support for concept-first or procedure-first theories.

Existence of potentially confounding variables (such as students' activities, motivation, prior education on visualization, socioeconomic status, parents' education level etc…) can be thought as a limitation. Neglecting these variables may have affected the results of this this study.

5.6 Recommendations for Future Research

The present study contributes to the gap in the research supporting the idea that spatial abilities play a significant role in 3D geometry performance. It represents a preliminary step in understanding the relationship among knowledge types of geometry and spatial abilities. As the connection between certain types of spatial abilities and knowledge of geometry becomes more widely known by researchers and educators, more specific research questions can be asked. Along with the clearer picture of the relations between spatial ability and geometry performance, new questions will arise.

- Future studies are required for cross-validation or replication of this study. Supplementary examination of relations for different subject areas and samples from different grade levels is likely to provide better understanding of the role of spatial abilities on knowledge of geometry.

- The present study modeled 12th grade students' knowledge on prisms and pyramids, spatial abilities, gender, and school type. The investigation of the relations among the variables of this study on other geometry topics would also be beneficial. Model tests across different grade levels would also be beneficial to provide further understanding about the relations among declarative, procedural, conditional knowledge, and spatial abilities. These efforts would provide further insight into the process of cognitive development and provide additional data to generalize the effect of spatial abilities on geometry performance.

- As mentioned before, this study was conducted with the 12th grade students who were trained based on previous mathematics and geometry curriculum in elementary and secondary education respectively. It is evident that future research with students who are educated based on the new curriculum must continue to examine the relations among components of geometry knowledge, spatial ability, gender, and school type. These efforts would provide further information about the outcomes of curriculum improvement efforts.

- An interesting research may be the investigation of the same variables within the nested structure. Since the students are in classrooms and the classrooms are in schools, nested models would provide deeper understanding of the relations among variables and provide to determine the explanatory variables for each level.

- Numerous variables have potential affect on students learning and performance. Future research that investigates the role of experiences, as well as attitudinal, affective, or motivational variables would also be beneficial. Such study gives opportunity to explain the reasons of individual differences in spatial ability and geometry performance.

- The superiority of students in Anatolian high schools pointed out the existence of different qualifications in different schools. This situation indicates the inequalities in educational opportunities. Opportunities that different type of schools have constitute the quality of school and the quality of instructions in schools. For instance, having a projector, hands on materials, a mathematics laboratory or qualified teachers are sample different elements that form learning environments. The investigation of the effects on such elements would provide data about the possible causes of school differences. The outcomes of such research could be used to establish equal educational opportunities for students in different types of schools.

- This study presents a model on relationships among knowledge types, spatial abilities, school type, and gender. Future research that uses qualitative or mixed data would provide further information about the reasons of the relationships revealed.

- This study was focused on students' declarative, conditional, and procedural knowledge on 3D geometry. Teachers were expected to teach students to construct this knowledge. Thus, the investigation of same variables with respect to teachers point provides additional information about the teachers' knowledge on 3D geometry.

- Research revealed that spatial ability can be developed by appropriate activities. This study is a cross-sectional survey study thus students' spatial abilities were assessed at a point. Studies show that the use of activities including spatial elements helps students to improve their spatial abilities. The effect of specific interventions designed to improve spatial skills could also be tested with different experimental designs. Considering the results of study, an experimental study investigating the effect of instruction including spatial elements regarding each spatial ability on geometry performance should be beneficial to determine the changes in students' knowledge and spatial abilities.

- The structure of the visual element (e.g. prototypical examples) might have negative effects on concept development. Additionally, presentation type of visual elements might have an effect on developing spatial abilities. The visual elements in geometry textbooks should be analyzed in terms of presentation type and the structure.

- The results of this study revealed gender differences, however, further research on model comparison for males and females with larger samples is recommended to determine the possible differences in relations among variables.

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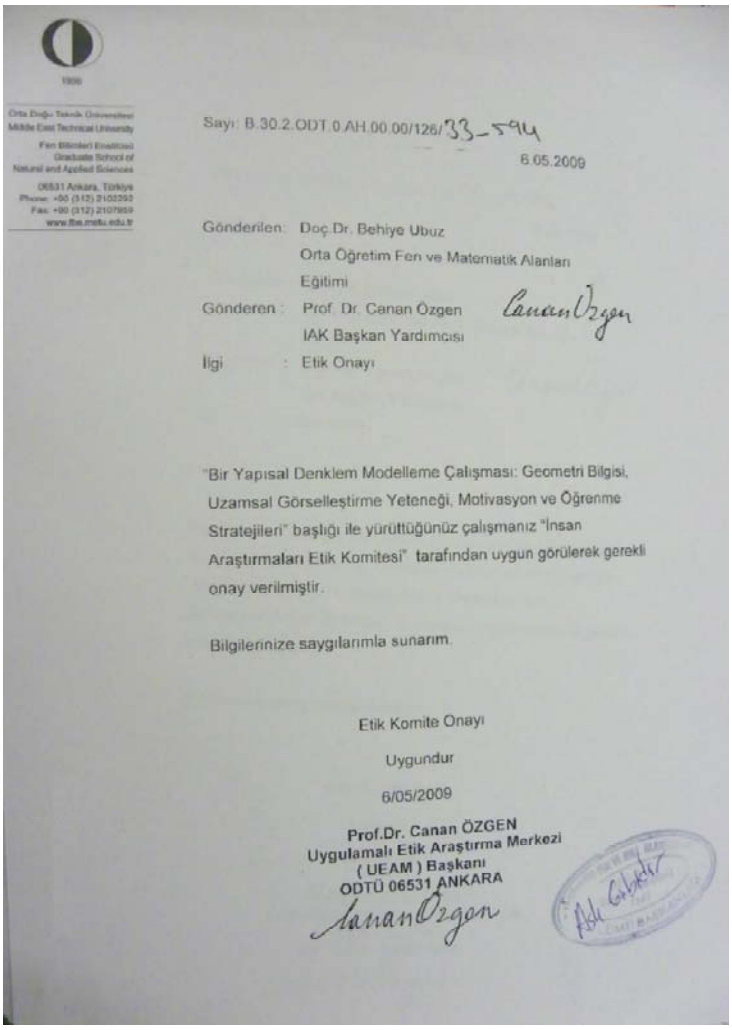
APPENDIX A

Ethics Committee Approval



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6.05.2009

Gönderilen: Doç.Dr. Behiye Ubuz
Orta Öğretim Fen ve Matematik Alanları Eğitimi

Gönderen : Prof. Dr. Canan Özgen
IAK Başkan Yardımcısı

İlgi : Etik Onayı

"Bir Yapısal Denklem Modelleme Çalışması: Geometri Bilgisi, Uzamsal Görselleştirme Yeteneği, Motivasyon ve Öğrenme Stratejileri" başlığı ile yürüttüğünüz çalışmanız "İnsan Araştırmaları Etik Komitesi" tarafından uygun görülerek gerekli onay verilmiştir.

Bilgilerinize saygılarımla sunarım.

Etik Komite Onayı

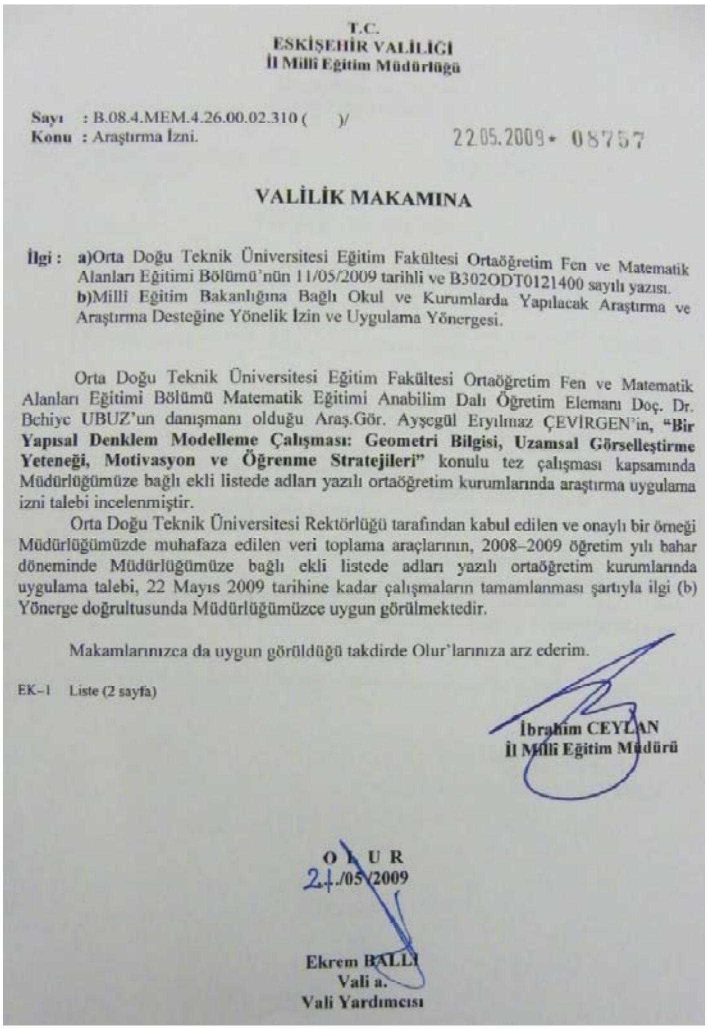
Uygundur

6/05/2009

Prof.Dr. Canan ÖZGEN
Uygulamalı Etik Araştırma Merkezi
(UEAM) Başkanı
ODTÜ 06531 ANKARA

APPENDIX B

Eskişehir National Education Directorate Approvals



T.C.
ESKİŞEHİR VALİLİĞİ
İl Milli Eğitim Müdürlüğü

Sayı : B.08.4.MEM.4.26.00.02.310 (   )/
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22.05.2009 * 08757

VALİLİK MAKAMINA

İlgi : **a)**Orta Doğu Teknik Üniversitesi Eğitim Fakültesi Ortaöğretim Fen ve Matematik Alanları Eğitimi Bölümü'nün 11/05/2009 tarihli ve B302ODT0121400 sayılı yazısı.
b)Milli Eğitim Bakanlığına Bağlı Okul ve Kurumlarda Yapılacak Araştırma ve Araştırma Desteğine Yönelik İzin ve Uygulama Yönergesi.

Orta Doğu Teknik Üniversitesi Eğitim Fakültesi Ortaöğretim Fen ve Matematik Alanları Eğitimi Bölümü Matematik Eğitimi Anabilim Dalı Öğretim Elemanı Doç. Dr. Behiye UBUZ'un danışmanı olduğu Araş.Gör. Ayşegül Eryılmaz ÇEVİRGEN'in, **"Bir Yapısal Denklem Modelleme Çalışması: Geometri Bilgisi, Uzamsal Görselleştirme Yeteneği, Motivasyon ve Öğrenme Stratejileri"** konulu tez çalışması kapsamında Müdürlüğümüze bağlı ekli listede adları yazılı ortaöğretim kurumlarında araştırma uygulama izni talebi incelenmiştir.

Orta Doğu Teknik Üniversitesi Rektörlüğü tarafından kabul edilen ve onaylı bir örneği Müdürlüğümüzde muhafaza edilen veri toplama araçlarının, 2008–2009 öğretim yılı bahar döneminde Müdürlüğümüze bağlı ekli listede adları yazılı ortaöğretim kurumlarında uygulama talebi, 22 Mayıs 2009 tarihine kadar çalışmaların tamamlanması şartıyla ilgi (b) Yönerge doğrultusunda Müdürlüğümüzce uygun görülmektedir.

Makamlarınızca da uygun görüldüğü takdirde Olur'larınıza arz ederim.

EK–1 Liste (2 sayfa)

İbrahim CEYLAN
İl Milli Eğitim Müdürü

O L U R
21./05/2009

Ekrem BALLI
Vali a.
Vali Yardımcısı

T.C.
ESKİŞEHİR VALİLİĞİ
İl Milli Eğitim Müdürlüğü

Sayı : B.08.4.MEM.4.26.00.02.310 (   )/ 10.06.2009- 09567
Konu : Araştırma İzni.

VALİLİK MAKAMINA

İlgi : **a)**Orta Doğu Teknik Üniversitesi Eğitim Fakültesi Ortaöğretim Fen ve Matematik Alanları Eğitimi Bölümü'nün 11/05/2009 tarihli ve B302ODT0121400 sayılı yazısı.
b)22/05/2009 tarihli ve 8757 sayılı Valilik Onayı.
c)Milli Eğitim Bakanlığına Bağlı Okul ve Kurumlarda Yapılacak Araştırma ve Araştırma Desteğine Yönelik İzin ve Uygulama Yönergesi.

Orta Doğu Teknik Üniversitesi Eğitim Fakültesi Ortaöğretim Fen ve Matematik Alanları Eğitimi Bölümü Matematik Eğitimi Anabilim Dalı Öğretim Elemanı Doç. Dr. Behiye UBUZ'un danışmanı olduğu Araş.Gör. Ayşegül Eryılmaz ÇEVİRGEN'in, **"Bir Yapısal Denklem Modelleme Çalışması: Geometri Bilgisi, Uzamsal Görselleştirme Yeteneği, Motivasyon ve Öğrenme Stratejileri"** konulu tez çalışması kapsamında Müdürlüğümüze bağlı ekli listede adları yazılı ortaöğretim kurumlarında araştırma uygulama izni talebi 22 Mayıs 2009 tarihine kadar çalışmaların tamamlanması şartıyla ilgi (c) Yönerge doğrultusunda Valilik Makamından alınan ilgi (b) onay ile uygun görülmüştü.

Ancak çalışmanın tamamlanamaması sebebiyle kişinin başvurusu yeniden değerlendirilmiş ve çalışma takviminde belirtilen tarihler olan Ekim/Kasım/Aralık 2009 tarihleri ile Nisan/Mayıs 2010 tarihleri arasında da uygulanmasında Müdürlüğümüzce bir sakınca görülmemektedir.

Makamlarınızca da uygun görüldüğü takdirde Olur'larınıza arz ederim.

İbrahim CEYLAN
İl Milli Eğitim Müdürü

O L U R
11/06/2009

Ekrem BALLI
Vali a.
Vali Yardımcısı

APPENDIX C

Purdue Spatial Visualization Test Cover Page and Information Page

009199-1

PSVT

PURDUE
SPATIAL VISUALIZATION
TEST

Roland Guay, PhD

Do NOT open this booklet until you are instructed to do so.

TC009199

Purdue Spatial Visualization Test by Roland Guay, 1976.

DESCRIPTION: The Purdue Spatial Visualization Test consists of three parts: Developments, Rotations, and Views. Developments consists of 12 questions designed to see how well subjects can visualize the folding of developments into three-dimensional objects. Rotations consists of 12 questions designed to see how well subjects can vsualize the rotation of three-dimensional objects. Views consists of 12 questions designed to see how well subjects can visualize what three-dimensional objects look like from various viewing positions. There are also three separate 30-item test booklets: one each for Developments, Rotations, and Views. The tests are suitable for use with subjects ages 13 or older.

ADMINISTRATION: The tests can be either group or individually administered.

SCORING AND INTERPRETATION: The scores are the number of items answered correctly. Ther is a scoring key available, listed under MATERIALS.

TECHNICAL CHARACTERISTICS: There is no information on technical characteristics.

MATERIALS: Test, Purdue Spatial Visualization Test; Test, Visualization of Developments; Test, Visualization of Rotations; Test, Visualization of Views; Answer Key

REFERENCES: Guay, Roland B. Spatial Ability Measurement: A Critique and an Alternative. April 1980. 19p. ED 189 166.

APPENDIX D

Purdue Spatial Visualization Directions and Sample Items

BÖLÜM 1: AÇILIMLAR

Testin ilk bölümü üç boyutlu nesnelerin açılımlarını ve katlama biçimlerini hayalinizde ne kadar iyi canlandırdığınızı ölçmek için tasarlanmış 12 adet sorudan meydana gelmektedir.

Sorularda bir geometrik cismin iç yüzlerini gösteren açılım verilmektedir. Açılımdaki taralı bölge ise nesnenin taban yüzeyini göstermektedir. Seçeneklerde de verilen açılım katlandıktan sonra oluşan üç boyutlu geometrik cisimlerin çizimleri verilmektedir. Sizden istenen açılımı verilen cismin katlanmasıyla oluşacak halini seçeneklerden seçmenizdir.

Bu bilgilere göre;

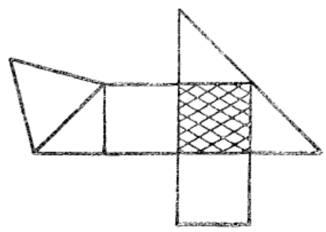
1. Açılım katlandığında meydana gelen üç boyutlu şekli hayalinizde canlandırın
2. Size verilen beş adet cisim arasından verilen açılımın katlanmasıyla meydana gelen cismi seçin (A, B, C, D veya E)

Testin bütün bölümlerindeki soruların sadece bir doğru cevabı vardır.

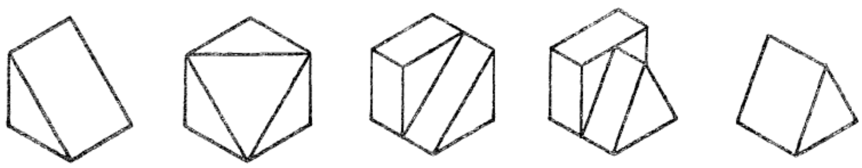
Sorularınızı cevaplarken açılımın nesnenin iç yüzeylerini gösterdiğini ve taralı alanın nesnenin taban yüzeyi olduğunu hatırlayın.

Cevabınızı, cevap kağıdı üzerine seçtiğiniz seçeneği belirgin bir şekilde işaretleyerek belirtiniz.

1



A B C D E



BÖLÜM 2: DÖNDÜRME

İkinci bölüm üç boyutlu nesnelerin döndürülmüş hallerini hayalinizde ne kadar iyi canlandırdığınızı ölçmek için tasarlanmış 12 adet sorudan meydana gelmektedir.

Sorularda öncelikle bir cismin iki hali gösterilmektedir. Birinci haldeki cisim belirli şekilde döndürülerek ikinci hali almaktadır. Sizden istenen uygulanan döndürme işlemini anlamanız ve verilen cismin aynı döndürme işleminden sonraki durumunu seçeneklerden seçmenizdir.

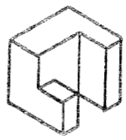
Verilen bilgilerle:

1. Soruda üst satırda verilen nesnenin nasıl döndürülmüş olduğunu anlamaya çalışın
2. Soruda verilen nesnenin aynı şekilde döndürülmüş halini hayalinizde canlandırın
3. Seçenekler arasından doğru yöne döndürülmüş olan nesneyi seçin.

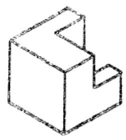
Her sorunun yalnız bir doğru cevabı olduğunu hatırlayınız.

Cevabınızı, cevap kağıdı üzerine seçtiğiniz seçeneği belirgin bir şekilde işaretleyerek belirtiniz.

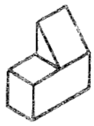
13



döndürüldüğünde



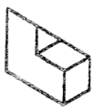
oluyorsa



aynı şekilde döndürüldüğünde

A

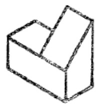
B



C

D

E



BÖLÜM 3: GÖRÜNÜŞLER

Üçüncü bölüm üç boyutlu nesnelerin değişik bakış açılarından nasıl göründüğünü ne kadar iyi hayal edebildiğinizi ölçmek için tasarlanmış 12 adet sorudan meydana gelmektedir.

Sorularda cam bir küpün ortasına yerleştirilmiş bir cisim ve bu cismin çeşitli bakış açılarından görüntülerini temsil eden beş adet seçenek verilmektedir. Cam küpün bir köşesindeki siyah nokta ise istenen bakış açısını göstermektedir. Sizden istenen küp içerisine yerleştirilerek verilen cismin, siyah nokta ile belirlenmiş bakış açısından nasıl göründüğünü seçenekler arasından seçmenizdir.

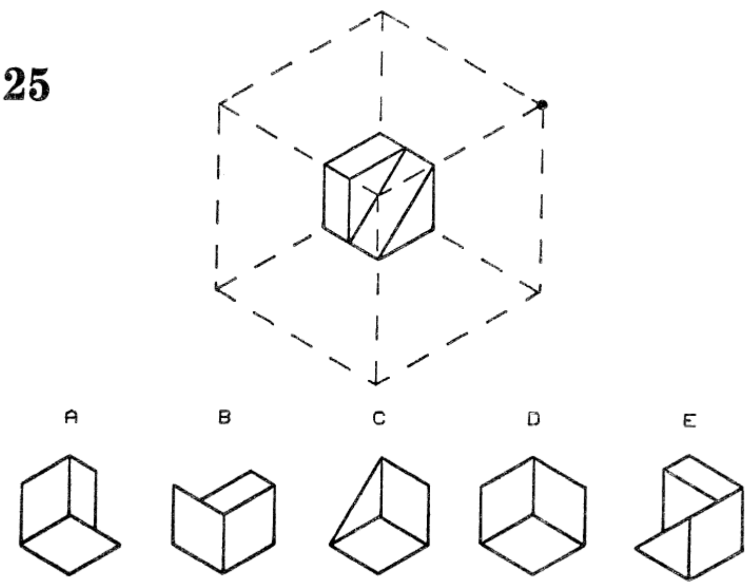
Verilen bilgiler ışığında;

1. Cam küpün etrafında siyah noktanın sizinle cisim arasında olacağı pozisyonu alacak şekilde hareket ettiğinizi hayal edin
2. Cam küpün içerisindeki nesnenin siyah noktanın bulunduğu bakış açısından nasıl göründüğünü hayalinizde canlandırın
3. A, B, C, D ve E seçeneklerinde verilen beş adet görüntüden nesnenin verilen bakış açısından görünen doğru görüntüsünü seçin.

Her sorunun sadece bir doğru cevaba sahip olduğunu hatırlayınız.

Nesnenin cam bir küpün ortasına yerleştirilmiş olduğunu ve siyah noktanın sizinle nesnenin arasında kalacak şekilde bakış doğrultusunu temsil ettiğini hatırlayınız. Bazı durumlarda siyah nokta cismin arkasında kaldığından gri olarak gözüktüğüne dikkat ediniz.

25



APPENDIX E

Hypothesized CFA Model for PSVT

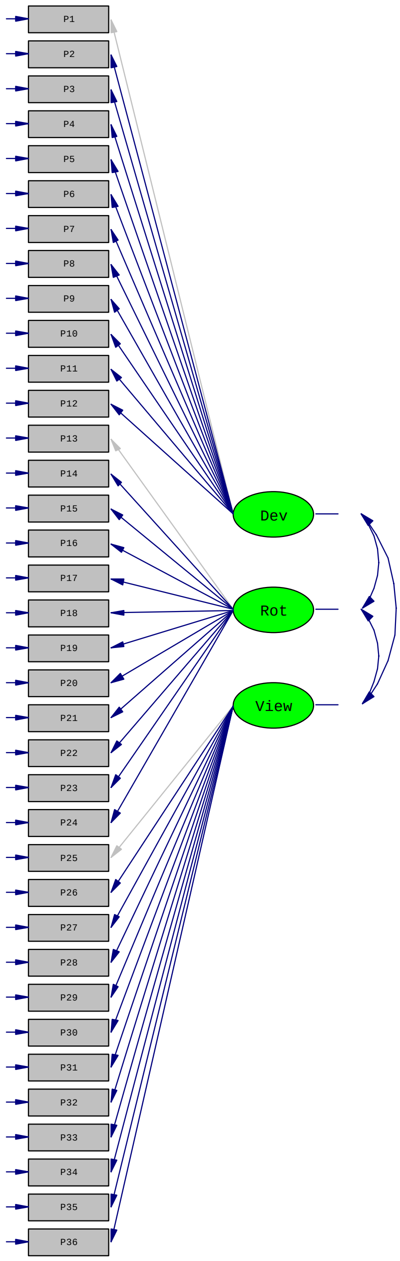


Figure A.1 Confirmatory factor model for PSVT

APPENDIX F

The SIMPLIS Syntax for the PSVT Model

```
36-Item Three-Factor Confirmatory Factor Analysis for PSVT

Observed Variables
P1 P2 P3 P4 P5 P6 P7 P8 P9 P10 P11 P12 P13 P14 P15 P16 P17 P18 P19 P20 P21 P22 P23
P24 P25 P26 P27 P28 P29 P30 P31 P32 P33 P34 P35 P36

Correlation matrix from File: cor.cor
Asymptotic Covariance Matrix from File: asymp.acm
Sample Size = 1067

Latent Variables
Dev Rot View

Relationships
P1 = 1*Dev
P2 P3 P4 P5 P6 P7 P8 P9 P10 P11 P12 = Dev
P13 = 1*Rot
P14 P15 P16 P17 P18 P19 P20 P21 P22 P23 P24 = Rot
P25 = 1*View
P26 P27 P28 P29 P30 P31 P32 P33 P34 P35 P36 = View

Path Diagram
Print Residuals
Admissibility Check = 1000
Iterations = 5000
Method of Estimation: Diagonally Weighted Least Squares
End of problem
```

APPENDIX G

Summary Statistics for Residuals and Steamleaf Plots of the PSVT Model

Smallest Fitted Residual = -0.16
Median Fitted Residual = 0.00
Largest Fitted Residual = 0.18

Stemleaf Plot

```
 -14|94
 -12|7322840
 -10|95422113110
  - 8|97665332210988885555431000
  - 6|99744444433332211009888766655443333222221100
 - 4|99877666554433322210000099999988887776555554444322221111000
 - 2|9998866665554444443332222211110000999998887777776665554444443333322222111+07
 - 0|9999988888888777777666555544444444443333222211111111100099999888887777666+69
   0|111111122222222333444444555666666777778888888999990000111111222223333334444+26
   2|000111111122333344444555666666777788889990000000011112233333444455666777999
   4|000011222244445555666666777778901111122234555555677788889999
   6|0111445566777777901123344456777
   8|0001122344466024678
 10|01124411355589
 12|3580
 14|09
 16|05
```

Smallest Standardized Residual = -3.19
Median Standardized Residual = 0.00
Largest Standardized Residual = 4.56

Stemleaf Plot

```
 - 3|22
 - 2|887655
 - 2|44443311111110
 - 1|99999988877777777777777766666665555555
 - 1|4444444444444333333333322222222222222221111111111110000000000000
 - 0|999999999999999999999988888888888888777777777777777777777666666666666666666+18
 - 0|444444444444444444444444444444444333333333333333333333333333333332222222222+94
   0|111111111111111111111111111122222222222222222222222222333333333333333333333333+24
   0|5555555555555666666666666666667777777777777777777777777777788888888888899999999
   1|0000000000000001111111111122222222222222222233333333333444444444
   1|55555566666677777777778888899999
   2|111222223444
   2|666778
   3|00123
   3|6
   4|11
   4|6
```

APPENDIX H

Prisms and Pyramids Knowledge Test Directions and Sample Items

GEOMETRİ BİLGİ TESTİ

Bu test sizin geometri alanında prizma ve piramitlerle ilgili bilginizi ölçmek için tasarlanmıştır. Test üç bölüm ve toplam 40 sorudan oluşmaktadır. Birinci Bölüm: Prizma mı, Piramit mi?, İkinci Bölüm: Doğru mu, yanlış mı?, Üçüncü Bölüm: Hangisi Doğru?. Her testin başında o testle ilgili açıklama yazılmıştır. Açıklamaları ve soruları dikkatle okumadan cevaplama işlemine geçmeyiniz.

Testte yer alan her sorunun yalnızca tek bir doğru cevabı bulunmaktadır. Bir soru için birden çok seçenek işaretlenmişse ya da hiç bir seçenek işaretlenmemişse o soru yanlış cevaplanmış sayılacaktır. Her soruyu cevaplamaya ve birden fazla seçenek işaretlememeye özen gösteriniz. Soruları çözerken kullandığınız tüm işlemleri ve çizimleri kağıdın boş olan yerlerine yapabilirsiniz.

Bu sınavda toplam cevaplama süresi 40 dakikadır.

Görevli tarafından sınavınız başlatılmadan soruları çözmeye başlamayınız.

DİKKAT! Cevap kağıdınızı başkalarının göremeyeceği şekilde tutunuz.

Başarılar Dilerim.

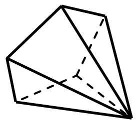
Ayşegül Eryılmaz Çevirgen
ODTÜ Eğitim Fakültesi OFMAE Bölümü
E-posta: eaysegul@metu.edu.tr

BİRİNCİ BÖLÜM

Bu bölümde üç seçenekli çoktan seçmeli 12 madde bulunmaktadır. Sizden istenen çizimleri verilen üç boyutlu cisimleri incelemeniz ve bu cisimlerin prizma veya piramit olup olmadığını belirlemenizdir.

PRİZMA MI, PİRAMİT Mİ?

Aşağıda verilen çizimleri verilen üç boyutlu cisimleri inceleyiniz. Verilen cisme uygun olan seçeneği işaretleyiniz. c seçeneğini işaretlediyseniz cismin genel adını yazınız.

1. 

a) prizma   b) piramit   c) Diğer : ....

İKİNCİ BÖLÜM

Bu bölümde doğru-yanlış tipinde 11 madde bulunmaktadır. Sizden istenen prizma ve piramitler hakkında verilen ifadelerin doğru veya yanlış olup olmadığını belirlemeniz ve doğru ise D yanlış ise Y işaretlemenizdir.

DOĞRU MU, YANLIŞ MI?

Aşağıda verilen ifadeleri inceleyiniz. Verilen ifadelerden **her zaman** doğru olanlar için **D** yanlış olanlar için **Y** harfini işaretleyiniz.

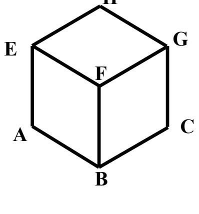
	DOĞRU	YANLIŞ
23. Tüm yüzleri eşkenar üçgen olan üçgen piramide düzgün dörtyüzlü denir.	(D)	(Y)

ÜÇÜNCÜ BÖLÜM

Bu bölümde beş seçenekli çoktan seçmeli 17 soru içermektedir. Sizden doğru olduğunu düşündüğünüz seçeneği işaretlemeniz istenmektedir.

HANGİSİ DOĞRU?

24. Şekildeki gibi bir ayrıtının uzunluğu 2 br olan küp üzerinde yürüyen bir örümceğin A noktasından G noktasına gitmek için alacağı en kısa mesafe aşağıdakilerden hangisidir?



a) 6
b) $2+2\sqrt{2}$
c) $2\sqrt{5}$
d) $2+2\sqrt{3}$
e) 4

28. “Bir dik kare prizmanın bir taban ayrıtının uzunluğu 4 br yüksekliği 5 br dir. Bu kare prizmanın içinde 60 br^3 su bulunmaktadır. Bu prizmanın içine bir kenarı 3 br olan küp şeklinde bir cisim atılır ve bu cisim tam olarak batarsa …..” cümlesini tamamlayan en doğru ifade aşağıdakilerden hangisidir?

a) Prizma içerisindeki su miktarı artar.
b) 7 br^3 su taşar.
c) Prizmadaki suyun yüksekliği 3 br artar.
d) Prizmadaki toplam su miktarı 87 br^3 olur.
e) Prizmadaki suyun yüksekliği değişmez.

APPENDIX I

Hypothesized CFA Model for PPKT

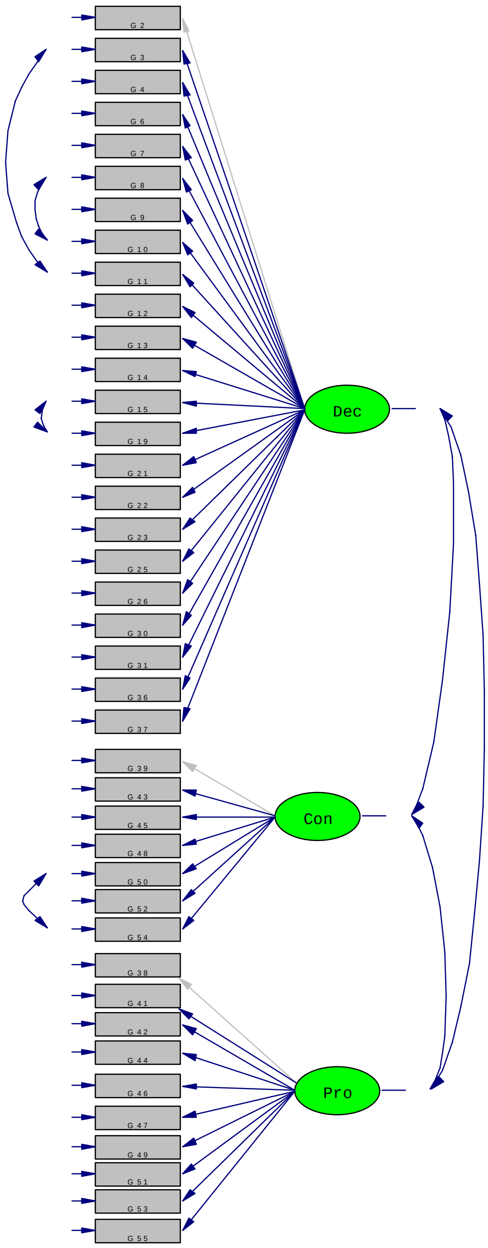


Figure A.2 Confirmatory factor model for PPKT

APPENDIX J

The SIMPLIS Syntax for the PPKT Model

```
40 Items, three factor PPKT Confirmatory Factor Analysis Model

Observed Variables
G2 G3 G4 G6 G7 G8 G9 G10 G11 G12 G13 G14 G15 G19 G21 G22 G23 G25 G26 G30 G31
G36 G37 G38 G39 G41 G42 G43 G44 G45 G46 G47 G48 G49 G50 G51 G52 G53 G54 G55

Correlation matrix from File: cor.cor
Asymptotic Covariance Matrix from File: asymp.acm
Sample Size = 784

Latent Variables
DecK ConK ProK

Relationships
G2=1*DecK
G3 G4 G6 G7 G8 G9 G10 G11 G12 G13 G14 G15 G19 G21 G22 G23 G25 G26 G30 G31 G36
G37 =DecK
G39=1*ConK
G43 G45 G48 G50 G52 G54 = ConK
G38=1*ProK
G41 G42 G44 G46 G47 G49 G51 G53 G55=ProK

Set Error Covariance Between G8 and G10 Free
Set Error Covariance Between G19 and G15 Free
Set Error Covariance Between G3 and G11 Free
Set Error Covariance Between G50 and G54 Free

Path Diagram
Print Residuals
Admissibility Check = 10000
Iterations = 50000
Method of Estimation: Diagonally Weighted Least Squares
End of problem
```

APPENDIX K

Summary Statistics for Residuals and Steamleaf Plots of the PPKT Model

Smallest Fitted Residual = -0.26
Median Fitted Residual = -0.01
Largest Fitted Residual = 0.34

Stemleaf Plot

- 2|655
- 2|2110
- 1|99888777666666665555555
- 1|444444444433333333332222222222222222111111111111111111111110000000000000000000
- 0|99999999999999988888888888888888888888888887777777777777777777777777777777777+72
- 0|4444444444444444444444444444444444444444444444444444444333333333333333333333333+81
0|11111111111111111111111111111111111112222222222222222222222222222222222222223+74
0|55555555555555555555555555555666666666666666666666667777777777777777777777777+40
1|0000000001111111111111112222334444
1|5555566777789999999999
2|0233
2|5689
3|04

Smallest Standardized Residual = -4.22
Median Standardized Residual = -0.12
Largest Standardized Residual = 7.02

Stemleaf Plot

- 4|2
- 3|633332110
- 2|87666655555444433333322222211111100000
- 1|99999999999999888888888777777777777777666666666666665555555555555555444444444+92
- 0|9999999999999999999999999999888888888888888777777777777777777777777777777777+99
0|1111111111111111122222222222222223333333333333333333333333334444444444444444+93
1|0000000000001111111111111111122222222222222333333333333333344444444444445555+36
2|000111122222344455666666677788999
3|0034445556677788
4|16
5|678
6|5
7|0

APPENDIX L

Purdue Spatial Visualization Test (PSVT) Ordering Information

From: <educationcs@digitalriver.com>
Date: Thu, Jan 1, 2009 at 12:56 PM
Subject: Educational Testing Service – Order confirmation for order #4475361614

Thank you for ordering from Educational Testing Service on January 1, 2009. The following email is a summary of your order. Please use this as your proof of purchase. If you paid by credit card, please look for ets on your credit card billing statement.

LOOKING UP YOUR ORDER
You can access your order at https://store.digitalriver.com/store/ets/DisplayOrderInformationPage by entering your e-mail address and last five digits of your credit card.

DOWNLOADABLE PRODUCTS
Downloadable products may be accessed by looking up your order. When the order summary appears, click on the Download link next to the product name. If you need assistance with the download of your product, please visit https://store.digitalriver.com/store/ets/DisplayHelpPage.

PHYSICAL PRODUCTS
You will receive a separate e-mail notification when your products have shipped.

Please note: This e-mail message was sent from a notification-only address that cannot accept incoming e-mail. Please do not reply to this message.

Sincerely,
ETS Store Customer Service

YOUR ORDER AND BILLING INFORMATION
Customer Number: 28184947308
Order Number: 4475361614
Order Date: Jan 1, 2009 11:55:08 AM

APPENDIX M

Measurement coefficients, squared multiple correlations, and reliability coefficients for PSVT calculated in CFA with main data

Table A.1 Measurement coefficients, R^2, and reliability coefficients for PSVT

Latent Variables	Observed Variables	Standardized Solutions	Estimates (λ_x)	Measurement Errors (δ)	R^2	Reliability coefficients (α)	
Spatial Visualization	P1	0.54	1.00	0.71	0.29	0.717	0.843
	P2	0.46	0.86	0.78	0.22		
	P3	0.53	0.99	0.72	0.28		
	P4	0.44	0.81	0.81	0.19		
	P5	0.52	0.97	0.73	0.27		
	P6	0.61	1.13	0.63	0.37		
	P7	0.66	1.23	0.56	0.44		
	P8	0.55	1.02	0.70	0.30		
	P9	0.63	1.16	0.61	0.39		
	P10	0.54	0.99	0.71	0.29		
	P11	0.61	1.12	0.63	0.37		
	P12	0.37	0.68	0.87	0.13		
Mental Rotation	P13	0.48	1.00	0.77	0.23	0.672	
	P14	0.55	1.13	0.70	0.30		
	P15	0.51	1.06	0.74	0.26		
	P16	0.47	0.98	0.77	0.23		
	P17	0.44	0.91	0.81	0.19		
	P18	0.41	0.84	0.84	0.16		
	P19	0.34	0.71	0.88	0.12		
	P20	0.52	1.08	0.73	0.27		
	P21	0.55	1.15	0.69	0.31		
	P22	0.37	0.77	0.86	0.14		
	P23	0.44	0.90	0.81	0.19		
	P24	0.37	0.74	0.86	0.14		
Spatial Perception	P25	0.60	1.00	0.64	0.36	0.744	
	P26	0.50	0.83	0.75	0.25		
	P27	0.51	0.84	0.74	0.26		
	P28	0.54	0.89	0.61	0.29		
	P29	0.58	0.97	0.66	0.34		
	P30	0.65	1.07	0.58	0.42		
	P31	0.69	1.15	0.52	0.48		
	P32	0.60	1.00	0.64	0.36		
	P33	0.53	0.88	0.72	0.28		
	P34	0.68	1.13	0.53	0.47		
	P35	0.55	0.91	0.70	0.30		
	P36	0.43	0.71	0.81	0.19		

APPENDIX N

Measurement coefficients, squared multiple correlations, and reliability coefficients for PPKT calculated in CFA with main data

Table A.2 Measurement coefficients, R^2, and reliability coefficients for PPKT

Latent Variables	Observed Variables	Standardized Solution	Estimates	Measurement Errors	R^2	Reliability coefficients (α)	
Declarative Knowledge	PPKT1	0.53	1.00	0.72	0.28	0.738	0.863
	PPKT2	0.37	0.69	0.87	0.13		
	PPKT3	0.51	0.95	0.74	0.26		
	PPKT4	0.63	1.18	0.61	0.39		
	PPKT5	0.62	1.16	0.62	0.38		
	PPKT6	0.61	1.14	0.63	0.37		
	PPKT7	0.54	1.01	0.71	0.29		
	PPKT8	0.54	1.02	0.71	0.29		
	PPKT9	0.38	0.72	0.85	0.15		
	PPKT10	0.42	0.79	0.82	0.18		
	PPKT11	0.39	0.74	0.85	0.15		
	PPKT12	0.31	0.57	0.90	0.10		
	PPKT13	0.36	0.68	0.87	0.13		
	PPKT14	0.52	0.98	0.73	0.27		
	PPKT15	0.32	0.60	0.90	0.10		
	PPKT16	0.38	0.72	0.85	0.15		
	PPKT17	0.60	1.12	0.65	0.35		
	PPKT18	0.41	0.77	0.83	0.17		
	PPKT19	0.43	0.81	0.81	0.19		
	PPKT20	0.48	0.90	0.77	0.23		
	PPKT21	0.51	0.96	0.74	0.26		
	PPKT22	0.41	0.77	0.83	0.17		
	PPKT23	0.46	0.86	0.79	0.21		
Conditional Knowledge	PPKT25	0.69	1.00	0.52	0.48	0.703	
	PPKT28	0.72	1.05	0.48	0.52		
	PPKT30	0.65	0.94	0.58	0.42		
	PPKT33	0.78	1.13	0.39	0.61		
	PPKT35	0.39	0.57	0.84.	0.16		
	PPKT37	0.54	0.77	0.71	0.29		
	PPKT39	0.53	0.76	0.72	0.28		
Procedural Knowledge	PPKT24	0.56	1.00	0.68	0.32	0.720	
	PPKT26	0.51	0.90	0.74	0.26		
	PPKT27	0.60	1.07	0.64	0.36		
	PPKT29	0.75	1.34	0.43	0.57		
	PPKT31	0.74	1.32	0.45	0.55		
	PPKT32	0.46	0.81	0.79	0.21		
	PPKT34	0.70	1.25	0.51	0.49		
	PPKT36	0.61	1.09	0.62	0.38		
	PPKT38	0.49	0.87	0.76	0.24		
	PPKT40	0.43	0.77	0.81	0.19		

APPENDIX O

The SIMPLIS Syntax for the Prisms and Pyramids Knowledge and Spatial Abilities, Gender, and School Type and Model

```
MIMIC Model gender-school-ability-knowledge
Observed Variables
SCHOOL GENDER DEV ROT VIEW DEC CON PRO PSVT PPKT

Covariance Matrix from File: cov.cov
Asymptotic Covariance Matrix from File: acm.acm
Sample Size = 1161

Relationships
DEC = SCHOOL GENDER DEV ROT VIEW PRO CON
CON = SCHOOL DEV ROT DEC PRO
PRO = SCHOOL DEV ROT VIEW DEC CON
DEV = SCHOOL VIEW ROT
ROT = SCHOOL GENDER VIEW DEV
VIEW = SCHOOL GENDER DEV ROT

Set the path from DEV to VIEW equal to the path from VIEW to DEV
Set the path from ROT to VIEW equal to the path from VIEW to ROT
Set the path from ROT to DEV equal to the path from DEV to ROT
Set the path from DEC to CON equal to the path from CON to DEC
Set the path from DEC to PRO equal to the path from PRO to DEC
Set the path from CON to PRO equal to the path from PRO to CON

Path Diagram
Admissibility Check = 1000
Iterations = 5000
Number of Decimal=3
Method of Estimation: Weighted Least Square
End of problem
```

CURRICULUM VITAE

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Education:

Degree	Institution	Year of Graduation
PhD.	Middle East Technical University, Faculty of Education, Secondary Science and Mathematics Education Department	2012
MS	Middle East Technical University, Faculty of Education, Secondary Science and Mathematics Education Department	2005
BS	Anadolu University, Faculty of Education, Secondary Science and Mathematics Education Department	2000
High School	Burdur Anatolian High School	1996

Work Experience

Enrollment		Year
Research Assistant	Middle East Technical University, Faculty of Education, Department of Secondary Science and Mathematics Education	June 2002 -
Research Assistant	Anadolu University, Faculty of Education, Department of Elementary Education	December 2001 June 2002
Mathematics and English Teacher	Bağyüzü Elementary School, Bağyüzü, Balıkesir	September 2000 December 2001