COLLABORATION AND COMPETITION IN PRESENCE OF IMPERFECT INFORMATION AND NON-LINEAR PRICING

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ŞÜKRİYE KARABAŞ

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submitted by **ŞÜKRİYE KARABAŞ** in partial fulfillment of the requirements for the degree of **Master of Science in Industrial Engineering Department, Middle East Technical University** by,

Prof. Dr. Canan Özgen
Dean, Graduate School of **Natural and Applied Sciences** ____________

Prof. Dr. Sinan Kayalıgil
Head of Department, **Industrial Engineering** ____________

Assist. Prof. Dr. Seçil Savaşaneril
Supervisor, **Industrial Engineering Department, METU** ____________

Examining Committee Members:

Assoc. Prof. Dr. Yasemin Serin
Industrial Engineering Dept., METU ____________

Assist. Prof. Dr. Seçil Savaşaneril
Industrial Engineering Dept., METU ____________

Assist. Prof. Dr. Sedef Meral
Industrial Engineering Dept., METU ____________

Assist. Prof. Dr. Sinan Gürel
Industrial Engineering Dept., METU ____________

Assist. Prof. Dr. Ferda Can Çetinkaya
Industrial Engineering Dept., Çankaya University ____________

Date: ____________

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last Name: ŞÜKRİYE KARABAŞ

Signature :

ABSTRACT

COLLABORATION AND COMPETITION IN PRESENCE OF IMPERFECT INFORMATION AND NON-LINEAR PRICING

Karabaş, Şükriye
M.Sc., Department of Industrial Engineering
Supervisor : Assist. Prof. Dr. Seçil Savaşaneril

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In this thesis, a market is assumed with n competing buyers where price is an inverse linear function of the quantity supplied to the market. The buyers get engaged in Cournot competition, but may also collaborate on purchasing decisions from a supplier. The supplier offers a quantity discount, as the quantity purchased increases unit price decreases. Furthermore, the demand base in the market is uncertain, but the buyers may get a signal of the demand. In this setting, the value of collaboration, information sharing and non-linear pricing is analyzed.

Keywords: Cournot Competition, Collaboration, Information Sharing, Non-linear Pricing

ÖZ

KESİN OLMAYAN BİLGİ VE DOĞRUSAL OLMAYAN FİYATLANDIRMA KARŞISINDA İŞBİRLİĞİ VE REKABET

Karabaş, Şükriye
Yüksek Lisans, Endüstri Mühendisliği Bölümü
Tez Yöneticisi : Yrd. Doç. Dr. Seçil Savaşaneril

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Bu tezde, rekabet eden n adet alıcı ve bir tedarikçinin bulunduğu bir pazarda alıcıların işbirliği kararları çalışılmaktadır. Ürün fiyatı pazara arz edilen miktar arttıkça azalmaktadır. Alıcılar pazarda Cournot rekabeti içinde olmakla beraber ortak bir tedarikçiden satın alma kararları üzerine işbirliği yapabilmektedirler. Tedarikçinin uyguladığı birim fiyat artan satın alma miktarı ile düşmektedir. Piyasadaki talep belirsiz olmasına rağmen, alıcılar talep hakkında öngörüde bulunabilmektedir. Bu piyasa koşulları altında, işbirliği, bilgi paylaşımı ve doğrusal olmayan fiyatlandırmanın alıcılar ve tedarikçi üzerindeki etkisi analiz edilmektedir.

Anahtar Kelimeler: Cournot Rekabet, İşbirliği, Bilgi Paylaşımı, Doğrusal Olmayan Fiyatlandırma

To My Husband and To My Nephew Efe

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CHAPTER 1

INTRODUCTION

Business climate has rapidly changed because of globalization. Firms should become more effective and efficient in operations to survive over many years when compared to the last century. The main goal of businesses is not only decreasing cost, but also becoming more responsive to the demand of end customers. In order to conquer difficulties of market, firms should collaborate with each other. Collaborative firms succeed to stand out in the market easily.

According to Prakash and Deshmukh [1], " collaboration is a negotiated cooperation between independent parties by exchanging capabilities and sharing burdens to improve collective responsiveness and profitability. Specifically, inter-organizational collaboration is defined as: a process in which organizations exchange information, alter activities, share resources and enhance each other's capacity for mutual benefits and a common purpose by sharing risks, responsibilities and rewards. Thus, companies tend to focus on streamlining the cross-company processes from an extended perspective of supply chain. The actual economic context forces the enterprises to collaborate together to survive against an increasingly aggressive competition. "

In the literature, collaboration is defined as being either horizontal or vertical. Types and differences of collaboration can be seen in Figure 1.1. Horizontal collaboration means that companies with similar characteristics (potential competitors) collaborate. For example, Unilever and Kimberly Clark came together because they had similar delivery addresses as 60-70 %. With this collaboration, combined deliveries to retail outlet decreased logistic cost of them and increased delivery frequencies of outlets, in-full and on-time performances. Vertical collaboration is coordination between the buyers and the supplier in a supply chain. For example, Shell Chemicals Europe and Bertschi AG, a Swiss Intermodal Transport company based in Dürrenäsch, successfully redesigned the supply network of Shell's petrochemical plant in Wilton/UK. System infrastructure that allows real-time exchange of information between the collaborative enterprises was an essential component of the collaborative relationship. By means of vertical collaboration strong innovative impact, lower costs were achieved.

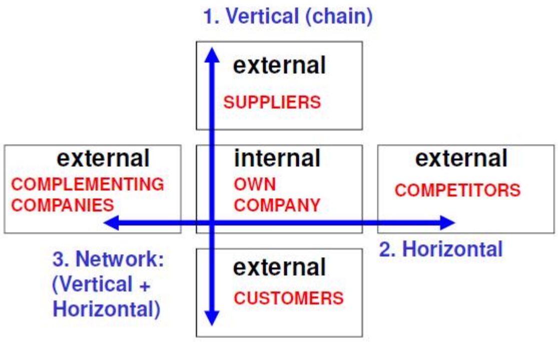

Figure 1.1: Form of supply chain collaboration adopted from [2]

Many firms hesitate to share information with other firms although such a collaboration provides important performance increase in the supply chain.The main reason of lack of sharing is scare about abusing of the information. In fact, many supply-chain related problems can be related to the lack of information sharing among supply chain members. For example, sharing of demand information enables each of supply chain firms to forecast accurately based on real demand. If all members of chain are willing to share information with the others, each member will have less uncertainties and more information about other parts of supply chain.

New systems and services that support collaboration and information sharing between members in supply chains are more famous nowadays. These systems are examplified as Quick Response (QR), Vendor Managed Inventory (VMI), Sales and Operations Planing (S&OP), Collaborative Forecasting (CF), Just-in-time Management (JITM), Collaborative Procurement, Collaborative Logistics, Collaborative Planning Forecasting Replenishment (CPFR) etc.

Focus in this thesis is on analyzing benefits of collaborative procurement among competing buyers in the presence of information sharing with the supplier. Information sharing is a type of vertical collaboration (between the buyers and the supplier) in this study. Through collaborative procurement, the buyers leverage their purchasing power and obtain quantity discount from the supplier. Monopoly, collaboration and no collaboration is analyzed separately.

The rest of the thesis is organized as follows. In Chapter 2, the related literature is reviewed. Next, the structures of the model depending on information type is characterized in Chapter 3. In this chapter, different settings such as deterministic, no information for buyers and supplier, imperfect information for buyers and supplier, imperfect information for buyers and no information for supplier is analyzed in subsections. During these settings, buyers always compete with each other. Finally summary of models and insights are pointed out in Chapter 4.

CHAPTER 2

LITERATURE REVIEW

In our study, we consider several concepts related to collaboration, competition and information sharing. In this chapter, the previous works on the concepts most of which are taken as a basis to this thesis are presented. They are grouped under separate titles such as collaborative practices in the supply chains, collaboration and competition, non-linear pricing and channel coordination and information sharing.

Before starting the discussion on the previous or prior studies, it can be stated that this thesis extends two papers in the literature. Firstly, it extends the study of Keskinocak and Savaşaneril [3] by determining the optimal procurement costs and information sharing with supplier. Secondly, it extends the study of Li [4] by investigating collaboration and non-linear pricing.

2.1 Collaborative Practices in the Supply Chains

There are different collaboration and information sharing reasons in the supply chains. For example, firms can collaborate for procurement, forecasting or logistics etc. In this section, different articles are reviewed.

Firstly, collaborative forecasting is reviewed. In the study of Aviv [5], the objective of Collaborative Planing, Forecasting and Replenishment (CPFR) is stated as "to provide trading partners with the potential for streamlining their supply chain operations, via the sharing of information, and the use of this information in operational planning and product replenishment." In the study of Aviv [5], the potential benefits of Collaborative Forecasting partnerships (one retailer- one manufacturer) are stated when production capacity is the important concern. Aviv [5] enables to study the potential benefits of collaborative forecasts by means of Linear Quadratic Gaussian model which consists of inventory holding, shortage penalty costs, etc. Results are listed as decreases in inventory levels and faster replenishment cycles.

Secondly, specific problems about collaborative logistics are studied in the paper of Ergun et al. [6] and Krajewska and Kopfer [7]. Shippers and carriers collaborate successively and they can be thought as buyers and suppliers. In [6], collaborative logistics takes place when two or more shippers share lanes. The authors focus on finding a set of tours connecting regularly executed truckload shipments with Lane Covering Problem in mathematical model. The authors state that collaborative logistics identifies and reduces costs that none of them are controlled individually in the logistic system. With collaboration, the empty movements of trucks can be decreased easily. For example, members in the network of Nistevo make 20% more profit when the routes are combined. Krajewska and Kopfer [7] is different from Ergun et al. [6], [7] deals with suppliers of operations while [6] deals with buyers. The paper presents a model for the collaboration among independent freight forwarding entities. Model is based on operational research game theory. The collaborative-integrated freight forwarding firms can reduce the costs by subcontraction. The collaborative freight carrier planning is of high practical importance in the modern transportation branch.

Finally, in the paper of Fry et al. [8], the model about Vendor Managed Inventory (VMI) agreements is reviewed. There are lots of reasons such as reducing lead times, delaying allocation of scarce products for implementing VMI. Retailers do not place orders to the supplier because supplier can observe the customers' demands and stocks of retailers in VMI. In other words, retailers share information about demands and stocks with supplier and this helps to increase performance of suppliers and retailers.

As stated in the introduction chapter, there are two types of collaboration such as horizontal and vertical. The paper of [5], [8] has vertical collaboration (between the buyers and supplier) , the other papers [7] and [6] have horizontal collaboration (between only the buyers or the suppliers). Collaboration between firms can be possible about different subjects such as forecasting, logistics and procurement also. This thesis makes a contribution about horizontal and vertical collaboration at the same time.

2.2 Collaboration and Competition

Game theoretic analysis of collaboration and competition is reviewed in the papers below. With substitutable product, competition between players is obtained. The results of cooperation between players are analyzed in detail.

In model of Parlar [9], there are two decision makers competing for substitutable products having random demands. Three different solutions are analyzed. These are Nash Solution for two player, Maximin solution for players and solution with cooperation between players. Firstly, in Nash solution, the players want to make maximum profit so they play rationally in the game. Namely, the player will not lower his objective function for the purpose of lowering the other player's objective function. Players know all the parameters in the problems. Secondly, in Maximin solution, players behave irrationally. Specifically, the player will lower his objective function for lowering the competitor's objective function. Damaging value of objective function is possible by acting irrationally. When first player wins the maximum value in game, the second player will be damaged with maximum possible value also. Finally, in cooperation between players, players cooperate to maximize a joint objective function of problem. Cooperation means that there is no penalty cost for unit that is not satisfied for customer's demand. The other player will help to satisfy the quantity of demand. To sum up, there are three solution examples for inventory problem with two substitutable products having random demands. There are different situations in solutions such as rationality, irrationality and cooperation between players.

Wang and Parlar [10] want to solve the lack of model in [9], in reality, the numbers of retailers are more than two. So that motivates them to model three-person game theory model. The authors study the substitutable product inventory problem when three or more retailers are present by using game theory concepts. When there are three or more retailers, there may be two way demand transfers and coalitions between players. This makes model very sophisticated so the purpose of introducing this model to see optimal ordering decisions when substitutability and competition exist instead of complexity. In non-cooperation part, players must make decisions independently. There is no communication among retailers. In this case, when there is substitution, the retailers order more than no substitution. When the difference between sales price and salvage price is big, optimal order quantity is also big. When m players act irrationally, the other players continue to game. In cooperation part, the supplier may not incur lost sales penalty cost. So the players can switch their excess inventory with other players to maximize joint objective functions. It can be stated that when there is no substitution, the joint profit equals the own independent profit, but when there is substitution, players save lost sales penalty cost by switching inventory. In this part, the important result is that the optimal order quantity is less than that when all the players work independently because of reduction of inventory. To sum up incentive to cooperation is the saving in lost sales penalty costs and the variability of demand of retailers. By means of three person game theory model, more person model can be introduced easily. Optimal order quantities are changing according to cooperation and side payments.

The paper of Keskinocak and Savaşaneril [3] deal with a game theoretical approach to study the interaction between two firms who are competitors at the end market. [3] helps to understand which companies collaborate under which conditions and in which time collaboration is attractive for companies. Horizontal collaborations are analyzed between uncapacitated and capacitated companies. Uncapacitated buyer means that there is no restriction on the procurement quantity. Supplier benefits from procurement through increased sales and revenues when buyers are uncapacitated. Capacitated buyer means that there is restriction on the procurement quantity of buyers. According to buyers' size, willingness of collaboration, impacts of collaborations, benefits change.

Li [4], Zhang [11], Ha et al. [12], Li [13], Shang et al. [14] will be reviewed in the information sharing part in detail, but it can be stated that, collaboration between firms is obtained with sharing information in these papers.

2.3 Non-linear Pricing and Channel Coordination

Quantity discounts are fundamental pricing strategies. Quantity discounts related with pricing has important roles about channel coordination in the supply chains. In this thesis, pricing function is non-linear like in [3].

In the paper of Keskinocak and Savaşaneril [3], cost, wholesale structure is similar when compared with this thesis. The procurement cost is composed of supply price for the first unit and the coefficient of discount. When the procurement quantity increases, total procurement cost decreases. So quantity discount provided by supplier results in collaborating procurement between two firms.

To develop pricing function and quantity discount model in this thesis, the papers below are important also. In the paper of Abad [15], supplier encourages buyers with quantity discounts. A model is formulated of the buyer's response when the supplier offers a temporary reduction in price. The buyer has price-sensitive demand like this thesis and buyers want to optimize selling price and procurement quantity simultaneously while supplier also want to maximize its own profit. In the paper of Weng and Wong [16], the model derives the optimal price schedule for a supplier whose customers face price-sensitive demand. In the other paper of Weng [17], coordination between the supplier with n buyers is modeled to analyze the impact of joint decisions of players.

During the channel coordination in the supply chain, quantity discounts and contracts have importance in the literature. Capacity- demand allocations are also important. Because when there is a capacity problem in the market, firms are more competitive for capacities and allocation rules are followed to solve the allocation rules fairly.

The paper of Heijboer [18] helps to allocate the profit or cost savings by modeling game theoretic cooperation. Contribution to development of collaborative game theory and clarification of member's savings are analyzed. By means of Cooperative Purchasing Game model, opening up the new prospects to players is very easy. The advantages can be seen and setting up consortium can be easier. It is important that the trust level will be increased by means of the model by reducing fear of setting up consortium.

2.4 Information Sharing

In literature, there are variety of papers related with information sharing, but papers that have vertical information sharing and game theory between supplier and buyers are reviewed.

Li [4] studies a model with vertical information in horizontal competition. The paper states that vertical information sharing has two effects namely direct and indirect effect. Direct effect occurs between parties which share the information. Indirect effect occurs between other competing firms which do not share information and may only infer the information from parties that involved in sharing information. The author proposes a model of a two level supply chain in which there is one upstream firm, the manufacturer, and many downstream firms, retailers. The downstream firms are engaged in a Cournot competition such as this thesis. The retailers can choose to share information or not. Moreover, the retailers behave different from each other but in this thesis, the retailers give the same decision about information sharing. The author states that the indirect effect of vertical information sharing is not analyzed before his own paper. The results can be listed as: First, indirect effect encourages the retailers if the shared information is about cost. In direct contradiction, when the information shared is about demand, the retailers are discouraged. Second, the direct effect always discourages the retailers from sharing the information. This thesis extends Li [4] by adding collaboration and non-linear pricing.

Model of Zhang [11] is more restrictive than Li [4] in that it deals with only two retailers while Li considers an arbitrary number of retailers. On the other hand, it allows for differentiated goods and/or Bertrand competition, while Li [4] assumes identical goods and Cournot competition. The products can be independent, complements or substitutes. After analysis, they have shown that, the type of downstream competition (Cournot or Bertrand) does not affect the optimal price of the manufacturer.

Different from [4], [11], this thesis, Ha et al. [12] study a setting with two manufacturers and two retailers. The authors study the incentive for information sharing in the competing supply chains with production technologies that exhibits diseconomies of scale (the forces that cause larger firms to produce goods and services at increased per-unit costs). Two supply chains each consisting of one manufacturer selling to one retailer and in this paper, Cournot and Bertrand competition is analyzed at the same paper such as [11]. The results part of this paper show that information sharing in one supply chain triggers a competitive reaction from the other supply chain and this reaction is damaging the first supply chain under Cournot competition but may be beneficial under Bertrand competition. Furthermore, for linear production costs, information sharing hurts a supply chain under Cournot competition but may benefit a supply chain under Bertrand competition when the information is accurate.

Li [13] studies the incentives for information sharing among firms in an oligopolistic industry (in which a particular market is controlled by a small group of firms). There is some uncertainty about either the demand function or the individual cost functions. That paper proposes a model of a two level supply chain in which there is one upstream firm, the manufacturer, and many downstream firms, retailers such as [4]. The downstream firms are engaged in a Cournot competition. In the model, effects of demand uncertainty, cost uncertainty are analyzed. As a result, when the uncertainty is about a cost, there is the unique equilibrium. When the uncertainty is about a demand, no information sharing is the unique equilibrium.

Shang et al. [14] consider the problem of sharing retailer's demand information in a supply chain with two competing manufacturers selling substitutable products through a common retailer. Different from this thesis, competition is between manufacturers not retailers. The paper proves that a larger production diseconomy (production cost is increasing in volume, due to the limited capacity or input) or higher competition intensity induces more information sharing. [14] is different from this thesis, because there is information contracting in the supply chain.

2.5 The Contribution of the Thesis

As stated in the start of this chapter, it can be stated that this thesis extends two papers in the literature. Giving the detailed similarities and dissimilarities between these helps us understand the contribution of this thesis to this area of research.

Firstly, it extends the study of Keskinocak and and Savaşaneril [3] by determining the optimal procurement costs and information sharing with supplier. In the study of Keskinocak and and Savaşaneril [3], similar price and wholesale functions are used. But different from this thesis, in wholesale functions, there is a spillover factor which determines the additional discount buyer gets from the quantity purchased by the other buyers. In this thesis, the spillover factor is equal to one. Procurement costs that compose the wholesale prices are obtained optimally in this thesis. And information sharing between supplier and buyers is one of the biggest differences.

Secondly, it extends the study of Li [4] by investigating collaboration and non-linear pricing. The sequences of the models, price functions are very similar. Two models have information sharing between supplier and buyers also. Dissimilarities are listed as follows: (i) While information sharing decisions of buyers can be different in the study of Li [4] (one buyer decides to share the information, the other decides to not share), all buyers have same decision in this thesis. (ii)In the study of Li [4], while there is a linear wholesale function, in this thesis with nonlinear wholesale function, collaboration is added to the models also.

CHAPTER 3

MODEL

In our model, there is one supplier and a set of $N = \{1, ..., n\}$ buyers. Here, n also denotes the cardinality of N. The buyers sell a homogenous product and they are engaged in single-period Cournot competition. It is assumed that total quantity supplied to the end market affects the price. As quantity in the market increases, the price decreases. Buyers are allowed to consolidate their purchasing power to have volume discount from the supplier.

Market price is modeled by $P = a + \theta - bQ$ when θ is a random variable with mean 0 and variance σ^2, $E[\theta] = 0$, $Var(\theta) = \sigma^2$. The maximum value θ can take is smaller than a. Total supply provided by the buyers in the market is equal to $Q = \sum_{i=1}^{n} q_i$. When price is equal to zero, total expected demand base in the market is $\frac{a}{b}$.

Each buyer purchases from the supplier exactly the amount it will sell in the end market. The only cost of buyers is the procurement cost from supplier. The supplier offers a quantity discount to the buyers, the unit wholesale price is of the form $w(q_1, q_2, \cdots, q_n) = c_1 - c_2(\sum_{i=1}^{n} q_i)$ in collaboration setting and $w(q_1, q_2, \cdots, q_n) = c_1 - c_2 q_i$ in no collaboration setting. The unit wholesale price decreases depending on both the procurement quantity of the buyer itself and the procurement quantity of the other buyers. When total quantity procured is increased, wholesale price decreases independent of which buyer increases the quantity (see Fig. 3.1)

The sequence of events is as follows:

1. Buyers may obtain a signal Y on the unknown parameter θ, and decide whether to share it with the supplier. It is assumed that the buyers simultaneously decide

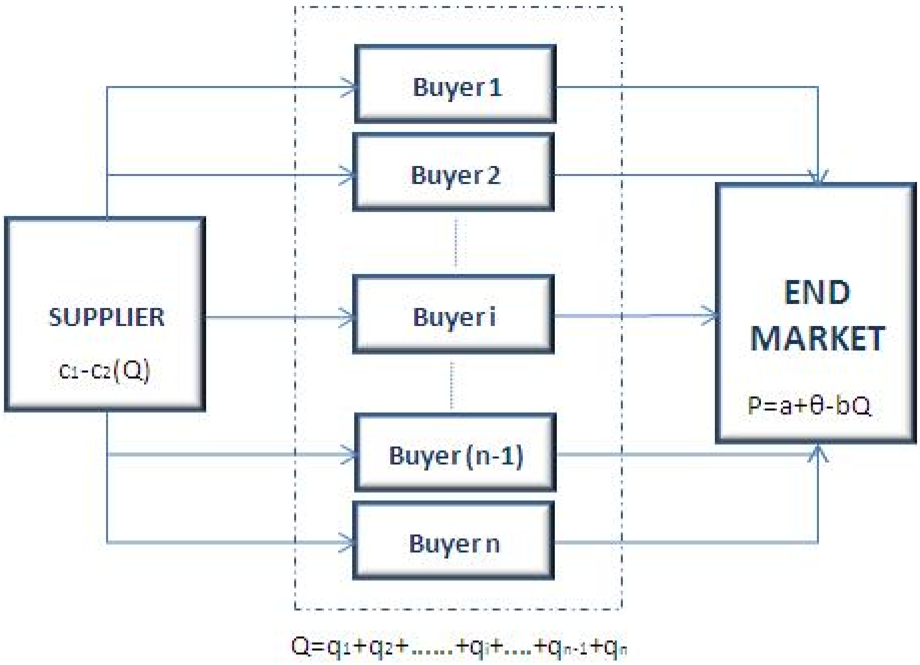

Figure 3.1: Supply prices under collaborative procurement

whether to share information or not, and their decisions are aligned. The signal received is the same for all buyers. Buyers also decide whether to collaborate or not.

2. The supplier offers c_1 and c_2 to the buyers.

3. Buyers decide on the corresponding quantities to purchase from the supplier. The quantities are determined considering that there is Cournot competition in the end market.

This is a three-stage game where the buyers and the supplier play sequentially and the buyers in stage-3 play simultaneously. Buyers are the first movers. In the thesis, stages 2 and 3 are studied as a two-stage Stackelberg game, called subgame, whereas stage 1 is analyzed under a combination of scenarios. There are four different scenarios depending on information type of the buyers and the supplier, and each scenario is composed of three subscenarios, namely; monopoly, collaboration and no collaboration. The outline of the scenarios is presented in Table 3.1. Under each scenario (for instance, perfect buyer- perfect supplier vs collaboration) the game is analyzed as

Table 3.1: Scenarios

	Deterministic	No Info Buyers-No Info Supplier	Imp. Buyers-Imp. Supplier	Imp. Buyers-No Info Supplier
Monopoly	D-M Section 3.1.1	NBNS-M Section 3.2.1	IBIS-M Section 3.3.1	IBNS-M Section 3.4.1
Under Coll.	D-C Section 3.1.2	NBNS-C Section 3.2.2	IBIS-C Section 3.3.2	IBNS-C Section 3.4.2
No-Coll.	D-NC Section 3.1.3	NBNS-NC Section 3.2.3	IBIS-NC Section 3.3.3	IBNS-NC Section 3.4.3

follows. In the first stage of the subgame, the supplier determines the wholesale price, i.e., c_1 and c_2 values, that will maximize her profit. The supplier's profit function consists of only the revenue obtained from the buyers. The supplier is uncapacitated and produces the total amount purchased by the buyers, $\sum_{i=1}^{n} q_i = Q$.
In the second-stage of the game under the corresponding c_1 and c_2 values the buyers get engaged in a game and decide how much to purchase. The profit function of a buyer i is a function of q_i and $q_j,\ j \neq i$. It is composed of Revenue-Cost, where the cost is due to the wholesale price of the supplier.

$$Revenue_i = (a + \theta - b\sum_{i=1}^{n} q_i)q_i \qquad \forall i \in N$$

$$Cost_i = (c_1 - c_2\sum_{i=1}^{n} q_i)q_i \qquad \forall i \in N$$

$$\Pi_i = (a + \theta - b\sum_{i=1}^{n} q_i)q_i - (c_1 - c_2\sum_{i=1}^{n} q_i)q_i \qquad \forall i \in N$$

The second stage of the subgame between the buyers is a simultaneous game. In the simultaneous game, for given c_1 and c_2 values the best response functions of the buyers, and the Nash equilibrium is determined. The equilibrium implies the quantities that will be purchased by the buyers to be sold in the end-market. In the two-stage subgame, first the decisions under the second stage are determined and then the decisions under the first stage are determined.

Under each scenario, the profit of the supplier and the buyers are as follows.Note that in supplier's profit function, depending on the information sharing scenario, q_i, $\forall i \in N$, can be a random variable.

Deterministic Model:

(i) Monopoly:

$$\Pi_m = (a - bq - c_1 + c_2 q)q \tag{3.1}$$
$$\Pi_s = (c_1 - c_2 q)q \tag{3.2}$$

(ii) Collaboration:

$$\Pi_i = (a - b\sum_{i=1}^{n} q_i - c_1 + c_2 \sum_{i=1}^{n} q_i)q_i \tag{3.3}$$
$$\Pi_s = (c_1 - c_2 \sum_i q_i) \sum_i q_i \tag{3.4}$$

(iii) No Collaboration:

$$\Pi_i = (a - b\sum_i q_i - (c_1 - c_2 q_i))q_i \tag{3.5}$$
$$\Pi_s = \sum_i (c_1 q_i - c_2 q_i^2) \tag{3.6}$$

No information for the Buyers and the Supplier:

(i) Monopoly:

$$E[\Pi_m] = E[(a + \theta - bq - c_1 + q)q] \tag{3.7}$$
$$E[\Pi_s] = (c_1 - c_2 q)q \tag{3.8}$$

(ii) Collaboration:

$$E[\Pi_i] = E[(a + \theta - b\sum_{i=1}^{n} q_i - c_1 + c_2 \sum_{i=1}^{n} q_i)q_i] \tag{3.9}$$
$$E[\Pi_s] = (c_1 - c_2 \sum_{i=1}^{n} q_i) \sum_{i=1}^{n} q_i \tag{3.10}$$

(iii) No Collaboration:

$$E[\Pi_i] = E[(a + \theta - b\sum_{i=1}^{n} q_i - c_1 + c_2 q_i)q_i] \tag{3.11}$$
$$E[\Pi_s] = \sum_{i=1}^{n} (c_1 - c_2 q_i)q_i \tag{3.12}$$

Imperfect Information for the Buyers and the Supplier:

(i) Monopoly:

$$E[\Pi_m|Y] = E[(a + \theta - bq - c_1 + c_2 q)q|Y] \tag{3.13}$$
$$E[\Pi_s|Y] = (c_1 - c_2(q|Y))(q|Y) \tag{3.14}$$

(ii) Collaboration:

$$E[\Pi_i|Y] = E[(a + \theta - b\sum_{i=1}^{n} q_i - c_1 + c_2 \sum_{i=1}^{n} q_i)q_i|Y] \tag{3.15}$$

$$E[\Pi_s|Y] = (c_1 - c_2 \sum_i q_i|Y) \sum_i q_i|Y \tag{3.16}$$

(iii) No Collaboration:

$$E[\Pi_i|Y] = E[(a + \theta - b\sum_{i=1}^{n} q_i - c_1 + c_2 q_i)q_i|Y] \tag{3.17}$$

$$E[\Pi_s|Y] = \sum_{i=1}^{n}(c_1 - c_2(q_i|Y))(q_i|Y) \tag{3.18}$$

Imperfect Information for the Buyers, No information for the Supplier:

(i) Monopoly:

$$E[\Pi_m|Y] = E[(a + \theta - bq - c_1 + c_2 q)q|Y] \tag{3.19}$$

$$E_Y[\Pi_s] = E_Y[(c_1 - c_2 q)q] = c_1 E_Y[q] - c_2 E_Y[q^2] \tag{3.20}$$

(ii) Collaboration:

$$E[\Pi_i|Y] = E[(a + \theta - b\sum_{i=1}^{n} q_i - c_1 + c_2 \sum_{i=1}^{n} q_i)q_i|Y] \tag{3.21}$$

$$E_Y[\Pi_s] = E_Y[(c_1 - c_2 \sum q) \sum q] = c_1 E_Y[\sum q] - c_2 E_Y[(\sum q)^2] \tag{3.22}$$

(iii) No Collaboration

$$E[\Pi_i|Y] = E[(a + \theta - b\sum_{i=1}^{n} q_i - c_1 + c_2 q_i)q_i|Y] \tag{3.23}$$

$$E_Y[\Pi_s] = E_Y[(c_1 - c_2 q) \sum q] \tag{3.24}$$

It is assumed that in Stage 1 of the main game the buyers may obtain a signal Y, where Y is an unbiased estimator of θ: $E[Y|\theta] = \theta$. It is further assumed that the expectation of θ given signal Y is a linear function of the signal. Ericson [19] has shown that $E[\theta|Y]$ is a weighted average of prior mean $E[\theta]$, if θ and Y follows certain distributions such as normal-normal. Below equation adopted from Shang et al.[14] is used in the thesis:

$$E[\theta|Y] = \frac{1}{1 + \alpha\sigma^2} E[\theta] + \frac{\alpha\sigma^2}{1 + \alpha\sigma^2} Y = \beta(\alpha, \sigma)Y$$

where $E[\theta] = 0$ assumed earlier. The signal accuracy is α defined as $\alpha = \frac{1}{E[Var[Y|\theta]]}$. It is assumed that the signal is imperfect. This implies, it is assumed that $E[\theta|Y] = \frac{\alpha\sigma^2}{1+\alpha\sigma^2} Y$. $\beta(\alpha, \sigma)$ is used instead of $\frac{\alpha\sigma^2}{1+\alpha\sigma^2}$ for simplicity. When α approaches to infinity, $\beta(\alpha, \sigma)$ goes to 1 and in direct contradiction, when α is equal to zero, $\beta(\alpha, \sigma)$ goes to 0. This means that $\beta(\alpha, \sigma) \in (0, 1)$.

In the analysis, the summary of the notation used is as follows.

P : Market price

a : Maximum reservation price in the market

b : the market sensitivity to quantity

q_i : Quantity that buyer i procures $\quad \forall i \in N$

q_{-i} : Quantity that other buyers procure $\quad -i = N \backslash i$

c_1 : Cost parameter (Supply price for the first unit)

c_2 : the coefficient of discount in wholesale price

r_i : the response of buyer i $\quad \forall i \in N$

Π_s : Profit of supplier

Π_i : Profit of buyer i $\quad \forall i \in N$

The following assumptions are made throughout the analysis:

A1 $a, b > 0$. These parameters that determine price are greater than zero. Buyers' goal is to get profit from sales, so to avoid trivial cases these parameters should not be zero or negative.

A2 $c_1, c_2 \geq 0$, Supplier's goal is to maximize own profit. The wholesale price should not be negative because of quantity discount. If c_2 is negative, the wholesale price will be increased with increasing procurement quantity.

A3 $b - \epsilon \geq c_2$, where $0 < \epsilon \leq b$. This assumption serves the purpose of allocating the profits to the supplier and the buyer. As the analysis show, if $\epsilon = 0$ then this may leave the buyer with zero profit. As ϵ increases so is the profit of the buyer. Here, ϵ can be thought of a agreed upon parameter that determines the "depth of the discount" provided by the supplier to the buyers. How the ϵ will be determined may depend on the power of the parties.

A4 $a \geq c_1$. Maximum reservation price in the market is greater than or equal to the supply price for the first unit to prevent negative procurement quantity.

A5 $P(\theta > -a) = 1$. The random variable θ can be negative or positive. It is assumed that in either case, a is greater than θ. If θ is normally distributed, it is assumed that σ^2 is sufficiently small.

3.1 Deterministic (D)

First as a building block, the analysis is made under the deterministic scenario. In this part, there is no uncertainty on θ i.e., $Var(\theta) = 0$. The procurement decisions of n buyers and wholesale price decision of the supplier under collaboration and no collaboration settings are analyzed successively.

3.1.1 Monopoly (D-M)

We first analyze the single supplier-single buyer (monopoly) model, where the supply price is $c_1 - c_2 q$. Under the deterministic monopoly setting, the profit of buyer is expressed in equation (3.1).

Proposition 1 *Under deterministic monopoly setting with one buyer, the purchasing quantity is* $q = \left(\frac{a-c_1}{2(b-c_2)}\right)$.

Proof. The objective function is concave in q, hence, first derivative of Π_m function is taken with respect to q to find optimal monopoly quantity.

$$\frac{\partial \Pi_m}{\partial q} = a - c_1 - (b - c_2)2q$$
$$q = \left(\frac{a - c_1}{2(b - c_2)}\right)$$

By assumptions $A3$ and $A4$, $q > 0$.

□

Next we move on to analyze the decisions of the supplier. Profit function of supplier is expressed as in equation (3.2).

Proposition 2 *Under the deterministic monopoly setting, optimal* c_1 *and* c_2 *values that maximizes the supplier's profit function are* $c_1^* = \frac{ab}{b+\epsilon}$ *and* $c_2^* = b - \epsilon$.

Proof. To find the c_1 and c_2 values that maximize Π_s under the monopoly setting, the first order conditions (FOC) and second order conditions (SOC) are analyzed.

The FOC yields,

$$\partial\Pi_s/\partial c_1 = \frac{ab + c_1(c_2 - 2b)}{2(b - c_2)^2} = 0,$$

observing that $2(b - c_2)^2 > 0,$ equivalently,

$$ab + c_1(c_2 - 2b) = 0$$

$$c_1 = \frac{ab}{2b - c_2} \tag{3.25}$$

$$\partial\Pi_s/\partial c_2 = -\frac{(a - c_1)(c_1(-3b + c_2) + a(b + c_2))}{4(b - c_2)^3} = 0$$

observing that $4(b - c_2)^3 > 0,$ equivalently,

$$-(a - c_1)(c_1(-3b + c_2) + a(b + c_2)) = 0$$

$$c_2 = b\big(3 - \frac{4a}{a + c_1}\big) \tag{3.26}$$

To find c_1 and c_2 that maximize Π_s, the following steps are taken. First note that, for a given c_1, say c_1^1, the c_2 value obtained from equation (3.26), say $c_2(c_1^1)$ will satisfy the following:

$$\Pi_s(c_1^1, c_2(c_1^1)) \geq \Pi_s(c_1^1, c_2).$$

Now consider the c_1 value obtained from equation (3.25) by placing $c_2(c_1^1)$ to the right hand side (RHS), call this $c_1^2 = c_1(c_2(c_1^1))$. It is obvious that c_1^2 will satisfy,

$$\Pi_s(c_1^2, c_2(c_1^1)) \geq \Pi_s(c_1, c_2(c_1^1)).$$

Iterating over c_1 and c_2 in this manner, in every iteration the value of Π_s will be non-decreasing. Observing this it is now shown that $c_1^{t+1} > c_1^t$ and $c_2^{t+1} > c_2^t$.

$$\begin{aligned} c_1^{t+1} = c_1(c_2(c_1^t)) =& \frac{ab}{2b - (b\big(3 - \frac{4a}{a+c_1}\big))} \\ =& \frac{a(a + c_1)}{3a - c_1} \\ c_2^{t+1} = c_2(c_1(c_2^t)) =& (b\big(3 - \frac{4a}{a + (\frac{ab}{2b-c_2})}\big)) \\ =& \frac{b(b + c_2)}{3b - c_2} \end{aligned}$$

$$
\begin{aligned}
&c_1(c_2(c_1^t)) >^? c_1^t \\
&\frac{a(a+c_1^t)}{3a-c_1^t} >^? c_1^t \\
&a(a+c_1) >^? c_1^t(3a-c_1^t)? \\
&a^2 - 2ac_1^t + (c_1^t)^2 > 0 \\
&(a-c_1^t)^2 > 0
\end{aligned}
$$

Note that, from equation (3.25) $c_1^t < a$ for any $c_2^{t-1} < b$. Next, we check whether c_2^{t+1}, $c_2(c_1(c_2^t)) > c_2^t$ holds.

$$
\begin{aligned}
&c_2(c_1(c_2^t)) > c_2^t \\
&\frac{b(b+c_2)}{3b-c_2} >^? c_2^t \\
&(b^2 - 2ac_2 + (c_2^t)^2) >^? 0 \\
&(b-c_2^t)(b-c_2^t) > 0
\end{aligned}
$$

Since $c_1^{t+1} > c_1^t$ and $c_2^{t+1} > c_2^t$, this implies c_i values assume values on their upper bounds (As stated in assumptions A3 and A4). However, note that it is possible that either c_1 or c_2 approaches the boundary closer than the other. To check whether c_1 or c_2 attains the boundary first, the following is checked.

1. Suppose, $c_1 = a$. Then c_2 is obtained as:

$$
\begin{aligned}
c_2 =& b\big(3 - \frac{4a}{a+c_1}\big) \\
=& b\big(3 - \frac{4a}{a+a}\big) \\
=& b
\end{aligned}
$$

Note that the upper bound on c_2 is $b - \epsilon < b$. This implies, c_2 reaches the boundary before c_1. This is verified in the following.

2. Suppose, $c_2 = b - \epsilon$. Then c_1 is obtained as:

$$\begin{aligned} c_1^* =& \frac{ab}{2b - c_2} \\ =& \frac{ab}{2b - (b - \epsilon)} \\ =& \frac{ab}{b + \epsilon} \end{aligned}$$

Note $\frac{ab}{b+\epsilon} < a$.

Thus the values that maximize Π_s are $c_2^* = b - \epsilon$ and $c_1^* = \frac{ab}{b+\epsilon}$.

□

Corollary 1 *Under the deterministic monopoly setting the supplier is always willing to give a discount, unless $\epsilon = b$.*

Proof. If optimal $c_2 > 0$, then this means supplier is willing to give a discount. For $c_2 < 0$ to happen, the condition $b\big(3 - \frac{4a}{a+c_1}\big) < 0$ must hold. This is equivalent to $c_1 < \frac{a}{3}$. However, note that when $c_2 = 0$, $c_1(c_2) = \frac{a}{2}$, which violates the condition $c_1 < \frac{a}{3}$ Thus c_2 is always greater than zero.

□

Under optimal c_1^* and c_2^*, $q^* = \frac{a}{2(b+\epsilon)}$, $P^* = \frac{a(b+2\epsilon)}{2(b+\epsilon)}$, $\Pi_s = \frac{a^2}{4b+4\epsilon}$, $\Pi_i = \frac{a^2\epsilon}{4(b+\epsilon)^2}$ as derived below,

$$\begin{aligned} q^* &= \frac{a - c_1}{2(b - c_2)} = \frac{a - (\frac{ab}{b+\epsilon})}{2(b - (b - \epsilon))} = \frac{a}{2(b + \epsilon)} \\ P^* &= a - b(q) = a - b(\frac{a}{2(b + \epsilon)}) = \frac{a(b + 2\epsilon)}{2(b + \epsilon)} \\ \Pi_s^* &= (c_1 - c_2 q)q = (c_1 q - c_2 q^2) = \frac{a^2}{4b + 4\epsilon} \\ \Pi_i^* &= (a - bq - c_1 + c_2 q)q = aq - bq^2 - c_1 q + c_2 q^2 = \frac{a^2\epsilon}{4(b + \epsilon)^2} \end{aligned}$$

Note that, $\lim_{\epsilon\to 0} q = \frac{a}{2b}$, $\lim_{\epsilon\to 0} P = \frac{a}{2}$, $\lim_{\epsilon\to 0} \Pi_s = \frac{a^2}{4b}$, $\lim_{\epsilon\to 0} \Pi_i = 0$ while $\lim_{\epsilon\to 0} c_1 = a$, $\lim_{\epsilon\to 0} c_2 = b$.

3.1.2 Collaboration (D-C)

We start the analysis with determining equilibrium quantities. As stated in introduction collaboration between buyers is horizontal. In collaboration model, the buyers pool their needs to obtain more reduction in wholesale price. When there is a collaboration, quantity discount is obtained according to total purchasing quantity. Under the deterministic collaboration setting, the profit of buyer i is expressed in equation(3.3).

Proposition 3 *Under deterministic collaboration setting with n buyers, there is only one equilibrium point expressed as* $(q_n) = \left(\frac{a-c_1}{(n+1)(b-c_2)}\right)$.

Proof. To obtain best response function of buyer i, first derivative of Π_i function is taken with respect to q_i.

$$\frac{\partial \Pi_i}{\partial q_i} = a - c_1 - (b - c_2)(2q_i + \sum_{j \neq i} q_j) = 0$$

The best response of buyer i, q_i, which maximizes Π_i given q_{-i},is as below:

$$q_i(q_{-i}) = r_i(q_{-i})^+$$

where $x^+ = \max\{x, 0\}$.

$$r_i(q_{-i}) = (\frac{a - c_1 - b\sum q_j + c_2 \sum q_j}{2(b - c_2)}) = (\frac{a - c_1}{2(b - c_2)} - \frac{\sum_{j=1}^{n} q_j}{2}) \quad \forall j \neq i \in N \quad (3.27)$$

Note that; when all buyers' purchasing quantities are non-negative, $r_i(q_{-i}) = \frac{a-c_1}{2b-2c_2}$ which is equal to monopoly purchasing quantity.

Solving the system of equations in (3.27) yields,

$$q_i = \frac{a - c_1}{(n + 1)(b - c_2)}$$

given that under equilibrium $\sum_{j \neq i} q_j < \frac{a-c_1}{b-c_2}$. Consider the set of buyers $C \subseteq N$. When the buyers get engaged in a game, the equilibrium quantity for buyer $i \in C$ is expressed as,

$$q_i = \frac{a - c_1}{(|C| + 1)(b - c_2)}.$$

Note that for any C, under the condition $b > c_2$ (it s assumed that in A3), $\sum_{j \in C} q_j = |C| \frac{a-c_1}{(|C|+1)(b-c_2)} < \frac{a-c_1}{b-c_2}$.This implies there cannot exist a group of buyers who will come together, get engaged in a game and procure a quantity that will leave the other buyers with zero procurement quantity at equilibrium. Thus, in equilibrium all buyers procure positive quantities as stated in the proposition.

Under 2 buyers setting, response functions can be seen in Figure 3.2

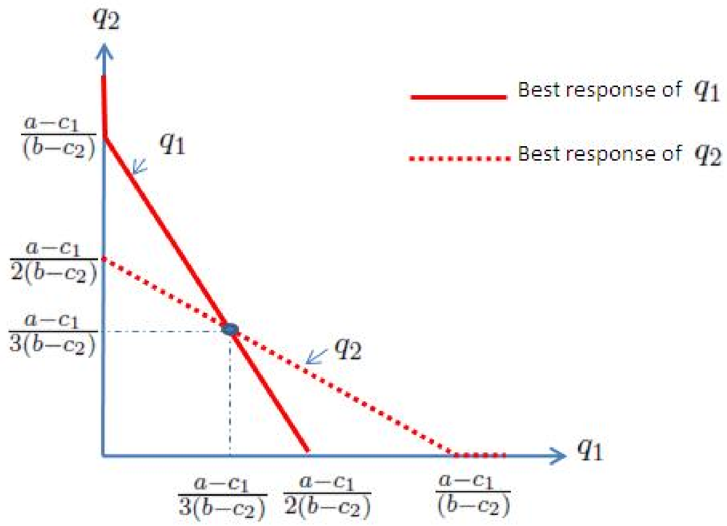

Figure 3.2: Best Response Functions under a two-buyer setting for Deterministic Model in Collaboration

□

When the q_i is analyzed with respect to a, b, c_1 and c_2:

- When b or c_1 increases, q_i decreases. High b implies lower demand base $(\frac{a}{b})$ in the market, and high c_1 implies high base wholesale price, both leading to low purchasing quantity of the buyers at equilibrium.
- Similarly, when a or c_2 increases, q increases. The parameter c_2 denotes the depth of the discount which leads to an increase in q_i.

Next we move on to first stage of subgame and analyze the decisions of the supplier. Profit function of supplier is expressed as in equation (3.4).

Proposition 4 *Under the deterministic collaboration setting, optimal c_1 and c_2 values that maximizes the supplier's profit function are $c_1^* = \frac{a}{2}\left(\frac{2bn-(n-1)\epsilon}{bn+\epsilon}\right)$ and $c_2^* = b - \epsilon$.*

Proof. To find the c_1 and c_2 values that maximize Π_s under the collaborative setting, the first order conditions (FOC) and second order conditions (SOC) are analyzed. The FOC yields,

$$\partial\Pi_s/\partial c_1 = \frac{n(-2c_1(b+bn-c_2)+a(b-c_2+bn-c_2n))}{(b-c_2)^2(1+n)^2} = 0,$$

observing that $(b-c_2)^2(1+n)^2 > 0,$ equivalently,

$$n(-2c_1(b+bn-c_2)+a(b-c_2+bn-c_2n)) = 0$$

$$c_1 = \frac{a}{2}(\frac{b-c_2+(b+c_2)n}{(b-c_2+bn)}) \tag{3.28}$$

$$\partial\Pi_s/\partial c_2 = -\frac{n(a-c_1)(c_2(c_1+an)-b(c_1-an+2c_1n))}{(b-c_2)^3(1+n)^2} = 0$$

observing that $(b-c)^3(1+n)^2 > 0,$ equivalently,

$$-n(a-c_1)(c_2(c_1+an)-b(c_1-an+2c_1n)) = 0$$

$$c_2 = b(\frac{c_1-an+2c_1n}{an+c_1}) \tag{3.29}$$

Note that the system of equations in (3.28) and (3.29) does not have a solution. When the SOC are analyzed, it is observed that the Π_s function is not necessarily convex, or concave. However it is conjectured that Π_s is unimodal in c_1 and c_2 (See Figure 3.3).

Iterating over c_1 and c_2 in this manner, in every iteration the value of Π_s will be non-decreasing. Observing this it is now shown that $c_1^{t+1} > c_1^t$ and $c_2^{t+1} > c_2^t$.

$$\begin{aligned} c_1^{t+1} = c_1(c_2(c_1^t)) &= \frac{a}{2}(\frac{b-c_2+(b+c_2)n}{(b-c_2+bn)}) \\ &= \frac{a}{2}(\frac{b-(b(\frac{c_1-an+2c_1n}{an+c_1}))+(b+(b(\frac{c_1-an+2c_1n}{an+c_1})))n}{(b-(b(\frac{c_1-an+2c_1n}{an+c_1}))+bn)}) \\ &= \frac{a(a+c_1n)}{a(2+n)-c_1} \end{aligned}$$

$$\begin{aligned} c_2^{t+1} = c_2(c_1(c_2^t)) &= b(\frac{c_1-an+2c_1n}{an+c_1}) \\ &= b(\frac{(\frac{a}{2}(\frac{b-c_2+(b+c_2)n}{2(b-c_2+bn)}))-an+2(\frac{a}{2}(\frac{b-c_2+(b+c_2)n}{2(b-c_2+bn)}))n}{an+(\frac{a}{2}(\frac{b-c_2+(b+c_2)n}{2(b-c_2+bn)}))}) \\ &= \frac{b(b+c_2(2n-1))}{b-c_2+2bn} \end{aligned}$$

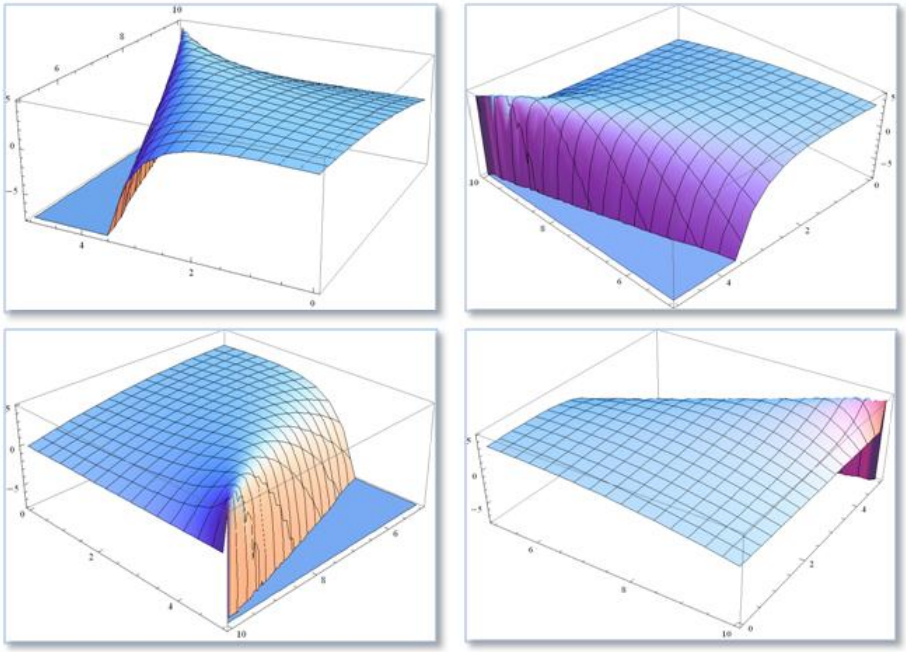

Figure 3.3: Π_s under D-C setting, $a = 10$, $b = 5$, and two-buyers

$$
\begin{aligned}
&c_1(c_2(c_1^t)) >^? c_1^t \\
&\frac{a(a + c_1^t n)}{a(2+n) - c_1^t} >^? c_1^t \\
&(a^2 - 2ac_1^t + (c_1^t)^2) >^? 0 \\
&(a - c_1^t)(a - c_1^t) > 0
\end{aligned}
$$

Note that, from expression 3.28 on page 15, $c_1^t < a$ for any $c_2^{t-1} < b$. Next, we check that c_2^{t+1}, $c_2(c_1(c_2^t)) > c_2^t$ is searched.

$$
\begin{aligned}
&c_2(c_1(c_2^t)) >^? c_2^t \\
&\frac{b(b + c_2^t(2n-1))}{b - c_2^t + 2bn} >^? c_2^t \\
&(b^2 - 2ac_2 + (c_2^t)^2) >^? 0 \\
&(b - c_2^t)(b - c_2^t) > 0
\end{aligned}
$$

Since $c_1^{t+1} > c_1^t$ and $c_2^{t+1} > c_2^t$, this implies c_i values assume values on their upper bounds (As stated in assumptions A3 and A4). However, note that it is possible that either c_1 or c_2 approaches the boundary closer than the other. To check whether c_1 or c_2 attains the boundary first, the following is checked.

1. Suppose, $c_1 = a$. Then c_2 is obtained as:

$$\begin{aligned} c_2 =& b(\frac{c_1 - an + 2c_1 n}{an + c_1}) \\ =& b(\frac{a - an + 2an}{an + a}) \\ =& b \end{aligned}$$

   Note that the upper bound on c_2 is $b - \epsilon < b$. This implies, c_2 reaches the boundary before c_1. This is verified in the following.

2. Suppose, $c_2 = b - \epsilon$. Then c_1 is obtained as:

$$\begin{aligned} c_1^* =& \frac{a}{2}(\frac{b - c_2 + (b + c_2)n}{2(b - c_2 + bn)}) \\ =& \frac{a}{2}(\frac{b - (b - \epsilon) + (b + b - \epsilon)n}{2(b - b + \epsilon + bn)}) \\ =& \frac{a}{2}\left(\frac{2bn - (n-1)\epsilon}{bn + \epsilon}\right) \end{aligned}$$

   Note $\frac{a}{2}\left(\frac{2bn-(n-1)\epsilon}{bn+\epsilon}\right) < a$.

Thus the values that maximize Π_s are $c_2^* = b - \epsilon$ and $c_1^* = \frac{a}{2}\left(\frac{2bn-(n-1)\epsilon}{bn+\epsilon}\right)$.

□

Corollary 2 *Under the deterministic collaboration setting the supplier is always willing to give a discount, unless $\epsilon = b$.*

Proof. If optimal $c_2 > 0$, then this means supplier is willing to give a discount. For $c_2 < 0$ to happen, the condition $b(\frac{c_1 - an + 2c_1 n}{an + c_1}) < 0$ must hold. This is equivalent to $c_1 < \frac{an}{1+2n}$. However, note that when $c_2 = 0$, $c_1(c_2) = \frac{a}{2}$, which violates the condition $c_1 < \frac{an}{1+2n}$. Thus c_2 is always greater than zero.

□

Under optimal c_1^* and c_2^*, $q^* = \frac{a}{2(bn+\epsilon)}$, $P^* = \frac{a(bn+2\epsilon)}{2(bn+\epsilon)}$, $\Pi_s = \frac{a^2 n}{4bn+4\epsilon}$, $\Pi_i = \frac{a^2\epsilon}{4(bn+\epsilon)^2}$ as derived below,

$$q^* = \frac{a-c_1}{(n+1)(b-c_2)} = \frac{a - (\frac{a}{2}\left(\frac{2bn-(n-1)\epsilon}{bn+\epsilon}\right))}{b-b+\epsilon} = \frac{a}{2(bn+\epsilon)}$$

$$P^* = a - b(q_1+q_2+...+q_n) = a - b(n\frac{a}{2(bn+\epsilon)}) = \frac{a(bn+2\epsilon)}{2(bn+\epsilon)}$$

$$\Pi_s^* = (c_1 - c_2(\sum_{i=1}^{n} q_i))(\sum_{i=1}^{n} q_i) = \frac{a^2 n}{4bn+4\epsilon}$$

$$\Pi_i^* = (a - b(\sum_{i=1}^{n} q_i) - (c_1 - c_2((\sum_{i=1}^{n} q_i))))q_i = \frac{a^2\epsilon}{4(bn+\epsilon)^2}$$

Note that, $\lim_{\epsilon\to 0} q = \frac{a}{2bn}$, $\lim_{\epsilon\to 0} P = \frac{a}{2}$, $\lim_{\epsilon\to 0} \Pi_s = \frac{a^2}{4b}$, $\lim_{\epsilon\to 0} \Pi_i = 0$ while $\lim_{\epsilon\to 0} c_1 = a$, $\lim_{\epsilon\to 0} c_2 = b$.

Observe that as ϵ increases, the supplier's profit decrease while the buyers' profit increase. For $\epsilon = b$, the supplier does not give any discount, wholesale price $w = c_1 = \frac{a}{2}$, and suppliers attains the lowest profit level. As ϵ approaches 0, the discount given by the supplier increases, however the buyers' profit approaches to 0. As the discount given by the supplier gets deeper ($\epsilon \to 0$) the quantity purchased by the buyers increase, and the maximum amount a buyer would purchase from the supplier is $\frac{a}{2bn}$.

3.1.3 No Collaboration (D-NC)

In this part, there is no collaboration between the buyers. No collaboration brings on the changes to functions, Π_s and Π_i. Buyers procure the quantities separately, so wholesale price for each buyer is obtained accordingly.

Proposition 5 *Under deterministic no collaboration setting, the number of equilibrium point changes depending on relation between b and c_2.*
(i)If $b > 2c_2$, then there is only one equilibrium point on $\frac{a-c_1}{(n+1)b-2c_2}$
(ii) If $b < 2c_2$, then there are $2^n - 1$ equilibria. The equilibrium points are characterized as $\frac{a-c_1}{(|C|+1)b-2c_2}$. ($C \subseteq N$, $C \neq \{\}$)

Proof. Profit function of buyer i is given in equation (3.5). To obtain the best response function of q_i, FOC is analyzed first.

$$\frac{\partial \Pi_i}{\partial q_i} = a - c_1 + 2c_2 q_i - b(2q_i + \sum_{j \neq i} q_j)$$

$$q_i(q_{-i}) = r_i(q_{-i})^+$$

The best response function of buyer i is then,

$$r_i(q_{-i}) = (\frac{a - c_1 - b\sum_{j \neq i} q_j}{2(b - c_2)} = \frac{a - c_1}{2(b - c_2)} - \frac{b\sum_{j \neq i} q_j}{2(b - c_2)}), \tag{3.30}$$

The equilibrium is characterized as follows.
(i) $b > 2c_2$. Solving the system of equations in (3.30) yields,

$$q_i = \frac{a - c_1}{(n+1)b - 2c_2},$$

given that under equilibrium $\sum_{j \neq i} q_j < \frac{a-c_1}{b}$. Consider the set of buyers $C \subseteq N$. When the buyers get engaged in a game, the equilibrium quantity for buyer $i \in C$ is expressed as,

$$q_i = \frac{a - c_1}{(|C| + 1)b - 2c_2}.$$

Note that for any C, under the condition $b > 2c_2$, $\sum_{j \in C} q_j = |C|\frac{a-c_1}{(|C|+1)b-2c_2} < \frac{a-c_1}{b}$. This implies there cannot exist a group of buyers who will come together, get engaged in a game and procure a quantity that will leave the other buyers with zero procurement quantity at equilibrium. Thus, in equilibrium all buyers procure positive quantities as stated in the proposition.
(ii) $b < 2c_2$. Under this condition, it is possible to show that for any $C \subset N$, except the empty set, the buyers in $|C|$ may procure the equilibrium quantity $\sum_{j \in C} q_j = |C|\frac{a-c_1}{(|C|+1)b-2c_2}$ which is greater than $\frac{a-c_1}{b}$. This quantity leaves the buyers $j \in N \backslash C$ with zero equilibrium quantity. This implies for every $C \subseteq N$ except $\{\}$, there will be an equilibrium. For $n = 2$ the possible equilibria under $b > 2c_2$ and $b < 2c_2$ is shown in Fig.3.4.

□

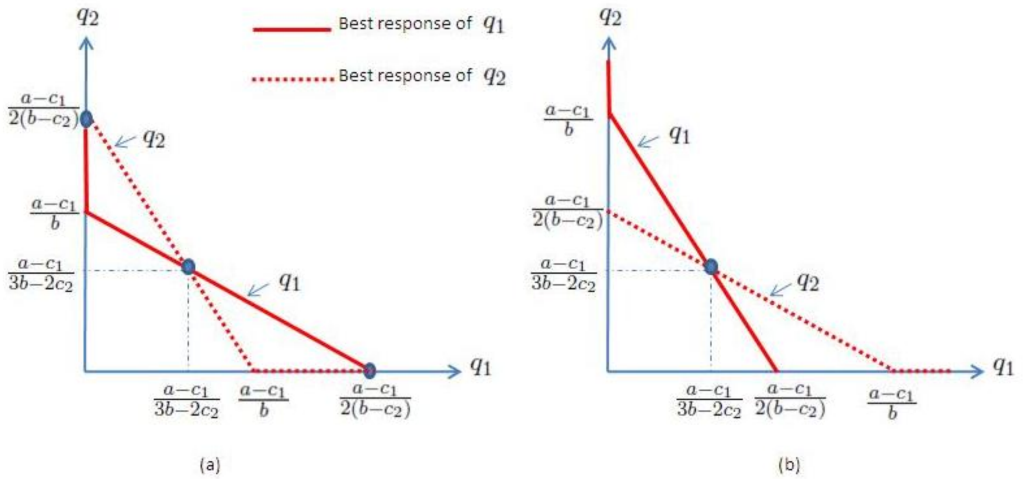

Figure 3.4: Best Response Functions of q_i for Deterministic Model in No Collaboration (a) Multiple Equilibriums when $b < 2c_2$ (b)Single Equilibrium when $b > 2c_2$

In the remainder of the thesis, it is assumed that under $b < 2c_2$ and $b > 2c_2$ the equilibrium quantity for buyer i is $q_i = \frac{a-c_1}{(n+1)b-2c_2}$.

Next, optimal c_1 and c_2 is determined for the supplier. Note that, Π_s, as expressed in equation (3.6).

Proposition 6 *Under the deterministic no collaboration setting, optimal* c_1 *and* c_2 *values that maximize the supplier's profit function are* $c_1^* = \frac{a}{2}\left(\frac{(n+1)b}{(bn+\epsilon)}\right)$ *and* $c_2^* = b - \epsilon$.

Proof. It is observed that, similar to the deterministic collaboration setting, Π_s is not a concave function, but is possibly unimodal. First order conditions yield, $\partial\Pi_s/\partial c_1 = 0$, $\partial\Pi_s/\partial c_2 = 0$.

$$\partial\Pi_s/\partial c_1 = \frac{n(ab(1+n) - 2c_1(b - c_2 + bn))}{((n+1)b - 2c_2)^2} = 0$$

observing that $((n+1)b - 2c_2)^2 > 0$, equivalently,

$$n(ab(1+n) - 2c_1(b - c_2 + bn)) = 0$$

$$c_1 = \frac{ab(n+1)}{2(b - c_2 + bn)} = \frac{a}{2}\left(\frac{b(n+1)}{b(n+1) - c_2}\right) \tag{3.31}$$

$$\partial\Pi_s/\partial c_2 = -\frac{(a-c_1)n(a(b+2c_2+bn)+c_1(2c_2-3b(1+n)))}{((n+1)b-2c_2)^3} = 0$$

observing that $((n+1)b-2c_2)^3 > 0$, equivalently,

$$-(a-c_1)n(a(b+2c_2+bn)+c_1(2c_2-3b(1+n))) = 0$$

$$c_2 = \frac{b(3c_1-a)(1+n)}{2(a+c_1)} \tag{3.32}$$

Note that the system of equations in (3.31) and (3.32) does not have a solution. The Π_s function is not necessarily convex, or concave. However it is conjectured that Π_s in Figure 3.5 is unimodal in c_1 and c_2. Note that for $c_2 = 0$, $c_1 = \frac{a}{2}$. When c_1 in no

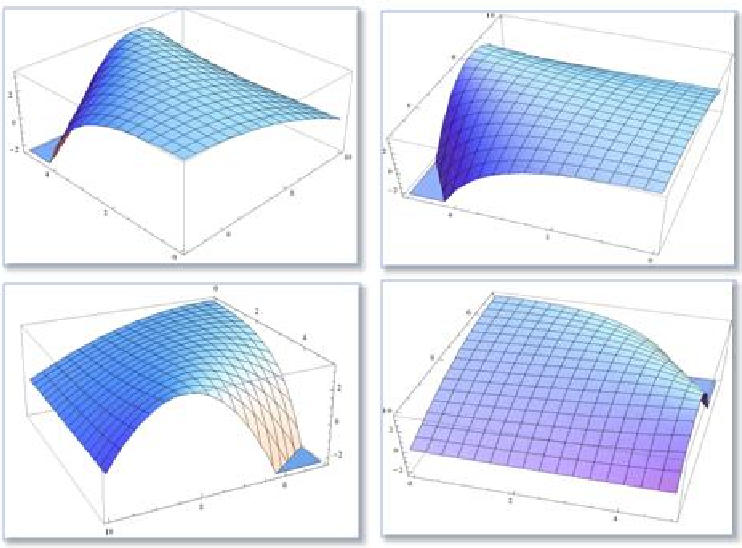

Figure 3.5: Π_s graph in D-NC model for given $a = 10$, $b = 5$, two-buyers

collaboration is compared with c_1 in collaboration model, there is a decrease. Iterating over c_1 and c_2 in the manner of deterministic collaboration, in every iteration the value of Π_s will be non-decreasing, also.

$$c_1^{t+1} = c_1(c_2(c_1^t)) = \frac{a(a+c_1^t)}{3a-c_1^t}$$

$$c_2^{t+1} = c_2(c_1(c_2^t)) = \frac{(1+n)b((n+1)b+2c_2^t)}{6b(1+n)-4c_2^t}$$

Observing this it is now shown that $c_1^{t+1} > c_1^t$ and $c_2^{t+1} > c_2^t$.

$$
\begin{aligned}
&c_1(c_2(c_1^t)) >^? c_1^t \\
&\frac{(a)(a+c_1^t)}{3a-c_1^t} >^? c_1^t \\
&(a)(a+c_1^t) >^? c_1^t(3a-c_1^t) \\
&a^2 - 2ac_1 + (c_1^t)^2 >^? 0 \\
&(a-c_1^t)^2 > 0
\end{aligned}
$$

Note that, from (3.31), $c_1^t < a$ for any $c_2^{t-1} < b$. $c_2^{t+1} = c_2(c_1(c_2^t)) > c_2^t$ is searched.

$$
\begin{aligned}
&c_2(c_1(c_2^t)) >^? c_2^t \\
&\frac{(1+n)b((n+1)b+2c_2^t)}{6b(1+n)-4c_2^t} >^? c_2^t \\
&(1+n)b((n+1)b+2c_2^t) >^? c_2^t(6b(1+n)-4c_2^t) \\
&((n+1)b-2c_2^t)((n+1)b-2c_2^t) > 0
\end{aligned}
$$

Note that it is possible that either c_1 or c_2 approaches the boundary closer than the other. To check which c_i attains the boundary first, the following is checked.

1. Suppose c_1 takes value at its upper bound: $c_1 = a$. Then,

$$
\begin{aligned}
c_2 =& \frac{b(3c_1-a)(1+n)}{2(a+c_1)} \\
=& \frac{b(3a-a)(1+n)}{2(a+a)} \\
=& (1.5)b
\end{aligned}
$$

   Since $c_1 = a$, c_2 must take value outside the boundary, it is concluded that c_2 attains a value at the boundary, whereas c_1 does not.

2. c_2 is set to its boundary: $c_2 = b - \epsilon$. Then,

$$
\begin{aligned}
c_1 =& \frac{a}{2}\left(\frac{b(n+1)}{b-c_2+bn}\right) = \frac{a}{2}\left(\frac{b(n+1)}{b-(b-\epsilon)+bn}\right) \\
=& \frac{a}{2}\left(\frac{(n+1)b}{2(bn+\epsilon)}\right)
\end{aligned}
$$

Note that $\frac{a}{2}\left(\frac{(n+1)b}{2(bn+\epsilon)}\right) < a$, given that $0 \leq \epsilon \leq b$. It is concluded that $c_2^* = b - \epsilon$ and $c_1^* = \frac{a}{2}\left(\frac{(n+1)b}{2(bn+\epsilon)}\right)$

□

Corollary 3 *Under the deterministic no collaboration setting the supplier is always willing to give a discount, unless $\epsilon = b$.*

Proof. Supplier is willing to give a discount, if optimal $c_2 > 0$. The condition $c_2 = \frac{b(3c_1-a)(1+n)}{2(a+c_1)} < 0$ must hold, for $c_2 < 0$ and this implies $c_1 < \frac{a}{3}$. However, note that $c_2 = 0$, $c_1(c_2) = \frac{a}{2}$. As a result, c_2 is always greater than zero.

□

Under optimal c_1^* and c_2^*, $q^* = \frac{a}{2(bn+\epsilon)}$, $P^* = \frac{a(bn+2\epsilon)}{2(bn+\epsilon)}$, $\Pi_s = \frac{a^2n}{4bn+4\epsilon}$, $\Pi_i = \frac{a^2\epsilon}{(4(bn+\epsilon)^2)}$ as shown below:

$$q^* = \frac{a - c_1}{((n+1)b - 2c_2)} = \frac{a - \frac{a}{2}\left(\frac{(n+1)b}{2(bn+\epsilon)}\right)}{(n+1)b - 2(b-\epsilon)} = \frac{a}{2(bn+\epsilon)}$$

$$P^* = a - b(q_1 + q_2 + ... + q_n) = a - \frac{nab}{2(bn+\epsilon)} = \frac{a(bn+2\epsilon)}{2(bn+\epsilon)}$$

$$\Pi_s^* = \sum_i^n (c_1 - c_2q_i)(q_i) = n(\frac{a}{2}(\frac{(n+1)b}{2(bn+\epsilon)}) - (b-\epsilon)(\frac{a}{2(bn+\epsilon)}))\frac{a}{2(bn+\epsilon)} = \frac{a^2n}{4bn+4\epsilon}$$

$$\Pi_i^* = (a - b\sum_i q_i - c_1 + c_2q_i)q_i = \frac{a^2\epsilon}{(4(bn+\epsilon)^2)}$$

Note as $\epsilon \to 0$, $q = \frac{a}{4b}$, $P = \frac{a}{2}$, $\Pi_s = \frac{a^2}{4b}$, $\Pi_i = 0$ while $\lim_{\epsilon \to 0} c_1 = \frac{a(n+1)}{4n}$, $\lim_{\epsilon \to 0} c_2 = b$

3.2 No Information for the Buyers and the Supplier (NBNS)

Under no information, buyers do not get any signal on θ and so does the supplier. The analysis shows that the results of this model is the same as the deterministic model. Under imperfect information, only $E[\theta]$ is defined.

Algebraic calculations are omitted in this section because of the same results in D-M, D-C and D-NC model. Only the important results are stated again.

3.2.1 Monopoly (NBNS-M)

Under no information, one buyer maximizes his expected profit ($E[\theta] = 0$) function in equation (3.7). Purchasing quantity for one buyer, q, is obtained as below.

$$q = \frac{a - c_1}{2(b - c_2)} \tag{3.33}$$

Note that q is not a random variable. Considering the buyer equilibrium purchasing quantity in equation (3.33), under imperfect info, supplier maximizes his (expected) profit in equation (3.8). Note that q and Π_s are invariant to realization of θ, thus these functions are the same with deterministic ones. The c_1 and c_2 values that maximize Π_s are $c_1^* = \frac{ab}{b+\epsilon}$ and $c_2^* = b - \epsilon$.

3.2.2 Collaboration (NBNS-C)

Under no information, buyer i maximizes his expected profit ($E[\theta] = 0$) function in equation (3.9). Equilibrium quantity for buyer i,q_i is obtained as below.

$$q_i = \frac{a - c_1}{(n + 1)(b - c_2)} \tag{3.34}$$

Considering the buyer equilibrium purchasing quantity in equation (3.34), under imperfect info, supplier maximizes his (expected) profit function in equation (3.10). Note that q_i and Π_s are invariant to realization of θ, thus these functions are the same with deterministic ones.The c_1 and c_2 values that maximize Π_s are $c_1^* = \frac{a}{2}\left(\frac{2bn-(n-1)\epsilon}{bn+\epsilon}\right)$ and $c_2^* = b - \epsilon$.

3.2.3 No Collaboration (NBNS-NC)

Under no information, buyer i maximizes his expected profit function in equation (3.11). Equilibrium quantity for buyer i, q_i is obtained as below.

$$q = \frac{a - c_1}{(n + 1)b - 2c_2} \tag{3.35}$$

Considering the buyer equilibrium purchasing quantity in equation (3.35), under no information, the supplier maximizes his expected profit function in equation (3.12).The c_1 and c_2 values that maximize Π_s are $c_1^* = \frac{a}{2}\left(\frac{(n+1)b}{2(bn+\epsilon)}\right)$ and $c_2^* = b - \epsilon$.

3.3 Imperfect Information for the Buyers and the Supplier (IBIS)

Buyers obtain a signal Y about θ, where Y is an unbiased estimator of θ: $E[Y|\theta] = \theta$. It is assumed that the signal is imperfect. This implies, it is assumed that $E[\theta|Y] = \frac{\alpha\sigma^2}{1+\alpha\sigma^2}Y = \beta(\alpha, \sigma)Y$. The buyers share the signal with the supplier.

3.3.1 Monopoly (IBIS-M)

Buyers have imperfect information so expected profit function of buyers in equation (3.13) can be expressed as:

$$\begin{aligned} E[\Pi_m|Y] =& E[a + \theta - b(q) - c_1 + c_2(q))q|Y] = \quad a + E[\theta|Y] - b(q) - c_1 + c_2(q))q \\ =& (a + \beta(\alpha, \sigma)Y - b(q) - c_1 + c_2(q))q \end{aligned}$$

Proposition 7 *Under imperfect information, given signal Y under monopoly the purchasing quantity is $\frac{a+\beta(\alpha,\sigma)Y-c_1}{2(b-c_2)}$ if $a + \beta(\alpha, \sigma)Y > c_1$ and otherwise quantity is equal to zero.*

Proof. To obtain the purchasing quantity for one buyer, function for buyer is obtained.

$$\frac{\partial \Pi_m|Y}{\partial q} = a + \beta(\alpha, \sigma)Y - c_1 - (b - c_2)(2q) = 0$$

FOC implies,

$$q_1|Y = \frac{a + \beta(\alpha, \sigma)Y - c_1}{2(b - c_2)}$$

$$q|Y = \begin{cases} \frac{a+\beta(\alpha,\sigma)Y-c_1}{2(b-c_2)} & \text{if } a + \beta(\alpha, \sigma)Y > c_1 \\ 0 & \text{otherwise} \end{cases}$$

□

Proposition 8 *Under the IBIS monopoly setting, optimal c_1 and c_2 values that maximize the supplier's profit function in (3.14) are $c_1^* = \frac{b(a+\beta(\alpha,\sigma)Y)}{b+\epsilon}$ and $c_2^* = b - \epsilon$.*

Proof. It is observed that, similar to deterministic scenario, Π_s is not concave function, but it is possibly unimodal. First order conditions yield,

$\partial\Pi_s/\partial c_1 = 0, \quad \partial\Pi_s/\partial c_2 = 0$

$$\partial\Pi_s/\partial c_1 = \frac{ab + c_1c_2 - b(-2c1 + \beta(\alpha,\sigma)Y)}{2(b-c_2)^2} = 0,$$

observing that $2(b-c_2)^2 > 0$, equivalently,

$$ab + c_1c_2 - b(-2c1 + \beta(\alpha,\sigma)Y) = 0$$

$$c_1 = \frac{b(a + \beta(\alpha,\sigma)Y)}{2b - c_2} \tag{3.36}$$

$$\partial\Pi_s/\partial c_2 =$$

$$-\frac{(a - c_1 + \beta(\alpha,\sigma)Y)(a(b + c_2) + b(-3c_1 + \beta(\alpha,\sigma)Y) + c_2(c_1 + \beta(\alpha,\sigma)Y))}{4(b-c_2)^3} = 0$$

observing that $4(b-c_2)^3 > 0$, equivalently,

$$-(a - c_1 + \beta(\alpha,\sigma)Y)(a(b + c_2) + b(-3c_1 + \beta(\alpha,\sigma)Y) + c_2(c_1 + \beta(\alpha,\sigma)Y)) = 0$$

$$c_2 = b\Big(-1 + \frac{4c_1}{a + c_1 + \beta(\alpha,\sigma)Y}\Big) \tag{3.37}$$

Iterating over c_1 in equation (3.36) and c_2 in equation (3.37) in this manner, in every iteration the value of Π_s will be non-decreasing. Observing this it is now shown that $c_1^{t+1} > c_1^t$ and $c_2^{t+1} > c_2^t$.

$$c_1^{t+1} = c_1(c_2(c_1^t)) = \frac{(a + \beta(\alpha,\sigma)Y)(a + \beta(\alpha,\sigma)Y + c_1)}{(a + \beta(\alpha,\sigma)Y)3 - c_1}$$

$$c_2^{t+1} = c_2(c_1(c_2^t)) = \frac{b(b + c_2)}{3b - c_2}$$

It is observed that in every iteration c_1 increases:

$$c_1(c_2(c_1^t)) >^? c_1^t$$

$$\frac{(a + \beta(\alpha,\sigma)Y)(a + \beta(\alpha,\sigma)Y + c_1^t)}{(a + \beta(\alpha,\sigma)Y)3 - c_1^t} >^? c_1^t$$

$$(a - c_1^t + \beta(\alpha,\sigma)Y)^2 > 0$$

Next, we check $c_2^{t+1} = c_2(c_1(c_2^t)) > c_2^t$.

$$\begin{aligned}
&c_2(c_1(c_2^t)) > c_2^t \\
&\frac{b(b+c_2^t)}{3b-c_2^t} >^? c_2^t \\
&(b^2 - 2bc_2^t + (c_2^t)^2) >^? 0 \\
&(b-c_2^t)(b-c_2^t) > 0
\end{aligned}$$

The increasing behavior of c_1 and c_2 implies, either c_1 or c_2, or both, will attain values at their upper bounds. Thus, the boundary conditions are checked as follows. Suppose c_1 takes value at its upper bound: $c_1 = a + \beta(\alpha,\sigma)Y$. Then,

$$\begin{aligned}
c_2^* =& b\big(-1 + \frac{4c_1}{a + c_1 + \beta(\alpha,\sigma)Y}\big)) \\
=& b\big(-1 + \frac{4(a+\beta(\alpha,\sigma)Y)}{a + (a+\beta(\alpha,\sigma)Y) + \beta(\alpha,\sigma)Y}\big) \\
=& b(2-1)
\end{aligned}$$

Since when $c_1 = a+\beta(\alpha,\sigma)Y$, c_2 must take value outside the boundary, it is concluded that c_2 attains a value at the boundary, whereas c_1 does not. To verify, c_2 is set to its boundary:$c_2 = b - \epsilon$. Then,

$$\begin{aligned}
c_1^* =& \frac{b(a+\beta(\alpha,\sigma)Y}{2b - c_2} \\
=& \frac{b(a+\beta(\alpha,\sigma)Y)}{2b - (b-\epsilon)} \\
=& \frac{b(a+\beta(\alpha,\sigma)Y)}{b+\epsilon}
\end{aligned}$$

It is concluded that $c_2^* = b - \epsilon$ and $c_1^* = \frac{b(a+\beta(\alpha,\sigma)Y)}{b+\epsilon}$.

□

When a signal is obtained on θ, it is relevant to discuss ex-post and ex-ante profits. Ex-post profit corresponds to the profit after the signal Y, whereas ex-ante profit is profit before signal is realized, considering the actions taken after the signal. Besides the profit, ex-post and ex-ante purchasing quantity and market price are also presented.

$$\begin{aligned}
q|Y =& \frac{a+\beta(\alpha,\sigma)Y - c_1}{2(b-c_2)} = \frac{a+\beta(\alpha,\sigma)Y}{2(b+\epsilon)} \\
P|Y =& \frac{(a+\beta(\alpha,\sigma)Y)(b+2\epsilon)}{2(b+\epsilon)} \\
\Pi_s|Y =& \frac{(a+\beta(\alpha,\sigma)Y)^2}{4b+4\epsilon} \\
\Pi_m|Y =& \frac{(a+\beta(\alpha,\sigma)Y)^2\epsilon}{4(b+\epsilon)^2}
\end{aligned}$$

Note that, $\lim_{\epsilon\to 0} q = \frac{a+\beta(\alpha,\sigma)Y}{2b}$, $\lim_{\epsilon\to 0} P = \frac{a+\beta(\alpha,\sigma)Y}{2}$, $\lim_{\epsilon\to 0} \Pi_s = \frac{(a+\beta(\alpha,\sigma)Y)^2}{4b}$, $\lim_{\epsilon\to 0} \Pi_m = 0$ while $\lim_{\epsilon\to 0} c_1 = a + \beta(\alpha,\sigma)Y$, $\lim_{\epsilon\to 0} c_2 = b$

Ex-ante values are expressed as follows:

$$E_Y[q] = \frac{a}{2(b+\epsilon)}$$
$$E_Y[P] = \frac{a(b+2\epsilon)}{2(b+\epsilon)}$$

Before finding ex-ante $E_Y[\Pi_s]$ and $E_Y[\Pi_i]$, the below equation helps.

$$E[E[Y^2|\theta]] = E[Var[Y|\theta]] + E[E[Y|\theta]^2] = \frac{1}{\alpha} + \sigma^2 \tag{3.38}$$

Using equation (3.38),

$$E_Y[\Pi_s] = \frac{a^2}{4b+4\epsilon} + \frac{(1+\sigma^2\alpha)\beta(\alpha,\sigma)^2}{\alpha(4b+4\epsilon)}$$
$$E_Y[\Pi_m|Y] = \frac{a^2\epsilon}{4(b+\epsilon)^2} + \frac{(1+\sigma^2\alpha)\beta(\alpha,\sigma)^2\epsilon}{\alpha(4(b+\epsilon)^2)}$$

3.3.2 Collaboration (IBIS-C)

Buyers have imperfect information so expected profit function of buyers in equation (3.15) can be expressed as:

$$\begin{aligned} E[\Pi_i|Y] =& E[a + \theta - b(\sum_{i=1}^{n} q_i) - c_1 + c_2(\sum_{i=1}^{n} q_i))q_i|Y] \qquad (i \in N) \\ =& a + E[\theta|Y] - b(\sum_{i=1}^{n} q_i) - c_1 + c_2(\sum_{i=1}^{n} q_i))q_i \\ =& (a + \beta(\alpha,\sigma)Y - b(\sum_{i=1}^{n} q_i) - c_1 + c_2(\sum_{i=1}^{n} q_i))q_i \end{aligned}$$

Proposition 9 *Under imperfect information, given signal Y under collaboration the equilibrium quantity is $\frac{a+\beta(\alpha,\sigma)Y-c_1}{(n+1)(b-c_2)}$ if $a + \beta(\alpha,\sigma)Y > c_1$ and otherwise quantity is equal to zero.*

Proof. To obtain the equilibrium quantity for buyer i, best response function for buyer i is obtained.

$$\frac{\partial \Pi_1|Y}{\partial q_1} = a + \beta(\alpha,\sigma)Y - c_1 - (b - c_2)(2q_i + \sum_{j=1}^{n} q_j) = 0$$

FOC implies,

$$q_i(q_i) = r_i(q_i)^+$$

$$r_i(q_i)|Y = \frac{a + \beta(\alpha,\sigma)Y - c_1}{2(b - c_2)} - \frac{(b - c_2)(\sum_{j=1}^{n} q_j))}{2(b - c_2)}$$
$$= \frac{a + \beta(\alpha,\sigma)Y - c_1}{2(b - c_2)} - \frac{\sum_{j=1}^{n} q_j}{2} \quad \forall i \in N$$

The equilibrium quantity is expressed as below:

$$q_i|Y = \begin{cases} \frac{a+\beta(\alpha,\sigma)Y - c_1}{(n+1)(b-c_2)} & \text{if } a + \beta(\alpha,\sigma)Y > c_1 \\ 0 & \text{otherwise} \end{cases}$$

Equilibrium follows from a similar analysis as in Prop. 3 and from the observation that if $q_i = 0$ and $a + \beta(\alpha,\sigma)Y < c_1$, then $q_j = 0$.

□

Proposition 10 *Under the IBIS collaboration setting, optimal c_1 and c_2 values that maximize the supplier's profit function in (3.16) are $c_1^* = \frac{a+\beta(\alpha,\sigma)Y}{2}\left(\frac{2bn-(n-1)\epsilon}{bn+\epsilon}\right)$ and $c_2^* = b - \epsilon$.*

Proof. It is observed that, similar to deterministic scenario, Π_s is not a concave function, but it is possibly unimodal. First order conditions yield,

$\partial \Pi_s/\partial c_1 = 0, \quad \partial \Pi_s/\partial c_2 = 0$

$\partial \Pi_s/\partial c_1 =$

$$\frac{n(2c_1c_2 + a(b - c_2 + (b + c_2)n) - b(1 + n)(2c_1 + \beta(\alpha,\sigma)Y) + c_2(n - 1)\beta(\alpha,\sigma)Y)}{(n + 1)^2(b - c_2)^2} = 0$$

observing that $(n + 1)^2(b - c)^2 > 0$, equivalently,

$$n(2c_1c_2 + a(b - c_2 + (b + c_2)n) - b(1 + n)(2c_1 + \beta(\alpha,\sigma)Y) + c_2(n - 1)\beta(\alpha,\sigma)Y) = 0$$

$$c_1 = \frac{a + \beta(\alpha,\sigma)Y}{2}\left(\frac{b - c_2 + (b + c_2)n}{(b - c_2 + bn)}\right)$$

$$\partial\Pi_s/\partial c_2 = -\frac{n(a-c_1+\beta(\alpha,\sigma)Y)(c_2(c_1+n(a+\beta(\alpha,\sigma)Y)))}{(n+1)^2(b-c_2)^3}$$
$$-\frac{n(a-c_1+\beta(\alpha,\sigma)Y)b(-c_1(2n+1)+n(a+\beta(\alpha,\sigma)Y))}{(n+1)^2(b-c_2)^3} = 0$$

observing that $(n+1)^2(b-c)^3 > 0,$ equivalently,

$$-n(a-c_1+\beta(\alpha,\sigma)Y)(c_2(c_1+n(a+\beta(\alpha,\sigma)Y))+b(-c_1(2n+1)$$
$$+n(a+\beta(\alpha,\sigma)Y)))=0$$

$$c_2 = b\left(\frac{c_1-(a+\beta(\alpha,\sigma)Y)n+2c_1n}{(a+\beta(\alpha,\sigma)Y)n+c_1}\right)$$

Iterating over c_1 and c_2 in this manner, in every iteration the value of Π_s will be non-decreasing. Observing this it is now shown that $c_1^{t+1} > c_1^t$ and $c_2^{t+1} > c_2^t$.

$$c_1^{t+1} = c_1(c_2(c_1^t)) = \frac{(a+\beta(\alpha,\sigma)Y)(a+\beta(\alpha,\sigma)Y+c_1n)}{(a+\beta(\alpha,\sigma)Y)(2+n)-c_1}$$
$$c_2^{t+1} = c_2(c_1(c_2^t)) = \frac{b(b+c_2(2n-1))}{b-c_2+2bn}$$

It is observed that in every iteration c_1 increases:

$$c_1(c_2(c_1^t)) >^? c_1^t$$
$$\frac{(a+\beta(\alpha,\sigma)Y)(a+\beta(\alpha,\sigma)Y+c_1^tn)}{(a+\beta(\alpha,\sigma)Y)(2+n)-c_1^t} >^? c_1^t$$
$$a^2-2ac_1^t+2a\beta(\alpha,\sigma)Y+(c_1^t)^2-2\beta(\alpha,\sigma)Yc_1^t+\beta(\alpha,\sigma)Y^2 >^? 0$$
$$(a-c_1^t+\beta(\alpha,\sigma)Y)^2 > 0$$

Next, we check $c_2^{t+1} = c_2(c_1(c_2^t)) > c_2^t$.

$$c_2(c_1(c_2^t)) > c_2^t$$
$$\frac{b(b+c_2^t(2n-1))}{b-c_2^t+2bn} >^? c_2^t$$
$$(b^2-2ac_2^t+(c_2^t)^2) >^? 0$$
$$(b-c_2^t)(b-c_2^t) > 0$$

The increasing behavior of c_1 and c_2 implies, either c_1 or c_2, or both, will attain values at their upper bounds. Thus, the boundary conditions are checked as follows. Suppose

c_1 takes value at its upper bound: $c_1 = a + \beta(\alpha, \sigma)Y$. Then,

$$\begin{aligned}
c_2^* =& b(\frac{c_1 - (a + \beta(\alpha, \sigma)Y)n + 2c_1 n}{(a + \beta(\alpha, \sigma)Y)n + c_1}) \\
=& b(\frac{a + \beta(\alpha, \sigma)Y - (a + \beta(\alpha, \sigma)Y)n + 2a + \beta(\alpha, \sigma)Yn}{(a + \beta(\alpha, \sigma)Y)n + a}) \\
=& b(2 - 1)
\end{aligned}$$

Since when $c_1 = a+\beta(\alpha, \sigma)Y$, c_2 must take value outside the boundary, it is concluded that c_2 attains a value at the boundary, whereas c_1 does not. To verify, c_2 is set to its boundary:$c_2 = b - \epsilon$. Then,

$$\begin{aligned}
c_1^* =& \frac{a + \beta(\alpha, \sigma)Y}{2}(\frac{b - c_2 + (b + c_2)n}{(b - c_2 + bn)}) \\
=& \frac{a + \beta(\alpha, \sigma)Y}{2}(\frac{b - (b - \epsilon) + (b + b - \epsilon)n}{(b - (b - \epsilon) + bn)}) \\
=& \frac{a + \beta(\alpha, \sigma)Y}{2}\left(\frac{2bn - (n - 1)\epsilon}{bn + \epsilon}\right)
\end{aligned}$$

It is concluded that $c_2^* = b - \epsilon$ and $c_1^* = \frac{a+\beta(\alpha,\sigma)Y}{2}\left(\frac{2bn-(n-1)\epsilon}{bn+\epsilon}\right)$. Note that $c_1^* < a + \beta(\alpha, \sigma)Y$, since otherwise $q_1 = q_2 = 0$, and the supplier makes 0 profit.

□

Ex-post and ex-ante values are also presented in IBIS-Collaboration.

$$q|Y = \frac{a + \beta(\alpha, \sigma)Y - c_1}{(n + 1)(b - c_2)} = \frac{a - c_1}{(n + 1)(b - c_2)} = \frac{a + \beta(\alpha, \sigma)Y - (\frac{a+\beta(\alpha,\sigma)Y}{2}\left(\frac{2bn-(n-1)\epsilon}{bn+\epsilon}\right))}{b - b + \epsilon}$$

$$= \frac{a + \beta(\alpha, \sigma)Y}{2(bn + \epsilon)}$$

$$P|Y = \frac{(a + \beta(\alpha, \sigma)Y)(bn + 2\epsilon)}{2(bn + \epsilon)}$$

$$\Pi_s|Y = \frac{(a + \beta(\alpha, \sigma)Y)^2 n}{4bn + 4\epsilon}$$

$$\Pi_i|Y = \frac{(a + \beta(\alpha, \sigma)Y)^2 \epsilon}{4(bn + \epsilon)^2}$$

Note that, $\lim_{\epsilon\to 0} q = \frac{a+\beta(\alpha,\sigma)Y}{2bn}$, $\lim_{\epsilon\to 0} P = \frac{a+\beta(\alpha,\sigma)Y}{2}$, $\lim_{\epsilon\to 0} \Pi_s = \frac{(a+\beta(\alpha,\sigma)Y)^2}{4b}$, $\lim_{\epsilon\to 0} \Pi_i = 0$ while $\lim_{\epsilon\to 0} c_1 = a + \beta(\alpha, \sigma)Y$, $\lim_{\epsilon\to 0} c_2 = b$

Ex-ante values are expressed as follows, $E_Y[\Pi_s]$, $E_Y[\Pi_i|Y]$ is analyzed by means of equation (3.38):

$$E_Y[q] = \frac{a}{2(bn+\epsilon)}$$
$$E_Y[P] = \frac{a(bn+2\epsilon)}{2(bn+\epsilon)}$$
$$E_Y[\Pi_s] = \frac{a^2 n}{4bn+4\epsilon} + \frac{(1+\sigma^2\alpha)\beta(\alpha,\sigma)^2}{\alpha(4bn+4\epsilon)}$$
$$E_Y[\Pi_i|Y] = \frac{a^2\epsilon}{4(bn+\epsilon)^2} + \frac{(1+\sigma^2\alpha)\beta(\alpha,\sigma)^2\epsilon}{\alpha(4(bn+\epsilon)^2)}$$

3.3.3 No Collaboration (IBIS-NC)

When there is no collaboration between the buyers, expected profit of buyers in equation (3.17) are obtained as follows:

$$\begin{aligned} E[\Pi_i|Y] =& E[a+\theta-b(\sum_{i=1}^{n} q_i) - c_1 + c_2(q_i))q_i|Y] \qquad (i \in N) \\ =& a + E[\theta|Y] - b(\sum_{i=1}^{n} q_i) - c_1 + c_2(q_i))q_i \\ =& (a+\beta(\alpha,\sigma)Y - b(\sum_{i=1}^{n} q_i) - c_1 + c_2(q_i))q_i \end{aligned}$$

Proposition 11 *Under the imperfect signal $\beta(\alpha,\sigma)Y$, under no collaboration equilibrium quantities are as follows:*
(i)If $b > 2c_2$, then there is only one equilibrium point on $\frac{a+\beta(\alpha,\sigma)Y-c_1}{(n+1)b-2c_2}$ if $a+\beta(\alpha,\sigma)Y > c_1$ and otherwise quantity is equal to zero. $(n+1)b-2c_2$ is always bigger than zero, because of assumption A3.
(ii) If $b < 2c_2$, then there are $2^n - 1$ equilibria. The equilibrium points are characterized as $\frac{a+\beta(\alpha,\sigma)Y-c_1}{(|C|+1)b-2c_2}$ if $a+\beta(\alpha,\sigma)Y > c_1$ and otherwise quantity is equal to zero.

Proof. Profit function of buyer i is given below. To obtain the best response function of q_i, FOC is analyzed first.

$$E[\Pi_i|Y] = (a+\beta(\alpha,\sigma)Y - b\sum_i q_i - (c_1 - c_2 q_i))q_i \qquad (i \in N)$$
$$\frac{\partial \Pi_i}{\partial q_i} = a + \beta(\alpha,\sigma)Y - c_1 + 2c_2 q_i - b(2q_i + \sum_{j\neq i} q_j)$$

$$q_i(q_{-i})|Y = (r_i(q_{-i})|Y)^+$$

The best response function of buyer i is then,

$$r_i(q_{-i})|Y = (\frac{a + \beta(\alpha,\sigma)Y - c_1 - b\sum_{j\neq i} q_j}{2(b-c_2)}) = (\frac{a + \beta(\alpha,\sigma)Y - c_1}{2(b-c_2)} - \frac{b\sum_{j\neq i}}{2(b-c_2)}) \tag{3.39}$$

(i) $b > 2c_2$. Solving the system of equations in (3.39) yields,

$$q_i = \frac{a + \beta(\alpha,\sigma)Y - c_1}{(n+1)b - 2c_2},$$

given that under equilibrium $\sum_{j\neq i} q_j < \frac{a+\beta(\alpha,\sigma)Y - c_1}{b}$. Consider the set of buyers $C \subseteq N$. When the buyers get engaged in a game, the equilibrium quantity for buyer $i \in C$ is expressed as,

$$q_i = \frac{a + \beta(\alpha,\sigma)Y - c_1}{(|C|+1)b - 2c_2}.$$

Note that for any C, under the condition $b > 2c_2$, $\sum_{j\in C} q_j = |C|\frac{a+\beta(\alpha,\sigma)Y-c_1}{(|C|+1)b-2c_2} < \frac{a+\beta(\alpha,\sigma)Y-c_1}{b}$. This implies, when $a + \beta(\alpha,\sigma)Y > c_1$ there cannot exist a group of buyers who will come together, get engaged in a game and procure a quantity that will leave the other buyers with zero procurement quantity at equilibrium. Thus, in equilibrium all buyers procure positive quantities as stated in the proposition.

(ii) $b < 2c_2$. Under this condition, it is possible to show that for any $C \subset N$, except the empty set, the buyers in $|C|$ may procure the equilibrium quantity $\sum_{j\in C} q_j = |C|\frac{a+\beta(\alpha,\sigma)Y-c_1}{(|C|+1)b-2c_2}$ which is greater than $\frac{a+\beta(\alpha,\sigma)Y-c_1}{b}$. This quantity leaves the buyers $j \in N\backslash C$ with zero equilibrium quantity. This implies for every $C \subseteq$ except $\{\}$, there will be an equilibrium.

Equilibrium follows from a similar analysis as in Prop. 5 and from the observation that if $q_i = 0$ and $a + \beta(\alpha,\sigma)Y < c_1$, then $q_j = 0$. Next, optimal c_1 and c_2 is determined for the supplier.

□

Proposition 12 *Under the IBIS no collaboration setting, optimal* c_1 *and* c_2 *values that maximize the supplier's profit function in (3.18) are* $c_1^* = \frac{a+\beta(\alpha,\sigma)Y}{2}\left(\frac{(n+1)b}{(bn+\epsilon)}\right)$ *and* $c_2^* = b - \epsilon$.

Proof. Similar to deterministic scenario, Π_s is not a concave function, but is possibly unimodal. First order conditions yield,

$\partial\Pi_s/\partial c_1 = 0$, $\partial\Pi_s/\partial c_2 = 0$

$$\partial\Pi_s/\partial c_1 = \frac{n(2c_1(b-2c_2+(b+c_2)n)+a(2c_2(n-1)+b(1+n))}{((n+1)b-2c_2)^2}$$
$$+\frac{n(2c_2(n-1)+b(1+n))\beta(\alpha,\sigma)Y}{((n+1)b-2c_2)^2} = 0$$

observing that $((n+1)b-2c_2)^2 > 0$, equivalently,

$$n(2c_1(b-2c_2+(b+c_2)n)+a(2c_2(n-1)+b(1+n))$$
$$+(2c_2(n-1)+b(1+n))\beta(\alpha,\sigma)Y) = 0$$
$$c_1^* = \frac{a+\beta(\alpha,\sigma)Y}{2}\left(\frac{b(n+1)}{b(n+1)-c_2}\right)$$

$$\partial\Pi_s/\partial c_2 =$$
$$-\frac{(a+\beta(\alpha,\sigma)Y-c_1)n((a+\beta(\alpha,\sigma)Y)(b+2c_2+bn)+c_1(2c_2-3b(1+n)))}{((n+1)b-2c_2)^3} = 0$$

observing that $((n+1)b-2c_2)^3 > 0$, equivalently,

$$-(a+\beta(\alpha,\sigma)Y-c_1)n((a+\beta(\alpha,\sigma)Y)(b+2c_2+bn)+c_1(2c_2-3b(1+n))) = 0$$
$$c_2^* = \frac{b(3c_1-a-\beta(\alpha,\sigma)Y)(1+n)}{2(a+\beta(\alpha,\sigma)Y+c_1)}$$

Iterating over c_1 and c_2,in the manner of deterministic collaboration, in every iteration the value of Π_s will be non-decreasing, also.

$$c_1^{t+1} = c_1(c_2(c_1^t)) = \frac{(a+\beta(\alpha,\sigma)Y)(a+\beta(\alpha,\sigma)Y+c_1^t)}{3a-c_1^t}$$
$$c_2^{t+1} = c_2(c_1(c_2^t)) = \frac{(1+n)b((n+1)b+2c_2^t)}{6b(1+n)-4c_2^t}$$

$$c_1(c_2(c_1^t)) >^? c_1^t$$
$$\frac{(a+\beta(\alpha,\sigma)Y)(a+\beta(\alpha,\sigma)Y+c_1^t)}{3a-c_1^t} >^? c_1^t$$
$$(a+\beta(\alpha,\sigma)Y)(a+c_1+\beta(\alpha,\sigma)Y) >^? c_1^t(3a-c_1+3\beta(\alpha,\sigma)Y)$$
$$a^2-2ac_1^t+(c_1^t)^2+2a\beta(\alpha,\sigma)Y-2\beta(\alpha,\sigma)Yc_1^t+(\beta(\alpha,\sigma)Y)^2 >^? 0$$
$$(a-c_1^t+\beta(\alpha,\sigma)Y)^2 > 0$$

It is observed that in every iteration c_2 increases:

$$
\begin{aligned}
&c_2(c_1(c_2^t)) >^? c_2^t \\
&\frac{(1+n)b((n+1)b+2c_2^t)}{6b(1+n)-4c_2^t} >^? c_2^t \\
&(1+n)b((n+1)b+2c_2^t) >^? c_2^t(6b(1+n)-4c_2^t) \\
&((n+1)b-2c_2^t)((n+1)b-2c_2^t) > 0
\end{aligned}
$$

The increasing behavior of c_1 and c_2 implies, either c_1 or c_2, or both, will attain values at their upper bounds. Thus, the boundary conditions are checked as follows. Suppose c_1 takes value at its upper bound:

1. $c_1 = a + \beta(\alpha, \sigma)Y$. Then,

$$
\begin{aligned}
c_2 =& \frac{b(3c_1 - a - \beta(\alpha,\sigma)Y)(1+n)}{2(a+\beta(\alpha,\sigma)Y + c_1)} \\
=& \frac{b(3(a+\beta(\alpha,\sigma)Y) - a - \beta(\alpha,\sigma)Y)(1+n)}{2(a+\beta(\alpha,\sigma)Y + a + \beta(\alpha,\sigma)Y)} \\
=& (1.5)b
\end{aligned}
$$

   Since $c_1 = a+\beta(\alpha,\sigma)Y$, c_2 must take value outside the boundary, it is concluded that c_2 attains a value at the boundary, whereas c_1 does not.

2. To verify, c_2 is set to its boundary: $c_2 = b - \epsilon$. Then,

$$
\begin{aligned}
c_1 =& \frac{a+\beta(\alpha,\sigma)Y}{2}\left(\frac{b(n+1)}{b-c_2+bn}\right) = \frac{a+\beta(\alpha,\sigma)Y}{2}\left(\frac{b(n+1)}{b-(b-\epsilon)+bn}\right) \\
=& \frac{a+\beta(\alpha,\sigma)Y}{2}\left(\frac{(n+1)b}{(bn+\epsilon)}\right)
\end{aligned}
$$

It is concluded that $c_2 = b - \epsilon$ and $c_1^* = \frac{a+\beta(\alpha,\sigma)Y}{2}\left(\frac{(n+1)b}{bn+\epsilon}\right)$. Note that $c_1^* = \frac{a+\beta(\alpha,\sigma)Y}{2}\left(\frac{(n+1)b}{(bn+\epsilon)}\right) < a + \beta(\alpha,\sigma)Y$, given that $0 < \epsilon \leq b$.

□

Under optimal c_1 and c_2, ex-post q^*, P^*, Π_s^* and Π_i^* are obtained as follows:

$$
\begin{aligned}
q|Y &= \frac{a+\beta(\alpha,\sigma)Y - c_1}{((n+1)b-2c_2)} = \frac{a+\beta(\alpha,\sigma)Y}{2(bn+\epsilon)} \\
P|Y &= \frac{(a+\beta(\alpha,\sigma)Y)(bn+2\epsilon)}{2(bn+\epsilon)} \\
\Pi_s|Y &= \frac{(a+\beta(\alpha,\sigma)Y)^2 n}{4bn+4\epsilon} \\
\Pi_i|Y &= \frac{(a+\beta(\alpha,\sigma)Y)^2 \epsilon}{(4(bn+\epsilon)^2}
\end{aligned}
$$

Note that, $\lim_{\epsilon\to 0} q = \frac{a+\beta(\alpha,\sigma)Y}{4b}$, $\lim_{\epsilon\to 0} P = \frac{a+\beta(\alpha,\sigma)Y}{2}$, $\lim_{\epsilon\to 0} \Pi_s = \frac{(a+\beta(\alpha,\sigma)Y)^2}{4b}$, $\lim_{\epsilon\to 0} \Pi_i = 0$ while $\lim_{\epsilon\to 0} c_1 = \frac{3(a+\beta(\alpha,\sigma)Y)}{4}$, $\lim_{\epsilon\to 0} c_2 = b$

Ex-ante values are obtained as follows, $E_Y[\Pi_s]$, $E_Y[\Pi_i|Y]$ is analyzed by means of equation (3.38):

$$E_Y[q] = \frac{a}{2(bn+\epsilon)}$$

$$E_Y[P] = \frac{a(bn+2\epsilon)}{2(bn+\epsilon)}$$

$$E_Y[\Pi_s] = \frac{a^2 n}{4bn+4\epsilon} + \frac{(1+\sigma^2\alpha)\beta(\alpha,\sigma)^2}{\alpha(4bn+4\epsilon)}$$

$$E_Y[\Pi_i|Y] = \frac{a^2\epsilon}{4(bn+\epsilon)^2} + \frac{(1+\sigma^2\alpha)\beta(\alpha,\sigma)^2\epsilon}{\alpha(4(bn+\epsilon)^2)}$$

3.4 Imperfect Information for the Buyers, No Information for the Supplier (IBNS)

In this scenario, the buyers have imperfect information about θ, but they do not share the signal with the supplier.

3.4.1 Monopoly (IBNS-M)

Under imperfect information, the monopoly maximizes the expected profit function in equation (3.19)given signal Y,

$$E[\Pi_m|Y] = (a + E[\theta|Y] - bq - c_1 + c_2 q)q = \quad (a + \beta(\alpha,\sigma)Y - bq - c_1 + c_2 q)q$$

Proposition 13 *Under IBNS monopoly setting the purchasing quantity is* $\frac{a+\beta(\alpha,\sigma)Y-c_1}{2(b-c_2)}$ *if* $a+\beta(\alpha,\sigma)Y > c_1$ *and otherwise quantity is equal to zero.*

Proof. To obtain the equilibrium quantity for one buyer, the function for buyer is obtained.

$$\frac{\partial \Pi_m|Y}{\partial q} = a + \beta(\alpha,\sigma)Y - c_1 - (b-c_2)(2q) = 0$$

FOC implies,

$$q_1|Y = \frac{a+\beta(\alpha,\sigma)Y - c_1}{2(b-c_2)}$$

The equilibrium quantity is expressed as below:

$$q|Y = \begin{cases} \frac{a+\beta(\alpha,\sigma)Y-c_1}{2(b-c_2)} & \text{if } a+\beta(\alpha,\sigma)Y > c_1 \\ 0 & \text{otherwise} \end{cases}$$

Equilibrium follows from a similar analysis as in Prop. 7, and from the observation that if $q_i = 0$ and $a + \beta(\alpha,\sigma)Y < c_1$, then $q_j = 0$.

□

Under no information sharing, the supplier's profit function in equation (3.20) is analyzed as follows.

$$\begin{aligned} E_Y[q_1] =& \int_{\frac{c_1-a}{\beta(\alpha,\sigma)}}^{\infty} \frac{a+\beta(\alpha,\sigma)y-c_1}{2(b-c_2)} f_Y(y)\,\mathrm{d}y + \int_{-\infty}^{\frac{c_1-a}{\beta(\alpha,\sigma)}} 0 f_Y(y)\,\mathrm{d}y \\ =& \frac{a-c_1}{2(b-c_2)} \overline{F}_Y(c_1-a) + \frac{1}{2(b-c_2)} \int_{\frac{c_1-a}{\beta(\alpha,\sigma)}}^{\infty} \beta(\alpha,\sigma) y f_Y(y)\,\mathrm{d}y \end{aligned}$$

Where $f_Y(y)= \int_\theta f_{Y,\theta}(y,\theta)\,\mathrm{d}\theta$, and $f_{Y,\theta}(y,\theta)=f_Y(y|\theta)f_\theta(\theta)$

$$\begin{aligned} E_Y[q_1^2] =& \int_{\frac{c_1-a}{\beta(\alpha,\sigma)}}^{\infty} (\frac{a+\beta(\alpha,\sigma)y-c_1}{2(b-c_2)})^2 f_Y(y)\,\mathrm{d}y \\ =& \frac{1}{4(b-c_2)^2} \int_{\frac{c_1-a}{\beta(\alpha,\sigma)}}^{\infty} ((a-c_1)^2 + 2\beta(\alpha,\sigma)y(a-c_1) + \beta(\alpha,\sigma)y^2) f_Y(y)\,\mathrm{d}y \\ =& \frac{(a-c_1)^2\overline{F}_Y(a-c_1)}{4(b-c_2)^2} + \frac{2(a-c_1)}{4(b-c_2)^2} \int_{\frac{c_1-a}{\beta(\alpha,\sigma)}}^{\infty} \beta(\alpha,\sigma) y f_Y(y)\,\mathrm{d}y \\ &+ \frac{1}{4(b-c_2)^2} \int_{\frac{c_1-a}{\beta(\alpha,\sigma)}}^{\infty} (\beta(\alpha,\sigma)y)^2 f_Y(y)\,\mathrm{d}y \end{aligned}$$

Finding optimal c_1 and c_2 of $E_Y[\Pi_s]$ in closed form is difficult, so $E_Y[\Pi_s^{LB}]$ is used in the analysis, which is defined below.

Proposition 14 *A lower bound on* $E[\Pi_s]$ *is the function* $E[\Pi_s^{LB}]$ *defined as follows:*

$$E[\Pi_s^{LB}] = \frac{2b(a-c_1)c_1\alpha - c_2((a-c_1)(a+c_1)\alpha + \beta(\alpha,\sigma)^2(1+\alpha\sigma^2))}{4\alpha(b-c_2)^2} \tag{3.40}$$

Proof.

$$\begin{aligned} E[\Pi_s^{LB}] =& c_1 E[\frac{a+\beta(\alpha,\sigma)Y-c_1}{2(b-c_2)}] - c_2 E[(\frac{a+\beta(\alpha,\sigma)Y-c_1}{2(b-c_2)})^2] \\ =& c_1(\frac{a-c_1}{2(b-c_2)}) - c_2(\frac{(a-c_1)^2 + \beta(\alpha,\sigma)^2 E[Y^2]}{4(b-c_2)^2} \\ =& c_1(\frac{a-c_1}{2(b-c_2)}) - c_2(\frac{(a-c_1)^2 + \beta(\alpha,\sigma)^2(\frac{1+\alpha\sigma^2}{\alpha})}{4(b-c_2)^2} \end{aligned}$$

Under the assumption that $q_i = \frac{a+\beta(\alpha,\sigma)Y-c_1}{2(b-c_2)}$ under equilibrium (i.e., ignoring $a + \beta(\alpha,\sigma)Y - c_1 < 0$), c_1^* , c_2^* and expected values of supplier profit is determined. Call Π_s obtained ignoring the probability of $q = 0$, Π_s^{LB}. Note that for all values of $\beta(\alpha,\sigma)Y$, $\Pi_s^{LB} \leq \Pi_s$. Thus $E[\Pi_s^{LB}] \leq E[\Pi_s]$.

□

Proposition 15 *Under IBNS monopoly setting, optimal c_1 and c_2 that maximizes the suppliers's profit in (3.40) are changing respect to a and σ.*
(i)If $\frac{a^2}{4} > \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{\alpha}$, then $c_1^* = \frac{a}{2}\left(\frac{2b}{b+\epsilon}\right)$ and $c_2^* = b - \epsilon$.
(ii)If $\frac{a^2}{4} < \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{\alpha}$, c_1^* is equal to $\frac{a}{2}$ and c_2^* is equal to 0.

Proof. To analyze c_1 and c_2, derivative of Π_s^{LB} is analyzed.

$$\partial\Pi_s/\partial c_1 = \frac{ab + c_1(c_2 - 2b)}{2(b - c_2)^2} = 0,$$
$$\text{observing that } 2(b-c_2)^2 > 0, \text{ equivalently,}$$
$$ab + c_1(c_2 - 2b) = 0$$

$$c_1 = \frac{ab}{2b - c_2} \tag{3.41}$$

$$\partial\Pi_s/\partial c_2 = \frac{-b((a-3c_1)(a-c_1)\alpha + \beta(\alpha,\sigma)^2(1+\alpha\sigma^2))}{4\alpha(b-c_2)^3}$$
$$+ \frac{c_2((a-c_1)(a+c_1)\alpha + \beta(\alpha,\sigma)^2(1+\alpha\sigma^2))}{4\alpha(b-c_2)^3} = 0$$
$$\text{observing that } 4\alpha(b-c_2)^3 > 0, \text{ equivalently,}$$
$$-b((a-3c_1)(a-c_1)\alpha + \beta(\alpha,\sigma)^2(1+\alpha\sigma^2))$$
$$+ c_2((a-c_1)(a+c_1)\alpha + \beta(\alpha,\sigma)^2(1+\alpha\sigma^2)) = 0$$

$$c_2 = b(-1 + \frac{4(a-c_1)c_1\alpha}{(a-c_1)(a+c_1)\alpha + \beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}) \tag{3.42}$$

Thus c_i in equations (3.41) and (3.42) that maximize Π_s may change depending on α, σ or $\beta(\alpha,\sigma)$ different from other settings (c_2 can be zero). So firstly limit value that

c_2 changes sign is obtained as:

$$c_2 = b(-1 + \frac{4(a-c_1)c_1\alpha}{(a-c_1)(a+c_1)\alpha + \beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}) < 0$$
$$4(a-c_1)c_1\alpha < (a-c_1)(a+c_1)\alpha + \beta(\alpha,\sigma)^2(1+\alpha\sigma^2)$$

Because of Assumption A2, the minimum value of c_2 is equal to zero, and when $c_2 = 0$, $c_1 = \frac{a}{2}$. Instead of c_1, $\frac{a}{2}$ is put, and below condition is obtained.

$$\frac{a^2}{4} < \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{\alpha}$$

As a result, when the above condition holds, $c_1 = a/2$ and $c_2 = 0$. When the above condition does not hold, it is possible that either c_1 or c_2 approaches the upper boundary closer than the other. To check whether c_1 or c_2 attains the boundary first, the following is checked.

1. Suppose, $c_1 = a$. Then c_2 is obtained as:

$$\begin{aligned} c_2 =& b(1 + \frac{4(a-c_1)c_1\alpha}{(a-c_1)(a+c_1)\alpha + \beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}) \\ =& b \end{aligned}$$

   Note that the upper bound on c_2 is $b - \epsilon < b$. This implies, c_2 reaches the boundary before c_1. This is verified in the following.

2. Suppose, $c_2 = b - \epsilon$. Then c_1 is obtained as:

$$\begin{aligned} c_1^* =& \frac{a}{2}(\frac{b - c_2 + (b + c_2)}{2(2b - c_2)}) \\ =& \frac{a}{2}(\frac{b - (b-\epsilon) + (b + b - \epsilon)}{2(2b - b + \epsilon)}) \\ =& \frac{a}{2}\left(\frac{2b}{b+\epsilon}\right) \end{aligned}$$

   Note $\frac{a}{2}\left(\frac{2b}{b+\epsilon}\right) < a$.

Thus the values that maximize Π_s under second condition ($\frac{a^2}{4} > \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{\alpha}$) are $c_2^* = b - \epsilon$ and $c_1^* = \frac{a}{2}\frac{2b}{b+\epsilon}$.

□

In the comparison of the scenarios, to avoid the complexity, $E[\Pi_s^{LB}]$ is used in place of $E[\Pi_s]$. Accordingly, when comparing the ex-post and ex-ante q_i, P, and Π_i, q_i is assumed to take negative values and c_1 and c_2 values that maximize $E[\Pi_s^{LB}]$ are used in the analysis.

$$E[\Pi_s^{LB}] = \begin{cases} \frac{\epsilon^2(a^2\alpha+\beta(\alpha,\sigma)^2(1+\alpha\sigma^2))-b^2(\beta(\alpha,\sigma)^2(1+\alpha\sigma^2))}{4\epsilon^2(b+\epsilon)\alpha} & \text{if } \frac{a^2}{4} > \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{\alpha} \\ \frac{a^2}{8b} & \text{if } \frac{a^2}{4} < \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{\alpha} \end{cases}$$

Firstly, the condition of $\frac{a^2}{4} > \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{\alpha}$ is analyzed. Under optimal c_1 and c_2, ex-post q^*, P^*, Π_s^* and Π_m^* can be obtained.

$$\begin{aligned} q^* &= \frac{a+\beta(\alpha,\sigma)Y-c_1}{2(b-c_2)} = \frac{a}{2(b+\epsilon)} + \frac{\beta(\alpha,\sigma)Y}{2\epsilon} \\ P^* &= \frac{a(b+2\epsilon)}{2(b+\epsilon)} + \frac{\beta(\alpha,\sigma)Y(2\epsilon-b)}{2\epsilon} \\ \Pi_s^* &= \frac{\epsilon^2(a^2\alpha+\beta(\alpha,\sigma)^2(1+\alpha\sigma^2))-b^2(\beta(\alpha,\sigma)^2(1+\alpha\sigma^2))}{4\epsilon^2(b+\epsilon)\alpha} \\ \Pi_m^* &= \frac{(a\epsilon)^2-((b+\epsilon)\beta(\alpha,\sigma)Y)^2}{4\epsilon(b+\epsilon)^2} \end{aligned}$$

To obtain ex-ante values under the first condition expected values of ex-post functions:

$$\begin{aligned} q^* &= \frac{a}{2(b+\epsilon)} \\ P^* &= \frac{a(b+2\epsilon)}{2(b+\epsilon)} \\ \Pi_m^* &= \frac{a^2\epsilon}{4(b+\epsilon)^2} - \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{4\alpha\epsilon} \end{aligned}$$

Secondly, the condition of $\frac{a^2}{4} < \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{\alpha}$ is analyzed. Under optimal $c_1 = \frac{a}{2}$ and $c_2 = 0$, ex-post q^*, P^*, Π_s^* and Π_m^* can be obtained.

$$\begin{aligned} q^* &= \frac{a+\beta(\alpha,\sigma)Y-c_1}{2(b-c_2)} = \frac{a+2\beta(\alpha,\sigma)Y}{4b} \\ P^* &= \frac{3a-2\beta(\alpha,\sigma)Y}{4} \\ \Pi_s^* &= \frac{a^2}{8b} \\ \Pi_m^* &= \frac{(a+2\beta(\alpha,\sigma)Y)(a-2\beta(\alpha,\sigma)Y)}{16b} \end{aligned}$$

To obtain ex-ante values, expected values of ex-post functions:

$$\begin{aligned} q^* &= \frac{a}{4b} \\ P^* &= \frac{3a}{4} \\ \Pi_m^* &= \frac{a^2}{16b} - \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{4b\alpha} \end{aligned}$$

3.4.2 Collaboration (IBNS-C)

Under imperfect information, buyer i maximizes expected profit function in equation (3.21) given signal ,

$$
\begin{aligned}
E[\Pi_i|Y] =& a + E[\theta|Y] - b(\sum_{i=1}^{n} q_i) - c_1 + c_2(\sum_{i=1}^{n} q_i))q_i \\
=& (a + \beta(\alpha,\sigma)Y - b(\sum_{i=1}^{n} q_i) - c_1 + c_2(\sum_{i=1}^{n} q_i))q_i
\end{aligned}
$$

Proposition 16 *Under IBNS collaboration setting, the purchasing quantity is* $\frac{a+\beta(\alpha,\sigma)Y-c_1}{(n+1)(b-c_2)}$ *if* $a + \beta(\alpha,\sigma)Y > c_1$ *and otherwise quantity is equal to zero.*

Proof. To obtain the equilibrium quantity for buyer i, best response function for buyer i is obtained.

$$\frac{\partial \Pi_1|Y}{\partial q_1} = a + \beta(\alpha,\sigma)Y - c_1 - (b - c_2)(2q_i + \sum_{j=1}^{n} q_j) = 0$$

FOC implies,

$$q_i(q_{-i})|Y = (r_i(q_{-i})|Y)^+$$

$$r_i(q_{-i})|Y = \frac{a + \beta(\alpha,\sigma)Y - c_1}{2(b - c_2)} - \frac{(b - c_2)(\sum_{j=1}^{n} q_j))}{2(b - c_2)}$$

$$= \frac{a + \beta(\alpha,\sigma)Y - c_1}{2(b - c_2)} - \frac{\sum_{j=1}^{n} q_j}{2} \quad \forall i \in N$$

The equilibrium quantity is expressed as below:

$$q_i|Y = \begin{cases} \frac{a+\beta(\alpha,\sigma)Y-c_1}{(n+1)(b-c_2)} & \text{if } a + \beta(\alpha,\sigma)Y > c_1 \\ 0 & \text{otherwise} \end{cases}$$

Equilibrium follows a similar analysis as in Prop. 9 and from the observation that if $q_i = 0$ and $a + \beta(\alpha,\sigma)Y < c_1$, then $q_j = 0$. □

Under no information sharing, the supplier's profit function in equation (3.22)is expressed as follows.

$$
\begin{aligned}
E_Y[q_1] =& \int_{\frac{c_1-a}{\beta(\alpha,\sigma)}}^{\infty} \frac{a + \beta(\alpha,\sigma)y - c_1}{(n+1)(b - c_2)} f_Y(y)\,\mathrm{d}y + \int_{-\infty}^{\frac{c_1-a}{\beta(\alpha,\sigma)}} 0 f_Y(y)\,\mathrm{d}y \\
=& \frac{a - c_1}{(n+1)(b - c_2)} \overline{F}_Y(c_1 - a) + \frac{1}{(n+1)(b - c_2)} \int_{\frac{c_1-a}{\beta(\alpha,\sigma)}}^{\infty} \beta(\alpha,\sigma) y f_Y(y)\,\mathrm{d}y
\end{aligned}
$$

Where $f_Y(y) = \int_\theta f_{Y,\theta}(y,\theta)\,\mathrm{d}\theta$, and $f_{Y,\theta}(y,\theta) = f_Y(y|\theta) f_\theta(\theta)$

$$\begin{aligned}
E_Y[q_1^2] &= \int_{\frac{c_1-a}{\beta(\alpha,\sigma)}}^{\infty} (\frac{a+\beta(\alpha,\sigma)y-c_1}{(n+1)(b-c_2)})^2 f_Y(y)\,\mathrm{d}y \\
&= \frac{1}{(n+1)^2(b-c_2)^2}\int_{\frac{c_1-a}{\beta(\alpha,\sigma)}}^{\infty} ((a-c_1)^2 + 2\beta(\alpha,\sigma)y(a-c_1) + \beta(\alpha,\sigma)y^2) f_Y(y)\,\mathrm{d}y \\
&= \frac{(a-c_1)^2 \overline{F}_Y(a-c_1)}{(n+1)^2(b-c_2)^2} + \frac{2(a-c_1)}{(n+1)^2(b-c_2)^2}\int_{\frac{c_1-a}{\beta(\alpha,\sigma)}}^{\infty} \beta(\alpha,\sigma) y f_Y(y)\,\mathrm{d}y \\
&\quad + \frac{1}{(n+1)^2(b-c_2)^2}\int_{\frac{c_1-a}{\beta(\alpha,\sigma)}}^{\infty} (\beta(\alpha,\sigma)y)^2 f_Y(y)\,\mathrm{d}y
\end{aligned}$$

Finding optimal c_1 and c_2 of $E_Y[\Pi_s]$ in closed form is difficult, so $E_Y[\Pi_s^{LB}]$ is used in the analysis.

Proposition 17 *A lower bound on $E[\Pi_s]$ is the function $E[\Pi_s^{LB}]$ defined as follows:*

$$= -\frac{n(b(-a+c_1)c_1(1+n)\alpha + c_2((a-c_1)c_1\alpha + n(a(a-c_1)\alpha + \beta(\alpha,\sigma)^2(1+\alpha\sigma^2))))}{(n+1)^2\alpha(b-c_2)^2} \tag{3.43}$$

Proof.

$$\begin{aligned}
E[\Pi_s^{LB}] &= nc_1 E[\frac{a+\beta(\alpha,\sigma)Y-c_1}{(n+1)(b-c_2)}] - c_2 E[(\frac{n(a+\beta(\alpha,\sigma)Y-c_1)}{(n+1)(b-c_2)})^2] \\
&= nc_1(\frac{a-c_1}{(n+1)(b-c_2)}) - c_2 n^2(\frac{(a-c_1)^2 + \beta(\alpha,\sigma)^2 E[Y^2]}{(n+1)^2(b-c_2)^2} \\
&= nc_1(\frac{a-c_1}{(n+1)(b-c_2)}) - c_2(\frac{n^2(a-c_1)^2 + n^2\beta(\alpha,\sigma)^2(\frac{1+\alpha\sigma^2}{\alpha})}{(n+1)^2(b-c_2)^2}
\end{aligned}$$

Under the assumption that $q_i = \frac{a+\beta(\alpha,\sigma)Y-c_1}{(n+1)(b-c_2)}$ under equilibrium (i.e., ignoring that for $a+\beta(\alpha,\sigma)Y-c_1 < 0$), c_1^* , c_2^* and expected values of supplier profit is determined. Call Π_s obtained ignoring the probability of $q_i = 0$, Π_s^{LB}. Note that for all values of $\beta(\alpha,\sigma)Y$, $\Pi_s^{LB} \leq \Pi_s$. Thus $E[\Pi_s^{LB}] \leq E[\Pi_s]$.

□

Proposition 18 *Under IBNS collaboration setting, optimal c_1 and c_2 that maximizes the suppliers's profit in (3.43) are changing respect to a and $\frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{\alpha}$.*
(i) If $\frac{a^2}{4n} > \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{\alpha}$, $c_1^* = \frac{a}{2}\left(\frac{2bn-(n-1)\epsilon}{bn+\epsilon}\right)$ and $c_2^* = b-\epsilon$.
(ii)If $\frac{a^2}{4n} < \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{\alpha}$ holds, c_1^* is equal to $\frac{a}{2}$ and c_2^* is equal to 0.

Proof. To analyze c_1 and c_2, derivative of Π_s^{LB} is analyzed.

$$\partial\Pi_s/\partial c_1 = \frac{n(-2c_1(b+bn-c_2)+a(b-c_2+bn-c_2n))}{(b-c_2)^2(1+n)^2} = 0,$$

observing that $(b-c_2)^2(1+n)^2 > 0,$ equivalently,

$$n(-2c_1(b+bn-c_2)+a(b-c_2+bn-c_2n)) = 0$$

$$c_1 = \frac{a}{2}\left(\frac{b-c_2+(b+c_2)n}{(b-c_2+bn)}\right) \tag{3.44}$$

$$\partial\Pi_s/\partial c_2 = \frac{n(-(b+c_2)n\beta(\alpha,\sigma)^2-(a-c_1)(c_2(c_1+an)-b(c_1-an+2c_1n))\alpha)}{(1+n)^2(b-c_2)^3\alpha}$$
$$+\frac{-n(b+c_2)n\alpha\sigma^2\beta(\alpha,\sigma)^2}{(1+n)^2(b-c_2)^3\alpha} = 0$$

observing that $(1+n)^2(b-c_2)^3\alpha > 0,$ equivalently,

$$n(-(b+c_2)n\beta(\alpha,\sigma)^2-(a-c_1)(c_2(c_1+an)-b(c_1-an+2c_1n))\alpha)$$
$$-n(b+c_2)n\alpha\sigma^2\beta(\alpha,\sigma)^2) = 0$$

$$c_2 = -\frac{b(c_1(-a+c_1)\alpha+n((a-2c_1)(a-c_1)\alpha+\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)))}{(a-c_1)c_1\alpha+n(a(a-c_1)\alpha+\beta(\alpha,\sigma)^2(1+\alpha\sigma^2))} \tag{3.45}$$

Thus c_i in equation (3.44) and (3.44) that maximize Π_s may change depending on α, σ or $\beta(\alpha,\sigma)$ different from other settings (c_2 can be zero). So firstly limit value that c_2 changes sign is obtained as:

$$c_2 = -\frac{b(c_1(-a+c_1)\alpha+n((a-2c_1)(a-c_1)\alpha+\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)))}{(a-c_1)c_1\alpha+n(a(a-c_1)\alpha+\beta(\alpha,\sigma)^2(1+\alpha\sigma^2))} < 0$$
$$\frac{a^2}{4n} < \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{\alpha}$$

As a result, when the above condition holds, $c_1 = a/2$ and $c_2 = 0$.When the above condition does not hold, it is possible that either c_1 or c_2 approaches the boundary closer than the other. To check whether c_1 or c_2 attains the boundary first, the following is checked.

1. Suppose, $c_1 = a$. Then c_2 is obtained as:

$$c_2 = -\frac{b(c_1(-a+c_1)\alpha+n((a-2c_1)(a-c_1)\alpha+\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)))}{(a-c_1)c_1\alpha+n(a(a-c_1)\alpha+\beta(\alpha,\sigma)^2(1+\alpha\sigma^2))}$$
$$=b\frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}$$
$$=b$$

Note that the upper bound on c_2 is $b - \epsilon < b$. This implies, c_2 reaches the boundary before c_1. This is verified in the following.

2. Suppose, $c_2 = b - \epsilon$. Then c_1 is obtained as:

$$\begin{aligned} c_1^* &= \frac{a}{2}\left(\frac{b - c_2 + (b + c_2)n}{2(b - c_2 + bn)}\right) \\ &= \frac{a}{2}\left(\frac{b - (b - \epsilon) + (b + b - \epsilon)n}{2(b - b + \epsilon + bn)}\right) \\ &= \frac{a}{2}\left(\frac{2bn - (n-1)\epsilon}{bn + \epsilon}\right) \end{aligned}$$

Note $\frac{a}{2}\frac{2bn-(n-1)\epsilon}{bn+\epsilon} < a$.

If $\frac{a^2}{4n} > \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{\alpha}$ holds, the values that maximize Π_s are $c_2^* = b - \epsilon$ and $c_1^* = \frac{a}{2}\frac{2bn-(n-1)\epsilon}{bn+\epsilon}$.

□

$$E[\Pi_s^{LB}] = \begin{cases} \frac{n(-4b^2n^2\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)+4b(n-1)n\epsilon\beta(\alpha,\sigma)^2(1+\alpha\sigma^2))}{4(1+n)^2\epsilon^2(bn+\epsilon)\alpha} & \\ +\frac{n\epsilon^2(a^2\alpha+n(a^2(2+n)\alpha+4\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)))}{4(1+n)^2\epsilon^2(bn+\epsilon)\alpha} & \text{if } \frac{a^2}{4n} > \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{\alpha} \\ \frac{a^2n}{4b(n+1)} & \text{if } \frac{a^2}{4n} < \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{\alpha} \end{cases}$$

Firstly, the condition of $\frac{a^2}{4n} > \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{\alpha}$ is analyzed. Under optimal c_1 and c_2, ex-post q^*, P^*, Π_s^* and Π_i^* can be obtained.

$$\begin{aligned} q^* &= \frac{a + \beta(\alpha,\sigma)Y - c_1}{(n+1)(b - c_2)} = \frac{a}{2(bn+\epsilon)} + \frac{\beta(\alpha,\sigma)Y}{(n+1)\epsilon} \\ P^* &= \frac{a(bn+\epsilon)}{2(bn+\epsilon)} + \frac{\beta(\alpha,\sigma)Y((n+1)\epsilon - nb)}{(n+1)\epsilon} \\ \Pi_s^* &= \frac{n(-4b^2n^2\beta(\alpha,\sigma)^2(1+\alpha\sigma^2) + 4b(n-1)n\epsilon\beta(\alpha,\sigma)^2(1+\alpha\sigma^2))}{4(1+n)^2\epsilon^2(bn+\epsilon)\alpha} \\ &+ \frac{n\epsilon^2(a^2\alpha + n(a^2(2+n)\alpha + 4\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)))}{4(1+n)^2\epsilon^2(bn+\epsilon)\alpha} \\ \Pi_i^* &= \frac{(2bn\beta(\alpha,\sigma)Y + (a + an + 2\beta(\alpha,\sigma)Y)\epsilon)(a\epsilon + n(a\epsilon - 2\beta(\alpha,\sigma)Y(bn+\epsilon)))}{4(1+n)^2\epsilon(bn+\epsilon)^2} \end{aligned}$$

To obtain ex-ante values under the first condition, expected values of ex-post functions:

$$q^* = \frac{a}{2(bn+\epsilon)}$$
$$P^* = \frac{a(bn+2\epsilon)}{2(bn+\epsilon)}$$
$$\Pi_i^* = \frac{a^2\epsilon}{4(bn+\epsilon)^2} - \frac{n\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{(1+n)^2\epsilon\alpha}$$

Secondly, the condition of $\frac{a^2}{4n} < \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{\alpha}$ is analyzed. Under optimal $c_1 = \frac{a}{2}$ and $c_2 = 0$, ex-post q^*, P^*, Π_s^* and Π_i^* can be obtained.

$$q^* = \frac{a+\beta(\alpha,\sigma)Y - c_1}{(n+1)(b-c_2)} = \frac{a+2\beta(\alpha,\sigma)Y}{2(n+1)b}$$
$$P^* = \frac{2a+an+2\beta(\alpha,\sigma)Y}{2+2n}$$
$$\Pi_s^* = \frac{a^2 n}{4b(1+n)}$$
$$\Pi_i^* = \frac{(a+2\beta(\alpha,\sigma)Y)(a-2n\beta(\alpha,\sigma)Y)}{4b(1+n)^2}$$

To obtain ex-ante values, expected values of ex-post functions:

$$q^* = \frac{a}{2(n+1)b}$$
$$P^* = \frac{2a+an}{2+2n}$$
$$\Pi_i^* = \frac{a^2}{4b(1+n)^2} - \frac{n\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{b\alpha(1+n)^2}$$

3.4.3 No Collaboration (IBNS-NC)

In this part, there is no collaboration between buyers. No collaboration brings on the changes between the function of Π_s and Π_i in the collaboration model. By using profit function of buyers in equation (3.23); the below analysis is done.

$$\begin{aligned} E[\Pi_i|Y] =& a + E[\theta|Y] - b(\sum_{i=1}^{n} q_i) - c_1 + c_2(q_i))q_i \\ =& (a + \beta(\alpha,\sigma)Y - b(\sum_{i=1}^{n} q_i) - c_1 + c_2(q_i))q_i \end{aligned}$$

Proposition 19 *Under the imperfect signal, under no collaboration equilibrium quantities are as follows:*

(i)If $b > 2c_2$ *, then there is only one equilibrium point on* $\frac{a+\beta(\alpha,\sigma)Y-c_1}{(n+1)b-2c_2}$

(ii) If $b < 2c_2$, then there are $2^n - 1$ equilibria. The equilibrium points are characterized as $\frac{a+\beta(\alpha,\sigma)Y-c_1}{(|C|+1)b-2c_2}$.

Proof. Profit function of buyer i is given below. To obtain the best response function of q_i, FOC is analyzed first.

$$\frac{\partial \Pi_i}{\partial q_i} = a + \beta(\alpha,\sigma)Y - c_1 + 2c_2 q_i - b(2q_i + \sum_{j\neq i} q_j)$$

The best response function of buyer i is then, $q_i(q_{-i})|Y = (r_i(q_{-i})|Y)^+$

$$r_i(q_{-i})|Y = \frac{a + \beta(\alpha,\sigma)Y - c_1 - b\sum_{j\neq i} q_j}{2(b-c_2)}) = (\frac{a + \beta(\alpha,\sigma)Y - c_1}{2(b-c_2)} - \frac{b\sum_{j\neq i}}{2(b-c_2)} \quad (3.46)$$

.

(i) $b > 2c_2$. Solving the system of equations in (3.46) yields,

$$q_i = \frac{a + \beta(\alpha,\sigma)Y - c_1}{(n+1)b - 2c_2},$$

given that under equilibrium $\sum_{j\neq i} q_j < \frac{a+\beta(\alpha,\sigma)Y-c_1}{b}$. Consider the set of buyers $C \subseteq N$. When the buyers get engaged in a game, the equilibrium quantity for buyer $i \in C$ is expressed as,

$$q_i = \frac{a + \beta(\alpha,\sigma)Y - c_1}{(|C|+1)b - 2c_2}.$$

Note that for any C, under the condition $b > 2c_2$, $\sum_{j\in C} q_j = |C|\frac{a+\beta(\alpha,\sigma)Y-c_1}{(|C|+1)b-2c_2} < \frac{a+\beta(\alpha,\sigma)Y-c_1}{b}$. This implies there cannot exist a group of buyers who will come together, get engaged in a game and procure a quantity that will leave the other buyers with zero procurement quantity at equilibrium. Thus, in equilibrium all buyers procure positive quantities as stated in the proposition.

(ii) $b < 2c_2$. Under this condition, it is possible to show that for any $C \subset N$, except the empty set, the buyers in $|C|$ may procure the equilibrium quantity $\sum_{j\in C} q_j = |C|\frac{a+\beta(\alpha,\sigma)Y-c_1}{(|C|+1)b-2c_2}$ which is greater than $\frac{a+\beta(\alpha,\sigma)Y-c_1}{b}$. This quantity leaves the buyers $j \in N\backslash C$ with zero equilibrium quantity. This implies for every $C \subseteq$ except $\{\}$, there will be an equilibrium.

At equilibrium point , buyers have positive procurement quantities.

$$q_i|Y = \begin{cases} \frac{a+\beta(\alpha,\sigma)Y-c_1}{(n+1)b-2c_2} & \text{if } a + \beta(\alpha,\sigma)Y > c_1 \\ 0 & \text{otherwise} \end{cases}$$

Note that if $q_i = 0$, and $a + \beta(\alpha,\sigma)Y < c_1$ then $q_j = 0$. It is assumed that under $b < 2c_2$ and $b > 2c_2$ the equilibrium quantity for buyer i is $q_i = \frac{a+\beta(\alpha,\sigma)Y-c_1}{(n+1)b-2c_2}$. Next, optimal c_1 and c_2 is determined for the supplier.

Equilibrium follows from the observation that if $q_i = 0$ and $a + \beta(\alpha,\sigma)Y < c_1$, then $q_j = 0$. Under no information sharing, the supplier's profit function in equation (3.24) is expressed as follows.

$$
\begin{aligned}
E_Y[q_1] =& \int_{\frac{c_1-a}{\beta(\alpha,\sigma)}}^{\infty} \frac{a+\beta(\alpha,\sigma)Y-c_1}{(n+1)b-2c_2} f_Y(y)\,\mathrm{d}y + \int_{-\infty}^{\frac{c_1-a}{\beta(\alpha,\sigma)}} 0 f_Y(y)\,\mathrm{d}y \\
=& \frac{a-c_1}{(n+1)b-2c_2}\overline{F}_Y(c_1-a) + \frac{1}{(n+1)b-2c_2}\int_{\frac{c_1-a}{\beta(\alpha,\sigma)}}^{\infty} \beta(\alpha,\sigma) y f_Y(y)\,\mathrm{d}y
\end{aligned}
$$

Where $f_Y(y)$= $\int_\theta f_{Y,\theta}(y,\theta)\,\mathrm{d}\theta$, and $f_{Y,\theta}(y,\theta)$=$f_Y(y|\theta)f_\theta(\theta)$

$$
\begin{aligned}
E_Y[q_1^2] =& \int_{\frac{c_1-a}{\beta(\alpha,\sigma)}}^{\infty} (\frac{a+\beta(\alpha,\sigma)y-c_1}{((n+1)b-2c_2)})^2 f_Y(y)\,\mathrm{d}y \\
=& \frac{1}{((n+1)b-2c_2)^2}\int_{\frac{c_1-a}{\beta(\alpha,\sigma)}}^{\infty} ((a-c_1)^2 + 2\beta(\alpha,\sigma)y(a-c_1) + \beta(\alpha,\sigma)y^2) f_Y(y)\,\mathrm{d}y \\
=& \frac{(a-c_1)^2\overline{F}_Y(a-c_1)}{((n+1)b-2c_2)^2} + \frac{2(a-c_1)}{((n+1)b-2c_2)^2}\int_{\frac{c_1-a}{\beta(\alpha,\sigma)}}^{\infty} \beta(\alpha,\sigma) y f_Y(y)\,\mathrm{d}y \\
&+ \frac{1}{((n+1)b-2c_2)^2}\int_{\frac{c_1-a}{\beta(\alpha,\sigma)}}^{\infty} (\beta(\alpha,\sigma)y)^2 f_Y(y)\,\mathrm{d}y
\end{aligned}
$$

Finding optimal c_1 and c_2 of $E_Y[\Pi_s]$ in closed form is difficult, so $E_Y[\Pi_s^{LB}]$ is used in the analysis.

Proposition 20 *A lower bound on $E[\Pi_s]$ is the function $E[\Pi_s^{LB}]$ defined as follows:*

$$
E[\Pi_s^{LB}] = \frac{n(b(a-c_1)c_1(1+n)\alpha - c_2((a-c_1)(a+c_1)\alpha + \beta(\alpha,\sigma)^2(1+\alpha\sigma^2)))}{((n+1)b-c_2)^2\alpha} \tag{3.47}
$$

Proof.

$$
\begin{aligned}
E[\Pi_s^{LB}] =& nc_1 E[\frac{a+\beta(\alpha,\sigma)Y-c_1}{(n+1)b-2c_2}] - c_2 E[(\frac{n(a+\beta(\alpha,\sigma)Y-c_1)}{(n+1)b-2c_2})^2] \\
=& nc_1(\frac{a-c_1}{(n+1)b-2c_2}) - c_2 n(\frac{(a-c_1)^2 + \beta(\alpha,\sigma)^2 E[Y^2]}{((n+1)b-2c_2)^2} \\
=& nc_1(\frac{a-c_1}{(n+1)(b-c_2)}) - c_2(\frac{n(a-c_1)^2 + n\beta(\alpha,\sigma)^2(\frac{1+\alpha\sigma^2}{\alpha})}{((n+1)b-2c_2)^2}
\end{aligned}
$$

Under the assumption that $q_i = \frac{a+\beta(\alpha,\sigma)Y-c_1}{(n+1)(b-c_2)}$under equilibrium (i.e., ignoring that for $a + \beta(\alpha,\sigma)Y - c_1 < 0$), c_1^* , c_2^* and expected values of supplier profit is determined.

Call Π_s obtained ignoring the probability of $q_i = 0$, Π_s^{LB}. Note that for all values of $\beta(\alpha,\sigma)Y$, $\Pi_s^{LB} \leq \Pi_s$. Thus $E[\Pi_s^{LB}] \leq E[\Pi_s]$.

□

Proposition 21 *Under IBNS no collaboration setting, optimal c_1 and c_2 that maximizes the suppliers's profit in (3.47) are obtained as follows. (i) If $\frac{a^2}{4} > \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{\alpha}$, $c_1^* = \frac{a}{2}\left(\frac{(n+1)b}{(bn+\epsilon)}\right)$ and $c_2^* = b - \epsilon$.*
(ii)If $\frac{a^2}{4} < \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{\alpha}$ holds, c_1^* is equal to $\frac{a}{2}$ and c_2^* is equal to 0.

Proof. Derivatives of Π_s with respect to c_1 and c_2 are analyzed.

$$\partial\Pi_s/\partial c_1 = \frac{n(ab(1+n) - 2c_1(b - c_2 + bn))}{((n+1)b - 2c_2)^2} = 0$$

observing that $((n+1)b - 2c_2)^2 > 0,$ equivalently,

$$n(ab(1+n) - 2c_1(b - c_2 + bn)) = 0$$

$$c_1 = \frac{ab(n+1)}{2(b - c_2 + bn)} = \frac{a}{2}\frac{b(n+1)}{b(n+1) - c_2} \tag{3.48}$$

$$\partial\Pi_s/\partial c_2 = \frac{n(-b(1+n)((a-3c_1)(a-c_1)\alpha + \beta(\alpha,\sigma)^2(1+\alpha\sigma^2)))}{((n+1)b - 2c_2)^3\alpha}$$
$$- \frac{n2c_2((a-c_1)(a+c_1)\alpha + \beta(\alpha,\sigma)^2(1+\alpha\sigma^2))}{((n+1)b - 2c_2)^3\alpha} = 0$$

observing that $((n+1)b - 2c_2)^3\alpha > 0,$ equivalently,

$$n(-b(1+n)((a-3c_1)(a-c_1)\alpha + \beta(\alpha,\sigma)^2(1+\alpha\sigma^2))]$$
$$- n2c_2((a-c_1)(a+c_1)\alpha + \beta(\alpha,\sigma)^2(1+\alpha\sigma^2)) = 0$$

$$c_2 = -\frac{b(1+n)((a-3c_1)(a-c_1)\alpha + \beta(\alpha,\sigma)^2(1+\alpha\sigma^2))}{2((a-c_1)(a+c_1)\alpha + \beta(\alpha,\sigma)^2(1+\alpha\sigma^2))} \tag{3.49}$$

Thus c_i in equations (3.48) and (3.49) that maximize Π_s may change depending on α, σ or $\beta(\alpha,\sigma)$ different from other settings (c_2 can be zero). So firstly limit value that c_2 changes sign is obtained as:

$$c_2 = -\frac{b(1+n)((a-3c_1)(a-c_1)\alpha + \beta(\alpha,\sigma)^2(1+\alpha\sigma^2))}{2((a-c_1)(a+c_1)\alpha + \beta(\alpha,\sigma)^2(1+\alpha\sigma^2))} < 0$$
$$\frac{a^2}{4} < \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{\alpha}$$

As a result, when the above condition holds, $c_1 = a/2$ and $c_2 = 0$. Note that it is possible that either c_1 or c_2 approaches the boundary closer than the other. To check which c_i attains the boundary first, the following is checked.

1. Suppose c_1 takes value at its upper bound: $c_1 = a$. Then,

$$
\begin{aligned}
c_2 =& \frac{b(3c_1 - a)(1+n)}{2(a+c_1)} \\
=& \frac{b(3a - a)(1+n)}{2(a+a)} \\
=& (1.5)b
\end{aligned}
$$

   Since $c_1 = a$, c_2 must take value outside the boundary, it is concluded that c_2 attains a value at the boundary, whereas c_1 does not.

2. c_2 is set to its boundary: $c_2 = b - \epsilon$. Then,

$$
\begin{aligned}
c_1 =& \frac{a}{2}\left(\frac{b(n+1)}{b - c_2 + bn}\right) = \frac{a}{2}\left(\frac{b(n+1)}{b-(b-\epsilon)+bn}\right) \\
=& \frac{a}{2}\left(\frac{(n+1)b}{(bn+\epsilon)}\right)
\end{aligned}
$$

If $\frac{a^2}{4} > \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{\alpha}$ holds, the values that maximize Π_s are $c_2^* = b - \epsilon$ and $c_1^* = \frac{a}{2}\frac{(n+1)b}{bn+\epsilon}$.

□

$$
E[\Pi_s^{LB}] = \begin{cases}
\frac{n(4b(n-1)\epsilon(a^2\alpha+\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)))}{4(bn+\epsilon)(b(n-1)+2\epsilon)^2\alpha} \\
+\frac{n4\epsilon^2(a^2\alpha+\beta(\alpha,\sigma)^2(1+\alpha\sigma^2))}{4(bn+\epsilon)(b(n-1)+2\epsilon)^2\alpha} \\
+\frac{nb^2(a^2\alpha+n(a^2(n-2)\alpha-4\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)))}{4(bn+\epsilon)(b(n-1)+2\epsilon)^2\alpha} & \text{if } \frac{a^2}{4} > \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{\alpha} \\
\frac{a^2 n}{4b(1+n)} & \text{if } \frac{a^2}{4} < \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{\alpha}
\end{cases}
$$

$$
\begin{aligned}
q^* &= \frac{a + \beta(\alpha,\sigma)Y - c_1}{(n+1)b - 2c_2} = \frac{a}{2(bn+\epsilon)} + \frac{\beta(\alpha,\sigma)Y}{b(n-1)+2\epsilon} \\
P^* &= \frac{a(bn+2\epsilon)}{2(bn+\epsilon)} + \frac{\beta(\alpha,\sigma)Y(b-2\epsilon)}{b-bn-2\epsilon} \\
\Pi_s^* &= \frac{n(4b(n-1)\epsilon(a^2\alpha+\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)) + 4\epsilon^2(a^2\alpha+\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)))}{4(bn+\epsilon)(b(n-1)+2\epsilon)^2\alpha} \\
&+ \frac{nb^2(a^2\alpha+n(a^2(n-2)\alpha-4\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)))}{4(bn+\epsilon)(b(n-1)+2\epsilon)^2\alpha} \\
\Pi_i^* &= \frac{\epsilon(ab(n-1)+2a\epsilon+2(bn+\epsilon)\beta(\alpha,\sigma)Y)^2}{4(bn+\epsilon)^2(b(n-1)+2\epsilon)^2}
\end{aligned}
$$

To obtain ex-ante values, expected values of ex-post functions:

$$q^* = \frac{a}{2(bn+\epsilon)}$$
$$P^* = \frac{a(bn+2\epsilon)}{2(bn+\epsilon)}$$

$$\Pi_i^* = \frac{\epsilon(ab(n-1)+2a\epsilon)}{4(bn+\epsilon)^2(b(n-1)+2\epsilon)^2} + \frac{\epsilon\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{(b(n-1)+2\epsilon)^2\alpha}$$

If $\frac{a^2}{4} < \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{\alpha}$ holds, the values that maximize Π_s are $c_2^* = 0$ and $c_1^* = \frac{a}{2}$.

$$q^* = \frac{a+\beta(\alpha,\sigma)Y - c_1}{(n+1)b - 2c_2} = \frac{a+2\beta(\alpha,\sigma)Y}{2(n+1)b}$$
$$P^* = \frac{2a+an+2\beta(\alpha,\sigma)Y}{2+2n}$$
$$\Pi_s^* = \frac{a^2 n}{4b(1+n)}$$
$$\Pi_i^* = \frac{(a+2\beta(\alpha,\sigma)Y)^2}{4b(n+1)^2}$$

To obtain ex-ante values, expected values of ex-post functions:

$$q^* = \frac{a}{2(n+1)b}$$
$$P^* = \frac{2a+an}{2+2n}$$
$$\Pi_i^* = \frac{a^2}{4b(n+1)^2} + \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{b\alpha(1+n)^2}$$

CHAPTER 4

RESULTS AND INSIGHTS

Four different models with two alternatives such as collaboration and no collaboration are compared in this section. To see the difference of models, all equations about q_i,c_i are shown in tables. Deterministic and NBNS is joined in one column always.
Condition column is only filled in IBNS scenario.
Abbreviation Cond. is as follows: $(\beta(\alpha,\sigma) = \frac{\alpha\sigma^2}{1+\alpha\sigma^2})$

$$= \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{\alpha} = (\frac{\alpha\sigma^2}{1+\alpha\sigma^2})^2(\frac{1+\alpha\sigma^2}{\alpha}) = \beta(\alpha,\sigma)\sigma^2$$

Note that $\frac{a^2}{4n}$ in condition part is valid for Collaboration settings and for Monopoly and No collaboration $\frac{a^2}{4}$ is true. To avoid more rows in tables, they are given in one space.

In Table (4.1), c_i values are listed for collaboration and no collaboration to maximize profit of supplier:

When we compare c_1 Collaboration and No Collaboration separately,

- IBIS-C c_1 = NBNS-C c_1 = IBNS-C c_1
- IBIS-NC c_1 = NBNS-NC c_1 = IBNS-NC c_1
- NBNS-C c_1 > NBNS-NC c_1 if $\epsilon < b$
- IBIS-C c_1 > IBIS-NC c_1 if $\epsilon < b$
- IBNS-C c_1 > IBNS-NC c_1 if $\epsilon < b$

When we compare c_2 Collaboration and No Collaboration, it can be stated that there is no difference between two settings.

Table 4.1: Comparison of c_i^*'s

	Condition	c_1^* Coll	c_1^* No-Coll	c_2^* Coll	c_2^* No-Coll
NBNS	——	$\frac{a}{2}\left(\frac{2bn-(n-1)\epsilon}{bn+\epsilon}\right)$	$\frac{a}{2}\frac{(n+1)b}{(bn+\epsilon)}$	$b-\epsilon$	$b-\epsilon$
IBIS ex-ante	——	$\frac{a}{2}\left(\frac{2bn-(n-1)\epsilon}{bn+\epsilon}\right)$	$\frac{a}{2}(n+1)b(bn+\epsilon)$	$b-\epsilon$	$b-\epsilon$
IBNS ex-ante	$\frac{a^2}{4n} > Cond.$	$\frac{a}{2}\left(\frac{2bn-(n-1)\epsilon}{bn+\epsilon}\right)$	$\frac{a}{2}\frac{(n+1)b}{bn+\epsilon}$	$b-\epsilon$	$b-\epsilon$
	$\frac{a^2}{4n} < Cond.$	$\frac{a}{2}$	$\frac{a}{2}$	0	0

In Table (4.2), purchasing quantities are listed for Monopoly, Collaboration and No Collaboration setting. When c_i^* values are used to obtain q_i^*, differences between collaboration and no collaboration models are disappeared because of quantity discounts except IBNS ex-post for first condition.

When it is explained that difference between q_i values in IBNS ex-post for first condition; $\frac{a}{2(bn+\epsilon)} + \frac{\beta(\alpha,\sigma)Y}{(n+1)\epsilon}$ is equal to $\frac{a}{2(bn+\epsilon)} + \frac{\beta(\alpha,\sigma)Y}{b(n-1)+2\epsilon}$ only $\epsilon = b$ holds. However as stated in $A3$, $0 < \epsilon \leq b$ holds,

- IBNS-C ex-post q_i > IBNS-NC ex-post q_i if $\epsilon < b$

Purchasing quantity in Monopoly setting is always greater than Collaboration and No Collaboration settings which have more than one buyer. Because in monopoly setting, one buyer has all purchasing power in the market. There is no other player to compete.

- NBNS-M > NBNS-C = NBNS-NC if $n \neq 1$
- IBIS-M ex-ante > IBIS-C ex-ante = IBIS-NC ex-ante if $n \neq 1$
- IBNS-M ex-ante > IBNS-C ex-ante = IBNS-NC ex-ante if $n \neq 1$

When Monopoly, Collaboration and No Collaboration for different settings are compared separately, below results are obtained.

- NBNS-M = IBIS-M ex-ante = IBNS-M ex-ante Cond.1>IBNS-M ex-ante Cond.2 if $\epsilon < b$
- NBNS-C = IBIS-C ex-ante = IBNS-C ex-ante Cond.1>IBNS-C ex-ante Cond.2 if $\epsilon < b$
- NBNS-NC = IBIS-NC ex-ante = IBNS-NC ex-ante Cond.1>IBNS-NC ex-ante Cond.2 if $\epsilon < b$

Table 4.2: Comparison of q_i^*'s

	Condition	Monopoly	Collaboration	No Collaboration
NBNS	——	$\frac{a}{2(b+\epsilon)}$	$\frac{a}{2(bn+\epsilon)}$	$\frac{a}{2(bn+\epsilon)}$
IBIS ex-post	——	$\frac{a+\beta(\alpha,\sigma)Y}{2(b+\epsilon)}$	$\frac{a+\beta(\alpha,\sigma)Y}{2(bn+\epsilon)}$	$\frac{a+\beta(\alpha,\sigma)Y}{2(bn+\epsilon)}$
IBIS ex-ante	——	$\frac{a}{2(b+\epsilon)}$	$\frac{a}{2(bn+\epsilon)}$	$\frac{a}{2(bn+\epsilon)}$
IBNS ex-post	$\frac{a^2}{4n} > Cond.$	$\frac{a}{2(b+\epsilon)} + \frac{\beta(\alpha,\sigma)Y}{2\epsilon}$	$\frac{a}{2(bn+\epsilon)} + \frac{\beta(\alpha,\sigma)Y}{(n+1)\epsilon}$	$\frac{a}{2(bn+\epsilon)} + \frac{\beta(\alpha,\sigma)Y}{b(n-1)+2\epsilon}$
	$\frac{a^2}{4n} < Cond.$	$\frac{a+2\beta(\alpha,\sigma)Y}{4b}$	$\frac{a+2\beta(\alpha,\sigma)Y}{2(n+1)b}$	$\frac{a+2\beta(\alpha,\sigma)Y}{2(n+1)b}$
IBNS ex-ante	$\frac{a^2}{4n} > Cond.$	$\frac{a}{2(b+\epsilon)}$	$\frac{a}{2(bn+\epsilon)}$	$\frac{a}{2(bn+\epsilon)}$
	$\frac{a^2}{4n} < Cond.$	$\frac{a}{4b}$	$\frac{a}{2(n+1)b}$	$\frac{a}{2(n+1)b}$

Changes of price can be seen below in Table (4.3). Depending on c_i values, q_i values are obtained. And for obtaining price, q_i values are used. Price function can be see below:

$P = a + \theta - b\sum_i q$

Price in Monopoly setting is always greater than Collaboration and No Collaboration settings which have more than one buyer. Because in monopoly setting, one buyer has all purchasing power in the market. There is no other player to compete. And one buyer decides the price.

$$\frac{a(b+2\epsilon)}{2(b+\epsilon)} >^{?} \frac{a(bn+2\epsilon)}{2(bn+\epsilon)}$$

$$a(b+2\epsilon)2(bn+\epsilon) >^{?} a(bn+2\epsilon)2(b+\epsilon)$$

$$b\epsilon(2n+1) > b\epsilon(n+2)$$

$$n > 1$$

As obtained as above, when $n > 1$ holds, price of monopoly setting is greater than others.

- NBNS-M > NBNS-C = NBNS-NC if $n \neq 1$
- IBIS-M ex-ante > IBIS-C ex-ante = IBIS-NC ex-ante if $n \neq 1$
- IBNS-M ex-ante > IBNS-C ex-ante = IBNS-NC ex-ante if $n \neq 1$

When Monopoly, Collaboration and No Collaboration for different settings are compared separately, below results are obtained.

- NBNS-M = IBIS-M ex-ante = IBNS-M ex-ante Cond.1<IBNS-M ex-ante Cond.2 if $\epsilon < b$
- NBNS-C = IBIS-C ex-ante = IBNS-C ex-ante Cond.1<IBNS-C ex-ante Cond.2 if $\epsilon < b$
- NBNS-NC = IBIS-NC ex-ante = IBNS-NC ex-ante Cond.1<IBNS-NC ex-ante Cond.2 if $\epsilon < b$

Table 4.3: Comparison of P^*'s

	Condition	Monopoly	Collaboration	No Collaboration
NBNS	——	$\frac{a(b+2\epsilon)}{2(b+\epsilon)}$	$\frac{a(bn+2\epsilon)}{2(bn+\epsilon)}$	$\frac{a(bn+2\epsilon)}{2(bn+\epsilon)}$
IBIS ex-post	——	$\frac{(a+\beta(\alpha,\sigma)Y)(b+2\epsilon)}{2(b+\epsilon)}$	$\frac{(a+\beta(\alpha,\sigma)Y)(bn+2\epsilon)}{2(bn+\epsilon)}$	$\frac{(a+\beta(\alpha,\sigma)Y)(bn+2\epsilon)}{2(bn+\epsilon)}$
IBIS ex-ante	——	$\frac{a(b+2\epsilon)}{2(b+\epsilon)}$	$\frac{a(bn+2\epsilon)}{2(bn+\epsilon)}$	$\frac{a(bn+2\epsilon)}{2(bn+\epsilon)}$
IBNS ex-post	$\frac{a^2}{4n} > Cond.$	$\frac{a(b+2\epsilon)}{2(b+\epsilon)} + \frac{\beta(\alpha,\sigma)Y(2\epsilon-b)}{2\epsilon}$	$\frac{a(bn+\epsilon)}{2(bn+\epsilon)} + \frac{\beta(\alpha,\sigma)Y((n+1)\epsilon-nb)}{(n+1)\epsilon}$	$\frac{a(bn+2\epsilon)}{2(bn+\epsilon)} + \frac{\beta(\alpha,\sigma)Y(b-2\epsilon)}{b-bn-2\epsilon}$
	$\frac{a^2}{4n} < Cond.$	$\frac{3a-2\beta(\alpha,\sigma)Y}{4}$	$\frac{2a+an+2\beta(\alpha,\sigma)Y}{2+2n}$	$\frac{2a+an+2\beta(\alpha,\sigma)Y}{2+2n}$
IBNS ex-ante	$\frac{a^2}{4n} > Cond.$	$\frac{a(b+2\epsilon)}{2(b+\epsilon)}$	$\frac{a(bn+2\epsilon)}{2(bn+\epsilon)}$	$\frac{a(bn+2\epsilon)}{2(bn+\epsilon)}$
	$\frac{a^2}{4n} < Cond.$	$\frac{3a}{4}$	$\frac{2a+an}{2+2n}$	$\frac{2a+an}{2+2n}$

Supplier's profit is calculated by using c_i and q_i in Table (4.4). To avoid complexity, monopoly setting is analyzed firstly and it can be generalized to the others:

- IBIS-M ex-ante> NBNS-M

  IBIS ex-ante monopoly profits are higher than the profit under no information (NBNS-M), when $\beta(\alpha, \sigma)$ converges to 1 (signal accuracy α is high), this means that demand signal becomes perfect and the difference between profits are higher. Furthermore, when $\beta(\alpha, \sigma)$ is replaced with $\frac{\alpha\sigma^2}{1+\alpha\sigma^2}$, $\frac{(1+\sigma^2\alpha)\beta(\alpha,\sigma)^2}{\alpha}$ becomes $\frac{\alpha\sigma^4}{1+\alpha\sigma^2}$. This means that increase in α and σ results in higher differences between IBIS-M and NBNS-M. (Increase in σ results in more drastic differences than increase in α) Same analysis is valid for others: IBIS-C ex-ante> NBNS-C, IBIS-NC ex-ante> NBNS-NC.

- NBNS-M>IBNS-M ex-ante Cond.1

  NBNS-M profits are higher than the profit under IBNS-M ex-ante Cond.1. While buyer has imperfect information, supplier has no information so this results in lower profit when compared with under no information for buyer and supplier. In the buyer's side, knowing more information than supplier reduces profit of supplier.

  (Π_s NBNS-M)- (Π_s IBNS-M ex-ante Cond.1)

  $$= (b-\epsilon)^2 \frac{\beta(\alpha,\sigma)^2(1+\sigma^2\alpha)}{4\alpha\epsilon^2(b+\epsilon)}$$

  When the difference between b and ϵ is higher, the difference between NBNS-M and IBNS-M ex-ante Cond.1 is higher also. (When $b = \epsilon$ holds, there is no difference) Same analysis is valid for collaboration and no collaboration setting also.

  (Π_s NBNS-C)- (Π_s IBNS-C ex-ante Cond.1)

  $$= \frac{n^2(b-\epsilon)\beta(\alpha,\sigma)^2(1+\sigma^2\alpha)}{(n+1)^2\epsilon)^2\alpha}$$

  (Π_s NBNS-NC)- (Π_s IBNS-NC ex-ante Cond.1)

  $$= n(b-\epsilon)\epsilon^2 \frac{\beta(\alpha,\sigma)^2(1+\sigma^2\alpha)}{(b(n-1)+2\epsilon)^2\alpha}$$

- IBIS-M ex-ante> NBNS-M >IBNS-M ex-ante Cond.1 When two above items are combined, imperfect information for all market has the highest profit.Same analysis is valid for collaboration and no collaboration setting also.

IBIS-C ex-ante> NBNS-C >IBNS-C ex-ante Cond.1

IBIS-NC ex-ante> NBNS-NC >IBNS-NC ex-ante Cond.1

- NBNS-M >IBNS-M ex-ante Cond.2 if $\epsilon < b$ holds.

  When Monopoly, Collaboration and No Collaboration for different settings are compared separately, below results are obtained. Same is valid for collaboration and no collaboration.

  NBNS-C >IBNS-C ex-ante Cond.2

  NBNS-NC >IBNS-NC ex-ante Cond.2

- IBNS-NC ex-ante Cond.1>IBNS-C ex-ante Cond.1, because of the below positive difference.

  (Π_s IBNS-NC ex-ante Cond.1)- (Π_s IBNS-NC ex-ante Cond.1)

  $$= \frac{(n-1)n(b-\epsilon)(b^2(n-1)n + 4bn\epsilon - (n-1)\epsilon^2)\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{(n+1)^2\epsilon^2(b(n-1)+2\epsilon)^2\alpha}$$

  This means that when buyers do not collaborate, profit of supplier is higher. So supplier do not want buyers collaborate to have more profit in the market.

- IBNS-NC ex-ante Cond.2=IBNS-C ex-ante Cond.2

  When $c_2 = 0$ holds (linear price function), collaboration and no collaboration settings have the same profits.

Table 4.4: Comparison of Π_s^*'s

	Condition	Monopoly	Collaboration	No Collaboration
NBNS	——	$\frac{a^2}{4b+4\epsilon}$	$\frac{a^2 n}{4bn+4\epsilon}$	$\frac{a^2 n}{4bn+4\epsilon}$
IBIS ex-post	——	$\frac{(a+\beta(\alpha,\sigma)Y)^2}{4b+4\epsilon}$	$\frac{(a+\beta(\alpha,\sigma)Y)^2 n}{4bn+4\epsilon}$	$\frac{(a+\beta(\alpha,\sigma)Y)^2 n}{4bn+4\epsilon}$
IBIS ex-ante	——	$\frac{a^2}{4b+4\epsilon}+\frac{(1+\sigma^2\alpha)\beta(\alpha,\sigma)^2}{\alpha(4b+4\epsilon)}$	$\frac{a^2 n}{4bn+4\epsilon}+\frac{(1+\sigma^2\alpha)\beta(\alpha,\sigma)^2}{\alpha(4bn+4\epsilon)}$	$\frac{a^2 n}{4bn+4\epsilon}+\frac{(1+\sigma^2\alpha)\beta(\alpha,\sigma)^2}{\alpha(4bn+4\epsilon)}$
IBNS	$\frac{a^2}{4n} > Cond.$	$\frac{\epsilon^2(a^2\alpha+\beta(\alpha,\sigma)^2(1+\alpha\sigma^2))}{4\epsilon^2(b+\epsilon)\alpha} - \frac{b^2(\beta(\alpha,\sigma)^2(1+\alpha\sigma^2))}{4\epsilon^2(b+\epsilon)\alpha}$	$\frac{n(-4b^2n^2\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)+4b(n-1)n\epsilon\beta(\alpha,\sigma)^2(1+\alpha\sigma^2))}{4(1+n)^2\epsilon^2(bn+\epsilon)\alpha} + \frac{n\epsilon^2(a^2\alpha+n(a^2(2+n)\alpha+4\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)))}{4(1+n)^2\epsilon^2(bn+\epsilon)\alpha}$	$\frac{n(4b(n-1)\epsilon(a^2\alpha+\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)))}{4(bn+\epsilon)(b(n-1)+2\epsilon)^2\alpha} + \frac{n4\epsilon^2(a^2\alpha+\beta(\alpha,\sigma)^2(1+\alpha\sigma^2))}{4(bn+\epsilon)(b(n-1)+2\epsilon)^2\alpha} + \frac{nb^2(a^2\alpha+n(a^2(n-2)\alpha-4\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)))}{4(bn+\epsilon)(b(n-1)+2\epsilon)^2\alpha}$
	$\frac{a^2}{4n} < Cond.$	$\frac{a^2}{8b}$	$\frac{a^2 n}{4b(1+n)}$	$\frac{a^2 n}{4b(1+n)}$

In Table (4.5), Π_i is analyzed under different settings.To avoid complexity, monopoly setting is analyzed firstly:

- IBIS-M ex-ante> NBNS-M

  IBIS-M ex-ante profits are higher than the profit under no information (NBNS-M), when $\beta(\alpha, \sigma)$ converges to 1 (signal accuracy α is high), this means that demand signal becomes perfect and the difference between profits are higher. Furthermore, when $\beta(\alpha, \sigma)$ is replaced with $\frac{\alpha\sigma^2}{1+\alpha\sigma^2}$, $\frac{(1+\sigma^2\alpha)\beta(\alpha,\sigma)^2}{\alpha}$ becomes $\frac{\alpha\sigma^4}{1+\alpha\sigma^2}$. This means that increase in α and σ results in higher differences between IBIS-M and NBNS-M. (Increase in σ results in more drastic differences than increase in α)

- NBNS-M>IBNS-M ex-ante Cond.1

  NBNS-M profits are higher than the profit under IBNS-M ex-ante. While buyer has imperfect information, supplier has no information so this results in lower profit when compared with under no information for buyer and supplier. In the buyer's side, knowing information reduces profit of buyer because of no information in the side of supplier.

  (Π_i NBNS-M)- (Π_i IBNS-M ex-ante Cond.1)

  $$= \frac{\beta(\alpha, \sigma)^2(1 + \sigma^2\alpha)}{4\alpha\epsilon}$$

  Same analysis is valid for collaboration setting.

  (Π_i NBNS-C)- (Π_i IBNS-C ex-ante Cond.1)

  $$= \frac{n\beta(\alpha, \sigma)^2(1 + \sigma^2\alpha)}{(n + 1)^2\epsilon)\alpha}$$

  However, when no collaboration setting is analyzed,

  (Π_i NBNS-NC)- (Π_i IBNS-NC ex-ante Cond.1)

  $$= -\epsilon\frac{\beta(\alpha, \sigma)^2(1 + \sigma^2\alpha)}{(b(n - 1) + 2\epsilon)^2\alpha} < 0$$

  As a result, IBNS-NC ex-ante Cond.1> NBNS-NC. This means that buyers have more profit during no collaboration when they have imperfect information instead of no information. IBNS-NC ex-ante Cond.2> NBNS-NC is also valid.

- IBIS-M ex-ante>NBNS-M >IBNS-M ex-ante Cond.1>IBNS-M ex-ante Cond.2 if $\epsilon < b$

IBIS-C ex-ante>NBNS-C >IBNS-C ex-ante Cond.1>IBNS-C ex-ante Cond.2 if $\epsilon < b$

When two above items are combined, imperfect information for all market has the highest profit. IBNS-NC ex-ante Cond.1>IBIS-NC ex-ante>NBNS-NC

Results of model under no collaboration setting differentiates from the others.

- IBNS-NC ex-ante Cond.1 >IBNS-C ex-ante Cond.1. (Π_i IBNS-C ex-ante Cond.1)-(Π_i IBNS-C ex-ante Cond.1)

$$= \frac{(b^2(n-1)^2 n + 4b(n-1)n\epsilon + (1+n(6+n))\epsilon^2)\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{(1+n)^2\epsilon(b(n-1)+2\epsilon)^2\alpha} > 0$$

This means that when buyers do not collaborate, profit of buyer is higher. So buyers do not want collaborate to have more profit in the market under IBNS setting.

Table 4.5: Comparison of Π_i^*'s

	Condition	Monopoly	Collaboration	No Collaboration
NBNS	——	$\frac{a^2\epsilon}{4(b+\epsilon)^2}$	$\frac{a^2\epsilon}{4(bn+\epsilon)^2}$	$\frac{a^2\epsilon}{4(bn+\epsilon)^2}$
IBIS ex-post	——	$\frac{(a+\beta(\alpha,\sigma)Y)^2\epsilon}{4(b+\epsilon)^2}$	$\frac{(a+\beta(\alpha,\sigma)Y)^2\epsilon}{4(bn+\epsilon)^2}$	$\frac{(a+\beta(\alpha,\sigma)Y)^2\epsilon}{4(bn+\epsilon)^2}$
IBIS ex-ante	——	$\frac{a^2\epsilon}{4(b+\epsilon)^2} + \frac{(1+\sigma^2\alpha)\beta(\alpha,\sigma)^2\epsilon}{\alpha(4(b+\epsilon)^2)}$	$\frac{a^2\epsilon}{4(bn+\epsilon)^2} + \frac{(1+\sigma^2\alpha)\beta(\alpha,\sigma)^2\epsilon}{\alpha(4(bn+\epsilon)^2)}$	$\frac{a^2\epsilon}{4(bn+\epsilon)^2} + \frac{(1+\sigma^2\alpha)\beta(\alpha,\sigma)^2\epsilon}{\alpha(4(bn+\epsilon)^2)}$
IBNS ex-post	$\frac{a^2}{4n} > Cond.$	$\frac{(a\epsilon)^2-((b+\epsilon)\beta(\alpha,\sigma)Y)^2}{4\epsilon(b+\epsilon)^2}$	$\frac{(2bn\beta(\alpha,\sigma)Y+(a+an+2\beta(\alpha,\sigma)Y)\epsilon)(a\epsilon+n(a\epsilon-2\beta(\alpha,\sigma)Y(bn+\epsilon)))}{4(1+n)^2\epsilon(bn+\epsilon)^2}$	$\frac{\epsilon(ab(n-1)+2a\epsilon+2(bn+\epsilon)\beta(\alpha,\sigma)Y)^2}{4(bn+\epsilon)^2(b(n-1)+2\epsilon)^2}$
	$\frac{a^2}{4n} < Cond.$	$\frac{(a+2\beta(\alpha,\sigma)Y)(a-2\beta(\alpha,\sigma)Y)}{16b}$	$\frac{(a+2\beta(\alpha,\sigma)Y)(a-2n\beta(\alpha,\sigma)Y)}{4b(1+n)^2}$	$\frac{(a+2\beta(\alpha,\sigma)Y)^2}{4b(n+1)^2}$
IBNS ex-ante	$\frac{a^2}{4n} > Cond.$	$\frac{a^2\epsilon}{4(b+\epsilon)^2} - \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{4\alpha\epsilon}$	$\frac{a^2\epsilon}{4(bn+\epsilon)^2} - \frac{n\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{(1+n)^2\epsilon\alpha}$	$\frac{\epsilon(ab(n-1)+2a\epsilon)}{4(bn+\epsilon)^2(b(n-1)+2\epsilon)^2} + \frac{\epsilon\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{(b(n-1)+2\epsilon)^2\alpha}$
	$\frac{a^2}{4n} < Cond.$	$\frac{a^2}{16b} - \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{4b\alpha}$	$\frac{a^2}{4b(1+n)^2} - \frac{n\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{b\alpha(1+n)^2}$	$\frac{a^2}{4b(n+1)^2} + \frac{\beta(\alpha,\sigma)^2(1+\alpha\sigma^2)}{b\alpha(1+n)^2}$

CHAPTER 5

CONCLUSION

The model setting in this thesis can be described as, a market is assumed with n competing buyers where price is an inverse linear function of the quantity supplied to the market. The buyers get engaged in Cournot competition, but may also collaborate on purchasing decisions from a supplier. The supplier offers a quantity discount, as the quantity purchased increases unit price decreases. Furthermore, the demand base in the market is uncertain, but the buyers may get a signal of the demand.
Firstly, when the profits of supplier for different settings are analyzed, the following important results are obtained:

- Imperfect buyers,imperfect supplier scenario has the highest profit. Increase in signal accuracy α and variance σ results in higher differences between IBIS Π_s and NBNS Π_s. (Increase in σ results in more drastic differences than increase in α). When α approaches to infinity, $\beta(\alpha, \sigma)$ goes to 1, and this means that demand signal becomes perfect.

- NBNS profits are higher than the profit under IBNS ex-ante Cond.1. While buyers have imperfect information, supplier has no information so this results in lower profit when compared with under no information for buyer and supplier. In the buyer's side, knowing more information than supplier reduces profit of supplier. Because the supplier is affected by order variability caused by demand uncertainty.

- In imperfect information for the buyers, no information for the supplier scenario, the profit of supplier in no collaboration is the higher than collaboration. This means that when buyers do not collaborate, profit of supplier is higher because there is no quantity discount. So supplier does not want buyers to collaborate to have more profit in the market.

Secondly, when the profits of buyers for different settings are analyzed, the following important results are obtained:

- Imperfect buyers,imperfect supplier scenario has the highest profit. Increase in signal accuracy α and variance σ results in higher differences between IBIS Π_i and NBNS Π_i. When the demand signal becomes perfect, profit of buyers will be increasing.

- NBNS profits are higher than the profit under IBNS ex-ante Cond.1 except no collaboration setting. For first part, while buyers have imperfect information, supplier has no information, this results in lower profit when compared with under no information for buyer and supplier. In the buyer's side, knowing more information than supplier reduces profit of buyer because of no information in the side of supplier. This means that not sharing information is worse than no information. For second part (no collaboration), this means that buyers have more profit during no collaboration when they have imperfect information instead of no information.

- In IBNS scenario, the profit of buyer in no collaboration is the higher than collaboration. This means that when buyers do not collaborate, profit of buyer is higher. So buyers do not want collaborate to have more profit in the market under this scenario.

Future researches that develop our model can be listed as follows:In this thesis, it is assumed that the buyers simultaneously decide whether to share information or not, and their decisions are aligned. With different decisions of buyers can be input to the our model also. Supplier is uncapacitated in our model. Total demands of buyers are supplied. With capacitated supplier, this thesis can be improved. Each buyer purchases from the supplier exactly the amount it will sell in the end market. There are no inventories in the buyers and this may move away our model from the real supply chain problems. The model in this thesis is a single-period model so it has some disadvantages when compared with multi-period models.

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