A GENETIC ALGORITHM FOR THE p-HUB CENTER PROBLEM WITH STOCHASTIC SERVICE LEVEL CONSTRAINTS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

ŞÜKRAN ERASLAN DEMİRCİ

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
INDUSTRIAL ENGINEERING

DECEMBER 2010

Approval of the thesis:

A GENETIC ALGORITHM FOR THE p-HUB CENTER PROBLEM WITH STOCHASTIC SERVICE LEVEL CONSTRAINTS

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ABSTRACT

A GENETIC ALGORITHM FOR p-HUB CENTER PROBLEM WITH STOCHASTIC SERVICE LEVEL CONSTRAINTS

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December 2010, 170 pages

The emphasis on minimizing the costs and travel times in a network of origins and destinations has led the researchers to widely study the hub location problems in the area of location theory in which locating the hub facilities and designing the hub networks are the issues. The p-hub center problem considering these issues is the subject of this study. p-hub center problem with stochastic service level constraints and a limitation on the travel times between the nodes and hubs is addressed, which is an uncapacitated, single allocation problem with a complete hub network.

Both a mathematical model and a genetic algorithm are proposed for the problem. We discuss the general framework of the genetic algorithm as well as the problem-specific components of algorithm. The computational studies of the proposed algorithm are realized on a number of problem instances from Civil Aeronautics Board (CAB) data set and Turkish network data set. The computational results indicate that the proposed genetic algorithm gives satisfactory results when compared with the optimum solutions and solutions obtained with other heuristic methods.

Keywords: Hub Location, P-Hub Center Problems, Stochastic Service Level Constraints, Genetic Algorithms

ÖZ

STOKASTİK HİZMET DÜZEYİ KISITLI p-ANA DAĞITIM ÜSSÜ MERKEZLİ PROBLEM İÇİN BİR GENETİK ALGORİTMA

Eraslan Demirci, Şükran
Yüksek Lisans, Endüstri Mühendisliği
Tez Yöneticisi: Yrd. Doç. Dr. Sedef Meral

Aralık 2010, 170 sayfa

Çıkış ve varış noktalarının oluşturduğu ağlarda maliyetleri ve ulaşım sürelerini enazlamaya verilen önem; araştırmacıları, ana dağıtım üslerinin konumlandırıldığı ve ana dağıtım ağının tasarlandığı yerleşim kuramı çerçevesindeki ana dağıtım üssü konumlandırma problemini daha kapsamlı çalışmaya yöneltmiştir. Bu konuyu ele alan p-ana dağıtım üssü merkezli problem bu çalışmanın konusudur. Stokastik hizmet düzeyi kısıtının ve ana dağıtım üsleri ile varış/çıkış noktaları arasındaki taşıma süresi üzerinde kısıt olan p-ana dağıtım üssü merkez problemi üzerinde çalışılmıştır. Bu problem tam bağlantılı ana dağıtım üssü ağında, kapasite kısıtı olmayan, tek atamalı p-ana dağıtım üssü merkez problemidir.

Üzerinde çalışılan problem ile ilgili olarak bir matematiksel model ve genetik algoritmaya dayalı bir sezgisel yöntem geliştirilmiştir. Genetik algoritmanın genel çerçevesi ile birlikte, problem-özgü bileşenler tartışılmıştır. Önerilen yöntem ile Amerika Sivil Havacılık Kurulu (CAB) veri kümesi ve Türkiye Ağı veri kümesinden elde edilen çeşitli problem örnekleri üzerinde önerilen yöntem ile sonuçlar elde edilmiştir. Genetik algoritma ile elde edilen sonuçların;

optimum sonuçlar ve diğer bazı sezgisel yöntemlerden elde edilen sonuçlar ile karşılaştırıldığında tatmin edici düzeyde olduğu görülmüştür.

Anahtar Kelimeler: Ana Dağıtım Üssü Konumlandırma, p-Ana Dağıtım Üssü Merkezli Problem, Stokastik Hizmet Düzeyi Kısıtı, Genetik Algoritma.

ACKNOWLEDGEMENTS

First of all, I would like to thank my supervisor Ass. Prof. Dr. Sedef Meral for her kind guidance, cooperation, encouragement and advice throughout the study. I would also like to express appreciation to her for her trust and supporting in a friendly atmosphere that inspired creativity and motivation.

I sincerely thank TÜBİTAK and the managers for giving me the chance to complete my Master of Science degree.

I would like to present special thanks to helpful Erhan Girgin for his support for the computational part of this work.

I express my deepest gratitude to my father Hakkı Eraslan, my mother Süheyla Eraslan and my sweet sister Yasemin Eraslan for their endless love, moral, and encouragement throughout my whole life.

Finally, I would like to thank my darling Yücel Demirci for his existence, trust, endurance and everlasting love. I always feel his encouragement and that he will always be with me whenever I need him.

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CHAPTER 1

1. INTRODUCTION

The aim of the hub location problems is to reduce the costs or transportation times of a given network. Locations for the hubs and allocations of the non-hub nodes to the hubs are determined in the hub location problems. The service is given by or at a facility in the location problems. The difference of the hub location problems from the other location problem is that, “service” means the transferring of the goods, people or information between an origin node and a destination node. Each pair of origin and destination nodes represents a different service that needs to be provided. Thus, packages moving from node A to node B are not interchangeable with packages moving from node C to node D (Daskin, 1995, p.349). Hub location problems are proper for the networks that aggregation and disaggregation of flows at certain locations are possible (Kara and Tansel, 2000). The hubs serve as connection, transshipment and classification points. Generally establishing a complete network for transporting the commodities directly between origin and destination nodes is extremely costly. Instead of having a complete network with direct links between each origin node and each destination node, hub facilities are constructed in order to benefit from the economies of scale of transferring larger flows. Flows belonging to the same origin node are aggregated and transferred to the same hub and then are consolidated with the flows from the remaining origin nodes that will be transferred to the same destination node. Some typical application areas of the hub location problem are airline passenger travel, cargo delivery and message delivery in computer communication networks (Kara and Tansel, 2000).

There are *n* nodes and the amount of flow between each pair of nodes is given. The aim is to determine the locations of the hubs among these *n* nodes and the allocations of these *n* nodes to the selected hubs so as to minimize the costs. The costs considered in these problems can be the transportation costs of the flows, the fixed costs of the network, the path length of the transportation or the time of the transportation. The assumptions often valid in the hub location literature are as follows: There are economies-of-scale for the inter-hub connections reflected by a discount factor, α, and there is a complete network for the hubs.

Although the basic environment of the hub location problem is as defined above, different characteristics of environments for hub locations and non-hub node allocations cause to diversify the hub location environment. The literature about the hub location problems is examined in a classification with regard to these problem characteristics in the following sections.

This study is concerned with the p-hub center problem, which considers locating *p* many hubs and allocating the remaining non-hub nodes to these located hubs. The p-hub center problem is similar to the p-center problem in minimizing the adverse affects of the worst case. The aim is to find p many hub locations and the remaining non-hub nodes allocations to these located hubs in order to minimize the maximum length of the links or maximum cost of the links. Center type of location problems resembles real life situations like the problem of emergency aid facilities or vehicles locations.

The p-hub center problem is NP-hard (Kara and Tansel, 2000) and classical mathematical formulation based approaches are usually not satisfactory especially for the large problems.

Our aim in this study is to propose a formulation for the p-hub center problem with stochastic service level constraints, considering that the travel times on the links may be stochastic rather than being deterministic. Firstly, we formulate the problem as an integer programming model. However, because of the incapability of the commercial solvers in solving especially the large size problems, we propose to the use of metaheuristics. For this purpose, we apply a genetic algorithm (GA) based solution approach for our p-hub center problem.

In the following chapters, the details of our study are presented.

In Chapter 2, an overview of the literature of the hub location problems is included based on a classification scheme.

In Chapter 3, the problem environment of our problem is explained in detail and the related integer programming formulation is presented.

Chapter 4 presents our solution method for the p-hub center problem with stochastic service level constraints. The solution procedure is based on Genetic Algorithms, and before describing the proposed algorithm, the brief overview and general components of the Genetic Algorithms are given.

Chapter 5 includes the computational study. The test problems used in the computational study are first defined. Then the GAMS results for small sized problems and aggregation and decomposition based heuristic results for large sized problems are obtained for evaluating the performance of our method.

Chapter 6 concludes our study by briefly indicating the significant parts of our study and pointing some directions for future researches.

CHAPTER 2

2. LITERATURE REVIEW

2.1 HUB LOCATION

The hub location problems are network design problems, which has *n* nodes and the flow among each origin and destination node pair is given. In a simplest network, these flows can be handled by connecting each pair of nodes with a complete network among them. However, this will result in a highly inefficient network in terms of both economical and managerial issues. For an *n* node network, if a fully connected network is constructed, then this will result in $n(n-1)/2$ connections, which will be a costly and a difficult network to manage. On the other hand, if a network with one hub among these *n* nodes is constructed, this will result in a network of $(n-1)$ connections. In Figure 1.1, a complete network for 5 nodes is illustrated, whereas in Figure 1.2, the network with one hub is illustrated. Although in these examples we have a few nodes, it is obvious that the fully connected network is more complicated.

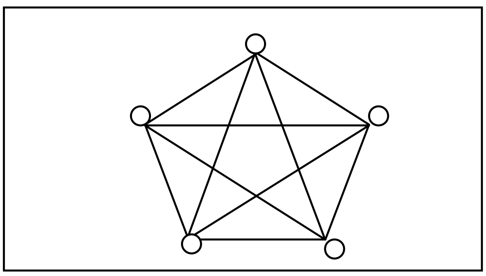


Figure 1.1 A Complete Network for 5 Nodes

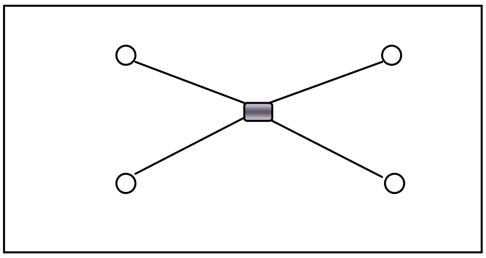


Figure 1.2 One Hub Network for 5 Nodes

There are also drawbacks of the hub location networks; the obvious one of which is the increasing transportation leg for an origin-destination pair; except the case in which either is a hub (Daskin, 1995, p.350). Consequently, the location of the hubs and the allocation of the nodes to these hubs are critical factors for the efficiency of a network and the aim is to locate the hubs among these *n* nodes and allocate these nodes to the selected hubs in order to minimize the total costs. The costs associated to these problems can be the fixed costs, the transportation costs, and the path length of the transportation or the time of transportation. The assumptions often valid in hub location literature can be stated as: There is a complete network for the hubs and there are economies-of-scale for the inter-hub connections reflected by a discount factor, α.

Hub location network problems are first defined in the paper of Goldman (1969). However, the literature on the hub location problem started with O'Kelly (1986a). There exists a wide literature about hub location problems, including several review papers that classify the hub location problems. In our paper, we classify the problem environment according to the classification of Alumur and Kara (2007). In this paper, the hub location problems are classified into four main categories: the p-hub median problem, the hub location problem with fixed costs, the p-hub center problem and the hub covering problem. This classification is mainly according to the objective of the hub location problems. However, there are also other variations according to the hub network structure. Some variations result from the assumption about the allocation of the non-hub nodes. According to these assumptions, the hub location problems can be classified as: single-allocation hub location problems, where the non-hub node is allowed to be assigned to only one hub and multiple-allocation hub location problems, where the non-hub node is allowed to be assigned to more than one single hub. Moreover, the hub location problem environment varies with respect to the links among non-hub nodes: direct links between non-hub node pairs are allowed or the transportation between the non-hub-nodes can be done only via the hubs. In addition, there exist the capacitated and uncapacitated versions of the hub location problems according to whether there exists a capacity constraint for the hubs or for the arcs, or not.

2.1.1 The p-Hub Median Problem

The p-hub median problem is one of the most commonly touched areas of the hub location problems. The number of hubs to be opened is given as p in this problem. There are n nodes and the flow among each pair of nodes is given. The aim is to determine the location of the hubs and the route of each pair of nodes denoted by an origin and a destination node so that the total transportation cost is minimized. The p-hub median problem is NP-hard. Furthermore, Kara (1999) shows that the allocation part of the p-hub median problem, for fixed hub locations, is also NP-hard. According to Alumur and Kara (2007), the studies regarding the p-hub median problem can be analyzed in two categories according to the allocation decisions: single allocation p-hub median problems and multiple allocation p-hub median problems.

2.1.1.1 Single Allocation p-Hub Median Problem

Each non-hub node is allowed to be connected to only a single hub in the single allocation p-hub median problem. The first formulation of the single allocation p-hub median problem is provided by O'Kelly (1987) as a quadratic integer program. He also proposes first two heuristics for that problem, called HEUR1 and HEUR2. In these heuristics, all possible alternatives for p-hub locations are enumerated. The assignment of the non-hubs to the nearest hub rule is used in HEUR1 and in HEUR2 to the better of the first and the second nearest hubs is used. Furthermore, to present the computational results for these two heuristics, O'Kelly introduces a data set referred as CAB data set. It is based on the airline passenger transportations of 25 US cities. This data set has been widely used in the hub location literature.

Aykın (1990) develops a technique for finding the optimal allocations of the nodes to the given hub locations. Klincewicz (1991) examines the single allocation p-hub median problem with a potential set of hubs and develops an exchange heuristic that substitutes other nodes based on local improvement taking account of one substitution and two substitutions at a time. He compares the heuristics with clustering and enumeration heuristics. Then Klincewicz (1992) presents a greedy randomized search procedure heuristic (GRASP) and a tabu search heuristic.

Skorin-Kapov and Skorin-Kapov (1994) develop another tabu search heuristic that emphasizes the allocation part of the problem. The results on the CAB data show that their heuristic is superior to the heuristics of O'Kelly (1987) and the tabu search heuristic of Klincewicz (1992). However, the CPU time requirement is greater. O'Kelly et al. (1995) linearize the quadratic objective function of O'Kelly (1987) for finding a lower bound technique where the triangle inequality is assumed to be satisfied. Later the solutions obtained by Skorin-Kapov and Skorin-Kapov (1994) are validated to be optimal in the work of Skorin-Kapov et al (1996) in which the authors proposes a mixed integer programming formulation. The LP relaxation of this formulation is shown to be tight by Skorin-Kapov et al. (1996).

The single allocation p-hub median problem is first proposed by Campbell (1994b) as a linear integer formulation. He also formulates the p-hub median problem with flow thresholds. After that, he obtains a formulation by setting the flow thresholds to their maximum values, because in that case each node is allocated to a single hub.

Then, with the obvious result that the multiple allocation p-hub median problem provides a lower bound for the single allocation p-hub median problem, two new heuristics for the same problem called MAXFLO and ALLFLO heuristics are proposed by Campbell (1996). The location decisions are the same in these heuristics; however the allocation decisions are made using different rules.

Ernst and Krishnamoorthy (1996) present a mixed integer LP formulation of the uncapacitated single allocation p-hub median problem (USApHMP). They propose a simulated annealing heuristic and an LP-based B&B algorithm. The B&B algorithm and the proposed heuristic and are both tested on the CAB data set and a newly introduced data set called the Australian Post (AP) data set, but they solve only the problems up to 50 nodes. AP data set is based on 200 postal districts of Australian postal delivery. The flows of the AP data set are asymmetrical as opposed to the CAB data set.

Then, Ernst and Krishnamoorthy (1998b) develop a B&B algorithm based on the shortest path approach. They state that their new algorithm is significantly faster and requires less memory than the algorithm of Ernst and Krishnamoorthy (1996) for small values of p. Problems with 100 nodes for $p=2$ and $p=3$ are solved in approximately 228 and 2629 seconds, respectively.

Later, Ebery (2001) presents a mathematical formulation for the single allocation two-hub median problem and for the single allocation three-hub median problem, and he also considers the situation where the hub nodes are fixed.

O'Kelly et al. (1996) discuss the sensitivity of the results to the discount factor α with a new formulation assuming a symmetric flow data.

Smith et al. (1996) study the modified Hopfield neural network of single allocation p-hub median problem using the quadratic integer formulation of O'Kelly (1987).

Sohn and Park (1997) provide a linear formulation and show that the two hub problem is polynomially solvable. Sohn and Park (1998) present a mixed integer formulation for given hub locations and also consider the fixed costs of the links between non-hub nodes and hubs. Later, Sohn and Park (2000) study the three-hub network single allocation problem with known hub locations as a mixed integer formulation. They show that even for the three hubs, the single allocation problem is NP-hard and study the polyhedral properties of the formulation.

Pirkul and Schilling (1998) construct a Lagrangean relaxation heuristic. They use subgradient optimization of the relaxed model. The constructed heuristic is the most effective heuristic for the single allocation p-hub median problem up to 2007 according to Alumur and Kara (2007).

Abdinnour-Helm (2001) proposes a simulated annealing based heuristic; however, it is shown to perform worse than that of Ernst and Krishnamoorthy (1996).

Aversa et al. (2005) investigate the hub networks in marine transportation. They propose a model for selecting a hub port among 11 potential hub locations. The model is based on the single allocation p-hub median model and the port of Santos in Brazil is selected in the model.

Elhedhli and Hu (2005) propose a model for the single allocation p-hub median problem that has a non-linear convex cost function and they consider congestion at the hubs. They use piecewise linear functions to linearize this model and then apply Lagrangean relaxation to the linearized model. They state that the solutions of the congestion model are more balanced in terms of the allocation of flows to the hubs when compared with the problem without congestion.

Pérez et al. (2005) provide a hybrid heuristic that combines GRASP (greedy random adaptive search procedure) heuristic and Path-Relinking heuristic for the problem with capacity restrictions. They benefit from the GRASP heuristic proposed by Klincewicz (1992) and GRASP is used for the generation of the population of the Path Relinking. In addition, they propose a Path Relinking heuristic for this problem.

Kratica et al. (2007) propose two genetic algorithm approaches called GAHUB1 and GAHUB2 in order to solve the uncapacitated single allocation p-hub median problem. The genetic algorithms are tested on the CAB and the AP data sets and GAHUB2 outperforms GAHUB1, and GAHUB2 gives all previously known optimal solutions for small and medium sized problems. In addition, the best-known solutions from the literature for large size problems are significantly improved by GAHUB2.

Yaman (2009) considers the three-level (hierarchical) hub network problem with a complete network in the first level, aiming to find the locations of the central hubs and the hubs, and the connections such that the total routing cost of the resulting network is minimized. Yaman (2009) proposes an MIP formulation for the hierarchical single assignment hub median problem; then she obtains a formulation for the hierarchical single assignment hub median problem with time restrictions. She solves these models by GAMS and CPLEX using CAB data set and the Turkish network data set to see the effects of various model parameters on the results.

A summary of the studies on the p-hub median problem is provided in Table 2.1.

Table 2.1 Summary of the Single Allocation p-Hub Median Problem Literature

Source	Characteristics
O'Kelly, 1987	First mathematical formulation; HEUR1 and HEUR2
Aykın, 1990	Optimal allocations with given hubs
Klincewicz, 1991	Exchange heuristic based on local improvement
Klincewicz, 1992	TABU Search and Greedy Randomized Search Procedure (GRASP)
Skorin-Kapov & Skorin-Kapov, 1994	TABU Search with emphasis on the allocation phase
Campbell, 1994b	First LP formulation
O'Kelly et al., 1995	Lower bounding technique
Skorin-Kapov et al., 1996	LP relaxation of an MIP shown to be tight
Campbell, 1996	MAXFLO and ALLFLO heuristics
Ernst & Krishnamoorthy, 1996	LP based B&B algorithm using the upper bound obtained from simulating annealing for USApHMP
O'Kelly et al., 1996	Sensitivity of solutions to α
Smith et al., 1996	SApHMP on a modified neural network
Sohn&Park, 1997	SApHMP with fixed hub locations
Sohn&Park, 1998	MIP with fixed hub locations
Ernst & Krishnamoorthy, 1998b	B&B algorithm solving shortest path based problems
Pirkul & Schilling, 1998	Subgradient lagrangean relaxation heuristic
Sohn&Park, 2000	SApHMP on a fixed three-hub network
Ebery, 2001	New mathematical formulation with 2 or 3 hubs
Abdinnour-Helm, 2001	Simulated annealing heuristic
Aversa et al., 2005	Marine transportation networks based on SApHMP
Eldelhi & Hu, 2005	Lagrangean relaxation to the linearized model of SApHMP with nonlinear convex cost function
Perez et al., 2005	A hybrid heuristic of GRASP and path-relinking for CSApHMP
Kratica et al., 2007	GAHUB1 & GAHUB2 for USApHMP
Yaman, 2009	3-level hierarchical single assignment hub median problem

2.1.1.2 Multiple Allocation p-Hub Median Problem

In the multiple allocation p-hub median problems, each node is allowed to be assigned to more than one hub. The multiple allocation p-hub median problem is first formulated by Campbell (1992) as a linear integer model. Then, Campbell (1994b) states that there is an optimal solution to the problem with no capacity constraints.

Skorin-Kapov et al. (1996) replace some constraints in the formulation of Campbell (1992) with the aggregate forms of them. The advantage of this formulation is that, it results in tighter LP relaxations. To obtain optimal solutions in the case of no integer solution from the LP relaxation, an implicit enumeration search tree is designated.

Ernst and Krishnamoorthy (1998a) define a new mixed integer formulation and show this formulation to be more effective than that of Skorin-Kapov (1996). Ernst and Krishnamoorthy (1998a) obtain a strong lower bound by adding the violated inequalities of B&B method to the LP. Also, Ernst and Krishnamoorthy (1998a) propose a shortest path based heuristic and an explicit enumeration based heuristic for fixed hub locations. Furthermore, Ernst and Krishnamoorthy (1998b) develop an alternative B&B algorithm obtaining lower bounds from the shortest path problems.

In order to overcome the weak lower bound deficiency of the formulation in Ernst and Krishnamoorthy (1998a), Boland et al. (2004) develop some tightening constraints and preprocessing techniques. The results are improved with these preprocessing techniques and tightening constraints.

An efficient shortest path based heuristic method is proposed by Sohn and Park (1998), for small p values of the multiple allocation p-hub median problem.

Sasaki et al. (1999) present the multiple allocation p-hub median problem with 1-stop in which the origin-destination pairs is allocated to one hub simultaneously. A greedy heuristic and a B&B algorithm are tested.

Campbell (2009) provides a time-definite model for the multiple allocation p-hub median problem and another time-definite model for the hub arc location problem. Service level constraints on the maximum travel distance between each origin node and destination node are imposed in those models. Both models are solved using CPLEX and the CAB data set. The effects of the time definite service levels are demonstrated on truck transportation in North America.

We summarize the literature on the multiple allocation p-hub median problem in Table 2.2.

Table 2.2 Summary of the Multiple Allocation p-Hub Median Problem Literature

Source	Characteristics
Campbell, 1992	First mathematical formulations
Campbell, 1996	Greedy interchange heuristic
Skorin-Kapov et al., 1996	Aggregation of some constraints resulting in tighter LP relaxations
Ernst & Krishnamoorthy, 1998a	Shortest path based heuristic; explicit enumeration heuristic and LP based B&B
Ernst & Krishnamoorthy, 1998b	Lower bounds obtained by shortest path problem and a B&B algorithm
Sohn & Park, 1998	An efficient shortest path based heuristic
Sasaki et al., 1999	A B&B algorithm and a greedy type heuristic for 1-stop multiple allocation p-hub median
Boland et al., 2004	Some preprocessing techniques and tightening constraints
Campbell, 2009	Multiple allocation p-hub median problem as time definite model

2.1.2 The Hub Location Problem with Fixed Costs

There is a fixed cost of opening a hub and the number of hubs becomes a decision variable in the hub location problems with fixed costs, unlike the p-hub median problem. The aim is to determine the number and location of hubs and the allocation of the nodes to these hubs in order to minimize the total of fixed and transportation costs.

A quadratic integer program is introduced by O'Kelly (1992a). Then, the first linear formulations of the both capacitated and uncapacitated versions of the single and multiple allocation hub location problems with fixed costs are presented by Campbell (1994b).

Aykın (1994) models the capacitated single allocation hub location problem with fixed costs in which the direct links are allowed. He proposes a B&B algorithm where lagrangean relaxation is used for obtaining the lower bounds. Aykın (1995a) analyzes the hub location problem with fixed costs for given *p*. Two hubbing policies are compared. The first one is called the non-strict hubbing policy and, he formulates this policy and solves it to optimality by enumeration. Also a greedy-interchange heuristic for this policy is proposed. The second policy is called the strict hubbing policy.

In Klincewicz (1996) a set of potential hubs is used for determining the hub locations and a dual-ascent and dual adjustment techniques based B&B algorithm is tested.

Abdinnour-Helm and Venkataramanan (1998) propose a quadratic integer formulation for the uncapacitated single allocation hub location problem with fixed costs making good use of the multicommodity flows in networks. Their formulation is denoted by the name MCUHP. They propose a B&B method with the lower bounds resulted from the structure of the network in order to get the optimal solution. In addition, they develop a genetic algorithm for that problem.

Abdinnour-Helm (1998) proposes a new hybrid heuristic called GATS which is a hybrid of genetic algorithms (GA) and tabu search (TS). First, the number and the location of the hubs are determined by the genetic algorithm and then, each node is allocated to the nearest hub. They start the tabu search with this solution and the hybrid of the tabu search and genetic algorithm results with better solutions than the pure genetic algorithm.

Two new formulations by modifying the mixed integer formulations of the p-hub median problem are presented in the study of Ernst and Krishnamoorthy (1999). A simulated annealing based heuristic and a random descent based heuristic are provided for the problem. They test their methodology on the AP data set. As a result, they see that both heuristics are quite efficient and, the random descent heuristic to be more efficient for small and medium sized problems. The simulated annealing heuristic is also found to be more efficient for large problem.

Ebery et al. (2000) survey the capacitated multiple allocation hub location problem with fixed costs. They propose a formulation similar to the formulation of Ernst and Krishnamoorthy (1998a) for the multiple-allocation p-hub median problem. They propose an LP based B&B technique in which an upper bound from a shortest path based heuristic is integrated. The results using the CAB and the AP data sets show that it is possible to solve large problems with their new formulations in a reasonable amount of time.

Mayer and Wagner (2002) develop a method called “Hub Locater” for the multiple allocation uncapacitated hub location problem which is based on a B&B technique. Since The Hub Locater obtains tight lower bounds for the problem, this formulation results in reduced computational effort. The comparison of the Hub Locater with the B&B algorithm of Klincewicz (1996) and with optimum or near optimum solutions of CPLEX shows that the hub locater is superior to the B&B algorithm of Klincewicz (1996).

Marianov and Serra (2002) present a formulation for the multiple allocation hub location problem with fixed costs that takes into account the congestion at the hubs. Their model has two versions. In the first version, the number of runways to open a hub is given, while in the second version this number is a decision variable of the model. Their model transforms the probabilistic constraint about the congestion in a hub to a deterministic linear constraint. They also propose a tabu search based heuristic to solve the problem.

Sasaki and Fukushima (2003) study the multiple allocation capacitated 1-stop hub location problem with fixed costs where they put constraints on both hubs and arcs. They propose a Lagrangean relaxation based B&B algorithm for their formulation.

Boland et al. (2004) develop some preprocessing techniques and some tightening constraints with the help of the properties of the optimal solutions. The aim of their study is to get some improvements of the linear relaxations of the mixed integer formulations. Some flow-cover constraints are also employed in their study to decrease the computation times.

Marı´n (2005a) uses some of the ideas presented in Marı´n et al. (2006) to reduce the size of the problem for the capacitated multiple allocation hub location problem with fixed. The computational experience on the AP data set shows that medium size problems can be solved more efficiently with this new formulation than the previous formulations. Marı´n (2005b) develops a relax-and-cut algorithm for the multiple allocation uncapacitated hub location problem with fixed costs by applying some polyhedral properties of the set-packing problem. Computational study is carried out using the AP data set and it is seen that the instances are solved to optimality with low computational effort by relax-and-cut algorithm. Marı´n et al. (2006) relax the assumption of the costs satisfying the triangle inequality and present a formulation for the uncapacitated multiple allocation hub location problem with fixed costs. They use some polyhedral results to tighten the constraints and decrease the number of them.

Topçuoğlu et al. (2003) propose a GA-based heuristic performs better than the GATS heuristic of Abdinnour-Helm (1998). Later, Cunha and Silva (2007) develop a hybrid of GA and simulated annealing heuristic for the same problem instance. In their problem, the discount factor α may vary with the total amount of load between the hubs. For this problem, Chen (2007) proposes another hybrid of simulated annealing heuristic, tabu search and improvement procedures.

For the uncapacitated multiple allocation problem, Ca´novas et al. (2007) carry out a dual-ascent technique based heuristic within a B&B algorithm. They obtain the best solutions for the uncapacitated multiple allocation hub location problem with fixed costs according to Alumur and Kara (2007).

Costa et al. (2007) propose a bi-criteria approach for the capacitated problem. Rather than imposing capacity constraints for the hubs, they introduce another objective function for the processing time in the hubs. They consider two formulations called BSAHLP-1 and BSAHLP-2. Both formulations have minimizing the total transportation cost and fixed cost of locating hubs as the first objective function. The second objective function in BSAHLP-1 is to minimize the total processing time in the hubs and the second objective function in BSAHLP-2 is to minimize the maximum processing time among the hubs. They propose an iterative solution procedure to find the non-dominated solutions.

Camargo et al. (2008) present a Benders decomposition based algorithm. Their algorithm is based on the formulation of Hamacher et al. (2004). In fact, they implement three variants of the Benders decomposition. In order to test these formulations, they use the AP data set and the CAB data set with minor modifications. Camargo et al. (2008) note that, instances of large sizes which are solved by the Benders decomposition have not been solved to optimality until that time.

Silva and Cunha (2009) propose an integrated tabu search heuristic with two-stages and three different multi-start tabu search heuristic. They perform computational studies using the CAB data set and the AP data set as well as other problem instances.

The Table 2.3 summarizes the literature on the hub location problem with fixed costs.

Table 2.3 Summary of the Hub Location Problem with Fixed Costs Literature

Source	Characteristics
O'Kelly, 1992a	Quadratic integer program for single allocation and uncapacitated cases
Campbell, 1994b	First LP formulations
Aykın, 1994	B&B algorithm and lower bounds obtained from lagrangean relaxation by subgradient optimization
Aykın, 1995a	Enumeration and greedy interchange heuristic for non-strict policy; B&B and greedy interchange heuristic based on simulated annealing for strict policy
Klincewicz, 1996	Dual ascent and dual adjustment techniques
Abdinnour-Helm & Venkataraman, 1998	B&B and GA based on the assumption of flows as multicommodities
Abdinnour-Helm, 1998	Hybrid of GA and TABU search called GATS
Ernst & Krishnamoorthy, 1999	Upper bounds from simulated annealing and random descent technique used in LP based B&B
Ebery et al., 2000	Upper bound from shortest path heuristic integrated to an LP based B&B algorithm
Mayer and Wagner, 2002	B&B method called hub locater
Marianov & Serra, 2002	A formulation taking into account congestion at the hubs and the runways in each hub
Sasaki & Fukushima, 2003	Lagrangean based B&B algorithm for 1-stop hub location problem
Boland et al., 2004	Preprocessing techniques and tightening constraints to improve linear relaxations
Labbè et al., 2005	Polyhedral properties of single allocation capacitated problem
Mari'n, 2005a	New formulation for multiple allocation capacitated case
Mari'n, 2005b	Facet defining valid inequalities for multiple allocation capacitated problem
Topçuoğlu et al., 2005	GA for single allocation uncapacitated problem
Mari'n et al., 2006	Relaxing the triangle inequality for multiple allocation uncapacitated problem
Cunha & Silva, 2007	Hybrid of GA and simulated annealing
Chen, 2007	Hybrid heuristic of simulated annealing and TABU search
Canovas et al., 2007	Implementation of dual ascent technique with B&B algorithm
Costa et al., 2007	Bi-criteria approach and BSAHLP-1 & BSAHLP-2 formulations
Camargo et al., 2008	Benders decomposition based algorithm
Silva & Cunha, 2009	Three variants of multistart TABU search and two stage integrated TABU search heuristics

2.1.3 The p-Hub Center Problem

The p-hub center problem is analogous to the p-center problem which is min-max type one to minimize the adverse effects of the worst case. There are *n* nodes and the demand between each origin-destination pair with the number of hubs to be opened, *p*, are given. The aim is to locate the hubs and allocate the non-hub nodes to these hubs with the objective of minimizing the maximum route length or maximum cost. Center type problem formulations are important both because of applications and worst case scenarios, like emergency service facilities locations or maximum travel times (Campbell, 1994b).

Campbell (1994b) first formulates and discusses the p-hub center problem in the literature. He examines the origin-destination path by treating the links in a path; origin-to-hub, hub-to-hub and hub-to-destination links; separately. According to this separation, he studies three types of p-hub center problem in terms of the link considered. In the first type of the p-hub center problem, the maximum cost (length) from a origin node to a destination node is minimized. A second type is minimizing the maximum cost (length) of any of the three single links mentioned above. In the third type of the problem, which is called vertex centers, the maximum cost (length) for a link between a hub and a non-hub node is taken into account. Campbell presents the first formulations of single and multiple allocation versions of all these types of problems.

Kara and Tansel (2000) study the single allocation p-hub center problem of Campbell's first type problem which has the objective of minimizing the maximum cost (length) of any origin-destination pair. They first give a combinatorial formulation of the problem and prove that it is NP-hard.

Kara and Tansel (2001) focus on models that are especially suitable for cargo companies. They propose new models for the latest arrival hub location problem. In those models, the time spent at the hubs called the transient times is included, since these times are significant for cargo delivery systems. They propose single and multiple allocation versions of minisum, minimax and covering type latest arrival hub location problem. They focus on the single allocation version of minimax type latest arrival hub location problem and show that this problem is NP-Hard. Later, Wagner (2004a) shows that the critical paths of the minimax type latest arrival hub location problem and of the p-hub center problem that ignores transient times are the same, if the objective functions depend just on the maximum travel time. However, the minisum type latest arrival problem is different. He also shows that the minimax type latest arrival hub location problem outperforms the p-hub center problem in terms of CPU time required.

Pamuk and Sepil (2001) propose a single exchange heuristic and superimpose a tabu search to prevent being trapped by the local optima. Three intuitively appealing heuristics are used for the locations of the initial hubs. They study the p-hub center problem with single allocation and strict hubbing policies.

Ernst et al. (2002a) develop a new formulation for the single allocation p-hub center problem which has more variables but fewer constraints than the formulation of Kara and Tansel (2000). When Ernst et al. (2002a) compare the two formulations, they see that their formulation requires less CPU time than the formulation of Kara and Tansel (2000). Ernst at al. (2000a) also provide two new formulations for the multiple allocation p-hub center problem and prove the NP-hardness of it. They develop a heuristic method for the single and multiple allocation of the p-hub center problems. Ernst et al. (2002b) focus on the allocation subproblem of the single allocation p-hub center problem for fixed hubs. They present linear programming formulations for this subproblem and prove the NP-Hardness of it. In addition, they propose five heuristics for the allocation subproblem and analyze the worst case performances of the heuristics.

Hamacher and Meyer (2006) propose a procedure to solve the uncapacitated single allocation p-hub center problem with a binary search algorithm

BS(HcoP) which is based on the inverse relationship of the p-hub center problem and the hub covering problem . The feasibility polyhedron and many classes of facet defining valid inequalities are analyzed. The two most efficient hub covering formulations and the radius formulation of USApHCP are compared with each other. Although all algorithms find the optimal solution, BS(HcoP) performs better mostly with respect to the computational time.

Kratica and Stanimirovic' (2006) propose a genetic algorithm with binary coding for the uncapacitated multiple allocation p-hub center problem. They construct and implement problem-specific genetic operators in their genetic algorithm. The computational studies on the CAB and the AP data sets indicate that their genetic algorithm obtains all solutions that have been proved to be optimal in a reasonable amount of time.

Juette et al. (2007) study the polyhedral analysis of the uncapacitated single allocation p-hub center problem. The analysis is based on a radius formulation of Ernst et al. (2000a) and they show which of the valid inequalities in the formulation are facet-defining and which present non-elementary classes of facets for which they propose separation problems.

Yaman et al. (2005) analyze the latest arrival hub location problem with stopovers and they include the transient times, the time spent for unloading, sorting and loading at hubs in their model. The model is strengthened with some valid inequalities. The model is verified with CAB data set and tested with a data set constructed from the Turkish map.

Gavriliouk (2008) considers aggregation heuristic procedures for the hub location problems and calculates bounds for errors from such heuristics. The computational performance of the heuristic is tested on the AP data set and a randomly generated, and it is shown that the heuristic results in shorter times than solving in CPLEX.

Meyer et al. (2008) present an exact 2-phase algorithm. In the first phase, a set of potential optimal hub locations is computed with a shortest path based B&B algorithm and in the second phase, allocation phase, the optimal allocations are computed accordingly. They also develop an ant colony optimization heuristic for the upper bound needed for the B&B. The solution approach is shown to be significantly faster than MIP solver and CPLEX using AP data set and a newly generated data set. They are able to provide exact solutions to the single allocation p-hub center problems up to 400 nodes.

Sim et al. (2009) present the stochastic p-hub center problem with chance constraints. He proposes a two stage heuristic approach. In the first stage of the heuristic, an initial feasible solution is generated by a radial heuristic. And then, in the second stage of the heuristic, the initial solution is improved by the one-opt best-improvement heuristic which is proposed by Teitz and Bart (1968). Computational results show that in many this heuristic ends up with better objective function values than that of the Teitz-Bart heuristic with randomized initialization.

Ernst et al. (2009) prove both the uncapacitated single and multiple allocation p-hub center problems are NP-Hard. Even the single allocation sub-problem is shown to be NP-Hard. They formulate both problems and propose a shortest path based B&B approach for the multiple allocation problem. The numerical experiments with CAB and AP data sets show that their new formulation for USApHCP is superior to Kara and Tansel in terms of computational time. Also the shortest path based B&B method is shown to be extremely efficient for solving UMApHCP.

In Table 2.4, we summarize the literature on the p-hub center problem.

Table 2.4 Summary of the p-Hub Center Problem Literature

Source	Characteristics
Campbell, 1994b	First formulations with considering links separately
Kara & Tansel, 2000	Proof of NP-Hardness of single allocation p-hub center problem
Kara & Tansel, 2001	Latest arrival hub location problem
Pamuk & Sepil, 2001	TABU search imposed single exchange heuristic
Ernst et al., 2002a	Proof of NP-Hardness of multiple allocation p-hub center problem
Ernst et al., 2002b	Proof of the NP-Hardness of the allocation subproblem of single allocation p-hub center problem with the given hubs
Hamacher & Meyer, 2006	A binary search algorithm, called BS(HcoP), for the uncapacitated single allocation problem
Kratica & Stanimirovic, 2006	GA for the uncapacitated multiple allocation problem
Juette et al., 2007	Polyhedral analyses of uncapacitated single allocation p-hub center problem
Yaman et al., 2007	Latest arrival hub location problem with stopovers
Gavriliouk, 2008	Heuristic procedures based on aggregation
Meyer et al., 2008	Optimal hub locations using a shortest path based B&B and optimal allocations based on reduced size allocation; and ant colony optimization heuristic
Sim et al., 2009	Stochastic p-hub center problems with chance constraints
Ernst et al., 2009	Shortest path based B&B algorithm and proof of the NP-Hardness of the problem

2.1.4 Hub Covering Problems

In the hub covering problems, there are *n* nodes and the flow between each pair of nodes is given. In the hub covering problem the aim is to find the minimum number of hubs (which is essential to find the minimum cost of establishing hubs) in order to satisfy a given service level.

In the set covering hub problem, the aim is to determine the number and places of hubs and the route an origin node and a destination node (in other words, the assignment of the remaining nodes to the hubs) so that all demand is covered and the total fixed cost of the hubs is minimized. In the maximal covering hub problem, the number of hubs to be opened is given as *p* and the objective is to maximize the covered demand.

Campbell (1994b) first defines and formulates the hub covering problems. He defines three coverage criteria. The first criterion is that the cost of the complete path from node *i* to node *j* via the hubs *k* and *m* should not exceed a predetermined value then the origin-destination pair *(i,j)* is assigned to hubs *k* and *m*. According to the second coverage criterion, the origin-destination pair *(i,j)* is covered by hubs *k* and *m* if the cost for each link (non-hub nodes to hubs or hub to hub links) does not exceed a predetermined value. According to the third coverage criterion, the origin destination pair *(i,j)* is covered by hubs *k* and *m* if the cost of each of the origin-hub and hub-destination links does not exceed separately the specified values.

Kara and Tansel (2003) prove the NP-hardness of the single allocation hub-set covering problem. They provide three different linearizations of the quadratic formulation of Campbell (1994b) and propose a new formulation of the problem and a linearized version of it called HC-Lin. The comparison of these four linearizations shows that the performance of the linearization of their model is superior to the performances of the linearizations of Campbell (1994b).

Wagner (2004b) reduces the formulation of Kara and Tansel (2003) with preprocessing techniques, thus reduces the number of variables and constraints of formulation with preprocessing techniques. Furthermore, by the aggregating some of the constraints, he improves his formulations.

Ernst et al. (2005) propose a new formulation based on a concept of cover radius β which is similar to the p-hub center problem formulation of Ernst et al. (2002a). Also, two new formulations and an enumeration based algorithm are proposed for the multiple allocation hub set covering problem in their study.

Hamacher and Meyer (2006) analyze the facet-defining valid inequalities for the hub set covering problem. They propose a procedure for solving the uncapacitated single allocation p-hub center problem by solving the hub covering problem for a given β combined with binary search.

Sim (2007) introduces the hub covering flow problem motivated by the air travel industry considering the amount of flow at the arcs as well. For the coverage criterion, the distance of each of the links between the hub nodes and non-hub nodes are considered. Each non-hub node is allocated to a single hub. He proposes two formulations. In the first formulation, coverage requirements are included in a constraint. In the second formulation, coverage requirements are included in the objective. In both formulations, only one aircraft-type is assumed to operate out of a hub. He also performs computational tests and sensitivity analysis for the hub covering flow problem using the AP data set. Sim (2007) states that alternative solution approaches should be proposed for the hub covering flow problem other than using commercial solvers especially for large problems.

Tan and Kara (2007) study the latest arrival hub covering problem for cargo delivery systems. In this problem, the departure from a hub does not occur until the latest arrival to the hub from the nodes that it serves or from the other hubs. So these departure times are in the model and a linear model called Latest Cover is written. Then, two variations of this model called Latest Cover-1 and Latest Cover-2 are presented. In Latest Cover-2, weights for each possible hub location are included in the model. In latest Cover-2, a tight service level for some cities and an extensive service level for the rest are imposed in the model. Tan and Kara (2007) test their models on the 81-node Turkish postal network and also on the CAB data set. Furthermore, they incorporate a limit on the driving time of a driver as a result of a legislative requirement and improved the CPU time of the models, but the number of opened hubs increased.

Alumur and Kara (2007) focus on a single allocation hub covering problem for cargo applications in Turkey. They relax the complete hub network assumption and a hub may not be directly linked to all other hubs in their model. However, the number of hubs that can be visited on a route is limited by three, because of safety reasons and the geographical structure of Turkey. The possible waiting times at the hubs are taken into consideration by their model. Each demand from an origin to a destination should be satisfied within a time limit. Firstly, a nonlinear MIP called 3-stop-0 is written to find the location of the hubs, allocation of the nodes to the located hubs and ready time of each hub in order to minimize the fixed hub costs and hub-to-hub link costs. Then a linearized version of it called 3-stop is written. Alumur and Kara (2007) solve their linear model by CPLEX 8.1 using data from the Turkish network and the CAB data set. They use the tightest possible time limits in their test data. In spite of this, their model gives optimal solutions in reasonable CPU times with both the Turkish network data and the CAB data set. They observe that the solutions are usually insensitive to the hub-to-hub link costs, although the low link costs require less CPU time. Another observation of them is that their model produces incomplete hub networks in most of the cases despite using the tightest possible time limits in their test data.

Wagner (2007) proposes a formulation for the single allocation hub set covering problem for the case where the discount factor between any two hubs is independent of the quantity transported between them. In this formulation, he uses some preprocessing techniques to determine the valid and incompatible assignments. In this way, he eliminates some assignments and decreases the number of variables and constraints compared to the formulation called HC-Lin of Kara and Tansel (2003). This formulation is called SAQI-W1. Then, he obtains the formulation called SAQI-W2 of the same problem by aggregating some constraints of SAQI-W1. Nets, a formulation of the single allocation hub set covering problem for the case where the discount factor between any two hubs is dependent of the quantity transported between them is given by Wagner (2007). This formulation is called SAQD-W. Wagner (2007) also proposes a formulation for the multiple allocation case of the same problem for the case where the discount factor between any two hubs is independent of the quantity transported between them. In this formulation, the number of constraints and variables are reduced compared to the formulation of Campbell (1994b) for the same problem with the help of using valid route sets. This formulation is called MAQI-W.

Calik et al. (2009), in their study, allow their model to have incomplete networks in the hub level and impose time restrictions on the delivery times. They develop a linear MIP for the single allocation hub covering problem with these constraints and configuration, and propose a metaheuristic based on the tabu search. The tabu search based heuristic is tested using data from the Turkish network and also the CAB data set.

Alumur et al. (2009) focus on the design of single allocation incomplete hub networks. They relax the complete hub network assumption and a hub may not be directly linked to all other hubs in their model like in Calik et al. (2009). They give an O(n3) formulation for the problem. They also propose four valid inequalities to increase the exact solution potential. Also, Alumur et al. (2009) propose that they are able to solve the largest instances of incomplete hub networks to optimality in the literature up to that time. Alumur et al. (2009) also provide O(n3) formulations for the single allocation incomplete p-hub median problem, hub location problem with fixed costs and p-hub center problem.

Qu and Weng (2008) focus on the multiple allocation hub maximal covering problem introduced by Campbell (1994b). The objective of the hub maximal covering problem is to maximize the covered demand while determining the places of hubs and the route of each demand denoted by an origin node and a destination node. They provide a new model for this problem and propose an evolutionary approach based on Path Relinking in order to solve it.

The Table 2.5 summarizes the hub covering problem literature.

Table 2.5 Summary of the Hub Covering Problem Literature

Source	Characteristics
Campbell, 1994b	First definition of the problem
Kara and Tansel, 2003	Proof of the NP-Hardness of the single allocation case and 4 linearizations of the problem
Wagner, 2004b	Reduction in the number of variables and constraints by preprocessing techniques
Ernst et al., 2005	New formulation based on the cover radius β
Hamacher & Meyer, 2006	Analysis of feasibility polyhedrons and facet defining inequalities
Sim, 2007	Hub covering problem motivated by the air travel industry
Tan & Kara, 2007	Latest arrival hub covering for cargo delivery systems
Alumur & Kara, 2008b	Hub covering problem for cargo delivery applications in Turkey
Wagner, 2008	New formulation for the single allocation case where the discount factor is independent of the quantity
Qu & Weng, 2008	Evolutionary approach based on path relinking for multiple allocation case, given p
Çalık et al., 2009	TABU search based heuristic relaxing the complete hub network assumption

2.1.5 Other Hub Location Studies

Kuby and Gray (1993) study the hub network design problem with stopovers and feeders. In the model, multiple node visits are allowed before arriving a hub and after departing from a hub and each such visit is called a stopover.

Considering the models for the capacitated airline networks in their study, Jaillet et al. (1996) do not force hubs to occur in their solutions; only when it is found to be cost efficient, the model results in a network with hubs.

Unlike most of the studies, O'Kelly and Bryan (1998) adopt a non-linear cost function which allows costs to change with the traffic on hub-to-hub links.. Then Bryan (1998) extends the formulation presented in O'Kelly and Bryan (1998) by adding capacities and flow thresholds for minimum flows on hub-to-hub links. She also extends the flow dependent discount factor to all links in the network, not just to hub-to-hub links. Horner and O'Kelly (2001) state that economies of scale can be seen on all links in a network and so they apply a piecewise-linear concave cost function like Bryan (1998).

Marianov et al (1998) propose a model called the hub location competitive model (HuLC). In that model, if the cost of satisfying the demand of an origin-destination pair for a firm is less than the cost of competitors, that demand is completely captured by that firm. The aim of the model is to maximize the sum of the captured demand for a firm. The number of hubs to be opened is given as p and the location of hubs, the flows that are captured and the route of the captured origin-destination flows are determined in the model. Then the model is refined for the case where the number of hubs to be opened is not pre-specified. Another modification is done for the case where relocation and/or addition of hubs are possible. A further modification is done for the case where capture of some fraction of a demand between two nodes is possible. A tabu search heuristic is proposed for the solution of the model and it is evaluated using randomly generated data as well as the AP data set. The heuristic obtains the optimal results in all test runs.

Drezner and Drezner (2001) propose a model for the airline hub selection problem. In their model, p numbers of hubs are opened in order to minimize the total miles travelled by passengers. Passengers use one hub on the way to their destination. They test their model by changing various parameters and comment on the results.

Nickel et al. (2001) present a network design problem for the urban public transportation networks. They relax the complete network assumption and impose a fixed cost for locating hub arcs in their model.

Sung and Jin (2001) partition the nodes into clusters. For each cluster, one node in the cluster should be determined as a hub for that cluster. Direct links between nodes within the same cluster are allowed in the model and the aim is to minimize the total fixed cost of locating hubs and the total transportation cost. Later, Wagner (2007) shows that the model of Sung and Jin (2001) is NP-hard and he proposes a new mixed integer programming formulation for that problem. There are fewer variables in the formulation of Wagner (2007) than the formulation of Sung and Jin (2001). He also proposes a constraint programming approach and some preprocessing techniques for using MIP solvers for that problem.

Klincewicz (2002) focuses on a generalization of the multiple allocation p-hub median problem where the unit cost of transportation between two hubs is dependent on the amount transported between them. Klincewicz (2002) proposes an enumeration method, a tabu search heuristic and a greedy random adaptive search procedure (GRASP) for this problem. He evaluates his solution procedures on the CAB data set and shows that the optimal hub locations depend on the cost function used.

O'Kelly and Bryan (2002) examine the optimal locations of the interactive facilities. They analyze the behavior of solutions to several alternative models.

Podnar et al. (2002) propose a network design problem in which hubs are not located. Instead, they determine the flows on the links and impose reduction on the unit transportation costs by a factor α, for the flows larger than a specified threshold value on a link.

Zapfel and Wasner (2002) analyze a hybrid hub-and-spoke system for a single product. The places of the hub and the non-hub nodes are fixed. There are two types of trucks with different capacities. First, they focus on the line haul problem. In this problem, both direct transports between non-hub nodes that do not use hubs and transports between non-hub nodes that use hubs are allowed. The aim is to determine the number of the two types of trucks used and the quantity transported between any two non-hubs and hubs and non-hubs, so that the total transportation cost is minimized. For the solution, a partial enumeration heuristic is developed that first decreases the number of possible solutions using rules of the expert system and then does enumeration among the remaining possibilities. Then, combined line haul and pickup/delivery problem is formulated that also takes the pickup and delivery tours between non-hub nodes and customers into consideration.

Lin et al. (2003) study the economic effects of hub and spoke networks with center to center directs. They model the hub-and-spoke network with center directs in the paths and their model determines the fleet sizes, fleet routes and freight paths so that the total costs are minimized while the service restrictions and operational restrictions are satisfied. They solve their model using CPLEX and compare their results with the Federal Express AsiaOne express network.

Carello et al. (2004) focus on the hub location problem in telecommunications network design. There are some access nodes that represent origins and destinations of traffic that are not connected directly. Transit nodes (hubs) are chosen among some candidate nodes and the hubs are fully connected. Hubs are capacitated. The problem is to select the number and location of hubs and the allocation of non-hub nodes to hubs in order to minimize the total cost. Cost components are the fixed cost of opening hubs and the installation cost of the capacity on the edges needed. Three different metaheuristics are developed for this problem, which are tabu search, iterated local search and random multistart. The most promising metaheuristic is found to be the tabu search as a result of the computational experiments.

Campbell et al. (2005a) present a model in which the locations of the hub arcs are determined. Campbell et al. (2005b) present integer programming formulations for the four different cases of the hub arc location problem and two optimal solution algorithms for them, one being the enumeration-based optimal algorithm and the other using the commercial MILP solver CPLEX 6.6.

In Kimms (2005) all the links in the network are assumed to have the possibility of having economies of scale like Bryan (1998) and Horner and O'Kelly (2001). His model has a piecewise-linear concave cost function and fixed cost for the usage of each link.

Yaman (2005) provides polyhedral analysis for the problem called the uncapacitated hub location problem with modular arc capacities. The problem is to locate the hubs, assign non-hub nodes to hubs and install capacities on the arcs so that the total cost of installing the hubs and capacity units on the arcs are minimized.

Yaman and Carello (2005) consider the problem called the capacitated hub location problem with modular link capacities. The hubs have a capacity which represents the maximum amount of traffic that can be routed through the hub. Integer-valued capacity units are installed on the edges. The problem is to locate the hubs, assign each non-hub node to a hub and install capacities on the edges so that total hub and link costs are minimized and capacity constraints are satisfied. They formulate this problem as a quadratic mixed integer program and then linearize it. In order to solve the problem, an exact branch-and-cut algorithm and a two-level local search metaheuristic are developed. The branch-and-cut algorithm taking the upper bound from the two-level local search metaheuristic and the combination of the two solution approaches are applied on a number of test instances.

Sim (2007) addresses the two-stage stochastic p-hub center problem for the communication networks. In the first-stage, the best locations for p hubs in the network are found and in the second stage, each node is assigned to only one of the given hubs in order to minimize the longest connection path in the network. He gives a formulation of the problem and uses a radial heuristic to solve the first stage and CPLEX to solve the second stage. He also proposes a simulated annealing heuristic to solve large problems. He performs computational tests on the CAB and the AP data sets for these solution approaches.

In this study we also address the p-hub center problem with stochastic service level constraints. Our study differs from theirs in two aspects:

- Objective function:
    - Sim (2007) defines the objective function as the minimization of the service level β (maximum travel time that should be satisfied with probability γ)
    - We define the objective function as the minimization of the maximum weighted travel time less than a given service level β that should be satisfied with probability γ at least for specified node pairs.

- Solution approach:

  - Sim proposes a 2-stage approach in which the two decisions, location and allocation, are made sequentially; first location of hubs, then allocation of non-hubs to hubs.
  - We solve these two problems simultaneously using a genetic algorithm.

CHAPTER 3

3. MATHEMATICAL MODELING OF THE p-HUB CENTER PROBLEM WITH SERVICE LEVEL CONSTRAINTS

In this chapter, we give the general description of the p-hub center problem and we define our problem characteristics with the assumptions in this study, followed with the related mathematical model of our formulation.

3.1 The p-Hub Center Problem

Airline systems, cargo delivery systems and telecommunication networks are some application areas of hub location problems. We mainly focus on the cargo delivery system and the Turkish network of towns in this study.

p-hub median problem, whose objective is the minimization of the total cost, is exclusively focused on among the existing literature (Kara and Tansel, 2000). Although the minimization of the total cost is a worthy objective, the minimax criterion is still very crucial in terms of minimizing the worst delivery time in cargo delivery systems, especially for delivery of perishable and time-sensitive items. In this competitive environment, cargo delivery companies need to offer reliable deliveries and offer guarantees on the delivery times. The companies can decide on what service guarantees they can offer according to their network designs. Among the hub location problems, the p-hub center problem is the most suited model in finding out the maximum time limit to be given to the customers for the deliveries.

The p-hub center problem, in general, has the following characteristics:

- There are *n* nodes in the network which are the origin-destination points,
- The distance (or travel time) between the nodes are given and satisfies the triangular inequality,
- The hub level network is complete,
- The number of hubs to be located is given as *p*,
- There is a discount on the hub-to-hub links incorporated with a discount factor α.

The decisions to be made are the locations of the hubs and the allocation of the non-hub nodes to the hubs with the objective of minimizing the maximum cost or the maximum travel time of the network.

p-hub center problem, having components of both location and allocation problems and both uncapacitated multiple and single allocation p-hub center problems, are proven to be NP-Hard (Ernst et al., 2009).

3.2 The Characteristics of the Problem Environment

Besides the general definition for the p-hub center problems above, we define our problem environment characteristics as follows:

Characteristics of the p-Hub Center Environment:

- The travel time is more crucial than the cost of the delivery, for the cargo delivery firms, hence the objective function is to minimize the maximum travel time of the paths between the origin-destination pairs.
- Cargo firms take into account mainly the amount of flows between the node pairs in decision making processes. In Turkey, although it may not be a long travel time between any origin-destination, it is highly likely that only a single delivery in a week occurs because of the low volume

of flow. Using only the travel time means giving equal importance for the node pairs having the same travel time. However, when we have two different node pairs having the same travel time, the one with the higher volume of flow will be more crucial for the cargo delivery firms. In order to reflect the effect of the flow density on the solution, the travel times between the node pairs are weighted with the flow weight among of the node pairs, as an attempt to give more importance to the nodes with high interactions.

- Single allocation of the nodes to the hubs is assumed; moreover no direct transportation between the non-hub nodes is allowed. As a result every node should be assigned to one and only one hub.
- In the study, there exist no capacity restrictions for the hubs and the arcs. So there is no limit on the number of nodes that can be assigned to a hub.
- In real life, the problem environment of the hub location issues is not deterministic as assumed in the mathematical models. The flow between the nodes may vary as a result of special days or events, like Christmas or the travel time between the nodes may vary because of the weather conditions. For representing the consequences of the variability in the problem environment, we have used the idea of stochastic service level constraints of Sim et al. (2009). As in the study of Sim et al. (2009), we try to introduce the variability in travel times only, since we deal with the travel times in the objective function when designing the hub network of our study and since we do not have any capacity restrictions. The travel times among the nodes are assumed to be uncertain and distributed with the normal distribution and the real travel times are taken as the mean of the travel times.
- Guaranteed overnight delivery has become an important condition for the high market shares. Customers want to have rapid and reliable deliveries. However it is not possible to guarantee overnight delivery

for all the node pairs. First of all, the travel time among the node pairs should be in a reasonable limit. Moreover, to be cost effective for the firm, the node pairs should have an interaction in between higher than a threshold level. For observing the resulting effects, we define a service level target, which can be 24-hour for overnight delivery case, to be achieved. As mentioned, this target cannot be reasonable for all node pairs because of both physical and financial reasons. We also define, $\boldsymbol{F_{ij}}$, a lower limit on the flow between a non-hub-node pair *i* and *j*, and $\boldsymbol{T_{ij}}$, an upper limit on the mean travel time between the non-hub node pair *i* and *j* that identifies the targeted service level applicability. Sim et al. (2009), in their study, try to find the minimum travel-time guarantee for the whole network with a predetermined level of probability with chance constraints. Instead of targeting to find a travel-time guarantee to the whole network, we determine a travel time guarantee in advance for certain node pairs, and try to find the resulting network that achieves the predetermined travel time guarantee for the predetermined node pairs *i* and *j* by the limits $\boldsymbol{F_{ij}}$ on the flow and $\boldsymbol{T_{ij}}$ on the mean travel time.

- A restriction on the maximum travel times between a non-hub node and a hub node is imposed in our model as in Tan and Kara (2007), which is a reflection of a real life restriction on the driving time of a commercial driver. However, this restriction is not imposed on the links between the hub nodes, since larger special trucks with more than one driver are used on these hub-hub links.

3.3 Mathematical Modeling of the Problem

We develop a linear integer programming model for our p-hub center problem. We have *N* nodes and aim to locate *p* number of hubs among these *N* nodes with the objective of minimizing the maximum weighted travel time between the node pairs via the hubs. By using the weighted travel times in our objective function, we try to give more importance to the node pairs with larger flows. In Figure 3.1, we have a 1-hub network with 5 nodes. The links between the nodes A-D and C-B via hub E have travel time of 10 hours; however the flow between A and D is 100 units, while it is 10 units between C and B. It is obvious that for a cargo firm transferring 100 units of flow within 10 hours is much more important than transferring 10 units within 10 hours.

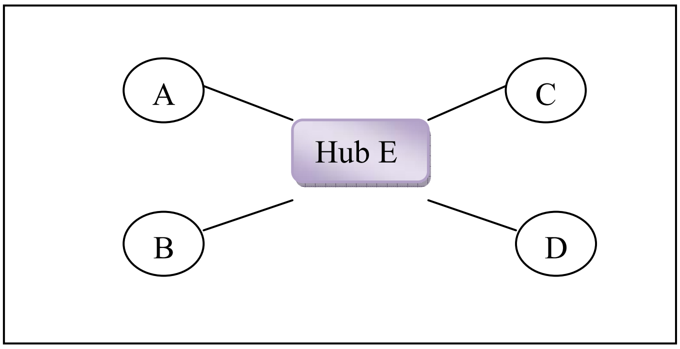


Figure 3.1 One-Hub Network with 5 Nodes

The center type location problems, as a course of their nature, may have alternative optimal solutions (Sim et al, 2009). When only the weighted travel times are used in the objective function without any constraints on the assignment of the nodes, only the maximum weighted travel time in the network becomes important. As a result other, locations and assignments do affect the solution in the model. However, with the service level constraint imposed on the specified nodes, we try to control the assignment of other nodes with high interactions and maintain a reasonable travel time between hub pairs. Moreover, the restriction on the maximum travel times on the node-hub links also eliminates some of the alternative solutions of the pure p-hub center problem and contributes to the enhancement of the resulting network structure.

We first describe the sets, parameters and decision variables, and then present the integer model for our p-hub center problem with service level constraints.

Sets

I set of nodes

K potential hub nodes

Parameters

F_{ij} amount of flow from origin node i to destination node j

W_{ij} weight of the flow between node i and node j: ratio of the flow between node i and node j to the total flow on the network

T_{ij} travel time (minute) on the link from node i to node j and $Tij \sim N(t_{ij}, \sigma_{ij}^2)$

t_{ij} mean travel time (minute) on the link from node i to node j

σ_{ij} standard deviation of the travel time on the link from node i to node j

p number of hubs to be located

α discount factor for the inter hub transportation

β targeted service level between the specified nodes

γ target service level probability

L restriction on the maximum travel time between a non-hub node and a hub

Q lower limit on the flow between any non-hub nodes for the service level constraint to be considered.

H upper limit on the mean travel time defined for any two origin-destination nodes for the service level constraints to be considered.

<u>Decision Variables</u>

$$X_{ijkm} = \begin{cases} 1, \text{ if there is a path from origin node } i \text{ to destination node } j \text{ via the hubs } k \text{ and } m \\ 0, \text{ otherwise} \end{cases}$$

$$Y_{ik} = \begin{cases} 1, \text{ if node } i \text{ is assigned to hub } k \\ 0, \text{ otherwise} \end{cases}$$

$$Y_{kk} = \begin{cases} 1, \text{ if hub } k \text{ is opened} \\ 0, \text{ otherwise} \end{cases}$$

Z= objective function value

<u>Model</u>

Min Z

Subject to

$$Z \geq W_{ij} \{X_{ijkm} (t_{ik} + \alpha\, t_{km} + t_{jm})\}, \qquad \forall i, j \in I \text{ and } \forall k, m \in K \quad (3.1)$$

$$P\{X_{ijkm} (T_{ik} + \alpha T_{km} + T_{jm}) \leq \beta\} \geq \gamma,$$

$$\forall (i, j, k, m) \text{ for } F_{ij} + F_{ji} \geq Q \text{ and for } t_{ij} \leq H \quad (3.2)$$

$$t_{ik} Y_{ik} \leq L, \qquad \forall i \in I \text{ and } \forall k \in K \quad (3.3)$$

$$\sum_k Y_{ik} = 1, \qquad \forall i \in I \quad (3.4)$$

$$\sum_k Y_{kk} = p, \qquad \forall k \in K \quad (3.5)$$

$$Y_{ik} \leq Y_{kk}, \qquad \forall i \in I \text{ and } \forall k \in K \quad (3.6)$$

$X_{ijkm} \leq Y_{ik} + Y_{jm} - 1,$ $\forall i, j \in I$ *and* $\forall k, m \in K$ (3.7)

$\sum_m \sum_k X_{ijkm} = 1,$ $\forall i, j \in I$ *and* $\forall k, m \in K$ (3.8)

$X_{ijkm}, Y_{ik}, \in \{0, 1\},$ $\forall i \in I$ *and* $\forall k \in K$ (3.9)

The constraint (3.1) satisfies the minimization of the maximum of the weighted travel times of the resulting network in the objective function. The travel times include the time for a complete link from a non-hub-node to another non-hub-node that constitutes all the links from a non-hub node (i) to a first hub, from the first hub (k) to the second hub (m) and from the second hub (m) to a non-hub node (j). The constant α in the equation corresponds to the discount on the travel time between the hub nodes k and m. Since in our study the travel times are uncertain, we use the mean values of the related times in the objective function. It can also be possible to construct this constraint such that instead of using the mean travel times, random values can be used. However, since we do not have distribution of the travel times of the network, we use the travel time data of the network as the mean travel times and calculate the related standard deviations through constant coefficient of variations (CV). This means that, whether using the mean travel times or random travel times for the objective function does not change the path that has the maximum weighted travel time. So, we conclude to use the mean travel times for the objective function calculations.

Constraint (3.2) requires that, for the non-hub-node pairs *i-j* that have a total flow greater than Q and that have a mean travel time lower than H; the probability of the resulting travel time from node i to node j via hubs k and m to be smaller than or equal to β, should be greater than or equal to the target service level probability γ. As mentioned before, the travel times are assumed to be normally distributed by $Tij \sim N\ (t_{ij},\sigma_{ij}^2)$; which results in $(T_{ik}+\alpha T_{km}+T_{jm}) \sim N(\ (t_{ik}+\alpha t_{km}+t_{jm}),(\sigma_{ik}^2+\alpha^2\sigma_{km}^2+\sigma_{jm}^2))$. The constraint (3.2) can then be expressed as;

$\beta \geq X_{ijkm}(t_{ik}+\alpha t_{km}+t_{jm}+z_\gamma\sqrt{\sigma_{ik}^2+\alpha^2\sigma_{km}^2+\sigma_{jm}^2})$ where z_γ is the z-value corresponding to the probability γ. This equation is used to substitute the constraint (3.2). We use the real travel times as the mean travel times and calculate the corresponding standard deviations from these mean travel times using the specified *CV* value.

Constraint (3.3) assures that the travel time between a non-hub node and a hub node does not exceed the specified restriction *L* and this constraint reflects the real life restriction on the maximum traveling time of a commercial driver.

Constraint (3.4) is the single-assignment constraint that guarantees the assignment of every non-hub-node to exactly one hub.

Constraint (3.5) guarantees that *p*- given as a parameter in the model- number of hubs are opened in the resulting network.

Constraint (3.6) prevents the assignment of a non-hub-node to a location where no hub is located.

Constraint (3.7) satisfies the existence of a path between an origin node and a destination node over two hubs, via only the hubs which the nodes are assigned to.

Constraint (3.8) guarantees a single path between an origin node and a destination node over two hubs.

Constraint (3.9) defines the variable types in the model as binary. Actually $X_{ijkm} \in \{0, 1\}$ constraint is satisfied in the model implicitly because of the single allocation requirement with no capacity restrictions.

3.4 Complexity of the Model

The p-hub center problems, both single and multiple allocation are proved to be NP-hard. Especially for the large problems, the number of possible location and allocation alternatives becomes very high. For instance, for an n-node network with p hubs; the number of possible hub locations is:

$$\binom{n}{p} = \frac{n!}{p!\,(n-p)!}$$

Moreover, for each hub location alternative, the model has $(n-p)^p$ allocation alternatives. Consequently, the number of total location and allocation alternatives for an n-node and p-hub hub center problem is:

$$\frac{n!}{p!\,(n-p)!}(n-p)^p$$

In the single assignment p-hub center problem, partial assignment is not allowed; so we have $(n^4 + n^2)$ binary variables, X_{ijkm} and Y_{ik}.

Although the p-hub center problem is NP-hard, some moderate size problems can be solved to optimality by the optimization software. We develop our p-hub center model in GAMS and utilize the cplex solver (Appendix A). By using the GMAS model, we can

- Validate our formulation, and
- Obtain the optimal solutions for the small-size problems at least to compare the performance of our GA based heuristic.

CHAPTER 4

4. THE GENETIC ALGORITHM-BASED METHOD

The p-hub center problems are proved to be NP-Hard (Ernst et al., (2009), even the allocation part of the problem is proved to be NP-Hard when the hub locations are given (Ernst et al., 2002b). Although the mathematical models guarantee the optimum solutions, only some small sized problems can be solved to optimality. However, the real life applications are usually large-sized ones and cannot be solved to optimality in a reasonable amount of time. Hence, heuristic solution approaches are developed. Greedy heuristics, tabu search, simulated annealing, lagrangean based heuristics and genetic algorithms are some of the heuristic approaches that have been developed for the location problems.

After reviewing the literature, we have decided to develop our own solution method based on the genetic algorithm (GA). In this chapter we explain GA-based method in detail. First, a brief review of the genetic algorithms with some basic concepts and definitions is given. Then, different phases and parameters of the proposed genetic algorithm are included in this chapter.

4.1 GA-Based Heuristics

Basically, GAs are one of the metaheuristic search methods based on the evolution process and natural selection principles in the nature with the concept of “survival of the fittest”. They are used for finding the near-optimal solutions by allowing solutions to evolve iteratively to good ones. GAs were first introduced by John Holland, his colleagues and students at the University of Michigan (Goldberg, 1989) and developed by Goldberg (1989). GAs can solve hard problems both quickly and reliably; they are easy to interface to existing simulation models and mathematical models, are extensible and they are easy to hybridize (Goldberg, 1994). In the last three decades, GAs have emerged as effective, robust optimization and search methods (Kratica et al., 2007). They have been successfully applied in solving a variety of optimization problems that are difficult to solve, including the travelling salesperson problem, job-shop scheduling problems, vehicle routing problems, airline crew scheduling problems, optimizing the sequence of advertisements within a commercial break at a British television station, and painting trucks at a General Motors production facility to name a few (Chaudhry et al., 2000). GAs are also used for solving the location and hub location problems such as simple plant location problem (Kratica et al., 2001), dynamic facility layout problem (Dunker et al., 2005), uncapacitated single allocation *p*-hub median problem-USApHMP (Kratica et al., 2007), uncapacitated hub location problem (Abdinnour-Helm, 1998), uncapacitated single allocation hub location problem-USAHLP (Topçuoğlu et al., 2005) and uncapacitated multiple allocation *p*-hub center problem-UMApHCP (Kratica and Stanimirovic, 2006).

It is important to design the genetic algorithms and its components based on the problem specific characteristics. The components of the GAs that must be defined properly are basically as follows:

- Representation of the chromosomes
- Parameter determination such as population size, crossover and mutation rates, maximum number of offspring to be generated
- Initial population generation
- Fitness function evaluation

- Evolutionary process of genetic operators, parent selection, crossover, mutation and replacement strategies.

4.2 The Proposed GA Method

The components of the GAs and how they are adopted to our *p*-hub center problem are explained in detail in the following sections.

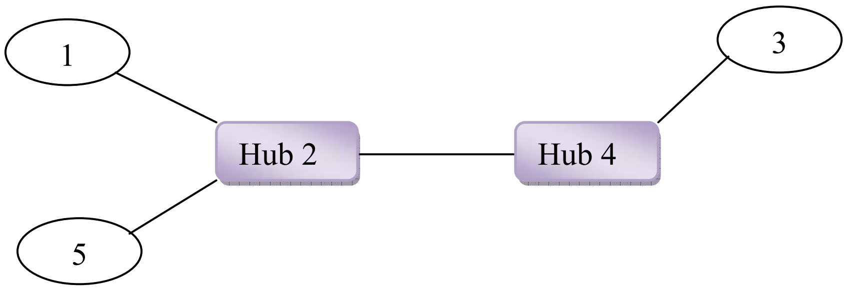
4.2.1 Representation of the Chromosomes

Chromosome representation is very critical for the success of the GA heuristic in terms of the ability to represent the possible solutions and to avoid the infeasible solutions in the population. For instance, in the early studies related to the *p*-median problem, representations which show only the open and closed facilities without restricting the number of facilities is used and this ends up with more or less facilities than the desired number of facilities (Özgönenç, 2006).

The chromosomes can be coded with two representation schemes: (i) binary scheme and (ii) non-binary scheme. In the binary scheme coding 0s and 1s are used and any integer value is converted to base 2 with bits of 0 and 1s. Since to apply most of the operators of GA is easy in binary coding, it is used for most types of problems.

However, in several problem environments, it is more appropriate to use integer representations in order to cover all the related information of the problem environment in a chromosome. In this case, more attention should be given while designing the problem-specific genetic operators according to the integer representation of the chromosomes.

In our study, we have chosen the non-binary coding for chromosome representation of our problem. For an *n*-node problem with *p* hubs, each chromosome of our GA method consists of *n* genes and each gene represents the nodes in our network. Each gene in the chromosome can take a value between *1* and *n* which shows the hub that the node is assigned to in that solution. A possible network scheme and the related chromosome coding for a feasible solution of 5 nodes and 2 hubs can be seen in Figure 4.1.



Node No	1	2	3	4	5
Assigned Hub No	2	2	4	4	2
Chromosome	2	2	4	4	2

Figure 4.1 Example: Chromosome Representation of the proposed GA

In the above example representation, we have 5 nodes, and 2 nodes are chosen among them as the hubs. The remaining non-hub nodes and the hub nodes themselves are assigned to these two hub nodes. As a result, nodes 2 and 4 are chosen as the hubs and nodes 1, 2 and 5 are assigned to hub node 2, and nodes 3 and 4 are assigned to hub node 4 in the example representation above. To represent the feasible solutions, the important points in our problem for the chromosome representation are that, the hub nodes chosen in each chromosome should be strictly equal to *p* and the chosen hub nodes should be assigned to themselves. These two conditions are satisfied while coding our GA method. The single assignment assumption of our *p*-hub center problem is satisfied implicitly with our chromosome representation.

4.2.2 Initial Population Generation

The search of the GA method starts with and continues by using the members of the initial population. The importance of the initial population is that, every possible solution to the problem can be attained from the solutions in the initial population by the genetic operators.

Usually, the initial population is generated randomly without any control on the diversity among the members of the population, so the individuals generated in this way may not cover the solution space entirely. So, the use of some control on the members of the population and the inclusion of problem specific knowledge to prevent a fully randomized population generation will provide some advantages.

The whole search space of our problem is very large especially for large problems, so narrowing the solution space according to our constraints is very crucial. Considering constraint (3.3) of our model, there are some assignments which can easily be shown to be infeasible. A non-hub node cannot be assigned to a hub having a travel time longer than L. So the potential hub locations that each node can be assigned to are obvious before the chromosomes are generated randomly. The non-hub-node pair i-j, having a total flow greater than the lower limit Q and having a mean travel time lower than H, that should satisfy constraint (3.2) are also known before the random generation.

After the elimination of some values using the information above, the values each gene in a chromosome can take are determined randomly. This process is repeated until the required population size is reached. During the population generation procedure, it is checked whether there is any duplication among the individuals in order to maintain the diversity in the population. If there are any duplicates, the copy of the existing individual is not included into the population and a new non-identical individual is inserted instead.

The population generation ends up with the predetermined number of feasible members without any duplication.

Another issue about the population generation phase is the population size. Small population size might prevent genetic variety in the population. On the other hand, a very large population size may result in long CPU times to solve the problem. So, it should be a moderate size so as to allow population diversity and to prevent long solution times. There are several strategies to determine the population size; according to a traditional approach, the population size can be (n/p) and according to another approach by Alp et al. (2003) the population size based on the density approach can be defined as:

$$P(n,p) = max\left\{2, \left\lceil \frac{n}{100} * \frac{\ln S}{d} \right\rceil\right\} * d$$

Where $S=\binom{n}{p}$ is the total number of possible hub locations for the problem, $d=\lceil n/p \rceil$ is the rounded density of the problem. In our GA method we use the second approach by Alp et al. (2003).

4.2.3 Fitness Function Evaluation

Although some other evaluation criteria can be used, the fitness function for a chromosome is generally the objective function value of the solution represented by the chromosome. As in many cases, the objective function values of the chromosomes are used as the fitness function value in our study. In our study, the objective function value is the maximum of the weighted travel times among all the node pairs, which is the summation of the weighted travel times from the nodes to the hubs that they are assigned to and the discounted weighted travel time between the hub pairs. The information of the assignments of the nodes to the hubs is taken by decoding a chromosome, and the related weighted travel times and the discounted weighted travel times are calculated by some intermediate calculations for each pair of nodes. The maximum of these calculated travel times for each node pair is taken as the fitness function value of the chromosome.

4.2.4 Parent Selection

After the generation of the initial population, the task of choosing parents to generate offspring that carries the properties from the parents as in the nature follows. There has been several parent selection techniques used in the literature. The selection techniques can be summarized as follows:

- Roulette-wheel Selection: Roulette is divided in slots and each individual possesses a part of the area of the roulette according to its fitness value. So, the individuals with higher fitness values have higher probability of selection.
- Stochastic Remainder Selection: In this selection method, a score is calculated for each individual in the population by dividing the fitness value of the individual with the average population fitness. Then each individual is replicated according to the integer part of these values and the fractional remainders are used as the probabilities of selection.
- Stochastic Universal Selection: Similar to the roulette wheel selection, again roulette is divided according to the fitness values. Equidistant markers are placed around the wheel as many as the number of required parents.
- Local Selection: Every individual is mated with an individual in the neighborhood defined with specific characteristics.
- Truncation Selection: All individuals are ranked according to their fitness values and only the fraction T (a predetermined threshold level)

best individuals can be selected having the same probability of selection.

- Tournament Selection: A number "Tour" of individuals are chosen randomly from the population and the individual with the best fitness value among these selected ones is chosen as a parent. The procedure continues until the required number of parents is chosen.
- Random Selection: Parents are selected randomly from the population and every individual has the same probability of selection.

In the nature, it is a fact that two genetically good parents yield a good child; it is not always true for the GAs. Sometimes two poor parents may generate an offspring with very good characteristics in terms of the solution to the problem. Although convergence of the algorithm takes time in the random selection method, the genetic diversity is kept in this method by giving the same importance to each chromosome. In this study we prefer to use random selection for our algorithm. Since the reproduction is repeated many times, many different individuals have the chance to be selected as parents and contribute to the diversification as well as the convergence of the population.

4.2.5 Crossover

Crossover is the phase where the genetic material, that is to say the information related to the problem, from two parents is combined to generate new offspring. Generally, either a single point crossover or two-point crossover is applied to the parents in GA applications. The parent chromosomes are split into two or three parts and these parts are exchanged between the parents creating new chromosomes. A classic two-point crossover is shown in Figure 4.2.

Parent 1	2	5	1	↓	7	11	15	8	↓	13	18
Parent 2	11	6	4	↓	12	17	9	2	↓	16	10
Offspring 1	2	5	1		12	17	9	2		13	18
Offspring 2	11	6	4		7	11	15	8		16	10

Figure 4.2 A Two-Point Crossover

Beside the traditional crossover explained, problem-specific knowledge can be inserted into the algorithm for the crossover phase. In our methodology, we design our own crossover for the *p*-hub center problem instead of using the traditional operators. After the parents are selected, the union of the hubs in both parents is taken and the common hubs repeated in both parents are taken as the frozen hubs for both children as in Alp et al. (2003). Say we have *n* nodes and *p* hubs in our problem, and m hubs are common. So we have a union of *(2p-m)* hubs and take *m* hubs as frozen. Then we need *(p-m)* more hubs to be selected for each child among the *(2p-2m)* remaining hubs in the union. In this case, we evaluate the alternatives for the hub selection; which yields C$\binom{(\boldsymbol{p}-\boldsymbol{m})}{(2\boldsymbol{p}-2\boldsymbol{m})}$ alternatives. For each alternative hub combination, we assign the hub nodes to themselves and evaluate the alternatives according to their fitness function. The two children with the better fitness functions are selected and replace the parents in the population. If no such child can be found with these parents, the parent pair is changed by the algorithm.

4.2.6 Mutation

Another component of the genetic algorithm that is used to maintain diversity in the population is mutation. It is used to avoid quick convergence or local optimum trap by randomly changing some existing genes of some chromosomes.

In our solution, to apply the mutation operator to our problem, a chromosome is selected randomly among the parents that do not contribute to the crossover part and one of the hubs of that chromosome is selected again randomly as in a standard mutation operator. This selected hub is replaced with the nearest non-hub node and the non-hub nodes assigned to the original hub are assigned to this new hub. At this step, we need to include some problem-specific corrections when needed. The necessary correction is done to assign this new hub to itself. For example, say the original hub node 2 is selected randomly and the non-hub nodes assigned to it are 1, 2 and 6. If the nearest non-hub node to 2 is 3, then instead of node 2, node 3 becomes the new hub and the nodes 1, 2 and 6 are assigned to it. Since node 3 was not formerly assigned to node 2, a correction is needed to assign new hub node 3 to itself.

4.2.7 Termination

Mainly, the GAs are terminated when a predetermined number of iterations is reached. One of the termination criterion used is $n\sqrt{(\boldsymbol{n}-\boldsymbol{p})}$ iteration counter. Purely using the predetermined number of iterations for termination may sometimes result in unnecessary computational time, since an exact convergence may occur earlier. For this reason, another criterion can be added to the algorithm for the successively repeated solutions.

4.2.8 The Overall Algorithm

The components of our GA method and how they are implemented are explained in detail; in the previous sections. The overall algorithm then can be summarized as:

(1) Read the problem data and generate the initial population with the determined chromosome representation and the feasible members according to the constraints without any duplication among the members.
(2) Evaluate the fitness function of each member in the initial population and record the best fitness value of the population,
(3) Apply crossover to the selected parents of the initial population. Check the feasibility of the offspring and check the fitness values of the offspring for inserting to the population.
(4) Apply mutation to the selected members of the initial population.
(5) Update the best fitness value of the population.
(6) Check the stopping criterion, if it is satisfied stop the algorithm, else to go to step 3.

The overall algorithm is also given in the following flowchart in Figure 4.3.

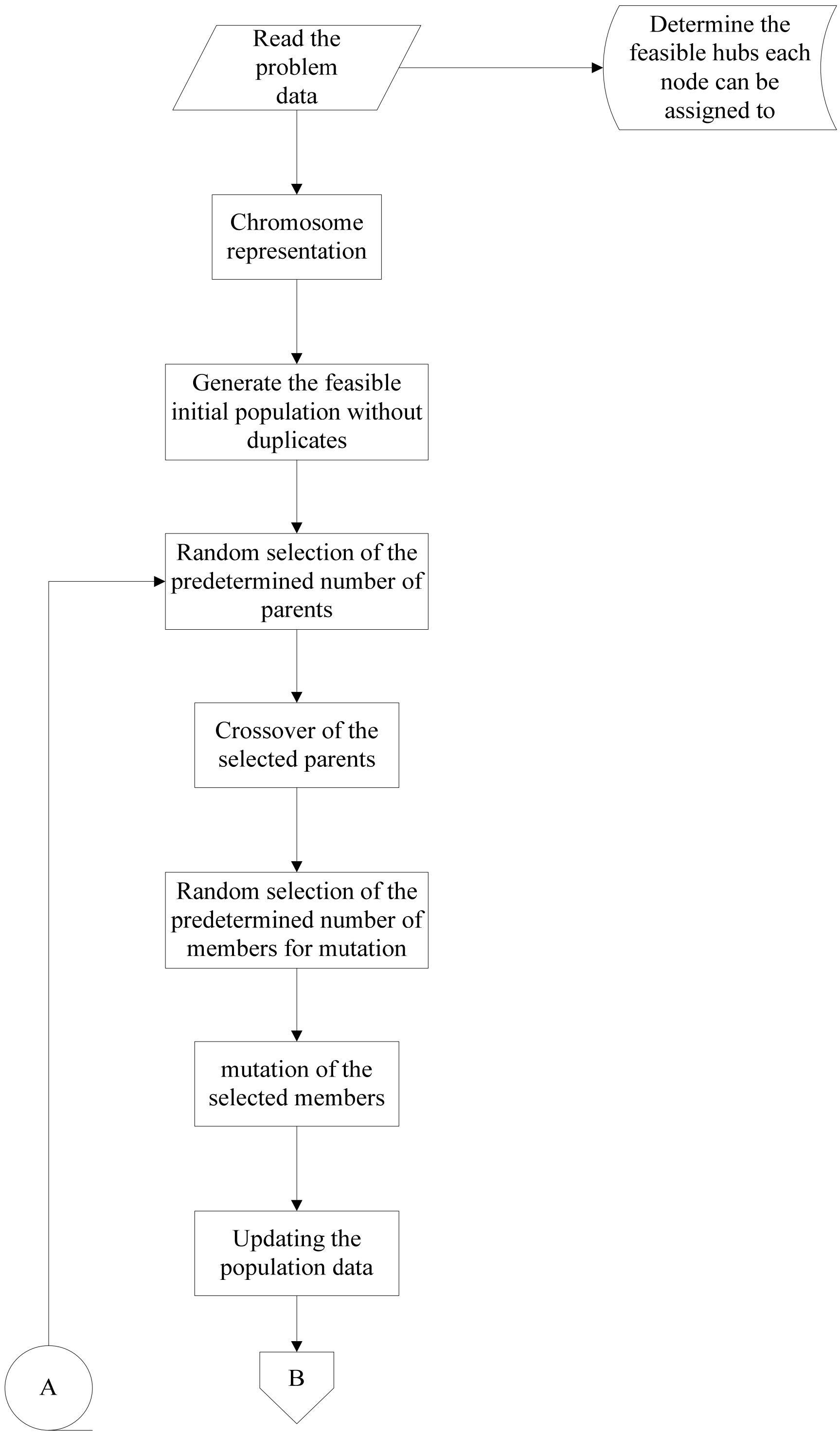


Figure 4.3 The Flowchart of the Algorithm

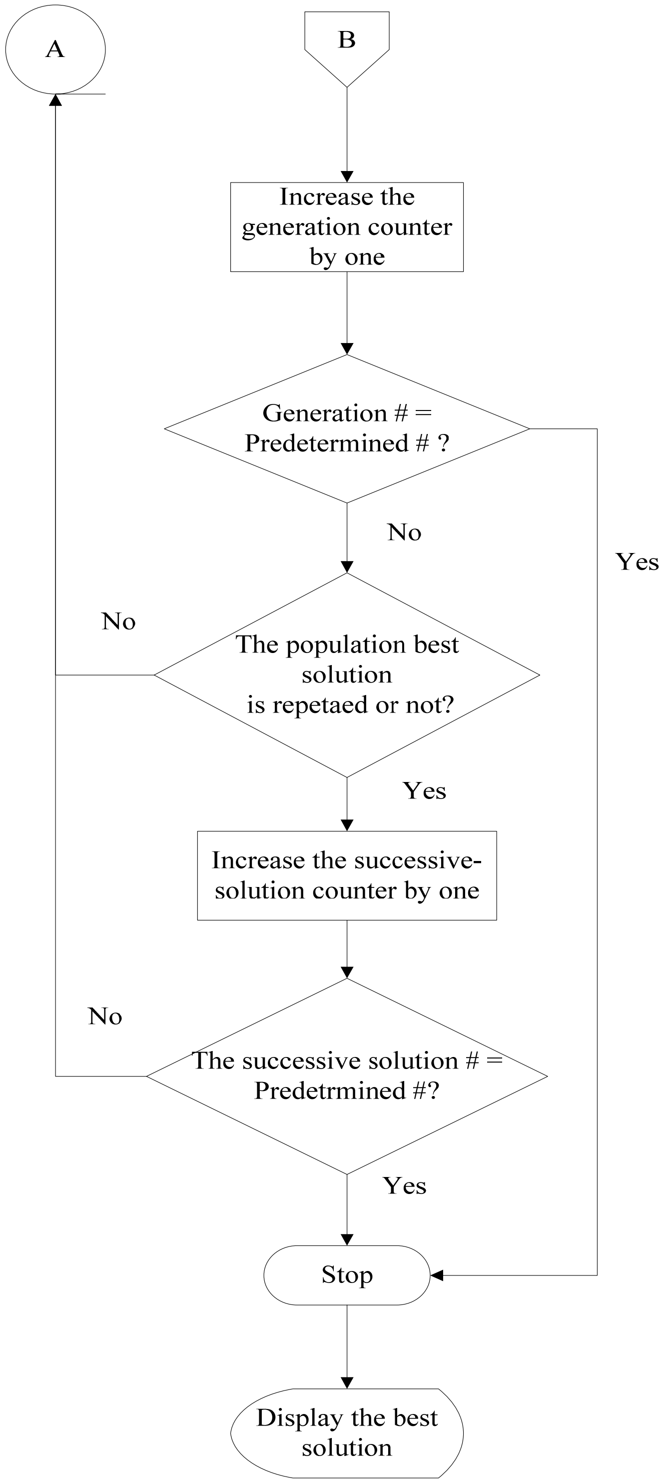


Figure 4.3 (cont’d)

CHAPTER 5

5. COMPUTATIONAL STUDY

In this chapter, the accuracy and the validity of the proposed GA method for our model are evaluated. This evaluation has three phases:

- Finding and generating appropriate problem instances
- Deriving the computational results with the proposed method
- Comparing the performance of the proposed method

5.1. Test Data and Problem Instances

As we describe, the elements of the problem environment that we deal with can be summarized as:

- Origin and destination node locations,
- Flow between each node pair,
- Travel time between each node pair,
- Potential hub locations.

As a result of our search on the test problems for hub the location problem, one of the most commonly used test problems are the CAB data set in which the network has 25 nodes with the flow data and distance data between each pair of nodes. We have used the distance data as the travel time data in our model.

Other than the CAB data set, the Turkish network data, with the distance and time data taken as in Tan and Kara (2007) and flow data taken as in Çetiner, Sepil and Süral (2006), is used for testing our solution approach.

The districts-cities with their node numbers, the flow data and distance data between each pair of nodes are given in Appendix C for both data sets.

We generate new problem instances using the CAB data set and Turkish network data set by four different ways:

(1) The first *n* entries of the data sets are taken in order to generate new *n*-node new networks.

(2) The last *n* entries of the data sets are taken in order to generate new *n*-node networks.

(3) *n*-nodes of the data sets are selected in order to generate new *n*-node networks.

(4) The *n*-nodes with the maximum flows are taken from the data sets in order to generate new *n*-node networks.

A problem instance can then be defined by the characteristics as:

- The number of nodes.
- The number of hubs to be located.
- The way the network is generated.

For instance, the problem instance "CAB_20_4(1)" is the problem instance of 20-node CAB data set with 4 hubs, generated by the way (i) defined above, which means that the network is generated by taking the first 20 nodes of the CAB data set.

5.2. The Evaluation of the Proposed Method

The GA-based solution method proposed should be evaluated in terms of the objective function values observed and the CPU time used. There are several ways to evaluate a proposed solution method such as;

- Comparing the solutions of the proposed method with the proven optimal values from the literature.
- Comparing the solutions of the proposed method with the best known solutions.
- Comparing the results from the proposed method with a lower bound or an upper bound.

The mathematical model proposed in Chapter 3, is coded in GAMS solver to get the optimal solutions (Appendix A). However, as mentioned before, the p-hub center problem is NP-hard, and with so many variables and constraints, large sized problems are hard to solve with mixed integer programming solvers.

Another point worth mentioning about the evaluation of the model studied is that, although there exist a number of studies in p-hub center literature, the problems studied are not exactly defined as our p-hub center problem.

When the conditions above are considered, we conclude that in order to evaluate the performance of our solution method, some heuristic solutions should be obtained for comparison. The model generated in Chapter 3 has got several binary variables, and the mathematical models with binary variables are hard to solve with the commercial solvers available. So, relaxing the binary variables Y_{ik} is a way to get an upper bound for our problem. When we relax the binary variables, actually the resulting problem becomes multiple allocation p-hub center problem; and the resulting assignment variables which will take on values between 0 and 1, will show the percentage of the flows that is assigned to a hub. However, relaxing binary variables is not enough for getting solutions for the large sized problems, since again multiple allocation p-hub center problems are NP-hard as mentioned in Ernst et al. (2002a).

Considering all of the above, in order to evaluate the performance of our solution method for the problems that cannot be solved by the GAMS solver, we have decided to use a sequential approach which handles the problem by solving two sub-problems sequentially. In this approach we first solve the constrained p-center problem for locating the hubs using GAMS and then determine the allocation of nodes to these given hubs. In the allocation phase of the approach; again we cannot use the single allocation p-hub center approach for given hubs; since even the allocation sub-problem of p-hub center problem is NP-hard when the hub locations are given (Ernst et al., 2002b). However, the single allocation phase is polynomially solvable, when $\alpha=0$ on a hub-complete graph (Campbell et al., 2007). So, for the location phase of the approach we use the p-center problem, and for the allocation phase we assume that $\alpha=0$ between the hubs and find the assignments of the nodes to the given hubs and call this overall approach as ALL. When α is assumed to be zero, this means the objective function of our model reduces to:

“$W_{ij}(t_{ik}\, Y_{ik} + \ t_{jm} Y_{jm})$”.

After finding the locations and allocations of the hubs and nodes using this approach, in order to be comparable with our objective function, we compute our original objective function which is $W_{ij}(t_{ik}\, Y_{ik} + \ \alpha t_{km} + t_{jm} Y_{jm})$ for the resulting network.

Another approach to get a solution to compare our method is the aggregation method. Aggregation is the assignment of several points to a single representative point (Gavriliouk, 2009). According to Gavriliouk, for an allocation problem that is difficult to solve in its original environment, the points in the original environment can be aggregated, and the same location problem can be solved in the aggregated environment with a less number of nodes. The solution of the aggregated environment can be transferred into the original environment. For finding another solution value we decide to aggregate our nodes. In the Turkish network data set, we apply an aggregation method according to the flow densities of the nodes and according to the travel times. In order to find a moderate sized network that we can model in GAMS, we order the nodes according to their flow densities in a descending manner, and determine the nodes that will be in the aggregated environment. Other nodes that will not be in the aggregated environment are assigned to the nearest node already determined to be in the aggregated environment. While doing the aggregation, we sum up the flows of the aggregated nodes. Then we use this aggregated environment to solve our original p-hub center problem. After finding the locations of the hubs and allocations of the aggregated nodes, we disaggregate the aggregated nodes and assign them to the hubs, we call this approach as AGG. Again as in the ALL heuristic explained above, we compute our original objective function value $W_{ij}(t_{ik} Y_{ik} + \alpha t_{km} + t_{jm} Y_{jm})$ for the resulting network.

As a result, for the small problem instances that can be solved to optimality with GAMS, we compare the performance of our solution method with the optimal results that we get from GAMS. For large problem instances that we cannot solve with the commercial optimization solvers, we use the results from the heuristic approaches AGG and ALL. We perform 10 runs with our GA method for each problem instance; the individual results of these runs for each problem instance are given in Appendix D.

5.2.1 Small Problem Instances

In this section we study on small problem instances from the Turkish network data set and CAB data set. The CAB data consist of 25 node network and we generate four different sized problem instances as CAB_25, CAB_20, CAB-15 and CAB_10. For the 20-node, 15-node and 10-node CAB data set instances we use the four generation methods mentioned above to generate new problem instances. On the other hand, the Turkish network data consists of 81 nodes and the whole network cannot be solved using GAMS. As a result, again we get small sized problem instances as Turk_30, Turk_25 and Turk_20 from Turkish network data set again by using the four problem generation methods.

We consider the results of the CAB data set and the Turkish network data set in the following sections below.

5.2.1.1 Small Problem Instances of the CAB Data Set

For the CAB data set, we construct four different sized problem instances. The parameters in the problem environments should be problem specific and should be determined accordingly. Because of the network structure of the CAB data set, we start our computational studies on CAB data set with the corresponding parameters:

- L=650 minutes
- H=960 minutes
- β=1,440 minutes
- Q=200,000
- α=0.90
- γ=0.95
- CV=0.80

However, for some problem instances with these parameters, the problem turns out to be infeasible because of the constraints. In these cases, we change the parameters one by one ceteris paribus if not mentioned, starting with the parameter γ and then the given *p* value.

The results obtained for the problem instances of the CAB data are given in Table 5.1.

For the problem instance, CAB_20_5(4), the problem is infeasible with the parameters above. So we decrease the γ value to 0.85 and then solve the resulting problem. Decreasing the probability level makes the problem feasible, because the higher the γ value, the higher the z_γ value; so the right hand side of the constraint (3.2) will be higher and it becomes larger than the specified β value for some node pairs:

$$\beta \geq X_{ijkm}(t_{ik} + \alpha t_{km} + t_{jm} + z_\gamma \sqrt{\sigma_{ik}^2 + \alpha^2 \sigma_{km}^2 + \sigma_{jm}^2}) \qquad (3.2)$$

For the problem instance CAB_15_4(2), the problem is again infeasible, and we decrease the γ value first to 0.85. However, the problem is still infeasible with these parameters. Actually the problem is infeasible even for γ=0.70. As a result, the given number of hubs is not sufficient to meet the service level constraint. We increase the *p* value from 4 to 5, other parameters being the same as they were at the beginning. With *p*=5 the service level constraint can be met and the problem becomes feasible. The results Table 5.1 are given for this problem instance as CAB_15_5(2).

The problem instance CAB_15_4(4) is again an infeasible problem for the given parameters. However, when the γ value is decreased to 0.85 the problem becomes feasible, and the results are given in Table 5.1 for this problem instance with γ=0.85.

Another infeasible problem instance with CAB data set is CAB_10_3(2), and it is even infeasible for γ=0.70. So, we increase the *p* value from 3 to 4; however the problem is still infeasible for γ=0.95. When the *p* value is increased to 5, the resulting problem become feasible and the solutions are given in Table 5.1 for CAB_10_5(2).

For the infeasible problem instance CAB_10_3(3), first we decrease the γ value until 0.70, but the problem is still infeasible. So, the *p* value is increased to 4 with the given parameters at the beginning of the section, and the solutions of the resulting problem instance are given in Table 5.1 for CAB_10_4(3).

The last small sized problem instance generated from the CAB data set is CAB_10_3(4), and it is again infeasible with the starting parameters. The problem instance remains infeasible although, the γ value is decreased to 0.70. For p=4 when other parameters remain unchanged, the problem is infeasible, but when the γ value is decreased to 0.85, the problem results with feasible solutions which are given in Table 5.1 as CAB_10_4(3). For this problem instance, when we increase the number of hubs to 5, we can get feasible solutions, although the γ value is not decreased with an objective function value of 51.172. There is a trade-off between the number of hubs and the probability of service level guarantee. If a higher service level guarantee, 0.95, is desired then the hub number should not be smaller than 5; however if the number of hubs cannot be more than 4, then the probability of service level guarantee cannot be greater than 0.85. The decision maker should decide on these conflicting decisions.

Table 5.1 Comparison of GAMS and GA Results for the CAB Data

	GAMS Results		GA Results		
Problem Instances	Obj. Function Value	Computational Time (Sec.)	Best Obj. Function Value	Average Obj. Function Value	Range of Comp. Time (Sec.)
CAB_25_5	32.993	2,139	32.993	33.599	[69,83]
CAB_20_5(1)	40.310	766	40.310	41.397	[64,127]
CAB_20_5(2)	46.252	405	46.252	47.229	[59,132]
CAB_20_5(3)	41.289	558	41.289	42.985	[82,217]
CAB_20_5(4)	32.486	268	32.486	36.633	[63,137]
CAB_15_4(1)	45.032	29	45.032	52.612	[31,44]
CAB_15_5(2)	73.137	32	73.137	73.816	[28,62]
CAB_15_4(3)	13.552	194	13.552	16.273	[28,67]
CAB_15_4(4)	35.247	81	35.247	40.245	[49,83]
CAB_10_3(1)	34.662	14	34.662	40.011	[14,27]
CAB_10_5(2)	120.028	27	120.028	120.719	[29,44]
CAB_10_4(3)	70.401	30	70.401	70.401	[34,41]
CAB_10_4(4)	54.175	27	54.175	54.175	[35,40]

5.2.1.2 Small Problem Instances of the Turkish Network Data Set

For the Turkish Network data set, we generate three different sized small problem instances. As for the CAB data set, the parameters in the problem environments should be problem specific and should be determined accordingly. Because of the network structure of the Turkish network data set, we start our computational studies on the CAB data set with the corresponding parameters:

- L=480 minutes
- H=960 minutes
- β=1,440 minutes
- Q=200,000
- α=0.90
- γ=0.95
- CV=0.80

However, for some problem instances with these parameters, the problem instances become infeasible because of the constraints. In these cases we change the parameters one by one ceteris paribus if not mentioned, starting with the parameter γ and then the given *p* value.

The results obtained on the small problem instances of the Turkish Network data are given in Table 5.2. In Table 5.2 below, it is obvious that the computational time may become high with a small increase in the network in GAMS solver.

For Turk_30_6(1) network the computational time is more than 12 hours and the solution cannot be proved to be optimal by GAMS and it is in a relative gap of 0.669 (and absolute gap of 10.810). As a result our GA result is better than the GAMS result for our problem.

Again for the Turk_30_6(2) the given GAMS solution is not proven optimal solution, it is in a relative gap of 0.196 and an absolute gap of 1.453 and the program ends up with out of memory warning.

For the problem instance, Turk_30_6(4), the problem is infeasible with the initial parameters. So, we decrease the γ value to 0.85 and then solve the resulting problem. Decreasing the probability level makes the problem feasible, because the higher the γ value, the higher the z_γ value. In Table 5.2, the solutions for the Turk_30_6(4) are given for γ=0.85 and the solution of the GAMS is not proven to be the optimal solution, although the program runs for more than 26 hours. The solution is in a relative gap of 0.504.

For the 25 node Turkish Network data set obtained by the problem generation method (3), the problem is infeasible for *p*=5 and γ=0.95. The problem becomes feasible for γ=0.85 and the related solutions are given in Table 5.2. According to the network characteristics, there is a lower limit on the number of hubs that should be satisfied in order for the problem to be feasible. The γ value is about the service level constraint, and decreasing it makes the constraint loose. Another constraint in our problem is the constraint (3.3) that forces to end up with the number of hubs greater than a threshold level. For instance, the Turk_25_5(3) problem is infeasible for L=480; however, when we increase L to 520, the problem becomes feasible.

The Turk_25_5(4) problem instance is feasible for the initial parameters, and it is solved for γ=0.80 which is a feasible network structure ceteris paribus.

The Turk_20_4(4) is a feasible network for γ=0.75 with the solutions given in Table 5.2. If the number of hubs for this problem is increased to 5, then the network structure becomes feasible for the service level probability γ=0.85. Again in the case of CAB_10_3(4) problem instance, we face with a tradeoff between the number of hub and the probability of service level guarantee. The decision should be made whether to have a network with 4 hubs and to be sure with 0.75 probability that the deliveries for the specified node pairs will be made in β minutes; or to have a network with 5 hubs and to be sure with 0.85 probability that the deliveries for the specified node pairs will be made in β minutes.

Table 5.2 Comparison of GAMS and GA Results for the Small Turkish Network Problem Instances

	GAMS Results		GA Results		
Problem Instances	Obj. Function Value	Computational Time (Sec.)	Best Obj. Function Value	Average Obj. Function Value	Range of Comp. Time (Sec.)
Turk_30_6(1)	16.161	45,808	10.590	12.751	[1,006;1,224]
Turk_30_6(2)	5.619	16,826	5.619	6.817	[783;1,023]
Turk_30_6(3)	7.398	1,321	6.549	8.054	[692,983]
Turk_30_6(4)	12.057	95,713	8.981	10.791	[1,024;1,802]
Turk_25_5(1)	10.483	5,382	10.483	12.251	[44;21]
Turk_25_5(2)	12.577	6,626	12.577	14.237	[157;203]
Turk_25_5(3)	22.922	1,812	22.922	24.664	[108;218]
Turk_25_5(4)	8.592	8,150	8.592	10.617	[145;287]
Turk_20_4(1)	12.926	684	12.926	13.360	[24;48]
Turk_20_4(2)	14.963	232	14.963	20.004	[37;54]
Turk_20_4(3)	19.471	25,243	19.471	21.786	[124;187]
Turk_20_4(4)	11.666	144	11.666	12.760	[78;92]

5.2.2 Large Problem Instances

For the large problem instances, we use the Turkish network data set. Again we use four different problem generation methods, as the small problem instances to generate different large sized problem instances. As we mention before, the large sized problems cannot be solved with GAMS to optimality. To compare the performance of our solution method for large problems, we use the heuristic approaches which we call AGG and ALL.

In the heuristic approach AGG, we aggregate the nodes to get smaller problem instances. We assign the nodes with small flows to the nearest node that will exist in the aggregated environment, and solve our problem in this aggregated environment. Then we disaggregate the resulting nodes and calculate the objective function value accordingly.

In ALL heuristic approach, in the first phase, we solve p-center problem to find the location of the hubs. The hubs found in the location phase are used as given in the allocation phase. For the allocation phase, we assume the discount factor $\alpha=0$ between the hubs, and find the assignments of the nodes to the given hubs.

For the large sized problem instances with the Turkish Network data set, we generate three different sized problem instances. The results obtained for the large sized problem instances of the Turkish Network data are given in Table 5.3.

Table 5.3 Comparison of GAMS and GA Results for the Large Turkish Network Problem Instances

	AGG Results		ALL Results		GA Results		
Problem Instances	Obj. Function Value	Computational Time (Sec.)	Obj. Function Value	Computational Time (Sec.)	Best Obj. Function Value	Average Obj. Function Value	Range of Comp. Time (Sec.)
Turk_81_12	7.836	47,225	7.957	32	4.890	6.357	[3,161;3,648]
Turk_60_9(1)	27.061	6,248	10.549	24	8.206	9.847	[2,137;2,633]
Turk_60_9(2)	16.317	7,347	14.337	35	8.481	9.833	[1,874;2,124]
Turk_60_9(3)	14.395	9,502	11.876	37	7.876	8.905	[1,584;1,894]
Turk_60_9(4)	9.430	4,347	10.147	39	6.429	7.747	[1,145;2,048]
Turk_50_8(1)	11.047	1,478	13.302	21	10.348	13.841	[1,084;1,478]
Turk_50_8(2)	19.439	2,038	21.239	24	13.318	15.197	[1,148;1,847]
Turk_50_8(3)	16.846	1,784	14.471	28	12.310	13.453	[1,245;1,905]
Turk_50_8(4)	10.806	1,547	9.904	31	6.096	7.557	[1,147;1,284]

The GA algorithm based heuristic method that is proposed results in very good solutions in a very short time. Although the algorithm does not always end up with the optimal solutions, instead of having a single solution to a problem, usually it is preferable to have alternative good solutions for the decision makers. The algorithm enables us to analyze a set of alternative good solutions for a network.

5.3 Extensions about the Model

After evaluating our solution method in the previous sections, we want to introduce the effects of the constraints and the parameters used in the solutions, we deal with the problem instance Turk_25_5.

First we solve our problem for different *p* values, the number of hub values. As can be seen in Table 5.4, the constraints we impose in our model, the constraint on the service level and on the travel time between the nodes and hubs indicate that there should be at least 4 hubs in order for the network to be feasible. When p=3, the constraints are not satisfied, and for p=4 and higher, the problem is feasible. After p=5, the objective function does not change.

Table 5.4 The Effects of the Number of Hubs, *p*,

p	Obj. Function Value	Computational Time	Selected Hubs
3	Infeasible	-	-
4	12,481	3.271	6,12,15,17
5	10,483	5.382	6, 7, 13, 14, 21
6	10,483	6.452	3,6,12,18,21,22
7	10,483	2.850	4,5,6,7,11,16,21
10	10,483	5248	10,11,18,21,22

As for the constant L, which is the restriction on the travel time between the hubs and non-hub nodes, the results for different values of L are given in Table 5.5. As in the case of p, infeasibility may be observed below a certain level for L values. This is because of the structure of the network that is there cannot be any hub assignments for some cities that satisfy L, for small values. The threshold level below which infeasibility results, depends on the structure of the network; so the level of L should be problem-specific. In our problems, increasing L decreases the value of the objective function, because small values of L eliminates some of the alternatives for the hubs; while decreasing the travel time between the hubs and non-hub nodes with L, the maximum weighted travel time is increased. Although L is increased, the objective function value does not change after a certain level; but we get different hubs selected. This is because of the alternative optima nature of the p-hub center problems. Actually, by varying the parameter L, we chose a well structured alternative optimal solution.

Table 5.5 The Effects of L on the Solution

L	Obj. Function Value	Computational Time (Sec.)	Selected Hubs
300	Infeasible		
360	15,195	6792	1,6,16,20,25
420	12,481	8784	1,3,6,11,12
480	10,483	5.382	6,7,13,14,21
540	10,483	7260	6,11,14,21,25
780	10,483	18.716	1,3,6,7,21

To sum up, the p-hub center problem has alternative optimal solutions and with the constraints we imposed, we try to get a well structured solution among the alternative optimal solutions. The p-hub center problems try to minimize the maximum travel times or costs. However, when minimizing the maximum path, many of the paths become trivial and actually the optimal solutions occur as a result of this. For instance in Figure 5.1 and Figure 5.2, networks with 5 nodes and 2 hubs are demonstrated. Assuming that the path from node A to node D is the longest one to be minimized, in terms of the objective function value, the two networks are similar. However, when the path form node E to node D is considered, the two networks have difference in terms of their travel time or cost. In Figure 5.1, the node E is assigned to the hub C, and the flows between E and other two nodes A and D should be transferred through hubs C and D. However, in Figure 5.2 the node E is assigned to hub B and the flows between node A and the other two nodes are transferred via hub B only. Although the two networks are indifferent in terms of the objective function values, the second one is a better structured network.

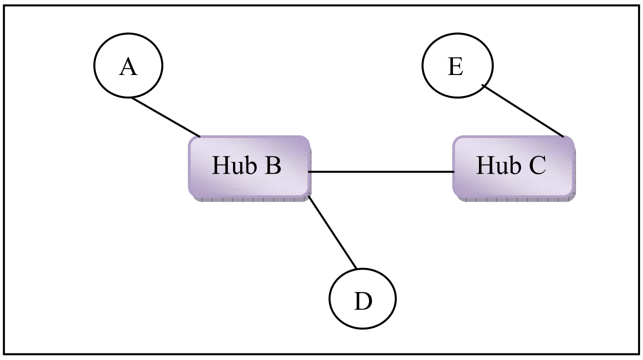


Figure 5.1 A Network with 2 Hubs and 5 Nodes (1)

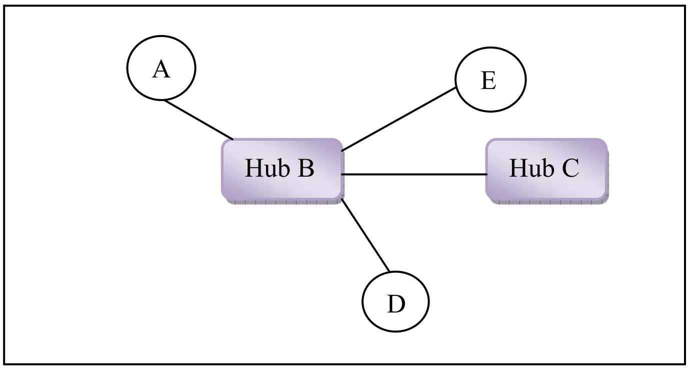


Figure 5.2 A Network with 2 Hubs and 5 Nodes (2)

5.4 The Effects of the Coefficient of Variation to the Solution

Up to now, for the computational analysis we use 0.80 for the value of the coefficient of variation. The real travel times of the data set sets are taken as the mean travel times; the standard deviations are calculated with these mean values and the coefficient variation value of 0.80. The value of the coefficient of variation changes the variability of the distributions. The greater the coefficient of variation, the more variable is the distribution. In order to see the effects of different coefficients of variations, we use two more values for it; 0.30 and 0.50 other than 0.80 which was originally used. The coefficient of variation value is used in constraint (3.2) in order to calculate the standard deviations from the mean travel times. Since $CV = \sigma/\mu$, the higher the coefficient of variation the higher the variability of the distribution and the higher the standard deviation. Higher coefficient of variation and consequently higher standard deviation cause constraint (3.2) to be tighter. In Table 5.6 we present the solutions obtained at three levels for CV, using GAMS.

Table 5.6 The Effects of Different Coefficient of Variation Values on the Solutions

Problem Instances	CV=0.80		CV=0.50		CV=0.30	
	Obj. Function Value	Selected Hubs	Obj. Function Value	Selected Hubs	Obj. Function Value	Selected Hubs
CAB_20_5(4)	Infeasible		32.486	1,11,12,20,23	32.486	1,11,12,20,23
CAB_15_4(4)	Infeasible		35.247	14,21,22,25	35.247	14,21,22,25
CAB_20_5(1)	40.310	8,10,12,14,17	40.310	1,11,12,17,20	40.310	1,2,11,12,17
CAB_20_5(2)	46.252	11,12,17,23,24	46.252	11,12,17,23,24	46.252	11,12,17,23,24
Turk_20_4(1)	12.926	6,7,12,14	12.926	6,9,13,15	12.926	6,12,14,15
Turk_20_4(4)	Infeasible		Infeasible		11.661	34,42,46,55

CHAPTER 6

6. CONCLUSIONS AND DIRECTIONS FOR FUTURE STUDY

In this study we have searched the solution techniques for the uncapacitated, single allocation p-hub center problem with service level constraints. First, we have presented the mathematical formulations for our problem environment. We try to represent the real life environment of the cargo delivery systems while constructing our model. The NP-hard nature our mathematical model and the difficulty in obtaining the optimal solutions for the reasonable sized problems led us to search on the applications of metaheuristics.

In our GA-based solution method, we have developed genetic algorithms based solution approach with problem specific characteristics. The developed algorithm has been implemented to problem instances constructed from CAB data set and the Turkish network data set. The results for small problem instances were compared with the optimal results we get from GAMS solutions. For the large sized problems, in order to get comparable solutions, we did aggregation and decomposition. In aggregation method we aggregated the nodes with few flows with the nearest nodes with higher flows and solved the aggregated problem with GAMS solver again. In the decomposition of the problem, we solve two sub-problems; location and allocation problems. In the allocation phase, we solve the *p*- center problem and find the locations of the hubs for the network. In the allocation phase, the hubs found from p-center problem are used as given and the nodes are allocated to the hubs assuming that $\alpha=0$. The computational results on different problem instances show that our GA based heuristic approach produces good results both in terms of the objective function value and the computational time. The heuristic approach proposed ends up with alternative optimal solutions and near optimal solutions, in a reasonable amount of time. We get a set of different good solutions and in many situations having a set of alternative good solutions is preferable by the decision makers.

For further research on the solution method proposed; other problem specific operators apart from those used in this study may be generated and evaluated. Moreover, additional improvements on the problem specific characteristics may be included in different phases of the solution method. The coding of the GA proposed here can be improved further.

For the stochastic travel times of the problem environment; different distributions other than the normal distribution can be used. Since in the real life applications, the stochastic behavior of the travel times change from link to link, different node pairs may be considered as having different variability. For the capacitated case of the problem, variability in the demand may also be incorporated to the problem.

As mentioned before, the *p*-hub center problems have alternative optimal solutions; however, the structures of the networks among these alternative optimal solutions are different. In this study, we try to select the well structured ones. Other criterion about this point can be introduced for p-hub center problems.

Incremental increases in the number of hubs do not contribute to our objective function value, which is the maximum weighted travel time, after a threshold level. However, having more hubs in a network may decrease the total travel times of the network. There is a trade-off between the number of hubs opened and total travel time of the networks. Additionally, relaxing the restriction on the travel times between the hub nodes and non-hub nodes decreases the maximum weighted travel time. The cost of travel between a non-hub node and a hub node is not identical with the cost of travel between the hubs. So, analysis considering these points can be done for the number of hubs to be opened, and the total network travel times and the traveling time between non-hub nodes and hub nodes and between the hub nodes.

In this study, we only considered the travel times. Multi-criteria models that take into account both the travel times and costs can be developed.

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APPENDIX A

GAMS MODEL

The mathematical model formulations translated to GAMS with the definition of the elements of the model, the objective function and the constraints are presented below.

<u>Linear Model for the P-Hub center Problem with Stochastic Service Level</u>

<u>Constraints:</u>

Sets

i origin nodes;
j destination nodes;
k possible origin hubs ;
m possible destination hubs ;

Parameters

W(i,j) the flow weight between origin node i to destination node j ;
F1(i,j) the flow from origin node i to destination node j ;
F2(j,i) the flow from origin node j to destination node i ;
T(i,j) standard travel time on the link from node i to node j ;
t1(i,k) travel time from node i to hub k ;
t2(k,m) travel time from hub k to hub m ;
t3(m,j) travel time from hub k to hub m ;

Display i,j,k,m,W,T,t1,t2,t3 ;

Scalars

scalar P number of hubs / / ;

scalar a discount factor / / ;

scalar b targeted service level for specified node pairs / / ;

scalar s standard normal z value / / ;

scalar cv coefficient of variation / / ;

scalar L restriction on the travel times between nodes and hubs / /;

scalar Q flow threshold for the service level applicability / /;

scalar H travel time threshold for the service level applicability / /;

Variables

z maksimum weigted travel time from node i to node j ;

binary variable X(i,j,k,m) ;

*=== 1, if there is a path from origin node i to destination node j via the hubs k and m / 0, otherwise

binary variable X1(i,k) ;

*=== 1, if node i is assigned to hub k / 0, otherwise

binary variable X2(j,m) ;

*=== 1, if node j is assigned to hub m / 0, otherwise

binary variable X3(k,k) ;

*=== 1, if node k is a hub/ 0, otherwise

binary variable X4(m,m) ;

*=== 1, if node m is a hub/ 0, otherwise

Equations

time(i,j,k,m) define the objective function ;

total_hub_number1 satisfies the given hub number for k

total_hub_number2 Satisfies the given hub number for m

node_hub_assign1(i,k) No node is assigned unless there is a hub for k

self_assign1(i,k) A hub is assigned to itself for k node_hub_assign2(j,m) No node is assigned there is a hub for m

self_assign2(j,m) A hub is assigned to itself for m

exactly_one_hub1(i) Satisfies the assignment of node to a hub for k

exactly_one_hub2(j) Satisfies the assignment of node to a hub for m

path_assignment1(i,j,k,m) Satisfies path assignment iff node to hub assignments exists

Single_path(i,j) Satisfies single path assignment for every node pair

serv_crit(i,j,k,m) Satifies the service level criterion for selected hub pairs

equal(i,j,k,m) Satisfies the symetry of the assignments

leg1(i,k) Restriction about the travel time between nodes and hubs i and k

leg2(j,m) restriction about the travel time between nodes and hubs j and m

```
time(i,j,k,m) .. z =g= (W(i,j)*X(i,j,k,m))*((t1(i,k)) + (a*t2(k,m)) + t3(m,j)) ;
total_hub_number1 .. sum((k), X3(k,k)) =e= p ;
total_hub_number2 .. sum((m), X4(m,m)) =e= p ;
node_hub_assign1(i,k) .. X1(i,k) =l= X3(k,k) ;
self_assign1(i,k)$(ord(i)=ord(k)) .. X1(i,k) =e= X3(k,k);
node_hub_assign2(j,m) .. X2(j,m) =l= X4(m,m) ;
self_assign2(j,m)$(ord(j)=ord(m)) .. X2(j,m) =e= X4(m,m);
exactly_one_hub1(i) .. sum((k),X1(i,k)) =e= 1 ;
exactly_one_hub2(j) .. sum((m),X2(j,m)) =e= 1 ;
path_assignment1(i,j,k,m) .. X(i,j,k,m) =g= X1(i,k) + X2(j,m) - 1 ;
single_path(i,j) .. sum((k,m),X(i,j,k,m)) =e= 1 ;
serv_crit (i,j,k,m) $((T(i,j) lt H) and ((F1(i,j) + F2(j,i)) gt Q)) .. b =g=
X(i,j,k,m)*((t1(i,k)+a*t2(k,m) + t3(m,j))+
(s*sqrt(sqr(cv*t1(i,k))+sqr(a)*sqr(cv*t2(k,m))+sqr(cv*t3(m,j))))) ;
equal(i,j,k,m)$(ord(i)=ord(j) and ord(k)=ord(m)) .. X1(i,k) =e= X2(j,m) ;
leg1(i,k) .. L =g= X1(i,k)*t1(i,k) ;
leg2(j,m) .. L =g= X2(j,m)*t3(m,j) ;
```

```
option optcr = 0.00 ;
Model transport  /all/ ;
solve transport using MIP minimizing z ;
```

APPENDIX B

THE PSEUDOCODES FOR GA-BASED HEURISTIC

The pseudocodes which are the basis of the code of the algorithm are presented below.

```
ENTER Number of Hubs
ENTER Population Size
ENTER Generation Number
ENTER Crossover Percentage
ENTER Mutation Percentage
ENTER Standard Normal Value
ENTER Coefficient of Variation
FOR i=1 TO NodeNo DO
FOR j=1 TO NodeNo DO
            ASSIGN Travel Time to array T[i][j]
      ENDFOR
ENDFOR
FOR i=1 TO NodeNo DO
    FOR j=1 NodeNo DO
             ASSIGN Flow to array F[i][j]
            COUNT total_flow by adding F[i][j]
ENDFOR
ENDFOR
FOR i=1 TO NodeNo DO
      FOR j=1 TO NodeNo DO
            CALCULATE W[i][j] by dividing F[i][j] to total_flow
ENDFOR
ENDFOR
```

```
FOR g=1 TO GenerationNo DO
        BEGIN:
        SET 0 TO d
        SET 1 TO r
        SET 1 TO o
        SET 1 TO v
        FOR t=1 TO BigNo DO
        TEKRR:
                FOR i=1 TO HubNo DO
                TEKRAR:
                ASSIGN random_hub[i] by generating random numbers in
[1,NodeNo]
                        FOR j=1 TO (i-1) DO
                                IF ( random_hub[i] = random_hub[j] )
                                        RETURN TEKRAR
                        END FOR
                END FOR
                FOR l=1 TO NodeNo DO
                        FOR m=1 TO HubNo DO
                                IF ( l = random_hub[m] )
                                                ASSIGN random_chromosome[l]
TO random_hub[m]
                                                ASSIGN HubNo + 1 TO m
                                        END IF
                                        ELSE ASSIGN 0 TO
random_chromosome[l]
                                END FOR
                        END FOR
                        FOR l=1 TO NodeNo DO
                                ASSIGN min_time = 480
```

```
IF (random_chromosome[l] = 0 )
                                        FOR m=1 TO HubNo DO
                                                IF (T[l][random_hub[m]] <=
min_time )
                                                        ASSIGN
T[l][random_hub[m]] TO          min_time
ASSIGN random_chromosome[l] TO                  random_hub[m]
                                                END IF
                                        END FOR
                                IF ( random_chromosome[l] = 0 )
                                        RETURN TEKRR
                        END IF
                END FOR
                IF ( l = NodeNo + 1 )
                        ASSIGN 1440 TO max_time
                        ASSIGN 200000 TO max_flow
                        ASSIGN 900 TO min_time
                        FOR i=1 TO NodeNo DO
                                FOR j=1 TO NodeNo DO
                                        ASSIGN random_chromosome[i] TO k
                                        ASSIGN random_chromosome[j] TO m
IF ( F[i][j] + F[j][i] >= max_flow AND T[i][j] <= min_time )
                                                CALCULATE
ServiceLevelConstraint
                                                IF ( ServiceLevelConstraint >=
max_time )
                                                        INCREASE t by 1
RETURN TEKRR
                                                END IF
                                        END IF
```

```
END FOR
                        END FOR
                END IF
                FOR i=1 TO HubNo DO
                        ASSIGN random_hub[i] TO population_hub[count][i]
                END FOR
                FOR i=1 TO NodeNo DO
                        ASSIGN random_chromosome[i] TO
population[count][i]
                END FOR
                IF ( count = PopulationSize )
                        t = BigNo + 1
                        INCREASE count by 1
                END IF
                ASSIGN  0 TO popu_best
                ASSIGN a BigNo TO population_best
                FOR i=1 TO PopulationSize DO
                        FOR k=1 TO NodeNo DO
                                FOR m=1 TO NodeNo DO
                                        CALCULATE fitness
                                        IF ( fitness > popu_best )
                                                ASSIGN fitness TO popu_best
                                                ASSIGN i TO bir
                                                ASSIGN k TO yat
                                                ASSIGN m TO dik
                                        END IF
                                END FOR
                        END FOR
                        IF ( popu_best < population_best )
                                ASSIGN popu_best TO population_best
```

```
                        ASSIGN bir TO birey
                        ASSIGN yat TO yatay
                        ASSIGN dik TOdikey
                    END IF
                END FOR
CALCULATE number of parents that CrossOver will be applied by
p_cross(percentage of CrossOver)
FOR c=1 TO GenerationNo DO
    SET u TO 1
    ASSIGN s TO x_num
    FOR i=1 TO PopulationSize DO
        TEKR:
        ASSIGN random number TO y
ASSIGN x[i] by generating random numbers y in [1,PopulationSize]
        FOR m=1 TO ( i-1 ) DO
            IF ( x[m] = y )
                RETURN TEKR
            IF ( m = i-1 )
                ASSIGN y TO x[i]
        END FOR
    END FOR
   FOR j=1 TO ParentNo/2 DO
        TEKiRA:
        ASSIGN 0 TO best_anne (fitness value of mother)
        FOR i=1 TO NodeNo DO
            FOR k=1 TO NodeNo DO
                CALCULATE fitness of mother
                IF (fitness > best_anne)
                    ASSIGN fitness TO best_anne
            END FOR
```

```
END FOR
ASSIGN 0 TO best_baba (fitness value of father)
FOR i=1 TO NodeNo DO
    FOR k=1 TO NodeNo DO
        CALCULATE fitness of father
        IF (fitness > best_baba)
            ASSIGN fitness TO best_baba
    END FOR
END FOR
IF (best_baba < best_anne )
    ASSIGN best_baba TO best_sol
ELSE ASSIGN best_anne TO best_sol
ASSIGN 1 TO counter
ASSIGN 1 TO countr
FOR i=1 TO HubNo DO
    ASSIGN population_hub[x[r]][i] (Hubs of mother ) TO
child_hub_all[countr]
    INCREASE countr by 1
    FOR k=1 TO HubNo DO
        IF ( Hub of mother equals to Hub of father )
            ASSIGN population_hub[x[r]][i] TO
child_hub[counter]
            INCREASE counter by 1
        END IF
    END FOR
END FOR
ASSIGN 1 TO countrr
FOR i=1 TO counter-1 DO
    FOR k=1 TO HubNo DO
        IF ( child_hub[i] = child_hub_all[k] )
```

```
                    ASSIGN child_hub_all[countrr] TO temp
                    ASSIGN child_hub[i] TO
child_hub_all[countrr]
                    ASSIGN temp TO child_hub_all[k]
                    INCREASE countrr by 1
                END IF
            END FOR
        END FOR
        FOR k=1 TO HubNo DO
            FOR j=1 TO countr-1 DO
                IF ( population_hub[x[r+v]][k] (Hubs of father)=
child_hub_all[i] )
                    ASSIGN countr TO i
                IF ( i = countr-1 )
                    ASSIGN population_hub[x[r+v]][k] TO
child_hub_all[countr]
                    INCREASE countr by 1
                END IF
            END FOR
        END FOR
        FOR i=1 TO countr-1 DO
        ASSIGN child_hub_all[i+counter-1] TO child_hub_all[i]
        ASSIGN 1 TO cou
        TEKiR:
        FOR i=counter TO HubNo DO
            TKRR:
            ASSIGN b by generating random number in [1,number
of uncommon hubs]
            ASSIGN child_hub_all[b] TO child_hub[i]
            FOR m=1 TO i-1 DO
                IF ( child_hub[i] = child_hub[m] )
```

```
                    RETURN TKRR
                END FOR
            END FOR
            FOR i=1 TO NodeNo DO
                FOR m=1 TO HubNo DO
                    IF ( i = child_hub[m] )
                        ASSIGN child_hub[m] TO
child_chromosome[i]
                        ASSIGN p+1 TO m
                    END IF
                END FOR
        END FOR
        FOR l=1 TO NodeNo DO
            ASSIGN 480 TO min_time
            IF ( child_chromosome[l] = 0 )
                FOR m=1 TO HubNo DO
                    IF (T[l][child_hub[m]] <= min_time )
                        ASSIGN T[l][child_hub[m]] TO
min_time
                            ASSIGN child_hub[m] TO
child_chromosome[l]
                        END IF
                    END FOR
                    IF ( child_chromosome[l] = 0 )
                        INCREASE cou by 1
                        RETURN TEKiR
                END IF
            END FOR
            ASSIGN 0 TO best_cocuk
            FOR i=1 TO NodeNo DO
                FOR k=1 TO NodeNo DO
```

```
                    CALCULATE fitness value of child
                    IF ( fitness > best_cocuk )
                         ASSIGN fitness TO best_cocuk
               END FOR
          END FOR
          IF ( cou > 1000 )
               IF ( o = 2 )
               FOR i=1 TO NodeNo DO
                         ASSIGN tem[i] TO population[x[r]][i]
                    FOR i=1 TO HubNo DO
                         ASSIGN temper[i] TO
population_hub[x[r]][i]
                    ASSIGN 1 TO o
               END IF
               ASSIGN 1 TO cou
               INCREASE v by 1
               IF ( r+v = x_num+1 )
                    ASSIGN x[r] TO mut[u]
                    INCREASE u by 1
                    INCREASE r by 1
                    ASSIGN 1 TO v
               END IF
               IF ( r = x_num )
                    ASSIGN 1 TO cou
                    ASSIGN 1 TO r
                    ASSIGN 1 TO v
                    IF ( g = Iteration_limit)
                         DiSPLAY "Calculation Time"
STOP
               END IF
               INCREASE g by 1
```

```
                    RETURN BEGİN
                END IF
            END IF
            RETURN TEKİRA
        END IF
        ELSE
            IF ( best_cocuk >= best_sol )
                INCREASE cou by 1
                RETURN TEKİR
            END IF
            IF ( o = 1)
                FOR i=1 TO NodeNo DO
                    ASSIGN population[x[r]][i] TO tem[i]
                    ASSIGN child_chromosome[i] TO population[x[r]][i]
                END FOR
                FOR i=1 TO HubNo DO
                    ASSIGN population_hub[x[r]][i] TO temper[i]
                    ASSIGN child_hub[i] TO population_hub[x[r]][i]
                END FOR
            END IF
            ASSIGN best_cocuk TO best_sol
            IF ( o = 2 )
                FOR i=1 TO NodeNo DO
                    ASSIGN child_chromosome[i] TO population[x[r+v]][i]
                FOR i=1 TO HubNo DO
                    ASSIGN child_hub[i] TO population_hub[x[r+v]][i]
                ASSIGN x[x_num] TO x[r+v]
                DECREASE x_num by 1
                INCREASE r by 1
                ASSIGN 1 TO v
            END IF
```

```
        INCREASE o by 1
        IF ( o <= 2 )
                RETURN TEKİR
        IF ( o = 3 )
                ASSIGN 1 TO o
        ASSIGN 1 TO cou
END ELSE
END FOR
CALCULATE number of member that Mutation will be applied by
p_mut(percentage of Mutation)
IF ( u > mutationNo + 1 )
        FOR i=1 TO mutationNo DO
                IF ( birey = mut[d+1] )
                        INCREASE d by 1
                STRTR:
                ASSIGN w by generating random number in [1,HubNo]
                ASSIGN population_hub[mut[d+1]][w] TO f
                ASSIGN f TO change_hub
                ASSIGN BigNo TO min_tim
                FOR k=1 TO NodeNo DO
                        STARTR:
                        FOR m=1 TO HubNo DO
                                IF( k = population_hub[mut[d+1]][m] )
                                        INCREASE k by 1
                                        RETURN STARTR
                                END IF
                        END FOR
                        IF ( T[f][k] < min_tim AND T[f][k] != 0 )
                                ASSIGN T[f][k] TO min_tim
                                ASSIGN k TO change_hub
                        END IF
```

```
                              IF ( k = NodeNo AND f = change_hub )
                                        RETURN STRTR
                              FOR m=1 TO NodeNo DO
                                        IF ( population[mut[d+1]][m] = f )
                                                  ASSIGN change_hub TO
population[mut[d+1]][m]
                                        ASSIGN change_hub TO
population[mut[d+1]][change_hub]
                              END FOR
                    END FOR
                    ASSIGN change_hub TO population_hub[mut[d+1]][w]
                    INCREASE d by 1
          END IF
          ASSIGN 0 TO d
END IF
ASSIGN 0 TO popu_best
ASSIGN BigNo TO population_best
FOR i=1 TO PopulationSize DO
          FOR k=1 TO NodeNo DO
                    FOR m=1 TO NodeNo DO
                              CALCULATE fitness
                              IF ( fitness > popu_best )
                                        ASSIGN fitness TO popu_best
                                        ASSIGN i TO bir
                                        ASSIGN k TO yat
                                        ASSIGN m TO dik
                              END IF
                    END FOR
          END FOR
          IF ( popu_best < population_best )
                    ASSIGN popu_best TO population_best
```

```
            ASSIGN bir TO birey
            ASSIGN yat TO yatay
            ASSIGN dik TO dikey
        END IF
END FOR
ASSIGN 1 TO r
ASSIGN 1 TO v
END FOR
RETURN 0
```

APPENDIX C

DATA SETS

CAB Data Set

Table C.1 Districts and Node Numbers for the CAB Data Set

Node #	District	Node #	District
1	Atlanta	14	Miami
2	Baltimore	15	Minneapolis
3	Boston	16	New Orleans
4	Chicago	17	New York
5	Cincinnati	18	Philadelphia
6	Cleveland	19	Phoneix
7	Dallas-Ft. Worth	20	Pittsburgh
8	Denver	21	St. Louis
9	Detroit	22	San Francisco
10	Houston	23	Seattle
11	Kansas City	24	Tampa
12	Los Angeles	25	Washington DC
13	Memphis		

Table C.2 Travel Times for the CAB Data Set

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0	577	946	598	374	560	709	1208	604	695	681	1937	332
2	577	0	370	613	429	313	1196	1502	406	1242	960	2318	787
3	946	370	0	858	750	556	1541	1765	621	1603	1251	2600	1137
4	598	613	858	0	255	311	790	907	237	932	406	1742	486
5	374	429	750	255	0	226	794	1080	239	880	533	1890	402
6	560	313	556	311	226	0	1010	1217	94	1105	695	2047	627
7	709	1196	1541	790	794	1010	0	664	983	221	448	1250	411
8	1208	1502	1765	907	1080	1217	664	0	1144	875	552	842	880
9	604	406	621	237	239	94	983	1144	0	1095	637	1979	620
10	695	1242	1603	932	880	1105	221	875	1095	0	642	1376	477
11	681	960	1251	406	533	695	448	552	637	642	0	1358	379
12	1937	2318	2600	1742	1890	2047	1250	842	1979	1376	1358	0	1608
13	332	787	1137	486	402	627	411	880	620	477	379	1608	0
14	593	950	1267	1187	947	1085	1098	1715	1152	964	1236	2336	858
15	909	939	1125	346	599	626	852	694	535	1046	405	1531	701
16	426	1000	1368	830	700	922	424	1067	936	305	674	1662	348
17	756	179	190	720	578	409	1363	1626	490	1417	1097	2453	956
18	673	96	274	675	512	366	1289	1575	453	1338	1039	2397	880
19	1590	2000	2299	1447	1571	1743	895	593	1682	1017	1049	358	1266
20	527	211	494	404	256	105	1049	1302	199	1125	768	2126	651
21	483	736	1043	256	307	491	538	781	450	677	229	1582	255
22	2141	2456	2703	1854	2036	2165	1494	956	2087	1650	1506	362	1809
23	2184	2340	2504	1733	1967	2027	1687	1025	1936	1891	1504	987	1873
24	408	844	1189	1006	775	933	912	1519	992	795	1039	2158	661
25	541	36	406	592	399	299	1162	1475	393	1206	932	2289	751

Table C.2 (cont'd)

	14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
1	593	909	426	756	673	1590	527	483	2141	2184	408	541
2	950	939	1000	179	96	2000	211	736	2456	2340	844	36
3	1267	1125	1368	190	274	2299	494	1043	2703	2504	1189	406
4	1187	346	830	720	675	1447	404	256	1854	1733	1006	592
5	947	599	700	578	512	1571	256	307	2036	1967	775	399
6	1085	626	922	409	366	1743	105	491	2165	2027	933	299
7	1098	852	424	1363	1289	895	1049	538	1494	1687	912	1162
8	1715	694	1067	1626	1575	593	1302	781	956	1025	1519	1475
9	1152	535	936	490	453	1682	199	450	2087	1936	992	393
10	964	1046	305	1417	1338	1017	1125	677	1650	1891	795	1206
11	1236	405	674	1097	1039	1049	768	229	1506	1504	1039	932
12	2336	1531	1662	2453	2397	358	2126	1582	362	987	2158	2289
13	858	701	348	956	880	1266	651	255	1809	1873	661	751
14	0	1501	676	1098	1022	1978	1015	1066	2591	2726	198	923
15	1501	0	1040	1018	988	1281	728	450	1590	1401	1311	922
16	676	1040	0	1178	1096	1304	919	602	1917	2090	496	963
17	1098	1018	1178	0	84	2144	329	881	2574	2415	1008	216
18	1022	988	1096	84	0	2082	273	818	2527	2389	927	133
19	1978	1281	1304	2144	2082	0	1815	1264	662	1129	1800	1969
20	1015	728	919	329	273	1815	0	552	2253	2129	875	195
21	1066	450	602	881	818	1264	552	0	1736	1712	872	707
22	2591	1590	1917	2574	2527	662	2253	1736	0	695	2405	2430
23	2726	1401	2090	2415	2389	1129	2129	1712	695	0	2528	2322
24	198	1311	496	1008	927	1800	875	872	2405	2528	0	814
25	923	922	963	216	133	1969	195	707	2430	2322	814	0

Table C.3 Flows for the CAB Data Set

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0	6469	7629	20036	4690	6194	11688	2243	8857	7248	3559	9221	10099
2	6469	0	12999	13692	3322	5576	3878	3202	6699	4198	2454	7975	1186
3	7629	12999	0	35135	5956	14121	5951	5768	16578	4242	3365	22254	1841
4	20036	13692	35135	0	19094	35119	21423	27342	51341	15826	28537	65387	12980
5	4690	3322	5956	19094	0	7284	3102	1562	7180	1917	2253	5951	1890
6	6194	5576	14121	35119	7284	0	5023	3512	10419	3543	2752	14412	2043
7	11688	3878	5951	21423	3102	5023	0	11557	6479	34261	10134	27350	6929
8	2243	3202	5768	27342	1562	3512	11557	0	5615	7095	10753	30362	1783
9	8857	6699	16578	51341	7180	10419	6479	5615	0	4448	5076	22463	4783
10	7248	4198	4242	15826	1917	3543	34261	7095	4448	0	4370	17267	3929
11	3559	2454	3365	28537	2253	2752	10134	10753	5076	4370	0	15287	3083
12	9221	7975	22254	65387	5951	14412	27350	30362	22463	17267	15287	0	5454
13	10099	1186	1841	12980	1890	2043	6929	1783	4783	3929	3083	5454	0
14	22866	7443	23665	44097	7097	15642	7961	3437	24609	8602	4092	15011	3251
15	3388	1162	6517	51525	2009	5014	4678	8897	9969	2753	7701	17714	1126
16	9986	5105	3541	14354	1340	2016	13511	2509	4224	20013	2809	10037	5926
17	46618	24817	205088	172895	25303	62034	29801	23273	79945	28080	17291	105507	10653
18	11639	6532	37669	37305	6031	15385	7549	5160	20001	5971	4462	20010	3062
19	1380	806	2885	15418	1041	2957	5550	8750	4291	2131	3239	31780	759
20	5261	8184	13200	26221	4128	5035	3089	2583	10604	3579	2309	10822	1255
21	5985	3896	7116	42303	5452	7482	9958	7288	11925	6809	16003	16450	6173
22	6731	7333	17165	35303	3344	6758	14110	17481	13091	8455	8381	92083	2974
23	2704	3719	4284	13618	1067	2191	4911	7930	4172	2868	3033	32908	1056
24	12250	2015	8085	17580	4608	6599	2722	1278	12891	2336	1755	3865	1504
25	16132	565	51895	40708	7050	14181	10802	8447	19500	5616	7266	24583	4588

Table C.3 (cont'd)

	14	15	16	17	18	19	20	21	22	23	24	25
1	7443	1162	5105	24817	6532	806	8184	3896	7333	3719	2015	565
2	23665	6517	3541	205088	37669	2885	13200	7116	17165	4284	8085	51895
3	44097	51525	14354	172895	37305	15418	26221	42303	35303	13618	17580	40708
4	7097	2009	1340	25303	6031	1041	4128	5452	3344	1067	4608	7050
5	15642	5014	2016	62034	15385	2957	5035	7482	6758	2191	6599	14181
6	7961	4678	13511	29801	7549	5550	3089	9958	14110	4911	2722	10802
7	3437	8897	2509	23273	5160	8750	2583	7288	17481	7930	1278	8447
8	24609	9969	4224	79945	20001	4291	10604	11925	13091	4172	12891	19500
9	8602	2753	20013	28080	5971	2131	3579	6809	8455	2868	2336	5616
10	4092	7701	2809	17291	4462	3239	2309	16003	8381	3033	1755	7266
11	15011	17714	10037	105507	20040	31780	10822	16450	92083	32908	3865	24583
12	3251	1126	5926	10653	3062	759	1255	6173	2974	1056	1504	4588
13	0	5550	9473	169397	25073	1170	14272	8543	8064	1840	20618	20937
14	5550	0	2152	26816	6931	4947	2676	8033	12692	6157	3065	12044
15	9473	26816	0	21806	4519	886	1742	4782	6453	2022	3546	5065
16	169397	26816	21806	0	9040	11139	63153	34092	70935	14957	28398	166694
17	25073	6931	4519	9040	0	2802	30224	7982	14964	4589	6227	12359
18	1170	4947	886	11139	2802	0	1869	3716	11510	3519	569	3520
19	14272	2676	1742	63153	30224	1869	0	5020	6610	2139	5431	13541
20	8543	8033	4782	34092	7982	3716	5020	0	9942	3276	3820	11799
21	8064	12692	6453	70935	14964	11510	6610	9942	0	35285	2566	19926
22	1840	6157	2022	14957	4589	3519	2139	3276	35285	0	940	4951
23	20618	3065	3546	28398	6227	569	5431	3820	2566	940	0	6237
24	20937	12044	5065	166694	12359	3520	13541	11799	19926	4951	6237	0
25	7443	1162	5105	24817	6532	806	8184	3896	7333	3719	2015	565

Turkish Network Data Set

Table C.4 Cities and Node Numbers for the Turkish Network Data Set

Node #	**City**	**Node #**	**City**
1	Adana	42	Konya
2	Adıyaman	43	Kütahya
3	Afyon	44	Malatya
4	Ağrı	45	Manisa
5	Amasya	46	Kahramanmaraş
6	Ankara	47	Mardin
7	Antalya	48	Muğla
8	Artvin	49	Muş
9	Aydın	50	Nevşehir
10	Balıkesir	51	Niğde
11	Bilecik	52	Ordu
12	Bingöl	53	Rize
13	Bitlis	54	Sakarya
14	Bolu	55	Samsun
15	Burdur	56	Siirt
16	Bursa	57	Sinop
17	Çanakkale	58	Sivas
18	Çankırı	59	Tekirdağ
19	Çorum	60	Tokat
20	Denizli	61	Trabzon
21	Diyarbakır	62	Tunceli
22	Edirne	63	Şanlıurfa
23	Elazığ	64	Uşak
24	Erzincan	65	Van
25	Erzurum	66	Yozgat
26	Eskişehir	67	Zonguldak
27	Gaziantep	68	Aksaray
28	Giresun	69	Bayburt

Table C.4 (cont'd)

29	Gümüşhane	70	Karaman
30	Hakkari	71	Kırıkkale
31	Hatay	72	Batman
32	Isparta	73	Şırnak
33	İçel	74	Bartın
34	İstanbul	75	Ardahan
35	İzmir	76	Iğdır
36	Kars	77	Yalova
37	Kastamonu	78	Karabük
38	Kayseri	79	Kilis
39	Kırklareli	80	Osmaniye
40	Kırşehir	81	Düzce
41	Kocaeli		

Table C.5 Travel Times for the Turkish Network Data Set

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	0	300	578	953	548	479	534	967	876	910	733	599	730	589	603	812	1063	565
2	300	0	871	680	571	687	833	694	1169	1192	984	326	428	801	896	1039	1276	708
3	578	871	0	1393	605	279	269	1271	337	332	212	1137	1301	371	165	265	545	411
4	953	680	1393	0	806	1118	1457	351	1726	1623	1416	353	233	1233	1467	1471	1707	1061
5	548	571	605	806	0	331	801	686	943	821	614	572	761	431	765	669	905	260
6	479	687	279	1118	331	0	516	1003	616	525	317	870	1080	134	442	372	609	129
7	534	833	269	1457	801	516	0	1378	376	516	477	1131	1262	608	112	524	750	648
8	967	694	1271	351	686	1003	1378	0	1609	1463	1255	380	463	1072	1388	1310	1547	926
9	876	1169	337	1726	943	616	376	1609	0	251	449	1434	1598	624	288	401	423	748
10	910	1192	332	1623	821	525	516	1463	251	0	248	1369	1579	390	412	145	194	621
11	733	984	212	1416	614	317	477	1255	449	248	0	1162	1371	183	373	96	376	414
12	599	326	1137	353	572	870	1131	380	1434	1369	1162	0	204	979	1176	1216	1453	827
13	730	428	1301	233	761	1080	1262	463	1598	1579	1371	204	0	1188	1326	1426	1662	1016
14	589	801	371	1233	431	134	608	1072	624	390	183	979	1188	0	535	238	474	202
15	603	896	165	1467	765	442	112	1388	288	412	373	1176	1326	535	0	420	646	574
16	812	1039	265	1471	669	372	524	1310	401	145	96	1216	1426	238	420	0	279	469
17	1063	1276	545	1707	905	609	750	1547	423	194	376	1453	1662	474	646	279	0	705
18	565	708	411	1061	260	129	648	926	748	621	414	827	1016	202	574	469	705	0
19	527	620	511	900	90	242	740	760	848	741	534	667	856	350	674	588	825	144
20	742	1035	218	1592	837	510	229	1503	133	307	363	1301	1465	546	142	356	541	642
21	512	211	1083	468	652	867	1045	519	1381	1366	1159	146	216	976	1108	1214	1450	827
22	931	1144	477	1575	774	477	736	1415	607	386	346	1321	1531	342	632	223	223	573
23	458	184	995	503	513	729	990	517	1293	1228	1020	139	359	837	1035	1075	1312	688
24	643	353	984	408	397	716	1049	345	1317	1215	1007	174	363	824	1059	1062	1299	652
25	771	498	1178	219	591	910	1243	205	1511	1408	1201	171	261	1018	1253	1256	1492	846
26	654	919	146	1361	574	247	426	1240	419	309	73	1107	1317	187	311	151	437	379
27	175	144	746	816	593	647	707	853	1044	1078	901	485	568	756	771	980	1231	690

Table C.5 (cont'd)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
28	686	532	898	564	334	630	1048	373	1236	1090	882	411	600	699	1058	937	1173	553
29	699	493	984	417	397	716	1089	311	1322	1215	1007	272	477	824	1099	1062	1298	652
30	844	652	1415	361	1015	1303	1377	691	1713	1748	1571	453	247	1411	1440	1649	1886	1263
31	139	338	707	991	646	608	669	1005	1005	1039	862	637	767	718	732	941	1192	695
32	576	869	158	1440	737	414	112	1360	305	414	375	1149	1298	507	29	422	648	546
33	48	349	547	1002	549	480	486	1015	842	879	734	647	778	590	563	813	1064	566
34	782	995	328	1426	624	328	587	1266	476	234	177	1172	1382	193	483	139	293	424
35	923	1216	344	1738	950	624	474	1616	106	191	437	1481	1645	580	386	341	292	756
36	989	714	1388	164	801	1120	1453	172	1721	1619	1411	388	374	1228	1463	1466	1703	1057
37	678	793	493	1060	254	228	730	896	831	643	436	826	1015	253	656	491	727	109
38	268	375	555	872	287	317	585	792	853	816	609	588	799	426	595	664	900	354
39	927	1140	473	1571	769	473	732	1411	641	420	341	1317	1526	338	628	218	241	569
40	341	514	420	1011	283	176	536	931	750	681	473	727	938	290	519	528	765	227
41	705	918	334	1349	547	251	593	1189	535	276	99	1095	1305	116	489	116	360	347
42	331	624	247	1197	549	289	259	1117	544	579	448	906	1054	398	272	513	793	426
43	683	976	104	1444	656	330	363	1322	348	228	107	1190	1399	269	257	159	740	762
44	357	116	895	596	471	664	889	609	1192	1163	956	242	452	773	934	1011	1247	627
45	908	1201	330	1723	936	609	459	1602	130	150	396	1467	1631	539	372	301	271	741
46	151	152	728	823	515	575	689	831	1025	1060	867	468	599	684	753	922	1158	612
47	511	318	1081	504	752	968	1043	606	1379	1414	1237	234	253	1076	1107	1314	1551	927
48	845	1138	358	1710	965	638	316	1630	107	358	497	1418	1567	680	238	490	531	770
49	719	417	1259	237	673	992	1251	375	1557	1491	1283	119	85	1100	1299	1338	1574	928
50	244	445	478	950	330	272	507	870	775	771	563	659	869	380	517	618	855	317
51	163	457	487	1000	408	357	481	921	785	819	618	688	887	466	512	696	940	403
52	699	584	847	615	248	578	1045	424	1185	1038	831	462	651	648	1010	886	1122	502
53	836	571	1120	429	535	852	1227	150	1458	1312	1104	335	511	921	1237	1159	1396	775
54	680	893	304	1324	522	226	590	1164	557	326	116	1070	1280	91	486	166	410	322

Table C.5 (cont'd)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
55	661	613	690	771	117	422	919	580	1028	882	674	583	772	491	853	729	966	345
56	710	409	1281	312	827	1066	1243	532	1579	1564	1357	243	78	1174	1306	1412	1648	1025
57	830	782	710	940	243	467	947	749	1048	836	628	751	940	445	873	683	920	271
58	431	357	721	672	201	453	785	592	1054	952	744	438	627	561	873	799	1035	412
59	892	1104	437	1536	734	437	696	1375	536	305	306	1282	1491	303	592	197	156	534
60	491	467	666	727	109	398	838	607	1004	897	689	493	683	506	809	744	980	334
61	760	569	1043	481	459	774	1150	228	1381	1234	1027	366	542	844	1160	1082	1318	697
62	531	257	1044	452	494	776	1063	425	1369	1275	1067	108	407	884	1111	1122	1359	735
63	310	115	881	667	666	782	842	718	1179	1213	1036	346	419	891	906	1114	1366	803
64	701	995	123	1516	729	402	288	1395	217	224	231	1260	1424	414	183	240	461	534
65	891	589	1462	199	902	1220	1423	520	1760	1719	1512	350	160	1328	1487	1566	1803	1157
66	425	526	484	909	161	209	639	812	821	714	507	655	865	323	603	561	798	173
67	674	886	456	1264	504	215	693	1073	711	477	270	1063	1265	117	619	325	561	245
68	238	525	402	1030	391	249	432	950	700	734	504	739	949	359	442	582	833	335
69	747	457	1072	336	485	803	1153	236	1409	1302	1095	221	396	911	1163	1149	1386	740
70	237	530	361	1179	587	402	302	1099	649	693	562	829	960	512	369	627	906	540
71	456	629	336	1056	269	61	573	935	674	566	359	802	1012	176	499	414	650	113
72	608	306	1179	382	748	963	1140	512	1476	1462	1255	172	133	1071	1204	1309	1546	923
73	695	503	1266	401	915	1153	1228	620	1564	1599	1422	342	151	1262	1291	1500	1736	1113
74	703	915	485	1203	486	247	723	1012	774	540	333	1014	1203	149	649	388	624	215
75	1014	741	1360	257	773	1092	1448	98	1698	1562	1355	426	463	1171	1458	1409	1646	1025
76	1039	766	1478	109	891	1209	1543	317	1811	1708	1501	439	316	1318	1553	1556	1792	1146
77	775	987	310	1419	617	320	569	1258	458	202	115	1165	1374	186	465	56	336	417
78	641	848	424	1207	401	176	661	1043	756	522	315	973	1163	132	587	370	606	144
79	190	201	806	874	653	707	768	913	1104	1138	961	546	625	817	831	1040	1291	750
80	61	241	634	894	559	535	596	907	932	966	789	540	670	645	659	868	1119	622
81	635	847	363	1279	477	180	622	1118	590	357	149	1025	1234	46	518	204	440	277

Table C.5 (cont'd)

	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
1	527	742	512	931	458	643	771	654	175	686	699	844	139	576	48	782	923	989
2	620	1035	211	1144	184	353	498	919	144	532	493	652	338	869	349	995	1216	714
3	511	218	1083	477	995	984	1178	146	746	898	984	1415	707	158	547	328	344	1388
4	900	1592	468	1575	503	408	219	1361	816	564	417	361	991	1440	1002	1426	1738	164
5	90	837	652	774	513	397	591	574	593	334	397	1015	646	737	549	624	950	801
6	242	510	867	477	729	716	910	247	647	630	716	1303	608	414	480	328	624	1120
7	740	229	1045	736	990	1049	1243	426	707	1048	1089	1377	669	112	486	587	474	1453
8	760	1503	519	1415	517	345	205	1240	853	373	311	691	1005	1360	1015	1266	1616	172
9	848	133	1381	607	1293	1317	1511	419	1044	1236	1322	1713	1005	305	842	476	106	1721
10	741	307	1366	386	1228	1215	1408	309	1078	1090	1215	1748	1039	414	879	234	191	1619
11	534	363	1159	346	1020	1007	1201	73	901	882	1007	1571	862	375	734	177	437	1411
12	667	1301	146	1321	139	174	171	1107	485	411	272	453	637	1149	647	1172	1481	388
13	856	1465	216	1531	359	363	261	1317	568	600	477	247	767	1298	778	1382	1645	374
14	350	546	976	342	837	824	1018	187	756	699	824	1411	718	507	590	193	580	1228
15	674	142	1108	632	1035	1059	1253	311	771	1058	1099	1440	732	29	563	483	386	1463
16	588	356	1214	223	1075	1062	1256	151	980	937	1062	1649	941	422	813	139	341	1466
17	825	541	1450	223	1312	1299	1492	437	1231	1173	1298	1886	1192	648	1064	293	292	1703
18	144	642	827	573	688	652	846	379	690	553	652	1263	695	546	566	424	756	1057
19	0	742	701	693	562	492	686	479	609	387	492	1109	629	646	527	544	856	896
20	742	0	1248	580	1160	1184	1377	331	910	1130	1216	1580	871	169	709	431	231	1588
21	701	1248	0	1318	146	301	312	1104	350	537	450	438	550	1081	561	1169	1428	528
22	693	580	1318	0	1180	1167	1361	392	1099	1042	1167	1754	1060	634	932	161	499	1571
23	562	1160	146	1180	0	168	321	966	344	399	282	582	495	1007	506	1031	1340	537
24	492	1184	301	1167	168	0	187	953	530	229	113	617	681	1032	692	1018	1329	404
25	686	1377	312	1361	321	187	0	1147	658	349	203	541	809	1225	820	1212	1523	210
26	479	331	1104	392	966	953	1147	0	822	867	953	1491	783	304	655	242	423	1357
27	609	910	350	1099	344	530	658	822	0	622	605	682	200	744	223	950	1091	874

Table C.5 (cont'd)

	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
28	387	1130	537	1042	399	229	349	867	622	0	146	853	739	1031	732	893	1243	542
29	492	1216	450	1167	282	113	203	953	605	146	0	731	756	1078	754	1024	1336	413
30	1109	1580	438	1754	582	617	541	1491	682	853	731	0	885	1416	896	1608	1763	517
31	629	871	550	1060	495	681	809	783	200	739	756	885	0	705	185	912	1052	1025
32	646	169	1081	634	1007	1032	1225	304	744	1031	1078	1416	705	0	540	485	403	1444
33	527	709	561	932	506	692	820	655	223	732	754	896	185	540	0	783	892	1036
34	544	431	1169	161	1031	1018	1212	242	950	893	1024	1608	912	485	783	0	416	1422
35	856	231	1428	499	1340	1329	1523	423	1091	1243	1336	1763	1052	403	892	416	0	1733
36	896	1588	528	1571	537	404	210	1357	874	542	413	517	1025	1444	1036	1422	1733	0
37	182	725	883	596	744	651	845	462	775	522	658	1272	796	637	679	446	833	1057
38	262	719	583	769	447	464	658	544	335	463	510	1008	356	575	269	619	900	869
39	689	575	1314	65	1176	1163	1356	388	1095	1037	1169	1753	1057	630	928	157	534	1568
40	217	617	722	633	586	603	797	407	475	595	649	1147	471	499	342	484	764	1008
41	467	480	1092	229	954	941	1135	166	873	816	947	1531	835	491	706	79	466	1346
42	488	411	836	741	764	789	982	369	499	788	835	1172	461	243	300	592	592	1094
43	562	261	1187	372	1048	1035	1229	82	851	949	1042	1523	813	259	652	237	340	1441
44	520	1059	236	1115	100	283	414	891	224	406	388	674	395	915	405	966	1239	631
45	841	216	1414	537	1326	1315	1509	408	1076	1229	1321	1749	1038	388	877	376	40	1720
46	530	892	382	1027	327	490	641	802	77	544	561	750	189	733	205	877	1072	858
47	801	1246	100	1419	246	409	399	1157	348	638	537	359	549	1087	559	1270	1426	616
48	870	129	1350	715	1277	1301	1495	459	1013	1258	1348	1685	975	276	798	564	214	1707
49	768	1423	199	1443	271	275	175	1229	557	512	389	344	757	1279	767	1294	1604	343
50	274	642	653	723	517	541	735	496	387	541	588	1060	377	498	245	574	822	947
51	364	651	669	809	546	592	786	539	332	591	638	1004	294	493	164	659	832	997
52	335	1078	589	991	450	275	400	816	674	51	197	908	791	991	738	841	1192	594
53	609	1352	483	1264	386	223	198	1089	748	222	159	767	888	1217	885	1115	1466	321
54	442	480	1067	278	929	916	1110	158	848	791	922	1506	810	488	681	129	516	1321

Table C.5 (cont'd)

	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
55	173	922	666	834	528	408	556	659	658	207	353	1028	750	834	681	685	1035	751
56	899	1445	189	1517	344	429	333	1303	548	665	547	225	748	1287	759	1367	1626	467
57	222	942	835	788	696	576	725	675	827	375	522	1197	918	854	848	639	1025	919
58	261	920	439	904	301	263	457	690	393	251	310	878	511	776	469	755	1066	669
59	653	540	1279	122	1140	1127	1321	352	1059	1002	1133	1717	1021	594	892	122	429	1533
60	179	898	547	849	409	319	512	635	488	235	325	939	580	789	531	700	1011	724
61	531	1274	514	1186	436	202	250	1011	682	144	99	798	511	1141	809	1037	1388	399
62	584	1236	206	1227	72	95	233	1013	417	332	209	664	569	1092	579	1078	1389	449
63	715	1245	197	1234	254	458	511	957	148	639	621	537	348	887	358	1085	1225	728
64	634	131	1207	468	1119	1108	1302	205	869	1022	1114	1542	831	185	670	319	224	1513
65	996	1626	381	1671	499	504	382	1457	729	726	580	165	929	1068	939	1522	1807	346
66	101	701	652	666	513	501	695	452	508	472	530	1091	529	584	426	517	829	906
67	424	633	1061	430	922	901	1049	305	841	700	846	1499	803	600	674	280	667	1243
68	330	567	733	702	597	621	815	425	406	621	668	1579	488	423	239	553	747	1027
69	579	1288	369	1254	272	108	122	1040	633	226	80	653	785	1144	796	1105	1417	333
70	542	515	742	855	688	771	964	482	405	770	817	1077	367	347	193	705	705	1176
71	174	567	799	518	661	648	842	305	589	562	654	1238	586	480	457	369	681	1053
72	797	1343	98	1414	242	351	313	1200	446	587	479	355	646	1184	656	1265	1523	480
73	987	1431	288	1605	432	517	445	1342	533	753	635	149	734	1272	744	1455	1611	527
74	416	696	1090	493	951	840	988	379	870	639	785	1460	832	629	704	343	730	1182
75	859	1583	566	1514	563	404	245	1329	900	492	369	606	1052	1439	1062	1365	1705	98
76	976	1677	537	1661	588	493	304	1446	885	649	502	392	1077	1533	1087	1511	1823	143
77	537	413	1162	167	1023	1010	1204	189	943	885	1017	1601	904	467	776	83	398	1416
78	321	656	1023	475	884	799	992	361	809	670	805	1419	771	568	642	325	712	1204
79	669	970	408	1159	404	590	718	882	59	683	687	743	167	812	284	1010	1151	935
80	541	799	453	988	398	584	712	710	115	642	659	788	101	640	109	838	979	929
81	396	513	1022	309	883	870	1064	195	802	745	877	1461	764	520	636	160	546	1276

Table C.5 (cont'd)

	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
1	678	268	927	341	705	331	683	357	908	151	511	845	719	244	163	699	836	680
2	793	375	1140	514	918	624	976	116	1201	152	318	1138	417	445	457	584	571	893
3	493	555	473	420	334	247	104	895	330	728	1081	358	1259	478	487	847	1120	304
4	1060	872	1571	1011	1349	1197	1444	596	1723	823	504	1710	237	950	1000	615	429	1324
5	254	287	769	283	547	549	656	471	936	515	752	965	673	330	408	248	535	522
6	228	317	473	176	251	289	330	664	609	575	968	638	992	272	357	578	852	226
7	730	585	732	536	593	259	363	889	459	689	1043	316	1251	507	481	1045	1227	590
8	896	792	1411	931	1189	1117	1322	609	1602	831	606	1630	375	870	921	424	150	1164
9	831	853	641	750	535	544	348	1192	130	1025	1379	107	1557	775	785	1185	1458	557
10	643	816	420	681	276	579	228	1163	150	1060	1414	358	1491	771	819	1038	1312	326
11	436	609	341	473	99	448	107	956	396	867	1237	497	1283	563	618	831	1104	116
12	826	588	1317	727	1095	906	1190	242	1467	468	234	1418	119	659	688	462	335	1070
13	1015	799	1526	938	1305	1054	1399	452	1631	599	253	1567	85	869	887	651	511	1280
14	253	426	338	290	116	398	269	773	539	684	1076	680	1100	380	466	648	921	91
15	656	595	628	519	489	272	257	934	372	753	1107	238	1299	517	512	1010	1237	486
16	491	664	218	528	116	513	159	1011	301	922	1314	490	1338	618	696	886	1159	166
17	727	900	241	765	360	793	740	1247	271	1158	1551	531	1574	855	940	1122	1396	410
18	109	354	569	227	347	426	762	627	741	612	927	770	928	317	403	502	775	322
19	182	262	689	217	467	488	562	520	841	530	801	870	768	274	364	335	609	442
20	725	719	575	617	480	411	261	1059	216	892	1246	129	1423	642	651	1078	1352	480
21	883	583	1314	722	1092	836	1187	236	1414	382	100	1350	199	653	669	589	483	1067
22	596	769	65	633	229	741	372	1115	537	1027	1419	715	1443	723	809	991	1264	278
23	744	447	1176	586	954	764	1048	100	1326	327	246	1277	271	517	546	450	386	929
24	651	464	1163	603	941	789	1035	283	1315	490	409	1301	275	541	592	275	223	916
25	845	658	1356	797	1135	982	1229	414	1509	641	399	1495	175	735	786	400	198	1110
26	462	544	388	407	166	369	82	891	408	802	1157	459	1229	496	539	816	1089	158
27	775	335	1095	475	873	499	851	224	1076	77	348	1013	557	387	332	674	748	848

Table C.5 (cont'd)

	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
28	522	463	1037	595	816	788	949	406	1229	544	638	1258	512	541	591	51	222	791
29	658	510	1169	649	947	835	1042	388	1321	561	537	1348	389	588	638	197	159	922
30	1272	1008	1753	1147	1531	1172	1523	674	1749	750	359	1685	344	1060	1004	908	767	1506
31	796	356	1057	471	835	461	813	395	1038	189	549	975	757	377	294	791	888	810
32	637	575	630	499	491	243	259	915	388	733	1087	276	1279	498	493	991	1217	488
33	679	269	928	342	706	300	652	405	877	205	559	798	767	245	164	738	885	681
34	446	619	157	484	79	592	237	966	376	877	1270	564	1294	574	659	841	1115	129
35	833	900	534	764	466	592	340	1239	40	1072	1426	214	1604	822	832	1192	1466	516
36	1057	869	1568	1008	1346	1094	1441	631	1720	858	616	1707	343	947	997	594	321	1321
37	0	439	591	340	369	520	544	701	793	697	983	853	927	430	515	471	745	344
38	439	0	764	139	542	324	626	346	885	246	671	837	711	77	128	469	641	517
39	591	764	0	629	224	737	445	1111	588	1022	1415	765	1438	719	804	986	1260	274
40	340	139	629	0	407	273	491	485	752	397	810	746	850	93	178	535	781	382
41	369	542	224	407	0	515	207	889	425	800	1193	614	1217	497	582	764	1038	52
42	520	324	737	273	515	0	352	664	577	481	834	514	1028	247	240	784	966	490
43	544	626	445	491	207	352	0	973	325	832	1186	396	1311	578	592	898	1171	224
44	701	346	1111	485	889	664	973	0	1225	204	336	1176	364	417	446	457	508	864
45	793	885	588	752	425	577	325	1225	0	1058	1412	238	1589	808	817	1178	1451	475
46	697	246	1022	397	800	481	832	204	1058	0	413	994	591	309	313	596	692	775
47	983	671	1415	810	1193	834	1186	336	1412	413	0	1348	267	723	667	689	570	1168
48	853	837	765	746	614	514	396	1176	238	994	1348	0	1541	759	755	1206	1479	614
49	927	711	1438	850	1217	1028	1311	364	1589	591	267	1541	0	781	810	563	423	1192
50	430	77	719	93	497	247	578	417	808	309	723	759	781	0	83	537	719	472
51	515	128	804	178	582	240	592	446	817	313	667	755	810	83	0	597	770	557
52	471	469	986	535	764	784	898	457	1178	596	689	1206	563	537	597	0	273	739
53	745	641	1260	781	1038	966	1171	508	1451	692	570	1479	423	719	770	273	0	1013
54	344	517	274	382	52	490	224	864	475	775	1168	614	1192	472	557	739	1013	0

Table C.5 (cont'd)

	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
55	314	422	830	402	608	668	741	487	1021	580	767	1050	684	458	540	156	429	583
56	1081	781	1512	920	1291	1034	1385	434	1611	602	210	1548	157	851	868	717	580	1266
57	174	589	784	557	562	737	742	655	985	749	935	1070	852	625	707	324	598	537
58	443	200	899	339	678	525	772	236	1052	314	540	1037	539	278	328	225	441	653
59	556	729	112	593	189	701	410	1076	483	987	1379	660	1403	683	769	951	1224	239
60	365	262	844	327	623	577	717	366	997	410	648	1026	594	330	390	185	456	598
61	667	565	1182	704	960	890	1094	443	1373	616	601	1402	454	643	693	195	77	936
62	748	516	1223	655	1001	840	1095	195	1375	422	641	1353	228	593	641	383	313	976
63	888	470	1230	609	1008	634	986	231	1211	202	200	1148	408	522	467	690	682	983
64	617	678	541	545	351	371	130	1018	209	851	1205	261	1382	601	611	971	1244	348
65	1156	939	1667	1078	1445	1215	1540	592	1792	783	417	1729	227	1009	1038	777	606	1421
66	259	164	662	122	440	388	535	451	814	430	752	829	777	173	255	425	661	416
67	217	511	425	375	198	483	383	858	627	769	1161	767	1176	465	551	648	922	174
68	448	157	697	107	476	171	507	497	733	388	742	684	861	80	94	631	799	451
69	739	568	1250	707	1028	893	1123	394	1402	594	456	1405	280	645	696	278	113	1004
70	634	306	850	320	629	112	466	587	691	386	740	607	949	262	178	789	948	604
71	221	253	514	118	292	321	387	599	666	511	900	695	924	208	293	511	784	268
72	979	678	1410	817	1188	932	1283	332	1509	500	133	1445	136	749	765	639	512	1164
73	1169	856	1600	995	1378	1019	1371	522	1597	598	206	1533	237	908	853	805	668	1354
74	158	540	488	404	267	513	446	887	690	798	1190	830	1115	494	580	587	861	239
75	995	863	1510	1004	1288	1188	1411	656	1691	883	652	1700	388	941	991	523	250	1264
76	965	957	1656	1096	1434	1282	1529	681	1809	908	573	1795	344	1035	1086	700	450	1410
77	439	612	240	476	72	558	205	959	358	870	1262	546	1266	566	652	834	1107	122
78	115	473	470	337	248	451	428	820	672	731	1123	783	1075	427	513	618	892	224
79	836	395	1155	535	933	559	911	284	1136	137	406	1073	615	447	393	734	808	909
80	708	262	983	397	761	387	739	298	965	92	451	901	660	303	220	694	790	737
81	299	472	304	336	83	445	257	819	506	730	1122	646	1146	426	512	694	967	58

Table C.5 (cont'd)

	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
1	661	710	830	431	892	491	760	531	310	701	891	425	674	238	747	237	456	608
2	613	409	782	357	1104	467	569	257	115	995	589	526	886	525	457	530	629	306
3	690	1281	710	721	437	666	1043	1044	881	123	1462	484	456	402	1072	361	336	1179
4	771	312	940	672	1536	727	481	452	667	1516	199	909	1264	1030	336	1179	1056	382
5	117	827	243	201	734	109	459	494	666	729	902	161	504	391	485	587	269	748
6	422	1066	467	453	437	398	774	776	782	402	1220	209	215	249	803	402	61	963
7	919	1243	947	785	696	838	1150	1063	842	288	1423	639	693	432	1153	302	573	1140
8	580	532	749	592	1375	607	228	425	718	1395	520	812	1073	950	236	1099	935	512
9	1028	1579	1048	1054	536	1004	1381	1369	1179	217	1760	821	711	700	1409	649	674	1476
10	882	1564	836	952	305	897	1234	1275	1213	224	1719	714	477	734	1302	693	566	1462
11	674	1357	628	744	306	689	1027	1067	1036	231	1512	507	270	504	1095	562	359	1255
12	583	243	751	438	1282	493	366	108	346	1260	350	655	1063	739	221	829	802	172
13	772	78	940	627	1491	683	542	407	419	1424	160	865	1265	949	396	960	1012	133
14	491	1174	445	561	303	506	844	884	891	414	1328	323	117	359	911	512	176	1071
15	853	1306	873	873	592	809	1160	1111	906	183	1487	603	619	442	1163	369	499	1204
16	729	1412	683	799	197	744	1082	1122	1114	240	1566	561	325	582	1149	627	414	1309
17	966	1648	920	1035	156	980	1318	1359	1366	461	1803	798	561	833	1386	906	650	1546
18	345	1025	271	412	534	334	697	735	803	534	1157	173	245	335	740	540	113	923
19	173	899	222	261	653	179	531	584	715	634	996	101	424	330	579	542	174	797
20	922	1445	942	920	540	898	1274	1236	1245	131	1626	701	633	567	1288	515	567	1343
21	666	189	835	439	1279	547	514	206	197	1207	381	652	1061	733	369	742	799	98
22	834	1517	788	904	122	849	1186	1227	1234	468	1671	666	430	702	1254	855	518	1414
23	528	344	696	301	1140	409	436	72	254	1119	499	513	922	597	272	688	661	242
24	408	429	576	263	1127	319	202	95	458	1108	504	501	901	621	108	771	648	351
25	556	333	725	457	1321	512	250	233	511	1302	382	695	1049	815	122	964	842	313
26	659	1303	675	690	352	635	1011	1013	957	205	1457	452	305	425	1040	482	305	1200
27	658	548	827	393	1059	488	682	417	148	869	729	508	841	406	633	405	589	446

Table C.5 (cont'd)

	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
28	207	665	375	251	1002	235	144	332	639	1022	726	472	700	621	226	770	562	587
29	353	547	522	310	1133	325	99	209	621	1114	580	530	846	668	80	817	654	479
30	1028	225	1197	878	1717	939	798	664	537	1542	165	1091	1499	1579	653	1077	1238	355
31	750	748	918	511	1021	580	511	569	348	831	929	529	803	488	785	367	586	646
32	834	1287	854	776	594	789	1141	1092	887	185	1068	584	600	423	1144	347	480	1184
33	681	759	848	469	892	531	809	579	358	670	939	426	674	239	796	193	457	656
34	685	1367	639	755	122	700	1037	1078	1085	319	1522	517	280	553	1105	705	369	1265
35	1035	1626	1025	1066	429	1011	1388	1389	1225	224	1807	829	667	747	1417	705	681	1523
36	751	467	919	669	1533	724	399	449	728	1513	346	906	1243	1027	333	1176	1053	480
37	314	1081	174	443	556	365	667	748	888	617	1156	259	217	448	739	634	221	979
38	422	781	589	200	729	262	565	516	470	678	939	164	511	157	568	306	253	678
39	830	1512	784	899	112	844	1182	1223	1230	541	1667	662	425	697	1250	850	514	1410
40	402	920	557	339	593	327	704	655	609	545	1078	122	375	107	707	320	118	817
41	608	1291	562	678	189	623	960	1001	1008	351	1445	440	198	476	1028	629	292	1188
42	668	1034	737	525	701	577	890	840	634	371	1215	388	483	171	893	112	321	932
43	741	1385	742	772	410	717	1094	1095	986	130	1540	535	383	507	1123	466	387	1283
44	487	434	655	236	1076	366	443	195	231	1018	592	451	858	497	394	587	599	332
45	1021	1611	985	1052	483	997	1373	1375	1211	209	1792	814	627	733	1402	691	666	1509
46	580	602	749	314	987	410	616	422	202	851	783	430	769	388	594	386	511	500
47	767	210	935	540	1379	648	601	641	200	1205	417	752	1161	742	456	740	900	133
48	1050	1548	1070	1037	660	1026	1402	1353	1148	261	1729	829	767	684	1405	607	695	1445
49	684	157	852	539	1403	594	454	228	408	1382	227	777	1176	861	280	949	924	136
50	458	851	625	278	683	330	643	593	522	601	1009	173	465	80	645	262	208	749
51	540	868	707	328	769	390	693	641	467	611	1038	255	551	94	696	178	293	765
52	156	717	324	225	951	185	195	383	690	971	777	425	648	631	278	789	511	639
53	429	580	598	441	1224	456	77	313	682	1244	606	661	922	799	113	948	784	512
54	583	1266	537	653	239	598	936	976	983	348	1421	416	174	451	1004	604	268	1164

Table C.5 (cont'd)

	55	**56**	**57**	**58**	**59**	**60**	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**
55	0	837	168	270	794	168	351	504	719	814	912	281	492	510	434	719	354	759
56	837	0	1007	639	1478	747	613	440	401	1406	195	852	1260	933	468	942	999	90
57	168	1007	0	440	748	340	522	674	890	834	1082	450	358	665	605	851	442	930
58	270	639	440	0	864	108	365	323	453	845	767	237	646	358	367	507	385	535
59	794	1478	748	864	0	809	1147	1187	1194	506	1631	626	390	662	1214	815	479	1374
60	168	747	340	108	809	0	380	415	561	790	823	202	591	390	406	568	330	643
61	351	613	522	365	1147	380	0	297	676	1167	644	585	844	723	128	872	706	543
62	504	440	674	323	1187	415	297	0	345	1168	548	560	969	673	199	782	708	274
63	719	401	890	453	1194	561	676	345	0	1004	580	621	976	541	568	540	723	297
64	814	1406	834	845	506	790	1167	1168	1004	0	1585	607	501	526	1195	484	460	1302
65	912	195	1082	767	1631	823	644	548	580	1585	0	1005	1405	1089	499	1121	1152	295
66	281	852	450	237	626	202	585	560	621	607	1005	0	408	227	605	441	147	748
67	492	1260	358	646	390	591	844	969	976	501	1405	408	0	444	927	597	261	1156
68	510	933	665	358	662	390	723	673	541	526	1089	227	444	0	725	197	216	829
69	434	468	605	367	1214	406	128	199	568	1195	499	605	927	725	0	875	735	398
70	719	942	851	507	815	568	872	782	540	484	1121	441	597	197	875	0	434	838
71	354	999	442	385	479	330	706	708	723	460	1152	147	261	216	735	434	0	895
72	759	90	930	535	1374	643	543	274	297	1302	295	748	1156	829	398	838	895	0
73	925	89	1096	725	1565	834	699	526	385	1390	195	938	1347	927	554	925	1086	176
74	431	1271	297	675	453	599	783	936	1005	564	1344	437	64	473	866	626	290	1186
75	679	547	850	662	1474	695	327	472	765	1484	435	900	1172	1021	294	1170	1024	525
76	856	409	1027	757	1621	812	528	537	736	1602	225	995	1349	1115	421	1264	1142	450
77	677	1362	631	747	192	692	1030	1070	1077	296	1515	510	273	545	1098	671	362	1258
78	462	1222	417	583	435	505	814	895	943	546	1303	370	104	412	886	565	219	1118
79	719	608	889	454	1120	549	742	499	188	929	787	575	902	466	694	465	650	504
80	662	652	832	414	948	492	714	493	251	758	832	448	730	294	688	293	512	548
81	537	1222	481	607	269	552	890	930	937	380	1375	369	122	405	958	558	222	1118

Table C.5 (cont'd)

	73	74	75	76	77	78	79	80	81
1	695	703	1014	1039	775	641	190	61	635
2	503	915	741	766	987	848	201	241	847
3	1266	485	1360	1478	310	424	806	634	363
4	401	1203	257	109	1419	1207	874	894	1279
5	915	486	773	891	617	401	653	559	477
6	1153	247	1092	1209	320	176	707	535	180
7	1228	723	1448	1543	569	661	768	596	622
8	620	1012	98	317	1258	1043	913	907	1118
9	1564	774	1698	1811	458	756	1104	932	590
10	1599	540	1562	1708	202	522	1138	966	357
11	1422	333	1355	1501	115	315	961	789	149
12	342	1014	426	439	1165	973	546	540	1025
13	151	1203	463	316	1374	1163	625	670	1234
14	1262	149	1171	1318	186	132	817	645	46
15	1291	649	1458	1553	465	587	831	659	518
16	1500	388	1409	1556	56	370	1040	868	204
17	1736	624	1646	1792	336	606	1291	1119	440
18	1113	215	1025	1146	417	144	750	622	277
19	987	416	859	976	537	321	669	541	396
20	1431	696	1583	1677	413	656	970	799	513
21	288	1090	566	537	1162	1023	408	453	1022
22	1605	493	1514	1661	167	475	1159	988	309
23	432	951	563	588	1023	884	404	398	883
24	517	840	404	493	1010	799	590	584	870
25	445	988	245	304	1204	992	718	712	1064
26	1342	379	1329	1446	189	361	882	710	195
27	533	870	900	885	943	809	59	115	802

Table C.5 (cont'd)

	73	74	75	76	77	78	79	80	81
28	753	639	492	649	885	670	683	642	745
29	635	785	369	502	1017	805	687	659	877
30	149	1460	606	392	1601	1419	743	788	1461
31	734	832	1052	1077	904	771	167	101	764
32	1272	629	1439	1533	467	568	812	640	520
33	744	704	1062	1087	776	642	284	109	636
34	1455	343	1365	1511	83	325	1010	838	160
35	1611	730	1705	1823	398	712	1151	979	546
36	527	1182	98	143	1416	1204	935	929	1276
37	1169	158	995	965	439	115	836	708	299
38	856	540	863	957	612	473	395	262	472
39	1600	488	1510	1656	240	470	1155	983	304
40	995	404	1004	1096	476	337	535	397	336
41	1378	267	1288	1434	72	248	933	761	83
42	1019	513	1188	1282	558	451	559	387	445
43	1371	446	1411	1529	205	428	911	739	257
44	522	887	656	681	959	820	284	298	819
45	1597	690	1691	1809	358	672	1136	965	506
46	598	798	883	908	870	731	137	92	730
47	206	1190	652	573	1262	1123	406	451	1122
48	1533	830	1700	1795	546	783	1073	901	646
49	237	1115	388	344	1266	1075	615	660	1146
50	908	494	941	1035	566	427	447	303	426
51	853	580	991	1086	652	513	393	220	512
52	805	587	523	700	834	618	734	694	694
53	668	861	250	450	1107	892	808	790	967
54	1354	239	1264	1410	122	224	909	737	58

Table C.5 (cont'd)

	73	74	75	76	77	78	79	80	81
55	925	431	679	856	677	462	719	662	537
56	89	1271	547	409	1362	1222	608	652	1222
57	1096	297	850	1027	631	417	889	832	481
58	725	675	662	757	747	583	454	414	607
59	1565	453	1474	1621	192	435	1120	948	269
60	834	599	695	812	692	505	549	492	552
61	699	783	327	528	1030	814	742	714	890
62	526	936	472	537	1070	895	499	493	930
63	385	1005	765	736	1077	943	188	251	937
64	1390	564	1484	1602	296	546	929	758	380
65	195	1344	435	225	1515	1303	787	832	1375
66	938	437	900	995	510	370	575	448	369
67	1347	64	1172	1349	273	104	902	730	122
68	927	473	1021	1115	545	412	466	294	405
69	554	866	294	421	1098	886	694	688	958
70	925	626	1170	1264	671	565	465	293	558
71	1086	290	1024	1142	362	219	650	512	222
72	176	1186	525	450	1258	1118	504	548	1118
73	0	1357	617	422	1448	1309	591	636	1308
74	1357	0	1111	1288	336	86	931	759	187
75	617	1111	0	232	1358	1142	962	956	1217
76	422	1288	232	0	1504	1292	942	979	1364
77	1448	336	1358	1504	0	318	1003	831	152
78	1309	86	1142	1292	318	0	869	697	178
79	591	931	962	942	1003	869	0	121	863
80	636	759	956	979	831	697	121	0	691
81	1308	187	1217	1364	152	178	863	691	0

Table C.6 Flows for the Turkish Network Data Set

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	0	17493	22782	14827	10242	112387	48225	5382	26661	30183	5449	7115	10899	7590	7201	59593	13039	7581
2	17174	0	7544	4910	3391	37216	15969	1782	8828	9995	1804	2356	3609	2513	2385	19733	4318	2510
3	22429	7565	0	6412	4429	48604	20856	2328	11530	13053	2357	3077	4714	3282	3114	25772	5639	3279
4	14536	4903	6385	0	2871	31499	13516	1508	7472	8459	1527	1994	3055	2127	2018	16702	3654	2125
5	10016	3378	4400	2864	0	21706	9314	1039	5149	5829	1052	1374	2105	1466	1391	11509	2518	1464
6	116190	39190	51038	33217	22945	0	108040	12058	59729	67619	12208	15941	24418	17003	16133	133508	29211	16985
7	48130	16234	21142	13760	9505	104299	0	4995	24742	28010	5057	6603	10115	7043	6683	55304	12100	7036
8	5250	1771	2306	1501	1037	11377	4882	0	2699	3055	552	720	1103	768	729	6033	1320	767
9	26302	8872	11554	7520	5194	56998	24458	2730	0	15307	2764	3609	5528	3849	3652	30223	6613	3845
10	29833	10062	13105	8529	5891	64649	27740	3096	15336	0	3135	4093	6270	4366	4142	34279	7500	4361
11	5316	1793	2335	1520	1050	11520	4943	552	2733	3094	0	729	1117	778	738	6108	1336	777
12	6947	2343	3052	1986	1372	15055	6460	721	3571	4043	730	0	1460	1017	965	7983	1747	1016
13	10663	3597	4684	3048	2106	23107	9915	1107	5482	6206	1120	1463	0	1560	1481	12252	2681	1559
14	7412	2500	3256	2119	1464	16062	6892	769	3810	4314	779	1017	1558	0	1029	8517	1863	1084
15	7031	2372	3089	2010	1389	15237	6538	730	3615	4092	739	965	1478	1029	0	8079	1768	1028
16	59843	20184	26287	17108	11818	129681	55645	6210	30763	34827	6288	8210	12576	8757	8309	0	15045	8748
17	12771	4307	5610	3651	2522	27674	11875	1325	6565	7432	1342	1752	2684	1869	1773	14674	0	1867
18	7404	2497	3252	2117	1462	16045	6885	768	3806	4309	778	1016	1556	1084	1028	8508	1861	0
19	16431	5542	7217	4697	3245	35606	15278	1705	8447	9562	1726	2254	3453	2404	2281	18880	4131	2402
20	23480	7920	10314	6713	4637	50883	21834	2437	12071	13665	2467	3221	4935	3436	3260	26980	5903	3432
21	37933	12794	16663	10845	7491	82201	35272	3937	19500	22076	3986	5204	7972	5551	5267	43587	9537	5545
22	11047	3726	4853	3158	2182	23940	10273	1146	5679	6429	1161	1516	2322	1617	1534	12694	2777	1615
23	15669	5285	6883	4480	3094	33955	14570	1626	8055	9119	1646	2150	3293	2293	2176	18004	3939	2290
24	8683	2929	3814	2482	1715	18816	8074	901	4464	5053	912	1191	1825	1271	1206	9977	2183	1269
25	25927	8745	11389	7412	5120	56185	24109	2691	13328	15089	2724	3557	5449	3794	3600	29792	6518	3790
26	19460	6564	8548	5563	3843	42171	18095	2020	10004	11325	2045	2670	4090	2848	2702	22361	4892	2845
27	35735	12053	15697	10216	7057	77438	33228	3708	18370	20797	3755	4903	7510	5229	4962	41061	8984	5224

Table C.6 (cont'd)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
28	14399	4857	6325	4117	2844	31204	13389	1494	7402	8380	1513	1976	3026	2107	1999	16546	3620	2105
29	5114	1725	2246	1462	1010	11081	4755	531	2629	2976	537	702	1075	748	710	5876	1286	747
30	6476	2184	2845	1851	1279	14033	6022	672	3329	3769	680	888	1361	948	899	7441	1628	947
31	34842	11752	15305	9961	6881	75503	32398	3616	17911	20277	3661	4780	7322	5099	4838	40035	8760	5093
32	14119	4762	6202	4036	2788	30595	13128	1465	7258	8217	1483	1937	2967	2066	1960	16223	3550	2064
33	46169	15573	20281	13199	9117	100050	42931	4791	23734	26869	4851	6334	9703	6756	6411	53051	11607	6749
34	320661	108156	140856	91673	63323	694878	298169	33277	164841	186616	33692	43993	67389	46926	44524	368455	80617	46874
35	96757	32635	42502	27662	19107	209674	89970	10041	49740	56310	10166	13275	20334	14159	13435	111178	24326	14144
36	8908	3005	3913	2547	1759	19304	8283	924	4579	5184	936	1222	1872	1304	1237	10236	2240	1302
37	10299	3474	4524	2944	2034	22318	9576	1069	5294	5994	1082	1413	2164	1507	1430	11834	2589	1505
38	29385	9911	12908	8401	5803	63678	27324	3049	15106	17101	3087	4031	6175	4300	4080	33765	7388	4295
39	9003	3037	3955	2574	1778	19510	8372	934	4628	5240	946	1235	1892	1318	1250	10345	2263	1316
40	6933	2339	3046	1982	1369	15025	6447	720	3564	4035	729	951	1457	1015	963	7967	1743	1014
41	33494	11297	14713	9576	6614	72582	31145	3476	17218	19493	3519	4595	7039	4902	4651	38486	8421	4896
42	61793	20842	27144	17666	12203	133907	57459	6413	31766	35962	6493	8478	12986	9043	8580	71003	15535	9033
43	18094	6103	7948	5173	3573	39209	16824	1878	9301	10530	1901	2482	3802	2648	2512	20790	4549	2645
44	23582	7954	10359	6742	4657	51103	21928	2447	12123	13724	2478	3235	4956	3451	3274	27097	5929	3447
45	35024	11813	15385	10013	6917	75899	32568	3635	18005	20383	3680	4805	7361	5125	4863	40245	8805	5120
46	27752	9361	12191	7934	5480	60140	25806	2880	14266	16151	2916	3807	5832	4061	3853	31889	6977	4057
47	19435	6555	8537	5556	3838	42116	18072	2017	9991	11311	2042	2666	4084	2844	2699	22332	4886	2841
48	19720	6651	8662	5638	3894	42734	18337	2046	10137	11476	2072	2705	4144	2886	2738	22659	4958	2883
49	12458	4202	5472	3561	2460	26996	11584	1293	6404	7250	1309	1709	2618	1823	1730	14314	3132	1821
50	8492	2864	3730	2428	1677	18403	7897	881	4366	4942	892	1165	1785	1243	1179	9758	2135	1241
51	9544	3219	4192	2728	1885	20681	8874	990	4906	5554	1003	1309	2006	1397	1325	10966	2399	1395
52	24537	8276	10778	7015	4845	53172	22816	2546	12614	14280	2578	3366	5157	3591	3407	28194	6169	3587
53	10036	3385	4408	2869	1982	21748	9332	1041	5159	5841	1054	1377	2109	1469	1393	11532	2523	1467
54	20859	7035	9162	5963	4119	45201	19395	2165	10723	12139	2192	2862	4384	3052	2896	23967	5244	3049

Table C.6 (cont'd)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
55	33580	11326	14751	9600	6631	72769	31225	3485	17263	19543	3528	4607	7057	4914	4663	38585	8442	4909
56	7220	2435	3172	2064	1426	15647	6714	749	3712	4202	759	991	1517	1057	1003	8297	1815	1055
57	6173	2082	2712	1765	1219	13378	5740	641	3174	3593	649	847	1297	903	857	7094	1552	902
58	20828	7025	9149	5955	4113	45136	19368	2162	10707	12122	2188	2858	4377	3048	2892	23933	5236	3045
59	17167	5790	7541	4908	3390	37202	15963	1782	8825	9991	1804	2355	3608	2512	2384	19726	4316	2510
60	22865	7712	10044	6537	4515	49549	21261	2373	11754	13307	2402	3137	4805	3346	3175	26273	5749	3342
61	26987	9102	11854	7715	5329	58481	25094	2801	13873	15706	2836	3702	5671	3949	3747	31009	6785	3945
62	2556	862	1123	731	505	5539	2377	265	1314	1488	269	351	537	374	355	2937	643	374
63	40228	13569	17671	11501	7944	87176	37407	4175	20680	23412	4227	5519	8454	5887	5586	46224	10114	5881
64	8834	2980	3880	2525	1744	19143	8214	917	4541	5141	928	1212	1856	1293	1227	10150	2221	1291
65	24250	8179	10652	6933	4789	52550	22549	2517	12466	14113	2548	3327	5096	3549	3367	27864	6097	3545
66	18817	6347	8266	5380	3716	40778	17498	1953	9673	10951	1977	2582	3955	2754	2613	21622	4731	2751
67	16945	5716	7444	4845	3346	36721	15757	1759	8711	9862	1780	2325	3561	2480	2353	19471	4260	2477
68	10867	3665	4774	3107	2146	23550	10105	1128	5587	6325	1142	1491	2284	1590	1509	12487	2732	1589
69	2659	897	1168	760	525	5763	2473	276	1367	1548	279	365	559	389	369	3056	669	389
70	6658	2246	2925	1903	1315	14428	6191	691	3423	3875	700	913	1399	974	924	7650	1674	973
71	10520	3548	4621	3008	2078	22798	9782	1092	5408	6123	1105	1443	2211	1540	1461	12088	2645	1538
72	12543	4231	5510	3586	2477	27180	11663	1302	6448	7300	1318	1721	2636	1836	1742	14412	3153	1833
73	9685	3267	4254	2769	1912	20987	9005	1005	4979	5636	1018	1329	2035	1417	1345	11128	2435	1416
74	5037	1699	2213	1440	995	10916	4684	523	2590	2932	529	691	1059	737	699	5788	1266	736
75	3656	1233	1606	1045	722	7922	3399	379	1879	2127	384	502	768	535	508	4201	919	534
76	4611	1555	2026	1318	911	9993	4288	479	2371	2684	485	633	969	675	640	5299	1159	674
77	4610	1555	2025	1318	910	9990	4287	478	2370	2683	484	632	969	675	640	5297	1159	674
78	6161	2078	2706	1761	1217	13350	5728	639	3167	3585	647	845	1295	902	855	7079	1549	901
79	3135	1057	1377	896	619	6793	2915	325	1611	1824	329	430	659	459	435	3602	788	458
80	12599	4250	5535	3602	2488	27303	11716	1308	6477	7333	1324	1729	2648	1844	1749	14477	3168	1842
81	8612	2905	3783	2462	1701	18663	8008	894	4427	5012	905	1182	1810	1260	1196	9896	2165	1259

Table C.6 (cont'd)

	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
1	16743	23836	38213	11290	15973	8885	26286	19798	36041	14689	5242	6634	35157	14405	46308	280943	94525	9114
2	5544	7893	12654	3738	5289	2942	8704	6556	11934	4864	1736	2197	11642	4770	15334	93030	31301	3018
3	7241	10308	16526	4882	6908	3842	11368	8562	15586	6352	2267	2869	15204	6229	20027	121499	40879	3942
4	4693	6681	10710	3164	4477	2490	7367	5549	10101	4117	1469	1859	9854	4037	12979	78741	26493	2554
5	3234	4604	7380	2180	3085	1716	5077	3824	6961	2837	1012	1281	6790	2782	8944	54259	18256	1760
6	37509	53401	85609	25293	35785	19905	58890	44354	80743	32908	11745	14863	78763	32271	103746	629407	211768	20418
7	15538	22121	35463	10477	14823	8245	24394	18373	33447	13632	4865	6157	32627	13368	42975	260724	87722	8458
8	1695	2413	3868	1143	1617	899	2661	2004	3649	1487	531	672	3559	1458	4688	28441	9569	923
9	8491	12089	19380	5726	8101	4506	13331	10041	18278	7450	2659	3365	17830	7305	23485	142482	47939	4622
10	9631	13711	21981	6494	9188	5111	15121	11388	20732	8449	3016	3816	20223	8286	26638	161607	54374	5243
11	1716	2443	3917	1157	1637	911	2694	2029	3694	1506	537	680	3604	1476	4747	28796	9689	934
12	2243	3193	5119	1512	2140	1190	3521	2652	4828	1968	702	889	4709	1930	6203	37633	12662	1221
13	3442	4901	7857	2321	3284	1827	5404	4070	7410	3020	1078	1364	7228	2962	9521	57762	19434	1874
14	2393	3407	5461	1614	2283	1270	3757	2829	5151	2099	749	948	5025	2059	6618	40152	13509	1303
15	2270	3232	5181	1531	2166	1205	3564	2684	4886	1991	711	899	4766	1953	6278	38090	12815	1236
16	19319	27504	44093	13027	18431	10252	30331	22844	41586	16949	6049	7655	40566	16621	53434	324172	109070	10516
17	4123	5869	9409	2780	3933	2188	6473	4875	8875	3617	1291	1634	8657	3547	11403	69179	23276	2244
18	2390	3403	5455	1612	2280	1268	3753	2826	5145	2097	748	947	5019	2056	6611	40108	13494	1301
19	0	7552	12106	3577	5060	2815	8328	6272	11418	4654	1661	2102	11138	4564	14671	89006	29947	2887
20	7580	0	17301	5111	7232	4023	11901	8963	16317	6650	2374	3004	15917	6522	20966	127195	42796	4126
21	12246	17434	0	8257	11683	6498	19226	14480	26360	10744	3834	4852	25714	10536	33870	205484	69136	6666
22	3566	5077	8140	0	3402	1893	5599	4217	7677	3129	1117	1413	7489	3068	9864	59845	20135	1941
23	5058	7202	11545	3411	0	2684	7942	5981	10889	4438	1584	2004	10622	4352	13991	84880	28558	2754
24	2803	3991	6398	1890	2674	0	4401	3315	6034	2459	878	1111	5886	2412	7753	47036	15826	1526
25	8370	11916	19104	5644	7985	4442	0	9897	18018	7343	2621	3317	17576	7201	23151	140451	47256	4556
26	6282	8944	14339	4236	5994	3334	9863	0	13523	5512	1967	2489	13192	5405	17376	105418	35468	3420
27	11536	16424	26330	7779	11006	6122	18112	13641	0	10121	3612	4571	24224	9925	31908	193578	65131	6280

Table C.6 (cont'd)

	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
28	4649	6618	10610	3135	4435	2467	7298	5497	10006	0	1456	1842	9761	3999	12857	78002	26244	2530
29	1651	2350	3768	1113	1575	876	2592	1952	3554	1448	0	654	3466	1420	4566	27701	9320	899
30	2091	2976	4771	1410	1994	1109	3282	2472	4500	1834	655	0	4390	1799	5782	35080	11803	1138
31	11248	16014	25672	7585	10731	5969	17659	13300	24213	9868	3522	4457	0	9677	31110	188741	63503	6123
32	4558	6489	10403	3073	4348	2419	7156	5390	9811	3999	1427	1806	9571	0	12606	76481	25733	2481
33	14905	21220	34018	10050	14220	7909	23401	17624	32084	13076	4667	5906	31297	12823	0	250103	84149	8114
34	103518	147377	236265	69803	98759	54934	162524	122407	222835	90819	32414	41018	217370	89061	286318	0	584437	56351
35	31236	44470	71291	21063	29800	16576	49040	36935	67239	27404	9781	12377	65590	26874	86394	524138	0	17003
36	2876	4094	6564	1939	2744	1526	4515	3401	6190	2523	900	1140	6039	2474	7954	48256	16236	0
37	3325	4733	7588	2242	3172	1764	5220	3931	7157	2917	1041	1317	6981	2860	9196	55789	18771	1810
38	9486	13505	21651	6397	9050	5034	14893	11217	20420	8323	2970	3759	19919	8161	26238	159179	53557	5164
39	2906	4138	6633	1960	2773	1542	4563	3437	6256	2550	910	1152	6103	2501	8039	48770	16409	1582
40	2238	3187	5109	1509	2135	1188	3514	2647	4818	1964	701	887	4700	1926	6191	37559	12637	1218
41	10813	15394	24679	7291	10316	5738	16976	12786	23276	9486	3386	4284	22705	9303	29907	181439	61046	5886
42	19949	28400	45530	13452	19032	10586	31319	23589	42942	17501	6246	7904	41888	17163	55175	334738	112625	10859
43	5841	8316	13331	3939	5573	3100	9171	6907	12574	5125	1829	2314	12265	5025	16156	98014	32977	3180
44	7613	10838	17375	5133	7263	4040	11952	9002	16388	6679	2384	3017	15986	6550	21056	127745	42981	4144
45	11307	16097	25806	7624	10787	6000	17752	13370	24339	9920	3540	4480	23742	9728	31273	189729	63836	6155
46	8959	12755	20448	6041	8547	4754	14066	10594	19286	7860	2805	3550	18813	7708	24780	150335	50581	4877
47	6274	8932	14320	4231	5986	3329	9850	7419	13506	5504	1965	2486	13175	5398	17353	105280	35422	3415
48	6366	9063	14530	4293	6073	3378	9995	7528	13704	5585	1993	2523	13368	5477	17608	106824	35942	3465
49	4022	5726	9179	2712	3837	2134	6314	4755	8657	3528	1259	1594	8445	3460	11123	67484	22705	2189
50	2742	3903	6257	1849	2616	1455	4304	3242	5902	2405	858	1086	5757	2359	7583	46003	15478	1492
51	3081	4386	7032	2077	2939	1635	4837	3643	6632	2703	965	1221	6469	2651	8521	51698	17394	1677
52	7921	11277	18079	5341	7557	4203	12436	9366	17051	6949	2480	3139	16633	6815	21909	132917	44721	4312
53	3240	4613	7394	2185	3091	1719	5087	3831	6974	2842	1014	1284	6803	2787	8961	54365	18291	1764
54	6734	9587	15369	4541	6424	3573	10572	7962	14495	5908	2108	2668	14140	5793	18625	112992	38017	3666

Table C.6 (cont'd)

	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
55	10841	15434	24742	7310	10342	5753	17020	12819	23336	9511	3394	4296	22763	9327	29984	181906	61204	5901
56	2331	3318	5320	1572	2224	1237	3660	2756	5018	2045	730	924	4895	2005	6447	39113	13160	1269
57	1993	2837	4549	1344	1901	1058	3129	2357	4290	1748	624	790	4185	1715	5512	33442	11252	1085
58	6724	9573	15347	4534	6415	3568	10557	7951	14474	5899	2105	2664	14119	5785	18598	112829	37962	3660
59	5542	7890	12649	3737	5287	2941	8701	6553	11930	4862	1735	2196	11638	4768	15329	92997	31290	3017
60	7382	10509	16847	4977	7042	3917	11589	8728	15890	6476	2311	2925	15500	6351	20416	123862	41674	4018
61	8712	12403	19884	5875	8312	4623	13678	10302	18754	7643	2728	3452	18294	7495	24097	146189	49186	4742
62	825	1175	1883	556	787	438	1296	976	1776	724	258	327	1733	710	2282	13847	4659	449
63	12987	18489	29641	8757	12390	6892	20389	15357	27956	11394	4066	5146	27270	11173	35920	217920	73320	7069
64	2852	4060	6509	1923	2721	1513	4477	3372	6139	2502	893	1130	5988	2453	7888	47853	16100	1552
65	7829	11145	17868	5279	7469	4154	12291	9257	16852	6868	2451	3102	16439	6735	21653	131363	44198	4262
66	6075	8649	13865	4096	5796	3224	9537	7183	13077	5330	1902	2407	12756	5226	16802	101935	34297	3307
67	5470	7788	12486	3689	5219	2903	8589	6469	11776	4799	1713	2168	11487	4706	15131	91795	30885	2978
68	3508	4995	8007	2366	3347	1862	5508	4148	7552	3078	1099	1390	7367	3018	9704	58869	19807	1910
69	859	1222	1959	579	819	456	1348	1015	1848	753	269	340	1803	739	2375	14406	4847	467
70	2149	3060	4906	1449	2051	1141	3374	2542	4627	1886	673	852	4513	1849	5945	36066	12135	1170
71	3396	4835	7752	2290	3240	1802	5332	4016	7311	2980	1063	1346	7132	2922	9394	56990	19175	1849
72	4049	5765	9242	2730	3863	2149	6357	4788	8716	3552	1268	1604	8502	3484	11199	67945	22860	2204
73	3126	4451	7136	2108	2983	1659	4909	3697	6730	2743	979	1239	6565	2690	8647	52462	17651	1702
74	1626	2315	3712	1097	1551	863	2553	1923	3501	1427	509	644	3415	1399	4498	27288	9181	885
75	1180	1680	2694	796	1126	626	1853	1395	2540	1035	370	468	2478	1015	3264	19803	6663	642
76	1489	2119	3398	1004	1420	790	2337	1760	3204	1306	466	590	3126	1281	4117	24980	8405	810
77	1488	2119	3397	1004	1420	790	2337	1760	3204	1306	466	590	3125	1280	4116	24973	8402	810
78	1989	2831	4539	1341	1897	1055	3122	2352	4281	1745	623	788	4176	1711	5501	33372	11228	1083
79	1012	1441	2310	682	965	537	1589	1197	2178	888	317	401	2125	871	2799	16980	5713	551
80	4067	5791	9283	2743	3880	2158	6386	4810	8756	3568	1274	1612	8541	3499	11250	68252	22964	2214
81	2780	3958	6345	1875	2652	1475	4365	3288	5985	2439	871	1102	5838	2392	7690	46652	15696	1513

Table C.6 (cont'd)

	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
1	10529	29736	9211	7101	33821	61472	18421	23938	35337	28109	19772	20059	12721	8691	9761	24894	10262	21204
2	3487	9847	3050	2351	11199	20356	6100	7927	11701	9308	6547	6642	4212	2878	3232	8243	3398	7022
3	4553	12860	3983	3071	14626	26585	7966	10352	15282	12156	8551	8675	5502	3758	4221	10766	4438	9170
4	2951	8334	2582	1990	9479	17229	5163	6709	9904	7878	5542	5622	3565	2436	2736	6977	2876	5943
5	2033	5743	1779	1371	6532	11872	3558	4623	6825	5429	3819	3874	2457	1678	1885	4808	1982	4095
6	23589	66620	20635	15909	75770	137718	41269	53629	79168	62973	44296	44939	28500	19470	21867	55772	22989	47505
7	9771	27596	8548	6590	31387	57048	17095	22215	32794	26086	18349	18615	11806	8065	9058	23103	9523	19678
8	1066	3010	932	719	3424	6223	1865	2423	3577	2846	2002	2031	1288	880	988	2520	1039	2147
9	5340	15081	4671	3601	17152	31176	9342	12140	17922	14255	10028	10173	6452	4407	4950	12625	5204	10754
10	6057	17105	5298	4085	19455	35361	10596	13770	20327	16169	11374	11539	7318	4999	5615	14320	5903	12197
11	1079	3048	944	728	3467	6301	1888	2454	3622	2881	2027	2056	1304	891	1000	2552	1052	2173
12	1410	3983	1234	951	4530	8234	2468	3207	4734	3765	2649	2687	1704	1164	1307	3335	1375	2840
13	2165	6114	1894	1460	6954	12639	3787	4922	7265	5779	4065	4124	2616	1787	2007	5118	2110	4360
14	1505	4250	1316	1015	4834	8786	2633	3421	5050	4017	2826	2867	1818	1242	1395	3558	1467	3031
15	1427	4032	1249	963	4585	8334	2497	3245	4791	3811	2681	2720	1725	1178	1323	3375	1391	2875
16	12149	34312	10628	8194	39025	70931	21255	27621	40775	32434	22815	23146	14679	10028	11263	28725	11840	24467
17	2593	7322	2268	1749	8328	15137	4536	5895	8701	6921	4869	4939	3132	2140	2403	6130	2527	5221
18	1503	4245	1315	1014	4828	8776	2630	3417	5045	4013	2823	2864	1816	1241	1393	3554	1465	3027
19	3336	9421	2918	2250	10715	19475	5836	7584	11195	8905	6264	6355	4030	2753	3092	7887	3251	6718
20	4767	13463	4170	3215	15312	27831	8340	10838	15999	12726	8952	9082	5759	3935	4419	11271	4646	9600
21	7701	21749	6737	5194	24737	44961	13473	17509	25846	20559	14462	14671	9304	6356	7139	18208	7505	15509
22	2243	6334	1962	1513	7204	13094	3924	5099	7527	5988	4212	4273	2710	1851	2079	5303	2186	4517
23	3181	8984	2783	2145	10218	18572	5565	7232	10676	8492	5974	6060	3843	2626	2949	7521	3100	6406
24	1763	4979	1542	1189	5662	10292	3084	4008	5916	4706	3310	3358	2130	1455	1634	4168	1718	3550
25	5264	14866	4605	3550	16908	30732	9209	11967	17666	14052	9885	10028	6360	4345	4880	12445	5130	10601
26	3951	11158	3456	2665	12691	23066	6912	8982	13260	10547	7419	7527	4773	3261	3663	9341	3850	7956
27	7255	20489	6346	4893	23304	42356	12692	16494	24349	19368	13624	13821	8765	5988	6725	17153	7071	14610

Table C.6 (cont'd)

	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
28	2923	8256	2557	1972	9390	17067	5114	6646	9811	7804	5490	5569	3532	2413	2710	6912	2849	5887
29	1038	2932	908	700	3335	6061	1816	2360	3484	2771	1950	1978	1254	857	962	2455	1012	2091
30	1315	3713	1150	887	4223	7676	2300	2989	4412	3510	2469	2505	1588	1085	1219	3108	1281	2648
31	7074	19977	6188	4771	22721	41298	12375	16082	23740	18884	13283	13476	8546	5838	6557	16724	6894	14245
32	2866	8095	2507	1933	9207	16735	5015	6517	9620	7652	5383	5461	3463	2366	2657	6777	2794	5772
33	9373	26472	8200	6322	30108	54724	16399	21310	31458	25023	17602	17857	11325	7737	8689	22162	9135	18877
34	65100	183857	56948	43906	209110	380075	113893	148006	218487	173792	122249	124023	78654	53733	60350	153920	63446	131104
35	19643	55477	17184	13248	63097	114685	34366	44660	65927	52441	36888	37423	23733	16213	18210	46444	19144	39560
36	1809	5108	1582	1220	5809	10559	3164	4112	6070	4828	3396	3445	2185	1493	1677	4276	1763	3642
37	0	5905	1829	1410	6716	12207	3658	4754	7017	5582	3926	3983	2526	1726	1938	4944	2038	4211
38	5966	0	5219	4024	19162	34830	10437	13563	20022	15926	11203	11365	7208	4924	5530	14105	5814	12014
39	1828	5162	0	1233	5871	10671	3198	4155	6134	4879	3432	3482	2208	1509	1694	4322	1781	3681
40	1408	3975	1231	0	4521	8218	2463	3200	4724	3758	2643	2682	1701	1162	1305	3328	1372	2835
41	6800	19204	5948	4586	0	39700	11896	15460	22822	18153	12769	12955	8216	5613	6304	16077	6627	13694
42	12545	35430	10974	8461	40297	0	21948	28522	42104	33491	23558	23900	15157	10355	11630	29661	12226	25264
43	3673	10374	3213	2477	11799	21446	0	8351	12328	9806	6898	6998	4438	3032	3405	8685	3580	7398
44	4788	13521	4188	3229	15378	27951	8376	0	16068	12781	8990	9121	5784	3952	4438	11320	4666	9642
45	7111	20082	6220	4796	22840	41514	12440	16166	0	18983	13353	13546	8591	5869	6592	16812	6930	14320
46	5634	15912	4929	3800	18098	32894	9857	12809	18909	0	10580	10734	6807	4650	5223	13321	5491	11347
47	3946	11143	3452	2661	12674	23036	6903	8971	13242	10533	0	7517	4767	3257	3658	9329	3845	7946
48	4003	11307	3502	2700	12860	23374	7004	9102	13436	10688	7518	0	4837	3304	3711	9466	3902	8063
49	2529	7143	2212	1706	8124	14766	4425	5750	8488	6752	4749	4818	0	2088	2345	5980	2465	5093
50	1724	4869	1508	1163	5538	10066	3016	3920	5786	4603	3238	3285	2083	0	1598	4076	1680	3472
51	1938	5472	1695	1307	6224	11312	3390	4405	6503	5172	3638	3691	2341	1599	0	4581	1888	3902
52	4981	14069	4358	3360	16001	29083	8715	11325	16718	13298	9354	9490	6019	4112	4618	0	4855	10032
53	2037	5754	1782	1374	6545	11895	3565	4632	6838	5439	3826	3882	2462	1682	1889	4817	0	4103
54	4235	11960	3704	2856	13602	24723	7409	9628	14212	11305	7952	8068	5116	3495	3926	10012	4127	0

Table C.6 (cont’d)

	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
55	6817	19254	5964	4598	21898	39802	11927	15500	22880	18200	12802	12988	8237	5627	6320	16119	6644	13729
56	1466	4140	1282	989	4709	8558	2565	3333	4920	3913	2753	2793	1771	1210	1359	3466	1429	2952
57	1253	3540	1096	845	4026	7317	2193	2849	4206	3346	2354	2388	1514	1034	1162	2963	1221	2524
58	4229	11942	3699	2852	13583	24688	7398	9614	14192	11289	7941	8056	5109	3490	3920	9998	4121	8516
59	3485	9843	3049	2351	11195	20348	6098	7924	11697	9304	6545	6640	4211	2877	3231	8241	3397	7019
60	4642	13110	4061	3131	14911	27102	8121	10554	15580	12393	8717	8844	5609	3831	4303	10975	4524	9349
61	5479	15473	4793	3695	17599	31987	9585	12456	18388	14626	10288	10438	6620	4522	5079	12954	5340	11034
62	519	1466	454	350	1667	3030	908	1180	1742	1385	975	989	627	428	481	1227	506	1045
63	8167	23066	7144	5508	26234	47682	14288	18568	27410	21803	15337	15559	9868	6741	7571	19310	7960	16448
64	1793	5065	1569	1210	5761	10470	3138	4077	6019	4788	3368	3417	2167	1480	1663	4240	1748	3612
65	4923	13904	4307	3320	15814	28743	8613	11193	16523	13143	9245	9379	5948	4064	4564	11640	4798	9915
66	3820	10789	3342	2577	12271	22304	6684	8685	12822	10199	7174	7278	4616	3153	3542	9033	3723	7694
67	3440	9716	3009	2320	11051	20085	6019	7821	11546	9184	6460	6554	4157	2840	3189	8134	3353	6928
68	2206	6231	1930	1488	7087	12881	3860	5016	7405	5890	4143	4203	2666	1821	2045	5216	2150	4443
69	540	1525	472	364	1734	3152	945	1228	1812	1441	1014	1029	652	446	501	1277	526	1087
70	1352	3817	1182	912	4342	7892	2365	3073	4536	3608	2538	2575	1633	1116	1253	3196	1317	2722
71	2136	6032	1868	1441	6861	12470	3737	4856	7168	5702	4011	4069	2581	1763	1980	5050	2082	4301
72	2546	7192	2228	1717	8179	14867	4455	5789	8546	6798	4782	4851	3077	2102	2361	6021	2482	5128
73	1966	5553	1720	1326	6316	11479	3440	4470	6599	5249	3692	3746	2376	1623	1823	4649	1916	3960
74	1023	2888	895	690	3285	5971	1789	2325	3432	2730	1920	1948	1236	844	948	2418	997	2060
75	742	2096	649	501	2384	4333	1298	1687	2491	1981	1394	1414	897	613	688	1755	723	1495
76	936	2644	819	631	3007	5466	1638	2128	3142	2499	1758	1784	1131	773	868	2213	912	1885
77	936	2643	819	631	3006	5464	1637	2128	3141	2499	1758	1783	1131	773	868	2213	912	1885
78	1251	3532	1094	844	4017	7302	2188	2843	4198	3339	2349	2383	1511	1032	1159	2957	1219	2519
79	636	1797	557	429	2044	3715	1113	1447	2136	1699	1195	1212	769	525	590	1505	620	1282
80	2558	7224	2238	1725	8216	14934	4475	5815	8585	6829	4803	4873	3090	2111	2371	6048	2493	5151
81	1748	4938	1529	1179	5616	10208	3059	3975	5868	4668	3283	3331	2112	1443	1621	4134	1704	3521

Table C.6 (cont'd)

	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
1	33906	7394	6325	21174	17487	23219	27345	2624	40476	9038	24607	19150	17262	11107	2730	6820	10754	12808
2	11228	2448	2095	7012	5790	7689	9055	869	13403	2993	8148	6341	5716	3678	904	2258	3561	4241
3	14663	3198	2736	9157	7562	10042	11826	1135	17505	3909	10642	8282	7465	4803	1181	2949	4651	5539
4	9503	2072	1773	5935	4901	6508	7664	736	11344	2533	6897	5367	4838	3113	765	1911	3014	3590
5	6548	1428	1222	4089	3377	4484	5281	507	7817	1746	4752	3699	3334	2145	527	1317	2077	2474
6	75962	16565	14171	47437	39176	52019	61261	5879	90680	20249	55129	42903	38674	24883	6116	15279	24093	28693
7	31466	6862	5870	19650	16228	21548	25377	2435	37563	8388	22836	17772	16020	10308	2534	6329	9980	11886
8	3432	749	640	2144	1770	2351	2768	266	4098	915	2491	1939	1748	1124	276	690	1089	1297
9	17196	3750	3208	10739	8868	11776	13868	1331	20528	4584	12480	9712	8755	5633	1385	3459	5454	6495
10	19504	4253	3639	12180	10059	13356	15729	1510	23283	5199	14155	11016	9930	6389	1570	3923	6186	7367
11	3475	758	648	2170	1792	2380	2803	269	4149	926	2522	1963	1769	1138	280	699	1102	1313
12	4542	990	847	2836	2342	3110	3663	352	5422	1211	3296	2565	2312	1488	366	914	1441	1716
13	6971	1520	1301	4353	3595	4774	5622	540	8322	1858	5059	3937	3549	2284	561	1402	2211	2633
14	4846	1057	904	3026	2499	3318	3908	375	5785	1292	3517	2737	2467	1587	390	975	1537	1830
15	4597	1002	858	2871	2371	3148	3707	356	5488	1225	3336	2596	2340	1506	370	925	1458	1736
16	39124	8532	7299	24432	20177	26792	31552	3028	46704	10429	28394	22097	19919	12816	3150	7869	12409	14778
17	8349	1821	1558	5214	4306	5718	6733	646	9967	2226	6059	4716	4251	2735	672	1679	2648	3154
18	4840	1056	903	3023	2496	3315	3904	375	5778	1290	3513	2734	2464	1586	390	974	1535	1828
19	10742	2342	2004	6708	5540	7356	8663	831	12823	2863	7796	6067	5469	3519	865	2161	3407	4058
20	15351	3348	2864	9586	7917	10512	12380	1188	18325	4092	11141	8670	7815	5029	1236	3088	4869	5799
21	24799	5408	4627	15487	12790	16983	20000	1919	29605	6611	17998	14007	12626	8124	1997	4988	7866	9368
22	7222	1575	1347	4510	3725	4946	5825	559	8622	1925	5242	4079	3677	2366	582	1453	2291	2728
23	10244	2234	1911	6397	5283	7015	8261	793	12229	2731	7434	5786	5215	3356	825	2061	3249	3869
24	5677	1238	1059	3545	2928	3887	4578	439	6777	1513	4120	3206	2890	1860	457	1142	1801	2144
25	16951	3696	3162	10585	8742	11608	13670	1312	20235	4518	12302	9574	8630	5553	1365	3410	5376	6403
26	12723	2774	2374	7945	6561	8713	10260	985	15188	3391	9233	7186	6477	4168	1024	2559	4035	4806
27	23362	5095	4358	14590	12049	15999	18841	1808	27889	6228	16955	13195	11894	7653	1881	4699	7410	8825

Table C.6 (cont'd)

	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
28	9414	2053	1756	5879	4855	6447	7592	729	11238	2509	6832	5317	4793	3084	758	1894	2986	3556
29	3343	729	624	2088	1724	2289	2696	259	3991	891	2426	1888	1702	1095	269	672	1060	1263
30	4234	923	790	2644	2183	2899	3414	328	5054	1129	3073	2391	2155	1387	341	852	1343	1599
31	22779	4967	4250	14225	11748	15599	18370	1763	27192	6072	16532	12865	11597	7462	1834	4582	7225	8604
32	9230	2013	1722	5764	4760	6321	7444	714	11019	2460	6699	5213	4699	3024	743	1857	2928	3487
33	30184	6582	5631	18850	15567	20670	24343	2336	36033	8046	21906	17048	15368	9888	2430	6071	9574	11402
34	209639	45716	39110	130917	108118	143562	169068	16225	250259	55882	152144	118404	106732	68673	16880	42167	66492	79188
35	63257	13794	11801	39503	32624	43319	51015	4896	75514	16862	45908	35727	32206	20721	5093	12724	20064	23894
36	5824	1270	1086	3637	3004	3988	4697	451	6952	1552	4227	3289	2965	1908	469	1171	1847	2200
37	6733	1468	1256	4205	3472	4611	5430	521	8038	1795	4887	3803	3428	2206	542	1354	2136	2543
38	19211	4189	3584	11997	9908	13156	15493	1487	22933	5121	13942	10850	9781	6293	1547	3864	6093	7257
39	5886	1284	1098	3676	3036	4031	4747	456	7026	1569	4272	3324	2997	1928	474	1184	1867	2223
40	4533	988	846	2831	2338	3104	3656	351	5411	1208	3290	2560	2308	1485	365	912	1438	1712
41	21897	4775	4085	13675	11293	14996	17660	1695	26140	5837	15892	12368	11148	7173	1763	4405	6945	8271
42	40399	8810	7537	25228	20835	27665	32580	3127	48226	10769	29319	22817	20568	13234	3253	8126	12813	15260
43	11829	2580	2207	7387	6101	8101	9540	916	14121	3153	8585	6681	6022	3875	952	2379	3752	4468
44	15417	3362	2876	9628	7951	10558	12434	1193	18405	4110	11189	8708	7849	5050	1241	3101	4890	5824
45	22898	4993	4272	14299	11809	15681	18467	1772	27335	6104	16618	12933	11658	7501	1844	4606	7263	8649
46	18144	3957	3385	11330	9357	12425	14632	1404	21659	4836	13168	10247	9237	5943	1461	3649	5755	6853
47	12706	2771	2370	7935	6553	8701	10247	983	15168	3387	9221	7176	6469	4162	1023	2556	4030	4800
48	12892	2811	2405	8051	6649	8829	10397	998	15390	3437	9357	7282	6564	4223	1038	2593	4089	4870
49	8144	1776	1519	5086	4200	5577	6568	630	9723	2171	5911	4600	4147	2668	656	1638	2583	3076
50	5552	1211	1036	3467	2863	3802	4478	430	6628	1480	4029	3136	2827	1819	447	1117	1761	2097
51	6239	1361	1164	3896	3218	4273	5032	483	7448	1663	4528	3524	3177	2044	502	1255	1979	2357
52	16041	3498	2993	10018	8273	10985	12937	1242	19150	4276	11642	9060	8167	5255	1292	3227	5088	6059
53	6561	1431	1224	4097	3384	4493	5291	508	7832	1749	4762	3706	3340	2149	528	1320	2081	2478
54	13637	2974	2544	8516	7033	9339	10998	1055	16279	3635	9897	7702	6943	4467	1098	2743	4325	5151

Table C.6 (cont'd)

	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
55	0	4787	4096	13710	11322	15034	17705	1699	26208	5852	15933	12400	11177	7192	1768	4416	6963	8293
56	4720	0	881	2948	2434	3233	3807	365	5635	1258	3426	2666	2403	1546	380	949	1497	1783
57	4036	880	0	2520	2082	2764	3255	312	4818	1076	2929	2280	2055	1322	325	812	1280	1525
58	13617	2969	2540	0	7023	9325	10982	1054	16256	3630	9883	7691	6933	4461	1096	2739	4319	5144
59	11224	2448	2094	7009	0	7686	9052	869	13398	2992	8145	6339	5714	3677	904	2258	3560	4240
60	14949	3260	2789	9335	7709	0	12056	1157	17845	3985	10849	8443	7611	4897	1204	3007	4741	5647
61	17643	3847	3291	11018	9099	12082	0	1366	21062	4703	12804	9965	8983	5779	1421	3549	5596	6664
62	1671	364	312	1044	862	1144	1348	0	1995	445	1213	944	851	547	135	336	530	631
63	26300	5735	4907	16424	13564	18011	21210	2036	0	7011	19087	14854	13390	8615	2118	5290	8342	9935
64	5775	1259	1077	3607	2978	3955	4658	447	6894	0	4191	3262	2940	1892	465	1162	1832	2182
65	15854	3457	2958	9901	8176	10857	12786	1227	18926	4226	0	8954	8072	5193	1277	3189	5028	5989
66	12302	2683	2295	7683	6345	8425	9921	952	14686	3279	8928	0	6263	4030	991	2475	3902	4647
67	11078	2416	2067	6918	5714	7587	8934	857	13225	2953	8040	6257	0	3629	892	2228	3514	4185
68	7105	1549	1325	4437	3664	4865	5730	550	8481	1894	5156	4013	3617	0	572	1429	2253	2684
69	1739	379	324	1086	897	1191	1402	135	2076	463	1262	982	885	570	0	350	551	657
70	4353	949	812	2718	2245	2981	3510	337	5196	1160	3159	2458	2216	1426	350	0	1381	1644
71	6878	1500	1283	4295	3547	4710	5547	532	8211	1833	4992	3885	3502	2253	554	1383	0	2598
72	8200	1788	1530	5121	4229	5615	6613	635	9789	2186	5951	4631	4175	2686	660	1649	2601	0
73	6331	1381	1181	3954	3265	4336	5106	490	7558	1688	4595	3576	3224	2074	510	1274	2008	2392
74	3293	718	614	2057	1698	2255	2656	255	3931	878	2390	1860	1677	1079	265	662	1045	1244
75	2390	521	446	1493	1233	1637	1927	185	2853	637	1735	1350	1217	783	192	481	758	903
76	3015	657	562	1883	1555	2065	2431	233	3599	804	2188	1703	1535	988	243	606	956	1139
77	3014	657	562	1882	1554	2064	2431	233	3598	803	2187	1702	1534	987	243	606	956	1138
78	4028	878	751	2515	2077	2758	3248	312	4808	1074	2923	2275	2051	1319	324	810	1277	1521
79	2049	447	382	1280	1057	1403	1653	159	2446	546	1487	1157	1043	671	165	412	650	774
80	8237	1796	1537	5144	4248	5641	6643	638	9833	2196	5978	4652	4194	2698	663	1657	2613	3111
81	5630	1228	1050	3516	2904	3856	4541	436	6721	1501	4086	3180	2867	1844	453	1133	1786	2127

Table C.6 (cont'd)

	73	74	75	76	77	78	79	80	81
1	9904	5165	3751	4729	4728	6312	3217	12865	8813
2	3280	1710	1242	1566	1565	2090	1065	4260	2918
3	4283	2234	1622	2045	2045	2730	1391	5564	3811
4	2776	1448	1051	1325	1325	1769	902	3606	2470
5	1913	997	724	913	913	1219	621	2485	1702
6	22189	11571	8403	10594	10592	14142	7207	28822	19743
7	9191	4793	3481	4388	4387	5858	2986	11939	8178
8	1003	523	380	479	479	639	326	1302	892
9	5023	2619	1902	2398	2398	3201	1632	6525	4469
10	5697	2971	2158	2720	2719	3631	1851	7400	5069
11	1015	529	384	485	485	647	330	1319	903
12	1327	692	502	633	633	846	431	1723	1180
13	2036	1062	771	972	972	1298	661	2645	1812
14	1416	738	536	676	676	902	460	1839	1259
15	1343	700	509	641	641	856	436	1744	1195
16	11428	5959	4328	5456	5455	7284	3712	14845	10169
17	2439	1272	924	1164	1164	1554	792	3168	2170
18	1414	737	535	675	675	901	459	1837	1258
19	3138	1636	1188	1498	1498	2000	1019	4076	2792
20	4484	2338	1698	2141	2140	2858	1457	5825	3990
21	7244	3777	2743	3459	3458	4617	2353	9410	6446
22	2110	1100	799	1007	1007	1345	685	2740	1877
23	2992	1560	1133	1429	1428	1907	972	3887	2662
24	1658	865	628	792	792	1057	539	2154	1475
25	4951	2582	1875	2364	2363	3156	1608	6432	4406
26	3716	1938	1407	1774	1774	2369	1207	4827	3307
27	6824	3559	2584	3258	3257	4349	2217	8864	6072

Table C.6 (cont'd)

	73	74	75	76	77	78	79	80	81
28	2750	1434	1041	1313	1313	1753	893	3572	2447
29	977	509	370	466	466	622	317	1268	869
30	1237	645	468	590	590	788	402	1606	1100
31	6654	3470	2520	3177	3176	4241	2161	8643	5920
32	2696	1406	1021	1287	1287	1718	876	3502	2399
33	8817	4598	3339	4210	4209	5619	2864	11453	7845
34	61237	31933	23190	29238	29230	39028	19891	79543	54487
35	18478	9635	6998	8822	8820	11776	6002	24002	16441
36	1701	887	644	812	812	1084	553	2210	1514
37	1967	1026	745	939	939	1253	639	2555	1750
38	5612	2926	2125	2679	2679	3576	1823	7289	4993
39	1719	897	651	821	821	1096	558	2233	1530
40	1324	690	501	632	632	844	430	1720	1178
41	6396	3335	2422	3054	3053	4077	2078	8309	5691
42	11801	6154	4469	5634	5633	7521	3833	15328	10500
43	3455	1802	1309	1650	1649	2202	1122	4488	3074
44	4503	2348	1705	2150	2150	2870	1463	5850	4007
45	6689	3488	2533	3193	3193	4263	2173	8688	5951
46	5300	2764	2007	2530	2530	3378	1721	6884	4716
47	3712	1935	1406	1772	1772	2365	1206	4821	3302
48	3766	1964	1426	1798	1798	2400	1223	4892	3351
49	2379	1241	901	1136	1136	1516	773	3090	2117
50	1622	846	614	774	774	1034	527	2107	1443
51	1823	950	690	870	870	1162	592	2367	1622
52	4686	2443	1775	2237	2237	2986	1522	6087	4169
53	1917	999	726	915	915	1221	623	2489	1705
54	3983	2077	1509	1902	1901	2539	1294	5174	3544

Table C.6 (cont'd)

	73	74	75	76	77	78	79	80	81
55	6413	3344	2429	3062	3061	4087	2083	8330	5706
56	1379	719	522	658	658	879	448	1791	1227
57	1179	615	446	563	563	751	383	1531	1049
58	3978	2074	1506	1899	1899	2535	1292	5167	3539
59	3278	1710	1242	1565	1565	2089	1065	4259	2917
60	4367	2277	1654	2085	2084	2783	1418	5672	3885
61	5154	2687	1952	2461	2460	3285	1674	6694	4586
62	488	255	185	233	233	311	159	634	434
63	7682	4006	2909	3668	3667	4896	2495	9979	6836
64	1687	880	639	805	805	1075	548	2191	1501
65	4631	2415	1754	2211	2211	2951	1504	6015	4121
66	3594	1874	1361	1716	1715	2290	1167	4668	3197
67	3236	1687	1226	1545	1545	2062	1051	4203	2879
68	2075	1082	786	991	991	1323	674	2696	1847
69	508	265	192	242	242	324	165	660	452
70	1271	663	482	607	607	810	413	1652	1131
71	2009	1048	761	959	959	1280	653	2610	1788
72	2395	1249	907	1144	1143	1527	778	3111	2131
73	0	964	700	883	883	1179	601	2402	1646
74	962	0	364	459	459	613	312	1250	856
75	698	364	0	333	333	445	227	907	621
76	881	459	333	0	420	561	286	1144	784
77	880	459	333	420	0	561	286	1144	783
78	1176	613	446	562	562	0	382	1528	1047
79	599	312	227	286	286	382	0	778	533
80	2406	1255	911	1149	1149	1533	782	0	2141
81	1645	858	623	785	785	1048	534	2136	0

APPENDIX D

Results for the Small Problem Instances with the CAB Data Set

Table D.1 GA Results For Instance CAB_25_5

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	33.501	77	6,7,9,14,22
Run 2	34.340	88	9,15,16,17,22
Run 3	34.061	71	1,6,7,22,25
Run 4	32.993	83	1,6,7,17,22
Run 5	34.711	74	2,7,9,22,24
Run 6	32.993	77	6,7,14,17,22
Run 7	33.511	78	11,14,18,20,22
Run 8	33.894	82	1,2,6,7,22
Run 9	32.993	69	6,7,9,14,22
Run 10	32.993	71	1,6,8,17,22

Table D.2 GA Results For Instance CAB_20_5(1)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	42.227	68	9,10,14,18,19
Run 2	41.100	74	6,12,14,16,19
Run 3	41.762	72	1,9,10,17,19
Run 4	40.310	127	8,10,12,14,17
Run 5	41.100	69	1,6,11,12,14
Run 6	40.917	67	1,8,9,12,18
Run 7	42.664	72	2,9,13,14,19
Run 8	40.917	64	1,9,11,12,18
Run 9	40.310	72	2,3,8,12,16
Run 10	42.664	68	2,9,10,14,19

Table D.3 GA Results For Instance CAB_20_5(2)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	48.652	64	6,11,12,23,24
Run 2	46.252	78	11,12,17,23,24
Run 3	46.252	62	11,12,17,23,24
Run 4	48.652	128	6,11,12,23,24
Run 5	46.969	59	11,12,20,23,24
Run 6	46.949	72	11,12,18,23,24
Run 7	47.675	132	11,12,23,24,25
Run 8	46.969	84	11,12,20,23,24
Run 9	46.252	89	11,12,17,23,24
Run 10	47.675	122	11,12,23,24,25

Table D.4 GA Results For Instance CAB_20_5(3)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	42.562	114	1,11,12,23,25
Run 2	42.358	127	2,11,12,23,24
Run 3	41.911	82	2,11,12,18,23
Run 4	51.705	102	11,22,23,24,25
Run 5	41.289	203	1,11,12,17,23
Run 6	41.911	217	1,11,12,18,23
Run 7	41.289	132	1,11,12,17,23
Run 8	42.562	142	3,11,12,23,24
Run 9	41.911	117	1,11,18,22,23
Run 10	42.358	182	1,2,11,12,23

Table D.5 GA Results For Instance CAB_20_5(4)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	39.398	84	1,11,20,22,23
Run 2	32.486	72	1,11,12,20,23
Run 3	39.398	63	1,11,20,22,23
Run 4	32.486	107	1,11,12,20,23
Run 5	39.398	92	1,8,20,22,23
Run 6	39.398	121	1,11,20,22,23
Run 7	39.398	137	1,11,20,22,23
Run 8	32.486	89	1,11,20,22,23
Run 9	39.398	95	1,11,12,20,23
Run 10	42.486	92	1,11,20,22,23

Table D.6 GA Results For Instance CAB_15_4(1)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	45.032	44	1,3,11,12
Run 2	59.535	38	1,6,11,12
Run 3	45.032	39	1,2,11,12
Run 4	59.535	44	1,6,11,12
Run 5	55.797	34	1,9,11,12
Run 6	55.797	38	9,11,12,14
Run 7	45.032	31	2,11,12,14
Run 8	59.535	37	6,11,12,14
Run 9	45.032	38	1,3,11,12
Run 10	55.797	31	1,9,11,12

Table D.7 GA Results For Instance CAB_15_5(2)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	74.276	28	12,20,21,23,24
Run 2	74.276	34	11,12,20,23,24
Run 3	73.137	54	12,17,21,23,24
Run 4	73.137	31	11,12,17,23,24
Run 5	73.137	62	11,12,17,23,24
Run 6	74.276	51	11,12,20,23,24
Run 7	74.240	47	11,12,18,23,24
Run 8	74.276	33	12,20,21,23,24
Run 9	73.137	30	11,12,17,23,24
Run 10	74.276	37	12,20,21,23,24

Table D.8 GA Results For Instance CAB_15_4(3)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	15.249	28	1,3,11,18
Run 2	15.249	32	1,8,11,17
Run 3	13.552	31	7,11,24,25
Run 4	20.436	67	9,11,24,25
Run 5	14.862	54	1,10,11,17
Run 6	15.249	52	5,11,17,24
Run 7	15.249	34	1,11,17,18
Run 8	14.862	31	1,10,11,18
Run 9	13.552	58	1,7,11,17
Run 10	24.472	52	1,11,18,24

Table D.9 GA Results For Instance CAB_15_4(4)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	41.850	52	11,14,20,22
Run 2	41.850	49	11,14,20,22
Run 3	35.247	63	11,14,20,25
Run 4	47.030	78	1,11,20,22
Run 5	35.247	83	11,14,22,25
Run 6	41.850	65	11,14,20,22
Run 7	47.030	78	1,11,20,22
Run 8	35.247	65	11,14,22,25
Run 9	41.850	54	11,14,20,22
Run 10	35.247	81	11,14,22,25

Table D.10 GA Results For Instance CAB_10_3(1)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	53.773	17	2,8,10
Run 2	34.662	15	8,9,10
Run 3	38.488	22	6,7,8
Run 4	38.488	27	6,8,10
Run 5	34.662	13	8,9,10
Run 6	34.662	14	7,8,9
Run 7	38.488	17	6,8,10
Run 8	53.733	14	2,8,10
Run 9	34.662	14	7,8,9
Run 10	38.488	23	6,7,8

Table D.11 GA Results For Instance CAB_10_5(2)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	122.101	44	16,19,20,22,23
Run 2	120.028	32	16,17,19,22,23
Run 3	122.101	41	16,19,20,22,23
Run 4	124.500	43	16,19,22,23,24
Run 5	122.101	38	19,20,22,23,24
Run 6	120.028	29	16,17,19,22,23
Run 7	122.101	36	19,20,22,23,24
Run 8	122.101	39	19,20,22,23,24
Run 9	122.101	35	19,20,22,23,24
Run 10	120.028	42	16,17,19,22,23

Table D.12 GA Results For Instance CAB_10_4(3)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	70.402	34	6,7,12,23
Run 2	70.402	41	6,7,12,23
Run 3	70.402	35	6,10,12,23
Run 4	70.402	34	6,12,16,23
Run 5	70.402	34	6,12,16,23
Run 6	70.402	37	6,7,12,23
Run 7	70.402	38	6,10,12,23
Run 8	70.402	37	6,7,12,23
Run 9	70.402	40	6,10,12,23
Run 10	70.402	39	6,10,12,23

Table D.13 GA Results For Instance CAB_10_4(4)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	54.175	37	7,14,22,25
Run 2	54.175	37	7,14,22,25
Run 3	54.175	35	7,14,22,25
Run 4	54.175	36	7,14,22,25
Run 5	54.175	40	7,14,22,25
Run 6	54.175	36	7,14,22,25
Run 7	54.175	36	7,14,22,25
Run 8	54.175	38	7,14,22,25
Run 9	54.175	35	7,14,22,25
Run 10	54.175	37	7,14,22,25

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Table D.14 GA Results For Instance Turk_30_6(1)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	13.130	1.207	15,11,12,18,19,30
Run 2	15.497	1.205	3,6,13,14,18,23
Run 3	16.161	1.184	3,4,26,27,29,30
Run 4	13.130	1.076	1,2,16,18,20,22,24
Run 5	10.926	1.157	6,7,10,21,26,30
Run 6	12.175	1.224	5,6,13,14,20,24
Run 7	10.590	1.180	1,12,14,23,26,28
Run 8	12.657	1.054	4,10,11,18,19,25
Run 9	10.590	1.208	1,7,14,16,23,30
Run 10	12.657	1.006	5,11,12,18,19,30

Table D.15 GA Results For Instance Turk_30_6(2)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	5.884	760	54,55,61,63,76,79
Run 2	7.111	854	55,63,75,77,78,80
Run 3	8.552	982	54,58,66,69,76,80
Run 4	7.381	783	53,60,68,73,79,81
Run 5	5.884	1.023	54,58,63,70,73,76
Run 6	8.552	784	58,69,72,76,77,80
Run 7	5.884	834	54,58,63,69,76,80
Run 8	5.619	795	55,56,63,69,71,79
Run 9	7.111	997	55,63,75,76,78,80
Run 10	6.187	906	58,63,64,76,77,80

Table D.16 GA Results For Instance Turk_30_6(3)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	7.976	902	3,12,16,45,46,67
Run 2	7.688	821	15,16,24,47,68,80
Run 3	9.739	692	27,30,37,45,61,64
Run 4	8.599	714	12,16,17,27,48,68
Run 5	10.011	872	27,37,45,61,65,80
Run 6	6.851	765	16,21,24,45,71,80
Run 7	7.976	983	1,46,61,64,67,80
Run 8	6.549	717	1,16,21,60,64,71,
Run 9	6.549	782	1,12,16,27,64,71
Run 10	8.599	804	12,16,24,45,71,80

Table D.17 GA Results For Instance Turk_30_6(4)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	8.981	1.802	10,21,38,41,42,55
Run 2	8.981	1.024	3,16,21,38,41,55
Run 3	12.057	1.104	3,38,41,42,55,65
Run 4	9.515	1.504	3,10,25,38,41,55
Run 5	12.364	1.384	3,20,21,38,41,60,
Run 6	12.026	1.235	3,21,38,55,60,65
Run 7	11.473	1.157	1,3,6,25,44,60
Run 8	11.473	1.128	3,6,21,38,42,60
Run 9	12.057	1.187	3,21,38,41,54,55
Run 10	8.981	1.205	6,21,38,41,45,48

Table D.18 GA Results For Instance Turk_25_5(1)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	12481	21	3,6,7,12,17
Run 2	10.483	37	6,7,8,14,21
Run 3	12.481	29	3,6,12,14,18
Run 4	15.960	29	3,4,6,13,16
Run 5	12.481	44	6,7,11,12,15
Run 6	12.481	22	1,6,12,15,14
Run 7	10.483	27	6,7,11,21,24
Run 8	10.483	24	6,12,14,15,21
Run 9	12.701	28	6,7,16,22,24
Run 10	12.481	38	6,7,9,12,18

Table D.19 GA Results For Instance Turk_25_5(2)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	12.577	201	58,61,72,77,80
Run 2	15.491	184	60,70,73,75,77
Run 3	12.577	157	57,58,65,70,77
Run 4	14.953	165	57,58,70,76,77
Run 5	15.491	198	60,68,69,71,73
Run 6	14.395	187	61,71,73,77,79
Run 7	12.577	164	62,64,65,71,77
Run 8	13.220	160	58,61,72,77,80
Run 9	15.491	195	60,70,73,75,77
Run 10	14.953	192	57,58,70,76,77

Table D.20 GA Results For Instance Turk_25_5(3)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	30.648	167	6,48,62,70,73
Run 2	25.487	152	6,30,42,48,62
Run 3	25.487	165	6,30,44,48,62
Run 4	22.922	108	6,42,48,62,65
Run 5	23.584	114	6,23,48,62,65
Run 6	23.584	184	6,23,48,65,73
Run 7	23.881	178	6,23,48,53,73
Run 8	23.584	218	6,44,48,53,65
Run 9	23.881	142	6,48,62,73,80
Run 10	23.584	113	6,23,48,53,65

Table D.21 GA Results For Instance Turk_25_5(4)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	11.220	194	1,7,21,41,55
Run 2	8.592	145	1,21,34,42,52
Run 3	11.302	272	1,7,25,41,55
Run 4	11.466	197	21,38,41,45,55
Run 5	8.592	245	7,21,34,42,52
Run 6	11.220	207	1,7,21,41,55
Run 7	10.587	287	7,25,33,41,52
Run 8	10.669	248	16,21,41,42,55
Run 9	11.302	192	7,25,33,41,55
Run 10	11.220	237	7,21,38,41,52

Table D.22 GA Results For Instance Turk_20_4(1)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	12.926	31	6,7,12,14
Run 2	12.926	24	6,12,14,15
Run 3	17.263	33	6,12,14,20
Run 4	12.926	37	3,6,11,13
Run 5	12.926	48	6,7,11,13
Run 6	12.926	34	6,7,13,16
Run 7	12.926	50	6,7,12,14
Run 8	12.926	27	6,7,11,13
Run 9	12.926	31	6,12,14,15
Run 10	12.926	38	6,7,13,16

Table D.23 GA Results For Instance Turk_20_4(2)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	14.963	39	62,63,71,73
Run 2	23.390	45	68,72,75,81
Run 3	20.762	42	63,68,71,76
Run 4	21.405	37	69,71,73,79
Run 5	21.913	49	69,71,72,79
Run 6	20.560	54	62,65,70,71
Run 7	20.762	41	63,70,71,76
Run 8	20.560	42	62,65,68,71
Run 9	14.963	52	63,71,73,76
Run 10	20.762	51	63,66,76,77

Table D.24 GA Results For Instance Turk_20_4(3)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	20.198	131	4,44,45,64
Run 2	21.017	147	13,15,33,44
Run 3	20.198	142	42,44,49,64
Run 4	19.471	187	13,33,50,64
Run 5	20.231	164	4,42,44,45
Run 6	23.357	129	5,15,33,49
Run 7	25.618	124	13,15,44,80
Run 8	21.915	134	4,15,42,44
Run 9	24.841	152	35,42,44,49
Run 10	21.017	143	5,13,15,33

Table D.25 GA Results For Instance Turk_20_4(4)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	12.694	83	7,34,46,55
Run 2	11.661	78	34,42,46,55
Run 3	11.661	81	34,42,46,55
Run 4	13.948	87	7,41,46,55
Run 5	12.694	89	7,34,46,55
Run 6	12.694	92	7,34,46,55
Run 7	11.661	78	34,42,46,55
Run 8	13.948	84	7,41,46,55
Run 9	13.948	83	7,41,46,55
Run 10	12.694	86	7,34,46,55

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Table D.26 GA Results For Instance Turk_81_12

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	6,649	3.519	8,23,26,37,40,47, 54,58,62,64,68,74
Run 2	7,009	3.549	2,11,13,15,23,35, 41,54,57,66,68,74
Run 3	6,542	3.233	10,32,35,36,38,40, 48,49,52,67,68,81
Run 4	6,649	3.222	19,24,32,38,39,40, 42,43,54,56,68,75
Run 5	4,890	3.215	6,8,25,26,38,40, 42,45,49,55,62,68
Run 6	5,745	3.161	9,18,22,38,41,52, 53,58,64,68,72,73
Run 7	5,745	3.328	9,12,18,19,20,41, 43,50,62,68,69,76
Run 8	5,745	3.202	11,13,28,44,54,64, 65,67,68,71,75,81
Run 9	6,542	3.648	15,17,19,24,29,35, 40,49,50,56,67,81
Run 10	6,680	3.510	12,14,19,21,26,43, 49,50,55,58,65,78

Table D.27 GA Results For Instance Turk_60_9(1)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	8,206	2.137	3,6,14,21,25, 40,45,47,60
Run 2	8,858	2.294	6,14,21,30,37, 43,50,53,55
Run 3	8,399	2.633	5,10,12,14,24, 37,40,43,50
Run 4	8,858	2.340	12,14,25,26,40, 43,44,47,53
Run 5	8,206	2.275	3,4,9,14,17, 24,45,50,60
Run 6	8,858	2.538	6,8,12,14,18, 37,40,43,55
Run 7	8,206	2.415	3,4,14,23,30, 36,40,45,55
Run 8	8,399	2.598	6,14,23,32,42, 45,49,50,57
Run 9	8,858	2,617	3,13,14,23,40, 43,50,57,60
Run 10	8,858	2.438	8,12,14,18,40, 43,49,53,57

Table D.28 GA Results For Instance Turk_60_9(2)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	8.873	1,907	24,25,33,45,49, 60,69,70,77
Run 2	11.193	1,874	30,33,36,38,45, 70,76,78,81
Run 3	8.947	2,028	29,30,33,34,35, 48,50,52,77
Run 4	8.873	2,124	26,33,35,40,60, 63,69,76,77
Run 5	10.570	1,978	36,38,40,41,43, 55,64,71,72
Run 6	9.821	2,117	23,25,34,45,47, 60,71,78,80
Run 7	11.378	2,185	23,32,38,40,49, 54,58,64,66
Run 8	9.821	1,984	34,40,43,47,52, 60,67,76,80
Run 9	8.481	1,916	29,30,33,34,35, 48,50,52,77
Run 10	8,947	2,194	40,47,51,64,66, 74,75,77,80

Table D.29 GA Results For Instance Turk_60_9(3)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	10.551	1,694	3,7,8,19,41, 51,55,66,72
Run 2	8.735	1,598	10,13,33,41,42, 52,62,64,74
Run 3	9.331	1,746	4,17,35,51,52, 57,64,75,81
Run 4	9.316	1,894	14,17,24,28,38, 42,45,56,76
Run 5	8.029	1,607	8,13,18,24,41, 50,52,64,81
Run 6	8.029	1,584	3,4,18,20,25, 33,34,57,74
Run 7	7.876	1,789	13,18,20,29,34, 40,52,64,67
Run 8	10.033	1,805	4,10,30,42,43, 48,51,55,81
Run 9	9.276	1,718	3,5,14,35,46, 55,58,61,72
Run 10	7.876	1,674	8,20,30,41,50, 55,57,58,65

Table D.30 GA Results For Instance Turk_60_9(4)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	8.286	1,169	3,24,25,26,32, 51,55,60,80
Run 2	8.199	1,985	3,6,21,38,49, 58,68,80,81
Run 3	7.323	1,145	1,6,20,31,41, 44,52,58,65
Run 4	7.218	1,897	3,13,24,26,33, 37,43,60,81
Run 5	7.363	1,354	3,4,5,47,51, 54,58,73,80
Run 6	8.360	2,007	3,33,36,46,47, 54,58,66,67
Run 7	7.369	1,187	3,25,26,51,53, 54,55,60,71
Run 8	7.817	2,048	1,20,24,42,44, 54,60,72,73
Run 9	6.429	1,934	5,25,26,41,45, 51,53,67,71
Run 10	8.840	1,787	21,38,47,48,54, 55,61,73,81

Table D.31 GA Results For Instance Turk_50_8(1)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	11,169	1.221	2,6,14,24,30, 38,43,50
Run 2	11,169	1.120	6,14,38,40,43, 47,49,50
Run 3	10,348	1.478	2,5,13,14,28, 43,45,50
Run 4	10,591	1.084	2,4,6,8,10, 14,43,50
Run 5	11,169	1.134	5,8,14,19,43, 44,47,50
Run 6	11,169	1.107	8,12,14,28,29, 43,44,50
Run 7	11,169	1.135	5,13,14,19,30, 36,43,50
Run 8	11,169	1067	4,8,13,14,25, 36,43,50
Run 9	11,169	1.108	3,14,28,29,38, 43,47,50
Run 10	10,591	1.403	3,6,10,13,14,25,36,40

Table D.32 GA Results For Instance Turk_50_8(2)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	13.318	1,213	35,38,41,45, 49,56,60,80
Run 2	16.812	1,765	43,47,51,52, 53,61,69,81
Run 3	15.480	1,148	34,38,40,45, 49,55,61,69
Run 4	15.416	1,324	43,47,50,54, 56,58,69,71
Run 5	14.886	1,565	35,50,51,55, 62,71,72,77
Run 6	15.067	1,683	32,43,58,66, 71,72,76,80
Run 7	16.953	1,312	36,45,52,56, 60,68,73,81
Run 8	13.641	1,847	33,35,48,51, 54,57,60,65
Run 9	13.713	1,764	33,38,42,45, 54,58,65,68
Run 10	16.690	1,654	50,56,57,58, 64,65,69,81

Table D.33 GA Results For Instance Turk_50_8(3)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	12.867	1,435	15,17,19,40, 41,49,73,78
Run 2	13.083	1,643	1,19,20,40, 41,47,62,70
Run 3	15.959	1,398	1,15,19,27, 36,41,47,77
Run 4	12.091	1,905	13,17,20,23, 49,51,66,77
Run 5	12.724	1,794	26,33,34,36, 48,49,66,68
Run 6	15.035	1,312	15,20,28,36, 47,51,52,81
Run 7	14.910	1,843	13,40,41,48, 52,53,66,77
Run 8	12.310	1,724	9,16,20,40, 49,51,52,77
Run 9	13.003	1,298	10,42,48,49, 66,68,73,81
Run 10	12.549	1,245	33,35,40,41, 57,58,65,80

Table D.34 GA Results For Instance Turk_50_8(4)

	Objective Value for the Best Solution	Computational Time (Sec.)	Selected Hubs
Run 1	8.759	1,187	1,13,20,23, 38,54,58,65
Run 2	6.096	1,209	7,25,34,49, 55,58,71,80
Run 3	6.850	1,284	3,7,16,19, 49,54,58,80
Run 4	6.604	1,246	16,23,26,35, 38,54,55,65
Run 5	7.493	1,302	6,25,38,41, 43,49,55,58
Run 6	7.686	1,147	4,17,19,26, 41,55,71,80
Run 7	8.256	1,283	3,26,38,54, 55,58,67,72
Run 8	8.206	1,267	16,20,31,41, 43,49,58,80
Run 9	8.256	1,166	3,19,33,49, 54,55,58,61
Run 10	7.371	1,196	9,38,41,52, 61,65,72,80