PERIODIC SEARCH STRATEGIES FOR ELECTRONIC COUNTERMEASURE RECEIVERS WITH DESIRED PROBABILITY OF INTERCEPT FOR EACH FREQUENCY BAND

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ABSTRACT

PERIODIC SEARCH STRATEGIES FOR ELECTRONIC COUNTERMEASURE RECEIVERS WITH DESIRED PROBABILITY OF INTERCEPT FOR EACH FREQUENCY BAND

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Radar systems have been very effective in gathering information in a battlefield, so that the tactical actions can be decided. On the contrary, self-protection systems have been developed to break this activity of radars, for which radar signals must be intercepted to be able to take counter measures on time. Ideally, interception should be done in a certain time with a 100% probability, but in reality this is not the case. To intercept radar signals in shortest time with the highest probability, a search strategy should be developed for the receiver. This thesis studies the conditions under which the intercept time increases and the probability of intercept decreases. Moreover, it investigates the performance of the search strategy of Clarkson with respect to these conditions, which assumes that *a priori* knowledge about the radars that will be intercepted is available. Then, the study identifies the cases where the search strategy of Clarkson may be not desirable according to tactical necessities, and proposes a probabilistic search strategy, in which it is possible to intercept radar signals with a specified probability in a certain time.

Keywords: Radar interception, probability of intercept, electronic countermeasure receivers, search strategy, synchronization with radar, Farey series

ÖZ

ELEKTRONİK KARŞI TEDBİR ALMAÇLARINDA HER FREKANS BANDINDA İSTENEN YAKALAMA OLASILIĞINI SAĞLAYAN PERİYODİK TARAMA STRATEJİLERİ

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Radar sistemleri, taktik kararların alınması için gerekli istihbarat bilgilerinin toplanmasında oldukça etkili olmuştur. Buna karşın, radarların bu etkinliğini kırmak için kendini koruma sistemleri geliştirilmiştir. Bu sistemler, radarlara karşı tedbirlerin alınabilmesi için radar sinyallerini yakalamalıdır. İdeal olarak bu işlem belirli bir zaman içerisinde %100 olasılıkla gerçekleşmelidir; fakat gerçekte bu mümkün değildir. Radar sinyallerini mümkün olan en kısa zaman ve en büyük olasılıkla yakalamak amacıyla, almaç için bir tarama stratejisi belirlenmelidir. Bu tezde, radar sinyalini yakalama zamanını artıran, olasılığını düşüren durumlar araştırılmıştır. Ayrıca, Clarkson'un önerdiği, almacın ilgilendiği radarlar için bir ön bilgiyi şart koşan tarama stratejisinin performansı sorgulanmıştır. Bunun sonucunda, taktik gereklere bağlı olarak bu stratejinin elverişsiz olabileceği

durumlar saptanmış ve belirli bir zamanda istenen bir olasılıkla radar sinyalini yakalamayı mümkün kılan bir olasılıksal tarama stratejisi önerilmiştir.

Anahtar Kelimeler: Radar sinyali yakalama, sinyal yakalama olasılığı, elektronik karşı tedbir alıcısı, tarama stratejisi, radarla senkron olma, Farey dizisi

To Maria

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

CW:	Continuous Wave
ECM:	Electronic Countermeasures
ELINT:	Electronic Intelligence
ESM:	Electronic Support Measures
EW:	Electronic Warfare
FSR:	Frequency Swept Receiver
IFM:	Instantaneous Frequency Measurements
POI:	Probability of Intercept
PRF:	Pulse Repetition Frequency
PRI:	Pulse Repetition Interval
PW:	Pulse Width
RF:	Radio Frequency
SHR:	Super Heterodyne Receiver

CHAPTER 1

INTRODUCTION

Especially beginning with the World War II, a new battlefield was created: Electromagnetic spectrum. This war is named as Electronic Warfare (EW). EW includes using electromagnetic sprectrum to determine enemy's order of battle, intensions and capabilities or prevent hostile use of the electromagnetic spectrum [12]. EW mainly includes two fields: Electronic Support Measures (ESM) and Electronic Countermeasures (ECM) [14]. ESM is defined as "action taken under direct control of an operational commander to search for, intercept, identify, and locate sources of radiated electromagnetic energy for the purpose of immediate threat recognition. ESM provide a source of information required for immediate decisions involving ECM, avoidance, targeting, and other tactical employment of forces" [13], whereas ECM is "action taken to prevent or reduce an enemy's effective use of the electromagnetic spectrum" [13].

Radar systems have been very effective in gathering information in electromagnetic spectrum. Basically, radars generate radio frequency (RF) energy, transmit, and collect and detect the reflected RF [15]. On the contrary, ECM systems have been developed to break this activity of radars, for which radar signals must be intercepted in a certain time to be able to take counter measures on time. Radars use a very wide spectrum and usually an ECM receiver must be able to intercept multiple radars operating in different frequency bands from different directions, simultaneously. Even if it is possible to intercept radar signals in a wide spectrum at the same time with some receivers, such as Instantaneous Frequency Measurements (IFM) receivers [9], with current technology these receivers can not be highly sensitive [2]. In many applications, there is need to use sensitive receivers but they can work only on a narrow band of spectrum at a time. To cover all the spectrum of interest, these receivers sweep all frequencies, which is why they are named as Frequency-Swept Receiver (FSR). Super heterodyne receivers (SHR) form a subclass of FSRs, which can tune their frequencies to a different value at each time [2]. That is, they are changing the band in which they receive signals depending on time, although the bandwidth is fixed.

Radars are faced with a similar problem; they can not be sensitive in both range and angular resolution [2], so that they have to use some methods like using directional antenna, emitting pulsed signals, scanning all the directions, etc. Thus, both systems' activity is intermittent and this makes the interception of radar signals probabilistic, rather than being deterministic. Ideally, the interception should be done in a certain time with a 100% probability, but in reality this is not the case. To intercept radar signals in shortest time with the highest probability, a search strategy should be developed for the receiver. The performance of SHR is directly related to its search characteristics [16]. Then, the question is, how to determine the best search strategy for a SHR; how to compare one search strategy with another one. This thesis studies the factors that affect the performance of search strategy, tests the performance of one search strategy proposed by Clarkson [4], and proposes another approach named as probabilistic search.

1.1 PREVIOUS WORK

The literature on this problem is quite sparse, and most of the articles are about probability of intercept calculations, with a limited number of studies on search strategy applications. Richards [6] was most probably the first researcher who studied the interception problem comprehensively for two strictly periodic and deterministic events. He modeled the problem as the coincidence of two pulse trains (or window functions) which is usually used by the followers, and also he was the one who linked the problem to the number theory, using Farey series. Miller and Schwarz [17] also investigated connections with the number theory. In their approach, periods and durations of events are quantized. Their work was simplified in [18] and [19]. Stein and Johansen [20] obtained some statistical information on coincidences of random pulse trains, which is extended in [10], but their methods were not applicable for strictly periodic pulse trains. Hatcher [8] carried out calculations for intercept time for a pre-selected probability for random-phase pulse trains. Wiley [9] also developed a formulation to calculate intercept time, by using average coincidence period and average coincidence duration. He was also maybe the first one who gave implications of his calculations to search strategies. Most of these studies ignored the synchronization problem, but in [7], methods developed by Richards [6] were extended and synchronization problem is examined with number theory. Clarkson presented a systematic study in which he questions the optimal periodic search strategy with respect to some parameters of emitters and receiver. In [11] and [21], this author also studied the synchronization problem with number theory, especially using Diophantine approximation and Farey series. In [1], he gave the mathematical model of the problem. With this model, he examined the calculation of the maximum intercept time with change in the sweep period, i.e. the period of the receiver's scan, with equal duration of sweeping for each frequency band. In addition, he proposed a method to find periodic search strategy for a SHR receiver with an optimum sweep period which minimizes the maximum intercept time. The continuation of his work was presented in [2], where it was shown that it was also possible to assign a different duration of sweeping for each frequency band. In [4] he gave summary and more detailed information about his work for selecting optimal search strategy with the min-max intercept time criterion.

1.2 SCOPE OF THE THESIS

This thesis aims to examine the role of radar interception problem in search strategy applications for a SHR receiver. It explores the factors that affect the performance of the search strategy by using the approach of Clarkson [4]. In the articles that appeared in the literature, Clarkson did not include simulations for his algorithm.

Thus, this thesis presents some simulations with different parameters and analyzes his approach against number of frequency bands, duty cycle of radars, PRI of radar signals, and diversity of scan period and duty cycle of radars. In addition, this thesis proposes another approach for the search strategy, named as probabilistic search, which provides a means to intercept radar signal with a pre-selected probability in a certain time.

Radars may use interpulse modulations, such as continuous wave (CW), pulsed, or chirp radars. Moreover, they can transmit more than one frequency; they can be frequency agile, or may employ frequency hopping [16]. In addition, radars may operate in many types of mode, such as switching, staggering, pulse repetition frequency (PRF) jittering [4]. Also they can make raster scan, spiral scan, or lobe-switching scan. Note that in this study it is assumed that the radars are circularly scanning and do not use any interpulse modulation, and also they emit with a constant frequency. Furthermore, throughout the thesis it is assumed that SHR receiver periodically sweep frequency bands with an omnidirectional antenna. Therefore, all of the search strategy considered in this study is periodic. Also it is assumed that a pre-knowledge about radars to be intercepted is available. Otherwise, there can not be any search strategy that guarantee finite intercept times [22]. Another assumption made here is that there is no noise, so that the interception is certain when coincidence between pulse trains happen. In the presence of noise, the interception problem must be modeled in terms of detection theory.

1.3 OUTLINE OF THE THESIS

In Chapter 2, a brief summary for intercept problem and pulse train model is given as well as definitions of the parameters that are used throughout the study.

Chapter 3 explains periodic search strategy for SHR receiver and gives a mathematical background to calculate the maximum intercept time which is used to find optimal search strategy. In addition, an iterative algorithm by Clarkson [4] to compute the optimal search strategy is explained.

Chapter 4 gives the method for generation of data to test the performance of the algorithm of Clarkson and clarifies in which way the test is done.

Results of simulations of the algorithm of Clarkson are given in Chapter 5 and also an analysis about the results is presented.

Chapter 6 proposes a new approach, named as probabilistic search, where for each frequency band different probabilities of intercept are considered during the search. The algorithm of this approach is also given in this chapter.

In Chapter 7 simulation results of probabilistic search are given and a comparison is made with the results of the algorithm of Clarkson.

Chapter 8 concludes the study.

CHAPTER 2

RADAR INTERCEPT

2.1 COINCIDENCE WITH RADAR SIGNALS

For an intercept to occur, the radar should direct its main beam to the receiver, and the receiver should be tuned to the frequency of the radar pulses. Therefore, the problem may include different type of interceptions; such as, beam-on-beam intercept, beam-on-frequency intercept, or frequency-on-frequency intercept [8].

a) **Beam-on-beam intercept:** In this problem, as indicated in Figure 2-1, the radar and the receiver have directional antennas that are rotating. Therefore, an intercept can occur only when the main beams of both antennas are coincident.

Figure 2-1: Beam-on-beam intercept [8]

b) Frequency-on-frequency intercept: In this case, the radar and receiver have omnidirectional antennas, and the radar is continuously emitting. But, the radar is sweeping or jumping in frequency. Thus, the receiver should be tuned to the frequency of the radar pulses for an intercept. This is illustrated in the Figure 2-2.

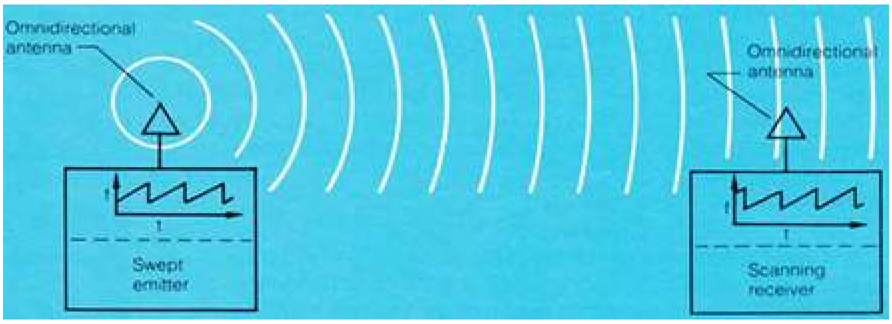

Figure 2-2: Frequency-on-frequency intercept [8]

c) Beam-on-frequency intercept: As illustrated in Figure 2-3, in this intercept problem, the radar antenna is directionally rotating, whereas the receiver has an omnidirectional antenna and searching in frequency. An intercept occurs when the receiver is tuned to the frequency of radar pulses and the main beam of radar is directed to the receiver antenna.

Figure 2-3: Beam-on-frequency intercept [8]

In this thesis, the beam-on-frequency intercept problem is studied. Therefore, throughout the study it is assumed that the radar is circularly scanning with a directional antenna, and the receiver is sweeping frequency bands with an omnidirectional antenna.

2.2 PULSE TRAIN (WINDOW FUNCTION) MODEL

Typically, radars make a search in angle with a directional antenna. In Figure 2-4, a typical antenna pattern of radar is shown.

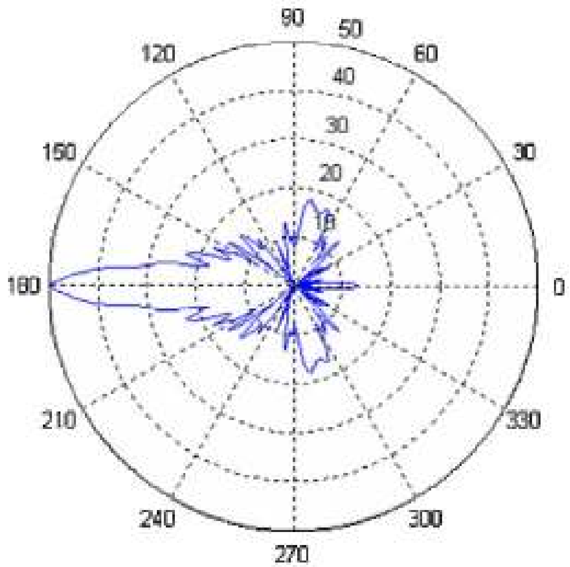

Figure 2-4: Example directional radar antenna pattern

Assuming that the radar is making a periodic search in angle and the receiver is tuned to the frequency of radar signals, the signals will be received with a highest power when the mean beam is directed to the receiver, while their power will be lower, even below the noise power, when the antenna of the radar is directed elsewhere. In this way, the graph of noise-free perceived radar signal powers on the receiver will be like in Figure 2-5:

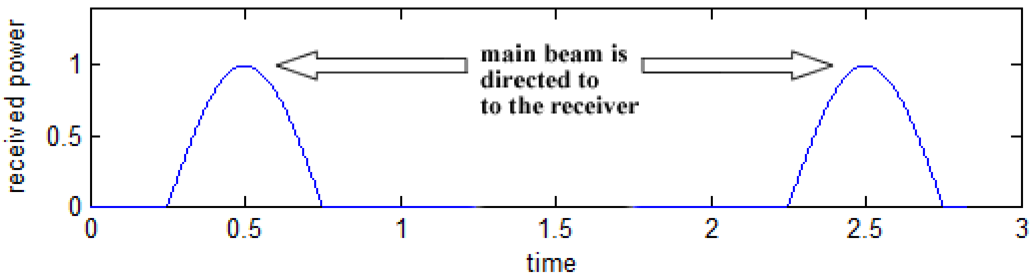

Figure 2-5: Circularly scanning radar signal interception at the receiver

For the sake of simplicity, radar interception may be modeled by a pulse train, so that the value is 1 when the radar signals can be received over the noise level and 0 when the received power would be under noise level if the receiver were tuned to the true frequency. With this model, Figure 2-5 will be replaced by Figure 2-6.

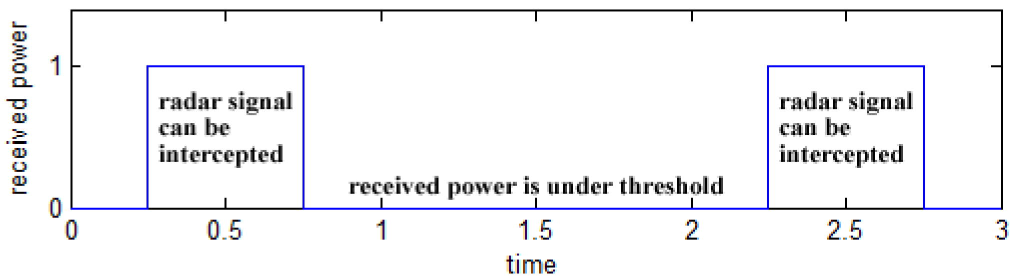

Figure 2-6: Interception model with pulse train

In this respect, Figure 2-6 shows the temporal regions where it is possible to intercept the signals of radar. So, in these regions the pulse train has a value of 1, but also the receiver has to be tuned to the frequency band to be able to intercept the signals. Note that we can also model the behavior of the receiver with a pulse train as follows: Let us assume that the receiver is also making a periodic search in frequency bands yielding a value of 1 when the receiver is tuned to the true frequency and 0 when the receiver is tuned to any other band. Then, we can plot the frequency sweeping behavior of the receiver as in Figure 2-7 b).

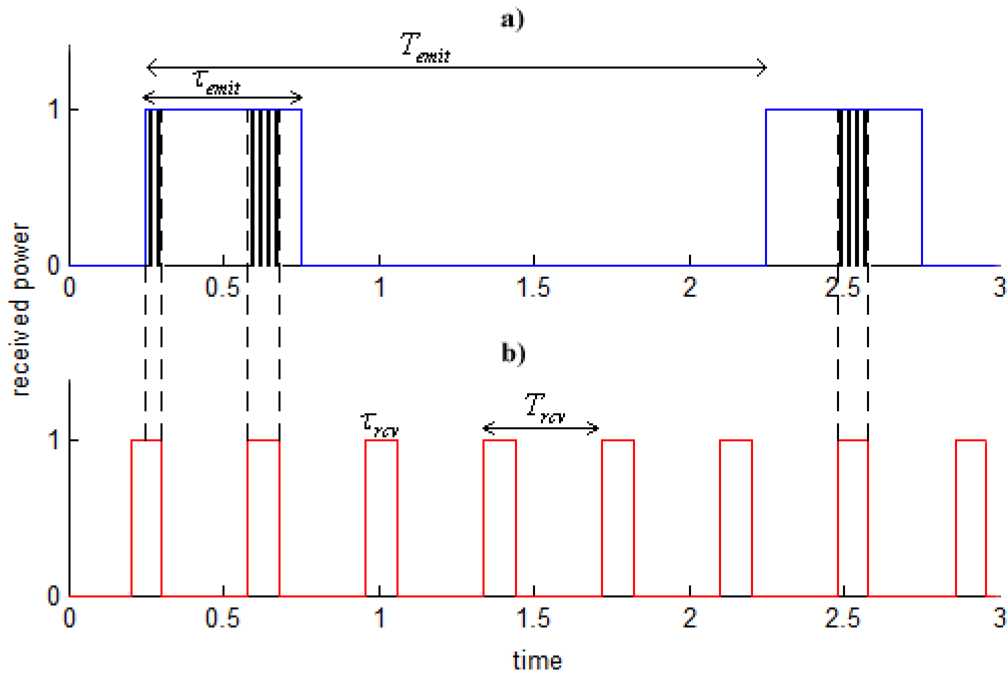

Figure 2-7: Coincidence of two pulse trains. a) Pulse train (1) of radar b) Pulse train (2) of receiver

If Figure 2-7 is examined, the dashed lines show the times where both pulse train 1 and pulse train 2 have a value of 1, namely both the radar is directed to the receiver and the receiver is tuned to the frequency of radar signals. Thus, interceptions occur in dashed regions.

2.3 DEFINITIONS

2.3.1 Scan Period

Search radar with an antenna that rotates with constant angular velocity directs its main beam to the receiver with regular time intervals. Thus, as seen in Figure 2-7 a), there will be a periodicity in radar's scan, which is named as *scan period*, T_{emit}.

2.3.2 Beamwidth

Beamwidth (in degree) defines the sight of the radar when it directs its main beam in a certain direction. The duration during which the radar directs its main beam to the receiver is proportional to the beamwidth. We can assign beamwidth to the pulse width in Figure 2-7 a), τ_{emit}, with the following formula:

$$\tau_{emit} = \frac{beamwidth}{360} \times T_{emit} \tag{2.1}$$

2.3.3 Pulse Repetition Interval (PRI)

During the time τ_{emit}, actually the radar emits not only one pulse, but many pulses one after another with an interval to be able to detect its targets in its range. The pulses in Figure 2-7 a) illustrate the time intervals when the radar directs its main beam to the receiver. Actual radar pulses are shown in Figure 2-8. The interval between these pulses is defined as PRI.

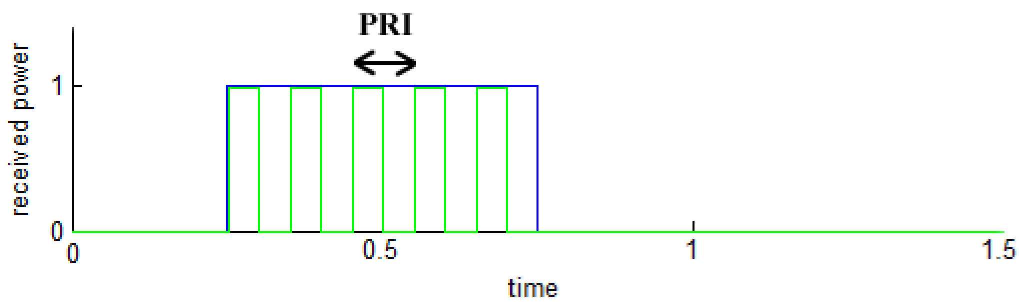

Figure 2-8: Pulse repetition interval

2.3.4 Sweep Period

A receiver that applies a periodic search strategy will tune its frequency to each band periodically. This period can be defined as the *sweep period*, T_{rcv}. It can be seen in Figure 2-7 b).

2.3.5 Dwell Time

Dwell time determines the amount of time during which the receiver stays tuned to a frequency band. It is seen in Figure 2-7 b) as the pulse width of the pulse train 2, τ_{rcv}.

The role of PRI in interception is important and must be clarified. It can be seen from Figure 2-7 that when interception occurs, it lasts at most for an amount of time equal to the dwell time. In order to detect the radar, at least a few pulses are necessary. This means that when an interception occurs, the receiver has to perceive at least some number of pulses. Therefore, we can assign the time to intercept a given minimum number of pulses to dwell time as follows:

$$\text{minimum dwell time} = \text{minimum number of pulses to receive} \times \text{PRI} \tag{2.2}$$

2.3.6 Duty Cycle

The duty cycle of a pulse train is the ratio of the pulse width to the period. So, for the receiver, it can be written as follows:

$$\text{duty cycle} = \frac{\text{dwell time}}{\text{sweep period}} = \frac{\tau_{rcv}}{T_{rcv}} \tag{2.3}$$

The duty cycle can show how busy is the receiver for a specific frequency band. To save time for other frequency bands, it will be necessary to consider duty cycle for one band.

CHAPTER 3

PERIODIC SEARCH STRATEGY

In the search strategy, intercept time has an important role. The aim is not only to get information about the electromagnetic spectrum as much as possible in a limited time, sweep period, but also to intercept the emitters as soon as possible. Therefore, the maximum time to intercept a radar signal is of interest.

Here, the intercept time refers to this possible maximum time to receive the signal of the emitter of interest. That is, during the intercept time, at least once the radar is directed to the receiver and the receiver is tuned to the frequency of the radar at the same time. If we use the model introduced in part 2.2, intercept time is the time to wait for at least one coincidence between pulse train 1 and pulse train 2 in the worst case. It is possible to receive signals before the intercept time, but at the end of intercept time, this is guaranteed.

It should be noted that when a coincidence occurs, intercept is accepted to be certain, due to the absence of noise.

3.1 THE ROLE OF SWEEP PERIOD AND DWELL TIME IN INTERCEPTION

Since one of the main parameters to be decided for the best search strategy is the sweep period, it is useful to observe the variation of the intercept time with the change in sweep period. This is provided in [1], where the author gives a calculation method for intercept time to easily see the effect of change in sweep period, assuming fixed sum of pulse widths ($\tau_{emit} + \tau_{rcv}$), or constant duty cycles, $\left(\frac{\tau_{emit}}{T_{emit}} \text{ and } \frac{\tau_{rcv}}{T_{rcv}}\right)$. Another assumption that is made in [1] is that the dwell time τ_{rcv} is the same for each frequency band. In Figure 3-1, such a simple periodic search strategy can be seen for an example of 10 frequency bands.

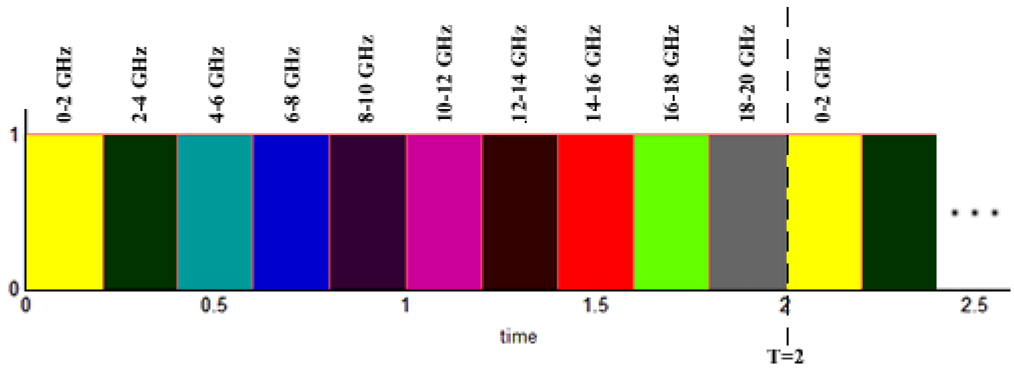

Figure 3-1: Simple periodic search strategy of a SHR for 10 frequency bands with $T_{rcv} = 2$

Obviously, here the question is to decide the sweep period T_{rcv} for the best search strategy which minimizes the maximum intercept time of all the emitters in the threat-emitter list. This will be explained in more detail later in this section, but for now it is convenient to show how the search strategy differs with T_{rcv} when pulse width or duty cycle is constant.

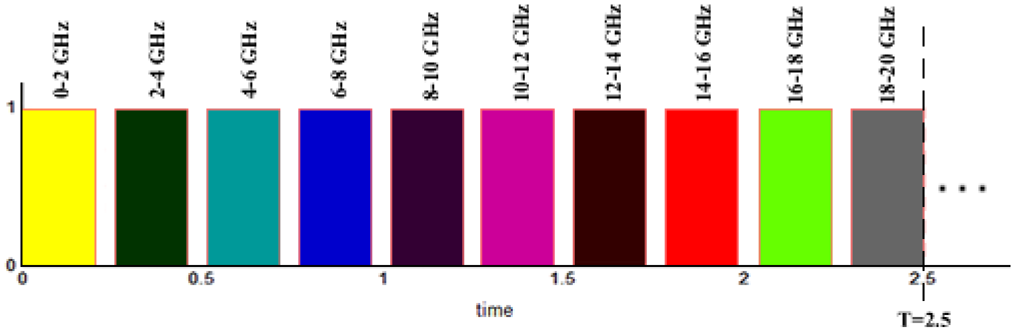

Figure 3-2: Simple periodic search strategy of a SHR with sum of dwell times less than T_{rcv}

In Figure 3-2, we see that if the pulse width is constant, when T_{rcv} increases, some gaps appear while sweeping during T_{rcv}. This is because pulse width for each frequency band is fixed while increasing T_{rcv} creates more idle time available for the receiver. The places of the gaps are actually not important unless the search of any frequency band becomes aperiodic. To answer how to fill this time is not a purpose of [1], but one of the useful results will be to show how intercept time for each emitter in the threat-emitter list changes with T_{rcv} when pulse width is fixed.

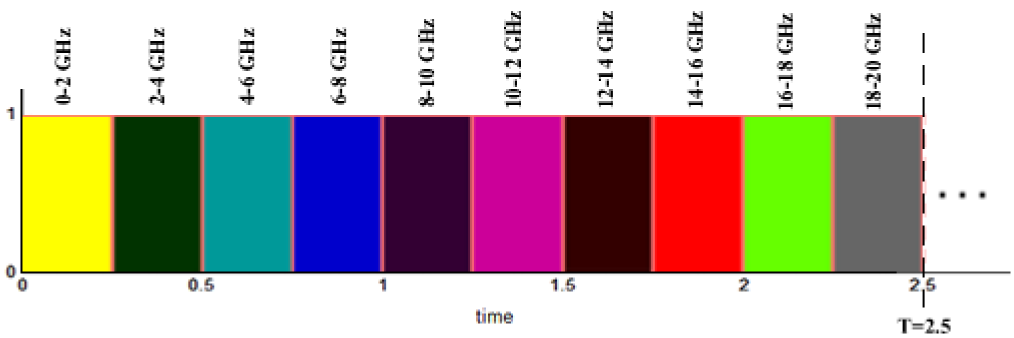

Figure 3-3: Gaps are filled equally when sweep period increased

Moreover, this new idle time can be automatically filled by increasing pulse width of each frequency band by the same proportion, as in Figure 3-3. In other words, the duty cycles remain the same. The variation of intercept time by changing T_{rcv} is also examined in [1] when duty cycle is fixed.

First of all, consider the equation

$$|t - kT - \phi| \le \frac{1}{2}\tau \tag{3.1}$$

which tells us that a pulse appears at all times t that satisfies the condition in (3.1), where T is period and τ is pulse width as stated before, k is defined as pulse index for k^{th} pulse, which is an integer ($k \in \mathrm{Z}$), and ϕ is the time delay. Therefore, for a coincidence between two pulse trains, for an emitter and the receiver, a necessary and sufficient condition is as follows,

$$\left|(k_{emit}T_{emit} + \phi_{emit}) - (k_{rcv}T_{rcv} + \phi_{rcv})\right| \le \frac{1}{2}(\tau_{emit} + \tau_{rcv} - 2d) \tag{3.2}$$

where d is the minimum time required for a coincidence. Also note that for a coincidence of duration d both $\tau_{emit} > d$ and $\tau_{rcv} > d$.

Intercept time is the required time to guarantee at least one coincidence between two pulse trains, which is independent from phases of pulse trains. Then, in the SHR problem the intercept time can be defined as integer multiple of T_{rcv}.

Now, it will be useful to examine the intercept time relative to the scan period of the emitter, T_{rcv}. Thus, define the values α, β, and, ε as follows:

$$\text{Ratio of periods,} \qquad \alpha = \frac{T_{rcv}}{T_{emit}} \tag{3.3}$$

$$\text{Normalized sum of pulse widths,} \qquad \varepsilon = \frac{\tau_{emit} + \tau_{rcv} - 2d}{T_{emit}} \tag{3.4}$$

$$\text{Normalized phase difference,} \qquad \beta = \frac{\phi_{rcv} - \phi_{emit}}{T_{emit}} \tag{3.5}$$

If we rewrite (3.2) in terms of α, β, and, ε as in (3.6) below,

$$\left|q\alpha - p + \beta\right| \le \frac{1}{2}\varepsilon \tag{3.6}$$

we can see that there is an interval for normalized phase difference β, in which a coincidence occurs between p^{th} pulse and q^{th} pulse of pulse train 1 and 2, respectively. From (3.6), this interval can be written as follows:

$$I_{p,q} = \left\{ x \in \Re \,\middle\|\, \left|q\alpha - p + x\right| \le \frac{1}{2}\varepsilon \right\}$$
$$= \left[p - q\alpha - \frac{1}{2}\varepsilon,\ p - q\alpha + \frac{1}{2}\varepsilon \right] \tag{3.7}$$

When β is a member of such an interval, then there is coincidence. More precisely, if a union is defined which consists of all these intervals, the condition for a coincidence is:

$$\beta \in \bigcup_{\substack{p,q \in Z \\ 0 \le q < n}} I_{p,q}$$

A characteristic function $C_n(\beta)$ of this union then can be defined, which takes the value 1 if some integers p, q can be found with $0 \le q < n$ such that $\beta \in I_{p,q}$ and it is 0 otherwise. Inherited from the periodicity of both pulse trains, $C_n(\beta)$ is periodic with $\beta = 1$.

Therefore, when $C_n(\beta)$ is always 1 for $0 \le \beta \le 1$, a coincidence must occur between pulse trains, within these n consecutive pulses. Then, the intercept time is $n \times T_{rcv}$.

In Figure 3-4 taken from [1] the values of characteristic function $C_n(\beta)$ are shown for $n = 5, n = 9, n = 14$ between $\beta = 0$ and $\beta = 1$, for an example with values $\alpha = 0.217$ and $\varepsilon = 0.1$. Here, the important thing is to see that $C_n(\beta)$ takes 0s for some values of β for $n < 14$. This means that when $n < 14$, there are some values of β, which makes the coincidence impossible. In other words, coincidence is still phase-dependent until $n = 14$. For $n = 14$, however, $C_{14}(\beta)$ becomes 1 for all values of β between 0 and 1, so that a coincidence is guaranteed. Thus, the receiver should be tuned to the frequency of the emitter for duration of 14 times T_{rcv} to ensure to intercept the signals of emitter. Obviously, this is the intercept time for that emitter: $n \times T_{rcv} = 14 \times T_{rcv}$.

Another result from Figure 3-4 is that as n increases, there are more intervals that makes $C_n(\beta)$ = 1. Note that some intervals form overlaps over the values of β. Because of this, $C_n(\beta)$ would be equal to 1 for all values of β most quickly, if there were not overlaps. Actually, this is the theoretical lower bound for the intercept time. Remembering that the width of each interval is ε, this bound is expressed as follows:

$$\text{intercept time} \geq \frac{T_{rcv}}{\varepsilon} = \frac{T_{emit} T_{rcv}}{\tau_{emit} + \tau_{rcv}} \tag{3.8}$$

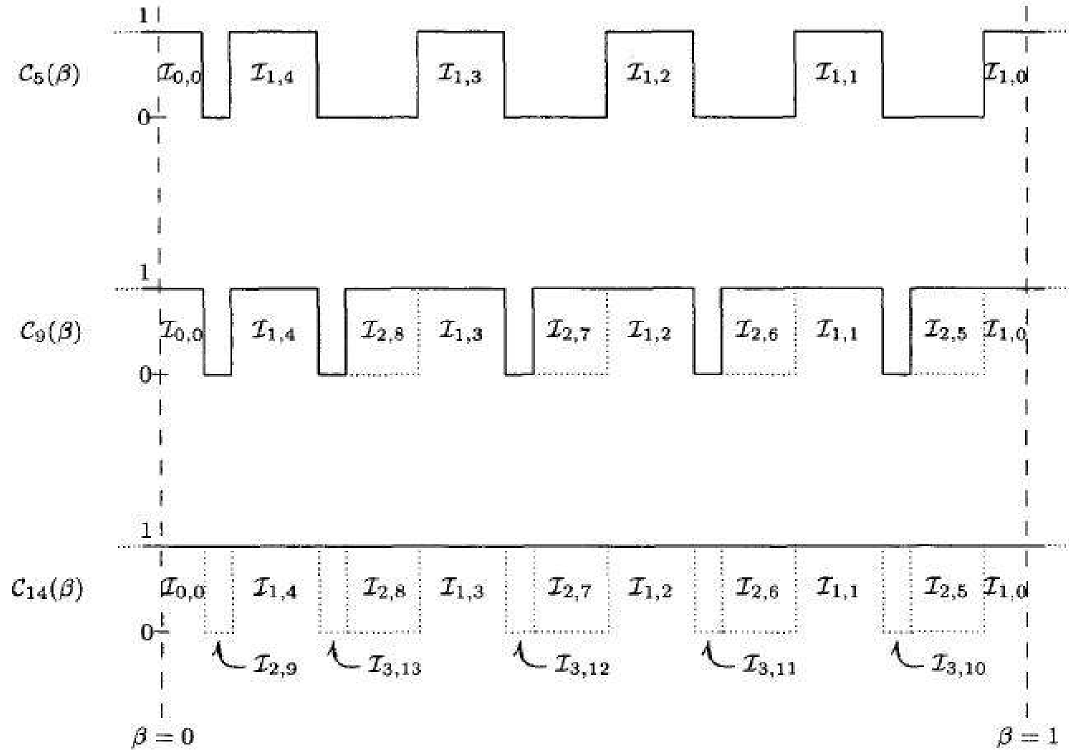

Figure 3-4: The value of the characteristic function $C_n(\beta)$ for $n = 5$, $n = 9$, $n = 14$ [1]

3.2 MAXIMUM INTERCEPT TIME

3.2.1 Diophantine Approximation

Diophantine approximation is a branch of number theory which deals with approximation of real numbers by rational numbers. In this context, it is important how to decide one approximation is better than the other. For our purpose, in [1] a best approximation p/q with $q \geq 0$ to α is defined as the one for which the following is true:

For all any other approximation p'/q' with $q' \geq 0$,

$$q' \leq q \quad \Rightarrow \quad |q'\alpha - p'| \geq |q\alpha - p| \tag{3.9}$$

and

$$|q'\alpha - p'| \le |q\alpha - p| \quad \Rightarrow \quad q' \ge q \tag{3.10}$$

where $|\eta| = |q\alpha - p|$ is the absolute approximation error.

For any real number α we can find a sequence of best approximations, like

$$\frac{p_1}{q_1}, \frac{p_2}{q_2}, \cdots, \frac{p_n}{q_n}, \cdots$$

such that the absolute approximation error $|\eta|$ is non-increasing. The sequence is infinite unless α is rational.

The best approximation of α within ε is the one that is the first in the sequence of best approximations to α with an absolute approximation error not greater than ε [1].

The sequence of best approximations can be ordered so that they have the following properties:

- $\eta_n \eta_{n+1} < 0$, where η_n and η_{n+1} are the approximation errors of successive elements of the sequence.
- Let p_n/q_n and p_{n+1}/q_{n+1} be two successive elements of the sequence. Then, they exhibit unimodularity property as follows:

$$p_{n+1}q_n - p_n q_{n+1} = \begin{cases} 1, & \text{if } \eta_\mathrm{n} > 0 \\ -1, & \text{otherwise} \end{cases} \tag{3.11}$$

Then, according to [1], Diophantine approximation can be used to find the intercept time of any two pulse trains with α and ε as defined in (2.3) and (2.4) by the following procedure:

1. Determine the best approximation of α to within ε, which is denoted as $p_{n(\varepsilon)}/q_{n(\varepsilon)}$.
2. If the approximation error of the best approximation found in 1 is zero, this is because α is rational and this corresponds to a synchronization ratio, so that the intercept time is infinite.

3. If not, determine the next element in the sequence: $p_{n(\varepsilon)+1} / q_{n(\varepsilon)+1}$.

4. Calculate k as follows:

$$k = \left\lfloor \frac{\varepsilon - \left|\eta_{n(\varepsilon)+1}\right|}{\left|\eta_{n(\varepsilon)}\right|} \right\rfloor$$

5. The intercept time is $T_{rcv} \times \left(q_{n(\varepsilon)+1} + q_{n(\varepsilon)} - kq_{n(\varepsilon)}\right)$.

3.2.2 Farey Series and Intercept Time

According to [1], to enumerate the synchronization ratios and to find the intercept time, it is suitable to use Farey series[1] of appropriate order. Farey series of order n, $\Im_n$, is such a series that its elements are fractions in lowest terms in ascending order, whose denominators are positive and less than or equal to the order n [3]. The Farey series between 0 and 1 up to order five can be seen in Table 3-1.

Table 3-1: The Farey series up to order 5 between 0 and 1

1. order	$\frac{0}{1}$,										$\frac{1}{1}$
2. order	$\frac{0}{1}$,					$\frac{1}{2}$,					$\frac{1}{1}$
3. order	$\frac{0}{1}$,			$\frac{1}{3}$,		$\frac{1}{2}$,		$\frac{2}{3}$,			$\frac{1}{1}$
4. order	$\frac{0}{1}$,		$\frac{1}{4}$,	$\frac{1}{3}$,		$\frac{1}{2}$,		$\frac{2}{3}$,	$\frac{3}{4}$,		$\frac{1}{1}$
5. order	$\frac{0}{1}$,	$\frac{1}{5}$,	$\frac{1}{4}$,	$\frac{1}{3}$,	$\frac{2}{5}$,	$\frac{1}{2}$,	$\frac{3}{5}$,	$\frac{2}{3}$,	$\frac{3}{4}$,	$\frac{4}{5}$,	$\frac{1}{1}$

[1] Although the Farey Series is in fact a "sequence", this terminology is used in the literature.

The Farey series have the following properties: Let h/k and h'/k' be two adjacent elements of the series. Their median is defined as $(h+h')/(k+k')$ and when the order reaches $k+k'$ it is added to the series between h/k and h'/k'. Moreover, the elements of the Farey series also obey unimodularity property, i.e. $h'k-hk'=1$. Then, the Farey series can be used to determine intercept time as follows [1]:

1. Calculate α and ε as in (2.3) and (2.4).
2. Find the appropriate order of Farey series, which is $\lceil 1/\varepsilon \rceil - 1$.
3. Find adjacent elements of the series, h/k and h'/k', such that $h/k \le \alpha \le h'/k'$. If α is equal to one of these elements, then this element corresponds to a synchronization ratio, so the intercept time is infinite and there is no further steps.
4. Calculate x_1 as follows:

$$x_1 = \begin{cases} \dfrac{h+\varepsilon}{k}, & \text{if } \ k < k' \\ \dfrac{h'-\varepsilon}{k'}, & \text{otherwise} \end{cases} \tag{3.12}$$

5. Calculate the values p, q, P, Q and κ as follows:

$$p, q, P, Q = \begin{cases} h, k, h', k' & \text{if } \ \alpha < x_1 \text{ or both } k < k' \text{ and } \alpha = x_1 \\ h', k', h, k & \text{otherwise} \end{cases} \tag{3.13}$$

$$\kappa = \left\lfloor \frac{\varepsilon - |Q\alpha - P|}{|q\alpha - p|} \right\rfloor \tag{3.14}$$

6. The intercept time is $T_{rcv} \times (Q + q - \kappa q)$.

With this procedure, holding ε as constant, the change in intercept time by varying T_{emit} or T_{rcv} can be easily seen. As α moves towards elements of Farey series, intercept time goes to infinity. Thus, the elements of the Farey series of order $\lceil 1/\varepsilon \rceil - 1$ give the complete series of synchronization ratios of relevant pulse trains. On the other hand, the intercept time reaches to its minimum value as α moves between adjacent elements [1].

3.2.3 Constant Duty Cycles and Intercept Time

We have shown the procedure to use Farey series to determine synchronization ratios and intercept times when the sum of pulse widths, i.e. ε, is constant. However, since the purpose here is to be able to intercept radars as quick as possible, the new idle time of receiver created by increasing T_{rcv} would be filled with some duties immediately. Therefore, it would be useful if we were also able to observe the effects of varying T_{rcv} on the intercept time when the duty cycle is constant.

In [1], there is a method for this purpose very similar to the constant sum of pulse widths case. The main difference is that in the constant duty cycle case, instead of Farey series, an augmented generalized Farey series is used, which is denoted by $\xi^*(\lambda_1, \lambda_2)$, where λ_1 and λ_2 are the duty cycles of two pulse trains. Then, the procedure to find the intercept time where the duty cycles are constant is as follows:

1. Calculate α as in (2.3).
2. Calculate the augmented, generalized Farey series, $\xi^*(\lambda_{emit}, \lambda_{rcv})$, where λ_{emit} and λ_{rcv} are the duty cycles for the emitter and the receiver, respectively.
3. Find the successive elements of $\xi^*(\lambda_{emit}, \lambda_{rcv})$ such that $h/k \le \alpha \le h'/k'$. If α is equal to one of the elements, the intercept time is infinite because of synchronization. Otherwise, follow next steps.
4. Calculate x_1 as follows:

$$x_1 = \begin{cases} \dfrac{h+\lambda_1}{k-\lambda_2}, & \text{if } \ k < k' \\ \dfrac{h'-\lambda_1}{k'+\lambda_2}, & \text{otherwise} \end{cases} \tag{3.15}$$

5. Calculate the values p, q, P, Q and κ as in (3.13) and (3.14).
6. The intercept time is $T_{rcv} \times (Q + q - \kappa q)$.

In Figure 3-5 below, we see how the intercept time changes with the sweep period T_{rcv}, when duty cycle of both the emitter and the receiver is constant. For the emitter a duty cycle of 0.13 is used whereas for the receiver the duty cycle is 0.26. T_{emit} is taken constant as 1, and T_{rcv} is varied between 0.1 and 4. As seen from the graph, intercept time sometimes goes to infinity. This corresponds to the synchronization ratios, and the augmented generalized Farey series give these ratios.

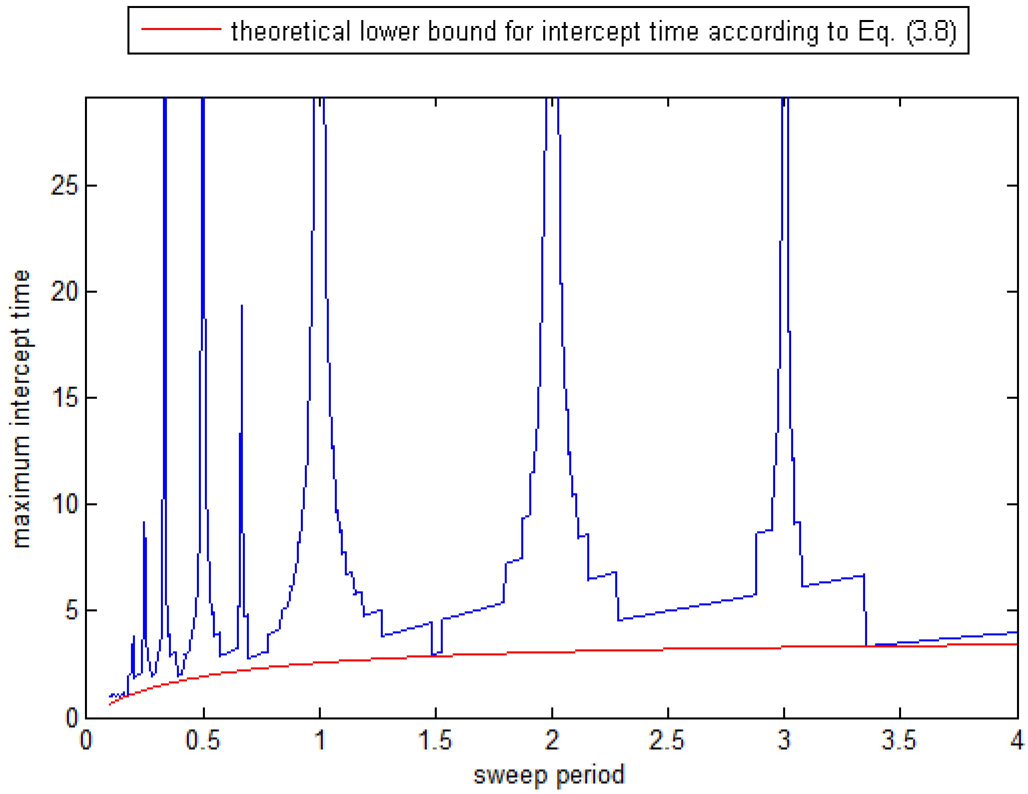

Figure 3-5: Intercept time vs. T_{rcv} with constant duty cycles

3.2.4 Geometric Construction of Intercept Time

Another result seen in Figure 3-5 is that there is a point where intercept time reaches a local minimum value between two infinite values, i.e. between two Farey ratios. Moreover, there are regions where the change in intercept time is piecewise linear, and there are jumps between these linear regions. These jumps are because of the change in intercept time as number of looks. This means that there is a tolerance for dwell time for which intercept time as the number of looks remain the same. These regions depend on the ratio of the periods of the pulse train, α as in (3.3), and the tolerance, $\varepsilon = (\tau_{emit} + \tau_{rcv} - 2d)/T_{emit}$, where d is the minimum required time for a coincidence. Note that ε is actually the normalized sum of pulse widths (3.4) when $d = 0$.

In fact, α-ε plane can be divided into these regions which shows us at once where the intercept time becomes infinite, where there are jumps in intercept time as the number of looks, and where it remains constant. To get subdivided α-ε plane as in [4], we need some theorems.

Theorem 1: The fractions h/k and h'/k' are adjacent elements of Farey series if and only if $h'k - hk' = 1$. The necessity and sufficiency part of the proof of this theorem can be found in [3] and [5], respectively.

Theorem 2: Let h/k and h'/k' be two adjacent elements in a Farey series, such that $h/k \le \alpha \le h'/k'$. If for some h''/k'', $k\alpha - h < k''\alpha - h'' < k'\alpha - h'$, then $k'' \ge k + k'$.

Theorem 3: Let α and ε be period ratio and tolerance as defined before, and let h/k and h'/k' be two fractions such that $h/k \le \alpha < h'/k'$. If $h - k\alpha \le \varepsilon$ and $k'\alpha - h' \le \varepsilon$, then the intercept time (the number of looks) is not greater than $k + k'$, i.e. int. time $\le k + k'$.

Theorem 4: Again consider α and ε as in previous theorem and let h/k and h'/k' be two adjacent elements in a Farey series such that $h/k \le \alpha < h'/k'$. If $(k'-k)\alpha - (h'-h) > \varepsilon$, then the intercept time as the number of looks is not less than $k + k'$, i.e. int. time $\ge k + k'$.

The proofs of Theorems 2, 3, and 4 can be found in [4].

Now, consider any two adjacent Farey elements such that $h/k \le \alpha < h'/k'$. From theorem 3 we know that the intercept time is not greater than $k + k'$, when $h - k\alpha \le \varepsilon$ and $k'\alpha - h' \le \varepsilon$. The intersection of these regions can be found by writing the equality $h - k\alpha = k'\alpha - h'$ and using Theorem 1. Then, α is found to be $(h + h')/(k + k')$, and $\varepsilon = 1/(k + k')$. Also note that at $\alpha = h/k$, $k'\alpha - h' \le \varepsilon$ reduces to $\varepsilon \ge 1/k$, and at $\alpha = h'/k'$, $h - k\alpha \le \varepsilon$ reduces to $\varepsilon \ge 1/k'$. Furthermore, from Theorem 4 we know that the intercept time is not less than $k + k'$, when $(k'-k)\alpha - (h'-h) > \varepsilon$, which reduces to $\varepsilon < 1/k$ at $\alpha = h/k$, and $\varepsilon < 1/k'$. As a result, these boundaries form a triangle in the α - ε plane, inside which the intercept time is constant, $k + k'$ [4]. The vertices of the triangle are as follows:

$$\left\{ \quad \left(\frac{h}{k}, \frac{1}{k}\right), \quad \left(\frac{h'}{k'}, \frac{1}{k'}\right), \quad \left(\frac{h + h'}{k + k'}, \frac{1}{k + k'}\right) \quad \right\} \tag{3.16}$$

Such a triangle is drawn in Figure 3-6 as an example. Except the dotted line, the intercept time is $k + k'$ inside and on the edges of this triangle.

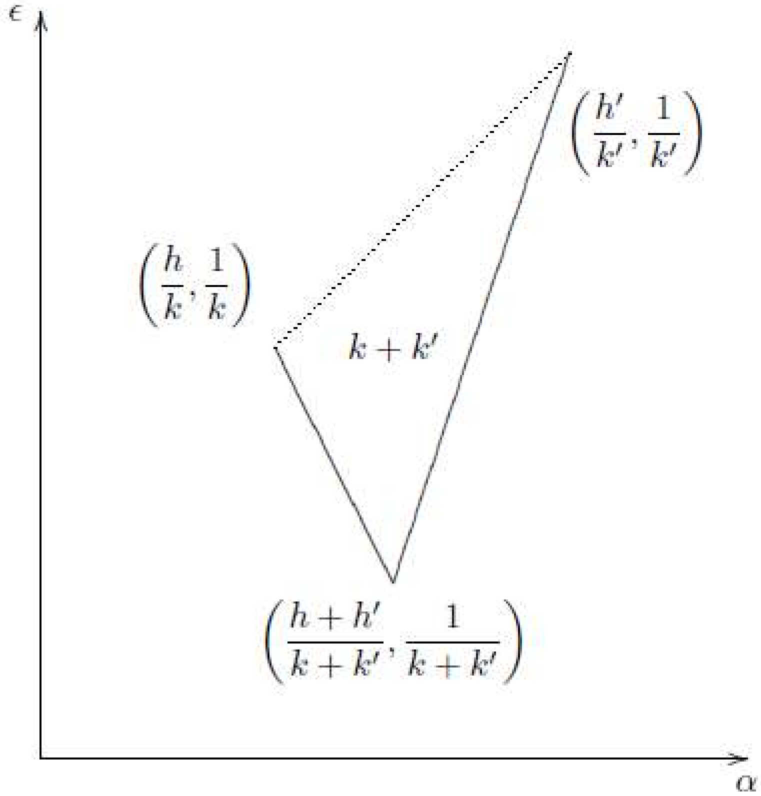

Figure 3-6: A triangle in α - ε plane inside which the intercept time is constant. [4]

The whole α - ε plane can be partitioned by these triangles which show us the value of intercept times depending on α and ε values. The region for which the intercept time is infinite, i.e. synchronization occurs, remains non-partitioned. Since these triangles are not used directly but instead just the idea is used in later chapters, the details of the procedure for this partition will not be given here but can be found in [4]. However, in Figure 3-7 this partitioned α - ε plane can be seen for intercept time up to 7. The numbers in the triangles show the intercept time for any two pulse trains which have the (α, ε) values that are inside the triangle. The graph repeats itself after $\alpha = 1$.

3.3 MIN-MAX INTERCEPT TIME OPTIMIZATION FOR SEARCH STRATEGY

The results of the previous chapter can now be used to develop a search strategy for the receiver. The assumption here made is that we have a priori information about the emitters that are expected to exist in the environment. This assumption is actually realistic, since the parameters of the radars can be extracted by ELINT systems. These parameters can not have always exact values, but their values are usually within a range. For now, let the parameters of the emitters have exact values for simplicity.

One simple example of threat-emitter list is given in Table 3-2 [4], in which there are 3 different bands and there is one emitter for each. The receiver should sweep

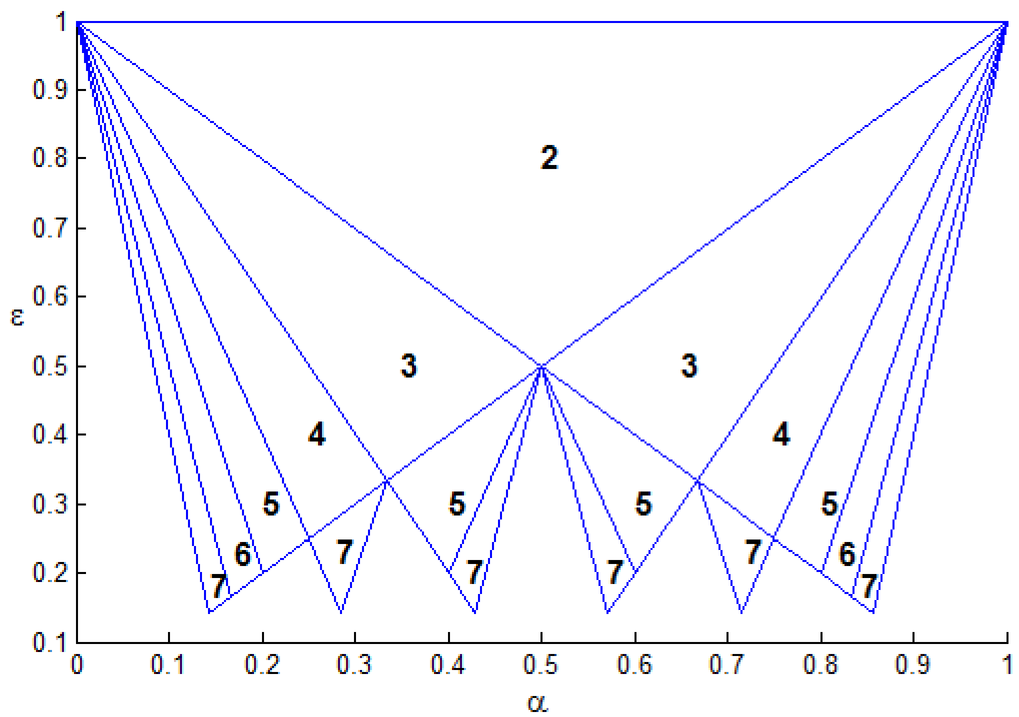

Figure 3-7: α - ε plane partitioned by triangles of constant intercept time

Table 3-2: Example threat-emitter list

Emitter number	Band	Scan period (μs)	PRI (μs)	Beamwidth ($^\circ$)
1	A	8.4×10^6	2.38633×10^3	1.3
2	B	2.97×10^6	1.37792×10^3	2.6
3	C	10.5×10^6	9.38	2.1

periodically these 3 bands and remain to be tuned to the frequency of each band during their respective dwell time. The dwell time together with T_{rcv} will result an intercept time for each emitter of the relevant band, in which at least one coincidence is guaranteed. In this manner, the aim is to find the sweep period of the receiver, T_{rcv}, and the dwell time τ_i for each band, such that the maximum of all the intercept times is minimized.

Due to the hardware and system restrictions, T_{rcv} may have a minimum and maximum acceptable value; we prefer to use $T_{rcv}\_\min$ and $T_{rcv}\_\max$ for these values in the rest of the thesis. Therefore, optimization is performed over $[T_{rcv}\_\min, T_{rcv}\_\max]$. Remember from the Sec. 2.3.5 that there is a minimum acceptable dwell time, since at least some number of pulses of the emitter should be received for interception to be possible. Let 5 be the minimum number of pulses, then, from (2.2) the minimum dwell time for a band is $5 \times \text{PRI}$. Note that if there is more than one emitter in the band, then here PRI actually refers the maximum of all PRIs of the emitters, in order to get a minimum of acceptable dwell time for all emitters in the band. Since in our example of Table 3-2 there is one emitter in each band, minimum dwell time of a band is equal to the PRI of the emitter in the band. Moreover, using (2.1) we can calculate the duration of the illumination of the emitters, τ_{emit}. The values that are discussed so far are added to Table 3-2 to form Table 3-3.

Table 3-3: Example threat-emitter list used in the algorithm

Emitter number	**Band**	T_{emit} (μs)	**min** τ_{rcv} (μs)	τ_{emit} (μs)
1	A	8.4×10^6	11931.65	30333.3333
2	B	2.97×10^6	6889.6	21450
3	C	10.5×10^6	46.9	61250

Obviously, the sum of dwell times of all bands can not exceed the sweep period. So, we can write this constraint of optimization as follows,

$$\sum_{i=1}^{n} \tau_{rcv}(i) \quad \leq \quad T_{rcv} \tag{3.17}$$

where n is the number of bands, and i is the index of the band, i.e. $\tau_{rcv}(1)$ is dwell time for band A, $\tau_{rcv}(2)$ is for band B, etc. It is clear that we want to allocate all the available time in T_{rcv} for dwells, since there is no benefit when the receiver is idle. Thus, the first aim will be to allocate all the time duration in $T_{rcv}\_\min$ to the dwell time of each band. We will refer this as optimization for a fixed sweep period. When all the time duration in $T_{rcv}\_\min$ is filled, it will be necessary to increase the sweep period to create more available time. This process is referred to as optimization over a range of sweep period.

3.3.1 Optimization for a Fixed Sweep Period

In this scenario, T_{rcv} is fixed at $T_{rcv}\_\min$ and only dwell times are varied, so that the maximum intercept time of all emitters is minimized, such that (3.17) is satisfied and dwell times are not less than their minimum acceptable values. Thus, initially we can assign the minimum acceptable values, min τ_{rcv}, to the dwell times. Just at this point it is possible that the sum of dwell times is greater than $T_{rcv}\_\max$, in which case it is not possible to find a feasible solution. Otherwise, optimization is carried out using a simple principle: As a function of dwell time, intercept times with emitters of a particular band is monotonically non-increasing [4]. In other words, if we assign more dwell time to a band, the intercept times for the emitters of that band can not increase. Conversely, if dwell time is reduced for a band, the intercept times for the emitters of that band can not decrease.

Beginning with the initial values of dwell times, optimization progresses iteratively. At each iteration, intercept time with each emitter is calculated using the method in Sec. 3.2.2. Then, for each band, its maximum intercept time is found. Since the aim is to minimize the maximum of all intercept times, optimization focuses on the band which has the maximum intercept time, i.e. the one that includes the emitter with maximum of all intercept times. According to the simple principle that has just been explained, we add some dwell time to the band with maximum intercept time. The amount of the dwell time to be added can be found by using the α-ε plane in Figure 3-7. For the emitter with maximum intercept time, α and ε are calculated by using (3.3) and (3.4). This (α, ε) point will belong to a triangle in α-ε plane, or it will be in a non-partitioned region, if the intercept time for this emitter is infinite. Then, while α is kept constant – since T_{rcv} is constant –, ε is increased until when (α, ε) point reaches to the upper triangle which represents a lower intercept time. This point can be found by the intersection of two lines, i.e. $\alpha = T_{rcv}$ line and the edge of upper triangle. However, in our simulations, the amount of dwell time to be added is found iteratively, i.e. by starting with some amount of dwell time and then increasing or decreasing it depending on whether the intercept time becomes worse (greater) or better (lower) than the current one. In this manner, the smallest dwell time to be added is found. If this extra amount of dwell does not cause (3.17) to fail, it is added to the related band. Now, intercept times of the emitters of that band may be changed, so they are recalculated. Since in this part of the optimization T_{rcv} is fixed at $T_{rcv}\_\min$, there will be no change in other bands. At this point, the first iteration is completed. For the next one, the band with the maximum intercept time is found again, since now it may have been changed. Then, the same steps are repeated for that band. Iterations will continue in the same way until when there is not enough available time left to be added to the dwell time of any band. The iteration steps are listed below:

1. All of the intercept times are calculated using the method in Sec. 3.2.2.
2. Maximum intercept time is decided for each band and the band with maximum intercept time between all bands is found. Choose it arbitrarily if there is a tie.
3. The smallest dwell time to be added to the band with maximum intercept time to decrease its intercept time is calculated by using α - ε plane as shown in Figure 3-7 or iteratively.
4. If there is available time in $T_{rcv} = T_{rcv}\_\min$, i.e. $\sum_{i=1}^{n} \tau_{rcv}(i) +$ extra dwell found in 3 $\leq$ T_{rcv}, this dwell time is added to the band with maximum intercept time, so that its intercept time is decreased.
5. Otherwise, try 3-4 with other bands with next maximum intercept time until 4 is applicable for a band.
6. If 4 or 5 is completed, return to 1 and repeat all the steps, otherwise optimization for a fixed sweep period is completed, since there is no more available time in T_{rcv}.

Now we can see the procedure with an example, again using the emitters of Table 3-3. Assume that $T_{rcv}\_\min = 1s = 1e6\mu s$, so that throughout all iterations T_{rcv} is fixed at $T_{rcv}\_\min$. At the end of the iterations, optimum dwell times for each band will be found such that maximum intercept time is minimized and (3.17) is satisfied. We can see the results of iterations in Table 3-4.

In iteration 0 we start with the minimum dwell times from Table 3-3 and we see that this results in infinite intercept time for Band A and Band C, because of synchronization. Thus, Band A is arbitrarily chosen and in the next iteration more dwell time is added to Band A, which reduces its maximum intercept time to 42 looks, i.e. only after $42 \times T_{rcv}\ \mu s$ it is guaranteed to intercept with all the emitters of Band A. Notice that dwell time of Band A is increased to 193530 μs. In the next iteration we will try to add more dwell time to Band C, which has the maximum intercept time now. We see that its intercept time is smaller than infinity only when its dwell time increased to 438844 μs. With these dwell times we see that there is 355296 μs left in $T_{rcv} = 1e6\ \mu s$ which can be used in the next iterations. In the rest of the iterations, this available time is used for Band B to reduce its intercept time to 65 looks. Finally there is no more available time to reduce intercept time of any band and optimization for a fixed sweep period is completed.

Table 3-4: Iteration results of optimization for a fixed sweep period

# of iteration	**Band A**		**Band B**		**Band C**		**available time**
	# of looks	**dwell (μs)**	**# of looks**	**dwell (μs)**	**# of looks**	**dwell (μs)**	$T_{rcv} - \sum_{i=1}^{n} \tau_{rcv}(i)$
0)	**Inf**	**11932**	297	6890	Inf	47	981131
1)	42	193530	297	6890	**Inf**	**47**	799533
2)	42	193530	**297**	**6890**	21	438844	360736
3)	42	193530	**199**	**12330**	21	438844	355296
4)	42	193530	**101**	**22330**	21	438844	345296
5)	42	193530	**98**	**32330**	21	438844	335296
6)	42	193530	**95**	**62330**	21	438844	305296
…	…	…	**…**	**…**	…	…	…
17)	42	193530	65	362330	21	438844	**5296**

Intercept times throughout all iterations for all bands can be seen in Figure 3-8 below. It is actually the graph of the results of Table 3-4. We can see that at the beginning Band A and Band C have infinite intercept times and they are reduced in the first iterations. After second iteration, however, we see how intercept time of Band B reduces, since it remains as the band with maximum intercept time until the end of optimization.

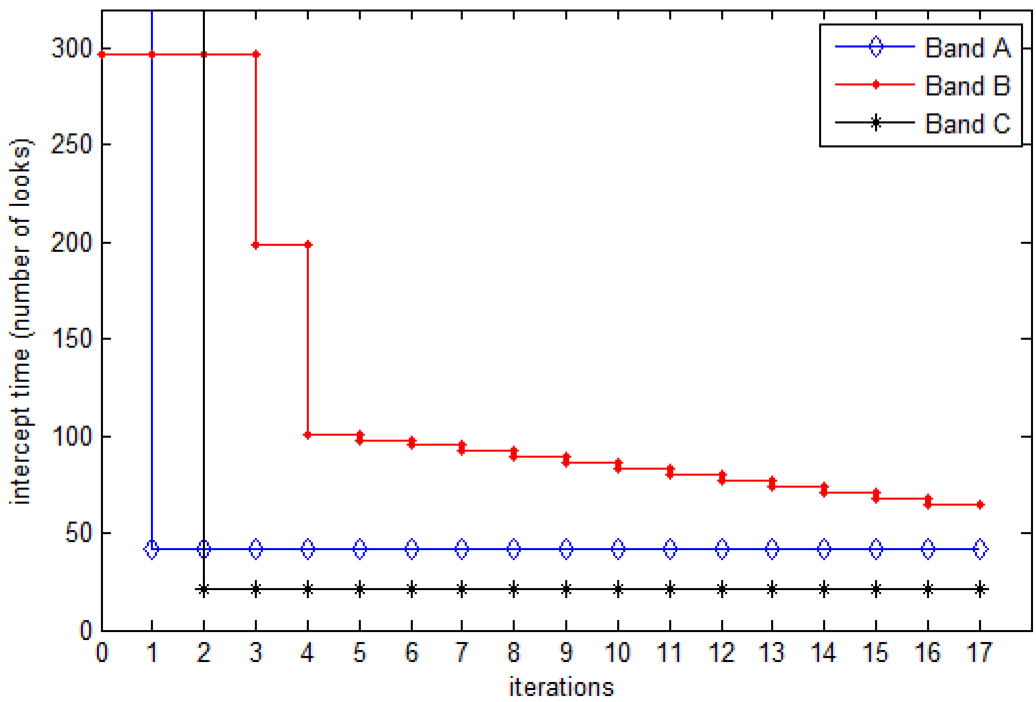

Figure 3-8: Optimization iterations with fixed sweep period T_{rcv}

3.3.2 Optimization over a Range of Sweep Period

In the previous section during optimization process we had fixed T_{rcv} at $T_{rcv}\_\min$ and we varied only dwell times. In this section, we will allow to vary T_{rcv} (between $T_{rcv}\_\min$ and $T_{rcv}\_\max$) and dwell times simultaneously [4]. The logic is again the same; at each iteration maximum intercept time is found and it is reduced by adding more dwell to that band. But, now there is not enough available time in sweep period, so we have to increase T_{rcv} if we want to add more dwell to a band. However, T_{rcv} is a common parameter in calculating intercept time for every emitter of any band and we have to be careful, since increasing T_{rcv} alters the situation for every emitter. Therefore, while reducing maximum intercept time with a particular emitter in its band, we have to be sure that we are not increasing intercept times with other emitters. To do this, we can use the information contained in the α - ε

plane shown in Figure 3-7, which plays an important role here. From that figure we see how intercept time changes if we increase α, therefore T_{rcv}. For example, we increase T_{rcv} by some number, let us call x. Calling the candidate of new sweep period as $T*_{rcv}$,

$$T*_{rcv} = T_{rcv} + x \tag{3.18}$$

Also we had some remaining time from optimization in previous part, and call this time as w. This means that now we have a total of $x + w \ \mu s$ to add to a dwell; which can be named as *idle time*, i.e. *idle time* = $x + w$. Now, return to Figure 3-7. Since we have increased α, obviously intercept times with emitters are affected; while some of them remain same, some of them may increase or decrease. Remember that the aim is to create more available time as much as possible to add to a dwell of the band with maximum intercept time. Therefore, we have to seek the smallest dwell times for each band, which reduces the maximum intercept time, whereas other intercept times are at worst kept the same. In other words, maximum intercept time must reduce, while other intercept times must not increase. To make it clear, let us explain it numerically. Let $\tau*_{rcv}(i)$ be the candidate of new dwell time for band i. Moreover, let $t\_\mathrm{int}(i, j)$ be the intercept time with j^{th} emitter of band i, calculated with current parameters T_{rcv} and $\tau_{rcv}(i)$, and similarly $t\_\mathrm{int}*(i, j)$ be the intercept time with that emitter, calculated with candidate parameters, $T*_{rcv}$ and $\tau*_{rcv}(i)$. Then, at the end of the iteration T_{rcv} is increased to $T*_{rcv}$ if the following conditions are true:

$$\sum_{i=1}^{n} \tau*_{rcv}(i) \quad \leq \quad T*_{rcv} \tag{3.19}$$

$$\max(t\_\mathrm{int}*(i, j)) \leq \max(t\_\mathrm{int}(i, j)), \quad \text{for } \forall(i, j),\ i \neq i\_\max \tag{3.20}$$

$$\max(t\_\mathrm{int}*(i\_\max, j)) < \max(t\_\mathrm{int}(i\_\max, j)), \quad \text{for } \forall j \tag{3.21}$$

where i represents the band, $i\_\max$ represents the band with maximum intercept time, and j refers to a particular emitter in band i.

The parameters T^*_{rcv} and $\tau^*_{rcv}(i)$ can be found by analytical calculations [4] using the triangles of α-ε plane as in Figure 3-7. However, in our implementations they will be found with another method, iteratively. T_{rcv} is increased to T^*_{rcv} step by step by an amount of time, x, which is actually the smallest resolution for the receiver, i.e. for the receiver it is not possible to schedule a dwell below that time because of hardware restrictions. At each step, by using the same procedure given in the previous section, the smallest dwell times for each band is calculated, such that they satisfy (3.20) and (3.21). Moreover, if (3.19) is also satisfied, T^*_{rcv} and $\tau^*_{rcv}(i)$ are found. Then, T_{rcv} is increased again by an amount x, and a better solution (the one for which maximum intercept time is lower than the last one) is searched again. When T_{rcv} exceeds $T_{rcv}\_\max$, optimization is completed and the optimum solution is the last solution. To make it clear, the algorithm is summarized below:

0. Start with dwell times found after previous optimization and with $T_{rcv}\_\min$. Dwell times and T_{rcv} are varied until $T_{rcv}\_\max$ to find a better solution, such that maximum intercept time is further decreased.
1. Increase T_{rcv} by an amount of x, to get $T^*_{rcv} = T_{rcv} + kx$, where k is the number of iterations, which is initially 1, and increments by 1 at the beginning of new iteration. T^*_{rcv} is the candidate to be the new sweep period.
2. Calculate all intercept times using the method in Sec. 3.2.2.
3. Decide maximum intercept time for each band, and find the band which has the maximum intercept time between all bands. Choose one of them arbitrarily if there is a tie.

4. Using T^*_{rcv}, find smallest dwell times which satisfy (3.20) and (3.21), using α - ε plane or iteratively. These dwell times, $\tau^*_{rcv}(i)$, are candidates to be new dwell times.
5. If $\tau^*_{rcv}(i)$ together with T^*_{rcv} satisfy (3.19), a better solution is found, since maximum intercept time is reduced. Thus, T^*_{rcv} and $\tau^*_{rcv}(i)$ are the parameters of the new solution. Update $T_{rcv} = T^*_{rcv}$ and $\tau_{rcv}(i) = \tau^*_{rcv}(i)$. Otherwise, a better solution can not be found with T^*_{rcv}, since available time in T^*_{rcv} is not enough to compensate the increase in sum of dwell times. T_{rcv} and $\tau_{rcv}(i)$ remain unchanged.
6. If T^*_{rcv} is equal to $T_{rcv}\_\max$, no further optimization is possible. T_{rcv} and $\tau_{rcv}(i)$ give the best solution between $T_{rcv}\_\min$ and $T_{rcv}\_\max$, so that maximum intercept time is minimized. Otherwise, go to 1 and repeat all the steps.

Table 3-5 together with Figure 3-9 shows iteration results for the emitters of Table 3-3 as a continuation to optimization for a fixed sweep period in the previous section. Assume that $T_{rcv}\_\min = 1000000\mu s$ and $T_{rcv}\_\max = 1050000\mu s$. Optimization starts with $T_{rcv} = T_{rcv}\_\min$ and dwell times are shown in Table 3-5 at 0^{th} iteration, which was the result of previous section. T_{rcv} is increased step by step with a resolution, $1\,\mu s$ in this example, and when T_{rcv} reaches to 1000772 μs, a new solution is found such that (3.19), (3.20), and (3.21) are satisfied. Maximum intercept time is reduced from 65 to 62 looks, while intercept times for Band A and Band C are not increased. Notice that there was need to add more dwell time to Band A and Band C in order to not increase their intercept times, while for Band B dwell time could be reduced to get a lower intercept time. Also note that these are smallest dwell times that give those intercept times, which ensures (3.19). Rest of the iterations continues similarly with an exception of 6. In that iteration we see that intercept time of both Band A and Band B is reduced. Actually, the iteration is taken just for Band B since it has maximum intercept time, 50 looks. However, remember that it is necessary to hold intercept time of other bands at most at their last values (for Band A this value is 42 looks in this case) to satisfy (3.20). For Band A, this is satisfied only when its dwell is $322154\,\mu s$, which yields the intercept time as 33 looks. There is no smaller dwell time which gives an intercept time not greater than or equal to 42 looks. For example, $322153\,\mu s$ results in 58 looks. Thus, $322154\,\mu s$ is the dwell time of Band A for the solution at 6th iteration and so intercept time of Band A is also decreased, although this was not the aim of this iteration.

When $T*_{rcv}$ exceeds $T_{rcv}\_\max = 1050000\mu s$, it can not be increased anymore and optimization is terminated. The solution is the last one, which is found at T_{rcv} = $1016491\,\mu s$ from Table 3-5. This is the optimum solution which ensures that for emitters of Table 3-3, maximum intercept time is minimized for T_{rcv} between $1000000\,\mu s$ and $1050000\,\mu s$ together with dwell times listed at 10th iteration of Table 3-5. Trace of intercept times for all bands can be seen graphically in Figure 3-9.

Table 3-5: Iteration results of optimization over a range of sweep period T_{rcv} between 1s and 1.05s

# of iteration	**Band A**		**Band B**		**Band C**		T_{rcv} (μs)
	# of looks	**dwell** (μs)	**# of looks**	**dwell** (μs)	**# of looks**	**dwell** (μs)	
0)	42	193530	65	362330	21	438844	1000000
1)	42	206654	62	346782	21	447336	1000772
2)	42	225609	59	316658	21	459601	1001887
3)	42	248916	56	279656	21	474682	1003258
4)	42	278326	53	232930	21	493712	1004988
5)	42	316542	50	172238	21	518440	1007236
6)	33	322154	47	149762	21	536986	1008922
7)	33	310506	44	146832	21	553002	1010378
8)	33	296922	41	143442	21	571680	1012076
9)	33	280874	38	139460	21	593746	1014082
10)	33	261602	35	134618	21	620245	1016491

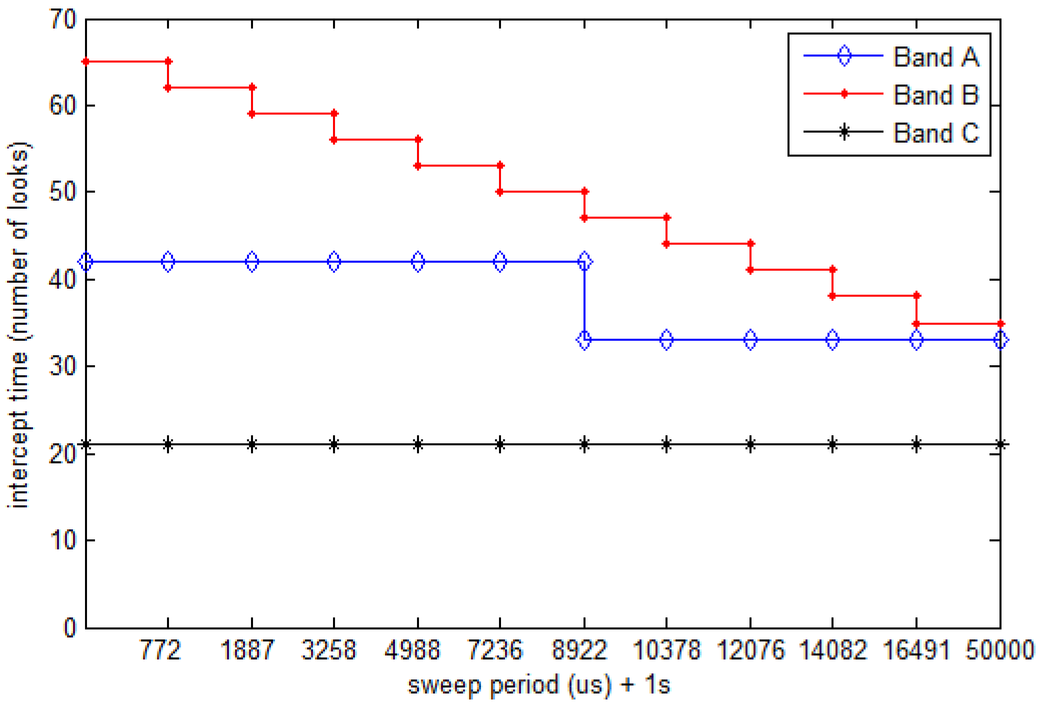

Figure 3-9: Optimization iterations with variable sweep period T_{rcv}

CHAPTER 4

TEST DATA GENERATION AND SIMULATIONS OF SEARCH STRATEGY OPTIMIZATION ALGORITHM

In this chapter, simulation results of the search strategy algorithm proposed in the previous chapter are included to find out the cases for which this search strategy works well and the cases for which the algorithm is not useful in practical applications.

Actually, it is not easy to propose generalized results valid for all similar cases, due to the diversity of radar types and their wide range of parameters. Moreover, if we remember that search strategy works on threat-emitter lists, in other words, on a group of radars, we notice that there can be infinitely many combinations of lists to be tested. It seems that this is why Clarkson did not include detailed analysis on the strategy, as he admitted in [4], but just a few restricted results.

Despite this difficulty, there is need to know the borders of the algorithm for practical purposes and even to propose new approaches on the search strategy, where the algorithm is insufficient. In fact, this and the following chapters fulfill this goal, covering the deficiency of analysis in the papers of Clarkson.

To analyze the algorithm, first of all, the parameters are determined for which the performance of the algorithm would be observed. The choice of these parameters is not complicated, if we pre-analyze the algorithm by deciding what is expected from it as an outcome. In this way, 4 parameters are found to test the search strategy:

Number of bands (to be scanned by the receiver), duty cycle of radars, PRI of radar signals, diversity of period and duty cycle of radars.

For simulations, Monte Carlo method is used. In the test employing each parameter, the range of the parameter is varied, while other parameters are hold within a constant and "easy" range. With "easy" range we mean a range of values which decrease the work load of the algorithm, i.e. the complexity of the problem. For example, when we test the algorithm for the number of bands, PRI of radar signals is chosen from a lower range of values, so that a coincidence between two pulse trains can be more easily found, since the minimum duration of a valid coincidence would be lower (as explained in Sec. 2.3.5). Thus, the effects of the other parameters can be either eliminated or controlled and the dependence of the results to the parameter which is being examined can be seen more easily.

The value of the parameter under focus is changed according to the specified values of the test cases. For each step, 100 tests are performed, with 100 different threat-emitter lists that include emitters with an appropriate range of parameters. The values of the parameters are chosen randomly from a uniform distribution over their ranges depending on the test case. To produce these random threat-emitter lists, a GUI is created with MATLAB. For parameters, the range of values is chosen realistically and accordingly simulations are carried out for a minimum sweep period of $1000\,ms$ and $1100\,ms$, i.e. the best search strategy is searched for a sweep period between these values. Finally, the average time, minimum, maximum, and standard deviation of sweep periods found as a result of the algorithm are calculated over 100 threat-emitter lists and analyzed. For the 4 parameters, at least 3 test cases for each, 100 simulations for each case, in other words, more than one thousand simulations are done in this work.

The range of values used in simulations can be seen below in Table 4-1. Notice that these ranges are minimum - maximum values considered throughout simulations, but in test cases restricted, particular ranges are used to test the algorithm.

Table 4-1: General range of values used throughout simulations

Number of bands	**Duty cycle of radars (%)**	**PRI of radar signals (ms)**	**Scan Period of radars (s)**
3 – 25	0.2 – 1.5	0.1 – 10	3 – 50

4.1 NUMBER OF BANDS

Since a super heterodyne receiver will find limited time to be tuned to a particular frequency when there are several frequency bands, it is obvious that an increase in the number of frequency bands to be scanned is a challenge for the algorithm. With more frequency bands, it is expected that maximum intercept time increases. If the number of bands is increased further, it will be even impossible to find a feasible solution, since the maximum possible time for receiver scan is not sufficient to cover all the bands.

In the search strategy proposed in the previous chapter, when there is more than one emitter in a band in threat-emitter list, maximum or minimum (depending on the parameter) parameter of all emitters in that band is taken and the algorithm processes as if there is one emitter with these parameters. Because of this, assuming that there is more than one emitter in a band does not change the process of the algorithm. Thus, in simulations it is assumed that there is one emitter in each band, for simplicity.

In order to see the effects of number of bands on algorithm performance, 4 test cases are created, each of which contains 100 threat-emitter lists including random emitters uniformly chosen depending on the range of values of the parameters. In each case, the number of bands is increased, while holding the values of other parameters within a constant range throughout all cases. To concentrate on the effects of number of bands, the range of values of the other parameters are chosen to lower the complexity of problem, as explained before. The range of values of the parameters used for this section can be seen below in Table 4-2.

Table 4-2: Range of values for test cases of the part 4.1

Test	Number of bands	Duty cycle of radars (%)	PRI of radar signals (μs)	Scan Period of radars (s)
Case 1.1	3	1 – 1.5	100 – 500	3 – 10
Case 1.2	8	1 – 1.5	100 – 500	3 – 10
Case 1.3	15	1 – 1.5	100 – 500	3 – 10
Case 1.4	25	1 – 1.5	100 – 500	3 – 10

We start to test the algorithm with 3 different bands to be scanned by the receiver, which is followed by the test case 1.2 with 8 bands. In fact, 3 or 8 bands can be thought rather small compared to a realistic application, but they are suitable to see the algorithm performance gradually. 15 different frequency bands are included in the threat-emitter lists for the test case 1.3, which is thought to be more difficult for the algorithm. The test case 1.4 with 25 emitters is thought to be a really difficult and time-wasting case for the algorithm.

With these test cases it is expected that in case 1.3, on the average maximum intercept time would be greater than in cases 1.1 and 1.2, and would reach at its maximum in the test case 1.4. Similarly, case 1.1 should have the lowest maximum intercept time. The results are shown in the next chapter.

4.2 DUTY CYCLE OF RADARS

Duty cycle of radars is thought to be another parameter that will directly affect the performance of the algorithm. Remember from Chapter 1 that in a pulse train duty cycle is defined as the ratio of pulse width to the period. Thus, the duty cycle of radar determines how much time the radar signal will be available in one period of radar signal. This means that together with the scan period of radar, duty cycle determines the pulse width of the pulse train of radar. For example, when the scan period is 10 seconds, 1% duty cycle means that in every 10 seconds there will be $100\,ms$ of available radar signal to be intercepted by the receiver. It should be noticed that because of noise and sensibility of the receiver, not all of the beam of the radar signal would be available at the receiver; but we can assume that the given duty cycle is calculated after considering this and thus all of the beam is thought to be available at the receiver.

Similar to the previous test, 100 random threat-emitter lists are created for each test case, but this time with the range of values of parameters given below in Table 4-3.

Table 4-3: Range of values for test cases of the part 4.2

Test	Number of bands	Duty cycle of radars (%)	PRI of radar signals (µs)	Scan Period of radars (s)
Case 2.1	5	1.2 – 1.5	100 – 500	3 – 10
Case 2.2	5	0.8 – 1.2	100 – 500	3 – 10
Case 2.3	5	0.3 – 0.6	100 – 500	3 – 10
Case 2.4	5	0.2 – 0.3	100 – 500	3 – 10

For simplicity, number of bands is fixed as 5 throughout all the test cases. Similarly, PRI of radar signals and scan period of radars are chosen from a range of lower values. On the other hand, in case 2.1 duty cycle of the radars is higher, while it is decreased in the cases 2.2 and 2.3, and even further decreased in the case 2.4.

Since the duty cycle of radars determines the pulse width of the pulse trains of our model, with a higher value for the duty cycle, it should more likely to have coincidences between two pulse trains. With lower duty cycles, however, radar signals available at the receiver would be decreased and so the coincidences would be sparser. Thus, with these test cases it is expected to see that average maximum intercept time would be smaller when duty cycle has a greater value and that it would increase when duty cycle is decreased.

4.3 PRI OF RADAR SIGNALS

PRI of radar signals does not affect the pulse train of our model, but it changes the minimum duration of a valid coincidence between two pulse trains. From (2.2) we see that minimum duration increases linearly with increase in PRI. When minimum duration of a valid coincidence increases, the coincidences with duration below that increased minimum duration but over the older (non-increased) minimum duration are now considered as invalid, although earlier they were obviously valid. This tells us that when PRI increases, number of valid coincidences decrease. Therefore, the time to wait for a first valid coincidence, in other words maximum intercept time, increases.

As seen in Table 4-4 below, the range of values of PRI is increased from 100 – 500 μs in the test case 3.1 to 6000 – 10000 μs in the case 3.3, while holding number of frequency bands at 5, duty cycle at 1% – 1.5% and scan period between $3\,s - 10\,s$.

Table 4-4: Range of values for test cases of the part 4.3

Test	Number of bands	Duty cycle of radars (%)	PRI of radar signals (µs)	Scan Period of radars (s)
Case 3.1	5	1 – 1.5	100 – 500	3 – 10
Case 3.2	5	1 – 1.5	2000 – 5000	3 – 10
Case 3.3	5	1 – 1.5	6000 – 10000	3 – 10

Through the test cases of this section, we should see that optimization in the search strategy will be more and more difficult with increasing PRI in the cases 3.2 and 3.3, because of the decrease in valid coincidences. For this reason, PRI is an important parameter to see the borders of the algorithm.

4.4 DIVERSITY OF SCAN PERIOD AND DUTY CYCLE

This test differs from the previous ones for its purpose. It is not focused on a parameter change, but on diversity of parameters (scan period and duty cycle). By testing the algorithm against threat-emitter lists including random emitters with a wide range of values of scan periods and duty cycles, we aim to see how the performance of the algorithm responds from one threat-emitter list to another with random, mixed emitters.

It should be remembered that in the previous tests, the range of values of one parameter changes from a test case to the other, while other parameters are kept fixed within a restricted range of values. In this way, in one particular test case, within a threat-emitter list the emitters have parameters with values that are close to each other. Therefore, although results may change from one test case to the other, this change may lie in a limited proportion. This could prevent us to see the tolerance of the algorithm to the change in threat-emitter list, i.e. dependency on the threat-emitter lists. For example, consider a threat-emitter list that consists of N emitters with scan periods about 3 seconds, with 1% duty cycle for each. When we add one more emitter to this threat-emitter list, but now with much a greater scan period, say 30 seconds, again with 1% duty cycle, the result of the algorithm may be very different compared to the older one. Because, the last emitter would have a much smaller PW in its pulse train and because of this in most of the steps of the algorithm, probably it would give maximum intercept time; thus it would dominate the optimization steps and most of the dwell time would be spent for that last emitter. Thus, since in this test threat-emitter lists include diverse emitters, this test is the one to see the effects of diversity of values of parameters on the algorithm performance.

As this test measures the reliability of the algorithm against diverse threat-emitter lists, it would be reasonable to look at the standard deviation of all the maximum intercept times as the result of the search strategy algorithm. Standard deviation for the cases in which the range of values of scan period and duty cycle of radar gradually increase will show us how the result of algorithm changes from one threat-emitter list to the other.

In order to achieve this purpose, 100 threat-emitter lists are created for each case as shown in Table 4-5. Again, 5 different bands are used for simplicity. In case 4.1, duty cycle and scan period of radars are restricted in a narrow range of values; 1.2% – 1.5%, and $3s - 10s$, respectively. Actually, this test case is the same with the case 2.1 of duty cycle of radars in Sec. 4.2. In the case 4.2, these ranges are wider, and in the case 4.3, duty cycles and scan periods are chosen between 0.3% – 1.5% and $3s - 50s$, respectively. With these cases, it is expected that in the case 4.3, standard deviation of maximum intercept times is higher, but more importantly, we will see the reliability of the algorithm.

Table 4-5: Range of values for test cases of the part 4.4

Test	Number of bands	Duty cycle of radars (%)	PRI of radar signals (µs)	Scan Period of radars (s)
Case 4.1	5	1.2 – 1.5	100 – 500	3 – 10
Case 4.2	5	0.8 – 1.5	100 – 500	3 – 25
Case 4.3	5	0.3 – 1.5	100 – 500	3 – 50

CHAPTER 5

RESULTS FOR SEARCH STRATEGY OPTIMIZATION ALGORITHM

In the previous chapter, we have defined test cases with particular parameters to see the performance and limits of the search strategy algorithm of Clarkson. For each test case, 100 different threat-emitter lists were created with the values of parameters particularly restricted for each case. For each threat-emitter list, the search strategy algorithm is run and the result, optimized minimum of maximum intercept time, is found.

The results are examined with respect to two main aspects. One of these aspects is comparing the result of the algorithm with the maximum intercept time of simple search. In simple search, all of the available time of the receiver is shared equally by each band, i.e. the dwell time for each band is equal. This approach does not need any *a priori* knowledge about the environment, such as threat-emitter list, but does not consider synchronization problem neither. Therefore, this comparison clarifies how far the strategic search algorithm minimizes the maximum intercept time and eliminates the synchronization problem. The comparisons are illustrated via the figures with the results of simple and strategic search.

The other aspect of the analysis of results is to show the borders of the algorithm, i.e. how the performance of the algorithm is affected by the change in parameters. For this purpose, in the previous chapter some test cases have been created, such that a particular parameter is changed gradually, while other parameters are kept within a range. Also, some anticipation has been made about the results. The validity of these anticipations is demonstrated in this chapter. To this end, output of the algorithm (minimums of maximum intercept times) are shown for each test case on the same graph (remember that there are N tries within each case).

Analysis of the results is given in four sections below with respect to the test cases.

5.1 NUMBER OF BANDS

In this test, to see the effect of number of bands on the performance of algorithm, number of bands is increased in each test case as seen in Table 4-2. The comparisons between the results and simple search are given in Figures 5-1 – 5-4.

As seen in Figure 5-1, when the number of bands is very small, the problem is relatively simple, so that even with simple search, for many of the threat-emitter lists, the maximum intercept time is very close to the one of strategic search. Nevertheless, there are some cases, in which for the simple search maximum intercept time is much larger, and there may be also synchronization problem (seen in the case where the maximum intercept time goes to infinity). The strategic search algorithm, however, can easily solve these synchronization problems and makes the larger maximum intercept times as small as the other cases successfully.

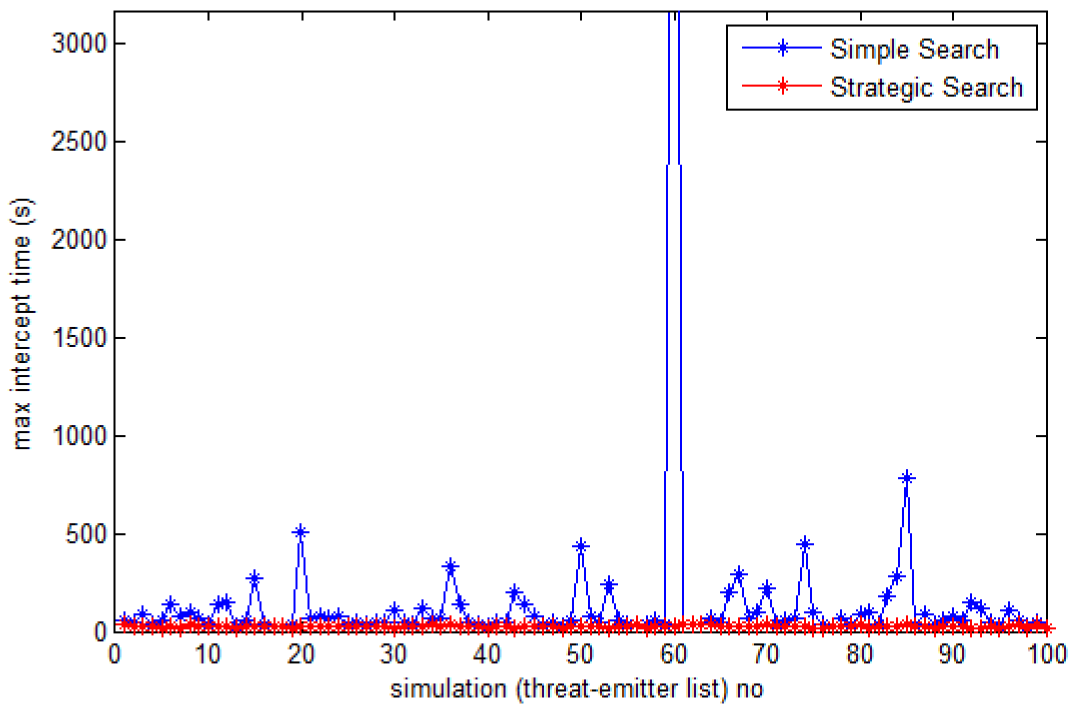

Figure 5-1: Comparison of simple search and test case 1.1 results

From Figure 5-2 we see that the simple search results in much larger maximum intercept time and more synchronization. The strategic search algorithm is again successful in decreasing the maximum intercept time.

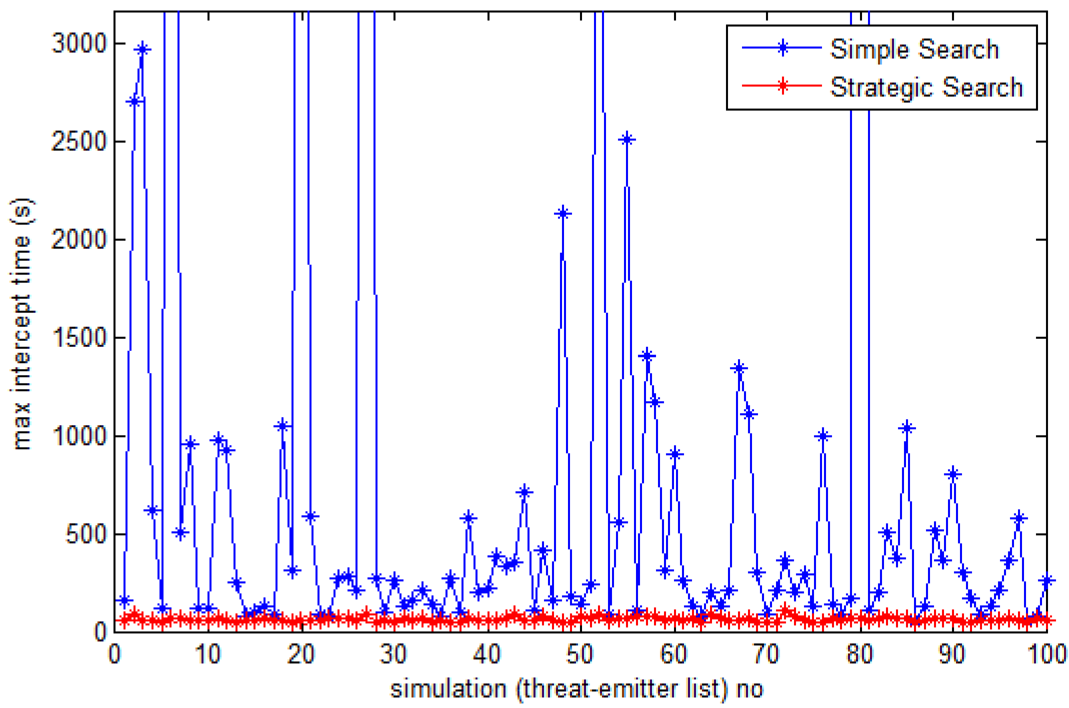

Figure 5-2: Comparison of simple search and test case 1.2 results

When the number of bands is further increased, the simple search becomes ineffective, as seen in Figure 5-3. With the most of the threat-emitter lists, the maximum intercept time goes to infinity (because of the synchronization problem), so that by applying simple search, the receiver would not be able to intercept all the radars in the threat-emitter lists. In the same figure it is seen that the strategic search algorithm is able to eliminate synchronization problem and minimized the maximum intercept time.

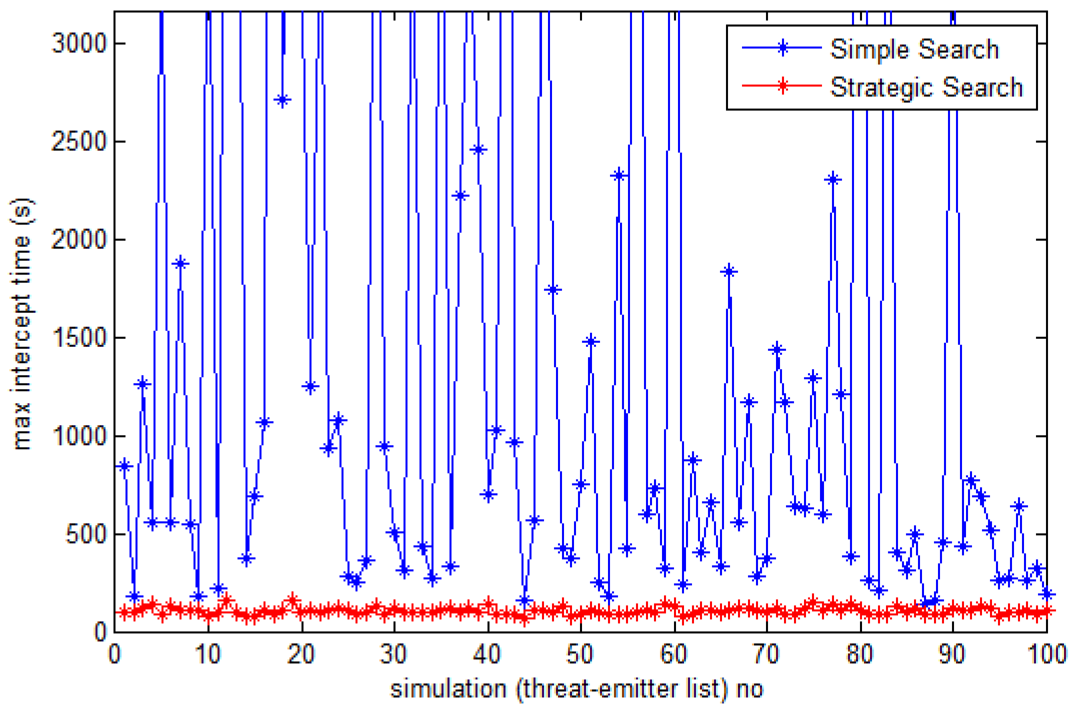

Figure 5-3: Comparison of simple search and test case 1.3 results

In the last test case, the number of bands is further increased to 25, which is actually a more realistic case, since the radars function in a much wider spectrum and the receiver should scan all the spectrum of its threats. As shown in Figure 5-4, simple search is as ineffective as in the previous case. Moreover, although the strategic search minimizes the maximum intercept time, there are some threat-emitter lists for which the maximum intercept time can not be decreased to acceptable values.

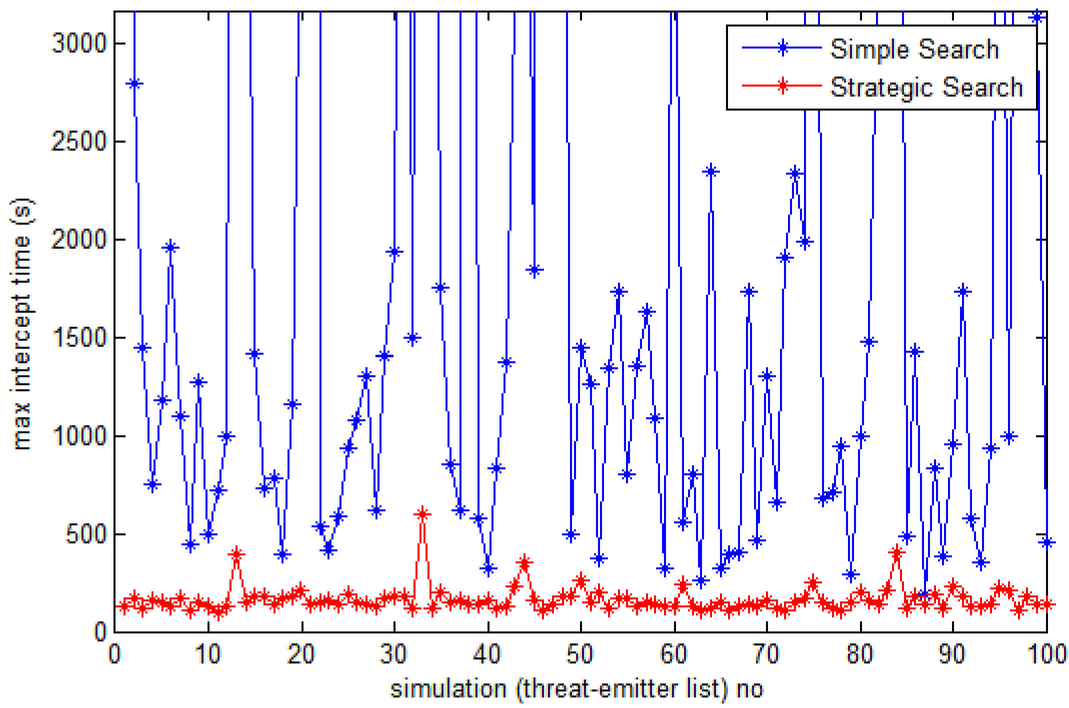

Figure 5-4: Comparison of simple search and test case 1.4 results

In Figure 5-5**,** the results of the algorithm are shown for all the cases. As we see, the minimized maximum of intercept time is increasing by increasing the number of bands. Moreover, for the case 1.4, with the maximum number of bands, we see that for some threat-emitter lists, the minimized maximum intercept time takes values greater than the average.

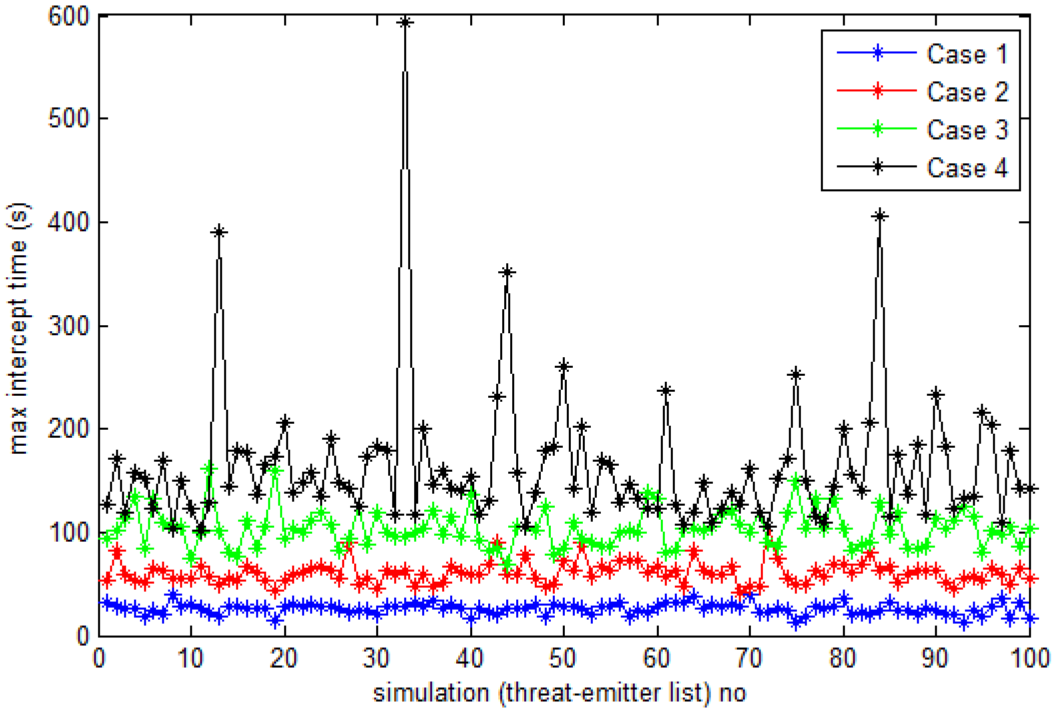

Figure 5-5: Results of all test cases together for number of bands

We can see this result numerically from Table 5-1. While with 3 bands, the average of the results is about 25 seconds, this gradually increases, and with 25 bands it becomes about 163 seconds. Furthermore, the minimum and the maximum values of the results are 100 and 592 s for 25 bands, while these values are 12 and 40 s for 3 bands. The standard deviation of the results show us that the strategic search algorithm becomes more unreliable when the number of bands increase, since in the first three cases the standard deviation changes between 5 and 18 seconds, while in the case 1.4 there is a rapid increase and it becomes 68 seconds. When the number of bands increases, the available time that the receiver can dedicate for a particular band decreases. Thus, the strategic search algorithm begins to become incapable to eliminate the synchronization problem.

Table 5-1: Results of test case for number of bands

Test	Minimum of Max Intercept Time (s)	Maximum of Max Intercept Time (s)	Average of Max Intercept Time (s)	Standard Deviation of Max Intercept Time (s)
Case 1.1	12.016	40.118	25.887	5.233
Case 1.2	42.236	105.276	60.780	10.834
Case 1.3	69.000	162.344	103.685	18.191
Case 1.4	100.884	592.350	163.443	67.889

5.2 DUTY CYCLE OF RADARS

From the previous chapter remember that in this test the values of Table 4-3 are used in which duty cycle of radars is gradually decreased. The comparisons between the results of the algorithm and the simple search are illustrated in Figure 5-6 - 5-9. As seen from these figures, in all the cases, the algorithm has superiority over simple search. Decreasing the duty cycle of radars did not dramatically lower the performance of the algorithm.

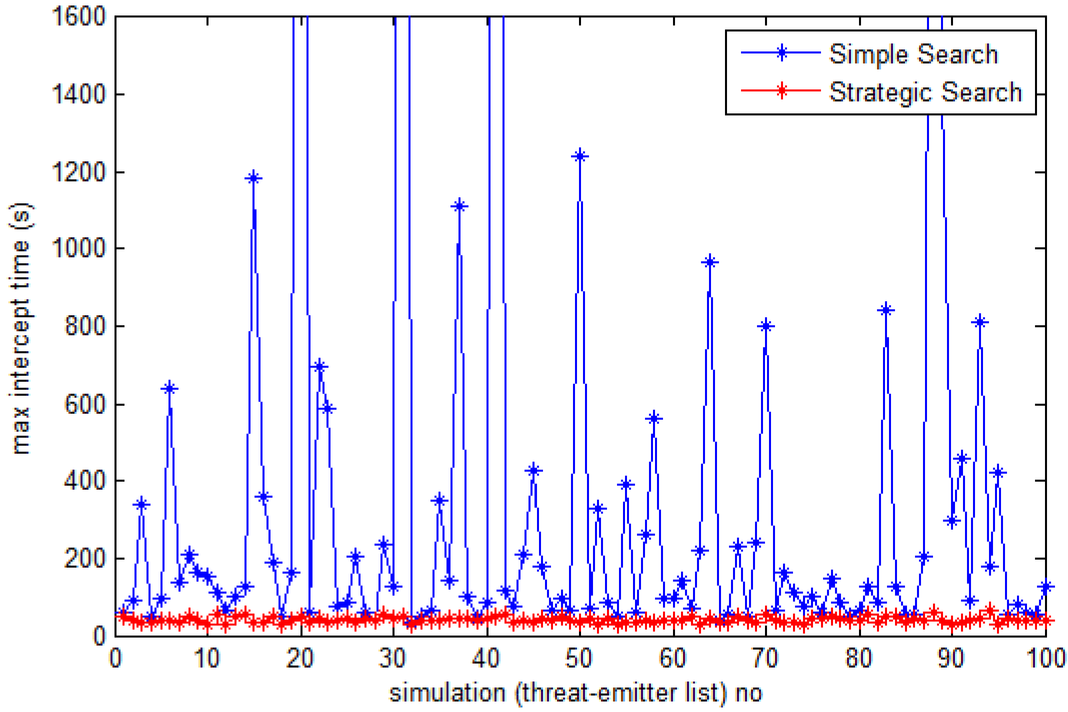

Figure 5-6: Comparison of simple search and test case 2.1 results

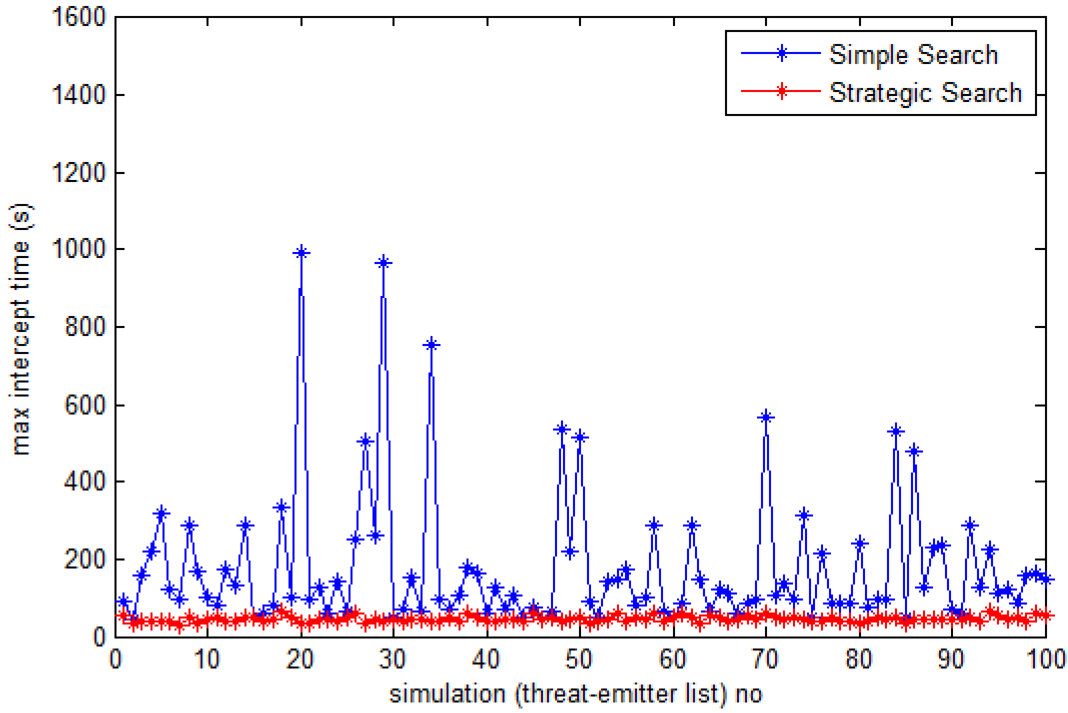

Figure 5-7: Comparison of simple search and test case 2.2 results

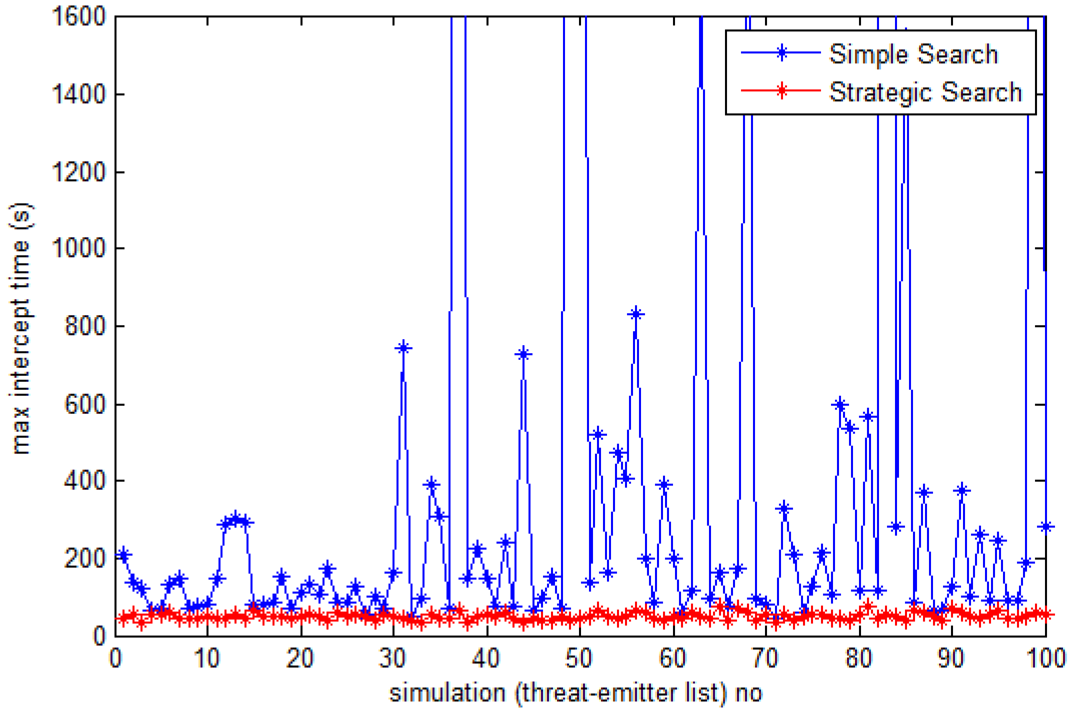

Figure 5-8: Comparison of simple search and test case 2.3 results

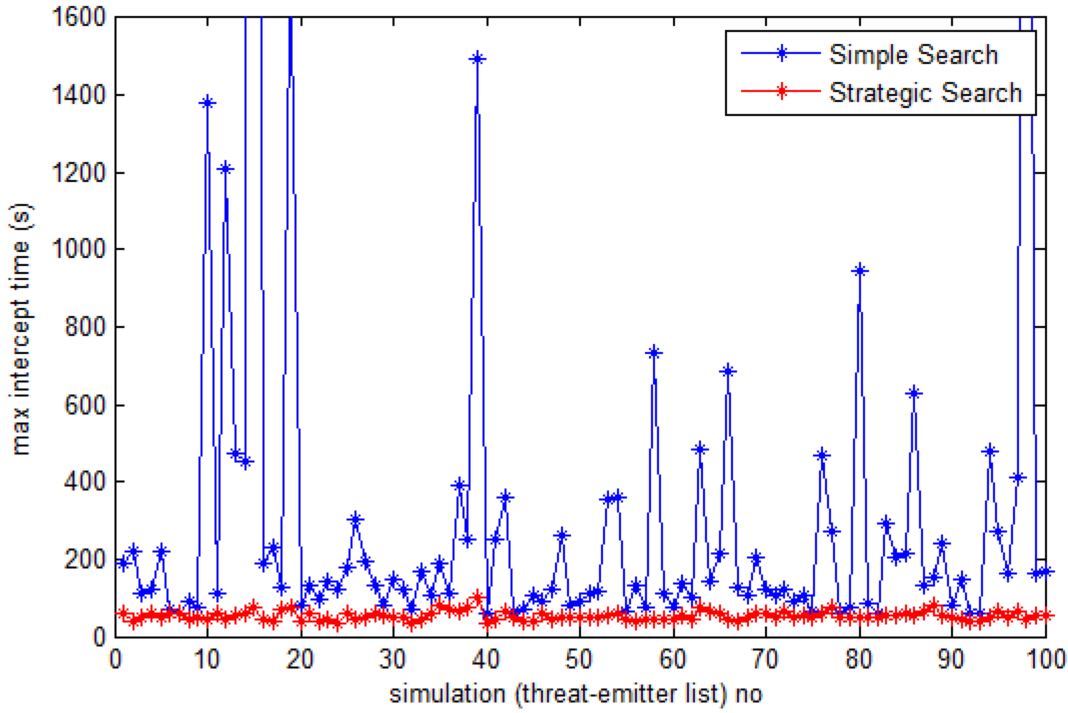

Figure 5-9: Comparison of simple search and test case 2.4 results

Nevertheless, when we look to the results all together in Figure 5-10, it is seen that on the average the minimized maximum intercept time increases while duty cycle is decreased, although this change is small as compared to the previous test.

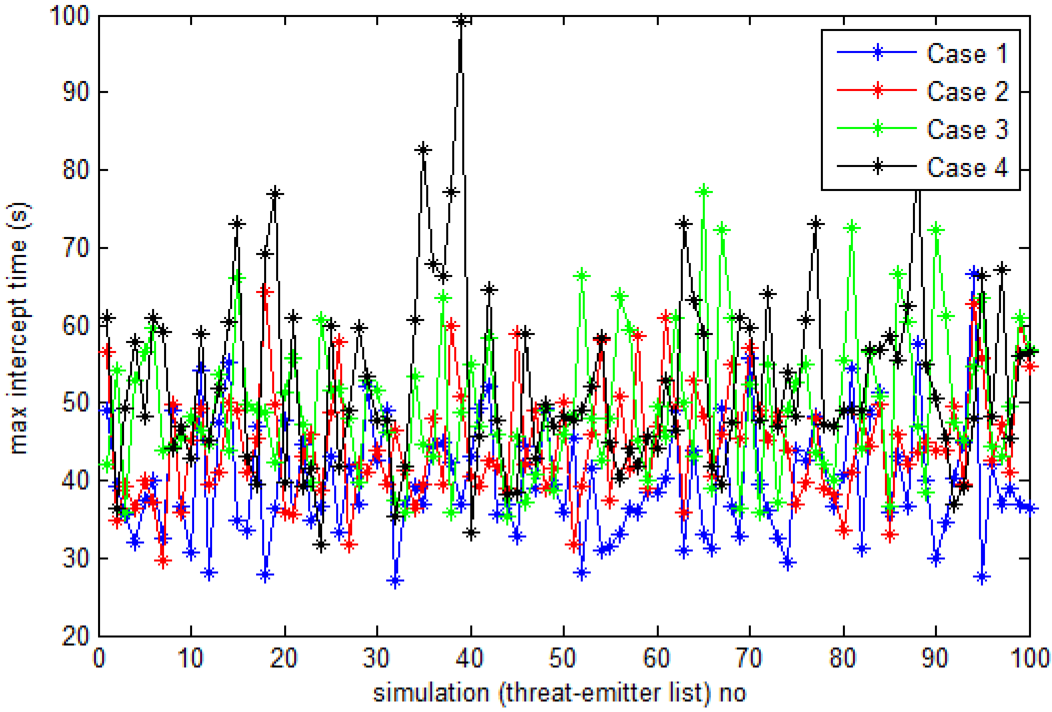

Figure 5-10: Results of all test cases together for duty cycle of radars

From Table 5-2, it is more clearly seen that the average of maximum intercept time increases starting from 40 seconds to about 53 seconds. In the same way, the maximum of the results increases from 66 to 99 seconds throughout the cases. Also we see that the standard deviation of the results increases. Remember that this test is done with 5 threats for simplicity. Although with 5 threats, decreasing the duty cycle to 0.2% – 0.3% did not cause any failure in the algorithm, it shows us that smaller percents of duty cycle of radars obviously increase the maximum intercept time on the average, and if we connect this result with the previous one, more number of bands with radars employing low duty cycles will get the algorithm into more trouble.

Table 5-2: Results of test cases for duty cycle of radars

Test	Minimum of Max Intercept Time (s)	Maximum of Max Intercept Time (s)	Average of Max Intercept Time (s)	Standard Deviation of Max Intercept Time (s)
Case 2.1	27.166	66.523	39.996	7.593
Case 2.2	29.753	64.316	44.809	7.477
Case 2.3	35.388	77.173	49.731	9.283
Case 2.4	31.726	99.000	52.765	11.820

5.3 PRI OF RADAR SIGNALS

The simulations for PRI of radar signals are carried out for three test cases, as shown in Table 4-4. The results of the algorithm and the simple search are compared in Figure 5-11 and Figure 5-12. Increasing PRI from 100 – 500 μs to 2 – 5 ms increased the average value by nearly 20%, as seen in Figure 5-14 and Table 5-3, and this does not seem to be a big performance loss for the algorithm. The algorithm eliminated the synchronization problem and it is clearly superior to simple search. However, notice that when the PRI is increased to 6 – 10 ms in the case 3.3, the maximum of maximum intercept time jumped to 109 s from 74.6 s. Furthermore, the simulation time increased excessively. This difficulty is caused by the increased order of Farey series used in the algorithm. Remember from Sec. 3.2.2 that the Farey order used in the algorithm is determined by the reciprocal of the normalized sum of pulse widths, i.e. $1/\varepsilon$. Also remember from (2.2) that increasing the PRI increases the minimum dwell time for a valid coincidence between two pulse trains. Since this minimum dwell time is subtracted from the sum of pulse widths while calculating ε, increasing the PRI causes ε to decrease. Therefore, the necessary Farey order increases considerably. This loaded excessive process on the algorithm and for many of the threat-emitter lists of the case 3.3, it could not produce results in a reasonable time. Increased Farey order also means that the maximum intercept time would become much larger. This is because, when the minimum acceptable time for a valid coincidence increases, it is much more difficult for a coincidence to happen. In this longer time, the possibility of synchronization increases, and actually this is another explanation for larger Farey order. Remember that Farey series include the ratios for synchronizations, and more number of synchronizations is obviously given by more number of Farey ratios, which implies larger Farey order.

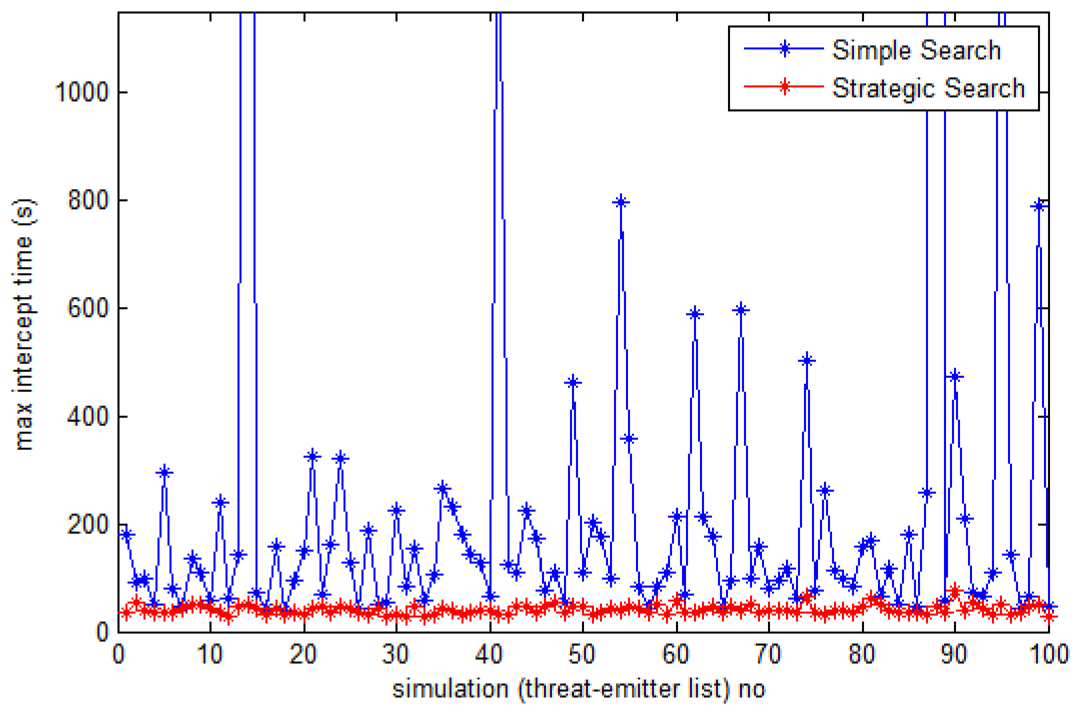

Figure 5-11: Comparison of simple search and test case 3.1 results

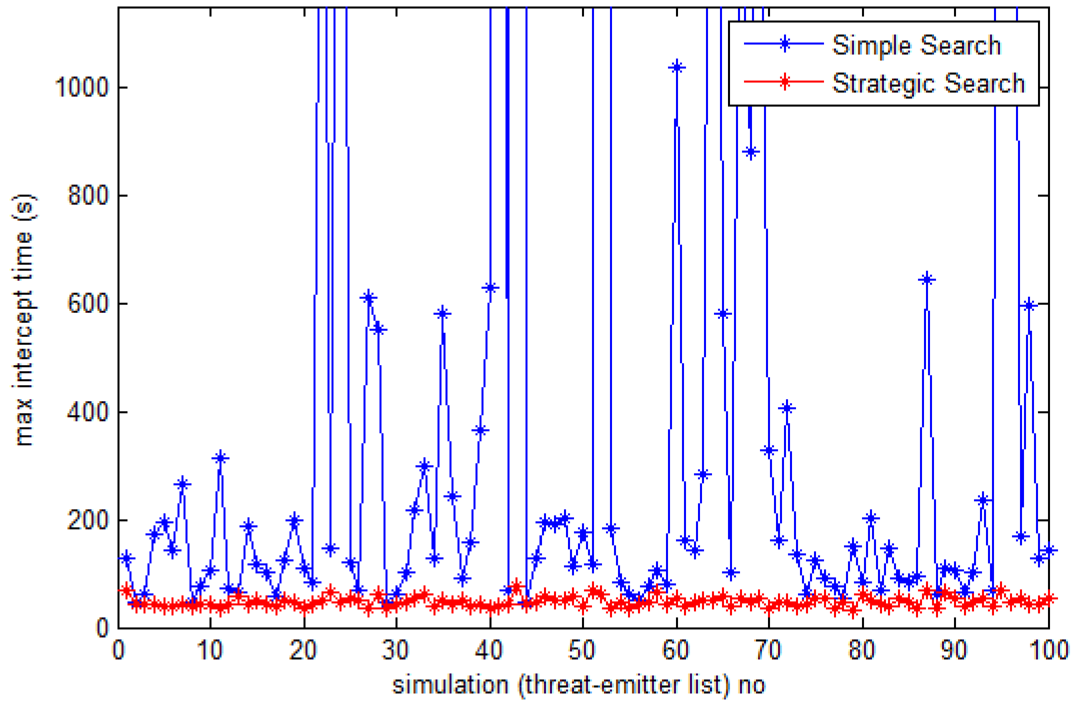

Figure 5-12: Comparison of simple search and test case 3.2 results

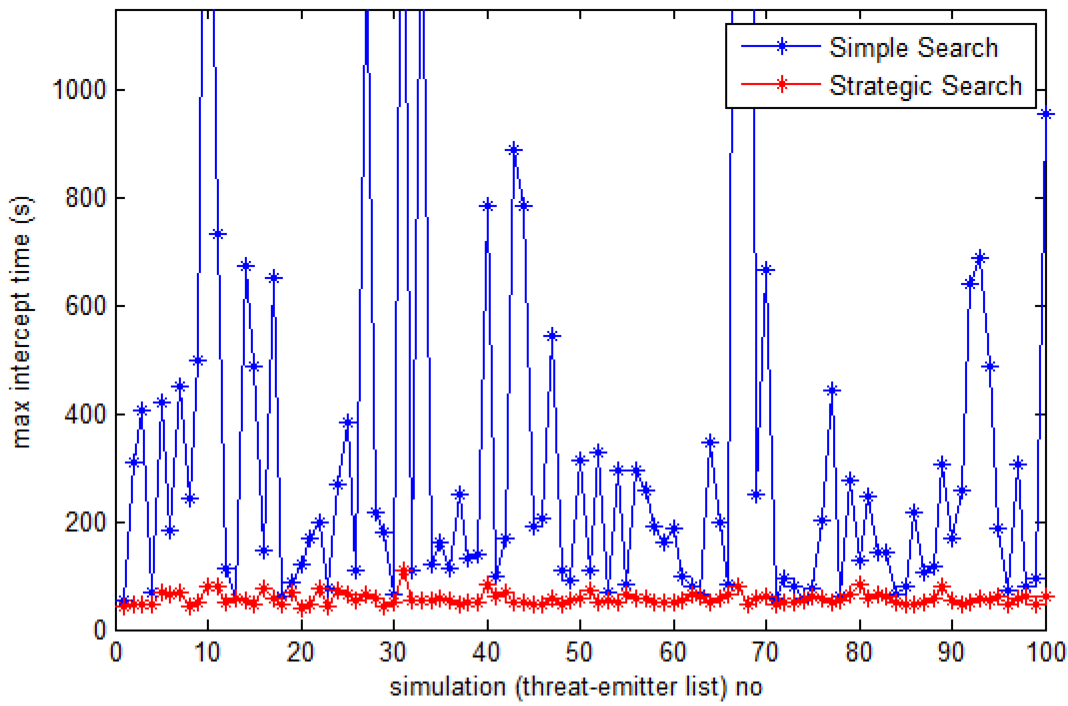

Figure 5-13: Comparison of simple search and test case 3.3 results

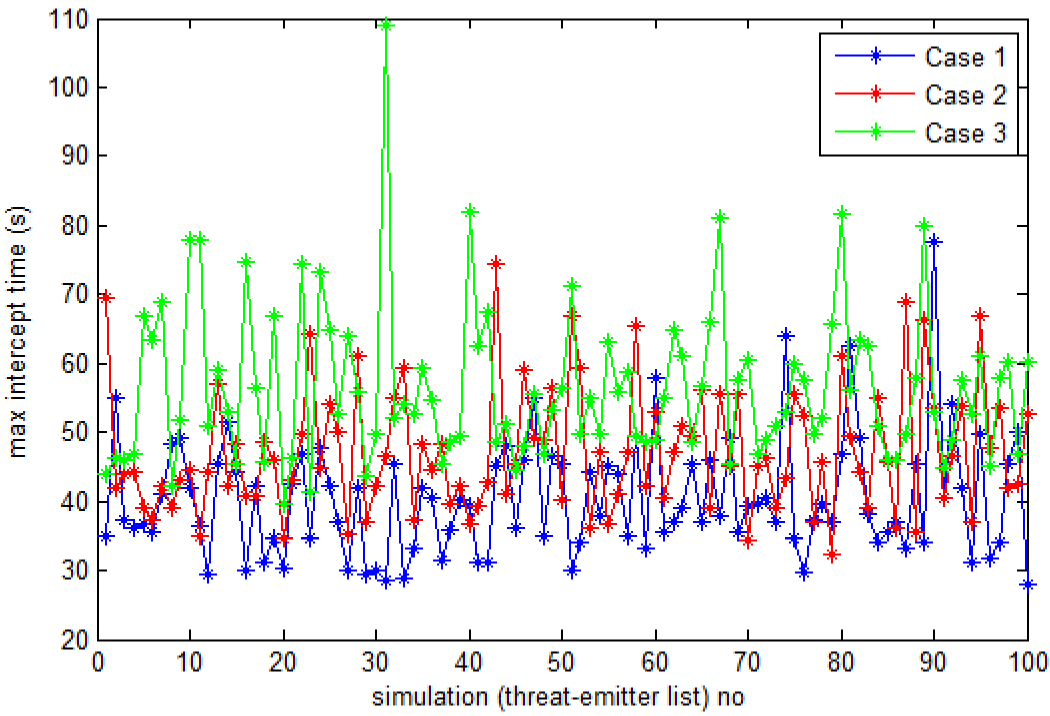

Figure 5-14: Results of all test cases together for PRI of radar signals

Table 5-3: Results of test cases for PRI of radar signals

Test	Minimum of Max Intercept Time (s)	Maximum of Max Intercept Time (s)	Average of Max Intercept Time (s)	Standard Deviation of Max Intercept Time (s)
Case 3.1	28.027	77.719	40.316	8.543
Case 3.2	32.295	74.574	47.208	9.149
Case 3.3	39.641	109.000	56.478	11.195

5.4 DIVERSITY OF SCAN PERIOD AND DUTY CYCLE

In this test, the range of values for the diversity of scan period and the duty cycle are increased throughout the 3 test cases of Table 4-5. In Figures 5-15 - 5-17, the comparisons between the simple search and the results of the algorithm are shown. With these comparisons we see that again the strategic search successfully eliminates the weakness of the simple search, i.e. it eliminates the synchronization problem and minimizes the maximum intercept time. However, when we consider Figure 5-18, we see that the maximum intercept time increases with increased diversity. From Table 5-4, we see this more clearly via the numerical results. While in the test case 4.1 the maximum intercept time changes between a minimum of 27 seconds and a maximum of 66 seconds, in case 4.3 it varies between 60 and 192 seconds. In this way, on the average, the result increases from 40 seconds to 98 seconds. The other important result is that the standard deviation increased from about 7.5 seconds to about 20 seconds. This shows that the strategic search algorithm loses its reliability when in the threat-emitter lists there are radars with a diverse range of values of scan period and duty cycle, which is actually the realistic case.

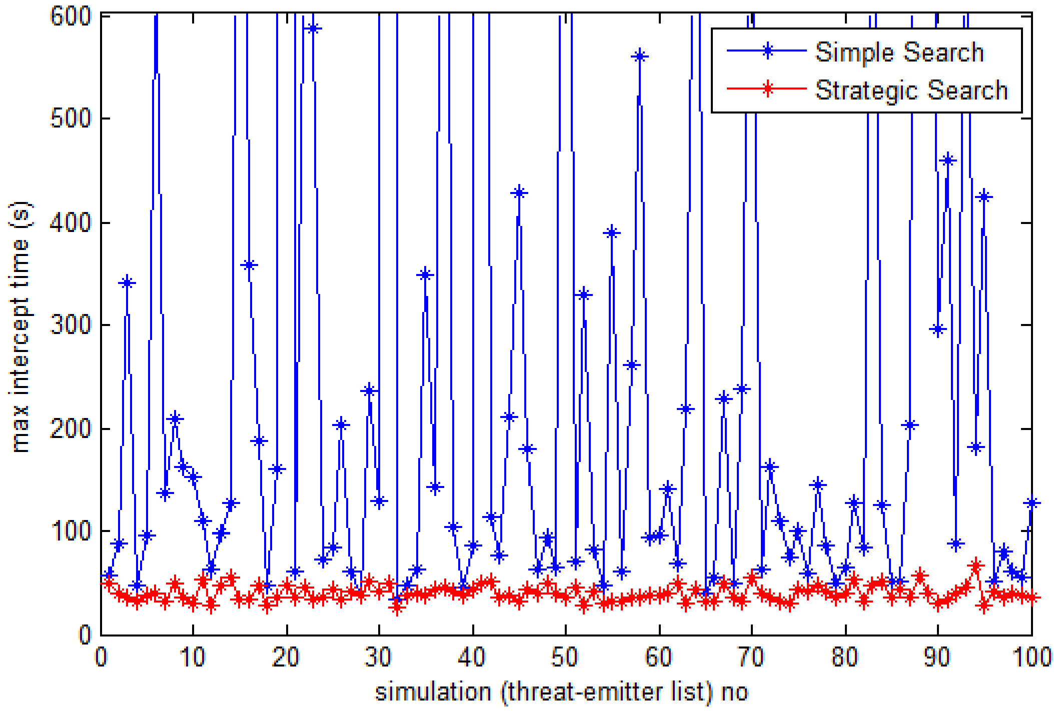

Figure 5-15: Comparison of simple search and test case 4.1 results

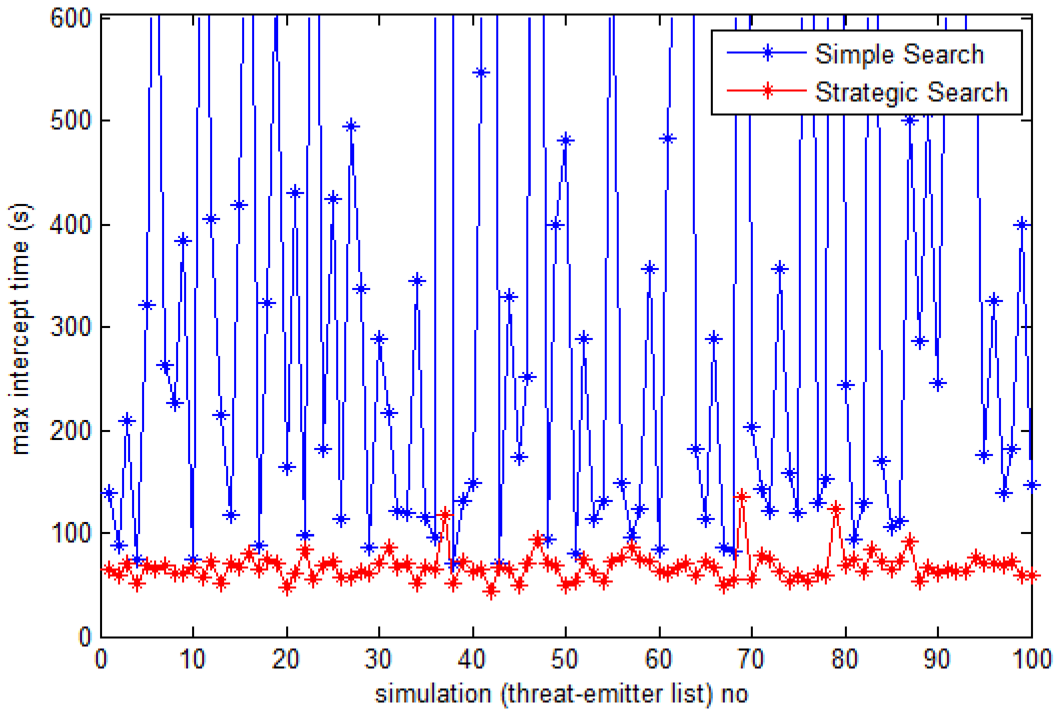

Figure 5-16: Comparison of simple search and test case 4.2 results

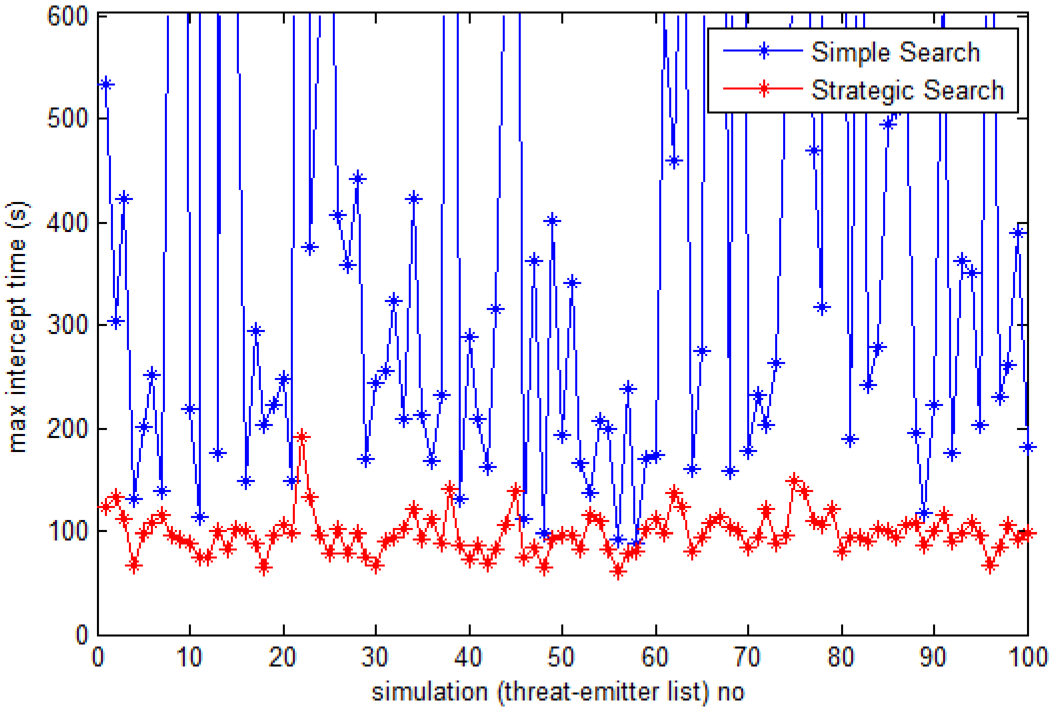

Figure 5-17: Comparison of simple search and test case 4.3 results

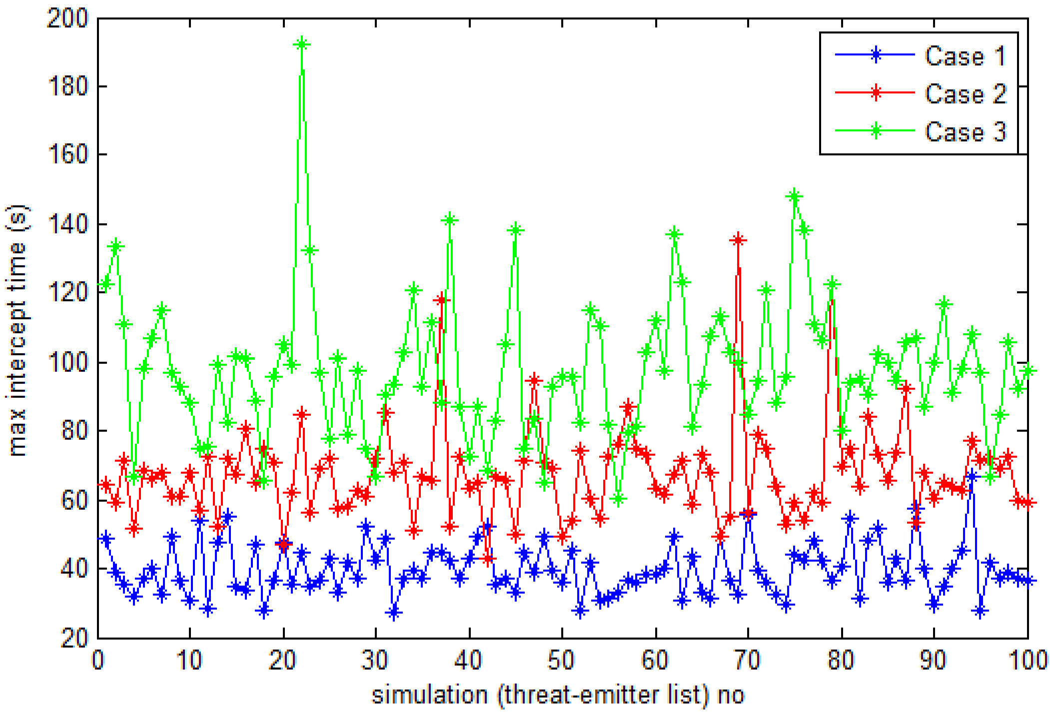

Figure 5-18: Results of all test cases of the part 5.4 together

Table 5-4: Results of test cases for diversity of scan period and duty cycle of radars

Test	Minimum of Max Intercept Time (s)	Maximum of Max Intercept Time (s)	Average of Max Intercept Time (s)	Standard Deviation of Max Intercept Time (s)
Case 4.1	27.166	66.523	39.996	7.593
Case 4.2	42.973	135.000	67.769	14.033
Case 4.3	60.370	192.318	98.343	20.288

From the simulations we have seen that in almost all the situations, strategic search results a maximum intercept time which is much smaller than the one of simple search. Moreover, in most of the cases, strategic search solves the synchronization problem, whereas the simple search cannot handle synchronization, which may cause the receiver not to intercept particular radar forever. In addition to these observations related to the strategic search, however, we have also seen the limits of the algorithm. Depending on the parameters that are just analyzed, the algorithm may become inefficient to minimize the maximum intercept time. Actually, these borders come from the nature of the problem, rather than the incapability of the algorithm; the receiver has a limited capacity, whereas the work load on it has no limit. However, this does not hide the fact that new approaches can be added to the strategic search logic. In the next chapter, a new algorithmic approach is considered. Probability of intercept is used to calculate the maximum intercept time for a particular band, so that it becomes possible to further decrease the maximum intercept time of a particular band, while being aware of the new risks that come as side-effects.

CHAPTER 6

PROBABILISTIC SEARCH STRATEGY

In the previous chapter, the analysis of the performance of the strategic search algorithm has shown that the success of the algorithm depends on the values of some parameters. We have seen that there are some cases for which the algorithm can not produce an acceptable search strategy, i.e. the maximum intercept time is too large, or there is no solution at all. Therefore, we need a new approach to get an acceptable search strategy, especially for cases where the strategic search algorithm failed. This new approach, which we named as probabilistic search, is given in this chapter. The probabilistic search provides more control on the search strategy by assigning more importance to a particular frequency band than the others. This method becomes effective especially when there is a priority threat, i.e. radar which is fatal and should be considered first, or there is an unsolved synchronization problem since the search strategy algorithm of Sec. 3.3 was unable to solve it.

6.1 PROBABILITY OF INTERCEPT

As has been underlined in the previous chapters, the most important parameter in a radar interception problem is the time to intercept, which is desired to be as small as possible to take the counter measures on time. Although not mentioned before, another important criterion is to intercept the radar with a high probability. The problem involves both the emitter and the receiver, and their functions somehow should coincide, depending on their parameters. Hatcher in [8] calls their functions as intermittent and he adds; "If the system has more than a single intermittent function, the resulting interception becomes one of probability, rather than being uniquely defined by the system parameters." [8].

Actually, the reason for not mentioning this before is the fact that these two criteria are always side by side; for example, the approach of Clarkson aims to obtain the maximum intercept time, at the end of which we guarantee to get at least one interception, which implicitly means a probability of 100%. However, the probability of intercept can be less than 100%. For example, a probability of intercept with 70% up to the time instant t means that at t the receiver may intercept the radar with the probability of 70%. For our new approach, we will consider the probability of intercept at a specific time instant. Hatcher worked on this problem and tried to find answers to these two questions:

i. "What is the probability an intercept will occur within a specified time after initiation of a particular activity?

ii. What is the observation time required after initiation of a particular activity to be assured a specific probability of intercept will be attained?" [8]

By using the pulse train model introduced in Sec. 2.2 and assuming that the starting time instants of the pulse trains are independent, he obtained the formulas given in Table 6.1.

Table 6-1: Probability of intercept (P_{12}) and intercept time (T_0) in terms of the pulse train parameters τ and T. [8]

$T_1 \le T_2$		
$P_{12}(T_1)$		
For	$\tau_2 \le T_1$	$T_1 \le \tau_2$
$\tau_1 \le T_2 - T_1$	$\frac{1}{T_2}\left[\tau_1 + \tau_2\left(1 - \frac{\tau_2}{2T_1}\right)\right]$	$\frac{1}{T_2}\left(\tau_1 + \frac{T_1}{2}\right)$
$T_2 - T_1 \le \tau_1$	$1 - \frac{(T_1 - \tau_2)^2 + (T_2 - \tau_1)^2}{2T_1T_2}$	$1 - \frac{(T_2 - \tau_1)^2}{2T_1T_2}$
Probability of intercept for a time T $P_{12}(T) = 1 - [1 - P_{12}(T_1)]^{T/T_1}$		
Observation time for a desired probability of intercept P_{01} $T_0 = T_1 \times \frac{\ln(1 - P_{01})}{\ln[1 - P_{12}(T_1)]}$		

In this table, $P_{12}(T_1)$ is the probability of a coincidence during the first period of the pulse train with shorter period, and τ_1, T_1, τ_2, T_2 are the pulse widths and periods of the pulse train 1 and pulse train 2, respectively.

Wiley [9] also worked on this problem by using a model in which the window functions are decomposed into elementary pulse trains, as shown in Figure 6-1. The pulse is divided into smaller unit pulses between which there is a phase difference equal to the width of unit pulse, thus forming the main pulse when they are superposed.

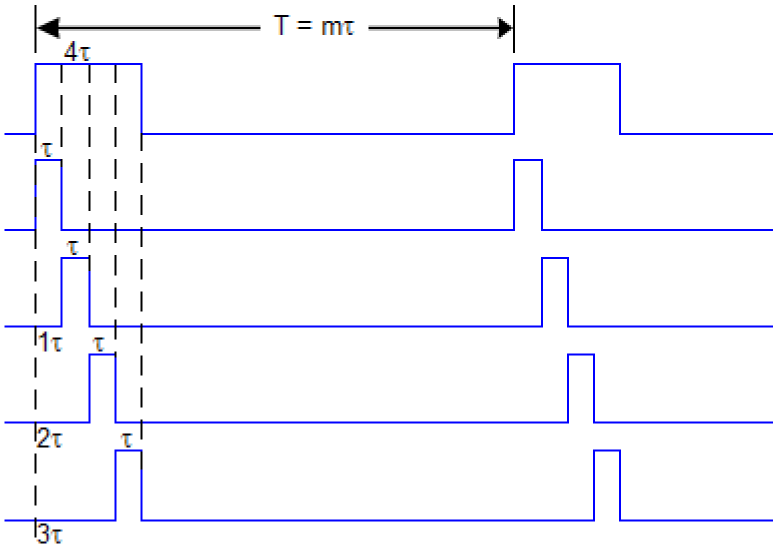

Figure 6-1: Decomposition of window functions [9]

Then, he gives the mean period of coincidence, T_0, and the mean duration of the coincidences, τ_0, as follows [10], where N is the number of pulse trains:

$$T_0 = \frac{\prod_{j=1}^{N}\left(\frac{T_j}{\tau_j}\right)}{\sum_{j=1}^{N}\left(\frac{1}{\tau_j}\right)} \tag{6.1}$$

$$\tau_0 = \frac{1}{\sum_{j=1}^{N}\left(\frac{1}{\tau_j}\right)} \tag{6.2}$$

Thus, the average coincidence fraction is [9]:

$$f = \tau_0 / T_0 = \prod_{j=1}^{N} \left(\frac{\tau_j}{T_j} \right) \tag{6.3}$$

Assuming that occurrence of coincidence is independent from one time increment to the next and that the probability of coincidence in a small increment Δt approaches the ratio of the increment to the average time between coincidences, T_0, he found that the probability of coincidence in time t, $P(t)$, is given by:

$$P(t) = 1 - (1 - f)e^{-t/T_0} \tag{6.4}$$

Furthermore, solving this for t, the required time to reach the desired probability $P(t)$ becomes:

$$t = -T_0 \ln[1 - P(t)] + \ln(1 - f) \tag{6.5}$$

Note that another important assumption Wiley made is that $P(\infty) = 1$, i.e. after a very long time a coincidence is certain [9]. Namely, synchronization problem is not considered in these formulas.

Different from these approaches, however, we need to find the minimum possible dwell time for which the radar may be intercepted with a pre-selected probability in a specific time which comes as an output of Clarkson's algorithm. In other words, minimum possible dwell time, pre-selected probability, and a fixed time to intercept should meet. This is solved with an iterative algorithm which is explained in Sec. 6.3.

6.2 SEARCH STRATEGY WITH PROBABILITY OF INTERCEPT

The probabilistic search algorithm begins at the point when Clarkson's algorithm terminates. The typical output of the algorithm of Clarkson is shown in Table 6-2. We have a dwell time and number of looks for each band. For simplicity, if we assume that there is one threat emitter in each band, we remember that this number of looks refers to the maximum time during which we guarantee at least one valid coincidence with the threat emitter, when the receiver waits as tuned to the frequency band for duration of dwell time in each period of receiver scan. In other words, if the receiver obeys the search strategy of the algorithm, at the end of the time given by number of looks, the radar will be intercepted with a probability of 100 %.

Table 6-2: Example input data for probabilistic search algorithm

Frequency Band	**Dwell Time (ms)**	**(Max) Number of Looks to Intercept all Radars in the Band**	**Probability to Intercept Radars in the Band**
A	201.868	43	100 %
B	378.696	14	100 %
C	251.139	37	100 %
D	71.007	59	100 %
E	126.522	40	100 %

As a result, until now the probabilities were not considered in the results, since for all the results the probabilities were implicitly 100%. However, now the probabilities will be the key part for the probabilistic search and thus for them we are adding a new field to the results, as shown in Table 6-2. In principle, the required dwell time is not higher when the desired probability is lower [4]. Probabilistic search is completely based on this principle. It will allow us to change the desired probability of any band, and according to this principle it will be possible to save more dwell time, since the required dwell times will be less. Then, the receiver will have more idle time, which can be used to reduce the intercept time of any band; for example, for a band whose intercept time goes to infinity. Consequently, the total dwell time in a scan period of the receiver can be used more effectively depending on the tactical necessities, i.e. synchronization problem can be solved, a high priority threat can be intercepted in a shorter time, etc. As an example, the information given in Table 6-2 is the input to the probabilistic search algorithm and results in an output as shown in Table 6-3.

Table 6-3: Example output of probabilistic search algorithm

Frequency Band	**Dwell Time (ms)**	**(Max) Number of Looks to Intercept all Radars in the Band**	**Probability of Intercept Radars in the Band**
A	230.392	24	100 %
B	220.487	14	70 %
C	148.835	37	70 %
D	340.289	43	100 %
E	82.370	40	70 %

From Table 6-2 we see that the algorithm of the Clarkson had produced the intercept times as 43, 14, 37, 59, and 40 looks, for the bands A-E, respectively.

Assume that the bands A and D are more critical than the other bands, i.e. include threat emitters which should be definitely (with a probability of 100%) intercepted in much less time. To do this, more dwell time should be added to the bands A and D. To create this extra dwell time, assume that the probabilities of intercept for bands B, C, and E are reduced to 70%. When we calculate the necessary dwell times of bands B, C, and E for these new probabilities, we see from Table 6-3 that the dwell times of these bands are reduced. About $305\,ms$ are saved and so it becomes possible to use this time for the bands A and D. Finally, from Table 6-3 we see that with this more dwell time, the intercepts times of the bands A and D are reduced to 24 and 43 looks, from 43 and 59 looks, respectively.

In order to explain the steps of the probabilistic search algorithm in more detail, it is appropriate to explain the probability of intercept concept and to show how to calculate the dwell time for a desired probability theoretically.

6.3 CALCULATION OF DWELL TIME FOR PROBABILISTIC SEARCH

In order to calculate the minimum dwell time which guarantees to intercept with a radar during a given time interval with a desired minimum probability of intercept, it is appropriate to use Clarkson's approach with some modification. In fact, we had already used this method in Sec. 3.2.2. Remember that, as shown in Figure 3-4, Clarkson uses the characteristic function to find the intercept time. When the characteristic function becomes entirely 1 between $\beta = 0$ and $\beta = 1$, for some value of α and ε, the interception between two pulse trains becomes independent from relative phases, i.e. the interception has the probability 100%. Then, the smallest value n that satisfies this condition gives the maximum intercept time. The method of Clarkson, given in Sec. 3.2.2, calculates this value of n directly. Now, instead of using this method, we can find another way to utilize characteristic function in our probabilistic search. In the probabilistic search, the main problem is to find the minimum dwell time to intercept radar with any desired probability, where n is known, and thus so is α. Notice that, since the phases are assumed to be uniformly distributed, this is equal to find the instant where the characteristic function is 1 with proportion equal to the desired probability. For example, if the desired probability is 80%, the characteristic function must be 1 in 80% of the time between $\beta = 0$ and $\beta = 1$. When this condition is satisfied for a minimum value of ε, then the minimum dwell time can be found easily using (3.4). Therefore, actually the problem is to find the minimum value of dwell time, and thus ε, for which the characteristic function becomes 1 with the desired probability, for some fixed value of n and α.

The problem is solved with an iterative algorithm, whose steps are given below:

0. The scan period and pulse duration of the radar (T_{emit}, τ_{emit}), sweep period of the receiver (T_{rcv}), maximum intercept time (n), desired probability of intercept, and minimum duration for a valid coincidence are given. From these, calculate α with (3.3), and assign an initial value to ε with (3.4), by substituting the minimum duration for the dwell time, τ_{rcv}, which is convenient since this will give the minimum possible value for ε. Moreover, calculate maximum value for q, which is $n - 1$ as seen from Figure 3-4 (For the meanings of p and q, see Sec. 3.2). Assign an initial value for the variation, which will be halved and added to or subtracted from the dwell time to approach to the solution. The initial value of the variation determines the upper limit of the search interval for the value of dwell time. Therefore, the value of dwell time will be searched in the interval $\left[\text{min duration, min duration+variation}\right]$.
1. Halve the value of variation.
2. For all the possible values of p and q, as in Figure 3-4, find all the intervals between $\beta = 0$ and $\beta = 1$ for which the characteristic function is 1, using (3.7).
3. Find the percentage of times when the characteristic function is 1, by dividing the sum of the durations of intervals that are found in 2 with the

duration of the total interval of β, which is actually 1. This gives us the probability of intercept for the current value of ε.

4. If the probability of intercept found in 3 is lower than the desired probability, add the variation to the value of dwell time, so that the probability can increase. Otherwise, subtract the variation from the dwell time to lower the probability.
5. Calculate the new value of ε with (3.4).
6. If the variation is lower than the desired resolution of the solution, iterations are finished, and the dwell time found in 4 is the solution. Otherwise, return to the step 1.

Calculation of the dwell time for any probability of intercept is the key part for the probabilistic search. Then, the algorithm that is explained in the next part becomes trivial.

6.4 COMPUTATION OF PROBABILISTIC SEARCH STRATEGY

After giving the background for the concept of probability of intercept and explaining the necessary calculations for the minimum dwell time for any probability, now it is convenient to give all the steps of probabilistic search. The probabilistic search takes the output of the algorithm of Clarkson, shown in Table 6-2, as input, in which all the probabilities of intercept are 100%. Therefore, before running the probabilistic search algorithm, one must determine the emitters whose maximum intercept time are needed to be decreased, and the emitters whose probability of intercept may be decreased, together with their desired probability. In other words, the emitters that need more resources (dwell time), the emitters that can tolerate reduction of their resources, and their amount of toleration (desired probability of intercept) are given. These decisions depend on the tactics of the warfare and are made according to the necessities. Then, the probabilistic search can be run.

The probabilistic search algorithm has 2 parts. In the first part, the new dwell times are calculated for the emitters whose desired probabilities are changed, i.e. decreased from 100% to create more resources, using the algorithm just explained in the previous part. Since the probabilities are decreased, the new dwell times will be also smaller [16]. Therefore, the sum of the differences between new and original dwell times give us the created resources, i.e. the idle time of the receiver which can be used for any emitter to decrease its maximum intercept time. Thus, we have some idle time of the receiver and we have some emitters for which we want to use this idle time to add to their dwell time, while the sweep period of the receiver is fixed. Notice that this is the same case given in Sec. 3.3.1. Thus, the rest of the algorithm is identical to the algorithm explained in Sec. 3.3.1. Finally, we have the results as given in Table 6-3.

With the probabilistic search, then, the maximum intercept times of the desired emitters can be further decreased and unsolved synchronization problems can be solved, while being aware of all the risks (lowered probability of intercepts for some other emitters). With this feature, probabilistic search provides more control and options for the search strategy of the receiver. In the next part, the capability of the periodic search will be demonstrated through test results.

CHAPTER 7

SIMULATIONS AND RESULTS FOR PROBABILISTIC SEARCH STRATEGY ALGORITHM

In Chapter 5, the simulation results for the Clarkson's search strategy algorithm were given together with a discussion about the conditions for which the algorithm becomes unable to produce a desirable search strategy for the receiver. In order to overcome this insufficiency, a new approach, probabilistic search was proposed in the previous chapter. Some tests are performed to investigate the performance of the probabilistic search algorithm and the results are given in this chapter.

The probabilistic search aims to perform better than Clarkson's algorithm. Thus, to see the performance better, the probabilistic search algorithm is run for the test cases where Clarkson's algorithm results were needed to improve; which are the cases 1.3, 1.4, 2.3, 2.4, 3.3, and 4.3. For all the test cases, the probabilistic search was run for all 100 threat-emitter lists and the results were obtained over all these runs, i.e. minimum, maximum, average, and the standard deviation of the maximum intercept time, just as has been done for Clarkson's algorithm.

In the test cases 1.3 and 1.4, there are 15 and 25 emitters in each threat-emitter list, respectively. For these test cases, 5 emitters with the smallest intercept time are chosen to decrease their probability of intercept to 70%. In each iteration of the algorithm, the emitter whose maximum intercept time is improved is chosen as the one that has the maximum of maximum intercept time between all other emitters, just as in the optimization for a fixed sweep period. The compared results for the test cases 1.3 and 1.4 are given in Table 7-1 and Table 7-2.

Table 7-1: Probabilistic vs. Strategic search for test case 1.3

Test	Minimum of Max Intercept Time (s)	Maximum of Max Intercept Time (s)	Average of Max Intercept Time (s)	Standard Deviation of Max Intercept Time (s)
Strategic	69.000	162.344	103.685	18.191
POI	61.000	126.043	88.034	13.434

Table 7-2: Probabilistic vs. Strategic search for test case 1.4

Test	Minimum of Max Intercept Time (s)	Maximum of Max Intercept Time (s)	Average of Max Intercept Time (s)	Standard Deviation of Max Intercept Time (s)
Strategic	100.884	592.350	163.443	67.889
POI	91.121	456.040	142.982	53.662

It can be noted that for the test case 1.3, minimum and maximum of maximum intercept time decreased considerably. Also, the average value reduced to 88 seconds from 103.7 seconds and the standard deviation becomes about 13.5 seconds, while it was previously about 18 seconds. Similar results are obtained for the test case 1.4, where the average of maximum intercept time reduced to 143 seconds from 163 and the standard deviation decreased to 53.6 s from 68 s. Also from the Figure 7-1 and Figure 7-2 it can be seen that the intercept times that are above the average are limited by the probabilistic search.

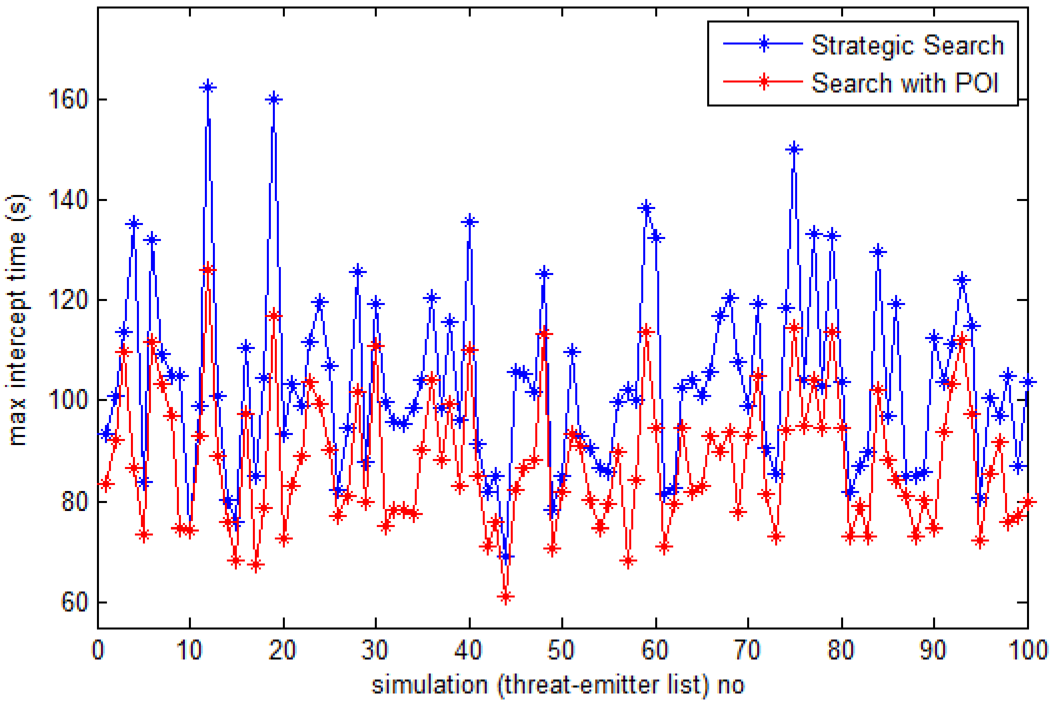

Figure 7-1: Probabilistic vs. Strategic search for test case 1.3

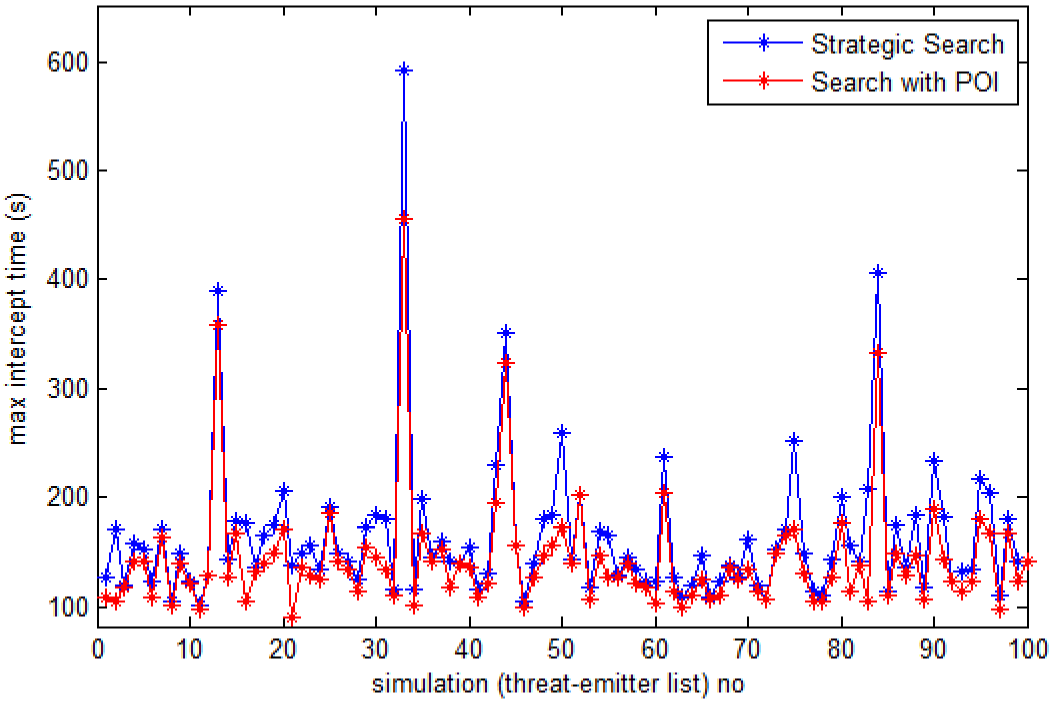

Figure 7-2: Probabilistic vs. Strategic search for test case 1.4

In the test cases 2.3 and 2.4, however, there were 5 emitters in each threat-emitter list. For their tests with the probabilistic search algorithm, 3 emitters with the smallest maximum intercept time are chosen to lower their probability of intercept to 70%. As seen from Table 7-3 and Table 7-4, on the average the maximum intercept time is decreased to 41.8 s and 44.8 s from 49.7 s and 52.8 s, respectively. In addition to this, the maximum intercept time becomes more predictable with the decreased standard deviation. The results can be seen for each threat-emitter list graphically in Figure 7-3 and Figure 7-4.

Table 7-3: Probabilistic vs. Strategic search for test case 2.3

Test	Minimum of Max Intercept Time (s)	Maximum of Max Intercept Time (s)	Average of Max Intercept Time (s)	Standard Deviation of Max Intercept Time (s)
Strategic	35.388	77.173	49.731	9.283
POI	28.000	56.519	41.787	6.597

Table 7-4: Probabilistic vs. Strategic search for test case 2.4

Test	Minimum of Max Intercept Time (s)	Maximum of Max Intercept Time (s)	Average of Max Intercept Time (s)	Standard Deviation of Max Intercept Time (s)
Strategic	31.726	99.000	52.765	11.820
POI	30.381	85.000	44.858	8.535

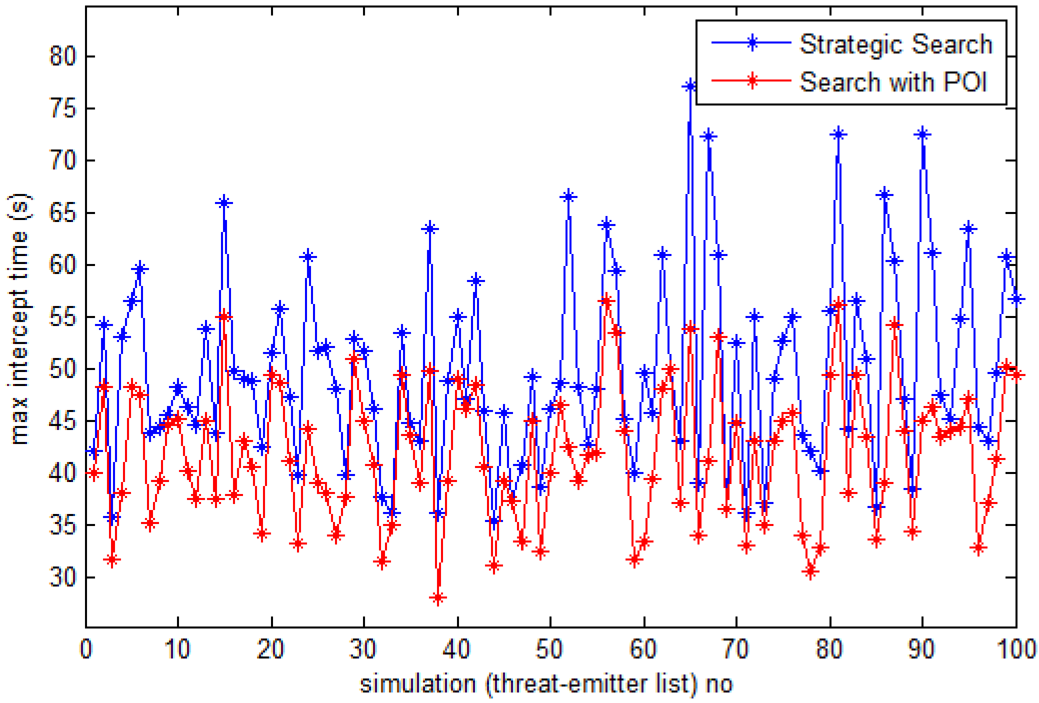

Figure 7-3: Probabilistic vs. Strategic search for test case 2.3

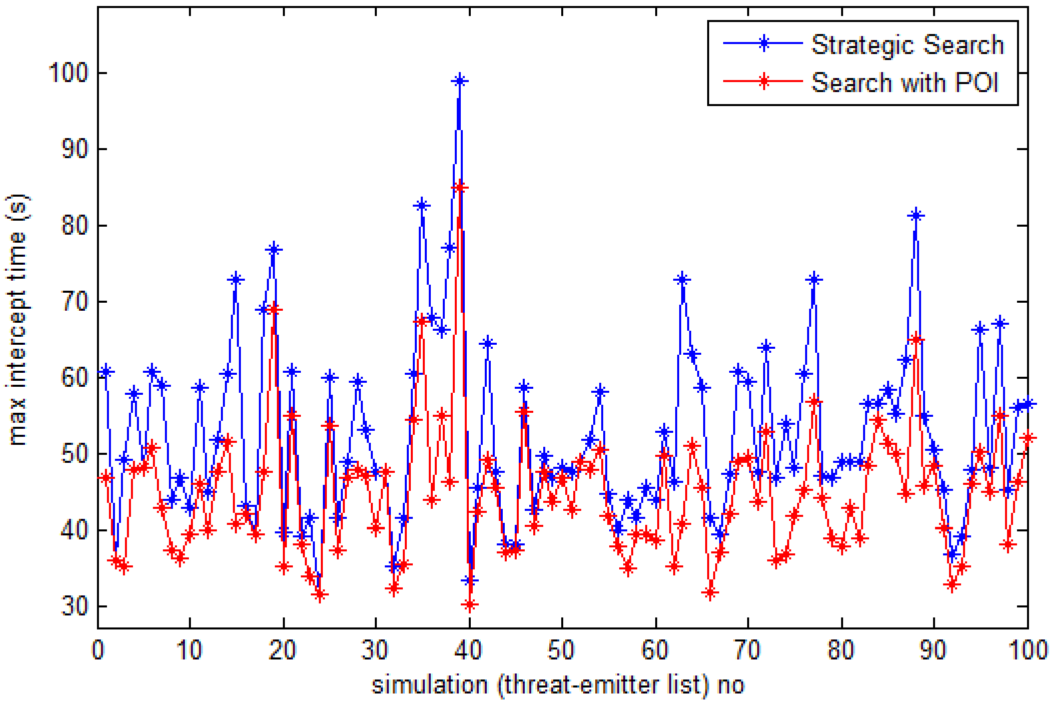

Figure 7-4: Probabilistic vs. Strategic search for test case 2.4

Similarly, there were 5 emitters in each threat list for the test cases 3.3 and 4.3. Again, 3 of them are chosen to adjust their probability of intercept at 70% for the probabilistic search. The created extra dwell time is used for the emitters with the maximum of maximum intercept time and the results are obtained as shown in Table 7-5 and Table 7-6. Like the other cases, we see that the maximum intercept times are successfully decreased, as well as the decreased standard deviation makes the results more consistent. The results for each threat-emitter list can be seen in Figure 7-5 and Figure 7-6.

Table 7-5: Probabilistic vs. Strategic search for test case 3.3

Test	**Minimum of Max Intercept Time (s)**	**Maximum of Max Intercept Time (s)**	**Average of Max Intercept Time (s)**	**Standard Deviation of Max Intercept Time (s)**
Strategic	39.641	109.000	56.478	11.195
POI	33.000	89.000	47.757	8.7682

Table 7-6: Probabilistic vs. Strategic search for test case 4.3

Test	Minimum of Max Intercept Time (s)	Maximum of Max Intercept Time (s)	Average of Max Intercept Time (s)	Standard Deviation of Max Intercept Time (s)
Strategic	60.370	192.318	98.343	20.288
POI	46.356	136.225	86.330	15.396

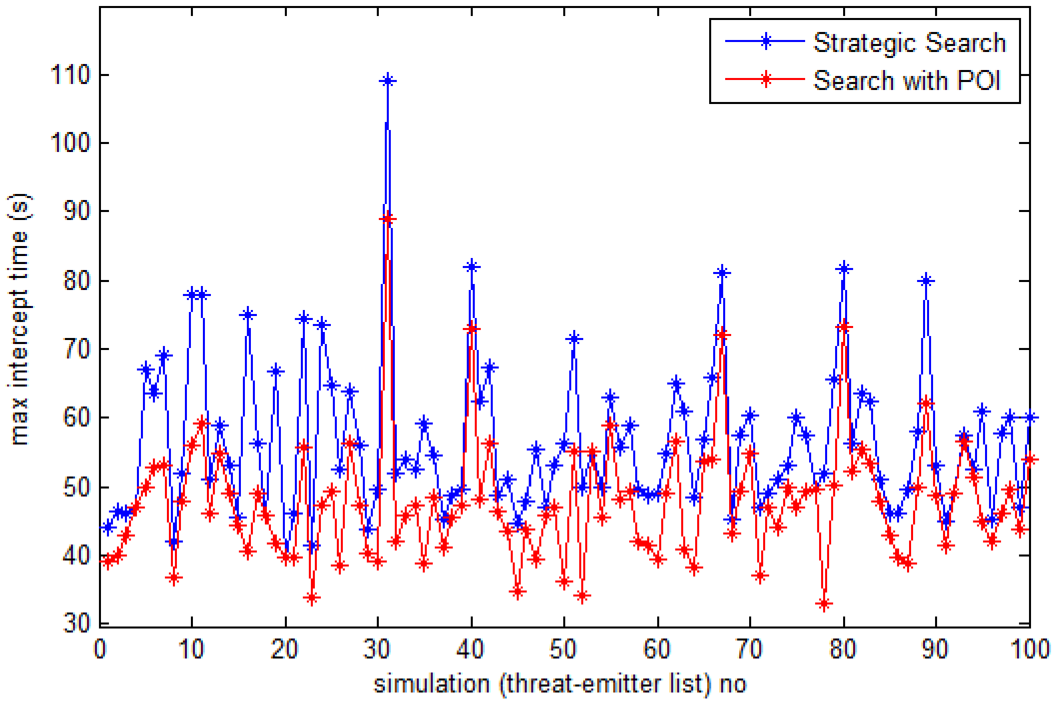

Figure 7-5: Probabilistic vs. Strategic search for test case 3.3

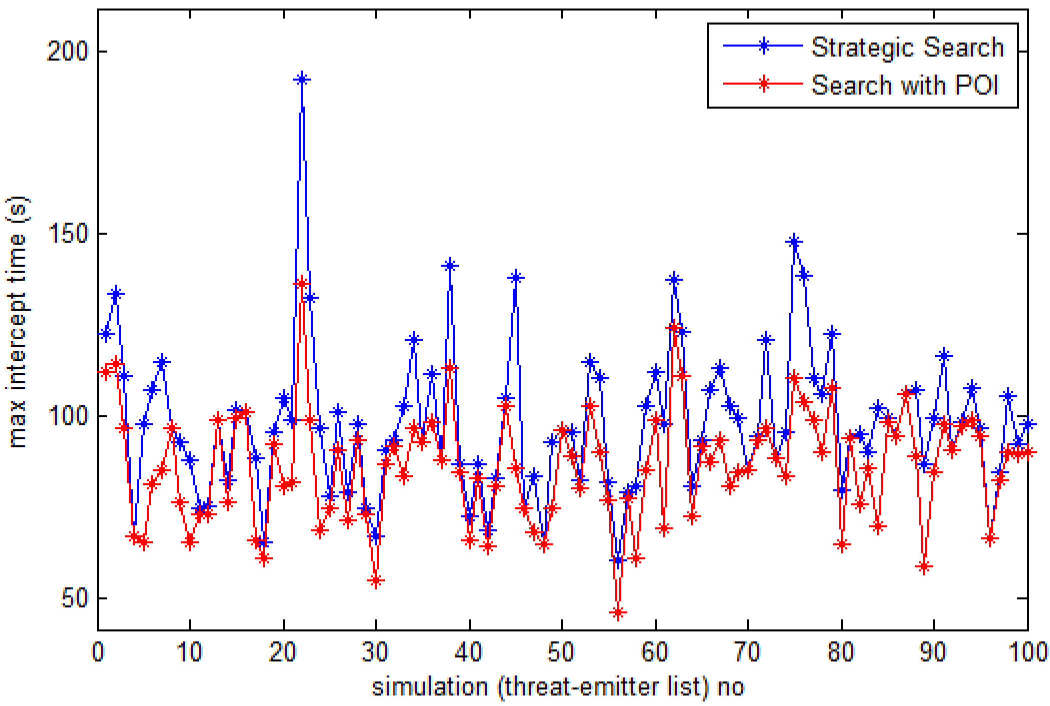

Figure 7-6: Probabilistic vs. Strategic search for test case 4.3

From all these results, we can claim that the probabilistic search algorithm succeeds in reaching its aim. It provides further control and introduces extra options for the search strategy of Clarkson's algorithm. In this chapter, the choice of the emitters whose maximum intercept time are planned to be decreased and the emitters with decreased probability of intercept, are carried out without any tactical knowledge, but just by looking at the largest and smallest maximum intercept time. In fact, the emitters that will play a role in the probabilistic search algorithm should be chosen according to the necessities, so that the algorithm could give a more meaningful search strategy for the receiver.

CHAPTER 8

CONCLUSION AND FUTURE WORK

8.1 CONCLUSION

This thesis is about the best search strategy for a frequency-swept SHR receiver in an environment with circularly scanning radars. First of all, the radar interception problem is modeled by two pulse trains. Then, the dependence of maximum intercept time on the radar and receiver parameters is studied. In this way, the connection between the synchronization problem and Farey series was clarified. Moreover, a method to select the best periodic search strategy is introduced, which finds the optimum sweep period for the receiver to minimize the maximum intercept time assuming a pre-knowledge about radars. However, there were no comprehensive simulations in literature for selecting the optimum search strategy. Thus, for this thesis more than a thousand threat-emitter lists were created to test the algorithm of Clarkson and simulation results are presented. As a result, it was found that the algorithm may produce search strategies which are not well suited to tactical necessities, when there is plenty of radars operating in different frequency bands, when duty cycle of radars is low, or the PRI of radar signals is high, and when the diversity of scan period and duty cycle of radars is high. Moreover, an algorithm to find the dwell time for an intercept time with a desired probability of intercept was given in this study. Thus, a new approach was proposed, named as probabilistic search, with remarkably more control on search strategy by being able to select different probability values to intercept the radar in a certain intercept time.

Then, it is concluded that the probabilistic search could be effective to decrease the maximum intercept time and to solve the synchronization problem, especially when there are many radars in the environment, some of which are specified to be more threatening. In this case, probabilistic search makes it possible to assign different priority to each frequency band and thus to decrease the maximum intercept time of radars in the bands with higher priority, at the expense of degrading the frequency bands with lower priority, by lowering the probability of intercept for radars of those bands. Even if the performance completely depends on the tactical necessities and therefore it can be application dependent, nevertheless some numerical results are presented in this thesis, in which the average maximum intercept time could be lowered by about 10% - 20%.

8.2 FUTURE WORK

In this thesis, it is assumed that the radars in the threat-emitter list are all circularly scanning. However, there may be radars with different scan type, such as raster scan, spiral scan etc. This case does not introduce any complexity to the search strategy finding method explained in this thesis. As long as a periodicity exists in the radar signals that will be intercepted, the algorithm should be able to optimize the search strategy as explained in the thesis.

In ECM systems, because of interference with jammers, sometimes the receiver should stop to listen radar signals, and the time and duration of this intermittency is not deterministic. This situation violates the periodicity of sweeping of the receiver and thus the calculations for intercept probability can not be used here, so that anohter approach should be proposed.

The search strategy considered in this thesis is periodic, and the period is the same for all frequency bands. Is it possible to find an aperiodic search strategy, or at least, one with different periods for each frequency bands? In this case, because of coincidences, it becomes impossible to schedule the dwells of all frequency bands when there are many frequency bands to be swept. For a few frequency bands, such periods may be found such that their dwells do not coincide, i.e. the receiver successfully schedules all of the dwells, however, this introduces severe constraints on the problem and possibly synchronization problem can not be eliminated. It is seen that the aperiodic case is much more difficult, and new techniques must be proposed to handle this problem efficiently.

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