MULTI-OBJECTIVE APPROACHES TO PUBLIC DEBT MANAGEMENT

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ABSTRACT

MULTI-OBJECTIVE APPROACHES TO PUBLIC DEBT MANAGEMENT

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Public debt managers have a certain range of borrowing instruments varying in their interest rate type, currency, maturity etc. at their disposal and have to find an appropriate combination of those while raising debt on behalf of the government. In selecting the combination of instruments to be issued, i.e. the borrowing strategy to be pursued for a certain period of time, debt managers need to consider several objectives that are conflicting by their nature, and the uncertainty associated with the outcomes of the decisions made. The objective of this thesis is to propose an approach to support the decision making process regarding sovereign debt issuance. We incorporate Multi-Criteria Decision Making (MCDM) tools using a multi-period stochastic programming model that takes into account sequential decisions concerned with debt issuance policies. The model is then applied for public debt management in Turkey.

Keywords: Public Debt Management, Risk Management, MCDM, Stochastic Programming

ÖZ

KAMU BORÇ YÖNETİMİNE ÇOK-AMAÇLI YAKLAŞIM

Balıbek, Emre
Doktora, Yöneylem Araştırması
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Kamu borç yönetimleri, kamu sektörünün borçlanma ihtiyacını karşılarken farklı faiz, döviz, vade yapısına sahip çeşitli finansal araçlar kullanmaktadır. Finansman ihtiyacının karşılanmasına yönelik olarak söz konusu finansal araçların hangi oranlarda kullanılacağına ve kamu borç portföyünün yapısına ilişkin stratejiler oluşturulurken; borç yöneticileri, kamunun ödünleşim içeren çeşitli borç yönetimi amaçlarını ve verilen kararların sonuçlarına ilişkin belirsizlikleri göz önünde bulundurmaktadır. Bu çalışmada, kamu borçlanmasında izlenecek stratejilere ilişkin karar verme sürecine yönelik olarak niceliksel bir yaklaşım önerilmektedir. Bu kapsamda, kamu borçlanmasına ilişkin ardışık kararları dikkate alan stokastik bir program geliştirilmiş, söz konusu model kullanılarak çok-amaçlı karar verme yöntemleri içeren bir karar destek süreci oluşturulmuştur. Geliştirilen öneriler, Türkiye uygulamasını baz alan bir örnek üzerinde somutlaştırılmaktadır.

Anahtar Kelimeler: Kamu Borç Yönetimi, Risk Yönetimi, Çok-Amaçlı Karar Verme, Stokastik Programlama

To My Family

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The ideas expressed in this study are only those of mine and do not necessarily reflect the views and policies of the Turkish Treasury.

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α	Confidence level parameter
p	Number of criteria
z_i	The ith objective function , i=1,…,p
x	Decision variable vector
X	Decision space
T	Decision horizon
S	Scenario set
s	Scenario Index, $s \in S$
p_s	Probability associated with scenario s.

CHAPTER 1

INTRODUCTION

Public debt management (PDM) is the process of raising funds for the financing needs of the government and managing the government's financial liabilities. Public debt managers have a certain range of borrowing instruments varying in their interest rate type, currency, maturity etc. at their disposal and have to find an appropriate combination of those while raising debt on behalf of the government. In selecting the combination of instruments to be issued, i.e. the borrowing strategy to be pursued for a certain period of time, debt managers need to consider several objectives that are conflicting by their nature, and the uncertainty associated with the outcomes of the decisions made.

Given the budgetary and other financial requirements of the government, one of the main objectives of PDM is to meet the funding needs with the lowest possible cost. On the other hand, the risks associated with the debt portfolio should be contained to avoid any adverse affect on the macroeconomic environment. Among the major risks that concern public debt managers, there are the market risk, which is defined as the risk of an increase in the cost of debt service due to fluctuations in market conditions and the liquidity (re-funding) risk that indicates the possibility to fail in finding the required funds in order to make debt re-payments.

Thanks to the nature of financial markets, in general, there exists a trade-off between return and risk. For a portfolio manager, achieving a higher return requires investing in high risk assets. From the government's point of view, considering the fact that the government is the issuer of financial assets (securities) the dilemma is between attaining a low cost portfolio and restricting the risks associated. Thus, the public debt management problem, i.e. formation of the financing strategies, is a multi-objective decision making problem.

On the other hand, the level of development of a country's financial markets, the government's credit ratings, its ability to access international markets and other macroeconomic environmental conditions impose several limitations on the activities of debt managers. Different levels of budget deficits induce different sizes of borrowing requirements and not every country can issue the same type of instruments. Some countries have developed pension fund systems that demand long-term government bonds, while others are struggling for finding customers for their medium-term securities. Government debt managers should all consider these constraints in developing their funding strategies.

An important characteristic of the multi-objective PDM problem is that decisions are made under uncertainty. Debt managers are not faced with choices that have deterministic outcomes. There is a degree of uncertainty associated with the evolution of economic factors such as interest and exchange rates that drive the cost of borrowing. Debt management decisions are concerned with future actions of the government and the outcomes of the decisions made depend on the realizations of relevant macroeconomic variables. The stochasticity of these factors needs to be taken into account while formulating the cost-risk structure of the debt portfolio.

Given its characteristics and significance on a country's economic life, the public debt management problem has drawn attention of both academicians and practitioners, including staffs of International Financial Institutions such as the International Monetary Fund or the World Bank and debt management offices. Risk management practices are gaining prime importance in public liability management operations and several approaches adapted from techniques applied by private financial institutions have been proposed for the case of the government. These are generally simulation or scenario analysis based methods that aim at quantifying the costs and risks of alternative strategies. However, to the best of our knowledge, there is little work on providing guidance to decision makers in comparing these quantities, the computed cost and risk metrics, and assisting them in finding efficient solutions to explore the consequences of different strategies in terms of costs and risks.

This thesis has two main objectives. First, we aim to develop an optimization approach for the debt strategy formulation problem. In this context, we show the applicability of a mathematical modelling paradigm, stochastic programming, in the field of public debt management. In developing the stochastic programming model, we adopt a multi-objective approach taking into account the multiple objectives associated.

We incorporate relevant criteria and develop a quantitative approach that take into account sequential decisions concerned with debt issuance policies, taking uncertainty into account making use of a scenario tree. We formulate the debt management problem as a deterministic equivalent linear programming model, in which the decision variables are the amounts of different types of bonds to be issued, accounting for the cash flow constraints for the government. In that setting, the government issues a certain set of treasury securities to meet its overall financing requirements that arise from its debt and non-debt obligations (net of tax receipts). The exact amount of each type of bond to be issued is determined by the model based on the decision makers' preferences with regard to the debt management objectives considering the scenario set available.

The second and ultimate objective is to develop an integrated decision support framework to guide debt managers in developing bond issuance strategies. We show how Multi-Criteria Decision Making (MCDM) approaches can be incorporated on the SP model and how the model can be used to assist decision makers in analyzing the trade-offs between alternative courses of action. In this context, we identify efficient solutions based on different preference structures and develop an interactive MCDM approach to guide the decision makers (DMs) in developing the debt strategy. We demonstrate how sovereign decision makers can experiment with such a tool in a practical setting, drawing on the case of Turkey.

While developing the MDCM framework for public debt management decisions, we bring forward the idea of constructing confidence regions around efficient solutions. We believe the concept can generically be applied in analyses regarding decisions under uncertainty. The stochastic interactive approach we develop in the context of public debt management is also original, to the best of our knowledge, in MCDM literature, and can well be adapted for decision making problems in other areas that involve multiple objectives and stochasticity.

The thesis is organized in four main chapters. Since we bring together ideas from different disciplines in the field of public debt management, literature reviews on several concepts and tools to which we make reference are spread over the chapters depending on the content.

Chapter 2 defines the public debt management problem and discusses its main features. In this section, we elaborate on the general objectives and constraints of PDM and introduce the main financial instruments available to debt managers. A literature review on various approaches to the PDM problem, both from practical and theoretical perspective is also provided.

In Chapter 3, we present our generic multi-stage SP model, developed to guide issuance decisions. The chapter begins with a discussion of the stochastic programming paradigm, touching on basic concepts including different types of models and scenario generation methods. This part also provides some literature review on the application of SP models to financial decision making problems. We then continue with the discussion of the mathematical formulation of the relevant objectives and constraints in the PDM context. After presenting the notation and formulation of the model, we include a simple illustrative model to concretize the discussion. The section ends with the Simulation/Optimization approach which is developed as a decision aid framework to assist strategy decisions in a dynamic environment.

Chapter 4 starts with a discussion of the relevance of Multi-Criteria Decision Making tools for the PDM problem and presents how we employ the SP model to develop a decision making framework. The section includes the methodology for obtaining efficient solutions. We then present an interactive algorithm by which the decision makers can experiment to explore alternative solutions. The algorithm makes use of multivariate statistical analysis tools to cope with the inherent uncertainty in the problem.

Chapter 5 is about the application of the developed methods in an example problem, i.e. the case of the Turkish Treasury, the institution in charge of PDM in Turkey. In this section, we also present a specific scenario generation mechanism that employs ideas from scenario clustering and reduction techniques developed to model the Turkish macroeconomic environment. In Chapter 6, we conclude and indicate prospects for future work.

CHAPTER 2

THE PUBLIC DEBT MANAGEMENT PROBLEM

Public debt management (PDM) is concerned with meeting the funding requirements of a country that arise from budgetary and other financial liabilities of the government. More specifically, it can be defined as the "process of establishing and executing a strategy for managing the government's debt to raise the required amount of funding, pursue its cost/risk objectives, and meet any other public debt management goals the government may have set, such as developing and maintaining an efficient and liquid market for government securities" (International Monetary Fund - World Bank, 2003, p.5).

Almost all countries, developed or under-developed, have a certain level of debt. States build up debt to fund extra expenditures at times of national troubles (wars, natural disasters etc.) or to undertake development projects (to construct roads, bridges, to finance social projects etc). Governments sometimes resort to borrowing even to finance current expenditures, when raising debt is technically and/or politically easier than to impose additional tax. Some countries also borrow in foreign currencies to build-up foreign exchange reserves or to finance their international payments. Figure 1 depicts the size of Central Government Debt in some selected OECD (Organization for Economic Co-Operation and Development) countries in comparison to their Gross Domestic Products (GDP).

In order to meet the financing requirements of the government, public debt management authorities (organized under the National Treasury, the Ministry of Finance, Central Bank or the Debt Management Office in different countries) issue short-term bills or longer-term bonds (securities) in the financial markets or use loans from banks or multi-national/governmental institutions. Once a certain level of debt stock is acquired, countries generally lack the funds or do not prefer to retire entire debt all at once, since this would require increasing the tax level or decreasing government expenditures substantially. Then, they have to roll-over existing debt to some extent by issuing new debt to finance re-payments. Thus, debt management is a continuous process and the governments' debt portfolio has a dynamic structure since there are bonds and bills entering and leaving the debt portfolio throughout time.

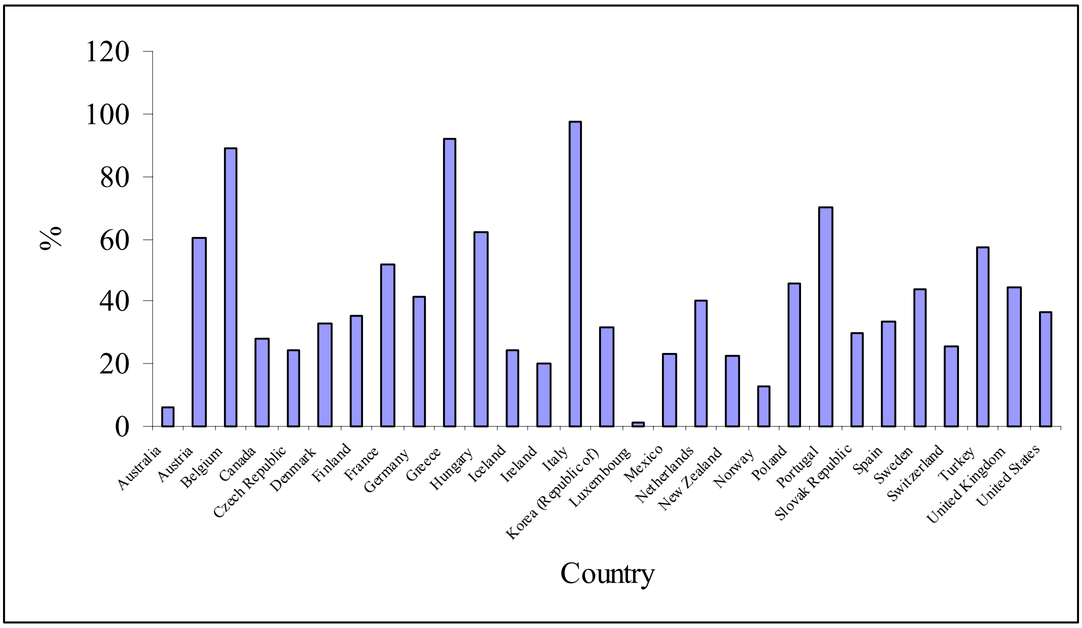

Figure 1 Total Central Government Debt, in Percent of GDP, in Selected OECD Countries, as of 2006. (source:OECD).

Especially in low and middle income countries, government debt is the largest financial liability portfolio in the country. The overall structure of public debt portfolio is key to a country's macroeconomic stability, given the exposure of public sector balances and the country's financial stability to public debt. This has been proved by a number of recent macroeconomic crises in several emerging countries such as Mexico (1994) and Turkey (2001), where the financial turmoil has been amplified by the highly vulnerable composition of the countries' debt liabilities.

Once a government is in a financial problem, i.e. facing difficulties in fulfilling fiscal liabilities or having to pay excessive costs when issuing debt, this has spillover effects on the entire economy. The banking sector, other financial institutions and individuals who have extended loans to the government are all affected by the government's financial troubles. The problems of intermediaries in the financial markets then hamper the functioning of the production sector that relies on funds provided by those institutions.

Generally, the government's cost of borrowing, i.e. the interest rate on government securities sets the basis for the level of interest rates in a country since the government is accepted as the least risky borrower in its own economy. The private institutions that are competing with the public sector to access funds have to pay a premium over the government's borrowing cost. If a government adopts a risky debt structure, this is reflected in its funding costs as lenders will price in a risk premium while extending credit to the public sector. This then affects the interest rates in the entire economy and hinders economic growth.

Thus, the financial liability portfolio of the government must be effectively managed. Public debt managers have to decide on a certain debt management strategy taking into account several policy targets. They have a range of financial instruments (securities) at their disposal and have to form a specific portfolio combination, in terms of maturity, currency and interest types, that would suit the government's debt management objectives.

2.1 Public Debt Management Objectives: Cost and Risks

Traditionally, the most important concern of public debt management had been the cost of borrowing, or even to be able to raise the necessary funds. Tobin (1963) states that "If anyone is in the position to be his own insurer, it is the Secretary of the Treasury" and thus argues the government should focus on cost minimization. For some countries, this cost minimization objective materialized into unbalanced debt structures, relying too much on short-term and/or foreign currency denominated debt, which turned out to be a source of risk in later years.

In 1994, Mexico did not hesitate to replace bulk of her public debt from *peso* denominated bonds to short-term dollar-linked *tesobonos* in pursuit of lower interest rates and attracting foreign investors, which led to an increased vulnerability of the country's economy to financial crisis. Mexican Crisis at the end of 1994 is partly attributable to the 29 billion United States Dollar (USD) *tesobonos* maturing in 1995, with 10 billion USD to be paid in the first 3 months, while the country's foreign reserves stood at a level of 6.3 billion USD (Cassard et al, 1997). Gill and Pinto (2005) found out that Argentina's debt increased by 41.7% of her Gross National Product (GDP) between 2001 and 2003, while that of Russia's by 40.4% in 1998-1999 after the financial crises in those countries due to the high share of foreign-denominated debt in those countries' debt stocks.

The increased volatility of international fund flows, the complexity of instruments used and the recent crises highlighted the importance of risk-related criteria, in addition to cost, while raising public debt. Most public debt managers are now concerned with the risks and macroeconomic issues associated as well as cost.

The public debt management objective in the United Kingdom, for example, is "to minimise, over the long term, the costs of meeting the Government's financing needs, taking into account risk, whilst ensuring that debt management policy is consistent with the aims of monetary policy" (HM Treasury, 2007). In its "Guidelines for Public Debt Management", Italian Treasury (2007) declares that "…during 2007 Government bond issues will be calibrated so as to meet the financing needs of the Central Government, with a medium-term view to further reducing the exposure to interest risks (nominal and real) and refinancing risk, while at the same time containing the dynamic of interest burden as a percentage of GDP".

The major risks public debt managers face are the market risk, which is defined as the risk of an increase in the cost of debt service as a result of unfavorable movements in market conditions and the liquidity (re-funding) risk that indicates the possibility to fail in finding the required funds in order to make debt re-payments.

2.1.1 Cost of Public Debt

It is possible to measure the cost associated with public debt in several ways. Debt management offices may adopt different cost measures depending on their perspectives in debt management and/or country characteristics such as the accounting standards.

The most common measure of cost in borrowing funds is the interest rate requested by lenders. Interest rate is the time value of money and lenders who extend loans to any borrower ask to be compensated for the duration of the loan since they will not be able to use their own funds in that period.

When a government issues debt, the cost of borrowing is reflected in the government's budget in terms of interest expenditures. Governments that employ cash accounting standards record interest expenditures when payments are actually made, while countries that follow an accrual accounting standard, depict interest expenses as they accrue.

For countries that issue debt in foreign currencies, the interest expenditure calculated in simple terms via multiplying the principal amounts of bonds by the interest rate is not the sole source of cost. The changes in the value of the debt, measured in the local currency, due to fluctuations in the exchange rate also adds to the cost of debt.

Debt management offices that engage in frequent secondary market activities such as debt buy-backs or bond exchanges may also follow the marked-to-market value of their debt portfolios. Marked-to-market value of a bond shows its value when measured with respect to current market indicators, i.e. the prevailing interest and exchange rates; and the difference between its issue price and market value is the buy-back cost for the government. For a country that redeems bonds at maturity at the interest rates or prices set at the time of issuance, the marked-to-market value is of no relevance.

When costs are distributed over a number of years, they can be measured in a present value basis. They can also be normalized with respect to a macroeconomic magnitude such as the GDP or the size of debt portfolio to allow period wise comparisons.

Even though the relevant cost definition may differ from country to country, the common aim in PDM is to minimize the cost of debt. It will be tax payers who will be paying back the debt and one of the main objectives of debt management offices is to find the necessary funds at the lowest possible cost in line with the citizens' expectations.

2.1.2 Market Risk

A well-known characteristic of financial markets is that there is a trade-off between return and risk. Generally, the higher the returns from an investment, the higher are the associated risks. Considering the fact that an investor's return on a financial instrument is a cost for the issuer, the "risk/return trade-off" concept has its mirror image for the government as the "cost-risk dilemma".

The cost and market risk objectives are generally conflicting by their nature, as short-term interest rates are usually lower than longer-term rates. This is also true in an economy where interest rates tend to decline. A good example to such a situation is the case of countries that went through a process of economic convergence as they were candidates for the European Union (EU). In the accession process, these countries saw the convergence of levels of the variables in their economies to EU standards.

In such a context, it would be less costly for the government to issue short-term debt to make use of lower or declining interest rates. The aim in issuing short term bills or longer term variable rate bonds indexed to short-term interest rates is to shorten the interest rate fixing period of the debt stock so that each time the interest rates are fixed there will be less cost on government debt. This policy will expectedly serve for cost minimization purposes. However, rolling debt too frequently or renewing the interest rate in short intervals will then increase exposure to changing market conditions. In case of a sudden climb in interest rates in financial markets due to some external or internal reason, the cost on a major portion of the government's debt will have to increase. This is the market risk in public debt management.

For countries that have liabilities denominated in foreign currencies, the volatility of exchange rates also constitute a major portion of the market risk. Considering the fact that, the main revenues of governments are taxes which are collected in the local currency, an increase in exchange rates, i.e. a depreciation in the value of the local currency may cause a significant rise in the debt service costs while the level of revenues remains constant.

2.1.3 Liquidity Risk

Liquidity risk or re-funding/re-financing risk as it is sometimes called is also a major concern for public debt managers, especially for those in developing countries. This type of risk often arises from concentration of debt re-payments at a certain point in time. If a country has to pay back or refinance the bulk of its debt within a short period, there is always a risk concerned with accessing the required amount of funds to fulfill liabilities. The risk may arise from the level of cash reserves of the government, for example due to a decline in tax revenues or from the lenders' reluctance in renewing their loans. The latter case is similar to liquidity risk faced by financial investors. For an investor who holds a certain financial instrument, liquidity risk is in failing to find potential buyers for this instrument when he decides to sell it. For a country debt office, re-funding risk is the possibility of falling short in finding lenders who would purchase government securities at a time the government is in a cash shortage.

In under-developed or developing countries that lack a well-functioning, liquid financial market with many actors, liquidity risk is more significant. At a time of financial turmoil, there will be limited amount of funds and a small number of lenders in the market, which in turn will amplify the level of refinancing risk.

Controlling liquidity risk is crucial for a government's reputation. If a government is seen struggling for finances, this has significant consequences. The realization of such a situation will ignite some turbulence or even panic in the entire economy. Lenders who have extended credits, other government institutions that depend on the central government to fulfil their own obligations, public employees who rely on salaries paid by the government, and in the end all the citizens will be affected from the government's liquidity crises.

Thus, controlling liquidity risk is an important objective of public debt management along with containing the level of costs and market risk. Unfortunately, there can also be a trade-off between the cost and liquidity risk objectives, since reducing the liquidity risk may require long-term borrowing at high costs and/or keeping a certain level of excess cash reserves which also induce a cost for the government. Aiming to minimize the market risk may dictate to borrow fixed rate long-term bonds whose repayments accumulate at a certain point in time which in turn induces a certain level of liquidity risk.

2.1.4 Macroeconomic Objectives

Debt management strategies need to be formed in harmony with the general macroeconomic policies of a country such as the monetary and fiscal policies. Monetary policy is generally concerned with a country's money supply and is aimed at objectives of maintaining price stability, caring for the health of money markets, providing sufficient liquidity to the financial system etc. Fiscal policy is about the government's actions and plans in setting the level and composition of its revenues and expenditures.

Central banks, the main institutions responsible for developing and implementing the monetary policy, may sometimes resort to conducting open market operations in financial markets in order to control the level of money supply and to reach their objectives with regard to growth and inflation rates. These operations are often in the form of auctions which aim at injecting (withdrawing) liquidity to (from) financial markets. Auctions in which bonds are tendered are also the main tools for public debt management offices to raise the liquidity needs of the government. Therefore, liquidity management requires a decent co-ordination between these organizations. PDM offices and central banks should abstain from engaging in contradictory actions in financial markets in order to avoid harming each other in reaching their objectives. This requires sharing of information on the cash flows of the government and the level of liquidity in the financial markets, and keeping away from conducting auctions at the same time.

Debt management should also take into account the general instruments of monetary policy specific to a country. In an economy where a fixed or pegged foreign exchange (FX) rate regime is implemented by the Central Bank, the government's too much reliance on foreign currency borrowing may impact the credibility of the regime even though this is less costly.

Harmonization of debt management strategies and fiscal policy is also of vital importance. Interest expenditures that arise from the debt obligations constitute an important part of a government's budget. On the other hand, tax revenues are the main source to fulfil debt obligations. Therefore, there is a mutual relationship between fiscal and debt management policies. The projections regarding revenues and non-debt outflows of the general government is a crucial input for debt management, while the structure of debt repayments must be known to adapt the relevant fiscal policies. Debt managers should also take into account current and planned taxing regimes regarding financial instruments while deciding on what instruments to issue.

2.2 Borrowing Instruments

Public debt managers have several instruments, different types of bonds and bills, at their disposal to raise funds for the government. These vary in interest rate, denomination currencies and maturities.

Formally, a bond "is a debt instrument requiring the issuer or the borrower to repay to the lender the amount borrowed plus interest over a specified period of time" (Fabozzi, 2000, p.1). A typical bond specifies a certain date, the maturity date, when the amount borrowed is due and the level and timing of interest which will be paid over the borrowed amount. The amount due at the maturity is known as the principal. The principal is also referred to as the face value, par value, maturity value, redemption or nominal value.

Creditors lend funds in return for a certain interest rate, either fixed at the start of the loan or allowed to vary throughout the life of debt. The interest on such variable or floating rate debt can be indexed to an external indicator such as the price index in case of inflation linked bonds. Interest payments can also be linked to some commonly accepted interest rate indicator such as the London Interbank Offered Rate (Libor) or to the interest on some other security. Turkish Treasury, for example, uses the rate on its three and six month Treasury Bills to set the rate for longer term floating rate notes (FRNs). Every three or six months, the interest rate on FRNs changes depending on the realizations in the most recent bill auctions. Debt management offices have to decide on the type of interest rates on their bonds taking into account their expectations and the government's preferences.

The timing of interest payments is also a decision variable for the issuer. Zero-coupon bonds pay the entire interest at maturity while coupon bonds have interest payment intervals in which a certain portion of interest on the bond is redeemed. Typically, the coupons are paid quarterly, semi-annually or yearly. The interest the bond issuer pays in each coupon period is known as the coupon rate. The amount paid in each coupon period is calculated by multiplying the face value of the bond by the coupon rate, adjusted for the coupon period.

The term to maturity of a bond is the number of months or years over which the bond issuer promises to meet her obligations. At the maturity date, the issuer redeems the bonds by paying the amount borrowed and debt ceases to exist. Securities can be issued in various maturities ranging from three months up to fifty years.

Interest rates charged for a loan generally differ with respect to the duration of the loan. Therefore, the interest or yield on a bond depends on its maturity. The graphical representation of this relationship between the yield on debt instruments of the same issuer and maturity is known as the "yield curve". The yield curve exhibits different shapes due to the structure of financial markets and the preferences of investors. Generally, investors perceive higher risks for credits they have extended for longer maturities and this causes the interest rates charged for longer terms to be higher than those for the near-term. However, the expectation of a reduction in interest rates may cause an inversion in the shape of the yield curve. If a majority of investors believe that the yields will decline in the future, they will tend to demand longer term bonds to able to fix their investments at the currently high yield levels and sell short term bonds. Then, by forces of demand and supply, the prices of longer term bonds will increase and yields for the long-term will be lower than those for the short-term. In countries that have a wide institutional investor base, such as pension funds who demand long-term securities to able to meet their long term liabilities, again the longer-term bonds may have lower yields. Public debt managers should consider the term structure of interest rates and the issues that affect it when making bond issuance decisions.

Bonds can be issued in domestic or foreign currencies. In general, developed countries that have developed domestic financial markets prefer to issue securities in their local currencies, while other countries resort to holding some foreign currency debt to increase maturities and/or to obtain cost savings. Some countries also choose to borrow in foreign currencies to diversify their liability portfolio and to achieve an improvement in the risk profile of public debt.

Table 1 includes the issues of consideration and the corresponding types of securities. A specific security includes a dimension from all these four decision issues (such as a zero-coupon, fixed rate, local currency bill with a maturity of 3 months).

Public debt management offices have several methods to issue their bonds. The most common technique is to conduct frequent auctions where investors quote price or interest rate bids along with the amount of bonds they would like to purchase. These bids are then evaluated and the amount of funds needed is covered by issuing the appropriate quantity of securities. The price of a bond issued depends on the expected cash flows from the bond and the yield required by the investors for lending funds for the maturity of the bond. Like other financial instruments, the price of a bond is the present value of the expected cash flows from the investment.

Therefore, if investors require a higher yield from a bond, they reduce the prices they bid in the auctions.

Table 1 Type of Treasury Securities.

Decision Issues	Type of Borrowing Instrument	Explanation
Timing of Interest Payment	Zero-coupon bills/bonds	Interest and principal paid at maturity
	Coupon Bonds	Interest paid in regular coupon periods, principal paid at maturity
Type of Interest	Fixed Rate Bills/Bonds	Interest fixed at issuance, remains constant until maturity
	Variable Rate Bonds	Interest based on an index such as Libor, inflation etc.
Currency Denomination	Local Currency Bills/Bonds	
	Foreign Currency Bills/Bonds	
Maturity	Bills (3-12 months)	
	Bonds (1 year and over)	

Governments often announce auction schedules or financing programs to publicize the amounts and timings with regard to planned bond issuance schedules. These issuance programs describe the types of bonds the government is planning to issue to meet the projected financing requirement in a certain period. The announcement frequency changes from country to country. Some countries use monthly programs while some announce the auction calendar for a whole year. Early announcement of issuance strategies leaves time for market participants, i.e. potential investors to absorb the information revealed and to adjust their cash-flow schemes if they would like to participate in the auctions.

Debt managers also use public offerings or direct sales techniques to convey their bonds to a specific group of investors without inviting them to auctions. These methods help them diversify the investor base.

Using the available set of instruments, public debt managers have to find a specific portfolio combination or develop a specific issuance program that would embody decisions on maturity, currency and interest type structures and timing of issuance, in line with government's debt management objectives. The PDM problem is about reflecting the government's cost and risk preferences to the public debt portfolio and selecting the appropriate combination of financial instruments, i.e. establishing "the debt management strategy".

With all the conflicting objectives to be considered, the public debt management problem, i.e. formation of the financing strategies, is a multi-objective decision making problem with several constraints. The solution to this problem would not be trivial even without the uncertainty associated.

2.3 Constraints in Public Debt Management

The size and efficiency of a country's financial markets, the government's ability to access international markets and other macroeconomic environmental conditions impose several limitations on the type of securities the debt managers can issue. That is, the set of instruments available to PDM offices may differ from country to country.

For under-developed or developing countries, where the level of domestic savings and efficiency of internal financial markets are limited, the main option is to opt for funds from international markets, often in the form of loans from international financial institutions such as the World Bank or the International Monetary Fund (IMF). Some developing countries have a functioning domestic financial market, but they also have access to international markets and issue debt in foreign currency to lengthen maturity, since domestic lenders generally prefer shorter maturities. The advanced economies, whose markets have largely integrated with international markets, have more options in selecting currency and maturity of loans.

Given a certain instrument set, public debt managers should also consider market constraints with regard to the availability of funds, demand for different types of securities etc. In a volatile environment, creditors may not be willing to extend long-term loans, and the government's insistence on lengthening maturities may result in a funding-crisis. Institutional investors, such as pension funds, may prefer longer-term bonds while individuals may be asking for liquid short-term bonds. Preferences of banks may be different than those of insurance companies. Thus, the characteristics of different segments in the market may impose different constraints on the size of bonds to be offered.

The amount of bonds to be issued is also constrained by the financing requirement of the government, i.e. the amount of funds raised should not be less than those required by the budget. Governments generally hold a cash account which serves a buffer to cover unexpected cash needs and this allows borrowing more or less than needed for a certain period of time. However, there are also limitations to the levels of this account, i.e. governments can not over or under borrow continuously. Thus, PDM offices should consider the inter-temporal budgetary and cash account constraints while issuing securities.

2.4 The Effect of Uncertainty in PDM Decisions

An important characteristic of the multi-objective PDM problem is that decisions are made under uncertainty. Debt managers are not faced with choices that have deterministic outcomes. Debt management decisions are concerned with future actions of the government and while making strategy decisions, debt managers are not certain about the future states of nature for the relevant macro-economic variables. There is a degree of uncertainty associated with the evolution of economic factors such as interest and exchange rates that drive the cost of borrowing.

For example, a debt management office may issue floating rate bonds assuming that the interest rates will fall in the future in order to achieve a cost reduction in the bond issuance program. However, the actual cost of borrowing through floating rate securities will be dependent on the interest rate realizations during the maturity of bonds. On the other hand, issuing fixed rate securities carries the risk of locking at high rates in case of a decline in interest rates, i.e. the risk of missing the chance to make use of more favourable market conditions.

Therefore, the actual outcomes of the decisions made while formulating the issuance strategy are contingent on realizations of macro-economic variables that exhibit different types of stochasticity. In fact, it is this uncertainty that raises the need to consider market and liquidity risk objectives.

Strategies developed need to ensure that the government's debt management objectives should be covered under different scenario realizations. Debt managers have to take into account the underlying stochasticity of macroecomic factors while formulating cost-risk structure of the debt portfolio.

On the other hand, the debt strategy is not a one-off decision. The PDM problem embodies a sequence of decisions that would allow the government's debt portfolio adjust to changing environmental conditions. That is, a decision made now for the portfolio structure is subject to revision in the future depending on changing outlooks for the macro-economy. Therefore, debt managers should incorporate this need for elasticity in their decision making processes. Decisions made as of now based on current states of nature and current projections about the future must be flexible enough to be changed when needed. Debt managers need to consider the effects of the potential for adjusting decisions in the future, since the future decisions will be contingent on the previous actions and prevailing market conditions.

2.5 Literature Review on PDM Strategy Formulation

Given its importance, the problem of designing the public debt management strategy, in terms of setting the maturity and the type of instruments to be used, draws attention of both practitioners and academicians from various perspectives.

Alesina et al (1990) elaborate on the choice of maturity of public debt and argue that issuing debt at long maturities and evenly concentrated in time will boost public confidence and reduce perceived likelihood of confidence crisis about debt default. Missale and Blanchard (1994) claim that government can use the maturity of debt to show her commitment to anti-inflationary policies and thus should prefer short-maturity or indexed debt.

The tax smoothing approach assumes that the main reason for the government to change taxes is to meet the long-term financing constraint, and the objective is to smooth taxes by choosing the optimal composition of debt with respect to maturity and contingencies. The assumption is that welfare loss from taxation is higher if taxes change from one period to other than the case they are constant. Thus taxes are distorting and government debt should be structured in a way that would minimize the need for changing taxes. There is uncertainty about macroeconomic variables such as public expenditures, tax base etc. and therefore, the composition of debt matters (Barro, 1995). The argument is that if the government can issue debt with costs that are lower when net tax receipts are also lower and vice versa, then debt can serve as a buffer. In that case, the government can keep the tax rate constant by adjusting the debt pay-offs. In Lucas and Stokey (1983), the government can issue debt contingent on the outcome of public revenues and spending. Barro (2003) proposes issuance of indexed securities (tied to interest rates, inflation rate etc) when such state-contingent debt is not available.

Debt management offices, Treasuries or other public institutions in charge of managing sovereign debt take a practical point of view and apply concepts and tools derived from those employed by private financial institutions. Danish Central Bank (Danmarks Nationalbank 2005) and Swedish National Debt Office (Bergström et al. 2002) are two organizations that make use of "Cost-at-Risk" simulation models by which cost and risk performances of alternative debt management strategies are tested under various macroeconomic simulation scenarios. Hahm and Kim (2003) apply the same approach to Korea; Turkish Treasury (2004) also uses a similar model. Bolder (2003) explains the simulation model for debt strategy analysis in Canada. More recently, Bolder and Rubin (2007) try to combine simulation and optimization approaches in debt strategy analysis. Their aim is to approximate the debt management objective function through simulations, using function approximation algorithms and to optimize on this approximation.

The simulation models of PDM offices are generally derived from the "Value at Risk" (VaR) concept widely used by banks and other financial firms. VaR models are developed to obtain an estimate of the maximum probable loss that the assets may suffer within a given period a certain confidence interval. For sovereigns, this approach is modified into a "Cost-at-Risk" or "Cash-Flow-at-Risk" model.

Countries also apply other methods like stress testing or scenario analysis to compare different PDM strategies (see IMF-WorldBank, 2003 and OECD, 2005 for discussions on debt management practices of selected countries). In these practical methods macroeconomic variables are not simulated, but several plausible scenarios are created by expert judgment. The general aim is to quantify costs and risks associated with policy choices in consideration. Giavazzi and Missale (2004) use the deviations between the survey of expectations and realizations as well as ordinary forecasting methods to judge unexpected movements in macro variables and their effects on government debt.

Debt management objectives are also defined in several different ways. Georges (2003) and Barro (2003) concentrate on the minimization of the fluctuations of the government budget, rather than the interest burden on the debt stock and try to smooth the budget balance by using bonds that will serve as hedges to the movements of public revenues or expenditures. Goldfajn (1998) considers the objective of minimizing inflation in addition to that of smoothing of the budget.

A good review of theoretical and practical concepts regarding public debt management can be found in Dornbush and Draghi (1990) and Leong (1999).

CHAPTER 3

A MULTI-OBJECTIVE STOCHASTIC PROGRAMMING APPROACH

Simulation models in use at PDM offices, generally compare “time-invariant” strategies, thus assume that the borrowing strategy will be kept constant until the end of the chosen time horizon whatever the macroeconomic conditions turn out to be in time. In real life, once a strategy is adopted, it may be subject to revision given the changes in environmental conditions. Therefore, it is useful to develop a mechanism that would allow strategies adjust to varying macroeconomic circumstances dynamically. Since, in theory, there is an infinite number of ways to construct a borrowing composition, the solution space of the problem is continuous. To simplify the solution, practitioners identify a certain number of plausible and applicable alternatives and choose to compare only these, thus convert the problem into a discrete case. This seems as a reasonable approach for the simulation framework. In this thesis, we try to adapt a continuous solution space approach by developing a stochastic programming model for PDM taking into account the associated objectives.

Stochastic programming (SP) has been widely used for modelling multi-period asset management problems in order to deal with the multi-stage decisions and the uncertainty involved in the parameters regarding economic factors such as interest rates, prices of securities etc. A seminal contribution was made by Bradley and Crane (1972) who proposed a multi-stage model for bond portfolio management. More recently, Carino et al (1994) applied SP to the asset-liability management problem of the insurance industry, and Zenios et al (1998) and Topaloglou et al (2004) formulated models for a portfolio of fixed income securities. Nielsen and

Poulsen (2004) proposed a multi-stage SP model for managing mortgage backed loans. Volosov et al (2004) developed a two-stage decision model for foreign exchange exposure management. Grill and Östberg (2003) have applied an optimization approach for debt management. Yu et al (2003) provide a bibliography of SP models in financial optimization. Extensive collections of stochastic programming models for financial optimization problems can also be found in Ziemba and Mulvey (1998) and Dupacova et al (2002).

While the classical Markowitz (1952) model considers the portfolio management problem as a single period case in which the decision on which instruments to include is made at the start of the period, taking into account expected return and variance over time; multi-stage SP models allow for changes in the structure of the portfolio as time evolves[1]. In the SP framework, the decision maker starts with a certain portfolio of assets, has knowledge on the current values of the economic/financial parameters and assesses the possible movements and co-movements of those parameters in the future. He has a longer horizon and has to consider the effects of the potential for adjusting his decisions in the future on his current decision, since the future decisions will be contingent on the previous actions and prevailing market conditions. The incorporation of adaptive decisions under changing conditions provides a more realistic approach for actual problems. Fleten et al (2002) compare the performance of a multi-stage SP model against a static approach and conclude that due its adaptive nature, the SP model dominates the fixed mix static model.

Multi-stage models can also integrate important practical issues such as transaction costs (in selling and purchasing securities), spreads between ask and bid prices, trading limits, taxes etc. and allow for modelling of derivative or hedging instruments (options, future contracts, interest rate caps or floors). The general approach of multi-stage SP models in representing uncertainty is forming a scenario tree that reflects the evolution of random variables in each stage of the decision horizon, by discretizing their joint probability distributions.

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[1] There are also multi-period extensions of the mean-variance framework of Markowitz. Two examples are Steinbach (2001) and Draviam and Chellathurai (2002).

SP framework is a useful approach for tackling the multi-objective debt management problem which is a real life multi-stage decision problem under uncertainty. Although much simplification may be required regarding the number of decision stages, and the size and scope of the scenario tree due to the complexity of the solution, the results from the SP model solution may well serve as a benchmark. Moreover, the scenario tree formulation embodied in SP may provide a more clear representation of uncertainty for the decision makers in terms of explaining the dependence between the states of stochastic variables and the decisions made at intermediate stages and thus can serve as part of the decision support process.

3.1 Basics of Stochastic Programming

Stochastic Programming models were formulated and proposed in mid 1950s independently by Dantzig (1955) and Beale (1955) and have been widely studied since then. Along with the development of conceptual modelling issues, the progress in computer technology and computational methods enabled handling of large scale real life problems with a high degree of reliability and SP techniques have become applicable to real life problems.

We will now introduce some special cases of stochastic programs and elaborate on scenario generation methods in the context SP model formulation.

3.1.1 Basic Stochastic Programming Models

Stochastic programming provides a general purpose framework to model decision making under uncertainty and is regarded as a powerful modelling paradigm for different fields of application. Anticipative and adaptive models are basic types of stochastic programs and their combination leads to the recourse model which is widely applied in financial decision making problems.

The discussion in this section is based on Birge and Louveaux (1997), Kouwenbeg and Zenios (2001) and Yu et al (2003).

3.1.1.1 Anticipative Models

Anticipative models, also known as static models, are developed for cases in which the decision does not depend on specific future observations of stochastic variables, but has to consider all possible realizations for prudent planning. Once a decision is made, there is no opportunity to adapt decisions. In such models, feasibility is articulated in the form of probabilistic (or chance) constraints.

For example, let us consider a case where a decision x must be made in an uncertain environment which is described by a random vector w with support Ω . If a reliability level, α $(0 < \alpha \leq 1)$ is specified, the constraints can be expressed in the following form:

$$P\left\{w \middle| f_j(x, w) = 0, j = 1,\ldots,n\right\} \geq \alpha \tag{3.1}$$

where x is the m-dimensional vector of decision variables and $f_j : R^m \times \Omega \rightarrow R, j = 1,\ldots,n$. The objective function can also be similar:

$$P\left\{w \middle| f_0(x, w) \leq \gamma\right\} \tag{3.2}$$

where $f_0 : R^m \times \Omega \rightarrow R \bigcup \{+\infty\}$ and γ is a constant.

An anticipative model identifies a decision that meets desirable characteristics of the constraints and the objective function. In the example given, it is required that the probability of constraint violation is less than the specified threshold level.

3.1.1.2 Adaptive Models

In adaptive models, information related to the uncertainty becomes partly available before the decision is made. The main difference to anticipative models is that decision making takes place in a learning environment.

Let A be the set of all relevant information available by observation. A is a subfield of all possible events and the decision x depends on the events that can be observed. x is termed $A-adapted$ or $A-measurable$. Using conditional expectation with respect to A, an adaptive SP can be formulated as follows:

Min $E\left[f_0(x(w),w)\middle|A\right]$ (3.3)

s.t. $E\left[f_j(x(w),w)\middle|A\right]=0, \quad j=1,...,n$ (3.4)

$x(w)\in X \quad almost\ surely$ (3.5)

The mapping $x:\Omega\to X$ is such that $x(w)$ is $A-measurable$.

The two extreme cases occur when there is complete and no information. In the absence of any information, the model reduces to an anticipative form. When there is full information about uncertainty, the model turns into what is known as a "distribution model" which characterizes the distribution of the objective function.

3.1.1.3 Recourse Models

The recourse model combines the anticipative and adaptive models. This framework tries to identify a strategy that not only anticipates future realizations but also takes into account temporarily available information about the state of stochastic variables. Thus, the model can adapt by taking recourse decisions. For example, in an asset management problem, to formulate the most profitable portfolio management strategy, a financial manager should consider the future movement of asset returns (anticipation) together with the requirement to rebalance the portfolio composition as prices change and cash flows from the assets are realized (adaptation).

A two-stage SP model with recourse can be formulated as follows:

Min $f(x)+E[\Psi(x,w)]$ (3.6)

s.t $Ax=b$ (3.7)

$x\in R_+^{m_0}$ (3.8)

where x is the m_0 dimensional vector of first stage decisions made before the random variables are observed (anticipative) and $\Psi(x,w)$ is the optimal value for the following program:

Min $g(y,w)$ (3.9)

s.t $W(w)y=h(w)-T(w)x$ (3.10)

$y\in R_+^{m_1}$ (3.11)

In this program, y is the m_1 dimensional vector of second stage decisions made after the random variables are observed, thus these decisions are adaptive. $g(y,w)$ denotes the cost function in the second stage. Parameters $T(w)$, $W(w)$ and $h(w)$ are functions of the random vector w. T stands for the technology matrix and contains the coefficients that convert the first stage decision x into resources for the second stage. W is the recourse matrix while h denotes the resource vector for the second stage.

In this formulation, the second stage problem tries to identify a decision, y that minimizes the cost in the second stage for a given value of x, the first stage decision. Once a first-stage decision is made, some realization of the random variables can also be observed. Then, the two-stage program with recourse is about optimizing the cost of the first-stage decision and the expected cost of the second-stage decisions. This can generally be formulated as follows:

$$\text{Min} \quad f(x) + E\left[\min_{y \in R_+^{m_1}} \left\{ g(y,w) \middle| T(w)x + W(w)y = h(w) \right\}\right] \tag{3.12}$$

$$\text{s.t} \quad Ax = b \tag{3.13}$$

$$x \in R_+^{m_0} \tag{3.14}$$

The recourse problem is not restricted to two-stage formulations. It is possible that observations about stochastic variables are made at different points in time and decisions are revised accordingly. This leads to the formulation of multi-stage problem where stages correspond to time instances when some information is revealed and a decision can be made.

3.1.1.4 Deterministic Equivalent Formulation

Deterministic equivalent formulations consider the cases where the random variable w has a discrete distribution with finite support $\Omega = \{w^1, w^2, ..., w^N\}$, which is called as the scenario set. If p^s denotes the probability of realization of the sth scenario w^s ($p^s > 0$ and $\sum_{l=1}^{N} p^s = 1$) , then the expected value of the second stage problem can be written as:

$$E[\Psi(x,w)] = \sum_{l=1}^{N} p^s \Psi(x, w^s) \tag{3.15}$$

A different second stage decision is made for each realization of the random vector, $w^s \in \Omega$, If this is denoted by y^s , the resulting second stage problem can be expressed as:

$$\text{Min} \quad g(y^s, w^s) \tag{3.16}$$

$$\text{s.t.} \quad W(w^s)y^s = h(w^s) - T(w^s)x \tag{3.17}$$

$$y^s \in R_+^{m_1} \tag{3.18}$$

Combining the above, the deterministic equivalent formulation of the two-stage model turns out to be as follows:

$$\text{Min} \quad f(x) + \sum_{l=1}^{N} p^s g(y^s, w^s) \tag{3.19}$$

$$\text{s.t} \quad Ax = b \tag{3.20}$$

$$W(w^s)y^s = h(w^s) - T(w^s)x \quad \text{for all } w^s \in \Omega \tag{3.21}$$

$$x \in R_+^{m_0} \tag{3.22}$$

$$y^s \in R_+^{m_1} \tag{3.23}$$

3.1.2 Scenario Generation in Stochastic Programming Models

A deterministic equivalent stochastic programming model is based on a scenario tree (or event tree) representation of the movement of stochastic variables in time. Each branch of the tree denotes a different path of evolution for the relevant random variables. The scenario tree has some decision nodes that represent the stages where the decision maker(s) decide on the courses of action to be pursued. The branches of the scenario tree disseminate from these decision nodes and correspond to alternative states of nature for the stochastic variables after each decision stage. The model is solved on this discretization and the solution determines an optimal decision for each node based on the information set available at that point. Therefore, constructing a "good" scenario tree that approximates the real stochastic process is a key issue for the success of the SP model.

3.1.2.1 Overview of Scenario Generation Methods

Scenario generation has been an active field of research within the SP context and several alternative methods have been proposed for creating "good" scenario trees. Yu et al (2003) and Kaut and Wallace (2003) provide brief overviews of some common methods available for scenario tree generation.

The simplest approach for generating scenario is to use historical data regarding random variables without any modeling and claim that future will replicate the past (e.g. sampling from past yields from different points in time for generating scenarios for bond returns). This method allows for scenario generation without assuming any specific distributional form for the random variables. Bootstrapping historical data is a common method employed in Value-at-Risk analysis known as "Historical Simulation". A drawback is that the approach is backward looking and does not represent expectations for the future. Thus, the results may be dominated by a "single, recent, specific crises and it is very difficult to test other assumptions" (Marrison, 2002, pp. 118)

Another approach that does not rely on distributional assumptions is to use the empirical characteristics of random variables and try to create scenarios that replicate those such as the moment matching method of Hoyland and Wallace (2001). In this approach, a scenario tree that matches the specified target values for the random variables, including correlations in-between, is generated. The users are allowed to specify the statistical properties (moments) that are relevant and the idea is to minimize some distance measure between these specified properties and the properties of the generated outcomes on the scenario tree. Hoyland et al (2003) propose an algorithm to speed up this scenario generation method.

A more sophisticated approach requires statistical or econometrical modelling that would capture the characteristics of the movements (and co-movements) of random variables in time. Boender (1997) uses a vector autoregressive (VAR) time series model to generate asset returns and wage increase scenarios for Dutch pension funds. Villaverde (2003) presents two VAR models including US, European and Japanese assets and exchange rates.

In Pflug (2001), the method to generate the discrete scenario tree is based on the objective of minimizing the "approximation error". This "optimal discretization method" tries to generate the discrete approximation in such a way that the "approximation error", i.e. the "difference between the optimal value of the underlying problem and the value found by inserting the solution of approximate problem" is smallest.

Research efforts in the field of scenario generation has also concentrated on reducing the number of scenarios in a given scenario tree to control model complexity while preserving the degree of approximation. The approach of Heitsch and Römisch (2005) is to bundle and delete some scenarios repeatedly from a pre-supplied multivariate scenario tree generated from historical or simulated data series. They employ a certain distance metric and proceed by uniting or deleting scenarios that are "close" to each other to obtain a tree, smaller than the given scenario fan, which maintains to be a "good" approximation.

Since a scenario tree representation contains a limited number of branches, the problem solved is only an approximation of the real problem and thus the "quality" of the scenario tree is extremely important for the "quality" of the solution. The model solutions can be hardly relied if the scenario tree we use is far from representing the true stochastic process. Naturally, the higher the number of scenarios on the scenario tree, the better is the degree of representation. However, that comes along with an amplification in the complexity of the model, i.e. an increase in solution times, and thus, we need to restrict the size of the tree in order to preserve the ability to solve the model. Here lies a trade-off between having a good approximation of the real stochastic process and controlling the dimension of the SP model.

3.1.2.2 Evaluation of Scenario Tree Generation Methods

Despite the importance of scenario tree generation in the SP framework, to the best of our knowledge, there has been little research on the assessment of the representative capacity of scenario trees. Kaut and Wallace (2003) focus on this issue and discuss the evaluation of the quality of scenario generation methods, defining some minimal requirements. Specifically, they propose two measures to test the suitability of a certain generation method for a given SP model: one related with the robustness of the tree generator (stability) and the other regarding the bias it contains.

If the scenario tree generation method is stochastic, it can generate different instances in different runs. In that case, we need to ensure that solving the SP model on different trees, generated by the same method, yields similar optimal values. Thus, by what they define as "in-sample" stability, Kaut and Wallace (2003) propose that the optimal objective values obtained in the SP model based on different scenario tree instances should be approximately identical. While "in-sample stability" is concerned with the variability of the optimal objective function value, "out-of-sample stability" is related with the performance of the optimal solutions in the decision space. In this regard, the authors propose the evaluation of the solutions of the SP model in the "true" problem and test whether solutions obtained on different scenario trees yield similar results when plugged in the real problem. However, this is not always possible since we may not have full information about the actual distributions that drive our stochastic variables.

To ensure that the scenario generation method contains no bias, we need to compare the optimal values in the scenario based problem to that of the true problem and see whether or not they are close to each other. This is again impossible in most cases, since this requires solving the true problem optimally. As a proxy, Kaut and Wallace (2003) recommend the employment of a larger "reference tree" which is believed to have a better representation of the true stochastic process and use the results from this as a benchmark to test for a possible bias.

3.2 The Multi-objective Public Debt Management Model

We now formulate the Public Debt Management Problem using the Stochastic Programming approach. We incorporate relevant criteria and develop a deterministic equivalent model based on a scenario tree representation of macroeconomic factors that affect the cost of public debt. The model has a multi-stage structure that takes into account sequential decisions concerned with debt issuance policies.

The government has to decide on the type of borrowing instruments (bonds) to be issued to meet the financing requirement in a given planning period. Our model aims to assist the formulation of the issuance calendar which includes the timing and amounts of bonds to be issued. The objective is to specify a sequence of bond issuance decisions at discrete points in time.

We formulate the debt management problem as a linear programming model. In simulation based approaches, the general methodology is to assume that the government is to meet its funding requirement by applying a fixed strategy which dictates the proportions of instruments to be issued in each period. Thus, the PDM office selects a certain set of weights for the bonds to be used and issues the same proportion of securities in each time step whatever the financing need is. Using these weights as decision variables in an optimization framework results in a non-linear and unfortunately non-convex problem structure. A bond issued in the first period is to be paid back in one of the following periods which adds up to the financing requirement in that phase which, in turn, is to be financed with the same (or another) set of weights. That induces a multiplicative form for the decision variables contained in the problem and to simplify the case and to ensure the optimality of the problem, we try to develop a linear program.

We present a general n-stage model in which each period is divided into several sub-periods, t. (If the periods correspond to years, sub-periods can be months or quarters). Scenarios unfold in each sub-period. Decisions are made at start of each period for the sub-periods contained in that period, i.e. issuance decisions are not revised in each sub-period, but only at decision stages, the scenarios between decision stages combine to form a sequence of joint realizations for a certain period. These sequences of scenarios are linked at the decision nodes and we have scenario paths covering the entire planning horizon.

We assume that at the beginning of each year the government sets a borrowing strategy, which embodies the timing and amount of bonds to be offered in each month (or quarter) of the following year and revises this strategy annually. The debt manager starts with a given liability cash flow scheme (arising from the current debt portfolio) and a set of anticipated scenarios about future states of relevant macroeconomic variables such as the interest and exchange rates. Based on the given scenario set, he decides on the issuance policy for each sub-period (months or quarters) within the next year and as a result, at the start of next year he has a new liability portfolio. He now has to make a new set of decisions incorporating this new portfolio structure, thus the updated cash flow scheme contingent on the scenario realization in the interim (first year) and the current scenario tree about the evolution of stochastic variables. Thus decisions, other than the first stage decision, are path-dependent and we have a stochastic programming problem with recourse. Figure 2 illustrates the structure of a problem with 3 periods each divided into 4 quarters.

One main assumption we make for our model is that the macroeconomic environment is independent of the government's policy actions with regard to public borrowing. That is, the amount and the type of the bonds the government decides to issue do not effect the level of prevailing interest rates in the market and the government can issue any amount of bonds without changing the interest rate. This is not an unrealistic assumption for countries that have deep and liquid bond markets with many issuers and lenders.

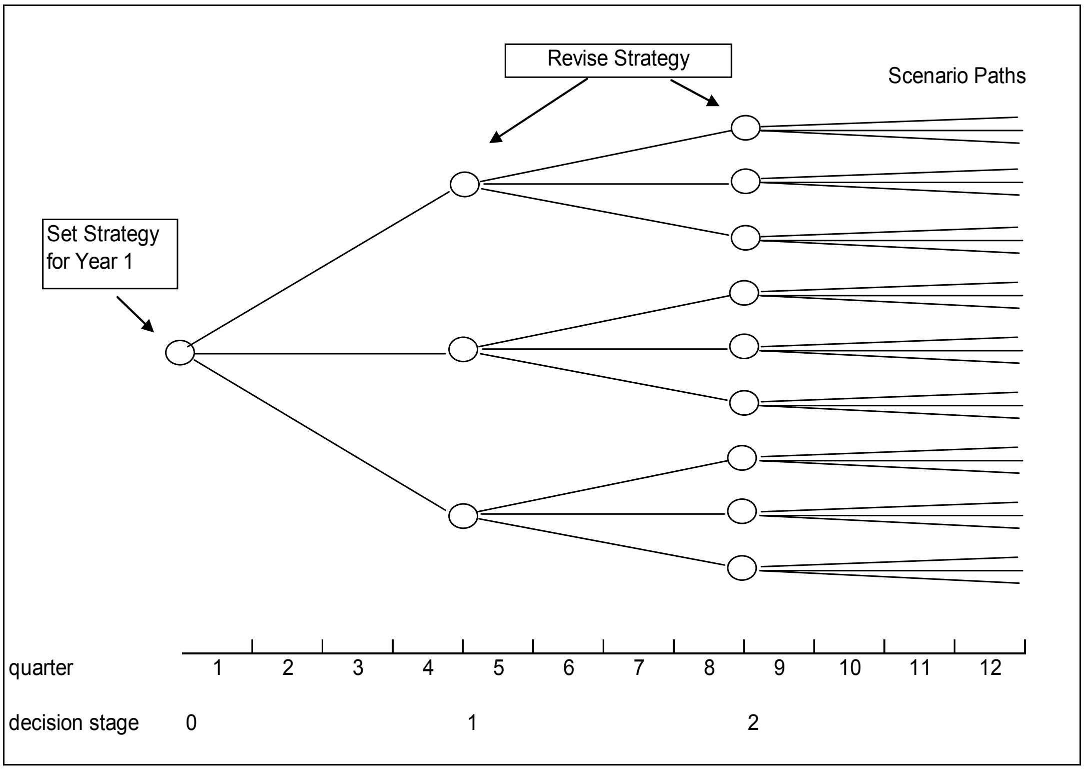

Figure 2 A Sample Scenario Tree.

3.2.1 Objectives of the Model

We formulate the PDM problem as a tri-criteria model, accounting for the objectives of minimizing the cost of raising debt, the market risk and the liquidity risk. One can also think of other objectives of debt management such as increasing the investor base for government securities, improving efficiency in the local markets, aligning borrowing strategies with other macroeconomic policies of the government such as the monetary and fiscal policies etc. Here, we adopt a financial point of view and see the problem as portfolio management exercise.

In the following sections, we elaborate on alternative formulations of the financial objectives of government debt management.

3.2.1.1 Cost

It is possible to define the costs and thus the variation of costs associated with public debt in several ways: The cost of debt can be measured by the market value of debt stock, present or nominal value of future interest cash flows, accrual based interest payments etc. Each debt office tracks several cost measures depending on the prevailing accounting principles and its own market activities. For countries that resort to debt buy-back and exchange operations frequently, the market-value of the bonds can be a relevant cost indicator, while for debt offices that prefer to redeem bonds at maturity, the interest expenditure is the appropriate measure.

We assume that the debt managers aim to minimize the expected value of their relevant "cost" measure over the decision horizon. In our model, we only account for the cost of bonds issued during the decision horizon as the cost of bonds already in the stock will be same for all alternative borrowing strategies. If the model is extended to include buybacks and debt exchanges that would allow for decisions on changing the structure of the starting debt stock, then the cost definition can be widened to include all liabilities including those fixed before time t=0.

3.2.1.2 Market Risk

Market risk is generally defined as the risk of an increase in costs, which again can be measured in several ways. Approximating this risk with the standard deviation as in the classical Markowitz model would result in a quadratic optimization problem. Thus, we consider other measures to preserve LP solvability (see Mansini et al. 2003 for linear risk measures used in portfolio optimization models).

The "Value-at-Risk" (VaR) measure is a very popular concept which is widely used by private banks and other financial firms. VaR models, used to obtain "a measure of the maximum potential change in value of a portfolio of financial instruments with a given probability over a pre-set horizon" (RiskMetrics, 1996, p.6) has even become part of the regulatory measures in the banking sector. Despite its popularity, it has been shown that VaR has undesirable mathematical characteristics such as non-convexity and non-subadditivity (VaR of a portfolio can be larger than the total of that of individual assets) and it is difficult to optimize when it is calculated from scenarios (see Pflug 2000). VaR does not either provide any information about the level of risk if the confidence level is exceeded.

The "Conditional Value-at-Risk" (CVaR) also referred as the "mean excess loss" or the "expected shortfall" emerged as an alternative risk measure as a response to the limitations of VaR. While the VaR of a portfolio is the maximum amount of loss expected over a certain horizon in a given confidence level, the portfolio's CVaR is the expected loss given that the loss is greater than (or equal to) its VaR. In other words, it is the expected value of 100α % worst costs over the entire scenario set at a given level of α. Pflug (2000) has shown that CVaR possesses the required properties of coherent risk measures in the sense identified by Artzner et al. (1999). Rockafellar and Uryasev (2000) show that CVaR, which quantifies the conditional expectation of losses when VaR is exceeded, can be efficiently minimized using linear programming in a scenario based framework. More discussion on the Conditional Value-at-Risk concept is provided in Appendix-A.

For a government who is concerned with the level of interest costs rather than the value of the debt portfolio, the CVaR measure can be turned into a Conditional Cost at Risk (CCaR) metric.

The worst-case cost can also be used as a measure of the market risk. The government might also have a target level for the debt service expenditures, and any deviation above this level due to market conditions can be a measure of market risk. If the high deviations are more important than lower ones, one can assign different weights to different levels of excess cost and invent a piece-wise linear objective function.

3.2.1.3 Liquidity Risk

While the cost and the market risk can be measured in accounting terms, liquidity or re-financing risk, as it is also called, is associated with the actual debt service (or total) cash flows of the government. It can precisely be defined as the threat that at the time of debt repayment, the government will lack the necessary funds, have difficulty in raising new debt and fail in fulfilling its objectives. To avoid such adversities, it is common practice for public debt managers to smooth out debt repayments and try to avoid concentration of paybacks in certain periods to control liquidity risk. For example, Sweden plans its borrowing in such a way that no more than a certain proportion of debt matures over a 12 month period (IMF-World Bank 2003).

This precaution can be matched with a measure that would quantify the variability of cash flows through time. In our modeling framework, we can compute the deviation of the cash flows received and paid by the government in each time step for every scenario and produce an expected variability magnitude over all scenarios in the problem with an approach similar to that of Bergström et al. (2002) Minimizing this "Mean In-Scenario Variation" measure, computed over debt amortization and interest payments may serve as a means for smoothing out debt repayments, while computing the same for net cash-flows of the government will help match the inflows and outflows. The government may also be interested in containing the maximum possible "in-scenario variation" of cash flows rather than its expected value and thus can also adopt a "minimax" type objective function.

Similar to market risk, the "in-scenario (net) cash flow variation" can be measured by the standard deviation of cash flows in each time step over the entire time horizon for a single scenario. To avoid non-linearity, we can again adopt an "expected excess cash flow" type of function for the refinancing risk. The "mean absolute deviation", which averages the absolute values of digressions from the mean or a target level can as well be used as a measure even though it attaches equal importance to all degrees of variation. A weighted absolute variation measure can be a remedy if different degrees of deviation from a target level bring different concerns for the government.

The highest possible debt service level in single time step over all covered scenarios (not in a single one) can also be treated as a signal of liquidity risk and a "minimax" type objective function accounting for this highest level may serve for preventing high concentration of debt in one period. Given that excessive amounts of debt re-payed in a small interval implies a critical level of refinancing risk for the government, the minimization of the highest possible cash outflow can be a notable intention for PDM units. We should note this formulation would result in a concentration of debt repayments in periods beyond the planning horizon if we are operating in a short decision horizon.

3.2.2 Notation

The model is based on cash-flow equations that guarantee that the total outflows of the government match the inflows. Since our model aims to determine the amounts of different types of bonds to be issued considering the given scenario set, the borrowing strategy should cover all possible scenarios. We include a cash account in our model that would absorb any excess or short borrowing that might occur when certain scenarios are realized since the cash outflows are based on some parameters that are scenario specific. Thus, the debt managers set the amount of each bond to be issued in all sub-periods of the following period considering the possibilities for the level of the financing requirement. If, in some cases, the total debt raised is more (less) than needed, the excess (short) amount is injected into (withdrawn from) the cash account of the government.

We first define the parameters of the model:

3.2.2.1 Parameters and Index Sets

T : decision horizon

I : number of periods

T_i : length of period i, i=1,…,I

t : time index (denoting subperiods), t=1,...,T

S : the scenario set

s : scenario index, $s \in S$

N : decision stages (the beginning of each period)

p_s : probability associated with scenario s.

J_1 : set of zero-coupon bonds/bills (bonds that pay interest at maturity)

J_2 : set of variable coupon bonds/bills (with interest fixing at the start of each coupon period)

J_3 : set of fixed coupon bonds/bills

J : set of all bonds ($J = J_1 \cup J_2 \cup J_3$)

m_j : maturity of instrument j, j $\in \cup J$,

c_j : coupon period of instrument j, j$\in J_2 \cup J_3$ (We assume all coupons are semi-annual)

$u_{t,j}$: upper bound for the issuance of bond j at time t.

PS_t : Primary surplus(net non-debt cash-flow) at time t, t=1,...,12

$Y_{\tau,t,j}$: Coupon payment indicator for instrument j (j$\in J_2 \cup J_3$) issued at time τ for time t.

($Y_{\tau,t,j}$ = 1 if instrument j issued at time τ pays coupon at time t.)

$n(s,t)$: decision node for scenario s, for period t

The decisions are made at the nodes of the scenario tree, thus nodes are where scenario paths disseminate. The parameter n(s,t) denotes the node in which the issuance decision is made for time t under scenario s. For all the scenarios in period 1, the decision is made in node 0.

$n(s,t) = 0$, for $\forall$ s$\in$S and t$\leq T_1$

The scenario dependent variables are given below:

3.2.2.2 Stochastic Variables:

$r_{t,j}^s$: interest rate prevailing at time t for instrument j under scenario s.

$e_{t,j}^s$: exchange rate prevailing at time t for instrument j under scenario s. ($e_{t,j}^s$=1 for local currency instruments)

L_t^s **:** Liability payments fixed before the decision horizon (which may be scenario specific) for time t under scenario s.

The decision variables are defined for each node of the scenario tree:

3.2.2.3 Decision Variables:

$X_{t,j}^{n(s,t)}$: amount of instrument j to be issued in period t under scenario s, decided at decision point n(s,t).

3.2.2.4 Auxiliary Variables:

I_t^s : total interest paid at time t under scenario s.

D_t^s : total principal (debt) paid at time t under scenario s.

B_t^s : borrowing requirement at time t under scenario s.

TC^s : total cost for scenario s.

C_t^s : withdrawal from cash account at time t, under scenario s.

CB_t^s : level of cash account (cash balance) at time t, under scenario s.

VR : variable used in the definition of CCaR – equals to VaR at the optimal solution.

cv^s : excess cost beyond VaR for scenario s.

3.2.3 Constraints

Some of the constraints of the model are for definitional purposes while some provide for the cash flow balance regarding the government's payments, including amortization and interest payments, and cash receipts. There is also an intertemporal balance equation for the cash account. We also include constraints regarding the marketability of the bonds, as there might be bond specific limitations for the amount of issuance due to the structure of market demand. Below are the constraints of our model:

- Total principal paid back at time t, scenario s:

$$D_t^s = \sum_{j \in J} X_{t-m_j,j}^{n(s,t-m_j)} \qquad \forall t : t - m_j > 0, \forall s \tag{3.24}$$

This equation sums the principal values of all the bonds that mature at time t for a specific scenario s.

- Total interest paid at time t, scenario s:

$$I_t^s = \sum_{j \in J} X_{t-m_j,j}^{n(s,t-m_j)} \left(\frac{e_{t,j}^s}{e_{t-m_j,j}^s} - 1\right) + \sum_{j \in J_1} X_{t-m_j,j}^{n(s,t-m_j)} \; r_{t-m_j,j}^s \cdot \frac{e_{t,j}^s}{e_{t-m_j,j}^s} +$$

$$\sum_{j \in J_2} \sum_{\tau = t-m_j}^{t-1} X_{\tau,j}^{n(s,\tau)} . Y_{\tau,t,j} \; r_{t-c_j,j}^s \cdot \frac{e_{t,j}^s}{e_{\tau,j}^s} + \sum_{j \in J_3} \sum_{\tau = t-m_j}^{t-1} X_{\tau,j}^{n(s,\tau)} . Y_{\tau,t,j} \; r_{\tau,j}^s \cdot \frac{e_{t,j}^s}{e_{\tau,j}^s}$$

$$\forall t : t - m_j > 0, \forall s \tag{3.25}$$

The interest cash-flow equation for scenario s, consists of the interest paid on maturing zero coupon bonds and the coupons paid for live fixed and floating rate bonds at time t, all adjusted for changes in the underlying exchange rate. The interest paid is computed by multiplying the principal value of a bond by the applicable interest rate, which is fixed at time of issuance for zero and fixed coupon bonds and at the start of coupon period for variable rate notes. The change in the market value of debt due to exchange rate fluctuations is also included in the interest definition. This is relevant for countries that have foreign currency denominated debt.

- The cash-flow balance:

$$\sum_{j=1}^{J} X_{t,j}^{n(s,t)} + C_t^s = D_t^s + I_t^s + L_t^s - PS_t \qquad \forall t, s \tag{3.26}$$

The cash-flow balance equation indicates that the total amount of bonds issued at time t (for scenario s) and the amount used from the government's cash account should equal the sum of debt repayments, including principal and interest, and the non-debt liabilities of the government, accounting for the primary surplus available for time t.

- Cash account balance:

$$CB_t^s = CB_{t-1}^s - C_t^s \qquad \forall t, s \tag{3.27}$$

In this equation, CB_0^s is the starting cash account balance.

The cash account balance should be adjusted after each time step taking into account in and out-flows.

- Non-negativity:

$$CB_t^s \geq 0 \quad \forall t, s \tag{3.28}$$

$$X_{t,j}^{n(s,t)} \geq 0 \qquad \forall t, s, j \tag{3.29}$$

We assume that the government does not allow its cash account to deplete. The amount of bonds issued can not as well be negative (no-buybacks are allowed).

- Marketability:

$$X_{t,j}^{n(s,t)} \leq u_{t,j} \quad \forall t, s, j \tag{3.30}$$

The marketability equation accounts for limitations on demand for different types of government bonds.

3.2.4 Objective Functions

Our tri-criteria model is based on minimizing the cost, and the market and re-financing risks associated with public debt. We include some alternative formulations for the objective functions based on the discussion in the previous sections. These objectives are all subject to the constraints described in the previous sections.

3.2.4.1 Expected Cost of Debt

The expected cost can be calculated by multiplying the cost associated in each scenario with the respective probability

$$\text{Min} \sum_{s=1}^{S} p_s \, TC^s \tag{3.31}$$

Here, the cost definition (TC^s) is to be determined taking into account the relevancy of possible alternative measures. For example, if a country's debt office is operating on cash-accounting principles, then the relevant cost indicator may be the total interest payments made in the planning horizon:

$$TC^s = \sum_{t=1}^{T} I_t^s \tag{3.32}$$

3.2.4.2 Market Risk

We provide formulations for two alternative market risk measures. The CCaR value can be computed in line with Rockafellar and Uryasev (2000). We define an auxiliary variable, cv^s, which takes positive values when a certain level, VR, is exceeded. In the optimal solution, the VR value equals the associated VaR level for a given α.

$$\text{Min } \text{VR} + \frac{1}{\alpha} \sum_{s \in S} (p_s \, . \, cv^s) \tag{3.33}$$

s.t.

$$cv^s \geq TC^s - VR \quad \forall s \tag{3.34}$$

$$cv^s \geq 0 \quad \forall s \tag{3.35}$$

The worst-case cost (wcc), which can be employed as another measure of market risk, is the highest level of cost that emerges across the entire scenario set.

Min wcc,

s.t.

$$\text{wcc} \geq TC^s \quad \forall s \tag{3.36}$$

3.2.4.3 Liquidity Risk

The liquidity risk can be measured by the maximum liability payment made in a single time step and the above objective tries to minimize this across the entire scenario set.

$$\text{Min } \max_{s,t} (D_t^s + I_t^s + L_t^s) \tag{3.37}$$

The below objective accounts for the primary surplus available for debt service and approximates the liquidity risk by taking into account the net cash outflow.

$$\text{Min } \max_{s,t} \left(D_t^s + I_t^s + L_t^s - PS_t \right) \quad (3.38)$$

Minimizing the expected absolute in scenario variation, as defined above, will ensure that liability payments will be smoothed out over the decision horizon as discussed before.

$$\text{Min } p_s \cdot \sum_{s \in S} \left(\sum_{t=1}^{T} \left| D_t^s + I_t^s + L_t^s - \mu^s \right| \right) \quad (3.39)$$

$$\text{where } \mu^s = \sum_{t=1}^{T} \frac{D_t^s + I_t^s + L_t^s}{T} \quad (3.40)$$

3.3 A Simple Illustrative Model

In this section, we present a two stage model based on a scenario tree with two branches at each stage to concretize our modeling approach. The model horizon is two years and the years are not divided into sub-periods (months or quarters). For sake of simplicity, we assume that there are two financial instruments at the disposal of debt managers: two zero-coupon bonds with maturities of one and two years respectively. The government does not have a starting debt stock, i.e. there are no pre-determined liabilities to be fulfilled during the period covered by the model, other than a borrowing requirement of 10 million TRY during year one that arises from non-debt obligations (e.g. salary payments for government employees).

Then in this example,

$S = \{1,2,3,4\}$, $J = \{1,2\}$, $T = \{1,2\}$, $m_1 = 1$, $m_2 = 2$.

Let the expected evolution of the one-year interest rate be as depicted on the following scenario tree:

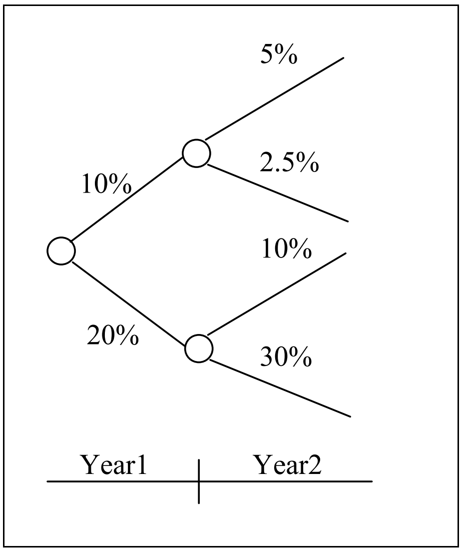

Figure 3 Scenario Tree for the One-Year Interest Rate.

Assuming a flat yield curve, the annual rate for the two-year instrument will be same as the rate for the one-year bond, i.e. the period rate for two years is twice as much as the one-year rate. Then the scenario specific values for our stochastic variables are as follows:

$$r_{1,1}^1 = r_{1,1}^2 = 10\%$$
$$r_{1,1}^3 = r_{1,1}^4 = 20\%$$
$$r_{2,1}^1 = 5\%$$
$$r_{2,1}^2 = 2.5\%$$
$$r_{2,1}^3 = 10\%$$
$$r_{2,1}^4 = 30\%$$
$$r_{t,2}^s = 2r_{t,1}^s \qquad \forall s,t$$

Assuming all four scenarios are equally likely, we have:

$$p_s = 0.25, \quad \forall s$$

Since there is a pre-fixed outflow of 10 TRY in year 1, $L_1^s = 10, \quad L_2^s = 0, \quad \forall s.$

The decision nodes corresponding to the scenarios and time periods are as follows:

n(s,1)=0, $\forall s$

n(s,2)=1, for s=1,2 and

n(s,2)=2, for s=3,4.

We have a total of six decision variables, which denote the amounts of the two bonds to be decided at each decision node:

$X_{1,1}^{0}$: amount of bond 1 to be issued in year 1, decided at node 0.

$X_{1,2}^{0}$: amount of bond 2 to be issued in year 1, decided at node 0.

$X_{2,1}^{1}$: amount of bond 1 to be issued in year 2, decided at node 1 (scenario specific decision)

$X_{2,2}^{1}$: amount of bond 2 to be issued in year 2, decided at node 1 (scenario specific decision)

$X_{2,1}^{2}$: amount of bond 1 to be issued in year 2, decided at node 2 (scenario specific decision)

$X_{2,2}^{2}$:amount of bond 2 to be issued in year 2, decided at node 2 (scenario specific decision)

The total principal to be re-paid in period 1 and 2 are calculated as follows:

$D_1^s = 0, \quad \forall s$

$D_2^s = X_{1,1}^{0}, \quad \forall s$

That is, the only maturing bond in our two year horizon is the one-year bond to be issued in the first year.

The interest payment equations are as follows:

$I_1^s = 0, \quad \forall s$

$I_2^s = 0.10 X_{1,1}^{0}$ for s=1,2.

$I_2^s = 0.20 X_{1,1}^{0}$, for s=3,4.

Then, the cash-flow balance equations will turn out to be as given:

$X_{1,1}^{0} + X_{1,2}^{0} + C_1^s = 10 \quad \forall s$ (since L_1^s is fixed for all scenarios, C_1^s is also fixed and this will reduce into one equation.)

$X_{2,1}^{1} + X_{2,2}^{1} + C_2^1 = D_2^1 + I_2^1$, for scenario 1

$X^1_{2,1}+X^1_{2,2}+C^2_2=D^2_2+I^2_2$, for scenario 2

$X^2_{2,1}+X^2_{2,2}+C^3_2=D^3_2+I^3_2$, for scenario 3

$X^2_{2,1}+X^2_{2,2}+C^4_2=D^4_2+I^4_2$, for scenario 4

Here, C^s_2 will serve as a buffer that mop up over or under financing that may arise due to scenario specific values for the interest expenditures, while the issuance amount for the bonds is made at the decision nodes without seeing the realization of interest rate.

In our model, C^s_t has to satisfy the cash-account balance equations:

$CB_0=0$,

$CB^s_1=CB_0-C^s_1 \quad \forall s$ (This will again be a single equation since C^s_1 is fixed for all scenarios).

$CB^s_2=CB^s_1-C^s_2 \ \forall s$

Assume that we would like to minimize the sum of expected interest paid in the two year period and the interest costs accrue for the bonds that do no mature at the end of the decision horizon. The bonds that are still alive at the end of year two are the two-year bond issued in year 1 and all the bonds that are issued in year two. Since the interest on these bonds is not paid within our planning period, we adjust our cost definition to include interest accrued on those instruments.

Let A^s denote the interest to accrue in scenario s. Then, assuming that interest costs accrue linearly and bonds are issued in the middle of each period, the equations for A_s are:

$$A^1=\frac{3}{4}X^0_{1,2}0.2+\frac{1}{2}X^1_{2,1}0.05+\frac{1}{4}X^1_{2,2}0.1$$

$$A^2=\frac{3}{4}X^0_{1,2}0.2+\frac{1}{2}X^1_{2,1}0.025+\frac{1}{4}X^1_{2,2}0.05$$

$$A^3=\frac{3}{4}X^0_{1,2}0.4+\frac{1}{2}X^2_{2,1}0.1+\frac{1}{4}0.\ X^2_{2,2}0.2$$

$$A^4=\frac{3}{4}X^0_{1,2}0.4+\frac{1}{2}X^2_{2,1}0.3+\frac{1}{4}0.\ X^2_{2,2}0.6$$

Then the total expected cost will be:

$$z_1 = \sum_{s=1}^{4} p_s.(A^s + I_1^s + I_2^s)$$

Solving the model by minimizing z_1, we find out the following first stage solutions (note that the second stage solutions are scenario specific):

$X_{1,1}^0$=10, $X_{1,2}^0$=0, z_1=2,2

That is, the model chooses to issue a short-term bond in the first year and roll this over in the second year. This is expected since in three of the four scenarios for year two, the interest rate is declining. Thus, to reduce the expected interest cost, the model chooses not to lock in a fixed cost for two years by not choosing the two-year bond.

We now solve the model with a risk management objective. Assume that the issuer aims to minimize the worst case cost (z_2) taking all scenarios into consideration.

$$z_2 = \max_{s \in S} \left\{ A^s + I_1^s + I_2^s \right\}$$

Solving the model minimizing z_2 yields the below solution:

$X_{1,1}^0$=0, $X_{1,2}^0$=10, z_1=2.25, z_2=3.0.

This time, the model proposes to issue a two-year bond in order not be affected by the high interest rate in scenario 4 (Note that the z_2 value in the first solution, minimization of cost, was 3.80. This is due to renewed borrowing in period 2 due to short-term borrowing in period 1).

3.4 An Integrated Simulation/Optimization Approach

Debt management is a continuous process and the government's debt portfolio has a dynamic structure since there are bonds and bills entering and leaving the debt portfolio throughout time. Public debt managers may renew their debt strategy decisions while maturing bonds and bills are rolled over by new debt issuance in line with the developments in the financial markets and in the macroeconomic environment.

Our SP-based framework takes this fact into consideration. The first stage decision is made by considering repercussions on the later stage decisions which are deemed to be scenario-specific. That is, in later stages, the decision maker can revise her strategy as scenarios are realized. However, the initial stage decision should be robust enough to meet possible outcomes contained in first-period of the scenario set and then to allow for policy changes in the upcoming periods.

In real-life, the PDM model is to be solved repeatedly in time. First the DM will solve the model based on the existing scenario tree that contains alternative future paths for stochastic variables, emanating from their current states. The initial stage decisions of the model will be implemented during the first upcoming period. Meanwhile, depending on the scenario realizations, the stochastic variables will move to new states of nature. The DM will then construct a new scenario tree that originates from those states and solve the model once again, this time starting with an updated liability portfolio based on the decisions from the previous stages. Again the current initial stage decision will be implemented and this process will be repeated in time.

In this context, we propose a method to test the "quality" of our initial stage decisions. To this aim, we adopt an integrated simulation/optimization approach similar to the method in Zenios et al (1998). In this setting, which they call as "dynamic games", alternative models are compared in a rolling-horizon environment. In a single "game", a scenario tree is generated at the beginning of the decision horizon (t_0) and the first stage decision is applied. Then the clock is moved on (to t_1) and a random scenario is assumed to be realized (for the period [t_0 t_1]). Then based on this assumed realization a new scenario tree is generated and the SP model is re-solved and the first stage decisions are implemented. The clock is moved on again and the process is repeated until the end of the decision horizon. This ends one run of the game and the game is repeated several times.

The simulation results provide for an assessment of the performance of the model in a dynamic setting similar to real life. The game can be played for alternative models for a comparison of performances in a simulation setting. Figure

4 summarizes an adoption of this approach to our PDM model on a two-stage example. The approach is illustrated in an application given in Chapter 5.

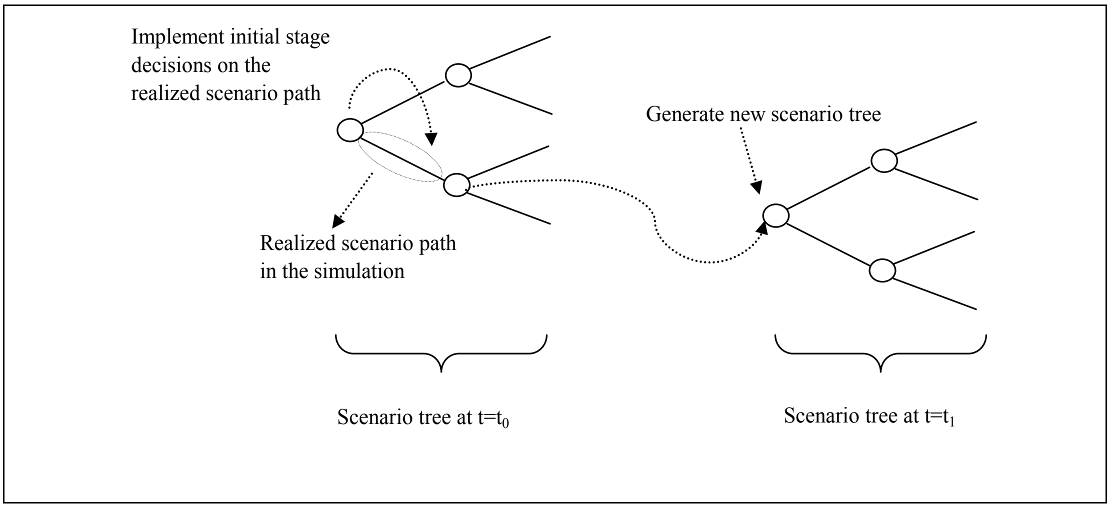

Figure 4 The Simulation/Optimization Framework.

CHAPTER 4

MULTI-CRITERIA DECISION MAKING ANALYSIS FOR THE PUBLIC DEBT MANAGEMENT PROBLEM

Several approaches derived from techniques applied by private financial institutions have been adopted for the case of the government in order to quantify the costs and risks associated in public debt management. However, debt management objectives are generally conflicting by their nature and thus public debt management is multi-objective decision making problem. Therefore there is a need for providing support to decision makers to find efficient solutions and to explore the consequences of different financing strategies.

Existing studies on public debt management formulate the case as a single or a bi-criteria problem taking into account a cost and/or a risk measure. For example, the widely used "Cost-at-Risk" simulation models of debt management offices come up with a set of alternative debt management strategies and present the degree of trade-off between the adopted cost and risk criteria, with risk defined as the variability of cost, omitting other decision criteria. However, measuring risk with a single measure embodies several problems in the context of an asset/liability management problem.

Hallerbach and Spronk (2002) argue that there are many factors that affect potential future variability in returns (costs for our case), extending from the state of interest and exchange rates to psychological factors such as the market sentiment. Even the change in interest rates is derived by changes in the level, slope and curvature of the yield curve. All these factors may inflict deviations in different directions. Moreover, dependent on the preferences of decision makers, other attributes such as the liquidity and taxability of instruments may be desirable in a portfolio. Thus, the two parameter risk-return perspective may not be sufficient and multi-criteria approaches may be of practical use.

Zopounidis (1999) encourages the use of a multiobjective vantage point in financial decision making problems arguing that speaking of optimality is illusory and narrows the view, and since decision makers are humans it is necessary to take into consideration their preferences, experiences and knowledge in order to solve these problems. Multicriteria decision making (MCDM) literature contains many examples which combine MCDM tools with the financial decision making process. Zopounidis (1999) and Steuer and Na (2003) present extensive bibliographies on the subject showing that methods like multiobjective/goal programming, outranking relations approaches, Analytical Hierarchical Process (AHP) etc. have been applied to the fields of portfolio analysis, financial planning, budgeting, risk analysis, corporate management etc.

In the previous section, we developed a multi-objective stochastic programming (SP) model that incorporates consecutive issuance decisions, taking uncertainty into account making use of a scenario tree. We now incorporate Multi-Criteria Decision Making tools on this deterministic equivalent SP model to assist decision makers (DMs) in analyzing the trade-offs between conflicting objectives. We identify some efficient solutions based on different preference structures and employ an interactive MCDM approach to guide DMs in making debt strategy solutions. We will illustrate the use of this tool in an application for PDM in Turkey.

4.1 Obtaining Efficient Solutions

We now would like to employ our model, which is of the following form, in a decision aid framework to guide policy analysis in the multi-objective PDM problem.

$$\text{"Min" } z = \{z_1(x), z_2(x), z_3(x)\} \tag{4.1}$$

$$\text{s.t } x \in X \tag{4.2}$$

where z_i, i=1,…p are the objective functions and $x \in X$ are the decision variables.

We use quotation marks since the minimization of a vector is not a well defined operation. When multiple criteria are considered, it is unusual to have a single solution that is best for all criteria. Typically, one needs to sacrifice in some criteria in order to improve in other criteria.

This is also the case for our PDM problem. Since, in general, short term rates are lower than long term rates, public debt offices find it less costly to borrow in short term maturities. However, as we have discussed, this then leads to an increased exposure to changing market conditions, i.e. a higher level of market risk. In order to contain market risk, the decision makers need to make some sacrifice from their cost reduction objectives. Thus, we need to identify the degree of trade-offs between our objective functions in order to assess alternative financing solutions.

On the other hand, these trade-offs do not exist for all possible solutions. There might be some solutions which are worse off than others in all criteria. Some solutions may be as good as others in most criteria, while being surpassed in one or more. This discussion leads us to the definition of "efficient solutions".

4.1.1 Definition of an Efficient Solution

In general, if we have p criteria, a solution $x \in X$ is said to be "efficient" if there does not exist $x' \in X$ such that $z_i(x') \leq z_i(x)$ for all i=1,…p, and $z_i(x') < z_i(x)$ for at least one i. If $x \in X$ is efficient then its image in the criterion space $\{z_1(x), z_2(x), ..., z_p(x)\}$ is said to be non-dominated.

$x \in X$ is weakly efficient if there does not exist some $x^* \in X$ such that $z_i(x^*) < z_i(x)$ for all i=1,…,p.

For an inefficient solution, there exists some solution which is equally as good in all criteria while being better in at least one. As a first step in our MCDM approach, we like to present the DMs a set of efficient solutions to be able to communicate the existent trade-offs between the objective functions

4.1.2 Identifying Efficient Solutions Using the PDM Model

In our framework, the SP model forms the basis on which the decision makers can experiment making use of MCDM approaches. We experiment with our model exploring possible achievements of the objectives and discuss the results from the model solutions to demonstrate how sovereign decision makers can employ these models as tools in making their decisions.

We first would like to present the decision makers the degree of trade-offs between alternative objectives to enable them to explore the outcomes of alternative issuance strategies. To this end, we try to obtain a set of non-dominated solutions i.e. a portion of the efficient frontier (E) by utilizing an achievement scalarizing program (See Steuer, 1986 pp. 400-405 for a discussion on achievement scalarizing functions). We first identify an ideal point (z^*) in the criterion space where each objective attains its respective minimum and then project this reference point onto the non-dominated surface. We employ a weighted Tchebycheff metric to discover the projected point on the surface, which is defined by the criterion vector that has the lowest valued weighted Tchebycheff distance to the ideal point. This projection is obtained by solving the following achievement scalarizing program:

$$\text{Min } \beta + \varepsilon \sum_{i=1}^{p} z_{i.}(x) \tag{4.3}$$

s.t.

$$\beta \geq \lambda_i . \left[z_i(x) - z_i^*(x) \right] \quad \text{i=1,…,p} \tag{4.4}$$

$$x \in X \tag{4.5}$$

where ε is a very small positive constant.

The approach is illustrated in Figure 5. The inclusion of ε in the objective guarantees that the solution obtained is non-dominated and the CCaR objective is properly computed, i.e. in line with Rockafellar and Uryasev (2000). Iteratively, by changing the values of λ_i, i.e. the weights assigned to the Tchebycheff distance with respect to the three criteria and solving the above program, we end up with a set of different points on the efficient surface.

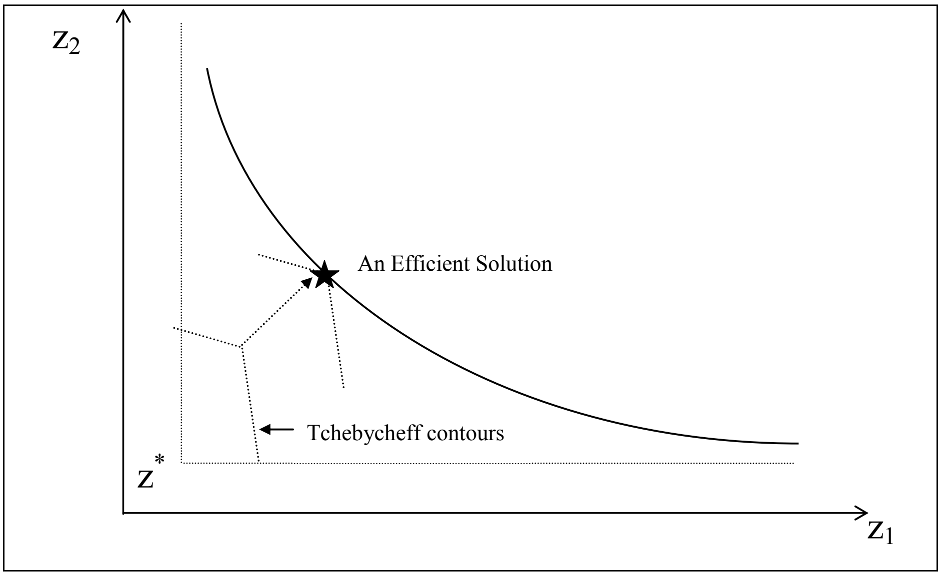

Figure 5 Illustration of the Tchebycheff Program.

4.1.3 Visual Interactive Approach of Korhonen and Laakso

It is practically not possible to identify all alternative efficient solutions in our continuous objective space. Therefore, getting decision maker involvement through the decision support process will help assess their preferences and explore distinct alternative solutions.

For example, in the visual interactive approach of Korhonen and Laakso (1986), the DMs can interact with the model solution process by specifying reference directions, d=($d_1,...,d_p$) that indicate the objectives they would like to improve based on a given solution, h=($h_1,...,h_p$). The DMs then select a preferred solution from a set of efficient solutions obtained along direction d. This provides an opportunity to explore parts of the non-dominated solution set according to decision maker choices and constitutes a learning environment for the DMs. The method is based on the solution of the following achievement scalarizing program:

$$\text{Min } \beta + \varepsilon \sum_{i=1}^{p} z_i(x) \tag{4.6}$$

s.t.

$$\beta \geq \lambda_i.\left[z_i(x) - h_i - \theta.d_i\right] \qquad \text{i=1,...,p} \tag{4.7}$$

$$x \in X \tag{4.8}$$

where ε is a very small positive constant and θ is the step size along direction d.

The DMs are assisted with a graphical display where the changes in the objective function values are depicted based on different d and θ values as illustrated in Figure 6. Korhonen and Laakso solve the achievement scalarizing program for θ going from 0 to ∞. The kinks of the objective function value trajectories occur at θ values which correspond to bases changes in the solution of the linear program.

The employment of these MCDM approaches on the SP model is illustrated in a real life application for the case of the Turkish Treasury.

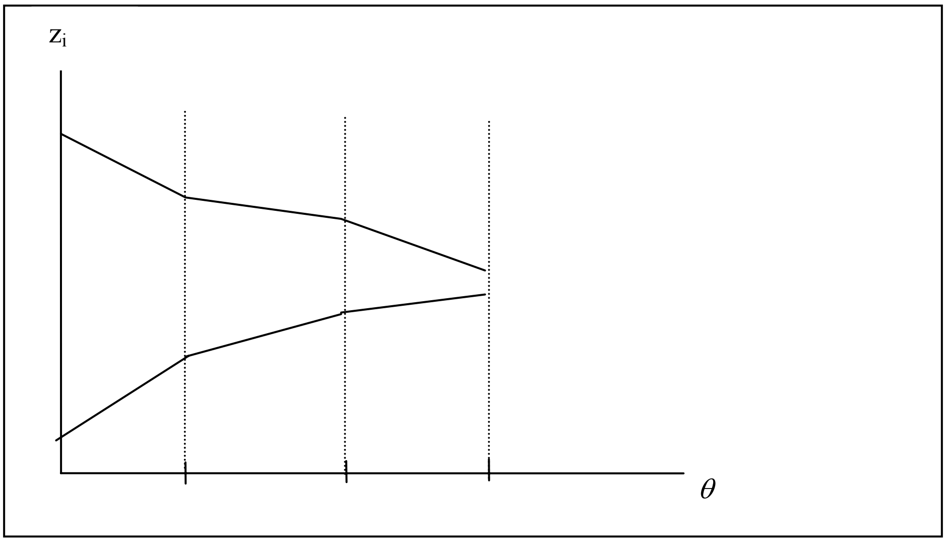

Figure 6 Illustration of the Visual Interactive Approach of Korhonen and Laakso (1986).

4.2 Accounting for Stochasticity in the SP Model

Our stochastic programming model relies on a scenario tree generated to reflect the uncertainty contained in real life such as the evolution of interest and exchange rates. In most real life cases, we do not know the underlying distributions of the relevant stochastic variables, and we need to approximate those by a scenario generator that mimics uncertainty in real life. Even if the actual processes are exactly known, the stochasticity contained in real life induces some randomness in the scenario generation mechanism. This then leads to some variation in the scenario tree instances and in turn, in the optimal objective function and decision variable values. This variation can be measured by the stability metric of Kaut and Wallace (2003) as discussed in Chapter 3.

Methods like conditional or selective sampling, scenario reduction or scenario bundling help obtain more representative scenario trees with a limited number of branches so that the model stability is maintained. However, in a scenario tree generation mechanism that involves some degree of randomness, it is not possible to remove the variation in the model outputs entirely without increasing the number of branches to infinity. In practical applications, the scenario generation mechanism is deemed to be of sufficient quality if the variation in the model outputs is contained within a certain range (see Di Domenica et al, 2003 for an example).

The dark-colored points in Figure 7 depicts a set of efficient solutions in a bi-criteria example we obtain from a single scenario tree for the case of Turkey (details of this example are given in Chapter 5). The light-colored points represent how an efficient solution may change when the same problem is solved using different scenario instances from the same generator.

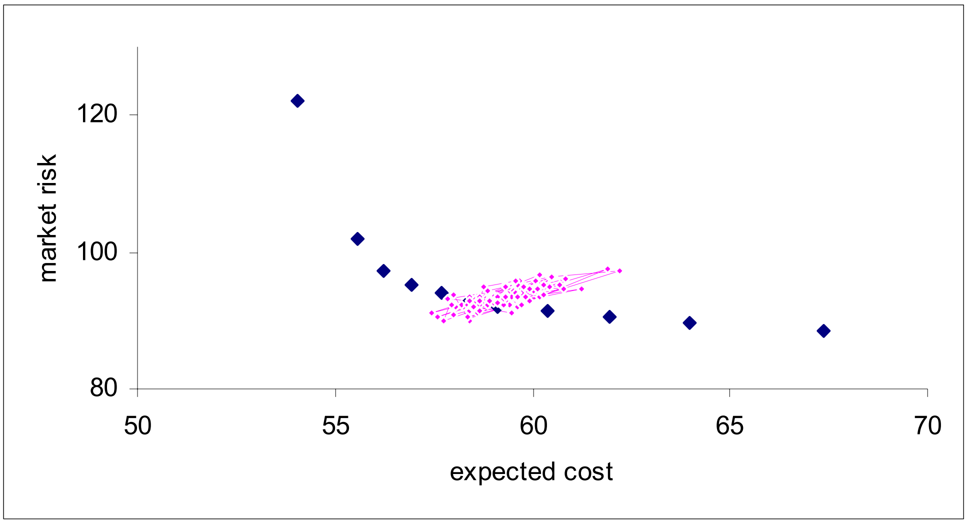

Figure 7 Variability of Efficient Solutions-Billion YTL (A Bi-criteria Example).

We incorporate the effect of stochasticity in the scenario generation mechanism into the decision making process. Our aim to make use of information obtained from solutions based on independent and identically distributed scenario trees and tools from multivariate statistical analysis to provide more guidance to decision makers. We present model results within a certain confidence interval, i.e. construct confidence regions around identified efficient solutions.

4.2.1 Multivariate Statistical Analysis

We employ multivariate statistical analysis techniques to make analyses about non-dominated solutions. The discussion in this section is not specific to our problem, but generally applicable. Since we do not know the actual multivariate distribution we are concerned with, we have to work with a large sample size, and apply large sample methods to make inferences.

It is known that large sample inferences about the mean are based on the χ^2 distribution. Let Y_j's (j=1,...,q) denote p-dimensional vectors independently sampled from the same distribution. Then $q(\overline{Y}-\mu)' S^{-1}(\overline{Y}-\mu)$ has an approximate

χ^2 distribution with p degrees of freedom, where vectors μ and $\overline{Y}$ are the population and sample means respectively, and S is the sample variance/covariance matrix (for necessary theory, especially on construction of confidence regions, see Johnson and Wichern, 2002, pp. 210-260).

$$\overline{Y} = \frac{1}{q}\sum_{j=1}^{q} Y_j \qquad (4.9)$$

$$S = \frac{1}{q-1}\sum_{j=1}^{q}(Y_j - \overline{Y})(Y_j - \overline{Y})' \qquad (4.10)$$

Thus, provided that n is large,

$$\mathrm{P}[\,q(\overline{Y}-\mu)' S^{-1}(\overline{Y}-\mu) \le \chi_p^2(\alpha)\,] = 1\text{-}\alpha \qquad (4.11)$$

Then, a 100(1-α)% confidence region for the mean of p-dimensional Y_j's is given by the ellipsoid defined in the above equality. The relative weights and direction of the axes of the confidence ellipsoid are determined by the eigenvalues and eigenvectors of the variance-covariance matrix, S of Y_i's. With $\overline{Y}$ at the center, the axes of the ellipsoid are given by:

$$\pm e_i \sqrt{\Lambda_i \chi_p^2(\alpha)}\ , \text{ i=1,...,p,} \qquad (4.12)$$

where $Se_i = \Lambda_i e_i$,i.e. e_i and Λ_i are the eigenvectors and eigenvalues of S.

We can also build simultaneous confidence intervals for the individual component means, μ_i, i.e. the means of our objective function values. One method to this end is to project the ellipsoid on the axes where we depict the objective function values. We then end up with the following 100(1-α)% simultaneous confidence statements,

$$\overline{y}^i \pm \sqrt{\chi_p^2 \frac{s_{ii}}{q}} \text{ contains } \mu_i,\ \text{i=1,...,p} \qquad (4.13)$$

where s_{ii} is the element in the ith row and ith column of S, i.e. the variance for component i. We will demonstrate these further in conjunction with the PDM problem of Turkey.

4.2.2 Constructing Confidence Regions around Efficient Solutions

The overall aim of the stochastic PDM model is to guide decision makers in making bond issuance decisions and to help them understand the trade-offs inherent in their decisions with regard to their objective criteria, z_i, i=1,...,p. With this aim, one of the tools we employ is to present the DMs with a set of non-dominated solutions that we obtain through a special case of an achievement scalarizing program, namely a Tchebycheff program, of the form given in section 4.1.2

In our stochastic setting, we exploit this approach as follows: We create a number of (q) independent scenario trees and employ our achievement scalarizing program on each of them to project their "ideal points", in terms of the optimal values of each objective function, onto their respective efficient surfaces. Figure 8 depicts this process in a bi-criteria case where z^{j*} and Y_j's, j=1,…,q denote ideal points and their projections onto the efficient frontier respectively. We first choose a direction, in terms of λ_i's and project all the ideal points in this direction. We repeat this several times changing the direction, thus ending up on different regions of the set of efficient frontiers.

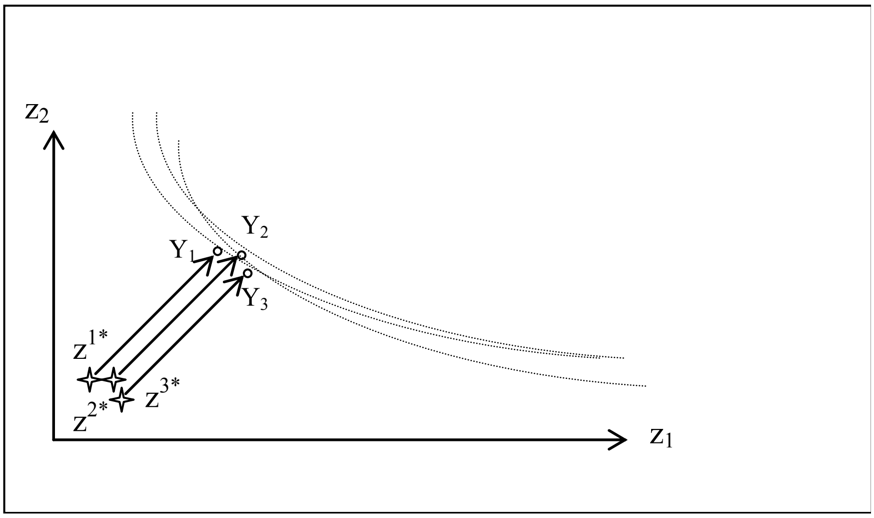

Figure 8 Projection onto the Efficient Surface.

As a result, we obtain sets of independent and identically distributed (as we are using independent instances from the same tree generator) multivariate observations (arrays of objective function values) in different parts of the "efficient frontier to be". We will describe these sets of points at different regions of the efficient surface as "efficient clusters". The light-coloured group of points in Figure 7 is an example for an efficient cluster.

We can now employ multivariate statistical tools to build our confidence regions for our multi-dimensional objective function to represent joint distributions of its components based on varying DM preferences. This framework is also illustrated in Chapter 5.

4.2.3 A Stochastic Interactive Approach

For our problem, we apply a modified version of the visual interactive approach of Korhonen and Laakso (1986) to account for uncertainty contained in the scenario generation process. In this approach we do not base our results on one scenario tree instance, but instead use the information from solutions on different scenario trees.

The procedure flows as follows: Using the achievement scalarizing program given in section 4.1.2, assuming a certain set of λ_i's, and working on 'q' scenario tree instances, we identify a starting solution for each scenario, i.e. $h_{0,i}^{st}$, st=1,...,q, i=1,…p.. Here, 'q' should be sufficiently high to be able to make large sample inferences. We then ask the DM to specify a desired direction. Given these, we solve the above program on every scenario tree instance starting from associated h_i^{st}'s for several specific θ values and present the trajectories to the DM. At this point, the DM decides whether to stop, to continue in the same direction or to change direction.

We repeat this process by replacing h_i^{st}'s with solutions of the current iteration when the DM changes direction. The method is illustrated in Figure 9. The dots correspond to independent observations for the objective function optimal values while the dashed arrows represent the direction we would like to move.

At each step, for each of the q scenarios, we have an array of observations for the values of the objective functions. That is, we have q independent observations for our p-dimensional arrays. Now, by means of multivariate statistical analysis, we can make simultaneous confidence statements for the means of the array components, i.e the objective function values. To this end, we use the variance/covariance information obtained from different scenario tree solutions. We then present the averages and confidence limits of objective function values in a graphical display to be examined by DMs. This approach is illustrated in Figure 10 where confidence bands are depicted with dashed lines.

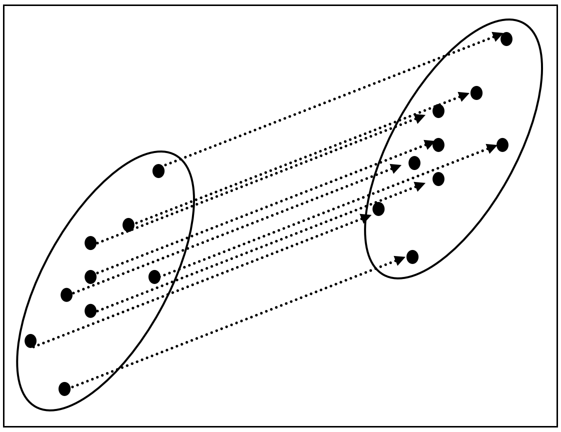

Figure 9 Illustration of the Visual Interactive Approach: Obtaining Scenario-Based Solutions as θ is incremented.

This visual representation will not only enable the DMs to evaluate the inherent trade-offs among decision criteria, but also help them have an idea about the degree of uncertainty associated with the objective function values.

The entire decision making process, a modified version of the Korhonen and Laakso (1986) method is summarized in Figure 11. The approach is applied for a real life example in the following Chapter.

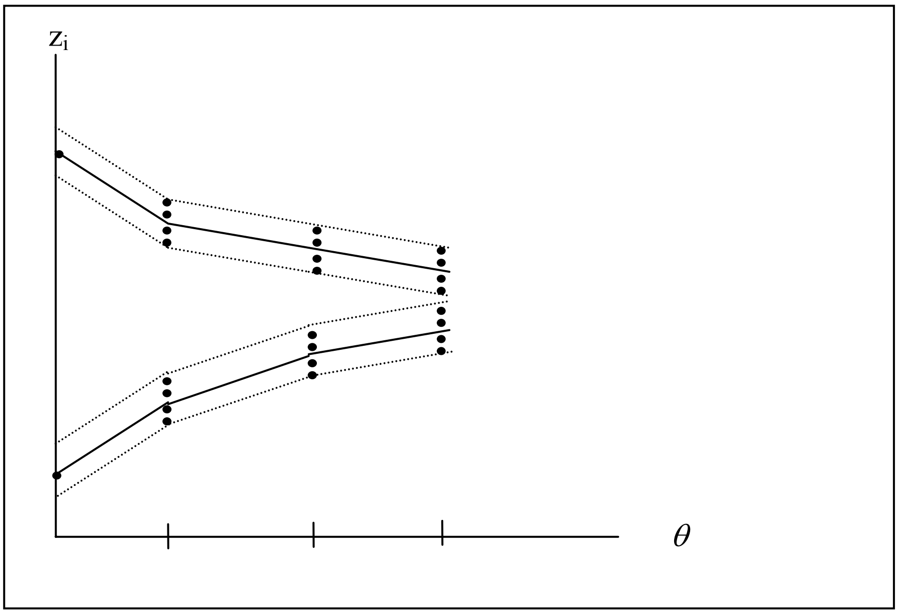

Figure 10 Illustration of the Visual Interactive Approach: Objective Function Values and Confidence Bands.

4.3 The Decision Aid Framework

In this section, we have discussed the applicability of some MCDM concepts and tools to the public debt management strategy formulation problem. In this context, the stochastic programming model we have developed constitutes the main instrument on which other methods are incorporated. We propose two main options for the employment of this deterministic equivalent model to construct a decision aid framework.

First alternative is to use the SP model on a single scenario tree instance. This single scenario tree, which depicts alternative states of nature for relevant stochastic variables, can be generated by the methods available in literature, such as moment-matching or statistical/econometrical modelling. These would both require quantitative analysis regarding historical time series of model inputs at different levels of detail.

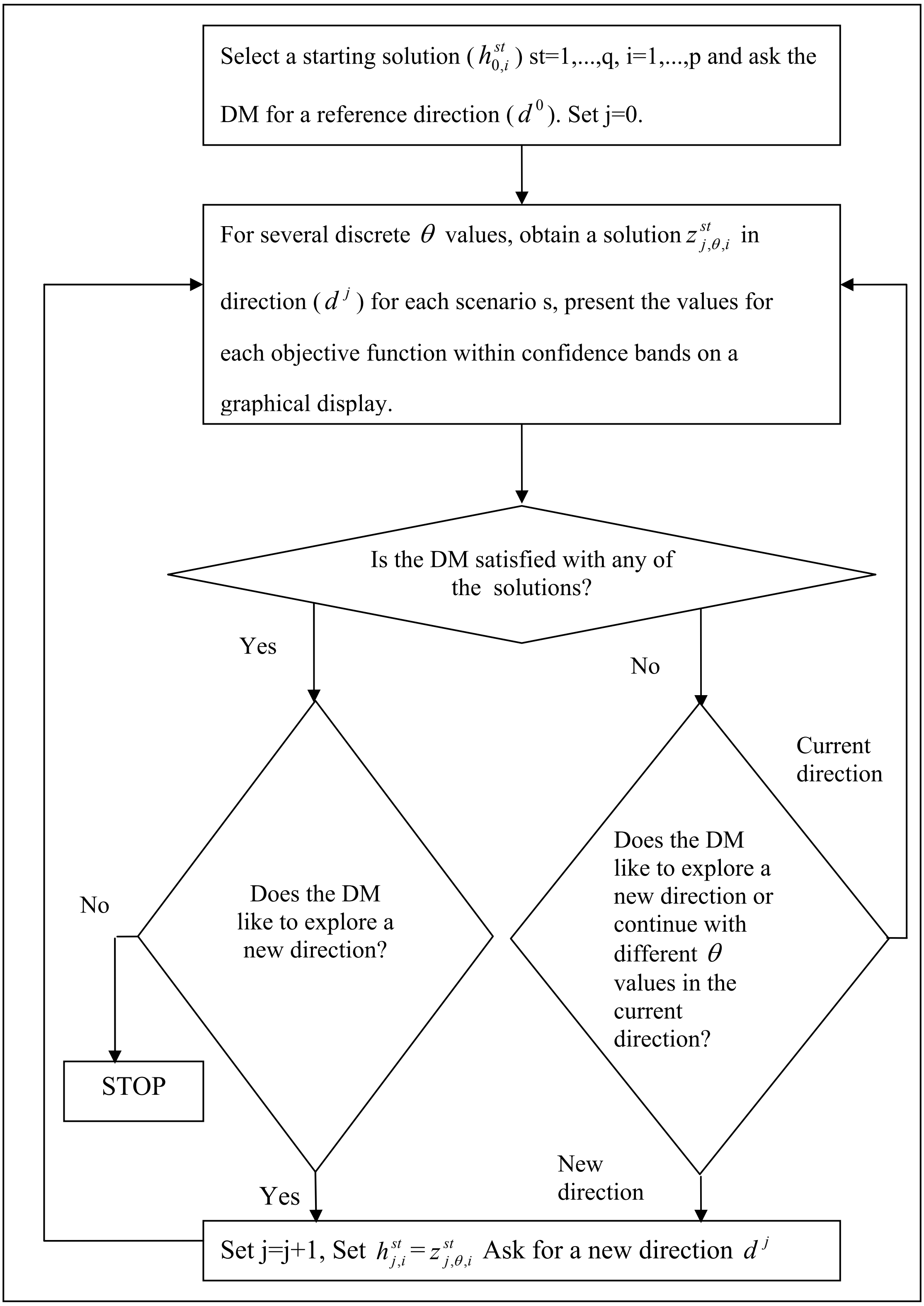

Figure 11 The Decision Making Process Using the Stochastic Interactive Algorithm.

Surveys done among market participants, i.e. bankers or representatives of other financial institutions to deduct market expectations can also be used in constructing a scenario tree. Financial data distribution companies or governmental organizations frequently conduct such surveys and announce detailed results about the general mean or median level and the range of expectations for future rates. Analysts can use this information to develop a scenario tree based on market expectations. In this regard, the moment-matching method of Hoyland and Wallace (2001) can also be employed to create a scenario tree with branches that match the values specified in these surveys.

Once a scenario tree is constructed, we can employ the achievement scalarizing program to identify portions of the efficient frontier and ask the decision makers to convey their views on alternative solutions.

If the decision makers need more guidance to compare alternative solutions or to move around solutions on the efficient frontier, the visual interactive approach can be implemented. In this method, the analysts will have the chance to get decision makers' involvement and to assess their preferences while exploring distinct alternative solutions.

When the decision makers are satisfied with a single solution, i.e. they are content with the trade-off structure they have obtained, the only thing that remains to be done is to identify the corresponding issuance strategy. This will then be implemented as part of the government's financing program.

A second alternative for the employment of the SP model is to use it in a stochastic setting. This would especially be useful if the scenario generation is stochastic. In that case, we propose to employ multivariate statistical analysis techniques to construct confidence regions around the efficient solutions identified to incorporate the inherent stochasticity. The stochastic interactive algorithm also relies on simulation and statistical analysis techniques to guide decision makers visually by presenting confidence bands around several objective function values. The decision makers will then be directed to a single preferred efficient cluster.

One main problem in using the stochastic methods is about the reflection of the selected objective function values, i.e. the preferred efficient cluster in the decision space. In most cases, different points within the selected efficient cluster will be reached by using varying issuance strategies, depending on the structure of the scenario tree instance that leads to this solution. We then need to identify a single bond issuance program which would reflect the preference structure of the decision maker in the objective space under different scenario tree instances. To this end, we propose to analyze the issuance decisions corresponding to the selected efficient cluster. In this regard, identifying common patterns in the solutions within the same cluster will be helpful to decide on a single strategy. This solution has to be robust enough to yield similar objective function values under different scenario tree instances. This argument is illustrated in the following Chapter.

We believe the developed simulation framework can also serve as a decision aid tool. Each replication of the simulation mimics the actual decision making process, and by carrying out several replications we can provide the expected cost and risk levels associated with a certain decision making "style" in rolling-horizon setting.

The rolling horizon setting we adapted replicates the real life decision making process. In Turkey, for example, debt strategy decisions made by the Debt Management Committee[2] are revised every year depending on the realizations of the previous year and on the current future outlook. The committee analyzes the existing debt position, the current state of the economic parameters and future expectations when revising the debt strategy. This strategy is implemented for a certain period, generally a year, and then is subject to revision if needed. This process can be generalized for many countries even though the revision periods or other dynamics of the decision making procedures may differ..

In the experiments in Chapter 5, the decision making style is based on an optimization approach which assumes that at each decision stage the DM aims to minimize a certain utility function in an SP model looking ahead over a set of years. The simulation results depict the possible performance of the optimization approach in a dynamic setting. One can also simulate the performances of other decision

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[2] The final decision making authority within the Treasury on strategic decisions regarding debt management.

making styles that can include basing strategy decisions on certain decision rules (or rules of thumb), using static decisions or even making random choices at each stage. Simulation results can then provide some insight for the DMs while adopting a certain decision making approach that is to be employed over and over in the dynamic environment of real life.

CHAPTER 5

APPLICATION OF THE MCDM APPROACHES: AN EXAMPLE FROM TURKISH CASE

We now illustrate the ideas discussed in the previous sections for the case of sovereign debt management in Turkey. In this implementation, we employ the generic stochastic programming model for PDM presented in Chapter 4, adopting the relevant cost and risk definitions for Turkey. The application also includes a scenario generation framework to construct scenario trees for the evolution of macroeconomic variables that affect government financing. The stochastic programming model based on the deterministic equivalent scenario tree mechanism is used as a platform for Multi-Criteria Decision Making analysis.

5.1 Background for Public Debt Management in Turkey

In Turkey, cash and debt management on behalf of the government is carried out by the Undersecretariat of Treasury. The Treasury issues debt securities to meet the financing requirement of the so-called "Central Government Budget" which covers the budgets of central government organizations such as the ministries and other government institutions like the State Highways Directorate, State Waterworks Administration etc. The financing requirement that arises from the budget deficits and re-payment of existing debt is mainly met by issuing Treasury Bills and Government Bonds in domestic financial markets and Eurobonds to international investors. There are also other forms of financing available from International Institutions such as the IMF and from foreign governmental institutions in addition to project-based credits that are specifically allocated to a public investment project rather than the current financing needs of the government cash position. This application will only focus on decision making for finances raised from financial markets in terms of security issuance.

While analyzing the accumulation of public debt in Turkey, we can identify two main reasons. One is the chronic budget deficits that became a regular phenomenon in the last decades (Figure 12) and paved the way for regular domestic bond auctions starting from 1985. As the Treasury had to finance maturing debt as well as the deficits of the current period, this led to increasing borrowing requirements, which in turn caused higher interest rates and higher amounts of debt maturing in the upcoming periods, creating some sort of a vicious circle or namely, a debt overhang. The typical phenomenon of the late 1990s was the issuance of short-term domestic debt at high real interest rates as a result of eroding public confidence.

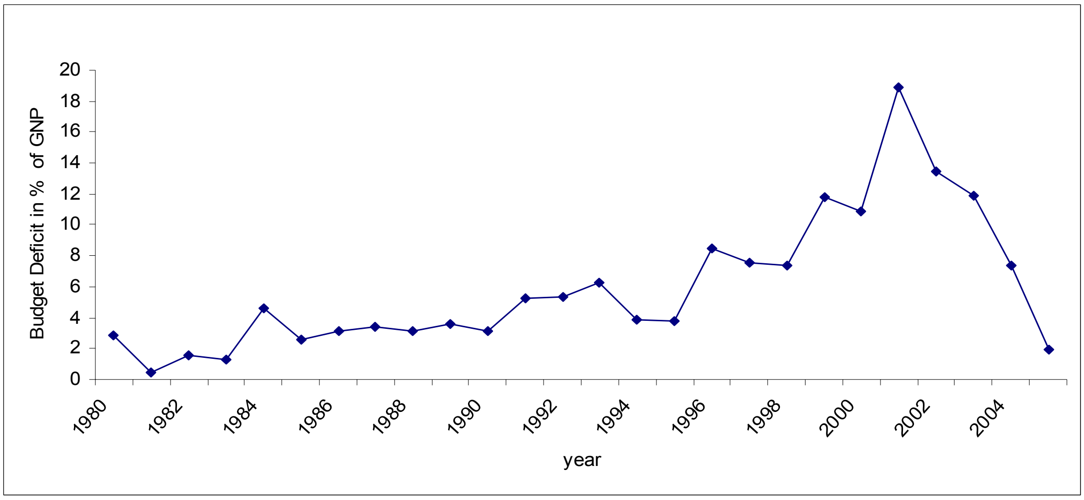


Figure 12 Budget Deficit in Turkey (in percent of GNP) Source: SPO[3].

Another reason that added to the government's financing deficit was the issuance of debt securities for bailing-out the losses incurred by public and private banks especially after the 2001 financial crisis which led to a sharp leap in the debt stock (Figure 13).

[3] State Planning Organization. www.dpt.gov.tr

As of August 2007, the total debt stock (in terms of principal value) the Treasury is responsible for managing stands at a level of 336 billion TRY (New Turkish Liras) and about 90% of this amount consists of debt raised through bond/bill issuance, with the remaining in the form of loans. Figure 14 depicts composition of the Treasury debt stock in terms of the currency and interest type structure.

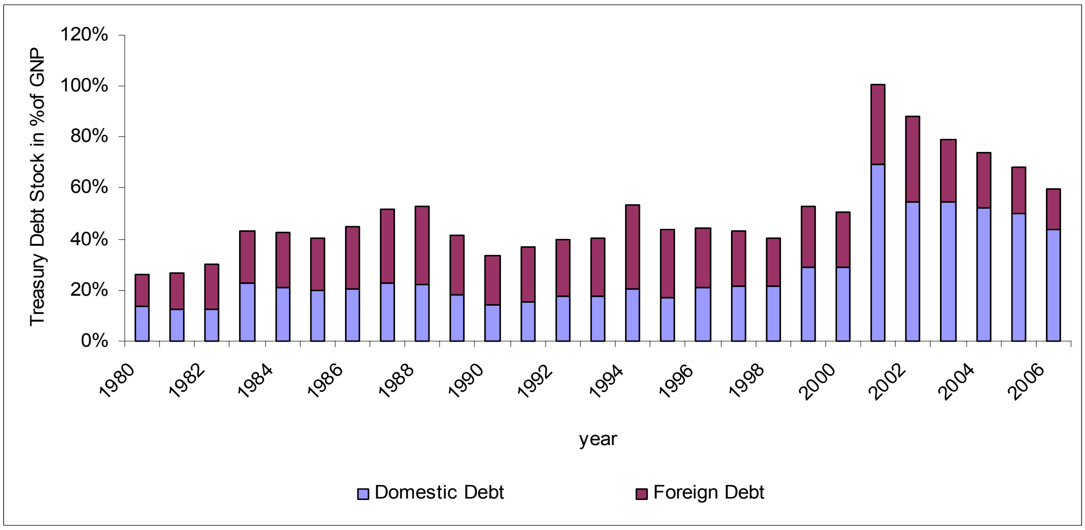


Figure 13 Central Government (Treasury) Debt Stock (in percent of GNP) Source: Turkish Treasury[4].

Despite the improvement in recent years, the level and structure of the debt stock still induces a sizeable vulnerability against adverse market movements Thus, when formulating the debt management strategy, Turkey should take into account the risk objectives in addition to cost.

This fact is also reflected in the legal framework for debt management and the "Regulation on the Principles and Procedures of Coordination and Execution of Debt and Risk Management" defines the core objective of public debt management as follows (Article 4).

[4] Source: www.hazine.gov.tr

"The execution of public debt and risk management shall be based on the following principles:

a) To follow a sustainable, transparent and accountable debt management policy that conforms to monetary and fiscal policies, considering macroeconomic balances,

b) To meet financing requirements at the lowest possible cost in the medium and long term, taking into account the risks, in addition to domestic and international market conditions."

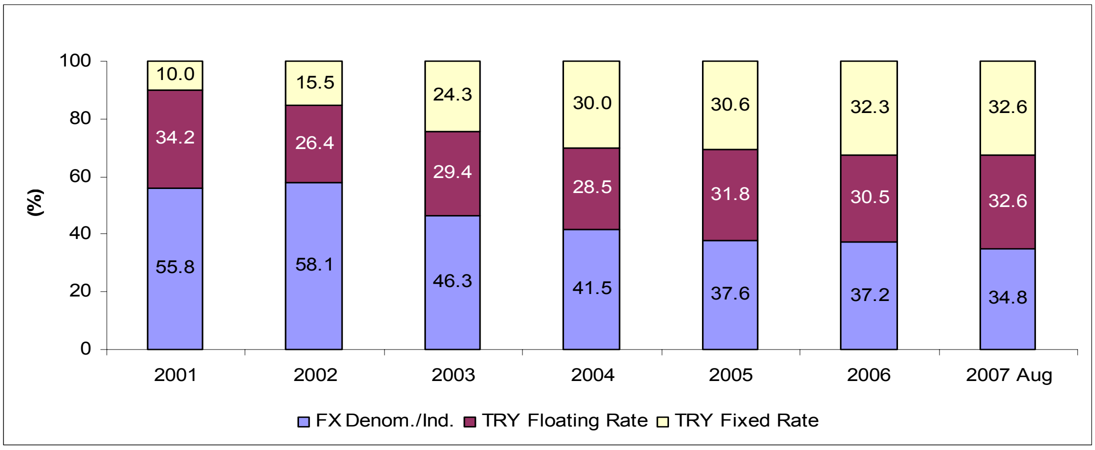


Figure 14 The Structure of the Treasury Debt Stock.

Source: Turkish Treasury.

5.2 The Stochastic Programming Model for Turkey

In the MCDM analysis for the debt strategy formulation problem of the Turkish Treasury, we employ the generic PDM model presented in Chapter 3, only modifying objective function formulations.

To adopt the relevant cost definition, we evaluate interest costs in accrual terms so that the interest payments can be attributed to the periods they are generated. However, for Turkey, the interest charge is not the sole source of cost for the government. There are bonds issued in currencies other than the numeraire currency, which is the legal tender for the country and any increase in debt repayments, including principal and interest components, due to changes in the exchange rates, adds up to the cost of debt. Thus, our cost definition covers not only the interest charges to accrue during the planning period but also the change in the market value of foreign currency denominated debt (see Turkish Treasury, 2004, p.59). Foreign currency linked debt that mature beyond the decision horizon are marked to market value at the end of the model period.

As a result, we include the following accrual cost measure in our model:

- Total accrued cost at T in scenario s:

$$A^s = \sum_{j \in J} \sum_{\tau = T - m_j + 1}^{T-1} X_{\tau,j}^{n(s,\tau)} \left(\frac{e_{T,j}^s}{e_{\tau,j}^s} \text{-}1 \right) + \sum_{j \in J_1} \sum_{\tau = T - m_j + 1}^{T-1} X_{\tau,j}^{n(s,\tau)} \frac{e_{T,j}^s}{e_{\tau,j}^s} . \left(\frac{T - \tau}{m_j} r_{\tau,j}^s \right) +$$

$$\sum_{j \in J_2} \sum_{\tau = T - m_j + 1}^{T-1} X_{\tau,j}^{n(s,\tau)} (1 - Y_{\tau,T,j}) \ r_{T - c_j + 1, j}^s . \frac{e_{T,j}^s}{e_{\tau,j}^s} . \frac{1}{c_j} +$$

$$\sum_{j \in J_3} \sum_{\tau = T - m_j + 1}^{T-1} X_{\tau,j}^{n(s,\tau)} (1 - Y_{\tau,T,j}) \ r_{\tau,j}^s . \frac{e_{T,j}^s}{e_{\tau,j}^s} . \frac{1}{c_j} \qquad \forall t : t - m_j > 0, \forall s \qquad (5.1)$$

(where c_j is 2, since the regular coupon period for Treasury bonds is 6 months)

The accrued cost is calculated for bonds and bills that have not yet matured at time T. It consists of the changes in the value of the bonds due to movements in the exchange rate and the interest that has accumulated on a bond since its issuance (for zero-coupon bonds) or its last coupon payment (for coupon bonds) up to time T. We compute the accrued interest rate by multiplying the effective interest rate by the fraction of days that have passed since the start of the coupon payment to the total number of days in the entire coupon period. We then adopt the following cost definition, which is the sum of actual interest payments made in cash and interest costs accrued.

$$TC^s = \sum_{t=1}^{T} I_t^s + A^s \qquad (5.2)$$

As the measure of market risk, we apply the Conditional Cost-at-Risk (CCaR) measure, by which we quantify the expected value of the cost of public borrowing given a certain threshold level is exceeded. In this application we calculate the CCaR values on the 10% tail distribution.

For the liquidity risk objective, we again adopt a Conditional "at-Risk" measure. We first identify the maximum cash outflow of the government for each scenario (cof^s) where

$$cof^s = \max_t (D_t^s + I_t^s + L_t^s) \tag{5.3}$$

We calculate the average of the highest 10% of the cof^s over the entire scenario set to obtain our Conditional Payment-at-Risk (CPaR) measure where α =10%.

CPaR $$\text{Min PR} + \frac{1}{\alpha} \sum_{s \in S} (p_s \, . \, cp^s) \tag{5.4}$$

s.t.

$$cp^s \geq cof^s - PR \quad \forall s \tag{5.5}$$

$$cp^s \geq 0 \quad \forall s \tag{5.6}$$

Here, PR is an auxiliary variable used in the definition of CPaR that equals to 100(1-α)% percentile value of cof^s at the optimal solution, and cp^s is the excess payment beyond PR for scenario s.

Thus our problem turns to be:

(P) "Min" z= $\{z_1, z_2, z_3\}$ where

$$z_1 = \sum_{s=1}^{S} p_s \, TC^s \tag{5.7}$$

$$z_2 = \text{VR} + \frac{1}{\alpha} \sum_{s \in S} (p_s \, cv^s) \tag{5.8}$$

$$z_3 = \text{PR} + \frac{1}{\alpha} \sum_{s \in S} (p_s \, cp^s) \tag{5.9}$$

subject to (5.1), (5.2), (5.3), (5.5), (5.6), (3.24), (3.25), (3.26), (3.27), (3.28), (3.29), (3.30), (3.34), (3.35).

The model covers a period of 3 years, in line with the central government medium term fiscal plan. We assume that the government prepares an annual borrowing program, thus we have a 3-period model. Each year is then divided into 4 quarters and at the beginning of each year the government decides on the issuance strategy for the following 4 quarters. This decision is then to be revised at the beginning of the next year. The application is based on data available as of December 2005.

As a simplification, we assume that the existing forward liabilities forecast as of that date are scenario-independent ($L_t^s = L_t \quad \forall s$) as well as the non-debt cash flows of the government ($PS_t^s = PS_t \quad \forall s$). The parameters L_t are based on debt stock data available at the Turkish Treasury web-site (www.hazine.gov.tr) and PS_t values are in line with the government's fiscal assumptions in the Medium Term Fiscal Plan (2006-2008) details of which can be found at the Official Gazette (no. 25863) dated 02.02.2005 (in Turkish).

The numeraire currency of the model is New Turkish Lira (TRY), i.e. all costs and risks are measured in that currency. Our model presents a selection of seven different kinds of bonds: four of which are TRY denominated zero-coupon bonds ranging from 3 months to 18 months in maturity. We also include in the model a 3 year TRY fixed rate coupon bond, and a 3 year TRY variable rate coupon bond indexed to the 6 month Treasury Bill yields, as well as a 3 year USD denominated fixed rate coupon bond (see Table 5). All coupons are semi-annually redeemed. The variable rate bond is assumed to be issued with a fixed spread over the prevailing 6-month interest rate. We include two marketability constraints: We assume that in one quarter the amount of 3 month bills the Treasury can issue is capped at 10 billion TRY and the market availability for the USD denominated bonds is 3 billion USD per quarter, considering the size and preferences of the lenders in those segments of the debt market.

5.3 Scenario Tree Generation

Our initial approach to scenario tree generation in the context of our debt management problem was to use a statistical model that describes the relationships among the random variables of concern, calibrated by historical data and then to sample randomly from the error term distribution of our model. To be able to maintain computational complexity, we need to work with sparsely branched multi-period scenario trees and the pure random sampling method we have adopted, led to unstable scenario trees when we kept the number of branches at a limited level. We tried to overcome this stability problem by increasing the dimension, i.e. the number of branches of the tree as much as we can, sacrificing from solution times.

We then improved our approach by incorporating ideas from research on scenario reduction/bundling. We again use our statistical model to create random scenarios, however we increase our sample size substantially, since we then try to identify similar outcomes and bundle them into discrete scenario clusters. This then allows us to reduce the number of branches in our scenario tree, and adjust the probabilities of remaining branches accordingly. This idea should be credited to the work of Grill and Östberg (2003).

5.3.1 The Statistical Model

The scenarios for the SP model were generated by a modified version of one of the macroeconomic simulation models of the Turkish Treasury, which is a vector autoregressive time series model containing the short and medium term local interest rate, the USD/TRY parity, the inflation rate (Consumer Price Index) and the Treasury's funding rate in USD denominated issues[5].

The vector autoregressive (VAR) approach models the co-movement of selected variables as functions of their own lagged values as well as the lagged values of the other variables. The following equation illustrates a VAR model for a vector Y_t of n variables, including l lagged values:

$$Y_t = \mathrm{C} + \sum_{i=1}^{l} A_i Y_{t-i} + e_t \qquad (5.10)$$

[5] The parameters of the model can not be disclosed due to the confidentiality reasons. However, estimation of the model parameters is a straightforward process which can be carried out by using commercially available econometrics packages.

where C is an nx1 vector of constants, A_i (i=1,…,l) are nxn matrices of coefficients and ε_t is the vector of error terms with the following properties:

$$E(e_t) = 0 \text{ for all t} \quad (5.11)$$

$$E(e_s . e_t^{'}) = \begin{cases} \Omega & for \quad s = t \\ 0 & otherwise \end{cases} \quad (5.12)$$

where Ω is the variance/covariance matrix assumed to be positive definite.

The parameters of the time series model are estimated based on a monthly data set from 2001 to 2005. Since Turkey had been implementing a pegged currency regime in year 2000 and moved to a floating regime afterwards, we start our dataset from 2001. The three month and twelve month interest rates reflect the rates that emerged in Treasury auctions for securities in those maturities. The inflation rate is the monthly rate of change in the 1994 based Consumer Price Index announced by the Turkish Statistical Institute. The USD/TRY exchange rate is the monthly average calculated over daily figures announced by the Central Bank of Turkey. We take the mid point of the official purchase and sell rates of the bank. At the end of 2005, the annualized 3 month and 12 month interest rates stood at a level of 14.2% and 14.1% respectively, while the average annual interest rate for one-year USD denominated bonds was about 4.8%. The monthly average value of the USD in December 2005 was1.35 TRY. The annual inflation rate for 2005 was recorded as 7.7%

We create random scenarios for our stochastic variables by making use of the VAR model via imposing correlated random shocks through the error term making use of the Cholesky decomposition of the variance/covariance matrix.

The random shocks are achieved by drawing five random variables from the standard normal distribution. To be able to create scenarios consistent with the empirical co-movements of our macroeconomic variables, one needs to obtain correlated random shocks. To this end, we make use of the Cholesky decomposition of the covariance vector Ω. Thus, we first find a matrix F such that

$$F'.F = \Omega \quad (5.13)$$

We then transform the 5x1 vector that contains the standard normal random variables by multiplying it by F and impose the resulting vector to the VAR model as a random shock. The monthly paths created by the model are then converted to quarterly figures by taking averages over three-month periods.

Once we obtain simulated values for the short and one year interest rate, we compute the yields for maturities in-between by linear interpolation. For maturities longer than a year, a flat yield curve is assumed. That is, interest rates are not allowed to vary with maturity, but taken constant for maturities over a year.

5.3.2 Scenario Clustering

In order to increase the approximation capacity or our scenario tree, we need to increase to number of sample scenarios obtained through the VAR model. However, there is also a need to reduce the number of branches on our scenario tree due to computational reasons. To this end we first create a high number of scenarios, then bundle similar ones together using the clustering approach, which has gained wide popularity in the field of data mining, a research area that tries to extract information from huge levels of data and to identify any meanings or patterns that are not evident at first sight.

In its simplest definition, clustering is to group similar items together. A more formal, mathematically elegant definition Graepel (1998) makes is as follows:

"Let $X \in R^{mxn}$ be a set of data items representing a set of m points x_i in R^n. The goal is to partition X into K groups C_k such that data that belong to the same group are more "alike" than data in different groups. Each of the K groups is called a cluster. The result of the algorithm is an injective mapping $X \to C$ of data items x_i to clusters C_k".

The measure of alikeness or similarity is based on a certain distance metric depending on the characteristics of data and/or modeler's objectives.

The clustering problem can be formulated in several ways. Data items can be clustered into disjoint or overlapping classes. The clusters can be deterministic so that each item is attached to a certain cluster or probabilistic, where each instance has an assigned probability to be a member of a certain group. We can also form hierarchical clusters such that low level clusters merge to form larger clusters at higher levels of the hierarchy.

In our case, i.e. macroeconomic scenario clustering for the stochastic debt management model, we are concerned with obtaining a disjoint set of scenarios. To this end, we would like to cluster a large number of scenarios that are obtained through Monte Carlo simulation into classes that will form our scenario set for the SP model. We are not interested in finding a hierarchy with regard to our obtained clusters, since the branches at the scenario tree are all at the same level. Since at the end of VAR simulation we only have the data points and do not have any a priori knowledge about the actual clusters these data points belong to, we are faced with an unsupervised learning/clustering problem (In supervised learning, there is a set of training data whose actual clusters are already known, and the objective is to understand the relation between the characteristics of data and being a member of a certain cluster, and then to predict a cluster when a new data item arrives.)

There are several clustering algorithms available. Among those, we adopt the K-means algorithm (MacQueen, 1967), which is one of the earliest, probably the simplest and the most widely used methods in practical applications. The aim in K-means is to cluster data points in K mutually exclusive classes, where the number of classes K is pre-supplied. Each cluster is identified by its centroid. That is the point to which the sum of distances from all data points in the cluster is minimum.

Given a set of initial clusters, the clustering algorithm moves items between clusters until the sum of distances within clusters can not be decreased below a certain level. The algorithm for K-mean clustering is as follows:

K-means Algorithm:

(a) *Find a set of initial cluster centroids,* $C_1^j ,..,. C_k^j$ *at iteration j=0.*

(b) *j=j+1. For each* x_i*, i=1,...,m, find the nearest cluster with respect to the chosen distance metric and assign* x_i *to that cluster*

(c) *For all k=1,...,K, re-compute cluster centroids,* $C_1^j ,..., C_k^j$ *as the mean of all points assigned to that cluster.*

(d) *If the number of points re-assigned to a different cluster is less than a certain level or the sum of distances could not significantly be decreased further, stop, else go to step (b)*

The K-means algorithm has several drawbacks: There is no clear guidance to find out the real number of clusters, i.e. K, and the results can be sensitive to the initial set of clusters. To choose the "right" K, we have tried the algorithm with different K values and tried to see the effects on obtained clusters and on our SP model results.

We did not code the algorithm from scratch, but used the built-in K-means function in MATLAB 6.5 Statistical Toolbox. This function offers some choices on the selection of initial clusters which can be completely random, uniformly selected over the data range or supplied by the user. We have selected the fourth option which performs a preliminary clustering analysis over a 10% sub-sample of the data set to find out the clusters to begin with.

Our econometric model contains five macroeconomic variables such as the interest and exchange rates which take values on different scales. Thus, to cluster our five-dimensional data points, we re-scaled the points on a 0-1 scale. For example, we assigned the data point (scenario) which has the highest exchange rate a value of 1 on that dimension and scaled the other points accordingly. We did this in every dimension so that every scenario turned into a vector of items on a 0-1 scale. We have then adopted the Euclidian distance metric as a measure of distance (similarity)

The following section explains the entire scenario generation-clustering process that we have adopted to create scenario trees for our multi-stage SP model.

5.3.3 Scenario Tree Generation Algorithm

Our debt management model is based on a multistage scenario tree of the form given in Figure 2. There are several decision stages (which may correspond to years) each divided into several sub-periods (which may correspond to quarters) in which the stochastic variables are set to evolve. That is, scenarios unfold in each sub-period, but decisions are only made or revised at certain decision stages. Then, the scenarios between decision stages combine to form a sequence of joint realizations for a certain period. These sequences of scenarios are linked at the decision nodes and we have scenario paths covering the entire planning horizon.

In this setting, our scenario generation algorithm proceeds as follows: We first create a populated set of scenario paths emanating from the initial (or current) decision node extending up to the beginning of next stage (or year) using the vector autoregressive model by means of random sampling through Monte Carlo simulation. We then cluster these into K distinct classes to form K decision nodes at the next stage using the method discussed in the previous section. Here, we only take into account the last elements of scenario paths (e.g. outcomes in sub-period 4 for scenarios in the first stage, sub-period 8 for scenarios of the second stage etc.) and run the clustering algorithm on these end points. We do not use data points regarding the interior sub-periods (sub-periods that are in-between decision nodes, e.g. sub-periods 1,2,3 for the first stage) covered by the scenario set and accept the loss of some information, as this reduces the dimension of our clustering problem.

Once we have obtained the clusters, we unite the scenario paths in the same cluster by finding the centroid in each dimension for all sub-periods covered. We keep track of the size of each cluster, i.e. the number of elements in each cluster and assign each cluster a probability based on its relative size. For example, if we have two clusters: one with 4 and the other with 6 elements, then the scenario path formed from cluster one is assigned with a probability of 40% while the path from cluster two is given a probability of 60%. We repeat this procedure for all consequent stages. The following algorithm summarizes this process in an orderly manner:

Scenario Tree Generation:

Let i=1,...,N denote the periods in our model, n=0,...,N-1 the decision stages, T the total time horizon and T_i the set of subperiods contained in period i. Then,

(a) n=0, i=1

(b) For j=1 to K^n (For each node at the current decision stage), repeat

i. Obtain a set of M multivariate scenario paths, covering all sub-periods in T_i, by means of random sampling through the VAR model *ii.* *Take the last observation in each scenario path, convert this M multivariate data points into a 0-1 scale over each dimension, and cluster them into K classes, scenario paths are allocated to clusters by their end (destination) points.*

iii. *Count the elements in the clusters and join the paths in the same cluster by finding the centroid for each sub-period in* T_i

iv. *Assign a probability to each newly formed k=1,...,K scenario path, by taking the ratio of the number of elements in cluster k to M.*

(c) n=n+1, i=i+1. If n=N, (if all stages are covered), stop. Else go to step (b)

Figure 15 illustrates this algorithm on a simple two-stage, two-cluster case.

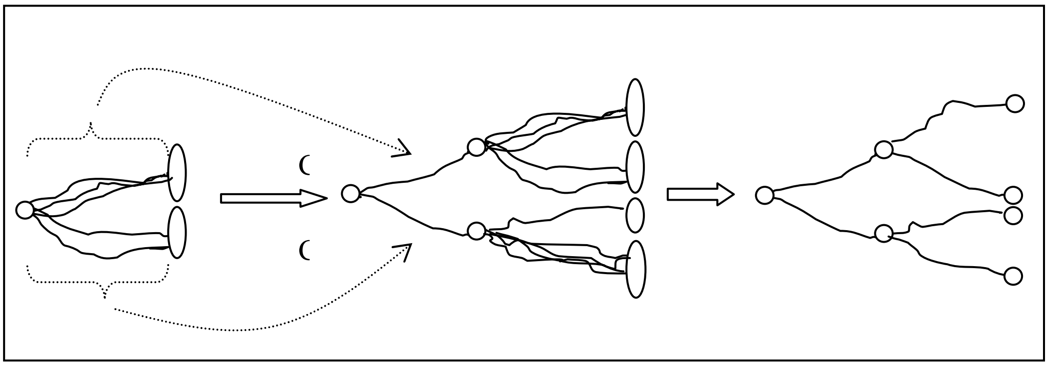


Figure 15 Generating a Scenario Tree.

The branch probabilities are accumulated to get the overall path probabilities from time 0 to T. These scenario probabilities are then fed into our SP model as well as other scenario characteristics and form the basis for our expected cost and risk calculations.

Figure 16 illustrates the development of a sample scenario tree for interest rates in a single stage problem. The probabilities associated with scenario paths, obtained through the number of elements in each cluster, are given to the right of the right pane. Note that the clustering algorithm is run on five dimensional data, while the figure only depicts one (the interest rate) dimension.

A drawback of the clustering approach is the loss of information regarding the worst-case scenario. This is a serious problem if we adopt the worst-case cost or payment as an objective criterion to be minimized. There are problems with worst-case scenario based measures, even without adopting a clustering approach, since the obtained result is based on a single observation obtained through a stochastic setting.

The following section comments on the results from our SP model when run on scenario trees of different sizes, with various choices for the clustering parameter K.

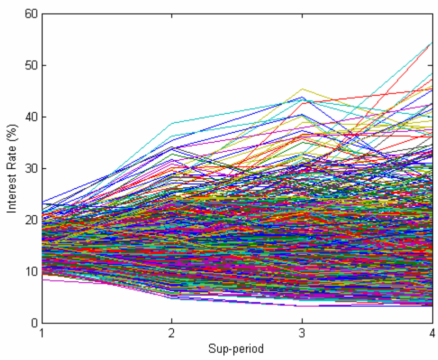
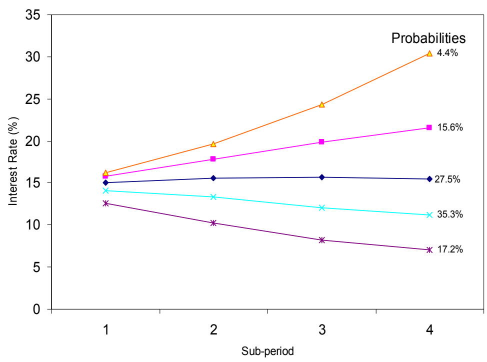
Original Scenario Paths	Scenario Tree with Associated Probabilities after Clustering
	

Figure 16 Generating a Scenario Tree - An Example with 5 Clusters.

5.3.4 Assessment of the Scenario Generation Algorithm

We assess the quality of our scenario tree generation approach in the framework of the discussion in 3.1.2.2. To test the "in-sample stability" of our method, we solved our SP model based on different instances obtained from the same scenario tree generator, trying to optimize our objective functions separately. We have generated 50 independent and identically distributed scenario trees and solved our model 50 times using these as inputs to see how the optimal value of each objective function varies due to the stochasticity included in our modelling framework.

In this setting, we have only accounted for the expected cost and market risk measures. We did not include our third objective. Since the SP model includes bonds with maturities as long as the decision horizon, the optimal solution for the liquidity risk minimizing model is independent of the scenario tree instance as the model chooses long maturities regardless of the tree.

The model is implemented on GAMS 2.0 using CPLEX as the linear programming solver. A sample code developed for the experiments is included in Appendix-B. Model parameters depend on debt stock realizations as of end-2005, and financing projections are based on the medium term fiscal plan for years 2006-2008.

We first present the results obtained without employing any clustering algorithm, i.e. by pure random sampling from the error term of the VAR model. Table 2 depicts the averages and standard deviations of optimal objective function values when models are solved on trees of different sizes. The results are all based on a three-stage model, only the numbers of branches differ. The notation of 10x10x10 corresponds to a three-stage tree with 10 branches disseminating from each node in each stage. Thus, in the final stage there are 1,000 branches.

The results in Table 2 provide evidence about two issues regarding "pure" random generation: First, a low-branch tree tends to underestimate the objectives, and second, we need to increase the dimensionality substantially to achieve a significant reduction in the variations of the optimal objective function values.

Table 2 Stability Results without Clustering
(Based on 50 Independent Replications).

Scenario Tree	**Objective: Min Cost**		**Objective: Min CCaR**	
Billion TRY	**Average**	**St. Dev.**	**Average**	**St. Dev.**
10x10x10	50.9	4.9	83.5	10.9
40x10x10	52.9	3.0	89.6	5.3
80x10x10	52.7	2.5	90.7	4.2

We can now compare these results to those from our scenario generation algorithm based on clustering of randomly generated data. We first run our model by generating 100 random paths from each node and clustering these into 5, 8 and 10 clusters consecutively (i.e. M=100 in the algorithm presented in Section 5.3.3). The results in Table 3 shows that averages and standard deviations of optimal function values obtained from trees of different sizes are quite close to each other, while the variances are less than those of higher dimensional trees in Table 2. Table 4 depicts the effect of increasing the number of scenarios paths (M), i.e. the data points to be clustered. Working with a higher number of paths before clustering provides significant improvement in "in-sample stability". As regards to measuring the "bias" in our model, the solutions in Table 4 are comparable to that of the largest (8,000 branch) tree given in Table 2.

Table 3 Stability Results after Clustering over 100 data points (Based on 50 Independent Replications).

Scenario Tree	**Objective: Min Cost**		**Objective: Min CCaR**	
Billion TRY	**Average**	**St. Dev.**	**Average**	**St. Dev.**
5x5x5	54.0	2.1	87.7	3.4
8x8x8	53.8	2.6	88.3	3.6
10x10x10	53.9	2.3	88.4	3.9

Table 4 Stability Results after Clustering over 1000 data points (Based on 50 Independent Replications).

Scenario Tree	**Objective: Min Cost**		**Objective: Min CCaR**	
Billion TRY	**Average**	**St. Dev.**	**Average**	**St. Dev.**
5x5x5	53.1	1.0	88.9	1.3
8x8x8	54.2	0.7	88.6	1.2
10x10x10	54.2	0.5	88.6	1.0

As discussed previously, the K-means algorithm does not provide any guidance to the "true" number of clusters. To this end, we have carried out visual and quantitative analysis of the scenarios generated, however since we are working with randomly generated data, it is hard to find a single K value that would be correct for all instances generated. Thus, we suggest using the results from our stability tests to assess the effect of using different K values. As depicted in Table 4, the averages for the optimal values of the two objective functions are quite similar, when we create our scenarios with different K values, while there is a slight improvement in variances as we increase the number of clusters. Even though it may be more appropriate to use 8 or more clusters to be able to obtain more stable results, there is a cost associated as the total number of scenarios is K^n. Considering the fact that our MCDM approaches require sequential solutions of the model (especially when approximating a portion of the efficient frontier), we present our illustrative results in the following sections based on a 125 branch (5^3) tree.

Our models are solved on a Pentium 4, 728MB RAM PC. We should note that it takes around 20 minutes to solve the 8,000 branch tree given in Table 2 . That also includes the time spent on the scenario generation process, based on pure random sampling. On the other hand, the total time used by our clustering-based algorithm to create a sample tree of 125 branches out of 1,000 paths at each node and by GAMS to solve a model of this size is around 100 seconds. These times quoted are for models in which a single objective is minimized. The time required for setting up and solving achievement scalarizing programs (of the form given in the following section) is around 120 seconds for a tree with 125 branches, Pentium 4, 728MB RAM PC capacity is not sufficient for getting solutions of scalarizing programs for 8,000 branch trees of the form given in Table 2.

The model size for a problem based on a 125-branch tree is about 20,000 rows and columns before any reduction is done by the CPLEX solver. When the tree size is increased to 8,000, the model size reaches to a level around 1.4 million rows and columns.

5.4 Experiments on the PDM Model using MCDM Tools

We now present findings from experiments on our model using the MCDM tools discussed in the previous sections. First set of experiments were done based on a single scenario instance, while the second part presents application of the stochastic interactive approach on a multi-scenario-tree framework. All experiments were conducted on a tree of 125 branches. That is, there are five branches disseminating from each node. These sets of 5 branches were created by clustering 1,000 paths down to five for each node.

5.4.1 Experiments on a Single Scenario Tree

We first provide the optimal borrowing strategies generated by our model when each decision criterion is optimized in isolation. Table 5 includes the issuance policy generated for 4 quarters of year 1 with respect to each objective. Since decisions for years 2 and 3 are scenario dependent, we only include the bonds to be issued in the first year.

As expected, the model chooses short-term securities for the first year when expected cost is to be minimized since our vector autoregressive models generally generated scenarios with declining interest rates in line with the macroeconomic environment in Turkey in the recent years. Short-term rates are also lower on average. However, as far as the market risk is concerned, all funding is raised through fixed rate bonds. The model aims to extend maturities to minimize liquidity risk, however floating rate bonds are also issued.

Figure 17 depicts several points identified on the efficient frontier and a fitted surface to those points which portrays the degree of trade offs between our three criteria. Presenting the efficient frontier may assist the decision makers in analyzing the trade-offs between alternative solutions and the sections of the frontier in which they are interested can be analyzed in more detail.

Table 5 Optimal Borrowing Strategies (Initial Stage Decisions).

Billion TRY	Bond / Quarter	3 month bill	6 month bill	12 month bill	18 month bill	3 year variable rate bond	3 year fixed rate bond	3 year USD bond
Minimize Cost	1			28.5				3.6
	2	10.0	22.7					
	3		42.4					
	4	10.0	36.9					
Minimize CCaR	1						128.0	
	2							
	3							
	4							
Minimize Liquidity Risk	1			1.4	12.7		14.4	3.6
	2				4.8	73.2		
	3					60.8		
	4							

Figure 18 displays the contours of the efficient frontier at different values of liquidity risk. The graph reflects the steepening nature of our efficient surface in z_3 dimension. In our illustrative problem, as the constraint on liquidity risk is released, we obtain a diminishing improvement in the levels of the other two objectives. Thus, it may be more preferable for the PDM decision makers to operate on relatively lower levels of re-financing risk since this would not require much sacrifice in the cost and market risk objectives.

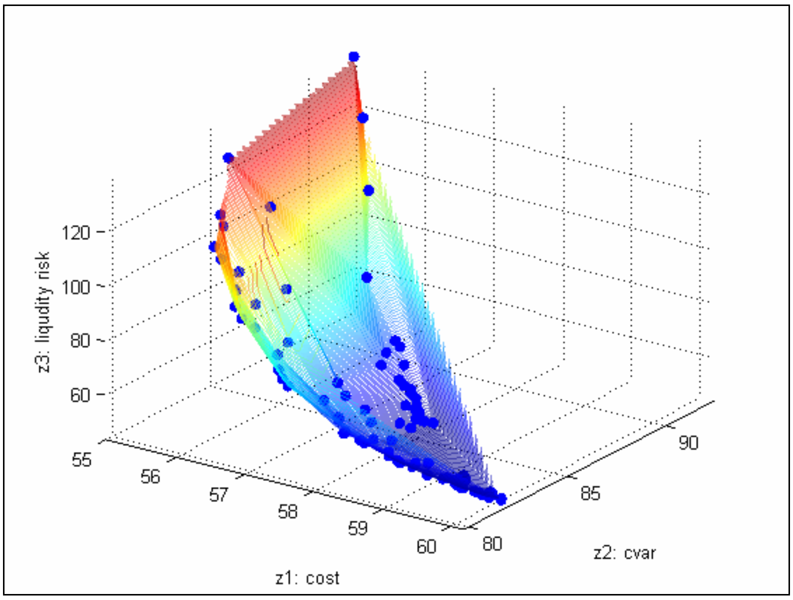


Figure 17 A Representation of the Efficient Frontier (billion TRY).

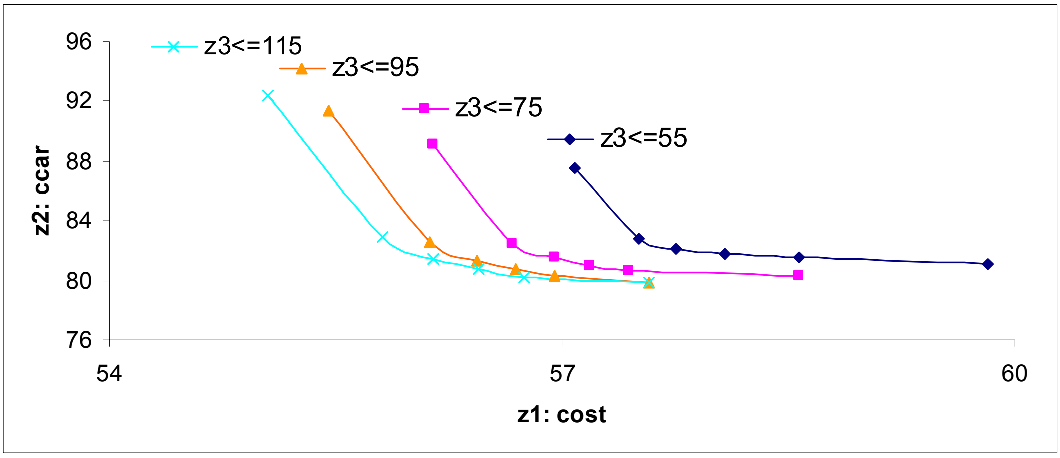


Figure 18 A Set of Efficient Solutions (billion TRY).

We now present an application of the visual interactive approach of Korhonen and Laakso (1986). Figure 19 displays the effect of altering the step size (θ) in moving from the point where liquidity risk is at its minimum in a direction to reduce cost, i.e. d=(-3.6, 8.8, 94.0), with λ=(1/3,1/3,1/3).

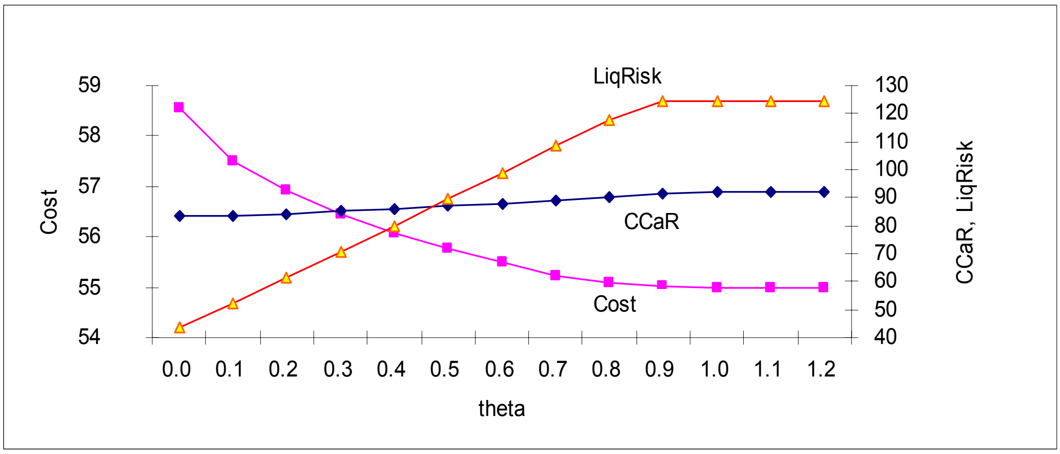


Figure 19 Criterion Value Trajectories (billion TRY).

Let us assume that the DM likes the solution at θ=0.3 (z_1=56.5, z_2=85.1, z_3=70.6) among the solutions in Figure 19. Table 6 contains the corresponding issuance strategy. As a compromise solution, we have a mixture of short term and long term bonds to be issued in the upcoming year.

Table 6 Optimal Borrowing Strategies (Initial Stage Decisions).

Bond / Quarter	3 month bill	6 month bill	12 month bill	18 month bill	3 year variable rate bond	3 year fixed rate bond	3 year USD bond
1						29.4	2.7
2	0.2		1.7			28.6	2.2
3	10.0	22.9					1.7
4	10.0	23.4					2.5

Changing the direction (d) and (θ) interactively with the decision makers will produce different trajectories on which the decision makers can analyze and experiment with their decisions. Let us know suppose the decision makers would like to explore solutions on the direction of reducing the market risk. Figure 20 plots the trajectories assuming that direction change occurs at θ=0.3 in Figure 19 in a way to reduce the market risk (z_2), for example with d=(1.0, -5.4, 73.8) (θ again starts from 0, since we are moving in a new direction).

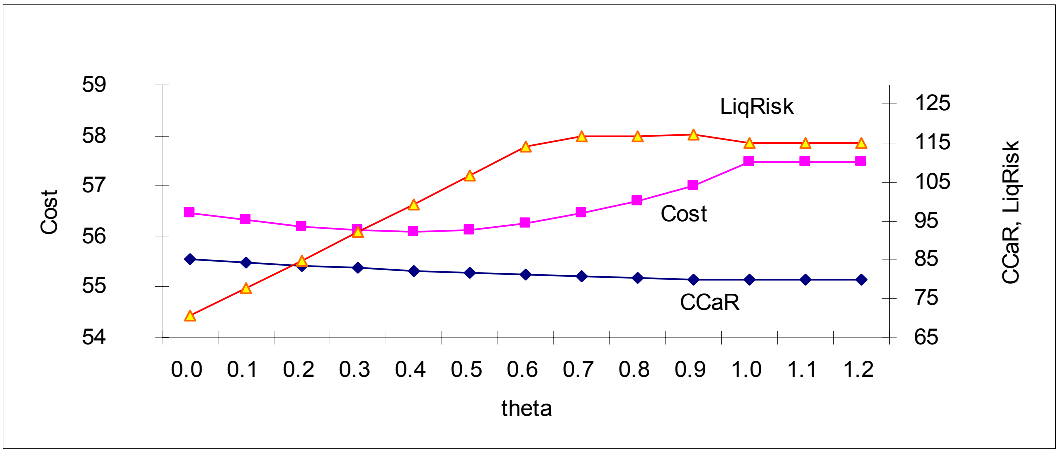


Figure 20 Criterion Value Trajectories (billion TRY).

Figure 21 displays the above trajectories on the efficient surface. The solid line is associated with Figure 19 while the dashed line corresponds to Figure 20 (reduction of in the CCaR measure). The experiments on the model can be extended even further in interaction with the decision makers.

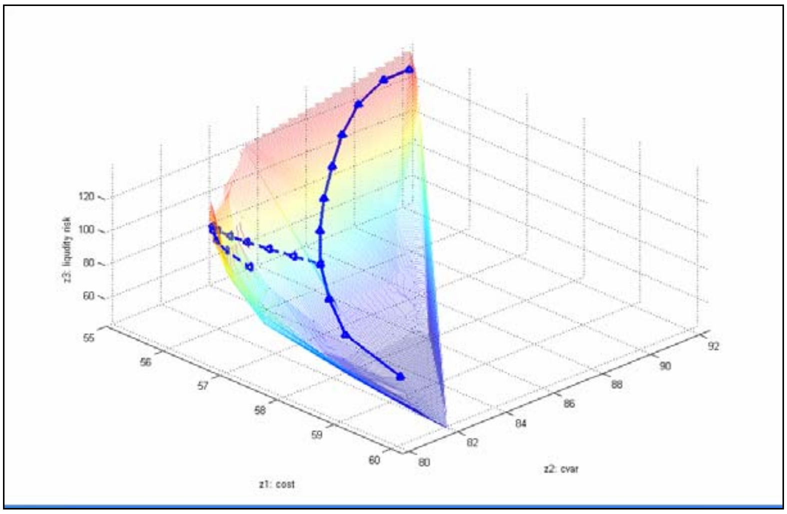


Figure 21 Movement on the Efficient Surface (billion TRY).

5.4.2 Application of the Stochastic Interactive Method

Like other scenario generation methods, our scenario generation mechanism for the Turkish model contains a degree of randomness which reflects the underlying stochasticity of the model parameters. Thus, guidance provided one scenario tree instance may be misleading due to the variability of the optimal solution on different trees. We now demonsrate an application of the modified interactive method given in section 4.2.3. taking into account this stochasticity.

Figure 22 depicts how stochasticity in the scenario generation mechanism effect the model solutions. The clusters of points represent efficient sets obtained by the method in section 4.2.2, based on selected a set of λ_i 's. One can observe different degrees of variation within each set. That is the variance/covariance structure between the objective function values varies depending on the region of the efficient surface we are operating at.

We can use these covariance structures to build confidence ellipsoids for each efficient cluster using ideas from section 4.2.1 where we discussed Multivariate Statistical Analysis tools. Figure 23 contains an example for a bi-criteria case. These confidence ellipsoids can be projected on the axes to obtain simultaneous confidence intervals for our objective function values.

Figure 24 represents an application of the visual interactive approach depicting simultaneous confidence intervals for each objective function. In this example, the DM starts from a given efficient cluster with centroid ([cost,ccar,liqrisk]=[63.0, 91.58, 51.72]) and moves in a direction to reduce CCaR iteratively. The figure depicts the change in objective functions as theta changes (In this example λ_i =1/3, i=1,…,3) .

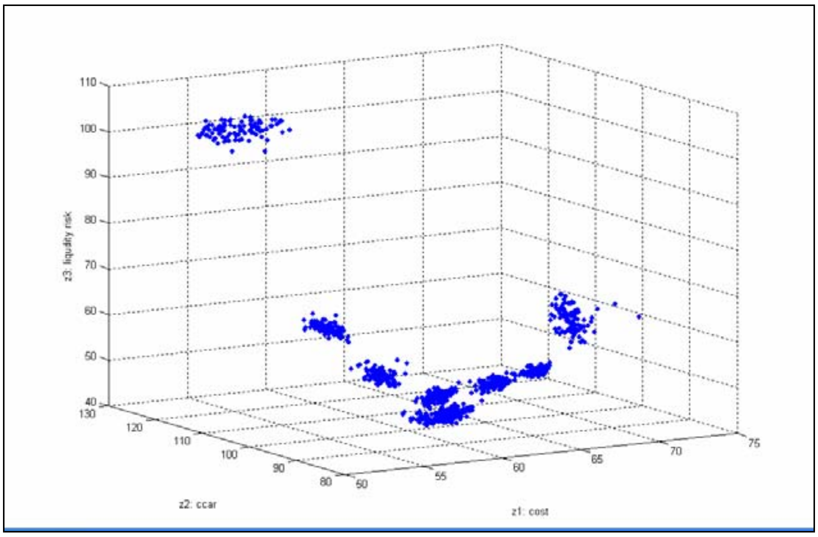


Figure 22 The Result of Randomness in the Scenario Generation Mechanism**.**

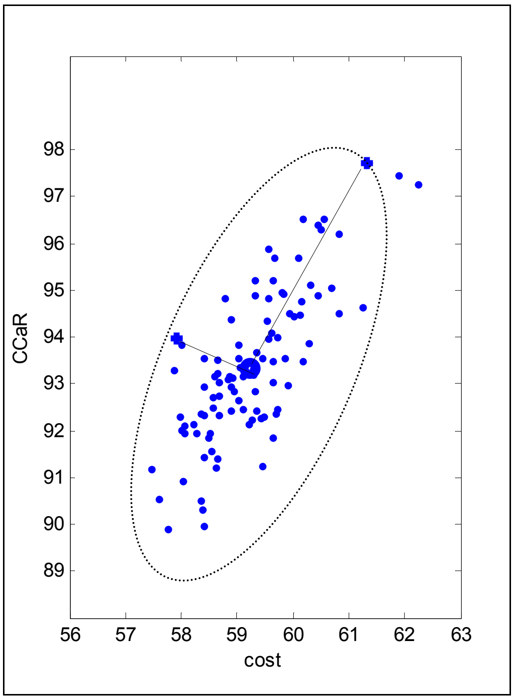


Figure 23 Confidence Ellipsoid around a Set of Efficient Solutions (Billion TRY).

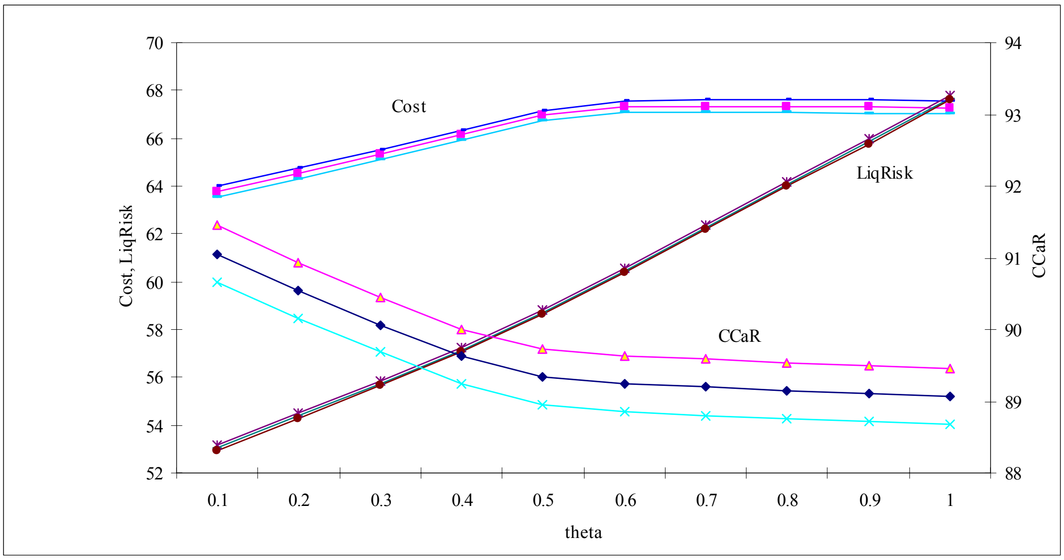


Figure 24 An application of the Visual Interactive Approach: Iteration 1 (Billion TRY).

Let us assume that the DM likes the solution at θ=0.5 and wishes to explore solutions that have a lower cost expectation. We then select a new direction accordingly and re-run our method starting from the current solution experimenting with different θ values. The results are depicted in Figure 25. Note that θ=0.6 corresponds to θ=0.1 after we change the direction.

Assume that at some point during this interactive process, the DM chooses a certain solution (an efficient cluster) evaluating the presented confidence intervals (such as those in Figures 24 and 25). One should also consider the reflection of this selection in the decision space. When we analyze decision variables in our scenario-based solutions, we see that there is a variation in terms of selected bonds and their issuance amounts depending on the scenario instance. Since the DM has to make a certain issuance decision at the start of the decision horizon, this solution has to be robust enough to yield similar objective function values under different scenario trees.

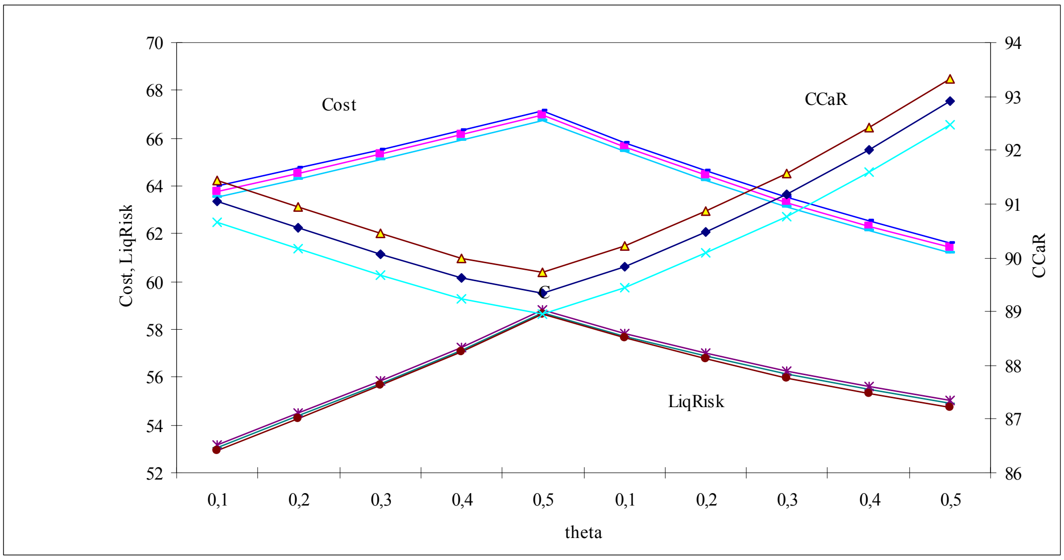


Figure 25 An Application of the Visual Interactive Approach: Iteration 2 (Billion TRY).

In this case, we would suggest the DM to implement the solution that has the shortest distance to the centroid of the selected efficient cluster, in terms of objective function values. Figure 26 depicts the performance of the pseudo-centroid solution under other random scenario tree instances in a bi-criteria example. Here, circles represent the actual objective function values in each scenario instance, while stars stand for the result of pseudo-centroid solution when plugged in other scenario tree instances.

Testing with several efficient clusters at different places of the frontier we see that on average the error caused by the pseudo-centroid is negligible. In Figure 26, for example, the ratio of root-mean square error in the cost dimension to average cost is less than 0.2%. Thus, in this example, our issuance strategy is sufficiently robust to yield the desired objective function values. In this context, once the DM chooses a certain efficient cluster, we recommend to carry out the robustness analysis to reflect about the actual decision variables.

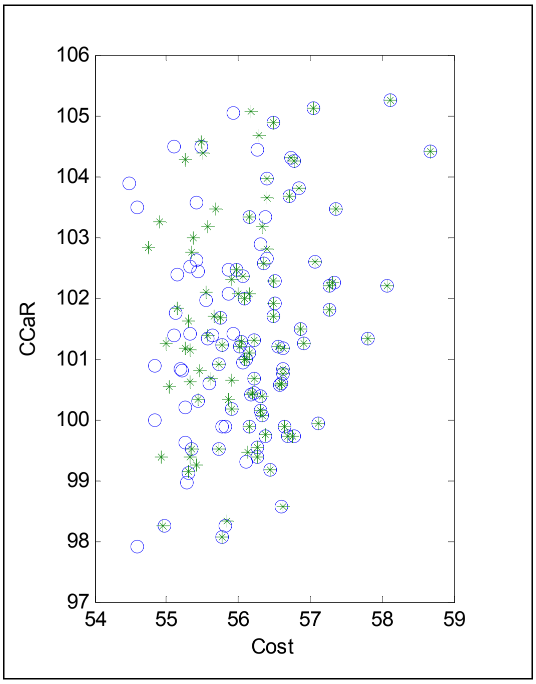


Figure 26 Performance of Pseudo-Centroid Solution is other Scenario Trees.

In summary, in our decision support procedure, we first apply the interactive algorithm to present the DMs several efficient solutions within confidence limits and experiment in different regions of the efficient surface based on DM preferences. Once they are content with a certain solution, we carry out an analysis on the decision variables corresponding to that solution to demonstrate how they perform under different scenario tree instances

5.4.3 Experimentation with the Simulation/Optimization Framework

We created the integrated testing framework through a MATLAB-GAMS interface. The scenario tree input for the SP model is generated in MATLAB 6.5 as described in the previous sections. The model is then solved in GAMS 2.0 and the resulting issuance decisions are exported to MATLAB environment to be included in our debt portfolio. The model in MATLAB then selects a random scenario, creates a scenario tree based on this assumed realization, scans the current debt stock matrix to evaluate the cash flows arising from previously made decisions and generates a cash-flow scenario tree. This information is then exported to GAMS for further decisions.

We carried out a simple test and compared the results from the three-stage model for Turkey to those of a myopic (single-stage) model in the simulation/optimization setting. The myopic model has a one year horizon, i.e. makes decisions considering only the first year scenario paths. The scenario tree for the myopic model does not re-branch after the first year, thus there are no scenario-specific solutions like those of the three stage model.

We have 125 scenarios in the myopic model so that the number scenarios in the two models are equalized. In order to be able to compare the solutions of the two models, we need to use the same set of DM preferences. For this purpose, we assume an additive utility function that is to be optimized in both models. As a starting point, we choose a utility function that combines our three objectives with equal weights.

In the experiment, the horizon of our simulation setting is three years for both models. At the start, the models are fed with their appropriate scenario trees generated on the same set of starting conditions. Once the decision for the first year is made, we generate a random scenario realization, implement this on the assumed scenario path and shift the models twice to obtain decisions for the remaining two years, re-generating the scenario trees based on the assumed realizations. One replication of the simulation ends at the end of the third year. The process is then repeated.

We calculate the expected objective function values over simulation realizations. The expected cost value is the average of all replications. The Conditional Cost-at-Risk is computed over the maximum 10% of the cost realizations in all replications. The liquidity risk measure is calculated similarly. Table 7 depicts the results based on 1,000 simulation replications.

The results from this simple experiment show that the three-stage model yields better results with respect to the assumed utility function when compared to the myopic model. In a sense, this result is expected since the three-stage model covers the entire decision horizon (in this case the simulation horizon) when making decisions. However, we should note that since the market risk value is calculated over the cost realizations in simulation replications, it is correlated with the expected cost computation. Then, it is more appropriate to consider the expected cost and liquidity risk objectives when comparing alternative simulation settings.

Table 7 Results of Dynamic Games. Experiment 1: An Additive Utility Function.

	3-Stage Model	**1-Stage Model**
Expected Cost	60.6	81.9
Market Risk (CCaR)	87.7	169.6
Liquidity Risk (CPaR)	63.8	92.2

We can use the results of simulation analysis to examine and compare the empirical distributions of objective function values. Figure 27 depicts the empirical cumulative distribution functions (cdf) of the "cost" value for our models based on the results of simulation analysis. The three-stage model "first order stochastically dominates" the myopic model in terms of their cost measures except for the small region at the lower end. That is, for any given cost value a, the three-stage model has a higher probability of being less than a.

We also compare the two models using a utility function based on Tchebycheff distances. In this setting, the objective is to minimize the maximum weighted Tchebycheff distance from a given point as explained in Chapter 4. Table 8 contains the results from 1000 simulation replications using an equally weighted Tchebycheff metric based on the ideal point [cost, ccar, liqrisk]=[20, 50, 40]. Simulation horizon is again three years.

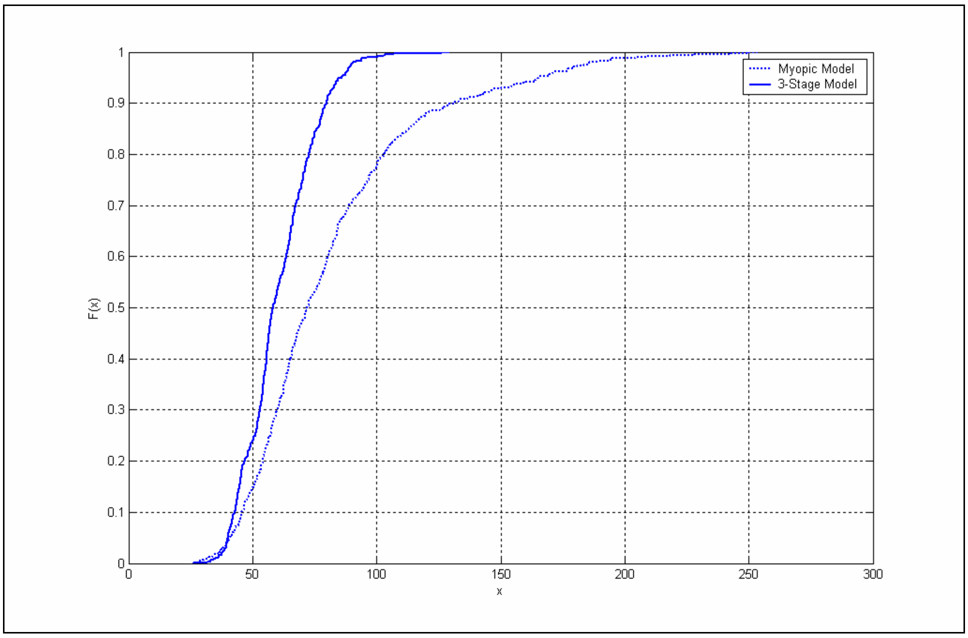


Figure 27 Empirical cdf Plots.

Table 8 Results of Dynamic Games. Experiment 2: A Tchebycheff-Type Utility Function.

	3-Stage Model	**1-Stage Model**
Expected Cost	64.3	83.7
Market Risk (CCaR)	89.0	175.4
Liquidity Risk (CPaR)	64.2	92.4

Table 9 depicts the results when the weights associated with the Tchebycheff distances are changed to [0.5, 0.1, 0.4] for cost, market risk and liquidity risk respectively. The three-stage model produces a better result in the cost dimension which is given more weight in the assumed utility function.

Table 9 Results of Dynamic Games. Experiment 3: A Tchebycheff-Type Utility Function.

	3-Stage Model	**1-Stage Model**
Expected Cost	50.1	83.2
Market Risk (CCaR)	81.6	170.5
Liquidity Risk (CPaR)	132.4	92.3

We now compare the two models using the additive equally weighted utility function of Table 7 in five year setting. In this experiments the three-year and one-year model are used five times at the beginning of each year to obtain the issuance strategy throughout these five years. The results are given in Table 10. While the cost values are comparable, the three-stage model yields lower risk values in this five year setting under the assumed utility function.

Table 10 Results of Dynamic Games. Experiment 4: An Additive Utility Function, A Decision Period of 5 years.

	3-Stage Model	**1-Stage Model**
Expected Cost	131.8	127.1
Market Risk (CCaR)	202.7	233.5
Liquidity Risk (CPaR)	141.5	151.7

With these outcomes, the experiments highlight the importance of selecting the "right" decision horizon when making debt strategy decisions or public policy decisions in general. For our experiments, we chose to employ a three-year model in line with the regulatory legal framework in Turkey[6]. However, there seems to be

6 "The Comminique on Principles and Procedures of Coordination and Execution of Debt and Risk Management" (Published in the Official Gazette dated September 1, 2002 and no. 24863) imposes

more value in experimenting with longer decision horizons. Basing decisions over a longer term analysis may yield better results over the medium and long term even though the costs measured in the short term may seem slightly higher.

The number of experiments in this simulation/optimization setting can easily be increased using alternative decision maker preference structures or various modelling frameworks.

that the debt strategy decisions are made for a three-year period. The budgetary framework is also based on a three-year horizon.

CHAPTER 6

CONCLUSIONS

In selecting the combination of instruments to be issued, i.e. the borrowing strategy, public debt managers have to take into account various objectives and the uncertainty associated with the outcomes of the decisions made. In debt management theory and practice, several approaches have been derived and proposed for the case of sovereign debt managers; however these are generally contented with the quantification of relevant cost and risk measures. Therefore, there is an additional need to assist decision makers in comparing alternative courses of action since targeted objectives are conflicting by their nature.

The objective of this thesis is to propose a framework to support the decision making process regarding sovereign debt issuance, drawing on the case of Turkish Treasury, the institution in charge of sovereign debt management in Turkey. We tried to incorporate relevant criteria and develop quantitative approaches that take into account sequential decisions concerned with debt issuance policies.

We first presented a multi-objective multi-period stochastic programming model that aims to support bond issuance decisions of public debt management units. It helps quantify the costs and risk associated with alternative courses of action under uncertainty, making use of a scenario tree. The deterministic-equivalent liability management model is concretized on an illustrative example.

This generic model can serve for different country characteristics with small modifications. There can be as many periods as relevant, and the number of sub-periods in each period can vary. For example, the model can consist of two dissimilar periods, the first period corresponding to year one and the second covering all remaining years in the model horizon. It is also possible to incorporate several different cost and risk measures to the model. SP model can well be extended to include debt management tools such as bond buy-backs and debt exchanges that have started to gain popularity among public debt management offices. The addition of such tools is straightforward and can be directly implemented if needed.

Having developed a generic decision model, we then incorporated some MCDM approaches to construct a generic decision aid tool. The aim was to assist decision makers in analyzing the trade-offs between conflicting objectives. In this context, we proposed to identify some efficient solutions based on different preference structures. Since, it was practically not possible to identify all alternative efficient solutions in our continuous objective space, we employed an interactive MCDM approach aimed at getting decision maker involvement through the decision support process. We believe this will help assess preferences, explore distinct alternative solutions and guide DMs in making debt strategy solutions. Since scenario generation mechanisms try to reflect the underlying stochasticity of the model parameters, such as interest and exchange rates, they contain a degree of randomness. Consequently, there is usually a variation in the optimal objective values when the model is solved on different scenario tree instances.

In this work, we modified the available MCDM approaches to account for the possible randomness in the scenario tree generation process and make statistical inferences. Specifically, we constructed confidence regions around our efficient solutions and modified the interactive procedure to cope with the uncertainty in the scenario generation method which represents the stochasticity in real life. To this end, we made use of tools from multi-variate statistical analysis.

This framework is then applied for the case of sovereign debt management in Turkey. In this illustrative application, we adapted the generic SP model in line with the structure of public debt management in the country. The three-objective three-stage model developed by taking into account the relevant objectives and constraints was used for further experimentation.

In this context, the high degree of variability of the scenario trees for macro economic variables, constructed by a vector autoregressive model, imposed stability problems. As a remedy to this problem, we adapted ideas from scenario reduction and clustering techniques and employed a specific scenario tree generation algorithm for the case of Turkish economy based on the K-means clustering algorithm. In this regard, it may also be worthwhile to try generate the tree with other clustering methods, to change to number of branches in each stage or to allow the number of clusters vary dynamically. That remains as a future work to be accomplished.

The scenario-tree based model was then used a basis for the illustration of MCDM approaches for the case of the government. Our experiments with the case of Turkey show that this framework can be of practical use in a real setting. The model suggests issuing short-term bonds to minimize expected cost and longer-term fixed rate securities to decrease the level of market risk as expected. The decision makers can solve the model attaching different weights to the objectives and gain insights about the resulting debt strategy compositions. With the help of such a quantitative tool, the sovereign debt issuers will have the means to see the effects of different risk and cost preferences on the debt issuance policy. Experimentation on the model can help the assessment of the decision makers' preferences with regard to associated criteria, which are not only crucial in debt management policies, but also in other financial decisions of the government.

Disclosing mainline results from the modelling work to general public opinion can also help the debt management offices convey their strategies to market participants and inform stakeholders, such as tax payers, about the objectives of the public debt management policy. Similar models can also be employed by independent organizations that conduct research on the macroeconomic policies of the government. This will provide them the means to comment on the actions and plans of the government in raising debt on behalf of the citizens.

The model is tested in simulation setting, where the model is solved iteratively on a rolling horizon setting. In this context, performances of alternative models were tested against each other in a simulation framework that mimics the dynamic macro-economic environment and the actual decision making process. We believe that the developed simulation framework can well serve as a decision aid tool by itself and the performances of alternative "decision making styles" can be analyzed in the proposed rolling-horizon setting.

Future research directions may include developing the stochastic programming model in a non-linear structure to cope with non-linear objective functions, such as non-linear risk measures. In this regard, the standard deviation of funding costs can be considered as a risk-related objective that the government would like to minimize.

The model can also be modified to allow decisions on the debt portfolio composition rather than targeting the issuance strategies. The main assumption of the modelling approach is that the evolution of stochastic variables is independent of the decisions of the government. Especially, in countries where the government is the largest financial actor and dominates the financial markets, the decisions of the government may have an impact on the states of nature for financial variables. In such cases, the volume of bonds issued in a specific maturity may affect the level of interest rates for that segment of the yield curve depending on the demand conditions. Future work may also concentrate on attaining means to cope with this issue. A simple solution to this issue would be to employ penalty functions that would penalize the cost of excess borrowing given the level of demand in the financial markets for government bonds. Providing more decision maker involvement in the scenario tree generation process can also be of interest as a future research prospect. The algorithms provided for MCDM analysis may also be enhanced to allow further analysis.

In general, this dissertation aimed to bring together ideas, concepts and methods from different disciplines, such as mathematical and financial modelling, risk management, simulation, clustering, statistical analysis and multi-criteria decision making in order to develop a quantitative framework for assisting debt strategy decisions of governments. A general objective was to demonstrate the relevance and applicability of these concepts in the realm of public debt management.

The existing methods employed for public debt strategy analysis rely on enumeration of costs and risks associated with given financing strategies under various different macro-economic scenarios. Since, these methods are limited with the user-supplied alternatives to be evaluated in a scenario-based analysis; they do not guarantee efficient solutions. In this study, we innovate an optimization approach for the PDM strategy problem using a multi-objective stochastic programming model. The developed framework helps identify efficient solutions and guides the decision makers in understanding the degree of trade-offs between different debt management objectives. Our experiments for the case of Turkey show that this tool can have important uses in a practical setting.

Even though the methods and tools are discussed in the context of sovereign debt management, we believe that the developed decision tools can be of practical use not only in debt strategy decision analysis, but also in other decisions involving multiple objectives and uncertainty. The concept of developing confidence regions around efficient solutions can provide an important input for decision analysis in general. Additionally, the stochastic interactive approach we developed as part of this work is novel in the MCDM literature and can be adapted for different multi-criteria decision making problems that involve stochasticity.

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APPENDIX-A

CONDITIONAL COST AT RISK

In financial investment decisions, there is a trade-off between cost and risk, inherent in the structure of financial markets, i.e. the higher the risk associated with returns is, the greater is the expected return. Risk, in this sense, is the threat that the selected investment portfolio will generate a loss or a lower return than expected due to unfavorable market conditions and is attributed to the uncertainty associated with variables that drive the portfolio returns. Given the dilemma in financial decision making problems, there has been substantial research and modelling effort to measure the degree of risks and to support investment decisions.

Markowitz (1952, 1959) in his seminal works, proposed to define this notion of risk as the variability (standard deviation) of returns of the elements (instruments) in a financial portfolio and developed the risk-return efficient portfolio concept. Even though this paradigm has prevailed for quite a long time and formed the basis for numerous applications, it is now widely accepted that its main assumption that returns of assets follow normal distributions does not generally hold in reality. In today's multifarious market conditions, returns are affected by many different factors and thus exhibit distributions that are more complex than the normal distribution.

To capture the asymmetries in the behaviour of portfolio returns, several metrics have been developed and proposed. Among those, the "Value-at-Risk" (VaR) measure has gained wide popularity and even become part of the regulatory measures in the banking sector. VaR is generally defined as the maximum probable loss (or minimum return) for a given portfolio for a specified time horizon within a certain confidence interval. If we let vector r denote the stochastic variables that drive the loss(return) of a portfolio and f(x,r) stand for the loss for a portfolio consisting of assets x, then the VaR value for this portfolio at a certain confidence level 100(1-α)% is given by the following equation:

$$\text{VaR}(x,1-\alpha) = \min\{u: P[f(x,r) \geq u] \geq 1-\alpha\} \qquad \text{(A.1)}$$

Despite its popularity, it has been shown that VaR has undesirable mathematical characteristics such as non-convexity and non-subadditivity (VaR of a portfolio can be larger than the total of that of individual assets) and it is difficult to optimize when it is calculated from scenarios (see Pflug, 2000). VaR does not either provide any information about the level of risk if the confidence level is exceeded, i.e about the nature of the tail of loss(return) distribution

The "Conditional Value-at-Risk" (CVaR) also referred as "the mean excess loss", "the expected shortfall" or the "the tail-VaR" emerged as an alternative risk measure as a response to the limitations of VaR. CVaR, which quantifies the conditional expectation of losses when VaR level is exceeded for a portfolio x, can be obtained by the given formula for specified level of α :

$$\text{CVaR}(x,\alpha) = E[\,f(x,r)\;|\;f(x,r) \geq \text{VaR}(x,1-\alpha)\,] \qquad \text{(A.2)}$$

That is, CVaR takes into account the tail of the loss distribution and is the conditional expectation of worst α*100% losses (Figure 28).

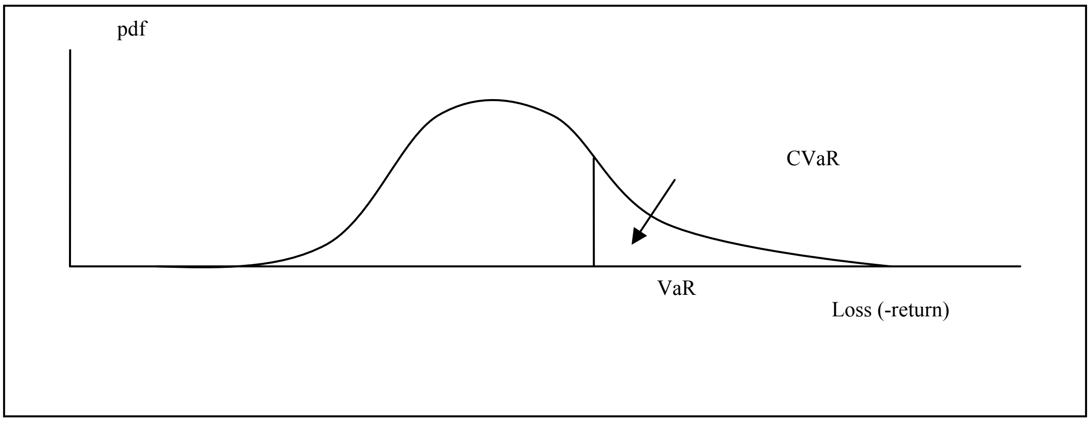

Figure 28 Conditional Value at Risk.

Pflug (2000) has shown that CVaR possesses the required properties of coherent risk measures in the sense identified by Artzner et al (1999). Rockafellar and Uryasev (2000) illustrate that CVaR can be efficiently minimized using linear programming in a scenario based framework. Their approach is pertinent in stochastic programming applications in which uncertainty is expressed in the form of a scenario tree consisting of a finite number of discrete scenarios. The main theme is to average the losses on scenarios that yield losses greater than the pre-specified VaR level.

To illustrate this method, let S={s : s=1,..., S} be the set of discrete scenarios that are built to express the uncertainty regarding the stochastic variable r and r_s denote value of r under scenario s. For each scenario s, we have an associated probability p_s and we can compute the loss of our portfolio of x, i.e. f(x, r_s) for each r_s, s=1,...,S. Then,

$$\text{CVaR}(x,\alpha) = \phi + \frac{1}{\alpha} \sum_{\{ s\in S: f(x,r_s)\geq z \}} f(x,r_s).p_s \tag{A.3}$$

where ϕ = VaR(x,1-α). Here, α is the probability that the VaR level is exceeded, that is f(x,r_s)$\geq \phi$, and thus, CVaR(x,α) turns out to be the conditional expectation of losses regarding portfolio x, given that the loss is greater than or equal to z.

If we define an auxiliary variable cv^s for each scenario s such that

$$cv^s = \max \{ 0, f(x,r_s) - \phi \} \tag{A.4}$$

then, CVaR can be expressed as follows:

$$\text{CVaR}(x,\alpha) = \phi + \frac{1}{\alpha} \sum_{s\in S} cv^s.p_s \tag{A.5}$$

We apply this approach to our scenario based stochastic programming application. Recently, Topaloglou et al (2004) have applied this measure for a portfolio management problem.

APPENDIX-B

SAMPLE PROGRAM CODE FOR THE PDM-SP MODEL

The developed deterministic equivalent PDM-SP model is implemented on GAMS 2.0 using the CPLEX solver for the case of Turkey. The program code given below is developed for the experiment regarding the application of the visual interactive approach of Korhonen and Laakso (1986):

```
$title Stochastic Programming Model for Public Debt Management

$ontext
The government has to decide on the type of borrowing instruments
(bonds) to be issued to meet the financing requirement in a given
planning period. The debt managers have a certain range of
instruments at their disposal, and they have to set a certain
borrowing strategy which embodies the proportion of each instrument
to be issued for the course of the decision horizon. We assume a
three-year period at the beginning of which the government
determines a borrowing mix to be implemented in the following year.
The strategy is revised at the start of year 2 and 3 depending on
the macroeconomic circumstances. Each year is divided into quarters,
thus the strategy set at the beginning of the year is pursued for 4
quarters

$offtext

* The sets of the model are defined below:

sets q dum /1/
     zz criteria /1*3/
     zzz /1/
     zzzz /1*3/
     t time /1*12/
     t1(t) sub-periods in the first period /1*4/
     j bonds /1*7/
     jtl(j) TRY bonds /1*6/
     j1(jtl) zero-coupons /1*4/
     j2(jtl) floating coupons /5/
     j3(jtl) fixed coupons /6/
     jf(j) fx-linked bond /7/
```

```
      s scenario /1*125/
      n nodes /n0*n30/
      alias(z,t);

sets

      tn(n,t,s) time-node-scenario mapping
      livezeros(j1,t) live zero-coupon bonds at end of horizon
       /2.11,3.(9*11),4.(7*11)/

      validfrns(j2,z,t) frn coupon payments
      livefrns(j2,t) live coupon bonds at end of horizon

      validfixed(j3,z,t) fixed coupon payments
      livefixed(j3,z) live coupon bonds at end of horizon

      validfx(jf,z,t) fixed coupon payments
      livefx(jf,z) live coupon bonds at end of horizon;

      validfrns(j2,z,t)=yes$((ord(z)<ord(t))and
       mod(ord(t)-ord(z),2)=0);
      livefrns(j2,t)=yes$((mod((card(t)-ord(t)),2)=1));

      validfixed(j3,z,t)=yes$((ord(z)<ord(t))and
       mod(ord(t)-ord(z),2)=0);
      livefixed(j3,z)=yes$((ord(z)<card(t))and
       mod(card(t)-ord(z),2)=1);

      validfx(jf,z,t)=yes$((ord(z)<ord(t))and
       mod(ord(t)-ord(z),2)=0);
      livefx(jf,z)=yes$((ord(z)<card(t))and
       mod(card(t)-ord(z),2)=1);

      tn(n,t,s)$(ord(t)<5)=yes$(ceil(ord(s)/125)=ord(n));
      tn(n,t,s)$(ord(t)<9 and ord(t)>4)=yes$(ceil(ord(s)/25)=
       ord(n)-1);
      tn(n,t,s)$(ord(t)<13 and ord(t)>8)=yes$(ceil(ord(s)/5)=
       ord(n)-6);


* The main parameters of the model are defined in the following
section

Parameters
       m(j) maturity /1 1,2 2,3 4, 4 6, 5 12, 6 12, 7 12/
       cp(j) coupon period /5 2,6 2,7 2/
       ps(t) primary balance  /1 8.8, 2 10.7,3 10.1,4 5.7,5 7.2,6
             10.4,7 9.9,8 3.6,9 7.8,10 11.2,11 10.7,12 3.9/
       r3(s,t) short interest rate
       r12(s,t) long interest rate
       fx(s,t) exchange rate
       fxr(s,t) fx interest rate
       alpha confidence level /0.9/
       l(t) current liabilities /1 40.9,2 43.4,3 41.9,4 27.7,5
             32.3,6 31.6,7 9.9,8 20.5,9 12.7,10 13.1,11 9.6,12 12.1/
       opt(zzzz,zzz)  current reference solution;
```

```
* The following section reads the scenario trees stored as text
files

table r3(s,t)
$batinclude 'data3.txt';
table r12(s,t)
$batinclude 'data12.txt';
table fx(s,t)
$batinclude 'datafx.txt';
table fxr(s,t)
$batinclude 'datafxr.txt';

* The following section reads the current reference solution

table opt(zzzz,zzz)
$batinclude 'datakorh.txt';

* The interest rates for maturities other than 3 and 12 months are
calculated in the following section

parameters
      r(s,t,jtl) interest rate for j;
      r(s,t,j1)$(ord(j1)<4)=(1+(r3(s,t)+((r12(s,t)-r3(s,t))/3)*
            (m(j1)-1)))**(m(j1)/4)-1;
      r(s,t,'4')=(1+r12(s,t))**(3/2)-1;
      r(s,t,j2)=r(s,t,'2')+0.01;
      r(s,t,j3)=(1+r(s,t,'3'))**(1/2)-1;

* The following section reads probabilities calculated for each
scenario branch

parameter   p(s,zzz) probability;
table  p(s,zzz)
$include 'datapr.txt'

* The following section reads current theta value from a text file

table teta(zzz,q)
$include 'tetat.txt';


* The variables of the model:

variables
      dist   distance measure
      x(n,t,j) amount of bond issued in type j decided at node n
            for time t
      y(s,t,j) node-scenario mapping for TRY bonds
      yf(s,t,j)  node-scenario mapping for foreign currency bonds
      i(s,t) interest paid
      d(s,t) principal paid
      b(s,t) borrowing requirement
      cb(s,t) cash account balance
      c(s,t) withdrawal from cash account
      cost   expected interest cost
      cst(s)  cost associated with one scenario branch
      ac(s) accrued coupons in scenario s
```

```
      az(s)  accrued zero-coupons at end of horizon in scenario s
      af(s)   accrued fx bonds in scenario s
      aff(s,t)  accrued fx differences in scenario s
      afc(s)    accrued fx coupons in scenario s
      a(s)     total accrued cost in scenario s
      py(s)    total interest paid in scenario s
      cvar   conditional value at risk for measuring market risk
      cv     the cost over var (dummy variable in cvar calculation)
      var    value at risk for measuring market risk
      liqcv(s,t) conditional value at risk for measuring liquidity
            risk
      liqcvar   the cost over liqvar while measuring liquidity risk
      liqvar   value at risk for measuring market risk
      myobj;

positive variables x,o,i,d,b,cb,y,var,cv,liqcv;

* the demand constraints
yf.up(s,t,'7')=3;
x.up(n,t,'1')=10;

* The equations that define the objective functions and constraints
of the model
equations
      obj  objective fucntion definition
      objcost  cost definition
      scenariocost(s)  total interest paid  in scenario s
      repay(t,s) principal payment
      intpay(t,s)   interest payment
      issue(n,t,s,j)  TRY bonds issued in node n
      issuefx(s,t,j) fx bonds issued
      cashbal(n,t,s) cash flow balance
      pay(s) total interest paid
      cashaccountbal(t,s) cash account balance
      accruedcoupons(s) accrued interest calculation
      accruedzeros(s)  accrued interest calculation
      accrued(s)       accrued interest calculation
      accruedfx(s)     accrued interest calculation
      accruedfxcoup(s)  accrued interest calculation
      accruedfxdiff(s,t) accrued interest calculation
      ConditionalVar    conditional value at risk calculation
      CondVar(s)        conditional value at risk calculation
      ConditionalVar2   conditional value at risk calculation
      CondVar2(s,t)     conditional value at risk calculation
      crit1             Thchebycheff distance calculation
      crit2             Thchebycheff distance calculation
      crit3             Thchebycheff distance calculation;


cashbal(tn(n,t,s)).. sum(j,x(n,t,j)) + c(s,t) =e=
      d(s,t)+i(s,t)+l(t)-ps(t);

issue(tn(n,t,s),j)..y(s,t,j)=e=x(n,t,j);

issuefx(s,t,jf)..yf(s,t,jf)=e=y(s,t,jf)/fx(s,t);

repay(t,s).. d(s,t) =e= sum(j,y(s,t-m(j),j));
```

```
intpay(t,s).. i(s,t) =e= sum(j1,y(s,t-m(j1),j1)*
      r(s,t-m(j1),j1)) + sum(validfrns(j2,z,t),y(s,z,j2)*
      r(s,t-cp(j2),j2)) + sum(validfixed(j3,z,t),y(s,z,j3)*
      r(s,z,j3))+ sum(validfx(jf,z,t),yf(s,z,jf)*
      fxr(s,z)/2*fx(s,t));

pay(s)..py(s)=e=sum(t,i(s,t));

cashaccountbal(t,s)..cb(s,t)=e=cb(s,t-1)- c(s,t);

accruedcoupons(s)..ac(s)=e=sum(livefrns(j2,t),y(s,t,j2)*
      r(s,'11',j2)*1/2)+sum(livefixed(j3,z),y(s,z,j3)*
      r(s,z,j3)*1/2);
accruedfxcoup(s)..afc(s)=e=sum(livefx(jf,z),yf(s,z,jf)*
      fxr(s,z)/2*1/2*fx(s,'12'));
accruedfxdiff(s,t)..aff(s,t)=e=sum(jf,(yf(s,t,jf)*
      fx(s,'12')-y(s,t,jf)));
accruedfx(s)..af(s)=e=afc(s)+sum(t,aff(s,t));
accruedzeros(s)..az(s)=e=sum(livezeros(j1,t),y(s,t,j1)*
      r(s,t,j1)*((card(t)-ord(t))/m(j1)));
accrued(s)..a(s)=e=ac(s)+az(s)+af(s);

scenariocost(s)..cst(s)=e=py(s)+a(s);
objcost..cost=e=sum(s,p(s,'1')*cst(s));

ConditionalVar..cvar=e=var+1/(1-alpha)*sum(s,p(s,'1')*cv(s));
CondVar(s)..cv(s)=g=(py(s)+a(s)-var);

ConditionalVar2..liqcvar=e=liqvar+
      1/(1-alpha)*sum((s,t),p(s,'1')*liqcv(s,t));
CondVar2(s,t)..liqcv(s,t)=g=(d(s,t)+i(s,t)+l(t)-liqvar);


crit1..dist=g=1/3*(cost-opt('1','1')-teta('1','1')*(7.234));
crit2..dist=g=1/3*(cvar-opt('2','1')-teta('1','1')*(-5.521));
crit3..dist=g=1/3*(liqcvar-opt('3','1')-teta('1','1')*(12.954));

obj.. myobj =e= dist+0.001*(cost+cvar+liqcvar);

model debt /all /;
file output /out.dat/;
file output2 /out2.dat/;

option profile=0;
option solprint=off;
option sysout=off;
option limcol=0;
option limrow=0;
option iterlim=1000000;
option reslim=1000000;
option lp=cplex;
```

```
solve debt using lp minimizing myobj;
put output;
put cost.l:9:4 @10, put cvar.l:9:4 @20, liqcvar.l:9:4 @30, put
myobj.l:9:4 @40 /;
put output2;
loop((t1,j), put x.l('n0',t1,j):7:4 /);
```

CURRICULUM VITAE

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