RATE OF PENETRATION ESTIMATION MODEL FOR DIRECTIONAL AND HORIZONTAL WELLS

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Approval of the thesis

RATE OF PENETRATION ESTIMATION MODEL FOR DIRECTIONAL AND HORIZONTAL WELLS

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ABSTRACT

RATE OF PENETRATION ESTIMATION MODEL FOR DIRECTIONAL AND HORIZONTAL WELLS

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Directional and horizontal drilling operations are increasingly conducted in all over the world, especially parallel to the growth of the technological developments in the industry. Common application fields for directional and horizontal drilling are in offshore and onshore when there is no way of drilling vertical wells. During directional and horizontal well drilling, many additional challenges occur when compared with vertical well drilling, such as limited weight on bit, harder hole cleaning, trajectory control, etc. This makes even harder to select the proper drilling parameters for increasing the rate of penetration. This study aims to propose a rate of penetration model considering many drilling parameters and conditions. The proposed model is a modified Bourgoyne & Young's model which considers formation compaction, formation pressure, equivalent circulating density, and effective weight on bit, rotation of the bit, bit wear, hole cleaning, inclination, fluid loss properties and bit hydraulics. Also, a bit wear model is developed for roller cones and PDCs. The model performance is tested using field data obtained from several directional and horizontal offshore wells drilled at Persian Gulf. It is observed that the model can estimate rate of penetration with an error of ±25 % when compared with the field data.

Keywords: Rate of Penetration, Multiple Regression Analysis, Optimization, Inclined and directional wells, Mathematical Model

ÖZ

YÖNLÜ VE YATAY KUYULARDA DELME HIZININ TESPİTİ İÇİN BİR MODEL

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Yönlü ve yatay sondaj uygulamaları, özellikle endüstrideki teknolojik gelişmelerle paralellik göstererek, bütün dünyada artarak gerçekleştirilmektedir. Yönlü ve yatay sondajlar genelde denizlerde ve çeşitli sebeplerden dolayı dik kuyu açmaya imkan vermeyen karasal ortamlarda gerçekleştirilmektedir. Dik kuyularla karşılaştırılıdığında, yönlü ve yatay sondajlar yapılırken, sınırlı matkap yükü, daha zor kuyu temizliği, yön kontrolü, vb,, gibi birçok faktörün gözönüne alınması gerekmektedir. Bu sebepten dolayı, sondaj delme hızını arttırabilmek için uygun sondaj parametrelerini seçmek daha da zorlaşmaktadır. Bu çalışmanın amacı, birçok sondaj parametresi ve koşulunu dikkate alan bir sondaj delme hızı modeli oluşturmaktır. Bu çalışmada oluşturulan model, Bourgoyne ve Young'a ait modelin geliştirilmişi olup, formasyon sıkışması, formasyon basıncı, eşdeğer sirkülasyon ağırlığı, etkin maktap yükü, matkap döndürme hızı, matkap aşınması, kuyu temizliği, kuyu eğimi, su kaybı ve matkap hidröliğini dikkate almaktadır. Ayrıca, döner konlu ve PDC matkaplar için uygulanabilen bir matkap aşınma modeli sunulmuştur. Oluşturulan modelin performansı, İran Körfezi'nde kazılmış olan birkaç yönlü ve yatay deniz sondajından elde edilen arazi verisi kullanılarak ölçülmüştür. Hesaplanan delme hızlarının gerçek değerlerle karşılaştırıldığında, ±%25'lik bir hata payı ile tespit edilebildiği gözlenmiştir.

Anahtar Kelimeler: Delme Hızı, Multiple Regresyon Analizi, Optimizasyon, Yönlü ve Yatay Kuyular, Matematiksel Modelleme

To My Wife

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NOMENCLATURE

A_{ann}	Area of annulus, ft^2
A_{bed}	Area of cuttings bed, ft^2
A_{cut}	Area occupied by cuttings in the annulus, ft^2
A_f	Formation abrasiveness parameter, hours
A_r	Archimedes number
A_{hole}	Area of the drilled hole, ft^2
A_{open}	Area open to flow above the cuttings bed, ft^2
A_{pipe}	Area of drill pipe or drill collars, ft^2
C_{ang}	Correction slip factor for angle of inclination
C_{bed}	Correction factor for cuttings concentration for sub-critical flow
$C_{geo(D)}$	Correction factor for geometry - hydraulic diameter
$C_{geo(PV)}$	Correction factor for geometry - mud rheology
$C_{geo(\theta)}$	Correction factor for geometry - angle
C_{mwt}	Correction slip factor for mud density
C_{size}	Correction slip factor for cuttings size
C_{conc}	Cuttings concentration by volume in the annulus
C_f	Drilling cost per foot drilled, $/ft
C_C	Cuttings concentration for a stationary bed (by volume), corrected for viscosity
D	well depth, ft
D_{hyd}	Hydraulic diameter, in
D_{pipe}	Diameter of pipe, in
D_{hole}	Diameter of drilled hole, in
EDC	Equivalent circulating mud density at the hole bottom, lb/gal
F_r	Froude number
G	Regression index of correlation
GPM	Mud flow rate, gpm
H	Hole or bit diameter, in
H_1,H_2,H_3	Constants that depend on bit type
K	Formation drillability parameter, ft/hr
N	Rotary speed, RPM
N_c	Critical rotary speed, rpm

N_{re}	Reynolds number
MD	Measured depth, ft
ΔP_b	Pressure drop across the bit, psi
ΔP_d	Pressure drop through the circulation system except the bit, psi
P_P	Pump Pressure, psi
PV	Plastic viscosity, cp
R_{dc}	Ratio of cuttings diameter to the annular hydraulic diameter
ROP	Rate of penetration, ft/hr
$(ROP)_c$	Calculated rate of penetration, ft/hrs
$(ROP)_{ob}$	Observed rate of penetration, ft/hrs
S_s	Ratio of the cuttings density to drilling fluid density
TVD	Total vertical depth, ft
T_a	Taylor number
V	Volume of rock removes
V_{cirt}	Critical transport velocity, ft/sec
V_c	Crater volume
V_{cut}	Particle travel velocity, ft/sec
V_{open}	Velocity in the open area above the cuttings bed, ft/sec
V_s	Particle volume
V_{slip}	Equivalent slips velocity, ft/sec
WOB	Bit weight, 1,000 lbf
WOB/d_b	Weight on bit per inch of bit diameter, 1000 lb/in
$(WOB/d_b)_{mech}$	Mechanical weight on bit per inch of bit diameter, 1000 lb/in
$(WOB/d_b)_c$	Critical weight on bit per inch of bit diameter, 1000 lb/in
$(WOB/d_b)_{cir}$	Normalized value for weight on bit per inch of bit diameter, 1000 lb/in
$(WOB/d_{b)\ max}$	Bit weight per inch of bit diameter at which the bit teeth would fail instantaneously, 1000 lb/in.
YP	Yield point, lb/100p
a_l	formation strength parameter
a_2	exponent of the normal compaction trend
a_3	under compaction exponent
a_4	pressure differential exponent
a_5	bit weight exponent
a_6	rotary speed exponent
a_7	tooth wear exponent

a_8	hydraulic exponent
a_9	hole cleaning exponent
a_{10}	hole cleaning exponent
a_{11}	hole cleaning exponent
d_b	bit diameter, in
d_s	particle diameter, in
d_n	bit nozzle diameter, in
d_1	outside diameter, in
d_2	inside diameter, in
f	fiction factor
f_s	volume fraction of cuttings in the annulus
g	acceleration of gravity
g_p	pore pressure gradient of the formation, lb/gal
h	bit tooth dullness, fraction of original tooth height worn away
h_f	final bit tooth dullness
i	summation index for ith data point
j	summation index for jth data drilling parameter
k	exponent on weight in drilling rate equation
n	number of data points used in regression analysis
q	flow rate, gal/min
q_m	mud flow rate, gal/min
r_i	residual error
t	rotating time, hours
t_b	bit rotating time, hours
t_f	final rotating time, hours
v_a	annular fluid velocity, ft/min
v_{sl}	particle slip velocity, ft/min
v_T	particle transport velocity, ft/min
v_{actual}	mud velocity in annulus, ft/s
$v_{critical}$	mud critical velocity in annulus, ft/s
x_1	formation strength factor
x_2	normal compaction drilling parameter
X_3	under compaction drilling parameter
X_4	pressure differential drilling parameter
X_5	bit weight drilling parameter
X_6	rotary speed drilling parameter
X_7	tooth wear drilling parameter

X_8	bit hydraulics drilling parameter
X_{9-11}	Hole cleaning parameters for horizontal, inclined, vertical section

Greek Letters

μ	Viscosity
μ_a	the apparent viscosity at 10,000 sec^{-1}
γ	density of rock cuttings, lb/ft^2
Φ	rock porosity
ρ	mud density, lb/gal
ρ_c	equivalent circulating mud density at the bottom hole, lb/gal
ρ_e	effective annular mud density, lb/gal
ρ_f	fluid density, lb/gal
ρ_s	particle density, lb/gal
τ_H	formation abrasiveness constant or life of teeth at standard conditions, hours
θ	angle of inclination of the well bore from vertical, degrees

Abbreviations

ann	annulus
ang	angle
bed	bed
cirt	critical
cir	normalized value
conc	concentration
f	Formation
geo	geometry
hyd	hydraulics
mwt	mud weight
mech	mechanical
Re	Reynolds
ob	observed

CHAPTER I

INTRODUCTION

1.1 What is a drilling operation?

By the name of a well (borehole) is meant a cylindrical mine opening made too small for man's access thereto, the diameter of the opening being many times less than its length. Drilling process is conducted by using machinery, called drill rig, which consists of a combination of numerous systems working together. It is the drill collars, screwed onto the bottom of the drill pipe assembly just above the bit, that provide the necessary weight, and prevent buckling of the drill pipes above them. Drill collars, along with drill pipe and bit all make up the drill string, which is rotated by the rotary table and the Kelly. The drill string component parts are hollow down the middle so that the drilling fluid can be circulated down to the bit. A fluid-tight rotary joint, the swivel, is located at the top of the Kelly and provides a connection between the mud pump discharge line and the inside of the drill string. A hoisting system is required to support the weight of the drill string, lower it into the hole and pull it out. This is the function of the derrick, the hook and the draw works.

The drilling rig is complete with facilities to treat the drilling fluid when it gets back to the surface, a storage area for tubular goods, shelters and offices on site.

In addition, when a well is being drilled, it is regularly cased. It is lined with steel pipe, or casing, which is lowered into the hole under its own weight in smaller and smaller diameters as the hole gets deeper. The first length of pipe is run in as soon as the bit has drilled the surface formation and is then cemented in the hole. A casing housing is connected to the top of the surface casing. All the following lengths of pipe are hung on the casing housing and cemented at the base to the walls of the hole.

After the first drilling phase is cased, drilling will be resumed with a bit with a diameter smaller than the inside diameter of the casing string that was run in and cemented. The deeper the borehole gets and the more casings are set in the well, the smaller the diameter of the bit must be.

In order to drill a well, three factors have to be established simultaneously; i) a certain load has to be applied on the bit, ii) the bit has to be rotated, and iii) a drilling fluid has to be circulated within the well bore.

Making a hole for the recovery of underground oil and gas is a process which requires two major constituents; i) man-power, and ii) hardware systems. The man power includes a drilling engineering group and a rig operator group. The first provides engineering support for optimum drilling operations, including rig selection, design of mud program, casing and cement programs, hydraulic program, drill bit program, drill string program and well control program. After drilling begins, the daily operations are handled by a rig operator group which consists of a tool pusher and several drilling crews. The hardware systems which make up a rotary drilling rig are i) power generation system, ii) hoisting system, iii) drilling fluid circulation system, iv) rotary system, v) well blowout control system, and vi) drilling data acquisition system and monitoring system.

As regards their purpose, boreholes drilled for geological exploration of the region, search for, prospecting and exploitation of deposits are classified into key or stratigraphic, extension or outpost, structure-exploratory, reconnaissance, prospecting production and special boreholes.

1.2 Factors Affecting Penetration Rate

The factors which are influencing ROP can be classified in two main groups: *i)* Controllable Factors, and *ii)* Environmental Factors. Table 1.1 lists these factors. The controllable factors can be altered more easily than environmental factors. Because of economical and geological conditions, the variation of environmental factors is impractical or expensive. The number of factors hints at the complexity of the bit/rock interaction, something which is compounded by interdependence and nonlinearity in some of these effects [11]. Since mud properties, such as type, density, etc, are all dependent on formation type, formation pressure, etc, mud properties are included in "Environmental Factors" in Table 1.1.

Table 1.1 FACTORS PROPOSED TO AFFECT ROP [11]

Enviromental Factors	Controllable Factors (Alterable)
Depth	Bit Wear State
Formation Properties	Bit Design
Mud Type	Weight on Bit
Mud Density	Rotary Speed
Other Mud Properties	Flow Rate
Overbalance Mud Pressure	Bit Hydraulic
Bottom hole Mud Pressure	Bit Nozzle Size
Bit Size	Motor/Turbine Geometry

Laboratory studies and modeling are, however, unraveling this complexity. For example, how ROP responds to changes in drilling parameters has been shown to depend strongly on rock properties. In permeable rocks, for example, overbalanced pressure influences ROP, giving way to a dependence on bottom hole pressure as permeability decreases. [6]

However, overbalanced pressure effects are subject to dynamic influences, either via filtration effects on pore pressure at the bit/rock interaction zone, or via stress effects on pore pressure around the well bore. Bit cleaning effects while drilling hydratable formations in water base drilling fluids (mud) may also override the effects of mechanical drilling parameters, so that rock mineralogy and mud chemistry are obviously significant factors. Nevertheless, these effects are directly influenced by bit design, and jet nozzle arrangement. In summary, rock properties that influence ROP include at least mineralogy, strength, density, porosity, and permeability of the formation to be drilled. Interdependence between controllable mechanical and hydraulic drilling parameter effects may also be significant, such as the response of ROP to weight on bit (WOB), rotary speed and flow rate is directly depend on absolute value of these parameters. Bit design effects are also not well understood. Differences in bit design effects on ROP with polycrystalline diamond compact (PDC) bits appear only to become significant when bit surface cleaning problems occur, or when cutters become worn, while with roller cone bits, also jet nozzle arrangement should be considered. [24]

Finally, complexity is increased by errors and inconsistencies in drilling data, meaning that correlations with ROP may be masked without extensive data treatment. Accuracy of the equipment used for data acquisition as well as heterogeneities and insufficient information about the formations cause such problems. This latter point may explain why, despite number of analytically derived ROP models published, none has yet become established as a comprehensive operational tool. [24]

The complexity of the bit/rock interaction, and the difficulties with implementation of analytical models, have encouraged professional people to adopt an empirical approach to optimize ROP in drilling operations. This methodology is usually conducted as follows: i) grouping of data according to the formations, i.e., analysis should be conducted for each drilled formation separately, ii) development of dimensionless groups, and iii) determination of the model constants using the collected data by the help of statistical tools. [24]

Emphasis has also been placed in this work on understanding the effects of controllable variables, i.e., those that can be readily changed to optimize ROP. Other environmental effects are however incorporated into another ROP modeling technique developed by Professional people and described elsewhere. [24]

In considering which variables to choose for developing an ROP model, experience and research suggest these eight variables: i) Mud Properties, ii) Hydraulics, iii) Bit type, iv) Weight on Bit, v) Drill string Rotation Speed, vi) Depth, vii) Bit tooth wear, and vii) Formation Properties. However, for horizontal and inclined well bores, hole cleaning is also a major factor influencing the ROP. The basic interactive effects between these variables were determined by design experiments. Variable interaction exists when the simultaneous increase of two or more variables does not produce an additive effect as compared with the individual effects. The meaning of variable interaction is illustrated in Figure 1.1. [24]

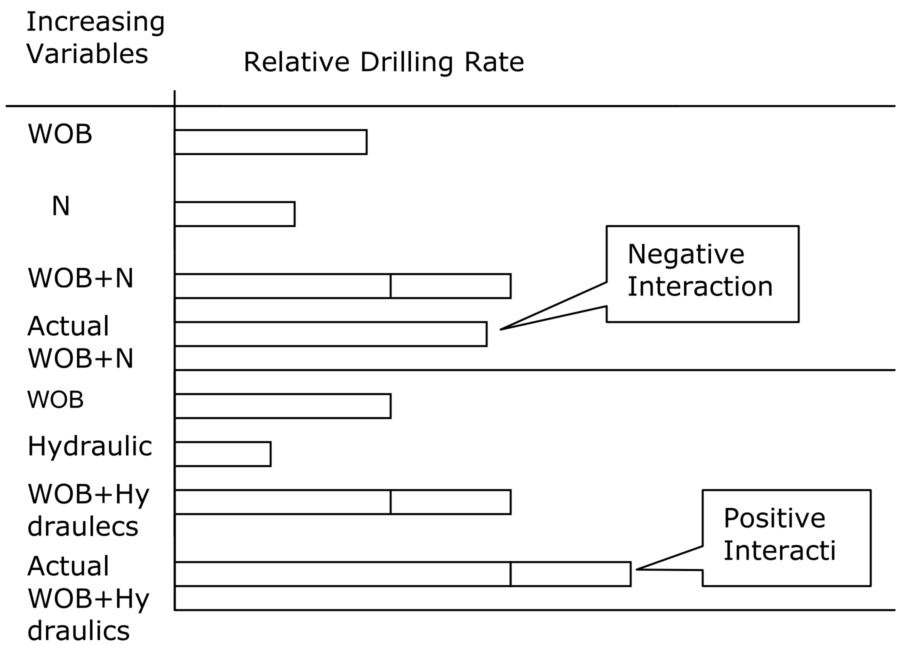

Figure 1.1 Positive and negative interaction [24]

This show the related responses in drilling rate when the variables are increased from one level to another; first individually, second simultaneously. A negative interaction exists when increasing both variables does not produce as high a drilling rate as expected, even though it may be higher than increasing either variable alone. A positive interaction exists when the drilling rate is higher than expected when both variables are increased; i.e., one helps the other. Table 1.2 shows typical interactions among the important drilling variables. Note that these results are not fixed, but may change if the levels at which the variables are being compared are changed.

Table 1.2 typical drilling variable interactions in Hard Rock [24]

Variable Combination	Interaction
WOB-RPM	Negative
WOB-Hydraulics	Positive
RPM- Hydraulics	None
Low solids- Hydraulics	Positive
Low solids-WOB	Positive
Bit type-Formation	Either
Low solids- Bit type	Positive
RPM- Formation	Negative

1.3 In-Depth Explanation of the Most Important Variables and Their Influences on ROP

The rate of penetration achieved with the bit as well as the rate of bit wear, has an obvious and direct bearing on the cost per foot drilled. The most important variables affecting penetration rate that have been identified and studied include (1) bit type. (2) Formation characteristic. (3) Drilling fluid properties. (4) Bit operating conditions (bit weight and rotary speed). (5) bit tooth wear, and (6) bit hydraulics. [6]

1.3.1 Bit Type

The bit type selected has a significant effect on penetration rate. For rolling cutter bits, the initial penetration rates for shallow depths are often highest when using bits with long teeth and a large cone offset angle. However, these bits are practical only in soft formations because of a rapid tooth wear and sudden decline in penetration rate in harder formations. The lowest cost per foot drilled usually is obtained when using the longest tooth bit that will give a tooth life consistent with the bearing life at optimum bit operating conditions. The diamond and PDC bits are designed for a given penetration per revolution by the selection of the size and number of diamonds or PDC blanks. The width and number of cutters can be used to compute the effective number of blades. The length of the cutters projecting from the face of the bit (less the bottom clearance) limited the depth of the cut. The PDC bits perform best in soft, firm, and medium-hard, nonabrasive formations that are not "gummy". [6]

1.3.2 Formation Characteristics

The elastic limit and ultimate strength of the formation are the most important formation properties affecting penetration rate. It is mentioned that the crater volume produced beneath a single tooth is inversely proportional to both the compressive strength of the rock and the shear strength of the rock. The permeability of the formation also has a significant effect on the penetration rate. In permeable rocks, the drilling fluid filtrate can move into the rock ahead of the bit and equalize the pressure differential acting on the chips formed beneath each tooth. It also can be argued that the nature of the fluid contained in the pore space of the rock also affects this mechanism since more filtrate volume would be required to equalize the pressure in a rock containing gas than in a rock containing liquid. The mineral composition of the rock also has some effect on penetration rate. [6]

1.3.3 Drilling Fluid Properties

The properties of the drilling fluid reported to affect the penetration rate include (l) density, (2) rheological flow properties, (3) filtration characteristics, (4) solids content and size distribution, and (5) chemical composition. Penetration rate tends to decrease with increasing fluid density, viscosity, and solids content, and tends to increase with increasing filtration rate. The density, solids content, and filtration characteristics of the mud control the pressure differential across the zone of crushed rock beneath the bit. The fluid viscosity controls the parasitic frictional losses in the drill string and, thus, the hydraulic energy available at the bit jets for cleaning. There is also experimental evidence that increasing viscosity reduces penetration rate even when the bit is perfectly clean. The chemical composition of the fluid has an effect on penetration rate, such that the hydration rate and bit ballling tendency of some clays are affected by the chemical composition of the fluid.

An increase in drilling fluid density causes a decrease in penetration rate for rolling cutter bit. An increase in drilling fluid density causes an increase in the bottom hole pressure beneath the bit and, thus, an increase in the pressure differential between the borehole pressure and the formation fluid pressure. [6]

1.3.4 Operating Conditions (WOB & Rotary Speed)

A typical plot of penetration rate versus bit weight obtained experimentally with all other drilling variables held constant is shown in Fig. 1.2. No significant penetration rate is obtained until the threshold bit weight is applied (Point a). Penetration rate, then, increases with increasing values of bit weight (Segment a-b). As the weight on bit values are increased, a higher increase in ROP is observed (Segment b-c). However, after a certain value of bit weight, subsequent increase in bit weight causes only slight improvements in penetration rate (Segment c-d). In some cases, a decrease in penetration rate is observed at extremely high values of bit weight (Segment d-e). This type of behavior often is called bit floundering. The poor response of penetration rate at high values of bit weight usually is attributed to less efficient bottom hole cleaning at higher rates of cuttings generation or to a complete penetration of the cutting elements of the bit into the well bore bottom. At this weight on bit values, wear on the bit is extremely high. [6]

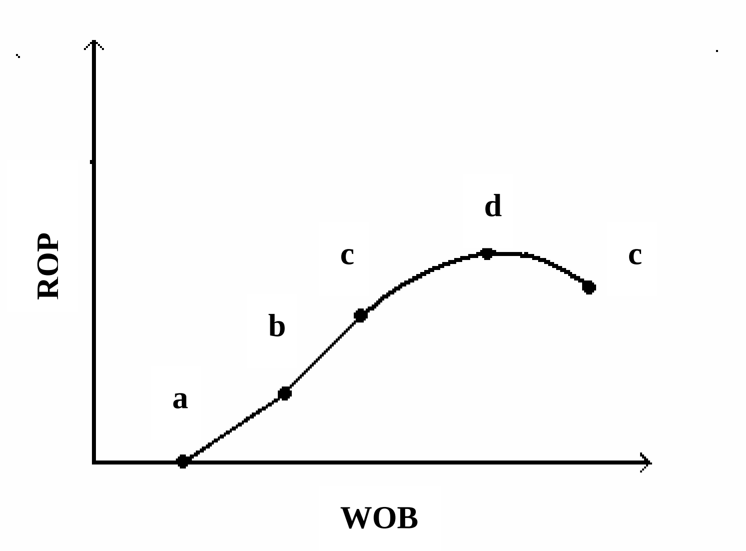

Figure 1.2 Typical response of penetration rate to increasing bit weight [6]

A typical plot of penetration rate versus rotary speed obtained with all other drilling variables held constant is shown in Fig. 1.3. Penetration rate usually increases linearly with an increase in rotary speed (Segment a-b). After a certain rotary speed value, the increase in ROP decelerates as rotation speed is increased (Segment b-c). After point-c, rotation speed has a very slight influence on ROP. The poor response of penetration rate at high values of rotary speed usually is also attributed to less wellborn stability and enlargement of the well bore. [6]

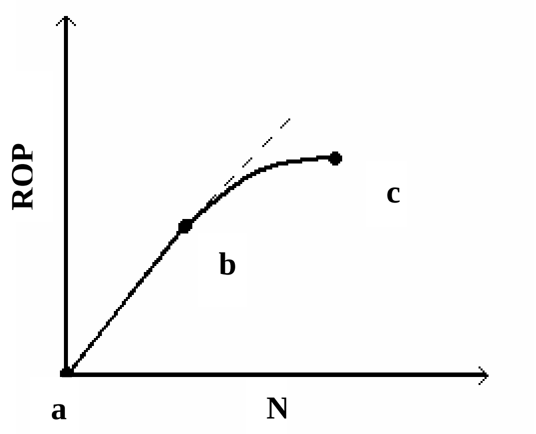

Figure 1.3 Typical response of penetration rate to increasing rotary speed [6]

1.3.5 Bit Tooth Wear

Most bits tend to drill slower as the drilling time elapses because of tooth wear. The tooth length of milled tooth rolling cutter bits is reduced continually by abrasion and chipping. The teeth are altered by hard facing or by case-hardening process to promote a self-sharpening type of tooth wear. However, while this tends to keep the tooth pointed, it does not compensate for the reduced tooth length. The teeth of tungsten carbide insert-type rolling cutter bits and PDC bits fail by breaking rather than by abrasion. Often, the entire tooth is lost when breakage occurs. Reductions in penetration rare due to bit wear usually are not as severe for insert bits as for milled tooth bits unless a large number of teeth are broken during the bit run. [6]

1.3.6 Bit Hydraulics

Significant improvements in penetration rate could be achieved by a proper jetting action at the bit. The improved jetting action promoted better cleaning of the bit face as well as the hole bottom. There exists an uncertainty on selection of the best proper hydraulic objective function to be used in characterizing the effect of hydraulics on penetration rate. Bit hydraulic horsepower, jet impact force, Reynolds number, etc, are commonly used objective functions for describing the influence of bit hydraulics on ROP. [6]

1.4 Directional and Horizontal Well Drilling

Recently, with the advancement of industrial techniques, the number of inclined and horizontal wells has been increased. Common application fields for directional and horizontal drilling are in offshore and onshore when there is no way of drilling vertical wells. The major application of directional drilling are 1) To develop the fields which located under population centers, 2) To drill wells where the reservoir is beneath a major natural obstruction, 3) To Sidetrack out of an existing well bore, 4) To elongate reservoir contact and thereby enhance well productivity.(see figs. 1.4, 1.5, 1.6 and 1.7).

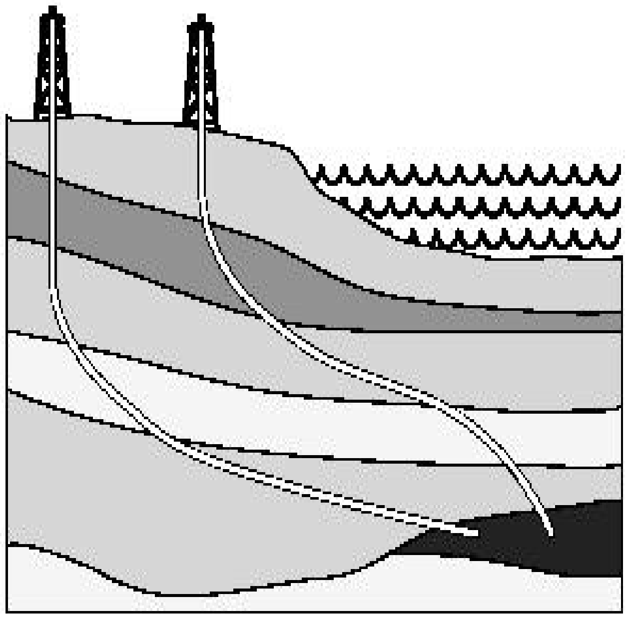

Figure 1.4 Drilling wells beneath a major surface obstruction [6]

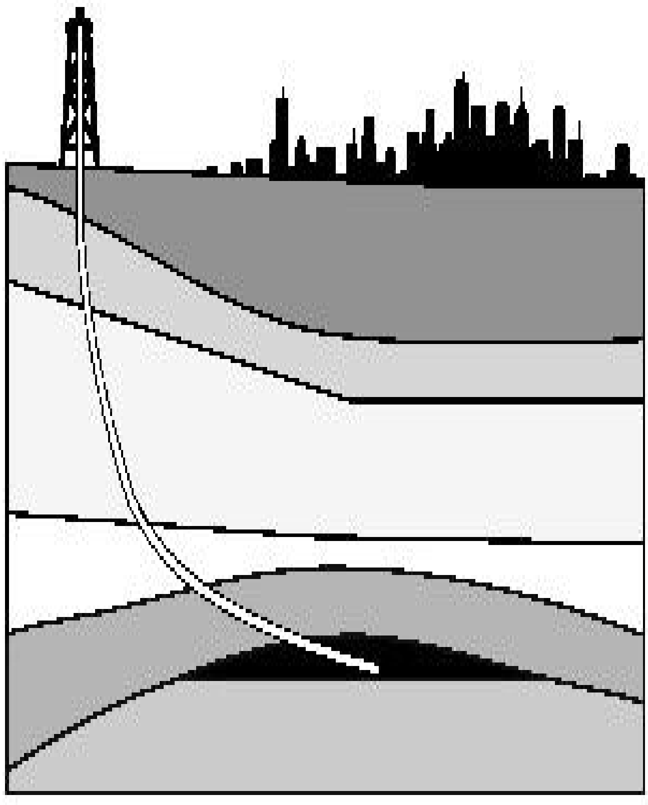

Figure 1.5 Developing a field under a city using directionally drilled well [6]

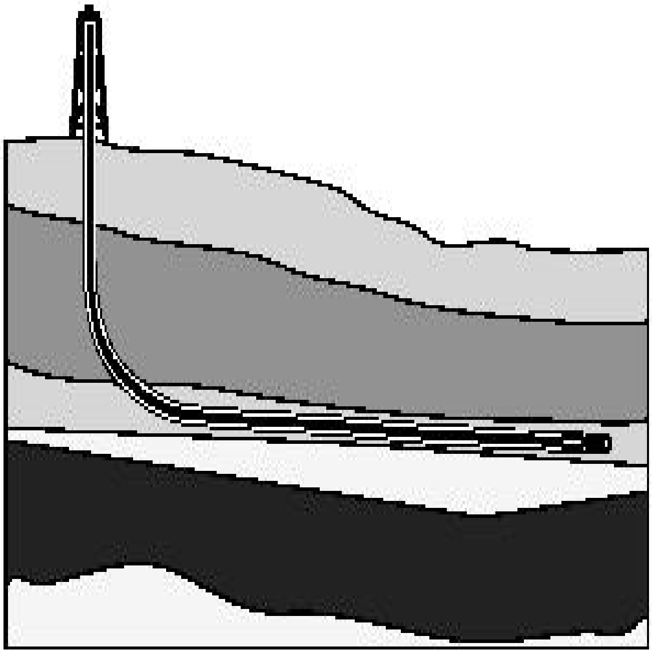

Figure 1.6 Elongating reservoir contacts and enhancing well productivity [6]

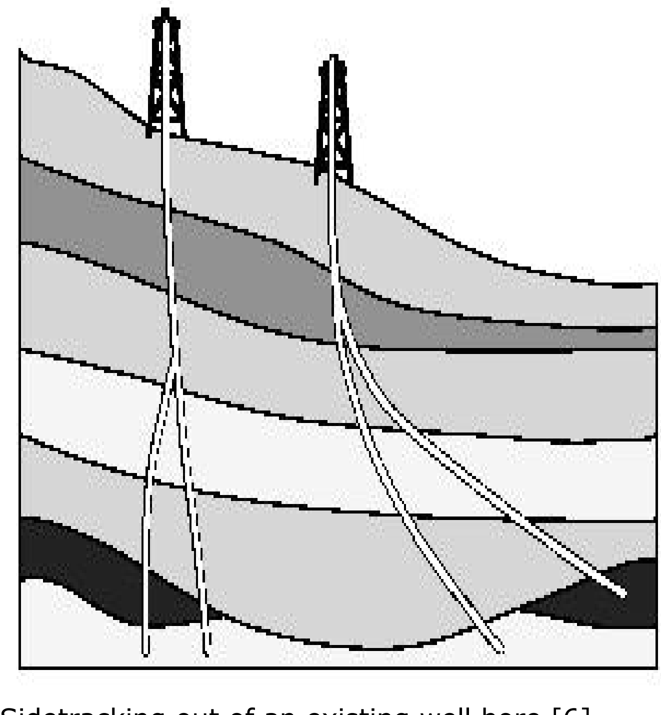

Figure 1.7 Sidetracking out of an existing well bore [6]

1.5 Importance of Estimating ROP & $/ft

The costs of horizontal well drilling are approximately 1.4 to 3 times more than a vertical well drilling depending on the drilling method [36].So, the careful estimating of rate of penetration and optimizing of cost per foot for a directional and horizontal project prior to the commencement of actual operations is probably the single most important factor of a project.

1.6 Need of ROP Model for Horizontal Wells

The major challenges in drilling inclined and horizontal wells are: 1) limited weight on bit, 2) Difficulty in hole cleaning, 3) trajectory control, 4) higher torque and drag, etc. Because there is friction force between drill string and well bore, the load transferred to the bit is less in directional and horizontal wells when compared with vertical wells. Also, especially at mid-range hole inclinations, considerably higher flow rates are required for effective hole cleaning, since a concept of cuttings bed development arise in such wells. The physical forces governing the movement of cuttings in directional and horizontal well bores are radically different than vertical wells. If minimum hydraulic requirements for the cuttings bed removal are not achieved, circulation can be ineffective, time consuming, and in some cases, detrimental to well bore stability. [6]

The problems such as abnormal torque and drag, lost circulation, difficulties in running casing, poor cement jobs, and the necessity of re-drilling and, in some cases, mechanical stuck may be caused by the excessive accumulation of solids in the annulus. Therefore, it is very important to clean the hole effectively during drilling of directional and horizontal wells. [6]

Although there exist numerous rate of penetration estimation models for vertical well drilling operations, very fewer studies have been conducted for directional and horizontal wells. Especially, the additional challenges are considered, serious modifications on existing ROP models for vertical wells are required to estimate ROP for directional and horizontal wells. [6]

CHAPTER II

LITERATURE REVIEW

An extensive literature review was carried out in order to determine the state of the art on the subject. Because of the vast number of articles in these areas, this literature review is limited to the most relevant and/or well known works.

2.1 Mechanical Parameters

Galle and Woods [13] presented a pioneer work that created a major breakthrough in drilling technology, mainly when referring to optimization aspects. They assumed that rate of penetration was affected by only two parameters, weight on bit and rotary speed. In their paper, also, it is assumed that all other variables involved, like bit selection, hydraulics, drilling fluid properties, etc., were properly selected. They defined an analytical model to predict rate of penetration (ROP) as a function of weight on bit, rotary speed, type of formation, and bit tooth wear.

Maurer [29] derived an equation for rate of penetration for roller-cone bits from rock cratering mechanisms. This equation holds for "perfect cleaning", which is defined as the condition where all of the rock debris is removed from the bottom hole.

Galle and Woods [14] followed the similar procedures that they used in their early 1960 paper. They presented procedures for determining the best combination of constant weight on bit and rotary speed, the best constant weight on bit for any given rotary speed, and the best constant rotary speed for any given bit weight analytically. For each of these procedures, they presented eight cases considering a combination of bit teeth and bearing life, such that drilling rate limits economical bit life. They established empirical equations for the effects of weight on bit, rotary speed, and cutter structure dullness on drilling rate, rate of tooth wear and bearing life.

Mechem and Fullerton [30] introduced a rate of penetration model based on formation drill ability, bit weight, rotary speed, well depth, mud pressure, and applied hydraulics. Their model is using a concept based on a single expression of drilling energy-level, the $WOB \times N$ product that can be related to those variables using graphical methods. These correlations provide the basis of for determining hydraulic requirements, estimating drilling cost, basic well planning, and drilling optimization.

Langston [23] described a methodology for managing daily drilling data as well as existing information collected from the very same field. He indicated that during analysis of actual drilling data, none of the drilling variables could be excluded due to the simultaneous interaction of each and every variable among themselves. In practice, procedures interrelate and depend upon each other as well as on personnel and mechanical factors.

Young [55] described a computer control system for collecting and analyzing the field data, and presented a real case application. He developed a solution for minimum-cost drilling assuming constant bit weight and rotary speed over the entire bit life for roller cone and PDC bits. The proposed solution is depended on four equations, i.e., drilling rate, bit bearing wear, bit tooth wear, and cost.

Lummus [24] presented the definition and philosophy of optimization. He discussed the influence of major drilling parameters on drilling performance. He also proposed a drilling optimization methodology. In this paper, data required for drilling optimization were obtained from *i*) Logs (preferably IES or sonic), *ii*) Bit records, *iii*) Mud records, *iv*) Recorded drilling data, such as torque, pump pressure, penetration rate, etc., *v*) Drilling program for proposed well, i.e., casing setting depths, hole size, expected problems, etc., *vi*) Rig specifications, and *vii*) Correlation of formation characteristics of the well.

Lummus [25] discussed the acquisition and analysis of data needed to plan, maintain, and appraise the drilling of a particular well. The data required for optimized drilling are classified as follows: *i*) Data needed for computer input to calculate optimum values for the controllable drilling conditions, *ii*) Data needed on a day-by-day basis to determine how efficiently drilling optimization is being applied and to provide the basis for suggested changes in mud, hydraulics, bits, etc., and *iii*) Data needed to evaluate the effectiveness of an optimum drilling program for a particular well and to develop definite recommendations for improving drilling efficiency on future wells.

Wilson and Bentsen [51] presented optimization techniques for minimizing drilling costs by restricting the number of parameters to be optimized to two, namely, the weight on the bit and the rotary speed. In this study, three methods of varying complexity have been developed. The first method seeks to minimize the cost per foot drilled during a bit run. The second method minimizes the cost of a selected interval, and the third method minimizes the cost over a series of intervals. The methods are listed in order to increase complexity. It was found that each of the methods gave a worthwhile cost saving and that the saving increased as the complexity of the method increased. The data requirements for the method increased with increasing method complexity.

In order to have the least cost per foot, Reed [40] developed a method to find the best combination of weight on bit and rotary speed in two cases, constant or variable parameters. His method agreed very well with results from Galle and Woods papers, but it was considered to be more precise because the equations were solved in a more rigorous way using a Monte Carlo Scheme. This paper also showed that there is little advantage in using variable weight speed technology over the simpler constant weight-speed method, if the formation is homogenous.

Hal B. Fullerton [12] followed the similar procedures that he used in their early 1965 paper. He presented relationship between weight on bit, rotary speed, rate of penetration, and apparent rock drill ability (K_f).In this study, it is assumed that ,within normal operating ranges, any $WOB \times N$ value may be considered a constant. Also, effect of hydrostatic pressure on apparent rock drill ability and effects of bit hydraulic horsepower and tooth wearing on

$WOB \times N$ are represented by related equations and graphs. Bit records obtained from wells in an area of interest are used to test the accuracy of model.

Bourgoyne and Young[5] developed a mathematical model, using a multiple regression analysis technique of detailed drilling data, to describe the drilling rate based on formation depth, formation strength, formation compaction, pressure differential across the bottom hole, bit diameter and bit weight, rotary speed, bit wear and bit hydraulics. As a function of these eight parameters, a mathematical model was developed in order to find the best constant weight on bit, rotary speed and optimum hydraulics for a single bit run in order to achieve minimum cost per foot. The method also predicts the drilling hours and bit wear. They considered that more emphasis had been placed on the collection of detailed drilling data to aid in the selection of improved drilling practices. Thus, the constants that appear in their model could be determined from a multiple regression analysis of field data (See Appendix A). The Bourgoyne& Young model has greater acceptance within the portion of the drilling industry that uses drilling models at all, because it is one of the most complete models.

E.Tanseu [48] presented a new approach in formulating and solving the optimal drilling problem. The approach is heuristic as it involves the interaction of raw data, regression and an optimization technique. From several bit runs, regression equations were established for predicting penetration rate and bit life. Three control variables are accounted for: weight on bit, rotary speed and bit hydraulic horsepower. The equations for penetration rate and bit life are incorporated into a drilling cost equation and the cost function is minimized over the control variables. These variables then dictate the optimal drilling of the next bit run.

Hoover and Middleton [17] tested experimentally five polycrystalline diamond Compact (PDC) bit designs in the laboratory at 100and 500 rpm in three different types of rock: Nugget sandstone, Crab Orchard sandstone, and Sierra White granite. This paper describes the testing procedures, summarizes bit performance and wears characteristics, and correlates these experimental results with specific design options such as rock angle, bit profile, and material selection. As the bits develop large wear flats in hard rock, it is concluded that the torque becomes much more sensitive to changes in the weight on the bit.

Hussein Rabia [18] presented a simplified approach to bit selection that uses the principle of specific energy. Specific energy (E_s) may be defined as the energy required removing a unit volume of rock. Comparison of bit selection, based on both cost per foot and specific energy, was made. It can be indicated that E_s can be used to select the proper bit type for any section of hole, and the switch over points for different bit types may be determined from the plots of Specific energy vs. depth. Specific energy also can be used as a criterion for ending the use of a current bit. For this application, Specific energy can be a more meaningful tool than any other available means, such as the cumulative cost per foot. The potential application of specific energy in development and exploration wells was discussed.

S.C, Malguarnera [27] formulated system of equations which describe the quantitative interactions of the most important parameters of the rotary drilling process. These equations are based on both laboratory and field observations. The equations were then incorporated into computer programs to provide bit run simulation, and operating condition of optimization. Drilling model which described in this paper provide a systematic way to use mathematical modeling and computer capability.

Ziaja and Miska [31] presented mathematical model of the polycrystalline diamond bit drilling process and its practical application. Expressions for bit torque and bit weight are obtained in terms of bit penetration rate. The model takes into account the reduction in penetration rate during drilling resulting from bit wear. Some teats in the field conditions have shown that theoretical results agree reasonably well with available experimental data. A graphical method for estimating so-called indexes of rock properties also has been established.

E.L.Simmons [47] illustrated a technique for synergistically coupling several optimization parameters, namely optimum hydraulics, weight on the bit and bit rotation, in order to achieve a higher degree of drilling efficiency. Formation drill ability and bit type selection are brought out and integrated with a generally accepted drilling rate equation. None of the technology or concepts brought out in this paper is new. What has been attempted however is an illustration of how several of these well known concepts should be sequentially coupled in order to achieve a system for true drilling optimization.

Reza and Alcocer [41] developed a drilling model using dimensional analysis. The parameters included in the three equations of penetration rate, rate of bit dulling and rate of bearing wear are weight on bit, rotary speed, flow rate, bit diameter, bit nozzle diameter, bearing diameter, mud kinematics viscosity, differential pressure, temperature, and heat transfer coefficient. They developed dimensionless models for roller cone, PDC and diamond bits.

Brett and Millheim [7] presented a method named Drilling Performance Curve (DPC) that is a simple powerful tool to assess the drilling performance in any given area where a consecutive series of similar wells have been drilled. All the information that is needed to perform the analysis is the sequence numbers of the well and the time to reach a given depth. This paper presents some typical examples of DPC'S covering a study of over 30 different areas (onshore and offshore) including over 2000 wells. From the data, a simple model for the overall drilling performance was derived It will be shown that the DPC can dictate the strategy for a drilling program and what the economics of drilling a sequence of wells should be in a given area.

T.M. Warren [50] developed a model for predicting ROP for roller-cone bits under low-borehole-pressure conditions. This model accounted for both cuttings generation and cuttings removal. Drilling data obtained under high-borehole-pressure conditions were analyzed to determine the reasons of the reduction in ROP as the borehole pressure increases. In some cases, the reduced ROP is caused by a buildup of rock debris under the bit. When this occurs, the ROP can be improved by an increased level of hydraulics. In other cases, the reduction in ROP seems to be caused by a local catering effect that is much less responsive to increases in hydraulics. Comparison of model predictions to the observed ROP can help to identify the mechanism that limits the ROP and provide insight into ways to improve it.

Winters, Warren and Onya [52] developed a model, which relates roller cone bit penetration rates to the bit design, the operating conditions, and the rock mechanics. Rock ductility is identified as a major influence on bit performance. Cone offset is recognized as an important design feature for drilling ductile rock. The model relates the effect of cone offset and rock ductility to predict the drilling response of each bit under reasonable combinations of operating conditions. Field data obtained with roller cone bits can be interpreted to generate a rock strength log. The rock strength log can be used in conjunction with the bit model to predict and interpret the drilling response of roller cone bits.

Wojtanowicz and Kuru [54] developed a new mechanistic drilling for both roller cone bits and PDC bits. The model was fully explicit with physical meanings given for al1 constants and functions. The response of the drilling model to weight-on-bit and cutters removal and the stability of constants were tested using some field and laboratory data. Also, the concept of maximum bit performance (UBP) curve was introduced in this paper. The curves represented maximum values of the average drilling rates for various pre-assumed footage values. In contrast to elaborate drilling models, the MBP curves are a single, comprehensive correlation representing drilling bit behavior in a formation. For calculating purposes, the curves were normalized and thus they became insensitive to drill ability change vs. depth as well as formation abrasiveness. The curves were plotted and analyzed for both roller cone bits and PDC bits. Also, the simple method for using the MBP curves for drilling optimization was presented.

Guo X.Z. [16] described a theoretical analysis of the penetration-cost objective function, specifically its first- and second-order differentiability, convexity, and presents the location and method of searching for optimum drilling parameter. The model used in this paper is basically the modified Young's model which was commonly used for unsealed-roller-cone bits. In this study, it is expended to insert-tooth bits by graphically processing. This paper discusses the features and practical significance of a combined isocost graph, such as determining the maximum economic results for each combination of bit weight and speed and providing a scientifically sound basis for modifying drilling parameters. A case study of the Zhong Yuan oil field also is given.

Bonet, Cunha and prado [4] analyzed the drilling cost for the operation of an entire drilling operation, from its initial to final depth, in homogeneous formations. The main objective of this work was to find the optimum drilling parameters for each bit used during the drilling operation, the number of bits to be used and the depth where each bit will be changed. A computer program was developed to simplify the use of the method.

Wojtanowicz and Kuru [53] presented a new methodology in drilling optimization using a dynamic programming (the dynamic drilling strategy). This strategy employs a two-stage optimization procedure, locally for each drill bit, and totally for the whole well. The program includes the distribution of bit footage along the well paths, depths of tripping operations, bit-control algorithms for all bits, and the optimum number of bits per well.

Barragan, Santos and Maidla, E.E. [2] indicated that optimization of multiple bit runs is more economical than optimization of single bit runs. They developed a method based on a heuristic approach to seek the optimum conditions using Monte Carlo Simulation. This method does not depend on a particular drilling model and has been tested with several models.

Parker, Collins, Pelli and Brancato [38] developed software to assist in the choice of roller bits and to estimate the optimum weight on bit and rotational speed. The analysis was based on prior drilling experience in a field, utilizing the Bourgoyne and Young method. The optimal weight on bit and rotational speed calculated based on the minimum cost per meter.

2.2 Cuttings Transport

Efficient removal of cuttings from the well bore is one of the major considerations during both design and operational stages of a drilling process. Inadequate hole cleaning may give rise to serious drilling problems, like increase in torque and drag, stuck pipe, loose control on density, difficulty when running and cementing casing, etc. [8,37]. If the situation is not handled properly, these problems can ultimately lead to the loss of a well. A single stuck pipe indecent may cost over million dollars [1]. To avoid such problems, generated cuttings have to be removed from the well bore by the help of the drilling fluid. The ability of the fluid to lift such cuttings is generally referred to as carrying capacity of the drilling fluid. The major factors affecting the carrying capacity of drilling fluids may be listed as fluid annular velocity, hole inclination, drilling fluid properties, penetration rate, pipe/hole eccentricity, hole geometry, cuttings properties, and drill pipe rotation speed[49]. In fact, fluid flow velocity is the dominant drilling variable on hole cleaning due to its direct relation with the shear stress acting on the cuttings bed [21]. It has been stated that in order to remove cuttings from a horizontal or a deviated well bore, a sufficient shear stress should be applied on the cuttings bed surface in order to lift the particles and erode the developed bed. Such a lifting process, of course is directly dependent on not only the fluid properties, but also the cuttings properties, like shape, compaction properties, etc[21,43,44]. Additionally, it is reported that due to the interaction between the drilling fluids and cuttings, gel formation within the developed cuttings bed occurs, which significantly increases the required shear force needed to erode the bed, and lift the cuttings particles up from the bed [43,44]. Studies on cuttings transport have been in progress during the past 50 years.[37] These studies can separated into two basic approaches: *i*) empirical and *ii*) theoretical. Tomren, Iyoho and Azar [49] investigated effects of pipe rotation and hole inclination angle, eccentricity, flow regimes on cuttings transport performance. Becker, Azar and Okrajni[8] conducted experimental study comparing the effects of fluid rheological parameters (fluid yield point (*YP*), plastic viscosity (*PV*), *YP/PV* ratio, power law exponent, consistency index, etc.) on annular hole cleaning using a large scale flow loop. They pointed out that turbulent flow improved cuttings transport for highly-inclined wellbores, and the effects of fluid rheology dominated at low inclinations. Sifferman and Becker[9] stated that the variables influencing cuttings bed thickness were mud annular velocity, mud density, inclination angle, and drillpipe rotation (with the first two being the most important). Sanchez [45] examined the effect of drillpipe rotation on hole cleaning during directional well drilling. He observed that bed erosion was improved with pipe rotation. He noted that pipe rotation also caused irregularities in bed thickness along the test section. Yu et al [56] proposed a new approach to improve the cuttings transport capacity of drilling fluid in horizontal and inclined wells by attaching gas bubbles to the surface of drilled cuttings using chemical surfactants.

Also, numerous theoretical and mechanistic models were introduced for describing the mechanism of bed development and cuttings transport in inclined and horizontal wells. Two and three layer models are introduced

[37,15,19]. Some of these model performances were tested using experimental data collected in different cuttings flow loops. Also, there were attempts for determining the critical fluid velocity for preventing bed development, either theoretically or experimentally. Larsen, Pilehvari and Azar [22] presented a new cuttings-transport model which predicted critical velocity needed to keep all cuttings moving for horizontal and high-angle wells. Cho, et al [9] developed a three-layer model similar to Nguyen and Rahman's [35] model. They developed a simulator and compared the results with existing models as well as the experimental data conducted by other researchers. They developed charts to determine the lowest possible pressure gradient to serve as an operational guide for drilling operations. They also observed the minimum critical velocity for preventing a stationary bed development using the simulator results. Masuda et al [28] conducted both experimental investigation and numerical simulation for different flow conditions to determine the critical fluid velocity in inclined annulus.

Ozbayoglu [36] presented an analysis of bed height in horizontal and highly-inclined wellbore by using artificial neural network. In this study, a dimensional analysis is conducted using basic drilling information such as pump rates, fluid densities and viscosity, drilling rate, and wellbore geometry. By using these drilling variables, three dimensionless groups (Reynolds Number, Froude Number and cuttings concentration at the bit) are developed for estimating the height of stationary cuttings beds deposited in horizontal and highly-inclined wellbores for a wide range of drilling fluids, including foams and compressible drilling fluids for underbalanced drilling. A series of cuttings transport tests were conducted within the annular test section of a flow loop in order to determine the equation constants.

Duan and Miska [32] investigated the effect of cutting size, drill pipe rotation, fluid rheology, flow rate and hole inclination in small cutting transport. The resulys shown significant difference in cuttings transport based on cuttings size.In this study, also, mathematical modeling was performed to develop correlations for cuttings concentration and bed height in an annulus for field applications.

2.3 Drilling Hydraulics Optimization

Several authors [20,33] have identified the drilling variables and drilling constraints used in the case of drilling hydraulics optimization. The variables are flow rate, which sets annular velocity and pressure losses in the system; pump pressure, which sets jet velocity through nozzles; flow rate-pump horsepower relationship, which sets hydraulic horsepower at bit; and the drilling fluid, which determines the pressure losses and cuttings transport rate. The constraints include (1) financial limits and (2) physical constraints such as the geometry of the wellbore, the performance of rig equipments such as mud pumps and riser booster pumps, the integrity of the wellbore and the removal of cuttings from the annulus.

Early published work on hydraulics optimization concentrated on maximizing bit hydraulic properties: bit hydraulic horsepower, bit jet velocity and jet impact force, examples include Kendall [22] and Moore [34] studies. Equations for each of these parameters were differentiated and solved to find a maximum value and hence the optimum flow rate for that condition. These techniques were translated into monograph and slide rule format. The optimization procedures included simple relationships for fluid in turbulent flow. Early studies paid very little attention to analysis of cuttings removal, while later procedures stated that bit hydraulics optimization was only valid within flow rate limits dictated by hole cleaning, hole erosion and ECD limitations.

CHAPTER III

Statement of the Problem and Scope

Drilling operations are the most expensive and money consuming processes in oil and gas industry. The companies are always interested in finding ways for drilling the safest as well as the most economical. Thus, drilling optimization becomes a very important issue for drilling companies.

The basic objective of drilling optimization is to achieve the greatest degree of efficiency possible under specified conditions, trying to get the highest or lowest outcome of an objective function. Thus, in general, the optimization technique involves the formulation of the objective function, identification of the controllable variables, dependent and independent, and some technical and technological limitations or constraints.

The concept of optimization is based on the fact that all drilling variables are interrelated; i.e., changes in one variable affect all the others, some positively, some negatively. During drilling horizontal and directional wells, even more variables arise when compared with vertical wells. Hole cleaning is a key parameter for such inclined wells, which influence ROP, hydraulics, torque and drag, etc. Therefore, in directional and horizontal wells, efficient hole cleaning must be considered during ROP estimations and optimization.

Drilling optimization is usually conducted using models for estimation of ROP as well as cost per foot. Altough there exist numerous models for optimization of vertical wells, very less is known for directional and horizontal wells, since very little attempts have been conducted for utilizing additional drilling parameters arisen during drilling horizontal and inclined wells with existing models. This study aims to fulfill this need.

The scope of this research is as follows:

- Literature reviews for all relevant past work.
- Analysis of existing mathematical (empirical and semi-empirical) ROP models.
- Investigation of drilling variables on ROP for horizontal and directional wells. Definition of the system of equations of all controllable variables and constraints. Development of a ROP model based on this analysis.
    - Conduct dimensionless analysis and develop dimensionless correlations to predict annular cuttings concentration, dimensionless equilibrium bed area, and dimensionless velocity for describing hole cleaning performance.
    - Development of a model to estimate tooth wear for insert roller cone bits and PDC bits.
- Determination of optimum values of some major drilling parameters using the proposed model.
- Testing the performance of the proposed model by using actual field data obtained from Persian Gulf.

CHAPTER IV

THEORY

4.1 ROP Models

There are three most widely used models for estimating rate of penetration; i) Maurer, ii) Galle and Woods, and iii) Bourgoyne and Young. Maurer [29] derived an equation for rate of penetration for roller-cone bits which is expressed as:

$$\frac{dD}{dt} = \frac{4}{\pi d_b^2}\frac{dV}{dt} \qquad (4.1)$$

Galle and Woods [14] established semi-empirical equations for the effects of weight on bit, rotary speed, and cutting structure dullness on drilling rate, rate of tooth wear and bearing life. These equations are shown below.

Drilling rate is defined as

$$\frac{dD}{dt} = K\frac{\overline{WOB^k}}{a^p}r \qquad (4.2)$$

Where "*r*" is a function of *N*, defined as

$$r = \left[e^{\frac{-100}{N^2}}N^{0.428} + 0.2N\left(1 - e^{\frac{-100}{N^2}}\right)\right] \text{for hard formation} \qquad (4.3)$$

$$r = \left[e^{\frac{-100}{N^2}}N^{0.750} + 0.5N\left(1 - e^{\frac{-100}{N^2}}\right)\right] \text{ for soft formation} \qquad (4.4)$$

and $\overline{WOB}$ is a function of *WOB* and d_b, such that

$$\overline{WOB} = \frac{7.88WOB}{d_b} \quad (4.5)$$

Bourgoyne and Young [5] developed a mathematical model.A summary of the equations is given below.

Rate of penetration is expressed as:

$$\frac{dD}{dt} = Exp(a_1 + \sum_{j=2}^{8} a_j x_i) \quad (4.6)$$

Where x_i is the set of dimensionless drilling parameters calculated from the actual collected drilling data, and a_j represents the set of constants that relates with each of the drilling parameters considered. Dinemsionless drilling parameters in this equation is described as following:

Formation Resistance:

$$x_1 = 1 \quad (4.7)$$

Consolidation Effects:

$$x_2 = 10{,}000 - TVD \quad (4.8)$$

Overpressure Effects:

$$x_3 = TVD^{0.69}(g_p - 9.0) \quad (4.9)$$

Differential Pressure:

$$x_4 = TVD(g_p - \rho_c) \quad (4.10)$$

Bit Diameter and WOB:

$$x_5 = \ln\left[\frac{\frac{WOB}{d_b} - \left[\frac{WOB}{d_b}\right]_t}{4.0 - \left[\frac{WOB}{d_b}\right]_t}\right] \quad (4.11)$$

Rotary Speed:

$$x_6 = \ln\left[\frac{N}{100}\right] \quad (4.12)$$

Tooth Wear:

$$x_7 = -h \quad (4.13)$$

Bit Hydraulic:

$$x_8 = \frac{\rho q}{350\mu d_n} \quad (4.14)$$

Bourgoyne and Young [5] also expressed bit wear by using certain assumptions. Tooth wear model is defined as following

$$\frac{dh}{dt} = \frac{H_3}{\tau_H}\left(\frac{N}{100}\right)^{H1}\left[\frac{\left(\frac{WOB}{d_b}\right)_{\max} - 4}{\left(\frac{WOB}{d_b}\right)_{\max} - \frac{WOB}{d_b}}\right]\left(\frac{1+\frac{H_2}{2}}{1+H_2 h}\right) \quad (4.15)$$

Bearing Wear Model:

$$\frac{dB}{dt} = \frac{1}{\tau_B}\left(\frac{N}{100}\right)\left(\frac{WOB}{4d_b}\right)^b \quad (4.16)$$

4.2 Drilling Model

The drilling model selected for predicting the rate of penetration, ROP, by considering the effect of the various drilling parameters is described as

$$ROP = (f_1)(f_2)(f_3)(f_4)(f_5)(f_6).....(f_n) \quad (4.17)$$

where f_1, f_2,f_3. . . . . . f_n represent the functional relations between penetration rate and various drilling variables. Each of these functions contains constants which are shown as a_1 through a_n. Determination of these constants is accomplished by using a multiple regression analysis of collected drilling data. In this study, Bourgoyne & Young's model is improved and enhanced for both PDC and insert-tooth – roller bits as well as for horizontal and directional wells. The major improvements are the consideration of additional drilling parameters occurring due to inclination as well as re-definition of same drilling parameters due to PDC's.

The proposed model for roller-cone bits is

$$ROP = (f_1)(f_2)(f_3)(f_4)(f_5)(f_6).....(f_{11}) \quad (4.18)$$

and for PDC bits

$$ROP = (f_1^{'})(f_2^{'})(f_3^{'})(f_4^{'})(f_5^{'})(f_6^{'}).....(f_{11}^{'}) \quad (4.19)$$

In the upcoming sections, the functions (f_1, f_2, f_3 . . . . . , f_n) are defined and presented for both type of bits.

Effect of formation strength (f_1) is defined by

$$f_1^{'} = f_1 = e^{a_1} \quad (4.20)$$

The functions of f_1 & $f_1^{'}$ primarily represent the effects of formation strength and bit type on the penetration rate. They also contain the effects of other parameters which are not included into consideration. The term f_1 & $f_1^{'}$ are expreared in the same units as penetration rate and commonly is called the drillability of the formation. The drillability is numerically equal to the penetration rate that would be observed in the given formation type (under normal compaction) when operating with a new bit at zero overbalance, a bit weight, a rotary speed, and a depth of the "normalization" values. The drillability of the various formations can be computed using drilling data obtained from previous wells in the area.

Effect of compaction (f_2) & (f_3) are defined by

$$f_2^{'} = f_2 = e^{a_2(8800-TVD)} \quad (4.21)$$

$$f_3^{'} = f_3 = e^{a_3 TVD^{0.69}(g_p-9)} \quad (4.22)$$

As seen from eq. 4.21, normalization depth used in this study is 8800 ft.

The functions f_2 & $f_2^{'}$ account for the rock strength increase due to the normal compaction with depth, and f_3 & $f_3^{'}$ model the effect of pore pressure gradient on penetration rate.

Effect of differential pressure (f_4) & ($f_4^{'}$) is defined by

$$f_4^{'} = f_4 = e^{a_4 TVD(g_p-ECD)} \quad (4.23)$$

Where measured depth is considered with determining *ECD.*

The functions f_4 & $f_4^{'}$ model the effect of overbalance on penetration rate, and, thus assume an exponential decrease in penetration rate with excessive bottom-hole pressure.

Effect of Bit Diameter and Bit Weight (f_5) & ($f_5^{'}$) is defined by

$$f_5 = \left[\frac{WOB/d_b}{WOB/d_b\big|_c} \right]^{a_5} \quad (4.24)$$

$$f_5^{'} = \left[\frac{WOB/d_b\big|_{mech}}{WOB/d_b\big|_c} \right]^{a_5} \quad (4.25)$$

and assumed that penetration rate is directly proportional to *(WOB/d_b)* as mentioned by several authors. Note that, the critical bit weight *($WOB/d_b)_c$* must be estimated by considering drill string properties, bit type and field data. In this study, normalization value for critical bit weight is assumed to be 4000 lb per inch of bit diameter.

The mechanical weight on bit *(WOB/d$_b$)$_{mech}$* is a concept usually observed when using PDC's and is defined as the difference between the applied weight on bit and pump-off force acting on the face of bit divided by the bit diameter. According to Duklet & Bates [10], the mechanical weight on bit is given by

$$\left({WOB}/{d_b}\Big|_{mech} \right) = \left(\frac{WOB\big|_{applied} - 0.942\,\Delta P_b (d_b - 1)}{d_b} \right) \qquad (4.26)$$

Where

$$\Delta P_b = \frac{q^2 \rho}{12031 (A_n)^2} \qquad (4.27)$$

Here A_n is the total nozzle area. The pump-off force is approximated by an empirical expression developed using previous Christensen tests. The pump-off force can be a substantial hydraulic force created by the differential pressures on the bit, due to the bit face pressure drop. This force tends to unload the cutting and is subtracted from the measured load to obtain the actual weight on bit.

Effect of Rotary Speed (f_6) & (f_6') is defined by

$$f_6' = f_6 = \left[\frac{N}{N_c} \right]^{a_6} \qquad (4.28)$$

and assumed that penetration rate is directly proportional to *N* as mentioned by several authors. Note that the critical rotary speed (N_c) must be estimated by considering drill string properties, bit type and field data. The normalization value is considered to be 100 rpm, as in Bourgoyne & Young's drilling model.

Effect of Tooth Wear (f_7) & (f_7') is defined by

$$f_7' = f_7 = e^{a_7(-h)} \qquad (4.29)$$

"h" is expressed in the next section.

Effect of Bit Hydraulic (f_8) & ($f_8^{'}$) is defined by

$$f_8^{'} = f_8 = \left[\frac{F_j}{F_{j_c}} \right]^{a_8} \tag{4.30}$$

The value of F_{j_c} depends on bit type, drilling mud property and pump pressure. The normalization value is assumed to be 1000 *lb*.

Effect of Hole Cleaning (f_9),(f_{10}), (f_{11}) & ($f_9^{'}$), ($f_{10}^{'}$), ($f_{11}^{'}$) is defined by

$$f_9^{'} = f_9 = \left(\frac{A_{bed} / A_{well}}{0.2} \right)^{a_9} \tag{4.31}$$

$$f_{10}^{'} = f_{10} = \left(\frac{V_{actual}}{V_{critical}} \right)^{a_{10}} \tag{4.32}$$

$$f_{11}^{'} = f_{11} = \left(\frac{C_c}{100} \right)^{a_{11}} \tag{4.33}$$

Today, one of the most common applications in the petroleum industry is drilling inclined and horizontal wells. One of the major problems in drilling horizontal and inclined well is hole cleaning. The technology applied successfully in cleaning vertical wells often does not apply directly in horizontal and inclined wells. So, hole cleaning plays an important role on developing the realistic functions to predict penetration rate.

The functions (f_9),(f_{10}), (f_{11}) & ($f_9^{'}$), ($f_{10}^{'}$), ($f_{11}^{'}$) define the effect of hole cleaning in horizontal, inclined and vertical sections of wells where roller cone bits as well as PDC bits are used. Note that, the equation (4.31) is a dimensionless function considering for horizontal section, equation (4.32)is simulating the inclined section and equation (4.33)is represented vertical section for proper hole cleaning for both PDC and roller cone bits.

4.3 Tooth Wear Model

As indicated in Bourgoyne & Young's drilling model, *(f_7) & ($f_7^{'}$)* have a value of 1.0 when totally new tungsten carbide insert bits (IADC code: 517 &

523) and PDC bits are used. In this study, it is assumed that bit cone offset selection is proper. So bearing wear is negligible. The developed model for estimating frictional tooth dullness, *h*, is given by

$$\frac{dh}{dt} = (g_1)(g_2)(g_3)(g_4) \tag{4.34}$$

The function g_1 describes the effect of formation abrasiveness on tooth wear and defined by

$$g_1 = \frac{H_3}{\tau_H} \tag{4.35}$$

In equation (4.35), the value of H_3 for tungsten carbide insert bits (IADC code: 517 & 523) and PDC bits is 0.02 according to Bourgoyne and Young and the value of τ_H depend on formation properties and it must be estimated using drill-off tests or previously drilled well data. The function g_2 considers the effect of weight on bit on tooth wear. This function is different for tungsten carbide insert bits (IADC code: 517 & 523) and PDC's. For tungsten carbide insert bits (IADC code: 517 & 523) it is defined by

$$g_2 = \left[\frac{\left({}^{WOB}\!/_{d_b} \right)_{\max} - 2.9}{\left({}^{WOB}\!/_{d_b} \right)_{\max} - {}^{WOB}\!/_{d_b}} \right] \tag{4.36}$$

Estes [57] has pointed out that the rate of bit wear will be excessive if a very high bit weight is used. His recommended maximum bit weights were used in this study. For PDC bits, the effect of weight on bit on tooth wear is defined by

$$g_2 = \left[\frac{{}^{WOB}\!/_{d_b}\Big|_{cir}}{{}^{WOB}\!/_{d_b}\Big|_{mech}} \right] \tag{4.37}$$

Note that the normalized bit weight $(WOB/d_b)_{cir}$ must be estimated by considering drill string properties, bit type and field data. In this study, the normalization value is assumed as 100 lb per inch of bit diameter.

The function g_3 describes the effect of pipe rotation on tooth wear and defined by

$$g_3 = \left[\frac{N}{N_c}\right]^{H_1} \tag{4.38}$$

The value of H_1 for tungsten carbide insert bits (IADC code: 517 & 523) and PDC bits is 1.50 according to Bourgoyne & Young and the value of N_c depend on drill string properties and bit type.

The function g_4 is used to emphasize the effect of tooth geometry on tooth wear. For all types of bits, tooth wear is proportional to the inverse of the contact area (*A*) if failing by fracturing of brittle tungsten carbide is ignored. Generally the shape of bits should be classified into three main shapes: cylindrical, triangular and spherical. For cylindrical shape, since there is no change in contact area, the effect of tooth geometry on tooth wear is given by

$$g_4 = 1 \tag{4.39}$$

For triangular shape, the effect of tooth geometry on tooth wear is given by

$$g_4 = \left(\frac{1 + {}^{H_2}\!/_{2}}{1 + H_2 h}\right) \tag{4.40}$$

The value of H_2 is 1.0 according to Bourgoyne & Young. Finally, for spherical shape, the effect of tooth geometry on tooth wear is given by

$$g_4 = \left(\frac{1}{d_c\sqrt{h(2-h)}}\right) \tag{4.41}$$

By substituting equations (4.35 through 4.41) into equation (4.34) and integrating for *h*, the value of frictional tooth dullness, *h*, can be calculated.

4.4 CUTTINGS TRANSPORT

One of the primary functions of drilling fluids in rotary drilling is to remove the rock fragments from beneath the bit and transport these cuttings to the surface. Insufficient hole cleaning cause enormous drilling problems, those may result in losing the overall well.

4.4.1 Vertical Wells

As a rule of thumb, from experiences, average cuttings concentration should not exceed 5 % in the annulus for vertical wells. [6] Therefore, flow rates required can be determined by using this criterion. An equation for the volumetric cuttings concentration, C_c, can be derived on the basis of mass balance. After simplification and in field units, volumetric cuttings concentration can be derived as

$$C_c = \frac{(ROP)D_b^2}{1466.95\left(1-\frac{v_s}{v_f}\right)Q} \tag{4.42}$$

In order for the fluid to lift the cuttings to the surface, the fluid annular velocity, *vf*, should be higher than cuttings slip velocity, *vs.*

v_f can be written as

$$v_f = \frac{Q}{\frac{\pi}{4}(D_o^2 - D_i^2)} \tag{4.43}$$

4.4.2 Inclined and Horizontal Wells

The transport of cuttings for inclined and horizontal wells is much more complicated than it is for vertical wells. There are two major problems that are not present in vertical wells. One problem is the existence and thickness of a bed of cuttings on the low side of the hole. The other problem is the sliding and/or saltating cuttings within the hole. At angles of inclination around 40^o-

60°, the bed of cuttings has the tendency to backslide, and the bed is said to be unstable. Having an unstable bed places the drill string in risk and especially so any time mud circulation is halted.

4.4.3 The Dimensionless Analysis to calculate C_c, A_{bed}/A_{well} & $V_{actual}/V_{critical}$

During the determination of rate of penetration constants different correlation are used to different inclination. For vertical and close to vertical inclinations equation (4.42), for higher inclinations up to 60 ° calculation of critical annular fluid velocity model and for inclination grater than 60° and horizontal section Ozbayoglu's cutting bed estimation model is selected. The results obtained from these equations was assumed as *observed data*.The major aspects of average volumetric cuttings concentration method, calculation of critical annular fluid velocity and cuttings concentration and Ozbayoglu's model are presented in appendix B.

With in the proper model for estimating rate of penetration, the following general and dimensionless equations for different inclination are selected by considering the effects of the four independent variables on cutting transport and hole cleaning.These variables namely are flow rate, hole angle, cuttings size and pipe rotation.[32]

For vertical section $(0 \le \theta \le 10)$

$$C_c = a_1 F_r^{a2} R_{dc}^{a3} \theta^{a4} (\tanh(1 - 9 \times 10^{-6} T_a))^{a5} \quad (4.44)$$

For inclined section $(10 \le \theta \le 60)$

$$V_{actual}/V_{critical} = b_1 F_r^{b2} R_{dc}^{b3} \theta^{b4} (\tanh(1 - 9 \times 10^{-6} T_a))^{b5} \quad (4.45)$$

For horizontal section $(60 \le \theta \le 90)$

$$A_{bed}/A_{well} = c_1 N_{re}^{c2} F_r^{c3} A_r^{c4} \theta^{c5} S_s^{c6} \quad (4.46)$$

By using these equations, Cc, A_{bed}/A_{well} and $V_{actual}/V_{critical}$ are calculated in order to predict ROP by using proposed model.

4.5 OPTIMIZATION OF THE PENETRATION RATE

The drilling optimization procedure considered in this study, is based on two objective functions *i*) maximizing the rate of penetration, and *ii*) minimizing cost per foot. In order to derive the optimum drilling parameters analytically, two separate differential equations are definded *i*) rate of penetration (eq 4.48) and *ii*) teeth wear as a function of time (eq 4.49). In a general form, they can be written as:

$$ROP = \frac{dD}{dt} = f_1(WOB/d_b, N, h) \tag{4.48}$$

$$\frac{dh}{dt} = f_2(WOB/d_b, N, h) \tag{4.49}$$

As seen in equations (4.48) & (4.49), only the operable parameters can be considered. So, it can be concluded that drilling optimization can be conducted to select the proper weight on bit and rotary speed. During analytical derivation of optimum value for weight on bit and rotary speed, some constraints due to practical application are introduced.

$$WOB_{\min} \le WOB \le WOB_{\max} \tag{4.50}$$

$$N_{\min} \le N \le N_{\max} \tag{4.51}$$

$$0 \le h \le 1.0 \tag{4.52}$$

Where for totally worn out teeth, the value of *h* is zero and for new teeth, it equal to one.

As mentioned earlier, bearing wear is ignored. So it can be considered that only, tooth wear limits bit life. The drilling cost per foot equation is given by

$$C_f = \frac{C_b + C_r(t_t + t_c + t_b)}{\Delta D} \tag{4.53}$$

The above equation can be rearranged to give

$$C_f = \frac{C_r}{\Delta D}\left(\frac{C_b}{C_r} + t_t + t_c + t_b\right) \tag{4.54}$$

From the equation (4.34), for insert roller cone bit

$$J_2 = \frac{\tau_H}{H_3}\left[\frac{\left(\frac{WOB}{d_b}\right)_{max} - \frac{WOB}{d_b}}{\left(\frac{WOB}{d_b}\right)_{max} - 2.9}\right]\left[\frac{N_c}{N}\right]^{H_1}\left(\frac{1}{1+\frac{H_2}{2}}\right) \tag{4.55}$$

By using the equations (4.48), (4.49) and (4.55), rotating time during bit run and footage drilled can be determined for insert roller cone bit.

$$t_b = J_2\int_0^{h_f}(1 + H_2 h)dh = J_2(h_f + H_2 h_f^2/2) \tag{4.56}$$

From the equation (4.34), for PDC bits

$$J_2' = \frac{\tau_H}{H_3}\left[\frac{\left.\frac{WOB}{d_b}\right|_{mech}}{\left.\frac{WOB}{d_b}\right|_{cir}}\right]\left[\frac{N_c}{N}\right]^{H_1}\left(\frac{1}{1+\frac{H_2}{2}}\right) \tag{4.57}$$

By using the equations (4.48), (4.49) and (4.57), rotating time during bit run and footage drilled can be determined for PDC bit.

$$t_b = J_2'\int_0^{h_f}(1 + H_2 h)dh = J_2'(h_f + H_2 h_f^2/2) \tag{4.58}$$

Composite drilling variables J_1 and $J_1^{\prime}$ are defined by using the equations (4.18) and (4.19).

$$J_1 = (f_1)(f_2)(f_3)(f_4)(f_5)(f_6)(f_8)(f_9)(f_{10})(f_{11}) \tag{4.59}$$

$$J_1^{'} = (f_1^{'})(f_2^{'})(f_3^{'})(f_4^{'})(f_5^{'})(f_6^{'})(f_8^{'})(f_9^{'})(f_{10}^{'})(f_{11}^{'}) \tag{4.60}$$

So, the equation (4.48) can be re-writen as below:

For tungsten carbide insert bits

$$\frac{dD}{dt} = J_1 f_7 = J_1 e^{-a_7 h} \tag{4.61}$$

For PDC bits

$$\frac{dD}{dt} = J_1^{'} f_7 = J_1^{'} e^{-a_7 h} \tag{4.62}$$

Separating variables in these equations yield.

For tungsten carbide insert bits

$$dD = J_1 e^{-a_7 h} dt \tag{4.63}$$

For PDC bits

$$dD = J_1^{'} e^{-a_7 h} dt \tag{4.64}$$

The integration of these equations requires a relation between time t and tooth wear h. by recall equations (4.34), (4.55) and (4.57) and substitut these expressions into equations. (4.63)& (4.64), respectively

For tungsten carbide insert bits

$$dD = J_2 J_1 e^{-a_7 h}(1 + H_2 h)dh \tag{4.65}$$

For PDC bits

$$dD = J_2^{'} J_1^{'} e^{-a_7 h}(1 + H_2 h)dh \tag{4.66}$$

If both side of equations (4.65) & (4.66) are integrated,the following equations can be derived as function of final tooth wear during a corresponding footage.

For tungsten carbide insert bits

$$\Delta D = J_2 J_1 \left[\frac{1 - e^{-a_7 h_f}}{a_7} + \frac{H_2 (1 - e^{-a_7 h_f} - a_7 h_f e^{-a_7 h_f})}{a_7^2} \right] \tag{4.67}$$

For PDC bits

$$\Delta D = J_2^{'} J_1^{'} \left[\frac{1 - e^{-a_7 h_f}}{a_7} + \frac{H_2 (1 - e^{-a_7 h_f} - a_7 h_f e^{-a_7 h_f})}{a_7^2} \right] \tag{4.68}$$

After defining footage, substituting equations (4.67) & (4.68) and equations (4.56) & (4.58) into cost per foot equation (4.54), the cost per foot equation can be expressed as following:

For tungsten carbide insert bits

$$C_f = \frac{C_r}{\int_0^{h_f} e^{-a_7 h}(1 + H_2 h)dh} \left[\frac{\frac{C_b}{C_r} + t_t + t_c}{J_1 J_2} + \frac{\int_0^{h_f} (1 + H_2 h)dh}{J_1} \right] \tag{4.69}$$

For PDC bits

$$C_f = \frac{C_r}{\int_0^{h_f} e^{-a_7 h}(1+H_2 h)dh}\left[\frac{\frac{C_b}{C_r}+t_t+t_c}{J_1^{'}J_2^{'}}+\frac{\int_0^{h_f}(1+H_2 h)dh}{J_1^{'}}\right] \qquad (4.70)$$

For optimizing weight on bit and rotary speed, $\delta C_f / [\delta(WOB/d_b)] = 0$ and $\delta C_f / \delta N = 0$ should be satisfied.The following expression for optimum bit weight can be obtained by satisfying these conditions.

For tungsten carbide insert bits

$$\left(WOB/d_b\right)_{opt} = \frac{a_5 H_1 \left(WOB/d_b\right)_{\max}}{a_5 H_1 + a_6} \qquad (4.71)$$

For PDC bits

$$\left(WOB/d_b\right)_{opt} = \frac{H_1\left[a_5\left(WOB/d_b\right)_c + \left(WOB/d_b\right)_{cir}\right]}{H_1 - a_6} \qquad (4.72)$$

Derivation of optimum weight on bit for roller cone and PDC bits is presented in appendix C.Taking $\delta C_f /(\delta N) = 0$ and solving yields.

For tungsten carbide insert bits

$$\left(\frac{C_b}{C_r}+t_t+t_c\right)\left(1-\frac{H_1}{a_6}\right)+J_2\int(1+H_2 h)dh = 0 \qquad (4.73)$$

For PDC bits

$$\left(\frac{C_b}{C_r}+t_t+t_c\right)\left(1-\frac{H_1}{a_6}\right)+J_2^{'}\int(1+H_2 h)dh = 0 \qquad (4.74)$$

The optimum bit life is obtained by solving either equation (4.73) or equation (4.74) for $J_2\int (1+H_2 h)dh$.

For tungsten carbide insert bits

$$t_b = (\frac{C_b}{C_r} + t_t + t_c)(\frac{H_1}{a_6} - 1) \tag{4.75}$$

For PDC bits

$$t_b = (\frac{C_b}{C_r} + t_t + t_c)(\frac{H_1}{a_6} - 1) \tag{4.76}$$

After determining the optimum weight on bit and bit life (t_b), the corresponding rotary speed can be calculated by integrating equation (4.56) for roller cone bits and equation (4.58) for PDC bits, using value of J_2 from equations (4.55) & (4.57) and assuming complete tooth wear. It gives:

For tungsten carbide insert bits

$$N_{opt} = 100\left[\frac{\tau_H}{t_b \times H_3}\frac{\left(\frac{WOB}{d_b}\right)_{max} - \left(\frac{WOB}{d_b}\right)_{opt}}{\left(\frac{WOB}{d_b}\right)_{max} - 2.9}\right]^{1/H_1} \tag{4.77}$$

For PDC bits

$$N_{opt} = \frac{100}{\left[\frac{t_b \times H_3}{\tau_H}\frac{\left(\frac{WOB}{d_b}\right)_{cir}}{\left(\frac{WOB}{d_b}\right)_{opt}}\right]^{1/H_1}} \tag{4.78}$$

CHAPTER V

DEVELOPMENT OF THE PROPOSED MODEL

A rate of penetration model is developed by considering many drilling parameters.The model can be used for horizontal and inclined wells with roller cone and PDC bits.Using the data provided from wells drilled at Persian Gulf, model parameters could be determined.Then, optimum weight and rotary speed is derived analiticaly using the proposed model.

5.1 Selecting and sorting relevant and proper field data

An Excel program has been provided to identify and store the field data. In this program firstly, data which were obtained from different wells and formations, were rearranged and then normalized by dividing them to their normalized value.These data which included depth, rate of penetration, weight on bit, drilling rotating time, flow rate, nozzles diameter, mud weight, pore pressure and pump pressure Sorted according to the IADC code, the bit diameter and formation type. This program also determines bit hydraulic information, bit wear condition, and hole cleaning performance. Finally, a data file created which can be used in multiple regression analysis.

5.2 Regression Analysis

A computer program has been developed to determine the coefficients of the proposed rate of penetration model using Multiple Regression Technique. The idea of using a regression analysis of past drilling data to evaluate constants in a drilling rate equation is known. However, most of the past work in this area has been hampered because obtaining large volumes of accurate field data were difficult and the effects of many of the drilling parameters discussed in section (4.2) were ignored. A derivation of the multiple regression-analysis procedure is presented in detail in appendix A.

After substituting the appropriate functions into equations (3.2) and (3.3), and by using multiple regression-analysis, in order to calculate the constants a_1 through a_{11}, the following linear equation system can be obtained by

$$\begin{bmatrix} n & \sum_{i=1}^{n} x_{2i} & \sum_{i=1}^{n} x_{3i} & \cdots & \sum_{i=1}^{n} x_{11i} \\ \sum_{i=1}^{n} x_{2i} & \sum_{i=1}^{n} x_{2i}^2 & \sum_{i=1}^{n} x_{2i}x_{3i} & \cdots & \sum_{i=1}^{n} x_{2i}x_{11i} \\ . & . & . & \cdots & . \\ . & . & . & \cdots & . \\ . & . & . & \cdots & . \\ \sum_{i=1}^{n} x_{11i} & \sum_{i=1}^{n} x_{11i}x_{2i} & \sum_{i=1}^{n} x_{11i}x_{3i} & \cdots & \sum_{i=1}^{n} x_{11i}^2 \end{bmatrix} \times \begin{bmatrix} a_1 \\ a_2 \\ .. \\ .. \\ .. \\ a_{11} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_{2i} y_i \\ .. \\ .. \\ .. \\ \sum_{i=1}^{n} x_{11i} y_i \end{bmatrix}$$

Where n is the number of data point which is used. Other functions are defined by

$$y = \ln(ROP) \quad (5.1)$$

$$x_2 = 8800 - TVD \quad (5.3)$$

$$x_3 = TVD^{0.69}(g_p - 9) \quad (5.4)$$

$$x_4 = TVD(g_p - ECD) \quad (5.5)$$

$$x_5 = \ln\left[\frac{WOB/d_b}{4}\right] \quad (5.6)$$

$$x_6 = \ln\left(\frac{N}{100}\right) \tag{5.7}$$

$$x_7 = -h \tag{5.8}$$

$$x_8 = \ln\left(\frac{F_j}{1000}\right) \tag{5.9}$$

For roller cone bit

$$x_9 = \ln\left(\frac{A_{bed}/A_{well}}{0.2}\right) \tag{5.10.a}$$

For PDC bit

$$x_9 = \ln\left(\frac{A_{bed}/A_{well}}{0.35}\right) \tag{5.10.b}$$

$$x_{10} = \ln\left(\frac{v_{actual}}{v_{critical}}\right) \tag{5.11}$$

For roller cone bit

$$x_{11} = \ln\left(\frac{c_c}{100}\right) \tag{5.12.a}$$

For PDC bit

$$x_{11} = \ln\left(\frac{c_c}{25}\right) \tag{5.12.b}$$

When regression analysis is conducted using the provided field data, some of the model coefficients end up with negative values which is mathematically correct, but physically does not make sense. A sensitivity analysis of the multiple regression-analysis procedure indicated that the number of data points required to give meaningful results depends not only on the accuracy of equations (4.18) and (4.19), but also on the range of values of the drilling parameters (x_1 through x_{11}).

Table (5.1.a) and (5.1.b) summarizes the recommended minimum ranges for each of the drilling parameters and the recommended minimum number of data points to be used in the analysis respectively. As the number of drilling parameters included in the analysis is decreased, the minimum number of data points required to calculate the remaining regression constants is also decreased. (See Table 5.1.b)

Table 5.1.a The recommended minimum ranges for regression analysis [57]

Parameter	Minimum Range*
X_2	2000
X_3	15000
X_4	15000
X_5	0.40
X_6	0.50
X_7	0.20
X_8	0.50
X_9	0.50
X_{10}	1.7
X_{11}	4.5

*Maximum observed value fewer minimums included in regression analysis, observed value

Table 5.1.b The recommended minimum number of data points for regression analysis [57]

Number of Parameter	Minimum Number of Points
11	45
10	40
9	35
8	30
7	25
6	20
5	15
4	10
3	7
2	5

The data available on the Bourgoyne and Young's paper [5] were used to check the accuracy of computer program. The results are shown in Table 5.2.

Table 5.2 Comparison of the Coefficients

coefficients	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
B&Y	3.78	0.00017	0.0002	0.00043	0.43	0.21	0.41	0.16
Proposed Model	3.77	0.000176	0.000189	0.00039	0.44	0.21	0.39	0.134

By comparing the results which presented in table 5.2, it can be concluded that the developed program in this work determine the coefficients of proposed model with a reasonable accuracy and model works properly.

5.3 Optimization of Mechanical Operational Parameters

Optimum weight on bit and rotary speed are determined analytically, and a computer program has been developed based on these equations. The coefficients obtained as described in the previous section are used in the derived equations, together with bit and rig cost information, rotary speed limits, weight on bit limits and tooth-wear parameters, to calculate the best weight on bit and rotary speed. The program also calculates the expected cost per foot, footage, drilling time and rate of penetration.The algorithm of the computer work used in this study is presented in figure 5.1

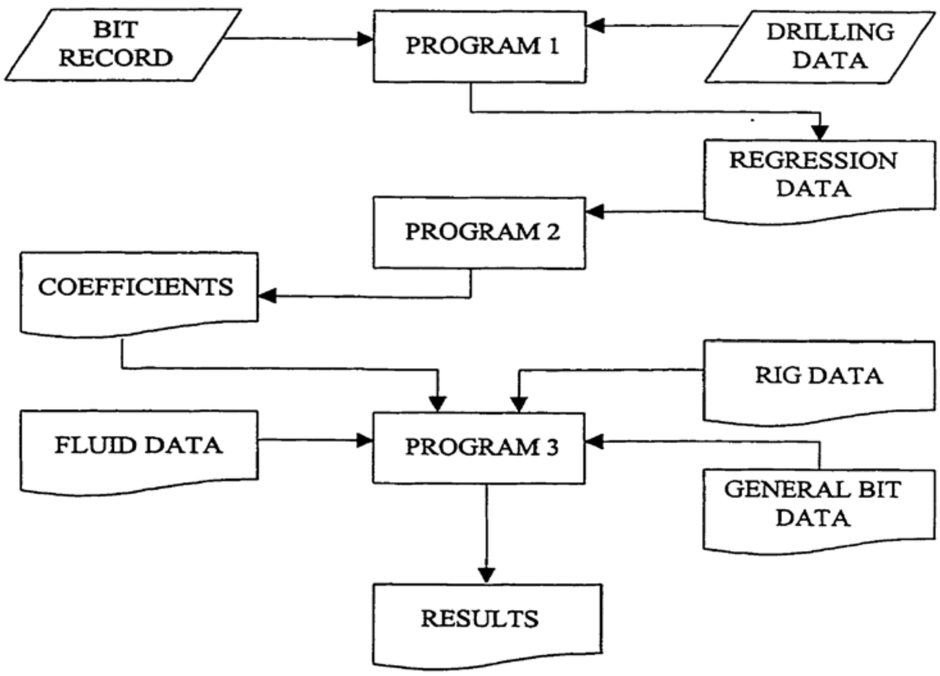

Figure 5.1 Model Flow Chart

CHAPTER VI

RESULTS AND DISCUSSIONS

Field data obtained from several directional and horizontal offshore wells drilled at Persian Gulf in 2004 were used in this study (Fig. 6.1). The location and the name of the field are confidential. So, they can not be mentioned directly. Also, the lithology formation available for this field is presented in figure 6.2.

Figure 6.1 Approximate Location of Field-Persian Gulf

Formation (Lithology)	Depth(ft)
ANHYDRITE – MARL- LIMESTONE	259
DOLOMITE - ANHYDRITE - LIMESTONE - CLAYSTONE - SANDSTONE	1476
DOLOMITE - ANHYDRITE - LIMESTONE - MARL - SANDSTONE	2904
CLAYSTONE - SHALE – SANDSTONE- LIMESTONE -MARL	5906
LIMESTONE-MARL	7635
LIMESTONE- MARL -SHALE	7848
LIMESTONE	8216
ANHYDRITE	9482
DOLOMITE & ANHYDRITE	9745
DOLOMITE- ANHYDRITE- LIMESTON	11008

Figure 6.2 Formation Lithology and depth

The majority of wells are similar in lithology, hole size, casing configuration, and casing setting points. They differ primarily in inclination and trajectory. Also, two types of bits (PDC and insert roller cone bits) were used through out the drilling operations in this field. Because of the similarity in lithology and well design among wells, any performance improvements that can be achieved are likely to apply uniformly to most of the wells to be drilled in the future and a significant economical improvment is expected on the total drilling program.

Because the formation strength accounted in the drilling model equations, the regression analysis should be applied to each individual lithology encountered in the well. Note that, all data recorded is representative of a single type of formation (Dolomite & Anhydrite). The general information of field data collected from dolomite & anhydrite formations from several directional and horizontal offshore Persian Gulf wells are presented in Table (6.1).

Table 6.1 The ranges of insert bit and PDC bit data (min&max) for Multiple Regression Analysis Taken in Dolomite & Anhydrite, Offshore Persian Gulf, 2004

Bit Type	***Roller cone Bit***	***PDC Bit***
MD(m)	2392-4130	3079-4081
TVD (m)	2389-2890	2476-2832
ROP (ft/hrs)	1.46-24.60	4.67-23.5
(WOB/db) (lb/in)	1037.5-3631	668-2833
Rotary Speed (rpm)	40-315	40-313
Fj(lbf)	93.2-953.1	83-603
Cc(%)	0.034-0.63	0.07-0.44
V/Vcrt	0.63-1.42	0.89-1.39
Abed/Aw	0.22-0.35	0.23-0.35
ECD (lb/gal)	10.17-11.69	10.61-11.69
pore Gradient (lb/gal)	8.61-10.71	8.92-10.2
Tooth wear	0.001-0.88	0.00108-0.38

For obtaining the hole cleaning performance, the equation constants of the correlations were obtained by using the Multiple Regression Analysis method and using several directional and horizontal offshore wells drilled at Persian Gulf in 2004 as presented in equations (6.1), (6.2) and (6.3). It is observed that the correlation can estimate annular cuttings concentration, C_c, equilibrium bed area, A_{bed}/A_{well}, and velocity, $V_{actual}/V_{critical}$ with a reasonable accuracy.

$$C_c = 1.24 \times 10^{-4} F_r^{-3.18} R_{dc}^{4.94} \theta^{-0.303} (\tanh(1 - 9 \times 10^{-6} T_a))^{-84.45} \qquad (6.1)$$

$$V_{actual} / V_{critical} = 0.55 \ F_r^{1.16} R_{dc}^{-.88} \theta^{0.03} (\tanh(1 - 9 \times 10^{-6} T_a))^{8.65} \qquad (6.2)$$

$$A_{bed} / A_{well} = e^{14} N_{re}^{-0.085} F_r^{-0.187} A_r^{0.42} \theta^{0.2} S_s^{-16} \qquad (6.3)$$

Figures (6.3), (6.4) and (6.5) show the calculated data which obtained by using the Multiple Regression Analysis versus observed data for both insert bits (IADC code: 517) and PDC bits for vertical, inclined and horizontal sections. The data which obtained from these equations were used to predict the rate of penetration by using Multiple Regression Analysis.

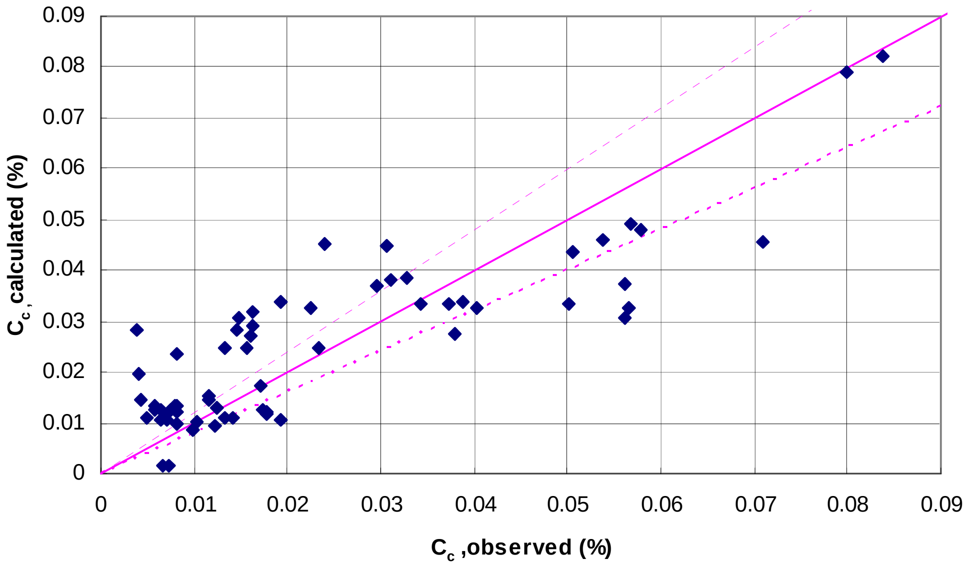

Figure 6.3 Calculated C_c versus Observed C_c for Example Data

In Figure (6.3) shows comparison of annular cuttings concentration between observed and model prediction is presented. Although the figure looks scattered, still the predictions from equation (6.1) are acceptable to be used in rate of penetration model.

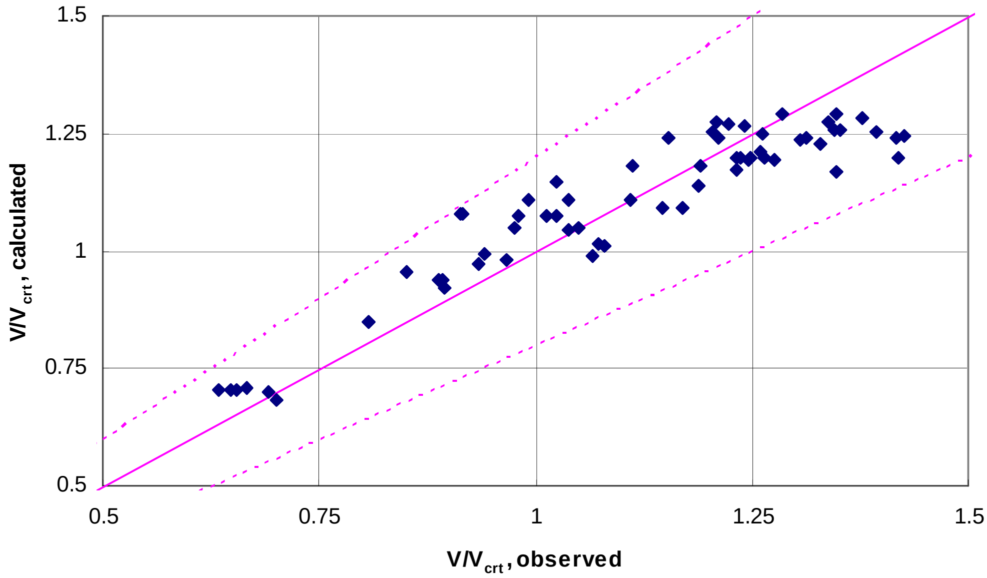

Figure 6.4 Calculated $V_{actual}/V_{critical}$ versus Observed $V_{actual}/V_{critical}$ for field Data

Figure (6.4) shows comparison of dimensionless velocity between observed and model prediction. By using equation 6.4 critical velocity can be predicted very accurately, as seen from Figure (6.4).

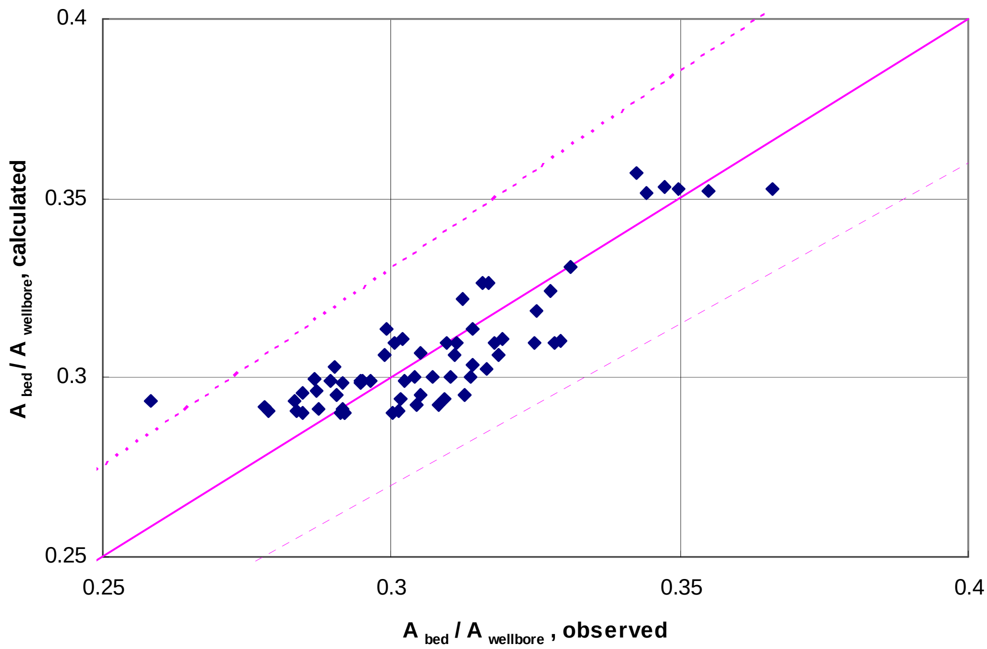

Figure 6.5 Calculated A_{bed}/A_{well} versus Observed A_{bed}/A_{well} for Example Data

Figure (6.5) shows the comparison of dimensionless bed area between observed and model predictions, as seen from 6.5, bed area can be estimated by using equation (6.3) with an error range of less than ±10%.

After determining all the drilling parameters required to be used in rate of penetration model, Multiple Regression analysis is applied to obtain model constant.Table 6.2 is the presentation of rate of penetration model constants for both roller cone and PDC bits.

Table 6.2 The Model Coefficients

Coefficient	***Insert Bit***	***PDC Bit***
a1	8.125	28.354
a2	5×10^{-7}	-6×10^{-3} *
a3	-5.24×10^{-4} *	-0.010 *
a4	-2.55×10^{-6} *	-4.7×10^{-7} *
a5	0.664	0.297
a6	0.087	0.021
a7	0.736	2.907
a8	0.376	1.333
a9	-1.138 *	-1.096 *
a10	0.436	12.786
a11	0.729	4.452

In Table (6.2), because corresponding drilling parameter did not vary over a wide enough range to be included in the regression analysis or because the related data did not be enough, negative values are calculated by using the proposed model, for some constants. After determining the model constants, rate of penetration for this field can be calculated for any given drilling condition. In order to test the performance of the proposed model, rate of penetrations are estimated for each data point provided for this field. The results are posted in Table (6.3) & (6.4) for PDC bits and roller cone bits, respectively. The regression index of correlation G is used to check the persistence of Multiple Regression Analysis. The regression index of correlation G is defined by

$$G = \sqrt{1.0 - \frac{\sum\left[\left(\ln(ROP)\right)_{observed} - \left(\ln(ROP)\right)_{calculated}\right]^2}{\sum\left[\left(\ln(ROP)\right)_{observed} - \left(\overline{\ln(ROP)}\right)\right]^2}} \quad (6.4)$$

By calculating (G) for both roller cone bit and PDC bit data, it is observed that the model can estimate rate of penetration with an error of ±25 % when compared with the field data.

Table 6.3 The result of Analysis for Field Data of PDC bits

ROP (ft/hrs) (measured)	ROP (ft/hrs) (calculated)	ROPob-ROPc
23.50	11.88	11.61
17.57	15.60	1.97
10.94	13.09	-2.15
12.92	12.64	0.28
10.74	10.92	-0.18
18.36	14.78	3.58
7.25	7.85	-0.60
21.17	17.39	3.78
15.11	18.37	-3.27
16.53	13.51	3.02
12.47	13.87	-1.40
9.97	10.11	-0.14
6.46	5.36	1.09
4.67	5.39	-0.72

Table 6.4 The result of Analysis for Field Data of roller cone bits

ROP (ft/hrs) (Measured)	**ROP (ft/hrs) (calculated)**	**ROPob-ROPc**
11.24	14.00	-2.76
10.29	7.99	2.29
14.76	9.75	5.01
13.24	11.23	2.01
10.06	7.60	2.46
6.98	8.71	-1.73
12.47	9.27	3.20
24.61	11.66	12.95
6.56	9.77	-3.21
14.74	10.20	4.53
7.46	6.77	0.69
5.34	8.20	-2.86
7.11	9.27	-2.16
6.64	6.85	-0.21
5.07	8.06	-2.99
3.68	5.37	-1.68
6.96	11.40	-4.44
12.22	13.00	-0.78
9.84	10.65	-0.80
8.99	13.84	-4.84
15.80	9.06	6.73
11.48	8.44	3.05
7.03	8.41	-1.38
9.51	8.25	1.27
10.78	8.49	2.29
11.46	6.32	5.14
14.68	10.52	4.17
5.13	7.79	-2.66
5.97	6.54	-0.58
5.34	6.60	-1.26
5.55	6.90	-1.36
6.63	5.51	1.12
5.49	6.77	-1.28
4.70	6.88	-2.18
4.26	6.94	-2.68
5.41	6.84	-1.44
5.72	5.77	-0.05
6.66	6.11	0.55
6.30	5.61	0.69
10.06	6.72	3.34
1.46	1.69	-0.23
5.84	5.77	0.07

The graphical representation of tables 6.3 and 6.4 are presented in Figures (6.6) and (6.7) for PDC and roller cone bits, rescectively.As seen from figures, it can be concluded that the proposed model can estimate rate of penetration with a reasonable accuracy.

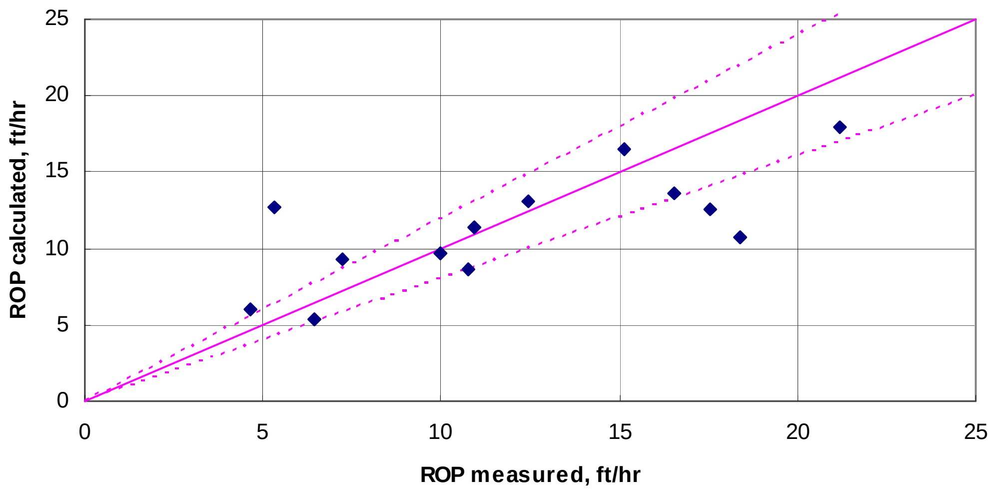

Figure 6.6 Calculated ROP versus Measured ROP for Field Data of PDC bits by using proposed model

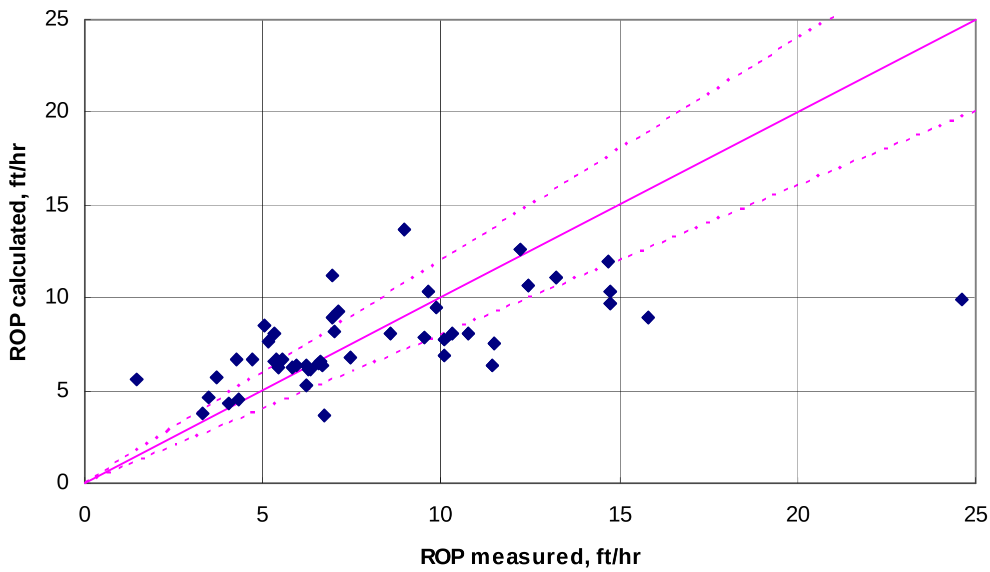

Figure 6.7 Calculated ROP versus Measured ROP for Field data of Insert bits by using proposed model

Also, the similar analysis is conducted by using Bourgoyne & Young's [4] model, which is developed for vertical wells with less number of drilling parameters than the proposed model. Figure 6.8 shows the comparison of ROP which were calculated by using proposed model and Bourgoyne & Young's model versus ROP measured in field. It is observed that, proposed model can predict ROP better than Bourgoyne & Young's model based on R2.

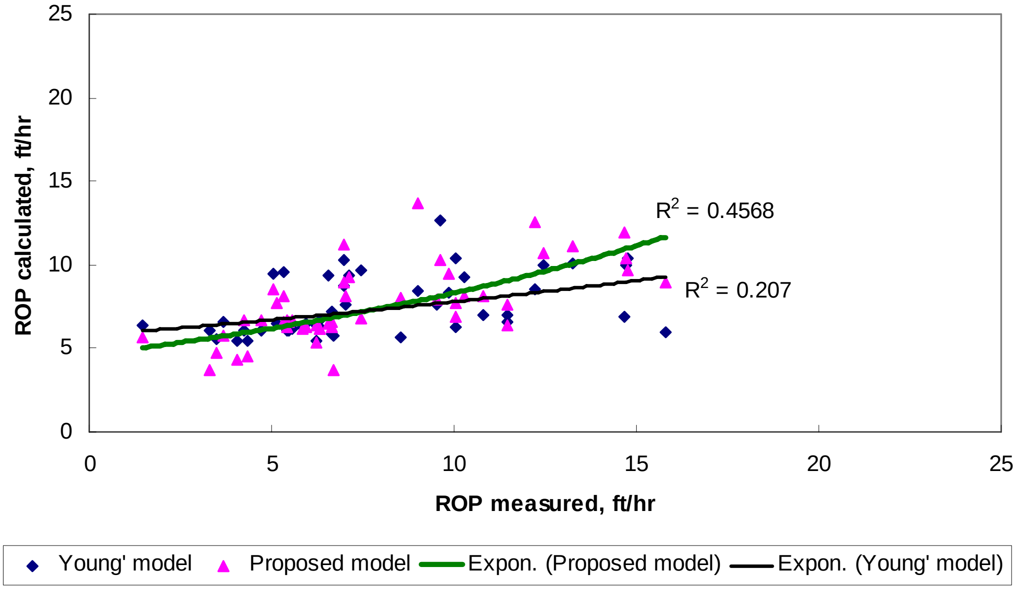

Figure 6.8 Calculated ROP versus Measured ROP for Field Data of Insert Bits by using younge's model and proposed model

6.1 Determination of Optimum Drilling Parameters

In order to calculate the optimum value of mechanical parameters (weight on bit and rotary speed), for the selected formation, equations from (4.77) to (4.82) are used. As mentioned earlier, minimizing cost per foot is the main objective of the optimization process. The results are presented in Table 6.5. The field data which is used in calculation of optimum weight on bit and rotary speed are shown as follows;

$$(WOB/d_b)_{max} = 3700 \ \ lb/inch$$

$$(WOB/d_b)_c = 4000 \ lb/inch$$

$$(WOB/d_b)_{cir} = 100 \ lb/inch$$

$$C_b = 400 \, \$$$

$$C_r = 500 \, \$/\text{hrs}$$

$$t_c = 3 \ \min$$

$$t_t = 6.5 \ hours$$

$$H_1 = 1.5$$

$$H_3 = 0.02$$

$$\tau_H = 7.18 \ \text{Hours}$$

Table 6.5 the results of optimization procedure

Bit type	Insert roller cone (517)	PDC bit
Optimum bit weight (lb per inch)	3404	1306
Optimum bit life (hrs)	121	517
Optimum rotary speed (rpm)	106.41	93.61

The application of optimum values which are presented in Table 6.5 for weight on bit and rotary speed is maximized rate of penetration and minimized cost per foot for both roller cone and PDC bit. After substituting the optimum values in the proposed model, the optimum value of penetration rate for roller cone bit and PDC bits are 8.46 ft/hrs 11.60 ft/hrs respectively.These optimum values are calculated for the single type of formation (Dolomite & Anhydrite) which were drilled at Persian Gulf in 2004.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

A comprehensive model was developed and successfully used to obtain a realistic solution to determine the penetration rate for both insert bits (IADC code: 517 & 523) and PDC bits for inclined and horizontal wells. The model provides an efficient tool for determining the combined effect of several variables on the rate of penetration within realistic technological constraints. At the present time, this approach has been tested only in the Persian Gulf area and the acceptable results have been obtained. The following conclusions resulted from this evaluation.

1. Proposed model can estimate rate of penetration as a function of many drilling variables such as weight on bit, rotary speed, flow rate, nozzle diameters, drilling fluid density and viscosity, bed height and cuttings concentration in the annulus with a reasonable accuracy.

2. By using modern well monitoring equipment, the Multiple Regression Analysis procedure can be applied to determine the regression coefficients present in the rate of penetration equation.

3. To increase the accuracy of model, it is necessary to use data from more than a single well. Also, these data should be from a single formation.

4. By using the mentioned Tooth Wear Model, the calculation of friction tooth dullness (h) is possible for both tungsten carbide insert bits (IADC code: 517 & 523) and PDC bits.

5. Because of the structure, geometry and the number and size of their nozzles of PDC bits, the pump-off force play an effective roll on the weight on bit.

6. By using the Dimensionless Analysis, the annular cuttings concentration, C_c, dimensionless equilibrium bed area, A_{bed}/A_{well}, and

dimensionless velocity, $V_{actual}/V_{critical}$ can be predicted with an acceptable accuracy.

The practical utilization of the model was illustrated by the use of field data which obtained from several horizontal and inclined wells in the Persian Gulf area. Analysis has shown that the simulator enhanced the evaluation of field data and the selection of optimum drilling parameters for a new well to be drilled in the field of interest. Results have also confined that the use of an optimization technique as suggested in this study can reduce drilling costs significantly.

Furthermore, inclusion of cuttings transport parameters in the drilling optimization of horizontal wells reduces the risk of problems such as abnormal torque and drag, lost circulation, difficulties in running casing, poor cement jobs, the necessity of redrilling, and sticking of the drill string.

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APPENDIX A

MULTIPLE RIEGRESSION ANALYSIS PROCEDURE

The equation of the proposed model is:

$$ROP = \frac{dD}{dt} = Exp\left(a_1 + \sum_{j=2}^{11} a_j x_j\right) \qquad \text{(A.1)}$$

Taking the logarithm of both sides of the above equation yields:

$$\ln\frac{dD}{dt} = \left(a_1 + \sum_{j=2}^{11} a_j x_j\right) \qquad \text{(A.2)}$$

If the residual error of the *i*th data point, r_i, is defined by

$$r_i = \left(a_1 + \sum_{j=2}^{11} a_j x_j\right) - \ln\frac{dD}{dt} \qquad \text{(A.3)}$$

In order to minimize the square of the residuals $\sum_{i=1}^{n} r_i^2$, the constants from a1 to a6 should be determined properly by taking derivative from the square of the residuals $\sum_{i=1}^{n} r_i^2$.

$$\frac{\partial\left(\sum_{i=1}^{n} r_i^2\right)}{\partial a_j} = \sum_{i=1}^{n} 2r_i \frac{\partial r_i}{\partial a_j} = \sum_{i=1}^{n} 2r_i x_j \qquad \text{(A.5)}$$

For j = 1, 2, 3, ...,11.

The constants a_1 through a_8 can be obtained by simultaneously solving the system of equations obtained by expanding $\sum_{i=1}^{n} r_i x_j$ for j = 1, 2, 3,...., 11.

The expansion of $\sum_{i=1}^{n} r_i x_j$ yields:

$$a_1 n + a_2 \sum x_2 + a_3 \sum x_3 + ...+ a_{11} \sum x_{11} = \sum \ln \frac{dD}{dt}$$

$$a_1 \sum x_2 + a_2 \sum x_2^2 + a_3 \sum x_2 x_3 + ...+ a_{11} \sum x_2 x_{11} = \sum x_2 \ln \frac{dD}{dt}$$

$$a_1 \sum x_3 + a_2 \sum x_3 x_2 + a_3 \sum x_3^2 + ...+ a_{11} \sum x_3 x_{11} = \sum x_3 \ln \frac{dD}{dt}$$

. . . . .

. . . . .

(A.6)

. . . . .

$$a_1 \sum x_{11} + a_2 \sum x_{11} x_2 + a_3 \sum x_{11} x_3 + ...+ a_{11} \sum x_{11}^2 = \sum x_{11} \ln \frac{dD}{dt}$$

When any of the regression constants are known, the corresponding terms $a_j x_j$ can be moved to the left side of Eq. A.1 and the previous analysis applied to the remaining terms.

APPENDIX B

Average Volumetric Cuttings Concentration model, Calculation of critical annular fluid Velocity and cuttings concentration and Ozbayoglu's model

Average Volumetric Cuttings Concentration

One of the most widely used methods for determining the required flow rates for effective cuttings transport in the annulus is calculating the volumetric concentration of the cuttings in the annulus. As a rule of thumb, from experiences, average cuttings concentration should not exceed 5 % in the annulus for trouble-free drilling. Therefore, flow rates required can be determined by using this criterion.An equation for the volumetric cuttings concentration can be derived on the basis of mass balance. When steady state conditions prevail, mass of cuttings generated by the drill bit per unit time must be equal to the mass of cuttings being transported out of the annulus.
Mathematically;

$$\rho_c (ROP) A_b = \rho_c C_C (v_f - v_s) A_w \qquad \text{(B.1)}$$

where *ROP* is the rate of penetration, *Ab* is the area of the hole (developed by the bit), *vf* is the average fluid velocity, *vs* is the slip velocity, ρ*c* is the cuttings density and *Aw* is the annular flow area (wellbore area). Transport velocity is defined as

$$v_T = v_f - v_s \qquad \text{(B.2)}$$

Solving this equation for volumetric cuttings concentration gives

$$C_C = \frac{(ROP)A_b}{(v_f - v_s)A_w} = \frac{(ROP)A_b}{v_f\left(1 - \frac{v_s}{v_f}\right)A_w} \quad \text{(B.3)}$$

vf can be written as

$$v_f = \frac{Q}{\frac{\pi}{4}(D_o^2 - D_i^2)} \quad \text{(B.4)}$$

where *Q* is the mud flow rate, *Do* and *Di* are outer and inner diameters, respectively. Most cases, *Do* is assumed to be equal to *Db*. Therefore, after simplification and in field units, Volumetric cuttings concentration can be derived as

$$C_c = \frac{(ROP)D_b^2}{1466.95\left(1 - \frac{v_s}{v_f}\right)Q} \quad \text{(B.5)}$$

where *ROP* is in ft/hr, *Db* is in inches and *Q* is in gpm. In order for the fluid to lift the cuttings to the surface, the fluid annular velocity, *vf*, should be higher than cuttings slip velocity, *vs*. A term, called cuttings transport ratio, is introduced by Sifferman as a measure of the effectiveness of the cuttings transport.

$$R_T = 1 - \frac{v_s}{v_f} \quad \text{(B.6)}$$

It is recommended that this ratio should be 0.5 to 0.55.

Slip Velocity for Newtonian Fluids

For Newtonian fluids, Stokes' derivation of *vs* can be used. In field units, slip velocity for any fluid type can be calculated in terms of drag coefficient, *CD*, as

$$v_s = 1.89\sqrt{\frac{d_c}{C_D}\left(\frac{\rho_c - \rho_f}{\rho_f}\right)} \tag{B.7}$$

where *dc* is in inches, ρ's are in ppg, and *vs* is in ft/s. *CD* is a function of Re*p N* , which also is a function of *vs*. Thus, an iterative calculation is required.

Slip Velocity for Non-Newtonian Fluids

For non-Newtonian fluids, different models have been proposed for slip velocity determination.

Bingham Plastic

For Bingham Plastic fluids, Chien proposed a model, which is used worldwide. According to this model, the effective viscosity term (cp) in N_{Re} equation can be computed as

$$\mu_e = \mu_p + \frac{5\tau_y d_c}{v_f} \tag{B.8}$$

Slip velocity including drag coefficient term can be used for Bingham Plastic fluids as well. However, it has been observed that when Chien's correlation is used for estimation of effective viscosity, it is underestimated. Thus, the particle Reynolds number should be calculated using the effective viscosity definition given for annular flow of Bingham Plastics in the previous chapters.

Power Law

Moore has proposed a formula for calculating the slip velocity (ft/s) for Power Law fluids. Calculation procedure is similar to Bingham Plastic fluids. Effective viscosity for Power Law type fluids is defined as:

$$\bar{\mu} = \frac{Kv^{n-1}}{144(D_o - D_i)^{n-1}}\left[\frac{2n+1}{0.0208n}\right]^n \tag{B.9}$$

where *dc* is in inches, *K* is in eq.cp, and *v* is in ft/sec. Since *CD* is a function of *vs*, iterative procedure is required for the solution. Particle Reynolds number should be determined using the effective viscosity.

Calculation of critical annular fluid velocity and cuttings concentration

Critical annular fluid velocity is defined as

$$v_{fcrt} = v_c + v_s \tag{B.10}$$

where vc is the cuttings rise velocity, s v is the cuttings slip velocity. Azar defined vc in field units as

$$v_c = \frac{1}{\left[1-\left(\frac{D_i}{D_o}\right)^2\right]\left[0.64+\frac{18.16}{ROP}\right]} \tag{B.11}$$

where vc is in ft/s, D's are in inches, and ROP is in ft/hr. vs is defined as

$$v_s = v_{cs} C_{inc} C_{size} C_{MW} \tag{B.12}$$

where

$$v_{cs} = 0.00516\mu_e + 3.006 \quad \text{For} \quad \mu_e \le 53cp \tag{B.13}$$

$$v_{cs} = 0.02554\mu_e + 3.28 \quad \text{For} \quad \mu_e \ge 53cp \tag{B.14}$$

μe is the effective viscosity of the mud, where the calculation method has been given in the previous chapters. C_{inc}, C_{size} and C_{MW} are correction factors for effects of hole angle, cuttings size and mud weight respectively. Empirically determined formulas for these correction factors are

$$C_{inc} = 0.0342\theta - 0.000233\theta^2 - 0.213 \tag{B.15}$$

where θ is the inclination angle from vertical in degrees;

$$C_{size} = -1.04 d_c + 1.286 \quad \text{(B.16)}$$

where d_C is cuttings diameter in inches, and

$$C_{MW} = 1 - 0.0333(\rho_f - 8.7) \quad \text{(B.17)}$$

where ρ_f is mud density in ppg.

Total cuttings concentration in the annulus for the operating flow rate can be determined as

$$C_{total}(\%) = C_{cl} C_{bed} \quad \text{(B.18)}$$

In this equation,

$$C_{cl} = \left[1 - \frac{v_f}{v_{fcrt}}\right](100 - \phi) \quad \text{(B.19)}$$

where v_f is the average annular fluid velocity, and φ is the bed porosity (25-48 %), and

$$C_{bed} = 0.97 - 0.00231 \mu_e \quad \text{(B.20)}$$

Ozbayoglu's model

Improvements in down hole motor technology have accelerated the possibilities of drilling longer horizontal wells. As the horizontal sections are getting longer, control on cuttings transport efficiency is essentially becomes more important. Different models have been proposed regarding with cuttings transport in horizontal wells. Most widely used technique is the layered modeling. However, solving a set of equations which are based on tough assumptions may result in inaccurate results. Thus, Ozbayoglu developed empirical equations for estimating the cuttings bed thickness in horizontal wells. Dimensionless groups are defined as a function of flow rate, rate of penetration, fluid properties and well bore geometry. Equations are summarized as:

For $N \geq 0.9$

$$A_{bed} / A_{well} = 4.1232(C_c)^{0.0035}(N_{\mathrm{Re}})^{-0.2198}(N_{Fr})^{-0.2164} \tag{B.21}$$

For $0.6 \leq N \leq 0.9$

$$A_{bed} / A_{well} = 0.7115(C_c)^{0.0697}(N_{\mathrm{Re}})^{-0.0374}(N_{Fr})^{-0.0681} \tag{B.22}$$

For $N \leq 0.6$

$$A_{bed} / A_{well} = 1.0484(C_c)^{0.0024}(N_{\mathrm{Re}})^{-0.1502}(N_{Fr})^{-0.0646} \tag{B.23}$$

where *N* is the generalized behavior index (for Newtonian fluids, *N* = 1), *Cc* is the cuttings concentration (%), *N*Re is the Reynolds number, and *N*Fr is the Froude number, which is defined as

$$N_{Fr} = \frac{v^2}{gD} \tag{B.24}$$

These equations can estimate cuttings bed thickness with an error less than 15 % of error.

APPENDIX C

Derivation of optimum weight on bit and rotary speed for roller cone and PDC bits

As mentioned previously, two differential equations (C.1) & (C.2) are defined in order to derive the optimum drilling parameters analytically for roller cone and PDC bits.

Assumptions:

$$ROP = \frac{dD}{dt} = f_1(WOB/d_b, N, h) \tag{C.1}$$

$$\frac{dh}{dt} = f_2(WOB/d_b, N, h) \tag{C.2}$$

Theses equations can be rewritten as follows

$$ROP = \frac{dD}{dt} = K \;\; g_1(WOB/d_b, N, h) \tag{C.3}$$

$$\frac{dh}{dt} = \frac{1}{\tau_H} g_2(WOB/d_b, N, h) \tag{C.4}$$

In the equations (C.3) and (C.4), the bit tooth dullness can be assumed as an independent variable, and t and D can be assumed as dependent variable. So the equations (C.3) and (C.4) can be rewritten as follows:

$$\frac{dt}{dh} = \tau_H g_1(WOB/d_b, N, h) \tag{C.5}$$

$$\frac{dD}{dh} = \tau_H K \;\; g_2(WOB/d_b, N, h) \tag{C.6}$$

By integrating from the equations (C.5) & (C.6) rotating time during bit run and footage drilled can be written in the following model.

$$t_b = \tau_H \int_0^{h_f} g_1(WOB/d_b, N, h)dh \tag{C.7}$$

$$\Delta D = \tau_H K \int_0^{h_f} g_2(WOB/d_b, N, h)dh \tag{C.8}$$

As mentioned earlier, the drilling cost per foot equation is given by

$$C_f = \frac{C_b + C_r(t_t + t_c + t_b)}{\Delta D} \tag{C.9}$$

The above equation can be rearranged to give

$$C_f = \frac{C_r}{\Delta D}\left(\frac{C_b}{C_r} + t_t + t_c + t_b\right) \tag{C.10}$$

By substituting Eqs. (C.7)& (C.8) into Eq. (C.10)

$$C_f = \frac{C_r}{\tau_H K \int g_2(WOB/d_b, N, h)}\left(\frac{C_b}{C_r} + t_t + t_c + \tau_H \int g_1(WOB/d_b, N, h)\right) \tag{C.11}$$

From equation (C.11) C_D can be defined as

$$C_D = \frac{C + \int g_1(WOB/d_b, N, h)}{\int g_2(WOB/d_b, N, h)} \tag{C.12}$$

where

$$C_D = \frac{C_f K}{C_r} \tag{C.13}$$

$$C = \left(\frac{C_b}{C_r} + t_t + t_c \right) \frac{1}{\tau_H} \qquad \text{(C.14)}$$

For minimizing Eq. (C.12), $\delta C_f / [\delta (WOB / d_b)] = 0$ and $\delta C_f / \delta N = 0$ should be satisfied.

$$\frac{\partial C_D}{\partial \left[WOB/d_b \right]} = \frac{\dfrac{\partial}{\partial \left[WOB/d_b \right]} \int_0^{h_f} f_1(WOB/d_b, N, h)dh - C_D \dfrac{\partial}{\partial \left[WOB/d_b \right]} \int_0^{h_f} f_2(WOB/d_b, N, h)dh}{\int_0^{h_f} f_2(WOB/d_b, N, h)dh} = 0$$

(C.15)

$$\frac{\partial C_D}{\partial N} = \frac{\dfrac{\partial}{\partial N} \int_0^{h_f} f_1(WOB/d_b, N, h)dh - C_D \dfrac{\partial}{\partial N} \int_0^{h_f} f_2(WOB/d_b, N, h)dh}{\int_0^{h_f} f_2(WOB/d_b, N, h)dh} = 0 \qquad \text{(C.16)}$$

From equations C.15 and C.16,it can be obtained that:

$$\frac{\dfrac{\partial}{\partial \left[WOB/d_b \right]} \int_0^{h_f} f_1(WOB/d_b, N, h)dh}{\dfrac{\partial}{\partial \left[WOB/d_b \right]} \int_0^{h_f} f_2(WOB/d_b, N, h)dh} = \frac{\dfrac{\partial}{\partial N} \int_0^{h_f} f_1(WOB/d_b, N, h)dh}{\dfrac{\partial}{\partial N} \int_0^{h_f} f_2(WOB/d_b, N, h)dh} \qquad \text{(C.17)}$$

or

$$\frac{\dfrac{\partial A}{\partial \left[WOB/d_b \right]}}{\dfrac{\partial B}{\partial \left[WOB/d_b \right]}} = \frac{\dfrac{\partial A}{\partial N}}{\dfrac{\partial B}{\partial N}} \qquad \text{(C.18)}$$

where

$$A = \int_0^{h_f} f_1(WOB/d_b, N, h)dh \qquad \text{(C.19)}$$

$$B = \int_0^{h_f} f_2(WOB/d_b, N, h)dh \qquad \text{(C.20)}$$

As mentioned earlier, the proposed model for roller-cone bits is

$$ROP = (f_1)(f_2)(f_3)(f_4)(f_5)(f_6).....(f_{11}) \qquad \text{(4.18)(C.21)}$$

and for PDC bits

$$ROP = (f_1^{'})(f_2^{'})(f_3^{'})(f_4^{'})(f_5^{'})(f_6^{'}).....(f_{11}^{'}) \qquad \text{(4.19) (C.22)}$$

and frictional tooth dullness, *h*, is given by

$$\frac{dh}{dt} = (g_1)(g_2)(g_3)(g_4) \qquad \text{(4.34) (C.23)}$$

From the equation (4.34), for insert roller cone bit

$$J_2 = \frac{\tau_H}{H_3}\left[\frac{\left(WOB/d_b\right)_{max} - WOB/d_b}{\left(WOB/d_b\right)_{max} - 2.9}\right]\left[\frac{N_c}{N}\right]^{H_1}\left(\frac{1}{1+H_2/2}\right) \qquad \text{(C.24)}$$

By using the equations (C.21),(C.23) and (C.24), rotating time during bit run and footage drilled can be determined for insert roller cone bit.

$$t_b = J_2\int_0^{h_f}(1+H_2h)dh = J_2(h_f + H_2h_f^2/2) \qquad \text{(C.25)}$$

$$\Delta D = J_1J_2\int_0^{h_f} e^{-a_7h}(1+H_2h)dh \qquad \text{(C.26)}$$

From the equation (C.23), for PDC bits

$$J_2^{'} = \frac{\tau_H}{H_3}\left[\frac{WOB/d_b\big|_{mech}}{WOB/d_b\big|_{cir}}\right]\left[\frac{N_c}{N}\right]^{H_1}\left(\frac{1}{1+H_2/2}\right) \quad \text{(C.27)}$$

By using the equations (C.22), (C.23)and (C.27) rotating time during bit run and footage drilled can be determined for PDC bit.

$$t_b = J_2^{'}\int_0^{h_f}(1+H_2 h)dh = J_2^{'}(h_f + H_2 h_f^2/2) \quad \text{(C.28)}$$

$$\Delta D = J_1^{'}J_2^{'}\int_0^{h_f} e^{-a_7 h}(1+H_2 h)dh \quad \text{(C.29)}$$

Composite drilling variables J_1 and $J_1^{\prime}$ are defined by using the equations (4.18) and (4.19).

$$J_1 = (f_1)(f_2)(f_3)(f_4)(f_5)(f_6)(f_8)(f_9)(f_{10})(f_{11}) \quad \text{(C.30)}$$

$$J_1^{'} = (f_1^{'})(f_2^{'})(f_3^{'})(f_4^{'})(f_5^{'})(f_6^{'})(f_8^{'})(f_9^{'})(f_{10}^{'})(f_{11}^{'}) \quad \text{(C.31)}$$

For roller cone bits, by comparing equations (C.25) and (C.26) with equations (C.7) and (C.8) and considering equations (C.19) and (C.20), the functions for A and B can be rewritten as follows:

$$A = \int_0^{h_f} f_1(WOB/d_b, N, h)dh = J_2\int_0^{h_f}(1+H_2 h)dh \quad \text{(C.32)}$$

$$B = \int_0^{h_f} f_2(WOB/d_b, N, h)dh = \frac{J_1 J_2}{K}\int_0^{h_f} e^{-a7h}(1+H_2 h)dh \quad \text{(C.33)}$$

Substituting equations (C.32) and (C.33) into equation (C.18) and after rearrangements, the below results can be obtained.

$$\frac{\frac{\partial A}{\partial\left[WOB/d_b\right]}}{\frac{\partial B}{\partial\left[WOB/d_b\right]}}=\frac{-\int_0^{h_f}(1+H_2h)dh}{\frac{\Delta D}{K}\left(\frac{N}{100}\right)^{a6}\left[\frac{WOB/d_b}{4}\right]^{a5}\left\{a_5\left[\frac{\left(WOB/d_b\right)_{max}-WOB/d_b}{WOB/d_b}\right]-1\right\}\int_0^{h_f}e^{-a7h}(1+H_2h)dh}$$

(C.34)

$$\frac{\frac{\partial A}{\partial N}}{\frac{\partial B}{\partial N}}=\frac{-H_1\int_0^{h_f}(1+H_2h)dh}{\frac{\Delta D}{K}\left(\frac{N}{100}\right)^{a6}\left[\frac{WOB/d_b}{4}\right]^{a5}(a_6-1)\int_0^{h_f}e^{-a7h}(1+H_2h)dh} \qquad (C.35)$$

Comparing equations (C.34) and (C.35), following equation can be obtained.

$$\frac{H_1}{H_1-a_6}=\frac{WOB/d_b}{WOB/d_b-a_5\left[\left(WOB/d_b\right)_{max}-WOB/d_b\right]} \qquad (C.36)$$

Solving equation C.36 for WOB/d_b,the optimum weight on bit can be derived as follow.

For roller cone bits

$$\left(WOB/d_b\right)_{opt}=\frac{a_5H_1\left(WOB/d_b\right)_{max}}{a_5H_1+a_6} \qquad (C.37)$$

For PDC bits, the same process should be repeated to derive optimum weight on bit by using equations (C.28) and (C.29).The equation is represented as follow.

$$\left(WOB/d_b\right)_{opt}=\frac{H_1\left[a_5\left(WOB/d_b\right)_c+\left(WOB/d_b\right)_{cir}\right]}{H_1-a_6} \qquad (C.38)$$