MODELLING THE EVOLUTION OF DEMAND FORECASTS IN A PRODUCTION-DISTRIBUTION SYSTEM

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ABSTRACT

MODELLING THE EVOLUTION OF DEMAND FORECASTS IN A PRODUCTION-DISTRIBUTION SYSTEM

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In this thesis, we focus on a forecasting tool, Martingale Model of Forecast Evolution (MMFE), to model the evolution of forecasts in a production-distribution system. Additive form is performed to represent the evolution process. Variance-Covariance (VCV) matrix is defined to express the forecast updates. The selected demand pattern is stationary and it is normally distributed. It follows an Autoregressive Order-1 (AR(1)) model. Two forecasting procedures are selected to compare the MMFE with. These are MA (Moving average) and ES (Exponential smoothing) methods. A production-distribution model is constructed to represent a two-stage supply chain environment. The performance measures considered in the analyses are the total costs, fill rates and forecast accuracy observed in the operation of the production-distribution system. The goal is to demonstrate the importance of good forecasting in supply chain environments.

Keywords: Martingale Model of Forecast Evolution, Additive Form, AR(1), Production-Distribution System, Forecast.

ÖZ

BİR ÜRETİM-DAĞITIM SİSTEMİNDE TALEP TAHMİNLERİ EVRİMİNİN MODELLENMESİ

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Bu çalışmada, bir üretim-dağıtım sistemindeki tahmin evrimini modellemek amacıyla Martingale talep tahmin evrimi modeli üzerinde durulmuştur. Evrim süreci anlatılırken toplamsal form uygulanmıştır. Talep tahmin değişiklikleri varyans-kovaryans matrisi ile tanımlanmıştır. Seçilen talep süreci sabittir ve normal dağılıma sahiptir. Otoregresif 1, AR(1), modelini takip eder. MMFE ile karşılaştırma yapmak üzere iki tahmin yöntemi seçilmiştir. Bunlar; hareketli ortalamalar ve üstel düzeltme yöntemleridir. İki aşamalı tedarik zinciri ortamını temsil etmek için bir üretim-dağıtım modeli yaratılmıştır. Analizlerde kullanılan performans ölçütleri; üretim-dağıtım sisteminin işletilmesinde gözlemlenen toplam maliyetler, talep karşılama oranı ve tahmin doğruluğudur. Amaç, tedarik zinciri çevrelerinde başarılı bir talep tahmininin önemini göstermektir.

Anahtar Kelimeler: Martingale Talep Tahmin Evrimi Modeli, Toplamsal Form, AR(1), Üretim-Dağıtım Sistemi, Tahmin.

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CHAPTER 1

INTRODUCTION

Supply chain management is a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses and stores so that product is produced and distributed at the right quantities, to the right locations and at the right time, in order to minimize systemwide costs while satisfying service level requirements (Simchi-Levi, Kaminsky, Simchi-Levi, 2000).

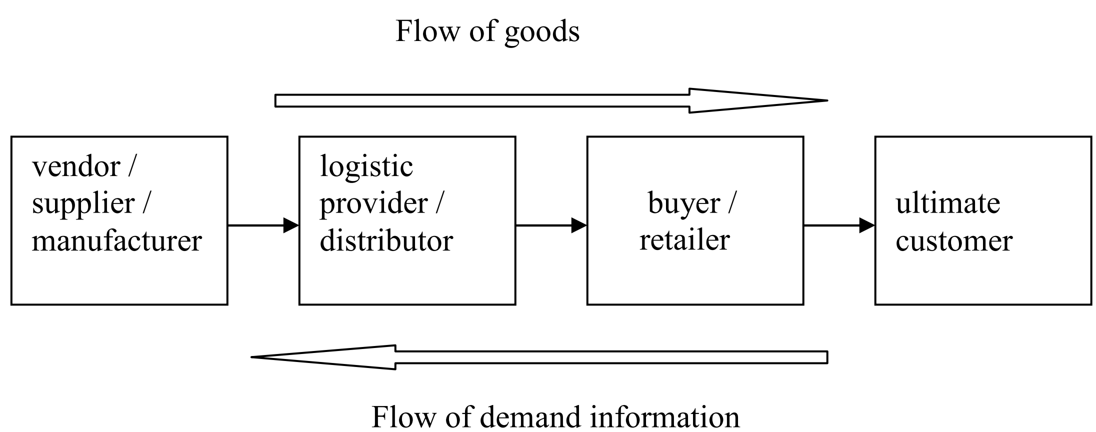

Figure 1 General structure of a supply chain

A simple supply chain consists of a manufacturer, a logistic service provider and the retailer (Figure 1). It coordinates all the materials, information and financial flows. A good coordination among the supply chain partners makes them feel better for the future. To integrate the organizations as partners, there are some problems that all of the organizations face. One problem is that the parties focus on different aims. Suppliers want stable volumes with flexible delivery dates. Manufacturer wants to have long production runs to meet changing customer demands. Retailer wants enough inventory levels to fulfill the ultimate customers' orders in order to have the planned customer service level. Trust is one of the most important terms that unites partners to achieve the shared goals. In this extent the collaboration starts when the companies are in the need of being together to compete against the other companies. Collaboration enables the partners to participate in the competition with their core strengths. As the collaboration increases, the partnership becomes more powerful.

In literature there are kinds of methods to make the forecasting decisions in the chain. Sometimes the manufacturer and buyer have their own forecasting techniques. They do forecasts according to their point of view and they take their expectations into account which are obtained by their own strategies. They are not aware of the benefits of making cooperation with the chain partners.

Recently, the importance of cooperation is understood (Maloni and Benton, 1997). New supply chains strongly depend on effective collaboration. Forecasting procedures are performed at once by collaboration and continuous information sharing. In this study a single forecasting is applied by the supply chain partners. Demand is defined as having normal distribution with known mean and standard deviation. New information and changes are reflected to the forecasts in the next period. The forecasts are required by production plans, thus they play an important role in the production decisions.

In this thesis, a supply chain environment which is a production-distribution system is simulated to show the importance and benefits of modeling forecast evolutions. The system consists of two partners. The upstream partner of the supply chain is the manufacturer and the downstream partner is the distribution center (DC). There are several DCs which act as the internal customers in the supply chain. There is a close relationship between the manufacturer and the DCs. The manufacturer has the facilities to produce certain products. These products are stated as product groups. There is a production plant which consists of several production lines that are capable of producing all product groups. The DCs are warehouses in different regions. They store each of the product groups in their inventories. The changes at the inventories in DCs are immediately monitored and recorded. The manufacturer has the responsibility to make the production plans. In other words the production plans are made by the upstream partner according to the information gathered from all DCs. Retailers' and external customers' demands are met from the DCs.

The production-distribution plans include all product groups at the DCs. Planning is typically done on a rolling horizon basis. A plan is created for the planning horizon, but only the decisions in the first few periods are implemented before a revised plan is issued. Indeed, the plan must be periodically revised due to the uncertainties in the demand forecasts and production. The manufacturer plans for a certain number of periods, but then revises this once in a period to incorporate new information on demand and production. In this study one period indicates one month. The length of the planning horizon is chosen as twelve months. Only the decisions of the first periods are taken for consideration. At any time, a forecast of the demand in all future periods in the horizon is maintained. In other words a new planning horizon is constructed at the beginning of each period.

At the end of each period, the DCs inform the manufacturer about the inventory status and the realized demand. At each DC, safety stocks are held. These safety stocks consist of two groups: at each DC there is a safety stock for product group 1 and there is another safety stock for product group 2. Safety stocks are held to cover excess demand. During the period, demand is realized and fulfilled as much as possible. Unsatisfied demand is fully backordered. It is served from the next period's production. Excess amount of products are held as inventories at the DCs. Transshipments among DCs are allowed. According to the inventory levels, there may be transshipments to satisfy the demand for any product group that is realized at any DC. At the end of the period, inventory holding, backorder and transshipment costs are charged, and demand forecasts are updated. In turn, the manufacturer makes the production decisions based on the forecasting procedure.

Two forecasting techniques are used to estimate the demand patterns for the product groups. These techniques are moving average (MA) and exponential smoothing (ES) methods. As a third forecasting system, martingale model of forecast evolution (MMFE) is implemented to the historical data. The additive form of MMFE is used to obtain the forecasts for the future demands. A comparison is made among the two forecasting techniques and MMFE system according to some performance criteria.

The research in this thesis consists of mainly three parts. In the first part, a brief information about the forecasting theory is presented. Common forecasting procedures are discussed. The selected forecasting techniques in the study are explained. Autoregressive moving average (ARMA) models are expressed and the most common types are included in this part.

In the second part MMFE system is introduced. The structure of the system is defined according to the study in Heath and Jackson (1994). By using historical data, forecast evolution is modeled as an additive process. To update the demand forecast, a standard forecasting tool such as time series models are used. Some well known processes of ARMA models are discussed by applying MMFE system. Then a frequently used process, autoregressive order 1 (AR(1)), is explained as the selected ARMA model.

In the third part, the application of forecast evolution modeling and other two forecasting techniques are made. The forecasts calculated by these methods are used in a linear programming (LP) model. Optimum production decisions are obtained. As a result of the LP model, the performance criteria and comparisons are presented in this last part. It is concluded that the MMFE system provides better performance results.

The aims of the research in this thesis are to show how a forecast evolution model solves forecast error variability, yields better results and integrates them in the manufacturer's production plans.

In this study, an implementation of martingale model of forecast evolution model is described to obtain better forecasts in a simulated environment. The forecast evolution modeling is integrated in the production plans in an attempt to have lower inventory levels, higher customer service levels and to save more money when compared to other two forecasting techniques.

The thesis is structured as follows : In Chapter 2, the related literature about the research is presented. In Chapter 3, the basics of forecasting methodology is explained and the selected forecasting techniques are expressed. In Chapter 4, the forecast evolution model, Martingale Model of Forecast Evolution, is stated. Using the additive form of the MMFE system, the selected autoregressive moving average model is defined. In Chapter 5, the structure of the problem is defined. Parameters of the study and how they are integrated in the problem are expressed. The results of the experimental runs are presented. Finally Chapter 6 contains a summary of the conclusions.

CHAPTER 2

LITERATURE SURVEY

There are many studies examining supply chains in various ways. Some are giving information about the structure and some are dealing with the methods used to cooperate the parties in the chain. The relationships among the manufacturer, buyer and distributor strongly affect the performance of a supply chain. Information sharing leads partners to make better or more adequate decisions for production, distribution and inventory status.

Min and Zhou (2002) synthesize past supply chain modeling efforts and identifies key challenges and opportunities associated with supply chain modeling. The paper also gives guidelines for successful development and implementation of supply chain models. The recommended guideline contains three structures in the supply chain network. These are: the type of a supply chain partnership, the structural dimensions of a supply chain network and the characteristics of process links among supply chain partners.

Identifying the partners in the structure of the supply chain is vital, because some of them will be primary partners and the others will be secondary (supporting) partners. The distinction between primary and supporting supply chain partners is not obvious and this allows the firm to decide on the upstream and downstream members of the supply chain. The horizontal structure indicates the number of tiers in the chain

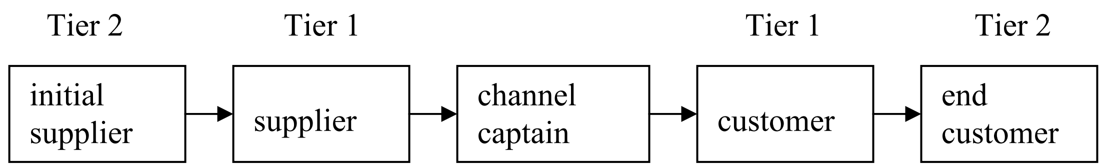

Figure 2 Horizontal structure of a supply chain

The vertical structure gives us information about the number of suppliers and retailers within each tier.

Min and Zhou (2002) foresee the growing needs of the research for future supply chain modeling efforts. Multi-echelon, multi-period issues should be studied when applying mathematical programming techniques to interfunctional integration (production/ distribution, production/ sourcing, location/ inventory, inventory/ transportation). The future models should include the issues like relationship management and conflict resolution between the partners. The supply chains need to include multi-objective treatments of joint procurement, production and inventory planning decisions. To simplify the complexity of supply chain, new methodologies like Theory of Constraints (TOC) can be applied. Instead of stand-alone mathematical models, the future research efforts should be supported by the design of model based Decision Support System that utilizes communication techniques (internet), knowledge discovery techniques (data mining).

The efficient coordination between the partners in the supply chain results in less misunderstandings, better planning decisions etc. Thomas and Griffin (1996) define the operational coordination in three categories. These are: Buyer-vendor coordination, production-distribution coordination and inventory-distribution coordination. Three issues are investigated in these categories. The issues can be expressed as: selection of batch size, choice of transportation mode and choice of production quantity.

In the buyer-vendor coordination most of the inventory models are defined by focusing on the optimal order quantities. This helps the chain to reduce the costs in the case of procurement. Investments in material handling and in data interchange technology can also provide significant savings.

For the category of production-distribution coordination, the subjects are chosen for examining the production and distribution phases simultaneously. The studied areas are: model for determining base stock levels and production lead times to minimize obsolete stock for products with explicitly defined life cycles, dynamic programming heuristics that minimize production and distribution costs, mixed integer programming to determine production and distribution batch sizes that minimize systemwide costs, combining the production planning problem with a vehicle routing problem etc.

In the last category, inventory-distribution coordination, research areas are focused on multi-echelon inventory systems. Research areas are: applicability of multi echelon methods in low demand systems, optimal ordering policies at a depot that distributes to multiple warehouses with correlated demand, optimal inventory levels of a component that is used in multiple end products, optimal solutions to multi echelon production or distribution networks etc.

If the companies know their core competencies and study on the related field, they can get more benefits. To solve the problems on certain fields, companies start building relationships with other companies. The level of the relationship can be advanced by increasing the amount of information shared.

Grean and Shaw (2001) describes the development of channel partnership between a manufacturer Procter & Gamble (P&G) and a retailer (Wal-Mart). At the beginning there were 12 different product divisions in the P&G. Each division had his own sales

manager and there was no communication between them when working with Wal-Mart. Efforts were made for day by day selling and the sales were planned irrespective of what the customer needs. There was no long term planning. Wal-Mart was not aware of the sales of all products in each of their stores. Then they (P&G and Wal-Mart) jointly develop an information sharing system named Data Delivery Highway. With the help of the scanners in all of the stores, Wal-Mart could track, measure and analyze the business. P&G could get the answers for the questions like why did she/he prefer that certain product or go to a certain store. These give useful information about the consumer. A scorecard is developed including the sale of P&G products at Wal-Mart, margin and profit results, inventory turns, and other financial logistics measurement.
Two partners gained important benefits from their information sharing mechanism. P&G reduced the order cycle time (amount of time from the order generation to delivery) by 3-4 days. This success increased inventory turns and resulted in a reduction in the inventory of the entire system.

Lee and Whang (1998) explain the types of information shared: inventory, sales, demand forecast, order status and production schedule. They study the way information is shared in the industry and discuss three alternative system models of information sharing. One alternative is information transfer model. Here one partner informs the other partner and that partner enters the information into the information system. The second alternative is the third party model. The valuable data coming from the partners are transferred to the information system through the third party processsor. The third model is the one in which both partners are immediately sending information as they get the data. A retail sale is processed simultaneously by multiple parties in a format of transactions. The inventory data at the warehouse is shared between the distributor and the retailer.

The biggest handicap of the information sharing in a supply chain is the aligning incentives of different partners. Instead of having good attitudes about the partners, most of them think the possibility of other partners abusing information and gaining all the benefits for their own. One way of defending a positive profit for the weaker side is to keep the cost hidden and maintain informational superiority. The profit obtained by superior information is called as "informational rent". The confidentiality of the information shared is critical for the manufacturer side, because it competes with other manufacturers in the final product market.

Technology is another constraint that the partners should agree on, like EDI standarts, how to split cost of investing in the system etc. The timeliness and accuracy of the shared information could be another hurdle. Since manufacturers are interested in aggregate sale through data of their products, they want the retailers to share their sales data nearly at the same time.

If the information is shared at a high level, the bullwhip effect can be reduced. Metters (1996) aims to determine the significance of the detrimental effect of the bullwhip effect on profitability. The benefits are obtained by caring about the amplified demand seasonality and variance that characterize the bullwhip effect. Any cure is likely to reduce both the induced seasonality and the variance to mean ratio concurrently. A reduction in seasonality from heavy to none and a reduction for variance to mean ratio from 4 to 0.5 can result in a profits increase by 32.8%.

Kok et al. (2005) study on reducing the results of bullwhip effect. Demand variability causes unnecessary inventory levels in the supply chain. This growth in the inventory amount becomes larger as one moves in the upstream direction. The company decided to form a steering committee and a project team. These teams decided to implement a Collaborative Planning (CP) project to solve the problems. The intentions are to improve customer service level, to increase sales and to reduce obsolescence and inventories. The intense working relationships among the teams lead Phillips company to success. Cooperative study caused a decrease in the supply amount and supply started to follow demand closely. This means that both obsolescence and inventories are reduced.

Forecasts are always wrong. However, implementing an appropriate forecasting technique in a supply chain yields better production decisions, less inventory levels and less costs.

Zhao et al. (2002) investigated the impact of forecasting model selection on the value of information sharing in a supply chain with one capacitated supplier and multiple retailers. Additionally different demand patterns are described and capacity tightness is used as a restriction for supplier. In this paper four demand patterns are used to represent different combinations of trends and seasonality. CON represents the demand without trend and seasonality. SEA is the pattern including seasonality without trend. SIT produces demand with seasonality and an increasing trend. Finally SDT produces demand with seasonality and a decreasing trend. One unit of resource is required by the supplier to produce exactly one unit of product. Capacity tightness is explained as the ratio of the total available capacity to the capacity needed.

Retailers' ordering decisions are made by using five typical forecasting models:

- A naive method (NAV)
- A simple moving average (SMA)
- A two parameter double exponential smoothing (DES)
- A no trend Winters' method (NTW)
- A three parameter Winters' model (WIN).

The supplier receives orders from different retailers and determines the production planning. The available information is very important when determining the production plan. There are three cases. One is the no information sharing (NIS) type. Here the production decision is based on the orders received from the retailers. In the second case, the retailers' forecasted net requirements are shared with the supplier. This is the demand information sharing (DIS) case. Third case is the order information sharing (OIS). Both the planned orders and placed orders are shared with the supplier. Then the supplier uses the planned orders as the gross requirements for its production planning. Future order plans of the retailers are taken into consideration in this case. A simulation program is carried out and an experimental design is performed. The independent variables are demand pattern (DP), capacity tightness (CT), forecasting model (FM) and information sharing (IS). The dependent variables are the performance criteria, such as total cost for retailers (TCR), total cost for supplier (TCS), total cost for the entire supply chain (TC), service level of the supply chain (SLS) and service level of the retailers (SLR).

Under all forecasting models OIS performs better than DIS and DIS performs better than NIS according to all five performance criteria (dependent variables). Information sharing results in greater cost reduction for the supplier and the amount of reduction for the supplier is more than that of the retailers. Winters' model takes care of trend and includes seasonality. Thus the forecasts become unbiased and the standard deviation in forecast error is very small under FM = WIN. The value of information sharing is lowest when NAV is shared with the supplier and the actual demand incurs the lowest total costs.

The paper of Zhao et al. (2002) show that the demand pattern and the forecasting model play an important role on the value of information sharing in terms of all performance criteria. One of the results is that higher benefits can be achieved by sharing information when retailers face decreasing trends. Improving the forecasting accuracy and sharing information on planed orders improve the performance of the supply chain. Another benefit of the information sharing is that the supplier has more room to improve capacity utilization with IS when CT is lower.

In the literature there are some studies trying to find out the best forecasting method from a group of methods and some studies explaining new forecasting techniques. Zhao et al. (2002) offer the use of other forecasting models. Their influences on the performance of the system and the value of information sharing can be beneficial for future research. Instead of caring about only the costs, the revenues should be taken into account to obtain a more complete understanding of the impact of information sharing.

In the paper of Xu et al. (2001) simple exponentially weighted moving average method is used to forecast the demand. The demand is defined as non-stationary, serially correlated demand. Here the aim is to reduce the volatility of manufacturer's order releases and forecast errors. As the amplification of bullwhip effect increases when moving up to the manufacturer in the supply chain, volatility gains power and causes extra costs. To get fluctuations under control, safety stocks are held at higher amounts. This causes important planning and capacity utilization problems.

Autoregressive moving average models are prefered frequently as a forecasting tool to represent the demand pattern. In the paper of Zhang (2004) the demand model is assumed as an ARMA process. An AIAO (ARMA in ARMA out) property is defined. It is stated that the demand process and the order quantity process both have the identical autoregressive (AR) structure. As the ARMA model is applied, the AR portion of the demand process remains the same, but the moving average (MA) portion is updated. The AIAO property results in coordinated forecasts, substantial cost savings and easily coordinated inventory control policies.

Gilbert (2005) uses ARIMA process to represent the supply chain model. The results of this study can be expressed as follows,

- The magnitude of bullwhip effect depends on the lead time and autocorrelation in the demand.
- Bullwhip effect does not depend on the number of stages in the supply chain.
- Improving forecast accuracy reduces but does not eliminate the bullwhip effect.
- Information sharing or sharing the point of sale (POS) data is not enough to eliminate the bullwhip effect.

Gaur et al. (2005) shows the value of demand information sharing in the supply chain. Time series structure (ARMA) of the demand process significantly affects the supply chain. According to the invertibility of demand and order processes, the necessity of information sharing between the manufacturer and retailer is explained. By comparing different values of time series parameters, the need for information sharing for ARMA demand is shown.

The AR(1) process has been adopted by several authors in the recent supply chain management literature to study the value of information sharing and collaborative forecasting. Aviv (2002) studies the benefits gained from joint forecasting and replenishment processes in the same supply chain structure. The difference in this paper appears when modeling the demand as an auto-regressive statistical time series of order 1 (AR (1)). This model of demand is issued to describe the settings in which intertemporal correlation in demand among consecutive periods exists. Early market signals are described in the model to learn more about future demands.

Forecast evolution models give companies the chance to revise their forecasts as the new information becomes available at the end of the each period. Heath and Jackson (1994) proposed a general probabilistic model for modeling the evolution of demand forecasts (Martingale Model of Forecast Evolution, MMFE). They carry out an application in a simulation study to analyze safety stock levels for a multi product/multi plant production/distribution system with seasonal stochastic demand. In the simulation study the evolution of inventory, production and shipment decisions and demand are tracked. SIMFORECAST program generates the forecasts. The production and shipment decisions for the current and future periods are generated by SIMLP (a multi-location, multi-time-period model). The result of the study is higher fill rates with less safety stock amounts. The simulation study is carried out with a variety of safety stock factor levels, percentages of current safety stock factor is used as a dimension to compare the contribution of forecasting models in the average annual cost. MMFE is proven as a good technique that is effective in reducing the forecast errors.

Lu et al. (2003) use AR(1) demand forecast model to represent both stationary and non-stationary demand patterns. A single item, periodic review inventory system with demand forecast updates following MMFE is examined. They develop tractable bounds on the optimal base stock levels and use these bounds to construct near optimal policies. A necessary and sufficient condition under which the myopic policy is optimal is identified. Their study shows that myopic policy is optimal or near optimal in many demand-forecast environments. T-horizon periodic-review inventory system with stochastic demand is considered. At the end of the each period, inventory holding and backorder-penalty costs are charged and demand forecasts are updated. A linear cost of for ordering, inventory-holding and backlogging are used. The objective is to minimize the total discounted expected costs. The study in this paper have similarities with our study. Demand follows an AR(1) process and the costs are considered as a performance criterion.

Aviv (2001) studies the effect of collaborative forecasting on supply chain performance in a single retailer, single supplier supply chain. Both retailer's and the supplier's inventories are replenished periodically. The demands are defined as independent and identically distributed (i.i.d.), each having a normal distribution with known mean and standard deviation. The demand is modeled with a constant, a residual forecast error and independent adjustments made in the periods leading up to the demand realization. This is a special case of the martingale model of forecast evolution, which allows for demand correlation across time periods. Since production and inventory plans are based on forecast demand, the correlations between changes in forecasts by region, product and the time period also become important. The forecast model proposed in Heath and Jackson (1994) captures these correlations.

In this thesis, we focus on demands of two product groups at several DCs. Different forecasting methods are used to obtain the forecasted demands. To update the demand forecasts, a standard forecasting tool, time series model, is used. AR (1) model represents the demand forecasts. Forecast revisions are made by implementing MMFE system. Main issues in this study are to investigate the performance of a production-distribution system by using MMFE and other forecasting techniques which affect directly the production decisions, thus inventory levels and backordering situations.

CHAPTER 3

FORECASTING METHODOLOGY

3.1. Basics of Forecasting

Forecast defines the requirements that supplier must care to do its production plans or must procure to satisfy the orders from downstream partners. On the other hand it indicates the amounts that retailers must hold to fulfill the customers' demand.

By forecasting the companies are trying to explain the future conditions. The components of the forecast should be well understood, because the forecasting model must represent all the factors that make the forecasters comfortable about the uncertainties. A general model includes these components (Bowersax et al., 2002):

- Base demand
- Seasonality factor
- Trend component
- Cyclical factor
- Promotional factor
- Noise factor

Base demand is the appropriate quantity when there is no seasonality, trend, cyclical or promotional factor. It is generally accepted as the average demand of the past data.

Seasonality stands for the increasing or decreasing movements in the demand pattern during a certain time (annually). Demand for a certain product can be high in summer time, but it can decline in fall and in winter time (swim wears).

Trend means a stable movement of the demand pattern across time. It can be positive, negative or neutral. It can be shown as a relationship between the base demands in the succeeding time periods,

B_t : base demand in period t,

T : trend,

$B_{t+1} = B_t \ . \ T$

If T is greater than one, the trend is an increasing trend. If T is less than one, then trend is a decreasing trend. The demand for a product can show both increasing or decreasing trends across time.

Cyclical factor indicates the swings in the demand pattern that last for a time period. The cycles may be upward or downward.

Promotional factor causes the demand pattern to swing across time due to the marketing activities (promotion, advertising etc.) determined by the company. In general, an increase is observed during a successful promotion period. Regular promotion factor can be considered as a seasonal factor, but this does not mean that the promotional factor is ineffective. Promotional sales play a vital role in the periodic volume variations.

Noise factor reflects the unpredictable quantity caused by the unexpected events or unknown factors. Due to the uncertainty of future events, it is not possible to forecast the actual demands. The noise factor represents the difference between the actual demand and the forecasted demand for a given period.

3.2. Forecasting Methods

There are many forecasting methods defined for satisfying the demands. In general the methods can be classified in two categories: qualitative and quantitative methods.

Qualitative methods heavily depend on expert judgments. When compared to other methods, they are costly and time consuming. The ideal situations to benefit from these methods are: when sufficient past data are not available and an expert judgment is required (sales in a new region, sales of a new product). Qualitative forecasts are developed using surveys, panels and consensus meetings (Bowersax et al., 2002).

Quantitative methods include: time series method, naive method, moving average method, exponential smoothing method, and regression method, etc.

If there is a judgment that past demand patterns will continue into the future, "time series" method is appropriate to estimate the future demands. If a stable relationship and a trend are observed in the historical data, time series method can be applied. Sometimes the trend changes significantly around a point (turning point). In this situation, the weighted average of past data depending on time series method fails. As a result other approaches can be more appropriate to determine the turning points.

Naive methods are developed to obtain simple models which assume that recent periods are best predictors of the future; for example:

F_{t+1}: forecast in period t+1,

D_t: demand in period t,

$F_{t+1} = D_t$

In general the relationship between the two succeeding periods are taken into consideration; like the difference between the last two demand periods,

$$F_{t+1} = D_t + (D_t - D_{t-1})$$

or the ratio between the last two periods' demand,

$$F_{t+1} = D_t \cdot \frac{D_t}{D_{t-1}} .$$

As new observations become available, a new mean can be computed by removing the oldest value and adding the new value. This method is described as the "moving average" method in which all the observations have equal weights.

Exponential smoothing method is based on averaging (smoothing) past values of a series in a decreasing manner (Hanke and Reitsch, 1995). Weighting is used and the more recent observation takes more weight than others. α (smoothing factor) weight is taken by the most recent observation. The next most recent one takes (α (1 - α)) as a weight, and (α^2(1 - α)) weight is taken by the next one. In smoothed form, the weighted sum of the most recent observation and the most recent forecast is used to obtain the new forecast:

$$F_{t+1} = \alpha\ D_t + (1-\alpha)\ F_t\ , \qquad 0 \leq \alpha \leq 1 .$$

An advantage for exponential smoothing method is that it is easier to calculate the new forecast without the need of substantial historical data. Adaptation to computerized forecasting is possible. High α values lead to quick response to the changes in the

demand, but can cause high forecast errors. Low α values lead to slow reactions to the changes in demand, thus minimize response to random fluctuations.
A line representing the best fits of x-y data points minimizes the sum of squared distances from the points to the line in the vertical (y) direction. This line is called the regression line and the equation is called the regression equation. The regression line is defined by these parameters,

b_o : y intercept b : slope y' : prediction for y value ;

and the regression line is

$$y' = b_o + b\ x$$

Regression method is based on the values of independent factors. By constructing a good relationship between the x-y data, good forecasts can be obtained. Including the external factors, events that take place during the forecasting process lead to more appropriate long term and aggregate forecasting (if y is the sales of a product, the sale of a related product can affect the sales of y).

3.3. Autoregressive Moving Average Models

Autoregressive moving average models are frequently preferred in expressing the time series. The model that defines the time series in terms of deviations from the mean is called as autoregressive model (AR).

$$\widetilde{X}_t = X_t - \mu$$

$$\widetilde{X}_t = \phi_1 \widetilde{X}_{t\text{-}1} + \phi_2 \widetilde{X}_{t\text{-}2} + \ldots + \phi_p \widetilde{X}_{t\text{-}p} + e_t$$

The model above is an autoregressive process of order p, AR(p). Here p denotes the number of autoregressive terms. ϕ is the parameter of the autoregressive model and e_t is the error term. One type of AR(p) process which is frequently used in modeling the time series is the first order autoregressive process, AR(1). It can be modeled as follows,

$$\widetilde{X}_t = \phi_1 . \widetilde{X}_{t-1} + e_t$$

$$X_t - \mu = \phi_1 (X_{t-1} - \mu) + e_t$$

Box and Jenkins methods (Johnson and Montgomery, 1974) showed that the process is stationary, if $|\phi| < 1$. If a model is constructed by giving q non-zero weights to the error terms, it is called a moving average model (MA).

$$\widetilde{X}_t = e_t - \theta_1 e_{t\text{-}1} - \theta_2 e_{t\text{-}2} - \ldots - \theta_q e_{t\text{-}q}$$

θ is the weight for the error term. The above model is a moving average process of order q, MA(q). Here q denotes the number of moving average terms. One frequently used type of the moving average process is the first order moving average model, MA(1).

$$\widetilde{X}_t = e_t - \theta_1 e_{t\text{-}1}$$

$$X_t - \mu = e_t - \theta_1 e_{t-1}$$

The model that includes both the autoregressive and moving average parameters is called as autoregressive moving average model, ARMA(p,q).

$$\tilde{X}_t = \phi_1 \tilde{X}_{t\text{-}1} + \phi_2 \tilde{X}_{t\text{-}2} + \ldots + \phi_p \tilde{X}_{t\text{-}p} + e_t - \theta_1 e_{t\text{-}1} - \theta_2 e_{t\text{-}2} - \ldots - \theta_q e_{t\text{-}q}$$

One useful model is the ARMA(1,1) process and it can be written as,

$$\tilde{X}_t = \phi_1 \tilde{X}_{t\text{-}1} - \theta_1 e_{t\text{-}1} + e_t$$

$$X_t - \mu = \phi(X_{t-1} - \mu) + e_t - \theta_1 e_{t-1}$$

3.4. Collaborative Forecasting

The more the information is obtained, the more effective forecasts can be made. According to the new information, the relevant factors can be taken into consideration. Retailers try to estimate the sales and the sales are influenced by pricing, promotions and release of new products. Some of these factors are controlled by retailers and some of them are controlled by distributor, supplier or competitors. Like the retailers' case, the distributor's and supplier's forecasts are influenced by factors which are under the control of retailer (like the design of a promotion). These consequences cause many supply chains to choose collaborative forecasting systems. Collaborative efforts in forecasting helps decreasing the bullwhip effect.

Supply chain management should choose the most appropriate forecasting technique to obtain the best results. Composite forecasting (Bowersax et al. 2002) is a result driven approach incorporating a number of techniques ranging from very simple to reasonably complex ones. At each time period, a forecast is generated for each stock keeping unit (sku) by using each technique. Then the results are combined by taking the average or giving weights to those forecasts. The combinations of different techniques are compared and best combinations are determined for the related period. The assumption is that the best combination for the next period is the one that would have been best for the most recent period.

The idea behind the concept of combining forecast results is that some method of combining the forecasts of separate small models will result in a better forecast than any other model by itself (Wilson and Keating, 1990). It must not be forgotten that combining results is not a method for eliminating bias in a forecast. Bias arises when forecaster's preconceived notions take the control. To overcome the bias problem, a forecaster will have to examine models that may contradict with his/her beliefs.

If two forecasting models' results and/or the related information to carry out these two models are on hand, removing one of the models can cause the loss of some valuable information. Instead of losing useful information, some method of combining those forecasts should be investigated. A combined forecast is a weighted average of the different forecasts. Here the problem is how the weights should be selected. Additionally a good forecast should contain low error when compared to the actual value. Calculating the root mean squared errors (RMSE) can give forecasters the chance to evaluate if the selected method works or not.

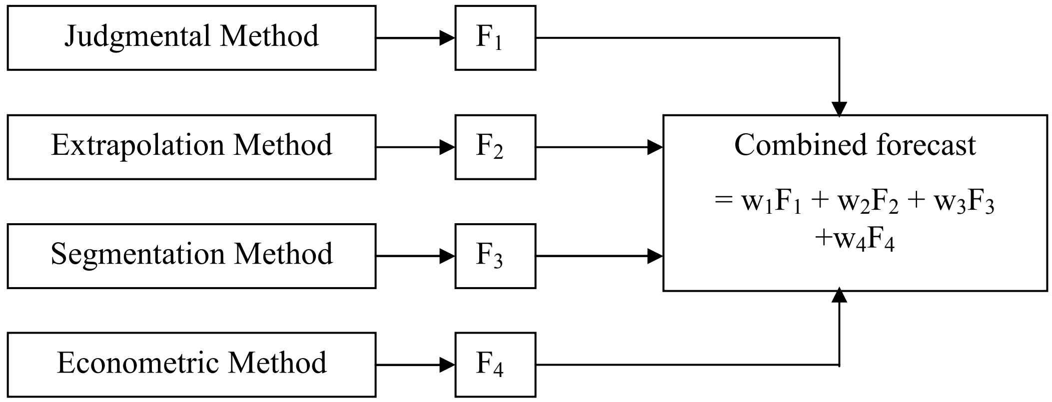

Figure 3 Combining the forecasts obtained with different methods

At the above figure: w_i denotes the weights and F_i denotes forecast made by the related method. Some researchers studying on the forecasting techniques use equal weights for the individual forecasts. Equal weighting has advantages due to its simplicity and its ability to preclude the forecaster's own bias in the selection of weighting factors. Instead of equal weights, sometimes the required weights should be given to the individual forecasts to use their relative accuracy in the combined method. By giving different weights to the selected forecasting model results, the RMSE values can be calculated, given the actual values. Then the combination which provides the lowest RMSE value can be selected for use in the calculation of future forecasts.

Xu et al. (2001) examine the influence of effective information exchange and consistent forecasting on the improvement of supply chain coordination (SCC). It is assumed that both the manufacturer and the retailer have decided to use simple exponentially weighted moving average to forecast the demand.

Forecast for the retailer can be shown as follows,

$$X_t = \alpha D_{t-1} + (1-\alpha) X_{t-1} \qquad (0 < \alpha < 1, \text{ retailer's smoothing factor})$$

and the forecast for the manufacturer is

$$Y_t = \beta D_{t-1} + (1-\beta)Y_{t-1} \qquad (0 < \beta < 1, \text{ manufacturer's smoothing factor})$$

Retailer's actual demand is stated as a non-stationary serially correlated (first order autoregressive) process,

$$D_t = d + \rho D_{t-1} + U_t$$

d : average of the demand

D_t : demand realized in period t

ρ : serial correlation coefficient

U_t : random error realized in period t

Before collaboration is done in the supply chain, the manufacturer relies on historical order data from the retailer to forecast the actual demand and future ordering patterns of the retailer.

In the collaboration phase, the manufacturer has the access to learn actual demand information. One forecast policy is applied for both parties (Xt = Yt). The smoothing factor that is used in the collaboration phase can be taken as a different value (γ) or one of the existing values (α,β). This decision is made by the whole supply chain according to the incentives of the partners. When trying to reduce the manufacturer's safety stocks, the collaboration is more effective if the demand is positively correlated. Otherwise the collaboration is effective only if the smoothing constants that are used before the collaboration exceed certain values.

To get rid of the doubts due to the use of non-stationary demand (it can cause excessive variations to the retailer's and manufacturer' forecast errors and order releases), a stationary (one lag correlated) demand pattern is evaluated. It is observed that having stationary and serially uncorrelated demand does not eliminate the safety stock amplifications and bullwhip effect in the supply chain. Non-stationary and serially correlated demands only increase the amplifications effect.

The collaborative program provides the manufacturer with more benefits than the retailer (in terms of safety stock and resource waste reduction). More active involvement of both parties can be examined to understand the benefits of collaboration.

Xu et al. (2001) use exponentially weighted moving average method, while comparing non-stationary and stationary demand processes. This forecasting technique is quite popular due to its simplicity, computational efficiency and ease of adjusting the forecast responsiveness. The collaboration is aimed in the forecasting stage. According to their own data, both the supplier and the retailer use exponentially weighted moving average method by including their own smoothing factors. Furthermore a collaboratively decided smoothing factor is implemented in a single forecasting stage and benefits of collaborative forecasting are discussed.

3.5. Selected Forecasting Techniques

In this study, the partners are assumed to use a single forecasting technique. It is aimed that by carrying out a single technique, one can avoid the volatility between the expectations of the partners.

The partners are going to estimate the demand pattern by using three different forecasting procedures. One of them is the moving average forecasting method. The idea behind the simple moving average method is that the forecaster records the data for a fixed length of time (time periods like 3 months) and then each data point is equally weighted. The average of the recorded data gives the next period's forecast. Here the parameter is the number of most recent periods that have to be taken into account.

For an N-period moving average model, the forecast for the next period is calculated as

$$M_t = \frac{1}{N} \sum_{i=t-N+1}^{t} D_i$$

$$F_{t+1} = M_t$$

M_t denotes the average of the N period data in the period t. Selecting a small N will cause a more responsive model. On the other hand a large N value will cause a slow reaction to the changes in the demand pattern.

The second one is the exponential smoothing method. It cares about the most recent demand point and the forecast estimated for that most recent period.

$$F_t = \alpha D_{t-1} + (1-\alpha)F_{t-1}$$

α denotes the smoothing factor (0 < α < 1). Here α is the only parameter in the forecasting method. Large values of α make the model more responsive to the changes in the demand. Small values of α result in lower forecast variances.

These two forecasting techniques explained above are appropriate to estimate stationary demand processes. The third one is the MMFE system. It will be discussed in the next chapter.

CHAPTER 4

MODELING THE FORECAST EVOLUTION

4.1. The Evolution Procedure

Martingale model of forecast evolution (MMFE) is a forecasting process. The evolution of forecasts can be obtained by taking new information into account. The changes in the demand pattern due to the new information (promotions, weather forecast, etc.) should be reflected to the new forecasts. In the case of forecasting, as well as past demand, the data like prices of competing goods, marketing, advertising, promotional plans and expert judgments can be taken into consideration.

There are two forms of MMFE system: Additive form and multiplicative form. The additive form is explained in the following paragraphs.

Let D_t denote the actual observed demand for a product at time period t and $D_{s,t}$ denotes the forecast of demand of period t made at time s, $s \leq t$. $D_{s,s}$ stands for the forecast for demand in the current period.

We assume that,

$$D_{s,t} = E[D_t \mid I_s \ ]$$

At each period a certain amount of information is available. I_s denotes all information available at time s. Then the finite horizon process $\{D_{s,t}; s = 1,2,...,t \ \}$ is a martingale (Jackson (2006)). In other words, the expectation values (predictions) for future random variable D_t is equal to $D_{s,t}$ when I_s is available. The successive predictions for random variable D_t form a martingale.

MMFE provides a unified way to quantify the impact of forecast variability on production and inventory decisions. Forecast variability costs money. MMFE allows us to make better decisions as to how to allocate scarce resources.

$\varepsilon_{s,t}$ denotes the change in forecast that occurred over the course of period $s-1$.

$$\varepsilon_{s,t} = D_{s,t} - D_{s-1,t} \qquad t \geq s$$

In the case of period $s = t$, it is the change that is realized from the last forecast to the actual demand. In this last period $s = t$, it measures the forecast error; otherwise it simply measures forecast change. It occurs because there is a change in the information. There is more information available at time s than there was at time $s-1$. Thus $I_{s-1} \subseteq I_s$. The forecasts will change in response to the new information.

The martingale assumption implies two things. These are as follows,

- $E[\ \varepsilon_{s,t}\ ] = 0$, and
- $E[\ \varepsilon_{s,t}\varepsilon_{r,t}\ ] = 0$ for $s > r$.

The expected value of the forecast change is zero and forecast changes are uncorrelated with the changes that occur in other periods. The change in forecasts are caused only by new information. All relevant old information is already reflected in the old forecasts.

The vector $\overline{D}_s$ indicates as a list of the current demands and the forecasts for future periods. The vector ε_s represents the change vector including changes to forecasts for demand in all future time periods:

$$\overline{D}_s = \begin{bmatrix} D_{s,s} \\ D_{s,s+1} \\ D_{s,s+2} \\ . \\ . \\ . \end{bmatrix} \qquad \overline{\varepsilon}_s = \begin{bmatrix} \varepsilon_{s,s} \\ \varepsilon_{s,s+1} \\ \varepsilon_{s,s+2} \\ . \\ . \\ . \end{bmatrix}$$

If $x = (x_1, x_2, ...)$, is an infinite vector and $s(x) \equiv (x_2, x_3, ...)$ is the shifted version of x, then

$$\overline{D}_s = s(\overline{D}_{s-1}) + \overline{\varepsilon}_s$$

A special case of the MMFE is to assume that $\{\overline{\varepsilon}_s\}_{s=1}^{\infty}$ is a series of independent, identically distributed multivariate normal random vectors with mean zero and variance covariance matrix Σ. The MMFE model is completely specified by the initial infinite horizon forecast vector, D_0, and the variance covariance matrix Σ.

4.2. Demand Models with MMFE Framework

A variety of commonly used demand models can be fit into the MMFE framework. One of them is the independent identically distributed (iid) demand model. This is the most common stationary demand model used in inventory theory. The demand, forecast and forecast update are formulated as follows,

$$D_t = \mu + \varepsilon_t ,$$

$$D_{1,t} = \mu ,$$

$$\varepsilon_{s,t} \approx 0 \quad s < t .$$

ε_t's are iid mean zero random variables with variance σ^2. This model implies that we are equally uncertain about the next period demand.

Another type is the independently distributed (id) demand model. This is the most common non-stationary demand model used in inventory theory.

$$D_t = \mu_t + \varepsilon_t ,$$

$$D_{1,t} = \mu_t ,$$

$$\varepsilon_{s,t} \approx 0 \quad s < t .$$

ε_t's are id mean zero random variables with variance σ^2. This model implies that we are equally uncertain about period s demand at any time period t prior to s.

Autoregressive moving average (ARMA) demand model with MMFE is constructed as follows,

$$D_1 = \mu(1-\rho) + \varepsilon_1$$

$$D_t = \mu(1-\rho) + \rho D_{t-1} + \varepsilon_t \qquad t \geq 2$$

ε_t's are iid mean zero random variables with variance σ^2 and $\rho \in (-1,1)$ is the correlation coefficient. This model fits into the MMFE framework as indicated below:

$$D_{s,t} = \mu(1-\rho^t),$$

$$\varepsilon_{s,t} = \rho^{t-s}\varepsilon_s \qquad t \geq s.$$

4.3. MMFE Frameworks Used in the Literature

Any appropriate stationary time series model can be approximated by a forecast evolution model (Güllü, 1993). Time series are constructed on historical data. They do not capture the impact of factors such as human judgment. On the other hand MMFE captures both historical data and potential information available. Under the assumptions of stationarity, Box and Jenkins method produces ARMA (p,q) forecasting models. For the representation of ARMA (p,q) process in the MMFE system, the simpler forms are AR(1) and MA(1) processes.

Güllü (1997) explains that for $i = 0,1,\ldots,H-1$ $D_{t,t+i}$ has MA(H-i-1) representation:

$$D_{t,t+i} - \mu = e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \ldots + \theta_{H-i-1} e_{t-H+i+1}$$

If $i = H-2$, then $H-i-1=1$ and $D_{t,t+H-2}$ has MA(1) representation.

$$D_{t,t+H-2} - \mu = e_t + \theta_1 e_{t-1}$$

Toktay and Wein (2001) implement the MA(q) according to the Box and Jenkins definition.

$$\widetilde{X}_t = e_t - \theta_1 e_{t\text{-}1} - \theta_2 e_{t\text{-}2} - \ldots - \theta_q e_{t\text{-}q}$$

If H = 1 and MA(1) process is considered, then the demand function becomes,

$$D_{tt} = \mu + e_t - \theta_1 e_{t-1}$$

The form of the forecast for period t+1 is defined as,

$$D_{t,t+1} = \mu - \theta_1 e_t$$

$$D_{t,t+i} = \mu \qquad \forall \;\; i > 1$$

Aviv (2001) recommends the MMFE technique in the collaborative forecasting. Here the forecast changes are defined as adjustments. During each period, some information (planned promotion, weather conditions etc.) is gathered and directly reflected to the next period's forecast. These forecast adjustments (updates) help the forecast evolution and decrease the forecast errors.

It is assumed that the demands are independent and identically distributed (i.i.d.), having a normal distribution with a mean μ and standard deviation σ. Demand is described as follows:

$$D_t = \mu + \varepsilon_t + \psi_{tt} + \psi_{t-1,t} + \psi_{t-2,t} + \ldots$$

D_t : demand realized in period t

ε_t : forecast error realized in period t

ψ_{tt} : adjustment made at the beginning of period t for meeting the demand in period t

The relationship between two successive forecasts can be shown as follows,

$$F_{t-1,t} = F_{t-2,t} + \psi_{t-1,t} \ ,$$

$$F_{t-1,t} + \varepsilon_{t-1} = D_{t-1} \quad ,$$

$F_{t-1,t}$: forecast made in period t-1 to meet the demand in period t.

It is stated that the forecast accuracy depends on these factors:

- Individual forecasting strength (η) $1 \le \eta \le 0$
- Standard deviation in period t (σ_t), $\sum_{t=1}^{\infty} \sigma_t^2 \le \sigma^2$
- Total variability (σ^2)

By combining the forecasting strength and standard deviation in a given period, forecasting capability is defined as $\eta\sigma_i$. In the model it is assumed that there exists correlation (ρ) between the pairs of adjustments (the adjustment made by the supplier and the adjustment made by the retailer for the same periods). ρ takes values in the range of $\Omega(\eta^r, \eta^s)$. Superscript r denotes the retailer and superscript s denotes the supplier. When combining the forecasts of supplier and retailer, these following weights are used to gather the adjustments of the partners to meet the demand in period t,

$$\frac{1-\rho\frac{\eta^s}{\eta^r}}{1-\rho^2}\sum_{i=0}^{\infty}\Psi_{it}^r$$: weighted part of the retailer's adjustment

$$\frac{1-\rho\frac{\eta^r}{\eta^s}}{1-\rho^2}\sum_{i=0}^{\infty}\Psi_{it}^s$$: weighted part of the supplier's adjustment

if $\eta^r, \eta^s > 0$, $\rho \in \Omega(\eta^r, \eta^s)$, $\rho < 1$.

In the collaborative forecasting process (CF process) the standard deviation for period t is defined as follows:

$$\sigma_t^{CF} = \sqrt{\sigma^2 - \left[(\eta^r)^2 + (\eta^s)^2 - 2\rho\eta^r\eta^s\right] / (1-\rho^2) \sum_{i=t}^{\infty} (\sigma_i)^2} \ .$$

The variance of the forecast error for each period causes the uncertainties. In each period there exists a variance of σ_t^2. The uncertainty in a certain period cannot be resolved more than σ_t^2. Thus σ_t^2 is named as the maximal uncertainty resolution pattern.

When demand is described as a linear regression model including constant term, sum of all adjustments and independent error term, the diversification of the forecasting data can be investigated. The correlation $(\hat{\rho})$ between the adjustment pairs $(\hat{\Psi}_{tt}^r, \hat{\Psi}_{tt}^s)$ can be defined as a measure of the diversification of forecasting capabilities. Lower values of correlation $(\hat{\rho})$ can lead to more benefits. If the forecastings of the supplier and retailer are done according to the same information, then fewer benefits can be gained in CF process than usual. No matter which type of process is used (as a forecasting process), the CF process is at least good as the best individual forecasting strength.

4.4. Obtaining the Variance-Covariance Matrix of Forecast Updates

All the recent and previous literature about the MMFE system state that the initial forecast vector and the variance-covariance matrix for the distribution of each forecast update vector are the only model parameters.

Basically it is known that the expectation of a random variable (x) can be shown as $E(x)$ or μ_x. The variance of the random variable x is defined as follows:

$$Var(x) = E[(x-\mu_x)(x-\mu_x)] = E[(x-\mu_x)^2].$$

Similarly the covariance of two random variables (x, y) is defined as:

$$Cov(x,y) = E[(x-\mu_x)(y-\mu_y)] \ .$$

To understand the covariance matrix representations of MA(q), AR(p) and ARMA(p,q) models, simple examples are defined below.

While obtaining the covariance terms of forecast updates, the statistical structure of error term should be well stated. In the MMFE system, it is assumed that the error term for any period (e_t) is an i.i.d. random variable with mean zero and standard deviation σ. Let us assume that the forecast horizon is one (H = 1).

In the AR(1) model, the forecast updates for the one period forecast horizon are:

$$\varepsilon_{tt} = \varepsilon_t \ ,$$

$$\varepsilon_{t,t+1} = \rho\varepsilon_{tt} = \rho\varepsilon_t \ .$$

The variances of these forecast updates are obtained as follows:

$$
\begin{aligned}
Var(\varepsilon_{tt}) &= E\left[(\varepsilon_t - 0)(\varepsilon_t - 0)\right] \\
&= E\left[\varepsilon_t^2\right] \\
&= Var(\varepsilon_t) \\
&= \sigma^2 .
\end{aligned}
$$

$$
\begin{aligned}
Var(\varepsilon_{t,t+1}) &= E\left[(\rho\varepsilon_t - 0)(\rho\varepsilon_t - 0)\right] \\
&= E\left[\rho^2\varepsilon_t^2\right] \\
&= \rho^2 E\left[\varepsilon_t^2\right] \\
&= \rho^2\sigma^2 .
\end{aligned}
$$

The covariance form of the consecutive (successive) forecast updates can be shown as,

$$
\begin{aligned}
Cov(\varepsilon_{tt}, \varepsilon_{t,t+1}) &= E\left[(\varepsilon_t - 0)(\rho\varepsilon_t - 0)\right] \\
&= E\left[\rho\varepsilon_t^2\right] \\
&= \rho E\left[\varepsilon_t^2\right] \\
&= \rho\sigma^2 .
\end{aligned}
$$

The forecast horizon is one ($H = 1$), thus a matrix of ($H+1$ x $H+1$) is going to be built. If the obtained variance and covariance forms are placed in the covariance matrix, then the below matrix can be constructed:

$$\Sigma = \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \rho^2\sigma^2 \end{bmatrix}$$ where Σ denotes the covariance matrix.

By applying the MA(1) process for the representation of the demand and forecasts, the forecast updates can be written as follows:

$\bar{\varepsilon}_t = [\varepsilon_t \ -\theta\varepsilon_t]$: forecast update vector

$\varepsilon_{tt} = \varepsilon_t$

$\varepsilon_{t,t+1} = -\theta\varepsilon_t$.

To construct the covariance matrix, the variance and covariance of the forecast updates can be written as,

$$
\begin{aligned}
Var(\varepsilon_{tt}) &= E[(\varepsilon_t - 0)(\varepsilon_t - 0)] \\
&= E[\varepsilon_t^2] \\
&= Var(\varepsilon_t) \\
&= \sigma^2 .
\end{aligned}
$$

$$
\begin{aligned}
Var(\varepsilon_{t,t+1}) &= E[(-\theta\varepsilon_t - 0)(-\theta\varepsilon_t - 0)] \\
&= E[\theta^2\varepsilon_t^2] \\
&= \theta^2 E[\varepsilon_t^2] \\
&= \theta^2\sigma^2 .
\end{aligned}
$$

$$
\begin{aligned}
Cov(\varepsilon_{tt}, \varepsilon_{t,t+1}) &= E[(\varepsilon_t - 0)(-\theta\varepsilon_t - 0)] \\
&= E[\ -\theta\varepsilon_t^2 \ ] \\
&= -\theta E[\ \varepsilon_t^2 \ ] \\
&= -\theta\sigma^2 .
\end{aligned}
$$

The covariance matrix of the forecast updates in the MA(1) process can be constructed as,

$$\Sigma = \begin{bmatrix} \sigma^2 & -\theta\sigma^2 \\ -\theta\sigma^2 & \theta^2\sigma^2 \end{bmatrix}$$

4.5. Selected Demand Model with MMFE Framework

The demand is assumed as independent and identically distributed. The statistical distribution of the demand is normal distribution. The demand process is assumed to be stationary with a mean μ and a standard deviation σ. If the demands are highly seasonal (periodic fluctuations exist), the stationarity assumption (a stationary process has the property that the mean and variance do not change over time) is violated. In this study we focus on the additive form of MMFE system.

We assume that ε_t is iid ~N$(0,\sigma\ )$. This assumption requires the demand process to be stationary and forecasts to be unbiased. Only a finite number of random variables exists and a finite number of uncertainty factors of the information set affects the forecasts.

We construct MMFE model by using AR(1) structure. If the demand process is AR(1), the demand is stated by mean of the demand plus correlation term and the error term. In this study it is assumed that the demand follows an autoregressive order 1 model. H indicates the forecast horizon. Indice i is used to express step ahead forecasts. The demand is formulated as follows,

$$D_t = \mu + \rho(D_{t-1} - \mu) + \varepsilon_t \qquad\qquad \mu > 0 \text{ and } \rho \le |1|,$$

$$D_{t,t+i} = \mu + \rho^i (D_t - \mu) \qquad\qquad 0 \le i \le H - t,$$

$$\varepsilon_{t,t+i} = \rho^i \varepsilon_{t,t} = \rho^i \varepsilon_t \ .$$

"ρ" is the autocorrelation coefficient between the two consecutive time periods. To express the relationship between the consecutive time periods a different formulation should be used. Basically autocorrelation coefficient is calculated as follows,

$$\rho_{i,k} = \frac{\sigma_{i,k}}{\sigma_{i,i}\sigma_{k,k}}$$

Here the two consecutive data in a sample will be taken into account, so autocorrelation coefficient will be calculated as follows,

$$\rho_1 = \frac{[E(D_t D_{t-1}) - \mu^2]}{\sigma^2}$$

Indice '1' means one lag correlation between the random variables.

MMFE process uses a forecast horizon while obtaining the forecast vector. This forecast vector includes the actual demand in that period and all the forecasts for the future periods. If the forecast horizon is H, then H forecasts will be made for the next H periods. The forecasts for other periods which are i > H will be indicated by μ (long run average of the demand).

If $i > H$, then $\varepsilon_{t,t+i} = 0$ and $D_{t,t+i} = \mu$, forecast vector is as follows ,

$$\overline{D_0} = \begin{bmatrix} D_{0,0} \\ D_{0,1} \\ D_{0,2} \\ \vdots \\ D_{0,H} \\ \mu \\ \mu \\ \vdots \end{bmatrix} \qquad \overline{D_1} = \begin{bmatrix} D_{1,1} \\ D_{1,2} \\ D_{1,3} \\ \vdots \\ D_{1,H+1} \\ \mu \\ \mu \\ \vdots \end{bmatrix}$$

D_0 is the initial forecast vector. D_1 is the forecast vector formed at period 1. Like the forecast vector, a forecast update vector should be constructed to understand the relation between the consecutive forecast vectors:

$$\overline{D_0} + \overline{\varepsilon}_1 = \overline{D_1}$$

$\overline{\varepsilon}_1$: forecast update vector constructed at the end of period 1.

The above equation is used in the additive model of the MMFE system and $\overline{\varepsilon}_1$ can be shown as follows:

$$\bar{\varepsilon}_1 = \begin{bmatrix} \varepsilon_{1,1} \\ \varepsilon_{1,2} \\ \vdots \\ \varepsilon_{1,\mathrm{H}+1} \end{bmatrix}.$$

Since the forecast horizon is H, $H+1$ terms are included in the forecast update vector. The values for all other future terms are zero.

The variance-covariance matrix (VCV) of this study ($H = 12$) can be formed as follows:

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{1,2} & \sigma_{1,3} & \sigma_{1,4} & \cdots & \sigma_{1,13} \\ \sigma_{2,1} & \sigma_2^2 & \sigma_{2,3} & \sigma_{2,4} & . & \vdots \\ \sigma_{3,1} & \sigma_{3,2} & \sigma_3^2 & . & . & . \\ \vdots & \vdots & \vdots & \sigma_4^2 & . & \vdots \\ . & . & . & \vdots & \ddots & . \\ \sigma_{13,1} & \sigma_{13,2} & \sigma_{13,3} & \sigma_{13,4} & \cdots & \sigma_{13}^2 \end{bmatrix}$$

Unbiased forecasts are required to estimate the demand. Therefore the expectation of forecast updates is approximately zero.

$$E(\varepsilon_{s,t}) \approx 0$$

Any two forecast update vectors are uncorrelated between each other. In other words $\varepsilon_{s,t}$ and $\varepsilon_{s',t}$ are independent for all s and s's. But, any two forecast updates ($\varepsilon_{s,t}, \varepsilon_{s,t+i}$) in the same forecast update vector are correlated.

The total forecast error variability over any number of periods is the sum of the diagonal elements of the VCV. Forecast error variability is calculated as follows,

$$Var(\varepsilon_{s,t}) = \sigma_{t-s+1}^2 \ .$$

Total forecast error over three periods is :

$$Var(D_{s+3,s+3} - D_{s,s+3}) = Var(D_{s+1,s+3} - D_{s,s+3}) + Var(D_{s+2,s+3} - D_{s+1,s+3}) + Var(D_{s+3,s+3} - D_{s+2,s+3})$$

$$= Var(\varepsilon_{s+1,s+3}) + Var(\varepsilon_{s+2,s+3}) + Var(\varepsilon_{s+3,s+3})$$

$$= \sigma_3^2 + \sigma_2^2 + \sigma_1^2 \ .$$

CHAPTER 5

COMPUTATIONAL STUDY AND COMPARISON

5.1. Problem Structure

In this study there is a single production plant. This production plant includes three production lines. Each production line is capable of manufacturing multiple products. There are two product groups. Manufacturer is responsible for producing the products and delivering them to the distribution centers (DC). There are three DCs. Each DC faces the demands of two products and tries to fullfill the customer orders.

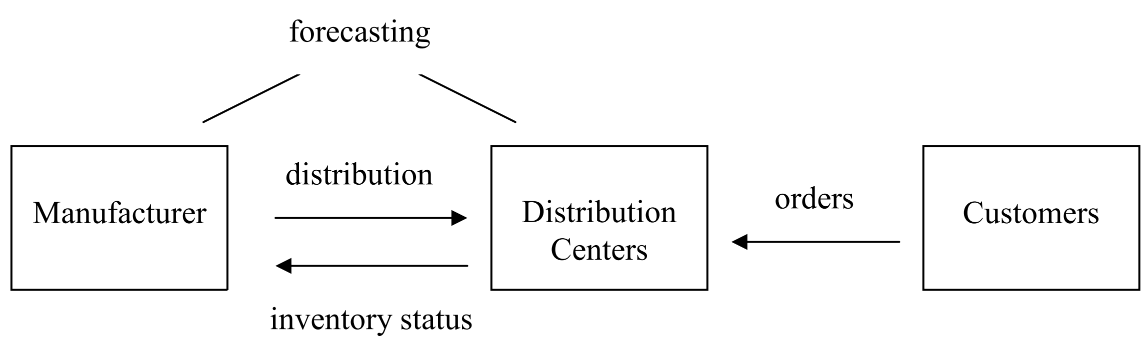

Figure 4 Structure of the supply chain

There are two types of demand processes according to time dependency of the demand. Stationary demand process means that the statistical distribution (mean and standard deviation of the demand) of the demand between the time periods do not change from time to time. Thus it is a time independent process. On the other hand, non-stationary demand process is the one that depends on time. Each time period can have a different statistical distribution of the demand. Then non-stationary demand process is stated as a time-dependent process.

X_t is the random variable denoting the demand in a certain time period t. If the mean or the autocovariances of the demand pattern do not depend on the time period t, then the X_t is said to be covariance stationary (weak stationary).

$E(X_t) = \mu$ for all t.

Covariance stationarity is the case that the properties of the demand distribution only depends on the first two moments of all variables. Under the assumptions of normality, covariance stationarity implies strong stationarity. Autoregressive moving average (ARMA) models are developed for stationary processes (Heath and Jackson, 1994).

Both product groups have time-dependent demand patterns with means μ_i (i indicates product group i). For each product group, demand has normal distribution. In normal distribution, the random variables take values around a mean value μ and the width of the probability distribution function is stated with standard deviation σ. X denotes the random variables:

$-\infty < X < \infty$.

Random variables can take any values under the area of a curve on the *x-y* coordinate system.

To be statistically reliable, a single mean with 3 standard deviation values are chosen. In other words, there will be 3 different data sets with the same mean and different standard deviation values for a single product group. This leads us to use coefficient of variation (CV) as a measure to form 3 types of problems. In the first problem type, low CV level is used. In this case, both product groups have low CV values. This means that the standard deviation of the samples will be little around the chosen mean values. In the second type, the product groups have medium level CV. The selected σ value will be larger than that of the first problem type. The third problem type has the highest CV level for the product groups' data sets. CV is calculated as follows:

$$CV_{i,j} = \frac{\sigma_{i,j}}{\mu_j} \qquad i = 1,2,3 \quad , \quad j = 1,2.$$

Here the index i stands for the number of problem type. Index j denotes the number of the product group. Each product group has three data sets. Thus there are 6 data sets. Relationship among the three CV levels for a single product group j can be shown as follows:

$\sigma_{1,j} < \sigma_{2,j} < \sigma_{3,j}$, where

$\sigma_{1,j}$: standard deviation of the low CV level demand pattern

$\sigma_{2,j}$: standard deviation of the medium CV level demand pattern

$\sigma_{3,j}$: standard deviation of the high CV level demand pattern.

By applying three different CV values, we can observe the behaviour of MMFE system and other forecasting techniques in different demand patterns. This will lead us to make more reliable comparisons.

Three forecasting procedures are chosen to meet the demands from the DCs. These are moving average, exponential smoothing and martingale model of forecast evolution system. Rolling horizon method is used to make the production planning decisions. 12-period planning horizon is selected. Unit of the periods is one month. At the beginning of each period 12 forecast values are calculated. These 12 forecast values are elements of the forecast vector obtained at that period. Each forecast vector contains the current demand at that period and the forecasts for the next 12 periods. The estimates in the forecast vector are used as the demand values of the related periods in a problem. After the realization of the actual demand, the forecasts are revised and calculated for the next 12 periods.

100 data points are used as the historical data to calculate the above measures. After the generation of the data, the forecasted demands will be calculated by the three forecasting procedures. These forecasted demands will appear in the LP and by running the LP, the production decisions can be made. Since 12-period rolling horizon is performed, at the beginning of each period, 12 new forecasts are calculated. The forecast vector acts as the demand values in that planning horizon (See Figures 5 and 6).

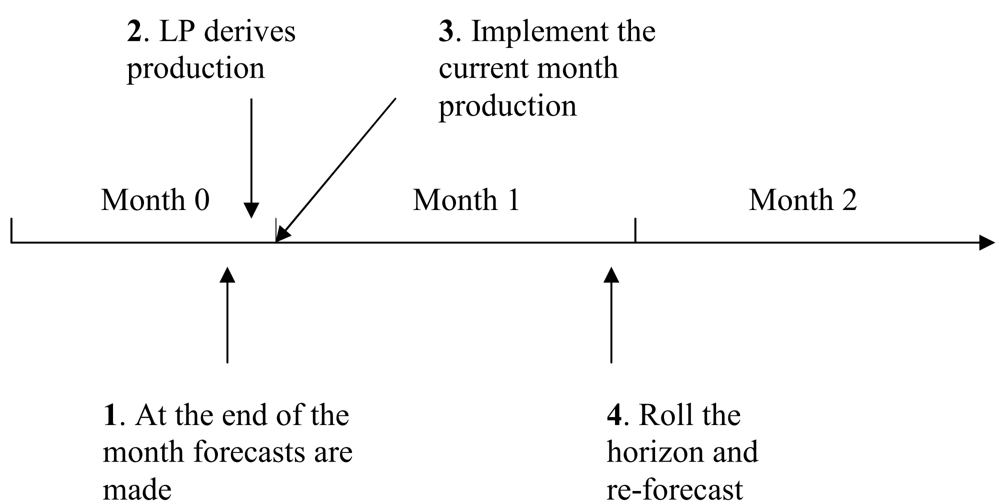

Figure 5 The planning sequence

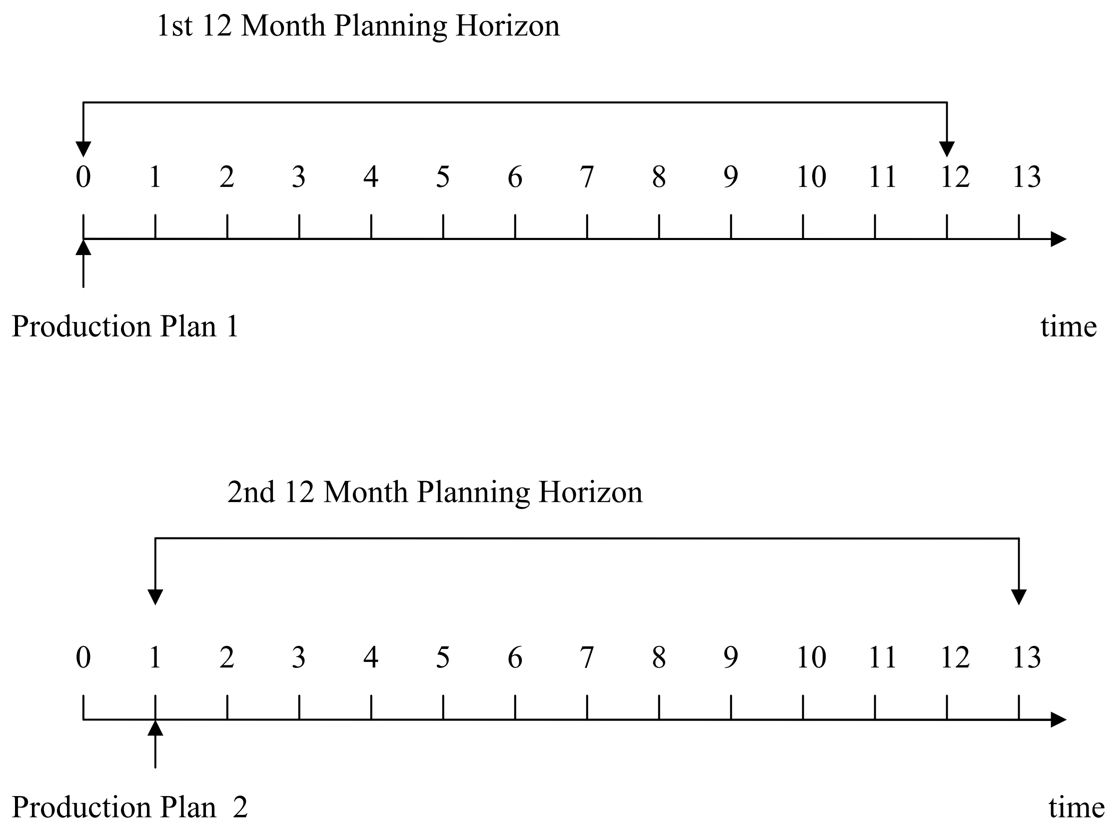

Figure 6 Implementation of the rolling horizon method

Moving average and exponential smoothing methods are frequently used forecasting techniques to estimate the stationary demand processes. By implementing MMFE system, it is aimed to obtain better forecasts than MA or ES methods.

MMFE system needs historical data to form its initial terms (initial forecast vector). Box-Jenkins methods require at least 72 data points as a historical data (Nahmias, 1997). With the help of this historical data, some important measures of the samples (mean, standard deviation and correlation coefficient) are calculated to use in the selected MMFE framework.

At each problem instance there are 6 different data samples. In other words, there are 3 DCs, and for each DC 2 different demand patterns are observed. One is for product group 1 and the other one is for product group 2. To have correct and trustworthy results, 10 problem instances are formed for each of the three CV levels. Therefore, 30 problem instances are generated. Each problem instance has different demand data and the same demand data set is used by the three forecasting procedures. For these 30 problems, 90 different forecast sets are obtained (30 sets for a single forecasting procedure). 12 runs are required for each forecast set. Totally 1080 runs are needed in the experimental study. Figure 7 outlines the steps in solving a problem instance.

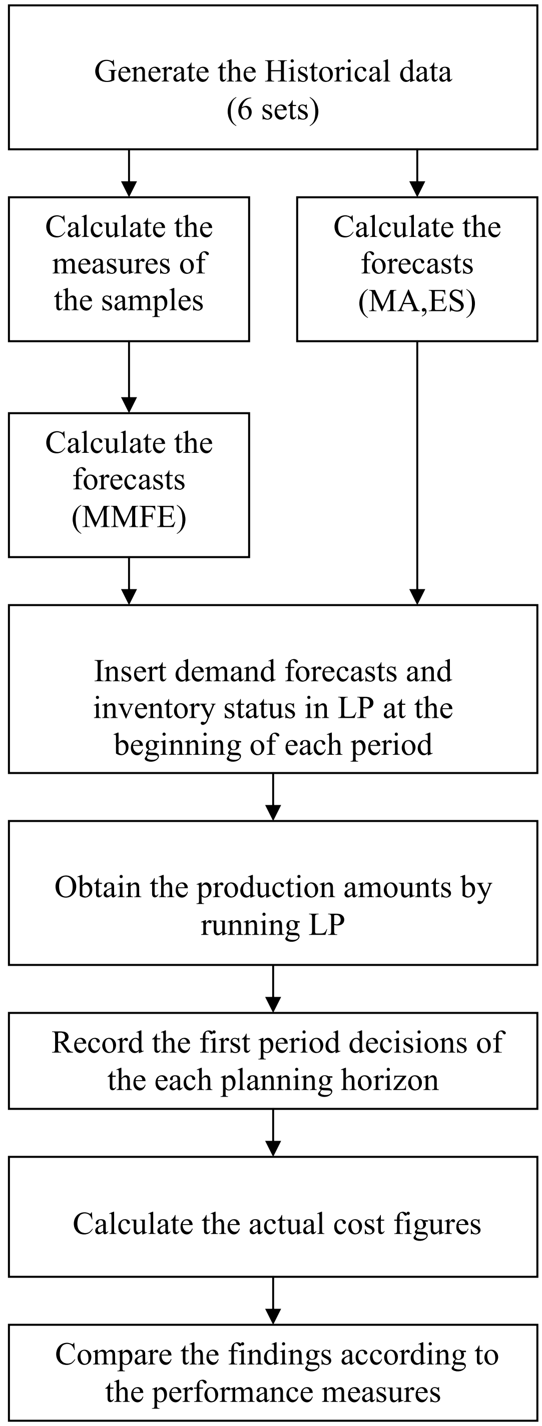

Figure 7 The major steps of a single problem instance

5.2. The Production-Distribution Model

When building the LP model, the supply chain environment is treated. The sets, variables, parameters, constants, cost coefficients, objective function and constraints are defined.

The following sets are used for parameter and variable definitions:

DC : set of DCs ($dc1, dc2, dc3$)

PG : set of product groups ($p1, p2$)

LN : set of production lines ($l1, l2, l3$)

T : set of time periods ($1,2,3,...,12$)

The production and inventory related decision variables are as follows:

P_{ijkt} : Amount of product group i produced on line j for DC k in period t

O_{jt} : Overtime on line j in period t

A_{ihkt} : Transshipment of product group i from DC h to DC k in period t

I_{ikt} : Inventory level of product group i at DC k at the end of period t

B_{ikt} : Backorders made for product group i at DC k at the end of period t

S_{ikt} : Shortage level of product group i at DC k at the end of period t

K : Total production cost (regular time, overtime, transshipment, inventory holding, backorder, shortage costs)

The following parameters and constants are used in the LP:

pr_{ij} : production rate of product group i on line j

r_{jt} : regular time hours available on line j in period t

ol_j : overtime hours limit on line j

d_{ikt} : forecast of demand for product group i at DC k in period t

mir_{ikt} : minimum inventory requirement of product group i at DC k in period t

A period is a month. Then the production rate of a product group is defined according to the regular working time in a month in the production plant. There are 3 production lines and each line has different production rate for a certain product group. Overtime is allowed and limited to a certain value at all of the production lines.

Minimum inventory requirement term can be thought as a safety stock. At each DC safety stocks are held for both product groups. Safety stocks are used to prevent DCs from having high amounts of backorders. Also safety stocks play an important role in obtaining higher service levels. For each CV level, different safety stock levels are defined. Safety stocks are calculated as follows:

$$SS = k\sigma .$$

For each of the CV levels, the related standard deviation ($\sigma_{1,j}, \sigma_{2,j}$ or $\sigma_{3,j}$) is used to calculate the safety stock amount. k is a constant value to calculate the amount to be held as a safety stock.

The cost coefficients in the LP are as follows,

c_{ij} : unit cost to produce product group i on line j

ac_{ihk} : unit cost of transshipment of product group i from DC h to DC k

h_{ik} : unit inventory holding cost of product group i at DC k

v_j : cost of running line j on overtime for one hour

sc_i : unit cost for shortage of product group i

bc_i : unit cost of back ordering product group i

Unit variable cost differs at each production line. Inventory holding cost is defined as the multiplication of unit variable cost and the monthly interest rate. Monthly interest rate (I) is accepted as %1. Inventory holding cost is calculated as follows:

$$h_{ik} = Ic_{ij}$$

Transshipment is allowed between all DCs. The transshipment costs have values less than unit variable costs and backorder costs, but more than inventory holding costs.

Backorder cost is defined relative to the unit variable cost, but its magnitude is larger than any of the unit variable cost. Cost for shortage is defined as a penalty cost for not meeting the minimum inventory requirement. It is incurred for any single unmet product. The relationship among the inventory holding, backorder and shortage costs is defined below:

$$h_{ik} < sc_i < bc_i .$$

The reason behind this relationship is especially for preventing the backorders. Since the customer order is not met, it is an undesired situation. On the other hand, cost of shortage is chosen larger than inventory holding cost, because safety stock is a vital factor to cover the fluctuations in the demand pattern.

The overtime cost on a production line is incurred for each extra work hour. It is obtained by using some constants and cost coeffficients. At first the average production rate for all product groups is determined. Secondly, the average of all unit variable costs is calculated. Then, these two findings are mutiplied and the cost of overtime on a line for one hour is obtained.

The linear programming model determines the production requirements, inventory, backorder, shortage and transshipment quantities for each of the 12 months in the related planning horizon. The objective is to minimize the total costs subject to the capacity, overtime, material balance and coverage constraints.

Minimize

$$K = \sum_{i \in PG} \sum_{j \in LN} \sum_{k \in DC} \sum_{t \in T} P_{ijkt}.c_{ij} + \sum_{j \in LN} v_j.O_{jt} + \sum_{i \in PG} \sum_{h \in DC} \sum_{k \in DC} \sum_{t \in T} A_{ihkt}.ac_{ihk} +$$

$$\sum_{i \in PG} \sum_{k \in DC} \sum_{t \in T} (h_{ik}.I_{ikt} + bc_i.B_{ikt} + sc_i.S_{ikt})$$

Subject to

$$\sum_{i \in PG} \sum_{k \in DC} pr_{ij}^{-1}.P_{ijkt} \leq r_{jt} + O_{jt} \qquad \forall \ j \in LN \text{ and } t \in T \qquad \text{(C1)}$$

$$I_{ikt} - B_{ikt} = I_{i,k,t-1} - B_{i,k,t-1} + \sum_{j \in LN} P_{ijkt} + \sum_{h \in DC} A_{ihkt} - \sum_{h \in DC} A_{ikht} - d_{ikt} \qquad \forall \ i \in PG \ , \ k \in DC \text{ and } t \in T \qquad \text{(C2)}$$

$$O_{jt} \leq ol_j \qquad \forall \ j \in LN \qquad \text{(C3)}$$

$$I_{ikt} - B_{ikt} + S_{ikt} \geq mir_{ikt} \qquad \forall \;\; i \in PG \;,\; k \in DC \text{ and } t \in T \qquad \text{(C4)}$$

$$P_{ijkt} \,,\, O_{jt} \,,\, A_{ihkt} \,,\, I_{ikt} \,,\, B_{ikt} \,,\, S_{ikt} \;\; \geq 0 \; and\; integer \quad \forall \;\; i \in PG \,,\; j \in LN \,,\; k \in DC \,,\; t \in T \qquad \text{(C5)}$$

At time 0, the initial inventory is taken as zero, but the inventory level or the backorder quantity at the end of period 1 is used as the initial inventory at the begining of period 2. Likewise this application is done till the last period.

Constraint (C1) is the capacity constraint. The total time hours needed to produce the required amounts at a certain period should be less than or equal to the sum of regular time hours available and the overtime used at that period.

Constraint (C2) is the material balance equation. As well as the current production decisions, the previous period's inventory and backorder amounts are taken into account. In every period either excess production is held as inventory or the missed amount is backordered. I_{ikt} and B_{ikt} both can not take values more than zero.

Constraint (C3) restricts the overtime at any period. The overtime required at any period should be less than or equal to the overtime limit at that period.

(C4) is the coverage constraint. With the help of this constraint, the missing amount to have the required minimum inventory level (safety stock) is determined. (C5) is the integrality constraint for the variables.

5.3. Obtaining the VCV Matrix

The selected ARMA model in this study is AR(1). The time series model with one-period lag is formulated as follows:

$$D_t = \mu(1-\rho) + \rho D_{t-1} + \varepsilon_t$$

ε_t ~N$(0, \sigma)$.

In general the pattern is:

$$D_{s,s+k} = \mu(1-\rho) + \rho D_{s,s+k-1}.$$

If we advance to the next period,

$$D_{s+1,s+1} = \mu(1-\rho) + \rho D_s + \varepsilon_{s+1}.$$

Similarly,

$$\begin{aligned} D_{s+1,s+2} &= \mu(1-\rho) + \rho D_{s+1} \\ &= \mu(1-\rho) + \rho(D_{s,s+1} + \varepsilon_{s+1}) \\ &= D_{s,s+2} + \rho\varepsilon_{s+1}. \end{aligned}$$

Similarly,

$$\begin{aligned} D_{s+1,s+3} &= \mu(1-\rho) + \rho D_{s+1,s+2} \\ &= \mu(1-\rho) + \rho D_{s,s+2} + \rho^2 \varepsilon_{s+1} \\ &= D_{s,s+3} + \rho^2 \varepsilon_{s+1}. \end{aligned}$$

Similarly,

$$\begin{aligned} D_{s+1,s+4} &= \mu(1-\rho) + \rho D_{s+1,s+3} \\ &= \mu(1-\rho) + \rho D_{s,s+3} + \rho^3 \varepsilon_{s+1} \\ &= D_{s,s+4} + \rho^3 \varepsilon_{s+1}. \end{aligned}$$

Seeing the pattern,

$$D_{s+1,s+k} = \mu(1-\rho) + D_{s,s+k} + a_k \varepsilon_{s+1},$$

where a_k is given by the recursive relation:

$$a_{k+1} = \rho a_k .$$

Starting from $a_1 = 1$, the other terms are $a_2 = \rho$, $a_3 = \rho^2$ ....

It follows that

$$\bar{\varepsilon}_{s+1} = \left(a_1\varepsilon_{s+1}, a_2\varepsilon_{s+1}, a_3\varepsilon_{s+1}, \ldots\right).$$

The new information that arrived as part of I_{s+1} is precisely ε_{s+1} and this information is used to update the forecasts of demands in all future periods. The MMFE in this study, has a variance-covariance matrix given by $\Sigma = (\sigma_{i,j})$ where

$$\begin{aligned}
\sigma_{i,j} &= E[\ \varepsilon_{s+1,s+i}\varepsilon_{s+1,s+j}\ ] \\
&= E[\ a_i . a_j (\varepsilon_{s+1})^2\ ] \\
&= a_i a_j \sigma^2 .
\end{aligned}$$

This study has an $13x13$ dimensional matrix:

$$(\sigma_{i,j}) = \begin{pmatrix} a_1^2\sigma^2 & a_1a_2\sigma^2 & a_1a_3\sigma^2 & \dots & a_1a_{13}\sigma^2 \\ a_1a_2\sigma^2 & a_2^2\sigma^2 & a_2a_3\sigma^2 & \dots & a_2a_{13}\sigma^2 \\ a_1a_3\sigma^2 & a_2a_3\sigma^2 & a_3^2\sigma^2 & \dots & a_3a_{13}\sigma^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ & & & & \\ a_1a_{13}\sigma^2 & a_2a_{13}\sigma^2 & a_3a_{13}\sigma^2 & \dots & a_{13}^2\sigma^2 \end{pmatrix}.$$

For the data set of product group 1 at DC 2, regarding the 8th problem instance of medium CV level demand pattern, calculated correlation coefficient (ρ) is -0,45737. If we investigate the total forecast error variability over three periods, it can be formulated as follows,

$$\begin{aligned} Var(D_{3,3} - D_{0,3}) &= Var(\varepsilon_{1,3}) + Var(\varepsilon_{2,3}) + Var(\varepsilon_{3,3}) \\ &= \sigma_3^2 + \sigma_2^2 + \sigma_1^2 \\ &= \rho^4\sigma^2 + \rho^2\sigma^2 + \sigma^2 \\ &= 0{,}044\sigma^2 + 0{,}21\sigma^2 + \sigma^2 \\ &= 1{,}254\sigma^2. \end{aligned}$$

For this case, we have that $\frac{0{,}044}{1{,}254} \times 100\% = 3{,}5\%$ of the variability is removed by advancing from period s to period $s+1$. $\frac{0{,}254}{1{,}254} \times 100\% = 20{,}3\%$ of the variability is removed by advancing from s to $s+2$, and 100% of the variability is removed by advancing from period s to $s+3$.

5.4. Experimental Runs and Performance Measures

Initially a single run requires 672 (6 x (100 + 12)) data points. These data points are generated for the 2 product groups and for the 3 DCs in the supply chain. MMFE system needs historical data to calculate its parameters. 112 data points are generated by random number generation according to the normal distribution. 100 of them are used as historical data. The rest 12 data points stand for the realized demand data in the next 12 time periods. After making the necessary calculations, 6 different mean, standard deviation and correlation coefficient values are obtained for the related samples. These parameters are used to obtain the forecasts. MMFE system calculates the forecasts according to the AR(1) process.

In the MA forecasting technique, the desired parameter is the number of previous demand points to be recorded. In this study it is selected as 3 periods' demands to be taken into account.

ES forecasting technique has the smoothing factor as a parameter. Smoothing factor is taken as 0,5 in the calculations. The reason is to give equal weights to the previous forecast and demand values.

Regular working time is 8 hours a day. Work days in a week is 5. Total regular time available in a month is 160 hours. Overtime is allowed and it is limited to 4 hours a day.

There are 3 CV levels. At first case, a problem instance is formed according to the low CV level. Product groups 1 and 2 are considered with low CV level. In the second case, they are obtained according to medium CV level. In the last case, both product groups are formed with high CV level. At each case there are 10 problem instances.

Totally there exist 30 problem instances. All these problem instances provide the results which are calculated by implementing the three forecasting techniques.

When comparing the forecasts of chosen procedures, a good performance measure is tracking the forecast accuracy. Forecast error is defined as the difference between the forecast value at a period and the actual demand for that period:

$$e_t = F_t - D_t .$$

There are three methods to measure the forecast accuracy. These are:

- Mean absolute deviation (MAD)
- Mean squared error (MSE)
- Mean absolute percentage error ($MAPE$).

MAD depends on the absolute value of forecast error. Since MSE depends on the squared errors. It is similar to the variance of a random sample. As a difference from the other measures of forecast accuracy, by implementing MAPE, the proportion of the forecast error and the related period’s demand are taken into account. This measure gives information about the forecast errors according to the magnitude of the demand value.

The formulas of these three measures are as follows:

$$MAD = \frac{1}{n}\sum_{i=1}^{n}|e_i|$$

$$MSE = \frac{1}{n}\sum_{i=1}^{n}e_i^2$$

$$MAPE = \left(\frac{1}{n} \sum_{i=1}^{n} \frac{e_i}{D_i} \right).100 \, .$$

In the following 6 tables, the results obtained for product 1 at all CV levels of demand patterns will be shown. Firstly the averages of the forecast accuracy measures considering the 30 data sets of each CV level (3 data sets for each problem instance) are calculated. These are presented in Tables 1, 3 and 5. Secondly, number of problem instances that result in the least forecast errors considering the 30 data sets of each CV level are given in Tables 2, 4 and 6. The comparisons among the three forecasting procedures are presented on the following pages. The other results for the performance measures are presented in Tables 19-24, Tables 25-30, and Tables 31-36 in Appendices A, B, and C for the forecast accuracy, fill rates, and costs, respectively.

Table 1 Averages of the forecast error measures for product group 1 at medium CV level demand pattern

	MA	**ES**	**MMFE**
MAD	1.527,999	1.593,309	1.359,548
MSE	3.807.871	4.173.135	3.119.355
MAPE	7,683696	8,030279	6,803884

In the above table the average values for MAD, MSE and MAPE are compared for the three forecasting procedures. In the table, the forecast errors caused by implementing MA and ES are larger than those of MMFE.

Table 2 Number of problem instances that result in the least forecast errors for product group 1 at medium CV level demand pattern

	MA	ES	MMFE
MAD	5	4	21
MSE	4	3	23
MAPE	5	4	21

If all of the problem instances are considered for product group 1 at medium CV level demand pattern, 21 of them show that MMFE system gives less MAD values. Only 5 of them indicate that MA results in less MAD values. In the rest 4 problems, ES has better values. When we look at the MSE results, MMFE yields in less forecast errors in 23 of the problems. MA has less errors in 4 of them and ES is the best in only 3 of the problems. When we take MAPE values into account, again MMFE leads in 21 problems. MA follows with 5 problem instances and ES is the worst with 4 problem instances. These show that MMFE system's predictions are more reliable than the other techniques.

Table 3 Averages of the forecast error measures for product group 1 at low CV level demand pattern

	MA	ES	MMFE
MAD	763,9993	796,6545	705,7442
MSE	951.967,8	1.043.284	841.322,4
MAPE	3,818371	3,986356	3,51807

Table 3 implies that the MMFE system calculates more closer results as forecasts.

Table 4 Number of problem instances that result in the least forecast errors for product group 1 at low CV level demand pattern

	MA	ES	MMFE
MAD	6	7	17
MSE	6	6	18
MAPE	6	6	18

In the Table 4, it can be noticed that number of problems that result in least forecast errrors according to the forecasting techniques MA and ES are slightly more than those of Table 2. At low CV level demand pattern, the related standard deviation is less and this causes closer forecasts for not only MMFE system, but also for MA and ES.

Table 5 Averages of the forecast error measures for product group 1 at high CV level demand pattern

	MA	ES	MMFE
MAD	3.056,915	3.187,574	2.706,46
MSE	15.240.629	16.702.560	12.225.539
MAPE	15,88826	16,63401	13,98619

At high CV level demand pattern, again the MMFE system leads among the three forecasting procedures. The values of MAD, MSE and MAPE are larger in this table, because the width that demand can take value is the largest of all. But there is no change in the comparison phase.

Table 6 Number of problem instances that result in the least forecast errors for product group 1 at high CV level demand pattern

	MA	**ES**	**MMFE**
MAD	4	4	22
MSE	3	5	22
MAPE	3	5	22

If we evaluate all of the results in the tables above, MMFE system yields in best results according to the forecast errors. All of the forecast accuracy measures indicate that no matter what the CV level is, the MMFE performs the better.

Another performance measure is the fill rate. At the end of the each period the supply chain partners should keep track of on-hand inventories. If they are sufficient to cover the demand, the rest will be inventory, but if they are not, the unmet part will be backordered. On-hand inventory includes the production amount at that period and the inventory amount at the recent period. Simply it is the percentage of demand that is met from the inventory.

In the following tables (Tables 7-9), fill rates are compared. In the tables, “p1dc1” stands for the the status of the product as “product group 1 at DC 1”. Likewise the status of each product group at each DC are obtained and shown for all ten problem instances of the related demand pattern.

In Tables 7-9, the values in each cell are the average fill rates of the related problem instances. At the last rows the values that are bold written are the overall average fill rates.

Table 7 Fill rates by applying MMFE for the medium CV level demand pattern

	p1dc1	p1dc2	p1dc3	p2dc1	p2dc2	p2dc3
Prob.1	0,992007	0,988804	0,996095	0,987415	0,994801	0,969625
Prob.2	1	1	0,987978	1	0,99006	0,994262
Prob.3	0,996783	0,991138	0,997193	0,999316	0,999968	0,985353
Prob.4	1	0,978878	0,984007	0,998596	0,990381	1
Prob.5	0,997575	0,998655	0,980883	0,996512	0,997123	0,992324
Prob.6	1	0,994502	0,98448	0,997711	0,993715	0,985764
Prob.7	0,992415	0,999779	1	0,998878	1	0,991721
Prob.8	0,987913	0,977938	0,995119	0,997198	0,995706	0,99829
Prob.9	1	0,99734	0,987329	1	1	0,984963
Prob.10	0,995723	1	0,979113	1	0,988826	0,996691
Average	**0,996242**	**0,992703**	**0,98922**	**0,997563**	**0,995058**	**0,989899**

Table 8 Fill rates by applying MA for the medium CV level demand pattern

	p1dc1	p1dc2	p1dc3	p2dc1	p2dc2	p2dc3
Prob.1	0,990744	0,992092	0,986913	0,987624	0,992932	0,979618
Prob.2	0,992991	0,999067	0,99227	0,99529	0,985539	0,989732
Prob.3	0,991667	0,990636	0,9882	0,992043	0,998935	0,991571
Prob.4	0,99825	0,989086	0,975762	0,999794	0,974067	0,992727
Prob.5	0,992126	0,997774	0,981178	0,996298	0,996244	0,986615
Prob.6	0,999663	0,997054	0,981378	0,991209	0,992141	0,983004
Prob.7	0,994796	0,998066	0,994323	0,996868	0,991649	0,991
Prob.8	0,98835	0,990023	0,995205	0,985063	0,981478	0,998206
Prob.9	0,995626	0,99405	0,981807	1	1	0,983941
Prob.10	0,994879	0,988836	0,986601	0,992417	0,987474	0,99122
Average	**0,993909**	**0,993668**	**0,986364**	**0,993661**	**0,990046**	**0,988763**

Table 9 Fill rates by applying ES for the medium CV level demand pattern

	p1dc1	**p1dc2**	**p1dc3**	**p2dc1**	**p2dc2**	**p2dc3**
Prob.1	0,990514	0,989438	0,98992	0,985076	0,990554	0,980347
Prob.2	0,992108	0,996437	0,989112	0,988334	0,984565	0,986105
Prob.3	0,996763	0,989845	0,986134	0,993156	0,997247	0,99216
Prob.4	0,996876	0,993833	0,974738	0,998554	0,97434	0,992514
Prob.5	0,994632	0,997446	0,979641	0,993589	0,995215	0,98504
Prob.6	0,999742	0,995906	0,9771	0,994066	0,990756	0,98611
Prob.7	0,995253	0,995921	0,996146	0,992368	0,995879	0,989584
Prob.8	0,979802	0,990944	0,994269	0,989978	0,986332	0,997644
Prob.9	0,997482	0,997425	0,97811	1	1	0,982978
Prob.10	0,996732	0,989573	0,987436	0,992307	0,992441	0,992373
Average	**0,99399**	**0,993677**	**0,985261**	**0,992743**	**0,990733**	**0,988485**

Since the selected demand process is stationary and safety stock factor is used, the obtained fill rates are high. If Tables 7, 8 and 9 are compared among each other, it is obvious that the fill rates with MMFE are slightly larger than other techniques. On the other hand, at some problem instances, 100% fill rate is achieved. There are more 100% fill rate achievements in Table 7. This shows that by applying MMFE system, good forecasting is made.

By forming a linear programming (LP) model, production decisions are made. Each LP requires forecast predictions as input for each planning horizon. Then LP gives the production amount for the first period of that planning horizon. Only the results those belong to the first period of the each horizon are used in the production plans. The cost figures in the LP model are,

- Production cost
- Inventory holding and back order costs
- Shortage cost
- Transshipment cost
- Overtime cost

These cost figures are also going to be used as a performance measure. Since good forecasting is so important, forecast errors may cause higher costs in the production / distribution problem. Especially inventory holding and backorder costs may be at lower values by applying MMFE system in the problems.

The cost figures obtained by implementing MMFE, MA and ES are shown in the following Tables 10-12.

Table 10 Costs by applying MMFE for the medium CV level demand pattern

	Production	Inv.-b/o	Shortage	Overtime	Total
Prob.1	102.840.800	6.296.363	4.047.660	13.694.484	126.879.307
Prob.2	102.623.000	2.545.685	2.390.199	13.689.408	121.248.292
Prob.3	102.982.400	2.808.781	2.984.634	13.147.037	121.922.852
Prob.4	101.080.350	4.999.103	3.190.697	14.817.916	124.088.066
Prob.5	102.414.400	3.629.768	3.401.922	14.423.567	123.869.657
Prob.6	101.342.300	4.147.390	3.036.580	12.839.263	121.365.532
Prob.7	102.664.050	1.651.241	2.241.845	13.466.543	120.023.680
Prob.8	101.253.350	4.823.404	4.041.003	14.506.729	124.624.485
Prob.9	101.196.650	2.915.742	2.147.104	12.782.411	119.041.907
Prob.10	100.731.750	3.895.228	3.181.796	13.848.371	121.657.145
Average	**101.912.905**	**3.771.271**	**3.066.344**	**13.721.573**	**122.472.092**

Table 11 Costs by applying MA for the medium CV level demand pattern

	Production	**Inv.-b/o**	**Shortage**	**Overtime**	**Total**
Prob.1	103.120.700	6.295.262	3.795.784	13.636.195	126.847.941
Prob.2	102.316.050	3.894.893	3.155.652	13.600.240	122.966.835
Prob.3	102.672.500	4.303.336	3.138.936	12.946.253	123.061.025
Prob.4	100.330.700	6.545.021	3.646.886	15.247.430	125.770.036
Prob.5	101.873.700	4.698.612	3.416.260	14.443.476	124.432.048
Prob.6	100.982.000	4.984.030	3.363.816	12.189.704	121.519.549
Prob.7	102.149.800	2.827.969	2.792.429	13.234.627	121.004.825
Prob.8	100.308.750	5.265.550	4.237.040	14.689.690	124.501.029
Prob.9	100.565.000	4.222.712	3.077.155	12.430.983	120.295.851
Prob.10	100.025.150	5.256.794	3.415.601	13.912.808	122.610.353
Average	**101.434.435**	**4.829.418**	**3.403.956**	**13.633.141**	**123.300.949**

Table 12 Costs by applying ES for the medium CV level demand pattern

	Production	**Inv.-b/o**	**Shortage**	**Overtime**	**Total**
Prob.1	102.997.550	6.575.957	3.947.310	13.739.350	127.260.167
Prob.2	102.004.400	5.299.798	3.380.897	13.428.614	124.113.709
Prob.3	102.765.950	4.074.136	2.956.688	12.951.781	122.748.554
Prob.4	100.431.100	6.347.241	3.637.526	15.448.129	125.863.997
Prob.5	101.698.000	5.046.492	3.660.847	14.409.269	124.814.609
Prob.6	101.164.000	5.152.235	3.404.777	12.968.419	122.689.431
Prob.7	102.185.750	2.946.294	3.047.234	13.184.374	121.363.653
Prob.8	100.580.700	5.467.744	4.195.240	14.533.096	124.776.780
Prob.9	100.549.650	4.115.666	2.950.989	12.574.690	120.190.995
Prob.10	99.924.050	4.472.980	3.723.482	14.093.542	122.214.054
Average	**101.430.115**	**4.949.854**	**3.490.499**	**13.733.126**	**123.603.595**

By examining the Tables 10, 11 and 12 above, it can be seen that the production costs are generally the same and around the 100.000.000 units of currency. However, the other cost figures can give more reliable information. Especially the inventory holding, backorder and shortage costs make sense, because the aim in this study is to obtain realistic forecast values. When the results of the three techniques are compared, the inventory holding and backorder costs of 9 problem instances according to MMFE have the least values. Only at one problem, MA has the least the inventory holding and backorder costs. If the shortage costs are considered, MMFE results in least costs at 8 of the problems. MA and ES have the least costs at 2 of the problems. When the total costs are compared, MMFE gives the least costs at 8 of the problem instances. At the rest 2 problems MA has the least costs. According to the cost figures, it can be said that by applying MMFE in the supply chain more money can be saved each year.

In order to see the performance of the MMFE system relative to the other forecasting procedures in a looser environment in terms of production capacity, the production capacity is increased by 25%, that is, regular time hours available are increased to 200 hours from 160 hours. The experimental runs are carried out for the medium CV level demand pattern. The fill rates and cost figures obtained are presented in Tables 13 -18.

Table 13 Fill rates by applying MMFE for the medium CV level demand pattern with 25% increased capacity

	p1dc1	**p1dc2**	**p1dc3**	**p2dc1**	**p2dc2**	**p2dc3**
Prob.1	0,993436	0,988804	0,996095	0,987415	0,994801	0,969625
Prob.2	1	1	0,987978	1	0,99006	0,994262
Prob.3	0,996783	0,991138	0,997193	0,999316	0,999968	0,985353
Prob.4	1	0,978878	0,984007	0,998596	0,990381	1
Prob.5	0,997575	0,998655	0,980883	0,996512	0,997123	0,992324
Prob.6	1	0,994502	0,98448	0,997711	0,993715	0,985764
Prob.7	0,992415	0,999779	1	0,998878	1	0,991721
Prob.8	0,988506	0,977938	0,995119	0,997198	0,995706	0,99829
Prob.9	1	0,99734	0,987329	1	1	0,984963
Prob.10	0,995723	1	0,979113	1	0,988826	0,996691
Average	**0,996444**	**0,992703**	**0,98922**	**0,997563**	**0,995058**	**0,989899**

Table 14 Fill rates by applying MA for the medium CV level demand pattern with 25% increased capacity

	p1dc1	**p1dc2**	**p1dc3**	**p2dc1**	**p2dc2**	**p2dc3**
Prob.1	0,991266	0,992092	0,986913	0,987624	0,992932	0,979618
Prob.2	0,992991	0,999067	0,99227	0,99529	0,985539	0,989732
Prob.3	0,991667	0,990636	0,9882	0,992043	0,998935	0,991571
Prob.4	1	0,989086	0,975762	0,999794	0,974067	0,992727
Prob.5	0,994706	0,997774	0,981178	0,996298	0,996244	0,986615
Prob.6	0,999663	0,997054	0,981378	0,991209	0,992141	0,983004
Prob.7	0,994796	0,998066	0,994323	0,996868	0,991649	0,991
Prob.8	0,989419	0,990023	0,995205	0,989106	0,981478	0,998206
Prob.9	0,995626	0,99405	0,981807	1	1	0,983941
Prob.10	0,997914	0,988836	0,986601	0,992417	0,987474	0,99122
Average	**0,994805**	**0,993668**	**0,986364**	**0,994065**	**0,990046**	**0,988763**

Table 15 Fill rates by applying ES for the medium CV level demand pattern with 25% increased capacity

	p1dc1	**p1dc2**	**p1dc3**	**p2dc1**	**p2dc2**	**p2dc3**
Prob.1	0,990514	0,989438	0,98992	0,985076	0,990554	0,980347
Prob.2	0,992108	0,996437	0,989112	0,988334	0,984565	0,986105
Prob.3	0,996763	0,989845	0,986134	0,993156	0,997247	0,99216
Prob.4	1	0,993833	0,974738	0,998554	0,97434	0,992514
Prob.5	0,994632	0,997446	0,979641	0,993589	0,995215	0,98504
Prob.6	0,999742	0,995906	0,9771	0,994066	0,990756	0,98611
Prob.7	0,995253	0,995921	0,996146	0,992368	0,995879	0,989584
Prob.8	0,979802	0,990944	0,994269	0,993575	0,986332	0,997644
Prob.9	0,997482	0,997425	0,97811	1	1	0,982978
Prob.10	0,999978	0,989573	0,987436	0,992307	0,992441	0,992373
Average	**0,994627**	**0,993677**	**0,985261**	**0,993102**	**0,990733**	**0,988485**

If Tables 13, 14 and 15 are compared with the related Tables 7, 8 and 9 corresponding to the tight capacity case, the increased production capacity results in equal or higher fill rates for all of the three forecasting procedures. Loose capacity gives the manufacturer the chance to cope with the variability in demand.

Table 16 Costs by applying MMFE for the medium CV level demand pattern with 25% increased capacity

	Production	**Inv.-b/o**	**Shortage**	**Overtime**	**Total**
Prob.1	93.982.625	6.162.757	3.830.273	1.503.060	105.478.715
Prob.2	93.795.725	2.545.740	2.390.199	989.735	99.721.400
Prob.3	93.783.350	2.808.781	2.984.634	913.313	100.490.078
Prob.4	92.853.225	5.006.018	2.939.227	1.365.980	102.164.450
Prob.5	93..094.825	3.629.768	3.401.922	2.340.910	102.467.425
Prob.6	91.624.325	4.147.390	3.036.580	1.481.515	100.289.810
Prob.7	94.082.725	1.664.424	2.006.353	1.029.977	98.783.479
Prob.8	92.283.825	4.764.660	3.981.415	2.010.096	103.039.996
Prob.9	91.345.250	2.916.041	2.102.179	1.366.290	97.729.760
Prob.10	91.703.075	3.895.519	3.175.196	1.525.084	100.298.874
Average	**92.854.895**	**3.754.110**	**2.984.798**	**1.452.596**	**101.046.399**

Table 17 Costs by applying MA for the medium CV level demand pattern with 25% increased capacity

	Production	**Inv.-b/o**	**Shortage**	**Overtime**	**Total**
Prob.1	92.895.650	6.250.155	3.785.659	2.688.419	105.619.883
Prob.2	93.096.975	3.896.162	3.155.652	1.391.585	101.540.375
Prob.3	93.327.500	4.303.336	3.138.936	895.435	101.665.206
Prob.4	91.120.550	6.388.422	3.427.959	3.095.740	104.032.671
Prob.5	92.863.600	4.438.882	3.227.017	2.597.869	103.127.368
Prob.6	91.208.975	4.984.710	3.363.816	1.571.191	101.128.692
Prob.7	92.586.250	2.828.847	2.792.429	1.457.743	99.665.269
Prob.8	91.546.600	4.901.249	3.979.428	2.397.846	102.825.123
Prob.9	89.979.550	4.228.772	3.077.155	1.942.049	99.227.527
Prob.10	91.303.250	4.949.592	3.226.786	1.778.941	101.258.568
Average	**91.992.890**	**4.717.013**	**3.317.484**	**1.981.682**	**102.009.068**

Table 18 Costs by applying ES for the medium CV level demand pattern with 25% increased capacity

	Production	**Inv.-b/o**	**Shortage**	**Overtime**	**Total**
Prob.1	92.582.300	6.584.674	3.925.014	3.029.921	106.121.908
Prob.2	92.715.125	5.300.068	3.380.897	1.308.001	102.704.091
Prob.3	93.223.225	4.074.768	2.956.688	1.346.832	101.601.513
Prob.4	91.232.600	6.060.385	3.407.616	4.463.327	105.163.927
Prob.5	92.066.800	5.049.320	3.552.229	3.057.923	103.726.272
Prob.6	90.959.500	5.152.146	3.403.637	1.996.757	101.512.040
Prob.7	92.492.150	2.948.241	3.037.616	1.570.458	100.048.465
Prob.8	92.315.450	5.237.120	3.816.315	2.282.875	103.651.759
Prob.9	89.214.825	4.117.470	2.950.989	3.180.227	99.463.510
Prob.10	91.687.750	4.143.196	3.495.485	1.969.967	101.296.399
Average	**91.848.973**	**4.866.739**	**3.392.649**	**2.420.629**	**102.528.989**

Tables 16, 17 and 18 show that the increased capacity do not affect the inventory, backorder and shortage amounts, thus the relevant cost figures are not affected. But the production cost is reduced by approximately 10%. For MMFE system, the overtime costs are reduced by 10% when capacity is increased by 25%. The decrease is 14% for MA method and 17% for ES method. When the three forecasting procedures are compared, again MMFE leads in all of the cost figures. At 8 of the 10 problems MMFE has the least total costs. These findings indicate that MMFE system results in more reliable forecast predictions among the three procedures. All performance measures prove that better forecasting and more cost savings are achieved by the implementation of MMFE.

CHAPTER 6

CONCLUSION

In this thesis it is shown that the forecast volatility can be solved better by applying MMFE system. When reaching this idea, an example supply chain structure is constructed. This supply chain consists of manufacturer and DCs as buyer (internal customer). The manufacturer is responsible for production plans and aware of every information that the DCs have. A single DC tries to fullfill the demand coming from the external customers. The upstream partner (manufacturer) and the downstream partners (DCs) are in close relationship and every single information is shared.

The demand process is stationary and normally distributed. It has an autoregressive order-1 structure. Three different coefficient of variation levels are used in the demand patterns. A single collaborative forecasting is made by the supply chain partners. The forecasts are made by using three different techniques (MMFE, MA and ES). These chosen techniques are the frequently used ones in forecasting stationary demand series. 12-month planning horizon is used when constructing the forecasting scheme.

The calculated forecasts are used as input in the LP which is run in general algebraic modeling system (GAMS). LP tries to minimize the total costs by determining the production, inventory, backorder, transshipment and shortage amounts in order to meet the 12 periods' demand values. The solutions of the LP are recorded and used to obtain the actual costs. Actual costs play an important role when comparing the three forecasting techniques. Other performance measures are forecast accuracy and fill rate.

As a performance measure, the forecast accuracy gives us vital information about the following findings. According to each measure of forecast accuracy (MAD, MSE and MAPE), the MMFE system gives the best results. In fact, generally this is not the case. Sometimes MAD and MSE values yield different results. In this study all of the three measures indicate that MMFE determines the closest values to the actual demands.

Another performance measure is the fill rate that serves as customer service level. Service level aims to meet the customer orders from the on-hand inventories. Although it seems that there is slight difference when compared to other techniques, MMFE has the best fill rates. In fact the results show that MA and ES techniques have difficulties to meet the demands on time. Although safety stocks are held in DCs, in some problem instances, the fill rates take values under 80%. MMFE has the highest fill rates in most of the time periods.

Actual costs are the important results when comparing MMFE with MA and ES. The production costs are nearly the same in all techniques. The overtime cost is slightly higher, but the inventory holding, backorder and shortage-related costs have the lowest values in MMFE system. There are high cost differences between the MMFE system and other techniques. Finally when we look at the total costs, MMFE has the lowest costs at all CV levels of the demand patterns.

In general the supply chain partners try to have less inventory and backorder costs, because of the volatility in forecasts. Here the importance of good forecasting comes into the picture. Good forecasting results in better forecast accuracy, higher fill rates and less chainwide total costs. In this study all these aims are achieved by the help of MMFE system. The above properties are greatly desired in production/distribution systems. MMFE system deserves more implementation areas in the supply chains as a successful forecasting tool.

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APPENDIX A

FORECAST ERROR MEASURES

Table 19 Averages of the forecast error measures for product group 2 at medium CV level demand pattern

	MA	ES	MMFE
MAD	2341,837	2394,612	2073,941
MSE	8796751	8916829	6910442
MAPE	8,686908	8,890382	7,758402

Table 20 Number of problem instances that result in the least forecast errors for product group 2 at medium CV level demand pattern

	MA	ES	MMFE
MAD	5	3	22
MSE	3	3	24
MAPE	4	3	23

Table 21 Averages of the forecast error measures for product group 2 at low CV level demand pattern

	MA	ES	MMFE
MAD	782,1926	816,3386	739,6309
MSE	984472,4	1093092	913386,6
MAPE	2,840941	2,969045	2,681129

Table 22 Number of problem instances that result in the least forecast errors for product group 2 at low CV level demand pattern

	MA	ES	MMFE
MAD	8	6	16
MSE	8	7	15
MAPE	8	5	17

Table 23 Averages of the forecast error measures for product group 2 at high CV level demand pattern

	MA	ES	MMFE
MAD	3735,796	3901,636	3365,402
MSE	21592830	22639989	17298581
MAPE	14,04872	14,581	12,70531

Table 24 Number of problem instances that result in the least forecast errors for product group 2 at high CV level demand pattern

	MA	ES	MMFE
MAD	6	2	22
MSE	4	3	23
MAPE	8	2	20

APPENDIX B

FILL RATES

Table 25 Fill rates by applying MMFE for the low CV level demand pattern

	p1dc1	**p1dc2**	**p1dc3**	**p2dc1**	**p2dc2**	**p2dc3**
Prob.1	0,9963	0,992914	0,997224	0,997997	0,996125	0,998338
Prob.2	1	1	0,991328	1	1	0,994849
Prob.3	0,997288	0,994025	0,997395	0,999146	0,996566	0,998272
Prob.4	0,999196	0,987025	0,989427	0,99938	0,992033	0,992737
Prob.5	0,996573	0,997426	0,987393	0,996416	0,999477	0,991912
Prob.6	0,999474	0,996352	0,98843	0,99973	0,997742	0,992524
Prob.7	0,995313	0,999167	0,99936	0,99702	0,999847	1
Prob.8	0,988346	0,983329	0,996138	0,991908	0,988339	0,997647
Prob.9	1	0,99777	0,992264	1	0,998939	0,995102
Prob.10	0,996052	0,999724	0,986233	0,99782	1	0,991823
Average	**0,996854**	**0,994773**	**0,992519**	**0,997942**	**0,996907**	**0,99532**

Table 26 Fill rates by applying MA for the low CV level demand pattern

	p1dc1	p1dc2	p1dc3	p2dc1	p2dc2	p2dc3
Prob.1	0,993863	0,99439	0,991746	0,996466	0,996776	0,994918
Prob.2	0,994742	0,997942	0,993911	0,997319	0,999649	0,996947
Prob.3	0,993637	0,993726	0,991841	0,996791	0,996327	0,995625
Prob.4	0,998357	0,992739	0,984271	0,99935	0,995557	0,990014
Prob.5	0,994588	0,997421	0,988393	0,990964	0,999154	0,992258
Prob.6	0,998742	0,997219	0,987815	0,999875	0,998683	0,992336
Prob.7	0,995892	0,998326	0,995697	0,997925	0,999269	0,997812
Prob.8	0,993454	0,996709	0,996104	0,991514	0,998716	0,987689
Prob.9	0,997067	0,995524	0,988884	0,998342	0,99768	0,992736
Prob.10	0,995483	0,992817	0,991342	0,996849	0,995687	0,994518
Average	**0,995583**	**0,995681**	**0,991**	**0,996539**	**0,99775**	**0,993485**

Table 27 Fill rates by applying ES for the low CV level demand pattern

	p1dc1	**p1dc2**	**p1dc3**	**p2dc1**	**p2dc2**	**p2dc3**
Prob.1	0,993741	0,992302	0,993362	0,996375	0,995713	0,996117
Prob.2	0,994572	0,996542	0,992272	0,997006	0,998693	0,995745
Prob.3	0,997651	0,993309	0,990785	0,998764	0,99602	0,997019
Prob.4	0,997033	0,995775	0,984458	0,998838	0,997502	0,989577
Prob.5	0,995574	0,997011	0,98734	0,990118	0,999033	0,991627
Prob.6	0,998947	0,99649	0,985557	0,999901	0,998385	0,990668
Prob.7	0,996155	0,996407	0,996655	0,998077	0,998452	0,998518
Prob.8	0,986708	0,993562	0,994981	0,992025	0,996518	0,997781
Prob.9	0,998026	0,997085	0,9868	0,999045	0,998998	0,991321
Prob.10	0,99693	0,993198	0,991778	0,99871	0,995968	0,99484
Average	**0,995534**	**0,995168**	**0,990399**	**0,996886**	**0,997528**	**0,994321**

If the tables 25, 26 and 27 are compared, it can be said that MMFE has slightly higher fill rates than other forecasting methods. Since the standard deviation around the mean value is low, MA and ES methods have closer fill rates to the MMFE.

Table 28 Fill rates by applying MMFE for the high CV level demand pattern

	p1dc1	p1dc2	p1dc3	p2dc1	p2dc2	p2dc3
Prob.1	0,990738	0,980501	0,993172	0,991981	0,994578	0,984562
Prob.2	1	1	0,979126	0,976624	0,991947	1
Prob.3	0,993674	0,984043	0,995078	0,997101	0,987429	0,988434
Prob.4	1	0,962877	0,973778	0,99315	0,993611	0,993103
Prob.5	0,994039	0,997103	0,96789	1	0,999426	0,984622
Prob.6	1	0,990382	0,974332	0,993711	0,998905	0,988714
Prob.7	0,986815	0,999679	1	1	0,975072	0,973946
Prob.8	0,980647	0,964476	0,991568	0,99488	0,994434	0,993338
Prob.9	1	0,99523	0,977552	0,999778	0,992379	0,985752
Prob.10	0,992697	0,998881	0,963527	0,977525	0,995048	0,982427
Average	**0,993861**	**0,987317**	**0,981602**	**0,992475**	**0,992283**	**0,98749**

Table 29 Fill rates by applying MA for the high CV level demand pattern

	p1dc1	**p1dc2**	**p1dc3**	**p2dc1**	**p2dc2**	**p2dc3**
Prob.1	1	0,986483	0,976084	0,992814	0,979669	0,988598
Prob.2	0,967598	1	0,980329	0,861894	0,952946	0,996535
Prob.3	0,984682	0,983193	0,977873	0,983433	0,989545	0,985365
Prob.4	0,996616	0,98124	0,959031	0,998674	0,98305	0,975476
Prob.5	0,995384	0,995824	0,968152	0,982966	0,99276	0,975718
Prob.6	0,999362	0,994794	0,968429	0,982968	0,98548	0,982927
Prob.7	0,990839	0,99632	0,989534	0,999295	0,969859	0,990762
Prob.8	0,983519	0,963761	0,991201	0,977018	0,992017	0,986764
Prob.9	0,991778	0,989238	0,967904	0,994499	0,977615	0,98892
Prob.10	0,99062	0,979579	0,977073	0,976819	0,986901	0,989579
Average	**0,99004**	**0,987043**	**0,975561**	**0,975038**	**0,980984**	**0,986064**

Table 30 Fill rates by applying ES for the high CV level demand pattern

	p1dc1	p1dc2	p1dc3	p2dc1	p2dc2	p2dc3
Prob.1	1	0,981809	0,981358	0,993321	0,982837	0,981669
Prob.2	0,912068	0,992957	0,908198	0,84302	0,971313	0,988639
Prob.3	0,969491	0,938103	0,978209	0,976168	0,9742	0,919045
Prob.4	1	0,860106	0,770341	0,927002	0,912715	0,755619
Prob.5	0,973902	0,980553	0,861684	0,915732	1	0,775013
Prob.6	0,840066	1	0,983561	0,968963	0,952933	0,958171
Prob.7	0,914782	0,968301	0,847082	0,944681	0,951147	0,841265
Prob.8	0,963653	0,85493	1	0,996979	0,927614	0,990085
Prob.9	0,993573	0,915638	0,980805	0,939714	0,993421	0,918817
Prob.10	0,966416	0,930979	0,870427	0,930416	0,962313	0,966417
Average	**0,953395**	**0,942338**	**0,918167**	**0,9436**	**0,962849**	**0,909474**

By examining the values in Tables 28, 29 and 30, again MMFE leads among the three methods. At high CV level, MA method has closer fill rates to the MMFE, but ES method has the lowest fill rates among the three forecasting procedures.

APPENDIX C

COST FIGURES

Table 31 Costs by applying MMFE for the low CV level demand pattern

	Production	Inv.-b/o	Shortage	Overtime	Total
Prob.1	102493550	1881163	1195909	13547590	119118212
Prob.2	102414900	1216399	1004762	13511184	118147245
Prob.3	102548100	1524068	1197909	13232342	118502419
Prob.4	103384350	3528417	1822918	14086577	122822262
Prob.5	102515200	2681444	1733303	13769496	120699443
Prob.6	101954200	2289601	1328139	13204735	118776675
Prob.7	102217300	877184	1917885	13392011	118404379
Prob.8	102979150	4598762	2198785	13829167	123605864
Prob.9	101692550	1445536	933895	13063086	117135068
Prob.10	102486500	2492367	1659547	13539553	120177968
Average	**102468580**	**2253494**	**1499305**	**13517574**	**119738953**

Table 32 Costs by applying MA for the low CV level demand pattern

	Production	**Inv.-b/o**	**Shortage**	**Overtime**	**Total**
Prob.1	102381500	2745109	1391663	13476188	119994460
Prob.2	102330250	1715595	1280738	13415783	118742366
Prob.3	102319800	2748119	1440847	13083249	119592015
Prob.4	103652850	3521739	1688435	14255382	123118405
Prob.5	102407200	3143192	1600020	13846764	120997176
Prob.6	101934600	2271995	1423569	13188181	118818345
Prob.7	102052300	1348961	928081	13259414	117588757
Prob.8	103650750	2849380	1584172	14854322	122938623
Prob.9	101436950	2580575	1444659	12884298	118346481
Prob.10	102566750	2868850	1574328	13618654	120628582
Average	**102473295**	**2579351**	**1435651**	**13588224**	**120076521**

Table 33 Costs by applying ES for the low CV level demand pattern

	Production	Inv.-b/o	Shortage	Overtime	Total
Prob.1	102455300	2800802	1387821	13512171	120156094
Prob.2	102194700	2164617	1362889	13332565	119054772
Prob.3	102439250	2312278	1223317	13118555	119093401
Prob.4	103816050	3264594	1682328	14366293	123129265
Prob.5	102265650	3302714	1760695	12707287	120036345
Prob.6	101840350	2669163	1462121	13126592	119098227
Prob.7	102003100	1410274	1076108	13216776	117706258
Prob.8	103076650	3318603	1795387	13886103	122076743
Prob.9	101534300	2501301	1423654	12928910	118388165
Prob.10	102754800	2504241	1513880	13720682	120493603
Average	**102438015**	**2624859**	**1468820**	**13391593**	**119923287**

The results of Tables 31, 32 and 33 show that the cost figures of the three forecasting procedures are slightly different among eachother. MMFE results in less costs at most of the problems.

Table 34 Costs by applying MMFE for the high CV level demand pattern

	Production	**Inv.-b/o**	**Shortage**	**Overtime**	**Total**
Prob.1	101824450	6729388	5546795	15076340	129176973
Prob.2	101749950	5153400	4543523	13856211	125303085
Prob.3	98568250	5507901	5101731	12739745	121917627
Prob.4	106367800	9826340	6775923	16024509	138994572
Prob.5	102230850	6653821	6443118	14837317	130165106
Prob.6	99333250	6334605	4651087	12535013	122853955
Prob.7	106384200	6114476	5038320	13422636	130959632
Prob.8	101151450	9125042	7037051	15234571	132548113
Prob.9	99723850	5469573	4445820	11842026	121481269
Prob.10	106897150	9351527	8018513	14176958	138444148
Average	**102423120**	**7026607**	**5760188**	**13974533**	**129184448**

Table 35 Costs by applying MA for the high CV level demand pattern

	Production	**Inv.-b/o**	**Shortage**	**Overtime**	**Total**
Prob.1	101924300	7583497	5216844	16380139	131104780
Prob.2	101914900	18764948	7759762	15779028	144218639
Prob.3	97147750	9200933	6258284	12356422	124963389
Prob.4	107877600	11450576	6471683	15327631	141127490
Prob.5	100820750	8989510	6794789	15267113	131872162
Prob.6	98384900	8784044	6030124	12642257	125841326
Prob.7	107191750	6196213	4864448	12566202	130818613
Prob.8	100113150	10456227	7496679	15276222	133342278
Prob.9	99755900	9151732	6305144	10197205	125409981
Prob.10	107643400	9974739	6658095	14289673	138565907
Average	**102277440**	**10055242**	**6385585**	**14008189**	**132726456**

Table 36 Costs by applying ES for the high CV level demand pattern

	Production	Inv.-b/o	Shortage	Overtime	Total
Prob.1	102168750	7751761	5425912	15308116	130654539
Prob.2	99054150	31742972	10866286	12390798	154054206
Prob.3	98780450	20880917	7038938	14268946	140969251
Prob.4	91922150	64117743	19959830	10940359	186940082
Prob.5	92053400	39067659	12882684	14914219	158917961
Prob.6	104795700	24006058	8981971	10784977	148568706
Prob.7	100939650	42314070	13862303	12100451	169216474
Prob.8	104917400	23940004	9018027	17574184	155449614
Prob.9	98260300	20439611	7751792	14290012	140741715
Prob.10	102404100	34619952	13416347	10905871	161346269
Average	**99529605**	**30888075**	**10920409**	**13347793**	**154685882**

Since the related magnitudes of the costs are more at high CV level demand pattern, the results are more obvious in this case. By comparing the tables 34, 35 and 36, it can be said that there are clear differences among the cost figures of MMFE and other two methods. MMFE has the least total costs at 8 of the 10 problems.