GENERALIZED RANDOM SPREADING PERFORMANCE ANALYSIS OF CDMA OVER GWSSUS FADING CHANNELS

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Özgür Ertuğ

ABSTRACT

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Since direct-sequence code-division multiple-access (DS-CDMA) is an interference-limited random multiple-access scheme, the reduction of co-channel interference with either interference suppression or interference cancellation multiuser receivers and/or power control to prevent detrimental near-far situations is vital for improved performance. Up to date, some contributions investigated randomly-spread asymptotically - large number of users and large bandwidth - large CDMA systems with multiuser receivers and power control via random matrix theoretic and free probability theoretic tools especially over Gaussian and single-path fading channels. As complement within this thesis, we analyze also within the generalized random spreading framework but at finite system sizes and without power control the capacity achievable with linear multichannel multiuser receivers; i.e. RAKE, zero-forcing decorrelator, linear minimum mean-squared error (LMMSE) multiuser receivers, within a single-cell setting over generalized time-varying GWSSUS - Rayleigh/Ricean - fading channels via random matrix theoretic tools. Assuming maximal-ratio combining (MRC) of resolved frequency - multipath - diversity channels due to wideband transmission, the signal-to-interference ratios (SIRs) with multichannel multiuser receivers that set the basis for further derivations are statistically characterized. The information-theoretic ergodic and outage sum-rates spectral efficiencies are then derived and analyzed.

Keywords: CDMA, multiuser detection, multiuser information theory, GWSSUS fading channels, performance analysis, random matrix theory.

ÖZ

ÇWSSUS SÖNÜMLEMELİ CDMA KANALLARININ GENELLEŞTİRİLMİŞ İSTATİSTİKİ-YAYMALI PERFORMANS ANALİZİ

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Direkt-dizin kod-bölmeli çoklu-erişim (DS-CDMA) enterferans-limitli istatistiki bir çoklu-erişim sistemi olduğundan dolayı zararlı uzak-yakın etkisinin giderilerek daha iyi performans elde edilebilmesi için aynı kanal enterferansının ya enterferans bastırım/çıkartma almaçları ile ve/veya güç kontrolü kullanımı ile giderilmesi son derece önemlidir. Bugüne kadar, bazı araştırmalar istatistiki-yaymalı asimptotik - çok sayıda kullanıcı ve geniş bant - CDMA sistemlerini çok-kullanıcı almaçları ve güç kontrolü ile Gaussian ve tek-yollu sönümlemeli kanallarda analizini yapmışlardır. Bu tezde eşlik eder şekilde, yine genelleştirilmiş istatistiki yayma çatısı altında ama sonlu sistem boyutları için ve güç kontrolü olmadan multikanal lineer çok-kullanıcılı almaçların, i.e. RAKE, dekorelatör, LMMSE, erişim kapasitesini tek-hücre formasyonu içinde genelleştirilmiş zamanla-değişken GWSSUS - Rayleigh/Ricean - sönümlemeli kanallar üzerinde istatistiki matris teorisi tabanlı araçlar ile incelenmektedir. Genişband gönderime bağlı olarak çözünen çok-gönderim yolu kanallarının maksimal-katsayı birleştirimini baz alarak, öncelikle çok-kullanıcılı multikanal almaçlarının sinyal-enterferans (SIR) oranları istatistiki olarak karakterize edilmektedir. Daha sonra, enformasyon-teorisi tabanlı ergodik ve yetmezlik toplam-oranlar spektral kazançları bulunmakta ve analiz edilmektedir.

Anahtar Kelimeler: CDMA, çok-kullanıcılı işaret belirleme, çok-kullanıcılı enformasyon teorisi, GWSSUS sönümlemeli kanallar, performans analizi, istatistiki matris teorisi.

Dedicated to My Family and Nurcan

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PREFACE

In this thesis, we focus on generalized random spreading performance analysis of code-division multiple-access systems with linear multichannel multiuser receivers over time-varying GWSSUS fading channels. Analysis through random spreading sequences both serves as an accurate model for large systems in which long signature sequences that span many symbol periods are used to breakdown the cyclostationarity and average out the multiple-access interference as well as provides a comparison basis for finite-dimensional short spreading sequence systems. In terms of spreading sequence sets, random spreading sequences, especially Gaussian, also well model the randomization behavior of tapped-delay line multipath channels on the transmitted signature sequences and they further provide average performances achievable over the spreading sequence sets with various correlation properties.

Many of the previous works on the random spreading performance analysis of CDMA systems are mainly constrained to the analysis over Gaussian or single-path fading channels. In this retrospect, the main contribution of this dissertation is the *random spreading performance analysis of CDMA over multipath fading channels at finite system sizes.* This is done when using linear multichannel multiuser receivers and maximal-ratio diversity combining (MRC) of output statistics with perfect knowledge of channel state information (CSI) and hence the work presented herein widely differs from that presented by Evans and Tse in [139] due to imperfect channel estimation considered.

Information-theoretic sum-rates ergodic spectral efficiencies and total outage spectral efficiencies are derived and analyzed in order. It is shown that the baseline correlated-waveform vector multiple-access channel model has ergodic capacity/spectral efficiency equivalent to the non-fading AWGN bound channel capacity for sufficiently fast fading scenarios as codeword length tends to infinity that validates Kennedy Law [149]/Telatar-Tse conjecture [150] in multiuser scenario. It is further mainly presented in terms of outage spectral efficiencies for slowly-fading coding delay limited situations that they converge to the ergodic capacities as the codeword length tends to infinity.

LIST OF SYMBOLS AND ACRONYMS

K	Number of users
k	User index
$\mathcal{K}$	User set
n	Signalling interval index
t	Time
T_s	Symbol period
A_k	Amplitude of kth user
$s_k(n)$	Information symbol of kth user
$c_k(t)$	Spreading signal of kth user
τ_k	Delay of kth user
L	Spreading factor
c_{kl}	lth chip of kth user
$v(t)$	Chip pulse-shaping filter
T_c	Chip period
$r(t)$	Received signal
l	Chip index
$n(t)$	Stochastic noise signal
N_0	White noise power spectral density level
$E_s^{(k)}$	Symbol energy of kth user
$\text{SNR}_s^{(k)}$	Symbol signal to noise ratio of kth user
σ_n^2	Noise variance
y_k	Sufficient statistic of kth user
k'	Interfering user index
$\rho_{kk'}$	Crosscorrelation coefficient between users kth and k'th
$A_{k'}$	Amplitude of k'th user
$s_{k'}(n)$	Information symbol of k'th user
w_k	Noise sample of k'th user
ρ_{kk}	Autocorrelation coefficient of kth user
$\mathbf{y}$	Stacked sufficient statistics vector for K users
$\mathbf{s}$	Stacked information symbols for K users
$\mathbf{A}$	$K \times K$ diagonal amplitudes matrix

$\mathbf{S}$	Spreading sequences matrix
$\mathbf{R}$	Crosscorrelation matrix
$\mathbf{r}$	Received signal vector
$\mathbf{w}$	Noise samples vector
$\in$	Error event
$P_k(\in)$	Probability of error for kth user
$Q(.)$	Complementary Gaussian error function
$\mathbf{z}$	Processed sufficient statistics vector
$sgn(.)$	Signum function
$\hat{b}_k(n)$	Estimate of kth user's nth bit
$\mathbf{M}^{(MUD)}$	Estimator function of multiuser receiver
$\mathbf{I}$	Identity matrix
$\mathrm{S}IR_k^{(MUD)}$	Signal-to-interference ratio of kth user with multiuser receiver
$\mathbf{c}_k$	kth user's spreading sequence
$\sum_k$	Covariance matrix of multiple access interference plus noise of kth user
$\mathbf{F}$	Upper Cholesky decomposition of $\mathbf{R}$
$\mathbf{y}_w$	Whitened sufficient statistics vector
$\mathbf{w}_w$	Whitened noise samples vector
$s(t)$	Bandpass signal
$s_l(t)$	Complex lowpass equivalent of bandpass signal
$\theta_n(t)$	Time-varying phase of nth path
f_c	Carrier frequency
$\tau_n(t)$	Time-varying delay of nth path
$\alpha_n(t)$	Time-varying gain of nth path
$h(t;\tau)$	Time-varying channel impulse response
$h_n(t)$	Time-varying channel impulse response of nth path
$(\Delta t)_c$	Coherence time
$B_d^{(n)}$	Doppler spread of nth path
V_m	Velocity of mobile user
λ_c	Wavelength
c	Speed of light
θ_n	Direction of arrival of nth path

$(\Delta f)_c$	Coherence bandwidth
T_m	Delay spread
S_f	Spread factor
σ_c^2	Variance of channel coefficients
D	Diversity order resolved
s^2	Non-centrality parameter
μ_i	Mean of channel coefficients
$I_\alpha(x)$	αth order modified Bessel function of first kind
$\Gamma_c(n)$	complete Gamma function
$Q_m(a;b)$	Marcum's Q function
$(x)_k$	Pochammer symbol for x
Γ	A subset of users
Γ^c	Complement of a subset of users
R_k	Rate of kth user
R_Γ	Rate of subset of users Γ
$\mathcal{X}_\mathcal{K}$	Input alphabet for set of users $\mathcal{K}$
$p(y\|x_k)$	Transition probability
$\mathcal{Y}$	Output alphabet
$\mathcal{M}_k$	Input symbols set
M_k	Number of input symbols
X_k	Encoding function
N	Codeword length
$P_{\in\|m_\mathcal{K}}$	Conditional probability of error
$\bar{P}_\in$	Average probability of error
$p(x_k)$	Input distribution
$I(;\|)$	Conditional mutual information
C_{sum}	Sum-rates capacity
$\Cap$	Convex hull (cover) operator
$\mathcal{E}.$	Expectation operator
P_Γ	Total powers of subset of users Γ
$\mathbf{H}$	Channel matrix
$\cup$	Union operator

$F_{\mathbf{A}}^{(o)}$	Cumulative distribution function of ordered eigenvalues of matrix A
$F_{\mathbf{A}}^{(\pi)}$	Cumulative distribution function of unordered eigenvalues of matrix A
$\lambda_{\mathbf{A}}$	Eigenvalue counting or averaged trace random variable of matrix A
$f_{\mathbf{A}}(\lambda_{\mathbf{A}})$	Density of eigenvalue counting random variable of matrix A
β	Load factor
λ_{MP}	Eigenvalue counting random variable of a Marhenko-Pastur matrix
T_{cs}	Channel symbol signalling period
α	Normalized delay spread
$\mathrm{S}NR_b$	Bit signal-to-noise ratio
h_{kd}	Channel coefficient of kth user for dth path
$\mathbf{h}_k$	Channel coefficient vector of kth user
$\mathbf{R}_{kk}$	kth diagonal submatrix of crosscorrelation matrix R
$\mathbf{R}_{kk'}$	kk'th offdiagonal submatrix of crosscorrelation matrix R
$\mathbf{w}_k^{(MUD)}$	Noise samples vector for kth user with multiuser receiver
(Ω, ξ, P)	Borel probability space
σ_{WG}^2	Common variance for Wigner distribution
λ_{WG}	Eigenvalue counting random variable of a Wigner matrix
$f_{WG}(\lambda_{WG})$	Density of eigenvalue counting random variable of a Wigner matrix
$\varphi_k(x)$	Hermite functions
$H_k(x)$	Hermite polynomials
$\mathbf{\Psi}_{\mathbf{H}}$	Channel power profile matrix
$\mathbf{W}$	Wishart matrix
λ_{WH}	Eigenvalue counting random variable of a Wishart matrix
$f_{WH}(\lambda_{WH})$	Density of eigenvalue counting random variable of a Wishart matrix
$\psi_k^{\alpha}(x)$	Laguerre functions
$L_k^{\alpha}(x)$	Laguerre polynomials
$\bar{\eta}_{sum}$	Total ergodic spectral efficiency
$\bar{\eta}_{\mathrm{u}b}$	Upper-bound total ergodic spectral efficiency
η_{sum}	Total spectral efficiency
$(\mathrm{S}NR_b)_{\mathrm{m}in}$	Minimum bit signal-to-noise ratio for reliable communications
$C_{sum}^{\mathrm{S}hannon}$	Shannon-sense sum-rates capacity
$I_{\max}^{sum}(n)$	Maximum conditional sum-rates mutual-information

$C^{out}_{sum}(N)$	Sum-rates outage capacity for codeword length N
R_{out}	Outage rate
P_{out}	Probability of outage
$\eta^{out}_{sum}(N)$	Sum-rates outage capacity random variable for codeword length N
$\eta^{*}_{out,sum}$	Outage spectral efficiency
FEC	Forward error correction
ARQ	Automatic repeat request
DS-CDMA	Direct-sequence code-division multiple-access
FDMA	Frequency-division multiple-access
TDMA	Time-division multiple-access
MAI	Multiple access interference
MUD	Multiuser detection
BS	Base station
SNR	Signal-to-noise ratio
GWSSUS	Gaussian wide sense stationary uncorrelated scatterer
RF	Radio frequency
PDMA	Polarization-division multiple-access
SDMA	Space-division multiple-access
VBR	Variable bit rate
LPI	Low probability of intercept
FH-CDMA	Frequency-hopping code-division multiple-access
TH-CDMA	Time-hopping code-division multiple-access
CH-CDMA	Code-hopping code-division multiple-access
MC-CDMA	Multicarrier code-division multiple-access
3GPP	Third generation partnership project
CT	Continous time
AWGN	Additive white Gaussian noise
PSD	Power spectral density
DT	Discrete time
BER	Bit error rate
SER	Symbol error rate
AME	Asymptotic multiuser efficiency

NFE	Near-far effectiveness
NFR	Near-far ratio
BPSK	Binary phase shift keying
CMF	Conventional matched filter receiver
MLSE	Maximum-likelihood sequence estimator
LMMSE	Linear minimum mean-squared error receiver
RAKE	RAKE receiver
MS	Multistage receiver
DecDF	Decorrelating decision-feedback receiver
DFE	Decision-feedback equalizer
WMF-DFE	Whitened matched-filter decision-feedback equalizer
ICI	Interchip interference
ISI	Intersymbol interference
CIR	Channel impulse response
WSS	Wide sense stationary
HF	High frequency
VHF	Very high frequency
SHF	Super high frequency
LOS	Line of sight
SIR	Signal to interference ratio
MEA	Multielement antenna array
MRC	Maximal-ratio combining
EGC	Equal-gain combining
SC	Selection combining
MAC	Multiple access channel
BAC	Binary adder channel
BMC	Binary multiplier channel
GMAC	Gaussian multiple access channel
UDE	Uniquely decodable
MIMO	Multiple-input multiple-output
ML	Maximum-likelihood
CW-VMAC	Correlated-waveform vector multiple-access channel

SGRM	Self-adjoint Gaussian random matrix

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Chapter 1

Introduction

The most beautiful thing
we can experience is the mysterious;
it is the source of all true art and all science...
Albert Einstein

1.1 Motivation

In the era of information, the efficient transmission, utilization and storage of information became an exceedingly important engineering task due to gradually increasing volumes and kinds of advanced communication and information processing technologies that turned into mass markets and became a significant part of people's lives. Fired up with Bell's phone and Marconi's telegraph, today communication systems offer all forms of fully-digital multipoint delivery of multimedia information such as file transfer, facsimile, e-mail, video conferencing and wireless voice/data that require high data rates with low delay and error rate.

In particular, the widespread deployment of wireless and mobile personal communication systems within the last three decades and the demand for increased capacity and quality especially for multimedia service support in the next generation wireless systems present communication engineers new challenges to meet these needs. This challenging nature of wireless and mobile communications stems from the facts that wireless mobile radio frequency (RF) channels are in general very harsh due to time-varying multipath fading, the bandwidth is very scarce and valuable, and the mobile handheld terminals are power-limited due to fixed battery sizes. Thus, given the set of resources such as bandwidth and power as well as channel characteristics and complexity constraints, it is vital to design systems with higher spectral and power efficiency which in turn results in higher information/user capacity and better error rate and delay performance in terms of quality experienced by all active users. This goal can be achieved via various transmission/reception strategies such as by efficient source coding/compression algorithms to remove redundancies in the information, by designing and employing powerful forward error correction (FEC) and automatic repeat request (ARQ) algorithms to remove transmission errors, by efficient cell-splitting, resource allocation, call admission and soft handovers, by utilizing inherent dimensions of diversity for statistical signal enhancement via combining and by designing reliable multiple-access techniques and efficient transceivers for them.

Direct-sequence code-division multiple-access (DS-CDMA) systems have especially been a focused area in multiuser communications due to the intrinsic advantageous properties such as large time-bandwidth product allowing wideband transmission for each active user at all times and robustness to signal fading effects resulting in enhanced capacity and quality with respect to the narrowband multiple-access schemes such as FDMA and TDMA. Besides its widespread deployment initially in the military/tactical networks for security purposes and in the 2nd generation commercial wireless communication systems such as IS-95 [IS95], CDMA is also proposed as the air-interface in the upcoming 3rd/4th generation personal communication system standards [Prasad1998] for high-capacity wideband wireless communication.

Since CDMA is an interference-limited random multiple-access technique unlike FDMA and TDMA based on scheduled single-user transmission within assigned frequency or time slots respectively, it is vitally important to enhance the desired signal components and to reduce the multiple-access interference (MAI) seen by each user due to the other users simultaneously accessing the channel to enhance reception performance. The main strategy for removal of MAI components is coined the term *multiuser detection* (MUD) and the receivers producing interference-reduced sufficient decision statistics at least up to the soft-output stage via multiuser demodulation are in general called *multiuser receivers.*

On the other hand, *diversity techniques* for efficiently combining desired signal replicas received in time, frequency and/or space domains for statistical signal enhancement are especially vital for enhanced reception performance over *fading channels* where the complex-envelope of the transmitted uplink signals from all users randomly fluctuates in time-varying manner and frequently goes into deep amplitude drops due to destructive signal combinations at the base-station (BS) resulting in significant effective received signal-to-noise ratio (SNR) losses and hence degraded reception performance. Various forms of diversity and combining techniques can also be used in CDMA systems by designing multichannel *diversity-reception multiuser receivers* to further enhance reception performance.

The topic of this thesis is the generalized performance analysis of linear multichannel diversity-reception multiuser receivers over GWSSUS multipath fading DS-CDMA channels via random spreading analysis techniques based on *random matrix theory.* Up to date, some contributions investigated randomly-spread asymptotic - large number of users and large bandwidth - CDMA systems with multiuser receivers and power control via random matrix theoretic and free probability theoretic tools especially over Gaussian and single-path fading channels. As complement within this thesis, we analyze also within the generalized random spreading framework but at finite system sizes and without power control the capacity achievable with linear multichannel multiuser receivers; i.e. RAKE, zero-forcing decorrelator, linear minimum mean-squared error (LMMSE) multiuser receivers, within a single-cell setting over generalized time-varying GWSSUS - Rayleigh/Ricean - fading channels via random matrix theoretic tools. As the introduction, multiple-access techniques in general and CDMA systems in specific are comparatively discussed in Section 1.2. The objectives of the thesis are then described in Section 1.3.

1.2 General Multiple-Access Techniques and CDMA

Multiple-access in communication systems refers in general to the techniques for efficient multiplexing/demultiplexing of digital information streams originating from multiple sources and destined to multiple destinations over a channel/transmission medium with constrained resources in the general multipoint-to-multipoint communications system setting. Since all users share simultaneously the resources of the common multiple-access channel within the same transmission medium such as the RF channels for wireless and mobile communications, users have to be reliably separated to access the resources of the channel with minimal interference.

The possible dimensions for user separation include time, frequency and space domains. Other more complicated alternatives include polarization-division multiple-access (PDMA) where users are separated via electromagnetic waves of different polarizations and space-division multiple-access (SDMA) where users at different locations are separated by sectorized multiple-antennas at BS directed towards various parts of each cell.

Based on signal-level separation, frequency-division multiple-access (FDMA), time-division multiple-access (TDMA) and code-division multiple-access are the three basic ones with possibility of hybrid usage [Wittman1967] and all these multiple-access strategies are competing techniques, each aiming at distributing the transmitted signal energy per user access to the channel within the constrained time-frequency plane resource available.

In FDMA, the time-frequency plane is divided into discrete frequency subchannels. Each user transmits signal energy in any of these frequency bands at all times and reception is obtained by simple bandpass filtering. FDMA as known to be the oldest multiple-access technique is suitable for both analog and digital transmission, and is employed commercially in the first generation analog wireless cellular standard AMPS in North America. Though simple in terms of implementation, FDMA has certain disadvantages such as high overhead control signalling requirement for slot assignment, inability to support variable-bit-rate (VBR) services, high frequency reuse factor requirements to reduce intercell interference and high sensitivity to carrier frequency/phase errors that are the byproducts of the baseband downconversion process at the receiver side RF-mixer local oscillators.

With the advance of digital modulation techniques, TDMA has gained further popularity for deployment. As time-dual to FDMA, time-frequency plane is divided into time slots of certain length in TDMA and each user transmits signal energy within any of these time slots using all the bandwidth of the channel available. First commercial deployment of TDMA appears in the second generation digital cellular standard IS-54 [IS54]. Despite its ability to support VBR services and lower frequency reuse factor than FDMA, TDMA like FDMA also has similar disadvantages of scheduling overhead and high sensitivity to carrier synchronization errors.

The development of spread spectrum single-user transmission techniques at first for low-probability of intercept (LPI) military communications has later led the way to the multiple-access based on code-division (CDMA) via multiuser spread spectrum signalling. A general overview of spread spectrum signalling can be found in the papers by Scholtz [Scholtz1977] , Milstein et al. [Milstein1982] and in books by Levitt et al. [Levitt1994], Dixon [Dixon1994], Ziemer et al. [Ziemer1995], Viterbi [Viterbi1995], Vucetic et al. [Vucetic1997] and Lee et al. [Lee1998].

In CDMA, unlike scheduled single-user narrowband multiple-access like FDMA and TDMA, each user transmits signal energy over the entire time-frequency plane by spreading their signal onto a much larger bandwidth than that of the actual information-bearing signal via uniquely assigned pseudo-random spreading codes. The spreading codes are composed of a certain number of symbols, called the *chips* and the number of chips per symbol is in general called the *spread factor*. The spread signals look like white noise in the frequency domain for an observer without the knowledge of the spreading codes and this form of CDMA is in general called direct-sequence code-division multiple-access (DS-CDMA).

Depending on the use of spreading codes and underlying spectral partitioning, CDMA can also be implemented in other ways such as frequency-hopping (FH-CDMA), time-hopping (TH-CDMA), code-hopping (CH-CDMA) [Viterbi1995] and multicarrier (MC-CDMA) [Hara1996]. On the other hand, depending on the length of the spreading codes, CDMA systems can also be classified as short-code and long-code systems. In short-code systems, spreading codes are of length equal to the symbol period and repeat at every symbol, thus MAI components are cyclostationary with symbol period. On the other hand, long-codes with periods of several symbol signalling intervals are in general used to break down the cyclostationarity and to induce "more randomness" into the signal to average out the MAI components such as in IS-95.

As elaborated deeper in Chapter 2, where the general system model and preliminaries will be presented, the existence of MAI in CDMA due to the correlation between spreading codes of users is in general not inherent to the spread spectrum transmission technique, but arises due to the system constraints and imperfections when using spread spectrum transmission for multiple-access. In fact, for a small set of users, it is possible to use a completely orthogonal set of spreading codes; i.e. Walsh-Hadamard codes, such that all users experience no MAI and this format is called *orthogonal CDMA* which is equivalent to FDMA and TDMA based on single-user transmission. However, for a large wideband CDMA system with high number of users and large processing gain, it is very hard to find sets of orthogonal codes. Furthermore, even if users transmit their signals in a completely chip and symbol synchronous manner, nonorthogonality between codes is almost always induced due to multipath propagation which is an impairment inherent to many radio frequency channels. Hence, this asynchronism induced nonorthogonality results in MAI to be experienced by all active users and degrades reception performance. However, due to the advantages as will be mentioned shortly, nonorthogonal CDMA is more flexible and robust with respect to orthogonal CDMA and we concentrate on nonorthogonal CDMA for uplink transmission in this thesis.

The performance degradation of nonorthogonal CDMA for uplink transmission due to interference-limitation becomes more significant if the received power levels of users at the BS are very dissimilar. In this case, the signals of users with weak received powers at the BS are almost completely swept out by the MAI from the signals of users with strong received powers at the BS, and this is the so called *near-far problem*. Three main and competing strategies for achieving near-far resistance in nonorthogonal CDMA systems are to use highly near-far resistant multiuser receivers with interference reduction capabilities, to use power control algorithms to control the received powers of users at the BS to have equal received powers and to design specific spreading sequence sets with desired high autocorrelation/low crosscorrelation properties to reduce the MAI seen by users [Levitt1994,Dixon1994,Ziemer1995,Viterbi1995]. However, power control algorithms if applied as open-loop, are not reliable especially over fading channels. On the other hand, if applied as closed-loop, power control algorithms require too much overhead feedback data from users frequently that reduces effective data rates achievable. Furthermore, even if crosscorrelation of spreading sequence sets are minimized by spreading sequence design, the nonzero levels of crosscorrelation values almost always indicate near-far problem if highly near-far resistant multiuser receivers are not further used. Hence, it is almost agreed in CDMA research and development community that for uplink multiple-access via nonorthogonal CDMA, some form of multiuser detection or hybrids with power control and spreading sequence design is the choice.

Up to date, efficiency of CDMA for multiple-access is widely questioned due to its interference-limited nature and the claim on the higher spectral efficiency of CDMA with respect to FDMA and TDMA strenghtened by information-theoretic capacity-maximization arguments due to white noise like input signals still remains controversial. Furthermore, it has recently been claimed successively by Medard and Gallager in [Medard2002] and Subramanian and Hajek in [Hajek2002] that too large bandwidths are actually not exploited well by code-division. However, CDMA has in general well-known agreed advantages with respect to narrowband FDMA and TDMA due to wideband transmission such as being the best format for VBR services, soft degradation in error rate performance for higher number of users to be supported, theoretical frequency reuse factor of unity for macrocellular systems, offering diversity to counteract fading, no overhead for slot scheduling due to random access and relatively less sensitivity to carrier synchronization errors. Furthermore, besides these advantages, CDMA systems with higher spectral and power efficiency are further possible by employing several transmission/reception strategies such as multiuser receivers with interference reduction capability, exploiting diversity in different dimensions and introducing error-control coding.

As commercial deployments of CDMA, IS-95 standard [IS95] in North America is the first example with a relatively narrow bandwidth of approximately 1.25 MHZ with respect to currently developing systems. Today for the development of upcoming 3rd and 4th generation wireless and mobile personal communication systems with VBR multimedia services provision, various forms of CDMA with large bandwidths of around 5 MHz and supported data rates up to about 1 Mbits/s are also proposed and standardized as the key air-interface technology such as in standards of CDMAOne and CDMA2000 in North America [Garg2000], UTRA and FRAMES in Europe [Ojanpera1996] and W-CDMA in Japan [Fisher1996]. A general overview of these standards under the worldwide consortium umbrella of 3rd Generation Partnership Project (3GPP) can be found in [Toskala2001] and all of these 3G/4G wideband CDMA standards include various forms of multiuser receivers for interference reduction in conjunction with utilization of diversity in several dimensions.

1.3 Thesis Objectives

The general goal of this thesis is to analyze in a generalized fashion the performance trade-offs and the fundamental limits/bounds on the capacity achievable with linear multichannel diversity-reception multiuser receivers over GWSSUS fading CDMA channels. Up to date, some contributions investigated randomly-spread asymptotic - large number of users and large bandwidth - CDMA systems with multiuser receivers and power control via random matrix theoretic and free probability theoretic tools; however these contributions were mainly limited to the Gaussian and single-path fading channels in the asymptotic scenario.

As supplement to the contributions of the thesis, multiuser detection and related performance measures as well as fading channel models and diversity/combining techniques are presented in Chapter 2. Multiuser communication theory is further briefly overviewed next in Chapter 3.

The objectives can be classified in two parts. First, via the theory of eigenvalue distribution of random matrices, a new approach for generalized random spreading performance analysis of DS-CDMA systems with linear multichannel diversity-reception multiuser receivers over GWSSUS fading channels is presented in Section 4.4. This approach assumes finite-size systems without power-control and is complementary to the *asymptotic limiting theory* based random spreading analysis methodology which is used up to date in the literature for generalized performance analysis of CDMA systems with power control in the large system limit as number of users and spread factor grow without bound but with a constant ratio. Achievable SIRs with linear multichannel diversity-reception multiuser receivers are then examined based on the presented finite-dimensional random spreading analysis methodology. Next within this analysis framework, the ergodic and outage sum-rates spectral efficiencies are derived and analyzed respectively in Sections 4.5 and 4.6. Finally, summary of contributions as well as further work is presented in Chapter 5.

Chapter 2

General System Model and Preliminaries

The general system model, mathematical introduction to the main topics and some preliminaries required for the analysis in the subsequent chapters are presented in this chapter. In Section 2.1, multiuser detection and related performance measures with emphasis on synchronous Gaussian CDMA channels are presented. The statistical fading channel models and their classification and characterization that are used throughout the thesis are elaborated in Section 2.2. Furthermore, a brief overview of diversity and combining techniques with applications in CDMA is given in Section 2.3.

2.1 Multiuser Detection and Related Performance Measures

Multiuser detection is in general the term coined for the efficient detection of digital information streams from multiple sources in the presence of multiple-access interference. Cellular telephony, satellite communication, high-capacity coaxial cable networks, digital broadcasting, and multitrack magnetic recording are some kinds of the communication systems subject to multiaccess interference. In some cases, the mutual coupling that causes the interference between transmitted signals may arise due to the nonidealities of the transmission media such as crosstalk in twisted-pair lines, while in the case of CDMA, multiaccess interference is an integral part of the multiplexing scheme. Multiuser detection receivers exploit the underlying structure of the multiuser interference in order to reduce its effects and to increase the spectral and power efficiency in the use of the CDMA channel for multiplexing. A broad introduction to the topic of multiuser detection with the historical development can be found in the book by Verdú [Verdu1998].

The basic block diagram of an uplink CDMA system with K users randomly-located in a single cell is depicted in Fig. 2.1. The user $k \in \mathcal{K}$, where $\mathcal{K} = \{1, 2, ..., K\}$ is the all-users set, transmits in the nth symbol interval of $t \in [(n-1)T_s, nT_s)$ the complex signal

$$s_k(t) = A_k s_k(n) c_k(t - \tau_k) \tag{2.1}$$

where T_s is the symbol period, $s_k(n)$ is the transmitted complex information symbol of kth user, A_k is the amplitude of kth user, τ_k is the delay of kth user's signal and $c_k(t)$ is the signature waveform of kth user. In uncoded transmission, the information symbols $s_k(n)$ can be symbols from any linear/nonlinear modulation alphabet set, while in coded transmission, they correspond to the letters of the codewords. Furthermore, *synchronous transmission* will be assumed throughout the thesis with equal delays for each user normalized to zero; i.e. $\tau_1 = \tau_2 = ... = \tau_K = 0$.

The signature waveforms of the users can be both real and complex depending on the use of real/complex spreading sequences. We will assume complex spreading sequences throughout that are energy-normalized such that $\int_0^{T_s} |c_k(t)|^2 \, dt = 1$. In a DS-CDMA system, the signature waveforms are in the following form:

$$c_k(t) = \sum_{l=0}^{L-1} c_{kl} v(t - lT_c) \tag{2.2}$$

where c_{kl} is the lth chip of kth user, T_c is the chip interval, $L = T_s/T_c$ is the processing gain and $v(t)$ is the unit-energy Nyquist chip pulse-shaping waveform. Then, the complex envelope of the continous-time (CT) received composite signal perturbed by additive white Gaussian noise (AWGN) after downconversion can be expressed in an arbitrary symbol period n as

$$r(t) = \sum_{k=1}^{K} A_k s_k(n) \sum_{l=0}^{L-1} c_{kl} v(t - nT_s - lT_c) + n(t), t \in [(n-1)T_s, nT_s) \tag{2.3}$$

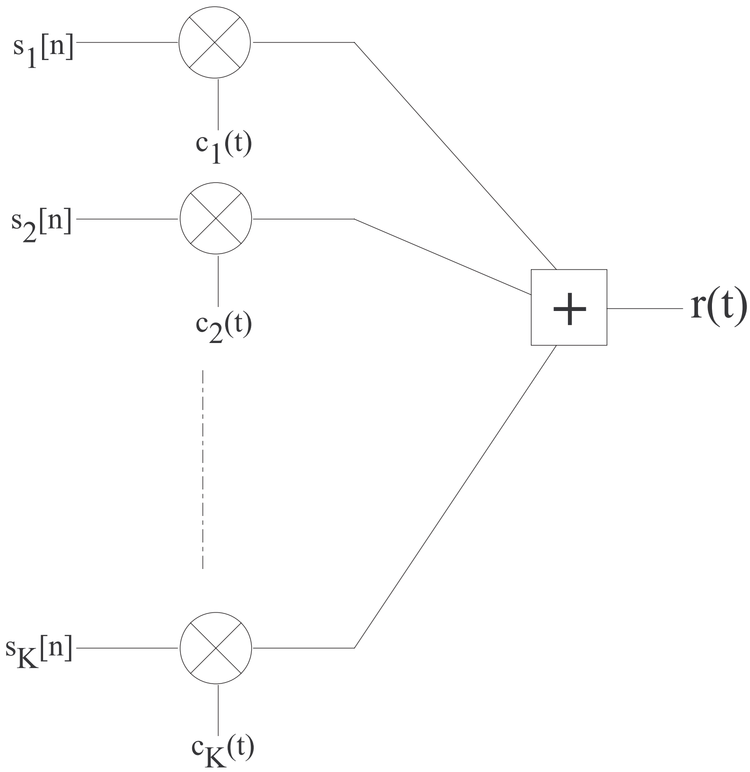

Figure 2.1: Basic block-diagram of an uplink CDMA system.

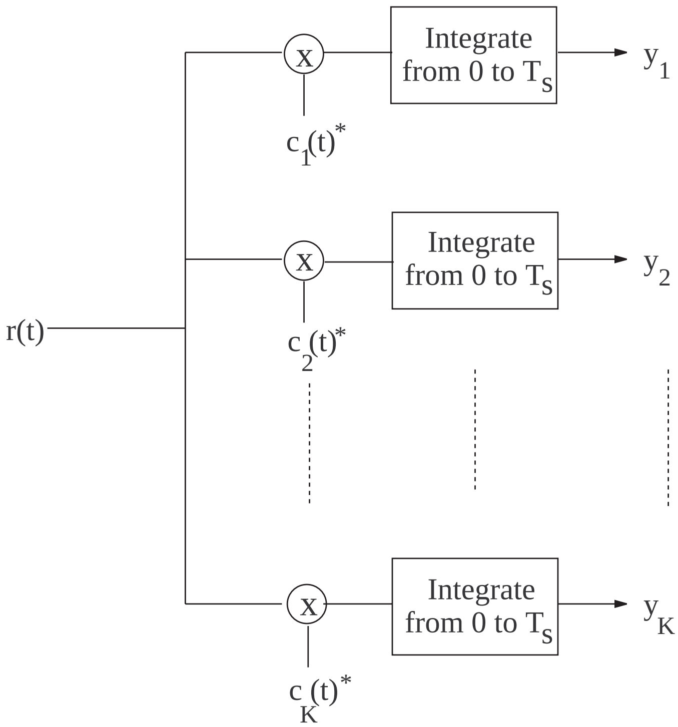

Figure 2.2: Sufficient decision statistics obtained via front-end chip/code matched-filter banks.

where $n(t)$ is the ergodic and stationary zero-mean circularly-symmetric complex AWGN process with baseband power spectral density (PSD) level of N_0 watts/Hz. Hence, in this scenario of single-rate transmission that we will keep throughout the thesis, the common symbol rate of all users are given by $R_s = 1/T_s$ symbols/sec and the system bandwidth after spreading is given by $W_s = 1/T_c = L/T_s$ Hz. In addition, further assuming that the noise PSD level is normalized to unity in watts/Hz; i.e. $N_0 = 1$, without loss of generality, and denoting the symbol energy of kth user by $E_s^{(k)}$, the symbol signal-to-noise ratio $SNR_s^{(k)}$ of kth user - as usual with respect to unity resistance in ohms - can be defined as

$$SNR_s^{(k)} = \frac{A_k^2(\text{watts})}{\sigma_n^2(\text{watts})} = \frac{E_s^{(k)}(\text{joules})}{N_0(\text{watts/Hz})} \tag{2.4}$$

where $\sigma_n^2 = N_0 \times W_s$ is the noise variance in watts.

Based on the received composite signal formulation in (2.3), the optimum single-user maximum-SNR front-end demodulation filtering for producing the sufficient decision statistics of each user corresponds to *matched-filtering* or *correlation* with the signature waveforms of each user (see Fig. 2.2) as

$$y_k = \int_0^{T_s} r(t) c_k^*(t) dt \tag{2.5}$$

where y_k is the sufficient decision statistic of kth user and $(.)^*$ is the complex-conjugation operator. The correlation operation in (2.5) that is mathematically equivalent to matched-filtering can be obtained by successive chip-matched and code-matched filtering with chip and symbol rate sampling respectively that is sufficient for optimality of one-shot detection. The sufficient decision statistic y_k of kth user can then be expanded as

$$y_k = A_k s_k + \sum_{\acute{k}=1, \acute{k} \neq k}^{K} \rho_{k\acute{k}} A_{\acute{k}} s_{\acute{k}} + w_k \tag{2.6}$$

where the first component in the summation is the desired signal of kth user, second term is the MAI component and the third term is the noise component. Here, $\rho_{k\acute{k}} = \int_0^{T_s} c_{\acute{k}}(t) c_k^*(t) dt$, $0 \leq \rho_{k\acute{k}} \leq 1$ and $\rho_{kk} = 1, \forall k, \acute{k}$, is the normalized crosscorrelation between the signature waveforms of kth user and the interfering users, and the background noise component $w_k = \int_0^{T_s} n(t) c_k^*(t) dt$ is a zero-mean circularly-symmetric complex-Gaussian random variable (rv) with variance $\sigma_{w_k}^2 = N_0 \times W_s$. Furthermore, after the matched-filtering operation, the noise components of sufficient statistics over all users become correlated, and the normalized crosscorrelation value between the noise components of kth and $\acute{k}th$ users is equal to $\rho_{w_k w_{\acute{k}}} = \rho_{k\acute{k}}$.

Via the discrete-time (DT) sufficient statistics formulation for each user after front-end matched filtering banks in (2.6), a DT equivalent of the synchronous Gaussian CDMA channel can be compactly written in the following matrix-vector notation as

$$\mathbf{y} = \mathbf{R}\mathbf{A}\mathbf{s} + \mathbf{w} \tag{2.7}$$

where $\mathbf{y} = [y_1 \ y_2 ... y_K]^T$ with $(.)^T$ being the transpose operator is the $K \times 1$ stacked sufficient statistics vector, $\mathbf{s} = [s_1 \ s_2 ... s_K]^T$ is the $K \times 1$ symbols vector of all users and $\mathbf{A} = \text{diag}\{A_1, A_2, ..., A_K\}$ is the $K \times K$ diagonal matrix of amplitudes. In addition, if we define the spreading sequences matrix as $\mathbf{S} = [\mathbf{c}_1 |\ \mathbf{c}_2 |\ ... |\ \mathbf{c}_K]$ where the kth column of $\mathbf{S}$ is the spreading sequence of kth user, $\mathbf{R} = \mathbf{S}^H \mathbf{S}$ with $(.)^H$ being the conjugate-transpose (Hermitian) operator is the normalized crosscorrelation matrix, $[\mathbf{R}_{k\acute{k}}] = \rho_{k\acute{k}}$. In the case of real spreading sequences, the crosscorrelation matrix $\mathbf{R}$ is symmetric; i.e. $\mathbf{R} = \mathbf{R}^T$, and in the case of complex spreading sequences, $\mathbf{R}$ is complex-symmetric/Hermitian; i.e. $\mathbf{R} = \mathbf{R}^H$, meaning that the off-diagonal elements of $\mathbf{R}$ obeys the complex-conjugate symmetry condition: $\rho_{k\acute{k}} = \rho^*_{\acute{k}k}, \forall k, \acute{k}$. Furthermore, $\mathbf{w} = [w_1\ w_2 ... w_K]^T$ is the zero-mean circularly-symmetric complex-Gaussian noise vector with covariance matrix $\mathbf{K_w} = \sigma^2_{w_k} \mathbf{R} = N_0 \times W_s \mathbf{R}$.

The performance of multiuser receivers can be measured via several metrics that are naturally interrelated on the power-bandwidth plane. The main user-level performance measure for a multiuser receiver is the bit error rate (BER) or symbol error rate (SER) experienced by each user. Furthermore, since the common BER experienced by equal-rate/equal-energy users in a CDMA system is a function of the number of active users K in the system due to the soft-degradation property, *user capacity* that is defined to be the number of users supportable at a certain BER/SER is another important system-level performance measure for a multiuser receiver.

On the other hand for coded transmission, besides coded BER/SER, the Shannon-sense total information capacity/spectral efficiency for large codeword block lengths achievable by multiuser receivers is further important to decipher the performance trade-offs on the power-bandwidth plane and give strong indications on the data rates achievable under coded transmission with well-designed finite-length codes.

In terms of uncoded error rates, the power efficiency performance can also be quantified via other simpler measures well suited for multiaccess communication such as *asymptotic multiuser efficiency* (AME) and *near-far effectiveness* (NFE)

[Verdu1998]. Though slightly different notions can be employed, AME via the definition of Verdú for a certain user as a function of *near-far ratio* (NFR); i.e. $\text{NFR}_k = \frac{SNR_s^{(j)}}{SNR_s^{(k)}}$, measures the slope of the error rate curves at high-SNR region as the noise variance tends to zero and quantifies the asymptotic power loss experienced by the user due to the existence of MAI with respect to the single-user channel. For instance, for coherent BPSK modulation over AWGN channel, AME for kth user is defined as

$$\text{AME}_k = \sup_{0 \leq r \leq 1} \lim_{\sigma_n^2 \to 0} \left\{ \frac{P_k(\in)}{Q\left(\frac{rE_s^{(k)}}{\sigma_n^2}\right)} \right\} < \infty \tag{2.8}$$

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt$ is the standard Gaussian complementary error function and $P_k(\in)$ is the BER of kth user with the considered multiuser receiver. Consequently, near-far effectiveness of the particular multiuser receiver is the AME of kth user for worst-case interfering energy combinations, that is in mathematical terms:

$$\text{NFE}_k = \inf_{\left\{E_s^{(l)}\right\}_{l=1}^{K}, l \neq k} \text{AME}_k \tag{2.9}$$

and this particular multiuser receiver is then said to be near-far resistant if $\text{NFE}_k > 0, \forall k$.

The simplest multiuser receiver, the *conventional matched-filter* multiuser receiver (CMF), detects users based on the sufficient statistics provided by the front-end matched-filter banks. For example with antipodal BPSK modulation, the bit estimate of kth user is simply obtained by taking the sign of the sufficient statistic of kth user as the result of the binary hypothesis test assuming MAI only contributes to the enhancement of effective background noise level: $\hat{b}_k = \text{sgn}\{y_k\}$. Though optimal in terms of SNR maximization and hence reception performance under single-user transmission, since conventional receiver does not take into account the structure of the MAI and employs no interference reduction, it is far from being optimal in the multiuser scenario and users experience high error rates especially under near-far situations due to the high-level of MAI existing in the sufficient statistics. Under Gaussian approximation for total MAI plus noise components and the use of corresponding effective root-mean-squared power that is a major methodology for performance analysis with multiuser receivers [21-23] which we will also employ throughout the thesis, the approximate probability of bit error for synchronous BPSK-CDMA under AWGN with conventional matched-filter multiuser receiver for equal-rate users is given by Eq. 3.93 of [Verdu1998]:

$$P_k^{(\mathrm{CMF})}(\in) \simeq Q\left(\frac{A_k}{\sqrt{\sum_{j\neq k} A_j^2 \rho_{jk}^2 + \sigma_n^2}}\right) \tag{2.10}$$

Motivated with the fact that conventional receiver is not near-far resistant, various forms of receivers employing multiuser detection are formulated based on different optimality criteria that reduces the MAI in the sufficient statistics provided by the front-end matched-filter banks.

Multiuser detection research for CDMA is in general agreed to be triggered by the formulation and analysis of the jointly-optimum multiuser receiver by Verdú in [Verdu1986] based on the maximum-likelihood sequence estimation (MLSE) that achieves minimum error rate and optimum near-far resistance for all users. Though Verdú's formulation is for the asynchronous Gaussian channels in [Verdu1986], the jointly-optimum MLSE multiuser receiver (JOPT-MLSE) in the scenario of synchronous Gaussian CDMA channel maximizes the log-likelihood function over a data burst of N symbols or under Gaussian noise assumption, equivalently minimizes the composite distance between the sufficient statistics vector and $2^{K\times N}$ likely transmitted sequences in the case of binary modulation, which can be expressed as

$$\{\hat{\mathbf{b}}(1), \hat{\mathbf{b}}(2), ..., \hat{\mathbf{b}}(N)\} = \max_{\{\mathbf{b}(1),\mathbf{b}(2),...,\mathbf{b}(N)\}} \sum_{n=1}^{N} |\mathbf{y}(n) - \mathbf{b}(n)|^2 \tag{2.11}$$

Though this exhaustive search can be dropped to the complexity of 2^K independent of the data size by dynamic programming type fixed-decoding depth trellis search such as Viterbi Algorithm, this is still a very huge complexity for large systems with high number of users and prohibits the real-time use of the jointly-optimum minimum error rate receivers in CDMA systems. Although some contributions in the literature also dealt with complexity reduction of the optimum receiver by iterative methods to compute the log-likelihood functions such as expectation-maximization [Fawer1995, Raphaeli2000], motivated with this high complexity problem of the optimum receiver, other forms of nearly-optimal linear interference suppression filtering multiuser receivers and nonlinear interference cancellation multiuser receivers with linear time-complexity per bit in the number of users are developed.

The well-known linear-interference suppression filtering multiuser receivers include the zero-forcing decorrelator and the linear minimum mean-squared error (LMMSE) receivers. Application of linear transformation based multiuser detection based on single-user optimization criteria is initiated by Schneider in [Schneider1979] where decorrelating detector based on zero-forcing criterion is presented. The following formulations of the decorrelator and LMMSE multiuser receivers in the literature are done by Lupas and Verdú in [Verdu1989] and Xie et al. in [Xie1990]. The analysis of the near-far resistance of decorrelator and LMMSE is presented by Lupas and Verdú in [Lupas1990] and a rigorous probability of error rate analysis of LMMSE multiuser receiver is given by Poor and Verdú in [Poor1997].

The decorrelating and LMMSE multiuser receivers are the multichannel analogues of the well-known zero-forcing and LMMSE equalizers for single-user ISI channels and they both aim at reducing the MAI in the sufficient statistics by applying front-end joint estimator filtering matrices on the sufficient statistics vector as

$$\mathbf{z}^{(\mathrm{MUD})} = \mathbf{M}^{(\mathrm{MUD})}\mathbf{y} \tag{2.12}$$

where $\mathbf{M}^{(\mathrm{MUD})}$ is the joint linear estimator matrix of the corresponding linear multiuser receiver. In fact, the conventional matched-filter multiuser receiver may also be put in this category of linear multiuser receivers with the joint estimator matrix as $\mathbf{M}^{(\mathrm{CMF})} = \mathbf{I}_{K\times K}$, hence having no interference suppression capability.

The zero-forcing criterion for the decorrelating receiver (Dec) for the complete inversion of the multi-access CDMA channel simply results in the estimator matrix that is the inverse of the crosscorrelation matrix: $\mathbf{M}^{(\mathrm{Dec})} = \mathbf{R}_{K\times K}^{-1}$. Hence, if the signature sequences of all users are mutually linearly-independent so that the crosscorrelation matrix is nonsingular and accurately invertible, the decorrelating receiver results in complete suppression of MAI components by projecting the signal of the desired user onto the subspace orthogonal to the space spanned by the interfering signature waveforms, however enhances background noise power similar to single-user zero-forcing equalization [Verdu1998][1]. Besides the advantage of having no need for estimation of received SNRs, the biggest advantage of the decorrelating receiver is that it can readily be decentralized due to the decorrelating FIR filters for each user being simply the corresponding row of the inverse crosscorrelation matrix: $z_k^{(\mathrm{Dec})} = \sum_{j=1}^{K} \left[\mathbf{R}^{-1}\right]_{kj} y_j$; i.e. $\mathbf{m}_k = \left\{\left[\mathbf{R}^{-1}\right]_{kj}\right\}_{j=1}^{K}$. Furthermore, for example for BPSK modulation, the exact bit error probability of desired user is given by:

$$P_k^{(\mathrm{Dec})}(\in) = Q\left(\frac{A_k}{\sqrt{\sigma_n^2\left[\mathbf{R}^{-1}\right]_{kk}}}\right) \tag{2.13}$$

that is irrespective of the interfering users'

amplitudes due to perfect MAI suppression, and the near-far effectiveness of the decorrelating detector is hence given by:

$$\mathrm{NFE}_k^{(\mathrm{Dec})} = \frac{1}{\left[\mathbf{R}^{-1}\right]_{kk}} \tag{2.14}$$

that is equivalent to the maximum near-far effectiveness achievable with jointly-optimum MLSE multiuser receiver [Verdu1998]. Hence, decorrelating receiver provides an optimally near-far resistant solution for multiuser demodulation at only linear time-complexity per bit. However, it is also a well-known fact that due to the noise enhancement, decorrelating receiver has poorer error rate performance with respect to conventional matched-filter multiuser receiver at low SNRs.

Decorrelating multiuser receiver is the reliable solution for optimally near-far resistant linear multiuser detection when the received SNRs are unknown.

[1] In fact, even if the crosscorrelation matrices are close to singular or badly conditioned, decorrelating multiuser detection is still possible via Moore-Penrose generalized inverse (Proposition 5.1 of [Verdu1998]).

However, if the received amplitudes of all users are known reliably, then LMMSE multiuser receiver based on Wiener type minimum mean-squared error criterion formulation can be derived based on knowledge of the received amplitudes and LMMSE maximizes the signal-to-interference ratio (SIR) achievable among all linear multiuser receivers. The joint linear front-end estimator filtering matrix of the LMMSE receiver is the solution of the following optimization problem:

$$\hat{\mathbf{M}}^{(\mathrm{LMMSE})} = \min_{\mathbf{M}^{(\mathrm{LMMSE})} \in \mathcal{C}^{K \times K}} E\left\{ \left\| \mathbf{s} - \mathbf{M}^{(\mathrm{LMMSE})}\mathbf{y} \right\|^2 \right\} \tag{2.15}$$

that is K-dimensional uncoupled minimization problems to minimize the mean-squared error between the data and the output of the linear transformation on the sufficient statistics. The optimum linear estimator matrix of the LMMSE multiuser receiver based on (2.15) has the neat analytical form [Verdu1998]:

$$\hat{\mathbf{M}}^{(\mathrm{LMMSE})} = \left(\mathbf{R} + \sigma_n^2 \mathbf{A}^{-2}\right)^{-1} \tag{2.16}$$

and for decentralized detection, each user's LMMSE FIR filter is simply the corresponding row of the joint estimator matrix; i.e. $\hat{\mathbf{m}}_k = \left\{ \left[\left(\mathbf{R} + \sigma_n^2 \mathbf{A}^{-2}\right)^{-1} \right]_{kj} \right\}_{j=1}^{K}$. Due to the received SNRs in the joint estimator matrix form, unlike decorrelating receiver, there is no concern on the linear-independence of the signature sequences of users and it is simple observation from the form of the joint estimator matrix of LMMSE that decorrelating receiver aiming at complete MAI suppression is the limiting case of LMMSE at high SNR as $A_k^2/\sigma_n^2 \to \infty$ and conventional matched-filter multiuser receiver designed only to combat background noise is the limiting case of LMMSE at low SNR as $A_k^2/\sigma_n^2 \to 0$. Thus, LMMSE receiver presents the inbetween optimal trade-off for MAI suppression and background noise enhancement.

The achievable SIR with LMMSE receiver is the maximum output SIR with any linear transformation and is given by [Verdu1998]:

$$\mathrm{SIR}_k^{(\mathrm{LMMSE})} = A_k^2 \mathbf{c}_k^H \boldsymbol{\Sigma}_k^{-1} \mathbf{c}_k \tag{2.17}$$

where $\boldsymbol{\Sigma}_k$ is the covariance matrix of the interference plus noise on the transformed sufficient statistics of kth user that has the following form:

$$\mathbf{\Sigma}_k = \sigma_n^2 \mathbf{I} + \sum_{j=1, j\neq k}^{K} A_j^2 \mathbf{c}_j \mathbf{c}_j^H \tag{2.18}$$

The performance analysis of the LMMSE multiuser receiver even for BPSK modulation is not as straightforward as that of decorrelating detector due to residual MAI components in the transformed sufficient statistics, the sum of which is a binomial random variable, however Gaussian approximations can also be further used to derive approximate solutions for the error probability of the LMMSE receiver. Based on Gaussian approximations for the total MAI plus noise components, the error rate performance of LMMSE is slightly better than that of decorrelating receiver at high SNRs and it does not seem reasonable to add higher complexity to the receiver for the estimation of received SNRs for such a slight performance improvement. However, the main advantage of LMMSE receiver is its ease for adaptive implementation in a decentralized fashion in cases of no knowledge on the signature sequences of the interfering users [Verdu1998]. A collection of papers on the design and analysis of adaptive and blind adaptive linear MMSE multiuser receivers can be found in [Madhow1994, Miller1996, Rapajic1994, Honig1995]. Furthermore, in terms of near-far resistance, the near-far effectiveness of LMMSE is equal to the optimal decorrelator near-far effectiveness:

$$\mathrm{NFE}_k^{(\mathrm{LMMSE})} = \mathrm{NFE}_k^{(\mathrm{Dec})} = \frac{1}{[\mathbf{R}^{-1}]_{kk}} \tag{2.19}$$

since LMMSE converges to decorrelator as received SNRs tend to infinity.

Besides linear interference suppression multiuser receivers, some contributions in the literature also considered nonlinear interference cancellation multiuser receivers that use internal tentative decisions to estimate and cancel the interference from the sufficient statistics. Such techniques are based on the successive decoding - also generally known as "onion peeling" - idea which originally dates back to Shannon. Since such schemes' performance strictly depends on the reliability of internal tentative decisions, they are particularly suitable for high SNR channels and the decoding order of users in such schemes is particularly important for better reception performance especially under high near-far situations since erroneous intermediate decisions affect the reliability of all successive decisions.

Just like linear multiuser receivers, the well-known nonlinear interference cancellation multiuser receivers, the multistage receiver (MS) proposed by Varanasi and Aazhang [Varanasi1990, Varanasi1991] and the decorrelating decision-feedback (DecDF) multiuser receiver proposed by Duel-Hallen [Duel-Hallen1995], are also the multichannel analogues of the conventional decision-feedback (DFE) and whitened matched-filter decision-feedback (WMF-DFE) equalizers for single-user ISI channels respectively. Following these contributions on decision-directed multiuser receivers, various other forms of interference cancellation multiuser receivers are successively designed and analyzed by Bar-Ness in [Bar-ness1999], Shi. et al in [Shi1996], Host-Madsen and Cho in [Cho1999] and Guo et al. in [Guo1999]. Furthermore, analysis of the asymptotic multiuser efficiencies of various decision-directed multiuser receivers are presented by Zhang and Brady in [Brady1998].

The principle of the multistage receiver of Varanasi and Aazhang in [Varanasi1990, Varanasi1991] is based on the use of internal tentative decisions provided by a first conventional matched-filter multiuser receiver stage to reproduce and cancel the interference on the sufficient statistics of all users. Final decisions are made upon a second matched-filter multiuser receiver stage. The number of cancellation stages can be increased infinitely, however it is also reported in [Varanasi1990, Varanasi1991] that the performance improvement achievable by increasing the number of cancellation stages beyond 3 is very slight and thus not judicious to incur the extra complexity.

On the other hand, the decorrelating decision-feedback multiuser receiver of Duel-Hallen [Duel-Hallen1995], is based on the whitening of the discrete-time CDMA channel. Since in the general case of complex signature sequences and positive-definite crosscorrelation matrices with non-negative eigenvalues, the crosscorrelation matrices can be lower-upper decomposed by Cholesky Decomposition [Lutkepohl1996, Golub1983] after received power ordering as

$$\mathbf{R} = \mathbf{F}^H \mathbf{F} \tag{2.20}$$

where $\mathbf{F}$ is lower-triangular, the postprocessing of the sufficient statistics with $\mathbf{F}^{-H}$ as

$$\mathbf{y}_w = \mathbf{F}^{-H}\mathbf{y} = \mathbf{F}\mathbf{A}\mathbf{s} + \mathbf{w}_w \tag{2.21}$$

results in the decorrelation of the noise components and the discrete-time equivalent whitened CDMA channel model due to the whitened zero-mean transformed noise vector having covariance matrix: $E\left\{\mathbf{w}_w\mathbf{w}_w^H\right\} = \sigma_n^2\mathbf{I}$. Based on this whitened equivalent model in (2.21), the decorrelating decision-feedback receiver then decodes the users in the order descending received powers for the efficiency of decision-feedback detection. The main advantage of the decorrelating decision-feedback multiuser receiver is that since the matrix $\mathbf{F}$ is lower-triangular, the kth strongest user has MAI components in the transformed decision statistics only from the first $k-1$ stronger users. Thus, the strongest user has no MAI and experiences a theoretical error rate that is equivalent to that achievable with decorrelator. On the other hand, the weakest user has MAI components from all stronger $K-1$ users and under the ideal condition of no internal error propagation, the weakest user experiences single-user performance. Hence, decorrelating decision-feedback multiuser receiver in the high SNR regime is a significant solution especially under high near-far situations. However, this is only a theoretical expectation at very high SNRs and the error propagation dominates the performance of the decorrelating decision-feedback receiver at low/medium SNRs. Hence, the performance experienced by weak users may widely deviate from the single-user performance.

2.2 Fading Channel Models and Statistical Characterization

2.2.1 Introduction

One of the major impairments for wireless communications over bandlimited RF channels is that the signals transmitted over such channels are subject to *amplitude fading* during uplink transmission to BS that occurs in many cases over multiple propagation paths due to random scatterers in the transmission media.

Such multipath propagation of the signals due to random scatterers in the transmission medium may result in the destructive combination of the replicas of the same signal received at BS and hence results in the fade of the magnitude of the complex received signal envelope. In addition, due to the mobility of users, fading channels are in general time-varying in nature. Such time-varying fading, if not compensated by counterfading measures, may result in deep received power drops and the users may experience poor reception performances and possibly for long periods.

Propagation through mobile and wireless RF fading channels is in general a very complex process and is random in nature due to the random scatterers in the transmission media and the mobile users continously moving at certain speeds within the cells of the network. Due to such inherent randomness, fading channels require statistical characterizations and hence, up to date, many works are devoted to the accurate statistical characterization of fading channels depending on the particular propagation environment and the communication system scenarios.

The main aim of this section is to briefly present the principal classification and characterization of fading channels. Wider treatments of the topic can also be found in the classic paper of Bello [Bello1963] and the book by Proakis [Proakis1995]. A seminal tutorial by Biglieri, Proakis and Shamai on both the modelling and communication/information-theoretic aspects of fading channels is [Proakis1998]. In the next subsection, we present the classification of fading channels based on the main characteristics. The statistical models for fading channels with bias on the ones we will be using throughout the thesis are undertaken in subsection 2.2.3.

2.2.2 Classification of Fading Channels

In general, a time-varying multipath fading channel is modelled to be composed of a certain number of propagation paths and each path has a certain propagation delay and attenuation factor that are time-varying. Thus, when a modulated bandpass signal $s(t) = \mathrm{Re}\left\{s_l(t)e^{j2\pi f_c t}\right\}$ is transmitted over such a channel at carrier frequency f_c, then the complex low-pass equivalent of the received bandpass signal after convolution with the channel impulse response (CIR) can be expressed in the following form as

$$r_l(t) = \sum_n \alpha_n(t)e^{-j\theta_n(t)}s_l(t - \tau_n(t)) \tag{2.22}$$

where $\alpha_n(t)$ is the real time-varying attenuation factor of the nth path, $\theta_n(t) = 2\pi f_c \tau_n(t)$ is the time-varying phase of the nth path in radians, and $\tau_n(t)$ is the time-varying propagation delay of the nth path. Thus, defining the complex stochastic fading process on the nth path as $h_n(t) = \alpha_n(t)e^{-j\theta_n(t)}$, the time-varying complex low-pass equivalent of the bandpass multipath fading channel can be written as

$$h(t;\tau) = \sum_n h_n(t)\delta(t - \tau_n(t)) \tag{2.23}$$

Since the form of the CIR of the time-varying multipath fading channel in (2.23) is a sum of randomly time-varying phasors, it is in general common to model the CIR $h(t;\tau)$ as a complex-Gaussian stochastic process when the number of paths is sufficiently large due to Central Limit Theorem. The fading phenomenon is also the product of the destructive combinations of these randomly time-varying phasors and when this happens, the resultant received signal $r_l(t)$ is attenuated in amplitude significantly that is detrimental for reliable communications.

Through the generally used assumption of wide-sense stationary (WSS) stochastic processes, the main physical characteristics of fading channels are mainly defined by the correlation and power spectra functions of their CIR that are Fourier transform pairs. With complex-Gaussian fading processes, if the WSS assumption are complemented with the assumption of uncorrelated scattering, that is the fading processes for each path are uncorrelated, then this commonly used model is in general referred as independent *Gaussian WSS uncorrelated scatterer* (GWSSUS) model.

The classification of the fading channels based on their temporal rate of change is significant in terms of both channel modelling and the design/analysis of communication systems over these channels. The rate of change of time-variations on a fading channel is defined by the *coherence time* in seconds denoted by $(\Delta t)_c$ of the channel, that is the minimum of the coherence times over all paths. Furthermore, due to Fourier duality, coherence time of a path is the inverse of the *Doppler spread* denoted in Hz of the particular path given by:

$$B_d^{(n)} = \frac{V_m}{\lambda_c}\cos(\theta_n), \tag{2.24}$$

where V_m is the mobile speed in m/sec, $\lambda_c = \frac{c_l}{f_c}$ is the carrier wavelength in meters for carrier frequency f_c, $c_l = 3 \times 10^8$ m/sec is the speed of light and θ_n is the direction-of-arrival (DOA) angle of the path in radians with respect to the BS normal. The Doppler spread of the channel denoted by B_d is then given by the maximum Doppler spread over all paths. The channel is then said to be *slowly-fading* if the coherence time of the channel is sufficiently larger than symbol period and otherwise, the channel is said to be *fast-fading*. In slowly-fading channel, a certain fade level affects many successive symbols resulting in burst errors. On the other hand, in fast-fading, fade level is independent from symbol to symbol.

Spectral selectivity is also another important property for classification of fading channels that is defined by both the physical properties of the CIRs and the type of signalling over these channels. Frequency-selectivity of a channel is defined based on the *coherence bandwidth* of the channel in Hz denoted by $(\Delta f)_c$, that is the inverse of the *multipath delay spread* of the channel denoted by T_m. Multipath delay spread is the maximum spread of the propagation delay of paths in temporal domain and also measures the range of frequencies over which the stochastic CIR process is correlated. Then based on the signalling structure, if the system is *narrowband,* i.e. the system bandwidth is much smaller than the coherence bandwidth of the channel, the channel is called to be *frequency-nonselective* or *frequency-flat*. On the other hand, if the system is wideband having much larger bandwidth than the coherence bandwidth of the channel, then different parts of the transmitted signal spectrum are subject to different fade levels. Hence, the channel is called to be *frequency-selective*.

Since signals transmitted over fading channels are spreaded in both time and frequency, the spread factor of a fading channel is also another important characteristic for classification of fading channels and the metric measuring the time-frequency dispersivity of a fading channel is given by the *spread factor* $S_f = T_m B_d$. If the spread factor of a fading channel is large so that the transmitted signals are largely spread in both time and frequency, then the channel is said to be *overspread*. Otherwise, it is called an *underspread* channel.

2.2.3 Statistical Characterization of Fading Channels

Up to date, many works in the literature have been devoted to the statistical characterization of the fading channels based on their envelope fluctuations that are also validated by experimental tests. Such statistical models for multipath fading channels are strictly dependent on the nature of radio propagation medium, and since multipath fading is relatively fast, such models account mainly for the short time-scale variations of the signals.

The four main statistical models for multipath fading channels that are frequently used in the literature are Rayleigh, Ricean, Log-normal and Nakagami-m models. Out of the later two, log-normal fading models are used to characterize both short and long term fading especially in areas with shadowing. On the other hand, Nakagami-m statistical fading model is a general variable model which incorporates both Rayleigh and Ricean statistical fading models as special cases. Since we are focused on complex-Gaussian fading processes and GWSSUS fading model - Rayleigh/Ricean models - throughout the thesis, let us now briefly overview this statistical fading model for multipath channels.

In the independent GWSSUS multipath fading model, when there are no fixed-scatterers or reflectors in the transmission medium, then the magnitude of the stochastic channel impulse response (CIR) process $|h(t;\tau)|$ is zero-mean. Hence, keeping the mean-squared value or the path-power of each path to unity; i.e. $E\left\{|h_n(t;\tau)|^2\right\} = 1$, the norm distribution of the stochastic CIR process obeys the Rayleigh distribution and the model is called the *Rayleigh fading model.* Rayleigh fading model agrees well with the experimental data for wireless and mobile systems where no line-of-sight (LOS) path exists between transmitter and receiver antennas [Stuber1996], as well as ionospheric HF/VHF/UHF communications [Wells1995], tropospheric milimeterwave SHF satellite communications [Sugar1955, Basu1987], and marine communications [TStaley1996, Staley1996, Staley1997] radio links.

On the other hand, again within the independent GWSSUS multipath fading model, if there exist fixed-scatterers and reflectors in addition to randomly moving users, the magnitude of the stochastic CIR process $|h(t;\tau)|$ is not zero-mean any more. Thus, the norm distribution of the stochastic CIR process obeys the Ricean distribution and the model is called the *Ricean fading model.* Ricean fading model typically models communication links with LOS paths such as urban/suburban land mobile [Sohrabi1995] and picocellular indoor [Mahmoud1989] radio links.

Keeping the unity path-power assumption for a total of D propagation paths, the analytic pdf of the norm of the circularly-symmetric stochastic CIR process under generalized independent GWSSUS fading model obeys the Ricean density of $2D$ degrees of freedom [Proakis1995] with individual component variances of $\frac{\sigma_c^2}{2}$ (Fig. 2.3), the analytic form of which is given by:

$$f_G^{(n)}(r) = \frac{2}{\sigma_c^2 s^{D-1}} r^D \exp\left(-\frac{r^2+s^2}{\sigma_c^2}\right) I_{D-1}\left(\frac{2rs}{\sigma_c^2}\right), \ r \geq 0 \qquad (2.25)$$

where $s^2 = D(1-\sigma_c^2)$ is the non-centrality parameter, μ_i and σ_c^2 are the mean and the variance of the ith path, respectively; i.e. $\mu_i = \sqrt{1-\sigma_c^2}$, $I_\alpha(x) = \sum_{k=0}^{\infty} \frac{(x/2)^{\alpha+2k}}{k!.\Gamma_c(\alpha+k+1)}$ is the αth-order modified Bessel function of the first kind and $\Gamma_c(n) = \int_0^\infty t^{n-1}\exp(-t)dt$ is the complete Gamma function. The corresponding cdf is also given by:

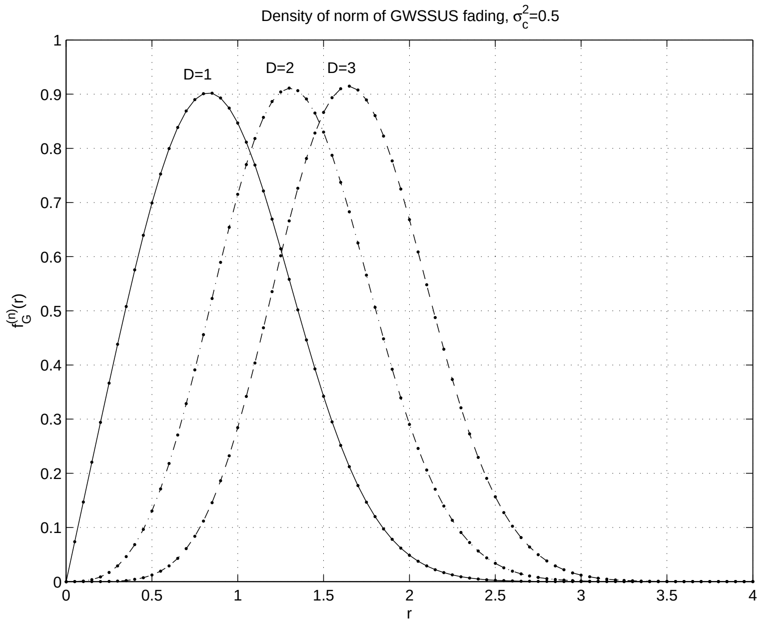

Figure 2.3: Pdf of the norm of the stochastic CIR processes under unity path-power D-path GWSSUS fading model for D=1,2,3 at σ_c^2 =0.5

$$F_G^{(n)}(r) = 1 - Q_m\left(\frac{\sqrt{2}s}{\sigma_c}, \frac{\sqrt{2}r}{\sigma_c}\right),\ r \geq 0 \tag{2.26}$$

where $Q_m(a,b) = Q_1(a,b) + \exp\left(\frac{a^2+b^2}{2}\right)\sum_{k=1}^{D-1}\left(\frac{b}{a}\right)^k I_k(ab)$, for $b > a > 0$, is the Marcum's Q-function [Simon1998] with $Q_1(a,b) = \exp\left(-\frac{a^2+b^2}{2}\right)\sum_{k=0}^{\infty}\left(\frac{a}{b}\right)^k I_k(ab)$. The central moments of the norm of the circularly-symmetric stochastic CIR process under unity path-power Ricean fading model is thus given by:

$$\begin{aligned} \mu_G^{(n)}[k] &= \mathcal{E}\left\{r^k\right\} = \left(2\sigma_c^2\right)^{\frac{k}{2}} \exp\left(-\frac{D(1-\sigma_c^2)}{2\sigma_c^2}\right)\frac{\Gamma_c\left(\frac{2D+k}{2}\right)}{\Gamma_c(D)} \\ &\times {}_1F_1\left(\frac{2D+k}{2}, D; \frac{D(1-\sigma_c^2)}{2\sigma_c^2}\right) \end{aligned} \tag{2.27}$$

for non-negative k, where ${}_1F_1(\alpha,\beta;x) = \sum_{k=0}^{\infty}\frac{(a)_k x^k}{(b)_k k!}$ is the Kummer confluent hypergeometric function and $(x)_k = \frac{\Gamma_c(x+k)}{\Gamma_c(x)}$ is the Pochammer symbol for value x.

Furthermore, the pdf of the norm-squared of the circularly-symmetric stochastic CIR process under unity path-power D-path GWSSUS fading model conse-

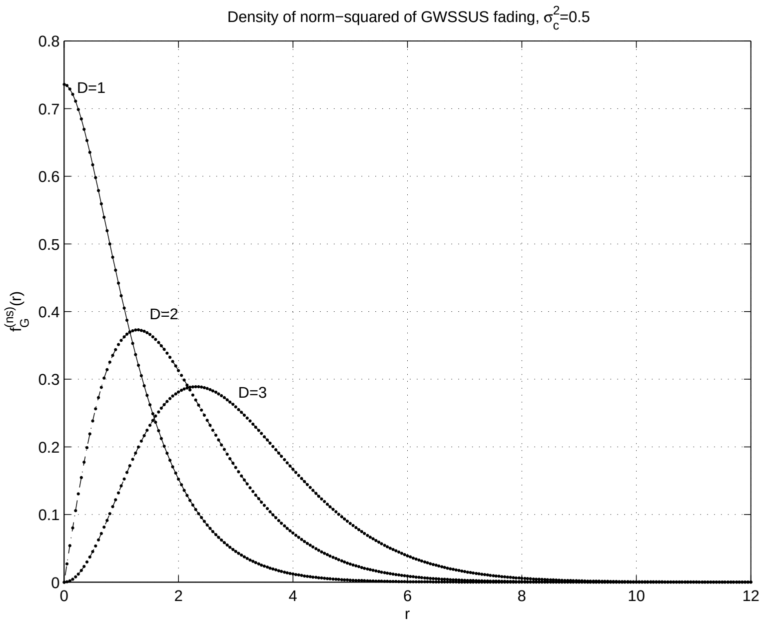

Figure 2.4: Pdf of the norm-squared of the stochastic CIR processes under unity path-power D-path GWSSUS fading model for D=1,2,3 at σ_c^2 =0.5

quently obeys the non-central chi-squared density [Proakis1995] of $2D$ degrees of freedom with individual component variances of $\frac{\sigma_c^2}{2}$; i.e. $\chi_{2D}^{2,nc}\left(\frac{\sigma_c^2}{2}\right)$, the analytic form of which is given by (Fig. 2.4.):

$$f_G^{(ns)}(r) = \frac{1}{\sigma_c^2}\left(\frac{r}{s^2}\right)^{\frac{D-1}{2}} \exp\left(-\frac{s^2+r}{\sigma_c^2}\right) I_{D-1}\left(\frac{2\sqrt{r}s}{\sigma_c^2}\right),\ r \geq 0 \tag{2.28}$$

and the corresponding cdf is given by:

$$F_G^{(ns)}(r) = 1 - Q_m\left(\frac{\sqrt{2}s}{\sigma_c}, \frac{\sqrt{2r}}{\sigma_c}\right),\ r \geq 0 \tag{2.29}$$

The mean and the variance of the norm-squared of the circularly-symmetric stochastic CIR processes under unity path-power D-path GWSSUS fading model is further given by:

$$\mu_G^{(ns)} = D,\ \sigma_G^{2,(ns)} = D(2\sigma_c^2 - \sigma_c^4) \tag{2.30}$$

These statistical characterizations for the norm and norm-squared distributions of the CIR random processes for GWSSUS multipath fading channels are vitally important for performance analysis over these channels and we will make use of them frequently throughout the thesis.

2.3 Diversity and Combining Techniques

Since fade of the received signals' amplitudes over fading channels results in degraded reception performances, diversity and combining techniques should be used for statistical signal enhancement as countermeasures for fading to achieve higher average received SNR/SIRs, and consequently better reception performances. In the context of CDMA systems design over fading channels, multiuser receivers with multichannel diversity-reception and combining can also be designed and used to enhance the capacity, quality and near-far resistance of the system.

The basis of statistical signal enhancement via diversity and combining techniques is the fact that transmission errors on a fading channel mainly occur when the amplitude attenuation due to channel is high, yielding very small received powers and hence, degraded reception performance. In such cases, the channel is then said to be in a deep fade. If appropriate transmission techniques in conjunction with the physical properties of the fading channels are used to provide the receivers with several replicas of the same information-bearing signal over independently fading channels in such deep fade situations, then the probability that all the signal components will fade simultaneously is reduced considerably by efficiently combining the received signal replicas. In mathematical terms, if the probability of a deep fade on one path is, say p, then the probability of D independently fading signal replicas will be in deep fade is $p^D << p$ due to independence. There are various ways for providing the receiver with independently fading many replicas of the information-bearing signals both via appropriate signalling/reception strategies and via increasing the physical degrees of freedom of the communications links.

One of the major methods to provide diversity is to send the same information-bearing signal on multiple carriers in frequency domain that are separated at least by the coherence bandwidth $(\Delta f)_c$ of the channel so that the signals on different carriers fade independently in the GWSSUS multipath fading model. The methodology within narrowband transmission is in general called the *frequency diversity*. A more elegant approach to obtain frequency diversity is to spread signals onto a much larger bandwidth than the coherence bandwidth of the fading channel such as in CDMA. In that manner, the receivers can resolve the multipath components, hence this type of broadband frequency diversity is in general also called *multipath diversity*. For a wideband signal with W_s Hz bandwidth, the number of resolvable multipath components is then given by $D = \lceil W_s/(\Delta f)_c \rceil$ where $\lceil . \rceil$ is the ceiling operator. The optimum receiver to resolve the multipath components by matched-filtering at the appropriate propagation delays is in general called the *RAKE matched-filter receiver* and is elaborated deeper in Chapter 4.

Based on appropriate signalling/reception strategies, another important methodology to obtain diversity is to transmit the information-bearing signal in different time slots that are at least separated by the coherence time $(\Delta t)_c$ of the channel for independence of fading on the signals in different time slots. Such methods are in general referred to as *time diversity* techniques. Besides transmission in multiple time slots, time diversity over fast-fading channels can also be obtained by appropriate signalling and reception during each symbol period since the fade levels affecting the transmission of a signal during a symbol interval are independent. Such a time diversity technique is in general called *Doppler diversity* since the rate of change of time variations of the channel is mainly identified by the Doppler spread of the channel that is the inverse of the coherence time.

Besides frequency and time diversity that can be resolved based on the signalling structure and the channel characteristics, diversity on wireless communication links can also be provided by increasing the physical degrees of freedom of the communication link such as the use of multiple antennas at the BSs and exploiting base station diversity in macrocellular systems. For efficient spatial diversity provision, the separation of successive antennas of the MEA should be sufficiently large such as at least $10\lambda_c$ for the signals received at different antenna elements of the array to fade independently. Furthermore, in macrocellular wireless and mobile communication systems, base station diversity can also be exploited by receiving the signals of users at more than one closeby BSs and efficiently combining them. Other than frequency, time and space diversity, there also exist other forms of diversity techniques such as angle-of-arrival diversity and polarization diversity that received attention in the literature, however the use of these techniques are not as wide as the frequency/multipath diversity that we mainly focus on throughout the thesis.

Combining techniques for the signals received over fading channels are also important in terms of the performance and complexity of diversity-reception systems. Focusing on linear combining techniques, the three major linear combining techniques that received high attention in the literature can be itemized as:

- Maximal-Ratio Combining (MRC)
- Equal-Gain Combining (EGC)
- Selection-Combining (SC)

and all these combining techniques represent a trade-off between performance and channel estimation complexity requirement. The performances of the ones that require channels to be known further strictly depend on the accuracy of channel estimation and their use should be judiciously decided based on the propagation environment, modulation types used, system operating point and the reliability of channel estimates.

Maximal-ratio combining (MRC) is the optimum linear diversity combining method that forms the sufficient decision statistic with the highest average SNR/SIR among all linear diversity combining schemes and requires both the magnitude and phase of all the complex channel gain coefficients for each path to be known reliably at the receiver. The basis of MRC is to form a weighted sum of matched-filter outputs by multiplying the matched-filter output for each path by the complex-conjugate of the corresponding channel coefficient as

$$y^{(\mathrm{MRC})} = \sum_{d=1}^{D} h_d^* y_d = \mathbf{h}^H \mathbf{y} \tag{2.31}$$

to form the final output sufficient decision statistic $y^{(\mathrm{MRC})}$, where $\mathbf{h} = [h_1\ h_2 ... h_D]^T$ is the vector of complex channel coefficients and $\mathbf{y} = [y_1\ y_2 ... y_D]^T$ is the vector of stacked matched-filter outputs of all paths. The reason for multiplying each path with the complex-conjugate of the corresponding channel coefficient for an optimal weighted sum is to compensate for the phase shift due to the channel gain and then to weight the signal by a factor that is proportional to the signal strength over that path. MRC as the optimal linear combining scheme can be used with almost all modulation types and over all propagation environments given that the complex channel coefficients are reliably available to the receivers. However, MRC is mainly used to combine signals with coherent modulation schemes since MRC is impractical for noncoherent or differentially-coherent modulations. This is due to the requirement of perfect knowledge on the channel gain phase estimates that is in general difficult to have with noncoherent and differentially coherent schemes. If the channel gain phase estimates are perfectly known, then coherent modulations with MRC can simply be used to achieve better reception performance. On the other hand, equal-gain combining (EGC) equally weights each path before combining, and therefore EGC does not require the estimation of complex channel fading coefficients. Though suboptimal, EGC is hence suitable for combining signals with noncoherent or differentially-coherent modulations. Furthermore, selection-combining (SC) technique only processes the path with highest SNR if such an information is available.

Chapter 3

Introduction to Multiaccess Communication Theory

In this dissertation, we focus on random spreading performance analysis in terms of capacity of code-division multiple-access systems with linear multichannel multiuser receivers over GWSSUS fading channels and our analysis heavily relies on the information theory of multiple-access channels. Thus, in this chapter, we both introduce the basic concepts in *multiuser information theory* with historical development as well as indicate the relationship between the research in this thesis and the current literature. In this retrospect, we will mainly present the results currently in the literature that let us further the research presented in this thesis in Chapter 4. Towards this goal, we will categorize our presentation in multiaccess research as:

- Multiuser information theory
- Multiaccess coding and joint-decoding
- Collision resolution

3.1 Multiuser Information Theory

Unlike a point-to-point communication system with a single source and a sink connected with a link established by a channel, a communication network in its most general view is a set of sources and sinks connected by channels within an arbitrary network topology. Though no complete and universal theory of such networks has been established yet, basic building blocks of such networks

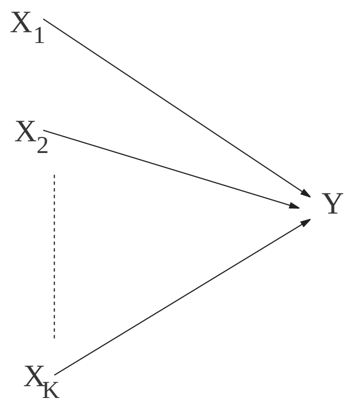

Figure 3.1: Multiple access channel.

such as multiple-access channels, broadcast channels, interference channels and relay channels have been firmly analyzed in the literature. Nice surveys of multiaccess information theory can be found in Gallager [Gallager1985], Van der Meulen [Meulen1977], Csiszár and Körner [Korner1981], Blahut [Blahut1987] and Cover and Gamal [Gamal1980]. Although our focus is mainly on spread-spectrum multiple-access channels, we now provide an overview of the results on analysis of these building blocks for background.

3.1.1 Multiple-Access Channels

Multiple-access channels (MAC) are multipoint-to-point channels where many sources try to send information to a single sink over channels such as in uplink mobile channels where each mobile user tries to send information to a single base-station or in uplink satellite communications where many ground stations try to send information to a single satellite. Multipoint-to-point multiple-access channels are schematically depicted in Fig. 3.1 where K sources $X_1, X_2, ..., X_K$ are transmitting their information to the common receiver and the received signal at the sink Y may be corrupted by AWGN as well as multiple-access interference.

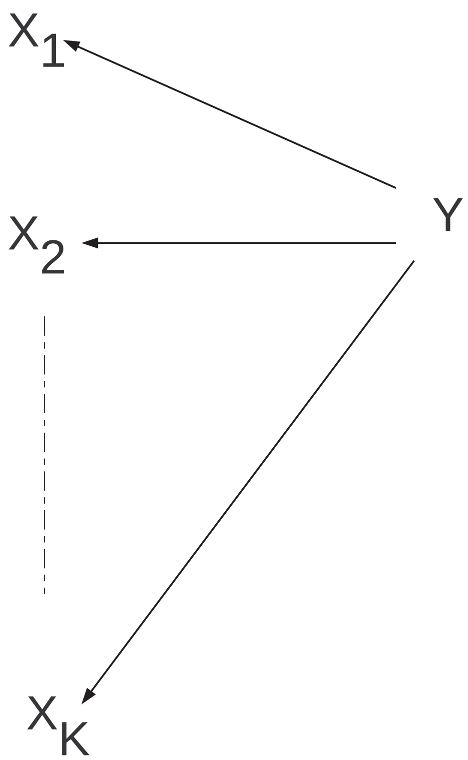

Figure 3.2: Broadcast channel.

3.1.2 Broadcast Channels

In broadcast channels, a single-transmitter sends information simultaneously to a set of users, such as in television broadcast from satellites or in the downlink of a wireless communications system for transmissions from base-station to mobile station (see Fig. 3.2).

3.1.3 Interference Channels

Interference channels are channel models in which two point-to-point links mutually interfere with each other. Fig. 3.3. depicts a degraded interference channel where Y_1 receives both X_1 and a degraded version of X_2 with a scale factor of a_{21} while Y_2 receives both X_2 and a degraded version of X_1 with a scale factor of a_{12}. Such transmission/reception scenarios can occur in communication systems such as wireless communications with cochannel interference as well as between

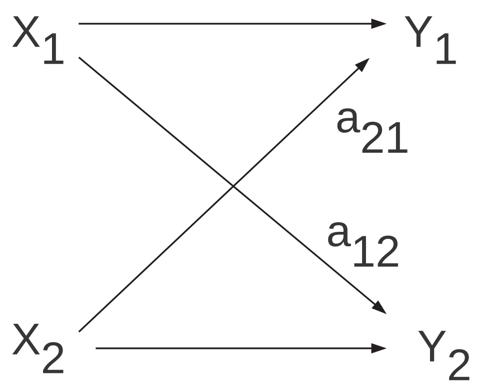

Figure 3.3: Interference channel.

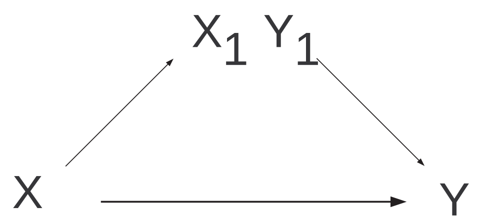

Figure 3.4: Relay channel.

different links of cable networks.

3.1.4 Relay Channels

The relay channels are in fact single-user point-to-point channels where additional links to the actual communication links exist (see Fig. 3.4.).

3.1.5 Capacity Region

Information theory mostly deals with the maximum coded information rate that can be transmitted over a channel with arbitrarily low probability of error that is in general called the *channel capacity* achievable by random-coding argument. By this rate, the rate below which there exists channel coding strategies such that by increasing codeword length, the probability of error can be made arbitrarily small is meant. Channel capacity via random-coding notion is first treated and explained in his seminal 1948 paper of Shannon [Shannon1948] for a large class of single-user channels.

The capacity of multiple-access channels is first determined by Ahlswede [Ahlswede1971] and Liao [Liao1972]. Let's first define a multiple-access channel:

Definition 3.1. Discrete Memoryless Multiple-Access Channel

Let $\mathcal{K} = \{1, 2, ..., K\}$ be the index set of K active users in the system and let $\Gamma \subseteq \mathcal{K}$ define a subset of users, whose complement is given by Γ^c. Let the information rate of user k be R_k and $R_\Gamma = \sum_{k \in \Gamma} R_k$. Then, a discrete memoryless multiple-access channel $(\mathcal{X}_k, p(y|x_k), \mathcal{Y})$ (DM-MAC) is defined by K input alphabets $\mathcal{X}_K$, a set of transition probabilities $p(y|x_k)$ and an output alphabet $\mathcal{Y}$.

For counteracting the noise and multiple-access interference as well as providing margin for fading, each user or source introduces error correction codes and a multiaccess code can be defined in the following way:

Definition 3.2. Multiaccess Codes and Joint Decoding

Let $\mathcal{M}_k = \{1, 2, ..., M_k\}$ be the set of symbols that user k may wish to transmit and $M_k = \lceil 2^{NR_k} \rceil$ where N is the codeword length, R_k is the information rate of user k in bits/symbol and symbol corresponds to a codeword letter. The encoding function of user k, X_k; maps each symbol in $\mathcal{M}_k$ to an N-dimensional codeword sequence drawn from the source alphabet as

$$X_k : \mathcal{M}_k \to \mathcal{X}_k^N \tag{3.1}$$

where the overall mapping over users is denoted via Cartesian product by:

$$X_\mathcal{K} : \mathcal{M}_\mathcal{K} \to \mathcal{X}_1 \times \mathcal{X}_2 \times ... \times \mathcal{X}_K \tag{3.2}$$

The information rate of user k is then given by $R_k = \frac{\log_2(M_k)}{N}$ in bits/symbol for codeword length N and the code is referred to as (N, R_k) multiaccess code. The decoding function for the multiaccess code is then defined as

$$Y : \mathcal{Y}^N \to \mathcal{M}_\mathcal{K} \tag{3.3}$$

Then, the error probability conditioned on a given set of messages $m_{\mathcal{K}}$transmitted is:

$$P_{\in|m_{\mathcal{K}}} = \Pr\{Y(\mathcal{Y}^N) \neq m_{\mathcal{K}}\} \tag{3.4}$$

and the average probability of error averaged over randomly-chosen codes for equiprobable messages is given by:

$$\bar{P}_{\in} = \frac{1}{2^{NR_k}} \sum_{m_{\mathcal{K}} \in \mathcal{M}_{\mathcal{K}}} P_{\in|m_{\mathcal{K}}} \tag{3.5}$$

Definition 3.3. Achievable Rate Region (Ahlswede [Ahlswede1971] and Liao [Liao1972])

A particular set of rates $R_{\mathcal{K}}$ is termed achievable if there exists a sequence of codes $(N, R_{\mathcal{K}})$ such that as codeword length $N \to \infty$, the average probability of error $\bar{P}_{\in} \to 0$. Achievable rate region for a multiple-access channel $(\mathcal{X}_k, p(y|x_k), \mathcal{Y})$ is a set of rate vectors $R_{\mathcal{K}}$ satisfying a particular product distribution $p(x_{\mathcal{K}}) = \prod_{k=1}^{K} p_k(x_k)$:

$$\sum_{k \in \Gamma} R_{\Gamma} \leq I(X_{\Gamma}; Y|X_{\Gamma^c}) \tag{3.6}$$

where $I(;|)$ is the conditional mutual information maximized over the input distribution X_{Γ} for subset of users Γ. Every point contained in this capacity region is achievable with some source distribution, while any point outside the capacity region is not achievable by any source distribution.

It is particularly interesting for us to determine the sum rates capacity in achievable region for K users:

$$C_{sum} = \max \sum_{k \in \mathcal{K}} R_k \tag{3.7}$$

that is the maximum rate at which users may transmit with vanishing error probability where each individual rate may in fact be unequal. Another important insight into the achievable rate region is also that the rate region is the convex hull (cover) of the union of all achievable rate regions over all the possible input distribution products:

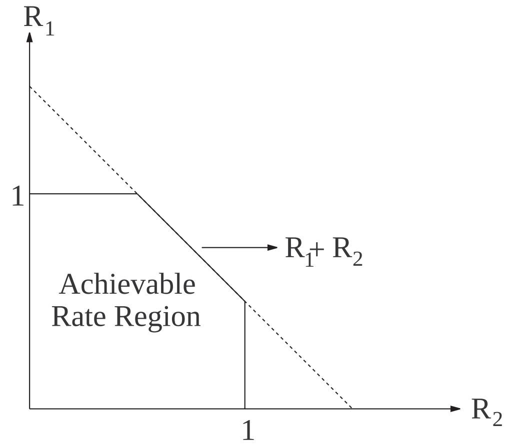

Figure 3.5: Capacity region of binary adder channel.

$$C(p(y|x_{\mathcal{K}})) = \Cap \left\{ \cup_{p(x_{\mathcal{K}})} R[p(x_{\mathcal{K}}), p(y|x_{\mathcal{K}})] \right\} \tag{3.8}$$

where $\Cap$ is the convex hull operator.

We now review some results on the achievable rate region of some multiple-access channels previously solved in the literature.

Example 3.1. Binary Adder Channel

In a binary adder channel (BAC), each user transmits using the alphabet $X_k \in \{0, 1\}$ and the output is $Y = \sum_k X_k \in \{0, 1, ..., K\}$. The capacity region of a two user BAC is given by:

$$\{(R_1, R_2) : 0 \leq R_1 \leq 1; 0 \leq R_2 \leq 1; 0 \leq R_1 + R_2 \leq 1.5\} \tag{3.9}$$

where the corresponding capacity region that is a pentagon is depicted in Fig. 3.5.

Example 3.2. Binary Multiplier Channel

The two user binary multiplier channel (BMC) performs the multiplication $Y = X_1 X_2$ where $X_1, X_2 \in \{0, 1\}$ and hence $Y \in \{0, 1\}$ either. Thus, the sum-rates capacity is restricted by:

$$\{(R_1, R_2) : R_1 + R_2 \leq 1\} \tag{3.10}$$

as depicted in Fig. 3.6.

Example 3.3. Gaussian Multiple Access Channel (Cover [Cover1975] and Wyner [Wyner1974])

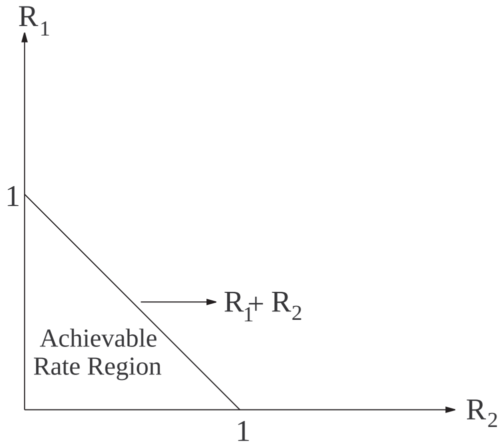

Figure 3.6: Capacity region of binary multiplier channel.

In a Gaussian multiple access channel, each user transmits from an infinite alphabet set, $X_k \in \mathbb{R}$, subject to an average power constraint $\mathcal{E}\{X_k^2\} \leq P_k$:

$$Y = \sum_k X_k + z \tag{3.11}$$

where z is a zero-mean σ^2 variance Gaussian random variable. The capacity region found as an extension of BAC capacity to infinite alphabets is given by:

$$R_\Gamma \leq \frac{1}{2}\log_2\left(1 + \frac{P_\Gamma}{\sigma^2}\right) \text{bits/symbol}, \forall\Gamma \in \mathcal{K} \tag{3.12}$$

Further results on the capacity of Gaussian multiple-access channels can also be found in [Cheng1989, ChengVerdu1989, Cheng1991, Verdu1993, Wyner1997, Shamai1997, Hanly 1998, Tse 1998].

3.2 Multiaccess Coding

Capacity regions provide bounds on the maximum achievable transmission rates with arbitrarily low probability error as the codeword length tends to infinity. However, to get close to these bounds practical and powerful error correction codes have to be designed. Codes for multiple-access channels not only should be designed to combat background noise but they should be eliminating multiple-access interference as well as provide margin for fading at the same time. A code that can perform the task of eliminating MAI as well as providing margin for noise and fading at the same time is called a one-to-one *uniquely decodable* (UDE) multiaccess code. Both block and trellis UDE codes for multiple access channels with particular emphasis on BAC are constructed and proposed in the literature. We will further present superposition coding for the achievability proof of the GMAC capacity region.

3.2.1. Block Codes

Much of the literature on multiaccess block coding in the literature is for BAC. The best known code for the 2-BAC until mid 1980s was the one that assigns the user 1 the codebook $C_1 = \{00, 11\}$ and the user 2 $C_2 = \{00, 01, 10\}$ presented by Kasami and Lin in [Lin1976]. For this code, $R_1 = 0.5, R_2 = 0.792$ and the sum-rate $R(\mathcal{K}) = 1.29$. They further give bounds on the achievable rates for certain block codes in [Lin1978] for the noisy channel. In [Kasami1978], they present a reduced complexity decoding scheme, however, it is still exponentially complex in the codeword length. Tilborg et. al in [Tilborg1978] further used graph-theoretic approaches to improve upon their previous lower bounds. They relate the code design problem to the independent set problem of graph theory, and use Turan theorem which gives a lower bound on the independence number in terms of the number of vertices and edges of the graph.

Chang and Weldon [Chang1979] constructed codes for K-BAC where they showed that their iterated construction is asymptotically good, $R(\mathcal{K})/C_{sum} \to 1$ as $K \to \infty$. Chang and Wolf [Chang1981] have later constructed codes for a generalization of K-BAC with larger input alphabets. They consider a channel in which each user in each symbol interval activates one of N frequencies. Depending upon what type of observation is made of each frequency, two channels are defined: the channel with intensity information and the channel without intensity information. For the former case, the number of users transmitting on each frequency is available to the receiver. The latter case provides only an active/nonactive output for each frequency. As for the Chang-Weldon codes, this code is also asymptotically optimal if one increases $K \to \infty$, when N is fixed.

Van den Braak and Van Tilborg in [Tilborg1985] have constructed codes for the 2-BAC. They construct code pairs at sum rates above 1.29 but only marginally. The highest rate code presented has $N = 7$, $R_1 = 0.512$ and $R_2 = 0.793$ giving a sum rate of 1.30565. Vanroose [Vanroose1988] further considers coding for the two-user binary switching MAC with output $Y = X_1/X_2$. Division by 0 results in ∞ symbol. This is the only ternary output binary input MAC form. The capacity region is determined to be equal to both the total co-operation zero-error capacity region.

3.2.2. Trellis Codes

Peterson and Costello have investigated convolutional codes in [Peterson1980]. They introduce the concept of a combined two-user trellis and define a distance measure, the L-distance between any two output channel sequences. They then go on to prove several results such as unique decodability and catastrophicity. In another important theorem, they prove that no convolutional code pair for the 2-BAC has sum rate above 1 bits/symbol that can already be achieved by no cooperation. Chevillat in [Chevillat1981] has further investigated trellis coding for the K-BAC where he finds his convolutional codes with large $d_{L,\text{free}}$ and gives a two user trellis code having sum rate 1.29.

3.2.3. Superposition Coding

Superposition coding for the Gaussian multiple access channel was first introduced by Carleial in [Carleial1975] and later investigated by Rimoldi and Urbanke in [Urbanke1994]. Therein, the superposition coding is used to show the achievability of the entire GMAC capacity region without any time-sharing argument. The original idea of achievability on the GMAC is due to Cover and Wyner as presented in Example 3.3. The idea that lies behind the achievability is the *successive cancellation* possibly after a re-indexing according to the powers:

$$R(\mathcal{K}) : R_k = \frac{1}{2}\log_2\left(1 + \frac{P_k}{\sigma^2 + \sum_{i=k+1} Pi}\right) \text{bits/symbol}, \forall k \in \mathcal{K} \qquad (3.13)$$

We should now remember that when the output is $Y = \sum_k X_k + Z$, information theory tells that when we first decode users in order treating other users as noise under the assumption of large number of users or Gaussian random codewords, the achievable rate of user 1 is:

$$R_1 \leq \frac{1}{2} \log_2 \left(1 + \frac{P_1}{\sigma^2 + \sum_{i=2} Pi}\right) \text{ bits/symbol} \tag{3.14}$$

that is the condition set in (3.13). Having successfully decoded the information for user 1, we re-encode it and subtract the signal from the output Y. This goes on till the Kth user is decoded. In this manner, it can be shown via a genie-aided argument that the probability of error may be forced to zero exponentially with the codeword length. This approach is also generally known as *iterative decoding* or *onion peeling*, which is an important concept and finds applications in the achievability proofs of broadcast channels, interference channels and relay channels either.

Returning now to the superposition coding scheme, assume that $R_{\mathcal{K}}$ is a point not a vertex already on the sum-rate constraint:

$$R_1 < \frac{1}{2} \log_2 \left(1 + \frac{P_1}{\sigma^2}\right) \text{ bits/symbol} \tag{3.15}$$

$$R_2 < \frac{1}{2} \log_2 \left(1 + \frac{P_2}{\sigma^2}\right) \text{ bits/symbol,} \tag{3.16}$$

$$R_1 + R_2 = \frac{1}{2} \log_2 \left(1 + \frac{P_1 + P_2}{\sigma^2}\right) \text{ bits/symbol} \tag{3.17}$$

Defining two independent pseudo-users, $X_1^{(a)}$ and $X_1^{(b)}$ with rates $R_1^{(a)} + R_1^{(b)} = R_1$ and powers $P_1^{(a)} + P_1^{(b)} = P_1$, it is always possible to set $P_1^{(b)} > 0$ such that:

$$R_2 = \frac{1}{2} \log_2 \left(1 + \frac{P_2}{\sigma^2 + P_1^{(b)}}\right) \text{ bits/symbol} \tag{3.18}$$

Now letting $X_1 = X_1^{(a)} + X_1^{(b)}$, then the rate triple $(R_1^{(a)}, R_2, R_1^{(b)})$, where:

$$R_1^{(a)} = \frac{1}{2} \log_2 \left(1 + \frac{P_1^{(a)}}{\sigma^2 + P_2 + P_1^{(b)}}\right) \text{ bits/symbol} \tag{3.19}$$

$$R_2 = \frac{1}{2}\log_2\left(1 + \frac{P_2}{\sigma^2 + P_1^{(b)}}\right) \text{ bits/symbol} \tag{3.20}$$

$$R_1^{(b)} = R_1 - R_1^{(a)} = \frac{1}{2}\log_2\left(1 + \frac{P_1^{(b)}}{\sigma^2}\right) \text{ bits/symbol} \tag{3.21}$$

satisfies the conditions defined by the Cover-Wyner capacity region in (3.13). In fact, it is a step-by-step decodable point (vertex) of the three user rate region defined by the two pseudo-users and user 2. For the general K users case, only $2K-1$ steps in decoding is required. This is a significant result since not only the system complexity is reduced in this manner, but also the requirement for frame-synchronism is removed. In this way, the multiaccess coding problem is transformed into a series of $2K-1$ single-user coding problems that are well understood in the Gaussian case. We will make extensive use of the Cover-Wyner capacity region of GMAC in Chapter 4.

3.3 Collision Resolution

The collision resolution approach to the multiuser communications has begun with the introduction of the ALOHA scheme by Abramson in [Abramson1970]. Since then the collision resolution approach has been favored for random access channels in which many users transmit packets of information in bursty fashion. Collision resolution schemes are generally characterized by the following assumptions:

- **Random transmissions:** Each user transmits packets at random times according to a distribution such as Poisson.
- **Collision:** If two users' packets collide in time they are assumed completely lost. Otherwise they are assumed to be received error free.
- **Feedback:** Feedback to the user from receiver exists whether the packets are received in collision or error free.
- **Retransmission:** Collided packets are retransmitted at a later time.

Collision resolution systems are primarily concerned with random arrival of messages and ignore the problem of noise. The problem of multiple-access interference is addressed by collision resolution algorithms or protocols which attempt to minimize the number of collisions or packets lost. On the other hand, an information theoretic approach would use source coding to let each user transmit at an average rate.

Chapter 4

Spectral Efficiency of Randomly-Spread CDMA with Linear Multiuser Receivers

4.1 Introduction and Preliminaries

In this chapter, we analyze the spectral efficiency of linear multichannel multiuser receivers for randomly-spread CDMA over time-varying GWSSUS multipath fading channels in an information-theoretic manner. There is in fact a quite large amount of research presented in the literature on randomly-spread CDMA; however, almost all of these contributions mainly concentrate at large system and asymptotic wideband regimes only and consider either Gaussian or at most flat-fading channels for analysis. We will also give an overview of present randomly-spread CDMA research literature in the next subsection; however, our work presented in this chapter mainly differs from those in the current literature in terms of:

- Finite-dimensional analysis
- Focus on multipath fading channels

Multichannel multiuser detection has been attacked vastly in the literature and optimum [Fazel1994, Kaiser1995, Borah1998], linear [Zvonar1994, Zvonar1995, Brady1996, Zvonar1996, Chen1996, Klein1996, Juntti1999, Yang1999, Miller2000] and nonlinear [Imai1993, Patel1994, Hui1998, Tahar1999, Weng2001] multichannel multiuser receivers have been derived and analyzed. Our focus will be mainly on the multichannel linear receivers as elaborated in [Zvonar1994, Zvonar1995, Brady1996, Zvonar1996].

Analysis through random spreading sequences both serves as an accurate model for large systems in which long signature sequences that span many symbol periods are used to breakdown the cyclostationarity and average out the multiple-access interference as well as provides a comparison baseline for finite-dimensional short spreading sequence systems. In terms of spreading sequence sets, random spreading sequences, especially Gaussian, also well model the randomization behavior of tapped-delay line multipath channels on the transmitted signature sequences and they further provide average performances achievable over the spreading sequence sets with various correlation properties.

Random spreading performance analysis of CDMA mainly draws upon tools in *random matrix theory* and *free probability theory* [Mehta1990, Haagerup1998, Harer1986, Wigner1965, Pastur1967, Pastur1999, Bai1995, Bai1998, Voiculescu1992, Petz2000, Speicher2001]. Random matrix theory identifies the finite or asymptotic eigenvalue densities of random matrices while free probability theory is a tool to find the densities for the sum or multiplication of random matrices via R or S transforms respectively.

Random matrix theory has been heavily influenced since its inception by its applications in physics, statistics and engineering. The major contributions in random matrix theory are motivated in general by practical experiments. Concurrently, random matrix theory finds applications in stochastic calculus, condensed matter physics, numerical linear algebra, neural networks, multivariate statistics, information theory and signal processing. Especially in the last decade, a significant body of work has been done in communications, signal processing and information theory that make use of tools out of random matrix theory and free probability theory. This is especially vast in the analysis of multichannel and multiuser systems where multivariate statistics comes into account as in wireless channels and as we focus on in this thesis. Some application examples other than randomly-spread CDMA channels are array signal processing

[Combettes1992], multiple-input multiple-output (MIMO) channels [Telatar1999, Popescu2000, Muller2002, RMuller2002, Scaglione2002, Chuah2002, Mestre2003, Mestre2002] and precoding [Loubaton2003].

Most of the information-theoretic literature that studies the effects of random matrix features on channel capacity mainly deals with the linear vector memoryless channels as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{4.1}$$

where $\mathbf{x}$ is the K-dimensional input vector, $\mathbf{y}$ is the N-dimensional output vector, and the N-dimensional vector $\mathbf{n}$ models the additive circularly-symmetric Gaussian noise. Furthermore, $\mathbf{H}$ is the $N \times K$ channel matrix. The nature of the application determines the meaning of K and N. In a multiantenna MIMO system, K is the number of input antennas, N is the number of output antennas and $\mathbf{H}$ models the channel matrix. On the other hand, in a CDMA system, K is the number of users, N is the spread factor and $\mathbf{H}$ is the spreading sequences matrix. In a precoding application, $\mathbf{H}$ is directly the precoding matrix.

In CDMA, MIMO or precoding applications, the performance depends on either the channel or spreading sequence matrix's covariance eigenvalue density - squared singular values of $\mathbf{H}$ - or the precoding matrix's eigenvalue density function. The *cumulative magnitude-ordered eigenvalue distribution function* for an $M \times M$ random matrix $\mathbf{A}$ is given by:

$$F_{\mathbf{A}}^{(O)} = \Pr\{\frac{1}{M}\sum_{i=1}^{M}\delta(\lambda - \lambda_i^{(O)}(\mathbf{A})) \leq \lambda_t\} \tag{4.2}$$

defines the probability for the percentage of magnitude-ordered eigenvalues of $\mathbf{A}$ being below a specific threshold λ_t, and the density of the corresponding *eigenvalue-counting* or the *average-trace* rv λ - *eigenvalue density* - is:

$$\frac{dF_{\mathbf{A}}^{(\pi)}}{d\lambda} = f_{\mathbf{A}}(\lambda) \leftrightarrow \lambda = \frac{1}{M}\sum_{i=1}^{M}\lambda_i^{(\pi)}(\mathbf{A}) = \frac{1}{M}Tr\{\mathbf{A}\} \tag{4.3}$$

where $F_{\mathbf{A}}^{(\pi)} = \frac{1}{K!}F_{\mathbf{A}}^{(O)}$ is the *cumulative magnitude-unordered eigenvalue distribution function* obtained by averaging $F_{\mathbf{A}}^{(O)}$ over all possible $K!$ permutations

[Haagerup1998]. Via this definition of the eigenvalue density, we also have the following important *averaging lemma:*

Lemma 4.1(Haagerup and Thornbjorsen, [Haagerup1998]): For an $M \times M$ random matrix $\mathbf{A}$ with eigenvalue density $f_{\mathbf{A}}(\lambda)$, let $g(x) : \mathbb{R} \to \mathbb{R}$ be a Borel function. Then,

$$\mathcal{E}\left\{\frac{1}{M}\sum_{i=1}^{M} g\left(\lambda_i(\mathbf{A})\right)\right\} = \int_{\mathbb{R}} g(\lambda) f_{\mathbf{A}}(\lambda) d\lambda \tag{4.4}$$

provided that the integral on the right-hand side of (4.4) is well-defined; i.e. $g(\lambda) \geq 0$ or $\int_{\mathbb{R}} |g(\lambda)| f_{\mathbf{A}}(\lambda) d\lambda < \infty$.

Proof: Assume first that $g(\lambda) \geq 0$. Since:

$$\sum_{i=1}^{M} g\left(\lambda_i(\mathbf{A})\right) = Tr\{g(\mathbf{A})\} \tag{4.5}$$

is a symmetric function of the eigenvalues of $\mathbf{A}$, it follows that:

$$\mathcal{E}\left\{\sum_{i=1}^{M} g\left(\lambda_i(\mathbf{A})\right)\right\} = \tag{4.6}$$

$$\int_{\mathbb{R}^M} \left(\sum_{i=1}^{M} g\left(\lambda_i(\mathbf{A})\right)\right) .F_{\mathbf{A}}^{(O)} d\lambda_1 ... d\lambda_M \tag{4.7}$$

Using then the fact that $F_{\mathbf{A}}^{(O)}$ is invariant under the permutations of the eigenvalues of $\mathbf{A}$, it follows that:

$$\mathcal{E}\left\{\sum_{i=1}^{M} g\left(\lambda_i(\mathbf{A})\right)\right\} = M. \int_{\mathbb{R}} g(\lambda) f_{\mathbf{A}}(\lambda) d\lambda \tag{4.8}$$

which proves that (4.4) holds whenever $g(\lambda) \geq 0$.♦

The most important random matrix types for which the finite and asymptotic eigenvalue densities are well known are Wigner, Wishart and Haar matrices. We will make use of Wigner and Wishart densities in our derivations and leave the elaboration to later. For deeper treatment of random matrix theory and free probability theory, one is referred to [Tulino2004].

4.2 Previous Parallel Work

Analysis of code-division multiple-access with random spreading was addressed by Madhow and Honig in [Honig1993] for MMSE detection on the AWGN channel. Grant and Alexander discussed random sequence multisets in [Grant1998]. Verdú and Shamai presented asymptotic results and bounds on the spectral efficiencies of several multiuser receivers with binary random spreading in [VerduShamai1999]. Mueller and Schramm, and Mueller derived the spectral efficiency and the SIR statistics with both linear and nonlinear multiuser receivers in [Schramm1999] and [RRMuller2001] respectively. Tse and Hanly further analyzed the user capacity with linear multiuser receivers using the concept of effective interference and effective bandwidth [DNCTse1999].

Following analysis on the capacity and error-rate of CDMA with random spreading and multiuser receivers is extended to fading channels in a number of recent contributions. In [VerduShamai2001], Verdú and Shamai analyzed the impact of frequency-flat fading on the spectral efficiency of synchronous CDMA with multiuser receivers. Biglieri et al. [Taricco2001] derived the ergodic capacity and outage capacity for conventional matched-filter and MMSE multiuser receivers on the flat-fading asymptotic CDMA channel. Further, Evans and Tse analyzed the SIR statistics of linear multiuser receivers over multipath fading channels taking into consideration the channel estimation errors for asymptotic uncoded CDMA with binary random spreading sequences in [Evans2000].

In these aforementioned contributions, the *asymptotic limiting theory* used depends on the averaging of performance metrics with respect to the asymptotic eigenvalue density of the random crosscorrelation matrix as number of users K and spread factor $L \to \infty$ but with fixed $\beta = \frac{K}{L}$ that is called the *load factor* in users/chip. Assumption is that asymptotic systems also present good models for finite-size systems. At this large system limit, the asymptotic eigenvalue density of random crosscorrelation matrices $\frac{1}{K}\mathbf{S}^H\mathbf{S}$, where the elements of the spreading sequences matrix are i.i.d. with zero-mean and unity variance, tend to

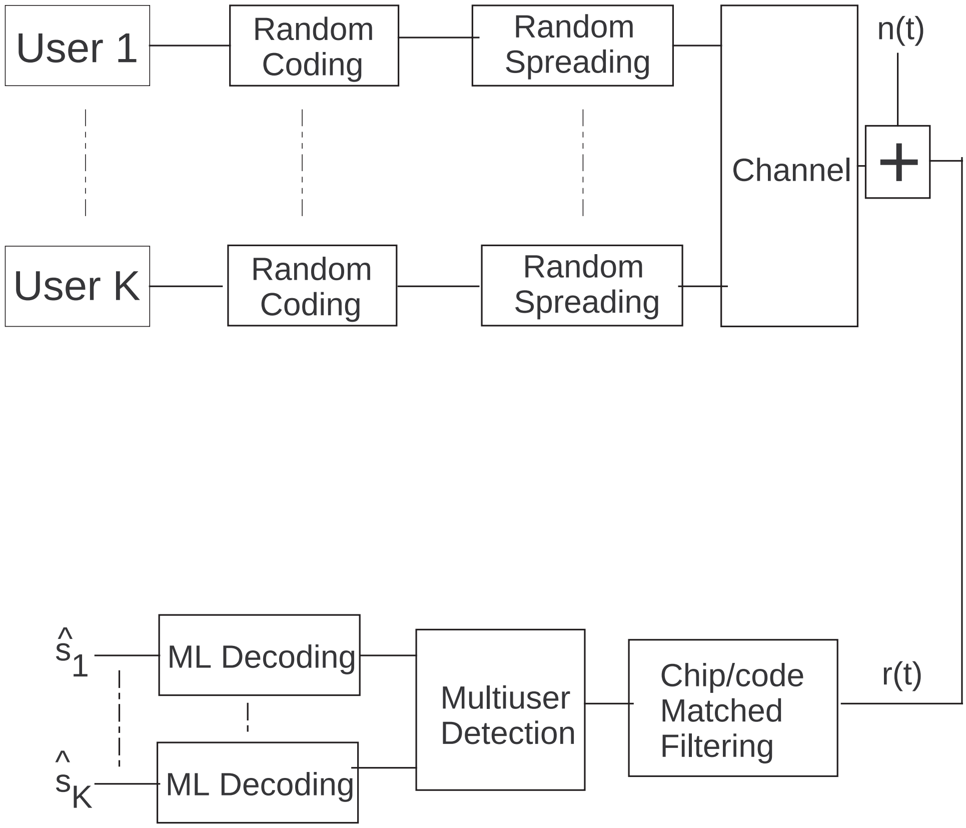

Figure 4.1: Overview of the general system model considered.

the *Marćhenko-Pastur* distribution, whose density function obeys:

$$f_{MP}(\lambda_{MP}) = \left[1 - \frac{1}{\beta}\right]^+ \delta(\lambda_{MP}) + \frac{\sqrt{\left(\lambda_{MP} - (1-\sqrt{\beta})^2\right)\left((1+\sqrt{\beta})^2 - \lambda_{MP}\right)}}{2\pi\beta\lambda_{MP}}, \tag{4.9}$$

$\forall (1-\sqrt{\beta})^2 \leq \lambda_{MP} \leq (1+\sqrt{\beta})^2 \cup \{0\}$ [Haagerup1998]. In this asymptotic scenario, it is also presented that signal-to-interference ratios (SIRs) tend to deterministic constants.

4.3 Overview of the General System Model

We consider a chip/symbol synchronous uplink DS-CDMA system of W_s Hz system bandwidth as depicted in Fig. 4.1. with K equal-rate equal-power users randomly located in a single isolated cell and a central base-station receiver with a single receive antenna. Each symbol of each user is then assumed to be coded with an (N, R_k) complex-Gaussian random block code with SNR_s average power constraint [CoverThomas1991] as $N \rightarrow \infty$:

$$\frac{1}{N}\sum_i x_k^2(i) \leq SNR_s \tag{4.10}$$

where $x_k(i)$ are the codeword letters of kth user.

Denoting the channel-symbol signalling period by T_{cs} and the chip period by T_c, the channel-symbols at the transmitters are assumed to be spread with normalized complex-Gaussian random spreading sequences of processing gain $L = \frac{T_{cs}}{T_c}$:

$$c_k = \frac{1}{\sqrt{L}}[c_{k1}c_{k2}...c_{kL}]^T \tag{4.11}$$

where each chip corresponds to a circularly-symmetric spherical complex-Gaussian, zero-mean, unity-variance random variable. Hence, the realization of the spreading sequences in each channel-symbol signalling period T_{cs} is asymptotically unit Euclidean-norm via law of large numbers as $L \rightarrow \infty$, i.e. desired signal loss due to random spreading sequences with randomly fluctuating energies vanishes at the asymptotic wideband limit.

The unit-energy pulse-shaped and spread signals of all users occupying the same bandwidth are transmitted over the time-varying waveform channel. We are considering statistically identical time-varying Gaussian wide-sense-stationary uncorrelated-scattering (GWSSUS) channels as elaborated in Chapter 2 for each user with *multipath delay spread* T_m, where $T_m << T_{cs}$ is assumed for negligible ISI and the optimality of one-shot detection. The inverse of the multipath delay spread reveals the *coherence bandwidth, i.e.* $(\Delta f)_c = \frac{1}{T_m}$, identifying the frequency-selectivity of the channel, and the *multipath diversity order* D resolvable by each user in conjunction with the spread system bandwidth W_s is then given by:

$$D = \lceil T_m W_s \rceil \simeq \frac{T_m}{T_c} \tag{4.12}$$

As common exercise for chip-synchronous systems, let's assume the propagation delays of each path of each user are integer multiples of the chip period with the first path of each user being at zero propagation delay. Then, by successive chip-matched/code-matched filtering at the appropriate delays for each path and chip-rate sampling at the receiver side, the discrete-time equivalent of the multichannel diversity-reception DS-CDMA system in an arbitrary channel-symbol signalling interval is obtained as

$$\mathbf{y} = \mathbf{RHs} + \mathbf{w} \tag{4.13}$$

where $\mathbf{y} = [y_{11}\ y_{12} \cdots y_{1D}\ y_{21} \cdots y_{KD}]^T \in \mathbb{C}^{KD\times 1}$ is the stacked sufficient statistics vector for each path of each user, $\mathbf{R} = \mathbf{S}^H\mathbf{S} \in \mathbb{C}^{KD\times KD}$ is the Hermitian crosscorrelation matrix with $\mathbf{S} \in \mathbb{C}^{L\times KD}$ being the matrix of the chip-shifted spreading sequences for each path of each user with the first path of each user at zero-offset. $\mathbf{H} = diag\{\mathbf{h}_1, \mathbf{h}_2, .., \mathbf{h}_K\} \in \mathbb{C}^{KD\times K}$ is the block-diagonal channel coefficients matrix of all users with $\mathbf{h}_k = [h_{k1}\, h_{k2} ... h_{kD}]^T \in \mathbb{C}^{D\times 1}$ being the channel coefficients vector of kth user and $\mathbf{s} \in \mathbb{C}^{K\times 1}$ is the channel-symbols vector of all K users. Furthermore, $\mathbf{w} \in \mathbb{C}^{KD\times 1}$ is the zero-mean complex-Gaussian background noise vector having crosscovariance matrix $\sigma_n^2\mathbf{R}$, where the noise variance $\sigma_n^2 = N_0 \times W_s$ - in watts - and N_0 (watts/Hz) is the background AWGN power spectral density. The multiple-access channel defined by (4.13) is in general called a *vector multiple-access channel* since there exists multiple degrees of freedom for the flow of mutual information of each user [Viswanath1999] and we will be using the notation correlated-waveform vector multiple-access channel (CW-VMAC) for the channel definition in (4.13) throughout the sequel.

Due to the GWSSUS assumption for the channels of each user, the complex-Gaussian channel coefficients are independent over paths for each user due to the Gaussianity and the uncorrelatedness of the scattering. Furthermore, we keep the mean-squared values of the complex channel coefficients normalized to unity throughout the sequel; i.e. $E\left\{|h_{kd}|^2\right\} = 1$, $\forall k, d$. The transmitters are also assumed to have no knowledge on the complex channel coefficients while the receiver has perfect information about the complex channel coefficients of all users for diversity combining purposes. Thus, based on the received signal model and further assuming that the noise PSD level is normalized to unity in watts/Hz;

i.e. $N_0 = 1$, without loss of generality, the transmit symbol signal-to-noise ratio of kth user is defined as

$$SNR_s^{(k)} = \frac{A_k^2}{\sigma_n^2} = \frac{E_s^{(k)}}{N_0} = E_s^{(k)} \tag{4.14}$$

Furthermore, the average received symbol signal-to-noise ratio per path of kth user is thus given by:

$$SNR_s^{(\mathrm{av},k)} = E\left\{|h_{kd}|^2\right\} \frac{E_s^{(k)}}{N_0} = E_s^{(k)} = SNR_s^{(k)} \tag{4.15}$$

In this finite-dimensional system scenario, it is assumed that as the system bandwidth is increased through higher spread factors L, the number of users K is also assumed to increase to keep the load factor β fixed. Another system dimensionality parameter assumed to remain fixed as spread factor is increased here is the normalized delay spread $\alpha \triangleq \frac{D}{L} = \frac{T_m}{T_{cs}}$ that unitlessly measures the delay spread of the channel with respect to the channel symbol signalling period. With these system dimensionality parameter definitions, the system operating point is then defined by:

- $\beta \triangleq \frac{K}{L}$, load factor
- $\alpha \triangleq \frac{D}{L}$, normalized delay spread
- D, diversity order
- SNR_s or SNR_b, Symbol or bit SNR (dB)
- 1-σ_c^2, Specularity level (dB)

where σ_c^2 is the variance of each channel coefficient in the GWSSUS channel definition of Chapter 2. This finite-dimensional random-spreading performance analysis methodology is also further published and can be found in [Ertug2003].

4.4 Analysis of SIRs with Linear Multichannel Multiuser Receivers

In this section, we attack the statistical characterization of SIRs with multiuser receivers. SIR being the major capacity and quality determining factor in interference limited multiaccess communication, we statistically characterize the random soft-output SIR of maximal-ratio diversity-combining multiuser receivers and derive consequently the mean of the SIRs in terms of the system parameters.

4.4.1 Linear RAKE

The signal-to-interference ratio $\mathrm{SIR_k}^{(\mathrm{MUD})}$ is statistically identical over the set of users $\mathcal{K}$. The soft-output conditional SIR for user k conditioned on the channel and the spreading sequences achievable by linear RAKE multiuser receiver, whose linear estimator matrix after chip-matched filtering by $\mathbf{S}^H$ is $\mathbf{M} = \mathbf{I}$ that is analogous to the conventional matched-filter receiver for AWGN channels resulting in no interference suppression or cancellation, and followed by optimal linear diversity combining method maximal-ratio-combining (MRC), that coherently combines the D diversity channels of each user in the maximal-SIR sense, is given by:

$$SIR_k^{(\mathrm{RAKE})} =_{\mathcal{D}} \frac{\mathbf{h}_k^H \mathbf{R}_{kk}^2 \mathbf{h}_k SNR_s^{(k)}}{\sum_{\acute{k}=1,\acute{k}\neq k}^{K} \mathbf{h}_{\acute{k}}^H \mathbf{R}_{k\acute{k}}^2 \mathbf{h}_{\acute{k}} SNR_s^{(k)} + P_{tot}^{(noise)}} \tag{4.16}$$

in terms of power by power ratio, where conditional total power of desired signal components $P_{tot}^{(desired)} = \mathbf{h}_k^H \mathbf{R}_{kk}^2 \mathbf{h}_k SNR_s^{(k)}$, conditional total power of multiaccess interference terms $P_{tot}^{(MAI)} = \sum_{\acute{k}=1,\acute{k}\neq k}^{K} \mathbf{h}_{\acute{k}}^H \mathbf{R}_{k\acute{k}}^2 \mathbf{h}_{\acute{k}} SNR_s^{(k)}$, total power of noise component $P_{tot}^{(noise)} = \mathcal{E}\left\{\left|\mathbf{h}_k^H \mathbf{w}_k^{(\mathrm{RAKE})}\right|^2\right\}$, $(=_{\mathcal{D}})$ is the distribution-equivalence operator and $\mathcal{E}\{.\}$ is the expectation operator. The total power of the zero-mean noise component in the denominator of (4.16) for $N_0 = 1$ can further be expanded via eigenvalue decomposition (EVD) as:

$$P_{tot}^{(noise)} = \mathcal{E}\left\{\left|\mathbf{h}_k^H \mathbf{w}_k^{(\mathrm{RAKE})}\right|^2\right\} =$$

$$\mathcal{E}\left\{\mathbf{h}_k^H\mathbf{w}_k^{(\mathrm{RAKE})}\left(\mathbf{w}_k^{(\mathrm{RAKE})}\right)^H\mathbf{h}_k\right\} =$$
$$\mathcal{E}\left\{\mathbf{h}_k^H\mathbf{R}_{kk}\mathbf{h}_k\right\} = \mathcal{E}\left\{\mathbf{h}_k^H\mathbf{U}_{kk}^H\mathbf{D}_{kk}\mathbf{U}_{kk}\mathbf{h}_k\right\} \tag{4.17}$$

where $\mathbf{D}_{kk} = diag\{\lambda_{kk1}, \lambda_{kk2}, ..., \lambda_{kkD}\}$ is the diagonal eigenvalues matrix of $\mathbf{R}_{kk}$ and $\mathbf{U}_{kk}$ is the unitary eigenvectors matrix of $\mathbf{R}_{kk}$. Since $\mathbf{U}_{kk}$ is unitary - $\mathbf{U}_{kk}^H = \mathbf{U}_{kk}^{-1}$ - and if we denote the transformed channel coefficients vector by $\acute{\mathbf{h}}_k = \mathbf{U}_{kk}\mathbf{h}_k$:

$$\acute{\mathbf{h}}_k^H\acute{\mathbf{h}}_k =_{\mathcal{D}} \mathbf{h}_k^H\mathbf{h}_k \tag{4.18}$$

Due to this distribution-equivalence property, the following expression from (4.17) is valid:

$$\mathcal{E}\left\{\mathbf{h}_k^H\mathbf{U}_{kk}^H\mathbf{D}_{kk}\mathbf{U}_{kk}\mathbf{h}_k\right\} =$$
$$\mathcal{E}\left\{\acute{\mathbf{h}}_k^H\mathbf{D}_{kk}\acute{\mathbf{h}}_k\right\} = \mathcal{E}\left\{\sum_{i=1}^{D}\lambda_{kki}\left|h_{ki}\right|^2\right\} \tag{4.19}$$

As it can be deduced, for the statistical characterization of the SIR with multiuser receivers, the finite-dimensional eigenvalue densities of the Hermitian diagonal $\mathbf{R}_{kk}$ and the Hermitian off-diagonal $\mathbf{R}_{k\acute{k}}$ submatrices of the Hermitian crosscorrelation matrix $\mathbf{R}$ have to be accurately characterized. Due to the normalized spherical complex-Gaussian random spreading sequences assumed where the real and imaginary parts of each chip is independently and identically distributed with variance $\frac{1}{2}$, the diagonal elements of the crosscorrelation matrix $\mathbf{R}$ are distributed according to the central chi-squared distribution $\chi_{2L}^{2,c}(1/2)$ of $2L$ degrees of freedom with unity mean and variance $\frac{1}{L}$ due to the scaling by $\frac{1}{L}$. Furthermore, when the chip-synchronism is complete and the path delays are integer multiples of the chip period T_{cs}, the off-diagonal elements of $\mathbf{R}$ converge by Central Limit Theorem to the circularly-symmetric complex-Gaussian distribution $N(0, \frac{1}{L})$ with zero mean and variance $\frac{1}{L}$. Thus, by the average trace-rv definition, the eigenvalue-counting rv λ_{kk} of $\mathbf{R}_{kk}$ is distributed according to the central chi-squared $\chi_{2DL}^{2,c}$ distribution with $2DL$ degrees of freedom scaled by $\frac{1}{DL}$ having unity mean and variance $\frac{1}{DL}$:

$$f_{\mathbf{R}_{kk}}(\lambda_{kk}) = \frac{(DL)^{DL}}{\Gamma_c(DL)}\lambda_{kk}^{DL-1}\exp(-DL\lambda_{kk}) \tag{4.20}$$

and moreover, using Lemma 4.1., the total power of the noise component by RAKE in (4.19) is given by:

$$P_{tot}^{(noise)} = \mathcal{E}\left\{\left|\mathbf{h}_k^H \mathbf{w}_k^{(\mathrm{RAKE})}\right|^2\right\} =$$

$$\mathcal{E}\left\{\sum_{i=1}^{D} \lambda_{kki} \left|h_{ki}\right|^2\right\} =$$

$$D\int_0^\infty \int_0^\infty \lambda_{kk} r f_{\mathbb{R}_{kk}}(\lambda_{kk}) f_G^{(ns)}(r) d\lambda_{kk} dr = D^2 \tag{4.21}$$

with the definition of the density of norm-squared pdf of CIR $f_G^{(ns)}(r)$ - for $\|\mathbf{h}_k\|^2$ - as defined in Chapter 2:

$$f_G^{(ns)}(r) = \frac{1}{\sigma_c^2}\left(\frac{r}{s^2}\right)^{\frac{D-1}{2}} \exp\left(-\frac{s^2+r}{\sigma_c^2}\right) I_{D-1}\left(\frac{2\sqrt{r}s}{\sigma_c^2}\right),\ r \geq 0 \tag{4.22}$$

with the following mean and variance:

$$\mu_G^{(ns)} = D,\ \sigma_G^{2,(ns)} = D(2\sigma_c^2 - \sigma_c^4) \tag{4.23}$$

Furthermore, the fact that the crosscorrelation matrix under the considered spreading scenario tending to the identity matrix $\mathbf{R} \to \mathbf{I}$ as $L \to \infty$ validates the averaging out of MAI components asymptotically at the wideband limit.

For the statistical characterization of the multi-access interference term with RAKE in (4.16) via the eigenvalue distribution of the off-diagonal random submatrices $\mathbf{R}_{k\acute{k}}$ of $\mathbf{R}$, $\mathbf{R}_{k\acute{k}}$ can be further partitioned into two matrices as

$$\mathbf{R}_{k\acute{k}} = \mathbf{A} + \mathrm{sqrt}(-1)\mathbf{B} \tag{4.24}$$

where $\mathbf{B}$ is a diagonal matrix of independently and identically distributed $N(0, \frac{1}{2L})$ random variables having eigenvalue density $N(0, \frac{1}{2DL})$ and $\mathbf{A}$ is a proper Hermitian complex-Gaussian random matrix. The set of proper Hermitian complex-Gaussian random matrices can be defined as [Haagerup1998]:

Definition 4.1. Let (Ω, ξ, P) be a probability space, let n be a positive integer and let $\mathbf{A}: \Omega \to M_n(\mathbb{C})$ be a complex random $n \times n$ matrix defined on Ω. For i, j in $\{1, 2, ..., 2p\}$, let $a(i, j)$ denote the entry at position (i, j) of $\mathbf{A}$. The matrix $\mathbf{A}$ is said to be a self-adjoint (Hermitian) Gaussian random $n \times n$ matrix with entries of variance σ^2, if the following conditions are satisfied:

(i) The entries $a(k,l)$, $1 \leq k \leq l \leq n$, form a set of $\frac{1}{2}n(n+1)$ independent, complex-valued random variables.

(ii) For each k in $\{1, 2, ..., n\}$, $a(k,k)$ is a real-valued random variable with distribution $N(0, \sigma^2)$.

(iii) When $k < l$, the real and imaginary parts $Re\{a(k,l)\}$ and $Im\{a(k,l)\}$ of $a(k,l)$ are i.i.d. random variables with distribution $N(0, \frac{1}{2}\sigma^2)$.

(iv) When $k > l$, $a(k,l) = a^*(l,k)$.

and all such random matrices $\mathbf{A}$ defined on Ω are denoted by the set **SGRM** (n, σ_{WG}^2). $\diamondsuit$

The eigenvalue distribution of proper Hermitian complex-Gaussian random matrices **SGRM** (n, σ_{WG}^2) is identified by the following theorem due to Wigner [Wigner1965]:

Theorem 4.1. Let $\mathbf{A}$ be an element of **SGRM** (n, σ_{WG}^2). Then the joint distribution of the magnitude-ordered independent real eigenvalues $\lambda_1(\mathbf{A}) \leq ... \leq \lambda_n(\mathbf{A})$ of $\mathbf{A}$, has *Wigner density* with respect to the Lebesque-measure on $\mathbb{R}^n$, given by:

$$\begin{aligned} f_{ordered}(\lambda_{ordered}) &= \left((2\pi)^{\frac{n}{2}} \sigma_{WG}^{n^2} (\prod_{j=1}^{n-1} j!)\right)^{-1} \\ &\times \left(\prod_{1 \leq j < k \leq n} (\lambda_j - \lambda_k)^2\right) \exp\left(-\frac{1}{2\sigma_{WG}^2} \sum_{j=1}^{n} \lambda_j^2\right) \end{aligned} \quad (4.25)$$

where $\lambda_{ordered} = [\lambda_1(\mathbf{A}) \ \lambda_2(\mathbf{A}) ... \lambda_n(\mathbf{A})]^T$.

Let $f_{unordered}(\lambda_1, \lambda_2, ..., \lambda_n) : \mathbb{R}^n \Rightarrow \mathbb{R}$, be the density function obtained by taking the average of (4.25) over all permutations of $(\lambda_1, \lambda_2, ..., \lambda_n)$ such that:

$$\begin{aligned} f_{unordered}(\lambda_1, \lambda_2, ..., \lambda_n) &= \frac{\left((2\pi)^{\frac{n}{2}} \sigma_{WG}^{n^2} (\prod_{j=1}^{n-1} j!)\right)^{-1}}{n!} \\ &\times \left(\prod_{1 \leq j < k \leq n} (\lambda_j - \lambda_k)^2\right) \end{aligned}$$

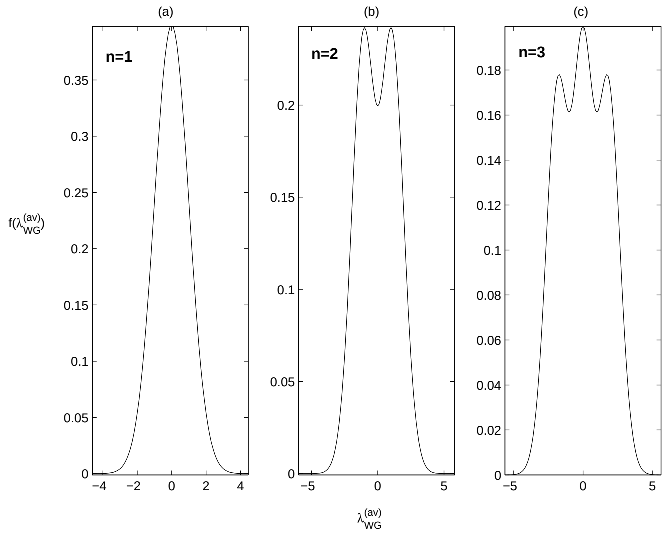

Figure 4.2: Wigner densities.

$$\times \exp\left(-\frac{1}{2\sigma_{WG}^2}\sum_{j=1}^{n}\lambda_j^2\right), \tag{4.26}$$

Then, the marginal density of the eigenvalue-counting or average-trace rv λ_{WG} obtained by integrating (4.26) over the densities of the rest $n-1$ eigenvalues is given by:

$$f(\lambda_{WG}) = \frac{1}{n\sigma_{WG}\sqrt{2}}\sum_{k=0}^{n-1}\left[\varphi_k\left(\frac{\lambda_{WG}}{\sigma_{WG}\sqrt{2}}\right)\right]^2, \ \bar{\lambda}_{WG} \in \mathbb{R}, \tag{4.27}$$

where $\varphi_k(.)$ is the sequence of Hermite functions:

$$\varphi_k(x) = \frac{1}{(2^k k!\sqrt{\pi})^{\frac{1}{2}}} H_k(x)\exp(-\frac{x^2}{2}), \ k \in \mathbb{N}^+, \tag{4.28}$$

and $H_k(.)$ is the sequence of Hermite polynomials:

$$H_k(x) = (-1)^k \exp(x^2).(\frac{d^k}{dx^k}\exp(-x^2)), \ k \in \mathbb{N}^+, \tag{4.29}$$

(see Fig. 4.2.). $\diamondsuit$

It is a general assumption for analysis with RAKE receivers that the total MAI components are distributed normally and to use the corresponding average total power in SIR expressions which is in general termed the Gaussian assumption [Rasmussen2000]. This further requires us to determine the mean-squared value of the eigenvalue-counting rv of a proper Hermitian complex-Gaussian random matrix $\mathbf{A}$, since by eigenvalue decomposition and averaging lemma, the average total power of the MAI components is given by:

$$P_{tot}^{(MAI,av)} = \mathcal{E}\left\{\sum_{\acute{k}=1,\acute{k}\neq k}^{K} \mathbf{h}_{\acute{k}}^{H}\mathbf{R}_{k\acute{k}}^{2}\mathbf{h}_{\acute{k}} SNR_{s}^{(k)}\right\} \tag{4.30}$$

$$= \mathcal{E}\left\{\sum_{\acute{k}=1,\acute{k}\neq k}^{K} \mathbf{h}_{\acute{k}}^{H}\mathbf{U}_{k\acute{k}}^{H}\mathbf{D}_{k\acute{k}}\mathbf{U}_{k\acute{k}}\mathbf{U}_{k\acute{k}}^{H}\mathbf{D}_{k\acute{k}}\mathbf{U}_{k\acute{k}}\mathbf{h}_{\acute{k}} SNR_{s}^{(k)}\right\} \tag{4.31}$$

$$= \mathcal{E}\left\{\sum_{\acute{k}=1,\acute{k}\neq k}^{K} \acute{\mathbf{h}}_{\acute{k}}^{H}\mathbf{D}_{k\acute{k}}^{2}\acute{\mathbf{h}}_{\acute{k}} SNR_{s}^{(k)}\right\} \tag{4.32}$$

$$= \mathcal{E}\left\{\sum_{\acute{k}=1,\acute{k}\neq k}^{K} \sum_{i=1}^{D} \lambda_{k\acute{k}i}^{2} \left|h_{\acute{k}i}\right|^{2} SNR_{s}^{(k)}\right\} \tag{4.33}$$

$$= \sum_{\acute{k}=1,\acute{k}}^{K} DSNR_{s}^{(k)} \int_{0}^{\infty}\int_{-\infty}^{\infty} \lambda_{k\acute{k}}^{2} r f_{\mathbf{R}_{k\acute{k}}}(\lambda_{k\acute{k}}) \tag{4.34}$$

$$\times f_{G}^{(ns)}(r) d\lambda_{k\acute{k}} dr \tag{4.35}$$

The mean value of the eigenvalue-counting rv of an $n \times n$ proper Hermitian complex-Gaussian random matrix can be calculated directly as 0 due to properties of Hermite functions [Ryzhik1994]. Similarly, the mean-squared value being equivalent to the variance is given by $n\sigma_{WG}^{2} = \frac{D}{L}$ in our case that can be derived via Harer-Zagier recursion defined as follows:

Theorem 4.2 (Harer and Zagier, Section 4, Proposition 1, [Harer1986]): Let $\mathbf{A}$ be an element of $\mathbf{SGRM}(n,1)$ and define:

$$C(p,n) = \mathcal{E}\left\{Tr_{n}[\mathbf{A}^{2n}]\right\}, p \in \mathbb{N}_{0} \tag{4.36}$$

Then $C(0,n) = n, C(1,n) = n^{2}$, and for fixed n in $\mathbb{N}$, the numbers $C(p,n)$ satisfy the recursion formula:

$$C(p+1,n) = n.\frac{4p+2}{p+2}.C(p,n) + \frac{p.(4p^{2}-1)}{p+2}C(p-1,n) \tag{4.37}$$

for $p \geq 1$.♦

Letting $K-1 \simeq K$ as $K \to \infty$ and following from (4.30), the average total power of MAI components is then given by:

$$P_{tot}^{(MAI,av)} = \beta(D^2 + \frac{1}{2})SNR_s^{(k)} \tag{4.38}$$

If we further expand the total conditional power of the desired signal components in the numerator of (4.16) using eigenvalue decomposition and the averaging lemma reversely as

$$\begin{aligned} P_{tot}^{(desired)} = \mathbf{h}_k^H \mathbf{R}_{kk}^2 \mathbf{h}_k SNR_s^{(k)} = \\ \mathbf{h}_k^H \mathbf{U}_{kk}^H \mathbf{D}_{kk} \mathbf{U}_{kk} \mathbf{U}_{kk}^H \mathbf{D}_{kk} \mathbf{U}_{kk} \mathbf{h}_k SNR_s^{(k)} = \\ \acute{\mathbf{h}}_k^H \mathbf{D}_{kk}^2 \acute{\mathbf{h}}_k SNR_s^{(k)} = \\ \sum_{i=1}^{D} \lambda_{kki}^2 \left| h_{ki} \right|^2 SNR_s^{(k)} =_{\mathcal{D}} D\lambda_{kk\cdot}^2 \left\| \mathbf{h}_k \right\|^2 SNR_s^{(k)} \end{aligned} \tag{4.39}$$

since $\acute{\mathbf{h}}_k{}^H \acute{\mathbf{h}}_k =_{\mathcal{D}} \mathbf{h}_k^H \mathbf{h}_k$ for $\acute{\mathbf{h}}_k = \mathbf{U}_{kk} \mathbf{h}_k$ and unitary $\mathbf{U}_{kk}$. Following, the SIR of kth user with RAKE receiver under Gaussian assumption is distributed as:

$$SIR_k^{(\mathrm{RAKE,GA})} =_{\mathcal{D}} \frac{D\lambda_{kk\cdot}^2 \left\| \mathbf{h}_k \right\|^2 SNR_s^{(k)}}{\beta(D^2 + \frac{1}{2})SNR_s^{(k)} + D^2} \tag{4.40}$$

where $\left\| \mathbf{h}_k \right\|^2$ is the norm-squared of the channel filter for desired user with density given in (4.22). Hence, the mean SIR with RAKE receiver under Gaussian approximation is given by:

$$\mathcal{E}\left\{ SIR_k^{(\mathrm{RAKE,GA})} \right\} = \frac{(D^2 + \alpha)SNR_s^{(k)}}{\beta(D^2 + \frac{1}{2})SNR_s^{(k)} + D^2} \tag{4.41}$$

since $\mathcal{E}\left\{ \lambda_{kk}^2 \right\} = 1 + \frac{1}{DL}$ and $\mathcal{E}\left\{ \left\| \mathbf{h}_k \right\|^2 \right\} = D$.

4.4.2 Linear Decorrelating Interference Supression

Linear decorrelating interference suppression multiuser receiver as identified in [Zvonar1994, Zvonar1995, Brady1996, Zvonar1996] applies the linear estimator matrix $\mathbf{M}^{(\mathrm{Dec})} = \mathbf{R}^{-1}$ after chip-matched filtering by $\mathbf{S}^H$, resulting in complete multi-access interference suppression via zero-forcing sense equalization. The soft-output conditional random SIR of user kth achievable by decorrelating multiuser detector followed by maximal-ratio-combining is given by:

$$SIR_k^{(\mathrm{Dec})} =_{\mathcal{D}} \frac{D\mathbf{h}_k^H \mathbf{h}_k SNR_s^{(k)}}{\mathcal{E}\left\{\left|\mathbf{h}_k^H \mathbf{w}_k^{(\mathrm{Dec})}\right|^2\right\}} \tag{4.42}$$

The total power of the noise term $\mathcal{E}\left\{\left|\mathbf{h}_k^H \mathbf{w}_k^{(\mathrm{Dec})}\right|^2\right\}$ can be further expanded via eigenvalue decomposition and averaging lemma (Lemma 4.1.) as:

$$\begin{aligned}
\mathcal{E}\left\{\left|\mathbf{h}_k^H \mathbf{w}_k^{(\mathrm{Dec})}\right|^2\right\} = \mathcal{E}\{\mathbf{h}_k^H \mathbf{R}_{kk}^{-1} \mathbf{h}_k\} = \\
\mathcal{E}\{\mathbf{h}_k^H \mathbf{U}_{kk}^{-1} \mathbf{D}_{kk}^{-1} \mathbf{U}_{kk}^{-H} \mathbf{h}_k\} = \\
\mathcal{E}\left\{\sum_{i=1}^{D} \lambda_{kki}^{-1} \left|\breve{h}_{ki}\right|^2\right\} = \\
D\int_0^\infty \int_0^\infty \lambda_{kk}^{-1} r f_{\mathbb{R}_{kk}}(\lambda_{kk}) f_G^{(ns)}(r) d\lambda_{kk} dr = \\
D^2 \int_0^\infty \lambda_{kk}^{-1} f_{\mathbb{R}_{kk}}(\lambda_{kk}) d\lambda_{kk} = \\
D^2 \frac{\left(\frac{D^2}{\alpha}\right)^{\left(\frac{D^2}{\alpha}\right)}}{\Gamma_{\mathrm{c}}(\frac{D^2}{\alpha})} \int_0^\infty \lambda_{kk}^{DL-2} \exp(-DL\lambda_{kk}) d\lambda_{kk} = \\
D^2 \frac{\left(\frac{D^2}{\alpha}\right)^{\left(\frac{D^2}{\alpha}\right)}}{\Gamma_{\mathrm{c}}(\frac{D^2}{\alpha})} \frac{\Gamma_{\mathrm{c}}(\frac{D^2}{\alpha}-1)}{\left(\frac{D^2}{\alpha}\right)^{\left(\frac{D^2}{\alpha}-1\right)}}
\end{aligned} \tag{4.43}$$

since the covariance matrix of the noise vector $\mathbf{w}_k^{(\mathrm{Dec})}$ is $\mathcal{E}\left\{\mathbf{w}_k^{(\mathrm{Dec})}\left(\mathbf{w}_k^{(\mathrm{Dec})}\right)^H\right\} = \mathbf{R}_{kk}^{-1}$, $\int_0^\infty t^{n-1}\exp(-at)dt = \frac{\Gamma_{\mathrm{c}}(n)}{a^{n+1}}$ and $\breve{\mathbf{h}}_k{}^H \breve{\mathbf{h}}_k =_{\mathcal{D}} \mathbf{h}_k^H \mathbf{h}_k$ for $\breve{\mathbf{h}}_k = \mathbf{U}_{kk}^{-H}\mathbf{h}_k$ and unitary $\mathbf{U}_{kk}$. Hence, the SIR of kth user with linear decorrelating interference suppression multiuser receiver followed by MRC is derived to be distributed as:

$$SIR_k^{(\mathrm{Dec})} =_{\mathcal{D}} \frac{\Gamma_{\mathrm{c}}(\frac{D^2}{\alpha}) \left(\frac{D^2}{\alpha}\right)^{\left(\frac{D^2}{\alpha}-1\right)} \|\mathbf{h}_k\|^2 SNR_s^{(k)}}{D\left(\frac{D^2}{\alpha}\right)^{\left(\frac{D^2}{\alpha}\right)} \Gamma_{\mathrm{c}}(\frac{D^2}{\alpha}-1)} \tag{4.44}$$

that is a scaled noncentral chi-squared rv. Simplifying the above expression (4.44):

$$SIR_k^{(\mathrm{Dec})} =_{\mathcal{D}} \frac{(\frac{D^2}{\alpha}-1)\|\mathbf{h}_k\|^2 SNR_s^{(k)}}{D\left(\frac{D^2}{\alpha}\right)} \tag{4.45}$$

Following, the mean of the SIR with decorrelating receiver and MRC is given by:

$$\mathcal{E}\left\{SIR_k^{(\mathrm{Dec})}\right\} = \frac{(\frac{D^2}{\alpha}-1)SNR_s^{(k)}}{\left(\frac{D^2}{\alpha}\right)} \tag{4.46}$$

since $\mathcal{E}\left\{\|\mathbf{h}_k\|^2\right\} = D$.

4.4.3 Linear Minimum Mean-Squared Error (LMMSE) Interference Suppression

Linear MMSE interference suppression multiuser receiver as identified in [Zvonar1994, Zvonar1995, Brady1996, Zvonar1996] applies the joint linear estimator matrix $\mathbf{M}^{(\mathrm{LMMSE})}=\left[\mathbf{R}+\left(SNR_s^{(k)}\mathbf{\Psi}\right)^{-1}\right]^{-1}$ after chip-matched filtering by $\mathbf{S}^H$ where the matrix $\mathbf{\Psi}=\mathrm{E}\{\mathbf{H}\mathbf{H}^H\}$ is the channel power profile matrix that is equivalent to the identity matrix $\mathbf{I}_{KD\times KD}$ due to the uniform multipath intensity profile assumption. Following and extending the derivation of Verdu [20] over Gaussian channels, the soft-output conditional random SIR of user kth achievable by LMMSE multiuser detector followed by maximal-ratio-combining is given by:

$$SIR_k^{(\mathrm{LMMSE})}=_{\mathcal{D}}\mathbf{h}_k^H\mathbf{S}_k^H\mathbf{\Sigma}_k^{-1}\mathbf{S}_k\mathbf{h}_k SNR_s^{(k)} \tag{4.47}$$

where:

$$\mathbf{\Sigma}_k=\sum_{\acute{k}=1,\acute{k}\neq k}^{K}\mathbf{S}_{\acute{k}}\mathbf{S}_{\acute{k}}^H SNR_s^{(k)}+\mathbf{I}_L \tag{4.48}$$

is the random total MAI plus noise covariance matrix and $\mathbf{S}_{\acute{k}}$ is the $L\times D$ chip-shifted spreading sequences matrix for $\acute{k}$th user. Since the direct derivation of the pdf of $\mathbf{\Sigma}_k$ is cumbersome, we employ a Gaussian type approximation on $\mathbf{\Sigma}_k$ for β sufficiently larger than unity and imposing a relaxation of independence on the entries of $\mathbf{S}_{\acute{k}}$ for every $\acute{k}$:

$$\tilde{\mathbf{\Sigma}}_k=\beta D\mathcal{E}\left\{\lambda_{\mathbf{W}}\right\}SNR_s^{(k)}\mathbf{I}_L+\mathbf{I}_L \tag{4.49}$$

where the random matrix in the form $\mathbf{W}=\mathbf{S}\mathbf{S}^H$ is well-known to belong in the *complex-Wishart Ensembles* $\mathcal{WH}(n,m)$ random matrix class, and the eigenvalue density of a complex-Wishart random matrix for finite n and m parameters with $n=L$ and $m=D$ in our case is known to obey *complex-Wishart density* [Theorem

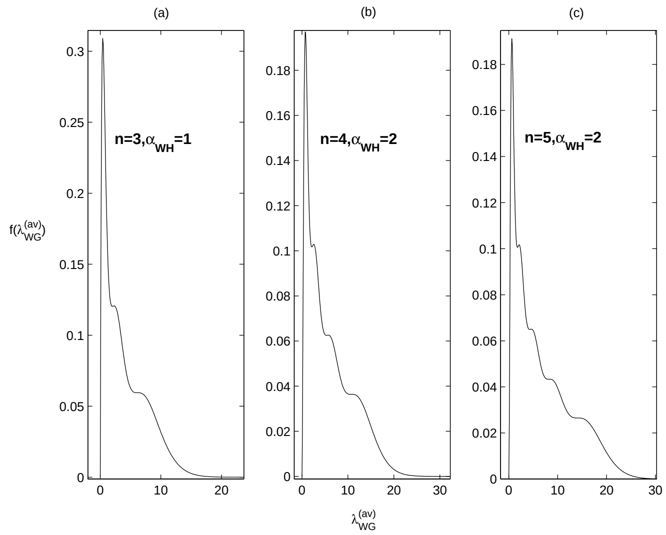

Figure 4.3: Wishart densities.

5.4, [Haagerup1998]] with the analytic form:

$$f_{WH(m,n,1)}(\lambda) = \frac{1}{m}\sum_{k=0}^{m-1}\left[\psi_k^{n-m}(\lambda)\right]^2,\ \lambda \geq 0 \tag{4.50}$$

in terms of the sequence of Laguerre functions; $\psi_k^{\alpha}(x) = \left[\frac{k!}{(k+\alpha)!}x^{\alpha}e^{-x}\right]^{\frac{1}{2}} L_k^{\alpha}(x),\ k \in \mathbb{N},\ x \geq 0$; where $L_k^{\alpha}(x) = \frac{1}{k!}x^{-\alpha}e^{x}\frac{d^k}{dx^k}\left[x^{k+\alpha}e^{-x}\right],\ k \in \mathbb{N},\ x \geq 0$, is the αth order kth Laguerre polynomial. Furthermore, by Hanlon-Stanley-Stembridge formula, we have the following result for the non-normalized moments of the complex-Wishart density:

Theorem 4.3 (Hanlon, Stanley and Stembridge, [Hanlon1992]): Let $\mathbf{W}(m,n,1)$ be a complex-Wishart random matrix. Then, for any p in $\mathbb{N}$:

$$\begin{aligned}\mathcal{E}\left\{Tr_n[\mathbf{W}^p]\right\} = mn(p-1)! \sum_{j=0}^{\lceil\frac{p-1}{2}\rceil} \frac{1}{j+1} \binom{m-1}{j} \\ \times \binom{n-1}{j}\binom{m+n+p-2j-2}{p-2j-1}\end{aligned} \tag{4.51}$$

♦

From Hanlon-Stanley-Stembridge formula, we find that:

$$\mathcal{E}\left\{\lambda_{\mathbf{W}}\right\} = \mathcal{E}\left\{\frac{1}{n}Tr_n[\mathbf{W}^1]\right\} = m = D \tag{4.52}$$

and inserting $\tilde{\mathbf{\Sigma}}_k$ in (4.47), the SIR of kth user with LMMSE receiver and MRC under Gaussian assumption used is distributed as:

$$SIR_k^{(\mathrm{LMMSE,GA})} =_{\mathcal{D}} \frac{D\lambda_{kk}. \left\|\mathbf{h}_k\right\|^2 SNR_s^{(k)}}{1+\beta D^2 SNR_s^{(k)}} \tag{4.53}$$

and hence, the mean SIR is:

$$\mathcal{E}\left\{SIR_k^{(\mathrm{LMMSE,GA})}\right\} = \frac{D^2 SNR_s^{(k)}}{1+\beta D^2 SNR_s^{(k)}} \tag{4.54}$$

since $\mathcal{E}\left\{\lambda_{kk}\right\} = 1$ and $\mathcal{E}\left\{\left\|\mathbf{h}_k\right\|^2\right\} = D$.

4.5 Ergodic Spectral Efficiencies

4.5.1 Ergodic Spectral Efficiency of CW-VMAC

Determination of spectral efficiency achievable by multiuser communication systems is especially important to determine the maximum data rate for transmission given a specific system bandwidth and conversely to determine the minimum bandwidth for transmission at a given data rate. In the ergodic case, the spectral efficiency over distribution ergodic CW-VMAC model identified in (4.13) can be formulated via the Cover-Wyner capacity region of the classical discrete-time memoryless Gaussian multiple-access channel model of Example 3.3. based on the optimal joint-ML multiuser decoding of all users' codebooks.

Let $\mathbf{x} = [x_1 \;\; x_2 ... x_K]^T$ be the vector of input symbols at channel symbol signalling interval n. Dropping the time-index due to ergodicity, the capacity region over CW-VMAC for the subset Γ of users out of all users subset $\mathcal{K}$ where $\mathcal{K} = \Gamma \cup \Gamma^c$ is given by the convex hull of the $|\Gamma|-$tuple rates of each user in $|\Gamma|-$dimensional space defined by $2^{|\Gamma|}-1$ linear sum constraints such that:

$$\sum_{i\in\Gamma} R_i \leq \mathcal{E}\left\{I_{\max}(x_\Gamma; r|x_{\Gamma^c})\right\}, \forall\Gamma \in \mathcal{K} \tag{4.55}$$

for the maximizing product distribution $\prod_{i\in\mathcal{K}} p(x_i)$.

Focusing on the sum-rates capacity $\sum_{i\in K} R_i$, the total ergodic spectral efficiency - sum-rates capacity per temporal degrees of freedom where temporal degrees of freedom is the spread factor L - over CW-VMAC based on Gaussian random coding that maximizes the mutual information is given by:

$$\bar{\eta}_{\text{sum}}^{(\text{CW}-\text{VMAC})} = \frac{1}{L}\mathcal{E}\left\{\log_2(\det(\mathbf{I}_{K\times K} + \frac{1}{D}\mathbf{H}^H\mathbf{R}\mathbf{H}SNR_s^{(k)}))\right\} \tag{4.56}$$

where the scaling of $SNR_s^{(k)}$ by the diversity order stems from the fact that codeword symbols of each user is transmitted over D independently flat-fading channels resulting in symbol signal-to-noise ratio per channel definition.

The ergodic capacity expression in (4.57) can be expressed as

$$\bar{\eta}_{\text{sum}}^{(\text{CW}-\text{VMAC})} = \frac{1}{L}\mathcal{E}\left\{\log_2(\prod_{i=1}^{K}(1 + \frac{1}{D}\mathbf{h}_k^H\mathbf{R}_{kk}\mathbf{h}_k SNR_s^{(k)}))\right\} \tag{4.57}$$

and further via EVD as

$$\bar{\eta}_{\text{sum}}^{(\text{CW}-\text{VMAC})} = \frac{1}{L}\mathcal{E}\left\{\sum_{i=1}^{K}\log_2(1 + \frac{1}{D}\sum_{d=1}^{D}\lambda_{kkd}\left|h_{kd}\right|^2 SNR_s^{(k)})\right\} \tag{4.58}$$

Since the statistics of random spreading sequences and the users' channels are identical over the set of users, an upper-bound on the ergodic capacity of CW-VMAC via converse Jensen's inequality due to concavity of log(.) function - $\mathcal{E}\left\{\log(f(x))\right\} \leq \log(f(\mathcal{E}\{x\}))$ - and the averaging lemma (Lemma 4.1) is found as

$$\bar{\eta}_{ub}^{(\text{CW}-\text{VMAC})} = \beta\log_2\left(1 + SNR_s^{(k)}\right) \tag{4.59}$$

after taking K out and since $\mathcal{E}\left\{\lambda_{kk}\right\} = 1$ and $\mathcal{E}\left\{\left|h_{kd}\right|^2\right\} = 1$.

4.5.2 Ergodic Spectral Efficiency with Linear Multiuser Receivers

Via similar arguments with CW-VMAC, the total ergodic spectral efficiency achievable with any linear multiuser receiver with suboptimal single-user ML decoding by Jensen's inequality is given by:

$$\bar{\eta}_{ub}^{(\text{MUD})} = \beta\mathcal{E}\left\{\log_2\left(1 + \mathcal{E}\left\{SIR_k^{(\text{MUD})}\right\}\right)\right\} \tag{4.60}$$

and hence, the upper-bound ergodic spectral efficiencies with linear multiuser receivers based on the SIR characterizations and the mean SIRs derived in Section 4.4 are as follows:

$$\bar{\eta}_{ub}^{(\mathrm{RAKE,GA})} = \beta \log_2 \left(1 + \frac{(D^2 + \alpha) SNR_s^{(k)}}{\beta (D^2 + \frac{1}{2}) SNR_s^{(k)} + D^2} \right) \tag{4.61}$$

$$\bar{\eta}_{ub}^{(\mathrm{Dec})} = \beta \log_2 \left(1 + \frac{(\frac{D^2}{\alpha} - 1) SNR_s^{(k)}}{(\frac{D^2}{\alpha})} \right) \tag{4.62}$$

$$\bar{\eta}_{ub}^{(\mathrm{LMMSE,GA})} = \beta \log_2 \left(1 + \frac{D^2 SNR_s^{(k)}}{1 + \beta D^2 SNR_s^{(k)}} \right) \tag{4.63}$$

4.5.3 Numerical Results and Discussion

We first provide a Monte-Carlo simulation study for the validation of the analytic upper-bound ergodic spectral efficiencies derived in Fig. 4.4. The results of the Monte-Carlo simulations made by 50 runs of length 100 for both CW-VMAC and with linear multiuser receivers presents close accordance with the derived analytic upper-bound ergodic spectral efficiencies.

At a first glance to the upper-bound ergodic spectral efficiency expressions for CW-VMAC, RAKE, decorrelator and LMMSE in (4.59), (4.61), (4.62) and (4.63) respectively, it can be seen that the maximum spectral efficiency over CW-VMAC is an upper-bound on the maximum spectral efficiency achievable by RAKE, decorrelating and LMMSE multiuser receivers for any finite load factor $\beta > 0$, finite diversity order D and normalized delay spread α as depicted in Fig. 4.4. On the other hand, upper-bound spectral efficiency of RAKE converges to that over CW-VMAC as $\beta \to 0$ that can be viewed as the asymptotic single-user case due to RAKE being the optimal single-user receiver. The upper-bound nature of the maximum spectral efficiency over CW-VMAC stems from the perfect successive-decoding via optimal joint-ML multiuser decoding of all users' codebooks and hence, the maximum spectral efficiency over CW-VMAC in (4.59) also presents a close upper-bound for the maximum spectral efficiency achievable by the optimum non-linear ML multiuser receiver over CW-VMAC. Furthermore,

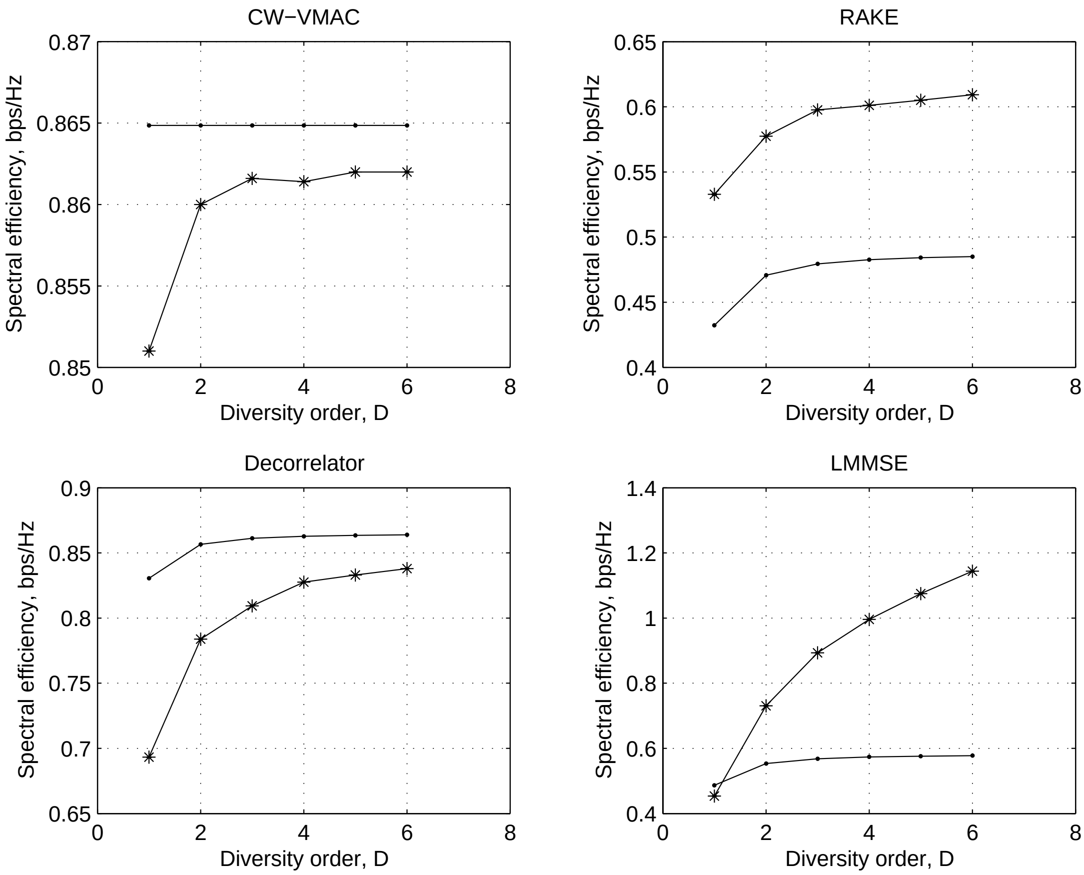

Figure 4.4: Simulated and analytic upper-bound ergodic spectral efficiencies versus diversity order at $\beta = 0.25, \alpha = 0.1$, and SNR_s=10 dB. Lines with * are simulation results.

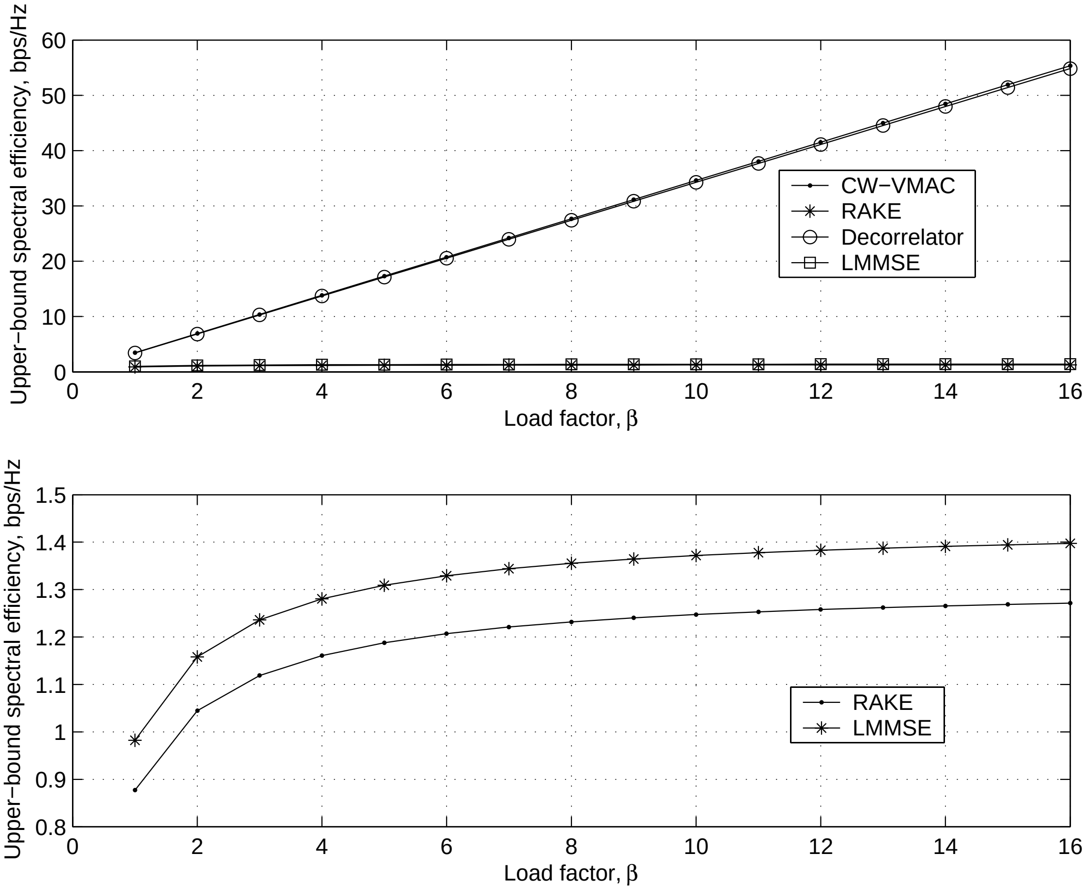

Figure 4.5: Upper-bound ergodic spectral efficiencies versus load factor β at $\alpha = 0.1$, SNR_s=10dB and $D = 2$.

at the asymptotic wideband limit as $D \to \infty$, the upper-bound spectral efficiency over CW-VMAC is equal to and with the decorrelating multiuser receiver converge to the non-fading AWGN-bound spectral efficiency $\beta \log_2(1 + \text{SNR}_\text{s})$ validating and extending to the multiuser case, the Kennedy Law [Kennedy1969] and Telatar-Tse conjecture [Telatar2000] that the capacity of a single-user wideband multipath fading channel converges asymptotically at the wideband limit to the capacity of the non-fading Gaussian channel with the same average received power.

The upper-bound spectral efficiency achievable with decorrelator is a strictly increasing function of the diversity order D due to perfect multi-access interference suppression and optimal linear diversity-combining via maximal-ratio-combiner. However, the maximum spectral efficiency of the decorrelator converges from below to the non-fading AWGN-bound spectral efficiency at the wideband limit due to the desired signal loss and noise enhancement stemming from the use of random spreading sequences and the equalization in the zero-forcing sense, respectively. Thus, the decorrelating multiuser receiver asymptotically equalizes the time-varying multipath fading CDMA channel at the wideband limit as $D \to \infty$ meriting one of the main philosophies of communication theory that the channel should be made look like AWGN if non-AWGN. On the other hand, the upper-bound spectral efficiencies achievable with linear RAKE and LMMSE multiuser receivers in (4.61) and (4.63) are strictly increasing functions of the diversity order D converging at the asymptotic wideband limit as $D \to \infty$:

$$\lim_{D\to\infty} \bar{\eta}_{ub}^{(\mathrm{RAKE})} = \beta \log_2 \left(1 + \frac{SNR_s^{(k)}}{\beta SNR_s^{(k)} + 1} \right) \tag{4.64}$$

$$\lim_{D\to\infty} \bar{\eta}_{ub}^{(\mathrm{LMMSE})} = \beta \log_2 \left(1 + \frac{1}{1+\beta} \right) \tag{4.65}$$

due to interference limitation.

Within a large-dimensional asymptotic CDMA system with multiuser receivers, the load factor at which the system operates has a significant impact on the spectral efficiency achievable especially when the interference limitation is substantially higher. Via (4.62), it turns out that arbitrarily reliable transmission of arbitrarily large sum-rates information per unit bandwidth is possible with the decorrelating receiver due to perfect multi-access interference suppression as the load factor $\beta \to \infty$, while the upper-bound spectral efficiency with RAKE and LMMSE converge to certain limits as $\beta \to \infty$ due to interference limitation as depicted in Fig. 4.5.

The level of time-dispersivity of the channel measured by the per-chip diversity order or normalized delay spread parameter $\alpha \in (0, 1)$ that is the time-dispersion per unit channel-symbol signalling period also has a substantial impact on the achievable spectral efficiencies with decorrelating multiuser receivers as depicted in Fig. 4.6. The upper-bound spectral efficiency with decorrelating multiuser

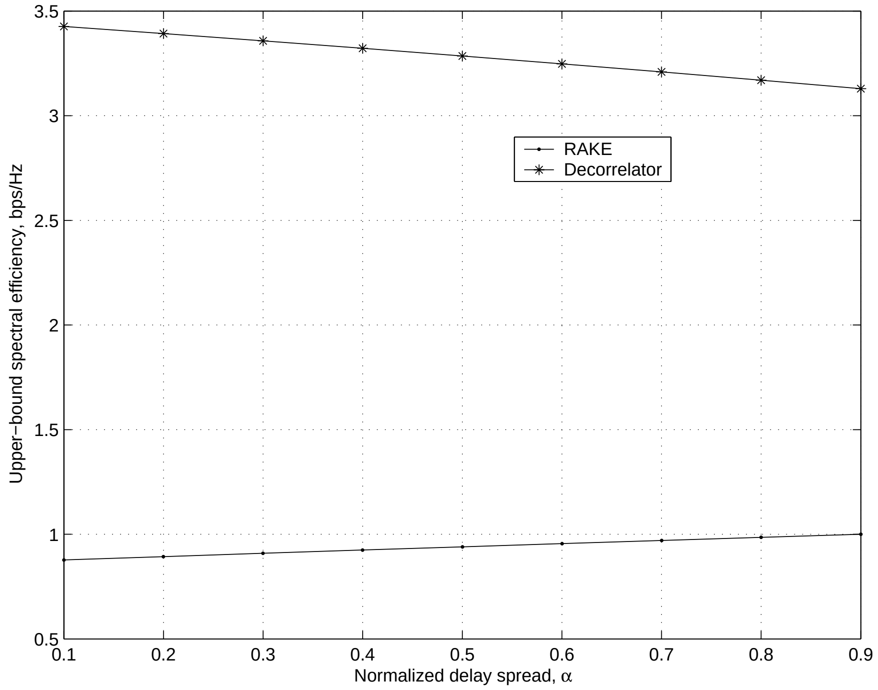

Figure 4.6: Upper-bound ergodic spectral efficiencies versus normalized delay spread α at $\beta = 1$, SNR_s=10dB and $D = 2$.

receivers is a strictly decreasing function of the normalized delay spread α since increased delay spread is detrimental for zero-forcing equalization. Furthermore, the upper-bound spectral efficiency with RAKE receiver slightly increases with increased delay spread.

The analysis of the upper-bound spectral efficiencies at high $SNR_s^{(k)}$ region, i.e. $SNR_s^{(k)} \to \infty$, also presents importance in terms of the power-efficiency comparison. As depicted in Fig. 4.7, it is possible to transmit arbitrarily large sum-rates information per unit processing gain over CW-VMAC and with the decorrelating multiuser receiver as $SNR_s^{(k)} \to \infty$, while the upper-bound ergodic spectral efficiencies with RAKE and LMMSE saturate as $SNR_s^{(k)} \to \infty$ converging to the following limits respectively:

$$\lim_{SNR_s^{(k)} \to \infty} \bar{\eta}_{ub}^{(\text{RAKE})} = \beta \log_2 \left(1 + \frac{(D^2 + \alpha)}{\beta(D^2 + \frac{1}{2})} \right) \tag{4.66}$$

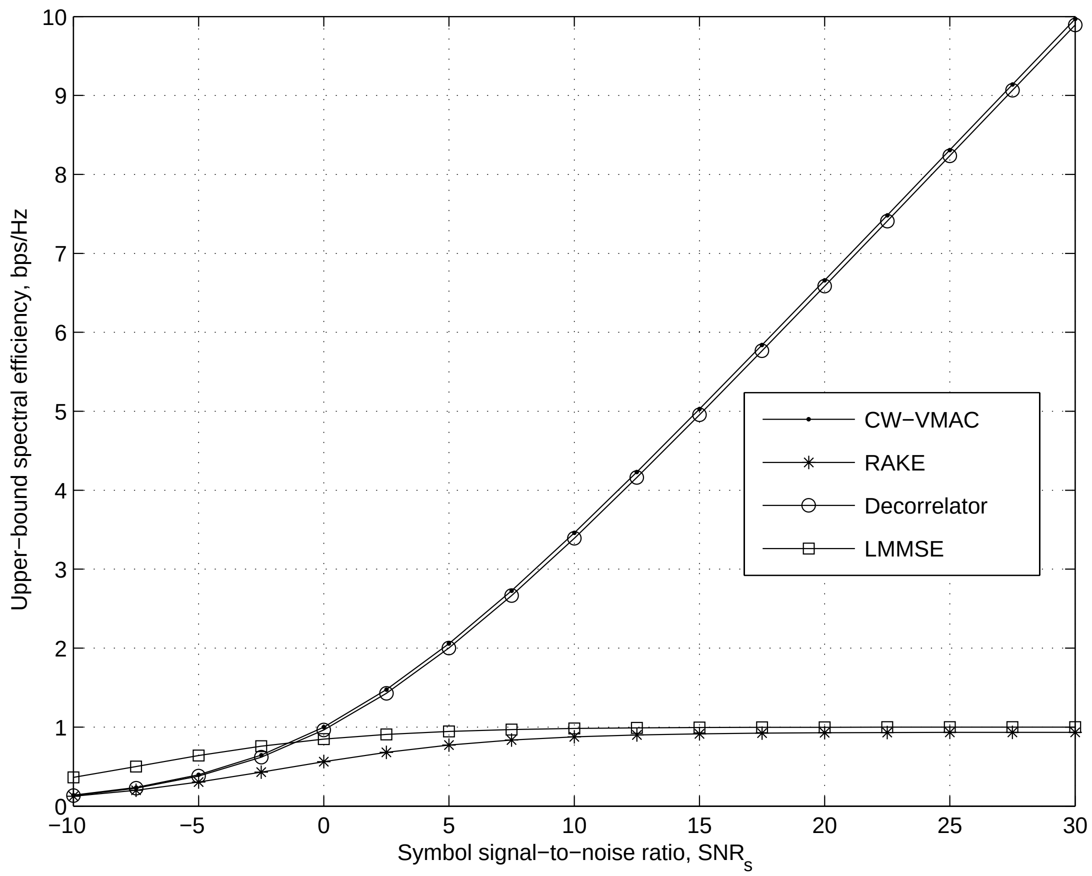

Figure 4.7: Upper-bound ergodic spectral efficiencies versus SNR_s at $\beta = 1$, $\alpha = 0.1$ and $D = 2$.

$$\lim_{SNR_s^{(k)}\to\infty} \bar{\eta}_{ub}^{(\mathrm{LMMSE})} = \beta \log_2\left(1+\frac{1}{\beta}\right) \tag{4.67}$$

due to interference limitation. Moreover, it turns out that both high and low SNR asymptote upper-bound spectral efficiency with LMMSE receiver converge to RAKE receiver in the multichannel scenario and can be higher than that of decorrelator at low SNR. Furthermore, for the power-bandwidth efficiency comparison based on spectral efficiency, the upper-bound spectral efficiency expressions further have to be expressed in terms of bit signal-to-noise ratio $SNR_b^{(k)} = \frac{E_b}{N_0}$ and the relation between $SNR_s^{(k)}$ and $SNR_b^{(k)}$ is given by [VerduShamai2001]:

$$SNR_b^{(k)} = \frac{\beta}{\eta_{\mathrm{sum}}} SNR_s^{(k)} \tag{4.68}$$

since spectral efficiency η_{sum} is the total number of bits transmitted reliably per unit processing gain for K users while the symbol energy for noise PSD level $N_0 = 1$ is equal to $SNR_s^{(k)}$. Thus, the minimum bit signal-to-noise ratio $\left(SNR_b^{(k)}\right)_{\min}$ required for reliable communication over CW-VMAC and by RAKE, decorrelating and LMMSE multiuser receivers are derived as $\mathrm{SNR}_s \to 0$ via L'Hopital's rule limit respectively as:

$$\left(SNR_b^{(k)}\right)_{\min}^{(\mathrm{CW-VMAC})} = \ln 2, \tag{4.69}$$

$$\left(SNR_b^{(k)}\right)_{\min}^{(\mathrm{RAKE})} = \frac{D^2}{D^2+\alpha} \ln 2, \tag{4.70}$$

$$\left(SNR_b^{(k)}\right)_{\min}^{(\mathrm{Dec})} = \frac{\left(\frac{D^2}{\alpha}\right)}{\left(\frac{D^2}{\alpha}-1\right)} \ln 2, \tag{4.71}$$

$$\left(SNR_b^{(k)}\right)_{\min}^{(\mathrm{LMMSE})} = \frac{1}{D^2} \ln 2, \tag{4.72}$$

Hence, as depicted in Fig. 4.8, $\left(SNR_b^{(k)}\right)_{\min}^{(\mathrm{CW-VMAC})}$ is equivalent to the single-user Shannon-bound $10\log_{10}(\ln(2)) \simeq -1.6$ dB irrespective of the diversity order. Furthermore, $\left(SNR_b^{(k)}\right)_{\min}^{(\mathrm{RAKE})}$ is a strictly increasing function of diversity order converging to single-user Shannon-bound as the diversity order tends to infinity.

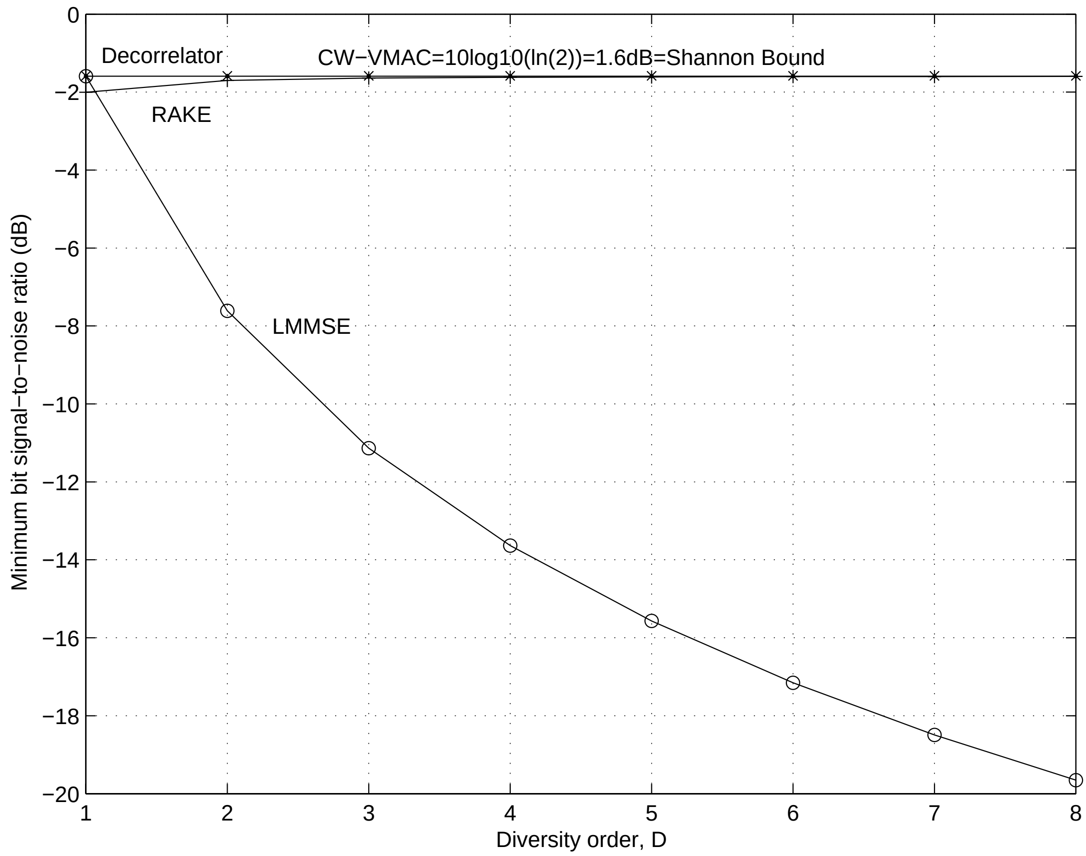

Figure 4.8: Minimum bit signal-to-noise ratios versus diversity order D at $\alpha = 0.1$.

On the other hand, $\left(SNR_b^{(k)}\right)_{\min}^{(\mathrm{Dec})}$ is strictly decreasing function of the diversity order converging to the single-user Shannon-bound $10\log_{10}(\ln(2)) \simeq -1.6$ dB at the asymptotic wideband limit as $D \to \infty$, while $\left(SNR_b^{(k)}\right)_{\min}^{(\mathrm{LMMSE})}$ strictly decreases as $D \to \infty$. Furthermore, the minimum bit signal-to-noise ratios required for reliable transmission over CW-VMAC and by RAKE, decorrelating and LMMSE multiuser receivers are independent of the load factor β.

4.6 Delay-limited Outage Spectral Efficiencies

The analysis of the outage capacity over fading channels and especially within CDMA context lacks quite attention in the literature. Out of the few previous contributions Ozarow et al. first analyzed the outage capacity for a TDMA system over two-path fading channel in [Ozarow1994]. Furthermore, Veerevalli and Mantravadi in [Veerevalli2002] analyzed the coding-spreading trade-off for CDMA systems with Gaussian noise.

When the ergodicity assumption holds, the Shannon-sense sum-rates capacity is observable by considering large block lengths and is equivalent to the infinite-length time-series averaging of the conditional maximum mutual-information random variables in each channel-symbol signalling interval as

$$C_{\mathrm{sum}}^{\mathrm{Shannon}} = \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} I_{\max}^{\mathrm{sum}}(n) \tag{4.73}$$

where $I_{\max}^{\mathrm{sum}}(n)$ is the conditional sum-rates maximum mutual-information random variable in the nth channel-symbol signalling interval. Via law of large numbers, (4.66) is equivalent to the expectation $\mathcal{E}\left\{I_{\max}^{\mathrm{sum}}\right\}$ in an arbitrary channel-symbol signalling interval due to ergodicity that results in the averaging out of the time-varying fading channel parameters as well as the noise in the system. However, in many practical situations, either the time variations on the fading processes are slow or the transmission is strictly coding-delay limited such as in real-time services so that the ergodicity assumption for the applicability of Shannon-sense capacity does not hold.

Within a slowly-fading coding-delay limited non-ergodic situations where the maximum mutual-information in each channel-symbol signalling interval fluctuates, the *outage capacity* for a finite block length N is the average flow of maximum mutual-information over the block length and the sum-rates outage capacity $C_{\text{sum}}^{\text{out}}(N)$ that is a random variable can be defined as

$$C_{\text{sum}}^{\text{out}}(N) = \frac{1}{N}\sum_{n=1}^{N} I_{\text{max}}^{\text{sum}}(n) \tag{4.74}$$

and an information outage is said to occur with probability-of-outage P_{out} if the sum-rates outage capacity $C_{\text{sum}}^{\text{out}}(N)$ over the block length N falls below a target information rate R_{out} where P_{out} is by definition:

$$\text{P}_{\text{out}} = \text{Prob}\left\{ C_{\text{sum}}^{\text{out}}(N) = \frac{1}{N}\sum_{n=1}^{N} I_{\text{max}}^{\text{sum}}(n) < R_{\text{out}} \right\} \tag{4.75}$$

that corresponds to the reliable transmission of information in the outage-sense over the block length of N channel-symbols at an average rate of R_{out} with probability $1 - \text{P}_{\text{out}}$.

4.6.1 Derivation of Total Outage Spectral Efficiencies

The sum-rates outage capacity based spectral efficiencies over CW-VMAC followed by optimal joint-ML multiuser decoding and with multiuser receivers followed by suboptimal single-user ML decoding can be further formulated with the following definition of the probability-of-outage as

$$\text{P}_{\text{out}} = \text{Prob}\left\{ \eta_{\text{sum}}^{\text{out}}(N) = \frac{1}{N}\sum_{n=1}^{N} \frac{1}{L} I_{\text{max}}^{\text{sum}}(n) < \eta_{\text{out,sum}}^{*} \right\}, \tag{4.76}$$

since spectral efficiency of a CDMA system is the capacity per temporal degrees of freedom or the spread factor L. The maximum sum-rates mutual information in bits per channel use maximized over the Gaussian input distribution for CW-VMAC is as follows:

$$I_{\max}^{(\mathrm{sum,CW-VMAC})}(n) = \sum_{k=1}^{K} \log_2\left(1 + \frac{1}{D}\mathbf{h}_k^H \mathbf{R}_{kk}\mathbf{h}_k SNR_s^{(k)}\right) \qquad (4.77)$$

Furthermore, based on the SIR characterizations with linear multiuser receivers for an arbitrary user that is statistically identical over the set of users $\mathcal{K}$ and via the approximate Gaussianity of MAI plus noise terms, the maximum sum-rates mutual information in bits per channel use with linear multiuser receivers is as follows:

$$I_{\max}^{(\mathrm{sum,MUD})}(n) = \sum_{k=1}^{K} \log_2\left(1 + \mathrm{SIR}_k^{(\mathrm{MUD})}\right) \qquad (4.78)$$

Thus, we express the total outage spectral efficiencies both over CW-VMAC and with linear multiuser receivers changing to natural basis as follows:

$$P_{\mathrm{out}}^{(\mathrm{CW-VMAC})} = \mathrm{Prob}\left\{ \begin{array}{c} \eta_{\mathrm{out,sum}}^{(\mathrm{CW-VMAC})}(N) = \frac{1}{NL\ln(2)} \\ \times \sum_{k=1}^{K}\sum_{n=1}^{N} \ln\left(1 + \mathbf{h}_k^H(n)\mathbf{R}_{kk}(n)\mathbf{h}_k(n)\frac{SNR_s^{(k)}}{D}\right) \\ < \eta_{\mathrm{out,sum}}^{*} \end{array} \right\} \qquad (4.79)$$

$$P_{\mathrm{out}}^{(\mathrm{MUD})} = \mathrm{Prob}\left\{ \begin{array}{c} \eta_{\mathrm{out,sum}}^{(\mathrm{MUD})}(N) = \frac{1}{NL\ln(2)} \\ \times \sum_{k=1}^{K}\sum_{n=1}^{N} \ln(1 + \mathrm{SIR}_k^{(\mathrm{MUD})}(n)) \\ < \eta_{\mathrm{out,sum}}^{*} \end{array} \right\} \qquad (4.80)$$

Following, using Gaussian assumption for large block lengths for the total outage spectral efficiency random variables, we define the probability of outage as follows:

$$P_{\mathrm{out}} = 1 - \frac{1}{2} erfc\left(\frac{\eta_{\mathrm{out,sum}}^{*} - \mu_{\eta_{\mathrm{out,sum}}(N)}}{\sqrt{2\sigma^2_{\eta_{\mathrm{out,sum}}(N)}}}\right) \qquad (4.81)$$

where $\mu_{\eta_{\mathrm{out,sum}}(N)}$ and $\sigma^2_{\eta_{\mathrm{out,sum}}(N)}$ are the mean and variance of the total outage spectral efficiency random variable and $erfc(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty}\exp(-t^2)dt$ is the complementary Gaussian error function.

Starting with CW-VMAC, the mean of the total outage spectral efficiency random variable of CW-VMAC is given by:

$$\mathcal{E}\left\{\eta_{\text{out,sum}}^{(\text{CW-VMAC})}(N)\right\}=\mathcal{E}\left\{\frac{1}{NL\ln(2)}\times\sum_{k=1}^{K}\sum_{n=1}^{N}\ln\left(1+\mathbf{h}_k^H(n)\mathbf{R}_{kk}(n)\mathbf{h}_k(n)\frac{SNR_s^{(k)}}{D}\right)\right\} \tag{4.82}$$

that is equivalent via EVD to:

$$\mathcal{E}\left\{\eta_{\text{out,sum}}^{(\text{CW-VMAC})}(N)\right\}=\frac{\beta}{\ln(2)}\mathcal{E}\left\{\ln\left(1+\sum_{d=1}^{D}\lambda_{kkd}\left|h_{kd}\right|^2\frac{SNR_s^{(k)}}{D}\right)\right\} \tag{4.83}$$

Since $\mathcal{E}\left\{\left|h_{kd}\right|^2\right\}=1$, this expression is equivalent using averaging lemma reversely to:

$$\mathcal{E}\left\{\eta_{\text{out,sum}}^{(\text{CW-VMAC})}(N)\right\}=\frac{\beta}{\ln(2)}\mathcal{E}\left\{\ln\left(1+\lambda_{kk}SNR_s^{(k)}\right)\right\} \tag{4.84}$$

Due to direct derivation being cumbersome, we make use of the Taylor Series method of Papoulis [Papoulis1991] for finding the mean and the variance of a function $y=g(x)$ of an rv x as

$$\mu_y=g(\mu_x),\sigma_y^2=\left|g'(\mu_x)\right|^2\sigma_x^2 \tag{4.85}$$

Hence, we derive approximately for the mean of the total outage spectral efficiency random variable of CW-VMAC as follows:

$$\mathcal{E}\left\{\eta_{\text{out,sum}}^{(\text{CW-VMAC})}(N)\right\}\cong\beta\log_2\left(1+SNR_s^{(k)}\right) \tag{4.86}$$

Via Taylor Series method, for the variance of the total outage spectral efficiency random variable of CW-VMAC, we have:

$$Var\left\{\eta_{\text{out,sum}}^{(\text{CW-VMAC})}(N)\right\}\cong\frac{1}{N}\frac{\beta^2}{L\left(\ln(2)\right)^2}\left(\frac{SNR_s^{(k)}}{1+SNR_s^{(k)}}\right)^2\frac{1}{DL} \tag{4.87}$$

Finally, we state that the total outage spectral efficiency random variable with CW-VMAC is approximately distributed as

$$\eta_{\text{out,sum}}^{(\text{CW-VMAC})}(N)\tilde{=}_{\mathcal{D}}N\left(\ \beta\log_2\left(1+SNR_s^{(k)}\right),\frac{\beta^2}{N(\ln(2))^2}\left(\frac{SNR_s^{(k)}}{1+SNR_s^{(k)}}\right)^2\frac{\alpha^2}{D^3}\ \right) \tag{4.88}$$

where $G(\mu, \sigma^2)$ stands for the real Gaussian distribution with mean μ and variance σ^2.

Based on the SIR characterization in Section 4.4, for the approximate mean of the total outage spectral efficiency random variable of RAKE under Gaussian assumption, we have after EVD and using averaging lemma reversely:

$$\mathcal{E}\left\{\eta_{\text{out,sum}}^{(\text{RAKE,GA})}(N)\right\} \cong \frac{\beta}{\ln(2)}\mathcal{E}\left\{\ln\left(1+\frac{D\lambda_{kk}^2\left\|\mathbf{h}_k\right\|^2 SNR_s^{(k)}}{\beta(D^2+\frac{1}{2})SNR_s^{(k)}+D^2}\right)\right\} \tag{4.89}$$

Via Taylor Series method, since $\mathcal{E}\left\{\lambda_{kk}^2\left\|\mathbf{h}_k\right\|^2\right\} = (1+\frac{1}{DL})\times D = D+\frac{\alpha}{D}$, we derive the approximate mean of the total outage spectral efficiency random variable of RAKE as

$$\mathcal{E}\left\{\eta_{\text{out,sum}}^{(\text{RAKE,GA})}(N)\right\} \cong \beta\log_2\left(1+\frac{(D^2+\alpha)\,SNR_s^{(k)}}{\beta(D^2+\frac{1}{2})SNR_s^{(k)}+D^2}\right) \tag{4.90}$$

Furthermore, for the variance of the total outage spectral efficiency random variable of RAKE, we have:

$$Var\left\{\eta_{\text{out,sum}}^{(\text{RAKE,GA})}(N)\right\} \cong \frac{\beta^2}{NL\left(\ln(2)\right)^2}Var\left\{\ln\left(1+\frac{D\lambda_{kk}^2\left\|\mathbf{h}_k\right\|^2 SNR_s^{(k)}}{\beta(D^2+\frac{1}{2})SNR_s^{(k)}+D^2}\right)\right\} \tag{4.91}$$

that is equivalent to:

$$Var\left\{\eta_{\text{out,sum}}^{(\text{RAKE,GA})}(N)\right\} \cong \frac{\beta^2}{NL\left(\ln(2)\right)^2}\left(\frac{DSNR_s^{(k)}}{1+DSNR_s^{(k)}}\right)^2 Var\{\lambda_{kk}^2\}Var\{\left\|\mathbf{h}_k\right\|^2\} \tag{4.92}$$

If we use Taylor Series method again as; $Var\{\lambda_{kk}^2\} \cong \frac{4}{DL}$, and since $Var\{\left\|\mathbf{h}_k\right\|^2\} = D(2\sigma_c^2-\sigma_c^4)$, the approximate variance of the total outage spectral efficiency random variable of RAKE is given by:

$$Var\left\{\eta_{\text{out,sum}}^{(\text{RAKE,GA})}(N)\right\} \cong \frac{\beta^2}{N\left(\ln(2)\right)^2}\left(\frac{DSNR_s^{(k)}}{1+DSNR_s^{(k)}}\right)^2\frac{4\alpha(2\sigma_c^2-\sigma_c^4)}{D} \tag{4.93}$$

Hence, we deduce that the total outage spectral efficiency random variable of RAKE is approximately distributed as:

$$\eta_{\text{out,sum}}^{(\text{RAKE,GA})}(N) \tilde{=}_{\mathcal{D}} N \left(\begin{array}{c} \beta \log_2 \left(1 + \frac{\left(D^2+\alpha\right) SNR_s^{(k)}}{\beta(D^2+\frac{1}{2})SNR_s^{(k)}+D^2} \right), \\ \frac{\beta^2}{N(\ln(2))^2} \left(\frac{DSNR_s^{(k)}}{1+DSNR_s^{(k)}} \right)^2 \frac{4\alpha(2\sigma_c^2-\sigma_c^4)}{D} \end{array} \right) \tag{4.94}$$

We have derived in Section 4.4 that the SIR of decorrelator with MRC is distributed as:

$$SIR_k^{(\text{Dec})} =_{\mathcal{D}} \frac{(\frac{D^2}{\alpha} - 1) \left\| \mathbf{h}_k \right\|^2 SNR_s^{(k)}}{D \left(\frac{D^2}{\alpha} \right)} \tag{4.95}$$

Hence, we have for the approximate mean of the total outage spectral efficiency random variable of decorrelator:

$$\mathcal{E} \left\{ \eta_{\text{out,sum}}^{(\text{Dec})}(N) \right\} \cong \beta \log_2 \left(1 + \frac{(\frac{D^2}{\alpha} - 1)SNR_s^{(k)}}{\left(\frac{D^2}{\alpha} \right)} \right) \tag{4.96}$$

Furthermore, via Taylor Series method, the approximate variance of the total outage spectral efficiency random variable of decorrelator is given by:

$$Var \left\{ \eta_{\text{out,sum}}^{(\text{Dec})}(N) \right\} \cong \frac{\beta^2}{NL\left(\ln(2)\right)^2} \left(\frac{(\frac{D^2}{\alpha} - 1)SNR_s^{(k)}}{1 + (\frac{D^2}{\alpha} - 1)SNR_s^{(k)}} \right)^2 D(2\sigma_c^2 - \sigma_c^4) \tag{4.97}$$

that is equivalent to:

$$Var \left\{ \eta_{\text{out,sum}}^{(\text{Dec})}(N) \right\} \cong \frac{\beta^2 \left(\alpha(2\sigma_c^2 - \sigma_c^4) \right)}{N \left(\ln(2)\right)^2} \left(\frac{(\frac{D^2}{\alpha} - 1)SNR_s^{(k)}}{1 + (\frac{D^2}{\alpha} - 1)SNR_s^{(k)}} \right)^2 \tag{4.98}$$

Hence, we deduce that the total outage spectral efficiency random variable of decorrelator is approximately distributed as:

$$\eta_{\text{out,sum}}^{(\text{Dec})}(N) \tilde{=}_{\mathcal{D}} N \left(\begin{array}{c} \beta \log_2 \left(1 + \frac{(\frac{D^2}{\alpha}-1)SNR_s^{(k)}}{\left(\frac{D^2}{\alpha}\right)} \right), \\ \frac{\beta^2\left(\alpha(2\sigma_c^2-\sigma_c^4)\right)}{N(\ln(2))^2} \left(\frac{(\frac{D^2}{\alpha}-1)SNR_s^{(k)}}{1+(\frac{D^2}{\alpha}-1)SNR_s^{(k)}} \right)^2 \end{array} \right) \tag{4.99}$$

We have further derived in Section 4.4 that the SIR of LMMSE with MRC under Gaussian approximation is approximately distributed as:

$$SIR_k^{(\text{LMMSE,GA})} =_{\mathcal{D}} \frac{D\lambda_{kk} \cdot \|\mathbf{h}_k\|^2 \, SNR_s^{(k)}}{1 + \beta D^2 SNR_s^{(k)}} \tag{4.100}$$

Hence, via Taylor Series method, we have for the approximate mean of the total outage spectral efficiency random variable of LMMSE:

$$\mathcal{E}\left\{\eta_{\text{out,sum}}^{(\text{LMMSE,GA})}(N)\right\} \cong \beta \log_2\left(1 + \frac{D^2 SNR_s^{(k)}}{1 + \beta D^2 SNR_s^{(k)}}\right) \tag{4.101}$$

since $\mathcal{E}\left\{\lambda_{kk} \|\mathbf{h}_k\|^2\right\} = 1 \times D = D$. Furthermore, for the approximate variance of the total outage spectral efficiency random variable of LMMSE, we have via Taylor Series method:

$$Var\left\{\eta_{\text{out,sum}}^{(\text{LMMSE,GA})}(N)\right\} \cong \frac{\beta^2}{NL\left(\ln(2)\right)^2}\left(\frac{DSNR_s^{(k)}}{1 + DSNR_s^{(k)}}\right)^2 \frac{1}{DL} D(2\sigma_c^2 - \sigma_c^4) \tag{4.102}$$

that is equivalent to:

$$Var\left\{\eta_{\text{out,sum}}^{(\text{LMMSE,GA})}(N)\right\} \cong \frac{\beta^2}{N\left(\ln(2)\right)^2}\left(\frac{DSNR_s^{(k)}}{1 + DSNR_s^{(k)}}\right)^2 \frac{\alpha^2(2\sigma_c^2 - \sigma_c^4)}{D^2} \tag{4.103}$$

since $Var\{\lambda_{kk}\} = \frac{1}{DL}$ and $Var\{\|\mathbf{h}_k\|^2\} = D(2\sigma_c^2 - \sigma_c^4)$. Hence, we deduce that the total outage spectral efficiency random variable of LMMSE is approximately distributed as:

$$\eta_{\text{out,sum}}^{(\text{LMMSE,GA})}(N) \,\tilde{=}_{\mathcal{D}}\, N\left(\begin{array}{c} \beta \log_2\left(1 + \frac{D^2 SNR_s^{(k)}}{1+\beta D^2 SNR_s^{(k)}}\right), \\ \frac{\beta^2}{N(\ln(2))^2}\left(\frac{DSNR_s^{(k)}}{1+DSNR_s^{(k)}}\right)^2 \frac{\alpha^2(2\sigma_c^2 - \sigma_c^4)}{D^2} \end{array}\right) \tag{4.104}$$

4.6.2 Numerical Results and Discussion

We first provide semi-analytic Monte-Carlo simulations for the validation of the derived total outage spectral efficiencies in Fig. 4.9. In all 4 cases, the Monte-Carlo simulation results made by 50 runs of length 100 closely match and validate the derived analytic total outage spectral efficiencies.

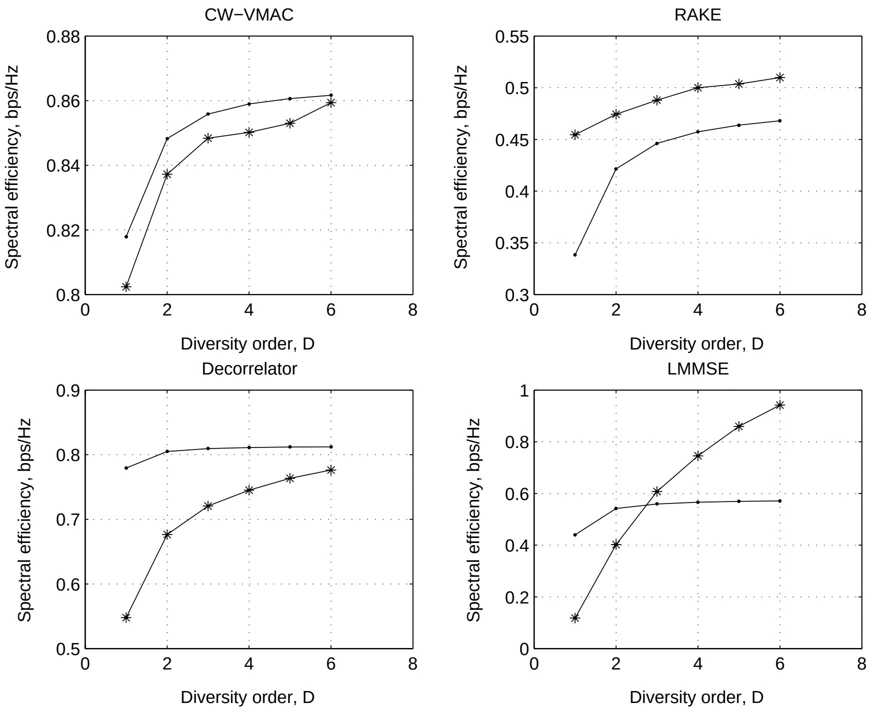

Figure 4.9: Simulated and analytic total outage spectral efficiencies versus codeword length at $\beta = 0.25, \alpha = 0.1$,$\text{SNR}_s$=10 dB, $N = 8$, $\sigma_c^2 = 1$ and $P_{out} = 0.1$. Lines with * are simulation results.

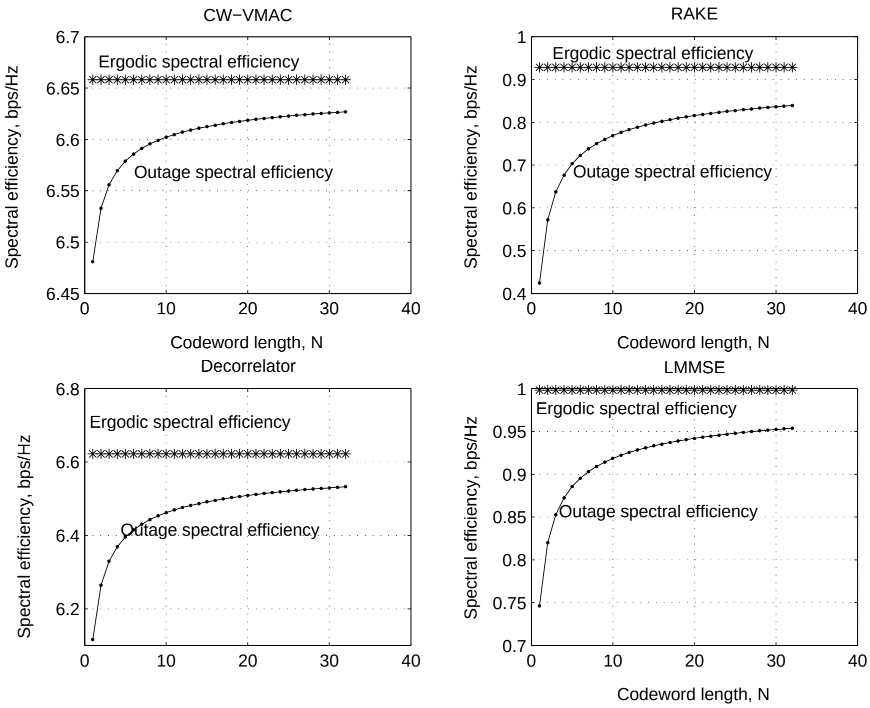

Figure 4.10: Total outage spectral efficiencies versus codeword length at $\beta = 1, \alpha = 0.1$,SNR$_s$=20 dB, $D = 4$, $\sigma_c^2 = 0.5$ and $P_{out} = 0.1$.

We provide the total outage spectral efficiencies versus codeword length N in Fig. 4.10. and versus diversity order D in Fig. 4.11. In both cases the total outage spectral efficiencies derived tend towards the upper-bound ergodic spectral efficiencies derived in Section 4.5.

Total outage spectral efficiencies are depicted versus the variance of the channel coefficients in Fig. 4.12. As expected, as the specularity of the channel increases - variance of the channel coefficients decrease -, the total outage spectral efficiencies tend to be close to the upper-bound ergodic spectral efficiencies derived in Section 4.5.

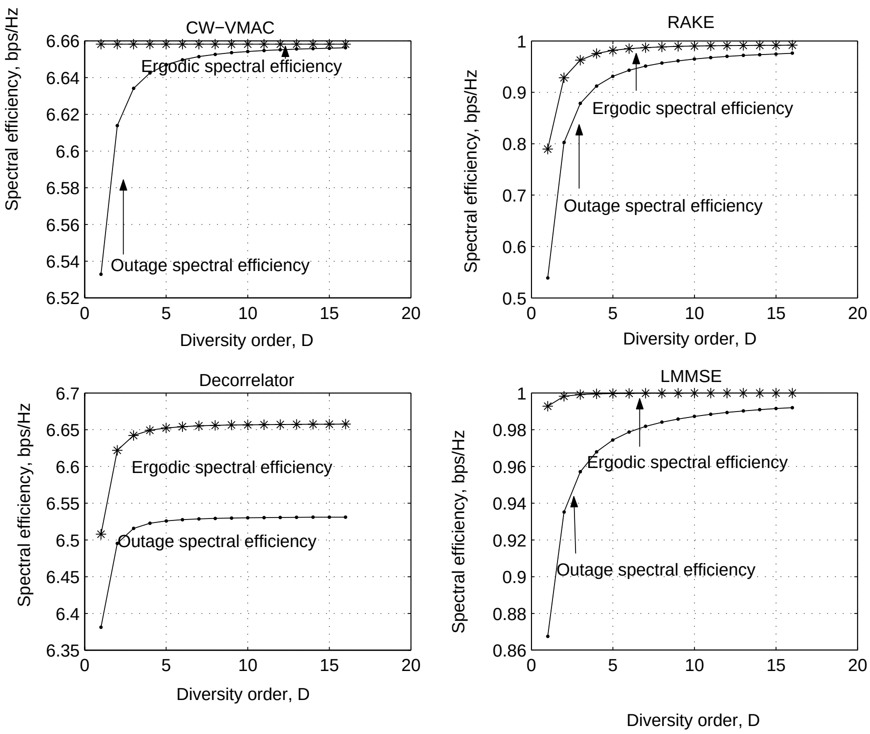

Figure 4.11: Total outage spectral efficiencies versus diversity order at $\beta = 1, \alpha = 0.1$,SNR_s=20 dB, $N = 16$, $\sigma_c^2 = 0.5$ and $P_{out} = 0.1$.

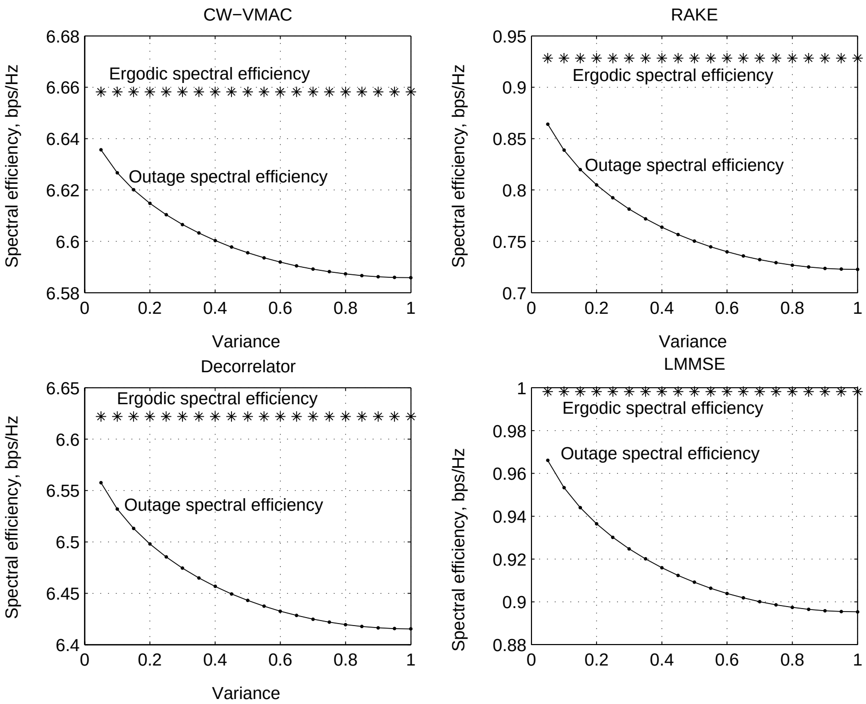

Figure 4.12: Total outage spectral efficiencies versus the variance of the channel coefficients at $\beta = 1, \alpha = 0.1$,SNR$_s$=20 dB, $D = 4$, $N = 16$ and $P_{out} = 0.1$.

Chapter 5

Summary of Contributions and Conclusions

In this thesis, we focused on generalized random spreading performance analysis of CDMA systems with linear multichannel multiuser receivers over time-varying GWSSUS fading channels. Keeping our attention on finite-dimensional analysis, we found spectral efficiencies achievable with linear multichannel multiuser receivers in terms of system parameters both in ergodic and delay-limited transmission situations. In terms of previous parallel work on randomly-spread CDMA, the work presented in this thesis mainly differs in

- Finite-dimensional analysis
- Focus on multipath fading channels

In Chapter 2 , we presented an overview of multiuser detection and related performance measures, fading channels and their statistical characterization, and diversity and combining techniques. In terms of multiuser detection, we presented the multiuser receiver structures and how they are derived as well as the asymptotic multiuser power efficiency performance metric. We later presented the classification of fading channels in terms of time dispersivity, frequency dispersivity and the statistical characterization of GWSSUS fading channels. We finally gave an overview of diversity techniques and different combining techniques with the trade-offs amongst them.

In Chapter 3, we gave an overview of multiuser communications theory with emphasis on forming a ground for our analysis in Chapter 4. We mainly introduced notions of multiaccess channel capacity and in special the Cover-Wyner capacity region for a Gaussian discrete memoryless multiple access channel, multiaccess coding and joint decoding and collision resolution tehcniques.

In Chapter 4, we first reviewed previous parallel work and gave an overview of the uplink CDMA system model we consider with multiuser detection. We defined the correlated-waveform multiple-access channel model for the multichannel CDMA system. Next, we first statistically characterized the SIRs achievable with linear multiuser receivers using the finite-dimensional eigenvalue densities of the submatrices of the random crosscorrelation matrix. Then, using the statistical characterization of SIRs, we derived the upper-bound total ergodic spectral efficiencies and total outage spectral efficiencies. Based on our derivations and analysis of the derivations in terms of the system parameters, our major findings and observations out of this thesis are as follows:

- CW-VMAC upper-bound ergodic spectral efficiency is equal to the non-fading AWGN bound spectral efficiency and an upper-bound on the spectral efficiencies with linear multiuser receivers.
- The upper-bound ergodic spectral efficiency with RAKE receiver is equal to that over CW-VMAC for $\beta = 0$ since RAKE is the optimum single-user receiver.
- Decorrelating receiver asymptotically equalizes the time-varying GWSSUS fading channel and turn it into a single-user channel as $D \to \infty$. This fact in conjunction with CW-VMAC upper-bound ergodic spectral efficiency being equal to the non-fading AWGN bound spectral efficiency validates and extends to the multiuser case, Kennedy Law [Kennedy1969] and Telatar-Tse conjecture [Telatar2000] that the capacity of a single-user wideband multipath fading channel converges asymptotically at the wideband limit to the capacity of the non-fading Gaussian channel with the same average received power.
- The spectral efficiencies with RAKE and LMMSE receivers converge to certain bounds or saturates as $\beta \to \infty$ and $\mathrm{SNR_s} \to \infty$ due to interference

limitation.

- Outage spectral efficiencies converge to the upper-bound ergodic spectral efficiencies as $N \to \infty$, $D \to \infty$ and as the specularity of the channel increases.

- The significant difference observed between the derived performances of LMMSE and decorrelating receivers requires further search for more accurate analysis methods for LMMSE receiver.

The work presented in this thesis is also considered to be extended in the near future along these lines below:

- Derivation of direct closed-form expressions for ergodic spectral efficiencies

- Derivation of block-fading outage spectral efficiencies

- Derivation of error-exponents for coded probability of error analysis

- Extension to other fading channel models

- Extension to nonlinear multiuser receivers

- Derivation of closed-form uncoded BER/SER expressions with linear modulations

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