

IDENTIFICATION OF
PERIODIC AUTOREGRESSIVE MOVING AVERAGE MODELS

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ABSTRACT

**IDENTIFICATION OF
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MODELS**

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In this thesis, identification of periodically varying orders of univariate Periodic Autoregressive Moving-Average (PARMA) processes is mainly studied.

The identification of the varying orders of PARMA process is carried out by generalizing the well-known Box-Jenkins techniques to a seasonwise manner. The identification of pure periodic moving-average (PMA) and pure periodic autoregressive (PAR) models are considered only. For PARMA model identification, the Periodic Autocorrelation Function (PeACF) and Periodic Partial Autocorrelation Function (PePACF), which play the same role as their ARMA counterparts, are employed.

For parameter estimation, which is considered only to refine model identification, the conditional least squares estimation (LSE) method is used which is applicable to PAR models. Estimation becomes very complicated,

difficult and may give unsatisfactory results when a moving-average (MA) component exists in the model. On account of overcoming this difficulty, seasons following PMA processes are tried to be modeled as PAR processes with reasonable orders in order to employ LSE. Diagnostic checking, through residuals of the fitted model, is also performed stating its reasons and methods.

The last part of the study demonstrates application of identification techniques through analysis of two seasonal hydrologic time series, which consist of average monthly streamflows. For this purpose, computer programs were developed specially for PARMA model identification.

Keywords: PARMA Process, Periodic Process, Identification, Autocorrelation Function, Partial Autocorrelation Function, Least Squares Estimation

ÖZ

PERİYODİK

OTOREGRESİF HAREKETLİ-ORTALAMALAR

MODELLERİNİN BELİRLENMESİ

Akgün, Burçin

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Bu tezde, Periyodik Otoresif Hareketli-Ortalamlar (PARMA) süreçlerinin periyodik olarak deęişen derecelerinin belirlenmesi esas olarak çalışılmıştır.

PARMA sürecinin deęişken derecelerinin belirlenmesi, iyi bilinen Box-Jenkins tekniklerinin mevsimsel bir usule genelleştirilmesiyle gerçekleştirilmiştir. Yalnızca periyodik hareketli-ortalamlar (PMA) ve periyodik otoresif (PAR) modellerinin belirlenmesi dikkate alınmıştır. PARMA model belirlenmesi için, ARMA modellerindeki karşılıklarıyla aynı rolü üstlenen Periyodik Otokorelasyon Fonksiyonu (PeACF) ve Periyodik Kısmi Otokorelasyon Fonksiyonu (PePACF) kullanılmıştır.

Model belirlenmesini iyileştirmek amacıyla uygulanan parametre tahmini için, PAR modellerine uygulanabilir olan koşullu en küçük kareler tahmini metodu kullanılmıştır. Modelde bir hareketli-ortalamlar (MA) kısmı

bulduğunda tahmin çok kompleks ve zor bir hale gelmekte ve tatminkar olmayan sonuçlar verebilmektedir. Bu problemi çözmek için, PMA süreci izleyen mevsimler makul dereceli PAR süreçlerine dönüştürülerek modellenmeye çalışılmıştır. Uygunluk incelenmesi, tahmin edilen modelden oluşan artık değerler aracılığıyla, sebep ve metotları açıklanarak yapılmıştır.

Çalışmanın son kısmı, aylık ortalama su akımlarından oluşan iki mevsimsel hidrolojik zaman serisinin analizi aracılığıyla belirlenme tekniklerinin uygulanmasını göstermektedir. Bu amaçla, özel olarak PARMA model belirlenmesi için bilgisayar programları geliştirilmiştir.

Anahtar Kelimeler: PARMA Süreci, Periyodik Süreç, Belirlenme, Otokorelasyon Fonksiyonu, Kısmi Otokorelasyon Fonksiyonu, En Küçük Kareler Tahmini

To My Daddy

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TABLE OF CONTENTS

ABSTRACT.....	iii
ÖZ.....	v
ACKNOWLEDGEMENTS.....	viii
TABLE OF CONTENTS.....	ix
LIST OF TABLES.....	xii
LIST OF FIGURES.....	xiii
CHAPTER	
1. INTRODUCTION.....	1
1.1 Introduction to Time Series.....	1
1.2 Preliminaries	2
1.3 PARMA Models	4
1.4 Aims and Scope of the Study.....	11
2. IDENTIFICATION OF PARMA PROCESSES.....	13
2.1 Periodic Autocorrelation Function.....	14
2.1.1 <i>Sample Periodic Autocorrelation Function</i>	16
2.2 Periodic Partial Autocorrelation Function	19
2.2.1 <i>Sample Periodic Partial Autocorrelation Function</i>	22
3. ESTIMATION OF PARMA MODELS.....	23
4. APPLICATION.....	30

4.1 Application on Atnos Creek.....	31
4.1.1 <i>Model Identification</i>	32
4.1.2 <i>Model Estimation</i>	37
4.1.3 <i>Diagnostic Checks</i>	41
4.2 Application on Nilüfer Creek.....	43
4.2.1 <i>Model Identification</i>	45
5. SUMMARY AND CONCLUSIONS	50
REFERENCES.....	55
APPENDICES	59
A. COMPUTER PROGRAM FOR COMPUTATION OF SAMPLE PERIODIC MEANS, AUTOCOVARIANCES AND AUTOCORRELATIONS.....	59
B. COMPUTER PROGRAM FOR IDENTIFICATION OF PURE PERIODIC MOVING AVERAGE MODELS.....	70
C. COMPUTER PROGRAM FOR IDENTIFICATION OF PURE PERIODIC AUTOREGRESSIVE MODELS.....	84
D. COMPUTER PROGRAM FOR LEAST SQUARES ESTIMATION OF PAR MODELS.....	95
E. APPLIED SERIES	101
E1. Atnos Creek.....	101
E2. Nilüfer Creek.....	102

F. PeACF AND PePACF GRAPHS.....	103
F1. PeACF and PePACF Graphs Of Atnos Creek.....	104
F2. PeACF and PePACF Graphs Of Nilüfer Creek.....	116

LIST OF TABLES

TABLE

4.1.1	Sample statistics for seasons of Atnos Creek.....	32
4.1.2	PeACF and PePACF of Atnos season 1.....	34
4.1.3	Model identification for Atnos Creek prior to estimation and diagnostic checks.....	36
4.1.4	Model identification for Atnos Creek after estimation.....	40
4.1.5	Summary of diagnostic checks on Atnos Creek.....	42
4.2.1	Sample statistics for seasons of Nilüfer Creek.....	45
4.2.2	Model identification for Nilüfer Creek.....	48

LIST OF FIGURES

FIGURES

4.1.1	Time plot of Atnos Creek.....	31
4.1.2	(a) Sample PeACF of season 1 of Atnos.....	33
	(b) Sample PePACF of season 1 of Atnos.....	33
4.2.1	Time plot of Nilüfer Creek.....	44

CHAPTER 1

INTRODUCTION

1.1 Introduction to Time Series

A time series $\{X_t; t \in T\}$ can be defined as an ordered sequence of random variables over time, where T denotes an index time points set. It is the historical record of some activity, with observations taken at equally spaced intervals.

Much of statistical methodology deals with models in which the observations are assumed be independent. However, a great deal of data in business, economics, engineering and natural sciences occur in the form of time series where observations are dependent and where the nature of this internal structure is of interest itself. This obvious correlation resulted from collecting the observations over time severely restrict the applicability of many traditional statistical methods depending on the assumption that these adjacent observations are independent and identically distributed. On the other hand, the body of techniques available for the analysis of such series of dependent observations is called *time series analysis*. In most statistical problems, estimating the properties of a population from a selected sample is of concern. In time series analysis, however, it is often impossible to have more than a single observation

at a given time point.

The number of areas in which time series are observed and analyzed is endless. Virtually any quantity recorded over time yields a time series. In

economics the recorded history of the economy is often in the form of time series, the natural sciences and social sciences also supply many examples of time series.

The fundamental aim of time series analysis is generally two-fold: to understand and identify the stochastic process that produced the observed series and, in turn, to forecast future values of a series from past values alone.

1.2 Preliminaries

Being a stochastic process consisting of sequential random variables, in time series analysis some assumptions had to be set in order to make statistical analysis based on a finite observed record. The most vital and common such assumption in time series techniques is the *stationarity*. The basic idea of stationarity is that the probability laws ruling the process do not change with time - that is, the process is in statistical equilibrium.

Specifically, a time series is said to be *strictly stationary* if the joint distribution of $X_{t_1}, X_{t_2}, \dots, X_{t_n}$ is the same as the joint distribution of $X_{t_1+k}, X_{t_2+k}, \dots, X_{t_n+k}$, hence its statistical properties are unaffected by a change of time origin. In addition, a series is said to be *covariance stationary* (or, second-order stationary) if its mean function is constant over time and its second order moments are functions of time difference (which is termed *lag*) only, i.e., its first and second order moments do not depend on time t . The version of strict stationarity is too strong for most applications, hence second-

order type of stationarity is the most common version and is often sufficient in practice. In addition, a strict stationary process is always covariance stationary, while the opposite is satisfied if and only if the covariance stationary process

is normally distributed. In this thesis, the term “stationary” when used alone is taken to mean “covariance stationary”.

Further, there is an important consequence of stationarity. The fact that the covariance of a stationary process do not depend on time but only on time lag allows us to define a fundamental quantity of time series analysis: the *autocovariance function* (ACVF). This leads to the *autocorrelation function* (ACF) and *partial autocorrelation function* (PACF). The autocorrelation functions are easier to interpret since they are unitless; they provide insight into the dependence between the observations of the series, besides, they are essential tools in model identification stage which will be analyzed later in detail.

One of the primary objectives of time series analysis is to develop mathematical models which describes the character of the process. A broad parametric family of such models is the *Autoregressive Moving-Average* (ARMA) models (or, Box-Jenkins models) which are stationary time series models. Before specifying the three special models of ARMA family, it should be noted that a time series generated from zero-mean, finite variance, uncorrelated variables is called a “white noise” series, it is not only an interesting model itself, but also many useful models can be constructed from white noise. If a series is represented as a linear combination of the present and q past terms of a white noise process, then it is the so-called Moving-Average (MA (q)) model, where q denotes the order of the model. Secondly, if a series is a linear combination of the p most recent lagged (past) values of itself plus a noise term is added, then it is called as an Autoregressive model of order p (AR (p)). And, the mixed type of series which are explained both

by its own lagged values and by lagged noise terms is called as Autoregressive Moving-Average models of order (p, q), denoted by ARMA (p, q). A general mathematical expression of an ARMA (p, q) model is given as

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \Lambda + \phi_p X_{t-p} + a_t - \theta_1 a_{t-1} - \Lambda - \theta_q a_{t-q} \quad (1.1)$$

for all integers t , where $\{a_t\}$ is a white noise process, and the ϕ and θ coefficients are the autoregressive (AR) and moving-average (MA) parameters respectively. Note that, when $p = 0$, the process becomes a pure moving-average (MA) model, and when $q = 0$, it becomes a pure autoregressive (AR) model.

This systematic class of stationary time series models carries great importance and usefulness especially in real-life situations. If the process is stationary, then one shall seek a suitable ARMA model to represent the data. If not, then first of all stationarity must be achieved by for example, differencing, and this leads to the *Autoregressive Integrated Moving Average* (ARIMA) models, which are nonstationary models that surely become stationary after appropriate differencing. All these models mentioned up to now are widely studied in the literature, and efficient methodologies for their identification and estimation are well developed.

1.3 PARMA Models

Time series which display a periodic structure are often faced in real life, for example, in economic, geophysical and hydrologic time series. If a strong and stable seasonal component causes a series to be nonstationary, it is usually passed through a filter or an appropriate seasonal differencing is applied.

An ARIMA extended class of models that accounts for modeling univariate seasonal time series is the widely used multiplicative seasonal autoregressive integrated moving average models (Box and Jenkins, 1976). Those are

nonstationary models which can be transformed into stationary ARMA processes after applying some appropriate differencing.

Seasonal ARIMA models considers a very simplified form of seasonality, which means that despite its usefulness, implicit in such models is the assumption of homogeneity or time invariance, that is, the seasonally differenced series is sure to become stationary. However, many seasonal time series cannot be filtered, standardized or differenced to achieve second-order stationarity because the series exhibits a strong seasonal behaviour such that the entire correlation structure of the series depends on the season, hence such homogeneity assumption sometimes fails (Tiao and Grupe, 1980). For instance, in a river where high runoffs occur in the spring and low flows occur in the summer, the streamflow correlations between spring months may be different from the correlations between summer months (Vecchia, 1985). A more realistic family of models characterizing those kind of seasonal time series is the *Periodic Autoregressive Moving Average* (PARMA) models, which are actually extensions of ARMA models in which the model orders and parameters are allowed to vary among seasons. It should be pointed out that, a PARMA model is again of PARMA type even after seasonal differencing. Shortly, it can be said that PARMA model is somewhat a periodic version of the ordinary ARMA model, yet any PARMA model can also be expressed as a vector ARMA model which will be shown fully later. Being completely different from seasonal ARIMA models, PARMA is a more realistic but complicated class of models which are fully suited to the nature of seasonal time series.

PARMA processes are receiving considerable attention in many different fields. For instance, they have found wide applications in modeling hydrological

time series (Delleur *et al.*, 1976; Salas *et al.*, 1980; Vecchia, 1985), in signal processing (Sakai, 1982), and in climatology (Hannan, 1955; Monin, 1963; Jones and Brelsford, 1967).

The univariate ω -period PARMA model with varying orders $(p(1),q(1); p(2),q(2); \dots ;p(\omega),q(\omega))$, denoted by $\text{PARMA}_\omega(p(v), q(v))$, is defined as

$$X_{k\omega+v} = \phi_1^{(v)} X_{k\omega+v-1} + \Lambda + \phi_{p(v)}^{(v)} X_{k\omega+v-p(v)} + a_{k\omega+v} - \theta_1^{(v)} a_{k\omega+v-1} - \Lambda - \theta_{q(v)}^{(v)} a_{k\omega+v-q(v)} \quad (1.2)$$

where v is the period index ($v=1, \dots, \omega$), k is the year index ($k=0, \pm 1, \pm 2, \dots$) and $\phi_i^{(v)}$ and $\theta_i^{(v)}$ are the autoregressive and moving-average coefficients, respectively, during season v . Further, the periodic sequence $\{a_{k\omega+v}\}$ is a white noise process with mean zero and periodic variance $\sigma_a^2(v)$. In this model, it is assumed that $E(X_{k\omega+v})=0$. If, however, $E(X_{k\omega+v})=\mu_v \neq 0$, then without loss of generality $X_{k\omega+v}$ is replaced by $X_{k\omega+v} - \mu_v$. It is seen apparently that, (1.2) is simply an extension of the standard ARMA difference equation with orders and coefficients varying among seasons. If $p(1) = \dots = p(\omega) = 0$, then it is called a pure *periodic moving-average model* and denoted by $\text{PMA}_\omega(q(v))$, and if $q(1) = \dots = q(\omega) = 0$, then it becomes a pure *periodic autoregressive model* which is denoted by $\text{PAR}_\omega(p(v))$. Furthermore, if the white noise terms are independently and normally distributed, then (1.2) is called a Gaussian PARMA process.

Writing the time index parameter t as $k\omega + v$ gives a more practical understanding besides displaying the periodicity notationally. For example, in the case of a quarterly data where $\omega = 4$ and say $(k + 1)$ and v represents the year and the season, ($v = 1, 2, \dots, \omega$), respectively, it is easily understood

that time point 11 means the 3rd season of year 3. The subscripts in (1.2), must be between 1 and ω . So, whenever $(v-l)$ is less than or equal to zero, number of periods ω is added to the subscript, so that when $v - l = 0$,

this time point in fact belongs to season ω , and $\nu - l = -1$ belongs to season $-1 + \omega$, and so on. Furthermore, the speciality in the formula (1.2) is that for a year, k , the model is represented by a number of ω equations, such that a different ARMA model is presented for each season ν of the process.

Any PARMA model can be expressed as a vector ARMA model which is the so-called “*lumped*” vector process, whose elements are the seasonal variables for all ω periods. Consider a periodic process $\{X_{k\omega+\nu}\}$, the lumped vector process is then denoted for all integers k by

$$Y_k = \begin{bmatrix} X_{k\omega+\omega} \\ \mathbf{M} \\ X_{k\omega+1} \end{bmatrix} \quad (1.3)$$

For a $\text{PARMA}_\omega(p(\nu), q(\nu))$ process with periodically varying orders, it is known that $\{Y_k\}$, which is referred as the *lumped-vector process*, follows a ω -variate ARMA (p^*, q^*) process (Tiao and Grupe, 1980), where

$$p^* = \max_\nu \{[(p(\nu) - \nu) / \omega]\} + 1 \quad (1.4)$$

$$q^* = \max_\nu \{[(q(\nu) - \nu) / \omega]\} + 1$$

where $[x]$ denotes the integral part of the x .

When modeling a time series with an ordinary ARMA model, recall that it is necessary to assume that the series is stationary, and if not, it should be

transformed into stationary type. In much the same manner, PARMA models requires *periodic covariance stationarity*, i.e., its mean and autocovariance function are periodic functions of time with period ω : that is, the first and second order moments of the process do depend on the period and the lag but not on the absolute time, and hence it imposes some constraints on AR parameters (Vecchia, 1985; Ula, 1990; Ula and Smadi, 1997). If it is shown mathematically,

$$\begin{aligned} E(X_t) &= \mu(t) = \mu(t+k\omega) \\ \text{Cov}(X_t, X_s) &= \gamma(t,s) = \gamma(t+k\omega, s+k\omega) \end{aligned} \tag{1.5}$$

The periodic stationarity conditions, however, are not easily stated in terms of the PARMA model as defined by (1.2) but can be deduced from its vector ARMA counterpart. Gladyšev (1961) proved that a periodic process with period ω is periodically stationary if and only if the ω -variate vector process is stationary. As a result, the periodic stationarity of a PARMA process is equivalent to the stationarity conditions of its corresponding lumped-vector process whose stationarity conditions depend only on the $\text{AR}(p^*)$ part and are readily available. The lumped $\text{ARMA}(p^*, q^*)$ process is stationary if and only if the ωp^* roots of the characteristic equation are all less than 1 in absolute value (Fuller 1976, p. 72-73). Similarly, the invertibility of a PARMA process is equivalent to the invertibility of its corresponding lumped-vector process, whose invertibility conditions depend only on the $\text{MA}(q^*)$ part. Invertibility is not a prerequisite to analysis of PARMA processes and not covered in this

study, however, it is important to state that an ARMA model can be written as an infinite order AR model if and only if it is invertible, and same statement holds for PARMA case too.

Analysis of PARMA models is usually based on some extension of Box-Jenkins approach to a periodic case. Periodic stationarity of the PARMA model leads to a *periodic autocovariance function* (PeACVF), *periodic autocorrelation function* (PeACF), and *periodic partial autocorrelation function* (PePACF) in analogy with the periodic versions of ordinary ACVF, ACF and PACF of a stationary process. PeACF and PePACF functions can be used for model identification.

Literature on PARMA models is scarce compared to univariate and multivariate ARMA models. Tiao and Grupe (1980) explored some properties of the periodic models investigating the consequences of applying standard analysis to periodic data and stated the relationships between periodic models and multiple autoregressive moving average models. Tiao and Grupe (1980), and Salas, *et al.* (1982) considered models containing both periodic autoregressive and moving average parts and investigated their correlation properties.

Gladyšev (1961), Vecchia (1985), Ula (1990), and Ula and Smadi (1997) investigated the various aspects of periodic stationarity conditions for PARMA models. Ula and Smadi (1997) showed that periodic covariance stationarity conditions for univariate and multivariate PARMA processes can be reduced to eigenvalue problems which are analytically easier to deal with. The ω -span lumping over all ω periods and the p -span lumping of the p^{th} order PAR process over p periods are investigated respectively and it is shown that the p -span lumping provides the periodic stationarity conditions in an analytically simpler form as compared to the ω -span lumping in case of $p < \omega$. Whatever the AR order p^* of the lumped vector is, the stationarity conditions can be

reduced to an eigenvalue problem (Fuller, 1976, p.50; Barone and Roy, 1983; Barone, 1987; Smadi and Ula, 1997).

Anderson and Vecchia (1993) developed some asymptotic results for the sample periodic autocorrelation function of univariate PARMA processes. Sakai (1982) studied partial autocorrelations of PAR processes.

Smadi (1994), Smadi and Ula (1998), and Ula and Smadi (2003a) studied the identification of periodic moving-average (PMA) models by using PeACF. Again Ula and Smadi (2003b) considered the identification of PAR models by making use of PePACF. Authors derived some useful results from available asymptotic theory for obtaining band limits on sample PeACF and sample PePACF.

The estimation of PARMA models has received considerable attention. Jones and Brelsford (1967) considered moment estimation for periodic autoregressive models. Vecchia (1985) gave an overview of some of the more useful results regarding correlation structure and parameter estimation presenting some correlation properties of PARMA processes, discussing the moment estimation and maximum likelihood estimation and also demonstrated an application of those techniques. Again Vecchia (1985) developed an approximation to the exact likelihood for Gaussian PARMA process, and a straightforward algorithm for its maximization is presented. Lund and Basawa (1998) explored recursive prediction and likelihood evaluation techniques for PARMA and developed a simple recursive scheme for computing one-step-ahead predictors and their mean squared errors using the innovations algorithm. Anderson, Meerschaert, and Vecchia (1999) developed the innovations algorithm for periodically stationary processes and showed how to obtain estimates for the parameters of the PARMA model. Again, Basawa and Lund (1999) studied the asymptotic properties of parameter estimates for casual and invertible PARMA

models and derived a general limit result for PARMA parameter estimates with a moving average component.

1.4 Aims and Scope of the Study

The primary aim of this thesis study is to contribute to the identification of the seasonally varying orders of the PARMA model and to develop a practical program which decides those orders. The absence of a real life application to the model identification process of a periodically stationary PARMA type series is the main motivation behind this thesis.

Analysis of PARMA models is established on the extension of Box-Jenkins methodology to a seasonwise case. The definition and a brief introduction of PARMA models was presented in this chapter. In addition, the determination of the periodic stationarity conditions of PARMA processes was explained. Since PARMA models are quite new compared to others of the similar kind, many aspects about them are still shaded.

Chapter II focuses on identification of PARMA models showing that periodic autocorrelation function, PeACF and periodic partial autocorrelation function, PePACF serve as essential tools for the choice of a suitable model, similar to the ACF and PACF as in ARMA model identification. Both PeACF and PePACF have similar cut-off and sampling properties with ordinary ACF and PACF in the case of pure PMA and PAR processes respectively. Emphasis is given on the properties and results of the sample periodic ACF and sample periodic PACF, and the asymptotic bands are developed for the decision of the cut-off situations of these periodic functions. The more complicated problem of mixed PARMA identification is not covered in this study, since no general method is available.

Although estimation of PARMA models is not a major objective in this thesis, Chapter III considers the model estimation in order to refine the model identification stage and achieve a final specification of the most appropriate model among suitable candidates. The estimation of PARMA models is not straightforward, especially the estimation of mixed PARMA models and PMA

models is technically difficult. Since it is not the intention to go far into the details of estimation algorithms in this study, Chapter III involves the least squares estimation (LSE) method for periodic autoregressive (PAR) models, both its versatility and pitfalls; re-expressing periodic moving-average (PMA) models as large order PAR models to perform LSE; and the basics of model diagnostics which are performed on the residuals of the fitted model.

In Chapter IV, application on two actual hydrologic series, namely Atnos Creek and Nilüfer Creek, is presented. The usage and results of the computer programs developed especially for model identification is demonstrated. Besides, a program which performs LSE for PAR models is given. The mentioned programs are created in Visual Fortran, Professional Edition 5.0A. All three stages of model development, i.e., model identification, estimation and diagnostic checking are illustrated with the Atnos series, such that the concluded orders for each season, the results of the diagnostic checks and the general model representing the whole process are given for Atnos series. For Nilüfer series, only model identification is proceeded again presenting a separate model for each season and a general model representing the whole process.

Finally, in Chapter V, summary and conclusions on the findings are given.

CHAPTER II

IDENTIFICATION OF PARMA PROCESSES

There are three basic stages to fitting a model to a time series data, these stages involve *model identification*, *parameter estimation* and *model diagnostics*.

Model identification is the identification of a possible model based on an available realization, i.e., determining the type of the model with appropriate orders. Parameter estimation is the estimation of the model parameters. At this stage, the orders of the model may be further reduced by significance tests on parameters. Diagnostic checks are directed to the residuals of the fitted model to verify the assumptions on the white noise terms such as independence and normality. If verifications fail, model identification stage is to be repeated leading to a new possible model.

For ARMA models, the autocorrelation function (ACF) and the partial autocorrelation function (PACF) serve as useful indicators of the correlation or of the dependence between the values of the series so that they play an important role in model identification (Box and Jenkins, 1976). This chapter considers the identification of the seasonally varying orders of PARMA processes making use of two similar functions, namely Periodic Autocorrelation Function (PeACF) and Periodic Partial Autocorrelation Function (PePACF). The choice of orders of the MA and AR parts requires a detailed analysis of these

functions, respectively, whose shape and value determine the order of the model. It should be pointed out that, this thesis considers the identification of pure PAR and pure PMA models only. No satisfactory generalized method is available for deciding the mixed PARMA model orders.

The determination of period ω is not considered in this study, since in practice, it is often known by the nature of the data collected.

2.1 Periodic Autocorrelation Function

For the univariate periodic stationary PARMA process $\{X_{k\omega+v}\}$ defined by (1.2), in which the white noise terms $\{a_{k\omega+v}\}$ are assumed to be independent, the *periodic autocovariance function* (PeACVF) is defined as

$$\begin{aligned}\gamma_l(v) &= \text{Cov}(X_{k\omega+v}, X_{k\omega+v-l}) \\ &= E[(X_{k\omega+v} - \mu_v)(X_{k\omega+v-l} - \mu_{v-l})]\end{aligned}\tag{2.1}$$

for season v at backward lag $l \geq 0$.

Then, the PeACF for season v at backward lag $l \geq 0$ is defined as

$$\begin{aligned}\rho_l(v) &= E\left[\left(\frac{X_{k\omega+v} - \mu_v}{\sqrt{\gamma_0(v)}}\right)\left(\frac{X_{k\omega+v-l} - \mu_{v-l}}{\sqrt{\gamma_0(v-l)}}\right)\right] \\ &= E(Z_{k\omega+v}, Z_{k\omega+v-l}) \\ &= \frac{\gamma_l(v)}{\sqrt{\gamma_0(v)\gamma_0(v-l)}}, \quad l \geq 0\end{aligned}\tag{2.2}$$

where $\gamma_0(v)$ is the variance for the v^{th} season and $\{Z_{k\omega+v}\}$ denotes the periodically standardized time series.

A delicate point is that, $\rho_l(v)$ is defined for nonnegative time lags only, since it is not symmetric with respect to time lag l , that is,

$\rho_l(v) \neq \rho_{-l}(v)$. This is because the periodic autocovariance function $\gamma_l(v)$ is not strictly equal to $\gamma_{-l}(v)$, unless l is a multiple of the period ω . However, it can be shown that (Smadi, 1994; p.29)

$$\rho_l(v) = E(Z_{k\omega+v} Z_{k\omega+v-l}) = E(Z_{k\omega+v-l} Z_{k\omega+v-l-(-l)}) = \rho_{-l}(v-l)$$

Another important point is that the ordinary ACF of a stationary series goes to zero as time lag increases. A similar property holds also for $\rho_l(v)$ that, for a univariate periodic stationary PARMA process, $\lim_{l \rightarrow \infty} \rho_l(v) = 0$ for any season v (Smadi, 1994; p.29).

From the point of identification, an important property of the PeACF is that, for a season following a pure PMA ($q(v)$) process, such that $q(v) > 0$ and $p(v) = 0$, which is given by,

$$X_{k\omega+v} = a_{k\omega+v} - \theta_1^{(v)} a_{k\omega+v-1} - \Lambda - \theta_{q(v)}^{(v)} a_{k\omega+v-q(v)} \quad (2.4)$$

then $\rho_l(v) = 0$ for all $l > q(v)$, which is namely the *cut-off property* of the autocorrelation function of pure MA processes. However, in applications only a finite series is available in hand, thus, only the estimate of the theoretical PeACF can be obtained and used as a tool to check whether v^{th} season follows a pure PMA or not and to decide its order $q(v)$.

2.1.1 Sample Periodic Autocorrelation Function

Let $\{X_1, X_2, \dots, X_{N\omega}\}$ be a series of size $N\omega$ (say, N years and ω periods) coming from a periodic stationary process $\{X_{k\omega+v}\}$. Then the sample estimate of $\rho_l(v)$ is called as the *sample periodic autocorrelation function* and it is given by,

$$r_l(v) = \frac{\hat{\gamma}_l(v)}{\sqrt{\hat{\gamma}_0(v)\hat{\gamma}_0(v-l)}}, \quad l \geq 0 \quad (2.5)$$

where $\hat{\gamma}_l(v)$ is the *sample periodic autocovariance function* calculated from

$$\hat{\gamma}_l(v) = \frac{1}{N} \sum_{k=0}^{N-1} (X_{k\omega+v} - \bar{X}_v)(X_{k\omega+v-l} - \bar{X}_{v-l}), \quad (2.6)$$

in which

$$\bar{X}_v = \frac{1}{N} \sum_{k=0}^{N-1} X_{k\omega+v} \quad (2.7)$$

is the sample mean for season v . In addition, the terms in (2.6) are set to zero whenever $k\omega + v - l < 1$. If it is assumed that the white noise terms are independent and normal, so that $\{X_{k\omega+v}\}$ is a Gaussian PARMA process, then proved by Pagano (1978), $\hat{\gamma}_l(v)$ are consistent, asymptotically independent, joint normal and unbiased estimates and converge almost surely to $\gamma_l(v)$. Besides, \bar{X}_v are also consistent and unbiased estimates of μ_v .

The asymptotic properties of the ordinary sample ACF of stationary ARMA processes, have been extensively studied in the literature, Bartlett (1946)

has originally derived the asymptotic normality and unbiasedness of the sample ACF and asymptotic variance-covariance matrix has been specified by Box and Jenkins (1976; p. 35). These results were then generalized to periodic stationary processes by Anderson and Vecchia (1993). The sampling properties of $r_l(v)$ can not be easily obtained. Even the expected value of $r_l(v)$ is difficult to find, so one might expect that the variance of the estimated PeACF is much more complicated and daunting.

An approximate solution for the first and second order moments of $r_l(v)$ is obtained by Smadi (1994), pretending that the sample means, \bar{X}_v , and variances, $\hat{\gamma}_0(v)$, in the formula of $r_l(v)$ are equal to their population counterparts, μ_v and $\gamma_0(v)$, respectively. Such an assumption will obviously be reasonable for large samples owing to the consistency of those estimates as mentioned before. Moreover, Smadi's result is somewhat an approximate version of the fairly complicated formula of Anderson and Vecchia (1993), it only differs in terms of third and higher orders, which are usually negligible, and more importantly, Smadi (1994) showed that his result is identical with that of Anderson and Vecchia (1993) for pure PMA processes and as applicable as theirs.

Consequently, $\{X_{k\omega+v}\}$ being a periodic stationary PARMA $_{\omega}(p(v), q(v))$ process, if an arbitrary season v follows a MA ($q(v)$) process as given by (2.4), then for positive integers $l > q(v)$, the following results are derived for $Var[r_l(v)]$ (Smadi, 1994);

(I) For $q(v) < \omega$,

a. $Var[r_l(v)] \cong \frac{1}{N} \{1 + [\rho_l(v)]^2\}, \quad l \leq q(v)$

b. $Var[r_l(v)] \cong \frac{1}{N}, \quad l > q(v)$

(II) For $k\omega \leq q(v) < (k+1)\omega$, $k=1,2,\dots$

a. if $1 \leq l \leq q(v) - k\omega$

$$Var[r_l(v)] \cong \frac{1}{N} \left(1 + [\rho_l(v)]^2 + 2 \sum_{m=1}^k \{ \rho_{m\omega}(v) \rho_{m\omega}(v-l) + \rho_{m\omega+l}(v) \rho_{m\omega-l}(v-l) \} \right),$$

b. if $q(v) - (j+1)\omega < l \leq q(v) - j\omega$, where $j = 1, \mathbf{K}, (k-1)$

$$Var[r_l(v)] \cong \frac{1}{N} \left(1 + [\rho_l(v)]^2 + 2 \left\{ \sum_{m=1}^k \rho_{m\omega}(v) \rho_{m\omega}(v-l) + \sum_{m=1}^j \rho_{m\omega+l}(v) \rho_{m\omega-l}(v-l) \right\} \right),$$

c. if $q(v) - \omega < l \leq q(v)$

$$Var[r_l(v)] \cong \frac{1}{N} \left(1 + [\rho_l(v)]^2 + 2 \sum_{m=1}^k \{ \rho_{m\omega}(v) \rho_{m\omega}(v-l) \} \right),$$

d. and if $l > q(v)$

$$Var[r_l(v)] \cong \frac{1}{N} \left(1 + 2 \sum_{m=1}^k \rho_{m\omega}(v) \rho_{m\omega}(v-l) \right), \quad (2.8)$$

Case (b) of part I and (d) of part II, which are both defined for $l > q(v)$, are relevant to the assessment of the cut-off property of the PeACF. Thus, these two cases will lead to the computation of band limits of the sample periodic autocorrelation function.

The result of Anderson and Vecchia (1993) about $r_l(v)$ being asymptotically normal with mean $\rho_l(v)$ and variance $Var[r_l(v)]$ as given in the

above formula is applied for identification of a season v which follows a PMA ($q(v)$) process. Hence, the large sample distribution of $r_l(v)$ is normal with mean zero and variance (2.8) - I(b) as long as $q(v) < \omega$ and $l > q(v)$, so that the approximate 95% band $(-1.96/\sqrt{N}, 1.96/\sqrt{N})$ is applied to the sample PeACF where -1.96 and 1.96 are the upper $\alpha/2$ -point and lower $\alpha/2$ -point of standard normal distribution when significance level α is equal to 0.05. And, as $q(v) \geq \omega$ and $l > q(v)$, (2.8) - II(d) should be used, so, the

approximate 95% band $(-1.96\sqrt{\text{Var}(r_l(v))}, 1.96\sqrt{\text{Var}(r_l(v))})$ should be applied with unknown periodic autocorrelations replaced by their estimates. Shortly, for a season v which follows a pure PMA process, successive values for $q(v)$ is checked initially with $q(v) = 1$ according to the appropriate band limits. Therefore, the final lag of which the value of sample PeACF falls outside the band limits indicates the order of the MA process of the corresponding season.

2.2 Periodic Partial Autocorrelation Function

In time series analysis, the partial autocorrelation function is well adapted to the identification of pure AR processes. The partial autocorrelation function, like the autocorrelation function, carries vital information about the dependence structure of a stationary series.

If v^{th} season of a univariate PARMA $_{\omega}(p(v), q(v))$ process $\{X_{k\omega+v}\}$ follows pure AR process such that $p(v) > 0$, and $q(v) = 0$, given by,

$$X_{k\omega+v} = \phi_1^{(v)} X_{k\omega+v-1} + \Lambda + \phi_{p(v)}^{(v)} X_{k\omega+v-p(v)} + a_{k\omega+v} \quad (2.9)$$

then this equation can be considered as a classical “regression” equation, in which the present value of the series, $X_{k\omega+v}$ is regressed on its own p most

recent past values $X_{k\omega+v-1}, \dots, X_{k\omega+v-p}$.

The *periodic partial autocorrelation function* (PePACF), $\phi_{ll}(v)$, is pursued as a measure of the exact relationship between $X_{k\omega+v}$ and $X_{k\omega+v-l}$, removing the effect of the intervening observations and it is defined for integers $l \geq 1$ as

$$(2.10)$$

$$\phi_{ll}(\nu) = \text{Corr}(X_{k\omega+\nu}, X_{k\omega+\nu-l} | X_{k\omega+\nu-1}, \mathbf{K}, X_{k\omega+\nu-l+1})$$

A regression interpretation of PePACF is given by Smadi (1994), and Ula and Smadi (submitted for publication to Environmetrics, 2003b).

It was mentioned that for a pure PMA process, the PeACF is zero for lags beyond $q(\nu)$ and the order of the process can be decided according to the sample PeACF. Likewise, for pure PAR processes, the PePACF becomes zero for lags beyond $p(\nu)$, so if the correct order is $p(\nu)$ for season ν , then $\phi_{ll}(\nu) = 0$ for all $l > p(\nu)$. This is the cut-off property of periodic partial autocorrelation function for PAR processes.

For the computation of the PePACF, $\phi_{ll}(\nu)$, Sakai's (1982) algorithm for calculating $\phi_{ll}(\nu)$ iteratively is used. This approach is as follows:

(1) Initial Conditions

$$\begin{aligned} \delta_v^2(0) &= \tau_v^2(0) = \gamma_0(\nu), \\ \alpha_v(p, 0) &= 1 \end{aligned} \quad p = 0, 1, 2, \mathbf{K}; \quad \nu = 1, \mathbf{K}, \omega$$

(2) Order update from p to $p+1$ ($p=0, 1, 2, \dots$)

i. Compute;

$$\Delta_v(p) = \sum_{m=0}^p \gamma_{p+1-m}(\nu - m) \alpha_v(p, m)$$

$$\alpha_v(p+1, p+1) = \frac{-\Delta_v(p)}{\tau_{v-1}^2(p)} \quad \beta_v(p+1, p+1) = \frac{-\Delta_v(p)}{\delta_v^2(p)}$$

ii. Update;

$$\delta_v^2(p+1) = \delta_v^2(p)[1 - \alpha_v(p+1, p+1)\beta_v(p+1, p+1)]$$

$$\tau_v^2(p+1) = \tau_{v-1}^2(p)[1 - \alpha_v(p+1, p+1)\beta_v(p+1, p+1)]$$

and for $i = 1, 2, \dots, p$,

$$\alpha_v(p+1, i) = \alpha_v(p, i) + \alpha_v(p+1, p+1)\beta_{v-1}(p, p+1-i)$$

$$\beta_v(p+1, i) = \beta_{v-1}(p, i) + \beta_v(p+1, p+1)\alpha_v(p, p+1-i)$$

iii. Calculate;

$$\phi_{(p+1)(p+1)}(v) = \frac{\Delta_v(p)}{\delta_v(p)\tau_{v-1}(p)} \quad (2.11)$$

where $\gamma_l(v)$ is the periodic autocovariance function as defined before and the subscript $v-l = 0$ is always replaced by ω again as mentioned in previous chapter.

2.2.1 Sample Periodic Partial Autocorrelation Function

Similar to that of PeACF, this time the sample PePACF, denoted by $\hat{\phi}_{ll}(v)$, is a valuable identification tool and can simply be obtained from (2.11) by replacing the periodic autocovariances, $\gamma_l(v)$, by their corresponding sample

estimates, $\hat{\gamma}_l(v)$. Besides developing the mentioned algorithm for the calculation of PePACF, Sakai (1982) also derived the distribution of $\hat{\phi}_l(v)$: If a season v follows a PAR ($p(v)$) process, then for all v and $l > p(v)$, $\hat{\phi}_l(v)$ are asymptotically independent, and normally distributed with zero mean and variance $1/N$. Therefore, for large N , for order identification, the approximate 95% band $(-1.96/\sqrt{N}, 1.96/\sqrt{N})$ should be applied for $\hat{\phi}_l(v)$ for all $l > p(v)$, again -1.96 and 1.96 representing the upper $\alpha/2$ -point and lower $\alpha/2$ -point of standard normal distribution when significance level α is equal to 0.05. Hence, the final lag of which the value of sample PePACF falls outside the band limits indicates the order of the AR process of the corresponding season.

As being a sequence of uncorrelated zero-mean and finite variance random variables, white noise is also one of the most important time series models and needs an identification rule either. Henceforth, for an arbitrary season v , if the values of both sample PeACF and sample PePACF all lie within $(-1.96/\sqrt{N}, 1.96/\sqrt{N})$ band limits, then that season is said to follow a white noise process.

CHAPTER III

ESTIMATION OF PARMA MODELS

The main motivation behind this thesis is to emphasize on model identification as stated many times. However, in order to refine model identification and achieve the selection of the most appropriate model, model estimation and diagnostic checks will also be handled in a simple manner.

Once an appropriate model is proposed for the periodic process using identification procedures, next stage is to estimate the model parameters. Estimation and likelihood evaluation methods for PARMA processes remain complicated (see various references in Section 1.3). Further, when the number of seasons, ω , is large, estimation becomes harder, since this large number of seasons results in a ω -fold increase in the number of parameters to be estimated.

The method of moments, least squares estimation method and maximum likelihood estimation method are widely used parameter estimation methods in statistical inference and in time series analysis as well. The method of moments is one of the most common in time series context but it has serious disadvantages for some situations. Although method of moments can produce good estimators in the case of pure AR processes, they lead to unsatisfactory or even infeasible estimates when the model involves an MA component.

Further, estimation becomes difficult because a number of non-linear equations arise which are to be solved simultaneously. Similarly, for PAR processes, method of moments is straightforward and satisfactory, but same problems arise for PARMA and PMA processes. Pagano (1978) showed that the moment estimators of a univariate Gaussian PAR process are almost surely consistent, asymptotically efficient and asymptotically joint normal and unbiased.

Secondly, it is known from standard AR model estimation that the conditional least squares (LS) procedures can be used to obtain efficient estimators for the parameters of AR model since AR equation is somewhat

similar to the classical regression equation. This fact also holds for PAR models adopting the LS method to a seasonwise case in order to obtain LS estimates of PAR parameters. The term conditional actually comes from the conditional likelihood function which will be explained later in brief. Moreover, LS estimates are superior to method of moments estimates for small samples, yet for large samples the difference between the estimates of two methods is negligible. A significant feature of the conditional LS is that, if the PAR process is Gaussian, then the conditional LS estimates are identical with the conditional maximum likelihood (ML) estimates.

Smadi (1994; pp.74-84) compared the three estimation methods, namely the method of moments, method of conditional LS, and method of conditional ML, theoretically and also through a simulation study for three different PAR models and investigated their behaviours for various cases. He showed that, all methods produce estimates which are close to the actual values of the parameters, and conditional LS estimates and conditional ML estimates of the PAR parameters are same; and also identical to moment estimates except for some initial seasons in which some observations are lost. Finally, he concluded that the conditional LS estimates are superior to moment estimation in terms of bias and mean squared error.

If, however, more accurate and efficient estimates are required, then exact (unconditional) MLE should be considered. On the other hand, it is very difficult to obtain the exact likelihood function as an explicit function of the parameters even for standard ARMA models, that it becomes indeed more tedious for PARMA family. Moreover, very complicated numerical optimization routines are required for maximization of the resulting non-linear equations.

Since it is not the intention to go far into the details of estimation algorithms in this study, only the least squares estimation technique for PAR processes will be discussed and used pointing some important problems.

Consider the univariate $\text{PAR}_{\omega}(p(v))$ process, $X_{k\omega+v} = \phi_1^{(v)} X_{k\omega+v-1} + \phi_2^{(v)} X_{k\omega+v-2} + \mathbf{K} + \phi_{p(v)}^{(v)} X_{k\omega+v-p(v)} + a_{k\omega+v}$. When this model is applied to a series of size $N\omega$ (say, N years and ω periods), then we have

$$\begin{aligned} X_1 &= \phi_1^{(1)} X_0 + \phi_2^{(1)} X_{-1} + \mathbf{K} + a_1 \\ X_2 &= \phi_1^{(2)} X_1 + \phi_2^{(2)} X_0 + \mathbf{K} + a_2 \end{aligned} \quad (3.1)$$

M

$$X_{N\omega} = \phi_1^{(\omega)} X_{N\omega-1} + \phi_2^{(\omega)} X_{N\omega-2} + \mathbf{K} + a_{N\omega}$$

If noise terms $\{a_{k\omega+v}\}$ are set alone on the left hand side of the equations above, then the conditional sum of squares is written as

$$\begin{aligned} S(\phi_1^{(1)}, \mathbf{K}, \phi_{p(1)}^{(1)}, \mathbf{K}, \phi_1^{(\omega)}, \mathbf{K}, \phi_{p(\omega)}^{(\omega)}) &= \sum_{v=1}^{\omega} \sum_{k=k_v}^{N-1} a_{k\omega+v}^2 \\ &= \sum_{v=1}^{\omega} \sum_{k=k_v}^{N-1} (X_{k\omega+v} - \phi_1^{(v)} X_{k\omega+v-1} - \mathbf{K} - \phi_{p(v)}^{(v)} X_{k\omega+v-p(v)})^2 \\ &= \sum_{v=1}^{\omega} S_v(\phi_1^{(v)}, \mathbf{K}, \phi_{p(v)}^{(v)}) \end{aligned} \quad (3.2)$$

where

$$k_v = \left[\frac{p(v) - v}{\omega} + 1 \right] \quad (3.3)$$

again $[x]$ representing the integral part of the real number x . Note that, the equations involving any unknown observations like X_0, X_{-1}, X_{-2} are omitted, so that the sum of squares for season v , S_v , contains only $N - k_v$ terms. The term ‘‘conditional’’ is related to this fact that, for example a_1 can not be

obtained since it involves unknown observations, so that the summation in formula (3.2) starts from k_v . Here, it should be noted that the conditional ML estimates can be obtained optimizing the conditional likelihood function, conditional with respect to X_1 in (3.2), which is assumed to be fixed.

It can be seen from (3.2) that, each season v has its own sum of squares of errors S_v and those sum up to the conditional sum of squares S . Thus, it is apparent that, for any season v , minimizing S with respect to phi coefficients in order to obtain least squares estimates (LSE) is equivalent to minimizing S_v for each season separately. In addition, if the process has a non-zero mean, then minimizing S_v loses its simplicity and becomes problematic. It is because the conditional sum of squares, S_v , involves parameters except seasonal ones (v^{th} season parameters), so that seasonwise minimization can not be performed. This non-zero mean problem is usually achieved by estimating μ_v by \bar{X}_v , the sample mean of season v and adding an intercept parameter in the regression equation for each season.

Generalization of the least squares method for regression to a seasonwise case is straightforward, yet it brings some vital assumptions. The classical

regression theory crucially assumes that the regressor (or, independent) variables are nonrandom. However, in many situations, the assumption about regressors being nonrandom is inappropriate. Autoregressive models in time series analysis is one such situation, where the regressors are observations from a stochastic process and they are the lagged - or, past - values of the response variable. Fortunately, most of the results concerning LS parameter estimation still holds for the situations where regressors are random provided that the response and regressor variables are distributed as multivariate joint normal. The multivariate joint normality is satisfied if the process is a Gaussian PAR process, i.e., if the white noise terms in the model are assumed to be independent and normal.

The problem of regressors being random is overcome this way. However, another pitfall is that, in PAR processes, the response variables, i.e., the dependent variables of regression are correlated, since one is a past or future value of another. This gravely violates the assumption of independence between response variables. On the other hand, this problem is negligible in a periodic process, since for any season ν , responses will be at least ω units far apart from each other, so that the correlation will be very weak by the nature of autocorrelation especially for large seasons ω , which means independence for a Gaussian process. Note that the uncorrelation of the variables also implies their independence if and only if the process is Gaussian, so a Gaussian PAR model satisfies the independence assumption of regression theory, too.

The overall difficulties of pursuing estimation to MA models were explained in the beginning stating the reasons, so that if any season ν is decided to follow a PMA model, again the least squares estimation will be applied. At first glance, it is not apparent how a LSE method can be applied to MA type models, since they do not have the form of a classical regression equation like AR models. However, it is known that any invertible MA model can be expressed as an infinite order AR model, but in practice that infinite

order is of course finite, yet reasonably large to represent the autocorrelation behaviour of the series. Thus, LS method can be meaningfully carried out by setting a maximal value for AR order. Hence, in this thesis, seasons which are found to follow pure PMA processes are re-expressed as PAR processes, setting some maximal orders $p(\nu)$ and fitting, in turn, the most suitable PAR model

according to the results of hypothesis tests which are similar to the tests on individual regression coefficients in multiple regression (Newbold, 1994; p.723). These tests concerning the individual PAR coefficients of any arbitrary season, $\phi_i^{(\nu)}$, ($i = 1, 2, \dots, p(\nu)$), have the following alternatives:

$$\begin{aligned} H_0 : \phi_i^{(v)} &= 0 \\ H_A : \phi_i^{(v)} &\neq 0 \end{aligned} \tag{3.4}$$

The mentioned maximal order may be set as a sufficiently large lag number and then significance testing is employed iteratively. Finally, this iterative procedure terminates when a value $p(v)$ is found for which the null hypothesis is rejected. An application of this method will be shown in Chapter IV.

Besides estimating PAR parameters with regression method, the model diagnostics, which are the the last and third stage of model development, are performed by using residuals of the fitted model.

Diagnostic checking, or model criticism, is concerned with the goodness-of-fit of a model, and, if the fit is poor, suggests necessary modifications which means that the whole process of model identification, estimation and diagnostic checking must be repeated. The analysis on residuals resulting from the fitted model is one approach for diagnostic checking.

Recall from the definition of PARMA process given in (1.2) that, the sequence of error terms, $\{a_{k\omega+v}\}$, is assumed to be zero-mean, finite variance, uncorrelated variables, which is named as white noise. In particular, independence and normality assumptions are added for estimation. Diagnostic checks, shortly, test if these assumptions on error terms are verified or not. Therefore, these checks include the following analysis on residuals of a fitted model:

1. if residuals are random (independent) or not
2. if residuals are white noise or not
3. if residuals are normally distributed or not

In order to test the randomness of residuals, Runs Test is performed. Runs Test is a nonparametric procedure for examining whether or not a set of observations are of a random nature, whose null hypothesis that residuals come from a random process is tested against the opposite alternative.

By the end of Chapter II, it was given that a series is said to be white noise if its ACF and PACF values all fall inside $\pm 1.96/\sqrt{N}$ limits. Hence, in order to test the white noise assumption on residuals, the ACF and PACF plot of the residuals are analyzed.

The normality of residuals, which provides stronger conclusions on the model, is tested through the Anderson – Darling Test for Normality. The null hypothesis that residuals follow normal distribution is tested against the opposite alternative. The critical values for the test are dependent on the specific distribution that is being tested, and the tabulated values are available for a few of them like normal, lognormal, exponential etc.. If the normality assumption is not achieved, then it means that transformation is needed, such as logarithmic transformation or some other. Transformation of a series in order to satisfy normality is however not covered in this study. All these tests concerning diagnostic checks are performed through MINITAB, Release 13.1.

CHAPTER IV

APPLICATION

This thesis is intended to emphasize on PARMA model identification and to offer practical computer programs which compute sample seasonal means, autocovariances and autocorrelations, mainly perform model identification using PeACF and PePACF, and estimate parameters of pure PAR processes using LSE method for any given actual series displaying periodic structure. Thus, in this chapter, the usage of the mentioned programs which are listed fully in appendices, is demonstrated by presenting applications on two seasonal hydrologic time series. The model identification is applied utilizing the programs provided in Appendix B and C.

The program in Appendix B computes the sample PeACF with corresponding band limits, specifies the significant lags and decides the suitable PMA model for each season of the series, and the other program in Appendix C computes the sample PePACF, and similarly decides the suitable PAR model for each season. The least squares estimation, hypothesis testing and diagnostic checking are performed through MINITAB statistical software, Release 13.1. Though, for seasons which surely follow PAR models, LSE can either be performed utilizing the program in Appendix D.

Both series considered consist of average monthly streamflows (in $\text{m}^3 / \text{sec.}$) which are presented in Appendix E. Data is obtained from Book of Average Monthly Streamflow of EIE which is available in Water Resources Division, Dept. Of Civil Eng., METU. First series is observed from Atnos Creek, Balıkesir, while the second one from Nilüfer Creek, Bursa and their time plot are given in Figure 4.1.1 and Figure 4.2.1 respectively.

All three stages of model development, i.e., identification, estimation and diagnostic checking are illustrated with the first series, while only the model identification is proceeded with the second.

4.1 Application on Atnos Creek

The series consists of average monthly streamflow (in $\text{m}^3 / \text{sec.}$) values for the Atnos Creek in Balikesir, Turkey from October 1963 to September 1995, where season 1 corresponds to October and season 12 corresponds to September. The number of observations is 396, and since observations are recorded monthly, the period ω is equal to 12. The time plot of the series is given in Figure 4.1.1 in which the periodic behaviour is apparent.

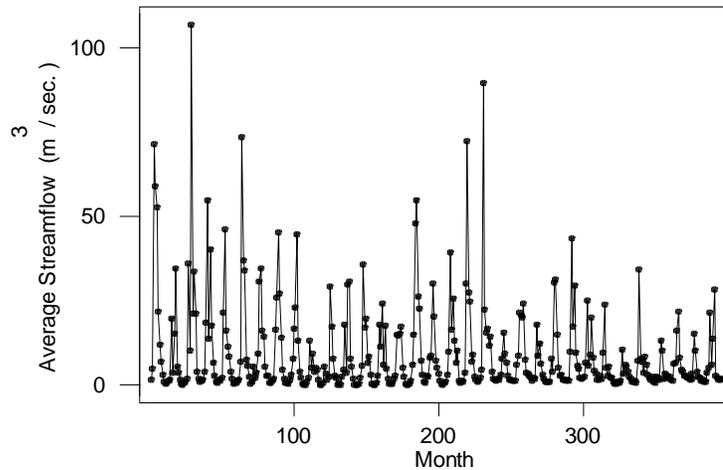


Figure 4.1.1 Time plot of Atnos Creek

In Table 4.1.1, the sample means and sample variances, which are computed through the program listed in Appendix A, is presented for each of 12 seasons.

Table 4.1.1 Sample statistics for seasons of Atnos Creek

Season ν	Sample Mean \bar{X}_ν	Sample Variance $\hat{\gamma}_0(\nu)$
1	1.50961	0.77816
2	3.56882	5.15689
3	17.29515	345.30518
4	23.66397	425.31482
5	23.05264	389.47153
6	18.99136	107.54240
7	10.94206	52.22260

Once deciding on a PARMA type model, the next step is the identification of suitable models for each 12 seasons of Atnos. This task is done by executing the programs listed in Appendix B and Appendix C, whose outputs are the values of PeACF and PePACF, respectively, with the corresponding band limits from which the significant lags are specified.

4.1.1 Model Identification

The sample PeACF and sample PePACF graphs for each 12 seasons are presented in Appendix F in which the autocorrelation and partial autocorrelation values are drawn versus time lag. In practice, maximum lag is taken as one fourth of the number of years, however, this value here is, $N / 4 = 33 / 4 \approx 8$, which is very small to give an appropriate decision on identification. Therefore, maximum lag number is taken as 16 in these graphs. Recall from the sampling properties of the sample PeACF that, its variance and hence the band limit changes only when the order, $q(v)$, of the PMA process is greater or equal to

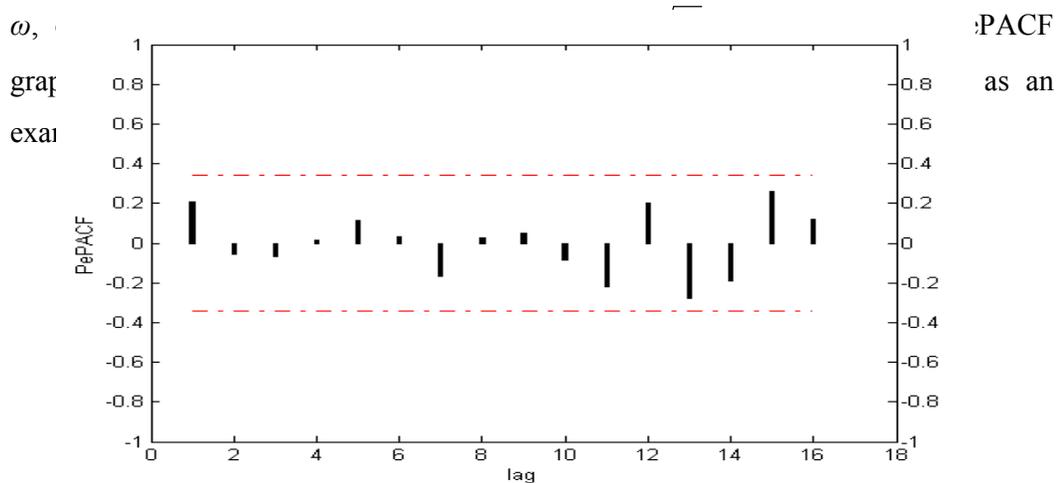


Figure 4.1.2 (b) Sample PePACF of season 1 of Atnos

Also, as an example of the computer output, Table 4.1.2 is given again for the first season of Atnos. Significant lags, if exists, are marked with a star.

Table 4.1.2 PeACF and PePACF of Atnos season 1

Lag							
	- Band	PeACF	+ Band		- Band	PePACF	+ Band
1	-0.34119	0.20607	0.34119		-0.34119	0.20607	0.34119
2	-0.34119	0.06580	0.34119		-0.34119	-0.05808	0.34119
3	-0.34119	0.00544	0.34119		-0.34119	-0.06865	0.34119
4	-0.34119	0.03297	0.34119		-0.34119	0.01659	0.34119
5	-0.34119	0.09381	0.34119		-0.34119	0.11492	0.34119
6	-0.34119	0.07441	0.34119		-0.34119	0.03028	0.34119
7	-0.34119	-0.08915	0.34119		-0.34119	-0.16695	0.34119
8	-0.34119	0.07272	0.34119		-0.34119	0.02569	0.34119
9	-0.34119	0.08802	0.34119		-0.34119	0.04751	0.34119
10	-0.34119	-0.01287	0.34119		-0.34119	-0.08444	0.34119
11	-0.34119	-0.17599	0.34119		-0.34119	-0.21865	0.34119
12	-0.34119	0.03313	0.34119		-0.34119	0.20096	0.34119
13	-0.34218	-0.17061	0.34218		-0.34119	-0.27924	0.34119
14	-0.34789	-0.21697	0.34789		-0.34119	-0.18766	0.34119
15	-0.34593	-0.11187	0.34593		-0.34119	0.25950	0.34119
16	-0.33975	0.03685	0.33975		-0.34119	0.12160	0.34119

Analyzing the PeACF and PePACF together, which are presented in Appendix F, respectively, it can be said that the first, second, third and the fourth seasons follow white noise process without any doubt, since all values fall inside the bands for both functions.

For season five, 6th lag in PeACF is outside the bands, but since that value is close to band limits, it may be ignorable. And in PePACF all values fall inside band limits. This indicates that the process may be an MA(6) or

white noise with a negligibly small correlation close to the band value, where the latter one is more plausible, but final decision will be given after estimation and diagnostic checks.

In season six, both PeACF and PePACF show cut-off after lag 13, but since PeACF shows better cut-off, this season is said to follow MA(13) model. On the other hand, since only PAR models is to be specified in this section in order to employ LSE, this MA (13) should be re-expressed as an AR model of sufficient order which will be investigated in the next section.

It is appearant that season seven follows a MA (2) model with its perfect cut-off after lag 2 of PeACF. Note that PePACF shows a very large value greater than the limits lately. That is, season seven follows a MA (2) model.

Analyzing the PeACF & PePACF of season eight, one may claim that this season can either be a MA (5) or an AR (5) since both functions show cut-off after lag 5. Since PePACF enjoys better cut-off and it is proper to LS estimation, AR (5) is preferred as the suitable model for this season.

In season nine, it is clear that the PeACF shows cut-off after lag 1 while PePACF insists not to because of its 14th lag. This indicates that season nine follows a MA (1) process.

Season ten and season eleven are indeed troublesome; for both seasons neither PeACF nor PePACF show satisfactory cut-offs. However, in season ten, compared to PePACF, it is seen that PeACF values lie within bands after lag 12, i.e the season may be claimed to follow a MA (12) model. In addition, for season eleven, MA(13) seems to be the suitable model.

Season twelve may follow a MA (2) model since PeACF displays satisfactory cut-off after lag 2, and PePACF cuts-off after lag 14. Hence, this season has a MA(2) model.

To sum up, PMA and PAR models are recommended for each twelve seasons of Atnos Creek indicating four white noise, one AR, and seven MA models. The results of initial models recommended in this section is summarized in Table 4.1.3. The final decisions on model specification, which are summarized in Table 4.1.4, will be made after estimation and diagnostic checking.

Table 4.1.3 Model identification for Atnos Creek
prior to estimation and diagnostic checks

Season v	$p(v)$	$q(v)$	Model
1	0	0	White Noise (WN)
2	0	0	WN
3	0	0	WN
4	0	0	WN
5	0	6	MA(6)
6	0	13	MA(13)
7	0	2	MA(2)
8	5	0	AR(5)
9	0	1	MA(1)
10	0	12	MA(12)
11	0	13	MA(13)
12	0	2	MA(2)

4.1.2 Model Estimation

Prior to PAR estimation for Atnos series, a short description on the method and hypothesis test should be given. As mentioned before, the equation of a PAR model can be considered as a multiple regression equation, so that estimation and testing of PAR coefficients, can be carried out similar to that of individual regression coefficients. Therefore, tests concerning the individual PAR coefficients of an arbitrary season, $\phi_i^{(v)}$ ($i = 1, 2, \dots, p(v)$), have the following alternatives

$$\begin{aligned} H_0 : \phi_i^{(v)} &= 0 \\ H_A : \phi_i^{(v)} &\neq 0 \end{aligned} \tag{4.1}$$

where the test is based on the fact that, the parameter estimator divided by its estimated standard error follows t-distribution when the null hypothesis is true. The test statistic, then, is

$$t_0 = \frac{\hat{\phi}_i^{(v)}}{se(\hat{\phi}_i^{(v)})} \tag{4.2}$$

where $\hat{\phi}_i^{(v)}$ is the parameter estimate and $se(\hat{\phi}_i^{(v)})$ is its standard error. The null hypothesis is rejected with a significance level of α if absolute value of the calculated t -stat is greater than the tabulated value.

In the previous section, models have been tentatively specified much or less for each season, and with regard to those specified models, this section considers parameter estimation by LSE. Also note that, since all seasons are found to have non-zero mean, an intercept parameter must be added into the models and estimated in turn.

Since for the first through fourth seasons, white noise process is found suitable which does not contain any parameters, but only uncorrelated terms, estimation is not of consideration.

For season five, we doubt of an MA (6) model or a white noise process with a negligibly small value of autocorrelation at 6th lag. This MA (6) model is expressed initially as an AR (15) model where maximum lag $l=15$ is

decided to be the maximal order set following the procedure explained a chapter before. To gain further understanding, the mentioned method is shown here with an example on season five. For instance, fixing the maximal order $p(5)$ as 15, i.e., $p(5)=15$, the fitted AR(15) is,

$$X_{12k+5} = 28.96 + 0.4212X_{12k+4} + K - 2.387X_{12k-10}, \quad k = 1,2,\dots \quad (4.3)$$

(0.117) (0.107) (0.186)

where the values in parantheses beneath the coefficient estimates are the corresponding p -values of the t -test. It is obvious that the null hypothesis that $\phi_{15}^{(5)} = 0$ is accepted by 0.05 significance level since $0.05 < 0.186$, so omitting this parameter from the model, next AR(14) model is fitted as,

$$X_{12k+5} = 16.50 + 0.4303X_{12k+4} + K + 0.1384X_{12k-9}, \quad k = 1,2,\dots \quad (4.4)$$

(0.298) (0.107) (0.637)

Since the null hypothesis that $\phi_{14}^{(5)} = 0$ is also accepted at 0.05 significance level, AR(13) should be tried in turn. Accordingly, the hypothesis tests will continue to proceed until the appropriate PAR order $p(5)$ is found for season five. Not all of the trials are presented, but as the procedure performed, all null hypotheses are accepted with 5% significance level. Thus, this season can

not be modeled with a PAR process and white noise claim is accepted to be proper.

Season six which is said to follow an MA (13) model, is here reconsidered as an AR (15) initially, whose starting null hypothesis is $\phi_{15}^{(6)}$ being equal to zero, is terminated at lag 13. Hence, AR (13) can possibly be suitable for this season.

Exactly the same statements cover season seven, which follows MA(2), too, but this time the AR model of maximal order 15 iterates until lag 2, i.e., season seven can be modeled as an AR (2) model.

Season eight is decided to be an AR (5) model and LSE is applied by MINITAB. Besides, since this season is claimed to be an AR(5) doubtless, the program listed in Appendix D can directly be utilized also in order to obtain LS estimates of $\phi_1^{(8)}$ through $\phi_5^{(8)}$ along with the resulting residuals.

For season nine, MA(1) model is also reconsidered as an AR model beginning with an order of 15 and ending at 1, so that season nine can possibly be modeled as AR(1).

For seasons ten and eleven, beginning with AR (15), the trials end at lag 10 for season 10, and at 12 for season eleven. Hence, AR (10) and AR(12) are said to be the appropriate models for seasons ten and eleven, respectively.

MA(2) model was specified for the 12th and the last season, and the iterative tests end at lag 8 this time, so that season twelve can possibly be modeled as an AR(8) model.

Table 4.1.4 Model identification for Atnos Creek after estimation

Season ν	$p(\nu)$	$q(\nu)$	Model
1	0	0	WN
2	0	0	WN
3	0	0	WN
4	0	0	WN
5	0	0	WN
6	13	0	AR(13)
7	2	0	AR(2)
8	5	0	AR(5)
9	14	0	AR(1)
10	11	0	AR(10)
11	11	0	AR(12)
12	8	0	AR(8)

In view of this section, some problems must be stated which especially arise because of trying to model all seasons as PAR processes. Firstly, it is seen that re-expressing a PMA process as a PAR process results in a significant increase in the number of parameters which is not a desired situation since it hardens the estimation and most of all violates the parsimony. In addition, when the order of PAR model is tried to be reduced more than the specified one according to significance tests, it is seen that diagnostic checks become unsatisfactory. It should be stated that, if PAR models which are found suitable and used instead of PMA model do have very large orders, then this method loses its meaning and usefulness. The problem of estimation of PMA processes may be achieved by other estimation methods like MLE, but then mathematical difficulties shall arise. However, a new method for the preliminary estimation of the MA parameters is proposed by Monti (1995), in which the estimator is called Random Initial Values Estimator (RIVE). The method is based on artificially generating white noise series which replace the

actual noise. The idea is that all information relating the parameter is conveyed by the observed series, so that the knowledge of the actual noise is not

essential for estimation. It is proved that this estimator is consistent, asymptotically normal and performs well also in small samples. Suitability of this procedure to periodic case may be taken into consideration in order to be able to perform preliminary estimation for PMA models, but needs investigation and detailed study on its applicability.

Secondly, mixed PARMA models may be more suitable for some of the seasons, but since standard ARMA identification procedure does not work for the identification of mixed PARMA model, and since there exists no other available method, further investigation should be made for this case.

4.1.3 Diagnostic Checks

In the previous sections, white noise process was found suitable for seasons one through five, while different PAR models are fitted for the rest. This section involves model diagnostics on these seasons in order to give final decisions about models.

Recall that, in order to test the randomness of residuals, Runs Test is performed, whose null hypothesis is that residuals are random. To test normality of residuals, Anderson-Darling test is applied where the null hypothesis is that residuals are normally distributed. For both tests, significance level α is accepted as 0.05, so that the null hypotheses are rejected (assumption is not verified) if α is greater than the p -value. And in order to test the white noise assumption on residuals, the ACF and PACF plot of the residuals are analyzed.

Diagnostic checks on Atnos series are mostly satisfactory, such as for seasons 6, 8, 10, 11 and 12, all tests on residuals give affirmative results for

the corresponding fitted models. However, for seasons 7 and 9, normality assumption fails which indicates the necessity of an appropriate transformation.

Consequently, model identification had to be repeated over the transformed series which may lead to a new possible model.

Table 4.1.5 presents the corresponding p -values and decisions of the tests on residuals for each 12 seasons separately.

Table 4.1.5 Summary of diagnostic checks on Atnos Creek

Season ν	Run Test for Independence	Being White Noise	Anderson-Darling Test for Normality
1	-	-	-
2	-	-	-
3	-	-	-
4	-	-	-
5	-	-	-
6	VERIFIED p-value = 0.2809	VERIFIED	VERIFIED p-value = 0.812
7	VERIFIED p-value = 0.1533	VERIFIED	NOT VERIFIED p-value = 0.002
8	VERIFIED p-value = 0.4961	VERIFIED	VERIFIED p-value = 0.197

Table 4.1.5 (cont'd) Summary of diagnostic checks on Atnos Creek

Season ν	Run Test for Independence	Being White Noise	Anderson-Darling Test for Normality
9	VERIFIED p-value = 0.0659	VERIFIED	NOT VERIFIED p-value = 0.000
10	VERIFIED p-value = 0.5226	VERIFIED	VERIFIED p-value = 0.278
11	VERIFIED p-value = 0.4567	VERIFIED	VERIFIED p-value = 0.482
12	VERIFIED p-value = 0.2099	VERIFIED	VERIFIED p-value = 0.089

Finally, the general model representing the Atnos series is found as $PAR_{12} (0; 0; 0; 0; 0; 0; 13; 2; 5; 1; 10; 12; 8)$.

4.2 Application on Nilüfer Creek

The series consists of average monthly streamflow (in $m^3 / sec.$) values for the Nilüfer Creek in Bursa, Turkey from October 1954 to September 1995,

where season 1 corresponds to October and season 12 corresponds to September. The number of observations is 504, and since observations are recorded monthly, the period ω is equal to 12.

The time plot of the series is given in Figure 4.2.1 in which the strong periodic behaviour is displayed.

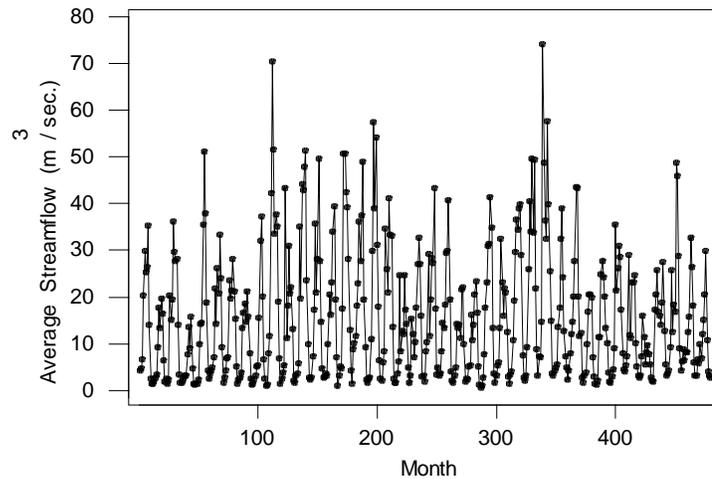


Figure 4.2.1 Time plot of Nilüfer Creek

Once deciding that the model should be of PARMA type, the next step is the identification of suitable models for each 12 seasons of Nilüfer. Again executing the programs listed in Appendix B and C whose outputs are the values of PeACF and PePACF, respectively, with the corresponding band limits from which the significant lags are specified.

In Table 4.2.1, the sample means and sample variances, which are computed through the program listed in Appendix A, is presented for each 12 seasons of the creek.

Table 4.2.1 Sample statistics for seasons of Nilüfer Creek

Season ν	Sample Mean \bar{X}_ν	Sample Variance $\hat{\gamma}_0(\nu)$
1	4.75200	10.49473
2	7.34255	17.24455
3	17.28931	182.38977
4	20.3324	184.89177
5	22.54998	148.53142
6	25.30845	99.66177
7	31.16897	141.49802
8	29.18558	118.99061
9	14.86316	37.61295
10	5.13607	8.99974
11	2.59871	3.41349
12	2.74981	2.34262

4.2.1 Model Identification

The PeACF and PePACF plots of Nilüfer Creek are presented in Appendix F, analyzing the plots together, PARMA model identification will be performed for each season.

For season one, the PeACF plot shows perfect cut-off after 1st lag, while PePACF shows cut-off after lag 5. This states that season one either follows an MA(1) or an AR(5) model. Since PeACF shows better cut-off and suits the rule of parsimony, MA(1) is more preferable.

For season two, both PeACF and PePACF values fall inside band limits after 1st lag, which indicates that this season may be an MA(1) or an AR(1).

The PePACF shows better cut-off at first glance, but it is seen that its late lags become larger again while PeACF keeps cut-off. Thus, MA(1) seems the suitable model for season two.

For season three, 11th lag in PeACF and 9th lag in PePACF are the final lags which are outside the bands. This indicates that the process may be an MA(11) or an AR(9), where the latter one is more plausible with its better cut-off and less number of order.

In the PeACF of season four, values fall inside band limits after lag 10, and in PePACF after lag 13. Neither PeACF nor PePACF show satisfactory cut-offs, so MA(10) or AR(13) are both suitable candidates for season four. A final decision requires diagnostic checks.

For season five, the PeACF shows perfect cut-off after lag 2, that is, this season may be an MA(2). And in PePACF, the 1st and 16th lags are significant, 16th lag is a sign that the cut-off property of PePACF is ruined. But still, AR(1) may be considered as another suitable model for this season.

For season six, MA(13) is suspected from PeACF, and AR(16) from PePACF. None of these models seem superior to others just by looking at autocorrelation plots, that the final decision on specification can only be given after estimation and diagnostic checks.

For season seven, PeACF shows cut-off after lag 4, and PePACF shows cut-off after lag 1. Though, 10th and 15th lags of PePACF are outside the limits, note that they are very close to band values so that they may be negligible. Therefore, among MA(4) and AR(1), the latter seems more suitable for season seven.

Analyzing the PeACF & PePACF of season eight, it is seen that PePACF shows nice cut-off after 2nd lag, while PeACF does not. However,

10th-13th lags of PePACF fall outside band limits ruining the cut-off. Also, for this season the suitable AR order should be decided after estimation and diagnostic checks.

In season nine, it is clear that the PePACF shows perfect cut-off after lag 1 while PeACF rather decays. This indicates that season nine almost surely follows an AR(1) model.

Season ten displays a decay rather than cut-off in PeACF, but it can be said that the model may be AR(16) or AR(10) by looking PePACF, AR(1) may be a choice, too. After testing these choices with the iterative testing procedure the most suitable model for this season should be found.

For season eleven, MA(4) is the most suitable with the cut-off of PeACF after lag 4, while PePACF insists on larger values than the band limits.

Season twelve may follow an MA (8) model as well as an AR (8) model, since both PeACF and PePACF display cut-off after lag 8. MA(8) is more suitable with a better cut-off in PeACF.

Consequently, for Nilüfer series, mostly PMA models are found suitable for the seasons. Unfortunately, for this series, the method of using PAR models with reasonable orders instead of PMA models does not work efficiently, that especially the normality test in diagnostic checks generally failed.

For season 1, MA(1) is tried to be re-expressed as an AR model with a suitable order, and the iterative significance tests stopped at AR(5), but

normality assumption can not be verified. For season 2, significance test continued until AR(1) but the normality assumption fails for this case, too. For season 4, 5, 11 and 12, same problem on normality arises if MA models are tried to be re-expressed as AR type. Only, season 6 which follows an MA(13)

model can be modeled as an AR(10) model, and satisfy all residual checks. For seasons 8 and 10, the least squares states AR(2) and AR(10) models are suitable, respectively.

To sum up, PMA and PAR models are recommended for each season of Nilüfer Creek and when PAR models are tried to be fitted, it is seen that the general violation is on normality assumption rather than independence or white noise. This indicates that Nilüfer series may itself need an appropriate transformation to achieve normality, or series does not satisfy periodic stationarity, although the nonstationary behaviour is surely not displayed in periodic autocorrelation or partial autocorrelation functions. The recommended models are summarized in Table 4.2.2.

Table 4.2.2 Model identification for Nilüfer Creek

Season v	$p(v)$	$q(v)$	Model
1	0	1	MA(1)
2	0	1	MA(1)
3	9	0	AR(9)
4	0	10	MA(10)
5	0	2	MA(2)
6	10	0	AR(10)
7	1	0	AR(1)
8	2	0	AR(2)
9	1	0	AR(1)
10	10	0	AR(10)
11	0	4	MA(4)
12	0	8	MA(8)

Consequently, the PARMA model representing the whole process for Nilüfer Creek is recommended as $\text{PARMA}_{12} (0,1; 0,1; 9,0; 0,10; 0,2; 10,0; 1,0; 2,0; 1,0; 10,0; 0,4; 0,8)$.

CHAPTER V

SUMMARY AND CONCLUSIONS

Periodic autoregressive moving-average (PARMA) models proved to be useful in practice and received considerable attention in many fields. They become popular with seasonal series recently as the techniques for their analysis develop.

The major aim of this thesis is the PARMA model identification, i.e., the determination of the seasonally varying orders of the PARMA model, and to develop a practical computer program which performs model identification for any given actual periodic stationary series.

In the first part of the thesis, posterior to a general information on time series analysis, the basics of PARMA process are explained. It is shown that any PARMA model can be expressed as a vector ARMA model which is named as the “lumped” vector process. Through this property, it is given that the periodic stationarity conditions and also the invertibility conditions of a univariate PARMA process can be obtained from its corresponding lumped vector process.

The determination of an appropriate $\text{PARMA}_\omega(p(v), q(v))$ model to represent an observed periodic stationary series involves a number of inter-

related subjects. These include the order identification, parameter estimation and diagnostic checks.

For PARMA model identification, which is the main body of this study, the Box-Jenkins methodology for the identification of ordinary ARMA processes is generalized to a seasonwise identification procedure. For identification of standard ARMA models, the autocorrelation function (ACF) and the partial autocorrelation function (PACF) serve as useful tools, similarly for PARMA processes, the periodic autocorrelation function (PeACF) and periodic partial autocorrelation function (PePACF) play the same role.

The PeACF is used for the identification of seasons which follow pure periodic moving-average (PMA) processes. It is given that PeACF becomes zero, namely cuts-off, for lags beyond the appropriate order, $q(v)$, of the PMA process. For the assesment of this cut-off property for a finite realization, the sample PeACF is utilized which is the sample estimate of the theoretical PeACF. The asymptotic results for the first and second order moments of the sample PeACF, and its asymptotic normality had already been proved, so that they are used together to obtain band limits for the application of cut-off property.

On the other hand, for the identification of seasons following pure autoregressive (PAR) processes, the PePACF is well adapted. The computation of partial autocorrelations as in standard AR context does not work with PePACF, so that a different algorithm is used for its computation . Similar to the PeACF of pure PMA processes, this time the PePACF shows cut-off for lags beyond the appropriate order, $p(v)$, of the PAR process. The sample PePACF, which is the sample estimate of its population counterpart, is employed for the assesment of cut-off property for a finite series coming from a pure PAR process.

The model estimation is considered in the simplest manner not for estimation purposes but for refining and developing the model identification stage. Surely, estimation together with significance tests may suggest which of the possible candidates is a suitable model for representing the data. Previous studies on estimation methods are summarized mentioning that the estimation of PARMA models with a MA part is very complicated and technically difficult. Only the conditional least squares estimation (LSE) method is considered in this study for pure PAR processes. It is shown that conditional LS estimation of PAR parameters is simply a generalization of the least squares method to a seasonwise case. For the estimation of PMA models, again LSE is proposed but first PMA model is re-expressed as a PAR model with a reasonable order which is able to represent the autocorrelation behaviour of the series. This proposed method may not be pretentious but at least useful for practical purposes. Diagnostic checking is also mentioned stating its reasons and methods. Through this thesis, diagnostic checks are made by residual analysis.

The last part of the study is devoted to application on two actual series of hydrologic processes. Both series consist of average monthly streamflow observations from Atnos Creek, Balikesir, and from Nilüfer Creek, Bursa. Their time plots both display the strong periodic structure and indicate the necessity of including periodically varying parameters in the model.

All three stages of model development, i.e., identification, estimation and diagnostic checking are illustrated with Atnos series by making use of the programs developed for identification purposes. The sample means, variances and sample autocovariances are computed and summarized for each twelve seasons, in addition the sample PeACF and PePACF of each season is obtained and represented graphically. Some seasons of the series are found to follow white noise, while some PMA or PAR models. The method of changing PMA models to PAR type is achieved and conditional LSE is performed for all

seasons except the ones which follow white noise. Results of the diagnostic checks are given for each fitted model, where nearly all seasons satisfy the tests on residuals, but normality assumption is failed for only two of them. The initial and final decisions on model identification of seasons of Atnos are presented through tables, so that 12 different models are specified separately for each season. Thus, the general model for Atnos Creek is recommended initially as $PARMA_{12}(0,0; 0,0; 0,0; 0,0; 0,6; 0,13; 0,2; 5,0; 0,1; 0,12; 0,13; 0,2)$, and after changing all seasons into PAR type, the general model becomes $PAR_{12}(0; 0; 0; 0; 0; 13; 2; 5; 1; 10; 12; 8)$.

For Nilüfer Creek, only model identification is considered. PMA models are found suitable for most of the seasons, and PAR for the rest. The final general model representing the whole series is found as $PARMA_{12}(0,1; 0,1; 9,0; 0,10; 0,2; 0,3; 1,0; 10,0; 0,4; 0,8)$.

The developed programs, which are implemented with Visual Fortran, Professional Edition 5.0A, are listed fully in appendices with input and output examples. For all programs, the series of interest can either be entered manually by the user, or the program reads it from a stored file itself. The execution of the model identification program is performed in two phases. Firstly, computation of PeACF values specifying significant lags according to band limits and proposition of suitable PMA processes for each season are made. Secondly, the computation of PePACF values specifying significant lags is made again and suitable PAR processes are proposed. The LSE and diagnostic checks are performed through MINITAB statistical software, Release 13.1. However, the program listed in Appendix D also performs LSE of the PAR coefficients along with the resulting residuals.

Since PARMA models are quite new, many questions about them are still unanswered and need further studies. Some problems faced throughout this

study should be stated such as identification of mixed PARMA processes, an applicable preliminary estimation method for mixed PARMA and PMA

processes. It is known that there exists general methods for the identification of standard ARMA models of mixed type, however no such satisfactory method is available for mixed PARMA processes. In the application part of this thesis, mixed PARMA models may be more suitable for some seasons. Moreover, in order to be able to perform a simple preliminary estimation, even PMA models are re-expressed as PAR models and this results in a significant increase in the number of parameters which violates parsimony. This study involves only the periodic stationarity case and the programs work only for periodic stationary processes. There exists well-known methods like differencing or filtering to achieve stationarity of standard ARMA models, such methods for achieving periodic stationarity should also be investigated.

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APPENDIX A

COMPUTER PROGRAM FOR COMPUTATION OF SAMPLE PERIODIC MEANS, AUTOCOVARIANCES AND AUTOCORRELATIONS

Description:

Programs listed in these appendices were all written in Visual Fortran, Professional Edition 5.0A.

This program computes the sample periodic means, sample periodic autocovariances, and sample periodic autocorrelation and partial autocorrelations for each season of any periodic process. User may enter the series manually or program can read the series from a stored file. The computations are performed up to a desired lag if specified, or up to a default lag, which is the one fourth of total number of years, if nothing is specified.

Inputs:

N: Number of years

IW: Number of periods

NOLAG: default number of lags ($N/4$)

CANS: Name of the text file in which the series of interest is stored

Outputs:

XSUM(): Sum of the observations in each season

XMEAN(): Sample mean for each season

XP0(): Sample variance for each season

XPACVF(): Sample periodic autocovariances for each season

XPACF(): Sample periodic autocorrelation for each season

PHI(): Sample periodic partial autocorrelation for each season

ALPHA() / BETA() / TAOSQ() / DELTASQ() / DELTA() / CDELTA()

Parameters used in Sakai's algorithm

Program Listing:

THE OUTPUTS OF THIS PROGRAM ARE STORED IN FILE 'STATatnos.txt'

```
C /* -----
C     program that finds PeAcf and PePacf
C
C     (c) 2002
C     by Burcin Akgun
C
C     ----- */
C     PROGRAM PeSTATS
C     REAL X(-500:500),XSUM(1000),XMEAN(-500:500),XP(12,500)
C     REAL XPACVF(12,1000),XP0(1000),XP00(-500:500),SUM,TEMP1,TEMP2
C     REAL TPACVF(-500:500,200)
C     REAL DELTSQ(-2:12,0:200),TAOSQ(-2:12,-1:200)
C     REAL ALPHA(-1:12,0:200,0:200),BETA(-1:12,0:200,0:200)
C     REAL CDELTA(-1:12,0:200),PHI(0:12,0:200,0:200)
C     REAL DSUM
C     INTEGER ICON,IV,IW,N,NW,NOLAG,NMAXLG,IP,IM
C     CHARACTER FNAME*20, CANS*1
C // open the output file
C     OPEN (2, FILE='STATatnos.txt')
C //
C     DO 3 I=1,100
C         XSUM(I)=0
C         XP0(I)=0
C     3     CONTINUE
C     DO 4 I=-100,100
C         X(I)=0
C         XMEAN(I)=0
C         XP00(I)=0
C     4     CONTINUE
```

```

DO 5 J=1,12
  DO 6 I=1,100
    XP(J,I)=0
    XPACVF(J,I)=0
6    CONTINUE
5    CONTINUE
PRINT*, ' ENTER YEAR(N), PERIOD(W):'
WRITE(2,*) ' ENTER YEAR(N), PERIOD(W):'
READ*, N,IW
WRITE(2,*) N, IW
PRINT*, ' ENTER NUMBER OF LAGS (NOLAG):'
WRITE(2,*) ' ENTER NUMBER OF LAGS (NOLAG):'
READ*, NOLAG
NW=N*IW
IF (NW.GE.40) NOLAG=N/4
PRINT*, '# OF LAGS :', NOLAG
WRITE(2,*) '# OF LAGS :', NOLAG
C // -----
PRINT 88, 'THERE ARE', NW, 'OBSERVATIONS'
WRITE(2,88) 'THERE ARE', NW, 'OBSERVATIONS'
PRINT*, 'THE MAXIMUM NUMBER OF LAGS IS', NOLAG
WRITE(2,*) 'THE MAXIMUM NUMBER OF LAGS IS', NOLAG
PRINT *, '!
WRITE(2,*) '!
PRINT*, ' READ INPUTS FROM A (F)ILE OR ENTER (M)ANUALLY ? F/M :'
WRITE(2,*) ' READ INPUTS FROM A (F)ILE OR ENTER (M)ANUALLY ? F/M :'
READ 81, CANS
WRITE(2,*) CANS
IF ((CANS.EQ.'F').OR.(CANS.EQ.'f')) THEN
  PRINT*, ' ENTER FILE NAME ?;'
  WRITE(2,*) ' ENTER FILE NAME ?;'
  READ 82, FNAME
  PRINT*, FNAME
  WRITE(2,*) FNAME
  OPEN (1, FILE=FNAME)
  REWIND 1
  READ(1,*,END=99) (X(I), I=1, NW)
  CLOSE (1)
ELSE
  PRINT 89, 'ENTER', NW, 'OBSERVATIONS IN TIME SEQUENCE;'
  WRITE(2,89) 'ENTER', NW, 'OBSERVATIONS IN TIME SEQUENCE;'
  READ*(X(I), I=1, NW)
ENDIF
DO 7 K=1,N
  PRINT 85, 'year', K, 'period'
  WRITE(2,85) 'year', K, 'period'
  PRINT 86, ((I-K*IW+IW), X(I), I=((K-1)*IW)+1, K*IW)
  WRITE(2,86) ((I-K*IW+IW), X(I), I=((K-1)*IW)+1, K*IW)
  PRINT *, ' ---'
  WRITE(2,*) ' ---'
7  CONTINUE
PRINT*, ' enter 0 to exit any integer to continue'
WRITE(2,*) ' enter 0 to exit any integer to continue'
READ *, ICON

IF (ICON.EQ.0) STOP
C // -----

```

```

C // finding the periodic means()
  DO 10 IV=1,IW
    DO 11 I=IV,NW,IW
      XSUM(IV)=XSUM(IV)+X(I)
      XMEAN(IV)=XSUM(IV)/N
11    CONTINUE
      PRINT 90,'SUM OF',IV,'TH PERIOD IS',XSUM(IV)
      WRITE(2,90) 'SUM OF',IV,'TH PERIOD IS',XSUM(IV)
      PRINT 90,'MEAN OF',IV,'TH PERIOD IS',XMEAN(IV)
      WRITE(2,90) 'MEAN OF',IV,'TH PERIOD IS',XMEAN(IV)
10    CONTINUE
      PRINT*, ' enter 0 to exit any integer to continue'
      WRITE (2,*) ' enter 0 to exit any integer to continue'
      READ*, ICON
      IF (ICON.EQ.0) STOP
C // end of finding means()
C // -----
C // finding the sample PeACVF of period v at lag l() --- Gamma(l)(v)
  SUM=0.0
  DO 15 IV=1,IW
    DO 16 L=1,NOLAG
      DO 17 I=IV,NW,IW
        IF ((I-L).LT.1) THEN
          TEMP1=0.0
        ELSE
          TEMP1=X(I-L)
        ENDIF
        IF ((IV-L).GE.1) THEN
          TEMP2=XMEAN(IV-L)
        ELSE
          DO 18 K=IV-L,0,IW
            IT=K+IW
18          CONTINUE
            TEMP2=XMEAN(IT)
          ENDIF
          SUM=SUM+((X(I)-XMEAN(IV))*(TEMP1-TEMP2))
C // if kw+v-l<1 set elements to zero
          IF ((I-L).LT.1) SUM=0.0
17        CONTINUE
          XPACVF(IV,L)=SUM / N
          SUM=0.0
          PRINT 91,'THE PEACVF AT',IV,'TH PERIOD AT LAG',L,
*            'IS',XPACVF(IV,L)
          WRITE(2,91) 'THE PEACVF AT',IV,'TH PERIOD AT LAG',L,
*            'IS',XPACVF(IV,L)
16        CONTINUE
15      CONTINUE
      PRINT*, ' enter 0 to exit any integer to continue'
      WRITE(2,*) ' enter 0 to exit any integer to continue'
      READ*, ICON
      IF (ICON.EQ.0) STOP
C //
C // end of finding sample PeACVF's()

C // -----
C // finding the sample PeACVF of period v at lag 0() - Gamma0(v)
  SUM=0.0

```

```

DO 20 IV=1,IW
  L=0
  DO 21 I=IV,NW,IW
    TEMP1=X(I-L)
    TEMP2=XMEAN(IV-L)
    SUM=SUM+((X(I)-XMEAN(IV))*(TEMP1-TEMP2))
21  CONTINUE
  XP0(IV)=SUM / N
  SUM=0.0
  PRINT 92,'PEACVF AT',IV,'TH PERIOD AT LAG',L,
*   'IS',XP0(IV)
  WRITE(2,92) 'PEACVF AT',IV,'TH PERIOD AT LAG',L,
*   'IS',XP0(IV)
20  CONTINUE
  PRINT*, ' enter 0 to exit any integer to continue'
  WRITE(2,*) ' enter 0 to exit any integer to continue'
  READ*, ICON
  IF (ICON.EQ.0) STOP

C //
C // end of finding sample PeACVF of period v at lag 0()
C // -----
C // finding the sample PeACVF of period v-1 at lag 0() - Gamma0(v-1)
C // for v-1 <= 0, gamma0(v-1) ==> gamma0(v-1+w)
DO 25 IV=1,IW
  DO 26 L=1,NOLAG
    IF ((IV-L).GT.0) THEN
      XP00(IV-L)=XP0(IV-L)
    ELSE
      IF ((IV-L+IW).GT.0) THEN
        XP00(IV-L)=XP0(IV-L+IW)
      ELSE
        XP00(IV-L)=XP00(IV-L+IW)
      ENDIF
    ENDIF
26  CONTINUE
25  CONTINUE
  XP00(IW)=XP0(IW)
C // end of finding sample PeACVF of period v-1 at lag 0()
C // -----
C // just for control.....
DO 30 K=IW-1,1-NOLAG,-1
  PRINT 93,'GAMMA 0 (V-L=',K,') IS',XP00(K)
  WRITE(2,93) 'GAMMA 0 (V-L=',K,') IS',XP00(K)
30  CONTINUE
  PRINT*, ' enter 0 to exit any integer to continue'
  WRITE(2,*) ' enter 0 to exit any integer to continue'
  READ*, ICON
  IF (ICON.EQ.0) STOP
C // end of controlling part.....
C // -----
C // finding the sample PeACF of period v at lag l - r(l)(v)
DO 35 IV=1,IW

  DO 36 L=1,NOLAG
  XP(IV,L)=XPACVF(IV,L)/(SQRT(XP0(IV)*XP00(IV-L)))
  PRINT 94,'THE SAMPLE PEACF AT',IV,'TH PERIOD AT LAG',
*   L,'IS',XP(IV,L)

```

```

                WRITE(2,94) 'THE SAMPLE PEACF AT',IV,'TH PERIOD AT LAG',
*           L,'IS',XP(IV,L)
36          CONTINUE
35          CONTINUE
            PRINT*, ' enter 0 to exit any integer to continue'
            WRITE(2,*) ' enter 0 to exit any integer to continue'
            READ*, ICON
            IF (ICON.EQ.0) STOP
C // end of finding sample PeACF of period v at lag l
C // -----
C // cloning xpacvf(iv,il) array to temporary tpacvf(jv,jl)
C //   where iv=1,...,iw but jv=1-nolag,...,iw
C //
            DO 40 JV=1-NOLAG,IW
              DO 41 JL=1,NOLAG
                IF (JV.GE.1) THEN
                  TPACVF(JV,JL)=XPACVF(JV,JL)
                ELSE
                  DO 42 I=JV,0,IW
                    IT=I+IW
42              CONTINUE
                  TPACVF(JV,JL)=XPACVF(IT,JL)
                ENDIF
              CONTINUE
            CONTINUE
40          CONTINUE
C //
C // end of cloning
C // -----
C //   initial values;
C //
            DO 45 IV=1,IW
              DO 46 IL=1,NOLAG
                ALPHA(IV,IL-1,0)=1
46              CONTINUE
                DELTSQ(IV,0)=XP0(IV)
                TAOSQ(IV,0)=XP0(IV)
45              CONTINUE
C //
C // -----
C //   calculating Updates;
C //   cdelta,alpha,beta,gamma,deltasq, and taosq
C //
            DSUM=0
            DO 50 IL=1,NOLAG
              DO 51 IV=1,IW
                DO 52 IM=0,IL-1
                  DSUM=DSUM+(TPACVF(IV-IM,IL-IM)*ALPHA(IV,IL-1,IM))
52              CONTINUE
                CDELTA(IV,IL-1)=DSUM
                DSUM=0
                IF ((IV-1).EQ.0) TAOSQ(IV-1,IL-1)=TAOSQ(IW,IL-1)

                ALPHA(IV,IL,IL)=(-CDELTA(IV,IL-1))/TAOSQ(IV-1,IL-1)
                BETA(IV,IL,IL)=(-CDELTA(IV,IL-1))/DELTSQ(IV,IL-1)
                DELTSQ(IV,IL)=DELTSQ(IV,IL-1)*
*           (1-ALPHA(IV,IL,IL)*BETA(IV,IL,IL))
                TAOSQ(IV,IL)=TAOSQ(IV-1,IL-1)*

```

```

*      (1-ALPHA(IV,IL,IL)*BETA(IV,IL,IL))
DO 53 I=1,IL-1
  IF ((IV-1).EQ.0) THEN
    BETA(IV-1,IL-1,IL-1)=BETA(IW,IL-1,IL-1)
    BETA(IV-1,IL-1,1)=BETA(IW,IL-1,1)
  ENDIF
  ALPHA(IV,IL,1)=ALPHA(IV,IL-1,1)+
*      (ALPHA(IV,IL,IL)*BETA(IV-1,IL-1,IL-1))
  BETA(IV,IL,1)=BETA(IV-1,IL-1,1)+
*      (BETA(IV,IL,IL)*ALPHA(IV,IL-1,IL-1))
    PRINT 80, 'alpha(',1,2,1,')',ALPHA(1,2,1)
    WRITE(2,80) 'alpha(',1,2,1,')',ALPHA(1,2,1)
    PRINT 80, 'alpha(',2,2,1,')',ALPHA(2,2,1)
    WRITE(2,80) 'alpha(',2,2,1,')',ALPHA(2,2,1)
    PRINT 80, 'alpha(',3,2,1,')',ALPHA(3,2,1)
    WRITE(2,80) 'alpha(',3,2,1,')',ALPHA(3,2,1)
53  CONTINUE
    PRINT 95, 'PERIOD= ',IV,'LAG= ',IL
    WRITE(2,95) 'PERIOD= ',IV,'LAG= ',IL
    PRINT 96, 'alpha(',IV,IL,IL,')',ALPHA(IV,IL,IL)
    WRITE(2,96) 'alpha(',IV,IL,IL,')',ALPHA(IV,IL,IL)
    PRINT 96, 'beta(',IV,IL,IL,')',BETA(IV,IL,IL)
    WRITE(2,96) 'beta(',IV,IL,IL,')',BETA(IV,IL,IL)
    PRINT 97, 'deltasq(',IV,IL,')',DELTSQ(IV,IL)
    WRITE(2,97) 'deltasq(',IV,IL,')',DELTSQ(IV,IL)
    IF (DELTSQ(IV,IL).LE.0) THEN
      PRINT*, ' enter 0 to exit any integer to continue'
      WRITE(2,*) ' enter 0 to exit any integer to continue'
      READ*, ICON
      ICON=1
    ENDIF
    PRINT 97, 'taosq(',IV,IL,')',TAOSQ(IV,IL)
    WRITE(2,97) 'taosq(',IV,IL,')',TAOSQ(IV,IL)
    IF (TAOSQ(IV,IL).LE.0) THEN
      PRINT*, ' enter 0 to exit any integer to continue'
      WRITE(2,*) ' enter 0 to exit any integer to continue'
      READ*, ICON
      ICON=1
    ENDIF
    PRINT 97, 'DELTA(',IV,IL-1,')',CDELTA(IV,IL-1)
    WRITE(2,97) 'DELTA(',IV,IL-1,')',CDELTA(IV,IL-1)
    IF (ICON.EQ.0) STOP
51  CONTINUE
50  CONTINUE
    PRINT*, ' enter 0 to exit any integer to continue'
    WRITE(2,*) ' enter 0 to exit any integer to continue'
    READ*, ICON
    ICON=1

C //
C // end of calculating Updates

C // -----
C // calculating phi();
C // PRINT*, ' calculating phi()'

C //
DO 110 IV=1,IW
  DO 111 IL=1,NOLAG

```

```

        PRINT 190,'CDELTA(',IV,IL-1,')','DELTASQ(',IV,IL-1,
*      ')','TAOSQ(',IV-1,IL-1,')'
        WRITE(2,190) 'CDELTA(',IV,IL-1,')','DELTASQ(',IV,IL-1,
*      ')','TAOSQ(',IV-1,IL-1,')'
        PRINT 191,CDELTA(IV,IL-1),DELTSQ(IV,IL-1),TAOSQ(IV-1,IL-1)

        PHI(IV,IL,IL)=CDELTA(IV,IL-1)/
*      ((SQRT(DELTSQ(IV,IL-1))) * (SQRT(TAOSQ(IV-1,IL-1))))
        WRITE(2,191) CDELTA(IV,IL-1),DELTSQ(IV,IL-1),TAOSQ(IV-1,IL-1)
        PHI(IV,IL,IL)=CDELTA(IV,IL-1)/
*      ((SQRT(DELTSQ(IV,IL-1))) * (SQRT(TAOSQ(IV-1,IL-1))))
        PRINT 98, PHI(IV,IL,IL), ' IS THE PEPACF AT LAG',
*      IL,'OF PERIOD',IV
        WRITE(2,98) PHI(IV,IL,IL), ' IS THE PEPACF AT LAG',
*      IL,'OF PERIOD',IV
111    CONTINUE
110    CONTINUE
C //
C //   end of calculating phi;
C // -----
C //
C // single result obtained manually()
        PRINT*,'DO YOU WANT TO OBTAIN PEPACF MANUALLY?'
        WRITE(2,*) 'DO YOU WANT TO OBTAIN PEPACF MANUALLY?'
        PRINT*,'IF YES TYPE 1, IF NO TYPE 0'
        WRITE(2,*) 'IF YES TYPE 1, IF NO TYPE 0'
        READ*,ID
200    IF (ID.EQ.1) THEN
            PRINT*,'ENTER THE PERIOD & THE LAG ; '
            WRITE(2,*) 'ENTER THE PERIOD & THE LAG ; '
            READ*,IV,L
                PRINT 94,'THE SAMPLE PEPACF AT',IV,'TH PERIOD AT LAG',
*              L,'IS',XP(IV,L)
                WRITE(2,94) 'THE SAMPLE PEPACF AT',IV,'TH PERIOD AT LAG',
*              L,'IS',XP(IV,L)
                GO TO 200
        ELSEIF (ID.EQ.0) THEN
            PRINT*,' enter 0 to exit any integer to continue'
            WRITE(2,*) ' enter 0 to exit any integer to continue'
            READ*,ICON
            IF (ICON.EQ.0) THEN
                STOP
            ELSE
                GOTO 200
            ENDIF
        ENDIF
C // end of single result obtained manually()
C // -----
C //

C // close the output file :
C
        CLOSE (2)
C
C // formatting
81    FORMAT(A1)
82    FORMAT(A20)

```

```

85   FORMAT(A4,I3,2X,A6)
86   FORMAT(9X,I3,' ',F15.5)
88   FORMAT(A10,2X,I3,3X,A14)
89   FORMAT(A10,2X,I3,3X,A32)
90   FORMAT(A8,3X,I2,A13,2X,F15.5)
91   FORMAT(A14,2X,I3,A17,1X,I3,2X,A2,1X,F15.5)
92   FORMAT(A14,2X,I3,A17,1X,I3,2X,A2,1X,F15.5)
93   FORMAT(A15,I3,A4,2X,F15.5)
94   FORMAT(A20,2X,I3,A17,1X,I3,2X,A2,1X,F15.5)
95   FORMAT(A10,I2,A7,I2)
96   FORMAT(A10,I3,I3,I3,A2,F15.5)
97   FORMAT(A10,I3,I3,A3,2X,F15.5)
80   FORMAT(A10,I2,I2,I2,A2,F15.5)
98   FORMAT(F15.5,A22,I2,A12,I2)
190  FORMAT(A11,I2,I2,A2,A12,I2,I2,A2,A9,I2,I2,A2)
191  FORMAT(F15.5,1X,F15.5,1X,F15.5)
99   ENDFILE 1
C //
C //
      STOP
      END

```

Output Example:

A PART OF THE OUTPUT IS GIVEN BELOW.

```

ENTER YEAR(N), PERIOD(W):
      33      12
ENTER NUMBER OF LAGS (NOLAG):
# OF LAGS :      8
THERE ARE 396 OBSERVATIONS
THE MAXIMUM NUMBER OF LAGS IS      8
.
READ INPUTS FROM A (F)ILE OR ENTER (M)ANUALLY ? F/M :
f
ENTER FILE NAME ?;
atnos.txt

---
enter 0 to exit any integer to continue

SUM OF  1 TH PERIOD IS      49.81700
MEAN OF  1 TH PERIOD IS      1.50961
SUM OF  2 TH PERIOD IS      117.77100

MEAN OF  2 TH PERIOD IS      3.56882
SUM OF  3 TH PERIOD IS      570.74005
MEAN OF  3 TH PERIOD IS      17.29515
SUM OF  4 TH PERIOD IS      780.91107
MEAN OF  4 TH PERIOD IS      23.66397
SUM OF  5 TH PERIOD IS      760.73700
MEAN OF  5 TH PERIOD IS      23.05264

```

SUM OF 6 TH PERIOD IS	626.71503
MEAN OF 6 TH PERIOD IS	18.99136
SUM OF 7 TH PERIOD IS	361.08795
MEAN OF 7 TH PERIOD IS	10.94206
SUM OF 8 TH PERIOD IS	179.33305
MEAN OF 8 TH PERIOD IS	5.43433
SUM OF 9 TH PERIOD IS	90.13300
MEAN OF 9 TH PERIOD IS	2.73130

SUM OF 10 TH PERIOD IS	37.70499
MEAN OF 10 TH PERIOD IS	1.14258
SUM OF 11 TH PERIOD IS	30.98600
MEAN OF 11 TH PERIOD IS	0.93897
SUM OF 12 TH PERIOD IS	42.18499
MEAN OF 12 TH PERIOD IS	1.27833

enter 0 to exit any integer to continue

THE PEACVF AT 6 TH PERIOD AT LAG 1 IS	59.22530
THE PEACVF AT 6 TH PERIOD AT LAG 2 IS	90.38964
THE PEACVF AT 6 TH PERIOD AT LAG 3 IS	2.45501
THE PEACVF AT 6 TH PERIOD AT LAG 4 IS	-4.92702
THE PEACVF AT 6 TH PERIOD AT LAG 5 IS	0.55248
THE PEACVF AT 6 TH PERIOD AT LAG 6 IS	-3.32227
THE PEACVF AT 6 TH PERIOD AT LAG 7 IS	-1.10589

enter 0 to exit any integer to continue

PEACVF AT 1 TH PERIOD AT LAG 0 IS	0.77816
PEACVF AT 2 TH PERIOD AT LAG 0 IS	5.15689
PEACVF AT 3 TH PERIOD AT LAG 0 IS	345.30518
PEACVF AT 4 TH PERIOD AT LAG 0 IS	425.31482
PEACVF AT 5 TH PERIOD AT LAG 0 IS	389.47153
PEACVF AT 6 TH PERIOD AT LAG 0 IS	107.54240
PEACVF AT 7 TH PERIOD AT LAG 0 IS	52.33360
PEACVF AT 8 TH PERIOD AT LAG 0 IS	14.23912
PEACVF AT 9 TH PERIOD AT LAG 0 IS	2.58686
PEACVF AT 10 TH PERIOD AT LAG 0 IS	0.79756
PEACVF AT 11 TH PERIOD AT LAG 0 IS	0.54926
PEACVF AT 12 TH PERIOD AT LAG 0 IS	1.13483

enter 0 to exit any integer to continue

THE SAMPLE PEACF AT 6 TH PERIOD AT LAG 1 IS	0.28939
---	---------

THE SAMPLE PEACF AT 6 TH PERIOD AT LAG 2 IS	0.42264
THE SAMPLE PEACF AT 6 TH PERIOD AT LAG 3 IS	0.01274
THE SAMPLE PEACF AT 6 TH PERIOD AT LAG 4 IS	-0.20922
THE SAMPLE PEACF AT 6 TH PERIOD AT LAG 5 IS	0.06039
THE SAMPLE PEACF AT 6 TH PERIOD AT LAG 6 IS	-0.30073
THE SAMPLE PEACF AT 6 TH PERIOD AT LAG 7 IS	-0.14389
THE SAMPLE PEACF AT 6 TH PERIOD AT LAG 8 IS	-0.06122

enter 0 to exit any integer to continue

PERIOD= 6 LAG= 1
alpha(6 1 1) -0.1520657986
beta(6 1 1) -0.5507158041
deltasq(6 1) 98.5362548828
taosq(6 1) 356.8552246094
DELTA(6 0) 59.2252960205

enter 0 to exit any integer to continue

CDELTA(6 0) DELTASQ(6 0) TAOSQ(5 0)
59.22530 107.54240 389.47153
0.28939 IS THE PEPACF AT LAG 1 OF PERIOD 6
CDELTA(6 1) DELTASQ(6 1) TAOSQ(5 1)
74.35857 98.53625 396.77930
0.37606 IS THE PEPACF AT LAG 2 OF PERIOD 6
CDELTA(6 2) DELTASQ(6 2) TAOSQ(5 2)
-20.33564 84.60106 317.83087
-0.12401 IS THE PEPACF AT LAG 3 OF PERIOD 6
CDELTA(6 3) DELTASQ(6 3) TAOSQ(5 3)
-4.39001 83.29993 4.75019
-0.22069 IS THE PEPACF AT LAG 4 OF PERIOD 6
CDELTA(6 4) DELTASQ(6 4) TAOSQ(5 4)
1.48797 79.24281 0.72223
0.19669 IS THE PEPACF AT LAG 5 OF PERIOD 6
CDELTA(6 5) DELTASQ(6 5) TAOSQ(5 5)
-2.64319 76.17721 1.01500
-0.30060 IS THE PEPACF AT LAG 6 OF PERIOD 6
CDELTA(6 6) DELTASQ(6 6) TAOSQ(5 6)
0.87473 69.29400 0.32601
0.18404 IS THE PEPACF AT LAG 7 OF PERIOD 6
CDELTA(6 7) DELTASQ(6 7) TAOSQ(5 7)
0.18407 66.94695 0.18982
0.05164 IS THE PEPACF AT LAG 8 OF PERIOD 6

APPENDIX B

**COMPUTER PROGRAM FOR
IDENTIFICATION OF
PURE PERIODIC MOVING-AVERAGE MODELS**

Description:

This program computes the sample periodic autocorrelations and corresponding band limits for each season of any periodic process entered as an input. User may enter the series manually or program can read the series from a stored file. The computations are performed up to a default lag, which is the one fourth of total number of years, and additionally up to a maximum lag which is twice the number of seasons. The PMA model identification for each season is given as output.

Inputs:

N: Number of years

IW: Number of periods

NOLAG: Default number of lags ($N/4$)

CANS: Name of the text file in which the series of interest is stored

MAXLAG: Maximum number of lags which is twice the period ω

Outputs:

XPACF(): Sample periodic autocorrelation for each season

FBAND (): Critical band limits for $q(v) < \omega$ case
 SBAND (): Critical band limits for $q(v) \geq \omega$ case
 PMA IDENTIFICATION when maximum lag number is MAXLAG
 PMA IDENTIFICATION when maximum lag number is NOLAG

Program Listing:

THE OUTPUTS OF THIS PROGRAM ARE STORED IN FILE 'ACFatnos.txt'

```

C /* -----
C     program that finds PeACF and confidence bands

C     Definitions :

C     X(): observations
C     XSUM(): sample sum of observations for period v
C     XMEAN: sample mean of period v
C     XP(): sample PeACF value for period v at lag l (r(l)(v));
C     XPACVF(): sample PeACVF value for period v at lag l (gamma(l)(v));
C     XP0(): sample PeACVF value for period v at lag 0 (gamma(0)(v));
C     XP00(): sample PeACF value for period v-1 at lag 0 (gamma(0)(v-1));
C     FBAND(): first band value for the PeACF
C     SBAND(): second band value for the PeACF
C     TEMP1,TEMP2: temporary values;
C     N: number of years
C     IV: season number
C     NW: total number of observations
C
C     (c) 2002
C     by Burcin Akgun
C
C ----- */
PROGRAM PeACFBAND
REAL X(-500:500),XSUM(1000),XMEAN(-500:500),XP(12,500),FBAND(12,500)
REAL XPACVF(12,1000),XP0(1000),XP00(-500:500),SUM,TEMP1,TEMP2
REAL SBAND(12,500),XPC(-500:500,500)
REAL XPSUM,XPCSUM
REAL DSUM
INTEGER ICON,N,IV,IW,NW,NOLAG,NMAXLG,IP,IM,MAXLAG
INTEGER XIOUT(12),XIOUT2(12),XFOUT(50),XSOUT
INTEGER Q(12)

CHARACTER FNAME*20, CANS*1
C
C // open the output file
OPEN (2, FILE='ACFatnos.txt')
C //
C

```

```

DO 3 I=1,100
    XSUM(I)=0
    XP0(I)=0
3 CONTINUE
DO 4 I=-100,100
    X(I)=0
    XMEAN(I)=0
    XP00(I)=0
4 CONTINUE
DO 5 J=1,12
    DO 6 I=1,100
        XP(J,I)=0
        XPACVF(J,I)=0
6 CONTINUE
5 CONTINUE
PRINT*,' ENTER YEAR(N), PERIOD(W):'
WRITE(2,*) ' ENTER YEAR(N), PERIOD(W):'
READ*, N,IW
WRITE(2,*) N, IW
PRINT*,' ENTER NUMBER OF LAGS (NOLAG):'
WRITE(2,*) ' ENTER NUMBER OF LAGS (NOLAG):'
READ*, NOLAG
NW=N*IW
IF (NW.GE.40) NOLAG=N/4
PRINT*,' # OF LAGS :', NOLAG
WRITE(2,*) '# OF LAGS :', NOLAG
C // -----
PRINT 88,'THERE ARE',NW,'OBSERVATIONS'
WRITE(2,88) 'THERE ARE',NW,'OBSERVATIONS'
PRINT*,'THE MAXIMUM NUMBER OF LAGS IS',NOLAG
WRITE(2,*) 'THE MAXIMUM NUMBER OF LAGS IS',NOLAG
PRINT*,'!'
WRITE(2,*)'!'
PRINT*,' READ INPUTS FROM A (F)ILE OR ENTER (M)ANUALLY ? F/M :'
WRITE(2,*) ' READ INPUTS FROM A (F)ILE OR ENTER (M)ANUALLY ? F/M :'
READ 81, CANS
WRITE(2,*) CANS
IF ((CANS.EQ.'F').OR.(CANS.EQ.'f')) THEN
    PRINT*,' ENTER FILE NAME ?;'
    WRITE(2,*) ' ENTER FILE NAME ?;'
    READ 82, FNAME
    PRINT*, FNAME
    WRITE(2,*) FNAME
    OPEN (1, FILE=FNAME)
    REWIND 1
    READ(1,*,END=99) (X(I), I=1, NW)
    CLOSE (1)
ELSE
    PRINT 89,'ENTER',NW,'OBSERVATIONS IN TIME SEQUENCE;'

    WRITE(2,89) 'ENTER',NW,'OBSERVATIONS IN TIME SEQUENCE;'
    READ*(X(I), I=1, NW)
ENDIF
DO 7 IK=1,N
    PRINT 85,'year',IK,'period'
    WRITE(2,85) 'year',IK,'period'
    PRINT 86, ((I-1)*IW+1), X(I), I=((IK-1)*IW)+1, IK*IW)

```

```

WRITE(2,86) ((I-IK*IW+IW), X(I), I=((IK-1)*IW)+1, IK*IW)
PRINT *, ' ---'
WRITE(2,*) ' ---'
7 CONTINUE
PRINT *, ' enter 0 to exit any integer to continue'
WRITE(2,*) ' enter 0 to exit any integer to continue'
READ *, ICON
IF (ICON.EQ.0) STOP
C // -----
C // finding the periodic means()
DO 10 IV=1,IW
DO 11 I=IV,NW,IW
XSUM(IV)=XSUM(IV)+X(I)
XMEAN(IV)=XSUM(IV)/N
11 CONTINUE
10 CONTINUE
C // end of finding means()
C // -----
C // finding the sample PeACVF of period v at lag l() --- Gamma(l)(v)
IF ((2*IW).GT.NOLAG) THEN
MAXLAG=2*IW
ELSE
MAXLAG=NOLAG
ENDIF

SUM=0.0
DO 15 IV=1,IW
DO 16 L=1,MAXLAG
DO 17 I=IV,NW,IW
IF ((I-L).LT.1) THEN
TEMP1=0.0
ELSE
TEMP1=X(I-L)
ENDIF
IF ((IV-L).GE.1) THEN
TEMP2=XMEAN(IV-L)
ELSE
DO 18 K=IV-L,0,IW
IT=K+IW
18 CONTINUE
TEMP2=XMEAN(IT)
ENDIF
SUM=SUM+((X(I)-XMEAN(IV))*(TEMP1-TEMP2))
C // if kw+v-l<1 set elements to zero
IF ((I-L).LT.1) SUM=0.0
17 CONTINUE
XPACVF(IV,L)=SUM / N
SUM=0.0

16 CONTINUE
15 CONTINUE
C //
C // end of finding sample PeACVF's()
C // -----
C // finding the sample PeACVF of period v at lag 0() - Gamma0(v)
SUM=0.0
DO 20 IV=1,IW

```

```

L=0
DO 21 I=IV,NW,IW
    TEMP1=X(I-L)
    TEMP2=XMEAN(IV-L)
    SUM=SUM+((X(I)-XMEAN(IV))*(TEMP1-TEMP2))
21    CONTINUE
    XP0(IV)=SUM / N
    SUM=0.0
20    CONTINUE
C //
C // end of finding sample PeACVF of period v at lag 0()
C // -----
C // finding the sample PeACVF of period v-1 at lag 0() - Gamma0(v-1)

```

```

DO 25 IV=1,IW
    DO 26 L=1,MAXLAG
        IF ((IV-L).GT.0) THEN
            XP00(IV-L)=XP0(IV-L)
        ELSE
            IF ((IV-L+IW).GT.0) THEN
                XP00(IV-L)=XP0(IV-L+IW)
            ELSE
                XP00(IV-L)=XP00(IV-L+IW)
            ENDIF
        ENDIF
26    CONTINUE
25    CONTINUE
    XP00(IW)=XP0(IW)
C // end of finding sample PeACVF of period v-1 at lag 0()
C // -----
C // -----
C // finding the sample PeACF of period v at lag l - r(l)(v) and 1. Band

```

```

PRINT 103, 'MODEL IDENTIFICATION WHEN MAX LAG IS',MAXLAG
WRITE(2,103) 'MODEL IDENTIFICATION WHEN MAX LAG IS',MAXLAG
PRINT* '*****'
WRITE(2,* ) '*****'

```

```

DO 35 IV=1,IW
    DO 36 L=1,MAXLAG
        XP(IV,L)=XPACVF(IV,L)/(SQRT(XP0(IV)*XP00(IV-L)))
        FBAND(IV,L)=1.96/SQRT(1.0*N)
    IF ((XP(IV,L).GT.FBAND(IV,L)).OR.(XP(IV,L).LT.(-1*FBAND(IV,L)))) THEN
        PRINT 102, 'PERIOD:',IV, 'LAG:',L,'RHO(',L,',',IV,'):'
    
```

```

*    ,XP(IV,L),'-->FBAND:',FBAND(IV,L),'*
        WRITE (2,102) 'PERIOD:',IV, 'LAG:',L,'RHO(',L,',',IV,'):'
*    ,XP(IV,L),'-->FBAND:',FBAND(IV,L),'*
    ELSE
        PRINT 97, 'PERIOD:',IV, 'LAG:',L,'RHO(',L,',',IV,'):'
*    ,XP(IV,L),'-->FBAND:',FBAND(IV,L)
        WRITE (2,97) 'PERIOD:',IV, 'LAG:',L,'RHO(',L,',',IV,'):'
*    ,XP(IV,L),'-->FBAND:',FBAND(IV,L)
    ENDIF

```

```

36     CONTINUE
35 CONTINUE

      PRINT*, ' enter 0 to exit any integer to continue'
      WRITE(2,*) ' enter 0 to exit any integer to continue'

      READ*, ICON
      IF (ICON.EQ.0) STOP
C
      DO 234 L=1,MAXLAG
        XFOUT(L)=0
234    CONTINUE
        DO 235 IV=1,IW
          XIOUT(IV)=0
235    CONTINUE
        DO 236 IV=1,IW
          DO 237 L=1,MAXLAG
            IF ((XP(IV,L).GT.FBAND(IV,L)).OR.(XP(IV,L).LT.(-1*FBAND(IV,L))
*          )) XIOUT(IV)=L
237    CONTINUE
236    CONTINUE

      SUM=0.0
      DO 37 IV=1,IW
        IF (XIOUT(IV).GE.IW) THEN
          K=N/(4*IW)
          IF (K.EQ.0) THEN
            K=1
          ELSE
            K=K
          ENDIF
          PRINT 101, 'FOR PERIOD',IV,'WE NEED TO CALCULATE 2. BAND,
* WHERE K HAS A MAXIMUM VALUE OF',K
          WRITE (2,101) 'FOR PERIOD',IV,'WE NEED TO CALCULATE 2. BAND
*, WHERE K HAS A MAXIMUM VALUE OF',K
          DO 526 L=IW,MAXLAG
            DO 527 I=IV,NW,IW
              IF ((I-L).LT.1) THEN
                TEMP1=0.0
              ELSE
                TEMP1=X(I-L)
              ENDIF
              IF ((IV-L).GE.1) THEN
                TEMP2=XMEAN(IV-L)
              ELSE

                DO 528 KZ=IV-L,0,IW
                  IT=KZ+IW
528    CONTINUE
                  TEMP2=XMEAN(IT)
                ENDIF
                SUM=SUM+((X(I)-XMEAN(IV))*(TEMP1-TEMP2))
C // if kw+v-l<1 set elements to zero
                IF ((I-L).LT.1) SUM=0.0

527    CONTINUE

```

```

        XPACVF(IV,L)=SUM / N
        SUM=0.0
    IF ((IV-L).GT.0) THEN
        XP00(IV-L)=XP0(IV-L)
    ELSE
        IF ((IV-L+(K*IW)).GT.0) THEN
            XP00(IV-L)=XP0(IV-L+(K*IW))
        ELSE
            IF ((IV-L+((K+1)*IW)).GT.0) THEN
                XP00(IV-L)=XP0(IV-L+((K+1)*IW))
            ENDIF
        ENDIF
    ENDIF
    ENDIF
526    CONTINUE
    DO 536 L=IW,MAXLAG
        XP(IV,L)=XPACVF(IV,L)/(SQRT(XP0(IV)*XP00(IV-L)))
536    CONTINUE
    DO 38 L=IW,MAXLAG
    IF ((IV-L).GT.0) THEN
        XPC(IV-L,IW)=XP(IV-L,IW)
    ELSE
        IF ((IV-L+IW).GT.0) THEN
            XPC(IV-L,IW)=XP(IV-L+IW,IW)
        ELSE
            XPC(IV-L,IW)=XP(IV-L+IW+IW,IW)
        ENDIF
    ENDIF
        XPSUM=0.0
        XPCSUM=0.0
    DO 202 M=1,K
        XPSUM=XPSUM+XP(IV,IW*M)
        XPCSUM=XPCSUM+XPC(IV-L,IW*M)
202    CONTINUE
        SBAND(IV,L)=1.96*SQRT((1/(1.0*N))*(1+
*         (2*XPSUM*XPCSUM)))
38    CONTINUE

XIOUT2(IV)=0
    DO 436 L=IW,MAXLAG
        XP(IV,L)=XPACVF(IV,L)/(SQRT(XP0(IV)*XP00(IV-L)))
        IF ((XP(IV,L).GT.SBAND(IV,L)).OR.(XP(IV,L).LT.(-1*SBAND(IV,L))
*    )) THEN
            PRINT 102, 'PERIOD:',IV, 'LAG:',L,'rho(',L,',',IV,'):'
*            ,XP(IV,L),'-->SBAND:',SBAND(IV,L),'*'
            WRITE (2,102) 'PERIOD:',IV, 'LAG:',L,'rho(',L,',',IV,'):'
*            ,XP(IV,L),'-->SBAND:',SBAND(IV,L),'*'

            ELSE
                PRINT 97, 'PERIOD:',IV, 'LAG:',L,'rho(',L,',',IV,'):'
*            ,XP(IV,L),'-->SBAND:',SBAND(IV,L)
                WRITE (2,97) 'PERIOD:',IV, 'LAG:',L,'rho(',L,',',IV,'):'
*            ,XP(IV,L),'-->SBAND:',SBAND(IV,L)
            ENDIF
436    CONTINUE
    PRINT*, ' enter 0 to exit any integer to continue'

    WRITE(2,*) ' enter 0 to exit any integer to continue'
    READ*, ICON

```

```

IF (ICON.EQ.0) STOP
DO 444 L=1,MAXLAG
  XFOUT(L)=0
444 CONTINUE
  XSOUT=0

DO 447 L=IW,MAXLAG
  IF ((XP(IV,L).GT.SBAND(IV,L)).OR.(XP(IV,L).LT.(-1*SBAND(IV,L))
* )) XIOUT2(IV)=L-IW
447 CONTINUE
ENDIF
37 CONTINUE

DO 39 IV=1,IW
  PRINT 90,'PERIOD',IV,':'
  WRITE(2,90) 'PERIOD',IV,':'
  IF(XIOUT(IV).LT.IW) THEN
    Q(IV)=XIOUT(IV)
  ELSE
    Q(IV)=IW+XIOUT2(IV)
  ENDIF
  PRINT 96, '*This is a MA(',Q(IV),') model for period ',IV
  WRITE (2,96) '*This is a MA(',Q(IV),') model for period ',IV
  PRINT*,'
  WRITE(2,*)' _____'
39 CONTINUE

IF (MAXLAG.GT.NOLAG) THEN
PRINT 103, 'MODEL IDENTIFICATION WHEN MAX LAG IS',NOLAG
WRITE(2,103) 'MODEL IDENTIFICATION WHEN MAX LAG IS',NOLAG
PRINT*, '*****'
WRITE(2,*) '*****'
DO 1035 IV=1,IW
  DO 1036 L=1,NOLAG
  IF ((XP(IV,L).GT.FBAND(IV,L)).OR.(XP(IV,L).LT.(-1*FBAND(IV,L))
* )) THEN
    PRINT 102, 'PERIOD:',IV, 'LAG:',L,'RHO(',L,',',IV,'):'
    * ,XP(IV,L),'-->FBAND:',FBAND(IV,L),'*'
    WRITE (2,102) 'PERIOD:',IV, 'LAG:',L,'RHO(',L,',',IV,'):'
    * ,XP(IV,L),'-->FBAND:',FBAND(IV,L),'*'
    ELSE
    PRINT 97, 'PERIOD:',IV, 'LAG:',L,'RHO(',L,',',IV,'):'
    * ,XP(IV,L),'-->FBAND:',FBAND(IV,L)
    WRITE (2,97) 'PERIOD:',IV, 'LAG:',L,'RHO(',L,',',IV,'):'

* ,XP(IV,L),'-->FBAND:',FBAND(IV,L)
ENDIF
1036 CONTINUE
1035 CONTINUE

DO 1234 L=1,NOLAG
  XFOUT(L)=0
1234 CONTINUE
DO 1235 IV=1,IW
  XIOUT(IV)=0
1235 CONTINUE
DO 1236 IV=1,IW

```

```

DO 1237 L=1,NOLAG
  IF ((XP(IV,L).GT.FBAND(IV,L)).OR.(XP(IV,L).LT.(-1*FBAND(IV,L))
* )) XIOUT(IV)=L
1237 CONTINUE
1236 CONTINUE

SUM=0.0
DO 1037 IV=1,IW
IF (XIOUT(IV).GE.IW) THEN
  K=N/(4*IW)
  IF (K.EQ.0) THEN
    K=1
  ELSE
    K=K
  ENDIF
  PRINT 101, 'FOR PERIOD',IV,'WE NEED TO CALCULATE 2. BAND,
* WHERE K HAS A MAXIMUM VALUE OF',K
  WRITE (2,101) 'FOR PERIOD',IV,'WE NEED TO CALCULATE 2. BAND
*, WHERE K HAS A MAXIMUM VALUE OF',K
  DO 1526 L=IW,NOLAG
    DO 1527 I=IV,NW,IW
      IF ((I-L).LT.1) THEN
        TEMP1=0.0
      ELSE
        TEMP1=X(I-L)
      ENDIF
      IF ((IV-L).GE.1) THEN
        TEMP2=XMEAN(IV-L)
      ELSE
        DO 1528 KZ=IV-L,0,IW
          IT=KZ+IW
1528 CONTINUE
          TEMP2=XMEAN(IT)
        ENDIF
        SUM=SUM+((X(I)-XMEAN(IV))*(TEMP1-TEMP2))
C // if kw+v-l<1 set elements to zero
        IF ((I-L).LT.1) SUM=0.0
1527 CONTINUE
        XPACVF(IV,L)=SUM / N
        SUM=0.0
      IF ((IV-L).GT.0) THEN
        XP00(IV-L)=XP0(IV-L)
      ELSE

        IF ((IV-L+(K*IW)).GT.0) THEN
          XP00(IV-L)=XP0(IV-L+(K*IW))
        ELSE
          IF ((IV-L+((K+1)*IW)).GT.0) THEN
            XP00(IV-L)=XP0(IV-L+((K+1)*IW))
          ENDIF
        ENDIF
      ENDIF
    CONTINUE
1526 DO 1536 L=IW,NOLAG
      XP(IV,L)=XPACVF(IV,L)/(SQRT(XP0(IV)*XP00(IV-L)))
1536 CONTINUE

```

```

DO 1038 L=IW,NOLAG
  IF ((IV-L).GT.0) THEN
    XPC(IV-L,IW)=XP(IV-L,IW)
  ELSE
    IF ((IV-L+IW).GT.0) THEN
      XPC(IV-L,IW)=XP(IV-L+IW,IW)
    ELSE
      XPC(IV-L,IW)=XP(IV-L+IW+IW,IW)
    ENDIF
  ENDIF
  XPSUM=0.0
  XPCSUM=0.0
  DO 1202 M=1,K
    XPSUM=XPSUM+XP(IV,IW*M)
    XPCSUM=XPCSUM+XPC(IV-L,IW*M)
1202  CONTINUE
    SBAND(IV,L)=1.96*SQRT((1/(1.0*N))*(1+
*   (2*XPSUM*XPCSUM)))
1038  CONTINUE

XIOUT2(IV)=0
DO 1436 L=IW,NOLAG
  XP(IV,L)=XPACVF(IV,L)/(SQRT(XP0(IV)*XP00(IV-L)))
  IF ((XP(IV,L).GT.SBAND(IV,L)).OR.(XP(IV,L).LT.(-1*SBAND(IV,L))
* )) THEN
    PRINT 102, 'PERIOD:',IV, 'LAG:',L,'rho(',L,',',IV,'):'
*   ,XP(IV,L),'-->SBAND:',SBAND(IV,L),'*'
    WRITE (2,102) 'PERIOD:',IV, 'LAG:',L,'rho(',L,',',IV,'):'
*   ,XP(IV,L),'-->SBAND:',SBAND(IV,L),'*'

    ELSE
    PRINT 97, 'PERIOD:',IV, 'LAG:',L,'rho(',L,',',IV,'):'
*   ,XP(IV,L),'-->SBAND:',SBAND(IV,L)
    WRITE (2,97) 'PERIOD:',IV, 'LAG:',L,'rho(',L,',',IV,'):'
*   ,XP(IV,L),'-->SBAND:',SBAND(IV,L)
  ENDIF
1436  CONTINUE
  PRINT*, ' enter 0 to exit any integer to continue'
  WRITE(2,*) ' enter 0 to exit any integer to continue'
  READ*, ICON
  IF (ICON.EQ.0) STOP
  DO 1444 L=1,NOLAG
    XFOUT(L)=0

1444  CONTINUE
    XSOUT=0

    DO 1447 L=IW,NOLAG
      IF ((XP(IV,L).GT.SBAND(IV,L)).OR.(XP(IV,L).LT.(-1*SBAND(IV,L))
* )) XIOUT2(IV)=L-IW
1447  CONTINUE
    ENDIF
1037  CONTINUE

DO 1039 IV=1,IW
  PRINT 90,'PERIOD',IV,':'
  WRITE(2,90) 'PERIOD',IV,':'
  IF(XIOUT(IV).LT.IW) THEN

```

```

        Q(IV)=XIOUT(IV)
    ELSE
        Q(IV)=IW+XIOUT2(IV)
    ENDIF
    PRINT 96, '*This is a MA(',Q(IV),') model for period ',IV
    WRITE (2,96) '*This is a MA(',Q(IV),') model for period ',IV
    PRINT*, '_____ '
    WRITE(2,*) '_____ '
1039 CONTINUE
    ELSE
    CLOSE (2)
    ENDIF

C //
C // close the output file :
C
        CLOSE (2)
C
C // formatting
81     FORMAT(A1)
82     FORMAT(A20)
85     FORMAT(A4,I3,2X,A6)
86     FORMAT(9X,I3,' ',F15.5)
88     FORMAT(A10,2X,I3,3X,A14)
89     FORMAT(A10,2X,I3,3X,A32)
90     FORMAT(A6,1X,I2,1X,A1)
91     FORMAT(A14,2X,I3,A17,1X,I3,2X,A2,1X,F15.5)
92     FORMAT(A14,2X,I3,A17,1X,I3,2X,A2,1X,F15.5)
93     FORMAT(A15,I3,A4,2X,F15.5)
94     FORMAT(A13,2X,I3,A17,1X,I3,2X,A2,1X,F15.5)
95     FORMAT(A5,2X,I3,2X,A28)
96     FORMAT(A14,I2,A18,1X,I3)
97     FORMAT(A7,I3,3X,A4,I3,3X,A4,I2,A1,I2,A2,F10.5,3X,A9,F10.5)
98     FORMAT(A3,I2,2X,A3,I2,2X,A4,I2,A1,I2,A3,F10.5,2X,A4,I2,A1,
        *I2,A1,I2,A3,F10.5,2X,A9,F10.5)
100    FORMAT(A55,1X,I3)
101    FORMAT(A10,1X,I2,1X,A61,1X,I3)
102    FORMAT(A7,I3,3X,A4,I3,3X,A4,I2,A1,I2,A2,F10.5,3X,A9,F10.5,2X,A1)
103    FORMAT(A36,2X,I3)
99     ENDFILE 1
C //

        STOP
        END

```

Output Example:

A PART OF THE OUTPUT IS GIVEN BELOW.

ENTER YEAR(N), PERIOD(W):
 33 12

ENTER NUMBER OF LAGS (NOLAG):
OF LAGS : 8
THERE ARE 396 OBSERVATIONS
THE MAXIMUM NUMBER OF LAGS IS 8
.
READ INPUTS FROM A (F)ILE OR ENTER (M)ANUALLY ? F/M :
f
ENTER FILE NAME ?;
atnos.txt

enter 0 to exit any integer to continue

MODEL IDENTIFICATION WHEN MAX LAG IS 24

.
.
PERIOD: 6 LAG: 1 RHO(1, 6): 0.28939 -->FBAND: 0.34119
PERIOD: 6 LAG: 2 RHO(2, 6): 0.42264 -->FBAND: 0.34119 *
PERIOD: 6 LAG: 3 RHO(3, 6): 0.01274 -->FBAND: 0.34119
PERIOD: 6 LAG: 4 RHO(4, 6): -0.20922 -->FBAND: 0.34119
PERIOD: 6 LAG: 5 RHO(5, 6): 0.06039 -->FBAND: 0.34119
PERIOD: 6 LAG: 6 RHO(6, 6): -0.30073 -->FBAND: 0.34119
PERIOD: 6 LAG: 7 RHO(7, 6): -0.14389 -->FBAND: 0.34119
PERIOD: 6 LAG: 8 RHO(8, 6): -0.06122 -->FBAND: 0.34119
PERIOD: 6 LAG: 9 RHO(9, 6): 0.03533 -->FBAND: 0.34119
PERIOD: 6 LAG: 10 RHO(10, 6): 0.14955 -->FBAND: 0.34119
PERIOD: 6 LAG: 11 RHO(11, 6): 0.19934 -->FBAND: 0.34119
PERIOD: 6 LAG: 12 RHO(12, 6): -0.01319 -->FBAND: 0.34119
PERIOD: 6 LAG: 13 RHO(13, 6): 0.52922 -->FBAND: 0.34119 *
PERIOD: 6 LAG: 14 RHO(14, 6): 0.13491 -->FBAND: 0.34119
PERIOD: 6 LAG: 15 RHO(15, 6): 0.06802 -->FBAND: 0.34119
PERIOD: 6 LAG: 16 RHO(16, 6): -0.16383 -->FBAND: 0.34119
PERIOD: 6 LAG: 17 RHO(17, 6): 0.13848 -->FBAND: 0.34119
PERIOD: 6 LAG: 18 RHO(18, 6): 0.00103 -->FBAND: 0.34119
PERIOD: 6 LAG: 19 RHO(19, 6): -0.24191 -->FBAND: 0.34119
PERIOD: 6 LAG: 20 RHO(20, 6): -0.03653 -->FBAND: 0.34119
PERIOD: 6 LAG: 21 RHO(21, 6): -0.06288 -->FBAND: 0.34119
PERIOD: 6 LAG: 22 RHO(22, 6): -0.02140 -->FBAND: 0.34119
PERIOD: 6 LAG: 23 RHO(23, 6): 0.07405 -->FBAND: 0.34119
PERIOD: 6 LAG: 24 RHO(24, 6): 0.04523 -->FBAND: 0.34119
.
.

FOR PERIOD 6 WE NEED TO CALCULATE 2. BAND, WHERE K HAS A MAXIMUM VALUE OF 1

PERIOD: 6 LAG: 12 rho(12, 6): -0.01319 -->SBAND: 0.34125
PERIOD: 6 LAG: 13 rho(13, 6): 0.52922 -->SBAND: 0.34125 *
PERIOD: 6 LAG: 14 rho(14, 6): 0.13491 -->SBAND: 0.33968
PERIOD: 6 LAG: 15 rho(15, 6): 0.06802 -->SBAND: 0.34086
PERIOD: 6 LAG: 16 rho(16, 6): -0.16383 -->SBAND: 0.34184
PERIOD: 6 LAG: 17 rho(17, 6): 0.13848 -->SBAND: 0.34104
PERIOD: 6 LAG: 18 rho(18, 6): 0.00103 -->SBAND: 0.34080
PERIOD: 6 LAG: 19 rho(19, 6): -0.24191 -->SBAND: 0.33849
PERIOD: 6 LAG: 20 rho(20, 6): -0.03653 -->SBAND: 0.33929
PERIOD: 6 LAG: 21 rho(21, 6): -0.06288 -->SBAND: 0.34177
PERIOD: 6 LAG: 22 rho(22, 6): -0.02140 -->SBAND: 0.34111
PERIOD: 6 LAG: 23 rho(23, 6): 0.07405 -->SBAND: 0.34184
PERIOD: 6 LAG: 24 rho(24, 6): 0.04523 -->SBAND: 0.34125

enter 0 to exit any integer to continue

PERIOD 1 :
*This is a MA(0) model for period 1

PERIOD 2 :
*This is a MA(0) model for period 2

PERIOD 3 :
*This is a MA(18) model for period 3

PERIOD 4 :
*This is a MA(0) model for period 4

PERIOD 5 :
*This is a MA(6) model for period 5

PERIOD 6 :
*This is a MA(13) model for period 6

PERIOD 7 :
*This is a MA(17) model for period 7

PERIOD 8 :
*This is a MA(5) model for period 8

PERIOD 9 :
*This is a MA(1) model for period 9

PERIOD 10 :
*This is a MA(12) model for period 10

PERIOD 11 :
*This is a MA(24) model for period 11

PERIOD 12 :
*This is a MA(2) model for period 12

MODEL IDENTIFICATION WHEN MAX LAG IS 8

.
.
PERIOD: 6 LAG: 1 RHO(1, 6): 0.28939 -->FBAND: 0.34119
PERIOD: 6 LAG: 2 RHO(2, 6): 0.42264 -->FBAND: 0.34119 *
PERIOD: 6 LAG: 3 RHO(3, 6): 0.01274 -->FBAND: 0.34119
PERIOD: 6 LAG: 4 RHO(4, 6): -0.20922 -->FBAND: 0.34119
PERIOD: 6 LAG: 5 RHO(5, 6): 0.06039 -->FBAND: 0.34119
PERIOD: 6 LAG: 6 RHO(6, 6): -0.30073 -->FBAND: 0.34119
PERIOD: 6 LAG: 7 RHO(7, 6): -0.14389 -->FBAND: 0.34119
PERIOD: 6 LAG: 8 RHO(8, 6): -0.06122 -->FBAND: 0.34119
.
.

*This is a MA(0) model for period 1

PERIOD 2 :
*This is a MA(0) model for period 2

PERIOD 3 :
*This is a MA(0) model for period 3

PERIOD 4 :
*This is a MA(0) model for period 4

PERIOD 5 :
*This is a MA(6) model for period 5

PERIOD 6 :
*This is a MA(2) model for period 6

PERIOD 7 :
*This is a MA(2) model for period 7

PERIOD 8 :
*This is a MA(5) model for period 8

PERIOD 9 :
*This is a MA(1) model for period 9

PERIOD 10 :
*This is a MA(1) model for period 10

PERIOD 11 :
*This is a MA(1) model for period 11

PERIOD 12 :
*This is a MA(2) model for period 12

APPENDIX C

COMPUTER PROGRAM FOR IDENTIFICATION OF PURE PERIODIC AUTOREGRESSIVE MODELS

Description:

This program computes the sample periodic partial autocorrelations using Sakai's algorithm and corresponding band limits for each season of any periodic process entered as an input. User may enter the series manually or program can read the series from a stored file. The computations are performed up to a default lag, which is one fourth of total number of years, and additionally up to a maximum lag which is twice the number of seasons. The PAR model identification for each season is given as output.

Inputs:

N: Number of years

IW: Number of periods

NOLAG: Default number of lags ($N/4$)

CANS: Name of the text file in which the series of interest is stored

MAXLAG: Maximum number of lags which is twice the period ω

Outputs:

PHI(): Sample periodic autocorrelation for each season

BAND(): Critical band limits

PAR IDENTIFICATION when maximum lag number is MAXLAG

PAR IDENTIFICATION when maximum lag number is NOLAG

Program Listing:

THE OUTPUTS OF THIS PROGRAM ARE STORED IN FILE 'PACFatnos.txt'

```
C /* -----
C      program that finds PePacf and Bands
C
C      (c) 2002
C      by Burcin Akgun
C
C ----- */
      PROGRAM PePACFBAND
      REAL X(-500:500),XSUM(1000),XMEAN(-500:500),XP(12,500)
      REAL XPACVF(12,1000),XP0(1000),XP00(-500:500),SUM,TEMP1,TEMP2
      REAL TPACVF(-500:500,200)
      REAL DELTSQ(-2:12,0:200),TAOSQ(-2:12,-1:200)
      REAL ALPHA(-1:12,0:200,0:200),BETA(-1:12,0:200,0:200)
      REAL CDELTA(-1:12,0:200),PHI(0:12,0:200,0:200)
      REAL DSUM
      REAL BAND(12,500)
      INTEGER ICON,IV,IW,N,NW,NOLAG,NMAXLG,IP,IM,MAXLAG
      INTEGER XIOUT(12),XIOUT2(12)
      INTEGER P(12)
      CHARACTER FNAME*20, CANS*1
C // open the output file
      OPEN (2, FILE='PACFatnos.txt')
C //
      DO 3 I=1,100
          XSUM(I)=0
          XP0(I)=0
3      CONTINUE
      DO 4 I=-100,100
          X(I)=0
          XMEAN(I)=0
          XP00(I)=0
4      CONTINUE
      DO 5 J=1,12

          DO 6 I=1,100
              XP(J,I)=0
              XPACVF(J,I)=0
6          CONTINUE
5      CONTINUE
      PRINT*,' ENTER YEAR(N), PERIOD(W):'
      WRITE(2,*) ' ENTER YEAR(N), PERIOD(W):'
      READ*, N,IW
      WRITE(2,*) N, IW
      PRINT*,' ENTER NUMBER OF LAGS (NOLAG):'
      WRITE(2,*) ' ENTER NUMBER OF LAGS (NOLAG):'
      READ*, NOLAG
```

```

NW=N*IW
IF (NW.GE.40) NOLAG=N/4
PRINT*, '# OF LAGS :', NOLAG
WRITE(2,*) '# OF LAGS :', NOLAG
C // -----
PRINT 88,'THERE ARE',NW,'OBSERVATIONS'
WRITE(2,88) 'THERE ARE',NW,'OBSERVATIONS'
PRINT*, 'THE MAXIMUM NUMBER OF LAGS IS',NOLAG
WRITE(2,*) 'THE MAXIMUM NUMBER OF LAGS IS',NOLAG
PRINT *,!'
WRITE(2,*) '!
PRINT*, ' READ INPUTS FROM A (F)ILE OR ENTER (M)ANUALLY ? F/M :
WRITE(2,*) ' READ INPUTS FROM A (F)ILE OR ENTER (M)ANUALLY ? F/M :
READ 81, CANS
WRITE(2,*) CANS
IF ((CANS.EQ.'F').OR.(CANS.EQ.'f')) THEN
  PRINT*, ' ENTER FILE NAME ?;'
  WRITE(2,*) ' ENTER FILE NAME ?;'
  READ 82, FNAME
  PRINT*, FNAME
  WRITE(2,*) FNAME
  OPEN (1, FILE=FNAME)
  REWIND 1
  READ(1,*,END=99) (X(I), I=1, NW)
  CLOSE (1)
ELSE
  PRINT 89,'ENTER',NW,'OBSERVATIONS IN TIME SEQUENCE;'
  WRITE(2,89) 'ENTER',NW,'OBSERVATIONS IN TIME SEQUENCE;'
  READ*(X(I), I=1, NW)
ENDIF
DO 7 IK=1,N
  PRINT 85,'year',IK,'period'
  WRITE(2,85) 'year',IK,'period'
  PRINT 86, ((I-1)*IW+1), X(I), I=((IK-1)*IW)+1, IK*IW
  WRITE(2,86) ((I-1)*IW+1), X(I), I=((IK-1)*IW)+1, IK*IW
  PRINT *,' ---'
  WRITE(2,*) ' ---'
7 CONTINUE
PRINT*, ' enter 0 to exit any integer to continue'
WRITE(2,*) ' enter 0 to exit any integer to continue'
READ *, ICON
IF (ICON.EQ.0) STOP
C // -----

C // finding the periodic means()
DO 10 IV=1,IW
  DO 11 I=IV,NW,IW
    XSUM(IV)=XSUM(IV)+X(I)
    XMEAN(IV)=XSUM(IV)/N
11 CONTINUE
10 CONTINUE

C // end of finding means()
C // -----
C // finding the sample PeACVF of period v at lag l() --- Gamma(l)(v)

IF ((2*IW).GT.NOLAG) THEN

```

```

        MAXLAG=2*IW
ELSE
        MAXLAG=NOLAG
ENDIF

SUM=0.0
DO 15 IV=1,IW
    DO 16 L=1,MAXLAG
        DO 17 I=IV,NW,IW
            IF ((I-L).LT.1) THEN
                TEMP1=0.0
            ELSE
                TEMP1=X(I-L)
            ENDIF
            IF ((IV-L).GE.1) THEN
                TEMP2=XMEAN(IV-L)
            ELSE
                DO 18 K=IV-L,0,IW
                    IT=K+IW
18                CONTINUE
                    TEMP2=XMEAN(IT)
                ENDIF
                SUM=SUM+((X(I)-XMEAN(IV))*(TEMP1-TEMP2))
C // if kw+v-l<1 set elements to zero
                IF ((I-L).LT.1) SUM=0.0
17                CONTINUE
                    XPACVF(IV,L)=SUM / N
                    SUM=0.0
16                CONTINUE
15                CONTINUE

C //
C //
C // end of finding sample PeACVF's()
C // -----
C // finding the sample PeACVF of period v at lag 0() - Gamma0(v)
        SUM=0.0
        DO 20 IV=1,IW
            L=0
            DO 21 I=IV,NW,IW
                TEMP1=X(I-L)
                TEMP2=XMEAN(IV-L)

                SUM=SUM+((X(I)-XMEAN(IV))*(TEMP1-TEMP2))
21                CONTINUE
                    XP0(IV)=SUM / N
                    SUM=0.0
20                CONTINUE
C //
C // end of finding sample PeACVF of period v at lag 0()
C // -----
C // finding the sample PeACVF of period v-1 at lag 0() - Gamma0(v-1)
C // for v-1 <= 0, gamma0(v-1) ==> gamma0(v-1+w)
        DO 25 IV=1,IW
            DO 26 L=1,MAXLAG
                IF ((IV-L).GT.0) THEN
                    XP00(IV-L)=XP0(IV-L)

```

```

ELSE
  IF ((IV-L+IW).GT.0) THEN
    XP00(IV-L)=XP0(IV-L+IW)
  ELSE
    XP00(IV-L)=XP00(IV-L+IW)
  ENDIF
ENDIF
26 CONTINUE
25 CONTINUE
XP00(IW)=XP0(IW)
C // end of finding sample PeACVF of period v-1 at lag 0()
C // -----
C // -----
C // finding the sample PeACF of period v at lag l - Rho(l)(v)
DO 35 IV=1,IW
  DO 36 L=1,MAXLAG
    XP(IV,L)=XPACVF(IV,L)/(SQRT(XP0(IV)*XP00(IV-L)))
36 CONTINUE
35 CONTINUE

C // end of finding sample PeACF of period v at lag l
C // -----
C // cloning xpacvf(iv,il) array to temporary tpacvf(jv,jl)
C // where iv=1,...,iw but jv=1-nolag,...,iw
C //
DO 40 JV=1-MAXLAG,IW
  DO 41 JL=1,MAXLAG
    IF (JV.GE.1) THEN
      TPACVF(JV,JL)=XPACVF(JV,JL)
    ELSE
      DO 42 I=JV,0,IW
        IT=I+IW
42 TPACVF(JV,JL)=XPACVF(IT,JL)
      ENDIF
41 CONTINUE
40 CONTINUE
C //
C // end of cloning
C // -----
C // initial values;

C //
DO 45 IV=1,IW
  DO 46 L=1,MAXLAG
    ALPHA(IV,L-1,0)=1
46 CONTINUE
    DELTSQ(IV,0)=XP0(IV)
    TAOSQ(IV,0)=XP0(IV)
45 CONTINUE
C //
C // -----
C // calculating Updates;
C // cdelta,alpha,beta,gamma,deltasq, and taosq
C //
DSUM=0
DO 50 L=1,MAXLAG

```

```

DO 51 IV=1,IW
DO 52 IM=0,L-1
  DSUM=DSUM+(TPACVF(IV-IM,L-IM)*ALPHA(IV,L-1,IM))
52 CONTINUE
  CDELTA(IV,L-1)=DSUM
  DSUM=0
  IF ((IV-1).EQ.0) TAOSQ(IV-1,L-1)=TAOSQ(IW,L-1)
  ALPHA(IV,L,L)=(-CDELTA(IV,L-1))/TAOSQ(IV-1,L-1)
  BETA(IV,L,L)=(-CDELTA(IV,L-1))/DELTSQ(IV,L-1)
  DELTSQ(IV,L)=DELTSQ(IV,L-1)*
*     (1-ALPHA(IV,L,L)*BETA(IV,L,L))
  TAOSQ(IV,L)=TAOSQ(IV-1,L-1)*
*     (1-ALPHA(IV,L,L)*BETA(IV,L,L))
DO 53 I=1,L-1
  IF ((IV-1).EQ.0) THEN
    BETA(IV-1,L-1,L-I)=BETA(IW,L-1,L-I)
    BETA(IV-1,L-1,I)=BETA(IW,L-1,I)
  ENDIF
  ALPHA(IV,L,I)=ALPHA(IV,L-1,I)+
*     (ALPHA(IV,L,L)*BETA(IV-1,L-1,L-I))
  BETA(IV,L,I)=BETA(IV-1,L-1,I)+
*     (BETA(IV,L,L)*ALPHA(IV,L-1,L-I))
53 CONTINUE
51 CONTINUE
50 CONTINUE

C //
C // end of calculating Updates
C // -----
C // calculating phi() AND Bands;
PRINT 103, 'MODEL IDENTIFICATION WHEN MAX LAG IS',MAXLAG
WRITE(2,103) 'MODEL IDENTIFICATION WHEN MAX LAG IS',MAXLAG
PRINT*, '*****'
WRITE(2,*) '*****'

DO 110 IV=1,IW
DO 111 L=1,MAXLAG
  PHI(IV,L,L)=CDELTA(IV,L-1)/
*     ((SQRT(DELTSQ(IV,L-1))) * (SQRT(TAOSQ(IV-1,L-1))))

  PHI(IV,L,L)=CDELTA(IV,L-1)/
*     ((SQRT(DELTSQ(IV,L-1))) * (SQRT(TAOSQ(IV-1,L-1))))
  BAND(IV,L)=1.96/SQRT(1.0*N)
  IF((PHI(IV,L,L).GT.BAND(IV,L)).OR.
* (PHI(IV,L,L).LT.(-1*BAND(IV,L))) ) THEN
    PRINT 194,'PERIOD:',IV,'LAG:',L,'PePACF:',PHI(IV,L,L),
* '->BAND:',BAND(IV,L),'*'
    WRITE(2,194) 'PERIOD:',IV,'LAG:',L,'PePACF:',PHI(IV,L,L),
* '->BAND:',BAND(IV,L),'*'
  ELSE
    PRINT 98,'PERIOD:',IV,'LAG:',L,'PePACF:',PHI(IV,L,L),
* '->BAND:',BAND(IV,L)
    WRITE(2,98) 'PERIOD:',IV,'LAG:',L,'PePACF:',PHI(IV,L,L),
* '->BAND:',BAND(IV,L)
  ENDIF
111 CONTINUE

```

```

110 CONTINUE
PRINT*, ' enter 0 to exit any integer to continue'
WRITE(2,*) ' enter 0 to exit any integer to continue'
READ*, ICON
IF (ICON.EQ.0) STOP

DO 116 IV=1,IW
    XIOUT(IV)=0
116 CONTINUE
DO 117 IV=1,IW
    DO 118 L=1,MAXLAG
        IF((PHI(IV,L,L).GT.BAND(IV,L)).OR.
* (PHI(IV,L,L).LT.(-1*BAND(IV,L))) ) XIOUT(IV)=L
118 CONTINUE
117 CONTINUE

DO 127 IV=1,IW
    PRINT*, 'PERIOD',IV,':'
    WRITE(2,*) 'PERIOD',IV,':'
        P(IV)=XIOUT(IV)
    PRINT 101, '*This is an AR(',P(IV),') model for period',IV
    WRITE(2,101) '*This is an AR(',P(IV),') model for period',IV
    PRINT*, '_____ '
    WRITE(2,*) '_____ '
127 CONTINUE

IF (MAXLAG.GT.NOLAG) THEN
PRINT 103, 'MODEL IDENTIFICATION WHEN MAX LAG IS',NOLAG
WRITE(2,103) 'MODEL IDENTIFICATION WHEN MAX LAG IS',NOLAG
PRINT*, '*****'
WRITE(2,*) '*****'

DO 1110 IV=1,IW
    DO 1111 L=1,NOLAG
        IF((PHI(IV,L,L).GT.BAND(IV,L)).OR.
* (PHI(IV,L,L).LT.(-1*BAND(IV,L))) ) THEN
            PRINT 194, 'PERIOD:',IV, 'LAG:',L, 'PePACF:',PHI(IV,L,L),
* '-->BAND:',BAND(IV,L), '*'
                WRITE(2,194) 'PERIOD:',IV, 'LAG:',L, 'PePACF:',PHI(IV,L,L),

*
                    '-->BAND:',BAND(IV,L), '*'
            ELSE
                PRINT 98, 'PERIOD:',IV, 'LAG:',L, 'PePACF:',PHI(IV,L,L),
* '-->BAND:',BAND(IV,L)
                WRITE(2,98) 'PERIOD:',IV, 'LAG:',L, 'PePACF:',PHI(IV,L,L),
*
                    '-->BAND:',BAND(IV,L)
            ENDIF
1111 CONTINUE
1110 CONTINUE

DO 1116 IV=1,IW
    XIOUT(IV)=0
1116 CONTINUE
DO 1117 IV=1,IW
    DO 1118 L=1,NOLAG
        IF((PHI(IV,L,L).GT.BAND(IV,L)).OR.
* (PHI(IV,L,L).LT.(-1*BAND(IV,L))) ) XIOUT(IV)=L

```

```

1118     CONTINUE
1117     CONTINUE

        DO 1127 IV=1,IW
          PRINT*,'PERIOD',IV,':'
          WRITE(2,*) 'PERIOD',IV,':'
                P(IV)=XIOUT(IV)
          PRINT 101,'*This is an AR(',P(IV),') model for period',IV
          WRITE(2,101) '*This is an AR(',P(IV),') model for period',IV
          PRINT*,'
          WRITE(2,*) '
1127     CONTINUE
        ELSE
          CLOSE (2)
        ENDIF

C //     end of calculating phi and bands;
C // -----

C // close the output file :
C
        CLOSE (2)
C
C // formatting
81     FORMAT(A1)
82     FORMAT(A20)
85     FORMAT(A4,I3,2X,A6)
86     FORMAT(9X,I3,' ',F15.5)
88     FORMAT(A10,2X,I3,3X,A14)
89     FORMAT(A10,2X,I3,3X,A32)
90     FORMAT(A8,3X,I2,A13,2X,F15.5)
91     FORMAT(A14,2X,I3,A17,1X,I3,2X,A2,1X,F15.5)
92     FORMAT(A14,2X,I3,A17,1X,I3,2X,A2,1X,F15.5)
93     FORMAT(A15,I3,A4,2X,F15.5)
94     FORMAT(A20,2X,I3,A17,1X,I3,2X,A2,1X,F15.5)
95     FORMAT(A10,I2,A7,I2)
96     FORMAT(A10,I3,I3,I3,A2,F15.5)
97     FORMAT(A10,I3,I3,A3,2X,F15.5)

80     FORMAT(A10,I2,I2,I2,A2,F15.5)
98     FORMAT(A7,I2,2X,A4,I2,2X,A7,F10.5,2X,A9,F10.5)
100    FORMAT(A5,2X,I3,2X,A26)
101    FORMAT(A15,I2,A18,2X,I3)
103    FORMAT(A36,1X,I3)
190    FORMAT(A11,I2,I2,A2,A12,I2,I2,A2,A9,I2,I2,A2)
191    FORMAT(F15.5,1X,F15.5,1X,F15.5)
192    FORMAT(A2,I2,2X,A2,I2,2X,A4,I2,A1,I2,A2,F10.5)
193    FORMAT(A55,1X,I3)
194    FORMAT(A7,I2,2X,A4,I2,2X,A7,F10.5,2X,A9,F10.5,2X,A1)
99     ENDFILE 1
C //
        STOP
        END

```

Output Example:

A PART OF THE OUTPUT IS GIVEN BELOW.

```
ENTER YEAR(N), PERIOD(W):
    33    12
ENTER NUMBER OF LAGS (NOLAG):
# OF LAGS :      8
THERE ARE 396  OBSERVATIONS
THE MAXIMUM NUMBER OF LAGS IS      8
.
READ INPUTS FROM A (F)ILE OR ENTER (M)ANUALLY ? F/M :
f
ENTER FILE NAME ?;
atnos.txt

enter 0 to exit any integer to continue
```

MODEL IDENTIFICATION WHEN MAX LAG IS 24

```
*****
.
.
PERIOD: 6 LAG: 1 PePACF: 0.28939 -->BAND: 0.34119
PERIOD: 6 LAG: 2 PePACF: 0.37606 -->BAND: 0.34119 *
PERIOD: 6 LAG: 3 PePACF: -0.12401 -->BAND: 0.34119
PERIOD: 6 LAG: 4 PePACF: -0.22069 -->BAND: 0.34119
PERIOD: 6 LAG: 5 PePACF: 0.19669 -->BAND: 0.34119
PERIOD: 6 LAG: 6 PePACF: -0.30060 -->BAND: 0.34119
PERIOD: 6 LAG: 7 PePACF: 0.18404 -->BAND: 0.34119
PERIOD: 6 LAG: 8 PePACF: 0.05164 -->BAND: 0.34119
PERIOD: 6 LAG: 9 PePACF: -0.05279 -->BAND: 0.34119
PERIOD: 6 LAG:10 PePACF: 0.09710 -->BAND: 0.34119
PERIOD: 6 LAG:11 PePACF: 0.10564 -->BAND: 0.34119
PERIOD: 6 LAG:12 PePACF: -0.03074 -->BAND: 0.34119
PERIOD: 6 LAG:13 PePACF: 0.70048 -->BAND: 0.34119 *
PERIOD: 6 LAG:14 PePACF: 0.24630 -->BAND: 0.34119

PERIOD: 6 LAG:15 PePACF: 0.21351 -->BAND: 0.34119
PERIOD: 6 LAG:16 PePACF: -0.29216 -->BAND: 0.34119
PERIOD: 6 LAG:17 PePACF: 0.21100 -->BAND: 0.34119
PERIOD: 6 LAG:18 PePACF: -0.11819 -->BAND: 0.34119
PERIOD: 6 LAG:19 PePACF: -0.01044 -->BAND: 0.34119
PERIOD: 6 LAG:20 PePACF: 0.49543 -->BAND: 0.34119 *
PERIOD: 6 LAG:21 PePACF: 0.02357 -->BAND: 0.34119
PERIOD: 6 LAG:22 PePACF: -0.33499 -->BAND: 0.34119
PERIOD: 6 LAG:23 PePACF: 0.02739 -->BAND: 0.34119
PERIOD: 6 LAG:24 PePACF: -0.07716 -->BAND: 0.34119
.
.
enter 0 to exit any integer to continue
```

```
PERIOD      1:
*This is an AR(24) model for period  1
```

PERIOD 2:
*This is an AR(24) model for period 2

PERIOD 3:
*This is an AR(21) model for period 3

PERIOD 4:
*This is an AR(21) model for period 4

PERIOD 5:
*This is an AR(0) model for period 5

PERIOD 6:
*This is an AR(20) model for period 6

PERIOD 7:
*This is an AR(16) model for period 7

PERIOD 8:
*This is an AR(23) model for period 8

PERIOD 9:
*This is an AR(24) model for period 9

PERIOD 10:
*This is an AR(22) model for period 10

PERIOD 11:
*This is an AR(23) model for period 11

PERIOD 12:
*This is an AR(14) model for period 12

MODEL IDENTIFICATION WHEN MAX LAG IS 8

.
.

PERIOD: 6 LAG: 1 PePACF: 0.28939 -->BAND: 0.34119
PERIOD: 6 LAG: 2 PePACF: 0.37606 -->BAND: 0.34119 *
PERIOD: 6 LAG: 3 PePACF: -0.12401 -->BAND: 0.34119
PERIOD: 6 LAG: 4 PePACF: -0.22069 -->BAND: 0.34119
PERIOD: 6 LAG: 5 PePACF: 0.19669 -->BAND: 0.34119
PERIOD: 6 LAG: 6 PePACF: -0.30060 -->BAND: 0.34119
PERIOD: 6 LAG: 7 PePACF: 0.18404 -->BAND: 0.34119
PERIOD: 6 LAG: 8 PePACF: 0.05164 -->BAND: 0.34119

.
.

PERIOD 1:
*This is an AR(0) model for period 1

PERIOD 2:
*This is an AR(0) model for period 2

PERIOD 3:

*This is an AR(0) model for period 3

PERIOD 4:

*This is an AR(0) model for period 4

PERIOD 5:

*This is an AR(0) model for period 5

PERIOD 6:

*This is an AR(2) model for period 6

PERIOD 7:

*This is an AR(2) model for period 7

PERIOD 8:

*This is an AR(5) model for period 8

PERIOD 9:

*This is an AR(1) model for period 9

PERIOD 10:

*This is an AR(1) model for period 10

PERIOD 11:

*This is an AR(1) model for period 11

PERIOD 12:

*This is an AR(8) model for period 12

APPENDIX D

COMPUTER PROGRAM FOR LEAST SQUARES ESTIMATION OF PAR MODELS

Description:

This program performs least squares estimation for PAR coefficients for any season of a given actual series. The multiple regression subroutine in the IMSL Stat Library is extended to periodic case for an ordered sequence. Besides computing the LS estimates of the coefficients, program also stores the residuals which are used further in diagnostic checks.

Inputs:

N: Number of years

IW: Number of periods

CANS: Name of the file in which series is stored

IV: Period performed LSE

INTCEP: This value states if there exists intercept or not in the model

NIND: The order of the PAR process for season v

Outputs:

B (): Estimated coefficients of the PAR process for season v

RES (): Residuals resulted from the fitted model

SST: Sum of squares of total

SSE: Sum of squares of errors

Note: If an intercept parameter is included in the model, it is automatically denoted with B(0) in the outputs.

Program Listing:

THE OUTPUTS OF THIS PROGRAM ARE STORED IN FILE 'LSEatnos.txt'

```
C//
C// program for lse
C
PROGRAM LSE
PARAMETER (INTCEP=1, NIND=8, NOBS=33, LDX=NOBS,IV=12,
*          NCOEF=INTCEP+NIND)

      INTEGER  NW, N, IW, I, J, T, INDICE
      INTEGER  IOFF, ICON
      INTEGER  NOUT
      REAL     B(NCOEF), SSE, SST, XSUB(LDX,NIND), YSUB(NOBS),RES(500)
      EXTERNAL RLSE, UMACH, WRRRN
      REAL     D(500),X(20,150),SUM
      CHARACTER FNAME*20, CANS*1

C// open the output file
      OPEN (2, FILE='LSEatnos.txt')
C
      PRINT*,' ENTER YEAR(N), PERIOD(W):'
      WRITE(2,*) ' ENTER YEAR(N), PERIOD(W):'
      READ*, N,IW
      WRITE(2,*) N,IW

C

      NW=N*IW

C
C
      PRINT*,' ENTER FILE NAME ?;'
      WRITE(2,*) ' ENTER FILE NAME ?;'
      READ 82, FNAME
      PRINT*, FNAME
      WRITE(2,*) FNAME
      OPEN (1, FILE=FNAME)
      REWIND 1
      READ(1,*,END=99) (D(I), I=1, NW)
      CLOSE (1)

C// fill the xsub matrix and ysub array for subroutine
      DO 4 I=1,N
```

```

INDICE=IW*(I-1)+IV
IF((INDICE-NIND).GT.0)THEN
    DO 5 J=1,NIND
        XSUB(I-IOFF,J)=D(INDICE-J)
5    CONTINUE
    ELSE
        IOFF=I
    ENDIF
4    CONTINUE

DO 6 I=1,N
INDICE=IW*(I-1)+IV
IF((INDICE-NIND).GT.0)THEN
    YSUB(I-IOFF)=D(INDICE)
    ELSE
        IOFF=I
    ENDIF
6    CONTINUE

CALL RLSE (NOBS, YSUB, NIND, XSUB, LDX, INTCEP, B, SST, SSE)
CALL WRRRN ('B', NCOEF, 1, B, NCOEF, 0)
WRITE (*,99999) 'SST = ', SST, ' SSE = ', SSE

DO 7 I=1,NOBS
SUM = B(1)
DO 8 J=1,NIND
SUM = SUM+ B(J+1)* XSUB(I,J)
8    CONTINUE

C// calculating residuals

7    RES(I)=YSUB(I)-SUM
    CONTINUE
DO 12 I=1,NCOEF
PRINT 84,'B( ', I, ')=' ,B(I)
WRITE (2,84) 'B( ', I, ')=' ,B(I)
12   CONTINUE

DO 13 I=1,NOBS
PRINT 83,'RES( ',I, ')=' ,RES(I)
WRITE (2,83) 'RES( ',I, ')=' ,RES(I)

13   CONTINUE

PRINT *,'SST = ', SST, ' SSE = ', SSE
WRITE (2,*) 'SST = ', SST, ' SSE = ', SSE

82   FORMAT(A20)
84   FORMAT(A2,I2,A2,F10.5)
83   FORMAT(A4,I3,A2,F10.5)
99   ENDFILE 1
END

```

Output Example:

THE OUTPUT OF THIS PROGRAM IS AS FOLLOWS,

ENTER YEAR(N), PERIOD(W):

33 12

ENTER FILE NAME ?;

atnos2.txt

B(1)= 11.88494

B(2)= 0.12092

B(3)= 0.16902

B(4)= 0.01306

B(5)= -0.99822

B(6)= 2.16752

B(7)= -3.34290

B(8)= 3.56133

B(9)= 0.54212

B(10)= 0.75493

B(11)= -1.17774

B(12)= -0.32088

B(13)= -0.14686

B(14)= 0.48753

RES(1)= 5.95337

RES(2)= -2.50174

RES(3)= 0.35670

RES(4)= 2.34907

RES(5)= 3.56994

RES(6)= 2.74192

RES(7)= -2.84329

RES(8)= 15.28082

RES(9)= -6.50660

RES(10)= -2.69373

RES(11)= 3.78721

RES(12)= -0.95144

RES(13)= -4.73891

RES(14)= -0.83926

RES(15)= 3.15578

RES(16)= -12.66762

RES(17)= 3.09277

RES(18)= -0.65772

RES(19)= -1.32405

RES(20)= 1.67526

RES(21)= 1.85140

RES(22)= -1.87665

RES(23)= 0.70174

RES(24)= -6.89446

RES(25)= 9.02119

RES(26)= 3.34445

RES(27)= -4.33182

RES(28)= -3.44289

RES(29)= 5.04392
RES(30)= 8.20067
RES(31)= -10.71815
RES(32)= -7.13802
SST = 3540.473 SSE = 1018.867

APPENDIX E

APPLIED SERIES

E1. Atnos Creek

YEAR	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUNE	JULY	AUG	SEP
1963	1.680	4.970	71.61	59.06	52.75	21.85	11.92	6.790	3.070	0.994	0.455	0.590
1964	1.110	1.530	19.62	3.550	15.18	34.60	5.560	3.710	1.670	0.383	0.160	0.560
1965	0.904	1.740	36.21	10.12	107.0	21.06	33.57	21.33	4.050	1.700	1.070	1.170
1966	1.600	3.820	18.44	54.85	13.76	40.13	17.76	6.600	2.840	0.916	0.573	1.640
1967	1.310	2.190	21.61	46.10	16.18	11.46	8.310	3.960	1.860	0.344	0.580	0.655
1968	1.270	1.480	7.000	73.51	36.90	34.05	7.490	5.810	2.570	0.500	0.982	5.410
1969	2.030	3.740	9.350	30.78	34.61	16.17	14.42	5.500	2.670	2.780	0.648	0.583
1970	1.200	1.840	16.30	26.01	45.23	27.16	14.18	4.560	1.860	0.530	0.719	0.261
1971	1.580	3.020	7.920	16.69	23.06	44.81	13.16	4.020	2.080	0.271	0.014	0.182
1972	0.838	2.164	13.16	5.010	9.251	4.035	5.230	4.434	1.560	0.090	0.000	0.757
1973	5.372	3.330	1.656	2.450	29.11	17.18	7.945	2.745	2.105	0.178	0.614	0.005
1974	2.412	4.529	17.99	3.654	29.82	30.58	7.706	5.427	1.726	0.183	0.000	0.097
1975	0.271	2.119	5.808	35.68	17.01	19.83	6.707	8.480	2.948	0.253	0.041	0.143
1976	0.683	2.830	17.90	11.30	24.08	6.129	17.53	4.835	1.982	0.232	0.248	0.362
1977	1.727	2.783	14.99	14.54	15.22	17.32	5.049	2.510	0.359	0.019	0.000	0.320
1978	1.275	6.014	15.02	47.95	54.88	26.25	22.72	7.296	2.985	0.872	0.689	2.778
1979	2.566	8.142	8.727	30.03	20.26	7.202	5.207	3.362	1.288	0.312	0.156	0.632
1980	1.045	2.997	9.828	39.49	16.31	25.80	13.10	6.595	10.09	1.112	0.645	1.164
1981	1.088	3.603	30.23	72.42	27.52	24.77	6.911	8.867	2.574	1.485	0.995	1.174
1982	2.142	4.671	89.55	22.31	15.57	16.85	11.58	14.43	4.061	1.791	1.591	1.188
1983	1.817	1.663	2.926	7.763	15.68	9.222	6.743	2.639	1.618	1.668	1.272	1.233
1984	1.140	6.132	8.725	21.52	20.51	19.87	24.25	7.524	3.547	3.222	2.597	2.012
1985	1.346	2.078	1.768	18.01	8.647	12.35	6.392	2.938	1.987	1.088	0.903	0.918
1986	0.932	7.687	3.825	30.45	31.36	15.05	4.998	2.853	3.078	1.518	1.596	1.244
1987	1.500	1.197	9.837	43.56	17.41	29.43	9.695	5.782	4.723	2.177	1.737	2.177
1988	2.496	6.603	24.99	5.687	8.996	20.00	8.063	4.330	3.325	1.557	1.645	2.596
1989	1.726	9.448	23.81	5.057	2.693	5.327	2.092	2.016	1.067	0.232	0.489	1.074
1990	0.640	1.400	10.50	3.430	6.060	5.560	3.970	2.350	1.880	0.838	0.837	1.320
1991	0.637	7.300	34.30	7.780	6.890	3.670	8.380	6.100	3.050	2.170	1.670	2.350
1992	0.813	0.601	2.390	2.220	1.530	13.20	10.30	1.940	3.450	2.680	2.080	2.540
1993	1.690	1.180	6.410	6.550	16.10	21.90	8.150	4.470	3.920	2.830	2.800	2.200
1994	2.030	1.450	3.340	1.980	15.20	10.20	3.800	2.380	2.040	1.230	1.570	1.100
1995	0.947	3.520	5.000	21.40	5.960	13.70	28.20	2.750	2.100	1.550	1.610	1.750

Location: Susurluk Basin , Balikesir / Turkey

Elavation: 94 m.

Drainage Area : 1384.0 Km²

Period of observation: 1963 – 1995

Long-term Mean Discharge: 9.116 m³/sn.

E2. Nilüfer Creek

YEAR	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUNE	JULY	AUG	SEP
1954	4.210	4.560	6.600	20.32	29.88	25.33	26.45	35.33	14.08	2.400	1.390	1.370
1955	1.830	2.700	3.420	9.220	17.71	13.40	19.65	16.41	6.300	1.870	2.440	1.420
1956	2.240	20.36	14.98	19.45	36.11	29.69	27.76	28.18	14.05	3.270	1.550	1.660
1957	2.150	2.870	3.100	3.110	7.650	13.63	8.950	15.66	4.690	1.340	1.250	1.360
1958	1.360	2.170	9.970	14.20	14.54	35.40	51.16	37.82	18.82	4.270	2.460	3.580

1959	4.280	4.940	6.990	21.73	14.16	26.07	20.73	33.39	24.02	9.290	1.660	2.760
1960	4.200	6.770	7.010	23.53	19.67	21.36	28.19	21.13	15.28	4.990	1.450	2.170
1961	2.780	3.790	13.30	16.60	18.60	14.70	21.20	15.70	7.910	2.490	1.220	1.420
1962	2.290	3.100	5.150	5.290	15.47	32.09	37.32	20.12	6.690	2.560	1.050	1.130
1963	7.920	11.55	42.31	70.54	51.60	33.56	37.63	35.08	18.95	6.910	1.370	2.520
1964	3.830	5.300	43.23	11.15	18.06	31.01	20.75	21.96	13.06	2.020	1.560	3.150
1965	3.490	5.780	35.07	19.66	44.14	42.96	47.77	51.41	23.59	9.890	2.650	2.190
1966	2.720	7.230	17.30	35.69	20.88	28.11	49.57	27.57	14.63	4.580	2.710	2.830
1967	3.650	3.130	9.910	20.45	16.10	23.09	33.99	39.37	19.49	7.010	1.020	3.130
1968	5.030	4.610	17.56	50.63	50.77	42.45	39.10	28.02	12.93	4.260	1.490	8.730
1969	10.11	11.57	18.10	22.82	36.04	27.73	37.37	48.86	19.43	9.150	2.310	1.600
1970	2.490	2.740	10.90	29.79	57.33	39.05	54.07	31.19	17.95	6.340	2.470	2.060
1971	6.000	8.290	34.68	25.86	20.96	41.15	33.24	33.14	13.53	2.500	1.590	1.660
1972	3.647	6.183	24.57	8.415	12.74	12.06	24.71	17.22	14.20	4.902	1.650	3.217
1973	15.28	11.98	7.094	10.27	17.71	26.95	32.58	27.00	15.92	2.850	3.064	1.910
1974	8.266	10.26	29.27	11.54	19.36	28.27	27.19	43.34	17.51	3.337	4.839	3.216
1975	3.559	8.305	14.39	13.43	18.27	29.37	29.83	40.63	19.38	4.056	2.109	1.571
1976	3.113	4.924	14.17	13.33	13.96	11.22	21.60	22.14	9.882	1.877	2.415	2.267
1977	5.193	5.212	16.08	10.66	14.02	20.58	23.38	16.63	4.992	1.189	0.574	1.079
1978	2.804	7.604	17.61	23.22	30.89	31.43	41.31	34.81	13.32	3.619	1.694	3.202
1979	5.212	5.916	13.24	32.50	23.22	15.87	21.84	20.88	12.45	2.834	1.388	3.322
1980	3.920	10.76	19.13	29.69	36.56	34.42	38.90	39.75	29.03	7.404	2.807	1.952
1981	3.094	9.164	25.84	40.45	34.05	49.51	33.86	49.32	21.87	8.848	3.248	7.235
1982	7.078	14.71	74.11	48.69	36.34	32.55	57.71	39.86	25.58	14.81	3.526	3.077
1983	4.243	4.806	5.493	13.55	17.76	32.47	38.90	24.17	12.68	7.312	4.910	2.231
1984	4.149	7.857	11.96	14.71	20.13	27.61	43.47	43.26	20.11	11.63	12.31	2.648
1985	1.713	3.138	3.740	9.871	16.73	20.48	20.46	19.80	7.077	2.909	1.302	1.276
1986	2.050	11.50	11.41	24.76	27.61	24.28	20.17	13.42	9.989	3.701	1.906	1.613
1987	2.983	4.343	9.004	35.59	21.35	26.06	30.83	28.53	17.32	7.923	4.513	3.992
1988	5.406	7.366	28.95	11.81	11.10	23.10	23.09	24.73	10.14	5.134	3.101	2.715
1989	3.234	7.209	15.89	11.41	5.439	8.035	9.567	7.764	5.573	2.610	2.010	1.939
1990	17.20	20.60	25.70	16.80	15.90	14.00	18.70	27.50	12.80	5.430	3.100	3.470
1991	4.240	9.320	25.70	12.50	18.30	16.80	48.70	45.90	28.80	8.940	4.250	6.100
1992	8.870	6.340	8.110	8.140	12.50	15.80	32.60	26.40	18.10	5.890	3.230	3.200
1993	6.620	5.830	9.950	6.890	12.00	15.10	20.60	29.90	10.80	4.000	3.090	2.750
1994	3.050	5.340	6.390	5.830	6.790	6.110	11.90	10.70	7.730	3.130	2.800	2.640
1995	4.080	8.260	8.770	19.90	14.70	20.10	42.30	31.80	13.60	6.240	3.670	4.130

Location: Susurluk Basin, Bursa / Turkey

Elevation: 63 m.

Drainage Area: 1290.8 Km²

APPENDIX F

PeACF AND PePACF GRAPHS

In this appendice, the PeACF and PePACf graphs for each season of Atnos creek and Nilüfer creek, which are drawn using MATLAB 6.1, are presented. The graphs are created from the values of periodic autocorrelations and periodic partial autocorrelations with the corresponding band limits from which the significant lags are specified. These values are obtained through the programs listed in appendices B and C. In the graphs, the black vertical lines at each lag show the value of periodic autocorrelation or periodic partial autocorrelation at that lag, and the red dotted lines show the band limits.

F1. PeACF and PePACF Graphs Of Atnos Creek

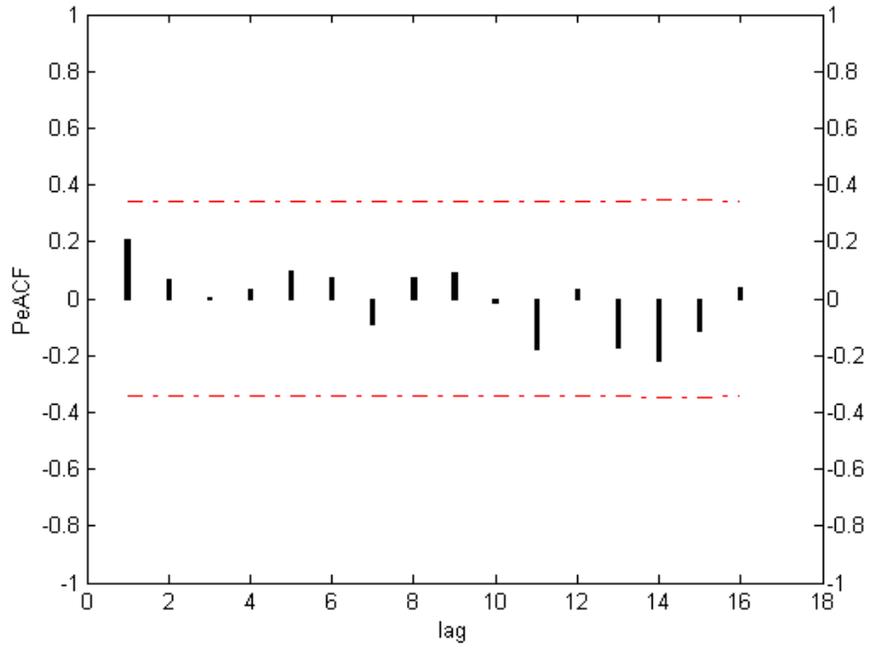


Figure F1.1 (a) PeACF graph of Atnos season 1

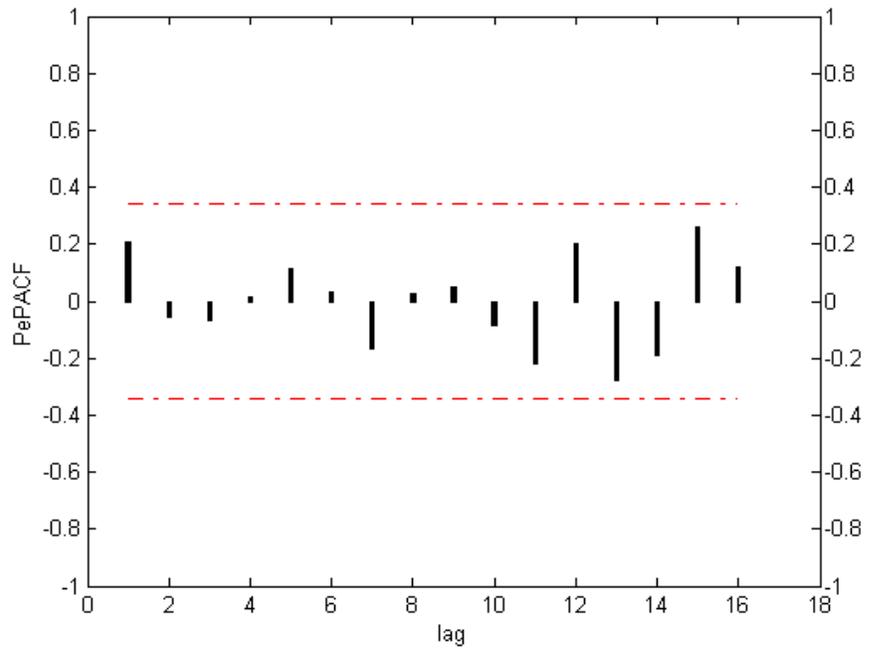


Figure F1.1 (b) PePACF graph of Atnos season 1

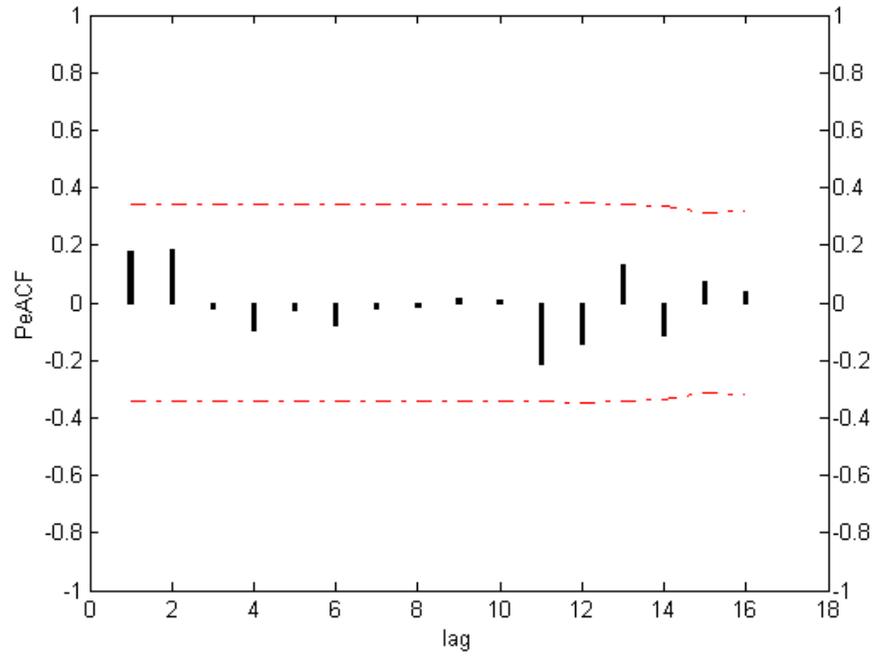


Figure F1.2 (a) PeACF graph of Atnos season 2

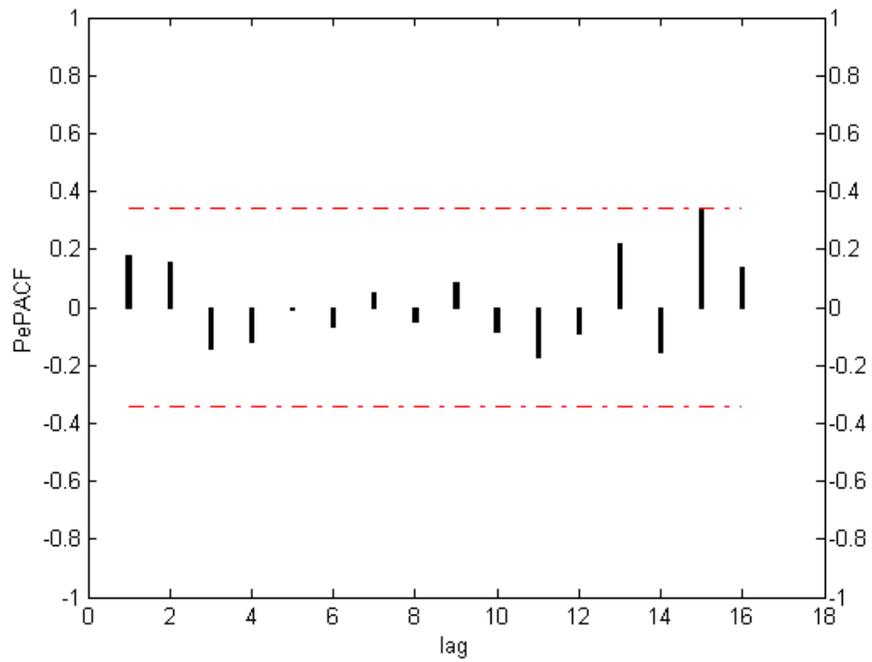


Figure F1.2 (b) PePACF graph of Atnos season 2

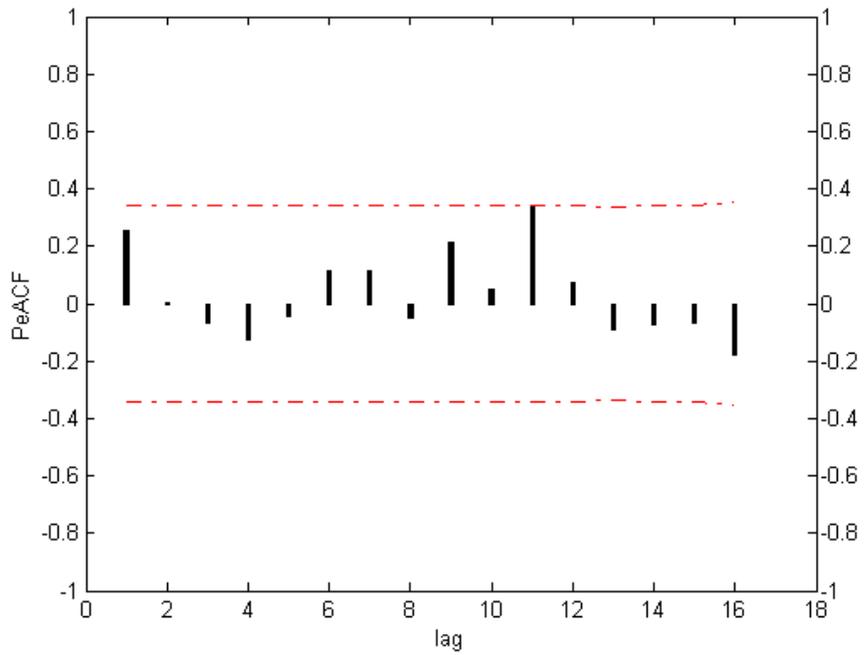


Figure F1.3 (a) PeACF graph of Atnos season 3

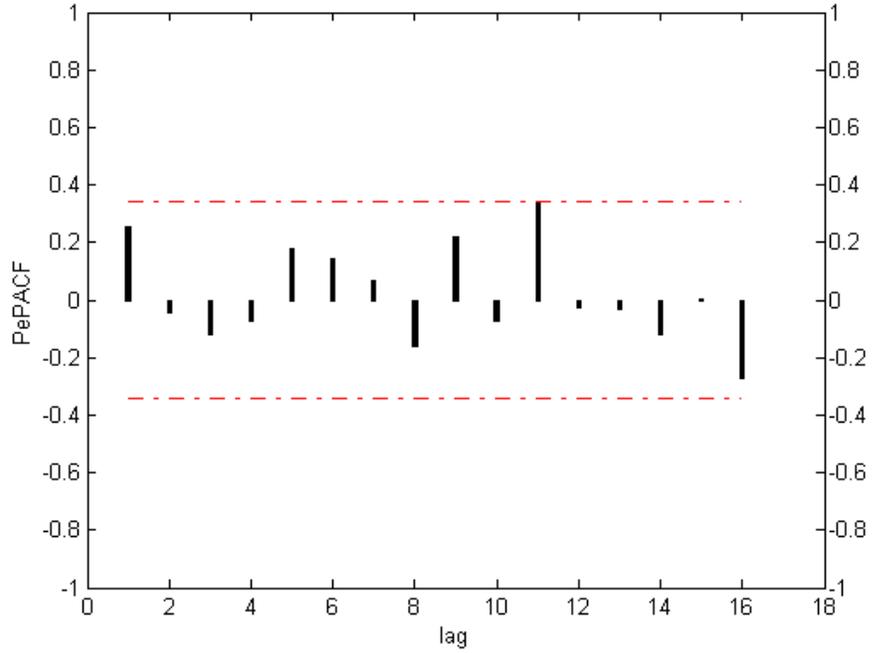


Figure F1.3 (b) PePACF graph of Atnos season 3

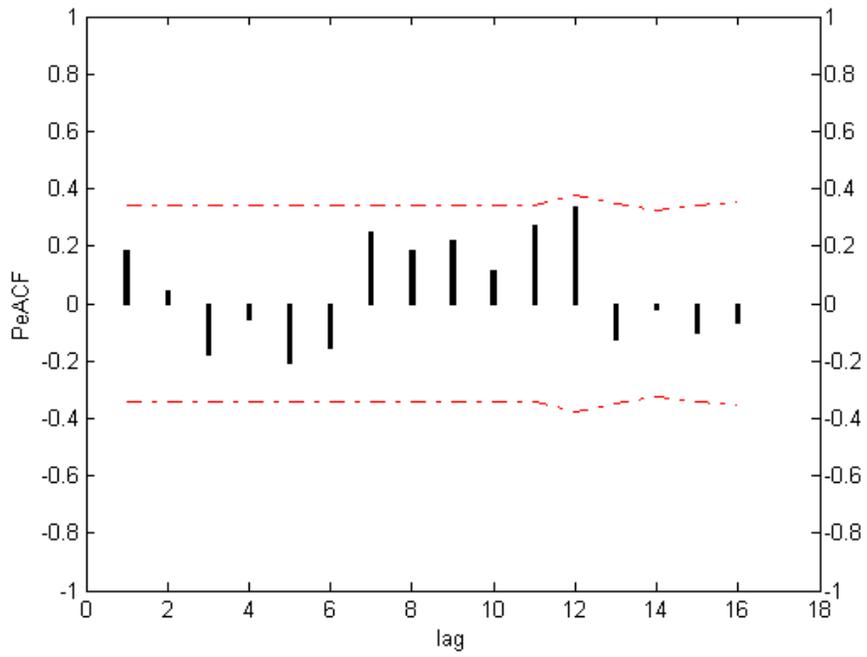


Figure F1.4 (a) PeACF graph of Atnos season 4

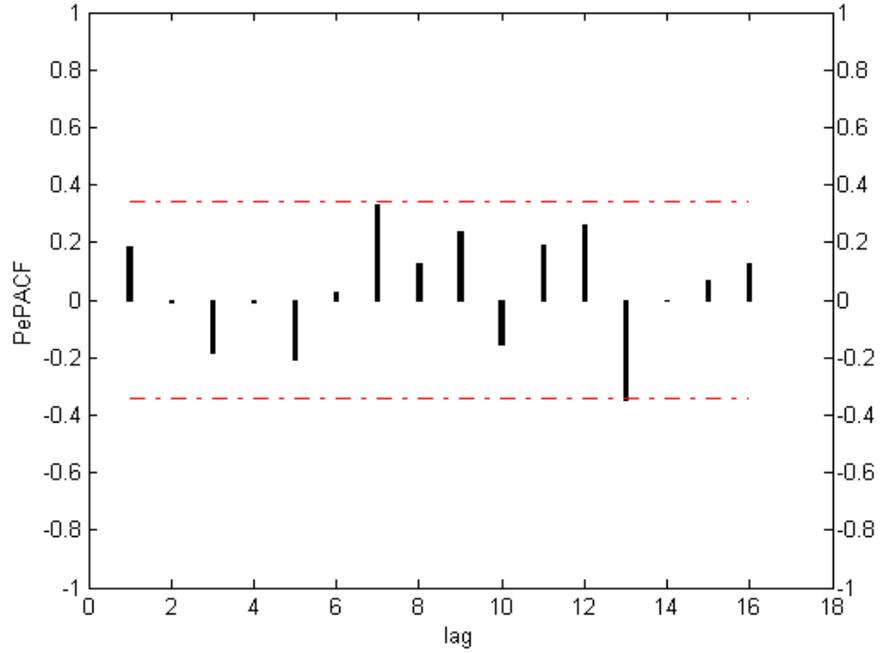


Figure F1.4 (b) PePACF graph of Atnos season 4

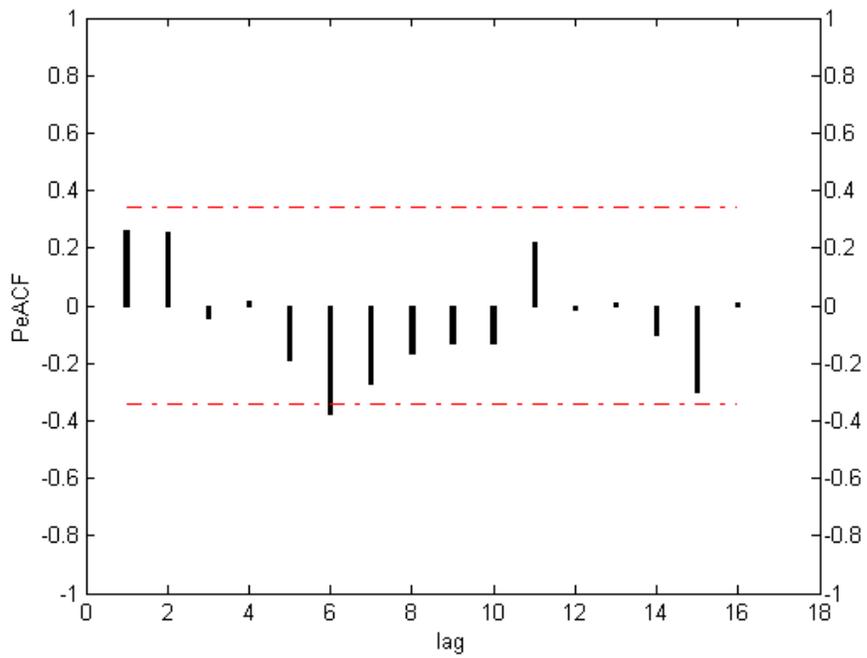


Figure F1.5 (a) PeACF graph of Atnos season 5

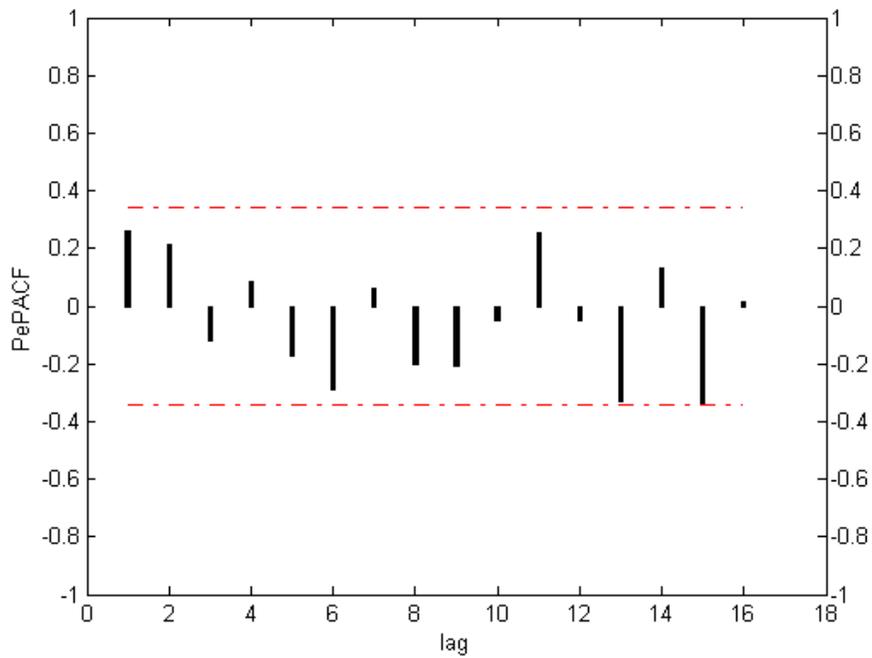


Figure F1.5 (b) PePACF graph of Atnos season 5

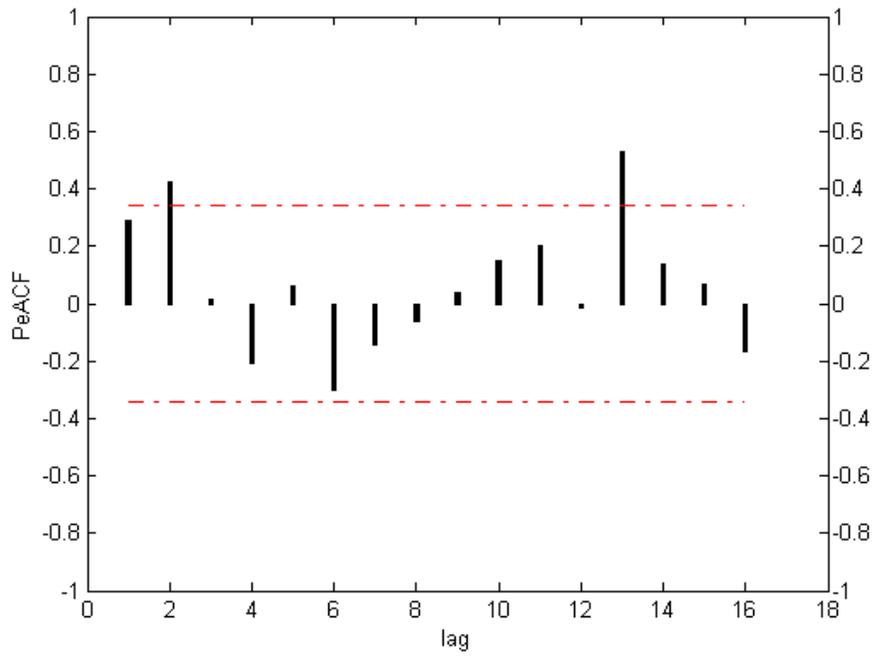


Figure F1.6 (a) PeACF graph of Atnos season 6

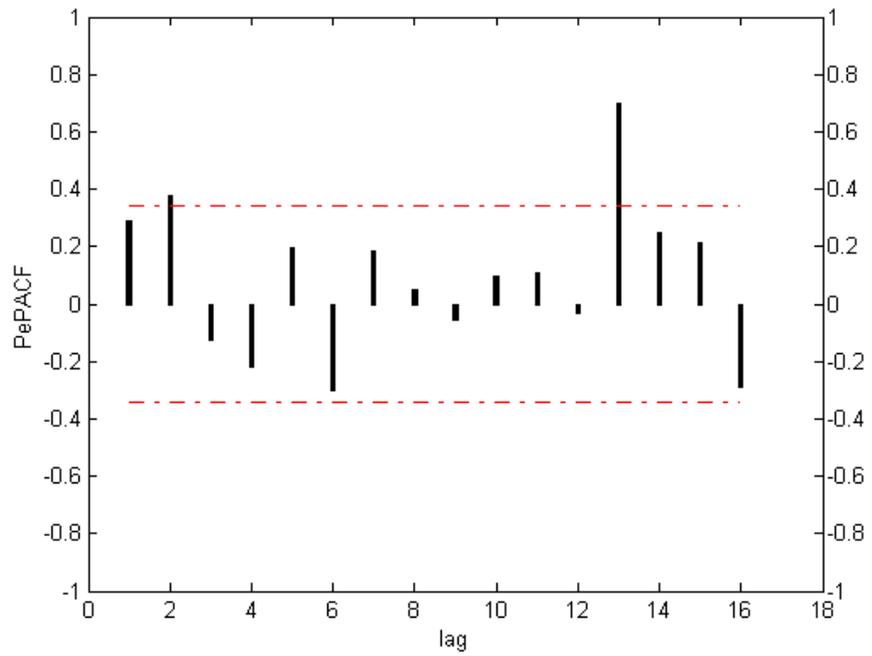


Figure F1.6 (b) PePACF graph of Atnos season 6

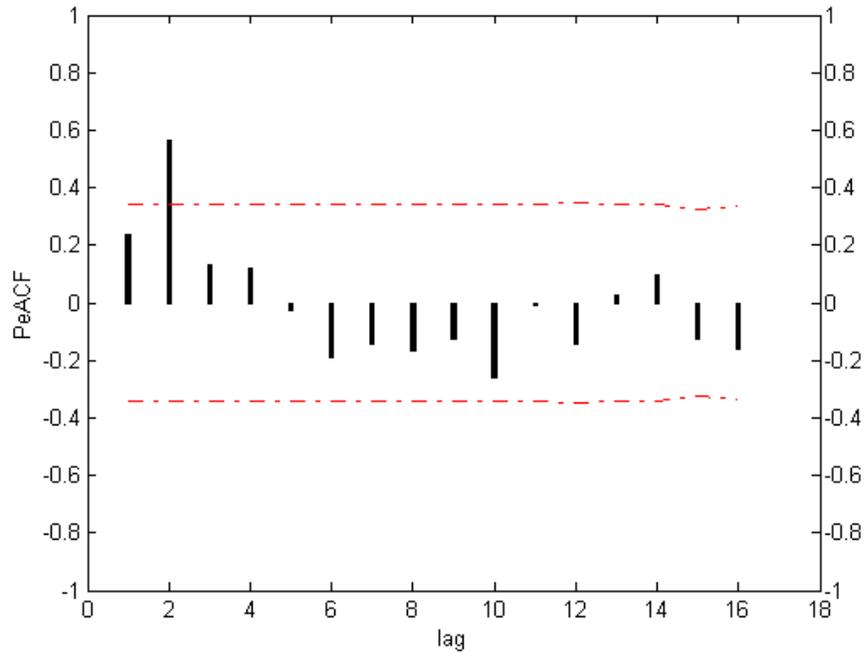


Figure F1.7 (a) PeACF graph of atnos season 7

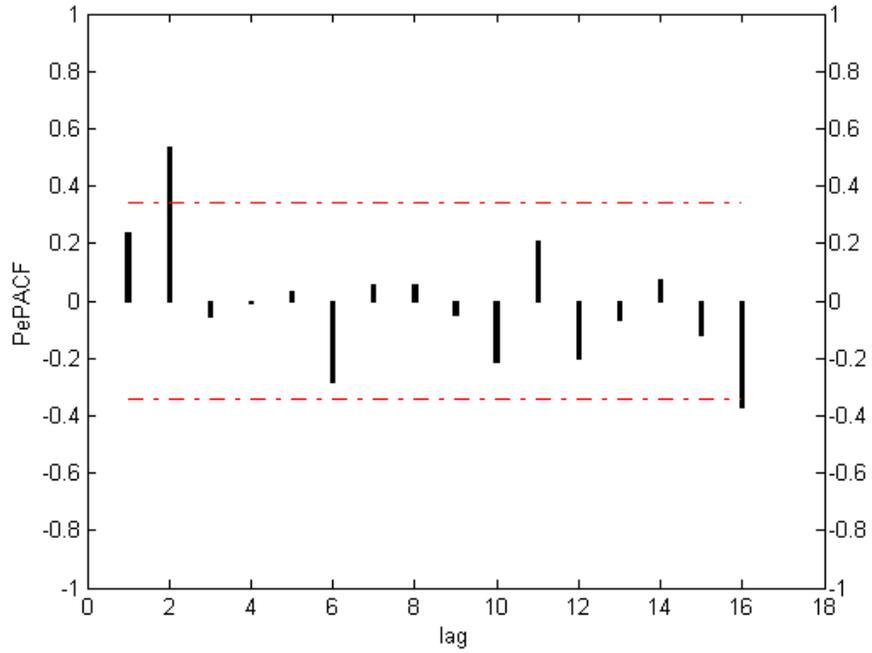


Figure F1.7 (b) PePACF graph of Atnos season 7

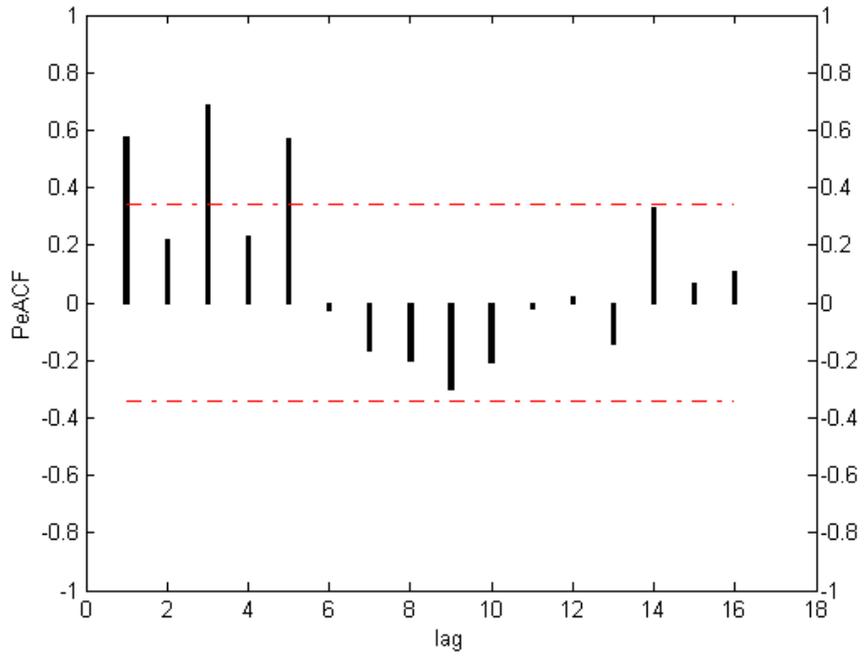


Figure F1.8 (a) PeACF graph of Atnos season 8

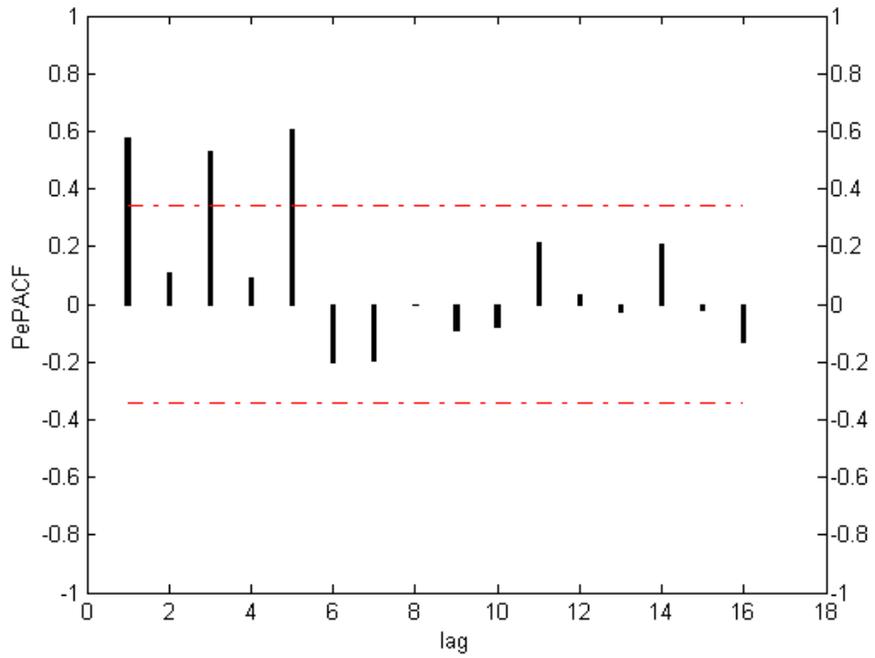


Figure F1.8 (b) PePACF graph of Atnos season 8

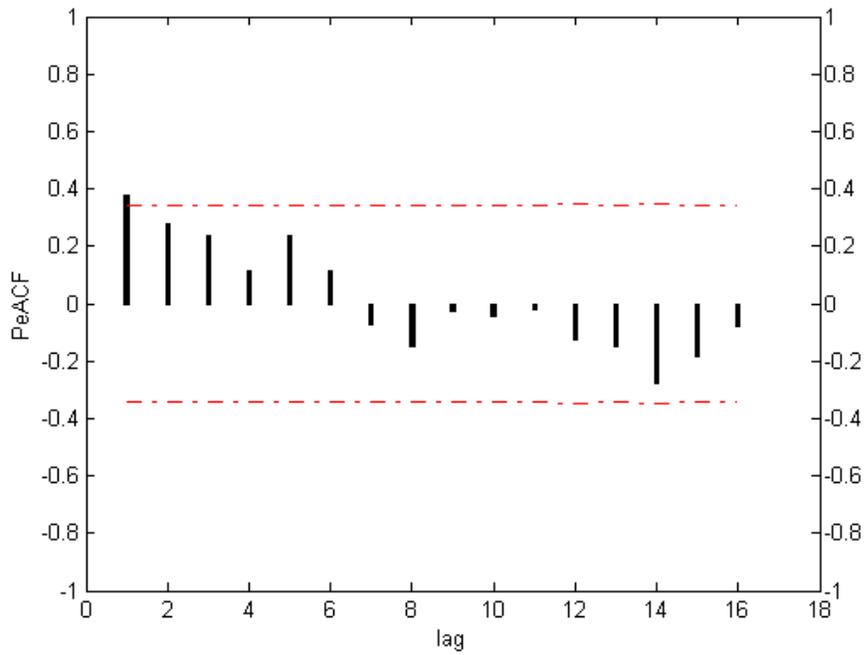


Figure F1.9 (a) PeACF graph of Atnos season 9

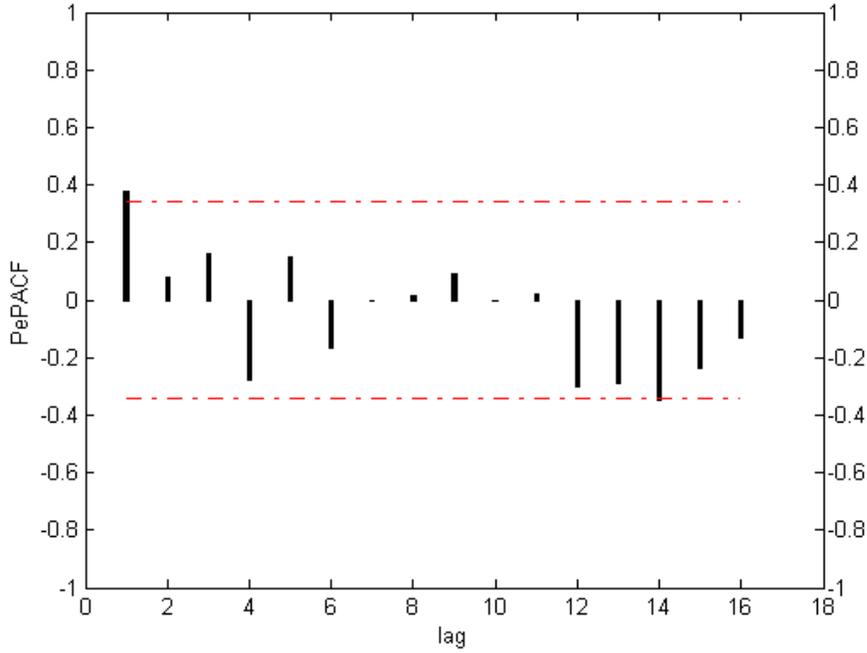


Figure F1.9 (b) PePACF graph of Atnos season 9

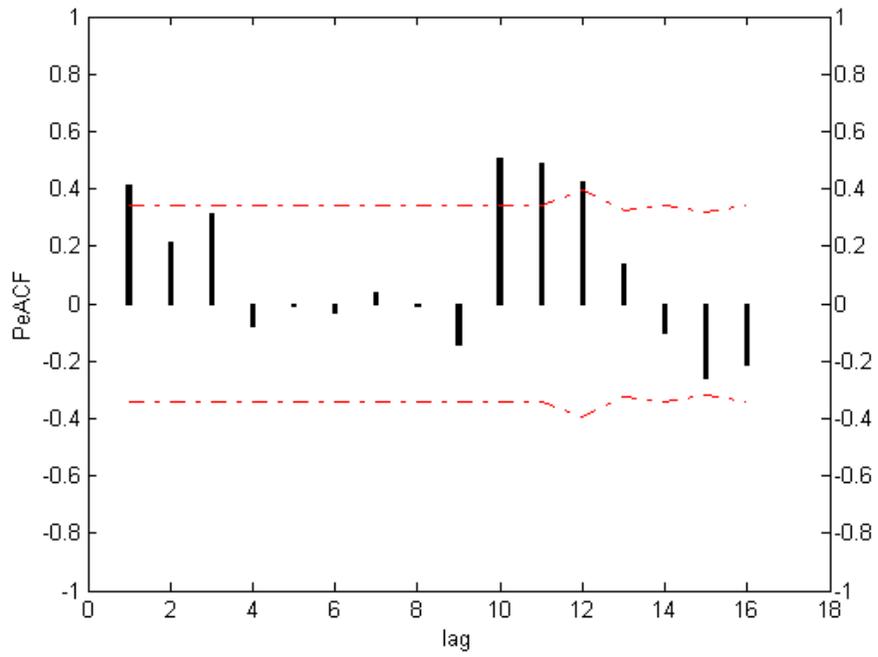


Figure F1.10 (a) PeACF graph of Atnos season 10

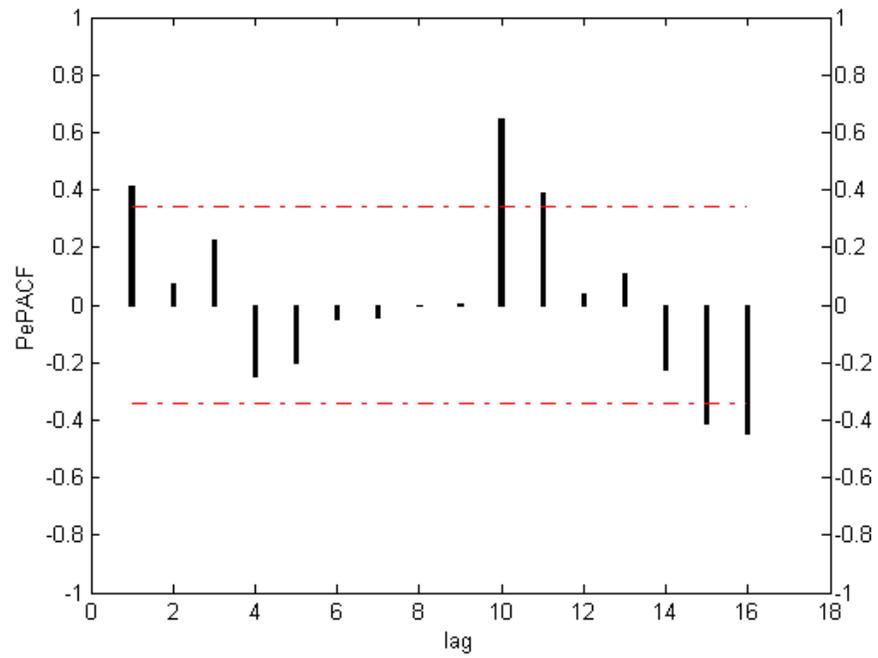


Figure F1.10 (b) PePACF graph of atnos season 10

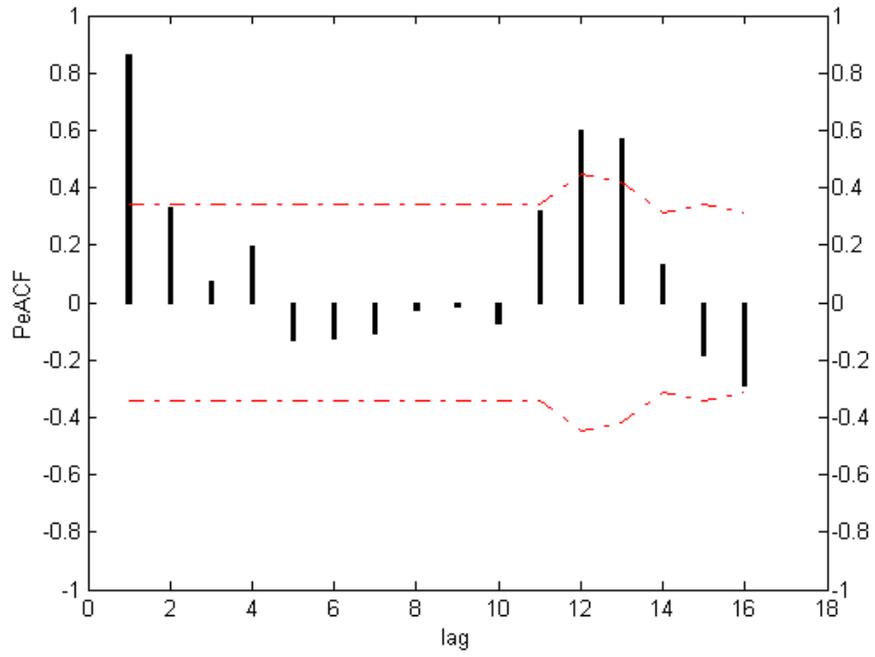


Figure F1.11 (a) PeACF graph of Atnos season 11

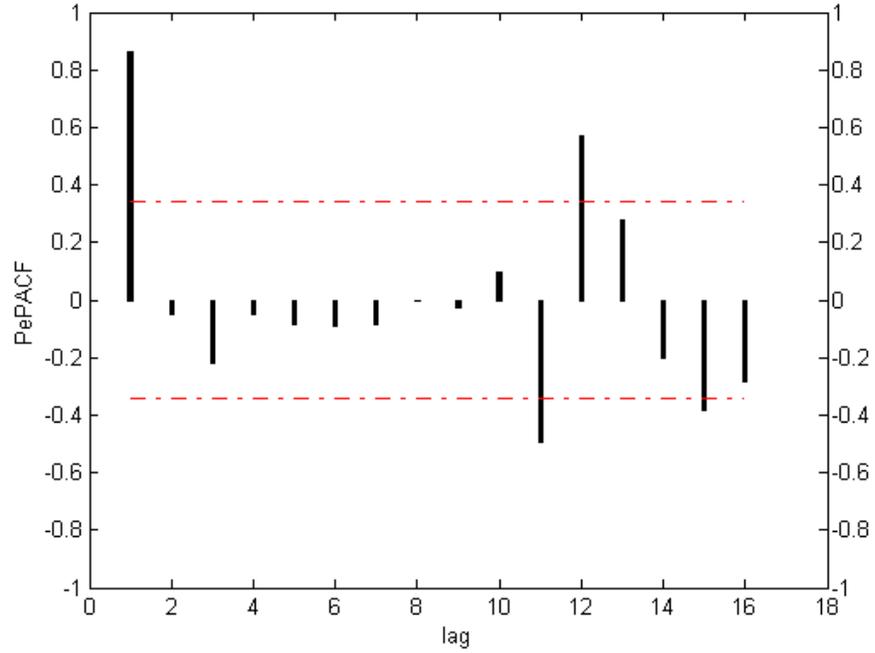


Figure F1.11 (b) PePACF graph of Atnos season 11

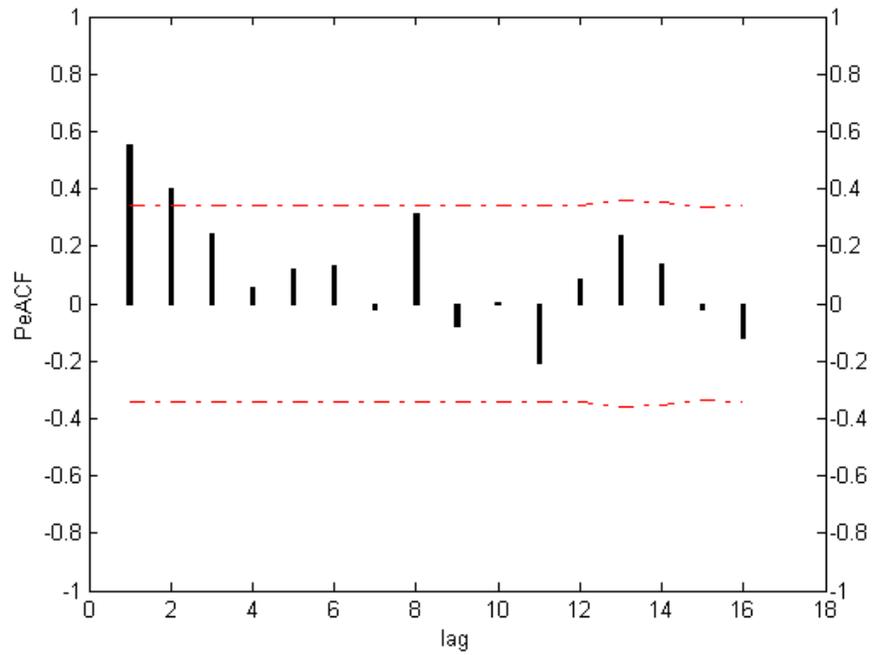


Figure F1.12 (a) PeACF graph of Atmos season 12

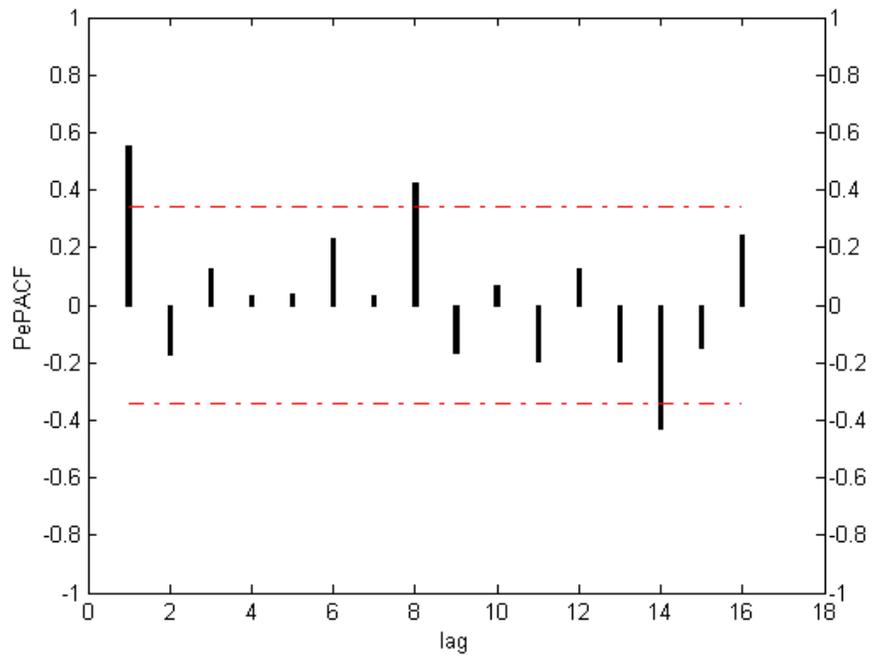


Figure F1.12 (b) PePACF graph of Atmos season 12

F2. PeACF and PePACF Graphs Of Nilufer Creek

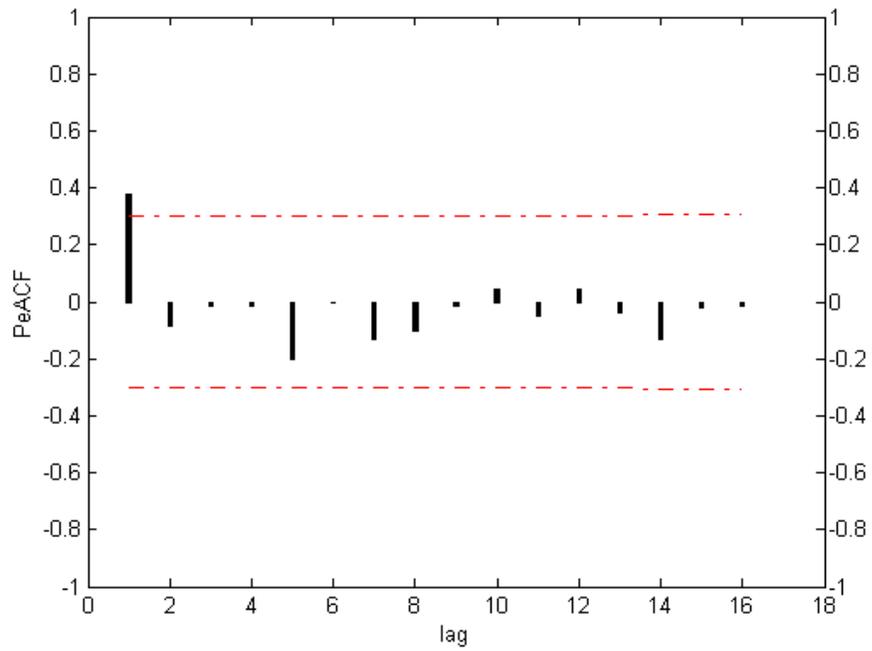


Figure F2.1 (a) PeACF graph of Nilufer season 1

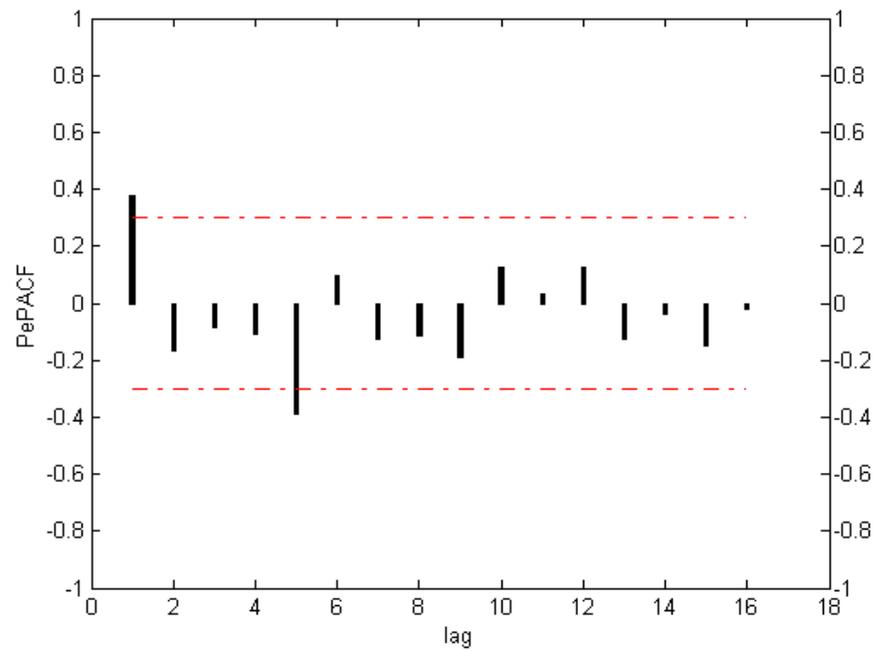


Figure F2.1 (b) PePACF graph of Nilufer season 1

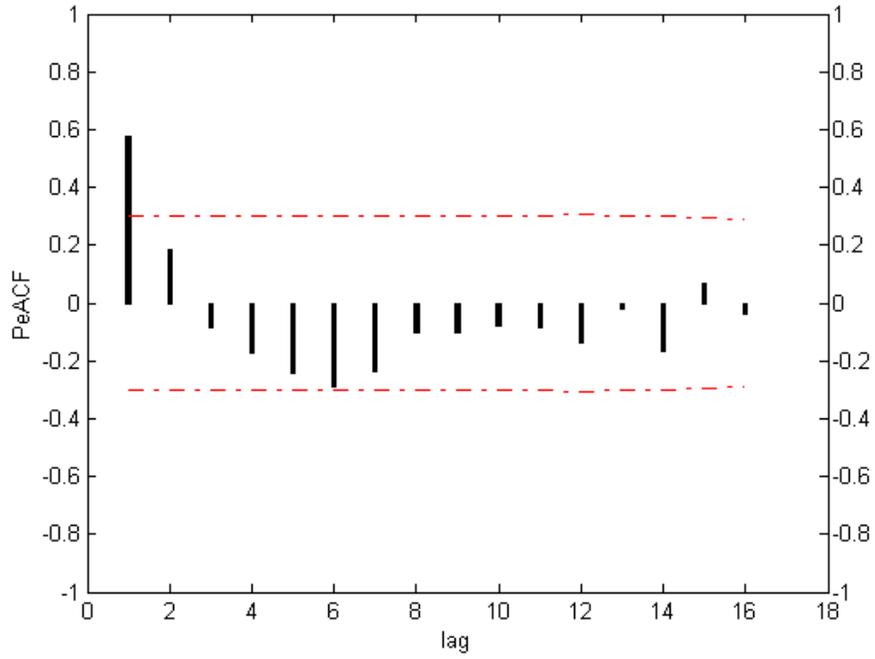


Figure F2.2 (a) PeACF graph of Nilüfer season 2

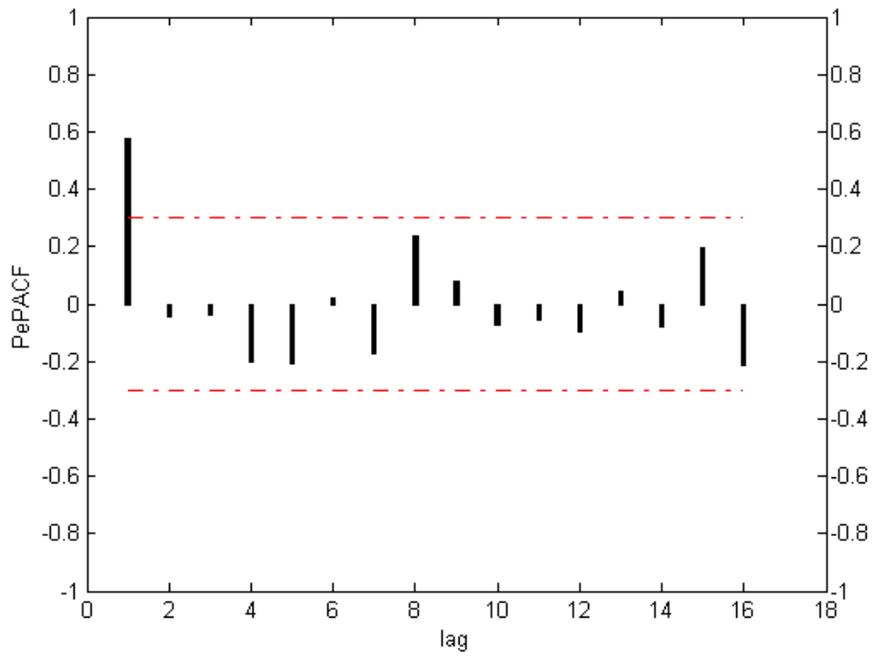


Figure F2.2 (b) PePACF graph of Nilüfer season 2

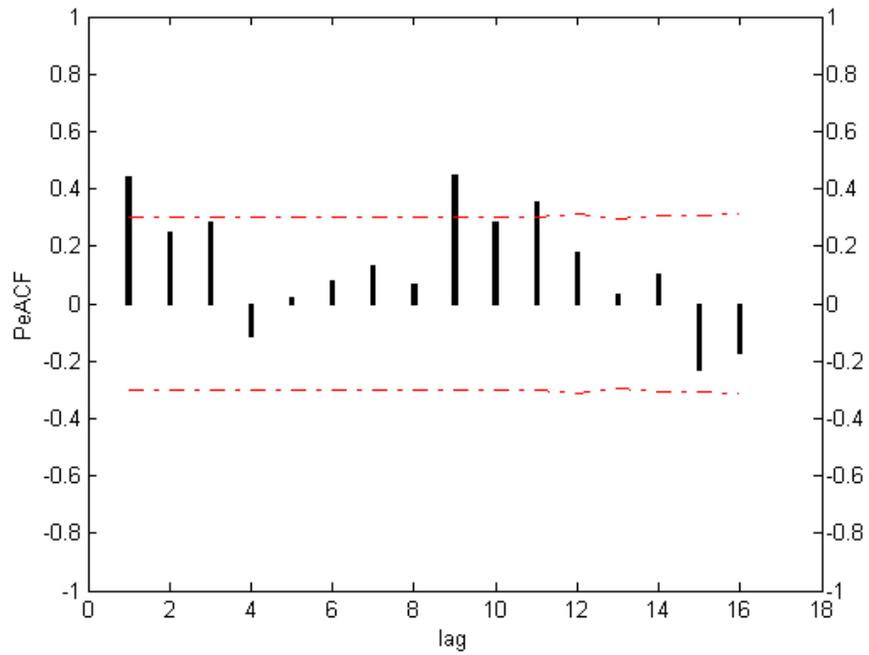


Figure F2.3 (a) PeACF graph of Nilüfer season 3

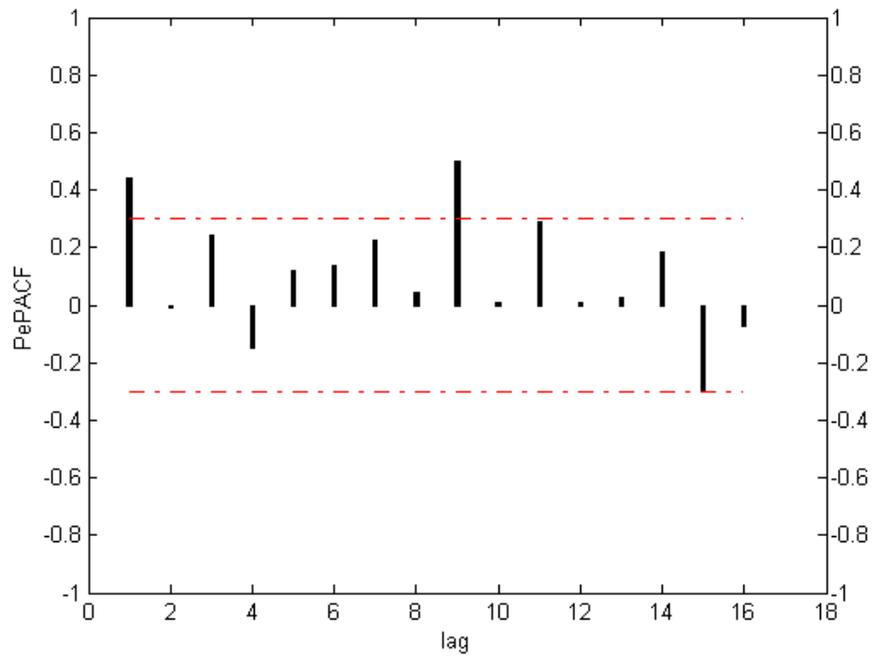


Figure F2.3 (b) PePACF graph of Nilüfer season 3

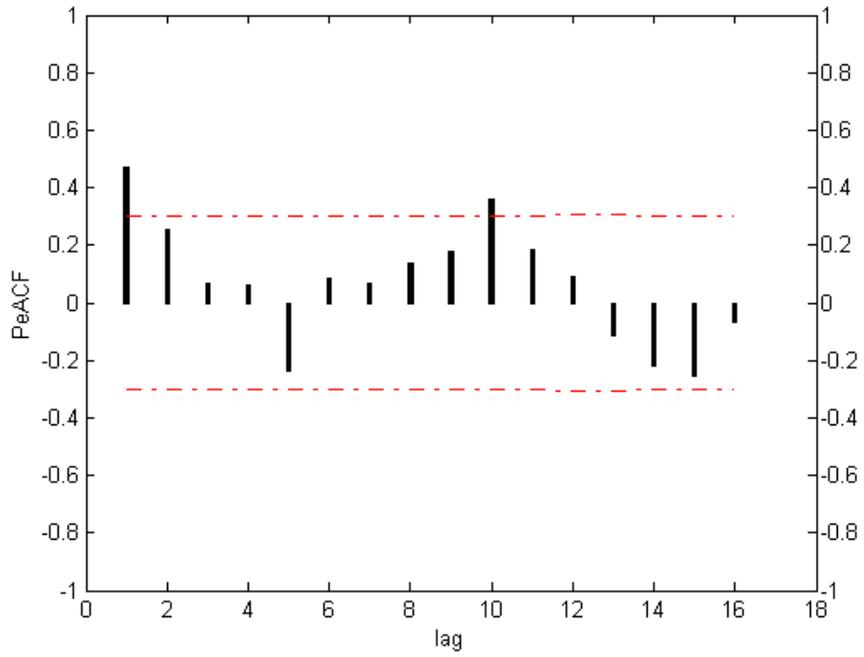


Figure F2.4 (a) PeACF graph of Nilüfer season 4

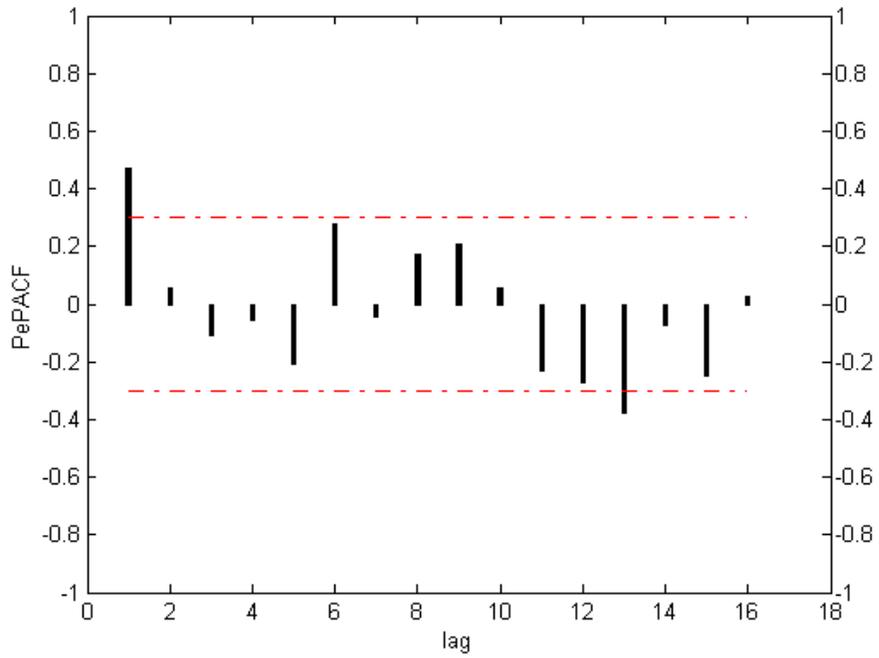


Figure F2.4 (b) PePACF graph of Nilüfer season 4

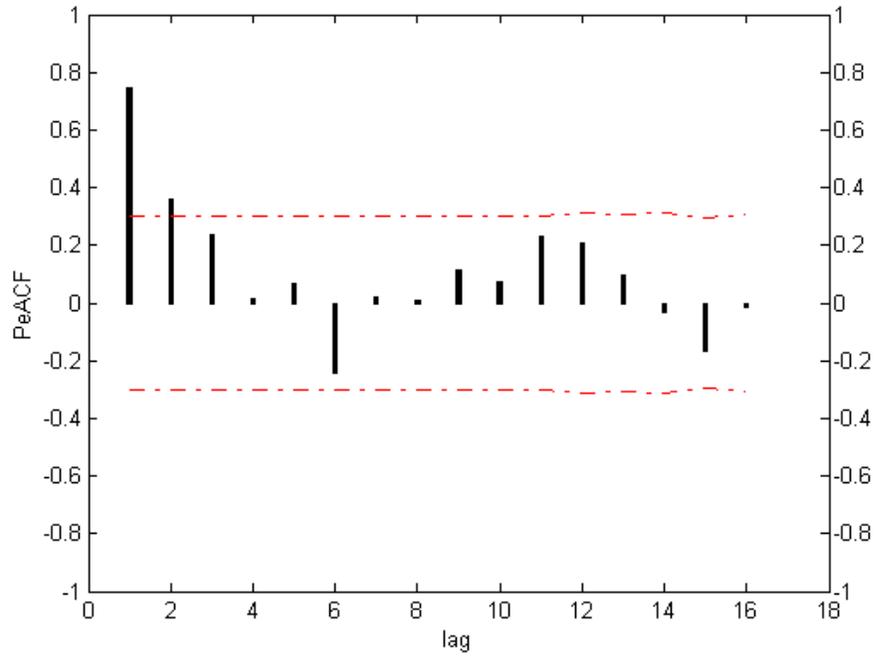


Figure F2.5 (a) PeACF graph of Nilüfer season 5

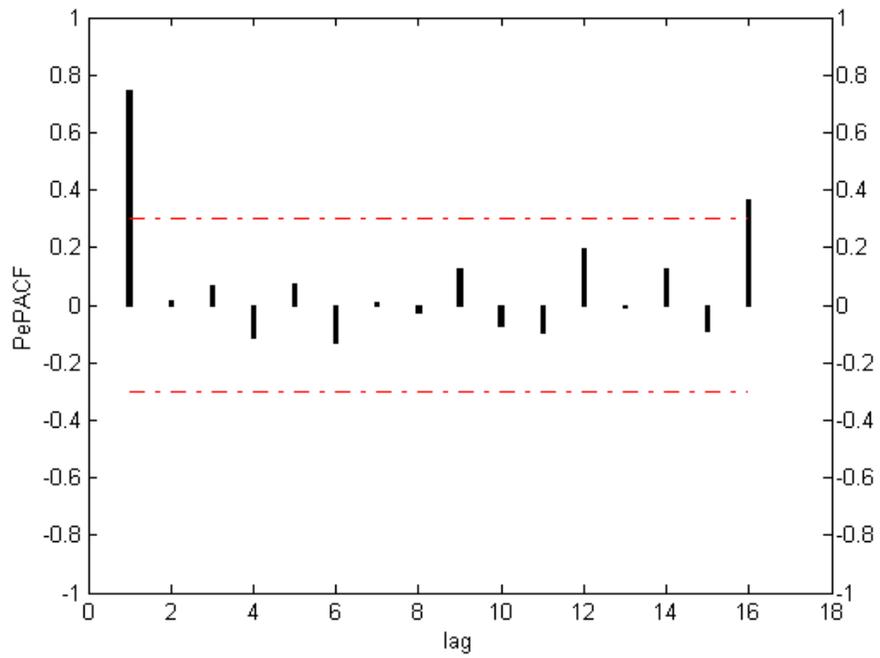


Figure F2.5 (b) PePACF graph of Nilüfer season 5

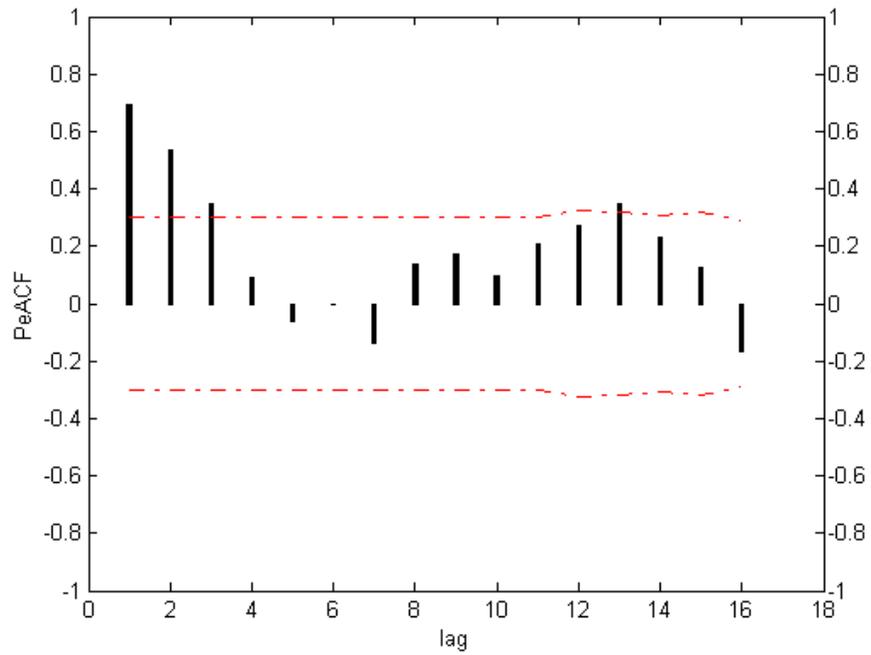


Figure F2.6 (a) PeACF graph of Nilüfer season 6

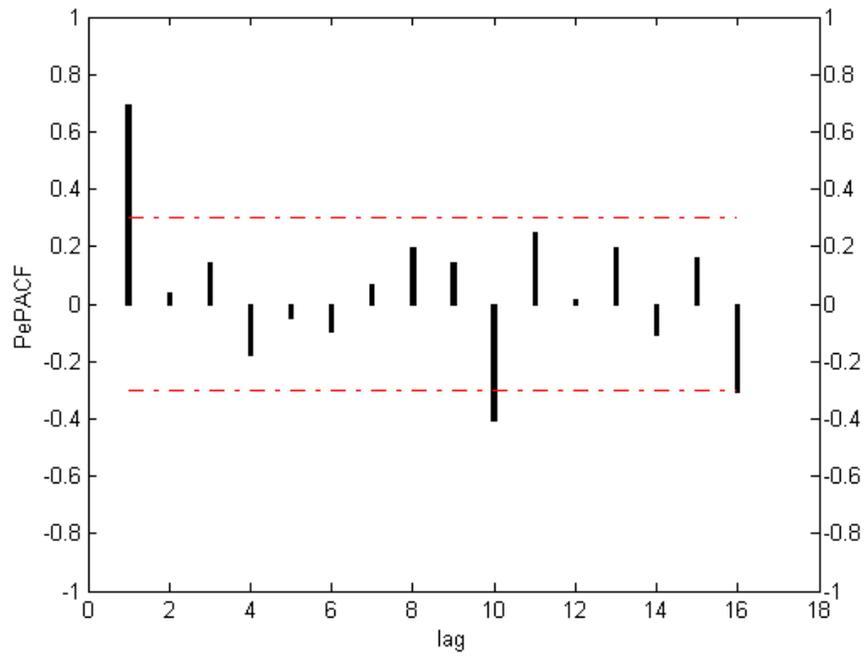


Figure F2.6 (b) PePACF graph of Nilüfer season 6

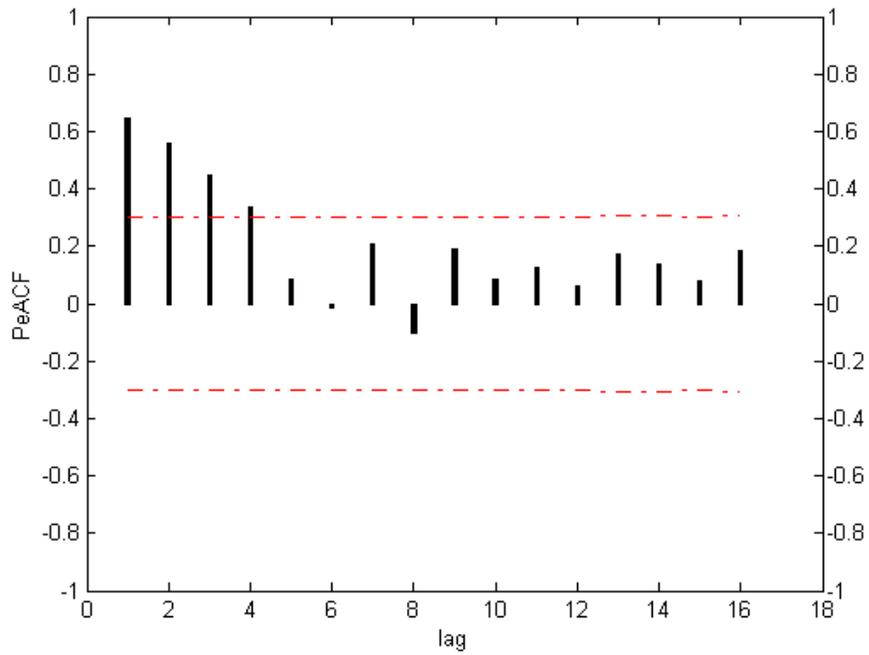


Figure F2.7 (a) PeACF graph of Nilüfer season 7

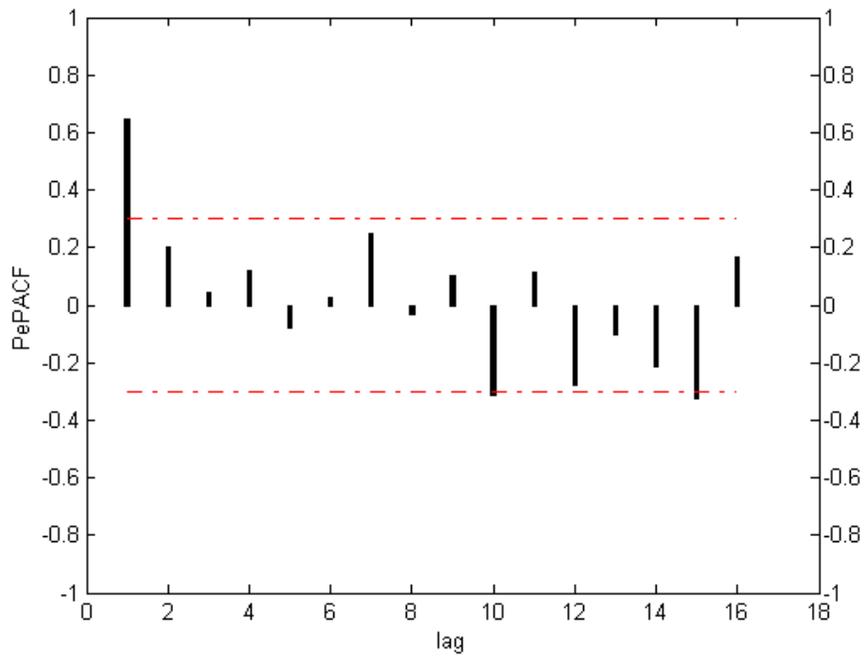


Figure F2.7 (b) PePACF graph of Nilüfer season 7

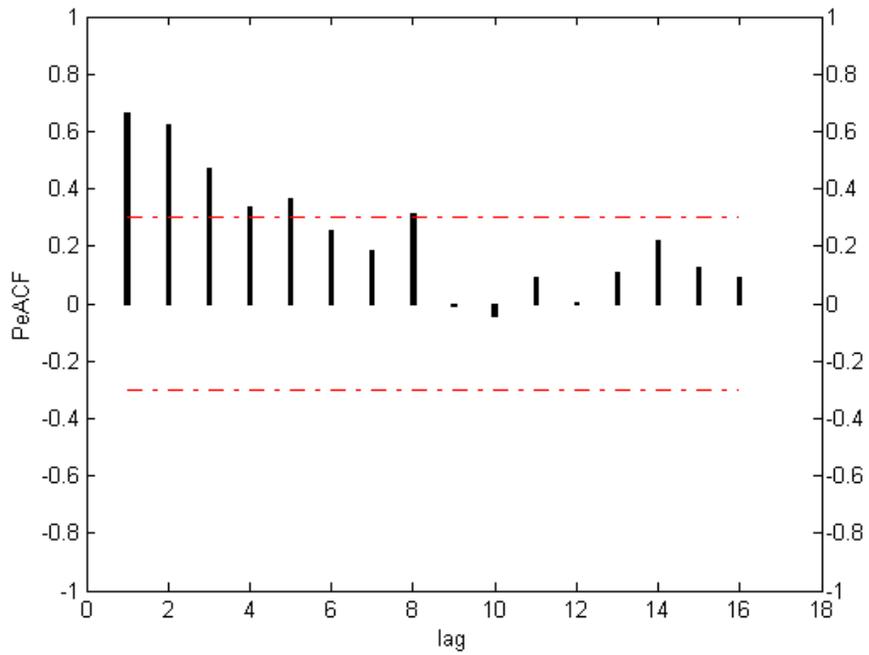


Figure F2.8 (a) PeACF graph of Nilüfer season 8

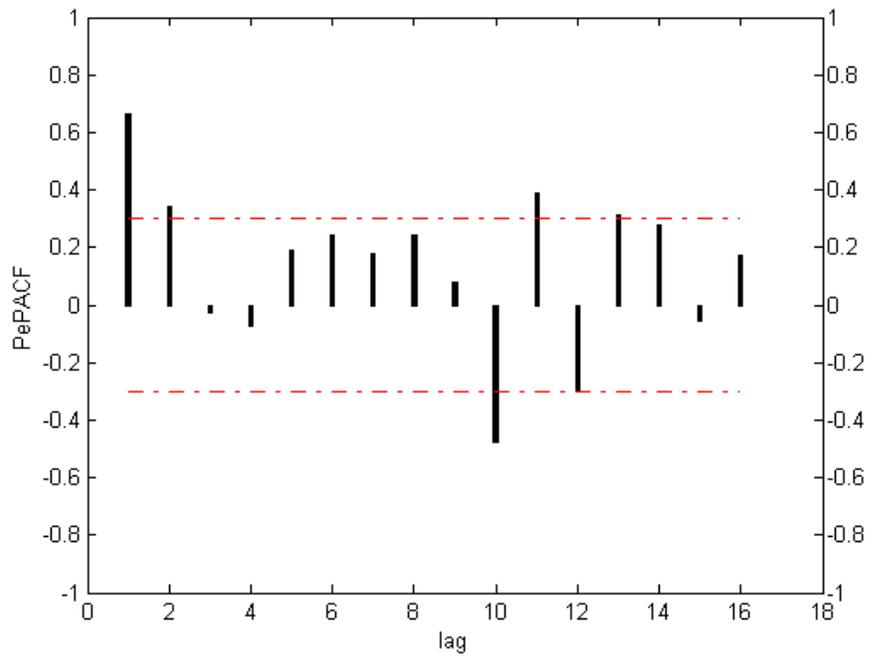


Figure F2.8 (b) PePACF graph of Nilüfer season 8

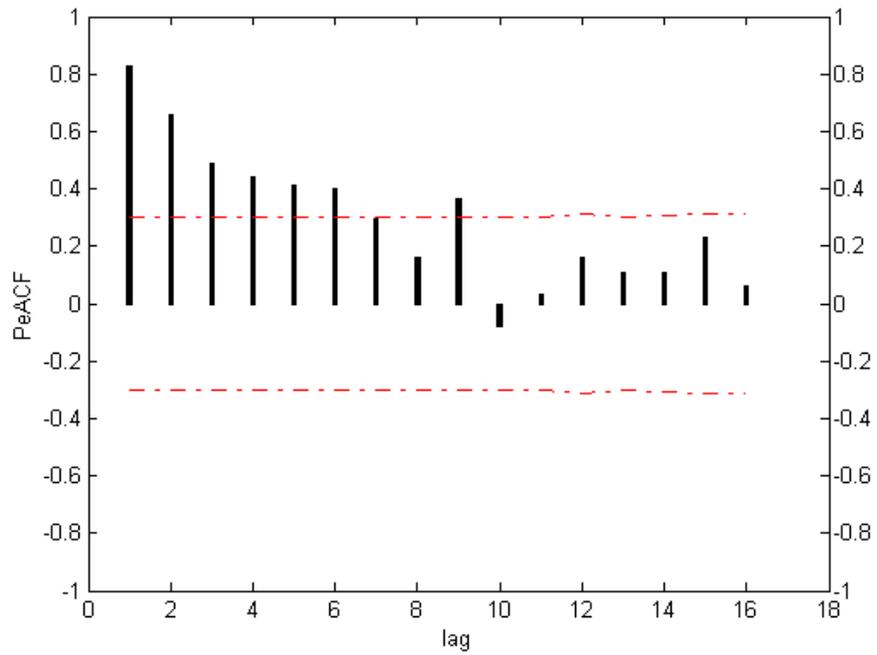


Figure F2.9 (a) PeACF graph of Nilüfer season 9

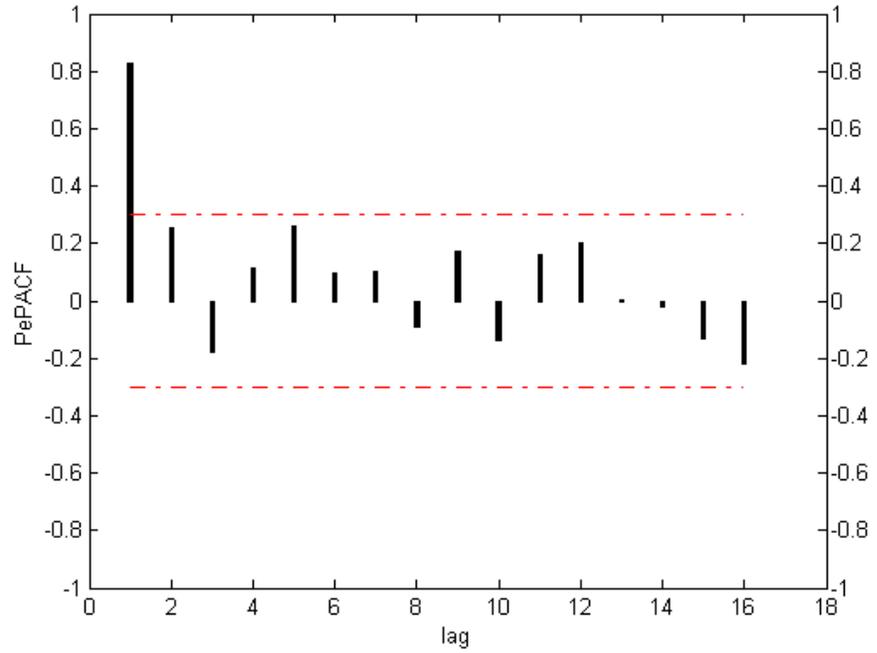


Figure F2.9 (b) PePACF graph of Nilüfer season 9

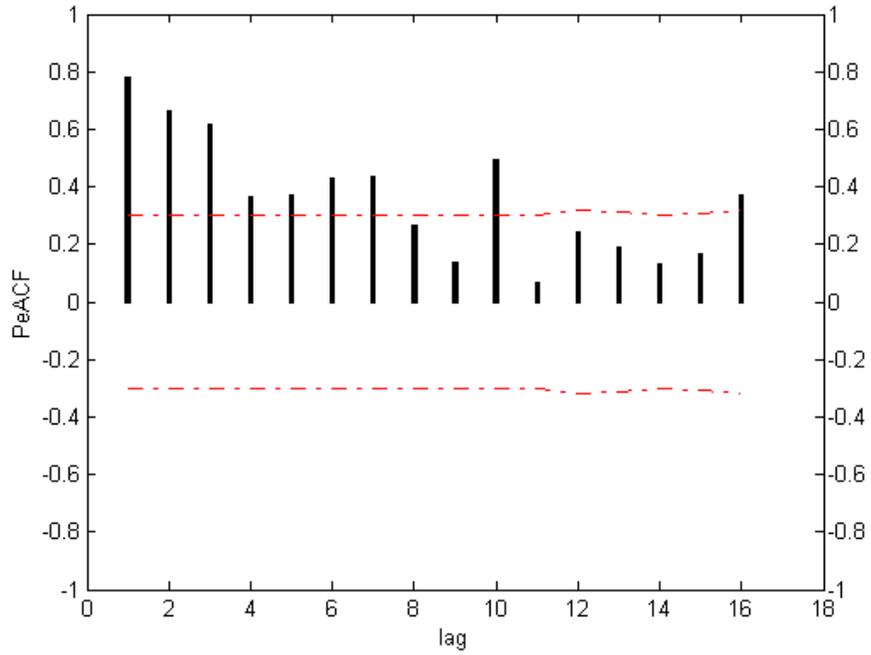


Figure F2.10 (a) PeACF graph of Nilüfer season 10

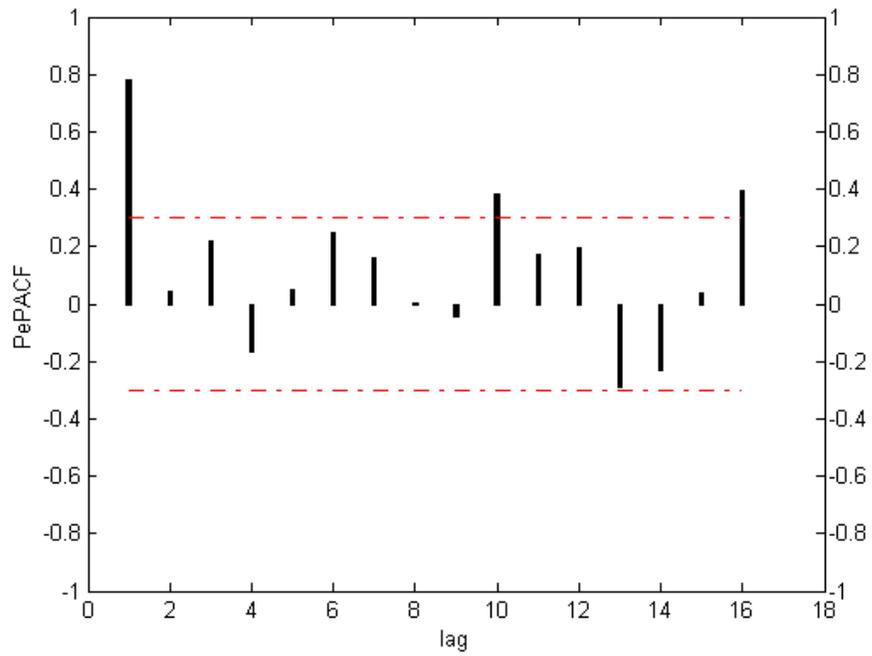


Figure F2.10 (b) PePACF graph of Nilüfer season 10

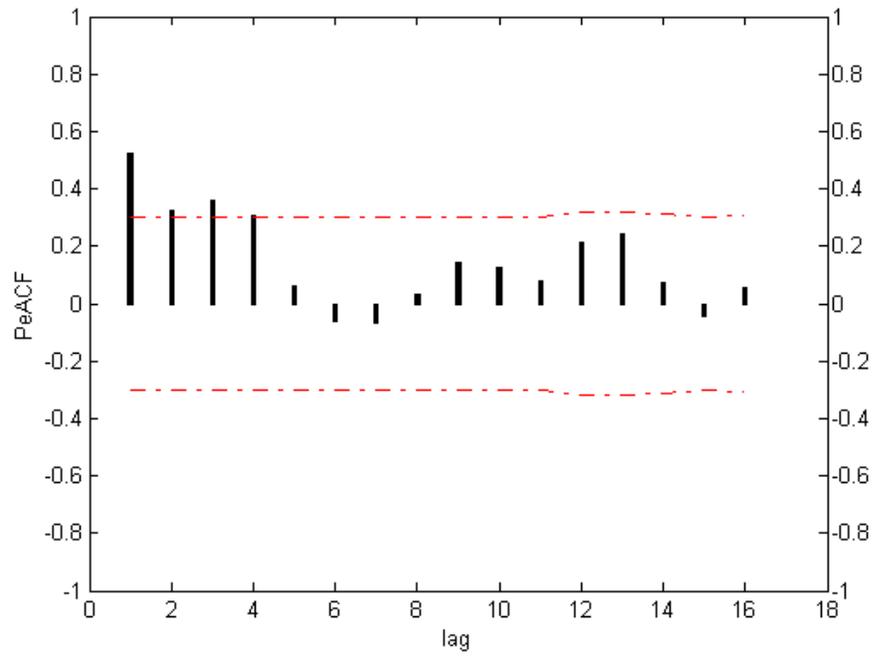


Figure F2.11 (a) PeACF graph of Nilüfer season 11

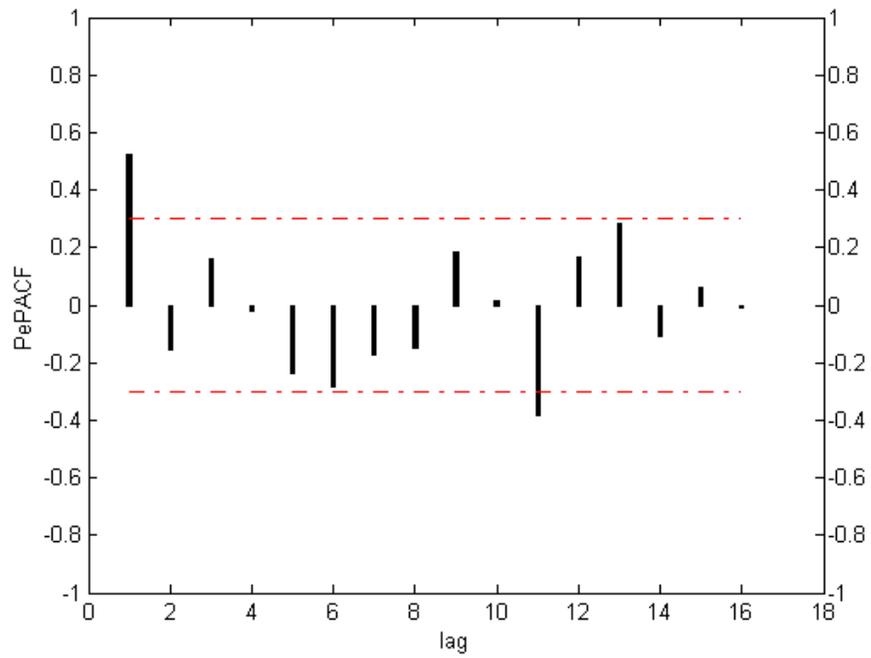


Figure F2.11 (b) PePACF graph of Nilüfer season 11

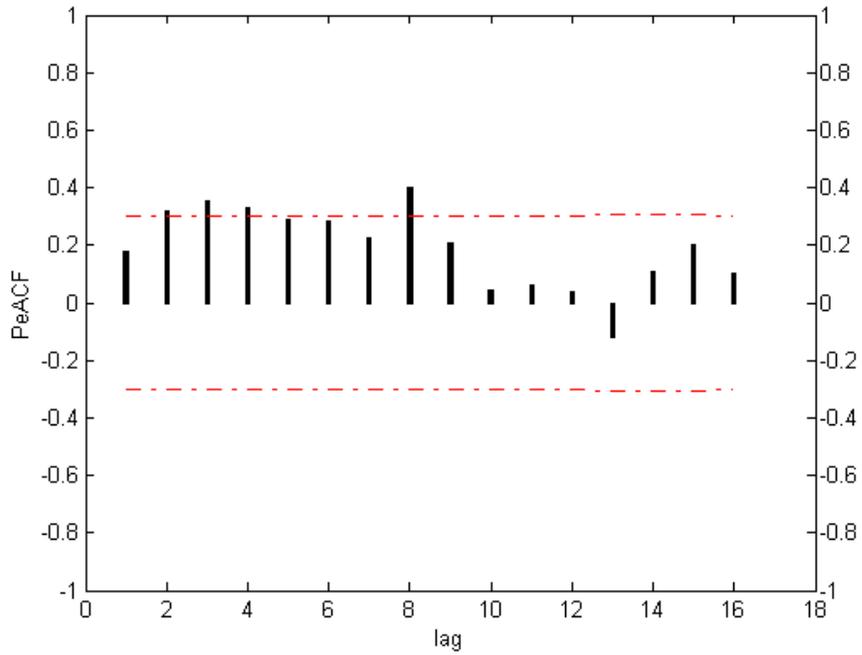


Figure F2.12 (a) PeACF graph of Nilüfer season 12

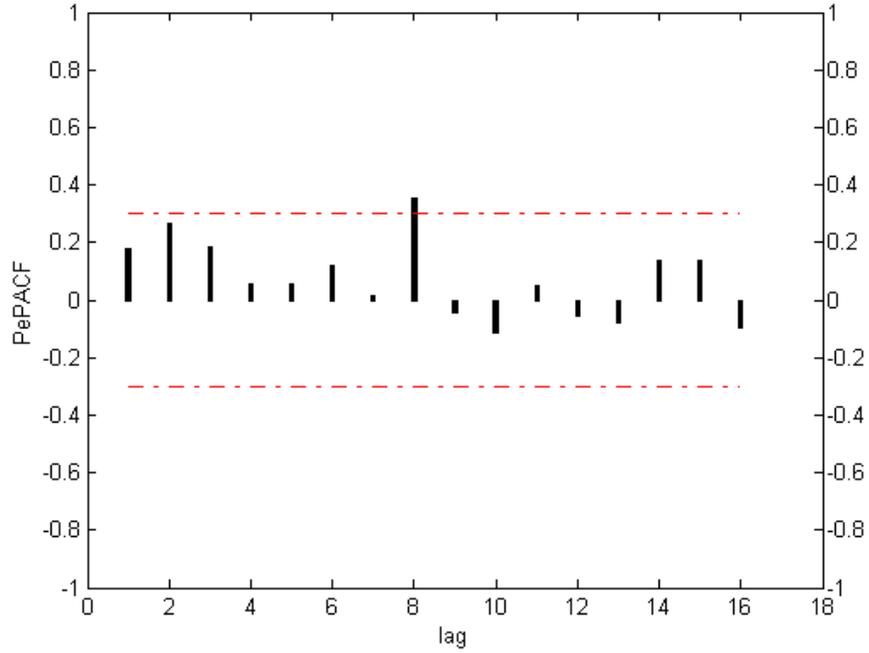


Figure F2.12 (b) PePACF graph of Nilüfer season 12