SCALAR MESON EFFECTS IN RADIATIVE DECAYS OF VECTOR MESONS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
THE MIDDLE EAST TECHNICAL UNIVERSITY

BY

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IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE

OF

DOCTOR OF PHILOSOPHY

IN

THE DEPARTMENT OF PHYSICS

OCTOBER 2003

Approval of the Graduate School of Natural and Applied Sciences.

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ABSTRACT

SCALAR MESON EFFECTS IN RADIATIVE DECAYS OF VECTOR MESONS

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October 2003, 98 pages.

The role of scalar mesons in radiative vector meson decays is investigated. The effects of scalar-isoscalar $f_0(980)$ and scalar-isovector $a_0(980)$ mesons are studied in the mechanism of the radiative $\phi \rightarrow \pi^+\pi^-\gamma$ and $\phi \rightarrow \pi^0\eta\gamma$ decays, respectively. A phenomenological approach is used to study the radiative $\phi \rightarrow \pi^+\pi^-\gamma$ decay by considering the contributions of σ-meson, ρ-meson and f_0-meson. The interference effects between different contributions are analyzed and the branching ratio for this decay is calculated. The radiative $\phi \rightarrow \pi^0\eta\gamma$ decay is studied within the framework of a phenomenological approach in which the contributions of ρ-meson, chiral loop and a_0-meson are considered. The interference effects between different contributions are examined and the coupling constants $g_{\phi a_0 \gamma}$ and $g_{a_0 K^+ K^-}$ are estimated using the experimental branching ratio for the $\phi \rightarrow \pi^0\eta\gamma$ decay. Furthermore, the radiative $\rho^0 \rightarrow \pi^+\pi^-\gamma$ and $\rho^0 \rightarrow \pi^0\pi^0\gamma$ decays are studied to investigate the role of scalar-isoscalar σ-meson. The branching ratios of the $\rho^0 \rightarrow \pi^+\pi^-\gamma$ and $\rho^0 \rightarrow \pi^0\pi^0\gamma$ decays are calculated using a phenomenological approach by adding to the amplitude calculated within the framework of chiral perturbation theory and vector meson dominance the amplitude of σ-meson intermediate state. In all the decays studied the scalar meson intermediate states make important contributions to the overall amplitude.

Keywords: Vector Meson Decay, Scalar-Isoscalar Meson, Scalar-Isovector Meson, Vector Meson Dominance, Chiral Perturbation Theory, Coupling Constant, Radiative Decay

ÖZ

VEKTÖR MEZONLARIN IŞINSAL BOZUNMALARINDA SKALER MEZON ETKİLERİ

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Ekim 2003, 98 sayfa.

Işınsal vektör mezon bozunmalarında skaler mezonların rolü araştırıldı. Skaler-izoskaler $f_0(980)$ ve skaler-izovektör $a_0(980)$ mezonlarının etkileri sırasıyla $\phi \rightarrow \pi^+\pi^-\gamma$ ve $\phi \rightarrow \pi^0\eta\gamma$ ışınsal bozunma mekanizmalarında çalışıldı. Işınsal $\phi \rightarrow \pi^+\pi^-\gamma$ bozunmasının incelenmesinde σ-mezon, ρ-mezon ve f_0-mezon katkılarının düşünüldüğü fenomenolojik bir yaklaşım kullanıldı. Farklı katkılar arasındaki girişim etkileri analiz edildi ve bu bozunma için dallanma oranı hesaplandı. ρ-mezon, chiral halkası ve a_0-mezon katkıları göz önüne alınarak fenomenolojik bir yaklaşım çerçevesinde ışınsal $\phi \rightarrow \pi^0\eta\gamma$ bozunması çalışıldı. Farklı katkılar arasındaki girişim etkileri incelendi ve $\phi \rightarrow \pi^0\eta\gamma$ bozunmasının deneysel dallanma oranı kullanılarak $g_{\phi a_0 \gamma}$ ve $g_{a_0 K^+ K^-}$ çiftlenim sabitleri hesap edildi. Ayrıca, skaler-izoskaler σ-mezonunun rolünü araştırmak için ışınsal $\rho^0 \rightarrow \pi^+\pi^-\gamma$ ve $\rho^0 \rightarrow \pi^0\pi^0\gamma$ bozunmaları çalışıldı. Chiral tedirgeme kuramı ve vektör mezon dominans çerçevesinde hesap edilen genliklere σ-mezon ara durum genliği eklenerek fenomenolojik bir yaklaşımla $\rho^0 \rightarrow \pi^+\pi^-\gamma$ ve $\rho^0 \rightarrow \pi^0\pi^0\gamma$ bozunmalarının dallanma oranı hesaplandı. Bütün bu bozunmalarda, skaler mezon ara durumlarının toplam genliğe önemli katkılar sağladığı bulundu.

Anahtar Sözcükler: Vektör Mezon Bozunması, Skaler-İzoskaler Mezon, Skaler-İzovektör Mezon, Vektör Mezon Dominans, Chiral Tedirgeme Kuramı, Çiftlenim Sabiti, Işınsal Bozunma

To My Mom and Dad...

ACKNOWLEDGMENTS

I am deeply indebted to my supervisor Prof. Dr. Ahmet Gökalp for his guidance, encouragement, invaluable comments, patience and friendly attitude during this work. He gave me valuable insights and moral support when I was lost. Thank you sincerely.

I would like to express my sincere gratitude to Prof. Dr. Osman Yılmaz for his valuable suggestions, friendly attitude and guidance especially about the problems related to the Pascal program.

There are no words to describe the appreciation and gratitude I feel for my parents. Their wonderful example of dedication and commitment has provided me with a lifetime of inspiration. I thank them for their optimism, beautiful spirits and belief in me. I also wish to express my appreciation to my sister Naime. She patiently and lovingly encouraged me to do my best.

I am also grateful to my best friend Ayşe who contributed greatly throughout this study. And Levent, special thanks are for you, I could not have completed this thesis without you.

Thank you all very much indeed.

TABLE OF CONTENTS

ABSTRACT iii

ÖZ v

DEDICATON vii

ACKNOWLEDGMENTS viii

TABLE OF CONTENTS ix

LIST OF TABLES xi

LIST OF FIGURES xii

CHAPTER

1 INTRODUCTION 1

2 FORMALISM 14

2.1 Scalar $f_0(980)$ meson in $\phi \to \pi^+\pi^-\gamma$ decay 14

2.2 Scalar $a_0(980)$ meson in $\phi \to \pi^0\eta\gamma$ decay 21

2.3 Scalar σ meson effects in radiative ρ^0 meson decays . . . 28

3 RESULTS AND DISCUSSION 35

3.1 Radiative $\phi \to \pi^+\pi^-\gamma$ decay 36

3.2 Radiative $\phi \to \pi^0\eta\gamma$ decay and the coupling constants $g_{\phi a_0 \gamma}$, $g_{a_0 K^+ K^-}$ 39

3.3 Radiative $\rho^0 \to \pi^+\pi^-\gamma$ and $\rho^0 \to \pi^0\pi^0\gamma$ decays 44

4 CONCLUSIONS 50

REFERENCES 54

APPENDICES 57

A TWO BODY DECAY RATES 57

B THREE BODY DECAY AND THE BOUNDARY OF DALITZ PLOT 65

C INVARIANT AMPLITUDE OF THE RADIATIVE $\phi \to \pi^+\pi^-\gamma$ DECAY 68

D INVARIANT AMPLITUDE OF THE RADIATIVE $\phi \to \pi^0\eta\gamma$ DECAY 76

D.1 Invariant Amplitude for the Decay $\phi \to \pi^0\eta\gamma$ in Model I 76

D.2 Invariant Amplitude for the Decay $\phi \to \pi^0\eta\gamma$ in Model II 80

E INVARIANT AMPLITUDE OF THE RADIATIVE $\rho^0 \to \pi^+\pi^-\gamma$ AND $\rho^0 \to \pi^0\pi^0\gamma$ DECAYS 85

E.1 Invariant Amplitude for the Decay $\rho^0 \to \pi^+\pi^-\gamma$ 85

E.2 Invariant Amplitude for the Decay $\rho^0 \to \pi^0\pi^0\gamma$ 90

VITA 98

LIST OF TABLES

TABLE

A.1 The experimental decay widths of various two body decays and the calculated coupling constants. 64

LIST OF FIGURES

2.1 Feynman diagrams for the decay $\phi \to f_0\gamma$ 15

2.2 Feynman diagrams for the decay $\phi \to \pi^+\pi^-\gamma$ 17

2.3 Feynman diagrams for the decay $\phi \to \pi^0\eta\gamma$ in model I 22

2.4 Feynman diagrams for the decay $\phi \to \pi^0\eta\gamma$ in model II 25

2.5 Feynman diagrams for the decay $\rho^0 \to \pi^+\pi^-\gamma$ 29

2.6 Feynman diagrams for the decay $\rho^0 \to \pi^0\pi^0\gamma$ 30

3.1 The $\pi\pi$ invariant mass spectrum for the decay $\phi \to \pi^+\pi^-\gamma$. The contributions of different terms are indicated. 37

3.2 The $\pi^0\eta$ invariant mass spectrum for the decay $\phi \to \pi^0\eta\gamma$ for $g_{\phi a_0\gamma} = 0.24$ in model I. The contributions of different terms are indicated. 40

3.3 The $\pi^0\eta$ invariant mass spectrum for the decay $\phi \to \pi^0\eta\gamma$ for $g_{a_0K^+K^-} = -1.5$ in model II. The contributions of different terms are indicated. 42

3.4 The $\pi^0\eta$ invariant mass spectrum for the decay $\phi \to \pi^0\eta\gamma$ for $g_{a_0K^+K^-} = 3.0$ in model II. The contributions of different terms are indicated. 43

3.5 The photon spectra for the branching ratio of $\rho^0 \to \pi^+\pi^-\gamma$ decay. The contributions of different terms are indicated. The experimental data taken from Ref. [25] are normalized to our results. . 45

3.6 The photon spectra for the branching ratio of $\rho^0 \to \pi^0\pi^0\gamma$ decay. The contributions of different terms are indicated. 47

CHAPTER 1

INTRODUCTION

Radiative decays of vector mesons offer the possibility of investigating new physics features about the interesting mechanism involved in these decays. One particular mechanism involves the exchange of scalar mesons. The scalar mesons, isoscalar σ and $f_0(980)$ and isovector $a_0(980)$, with vacuum quantum numbers $J^{PC} = 0^{++}$ are known to be crucial for a full understanding of the low energy QCD phenomenology and the symmetry breaking mechanisms in QCD. The existence of the σ-meson as a broad $\pi\pi$ resonance has been the subject of a long standing controversy although the $f_0(980)$ and the $a_0(980)$ mesonic states are well established experimentally. Recently, on the other hand, new theoretical and experimental studies find a σ-pole position near $(500 - i250)$ MeV [1, 2]. An experimental evidence for a light and broad scalar σ resonance, of mass $M_\sigma = 478$ MeV and width $\Gamma_\sigma = 324$ MeV, was found by the Fermilab E791 collaboration in $D^+ \to \pi^-\pi^+\pi^+$ decay [3]. From the other side there is some debate about the nature and the quark substructure of these scalar mesons. Several proposals have been made about the nature of these states: $q\bar{q}$ states

[4], $\pi\bar{\pi}$ in case of σ [5] and $K\bar{K}$ molecules in case of f_0 and a_0 [6] or multi-quark $q^2\bar{q}^2$ states [7, 8]. The scalar mesons have been a persistent problem in hadron spectroscopy. In addition to the identification of their nature, the role of scalar mesons in hadronic processes is of extreme importance and the study of radiative decays of vector mesons may provide insights about their role. The radiative decay processes of the type $V \rightarrow PP\gamma$ where V and P stand for the lowest multiplets of vector (V) and pseudoscalar (P) mesons have been studied extensively. The studies of such decays may serve as tests for the theoretical ideas about the nature of the intermediate states and the interesting mechanisms of these decays and they may thus provide information about the complicated dynamics of meson physics in the low energy region.

In particular, radiative ϕ meson decays, $\phi \rightarrow \pi\pi\gamma$ and $\phi \rightarrow \pi^0\eta\gamma$, can play a crucial role in the clarification of the structure and properties of scalar $f_0(980)$ and $a_0(980)$ mesons since these decays primarily proceed through processes involving scalar resonances such as $\phi \rightarrow f_0(980)\gamma$ and $\phi \rightarrow a_0(980)\gamma$, with the subsequent decays into $\pi\pi\gamma$ and $\pi^0\eta\gamma$ [9, 10]. Achasov and Ivanchenko [9] showed that if the $f_0(980)$ and $a_0(980)$ resonances are four-quark ($q^2\bar{q}^2$) states the processes $\phi \rightarrow f_0(980)\gamma$ and $\phi \rightarrow a_0(980)\gamma$ are dominant and enhance the decays $\phi \rightarrow \pi\pi\gamma$ and $\phi \rightarrow \pi^0\eta\gamma$ by at least an order of magnitude over the results predicted by the Wess-Zumino terms. Then Close et al. [10] noted that the study of the scalar states in $\phi \rightarrow S\gamma$, where $S = f_0$ *or* a_0, may offer unique insights into the nature of the scalar mesons. They have shown that although the transition rates $\Gamma(\phi \to f_0\gamma)$ and $\Gamma(\phi \to a_0\gamma)$ depend on the unknown dynamics, the ratio of the decay rates $\Gamma(\phi \to a_0\gamma)/\Gamma(\phi \to f_0\gamma)$ provides an experimental test which distinguishes between alternative explanations of their structure. On the experimental side, the Novosibirsk CMD-2 [11, 12] and SND [13] collaborations give the following branching ratios for $\phi \to \pi^+\pi^-\gamma$ and $\phi \to \pi^0\eta\gamma$ decays: $BR(\phi \to \pi^+\pi^-\gamma) = (0.41 \pm 0.12 \pm 0.04) \times 10^{-4}$ [11], $BR(\phi \to \pi^0\eta\gamma) = (0.90 \pm 0.24 \pm 0.10) \times 10^{-4}$ [12], $BR(\phi \to \pi^0\eta\gamma) = (0.88 \pm 0.14 \pm 0.09) \times 10^{-4}$ [13], where the first error is statistical and the second one is systematic. Theoretically, the role of $f_0(980)$-meson in the radiative decay processes $\phi \to \pi\pi\gamma$ was also investigated by Achasov et al. [14]. They calculated the branching ratio for this decay by considering only $f_0(980)$-meson contribution. In their study, they used two different models of $f_0(980)$-meson: the four-quark model and $K\bar{K}$ molecular model. In the four-quark model they obtained the value for the branching ratio as $BR(\phi \to f_0\gamma \to \pi\pi\gamma) = 2.3 \times 10^{-4}$ and in case of the $K\bar{K}$ molecular model, the branching ratio was $BR(\phi \to f_0\gamma \to \pi\pi\gamma) = 1.7 \times 10^{-5}$. Marco et al., later considered the radiative ϕ meson decays [15] as well as other radiative vector meson decays within the framework of chiral unitary theory developed earlier by Oller [16]. Using a chiral unitary approach, they included the final state interactions of the two pions by summing the pion-loops through Bethe-Salpeter equation and they obtained the result $BR(\phi \to \pi^+\pi^-\gamma) = 1.6 \times 10^{-4}$ for the branching ratio of the $\phi \to \pi^+\pi^-\gamma$ decay. In their calculation they emphasized that the branching ratio for $\phi \to \pi^+\pi^-\gamma$ decay is twice the one for

$\phi \to \pi^0\pi^0\gamma$ decay. Recently, the radiative $\phi \to \pi^0\pi^0\gamma$ decay, where the scalar $f_0(980)$-meson plays an important role was studied by Gökalp and Yılmaz [17] within the framework of a phenomenological approach in which the contributions of σ-meson, ρ-meson and f_0-meson are considered. They analyzed the interference effects between different contributions. By employing the experimental branching ratio for this decay, they calculated the coupling constant $g_{\phi\sigma\gamma}$. Their analysis showed that $f_0(980)$-meson amplitude makes a substantial contribution to the branching ratio of this decay.

On the other hand, Fajfer and Oakes [18] studied the radiative decay processes of the type $V^0 \to P^0P^0\gamma$ by a low energy effective Lagrangian with the gauged Wess-Zumino terms. Using such an effective Lagrangian they calculated the branching ratios for these decays in which scalar meson contributions were neglected and the branching ratio for the radiative $\phi \to \pi^0\eta\gamma$ decay was found as $BR(\phi \to \pi^0\eta\gamma) = 5.18 \times 10^{-5}$. The contributions of intermediate vector mesons to the decays $V^0 \to P^0P^0\gamma$ were later considered by Bramon et al. [19] using standard Lagrangians obeying SU(3) symmetry, and they obtained the result $BR(\phi \to \pi^0\eta\gamma) = 5.4 \times 10^{-6}$ for the branching ratio of the $\phi \to \pi^0\eta\gamma$ decay. This result was not in agreement with the numerical prediction quoted in Ref. [18] even if the initial expressions for the Lagrangians were the same. Later, Bramon et al. [20] studied these decays within the framework of chiral effective Lagrangians enlarged to include on-shell vector mesons using chiral perturbation theory, and they calculated the branching ratio for $\phi \to \pi^0\eta\gamma$ decay as well as other radiative vector meson decays of the type $V^0 \to P^0P^0\gamma$ at the one-loop level. They showed that the one-loop contributions are finite and to this order no counterterms are required. In this approach, the decay $\phi \to \pi^0\eta\gamma$ proceeds through the charged kaon-loops and they obtained the contribution of charged kaon-loops to this decay rate as $\Gamma(\phi \to \pi^0\eta\gamma)_K = 131\ eV$. They noted that the intermediate vector meson state (VMD) contribution is much smaller than the contribution of charged kaon-loops due to the OZI rule. Adding VMD contribution to the contribution of charged kaon-loops they obtained for the decay rate the value $\Gamma(\phi \to \pi^0\eta\gamma) = 157.5\ eV$ and their result for the branching ratio was $BR(\phi \to \pi^0\eta\gamma) = 36 \times 10^{-6}$. Moreover they stated that OZI allowed kaon-loops are seen to dominate both photonic spectrum and decay rate. As we mentioned before, ϕ meson decays into $\pi^+\pi^-\gamma$, $\pi^0\pi^0\gamma$ and $\pi^0\eta\gamma$ was also investigated by Marco et al. [15] using a chiral unitary approach. The branching ratio, they obtained for the case $\phi \to \pi^0\eta\gamma$, was $BR(\phi \to \pi^0\eta\gamma) = 0.87 \times 10^{-4}$ in agreement with the SND data. They also noted that the branching ratio is dominated by the a_0 contribution. Bramon et al., later discussed the radiative $\phi \to \pi^0\eta\gamma$ decay emphasizing the effects of the $a_0(980)$ scalar resonance [21]. In their previous approaches [19, 20] the scalar resonance effects were not contemplated. They noted that the observed invariant mass distribution shows a significant enhancement at large $\pi^0\eta$ invariant mass according to Refs. [12, 13] so this could be interpreted as a manifestation of a sizeable contribution of $a_0(980)$ intermediate state. In order to take explicitly into account scalar resonances and their pole effects, they proposed to use the linear sigma model ($L\sigma M$) because of the fact that chiral perturbation theory is not reliable at energies of a typical vector meson mass and scalar resonance poles are not explicitly included. Moreover they stated that in the case of $\phi \to \pi^0\eta\gamma$ decay, the dominant contributions arise exclusively from loops of charged kaons since contributions from charged pion loops are highly suppressed due to the isospin violation and OZI rule forbidden $\phi\pi\pi$ coupling. Indeed they noted that the $L\sigma M$ amplitude is obtained by adding the contribution of $a_0(980)$-meson, generated through a loop of charged kaons, to the one coming from chiral loop amplitudes. They predicted a value of branching ratio to be in the range $BR(\phi \to \pi^0\eta\gamma) = (0.75 - 0.95) \times 10^{-4}$, compatible with the experimental data. They also showed that $a_0(980)$ scalar resonance dominates the high values of the $\pi^0\eta$ invariant mass spectrum. Recently, the radiative $\phi \to \pi^0\eta\gamma$ decay in addition to the radiative $\phi \to \pi^0\pi^0\gamma$ decay has been considered by Achasov and Gubin [22] taking into account the contributions of ρ-meson and a_0-meson. They analyzed the interference effects between $\phi \to a_0\gamma \to \pi^0\eta\gamma$ and $\phi \to \rho\pi^0 \to \pi^0\eta\gamma$ processes and obtained the branching ratio for this decay as $BR(\phi \to \pi^0\eta\gamma) = (0.79 \pm 0.2) \times 10^{-4}$. Their analysis showed that a_0-meson amplitude makes a substantial contribution to the branching ratio of this decay.

The radiative decays of ρ^0 meson into $\pi^+\pi^-\gamma$ and $\pi^0\pi^0\gamma$ have been studied extensively up to now. The Novosibirsk SND collaboration have reported very recently the branching ratio for the $\rho^0 \to \pi^0\pi^0\gamma$ decay the value $BR(\rho^0 \to$

$\pi^0\pi^0\gamma) = (4.1^{+1.0}_{-0.9} \pm 0.3) \times 10^{-5}$ [23], thus their preliminary study [24] including the measurement of $BR(\rho^0 \to \pi^0\pi^0\gamma) = (4.8^{+3.4}_{-1.8} \pm 0.2) \times 10^{-5}$, has been improved. On the other hand, the branching ratio for the $\rho^0 \to \pi^+\pi^-\gamma$ decay, obtained from Novosibirsk group, was $BR(\rho^0 \to \pi^+\pi^-\gamma) = (9.9 \pm 1.6) \times 10^{-3}$ [25, 26]. Moreover it was concluded that the $\rho^0 \to \pi^+\pi^-\gamma$ decay is dominated by the pion bremsstrahlung mechanism and the contribution of the structural radiation, proceeding trough the intermediate scalar resonance, to the branching ratio of this decay is one order of magnitude lower than the total branching ratio [25]. The theoretical study of ρ meson decays was begun by Singer [27] who considered only the bremsstrahlung contribution for the radiative $\rho^0 \to \pi^+\pi^-\gamma$ decay, and he suggested that the $\rho^0 \to \pi^0\pi^0\gamma$ decay proceeds via $\omega\pi^0$ intermediate state. Later, Renard [28] studied radiative decays $V \to PP'\gamma$ in a gauge invariant way with current algebra, hard-pion and Ward identities techniques. He concluded that the main contribution to the $\rho^0 \to \pi^+\pi^-\gamma$ decay comes from the pion bremsstrahlung term and the σ contribution modifies the shape of the photon spectrum for high momenta differently depending on the mass of the σ-meson. Moreover, he observed that for the radiative $\rho^0 \to \pi^0\pi^0\gamma$ decay, the intermediate σ and ω meson contributions are dominant and the ω peaks the photon spectrum toward the high momenta. The radiative $\rho^0 \to \pi^0\pi^0\gamma$ decay was also considered by Fajfer and Oakes [18] using a low energy effective Lagrangian approach with gauged Wess-Zumino terms and they calculated the branching ratio for this decay as $BR(\rho^0 \to \pi^0\pi^0\gamma) = 2.89 \times 10^{-5}$. The vector meson dominance (VMD) calculation of Bramon et al. [19] with $\omega\pi$ intermediate state resulted in the branching ratio $BR(\rho^0 \to \pi^0\pi^0\gamma) = 1.1 \times 10^{-5}$ for $\rho^0 \to \pi^0\pi^0\gamma$ decay. This result was not in agreement with the one obtained by Fajfer and Oakes although the initial expressions for the Lagrangians were the same. Bramon et al. [20] later considered the radiative vector meson decays within the framework of chiral effective Lagrangians and using chiral perturbation theory they calculated the branching ratios for various decays at the one-loop level, including both $\pi\pi$ and $K\bar{K}$ intermediate loops. In this approach, the decay $\rho^0 \to \pi^0\pi^0\gamma$ proceeds mainly through charged pion-loops, contribution of kaon-loops being three orders of magnitude smaller, resulting in the decay rate $\Gamma(\rho^0 \to \pi^0\pi^0\gamma)_\chi = 1.42\ keV$ which is of the same order of magnitude as the VMD contribution. They noted that the interference between pion-loop contribution and the VMD amplitude is constructive and the total branching ratio is $BR(\rho^0 \to \pi^0\pi^0\gamma)_{VMD+\chi} = 2.6 \times 10^{-5}$. Furthermore, ρ meson decays were also considered by Marco et al. [15] in the framework of unitarized chiral perturbation theory. They noted that the energies of two pion system are too big to be treated with standard chiral perturbation theory. They used the techniques of chiral unitary theory to include the final state interactions of two pions by summing the pion loops through the Bethe-Salpeter equation. The branching ratios for $\rho^0 \to \pi^+\pi^-\gamma$ and $\rho^0 \to \pi^0\pi^0\gamma$, they obtained, were $BR(\rho^0 \to \pi^+\pi^-\gamma) = 1.18 \times 10^{-2}$ for $E_\gamma > 50\ MeV$ and $BR(\rho^0 \to \pi^0\pi^0\gamma) = 1.4 \times 10^{-5}$ respectively. They showed that the branching ratio for the radiative $\rho^0 \to \pi^+\pi^-\gamma$ decay agrees well with the experimental number $BR(\rho^0 \to \pi^+\pi^-\gamma) = (0.99 \pm 0.04 \pm 0.15) \times 10^{-2}$ for $E_\gamma > 50\ MeV$ [25]. Indeed, they concluded that the obtained result for the branching ratio of the $\rho^0 \to \pi^0\pi^0\gamma$ decay could be interpreted as the result of the mechanism $\rho^0 \to (\sigma)\gamma \to (\pi^0\pi^0)\gamma$ since $\pi^0\pi^0$ interaction is dominated by the σ-pole in the relevant energy regime of this decay. The role of the σ meson in radiative decays $\rho^0 \to \pi^+\pi^-\gamma$ and $\rho^0 \to \pi^0\pi^0\gamma$ was investigated in detail by Gökalp and Yılmaz [29, 30]. They calculated the branching ratio for the radiative decay $\rho^0 \to \pi^+\pi^-\gamma$ by considering the bremsstrahlung amplitude and σ-pole amplitude [29]. Using the experimental value for this branching ratio they estimated the coupling constant $g_{\rho\sigma\gamma}$. Their analysis showed that, the contribution of the σ-term becomes increasingly important in the region of high photon energies dominating the contribution of the bremsstrahlung amplitude. Then, using this coupling constant, they calculated the branching ratio for the $\rho^0 \to \pi^0\pi^0\gamma$ decay [30]. In this approach, the contributions of σ-meson, ω-meson intermediate states and of the pion-loops are considered. They concluded that the σ-meson amplitude makes a substantial contribution to the branching ratio and this contribution strongly depends on the value of the coupling constant $g_{\rho\sigma\gamma}$. Moreover the branching ratio $BR(\rho^0 \to \pi^0\pi^0\gamma)$ obtained this way for $M_\sigma = 478\ MeV$ and $\Gamma_\sigma = 324\ MeV$ was more than an order of magnitude larger than the experimental value. This unrealistic value was the result of the constant $\rho \to \sigma\gamma$ amplitude used and consequently the large coupling constant $g_{\rho\sigma\gamma}$ extracted from the experimental value of the branching ratio of the $\rho^0 \to \pi^+\pi^-\gamma$ decay. Furthermore, they noted that, the measurement of the $\rho^0 \to \pi^0\pi^0\gamma$ decay rate may help to clarify the mechanism of the $\rho^0 \to \pi^0\pi^0\gamma$ decay and the role of the σ-meson in this process. Additionally, in a very recent paper of Palomar et al. [31] the branching ratios of the radiative ρ^0 and ω meson decays into $\pi^0\pi^0$ and $\pi^0\eta$ are evaluated using the sequential vector decay mechanisms in addition to chiral loops and $\rho - \omega$-mixing. They observed that for the $\rho^0 \to \pi^0\pi^0\gamma$ decay $\rho - \omega$ mixing is negligible but the branching ratio coming from the sum of the sequential and loop mechanisms is almost three times larger than either mechanism alone. The obtained value of this branching ratio was $BR(\rho^0 \to \pi^0\pi^0\gamma) = 4.2 \times 10^{-5}$ [31], which agrees well with the experimental value [23]. Finally, a consistent description of $\sigma(500)$ meson effects in $\rho^0 \to \pi^0\pi^0\gamma$ and $\rho^0 \to \pi^+\pi^-\gamma$ decays have been proposed by Bramon and Escribano [32] in terms of reasonably simple amplitudes which reproduce the expected chiral loop behaviour for large M_σ values. They have shown that for the neutral case, in addition to the well known ω exchange, there is an important contribution from the $\sigma(500)$ meson and for the charged case, where the dominant contribution comes from bremsstrahlung, the effects of the $\sigma(500)$ meson are relevant only at high values of the photon energy. In order to include the σ-meson effects, they have multiplied the four pseudoscalar amplitudes $\mathcal{A}(\pi^+\pi^- \to \pi^0\pi^0)_\chi$ and $\mathcal{A}(\pi^+\pi^- \to \pi^+\pi^-)_\chi$ with an additional factor $F_\sigma(s)$, where s denotes the invariant mass of the final dipion system and they have considered two possible values for the free parameter k incorporated in the additional factor $F_\sigma(s)$. In their study the first value for the parameter $k = 1$ corresponds to $L\sigma M$ and the second one $k \simeq -2.5$ represents the phenomenological σ amplitude. They have calculated the following values: $BR(\rho^0 \to \pi^0\pi^0\gamma)_{\chi+\omega} = 2.95 \times 10^{-5}$ from chiral loops and VMD amplitudes, $BR(\rho^0 \to \pi^0\pi^0\gamma)_{L\sigma M+\omega} = 4.21 \times 10^{-5}$ from $L\sigma M$ and VMD amplitudes and $BR(\rho^0 \to \pi^0\pi^0\gamma)_{\sigma-phen+\omega} = 3.42 \times 10^{-5}$ from phenomenological σ-meson and VMD amplitudes. In the same way for the branching ratios for different contributing reactions to the radiative decay $\rho^0 \to \pi^+\pi^-\gamma$ the values $BR(\rho^0 \to \pi^+\pi^-\gamma)_{\chi+backg} = 1.171 \times 10^{-2}$, $BR(\rho^0 \to \pi^+\pi^-\gamma)_{L\sigma M+backg} = 1.138 \times 10^{-2}$ and $BR(\rho^0 \to \pi^+\pi^-\gamma)_{\sigma-phen+backg} = 1.136 \times 10^{-2}$ are obtained where *backg* denotes the contributions from the background amplitude [32]. Also they have noted that their results are quite compatible with the experimental values. Furthermore, recently Escribano studied the scalar meson exchange in $V \to \pi^0\pi^0\gamma$ decays [33]. He discussed the scalar contributions to the $\phi \to \pi^0\pi^0\gamma$, $\phi \to \pi^0\eta\gamma$ and $\rho^0 \to \pi^0\pi^0\gamma$ decays in the framework of the $L\sigma M$. He obtained the branching ratios for the decays $\phi \to \pi^0\pi^0\gamma$, $\phi \to \pi^0\eta\gamma$ and $\rho^0 \to \pi^0\pi^0\gamma$ as $BR(\phi \to \pi^0\pi^0\gamma) = 1.16 \times 10^{-4}$, $BR(\phi \to \pi^0\eta\gamma) = 8.3 \times 10^{-5}$ and $BR(\rho^0 \to \pi^0\pi^0\gamma) = 3.8 \times 10^{-5}$ and noted that, the branching ratios for $\phi \to \pi^0\pi^0\gamma$, $\phi \to \pi^0\eta\gamma$ and $\rho^0 \to \pi^0\pi^0\gamma$ decays are dominated by $f_0(980)$, $a_0(980)$ and $\sigma(500)$ exchanges respectively.

In this thesis, we study the radiative vector meson decays $\phi \to \pi^+\pi^-\gamma$, $\phi \to \pi^0\eta\gamma$, $\rho^0 \to \pi^+\pi^-\gamma$ and $\rho^0 \to \pi^0\pi^0\gamma$ to investigate the role of the low mass scalar mesons and to extract the relevant information on the properties of these scalar mesons. We follow a phenomenological approach. In this approach we use the effective Lagrangians to describe the vertices for the considered processes. The invariant amplitudes resulting from these effective Lagrangians do not include any form factor as in point-like effective field theory calculations. Also, we extract the relevant coupling constants from the experimentally measured quantities. Theoretically, the radiative $\phi \rightarrow \pi^+\pi^-\gamma$ decay has not been studied extensively up to now. One of the rare studies of this decay was by Marco et al. [15] who neglected the contributions coming from intermediate vector meson states. Therefore, this decay should be reconsidered and the VMD amplitude should be added to the f_0-meson and σ-meson amplitudes. Firstly, we attempt to explain the effect of the scalar f_0-meson in the radiative $\phi \rightarrow \pi^+\pi^-\gamma$ decay. In our phenomenological approach for this decay we consider the contributions of σ-meson, ρ-meson and f_0-meson. We analyze the interference effects between different contributions and calculate the branching ratio of this decay. Then we study the role of the scalar a_0-meson in the mechanism of the radiative $\phi \rightarrow \pi^0\eta\gamma$ decay employing a phenomenological framework in which the contribution of the a_0-meson in addition to the contributions of the ρ-meson and chiral loops is considered. In our calculation we try to assess the roles of different processes and the contributions to the decay rate coming from their amplitude in the mechanism of this decay and utilizing the experimental branching ratio for this decay we estimate the coupling constants $g_{\phi a_0 \gamma}$ and $g_{a_0 K^+ K^-}$. We try to estimate the branching ratio $BR(\phi \to a_0(980)\gamma)$ of the decay $\phi \to a_0(980)\gamma$ from the experimental data of the radiative $\phi \to \pi^0\eta\gamma$ decay. It is also useful to state that in earlier calculations, the obtained values for the branching ratio of the radiative $\rho^0 \to \pi^0\pi^0\gamma$ decay are smaller than the latest experimental result so the mechanism of the decay $\rho^0 \to \pi^0\pi^0\gamma$ should be reexamined and additional contributions should be investigated. Therefore, we study the contribution of the σ-meson intermediate state amplitude to $\rho^0 \to \pi^+\pi^-\gamma$ and $\rho^0 \to \pi^0\pi^0\gamma$ decays and calculate their branching ratios using a phenomenological approach by adding to the amplitude, calculated within the framework of chiral perturbation theory and vector meson dominance, the amplitude of σ-meson intermediate states. In our calculations of the branching ratios, the coupling constants that we use are determined from the relevant experimental quantities.

CHAPTER 2

FORMALISM

In this chapter, we introduce the theoretical framework that we used to investigate the effects of scalar mesons in radiative vector meson decays. First of all, we describe the theoretical details of our calculation about the role of the scalar-isoscalar $f_0(980)$ meson in the mechanism of the radiative $\phi \to \pi^+\pi^-\gamma$ decay. Next we deal with the effect of scalar-isovector $a_0(980)$ meson in $\phi \to \pi^0\eta\gamma$ decay for which two different approaches are used. We then discuss the radiative $\rho^0 \to \pi^+\pi^-\gamma$ and $\rho^0 \to \pi^0\pi^0\gamma$ decays to include the effect of the scalar-isoscalar σ-meson.

2.1 Scalar $f_0(980)$ meson in $\phi \to \pi^+\pi^-\gamma$ decay

We study the radiative decay $\phi \to \pi^+\pi^-\gamma$ within the framework of a phenomenological approach in which the contributions of σ-meson, ρ-meson and f_0-meson are considered and we do not make any assumption about the structure of the f_0 meson. We note that ϕ and f_0 mesons both couple strongly to the K^+K^- system. In our phenomenological approach we describe the ϕKK-vertex

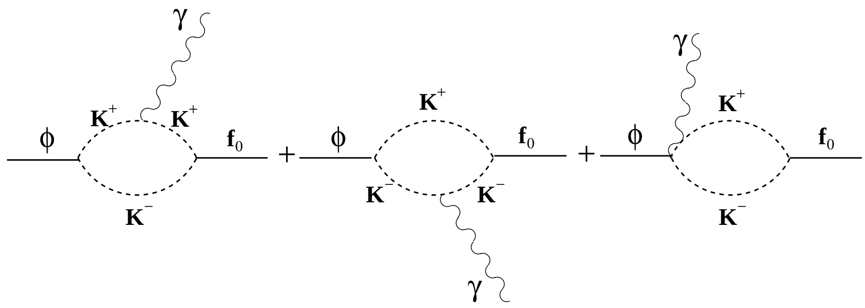

Figure 2.1: Feynman diagrams for the decay $\phi \to f_0\gamma$

by the effective Lagrangian

$$\mathcal{L}^{eff.}_{\phi KK} = -ig_{\phi KK}\phi^{\mu}\left(K^{+}\partial_{\mu}K^{-} - K^{-}\partial_{\mu}K^{+}\right) \quad , \tag{2.1}$$

which results from the standard chiral Lagrangians in the lowest order of the chiral perturbation theory [34] and for the f_0KK-vertex we use the phenomenological Lagrangian

$$\mathcal{L}^{eff.}_{f_0KK} = g_{f_0KK}M_{f_0}K^{+}K^{-}f_0 \quad . \tag{2.2}$$

The effective Lagrangians for the ϕKK- and f_0KK-vertices also serve to define the coupling constants $g_{\phi KK}$ and g_{f_0KK} respectively. The decay width for the $\phi \to K^{+}K^{-}$ decay, whose derivation we present in Appendix A, is obtained from the Lagrangian given in Eq. 2.1 and this decay width is

$$\Gamma(\phi \to K^{+}K^{-}) = \frac{g^2_{\phi KK}}{48\pi}M_{\phi}\left[1 - \left(\frac{2M_K}{M_{\phi}}\right)^2\right]^{3/2} \quad . \tag{2.3}$$

The coupling constant $g_{\phi KK}$ is determined as $g_{\phi KK} = (4.59 \pm 0.05)$ by using the experimental value for the branching ratio $BR(\phi \to K^{+}K^{-}) = (0.492 \pm 0.007)$

for the decay $\phi \rightarrow K^+K^-$ [26]. The amplitude of the radiative decay $\phi \rightarrow f_0\gamma$ is obtained from the diagrams shown in Fig. 2.1 where the last diagram assures gauge invariance [9, 35]. This amplitude is

$$\mathcal{M}\left(\phi \rightarrow f_0\gamma\right) = -\frac{1}{2\pi^2 M_K^2}\left(g_{f_0KK}M_{f_0}\right)\left(eg_{\phi KK}\right)I(a,b)\left[\epsilon\cdot u\ k\cdot p - \epsilon\cdot p\ k\cdot u\right] \quad (2.4)$$

where (u,p) and (ϵ,k) are the polarizations and four-momenta of the ϕ meson and the photon respectively, and also $a = M_\phi^2/M_K^2$, $b = M_{f_0}^2/M_K^2$. The $I(a,b)$ function has been calculated in different contexts [10, 16, 36] and is defined as

$$I(a,b) = \frac{1}{2(a-b)} - \frac{2}{(a-b)^2}\left[f(\frac{1}{b}) - f(\frac{1}{a})\right] + \frac{a}{(a-b)^2}\left[g(\frac{1}{b}) - g(\frac{1}{a})\right] \quad , \quad (2.5)$$

where

$$f(x) = \begin{cases} -\left[\arcsin(\frac{1}{2\sqrt{x}})\right]^2 \ , & x > \frac{1}{4} \\ \frac{1}{4}\left[\ln(\frac{\eta_+}{\eta_-}) - i\pi\right]^2 \ , & x < \frac{1}{4} \end{cases}$$

$$g(x) = \begin{cases} (4x-1)^{\frac{1}{2}}\arcsin(\frac{1}{2\sqrt{x}}) \ , & x > \frac{1}{4} \\ \frac{1}{2}(1-4x)^{\frac{1}{2}}\left[\ln(\frac{\eta_+}{\eta_-}) - i\pi\right] \ , & x < \frac{1}{4} \end{cases}$$

$$\eta_\pm = \frac{1}{2x}\left[1 \pm (1-4x)^{\frac{1}{2}}\right] \ . \quad (2.6)$$

The decay rate for the $\phi \rightarrow f_0\gamma$ decay that is obtained from the amplitude $\mathcal{M}(\phi \rightarrow f_0\gamma)$ is

$$\Gamma(\phi \rightarrow f_0\gamma) = \frac{\alpha}{6(2\pi)^4}\frac{M_\phi^2 - M_{f_0}^2}{M_\phi^3}g_{\phi KK}^2\left(g_{f_0KK}M_{f_0}\right)^2\left|(a-b)I(a,b)\right|^2 \quad . \quad (2.7)$$

The derivation of $\Gamma(\phi \rightarrow f_0\gamma)$ given in Eq. 2.7 is presented in Appendix A. Utilizing the experimental value for the branching ratio $BR(\phi \rightarrow f_0\gamma) = (3.4 \pm$

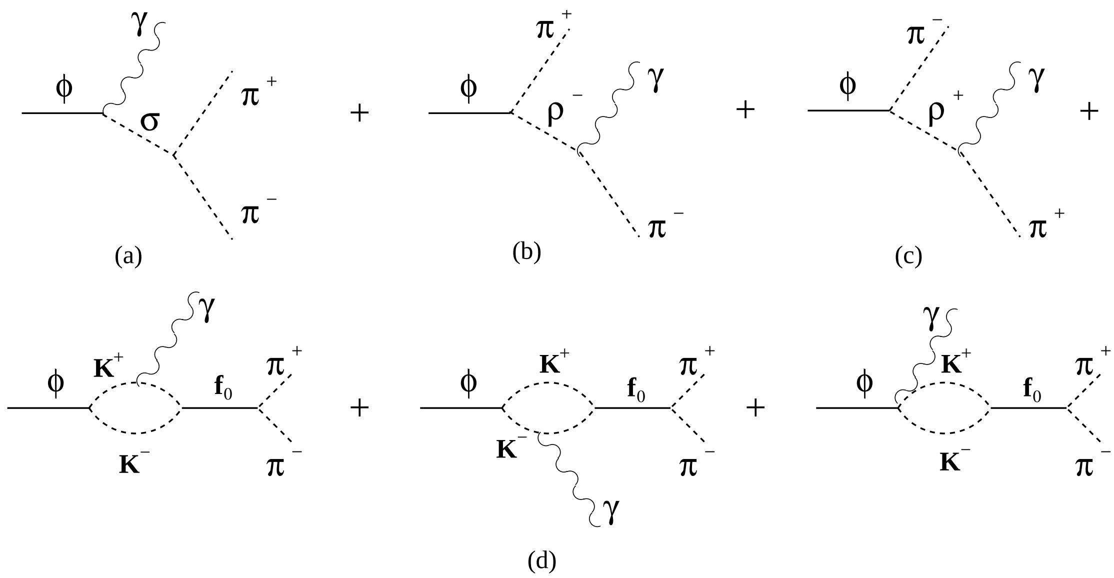

Figure 2.2: Feynman diagrams for the decay $\phi \to \pi^+\pi^-\gamma$

0.4) $\times\ 10^{-4}$ for the decay $\phi \to f_0\gamma$ [26], we determine the coupling constant g_{f_0KK} as $g_{f_0KK} = (4.13 \pm 1.42)$. In our calculation, we assume that the radiative decay $\phi \to \pi^+\pi^-\gamma$ proceeds through the reactions $\phi \to \sigma\gamma \to \pi^+\pi^-\gamma$, $\phi \to \rho^{\mp}\pi^{\pm} \to \pi^+\pi^-\gamma$ and $\phi \to f_0\gamma \to \pi^+\pi^-\gamma$. Therefore, our calculation is based on the Feynman diagrams shown in Fig. 2.2. For the $\phi\sigma\gamma$-vertex, we use the effective Lagrangian [37]

$$\mathcal{L}^{eff.}_{\phi\sigma\gamma} = \frac{e}{M_\phi} g_{\phi\sigma\gamma}[\partial^\alpha\phi^\beta\partial_\alpha A_\beta - \partial^\alpha\phi^\beta\partial_\beta A_\alpha]\sigma \quad , \tag{2.8}$$

which also defines the coupling constant $g_{\phi\sigma\gamma}$. The coupling constant $g_{\phi\sigma\gamma}$ is determined by Gökalp and Yılmaz [17] as $g_{\phi\sigma\gamma} = (0.025 \pm 0.009)$ using the experimental value of the branching ratio for the radiative decay $\phi \to \pi^0\pi^0\gamma$

[38]. We describe the $\sigma\pi\pi$-vertex by the effective Lagrangian [39]

$$\mathcal{L}^{eff.}_{\sigma\pi\pi} = \frac{1}{2} g_{\sigma\pi\pi} M_\sigma \vec{\pi} \cdot \vec{\pi} \sigma \quad . \tag{2.9}$$

The decay width of the σ-meson that results from this effective Lagrangian is given as

$$\Gamma_\sigma \equiv \Gamma(\sigma \to \pi\pi) = \frac{g^2_{\sigma\pi\pi}}{4\pi} \frac{3M_\sigma}{8} \left[1 - \left(\frac{2M_\pi}{M_\sigma} \right)^2 \right]^{1/2} \quad . \tag{2.10}$$

For given values of M_σ and Γ_σ, we use this expression to determine the coupling constant $g_{\sigma\pi\pi}$. Therefore, using the experimental values for M_σ and Γ_σ [3], given as $M_\sigma = (478 \pm 17)\ MeV$ and $\Gamma_\sigma = (324 \pm 21)\ MeV$, we obtain the coupling constant $g_{\sigma\pi\pi} = (5.25 \pm 0.32)$. The derivation of Γ_σ given in Eq. 2.10 is presented in Appendix A. The $\phi\rho\pi$-vertex in Fig. 2.2(b) and in Fig. 2.2(c) is conventionally described by the effective Lagrangian [40]

$$\mathcal{L}^{eff.}_{\phi\rho\pi} = \frac{g_{\phi\rho\pi}}{M_\phi} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \phi_\nu \partial_\alpha \vec{\rho_\beta} \cdot \vec{\pi} \quad . \tag{2.11}$$

The coupling constant $g_{\phi\rho\pi}$ is calculated as $g_{\phi\rho\pi} = (0.811 \pm 0.081)\ GeV^{-1}$ by Achasov and Gubin [22] using the data on the decay $\phi \to \rho\pi \to \pi^+\pi^-\pi^0$ [26]. For the $\rho\pi\gamma$-vertex the effective Lagrangian [41]

$$\mathcal{L}^{eff.}_{\rho\pi\gamma} = \frac{e}{M_\rho} g_{\rho\pi\gamma} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \vec{\rho_\nu} \cdot \vec{\pi}\ \partial_\alpha A_\beta \quad , \tag{2.12}$$

is used. At present there is a discrepancy between the experimental widths of the $\rho^0 \to \pi^0\gamma$ and $\rho^+ \to \pi^+\gamma$ decays. We use the experimental rate for the decay $\rho^0 \to \pi^0\gamma$ [26] to extract the coupling constant $g_{\rho\pi\gamma}$ as $g_{\rho\pi\gamma} = (0.69 \pm 0.35)$ since the experimental value for the decay rate of $\phi \to \pi^0\pi^0\gamma$ was used by Gökalp and Yılmaz [17] to estimate the coupling constant $g_{\phi\sigma\gamma}$. Finally, the $f_0\pi\pi$-vertex is described conventionally by the effective Lagrangian

$$\mathcal{L}^{eff.}_{f_0\pi\pi} = \frac{1}{2} g_{f_0\pi\pi} M_{f_0} \vec{\pi} \cdot \vec{\pi} f_0 \quad . \tag{2.13}$$

The decay width of the f_0-meson that follows from this Lagrangian is given by a similar expression as for Γ_σ and this expression can be used to obtain the coupling constant $g_{f_0\pi\pi}$ for a given value of Γ_{f_0}. Furthermore, it is useful to state that, in the calculation of invariant amplitudes we make the replacement $q^2 - M^2 \to q^2 - M^2 + iM\Gamma$, where q and M are four-momentum and mass of the virtual particles respectively, in ρ-, σ- and f_0- propagators in order to take into account the finite widths of these unstable particles and use the experimental values $\Gamma_\rho = (150.2 \pm 0.8)\ MeV$ [26] for ρ-meson and $\Gamma_\sigma = (324 \pm 21)\ MeV$ [3] for σ-meson. However, since the mass $M_{f_0} = 980\ MeV$ of f_0-meson is very close to the K^+K^- threshold this gives rise to a strong energy dependence on the width of the f_0-meson and to include this energy dependence different expressions for Γ_{f_0} can be used. First option is to use an energy dependent width for f_0

$$\Gamma_{f_0}(q^2) = \Gamma^{f_0}_{\pi\pi}(q^2)\ \theta\left(\sqrt{q^2} - 2M_\pi\right) + \Gamma^{f_0}_{K\overline{K}}(q^2)\ \theta\left(\sqrt{q^2} - 2M_K\right) \quad , \tag{2.14}$$

where q^2 is the four-momentum square of the virtual f_0-meson and the width $\Gamma^{f_0}_{\pi\pi}(q^2)$, derived in Appendix A, is given as

$$\Gamma^{f_0}_{\pi\pi}(q^2) = \Gamma^{f_0}_{\pi\pi}\ \frac{M^2_{f_0}}{q^2} \sqrt{\frac{q^2 - 4M^2_\pi}{M^2_{f_0} - 4M^2_\pi}} \quad . \tag{2.15}$$

We use the experimental value for $\Gamma^{f_0}_{\pi\pi}$ as $\Gamma^{f_0}_{\pi\pi} = 40 - 100\ MeV$ [26]. The width $\Gamma^{f_0}_{K\overline{K}}(q^2)$ which can be calculated from the effective Lagrangian given in Eq. 2.2 is given by a similar expression as for $\Gamma^{f_0}_{\pi\pi}(q^2)$. Another and widely accepted option is the work of Flatté [42]. In his work, the expression for $\Gamma^{f_0}_{K\overline{K}}(q^2)$ is extended below the $K\overline{K}$ threshold where $\sqrt{q^2 - 4M_K^2}$ is replaced by $i\sqrt{4M_K^2 - q^2}$ so $\Gamma^{f_0}_{K\overline{K}}(q^2)$ becomes purely imaginary. However in our work, we take into account both options. The invariant amplitude $\mathcal{M}(E_\gamma, E_1)$ which we present in Appendix C is expressed as $\mathcal{M}(E_\gamma, E_1) = \mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c + \mathcal{M}_d$ where $\mathcal{M}_a$, $\mathcal{M}_b$, $\mathcal{M}_c$ and $\mathcal{M}_d$ are the invariant amplitudes resulting from the diagrams (a), (b), (c) and (d) in Fig. 2.2 respectively. Therefore, the interference between different reactions contributing to the decay $\phi \to \pi^+\pi^-\gamma$ is taken into account. Then, in terms of the invariant amplitude $\mathcal{M}(E_\gamma, E_1)$ the differential decay probability of $\phi \to \pi^+\pi^-\gamma$ decay for an unpolarized ϕ-meson at rest is given as

$$\frac{d\Gamma}{dE_\gamma dE_1} = \frac{1}{(2\pi)^3}\,\frac{1}{8M_\phi}\,\mid \mathcal{M} \mid^2 \quad , \tag{2.16}$$

where E_γ and E_1 are the photon and pion energies respectively. An average over the spin states of ϕ-meson and a sum over the polarization states of the photon is performed. The decay width $\Gamma(\phi \to \pi^+\pi^-\gamma)$ is then obtained by taking the integral and this is given by

$$\Gamma = \int_{E_{\gamma,min.}}^{E_{\gamma,max.}} dE_\gamma \int_{E_{1,min.}}^{E_{1,max.}} dE_1 \frac{d\Gamma}{dE_\gamma dE_1} \quad , \tag{2.17}$$

where the minimum photon energy is $E_{\gamma,min.} = 0$ and the maximum photon energy is given as $E_{\gamma,max.} = (M_\phi^2 - 4M_\pi^2)/2M_\phi = 471.8\ MeV$. The maximum and minimum values for the pion energy E_1, derived in Appendix B, are given by

$$\frac{1}{2(2E_\gamma M_\phi - M_\phi^2)}[-2E_\gamma^2 M_\phi + 3E_\gamma M_\phi^2 - M_\phi^3 \pm E_\gamma \sqrt{(-2E_\gamma M_\phi + M_\phi^2)(-2E_\gamma M_\phi + M_\phi^2 - 4M_\pi^2)}\ ] \quad . \tag{2.18}$$

2.2 Scalar $a_0(980)$ meson in $\phi \to \pi^0\eta\gamma$ decay

In order to investigate the role of scalar-isovector a_0 (980) meson in radiative $\phi \to \pi^0\eta\gamma$ decay, we consider two different approaches and in both approaches, where the contributions of the ρ-meson, chiral loop and a_0-meson are considered, we do not make any assumption about the structure of scalar a_0 meson. In our first approach, called model I, the contribution of the a_0-meson is considered as resulting from an a_0-pole intermediate state. In this approach, we assume that the mechanism of the $\phi \to \pi^0\eta\gamma$ decay consists of the reactions shown by Feynman diagrams in Fig. 2.3. We describe the $\phi\rho\pi$-vertex by the effective Lagrangian given in Eq. 2.11 and as we mentioned before the coupling constant $g_{\phi\rho\pi}$ was determined by Achasov and Gubin as $g_{\phi\rho\pi} = (0.811 \pm 0.081)\ GeV^{-1}$ [22]. The $\rho\eta\gamma$-vertex is described by the effective Lagrangian [43]

$$\mathcal{L}^{eff.}_{\rho\eta\gamma} = \frac{e}{M_\rho} g_{\rho\eta\gamma} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \rho_\nu \partial_\alpha A_\beta \eta \quad , \tag{2.19}$$

which also defines the coupling constant $g_{\rho\eta\gamma}$. Then, the coupling constant $g_{\rho\eta\gamma}$ is obtained from the experimental partial width of the radiative decay $\rho \to \eta\gamma$ [26] as $g_{\rho\eta\gamma} = (1.14{\pm}0.18)$. We describe the ϕKK-vertex by the effective Lagrangian

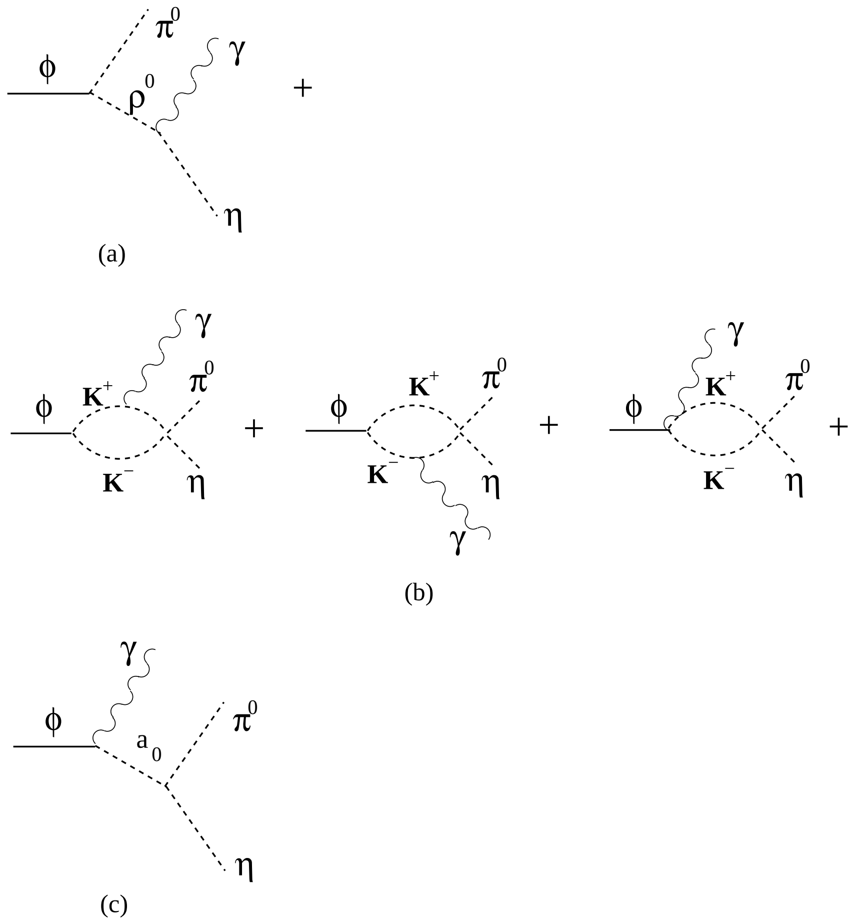

Figure 2.3: Feynman diagrams for the decay $\phi \to \pi^0\eta\gamma$ in model I

given in Eq. 2.1 and also the decay rate for the ϕK^+K^- decay resulting from this Lagrangian is given in Eq. 2.3. For the four pseudoscalar $KK\pi\eta$ amplitude, we use the result obtained by Bramon et al. using the standard chiral perturbation theory [21] which is

$$\mathcal{M}\left(K^+K^- \to \pi^0\eta\right) = \frac{\sqrt{3}}{4f_\pi^2}\left(M^2_{\pi^0\eta} - \frac{4}{3}M_K^2\right) \quad , \tag{2.20}$$

where $\eta-\eta'$ mixing effects are neglected. Here, $M_{\pi^0\eta}$ is the invariant mass of the $\pi^0\eta$ system and f_π is the pion decay constant having the value $f_\pi = 92.4\ MeV$. Therefore, the amplitude of the radiative decay $\phi \to \pi^0\eta\gamma$ obtained from the diagram in Fig. 2.3(b) is

$$\mathcal{M} = \left(\frac{ieg_{\phi KK}}{2\pi^2 M_K^2}\right) \mathcal{M}\left(K^+K^- \to \pi^0\eta\right) I(a,b)\left[\epsilon\cdot u\ k\cdot p - \epsilon\cdot p\ k\cdot u\right] \quad , \tag{2.21}$$

where (u, p) and (ϵ, k) are the polarizations and four-momenta of the ϕ meson and the photon respectively, and also $a = M_\phi^2/M_K^2$, $b = M_{\pi^0\eta}^2/M_K^2$. The invariant mass of the final $\pi^0\eta$ system is given by $M_{\pi^0\eta}^2 = (q_1+q_2)^2 = (p-k)^2$ where q_1 and q_2 are the four-momenta of the final pseudoscalar π^0 and η mesons respectively. Also the $I(a,b)$ function is given in Eq. 2.5. For the $\phi a_0\gamma$- and $a_0\pi^0\eta$-vertices we use the the effective Lagrangians [37]

$$\mathcal{L}^{eff.}_{\phi a_0\gamma} = \frac{e}{M_\phi} g_{\phi a_0\gamma}[\partial^\alpha\phi^\beta\partial_\alpha A_\beta - \partial^\alpha\phi^\beta\partial_\beta A_\alpha]a_0 \quad , \tag{2.22}$$

and

$$\mathcal{L}^{eff.}_{a_0\pi\eta} = g_{a_0\pi\eta}\vec{\pi}\cdot\vec{a_0}\eta \quad , \tag{2.23}$$

respectively, which can be used to determine the coupling constants $g_{\phi a_0\gamma}$ and $g_{a_0\pi\eta}$. Since there are no direct experimental results relating to the $\phi a_0\gamma$-vertex, but only an upper limit for the branching ratio of the decay $\phi \to a_0\gamma$ is mentioned in " Review of Particle Properties " as $BR(\phi \to a_0\gamma) < 5\times10^{-3}$ [26] we estimate the coupling constant $g_{\phi a_0\gamma}$ in our calculation utilizing the experimental branching ratio of the $\phi \to \pi^0\eta\gamma$ decay. The decay rates for the $\phi \to a_0\gamma$ and

$a_0 \to \pi^0\eta$ decays, given in detail in Appendix A, which follow from these effective Lagrangians are given as

$$\Gamma(\phi \to a_0\gamma) = \frac{\alpha}{24}\frac{(M_\phi^2 - M_{a_0}^2)^3}{M_\phi^5} g_{\phi a_0\gamma}^2 \quad , \tag{2.24}$$

and

$$\Gamma(a_0 \to \pi^0\eta) = \frac{g_{a_0\pi\eta}^2}{16\pi M_{a_0}}\sqrt{\left[1 - \frac{(M_{\pi^0} + M_\eta)^2}{M_{a_0}^2}\right]\left[1 - \frac{(M_{\pi^0} - M_\eta)^2}{M_{a_0}^2}\right]} \quad , \tag{2.25}$$

respectively. We obtain the coupling constant $g_{a_0\pi\eta}$ as $g_{a_0\pi\eta} = (2.34 \pm 0.18)$ by using the value $\Gamma_{a_0} = (0.069 \pm 0.011)\ GeV$ which was determined by the $E852$ collaboration at BNL [44]. Since the diagram in Fig. 2.3(c) implies a direct quark transition and it has a very small contribution as a result of the OZI suppression, we conclude that the introduction of the a_0 amplitude as in Fig. 2.3(c) can not be very realistic. Therefore, we reconsider the approach which we name model I. It has been shown that the scalar resonances f_0 (980) and a_0 (980) can be excited from the chiral loops, with the loop iteration provided by the Bethe-Salpeter equation using a kernel from the lowest order chiral Lagrangian [15] and also the experimental data obtained in Novosibirsk give reasonable arguments in favour of the one-loop mechanism for $\phi \to K^+K^- \to f_0\gamma$ and $\phi \to K^+K^- \to a_0\gamma$ decays [45]. Then, we develop a second approach to study the radiative $\phi \to \pi^0\eta\gamma$ decay, called model II, in the light of the works of Marco et al. and Achasov [15, 45]. In our second approach we assume that this decay proceeds through the reactions $\phi \to \rho^0\pi^0 \to \pi^0\eta\gamma$, $\phi \to K^+K^-\gamma \to \pi^0\eta\gamma$ and $\phi \to a_0\gamma \to \pi^0\eta\gamma$ where the last reaction proceeds by a two-step mechanism with

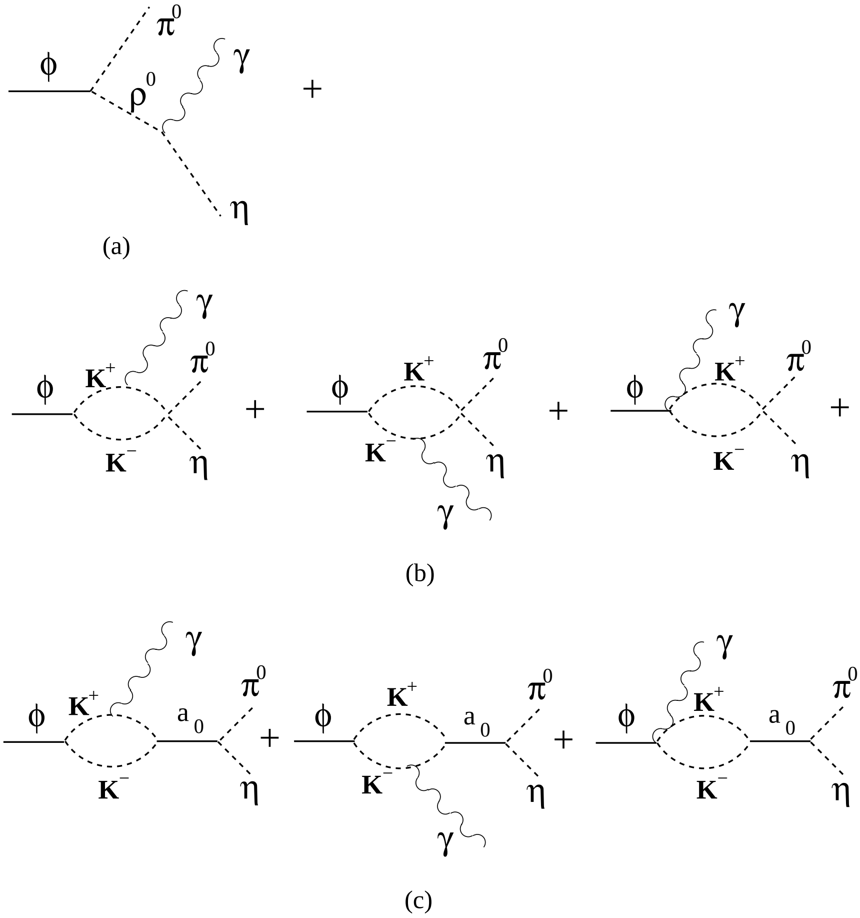

Figure 2.4: Feynman diagrams for the decay $\phi \to \pi^0\eta\gamma$ in model II

a_0 coupling to ϕ with intermediate $K\overline{K}$ states. The processes contributing to the $\phi \to \pi^0\eta\gamma$ decay amplitude is shown diagrammatically in Fig. 2.4. We proceed within a phenomenological framework and we do not make any assumption about the structure of the a_0-meson. We note that the ϕ and a_0 mesons both couple strongly to the K^+K^- system and therefore there is an amplitude for the decay $\phi \to \gamma a_0$ to proceed through the charged kaon-loop independent of the nature and the dynamical structure of the a_0-meson. In our second approach, we need also the effective Lagrangian which is used to describe the $K^+K^-a_0$-vertex in addition to the other Lagrangians used in model I

$$\mathcal{L}^{eff.}_{a_0K^+K^-} = g_{a_0K^+K^-} M_{a_0} K^+ K^- a_0 \quad . \tag{2.26}$$

The decay width of the a_0-meson that follows from this effective Lagrangian is given as

$$\Gamma(a_0 \to K^+K^-) = \frac{g^2_{a_0K^+K^-}}{16\pi} M_{a_0} \left[1 - \left(\frac{2M_k}{M_{a_0}}\right)^2\right]^{1/2} \quad . \tag{2.27}$$

Derivation of $\Gamma(a_0 \to K^+K^-)$ given in Eq. 2.27 is presented in Appendix A. This expression is used to determine the coupling constant $g_{a_0K^+K^-}$. In model II, we calculate the decay rate of the $\phi \to \pi^0\eta\gamma$ decay using the diagrams shown in Fig. 2.4 and determine the coupling constant $g_{a_0K^+K^-}$ by employing the experimental branching ratio of this decay. Furthermore, in our calculation of the invariant amplitudes for both models, we make the replacement $q^2 - M^2 \to q^2 - M^2 + iM\Gamma$ in a_0- and ρ^0- propagators in order to take into account the finite widths of these unstable particles. We use the experimental value $\Gamma_\rho = (150.2 \pm 0.8)\ MeV$ [26] for ρ^0-meson, because using a q^2-dependent width did not influence our results considerably. However, the mass M_{a_0} of a_0-meson is very close to the K^+K^- threshold and this induces a strong energy dependence on the width Γ_{a_0} of a_0-meson. In order to take that into account, we follow the widely accepted option that was proposed by Flatté [42] based on a coupled channel $(\pi\eta, K\overline{K})$ description of the a_0 resonance, and parametrize the a_0 width as

$$\Gamma_{a_0}(q^2) = \quad \Gamma^{a_0}_{\pi^0\eta}(q^2)\ \theta\left(\sqrt{q^2} - (M_{\pi^0} + M_\eta)\right)$$

$$+ig_{K\overline{K}}\sqrt{M_K^2 - q^2/4}\;\theta\left(2M_K - \sqrt{q^2}\right)$$
$$+g_{K\overline{K}}\sqrt{q^2/4 - M_K^2}\;\theta\left(\sqrt{q^2} - 2M_K\right) \quad , \tag{2.28}$$

where

$$\Gamma^{a_0}_{\pi^0\eta}(q^2) = \frac{g^2_{a_0\pi\eta}}{16\pi(q^2)^{3/2}}\sqrt{[q^2 - (M_{\pi^0} + M_\eta)^2]\,[q^2 - (M_{\pi^0} - M_\eta)^2]} \quad . \tag{2.29}$$

We use the Flatté parameter $g_{K\overline{K}}$ as $g_{K\overline{K}} = (0.22 \pm 0.04)$ which was determined by the $E852$ collaboration at BNL [44] by a fit to the data in their experiment in which they determined the parameters of the a_0-meson. In all our calculations we use the experimental value $M_{a_0} = (0.991 \pm 0.0025)\ GeV$ [44]. In model I, the invariant amplitude $\mathcal{M}(E_\gamma, E_1)$, given in Appendix D.1, for the decay $\phi \rightarrow \pi^0\eta\gamma$ is expressed as $\mathcal{M}(E_\gamma, E_1) = \mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c$ where $\mathcal{M}_a$, $\mathcal{M}_b$ and $\mathcal{M}_c$ are the invariant amplitudes resulting from the diagrams (a), (b) and (c) in Fig. 2.3 respectively. In model II, the invariant amplitude $\mathcal{M}(E_\gamma, E_1)$ which we present in Appendix D.2, for this decay is expressed as $\mathcal{M}(E_\gamma, E_1) = \mathcal{M}_a{}' + \mathcal{M}_b{}' + \mathcal{M}_c{}'$ where $\mathcal{M}_a{}'$, $\mathcal{M}_b{}'$ and $\mathcal{M}_c{}'$ are the invariant amplitudes corresponding to the diagrams (a), (b) and (c) in Fig. 2.4, respectively. This way, the interference between different reactions contributing to the decay $\phi \rightarrow \pi^0\eta\gamma$ is taken into account. In terms of the invariant amplitude $\mathcal{M}(E_\gamma, E_1)$ the differential decay probability of $\phi \rightarrow \pi^0\eta\gamma$ decay for an unpolarized ϕ-meson at rest is given in Eq. 2.16 where E_γ and E_1 are the photon and pion energies respectively. Then we perform an average over the spin states of ϕ-meson and a sum over the polarization states of the photon. The decay width

$\Gamma(\phi \to \pi^0\eta\gamma)$ is obtained by taking the integral and given in Eq. 2.17 where the minimum photon energy is $E_{\gamma,min.} = 0$ and the maximum photon energy is given as $E_{\gamma,max.} = \left[M_\phi^2 - (M_{\pi^0} + M_\eta)^2\right]/2M_\phi$. The maximum and minimum values for the pion energy E_1 which are derived in Appendix B, are given by

$$\frac{1}{2(2E_\gamma M_\phi - M_\phi^2)} \{-2E_\gamma^2 M_\phi - M_\phi(M_\phi^2 + M_{\pi^0}^2 - M_\eta^2) + E_\gamma(3M_\phi^2 + M_{\pi^0}^2 - M_\eta^2)$$
$$\pm E_\gamma[\ 4E_\gamma^2 M_\phi^2 + M_\phi^4 + (M_{\pi^0}^2 - M_\eta^2)^2 - 2M_\phi^2(M_{\pi^0}^2 + M_\eta^2)$$
$$+4E_\gamma M_\phi(-M_\phi^2 + M_{\pi^0}^2 + M_\eta^2)\ ]^{1/2}\ \}\ . \qquad (2.30)$$

2.3 Scalar σ meson effects in radiative ρ^0 meson decays

The effects of the scalar σ-meson are investigated in the radiative $\rho^0 \to \pi^+\pi^-\gamma$ and $\rho^0 \to \pi^0\pi^0\gamma$ decays. In our approach we assume that the σ-meson couples to the ρ^0 meson through the pion-loop; in other words, we consider that the amplitude $\rho^0 \to \sigma\gamma$ results from the sequential $\rho^0 \to (\pi^+\pi^-)\gamma \to \sigma\gamma$ mechanism as suggested by the unitarized chiral perturbation theory in which the σ-meson is generated dynamically by unitarizing the one-loop pion amplitudes [15, 16]. In our analysis of the radiative $\rho^0 \to \pi^+\pi^-\gamma$ and $\rho^0 \to \pi^0\pi^0\gamma$ decays, we proceed within a phenomenological framework and our calculation is based on the Feynman diagrams shown in Fig. 2.5 for $\rho^0 \to \pi^+\pi^-\gamma$ decay and in Fig. 2.6 for $\rho^0 \to \pi^0\pi^0\gamma$ decay. It is useful to state that, the last diagrams in Figs. 2.5(d),(e) and in Figs. 2.6(c),(d) and also the diagram in Fig. 2.5(c) are the direct terms which are required to establish the gauge invariance. For the

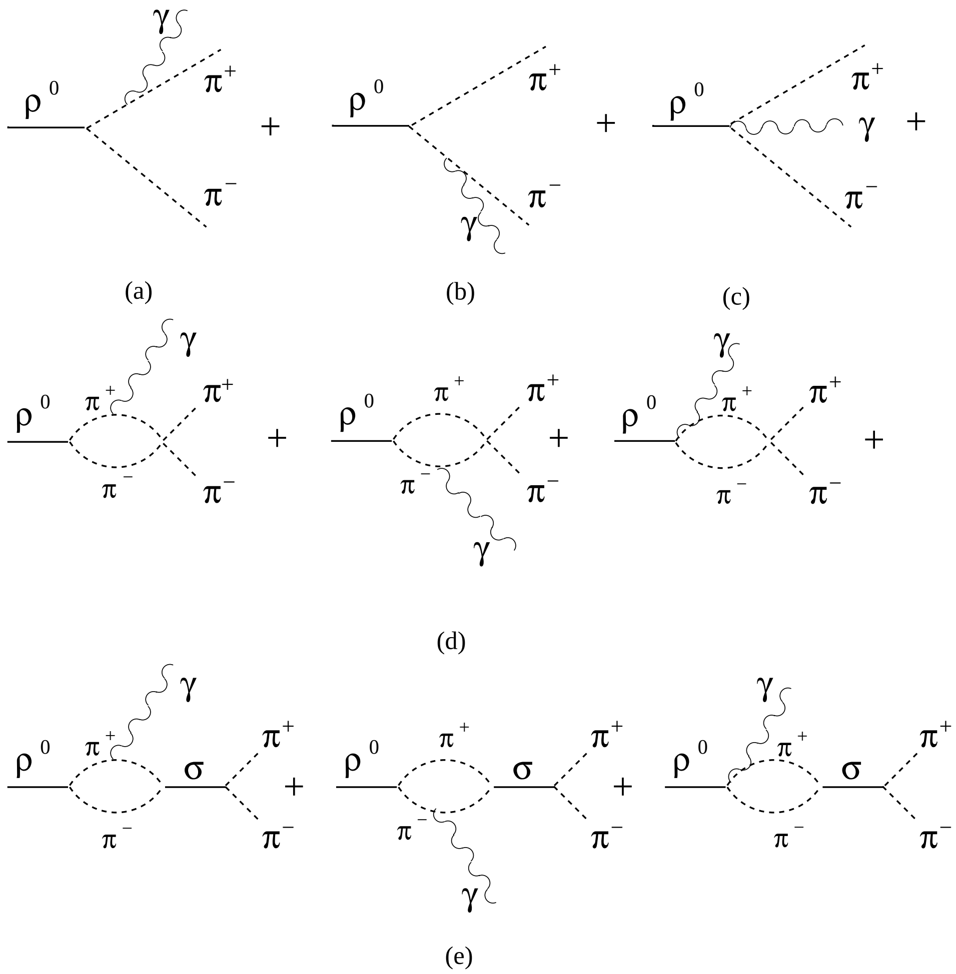

Figure 2.5: Feynman diagrams for the decay $\rho^0 \to \pi^+\pi^-\gamma$

$\rho\pi\pi$-vertex we use the effective Lagrangian [46]

$$\mathcal{L}^{eff.}_{\rho\pi\pi} = g_{\rho\pi\pi}\vec{\rho}_\mu \cdot (\partial^\mu \vec{\pi} \times \vec{\pi}) \quad . \tag{2.31}$$

As it is shown in Appendix A, the decay width of ρ-meson that follows from this effective Lagrangian is

$$\Gamma(\rho \to \pi\pi) = \frac{g^2_{\rho\pi\pi}}{48\pi} M_\rho \left[1 - \left(\frac{2M_\pi}{M_\rho}\right)^2\right]^{3/2} \quad . \tag{2.32}$$

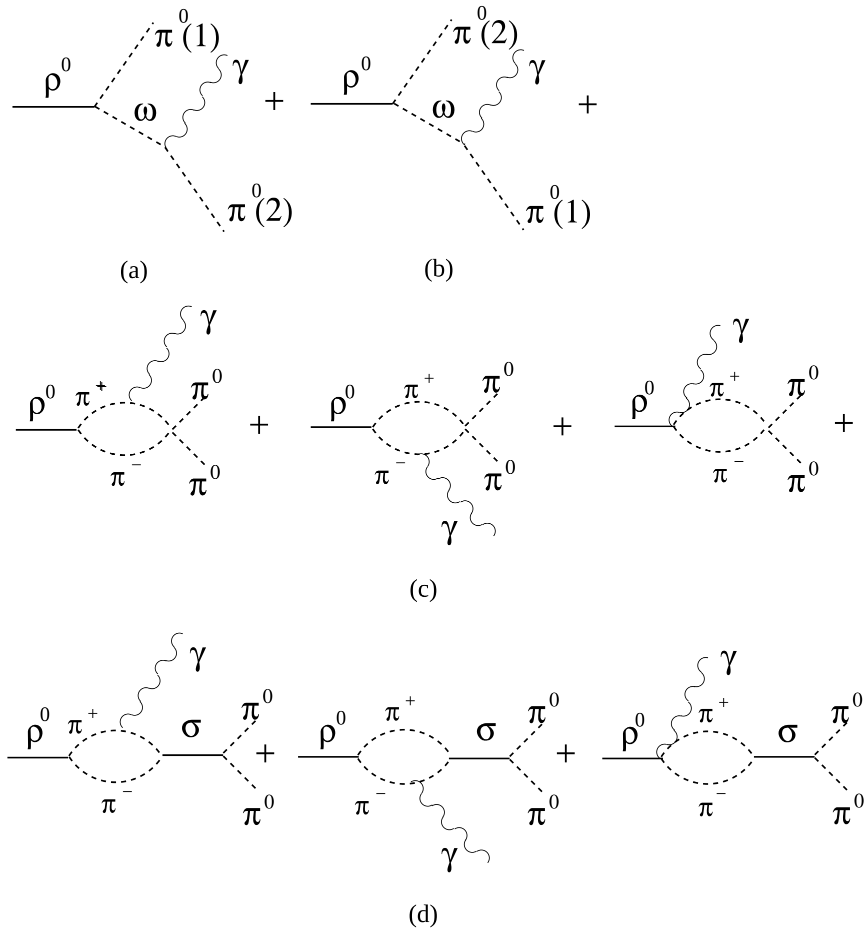

Figure 2.6: Feynman diagrams for the decay $\rho^0 \to \pi^0\pi^0\gamma$

We obtain the coupling constant $g_{\rho\pi\pi}$ from the experimental decay width of the decay $\rho \to \pi\pi$ [26] as $g_{\rho\pi\pi} = (6.03 \pm 0.02)$. We describe the $\sigma\pi\pi$-vertex by the effective Lagrangian given in Eq. 2.9. The decay width of the σ-meson that results from this effective Lagrangian is given in Eq. 2.10. Note that our effective Lagrangians $\mathcal{L}^{eff.}_{\sigma\pi\pi}$ and $\mathcal{L}^{eff.}_{\rho\pi\pi}$ result from an extension of the σ model to include the isovector ρ through a Yang-Mills local gauge theory based on isospin with the vector meson mass generated through the Higgs mechanism [47]. Meson-meson interactions were studied by Oller and Oset [5] using the standart chiral Lagrangian in the lowest order of chiral perturbation theory which contains the most general low energy interactions of the pseudoscalar meson octet in this order. Therefore, we used their results for the four pseudoscalar amplitudes $\pi^+\pi^- \to \pi^+\pi^-$ and $\pi^+\pi^- \to \pi^0\pi^0$ that is needed in the loop diagrams in Fig. 2.5(d) and Fig. 2.6(c). As it is shown by Oller [16] due to gauge invariance the off-shell parts of the amplitudes, which should be kept inside the loop integration, do not contribute and as a result of this, the amplitudes $\mathcal{M}_\chi(\pi^+\pi^- \to \pi^+\pi^-)$ and $\mathcal{M}_\chi(\pi^+\pi^- \to \pi^0\pi^0)$ factorize in the expressions for the loop diagrams. For the loop integrals appearing in Fig. 2.5(d), in Fig. 2.5(e), in Fig. 2.6(c) and in Fig. 2.6(d) we use the results of Lucio and Pestiau [36]. The contribution of the pion-loop amplitude corresponding to the $\rho^0 \to (\pi^+\pi^-)\gamma \to \pi^+\pi^-\gamma$ reaction can be written as

$$\mathcal{M}_\pi = \frac{e g_{\rho\pi\pi} \mathcal{M}_\chi(\pi^+\pi^- \to \pi^+\pi^-)}{2\pi^2 M_\pi^2} \; I(a,b) \, [\epsilon \cdot u \; k \cdot p - \epsilon \cdot p \; k \cdot u] \quad , \qquad (2.33)$$

where $a = M_\rho^2/M_\pi^2$, $b = (p-k)^2/M_\pi^2$, (u,p) and (ϵ,k) are the polarizations and four-momenta of the ρ^0 meson and the photon, respectively. Also the four pseudoscalar $\pi^+\pi^-\pi^+\pi^-$amplitude is given by

$$\mathcal{M}_\chi(\pi^+\pi^- \to \pi^+\pi^-) = -\frac{2}{3}\,\frac{1}{f_\pi^2}\left(s - \frac{7}{2}M_\pi^2\right) \quad , \qquad (2.34)$$

where $s = M_{\pi\pi}^2$, $f_\pi = 92.4\ MeV$. The contribution of the pion-loop amplitude corresponding to the $\rho^0 \to (\pi^+\pi^-)\gamma \to \pi^0\pi^0\gamma$ reaction is given by a similar expression as for the amplitude of the radiative $\rho^0 \to (\pi^+\pi^-)\gamma \to \pi^+\pi^-\gamma$ decay and this is given by

$$\mathcal{M}_\pi = \frac{e g_{\rho\pi\pi} \mathcal{M}_\chi(\pi^+\pi^- \to \pi^0\pi^0)}{2\pi^2 M_\pi^2} \, I(a,b) \, [\epsilon \cdot u \; k \cdot p - \epsilon \cdot p \; k \cdot u] \quad , \tag{2.35}$$

where $a = M_\rho^2/M_\pi^2$, $b = (p-k)^2/M_\pi^2$, $\mathcal{M}_\chi(\pi^+\pi^- \to \pi^0\pi^0) = -(1/f_\pi^2)\,(s - M_\pi^2)$, $s = M^2_{\pi^0\pi^0}$, $f_\pi = 92.4\ MeV$, (u,p) and (ϵ,k) are the polarizations and four-momenta of the ρ^0 meson and the photon respectively. The amplitudes resulting from the diagrams in Fig. 2.5(e) and in Fig. 2.6(d) can be written in a similar way. The $I(a,b)$ function is given in Eq. 2.5. The $\omega\rho\pi$-vertex, in Fig. 2.6(a) and in Fig. 2.6(b), is described by the effective Lagrangian [41]

$$\mathcal{L}^{eff.}_{\omega\rho\pi} = \frac{g_{\omega\rho\pi}}{M_\omega} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \omega_\nu \partial_\alpha \vec{\rho}_\beta \cdot \vec{\pi} \quad , \tag{2.36}$$

which also conventionally defines the coupling constant $g_{\omega\rho\pi}$. The coupling constant $g_{\omega\rho\pi}$ was determined by the Novosibirsk SND collaboration [48]. In their work, they assumed that $\omega \to 3\pi$ decay proceeds with the intermediate $\rho\pi$ state as $\omega \to (\rho)\pi \to \pi\pi\pi$ and using experimental value of the $\omega \to 3\pi$ decay width they obtained this coupling constant as $g_{\omega\rho\pi} = (14.3 \pm 0.2)\ GeV^{-1}$. We describe the $\omega\pi\gamma$-vertex by the effective Lagrangian [41]

$$\mathcal{L}^{eff.}_{\omega\pi\gamma} = \frac{e}{M_\omega} g_{\omega\pi\gamma} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \omega_\nu \partial_\alpha A_\beta \pi \quad , \tag{2.37}$$

and the coupling constant $g_{\omega\pi\gamma}$ is obtained as $g_{\omega\pi\gamma} = (1.82 \pm 0.05)$ by using the experimental partial width [26] of the radiative decay $\omega \to \pi^0\gamma$. In our calculation of the invariant amplitude, in the σ-meson propagator the replacement

$q^2 - M^2 \to q^2 - M^2 + iM\Gamma$ is made to take into account the finite lifetime of unstable σ-meson. The energy dependent width for the σ-meson, presented in Appendix A, is given as

$$\begin{aligned} \Gamma^{\sigma}_{\pi\pi}(q^2) &= \frac{2}{3}\Gamma^{\sigma}_{\pi\pi}(q^2 = M_\sigma^2)\frac{M_\sigma^2}{q^2}\sqrt{\frac{q^2 - 4M_{\pi^+}^2}{M_\sigma^2 - 4M_{\pi^+}^2}}\ \theta\left(q^2 - 4M_{\pi^+}^2\right) \\ &+\frac{1}{3}\Gamma^{\sigma}_{\pi\pi}(q^2 = M_\sigma^2)\frac{M_\sigma^2}{q^2}\sqrt{\frac{q^2 - 4M_{\pi^0}^2}{M_\sigma^2 - 4M_{\pi^0}^2}}\ \theta\left(q^2 - 4M_{\pi^0}^2\right). \end{aligned} \quad (2.38)$$

The invariant amplitude $\mathcal{M}(E_\gamma, E_1)$ which we present in Appendix E.1, for the decay $\rho^0 \to \pi^+\pi^-\gamma$ is expressed as $\mathcal{M}(E_\gamma, E_1) = \mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c + \mathcal{M}_d + \mathcal{M}_e$ where $\mathcal{M}_a$, $\mathcal{M}_b$, $\mathcal{M}_c$, $\mathcal{M}_d$ and $\mathcal{M}_e$ are the invariant amplitudes resulting from the diagrams (a), (b), (c), (d) and (e) in Fig. 2.5 respectively. Also, the invariant amplitude $\mathcal{M}(E_\gamma, E_1)$, given in detail in Appendix E.2, for the decay $\rho^0 \to \pi^0\pi^0\gamma$ is expressed as $\mathcal{M}(E_\gamma, E_1) = \mathcal{M}_a{}' + \mathcal{M}_b{}' + \mathcal{M}_c{}' + \mathcal{M}_d{}'$ where $\mathcal{M}_a{}'$, $\mathcal{M}_b{}'$, $\mathcal{M}_c{}'$ and $\mathcal{M}_d{}'$ are the invariant amplitudes corresponding to the diagrams (a), (b), (c) and (d) in Fig. 2.6, respectively. Then in terms of the invariant amplitude $\mathcal{M}(E_\gamma, E_1)$ the differential decay probability for an unpolarized ρ^0-meson at rest is given as

$$\frac{d\Gamma}{dE_\gamma dE_1} = \frac{1}{(2\pi)^3}\,\frac{1}{8M_\rho}\ \mid \mathcal{M} \mid^2 \quad , \quad (2.39)$$

where E_γ and E_1 are the photon and pion energies respectively. An average over the spin states of ρ^0-meson and a sum over the polarization states of the photon is performed and the decay width is then obtained by integration

$$\Gamma = \frac{1}{2}\int_{E_{\gamma,min.}}^{E_{\gamma,max.}} dE_\gamma \int_{E_{1,min.}}^{E_{1,max.}} dE_1 \frac{d\Gamma}{dE_\gamma dE_1} \quad , \quad (2.40)$$

where the factor $(\frac{1}{2})$ is included for the calculation of the decay rate for $\rho^0 \to \pi^0\pi^0\gamma$ because of the identity of π^0 mesons in the final state. The minimum photon energy for the decay $\rho^0 \to \pi^0\pi^0\gamma$ is $E_{\gamma,min.} = 0$ and the maximum photon energy is given as $E_{\gamma,max.} = (M_\rho^2 - 4M_{\pi^0}^2)/2M_\rho = 338\ MeV$. On the other hand, the minimum photon energy for the decay $\rho^0 \to \pi^+\pi^-\gamma$ is $E_{\gamma,min.} = 50\ MeV$ since the experimental value of the branching ratio is determined for this range of photon energies [25] and the maximum value of the photon energy is given as $E_{\gamma,max.} = (M_\rho^2 - 4M_{\pi^+}^2)/2M_\rho = 334\ MeV$. Finally the maximum and minimum values for the pion energy E_1 of the $\rho^0 \to \pi^+\pi^-\gamma$ and $\rho^0 \to \pi^0\pi^0\gamma$ decays, derived in Appendix B, are given by

$$\frac{1}{2(2E_\gamma M_\rho - M_\rho^2)}[-2E_\gamma^2 M_\rho + 3E_\gamma M_\rho^2 - M_\rho^3 \pm E_\gamma\sqrt{(-2E_\gamma M_\rho + M_\rho^2)(-2E_\gamma M_\rho + M_\rho^2 - 4M_\pi^2)}\ ]\ . \qquad (2.41)$$

CHAPTER 3

RESULTS AND DISCUSSION

In this chapter we first give the parameters that are needed in our calculation of the branching ratio and discuss the invariant mass distribution for the reaction $\phi \rightarrow \pi^+\pi^-\gamma$. Later, we indicate that the spectrum for this decay is dominated by the f_0-amplitude. We then present our calculation for the branching ratio of the $\phi \rightarrow \pi^+\pi^-\gamma$ decay and compare it with the previous calculations. Then for the radiative $\phi \rightarrow \pi^0\eta\gamma$ decay we give the details of our calculations about the coupling constants $g_{\phi a_0 \gamma}$ in model I and $g_{a_0 K^+ K^-}$ in model II. Afterwards, we indicate the invariant mass distributions of the radiative decay $\phi \rightarrow \pi^0\eta\gamma$ using the values of the coupling constants $g_{\phi a_0 \gamma}$ for model I and $g_{a_0 K^+ K^-}$ for model II. We then compare our results with the experimental invariant mass spectrum for the decay $\phi \rightarrow \pi^0\eta\gamma$. Finally, we present our results for the decay mechanism of ρ^0-meson radiative decays including the contribution coming from the σ-meson intermediate state as well as VMD- and chiral pion-loop contributions. Our results for the branching ratios are compared with the experimental values.

3.1 Radiative $\phi \to \pi^+\pi^-\gamma$ decay

In order to determine the coupling constant $g_{f_0\pi\pi}$, we choose for the f_0-meson parameters the values $M_{f_0} = 980\ MeV$ and $\Gamma_{f_0} = (70 \pm 30)\ MeV$. Therefore, through the decay rate that results from the effective Lagrangian given in Eq. 2.13 we obtain the coupling constant $g_{f_0\pi\pi}$ as $g_{f_0\pi\pi} = (1.58 \pm 0.30)$. If we use the form for $\Gamma^{f_0}_{K\bar{K}}(q^2)$, proposed by Flatté [42], the desired enhancement in the invariant mass spectrum appears in its central part rather than around the f_0 pole. Bramon et al. [21] also encountered a similar problem in their study of the effects of the $a_0(980)$ meson in the $\phi \to \pi^0\eta\gamma$ decay. Therefore, in the analysis which we present below for $\Gamma_{f_0}(q^2)$ we use the form given in Eq. 2.14. The invariant mass distribution $dB/dM_{\pi\pi} = (M_{\pi\pi}/M_\phi)dB/dE_\gamma$ for the radiative decay $\phi \to \pi^+\pi^-\gamma$ is plotted in Fig. 3.1 as a function of the invariant mass $M_{\pi\pi}$ of $\pi^+\pi^-$ system. In this figure we indicate the contributions coming from different reactions $\phi \to \sigma\gamma \to \pi^+\pi^-\gamma$, $\phi \to \rho^\mp\pi^\pm \to \pi^+\pi^-\gamma$ and $\phi \to f_0\gamma \to \pi^+\pi^-\gamma$ as well as the contribution of the total amplitude which includes the interference terms as well. It is clearly seen from Fig. 3.1 that the spectrum for the decay $\phi \to \pi^+\pi^-\gamma$ is dominated by the f_0-amplitude. On the other hand the contribution coming from σ-amplitude can only be noticed in the region $M_{\pi\pi} < 0.7\ GeV$ through interference effects. Likewise ρ-meson contribution can be seen in the region $M_{\pi\pi} < 0.8\ GeV$ so we can say that the contribution of the f_0-term is much larger than the contributions of the

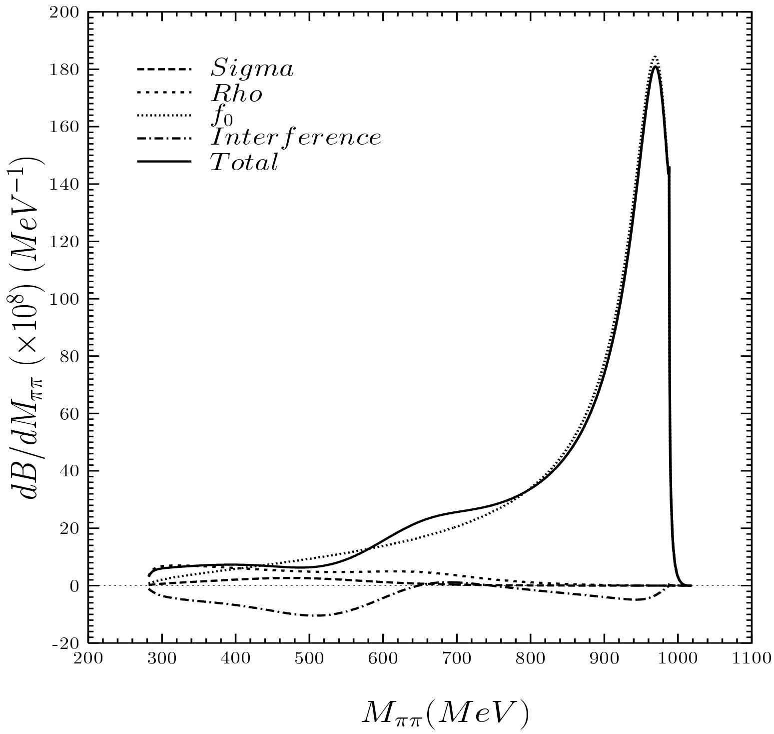

Figure 3.1: The $\pi\pi$ invariant mass spectrum for the decay $\phi \to \pi^+\pi^-\gamma$. The contributions of different terms are indicated.

σ-term and ρ-term as well as the contribution of the total interference term having opposite sign. The dominant f_0-term characterizes the invariant mass distribution in the region where $M_{\pi\pi} > 0.7\ GeV$. In our study contributions of different amplitudes to the branching ratio of the decay $\phi \to \pi^+\pi^-\gamma$ are $BR(\phi \to f_0\gamma \to \pi^+\pi^-\gamma) = 2.54 \times 10^{-4}$, $BR(\phi \to \sigma\gamma \to \pi^+\pi^-\gamma) = 0.07 \times 10^{-4}$, $BR(\phi \to \rho^{\mp}\pi^{\pm} \to \pi^+\pi^-\gamma) = 0.26\times10^{-4}$, $BR(\phi \to (f_0\gamma+\pi^{\pm}\rho^{\mp}) \to \pi^+\pi^-\gamma) = 2.74\times10^{-4}$, $BR(\phi \to (f_0\gamma+\sigma\gamma) \to \pi^+\pi^-\gamma) = 2.29\times10^{-4}$ and for the total interference term $BR(\text{interference}) = -0.29 \times 10^{-4}$. We then calculate the total branching ratio as $BR(\phi \to \pi^+\pi^-\gamma) = 2.57 \times 10^{-4}$. Our result is twice the theoretical result for $\phi \to \pi^0\pi^0\gamma$ decay, obtained by Gökalp and Yılmaz [17], as it should be. They obtained the following values: $BR(\phi \to f_0\gamma \to \pi^0\pi^0\gamma) = 1.29 \times 10^{-4}$, $BR(\phi \to \sigma\gamma \to \pi^0\pi^0\gamma) = 0.04 \times 10^{-4}$, $BR(\phi \to \rho^0\pi^0 \to \pi^0\pi^0\gamma) = 0.14 \times 10^{-4}$, $BR(\phi \to (f_0\gamma+\pi^0\rho^0) \to \pi^0\pi^0\gamma) = 1.34\times10^{-4}$, $BR(\phi \to (f_0\gamma+\sigma\gamma) \to \pi^0\pi^0\gamma) = 1.16\times10^{-4}$ and $BR(\text{interference}) = -0.25\times10^{-4}$. Moreover, our calculation for the branching ratio of the radiative decay $\phi \to \pi^+\pi^-\gamma$ is nearly twice the value for the branching ratio of the radiative decay $\phi \to \pi^0\pi^0\gamma$ that was obtained by Achasov and Gubin [22]. Besides, $\phi \to \pi^+\pi^-\gamma$ decay was considered by Marco et al. [15] in the framework of unitarized chiral perturbation theory. The branching ratio for $\phi \to \pi^+\pi^-\gamma$, they obtained, was $BR(\phi \to \pi^+\pi^-\gamma) = 1.6 \times 10^{-4}$ and for $\phi \to \pi^0\pi^0\gamma$ was $BR(\phi \to \pi^0\pi^0\gamma) = 0.8 \times 10^{-4}$. As we mentioned above, they noted that the branching ratio for $\phi \to \pi^0\pi^0\gamma$ is one half of $\phi \to \pi^+\pi^-\gamma$. Therefore our calculation for the branching ratio of $\phi \to \pi^+\pi^-\gamma$ decay is in accordance with the theoretical expectations. A similar relation can be seen between the decay rates of $\omega \to \pi^+\pi^-\gamma$ and $\omega \to \pi^0\pi^0\gamma$ [27]. It was noticed that $\Gamma(\omega \to \pi^0\pi^0\gamma) = 1/2\Gamma(\omega \to \pi^+\pi^-\gamma)$ and the factor 1/2 is a result of charge conjugation invariance to order α which imposes pion pairs of even angular momentum. The experimental value of the branching ratio for $\phi \to \pi^+\pi^-\gamma$, measured by Akhmetshin et al., is $BR(\phi \to \pi^+\pi^-\gamma) = (0.41\pm0.12\pm0.04)\times10^{-4}$ [11]. So the value of the branching ratio that we obtained is approximately six times larger than the value of the measured branching ratio. As it was stated by Marco et al. [15], we should not compare our calculation for the branching ratio of the radiative decay $\phi \to \pi^+\pi^-\gamma$ directly with experiment since the experiment is done using the reaction $e^+e^- \to \phi \to \pi^+\pi^-\gamma$, which interferes with the off-shell ρ dominated amplitude coming from the reaction $e^+e^- \to \rho \to \pi^+\pi^-\gamma$ [49]. Also the result in [11] is based on model dependent assumptions.

3.2 Radiative $\phi \to \pi^0\eta\gamma$ decay and the coupling constants $g_{\phi a_0\gamma}$, $g_{a_0K^+K^-}$

In order to determine the coupling constants $g_{\phi a_0\gamma}$ in model I and $g_{a_0K^+K^-}$ in model II, we use the experimental value of the branching ratio for the radiative decay $\phi \to \pi^0\eta\gamma$ [26] in our calculation of this decay rate. As a result of this we arrive at a quadric equation for the coupling constant $g_{\phi a_0\gamma}$ in model I and another quadric equation for the coupling constant $g_{a_0K^+K^-}$ in model II. In the first quadric equation for the coupling constant $g_{\phi a_0\gamma}$ the coefficient of the quadric term results from a_0-meson contribution of Fig. 2.3(c) and the coefficient of the linear term from the interference of the a_0-meson with the vector meson dominance term of Fig. 2.3(a) and the kaon-loop terms of Fig. 2.3(b). In the other quadric equation for the coupling constant $g_{a_0K^+K^-}$, the coefficient of the quadric term results from the a_0-meson amplitude contribution shown in Fig. 2.4(c) and the coefficient of the linear term from the interference of the a_0-meson amplitude with the vector meson dominance and the kaon-loop amplitudes shown in Figs. 2.4(a) and 2.4(b) respectively. Therefore, our analysis results in two values for each of the coupling constants stated above. In model I, we obtain for the coupling constant $g_{\phi a_0\gamma}$ the values $g_{\phi a_0\gamma} = (0.24 \pm 0.06)$

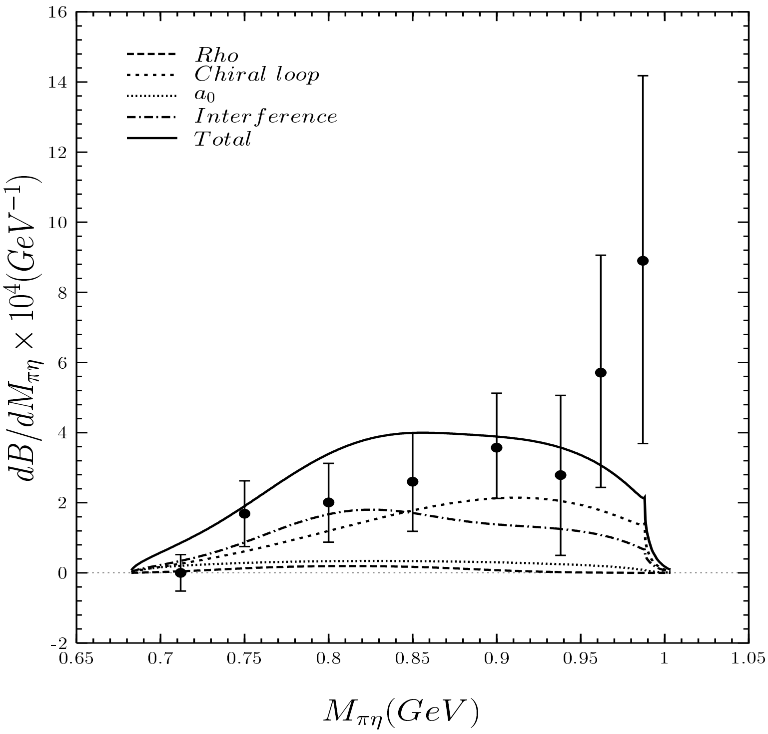

Figure 3.2: The $\pi^0\eta$ invariant mass spectrum for the decay $\phi \to \pi^0\eta\gamma$ for $g_{\phi a_0 \gamma} = 0.24$ in model I. The contributions of different terms are indicated.

and $g_{\phi a_0\gamma} = (-1.3 \pm 0.3)$ [50]. We then study the invariant mass distribution $dB/dM_{\pi^0\eta} = (M_{\pi^0\eta}/M_\phi)dB/dE_\gamma$ for the reaction $\phi \to \pi^0\eta\gamma$ in model I. In Fig. 3.2 we plot the invariant mass spectrum for the radiative decay $\phi \to \pi^0\eta\gamma$ in our phenomenological approach choosing the coupling constant $g_{\phi a_0\gamma} = (0.24 \pm 0.06)$. In this figure we indicate the contributions coming from different reactions $\phi \to \rho^0\pi^0 \to \pi^0\eta\gamma$, $\phi \to K^+K^-\gamma \to \pi^0\eta\gamma$ and $\phi \to a_0\gamma \to \pi^0\eta\gamma$ as well as the contribution of the total amplitude which includes the interference terms as well. Our results are in accordance with the experimental values [13] only in lower part of the invariant mass. It is expected that, the spectrum for the decay

$\phi \to \pi^0\eta\gamma$ is dominated by the a_0-amplitude but the expected enhancement due to the contribution of the a_0 resonance in the higher part of the invariant mass is not produced. Since the distribution $dB/dM_{\pi^0\eta}$ we obtain for the other root, that is for $g_{\phi a_0\gamma} = (-1.3 \pm 0.3)$, is worse than the distribution shown in Fig. 3.2 we do not show this in any figure. So model I does not produce a satisfactory description of the experimental invariant $M_{\pi^0\eta}$ mass spectrum for the decay $\phi \to \pi^0\eta\gamma$ and as a result of this the value of the coupling constant $g_{\phi a_0\gamma} = (0.24 \pm 0.06)$ can not be considered seriously [50]. Indeed, Gökalp et al. [51], used the same model in their study of scalar meson effects in radiative $\phi \to \pi^0\eta\gamma$ decay, noted that this approach does not give a reasonable a_0 contribution since the expected enhancement in the higher part of the invariant mass spectrum due to the contribution of a_0 resonance is not produced. On the other hand the value of the coupling constant $g_{\phi a_0\gamma}$ has been calculated by Gökalp and Yılmaz [52] in their study of the $\phi a_0\gamma$ and $\phi\sigma\gamma$ vertices in the light cone QCD. Utilizing $\omega\phi$-mixing, they estimated the coupling constant $g_{\phi a_0\gamma}$ as $g_{\phi a_0\gamma} = (0.11 \pm 0.03)$. Moreover, the ρ^0-meson photoproduction cross-section on proton targets near threshold is given mainly by σ-exchange [41]. Friman and Soyeur calculated $\rho\sigma\gamma$-vertex assuming vector meson dominance of the electromagnetic current and obtained the value of the coupling constant $g_{\rho\sigma\gamma}$ as $g_{\rho\sigma\gamma} \approx 2.71$. Later, Titov et al. [37] in their study of the structure of the ϕ-meson photoproduction amplitude based on one-meson exchange and Pomeron exchange mechanism used this value of the coupling constant $g_{\rho\sigma\gamma}$ to calculate the coupling constant $g_{\phi a_0\gamma}$. Their

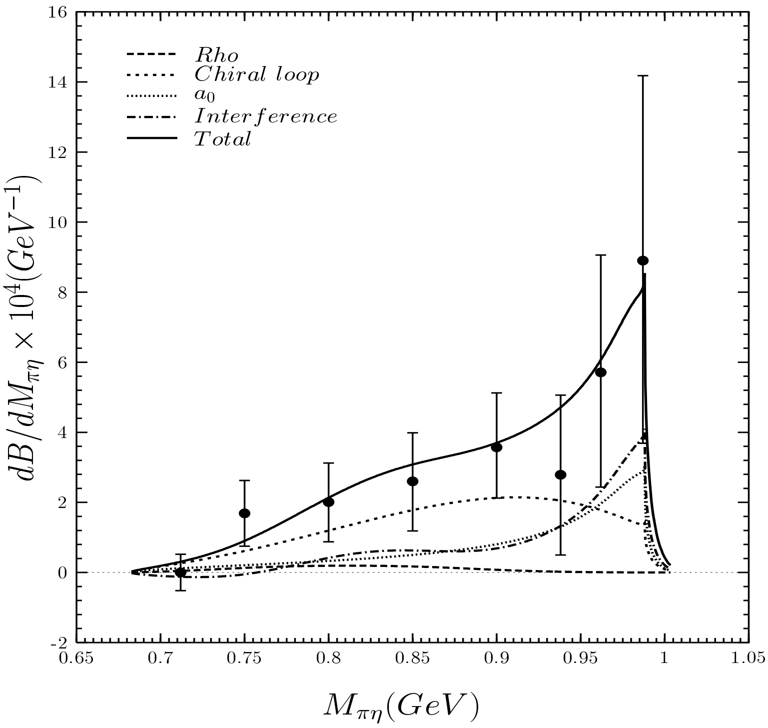

Figure 3.3: The $\pi^0\eta$ invariant mass spectrum for the decay $\phi \rightarrow \pi^0\eta\gamma$ for $g_{a_0K^+K^-} = -1.5$ in model II. The contributions of different terms are indicated.

result for this coupling constant was $|g_{\phi a_0\gamma}| = 0.16$. Our results for the coupling constant $g_{\phi a_0\gamma}$ are different than the values used in literature. Consequently, the contribution of the a_0-meson to the decay mechanism of $\phi \rightarrow \pi^0\eta\gamma$ decay should not be considered as resulting from a_0-pole intermediate state [50, 51]. Therefore, another model, called model II, is developed to obtain a reasonable a_0 contribution to the decay mechanism of this decay. The same procedure is followed in model II and utilizing the experimental value of the $\phi \rightarrow \pi^0\eta\gamma$ decay rate, the values for the coupling constant $g_{a_0K^+K^-}$ are obtained as $g_{a_0K^+K^-} = (-1.5 \pm 0.3)$ and $g_{a_0K^+K^-} = (3.0 \pm 0.4)$ [50]. We plot the distribution $dB/dM_{\pi^0\eta}$

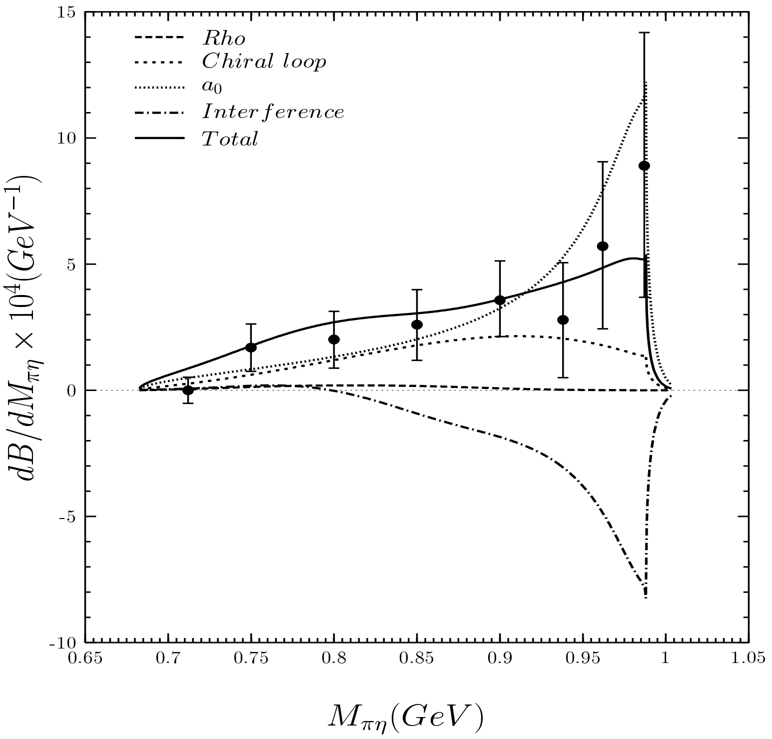

Figure 3.4: The $\pi^0\eta$ invariant mass spectrum for the decay $\phi \to \pi^0\eta\gamma$ for $g_{a_0K^+K^-} = 3.0$ in model II. The contributions of different terms are indicated.

for the radiative decay $\phi \to \pi^0\eta\gamma$ choosing coupling constants $g_{a_0K^+K^-} = -1.5$ in Fig. 3.3 and $g_{a_0K^+K^-} = 3.0$ in Fig. 3.4 as a function of the invariant mass $M_{\pi^0\eta}$ of the $\pi^0\eta$ system. In these figures we indicate the contributions coming from different reactions shown diagrammatically in Fig. 2.4 as well as the contribution of the total amplitude which includes the interference term as well. On the same figures we also show the experimental data points taken from Ref. [13]. As it can be seen in Fig. 3.3 the shape of the invariant mass distribution is reproduced well. As expected the enhancement caused by the contribution of the a_0 resonance is well produced on this figure. On the other hand, $\pi^0\eta$ invariant mass spectrum for $g_{a_0K^+K^-} = 3.0$ is not in good agreement with the experimental result. Therefore from the analysis of the spectrum obtained with the coupling constants $g_{a_0K^+K^-} = -1.5$ and $g_{a_0K^+K^-} = 3.0$ in Figs. 3.3 and 3.4 respectively, we may decide in favour of the value $g_{a_0K^+K^-} = -1.5$. Furthermore, we note that model II provides a better way, as compared to model I, in order to include the a_0-meson into the mechanism of the $\phi \rightarrow \pi^0\eta\gamma$ decay and thus our result supports the approach in which the a_0-meson state arises as a dynamical state. Consequently, a_0-meson should be considered to couple to the ϕ meson through a kaon-loop. Moreover it is possible to estimate the decay rate $\Gamma(\phi \rightarrow a_0\gamma)$ of the decay $\phi \rightarrow a_0(980)\gamma$. Using the coupling constant $g_{a_0K^+K^-} = -1.5$ we obtain the decay rate $\Gamma(\phi \rightarrow a_0\gamma)$, the expression of which is given in detail in Appendix A, as $\Gamma(\phi \rightarrow a_0\gamma) = (0.51 \pm 0.09)\ keV$, so the branching ratio is $BR(\phi \rightarrow a_0\gamma) = (1.1 \pm 0.2) \times 10^{-4}$. If we compare our result with the experimental value $BR(\phi \rightarrow a_0\gamma) = (0.88 \pm 0.17) \times 10^{-4}$ [13], we observe that our result does not contradict the experimental one.

3.3 Radiative $\rho^0 \rightarrow \pi^+\pi^-\gamma$ and $\rho^0 \rightarrow \pi^0\pi^0\gamma$ decays

The photon spectra for the branching ratio of the decay $\rho^0 \rightarrow \pi^+\pi^-\gamma$ is plotted in Fig. 3.5 as a function of photon energy E_γ. In this figure, the contributions from the pion-bremsstrahlung, pion-loop and σ-meson intermediate state amplitudes as well as the contribution of the interference term are indicated as a function of the photon energy. We take the minimum photon energy

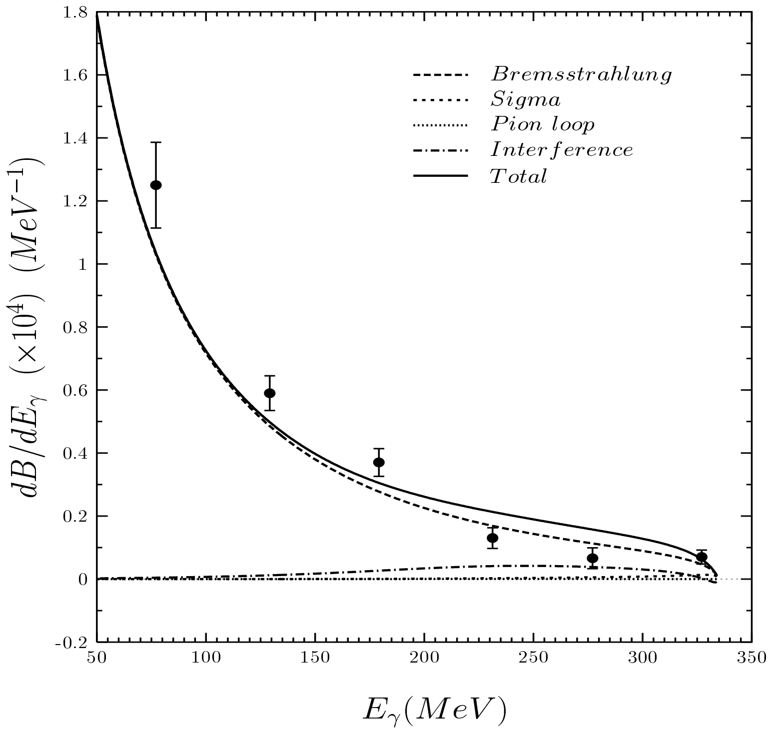

Figure 3.5: The photon spectra for the branching ratio of $\rho^0 \to \pi^+\pi^-\gamma$ decay. The contributions of different terms are indicated. The experimental data taken from Ref. [25] are normalized to our results.

as $E_{\gamma,min} = 50\ MeV$ since the experimental value of the branching ratio is determined for this range of photon energies [25]. We show also the experimental data points [25] on this figure. As shown in Fig. 3.5 the shape of the photon energy distribution is in good agreement with the experimental spectrum. In our calculation we observe that the contribution of the pion-bremsstrahlung amplitude to the branching ratio is much larger than the contributions of the rest. It is clearly seen that contributions of the pion-loop and σ-meson intermediate states can be noticed only in the region of high photon energies. It is useful to state that, if a σ-meson pole model is used as in Ref. [29], the contribution of the sigma term becomes larger at high photon energies and this enhancement, dominating the contribution of the bremsstrahlung amplitude, conflicts with the experimental spectrum. The contributions of bremsstrahlung amplitude, pion-loop amplitude and σ-meson intermediate state amplitude to the branching ratio of the decay are $BR(\rho^0 \to \pi^+\pi^-\gamma)_\gamma = (1.14 \pm 0.01) \times 10^{-2}$, $BR(\rho^0 \to \pi^+\pi^-\gamma)_\pi = (0.45 \pm 0.08) \times 10^{-5}$ and $BR(\rho^0 \to \pi^+\pi^-\gamma)_\sigma = (0.83 \pm 0.16) \times 10^{-4}$, respectively. If the interference term is considered between the pion-loop and σ-meson amplitudes, then the contribution coming from the structural radiation which includes the pion-loop and σ-meson intermediate state amplitudes as well as their interference is obtained as $BR(\rho^0 \to \pi^+\pi^-\gamma) = (0.83 \pm 0.14) \times 10^{-4}$. As a consequence this result agrees well with the experimental limit $BR(\rho^0 \to \pi^+\pi^-\gamma) < 5 \times 10^{-3}$ [25] for the structural radiation. Also our result for the contribution of the σ-meson intermediate state $BR(\rho^0 \to \pi^+\pi^-\gamma)_\sigma = (0.83 \pm 0.16) \times 10^{-4}$ is in accordance with the experimental limit $BR(\rho^0 \to \epsilon(700)\gamma \to \pi^+\pi^-\gamma) < 4 \times 10^{-4}$ where the transition proceeds through the intermediate scalar resonance [25]. For the total branching ratio, including the interference terms, we obtain the result $BR(\rho^0 \to \pi^+\pi^-\gamma) = (1.22 \pm 0.02) \times 10^{-2}$ for $E_\gamma > 50\ MeV$ [53]. Therefore, our result for the decay $\rho^0 \to \pi^+\pi^-\gamma$ is in good agreement with the experimental number $BR(\rho^0 \to \pi^+\pi^-\gamma) = (0.99 \pm 0.16) \times 10^{-2}$ [25].

The photon spectra for the branching ratio of the decay $\rho^0 \to \pi^0\pi^0\gamma$ is shown in Fig. 3.6. In this figure the contributions of VMD amplitude, the pion-loop amplitude and σ-meson intermediate state amplitude as well as the

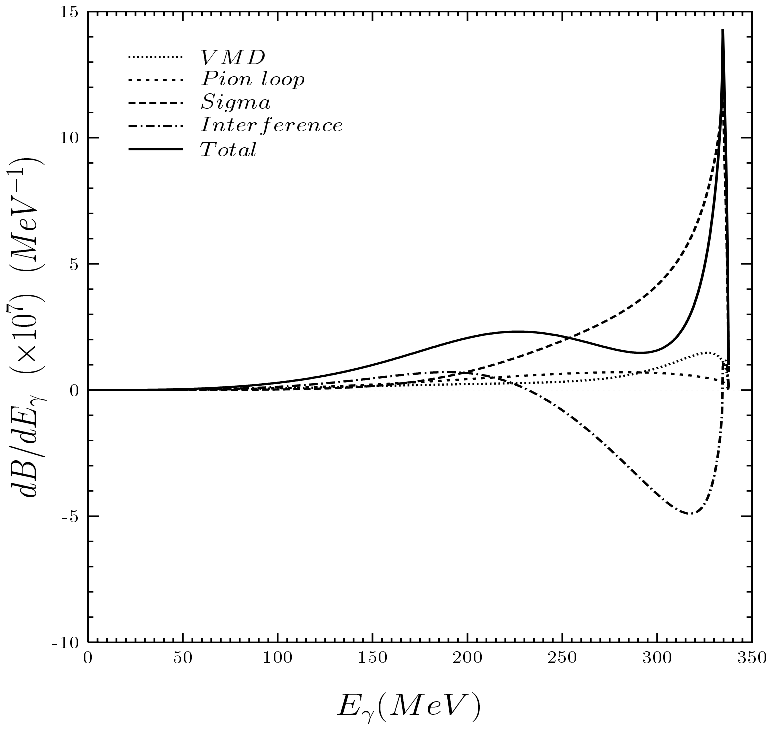

Figure 3.6: The photon spectra for the branching ratio of $\rho^0 \to \pi^0\pi^0\gamma$ decay. The contributions of different terms are indicated.

contributions of the interference terms are indicated. As it can be seen in Fig. 3.6 σ-meson amplitude contribution to the overall branching ratio for this decay is quite significant. This figure clearly shows the importance of the σ-meson amplitude term. We see that the dominant σ- term characterizes the photon spectrum which peaks at high photon energies and the contributions of vector meson intermediate state and pion-loop amplitudes are only noticeable in the region of high photon energies. Also it is clearly seen from this figure that total interference term is destructive for this decay. For the contribution of different amplitudes to the branching ratio the following results are obtained; $BR(\rho^0 \to \pi^0\pi^0\gamma)_{VMD} = (1.03 \pm 0.02) \times 10^{-5}$ from the VMD amplitude, $BR(\rho^0 \to \pi^0\pi^0\gamma)_{\pi} = (1.07 \pm 0.02) \times 10^{-5}$ from the pion-loop amplitude and $BR(\rho^0 \to \pi^0\pi^0\gamma)_{\sigma} = (4.96 \pm 0.18) \times 10^{-5}$ from the σ-meson intermediate state amplitude [53]. In our calculation we observe that the contribution of the σ-meson intermediate state amplitude is much larger than the contributions of VMD and pion-loop amplitudes. We also notice that the values for $BR(\rho^0 \to \pi^0\pi^0\gamma)_{VMD}$ and for $BR(\rho^0 \to \pi^0\pi^0\gamma)_{\pi}$ are in agreement well with previous calculations [19, 20]. Further the value, we obtain for the total branching ratio $BR(\rho^0 \to \pi^0\pi^0\gamma) = (4.95 \pm 0.82) \times 10^{-5}$, agrees well with the experimental result $BR(\rho^0 \to \pi^0\pi^0\gamma) = (4.1^{+1.0}_{-0.9} \pm 0.3) \times 10^{-5}$ [23] and the theoretical result $BR(\rho^0 \to \pi^0\pi^0\gamma) = 4.2 \times 10^{-5}$ [31]. Indeed, the total branching ratios that we obtained for $\rho^0 \to \pi^+\pi^-\gamma$ and $\rho^0 \to \pi^0\pi^0\gamma$ decays are quite compatible with the results obtained by Bramon and Escribano [32] in their study of $\rho^0 \to \pi\pi\gamma$ decays including $\sigma(500)$ meson effects. However, in the limit of high M_σ their results for the branching ratios of the radiative ρ^0-meson decays are in conflict with our results as well as with the conclusions of Marco et al. [15] and Palomar et al. [31]. In our calculations of the branching ratios, the coupling constants are determined from the relevant experimental quantities. Our results for the branching ratios are in accordance with the experimental values. Therefore, we propose that the contribution coming from σ-meson intermediate state amplitude should be included in the analysis of radiative ρ^0-meson decays and also σ-meson should be considered to couple to the ρ^0-meson through a pion-loop. In addition to the radiative ρ^0-meson decays, the effect of σ-meson in the radiative $\omega \rightarrow \pi^+\pi^-\gamma$ decay has been studied by Gökalp et al. [54] and it is noted that the σ-meson intermediate state amplitude makes a substantial contribution to the branching ratio of this decay.

CHAPTER 4

CONCLUSIONS

In our work we obtain the following conclusions:

• It is shown that for the radiative $\phi \to \pi^+\pi^-\gamma$ decay, the dominant f_0-meson amplitude term characterizes the invariant mass distribution in the region where $M_{\pi\pi} > 0.7\ GeV$ and the contributions coming from σ-meson and ρ-meson amplitudes are much smaller than the f_0-meson contribution.

• It is observed that there is a discrepancy between the experimental and the theoretical results for the branching ratio of the radiative $\phi \to \pi^+\pi^-\gamma$ decay. However, the theoretical result for the branching ratio of this decay should not be compared directly with the experimental one because of the fact that this experiment is done using the reaction $e^+e^- \to \phi \to \pi^+\pi^-\gamma$, which interferes with the off-shell ρ dominated amplitude coming from the reaction $e^+e^- \to \rho \to \pi^+\pi^-\gamma$.

• Two different values, one being positive and the other one negative, have been obtained for the coupling constant $g_{\phi a_0 \gamma}$ for the decay $\phi \to \pi^0\eta\gamma$ in model I.

• It is shown that for the positive value of the coupling constant $g_{\phi a_0 \gamma}$, the

$\pi^0\eta$ invariant mass spectrum is dominated by the a_0-meson amplitude but the expected enhancement due to the a_0-meson contribution in the higher part of the invariant mass is not produced. Also the distribution $dB/dM_{\pi^0\eta}$ for the negative value of the coupling constant $g_{\phi a_0\gamma}$ is worse than the one for the positive value. This can be interpreted as that the obtained values for the coupling constant $g_{\phi a_0\gamma}$ should not be considered too seriously and the contribution of the a_0-meson to the decay mechanism of the $\phi \to \pi^0\eta\gamma$ decay can not be considered as resulting from a_0-pole intermediate state.

• For the coupling constant $g_{a_0K^+K^-}$ in model II where a_0-meson state arises as a dynamical state, two different values having positive and negative signs have been obtained. It is demonstrated that for the negative value of the coupling constant $g_{a_0K^+K^-}$, our prediction for the invariant mass spectrum is in accordance with the experimental result and both the overall shape and the expected enhancement due to the contribution of the a_0 resonance has been well produced.

• From the analysis of the invariant mass spectrum, plotted for both values of the coupling constant $g_{a_0K^+K^-}$, the negative value of the coupling constant $g_{a_0K^+K^-}$ is suggested.

• It is concluded that a_0-meson should be considered to couple to the ϕ meson through the charged kaon-loops in radiative $\phi \to \pi^0\eta\gamma$ decay.

• It is observed that for the radiative $\rho^0 \to \pi^+\pi^-\gamma$ decay the main contribution comes from pion-bremsstrahlung term and the contributions of the pion-loop and the σ-meson intermediate state amplitudes can only be noticed in the region of high photon energies. This agrees well with the experimental result.

• It is demonstrated that for the radiative $\rho^0 \to \pi^0\pi^0\gamma$ decay the dominant σ term characterizes the photon spectrum which peaks at high photon energies. The contributions coming from vector meson intermediate (VMD) state and pion-loop amplitudes are much smaller than the one coming from σ-meson amplitude.

• It is shown that the values for the branching ratios of the radiative $\rho^0 \to \pi^+\pi^-\gamma$ and $\rho^0 \to \pi^0\pi^0\gamma$ decays are in good agreement with the experimental values.

• It is concluded that the σ-meson should be considered to couple to the ρ^0-meson through the charged pion-loops in radiative $\rho^0 \to \pi^+\pi^-\gamma$ and $\rho^0 \to \pi^0\pi^0\gamma$ decays.

• In the future further experiments such as the measurements of the invariant mass distributions will enable us to understand the mechanism of the $\rho^0 \to \pi^0\pi^0\gamma$ decay and to obtain insight into the nature and the properties of the σ-meson, and the role it plays in the dynamics of low energy meson physics.

• The radiative decays of light vector mesons into a photon and two pseudoscalar mesons, $V \to PP'\gamma$, are becoming an area of active experimental research in laboratories such as Novosibirsk and Frascati to investigate the nature and extract the properties of light scalar meson resonances. These decays will provide valuable information on the properties of the $f_0(980)$, $a_0(980)$ and $\sigma(500)$ scalar mesons.

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APPENDIX A

TWO BODY DECAY RATES

If an initial state, defined by the state vector $\mid i >$, undergoes to a final state $\mid f >$ then the transition probability is given by $|S_{fi}|^2 =|< f|S \mid i > |^2$. The corresponding probability amplitude is

$$< f|S \mid i >= S_{fi} \quad . \tag{A.1}$$

The S-matrix element is defined as

$$< f|S \mid i >= \delta_{fi}+(2\pi)^4\delta^{(4)}\left(\sum p'_f - \sum p_i\right)\mathcal{M}_{fi}\prod_i\left(\frac{1}{2VE_i}\right)^{1/2}\prod_f\left(\frac{1}{2VE'_f}\right)^{1/2} \quad , \tag{A.2}$$

where $\mathcal{M}_{fi}$ is the invariant matrix element $p_i = (E_i,\ \vec{p_i})$ and $p'_f = (E'_f,\ \vec{p'_f})$ are the four momenta of the initial and final particles respectively. In this case transition probability per unit time becomes

$$\Gamma = V(2\pi)^4\delta^{(4)}\left(\sum p'_f - \sum p_i\right)|\mathcal{M}_{fi}|^2\prod_i\left(\frac{1}{2VE_i}\right)\prod_f\left(\frac{1}{2VE'_f}\right) \quad . \tag{A.3}$$

This is the transition rate to one definite final state. To obtain the transition rate to a group of final states with momenta in the intervals $(\vec{p'_f},\ \vec{p'_f} + d\vec{p'_f})$, $f = 1, ..., N$ we must multiply Γ by the number of these states which is

$$\prod_f\left(\frac{Vd^3p'_f}{(2\pi)^3}\right) \quad , \tag{A.4}$$

therefore, the differential decay rate becomes

$$d\Gamma = V(2\pi)^4\delta^{(4)}\left(\sum p'_f - \sum p_i\right)|\mathcal{M}_{fi}|^2\prod_i\left(\frac{1}{2VE_i}\right)\prod_f\left(\frac{d^3p'_f}{(2\pi)^3 2E'_f}\right) \quad . \tag{A.5}$$

For the decay of particle of mass M and energy E into any number of particles 1,2,...,N the differential decay rate is

$$d\Gamma = (2\pi)^4\delta^{(4)}\left(\sum p_f - \sum p_i\right)|\mathcal{M}_{fi}|^2\frac{1}{2E}\prod_f\left(\frac{d^3p_f}{(2\pi)^3 2E_f}\right) \quad . \tag{A.6}$$

If we consider the two body decay in which the decay produces two particles, then in the rest frame of decaying particle $\vec{p_1} = -\vec{p_2} \equiv \vec{p}$, $E_1 + E_2 = M$, the differential decay rate is given by

$$d\Gamma = \frac{1}{(2\pi)^2}|\mathcal{M}_{fi}|^2\frac{1}{2M}\frac{1}{4E_1E_2}\delta^{(3)}(\vec{p_1}+\vec{p_2})\delta(E_1+E_2-M)d^3p_1d^3p_2 \quad . \tag{A.7}$$

Integration over d^3p_2, eliminates the first delta function and the differential d^3p_1 is written as

$$d^3p = p^2d|\vec{p}|d\Omega = |\vec{p}|d\Omega\frac{E_1E_2d(E_1+E_2)}{E_1+E_2} \quad , \tag{A.8}$$

since $E_1^2 - M_1^2 = E_2^2 - M_2^2 = \vec{p}^2$. The second delta function is eliminated by integration over $(E_1 + E_2)$, and this gives

$$d\Gamma = \frac{1}{32\pi^2M^2}|\mathcal{M}_{fi}|^2|\vec{p}|d\Omega \quad . \tag{A.9}$$

Therefore, the decay rate is obtained as

$$\Gamma = \frac{1}{8\pi M^2}|\mathcal{M}_{fi}|^2|\vec{p}| \quad . \tag{A.10}$$

In the rest frame of decaying particle, $|\vec{p}|$ is given as

$$|\vec{p}| = \frac{1}{2M}\sqrt{[M^2-(M_1+M_2)^2][M^2-(M_1-M_2)^2]} \quad . \tag{A.11}$$

For the decay $M \to M_1 + M_2$ where $M_1 = M_2$

$$|\vec{p}| = \frac{1}{2}M\sqrt{1 - \left(\frac{2M_1}{M}\right)^2} \quad , \tag{A.12}$$

and for the decay $M \to M_1 + \gamma$

$$|\vec{p}| = \frac{1}{2}M\left[1 - \left(\frac{M_1}{M}\right)^2\right] \quad . \tag{A.13}$$

For the decay $\phi \to K^+K^-$, the invariant matrix element that follows from the effective Lagrangian

$$\mathcal{L}^{eff.}_{\phi K^+K^-} = -ig_{\phi K^+K^-}\phi^\mu(K^+\partial_\mu K^- - K^-\partial_\mu K^+) \quad , \tag{A.14}$$

is given by $\mathcal{M}(\phi \to K^+K^-) = -ig_{\phi K^+K^-}(2q_1 - p)_\mu u^\mu$, where q_1 is the four-momentum of the kaon having plus sign and p(u) is the four-momentum (polarization) of the decaying ϕ-meson. Therefore,

$$\Gamma(\phi \to K^+K^-) = \frac{g^2_{\phi K^+K^-}}{48\pi}M_\phi\left[1 - \left(\frac{2M_K}{M_\phi}\right)^2\right]^{3/2} \quad . \tag{A.15}$$

For the decay $\phi \to S\gamma$ (where $S = f_0$ or a_0), in which the ϕ and the S each couple strongly to $K\bar{K}$, with the couplings $g_{\phi K^+K^-}$ and $g_{SK^+K^-}$ for ϕK^+K^- and SK^+K^-, the invariant amplitude is obtained as

$$\mathcal{M}(\phi \to S\gamma) = u^\mu\epsilon^\nu(k_\mu p_\nu - g_{\mu\nu}k\cdot p)\frac{eg_{\phi K^+K^-}(g_{SK^+K^-}M_S)}{2\pi^2M_K^2}I(a,b) \quad , \tag{A.16}$$

where (u, p) and (ϵ, k) are the polarizations and four-momenta of the decaying vector meson and the photon respectively and $a = M_\phi^2/M_K^2$, $b = M_S^2/M_K^2$. Then,

$$\Gamma(\phi \to S\gamma) = \frac{\alpha}{6(2\pi)^4}\frac{M_\phi^2 - M_S^2}{M_\phi^3}g^2_{\phi K^+K^-}(g_{SK^+K^-}M_S)^2|(a-b)I(a,b)|^2 \quad . \tag{A.17}$$

For the decay $S \rightarrow \varphi\varphi$, the invariant matrix element that follows from the effective Lagrangian

$$\mathcal{L}^{eff.}_{S\varphi\varphi} = \frac{1}{2} g_{S\varphi\varphi} M_S \vec{\varphi} \cdot \vec{\varphi} S \quad , \tag{A.18}$$

is given by $\mathcal{M}(S \rightarrow \varphi\varphi) = i g_{S\varphi\varphi} M_S$, where S and φ denote scalar σ or f_0 meson and pseudoscalar π meson respectively. Therefore

$$\Gamma(S \rightarrow \varphi\varphi) = \frac{g^2_{S\varphi\varphi}}{4\pi} \frac{3M_S}{8} \left[1 - \left(\frac{2M_\varphi}{M_S} \right)^2 \right]^{1/2} \quad . \tag{A.19}$$

The radiative decay $V \rightarrow \varphi\gamma$ is described by the effective Lagrangian

$$\mathcal{L}^{eff.}_{V\varphi\gamma} = \frac{e}{M_V} g_{V\varphi\gamma} \epsilon^{\mu\nu\alpha\beta} \partial_\mu V_\nu \partial_\alpha A_\beta \varphi \quad , \tag{A.20}$$

where V denotes the decaying vector meson and φ denotes the pseudoscalar meson. The invariant amplitude is obtained as

$$\mathcal{M}(V \rightarrow \varphi\gamma) = i \frac{e}{M_V} g_{V\varphi\gamma} \epsilon^{\mu\nu\alpha\beta} p_\mu u_\nu k_\alpha \epsilon_\beta \quad , \tag{A.21}$$

where (u, p) and $(\epsilon,\ k)$ are the polarizations and four-momenta of the decaying vector meson and the photon respectively. Then, the decay rate is

$$\Gamma(V \rightarrow \varphi\gamma) = \frac{\alpha}{24} \frac{(M_V^2 - M_\varphi^2)^3}{M_V^5} g^2_{V\varphi\gamma} \quad . \tag{A.22}$$

For the decay $\phi \rightarrow a_0\gamma$, the invariant matrix element that is obtained by using the effective Lagrangian

$$\mathcal{L}^{eff.}_{\phi a_0\gamma} = \frac{e}{M_\phi} g_{\phi a_0\gamma} [\partial^\alpha \phi^\beta \partial_\alpha A_\beta - \partial^\alpha \phi^\beta \partial_\beta A_\alpha] a_0 \quad , \tag{A.23}$$

is given by $\mathcal{M}(\phi \to a_0\gamma) = i\frac{e}{M_\phi} g_{\phi a_0 \gamma} p^\alpha u^\beta (k_\alpha \epsilon_\beta - k_\beta \epsilon_\alpha)$, where (u, p) and ($\epsilon,\ k$) are the polarizations and four-momenta of the ϕ meson and the photon respectively. Therefore, the decay rate for the $\phi \to a_0\gamma$ decay is

$$\Gamma(\phi \to a_0\gamma) = \frac{\alpha}{24}\frac{(M_\phi^2 - M_{a_0}^2)^3}{M_\phi^5} g_{\phi a_0 \gamma}^2 \quad . \tag{A.24}$$

The decay rate for $a_0 \to \pi^0\eta$ decay resulting from the effective Lagrangian

$$\mathcal{L}^{eff.}_{a_0\pi\eta} = g_{a_0\pi\eta} \vec{\pi} \cdot \vec{a_0} \eta \quad , \tag{A.25}$$

is

$$\Gamma(a_0 \to \pi^0\eta) = \frac{g_{a_0\pi\eta}^2}{16\pi M_{a_0}} \sqrt{\left[1 - \frac{(M_{\pi^0} + M_\eta)^2}{M_{a_0}^2}\right]\left[1 - \frac{(M_{\pi^0} - M_\eta)^2}{M_{a_0}^2}\right]} \quad . \tag{A.26}$$

For the decay $a_0 \to K^+K^-$, described by the effective Lagrangian,

$$\mathcal{L}^{eff.}_{a_0 K^+K^-} = g_{a_0 K^+K^-} M_{a_0} K^+ K^- a_0 \quad , \tag{A.27}$$

the invariant matrix element is $\mathcal{M}(a_0 \to K^+K^-) = i g_{a_0K^+K^-} M_{a_0}$, therefore the decay rate is given by

$$\Gamma(a_0 \to K^+K^-) = \frac{g_{a_0K^+K^-}^2}{16\pi} M_{a_0} \left[1 - \left(\frac{2M_K}{M_{a_0}}\right)^2\right]^{1/2} \quad . \tag{A.28}$$

For the decay $\rho^0 \to \pi\pi$ which is described by the effective Lagrangian

$$\mathcal{L}^{eff.}_{\rho\pi\pi} = g_{\rho\pi\pi} \vec{\rho}_\mu \cdot (\partial^\mu \vec{\pi} \times \vec{\pi}) \quad , \tag{A.29}$$

the invariant matrix element is $\mathcal{M}(\rho \to \pi\pi) = i g_{\rho\pi\pi}(2q_1 - p)_\mu u^\mu$, where q_1 is the four momentum of one of the pions and p(u) is the four momentum (polarization) of the decaying ρ-meson. Then, the decay rate is obtained as

$$\Gamma(\rho \to \pi\pi) = \frac{g_{\rho\pi\pi}^2}{48\pi} M_\rho \left[1 - \left(\frac{2M_\pi}{M_\rho}\right)^2\right]^{3/2} \quad . \tag{A.30}$$

If the particle is off-shell, then two body decay rate becomes

$$\Gamma = \frac{1}{8\pi q^2}|\mathcal{M}_{fi}|^2|\vec{p}| \quad , \tag{A.31}$$

where q^2 is the four-momentum square of the off-shell particle. In the rest frame of decaying off-shell particle $\vec{p_1} = -\vec{p_2} \equiv \vec{p}$, $E_1 + E_2 = \sqrt{q^2}$, and $|\vec{p}|$ is given as

$$|\vec{p}| = \frac{1}{2\sqrt{q^2}}\left\{\left[q^2 - (M_1 + M_2)^2\right]\left[q^2 - (M_1 - M_2)^2\right]\right\}^{1/2} \quad . \tag{A.32}$$

For the decay $\sqrt{q^2} \rightarrow M_1 + M_2$ where $M_1 = M_2$

$$|\vec{p}| = \frac{1}{2}\sqrt{q^2}\left[1 - \left(\frac{2M_1}{\sqrt{q^2}}\right)^2\right]^{1/2} \quad . \tag{A.33}$$

So, for the $\sigma \rightarrow \pi\pi$ decay where σ meson is an off-shell particle the matrix element is $ig_{\sigma\pi\pi}M_\sigma$ that follows from the effective Lagrangian

$$\mathcal{L}^{eff.}_{\sigma\pi\pi} = \frac{1}{2}g_{\sigma\pi\pi}M_\sigma\vec{\pi}\cdot\vec{\pi}\sigma \quad . \tag{A.34}$$

The energy dependent $\Gamma^\sigma_{\pi\pi}(q^2)$ decay width is given by

$$\begin{aligned}
\Gamma^\sigma_{\pi\pi}(q^2) &= \Gamma^\sigma_{\pi^+\pi^-}(q^2) + \Gamma^\sigma_{\pi^0\pi^0}(q^2) \\
&= \Gamma^\sigma_{\pi^+\pi^-}(q^2 = M_\sigma^2)\frac{M_\sigma^2}{q^2}\sqrt{\frac{q^2 - 4M^2_{\pi^+}}{M^2_\sigma - 4M^2_{\pi^+}}} \\
&\quad + \Gamma^\sigma_{\pi^0\pi^0}(q^2 = M_\sigma^2)\frac{M_\sigma^2}{q^2}\sqrt{\frac{q^2 - 4M^2_{\pi^0}}{M^2_\sigma - 4M^2_{\pi^0}}} \quad ,
\end{aligned} \tag{A.35}$$

or

$$\begin{aligned}
\Gamma^\sigma_{\pi\pi}(q^2) &= \frac{2}{3}\Gamma^\sigma_{\pi\pi}(q^2 = M_\sigma^2)\frac{M_\sigma^2}{q^2}\sqrt{\frac{q^2 - 4M^2_{\pi^+}}{M^2_\sigma - 4M^2_{\pi^+}}}\ \theta\left(q^2 - 4M^2_{\pi^+}\right) \\
&\quad + \frac{1}{3}\Gamma^\sigma_{\pi\pi}(q^2 = M_\sigma^2)\frac{M_\sigma^2}{q^2}\sqrt{\frac{q^2 - 4M^2_{\pi^0}}{M^2_\sigma - 4M^2_{\pi^0}}}\ \theta\left(q^2 - 4M^2_{\pi^0}\right) .
\end{aligned} \tag{A.36}$$

For $\rho \to \pi\pi$ decay where ρ meson is an off-shell particle, the absolute square of the matrix element is

$$|\mathcal{M}(\rho \to \pi\pi)|^2 = \frac{1}{3} g^2_{\rho\pi\pi}(q^2 - 4M_\pi^2) \quad , \tag{A.37}$$

this results from the effective Lagrangian $\mathcal{L}^{eff.}_{\rho\pi\pi} = g_{\rho\pi\pi}\vec{\rho}_\mu \cdot (\partial^\mu \vec{\pi} \times \vec{\pi})$ so the energy dependent $\Gamma^\rho_{\pi\pi}(q^2)$ decay width is

$$\begin{aligned} \Gamma^\rho_{\pi\pi}(q^2) &= \frac{1}{48\pi q^2} g^2_{\rho\pi\pi}(q^2 - 4M_\pi^2)^{3/2} \\ &= \Gamma^\rho_{\pi\pi}(q^2 = M_\rho^2)\frac{M_\rho^2}{q^2}\left(\frac{q^2 - 4M_\pi^2}{M_\rho^2 - 4M_\pi^2}\right)^{3/2} \quad . \end{aligned} \tag{A.38}$$

For $a_0 \to \pi\eta$ decay where a_0 meson is an off-shell particle the absolute square of the matrix element is

$$|\mathcal{M}(a_0 \to \pi\eta)|^2 = g^2_{a_0\pi\eta} \quad , \tag{A.39}$$

which follows from the effective Lagrangian $\mathcal{L}^{eff.}_{a_0\pi\eta} = g_{a_0\pi\eta}\vec{\pi} \cdot \vec{a_0}\eta$, then the energy dependent $\Gamma^{a_0}_{\pi\eta}(q^2)$ decay width is given by

$$\Gamma^{a_0}_{\pi\eta}(q^2) = \frac{g^2_{a_0\pi\eta}}{16\pi(q^2)^{3/2}}\sqrt{[q^2 - (M_{\pi^0} + M_\eta)^2]\,[q^2 - (M_{\pi^0} - M_\eta)^2]} \quad . \tag{A.40}$$

For $f_0 \to \pi\pi$ decay where f_0 meson is an off-shell particle the matrix element that follows from the effective Lagrangian

$$\mathcal{L}^{eff.}_{f_0\pi\pi} = \frac{1}{2} g_{f_0\pi\pi} M_{f_0} \vec{\pi} \cdot \vec{\pi} f_0 \quad , \tag{A.41}$$

is $i g_{f_0\pi\pi} M_{f_0}$ and the energy dependent $\Gamma^{f_0}_{\pi\pi}(q^2)$ decay width is given by

$$\Gamma^{f_0}_{\pi\pi}(q^2) = \Gamma^{f_0}_{\pi^+\pi^-}(q^2) + \frac{1}{2}\Gamma^{f_0}_{\pi^0\pi^0}(q^2)$$

$$
\begin{aligned}
&= \frac{1}{16\pi q^2} g^2_{f_0\pi\pi} M^2_{f_0} [q^2 - 4M^2_\pi]^{1/2} \left(1 + \frac{1}{2}\right) \\
&= \frac{3}{32\pi q^2} g^2_{f_0\pi\pi} M^2_{f_0} [q^2 - 4M^2_\pi]^{1/2} \\
&= \Gamma^{f_0}_{\pi\pi}(q^2 = M^2_{f_0}) \frac{M^2_{f_0}}{q^2} \left[\frac{q^2 - 4M^2_\pi}{M^2_{f_0} - 4M^2_\pi}\right]^{1/2} \quad . \qquad \text{(A.42)}
\end{aligned}
$$

A similar expression is used for $\Gamma^{f_0}_{K\overline{K}}(q^2)$

$$
\Gamma^{f_0}_{K\overline{K}}(q^2) = \Gamma^{f_0}_{K\overline{K}}(q^2 = M^2_{f_0}) \frac{M^2_{f_0}}{q^2} \left[\frac{q^2 - 4M^2_K}{M^2_{f_0} - 4M^2_K}\right]^{1/2} \quad . \qquad \text{(A.43)}
$$

Utilizing the experimental values of different two body decay rates, the various coupling constants are determined. The calculated coupling constants are given in Table 1.

Table A.1: The experimental decay widths of various two body decays and the calculated coupling constants.

Meson	Mass (MeV)	Γ (MeV)	Decay Mode	BR	Coupling Constant
ϕ	1020	4.46	K^+K^-	4.9×10^{-1}	$g_{\phi K^+K^-}= 4.59$
			$f_0\gamma$	3.4×10^{-4}	$g_{f_0 K^+K^-}= 4.13$
σ	478	324	$\pi\pi$	dominant	$g_{\sigma\pi\pi}$=5.25
f_0	980	70	$\pi\pi$	dominant	$g_{f_0\pi\pi}= 1.58$
ρ^0	770	150.2	$\pi^0\gamma$	6.8×10^{-4}	$g_{\rho\pi\gamma}= 0.69$
			$\eta\gamma$	2.4×10^{-4}	$g_{\rho\eta\gamma}= 1.14$
			$\pi\pi$	~ 100 %	$g_{\rho\pi\pi}= 6.03$
a_0	980	69	$\eta\pi^0$	dominant	$g_{a_0\pi\eta}= 2.34$
ω	782	8.44	$\pi^0\gamma$	8.5×10^{-2}	$g_{\omega\pi\gamma}= 1.82$

APPENDIX B

THREE BODY DECAY AND THE BOUNDARY OF DALITZ PLOT

The differential decay rate for the decay of a particle P with four momentum $p = (E_p,\ \vec{p})$ into N particles with four-momenta $p'_f = (E'_f,\ \vec{p'}_f)$ is given by

$$d\Gamma = (2\pi)^4 \delta^{(4)} \left(\sum p'_f - p\right) \frac{1}{2E_p} \prod_f \frac{d^3\vec{p'}_f}{(2\pi)^3(2E'_f)} \overline{|\mathcal{M}_{fi}|^2} \quad . \tag{B.1}$$

The differential decay rate for the three particle decay $M(p) \to M_1(q_1)+M_2(q_2)+\gamma(k)$ becomes

$$d\Gamma = (2\pi)^4\delta^{(4)}(q_1+q_2+k-p)\frac{1}{2E_p}\frac{d^3q_1}{(2\pi)^3(2E_1)}\frac{d^3q_2}{(2\pi)^3(2E_2)}\frac{d^3k}{(2\pi)^3(2E_\gamma)}\overline{|\mathcal{M}_{fi}|^2} \quad , \tag{B.2}$$

where $\overline{|\mathcal{M}_{fi}|^2}$ is the average over spin states of the absolute square of the decay invariant matrix element. In the rest frame of decaying particle the $\delta^{(4)}$ function can be written as $\delta^{(4)}(q_1 + q_2 + k - p) = \delta(E_1 + E_2 + E_\gamma - M)\delta^{(3)}(\vec{q_1} + \vec{q_2} + \vec{k})$ and to eliminate the $\delta^{(4)}$ function first integrate over the (three-) momentum of the final-state particle with momentum q_2. Using

$$\frac{d^3q_1}{2E_1} = \frac{|\vec{q_1}|^2 dq_1 d\Omega_1}{2E_1} = \frac{1}{2}|\vec{q_1}|dE_1 d\Omega_1 \quad , \tag{B.3}$$

and

$$\frac{d^3k}{2E_\gamma} = \frac{|\vec{k}|^2 dk d\Omega_\gamma}{2E_\gamma} = \frac{1}{2}E_\gamma dE_\gamma d\Omega_\gamma \quad , \tag{B.4}$$

we obtain

$$\frac{d\Gamma}{dE_\gamma dE_1} = \frac{|\vec{q_1}|E_\gamma\overline{|\mathcal{M}_{fi}|^2}}{16M(2\pi)^5}\int d\Omega_\gamma d\Omega_1 \frac{\delta(E_\gamma + E_1 - M + \sqrt{(\vec{k}+\vec{q_1})^2 + M_2^2})}{\sqrt{(\vec{k}+\vec{q_1})^2 + M_2^2}} \quad . \tag{B.5}$$

If the integral I is defined by

$$I = |\vec{q_1}|E_\gamma \int d\Omega_\gamma d\Omega_1 \frac{\delta(E_\gamma + E_1 - M + \sqrt{(\vec{k}+\vec{q_1})^2 + M_2^2})}{\sqrt{(\vec{k}+\vec{q_1})^2 + M_2^2}} \quad , \tag{B.6}$$

and the angular integrals are performed then we obtain

$$I = 8\pi^2 \int_{-1}^{1} d(\cos\theta)|\vec{q_1}|E_\gamma \frac{\delta(E_\gamma + E_1 - M + \sqrt{E_\gamma^2 + E_1^2 - M_1^2 + 2E_\gamma|\vec{q_1}|\cos\theta + M_2^2})}{\sqrt{E_\gamma^2 + E_1^2 - M_1^2 + 2E_\gamma|\vec{q_1}|\cos\theta + M_2^2}} \quad , \tag{B.7}$$

where θ is defined by $\vec{k}\cdot\vec{q_1} = |\vec{k}||\vec{q_1}|\cos\theta$. Change of variable

$$\xi = \sqrt{E_\gamma^2 + E_1^2 - M_1^2 + 2E_\gamma|\vec{q_1}|\cos\theta + M_2^2} \quad , \tag{B.8}$$

gives

$$I = 8\pi^2 \int d\xi \delta(E_\gamma + E_1 - M + \xi) = 8\pi^2 \quad , \tag{B.9}$$

with the condition $E_\gamma + E_1 - M + \xi = 0$. Therefore the double differential decay rate is obtained as

$$\frac{d\Gamma}{dE_\gamma dE_1} = \frac{1}{(2\pi)^3}\frac{1}{8M}\overline{|\mathcal{M}_{fi}|^2} \quad . \tag{B.10}$$

The limits of integral are defined by the condition

$$M - E_\gamma - E_1 = \sqrt{E_\gamma^2 + E_1^2 - M_1^2 + 2E_\gamma|\vec{q_1}|\cos\theta + M_2^2} \quad , \tag{B.11}$$

or

$$-1 \leq \frac{(M - E_\gamma - E_1)^2 - (E_\gamma^2 + E_1^2 - M_1^2 + M_2^2)}{2E_\gamma\sqrt{E_1^2 - M_1^2}} \leq 1 \quad . \tag{B.12}$$

APPENDIX C

INVARIANT AMPLITUDE OF THE RADIATIVE $\phi \to \pi^+\pi^-\gamma$ DECAY

For the radiative decay $\phi(p) \to \pi^+(q_1)\pi^-(q_2)\gamma(k)$, the invariant amplitude $\mathcal{M}(E_\gamma, E_1)$ is expressed as

$$\mathcal{M} = \mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c + \mathcal{M}_d \quad , \tag{C.1}$$

where $\mathcal{M}_a$, $\mathcal{M}_b$, $\mathcal{M}_c$ and $\mathcal{M}_d$ are the invariant amplitudes resulting from the diagrams (a), (b), (c) and (d) in Fig. 2.2 respectively,

$$\begin{aligned} \mathcal{M}_a &= -\left(\frac{e}{M_\phi} g_{\phi\sigma\gamma}\right)(g_{\sigma\pi\pi} M_\sigma)\, p^\alpha u^\beta (k_\alpha \epsilon_\beta - k_\beta \epsilon_\alpha) \\ &\quad \times \left\{ i[(p-k)^2 - M_\sigma^2] + \Gamma_\sigma M_\sigma \right\} \Delta_\sigma^0(p-k) \quad , \end{aligned} \tag{C.2}$$

$$\begin{aligned} \mathcal{M}_b &= -\left(\frac{g_{\phi\rho\pi}}{M_\phi}\right)\left(\frac{e}{M_\rho} g_{\rho\pi\gamma}\right) \epsilon^{\mu\nu\alpha\beta} p_\mu u_\nu (p-q_1)_\alpha \epsilon^{\mu'\nu'\alpha'\beta'} (p-q_1)_{\mu'} k_{\alpha'} \epsilon_{\beta'} \\ &\quad \times \left\{ i[(p-q_1)^2 - M_\rho^2] + \Gamma_\rho M_\rho \right\} R^\rho_{\beta\nu'}(p-q_1) \quad , \end{aligned} \tag{C.3}$$

$$\begin{aligned} \mathcal{M}_c &= -\left(\frac{g_{\phi\rho\pi}}{M_\phi}\right)\left(\frac{e}{M_\rho} g_{\rho\pi\gamma}\right) \epsilon^{\mu\nu\alpha\beta} p_\mu u_\nu (p-q_2)_\alpha \epsilon^{\mu'\nu'\alpha'\beta'} (p-q_2)_{\mu'} k_{\alpha'} \epsilon_{\beta'} \\ &\quad \times \left\{ i[(p-q_2)^2 - M_\rho^2] + \Gamma_\rho M_\rho \right\} R^\rho_{\beta\nu'}(p-q_2) \quad , \end{aligned} \tag{C.4}$$

$$
\begin{aligned}
\mathcal{M}_d &= -\left(e g_{\phi K K}\right)\left(g_{f_0 \pi \pi} M_{f_0}\right)\left(g_{f_0 K K} M_{f_0}\right) \\
&\quad \times\left\{\left[(p-k)^2-M_{f_0}^2\right]-i \Gamma_{f_0} M_{f_0}\right\} \Delta_{f_0}^0(p-k) \\
&\quad \times 2 u^\mu \epsilon^\nu \int \frac{d^4 q}{(2 \pi)^4}\left\{\frac{-4 q_\mu q_\nu+g_{\mu \nu}\left(q^2-M_K^2\right)}{\left(q^2-M_K^2\right)\left[(q-k)^2-M_K^2\right]\left[(p-q)^2-M_K^2\right]}\right\} \\
&= \frac{\left(e g_{\phi K K}\right)\left(g_{f_0 \pi \pi} M_{f_0}\right)\left(g_{f_0 K K} M_{f_0}\right)}{2 \pi^2 M_K^2}\left\{\left[(p-k)^2-M_{f_0}^2\right]-i \Gamma_{f_0} M_{f_0}\right\} \\
&\quad \times \Delta_{f_0}^0(p-k)\ I(a, b)\ [(\epsilon \cdot u)(k \cdot p)-(\epsilon \cdot p)(k \cdot u)] \quad , \qquad \text{(C.5)}
\end{aligned}
$$

$$
\Delta_\sigma^0(q) = \frac{1}{\left(q^2-M_\sigma^2\right)^2+\left(\Gamma_\sigma M_\sigma\right)^2} \quad , \qquad \text{(C.6)}
$$

$$
\Delta_{f_0}^0(q) = \frac{1}{\left(q^2-M_{f_0}^2\right)^2+\left(\Gamma_{f_0} M_{f_0}\right)^2} \quad , \qquad \text{(C.7)}
$$

$$
\begin{aligned}
R_{\mu \nu}^\rho(q) &= \frac{1}{\left(q^2-M_\rho^2\right)^2+\left(\Gamma_\rho M_\rho\right)^2}\left[-g_{\mu \nu}+\frac{q_\mu q_\nu}{M_\rho^2}\right] \\
&= R_\rho^0(q)\left[-g_{\mu \nu}+\frac{q_\mu q_\nu}{M_\rho^2}\right] \quad . \qquad \text{(C.8)}
\end{aligned}
$$

The complex amplitudes are parametrized with $\mathcal{M}_i = M_i''+iM_i'$ and the absolute value of the square of the invariant amplitude is obtained as

$$
\begin{aligned}
\mid \mathcal{M} \mid^2 &= {M_1'}^2+{M_2'}^2+{M_3'}^2+{M_4'}^2+{M_1''}^2+{M_2''}^2+{M_3''}^2+{M_4''}^2 \\
&\quad +2\left({M_{12}'}^2+{M_{13}'}^2+{M_{14}'}^2+{M_{23}'}^2+{M_{24}'}^2+{M_{34}'}^2\right. \\
&\quad \left.+{M_{12}''}^2+{M_{13}''}^2+{M_{14}''}^2+{M_{23}''}^2+{M_{24}''}^2+{M_{34}''}^2\right) \quad , \qquad \text{(C.9)}
\end{aligned}
$$

where

$$
\begin{aligned}
M_1'^2 &= \left(\frac{e}{M_\phi} g_{\phi\sigma\gamma}\right)^2 (g_{\sigma\pi\pi} M_\sigma)^2 \Big\{[(p-k)^2 - M_\sigma^2]\Delta_\sigma^0(p-k)\Big\}^2 \\
&\times \frac{2}{3}(k\cdot p)^2 \quad ,
\end{aligned}
\tag{C.10}
$$

$$
\begin{aligned}
M_1''^2 &= \left(\frac{e}{M_\phi} g_{\phi\sigma\gamma}\right)^2 (g_{\sigma\pi\pi} M_\sigma)^2 \Big\{(\Gamma_\sigma M_\sigma)\Delta_\sigma^0(p-k)\Big\}^2 \\
&\times \frac{2}{3}(k\cdot p)^2 \quad ,
\end{aligned}
\tag{C.11}
$$

$$
\begin{aligned}
M_2'^2 &= \left(\frac{e}{M_\rho} g_{\rho\pi\gamma}\right)^2 \left(\frac{g_{\phi\rho\pi}}{M_\phi}\right)^2 \Big\{[(p-q_1)^2 - M_\rho^2]R_\rho^0(p-q_1)\Big\}^2 \\
&\times \frac{1}{3}\{-2k\cdot p\ k\cdot q_1[p^2(p\cdot q_1 - 2q_1^2) + p\cdot q_1\ q_1^2] \\
&+ (k\cdot p)^2[2(p\cdot q_1)^2 - p^2 q_1^2 - 2p\cdot q_1\ q_1^2 + q_1^4] \\
&+ (k\cdot q_1)^2[p^4 + 2(p\cdot q_1)^2 - p^2(2p\cdot\ q_1 + q_1^2)]\} \quad ,
\end{aligned}
\tag{C.12}
$$

$$
\begin{aligned}
M_2''^2 &= \left(\frac{e}{M_\rho} g_{\rho\pi\gamma}\right)^2 \left(\frac{g_{\phi\rho\pi}}{M_\phi}\right)^2 \Big\{(\Gamma_\rho M_\rho)R_\rho^0(p-q_1)\Big\}^2 \\
&\times \frac{1}{3}\{-2k\cdot p\ k\cdot q_1[p^2(p\cdot q_1 - 2q_1^2) + p\cdot q_1\ q_1^2] \\
&+ (k\cdot p)^2[2(p\cdot q_1)^2 - p^2 q_1^2 - 2p\cdot q_1\ q_1^2 + q_1^4] \\
&+ (k\cdot q_1)^2[p^4 + 2(p\cdot q_1)^2 - p^2(2p\cdot\ q_1 + q_1^2)]\} \quad ,
\end{aligned}
\tag{C.13}
$$

$$
\begin{aligned}
M_3'^2 &= \left(\frac{e}{M_\rho} g_{\rho\pi\gamma}\right)^2 \left(\frac{g_{\phi\rho\pi}}{M_\phi}\right)^2 \Big\{[(p-q_2)^2 - M_\rho^2]R_\rho^0(p-q_2)\Big\}^2 \\
&\times \frac{1}{3}\{-2k\cdot p\ k\cdot q_2[p^2(p\cdot q_2 - 2q_2^2) + p\cdot q_2\ q_2^2] \\
&+ (k\cdot p)^2[2(p\cdot q_2)^2 - p^2 q_2^2 - 2p\cdot q_2\ q_2^2 + q_2^4]
\end{aligned}
$$

$$+ (k\cdot q_2)^2[p^4 + 2(p\cdot q_2)^2 - p^2(2p\cdot\ q_2 + q_2^2)]\} \quad , \tag{C.14}$$

$$\begin{aligned} M_3''^2 &= \left(\frac{e}{M_\rho} g_{\rho\pi\gamma}\right)^2 \left(\frac{g_{\phi\rho\pi}}{M_\phi}\right)^2 \Big\{(\Gamma_\rho M_\rho) R_\rho^0(p-q_2)\Big\}^2 \\ &\times \frac{1}{3}\{-2k\cdot p\ k\cdot q_2[p^2(p\cdot q_2 - 2q_2^2) + p\cdot q_2\ q_2^2] \\ &+ (k\cdot p)^2[2(p\cdot q_2)^2 - p^2 q_2^2 - 2p\cdot q_2\ q_2^2 + q_2^4] \\ &+ (k\cdot q_2)^2[p^4 + 2(p\cdot q_2)^2 - p^2(2p\cdot\ q_2 + q_2^2)]\} \quad , \end{aligned} \tag{C.15}$$

$$\begin{aligned} M_4'^2 &= \left(\frac{(e g_{\phi KK})(g_{f_0\pi\pi} M_{f_0})(g_{f_0 KK} M_{f_0})}{2\pi^2 M_K^2}\right)^2 \\ &\times \Big\{[(p-k)^2 - M_{f_0}^2] ImI - (\Gamma_{f_0} M_{f_0}) ReI\Big\}^2 \Big\{\Delta_{f_0}^0(p-k)\Big\}^2 \\ &\times \frac{2}{3}(k\cdot p)^2 \quad , \end{aligned} \tag{C.16}$$

$$\begin{aligned} M_4''^2 &= \left(\frac{(e g_{\phi KK})(g_{f_0\pi\pi} M_{f_0})(g_{f_0 KK} M_{f_0})}{2\pi^2 M_K^2}\right)^2 \\ &\times \Big\{[(p-k)^2 - M_{f_0}^2] ReI + (\Gamma_{f_0} M_{f_0}) ImI\Big\}^2 \Big\{\Delta_{f_0}^0(p-k)\Big\}^2 \\ &\times \frac{2}{3}(k\cdot p)^2 \quad , \end{aligned} \tag{C.17}$$

$$\begin{aligned} M_{12}'^2 &= \left(\frac{e}{M_\phi} g_{\phi\sigma\gamma}\right)(g_{\sigma\pi\pi} M_\sigma)\left(\frac{e}{M_\rho} g_{\rho\pi\gamma}\right)\left(\frac{g_{\phi\rho\pi}}{M_\phi}\right) \\ &\times \Big\{[(p-k)^2 - M_\sigma^2]\Delta_\sigma^0(p-k)\Big\}\Big\{[(p-q_1)^2 - M_\rho^2] R_\rho^0(p-q_1)\Big\} \\ &\times \frac{1}{3}\{2k\cdot p\ k\cdot q_1\ p^2 - (k\cdot q_1)^2\ p^2 + (k\cdot p)^2(-2p\cdot q_1 + q_1^2)\} \quad , \end{aligned} \tag{C.18}$$

$$
\begin{aligned}
M''^{\,2}_{12} \;=\; & \left(\frac{e}{M_\phi} g_{\phi\sigma\gamma}\right)(g_{\sigma\pi\pi}M_\sigma)\left(\frac{e}{M_\rho} g_{\rho\pi\gamma}\right)\left(\frac{g_{\phi\rho\pi}}{M_\phi}\right) \\
& \times \Big\{(\Gamma_\sigma M_\sigma)\Delta^0_\sigma(p-k)\Big\}\Big\{(\Gamma_\rho M_\rho)R^0_\rho(p-q_1)\Big\} \\
& \times \frac{1}{3}\{2k\cdot p\; k\cdot q_1\; p^2-(k\cdot q_1)^2\; p^2+(k\cdot p)^2(-2p\cdot q_1+q_1^2)\} \quad , \qquad \text{(C.19)}
\end{aligned}
$$

$$
\begin{aligned}
M'^{\,2}_{13} \;=\; & \left(\frac{e}{M_\phi} g_{\phi\sigma\gamma}\right)(g_{\sigma\pi\pi}M_\sigma)\left(\frac{e}{M_\rho} g_{\rho\pi\gamma}\right)\left(\frac{g_{\phi\rho\pi}}{M_\phi}\right) \\
& \times \Big\{[(p-k)^2-M_\sigma^2]\Delta^0_\sigma(p-k)\Big\}\Big\{[(p-q_2)^2-M_\rho^2]R^0_\rho(p-q_2)\Big\} \\
& \times \frac{1}{3}\{2k\cdot p\; k\cdot q_2\; p^2-(k\cdot q_2)^2\; p^2+(k\cdot p)^2(-2p\cdot q_2+q_2^2)\} \quad , \qquad \text{(C.20)}
\end{aligned}
$$

$$
\begin{aligned}
M''^{\,2}_{13} \;=\; & \left(\frac{e}{M_\phi} g_{\phi\sigma\gamma}\right)(g_{\sigma\pi\pi}M_\sigma)\left(\frac{e}{M_\rho} g_{\rho\pi\gamma}\right)\left(\frac{g_{\phi\rho\pi}}{M_\phi}\right) \\
& \times \Big\{(\Gamma_\sigma M_\sigma)\Delta^0_\sigma(p-k)\Big\}\Big\{(\Gamma_\rho M_\rho)R^0_\rho(p-q_2)\Big\} \\
& \times \frac{1}{3}\{2k\cdot p\; k\cdot q_2\; p^2-(k\cdot q_2)^2\; p^2+(k\cdot p)^2(-2p\cdot q_2+q_2^2)\} \quad , \qquad \text{(C.21)}
\end{aligned}
$$

$$
\begin{aligned}
M'^{\,2}_{14} \;=\; & -\left(\frac{e}{M_\phi} g_{\phi\sigma\gamma}\right)(g_{\sigma\pi\pi}M_\sigma)\left(\frac{(eg_{\phi KK})(g_{f_0\pi\pi}M_{f_0})(g_{f_0KK}M_{f_0})}{2\pi^2M_K^2}\right) \\
& \times \Big\{[(p-k)^2-M_\sigma^2]\Delta^0_\sigma(p-k)\Big\}\Big\{[(p-k)^2-M_{f_0}^2]ImI-(\Gamma_{f_0}M_{f_0})ReI\Big\} \\
& \times \Big\{\Delta^0_{f_0}(p-k)\Big\}\frac{2}{3}(k\cdot p)^2 \quad , \qquad \text{(C.22)}
\end{aligned}
$$

$$
\begin{aligned}
M''^{\,2}_{14} \;=\; & -\left(\frac{e}{M_\phi} g_{\phi\sigma\gamma}\right)(g_{\sigma\pi\pi}M_\sigma)\left(\frac{(eg_{\phi KK})(g_{f_0\pi\pi}M_{f_0})(g_{f_0KK}M_{f_0})}{2\pi^2M_K^2}\right) \\
& \times \Big\{(\Gamma_\sigma M_\sigma)\Delta^0_\sigma(p-k)\Big\}\Big\{[(p-k)^2-M_{f_0}^2]ReI+(\Gamma_{f_0}M_{f_0})ImI\Big\} \\
& \times \Big\{\Delta^0_{f_0}(p-k)\Big\}\frac{2}{3}(k\cdot p)^2 \quad , \qquad \text{(C.23)}
\end{aligned}
$$

$$
\begin{aligned}
{M'_{23}}^2 &= \left(\frac{e}{M_\rho} g_{\rho\pi\gamma}\right)^2 \left(\frac{g_{\phi\rho\pi}}{M_\phi}\right)^2 \\
&\times \Big\{[(p-q_1)^2 - M_\rho^2] R_\rho^0(p-q_1)\Big\} \Big\{[(p-q_2)^2 - M_\rho^2] R_\rho^0(p-q_2)\Big\} \\
&\times \frac{1}{3}\{(k\cdot q_2)^2[(p\cdot q_1)^2 - p^2\ q_1^2] + k\cdot p\ k\cdot q_2[p\cdot q_2\ q_1^2 - 2p\cdot q_1\ q_1\cdot q_2 \\
&+ p^2(-p\cdot q_1 + q_1^2 + q_1\cdot q_2)] + (k\cdot p)^2[-p\cdot q_2\ q_1^2 + q_1\cdot q_2(-p^2 + q_1\cdot q_2) \\
&+ p\cdot q_1(2p\cdot q_2 - q_2^2)] + (k\cdot q_1)^2[(p\cdot q_2)^2 - p^2\ q_2^2] \\
&+ k\cdot q_1[k\cdot q_2\ p^2(p^2 - p\cdot q_1 - p\cdot q_2 + q_1\cdot q_2) \\
&+ k\cdot p(-2p\cdot q_2\ q_1\cdot q_2 + p\cdot q_1\ q_2^2 + p^2(-p\cdot q_2 + q_1\cdot q_2 + q_2^2))]\} \quad , \text{(C.24)}
\end{aligned}
$$

$$
\begin{aligned}
{M''_{23}}^2 &= \left(\frac{e}{M_\rho} g_{\rho\pi\gamma}\right)^2 \left(\frac{g_{\phi\rho\pi}}{M_\phi}\right)^2 \\
&\times \Big\{(\Gamma_\rho M_\rho) R_\rho^0(p-q_1)\Big\} \Big\{(\Gamma_\rho M_\rho) R_\rho^0(p-q_2)\Big\} \\
&\times \frac{1}{3}\{(k\cdot q_2)^2[(p\cdot q_1)^2 - p^2\ q_1^2] + k\cdot p\ k\cdot q_2[p\cdot q_2\ q_1^2 - 2p\cdot q_1\ q_1\cdot q_2 \\
&+ p^2(-p\cdot q_1 + q_1^2 + q_1\cdot q_2)] + (k\cdot p)^2[-p\cdot q_2\ q_1^2 + q_1\cdot q_2(-p^2 + q_1\cdot q_2) \\
&+ p\cdot q_1(2p\cdot q_2 - q_2^2)] + (k\cdot q_1)^2[(p\cdot q_2)^2 - p^2\ q_2^2] \\
&+ k\cdot q_1[k\cdot q_2\ p^2(p^2 - p\cdot q_1 - p\cdot q_2 + q_1\cdot q_2) \\
&+ k\cdot p(-2p\cdot q_2\ q_1\cdot q_2 + p\cdot q_1\ q_2^2 + p^2(-p\cdot q_2 + q_1\cdot q_2 + q_2^2))]\} \quad , \text{(C.25)}
\end{aligned}
$$

$$
\begin{aligned}
{M'_{24}}^2 &= -\left(\frac{e}{M_\rho} g_{\rho\pi\gamma}\right) \left(\frac{g_{\phi\rho\pi}}{M_\phi}\right) \left(\frac{(e g_{\phi KK})(g_{f_0\pi\pi} M_{f_0})(g_{f_0 KK} M_{f_0})}{2\pi^2 M_K^2}\right) \\
&\times \Big\{[(p-q_1)^2 - M_\rho^2] R_\rho^0(p-q_1)\Big\} \\
&\times \Big\{[(p-k)^2 - M_{f_0}^2] ImI - (\Gamma_{f_0} M_{f_0}) ReI\Big\} \Big\{\Delta_{f_0}^0(p-k)\Big\}
\end{aligned}
$$

$$\times \frac{1}{3}\{2k \cdot q_1 \ k \cdot p \ p^2 - p^2(k \cdot q_1)^2 + (k \cdot p)^2(-2p \cdot q_1 + q_1^2)\} \quad , \qquad \text{(C.26)}$$

$$\begin{aligned} {M''_{24}}^2 &= -\left(\frac{e}{M_\rho} g_{\rho\pi\gamma}\right)\left(\frac{g_{\phi\rho\pi}}{M_\phi}\right)\left(\frac{(eg_{\phi KK})(g_{f_0\pi\pi}M_{f_0})(g_{f_0KK}M_{f_0})}{2\pi^2 M_K^2}\right) \\ &\times \left\{(\Gamma_\rho M_\rho) R_\rho^0(p-q_1)\right\} \\ &\times \left\{[(p-k)^2 - M_{f_0}^2] ReI + (\Gamma_{f_0} M_{f_0}) ImI\right\}\left\{\Delta_{f_0}^0(p-k)\right\} \\ &\times \frac{1}{3}\{2k \cdot q_1 \ k \cdot p \ p^2 - p^2(k \cdot q_1)^2 + (k \cdot p)^2(-2p \cdot q_1 + q_1^2)\} \quad , \end{aligned} \qquad \text{(C.27)}$$

$$\begin{aligned} {M'_{34}}^2 &= -\left(\frac{e}{M_\rho} g_{\rho\pi\gamma}\right)\left(\frac{g_{\phi\rho\pi}}{M_\phi}\right)\left(\frac{(eg_{\phi KK})(g_{f_0\pi\pi}M_{f_0})(g_{f_0KK}M_{f_0})}{2\pi^2 M_K^2}\right) \\ &\times \left\{[(p-q_2)^2 - M_\rho^2] R_\rho^0(p-q_2)\right\} \\ &\times \left\{[(p-k)^2 - M_{f_0}^2] ImI - (\Gamma_{f_0} M_{f_0}) ReI\right\}\left\{\Delta_{f_0}^0(p-k)\right\} \\ &\times \frac{1}{3}\{2k \cdot q_2 \ k \cdot p \ p^2 - p^2(k \cdot q_2)^2 + (k \cdot p)^2(-2p \cdot q_2 + q_2^2)\} \quad , \end{aligned} \qquad \text{(C.28)}$$

$$\begin{aligned} {M''_{34}}^2 &= -\left(\frac{e}{M_\rho} g_{\rho\pi\gamma}\right)\left(\frac{g_{\phi\rho\pi}}{M_\phi}\right)\left(\frac{(eg_{\phi KK})(g_{f_0\pi\pi}M_{f_0})(g_{f_0KK}M_{f_0})}{2\pi^2 M_K^2}\right) \\ &\times \left\{(\Gamma_\rho M_\rho) R_\rho^0(p-q_2)\right\} \\ &\times \left\{[(p-k)^2 - M_{f_0}^2] ReI + (\Gamma_{f_0} M_{f_0}) ImI\right\}\left\{\Delta_{f_0}^0(p-k)\right\} \\ &\times \frac{1}{3}\{2k \cdot q_2 \ k \cdot p \ p^2 - p^2(k \cdot q_2)^2 + (k \cdot p)^2(-2p \cdot q_2 + q_2^2)\} \quad , \end{aligned} \qquad \text{(C.29)}$$

where ImI and ReI are given in the text in Eq. 2.5 and in Eq. 2.6, respectively.

In the rest frame of the decaying ϕ-meson

$$k \cdot p \ = \ M_\phi E_\gamma \quad ,$$

$$
\begin{aligned}
k \cdot q_1 &= \frac{1}{2}(M_\phi^2 - 2M_\phi E_2) \quad , \\
k \cdot q_2 &= \frac{1}{2}(M_\phi^2 - 2M_\phi E_1) \quad , \\
p \cdot p &= p^2 = M_\phi^2 \quad , \\
p \cdot q_1 &= M_\phi E_1 \quad , \\
p \cdot q_2 &= M_\phi E_2 \quad , \\
q_1 \cdot q_1 &= q_1^2 = M_\pi^2 \quad , \\
q_2 \cdot q_2 &= q_2^2 = M_\pi^2 \quad , \\
q_1 \cdot q_2 &= \frac{1}{2}(M_\phi^2 - 2M_\phi E_\gamma - 2M_\pi^2) \quad .
\end{aligned}
\tag{C.30}
$$

APPENDIX D

INVARIANT AMPLITUDE OF THE RADIATIVE $\phi \to \pi^0\eta\gamma$ DECAY

D.1 Invariant Amplitude for the Decay $\phi \to \pi^0\eta\gamma$ in Model I

For the radiative decay $\phi(p) \to \pi^0(q_1)\eta(q_2)\gamma(k)$, the invariant amplitude $\mathcal{M}(E_\gamma, E_1)$ is expressed as

$$\mathcal{M} = \mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c \quad , \tag{D.1}$$

where $\mathcal{M}_a$, $\mathcal{M}_b$ and $\mathcal{M}_c$ are the invariant amplitudes resulting from the diagrams (a), (b) and (c) in Fig. 2.3 respectively,

$$\begin{aligned} \mathcal{M}_a &= -\left(\frac{e}{M_\rho} g_{\rho\eta\gamma}\right)\left(\frac{g_{\phi\rho\pi}}{M_\phi}\right)\epsilon^{\mu\nu\alpha\beta} p_\mu u_\nu (p-q_1)_\alpha \epsilon^{\mu'\nu'\alpha'\beta'}(p-q_1)_{\mu'} k_{\alpha'}\epsilon_{\beta'} \\ &\quad \times \left\{ i[(p-q_1)^2 - M_\rho^2] + \Gamma_\rho M_\rho \right\} R^\rho_{\beta\nu'}(p-q_1) \quad , \end{aligned} \tag{D.2}$$

$$\begin{aligned} \mathcal{M}_b &= -(ieg_{\phi KK})\mathcal{M}(K^+K^- \to \pi^0\eta) 2u^\mu \epsilon^\nu \\ &\quad \times \int \frac{d^4q}{(2\pi)^4} \left\{ \frac{-4q_\mu q_\nu + g_{\mu\nu}(q^2 - M_K^2)}{(q^2 - M_K^2)[(q-k)^2 - M_K^2][(p-q)^2 - M_K^2]} \right\} \\ &= \left(\frac{ieg_{\phi KK}}{2\pi^2 M_K^2}\right)\mathcal{M}(K^+K^- \to \pi^0\eta) I(a,b) \\ &\quad \times [(\epsilon\cdot u)(k\cdot p) - (\epsilon\cdot p)(k\cdot u)] \quad , \end{aligned} \tag{D.3}$$

$$\mathcal{M}_c = -\left(\frac{e}{M_\phi} g_{\phi a_0 \gamma}\right) (g_{a_0 \pi\eta})\, p^\alpha u^\beta (k_\alpha \epsilon_\beta - k_\beta \epsilon_\alpha) \times \left\{ i[(p-k)^2 - M_{a_0}^2] + \Gamma_{a_0} M_{a_0} \right\} \Delta_{a_0}^0 (p-k) \quad , \tag{D.4}$$

$$R_{\mu\nu}^\rho(q) = \frac{1}{(q^2 - M_\rho^2)^2 + (\Gamma_\rho M_\rho)^2} \left[-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_\rho^2} \right] = R_\rho^0(q) \left[-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_\rho^2} \right] \quad , \tag{D.5}$$

$$\mathcal{M}(K^+ K^- \to \pi^0 \eta) = \frac{\sqrt{3}}{4 f_\pi^2} \left({M_{\pi^0\eta}}^2 - \frac{4}{3} M_K^2 \right) \quad , \tag{D.6}$$

$$\Delta_{a_0}^0(q) = \frac{1}{(q^2 - M_{a_0}^2)^2 + (\Gamma_{a_0} M_{a_0})^2} \quad . \tag{D.7}$$

The complex amplitudes are parametrized with $\mathcal{M}_i = M_i'' + i M_i'$ and the absolute value of the square of the invariant amplitude is obtained as

$$| \mathcal{M} |^2 = {M_1'}^2 + {M_2'}^2 + {M_3'}^2 + {M_1''}^2 + {M_2''}^2 + {M_3''}^2 + 2({M_{12}'}^2 + {M_{13}'}^2 + {M_{23}'}^2 + {M_{12}''}^2 + {M_{13}''}^2 + {M_{23}''}^2) \quad , \tag{D.8}$$

where

$$\begin{aligned} {M_1'}^2 = & \left(\frac{e}{M_\rho} g_{\rho\eta\gamma}\right)^2 \left(\frac{g_{\phi\rho\pi}}{M_\phi}\right)^2 \left\{ [(p-q_1)^2 - M_\rho^2] R_\rho^0(p-q_1) \right\}^2 \\ & \times \frac{1}{3} \{ -2k\cdot p\ k \cdot q_1 [p^2 (p\cdot q_1 - 2q_1^2) + p\cdot q_1\ q_1^2] \\ & + (k\cdot p)^2 [2(p\cdot q_1)^2 - p^2 q_1^2 - 2p\cdot q_1\ q_1^2 + q_1^4] \end{aligned}$$

$$+ (k\cdot q_1)^2[p^4+2(p\cdot q_1)^2-p^2(2p\cdot\ q_1+q_1^2)]\} \quad , \tag{D.9}$$

$$\begin{aligned} M_1''^2 &= \left(\frac{e}{M_\rho}g_{\rho\eta\gamma}\right)^2\left(\frac{g_{\phi\rho\pi}}{M_\phi}\right)^2\left\{(\Gamma_\rho M_\rho)R_\rho^0(p-q_1)\right\}^2 \\ &\times\frac{1}{3}\{-2k\cdot p\ k\cdot q_1[p^2(p\cdot q_1-2q_1^2)+p\cdot q_1\ q_1^2] \\ &+(k\cdot p)^2[2(p\cdot q_1)^2-p^2q_1^2-2p\cdot q_1\ q_1^2+q_1^4] \\ &+(k\cdot q_1)^2[p^4+2(p\cdot q_1)^2-p^2(2p\cdot\ q_1+q_1^2)]\} \quad , \end{aligned} \tag{D.10}$$

$$M_2'^2 = \left(\frac{eg_{\phi KK}}{2\pi^2M_K^2}\right)^2\left\{\mathcal{M}(K^+K^-\to\pi^0\eta)\right\}^2(ReI)^2\ \frac{2}{3}(k\cdot p)^2 \quad , \tag{D.11}$$

$$M_2''^2 = \left(\frac{eg_{\phi KK}}{2\pi^2M_K^2}\right)^2\left\{\mathcal{M}(K^+K^-\to\pi^0\eta)\right\}^2(ImI)^2\ \frac{2}{3}(k\cdot p)^2 \quad , \tag{D.12}$$

$$\begin{aligned} M_3'^2 &= \left(\frac{e}{M_\phi}g_{\phi a_0\gamma}\right)^2(g_{a_0\pi\eta})^2\left\{[(p-k)^2-M_{a_0}^2]\Delta_{a_0}^0(p-k)\right\}^2 \\ &\times\frac{2}{3}(k\cdot p)^2 \quad , \end{aligned} \tag{D.13}$$

$$\begin{aligned} M_3''^2 &= \left(\frac{e}{M_\phi}g_{\phi a_0\gamma}\right)^2(g_{a_0\pi\eta})^2\left\{(\Gamma_{a_0}M_{a_0})\Delta_{a_0}^0(p-k)\right\}^2 \\ &\times\frac{2}{3}(k\cdot p)^2 \quad , \end{aligned} \tag{D.14}$$

$$\begin{aligned} {M_{12}'}^2 &= -\left(\frac{e}{M_\rho}g_{\rho\eta\gamma}\right)\left(\frac{g_{\phi\rho\pi}}{M_\phi}\right)\left(\frac{eg_{\phi KK}}{2\pi^2M_K^2}\right)\left\{\mathcal{M}(K^+K^-\to\pi^0\eta)\right\} \\ &\times\left\{[(p-q_1)^2-M_\rho^2]R_\rho^0(p-q_1)\right\}(ReI) \\ &\times\frac{1}{3}\{2k\cdot p\ k\cdot q_1\ p^2-(k\cdot q_1)^2\ p^2 \end{aligned}$$

$$+ (k\cdot p)^2(-2p\cdot q_1 + q_1^2)\} \quad , \tag{D.15}$$

$$\begin{aligned} {M''_{12}}^2 \quad = \quad & \left(\frac{e}{M_\rho} g_{\rho\eta\gamma}\right)\left(\frac{g_{\phi\rho\pi}}{M_\phi}\right)\left(\frac{e g_{\phi KK}}{2\pi^2 M_K^2}\right)\left\{\mathcal{M}(K^+K^- \to \pi^0\eta)\right\} \\ & \times \left\{(\Gamma_\rho M_\rho) R_\rho^0(p-q_1)\right\}(ImI) \\ & \times \frac{1}{3}\{2k\cdot p\ k\cdot q_1\ p^2 - (k\cdot q_1)^2\ p^2 \\ & + (k\cdot p)^2(-2p\cdot q_1 + q_1^2)\} \quad , \end{aligned} \tag{D.16}$$

$$\begin{aligned} {M'_{13}}^2 \quad = \quad & \left(\frac{e}{M_\rho} g_{\rho\eta\gamma}\right)\left(\frac{g_{\phi\rho\pi}}{M_\phi}\right)\left(\frac{e}{M_\phi} g_{\phi a_0\gamma}\right)(g_{a_0\pi\eta}) \\ & \times \left\{[(p-q_1)^2 - M_\rho^2] R_\rho^0(p-q_1)\right\}\left\{[(p-k)^2 - M_{a_0}^2]\Delta_{a_0}^0(p-k)\right\} \\ & \times \frac{1}{3}\{2k\cdot p\ k\cdot q_1\ p^2 - (k\cdot q_1)^2\ p^2 \\ & + (k\cdot p)^2(-2p\cdot q_1 + q_1^2)\} \quad , \end{aligned} \tag{D.17}$$

$$\begin{aligned} {M''_{13}}^2 \quad = \quad & \left(\frac{e}{M_\rho} g_{\rho\eta\gamma}\right)\left(\frac{g_{\phi\rho\pi}}{M_\phi}\right)\left(\frac{e}{M_\phi} g_{\phi a_0\gamma}\right)(g_{a_0\pi\eta}) \\ & \times \left\{(\Gamma_\rho M_\rho) R_\rho^0(p-q_1)\right\}\left\{(\Gamma_{a_0} M_{a_0})\Delta_{a_0}^0(p-k)\right\} \\ & \times \frac{1}{3}\{2k\cdot p\ k\cdot q_1\ p^2 - (k\cdot q_1)^2\ p^2 \\ & + (k\cdot p)^2(-2p\cdot q_1 + q_1^2)\} \quad , \end{aligned} \tag{D.18}$$

$$\begin{aligned} {M'_{23}}^2 \quad = - \quad & \left(\frac{e}{M_\phi} g_{\phi a_0\gamma}\right)(g_{a_0\pi\eta})\left(\frac{e g_{\phi KK}}{2\pi^2 M_K^2}\right)\left\{\mathcal{M}(K^+K^- \to \pi^0\eta)\right\} \\ & \times (ReI)\left\{[(p-k)^2 - M_{a_0}^2]\Delta_{a_0}^0(p-k)\right\}\frac{2}{3}(k\cdot p)^2 \quad , \end{aligned} \tag{D.19}$$

$$M_{23}''^{\,2} = \left(\frac{e}{M_\phi} g_{\phi a_0 \gamma}\right) (g_{a_0 \pi\eta}) \left(\frac{e g_{\phi KK}}{2\pi^2 M_K^2}\right) \left\{\mathcal{M}(K^+K^- \to \pi^0\eta)\right\}$$
$$\times (ImI) \left\{(\Gamma_{a_0} M_{a_0}) \Delta^0_{a_0}(p-k)\right\} \frac{2}{3}(k\cdot p)^2 \quad . \qquad \text{(D.20)}$$

D.2 Invariant Amplitude for the Decay $\phi \to \pi^0\eta\gamma$ in Model II

For the radiative decay $\phi(p) \to \pi^0(q_1)\eta(q_2)\gamma(k)$, the invariant amplitude $\mathcal{M}(E_\gamma, E_1)$ is expressed as

$$\mathcal{M} = \mathcal{M}_a{}' + \mathcal{M}_b{}' + \mathcal{M}_c{}' \quad , \qquad \text{(D.21)}$$

where $\mathcal{M}_a{}'$, $\mathcal{M}_b{}'$ and $\mathcal{M}_c{}'$ are the invariant amplitudes resulting from the diagrams (a), (b) and (c) in Fig. 2.4 respectively,

$$\mathcal{M}_a{}' = -\left(\frac{e}{M_\rho} g_{\rho\eta\gamma}\right) \left(\frac{g_{\phi\rho\pi}}{M_\phi}\right) \epsilon^{\mu\nu\alpha\beta} p_\mu u_\nu (p-q_1)_\alpha \epsilon^{\mu'\nu'\alpha'\beta'} (p-q_1)_{\mu'} k_{\alpha'} \epsilon_{\beta'}$$
$$\times \left\{i[(p-q_1)^2 - M_\rho^2] + \Gamma_\rho M_\rho\right\} R^\rho_{\beta\nu'}(p-q_1) \quad , \qquad \text{(D.22)}$$

$$\mathcal{M}_b{}' = -(ieg_{\phi KK})\mathcal{M}(K^+K^- \to \pi^0\eta) 2u^\mu \epsilon^\nu$$
$$\times \int \frac{d^4q}{(2\pi)^4} \left\{\frac{-4q_\mu q_\nu + g_{\mu\nu}(q^2 - M_K^2)}{(q^2 - M_K^2)[(q-k)^2 - M_K^2][(p-q)^2 - M_K^2]}\right\}$$
$$= \left(\frac{ieg_{\phi KK}}{2\pi^2 M_K^2}\right) \mathcal{M}(K^+K^- \to \pi^0\eta) I(a,b)$$
$$\times [(\epsilon\cdot u)(k\cdot p) - (\epsilon\cdot p)(k\cdot u)] \quad , \qquad \text{(D.23)}$$

$$\mathcal{M}_c{}' = -(eg_{\phi KK})(g_{a_0 K^+K^-} M_{a_0})(g_{a_0\pi\eta})$$
$$\times \left\{[(p-k)^2 - M_{a_0}^2] - i\Gamma_{a_0} M_{a_0}\right\} \Delta^0_{a_0}(p-k) 2u^\mu \epsilon^\nu$$

$$\times \int \frac{d^4q}{(2\pi)^4} \left\{ \frac{-4q_\mu q_\nu + g_{\mu\nu}(q^2 - M_K^2)}{(q^2 - M_K^2)[(q-k)^2 - M_K^2][(p-q)^2 - M_K^2]} \right\}$$

$$= \left(\frac{e g_{\phi KK} (g_{a_0 K^+ K^-} M_{a_0})(g_{a_0 \pi\eta})}{2\pi^2 M_K^2} \right) \left\{ [(p-k)^2 - M_{a_0}^2] - i\Gamma_{a_0} M_{a_0} \right\}$$

$$\times \Delta_{a_0}^0 (p-k)$$

$$\times I(a,b) \left[(\epsilon \cdot u)(k \cdot p) - (\epsilon \cdot p)(k \cdot u) \right] \quad . \tag{D.24}$$

The complex amplitudes are parametrized with $\mathcal{M}_i = M_i'' + iM_i'$ and the absolute value of the square of the invariant amplitude is obtained as

$$| \mathcal{M} |^2 = {M_1'}^2 + {M_2'}^2 + {M_3'}^2 + {M_1''}^2 + {M_2''}^2 + {M_3''}^2$$

$$+ 2({M_{12}'}^2 + {M_{13}'}^2 + {M_{23}'}^2 + {M_{12}''}^2 + {M_{13}''}^2 + {M_{23}''}^2) \quad , \tag{D.25}$$

where

$${M_1'}^2 = \left(\frac{e}{M_\rho} g_{\rho\eta\gamma} \right)^2 \left(\frac{g_{\phi\rho\pi}}{M_\phi} \right)^2 \left\{ [(p-q_1)^2 - M_\rho^2] R_\rho^0 (p-q_1) \right\}^2$$

$$\times \frac{1}{3} \{ -2k \cdot p \; k \cdot q_1 [p^2 (p \cdot q_1 - 2q_1^2) + p \cdot q_1 \; q_1^2]$$

$$+ (k \cdot p)^2 [2(p \cdot q_1)^2 - p^2 q_1^2 - 2p \cdot q_1 \; q_1^2 + q_1^4]$$

$$+ (k \cdot q_1)^2 [p^4 + 2(p \cdot q_1)^2 - p^2 (2p \cdot \; q_1 + q_1^2)] \} \quad , \tag{D.26}$$

$${M_1''}^2 = \left(\frac{e}{M_\rho} g_{\rho\eta\gamma} \right)^2 \left(\frac{g_{\phi\rho\pi}}{M_\phi} \right)^2 \left\{ (\Gamma_\rho M_\rho) R_\rho^0 (p-q_1) \right\}^2$$

$$\times \frac{1}{3} \{ -2k \cdot p \; k \cdot q_1 [p^2 (p \cdot q_1 - 2q_1^2) + p \cdot q_1 \; q_1^2]$$

$$+ (k \cdot p)^2 [2(p \cdot q_1)^2 - p^2 q_1^2 - 2p \cdot q_1 \; q_1^2 + q_1^4]$$

$$+ (k \cdot q_1)^2 [p^4 + 2(p \cdot q_1)^2 - p^2 (2p \cdot \; q_1 + q_1^2)] \} \quad , \tag{D.27}$$

$$M_2'^2 = \left(\frac{eg_{\phi KK}}{2\pi^2 M_K^2}\right)^2 \left\{\mathcal{M}(K^+K^- \to \pi^0\eta)\right\}^2 (ReI)^2 \ \frac{2}{3}(k\cdot p)^2 \quad , \tag{D.28}$$

$$M_2''^2 = \left(\frac{eg_{\phi KK}}{2\pi^2 M_K^2}\right)^2 \left\{\mathcal{M}(K^+K^- \to \pi^0\eta)\right\}^2 (ImI)^2 \ \frac{2}{3}(k\cdot p)^2 \quad , \tag{D.29}$$

$$\begin{aligned} M_3'^2 &= \left(\frac{(eg_{\phi KK})(g_{a_0K^+K^-}M_{a_0})(g_{a_0\pi\eta})}{2\pi^2 M_K^2}\right)^2 \\ &\times \left\{[(p-k)^2 - M_{a_0}^2]ImI - (\Gamma_{a_0}M_{a_0})ReI\right\}^2 \\ &\times \left\{\Delta_{a_0}^0(p-k)\right\}^2 \frac{2}{3}(k\cdot p)^2 \quad , \end{aligned} \tag{D.30}$$

$$\begin{aligned} M_3''^2 &= \left(\frac{(eg_{\phi KK})(g_{a_0K^+K^-}M_{a_0})(g_{a_0\pi\eta})}{2\pi^2 M_K^2}\right)^2 \\ &\times \left\{[(p-k)^2 - M_{a_0}^2]ReI + (\Gamma_{a_0}M_{a_0})ImI\right\}^2 \\ &\times \left\{\Delta_{a_0}^0(p-k)\right\}^2 \frac{2}{3}(k\cdot p)^2 \quad , \end{aligned} \tag{D.31}$$

$$\begin{aligned} M_{12}'^{\,2} &= -\left(\frac{e}{M_\rho}g_{\rho\eta\gamma}\right)\left(\frac{g_{\phi\rho\pi}}{M_\phi}\right)\left(\frac{eg_{\phi KK}}{2\pi^2 M_K^2}\right)\left\{\mathcal{M}(K^+K^- \to \pi^0\eta)\right\} \\ &\times \left\{[(p-q_1)^2 - M_\rho^2]R_\rho^0(p-q_1)\right\}(ReI) \\ &\times \frac{1}{3}\{2k\cdot p\ k\cdot q_1\ p^2 - (k\cdot q_1)^2\ p^2 \\ &+ (k\cdot p)^2(-2p\cdot q_1 + q_1^2)\} \quad , \end{aligned} \tag{D.32}$$

$$\begin{aligned} M_{12}''^{\,2} &= \left(\frac{e}{M_\rho}g_{\rho\eta\gamma}\right)\left(\frac{g_{\phi\rho\pi}}{M_\phi}\right)\left(\frac{eg_{\phi KK}}{2\pi^2 M_K^2}\right)\left\{\mathcal{M}(K^+K^- \to \pi^0\eta)\right\} \\ &\times \left\{(\Gamma_\rho M_\rho)R_\rho^0(p-q_1)\right\}(ImI) \end{aligned}$$

$$\times \frac{1}{3}\{2k \cdot p \; k \cdot q_1 \; p^2 - (k \cdot q_1)^2 \; p^2$$

$$+ (k \cdot p)^2(-2p \cdot q_1 + q_1^2)\} \quad , \tag{D.33}$$

$$\begin{aligned} {M'_{13}}^2 &= -\left(\frac{e}{M_\rho} g_{\rho\eta\gamma}\right)\left(\frac{g_{\phi\rho\pi}}{M_\phi}\right)\left(\frac{(eg_{\phi KK})(g_{a_0K^+K^-}M_{a_0})(g_{a_0\pi\eta})}{2\pi^2 M_K^2}\right) \\ &\times \left\{[(p-q_1)^2 - M_\rho^2] R_\rho^0(p-q_1)\right\} \\ &\times \left\{[(p-k)^2 - M_{a_0}^2] ImI - (\Gamma_{a_0} M_{a_0}) ReI\right\}\left\{\Delta_{a_0}^0(p-k)\right\} \\ &\times \frac{1}{3}\{2k \cdot p \; k \cdot q_1 \; p^2 - (k \cdot q_1)^2 \; p^2 \\ &+ (k \cdot p)^2(-2p \cdot q_1 + q_1^2)\} \quad , \end{aligned} \tag{D.34}$$

$$\begin{aligned} {M''_{13}}^2 &= -\left(\frac{e}{M_\rho} g_{\rho\eta\gamma}\right)\left(\frac{g_{\phi\rho\pi}}{M_\phi}\right)\left(\frac{(eg_{\phi KK})(g_{a_0K^+K^-}M_{a_0})(g_{a_0\pi\eta})}{2\pi^2 M_K^2}\right) \\ &\times \left\{(\Gamma_\rho M_\rho) R_\rho^0(p-q_1)\right\} \\ &\times \left\{[(p-k)^2 - M_{a_0}^2] ReI + (\Gamma_{a_0} M_{a_0}) ImI\right\}\left\{\Delta_{a_0}^0(p-k)\right\} \\ &\times \frac{1}{3}\{2k \cdot p \; k \cdot q_1 \; p^2 - (k \cdot q_1)^2 \; p^2 \\ &+ (k \cdot p)^2(-2p \cdot q_1 + q_1^2)\} \quad , \end{aligned} \tag{D.35}$$

$$\begin{aligned} {M'_{23}}^2 &= \left(\frac{eg_{\phi KK}}{2\pi^2 M_K^2}\right)^2 \{(g_{a_0K^+K^-}M_{a_0})(g_{a_0\pi\eta})\}\left\{\mathcal{M}(K^+K^- \to \pi^0\eta)\right\} \\ &\times (ReI)\left\{[(p-k)^2 - M_{a_0}^2] ImI - (\Gamma_{a_0} M_{a_0}) ReI\right\} \\ &\times \left\{\Delta_{a_0}^0(p-k)\right\}\frac{2}{3}(k \cdot p)^2 \quad , \end{aligned} \tag{D.36}$$

$$
\begin{aligned}
{M''_{23}}^2 &= -\left(\frac{eg_{\phi KK}}{2\pi^2 M_K^2}\right)^2 \{(g_{a_0 K^+K^-} M_{a_0})(g_{a_0\pi\eta})\} \left\{\mathcal{M}(K^+K^- \to \pi^0\eta)\right\} \\
&\quad \times (ImI) \left\{[(p-k)^2 - M_{a_0}^2] ReI + (\Gamma_{a_0} M_{a_0}) ImI\right\} \\
&\quad \times \left\{\Delta_{a_0}^0 (p-k)\right\} \frac{2}{3} (k \cdot p)^2 \quad , \qquad \text{(D.37)}
\end{aligned}
$$

where ImI and ReI are given in the text in Eq. 2.5 and in Eq. 2.6, respectively. In the rest frame of the decaying ϕ-meson

$$
\begin{aligned}
k \cdot p &= M_\phi E_\gamma \quad , \\
k \cdot q_1 &= \frac{1}{2}(M_\phi^2 - 2M_\phi E_2 + M_\eta^2 - M_{\pi^0}^2) \quad , \\
k \cdot q_2 &= \frac{1}{2}(M_\phi^2 - 2M_\phi E_1 + M_{\pi^0}^2 - M_\eta^2) \quad , \\
p \cdot p &= p^2 = M_\phi^2 \quad , \\
p \cdot q_1 &= M_\phi E_1 \quad , \\
p \cdot q_2 &= M_\phi E_2 \quad , \\
q_1 \cdot q_1 &= {q_1}^2 = M_{\pi^0}^2 \quad , \\
q_2 \cdot q_2 &= {q_2}^2 = M_\eta^2 \quad , \\
q_1 \cdot q_2 &= \frac{1}{2}(M_\phi^2 - 2M_\phi E_\gamma - M_{\pi^0}^2 - M_\eta^2) \quad . \qquad \text{(D.38)}
\end{aligned}
$$

APPENDIX E

INVARIANT AMPLITUDE OF THE RADIATIVE $\rho^0 \to \pi^+\pi^-\gamma$ AND $\rho^0 \to \pi^0\pi^0\gamma$ DECAYS

E.1 Invariant Amplitude for the Decay $\rho^0 \to \pi^+\pi^-\gamma$

For the radiative decay $\rho^0(p) \to \pi^+(q_1)\pi^-(q_2)\gamma(k)$, the invariant amplitude $\mathcal{M}(E_\gamma, E_1)$ is expressed as

$$\mathcal{M} = \mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c + \mathcal{M}_d + \mathcal{M}_e \quad , \tag{E.1}$$

where $\mathcal{M}_a$, $\mathcal{M}_b$, $\mathcal{M}_c$, $\mathcal{M}_d$ and $\mathcal{M}_e$ are the invariant amplitudes resulting from the diagrams (a), (b), (c), (d) and (e) in Fig. 2.5 respectively,

$$\mathcal{M}_a = -i\,(4eg_{\rho\pi\pi})\,u_\mu q_2^\mu q_{1\nu}\epsilon^\nu D_\pi{}^0(p-q_2) \quad , \tag{E.2}$$

$$\mathcal{M}_b = -i\,(4eg_{\rho\pi\pi})\,u_\mu q_1^\mu q_{2\nu}\epsilon^\nu D_\pi{}^0(p-q_1) \quad , \tag{E.3}$$

$$\mathcal{M}_c = -i\,(2eg_{\rho\pi\pi})\,u_\mu\epsilon^\mu \quad , \tag{E.4}$$

$$\mathcal{M}_d = -(e\lambda)(g_{\rho\pi\pi})2u^\mu\epsilon^\nu$$

$$
\begin{aligned}
&\times \int \frac{d^4q}{(2\pi)^4} \left\{ \frac{-4q_\mu q_\nu + g_{\mu\nu}(q^2 - M_\pi^2)}{(q^2 - M_\pi^2)[(q-k)^2 - M_\pi^2][(p-q)^2 - M_\pi^2]} \right\} \\
= & \left(\frac{e g_{\rho\pi\pi} \lambda}{2\pi^2 M_\pi^2} \right) I(a,b) \left[(\epsilon \cdot u)(k \cdot p) - (\epsilon \cdot p)(k \cdot u) \right] \quad , \qquad \text{(E.5)}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_e = & -(ie g_{\rho\pi\pi})(g_{\sigma\pi\pi} M_\sigma)^2 \left\{ i[(p-k)^2 - M_\sigma^2] + \Gamma_\sigma M_\sigma \right\} \\
& \times \Delta_\sigma^0(p-k) \; 2u^\mu \epsilon^\nu \\
& \times \int \frac{d^4q}{(2\pi)^4} \left\{ \frac{-4q_\mu q_\nu + g_{\mu\nu}(q^2 - M_\pi^2)}{(q^2 - M_\pi^2)[(q-k)^2 - M_\pi^2][(p-q)^2 - M_\pi^2]} \right\} \\
= & -\left(\frac{(e g_{\rho\pi\pi})(g_{\sigma\pi\pi} M_\sigma)^2}{2\pi^2 M_\pi^2} \right) \left\{ [(p-k)^2 - M_\sigma^2] - i\Gamma_\sigma M_\sigma \right\} \\
& \times \Delta_\sigma^0(p-k) \; I(a,b) \left[(\epsilon \cdot u)(k \cdot p) - (\epsilon \cdot p)(k \cdot u) \right] \quad , \qquad \text{(E.6)}
\end{aligned}
$$

$$
D_\pi^0(q) = \frac{1}{q^2 - M_\pi^2} \quad , \qquad \text{(E.7)}
$$

$$
\Delta_\sigma^0(q) = \frac{1}{(q^2 - M_\sigma^2)^2 + (\Gamma_\sigma M_\sigma)^2} \quad , \qquad \text{(E.8)}
$$

$$
\lambda = \mathcal{M}_\chi(\pi^+\pi^- \to \pi^+\pi^-) = -\frac{2}{3}\frac{1}{f_\pi^2}\left(s - \frac{7}{2} M_\pi^2 \right) \quad , \qquad \text{(E.9)}
$$

$$
s = (p-k)^2 = M_{\pi\pi}^2 \quad . \qquad \text{(E.10)}
$$

The complex amplitudes are parametrized with $\mathcal{M}_i = M_i'' + iM_i'$ and the absolute value of the square of the invariant amplitude is obtained as

$$
|\, \mathcal{M} \,|^2 = {M_1'}^2 + {M_2'}^2 + {M_3'}^2 + {M_4'}^2 + {M_5'}^2 + {M_4''}^2 + {M_5''}^2
$$

$$+ 2({M'_{12}}^2 + {M'_{13}}^2 + {M'_{14}}^2 + {M'_{15}}^2 + {M'_{23}}^2 + {M'_{24}}^2$$

$$+ {M'_{25}}^2 + {M'_{34}}^2 + {M'_{35}}^2 + {M'_{45}}^2 + {M''_{45}}^2) \quad , \tag{E.11}$$

where

$$M_1'^2 = (4eg_{\rho\pi\pi})^2 \left\{D_\pi^0(p-q_2)\right\}^2 \frac{1}{3}\left\{q_1^2\left[q_2^2 - \frac{(p\cdot q_2)^2}{M_\rho^2}\right]\right\} \quad , \tag{E.12}$$

$$M_2'^2 = (4eg_{\rho\pi\pi})^2 \left\{D_\pi^0(p-q_1)\right\}^2 \frac{1}{3}\left\{q_2^2\left[q_1^2 - \frac{(p\cdot q_1)^2}{M_\rho^2}\right]\right\} \quad , \tag{E.13}$$

$$M_3'^2 = (2eg_{\rho\pi\pi})^2 \quad , \tag{E.14}$$

$$M_4'^2 = \left(\frac{g_{\rho\pi\pi}e\lambda}{2\pi^2 M_\pi^2}\right)^2 (ImI)^2 \frac{2}{3}(k\cdot p)^2 \quad , \tag{E.15}$$

$$M_4''^2 = \left(\frac{g_{\rho\pi\pi}e\lambda}{2\pi^2 M_\pi^2}\right)^2 (ReI)^2 \frac{2}{3}(k\cdot p)^2 \quad , \tag{E.16}$$

$$\begin{aligned} M_5'^2 = & \left(\frac{eg_{\rho\pi\pi}}{2\pi^2 M_\pi^2}\right)^2 (g_{\sigma\pi\pi}M_\sigma)^4 \\ & \times\left\{[(p-k)^2 - M_\sigma^2]ImI - (\Gamma_\sigma M_\sigma)ReI\right\}^2 \\ & \times\left\{\Delta_\sigma^0(p-k)\right\}^2 \frac{2}{3}(k\cdot p)^2 \quad , \end{aligned} \tag{E.17}$$

$$\begin{aligned} M_5''^2 = & \left(\frac{eg_{\rho\pi\pi}}{2\pi^2 M_\pi^2}\right)^2 (g_{\sigma\pi\pi}M_\sigma)^4 \\ & \times\left\{[(p-k)^2 - M_\sigma^2]ReI + (\Gamma_\sigma M_\sigma)ImI\right\}^2 \end{aligned}$$

$$\times \left\{ \Delta^0_\sigma(p-k) \right\}^2 \frac{2}{3}(k\cdot p)^2 \quad , \tag{E.18}$$

$$\begin{aligned} {M'_{12}}^2 &= (4eg_{\rho\pi\pi})^2 \left\{ D^0_\pi(p-q_2) \right\} \left\{ D^0_\pi(p-q_1) \right\} \\ &\quad \times \frac{1}{3} \left\{ q_1\cdot q_2 \left[q_1\cdot q_2 - \frac{(p\cdot q_1)(p\cdot q_2)}{M_\rho^2} \right] \right\} \quad , \end{aligned} \tag{E.19}$$

$$\begin{aligned} {M'_{13}}^2 &= 8\,(eg_{\rho\pi\pi})^2 \left\{ D^0_\pi(p-q_2) \right\} \\ &\quad \times \frac{1}{3} \left\{ q_1\cdot q_2 - \frac{(p\cdot q_1)(p\cdot q_2)}{M_\rho^2} \right\} \quad , \end{aligned} \tag{E.20}$$

$$\begin{aligned} {M'_{14}}^2 &= -\left(\frac{4(eg_{\rho\pi\pi})^2\lambda}{2\pi^2 M_\pi^2} \right) \left\{ D^0_\pi(p-q_2) \right\} ImI \\ &\quad \times \frac{1}{3}\{ k\cdot p\ q_1\cdot q_2 - p\cdot q_1\ k\cdot q_2 \} \quad , \end{aligned} \tag{E.21}$$

$$\begin{aligned} {M'_{15}}^2 &= \left(\frac{4(eg_{\rho\pi\pi})^2(g_{\sigma\pi\pi}M_\sigma)^2}{2\pi^2 M_\pi^2} \right) \left\{ D^0_\pi(p-q_2) \right\} \\ &\quad \times \left\{ [(p-k)^2 - M_\sigma^2] ImI - (\Gamma_\sigma M_\sigma) ReI \right\} \left\{ D^0_\sigma(p-k) \right\} \\ &\quad \times \frac{1}{3}\{ k\cdot p\ q_1\cdot q_2 - p\cdot q_1\ k\cdot q_2 \} \quad , \end{aligned} \tag{E.22}$$

$$\begin{aligned} {M'_{23}}^2 &= 8\,(eg_{\rho\pi\pi})^2 \left\{ D^0_\pi(p-q_1) \right\} \\ &\quad \times \frac{1}{3} \left\{ q_1\cdot q_2 - \frac{(p\cdot q_1)(p\cdot q_2)}{M_\rho^2} \right\} \quad , \end{aligned} \tag{E.23}$$

$$ {M'_{24}}^2 = -\left(\frac{4(eg_{\rho\pi\pi})^2\lambda}{2\pi^2 M_\pi^2} \right) \left\{ D^0_\pi(p-q_1) \right\} ImI $$

$$\times \frac{1}{3}\{k \cdot p \; q_1 \cdot q_2 - p \cdot q_2 \; k \cdot q_1\} \quad , \tag{E.24}$$

$$\begin{aligned} {M'_{25}}^2 &= \left(\frac{4(eg_{\rho\pi\pi})^2(g_{\sigma\pi\pi}M_\sigma)^2}{2\pi^2 M_\pi^2}\right)\left\{D_\pi^0(p-q_1)\right\} \\ &\times \left\{[(p-k)^2 - M_\sigma^2]ImI - (\Gamma_\sigma M_\sigma)ReI\right\}\left\{D_\sigma^0(p-k)\right\} \\ &\times \frac{1}{3}\{k \cdot p \; q_1 \cdot q_2 - p \cdot q_2 \; k \cdot q_1\} \quad , \end{aligned} \tag{E.25}$$

$${M'_{34}}^2 = -\left(\frac{2(eg_{\rho\pi\pi})^2\lambda}{2\pi^2 M_\pi^2}\right) ImI\,(k \cdot p) \quad , \tag{E.26}$$

$$\begin{aligned} {M'_{35}}^2 &= \left(\frac{2(eg_{\rho\pi\pi})^2(g_{\sigma\pi\pi}M_\sigma)^2}{2\pi^2 M_\pi^2}\right)\left\{[(p-k)^2 - M_\sigma^2]ImI - (\Gamma_\sigma M_\sigma)ReI\right\} \\ &\times \left\{\Delta_\sigma^0(p-k)\right\}(k \cdot p) \quad , \end{aligned} \tag{E.27}$$

$$\begin{aligned} {M'_{45}}^2 &= -\left(\frac{(eg_{\rho\pi\pi})(g_{\sigma\pi\pi}M_\sigma)}{2\pi^2 M_\pi^2}\right)^2 \lambda\left\{[(p-k)^2 - M_\sigma^2]ImI - (\Gamma_\sigma M_\sigma)ReI\right\} \\ &\times \left\{\Delta_\sigma^0(p-k)\right\} ImI\;\frac{2}{3}(k \cdot p)^2 \quad , \end{aligned} \tag{E.28}$$

$$\begin{aligned} {M''_{45}}^2 &= -\left(\frac{(eg_{\rho\pi\pi})(g_{\sigma\pi\pi}M_\sigma)}{2\pi^2 M_\pi^2}\right)^2 \lambda\left\{[(p-k)^2 - M_\sigma^2]ReI + (\Gamma_\sigma M_\sigma)ImI\right\} \\ &\times \left\{\Delta_\sigma^0(p-k)\right\} ReI\;\frac{2}{3}(k \cdot p)^2 \quad , \end{aligned} \tag{E.29}$$

where ImI and ReI are given in the text in Eq. 2.5 and in Eq. 2.6, respectively. In the rest frame of the decaying ρ^0-meson

$$k \cdot p = M_\rho E_\gamma \quad ,$$

$$
\begin{aligned}
k \cdot q_1 &= \frac{1}{2}(M_\rho^2 - 2M_\rho E_2) \quad , \\
k \cdot q_2 &= \frac{1}{2}(M_\rho^2 - 2M_\rho E_1) \quad , \\
p \cdot p &= p^2 = M_\rho^2 \quad , \\
p \cdot q_1 &= M_\rho E_1 \quad , \\
p \cdot q_2 &= M_\rho E_2 \quad , \\
q_1 \cdot q_1 &= {q_1}^2 = M_\pi^2 \quad , \\
q_2 \cdot q_2 &= {q_2}^2 = M_\pi^2 \quad , \\
q_1 \cdot q_2 &= \frac{1}{2}(M_\rho^2 - 2M_\rho E_\gamma - 2M_\pi^2) \quad .
\end{aligned}
\tag{E.30}
$$

E.2 Invariant Amplitude for the Decay $\rho^0 \to \pi^0\pi^0\gamma$

For the radiative decay $\rho^0(p) \to \pi^0(q_1)\pi^0(q_2)\gamma(k)$, the invariant amplitude $\mathcal{M}(E_\gamma, E_1)$ is expressed as

$$
\mathcal{M} = {\mathcal{M}_a}' + {\mathcal{M}_b}' + {\mathcal{M}_c}' + {\mathcal{M}_d}' \quad , \tag{E.31}
$$

where ${\mathcal{M}_a}'$, ${\mathcal{M}_b}'$, ${\mathcal{M}_c}'$ and ${\mathcal{M}_d}'$ are the invariant amplitudes resulting from the diagrams (a), (b), (c) and (d) in Fig. 2.6 respectively,

$$
\begin{aligned}
{\mathcal{M}_a}' &= -\left(\frac{e}{M_\omega} g_{\omega\pi\gamma}\right)\left(\frac{g_{\omega\rho\pi}}{M_\omega}\right) \epsilon^{\mu\nu\alpha\beta} p_\mu u_\nu (p-q_1)_\alpha \epsilon^{\mu'\nu'\alpha'\beta'} (p-q_1)_{\mu'} k_{\alpha'} \epsilon_{\beta'} \\
&\quad \times \left\{ i[(p-q_1)^2 - M_\omega^2] + \Gamma_\omega M_\omega \right\} R^\omega_{\beta\nu'}(p-q_1) \quad ,
\end{aligned}
\tag{E.32}
$$

$$
{\mathcal{M}_b}' = -\left(\frac{e}{M_\omega} g_{\omega\pi\gamma}\right)\left(\frac{g_{\omega\rho\pi}}{M_\omega}\right) \epsilon^{\mu\nu\alpha\beta} p_\mu u_\nu (p-q_2)_\alpha \epsilon^{\mu'\nu'\alpha'\beta'} (p-q_2)_{\mu'} k_{\alpha'} \epsilon_{\beta'}
$$

$$\times \left\{ i[(p-q_2)^2 - M_\omega^2] + \Gamma_\omega M_\omega \right\} R^\omega_{\beta\nu'}(p-q_2) \quad , \tag{E.33}$$

$$\begin{aligned}
\mathcal{M}_c{}' &= -(eg_{\rho\pi\pi}\lambda) 2u^\mu \epsilon^\nu \int \frac{d^4q}{(2\pi)^4} \left\{ \frac{-4q_\mu q_\nu + g_{\mu\nu}(q^2 - M_\pi^2)}{(q^2 - M_\pi^2)[(q-k)^2 - M_\pi^2][(p-q)^2 - M_\pi^2]} \right\} \\
&= \left(\frac{eg_{\rho\pi\pi}\lambda}{2\pi^2 M_\pi^2} \right) \; I(a,b) \left[(\epsilon\cdot u)(k\cdot p) - (\epsilon\cdot p)(k\cdot u) \right] \quad ,
\end{aligned} \tag{E.34}$$

$$\begin{aligned}
\mathcal{M}_d{}' &= (eg_{\rho\pi\pi})(g_{\sigma\pi\pi}M_\sigma)^2 \left\{ [(p-k)^2 - M_\sigma^2] - i\Gamma_\sigma M_\sigma \right\} \Delta^0_\sigma(p-k) \\
&\quad \times 2u^\mu \epsilon^\nu \int \frac{d^4q}{(2\pi)^4} \left\{ \frac{-4q_\mu q_\nu + g_{\mu\nu}(q^2 - M_\pi^2)}{(q^2 - M_\pi^2)[(q-k)^2 - M_\pi^2][(p-q)^2 - M_\pi^2]} \right\} \\
&= -\left(\frac{(eg_{\rho\pi\pi})(g_{\sigma\pi\pi}M_\sigma)^2}{2\pi^2 M_\pi^2} \right) \left\{ [(p-k)^2 - M_\sigma^2] - i\Gamma_\sigma M_\sigma \right\} \\
&\quad \times \Delta^0_\sigma(p-k) \; I(a,b) \left[(\epsilon\cdot u)(k\cdot p) - (\epsilon\cdot p)(k\cdot u) \right] \quad ,
\end{aligned} \tag{E.35}$$

$$\begin{aligned}
R^\omega_{\mu\nu}(q) &= \frac{1}{(q^2 - M_\omega^2)^2 + (\Gamma_\omega M_\omega)^2} \left[-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_\omega^2} \right] \\
&= R^0_\omega(q) \left[-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_\omega^2} \right] \quad ,
\end{aligned} \tag{E.36}$$

$$\Delta^0_\sigma(q) = \frac{1}{(q^2 - M_\sigma^2)^2 + (\Gamma_\sigma M_\sigma)^2} \quad , \tag{E.37}$$

$$\lambda = \mathcal{M}_\chi(\pi^+\pi^- \to \pi^0\pi^0) = -\frac{1}{f_\pi^2}\left(s - M_\pi^2 \right) \quad , \tag{E.38}$$

$$s = (p-k)^2 = M^2_{\pi^0\pi^0} \quad . \tag{E.39}$$

The complex amplitudes are parametrized with $\mathcal{M}_i = M_i'' + iM_i'$ and the absolute value of the square of the invariant amplitude is obtained as

$$
\begin{aligned}
\mid \mathcal{M} \mid^2 \;=\;& M_1'^2 + M_2'^2 + M_3'^2 + M_4'^2 + M_1''^2 + M_2''^2 + M_3''^2 + M_4''^2 \\
&+ 2({M_{12}'}^2 + {M_{13}'}^2 + {M_{14}'}^2 + {M_{23}'}^2 + {M_{24}'}^2 + {M_{34}'}^2 \\
&+ {M_{12}''}^2 + {M_{13}''}^2 + {M_{14}''}^2 + {M_{23}''}^2 + {M_{24}''}^2 + {M_{34}''}^2) \quad , \qquad \text{(E.40)}
\end{aligned}
$$

where

$$
\begin{aligned}
M_1'^2 \;=\;& \left(\frac{e}{M_\omega} g_{\omega\pi\gamma}\right)^2 \left(\frac{g_{\omega\rho\pi}}{M_\omega}\right)^2 \Big\{[(p-q_1)^2 - M_\omega^2] R_\omega^0(p-q_1)\Big\}^2 \\
&\times \frac{1}{3}\{-2k\cdot p\; k\cdot q_1[p^2(p\cdot q_1 - 2q_1^2) + p\cdot q_1\; q_1^2] \\
&+ (k\cdot p)^2[2(p\cdot q_1)^2 - p^2 q_1^2 - 2p\cdot q_1\; q_1^2 + q_1^4] \\
&+ (k\cdot q_1)^2[p^4 + 2(p\cdot q_1)^2 - p^2(2p\cdot\; q_1 + q_1^2)]\} \quad , \qquad \text{(E.41)}
\end{aligned}
$$

$$
\begin{aligned}
M_1''^2 \;=\;& \left(\frac{e}{M_\omega} g_{\omega\pi\gamma}\right)^2 \left(\frac{g_{\omega\rho\pi}}{M_\omega}\right)^2 \Big\{(\Gamma_\omega M_\omega) R_\omega^0(p-q_1)\Big\}^2 \\
&\times \frac{1}{3}\{-2k\cdot p\; k\cdot q_1[p^2(p\cdot q_1 - 2q_1^2) + p\cdot q_1\; q_1^2] \\
&+ (k\cdot p)^2[2(p\cdot q_1)^2 - p^2 q_1^2 - 2p\cdot q_1\; q_1^2 + q_1^4] \\
&+ (k\cdot q_1)^2[p^4 + 2(p\cdot q_1)^2 - p^2(2p\cdot\; q_1 + q_1^2)]\} \quad , \qquad \text{(E.42)}
\end{aligned}
$$

$$
\begin{aligned}
M_2'^2 \;=\;& \left(\frac{e}{M_\omega} g_{\omega\pi\gamma}\right)^2 \left(\frac{g_{\omega\rho\pi}}{M_\omega}\right)^2 \Big\{[(p-q_2)^2 - M_\omega^2] R_\omega^0(p-q_2)\Big\}^2 \\
&\times \frac{1}{3}\{-2k\cdot p\; k\cdot q_2[p^2(p\cdot q_2 - 2q_2^2) + p\cdot q_2\; q_2^2] \\
&+ (k\cdot p)^2[2(p\cdot q_2)^2 - p^2 q_2^2 - 2p\cdot q_2\; q_2^2 + q_2^4]
\end{aligned}
$$

$$+ (k\cdot q_2)^2[p^4 + 2(p\cdot q_2)^2 - p^2(2p\cdot\ q_2 + q_2^2)]\} \quad , \tag{E.43}$$

$$\begin{aligned}
M_2''^2 &= \left(\frac{e}{M_\omega} g_{\omega\pi\gamma}\right)^2 \left(\frac{g_{\omega\rho\pi}}{M_\omega}\right)^2 \Big\{(\Gamma_\omega M_\omega) R^0_\omega(p-q_2)\Big\}^2 \\
&\times \frac{1}{3}\{-2k\cdot p\ k\cdot q_2[p^2(p\cdot q_2 - 2q_2^2) + p\cdot q_2\ q_2^2] \\
&+ (k\cdot p)^2[2(p\cdot q_2)^2 - p^2 q_2^2 - 2p\cdot q_2\ q_2^2 + q_2^4] \\
&+ (k\cdot q_2)^2[p^4 + 2(p\cdot q_2)^2 - p^2(2p\cdot\ q_2 + q_2^2)]\} \quad ,
\end{aligned} \tag{E.44}$$

$$M_3'^2 = \left(\frac{e g_{\rho\pi\pi}\lambda}{2\pi^2 M_\pi^2}\right)^2 (ImI)^2\, \frac{2}{3}(k\cdot p)^2 \quad , \tag{E.45}$$

$$M_3''^2 = \left(\frac{e g_{\rho\pi\pi}\lambda}{2\pi^2 M_\pi^2}\right)^2 (ReI)^2\, \frac{2}{3}(k\cdot p)^2 \quad , \tag{E.46}$$

$$\begin{aligned}
M_4'^2 &= \left(\frac{e g_{\rho\pi\pi}}{2\pi^2 M_\pi^2}\right)^2 (g_{\sigma\pi\pi} M_\sigma)^4 \Big\{[(p-k)^2 - M_\sigma^2] ImI - (\Gamma_\sigma M_\sigma) ReI\Big\}^2 \\
&\times \Big\{\Delta^0_\sigma(p-k)\Big\}^2 \frac{2}{3}(k\cdot p)^2 \quad ,
\end{aligned} \tag{E.47}$$

$$\begin{aligned}
M_4''^2 &= \left(\frac{e g_{\rho\pi\pi}}{2\pi^2 M_\pi^2}\right)^2 (g_{\sigma\pi\pi} M_\sigma)^4 \Big\{[(p-k)^2 - M_\sigma^2] ReI + (\Gamma_\sigma M_\sigma) ImI\Big\}^2 \\
&\times \Big\{\Delta^0_\sigma(p-k)\Big\}^2 \frac{2}{3}(k\cdot p)^2 \quad ,
\end{aligned} \tag{E.48}$$

$$\begin{aligned}
M_{12}'^{\,2} &= \left(\frac{e}{M_\omega} g_{\omega\pi\gamma}\right)^2 \left(\frac{g_{\omega\rho\pi}}{M_\omega}\right)^2 \Big\{[(p-q_1)^2 - M_\omega^2] R^0_\omega(p-q_1)\Big\} \\
&\times \Big\{[(p-q_2)^2 - M_\omega^2] R^0_\omega(p-q_2)\Big\} \\
&\times \frac{1}{3}\{(k\cdot q_2)^2[(p\cdot q_1)^2 - p^2\ q_1^2] + k\cdot p\ k\cdot q_2[p\cdot q_2\ q_1^2 - 2p\cdot q_1\ q_1\cdot q_2
\end{aligned}$$

$$+ p^2(-p\cdot q_1 + q_1^2 + q_1\cdot q_2)] + (k\cdot p)^2[-p\cdot q_2\ q_1^2 + q_1\cdot q_2(-p^2 + q_1\cdot q_2)$$

$$+ p\cdot q_1(2p\cdot q_2 - q_2^2)] + (k\cdot q_1)^2[(p\cdot q_2)^2 - p^2\ q_2^2]$$

$$+ k\cdot q_1[k\cdot q_2\ p^2(p^2 - p\cdot q_1 - p\cdot q_2 + q_1\cdot q_2)$$

$$+ k\cdot p(-2p\cdot q_2\ q_1\cdot q_2 + p\cdot q_1\ q_2^2$$

$$+ p^2(-p\cdot q_2 + q_1\cdot q_2 + q_2^2))]\} \quad , \tag{E.49}$$

$$\begin{aligned} {M''_{12}}^2 &= \left(\frac{e}{M_\omega} g_{\omega\pi\gamma}\right)^2 \left(\frac{g_{\omega\rho\pi}}{M_\omega}\right)^2 \Big\{(\Gamma_\omega M_\omega) R^0_\omega(p-q_1)\Big\} \Big\{(\Gamma_\omega M_\omega) R^0_\omega(p-q_2)\Big\} \\ &\times \frac{1}{3}\{(k\cdot q_2)^2[(p\cdot q_1)^2 - p^2\ q_1^2] + k\cdot p\ k\cdot q_2[p\cdot q_2\ q_1^2 - 2p\cdot q_1\ q_1\cdot q_2 \\ &+ p^2(-p\cdot q_1 + q_1^2 + q_1\cdot q_2)] + (k\cdot p)^2[-p\cdot q_2\ q_1^2 + q_1\cdot q_2(-p^2 + q_1\cdot q_2) \\ &+ p\cdot q_1(2p\cdot q_2 - q_2^2)] + (k\cdot q_1)^2[(p\cdot q_2)^2 - p^2\ q_2^2] \\ &+ k\cdot q_1[k\cdot q_2\ p^2(p^2 - p\cdot q_1 - p\cdot q_2 + q_1\cdot q_2) \\ &+ k\cdot p(-2p\cdot q_2\ q_1\cdot q_2 + p\cdot q_1\ q_2^2 \\ &+ p^2(-p\cdot q_2 + q_1\cdot q_2 + q_2^2))]\} \quad , \end{aligned} \tag{E.50}$$

$$\begin{aligned} {M'_{13}}^2 &= -\left(\frac{e}{M_\omega} g_{\omega\pi\gamma}\right) \left(\frac{g_{\omega\rho\pi}}{M_\omega}\right) \left(\frac{e g_{\rho\pi\pi}\lambda}{2\pi^2 M_\pi^2}\right) \Big\{[(p-q_1)^2 - M_\omega^2] R^0_\omega(p-q_1)\Big\} (ImI) \\ &\times\ \frac{1}{3}\{2k\cdot p\ k\cdot q_1\ p^2 - (k\cdot q_1)^2\ p^2 + (k\cdot p)^2(-2p\cdot q_1 + q_1^2)\} \quad , \end{aligned} \tag{E.51}$$

$$\begin{aligned} {M''_{13}}^2 &= -\left(\frac{e}{M_\omega} g_{\omega\pi\gamma}\right) \left(\frac{g_{\omega\rho\pi}}{M_\omega}\right) \left(\frac{e g_{\rho\pi\pi}\lambda}{2\pi^2 M_\pi^2}\right) \Big\{(\Gamma_\omega M_\omega) R^0_\omega(p-q_1)\Big\} (ReI) \\ &\times\ \frac{1}{3}\{2k\cdot p\ k\cdot q_1\ p^2 - (k\cdot q_1)^2\ p^2 + (k\cdot p)^2(-2p\cdot q_1 + q_1^2)\} \quad , \end{aligned} \tag{E.52}$$

$$
\begin{aligned}
{M'_{14}}^2 &= \left(\frac{e}{M_\omega} g_{\omega\pi\gamma}\right)\left(\frac{g_{\omega\rho\pi}}{M_\omega}\right)\left(\frac{(eg_{\rho\pi\pi})(g_{\sigma\pi\pi}M_\sigma)^2}{2\pi^2 M_\pi^2}\right) \\
&\times \left\{[(p-q_1)^2 - M_\omega^2] R_\omega^0(p-q_1)\right\}\left\{[(p-k)^2 - M_\sigma^2] ImI - (\Gamma_\sigma M_\sigma) ReI\right\} \\
&\times \left\{\Delta_\sigma^0(p-k)\right\} \\
&\times \ \frac{1}{3}\{2k\cdot p \ k\cdot q_1 \ p^2 - (k\cdot q_1)^2 \ p^2 + (k\cdot p)^2(-2p\cdot q_1 + q_1^2)\} \quad , \qquad \text{(E.53)}
\end{aligned}
$$

$$
\begin{aligned}
{M''_{14}}^2 &= \left(\frac{e}{M_\omega} g_{\omega\pi\gamma}\right)\left(\frac{g_{\omega\rho\pi}}{M_\omega}\right)\left(\frac{(eg_{\rho\pi\pi})(g_{\sigma\pi\pi}M_\sigma)^2}{2\pi^2 M_\pi^2}\right) \\
&\times \left\{(\Gamma_\omega M_\omega) R_\omega^0(p-q_1)\right\}\left\{[(p-k)^2 - M_\sigma^2] ReI + (\Gamma_\sigma M_\sigma) ImI\right\} \\
&\times \left\{\Delta_\sigma^0(p-k)\right\} \\
&\times \ \frac{1}{3}\{2k\cdot p \ k\cdot q_1 \ p^2 - (k\cdot q_1)^2 \ p^2 + (k\cdot p)^2(-2p\cdot q_1 + q_1^2)\} \quad , \qquad \text{(E.54)}
\end{aligned}
$$

$$
\begin{aligned}
{M'_{23}}^2 &= -\left(\frac{e}{M_\omega} g_{\omega\pi\gamma}\right)\left(\frac{g_{\omega\rho\pi}}{M_\omega}\right)\left(\frac{eg_{\rho\pi\pi}\lambda}{2\pi^2 M_\pi^2}\right)\left\{[(p-q_2)^2 - M_\omega^2] R_\omega^0(p-q_2)\right\}(ImI) \\
&\times \ \frac{1}{3}\{2k\cdot p \ k\cdot q_2 \ p^2 - (k\cdot q_2)^2 \ p^2 + (k\cdot p)^2(-2p\cdot q_2 + q_2^2)\} \quad , \qquad \text{(E.55)}
\end{aligned}
$$

$$
\begin{aligned}
{M''_{23}}^2 &= -\left(\frac{e}{M_\omega} g_{\omega\pi\gamma}\right)\left(\frac{g_{\omega\rho\pi}}{M_\omega}\right)\left(\frac{eg_{\rho\pi\pi}\lambda}{2\pi^2 M_\pi^2}\right)\left\{(\Gamma_\omega M_\omega) R_\omega^0(p-q_2)\right\}(ReI) \\
&\times \ \frac{1}{3}\{2k\cdot p \ k\cdot q_2 \ p^2 - (k\cdot q_2)^2 \ p^2 + (k\cdot p)^2(-2p\cdot q_2 + q_2^2)\} \quad , \qquad \text{(E.56)}
\end{aligned}
$$

$$
\begin{aligned}
{M'_{24}}^2 &= \left(\frac{e}{M_\omega} g_{\omega\pi\gamma}\right)\left(\frac{g_{\omega\rho\pi}}{M_\omega}\right)\left(\frac{(eg_{\rho\pi\pi})(g_{\sigma\pi\pi}M_\sigma)^2}{2\pi^2 M_\pi^2}\right) \\
&\times \left\{[(p-q_2)^2 - M_\omega^2] R_\omega^0(p-q_2)\right\}\left\{[(p-k)^2 - M_\sigma^2] ImI - (\Gamma_\sigma M_\sigma) ReI\right\} \\
&\times \left\{\Delta_\sigma^0(p-k)\right\}
\end{aligned}
$$

$$\times \ \frac{1}{3}\{2k\cdot p\ k\cdot q_2\ p^2 - (k\cdot q_2)^2\ p^2 + (k\cdot p)^2(-2p\cdot q_2 + q_2^2)\} \quad , \qquad \text{(E.57)}$$

$$\begin{aligned} {M''_{24}}^2 &= \left(\frac{e}{M_\omega} g_{\omega\pi\gamma}\right)\left(\frac{g_{\omega\rho\pi}}{M_\omega}\right)\left(\frac{(eg_{\rho\pi\pi})(g_{\sigma\pi\pi}M_\sigma)^2}{2\pi^2 M_\pi^2}\right) \\ &\times \left\{(\Gamma_\omega M_\omega)R_\omega^0(p-q_2)\right\}\left\{[(p-k)^2 - M_\sigma^2]ReI + (\Gamma_\sigma M_\sigma)ImI\right\} \\ &\times \left\{\Delta_\sigma^0(p-k)\right\} \\ &\times \ \frac{1}{3}\{2k\cdot p\ k\cdot q_2\ p^2 - (k\cdot q_2)^2\ p^2 + (k\cdot p)^2(-2p\cdot q_2 + q_2^2)\} \quad , \end{aligned} \qquad \text{(E.58)}$$

$$\begin{aligned} {M'_{34}}^2 &= -\left(\frac{(eg_{\rho\pi\pi})(g_{\sigma\pi\pi}M_\sigma)}{2\pi^2 M_\pi^2}\right)^2 \lambda\left\{[(p-k)^2 - M_\sigma^2]ImI - (\Gamma_\sigma M_\sigma)ReI\right\} \\ &\times \left\{\Delta_\sigma^0(p-k)\right\}(ImI)\,\frac{2}{3}(k\cdot p)^2 \quad , \end{aligned} \qquad \text{(E.59)}$$

$$\begin{aligned} {M''_{34}}^2 &= -\left(\frac{(eg_{\rho\pi\pi})(g_{\sigma\pi\pi}M_\sigma)}{2\pi^2 M_\pi^2}\right)^2 \lambda\left\{[(p-k)^2 - M_\sigma^2]ReI + (\Gamma_\sigma M_\sigma)ImI\right\} \\ &\times \left\{\Delta_\sigma^0(p-k)\right\}(ReI)\,\frac{2}{3}(k\cdot p)^2 \quad , \end{aligned} \qquad \text{(E.60)}$$

where ImI and ReI are given in the text in Eq. 2.5 and in Eq. 2.6, respectively.

In the rest frame of the decaying ρ^0-meson

$$\begin{aligned} k\cdot p &= M_\rho E_\gamma \quad , \\ k\cdot q_1 &= \frac{1}{2}(M_\rho^2 - 2M_\rho E_2) \quad , \\ k\cdot q_2 &= \frac{1}{2}(M_\rho^2 - 2M_\rho E_1) \quad , \\ p\cdot p &= p^2 = M_\rho^2 \quad , \\ p\cdot q_1 &= M_\rho E_1 \quad , \\ p\cdot q_2 &= M_\rho E_2 \quad , \end{aligned}$$

$$
\begin{aligned}
q_1 \cdot q_1 &= {q_1}^2 = M_{\pi^0}^2 \quad , \\
q_2 \cdot q_2 &= {q_2}^2 = M_{\pi^0}^2 \quad , \\
q_1 \cdot q_2 &= \frac{1}{2}(M_\rho^2 - 2M_\rho E_\gamma - 2M_{\pi^0}^2) \quad .
\end{aligned} \tag{E.61}
$$

VITA

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