LIMITED QUANTITY FLEXIBILITY IN A DECENTRALIZED SUPPLY CHAIN

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ABSTRACT

LIMITED QUANTITY FLEXIBILITY IN A DECENTRALIZED SUPPLY CHAIN

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In this study, we analyze a decentralized supply chain with a single retailer and a single manufacturer where the retailer sells two products in a single period. The products offered by the retailer consist of families of closely related products, which differ from each other in terms of a limited number of features only. The retailer places initial orders based on preliminary demand forecasts at the beginning of the period and has an opportunity to modify his initial order after receiving perfect demand information. However, the final orders of the retailer are constrained by his initial orders. Furthermore, the manufacturer is obligated to fill the retailer's final order for each product. The manufacturer has two options for procurement. The first procurement option is regular delivery at the beginning of the period, after the initial orders of the retailer. The next one is expedited delivery, after the updated orders of the retailer are received. The expedited delivery is more expensive than regular. In this setting, our objective is to present an analytical model for this contract and characterize the optimal policies for the retailer and the manufacturer. We analyze three different levels of order adjustment flexibility settings: (i) no order adjustment, (ii) unlimited order adjustment and (iii) limited order adjustment.

Keywords: Quantity flexibility, Decentralized supply chain

ÖZ

MERKEZİ OLMAYAN BİR TEDARİK ZİNCİRİNDE SINIRLI MİKTAR ESNEKLİĞİ

Karakaya, Selçuk
Yüksek Lisans, Endüstri Mühendisliği Bölümü
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Bu çalışmada, tek dönemde piyasaya iki ürün satan bir perakendeci ve bir üreticiden oluşan merkezi olmayan bir tedarik zinciri analiz edilmektedir. Perakendeci tarafından piyasaya sunulan ürünler yakın ürün ailelerinden olup sınırlı sayıdaki özellikleri birbirinden farklılık göstermektedir. Perakendeci ilk siparişini ön talep tahminlerine dayalı olarak dönemin başında yapar ve kesin talep bilgisini elde ettikten sonra ilk siparişini güncelleme fırsatına sahiptir. Ancak, perakendecinin son sipariş miktarları ilk siparişleri tarafından sınırlandırılır. Ayrıca, üretici her bir ürün için perakendecinin son siparişini yerine getirmekle yükümlüdür. Üreticinin ürününü temin etmek için iki seçeneği vardır. İlk satınalma seçeneği olağan teslimat olup dönemin başında, perakendecinin ilk siparişinden sonradır. Diğer temin seçeneği ise hızlandırılmış teslimat olup perakendecinin güncellenmiş siparişi alındıktan sonradır. Hızlandırılmış teslimat, olağan teslimata göre daha pahalıdır. Bu ortamda, amacımız bu tip bir sözleşme için analitik bir model sunmak ve perakendeci ve üretici için en uygun politikaları tanımlamaktır. Üç farklı düzeyde sipariş ayarlama esnekliği değerlendirilmiştir: (i) sipariş ayarlama olmadan, (ii) sınırsız sipariş ayarlama ve (iii) sınırlı sipariş ayarlama.

Anahtar Kelimeler: Miktar esnekliği, Merkezi Olmayan Tedarik Zinciri

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CHAPTER 1

INTRODUCTION

Customer satisfaction can be achieved by providing rapid delivery of a wide variety of products. In a supply chain, the retailers usually sell products with the same structure in the market. The product portfolio offered by a retailer consists of families of closely related products, which differ from each other in terms of a limited number of differentiating features only. Consider for example, automobile manufacturers offering a virtually endless variety of model configurations. Automobiles may have different audio systems: (i) CD player-changer and navigation, (ii) CD player-changer or (iii) CD player. As a result, the retailers are exposed to more risks associated with product variety and more uncertain demands in the market. In order to reduce mismatches between supply and demand, the retailers are moving to improve efficient coordination schemes across their supply chains. One of the mechanisms employed to balance the risk in a decentralized supply chain is quantity flexibility contract. The quantity flexibility contract is a coordination tool that provides a revision opportunity of the initial order or forecast in a planning zone. A maximum percentage revision of initial order is determined by the participants and the supplier is obliged to cover any requests that remain within the upside limits. This study is motivated by the supply contracts used by the retailers who want to reduce the impact of the fluctuations in customer demands as much as possible and transfer some part of the risk to their manufacturers.

In this study, we analyze a decentralized supply chain with a single retailer and a single manufacturer where the retailer sells two products to an end market in a single period. Furthermore, we assume that the manufacturer is obliged to fill the retailer's final order for each product and the manufacturer cannot sell the products directly to the final customer (i.e., the retailer's customer) in terms of the contract.

In the model, the retailer places an initial order for each product, which is based on preliminary demand forecasts. The supply contract provides the retailer with an opportunity to modify his initial order after observing the realization of the demands. However, the final order of the retailer is constrained by his initial ordering quantities. Order adjustment flexibility parameter is determined by the retailer and the manufacturer at the beginning of the contract and the parameter is exogenous to our model. According to the supply contract, the aggregate of initial order quantities determines the total order of the retailer for two products, whereas the retailer can modify the order of each product to the extent determined by the order adjustment flexibility parameter. The initial order of the retailer gives the manufacturer an idea about the quantity that the retailer intends to purchase. The manufacturer has two options to procure the components of the products. The first procurement option is regular delivery that takes place at the beginning of the period after the retailer's initial orders. The next one is expedited delivery, which can be utilized after the retailer's final orders. The expedited delivery is more expensive than regular delivery and provides a shorter lead time which is enough for the manufacturer to utilize during his production. The manufacturer produces the products after the retailer's final order of each product.

The objective of this study is to present an analytical model for this contract and characterize the optimal policies for the retailer and manufacturer to determine the ordering quantities that maximize their expected profits, separately. In our study, we analyze three different levels of order adjustment flexibility settings: (i) No order adjustment: The retailer is not allowed to modify the final order quantities; (ii) Unlimited order adjustment: The retailer determines the aggregate order quantity at the beginning of the period and final order quantity of each product at the end of the period; (iii) Limited order adjustment: The retailer determines the final order quantity of each product at the end of the period within the upper and lower limits which are determined based on initial orders and order adjustment flexibility parameter. Note that the retailer may have to order excess inventory in order to satisfy the contract requirements. The type of product to be ordered in excess depends on the profitability of the products when sold at the end of the season. Our model will explicitly consider each case.

The rest of the study is organized as follows: In Chapter 2, we summarize the related work in the literature. In Chapter 3, we give the environment, basic model description and define the necessary parameters, variables and functions. Chapter 4 is solely devoted to the analysis of the model and divided into three main sections for each kind of order adjustment model. Chapter 5 is related to the numerical analysis. In addition, the impacts of several parameters on the expected profit of the retailer, the manufacturer and the supply chain are analyzed in this chapter. In Chapter 6, we conclude with the summary of our study and propose further research directions.

CHAPTER 2

LITERATURE REVIEW

Quantity flexibility (QF) contract is a coordination tool for forecast information and order decisions between a retailer and a manufacturer, which is closely related to our work. Therefore, Section 2.1 is dedicated to the studies in the literature that discuss QF contracts. We are considering not only the simple QF contracts, but also the mechanisms that allow some form of order modification opportunity to the retailer in this section. The use of delayed product differentiation and quick response programs in supply chains provides the retailers the ability to delay their ordering decisions rather than revisions. The related literature for these topics is discussed shortly in Section 2.2. After discussing the relevant studies in the literature, their relation to our work is provided at the end of each section.

2.1. Quantity Flexibility Contracts

In this section, we present a number of papers that deal with quantity flexibility contracts in a supply chain. Our aim is to provide a general understanding of different types of quantity flexibility contracts and their effect on the performance of both the supply chain as a whole and the parties involved individually.

Tsay (1999) studies quantity flexibility (QF) contract to coordinate a decentralized supply chain consisting of a manufacturer and a retailer that faces an uncertain market demand. The retailer is able to revise purchase quantity in response to improved demand information under QF contract in the form of $\{c, (\alpha, \omega)\}$ where c is the unit transfer price. The retailer commits to purchase no less than a certain percentage (ω) below the initial forecast while the manufacturer guarantees to deliver up to a certain percentage above (α). He models the centralized system as the optimal performance benchmark. Then, he characterizes the decentralized system with no commitment under asymmetric information and common beliefs between the retailer and the manufacturer about market demand in order to identify the causes of inefficiency, respectively. He characterizes each party's preferences towards the QF contract parameters. However, as the QF contract does not guarantee efficiency by itself, he defines certain conditions that system-wide efficiency can be achieved with appropriate choice of the contract parameters. Finally, he states that there is a trade off between flexibility and unit price; the retailer prefers to pay more for increased flexibility.

Tsay and Lovejoy (1999) consider a Quantity Flexibility (QF) contract to coordinate materials and information flows in a multi-echelon supply chain operating under a rolling-horizon planning basis. The contract places bounds on how the supply chain agents may revise their replenishment schedule in time. That is, the estimate for future periods and purchase quantities cannot be revised upward or downward more than a certain fraction at each planning iteration. Under the QF contract, the supplier is obliged to cover any requests within the upward flexibility limits since he formally guarantees the buyer a safety buffer in excess of requirements. Additionally, the buyer agrees to limit its order reductions, essentially a form of minimum purchase agreement which discourages the buyer from overstating its needs. They analyze the supply chain by decomposing into modules of simpler structure (called node) and suggest an ordering policy considering the market demand dynamics, the costs of holding and shortage and the input flexibility parameters. They analyze the impacts of flexibility characteristics on both inventory and service levels and how order variability is spread in the supply chain. In particular, inventory is the cost incurred in overcoming the inflexibility of a supplier to meet a customer's desire for flexible response. Increasing a node's input flexibility reduces its costs whereas promising more output flexibility comes at the expense of greater inventory costs. They found that QF contracts can reduce the transmission of order variability throughout the chain. Finally, they study the design of QF contracts and provide choice of flexibility parameters for the greatest benefit and discuss how much to pay for it. Flexibility increases in value as the market environment becomes more volatile, and that flexibility observes a principle of diminishing returns.

Bassok et al. (1997) study a supply contract problem with periodic commitments and update flexibility from the buyer's perspective. At the beginning of the horizon, the buyer makes purchasing commitments for each period in the horizon to the supplier and purchase flexibility *(α)* is set. The actual purchasing quantity of the period may deviate from the previous commitment according to the flexibility permitted. Since demand information is updated based on the actual demand realization, the buyer may modify the previous commitments of the next periods and update purchase flexibility *(β).* They present a heuristic to compute nearly optimal initial commitments and purchasing quantities for each period and also a mechanism to update the commitments. They demonstrate the performance of the heuristic for different flexibilities and demand uncertainties and show that the results by the heuristic approach the optimal solution as the flexibility approaches zero. Finally, they use the heuristic to evaluate the worth of flexibility to adjust the commitments and purchase quantities according to the market conditions.

Donohue (2000) examines supply contracts for the coordination of two-mode production decisions with forecast information updates. The manufacturer manages production and sells products to the retailer who supplies these products to retail outlets for a single selling season. The manufacturer has two production modes for the retailer's orders. The first production mode is cheaper with a long lead time whereas the second one has a shorter lead time with a high cost. Additionally, the retailer determines her order quantity in two stages: In the first stage, she decides on the initial order size. ''New market information" becomes available in the second stage and the retailer utilizes this information to revise her forecast. In this setting, Donohue focuses on the coordination between the manufacturer and retailer via contracts to maximize total profit. The contract parameters are (w_1, w_2, b), where w_i is the wholesale price per unit offered for production mode i and b is a return price per unit offered for items left at the end of the selling season. The centralized system case is analyzed as a benchmark and in the decentralized system, efficient contract prices (w_1, w_2, b) and the impact of early market information are discussed. The results show that such a contract can coordinate the manufacturer and retailer to act in the best interest of the channel. The efficient conditions vary depending on the degree of demand forecast improvement between stages and the manufacturer's access to forecast information. The coordinated solution may not be a Pareto optimal one and the manufacturer's expected profit can reduce in some extreme cases.

Miltenburg and Pong (2007a) consider two order opportunities for a family of products under uncertain demand. The first order has a long lead time with low cost whereas the second one has a shorter lead time but higher cost. The demand information between two orders is updated by using Bayesian estimation process. Capacity restriction is not considered in this study. They consider simple procedures for order quantities when demand forecasts are not revised and complex procedures for order quantities when demand forecasts are revised by using Bayesian estimation. In both cases, production costs are considered as constant and variable. Algorithms that calculate best order quantities are presented with numerical examples. The company groups products into three categories according to annual sales as A, B and C. The A items have highest contribution margins. They conclude that complex procedures are appropriate for the most important A items whereas simple procedures are best for other A items, B and C items. Miltenburg and Pong (2007b) extend the same problem to the case with capacity constraints for each order.

Barnes–Schuster et al. (2002) investigate a system that consists of a supplier and a buyer who sells a short life-cycle product to consumers at a fixed market price in season. The season is divided into two periods of possibly unequal lengths with correlated demands between periods. The buyer determines his firm orders (Q_i *units*, $i = \{1, 2\}$), maximum option orders (M *units*) at the beginning of the season. The buyer updates the second period demand and may place additional option orders (m *units* $\leq M$) after observing first period actual demand. The supplier commits to produce for the second period up to the number of option orders (M *units*). The supplier has two production opportunities. She can produce at a cheaper cost in the first period, and at a more expensive cost before the beginning of the second period but after observing the number of options exercised by the buyer. The capacity of the supplier is unconstrained that she is able to produce what the buyer requests. They show that channel coordination can be achieved only if price is piecewise linear, and they derive the appropriate prices that achieve coordination. The joint profit of the channel always increases with options in decentralized and centralized systems. The benefits of options in improving channel performance are quantified via numerical studies and they compare the results to the no coordination case. Additionally, they illustrate how return policies can be used to coordinate the channel with linear prices. Numerical tests show that the optimal demand update timing is usually later in the decentralized system with linear prices than in the centralized system.

Yan et al. (2003) develop a model where a manufacturer orders raw material from two suppliers with demand forecast update. One supplier is fast but expensive and the other is cheap but slow. They derive optimal order quantities for the fast and the slow modes by dynamic optimization problem. They evaluate the benefits of demand information updates and extend the single-period model to a multiple-period model for the uniform and normally distributed demand cases. They demonstrate that the single-period policy is optimal with some regularity conditions on the demand process. Finally, they present data collected from a manufacturer, which support the structure and conclusions of the model and comment that the model can also be applied to production and out-bound logistics decisions.

Sethi et al. (2001) consider a discrete-time periodic review inventory problem with fast and slow delivery options under demand information updates. They analyze multiple demand forecast updates with multiple delivery modes but they focus only on one demand forecast update of each period and two delivery modes for simplicity in this study. When an order issued at the beginning of a period, fast and slow deliveries are realized at the end of the current period and at the end of the next period, respectively. Fast orders are naturally more expensive. At the beginning of each period, inventory and backlog levels are reviewed and the demand information to be realized at the end of the period is updated. With the information of the slow order in the previous period, the quantities to be ordered by the slow and fast modes are decided. They obtain the optimal policy for the finite horizon problem and extend the optimality results to the infinite horizon case. The policy is defined by the base-stock levels for the fast and the slow delivery modes. Sethi et al. (2003) extend their study to allow fixed ordering costs regarding each delivery mode and show the optimality of a forecast-update-dependent *(s, S)* type policy.

Feng at al. (2005) consider a similar system with three delivery modes and demand information updates before demand realization. The lead-times of the fast, medium and slow modes are one, two and three periods, respectively. At the beginning of each period, inventory levels are reviewed, the demand information to be realized at the end of the period is updated and the quantities to be ordered by each delivery mode is decided with the information of the orders in the previous periods. They prove that a state-dependent base-stock policy is optimal for the inventory replenishment policy of fast and medium modes but the slow mode does not generally follow a base-stock policy.

Eppen and Iyer (1997) focus on backup agreements between a catalog company and a manufacturer. According to the backup agreement, the catalog company commits to y units for the season. The manufacturer holds ρ*%* of the commitment and delivers the remaining $y(1-\rho)$ units before the season. After initial demand information, the catalog company has the option to buy as many of the $y\rho$ units with the original cost but will pay b for each unit held at in the backup that it does not buy. They develop a stochastic dynamic programming model of backup agreements, and analyze the impact of the contract conditions (b, ρ), and demand distribution on the performance of backup agreements. Increasing values of b decreases the advantage of using a backup agreement. They present that the existence of a backup agreement motivates the catalog company to increase the committed quantity and helps the catalog company to reduce the impact of the uncertainty in demand. Additionally, adjusting the order commitment in response to the offered ρ can have a significant impact on the expected profits. Finally, they show that the catalog company's and manufacturer's expected profits may improve together at a certain range of parameters.

Milner and Kouvelis (2005) study the impact of different demand characteristics on the quantity and timing flexibility at supply chain consisting of a firm that obtains supply from a supplier and transforms the material into a finished good. They model the ordering policy of a firm at a single season where two orders may be placed and the second order reflects the updated demand information. Depending on the case of flexibility, the second order will be placed (a) at a pre-specified time and quantity (the static case), (b) at a pre-specified time for any quantity (the quantity flexible case), (c) at any time for a pre-specified quantity (the timing flexible case), or (d) at any time for any quantity (the fully dynamic case). They observe that timing flexibility provides greatest benefits in the standard demand case for functional goods, especially with high holding cost. Quantity flexibility is the most beneficial at the Bayesian demand case for fashion driven products for all lead times. Both quantity and timing flexibility are needed to reduce the supply chain costs for the evolving demand products with long lead times.

Gurnani and Tang (1999) study the procurement policy for a retailer that orders from a manufacturer at two instants before the selling season. The total quantity ordered at both instants arrives before the start of the season. The demand information can be improved via market signals until the second instant. The unit cost at the second instant is uncertain and thus, the retailer has to consider the cost of improving forecast accuracy and a potentially higher unit cost at the second instant to maximize its profits. They also consider the special cases that the value of demand information observed between the first and second instants is worthless or perfect. They determine the optimal order quantity at each instant and the optimal conditions for the retailer to delay its order until the second instant.

Choi et al. (2003) consider a similar setting where a retailer orders from a manufacturer at two stages before the selling season. The demand information is updated for the second stage with the market data observed in the first stage by Bayesian approach. The ordering cost at the first stage is known but the ordering cost at the second stage is unknown. The first order has more uncertainty in demand but a deterministic ordering cost while the late order results in less uncertainty in demand but an uncertain ordering cost. They derive the optimal policy by using dynamic programming and study the service level and the variance of profit under the optimal policy by comparing with single stage ordering policies. Finally, they present an extensive numerical analysis and state that optimal ordering policy can improve the expected profit and reduce the profit uncertainty level.

Chen et al. (2006) study a contract in order to coordinate a supply chain with long lead time and demand information update. The selling season is divided into two periods which can have possibly unequal length. The manufacturer decides the initial production quantity before the retailer places the final order thus, he realizes the material procurement or product manufacturing in first period under limited market demand information. The demand information is improved during the first period and the retailer specifies her order quantity in the second stage under more accurate demand information so that some costs can be saved. They propose a bi-directional return policy contract for risk sharing and coordination. Under this contract, the retailer compensates the overproduction of the manufacturer at a pre-specified rate whereas the manufacturer will buy back the overstock products of the retailer at the end of the season at a pre-specified price. The centralized supply chain is analyzed as benchmark and they extend the basic model to a multiple-retailer setting. Numerical examples show that the contract discussed has a better supply chain efficiency performance than the standard return contracts which are only associated with overstocking risk. Finally, this contract can maximize supply chain profit and can modify the allocation of the total supply chain profit between the members by tuning the contract parameters.

Wang et al. (2007) consider a single-period supply chain model with downward substitution and forecast updating to reduce demand uncertainty. The manufacturer sells two products to the retailer where product 1 may be used to satisfy demand for product 1 or product 2 because of better its quality, but not vice versa. The manufacturer has two types of production process where the traditional production is cheaper than the mass customization and both processes can be utilized for each product. In the first stage of the production, the common parts are produced. Next stage is the production of individual products. The order quantities are decided in three stages. At the beginning of a period, the retailer determines the order quantities of each product and then the manufacturer starts production accordingly. The retailer can reorder based on the demand information update and the downward substitution between products before the production of the individual products starts. Finally, the products are allocated to satisfy actual demands. They model the problem as a stochastic linear programming model and provide the optimal ordering policy that maximizes the total system profits. They analyzed four cases in numerical example and show that demand information update and

substitution both reduce demand uncertainty and improve profit individually, but the effectiveness of the system is largest when two means are used simultaneously.
Wu (2005) develops a quantity flexibility contract, which coordinates a decentralized supply chain consisting of a manufacturer and a retailer. First, the retailer makes quantity commitment of q units and flexibility ω under this quantity flexibility contract. The manufacturer arranges his production according to this forecast. During the following periods, the retailer updates demand information through n observations (by using Bayesian procedure) and makes her ultimate order which must be at least $(1-\omega)q$ units. The manufacturer guarantees maximum q units since its production is already completed before the ultimate order. He models how the manufacturer and retailer decide the production quantity and purchase quantity, respectively and show the impact of flexibility ω, lead time for Bayesian updating n and transfer price c on the behaviors and profits of both parties. Numerical analysis shows that the retailer prefers more flexibility and lower transfer price, whereas the manufacturer prefers less flexibility in this type of contract. Additionally, both parties benefit from demand information update.

Liu and Ma (2008) also consider a decentralized supply chain with a manufacturer and a retailer under a standard quantity flexibility contract, which limits the fluctuation of the order quantity. The retailer has two order opportunities in the contract form of (q_1, q_2, η) where η is the fluctuation of ordering quantity. First, the retailer forecast the order quantity q_1 and the manufacturer builds production plan $(1+\eta)q_1$ units while the retailer has to purchase minimum $(1-\eta)q_1$ units. The retailer can revise her order to the amount q_2 based on the actual demand information. They consider several parameters that affect that affect the expected profits of supply chain, such as deposit cost, transport cost and remainder value. They derive the optimal quantity and flexibility of the contract and investigate the effect of flexibility η on the expected profits.

Zhou and Li (2007) consider a supply chain consisting of a manufacturer and a retailer that faces a random demand for a single product with short lifecycle. The retailer places his first order at the beginning of the period and if the actual demand exceeds this order, the retailer will place a second order with a higher unit cost to satisfy all demand. On the contrary, if the actual demand is less than the first order quantity, the manufacturer will take all the residual items at a return price at the end of the period. They derive the expected profit model of the retailer, the manufacturer and all supply chain, separately and compare the coordinated optimal ordering quantities and the retailer's optimal ordering quantities without coordination. They also analyze the impact of these ordering strategies on the order quantity and the expected profit of each individual player and the system expected profit. They present that the expected profit of the retailer and the whole supply chain increase under this kind of contract. Additionally, the manufacturer may induce the retailer to order the coordinated quantity by adjusting the unit return price. As a result, the supply chain is expected to achieve the optimal expected profit.

Ozer et al. (2007) analyze two types of contracts in a supply chain where a manufacturer sells to a procure-to-stock retailer. In the wholesale price contract, the retailer waits enough to observe the market for updating demand information and places her optimal order quantity at a price of *w per unit*. In the dual purchase contract, the retailer has two order opportunities, before and after obtaining the final demand information. The manufacturer provides a discount for advance orders. They characterize optimal policies under the wholesale price contract and the dual purchase contract. They show that dual purchase contract increases the expected profit of the retailer and the supply chain. They also determine the conditions that improve the expected profit of the manufacturer. Additionally, they study the conditions that the dual purchase contract reduces profit variability and how it can be used by the manufacturer as a tool to avoid any risk. The numerical study shows that dual purchase contract creates a strict Pareto improvement over the wholesale price contract when (1) low cost early production is available, (2) the manufacturer avoids from risk and (3) the market uncertainty is high.

Huang et al. (2005) study a buyer's problem involving a two-stage purchase contract with a demand forecast update. In the first stage, the buyer places an initial order based on a preliminary forecast and is able to adjust his initial commitment based on an updated demand forecast in the second stage. Any adjustment incurs a fixed and a variable cost where upward adjustment is no less than the initial cost and downward adjustment is a refund value that is lower than the initial cost. They formulate a two-stage dynamic programming problem to determine the optimal initial order policy at stage 1 and the optimal adjustment policy in view of the improved demand information at stage 2. They obtain an optimal solution for the contract management for general demand distributions and the optimal policy at stage 2 is a generalized (s, S) policy whereas the problem at stage 1 can be solved numerically. Finally, they provide a sensitivity analysis to decide regarding the direction of further improvement in the demand forecast quality and determine a critical contract exercise price above which the contract is not desirable.

Sethi et al. (2004) consider a single-period, two-stage quantity flexibility contract between a buyer and a supplier. The buyer purchases q units of a product at price p at the beginning of a period and then the demand information is updated within period. Under this contract, the buyer has an option to revise the order up to additional δq units at price $p_c > p$ at a time before the demand realizes at the end of the period where δ is known as the flexibility limit ($0 < \delta \leq 1$). Additionally, the buyer has another option for further purchases in the spot market at market price before the demand is realized. They provide optimal quantities to be purchased on contract flexibility and from the spot market after the forecast revision and before the demand is realized. They also obtain optimal initial order for the cases of worthless and perfect information updates. They analyze the impact of the flexibility factor δ on the optimal expected profit and the optimal initial order quantity and investigate the impact of the forecast accuracy on the ordering decisions. Finally, the model is extended to the multi-period case.

Bassok and Anupindi (2008) study a rolling horizon flexibility (RHF) contract and focus on the procurement problem of the buyer. Under this type of contract, the buyer makes order quantity commitments for each period at the beginning of the horizon. The supplier provides a limited flexibility to the buyer to adjust the current order of the period and the commitments of the next periods based on the latest demand information in a rolling horizon manner. They present a general model for an RHF contract for procurement and state that optimal solution to the general RHF contract is perhaps complex. They develop two heuristics to solve the general RHF contract. They propose two metrics; (i) the coefficient of variation to capture the variability in the order process; (ii) the mean absolute deviation to measure the accuracy of advance information shared between the supplier and buyer through the commitments. They compare the performance of the heuristics under various stationary and non-stationary demand patterns in the computational studies and provide several managerial insights for the buyer. They show that the unlimited flexibility is not required to achieve the performance of a newsvendor model but the order process variability decreases significantly as flexibility decreases and the larger value of flexibility allows the higher service level. Additionally, they analyze the value for a buyer to consider his “willingness-to pay” for additional increase in flexibility from a supplier. Finally, they observe that the variability in the order process is lower than the variability in the demand process.

Lian and Deshmukh (2009) also study the Rolling Horizon Planning (RHP) contracts between a supplier and the buyer for a single product with quantity flexibility. Under this supply contract, a buyer receives discounts for committing to purchase quantity in advance and the earlier the commitment is made, the larger the discount is obtained by the buyer. At the beginning of the horizon, the buyer confirms the order for current period and commits orders for the future periods. Based on the latest inventory status and the updated demand information, the buyer can increase the order quantities in future periods on a rolling horizon basis with an additional cost for the extra units. They formulate the problem as a finite-horizon dynamic programming model to minimize the total expected cost per period and present two heuristic solutions to calculate the order volume in each period of the rolling horizon that are called as frozen ordering planning (FOP) and second-level frozen ordering planning (FOPII). In numerical studies, they compare the results of the FOP and FOPII policies among the order-up-to policy under normally distributed demands and show that the expected costs from the heuristics are lower than the cost from the order-up-to policy, while the costs from both heuristics are nearly the same.

Our model differs from most of the existing models of quantity flexibility contracts in the following ways: (i) we propose a contract that coordinates the ordering of two products simultaneously; (ii) we propose that a contract has an order adjustment flexibility level which specifies the maximum and minimum amounts that can be ordered for each product whereas the aggregate order has to be equal to original aggregate order.

The aggregate order quantity is determined at the beginning of the period; however, most of the studies in the literature allow the quantity change until the final order timing. Our model includes an applicable ordering policy where the retailer and the manufacturer act independently but are owned by the same company.

2.2. Delayed Product Differentiation and Quick Response

In this section, we demonstrate a number of studies that explain delayed ordering decisions rather than updating the original orders in the supply chain. Delayed product differentiation and quick response are two of the most beneficial strategic mechanisms to manage the risks associated with product variety and uncertain sales. That is, our aim is to provide a general understanding of delayed product differentiation and quick response programs, and their effects on the performance of both the supply chain as a whole and the parties involved individually. In recent years, the order decoupling point has gained increased acceptance as an important concept when organizing value-adding activities in production and logistics. However, this stream of literature relies generally on qualitative analysis that focuses on concepts.

Fisher and Raman (1996) focus on a Quick Response (QR) system where a major part of production commitment is made after the initial demand is observed to shorten the lead-time. It is assumed that production commitments are made in two points. The first commitment is before any demand occurred and the second one is at given time after initial demand is observed. The manufacturer has two production periods to achieve efficient capacity utilization. The first period is before receiving initial demand information and the second one is during the season after initial demand but before receiving all orders. The second production period has a limited capacity. They formulate the production planning decisions required under QR as a two stage stochastic program. They obtain minimum production quantities and explain a method for estimating the demand probability distributions. Finally, they present the results of the application at a fashion skiwear firm.

Iyer and Bergen (1997) also examine the Quick response (QR) system on the production and marketing variables at a manufacturer-retailer channel in the apparel industry. They characterize inventory levels and expected profits for the manufacturer and retailer with and without a QR system and address that the manufacturer may not be in a better position with QR. They investigate how to make QR profitable for the whole channel and discuss the actions such as use of service level commitments, wholesale price commitments and volume commitments to make QR Pareto improving. Finally, they provide information from industry sources as supporting evidence for their study.

Aviv and Federgruen (2001a) explain and quantify the benefits of delayed differentiation and quick response programs. They consider a company offering a product line of J final products with exogenous, random demands. Inventories are reviewed and decisions are taken periodically. The J items are produced in two stages. First, a common intermediate product is manufactured; this stage requires a lead time of L periods. In the second stage of l periods, the common intermediate product is differentiated into the finished goods. In this study, they characterize the benefits in more general settings, where the demand distributions are unknown and consecutively correlated. They analyze this system in a Bayesian framework, assuming that the estimates of the parameters of the demand distributions are revised on the basis of observed demand data. They consider the problem of allocating an incoming order of the intermediate product among the finished items and also characterize the structure of close-to-optimal ordering policy (for the common intermediate product) in these systems for a variety of types of order cost functions. Finally, they discuss extensions of the basic model in which the demand processes are correlated across the different items or where the period-by-period deviations from the mean demands are correlated across time. They show that the learning effect always results in increased benefits of delayed differentiation as well as lead time reductions through quick response programs, and that the incremental benefits can be very significant indeed.

Aviv and Federgruen (2001b) address multi-item inventory systems with delayed differentiation strategy. Demands in each period follow a given multivariate distribution with arbitrary correlations between items. The items are produced in two stages, each with its own lead-time; in the first stage a common intermediate product is manufactured. The production volumes in the first stage are limited by given capacity constraints whereas no capacity limit is imposed to the second phase. They first develop a lower bound approximation and close-to-optimal heuristic strategies for the single stage production process where products need to be differentiated from the onset. They extend their strategies to general case where product differentiation can be postponed and production occurs in two stages. They use the model to investigate the benefits of various delayed product differentiation (postponement) strategies, including (i) the benefits of flexible versus dedicated production facilities; (ii) the trade-off between capacity and inventory investments; and (iii) the trade-off between capacity investments and service levels. They analyze HP DeskJet Printer supply chain case according to their model. Finally, they discuss two extensions of their basic model where intermediate products can be kept in stock and the estimates of the parameters of the demand distributions are revised on the basis of observed demand data.

Graman and Magazine (2002) study the impact of this postponement capacity on the ability to achieve the benefits of delayed product differentiation. They construct a single-period capacitated inventory - service level model and consider a manufacturing system that produces a single item that is finished into multiple products. After finishing, some of the common generic item is completed as non-postponed products and sent to the warehouse to be stored as finished-goods inventory. In addition, some of the common item is kept as in-process inventory, thereby postponing the commitment to a specific product. Combination of non-postponed and postponed inventories may be used to satisfy the demand. The non-postponed finished-goods inventory is used first to meet demand. Demand in excess of this inventory is met, if possible, through the completion of the common items. In results, the benefit of postponement is defined as the percent reduction in total non-postponed inventory attributed to postponement while the customer service level is held constant. Additionally, the results indicate that a relatively small amount of postponement capacity is needed to achieve all of the benefits of completely delaying product differentiation for all customer demand.

Our model differs from the most of the existing models mentioned in this section in the following ways: (i) The inventory of the intermediate product can be carried in most of the models of delayed product differentiation, however, all of the original aggregate order has to be allocated to one of each product in our model; (ii) The final order of the retailer is constrained by his initial orders in our limited order adjustment model whereas the allocation of intermediate product is not restricted in the most models of delayed product differentiation; (iii) Although, the unlimited order adjustment model is similar to delayed product differentiation in view point of the retailer, we also analyze the manufacturer case.

CHAPTER 3

PROBLEM FORMULATION

3.1. Environment

In this study, we consider a decentralized supply chain consisting of a single retailer and a single manufacturer who provides two products. Two products offered to the market consist of families of closely related products, which differ from each other in terms of a limited number of differentiating features only. Consider for example, automobile manufacturers offer a virtually endless variety of model configurations, that is, two automobiles may have all configurations same but only different colors. The retailer sells the products to an end market in a single period and the demand of each product is uncertain. Additionally, we assume the manufacturer cannot sell directly to the final customer (i.e., the retailer's customer). In this setting, the retailer first quotes an initial order for each product based on the demand forecast. The sum of initial orders determines the total final order quantity for the selling period. Given the initial order, the manufacturer begins to install its production capacity and procure components by regular delivery mode for production. Since the some components are common and used in any kind of product, the manufacturer will utilize regular delivery for these components. Meanwhile, the retailer collects more market demand information before committing the final order of each product. In the second decision stage, although the retailer is not allowed to change the total quantity of the order; he can adjust the order quantity of each product based on the flexibility ratio on the contract. While the manufacturer is obliged to fill the retailer's final order for each product, the retailer is not also allowed to violate the terms of the contract (i.e. flexibility, exceeding total order quantity or limits). The manufacturer utilizes expedited delivery if required, to procure additional components. The expedited delivery provides shorter procurement lead time but its cost is higher than regular delivery.

We illustrate the dynamics of such a contract in Figure 1. Suppose that the manufacturer provides 10% flexibility on the contract to the retailer to adjust the final order. The initial order of the retailer for product 1 and 2 are 80 and 120 units, respectively. Thus, the final order could be between [60, 100] and [100, 140] for product 1 and 2, respectively while total order quantity should be exactly equal to 200 units.

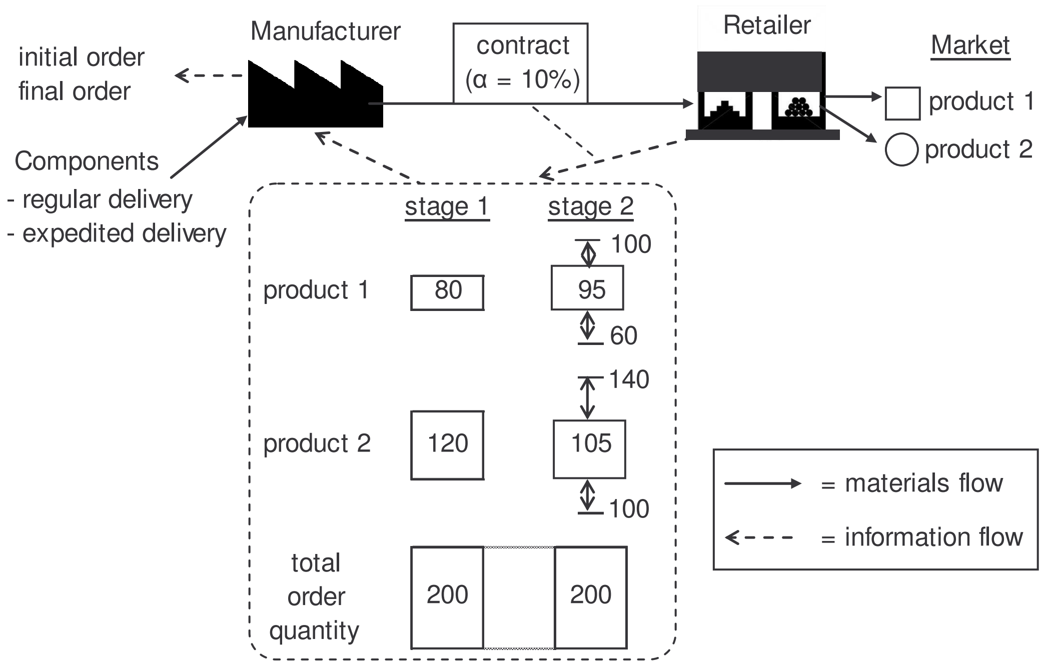

Figure 1 The environment of the model and the dynamics of the contract

In this study, we consider the problem in two phases. First, the problem from the retailer's point of view is formulated to determine the optimal commitment and purchasing policy based on the terms of the contract agreed with the manufacturer to maximize his total profit. Secondly, the manufacturer's problem is considered to determine optimal ordering policy to minimize his total cost.

3.2. Basic Model

The summary of main characteristics and assumptions of the model constructed are as follows:

- The problem is analyzed as a single period which is divided into two stages. The orders at stage 1 and 2 are considered as initial and final orders, respectively.

- The order adjustment flexibility α is negotiated between the retailer and the manufacturer before the beginning of stage 1. Our model assumes an exogenous α value.

- The retailer and the manufacturer do not have any inventory at the beginning of the period.

- The demand of each product is assumed to be independent of the demand of the other product. The retailer utilizes two stages of demand information in determining his order quantities. The demand is random at stage 1 with a distribution function, which is common information for both parties. Furthermore, we assume that the retailer acquires perfect demand information at the beginning of stage 2 before submitting final order.

- The retailer's cost includes only the unit purchase cost. The retailer's revenue includes the unit selling price from the products sold during the selling season. At the end of the selling season, if there are any items in inventory, they are cleared out at a discounted price. All costs and revenue are assumed to be proportional.

- The manufacturer has two options to procure components of products. The first procurement option is regular delivery which can be decided at the beginning of stage 1. The next one is expedited delivery which can be decided at the beginning of stage 2. The expedited delivery is more expensive than regular delivery and provides a shorter lead time which is enough for the manufacturer to utilize during his production. Additionally, the manufacturer's production process is realized during stage 2 according to the retailer's final order of each product.

- The manufacturer does not allow the retailer to violate the terms of the contract (i.e. exceeding total order quantity or limits) even if it is beneficial for him to do so.

- The manufacturer's cost includes unit production cost and unit procurement cost for regular and expedited delivery. The manufacturer's revenue includes the unit wholesale price from the products sold to the retailer. Any unsold item will be cleared out at a discounted price. All costs and revenue parameters are assumed to be proportional.

Below, we introduce the parameters and decisions variables used throughout our analysis.

Cost and revenue parameters:

p_i : the retailer's unit regular price of product *i* sold at the end of the period

s_i : the retailer's unit discounted sales price of product *i* not sold at the end of the period

w_i : the manufacturer's unit wholesale price of product *i*

m_i : the manufacturer's unit procurement cost for product *i*

d_t : the manufacturer's unit delivery cost at stage *t*, d_1 refers to regular delivery whereas d_2 refers to expedited delivery

r_i : the manufacturer's unit discounted sales price for product *i*

The cost and revenue parameters are assumed to be as $p_i > w_i > s_i$ for the retailer and similarly as $w_i > m_i + d_1 > r_i$ for the manufacturer. The delivery cost parameters are assumed to be $d_2 > d_1$ as mentioned in the model.

Flexibility parameter:

α : the order adjustment flexibility, where $\alpha \geq 0$

Demand parameters:

X_i : the random variable with a probability density function $f_i(x_i)$ and a cumulative distribution function $F_i(x_i)$ denoting the demand at stage 1

x_i : the realization of X_i

Decision variables:

q^r_{ti} : the retailer's order quantity of product *i* ordered at the beginning of stage *t*

q^m_{ti} : the manufacturer's procurement quantity for the components of product *i* ordered at stage *t*

Q^r : the retailer's aggregate order quantity for both products at the beginning of stage 1

Timeline of the system dynamics and the ordering decisions is illustrated in Figure 2. The sequence of events and the information structure are as follows:

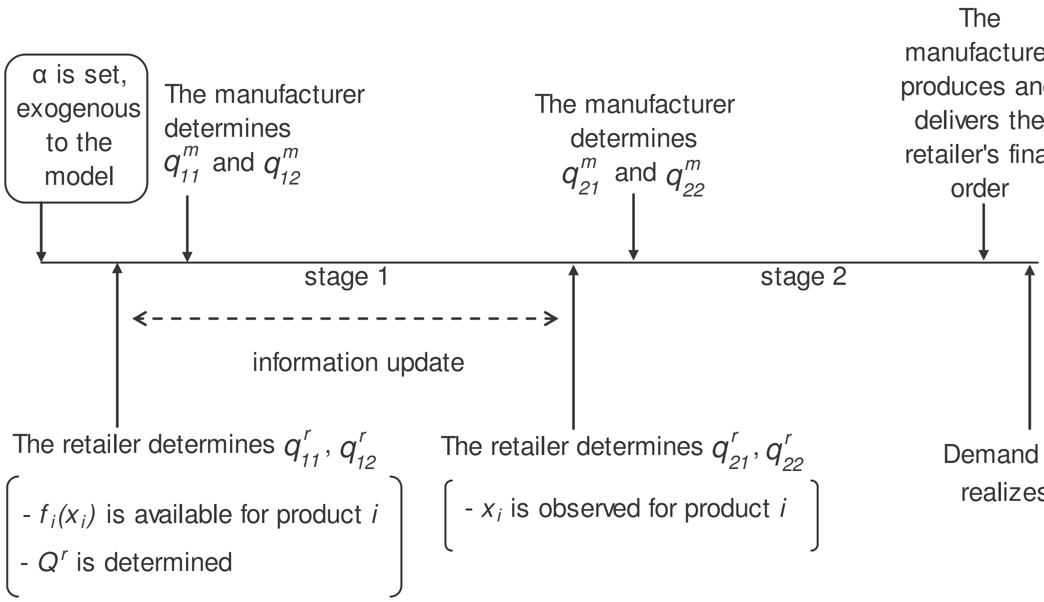

Figure 2 Timeline of the system dynamics and the ordering decisions

1. At the beginning of stage 1, the retailer states the initial order of product *i*, q^r_{1i}, based on the available information about the uncertain market demand, $f_i(x_i)$. The retailer's total order quantity Q^r is fixed at stage 1.

2. The manufacturer determines the regular procurement quantity for each product *i*, q^m_{1i} based on the retailer's initial order, q^r_{1i} and the available information about the uncertain market demand, $f_i(x_i)$.

3. At the beginning of stage 2, the retailer has updated information about the market and observes actual demand of product i, x_i. Based on this, the retailer has an option to adjust his initial order for each product i and decides the final order quantity q_{2i}^{r} for each product i. The initial order quantity q_{1i}^{r} and the final order quantity q_{2i}^{r} of product i and the total order quantity Q^r must satisfy the set of constraints below.

$$\max\{0, q_{1i}^{r} - \alpha Q^{r}\} \leq q_{2i}^{r} \leq \min\{Q^{r}, q_{1i}^{r} + \alpha Q^{r}\} \quad \text{for } i = 1,2$$

$$q_{11}^{r} + q_{12}^{r} = q_{21}^{r} + q_{22}^{r} = Q^{r}$$

4. The manufacturer determines the expedited procurement quantity for each product, q_{2i}^{m}, based on the retailer's final order, q_{2i}^{r}, and his regular procurement quantity, q_{1i}^{m}. The manufacturer delivers the retailer's final order quantity, q_{2i}^{r}, to the retailer and the leftovers are sold at a discounted price.

5. Market demand for each product is realized at the end of the period and is filled by the retailer's final order as much as possible. The retailer receives revenue p_i for unit sold and the items that are not sold at the regular selling season are cleared out a discounted sales price, s_i. Similarly, the manufacturer receives revenue w_i for each unit sold and the items that are not bought by the retailer at the regular selling season are cleared out a discounted sales price, r_i.

In this study, the decentralized supply chain is managed by the individual decisions of the retailer and the manufacturer. That is, the retailer optimizes its ordering policy by only considering its own profit whereas the manufacturer maximizes its own profit under the retailer's ordering behavior. Our main objective is to analyze how the order flexibility affects the expected profit of the parties and characterize the conditions under which it is beneficial to either or both of them.

CHAPTER 4

ANALYSIS OF THE MODEL

In this chapter, we consider three different flexibility settings:

- No order adjustment: The retailer is not allowed to change the final order quantity of each product from the initial order determined at the beginning of the period.
- Unlimited order adjustment: The retailer determines the final order quantity of each product at stage 2 but he should have fixed the total order quantity at the beginning of the period.
- Limited order adjustment: The retailer determines the final order quantity of each product at stage 2 according to the upper and lower limits based on flexibility parameter.

Note that the retailer will take the profitability of the items and their discounted prices into account while making its decision. Without loss of generality, we assume that the product 1 has a larger profit margin. That is, the retailer will first try to satisfy the demand of product 1. Furthermore, since the total order quantity is fixed, the retailer may end up with excess inventory. Noting that it has perfect demand information at the time of the final order; the retailer will choose the item that has the larger discounted profit to carry. Hence we consider two settings;

- "Choose Product II" Scenario: If $w_1 - s_1 > w_2 - s_2$, the retailer will choose to carry product 2 in excess.
- "Choose Product I" Scenario: If $w_1 - s_1 < w_2 - s_2$, the retailer will choose to carry product 1 in excess.

4.1. No Order Adjustment

In order to provide a benchmark for the effects of order flexibility, we first consider the no order adjustment case; that is, *α= 0.* In this case, the final order of the retailer, q^r_{2i}, can not be different from the initial order, q^r_{1i}. The timeline of the system dynamics and the ordering decisions for this case is illustrated in Figure 3.

As indicated in Figure 3, the retailer takes all the risks associated with demand uncertainty in the market. That is, the retailer its order quantity under uncertainty whereas the manufacturer has precise information about its demand. As a benchmark, this case will provide information for comparing the gain or loss from flexibility for the retailer and manufacturer, separately.

4.1.1. Retailer's Problem

In this case, the retailer's problem becomes simply newsvendor problem with two products. Let the retailer's expected profit function be denoted by $\prod^r_1(q^r_{11}, q^r_{12})$. Then, we have;

$$\prod^r_1(q^r_{11}, q^r_{12}) = -w_1 q^r_{11} - w_2 q^r_{12} + \int_{x_1=0}^{q^r_{11}} \left[p_1 x_1 + s_1(q^r_{11} - x_1)\right] f_1(x_1)dx_1 + \int_{x_1=q^r_{11}}^{\infty} p_1 q^r_{11}\ f_1(x_1)dx_1$$
$$+ \int_{x_2=0}^{q^r_{12}} \left[p_2 x_2 + s_2(q^r_{12} - x_2)\right] f_2(x_2)dx_2 + \int_{x_2=q^r_{12}}^{\infty} p_2 q^r_{12}\ f_2(x_2)dx_2$$

Recall that we assumed the demand of each product is independent of the other one. Therefore, we only need to consider the individual probability distribution function of the demand of each product.

Proposition 1 *The optimal order quantities of the retailer are given by;*

$$q^r_{11} = F_1^{-1}\left[\frac{p_1 - w_1}{p_1 - s_1}\right] \quad and \quad q^r_{12} = F_2^{-1}\left[\frac{p_2 - w_2}{p_2 - s_2}\right]$$

Proof: The first derivatives of $\prod^r_1(q^r_{11}, q^r_{12})$ with respect to q^r_{11} and q^r_{12} are provided in Equation (1) and Equation (2), respectively.

$$\frac{\partial \prod^r_1}{\partial q^r_{11}} = -w_1 + s_1 F_1(q^r_{11}) + p_1\left[1 - F_1(q^r_{11})\right] \qquad (1)$$

$$\frac{\partial \prod_1^r}{\partial q_{12}^r} = -w_2 + s_2 F_2(q_{12}^r) + p_2\left[1 - F_2(q_{12}^r)\right] \quad (2)$$

Since the second derivatives with respect to q_{11}^r and q_{12}^r, respectively are both less than zero (Equation (3) and Equation (4)), the expected profit of the retailer is concave in each decision variable. Hence, the optimal order quantities of the retailer are characterized by the unique solution of the first derivates.

$$\frac{\partial^2 \prod_1^r}{\partial (q_{11}^r)^2} = (s_1 - p_1) f_1\left(q_{11}^r\right) < 0 \quad (3)$$

$$\frac{\partial^2 \prod_1^r}{\partial (q_{12}^r)^2} = (s_2 - p_2) f_2\left(q_{12}^r\right) < 0 \quad (4)$$

□

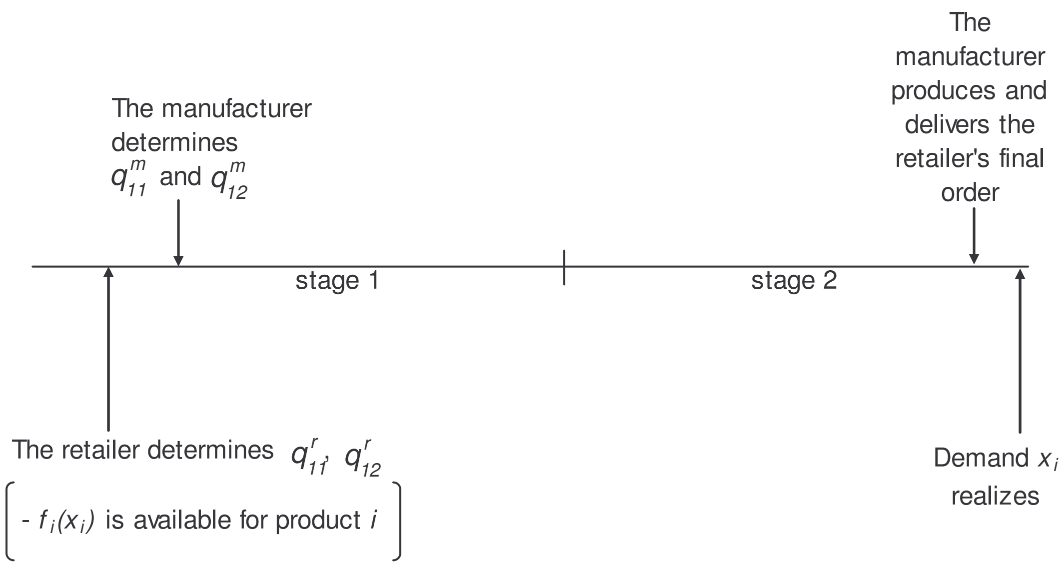

Figure 3 No order adjustment: Timeline of the system dynamics and the ordering decisions

4.1.2. Manufacturer’s Problem

Since there is no order update opportunity for the retailer, the manufacturer basically follows the retailer’s order quantities without taking any risks, that is, $q_{11}^m = (q_{11}^r)^*$ and $q_{12}^m = (q_{12}^r)^*$. The manufacturer’s profit function is then given by:

$$\prod_1^m(q_{11}^m, q_{12}^m) = (w_1 - m_1 - d_1)q_{11}^m + (w_2 - m_2 - d_1)q_{12}^m$$

$$\text{where} \quad q_{11}^m = F_1^{-1}\left[\frac{p_1 - w_1}{p_1 - s_1}\right] \quad \text{and} \quad q_{12}^m = F_2^{-1}\left[\frac{p_2 - w_2}{p_2 - s_2}\right]$$

4.2. Unlimited Order Adjustment

When the retailer has unlimited order adjustment opportunity, the final order quantity, q_{2i}^r, can be between 0 and the total order quantity Q^r in stage 1. The only constraint is that the sum of the final order should be equal to the original order quantity. The timeline of the system dynamics and the ordering decisions for this case is illustrated in Figure 4.

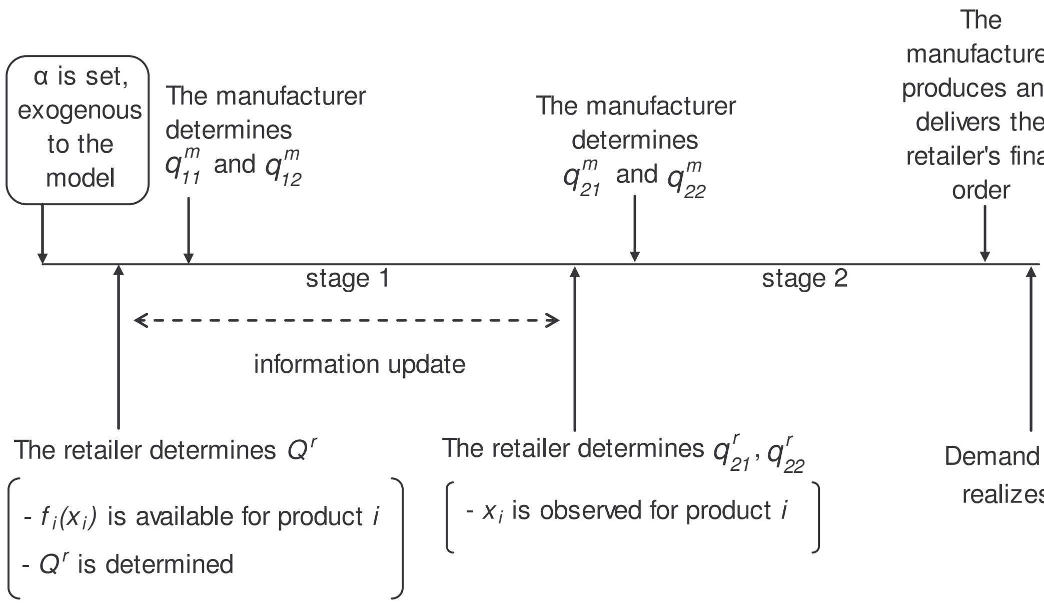

Figure 4 Unlimited order adjustment: Timeline of the system dynamics and the ordering decisions

In this setting, we consider the unlimited order adjustment case, that is to say, the retailer and the manufacturer share the risk of market and both operate under similar uncertain conditions. Some of the risk due to uncertainty is transferred from the retailer to the manufacturer compared to the model in section 4.1. This model will provide information about the maximum the gain and loss from flexibility for the retailer and manufacturer, separately.

Recall that we assume product 1 to be the more profitable to sell without loss of generality. However, in order to determine the final order of the retailer, we need to know which product is more profitable to sell at a discounted price. Note that the retailer would prefer to sell product 1 (2) at a discounted price if $w_1 - s_1 < w_2 - s_2$ ($w_1 - s_1 > w_2 - s_2$). We examine both cases in Section 4.2.1 and Section 4.2.2, respectively.

4.2.1. "Sell Product II At The Discounted Price"

In this section, we consider the setting where $p_1 - w_1 > p_2 - w_2$ and $w_1 - s_1 > w_2 - s_2$. Namely, the retailer prefers to sell product 1 in the regular season and product 2 at the end of the season at a discounted price. We start our analysis with the retailer's problem.

4.2.1.1. Retailer's Problem

In order to solve the retailer's problem, we first characterize its final order at stage 2 in relation to stage 1 total order and demand realization x_i.

Since selling product 1 is more profitable for the retailer, the retailer primarily meets the demand of product 1, x_1, to the extent possible. In addition, if $x_1 < Q^r$, the remaining order will be utilized entirely by product 2 since it is profitable to sell at a discounted price. Hence, we can simply say that the demand of product 1 mostly determines the optimal ordering policy, which is given by Equation (5).

$$(q^r_{21}, q^r_{22}) = \begin{cases} (Q^r, \quad 0) & \text{if} \quad x_1 \geq Q^r \\ (x_1, \quad Q^r - x_1) & \text{if} \quad x_1 < Q^r \end{cases} \tag{5}$$

We now consider the initial order of the retailer; that is stage I problem. Note that specifying individual orders for each product type does not restrict the retailer's order quantities in stage 2. Furthermore, it does not provide additional information to the manufacturer since the flexibility is unlimited. The retailer's conditional profits are given by:

$$\prod_1^r(Q^r/x_1,x_2)=\begin{cases}(p_1-w_1)Q^r & \text{if} \quad x_1\geq Q^r\\ (p_1-w_1)x_1+(p_2-w_2)(Q^r-x_1)+s_2(Q^r-x_1-x_2) & \\ \qquad \text{if} \quad x_1<Q^r, \quad x_2<Q^r-x_1 & \\ (p_1-w_1)x_1+(p_2-w_2)(Q^r-x_1) & \text{if} \quad x_1<Q^r, \quad x_2>Q^r-x_1\end{cases}$$

Hence, regardless of the individual initial order quantities, the expected profit function of the retailer can be expressed in terms of the aggregate order, Q^r, which is given in Equation (6).

$$\prod_1^r(Q^r)=\int_{x_1=0}^{Q^r}\Bigg[(p_1-w_1+w_2)x_1-w_2Q^r+\int_{x_2=0}^{Q^r-x_1}\{\,p_2x_2+s_2(Q^r-x_1-x_2)\,\}f_2(x_2)dx_2 + \int_{x_2=Q^r-x_1}^{\infty}p_2(Q^r-x_1)f_2(x_2)dx_2\Bigg]f_1(x_1)dx_1+\int_{x_1=Q^r}^{\infty}(p_1-w_1)Q^rf_1(x_1)dx_1 \qquad (6)$$

Proposition 2 *The optimal initial order quantity of the retailer, Q^r, is characterized by the unique solution of $d\prod_1^r/dQ^r=0$.*

Proof: The first and second derivatives of $\prod_1^r(Q^r)$ with respect to Q^r are given in Equation (7) and Equation (8), respectively.

$$\frac{d\prod_1^r}{dQ^r}=p_1-w_1-(p_1-w_1-p_2+w_2)F_1(Q^r)-\int_{x_1=0}^{Q^r}(p_2-s_2)F_2(Q^r-x_1)\;f_1(x_1)dx_1 \qquad (7)$$

$$\frac{d^2\prod_1^r(Q^r)}{d(Q^r)^2}=-(p_1-w_1-p_2+w_2)f_1(Q^r)-\int_{x_1=0}^{Q^r}(p_2-s_2)f_2(Q^r-x_1)\;f_1(x_1)dx_1<0 \qquad (8)$$

As the second derivative is less than zero, we can conclude that the expected profit is concave. Since $\lim_{Q^r\to 0}\frac{d\prod_1^r}{dQ^r}=p_1-w_1>0$ and $\lim_{Q^r\to\infty}\frac{d\prod_1^r}{dQ^r}=-w_2+s_2<0$, the optimal Q^r is the unique solution of $d\prod_1^r/dQ^r=0$. □

Note that the unique solution of the retailer's problem is independent of s_1, the discounted sales price of product 1. Since the retailer chooses product 2 to sell at a discounted price, the retailer will not have any product 1 leftover at the end of the selling season. Thus, s_1 does not have any impact on the decision of the retailer.

4.2.1.2. Manufacturer's Problem

The manufacturer's problem is to choose the optimal procurement quantities, q_{12}^m and q_{12}^m that maximize its expected profit subject to the retailer's ordering behavior. Similar to the retailer's problem, we first characterize stage 2 optimal procurement quantities based on the final order quantities of the retailer (which is determined by the demand realizations as observed in Equation (5)), and initial order quantities of the manufacturer. We then characterize the expected profit function of the manufacturer and obtain optimal order quantities.

The manufacturer is obliged to fill the retailer's final order for each product in terms of the contract. As a result, given the manufacturer's initial procurement quantity, the manufacturer will set the second procurement level to meet the retailer's final order quantities.

Note that the manufacturer's initial procurement decisions, q_{11}^m and q_{12}^m are expected to be less than the retailer's total order quantity Q^r as the manufacturer knows that the final order of the retailer for each product can be at most Q^r. To be precise, $q_{11}^m \le Q^r$ and $q_{12}^m \le Q^r$.

Proposition 3 *Given q_{11}^m, q_{12}^m and Q^r, the manufacturer's optimal procurement quantities for stage 2 are characterized as follows;*

(i) *if $q_{11}^m + q_{12}^m \ge Q^r$, then*

$$(q_{21}^m, q_{22}^m) = \begin{cases} (Q^r - q_{11}^m, \quad 0) & \text{if} \quad x_1 > Q^r \\ (x_1 - q_{11}^m, \quad 0) & \text{if} \quad q_{11}^m < x_1 < Q^r \\ (0, \quad 0) & \text{if} \quad Q^r - q_{12}^m < x_1 < q_{11}^m \\ (0, \quad Q^r - x_1 - q_{12}^m) & \text{if} \quad x_1 < Q^r - q_{12}^m \end{cases}$$

(ii) *if* $q_{11}^m + q_{12}^m < Q^r$, *then*

$$(q_{21}^m, q_{22}^m) \quad = \quad \begin{cases} (Q^r - q_{11}^m, \quad 0) & if \quad x_1 > Q^r \\ (x_1 - q_{11}^m, \quad 0) & if \quad Q^r - q_{12}^m < x_1 < Q^r \\ (x_1 - q_{11}^m, \quad Q^r - x_1 - q_{12}^m) & if \quad q_{11}^m < x_1 < Q^r - q_{12}^m \\ (0, \quad Q^r - x_1 - q_{12}^m) & if \quad x_1 < q_{11}^m \end{cases}$$

Proof: We begin with (i) $q_{11}^m + q_{12}^m \geq Q^r$. Recall that the retailer's optimal stage 2 orders are given by Equation (5). Also note that the manufacturer has to satisfy retailer's order in full. When $x_1 > q_{11}^m$, the manufacturer's initial order is insufficient to cover the retailer's order of product 1. Hence, it has to place an expedited order for product 1. If $Q^r - q_{12}^m < x_1 < q_{11}^m$, the retailer's final orders are given by $(q_{21}^r, q_{22}^r) = (x_1, Q^r - x_1)$. Noting that $x_1 < q_{11}^m$ and $Q^r - x_1 < q_{12}^m$, the manufacturer does not place any expedited orders. If $x_1 < Q^r - q_{12}^m$, the retailer's final orders are still $(q_{21}^r, q_{22}^r) = (x_1, Q^r - x_1)$. Noting that $x_1 < q_{11}^m$ and $Q^r - x_1 > q_{12}^m$, the manufacturer places an expedited order of size $q_{22}^m = Q^r - x_1 - q_{12}^m$ for product 2. Similar arguments are valid for the case where $q_{11}^m + q_{12}^m < Q^r$. □

We now characterize the expected profit of the manufacturer in stage 1. The manufacturer needs to decide the optimal procurement quantities to maximize the expected profit subject to the retailer's ordering policy. The manufacturer's conditional profits are given by:

(i) *if* $q_{11}^m + q_{12}^m \geq Q^r$, *then*

$$\prod\nolimits_1^m (q_{11}^m, q_{12}^m) = -(m_1 + d_1) q_{11}^m - (m_2 + d_1) q_{12}^m$$

$$\begin{cases} -(m_1 + d_2)(Q^r - q_{11}^m) + r_2 q_{12}^m + w_1 Q^r & if \quad x_1 > Q^r \\ -(m_1 + d_2)(x_1 - q_{11}^m) + r_2 (q_{12}^m - Q^r + x_1) + w_1 x_1 + w_2 (Q^r - x_1) & \\ \qquad if \quad q_{11}^m < x_1 < Q^r & \end{cases}$$

$$\begin{cases} +r_1(q_{11}^m - x_1) + r_2(q_{12}^m - Q^r + x_1) + w_1 x_1 + w_2(Q^r - x_1) \\ \qquad if \quad Q^r - q_{12}^m < x_1 < q_{11}^m \\ -(m_2 + d_2)(Q^r - x_1 - q_{12}^m) + r_1(q_{11}^m - x_1) + w_1 x_1 + w_2(Q^r - x_1) \\ \qquad if \quad x_1 < Q^r - q_{12}^m \end{cases}$$

(ii) *if* $q_{11}^m + q_{12}^m < Q^r$ *, then*

$$\prod_1^m(q_{11}^m, q_{12}^m) = -(m_1 + d_1)q_{11}^m - (m_2 + d_1)q_{12}^m$$

$$\begin{cases} -(m_1 + d_2)(Q^r - q_{11}^m) + r_2 q_{12}^m + w_1 Q^r & if \ x_1 > Q^r \\ -(m_1 + d_2)(x_1 - q_{11}^m) + r_2(q_{12}^m - Q^r + x_1) + w_1 x_1 + w_2(Q^r - x_1) & \\ \qquad if \quad Q^r - q_{12}^m < x_1 < Q^r & \\ -(m_1 + d_2)(x_1 - q_{11}^m) - (m_2 + d_2)(Q^r - x_1 - q_{12}^m) + w_1 x_1 + w_2(Q^r - x_1) & \\ \qquad if \quad q_{11}^m < x_1 < Q^r - q_{12}^m & \\ -(m_2 + d_2)(Q^r - x_1 - q_{12}^m) + r_1(q_{11}^m - x_1) + w_1 x_1 + w_2(Q^r - x_1) & if \ x_1 < q_{11}^m \end{cases}$$

Considering the optimal expedited delivery orders of the manufacturer given in Proposition 3, the manufacturer's expected profit in stage 1 is given in Equation (9).

$$\prod_1^m(q_{11}^m, q_{12}^m) \quad = \quad \begin{cases} \prod_{11}^m(q_{11}^m, q_{12}^m) & if \quad q_{11}^m + q_{12}^m \geq Q^r \\ \prod_{12}^m(q_{11}^m, q_{12}^m) & if \quad q_{11}^m + q_{12}^m < Q^r \end{cases} \quad \text{where} \qquad (9)$$

$$\begin{aligned} \prod_{11}^m(q_{11}^m, q_{12}^m) = & -(m_1 + d_1)q_{11}^m - (m_2 + d_1)q_{12}^m \\ & + \int_{x_1 = Q^r}^{\infty} \left[w_1 Q^r - (m_1 + d_2)(Q^r - q_{11}^m) + r_2 q_{12}^m \right] f_1(x_1)dx_1 \\ & + \int_{x_1 = 0}^{Q^r - q_{12}^m} \left[w_1 x_1 + w_2(Q^r - x_1) - (m_2 + d_2)(Q^r - x_1 - q_{12}^m) + r_1(q_{11}^m - x_1) \right] f_1(x_1)dx_1 \\ & + \int_{x_1 = Q^r - q_{12}^m}^{q_{11}^m} \left[w_1 x_1 + w_2(Q^r - x_1) + r_1(q_{11}^m - x_1) + r_2(q_{12}^m - Q^r + x_1) \right] f_1(x_1)dx_1 \\ & + \int_{x_1 = q_{11}^m}^{Q^r} \left[w_1 x_1 + w_2(Q^r - x_1) - (m_1 + d_2)(x_1 - q_{11}^m) + r_2(q_{12}^m - Q^r + x_1) \right] f_1(x_1)dx_1 \end{aligned}$$

and

$$
\begin{aligned}
\prod\nolimits_{12}^{m}(q_{11}^{m},q_{12}^{m}) &= -(m_1+d_1)q_{11}^{m}-(m_2+d_1)q_{12}^{m} \\
&+\int_{x_1=Q^r}^{\infty}\left[w_1Q^r-(m_1+d_2)(Q^r-q_{11}^{m})+r_2q_{12}^{m}\right]\, f_1(x_1)dx_1 \\
&+\int_{x_1=0}^{q_{11}^{m}}\left[w_1x_1+w_2(Q^r-x_1)-(m_2+d_2)(Q^r-x_1-q_{12}^{m})+r_1(q_{11}^{m}-x_1)\right]\, f_1(x_1)dx_1 \\
&+\int_{x_1=q_{11}^{m}}^{Q^r-q_{12}^{m}}\left[w_1x_1+w_2(Q^r-x_1)-(m_1+d_2)(x_1-q_{11}^{m})-(m_2+d_2)(Q^r-x_1-q_{12}^{m})\right] f_1(x_1)dx_1 \\
&+\int_{x_1=Q^r-q_{12}^{m}}^{Q^r}\left[w_1x_1+w_2(Q^r-x_1)-(m_1+d_2)(x_1-q_{11}^{m})+r_2(q_{12}^{m}-Q^r+x_1)\right]\, f_1(x_1)dx_1.
\end{aligned}
$$

After rearranging the terms in $\prod_1^m(q_{11}^m,q_{12}^m)$ and $\prod_1^m(q_{11}^m,q_{12}^m)$, we observe that these functions are actually identical and can be rewritten as in Equation (10).

$$
\begin{aligned}
\prod\nolimits_{1}^{m}(q_{11}^{m},q_{12}^{m}) = &-(m_1+d_1)q_{11}^{m}-(m_2+d_1)q_{12}^{m}+\int_{x_1=0}^{Q^r}\left[w_1x_1+w_2(Q^r-x_1)\right]\, f_1(x_1)dx_1 \\
&+\int_{x_1=Q^r}^{\infty}\left[w_1Q^r-(m_1+d_2)(Q^r-q_{11}^{m})+r_2q_{12}^{m}\right]\, f_1(x_1)dx_1 \\
&+\int_{x_1=0}^{q_{11}^{m}} r_1(q_{11}^{m}-x_1)\, f_1(x_1)dx_1-\int_{x_1=q_{11}^{m}}^{Q^r}(m_1+d_2)(x_1-q_{11}^{m})\, f_1(x_1)dx_1 \\
&+\int_{x_1=Q^r-q_{12}^{m}}^{Q^r} r_2(q_{12}^{m}-Q^r+x_1)\, f_1(x_1)dx_1-\int_{x_1=0}^{Q^r-q_{12}^{m}}(m_2+d_2)(Q^r-x_1-q_{12}^{m})\, f_1(x_1)dx_1
\end{aligned}
\qquad (10)
$$

Proposition 4 *Let $(q_{11}^m)'$ and $(q_{12}^m)'$ be the unique solution of $\partial\prod_1^m/\partial q_{11}^m=0$ and $\partial\prod_1^m/\partial q_{12}^m=0$, respectively. Then, the optimal initial procurement quantities of the manufacturer are $\left((q_{11}^m)^*,(q_{12}^m)^*\right)=\left(\min(Q^r,(q_{11}^m)'),(q_{12}^m)'\right)$.*

Proof: The first derivatives of $\prod_1^m(q_{11}^m,q_{12}^m)$ with respect to q_{11}^m and q_{12}^m are given in Equation (11) and Equation (12), respectively.

$$
\frac{\partial\prod_1^m}{\partial q_{11}^m}=d_2-d_1-(m_1+d_2-r_1)F_1(q_{11}^m) \qquad (11)
$$

$$
\frac{\partial\prod_1^m}{\partial q_{12}^m}=-m_2-d_1+r_2-(r_2-m_2-d_2)F_1(Q^r-q_{12}^m) \qquad (12)
$$

The second derivatives of $\prod_1^m(q_{11}^m, q_{12}^m)$ in Equation (13), Equation (14) and Equation (15) indicate that the determinant of the hessian matrix is positive.

$$\frac{\partial^2 \prod_1^m}{\partial (q_{11}^m)^2} = -(m_1 + d_2 - r_1) f_1(q_{11}^m) < 0 \tag{13}$$

$$\frac{\partial^2 \prod_1^m}{\partial (q_{11}^m)^2} = (r_2 - m_2 - d_2) f_1(Q^r - q_{12}^m) < 0 \tag{14}$$

$$\frac{\partial^2 \prod_1^m}{\partial q_{11}^m \partial q_{12}^m} = \frac{\partial^2 \prod_1^m}{\partial q_{12}^m \partial q_{11}^m} = 0 \tag{15}$$

If $(q_{11}^m)' \leq Q^r$, then the optimal solution is given by first order conditions. Otherwise, $(q_{11}^m)^* = Q^r$ and $(q_{12}^m)^* = (q_{12}^m)'$. □

Note that the optimal initial order quantities of the manufacturer do not depend explicitly on w_1 and w_2. This is reasonable since the manufacturer's sales quantities depend on the order of the retailer. However, it should be noted that they depend on Q^r, which depends on both w_1 and w_2. It should be also noted that optimal q_{11}^m and q_{12}^m depend on the only distribution of the demand of product 1. This results from the fact that the final order quantities of the retailer are determined by the demand of product 1. That is, if Q^r is sufficient to cover the demand for product 1, the remaining portion of the order is filled by product 2 regardless of its demand realization. Furthermore, note that the optimal initial order quantities of the manufacturer, q_{11}^m and q_{12}^m, are independent of each other.

4.2.2. "Sell Product I At The Discounted Price"

In this section, we consider the setting where $p_1 - w_1 > p_2 - w_2$ and $w_1 - s_1 < w_2 - s_2$. Specifically, the retailer prefers to sell product 1 at both the regular price and the discounted price. We start with our analysis with the retailer's problem.

4.2.2.1. Retailer's Problem

In order to solve the retailer's problem, we follow a similar approach as in Section 4.2.1.1. We first characterize final order of the retailer at stage 2 with regard to stage 1 total order and demand realization, x_i, and then characterize the expected profit function of stage 1, and derive the optimal total order quantity.

The retailer will first satisfy the demands to the extent possible starting with product 1 since it is more profitable. If Q^r is sufficient to satisfy the aggregate demand, the retailer will order product 1 in order to reach a total order quantity of Q^r since selling product 1 at a discounted price is more profitable when compared to product 2. Hence, given that the demand realizations, x_1 and x_2, are perfectly known and $q_{11}^r + q_{12}^r = Q^r$, the retailer's optimal order quantities for stage 2 are given in Equation (16).

$$(q_{21}^r, q_{22}^r) = \begin{cases} (Q^r, \quad 0) & \text{if} \quad x_1 \geq Q^r \\ (Q^r - x_2, \quad x_2) & \text{if} \quad x_1 < Q^r \quad \text{and} \quad x_2 \leq Q^r - x_1 \\ (x_1, \quad Q^r - x_1) & \text{if} \quad x_1 < Q^r \quad \text{and} \quad x_2 > Q^r - x_1 \end{cases} \tag{16}$$

The retailer's conditional profits are given by:

$$\prod_1^r (Q^r / x_1, x_2) = \begin{cases} (p_1 - w_1)Q^r & \text{if} \quad x_1 \geq Q^r \\ p_1 x_1 - w_1(Q^r - x_2) + (p_2 - w_2)x_2 & \\ \quad + s_1(Q^r - x_2 - x_1) & \text{if} \quad x_1 < Q^r, \quad x_2 \leq Q^r - x_1 \\ (p_1 - w_1)x_1 + (p_2 - w_2)(Q^r - x_1) & \text{if} \quad x_1 < Q^r, \quad x_2 > Q^r - x_1 \end{cases}$$

Considering the optimal stage 2 orders of the retailer given in Equation (16), the retailer's expected profit function is given by;

$$\prod_1^r (Q^r) = \int_{x_1=0}^{Q^r} \left[\int_{x_2=0}^{Q^r - x_1} \left\{ (p_1 x_1 + (p_2 - w_2)x_2 - w_1(Q^r - x_2) + s_1(Q^r - x_2 - x_1) \right\} f_2(x_2) dx_2 \right.$$

$$+ \int_{x_2=Q^r-x_1}^{\infty} \{ (p_1 - w_1)x_1 + (p_2 - w_2)(Q^r - x_1) \} f_2(x_2)dx_2 \Bigg] f_1(x_1)dx_1$$

$$+ \int_{x_1=Q^r}^{\infty} (p_1 - w_1)Q^r f_1(x_1)dx_1.$$

Proposition 5 *The optimal initial order quantity of the retailer, Q^r, is characterized by the unique solution of* $d\prod_1^r / dQ^r = 0$.

Proof: The first and second derivatives of $\prod_1^r(Q^r)$ with respect to Q^r are given in Equation (17) and Equation (18), respectively.

$$\frac{d\prod_1^r}{dQ^r} = p_1 - w_1 - (p_1 - w_1 - p_2 + w_2)F_1(Q^r) + \int_{x_1=0}^{Q^r} (s_1 - w_1 - p_2 + w_2)F_2(Q^r - x_1)f_1(x_1)dx_1 \tag{17}$$

$$\frac{d^2\prod_1^r}{d(Q^r)^2} = -(p_1 - w_1 - p_2 + w_2)f_1(Q^r) + \int_{x_1=0}^{Q^r} (s_1 - w_1 - p_2 + w_2)f_2(Q^r - x_1)f_1(x_1)dx_1 < 0 \tag{18}$$

As the second derivative is less than zero, we can conclude that the expected profit is concave.

Since $\lim_{Q^r \to 0} \frac{d\prod_1^r}{dQ^r} = p_1 - w_1 > 0$ and $\lim_{Q^r \to \infty} \frac{d\prod_1^r}{dQ^r} = -w_1 + s_1 < 0$, the optimal Q^r is the unique solution of $d\prod_1^r / dQ^r = 0$.

□

The unique solution of the retailer's problem is independent of s_2, the discounted sales price of product 2. Since the retailer chooses product 1 to sell at a discounted price, it will not have any product 2 leftover at the end of the selling season. Thus, s_2 does not have any impact on the decision of the retailer.

4.2.2.2. Manufacturer's Problem

The manufacturer's problem is same as in previous setting, that is, to decide the optimal procurement quantities (q_{12}^m and q_{12}^m) that maximize his expected profit subject to the retailer's ordering strategy. We follow a similar approach as in Section 4.2.1.2. That is, we first characterize stage 2 optimal procurement quantities based on the final order of the retailer which is determined by demand realizations x_i, and the initial order quantities of the manufacturer, and subsequently characterize the profit function of stage 1 to derive the optimal initial procurement quantities of the manufacturer.

Recall that the manufacturer is required to fill the retailer's final order quantity for each product in terms of the contract. Given the manufacturer's initial procurement quantities, the manufacturer will set the second procurement order to cover the retailer's final order quantity. Also note that the final order of the retailer for each product can be at most Q^r and the manufacturer's initial procurement decisions, q_{11}^m and q_{12}^m, are to be less than the retailer's total order quantity Q^r ($q_{11}^m \le Q^r$ and $q_{12}^m \le Q^r$).

Proposition 6 *Given q_{11}^m, q_{12}^m and Q^r, the manufacturer's optimal order quantities for stage 2 are characterized as follows;*

(i) *if $q_{11}^m + q_{12}^m \ge Q^r$, then*

$$(q_{21}^m, q_{22}^m) = \begin{cases} (Q^r - q_{11}^m, \quad 0) & \text{if} \quad x_1 > Q^r \\ (0, \quad x_2 - q_{12}^m) & \text{if} \quad x_1 + x_2 < Q^r, \quad q_{12}^m < x_2 \\ (0, \quad 0) & \text{if} \quad x_1 + x_2 < Q^r, \quad Q^r - q_{11}^m < x_2 < q_{12}^m \\ (Q^r - x_2 - q_{11}^m, \quad 0) & \text{if} \quad x_1 + x_2 < Q^r, \quad x_2 < Q^r - q_{11}^m \\ (x_1 - q_{11}^m, \quad 0) & \text{if} \quad x_1 + x_2 > Q^r, \quad q_{11}^m < x_1 \\ (0, \quad 0) & \text{if} \quad x_1 + x_2 > Q^r, \quad Q^r - q_{12}^m < x_1 < q_{11}^m \\ (0, \quad Q^r - x_1 - q_{12}^m) & \text{if} \quad x_1 + x_2 > Q^r, \quad x_1 < Q^r - q_{12}^m \end{cases}$$

(ii) *if $q_{11}^m + q_{12}^m < Q^r$, then*

$$(q_{21}^m, q_{22}^m) = \begin{cases} (Q^r - q_{11}^m, \quad 0) & \text{if} \quad x_1 > Q^r \\ (0, \quad x_2 - q_{12}^m) & \text{if} \quad x_1 + x_2 < Q^r, \quad Q^r - q_{11}^m < x_2 \\ (Q^r - x_2 - q_{11}^m, \quad x_2 - q_{12}^m) & \text{if} \quad x_1 + x_2 < Q^r, \quad q_{12}^m < x_2 < Q^r - q_{11}^m \\ (Q^r - x_2 - q_{11}^m, \quad 0) & \text{if} \quad x_1 + x_2 < Q^r, \quad x_2 < q_{12}^m \end{cases}$$

$$(q_{21}^m, q_{22}^m) = \begin{cases} (x_1 - q_{11}^m, \quad 0) & \text{if} \quad x_1 + x_2 > Q^r, \quad Q^r - q_{12}^m < x_1 \\ (x_1 - q_{11}^m, \quad Q^r - x_1 - q_{12}^m) & \text{if} \quad x_1 + x_2 > Q^r, \quad q_{11}^m < x_1 < Q^r - q_{12}^m \\ (0, \quad Q^r - x_1 - q_{12}^m) & \text{if} \quad x_1 + x_2 > Q^r, \quad x_1 < q_{11}^m \end{cases}$$

Proof: We begin with (i) $q_{11}^m + q_{12}^m \geq Q^r$. Recall that the retailer's optimal stage 2 orders are given by Equation (16). Also note that the manufacturer has to satisfy retailer's final order in full. When $x_1 > Q^r$, the manufacturer's initial order is insufficient to cover the retailer's order of product 1 and then it places an expedited order for product 1, $Q^r - q_{11}^m$.

If $x_1 + x_2 < Q^r$, the retailer's final orders are given by $(q_{21}^r, q_{22}^r) = (Q^r - x_2, x_2)$. When $q_{12}^m < x_2$, the manufacturer places an expedited order of size $q_{22}^m = x_2 - q_{12}^m$ for product 2. In case $x_2 < q_{12}^m$ and $Q^r - x_2 < q_{11}^m$, the manufacturer does not place any expedited orders. When $x_2 < q_{12}^m$ and $q_{11}^m < Q^r - x_2$, the manufacturer places an expedited order of size $q_{21}^m = Q^r - x_2 - q_{11}^m$ for product 1.

If $x_1 < Q^r$ and $x_1 + x_2 > Q^r$, the retailer's final orders are given by $(q_{21}^r, q_{22}^r) = (x_1, Q^r - x_1)$. When $x_1 > q_{11}^m$, the manufacturer's initial order is insufficient to cover the retailer's order of product 1 and then it places an expedited order of size $q_{21}^m = x_1 - q_{11}^m$ for product 1. When $x_1 < q_{11}^m$ and $Q^r - x_1 < q_{12}^m$, the manufacturer does not place any expedited orders. When $x_1 < q_{11}^m$ and $Q^r - x_1 > q_{12}^m$, the manufacturer places an expedited order of size $q_{22}^m = Q^r - x_1 - q_{12}^m$ for product 2. Similar arguments are also valid for the case where $q_{11}^m + q_{12}^m < Q^r$. □

We now characterize the expected profit of the manufacturer in stage 1. The manufacturer's conditional profits are given by:

(i) *if* $q_{11}^m + q_{12}^m \geq Q^r$, *then*

$$\prod_1^m(q_{11}^m, q_{12}^m) = -(m_1 + d_1)q_{11}^m - (m_2 + d_1)q_{12}^m$$

$$\begin{cases} -(m_1+d_2)(Q^r - q_{11}^m) + w_1 Q^r + r_2 q_{12}^m & \text{if} \quad x_1 > Q^r \\ -(m_2+d_2)(x_2 - q_{12}^m) + w_2 x_2 + w_1(Q^r - x_2) + r_1(q_{11}^m - Q^r + x_2) & \\ \qquad \text{if} \quad x_1 + x_2 < Q^r, \quad q_{12}^m < x_2 & \\ +w_2 x_2 + w_1(Q^r - x_2) + r_1(q_{11}^m - Q^r + x_2) + r_2(q_{12}^m - x_2) & \\ \qquad \text{if} \quad x_1 + x_2 < Q^r, \quad Q^r - q_{11}^m < x_2 < q_{12}^m & \\ -(m_1+d_2)(Q^r - x_2 - q_{11}^m) + w_2 x_2 + w_1(Q^r - x_2) + r_2(q_{12}^m - x_2) & \\ \qquad \text{if} \quad x_1 + x_2 < Q^r, \quad x_2 < Q^r - q_{11}^m & \\ -(m_1+d_2)(x_1 - q_{11}^m) + w_1 x_1 + w_2(Q^r - x_1) + r_2(q_{12}^m - Q^r + x_1) & \\ \qquad \text{if} \quad x_1 + x_2 > Q^r, \quad q_{11}^m < x_1 & \\ +w_1 x_1 + w_2(Q^r - x_1) + r_1(q_{11}^m - x_1) + r_2(q_{12}^m - Q^r + x_1) & \\ \qquad \text{if} \quad x_1 + x_2 > Q^r, \quad Q^r - q_{12}^m < x_1 < q_{11}^m & \\ -(m_2+d_2)(Q^r - x_1 - q_{12}^m) + w_1 x_1 + w_2(Q^r - x_1) + r_1(q_{11}^m - x_1) & \\ \qquad \text{if} \quad x_1 + x_2 > Q^r, \quad x_1 < Q^r - q_{12}^m & \end{cases}$$

(ii) *if* $q_{11}^m + q_{12}^m < Q^r$, *then*

$$\prod_1^m(q_{11}^m, q_{12}^m) = -(m_1 + d_1)q_{11}^m - (m_2 + d_1)q_{12}^m$$

$$\begin{cases} -(m_1+d_2)(Q^r - q_{11}^m) + w_1 Q^r + r_2 q_{12}^m & \text{if} \quad x_1 > Q^r \\ -(m_2+d_2)(x_2 - q_{12}^m) + w_2 x_2 + w_1(Q^r - x_2) + r_1(q_{11}^m - Q^r + x_2) & \\ \qquad \text{if} \quad x_1 + x_2 < Q^r, \quad Q^r - q_{11}^m < x_2 & \\ -(m_1+d_2)(Q^r - x_2 - q_{11}^m) - (m_2+d_2)(x_2 - q_{12}^m) + w_2 x_2 + w_1(Q^r - x_2) & \\ \qquad \text{if} \quad x_1 + x_2 < Q^r, \quad q_{12}^m < x_2 < Q^r - q_{11}^m & \\ -(m_1+d_2)(Q^r - x_2 - q_{11}^m) + w_2 x_2 + w_1(Q^r - x_2) + r_2(q_{12}^m - x_2) & \\ \qquad \text{if} \quad x_1 + x_2 < Q^r, \quad x_2 < q_{12}^m & \\ -(m_1+d_2)(x_1 - q_{11}^m) + w_1 x_1 + w_2(Q^r - x_1) + r_2(q_{12}^m - Q^r + x_1) & \\ \qquad \text{if} \quad x_1 + x_2 > Q^r, \quad Q^r - q_{12}^m < x_1 & \\ -(m_1+d_2)(x_1 - q_{11}^m) - (m_2+d_2)(Q^r - x_1 - q_{12}^m) + w_1 x_1 + w_2(Q^r - x_1) & \\ \qquad \text{if} \quad x_1 + x_2 > Q^r, \quad q_{11}^m < x_1 < Q^r - q_{12}^m & \\ -(m_2+d_2)(Q^r - x_1 - q_{12}^m) + w_1 x_1 + w_2(Q^r - x_1) + r_1(q_{11}^m - x_1) & \\ \qquad \text{if} \quad x_1 + x_2 > Q^r, \quad x_1 < q_{11}^m & \end{cases}$$

Considering the optimal expedited delivery orders of the manufacturer given in Proposition 6, the manufacturer's expected profit is given in Equation (19).

$$\prod_{1}^{m}(q_{11}^{m},q_{12}^{m}) = \begin{cases} \prod_{11}^{m}(q_{11}^{m},q_{12}^{m}) & if \quad q_{11}^{m}+q_{12}^{m} \geq Q^{r} \\ \prod_{12}^{m}(q_{11}^{m},q_{12}^{m}) & if \quad q_{11}^{m}+q_{12}^{m} < Q^{r} \end{cases} \quad \text{where} \qquad (19)$$

$$\prod_{11}^{m}(q_{11}^{m},q_{12}^{m}) = -(m_1+d_1)q_{11}^{m}-(m_2+d_1)q_{12}^{m}+\int_{x_1=Q^r}^{\infty}\Big[w_1Q^r-(m_1+d_2)(Q^r-q_{11}^{m})$$
$$+r_2q_{12}^{m}\Big]\,f_1(x_1)dx_1+\int_{x_2=0}^{Q^r-q_{11}^{m}}\int_{x_1=0}^{Q^r-x_2}\Big[-(m_1+d_2)(Q^r-x_2-q_{11}^{m})+w_2x_2$$
$$+w_1(Q^r-x_2)+r_2(q_{12}^{m}-x_2)\Big]\,f_1(x_1)dx_1f_2(x_2)dx_2+\int_{x_2=Q^r-q_{11}^{m}}^{q_{12}^{m}}\int_{x_1=0}^{Q^r-x_2}\Big[w_2x_2$$
$$+w_1(Q^r-x_2)+r_1(q_{11}^{m}-Q^r+x_2)+r_2(q_{12}^{m}-x_2)\Big]\,f_1(x_1)dx_1f_2(x_2)dx_2$$
$$+\int_{x_2=q_{12}^{m}}^{Q^r}\int_{x_1=0}^{Q^r-x_2}\Big[-(m_2+d_2)(x_2-q_{12}^{m})+w_2x_2+w_1(Q^r-x_2)$$
$$+r_1(q_{11}^{m}-Q^r+x_2)\Big]\,f_1(x_1)dx_1f_2(x_2)dx_2+\int_{x_1=0}^{Q^r-q_{12}^{m}}\int_{x_2=Q^r-x_1}^{\infty}\Big[-(m_2+d_2)(Q^r-x_1-q_{12}^{m})$$
$$+w_1x_1+w_2(Q^r-x_1)+r_1(q_{11}^{m}-x_1)\Big]\,f_2(x_2)dx_2f_1(x_1)dx_1$$
$$+\int_{x_1=Q^r-q_{12}^{m}}^{q_{11}^{m}}\int_{x_2=Q^r-x_1}^{\infty}\Big[w_1x_1+w_2(Q^r-x_1)+r_1(q_{11}^{m}-x_1)$$
$$+r_2(q_{12}^{m}-Q^r+x_1)\Big]\,f_2(x_2)dx_2f_1(x_1)dx_1+\int_{x_1=q_{11}^{m}}^{Q^r}\int_{x_2=Q^r-x_1}^{\infty}\Big[-(m_1+d_2)(x_1-q_{11}^{m})$$
$$+w_1x_1+w_2(Q^r-x_1)+r_2(q_{12}^{m}-Q^r+x_1)\Big]\,f_2(x_2)dx_2f_1(x_1)dx_1$$

and

$$\prod_{12}^{m}(q_{11}^{m},q_{12}^{m}) = -(m_1+d_1)q_{11}^{m}-(m_2+d_1)q_{12}^{m}+\int_{x_1=Q^r}^{\infty}\Big[w_1Q^r-(m_1+d_2)(Q^r-q_{11}^{m})$$
$$+r_2q_{12}^{m}\Big]\,f_1(x_1)dx_1+\int_{x_2=0}^{q_{12}^{m}}\int_{x_1=0}^{Q^r-x_2}\Big[-(m_1+d_2)(Q^r-x_2-q_{11}^{m})+w_2x_2$$
$$+w_1(Q^r-x_2)+r_2(q_{12}^{m}-x_2)\Big]\,f_1(x_1)dx_1f_2(x_2)dx_2$$

$$+\int_{x_2=q_{12}^m}^{Q^r-q_{11}^m}\int_{x_1=0}^{Q^r-x_2}\Big[-(m_1+d_2)(Q^r-x_2-q_{11}^m)-(m_2+d_2)(x_2-q_{12}^m)+w_2x_2$$

$$+w_1(Q^r-x_2)\Big]\ f_1(x_1)dx_1f_2(x_2)dx_2+\int_{x_2=Q^r-q_{11}^m}^{Q^r}\int_{x_1=0}^{Q^r-x_2}\Big[-(m_2+d_2)(x_2-q_{12}^m)$$

$$+w_2x_2+w_1(Q^r-x_2)+r_1(q_{11}^m-Q^r+x_2)\Big]\ f_1(x_1)dx_1f_2(x_2)dx_2$$

$$+\int_{x_1=0}^{q_{11}^m}\int_{x_2=Q^r-x_1}^{\infty}\Big[-(m_2+d_2)(Q^r-x_1-q_{12}^m)+w_1x_1+w_2(Q^r-x_1)$$

$$+r_1(q_{11}^m-x_1)\Big]\ f_2(x_2)dx_2f_1(x_1)dx_1+\int_{x_1=q_{11}^m}^{Q^r-q_{12}^m}\int_{x_2=Q^r-x_1}^{\infty}\Big[-(m_1+d_2)(x_1-q_{11}^m)$$

$$-(m_2+d_2)(Q^r-x_1-q_{12}^m)+w_1x_1+w_2(Q^r-x_1)\Big]\ f_2(x_2)dx_2f_1(x_1)dx_1$$

$$+\int_{x_1=Q^r-q_{12}^m}^{Q^r}\int_{x_2=Q^r-x_1}^{\infty}\Big[-(m_1+d_2)(x_1-q_{11}^m)+w_1x_1+w_2(Q^r-x_1)$$

$$+r_2(q_{12}^m-Q^r+x_1)\Big]\ f_2(x_2)dx_2f_1(x_1)dx_1\quad .$$

After rearranging the terms in $\prod_1^m(q_{11}^m,q_{12}^m)$ and $\prod_1^m(q_{11}^m,q_{12}^m)$, we observe that these functions are actually identical and can be rewritten as in Equation (20).

$$\prod_1^m(q_{11}^m,q_{12}^m)=-(m_1+d_1)q_{11}^m-(m_2+d_1)q_{12}^m$$

$$+\int_{x_1=Q^r}^{\infty}\Big[w_1Q^r-(m_1+d_2)(Q^r-q_{11}^m)+r_2q_{12}^m\Big]\ f_1(x_1)dx_1$$

$$+\int_{x_2=0}^{Q^r}\int_{x_1=0}^{Q^r-x_2}\Big[w_2x_2+w_1(Q^r-x_2)\Big]\ f_1(x_1)dx_1f_2(x_2)dx_2$$

$$+\int_{x_2=Q^r-q_{11}^m}^{Q^r}\int_{x_1=0}^{Q^r-x_2}r_1(q_{11}^m-Q^r+x_2)\ \ f_1(x_1)dx_1f_2(x_2)dx_2$$

$$-\int_{x_2=0}^{Q^r-q_{11}^m}\int_{x_1=0}^{Q^r-x_2}(m_1+d_2)(Q^r-x_2-q_{11}^m)\ \ f_1(x_1)dx_1f_2(x_2)dx_2$$

$$+\int_{x_2=0}^{q_{12}^m}\int_{x_1=0}^{Q^r-x_2}r_2(q_{12}^m-x_2)\ \ f_1(x_1)dx_1f_2(x_2)dx_2$$

$$
\begin{aligned}
&- \int_{x_2=q_{12}^m}^{Q^r} \int_{x_1=0}^{Q^r-x_2} (m_2+d_2)(x_2-q_{12}^m)\ f_1(x_1)dx_1 f_2(x_2)dx_2 \\
&+ \int_{x_1=0}^{Q^r} \int_{x_2=Q^r-x_1}^{\infty} \left[w_1x_1+w_2(Q^r-x_1)\right]\ f_2(x_2)dx_2 f_1(x_1)dx_1 \\
&+ \int_{x_1=0}^{q_{11}^m} \int_{x_2=Q^r-x_1}^{\infty} r_1(q_{11}^m-x_1)\ f_2(x_2)dx_2 f_1(x_1)dx_1 \\
&- \int_{x_1=q_{11}^m}^{Q^r} \int_{x_2=Q^r-x_1}^{\infty} (m_1+d_2)(x_1-q_{11}^m)\ f_2(x_2)dx_2 f_1(x_1)dx_1 \\
&+ \int_{x_1=Q^r-q_{12}^m}^{Q^r} \int_{x_2=Q^r-x_1}^{\infty} r_2(q_{12}^m-Q^r+x_1)\ f_2(x_2)dx_2 f_1(x_1)dx_1 \\
&- \int_{x_1=0}^{Q^r-q_{12}^m} \int_{x_2=Q^r-x_1}^{\infty} (m_2+d_2)(Q^r-x_1-q_{12}^m)\ f_2(x_2)dx_2 f_1(x_1)dx_1
\end{aligned} \tag{20}
$$

Proposition 7 *Let $(q_{11}^m)'$ and $(q_{12}^m)'$ be the unique solution of $\partial\prod_1^m/\partial q_{11}^m = 0$ and $\partial\prod_1^m/\partial q_{12}^m = 0$, respectively. Then, the optimal initial procurement quantities of the manufacturer are $\left((q_{11}^m)^*,(q_{12}^m)^*\right)=\left(\min(Q^r,(q_{11}^m)'),(q_{12}^m)'\right)$.*

Proof: The first derivatives of $\prod_1^m(q_{11}^m,q_{12}^m)$ with respect to q_{11}^m and q_{12}^m are given in Equation (21) and Equation (22), respectively.

$$
\begin{aligned}
\frac{\partial\prod_1^m}{\partial q_{11}^m} &= d_2-d_1-(m_1+d_2-r_1)F_1(q_{11}^m)+\int_{x_2=0}^{Q^r-q_{11}^m}(m_1+d_2)F_1(Q^r-x_2)f_2(x_2)dx_2 \\
&+ \int_{x_2=Q^r-q_{11}^m}^{Q^r} r_1F_1(Q^r-x_2)f_2(x_2)dx_2 - \int_{x_1=0}^{q_{11}^m} r_1F_2(Q^r-x_1)f_1(x_1)dx_1 \\
&- \int_{x_1=q_{11}^m}^{Q^r}(m_1+d_2)F_2(Q^r-x_1)f_1(x_1)dx_1
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
\frac{\partial\prod_1^m}{\partial q_{12}^m} &= -m_2-d_1+r_2-(r_2-m_2-d_2)F_1(Q^r-q_{12}^m)+\int_{x_2=0}^{q_{12}^m} r_2F_1(Q^r-x_2)f_2(x_2)dx_2 \\
&+ \int_{x_2=q_{12}^m}^{Q^r}(m_2+d_2)F_1(Q^r-x_2)f_2(x_2)dx_2 - \int_{x_1=Q^r-q_{12}^m}^{Q^r} r_2F_2(Q^r-x_1)f_1(x_1)dx_1 \\
&- \int_{x_1=0}^{Q^r-q_{12}^m}(m_2+d_2)F_2(Q^r-x_1)f_1(x_1)dx_1
\end{aligned} \tag{22}
$$

The second derivatives of $\prod_1^m(q_{11}^m, q_{12}^m)$ in Equation (23), Equation (24) and Equation (25) indicate that the determinant of the hessian matrix is positive.

$$\frac{\partial^2 \prod_1^m}{\partial (q_{11}^m)^2} = (r_1 - m_1 - d_2) f_1(q_{11}^m) < 0 \tag{23}$$

$$\frac{\partial^2 \prod_1^m}{\partial (q_{11}^m)^2} = (r_2 - m_2 - d_2) f_1(Q^r - q_{12}^m) < 0 \tag{24}$$

$$\frac{\partial^2 \prod_1^m}{\partial q_{11}^m \partial q_{12}^m} = \frac{\partial^2 \prod_1^m}{\partial q_{12}^m \partial q_{11}^m} = 0 \tag{25}$$

If $(q_{11}^m)' \leq Q^r$, then the optimal solution is given by first order conditions. Otherwise, $(q_{11}^m)^* = Q^r$ and $(q_{12}^m)^* = (q_{12}^m)'$. □

Note that the optimal initial order quantities of the manufacturer depend implicitly on w_1 and w_2 through the retailer's optimal order quantity, Q^r, as in the case where the retailer prefers to sell product 2 at a discounted price (Section 4.2.1.2). However, it should be noted that, contrary to Section 4.2.1.2, the optimal order quantities depend on the distribution of the demand of both products. Moreover, q_{11}^m and q_{12}^m are independent of each other as in Section 4.2.1.2.

4.3. Limited Order Adjustment

In this section, we consider a limited order adjustment scheme where $0 < \alpha < 1$. In this setting, the sum of the final orders of the retailer should still be equal to the sum of original order quantities as in Section 4.2. However, limited order flexibility introduces lower and upper bounds on the final order quantities in terms of the initial orders. That is, the final order quantity of the retailer, q_{2i}^r, should be between L_i and U_i where $L_i = \max(0, q_{1i}^r - \alpha Q^r)$ and $U_i = \min(Q^r, q_{1i}^r + \alpha Q^r)$. Note that we have $L_1 + U_2 = L_2 + U_1 = Q^r$. The timeline of the system dynamics and the ordering decisions for this case is illustrated in Figure 5.

In this setting, the retailer and the manufacturer distribute the risk caused by uncertainty. Although the retailer makes an initial decision under uncertainty, it has the opportunity to revise the order quantities to a certain extent once the uncertainty is resolved. Similarly, although the retailer's order quantities are not fixed until stage 2, the manufacturer has a certain degree of information about the orders through the initial orders of the retailer. The share of the risk transferred from the retailer to the manufacturer depends on the order adjustment flexibility parameter. Our analysis in this section will characterize the effects of such a flexibility scheme for the retailer and the manufacturer, respectively.

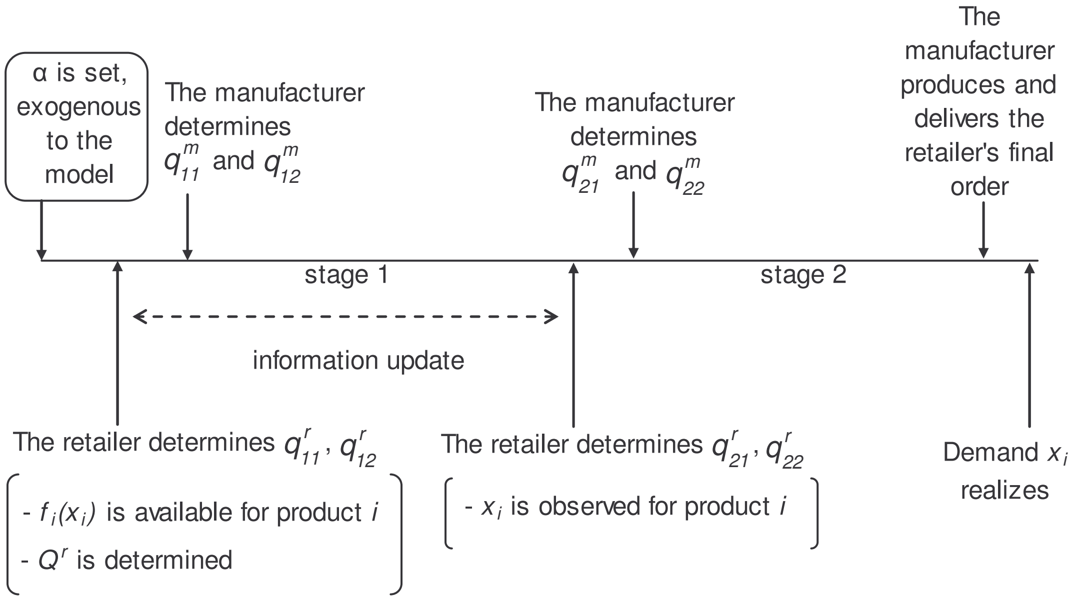

Figure 5 Limited order adjustment: Timeline of the system dynamics and the ordering decisions

As in Section 4.2., we start our analysis with the case $w_1 - s_1 > w_2 - s_2$; that is, it is more profitable to sell product 2 at a discounted price.

4.3.1. "Sell Product II At The Discounted Price"

In this section, we consider the setting where $p_1 - w_1 > p_2 - w_2$ and $w_1 - s_1 > w_2 - s_2$. Specifically, the retailer prefers to sell product 1 in the regular season and product 2 at the end of the season at a discounted price. We start our analysis with the retailer's problem.

4.3.1.1. Retailer's Problem

We first characterize the final order of the retailer at stage 2 in relation to stage 1 order of each product and demand realization, x_i. Since selling product 1 is more profitable for the retailer, the retailer first meets the demand of product 1 to the extent possible between L_1 *and* U_1. The remaining order will be utilized entirely by product 2 since it is profitable to sell at a discounted price. If $x_1 < L_1$, the retailer modifies the final order of product 1 and product 2 as L_1 *and* U_2, respectively, in order to minimize the quantity of product 1 leftover at the end of selling season. If $L_1 < x_1 < U_1$, the retailer primarily meets the demand of product 1, x_1, and the remaining order will be utilized entirely by product 2. If $x_1 > U_1$, the retailer primarily meets the demand of product 1 to the extent possible, U_1, and the remaining order will be utilized entirely by product 2, L_2.

We can simply say that the demand of product 1 determines the optimal ordering policy. Hence, given x_1 and the initial order of the retailer for each product, q_{11}^r and q_{12}^r, retailer's optimal order quantities for stage 2 is given by Equation (26).

$$(q_{21}^r, q_{22}^r) \quad = \quad \begin{cases} (L_1, \quad U_2) & \text{if} \quad x_1 < L_1 \\ (x_1, \quad Q^r - x_1) & \text{if} \quad L_1 < x_1 < U_1 \\ (U_1, \quad L_2) & \text{if} \quad x_1 > U_1 \end{cases} \tag{26}$$

The retailer's conditional profits are given by:

$$\prod\nolimits_1^r(q_{11}^r, q_{12}^r / x_1, x_2) = \begin{cases} p_1x_1 - w_1L_1 + s_1(L_1 - x_1) + p_2x_2 - w_2U_2 + s_2(U_2 - x_2) \\ \qquad \text{if} \quad x_1 < L_1, \quad x_2 < U_2 \\ p_1x_1 - w_1L_1 + s_1(L_1 - x_1) + (p_2 - w_2)U_2 \quad \text{if } x_1 < L_1, \ x_2 > U_2 \\ (p_1 - w_1)x_1 + p_2x_2 - w_2(Q^r - x_1) + s_2(Q^r - x_1 - x_2) \\ \qquad \text{if} \quad L_1 < x_1 < U_1, \quad x_2 < Q^r - x_1 \\ (p_1 - w_1)x_1 + (p_2 - w_2)(Q^r - x_1) \quad \text{if } L_1 < x_1 < U_1, \ x_2 > Q^r - x_1 \\ (p_1 - w_1)U_1 + p_2x_2 - w_2L_2 + s_2(L_2 - x_2) \quad \text{if } x_1 > U_1, \ x_2 < L_2 \\ (p_1 - w_1)U_1 + (p_2 - w_2)L_2 \qquad \text{if } x_1 > U_1, \ x_2 > L_2 \end{cases}$$

As observed in Equation (26), the final order quantities of the retailer depend on L_1 and U_1. Note that we define $L_1 = \max(0, q_{11}^r - \alpha Q^r)$ since $q_{11}^r - \alpha Q^r$ may be negative. However, when $q_{11}^r - \alpha Q^r < 0$, we have $P\{x_1 < q_{11}^r - \alpha Q^r\} = P\{x_1 < 0\}$ $= P\{x_1 < L_1\} = 0$. Hence, we can substitute $L_1 = q_{11}^r - \alpha Q^r$ instead of $L_1 = \max(0, q_{11}^r - \alpha Q^r)$. Noting that $U_1 = q_{11}^r + q_{12}^r$ if $(1-\alpha)q_{12}^r < \alpha q_{11}^r$ and $U_1 = (1+\alpha)q_{11}^r + \alpha q_{12}^r$ if $(1-\alpha)q_{12}^r > \alpha q_{11}^r$, we can characterize the optimal final order quantities of the retailer as follows;

(i) *If* $(1-\alpha)q_{12}^r < \alpha q_{11}^r$, *then*

$$(q_{21}^r, q_{22}^r) = \begin{cases} ((1-\alpha)q_{11}^r - \alpha q_{12}^r, \quad (1+\alpha)q_{12}^r + \alpha q_{11}^r)) & \text{if} \quad x_1 < (1-\alpha)q_{11}^r - \alpha q_{12}^r \\ (x_1, \quad q_{11}^r + q_{12}^r - x_1) & \text{if} \quad (1-\alpha)q_{11}^r - \alpha q_{12}^r < x_1 < q_{11}^r + q_{12}^r \\ (q_{11}^r + q_{12}^r, \quad 0) & \text{if} \quad x_1 > q_{11}^r + q_{12}^r \end{cases}$$

(ii) *If* $(1-\alpha)q_{12}^r > \alpha q_{11}^r$, *then*

$$(q_{21}^r, q_{22}^r) = \begin{cases} ((1-\alpha)q_{11}^r - \alpha q_{12}^r, \quad (1+\alpha)q_{12}^r + \alpha q_{11}^r)) & \text{if} \quad x_1 < (1-\alpha)q_{11}^r - \alpha q_{12}^r \\ (x_1, \quad q_{11}^r + q_{12}^r - x_1) & \text{if} \quad (1-\alpha)q_{11}^r - \alpha q_{12}^r < x_1 < (1+\alpha)q_{11}^r + \alpha q_{12}^r \\ ((1+\alpha)q_{11}^r + \alpha q_{12}^r, \quad (1-\alpha)q_{12}^r - \alpha q_{11}^r) & \text{if} \quad x_1 > (1+\alpha)q_{11}^r + \alpha q_{12}^r \end{cases}$$

Incorporating the optimal final orders of the retailer in its stage 1 problem, the expected profit of the retailer can be characterized as in Equation (27).

$$\prod_1^r(q_{11}^r, q_{12}^r) \quad = \quad \begin{cases} \prod_{11}^r(q_{11}^r, q_{12}^r) & \text{if} \quad (1-\alpha)q_{12}^r < \alpha q_{11}^r \\ \prod_{12}^r(q_{11}^r, q_{12}^r) & \text{if} \quad (1-\alpha)q_{12}^r > \alpha q_{11}^r \end{cases} \quad \text{where} \qquad (27)$$

$$\prod_{11}^r(q_{11}^r, q_{12}^r) = \int_{x_1=0}^{(1-\alpha)q_{11}^r - \alpha q_{12}^r} \Bigg[p_1 x_1 + s_1((1-\alpha)q_{11}^r - \alpha q_{12}^r - x_1) - w_1((1-\alpha)q_{11}^r - \alpha q_{12}^r)$$

$$- w_2((1+\alpha)q_{12}^r + \alpha q_{11}^r) + \int_{x_2=0}^{(1+\alpha)q_{12}^r + \alpha q_{11}^r} \left[(p_2 x_2 + s_2((1+\alpha)q_{12}^r + \alpha q_{11}^r - x_2)\right]$$

$$f_2(x_2)dx_2 + \int_{x_2=(1+\alpha)q_{12}^r + \alpha q_{11}^r}^{\infty} p_2((1+\alpha)q_{12}^r + \alpha q_{11}^r) \; f_2(x_2)dx_2 \quad \Bigg] \; f_1(x_1)dx_1$$

$$+\int_{x_1=(1-\alpha)q_{11}^r-\alpha q_{12}^r}^{q_{11}^r+q_{12}^r}\Bigg[\;p_1x_1-w_1x_1-w_2(q_{11}^r+q_{12}^r-x_1)+$$

$$\int_{x_2=0}^{q_{11}^r+q_{12}^r-x_1}\big[(p_2x_2+s_2(q_{11}^r+q_{12}^r-x_1-x_2)\big]\;f_2(x_2)dx_2$$

$$+\int_{x_2=q_{11}^r+q_{12}^r-x_1}^{\infty}p_2(q_{11}^r+q_{12}^r-x_1)\;f_2(x_2)dx_2\;\Bigg]\;f_1(x_1)dx_1$$

$$+\int_{x_1=q_{11}^r+q_{12}^r}^{\infty}(p_1-w_1)(q_{11}^r+q_{12}^r)f_1(x_1)dx_1$$

and

$$\prod\nolimits_{12}^r(q_{11}^r,q_{12}^r)=\int_{x_1=0}^{(1-\alpha)q_{11}^r-\alpha q_{12}^r}\Bigg[\;p_1x_1+s_1((1-\alpha)q_{11}^r-\alpha q_{12}^r-x_1)-w_1((1-\alpha)q_{11}^r-\alpha q_{12}^r)$$

$$-w_2((1+\alpha)q_{12}^r+\alpha q_{11}^r)+\int_{x_2=0}^{(1+\alpha)q_{12}^r+\alpha q_{11}^r}\big[(p_2x_2+s_2((1+\alpha)q_{12}^r+\alpha q_{11}^r-x_2)\big]$$

$$f_2(x_2)dx_2+\int_{x_2=(1+\alpha)q_{12}^r+\alpha q_{11}^r}^{\infty}p_2((1+\alpha)q_{12}^r+\alpha q_{11}^r)\;f_2(x_2)dx_2\;\Bigg]\;f_1(x_1)dx_1$$

$$+\int_{x_1=(1-\alpha)q_{11}^r-\alpha q_{12}^r}^{(1+\alpha)q_{11}^r+\alpha q_{12}^r}\Bigg[\;p_1x_1-w_1x_1-w_2(q_{11}^r+q_{12}^r-x_1)$$

$$+\int_{x_2=0}^{q_{11}^r+q_{12}^r-x_1}\big[(p_2x_2+s_2(q_{11}^r+q_{12}^r-x_1-x_2)\big]\;f_2(x_2)dx_2$$

$$+\int_{x_2=q_{11}^r+q_{12}^r-x_1}^{\infty}p_2(q_{11}^r+q_{12}^r-x_1)\;f_2(x_2)dx_2\;\Bigg]\;f_1(x_1)dx_1$$

$$+\int_{x_1=(1+\alpha)q_{11}^r+\alpha q_{12}^r}^{\infty}\Bigg[\;(p_1-w_1)((1+\alpha)q_{11}^r+\alpha q_{12}^r)-w_2((1-\alpha)q_{12}^r-\alpha q_{11}^r)$$

$$+\int_{x_2=0}^{(1-\alpha)q_{12}^r-\alpha q_{11}^r}\big[(p_2x_2+s_2((1-\alpha)q_{12}^r-\alpha q_{111}^r-x_2)\big]\;f_2(x_2)dx_2$$

$$+\int_{x_2=(1-\alpha)q_{12}^r-\alpha q_{11}^r}^{\infty}p_2((1-\alpha)q_{12}^r-\alpha q_{11}^r)\;f_2(x_2)dx_2\;\Bigg]\;f_1(x_1)dx_1\;.$$

Lemma 1 $\prod_1^r(q_{11}^r,q_{12}^r)$ *is jointly continuous in* q_{11}^r *and* q_{12}^r.

Proof: We consider the case that $(1-\alpha)q_{12}^r = \alpha q_{11}^r$ and that is, $q_{12}^r = \frac{\alpha}{(1-\alpha)} q_{11}^r$.

Then, $\prod_{11}^r \left(q_{11}^r,\ q_{12}^r = \frac{\alpha}{(1-\alpha)} q_{11}^r \right) = \prod_{12}^r \left(q_{11}^r,\ q_{12}^r = \frac{\alpha}{(1-\alpha)} q_{11}^r \right) = \prod_1^r (q_{11}^r)$ and the retailer's expected profit function is given by;

$$\begin{aligned}
\prod_1^r (q_{11}^r) = \int_{x_1=0}^{\frac{1-2\alpha}{1-\alpha}q_{11}^r} & \Bigg[p_1 x_1 + s_1 (\frac{1-2\alpha}{1-\alpha} q_{11}^r - x_1) - w_1 (\frac{1-2\alpha}{1-\alpha} q_{11}^r) - w_2 (\frac{2\alpha}{1-\alpha} q_{11}^r) \\
&+ \int_{x_2=0}^{\frac{2\alpha}{1-\alpha}q_{11}^r} \left[(p_2 x_2 + s_2 (\frac{2\alpha}{1-\alpha} q_{11}^r - x_2) \right] f_2(x_2) dx_2 \\
&+ \int_{x_2=\frac{2\alpha}{1-\alpha}q_{11}^r}^{\infty} p_2 (\frac{2\alpha}{1-\alpha} q_{11}^r)\ f_2(x_2) dx_2 \Bigg] f_1(x_1) dx_1 \\
&+ \int_{x_1=\frac{1-2\alpha}{1-\alpha}q_{11}^r}^{\frac{1}{1-\alpha}q_{11}^r} \Bigg[p_1 x_1 - w_1 x_1 - w_2 (\frac{1}{1-\alpha} q_{11}^r - x_1) \\
&+ \int_{x_2=0}^{\frac{1}{1-\alpha}q_{11}^r - x_1} \left[(p_2 x_2 + s_2 (\frac{1}{1-\alpha} q_{11}^r - x_1 - x_2) \right] f_2(x_2) dx_2 \\
&+ \int_{x_2=\frac{1}{1-\alpha}q_{11}^r - x_1}^{\infty} p_2 (\frac{1}{1-\alpha} q_{11}^r - x_1)\ f_2(x_2) dx_2 \Bigg] f_1(x_1) dx_1 \\
&+ \int_{x_1=\frac{1}{1-\alpha}q_{11}^r}^{\infty} (p_1 - w_1)(\frac{1}{1-\alpha} q_{11}^r) f_1(x_1) dx_1
\end{aligned}$$

Hence, $\prod_1^r (q_{11}^r, q_{12}^r)$ is jointly continuous in q_{11}^r and q_{12}^r. □

Lemma 2 $\prod_{11}^r (q_{11}^r, q_{12}^r)$ *is jointly concave in* q_{11}^r *and* q_{12}^r.

Proof: See Appendix A.

Lemma 3 $\prod_{12}^r (q_{11}^r, q_{12}^r)$ *is jointly concave in* q_{11}^r *and* q_{12}^r.

Proof: See Appendix B.

Lemma 4 $\prod_1^r(q_{11}^r)$ *is concave in* q_{11}^r.

Proof: See Appendix C.

Proposition 8 *Let* $(q_{11}^r)'$, $(q_{12}^r)'$ *be the solution to* $\partial\prod_{11}^r(q_{11}^r,q_{12}^r)/\partial q_{11}^r = 0$ *and* $\partial\prod_{11}^r(q_{11}^r,q_{12}^r)/\partial q_{12}^r = 0$. *Similarly, let* $(q_{11}^r)''$, $(q_{12}^r)''$ *be the solution to* $\partial\prod_{12}^r(q_{11}^r,q_{12}^r)/\partial q_{11}^r = 0$ *and* $\partial\prod_{12}^r(q_{11}^r,q_{12}^r)/\partial q_{12}^r = 0$. *Furthermore, let* $(q_{11}^r)'''$ *be the solution to* $d\prod_1^r(q_{11}^r)/dq_{11}^r = 0$. *Then, the optimal initial order quantities of the retailer are given by;*

(i) If $(1-\alpha)(q_{12}^r)' \le \alpha(q_{11}^r)'$ *and* $(1-\alpha)(q_{12}^r)'' \ge \alpha(q_{11}^r)''$ *then*

$$\left((q_{11}^r)^*,(q_{12}^r)^*\right) = \begin{cases} \left((q_{11}^r)',(q_{12}^r)'\right) & \text{if } \prod_{11}^r\left((q_{11}^r)',(q_{12}^r)'\right) > \prod_{12}^r\left((q_{11}^r)'',(q_{12}^r)''\right) \\ \left((q_{11}^r)'',(q_{12}^r)''\right) & \text{if } \prod_{11}^r\left((q_{11}^r)',(q_{12}^r)'\right) < \prod_{12}^r\left((q_{11}^r)'',(q_{12}^r)''\right) \end{cases}$$

(ii) If $(1-\alpha)(q_{12}^r)' \le \alpha(q_{11}^r)'$ *and* $(1-\alpha)(q_{12}^r)'' < \alpha(q_{11}^r)''$ *then*

$$\left((q_{11}^r)^*,(q_{12}^r)^*\right) = \left((q_{11}^r)',(q_{12}^r)'\right)$$

(iii) If $(1-\alpha)(q_{12}^r)' > \alpha(q_{11}^r)'$ *and* $(1-\alpha)(q_{12}^r)'' \ge \alpha(q_{11}^r)''$ *then*

$$\left((q_{11}^r)^*,(q_{12}^r)^*\right) = \left((q_{11}^r)'',(q_{12}^r)''\right)$$

(iv) If $(1-\alpha)(q_{12}^r)' > \alpha(q_{11}^r)'$ *and* $(1-\alpha)(q_{12}^r)'' < \alpha(q_{11}^r)''$ *then*

$$\left((q_{11}^r)^*,(q_{12}^r)^*\right) = \left((q_{11}^r)''',\frac{\alpha}{(1-\alpha)}(q_{11}^r)'''\right)$$

Proof: Note that $(q_{11}^r)'$, $(q_{12}^r)'$ denote the unconstrained optimal solution to $\prod_{11}^r(q_{11}^r,q_{12}^r)$ by Lemma 2. Similarly, $(q_{11}^r)''$, $(q_{12}^r)''$ denote the unconstrained optimal solution to $\prod_{12}^r(q_{11}^r,q_{12}^r)$ by Lemma 3.

(i) If these conditions are satisfied, then $\prod^r$ has two local maxima and the greater one is the optimal solution.

(ii) If these conditions are satisfied, then $\prod^r$ has one local maximum point which is $(q^r_{11})'$, $(q^r_{12})'$ and hence it is the optimal solution.

(iii) If these conditions are satisfied, then $\prod^r$ has one local maximum point which is $(q^r_{11})''$, $(q^r_{12})''$ and hence it is the optimal solution.

(iv) If these conditions are satisfied, the optimal will occur at the boundary, i.e. $(1-\alpha)q^r_{12} = \alpha q^r_{11}$. Hence, the optimal will be given by $(q^r_{11})''', \frac{\alpha}{(1-\alpha)}(q^r_{11})'''$ due to Lemma 4. □

4.3.1.2. Manufacturer's Problem

The manufacturer's problem is to choose the optimal procurement quantities, q^m_{12} and q^m_{12} that maximize his expected profit subject to the retailer's ordering behavior. Similar to the retailer's problem, we first characterize stage 2 optimal procurement quantities based on the final order quantities of the retailer (which are determined by the demand realizations as observed in Equation (26)), and initial order quantities of the manufacturer. We then characterize the expected profit function of the manufacturer and obtain optimal order quantities.

The manufacturer is obliged to fill the retailer's final order for each product in terms of the contract. As a result, given the manufacturer's initial procurement quantity, the manufacturer will set stage 2 procurement level to meet the retailer's final order quantities. Note that the retailer's final order is constrained by upper and lower limits for each product and it is obvious that the manufacturer's initial order quantities are to be $L_1 \leq q^m_{11} \leq U_1$ and $L_2 \leq q^m_{12} \leq U_2$.

Proposition 9 *Given* α, (q^m_{11}, q^m_{12}) *and* (q^r_{11}, q^r_{12}), *the manufacturer's optimal procurement quantities for stage 2 are characterized as follows;*

$$
(q_{21}^m, q_{22}^m) = \begin{cases} (0, \quad U_2 - q_{12}^m) & \text{if} \quad x_1 < L_1 \\ (x_1 - q_{11}^m, \ Q^r - x_1 - q_{12}^m) & \text{if} \quad L_1 < x_1 < U_1 \quad \text{and} \quad q_{11}^m < x_1 \quad \text{and} \quad q_{12}^m < Q^r - x_1 \\ (x_1 - q_{11}^m, \ 0) & \text{if} \quad L_1 < x_1 < U_1 \quad \text{and} \quad q_{11}^m < x_1 \quad \text{and} \quad q_{12}^m > Q^r - x_1 \\ (0, \ Q^r - x_1 - q_{12}^m) & \text{if} \quad L_1 < x_1 < U_1 \quad \text{and} \quad q_{11}^m > x_1 \quad \text{and} \quad q_{12}^m < Q^r - x_1 \\ (0, \ 0) & \text{if} \quad L_1 < x_1 < U_1 \quad \text{and} \quad q_{11}^m > x_1 \quad \text{and} \quad q_{12}^m > Q^r - x_1 \\ (U_1 - q_{11}^m, \quad 0) & \text{if} \quad x_1 > U_1 \end{cases} \tag{28}
$$

Proof: Recall that the retailer's optimal stage 2 orders are given by Equation (26) and note that the manufacturer has to satisfy retailer's final order in full. If $x_1 < L_1$, the retailer's final orders are given by $(q_{21}^r, q_{22}^r) = (L_1, U_2)$ and the manufacturer places an expedited order of size $q_{22}^m = U_2 - q_{12}^m$ for product 2. If $L_1 < x_1 < U_1$, the retailer's final orders are given by $(q_{21}^r, q_{22}^r) = (x_1, Q^r - x_1)$. Noting that $q_{11}^m < x_1$, the manufacturer places an expedited order of size $q_{21}^m = x_1 - q_{11}^m$ for product 1. Additionally, noting that $q_{12}^m < Q^r - x_1$, the manufacturer places an expedited order of size $q_{22}^m = Q^r - x_1 - q_{12}^m$ for product 2. $x_1 > U_1$, the retailer's final orders are given by $(q_{21}^r, q_{22}^r) = (U_1, L_2)$ and the manufacturer places an expedited order of size $q_{21}^m = U_1 - q_{11}^m$ for product 1. □

We now characterize the expected profit of the manufacturer in stage 1. The manufacturer needs to decide the optimal procurement quantities to maximize the expected profit subject to the retailer's ordering policy. The manufacturer's conditional profits are given by:

$$\prod_1^m(q_{11}^m,q_{12}^m) = -(m_1+d_1)q_{11}^m - (m_2+d_1)q_{12}^m$$

$$\begin{cases} +w_1L_1+w_2U_2+r_1(q_{11}^m-L_1)-(m_2+d_2)(U_2-q_{12}^m) & \text{if } \ x_1<L_1 \\ +w_1x_1+w_2(Q^r-x_1)-(m_1+d_2)(x_1-q_{11}^m)-(m_2+d_2)(Q^r-x_1-q_{12}^m) & \\ \qquad \text{if } \ L_1<x_1<U_1 \ \text{ and } \ q_{11}^m<x_1 \ \text{ and } \ q_{12}^m<Q^r-x_1 & \\ +w_1x_1+w_2(Q^r-x_1)-(m_1+d_2)(x_1-q_{11}^m)+r_2(q_{12}^m-Q^r+x_1) & \\ \qquad \text{if } \ L_1<x_1<U_1 \ \text{ and } \ q_{11}^m<x_1 \ \text{ and } \ q_{12}^m>Q^r-x_1 & \\ +w_1x_1+w_2(Q^r-x_1)-(m_2+d_2)(Q^r-x_1-q_{12}^m)+r_1(q_{11}^m-x_1) & \\ \qquad \text{if } \ L_1<x_1<U_1 \ \text{ and } \ q_{11}^m>x_1 \ \text{ and } \ q_{12}^m<Q^r-x_1 & \\ +w_1x_1+w_2(Q^r-x_1)+r_1(q_{11}^m-x_1)+r_2(q_{12}^m-Q^r+x_1) & \\ \qquad \text{if } \ L_1<x_1<U_1 \ \text{ and } \ q_{11}^m>x_1 \ \text{ and } \ q_{12}^m>Q^r-x_1 & \\ +w_1U_1+w_2L_2-(m_2+d_2)(U_1-q_{11}^m)+r_2(q_{12}^m-L_2) & \text{if } \ x_1>U_1 \end{cases}$$

Considering the optimal expedited delivery orders of the manufacturer given in Proposition 9, the manufacturer's expected profit is given by;

$$\begin{aligned} \prod_1^m(q_{11}^m,q_{12}^m) &= -(m_1+d_1)q_{11}^m - (m_2+d_1)q_{12}^m \\ &+ \int_{x_1=0}^{L_1} \left[w_1L_1+w_2(Q^r-L_1)+r_1(q_{11}^m-L_1)-(m_2+d_2)(Q^r-L_1-q_{12}^m)\right] f_1(x_1)dx_1 \\ &+ \int_{x_1=L_1}^{U_1} \left[w_1x_1+w_2(Q^r-x_1)\right] f_1(x_1)dx_1 + \int_{x_1=L_1}^{q_{11}^m} r_1(q_{11}^m-x_1)\ f_1(x_1)dx_1 \\ &- \int_{x_1=q_{11}^m}^{U_1} (m_1+d_2)(x_1-q_{11}^m)\ f_1(x_1)dx_1 \\ &- \int_{x_1=L_1}^{Q^r-q_{12}^m} (m_2+d_2)(Q^r-x_1-q_{12}^m)\ f_1(x_1)dx_1 \\ &+ \int_{x_1=Q^r-q_{12}^m}^{U_1} r_2(q_{12}^m-Q^r+x_1)\ f_1(x_1)dx_1 \\ &+ \int_{x_1=U_1}^{\infty} \left[w_1U_1+w_2(Q^r-U_1)-(m_2+d_2)(U_1-q_{11}^m)+r_2(q_{12}^m-Q^r+U_1)\right] f_1(x_1)dx_1 \end{aligned}$$

Proposition 10 *Let* $(q_{11}^m)'$ *and* $(q_{12}^m)'$ *be the unique solution of* $\partial\prod_1^m / \partial q_{11}^m = 0$ *and* $\partial\prod_1^m / \partial q_{12}^m = 0$. *Then, the optimal initial procurement quantities of the manufacturer are characterized by;*

$$\left((q_{11}^m)^*, (q_{12}^m)^*\right) = \left(\min\ \left\{\max(L_1, (q_{11}^m)'), U_1\right\}\ \min\ \left\{\max(L_2, (q_{12}^m)'), U_2\right\}\right)$$

Proof: The first derivatives of $\prod_1^m(q_{11}^m, q_{12}^m)$ with respect to q_{11}^m and q_{12}^m are given in Equation (29) and Equation (30), respectively.

$$\frac{\partial\prod_1^m}{\partial q_{11}^m} = d_2 - d_1 - (m_1 + d_2 - r_1)F_1(q_{11}^m) \tag{29}$$

$$\frac{\partial\prod_1^m}{\partial q_{12}^m} = -m_2 - d_1 + r_2 - (r_2 - m_2 - d_2)F_1(Q^r - q_{12}^m) \tag{30}$$

The second derivatives of $\prod_1^m(q_{11}^m, q_{12}^m)$ in Equation (31), Equation (32) and Equation (33) indicate that the determinant of the hessian matrix is positive.

$$\frac{\partial^2\prod_1^m}{\partial(q_{11}^m)^2} = -(m_1 + d_2 - r_1)f_1(q_{11}^m) < 0 \tag{31}$$

$$\frac{\partial^2\prod_1^m}{\partial(q_{11}^m)^2} = (r_2 - m_2 - d_2)f_1(Q^r - q_{12}^m) < 0 \tag{32}$$

$$\frac{\partial^2\prod_1^m}{\partial q_{11}^m \partial q_{12}^m} = \frac{\partial^2\prod_1^m}{\partial q_{12}^m \partial q_{11}^m} = 0 \tag{33}$$

If $L_1 \le (q_{11}^m)' \le U_1$, then the optimal solution is given by first order conditions. If $(q_{11}^m)' \le L_1$, then $(q_{11}^m)^* = L_1$ and otherwise, $(q_{11}^m)^* = U_1$. Similar arguments are also valid for $(q_{12}^m)^*$. □

Note that the optimal initial order quantities of the manufacturer do not depend explicitly on w_1 and w_2. This is reasonable since the manufacturer's sales quantities depend on the order of the retailer. However, it should be noted that they depend on Q^r, which depends on both w_1 and w_2. It should be also noted that optimal q_{11}^m and q_{12}^m depend on the only distribution of the demand of product 1. This results from the fact that the final order quantities of the retailer are determined by the demand of product 1. That is, if Q^r is sufficient to cover the demand for product 1, the remaining portion of the order is filled by product 2 regardless of its demand realization. Furthermore, note that the optimal initial order quantities of the manufacturer, q_{11}^m and q_{12}^m, are independent of each other.

4.3.2. "Sell Product I At The Discounted Price"

In this section, we consider the setting where $p_1 - w_1 > p_2 - w_2$ and $w_1 - s_1 < w_2 - s_2$. Specifically, the retailer prefers to sell product 1 in the regular season and at the end of the season at a discounted price. We start our analysis with the retailer's problem.

4.3.2.1. Retailer's Problem

We follow a similar approach as in Section 4.3.1.1 to solve the retailer's problem. We first characterize the final order of the retailer at stage 2, given stage 1 order of each product and demand realization, x_i. We then characterize the profit function of stage 1 and obtain the optimal initial order quantities.

Since selling product 1 is more profitable for the retailer, the retailer first fills the demand of product 1 to the extent possible between L_1 and U_1. If Q^r is not sufficient to the aggregate demand, the retailer will order product 2 to meet the demand of product 2, x_2, to the extent possible. If Q^r is sufficient to the aggregate demand, the retailer will order product 1 in order to reach the total order quantity of Q^r since selling product 1 at a discounted price is more profitable than product 2. Hence, given the demand x_1 and x_2, and the initial order of the retailer for each product, q_{11}^r and q_{12}^r, retailer's optimal order quantities for stage 2 are given by Equation (34).

$$
(q^r_{21}, q^r_{22}) = \begin{cases} (U_1, \quad L_2) & \text{if} \quad x_1 < L_1 \quad \text{and} \quad x_2 < L_2 \\ (Q^r - x_2, \quad x_2) & \text{if} \quad x_1 < L_1 \quad \text{and} \quad L_2 < x_2 < U_2 \\ (L_1, \quad U_2) & \text{if} \quad x_1 < L_1 \quad \text{and} \quad x_2 > U_2 \\ (U_1, \quad L_2) & \text{if} \quad L_1 < x_1 < U_1 \quad \text{and} \quad x_2 < L_2 \\ (Q^r - x_2, \quad x_2) & \text{if} \quad L_1 < x_1 < U_1 \quad \text{and} \quad L_2 < x_2 < Q^r - x_1 \\ (x_1, \quad Q^r - x_1) & \text{if} \quad L_1 < x_1 < U_1 \quad \text{and} \quad x_2 > Q^r - x_1 \\ (U_1, \quad L_2) & \text{if} \quad x_1 > U_1 \end{cases} \tag{34}
$$

The retailer's conditional profits are given by:

$$
\prod\nolimits_1^r (q^r_{11}, q^r_{12} / x_1, x_2) = \begin{cases} p_1 x_1 - w_1 U_1 + s_1(U_1 - x_1) + p_2 x_2 - w_2 L_2 + s_2(L_2 - x_2) \\ \qquad \text{if} \quad x_1 < L_1 \quad \text{and} \quad x_2 < L_2 \\ p_1 x_1 - w_1(Q^r - x_2) + s_1(Q^r - x_2 - x_1) + (p_2 - w_2) x_2 \\ \qquad \text{if} \quad x_1 < L_1 \quad \text{and} \quad L_2 < x_2 < U_2 \\ p_1 x_1 - w_1 L_1 + s_1(L_1 - x_1) + (p_2 - w_2) U_2 \\ \qquad \text{if} \quad x_1 < L_1 \quad \text{and} \quad x_2 > U_2 \\ p_1 x_1 - w_1 U_1 + s_1(U_1 - x_1) + p_2 x_2 - w_2 L_2 + s_2(L_2 - x_2) \\ \qquad \text{if} \quad L_1 < x_1 < U_1 \quad \text{and} \quad x_2 < L_2 \\ p_1 x_1 - w_1(Q^r - x_2) + s_1(Q^r - x_2 - x_1) + (p_2 - w_2) x_2 \\ \qquad \text{if} \quad L_1 < x_1 < U_1 \quad \text{and} \quad L_2 < x_2 < Q^r - x_1 \\ (p_1 - w_1) x_1 + p_2 x_2 - w_2(Q^r - x_1) + s_2(Q^r - x_1 - x_2) \\ \qquad \text{if} \quad L_1 < x_1 < U_1 \quad \text{and} \quad x_2 > Q^r - x_1 \\ (p_1 - w_1) U_1 + p_2 x_2 - w_2 L_2 + s_2(L_2 - x_2) \\ \qquad \text{if} \quad x_1 > U_1 \quad \text{and} \quad x_2 < L_2 \\ (p_1 - w_1) U_1 + (p_2 - w_2) L_2 \\ \qquad \text{if} \quad x_1 > U_1 \quad \text{and} \quad x_2 > L_2 \end{cases}
$$

As observed in Equation (34), the final order quantities of the retailer depend on the L_1 and U_1 . We will use the same approach for upper and lower limits as in Section 4.3.1.1. Hence, we can characterize the optimal final order quantities of the retailer as follows;

(i) *If* $(1-\alpha)q_{12}^{r} < \alpha q_{11}^{r}$, *then*

$$(q_{21}^{r}, q_{22}^{r}) = \begin{cases} (q_{11}^{r} + q_{12}^{r} - x_2, \quad x_2) \\ \qquad \text{if} \quad x_1 < (1-\alpha)q_{11}^{r} - \alpha q_{12}^{r} \quad \text{and} \quad 0 < x_2 < (1+\alpha)q_{12}^{r} + \alpha q_{11}^{r} \\ ((1-\alpha)q_{11}^{r} - \alpha q_{12}^{r}, \quad (1+\alpha)q_{12}^{r} + \alpha q_{11}^{r}) \\ \qquad \text{if} \quad x_1 < (1-\alpha)q_{11}^{r} - \alpha q_{12}^{r} \quad \text{and} \quad x_2 > (1+\alpha)q_{12}^{r} + \alpha q_{11}^{r} \\ (q_{11}^{r} + q_{12}^{r} - x_2, \quad x_2) \\ \qquad \text{if} \quad (1-\alpha)q_{11}^{r} - \alpha q_{12}^{r} < x_1 < q_{11}^{r} + q_{12}^{r} \quad \text{and} \quad 0 < x_2 < q_{11}^{r} + q_{12}^{r} - x_1 \\ (x_1, \quad q_{11}^{r} + q_{12}^{r} - x_1) \\ \qquad \text{if} \quad (1-\alpha)q_{11}^{r} - \alpha q_{12}^{r} < x_1 < q_{11}^{r} + q_{12}^{r} \quad \text{and} \quad x_2 > q_{11}^{r} + q_{12}^{r} - x_1 \\ (q_{11}^{r} + q_{12}^{r}, \quad 0) \qquad\qquad \text{if} \quad x_1 > q_{11}^{r} + q_{12}^{r} \end{cases}$$

(ii) *If* $(1-\alpha)q_{12}^{r} > \alpha q_{11}^{r}$, *then*

$$(q_{21}^{r}, q_{22}^{r}) = \begin{cases} ((1+\alpha)q_{11}^{r} + \alpha q_{12}^{r}, \ (1-\alpha)q_{12}^{r} - \alpha q_{11}^{r}) \\ \qquad \text{if} \quad x_1 < (1-\alpha)q_{11}^{r} - \alpha q_{12}^{r} \quad \text{and} \quad x_2 < (1-\alpha)q_{12}^{r} - \alpha q_{11}^{r} \\ (q_{11}^{r} + q_{12}^{r} - x_2, \quad x_2) \qquad \text{if} \quad x_1 < (1-\alpha)q_{11}^{r} - \alpha q_{12}^{r} \\ \qquad\qquad \text{and} \quad (1-\alpha)q_{12}^{r} - \alpha q_{11}^{r} < x_2 < (1+\alpha)q_{12}^{r} + \alpha q_{11}^{r} \\ ((1-\alpha)q_{11}^{r} - \alpha q_{12}^{r}, \quad (1+\alpha)q_{12}^{r} + \alpha q_{11}^{r}) \\ \qquad \text{if} \quad x_1 < (1-\alpha)q_{11}^{r} - \alpha q_{12}^{r} \quad \text{and} \quad x_2 > (1+\alpha)q_{12}^{r} + \alpha q_{11}^{r} \\ ((1+\alpha)q_{11}^{r} + \alpha q_{12}^{r}, \ (1-\alpha)q_{12}^{r} - \alpha q_{11}^{r}) \qquad \text{if} \quad x_2 < (1-\alpha)q_{12}^{r} - \alpha q_{11}^{r} \\ \qquad\qquad \text{and} \quad (1-\alpha)q_{11}^{r} - \alpha q_{12}^{r} < x_1 < (1+\alpha)q_{11}^{r} + \alpha q_{12}^{r} \\ (q_{11}^{r} + q_{12}^{r} - x_2, \quad x_2) \qquad \text{if} \quad (1-\alpha)q_{12}^{r} - \alpha q_{11}^{r} < x_2 < q_{11}^{r} + q_{12}^{r} - x_1 \\ \qquad\qquad \text{and} \quad (1-\alpha)q_{11}^{r} - \alpha q_{12}^{r} < x_1 < (1+\alpha)q_{11}^{r} + \alpha q_{12}^{r} \\ (x_1, \quad q_{11}^{r} + q_{12}^{r} - x_1) \qquad \text{if} \quad x_2 > q_{11}^{r} + q_{12}^{r} - x_1 \\ \qquad\qquad \text{and} \quad (1-\alpha)q_{11}^{r} - \alpha q_{12}^{r} < x_1 < (1+\alpha)q_{11}^{r} + \alpha q_{12}^{r} \\ ((1+\alpha)q_{11}^{r} + \alpha q_{12}^{r}, \ (1-\alpha)q_{12}^{r} - \alpha q_{11}^{r}) \quad \text{if} \quad x_1 > (1+\alpha)q_{11}^{r} + \alpha q_{12}^{r} \end{cases}$$

Incorporating the optimal final orders of the retailer in stage 1 problem, the expected profit of the retailer can be characterized as in Equation (35).

$$\prod_{1}^{r}(q_{11}^{r},q_{12}^{r}) \quad = \quad \begin{cases} \prod_{11}^{r}(q_{11}^{r},q_{12}^{r}) & if \quad (1-\alpha)q_{12}^{r} < \alpha q_{11}^{r} \\ \prod_{12}^{r}(q_{11}^{r},q_{12}^{r}) & if \quad (1-\alpha)q_{12}^{r} > \alpha q_{11}^{r} \end{cases} \quad \text{where} \tag{35}$$

$$\begin{aligned}
\prod_{11}^{r}(q_{11}^{r},q_{12}^{r}) = & \int_{x_1=0}^{(1-\alpha)q_{11}^{r}-\alpha q_{12}^{r}} \Bigg[\int_{x_2=0}^{(1+\alpha)q_{12}^{r}+\alpha q_{11}^{r}} \Big[(p_1-w_1)x_1-(w_1-s_1)(q_{11}^{r}+q_{12}^{r}-x_2-x_1) \\
& +(p_2-w_2)x_2\Big] \; f_2(x_2)dx_2 + \int_{x_2=(1+\alpha)q_{12}^{r}+\alpha q_{11}^{r}}^{\infty} \Big[(p_1-w_1)x_1-(w_1-s_1)((1-\alpha)q_{11}^{r}-\alpha q_{12}^{r}-x_1) \\
& +(p_2-w_2)((1+\alpha)q_{12}^{r}+\alpha q_{11}^{r})\Big] \; f_2(x_2)dx_2 \Bigg] + \int_{x_1=(1-\alpha)q_{11}^{r}-\alpha q_{12}^{r}}^{q_{11}^{r}+q_{12}^{r}} \Bigg[\int_{x_2=0}^{q_{11}^{r}+q_{12}^{r}-x_1} \Big[(p_1-w_1)x_1 \\
& -(w_1-s_1)(q_{11}^{r}+q_{12}^{r}-x_2-x_1)+(p_2-w_2)x_2 \; \Big] \; f_2(x_2)dx_2 \\
& + \int_{x_2=q_{11}^{r}+q_{12}^{r}-x_1}^{\infty} \Big[(p_1-w_1)x_1+(p_2-w_2)(q_{11}^{r}+q_{12}^{r}-x_1)\Big] \; f_2(x_2)dx_2 \; \Bigg] \; f_1(x_1)dx_1 \\
& + \int_{x_1=q_{11}^{r}+q_{12}^{r}}^{\infty} (p_1-w_1)(q_{11}^{r}+q_{12}^{r}) f_1(x_1)dx_1
\end{aligned}$$

and

$$\begin{aligned}
\prod_{12}^{r}(q_{11}^{r},q_{12}^{r}) = & \int_{x_1=0}^{(1-\alpha)q_{11}^{r}-\alpha q_{12}^{r}} \Bigg[\int_{x_2=0}^{(1-\alpha)q_{12}^{r}-\alpha q_{11}^{r}} \Big[(p_1-w_1)x_1-(w_1-s_1)((1+\alpha)q_{11}^{r}+\alpha q_{12}^{r}-x_1) \\
& +(p_2-w_2)x_2-(w_2-s_2)((1-\alpha)q_{12}^{r}-\alpha q_{11}^{r}-x_2)\Big] \; f_2(x_2)dx_2 \\
& + \int_{x_2=(1-\alpha)q_{12}^{r}-\alpha q_{11}^{r}}^{(1+\alpha)q_{12}^{r}+\alpha q_{11}^{r}} \Big[(p_1-w_1)x_1-(w_1-s_1)(q_{11}^{r}+q_{12}^{r}-x_2-x_1)+(p_2-w_2)x_2\Big] \; f_2(x_2)dx_2 \\
& + \int_{x_2=(1+\alpha)q_{12}^{r}+\alpha q_{11}^{r}}^{\infty} \Big[(p_1-w_1)x_1-(w_1-s_1)((1-\alpha)q_{11}^{r}-\alpha q_{12}^{r}-x_1) \\
& +(p_2-w_2)((1+\alpha)q_{11}^{r}+\alpha q_{12}^{r})\Big] \; f_2(x_2)dx_2 \; \Bigg] \; f_1(x_1)dx_1 \\
& + \int_{x_1=(1-\alpha)q_{11}^{r}-\alpha q_{12}^{r}}^{(1+\alpha)q_{11}^{r}+\alpha q_{12}^{r}} \Bigg[\int_{x_2=0}^{(1-\alpha)q_{12}^{r}-\alpha q_{11}^{r}} \Big[(p_1-w_1)x_1-(w_1-s_1)((1+\alpha)q_{11}^{r}+\alpha q_{12}^{r}-x_1) \\
& +(p_2-w_2)x_2-(w_2-s_2)((1-\alpha)q_{12}^{r}-\alpha q_{11}^{r}-x_2)\Big] \; f_2(x_2)dx_2 \\
& + \int_{x_2=(1-\alpha)q_{12}^{r}-\alpha q_{11}^{r}}^{q_{11}^{r}+q_{12}^{r}-x_1} \Big[(p_1-w_1)x_1-(w_1-s_1)(q_{11}^{r}+q_{12}^{r}-x_2-x_1)+(p_2-w_2)x_2\Big] \; f_2(x_2)dx_2 \\
& + \int_{x_2=q_{11}^{r}+q_{12}^{r}-x_1}^{\infty} \Big[(p_1-w_1)x_1+(p_2-w_2)(q_{11}^{r}+q_{12}^{r}-x_1)\Big] \; f_2(x_2)dx_2 \; \Bigg] \; f_1(x_1)dx_1
\end{aligned}$$

$$+ \int_{x_1=(1+\alpha)q_{11}^r+\alpha q_{12}^r}^{\infty} \Bigg[(p_1 - w_1)((1+\alpha)q_{11}^r + \alpha q_{12}^r)$$

$$+ \int_{x_2=0}^{(1-\alpha)q_{12}^r-\alpha q_{11}^r} \left[(p_2 - w_2)x_2 - (w_2 - s_2)((1-\alpha)q_{12}^r - \alpha q_{11}^r - x_2)\ \right]\ f_2(x_2)dx_2$$

$$+ \int_{x_2=(1-\alpha)q_{12}^r-\alpha q_{11}^r}^{\infty} \left[(p_2 - w_2)((1-\alpha)q_{12}^r - \alpha q_{11}^r)\right]\ f_2(x_2)dx_2\ \Bigg]\ f_1(x_1)dx_1.$$

Lemma 5 *$\prod_1^r(q_{11}^r, q_{12}^r)$ is jointly continuous in q_{11}^r and q_{12}^r.*

Proof: We consider the case that $(1-\alpha)q_{12}^r = \alpha q_{11}^r$ and that is, $q_{12}^r = \frac{\alpha}{(1-\alpha)} q_{11}^r$.

Then, $\prod_{11}^r\left(q_{11}^r,\ q_{12}^r = \frac{\alpha}{(1-\alpha)} q_{11}^r\right) = \prod_{12}^r\left(q_{11}^r,\ q_{12}^r = \frac{\alpha}{(1-\alpha)} q_{11}^r\right) = \prod_1^r(q_{11}^r)$ and the retailer's expected profit function is given by;

$$\prod_{.1}^r(q_{11}^r) = \int_{x_1=0}^{\frac{1-2\alpha}{1-\alpha}q_{11}^r} \left[\int_{x_2=0}^{\frac{2\alpha}{1-\alpha}q_{11}^r} [\ (p_1 - w_1)x_1 - (w_1 - s_1)(\frac{1}{1-\alpha} q_{11}^r - x_2 - x_1)\right.$$

$$+ (p_2 - w_2)x_2\ ]\ f_2(x_2)dx_2 + \int_{x_2=\frac{2\alpha}{1-\alpha}q_{11}^r}^{\infty} [\ (p_1 - w_1)x_1 - (w_1 - s_1)(\frac{1-2\alpha}{1-\alpha} q_{11}^r - x_1)$$

$$\left. + (p_2 - w_2)(\frac{2\alpha}{1-\alpha} q_{11}^r)\ ]\ f_2(x_2)dx_2\ \right]\ f_1(x_1)dx_1 + \int_{x_1=\frac{1-2\alpha}{1-\alpha}q_{11}^r}^{\frac{1}{1-\alpha}q_{11}^r} \left[\int_{x_2=0}^{\frac{1}{1-\alpha}q_{11}^r - x_1} [\ (p_1 - w_1)x_1 \right.$$

$$- (w_1 - s_1)(\frac{1}{1-\alpha} q_{11}^r - x_2 - x_1) + (p_2 - w_2)x_2\ ]\ f_2(x_2)dx_2 + \int_{x_2=\frac{1}{1-\alpha}q_{11}^r - x_1}^{\infty} [\ (p_1 - w_1)x_1$$

$$\left. + (p_2 - w_2)(\frac{1}{1-\alpha} q_{11}^r - x_1)\ ]\ f_2(x_2)dx_2\ \right]\ f_1(x_1)dx_1$$

$$+ \int_{x_1=\frac{1}{1-\alpha}q_{11}^r}^{\infty} (p_1 - w_1)(\frac{1}{1-\alpha} q_{11}^r) f_1(x_1)dx_1$$

Hence, $\prod_1^r(q_{11}^r, q_{12}^r)$ is jointly continuous in q_{11}^r and q_{12}^r. □

Lemma 6 $\prod_{11}^{r}(q_{11}^{r},q_{12}^{r})$ *is jointly concave in* q_{11}^{r} *and* q_{12}^{r}.

Proof: See Appendix D.

Lemma 7 $\prod_{12}^{r}(q_{11}^{r},q_{12}^{r})$ *is jointly concave in* q_{11}^{r} *and* q_{12}^{r}.

Proof: See Appendix E.

Lemma 8 $\prod_{1}^{r}(q_{11}^{r})$ *is concave in* q_{11}^{r}.

Proof: See Appendix F.

Proposition 11 *Let* $(q_{11}^{r})'$, $(q_{12}^{r})'$ *be the solution to* $\partial\prod_{11}^{r}(q_{11}^{r},q_{12}^{r})/\partial q_{11}^{r}=0$ *and* $\partial\prod_{11}^{r}(q_{11}^{r},q_{12}^{r})/\partial q_{12}^{r}=0$. *Similarly, let* $(q_{11}^{r})''$, $(q_{12}^{r})''$ *be the solution to* $\partial\prod_{12}^{r}(q_{11}^{r},q_{12}^{r})/\partial q_{11}^{r}=0$ *and* $\partial\prod_{12}^{r}(q_{11}^{r},q_{12}^{r})/\partial q_{12}^{r}=0$. *Furthermore, let* $(q_{11}^{r})'''$ *be the solution to* $d\prod_{1}^{r}(q_{11}^{r})/dq_{11}^{r}=0$. *Then, the optimal initial order quantities of the retailer are given by;*

(i) If $(1-\alpha)(q_{12}^{r})'\leq\alpha(q_{11}^{r})'$ *and* $(1-\alpha)(q_{12}^{r})''\geq\alpha(q_{11}^{r})''$ *then*

$$\left((q_{11}^{r})^{*},(q_{12}^{r})^{*}\right)=\begin{cases}\left((q_{11}^{r})',(q_{12}^{r})'\right) & \text{if}\quad \prod_{11}^{r}\left((q_{11}^{r})',(q_{12}^{r})'\right)>\prod_{12}^{r}\left((q_{11}^{r})'',(q_{12}^{r})''\right)\\ \left((q_{11}^{r})'',(q_{12}^{r})''\right) & \text{if}\quad \prod_{11}^{r}\left((q_{11}^{r})',(q_{12}^{r})'\right)<\prod_{12}^{r}\left((q_{11}^{r})'',(q_{12}^{r})''\right)\end{cases}$$

(ii) If $(1-\alpha)(q_{12}^{r})'\leq\alpha(q_{11}^{r})'$ *and* $(1-\alpha)(q_{12}^{r})''<\alpha(q_{11}^{r})''$ *then*

$$\left((q_{11}^{r})^{*},(q_{12}^{r})^{*}\right)=\left((q_{11}^{r})',(q_{12}^{r})'\right)$$

(iii) If $(1-\alpha)(q_{12}^{r})'>\alpha(q_{11}^{r})'$ *and* $(1-\alpha)(q_{12}^{r})''\geq\alpha(q_{11}^{r})''$ *then*

$$\left((q_{11}^{r})^{*},(q_{12}^{r})^{*}\right)=\left((q_{11}^{r})'',(q_{12}^{r})''\right)$$

(iv) If $(1-\alpha)(q_{12}^{r})'>\alpha(q_{11}^{r})'$ *and* $(1-\alpha)(q_{12}^{r})''<\alpha(q_{11}^{r})''$ *then*

$$\left((q_{11}^{r})^{*},(q_{12}^{r})^{*}\right)=\left((q_{11}^{r})''',\frac{\alpha}{(1-\alpha)}(q_{11}^{r})'''\right)$$

Proof: Note that $(q_{11}^{r})'$, $(q_{12}^{r})'$ denote the unconstrained optimal solution to $\prod_{11}^{r}(q_{11}^{r},q_{12}^{r})$ by Lemma 6. Similarly, $(q_{11}^{r})''$, $(q_{12}^{r})''$ denote the unconstrained optimal solution to $\prod_{12}^{r}(q_{11}^{r},q_{12}^{r})$ by Lemma 7.

(i) If these conditions are satisfied, then $\prod^r$ has two local maxima and the greater one is the optimal solution.

(ii) If these conditions are satisfied, then $\prod^r$ has one local maximum point which is $(q_{11}^r)'$, $(q_{12}^r)'$ and hence it is the optimal solution.

(iii) If these conditions are satisfied, then $\prod^r$ has one local maximum point which is $(q_{11}^r)''$, $(q_{12}^r)''$ and hence it is the optimal solution.

(iv) If these conditions are satisfied, the optimal will occur at the boundary, i.e. $(1-\alpha)q_{12}^r = \alpha q_{11}^r$. Hence, the optimal will be given by $(q_{11}^r)''', \frac{\alpha}{(1-\alpha)}(q_{11}^r)'''$ due to Lemma 8. □

4.3.2.2. Manufacturer's Problem

The manufacturer's problem is to choose the optimal procurement quantities, q_{12}^m and q_{12}^m that maximize his expected profit subject to the retailer's ordering behavior. Similar to the retailer's problem, we first characterize stage 2 optimal procurement quantities based on the final order quantities of the retailer (which is determined by the demand realizations as observed in Equation (34)), and initial order quantities of the manufacturer. We then characterize the expected profit function of the manufacturer and obtain optimal order quantities.

Recall that the manufacturer is required to fill the retailer's final order quantity for each product in terms of the contract. Given the manufacturer's initial procurement quantities, the manufacturer will set the second procurement order to cover the retailer's final order quantity. Moreover, recall that the initial procurement quantity of the manufacturer is to be $L_1 \le q_{11}^m \le U_1$ and $L_2 \le q_{12}^m \le U_2$ as mentioned in Section 4.3.1.2.

Proposition 12 *Given* α, (q_{11}^m, q_{12}^m) *and* (q_{11}^r, q_{12}^r), *the manufacturer's optimal procurement quantities for stage 2 are characterized as follows;*

$$
(q_{21}^m, q_{22}^m) = \begin{cases}
(U_1 - q_{11}^m, \quad 0) & \text{if} \quad x_1 < L_1 \quad \text{and} \quad x_2 < L_2 \\
(Q^r - x_2 - q_{11}^m, \quad x_2 - q_{12}^m) & \text{if} \quad x_1 < L_1 \quad \text{and} \quad L_2 < x_2 < U_2 \quad \text{and} \\
 & \quad q_{11}^m < Q^r - x_2 \quad \text{and} \quad q_{12}^m < x_2 \\
(Q^r - x_2 - q_{11}^m, \quad 0) & \text{if} \quad x_1 < L_1 \quad \text{and} \quad L_2 < x_2 < U_2 \quad \text{and} \\
 & \quad q_{11}^m < Q^r - x_2 \quad \text{and} \quad q_{12}^m > x_2 \\
(0, \quad x_2 - q_{12}^m) & \text{if} \quad x_1 < L_1 \quad \text{and} \quad L_2 < x_2 < U_2 \quad \text{and} \\
 & \quad q_{11}^m > Q^r - x_2 \quad \text{and} \quad q_{12}^m < x_2 \\
(0, \quad 0) & \text{if} \quad x_1 < L_1 \quad \text{and} \quad L_2 < x_2 < U_2 \quad \text{and} \\
 & \quad q_{11}^m > Q^r - x_2 \quad \text{and} \quad q_{12}^m > x_2 \\
(0, \quad U_2 - q_{12}^m) & \text{if} \quad x_1 < L_1 \quad \text{and} \quad x_2 > U_2 \\
(U_1 - q_{11}^m, \quad 0) & \text{if} \quad L_1 < x_1 < U_1 \quad \text{and} \quad x_2 < L_2 \\
(Q^r - x_2 - q_{11}^m, \quad x_2 - q_{12}^m) & \text{if} \quad L_1 < x_1 < U_1 \quad \text{and} \quad L_2 < x_2 < Q^r - x_1 \\
 & \quad \text{and} \quad q_{11}^m < Q^r - x_2 \quad \text{and} \quad q_{12}^m < x_2 \\
(Q^r - x_2 - q_{11}^m, \quad 0) & \text{if} \quad L_1 < x_1 < U_1 \quad \text{and} \quad L_2 < x_2 < Q^r - x_1 \\
 & \quad \text{and} \quad q_{11}^m < Q^r - x_2 \quad \text{and} \quad q_{12}^m > x_2 \\
(0, \quad x_2 - q_{12}^m) & \text{if} \quad L_1 < x_1 < U_1 \quad \text{and} \quad L_2 < x_2 < Q^r - x_1 \\
 & \quad \text{and} \quad q_{11}^m > Q^r - x_2 \quad \text{and} \quad q_{12}^m < x_2 \\
(0, \quad 0) & \text{if} \quad L_1 < x_1 < U_1 \quad \text{and} \quad L_2 < x_2 < Q^r - x_1 \\
 & \quad \text{and} \quad q_{11}^m > Q^r - x_2 \quad \text{and} \quad q_{12}^m > x_2 \\
(x_1 - q_{11}^m, \quad Q^r - x_1 - q_{12}^m) & \text{if} \quad L_1 < x_1 < U_1 \quad \text{and} \quad x_2 > Q^r - x_1 \\
 & \text{and} \quad q_{11}^m < x_1 \quad \text{and} \quad q_{12}^m < Q^r - x_1 \\
(x_1 - q_{11}^m, \quad 0) & \text{if} \quad L_1 < x_1 < U_1 \quad \text{and} \quad x_2 > Q^r - x_1 \\
 & \text{and} \quad q_{11}^m < x_1 \quad \text{and} \quad q_{12}^m > Q^r - x_1 \\
(0, \quad Q^r - x_1 - q_{12}^m) & \text{if} \quad L_1 < x_1 < U_1 \quad \text{and} \quad x_2 > Q^r - x_1 \\
 & \text{and} \quad q_{11}^m > x_1 \quad \text{and} \quad q_{12}^m < Q^r - x_1 \\
(0, \quad 0) & \text{if} \quad L_1 < x_1 < U_1 \quad \text{and} \quad x_2 > Q^r - x_1 \\
 & \text{and} \quad q_{11}^m > x_1 \quad \text{and} \quad q_{12}^m > Q^r - x_1 \\
(U_1 - q_{11}^m, \quad 0) & \text{if} \quad x_1 > U_1
\end{cases} \tag{36}
$$

Proof: Recall that the retailer's optimal stage 2 orders are given by Equation (34). Also note that the manufacturer has to satisfy retailer's final order in full. If $x_1 < L_1$ and $x_2 < L_2$, the retailer's final orders are given by $(q^r_{21}, q^r_{22}) = (U_1, L_2)$ and the manufacturer places an expedited order of size $q^m_{21} = U_1 - q^m_{11}$ for product 1. If $x_1 < L_1$ and $L_2 < x_2 < U_2$, the retailer's final orders are given by $(q^r_{21}, q^r_{22}) = (Q^r - x_2, x_2)$. Noting that $q^m_{11} < Q^r - x_2$, the manufacturer places an expedited order of size $q^m_{21} = Q^r - x_2 - q^m_{11}$ for product 1. Additionally, noting that $q^m_{12} < x_2$, the manufacturer places an expedited order of size $q^m_{22} = x_2 - q^m_{12}$ for product 2. If $x_1 < L_1$ and $x_2 > U_2$, the retailer's final orders are given by $(q^r_{21}, q^r_{22}) = (L_1, U_2)$ and the manufacturer places an expedited order of size $q^m_{22} = U_2 - q^m_{12}$ for product 2. If $L_1 < x_1 < U_1$ and $x_2 < L_2$, the retailer's final orders are given by $(q^r_{21}, q^r_{22}) = (U_1, L_2)$ and the manufacturer places an expedited order of size $q^m_{21} = U_1 - q^m_{11}$ for product 1. If $L_1 < x_1 < U_1$ and $L_2 < x_2 < Q^r - x_1$, the retailer's final orders are given by $(q^r_{21}, q^r_{22}) = (Q^r - x_2, x_2)$. Noting that $q^m_{11} < Q^r - x_2$, the manufacturer places an expedited order of size $q^m_{21} = Q^r - x_2 - q^m_{11}$ for product 1. Additionally, noting that $q^m_{12} < x_2$, the manufacturer places an expedited order of size $q^m_{22} = x_2 - q^m_{12}$ for product 2. If $L_1 < x_1 < U_1$ and $x_2 > Q^r - x_1$, the retailer's final orders are given by $(q^r_{21}, q^r_{22}) = (x_1, Q^r - x_1)$. Noting that $q^m_{11} < x_1$, the manufacturer places an expedited order of size $q^m_{21} = x_1 - q^m_{11}$ for product 1. Additionally, noting that $q^m_{12} < Q^r - x_1$, the manufacturer places an expedited order of size $q^m_{22} = Q^r - x_1 - q^m_{12}$ for product 2. Finally, if $x_1 > U_1$, the retailer's final orders are given by $(q^r_{21}, q^r_{22}) = (U_1, L_2)$ and the manufacturer places an expedited order of size $q^m_{21} = U_1 - q^m_{11}$ for product 1. □

The manufacturer's conditional profits are given by:

$$
\prod\nolimits_1^m(q_{11}^m,q_{12}^m) = -(m_1+d_1)q_{11}^m-(m_2+d_1)q_{12}^m
$$

$$
\begin{cases}
+w_1U_1+w_2L_2-(m_1+d_2)(U_1-q_{11}^m)+r_2(q_{12}^m-L_2) \quad \text{if} \quad x_1<L_1 \text{ and } x_2<L_2 \\
+w_1(Q^r-x_2)+w_2x_2-(m_1+d_2)(Q^r-x_2-q_{11}^m)-(m_2+d_2)(x_2-q_{12}^m) \\
\quad \text{if} \quad x_1<L_1 \text{ and } L_2<x_2<U_2 \text{ and } q_{11}^m<Q^r-x_2 \text{ and } q_{12}^m<x_2 \\
+w_1(Q^r-x_2)+w_2x_2-(m_1+d_2)(Q^r-x_2-q_{11}^m)+r_2(q_{12}^m-x_2) \\
\quad \text{if} \quad x_1<L_1 \text{ and } L_2<x_2<U_2 \text{ and } q_{11}^m<Q^r-x_2 \text{ and } q_{12}^m>x_2 \\
+w_1(Q^r-x_2)+w_2x_2-(m_2+d_2)(x_2-q_{12}^m)+r_1(q_{11}^m-Q^r+x_2) \\
\quad \text{if} \quad x_1<L_1 \text{ and } L_2<x_2<U_2 \text{ and } q_{11}^m>Q^r-x_2 \text{ and } q_{12}^m<x_2 \\
+w_1(Q^r-x_2)+w_2x_2+r_1(q_{11}^m-Q^r+x_2)+r_2(q_{12}^m-x_2) \\
\quad \text{if} \quad x_1<L_1 \text{ and } L_2<x_2<U_2 \text{ and } q_{11}^m>Q^r-x_2 \text{ and } q_{12}^m>x_2 \\
+w_1L_1+w_2U_2-(m_2+d_2)(U_2-q_{12}^m)+r_1(q_{11}^m-L_1) \quad \text{if} \quad x_1<L_1 \text{ and } x_2>U_2 \\
+w_1U_1+w_2L_2-(m_1+d_2)(U_1-q_{11}^m)+r_2(q_{12}^m-L_2) \quad \text{if} \quad L_1<x_1<U_1 \text{ and } x_2<L_2 \\
+w_1(Q^r-x_2)+w_2x_2-(m_1+d_2)(Q^r-x_2-q_{11}^m)-(m_2+d_2)(x_2-q_{12}^m) \\
\quad \text{if} \quad L_1<x_1<U_1 \text{ and } L_2<x_2<Q^r-x_1 \text{ and } q_{11}^m<Q^r-x_2 \text{ and } q_{12}^m<x_2 \\
+w_1(Q^r-x_2)+w_2x_2-(m_1+d_2)(Q^r-x_2-q_{11}^m)+r_2(q_{12}^m-x_2) \\
\quad \text{if} \quad L_1<x_1<U_1 \text{ and } L_2<x_2<Q^r-x_1 \text{ and } q_{11}^m<Q^r-x_2 \text{ and } q_{12}^m>x_2 \\
+w_1(Q^r-x_2)+w_2x_2-(m_2+d_2)(x_2-q_{12}^m)+r_1(q_{11}^m-Q^r+x_2) \\
\quad \text{if} \quad L_1<x_1<U_1 \text{ and } L_2<x_2<Q^r-x_1 \text{ and } q_{11}^m>Q^r-x_2 \text{ and } q_{12}^m<x_2 \\
+w_1(Q^r-x_2)+w_2x_2+r_1(q_{11}^m-Q^r+x_2)+r_2(q_{12}^m-x_2) \\
\quad \text{if} \quad L_1<x_1<U_1 \text{ and } L_2<x_2<Q^r-x_1 \text{ and } q_{11}^m>Q^r-x_2 \text{ and } q_{12}^m>x_2 \\
+w_1x_1+w_2(Q^r-x_1)-(m_1+d_2)(x_1-q_{11}^m)-(m_2+d_2)(Q^r-x_1-q_{12}^m) \\
\quad \text{if} \quad L_1<x_1<U_1 \text{ and } x_2>Q^r-x_1 \text{ and } q_{11}^m<x_1 \text{ and } q_{12}^m<Q^r-x_1 \\
+w_1x_1+w_2(Q^r-x_1)-(m_1+d_2)(x_1-q_{11}^m)+r_2(q_{12}^m-Q^r+x_1) \\
\quad \text{if} \quad L_1<x_1<U_1 \text{ and } x_2>Q^r-x_1 \text{ and } q_{11}^m<x_1 \text{ and } q_{12}^m>Q^r-x_1 \\
+w_1x_1+w_2(Q^r-x_1)-(m_2+d_2)(Q^r-x_1-q_{12}^m)+r_1(q_{11}^m-x_1) \\
\quad \text{if} \quad L_1<x_1<U_1 \text{ and } x_2>Q^r-x_1 \text{ and } q_{11}^m>x_1 \text{ and } q_{12}^m<Q^r-x_1 \\
+w_1x_1+w_2(Q^r-x_1)+r_1(q_{11}^m-x_1)+r_2(q_{12}^m-Q^r+x_1) \\
\quad \text{if} \quad L_1<x_1<U_1 \text{ and } x_2>Q^r-x_1 \text{ and } q_{11}^m>x_1 \text{ and } q_{12}^m>Q^r-x_1 \\
+w_1U_1+w_2L_2-(m_1+d_2)(U_1-q_{11}^m)+r_2(q_{12}^m-L_2) \quad \text{if } x_1>U_1
\end{cases}
$$

Considering the optimal stage 2 orders of the manufacturer given in Equation (36), the manufacturer's expected profit function is given by;

$$
\begin{aligned}
\Pi_1^m(q_{11}^m,q_{12}^m) &= -(m_1+d_1)q_{11}^m-(m_2+d_1)q_{12}^m \\
&+\int_{x_1=0}^{U_1}\int_{x_2=0}^{L_2}\left[w_1U_1+w_2L_2-(m_1+d_2)(U_1-q_{11}^m)+r_2(q_{12}^m-L_2)\right] f_2(x_2)dx_2f_1(x_1)dx_1 \\
&+\int_{x_1=U_1}^{\infty}\left[w_1U_1+w_2L_2-(m_1+d_2)(U_1-q_{11}^m)+r_2(q_{12}^m-L_2)\right] f_1(x_1)dx_1 \\
&+\int_{x_1=0}^{L_1}\int_{x_2=U_2}^{\infty}\left[w_1L_1+w_2U_2-(m_2+d_2)(U_2-q_{12}^m)+r_1(q_{11}^m-L_1)\right] f_2(x_2)dx_2f_1(x_1)dx_1 \\
&+\int_{x_1=0}^{L_1}\Bigg[\int_{x_2=L_2}^{U_2}\left[w_1(Q^r-x_2)+w_2x_2\right] f_2(x_2)dx_2 \\
&-\int_{x_2=L_2}^{Q^r-q_{11}^m}\left[(m_1+d_2)(Q^r-x_2-q_{11}^m)\right] f_2(x_2)dx_2 \\
&+\int_{x_2=Q^r-q_{11}^m}^{U_2}\left[r_1(q_{11}^m-Q^r+x_2)\right] f_2(x_2)dx_2+\int_{x_2=L_2}^{q_{12}^m}\left[r_2(q_{12}^m-x_2)\right] f_2(x_2)dx_2 \\
&-\int_{x_2=q_{12}^m}^{U_2}\left[(m_2+d_2)(x_2-q_{12}^m)\right] f_2(x_2)dx_2\Bigg] f_1(x_1)dx_1 \\
&+\int_{x_2=L_2}^{U_2}\int_{x_1=L_1}^{Q^r-x_2}\left[w_1(Q^r-x_2)+w_2x_2\right] f_1(x_1)dx_1f_2(x_2)dx_2 \\
&-\int_{x_2=L_2}^{Q^r-q_{11}^m}\int_{x_1=L_1}^{Q^r-x_2}(m_1+d_2)(Q^r-x_2-q_{11}^m)\ f_1(x_1)dx_1f_2(x_2)dx_2 \\
&+\int_{x_2=Q^r-q_{11}^m}^{U_2}\int_{x_1=L_1}^{Q^r-x_2}r_1(q_{11}^m-Q^r+x_2)\ f_1(x_1)dx_1f_2(x_2)dx_2 \\
&+\int_{x_2=L_2}^{q_{12}^m}\int_{x_1=L_1}^{Q^r-x_2}r_2(q_{12}^m-x_2)f_1(x_1)dx_1f_2(x_2)dx_2 \\
&-\int_{x_2=q_{12}^m}^{U_2}\int_{x_1=L_1}^{Q^r-x_2}(m_2+d_2)(x_2-q_{12}^m)f_1(x_1)dx_1f_2(x_2)dx_2 \\
&+\int_{x_1=L_1}^{U_1}\int_{x_2=Q^r-x_1}^{\infty}\left[w_1x_1+w_2(Q^r-x_1)\right] f_2(x_2)dx_2f_1(x_1)dx_1 \\
&+\int_{x_1=L_1}^{q_{11}^m}\int_{x_2=Q^r-x_1}^{\infty}r_1(q_{11}^m-x_1)f_2(x_2)dx_2f_1(x_1)dx_1
\end{aligned}
$$

$$
\begin{aligned}
&-\int_{x_1=q_{11}^m}^{U_1}\int_{x_2=Q^r-x_1}^{\infty}(m_1+d_2)(x_1-q_{11}^m)f_2(x_2)dx_2f_1(x_1)dx_1\\
&-\int_{x_1=L_1}^{Q^r-q_{12}^m}\int_{x_2=Q^r-x_1}^{\infty}(m_2+d_2)(Q^r-x_1-q_{12}^m)\ f_2(x_2)dx_2f_1(x_1)dx_1\\
&+\int_{x_1=Q^r-q_{12}^m}^{U_1}\int_{x_2=Q^r-x_1}^{\infty}r_2(q_{12}^m-Q^r+x_1)\ f_2(x_2)dx_2f_1(x_1)dx_1
\end{aligned}
$$

Proposition 13 *Let $(q_{11}^m)'$ and $(q_{12}^m)'$ be the unique solution of $\partial\prod_1^m/\partial q_{11}^m=0$ and $\partial\prod_1^m/\partial q_{12}^m=0$, respectively. Then, the optimal initial procurement quantities of the manufacturer are characterized by;*

$$
\left((q_{11}^m)^*,(q_{12}^m)^*\right)=\left(\min\ \left\{\max(L_1,(q_{11}^m)'),U_1\right\},\ \min\ \left\{\max(L_2,(q_{12}^m)'),U_2\right\}\right)
$$

Proof: The first derivatives of $\prod_1^m(q_{11}^m,q_{12}^m)$ with respect to q_{11}^m and q_{12}^m are given in Equation (37) and Equation (38), respectively.

$$
\begin{aligned}
\frac{\partial\prod_1^m}{\partial q_{11}^m}=&\ d_2-d_1+(m_1+d_2)\ F_1(U_1)\ F_2(L_2)-r_1\ F_2(U_2)\ F_1(L_1)+(r_1-m_1-d_2)\ F_1(q_{11}^m)\\
&+\int_{x_2=L_2}^{Q^r-q_{11}^m}(m_1+d_2)F_1(Q^r-x_2)\ f_2(x_2)dx_2+\int_{x_2=Q^r-q_{11}^m}^{U_2}r_1F_1(Q^r-x_2)\ f_2(x_2)dx_2\\
&-\int_{x_1=L_1}^{q_{11}^m}r_1F_2(Q^r-x_1)\ f_1(x_1)dx_1-\int_{x_1=q_{11}^m}^{U_1}(m_1+d_2)F_2(Q^r-x_1)\ f_1(x_1)dx_1 \qquad (37)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial\prod_1^m}{\partial q_{12}^m}=&\ r_2-m_2-d_1+r_2F_1(U_1)\ F_2(L_2)-(m_2+d_2)F_2(U_2)\ F_1(L_1)\\
&+(m_2+d_2-r_2)\ F_1(Q^r-q_{12}^m)+\int_{x_2=L_2}^{q_{12}^m}r_2F_1(Q^r-x_2)\ f_2(x_2)dx_2\\
&+\int_{x_2=q_{12}^m}^{U_2}(m_2+d_2)F_1(Q^r-x_2)\ f_2(x_2)dx_2 \qquad (38)\\
&-\int_{x_1=L_1}^{Q^r-q_{12}^m}(m_2+d_2)F_2(Q^r-x_1)\ f_1(x_1)dx_1-\int_{x_1=Q^r-q_{12}^m}^{U_1}r_2F_2(Q^r-x_1)\ f_1(x_1)dx_1
\end{aligned}
$$

The second derivatives of $\prod_1^m(q_{11}^m, q_{12}^m)$ in Equation (39), Equation (40) and Equation (41) indicate that the determinant of the hessian matrix is positive.

$$\frac{\partial^2 \prod_1^m}{\partial (q_{11}^m)^2} = (r_1 - m_1 - d_2)\ F_1(q_{11}^m) f_2(Q^r - q_{11}^m) + (r_1 - m_1 - d_2)\ f_1(q_{11}^m)\left[1 - F_2(Q^r - q_{11}^m)\right] \tag{39}$$

$$\frac{\partial^2 \prod_1^m}{\partial (q_{12}^m)^2} = (r_2 - m_2 - d_2) f_1(Q^r - q_{12}^m)\left[1 - F_2(q_{12}^m)\right] + (r_2 - m_2 - d_2) F_1(Q^r - q_{12}^m) f_1(q_{12}^m) \tag{40}$$

$$\frac{\partial^2 \prod_1^m}{\partial q_{11}^m \partial q_{12}^m} = \frac{\partial^2 \prod_1^m}{\partial q_{12}^m \partial q_{11}^m} = 0 \tag{41}$$

If $L_1 \le (q_{11}^m)' \le U_1$, then the optimal solution is given by first order conditions. If $(q_{11}^m)' \le L_1$, then $(q_{11}^m)^* = L_1$ and otherwise, $(q_{11}^m)^* = U_1$. Similar arguments are also valid for $(q_{12}^m)^*$. □

Note that the optimal initial order quantities of the manufacturer depend implicitly on w_1 and w_2 through the retailer's optimal order quantities, q_{11}^r and q_{12}^r, as in the case where the retailer prefers to sell product 2 at a discounted price (Section 4.3.1.2). However, it should be noted that, contrary to Section 4.3.1.2, the optimal order quantities depend on the distribution of the demand of both products. Moreover, q_{11}^m and q_{12}^m do not depend on each other.

CHAPTER 5

COMPUTIONAL ANALYSIS

In this chapter, our goal is to gain insights through computational experiments rather than doing a full experimental design and statistically attempt to verify hypotheses. We present the impact of different parameters on the ordering decisions of the retailer and manufacturer and try to reveal the intuition behind their responses to a change in these parameters. We also want to get an understanding of the impact of order adjustment flexibility on the performance of the retailer, the manufacturer and the supply chain.

We realize numerical analysis for both cases “Sell Product II at the Discounted Price” and “Sell Product I at the Discounted Price” as described in Chapter 4. Note that all set of parameters throughout this numerical analysis are inline with the assumptions in Chapter 3. Furthermore, demands for both products are considered to be normally distribution in the analysis. To be specific, we carry out computer runs with different values of the parameters. A set of parameter values will be used as a baseline and we then vary a particular parameter every run. The baseline parameter settings for the case “Sell Product II at the Discounted Price” and the case “Sell Product I at the Discounted Price” are given in Table 1.

Table 1 Baseline parameter settings

		p	*w*	*s*	*m*	*r*	*μ*	*σ*
Sell Product II at the Discounted Price	*product 1*	120	30	0	7	0	50	10
	product 2	100	25	0	7	0	50	10
Sell Product I at the Discounted Price	*product 1*	120	30	10	7	0	50	10
	product 2	100	25	0	7	0	50	10

The manufacturer related delivery costs are set as $d_1 = 3$, $d_2 = 5$.

$\prod^r$, $\prod^m$ and $\prod^s$ denote the expected profit of the retailer, the manufacturer and the supply chain throughout numerical analysis, respectively. In addition to the expected profit levels and order quantities, we also consider the percentage improvements in expected profits to evaluate the value of order adjustment flexibility and understand the effects of order adjustment flexibility among the retailer, the manufacturer and the supply chain.

$\%\prod^r$: Percentage improvement in the expected profit of the retailer compared to the no order adjustment case

$\%\prod^m$: Percentage improvement in the expected profit of the manufacturer compared to the no order adjustment case

$\%\prod^s$: Percentage improvement in the expected profit of the supply chain compared to the no order adjustment case

5.1. Sell Product II at the Discounted Price

Recall that the setting for this case is $p_1 - w_1 > p_2 - w_2$ and $w_1 - s_1 > w_2 - s_2$. In the following subsections, we analyze the effect of order adjustment flexibility, parameters related with demand and then cost and revenue parameters.

5.1.1. Order Adjustment Flexibility

The model is solved for different levels of order adjustment flexibility, α, (from 0.05 to 0.35), and also for the no order adjustment case (which is considered as benchmark) and the unlimited order adjustment case. Summary of the results obtained for the baseline and percentage improvements compared to benchmark for different levels of order adjustment flexibility are given in Table 2 and Table 3, respectively. Moreover, the profits of the retailer and manufacturer for the baseline for different levels of order adjustment flexibility are presented in Figure 6. Furthermore, order quantities of the retailer and the manufacturer are presented in Figure 7.

- The retailer will have a wider range between upper and lower limits for each product to fill the demand when α is increased. Due to larger range of modification opportunity, the retailer reduces Q^r as order adjustment flexibility, α, increases. Additionally, 98% of decrease in Q^r compared to the no order adjustment case is realized until $\alpha = 0.15$. The retailer also reduces q_{11}^r and q_{12}^r at $\alpha = 0.05$ relative to the no order adjustment case. Since the retailer chooses product 2 to have in excess at the end of the period, it continues decreasing q_{11}^r but starts increasing q_{12}^r as α increases.

Table 2 Summary of the results for different levels of α

	q_{11}^r	q_{12}^r	Q^r	$\prod^r$	q_{11}^m	q_{12}^m	$\prod^m$	$\prod^s$
No order adjustment	56.7	56.7	113.5	7533.0	56.7	56.7	1986.1	9519.1
α =0.05	55.3	55.5	110.8	7690.3	49.7	51.1	1903.8	9594.1
α =0.10	54.5	55.3	109.8	7757.6	43.5	50.2	1863.8	9621.4
α =0.15	53.8	55.8	109.6	7784.1	40.3	49.9	1841.2	9625.3
α =0.20	52.8	56.7	109.6	7790.6	40.3	49.9	1835.3	9625.9
α =0.25	52.3	57.2	109.5	7793.1	40.3	49.9	1833.5	9626.6
α =0.30	51.2	58.3	109.5	7794.4	40.3	49.9	1833.2	9627.6
α =0.35	50.4	59.2	109.5	7795.2	40.3	49.9	1833.1	9628.4
Unlimited	-	-	109.5	7795.8	40.3	49.9	1833.1	9628.9

Table 3 % Improvement in profits due to order adjustment: effects of α

	$\%\prod^r$	$\%\prod^m$	$\%\prod^s$
α =0.05	2.1	-4.1	0.8
α =0.10	3.0	-6.2	1.1
α =0.15	3.3	-7.2	1.1
α =0.20	3.4	-7.6	1.1
α =0.25	3.5	-7.7	1.1
α =0.30	3.5	-7.7	1.1
α =0.35	3.5	-7.7	1.1
Unlimited	3.5	-7.7	1.2

- The initial order quantity of the manufacturer for each product decreases significantly relative to the no order adjustment case. Additionally, $q_{12}^m > q_{11}^m$ although their distributions are the same, since the retailer chooses to sell product 2 at a discounted price. In the no order adjustment case, q_{11}^m and q_{12}^m are 56.7; q_{11}^m and q_{12}^m become 49.7 and 51.1 at $\alpha = 0.05$, respectively.

Above $\alpha = 0.15$, q_{11}^m and q_{12}^m are constant since α does not affect q_{11}^m and q_{12}^m explicitly in Equation (29) and (30), respectively. However, α has impact on Q^r which is a determinant of q_{12}^m in Equation (30). Furthermore, the manufacturer's initial order quantities are to be $L_1 \le q_{11}^m \le U_1$ and $L_2 \le q_{12}^m \le U_2$ (recall Proposition 10); the optimal q_{11}^m is determined by L_1 and Equation (29) for $\alpha \le 0.10$ and $\alpha \ge 0.15$, respectively.

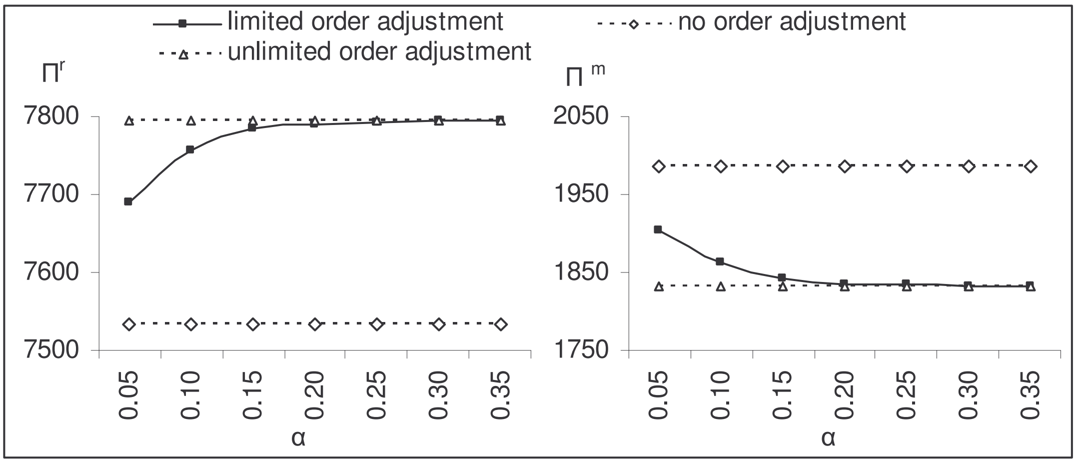

Figure 6 Effects of α on the expected profits

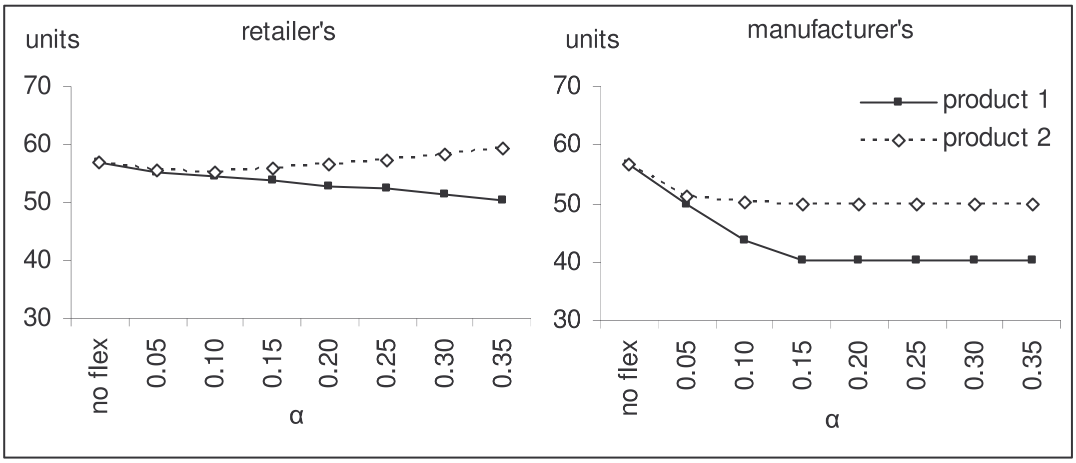

Figure 7 Effects of α on the order quantities

- The expected profit of the retailer increases as α increases and the maximum profit improvement that can be achieved because of order adjustment flexibility is 3.5%. The retailer obtains 98% of maximum profit improvement (which is in the unlimited order adjustment case) when $\alpha = 0.20$. The expected profit of the manufacturer decreases as α increases and the maximum profit loss that can be faced because of order adjustment flexibility is 7.7%. The manufacturer also experiences 98.6% of maximum profit loss (which is in the unlimited order adjustment case) when $\alpha = 0.20$. Although, the retailer determines ordering quantities only considering its expected profit, the expected profit of supply chain increases under this contract and 1.1% improvement is achieved when $\alpha = 0.10$.

Observation 1 A limited level of order adjustment flexibility (which depends on the coefficient of variation of demands) is required for the retailer and the supply chain to achieve almost all of the benefits of order adjustment flexibility.

5.1.2. Demand Parameters

In this section, the effects of demand variability and expected demand on the expected profits are examined. We conduct experiments by using sample data sets representing different demand parameters of each product. Specifically, we focus on different levels of coefficient of variation with equal mean, and different mean levels with the same level of coefficient of variation.

Coefficient of Variation of Product 1:

We will analyze the no order adjustment case, the unlimited order adjustment case and the limited order adjustment case, which will be represented by $\alpha = 0.10$. The model is solved for different coefficient of variation values of product 1, cv_1, which are *1/10, 1/7, 1/5 and 1/3* with equal mean, $\mu_1 = 50$. Summary of the results obtained and percentage improvements compared to the benchmark case are given in Table 4 and Table 5, respectively. The profits of the retailer and the manufacturer for different levels of cv_1 are presented in Figure 8, respectively.

- We observe that Q^r in the limited order adjustment case is less than Q^r in the no order adjustment case for all particular cv_1 values. In all flexibility levels, Q^r increases in cv_1, however, the increase is less steep in the limited order adjustment case when compared to the no order adjustment case. For instance, the difference between Q^r in the no order adjustment case and at

$\alpha = 0.10$ is 2.5 and 4.2 when $cv_1=1/10$ and $cv_1=1/3$, respectively. Moreover, the increase in Q^r is due to q^r_{11} which increases in cv_1. Although, demand parameters of product 2 are not modified, q^r_{12} is decreasing as cv_1 increases. Intuitively, an increase in q^r_{11} will provide more flexibility to the retailer in terms of product 2 as well. Hence, it is reasonable to have a decreasing q^r_{12} when q^r_{11} increases.

Table 4 Summary of the results for different levels of cv_1

	cv_1	q^r_{11}	q^r_{12}	Q^r	$\prod^r$	q^m_{11}	q^m_{12}	$\prod^m$	$\prod^s$
No order adjustment	*1/10*	53.4	56.7	110.1	7724.3	-	-	1918.6	9642.9
	1/7	54.8	56.7	111.6	7646.5	-	-	1947.5	9594.0
	1/5	56.7	56.7	113.5	7533.0	-	-	1986.1	9519.1
	1/3	61.2	56.7	118.0	7283.5	-	-	2076.0	9359.5
$\alpha=0.10$	*1/10*	51.6	56.0	107.6	7883.1	45.2	52.7	1834.6	9717.6
	1/7	52.8	55.6	108.4	7837.0	43.1	51.5	1840.9	9678.0
	1/5	54.5	55.3	109.8	7757.6	43.5	50.2	1863.8	9621.4
	1/3	58.9	54.8	113.8	7558.9	47.6	47.7	1935.6	9494.4
Unlimited	*1/10*	-	-	107.5	7888.2	45.2	52.7	1833.1	9721.3
	1/7	-	-	108.3	7853.9	43.1	51.4	1831.5	9685.4
	1/5	-	-	109.5	7795.8	40.3	49.9	1833.1	9628.9
	1/3	-	-	113.2	7637.8	33.9	47.1	1845.3	9483.2

Table 5 % Improvement in profits due to order adjustment: effects of cv_1

	c_{v1}	$\%\prod^r$	$\%\prod^m$	$\%\prod^s$
$\alpha=0.10$	*1/10*	2.1	-4.4	0.8
	1/7	2.5	-5.5	0.9
	1/5	3.0	-6.2	1.1
	1/3	3.8	-6.8	1.4
Unlimited	*1/10*	2.1	-4.5	0.8
	1/7	2.7	-6.0	1.0
	1/5	3.5	-7.7	1.2
	1/3	4.9	-11.1	1.3

- The initial order quantity of the manufacturer for each product decreases as cv_1 increases if $cv_1 \leq 1/7$; and the optimal q^m_{11} and q^m_{11} are determined by Equation (29) and (30), respectively. Moreover, the reduction in q^m_{11} is higher than the reduction in q^m_{12} since the retailer will keep product 2 in excess at the end of period. When $cv_1 \geq 1/5$, the optimal q_{11}^m is bounded by L_1 and changes in q_{11}^r and Q^r values. In the unlimited order adjustment case, q_{11}^m decreases as cv_1 increases.

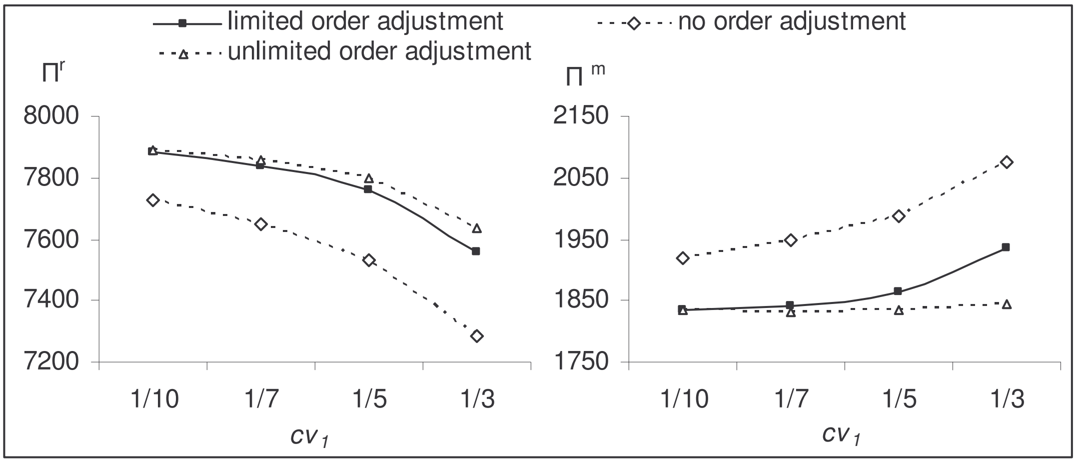

Figure 8 Effects of cv_1 on the expected profits

- The expected profit of the retailer decreases as the coefficient of variation increases. The percentage improvement in the expected profit of the retailer because of the order adjustment is higher when cv_1 is higher. The difference between $\prod^r$ in the no order adjustment case and at $\alpha = 0.10$ is 2.1% and 3.8% when $cv_1=1/10$ and when $cv_1=1/3$, respectively. The expected profit of the manufacturer increases as the initial order of the retailer increases due to high cv_1, however; the percentage loss due to order adjustment increases as well. Furthermore, the response of $\prod^s$ to a change in cv_1 is similar to the response of $\prod^r$.

Coefficient of Variation of Product 2:

We now consider the effects of uncertainty in the demand of product 2, cv_2. Summary of the results obtained are given in Table 6 and Table 7, respectively. The profits of the retailer and the manufacturer for different levels of cv_2 are presented in Figure 9, respectively.

- We observe that Q^r values in experiments of cv_2 are very close to the values in experiments of cv_1, and similar arguments are valid here. Moreover, the increase in Q^r is due to q^r_{12} which increases in cv_2 and q^r_{11} slightly changes in cv_2.

Table 6 Summary of the results for different levels of cv_2

	cv_2	q^r_{11}	q^r_{12}	Q^r	$\prod^r$	q^m_{11}	q^m_{12}	$\prod^m$	$\prod^s$
No order adjustment	*1/10*	56.7	53.4	110.1	7692.4	-	-	1935.5	9627.9
	1/7	56.7	54.8	111.6	7627.6	-	-	1957.2	9584.8
	1/5	56.7	56.7	113.5	7533.0	-	-	1986.1	9519.1
	1/3	56.7	61.2	118.0	7325.1	-	-	2053.5	9378.6
α =0.10	*1/10*	54.4	53.1	107.6	7869.5	43.7	47.9	1830.3	9699.8
	1/7	54.6	53.7	108.4	7834.2	43.8	48.7	1842.5	9676.8
	1/5	54.5	55.3	109.8	7757.6	43.5	50.2	1863.8	9621.4
	1/3	54.2	59.6	113.8	7593.1	42.8	54.2	1921.4	9514.6
Unlimited	*1/10*	-	-	107.5	7888.7	40.3	47.9	1803.2	9691.8
	1/7	-	-	108.3	7854.2	40.3	48.6	1814.4	9668.5
	1/5	-	-	109.5	7795.8	40.3	49.9	1833.1	9628.9
	1/3	-	-	113.1	7632.0	40.3	53.4	1886.7	9518.7

Table 7 % Improvement in profits due to order adjustment: effects of cv_2

	cv_2	$\%\prod^r$	$\%\prod^m$	$\%\prod^s$
α =0.10	*1/10*	2.3	-5.4	0.7
	1/7	2.7	-5.9	1.0
	1/5	3.0	-6.2	1.1
	1/3	3.7	-6.4	1.4
Unlimited	*1/10*	2.6	-6.8	0.7
	1/7	3.0	-7.3	0.9
	1/5	3.5	-7.7	1.2
	1/3	4.2	-8.1	1.5

- Although demand parameters of the product 2 has no explicit effect on q^m_{11} and the solution of Equation (29) is constant; the optimal q^m_{11} is bounded by L_1 and changes in q^r_{11} and Q^r values. The manufacturer reflects the increase in the retailer's aggregate order to q^m_{12} since the retailer will keep product 2 in excess at the end of period.

- The expected profit of the retailer, the manufacturer and the supply chain have similar trend as in experiments of cv_2.

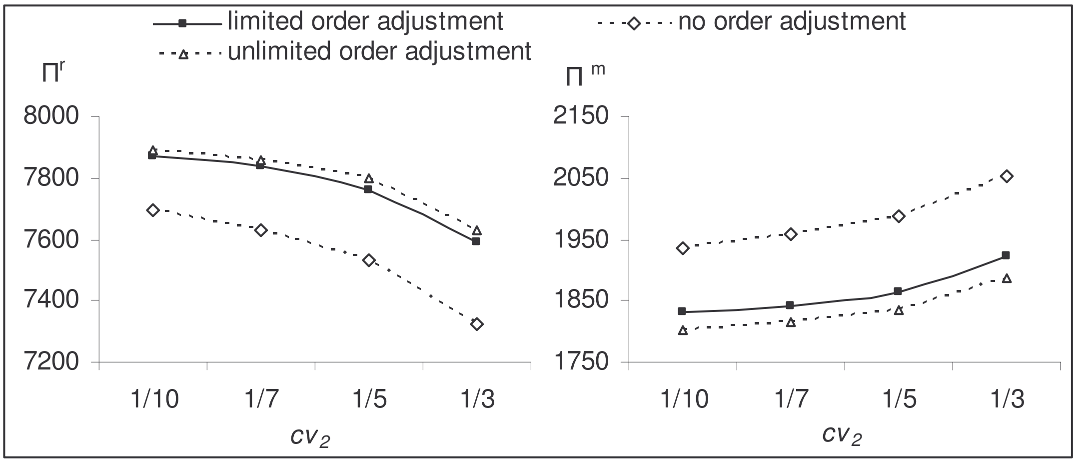

Figure 9 Effects of cv_2 on the expected profits

Observation 2 The order adjustment flexibility is more valuable to the retailer and the supply chain when demand uncertainty is high (as coefficient of variation increases).

<u>Mean Demand of Product 1:</u>

We now consider the effects of mean demand of product 1 keeping the coefficient of variation constant at $cv_1=cv_2=1/5$. Summary of the results obtained are given in Table 8 and Table 9, respectively. The expected profits of the retailer and the manufacturer for different levels of mean demand are presented in Figure 10, respectively.

- q_{11}^r and Q^r increases in parallel to μ_1 with nearly the same rate in all cases. Although, q_{12}^r is constant in the no order adjustment case and demand parameter of product 2 is not modified, q_{12}^r decreases slightly as μ_1 increases. Intuitively, an increase in q_{11}^r will provide more flexibility to the retailer in terms of product 2 as well. Hence, it is reasonable to have a decreasing q_{12}^r when q_{11}^r increases.

Table 8 Summary of the results for different levels of μ_1

	μ_1	q^r_{11}	q^r_{12}	Q^r	$\prod^r$	q^m_{11}	q^m_{12}	$\prod^m$	$\prod^s$
No order adjustment	*30*	34.0	56.7	90.8	5885.1	-	-	1532.1	7417.2
	40	45.4	56.7	102.1	6709.1	-	-	1759.1	8468.2
	50	56.7	56.7	113.5	7533.0	-	-	1986.1	9519.1
	60	68.1	56.7	124.8	8357.0	-	-	2213.1	10570.0
	70	79.4	56.7	136.2	9180.9	-	-	2440.0	11621.0
α =0.10	*30*	32.3	55.7	88.0	6059.0	24.2	52.2	1442.2	7501.2
	40	43.4	55.5	98.9	6924.1	33.5	51.2	1651.4	8575.5
	50	54.5	55.3	109.8	7757.6	43.5	50.2	1863.8	9621.4
	60	65.7	55.2	120.8	8604.9	53.6	49.2	2077.1	10681.9
	70	76.8	55.1	131.9	9463.8	63.7	48.4	2291.0	11754.8
Unlimited	*30*	-	-	87.9	6077.0	24.2	52.1	1432.0	7509.0
	40	-	-	98.6	6937.9	32.3	50.9	1631.6	8569.5
	50	-	-	109.5	7795.8	40.3	49.9	1833.1	9628.9
	60	-	-	120.5	8649.6	48.4	48.9	2036.1	10685.7
	70	-	-	131.6	9502.8	56.5	48.1	2240.1	11743.0

Table 9 % Improvement in profits due to order adjustment: effects of μ_1

	μ_1	$\%\prod^r$	$\%\prod^m$	$\%\prod^s$
α =0.10	*30*	3.0	-5.9	1.1
	40	3.2	-6.1	1.3
	50	3.0	-6.2	1.1
	60	3.0	-6.1	1.1
	70	3.1	-6.1	1.2
Unlimited	*30*	3.3	-6.5	1.2
	40	3.4	-7.2	1.2
	50	3.5	-7.7	1.2
	60	3.5	-8.0	1.1
	70	3.5	-8.2	1.0

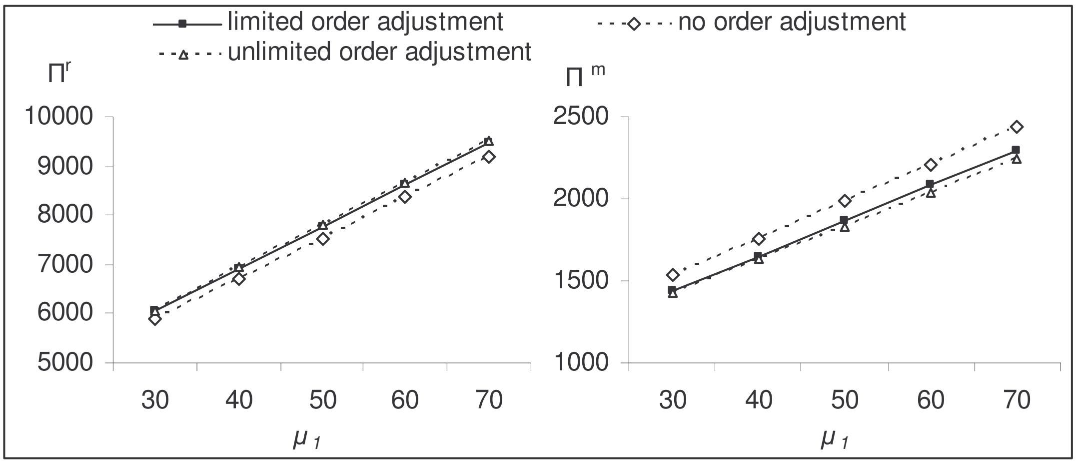

Figure 10 Effects of μ_1 on the expected profits

- The manufacturer increases q_{11}^{m} in parallel to μ_1 and q_{12}^{m} decreases as μ_1 increases. The difference between q_{12}^{r} and q_{12}^{m} increases in μ_1 as well.

- Increasing aggregate order quantity of the retailer due to μ_1 increases the expected profit of the retailer, the manufacturer and the supply chain.

Mean Demand of Product 2:

We now consider the effects of mean demand of product 2 keeping the coefficient of variation constant as $cv_1=cv_2=1/5$. Summary of the results obtained are given in Table 10 and Table 11, respectively. The expected profits of the retailer and the manufacturer for different levels of mean demand are presented in Figure 11, respectively.

Table 10 Summary of the results for different levels of μ_2

	μ_2	q_{11}^{r}	q_{12}^{r}	Q^r	$\prod^r$	q_{11}^{m}	q_{12}^{m}	$\prod^m$	$\prod^s$
No order adjustment	*30*	56.7	34.0	90.8	6159.8	-	-	1645.6	7805.4
	40	56.7	45.4	102.1	6846.4	-	-	1815.8	8662.2
	50	56.7	56.7	113.5	7533.0	-	-	1986.1	9519.1
	60	56.7	68.1	124.8	8219.7	-	-	2156.3	10376.0
	70	56.7	79.4	136.2	8906.3	-	-	2326.5	11232.8
α =0.10	*30*	55.0	33.0	88.0	6345.7	46.2	28.3	1547.1	7892.8
	40	54.8	44.1	98.9	7050.7	44.9	39.2	1704.0	8754.7
	50	54.5	55.3	109.8	7757.6	43.5	50.2	1863.8	9621.4
	60	54.3	66.6	120.9	8478.5	42.2	61.2	2024.4	10502.9
	70	54.1	77.9	131.9	9164.7	40.9	72.3	2185.9	11350.6
Unlimited	*30*	-	-	87.9	6378.0	40.3	28.2	1508.0	7886.0
	40	-	-	98.6	7089.0	40.3	39.0	1669.6	8758.6
	50	-	-	109.5	7795.8	40.3	49.9	1833.1	9628.9
	60	-	-	120.5	8498.8	40.3	60.9	1998.1	10496.8
	70	-	-	131.6	9202.6	40.3	71.9	2164.1	11366.7

- Similar conditions are observed in this experiment compared to the experiment of μ_1 but the behaviors of product 1 and 2 replace each other. Additionally, q_{11}^{r} in the experiment of μ_1 is less than q_{12}^{r} in the experiment of μ_2 for all μ values since the retailer chooses product 2 to keep in excess at the end of period.

Table 11 % Improvement in profits due to order adjustment: effects of μ_2

	μ_2	$\%\prod^r$	$\%\prod^m$	$\%\prod^s$
α =0.10	*30*	3.0	-6.0	1.1
	40	3.0	-6.2	1.1
	50	3.0	-6.2	1.1
	60	3.1	-6.1	1.2
	70	2.9	-6.0	1.0
Unlimited	*30*	3.5	-8.4	1.0
	40	3.5	-8.1	1.1
	50	3.5	-7.7	1.2
	60	3.4	-7.3	1.2
	70	3.3	-7.0	1.2

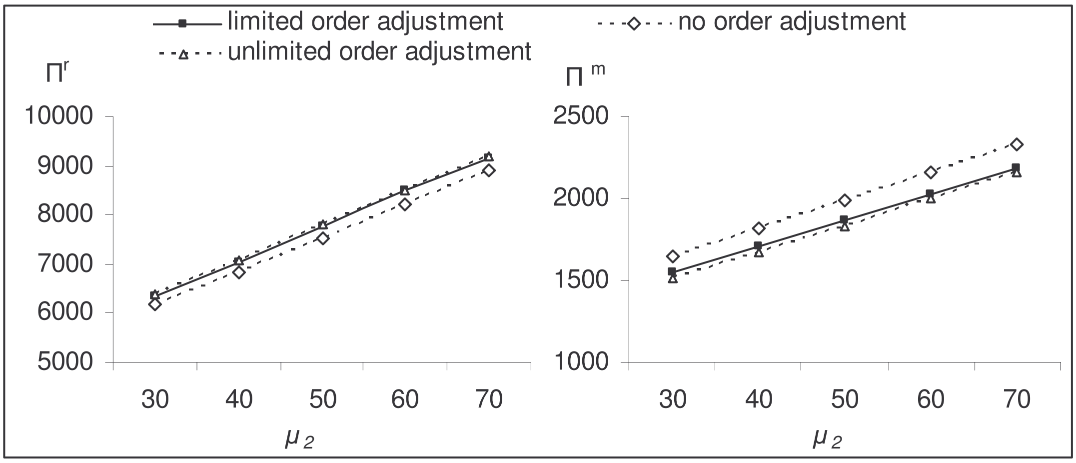

Figure 11 Effects of μ_2 on the expected profits

- Although demand parameters of the product 2 has no explicit effect on q_{11}^m and the solution of Equation (29) is constant for all μ_2; the optimal q_{11}^m is bounded by L_1 and changes according to q_{11}^r and Q^r values. The manufacturer reflects the increase in the retailer's aggregate order to q_{12}^m since the retailer will keep product 2 in excess at the end of period.

- Increasing aggregate order quantity of the retailer due to μ_2 increases the expected profit of the retailer, the manufacturer and the supply chain.

Observation 3 Different levels of mean demand with the same coefficient of variation values generate almost equal percentage improvement in profits due to order adjustment to the retailer and the supply chain.

5.1.3. Revenue and Cost Parameters

We now consider the effects of revenue and cost parameters of the retailer and the manufacturer in this and following sections.

Regular Sales Price of Product 1:

First, we analyze the impact of regular sales price of product 1, p_1. The results obtained are given in Table 12 and Table 13, respectively.

- We observe that Q^r in the limited order adjustment case is less than Q^r in the no order adjustment case for all particular p_1. At $\alpha = 0.10$, Q^r increases with p_1, however, the increase is less steep compared to the no order adjustment case. Similar arguments are also valid for q_{11}^r. On the other hand, q_{12}^r decreases as p_1 increases since an increase in q_{11}^r will provide more flexibility to the retailer in terms of product 2 as well.

Table 12 Summary of the results for different p_1

		q_{11}^r	q_{12}^r	Q^r	$\prod^r$	q_{11}^m	q_{12}^m	$\prod^m$	$\prod^s$
No order adjustment	p_1 =105	55.7	56.7	112.4	6815.3	-	-	1964.4	8779.7
	p_1 =120	56.7	56.7	113.5	7533.0	-	-	1986.1	9519.1
	p_1 =135	57.6	56.7	114.4	8261.1	-	-	2004.1	10265.2
	p_1 =150	58.4	56.7	115.2	9000.2	-	-	2019.5	11019.7
α =0.10	p_1 =105	53.7	56.0	109.6	7024.8	42.7	50.0	1858.0	8882.8
	p_1 =120	54.5	55.3	109.8	7757.6	43.5	50.2	1863.8	9621.4
	p_1 =135	55.2	54.8	110.0	8511.0	44.2	50.3	1868.7	10379.7
	p_1 =150	55.7	54.4	110.1	9262.8	44.7	50.5	1871.6	11134.4
Unlimited	p_1 =105	-	-	109.5	7045.8	40.3	49.9	1833.1	8878.9
	p_1 =120	-	-	109.5	7795.8	40.3	49.9	1833.1	9628.9
	p_1 =135	-	-	109.5	8545.8	40.3	49.9	1833.1	10378.9
	p_1 =150	-	-	109.5	9295.8	40.3	49.9	1833.1	11128.9

- The optimal q_{11}^m is bounded by L_1 and changes according to q_{11}^r and Q^r values. The manufacturer reflects the increase in the retailer's aggregate order to q_{12}^m since the retailer will keep product 2 in excess at the end of period. In the unlimited order adjustment case, p_1 does not have any significant impact on ordering policy of the retailer and manufacturer since $F_1(Q^r) \approx 1$ in our analysis.

Table 13 % Improvement in profits due to order adjustment: effects of p_1

	p_1	$\%\prod^r$	$\%\prod^m$	$\%\prod^s$
α =0.10	105	3.1	-5.4	1.2
	120	3.0	-6.2	1.1
	135	3.0	-6.8	1.1
	150	2.9	-7.3	1.0
Unlimited	105	3.4	-6.7	1.1
	120	3.5	-7.7	1.2
	135	3.4	-8.5	1.1
	150	3.3	-9.2	1.0

- Increasing p_1 increases the expected profit of the retailer; however, percentage improvement compared to the no order adjustment case slightly decreases when p_1 gets higher. Furthermore, the expected profit of the manufacturer increases with the increased order quantities of the retailer, however; the percentage loss due to order adjustment increases as well.

<u>Regular Sales Price of Product 2:</u>

We now analyze the impact of regular sales price of product 2, p_2. The results obtained are given in Table 14 and Table 15, respectively.

Table 14 Summary of the results for different p_2

		q_{11}^r	q_{12}^r	Q^r	$\prod^r$	q_{11}^m	q_{12}^m	$\prod^m$	$\prod^s$
No order adjustment	p_2 =70	56.7	53.7	110.4	6096.2	-	-	1939.8	8036.0
	p_2 =85	56.7	55.4	112.2	6813.8	-	-	1966.1	8779.9
	p_2 =100	56.7	56.7	113.5	7533.0	-	-	1986.1	9519.1
	p_2 =115	56.7	57.8	114.6	8268.7	-	-	2002.1	10270.8
α =0.10	p_2 =70	54.7	51.4	106.1	6337.2	44.1	46.4	1809.4	8146.7
	p_2 =85	54.6	53.6	108.2	7046.3	43.8	48.5	1839.4	8885.8
	p_2 =100	54.5	55.3	109.8	7757.6	43.5	50.2	1863.8	9621.4
	p_2 =115	54.4	56.8	111.2	8487.1	43.3	51.5	1882.2	10369.3
Unlimited	p_2 =70	-	-	105.2	6378.0	40.3	45.5	1767.7	8145.7
	p_2 =85	-	-	107.7	7085.1	40.3	48.0	1804.9	8890.0
	p_2 =100	-	-	109.5	7795.8	40.3	49.9	1833.1	9628.9
	p_2 =115	-	-	111.0	8515.7	40.3	51.4	1855.7	10371.4

- We observe that Q^r in the limited order adjustment case is less than Q^r in the no order adjustment case for all particular p_2 values. At $\alpha = 0.10$, Q^r increases with p_2, and the increase is more steep compared to the no order adjustment case. Similar arguments are also valid for q^r_{12}. On the other hand, q^r_{11} slightly decreases as p_2 increases since an increase in q^r_{12} will provide more flexibility to the retailer in terms of product 1 as well.

Table 15 % Improvement in profits due to order adjustment: effects of p_2

	p_2	$\%\prod^r$	$\%\prod^m$	$\%\prod^s$
α =0.10	70	4.0	-6.7	1.4
	85	3.4	-6.4	1.2
	100	3.0	-6.2	1.1
	115	2.6	-6.0	1.0
Unlimited	70	4.6	-8.9	1.4
	85	4.0	-8.2	1.3
	100	3.5	-7.7	1.2
	115	3.0	-7.3	1.0

- The optimal q^m_{11} is bounded by L_1 and changes according to q^r_{11} and Q^r values for all p_2 values. The manufacturer reflects the increase in the retailer's aggregate order to q^m_{12}.

- Increasing p_2 increases the expected profit of the retailer; however, the percentage improvement compared to the no order adjustment case decreases when p_2 gets higher. The expected profit of the manufacturer increases with the increased order quantities of the retailer. Furthermore, the percentage loss due to order adjustment decreases in when p_2.

Observation 4 The benefits of order adjustment flexibility for the retailer decrease as the regular sales price of product 1 or 2 , p_1 or p_2, increases. The decrease in p_2 is steeper than the decrease in p_1.

Discounted Sales Price of Product 1:

We now focus on discounted sales prices of product 1 and product 2, consecutively. The results obtained for different discounted sales prices of product 1 are given in Table 16 and Table 17, respectively.

- We observe that Q^r in the limited order adjustment case is less than Q^r in the no order adjustment case for all particular s_1 values. At $\alpha = 0.10$, Q^r slightly increases in s_1, however, the increase is less steep compared to the no order adjustment case. Also q^r_{11} increases as s_1 increases. On the other hand, q^r_{12} decreases as s_1 increases although it is constant in the no order adjustment case since an increase in q^r_{11} will provide more flexibility to the retailer in terms of product 2 as well. The aggregate order of the retailer is constant in the unlimited case since s_1 does not affect the ordering policy of retailer as observed in Equation (7). This is reasonable since the order quantity of product 1 will never exceed its demand realization, as selling product 2 at a discounted price is more profitable.

Table 16 Summary of the results for different s_1

		q^r_{11}	q^r_{12}	Q^r	$\prod^r$	q^m_{11}	q^m_{12}	$\prod^m$	$\prod^s$
No order adjustment	$s_1 = 0$	56.7	56.7	113.5	7533.0	-	-	1986.1	9519.1
	$s_1 = 1$	56.9	56.7	113.7	7541.6	-	-	1990.1	9531.7
	$s_1 = 2$	57.2	56.7	113.9	7559.4	-	-	1994.2	9553.6
	$s_1 = 3$	57.4	56.7	114.1	7565.5	-	-	1998.4	9563.9
	$s_1 = 4$	57.6	56.7	114.3	7570.4	-	-	2002.8	9573.2
	$s_1 = 5$	57.8	56.7	114.6	7583.9	-	-	2007.4	9591.3
$\alpha = 0.10$	$s_1 = 0$	54.5	55.3	109.8	7757.6	43.5	50.2	1863.8	9621.4
	$s_1 = 1$	54.7	55.1	109.9	7763.1	43.8	50.2	1864.7	9627.8
	$s_1 = 2$	55.0	54.9	109.9	7767.5	44.0	50.2	1865.6	9633.1
	$s_1 = 3$	55.2	54.7	109.9	7771.3	44.2	50.3	1866.7	9638.0
	$s_1 = 4$	55.5	54.5	110.0	7774.3	44.5	50.3	1867.9	9642.2
	$s_1 = 5$	55.7	54.3	110.0	7776.6	44.7	50.4	1870.6	9647.2
Unlimited	$s_1 = 0$	-	-	109.5	7795.8	40.3	49.9	1833.1	9628.9
	$s_1 = 1$	-	-	109.5	7795.8	40.3	49.9	1833.1	9628.9
	$s_1 = 2$	-	-	109.5	7795.8	40.3	49.9	1833.1	9628.9
	$s_1 = 3$	-	-	109.5	7795.8	40.3	49.9	1833.1	9628.9
	$s_1 = 4$	-	-	109.5	7795.8	40.3	49.9	1833.1	9628.9
	$s_1 = 5$	-	-	109.5	7795.8	40.3	49.9	1833.1	9628.9

- The manufacturer increases q^m_{11} in parallel to q^r_{11} since the optimal q^m_{11} is determined by L_1. Although the retailer reduces q^r_{12}, the manufacturer slightly increases q^m_{12} in order to reflect the increase in the retailer's aggregate order since the retailer will keep product 2 in excess at the end of period. Additionally, both q_{11}^{m} and q_{12}^{m} are constant in the unlimited order adjustment case since s_1 does not affect Q^r.

Table 17 % Improvement in profits due to order adjustment: effects of s_1

	s_1	$\%\prod^{r}$	$\%\prod^{m}$	$\%\prod^{s}$
α =0.10	0	3.0	-6.2	1.1
	1	2.9	-6.3	1.0
	2	2.8	-6.4	0.8
	3	2.7	-6.6	0.8
	4	2.7	-6.7	0.7
	5	2.5	-6.8	0.6
Unlimited	0	3.5	-7.7	1.2
	1	3.4	-7.9	1.0
	2	3.1	-8.1	0.8
	3	3.0	-8.3	0.7
	4	3.0	-8.5	0.6
	5	2.8	-8.7	0.4

- Increasing discounted sales price of product 1 increases the expected profit of the retailer. The percentage improvement compared to the no order adjustment case decreases as s_1 increases. Furthermore, the expected profit of the manufacturer increases with the increased order quantities of the retailer. However, the percentage loss due to order flexibility increases in s_1.

Discounted Sales Price of Product 2:

We now consider s_2 and the results obtained are given in Table 18 and Table 19, respectively.

- We observe that Q^r in the limited order adjustment case is less than in the no order adjustment case for all particular s_2 values. At $\alpha = 0.10$, Q^r increases in s_2 and the increase is steeper compared to the no order adjustment case. Similar arguments are also valid for q_{12}^{r}. Although it is constant In the no order adjustment case, q_{11}^{r} slightly decreases as s_2

increases since an increase in q^r_{12} will provide more flexibility to the retailer in terms of product 1 as well.

Table 18 Summary of the results for different s_2

		q^r_{11}	q^r_{12}	Q^r	$\prod^r$	q^m_{11}	q^m_{12}	$\prod^m$	$\prod^s$
No order adjustment	$s_2=0$	56.7	56.7	113.5	7533.0	-	-	1986.1	9519.1
	$s_2=1$	56.7	57.0	113.7	7543.2	-	-	1989.7	9532.9
	$s_2=2$	56.7	57.2	114.0	7552.0	-	-	1993.4	9545.4
	$s_2=3$	56.7	57.5	114.2	7559.3	-	-	1997.3	9556.6
	$s_2=4$	56.7	57.8	114.5	7573.3	-	-	2001.4	9574.6
	$s_2=5$	56.7	58.0	114.8	7577.4	-	-	2005.6	9583.0
$\alpha=0.10$	$s_2=0$	54.5	55.3	109.8	7757.6	43.5	50.2	1863.8	9621.4
	$s_2=1$	54.4	55.7	110.2	7775.9	43.4	50.5	1867.6	9643.5
	$s_2=2$	54.3	56.1	110.5	7792.9	43.3	50.8	1871.6	9664.5
	$s_2=3$	54.3	56.5	110.8	7799.9	43.2	51.1	1877.3	9677.2
	$s_2=4$	54.2	57.0	111.2	7818.0	43.1	51.5	1881.5	9699.6
	$s_2=5$	54.1	57.4	111.5	7820.1	42.9	51.9	1887.6	9707.8
Unlimited	$s_2=0$	-	-	109.5	7795.8	40.3	49.9	1833.1	9628.9
	$s_2=1$	-	-	109.9	7808.9	40.3	50.2	1838.2	9647.1
	$s_2=2$	-	-	110.2	7820.7	40.3	50.6	1843.5	9664.2
	$s_2=3$	-	-	110.6	7831.2	40.3	50.9	1849.0	9680.2
	$s_2=4$	-	-	111.0	7845.9	40.3	51.3	1854.7	9700.7
	$s_2=5$	-	-	111.4	7859.1	40.3	51.7	1860.7	9719.8

Table 19 % Improvement in profits due to order adjustment: effects of s_2

	s_2	$\%\prod^r$	$\%\prod^m$	$\%\prod^s$
$\alpha=0.10$	*0*	3.0	-6.2	1.1
	1	3.1	-6.1	1.2
	2	3.2	-6.1	1.2
	3	3.2	-6.0	1.3
	4	3.2	-6.0	1.3
	5	3.2	-5.9	1.3
Unlimited	*0*	3.5	-7.7	1.2
	1	3.5	-7.6	1.2
	2	3.6	-7.5	1.2
	3	3.6	-7.4	1.3
	4	3.6	-7.3	1.3
	5	3.7	-7.2	1.4

- The manufacturer decreases q^m_{11} in parallel to q^r_{11} since the optimal q^m_{11} is determined by L_1. The manufacturer reflects the increase in the retailer's aggregate order to q_{12}^{m} since the retailer will keep product 2 in excess at the end of period.

- Increasing discounted sales price of product 2 increases the expected profit of the retailer and the percentage improvement compared to the no order adjustment case slightly increases as s_2 increases. Furthermore, the expected profit of the manufacturer increases with the increased order quantities of the retailer.

Observation 5 The benefits of order adjustment flexibility for the retailer decrease as the discounted sales price of product 1, s_1, increases and slightly increase as s_2 increases. Additionally, when the order adjustment flexibility is large enough, s_1 does not affect the profitability of the supply chain and individual parties.

Expedited Delivery Cost:

The expedited delivery cost does not have any impact on the retailer's ordering policy; however, it directly affects the decisions of the manufacturer. Summary of the results obtained in the numerical analysis are given in Table 20 and Table 21, respectively.

- When $d_2 \leq 6$, the optimal q_{11}^{m} is determined by L_1 and it is constant since the order of the retailer does not change. Additionally, q_{12}^{m} increases in d_2. When $d_2 \geq 7$, q_{11}^{m} and q_{12}^{m} increase as d_2 increases. Although, the manufacturer has equal q_{11}^{m} and q_{12}^{m} in the no order adjustment case, it reduces q_{11}^{m} more than q_{12}^{m} since the retailer chooses product 2 to have in excess at the end of the period.

- The expected profit of the manufacturer and the supply chain decrease as d_2 increases.

Table 20 Summary of the results for different d_2

		q'^{r}_{11}	q'^{r}_{12}	Q^r	$\prod^r$	q^m_{11}	q^m_{12}	$\prod^m$	$\prod^s$
No order adjustment	$d_2=4$	56.7	56.7	113.5	7533.0	-	-	1986.1	9519.1
	$d_2=5$	56.7	56.7	113.5	7533.0	-	-	1986.1	9519.1
	$d_2=6$	56.7	56.7	113.5	7533.0	-	-	1986.1	9519.1
	$d_2=7$	56.7	56.7	113.5	7533.0	-	-	1986.1	9519.1
	$d_2=8$	56.7	56.7	113.5	7533.0	-	-	1986.1	9519.1
	$d_2=9$	56.7	56.7	113.5	7533.0	-	-	1986.1	9519.1
	$d_2=10$	56.7	56.7	113.5	7533.0	-	-	1986.1	9519.1
$\alpha=0.10$	$d_2=4$	54.5	55.3	109.8	7757.6	43.5	46.5	1882.0	9639.5
	$d_2=5$	54.5	55.3	109.8	7757.6	43.5	50.2	1863.8	9621.4
	$d_2=6$	54.5	55.3	109.8	7757.6	43.5	52.5	1848.0	9605.6
	$d_2=7$	54.5	55.3	109.8	7757.6	44.3	54.2	1834.0	9591.5
	$d_2=8$	54.5	55.3	109.8	7757.6	45.7	55.5	1821.9	9579.5
	$d_2=9$	54.5	55.3	109.8	7757.6	46.8	56.7	1811.4	9568.9
	$d_2=10$	54.5	55.3	109.8	7757.6	47.8	57.6	1802.4	9560.0
Unlimited	$d_2=4$	-	-	109.5	7795.8	36.6	46.2	1857.1	9652.9
	$d_2=5$	-	-	109.5	7795.8	40.3	49.9	1833.1	9628.9
	$d_2=6$	-	-	109.5	7795.8	42.6	52.2	1814.0	9609.8
	$d_2=7$	-	-	109.5	7795.8	44.3	53.9	1797.9	9593.7
	$d_2=8$	-	-	109.5	7795.8	45.7	55.2	1784.0	9579.8
	$d_2=9$	-	-	109.5	7795.8	46.8	56.4	1771.7	9567.6
	$d_2=10$	-	-	109.5	7795.8	47.8	57.3	1760.8	9556.6

Table 21 % Improvement in profits due to order adjustment: effects of d_2

		$\%\prod^m$	$\%\prod^s$
$\alpha=0.10$	$d_2=4$	-5.2	1.3
	$d_2=5$	-6.2	1.1
	$d_2=6$	-7.0	0.9
	$d_2=7$	-7.7	0.8
	$d_2=8$	-8.3	0.6
	$d_2=9$	-8.8	0.5
	$d_2=10$	-9.2	0.4
Unlimited	$d_2=4$	-6.5	1.4
	$d_2=5$	-7.7	1.2
	$d_2=6$	-8.7	1.0
	$d_2=7$	-9.5	0.8
	$d_2=8$	-10.2	0.6
	$d_2=9$	-10.8	0.5
	$d_2=10$	-11.3	0.4

Observation 6 The loss of the manufacturer due to order adjustment flexibility increases as the expedited delivery cost, d_2, increases.

Procurement Costs of Product 1 and Product 2:

The procurement costs of product 1 and 2 only affect the decisions of the manufacturer. The results obtained are given in Table 22 and Table 23, respectively.

Table 22 Summary of the results for different m_1=m_2

		q^r_{11}	q^r_{12}	Q^r	$\prod^r$	q^m_{11}	q^m_{12}	$\prod^m$	$\prod^s$
No order adjustment	m_1=m_2=3	56.7	56.7	113.5	7533.0	-	-	2440.0	9973.1
	m_1=m_2=5	56.7	56.7	113.5	7533.0	-	-	2213.1	9746.1
	m_1=m_2=7	56.7	56.7	113.5	7533.0	-	-	1986.1	9519.1
	m_1=m_2=9	56.7	56.7	113.5	7533.0	-	-	1759.1	9292.1
α =0.10	m_1=m_2=3	54.5	55.3	109.8	7757.6	43.5	53.1	2306.7	10064.3
	m_1=m_2=5	54.5	55.3	109.8	7757.6	43.5	51.4	2084.9	9842.5
	m_1=m_2=7	54.5	55.3	109.8	7757.6	43.5	50.2	1863.8	9621.4
	m_1=m_2=9	54.5	55.3	109.8	7757.6	43.5	49.2	1643.0	9400.6
Unlimited	m_1=m_2=3	-	-	109.5	7795.8	43.3	52.8	2280.4	10076.2
	m_1=m_2=5	-	-	109.5	7795.8	41.6	51.1	2056.2	9852.0
	m_1=m_2=7	-	-	109.5	7795.8	40.3	49.9	1833.1	9628.9
	m_1=m_2=9	-	-	109.5	7795.8	39.3	48.9	1610.8	9406.6

Table 23 % Improvement in profits due to order adjustment: effects of m_1=m_2

		$\%\prod^m$	$\%\prod^s$
α =0.10	m_1=m_2=3	-5.5	0.9
	m_1=m_2=5	-5.8	1.0
	m_1=m_2=7	-6.2	1.1
	m_1=m_2=9	-6.6	1.2
Unlimited	m_1=m_2=3	-6.5	1.0
	m_1=m_2=5	-7.1	1.1
	m_1=m_2=7	-7.7	1.2
	m_1=m_2=9	-8.4	1.2

- The optimal q^m_{11} is determined by L_1 and it is constant since the order of the retailer does not change. Additionally, q^m_{12} decreases as the procurement costs increase. Although, the manufacturer has equal q^m_{11} and q^m_{12} in the no order adjustment case, the manufacturer reduces q^m_{11} more than q^m_{12} since the retailer chooses product 2 to have in excess at the end of the period.

- The expected profit of the manufacturer and the supply chain decrease as m_1=m_2 increase.

Observation 7 The loss of the manufacturer due to order adjustment flexibility increases as the procurement costs, m_1=m_2, increase. Additionally, the profit of supply chain decreases as m_1=m_2 increase, however; the percentage improvement increases in m_1=m_2.

5.2. Sell Product I at the Discounted Price

Recall that the setting for this case is $p_1 - w_1 > p_2 - w_2$ and $w_1 - s_1 < w_2 - s_2$. In the following subsections, we analyze the effect of order adjustment flexibility, parameters related with demand and then cost and revenue parameters. The baseline parameter setting for the “Sell Product I at the Discounted Price” is given in Table 1.In order to satisfy the assumptions, s_1 is set as *10*.

5.2.1. Order Adjustment Flexibility

The model is solved for the baseline settings from $\alpha = 0.05$ to $\alpha = 0.35$, the no order adjustment case and the unlimited order adjustment case. Summary of the results obtained are given in Table 24 and Table 25. The profits of the manufacturer and retailer for different levels of order adjustment flexibility are presented in Figure 12. Moreover, the order quantities of the retailer and the manufacturer are presented in Figure 13.

- Similar arguments are valid for the order quantities of the retailer (Q^r, q_{11}^r and q_{12}^r) as in Section 5.1.1. However, the behaviors of product 1 and product 2 replace each other in the “sell product 2 at a discounted price” case and “sell product 1 at a discounted price” case. This is reasonable since the retailer chooses product 2 and product 1 in excess at the end of the period in the first and second cases, respectively.

- Similar arguments are valid for the order quantities of the manufacturer (q_{11}^m, q_{12}^m) as in Section 5.1.1 but the behaviors of product 1 and product 2 replaces each other as in the retailer's order quantities. Similarly, q_{11}^m and q_{12}^m decrease significantly relative to the no order adjustment case. Additionally, $q_{11}^m > q_{12}^m$ although their distributions are the same, since that the retailer chooses to sell product 1 at a discounted price. The optimal q_{12}^m is determined by L_2 and Equation (30) for $\alpha \leq 0.10$ and $\alpha \geq 0.15$, respectively as mentioned in Proposition 10.

Table 24 Summary of the results for different levels of α

	q_{11}^r	q_{12}^r	Q^r	$\prod^r$	q_{11}^m	q_{12}^m	$\prod^m$	$\prod^s$
No order adjustment	59.1	56.7	115.8	7627.7	-	-	2032.9	9660.5
α =0.05	57.6	55.0	112.7	7779.8	55.4	49.4	1970.2	9750.0
α =0.10	57.7	53.9	111.6	7821.9	54.7	42.8	1947.0	9768.9
α =0.15	58.7	52.7	111.4	7848.7	54.5	39.8	1936.5	9785.2
α =0.20	60.2	51.2	111.4	7855.0	54.5	39.8	1933.7	9788.7
α =0.25	61.9	49.4	111.4	7857.8	54.5	39.8	1933.1	9790.8
α =0.30	65.4	46.0	111.4	7858.2	54.5	39.8	1933.1	9791.3
α =0.35	70.8	40.6	111.4	7858.5	54.5	39.8	1933.1	9791.6
Unlimited	-	-	111.4	7859.1	54.5	39.8	1933.1	9792.2

Table 25 % Improvement in profits due to order adjustment: effects of α

	$\%\prod^r$	$\%\prod^m$	$\%\prod^s$
α =0.05	2.0	-3.1	0.9
α =0.10	2.5	-4.2	1.1
α =0.15	2.9	-4.7	1.3
α =0.20	3.0	-4.9	1.3
α =0.25	3.0	-4.9	1.3
α =0.30	3.0	-4.9	1.4
α =0.35	3.0	-4.9	1.4
Unlimited	3.0	-4.9	1.4

- The expected profit of the retailer, the manufacturer and the supply chain have similar behaviors under different levels of order adjustment flexibility, α, as in Section 5.1.1. The percentage improvement in the profit of the retailer in the "sell product 1 at a discounted price" case is less then the one in the "sell product 2 at a discounted price" case. In addition, the percentage loss in the profit of the manufacturer in this case is less then the one in

Section 5.1.1. The percentage improvement in the profit of the supply chain is better in the “sell product 1 at a discounted price” case.

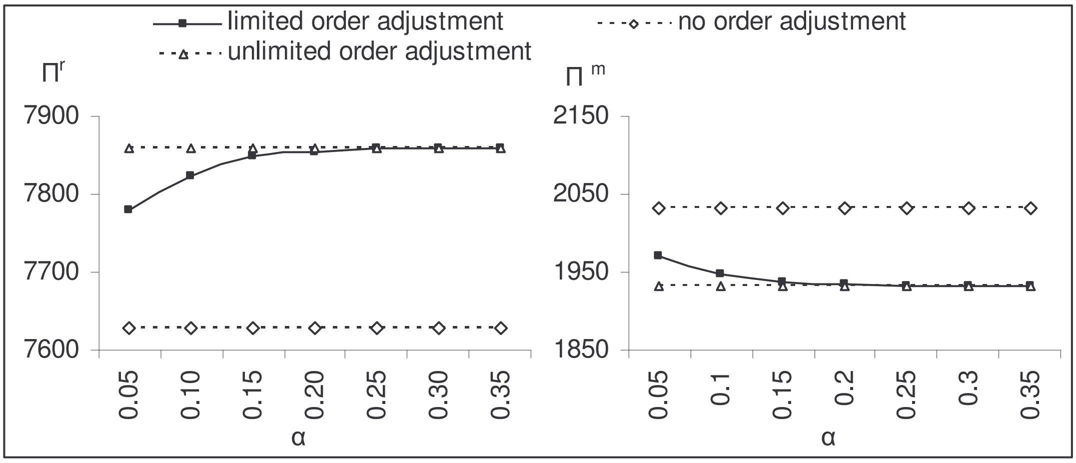

Figure 12 Effects of α on the expected profits

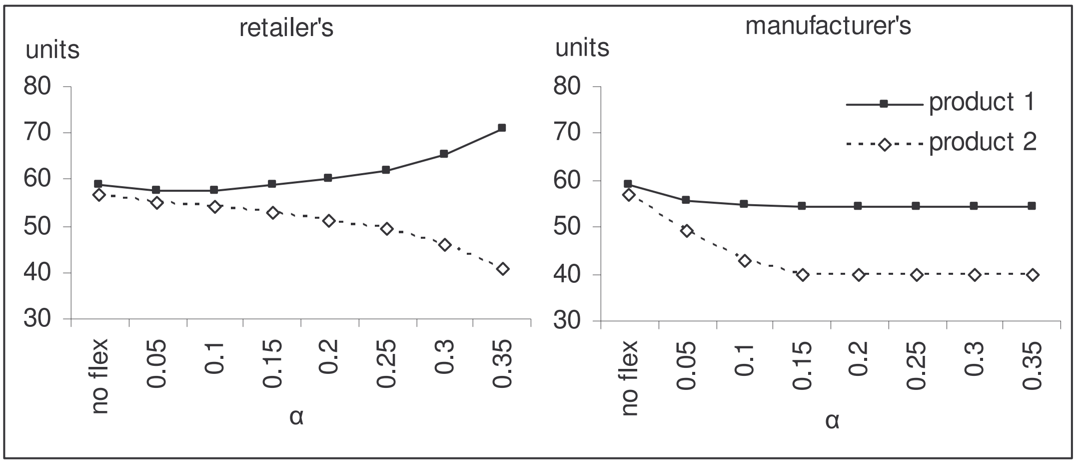

Figure 13 Effects of α on the order quantities

5.2.2. Demand Parameters

In this section, we conduct experiments by using sample data sets representing different demand parameters of each product as in Section 5.1.2.

<u>Coefficient of Variation of Product 1:</u>

We will utilize the same set of settings to analyze the effect of coefficient of variation values of product 1, cv_1, which are *1/10, 1/7, 1/5 and 1/3* with equal mean, $\mu_1 = 50$. Summary of the results obtained and percentage improvements are given in

Appendix G (Table G1 and Table G2). The profits of the retailer and the manufacturer for different levels of cv_1 are presented in Figure 14, respectively.

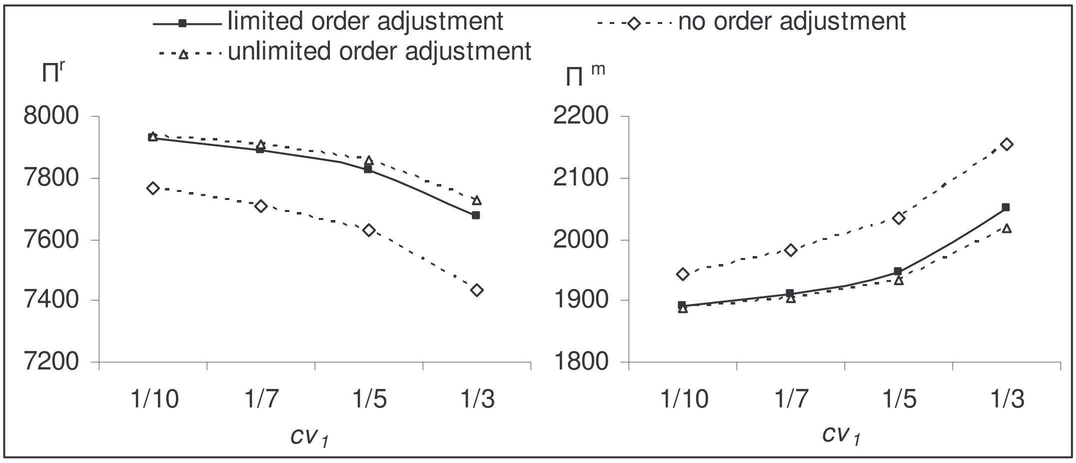

Figure 14 Effects of cv_1 on the expected profits

- We observe similar behaviors for Q^r and q^r_{11} with cv_1 in the limited order adjustment case as in Section 5.1.2. Although, demand parameters of product 2 are not modified, q^r_{12} slightly changes in cv_1.

- Additionally, q^m_{11} increases in cv_1 since the retailer increases q^r_{11} and will choose product 1 in excess at the end of period. Now, the optimal q^m_{12} is determined by L_2, q^m_{12} decreases according to the changes in Q^r and q^r_{12} values.

- The expected profits of the retailer, the manufacturer and the supply chain have similar behaviors under different levels of cv_1 as in Section 5.1.2. Although the percentage loss in the profit of the manufacturer is more, the order adjustment is more valuable to the retailer and the supply chain in the "sell product 2 at a discounted price" case.

Coefficient of Variation of Product 2:

We now consider the effects of uncertainty in the demand of product 2, cv_2. Summary of the results obtained are given in Appendix G (Table G3 and Table G4).

The profits of the retailer and the manufacturer for different levels of cv_2 are presented in Figure 15, respectively.

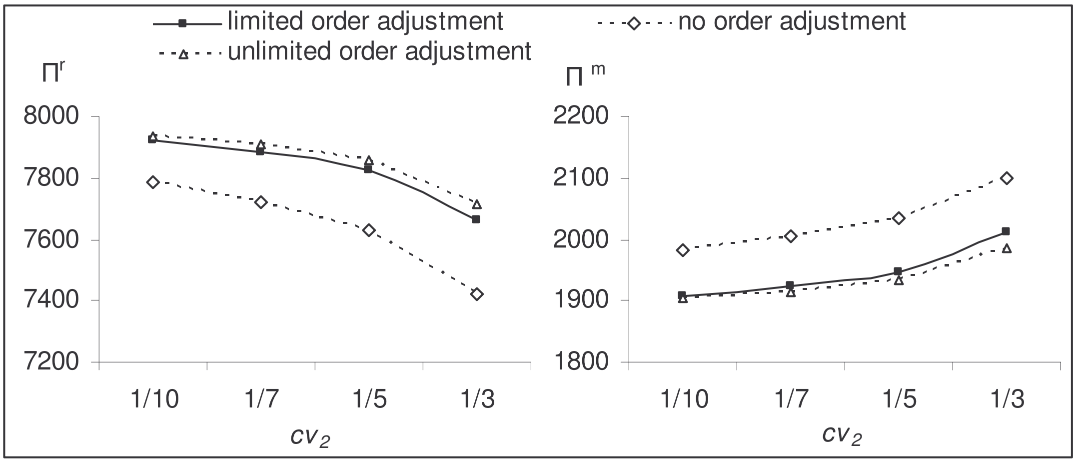

Figure 15 Effects of cv_2 on the expected profits

- We observe similar behaviors for Q^r and q^r_{12} with cv_2 in the limited order adjustment case as in Section 5.1.2. Although, demand parameters of product 1 are not modified, q^r_{11} slightly decreases in cv_2. Intuitively, an increase in q^r_{12} will provide more flexibility to the retailer in terms of product 1 as well. Hence, it is reasonable to have a decreasing q^r_{11} when q^r_{12} increases.

- The initial order quantity of the manufacturer for each product decreases as cv_2 increases if $cv_2 \leq 1/7$. Moreover, the reduction in q^m_{12} is higher than the reduction in q^m_{11} since the retailer will keep product 1 in excess at the end of period. When $cv_2 \geq 1/5$, the optimal q^m_{12} is bounded by L_2 and changes according to q^r_{12} and Q^r values. In the unlimited order adjustment case, q^m_{12} decreases as cv_2 increases.

- The expected profits of the retailer, the manufacturer and the supply chain have similar behaviors under different levels of cv_2 as in Section 5.1.2. Not only the percentage improvement in the profit of the retailer but also the percentage loss of the manufacturer is more in the "sell product 2 at a discounted price" case.

Mean Demand of Product 1:

We now consider the effects of mean demand of product 1 keeping the coefficient of variation constant at $cv_1=cv_2=1/5$. Summary of the results obtained are given in Appendix G (Table G5 and Table G6). The expected profit of the retailer and the manufacturer for different levels of mean demand are presented in Figure 16, respectively.

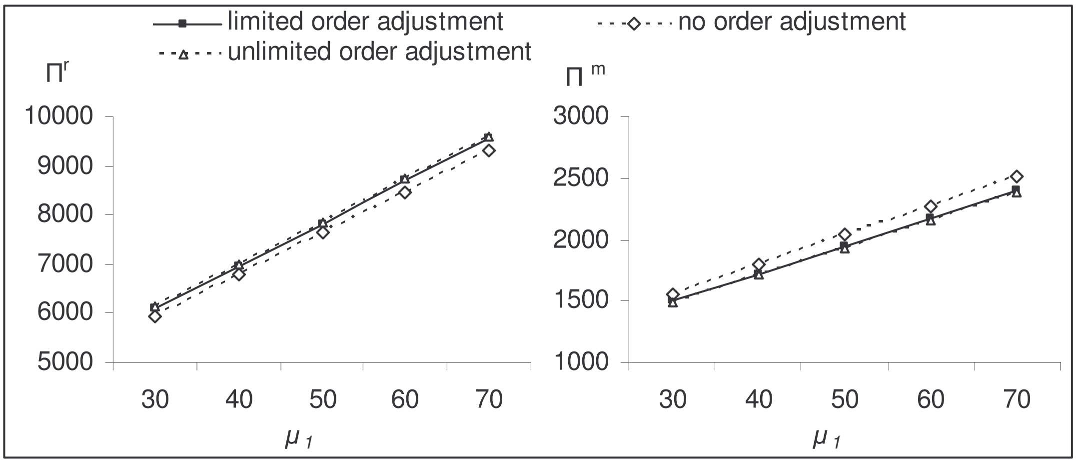

Figure 16 Effects of μ_1 on the expected profits

- We observe similar behaviors for the order of the retailer (q^r_{11} and q^r_{12}) and the order of the manufacturer (q^m_{11} and q^m_{12}) with μ_1 in the limited order adjustment case as in Section 5.1.2.
- The expected profits of the retailer, the manufacturer and the supply chain have similar behaviors under different levels of cv_2 as in Section 5.1.2. Not only the percentage improvement in the profit of the retailer but also the percentage loss of the manufacturer is more in the "sell product 2 at a discounted price" case.

Mean Demand of Product 2:

We analyze the effects of mean demand of product 2 keeping the coefficient of variation constant as $cv_1=cv_2=1/5$. Summary of the results obtained are given in

Appendix G (Table G7 and Table G8). The expected profit of the retailer and the manufacturer for different levels of mean demand are presented in Figure 17, respectively.

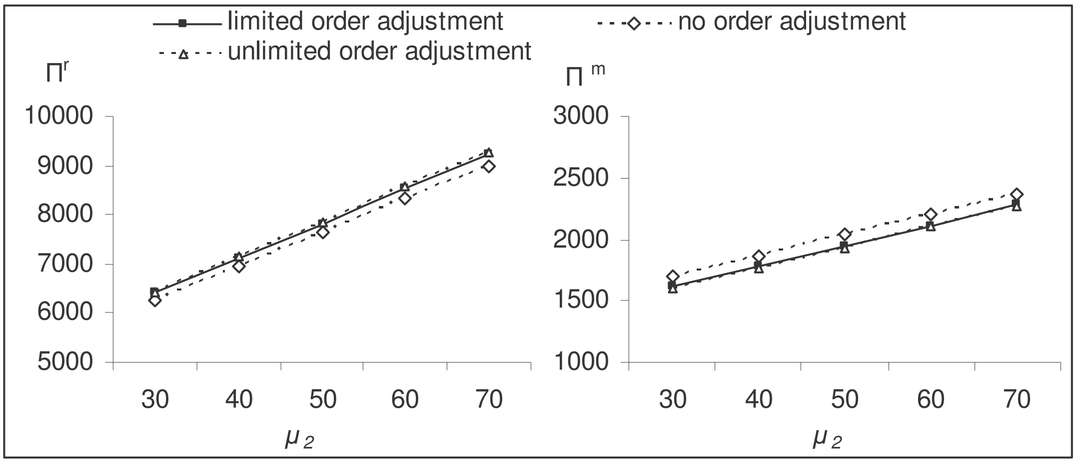

Figure 17 Effects of μ_2 on the expected profits

- We observe similar condition for the order of the retailer (q^r_{11} and q^r_{12}) and the order of the manufacturer (q^m_{11} and q^m_{12}) with μ_1 in the limited order adjustment case as in Section 5.1.2. The decrease in q^m_{11} in this case is less than the decrease in q^m_{11} in "sell product 2 at a discounted price" case since the retailer chooses product 1 to keep in excess at the end of the period now.

- The expected profits of the retailer, the manufacturer and the supply chain have similar behaviors under different levels of cv_2 as in Section 5.1.2. Not only the percentage improvement in the profit of the retailer but also the percentage loss of the manufacturer is more in the "sell product 2 at a discounted price" case.

5.2.3. Revenue and Cost Parameters

We now consider the effects of revenue and cost parameters of the retailer and the manufacturer in this and following sections.

Regular Sales Price of Product 1:

First, we analyze the impact of regular sales price of product 1, p_1. The results obtained are given in Appendix G (Table G9 and Table G10).

- We observe similar condition for the order of the retailer (q^r_{11} and q^r_{12}) with p_1 as in Section 5.1.3.
- The manufacturer reflects the increase in the order of the retailer to q^m_{11} since the retailer will keep product 1 in excess at the end of the period. The optimal q^m_{12} is bounded by L_2 in this case and changes according to q^r_{12} and Q^r values. In the unlimited order adjustment case, p_1 does not have any significant impact on ordering policy of the retailer and manufacturer since $F_1(Q^r) \approx 1$ in our analysis.
- The expected profit of the retailer, the manufacturer and the supply chain has similar behaviors under different levels of cv_2 as in Section 5.1.3.

Regular Sales Price of Product 2:

We now analyze the impact of regular sales price of product 2, p_2. The results obtained are given in Appendix G (Table G11 and Table G12).

- We observe similar behaviors for Q^r and q^r_{12} with p_2 in the limited order adjustment case as in Section 5.1.3. Although, q^r_{11} slightly decreases as p_2 increases in the previous case since an increase in q^r_{12} provides more flexibility in terms of product 1; q^r_{11} slightly increases in p_2 since the retailer will now keep product 1 in excess at the end of the period.
- The optimal q^m_{12} is bounded by L_2 and increases according to q^r_{12} and Q^r values for p_2 values. In contrary to Section 5.1.3, the manufacturer

increases q_{11}^{m} since the retailer will now keep product 1 in excess at the end of the period.

- The expected profits of the retailer, the manufacturer and the supply chain have similar behaviors under different levels of p_2 as in Section 5.1.3.

<u>Discounted Sales Price of Product 1:</u>

We now focus on discounted sales prices of product 1 and product 2, consecutively. The results obtained for s_1 are given in Appendix G (Table G13 and Table G14).

- We observe similar condition for the order of the retailer (q_{11}^{r} and q_{12}^{r}) with s_1 in the limited order adjustment case as in Section 5.1.3. On contrary to Section 5.1.3, Q^{r} increases in the unlimited case since s_1 affects the ordering policy of retailer as observed in Equation (17) in "sell product 1 at a discounted price" case.

- The manufacturer increases q_{12}^{m} in parallel to q_{12}^{r} since the optimal q_{12}^{m} is determined by L_2. The manufacturer slightly increases q_{11}^{m} in order to reflect the increase in the retailer's aggregate order since the retailer will keep product 1 in excess at the end of period. On contrary to Section 5.1.3, both q_{11}^{m} and q_{12}^{m} increase in the unlimited order adjustment case.

- Increasing discounted sales price of product 1 increases the expected profit of the retailer. The percentage improvement compared to the no order adjustment case is nearly the same as s_1 increases. Furthermore, the expected profit of the manufacturer increases with the increased order quantities of the retailer. However, the percentage loss due to order flexibility is again nearly the same in s_1.

Discounted Sales Price of Product 2:

We now consider s_2 and the results obtained are given in Appendix G (Table G15 and Table G16).

- We observe similar condition for the order of the retailer (q^r_{11} and q^r_{12}) with s_2 in the limited order adjustment case as in Section 5.1.3. On contrary to Section 5.1.3, Q^r is constant in the unlimited case since s_2 does not affect the ordering policy of retailer as observed in Equation (17). This is reasonable since the order quantity of product 2 will never exceed its demand realization, as selling product 1 at a discounted price is more profitable.

- The manufacturer increases q^m_{12} in parallel to q^r_{12} since the optimal q^m_{12} is determined by L_2. The manufacturer slightly increases q^m_{11} in order to reflect the increase in the retailer's aggregate order since the retailer will keep product 1 in excess at the end of period. On contrary to section 5.1.3, both q^m_{11} and q^m_{12} are constant at the unlimited case since s_2 does not affect Q^r.

- The expected profits of the retailer, the manufacturer and the supply chain have similar behaviors under different levels of s_2 as in Section 5.1.3.

Expedited Delivery Cost:

Summary of the results obtained in the numerical analysis are given in Appendix G (Table G17 and Table G18).

- We observe that similar condition for the order of manufacturer as in Section 5.1.3 but the behaviors of q^m_{11} and q^m_{12} replaces each other. The optimal q^m_{12} is determined by L_2 in this case. Although, the manufacturer has equal q^m_{11} and q^m_{12} in the no order adjustment case, it reduces q^m_{12} more than q^m_{11} since the retailer chooses product 1 to have in excess at the end of period.

- The expected profits of the manufacturer and the supply chain decrease as d_2 increases as Section 5.3.1.

Procurement Costs of Product 1 and Product 2:

Summary of the results obtained in the numerical analysis are given in Appendix G (Table G19 and Table G20).

- We observe that similar conditions for the order of the manufacturer as in Section 5.1.3 but the behaviors of the q_{11}^m and q_{12}^m replaces each other. The optimal q_{12}^m is determined by L_2 and it is constant since the order of the retailer does not change. Additionally, q_{11}^m decreases as the procurement costs increase. The manufacturer reduces q_{12}^m more than q_{11}^m since the retailer chooses product 1 to have in excess at the end of the period.

- The expected profits of the manufacturer and the supply chain decrease as $m_1=m_2$ increase as Section 5.3.1.

CHAPTER 6

CONCLUSION

In this study, we analyze a decentralized supply chain with a single retailer and a single manufacturer where the retailer sells two products in a single period. The products offered by the retailer consist of families of closely related products, which differ from each other in terms of a limited number of features only. The demand of each product is uncertain. The retailer first quotes an initial order for each product based on the demand forecast. The sum of initial orders determines the total final order quantity for the selling period. Given the initial order, the manufacturer procures products by regular delivery mode. Meanwhile, the retailer collects more market demand information before committing the final order of each product. In the second decision stage, although the retailer cannot change the total quantity of the order, he can adjust the order quantity of each product based on the flexibility ratio on the contract. The manufacturer utilizes expedited delivery if required, to procure additional products. The expedited delivery provides shorter procurement lead time but its cost is higher than regular delivery.

We present an analytical model for this contract and characterize the optimal solution for the retailer and manufacturer to determine ordering quantities that maximize their expected profits, separately. In this study, we analyze three different levels of order adjustment flexibility settings: (i) No order adjustment, (ii) Unlimited order adjustment, (iii) Limited order adjustment. Note that the retailer may have to order excess inventory in order to satisfy the contract requirements. The type of product to be ordered in excess depends on the profitability of the products when sold at the end of the season. Our model considers each case explicitly.

We gain insights through computational experiments and present the impact of different parameters on the ordering decisions of the retailer and manufacturer. We observe that such a contract can improve the expected profit of the retailer and the supply chain even though the decisions are made by only considering the retailer. Moreover, we observe that the retailer's product choice to sell a discounted price affects the order quantities of the retailer and the manufacturer. Another observation is that limited amount (which depends on the demand uncertainty) of order adjustment flexibility is required for the retailer and the supply chain to achieve almost all of the benefits of order adjustment flexibility. The retailer obtains 98% of maximum profit improvement (which is in the unlimited order adjustment case) when $\alpha = 0.20$. The order adjustment flexibility is more useful when demand uncertainty is high.

The first possible extension of this study that comes into mind is to include the manufacturer in the analysis as a player. Recall that the environment in the model is a decentralized supply chain. Note that the retailer does not pay for the order adjustment flexibility that he uses. The manufacturer can be included in the model by determining a unit variable cost or a fixed cost for the use of order adjustment flexibility to maximize his expected profit. This model can be more interesting for independent companies; however, our model is more valid where the retailer and manufacturer act independently, but belong to the same company.

Another extension may be about the demand structure. In the analysis, we assume independent products; however, products that have correlated demands can also be taken into consideration. Additionally, the analysis of the impact of different kinds of demand distributions may have interesting results. Recall that we assume that the retailer will have perfect information to decide on the final orders at stage 2. As part of the extension of this study, it is possible to modify the information type received by the retailer. For example, only one product may have perfect information at the decision timing.

There may also be extensions where the problem is addressed in a multi-period setting or where the number of products may be increased to more than two. However, the analysis of the problem may be very difficult to handle.

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APPENDIX A

PROOF OF LEMMA 2

Proof: The first derivatives of $\prod_{11}^{r}(q_{11}^{r},q_{12}^{r})$ with respect to q_{11}^{r} and q_{12}^{r} are given in Equation (1) and Equation (2), respectively.

$$\begin{aligned}\frac{\partial \prod_{11}^{r}}{\partial q_{11}^{r}} &= p_1 - w_1 + (-p_1 + w_1 + p_2 - w_2)F_1(q_{11}^{r} + q_{12}^{r}) \\ &+ (1-\alpha)(s_1 - w_1 - p_2 + w_2)F_1\left[(1-\alpha)q_{11}^{r} - \alpha q_{12}^{r}\right] \\ &+ \alpha(s_2 - p_2)F_2\left[(1+\alpha)q_{12}^{r} + \alpha q_{11}^{r})\right]\ F_1\left[(1-\alpha)q_{11}^{r} - \alpha q_{12}^{r}\right] \\ &+ \int_{x_1=(1-\alpha)q_{11}^{r}-\alpha q_{12}^{r}}^{q_{11}^{r}+q_{12}^{r}} (s_2 - p_2)F_2(q_{11}^{r} + q_{12}^{r} - x_1)\ f_1(x_1)dx_1\end{aligned} \qquad (1)$$

$$\begin{aligned}\frac{\partial \prod_{11}^{r}}{\partial q_{12}^{r}} &= p_1 - w_1 + (-p_1 + w_1 + p_2 - w_2)F_1(q_{11}^{r} + q_{12}^{r}) \\ &- \alpha(s_1 - w_1 - p_2 + w_2)F_1\left[(1-\alpha)q_{11}^{r} - \alpha q_{12}^{r}\right] \\ &+ (1+\alpha)(s_2 - p_2)F_2\left[(1+\alpha)q_{12}^{r} + \alpha q_{11}^{r})\right]\ F_1\left[(1-\alpha)q_{11}^{r} - \alpha q_{12}^{r}\right] \\ &+ \int_{x_1=(1-\alpha)q_{11}^{r}-\alpha q_{12}^{r}}^{q_{11}^{r}+q_{12}^{r}} (s_2 - p_2)F_2(q_{11}^{r} + q_{12}^{r} - x_1)\ f_1(x_1)dx_1\end{aligned} \qquad (2)$$

The second derivatives of $\prod_{11}^{r}(q_{11}^{r},q_{12}^{r})$ in Equation (3), Equation (4) and Equation (5) indicate that the determinant of the hessian matrix is positive. Hence, the first derivatives of the retailer's profit will provide the optimal initial order quantities.

$$\frac{\partial^2 \prod_{11}^r}{\partial (q_{11}^r)^2} = (-p_1 + w_1 + p_2 - w_2) f_1(q_{11}^r + q_{12}^r)$$
$$+\alpha^2 (s_2 - p_2) f_2\left[(1+\alpha) q_{12}^r + \alpha q_{11}^r)\right] \; F_1\left[(1-\alpha) q_{11}^r - \alpha q_{12}^r\right]$$
$$+(1-\alpha)^2 \left\{ s_1 - w_1 - s_2 + w_2 + (s_2 - p_2)\left(1 - F_2\left[(1+\alpha) q_{12}^r + \alpha q_{11}^r\right] \right) \right\} f_1\left[(1-\alpha) q_{11}^r - \alpha q_{12}^r\right]$$
$$+ \int_{x_1=(1-\alpha) q_{11}^r - \alpha q_{12}^r}^{q_{11}^r + q_{12}^r} (s_2 - p_2) f_2(q_{11}^r + q_{12}^r - x_1) \; f_1(x_1) dx_1 < 0 \qquad (3)$$

$$\frac{\partial^2 \prod_{11}^r}{\partial (q_{12}^r)^2} = (-p_1 + w_1 + p_2 - w_2) f_1(q_{11}^r + q_{12}^r)$$
$$+(1+\alpha)^2 (s_2 - p_2) f_2\left[(1+\alpha) q_{12}^r + \alpha q_{11}^r)\right] \; F_1\left[(1-\alpha) q_{11}^r - \alpha q_{12}^r\right]$$
$$+\alpha^2 \left\{ s_1 - w_1 - s_2 + w_2 + (s_2 - p_2)\left(1 - F_2\left[(1+\alpha) q_{12}^r + \alpha q_{11}^r\right] \right) \right\} f_1\left[(1-\alpha) q_{11}^r - \alpha q_{12}^r\right]$$
$$+ \int_{x_1=(1-\alpha) q_{11}^r - \alpha q_{12}^r}^{q_{11}^r + q_{12}^r} (s_2 - p_2) f_2(q_{11}^r + q_{12}^r - x_1) \; f_1(x_1) dx_1 < 0 \qquad (4)$$

$$\frac{\partial^2 \prod_{11}^r}{\partial q_{11}^r \partial q_{12}^r} = (-p_1 + w_1 + p_2 - w_2) f_1(q_{11}^r + q_{12}^r)$$
$$+\alpha(1+\alpha) (s_2 - p_2) f_2\left[(1+\alpha) q_{12}^r + \alpha q_{11}^r)\right] \; F_1\left[(1-\alpha) q_{11}^r - \alpha q_{12}^r\right]$$
$$-\alpha(1-\alpha) \left\{ s_1 - w_1 - s_2 + w_2 + (s_2 - p_2)\left(1 - F_2\left[(1+\alpha) q_{12}^r + \alpha q_{11}^r\right] \right) \right\} f_1\left[(1-\alpha) q_{11}^r - \alpha q_{12}^r\right]$$
$$+ \int_{x_1=(1-\alpha) q_{11}^r - \alpha q_{12}^r}^{q_{11}^r + q_{12}^r} (s_2 - p_2) f_2(q_{11}^r + q_{12}^r - x_1) \; f_1(x_1) dx_1 \qquad (5)$$

Furthermore, the determinant of hessian matrix is given in Equation (6).

$$IHI = AB + AC + BC + BD + CD > 0 \qquad \text{where} \qquad (6)$$

$$A = (-p_1 + w_1 + p_2 - w_2) f_1 (q_{11}^r + q_{12}^r)$$

$$B = (s_2 - p_2) f_2 \left[(1+\alpha) q_{12}^r + \alpha q_{11}^r) \right] \ F_1 \left[(1-\alpha) q_{11}^r - \alpha q_{12}^r \right]$$

$$C = s_1 - w_1 - s_2 + w_2 + (s_2 - p_2) \left(1 - F_2 \left[(1+\alpha) q_{12}^r + \alpha q_{11}^r \right] \right) \ f_1 \left[(1-\alpha) q_{11}^r - \alpha q_{12}^r \right]$$

$$D = \int_{x_1 = (1-\alpha) q_{11}^r - \alpha q_{12}^r}^{q_{11}^r + q_{12}^r} (s_2 - p_2) f_2 (q_{11}^r + q_{12}^r - x_1) \ f_1 (x_1) dx_1$$

APPENDIX B

PROOF OF LEMMA 3

Proof: The first derivatives of $\prod_{12}^{r}(q_{11}^{r},q_{12}^{r})$ with respect to q_{11}^{r} and q_{12}^{r} are given in Equation (1) and Equation (2), respectively.

$$
\begin{aligned}
\frac{\partial \prod_{12}^{r}}{\partial q_{11}^{r}} &= p_1 - w_1 + \alpha(p_1 - w_1 - p_2 + w_2) - \alpha(s_2 - p_2)F_2\left[(1-\alpha)q_{12}^{r} - \alpha q_{11}^{r})\right] \\
&+ (1-\alpha)(s_1 - w_1 - p_2 + w_2)\ \ F_1\left[(1-\alpha)q_{11}^{r} - \alpha q_{12}^{r}\right] \\
&+ (1+\alpha)(-p_1 + w_1 + p_2 - w_2)\ \ F_1\left[(1+\alpha)q_{11}^{r} + \alpha q_{12}^{r}\right] \\
&+ \alpha(s_2 - p_2)F_2\left[(1+\alpha)q_{12}^{r} + \alpha q_{11}^{r})\right]\ \ F_1\left[(1-\alpha)q_{11}^{r} - \alpha q_{12}^{r}\right] \\
&+ \alpha(s_2 - p_2)F_2\left[(1-\alpha)q_{12}^{r} - \alpha q_{11}^{r})\right]\ \ F_1\left[(1+\alpha)q_{11}^{r} + \alpha q_{12}^{r}\right] \\
&+ \int_{x_1=(1-\alpha)q_{11}^{r}-\alpha q_{12}^{r}}^{(1+\alpha)q_{11}^{r}+\alpha q_{12}^{r}} (s_2 - p_2)F_2(q_{11}^{r} + q_{12}^{r} - x_1)\ \ f_1(x_1)dx_1
\end{aligned} \tag{1}
$$

$$\frac{\partial \prod_{12}^{r}}{\partial q_{12}^{r}} = p_2 - w_2 + \alpha(p_1 - w_1 - p_2 + w_2) + (1-\alpha)(s_2 - p_2)F_2\left[(1-\alpha)q_{12}^{r} - \alpha q_{11}^{r})\right]$$
$$-\alpha(s_1 - w_1 - p_2 + w_2) \ \ F_1\left[(1-\alpha)q_{11}^{r} - \alpha q_{12}^{r}\right]$$
$$+\alpha(-p_1 + w_1 + p_2 - w_2) \ \ F_1\left[(1+\alpha)q_{11}^{r} + \alpha q_{12}^{r}\right]$$
$$+(1+\alpha)(s_2 - p_2)F_2\left[(1+\alpha)q_{12}^{r} + \alpha q_{11}^{r})\right] \ \ F_1\left[(1-\alpha)q_{11}^{r} - \alpha q_{12}^{r}\right] \tag{2}$$
$$-(1-\alpha)(s_2 - p_2)F_2\left[(1-\alpha)q_{12}^{r} - \alpha q_{11}^{r})\right] \ \ F_1\left[(1+\alpha)q_{11}^{r} + \alpha q_{12}^{r}\right]$$
$$+\int_{x_1=(1-\alpha)q_{11}^{r}-\alpha q_{12}^{r}}^{(1+\alpha)q_{11}^{r}+\alpha q_{12}^{r}} (s_2 - p_2)F_2(q_{11}^{r} + q_{12}^{r} - x_1) \ \ f_1(x_1)dx_1$$

The second derivatives of $\prod_{12}^{r}(q_{11}^{r}, q_{12}^{r})$ in Equation (3), Equation (4) and Equation (5) indicate that the determinant of the hessian matrix is positive. Hence, the first derivatives of the retailer's profit will provide the optimal initial order quantities.

$$\frac{\partial^2 \prod_{12}^{r}}{\partial (q_{11}^{r})^2} = (1+\alpha)^2 \left(w_1 - p_1 + p_2 - w_2 + (s_2 - p_2)F_2\left[(1-\alpha)q_{12}^{r} - \alpha q_{11}^{r}\right] \right) \ f_1\left[(1+\alpha)q_{11}^{r} + \alpha q_{12}^{r}\right]$$
$$+(1-\alpha)^2 \left\{ s_1 - w_1 - s_2 + w_2 + (s_2 - p_2)\left(1 - F_2\left[(1+\alpha)q_{12}^{r} + \alpha q_{11}^{r}\right] \right) \right\} f_1\left[(1-\alpha)q_{11}^{r} - \alpha q_{12}^{r}\right]$$
$$+\alpha^2 (s_2 - p_2) f_2\left[(1-\alpha)q_{12}^{r} - \alpha q_{11}^{r})\right] \ \left(1 - F_1\left[(1+\alpha)q_{11}^{r} + \alpha q_{12}^{r}\right] \right)$$
$$+\alpha^2 (s_2 - p_2) f_2\left[(1+\alpha)q_{12}^{r} + \alpha q_{11}^{r})\right] \ \ F_1\left[(1-\alpha)q_{11}^{r} - \alpha q_{12}^{r}\right] \tag{3}$$
$$+\int_{x_1=(1-\alpha)q_{11}^{r}-\alpha q_{12}^{r}}^{(1+\alpha)q_{11}^{r}+\alpha q_{12}^{r}} (s_2 - p_2) f_2(q_{11}^{r} + q_{12}^{r} - x_1) \ \ f_1(x_1)dx_1 < 0$$

$$\frac{\partial^2 \prod_{12}^r}{\partial (q_{12}^r)^2} = \alpha^2 \Big(w_1 - p_1 + p_2 - w_2 + (s_2 - p_2) F_2\big[(1-\alpha) q_{12}^r - \alpha q_{11}^r\big]\Big)\ f_1\big[(1+\alpha) q_{11}^r + \alpha q_{12}^r\big]$$

$$+ \alpha^2 \Big\{ s_1 - w_1 - s_2 + w_2 + (s_2 - p_2)\Big(1 - F_2\big[(1+\alpha) q_{12}^r + \alpha q_{11}^r\big]\Big)\Big\}\ f_1\big[(1-\alpha) q_{11}^r - \alpha q_{12}^r\big]$$

$$+ (1-\alpha)^2 (s_2 - p_2) f_2\big[(1-\alpha) q_{12}^r - \alpha q_{11}^r)\big]\ \Big(1 - F_1\big[(1+\alpha) q_{11}^r + \alpha q_{12}^r\big]\Big)$$

$$+ (1+\alpha)^2 (s_2 - p_2) f_2\big[(1+\alpha) q_{12}^r + \alpha q_{11}^r)\big]\ F_1\big[(1-\alpha) q_{11}^r - \alpha q_{12}^r\big] \tag{4}$$

$$+ \int_{x_1 = (1-\alpha) q_{11}^r - \alpha q_{12}^r}^{(1+\alpha) q_{11}^r + \alpha q_{12}^r} (s_2 - p_2) f_2(q_{11}^r + q_{12}^r - x_1)\ f_1(x_1) dx_1 < 0$$

$$\frac{\partial^2 \prod_{12}^r}{\partial q_{11}^r \partial q_{12}^r} = \alpha(1+\alpha)\Big(w_1 - p_1 + p_2 - w_2 + (s_2 - p_2) F_2\big[(1-\alpha) q_{12}^r - \alpha q_{11}^r\big]\Big) f_1\big[(1+\alpha) q_{11}^r + \alpha q_{12}^r\big]$$

$$- \alpha(1-\alpha)\Big\{ s_1 - w_1 - s_2 + w_2 + (s_2 - p_2)\Big(1 - F_2\big[(1+\alpha) q_{12}^r + \alpha q_{11}^r\big]\Big)\Big\} f_1\big[(1-\alpha) q_{11}^r - \alpha q_{12}^r\big]$$

$$- \alpha(1-\alpha)(s_2 - p_2) f_2\big[(1-\alpha) q_{12}^r - \alpha q_{11}^r)\big]\ \Big(1 - F_1\big[(1+\alpha) q_{11}^r + \alpha q_{12}^r\big]\Big)$$

$$+ \alpha(1+\alpha)(s_2 - p_2) f_2\big[(1+\alpha) q_{12}^r + \alpha q_{11}^r)\big]\ F_1\big[(1-\alpha) q_{11}^r - \alpha q_{12}^r\big] \tag{5}$$

$$+ \int_{x_1 = (1-\alpha) q_{11}^r - \alpha q_{12}^r}^{(1+\alpha) q_{11}^r + \alpha q_{12}^r} (s_2 - p_2) f_2(q_{11}^r + q_{12}^r - x_1)\ f_1(x_1) dx_1$$

Furthermore, the determinant of hessian matrix is given in Equation (6).

$$\begin{aligned} IHI = {} & 4\alpha^2 AB + (1-2\alpha)^2 AC + AD + AE + BC + (1+2\alpha)^2 BD + BE \\ & + 4\alpha^2 CD + CE + DE \end{aligned} \quad \text{where} \tag{6}$$

$$A = \Big\{ s_1 - w_1 - s_2 + w_2 + (s_2 - p_2)\Big(1 - F_2\big[(1+\alpha) q_{12}^r + \alpha q_{11}^r\big]\Big)\Big\}\ f_1\big[(1-\alpha) q_{11}^r - \alpha q_{12}^r\big]$$

$$B = \Big(w_1 - p_1 + p_2 - w_2 + (s_2 - p_2) F_2\big[(1-\alpha) q_{12}^r - \alpha q_{11}^r\big]\Big)\ f_1\big[(1+\alpha) q_{11}^r + \alpha q_{12}^r\big]$$

$$C = (s_2 - p_2) f_2\big[(1-\alpha) q_{12}^r - \alpha q_{11}^r)\big]\ \Big(1 - F_1\big[(1+\alpha) q_{11}^r + \alpha q_{12}^r\big]\Big)$$

$$D = (s_2 - p_2) f_2\big[(1+\alpha) q_{12}^r + \alpha q_{11}^r)\big]\ F_1\big[(1-\alpha) q_{11}^r - \alpha q_{12}^r\big]$$

$$E = \int_{x_1 = (1-\alpha) q_{11}^r - \alpha q_{12}^r}^{(1+\alpha) q_{11}^r + \alpha q_{12}^r} (s_2 - p_2) f_2(q_{11}^r + q_{12}^r - x_1)\ f_1(x_1) dx_1$$

APPENDIX C

PROOF OF LEMMA 4

Proof: The first and second derivatives of $\prod_1^r(q_{11}^r)$ with respect to q_{11}^r is given in Equation (1) and Equation (2), respectively.

$$\frac{d\prod_1^r}{dq_{11}^r} = \frac{1}{1-\alpha}(p_1 - w_1) + \frac{1}{1-\alpha}(-p_1 + w_1 + p_2 - w_2)F_1\left[\frac{1}{1-\alpha}q_{11}^r\right]$$
$$+\frac{1-2\alpha}{1-\alpha}(s_1 - w_1 - p_2 + w_2)F_1\left[\frac{1-2\alpha}{1-\alpha}q_{11}^r\right] + \frac{2\alpha}{1-\alpha}(s_2 - p_2)F_2\left[\frac{2\alpha}{1-\alpha}q_{11}^r\right] \; F_1\left[\frac{1-2\alpha}{1-\alpha}q_{11}^r\right]$$
$$+\int_{x_1=\frac{1-2\alpha}{1-\alpha}q_{11}^r}^{\frac{1}{1-\alpha}q_{11}^r} \frac{1}{1-\alpha}(s_2 - p_2)F_2\left[\frac{1}{1-\alpha}q_{11}^r - x_1\right] \; f_1(x_1)dx_1 \tag{1}$$

$$\frac{d^2\prod_1^r}{d(q_{11}^r)^2} = \left(\frac{1}{1-\alpha}\right)^2(-p_1 + w_1 + p_2 - w_2) \;\; f_1\left[\frac{1}{1-\alpha}q_{11}^r\right]$$
$$+\left(\frac{2\alpha}{1-\alpha}\right)^2(s_2 - p_2)f_2\left[\frac{2\alpha}{1-\alpha}q_{11}^r\right] \;\; F_1\left[\frac{1-2\alpha}{1-\alpha}q_{11}^r\right]$$
$$+\left(\frac{1-2\alpha}{1-\alpha}\right)^2\left\{s_1 - w_1 - s_2 + w_2 + (s_2 - p_2)\left(1 - F_2\left[\frac{2\alpha}{1-\alpha}q_{11}^r\right] \;\right) \right\} \; f_1\left[\frac{1-2\alpha}{1-\alpha}q_{11}^r\right]$$
$$+\int_{x_1=\frac{1-2\alpha}{1-\alpha}q_{11}^r}^{\frac{1}{1-\alpha}q_{11}^r} \left(\frac{1}{1-\alpha}\right)^2(s_2 - p_2)f_2\left[\frac{1}{1-\alpha}q_{11}^r - x_1\right] \; f_1(x_1)dx_1 < 0 \tag{2}$$

Its second derivative is less than zero; we can conclude that the expected profit is concave.

APPENDIX D

PROOF OF LEMMA 6

Proof: The first derivatives of $\prod_{11}^{r}(q_{11}^{r},q_{12}^{r})$ with respect to q_{11}^{r} and q_{12}^{r} are given in Equation (1) and Equation (2), respectively.

$$\frac{\partial\prod_{1}^{r}}{\partial q_{11}^{r}} = p_1 - w_1 + (-p_1 + w_1 + p_2 - w_2)F_1(q_{11}^{r} + q_{12}^{r})$$
$$+ (1-\alpha)(s_1 - w_1 - p_2 + w_2)F_1\left[(1-\alpha)q_{11}^{r} - \alpha q_{12}^{r}\right]$$
$$-\alpha(w_1 - s_1 + p_2 - w_2)F_2\left[(1+\alpha)q_{12}^{r} + \alpha q_{11}^{r})\right] \ F_1\left[(1-\alpha)q_{11}^{r} - \alpha q_{12}^{r}\right]$$
$$- \int_{x_1=(1-\alpha)q_{11}^{r}-\alpha q_{12}^{r}}^{q_{11}^{r}+q_{12}^{r}} (w_1 - s_1 + p_2 - w_2)F_2(q_{11}^{r} + q_{12}^{r} - x_1) \ f_1(x_1)dx_1 \tag{1}$$

$$\frac{\partial\prod_{1}^{r}}{\partial q_{12}^{r}} = p_1 - w_1 + (-p_1 + w_1 + p_2 - w_2)F_1(q_{11}^{r} + q_{12}^{r})$$
$$-\alpha(s_1 - w_1 - p_2 + w_2)F_1\left[(1-\alpha)q_{11}^{r} - \alpha q_{12}^{r}\right]$$
$$-(1+\alpha)(w_1 - s_1 + p_2 - w_2)F_2\left[(1+\alpha)q_{12}^{r} + \alpha q_{11}^{r})\right] \ F_1\left[(1-\alpha)q_{11}^{r} - \alpha q_{12}^{r}\right]$$
$$- \int_{x_1=(1-\alpha)q_{11}^{r}-\alpha q_{12}^{r}}^{q_{11}^{r}+q_{12}^{r}} (w_1 - s_1 + p_2 - w_2)F_2(q_{11}^{r} + q_{12}^{r} - x_1) \ f_1(x_1)dx_1 \tag{2}$$

The second derivatives of $\prod_{11}^{r}(q_{11}^{r},q_{12}^{r})$ in Equation (3), Equation (4) and Equation (5) indicate that the determinant of the hessian matrix is positive. Hence, the first derivatives of the retailer's profit will provide the optimal initial order quantities.

$$\frac{\partial^2 \prod_1^r}{\partial (q_{11}^r)^2} = (-p_1 + w_1 + p_2 - w_2) f_1 (q_{11}^r + q_{12}^r)$$
$$+ \alpha^2 (s_1 - w_1 - p_2 + w_2) f_2 \left[(1+\alpha) q_{12}^r + \alpha q_{11}^r) \right] \ F_1 \left[(1-\alpha) q_{11}^r - \alpha q_{12}^r \right]$$
$$+ (1-\alpha)^2 (-p_2 + w_2 - w_1 + s_1) \left(1 - F_2 \left[(1+\alpha) q_{12}^r + \alpha q_{11}^r \right] \right) \ f_1 \left[(1-\alpha) q_{11}^r - \alpha q_{12}^r \right]$$
$$+ \int_{x_1 = (1-\alpha) q_{11}^r - \alpha q_{12}^r}^{q_{11}^r + q_{12}^r} (s_1 - w_1 - p_2 + w_2) \ f_2 (q_{11}^r + q_{12}^r - x_1) \ f_1 (x_1) dx_1 < 0 \quad (3)$$

$$\frac{\partial^2 \prod_1^r}{\partial (q_{12}^r)^2} = (-p_1 + w_1 + p_2 - w_2) f_1 (q_{11}^r + q_{12}^r)$$
$$+ (1+\alpha)^2 (s_1 - w_1 - p_2 + w_2) f_2 \left[(1+\alpha) q_{12}^r + \alpha q_{11}^r) \right] \ F_1 \left[(1-\alpha) q_{11}^r - \alpha q_{12}^r \right]$$
$$+ \alpha^2 (-p_2 + w_2 - w_1 + s_1) \left(1 - F_2 \left[(1+\alpha) q_{12}^r + \alpha q_{11}^r \right] \right) \ f_1 \left[(1-\alpha) q_{11}^r - \alpha q_{12}^r \right]$$
$$+ \int_{x_1 = (1-\alpha) q_{11}^r - \alpha q_{12}^r}^{q_{11}^r + q_{12}^r} (s_1 - w_1 - p_2 + w_2) \ f_2 (q_{11}^r + q_{12}^r - x_1) \ f_1 (x_1) dx_1 < 0 \quad (4)$$

$$\frac{\partial^2 \prod_1^r}{\partial q_{11}^r \partial q_{12}^r} = (-p_1 + w_1 + p_2 - w_2) f_1 (q_{11}^r + q_{12}^r)$$
$$+ \alpha (1+\alpha) (s_1 - w_1 - p_2 + w_2) f_2 \left[(1+\alpha) q_{12}^r + \alpha q_{11}^r) \right] \ F_1 \left[(1-\alpha) q_{11}^r - \alpha q_{12}^r \right]$$
$$- \alpha (1-\alpha) (s_1 - w_1 - p_2 + w_2) \left(1 - F_2 \left[(1+\alpha) q_{12}^r + \alpha q_{11}^r \right] \right) \ f_1 \left[(1-\alpha) q_{11}^r - \alpha q_{12}^r \right]$$
$$+ \int_{x_1 = (1-\alpha) q_{11}^r - \alpha q_{12}^r}^{q_{11}^r + q_{12}^r} (s_1 - w_1 - p_2 + w_2) \ f_2 (q_{11}^r + q_{12}^r - x_1) \ f_1 (x_1) dx_1 \quad (5)$$

Furthermore, the determinant of hessian matrix is given in Equation (6).

$$IHI = AB + AC + BC + BD + CD \quad \text{where} \quad (6)$$

$$A = (-p_1 + w_1 + p_2 - w_2) f_1 (q_{11}^r + q_{12}^r)$$

$$B = (s_1 - w_1 - p_2 + w_2) f_2 \left[(1+\alpha) q_{12}^r + \alpha q_{11}^r)\right] \; F_1 \left[(1-\alpha) q_{11}^r - \alpha q_{12}^r\right]$$

$$C = (-p_2 + w_2 - w_1 + s_1) \left(1 - F_2 \left[(1+\alpha) q_{12}^r + \alpha q_{11}^r\right]\right) \; f_1 \left[(1-\alpha) q_{11}^r - \alpha q_{12}^r\right]$$

$$D = \int_{x_1 = (1-\alpha) q_{11}^r - \alpha q_{12}^r}^{q_{11}^r + q_{12}^r} (s_1 - w_1 - p_2 + w_2) \; f_2 (q_{11}^r + q_{12}^r - x_1) \; f_1 (x_1) dx_1$$

APPENDIX E

PROOF OF LEMMA 7

Proof: The first derivatives of $\prod_{12}^{r}(q_{11}^{r},q_{12}^{r})$ with respect to q_{11}^{r} and q_{12}^{r} are given in Equation (1) and Equation (2), respectively.

$$\frac{\partial\prod_{1}^{r}}{\partial q_{11}^{r}} = p_1 - w_1 + \alpha(p_1 - w_1 - p_2 + w_2) + \alpha(p_2 - s_2)F_2\left[(1-\alpha)q_{12}^{r} - \alpha q_{11}^{r}\right]$$
$$+F_1\left[(1-\alpha)q_{11}^{r} - \alpha q_{12}^{r}\right]\left[\,(1-\alpha)(s_1 - w_1 - p_2 + w_2) + \alpha(s_1 - w_1 - p_2 + w_2)F_2\left[(1+\alpha)q_{12}^{r} + \alpha q_{11}^{r})\right]\,\right]$$
$$+F_1\left[(1+\alpha)q_{11}^{r} + \alpha q_{12}^{r}\right]\left[\,(1+\alpha)(p_2 - w_2 - p_1 + w_1) + \alpha(s_1 - w_1 - p_2 + w_2)F_2\left[(1-\alpha)q_{12}^{r} - \alpha q_{11}^{r})\right]\,\right]$$
$$-\int_{x_1=(1-\alpha)q_{11}^{r}-\alpha q_{12}^{r}}^{(1+\alpha)q_{11}^{r}+\alpha q_{12}^{r}} (w_1 - s_1 + p_2 - w_2)F_2(q_{11}^{r} + q_{12}^{r} - x_1)\; f_1(x_1)dx_1 \qquad (1)$$

$$\frac{\partial\prod_{1}^{r}}{\partial q_{12}^{r}} = p_2 - w_2 + \alpha(p_1 - w_1 - p_2 + w_2) - (1-\alpha)(p_2 - s_2)F_2\left[(1-\alpha)q_{12}^{r} - \alpha q_{11}^{r}\right]$$
$$+F_1\left[(1-\alpha)q_{11}^{r} - \alpha q_{12}^{r}\right]\Big[\; -\alpha(s_1 - w_1 - p_2 + w_2)$$
$$+(1+\alpha)(s_1 - w_1 - p_2 + w_2)F_2\left[(1+\alpha)q_{12}^{r} + \alpha q_{11}^{r})\right]\Big]$$
$$+F_1\left[(1+\alpha)q_{11}^{r} + \alpha q_{12}^{r}\right]\Big[\;\alpha(p_2 - w_2 - p_1 + w_1)$$
$$-(1-\alpha)(s_1 - w_1 - p_2 + w_2)F_2\left[(1-\alpha)q_{12}^{r} - \alpha q_{11}^{r})\right]\Big] \qquad (2)$$
$$-\int_{x_1=(1-\alpha)q_{11}^{r}-\alpha q_{12}^{r}}^{(1+\alpha)q_{11}^{r}+\alpha q_{12}^{r}} (w_1 - s_1 + p_2 - w_2)F_2(q_{11}^{r} + q_{12}^{r} - x_1)\; f_1(x_1)dx_1$$

The second derivatives of $\prod_{12}^{r}(q_{11}^{r},q_{12}^{r})$ in Equation (3), Equation (4) and Equation (5) indicate that the determinant of the hessian matrix is positive. Hence, the first derivatives of the retailer's profit will provide the optimal initial order quantities.

$$\begin{aligned}
\frac{\partial^2 \prod_1^r}{\partial (q_{11}^r)^2} &= (1+\alpha)^2(-p_1+w_1+p_2-w_2)f_1\left[(1+\alpha)q_{11}^r+\alpha q_{12}^r)\right] \\
&+\alpha^2(s_1-w_1-p_2+w_2)f_2\left[(1+\alpha)q_{12}^r+\alpha q_{11}^r)\right]\ F_1\left[(1-\alpha)q_{11}^r-\alpha q_{12}^r\right] \\
&+(1+\alpha)^2(s_1-w_1-p_2+w_2)f_1\left[(1+\alpha)q_{11}^r+\alpha q_{12}^r)\right]\ F_2\left[(1-\alpha)q_{12}^r-\alpha q_{11}^r\right] \\
&-\alpha^2\Bigg[\ (s_1-w_1+w_2-s_2) \\
&+(p_2-w_2+w_1-s_1)\left(1-F_1\left[(1+\alpha)q_{11}^r+\alpha q_{12}^r\right]\ \right)\Bigg]f_2\left[(1-\alpha)q_{12}^r-\alpha q_{11}^r\right] \\
&+(1-\alpha)^2\left[\ (-p_2+w_2-w_1+s_1)\left(1-F_2\left[(1+\alpha)q_{12}^r+\alpha q_{11}^r\right]\ \right)\ \right]\ f_1\left[(1-\alpha)q_{11}^r-\alpha q_{12}^r\right] \\
&-\int_{x_1=(1-\alpha)q_{11}^r-\alpha q_{12}^r}^{(1+\alpha)q_{11}^r+\alpha q_{12}^r}(w_1-s_1+p_2-w_2)f_2(q_{11}^r+q_{12}^r-x_1)\ f_1(x_1)dx_1<0
\end{aligned} \tag{3}$$

$$\begin{aligned}
\frac{\partial^2 \prod_1^r}{\partial (q_{12}^r)^2} &= \alpha^2(-p_1+w_1+p_2-w_2)f_1\left[(1+\alpha)q_{11}^r+\alpha q_{12}^r)\right] \\
&+(1+\alpha)^2(s_1-w_1-p_2+w_2)f_2\left[(1+\alpha)q_{12}^r+\alpha q_{11}^r)\right]\ F_1\left[(1-\alpha)q_{11}^r-\alpha q_{12}^r\right] \\
&+\alpha^2(s_1-w_1-p_2+w_2)f_1\left[(1+\alpha)q_{11}^r+\alpha q_{12}^r)\right]\ F_2\left[(1-\alpha)q_{12}^r-\alpha q_{11}^r\right] \\
&-(1-\alpha)^2\Bigg[\ (s_1-w_1+w_2-s_2) \\
&+(p_2-w_2+w_1-s_1)\left(1-F_1\left[(1+\alpha)q_{11}^r+\alpha q_{12}^r\right]\ \right)\Bigg]f_2\left[(1-\alpha)q_{12}^r-\alpha q_{11}^r\right] \\
&+\alpha^2\left[\ (-p_2+w_2-w_1+s_1)\left(1-F_2\left[(1+\alpha)q_{12}^r+\alpha q_{11}^r\right]\ \right)\ \right]\ f_1\left[(1-\alpha)q_{11}^r-\alpha q_{12}^r\right] \\
&-\int_{x_1=(1-\alpha)q_{11}^r-\alpha q_{12}^r}^{(1+\alpha)q_{11}^r+\alpha q_{12}^r}(w_1-s_1+p_2-w_2)f_2(q_{11}^r+q_{12}^r-x_1)\ f_1(x_1)dx_1<0
\end{aligned} \tag{4}$$

$$
\begin{aligned}
\frac{\partial^2 \prod_1^r}{\partial q_{11}^r \partial q_{12}^r} &= \alpha(1+\alpha)(-p_1 + w_1 + p_2 - w_2) f_1\left[(1+\alpha)q_{11}^r + \alpha q_{12}^r)\right] \\
&+ \alpha(1+\alpha)(s_1 - w_1 - p_2 + w_2) f_2\left[(1+\alpha)q_{12}^r + \alpha q_{11}^r)\right] \; F_1\left[(1-\alpha)q_{11}^r - \alpha q_{12}^r\right] \\
&+ \alpha(1+\alpha)(s_1 - w_1 - p_2 + w_2) f_1\left[(1+\alpha)q_{11}^r + \alpha q_{12}^r)\right] \; F_2\left[(1-\alpha)q_{12}^r - \alpha q_{11}^r\right] \\
&+ \alpha(1-\alpha)\Bigg[\; (s_1 - w_1 + w_2 - s_2) \\
&+ (p_2 - w_2 + w_1 - s_1)\left(1 - F_1\left[(1+\alpha)q_{11}^r + \alpha q_{12}^r\right] \right) \Bigg] f_2\left[(1-\alpha)q_{12}^r - \alpha q_{11}^r\right] \\
&- \alpha(1-\alpha)\left[\; (-p_2 + w_2 - w_1 + s_1)\left(1 - F_2\left[(1+\alpha)q_{12}^r + \alpha q_{11}^r\right] \right) \right] \; f_1\left[(1-\alpha)q_{11}^r - \alpha q_{12}^r\right] \\
&- \int_{x_1=(1-\alpha)q_{11}^r - \alpha q_{12}^r}^{(1+\alpha)q_{11}^r + \alpha q_{12}^r} (w_1 - s_1 + p_2 - w_2) f_2(q_{11}^r + q_{12}^r - x_1) \; f_1(x_1) dx_1
\end{aligned} \tag{5}
$$

Furthermore, the determinant of hessian matrix is given in Equation (6).

$$
\begin{aligned}
IHI = AB &+ (1+2\alpha)^2 AC + AE + 4\alpha^2 AF + BC + BD + BE + BF + (1+2\alpha)^2 CD \\
&+ 4\alpha^2 CE + CF + DE + 4\alpha^2 DF + (1-2\alpha)^2 EF
\end{aligned} \quad \text{where} \tag{6}
$$

$$
\begin{aligned}
A &= (-p_1 + w_1 + p_2 - w_2) f_1\left[(1+\alpha)q_{11}^r + \alpha q_{12}^r)\right] \\
C &= (s_1 - w_1 - p_2 + w_2) f_2\left[(1+\alpha)q_{12}^r + \alpha q_{11}^r)\right] \; F_1\left[(1-\alpha)q_{11}^r - \alpha q_{12}^r\right] \\
D &= (s_1 - w_1 - p_2 + w_2) f_1\left[(1+\alpha)q_{11}^r + \alpha q_{12}^r)\right] \; F_2\left[(1-\alpha)q_{12}^r - \alpha q_{11}^r\right] \\
E &= \Bigg[\; -(s_1 - w_1 + w_2 - s_2) \\
&\quad - (p_2 - w_2 + w_1 - s_1)\left(1 - F_1\left[(1+\alpha)q_{11}^r + \alpha q_{12}^r\right] \right) \Bigg] f_2\left[(1-\alpha)q_{12}^r - \alpha q_{11}^r\right] \\
F &= (-p_2 + w_2 - w_1 + s_1)\left(1 - F_2\left[(1+\alpha)q_{12}^r + \alpha q_{11}^r\right] \right) \; f_1\left[(1-\alpha)q_{11}^r - \alpha q_{12}^r\right] \\
B &= -\int_{x_1=(1-\alpha)q_{11}^r - \alpha q_{12}^r}^{(1+\alpha)q_{11}^r + \alpha q_{12}^r} (w_1 - s_1 + p_2 - w_2) f_2(q_{11}^r + q_{12}^r - x_1) \; f_1(x_1) dx_1
\end{aligned}
$$

APPENDIX F

PROOF OF LEMMA 8

Proof: The first and second derivatives of $\prod_1^r(q_{11}^r)$ with respect to q_{11}^r is given in Equation (1) and Equation (2), respectively.

$$
\begin{aligned}
\frac{d\prod_1^r}{dq_{11}^r} &= \frac{1}{1-\alpha}(p_1-w_1)+\frac{1}{1-\alpha}(-p_1+w_1+p_2-w_2)F_1\left[\frac{1}{1-\alpha}q_{11}^r\right] \\
&+\frac{1-2\alpha}{1-\alpha}(s_1-w_1-p_2+w_2)F_1\left[\frac{1-2\alpha}{1-\alpha}q_{11}^r\right] \\
&+\frac{2\alpha}{1-\alpha}(-w_1+s_1-p_2+w_2)F_2\left[\frac{2\alpha}{1-\alpha}q_{11}^r\right]\ F_1\left[\frac{1-2\alpha}{1-\alpha}q_{11}^r\right] \\
&+\int_{x_1=\frac{1-2\alpha}{1-\alpha}q_{11}^r}^{\frac{1}{1-\alpha}q_{11}^r}\frac{1}{1-\alpha}(-w_1+s_1-p_2+w_2)F_2\left[\frac{1}{1-\alpha}q_{11}^r-x_1\right]\ f_1(x_1)dx_1
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
\frac{d^2\prod_1^r}{d(q_{11}^r)^2} &= \left(\frac{1}{1-\alpha}\right)^2(-p_1+w_1+p_2-w_2)\ \ f_1\left[\frac{1}{1-\alpha}q_{11}^r\right] \\
&+\left(\frac{2\alpha}{1-\alpha}\right)^2(s_1-w_1-p_2+w_2)f_2\left[\frac{2\alpha}{1-\alpha}q_{11}^r\right]\ F_1\left[\frac{1-2\alpha}{1-\alpha}q_{11}^r\right] \\
&+\left(\frac{1-2\alpha}{1-\alpha}\right)^2(s_1-w_1-p_2+w_2)\ \left(1-F_2\left[\frac{2\alpha}{1-\alpha}q_{11}^r\right]\right)\ f_1\left[\frac{1-2\alpha}{1-\alpha}q_{11}^r\right] \\
&+\int_{x_1=\frac{1-2\alpha}{1-\alpha}q_{11}^r}^{\frac{1}{1-\alpha}q_{11}^r}\left(\frac{1}{1-\alpha}\right)^2(s_1-w_1-p_2+w_2)\ \ f_2\left[\frac{1}{1-\alpha}q_{11}^r-x_1\right]\ f_1(x_1)dx_1
\end{aligned} \tag{2}
$$

Its second derivative is less than zero; we can conclude that the expected profit is concave.

APPENDIX G

SUMMARY OF RESULTS

Coefficient of Variation of Product 1:

Table G1 Summary of the results for different levels of cv_1

	cv_1	q^r_{11}	q^r_{12}	Q^r	$\prod^r$	q^m_{11}	q^m_{12}	$\prod^m$	$\prod^s$
No order adjustment	*1/10*	54.5	56.7	111.3	7765.4	-	-	1942.0	9707.4
	1/7	56.5	56.7	113.2	7707.6	-	-	1981.0	9688.6
	1/5	59.1	56.7	115.8	7627.7	-	-	2032.9	9660.5
	1/3	65.1	56.7	121.9	7436.0	-	-	2154.0	9590.0
α =0.10	*1/10*	55.6	53.2	108.8	7925.4	52.1	42.3	1891.8	9817.2
	1/7	55.9	53.9	109.8	7888.5	53.1	42.9	1911.8	9800.3
	1/5	57.7	53.9	111.6	7821.9	54.7	42.8	1947.0	9768.9
	1/3	63.3	53.5	116.8	7672.2	59.5	41.8	2049.4	9721.5
Unlimited	*1/10*	-	-	109.0	7937.0	52.2	40.3	1886.8	9823.8
	1/7	-	-	109.9	7908.3	53.1	40.2	1904.0	9812.3
	1/5	-	-	111.4	7859.1	54.5	39.8	1933.1	9792.2
	1/3	-	-	115.7	7723.9	58.6	38.4	2017.5	9741.4

Table G2 % Improvement in profits due to order adjustment: effects of cv_1

	c_{v1}	$\%\prod^r$	$\%\prod^m$	$\%\prod^s$
α =0.10	*1/10*	2.1	-2.6	1.1
	1/7	2.3	-3.5	1.2
	1/5	2.5	-4.2	1.1
	1/3	3.2	-4.9	1.4
Unlimited	*1/10*	2.2	-2.8	1.2
	1/7	2.6	-3.9	1.3
	1/5	3.0	-4.9	1.4
	1/3	3.9	-6.3	1.6

Coefficient of Variation of Product 2:

Table G3 Summary of the results for different levels of cv_2

	cv_2	q^r_{11}	q^r_{12}	Q^r	$\prod^r$	q^m_{11}	q^m_{12}	$\prod^m$	$\prod^s$
No order adjustment	*1/10*	59.1	53.4	112.5	7787.0	-	-	1982.3	9769.3
	1/7	59.1	54.8	113.9	7722.2	-	-	2004.0	9726.2
	1/5	59.1	56.7	115.8	7627.7	-	-	2032.9	9660.5
	1/3	59.1	61.2	120.3	7419.7	-	-	2100.3	9520.0
α =0.10	*1/10*	58.8	50.3	109.2	7922.4	55.5	43.9	1908.9	9831.4
	1/7	58.1	52.1	110.1	7880.3	55.1	42.1	1922.3	9802.7
	1/5	57.7	53.9	111.6	7821.9	54.7	42.8	1947.0	9768.9
	1/3	57.4	58.3	115.7	7663.7	54.4	46.8	2010.5	9674.2
Unlimited	*1/10*	-	-	109.0	7937.6	55.4	43.8	1905.6	9843.2
	1/7	-	-	109.9	7908.5	54.9	42.1	1915.0	9823.5
	1/5	-	-	111.4	7859.1	54.5	39.8	1933.1	9792.2
	1/3	-	-	115.6	7714.3	54.3	33.8	1986.7	9701.0

Table G4 % Improvement in profits due to order adjustment: effects of cv_2

	cv_2	$\%\prod^r$	$\%\prod^m$	$\%\prod^s$
α =0.10	*1/10*	1.7	-3.7	0.6
	1/7	2.0	-4.1	0.8
	1/5	2.5	-4.2	1.1
	1/3	3.3	-4.3	1.6
Unlimited	*1/10*	1.9	-3.9	0.8
	1/7	2.4	-4.4	1.0
	1/5	3.0	-4.9	1.4
	1/3	4.0	-5.4	1.9

Mean Demand of Product 1:

Table G5 Summary of the results for different levels of μ_1

	μ_1	q^r_{11}	q^r_{12}	Q^r	$\prod^r$	q^m_{11}	q^m_{12}	$\prod^m$	$\prod^s$
No order adjustment	*30*	35.5	56.7	92.2	5949.7	-	-	1560.2	7509.9
	40	47.3	56.7	104.0	6788.7	-	-	1796.5	8585.2
	50	59.1	56.7	115.8	7627.7	-	-	2032.9	9660.5
	60	70.9	56.7	127.6	8475.3	-	-	2269.2	10744.5
	70	82.7	56.7	139.5	9314.3	-	-	2505.5	11819.8
α =0.10	*30*	34.8	54.4	89.2	6098.3	32.5	45.5	1501.9	7600.2
	40	46.2	54.2	100.4	6964.4	43.5	44.2	1723.5	8687.8
	50	57.7	53.9	111.6	7821.9	54.7	42.8	1947.0	9768.9
	60	69.4	53.6	123.0	8689.2	65.9	41.3	2171.5	10860.6
	70	81.1	53.3	134.4	9553.3	77.2	39.8	2398.1	11951.4
Unlimited	*30*	-	-	89.4	6125.8	32.6	40.3	1494.2	7620.0
	40	-	-	100.3	6998.4	43.5	40.1	1712.1	8710.5
	50	-	-	111.4	7859.1	54.5	39.8	1933.1	9792.2
	60	-	-	122.6	8719.9	65.6	39.4	2156.3	10876.2
	70	-	-	133.8	9574.5	76.8	38.9	2381.0	11955.5

Table G6 % Improvement in profits due to order adjustment: effects of μ_1

	μ_1	$\%\prod^r$	$\%\prod^m$	$\%\prod^s$
α =0.10	*30*	2.5	-3.7	1.2
	40	2.6	-4.1	1.2
	50	2.5	-4.2	1.1
	60	2.5	-4.3	1.1
	70	2.6	-4.3	1.1
Unlimited	*30*	3.0	-4.2	1.5
	40	3.1	-4.7	1.5
	50	3.0	-4.9	1.4
	60	2.9	-5.0	1.2
	70	2.8	-5.0	1.1

Mean Demand of Product 2:

Table G7 Summary of the results for different levels of μ_2

	μ_2	q^r_{11}	q^r_{12}	Q^r	$\prod^r$	q^m_{11}	q^m_{12}	$\prod^m$	$\prod^s$
No order adjustment	*30*	59.1	34.0	93.1	6254.4	-	-	1692.4	7946.8
	40	59.1	45.4	104.5	6941.0	-	-	1862.6	8803.7
	50	59.1	56.7	115.8	7627.7	-	-	2032.9	9660.5
	60	59.1	68.1	127.2	8314.3	-	-	2203.1	10517.4
	70	59.1	79.4	138.5	9000.9	-	-	2373.3	11374.3
α =0.10	*30*	58.1	31.7	89.8	6412.8	55.4	23.1	1620.6	8033.4
	40	57.8	42.8	100.6	7118.2	55.0	32.7	1782.4	8900.6
	50	57.7	53.9	111.6	7821.9	54.7	42.8	1947.0	9768.9
	60	57.7	65.0	122.7	8530.8	54.5	52.7	2112.5	10643.4
	70	57.7	76.2	133.8	9237.9	54.3	62.7	2280.1	11517.9
Unlimited	*30*	-	-	89.4	6427.7	55.1	23.0	1609.5	8037.1
	40	-	-	100.3	7148.5	54.8	31.4	1769.9	8918.4
	50	-	-	111.4	7859.1	54.5	39.8	1933.1	9792.2
	60	-	-	122.6	8569.5	54.4	48.1	2098.5	10668.0
	70	-	-	133.8	9272.8	54.3	56.3	2265.6	11538.4

Table G8 % Improvement in profits due to order adjustment: effects of μ_2

	μ_2	$\%\prod^r$	$\%\prod^m$	$\%\prod^s$
α =0.10	*30*	2.5	-4.2	1.1
	40	2.6	-4.3	1.1
	50	2.5	-4.2	1.1
	60	2.6	-4.1	1.2
	70	2.6	-3.9	1.3
Unlimited	*30*	2.8	-4.9	1.1
	40	3.0	-5.0	1.3
	50	3.0	-4.9	1.4
	60	3.1	-4.7	1.4
	70	3.0	-4.5	1.4

Regular Sales Price of Product 1:

Table G9 Summary of the results for different p_1

		q_{11}^r	q_{12}^r	Q^r	$\prod^r$	q_{11}^m	q_{12}^m	$\prod^m$	$\prod^s$
No order adjustment	$p_1=105$	58.0	56.7	114.8	6892.6	-	-	2012.1	8904.7
	$p_1=120$	59.1	56.7	115.8	7627.7	-	-	2032.9	9660.5
	$p_1=135$	59.9	56.7	116.7	8361.8	-	-	2050.1	10411.9
	$p_1=150$	60.7	56.7	117.4	9103.3	-	-	2064.7	11167.9
$\alpha=0.10$	$p_1=105$	57.2	54.3	111.5	7073.2	54.6	43.1	1943.5	9016.6
	$p_1=120$	57.7	53.9	111.6	7821.9	54.7	42.8	1947.0	9768.9
	$p_1=135$	58.1	53.6	111.8	8574.7	54.8	42.5	1950.2	10524.8
	$p_1=150$	58.5	53.4	111.9	9329.9	54.9	42.2	1951.7	11281.6
Unlimited	$p_1=105$	-	-	111.4	7109.1	54.5	39.8	1933.1	9042.2
	$p_1=120$	-	-	111.4	7859.1	54.5	39.8	1933.1	9792.2
	$p_1=135$	-	-	111.4	8609.1	54.5	39.8	1933.1	10542.2
	$p_1=150$	-	-	111.4	9359.1	54.5	39.8	1933.1	11292.2

Table G10 % Improvement in profits due to order adjustment: effects of p_1

	p_1	$\%\prod^r$	$\%\prod^m$	$\%\prod^s$
$\alpha=0.10$	*105*	2.6	-3.4	1.3
	120	2.5	-4.2	1.1
	135	2.5	-4.9	1.1
	150	2.5	-5.5	1.0
Unlimited	*105*	3.1	-3.9	1.5
	120	3.0	-4.9	1.4
	135	3.0	-5.7	1.3
	150	2.8	-6.4	1.1

Regular Sales Price of Product 2:

Table G11 Summary of the results for different p_2

		q_{11}^r	q_{12}^r	Q^r	$\prod^r$	q_{11}^m	q_{12}^m	$\prod^m$	$\prod^s$
No order adjustment	$p_2=70$	59.1	53.7	112.7	6190.9	-	-	1986.6	8177.5
	$p_2=85$	59.1	55.4	114.5	6908.4	-	-	2012.9	8921.3
	$p_2=100$	59.1	56.7	115.8	7627.7	-	-	2032.9	9660.5
	$p_2=115$	59.1	57.8	116.9	8363.4	-	-	2048.8	10412.2
$\alpha=0.10$	$p_2=70$	57.1	50.8	107.9	6394.2	52.2	40.0	1877.6	8271.8
	$p_2=85$	57.4	52.6	110.0	7108.8	53.6	41.6	1917.5	9026.3
	$p_2=100$	57.7	53.9	111.6	7821.9	54.7	42.8	1947.0	9768.9
	$p_2=115$	58.0	55.0	112.9	8551.9	55.7	43.7	1971.4	10523.3
Unlimited	$p_2=70$	-	-	107.1	6426.3	51.7	39.0	1855.0	8281.4
	$p_2=85$	-	-	109.5	7136.7	53.2	39.5	1899.3	9035.9
	$p_2=100$	-	-	111.4	7859.1	54.5	39.8	1933.1	9792.2
	$p_2=115$	-	-	112.8	8583.7	55.6	39.9	1960.3	10544.0

Table G12 % Improvement in profits due to order adjustment: effects of p_2

	p_2	$\%\prod^r$	$\%\prod^m$	$\%\prod^s$
α =0.10	*70*	3.3	-5.5	1.2
	85	2.9	-4.7	1.2
	100	2.5	-4.2	1.1
	115	2.3	-3.8	1.1
Unlimited	*70*	3.8	-6.6	1.3
	85	3.3	-5.6	1.3
	100	3.0	-4.9	1.4
	115	2.6	-4.3	1.3

Discounted Sales Price of Product 1:

Table G13 Summary of the results for different s_1

		q^r_{11}	q^r_{12}	Q^r	$\prod^r$	q^m_{11}	q^m_{12}	$\prod^m$	$\prod^s$
No order adjustment	s_1 =5	57.8	56.7	114.6	7583.9	-	-	2007.4	9591.3
	s_1 =6	58.0	56.7	114.8	7586.3	-	-	2012.1	9598.4
	s_1 =7	58.3	56.7	115.0	7596.8	-	-	2017.0	9613.8
	s_1 =8	58.5	56.7	115.3	7605.8	-	-	2022.1	9627.9
	s_1 =9	58.8	56.7	115.6	7622.1	-	-	2027.4	9649.5
	s_1 =10	59.1	56.7	115.8	7627.7	-	-	2032.9	9660.5
α =0.10	s_1 =5	55.7	54.3	110.0	7770.6	53.6	43.3	1918.1	9740.0
	s_1 =6	56.1	54.2	110.3	7783.7	53.8	43.2	1923.2	9740.1
	s_1 =7	56.5	54.1	110.6	7793.4	54.0	43.1	1928.5	9727.4
	s_1 =8	56.9	54.1	110.9	7799.0	54.2	43.0	1933.9	9727.3
	s_1 =9	57.3	54.0	111.3	7816.9	54.5	42.9	1941.1	9724.8
	s_1 =10	57.7	53.9	111.6	7821.9	54.7	42.8	1947.0	9717.6
Unlimited	s_1 =5	-	-	109.5	7795.8	53.2	39.5	1899.3	9695.1
	s_1 =6	-	-	109.9	7808.9	53.5	39.6	1905.5	9714.4
	s_1 =7	-	-	110.2	7820.7	53.7	39.6	1912.0	9732.7
	s_1 =8	-	-	110.6	7831.2	54.0	39.7	1918.7	9749.9
	s_1 =9	-	-	111.0	7845.9	54.2	39.7	1925.7	9771.7
	s_1 =10	-	-	111.4	7859.1	54.5	39.8	1933.1	9792.2

Table G14 % Improvement in profits due to order adjustment: effects of s_1

	s_1	$\%\prod^r$	$\%\prod^m$	$\%\prod^s$
α =0.10	*5*	2.5	-4.4	1.6
	6	2.6	-4.4	1.5
	7	2.6	-4.4	1.2
	8	2.5	-4.4	1.0
	9	2.6	-4.3	0.8
	10	2.5	-4.2	0.6
Unlimited	*5*	2.8	-5.4	1.1
	6	2.9	-5.3	1.2
	7	2.9	-5.2	1.2
	8	3.0	-5.1	1.3
	9	2.9	-5.0	1.3
	10	3.0	-4.9	1.4

Discounted Sales Price of Product 2:

Table G15 Summary of the results for different s_2

		q^r_{11}	q^r_{12}	Q^r	$\prod^r$	q^m_{11}	q^m_{12}	$\prod^m$	$\prod^s$
No order adjustment	$s_2=0$	59.1	56.7	115.8	7627.7	-	-	2032.9	9660.5
	$s_2=1$	59.1	57.0	116.1	7637.9	-	-	2036.5	9674.4
	$s_2=2$	59.1	57.2	116.3	7646.6	-	-	2040.2	9686.8
	$s_2=3$	59.1	57.5	116.6	7653.9	-	-	2044.1	9698.0
	$s_2=4$	59.1	57.8	116.8	7667.9	-	-	2048.2	9716.1
	$s_2=5$	59.1	58.0	117.1	7672.0	-	-	2052.4	9724.4
α =0.10	$s_2=0$	57.7	53.9	111.6	7821.9	54.7	42.8	1947.0	9768.9
	$s_2=1$	57.5	54.1	111.7	7830.6	54.7	43.0	1947.6	9778.2
	$s_2=2$	57.3	54.4	111.7	7833.5	54.8	43.2	1948.2	9781.7
	$s_2=3$	57.1	54.6	111.8	7834.6	54.8	43.5	1950.2	9784.8
	$s_2=4$	56.9	54.9	111.8	7835.9	54.8	43.7	1950.9	9786.8
	$s_2=5$	56.6	55.2	111.9	7837.9	54.9	44.1	1951.8	9789.7
Unlimited	$s_2=0$	-	-	111.4	7859.1	54.5	39.8	1933.1	9792.2
	$s_2=1$	-	-	111.4	7859.1	54.5	39.8	1933.1	9792.2
	$s_2=2$	-	-	111.4	7859.1	54.5	39.8	1933.1	9792.2
	$s_2=3$	-	-	111.4	7859.1	54.5	39.8	1933.1	9792.2
	$s_2=4$	-	-	111.4	7859.1	54.5	39.8	1933.1	9792.2
	$s_2=5$	-	-	111.4	7859.1	54.5	39.8	1933.1	9792.2

Table G16 % Improvement in profits due to order adjustment: effects of s_2

	s_2	$\%\prod^r$	$\%\prod^m$	$\%\prod^s$
α =0.10	0	2.5	-4.2	1.1
	1	2.5	-4.4	1.1
	2	2.4	-4.5	1.0
	3	2.4	-4.6	0.9
	4	2.2	-4.7	0.7
	5	2.2	-4.9	0.7
Unlimited	0	3.0	-4.9	1.4
	1	2.9	-5.1	1.2
	2	2.8	-5.3	1.1
	3	2.7	-5.4	1.0
	4	2.5	-5.6	0.8
	5	2.4	-5.8	0.7

Expedited Delivery Cost:

Table G17 Summary of the results for different d_2

		q^r_{11}	q^r_{12}	Q^r	$\prod^r$	q^m_{11}	q^m_{12}	$\prod^m$	$\prod^s$
No order adjustment	d_2 =4	59.1	56.7	115.8	7627.7	-	-	2032.9	9660.5
	d_2 =5	59.1	56.7	115.8	7627.7	-	-	2032.9	9660.5
	d_2 =6	59.1	56.7	115.8	7627.7	-	-	2032.9	9660.5
	d_2 =7	59.1	56.7	115.8	7627.7	-	-	2032.9	9660.5
	d_2 =8	59.1	56.7	115.8	7627.7	-	-	2032.9	9660.5
	d_2 =9	59.1	56.7	115.8	7627.7	-	-	2032.9	9660.5
	d_2 =10	59.1	56.7	115.8	7627.7	-	-	2032.9	9660.5
α =0.10	d_2 =4	57.7	53.9	111.6	7821.9	51.7	42.8	1963.0	9784.9
	d_2 =5	57.7	53.9	111.6	7821.9	54.7	42.8	1947.0	9768.9
	d_2 =6	57.7	53.9	111.6	7821.9	56.6	42.8	1933.0	9754.9
	d_2 =7	57.7	53.9	111.6	7821.9	58.1	43.5	1920.4	9742.3
	d_2 =8	57.7	53.9	111.6	7821.9	59.2	44.8	1909.7	9731.6
	d_2 =9	57.7	53.9	111.6	7821.9	60.2	45.8	1900.4	9722.3
	d_2 =10	57.7	53.9	111.6	7821.9	61.0	46.6	1892.2	9714.2
Unlimited	d_2 =4	-	-	111.4	7859.1	51.5	36.3	1954.2	9813.4
	d_2 =5	-	-	111.4	7859.1	54.5	39.8	1933.1	9792.2
	d_2 =6	-	-	111.4	7859.1	56.4	41.9	1916.2	9775.4
	d_2 =7	-	-	111.4	7859.1	57.9	43.5	1902.1	9761.2
	d_2 =8	-	-	111.4	7859.1	59.0	44.7	1889.8	9748.9
	d_2 =9	-	-	111.4	7859.1	60.0	45.8	1879.0	9738.1
	d_2 =10	-	-	111.4	7859.1	60.8	46.6	1869.3	9728.5

Table G18 % Improvement in profits due to order adjustment: effects of d_2

		$\%\prod^{m}$	$\%\prod^{s}$
α =0.10	d_2 =4	-3.4	1.3
	d_2 =5	-4.2	1.1
	d_2 =6	-4.9	1.0
	d_2 =7	-5.5	0.8
	d_2 =8	-6.1	0.7
	d_2 =9	-6.5	0.6
	d_2 =10	-6.9	0.6
Unlimited	d_2 =4	-3.9	1.6
	d_2 =5	-4.9	1.4
	d_2 =6	-5.7	1.2
	d_2 =7	-6.4	1.0
	d_2 =8	-7.0	0.9
	d_2 =9	-7.6	0.8
	d_2 =10	-8.0	0.7

Procurement Costs of Product 1 and Product 2:

Table G19 Summary of the results for different m_1=m_2

		q_{11}^{r}	q_{12}^{r}	Q^{r}	$\prod^{r}$	q_{11}^{m}	q_{12}^{m}	$\prod^{m}$	$\prod^{s}$
No order adjustment	m_1=m_2=3	59.1	56.7	115.8	7627.7	-	-	2496.2	10123.9
	m_1=m_2=5	59.1	56.7	115.8	7627.7	-	-	2264.5	9892.2
	m_1=m_2=7	59.1	56.7	115.8	7627.7	-	-	2032.9	9660.5
	m_1=m_2=9	59.1	56.7	115.8	7627.7	-	-	1801.2	9428.9
α =0.10	m_1=m_2=3	57.7	53.9	111.6	7821.9	57.1	42.8	2397.0	10218.9
	m_1=m_2=5	57.7	53.9	111.6	7821.9	55.7	42.8	2171.7	9993.6
	m_1=m_2=7	57.7	53.9	111.6	7821.9	54.7	42.8	1947.0	9768.9
	m_1=m_2=9	57.7	53.9	111.6	7821.9	53.9	42.8	1722.6	9544.5
Unlimited	m_1=m_2=3	-	-	111.4	7859.1	57.0	42.5	2386.7	10245.8
	m_1=m_2=5	-	-	111.4	7859.1	55.6	41.0	2159.4	10018.5
	m_1=m_2=7	-	-	111.4	7859.1	54.5	39.8	1933.1	9792.2
	m_1=m_2=9	-	-	111.4	7859.1	53.7	38.8	1707.5	9566.6

Table G20 % Improvement in profits due to order adjustment: effects of m_1=m_2

		$\%\prod^{m}$	$\%\prod^{s}$
α =0.10	m_1=m_2=3	-4.0	0.9
	m_1=m_2=5	-4.1	1.0
	m_1=m_2=7	-4.2	1.1
	m_1=m_2=9	-4.4	1.2
Unlimited	m_1=m_2=3	-4.4	1.2
	m_1=m_2=5	-4.6	1.3
	m_1=m_2=7	-4.9	1.4
	m_1=m_2=9	-5.2	1.5