EFFECT OF FOUNDATION RIGIDITY ON CONTACT STRESS DISTRIBUTION IN SOILS WITH VARIABLE STRENGTH / DEFORMATION PROPERTIES

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EFFECT OF FOUNDATION RIGIDITY ON CONTACT STRESS DISTRIBUTION IN SOILS WITH VARIABLE STRENGTH / DEFORMATION PROPERTIES

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ABSTRACT

EFFECT OF FOUNDATION RIGIDITY ON CONTACT STRESS DISTRIBUTION IN SOILS WITH VARIABLE STRENGTH / DEFORMATION PROPERTIES

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In this study, a typical mat foundation and structural loading pattern is considered. Three dimensional finite element analyses, PLAXIS 3D, is performed to determine the soil / foundation contact stress distribution, settlement distribution, distribution of modulus of subgrade reaction as a function of column spacing, stiffness of the soil and thickness of the foundation. A parametric study is performed to demonstrate the dependence of those distributions on various parameters. Moreover, a relationship between size of the foundation, deformation modulus of foundation soil and modulus of subgrade reaction is proposed. Depending on the variations in those parameters, obtained shear force and bending moment distributions are compared. Consistency between the resulting shear forces and bending moments of a typical foundation, modeled in two different three dimensional finite element programs, PLAXIS 3D and SAP 2000, is discussed.

It is found that the variation in the aforementioned parameters cause different influences on contact stress distribution, settlement distribution, distribution of modulus of subgrade reaction. The importance of those variations in beforementioned parameters, under different situations is discussed. A relationship between modulus of subgrade reaction and deformation modulus of foundation soil is proposed.

Keywords: raft(mat) foundation, finite element model, contact stress distribution, settlement, modulus of subgrade reaction.

ÖZ

TEMEL RİJİTLİĞİNİN DEĞİŞKEN MUKAVEMET / DEFORMASYON ÖZELLİĞİNE SAHİP ZEMİNLERDEKİ STRES DAĞILIMINA OLAN ETKİSİ

ÇEKİNMEZ, Zeynep
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Tez Yöneticisi: Prof. Dr. Orhan EROL

Ocak 2010, 100 Sayfa

Bu çalışmada, tipik radye temel ve yapısal yükleme modelleri dikkate alınmıştır. Zemin / temel temas stres dağılımı, oturma dağılımı, yatak katsayısı dağılımı; kolon açıklığına, zemin elastik modülüne ve temel kalınlığına bağlı olarak üç boyutlu sonlu elemanlar programı, PLAXIS 3D, ile analizler yapılarak belirlenmiştir. Bu dağılımların değişik parametrelerle olan bağlantısı parametrik çalışmalarla gösterilmiştir. Ayrıca, radye boyutu, zemin elastik modülü ve yatak katsayısı arasında ilişki önerilmiştir. Parametrelerin değişimine bağlı olarak elde edilmiş olan kesme ve eğilme moment dağılımları karşılaştırılmıştır. İki farklı sonlu eleman programı olan PLAXIS 3D ve SAP 2000 'de modellenmiş tipik bir radyenin kesme ve eğilme moment dağılımlarının tutarlılığı karşılaştırılmıştır.

Bahsi geçen parametrelerdeki değişimin, zemin/temel temas stres dağılımı, oturma dağılımı, yatak katsayısı dağılımı üzerinde farklı etkileri olduğu bulunmuştur. Bu değişimlerin hangisinin hangi koşullarda önemli olduğu tartışılmıştır.

Anahtar Kelimeler: radye temel, sonlu elemanlar modeli, temas basınç dağılımı, oturma, yatak katsayısı.

To Founder of Turkish Republic
Mustafa Kemal Atatürk

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CHAPTER 1.

INTRODUCTION

Mat foundations are designed in order to satisfy both bearing capacity and settlement limitations. Thus, contact stresses developed under the mat foundation and settlement of the mat foundation should be obtained in most accurate way by studying the problem compatible with the real case.

The process of superstructure load transfer through the columns via foundation system to the soil. So that, problem should be analyzed according to appropriate pattern of load application. However, in general pattern loading distribution on the foundation is assumed as uniform.

In recent years, many problems in foundation engineering field are solved by using finite element method (F.E.M.) softwares in order to assess stresses and deformations. The main reason behind the wide spread use of finite element programs is high speed of calculation time of the problem. PLAXIS is one of the most commonly used finite element program since it involves various soil constitutive models in addition to high speed of calculation.

In this study, differences in the results of two different loading cases: uniform and column loading are investigated. Both patterns are handled seperately and effects of various parameters on contact stresses, foundation settlement, modulus of subgrade reaction, shear forces and bending moments are discussed.

Chapter 2 presents a literature survey on the relevant subjects.

Uniform loading condition (pattern A) is analysed and discussed in Chapter 3. The effect of stiffness of foundation soil, magnitude of load and rigidity (thickness of mat) on soil pressures and deformation patterns are discussed. Concentrated load case where the loads are applied through column is analysed and compared to uniform load case. The effect of column spacing and, stiffness of raft and supporting soil on soil stress and strain are emphasized.

Chapter 4 presents mainly the comparison of uniform and concentrated load cases or stresses and strains in the foundation soil.

The conclusions are presented in Chapter 5.

CHAPTER 2.

LITERATURE REVIEW OF PARAMETERS AFFECTING SOIL – STRUCTURE - FOUNDATION SYSTEM

Both the stresses and the deformations developed in the system can only be obtained through interactive analysis of the soil-structure-foundation system (Dutta and Roy, 2002). This explains the importance of considering soil-structure interaction. This interaction issue depends on the constitutive model used for soil media and foundation (Wang et. al., 2005). Dutta and Roy (2002) stated that “Emphasis has been given on the physical modeling of the soil media, since it appears that the modeling of the structure is rather straight forward.” Thus, the constitutive model should be selected by considering accurate simulation of the action of the soil media (Wang et. al., 2005). Fang, H.Y. (1991) stated that since the loading is below from the yielding load level with a high factor of safety accurate vertical stresses would be obtained with acceptable errors from the linear elastic solutions. Moreover, Moayed and Janbaz (2008) stated that Winkler approach and the elastic continum model are sufficiently accurate model used by the researchers and the engineers.

2.1 Previously Proposed Methods for Foundation Modelling

Although it is more important to accurately model the soil media, it is known that neither assuming foundations to be as perfectly rigid nor perfectly flexible is indeed true. To design mat foundations there are many methods that can be cathegorized under two topics (Coduto, 2001):

i) Rigid Methods

According to rigid methods, soil bearing pressure under loaded portions, i.e. column and wall locations for rigid and flexible foundations are shown in Figure 2.1. From the figure, it is obvious that bearing capacity under those zones are larger than the span locations.

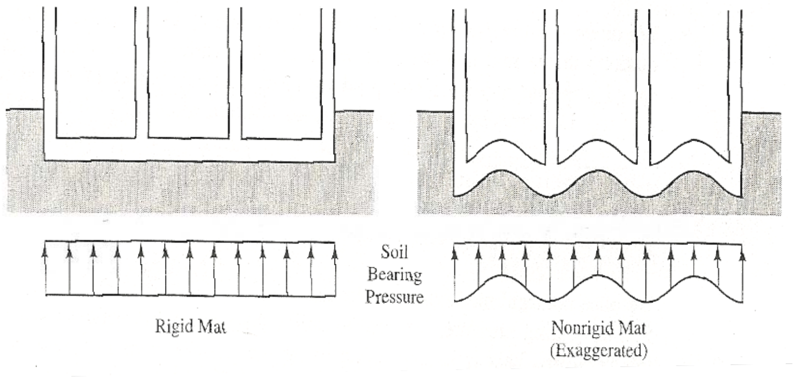

Figure 2.1 The rigid method assumes there are no flexural deflections in the mat, so the distribution of soil bearing pressure is simple to define. However, these deflections are important because they influence the bearing pressure distribution (Coduto, 2001)

The redistribution of contact pressure occurs if the soil is more stiff than the mat as illustrated in Figure 2.2.

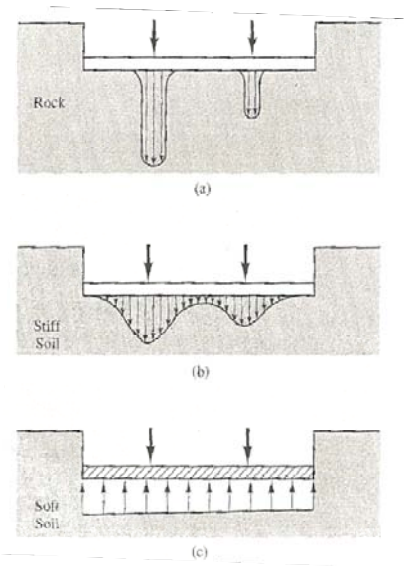

Figure 2.2 Distribution of bearing pressure under a mat foundation; (a) on bedrock or very hard soil; (b) on stiff soil; (c) on soft soil (Adapted from Teng, 1962 by Coduto, 2001)

Note that, since mat foundations have smaller t (=foundation thickness) / B (= foundation width) ratio than the spread footings they do not accurately model mat foundations.

ii) Nonrigid methods

Through nonrigid methods, only Winkler method and FEM will be briefly explained in the followings since they are related with the scope of the study.

1) Winkler Method

According to Winkler hypothesis the complex soil behavior is modelled by much simpler linear elastic system: subgrade model. Subgrade model is based on mutually independent, closely spaced, discrete and linearly elastic springs having stiffness k_s

(Sadrekarimi and Akbarzad, 2009). Spring stiffness is named as subgrade reaction coefficient (modulus of subgrade reaction). Subgrade reaction coefficient represents the required load for unit settlement over unit square area (Dutta and Roy, 2002). So, subgrade reaction coefficient is given as:

$$k_s = \frac{P}{y} \tag{2.1}$$

Where;

P: Contact pressure
y: Settlement

at any point.

However, this simplified approach is based on some approximations; for example it does not consider shear stresses under foundation, or coupling of springs. Moreover, because of the nonlinear, stress-dependent, anisotropic and heteregeneous nature of soil this model is insufficient to model the soil (Moayed and Janbaz, 2008).

2) Finite Element Method

Finite element method (FEM) is commonly used by engineers to model the soil-foundation-structure system. Reasons for the spread usage of FEM is, the possibility of modeling complex ground conditions with high degree of accuracy by including nonlinear stress-strain behavior of soil, non-homogeneous material conditions, changes in geometry and so on. In addition, FEM provides the option of three-dimensional modelling of the system and the option of considering discontinuous behavior at interfaces. Discretizing the system into a number of elements and using FEM has become the most widely used tool for solving soil-foundation interaction problems because of the benefits beforementioned (Dutta and Roy, 2002).

Small (2001) compared deformation of the foundation obtained from three-dimensional finite element analysis with the values measured at an instrumented foundation and proposed that results are compatible with each other. (Natarajan and Vidivelli, 2009).

Dutta and Roy mentioned in 2002 that to use elasto-plastic stress-strain behavior is important in soil foundation interaction problem, since when the load is applied on soil the strains may fall into elastic range up to certain stress level, after this it may enter in the plastic range depending on the magnitude of the applied load.

Because of the several prescribed reasons, it is necessary to use FEM to simulate the actual behavior of soil and soil-foundation interaction under the applied loads. PLAXIS 3D is a finite element code for soil and rock analyses, originally developed for analysing deformation and stability of the soil-foundation system in geotechnical engineering projects (Cui et. al., 2006). PLAXIS 3D allows the user to select an appropriate model for the soil layer in the problem. For example, soft soil, creep soft soil, hardening soil and Mohr-Coulomb models (as stated in Sadrekarimi and Akbarzad, 2009 according to PLAXIS 3D manual). To select the most appropriate model, one should also be careful about the accuracy with which the parameters involved with the model can be evaluated (Dutta and Roy, 2002).

One of the most commonly used model is Mohr-Coulomb model in order to generate the elasto-plastic behavior of soil media (Cui et. al., 2006). Yield criteria of the model is the extension of Coulomb's friction law to general states of stress (PLAXIS). Although Plaxis software allows to define the dependence of modulus of elasticity on the stress level in some of the other models, Mohr-Coulomb model can allow only to insert the increase of Young's modulus per unit depth. Note that, variation of Young's modulus with depth and with stress level is not same since the effect of specific volume (or void ratio) on Young's modulus is not represented by relating Young's modulus to depth. Thus, Mohr-Coulomb model is applicable to the conditions where the assumption of no dependence between effective stress and Young's modulus is realistic (Sadrekarimi and Akbarzad, 2009).

Moreover, Mohr-Coulomb model is generally used for drained conditions since it follows effective stress path (PLAXIS 3D Foundation Manual, 2007).

As previously explained Mohr-Coulomb method is based on elastic-perfectly plastic yield criteria. An elastic-perfectly plastic constitutive model consists of fixed yield surface that is not affected from the plastic straining. Furthermore, strains beneath the plastic strains is purely elastic and all are reversible as shown in Figure 2.3.

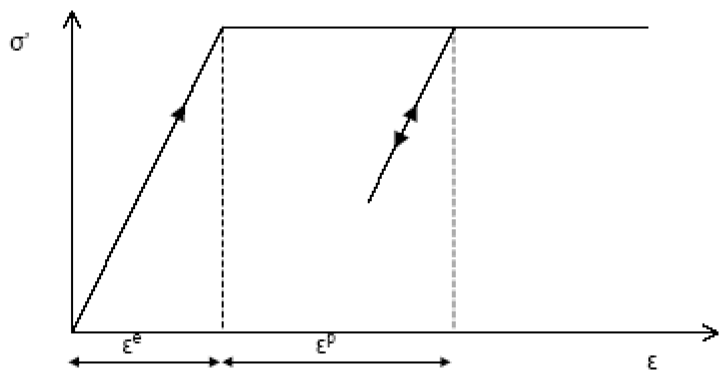

Figure 2.3 Basic idea of an elastic perfectly plastic model (PLAXIS 3D Foundation Manual, 2007)

The Mohr Coulomb model involves five input parameters: E (Young's modulus) and ν (Poisson's ratio) for soil elasticity; φ (angle of shearing resistance) and c (cohesion) for soil plasticity and ψ (angle of dilatancy) (PLAXIS 3D Foundation Manual, 2007).

2.2 Factors Affecting the Foundation-Soil System Behavior under Uniform Loading

2.2.1 Factors Affecting Contact Stresses at Soil-Foundation Interaction under Uniform Loading

As previously stated, contact stress distribution is the essential parameter at soil-foundation interface. For ideal modeling the foundation system, realistic contact stress distributions should be considered. Contact stress distribution depends on the foundation behavior (whether rigid or flexible: two extreme cases) and nature of soil deposit (cohesive or cohesionless soil) (Dutta and Roy, 2002). The contact stress distribution under the base of shallow foundations subjected to uniform loading under clayey and sandy soils for two extreme cases of foundation rigidity are given in Figure 2.4.

As seen from Figure 2.4 (a) and (b), for flexible foundations uniform bearing pressure with variable settlements and from Figure 2.4 (c) and (d) for rigid foundations uniform settlement with variable contact stresses are developed. Moreover, since real spread footings close to perfectly rigid, contact stress distribution is not uniform. Nevertheless, for simplicity contact stress distribution is assumed to be uniform to ease the calculation of bearing capacity and settlement (Figure 2.4(e)). The error due to this assumption is not significant (Coduto, 2001). However, this is obviously incorrect from a soil mechanics point of view (Fang, 1991).

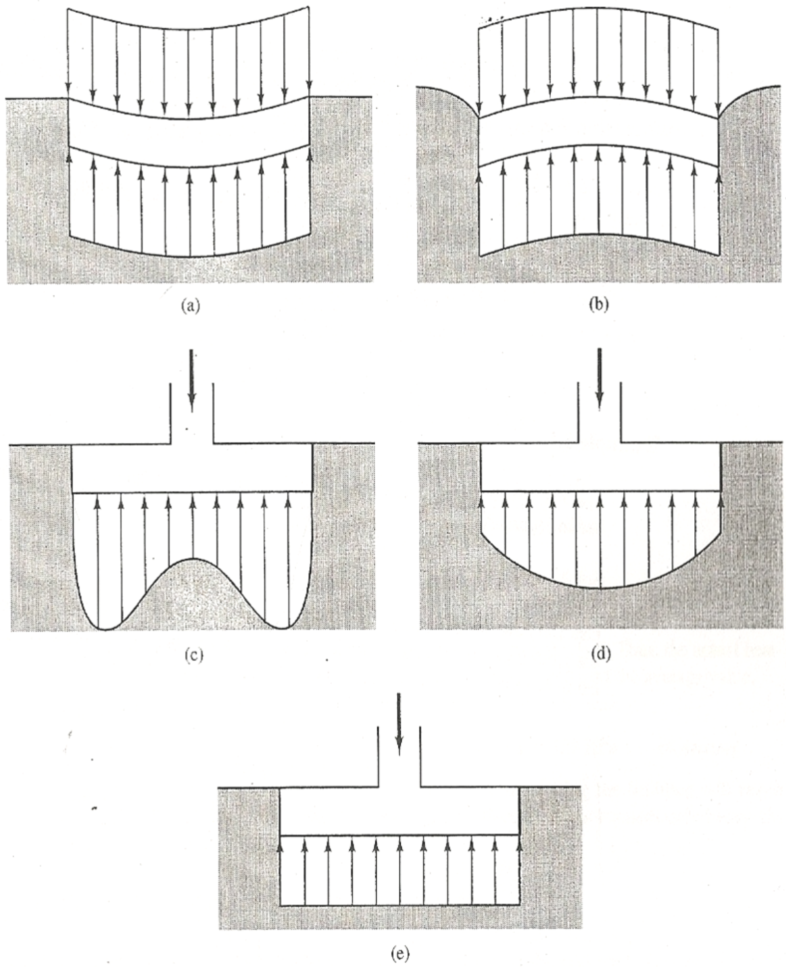

Figure 2.4 Distribution of bearing pressure along the base of shallow foundations subjected to concentric vertical loads: (a) flexible foundation on clay, (b) flexible foundation on sand, (c) rigid foundation on clay, (d) rigid foundation on sand and (e) simplified distribution (Taylor, 1948 by Coduto, 2001)

Fang (1991) explained the behavior illustrated in Figure 2.4 in more detail. Distortion settlement of foundation is the result of change in shape of soil mass rather than the change in volume where the shape of soil mass is related to whether the soil is cohesive or cohesionless and whether the foundation is rigid or flexible.

For rigid foundations resting over cohesive soils at the outer edges of the foundation in actuality stresses are limited by the shear strength of the soil. Whereas, for the rigid foundations resting over cohesionless soil, since the confinement is less at the outer edges, the stresses are also less. For this case, under very wide footings, settlements would be fairly uniform where contact stresses would be quite uniform. On the other hand, for flexible foundations resting over the cohesive soils, settlement profile would be concave upwards as shown in (Figure 2.4(a)). Oppositely, for flexible foundations resting on cohesionless soils settlement profile would be concave downward due to the less stress confinement at edge locations and relatively higher degree of confinement in the center. For this case, under very wide footings, settlements would be much uniform.

One should note that the deformation characteristics of the sand are a function of depth, because the modulus of elasticity of sand inreases with increasing depth (Terzaghi 1955). This concept is one of the main reasons of the difference between sandy and clayey soils that should be considered while modelling the soil media.

It is understood that altough the total of contact stresses under the area of shallow foundations must be equal to applied force, the pressure is not distributed evenly. As Coduto (2001) states, indeed actual contact stress distribution depends on many factors, including:

- Stress-strain properties of the soil
- Structural rigidity of the foundation
- Eccentricity, if any, of the applied load
- Magnitude of the applied moment, if any
- Roughness of the bottom of the foundation

2.2.1.1 Effect of Soil Stiffness (Stress-Strain Properties of the Soil) under Uniform Loading

As previously stated, the contact stress distribution is related to the stress-strain properties of the soil. For instance, since in cohesionless soils mostly which the drained behavior is commonly experienced, modulus of elasticity of soil is affected from the variation in effective average stress. This variation leads to diffences in contact stress distribution within cohesionless soils and cohesive soils (under undrained conditions mostly).

Moreover, instead of using terms such as“rigid foundation” or “flexible foundation”, it is more meaningful and realistic to classify the foundation relatively rigid or flexible with respect to subgrade soil. This concept is studied by many researchers in terms of a relative stiffness factor (Horikoshi and Randolph, 1997). The importance of the modulus of elasticity of soil in order to define whether the foundation is “relatively” rigid or flexible, is obvious in those relative stiffness factor definitions. Furthermore, definition of relative stiffness (K_r) also involves foundation thickness for the foundation stiffness which also affects the contact stress distribution (Chandrashekhara and Anony, 1996).

2.2.1.2 Effect of Foundation Thickness (Structural Rigidity of Foundation) under Uniform Loading

Dutta and Roy (2002) stated that, contact stress distribution depends on the rigidity of the structure (including foundation) in addition to the load-settlement characteristics of soil.

As mentioned above, there are two extreme cases: if the foundation can be considered as behaving flexible, loads are fixed and not depend on the foundation;

oppositely, if the structure can be considered as rigid, where settlements can be easily calculated (Breysse et. al., 2004).

Most of the structural design codes and specifications suggest a linear uniform contact stress distribution under the rigid spread footings. Nonetheless, shallow foundations may also be flexible generally if the footing is excesively long/wide and thin. However, as foundation rigidity increases with respect to underlying soil, maximum pressure and minimum pressure approaches to each other on the observed section, in other words soil pressure is uniformly distributed for rigid footings as seen in Figure 2.5 where K'_r is the ratio of foundation stiffness to the soil stiffness (Tabsh and Al-Shawa, 2005).

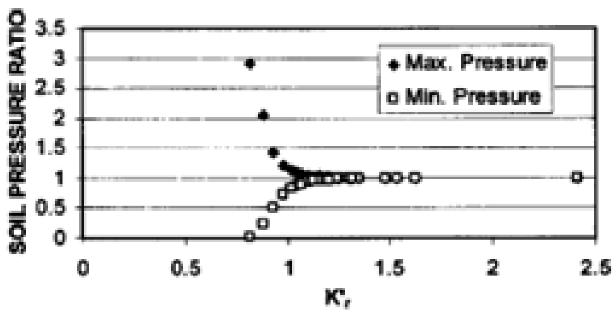

Figure 2.5 Results for concentratedly loaded square footings soil pressure (Tabsh and Al-Shawa, 2005)

Cui et. al. (2006) studied a footing having width of 1m over clayey and sandy soils for different soil properties, foundation stiffnesses and load levels in PLAXIS 2D software. They obtained different contact stress distributions at soil surface under varying flexural rigidity. PLAXIS analysis show that the soil modifies the shape of the stress distribution at the edges of the soil-foundation interface which: a parabolic shape is obtained for sand, on the other hand a U-shaped distribution is obtained for clay. Moreover, the flexural rigidity of the beam affects the shape of contact stress distribution which alters from a homogeneous (uniform) in rigid foundations to an inhomogeneous distribution (parabolic or U-shaped) in flexible foundations having zero stiffness. These results exactly agree with the theoretical contact stress distributions for different soil types (Cui et. al., 2006).

Contact stress distribution under rigid and flexible footings over sand and clay are illustrated in Figure 2.6.

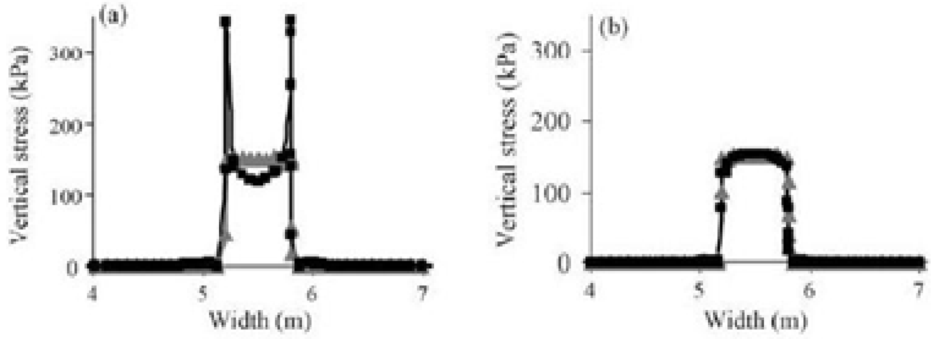

Figure 2.6 Calculated vertical stress distributions on the soil surface with a very soft plate (grey triangles) or a very rigid plate (black squares) on (a) a clay, (b) a sand (Cui et. al., 2006)

Cui et. al. (2006) also state that "as foundation flexural rigidity increases the position of maximum stress moves from the center towards the edge of the loading area" for clayey soils. Parallel to Cui et. al. (2006), Borowicka (1936) obtained the same behavior that for an absolutely rigid footing the contact distribution is saddle-shaped with minimum stress at the center and maximum at the edge of the foundation (Bose and Das, 1995) for clayey soils.

A three-dimensional plot of contact stress / applied average load pressure is obtained by (Wang et. al. ,2003) as seen in Figure 2.7. It is obvious that there is stress concentration along the edges of the plate, especially at corners of the plate. For internal points, contact stress is almost uniform.

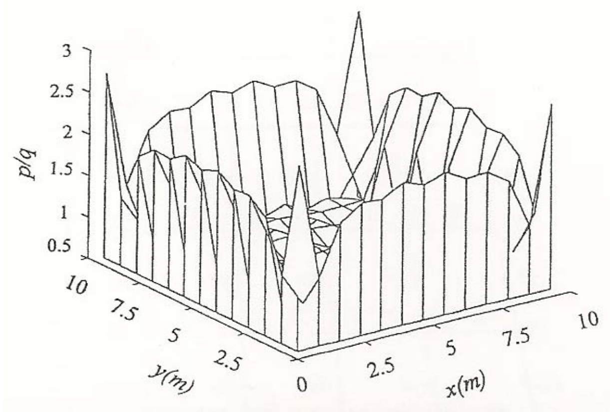

Figure 2.7 Contact pressure distribution beneath a square plate on a stratum (Wang et. al. ,2003)

2.2.1.3 Effect of Level of Applied Loading under Uniform Loading

As loading level increases, only the values of contact stresses increase where the distribution is same (Bose and Das, 1995).

Cui et. al. (2006) justifies this statement that they obtained same contact stress distributions at the surface of clay with a very rigid circular plate under 150 kPa and 180 kPa. The only difference is the difference between maximum stress and minimum stress is greater for 180 kPa than 150 kPa loading (Figure 2.8).

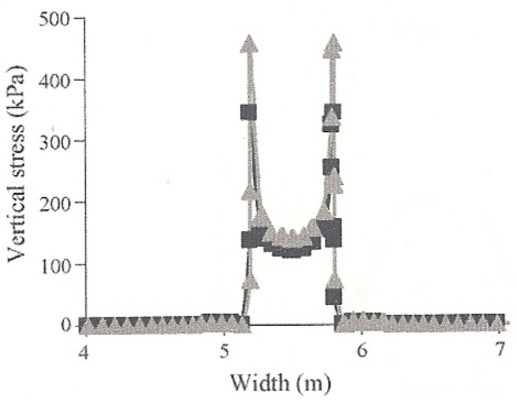

Figure 2.8 Calculated vertical stress distributions on a clay surface with a very rigid plate using a uniform applied stress q = 150 kPa (black squares) and a higher value 180 kPa (grey triangles) (Cui et. al., 2006)

Moreover, they noted that as the applied stress increases, more plastic points appear at the edges. This result is agreeing with the mechanics of the contact since for the elastic solids the influence of the solid by a rigid flat punch leads to stresses which the maxima develops at the edge of the punch as Johnson stated in 1985 (Cui et. al., 2006).

2.2.1.4 Effect of Point Loading Instead of Uniform Loading

Effect of column (point) loading instead of uniform loading on the contact stresses would be briefly explained under Section 2.3.

2.2.2 Factors Affecting Foundation Settlement under Uniform Loading

As Reznik (1998) mentioned, footing settlements depend on many variables which include mechanical properties of footing materials, footing shapes and dimensions, strength and deformation characteristics of supporting subgrades, and the depth of footing installation.

Mayne and Poulos (1999) and Bowles (1982) stated that for the simple case of a uniformly loaded (flexible) square footing having width of B and smooth base resting over a semiinfinite elastic half-space with constant Young's modulus with depth, the maginitude of settlement at the centerpoint is given by (e.g., Brown, 1969):

$$\rho = \frac{qBI(1-\upsilon^2)}{E_s} \tag{2.2}$$

Where, *I,* influence factor is the product of several influence factors depending on finite layer thickness, foundation rigidity and foundation embedment.

From the elastic settlement equation it can be understood that for a uniformly distributed foundation, settlement decreases by the increase of foundation thickness under any point. Moreover, at infinite rigidity, settlement under all points becomes equal to each other (Wang et. al., 2000).

It is found that relative stiffness of foundation to the stiffness of soil also affects vertical footing displacements besides the contact stresses. As foundation rigidity increases with respect to underlying soil, difference between the maximum and minimum settlement decreases under the footing for the section and becomes uniform as seen in Figure 2.9 (Tabsh and Al-Shawa, 2005).

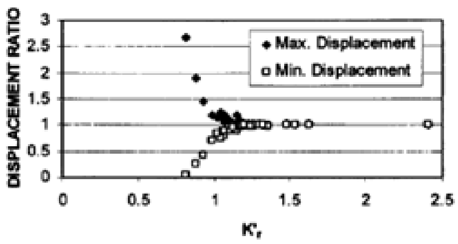

Figure 2.9 Results for concentratedly loaded square footings vertical displacement (Tabsh and Al-Shawa, 2005)

Here it is obvious that if a flexible footing is analyzed as rigid, the maximum soil pressure and vertical footing displacement would be underestimated (Tabsh and Al-Shawa, 2005).

Wang et. al. (2003) studied the effect of foundation thickness on the foundation settlement by assuming the other parameters are unchanged for foundation width 10 m . Two extreme thicknesses are studied: t = 0.1 m (very flexible plate) and t = 3 m (rather thick plate). Figure 2.10 illustrates the variations of w_A (the deflection at Point A, the center of the plate), w_B (the deflection at Point B, the mid-edge of the plate) and w_C (the deflection at Point C, the corner of the plate) with t. Consequently it is found that w_A decreases as t increases whereas, w_B and w_C increases as t increases. Furtermore when t is rather large (≥ 1.5 m), the settlement of the foundation is almost uniform. Moreover, when the thickness is smaller than 1.5 m, variation in settlement is more significant whereas, as thickness increases settlement distribution converge to the uniform.

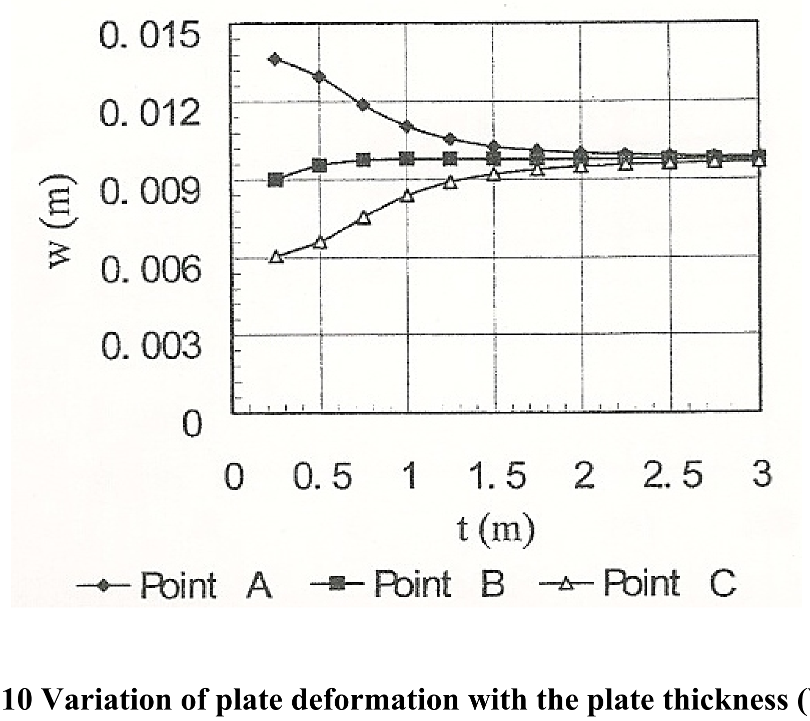

Figure 2.10 Variation of plate deformation with the plate thickness (Wang et. al. ,2003)

Wang et. al. (2003) plotted the deflection of foundation as seen in Figure 2.11 and stated that the deflection at the center of the plate has the maximum value and those at the corners are smallest. Moreover, as the foundation thickness increases, the settlement is more uniform.

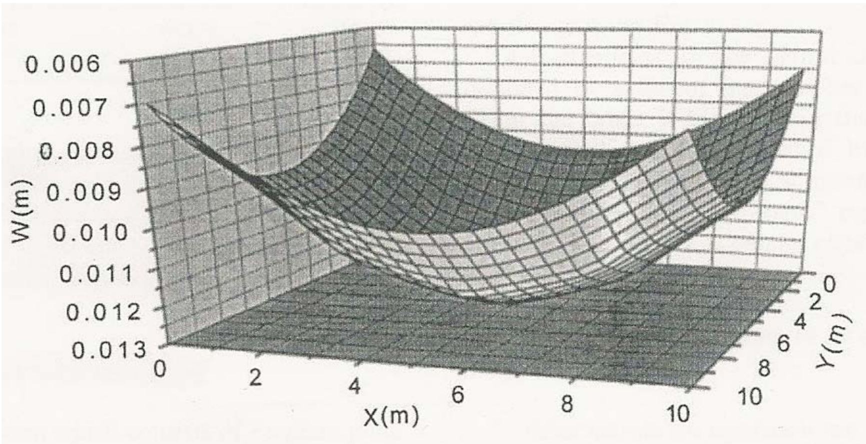

Figure 2.11 Three dimensional deformation of a square plate on a stratum (Wang et. al. ,2003)

Davis and Poulos (1968) mentioned that one may obtain an approximation to the uniform displacement of a rigid footing from the maximum and minimum displacements of a uniformly loaded area of the same shape as footing since the rigid footing settlement is known to be close to the mean displacement of the uniformly loaded area.

As Horikoshi and Randolph (1997) stated according to Small and Booker (1986) the average settlement of the raft is largely independent from the raft thickness and can be estimated by elastic and non-linear approaches.

To sum up as Reznik (1998) stated “Footing settlements depend not only on physical and mechanical properties of base soil, but also on applied load intensities and their distributions with depth, as well as on footing rigidity, shape and dimensions”.

2.2.3 Factors Affecting Subgrade Reaction Coefficient under Uniform Loading

The value of the coefficient of subgrade reaction depends on various factors such as (Coduto, 2001):

-The width of the loaded area: settlement of wider mat will be more than a narrower one for same applied load since it mobilizes the soil to a greater depth.

-The shape of the loaded area: contact stresses below long narrow loaded areas are different from those below square loaded areas.

-The depth of the loaded area below the ground surface: At greater depths, the change in stress in soil due to applied load is a smaller percentage of the initial stress, so the settlement is also smaller and k_s is greater.

-The position on the mat: to model the soil accurately k_s needs to be larger near the edges of the mat and smaller near the center.

Bowles (1982) also added that there is a direct relationship between E_s and k_s.

There are many different techniques to calculate k_s that some are based on plate load tests for in-situ estimation. Many researchers studied on evaluation of subgrade reaction coefficient (modulus of subgrade reaction), k_s. Terzaghi (1955) recommended ks values for a 0.305 x 0.305 m (1 x 1 ft) rigid slab placed on a soil medium. According to Terzaghi (1955), the coefficient of subgrade reaction is not a fundamental soil property and it is “problem-specific”. Furthermore the coefficient of subgrade reaction depends on elastic characteristics of subgrade soil, the geometry of the footing and loading scheme (Sadrekarimi and Akbarzad, 2009). Moreover, (Coduto, 2001) noted that plate load tests are not good estimator of k_s for design of mat foundations, since:

- it is not accurate to compate the shallow zone of influence under the plate of plate load test with the much deeper zone below the mat foundation
- some correction factors should be used for differences in width, shape and depth of the mat for the Terzaghi equation (Equation 2.4)

In addition to those factors, Sadrekarimi and Akbarzad (2009) mentioned “if the rate of the variation of E_s with respect to depth is considerable, results of plate-load test cannot be reliable.”

Moayed and Janbaz (2008) stated that the subgrade reaction coefficient depends mainly on parameters like soil type, size, shape and type of foundation. A plate load test over 30 - 100 cm diameter circular plate or equivalent rectangular plate is used to estimate the subgrade reaction coefficient directly. The estimated k_s values should be extrapolated for the exact foundation dimension. Although in practice Terzaghi equation is commonly used in order to estimate k_s values, there are some uncertainities in utilizing the equation (Moayed and Janbaz, 2008). Similarly, Daloğlu and Vallabhan (2000) stated that the implementation and the procedure to evaluate a k_s value in a larger slabs is not specific.

Moreover, as Bowles (1982) k_s can be obtained from elasticity theory by rewriting the elastic settlement equation of rectangular plates overlying on elastic half-space as:

$$k_s = \frac{E_s}{B(1-\upsilon_s^2)} \tag{2.3}$$

Sadrekarimi and Akbarzad (2009) found out the Biot and Vesic relations, the equation obtained from elastic theory are appropriate for calculation of ks. Moreover, contact stresses and settlements under the foundation calculated from theory of elasticity and Biot relation are so similar.

Daloğlu and Vallabhan (2000) deducted that for the analysis of slabs loaded by uniformly distributed loads and studied for constant value of subgrade reaction coefficient, displacements would be uniform and there would be no bending moments and shear forces, which is far from the reality. Thus, the variation of modulus of subgrade reaction should be considered Moreover, it is added Bowles (1988) and Coduto (1994) stated that the k_s has to be increased on the edges of the slab and more research is needed on this issue (Daloğlu and Vallabhan, 2000). Thus Daloğlu and Vallabhan noted in 2000, "if one uses a constant value of the modulus of subgrade reaction for a uniformly distributed load, the displacements are uniform and there are no bending moments and shear forces in the slab, in order to get realistic results, higher values of k_s have to be used closer to the edges of the slab."

Moayed and Janbaz (2008) studied the effect of size of foundation on clayey soil by using finite element software, Plaxis 3D and compared their results with the formulation recommended by Terzaghi (1955) which is:

$$k_s = k_{sp}\frac{B_1}{B} \tag{2.4}$$

Where

B_1: side dimension of square base used in the plate load test to produce k_s
B: side dimension of full size foundation
k_{sp}: the value of k_s for 0.3 x 0.3 bearing plate or other size load plate
k_s: desired value of the modulus of subgrade reaction for the full size foundation.

Terzaghi (1955) stated this equation becomes inaccurate when B/B1≥3. Moreover, Bowles (1977) added that this equation is almost inaccurate under every condition that k_s (subgrade reaction coefficient) of a footing having 3 m width is never be the 10 % of a 0.30 m plate (Moayed and Janbaz, 2008).

In the article of Moayed and Janbaz (2008), authors concluded that there is a good compatibility between finite element results and results obtained from in-situ plate load test and the k_s is decreased as side dimension of plate increases. However, the equation is failing for larger foundation width that it underestimates with respect to finite element results.

Kany (1974) found out that the settlement of foundation is same for both square and strip foundations at surface level whereas, the difference increases as investigated depth / foundation width increases.

2.2.4 Factors Affecting Shear Forces and Bending Moment under Uniform Loading

It is found that as the raft-soil stiffness ratio increases, differential settlements and the bending moments increase (Horikoshi and Randolph, 1997).

Tabsh and Al-Shawa (2005) studied on the same issue and proposed that since the flexibility of spread foundation is less affected from the applied load, foundation can be assumed as rigid so that shear forces and bending moments can be calculated easily and conservatively. They also claimed shear forces are less affected than the bending moments from the variation in foundation stiffness. Moreover, it is found that, relative stiffness of foundation to the stiffness of soil affects soil pressures,, vertical footing displacements, shear forces and bending moments. On the other hand, shear forces increase and bending moments are less affected from the variation in relative stiffness as shown in Figure 2.12 (Tabsh and Al-Shawa, 2005).

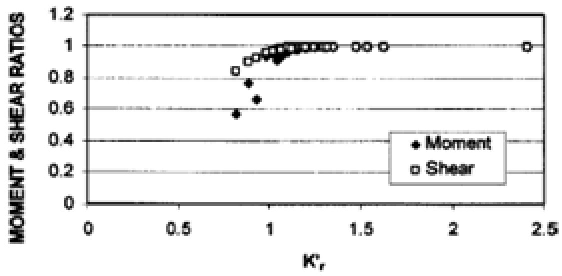

Figure 2.12 Results for concentiracally loaded square footings for shear and moment (Tabsh and Al-Shawa, 2005)

Chandrashekhara and Anony (1996) stated that settlement of foundation and the developed bending moments on it also depend on the soil behavior.

Bowles (1982) added that, bending moment is not affected from the variations in the modulus of subgrade reaction due to the fact that the flexural rigidity of the foundation is so larger than the soil. Furthermore, because coefficient of depth is zero in the evaluation of modulus of subgrade reaction, the effect of depth of foundation is not significant (Bowles, 1982).

2.3 Factors Affecting the Foundation-Soil System Behavior under Column Loading

Terzaghi (1955) stated that to explain the influence of the area of application of the load on the foundation on the value of subgrade reaction coefficient, bulb pressure concept can be used. The bulb pressure is arbitrarily defined as the space within the vertical normal stresses in soil are greater than the quarter of the normal applied pressure. However, replacing quarter with another value does not change the conclusions since the concept is used to visualize the actual stress condition in the loaded soil.

According to Terzaghi (1955) the most of the load is transferred on to the subgrade soil within a distance of R from the point of load application and beyond this distance the settlement of the base of the slab is very small so the disturbtion of foundation is very small. Thus, beyond this distance influence on the maximum bending moment in the slab is so small. R is defined as (Terzaghi, 1955):

$$R = \left(\frac{10Eh^3}{3(1-\upsilon^2)k_s}\right)^{0.25} \tag{2.5}$$

and R is “referred to as the range of influence of the concentrated load an that portion of the mat which is located within a distance R from the point of load application isthe equivalent circular footing”.

Figure 2.13 (a) shows a vertical section through a concrete mat having area of mB and nB carrying concentrated loads Q such as column loads spaced B in both directions over a deposit of stiff clay. The spacing B is assumed to be greater than twice of R. In this case the distribution of the stresses in the bulb of pressure of the load and the bending moments under the mat foundation is not changed.

On the other hand, if B is smaller than 2R, the bulb of pressure having 2R top diameter is illustrated in Figure 2.13 (b). As a result, it is seen that the level which the stresses become uniform, I-I, is high above than the bottom of bulbs. According to this, the compression of the soil below the I-I level has no influence on the deformation of raft. Thus, it would be reasonable to compute stresses by assuming that the range of influence of each load is B/2 and not R. Moreover, Terzaghi (1955) noted that the soil reactions on the interface would decrease from the points of load application towards the areas located between these points.

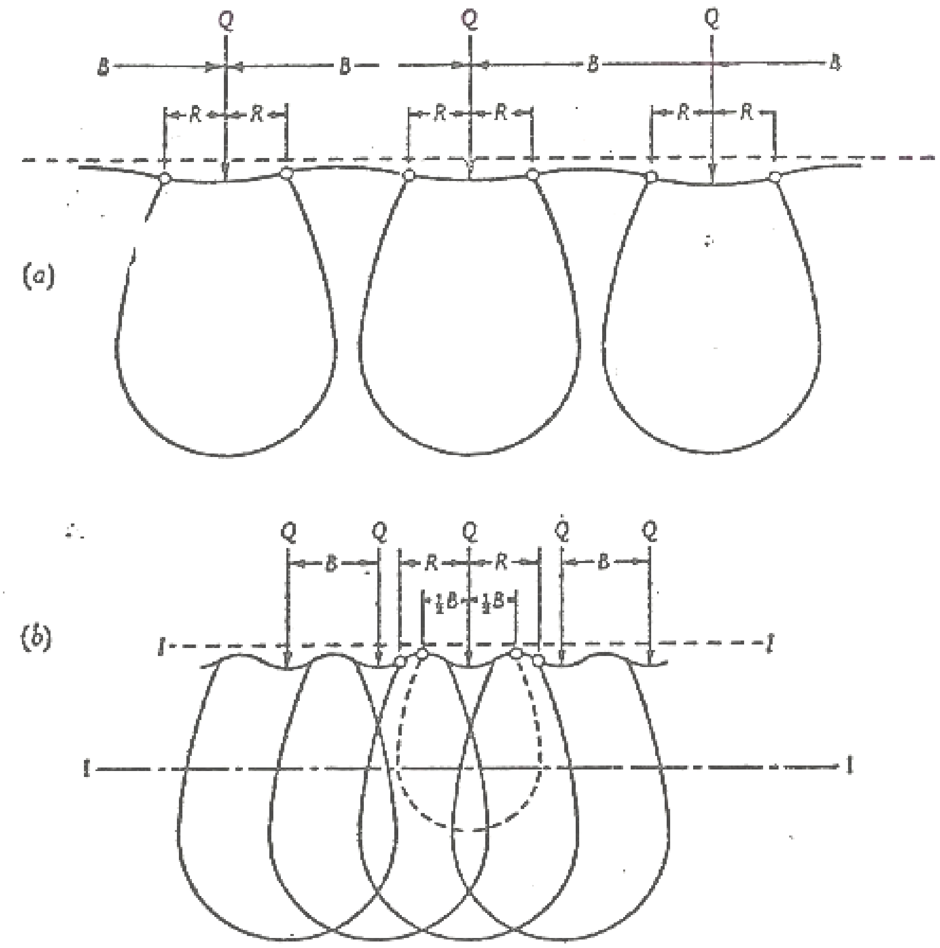

Figure 2.13 (a) and (b) bulb of pressure beneath the concentrated loads Q, equally spaced both ways acting on rectangular concrete mat (Terzaghi, 1955)

Furthermore, (Birand, 2001) stated that the overlapping stress bulbs under the support locations loaded by concentric loads lead to higher settlement.

Moreover, Cui et. al. (2006) states that the contact stress distribution is affected from the combination of soil properties, applied load level and beam characteristics.

2.3.1 Factors Affecting Contact Stresses at Soil-Foundation Interaction under Column Loading

Natarajan and Vidivelli (2009) studied a space frame-raft-soil system under static loads for different column spacings in order to comprehend the effect of it on contact stress, settlement an bending moment distribution at the interface.

Natarajan and Vidivelli (2009) concluded that:

- Effect of variation in column spacing on contact stress distribution is not so important.
- Since, contact stress distribution shows similar distributions for any foundation thickness, there is no effect of foundation thickness on contact stress distribution.
- For larger modulus of elasticity of soil, larger contact stresses develop under column support locations.
- Among foundation thickness and modulus elasticity of soil parameters, contact stresses are under greater influence of variation in modulus of elasticity of soil.

2.3.2 Factors Affecting Foundation Settlement under Column Loading

Natarajan and Vidivelli (2009) stated that, as column spacing increases foundation settlement increases significantly. In addition, for every column spacing the settlement at the centre of the raft was higher than the edge of the raft. The foundation settlement increases gradually as the column spacing increases from 3 m to 7.5 m. Thus column spacing has a major effect on settlement (Natarajan and Vidivelli, 2009).

For any column spacing, as modulus of elasticity of soil (E_s) increases settlement decreases at both edge and centre of the foundation. Settlement profiles showed similar trends for E_s=23 MPa and E_s=135 MPa. Whereas, settlements under each point are lower for E_s=135MPa than the settlements obtained for E_s=23 MPa. Moreover for larger E_s=135 MPa settlement under center and settlement under edge are almost same to each other irrespective to the column spacing. As a result, for higher modulus of elasticity of soil, lesser settlement occurs at mat foundation (Natarajan and Vidivelli, 2009).

Although settlement increases by the increase in E_s and/or decrease in the foundation stiffness, it is concluded that E_s has a dominant affect on the foundation settlement (Natarajan and Vidivelli, 2009).

Noorzaei et al (1991, 1995a, b) and Maharaj et al(2004) stated that by the increase of foundation rigidity, differential settlements significantly decreases (Natarajan and Vidivelli, 2009). However, foundation settlements decrease significantly as foundation thickness increases in the study of Natarajan and Vidivelli and they stated that the obtained results are parallel to the analysis of Viladkar et al (1991), Maharaj et al (2004) and Daniel and Illamparuthi (2007). This implies the importance of foundation for the settlement.

2.3.3 Factors Affecting Bending Moment under Column Loading

Natarajan and Vidivelli (2009) found out by the increase in column spacing, support moments increase considerably. For smaller column spacing, difference between the support moments and the span moments are lesser than the larger column spacing. As column spacing increases, moments at inner column locations increase.

On the other hand, Natarajan and Vidivelli (2009) stated that although bending moment variations show similar trends for both E_s=23 MPa and E_s=135 MPa, lower bending moments are encountered for E_s=135 MPa.

Change in foundation thickness leads to redistribution of contact stresses and bending moments. Span moments and edge moments are lower for smaller foundation stiffnesses regardless of the column spacing. Thus, as foundation thickness increases bending moments increase (Natarajan and Vidivelli, 2009).

CHAPTER 3.

PLAXIS ANALYSES OF PATTERN A AND PATTERN B

3.1 Finite Element Model

Three dimensional finite element model is built up by using PLAXIS 3D Foundation. The element use in analysis of three dimensional models is the 15-node wedge element that is composed of 6-node triangles for the entire model. For all the analysis homogeneous soil profile is defined as three-dimensional continuous isotropicly elastic layer in half-space.

In this study two different loading patterns are considered: uniform loading (Pattern A) and column loading (Pattern B) over a typical 42 m x 42 m square mat foundation which is overlying on soil under drained conditions. This main model is valid throughout all analyses unless any other information is given. Soil is modeled as Mohr-Coulomb material which demonstrates elastic perfectly plastic behavior. Since immediate settlements are considered as elastic settlements and there is not any loading-unloading cycle, the model is appropriate to be used (Plaxis 3D Foundation Materials Manual ver.2, 2007).

The Mohr-Coulomb soil parameters are illustrated in Table 3.1 and the parameters are changed within the ranges given in Table 3.2.

Table 3.1 Mohr-Coulomb model soil parameters

Soil Parameters
Unsaturated unit weight, γ_{unsat} = 19 kN/m^3
Saturated unit weight, γ_{sat} = 20 kN/m^3
Poisson's ratio, ν = 0.3
Cohesion, c_{ref} = 5 kPa
Angle of shearing resistance, φ = 30°

Table 3.2 Ranges of varying parameters

Parameter	Range of Variation
Modulus of elasticity, E	10 MPa – 100 MPa
Foundation thickness, t	0.30 m – 2.00 m
Loading, q	50 kPa – 300 kPa
Column spacing, s	5 m – 10 m

In order to determine the effects of those factors, for each analysis only one parameter is changed where others are kept constant. Furthermore, the results from various analyses are compared and interpreted.

Note that, since there is no water table in the studied conditions, it is not necessary to seperate the undrained and drained behavior from each other. Thus, the soil is not named as whether "sandy" or "clayey". Moreover, since it is found out that there is not a significant difference in numerical values and no difference in shape of the contact stress and settlement distributions, between constant modulus of elasticity of soil and variable modulus of elasticity of soil with respect to depth, in all analyses modulus of elasticity of soil with respect to depth is assumed to be constant (Figures 3.1 and 3.2).

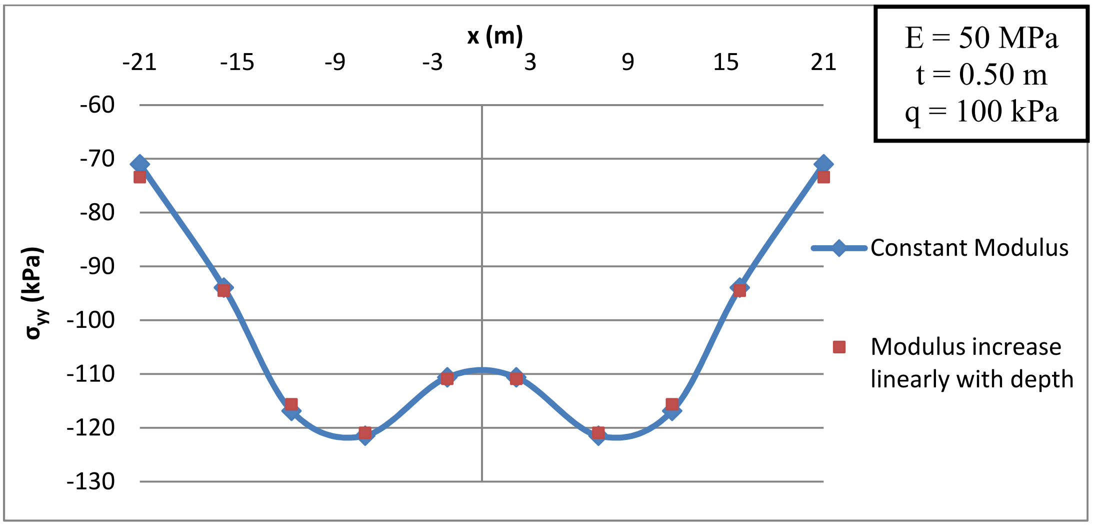

Figure 3.1 Comparison of contact stress distribution between constant E and variable E depending on depth (500 kPa / m) analysis in Mohr-Coulomb model

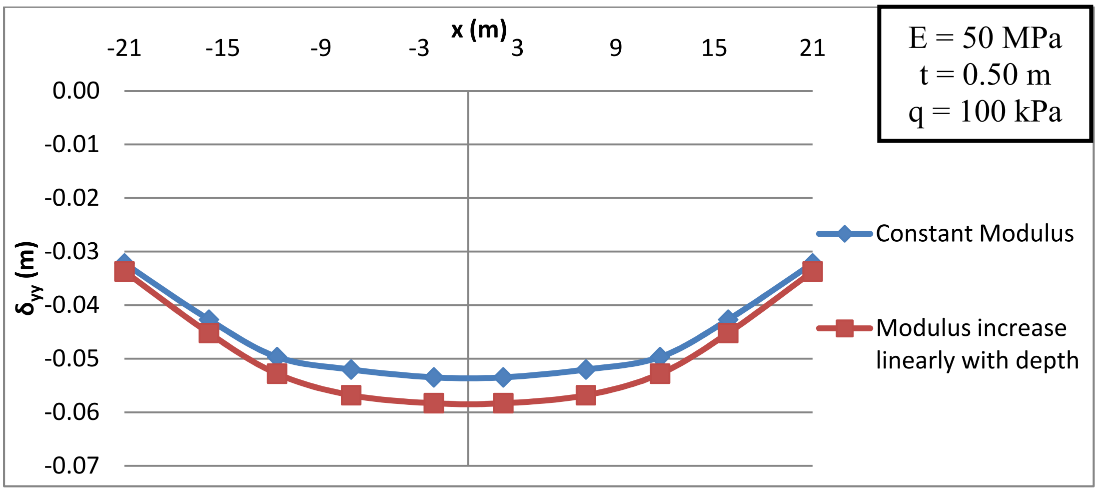

Figure 3.2 Comparison of settlement distribution between constant E and variable E depending on depth (500 kPa / m) analysis in Mohr-Coulomb model

3.2 Uniform Loading Case : Pattern A

Loading is distributed uniformly over the square mat foundation which is resting on the soil having prescribed properties. The model is performed step by step in 3 construction stages. Those stages are defined as:

Phase 0 : Initial phase
Phase 1 : Foundation construction
Phase 2 : Application of uniform loading (the distributed loading is activated by introducing the relevant value)

The calculated contact stresses and the developed settlements at each node are taken from different cross sections. Those cross sections for Pattern A are demonstrated in Figure 3.3.

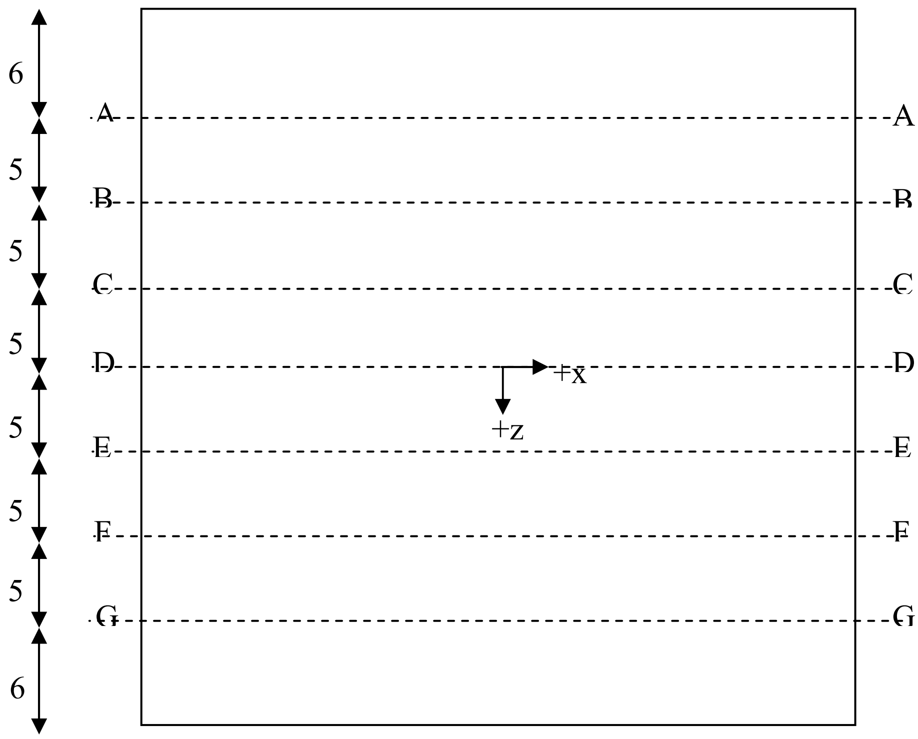

Figure 3.3 Plan view of the foundation model for Pattern A

For each cross section specified in Figure 3.3, the contact stresses σ_{yy} and settlements δ_{yy} for nodes located on each section are obtained and modulus of subgrade reaction , k is calculated by the Equation 3.1:

$$k = \frac{\sigma_{yy}}{\delta_{yy}} \tag{3.1}$$

Eventually, modulus of subgrade reaction values, k, are obtained for each node. By taking the mean of those values the average modulus of subgrade reaction, k_{ave}, are obtained. Although average modulus of subgrade reaction is calculated by considering various cross sections as illustrated in Figure 3.3, for comparison only the mid-cross section, D-D section, is considered in each analysis.

3.2.1 Effect of Deformation Modulus on Soil - Mat Interaction for Uniform Loading

As previously indicated, for the purpose of implying the effect of the modulus of elasticity (deformation modulus) of the subgrade soil, the following cases are analysed:

Applied uniform load : 100 kPa
Raft thickness : 0.50 m
Soil deformation modulus : Variable

Case 1-1: E = 10 MPa
Case 1-2: E = 25 MPa
Case 1-3: E = 50 MPa
Case 1-4: E = 100 MPa

The contact stress distribution obtained from the Plaxis analysis for Case 1-3 is illustrated in Figure 3.4.

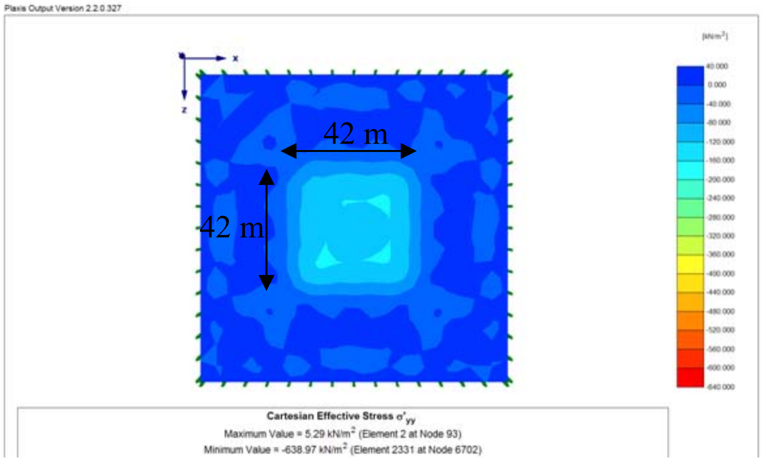

Figure 3.4 Contact stress distribution of Pattern A for Case 1-3

For each case, contact stress distribution, settlement distribution and modulus of subgrade reaction distribution through the mid-section are given in Figure 3.5, Figure 3.6 and Figure 3.7, respectively.

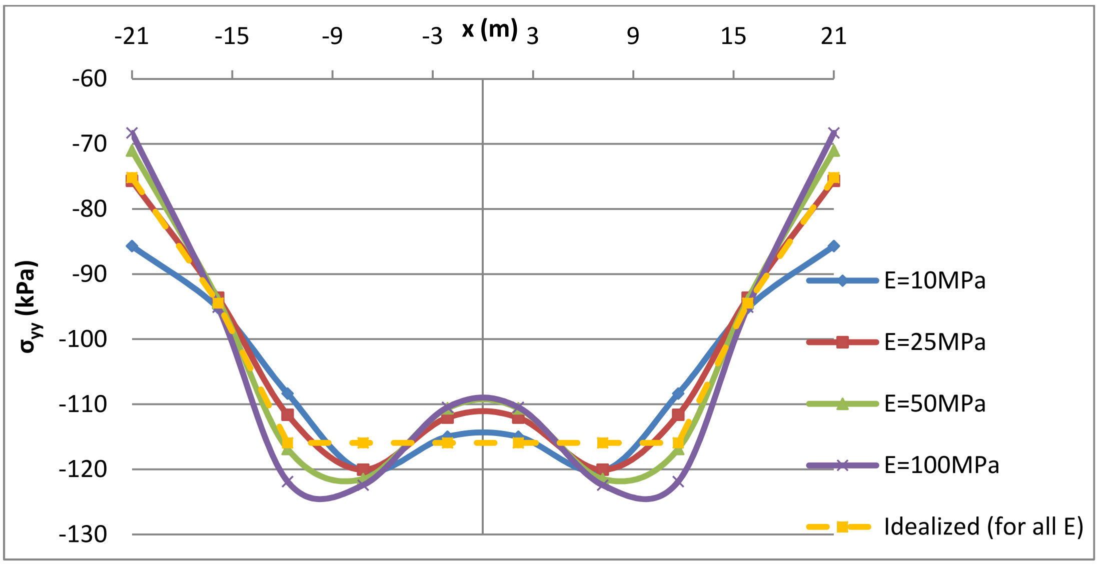

Figure 3.5 Comparison of contact stress distribution of Pattern A for Cases 1-1, 1-2, 1-3 and 1-4

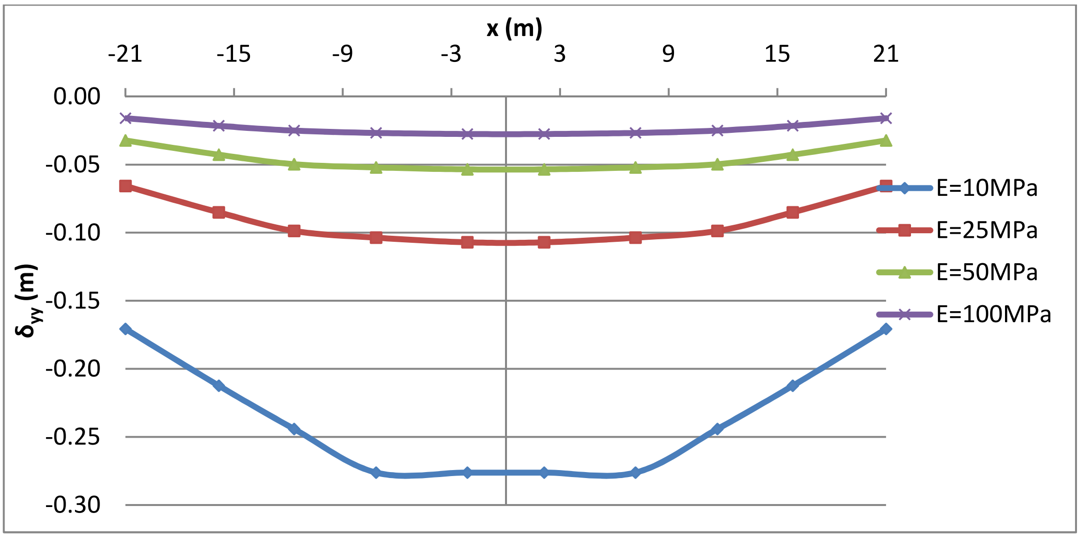

Figure 3.6 Comparison of settlement distribution of Pattern A for Cases 1-1, 1-2, 1-3 and 1-4

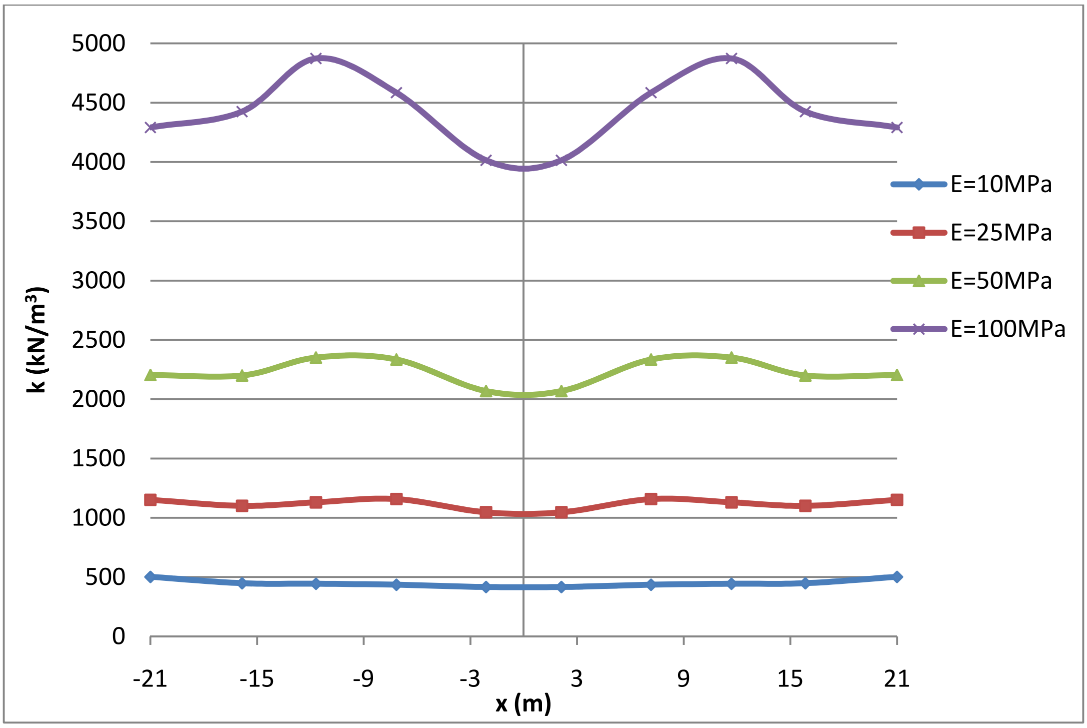

Figure 3.7 Comparison of modulus of subgrade rection of Pattern A for Cases 1-1, 1-2, 1-3 and 1-4

As seen in Figure 3.5, for idealized distribution average stress within -0.35B < x < 0.35B is 15% higher than applied stress (i.e. 115 kPa) irrespective of the soil modulus value. Moreover, modulus value significantly effects the contact stress distribution at points -0.35B < x and x > 0.35B. General trend is similar to the intermediate soil type proposed by Coduto (2001). This is expected since the soil is neither can be considered as cohesionless (c = 5 kPa) nor cohesive (φ_u = 0). In addition the figure implies that as deformation modulus increases, the contact stress difference between the points near to the edge (-0.35B < x and x > 0.35B) and points near to the center (-0.35B < x < 0.35B) of the foundation increases.

Figure 3.6 implies that for stiffer soil, the strains in the foundation soil is more uniform. The average foundation settlement decreases as deformation modulus of soil increases. For relatively softer soil (i.e.:E = 10 MPa) the differential settlements becomes larger (i.e.:angular rotations are being in the order of 6‰), whereas for relatively stiffer soil (i.e.:E = 100 MPa) differential settlements are significantly lower (i.e.:angular rotations are being in the order of 0.6‰. Thus it may be stated that angular rotations decrease with the increasing deformation modulus.

From Figure 3.7, it is obvious that the modulus of subgrade reaction at edges are less than the average of the modulus for E = 100 MPa. This variation in subgrade reaction coefficient values are more pronounced as the soil stiffness is increased. It is observed that the average modulus of subgrade reaction is directly influenced by the change in the average elastic settlement where the shape of the modulus of subgrade reaction distribution is significantly affected from the shape of contact stress distribution.

Figure 3.7 clearly shows that, modulus of subgrade reaction is not uniform under the mat foundation. It is seen that starting from center of the mat foundation subgrade reaction tends to increase within central zone of -0.30B < x < 0.30B, and beyond this region it tends to decrease with a flatter slope. This behavior is illustrated in Figure

3.8 and Table 3.3 that the variations in the modulus of subgrade reaction are idealized to regions. It is found that, unlike the footing having small plan dimensions with constant subgrade modulus, the modulus of subgrade reaction is not constant under the mat foundations.

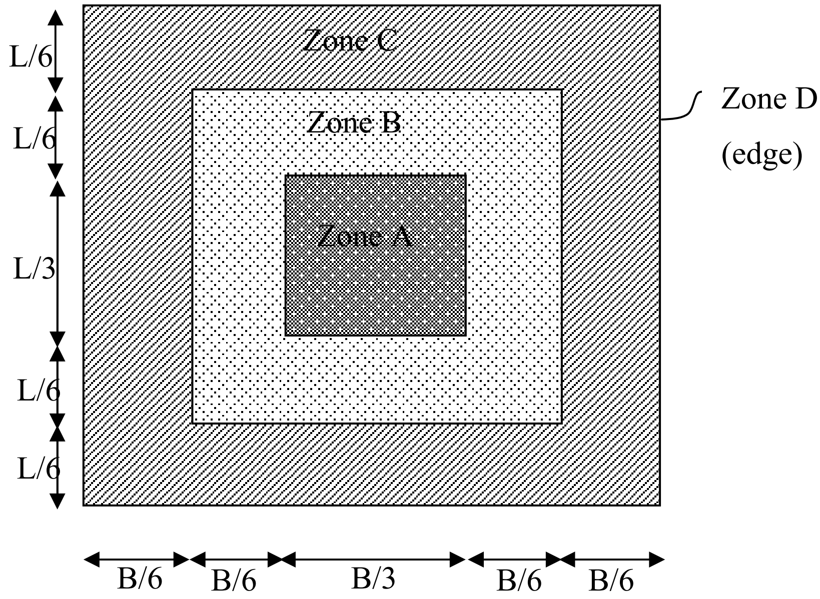

Figure 3.8 Modulus of subgrade reaction distribution over the mat foundation for Pattern A

Table 3.3 α values of zones defined in Figure 3.8 for Pattern A for t = 0.50 m

	α			
Zone	**E=10MPa**	**E=25MPa**	**E=50MPa**	**E=100MPa**
A	1.00	1.00	1.00	1.00
B	1.06	1.08	1.09	1.15
C	1.14	1.08	1.08	1.09
D	1.17	1.10	1.07	1.07

Where;

$$k = \alpha k_{ZoneA} \tag{3.2}$$

α: Ratio of average subgrade reaction coefficient in zones defined in Figure 3.8 to average subgrade reaction coefficient in Zone A for variable deformation modulus.

Comparision of Figure 3.5, Figure 3.6 and Figure 3.7, are summarized in Table 3.4

The variation in the contact stresses, settlements and modulus of subgrade reactions are summarized in Table 3.4 as a function of modulus of deformation of the foundation soil.

Table 3.4 Values of σ, δ and k depending on the variation in E for Pattern A

E (MPa)	σ/(q=100kPa)		$\sigma_{edge}/\sigma_{center}$	s_{edge}/s_{center}	k_{edge}/k_{center}	k_{max}/k_{min}
	Center	**Edge**				
10	1.15	0.86	0.75	0.62	1.21	1.21
25	1.12	0.76	0.68	0.61	1.10	1.11
50	1.11	0.71	0.64	0.60	1.07	1.14
100	1.10	0.68	0.62	0.58	1.07	1.21

3.2.2 Effect of Foundation Thickness on Soil - Mat Interaction for Uniform Loading

In order to determine the effect of foundation thickness, the following cases are considered:

Applied uniform load : 100 kPa
Soil deformation modulus : 50 MPa

Raft thickness : Variable

Case 2-1: t = 0.30 m

Case 2-2: t = 0.50 m

Case 2-3: t = 1.00 m

Case 2-4: t = 2.00 m

Note that, in order to implement only the effect of foundation rigidity, the weight of foundation is neglected in the analysis of Cases 2-1 to 2-4.

For each case, contact stress distribution, vertical deformation (settlement) distributions and modulus of subgrade reaction through the mid-section are all plotted in Figure 3.9, Figure 3.10 and Figure 3.11, respectively.

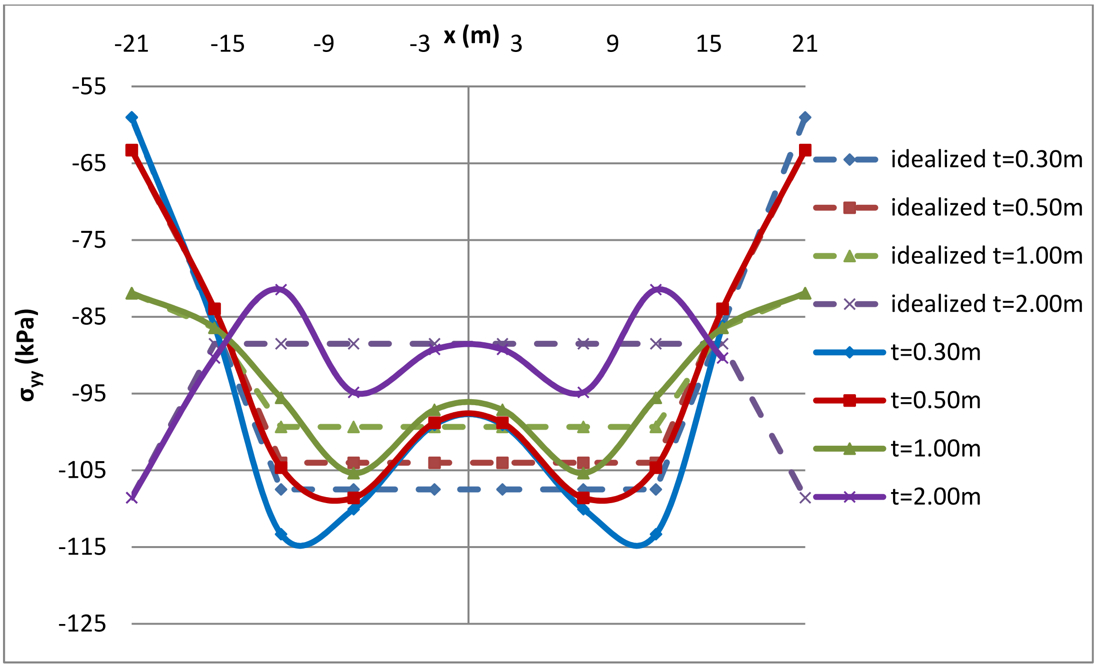

Figure 3.9 Comparison of contact stress distribution of Pattern A for Cases 2-1, 2-2, 2-3 and 2-4

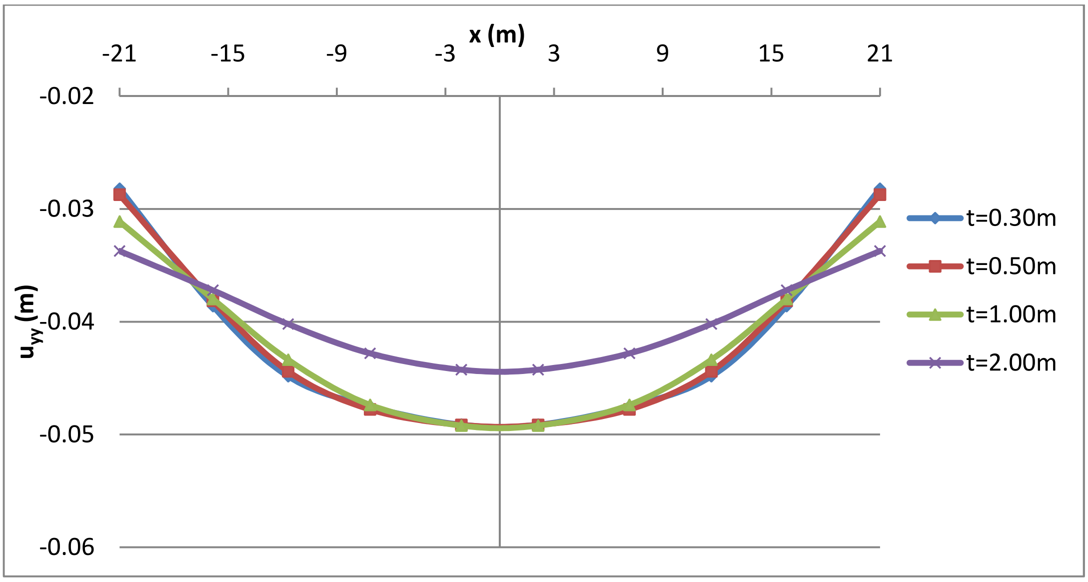

Figure 3.10 Comparison of settlement distribution of Pattern A for Cases 2-1, 2-2, 2-3 and 2-4

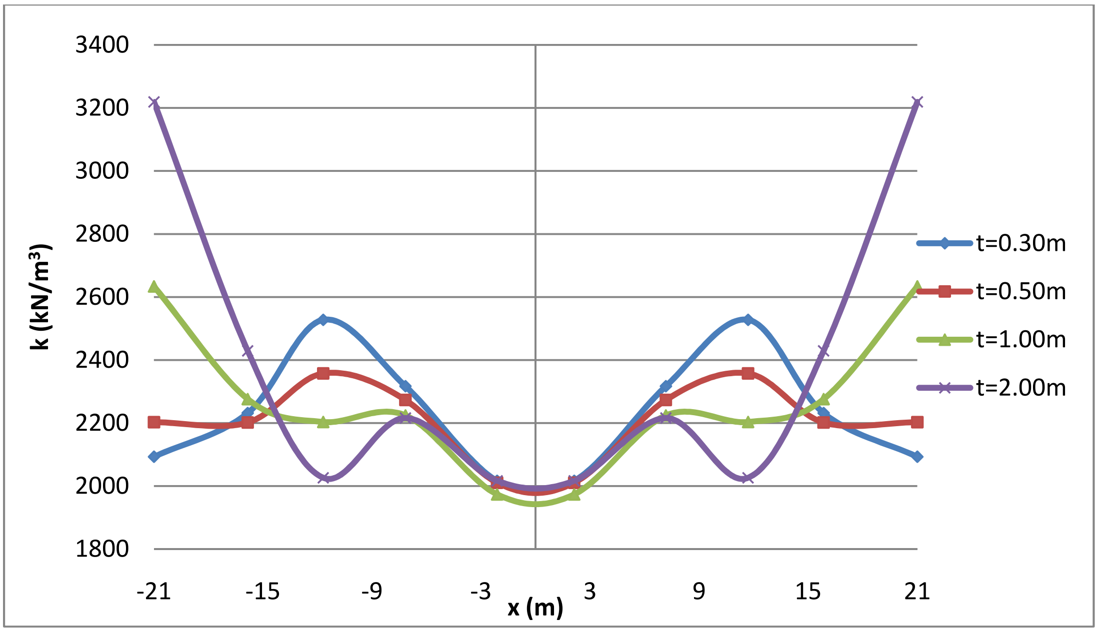

Figure 3.11 Comparison of modulus of subgrade reaction of Pattern A for Cases 2-1, 2-2, 2-3 and 2-4

As foundation thickness increases, through the region within $-0.15B < x < 0.15B$ average contact stress is almost constant for foundation thickness one meter and less, about q, i.e. 100 kPa (Figure 3.9). On the other hand, contact stress decrease through the section within $-0.35B < x < -0.15B$ and $0.15B < x < 0.35B$ as foundation thickness increases. Within $-0.50B < x < -0.35B$ and $0.35B < x < 0.50B$ contact stresses are increasing by the increase of foundation thickness. Note that, this behaviour is become more definite under the foundations having larger foundation thicknesses.

The shape of the soil pressure distribution is similar under the mat foundation having thicknesses one meter or less. In these cases the stresses at the edges are less than the ones at the center, the ratio being in the order of 60 ~ 85 %. This type of behaviour is typical for flexible foundations as Cui et. al. recommended in 2006.

However this trend is reversed in the case where t = 2.00m. For this case the edge stresses are %22 higher than the stresses at the center. This type of behaviour is typical for rigid foundation on soils having constant deformation modulus through depth (Coduto, 2001). Since, the confinement at the edges get larger which is similar to the behavior of clayey soils under infinitely rigid foundations (Coduto, 2001), stresses tend to significantly increase at edges with respect to center values. This observation indicates that 2.00 m thick raft behaves as an infinitely rigid foundation under the given analyses. Furthermore for all cases (Case 2-1 to 2-4), the shape of contact stress distribution within central zone $-0.35B < x < 0.35B$ is similar to the one proposed in literature for intermediate soil type.

In general pattern, increase in foundation thickness leads to increase in flexural stiffness of the foundation, i.e. EI, so that under same loading, settlements decrease according to elastic bending theory. Figure 3.10 demonstrates that as foundation thickness increases, foundation settlement seems to be same for t = 0.30 m, t = 0.50 m and t = 1.00 m since there is no significant change in *average* contact stress. On the other hand, for t = 2.00 m, there is decrease in foundation settlement due to the considerable rearrangement in contact stress within -0.35B < x < 0.35B where the maximum settlement is reached. Similarly, under the edge locations settlement slightly increases. By the combination of those, under thicker foundation, settlement decreases in average. In addition, by the increase in foundation thickness, settlement through the cross section becomes more uniform which is the main reason for prefering rigid mat foundations, since differential settlements tend to decrease.

Consequently, modulus of subgrade reaction increases under the points near to the edge of the foundation within –0.50B < x < -0.35B and 0.35B < x < 0.50B as foundation rigidity increases, due to the behavior prescribed for contact stress behavior. In other words, Figure 3.11 implies that distribution of modulus of subgrade reaction can not be independent from the foundation thickness. According to those observations, modulus of subgrade reaction distribution is shown in Table 3.5.

Table 3.5 β values of zones defined in Figure 3.8 for Pattern A for t = 0.50 m

	β			
Zone	**t=0.30m**	**t=0.50m**	**t=1.00m**	**t=2.00m**
A	1.00	1.00	1.00	1.00
B	1.09	1.06	1.06	1.05
C	1.00	1.03	1.17	1.33
D	0.97	1.03	1.25	1.52

Where;

$$k = \beta k_{Zone\ A} \qquad (3.3\ )$$

β: Ratio of average subgrade reaction coefficient in zones defined in Figure 3.8 to average subgrade reaction coefficient in Zone A for variable foundation thickness.

Comparision of Figure 3.9, Figure 3.10 and Figure 3.11, are summarized in Table 3.6.

Table 3.6 Values of σ, δ and k depending on the variation in t for Pattern A

t (m)	σ/(q=100kPa)		$\sigma_{edge}/\sigma_{center}$	s_{edge}/s_{center}	k_{edge}/k_{center}	k_{max}/k_{min}
	Center	Edge				
0.3	0.99	0.59	0.60	0.57	1.04	1.25
0.5	0.99	0.63	0.64	0.58	1.10	1.17
1.0	0.97	0.82	0.85	0.63	1.33	1.33
2.0	0.89	1.09	1.22	0.76	1.60	1.60

3.2.3 Effect of Loading Magnitude on Soil - Mat Interaction for Uniform Loading

To observe the effect of amount of loading, the following cases are analysed:

Soil deformation modulus : 50 MPa
Raft thickness : 0.50 m
Applied uniform load : Variable

Case 3-1: q = 50 kPa
Case 3-2: q = 100 kPa
Case 3-3: q = 300 kPa

For each case, contact stress distribution, vertical deformation (settlement) distributions and modulus of subgrade reaction through the mid-section are all plotted as in Figure 3.12, Figure 3.13 and Figure 3.14, respectively.

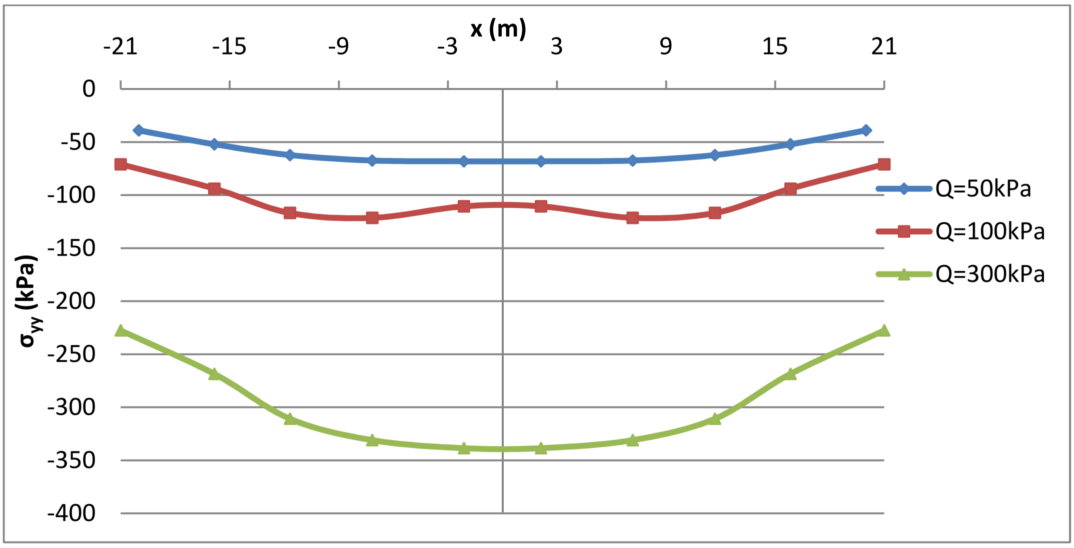

Figure 3.12 Comparison of contact stress distribution of Pattern A for Cases 3-1, 3-2 and 3-3

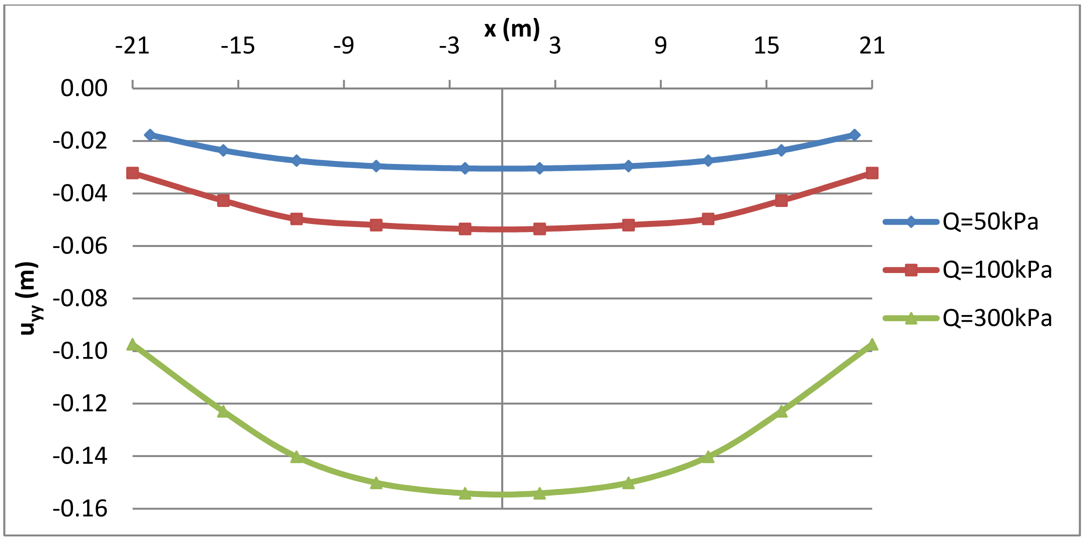

Figure 3.13 Comparison of settlement distribution of Pattern A for Cases 3-1, 3-2 and 3-3

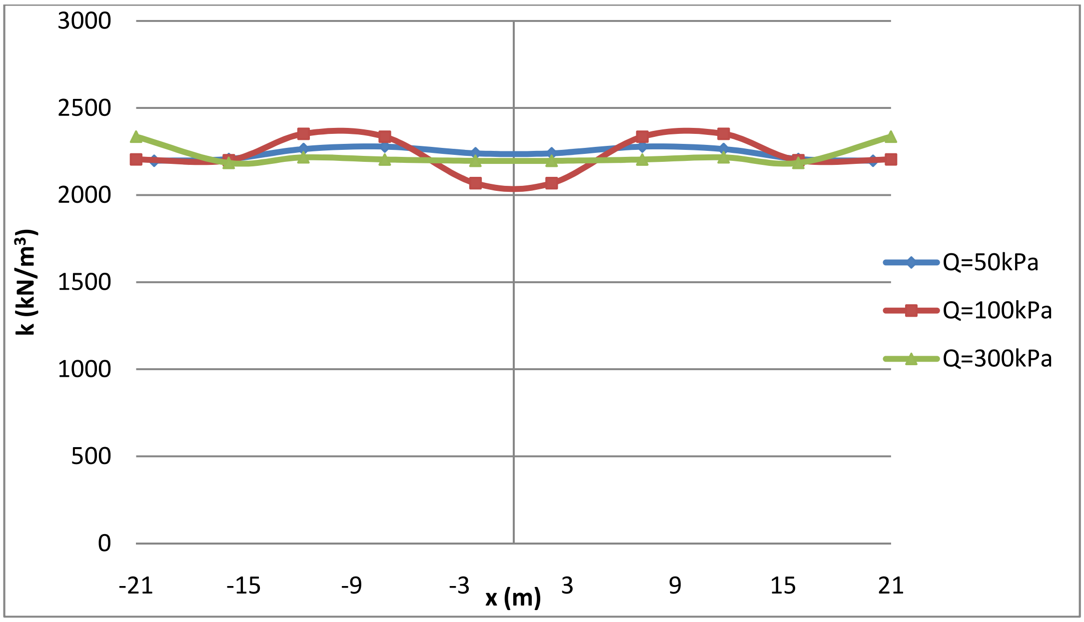

Figure 3.14 Comparison of modulus of subgrade reaction of Pattern A for Cases 3-1, 3-2 and 3-3

From Figure 3.12, it is noticed that the amount of the uniform load applied on the mat foundation is approximately same with the average contact stress developed under the foundation. The shape of the contact stress distributions resemble each other whatever the value of the uniform load is. The difference between maximum stress and the minimum stress increases as the amount of applied loading increases. This behavior is similar to the one stated in Cui et al (2007).

Moreover, the settlement of the foundation under superstructure load is directly related to amount of applied load (Figure 3.13). In other words, as load doubles foundation settlement also doubles and the settlement curve.

Evantually, modulus of subgrade reaction is not sensitive to the variation in the amount of superstructure load for pattern A (Figure 3.14). This is also explicit from the equation given by Bowles (1982).

Comparision of Figure 3.12, Figure 3.13 and Figure 3.14, are summarized in Table 3.7.

Table 3.7 Values of σ, δ and k depending on the variation in q for Pattern A

q (kPa)	σ/q		$\sigma_{edge}/\sigma_{center}$	s_{edge}/s_{center}	k_{edge}/k_{center}	k_{max}/k_{min}
	Center	Edge				
50	1.36	0.78	0.57	0.58	0.98	1.10
100	1.11	0.71	0.64	0.60	1.07	1.13
300	1.13	0.76	0.67	0.63	1.06	1.17

3.2.4 Effect of Foundation Size on Subgrade Modulus for Uniform Loading

Foundation size is another important factor that effects the behavior of the foundation, since the area and the shape of the foundation determines the distribution of the load both in vertical and horizontal directions.

To observe the effect of foundation size, the following cases are analysed:

Soil deformation modulus : 50 MPa
Raft thickness : 0.50 m
Applied uniform load : 100 kPa
Foundation width : Variable

Typical sizes for footing : 0.305 m < (B = L) < 10 m

Typical sizes for raft foundation : 10 m < (B = L) < 50 m

According to the various analysis, for uniform loading and square foundation the relationship between foundation size (B), deformation modulus of soil (E) and subgrade reaction coefficient (k) is established and illustrated in **Error! Reference source not found.**.

As size of the foundation increases, the influence zone of stresses beneath the foundation (i.e.:depth of pressure bulbs) increases. This effect causes larger settlements beneath the foundation. As a result, modulus of subgrade reaction decreases by the increase in foundation size as shown in **Error! Reference source not found.**. This behavior is the one which Moayed and Janbaz proposed in 2008 that modulus of subgrade reaction coefficient is inversely proportional to the foundation size as shown in **Error! Reference source not found.** but with different power. As Coduto (2001) and Moayed and Janbaz (2008) proposed for foundation size, power is different from 1, where it is found that 0.85 in average. The relationship between foundation size, modulus of elasticity of subgrade soil and modulus of subgrade reaction may be determined by the following expression :

$$k_{ave} = \frac{1.1E}{B^{0.85}} \tag{3.5}$$

where k_{ave} is in kN/m^3, E is in kPa and B is in meter units.

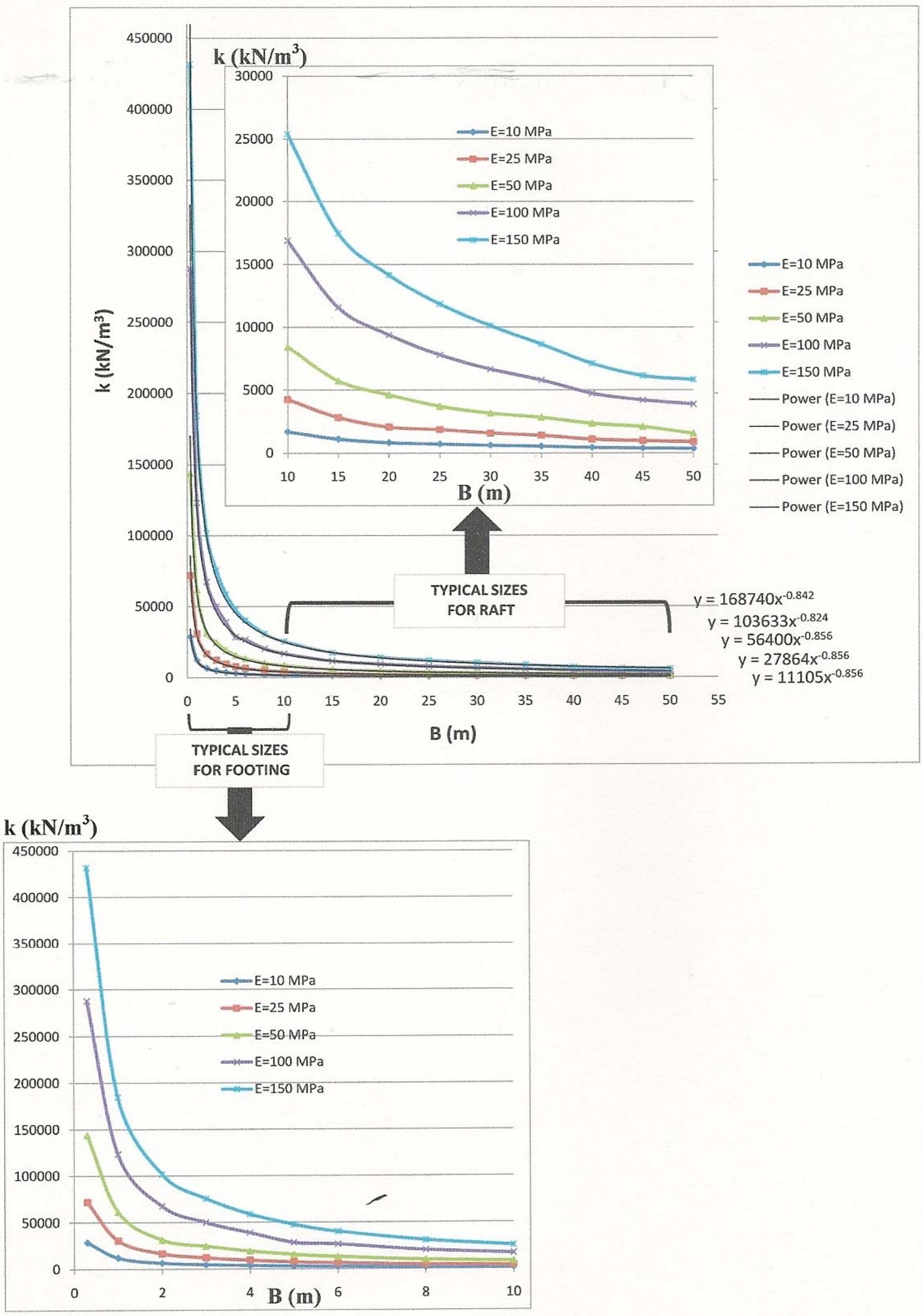

Figure 3.15 Relationship between B, E and k of Pattern A

3.3 Column Loading Case : Pattern B

Superstructure load is applied through columns as point loads over the square mat foundation which is overlying on the soil having prescribed properties. The model is performed step by step with construction stages. Those stages are defined as:

Phase 0: Initial phase
Phase 1: Foundation construction
Phase 2: Column construction
Phase 3: Application of column loading (the point loads are activated by introducing the relevant value)

Similar to the Pattern A, the calculated contact stresses and the developed settlements at each node are taken from different cross sections. For Pattern B, three different column spacings are studied: s = 5m, s = 8m and s = 10m. For each model, the effect of modulus elasticity of soil, foundation thickness and the magnitude of loading to the contact stress distribution, foundation settlement, modulus of subgrade reaction, shear force distribution and bending moment distribution are all examined. It is found that, although the numerical values are different for various column spacings, the behavior against the variations in parameters and their effects are similar. Thus, in order to summarize the general behavior only the analysis related to s = 5 m are represented in this chapter. For columns spacings; s = 8 m and s = 10 m, similar trends are observed.

The column loads are calculated for each column depending on the tributary areas as shown in Equation 3.6:

$$P = q_{uni,ave} * Area_{tributary} \tag{3.6}$$

Note that, for comparison figures only the mid-section are shown although all the cross sections under the column axes are considered throughout the calculations.

3.3.1 Column Spacing: s = 5 m

For column spacing 5 m the plan view of the foundation showing the considered cross sections is given in Figure 3.15.

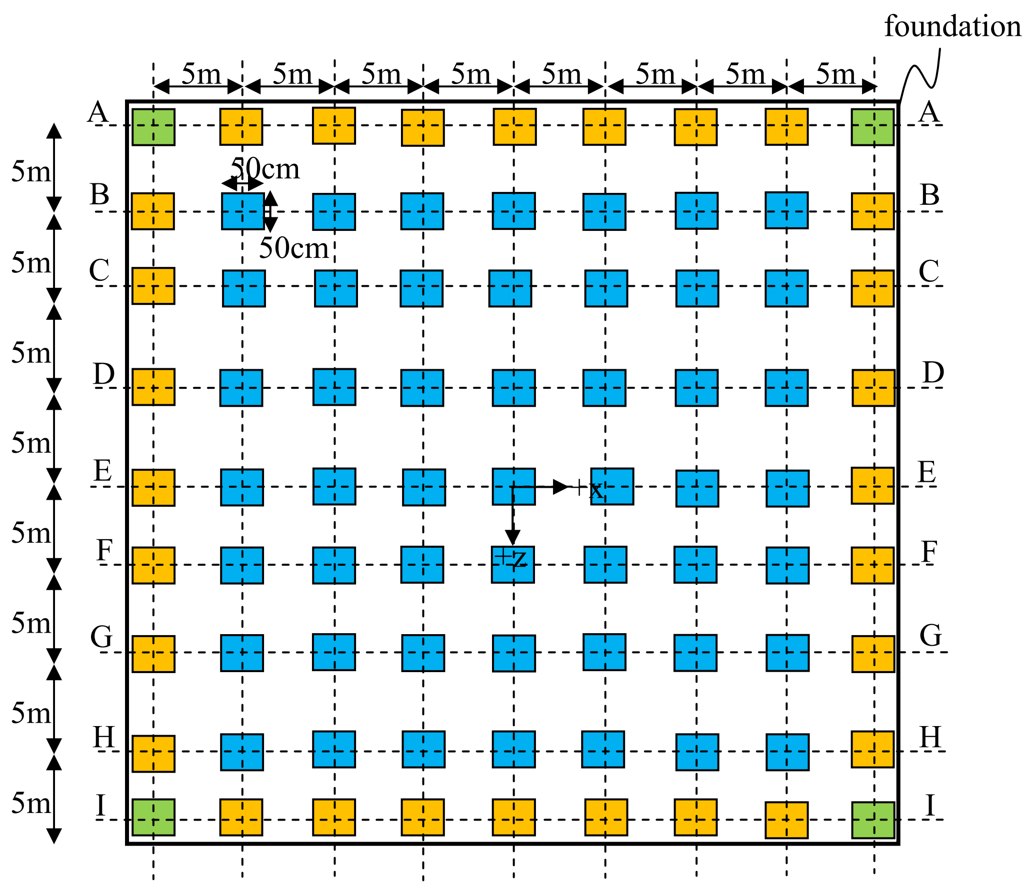

Figure 3.15 Plan view of the foundation model of Pattern B - s = 5 m

As previously stated to ease the comparison of the behavior of Pattern B with Pattern A, the average load pressure is given as 100 kPa and distributed to the columns according to their tributary areas. Finally the loads are given as point loads to the columns as:

For blue shaded columns : **2500 kN**
For orange shaded columns : **1250 kN**
For green shaded columns : **625 kN**

Note that soil conditions are same with the ones valid for Pattern A.

3.3.1.1 Effect of Deformation Modulus on Soil - Mat Interaction for Column Spacing, s = 5 m

As previously indicated, for the purpose of implying the effect of the modulus of elasticity (deformation modulus) of the subgrade soil, the following cases are analysed:

Column Spacing : 5 m
Applied uniform load : 100 kPa
Raft thickness : 0.50 m
Soil deformation modulus : Variable

Case 1-1: E = 10 MPa
Case 1-2: E = 25 MPa
Case 1-3: E = 50 MPa
Case 1-4: E = 100 MPa

The contact stress distribution obtained from the Plaxis analysis for Case 1-3 of Pattern B is illustrated in Figure 3.16.

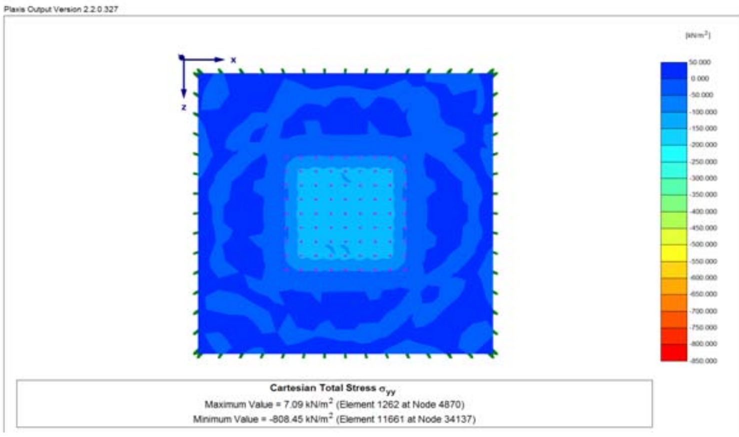

Figure 3.16 Contact stress distribution of Pattern B-s = 5 m for Case 1-3

For each case, contact stress distribution, settlement distribution and modulus of subgrade reaction through the mid-section, are all plotted and shown in Figure 3.17, Figure 3.18 and Figure 3.19, respectively.

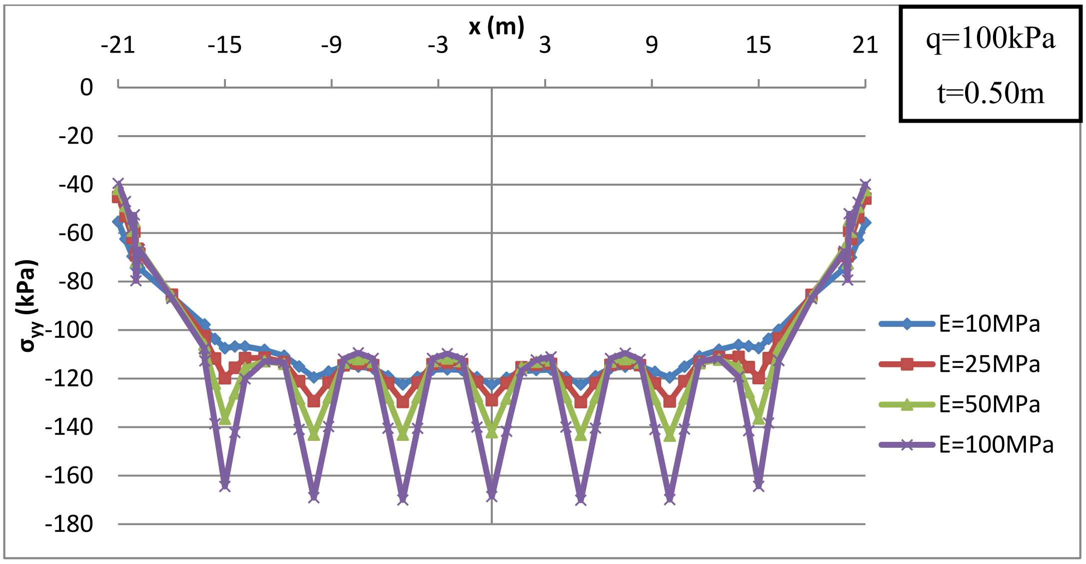

Figure 3.17 Comparison of contact stress distribution of Pattern B-s = 5m for Cases 1-1, 1-2, 1-3 and 1-4

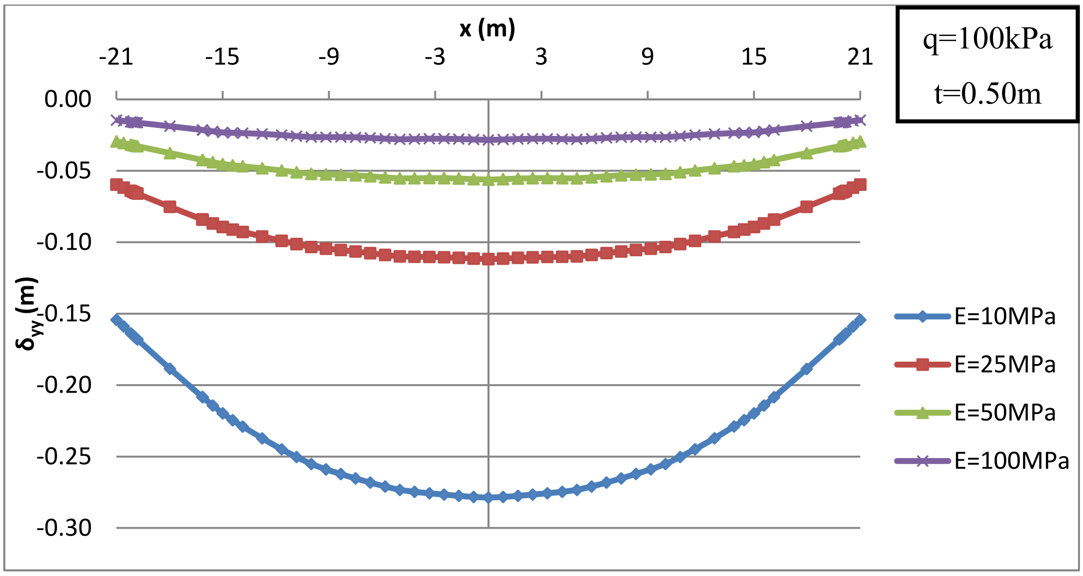

Figure 3.18 Comparison of settlement distribution of Pattern B-s = 5 m for Cases 1-1, 1-2, 1-3 and 1-4

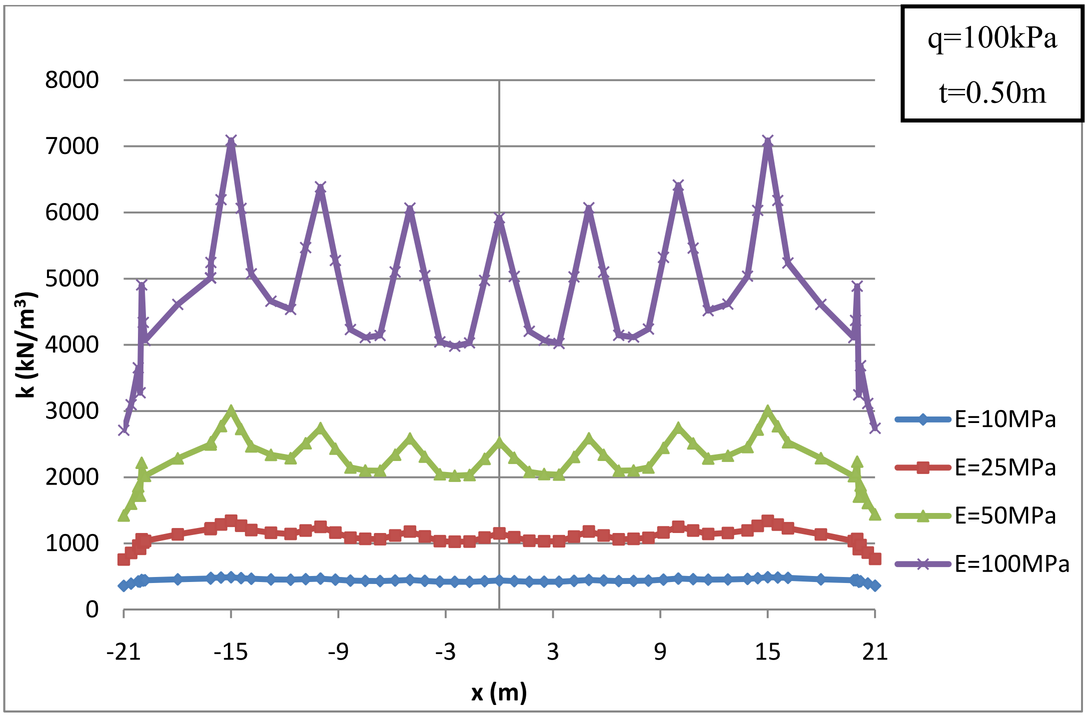

Figure 3.19 Comparison of modulus of subgrade reaction distribution of Pattern B-s = 5 m for Cases 1-1, 1-2, 1-3 and 1-4

As shown in Figure 3.17, the contact stress difference between mid-span soil pressures and the soil pressures under the columns are higher for stiffer soil as compared to relatively softer soil. For instance for the case E = 100 MPa, soil pressure under the column is 170 kPa, whereas in the mid-span the soil pressures are on the order of 110 kPa. This difference however is not even noticable for soft soil represented by E = 10 MPa. This finding clearly shows that as the soil gets softer, the column load is more evenly distributed under the foundation.

Just as the behavior of the uniformly loaded mat foundation, also for the foundation exposed to column loading, foundation settlement directly depends on the modulus elasticity of the soil that decreases by the increase in the modulus (Figure 3.18).

The variation of modulus of subgrade reaction throughout the mat foundation as a function of deformation modulus of foundation soil shows a similar trend to uniform loading case. The obtained subgrade reaction coefficent distribution behavior for variable deformation modulus of soil under loading Pattern A is also observed for Pattern B. For the soil having larger values of modulus of elasticity larger modulus of subgrade reaction are obtained under both column locations and span locations, i.e. through entire cross-section (Figure 3.19).

3.3.1.2 Effect of Foundation Thickness on Soil - Mat Interaction for Column Spacing, s = 5 m

In order to comprehend the effect of foundation rigidity on the subgrade soil, the following cases are analysed:

Column Spacing	: 5 m
Applied uniform load	: 100 kPa
Soil deformation modulus	: 50 MPa
Raft thickness	: Variable

Case 2-1: t = 0.30 m

Case 2-2: t = 0.50 m

Case 2-3: t = 1.00 m

Case 2-4: t = 2.00 m

Note that, in order to implement the only effect of foundation rigidity, in the analyses of Cases 2-1 to 2-4 weight of foundation is neglected different from the other analyses stated in section 3.3.1.1.

For each case, contact stress distribution, settlement distribution and modulus of subgrade reaction through the mid-section are all plotted as in Figure 3.20, Figure 3.21 and Figure 3.22, respectively.

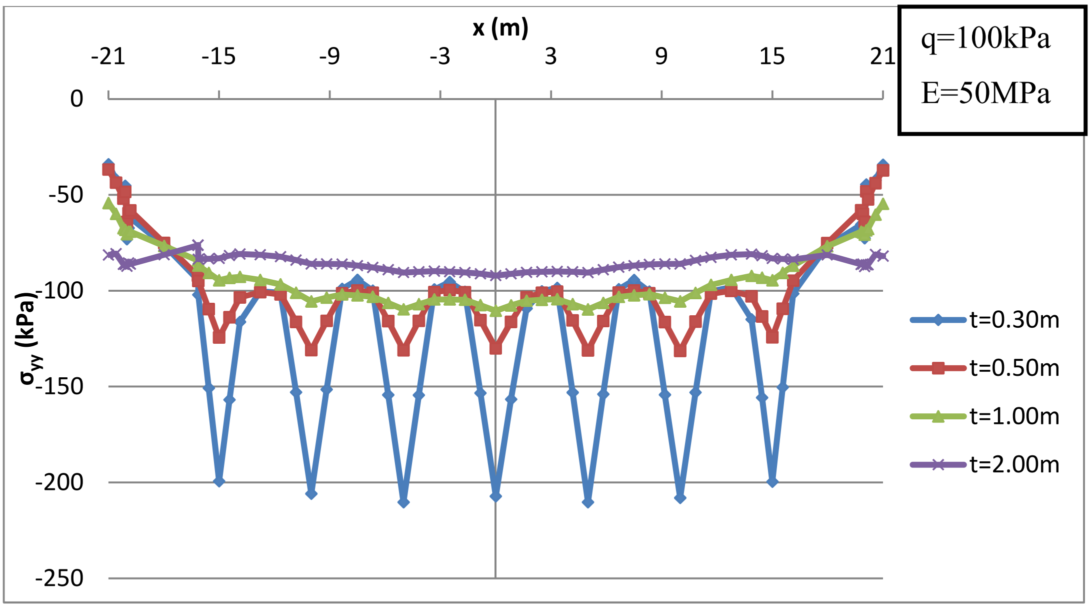

Figure 3.20 Comparison of contact stress distribution of Pattern B - s = 5 m for Cases 2-1, 2-2, 2-3 and 2-4

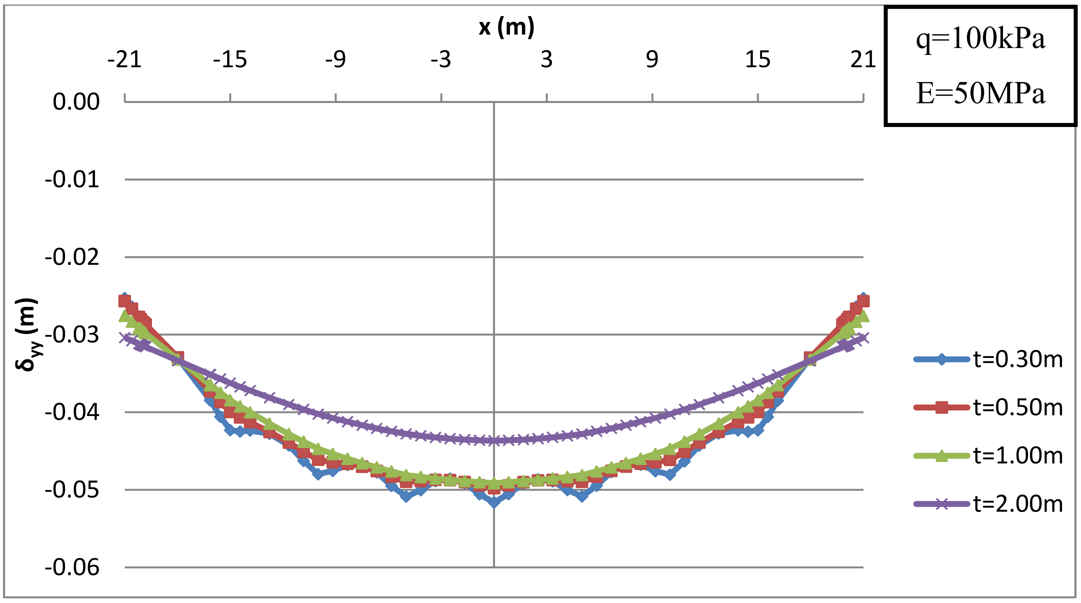

Figure 3.21 Comparison of settlement distribution of Pattern B - s = 5 m for Cases 2-1, 2-2, 2-3 and 2-4

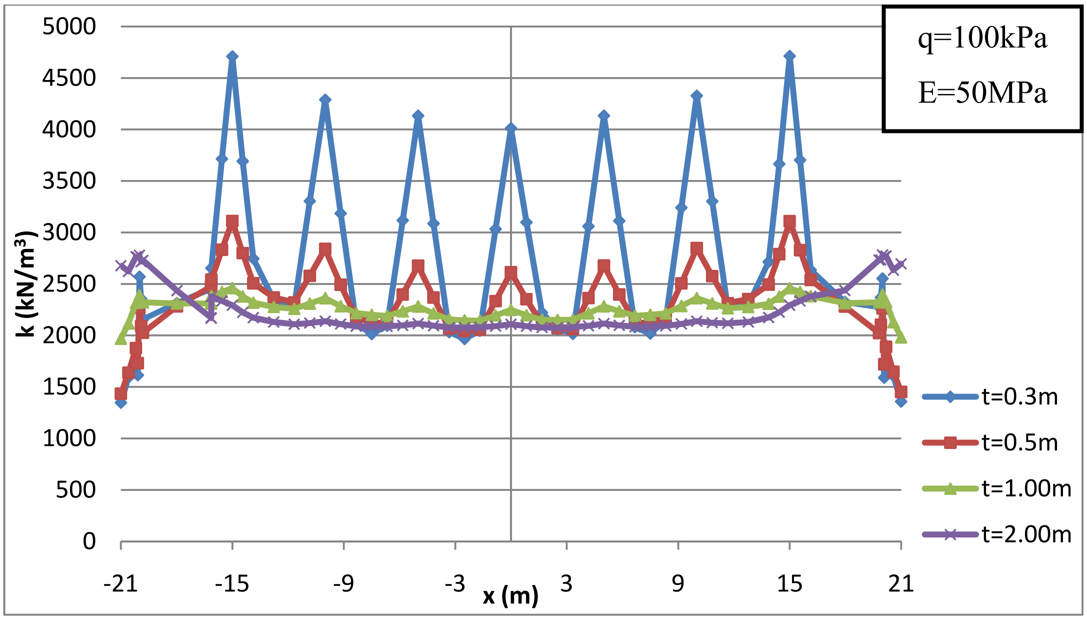

Figure 3.22 Comparison of modulus of subgrade reaction distribution of Pattern B - s = 5 m for Cases 2-1, 2-2, 2-3 and 2-4

As seen from Figure 3.20 contact stress beneath the foundation is uniform and approximately equal to the applied load pressure, i.e. 100 kPa, for t = 2.00 m. On the other hand, as foundation rigidity decrease the stress differences between the mid-span and column locations increase. For instance, for t = 0.30 m foundation under column locations stress is larger than the twice of the applied load (210 kPa) whereas under mid-span locations approximately equal to the applied load pressure (100kPa). In brief, as foundation becomes more rigid which is loaded by column loads, contact stress distribution becomes more uniform under the cross section. This behavior is also consistent with the generally known behavior which for rigid foundations the contact stress distribution differs from the shape(pattern) of application of the loading Coduto (2001).

Figure 3.21, shows that the foundation settlement decreases as foundation thickness increases. Furthermore, the case having t = 0.30 m, shows a flexible behavior that the settlement curve is parallel to the loading pattern where at column locations there are noticable peaks due to the point loading. However those peaks are not seen in thicker foundations. As foundation thickness increases, settlement under foundation gets uniform distribution that differential settlement decreases.

As a result, in general pattern by the considerable decrease in contact stress and marginal decrease in the foundation settlement, the modulus of subgrade reaction definitely decreases as the foundation thickness increases for column loading pattern of s = 5 m (Figure 3.22).

CHAPTER 4.

COMPARISONS OF THE RESULTS OBTAINED FROM UNIFORM LOADING AND CONCENTRATED LOADING

4.1 Loads are Applied Through Columns: Concentrated Loading Case

4.1.1 Effects of Change in Modulus of Elasticity

Under same loading applied on the foundation having same thickness for different column spacing cases over soil having different modulus of elasticity, developed contact stresses under the columns and mid-spans are compared as given in the Figure 4.1.

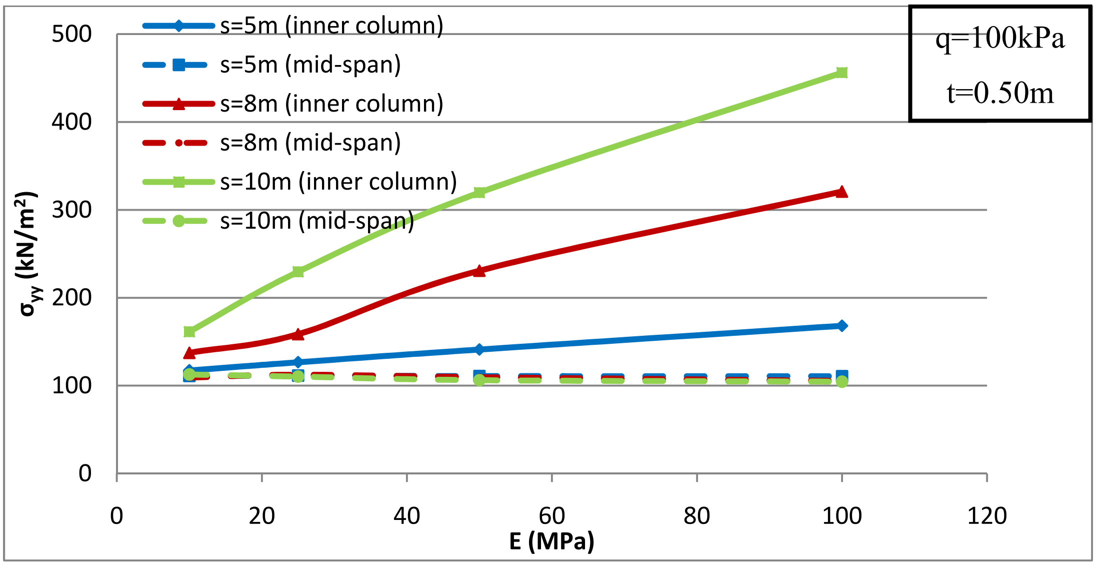

Figure 4.1 σ_{yy} vs E for various spacings under the columns and mid-spans

The contact stresses significantly increase at column locations as the column spacing increases as shown in Figure 4.1. Contrast to the column locations, mid-spans stresses appear to be constant for different modulus of elasticity of soil irrespective of the column spacings.

The relationship between the settlements and the modulus of elasticity of the soil for various column spacings is shown in Figure 4.2. In this figure both the settlements at mid-span and under columns are considered. It is found that the relationship between settlements and elastic modulus of soil is unique being independent of column spacings as well as being at mid-span or under column.

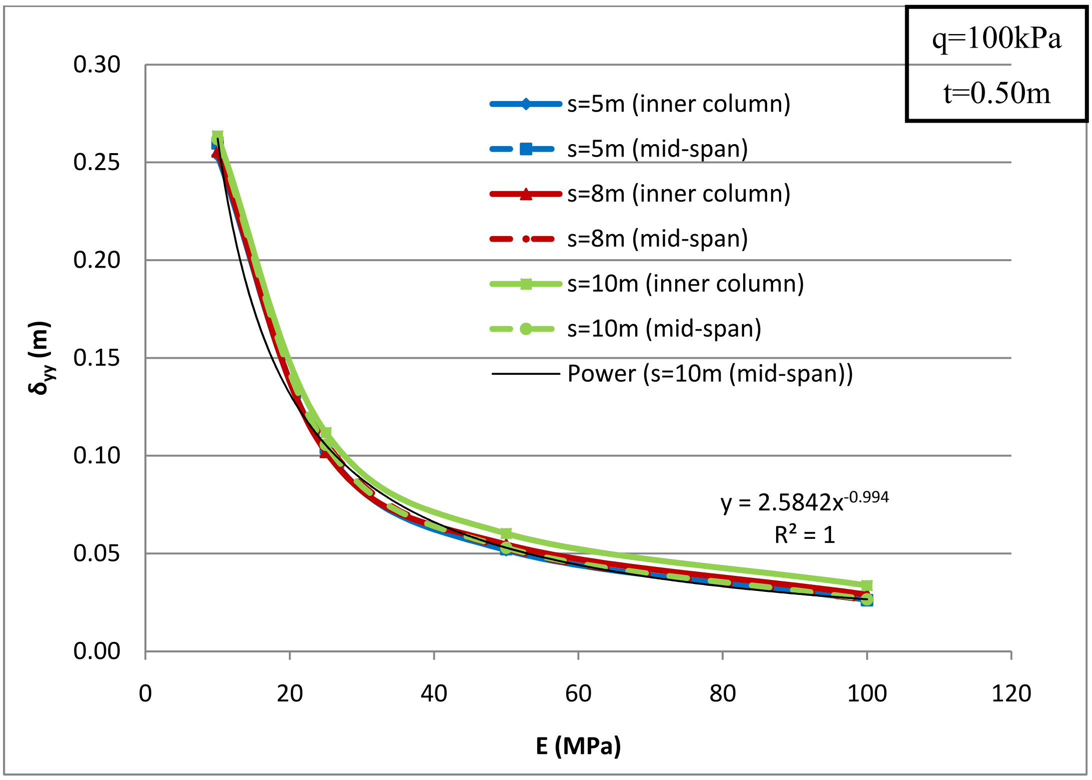

Figure 4.2 δ_{yy} vs E for various spacings under the columns and mid-spans

This implies that, average settlement is independent from the column spacing but directly depends on modulus of elasticity of soil as stated by Mayne & Poulos (1999) related to elastic settlement theory :

$$\delta_{yy} = \frac{q_{ave} d I_p}{E} \tag{4.1}$$

Where;

d: diameter of the equivalent circular footing

$$d = \sqrt{(4BL)/\pi} \tag{4.2}$$

I_p: Influence factor depending on the finite layer thickness and foundation rigidity

The modulus of subgrade reaction values determined both under the column and at midspans are shown in Figure 4.3 as a function of deformation modulus of foundation soil.

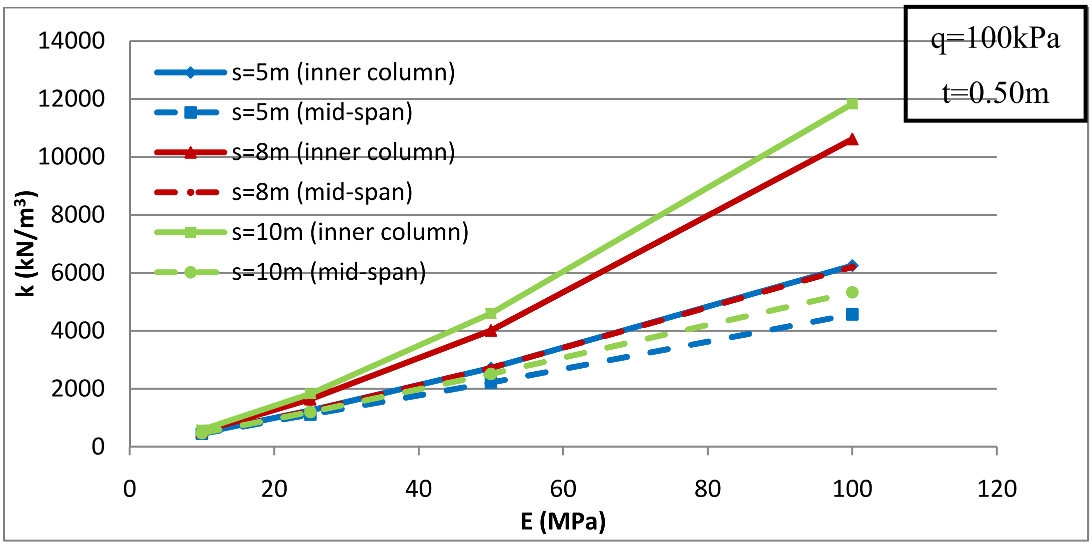

Figure 4.3 k vs E for various spacings under the columns and mid-spans

Figure 4.3 shows that the modulus of subgrade reaction under the column locations depends on the column spacing. The modulus of subgrade reaction increases with increase in column spacing. Whereas, at mid-span locations the dependence of subgrade modulus on column spacing is not so obvious. At mid-span locations, there is no significant change between different column spacings, maximum change being in the order of ±5 %.

Since increase in the modulus of elasticity means stiffer soil, subgrade reaction coefficient increases significantly. Variation in the deformation modulus of soil greatly influences with the modulus of subgrade reaction beneath the column locations with respect to the mid-span locations; and this effect increases at larger column spacing since the overlapping contact pressure zones dissapear.

For different column spacing over soil having different modulus of elasticity, shear forces (Q) developed under columns and under mid-spans are compared as given in Figure 4.4.

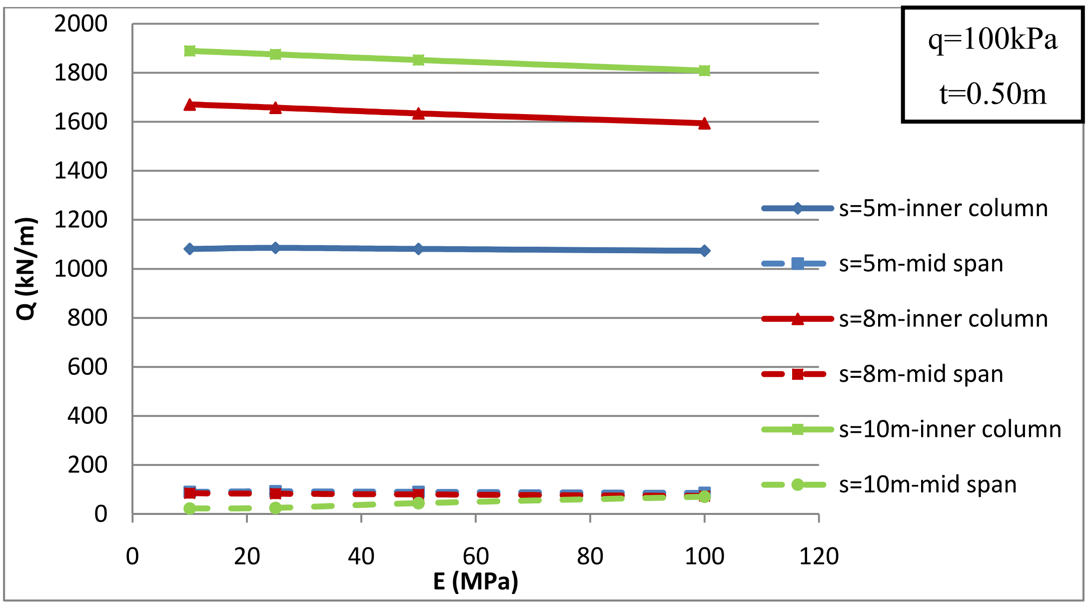

Figure 4.4 Q vs E for various spacings under the columns and mid-spans

As seen in Figure 4.4 the difference in the shear forces at column locations arise from the differences in the column loads, since higher magnitudes of point loads are applied through the columns with increasing column spacing.

For different column spacing cases over soil having different modulus of elasticity, bending moment (M) developed under columns and under mid-spans are compared as given in the Figure 4.5.

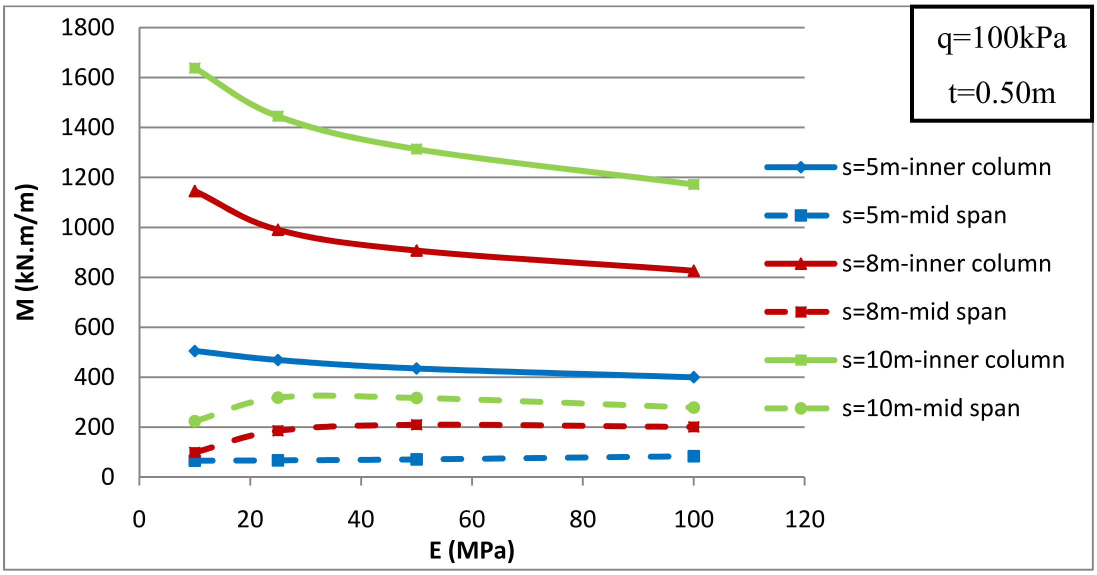

Figure 4.5 M vs E for various spacings under the columns and mid-spans

Contrary to shear forces, there is slight variation in bending moment under column and mid-span locations over the foundation with constant column spacing over soil having different modulus of elasticity. Furthermore, as deformation modulus of subgrade soil increases, in other words as relative stiffness of soil-raft interaction decreases, smaller bending moment develop at the foundation as seen shown in Figure 4.5. This behavior is similar to the one proposed by Natarajan and Vidivelli (2009).

Moreover, for smaller column spacing, difference between the support moments and the span moments is less than the larger column spacing, as stated by Natarajan and Vidivelli (2009).

The ratio of contact stress under columns to span locations for various modulus of elasticity of soil are calculated. The relation is shown in Figure 4.6.

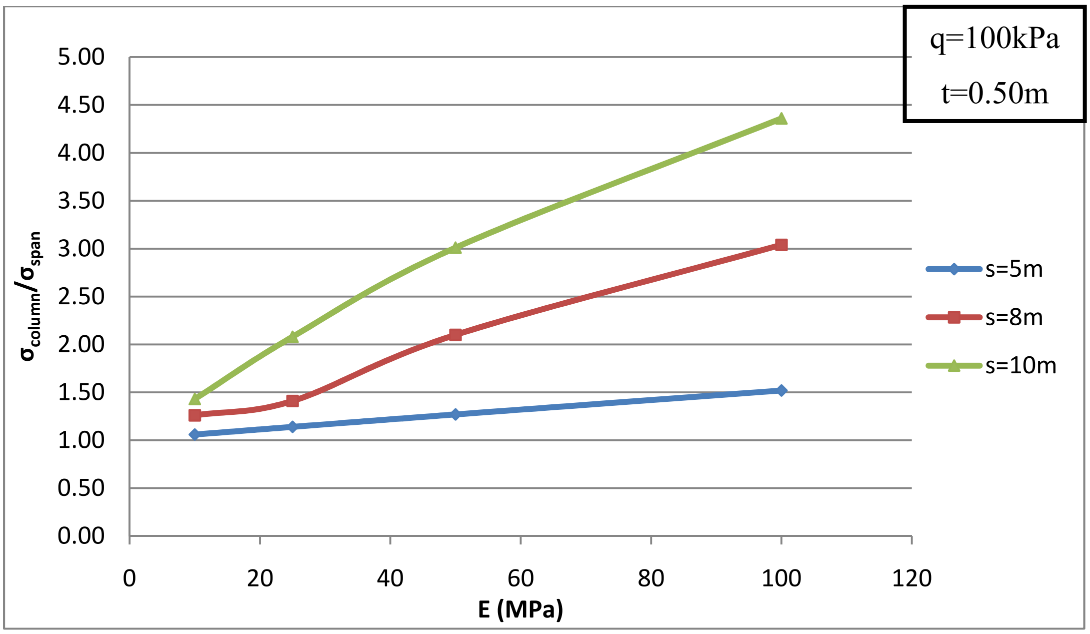

Figure 4.6 $\sigma_{column}/\sigma_{span}$ vs E for various column spacings

As previously noted, ratio of contact stress under column locations to the contact stress under span locations increases as soil becomes stiffer. This behavior is more obvious as column spacing increases.

The ratio of foundation settlement between column and span locations for various modulus of elasticity of soil are calculated. The relation is given in Figure 4.7.

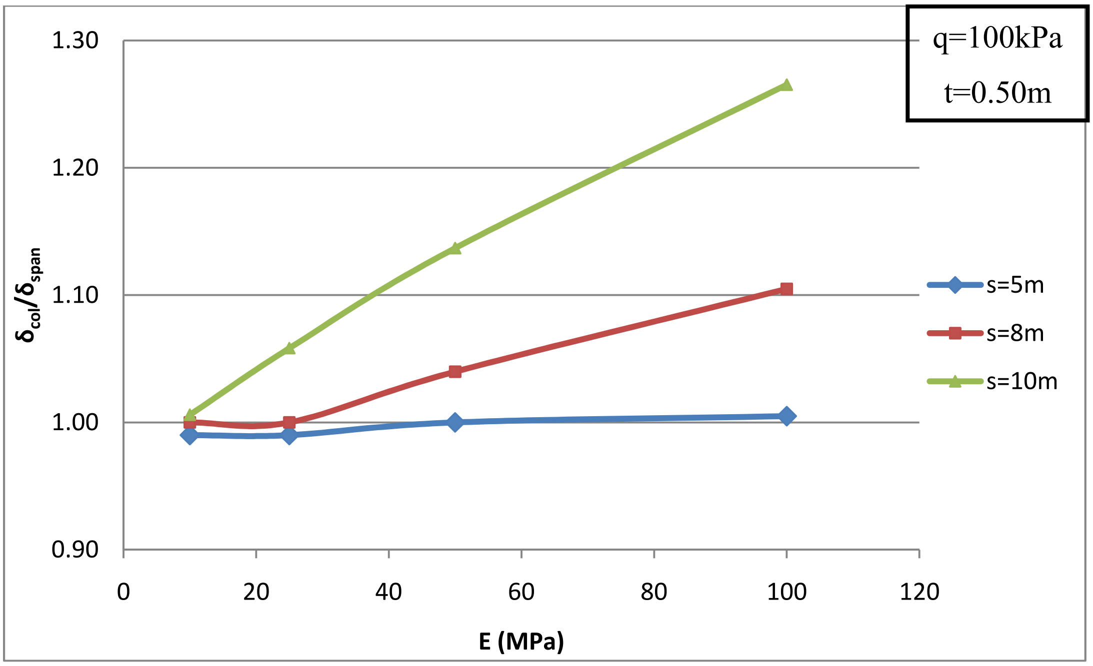

Figure 4.7 $\delta_{column}/\delta_{span}$ vs E for various column spacings

Foundation settlement under column locations is equal to or larger than the one at span locations. Besides, the ratio between settlement under column locations to the settlement under span location increases as soil stiffens. Whereas, this increase is not as obvious as the contact stress ratio. Moreover, as column spacing increases, the ratio increases for a specific deformation modulus of soil.

The ratio of modulus of subgrade reaction between column and span locations for various modulus of elasticity of soil are calculated. The relation is given in Figure 4.8.

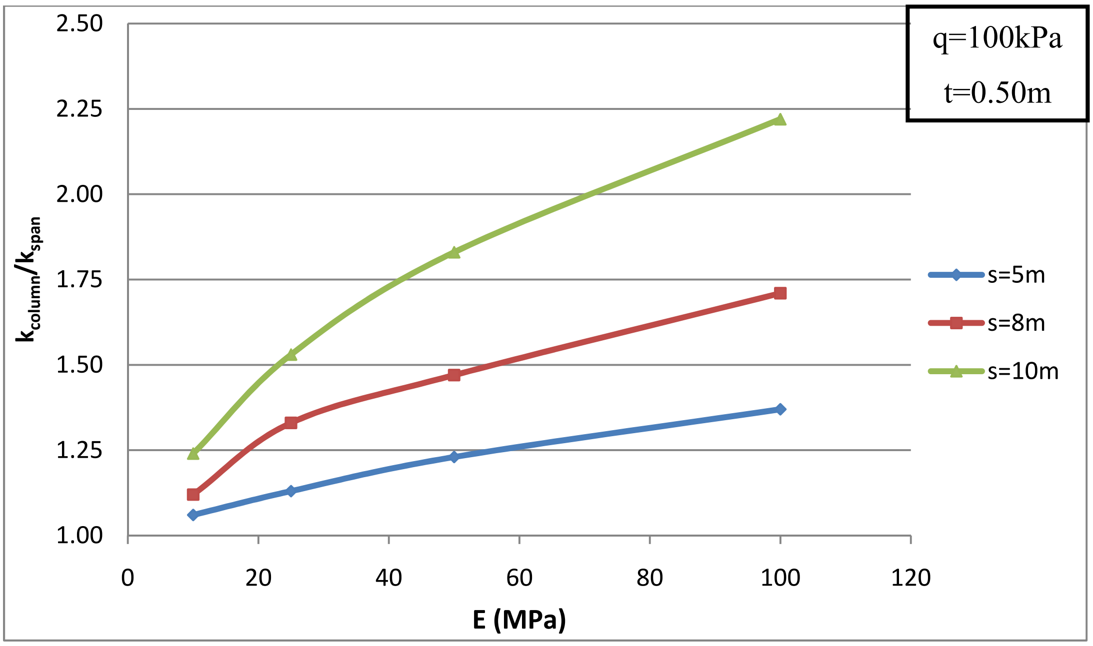

Figure 4.8 k_{column}/k_{span} vs E for various column spacings

As it is obvious, as modulus of elasticity increases, $k_{column}/k_{mid\text{-}span}$ ratio increases and always larger than 1 as expected since the increase in the ratio of contact stress is so larger than the increase in the ratio of settlement. This increase is more pronounced at higher column spacing values.

Over the foundation of same thickness, for different modulus elasticities of the underlying soil, contact stress (σ), settlement (δ) and subgrade reaction coefficient (k) ratios at column and at mid-span locations are given in Table 4.1.

Table 4.1 Comparison between the cases having different modulus of elasticity of subgrade soil

t = 0.50m and q = 100 kPa					
Spacing	**E ratios**	**Locations**	**σ ratios**	**δ ratios**	**k ratios**
s=5m	25 MPa / 10 MPa = 2.5	Column	1.05	0.40	2.61
		Mid-span	1.00	0.40	2.50
	50 MPa / 10 MPa = 5.0	Column	1.15	0.20	5.70
		Mid-span	1.01	0.20	4.99
	100 MPa / 10 MPa = 10.0	Column	1.34	0.10	13.07
		Mid-span	1.00	0.10	9.90
s=8m	25 MPa / 10 MPa = 2.5	Column	1.23	0.40	3.10
		Mid-span	0.99	0.41	2.46
	50 MPa / 10 MPa = 5.0	Column	1.59	0.21	7.58
		Mid-span	0.97	0.20	4.87
	100 MPa / 10 MPa = 10.0	Column	2.22	0.11	20.06
		Mid-span	0.96	0.10	9.61
s=10m	25 MPa / 10 MPa = 2.5	Column	1.29	0.41	3.11
		Mid-span	0.96	0.40	2.4
	50 MPa / 10 MPa = 5.0	Column	1.72	0.22	7.81
		Mid-span	0.93	0.20	4.69
	100 MPa / 10 MPa = 10.0	Column	2.41	0.12	20.07
		Mid-span	0.9	0.10	9.13

Between two analyses having different deformation modulus of foundation soil (analysis 1 and analysis 2), modulus of elasticity ratio versus average contact stress ratio under column and mid-span locations for various column spacings are plotted as seen in Figure 4.9. Here, it is seen that, as modulus of elasticity of subgrade soil increases, stress increase between column and mid-span locations increases. Moreover, this difference is larger for the foundation loaded through the columns having larger spacings.

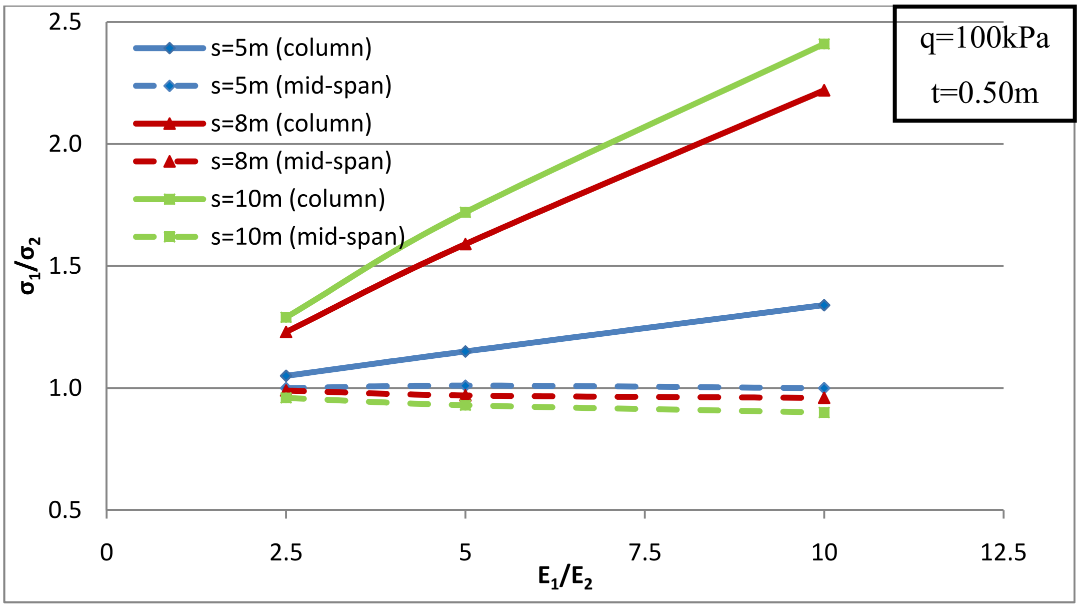

Figure 4.9 Relation between σ_1/σ_2 and E_1/E_2 for various column spacings under column and mid-spans

It is clear from the Table 4.1 that the settlement ratio between two soil type having different modulus of elasticity for every column spacing is same since elastic settlement is independent from the load pattern but only depend on the average pressure. Thus, the difference in modulus of subgrade reaction is only caused by the variations in the contact stress distributions. So;

$$\frac{\delta_1}{\delta_2} = \frac{E_2}{E_1} \tag{4.3}$$

Between two different analyses (analyse 1 and analyse 2) having different deformation modulus of foundation soil, modulus of elasticity ratio versus average modulus of subgrade reaction ratio under column and mid-span locations for various column spacings are plotted as shown in Figure 4.10. Here it is seen that, increase in modulus of subgrade reaction for any E_1/E_2 ratio at column locations for s = 8 m and s = 10 m are nearly same, whereas for s = 5 m the ratio is significantly lower. This implies, the sensitivity of modulus of subgrade reaction to column spacing. In addition as previously noted there is no significant difference in the increment ratio for at span locations between different column spacings.

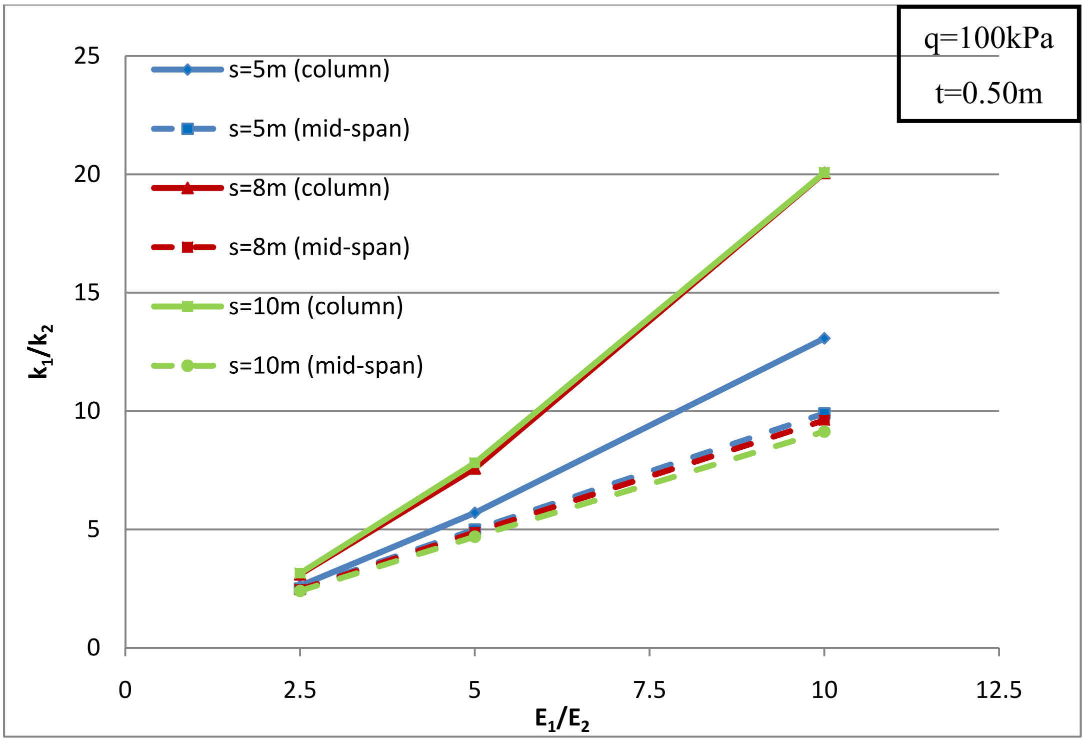

Figure 4.10 Relation between k_1/k_2 and E_1/E_2 for various column spacings under column and mid-spans

Several cases are studied for constant foundation thickness, t = 0.5 m, and constant uniform load, q = 100 kPa, to generalize the contact stress distribution under the mat foundation loaded by the columns having different column spacings over the soil having modulus of elasticity of 10 MPa, 25 MPa, 50 MPa. Those comparisons are illustrated in Figure 4.11, Figure 4.12 and Figure 4.13 by means of normalized contact stress with respect to applied load pressure.

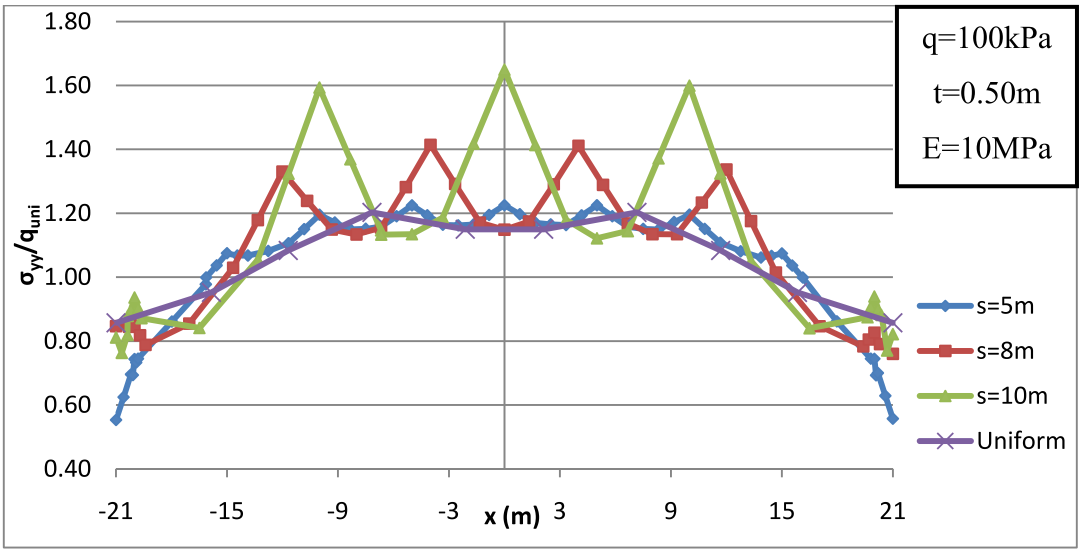

Figure 4.11 Normalized contact stress distribution for various column spacings for E = 10 MPa

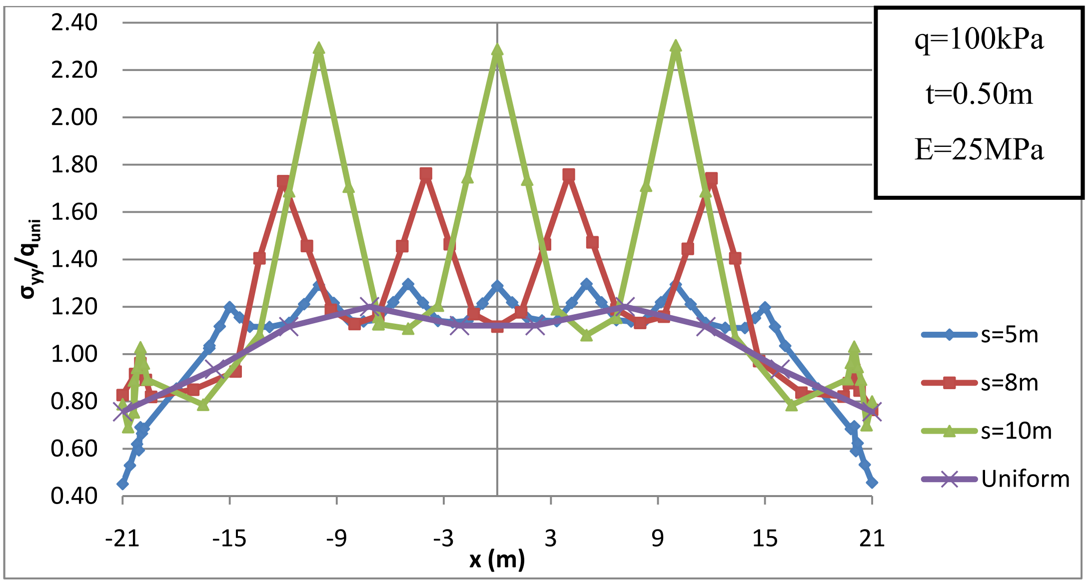

Figure 4.12 Normalized contact stress distribution for various column spacings for E = 25 MPa

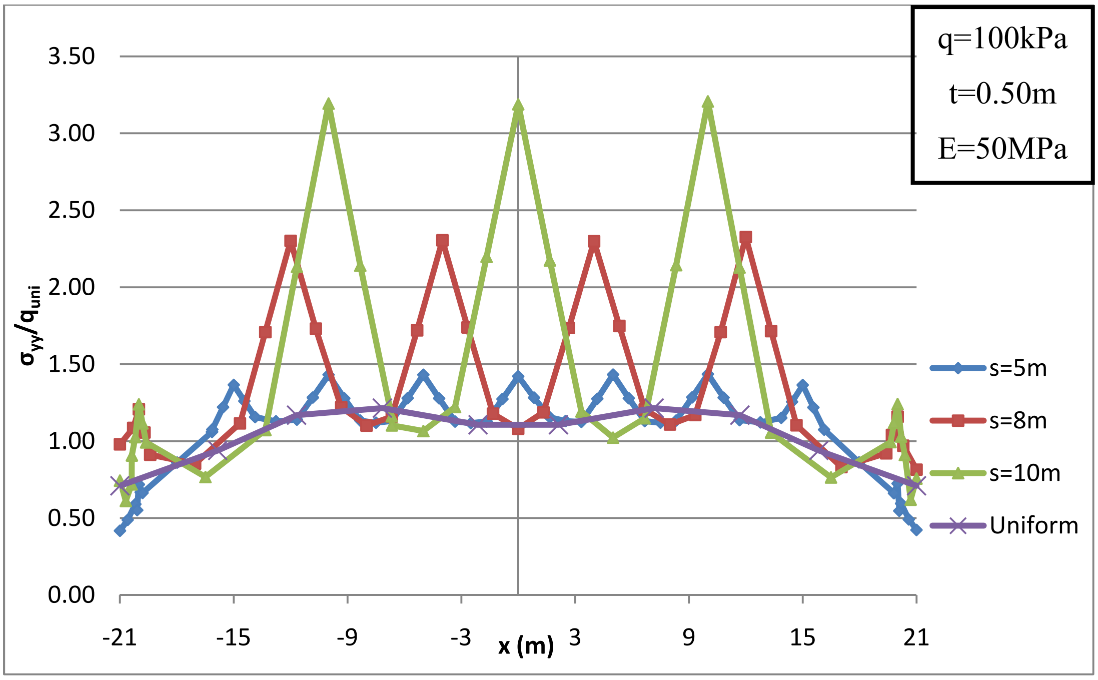

Figure 4.13 Normalized contact stress distribution for various column spacings for E = 50 MPa

All normalized stress distributions demonstrated in Figure 4.11, Figure 4.12 and Figure 4.13 are idealized and the general stress distribution is summarized as in Figure 4.14 and Table 4.2. Note that for s = 5 m and s = 8 m, contact stress zones are similar to the given case of s = 10 m in Figure 4.14.

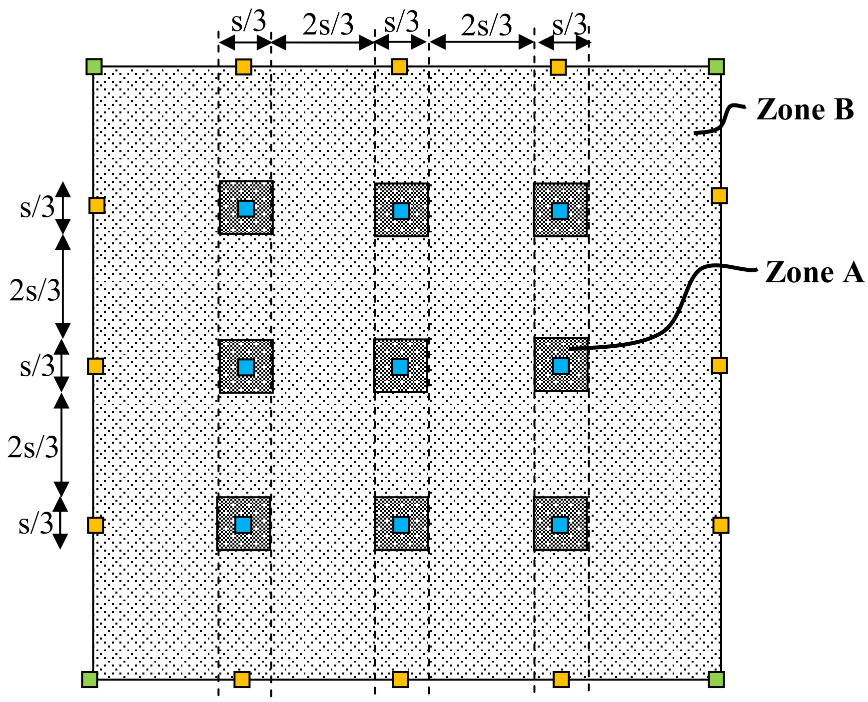

Figure 4.14 Zones for contact stress distribution for variation in foundation thickness of Pattern B – s = 10 m

Table 4.2 Summary of normalized contact stresses at zones shown in Figure 4.14 for different columns spacings over soil having different E

λ									
Zones	**E (MPa) (t = 0.5 m; q = 100 kPa)**								
	s = 5 m			**s = 8 m**			**s = 10 m**		
	10	**25**	**50**	**10**	**25**	**50**	**10**	**25**	**50**
A	1.20	1.30	1.40	1.35	1.75	2.30	1.60	2.30	3.20
B	1.15	1.12	1.13	1.15	1.12	1.10	1.15	1.10	1.10

Where;

$$\sigma_{yy} = \lambda * (q_{uni} = 100kPa) \quad (4.4)$$

λ: Ratio of average contact stress in zones defined in Figure 4.14 to average applied load (i.e. 100 kPa) for variable deformation modulus of foundation soil.

From Table 4.2, it is obvious that as column spacing increases the individual effect of a column is increasing so that the increase in contact stress occurs at larger area around the columns.

4.1.2 Effects of Change in Foundation Thickness

Under same loading applied on the same column spacing, same modulus of elasticity of the underlying soil, for different foundation thicknesses it is seen that contact stresses under column areas are decreasing to the stress levels of span locations, and distribution is getting more uniform. In other words, since as thickness increases the foundation system becomes more rigid than the underlying soil, so the stress concentration does not occur under the columns. Moreover, settlement tends to decrease as foundaton thickness increases since flexural stiffness (EI) increases, so that rotations and deformations of the mat foundation decrease. Furthermore, modulus of subgrade reaction decreases as foundation thickness increases where other variables are kept constant.

Under same loading applied on the foundation over soil with same properties, for different column spacing cases over various thickness of foundation contact stresses under inner columns and under mid-spans are compared and shown in Figure 4.15.

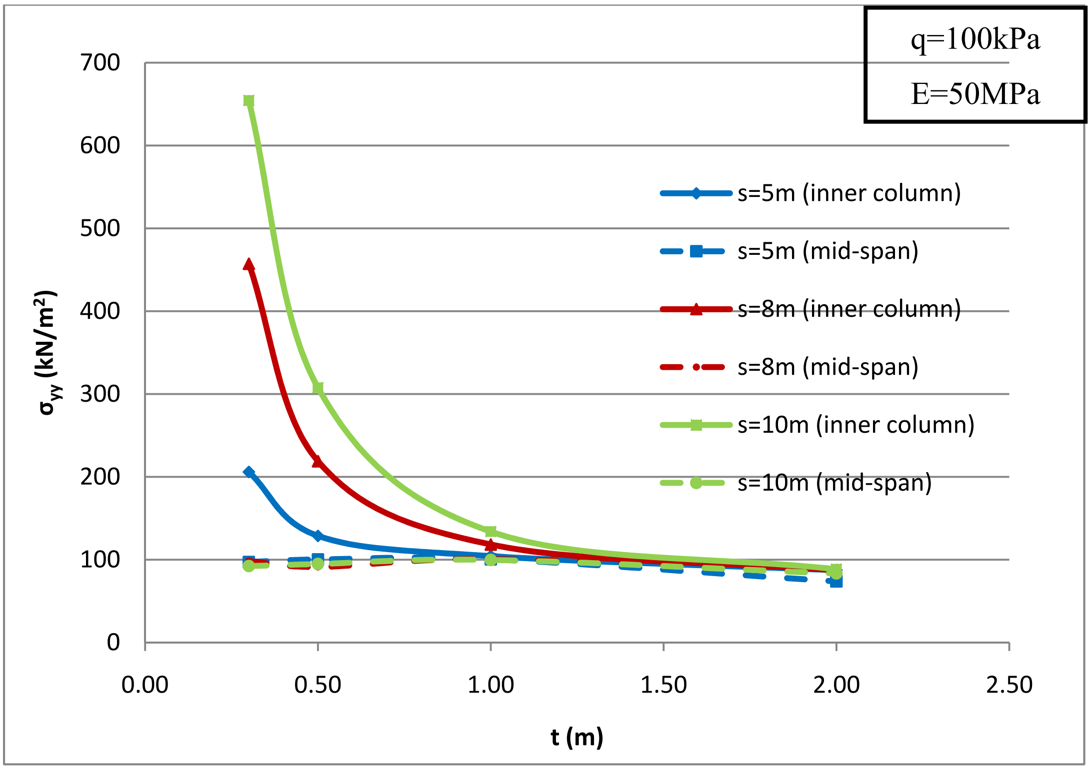

Figure 4.15 σ_{yy} vs t for various column spacings under the columns and mid-spans

Figure 4.15 demonstrates that as foundation thickness increases, contact stresses under column locations decrease rapidly and the trend is marginal for small foundation thicknesses. On the other hand, contact stresses under mid-span locations decreases at a smaller rate with respect to contact stresses under column locations, and may be considered as constant.

The variation of foundation displacements as a function of foundation thickness for both mid-span and column locations are shown in Figure 4.16 for different column spacings.

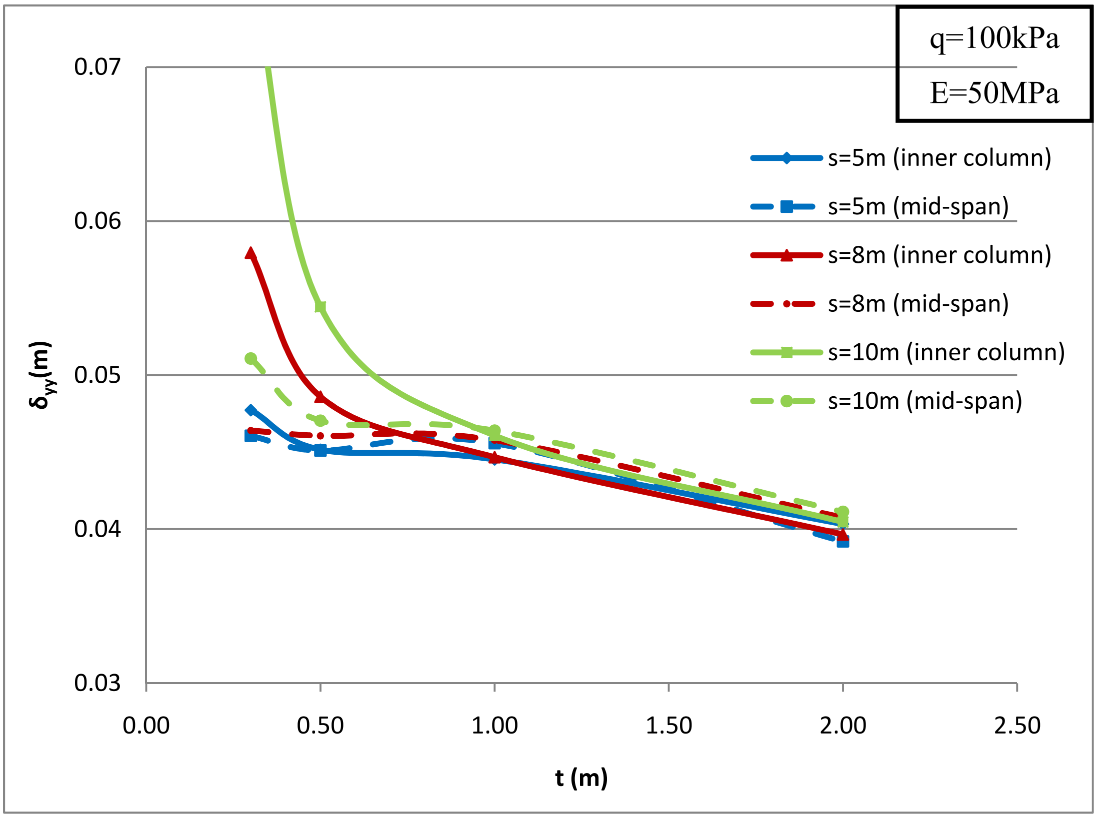

Figure 4.16 δ_{yy} vs t for various column spacings under the columns and mid-spans

As shown in Figure 4.16, there is not a significant difference between settlement under column locations and span locations for a specific column spacing over the foundation thickness larger than 1.00 m. The difference becomes obvious as foundation thickness decreases. Moreover, the difference in settlement under column locations between different column spacings is more pronounced as foundation thickness decreases. In addition, settlement is not sensitive to the variations in foundation thickness as much as the one affected by the variation in modulus of elasticity of soil.

For different column spacing cases over various thickness of foundation developed modulus of subgrade reactions under columns and under mid-spans are compared as given in the Figure 4.17.

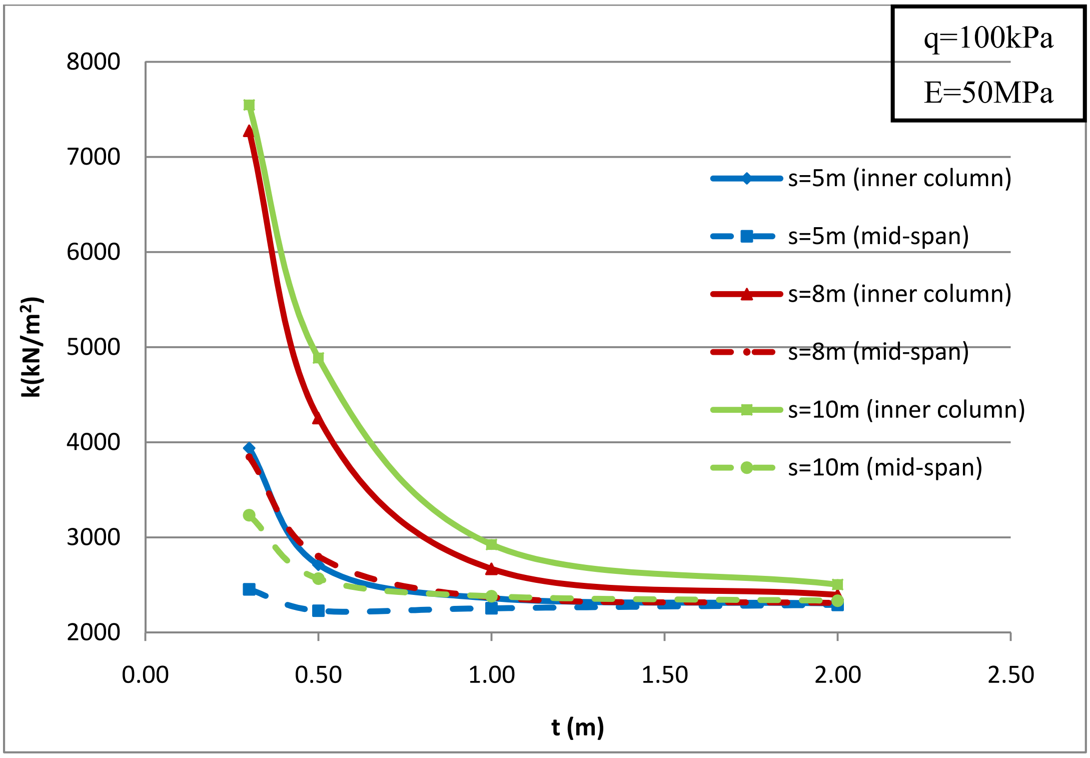

Figure 4.17 t vs k for various column spacings under the columns and mid-spans

Figure 4.17 shows that as foundation thickness increases, modulus of subgrade reaction decreases for all cases. This trend is more obvious in thinner foundations, but becomes more marginal as foundation becomes thicker. Decrement of modulus of subgrade reaction at column locations are more considerable than the ones at mid-span locations. General trend is similar to behavior of contact stress, since settlement depends less on foundation thickness but highly depends on the modulus of elasticity of soil.

For different column spacing cases over various thickness of foundation developed shear forces (Q) under columns and under mid-spans are compared as given in the Figure 4.18.

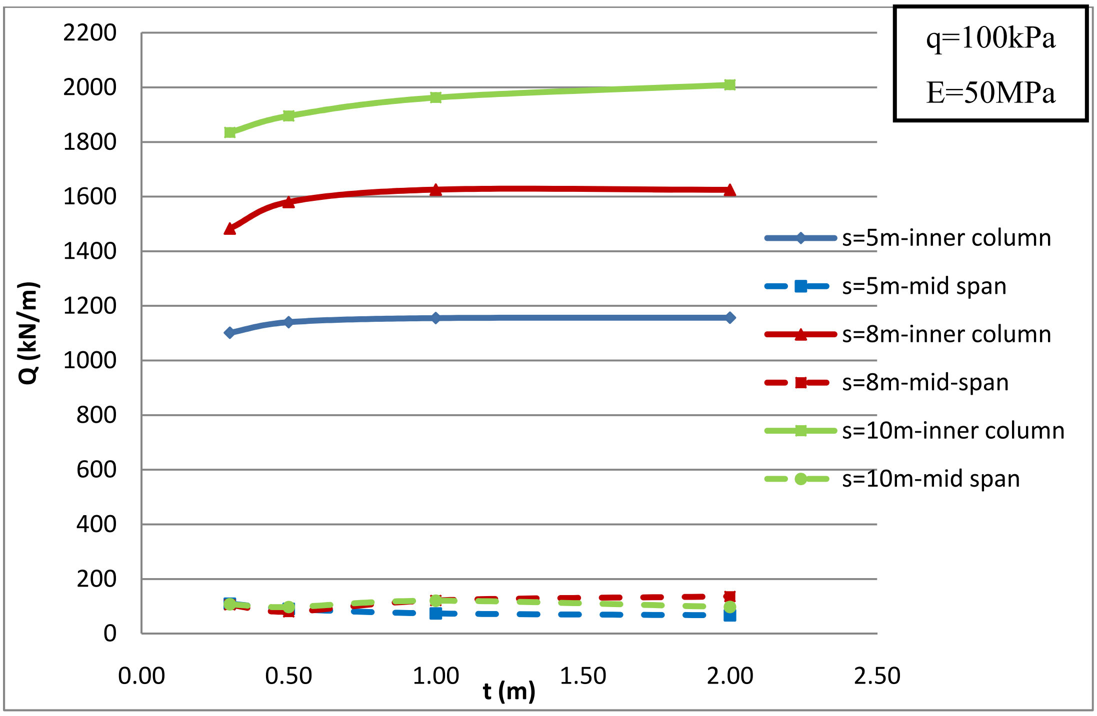

Figure 4.18 Q vs t for various column spacings under the columns and mid-spans

Shear forces under mid-span locations are said to be constant under foundations having different thicknesses. On the other hand, under columns shear forces increase as foundation becomes thicker irrespective to the column spacing. Moreover, shear forces under column locations increase as column spacing increases over specific foundation thickness due to the increase of applied loading through the columns. Whereas, average shear force under mid-span locations is said to be constant for irrespective of the column spacing and the foundation thickness.

Under same loading applied on the foundation over soil with same properties, for different column spacing cases over various thickness of foundation developed bending moment (M) under inner columns and under mid-spans are compared as given in the Figure 4.19.

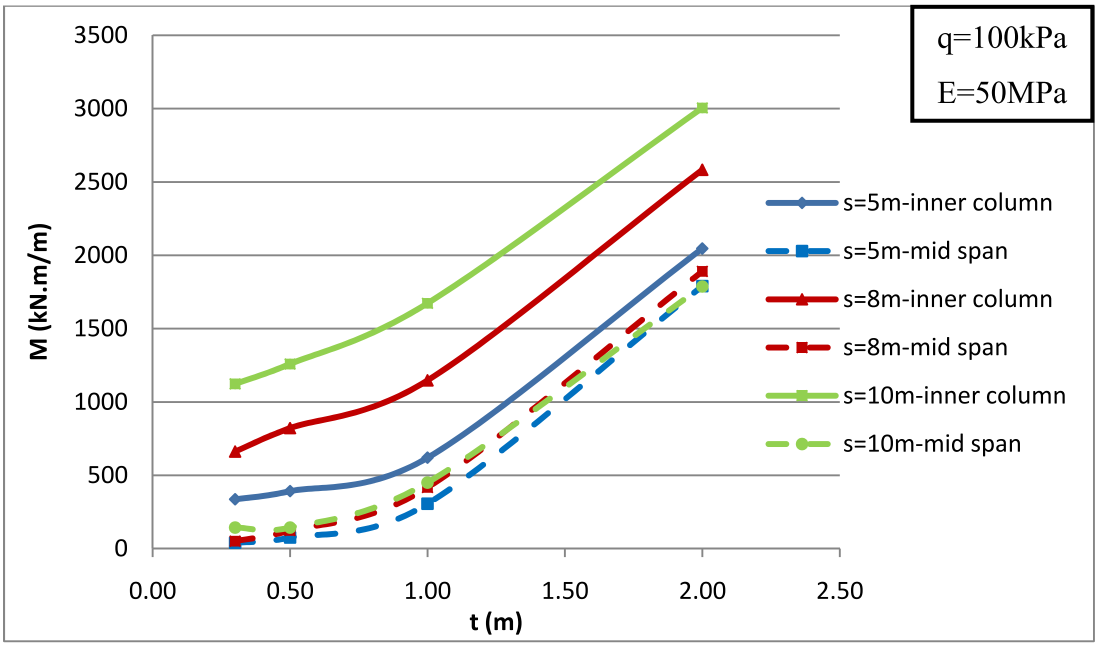

Figure 4.19 M vs t for various column spacings under the columns and mid-spans

Increase in foundation thickness leads to increase in bending moment under both inner column locations and mid span locations. This is expected since according to simple bending theory, as foundation thickness increases, bending moment increases at same unit rotation.

Contrary to shear forces, bending moments increase as foundation thickness increases both under column and mid-span locations. Opposite to the effect of variation in deformation modulus, variation in foundation thickness greatly affects the bending moment beneath both the column and mid-span locations.

The ratio of contact stress between column and span locations for various foundation thicknesses are calculated. The relation is given in Figure 4.20.

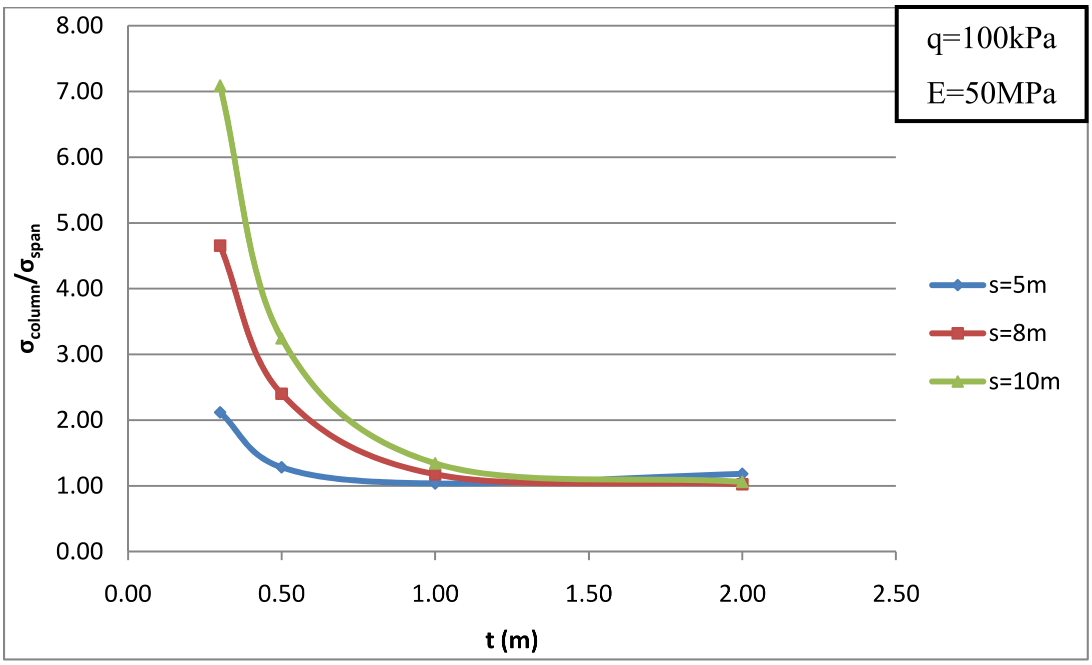

Figure 4.20 $\sigma_{column}/\sigma_{span}$ vs t for various column spacings

As it is illustrated in Figure 4.20, the $\sigma_{column}/\sigma_{span}$ ratio decreases as foundation thickness increases. In addition this trend is more obvious for larger column spacings. Since, more rigid foundation leads to a more uniform distribution of contact stress, the differences between column locations and span locations decreases.

The ratio of foundation settlement between column and span locations for various foundation thicknesses are calculated. The relation is given in Figure 4.21.

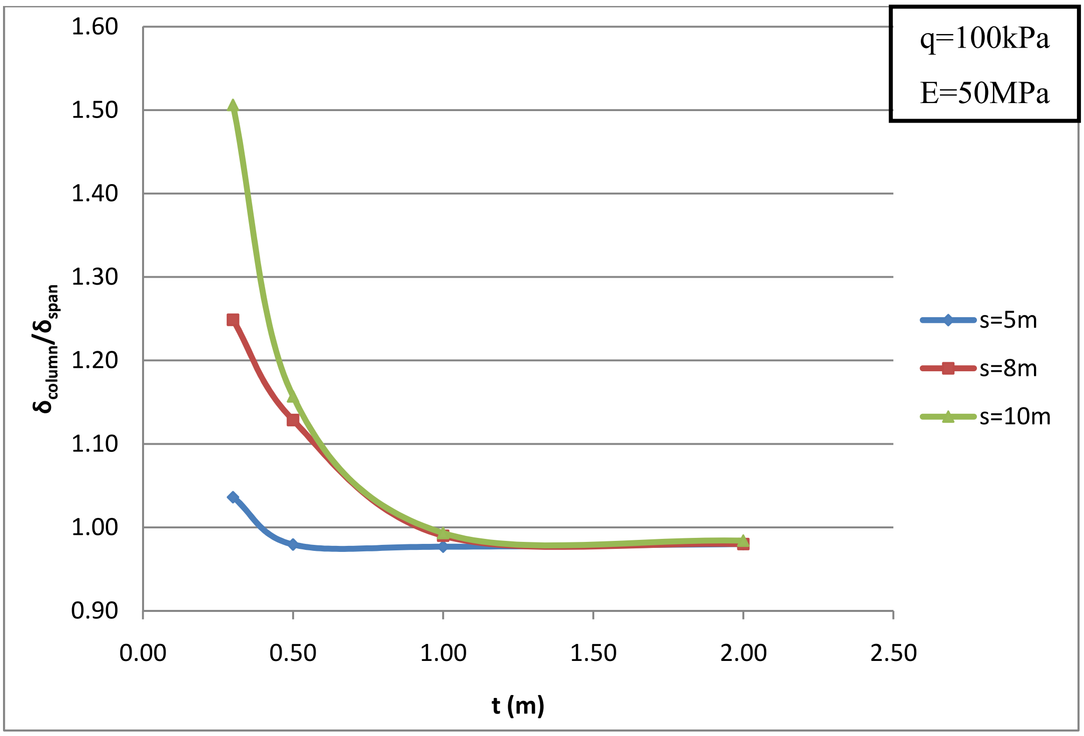

Figure 4.21 $\delta_{column}/\delta_{span}$ vs t for various column spacings

As demonstrated in Figure 4.21, settlement ratio between column and span locations is parallel to the previous stress that the ratio decreases as foundation thickness increases for same column spacing and the decaying curve gets much steeper as column spacing increases, since foundation is less rigid so that the same amount of increase in rigidity is more effective on larger column spacing.

The ratio of modulus of subgrade reaction between column and span locations for various foundation thicknesses are calculated. The relation is given in Figure 4.22.

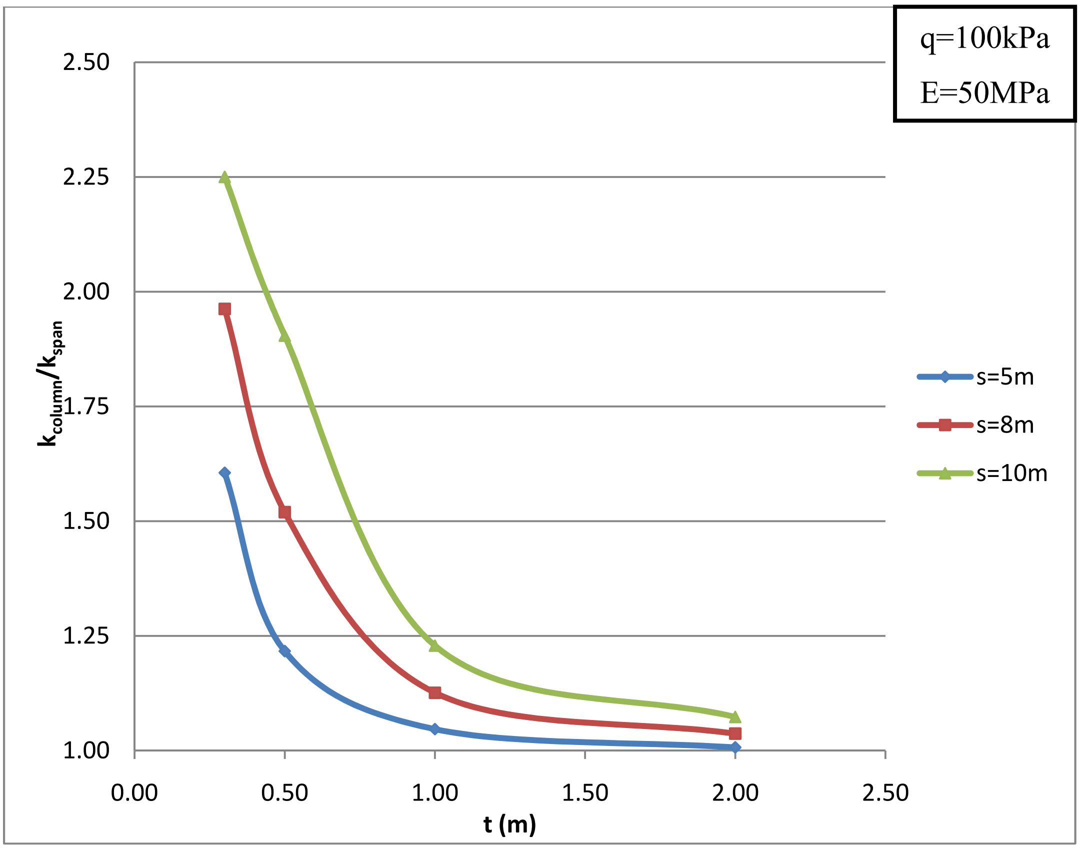

Figure 4.22 k_{column}/k_{span} vs t for various column spacings

Since decrease in contact stress is dominant than the decrease in foundation settlement through the entire cross-section, consequently modulus of subgrade reaction under column to the span decreases by the increase of foundation thickness. Moreover, as column spacing increases this decay is more rapid. Furthermore, as foundation rigidity inceases the ratio approaches to 1, that at infinite rigidity the contact stresses developed under the column locations are just same with ones developed under span locations. In other words, the modulus of subgrade reaction distribution would be uniform through the entire cross section as foundation thickness increases.

Several cases are studied for constant deformation modulus of soil, E = 50 MPa, and constant load pressure, q = 100 kPa, to generalize the contact stress distribution under the mat foundation having different thicknesses and loaded by the columns having different column spacings. Those comparisons are illustrated in Figure 4.23, Figure 4.24, Figure 4.25 and Figure 4.26 by means of normalized contact stress with respect to applied pressure.

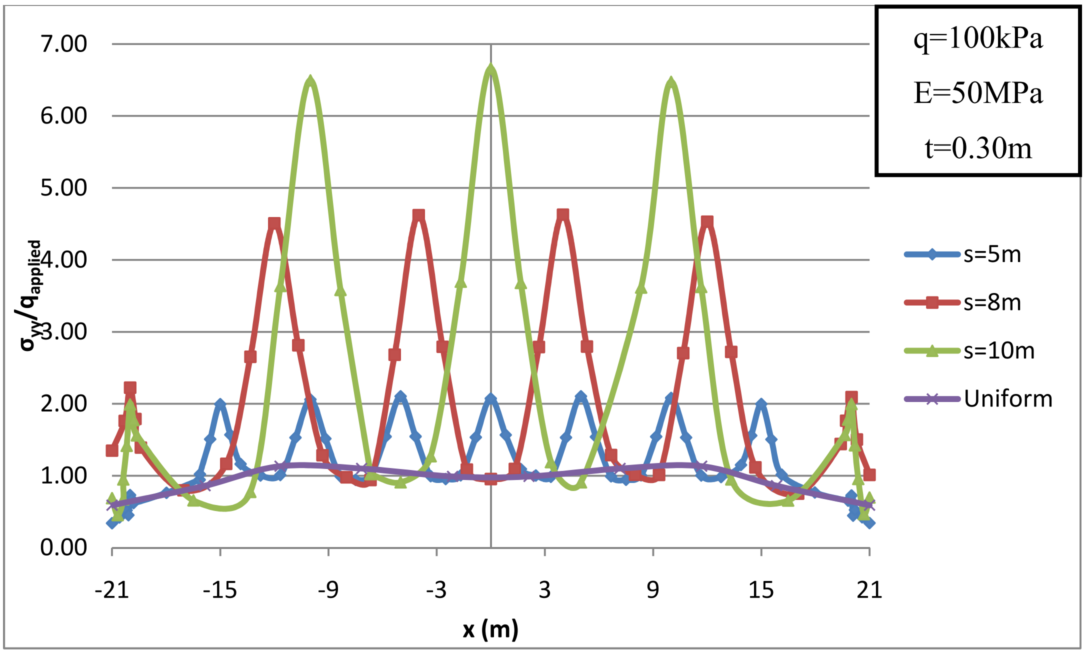

Figure 4.23 Normalized contact stress distribution for various column spacings for t = 0.30 m

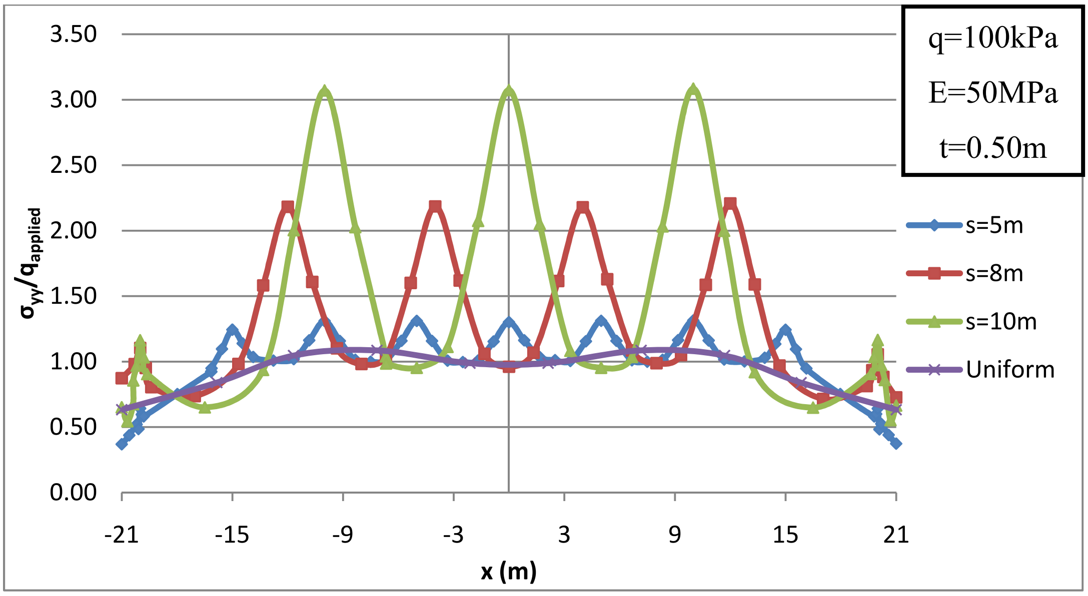

Figure 4.24 Normalized contact stress distribution for various column spacings for t = 0.50 m

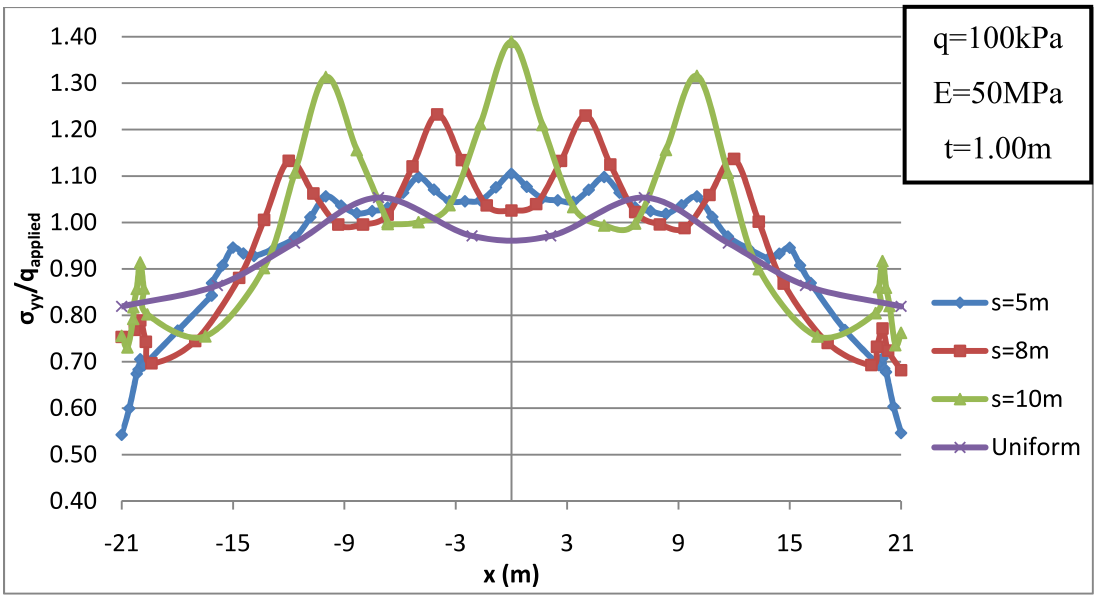

Figure 4.25 Normalized contact stress distribution for various column spacings for t = 1.00 m

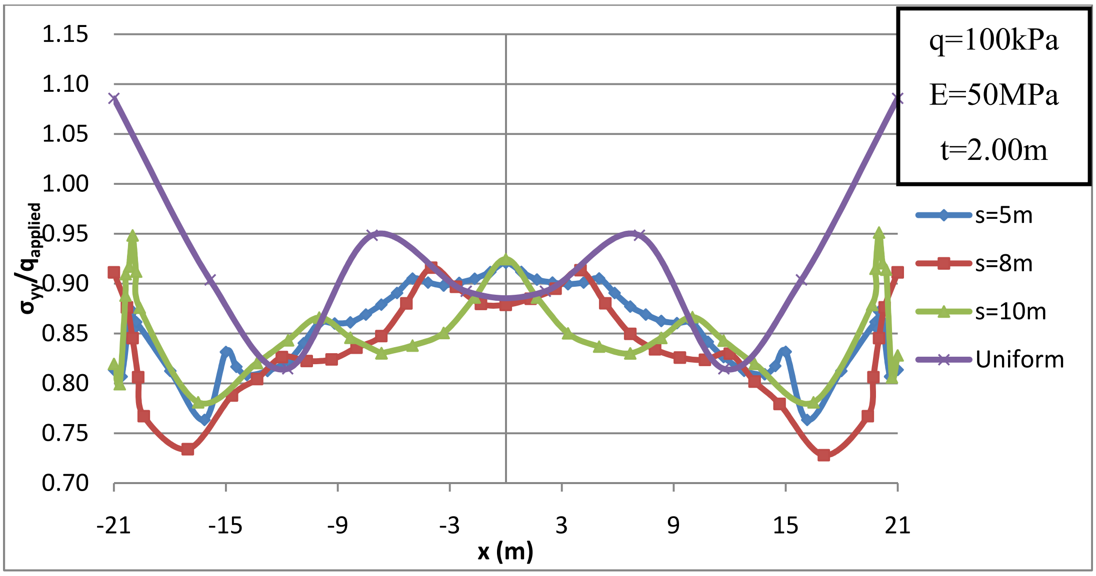

Figure 4.26 Normalized contact stress distribution for various column spacings for t = 2.00 m

All normalized stress distributions demonstrated in Figure 4.23, Figure 4.24, Figure 4.25 and Figure 4.26 are idealized and the general stress distribution is summarized as in Figure 4.27 and Table 4.3. Note that for s = 5 m and s = 8 m, contact stress zones are similar to the given case of s = 10 m in Figure 4.27.

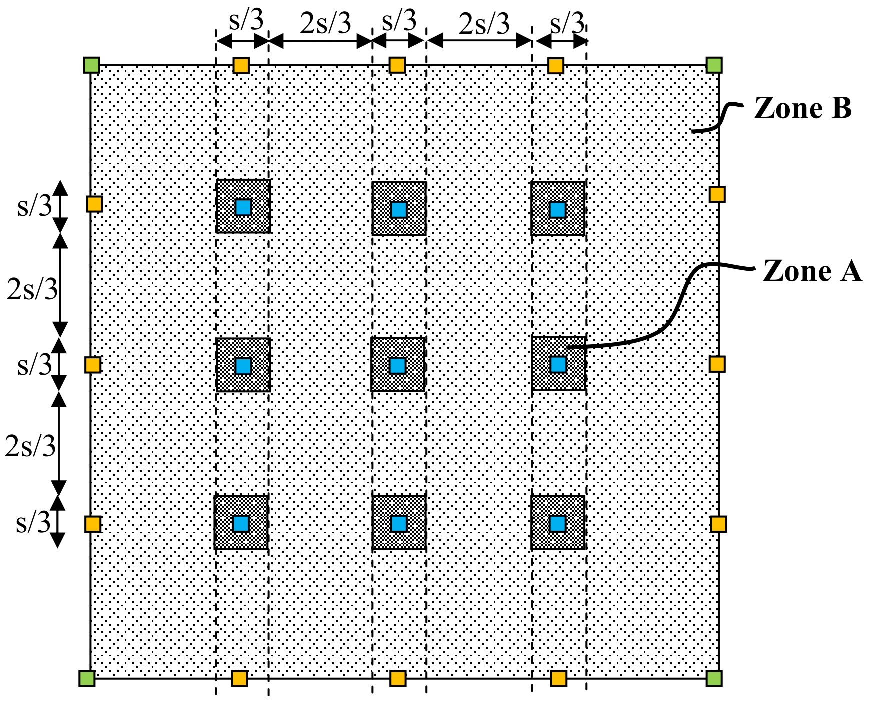

Figure 4.27 Zones for contact stress distribution for variation in foundation thickness of Pattern B – s = 10 m

Table 4.3 Summary of normalized contact stresses at zones shown in Figure 4.27 for different columns spacings over soil having different "t"

η												
Zones	**t (m) (E = 50 MPa; q = 100 kPa)**											
	s = 5 m				**s = 8 m**				**s = 10 m**			
	0.30	**0.50**	**1.00**	**2.00**	**0.30**	**0.50**	**1.00**	**2.00**	**0.30**	**0.50**	**1.00**	**2.00**
A	2.10	1.30	1.05	0.88	4.60	2.20	1.18	0.88	6.70	3.10	1.35	0.90
B	1.00	1.00	1.03	0.85	0.95	1.00	1.00	0.85	0.91	0.95	1.00	0.84

Where;

$$\sigma_{yy} = \eta * (q_{uni} = 100kPa) \qquad \text{(Equation 4.5)}$$

η: Ratio of average contact stress in zones defined in Figure 4.27 to average applied load (i.e. 100 kPa) for variable foundation thickness.

From Figures 4.28 to 4.31, it is obvious that as column spacing increases the individual effect of a column is increasing so that the increase in contact stress occurs at larger area around the columns.

Moreover, Table 4.3 shows that, as foundation rigidity increases contact stress distribution becomes uniformer that the stresses under columns decrease and stress difference between mid-span and column locations decrease, irrespective of the column spacing.

From Table 4.2 and Table 4.3 it is understood that as deformation modulus decreases and/or foundation thickness increases, contact stress distribution under the foundation becomes uniform. This means the differences between the stresses beneath the column locations (Zone A) and stresses beneath the span locations (Zone B) decrease and approaches to the applied load pressure. Thus, in order to obtain uniform contact stress pressure under the foundation, the combined effect of deformation modulus of soil and foundation rigidity should be considered.

Table 4.3 shows that there is no significant change in the stresses beneath the span locations (Zone B) opposite to the column locations (Zone A). Thus, other than increasing the entire foundation thickness, only increasing the foundation thickness at Zone A is also studied.

4.2 Comparision of Uniform and Concentrated Loading Cases

In general the variation of contact stress, settlements and modulus of subgrade reactions of uniform loading condition (Pattern: A) is very similar to the behavior of mat foundation with concentrated loading through columns at mid-span locations. This behaviour is illustrated in Figure 4.28 to Figure 4.32.

For different column spacings and uniform loading for E = 50 MPa, q = 100 kPa and t = 0.5 m contact stress, settlement, modulus of subgrade reaction, shear forces and bending moment distributions for the mid-section of the mat foundation are given in Figure 4.28, Figure 4.29, Figure 4.30, Figure 4.31 and Figure 4.32, respectively.

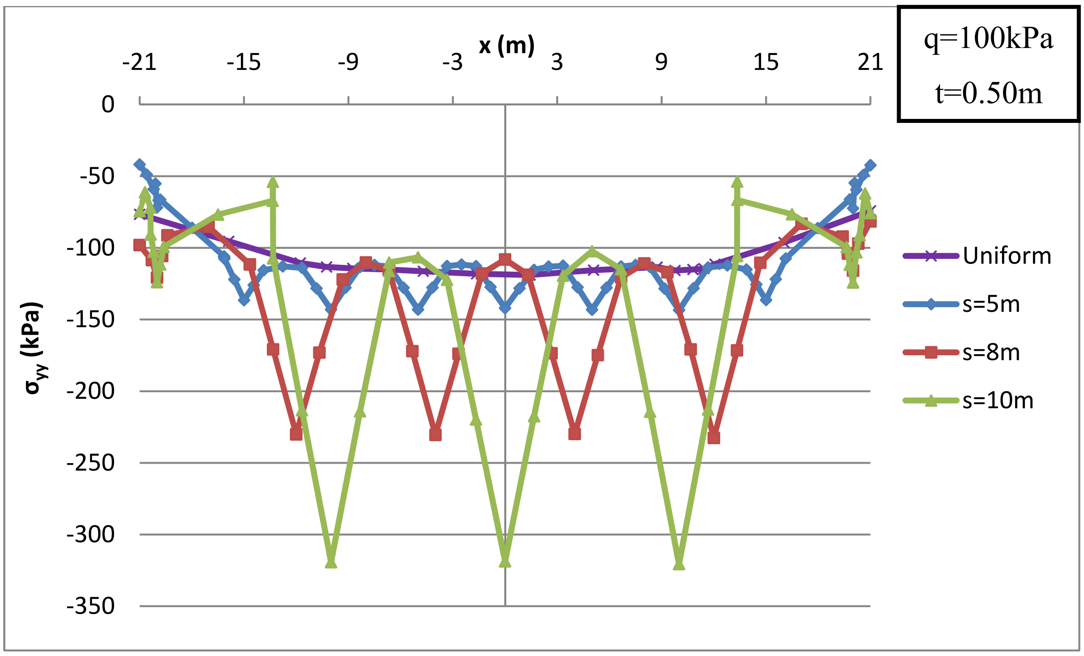

Figure 4.28 Contact stress distributions for pattern A and pattern B

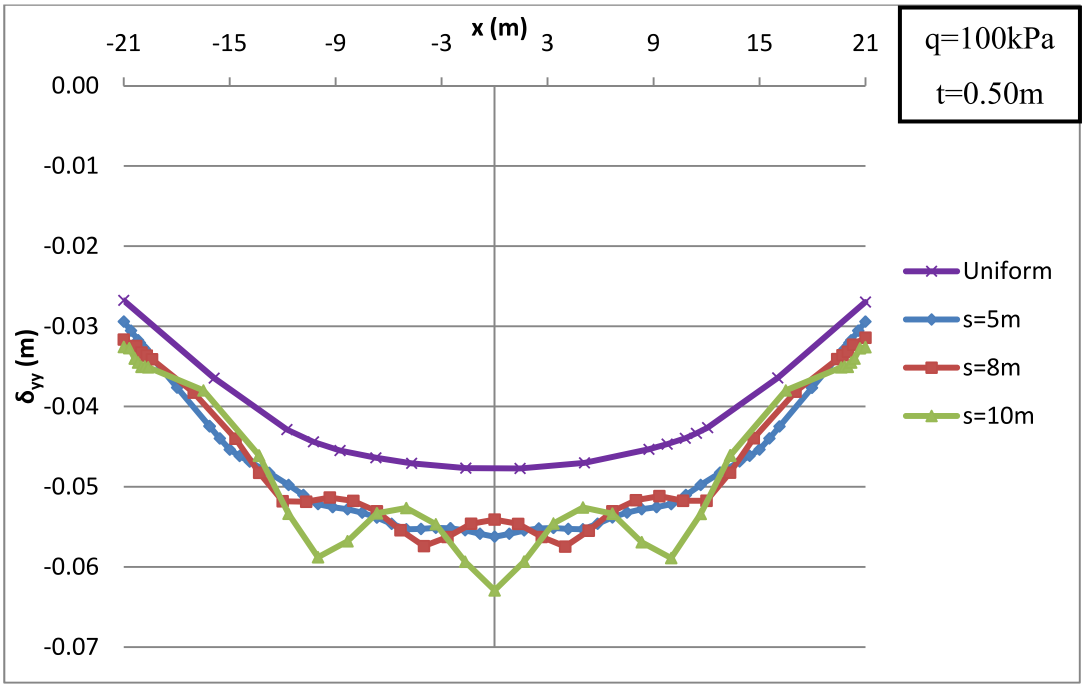

Figure 4.29 Settlement distributions for pattern A and pattern B

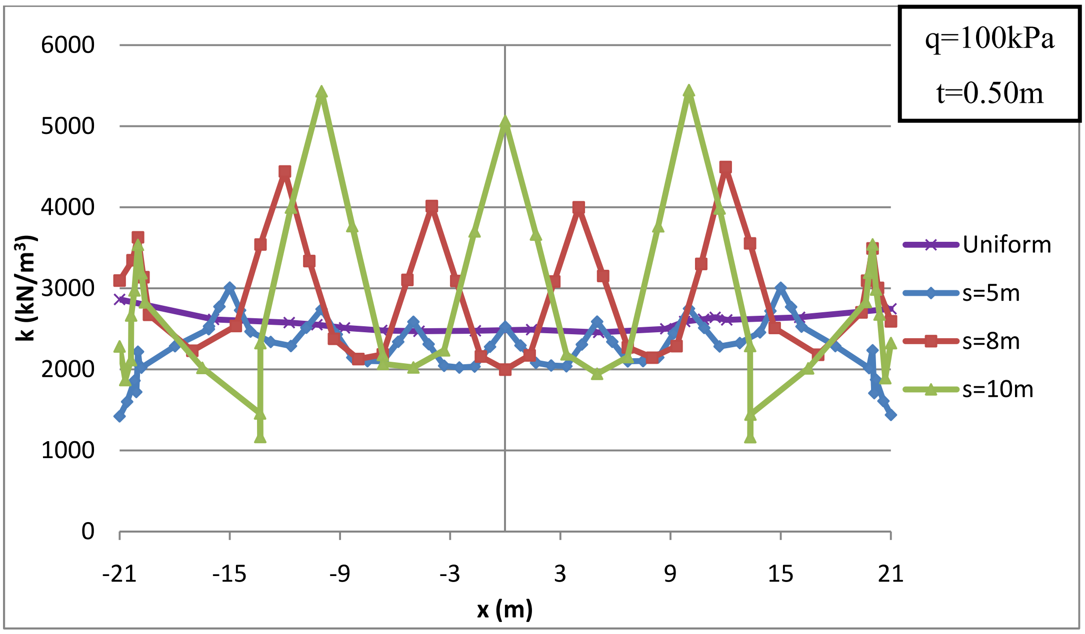

Figure 4.30 Modulus of subgrade reaction distributions for pattern A and pattern B

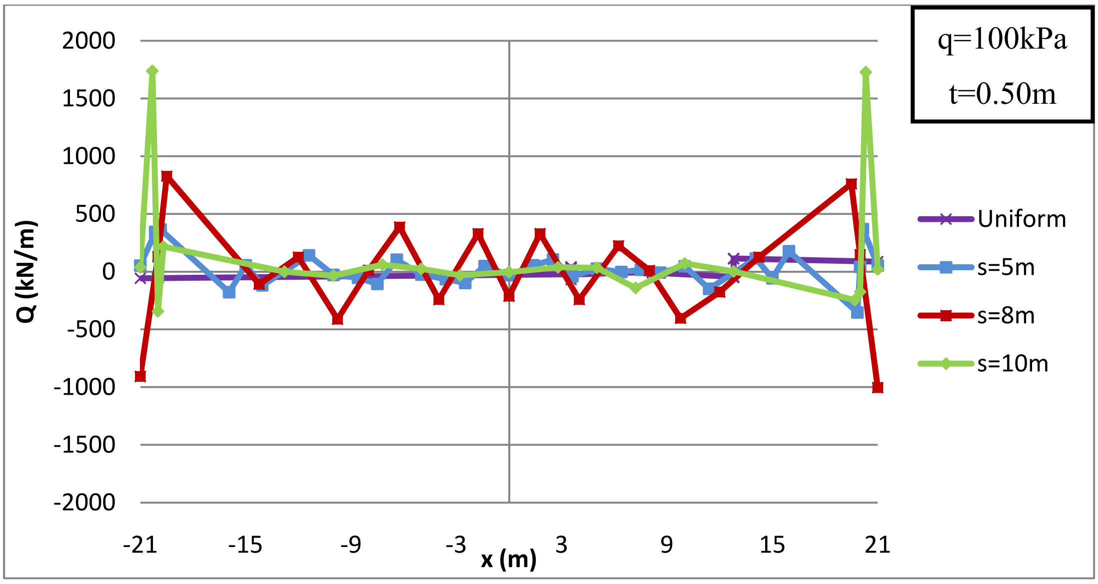

Figure 4.31 Shear force distributions for pattern A and pattern B

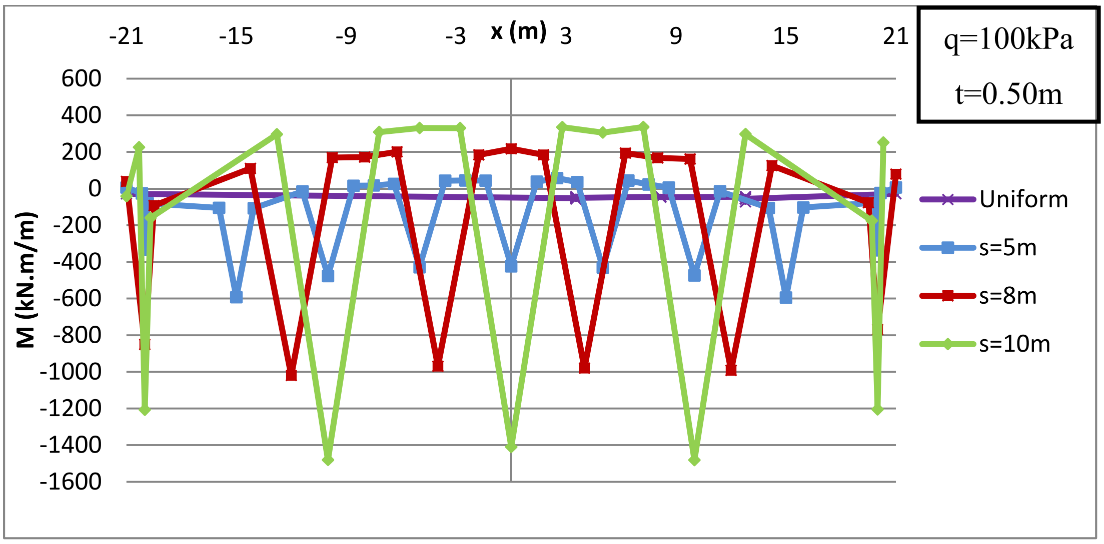

Figure 4.32 Bending moment distributions for pattern A and pattern B

It is noted that, moment and shear diagrams are independent from the modulus of elasticity of the underlying soil, thus only E = 50 MPa will be sufficient to illustrate the actual general behavior of the foundation under prescribed conditions.

4.2.5 Comparison between PLAXIS and SAP

In practice mat foundation are commonly designed using SAP 2000 computer software using a constant modulus of subgrade reaction value. This is a discrete model resembling the soil support as individual springs, modulus of subgrade reaction being the spring constant. Whereas, a more realistic approach could be a continuum model where soil is represented by a constant or variable deformation modulus. Such analysis may be done by using Plaxis 3D finite element computer software.

In first trial, the modulus of subgrade reaction values were assigned in accordance with the changes in "*k*" values at different locations (i.e. corner, edges and mid-span *k* values).

A typical case is considered in this section to compare the two approaches. The following case is analysed using SAP 2000 and Plaxis 3D:

Foundation thickness	: t = 0.50 m
Deformation modulus	: E = 25 MPa
Column spacing	: s = 8 m
Uniform loading	: q = 100 kPa

The bending moment and the shear diagrams shown in Figure 4.33 and Figure 4.34 indicate that both analysis reveal same distribution of bending moment and shear throughout the raft once proper values of modulus of subgrade reaction are assigned at different regions of the raft.

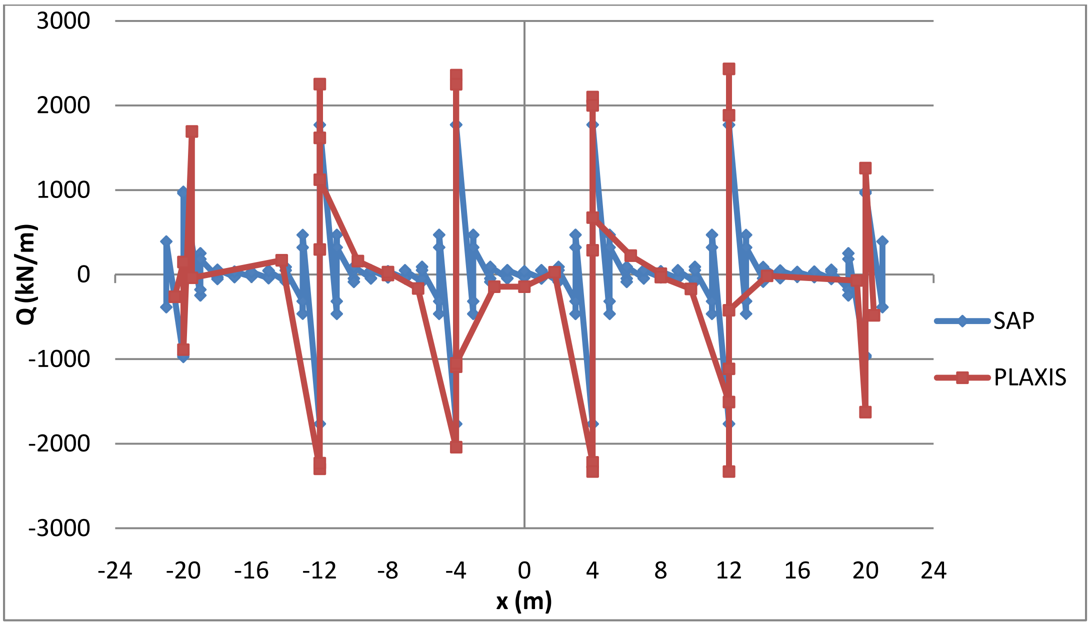

Figure 4.33 Comparison of shear force distributions obtained from PLAXIS and SAP

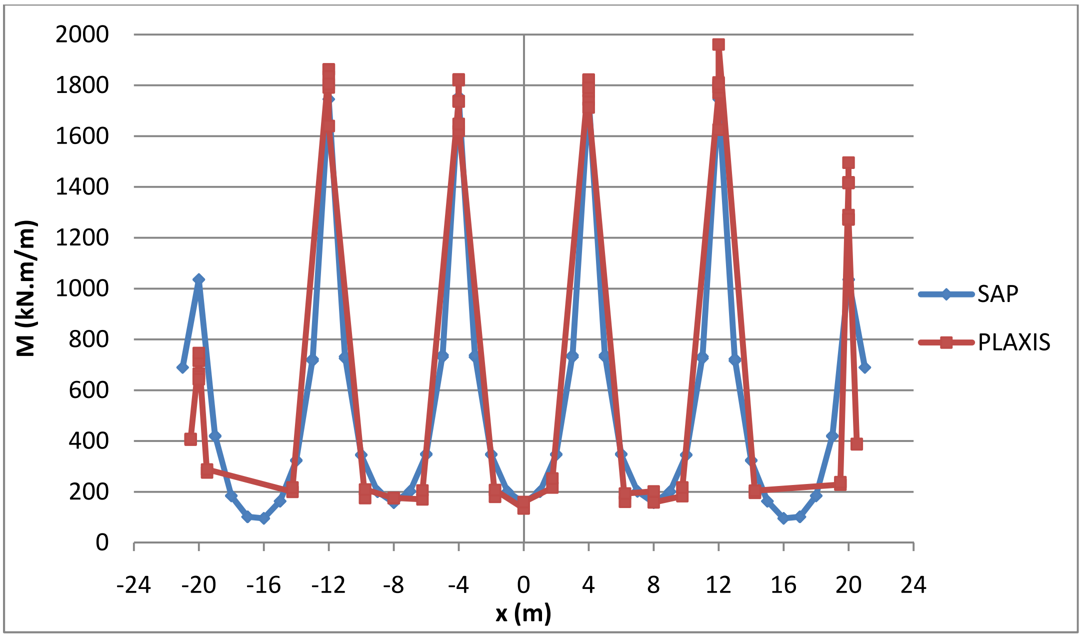

Figure 4.34 Comparison of bending moment distributions obtained from PLAXIS and SAP

As it is seen in Figure 4.33 and Figure 4.34, two different finite element softwares give similar shear force and bending moment distributions for the proceeded analysis.

In the second trial an average modulus of subgrade reaction is assigned to the raft ignoring the local variation of "*k*" values within the raft.

The results of the analyses are show in Figure 4.35 and Figure 4.36. The two analyses reveal almost identical results indicating that the spring support idealization of soil media is not sensitive to local variations in the "*k*" values.

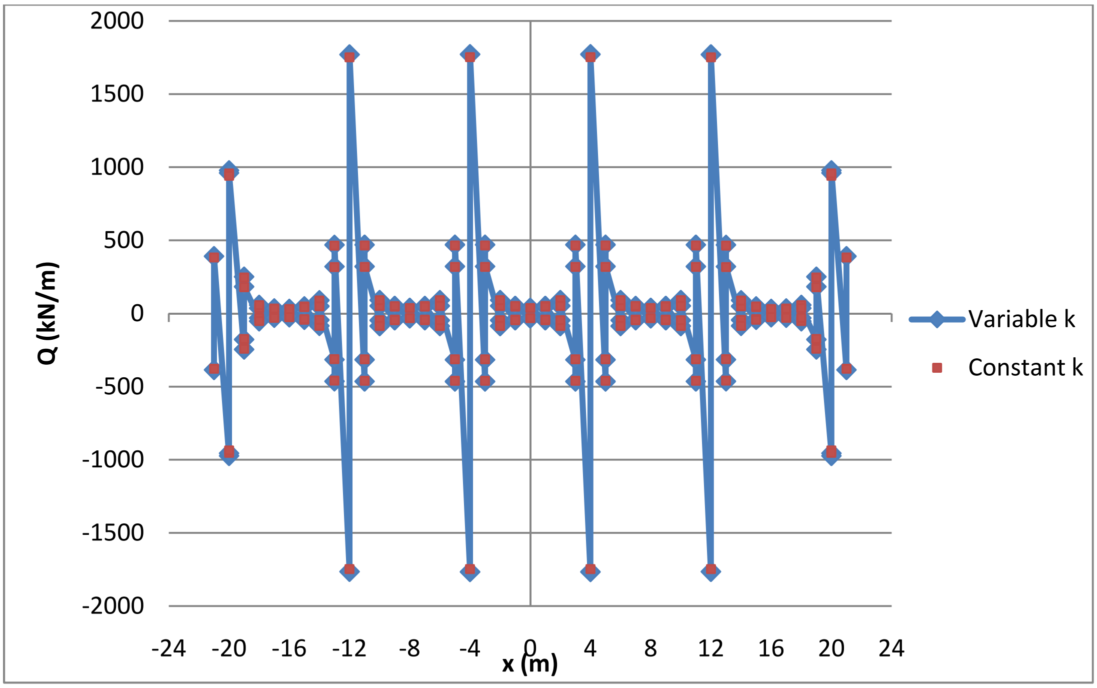

Figure 4.35 Comparison of shear force distribution between *Variable k* and *Constant k* analyses

As seen from Figure 4.35, for each point on the cross section the shear diagram totally overlaps for both analysis.

Similar to shear force distribution, also bending moments are insensitive to variation in modulus of subgrade reaction due to difference between Pattern A and Pattern B, as shown in Figure 4.36.

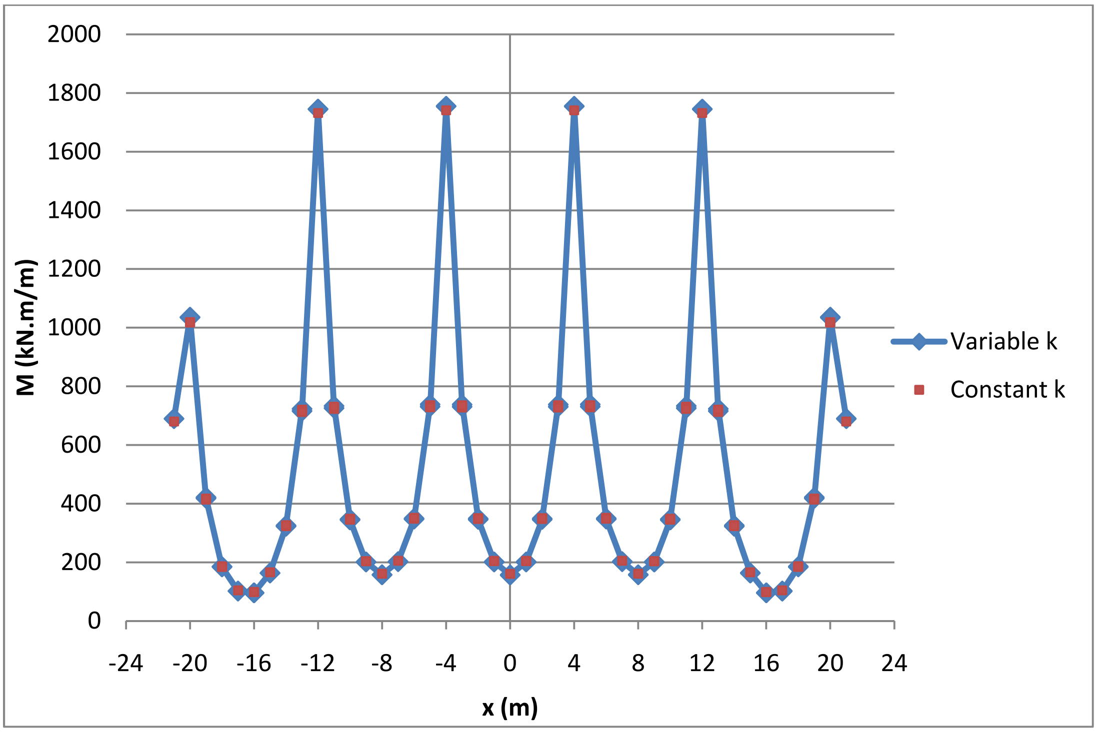

Figure 4.36 Comparison of bending moment distribution between *Variable k* and *Constant k* analyses

Figure 4.36 supports the statement given by Bowles (1982) that the bending moments are relatively insensitive to variation of modulus of subgrade reaction due to difference between Pattern A and Pattern B, since the flexural rigidity of the member is so much larger than the effective rigidity of the soil.

As modulus elasticity of soil increases, column spacing increases and foundation thickness decreases the difference between two analyses would get more importance where the difference between modulus of subgrade reactions at span locations and average modulus of subgrade reaction for uniform loading would increase.

CHAPTER 5.

CONCLUSIONS

The behavior of mat foundation is analysed using 3D finite element program. Parametric study is performed varying foundation rigidity, soil stiffness and loading pattern for constant deformation modulus with respect to depth. The following conclusions are driven:

- For Patterns A and B, variations in contact stress (σ_{yy}), settlement (δ_{yy}), angular rotation and modulus of subgrade reaction (k) depending on the increase in deformation modulus of subgrade soil (E) are all summarized in Table 5.1.

Table 5.1 Variations in σ_{yy}, δ_{yy}, angular rotation and k depending on increase in E for Patterns A and B

Uniform Loading (Pattern A)		Concentric Loading (Pattern B)	
$\sigma_{central\ zone}$	$\leftrightarrow$	σ_{col} / $\sigma_{mid\text{-}span}$	$\uparrow$
σ_{edge}	$\downarrow$		
δ_{ave}	$\downarrow$	δ_{ave}	$\downarrow$
Angular Rotation	$\downarrow$	**Angular Rotation**	$\downarrow$
k_{edge} / k_{center}	$\downarrow$	k_{col} / $k_{mid\text{-}span}$	$\uparrow$

- For Patterns A and B, variations in contact stress (σ_{yy}), settlement (δ_{yy}), angular rotation and modulus of subgrade reaction (k) depending on the increase in foundation thickness (t) are all summarized in Table 5.2.

Table 5.2 Variations in σ_{yy}, δ_{yy}, angular rotation and k depending on increase in t for Patterns A and B

Uniform Loading (Pattern A)		Concentrated Loading (Pattern B)	
$\sigma_{central\ zone}$	$\downarrow$	σ_{col} / $\sigma_{mid\text{-}span}$	$\downarrow$
σ_{edge}	$\uparrow$		
δ_{ave} (t ≤ 1m)	$\leftrightarrow$	δ_{col} / $\delta_{mid\text{-}span}$	$\downarrow$
δ_{ave} (t > 1m)	$\downarrow$		
Angular Rotation	$\downarrow$	**Angular Rotation**	$\downarrow$
k_{edge} / k_{center}	$\uparrow$	k_{col} / $k_{mid\text{-}span}$	$\downarrow$

- In uniform loading case the contact stresses in the middle of the raft is more or less uniform. The contact stresses decrease rapidly towards edges. This trend is reversed in case of rigid foundation (i.e. 2.00 m thick). The ratio of edge to center stresses for 0.30 m foundation thickness is approximately 0.60 where, for 2.00 m foundation thickness is approximately 1.20.

- The variation of modulus of subgrade reaction is not significant (i.e. less than 15%) at different regions of the raft irrespective of the value of deformation modulus of subgrade soil. The variation is somehow noticable as foundation thickness increases.

- The modulus of subgrade reaction depends on size of the foundation and the deformation modulus of the supporting soil. The following correlation is proposed:

$$k_{ave} = \frac{1.1E}{B^{0.85}}$$

- It is found that stress concentration occurs under the columns, the magnitude of the contact stress exceeds the mid-span stresses upto 7 times. The stress concentration effects are more pronounced in stiffer subgrade soil and in thin foundation plates.

- The modulus of subgrade reaction is uniform throughout the raft in softer subgrade soil, but highly variable (i.e higher values under columns, as compared to mid-span) in stiffer subgrade soils. The magnitude of average subgrade modulus significantly low in soft soil conditions as compared to stiffer subgrades.

- Distribution of modulus of subgrade reaction is more uniform for stiffer foundations loaded by columns having smaller spacing over the softer soil.

- In general the variations of modulus of subgrade reactions in mid-spans for concentrated loading case is very similar to uniform loading case.

- It is found that the analysis of rafts with spring support model is not sensitive to variations of modulus of subgrade reaction in the different parts of the rafts.

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