

A STUDY ON SIXTH GRADE STUDENTS' SPATIAL REASONING
REGARDING 2D REPRESENTATIONS OF 3D OBJECTS

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ABSTRACT

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The main purpose of this study was to investigate the contributions of spatial visualization and spatial orientation tasks regarding 2D representations of 3D objects and isometric drawing to the development of sixth grade students' spatial reasoning. The study also aimed to investigate students' performances on spatial tasks during the classes and to explore their views and feelings about the spatial tasks.

Data were collected from 24 6th grade students in a private school located in Ankara. The design of the research was first person inquiry where the researcher was also the teacher at the same time. During five class hour the topic was covered with spatial tasks developed by the researcher. Spatial Orientation Test (SOT) and Achievement Test on 2D Representations of 3D Objects and Isometric Drawing (AT) were administered to the students before and after the task to evaluate the effect of instruction. Wilcoxon signed rank test was run to test data. Statistical analyses revealed that there was a statistically significant difference in students' spatial reasoning development between pre-test and post-test.

Another finding of the study was related to the progress of students' performances in the visual reasoning tasks. As the time passed, students got more successful in completing even the more advanced tasks. The findings of the study indicated that in order to develop students' visual reasoning abilities teachers need to

provide them opportunities to practice with the visual tasks supported with the effective use of manipulative. In addition to this, teachers should create activities and design their lessons in a way that where the students are the actors and actively participated in the class.

Keywords: Spatial reasoning, spatial visualization, spatial orientation, isometric drawing, 2D representations

ÖZ

6.SINIF ÖĞRENCİLERİNİN 3B NESNELERİN 2B GÖSTERİMLERİ HAKKINDAKİ UZAMSAL MUHAKEMELERİ ÜZERİNE BİR ÇALIŞMA

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Bu çalışmada 3B nesnelerin 2B gösterimlerine ilişkin olarak uzamsal görselleştirme ve uzamsal yönelim etkinliklerinin 6.sınıf öğrencilerinin uzamsal muhakemelerine katkısı araştırılmaktadır. Öğrencilerin uzamsal etkinliklerdeki performanslarını gözlemek ve bu etkinlikler hakkındaki görüş ve duygularını ortaya çıkarmak çalışmanın diğer amaçları arasında yer almaktadır. Çalışmaya Ankara'da bir özel okuldan 24 6. sınıf öğrencisi katılmıştır. Çalışmada araştırmacı aynı zamanda öğretmendir. Bu bağlamda, araştırmacı tarafından geliştirilen uzamsal görselleştirme ve uzamsal yönelim etkinlikleri 5 ders saati boyunca öğrencilere uygulanmıştır. Etkinliklerden önce ve sonra öğrencilere Uzamsal Yönelim Testi ve 3B Nesnelerin 2B Gösterimleri ve İzometrik Çizim soruları içeren bir başarı testi uygulanmıştır.

Verileri test etmek üzere Wilcoxon signed rank test kullanılmıştır. İstatistiksel analiz sonucunda öğrencilerin uzamsal muhakemelerinde ön test ve son test arasında istatistiksel olarak anlamlı bir fark bulunmuştur. Çalışmanın diğer bir bulgusu ise öğrencilerin görsel muhakeme gerektiren etkinliklerde gelişme kaydettikleridir. Sayısal sonuçlar da etkinliklerden sonra öğrencilerin uzamsal görselleştirme ve yönelimde kendilerini geliştirdiklerini göstermektedir. Çalışmanın sonucu şunları göstermektedir: Öğrencilerin uzamsal becerilerini geliştirmek için öğretmenler amaca uygun materyallerle desteklenen görsel etkinliklerle dersi işlemelidir.

Etkinliklerini ve dersleri öğrencileri merkeze alacak şekilde tasarlamalıdır ve etkinliklere öğrencilerin etkin katılımını sağlamalıdır.

Anahtar Kelimeler: Uzamsal muhakeme, uzamsal görselleştirme, uzamsal yönelim, izometrik çizim, 2B gösterimler

To The Head Teacher Mustafa Kemal Atatürk and To Children

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LIST OF ABBREVIATIONS

AT: Achievement Test on 2D Representation of 3D Objects and Isometric Drawing

CCT: Cube Comparison Test

CRT: Card Rotation Test

DAT: Differential Aptitude Test

ETS: Educational Testing Service

MGMP: Middle Grades Mathematics Project

METU: Middle East Technical University

MNE: Ministry of National Education

NAEP: The National Assessment of Educational Progress

NCTM: National Council of Teachers of Mathematics

SAT: Spatial Ability Test

SOT: Spatial Orientation Test

SBS: Seviye Belirleme Sınavı

3D: Three dimensional

2D: Two dimensional

CHAPTER 1

INTRODUCTION

With the renewed curriculum in 2005, the concept spatial ability gained more importance in Turkey. The renewed curriculum has clear emphasis on the concepts related to spatial reasoning. Moreover, the possibility of questions measuring spatial reasoning appearing in the national examination (Seviye Belirleme Sınavı-SBS-) which was first administered to the 6th, 7th and 8th grade students in 2007 makes these concepts more significant for students. In the former curriculum, the attention given to spatial reasoning was limited to drawing a few specific 3D objects (cube, rectangular, square and triangular prism, cylinder, etc...) and finding the volume of 3D objects in middle grades. Students had to simply identify the elements of the 3D objects and find the areas and volumes of them--even the most difficult ones. However, obviously, they were mostly dealing with the algebraic or numerical aspects of space geometry. For instance, in the former curriculum, the objective of 6th grade regarding these concepts was to calculate the volume of cube and rectangular prism. There were 10 actions under this objective and all of them were related to finding and calculating the formula for cube and rectangular prism (MNE, 2002). Similarly, in the former 7th grade curriculum the objectives were to comprehend the properties of orthogonal cylinder and calculate the area and volume of it. Finally, in 8th grade the objectives were to comprehend the properties of right prisms, pyramid, cone and sphere and to calculate their area and volumes (MNE, 2002). In the former curriculum the emphasis, as understood from the objectives, was on calculation of volumes and areas of 3D objects. These types of activities or expectations did not intend to develop students' power in visualization.

In 2005, with the reform in Turkish national mathematics curriculum, new concepts related to spatial reasoning appeared in textbooks such as two dimensional

(2D) representations of three dimensional (3D) objects, 2D views of 3D objects from different angles (top, bottom, front and sides) constructing 3D objects given their 2D views and isometric drawings. These terms are new for the students, but they are also new for the teachers. Most of the teachers were not familiar to these terms previously since there were no such objectives in the former curriculum. The activities and questions in the new textbooks point out that in the new curriculum, the “visualization in space geometry” is more emphasized compared to former curriculum. One of the essential goals of the renewed mathematics curriculum in Turkey is to facilitate students’ comprehension of mathematical concepts and systems and create connections between them and utilize these concepts and systems in daily life and within the other disciplines (MNE, 2006). Through this revitalization of the curriculum, the understanding and conceptualization of our three dimensional world becomes one of the most essential points in geometry. Modeling, representing and participating in activities about spatial concepts can facilitate students to understand, discover, visualize, and represent concepts and properties of geometric figures in our physical world. Developing spatial reasoning in students make them function effectively in a three dimensional world (NCTM, 2000). Therefore, one of the purposes of the present study is to make students and teachers aware of the existence and importance of spatial tasks in school geometry in the middle grades.

According to Clements (1982) the spatial concepts which allow one to formulate mental images and to manipulate these images in mind have two major dimensions, which are spatial visualization and spatial orientation. Spatial visualization refers grasping and acting upon imaginary movements of two and three dimensional objects and spatial orientation refers understanding relationship between the positions of the object with respect to the one’s own position (Clements, 1998). Spatial visualization includes 2D to 3D transformations. Therefore 2D representations of 3D objects are an example of spatial visualization (Ben-Chaim, Lappan & Houang, 1985; Olkun & Sinoplu 2008). In spatial orientation, the emphasis is on understanding the relationship between the positions of the object with respect to the one’s own position and comparing the two representations of an object. (Educational Testing Service) ETS’s Card Rotations Test is an example of

spatial orientation test (Appendix B). In regard to teaching these concepts -spatial visualization and spatial orientation- there are a great deal of research studies presented in detail in the following sections. Briefly, what the studies claimed was that spatial ability can be improved with appropriate instruction.

The new Turkish mathematics curriculum brought the constructivist and student centered approach into the classrooms. Clearly, this approach can only be consolidated by goal directed tasks (MNE, 2005). The tasks in the current study were designed in such a way that students are the actors in the class-not the spectators. They best learn when they “do” mathematics; this includes thinking about the concepts, touching educational materials, utilizing them effectively, making inferences, discussing, peer tutoring and making reflections.

To illustrate this point, I planned an instructional unit in such a way that students can utilize the manipulative, and build and construct blocks when necessary. In this unit students are also able to effectively work independently and with peers, make drawings, reason about tasks, as well as make inferences and reflect and discuss tasks. In addition to this, I wanted to demonstrate that mathematics is not a subject consisting of rules, numbers, symbols and shapes dictated on a blackboard rather it is best internalized when you “do” it. In this sense, I developed a variety of spatial visualization and spatial orientation tasks regarding 2D representations of 3D objects. The main purpose of my study was to investigate the contributions of these tasks to the students’ spatial reasoning and to observe their performances on spatial tasks during the classes. In addition to this, I wanted to explore their views and feelings about the spatial tasks.

1.1 Main and Sub-problems of the Study

M. How do the spatial visualization and spatial orientation tasks regarding 2D representations of 3D objects contribute to the development of sixth grade students’ spatial reasoning?

1.1 Is there a statistically significant difference in students' spatial reasoning abilities between pre-test and post-test?

1.2 What are the students' performances in spatial tasks prior, during, and after the application of activities?

1.3 What are the students' views about the spatial reasoning activities and their feelings about the activities?

1.2 Significance of the Study

With the reform in national curriculum in 2005, the new objectives regarding spatial reasoning were added as an expectation. The topics include improving students' spatial reasoning ability, spatial visualization and orientation ability regarding 2D representations of 3D objects, given 2D views; isometric drawing the 3D objects. The new curriculum emphasizes the active participation of students in the instructional process. The environment required is that in which students can create their own knowledge, and can learn by doing and experiencing things on their own (MNE, 2006). The questions measuring students' spatial ability started to appear in the national examination (SBS) which was first administered in 2007 for 6th, 7th and 8th grade students. This examination is important for both the students and their parents because it has considerable effect in determining the type of high school they may attend after middle school. The more successful they are on this examination, the higher the chance they have to attend the schools of their choice. In this way the instructional unit developed in this study will aid the students, not only progressing in the concepts related to spatial reasoning; but also in improving their practical performance on such tasks.

Furthermore, students who start to appreciate their visual reasoning capacities may develop better intentions for their future plans. For instance they may head towards architecture, engineering or design, as their career goal, fostered by means of

their improved spatial abilities.

There is a common misconception that some people are inherently incapable of doing spatial tasks. Naturally, intelligence inherited innately plays a role in our potential of spatial ability however is it the unique factor determining our overall potential? The problem of current study is therefore significant to break this bias.

It is foreseen that this study will have significant contributions to the literature in terms of investigating the effects of the spatial tasks within the new national curriculum in Turkey. There have not been many research studies regarding this field conducted in Turkey. Considering this and the design of the current study (first person inquiry), it will contribute significantly to the literature. It is also expected that this study will give guidance to elementary school teachers in the future and present them hands-on tasks and ideas which they can use in the field practically.

1.3 Definition of Terms

Spatial ability: Spatial ability means “to formulate mental images and to manipulate these images in mind” (Clements, 1982, p.36). In this study spatial ability refers to be able to create and use mental manipulation mainly rotation and transformation.

Spatial reasoning: Spatial reasoning is “the ability to see, inspect and reflect on spatial objects, images, relationships and transformations” (Battista, 2007, p.843). In this study spatial reasoning is considered one of the dimensions of spatial ability. It is the ability of reflecting on mental manipulations, representations, relationships and transformations.

Spatial visualization: It is the ability “to mentally manipulate, rotate, twist, or invert a pictorially presented stimulus object”. (McGee, 1979, p.893). In this study spatial visualization refers grasping and acting upon imaginary movements of two and three dimensional objects. It was measured by Achievement Test on 2D Representation of

3D Objects and Isometric Drawing (AT), developed by the researcher.

Spatial orientation: Spatial orientation refers “understanding and operating on relationships between different positions in space, especially with respect to one’s own position.” (Clements, 1998, p.11). In this study spatial orientation refers to understanding relationship between the positions of the object with respect to the one’s own position (Clements, 1998) and comprehending the change between two representations of an object (Tartre, 1990). It was measured by Spatial Orientation Test (SOT) developed by Ekstrom et al. (as cited in Delialioğlu, 1996).

Spatial visualization task: Although this task has broad meaning, in the current study this kind of task includes 2D-3D transformations (2D representations of 3D objects, isometric drawing, drawing of 3D objects given its 2D views from different angles) and mental rotations.

Spatial orientation task: The task is related to understanding relationship between the positions of the object with respect to the one’s own position, and comparing the two representations of an object.

1.4 My Motivation for the Study

Considering my profession, one of the purposes of carrying out this study is to improve my subject matter knowledge as a teacher. While deciding the topic of this study, it was very important for me to internalize the concepts of study to be conducted. The fact that the spatial concepts were newly integrated in the curriculum was also a challenge for me to investigate.

I first met spatial ability test at the university years and since then, I always had a prejudice towards spatial concepts and related tasks. Although I was aware of the fact that having prejudice towards anything is not appropriate, I was thinking that I was a “spatially disabled” person. However, the problem is that a teacher with prejudices can only bring up students with prejudices. So I decided to get rid of my prejudice and decided to investigate spatial concepts. Gradually I had acquisition of

deep understanding of spatial concepts and my perspective was broadened. I planned to prepare a variety of activities for students in order them not to have difficulty that I had in these concepts. I have learnt how spatial ability can be improved from the literature during my investigations and implementation of my own study. Then I realized that spatial ability can be improved with an appropriate instruction. My investigation was also a kind of instruction for me.

CHAPTER 2

LITERATURE REVIEW

In this chapter the related literature is reviewed. This chapter is divided into two main parts. In the first part the theoretical framework is elaborated and in the second part empirical research studies were reviewed.

2.1 Theoretical Framework

In this section, firstly, the concepts of visualization and spatial reasoning are presented and subsequently spatial ability is introduced. After that, types of spatial ability are discussed and finally definitions and several approaches about spatial visualization and spatial orientation are given.

2.1.1 Geometry and Spatial Reasoning

When entering the field of spatial reasoning several common terms appear in the research papers. Some of them are: spatial thinking, spatial sense, spatial visualization, visual perception, spatial relation, and spatial orientation. Many research articles can be found about the relationship of visualization with drawing, writing, constructing and dealing with 3-dimensional objects; school geometry; and mathematics education. But one can also find articles related to art, engineering, architecture, medicine, and some other diverse range of specialties. According to Guitérrez (1996), people doing different activities may have developed different meanings for the same words. Therefore, a clarification of these terms is necessary before actually working with them.

Considering the studies carried out; one can conclude that there is no general

consensus about the terminology used in this field. While one researcher uses the term “visualization” another may use “spatial thinking” but assign similar meanings to such different terms.

Hershkowitz (as cited in Jones, 2001, p.55) described visualization as “the ability to represent, transform, generate, communicate, document, and reflect on visual information”. Visualization is a core element in problem solving and spatial reasoning as it facilitates people to use concrete means to tackle with abstract images. In geometry the process of visualization involves the process of forming and manipulating images (Jones, 2001).

Guitérrez (1996) argues that research findings show that although geometry can be considered as the origin of visualization in mathematics, many of the research studies of curriculum are focusing on the plane geometry, only a few of them focusing on space geometry. People start to deal with geometry in early childhood education, even earlier in the family with the toys. However when the term ‘school geometry’ is used, it is often commonly refers to the Euclidian geometry (Battista & Clements, 1992). When asked to a person, what is the first thing coming to your mind about geometry; it will be certainly the geometric shapes like square, triangle, and rectangle and measurement issue. The reason may be that the plane geometry is usually overemphasized at schools and the space geometry is usually neglected. The difficulty is to make someone ‘imagine’. However, with technological improvements, the use of manipulative and computers in math education visualization in geometry becomes easier (Gutierrez, 1996).

Battista (2007) described geometry as a “complex interconnected network of concepts, ways of reasoning, and representation systems that is used to conceptualize and analyze physical and imagined spatial environments.” Basically, geometry investigates shape and space. Battista (2007) expressed that spatial reasoning is essential in geometric thought. It is the ability to “see”, inspect and reflect on spatial objects, images, relationships and transformations. It consists of the set of cognitive processes by which mental representations for spatial objects, relationships, and transformations are constructed and manipulated (Battista, 2007). Spatial reasoning includes constructing and manipulating mental representations of spatial objects,

relationships and transformations (Clements, 1998). Obviously, spatial reasoning and geometry is strongly integrated with each other. It is the reason why most mathematics educators want to include spatial reasoning as the part of the geometry in the curriculum. Usiskin (1987) has illustrated four dimensions of geometry which are 1) visualization, drawing, and construction of figures; 2) study of the spatial aspects of the physical world; 3) use as a vehicle for representing nonvisual mathematical concepts and relationships; and 4) representation as a formal mathematical system. The first three of these dimensions necessitate the use of spatial reasoning in the study of geometry.

National mathematics education standards in USA states that spatial understandings are necessary for interpreting, understanding, and appreciating our inherently geometric world (NCTM, 1989, p.48), and according to Clements (1998) this is the core reason of developing students spatial sense. Furthermore, NCTM (1989) proposed that:

The mathematics curriculum for grade 5-8 should include the study of the geometry of one, two, and three dimensions in a variety of situations, so that students can visualize and represent geometric figures with special attention to developing spatial sense (p.48).

More recently NCTM (2000) Principles and Standards document emphasizes the importance of visualization and spatial reasoning by comprising a geometry standard reliable for all students from pre-kindergarten to grade 12 recommending that students “use visualization, spatial reasoning and geometric models to solve problems” (p.41). For example, in grades 3-5, students should be able to:

Identify and build a three-dimensional object from two-dimensional representations of that object; and identify and draw a two-dimensional representation of a three-dimensional object (NCTM, 2000, p.164).

As understood from the standards, students should be dealing with spatial tasks at the beginning of early years of elementary education.

In addition to NCTM in USA, Singapore curriculum emphasizes the importance of developing spatial sense in students. The objectives related to this concept include:

- Identify, describe and place an object in a specific location: above,

below, next to, top, bottom, on or off.

- Describe an object as being left or right with another object.
- Identify two dimensional shapes as faces of three dimensional figures.
- Create patterns of two- and three dimensional objects using positional, locational, and directional relationships.
- Identify stated figure in different orientations.

In England curriculum also, there are standards related to visualization of 3D objects given 2D representations.

Based upon the standards students should become experienced in using a variety of representations for three-dimensional shapes (NCTM, 1989, p.168), such as isometric drawings, a set of views from different angles (e.g. front, top, back and sides), and building plans (Christou *et al.* 2007). It means that school geometry curriculum should give this field the importance that the field deserves. However, research findings show that 3D shapes and 3D geometry have been neglected for considerable long time. With the reform in Turkish national curriculum in 2005, this field gained more importance but still the space geometry is delayed until the end of the school year. With the new curriculum, new terms about spatial issues appeared in the textbooks like isometric drawing or 2D views of a 3D object from different angles (top, front and side views). These terms are new for students and they are also new for the teachers. Therefore there isn't much resource in practice about this field.

In the next section the theory of spatial ability and its sub-dimensions are presented.

2.1.2 Spatial Ability

A great deal of thinking needed in mathematics includes spatial thinking in nature. Einstein noted that his elements of thought were not words but “certain signs and more or less clear images which can be voluntarily reproduced or combined” (Gardner, 1983, p.190).

Research findings show that there is a positive correlation between spatial ability and mathematics achievement and often recommended that contribution of

spatial tasks can improve students' mathematical thinking. Battista (1994, p.92) stated how one's algebraic thinking might advance from spatial operations on sets of objects. While doing the simple calculations a child uses first in action, later imagery spatial/kinesthetic manipulations of combining and separating configurations of objects. After much experience these manipulations are internalized so that they can be performed without perceptual input. For instance, a child may find out that $5 + 8$ is 13 by separating 8 into 5 and 3, combining 5 and 5 to make 10, and then adding 3. In doing calculations with larger numbers the child uses the same mental tokens he/she used in the smaller numbers. For instance in solving $55 + 18$ as fifty plus ten is sixty, plus ten ($5 + 5$) is seventy, plus three is seventy-three (Battista, 1994, p.92).

Although spatial ability is well-known and challenging field to study on, there is no one exact definition of it. Furthermore the review of the literature shows that there is no general agreement about the terminology used in this field. There are quite a few approaches about the definition and classification of spatial ability.

Clements (1982, p.36) defined spatial ability as "the ability to formulate mental images and to manipulate these images in mind." By mental imagery he means that "the occurrence of mental activity corresponding to the perception of an object, but when the object is not presented to the sense organ."

According to Linn and Petersen (1985) spatial ability is the mental process used to perceive, store, recall, create, edit, and communicate spatial images. For Yakimanskaya (as cited in Gutierrez, 1996) a "spatial image" is produced from sensory cognition of spatial relationships, and it may be expressed in a variety of verbal or graphical forms including diagrams, pictures, drawings, outlines, etc.

Tartre (1990, p.216) stated that "spatial skills are considered to be those mental skills concerned with understanding, manipulating, reorganizing or interpreting relationships visually". Another definition of spatial ability made by Lohman (1993) is the ability to generate, retain, retrieve, and transform well-structured visual images. Olkun (2003) described spatial ability as the mental manipulation of objects and their parts in 2D and 3D space.

As seen from the above definitions there are several approaches to the concept of spatial ability. However, the common point in all of them is that spatial

ability includes mental manipulation of objects.

In the coming section the types of spatial ability are discussed.

2.1.2.1 Types of Spatial Ability

In the previous sections it was mentioned that there is no unity of ideas among researchers about this field. It is also the case in categorization of spatial ability.

Linn and Peterson (1985) made a classification of spatial ability by dividing it into three parts “spatial perception”, “mental rotation” and “spatial visualization”. Furthermore, Lohman (1988) suggests also a three factor model for spatial ability, including “spatial visualization”, “spatial orientation”, and “spatial relations”. On the other hand, Pellegrino and Kail (1982) proposed another classification of spatial ability which is “spatial relation” and “spatial visualization.” McGee (1979) distinguished two major types of spatial skills: spatial visualization; “the ability to mentally manipulate, rotate, twist, or invert a pictorially presented stimulus object” (p.893) and spatial orientation; “understanding and operating on relationships between different positions in space, especially with respect to one’s own position” (Clements, 1998, p.11). Connor and Serbin (1980) and Kersh and Cook (1979) also supported this categorization. Furthermore, Clements (1998) also made the same categorization. The present study is based on the categorization expressed by McGee (1979), Connor and Serbin (1980), Kersh and Cook (1979) and Clements (1998).

In the next sections these two major types of spatial ability -spatial visualization and spatial orientation- are presented in detail.

2.1.3 Spatial Visualization and Spatial Orientation

There are several different approaches among researchers to spatial visualization. McGee (1979) described spatial visualization as “the ability to mentally manipulate, rotate, twist, or invert a pictorially presented stimulus object” (p. 893). Whereas one group of researchers emphasizes the mental manipulation as a

core element of spatial visualization (Fennema & Sherman, 1977); another group of researchers highlights the requirement for complex, multi-step analytic processing of spatially represented information (Linn & Peterson, 1985). Linn and Petersen (1985) believed that the only universal feature of spatial visualization tasks was that the solutions needed more than one step.

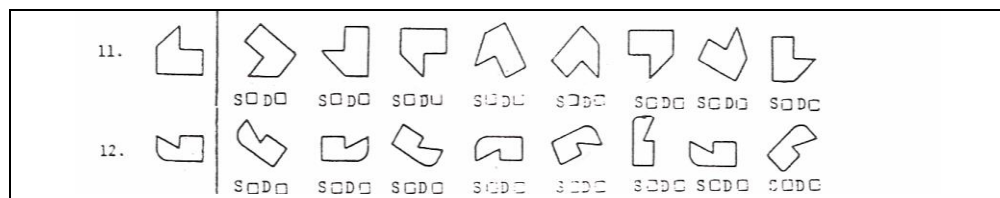
According to Lohman (1988) “Spatial visualization” is the ability to comprehend imaginary movements in a three-dimensional space or the ability to manipulate objects in imagination. Tartre (1990), on the other hand, defined spatial visualization as the ability to predict specified transformations of geometric figures. In addition, Burnet and Lane (1980) described spatial visualization as the ability to imagine mental rotations of objects or their parts in 3D space. Pointing out the descriptions, it can be concluded that spatial visualization includes imaginary movements of two and three dimensional objects (Clements, 1998). The ability to ‘read’ and ‘understand’ the 2D representations of 3D objects are also a part of spatial visualization (Ben-Chaim, Lappan & Houang, 1985; Olkun, 2008); since spatial visualization also includes 2D-3D transformations.

Furthermore spatial visualization covers mental combination of different views of an object, such as orthogonal views (Olkun, 2008) in such a way that transforming 2D orthogonal views into 3D objects and finding the different orthogonal views of a 3D object. Orthogonal view is an “image of an object as can be seen from a direction perpendicular to the viewer”. Orthogonal view is limited to the three coordinate axes (x, y and z) to produce front orthogonal view, side orthogonal view, and top orthogonal view.

Another major type of spatial ability is spatial orientation, which is defined as “understanding and operating on relationships between different positions in space, especially with respect to one’s own position” (Clements, 1998, p.11). For instance building and reading maps are related to spatial orientation ability because it involves understanding relationships between different positions in space. However, reading maps is also related to spatial visualization ability in such a way that it requires 3D to 2D transformations. Different than spatial visualization, spatial orientation task do not necessitate mental movement of the object. Spatial orientation tasks propose that

the person comprehend a change between two representations of an object (Tartre, 1990). Such tasks include reorganizing and comparing visual representation with another one, or seeing it from different angles but not mentally moving the object.

A typical spatial orientation test in two dimensions is ETS's Card Rotations Test which was translated into Turkish by Delialioğlu (1996) and also used in the present study as a measuring instrument. In the test the respondent is asked to decide whether each of the figure to the right of the line is same or not with respect to the figure to the left of the line. (Lynn, 1992). Sample items are shown below:



Source: Lynn, F. (1992). A Meta-Analysis of Correlations of Spatial and Mathematical Tasks. *Information Analyses-070-Reports*. (ERIC No. ED353270)

Figure 2.1 Sample Items from ETS's Card Rotations Test

As mentioned previously, there is no general agreement about the classification of the types of spatial abilities as well as the terms used in the categories. Lohman (1988) defined “spatial orientation” as a measure of one’s ability to remain unconfused by the changes in the orientation of visual stimuli that requires only a mental rotation of configuration. He also proposed one more term called “spatial relation” which is speed in manipulating simple visual patterns such as mental rotations and describes the ability to mentally rotate a spatial object fast and correctly. On the other hand Olkun (2008) used the term “spatial relations“ by resembling to spatial orientation and stated that spatial relation tasks involve 2D and 3D rotations and cube comparisons.

The definitions of Clements (1998) about spatial visualization and spatial orientation are used as a base in the present study. In these definitions the terms are given with their essential characteristics and the descriptions overlap the descriptions in my mind. In the present study spatial visualization refers grasping and acting upon imaginary movements of two and three dimensional objects and spatial orientation

refers understanding relationship between the positions of the object with respect to the one's own position (Clements, 1998).

In the next section a review of studies carried out about the field are presented.

2.2 Review of Related Research Studies

In this section several studies conducted about the field related to the present study are presented.

2.2.1 The Effect of Instruction on Spatial Ability

A great deal of studies in the literature indicates that spatial ability can be improved through training if appropriate materials are provided (e.g. Ben-Chaim *et al.*, 1988; Lord, 1985; Burnett & Lane, 1980). Olkun (2003) found that engineering drawing activities play a significant role to improve spatial ability. Engineering drawing activities, which enable graphical communication involved some technical rules or drawing conventions and visual skills are used in this kind of activities. Representing three dimensional objects in two dimensions or isometric drawings are also included in these activities. In a geometry lesson these types of activities can be done from the familiar and more concrete ones to the more complex ones. Engineering drawing is important because it has a deal with real life situations. This ability includes representing objects in graphical form and visualizing objects from their drawings. Moreover, representing objects in two dimensions is beneficial for students to advance their performance in spatial visualization. (Olkun, 2003)

Recently, Olkun and Sinoplu (2008) conducted a similar study to investigate the effect of pre-engineering activities on 4th and 5th grade students understanding of rectangular solids made of small cubes. The participants of the study were 121 students coming from different socioeconomic regions in Ankara, the capital city of Turkey. The design of the study was pretest-posttest experimental design. The experimental group was exposed to a 2 class- hour instruction in which they built up

cars, ships, and trucks out of wooden cubes and triangular prisms whereas the control group went on with the regular class activities. In the tests there were open-ended questions asking the students to find the number of cubes in the rectangular buildings represented pictorially. The materials used in the instruction consisted of 11 activities in which students are first expected to predict the number of cubes to build up the toys out of identical unit cubes and triangular prisms by looking at their graphical representations which involves both orthographic and perspective drawing of toys. After that they were asked to construct the buildings using cubes and triangular prisms. The activities were created from a simple structure to a more complex one.

The results of the study indicate that two-hour instructional activities were effective and beneficial for all students. Furthermore, construction activities in engineering context were very helpful in improving students' understanding of rectangular solids made of cubes.

Considering the research in spatial reasoning, there has been studies conducted as early as 1960s. These studies formed a basis for the more recent studies. Some of the past studies are presented below briefly.

Brinkmann (1966) conducted a study about the changes in spatial ability as shown on the DAT (Differential Aptitude Test) Spatial Relations Test. In this way, the participants were administered a 3-week training program that emphasizes pattern folding and object manipulation. The findings show that there is a significant gain by the experimental group and that the spatial visualization skills can be improved.

Similarly, Wolfe (1970) designed a training program for the students in grades seven, eight and nine (as cited in Ben-Chaim, Lappan, Houang, 1985). In a four weeks period videotaped lessons were analyzed together with students' activities. These activities were parallel to the tasks in spatial tests, like cube comparison, cube counting. Finally, there found to be a significant improvement on problem-related tests. However, there was a little improvement in scores on a "transfer" test

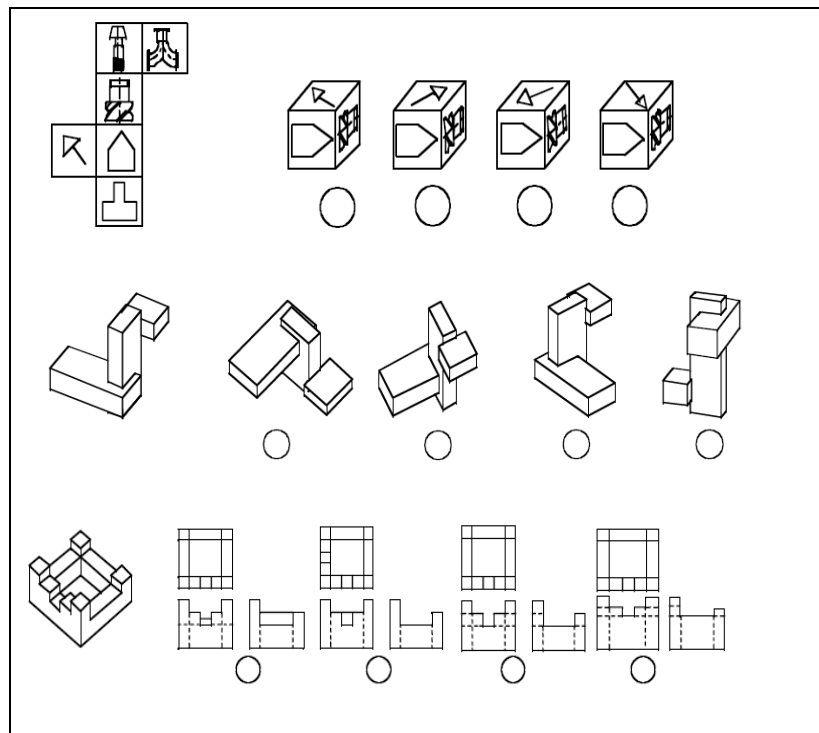
On the contrary, Sedgwick (1961) found that instruction in descriptive geometry¹ does not play a role to improve spatial visualization. In his study, fifty-one matched pairs of engineering, industrial education and industrial supervision students were separated into experimental and control groups. The students were matched according their preterm score on the DAT Space Relations Test. The preterm and post term scores of the same test. Research findings indicate no statistically significant change occurred. Based upon this result, he concluded that visualization is possibly an inborn capability and not changeable by a specific instruction. In fact the instructional method is crucial at this point.

Ben-Cahim, Lappan and Houang (1985) conducted a study to investigate the effect of instruction on spatial visualization performance of students in grades 5, 6, 7 and 8. 1000 students coming from different socio economic background in grades 5 through 8 were administered a unit of instruction on spatial visualization for the middle grades. Changes in the performances of the students were measured with spatial visualization test before and after the instruction and the retention of the effect of instruction. The spatial visualization test and the unit of instruction were developed by the Middle Grades Mathematics Project (MGMP). During a three week period the students were engaged concrete activities with small unit cubes. In these tasks students construct, represent and draw “buildings” formed with cubes. Various kinds of activities were presented to students such as creating the building and then representing it in two and three dimensions or starting with two dimensional views or isometric drawings of the buildings and have students create a well matched building. The spatial visualization test is consisted of similar visualization tasks to the ones used in the unit of instruction. The most significant result of this study was that after the implementation of the instruction there was a significant improvement in middle school students in spatial visualization tasks. The results indicated that students in general made a successful increase in posttest items even in the difficult ones despite the items in the test were not emphasized in the lessons. Nevertheless, Stringer (1975) examined the hypothesis that the more the training context is similar to the items in the test, the higher the scores in the spatial tests. He claimed that the

¹ Descriptive geometry is the technique the technique of accurately representing objects by means of drawing and solving graphically all problems related to their form and position (Watts, 1946, p.1)

scores are specific and closely related to the test item. As a consequence the results of the study matched up with the inference of Brinkman (1966) states that “spatial visualization can be improved when appropriate training is provided.” (p.184)

Alias, Black and Gray (2002) investigated in their studies the effect of manipulatives and sketching activities on spatial visualization ability in engineering students. The engineering students of Malaysian Polytechnics constituted the sample of the study and a pretest-posttest quasi experimental design was utilized in the study. Spatial visualization ability instrument which was specifically developed for the study and consisted of spatial tasks including cube construction, engineering drawing and mental rotation tasks was administered to the both experimental and control groups as a pretest (Figure 2.2). Afterwards, experimental group had experienced some sort of spatial tasks as a part of their structural design course whereas the control group continued their regular structural design class. Finally, both groups were exposed to spatial visualization ability test as a posttest.



Source: Alias, M., Black, T. R., & Gray, D. E. (2002). Effect of instructions on spatial visualisation ability in civil engineering students. *International Education Journal*, 3(1), 1-12

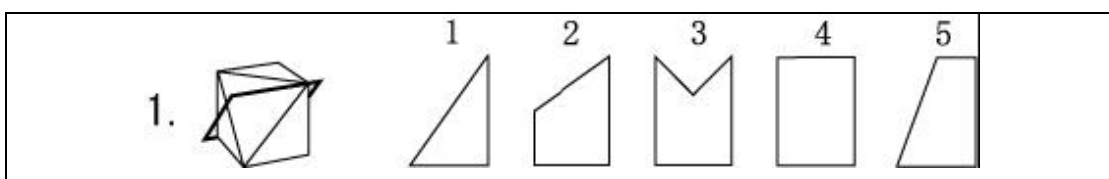
Figure 2.2 Sample Items in Spatial Visualization Ability Instrument Developed By Alias, Black and Gray (2002)

As a result, Alias Black and Gray (2002) found that there is an improvement in spatial ability in general after teaching and learning spatial visualization activities. The largest gain was observed in engineering drawing tasks and the least gain was in mental rotation task. Furthermore, the general gain in skills for the experimental group was not very large. The researchers concluded that this gain emphasizes the significance of the concrete materials in development of spatial visualization ability of engineering students. They highlighted that role of spatial visualization ability in problem solving. Thus, they recommended that spatial skills training should be integrated across the curriculum which would increase the students' awareness of its importance.

In another research Gittler & Glück (1998) were questioning the effectiveness of training programs on spatial ability. They conducted a study to evaluate the transfer effect of non-specific training program. The purpose of this study was to determine the effect of the instruction in descriptive geometry on spatial visualization performance of learners. Three-Dimensional Cube Test (3DC), a cube comparison test was administered to 275 students coming from different provinces of Austria twice. Between the two testing time intervals a part of the students attended the Descriptive Geometry courses. Since the "treatment" had been the piece of Austrian school system, it could not be counted as a deliberately selected design for this study. Besides, students decided on attending or not attending the descriptive geometry courses by themselves. Hence, it was also not experimental design. However, students who attended descriptive geometry course were referred as "experimental group" and the other group was called as "control group". As a consequence, they found that there was a significant effect of Descriptive Geometry instruction on students' performance in spatial ability tasks. Moreover, it was concluded that this school subject arouses the improvement of spatial ability by providing a significant transfer of learning beyond the training of a single skill which is only required to pass the school examinations.

More recently, a similar study was conducted by Tsutsumi, Schröcker, Stachel and Weiss (2005) in Austrian and German universities to determine the

effect of Descriptive Geometry on the performance of students in spatial ability tasks. The Mental Cutting Test was administered to the students in five Descriptive Geometry courses at universities in Austria and Germany. One group in Austria was referred as control group. Within the five courses, students in two courses had already been experienced of basic Descriptive Geometry learning at the beginning of the course. Therefore they were called ‘experienced group’ in this study.



Source: Tsutsumi, E. Schröcker, H.P., Stachel, H., & Weiss, G. (2005). Evaluation of Students' Spatial Abilities in Austria and Germany. *Journal for Geometry and Graphics*, 9(1), 107-117

Figure 2.3 An Example of the Mental Cutting Test

The results of the study indicate that there was a significant difference between the experienced and non-experienced groups. Furthermore, by means of the Descriptive Geometry course, not only students’ spatial recognition ability was improved but also logical thinking ability was also advanced because the experienced groups had solved some of the difficult problems which necessitate the process of logical reasoning.

In conclusion, the previous research results mostly supported that the hypothesis that spatial ability is affected and can be improved by the instruction and the spatial experiences in general. In the next section students’ strategies and the difficulties they face while dealing with spatial tasks are presented.

2.2.2 Students’ Strategies and Difficulties

Research findings show that students have difficulties in the activities in which students are asked to find the number of cubes to make rectangular solids made of small cubes and to determine the volume of those solids. For instance, Ben Chaim, Lappan, and Houang (1985)

claimed that less than 50 % of middle grade students could achieve this kind of problems. Furthermore the results of The National Assessment of Educational Progress (NAEP) showed that less than 40% of 17-year-olds could solve such problems (Hirstein, 1981).

Ben-Chaim et al. (1985) stated four types of errors that students in grades 5-8 made on these problems which ask the number of cubes necessary to make rectangular solids made of small cubes and to determine the volume of those solids : “type 1, counting the cube faces shown in the diagram; type 2, counting the cube faces shown in the diagram and doubling that number; type 3, counting the number of cubes showing in the diagram; and type 4, counting the number of cubes showing in the diagram and doubling that number. “ They recommended that errors about counting the faces derived from the students’ strategy that they are dealing with the picture strictly as a two-dimensional object because it is drawn on the paper. Besides, students who did not double their counts seem to have difficulty in visualizing the unseen sections of the object. And finally, students had difficulty to see the connection between the isometric types of drawings and rectangular solids they represent. As a consequence, Ben-Chaim et al. concluded that the two types of errors-dealing with two dimensions rather than three and not counting the unseen cubes “are clearly related to some aspects of spatial ability” (p.406)

Battista and Clements (1996) conducted a study to investigate students’ solution strategies and errors while dealing with rectangular solids made of small cubes. Quantitative and observational data were collected and two interviews were conducted. The first was done with 45 thirds graders and 78 fifth graders and the second one with 15 fifth graders. As a result they made a categorized five different strategies students used. Students who conceptualize the set of cubes as forming a rectangular array organized in layers used A strategy; students who conceptualize the set of cubes as space filling but do not utilize layers used B strategy; students who conceptualize the set of cubes in terms of its faces used C strategy; students who use the formula $L \times W \times H$ used D strategy and the others belong to D strategy (Battista & Clemets, 1996).

Their research findings show that students who used C strategy initially

perceived a 3-D rectangular array of cubes as an uncoordinated set of faces. The students who had a total restructuring of array used A strategies. Those who had a piece-by piece restructuring used B strategies. They also recommended that the development of students' enumeration of cubes in 3-D arrays was a basic idea in understanding the measurement of volume. Furthermore, they also suggested that curriculum developers and teachers to take into account the different sophistication levels of students in structuring and enumeration of 3-D cubes while designing an instruction.

In another research Gutierrez (1992) described different plane representations of 3-D objects and he reported on a research experiment purposed to analyze how students use these representations. In his paper he came up with five different representations:

- 1) Perspective drawing: It is the kind of the drawing as always made by children.

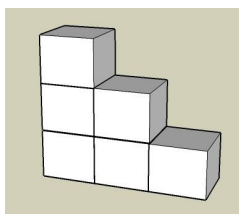


Figure 2.4 Perspective Drawing

- 2) Layers: This representation consists of some horizontal sections of the object at particular heights in order to give an idea of discrepancies in shape.

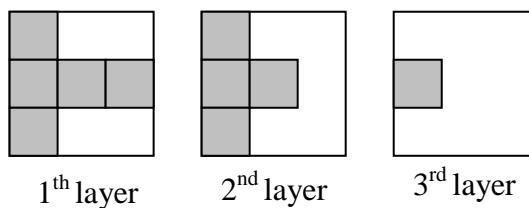


Figure 2.5 Layers

3) Orthogonal or Side Views: This representation is usually used in technical drawing. The object supposed to be into a cube and projected orthogonally on the six faces of the cube. Front, top and right (or left) view is most commonly used views since each pair of opposite views (front/back, top/bottom, right/left) are symmetric.

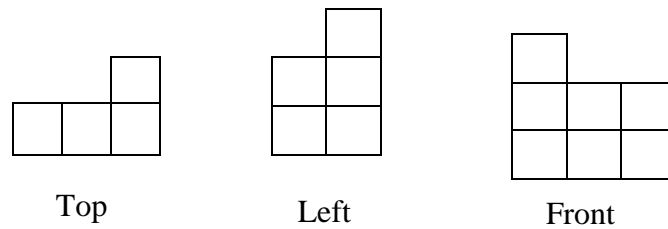


Figure 2.6 Orthogonal Views

4) Coded orthogonal views: Different than Orthogonal views, in this representation the number of cubes in each column is presented. This view is also called numeric view.

0	0	2
1	1	2

Top

Figure 2.7 Coded Orthogonal Views

5) Isometric: This is a type of parallel projection in which the three Cartesian axes form angles of 120° . Isometric drawings are usually made on a net of equilateral triangles with the convention that the vertices of the solids have to match the points of the net.

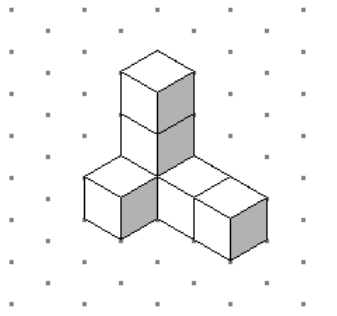


Figure 2.8 Isometric Drawing

In his study students were expected to construct the building given different plane representation and given several structures students were asked to draw different types of their plane representations and finally students are asked to establish the relationships between different types of representations without constructing the module. Students were selected from the grades 2, 4, 6 and 8 from an elementary school in Valencia, in Spain.

The results show that there are significant differences in the difficulty between building solids and drawing their plane representations. Nonetheless, it cannot be inferred that drawing is easier than building or vice versa. To illustrate, drawing side views is easier than building from side views, but isometric drawing is more difficult than building from an isometric representation. In addition to this, building from (coded) side views is the most difficult task; indeed even the 8th graders had difficulty in it. On the other hand, the easiest representation was the layers. As a consequence, 2nd and 4th graders had failures in achieving each kind of representations; hence the first grade level who has a sound success was the 6th graders.

2.3 Summary of the Literature Review

This chapter has reviewed theoretical background of the concepts spatial ability, spatial reasoning, spatial visualization and spatial orientation and research studies about effect of instruction on spatial ability and students' strategies and difficulties.

The findings of past and present researches about the effect of instruction on spatial ability show that spatial ability can be improved with an appropriate instruction. In other words, instructional method plays a crucial role in improving spatial ability.

The research studies about students' strategies and difficulties indicate that students had difficulty in activities which asks the number of cubes needed to make a rectangular solids made of small cubes and to determine the volume of those solids and counting the unseen cubes in a block of cubes (Ben-Chaim, Lappan and Houang, 1985; Battista & Clements, 1996). In his research study about different plane representations of 3D objects Gutierrez (1992) found that drawing side view is easier for students than building from side views and isometric drawing is more difficult than building from isometric representation.

CHAPTER 3

METHOD

In this chapter firstly, main and sub-problems of the study are reminded to the readers. Afterward, the design and participants of the study are presented and finally data collection tools are elaborated.

3.1 Main and Sub-problems of the Study

M. How do the spatial visualization and spatial orientation tasks regarding 2D representations of 3D objects contribute to the development of sixth grade students' spatial reasoning?

1.1 Is there a statistically significant difference in students' spatial reasoning abilities between pre-test and post-test?

1.2 What are the students' performances in spatial tasks prior, during, and after the application of activities?

1.3 What are the students' views about the spatial reasoning activities and their feelings about the activities?

3.2 Design of the Study

The main purpose of the study was to investigate the effect of instruction on students' performances on spatial ability tasks. The design of the present study is

first person inquiry with one group. The teacher who gives the instruction was also the researcher of the present study. Being teacher and researcher at once, is referred to as a 'first-person' inquiry and it also involves multiple forms of research on teaching like action research, teacher research, reflection in and on teaching, teacher narratives and researcher-teacher (Ball, 2000). First-person inquiry is distinguished from other types of research in some aspects that in this one it is the teacher who has the questions in mind, answers the questions, designs the study, and makes inferences.

Ball (2000) explains that in a classroom research, where there is a researcher other than the teacher; researcher sits and takes notes and observes the classroom and the teacher in general. The researcher tries to understand, analyze and give meaning to what is going on there. According to Ball (2000), as a person looking from "outside"; researcher has a broad perspective than a person looking from "inside" lacks. On one hand, as an eye looking from outside, he\she can see and realize the things what the eye looking from inside already internalized and accepts without questioning. On the other hand the outsiders cannot totally understand the meanings and the classroom language. In other words, they cannot feel the classroom spirit, and hence they miss the niceties.

In her experience, Ball (2000) stated that, being researcher and teacher at the same time gives her the opportunity to examine the needs and motivations of the students to reach the mathematical goals that she wants them to. Lampert (1986), (as cited in Ball, 2000) conducted a study where she is the teacher-researcher and studied student learning in the context of teaching. In her study, she investigated whether more computational ways of knowing and doing multiplication could be integrated beneficially in teaching fourth grade. She tried to develop an approach which links different elements of knowing and doing multiplication and to see how it works. This experiment let her to explore the capability of students in alternative instructional settings. Furthermore, being teacher-researcher provides her a space to work and added her study and her experiences as a teacher many valuable things that cannot be available otherwise. She had created a context for her research and raised questions and then she tried to find answers to these questions. In review of

Lampert's study, Ball (2000) argues that as a researcher she treated herself as a teacher and as a teacher she opened new doors not only for practice, but also for research.

According to Ball (2000), first-person perspective provides a special kind of inside view which cannot be gained from an outsider's view. To illustrate, some parts of experience are inexpressible, in other words you cannot express your feelings completely to an observer but you know you feel it. The question is: how can teacher-researcher talk to "the self"? Besides it is not that easy to stay at a certain distance from the "inside", from "the self", and criticize "the self". Heaton (1994) overcame this issue by separating herself into "multiple selves". Becoming multiple selves allowed her to reach different perspectives and experiences that she had across time and also gave her a standpoint in analyzing the data. Sometimes, a teacher's commitment to absolute help to the children can hinder the capability to realize problems and difficulties.

Considering the present study, where the researcher –namely I- investigated whether the instruction affects the performance of students on spatial ability tasks, first-person perspective had significant contributions to the study. Apparently, as a teacher, I have continuing relationships with students more closely than would someone unfamiliar to the class. This ensures me to understand them, to be more sensitive to them, and to be able to make inferences and explore meanings from their words and gestures. I can understand my students' talk and body language, but others would not. Since we have a shared history, problems, examples and discussions I can use them to explore my children's experiences and ideas (Ball, 2000).

3.3 Participants

The study was conducted in a private school in Ankara. The participants consisted of 24 sixth graders coming from relatively high socio-economic status. The data was collected only from one group of students. In other words, there was no control group in the study due to ethical issues. The topic in this study is one of the

current topics in the math curriculum and naturally students are responsible of this topic for the national high school states tests. Although I am the researcher of this study, I am a teacher at the same time and I am responsible of my students' learning. Therefore, I find it unethical not to give the instruction to a group of students that they have a right to. If I believe that this type of instruction improves my students' learning, then I should apply it all my classes. At this point I think that I don't have a right to deprive off my students from this instruction.

The general mathematics exam results administered in the school indicate that the overall mathematics achievement of students is about the same as the school average. The class is heterogeneous, namely there are low and high achievers together in the class and the classes are formed randomly. In general students have a sound mathematical background from previous elementary education. However, before starting the activities, when I asked my students what they know about spatial issues, isometric paper, isometric drawing, 2D views, etc. only a few of them mentioned that they had experience beforehand.

3.4 Data Collection Tools

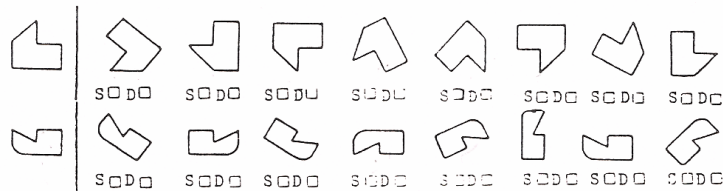
The participants were administered a test measuring spatial ability before and after the instruction. The test consists of two parts. The first part was designed to measure spatial orientation ability of the students and the second part was designed to measure the spatial visualization and drawing ability of the students. In addition to this, throughout the instruction students were given task sheets which are designed to explore students' spatial skills in general. In the next sections the tests and the tasks are presented in detail.

3.4.1 Spatial Orientation Test

As mentioned in the previous chapter, spatial orientation refers to understanding relationship between the positions of the object with respect to the one's own position (Clements, 1998). To measure the spatial orientation ability of the students Card Rotation Test (CRT)

having a reliability coefficient of 0.80 and Cube Comparison Test (CCT) having a reliability coefficient of 0.84 were administered to the students (Delialioğlu, 1996). The tests are given in Appendix B. These tests are the sub-tests of Spatial Ability Test (SAT) developed by Ekstrom et al. (as cited in Delialioğlu, 1996) and were translated into Turkish by Delialioğlu (1996). Each test is designed to be completed in 6 minutes for 15 year-olds and older ones. Since the participants are 12-13 year-olds, they were given twice as the time by consulting on expert opinion Dr. Ömer Delialioğlu, who translated the SAT into Turkish and also an instructor in Department of Computer Education and Instructional Technologies at Middle East Technical University (METU).

- (i) **Card Rotation Test (CRT):** In this test the respondent is asked to decide whether the each of the figure to the right of the line is same or not with respect to the figure to the left of the line. In each question there are 8 items that students need to respond as “same” or “different” and each true item was scored as 1. Since there are 20 questions the total score is 160. Sample items of the test are shown below:

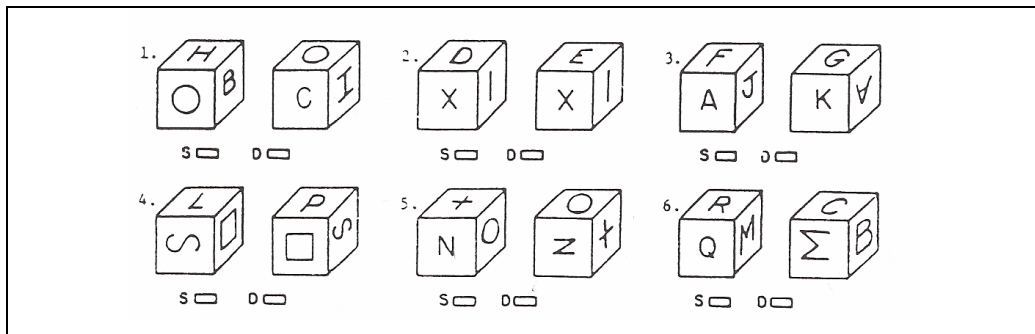


Source: Lynn, F. (1992). A Meta-Analysis of Correlations of Spatial and Mathematical Tasks. *Information Analyses-070-Reports*. (ERIC No. ED353270)

Figure 3.1 Sample items from Card Rotation Test

- (ii) **Cube Comparison Test (CCT):** In each item of the test there are two cubes with six faces and having different letters, numbers or figures on each face. The respondent is asked to determine whether the given cubes are same or not. Each correct answer was scored as 1 and incorrect one as 0. There were 42 items. Hence the test was scored over 42. Sample items

are shown below:



Source: Lynn, F. (1992). A Meta-Analysis of Correlations of Spatial and Mathematical Tasks. *Information Analyses-070-Reports*. (ERIC No. ED353270)

Figure 3.2 Sample Items from Cube Comparison Test

3.4.2 Achievement Test on 2D Representations of 3D Objects and Isometric Drawing (AT)

As mentioned previously, in the present study spatial visualization refers grasping and acting upon imaginary movements of two and three dimensional objects (Clements, 1998). To measure the spatial visualization ability of the student's spatial visualization tasks developed by the researcher were administered to the students (Appendix A). The tasks were developed through a process of reviewing of resources from literature. After the tasks were completed, they were reviewed by a graduate student and a faculty member specialized in elementary mathematics education and then updated according to the reviews. The Cronbach alpha coefficient of this test is found to be 0.88.

The test consists of three parts. In the first part students are asked to draw the three dimensional (3D) building made of small cubes onto the isometric dot paper. In the second part the students are expected to draw the two dimensional (2D) views-top, right and front views- of the 3D buildings made of small cubes. Finally, in the last part given 2D representations (views) students are expected to draw the 3D objects made of small cubes. The main reason of designing this kind of tasks is curricular needs. The task covers the 5th, 6th and 7th grade objectives since the study was designed in such a way that to review the prerequisite learning, to fulfill the

present learning and to make extension to future learning. In Table 3.1 the objectives matching with the test items are presented.

Table 3.1 Test Items and Corresponding Objectives

AT Items	Grade Level	Objectives
1	5	Draw the 3D buildings made of small cubes onto the isometric dot paper.
2	6	Draw 2D views (top, front, back, bottom and sides) of 3D buildings made of small cubes.
3	6	Draw 2D views (top, front, back, bottom and sides) of 3D buildings made of small cubes.
4	7	Given the different 2D views of the 3D buildings construct them with the unit cubes and draw them onto the isometric dot paper.

3.4.3 The Process of Instruction

During five class hour the topic was covered with spatial tasks developed by the researcher (Appendix C). The activities were developed through a process of reviewing of resources from literature. After the tasks were developed, they were reviewed by a graduate student and a faculty member specialized in elementary mathematics education and then updated according to the reviews.

The sequence of this topic in the national curriculum and in the annual plan of 6th graders is in spring time at the end of the semester. Hence the study was carried out in the spring semester of 2009, at the beginning of June. The topic was given 2-3 class hours in the annual plan. However, as expected, the time was not enough for the instruction planned to be administered for the present study. Therefore, the instruction continued five class hours long and the researcher made a great effort to reach the curriculum pace.

As stated previously, the activities were designed in a way that to review

the prerequisite learning, to fulfill the present learning and to make extension to future learning. They intended to scaffold students' spatial reasoning as a whole. In Table 3.2 activities and corresponding objectives are presented.

Table 3.2 Activities Used In Instruction and Corresponding Objectives

Activities	Objectives	Duration
Activity 1: Cube Counting and Isometric Drawing	Draw the 3D buildings made of small cubes onto the isometric dot paper.	40'
Activity 2: Drawing 2D Views of 3D Objects	Draw 2D views (top, front, back, bottom and sides) of 3D buildings made of small cubes.	40'
Activity 3: Given 2D Views Drawing 3D Objects	Given the different 2D views of the 3D buildings construct them with the unit cubes and draw them onto the isometric dot paper.	40'
Activity 4: Finding back view	Draw 2D views (top, front, back, bottom and sides) of 3D buildings made of small cubes.	20'
Activity 5: Coded Side View	Draw 2D views (top, front, back, bottom and sides) of 3D buildings made of small cubes.	40'
Activity 6: Constructor-architecture Ga Game	Draw 2D views (top, front, back, bottom and sides) of 3D buildings made of small cubes.	40'

The starting activity consists of three parts. The first part is about estimating the number of cubes in a block made of small cubes. The second one is counting the faces of the cubes in a more

complex block compared to former ones. And the final task is isometric drawing activity which reviews the prerequisite learning of the students from 5th grade. The instruction of the first activity was covered as in the following. The procedures specific to this activity actually reflects the approach and procedures in the other activities as well.

“Before I distributed the activity sheets to the students, I explained them the purpose of the lesson clearly and what we will learn in this lesson. I asked them whether they had heard about isometric paper or they had any drawing experiences with isometric paper or not. According to the responses, only some of the students had an idea about isometric paper. Indeed students had not enough experiences about 3D drawing either. Hence, I made a brief demonstration of isometric paper by explaining its properties and then distributed the activity sheets and the unit cubes. Afterwards, I told them to read what they were expected to do in the first tasks. What I expected from my students was that they could understand on their own what they supposed to in an activity by reading carefully the instructions. However, there were always some students asking to me what they supposed to do in the task. Therefore I made a brief explanation about the task and then they started doing the task. While they were dealing with the task I observed them by wandering around them. I observed while some students were using unit cubes, others did not need them. When I asked them how they utilized unit cubes, some students said that they could easily do the activities with unit cubes and some other said that they could do the activities without unit cubes. While they were dealing with the activities I was looking at their work and then giving feedback about it one by one. In this way I had an idea about the mistakes or misconceptions or strategies of my students. After a while, when they were at 4th question of the first part, some questions arouse. The most frequently asked question was “Do we also count the unseen cubes in the block?” or “how do we count them?” Instead of answering the question I asked these questions to the class and a discussion started between the students and I was leading the discussion. It was such an environment where the students shared their ideas and strategies. In the end as a class we reached some conclusions about the unseen cubes. Some students proposed to construct the block with unit cubes and see how many cubes

were unseen. Without constructing, some of them found the number of unseen cubes by counting the layers and some of them constructed and then visualized the block in their minds. I observed that after the discussion all students had an idea about dealing with unseen cubes in a complicated block. I didn't restrict the students in advancing the further tasks. I told them after they completed one task they could continue the next one. Indeed some students were faster compared to others. After all of the students completed the first part we discussed the answers as a class. In the second part of the activity, there was a more complicated block and what students were expected to find was the faces showing of the cubes in this block. The first question was to find the total number of cubes in the block. Almost all of the students could easily find the number of cubes. I gave students time to answer all of the questions. After they finished the task, they shared their responses with the class.”

In the second activity students are expected to draw 2D views (top, front and sides) of 3D buildings made of small cubes and solids; which is the intended goal to be achieved in the 6th grade.

The third activity includes three parts. Given the different 2D views of the 3D buildings in the first part students are asked to construct them with the unit cubes and draw them onto the isometric dot paper. In the second part they repeat the action but this time without using the unit cubes. Finally, in the third part they are asked the same questions as in the first and second part but in this part the 2D views are given in a way that with only one cube having different color.

In the fourth activity students are expected to find the back view of the 3D buildings made of small cubes given front, top, left and right views. At the end of the activity students are asked their strategies to find the back view. This task is related to spatial orientation since it is about understanding relationship between the positions of the object with respect to the one's own position (Clements, 1998) and comparing two different positions of the object as well.

The next activity consists of two parts. In the first part, the coded plan or coded orthogonal views (Gutiérrez, 1992) of 3D buildings are given and students are asked to construct the 3D buildings by using the unit cubes. In the second part the six different views of a relatively complex 3D building is given and students are asked to

construct as many buildings as possible with the unit cubes to be fit to the views.

The final activity is designed to be a game. It is a pair game in which one student pretends to be an architect having a 3D building made of unit cubes and his\her duty is to make construct the same building to his\her pair who is the constructor. The duty of the constructor is to construct the building explained by the architect without looking at the building. Students are playing the game by changing the roles. In the end of the task there are questions about the activity to be answered by the students.

The first three activities and fourth and sixth together are related to spatial visualization because the ability to “read” and “understand” the 2-D representations of 3D objects are a part of spatial visualization (Ben-Chaim, Lappan & Houang, 1985; Olkun & Sinoplu, 2008), since spatial visualization also includes 2D-3D transformations. In the next chapter the analysis of activities are presented in detail.

3.4.4 Observation Notes

During the instruction, since I am the teacher and also the researcher at the same time I took notes about the activities. I recorded some students’ strategies, difficulties, common mistakes, and common misconceptions. After each activity, we discussed as a class what they did in the activities and generally we talked about the significant points in the activities. Students shared their ideas and strategies and I took notes of them. I used my notes to analyze the data of the study and to make inferences from students’ activity sheets.

3.4.5 Reflective Paragraphs

After all activities were finished, I want my students to write reflective paragraphs to express their feelings and ideas about the activities and instruction process. I wanted to learn how they perceive the activities and evaluate the effectiveness of the activities for their learning and whether they enjoyed the lessons or not. These would be a good feedback for me to plan my further lessons and for my

study to see how it is perceived by the sample. In the next chapter some of students' reflective paragraphs are presented.

3.5 Limitations of the Study

One of the limitations of the current study is that the selection of the subjects for the study did not comprise a random sampling. It was one of my classes. However, the classes in the school are formed randomly and they are all heterogeneous. There are low achievers and high achievers together in the class.

During the implementation of the activities I took some notes and I tried not to include my own opinions into these notes. The design of the research was first person inquiry (Ball, 2000), namely the researcher is the teacher at the same time. This kind of research is open to be subjective but I paid attention to this point and tried to be as objective as possible while taking notes.

There were two students who didn't come to school during the implementation of the study. Therefore they had one missing activity.

The time interval between pre- and posttest was ten days. Therefore there was a possibility that students could have remembered the test items. However, there were so many items, hence; it is not possible to memorize all of them. In addition to this, isometric drawing is not such an activity which can be done by memorizing. Independent from the time interval between pre and post test, obviously, students could have felt themselves more comfortable in the posttest since they were familiar to the test before.

There was no control group due to ethical reasons which I explained previously.

CHAPTER 4

RESULTS

In this chapter the results of the study are explained. In the first part, descriptive statistics used in this study was elaborated and in the second part, students' performances on the activities are explained in detail. In the next part, students' views and feelings about the activities are presented and in the last part overall results are explained.

4.1 Descriptive Statistics

One of the research questions was about whether there was a statistically significant difference between students' pre-test and post-test scores in spatial ability test. Initially it was planned to use paired-samples t-test to evaluate the impact of the spatial ability tasks implemented during the class hours. However, since normality assumption of parametric test was not met, non-parametric equivalent test, Wilcoxon signed rank test was run to test data. After the data was analyzed, it was found that there was a statistically significant difference in students' spatial reasoning development between pre-test and post-test.

Table 4.1 presents the descriptive statistics data of spatial orientation test (SOT) and Achievement Test on 2D Representations of 3D Objects and 3D Drawings (AT).

Table 4.1 Descriptive Statistics Data of the Test Scores

		Mean	SD	Z
Spatial Orientation Test (SOT)	Pretest	149,83	26,3036	-4,28 *
	Posttest	170,83	21,2268	
Achievement Test on 2D Representations of 3D objects and 3D drawings (AT)	Pretest	5,54	3,176	-3,93 *
	Posttest	8,67	1,926	

Note 1. N=24

Note 2. Highest possible score for SOT is 202 and the lowest possible score is 0. For AT highest possible score is 10 and the lowest score is 0.

* $p < .05$

In Figure 4.1 the histograms representing the pre and posttest scores of students from AT are presented. As understood from the representations, there were considerably more students in the post test who had higher scores compared to pre test. In other words, in the posttest students were cumulated on the high scores mostly.

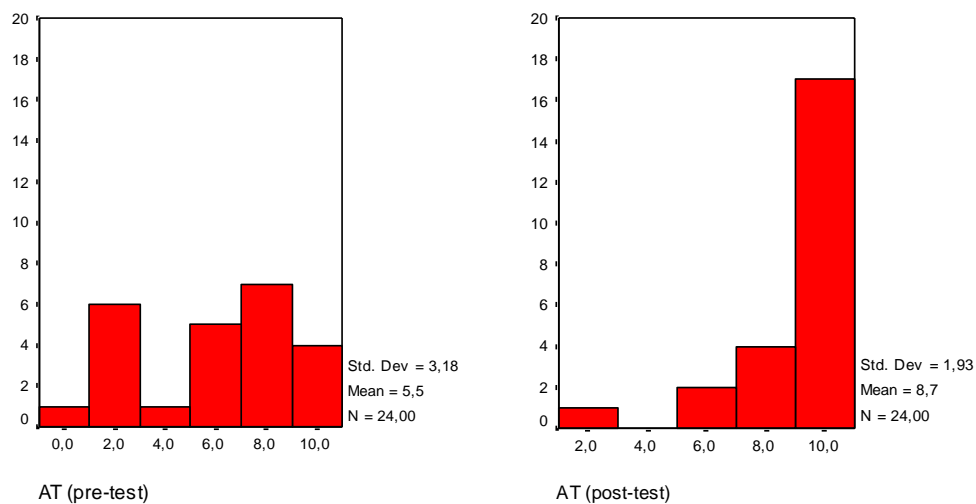


Figure 4.1 Histograms Representing Pre and Post AT Scores

Similarly, considering the SOT scores (Figure 4.2), it can be concluded that there were more students who had higher scores in post test. Namely most of the students

were cumulated between the scores 160 and 200 in posttest, whereas there was a distribution of scores in the pretest.

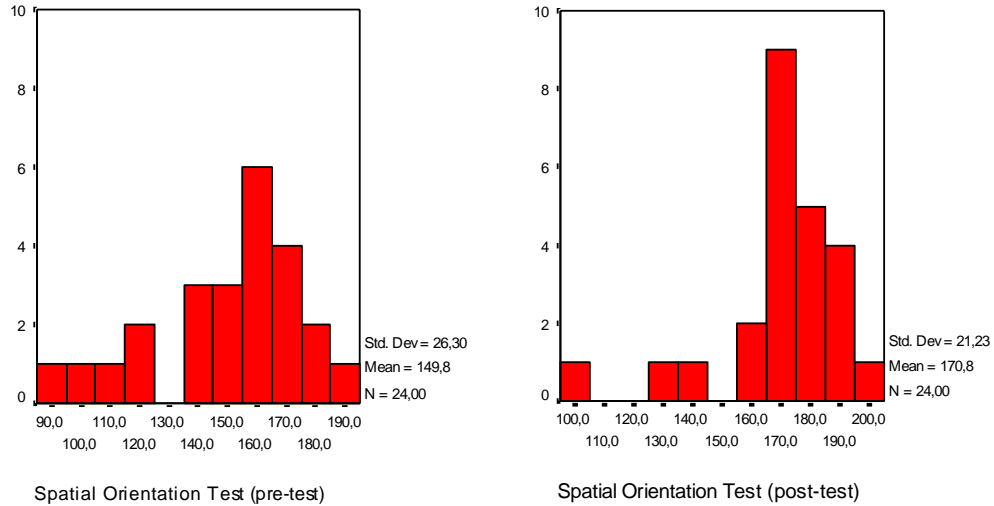


Figure 4.2 Histograms Representing Pre and Post SOT Scores

4.2 Students' Performances on the Activities

The other research questions were what the students' performances are during, and after the application of activities and what the students' views are about the spatial reasoning activities and their feelings about the activities. In this section the data analysis about these questions are given in detail.

Activity 1: Counting Cubes and Isometric Drawing

In this task students reviewed their previous learning from 5th grade. The task was a warming up activity. Another intention was to make sure that students recall or learn prerequisite skills.

Counting Cubes

Cube counting activity consists of two parts. In the first part there were four blocks composed of unit cubes and the students were expected to determine the number of cubes in each of the cubic blocks. Out of 23 students 13 found the number of cubes in all structures correctly; and 9 of them had only one mistake. During the lesson students had the unit cubes as manipulative in case they need them. They didn't have to use manipulative in the tasks. In the last block there were some cubes which could not be seen in the paper (Figure 4.3). Hence, some of the students – s3, s10, s14, s15 and s19- said that they could easily count the cubes when they constructed the block with the unit cubes. Some students constructed the block without using the unit cubes. For instance s23 expressed his strategy for constructing as following:

“In order to find the number of cubes, firstly I am counting the layers in the structure.”

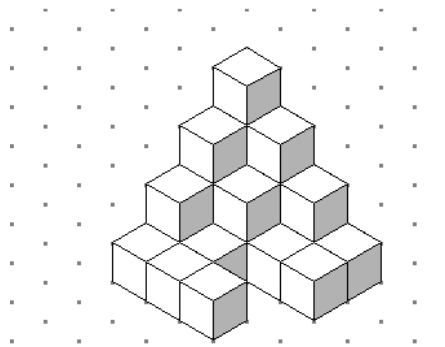


Figure 4.3 Fourth Question of the First Activity

In the second part of the activity there was a complicated block of cubes compared to first part, again composed of unit cubes (Figure 4.4) .There were six questions about this block and the results show that 50% of the students had at least four correct answers out of six. 30% of the students had two and three correct answers, 20% of the students had one correct answer (Table 4.2). First question was “*How many cubes are in the block?*” All students have the correct answer for this question. S10 expressed her experience as following:

“In the first activity I needed the unit cubes but now, in the second part even if the structure was more complicated I could count the cubes without constructing the structure with the materials”

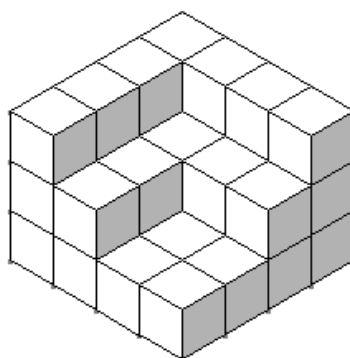


Figure 4.4 Second Part of the First Activity

Table 4.2 Results of the question in the second part of the first activity

	Number of correct answers					
	Six	Five	Four	Three	Two	One
Number of students (out of 22)	2	6	3	3	3	5

The rest of the questions were about the faces of the cubes showing in the block when looked from the front side of the block. This task was more difficult for the students. It was also a kind of activity that they never met before. According to my observation notes the most frequently asked question was: "Do we also count the faces behind the block which we cannot see?" After the activity, while we were discussing the answers as a class, students shared their strategies with the class. For instance S18 counted the one face showing by putting a point on each cube; two faces showing by drawing a line passing through two faces; and three faces showing by circling the corner where the three faces meet. His strategy can be seen from his paper below (Figure 4.5).

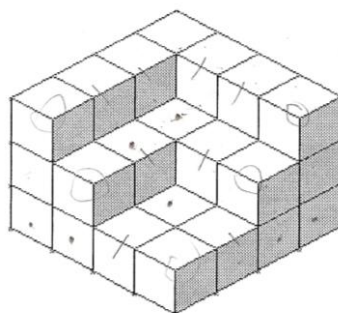


Figure 4.5 Work of S18 Where His Strategy Can Be Seen

Determining the number of cubes and counting the faces showing and were warming up activities for the students. According to my observation in the class the most common difficulty that students face in the cube counting was counting of unseen cubes. This is closely related to visualizing the 2D representation of a 3D object in mind.

Isometric Drawing

In this part, students were expected to draw blocks built up with unit cubes into the isometric paper. My observation notes show that some of the students were familiar to the isometric paper and isometric drawing from 5th grade. For those who do not know the isometric paper I made a demonstration before the task began. In fact the students should have been familiar to this task because it was the 5th grade's objective which was: "The students draw the structures built up with unit cubes into the isometric paper". (MNE, 2005)

There were four tasks each of which included a block composed of unit cubes. As seen in Table 4.3 out of 23 students 14 of the students did all of the four drawings totally correct; 8 of them didn't have correct drawings and 5 of them had two or three correct drawings.

During the task, I asked students to share their drawing strategies with the class. Some of them are below:

S11: First I am drawing the squares and then converting it into a cube.

S17: First I am drawing the top view of the block.

S18: If the block is multi-storey, first I am drawing the main line passing through the middle of the block.

Table 4.3 Results of the Isometric Drawing Activity

	Number of correct drawings (out of 4)		
	ALL	2-3	NO
Number of students (out of 23)	10	5	8

The most striking point in this isometric drawing activity was that 4 of the students made correct 3-D drawing but they didn't pay attention on the isometric characteristics of the paper (Figure 4.6). After the activity while we were discussing the answers as a class I made them to understand what they did with the isometric paper and what actually they were supposed to do. They didn't change their works on the paper but I took notes about them to follow their work in the next isometric drawing activity.

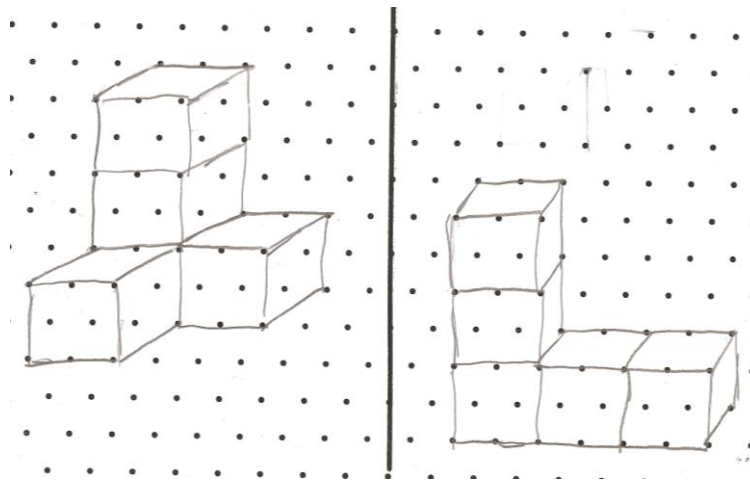


Figure 4.6 Students Responses That Did Not Pay Attention to Isometric Characteristics of Paper

Activity 2: Drawing 2D Views of 3D Objects

In this activity students were expected to draw 2D views (front, top and sides) of 3D objects. It consists of three parts. In the first part students should draw the 2D views of the objects built from unit cubes into the squared paper; in the second paper they should draw the views into the empty space provided under the objects. In the last part there were two different 3D objects but in this time not built from the unit cubes. Again students were expected to draw the 2D views of these objects into the empty space provided.

Before they began the task, we discussed what they understood from “front, top and side views”. Some of the students thought that they will draw the views in 3D. After the discussion the task was clear in their minds.

In the first part of the task 16 of the 23 students had all the drawings totally correct; 5 of the students had one incorrect drawing and 2 students had two or three incorrect drawings. There was an interesting common mistake that s9, s13 and s15 made empty squares in their drawing as if there weren't any cubes in a layer (Figure 4.7). After the task, during the discussion they realized their mistakes and didn't repeat the mistake it in the second part.

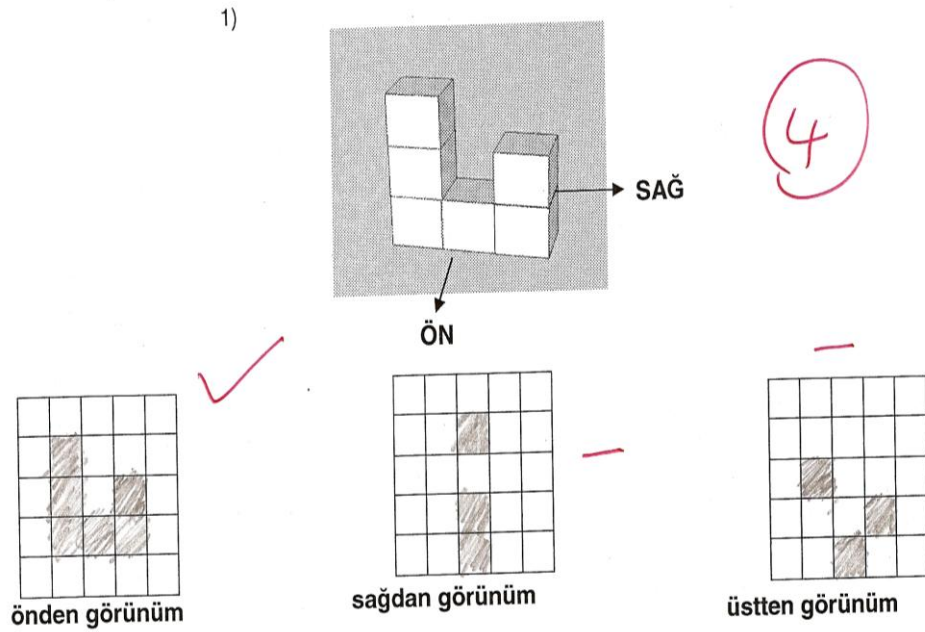


Figure 4.7 Students Who Made Empty Squares

In the second part, as seen in Table 4.4, 12 of 23 students had all the drawings totally correct; 2 of the students had one

incorrect drawing and 9 of the students had at least two incorrect drawings. One reason that the results became worse compared to first part was that there was no squared paper leading the students to draw. S10 expressed her feeling: “I feel myself more comfortable while drawing on the squared paper”. Another reason was that the blocks were more complicated than the first part. Some of the students (s1, s3, s10 and s15) said that they could be better if they could construct the objects with the unit cubes. The common mistake arose from the drawing of side views. The students who had mistake in side views reported that they could not imagine the projection in their minds. If they could construct the object and looked from the sides to the concrete object then they could draw the side views.

Table 4.4 Results of the Activity 2

	All drawings correct	Number of students	
		One incorrect	Two or more incorrect
Part 1	16	4	2
Part 2	12	2	9

The last part was more challenging for the students. The results show that 16 of the students had at most one incorrect drawing. The common mistake in the first question was to draw the side view of the object parallelogram as it is seen in 3D (Figure 4.8). After we discussed it as a class, there was no parallelogram shape in the second question (Figure 4.9). There is also another point that students had in common in the first question which was to draw the dimensions of the rectangle in top and side views same. The width of the rectangle in the side view should be shorter than in the top view. However, in the second question they paid attention on both issue.

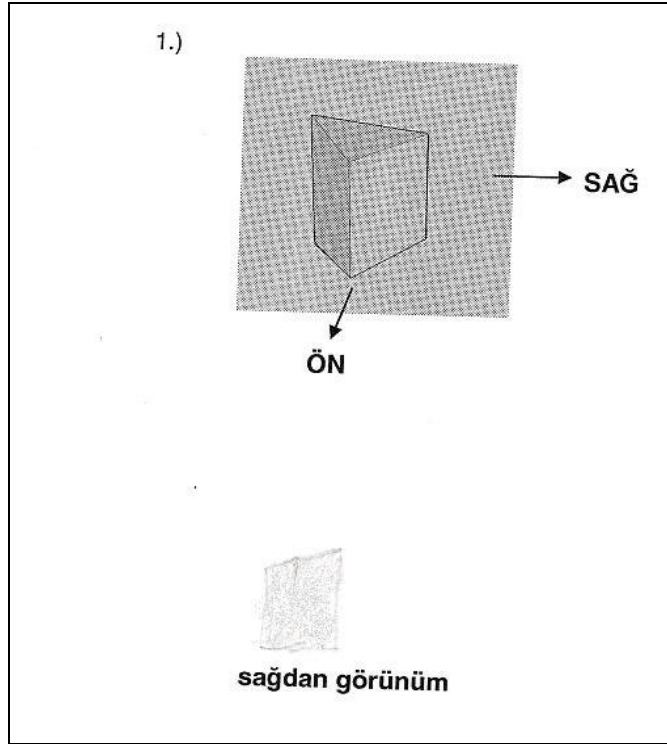


Figure 4.8 Student Responses Who Drew Parallelogram

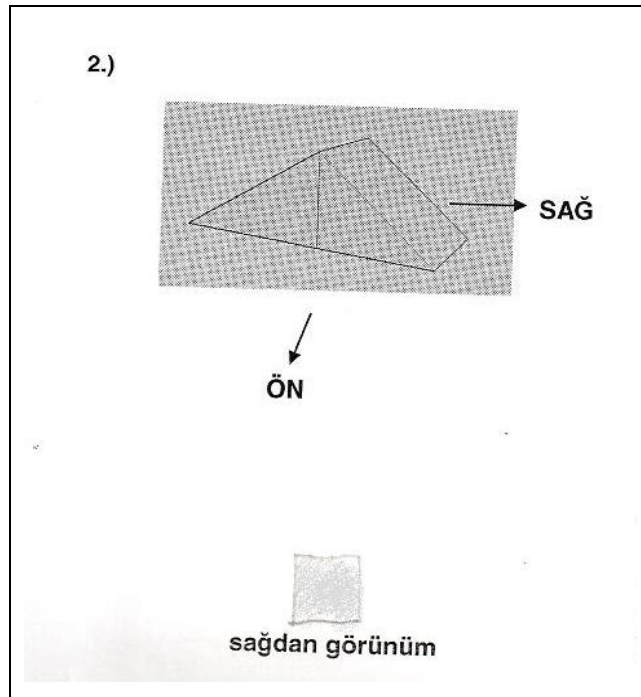


Figure 4.9 Same Student Responses Who didn't draw a parallelogram in the

Second Question

The attractive point in the task in general is that the different orientations of the students while drawing the top view. Most of the students understood from top view “stay in front of the object and then look from the top”. However, some students (s3, s5, s14, s15, s16, s19, s20 and s22) drew the top views by looking from the sides and top. It is a surprising result for me that this issue was quite common in the task.

Activity 3: Given 2D Views Drawing 3D Objects

In this activity students were expected to draw 3D objects given their 2D views from different angles (top, front and side views). The activity consists of three parts. In the first part students should construct the 3D objects using the unit cubes in same color and then draw them onto the isometric paper. In the second part they should draw without using the unit cubes. In the last part the views were given same as the first part but with one different colored cube in each block. In this part students were asked whether it was easier to construct the blocks in the first part or in the last part.

The results show that, out of 23 students 16 had at most one incorrect drawing; 3 students had two correct drawings and 4 students had one correct drawing. Compared to first isometric drawing activity, it was observed that students improved. According to my observation notes students got used to draw on isometric paper and in this activity they felt comfortable.

In the first activity, there were students who had no correct drawings at all. For instance while s1 had only one correct drawing out of four in the first activity, she had all the drawing correct in this activity. Similarly S21 had no correct drawings in the first activity. He had trials but there were no correct drawing. However, in this activity had had all the drawings correct (Figure 4.10) When I asked him the reason, he answered:

“Isometric paper was new for me. I see it for the first time with the first activity. I was confused. I couldn’t get the pattern between the dots. But when I get used to use it I see that I have better drawings.”

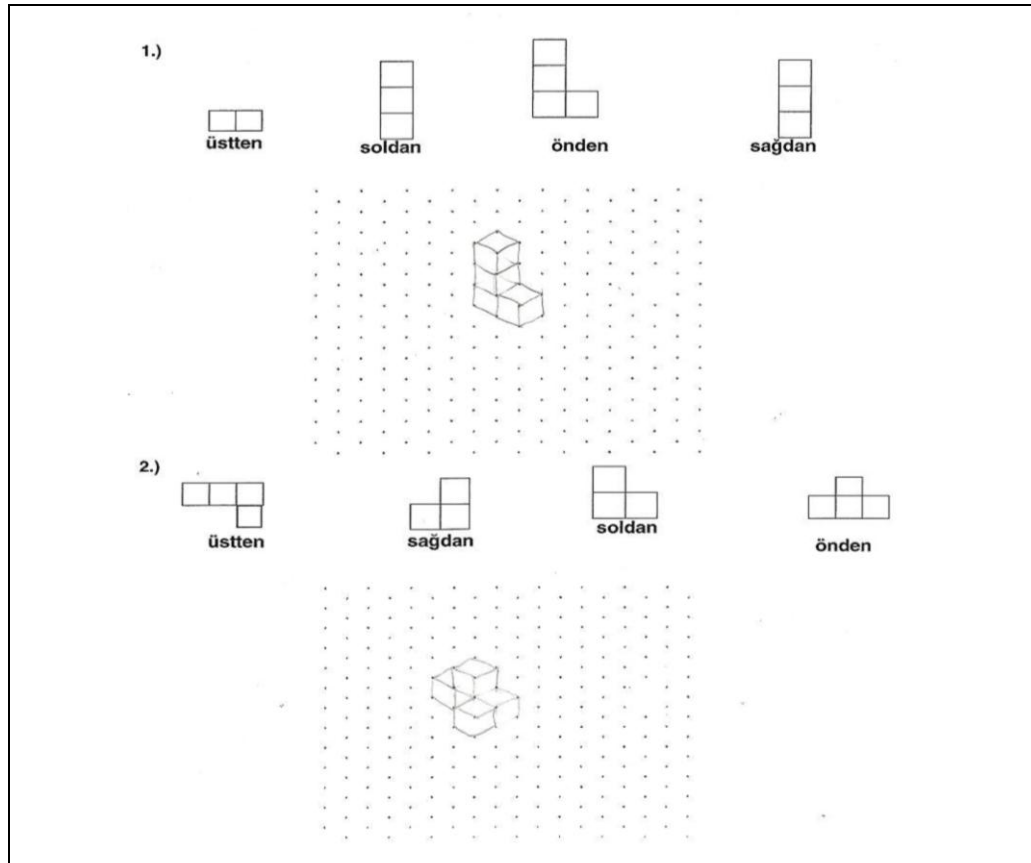


Figure 4.10 A Piece of Work of S21 who had all drawings Correct in Activity 3

Another student, S2, had two correct drawings in this activity while she had no correct drawings in the previous drawing activity. She told that she could draw when she used unit cubes as a manipulative. In without unit cubes part she tried to draw but her drawings were not correct at all. She expressed her feelings:

“At least I can make isometric drawing when I use unit cubes. Beforehand I

even couldn't do that and I couldn't understand to make drawings on an isometric paper. Now I know isometric paper. “

An interesting mistake was that two of the students couldn't understand what they supposed to do with the different views and they tried to draw 3D version of all the views separately.

When we discuss the activity as a class some students express their ideas and strategies as in the following:

S10: I don't need the unit cubes anymore in 3D drawing.

S18: I can see 3D object when I merge the 2D views in my mind.

S19: I can easily visualize the 3D object in my mind.

An interesting point is that some students think that instead of copying the 3D objects into the isometric paper, drawing it from the 2D views is more challenging and beneficial for improving visualization in mind.

According to the results one of the students, who didn't pay attention on the isometric characteristics of the paper in the first activity, understood her mistake and corrected herself in this activity. The interesting point is that she said that she could better draw by visualizing the object in her mind rather to construct it with the unit cubes.

The results indicate that only four of the students think that constructing was easier in the first part. The rest of the students think that constructing was easier when a precise cube is colored in all the views.

Activity 4: Finding back view

In this activity, students were expected to find the back view of the object when the other views (top, left, right, front) are given. The purpose of this activity was to observe that whether or not the students can find any relationships among the different views of the objects. Therefore there was a question in the last part which asks students' strategies while finding the back view and whether there was a relationship between the different views of objects.

The results show that out of 23 students 9 of them answered the question as

“exact opposite of the front view” and 13 of them used the term “symmetric with the front view”. Only one of the students used unit cubes to answer the question.

Generally this activity didn’t take much time, because almost all of the students found the pattern that the back view is the symmetric of the front view and they could easily draw the back views of the objects. When I asked them how they found the activity almost all answered it was very easy after the first three activities. For instance s16 expressed his opinion as following: (Figure 4.11)

“Now I have a better perspective to the 3D objects and their 2D views. I feel myself like my 3D part of my brain turns on. I could better imagine objects in my mind.”

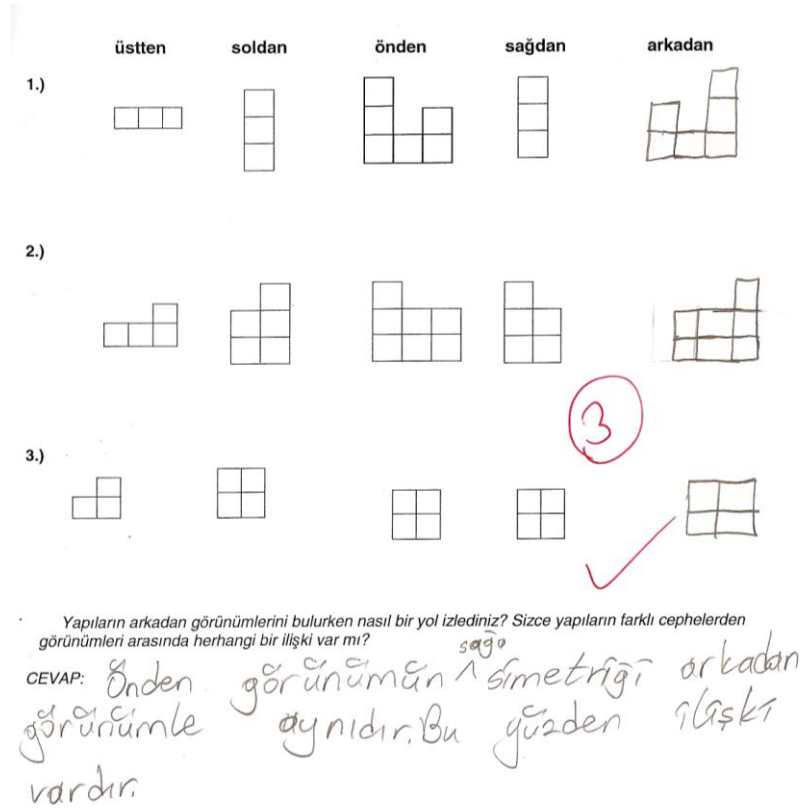


Figure 4.11 Work of S16 in Activity 4

After this activity, as a researcher and a teacher I could realize the improvement in my students. There was an observable progress in their expressions, questions and drawings ever since the first activity. In every class hour before I say a

word they were asking to me” We are going to do activities in this hour, aren’t we?” I could see that they were so enthusiastic and motivated about the activities.

Activity 5: Coded Side View

In this activity students were expected to construct 3D blocks whose coded side views are given. The coded side views were top views in all questions.

The results show that 18 of the 23 students constructed the block in four of the questions. The rest of the students had three correct constructions and drawings. Some of the students even drew the views on the paper. According to my observation, students did not live any serious difficulty in this activity because it was a construction activity but not drawing. From their activity papers and works in the classroom it can be understood that construction type tasks were easier than drawing. Despite that I told them they didn’t have to draw the views some students were very fast in constructing and after they finished they started to draw the views they constructed with the unit cubes. S20 expressed her opinions as following:

“I do not live difficulty in 3D drawing anymore. It’s fun”

At the end of this activity there was a question in which the different 2D views of a 3D object was given in Figure 4.12. The question was asking to construct as many different 3D blocks as possible fitting the views.

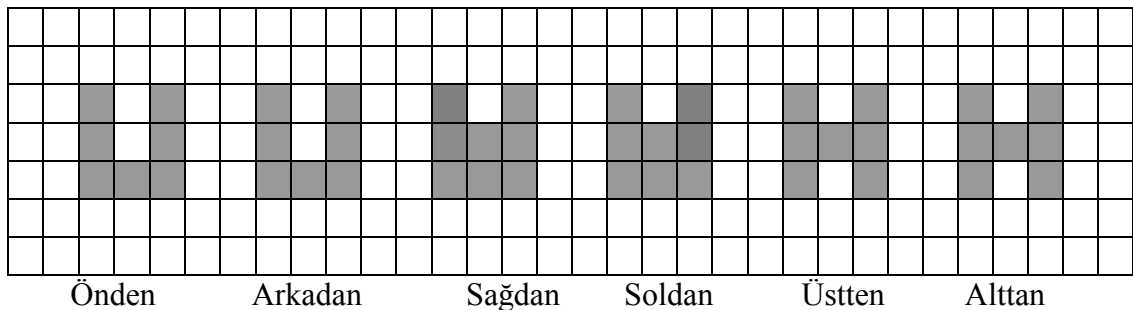


Figure 4.12 the Question Second Part Activity 5

The results show that 7 of the 23 students constructed one block; 2 students constructed 2 blocks ; 1 students 4 blocks; 3 students 5 blocks; 2 students 6 blocks and 2 students constructed totally 9 blocks. While the students were dealing with the activity, I observed how they constructed. One of the students who had 9 constructions expressed his strategy as in the following:

“First, I constructed a block fitting these views then by replacing a cube in the block I obtained different blocks. “

According to my observation notes this question was challenging for the students. They all studied on the task as deeply concentrated and motivated even they competed against each other for constructing more blocks. Some of the students got frustrated when they couldn't construct anything. The students, who finished earlier, accompanied them while constructing. By peer tutoring they felt better and dealt with the activities.

Activity 6: Constructor-Architecture Game

This activity is designed to be a game. It is a pair game in which one student pretends to be an architecture having a 3D building made of unit cubes and his\her duty is to make construct the same building to his\her pair who is the constructor. The duty of the constructor is to construct the building explained by the architecture without looking at the building. Students were playing the game by changing the roles. In the end of the task there were questions about the activity to be answered by the students.

This activity was not evaluated numerically because there were no correct or incorrect answers. All students played with their peers and answered the questions individually. For instance, there was a question which asked: “Which one was easier, being an architecture or constructor?” There wasn't a common answer. Whereas, some students thought that explaining was easier, others thought that constructing was easier. I realized that students had different skills and I should pay attention to their individual differences in their skills.

My observation notes show that students enjoyed this game. They even didn't hear the break bell. They were so focused on the task.

As a summary, Table 4.5 presents overall results of the activities in percentage. As seen from the table, there is a tendency to increase in students' true responses from the first to the last activity. Especially when compared the third part of the first activity and the third activity, which are both isometric drawing activities, it can be concluded that there is a considerable increase in students' isometric drawing compared to the beginning of the activities.

Table 4.5 True Response Percentage

Activities	True Response Percentage
1.1&1.2	52%
1.3	56%
2	88 %
3	75 %
4	99 %
5	96 %

4.3 Students' Views and Feelings about the Activities

After the activities were finished I asked my students to write a reflective paragraph to express their opinions and feelings about the activities. All of the students wrote a reflective paragraph consisted of almost five- six sentences in the first class hour after the activities were finished. The purpose of doing this writing was to learn their views and ideas about the activities with their own expressions. While analyzing these papers I paid attention to the words they used for the nature of the activities and also some strategies they used while doing the tasks. Almost all of the students wrote that they found the activities enjoyable and beneficial.

S1: These activities were the most beneficial and enjoyable activities that we have ever done in this year. We learnt a lot of things and besides we had fun a lot. It was beneficial for me to learn from my peer. The attractive thing was that I could better concentrate when I worked with my peer. To sum up I participated in the activities enthusiastically. Sometimes they were difficult but they were so fun.

S19: I think that my spatial ability really improved with these activities.

S20: I think that during a week we had a really effective and enjoyable math. I plan to be architecture in the future and these activities were really effective for me, for my future plans. Now I have an idea about drawing and I saw that architecture is just for me.

S11: During a week I saw activities that I had never seen before. In the activities I often used my imagination and didn't use unit cubes. There were times that I had difficulties but in general I really had a lot of fun with the activities.

Students had a week full of with activities and in all the class hours they actively participated in the class. They showed their work by drawing, constructing and discussing. Each student had the opportunity to show his\her work individually. Their motivation for the activities and their feelings were important for the quality of my study as a researcher and also the quality of my instruction as a teacher. The writings show that the students enjoyed the activities in general and they think that the lessons should always be covered with such activities. Their written and verbal responses, their actions and their participations in the classes indicated that they were highly motivated for the activities.

4.4 Summary of Findings

This study indicated that the students did not have some of the necessary prerequisite knowledge related to 2D representations of 3D objects. For instance, at the beginning of the instruction most of the students did not know about isometric drawing, although it was included in the 5th grade curriculum. At the beginning of instruction, isometric drawing

was one of the most difficult activities for students, compared to other activities.

Another finding was related to the progress of students' performances in the visual reasoning tasks. As the time passed, students got more successful in completing even the more advanced tasks. Quantitative findings demonstrated this improvement clearly, where students' performances in spatial visualization improved significantly after the instruction. Class observations and students' written responses to tasks in the instructional activities supported this view.

The observations, students' expressions during the activities, and their statements in the reflective paragraphs indicated that the activities also made them more comfortable and confident with the visual tasks being studied. This means that the activities were also motivationally support students. They always had positive attitudes towards the activities. Students were better concentrated and motivated on mathematics when they "do" by themselves.

There were also some tasks in which students had difficulties. For instance in the cube counting activities, the most common difficulty was to imagine and count the unseen cubes. In addition, drawing 2D views of the 3D object was easier than drawing 3D object given the 2D views.

CHAPTER 5

CONCLUSIONS AND DISCUSSION

In this chapter the results of the study are discussed. The chapter consists of four parts. In the first part the process of instruction and in the second part effect of instruction is discussed. In the last part implications and recommendations for further studies are given.

5.1 Process of Instruction

In the previous chapter the results of the study was analyzed activity by activity and overall conclusions were made based upon these results. One of these conclusions were about to cube counting activity. It was found that in this activity the most common difficulty was faced in counting the unseen cubes in a diagram on the paper. Indeed this result was consistent with what Ben-Chaim, Lappan, and Houang (1985) found in their research. According to Ben-Chaim et al. (1985) less than 50 % of middle grade students could achieve this kind of problems which involve block of unit cubes with unseen cubes on a paper. Furthermore the results of The National Assessment of Educational Progress (NAEP) showed that less than 40% of 17-year-olds could solve such problems (Hirstein, 1981). They recommended that errors about counting the faces derived from the students' strategy that they are dealing with the picture strictly as a two-dimensional object because it is drawn on the paper. Besides, students who did not double their counts seem to have difficulty in visualizing the unseen sections of the object. Consequently Ben-Chaim et al. (1985) concluded that not counting the unseen cubes "are clearly related to some aspects of spatial ability" (p.406). It was closely related to visualizing the 2D representation of a 3D object in mind. However, it was observed that when students utilize

manipulative, they felt themselves comfortable in doing this kind of activities. For this reason, it was effective first using manipulative to see the blocks in 3D world and then advancing to visualizing the blocks in mind without using manipulative.

In the isometric drawing activity the most striking result was that some (4 out of 23) of the students had 3D drawing but they were not “isometric” at all. They could draw the 3D object into the isometric paper but the drawing was not isometric. Besides, 8 of the students had no correct isometric drawings, even no 3D drawings. This means that 50% of the class had failed in isometric drawing. The main reason of failure may arise from the lack of prerequisite concepts of students from 5th grades. Indeed, students should have had the knowledge of isometric paper and isometric drawing from 5th grades and the isometric drawing activity was a kind of review task for the students about their knowledge of spatial tasks. However, it was observed that many students had no experience with isometric paper before, and they could not achieve the task. They got to know the isometric paper with this activity and in the next isometric drawing task they did better works compared to first one.

Drawing 2D views of 3D objects from different angles (top, front and sides) was an activity in which students had no serious difficulty at all. It was pleasing that they did not use manipulatives and forced themselves to visualize 3D blocks in their minds and draw 2D representations. This means that they progressed in spatial thinking. Gutierréz (1992) carried out a research about different plane representations and 3D objects and he found that drawing side views is easier than building from side views, but isometric drawing is more difficult than building from isometric representation. Indeed I would also expect that students would have more difficulty in activity which is building and drawing 3D objects from 2D views. However, according to the results they had almost the same performance in both activities. It was observed that there was a considerable improvement their isometric drawing ability since the first isometric drawing activity. It can be concluded that the first isometric drawing activity contributed students’ drawing skills.

In general, students enjoyed the lessons during the implementation of activities. They said that they were so motivated and enthusiastically waiting for the next activity and whenever I came to the class, they were asking to me whether we

will do activities or not. The class environment was a learning environment in which every single individual ask questions, discuss ideas, construct 3D blocks with manipulative, make drawings and conclusions, and work cooperatively with their peers. I saw that all students participated in the class as active learners. I observed that my students had a great self confidence while dealing with the activities. They were aware of that they were the actors in the class. When looking from my side, I was a real facilitator by engaging students to participate in the activities, by asking leading questions and by leading discussions.

5.2 Effect of Instruction

To evaluate the impact of the spatial ability tasks implemented during the class hours, a spatial ability test was administered to the students before and after the instruction. As stated previously, spatial ability test consists of two parts; one is spatial orientation test and the other one is achievement test on 2D representations and 3D drawings. The results showed that there was an increase in students' spatial ability test scores from pre - test to post.

The most striking result was that, although the students did not deal with the tasks similar in the spatial orientation test, they had progress in the post test. This means that the activities, the instruction improved students' overall spatial skills. However, there is a limitation for this result. There was ten days of duration between pre-test and post test administration of the spatial ability test. It was possible that students could remember the questions in the test. Another limitation may be they feel themselves more comfortable in the post test because they have seen the items before. Therefore, this result should be carefully discussed. The reason why they got higher scores from the post test might be the cognitive and motivational effect of the activities. During five class hours students studied on the activities as highly motivated and continuously they shared ideas and they internalized spatial concepts with the help of the activities. They dealt with a variety of activities in each of them they improved a different skill. For instance in one of the activities they made drawing, in another they made construction and so they create a spatial structure in

their mind and this helped them to achieve the post test. As a consequence, the activities contributed to the development of students' spatial reasoning in general.

When we look at the literature, a great number of studies in the literature indicates that spatial ability can be improved through training, if appropriate materials are provided (Ben-Chaim *et al.*, 1988). Considerably old studies and also newest ones support this inference. For instance Brinkmann (1966) conducted a study about the changes in spatial ability as shown on the Differential Aptitude Spatial Relations Test. In this way, the participants were administered a 3-week training program that emphasizes pattern folding and object manipulation. The findings showed that there was a significant gain by the experimental group, and that the spatial visualization skills could be improved. Similarly, Ben-Cahim, Lappan and Houang (1985) conducted a study to investigate the effect of instruction on spatial visualization performance of students in grades 5, 6, 7 and 8. As a consequence, they also found that spatial ability could be improved by appropriate instruction.

Similar to present study, Alias, Black and Gray (2002) investigated in their studies the effect of manipulative and sketching activities on spatial visualization ability in engineering students. As a result, Alias *et al.* (2002) found that there was an improvement in spatial ability in general after teaching and learning spatial visualization activities. In addition to this, Olkun (2003) found that engineering drawing activities play a significant role to improve spatial ability. In his recent study, Olkun and Sinoplu (2008) investigated the effect of pre-engineering activities on 4th and 5th grade students understanding of rectangular solids made of small cubes. The findings indicate that two-hour instructional activities were effective and beneficial for all students. Furthermore, construction activities in engineering context are very helpful in improving students' understanding of rectangular solids made of cubes.

What literature tells about the effect of instruction was actually what I expected from the present study. This study indicates that the more one faces spatial experience the more he\she progresses in spatial tasks. When looking at the literature, it can be foreseen that the instruction with spatial tasks would have a positive effect on students' spatial reasoning development. This significant

difference can be derived from the nature and application of the activities. There is a common prejudice about spatial issues in people's mind including me. However, with an appropriate instruction, the goals can be easily achieved. The results of the study indicate that visualization in mind develops eventually. At the very beginning of the activities, students feel themselves more comfortable with concrete materials and gradually they advance to visualization without manipulative. For this reason, for a deep understanding of spatial tasks, use of manipulative as scaffolds is inevitable.

5.3 Implications of the Study

Based upon the findings of the present study and previous research, some suggestions can be made for the educators. Firstly, the activities used in this study will be a resource for the teachers, which could be used in practice. Since spatial concepts have recently integrated in Turkish elementary mathematics curriculum, there are not many resources in this field. Therefore, these activities set an example for the teachers to create their own activities. They can prepare their own activities by considering their own class environment. In addition to this, utilizing manipulative at the beginning of the topic is a beneficial way to conceptualize spatial topics.

Moreover, taking into account of the class hour given to this topic in curriculum, it is observed that the time is not enough for meaningfully covering the topic and in this way this study has logical implications and recommendations to the curriculum developers.

Furthermore, this study indicates that we should get rid of our prejudices which prevent us from deeply understanding a topic. For instance some people may think that they will have some difficulties in certain topics before they have sufficient information about them. They believe that they won't be able to succeed in that topic and mostly this prejudice stem from the previous experiences about this topic. With this study I realized that with an appropriate instruction, the prejudices can be broken. If teachers provide environments in which students can fulfill their learning about spatial issues then students reach success in this field.

In addition to this, teachers should create activities and design their lessons in a way that where the students are the actors. They should create a classroom environment which allows students more responsibility and freedom. For a sound understanding of concepts, especially spatial concepts, standing on the board and lecturing are not enough. Teachers should be facilitator during the lesson and students should “do” mathematics.

As a consequence, in the light of the previous research findings and based on the findings of the present study it can be inferred that spatial reasoning can be improved with an appropriate instruction. In this context, to develop students’ visual reasoning abilities teachers need to provide them opportunities to practice with the visual tasks supported with the effective use of manipulative.

5.4 Recommendations for Further Studies

Present study suggests a variety of research topics for further studies. This study tells us spatial ability can be improved with appropriate instruction. Hence, in another research computer technology can be used as an instructional material. Computer activities can be designed or existing computer activities can be used during the instruction. In addition to this, the effect of manipulative and computer can be compared in further studies.

Current study is only administered in one class in a school. In later studies, it can be conducted in all the classes of the school and in addition to instruction effect; teacher efficacy can also be measured.

Moreover in further studies the relationship between spatial ability and math achievement can be tested. Similarly, the effect of spatial ability on mathematical problem solving can be studied in another research.

Finally, researches measuring the effect of spatial anxiety (Lawton, 1994) on students’ achievement can be conducted and similarly students’ attitudes to spatial concepts and students’ success in spatial issues can be studied in further researches.

5.5 Implications for my Future Practice

This study made me, once more; believe that spatial concepts could be best internalized with visual tasks and by the help of manipulatives. After this study, I became a teacher who has an improved spatial sense. I will use the activities of this study in my future practices. Moreover, I will try to develop new activities. In addition to this, I will use manipulatives more often in my classes, not only in spatial concepts but also in a variety of topics in mathematics curriculum. I have learnt from this study that before starting a new concept, prerequisites learning of the students must be absolutely questioned. Besides, I realized with this study that the students, - even the most indifferent ones-are more concentrated on the topic when they are involved in it. Being aware of it, I will create a learning environment in my classes, where the students talk more than me and where I will be a guide for their learning.

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APPENDICES

APPENDIX A

Achievement Test on 2D Representations of 3D Objects and Isometric Drawing

Sevgili öğrenciler:

Bu testin amacı sizlerin üç boyutlu düşünme becerinizi ölçmektir. Testin sonuçları sadece bilimsel bilgi edinmek amacıyla kullanılacaktır. Herhangi bir şekilde not ile değerlendirme amacıyla kullanılmayacaktır. Bu amaçla:

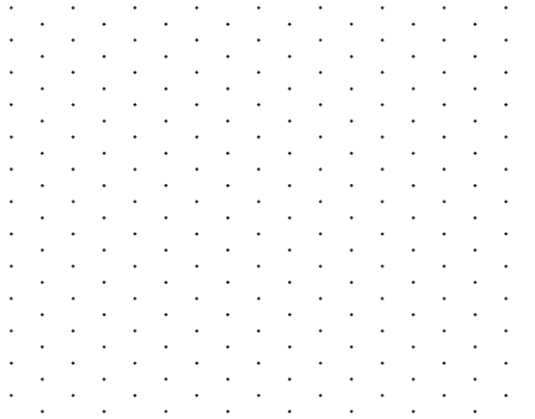
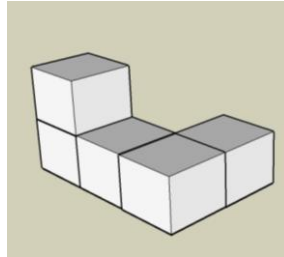
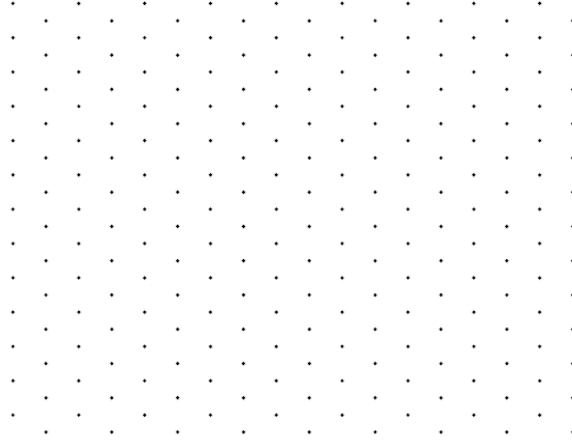
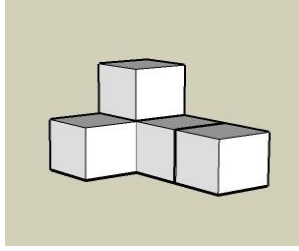
1. Aşağıda size ait bilgileri eksiksiz olarak doldurunuz.
2. Testi tamamlamak için 30 dakika süreniz vardır.

Teşekkürler.

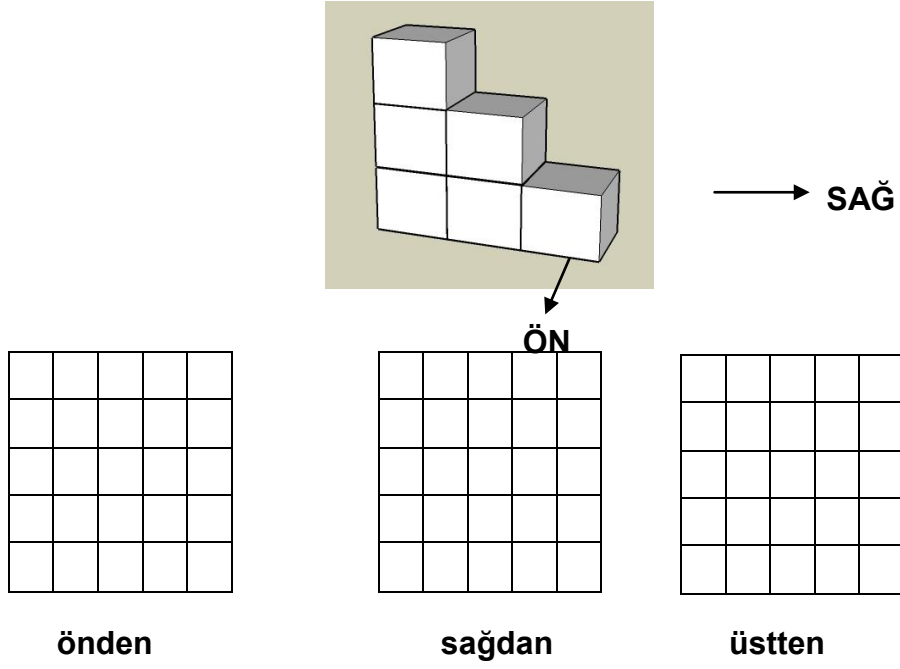
Zeynep ERYAMAN
zeryaman@yahoo.com

ADI SOYADI :

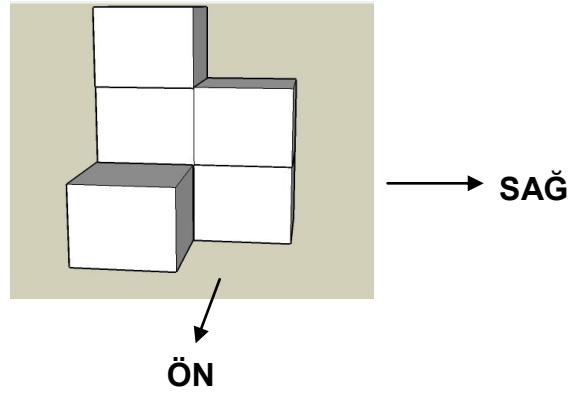
1.) Aşağıda eş küplerle oluşturulmuş 2 farklı yapının çizimlerini görüyorsunuz. Bu yapıları şeklin altındaki izometrik noktalı alana çiziniz.



2.) Eş küplerle oluşturulmuş yapının önden, sağdan, üstten görünümünü kareli kâğıt üzerindeki kareleri boyayarak gösteriniz.



3.) Eş küplerle oluşturulmuş yapının önden, sağdan, üstten görünümünü şeklin altındaki boş alana çiziniz.



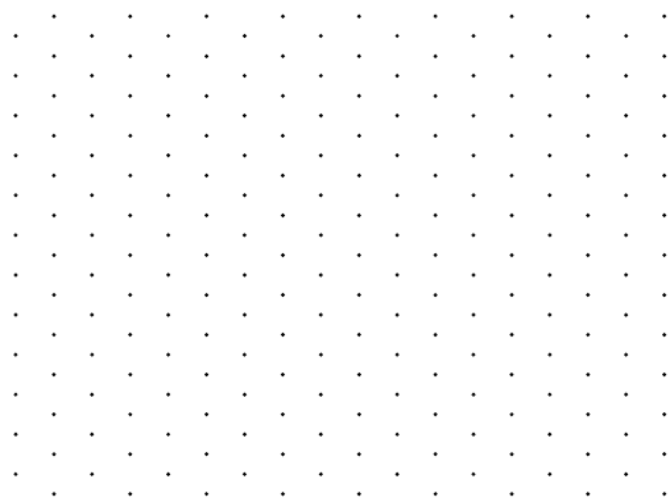
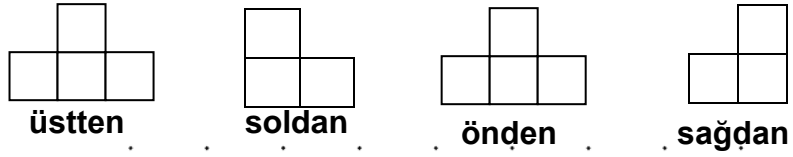
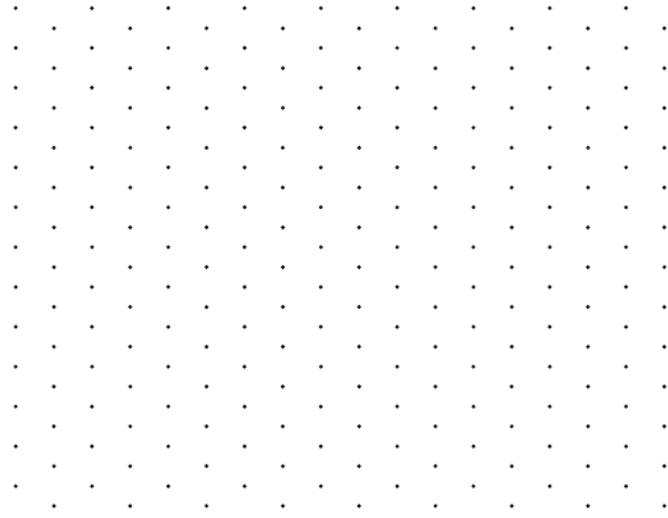
önden görünüm

sağdan görünüm

üstten görünüm

4.) Aynı renkte küpler kullanarak farklı cephelerden görünümü verilen küplü yapıları oluşturunuz ve yapılarınızı soruların altındaki izometrik noktalı alana çiziniz.

1.)



APPENDIX B

Spatial Orientation Test

Adı Soyadı: _____

KART ÇEVİRME TESTİ

Bu test şekiller arasındaki farkı görebilme yeteneğini ölçmek için geliştirilmiştir. Aşağıdaki üçgen şeklindeki 5 kartı inceleyiniz.



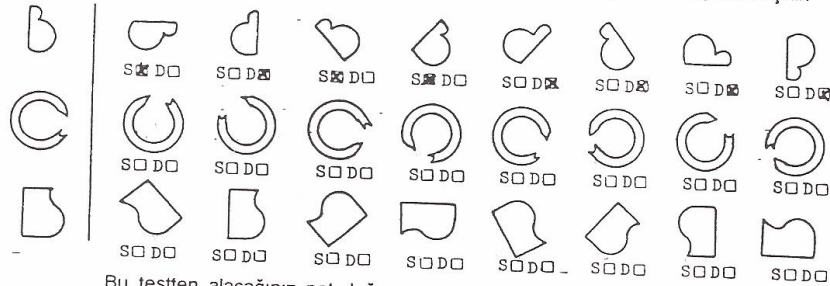
Fark edeceğiniz gibi tüm şekiller baştaki kartın döndürülmüş (yuvarlanmış) halleridir. Şimdi aşağıdaki iki kartı inceleyiniz.



Gördüğünüz gibi bu kartlar aynı değildir. İlk kart döndürme (yuvarlama) yoluyla ikincisine dönüştürülemez. Ancak yüzü tam çevrilirse ikine dönüşebilir. Dolayısıyla bu kartlar farklıdır diyebilirsiniz.

Bu testte yapmanız gereken dikey çizginin solundaki şekille sağdaki sekiz şekili karşılaştırıp aynı olup olmadıklarını tespit etmektir. Sağdaki şekillerden herhangi birisi soldakiyle aynı ise şeklin altındaki **S** (Sabit); farlı ise **D** (Değişik) şıklarını işaretleyiniz.

Aşağıdaki örnekleri inceleyip çözünüz. İlk sıra sizin için doğru olarak çözülmüştür.





















Bu testten alacağınız not doğru cevaplarınızdan yanlış cevaplarınız çıkarılarak elde edileceğinden, bir fikriniz olmadan tahminde bulunmamanız lehinize olacaktır.










Test iki bölümden oluşmaktadır ve her bölüm için 3 dakikanız vardır. Süre dolduğunda lütfen 1. Bölümü cevaplandırmayı bırakıp 2. Bölümün dağıtılmasını bekleyiniz. Başarılar;










LÜTFEN SÖYLENMEDEN SAYFAYI ÇEVİRMEYİNİZ.










1. BÖLÜM (3 Dakika)










1.         
SODD SODD SODD SODD SODD SODD SODD SODD SODD










2.         
SODD SODD SODD SODD SODD SODD SODD SODD SODD










3.         
SODD SODD SODD SODD SODD SODD SODD SODD SODD










4.         
SODD SODD SODD SODD SODD SODD SODD SODD SODD










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9.         
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10.         
SODD SODD SODD SODD SODD SODD SODD SODD SODD

LÜTFEN 2.BÖLÜMÜN DAĞITILMASINI BEKLEYİNİZ

2. BÖLÜM (3 Dakika)

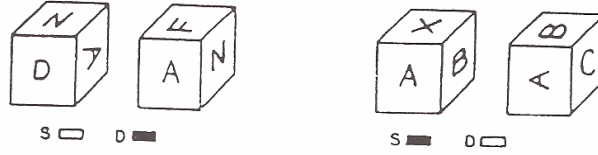
11.									
	SODD	SODD	SODD	SODD	SODD	SODD	SODD	SODD	SODD
12.									
	SODD	SODD	SODD	SODD	SODD	SODD	SODD	SODD	SODD
13.									
	SODD	SODD	SODD	SODD	SODD	SODD	SODD	SODD	SODD
14.									
	SODD	SODD	SODD	SODD	SODD	SODD	SODD	SODD	SODD
15.									
	SODD	SODD	SODD	SODD	SODD	SODD	SODD	SODD	SODD
16.									
	SODD	SODD	SODD	SODD	SODD	SODD	SODD	SODD	SODD
17.									
	SODD	SODD	SODD	SODD	SODD	SODD	SODD	SODD	SODD
18.									
	SODD	SODD	SODD	SODD	SODD	SODD	SODD	SODD	SODD
19.									
	SODD	SODD	SODD	SODD	SODD	SODD	SODD	SODD	SODD
20.									
	SODD	SODD	SODD	SODD	SODD	SODD	SODD	SODD	SODD

LÜTFEN SÜRENİZ BITENE KADAR BEKLEYİNİZ

Adı Soyadı: _____

KÜP KARŞILAŞTIRMA TESTİ

Bu testteki tüm problemlerde üzerlerinde harf, rakam veya şekil bulunan 6 yüzü (alt yüz, üst yüz ve dört yan yüz) olan küpler verilmiştir ve küplerin birbirlerinin aynı olup olmadığını bulmanız istenmektedir. Aşağıdaki iki küp çiftini inceleyiniz.

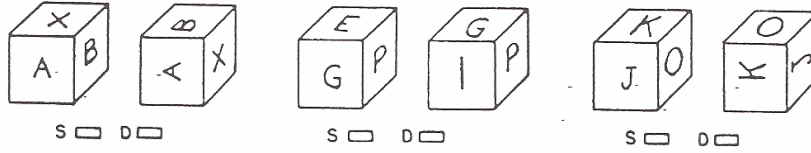


İlk çift için D şıkkı seçilmiştir çünkü küpler birbirinden farklıdır (Değişik). Soldaki küpün A harfi bulunan yüzü size bakacak şekilde çevrildiğinde, N harfi bulunan yüzü A harfi bulunan yüzün soluna ve görünmeyecek konuma gelir. Oysa sağdaki küpün N harfli yüzü A harfli yüzün sağında ve görünür haldedir, dolayısıyla bu küpler farklıdır.

İkinci çiftte ise S şıkkı seçilmiştir çünkü küpler aynı olabilir. A harfli yüzey yana çevrildiğinde X harfli yüzey görünmez konuma, B harfli yüzey üste gelir ve görünmez konumdaki C harfli yüzey görünür konuma gelir. Buda küplerin aynı olabileceğini gösterir.

Not: Bütün harf, rakam ve şekiller bir küpte birden fazla bulunamaz, fakat görünmeyecek konumda olabilir.

Aşağıdaki üç örneği inceleyiniz.



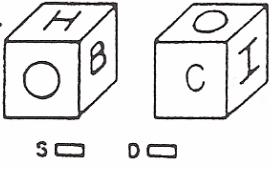
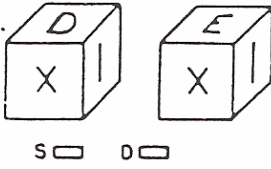
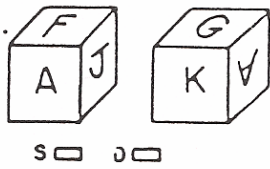
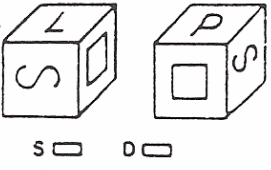
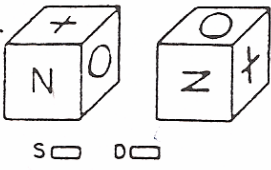
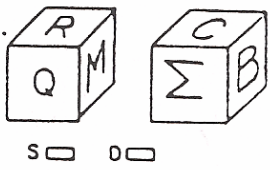
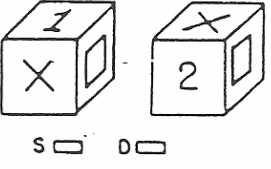
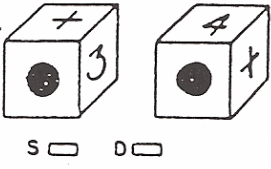
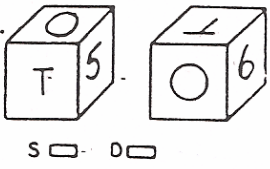
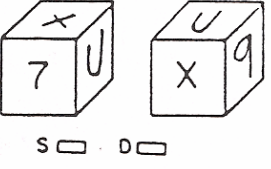
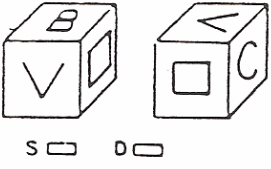
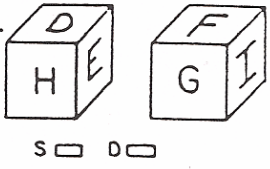
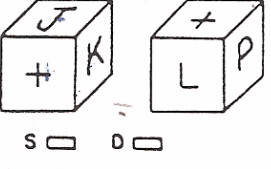
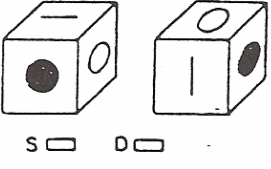
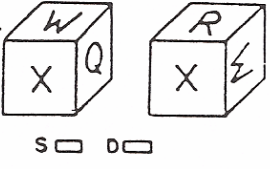
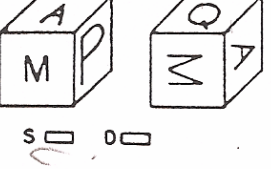
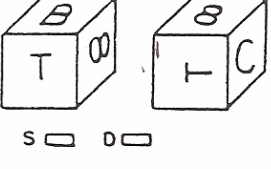
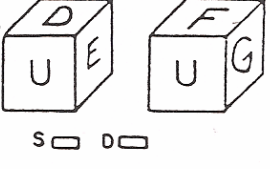
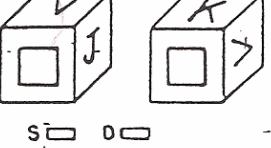
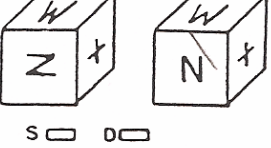
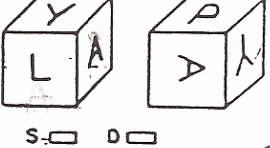
İlk çift hemen D işaretlenmelidir çünkü X harfi bir küpte iki defa bulunamaz. İkinci ve üçüncü çiftleri inceleyip cevaplandırınız.

-Bu testten alacağınız not doğru cevaplarınızdan yanlış cevaplarınız çıkarılarak elde edileceğinden, bir fikriniz olmadan tahminde bulunmanız lehinize olacaktır.

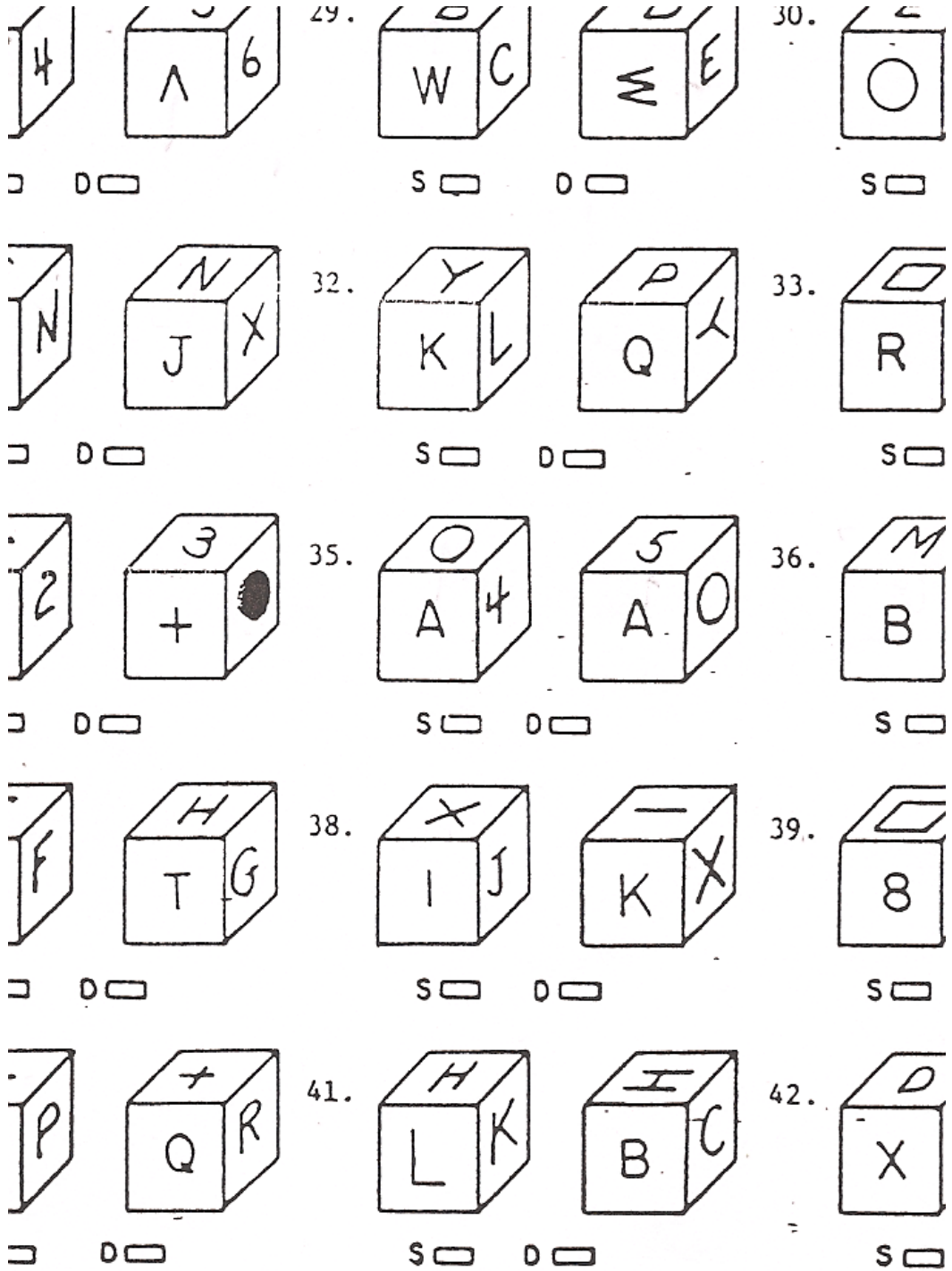
Test iki bölümden oluşmaktadır ve her bölüm için 3 dakikanız vardır. Süre dolduğunda lütfen 1. Bölümü cevaplandırmayı bırakıp 2. Bölümün dağıtılmasını bekleyiniz. Başarılar,

LÜTFEN SÖYLENMEYEN SAYFAYI ÇEVİRMİYİNİZ.

1. BÖLÜM (3 Dakika)

1. 
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21. 

LÜTFEN 2. BÖLÜMÜN DAĞITILMASINI BEKLEYİNİZ



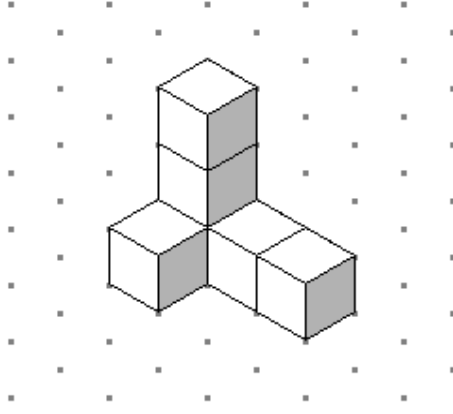
LÜTFEN SÜRENİZ BİTFNE KADAR RFKI FYİNİZ

APPENDIX C

Spatial Tasks (Activities)

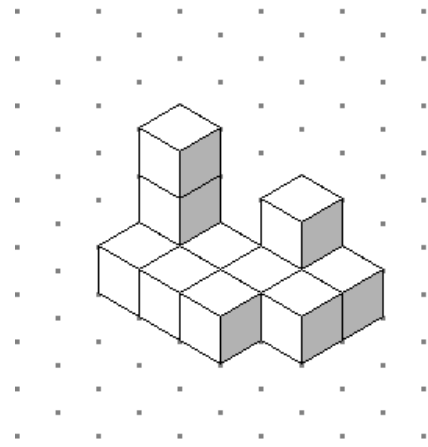
1.) İzometrik kâğıt üzerinde çizimi verilen yapıları eş küplerle oluşturunuz ve yapılardaki küp sayılarını şekillerin altındaki boşluklara yazınız.

a)



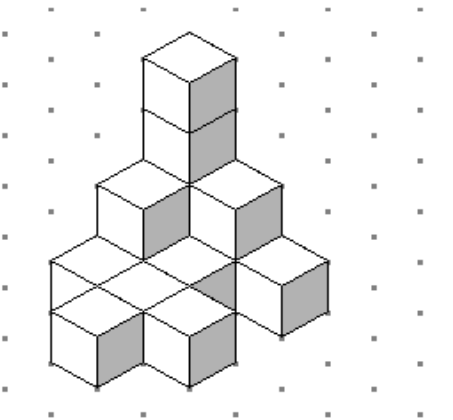
..... küp

b)



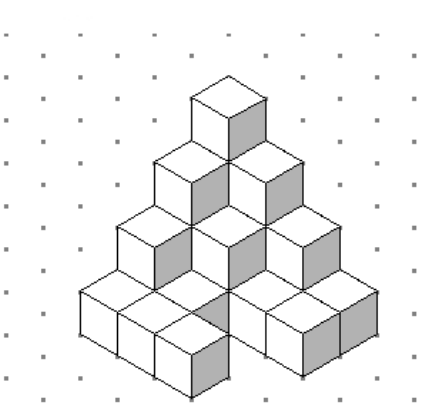
..... küp

c)



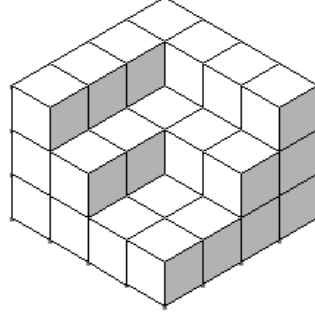
..... küp

d)



..... küp

Aşağıdaki şekilde çok küplüleriyle oluşturulmuş bir yapı görüyorsunuz. Bu yapıya bakarak şeklin altındaki sorulara cevap veriniz.

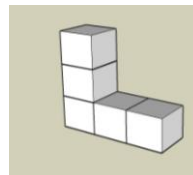
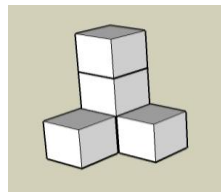
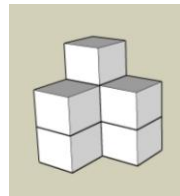
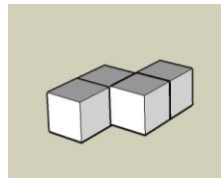


Bu açıdan bakıldığında;

- Bu yapıda kaç küp vardır? _____
- Yapıda 4 yüzü görünen kaç küp vardır? _____
- Yapıda 3 yüzü görünen kaç küp vardır? _____
- Yapıda 2 yüzü görünen kaç küp vardır? _____
- Yapıda 1 yüzü görünen kaç küp vardır? _____
- Yapıda hiçbir yüzü görünmeyen kaç küp vardır? _____

3.) 4 eş küp kullanarak 4 farklı yapı oluşturunuz ve bu yapıları izometrik kâğıda çiziniz.

4.) Aşağıda eş küplerle oluşturulmuş 4 farklı yapının çizimlerini görüyorsunuz. Bu yapıları izometrik kâğıda çiziniz.

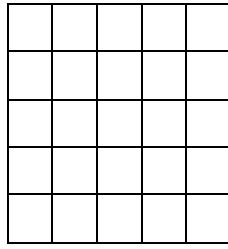
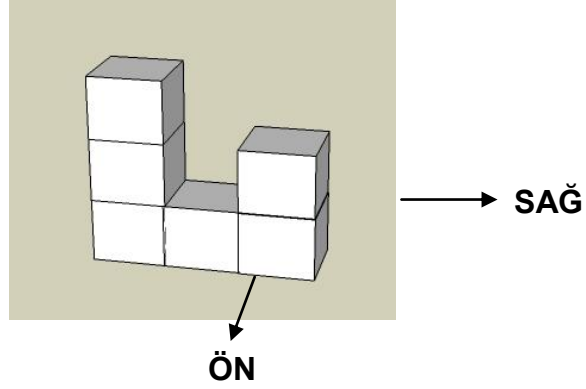


Etkinlik-2

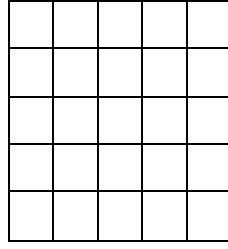
1.Bölüm

Eş küplerle oluşturulmuş yapıların önden, yandan ve üstten görünümünü kareli kâğıt üzerindeki kareleri boyayarak gösteriniz.

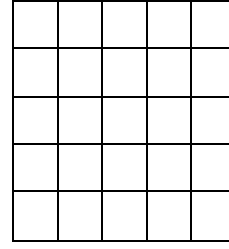
1)



önden

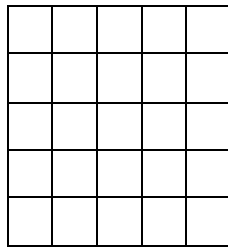
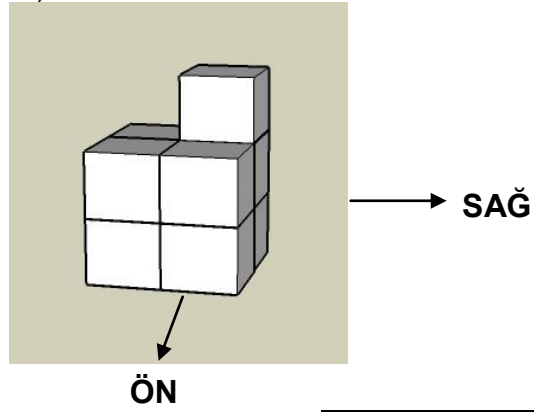


sağdan

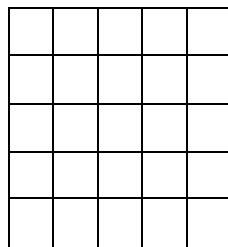


üstten

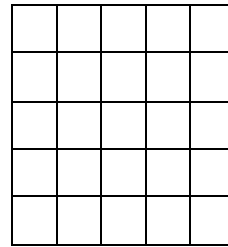
2)



önden



sağdan

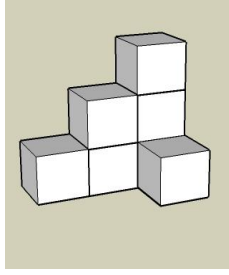


üstten

2. Bölüm

Eş küplerle oluşturulmuş yapının önden, sağdan, üstten görünümünü şeklin altındaki boş alana çizin.

1.)



SOL ←

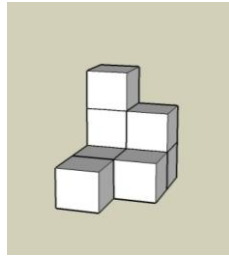
↓
ÖN

önden

soldan

üstten

4)



→ SAĞ

↓
ÖN

önden

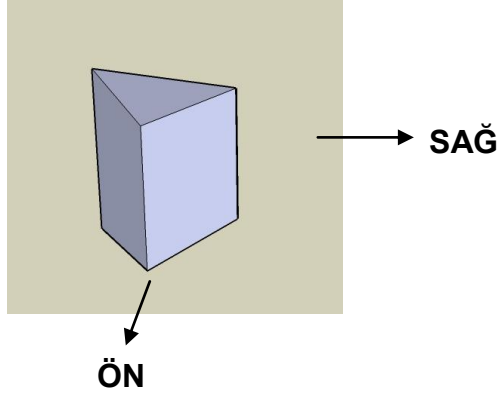
soldan

üstten

3. Bölüm

Aşağıdaki üç boyutlu cisimlerin önden, sağdan, üstten görünümünü şeklin altındaki boş alana çizin.

1.)

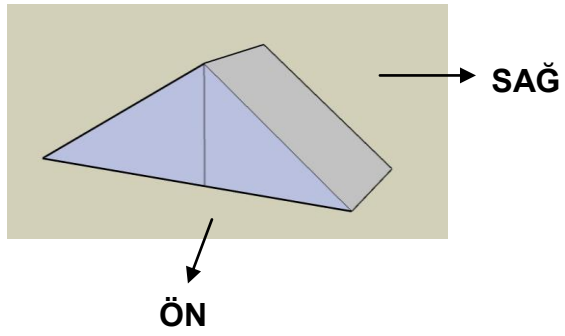


önden

sağdan

üstten

2.)



önden

sağdan

üstten

Etkinlik-3

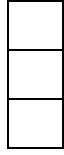
1. Bölüm

Aynı renkte küpler kullanarak aşağıda farklı cephelerden görünümü verilen küplü yapıları oluşturunuz ve yapılarınızı soruların altındaki izometrik noktalı alana çiziniz. **Dikkat! Çizdiğiniz yapıda herhangi bir cepheyi de belirtiniz.**

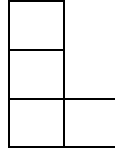
1.)



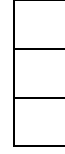
üstten



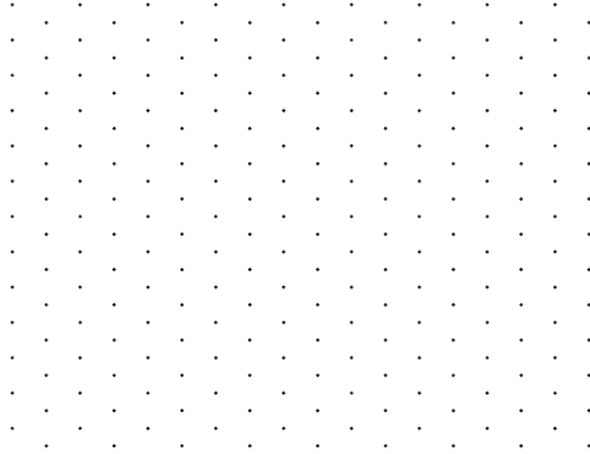
soldan



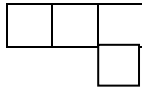
önden



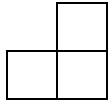
sağdan



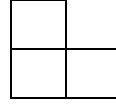
2.)



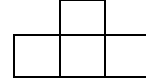
üstten



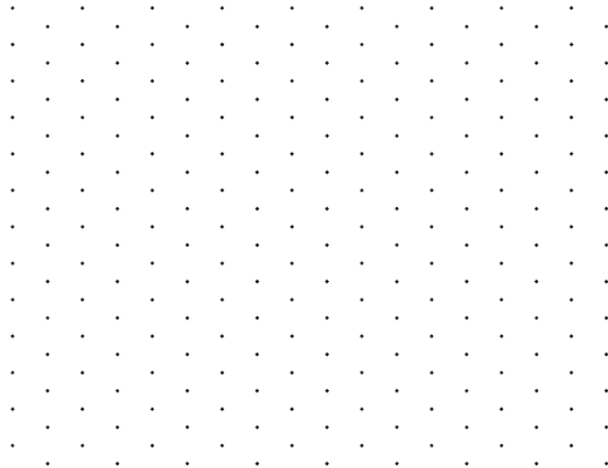
sağdan



soldan



önden



2. Bölüm

Eş küpleri kullanmadan, farklı cephelerden görünümü verilmiş olan üç boyutlu yapıları soruların altındaki izometrik noktalı alana çiziniz.

1.)



üstten



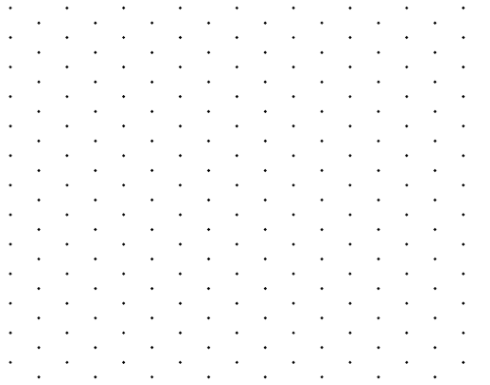
soldan



önden



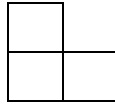
sağdan



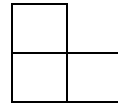
2.)



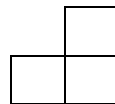
üstten



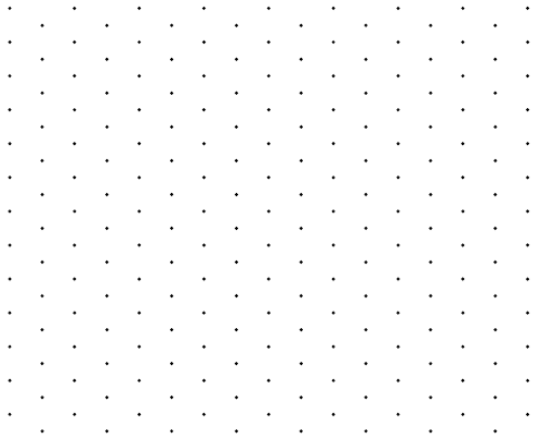
soldan



önden



sağdan



3. Bölüm

Aşağıda farklı cephelerden görünümü verilmiş olan küplü yapılar 1. ve 2. bölümdekilerle aynıdır. Bu bölümde her bir küplü yapıda farklı renkte sadece bir küp kullanılmıştır. Bu küplü yapıları küplerle oluşturunuz.

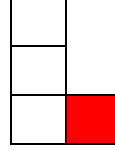
1.)



üstten



soldan

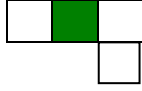


önden

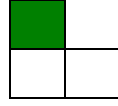


sağdan

2.)



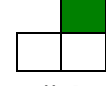
üstten



soldan



önden



sağdan

3.)



üstten



soldan



önden

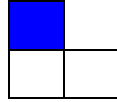


sağdan

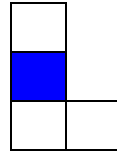
4.)



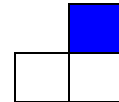
soldan



üstten



önden



sağdan

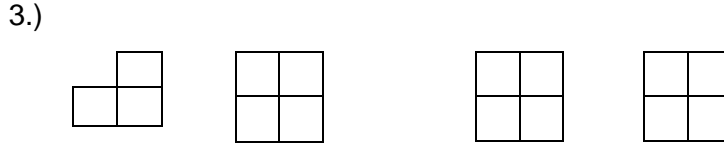
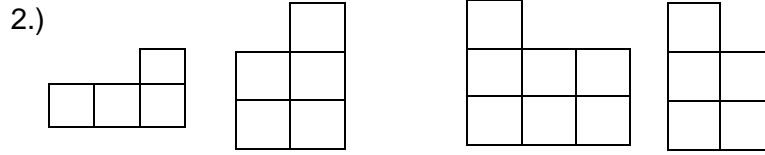
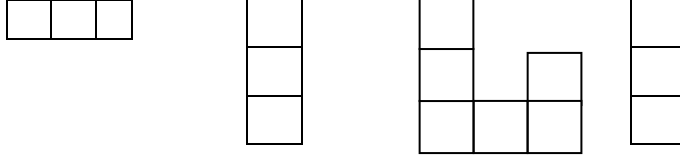
Küplü yapıları oluşturmak 1. bölümde mi, 3. bölümde mi daha kolaydı? Nedenini açıklayınız.

CEVAP:

Etkinlik-4

Üstten, soldan, sağdan ve önden görünümü verilen yapıların arkadan görünümünü bulunuz.

1.) **üstten** **soldan** **önden** **sağdan** **arkadan**



Yapıların arkadan görünümünü bulurken nasıl bir yol izlediniz? Sizce yapıların farklı cephelerden görünümü arasında herhangi bir ilişki var mı?

CEVAP:

Etkinlik-5

Aşağıda kareler ve karelerin üzerine yazılmış rakamlar görüyorsunuz. Karelerin üzerindeki rakamlar yapının sütunlarındaki küp sayısını göstermektedir. Bu şekil eş küplülerle oluşturulacak yapılar için bir çizim planıdır. Buna göre elinizdeki eş küpleri kullanarak verilen çizim planlarına göre küplü yapıları oluşturunuz.

Dikkat! Şekillerin üstten görünümü verilmektedir.

1.)

3	1
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3.)

1	2	1
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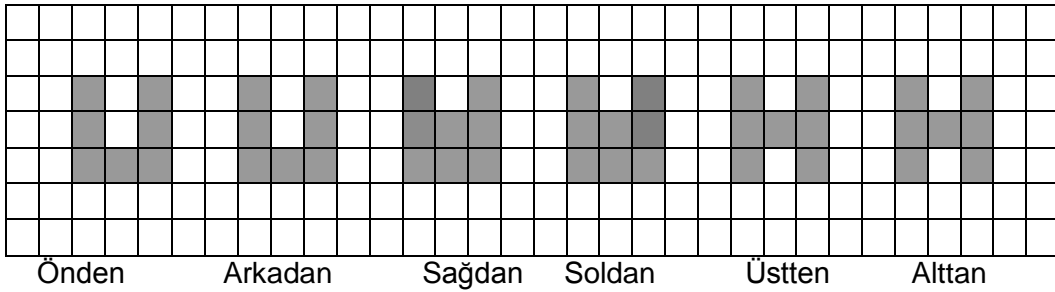
2.)

0	0	2
1	1	2

4.)

0	0	1	2
1	1	2	3

Eş küpleri kullanarak altı farklı cepheden görünümü aşağıdaki gibi olan oluşturabileceğiniz kadar farklı yapılar oluşturunuz.



Etkinlik-6

Bu etkinlikte sıra arkadaşınızla farklı rollere girip bir oyun oynayacaksınız.

Bu oyunun kuralları şöyle:

- Öncelikle rollerinizi seçin. Kimin mimar, kimin müteahhit olacağına karar verin.
- Birbirinizin çalışma alanını görememeniz için aranızda bir engel koyun.
- Mimar 8–12 küp kullanarak bir yapı oluşturacak ve daha sonra bunu müteahhide kafasında canlandırabileceği bir şekilde ayrıntılı olarak tarif etmeli.
- Müteahhit mimarın tarifini tam dinlemeli ve anlattığına tıpatıp uyan yapıyı kendi çalışma alanında oluşturmalı. Bu arada müteahhidin herhangi bir soru sorma hakkı olmamalı.
- Müteahhit yapıyı oluşturduktan sonra engeli kaldırıp yapıların aynı olup olmadığına bakınız ve değilse bunun nedenlerini tartışınız.
- Etkinliği tekrarlayınız, ancak bu sefer müteahhidin soru sorma hakkı olmalı.
- Rollerini değiştirip etkinliği bir kez daha yapınız.

Etkinlikle ilgili aşağıdaki sorulara cevap veriniz.

1. *Mimar mı olmak kolaydı, yoksa müteahhit olmak mı? Neden?*

CEVAP:

2. *Müteahhidin soru sorma hakkının olması yapıyı oluşturmada herhangi bir fark yarattı mı?*

CEVAP:

3. *Yapıyı tarif ederken anlatması zor ve kolay olan şeyler nelerdi?*

CEVAP:

4. *Yapıyı tarif ederken anlamayı kolaylaştıracak matematik terimleri bulabildiniz mi? Yazınız.*

CEVAP: