AN INVESTIGATION OF SEVENTH GRADE STUDENTS' COMPUTATIONAL ESTIMATION STRATEGIES AND FACTORS ASSOCIATED WITH THEM

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

BURÇAK BOZ

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY
IN
SECONDARY SCIENCE AND MATHEMATICS EDUCATION

NOVEMBER 2009

Approval of the thesis:

AN INVESTIGATION OF SEVENTH GRADE STUDENTS' COMPUTATIONAL ESTIMATION STRATEGIES AND FACTORS ASSOCIATED WITH THEM

submitted by **BURÇAK BOZ** in partial fulfillment of the requirements for the degree of **Doctor of Philosophy in Secondary Science and Mathematics Education, Middle East Technical University** by,

Prof. Dr. Canan Özgen _____________________
Dean, Graduate School of **Natural and Applied Sciences**

Prof. Dr. Ömer Geban _____________________
Head of Department, **Secondary Science and Mathematics Education**

Assoc. Prof. Dr. Safure Bulut _____________________
Supervisor, **Secondary Science and Mathematics Education Dept., METU**

Examining Committee Members:

Prof. Dr. Aysun Umay _____________________
Elementary Education Dept., Hacettepe University

Assoc. Prof. Dr. Safure Bulut _____________________
Secondary Science and Mathematics Education Dept, METU

Assoc. Prof. Dr. Ahmet Arıkan _____________________
Secondary Science and Mathematics Education Dept., Gazi University

Assoc. Prof. Dr. Erdinç Çakıroğlu _____________________
Elementary Education Dept., METU

Assist. Prof. Dr. Ayhan Kürşat Erbaş. _____________________
Secondary Science and Mathematics Education Dept., METU

Date: 23.11.2009

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name: Burçak BOZ

Signature:

ABSTRACT

AN INVESTIGATION OF SEVENTH GRADE STUDENTS′ COMPUTATIONAL ESTIMATION STRATEGIES AND FACTORS ASSOCIATED WITH THEM

Boz, Burçak

Ph.D., Department of Secondary Science and Mathematics Education
Supervisor: Assoc. Prof. Dr. Safure Bulut

November 2009, 281 pages

The purpose of this study was to identify seventh grade students' computational estimation strategies and factors associated with these strategies. A case study was conducted with five students. They were selected among 116 seventh grade students from a public elementary school in Aegean region. Two sessions of clinical interviews were carried out with each participant. In the first interview session, the Computational Estimation Test, which was consisted of 15 estimation questions, was administered to students with requesting explanations of solving procedure. In the second interview session, students answered to semi-structured questionnaire prepared by the researcher to understand their feelings and thoughts on estimation.

The results of the study indicated that students used three kinds of computational estimation strategies, which were reformulation, translation, and compensation. Reformulation was the most used types of estimation and by all interviewees. It was divided into four sub-strategies, which were observed during the interviews, among them rule based rounding was the most preferred one. The most sophisticated strategy was compensation, which was used least frequently by the participants. The other kind of computational estimation strategy was translation, which means changing the operation for handling the questions more easily. Translation strategy was used students who performed well in number sense. Based on interviews and observations, there were some cognitive and affective factors, which were associated with the specified strategies. Number sense and mental computation were two sub categories of the cognitive factors. Besides these cognitive factors, confidence in ability to do mathematics, perception of mathematics, confidence in ability to do estimation, perception of estimation and tolerance for error, which were identified as affective factors, played important role for strategy selection and computational estimation.

Good number sense may lead to use of multiple representations of numbers and use of translation strategies. Moreover, mental computation ability may enable students both to conduct reformulation and use compensation strategy easily. Interviewees who had both high confidence in ability to do mathematics and low confidence in ability to do estimation, preferred exact computation and more rule dependent estimation strategies, like rule based rounding. Low tolerance for error may influence students' answers, in order to produce them in a narrow interval. Additionally, perception of estimation may lead students recognize estimation as useful and use of variety of computational estimation strategies.

According to data analysis, feelings and thoughts about computational estimation may influence interviewees' strategy usage, such as students, who had negative feelings on estimation and thoughts about mathematics wanted exactness, generally preferred exact computation process and did not use diverse computational estimation strategies. Students who had poor in number sense and mental computation could not conduct computational estimation strategies.

Therefore, the research study may lead to better understanding of students' perspectives on computational estimation. With understanding used strategies, and related factors are affecting computational estimation strategies, it might be produce effective instructional designs for teaching computational estimation.

Keywords: Mathematics education, Computational Estimation Strategies, Cognitive factors, Affective factors, Clinical Interview

ÖZ

YEDİNCİ SINIF ÖĞRENCİLERİNİN TAHMİNİ HESAPLAMA STRATEJİLERİ VE BUNA BAĞLI FAKTÖRLERİNİN İNCELENMESİ

Boz, Burçak

Doktora, Orta Öğretim Fen ve Matematik Alanları Eğitimi Bölümü

Tez Yöneticisi: Doç. Dr. Safure Bulut

Kasım 2009, 281 sayfa

Bu çalışmanın amacı yedinci sınıf öğrencilerinin hesaplamalı tahmin stratejilerini ve buna bağlı faktörleri belirlemektir. Beş öğrenci ile bir durum (örnek olay) çalışması yürütülmüştür. Ege bölgesinde bulunan bir ilköğretim okulunun 116 öğrencisi arasından bu beş öğrenci seçilmiştir. Her bir görüşmeci ile iki seans klinik görüşme yapılmıştır. Birinci görüşmede 15 maddelik Hesaplamalı Tahmin Testinin soruları sorulmuş ve çözüm aşamalarının açıklanması istenmiştir. İkinci görüşmede öğrencilerin tahmin etmeye karşı duygu ve düşüncelerini anlamak üzere araştırmacı tarafından hazırlanan yarı yapılandırılmış görüşme formu uygulanmıştır.

Çalışmanın sonuçlarına göre, öğrenciler sayıların yeniden yapılandırılması, işlemlerin yeniden yapılandırılması ve düzenleme ve düzeltme olmak üzere üç çeşit hesaplamalı tahmin stratejisi kullanmaktadırlar. Sayıların yeniden yapılandırıması tüm sayı çeşitlerinde bütün görüşmeciler tarafından en çok kullanılan strateji olmuştur. En gelişmiş ve karışık strateji olarak belirlenen düzenleme ve düzeltme stratejisi ise en az sıklıkta kullanılan strateji olmuştur. Diğer bir hesaplamalı tahmin stratejisi olan işlemlerin yeniden yapılandırılması, sorularla başedilebilmesi için işlemlerin değiştirilmesi anlamına gelmektedir.

Öğrenciler arasında sayı algısı iyi olanlar bu stratejiyi kullanmışlardır. Görüşme ve gözlemlere dayanarak belirlenen stratejilerle ilgili olarak bazı bilişsel ve duyuşsal faktörler belirlenmiştir. Sayı algısı ve zihinden işlem yapma bilişsel faktörün iki alt

kategorisidir. Bilişsel faktörlerin yanısıra, matematikte kendine güvenmek, matematiğe karşı algı, tahmin etmeye dair kendine güven, tahmin etmeye dair algı ve hataya karşı tolerans; hesaplamalı tahmin stratejilerini seçme ve kullanmada önemli rol oynamaktadır.

Sayısal algıya sahip olmak sayıların çoklu gösterimlerini kullanabilmeyi ve işlemlerin yeniden düzenlenmesini sağlayabilir. Hatta zihinden işlem yapabilme öğrencilerin hem sayıların yeniden düzenlenmesini hem de düzenleme ve düzeltme stratejilerini kullanabilmelerini sağlayabilir. Matematikte kendine güvenirken, tahmin etmede kendine daha az güvenen öğrencilerin, net hesaplamaların yanısıra, kurala dayalı yuvarlama gibi daha sıklıkla kural tabanlı tahmin stratejilerini tercih ettikleri görülmüştür. Hataya karşı düşük tolerans, öğrencilerin cevaplarının dar bir aralıkta olmasına etki edebilmektedir. Buna ek olarak, tahmin etmeye karşı algı, öğrencilerin tahmini yararlı bulmalarını ve değişik tahmin stratejileri kullanmalarını sağlayabilmektedir.

Veri analizine göre, görüşmecilerin strateji kulllanmaları onların duygu ve düşüncelerinden etkilenebilmektedir. Örneğin, tahmin etmeye karşı negatif duyguları olan ve matematiğin net cevaplar istediğini düşünen öğrenciler, genellikle net hesaplamalar yapmaya çabalamakta ve farklı tahmin stratejileri kullanamamaktadırlar. Zihinden heaplama becerisi ve sayı algısı kötü olan öğrenciler hesaplamalı tahmin stratejilerini kullanamamaktadırlar.

Bu nedenle, bu çalışma öğrencilerin hesaplamalı tahmine karşı bakış açılarını daha iyi anlamayı sağlayabilir. Kullanılan hesaplamalı tahmin stratejilerini ve bunlarla ilgili faktörleri anlamak, hesaplamalı tahmin üzerine daha verimli bir öğretim planlanmasına yardımcı olabilir.

Anahtar Kelimeler: Matematik Eğitimi, Hesaplamalı Tahmin Stratejileri, Bilişsel faktörler, Duyuşsal faktörler, Klinik Görüşme

To my family

Semiha, Salih & Mustafa Burak BOZ

ACKNOWLEDGEMENTS

I would like to express my appreciation to my supervisor Assoc. Prof. Dr. Safure BULUT for her thoughtful direction. Her encouragement and feedback were invaluable. I wish to express my sincere gratitude to Assoc. Prof. Dr. Erdinç ÇAKIROĞLU, and Assist. Prof. Dr. Ayhan Kürşat ERBAŞ for their suggestions and guidance. This dissertation could not have been completed without your supports.

I am grateful to my parents who provided moral support and encouragement throughout the process and never stopped believing in me. Thank you for your patience, optimism, and support. Especially I wish to thank my little brother Mustafa Burak BOZ for sharing all stress with me and encouraging me. I also wish to thank to my best friend, my cousin Özlenen ÖZDİYAR for her open-ended support, encouragement, and endless hopefulness.

I would like to thank students who agreed to participate in this research, school principals, and teachers.

I would like to thank my colleagues for valuable help during the data analysis part.

TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

LIST OF ABBREVATIONS

CET	Computational Estimation Test
MoNE	Ministry of National Education
EX	Excerpt
NCTM	National Council of Teachers of Mathematics

CHAPTER 1

INTRODUCTION

About thirty years ago, being able to perform paper-pencil calculations or mental calculations as quickly, neatly, and accurately was a valuable skill for society. Now, with advances in technology, the society needs people who can estimate the reasonableness of the exact answer they may obtain. Therefore, estimation has become important when determining reasonableness of an answer, particularly when using a calculator (Hope, 1986). Usiskin (1986) stated that the computation of single correct answer covers only a part of mathematics; the other problems require estimation. Moreover, it was suggested by Reys (1992, p.142), "over 80% of all mathematical applications call for estimation, rather than exact computation."

Although exact computation could be performed easily with the aid of computers or calculators, most of the time, it is not enough for making decisions. For instance, when someone says that "budget for colleges with 62 772 pupils is $148 309 563" may be less meaningful to us. On the other hand, when this statement redesigned as "colleges with about 63 000 pupils, has budget approximately $150 million" gives a clear understanding about the college's budget. Therefore, sometimes exact-solutions in mathematics may make the situation more difficult to understand. In daily life applications, approximation and estimation may help more than exact answers. Since these concepts are important for daily life situations, they should be taught at schools. Therefore, schools curricula should contain estimation as much as exact computation. The goals of mathematics curricula should prepare students to handle daily life problems. Although daily life computation can be made with the aid of calculators and computers, we should be able to use our brain before using these kinds of tools for simple calculations. As Maier (1977) stated, "Other computation tools may not always be available, but people always carry their brains with them" (p.47). Still someone may think that with developing technology why we need estimation or approximate calculation. According to Reys (1986), the current emphasis on estimation in the mathematics curriculum has been fostered by the widespread availability and use of technology. Results obtained with the help of technological devices should be evaluated and interpreted by a human. Therefore, students should be taught how to use estimation in their computation or judging the exact computation.

Many elementary school mathematics curricula (i.e., The Turkish Ministry of National Education-MoNE, NCTM, The Spanish Ministry of Education-MEC, The England Department for Education and Skills- DfES) make it clear that mathematics today is more than just computation. Mathematics students today are expected to learn estimation as a means of checking answers in computation and problem-solving situations where an exact answer may not be needed (Usiskin, 1986).

In Turkey, estimation has become an explicit part of the mathematics curriculum since 2005. Before 2005 renewed mathematics curriculum, not much attention was paid to estimation in mathematics classes. However, since that time, MoNE underlined that students should be encouraged to use mental computation and estimation in mathematics lessons. Additionally, in the current mathematics curriculum, it is emphasized that students should be trained not only to know estimation and its strategies, but also the importance of judgments on the choice of different estimation strategies appropriate for the situations (i.e., estimating distance in meter or kilometer; giving more or less precise computation results) and judgments on more reasonable estimated answers. MoNE (2005) particularly claimed that estimation as a tool improves students' reasoning ability and critical thinking ability.

Although, in Turkey estimation is a fresh topic, in the United States, interest in the topic of estimation is not recent. 1986 yearbook of NCTM was devoted entirely to estimation, which was discussed in different perspectives, like meaning of estimation, strategies, types of estimation, estimation in specific mathematics topics, etc. NCTM (2000) acknowledged that students should develop and adapt procedures for mental computation and computational estimation with fractions, decimals, and integers.

Mathematics educators and curriculum developers altered the countries' mathematics curricula (e.g., MoNE, MEC, and DfES) by incorporating estimation and related concepts into the mathematics topics since estimation is an important concept with many applications in a person's life. On one hand, it relates to many mathematical areas, such as geometry, numbers, probability and statistics, among the other mathematical domains, and it helps foster students' understanding of mathematical concepts. On the other hand, concepts involving estimation are an integral part of practical applications in such fields as commerce, and industry. Therefore, estimation is essential not only for developing mathematical proficiency among students, but also for ensuring their success in other disciplines.

Curriculum developers and mathematics education researchers agree on the importance of estimation. For instance Reys (1988) underlined the importance of estimation by stating that "one of the exciting benefits of teaching estimation is the opportunities it provides for individual thinking to occur (p. 29)." He has pointed out "estimation skills are essential and must be given high priority within every school program…only a few mathematical topics provide the wealth of benefits both immediate as well as long-term as does estimation" (1988, p. 41). According to Dolma (2002) having the ability of estimation can help students achieve some very important goals, such as valuing mathematics, being a confident problem solver, communicating mathematically, and learning to reason and discuss reasonableness in mathematics.

Researchers (e.g., Hanson, & Hogan, 2000; Munakata, 2002; Reys, Reys, & Penafiel, 1991; Sowder, 1992; Volkova, 2006) generally investigated the estimation under three main categories: measurement estimation, numerosity estimation and computational estimation. Measurement estimation is related with the length, weight, or time of estimation in daily life applications. Numerosity concerns the amount of quantity, for example, the number of boxes in the storage. The last one is the computational estimation, which concerns the approximate computations. Although each of them has many applications in real life, there are more research studies conducted on computational estimation than other types of estimation.

The research studies on computational estimation have investigated many aspects of this type of estimation. For instance, most of the researchers (e.g., Baroody & Gatzke, 1991; Bestgen, Reys, Rybolt & Wyatt, 1980; Berry, 1998; Boz, 2004; Case & Sowder, 1990; Goodman, 1991; Reys, Reys, & Penafiel, 1991) were interested in general achievement levels of computational estimation in any age groups. Some of the researchers (e.g., Bestgen et al., 1980; Cilingir & Turnuklu, 2009; Rubenstein, 1982; Sowder, 1992; Goodman, 1991) investigated the achievement levels of computational estimation according to the formats of the question. Moreover, some other researchers (Blair, 2001; Goodman, 1991; Rubenstein, 1982, 1985) investigated computational estimation performance according to types of questions (i.e., multiple choices, reference number, open-ended, order of magnitudes). Likewise, considerable amount of attention is given to subjects' performance of computational estimation on specific topics (i.e., whole numbers, fractions, decimals, percents) of mathematics by the researchers (Bobis, 1991; Goodman, 1991; Hanson & Hogan, 2000; Reys, Reys, Nohda, Ishida, & Shimizu, 1991; Reehm, 1992; Rubenstein, 1985; Volkova, 2006).

With the hope of contributing to the knowledge of computational estimation strategies, and students' strategy use, this study will include the analysis of computational estimation strategies and factors associated with them. To identify the students' strategies and thinking process, qualitative research design is used. It facilitates the investigation of complicated thinking processes rather than just the end products (Bogdan & Biklen, 1992; Strauss & Corbin, 1990). Two interview sessions, classroom observations, and teachers' interviews are conducted to understand the students' thinking procedures and thoughts on computational estimation, usage of strategy and related factors, which are affecting students' preferences of the strategies.

1.1 Research Problems

In the study, two main problems and sub problems are examined. These are stated as follows:

P 1. Which strategies do seventh grade students use in computational estimation tasks?

SP 1.1 Which strategies are used in computational estimation tasks in whole numbers?

SP 1.2 Which strategies are used in computational estimation tasks in decimals?

SP 1.3 Which strategies are used in computational estimation tasks in fractions?

P 2. Which factors are associated with computational estimation strategies of seventh grade students?

SP 2.1 Which cognitive factors are associated with computational estimation strategies?

SP 2.2 Which affective factors are associated with computational estimation strategies?

1.2 Definition of the Terms

Computational estimation refers to the process of simplifying an arithmetic problem using same set of rules or procedures to produce an approximation but satisfactory answer through mental calculation (LeFevre, Greenham, & Waheed, 1993, p 95).

Mental computation is defined as "the process of carrying out arithmetic calculations without the aid of external devices" (Sowder-Threadgill, 1988, p. 182).

Number sense refers to a person's general understanding of numbers and operations and his ability to handle daily-life situations that include numbers (Yang, 2009, p.93)

In the current study, ***estimation strategies*** refer to reformulation, translation, and compensation.

Reformulation means the process of altering numeric data to produce more mentally manageable form (Heinrich, 1998, p. 15).

Rounding is a kind of reformulation strategy. Rounding means the process of changing a number with more manageable one, which is the nearest desired place value. For instance, 47 rounded to 50.

Truncation is a type of reformulation strategy. Truncation strategy could be performed by changing the number with a lower form of itself. For example, 47 might be truncated to 45; 3.54 might be truncated to 3.

Compatible numbers strategy refers to a set of numbers that can be easily "fit together" (i.e., are easy to manipulate mentally) (Reys, 1986, p.41).

Translation refers to process of mentally changing the mathematical structure of the problem to a more mentally manageable form (Reys, Reys, & Penafiel, 1991, p.353).

Compensation refers to process of altering numeric data to produce a more mentally manageable form. This strategy is also divided into two sub methods; these are final compensation and intermediate compensation. The first one is adjusting an initial estimate to more closely convey the user's knowledge of the error introduced by the strategy employed. The second one is adjusting numerical values prior to their being operated to systematically correct an error (Reys, Reys, & Penafiel, 1991, p. 354).

Tolerance for error refers to feeling comfortable with some pay off and not disturbed with approximate solutions. In their study, Reys et al. (1980) tried to explain error tolerance as follows:

> A knowledge of the meaning and intent of estimation was found to permeate the thinking of good estimators. This understanding of the concept of an estimate enabled them to be comfortable with some error. They frequently noted the importance of an efficient, reasonably accurate computational tool and felt that their ability to estimate filled this need. In other words, they saw estimation as an important tool when dealing with numbers and did not see themselves as being "wrong" when using estimates. (p.198)

According to quoted passage from the study of Reys et al. (1980), tolerance for error means not being disturbed with pay off estimated results. The operational definition is that feeling comfortable with pay off while giving estimated answers. When someone reacts to a computational estimation, if he/she hesitates about giving not exact result, this person has not tolerance for error. On the other hand, if someone has tolerance for error, it means that (s)he feels not being disturbed by this vague result.

SBS refers to an exam conducted at the end of each year of the secondary school. The Ministry of the National Education conducts the exam entitled "The Exam of Determination of Level (SBS)." The results give achievement level of students, classes, schools, regions and cities of Turkey. The other reason for applying the exam is to determine which high school the students will attend. In Turkey, there are many types of high schools, some of which give technical education, some science education, and so on. Students choose their schools according to both their interest and their SBS scores. The exam consists of questions, which are related with the each grade's courses given at secondary school period. These are Mathematics, Turkish, English, and Science.

Dersane refers to a private institute where students are prepared for the exams. Since SBS exam requires a kind of competition among the elementary school students, they feel that they have to enroll in a "dersane" to get higher scores from the SBS.

Computational Estimation Test (CET) refers to 15 item open-ended computational estimation ability test. The test was prepared in parallel forms as both numerical and word formats and used in whole class application session.

Pay off is used as an amount of distance between exact answer and estimated one in the current study. For example, if exact answer is 37 of an operation and someone obtained 50 as an estimated answer, 13 is pay off of the operation.

Whole class application session represents the procedures involved in the first part of the study. This part includes two measurements. In these measurements, word format and numeric format of CET were applied to classes A, B, C and D. According to these test scores, the interviewers were selected.

Multiple Representations of the numbers is used in the current study as converted version of the numbers; fraction to decimal or decimal to fraction. For instance, 0.5 can be used as $\frac{1}{2}$, or $\frac{3}{4}$ can be used as 0.75 in the questions. When a student uses these conversions, it is said that student can use the multiple representations of the numbers.

1.3 Significance of the Study

Although MoNE (2005) put into practice the new elementary mathematics curriculum, which, consisting of the computational estimation and measurement estimation in 2005, computational estimation has not been widely investigated by researchers in Turkey. MoNE (2005) were confirming that computational estimation was valuable ability, which should be improved through elementary school to higher education. It was also emphasized that measurement estimation should be taught to students in order to train the ability of measure of students without using any standard tool. While using measurement estimation, computational estimation is also used in order to obtain estimated measures by computing approximately. Therefore, even using measurement estimation, computational estimation process could be used.

In addition, research studies show that computational estimation is essential concept for mathematics education but there are few research studies addressing this concept. In those studies, achievement levels of students and strategies used during the estimating processes are mainly discussed. However, there are few research studies, which look at the reasons for choosing estimation strategies, and factors affecting those strategies of students. These factors associated with the computational estimation should be deeply investigated to understand students' strategy use and achievement on computational estimation.

Case (1996) claimed that computational estimation is a complex construct where a child tries to accomplish more than one task at a time during the estimation process. The constructs underlying of estimation skill may dependent on two-way explanations, which are cognitive and affective perspectives. Those constructs have not so far clearly emphasized as potential variables, which may contribute to students' understanding and achievement of computational estimation, number sense, and mental computation. Aiken (1976), and Ma and Kishor (1997) agreed that none of the cognitive constructs is free from feelings and thoughts. Therefore, computational estimation should be investigated through both affective and cognitive factors. For the reasons already discussed, it can be confirmed that there is a necessity for specifying the role of factors related with computational estimation strategies. Hence, current study will attempt to fill the gap in literature related with the topic. Some researchers have been interested in cognitive components of the computational estimation (Case, 1996; Crites, 1992; Dowker, 1992; Goodman, 1991; Rubenstein, 1982; Sowder, 1992; Volkova, 2006). These cognitive components were tested either statistically (Rubenstein, 1982) or observed by interview sessions (Reys, Reys, & Penafiel, 1991; Reys, Reys, Nohda, Ishida & Shimizu, 1991). A few of the studies identified that computational estimation include affective factors, too (Hogan, & Parlapiano, 2008; Hogan, et al., 2004; Sowder, 1992; Reys, et al., 1982). Some of the researchers conducted statistical investigations (Hogan, et al., 2004; Hogan, & Parlapiano, 2008), and some of them conducted qualitative inquiries (Reys, Rybolt, Bestgen, & Wyatt, 1982; Sowder, 1992).

The understanding of computational estimation strategies is important in mathematics education. The reviewed literature suggests that diversity of computational estimation strategies, students use enable us to better the students' conceptual knowledge about the computational estimation (e.g., Reys, Reys, Nohda, Ishida, & Shimizu, 1991; Reys, Rybolt, Bestgen, & Wyatt, 1982; Sowder, 1992). Since there is not much research examining students' computational estimation strategies in our country, this research may have implications for teachers and curriculum developers in order to improve students' computational estimation skills through teaching and use of variety of strategies. This research study is also important since knowing and understanding the reasons for students' choosing and using of strategies may contribute to the development of positive feelings and better understanding of computational estimation and mathematics (Bestgen, Reys, Rybolt, & Wyatt, 1980). Being able to understand what factors are related with the use of computational estimation strategies by students may lead us to produce effective instructions by which students can be trained as powerful estimators.

One additional contribution of this research to the related literature is that it identifies factors associated with the computational estimation in Turkish education context. Though existing research studies have already put forward some universal factors that are influencing students' use and selection of computational estimation strategy (Reys, Rybolt, Bestgen, & Wyatt, 1982), this study revealed additional factors based on our educational system requirements and culture. These differences based on culture and educational systems may guide us to better understanding of students' achievement on computational estimation and may give us opportunity to produce more effective instructions about the topic.

The existing literature about the computational estimation focused on testing the different age groups for understanding the achievement level on computational estimation (e.g., Bana, & Dolma, 2004; Berry, 1998; Hanson, & Hogan, 2000). However, related literature confirmed that testing procedure is not an appropriate way to asses the computational estimation achievement of the students since it is too complicated to understand whether students estimate or compute the questions exactly (e.g., Rubenstein, 1982; Sowder, 1992). Dowker (1992) stated that observing students' estimation strategies might provide information not only about estimation itself, but also about people's understandings of estimation and feelings about the computational estimation. Therefore, in the current study, clinical interview sessions are designed to investigate how could students estimate and how they could produce computational estimation strategies.

Because of aforementioned reasons, the present study paid attention to understanding of the strategies of computational estimation and factors that are associated with computational estimation. The current research study has an expected valuable contribution to mathematics education by expanding the knowledge base about the computational estimation.

1.3.1 Personal Significance of the Study

As a researcher, I have been interested in computational estimation since 2001. I have realized that there are a few research studies on estimation both in my country and abroad. The conducted studies reveal that students and adults have poor estimation performance. In 2002, I conducted a study with preservice elementary mathematics and science teachers and kindergarten teachers. The results showed that they could not obtain estimated solutions and even they did not know what estimation is. I observed that preservice teachers preferred exact solutions to the estimation of the problems. Mathematics and science preservice teachers were more successful than kindergarten teachers. According to unrecorded observations and interviews with the subjects presented to me, the pre-service teachers had negative feelings about estimation. These observations made me curious about the feelings and thoughts of subjects on estimation.

Two years later, I had completed my master thesis, in which I aimed to understand computational estimation performance of ninth grade students. Although time was restricted to make students estimate the answers rather than compute exactly, some of them could produce exact answers. According to unrecorded observations, since students had high mental computation skills, and did not have conceptual knowledge about estimation, they preferred exact computation.

In both experiences, I have observed that there are some factors, which might be affective and cognitive, related with the students' estimation performance and use of strategies. Therefore, in the current study, I investigate these factors, which are associated with computational estimation strategies. For this purposes, first, I should identify the strategies and then should specify the related factors

CHAPTER 2

REVIEW OF RELATED LITERATURE

Four fields of research literature are important to the background of this project. These are studies on estimation, the research studies on number sense, the research literature on mental computation and reviews of clinical interviews.

The first section of this chapter examines the studies on estimation and related concepts. This section is divided into three sub contents; these are the general overview on estimation studies where it is aimed to give a top view for estimation, the strategies used for estimation questions, which are the main concerns of the current study, and the components of the computational estimation ability, where categorized in affective and cognitive components.

In the second section, it is summarized the studies on number sense and related concepts. The relations between number sense and computational estimation are presented through the research studies. Then brief literature reviews are given about mental calculation and place of mental computation in the computational estimation processes in the next section.

Finally, the last section examines the research literature relevant to clinical interview that conducted on the mathematics education studies. The clinical interview history on mathematics education and how might be used are examined in the last section of this chapter.

2.1 Estimation Placed in Mathematics Curricula

Since this section aimed that giving a general view of research studies on estimation, it is started with the definition of the estimation. Then importance of the estimation ability is tried to explain through findings of research studies. In addition, there are some research studies about the computational estimation placed in mathematics curriculum of different countries. Particularly, the mathematics curriculum of Turkey is examined, since it has been revised and estimation ability is embedded into it in the recent years. Through the section it is examined some research studies which are served the types of the estimation abilities. The remaining of the section contains some studies on computational estimation, which are related with following answers of the questions:

- What are the successful levels of the subject on different age groups (kindergarten, elementary, secondary, higher education, or adults)?
- In which types of operations, or numbers the achievement level is low or high?
- How does the content of the questions or type of the questions affect the subjects' achievement level?
- Whether this ability could be improved.

Estimation is a critically useful skill in everyday life and in mathematics. The researchers claimed that estimation is important skill in three perspectives, first, it is used for more often than paper-pencil skills in everyday life; then, it is particularly important as both adults and children do more work with calculators and computers; and lastly, it is the way to check the reasonableness of results are vital (Glasgow, 1998; Reys & Reys, 1998; Star & Rittle-Johnson; 2009; Suydam, 1985).

For many years, educators and curriculum developers emphasize the importance of estimation in mathematics education. Especially reports of National Council of

Teachers of Mathematics (NCTM, 1989) and conducted nationwide studies (National Assessment of Educational Progress-NAEP) emphasized that estimation and computational estimation abilities critical topics for mathematics education and students had difficulties to perform this ability (Carpenter, Coburn, Reys & Wilson, 1976). Although the importance of the topic underlined by researchers, some countries' curricula (e.g., Kuwaiti, Mexico) still do not contain estimation as a topic or a concept embedded into other topics (Alajmi, 2009; Reys, Reys, & Penafiel, 1991). Alajmi (2009) reported mathematics teachers' understanding of the meaning of computational estimation and their views about its significance in the elementary and middle school curricula in Kuwait with underlying that the computational estimation has not yet established a place in the Kuwaiti national curriculum. Similarly, Reys et al. (1991) specified that the fifth and eighth grade students' low computational estimation ability with emphasizing they were not taught the estimation topics in any education level of the school life. Nevertheless, some of the countries (for example, Spain and Turkey) had been conducted the estimation as an explicit component of curriculum not for a long time ago. Segovia and Castro (2009) reported that estimation became an explicit part of curriculum plans for Primary Education and Secondary Education since mid 1990s. In Turkey, the MoNE put into practice the new mathematics curriculum with estimation ability applications in 2005. MoNE (2005) emphasized the elementary education (grades from 1 through 8) students' number sense, attitudes towards mathematics and mathematics achievement can be improved through estimation and mental computation. In the renewed curriculum in Turkey, estimation is underlined not only in algebra but also in measurement process of the mathematics (MoNE, 2005). Especially, it is regarded that the strategy using and producing of students both in computational estimation and measurement applications.

Similar to MoNE (2005), in the United States, both in 1977 and in 1989, the National Council of Supervisors of Mathematics recommended that estimation should be a part of the mathematics education since students be able to judge whether a calculation is reasonable or not. Estimating the solutions to the problems was recognized as one of the most useful topics in mathematics education (Trafton, 1986) and through out the 1980s estimation including as a topic in mathematics curriculum in USA (Mottram, 1995). According to NCTM (1989), estimation accepted as a standard for mathematics education for elementary, middle and secondary school students:

> Students should be able to carry out rapid approximate calculations through the use of mental arithmetic and variety of computational estimation techniques. When computing is needed in a problem or consumer setting, an estimate can be used to check reasonableness, examine a conjecture, or make a decision (students) should be able to decide when a particular result is precise enough for the purpose at hand. (p.8)

NCTM (2000, p.155) acknowledged the role of estimation "as an important companion to computation" and as "a tool for judging the reasonableness of calculator, mental and paper-pencil computations." Furthermore, NCTM (2000, p.78) stated that "instructional programs from kindergarten through grade twelve should enable all students to understand numbers, understanding meaning of operations, and how they relate to one another, compute fluently and make reasonable estimate."

Although in recent years, many countries' (e.g., USA, Spain, Turkey) mathematics curricula contain estimation and it is known as very important concept in mathematics, researchers are not paid attention as other mathematical topics (Sowder, 1992; Reys, Reys, & Penafiel, 1991). When the conducted research studies are investigated, it is observed that in generally estimation studies are made in three types of estimation categories. The related research literatures on these categories are presented the next section.

2.1.1 Types of Estimation

Numerosity estimation, measurement estimation and computational estimation are the types of estimation, which are defined and accepted most of the researchers (e.g., Hanson & Hogan, 2000; Munakata, 2002; Reys et. al., 1991; Sowder, 1992). Besides these three kinds of estimation there are some other forms of estimation, for example, Smart (1982) described estimating trigonometric functions, and estimating numerical values of derivative for a graph of a function. Additionally, probability and statistics are other areas where estimating will be useful and can lead to better understanding. However, these forms are not concerned as type of estimation; therefore, the three types are mainly discussed in the mathematics education.

Among the three types estimation, computational estimation has been the most frequently studied while the research literature on numerosity estimation and measurement estimation remain sparse (Sowder, 1992; Munakata, 2002; Volkova, 2006, Hanson, & Hogan, 2000). Through the current section, these three types of estimation are reviewed. After reporting some information on numerosity and measurement estimation briefly, computational estimation will be deeply concerned. Although numerosity and measurement estimation is not concerned of the current study, the reporting of the research studies about them is aimed to clarify the distinction and similarities among the estimation types.

Numerosity is defined as estimating the number of objects, usually dots in an array (Hanson & Hogan, 2000; Sowder, 1992). The answer of the question “approximately how many” gives an approximate number of the items in a set which is called the numerosity estimation. In many cases approximate number which is estimated sufficient perhaps even more reasonable and usable than the exact numbers. For instance, the questions “how many watchers are there in the stadium, how many bean are there in a jar, how many cars are there at the parking lot, how many books are there on a library self, etc." is asked for not exact numbers, the approximate one enough for making judgments.

Although numerosity is usable in many areas in daily life, a few studies conducted on it (Montague & Garderen, 2003; Baroody & Gatzke, 1991; Crites, 1992; Hogan & Brezinski, 2003; Siegel, Goldsmith, & Madson, 1982). The most inclusive study of all conducted up to now was Baroody and Gatzke's (1991) research. They investigated the ability and used strategies of gifted students by qualitative inquire. It was provided an excellent history research on numerosity estimation. In the research, they interviewed 18 potentially gifted kindergarten children about their ability to perform three tasks:

(a) Estimation tasks, where children were to estimate the number of dots in a set

(b) Number-referent task, where children decided whether a set of dots was larger or smaller than given reference numbers

(c) Order-of-magnitude task, where children decided where a set of dots fit in relation to two reference numbers.

According to the result of Baroody and Gatzke' study (1991), a majority of the children was successful on the number-referent task, but the performance of students varied about magnitude task. The researchers concluded that according to type of tasks, students' successes were changed. Moreover, Montague and van Garderen (2003) examined the different ability levels of the students' numerosity estimation. They examined that the fourth, sixth and eighth graders' numerosity estimation, relationships among the mathematics achievement, estimation skills, and academic self-perceptions. Despite the differences among the ability groups, it was evident that all students did quite poorly on the estimation test. When compared with the other ability groups, the intellectually gifted students significantly performed better on estimation measures. However, they still did not perform well when their overall percentage correct was calculated. The researchers concluded that estimating discrete quantities correlated with more than general intellectual ability and mature number sense to the acquisition of basic math skills (Montague & Van Garderen, 2003).

O'Daffer (cited in Hogan & Brezinski, 2003) was the first author to distinguish explicitly between numerosity, measurement, and computational estimation. However, in some research studies numerosity estimation embedded in measurement estimation (e.g., Hogan and Brezinski, 2003). Such as, Hogan and Brezinski (2003) served numerosity and measurement estimation in a unique form of estimation skill, which was separated from computational estimation and it was concerned that a general mathematical ability. They performed a research with 53 undergraduate in a Fundamental of Psychology course of a university through the five tests. Participants completed five tests: number facility, quantitative reasoning, computational estimation, measurement estimation, and numerosity estimation. The principal components analysis was applied to identify the components loadings and correlations among the components. As a result, the researchers concluded that numerical facility, computational estimation, and quantitative reasoning factors loaned in a factor, which was called as general mathematical ability. On the other hand, measurement estimation and numerosity loaned in another factor, which was thought that there should be relation with spatial ability by the researchers.

Some other researchers made clear distinction among the three types of the estimation (e.g., Schoen & Zweng, 1986; Sowder, 1992). For example, Schoen and Zweng (1986), in the preface of the NCTM 1986 Yearbook, distinguished between numerosity, measurement, and computational estimation. Moreover, Sowder (1992) adopted this many-sided distinction in her comprehensive summary of research on estimation with claiming that the skills required by these tasks were different from each other. Even though the strategy of numerosity and measurement estimation is the same, which is the "benchmark" strategy, the context of both estimation types is different from each other. Sowder (1992) explained the different skill requirements of each type as follows:

> Estimating result computations, estimating measures, and estimating numerosity……[e]ach requires different kinds of understandings and different sets of skills…..Estimating measures and estimating numerosity call upon some of the same skills….. Estimating the length of [a] tile, however, calls for a very different type of skill than estimating numerosity (p.371).

Other type of estimation, which has very similar strategy (that is benchmark strategy) with numerosity, is "measurement estimation." This type of estimation contains everyday situations such as the weight of a typical car, the length of the time for a normal adult to walk a kilometer. There are some studies on both types of estimation like the study of Siegel, Goldsmith, and Madson (1982). The researchers studied on both numerosity and measurement estimation with respect to the strategies of second through eighth grade students (Siegel, et al, 1982). The researchers tried to assess developmental differences in estimation strategies of the children on these types of estimation. In contradiction to researchers Crites (1992), Montague et al. (2003) and Mottram (1995); Siegel et al. (1982) stated that there was a weak relationship between accuracy in estimation and used strategies. They also found age differences for measurement estimates. According to result of the study, the more grade-level the more different sophisticated estimation strategies in both numerosity and measurement estimation (Siegel et al., 1982).

Likewise the findings of Montague and van Garderen (2003) on numerosity, Taylor, Simms, Kim and Reys (2001) stated that students were poor on measurement estimation. In order to results of Trends in the International Mathematics and Science Study (TIMSS), Taylor, et al.,(2001) investigated why American third- and fourth-grade students scored lower than the international average in the measurement estimation and number sense. The surveys were distributed to 110 students to inquire about the use of estimation on metric measurement in the classroom. The researchers served some recommendations to improve the students' measurement estimation ability, for example according to researchers teachers should help the students to produce own strategies and benchmarks for estimation.

While Taylor et al. (2001) discussed the conducted surveyed on measurement estimation, Forrester and Pike (1998) dealt with the same topic with conducting a different research method. The researcher conducted a conversation-analytic approach with classroom observation to identify children and teachers' acts on measurement estimation topic. They concluded that the significance of rough measurement concerning estimation was clearly evidenced in the children's activities although they didn't find explicit instructions or using a nonstandard measuring tool in any teachers' talk.

In order to results of these studies researchers agreed that for students to acquire skill in estimation, they must have practical experiences in making measurement estimates so that they can develop their own individual frames of reference for estimating the quantity of various types of measurement such as weight, time, length (Forrester & Pike, 1998; Sowder, 1992; Taylor et. al., 2001). Crites (1992) stated that to improve the measurement and numerosity estimation ability of students, several suggestions could be made. These are:

- Some opportunities should be provided for students to develop their own benchmarks.
- Students should observe their teachers while teachers' make use of benchmarks and the benchmark and decomposition-recomposition strategies to estimate discrete quantities.
- Students can develop their own estimation skills by frequently making estimates in practical-application situations.

The third type of estimation is the computational estimation. Because a universal definition does not exist for the computational estimation, every researcher defines the concept in his or her own style. Dowker (1992) defined computational estimation as making reasonable guesses as approximate answers to arithmetic problems, without or before actually doing the calculations. Reys, Rybolt, Bestgen, and Wyatt (1982) produced the most popular definition, which was referred in many studies;

> The interaction of mental computation, number concepts, arithmetic skills including rounding, place value and mental compensation that rapidly and consistently result in answers that are reasonably close to a correctly computed result. This process is done internally without the external use of a calculating or recording tool (p. 307).

As seen from the definition of computational estimation, different from measurement estimation and numerosity estimation. One of the earliest works in the area of estimation dealing with both measurement estimation and computational estimation was Paull's (1971) doctoral dissertation. He tested 196 pupils, aged sixteen, in college preparatory classes from an upper middle class community. One of his conclusions was that the ability to estimate is not a unitary ability. In other words, estimation tasks of different types (i.e. measurement estimation and computational estimation) appear to require different abilities and there did not appear to be an easy transfer of ability between the various estimation tasks. Therefore, computational estimation considered as a unique construct in the following studies.

Rubenstein (1982) stated that computational estimation is the finding of an approximate answer to a one-step verbal or numerical arithmetic exercise involving whole or decimals without the use of calculating or recording tools, using computation, arrived at quickly, and producing an answer adequate to make necessary decisions. In the current study, it is focused on computational estimation, which is defined as the process of mentally generating an approximate calculation for a given arithmetic problem (Rubenstein, 1985).

In his study Heinrich (1998) explained that the computational estimation is a multistep process performed mentally, which requires that a number be rounded off and then used to calculate an answer using one of the four basic mathematical applications of addition, subtraction, multiplication or division. Sowder (1992) confirmed the complexity of the computational estimation process and specified the computational estimation as stating that performing some mental computation on approximations of the original numbers. When calculating the approximate answers it should be considered that the correctness of the results as the answer must be fall within a certain interval, as determined by the problem itself or some outside source.

It can be seen from the definitions above that the set of mental arithmetic skills, approximations, and reasonableness of the results are intersecting of computational estimation. Since computational estimation is a complex ability, the success in this ability is rare within different groups of age levels. For instance, Goodman (1991) conducted a computational estimation study with preservice elementary school teachers and concluded that they had relatively low achievement on the estimation items. Among the studies with small age groups, Rubenstein (1985) produced a study to identify the computational estimation achievement within several dimension with three hundred eight graders. She specified that generally in all dimensions (open-ended, order of magnitude, reasonableness vs unreasonableness and reference number) students had low performance but in specifically the worst performance of the students on the open-ended type of the computational estimation questions. Siegler and Booth (2005) served that surprisingly even adults are far from good at it. However, Dowker (1992) examined estimates of four groups of adults who were mathematicians, accountants, psychology students and English students, and among these adults she concluded that mathematicians and accountants had good estimation abilities with a notable accuracy. Although the researcher tried to find out what strategies were used rather than how good or bad performance at estimating, except from the mathematicians and accountants, the other subjects were relatively bad at the estimating the arithmetic problems. According to the research studies age had a strong relationship to estimation performance (Case & Sowder, 1990; LeFevre, Greenham & Waheed, 1993; Sowder & Wheeler, 1989). Case and Sowder (1990) tried to build a developmental model of the concepts and processes involved in one of computational estimation in the study, which was conducted with twelve children at each grade K, 2, 4, 7, 9, and 11, 12. They concluded that the computational estimation performance could be improved through the age levels grower. Similarly, LeFevre et al. (1993) observed the difference among the fourth, sixth, eighth graders, and adults' computational estimation performance. The researchers concluded that older students produced estimates that were closer to the exact answer than younger students. According to LeFevre and colleagues (1993), arithmetic skill contributed to solutions' accuracy, and more complex problems were solved less accurately than simpler problems. Confirming to findings of the results of the studies of Case and Sowder (1990), and LeFevre et al. (1993); Sowder and Wheeler (1989) conducted a study with forty-eight students to understand performance on computational estimation. The researchers gave tasks that presented problems with solutions from hypothetical students, to the twelve subjects in grades 3, 5, 7 and 9 individually and asked them to contrast and compare the solutions (Sowder &Wheeler, 1989). According to findings, the older children understood better than the younger children what was asked but were uncomfortable with estimation processes and outcomes. It was stated that according to maturation of the subjects estimation skills might mature over time too. Moreover, Bestgen, Reys, Rybolt, and Wyatt (1980) found definite trend of improved performance in computational estimation from grade 7 to adult. As a contradiction of these findings, Reys, Reys, and Penafiel (1991) explained that there was no significant grade level difference between fifth and seventh grade students on computational estimation performance. According to NAEP reviewers (Carpenter, Coburn, Reys, & Wilson, 1976), the results showed that young adults could estimate much better than 17 year olds. By contrast, Forrester and Pike (1998), who studied on the measurement estimation with age 9-11, found that age did not affect length and area estimation.

2.1.2 Types and Contents of Computational Estimation Questions

Some of the studies identified in specific topics students are unsuccessful on estimation-required problems (Bestgen, Reys, Rybolt & Wyatt, 1980; Goodman, 1991; Hanson & Hogan, 2000; Levine, 1982; Rubenstein, 1985). Rubenstein (1982) investigated to eighth graders' estimation ability by developing an instrument and stated that students had found items on decimals were more difficult than items on whole numbers. She added that division is the most difficult operations, and then multiplication is the second difficult operation among four type operations. Bestgen et al. (1980) in a study with 187 preservice elementary teachers enrolled in one of two mathematics preparatory courses found that they did better on addition and subtraction problems involving estimation than on multiplication and division problems. They were also more successful with whole number estimation problems than with decimals, which was consistent with that of findings of Rubenstein's (1982) study. Estimation with decimal number was proved more difficult than estimation with whole numbers, which was a finding of both studies of Rubenstein (1985) and Bestgen et al. (1980). Giving the similar results, Levine (1982) conducted a study with undergraduate students, and suggested that estimating multiplication and division of whole numbers was difficult tasks for college students who were not mathematics majors particularly those of low quantitative ability. Additionally, Hanson and Hogan (2000) asserted that undergraduate students performed poorly on multiplication of decimals subtraction of fractions and division of decimals.

Researchers agreed that students had difficulty more on the questions related with fraction and decimal than whole numbers (Reys et al., 1991; Goodman, 1991; Hanson & Hogan, 2000). Reys and his colleagues (1991) pointed out that students' inability to estimate fractions may have more to do with not understanding the concept of fractions than with lack of estimation ability. This reason confirmed also by Hanson and Hogan (2000) and Carpenter and his colleagues (1976) who claimed that the low performance on fraction and decimal might reflect the lack of deep understanding of these concepts rather than inability to solve problems involving fractions and decimals. Similarly, Boz (2004) found that the ninth grade high school students had difficulty on fractions and decimals where the findings of Goodman's (1991) study were confirmed. Additionally, Reys and Bestgen (1981) reported that students had difficulty mostly on sum of three decimals estimation according to their findings.

It was claimed that the problem with fraction was founded to have relationship with the conceptual knowledge of fraction and Bobis (1991) served an alternative way to solve this problem. The study reported by Bobis (1991), it was claimed that when fraction concept first introduced with estimation applications the problems might be solved. In the study, it was conducted an experimental inquiry and obtained statistically significant results on 101 fifth grade boys from two primary schools. She observed that the most difficult obstacle for students during the estimation was that to overcome the reliance on paper-pencil techniques. However, she concluded that the conceptual problems did not appear when the teaching process was redesign with estimation. As a result, she suggested that when a new material like fractions should be introduced by way of estimation strategies to improve the students' achievement.

In some other research studies the low performance of estimation was discussed in different perspectives, such as types and context of the estimation questions (Blair, 2001; Mitchell, Hawkins, Stancavage, & Dossey, 1999). The results of three NAEP assessments cycles (1990, 1992, and 1996) have shown poor results in estimation (Mitchell et al., 1999). According to the results, students' errors in computational estimation problems seemed more commonly result from misinterpretation of problems more than errors in estimation strategies or mental computation (Blair, 2001).

It can be said that the assessment procedure is important in computational estimation studies, and many researchers addressed the difficulties of assessment of estimation performance (e.g., Goodman, 1991; Dowker, 1997; Rubenstein, 1985). Goodman (1991) assessed the preservice teachers' computational estimation performance with questions in three formats, reference number questions, open-ended questions and order of magnitude questions. According to findings of the study open-ended questions found more difficult than other types. Similarly, Rubenstein (1985) conducted four different types of estimation question tests, which were the questions served in open-ended estimation scale, reasonable vs unreasonable estimation scale, reference number estimation scale, and order of magnitude estimation scale, to understand the achievement difference among these types of question. In examining tasks within these four types, she found that tasks presented in open-ended estimation scale were more difficult that tasks presented in other forms. Goodman (1991) and Rubenstein (1985) findings show agreement on the conclusions that is students performed less successful on open-ended questions than reference number questions. However, Boz (2004) claimed a contradictory result according to students' achievement on open-ended, order of magnitude, and reference number questions in her study. She claimed that the subjects of her study performed more successful on reference number questions than open-ended questions. She argued the findings according to subjects' lack of regular instruction on estimation. According to researcher (Boz, 2004) one reasonable explanation of the unexpected finding was that students' dependency of exact answers and their high computational ability, which was performed on open-ended questions. However, the questions in the reference number category, there were two options (yes-no) significantly most of students did not try to estimate; they only made up an answer and passed the other question.

Estimation related questions were designed not only according to aforementioned four formats (open-ended, reference number, reasonable vs unreasonableness, and order of magnitude) but also designed according to the context, which were in application format and numerical formats. There are some amount of studies, which are discussed the achievement differences between these two kinds of the questions' formats where the numerical format served the problems in numbers, and the application questions served in the word formats (e.g., Gliner, 1991; Reys et al., 1982; Rubenstein, 1983).

The study reported by Gliner (1991) served the computational estimation performance of 141 elementary education students. The researcher tired to understand students' performance on estimation problems involving various operations and types of numbers, which are presented in word problem and computational (numerical format) formats. He stated that the word format (application format) of the estimation performance was greater than the number only format (numerical format) of the estimation tasks. As giving the similar results, Goodman (1991) and Morgan (1990) stated that numbers only format's questions were more difficult than application format's questions for the subjects. Although this result also confirmed by Reys, Reys and Penafiel (1991) and Bestgen et al. (1980); in her study Rubenstein (1985) disagreed with them and claimed that there was no difference between the types of questions (word and numeric formats). Goodman (1991) discussed this contradictory result by pointing out age level of the subjects. The subjects of study conducted by Bestgen et al. (1980) and Goodman (1991) were preservice teachers who were get used to application items in their everyday situations, on the other hand, Rubenstein (1985) studied on eighth graders who were relatively familiar to daily situations as estimation used.

As a different perspective, Reys et al. (1982), who were disagreeing with Rubenstein (1985), claimed that estimation items in context were easier than not in context form. To explain the reason of this disagreement, Rubenstein (1985) noted that her sample included average students where in Reys' (1982) study the subjects were above the average achievement. The difference in samples could cause the difference in results, but one would think that average students would find contextual problems more meaningful and therefore easier to compute. Students with above average mathematical skills should be able to estimate solutions to problems with little difficulty regardless of the problem format.

To understand the difference of students' computational estimation achievement on word and numeric formats, Reehm (1992) also conducted a research study with 238 eighth grade students by using parallel forms of word problems and numeric problems that required open-response answers. The researcher randomly selected fourteen students in each performance level (low, middle, high) and asked them to ten estimation questions. She found that students of higher ability performed better when estimating answers to word problems, and students of average and lower ability performed better when making estimates answers to numerical problems.

The reason of the poor ability on word problems may be related with lower ability students' inadequate reading skills. The reading ability of the subject during these tests interfered with his/her ability to estimate in as much as the time required to read and understand the question often impeded his/her progress. Children might not understand the words and structure of a problem and/or might have trouble accessing mental representations of quantities when physical referents were not provided. Levine, Jordan and Huttenlocher (1992) developed a nonverbal calculation tasks that eliminated these sources of difficulties and conducted to children between 4 and 6 years of age. In the study, addition and subtraction calculations were presented in three problem type formats, which were nonverbal problems, story problems, and number-fact problems. According to research result, children as young as 4 years of age had some success on the nonverbal problems. In contrast, children did not achieve on the story problems or number-fact problems until five and a half years of age. Moreover, throughout the age range tested, children performed better on nonverbal problems than on either story problems or number-fact problems. These results suggested that children's earliest ability to add and subtract was based on experiences combining and separating sets of objects in the world and that this ability came before the development of conventional verbal methods of calculating. The researchers stated that the task required a child to reach an exact solution to a calculation problem rather than to make a judgment about the effects of the addition or subtraction transformation in the numerical estimation questions.

Another distinction point in the estimation requested questions are the multiple-choice questions' effectiveness. That is, the traditional standardized test of multiple-choice items given over a single of time has not proved successful for testing estimation performance. As reviewers of NAEP, Carpenter et al. (1976, 1980) found that the use of that format allowed students to compute exactly and then rounded. Most of the researchers used special timing and open- ended questions to assess the performance of the subjects on estimation questions (e.g., Dowker, 1992; Goodman, 1991; Reys et al, 1991). On the other hand, different from other applications, in the study of Schoen, Friesen, Jarrett, and Urbatsch (1981), it was used individually administered oral tests. However, this requires considerable time. Many researchers gave briefly timed pencil and paper tests (e.g., Bestgen et al., 1980; Paull, 1971; Reys et al., 1991). A difficulty with this method was that items involving certain operations, for example, division were frequently avoids. In addition, the possibility of computing skill still existed. To handle with this obstacle, Paull (1971) used numbers with several decimal places to discourage computing. Most of the researchers preferred to time every item separately using a slide or overhead projector (e.g., Hogan, Wyckoff, Krebs, Jones, & Fitzgerald, 2004; Reys et al. 1982; Reys, Reys, & Penafiel, 1991; Reys, Reys, Nohda, Ishida, & Shimizu, 1991; Rubenstein, 1982; Rubenstein, 1985). The studies of Reys and his colleagues also restricted students to a very small answer sheet to guard against scratch work (Reys, Reys, & Penafiel, 1991; Reys, Reys, Nohda, Ishida, & Shimizu, 1991).

Whatever the reasons of the low achievement on computational estimation in all age groups it was investigated that whether computational ability performance could be improved. In the remaining of the section, it is presented the studies related with how could be improved the computational estimation performance of the subjects.

Murphy (1989) conducted an experimental study to understand the effectiveness of systematic instruction of computational estimation skills on two hundred forty five secondary school students. The experimental groups were taught a seven-lesson unit on estimation based on materials. The results provided that systematic instruction in estimation improved students' performance on standardized tests. Similarly, Bestgen et al. (1980) conducted an experimental study with 187 preservice teachers to identify the effects of instructional lessons on computational estimation performance. In the instruction sessions, the techniques and strategies that could be used to estimate solutions to computational estimation were taught by weekly practice and quizzes were applied one of the experimental groups. According to result of the study, the group who received weekly quizzes showed significantly greater gains in estimation performance than did the group receiving no practice. In a different perspective, Damarin et al. (1988) investigated whether estimation could be taught that using a sequence of computer based activities. They concluded that appropriately designed computer programs can helps students improve their estimation skills with a relatively small investment of instructional time.

Bobis (1991) investigated the effect of instruction on the development of the computational estimation strategies, and the degree of success in determining a close estimate after instruction. Bobis (1991) obtained that after training in estimation students tended to adopt the valid estimation strategies they had been taught. Therefore, the results of the studies Bestgen et al. (1980), Bobis (1991); and Damarin, Dziak, Stull and Whiteman (1988) concurred with Murphy's (1989) results which was improvement of the performance on computational estimation can be obtained by systematic instructions.

To improve the students' performance on the computational estimation it should be identified the strategies of this ability as a first step. The next section of the chapter presents the research studies on the strategies that are used for the computational estimation questions.

2.1.3 Strategies of Computational Estimation

A strategy can be defined as "a procedure or a set of procedures to achieve a higher level goal or a task" (Lemaire & Reder, 1999, p. 365). Although an assessment of the procedure is difficult for estimation, to identify the strategies, which are used by subjects during the estimation problems has been concern of many research studies (Brame, 1986, Cilingir & Turnuklu, 2009; Crites, 1992; Dowker, 1992; Jurdak & Shahin, 1999; Lemaire, Mireille, & Farioli, 2000; Levine, 1982; Morgan, 1990; Reys, Rybolt, Bestgen, & Wyatt, 1982; Reys B., 1986; Reys, Reys, & Penafiel, 1991, Reys, Reys, Nohda, Ishida, & Shimizu, 1991). According to these research studies, it were identified many strategies, such as compatible numbers, truncation, front-end strategies, reformulation, compensation, translation, nice numbers, matching pairs, comparing whole numbers, comparing fractions with a whole and a half, grouping, averaging or clustering, even standard computation procedure.

Of the work that has been done, most extensive research studies have been conducted by Reys and his colleagues (Bestgen, Reys, Rybolt, &Wyatt; 1980; Reys, Reys, Nohda, Ishida, Yoshikawa, & Shimizu; 1991; Reys, Reys, & Penafiel, 1991). These were translation, reformulation, and compensation. They followed generally same procedure in all studies. A large group of students were tested by a "Computational Estimation Test" by using overhead projector in limited time period. According to tests' results, the most successful students were selected and interviewed with them to indentify their strategies. In their studies, three general categories of strategies were identified.

Reformulation is a changing the numerical data into more mentally manageable form (Reys et al., 1982). One example of reformulation, which is most known one, is rounding numbers. This is the simplest strategy to teach or learn, and therefore, it is often the only strategy taught in the classroom (Levine, 1982; Trafton, 1986). A misconception could be seen among students and teachers, that is they thought that rounding is the only strategy to find the solution of estimation questions. Similar to this belief, Reys (1993) stated that students thought that estimation and rounding are synonymous.

However, there are many standard rules for rounding numbers such as rounding to nearest whole number, rounding the nearest ten, front-end rounding. Front-end rounding is usually employed when dealing with addition of the numbers. Reys et al. (1982) identified four forms of this strategy in their study. As follows, these four forms of front-end rounding are presented with an example;

To add 4792 + 5430 + 6452;

(a) By rounding and operating with rounded numbers using the same number of digits, so you should conduct 5000 + 5000+ 6000 → 16 000

(b) By rounding and operating with extracted portions of rounded numbers, so you should conduct 5+5+6→16 so the estimate is 16000

(c) By truncating and replacing the right hand digits with zeros and operating on the revised numbers using the same number of digits so you should conduct 4000 + 5000 + 6000→ 15 000

(d) By truncating and operating on extracted front-end digits so you should conduct 4 + 5 + 6 → 15, so the estimate is 15 000

The uses of “nice” numbers or “compatible numbers” are another example of reformulation strategy. Levine (1982) and Dowker (1992) called this strategy “known numbers” in their studies. Compatible numbers proposed many researchers as a reformulation strategy (e.g., Murphy, 1989; Reys et. al, 1982; Reys, 1986). Compatible numbers are those groups of numbers, which used in combination and then being operated on the procedure. Murphy (1989) gave some examples to compatible number in her research:

- when you are conducting the division of 5657÷28 it could be change by 6000÷30
- for 15% of 28.75 dollar could be changed by $\frac{1}{7}$ of 28 dollar or $\frac{1}{6}$ of 30 dollar

Similar to reformulation on whole numbers, in fraction and decimal related questions reformulation could be performed by converting the numbers to fractions and/or decimals equivalents. Reys (1986) called this kind of reformulation strategy in the decimal and fraction as "special numbers strategy." For example, rounding fractions could be done by controlling the nearness to 1, $\frac{1}{2}$ or 0, so that $3\frac{5}{12}$ might be thought as $3\frac{1}{2}$ or the operation 3.65 x 0.75 might be thought as $3\frac{1}{2}$ x $\frac{3}{4}$. According to researchers reformulation strategy was used by in all achievement levels of students for problems both in numerical and application formats (Dowker, 1992; Levine, 1982; Reys et al., 1982; Reys, Reys, Nohda, Ishida, & Shimizu, 1991; Reys, Reys, & Penafiel, 1991). The other strategy might be used in the computational estimation questions is "translation."

Translation is changing the equation or mathematical structure of the problem to a more mentally manageable form (Reys et al., 1982). The order of operations may be changed to make the problem more manageable; which means that addition may be converted in multiplication; division may be inverted to a fraction. For example, the addition of the five numbers, 253 + 248 + 198 + 204 + 186 can be converted to multiplication of 200 x 5 by conducting the translation strategy. Translation is more sophisticated technique than reformulation. As an observation of Reys et al. (1982), translation is more flexible than reformulation and may require an advanced level of conceptual knowledge. However, among the three of the strategies, the last one, compensation strategy, is the most complex strategy and the percentage of the usability of this strategy is lower than others.

Compensation is the process of the adjustments made into the intermediate and final estimate to reflect and awareness of the relationship of the estimate to the exact answer (Reys, et al., 1982). According to Reys et al. (1982), good estimators used compensation frequently and identified as essential to successful estimation. Lemaire, Lecacheur and Farioli (2000) concluded that the fastest strategy was reformulation and the slowest was the compensation strategy. According to the performance of the subjects and age level, the using of compensation is changed. LeFevre et al. (1993) reported that children used so—called prior-compensation strategies more frequently than post-compensation strategies whereas adults did the reverse. Reys et al. (1991) observed the common points of Japanese and American students according to computational estimation strategies. The researchers stated that the most common process applied by Japanese and American students was reformulation and lesser extent was compensation.

Sowder and Wheeler (1989) found that most fifth graders recognized the value of compensation but did not use it when generating computational estimates. As grade levels increased, the use of the strategies also increased. According to Sowder and Wheeler (1989), this age-related improvement in computational estimation and strategy using was because of the working memory capacity. Hunter (cited in Heirdsfield, 2000) suggested that the demand for retrieval of facts and strategies was met by long-term memory. Case and Sowder (1990) proposed that age-related increases in working memory allow children to maintain an increasing number of representations simultaneously. The researchers found that children of a wide variety of ages succeeded at estimation tasks for which their working memory capacities appeared sufficient and not tasks for which their memory capacities appeared insufficient (Case & Sowder, 1990).

Brame (1986) investigated the computational estimation strategies used by high-school students of limited computational estimation ability. The Assessing Computational Estimation (ACE) Test was administered 460 students, and 40 of them were selected for interviews. Each students interviewed was asked to estimate the answers to 14 computation and application problems. A comparison of the interview results and ACE Test results showed that removing the time pressure did improve performance. Students used wide variety of estimation strategies; however sometimes they had no strategy for estimation and attempted to use exact calculation. Although Brame (1986) did not classified exact calculation as a strategy, Levine (1982) and Dowker (1992) labeled the proceeding algorithmically as an estimation strategy in their studies.

In his study, Brame (1986) identified that all but one of the students used some form of the front-end strategies rounding and truncation in making estimates. It was observed that truncation was replaced by the use of rounding and compensation by the better estimators of the study. Although many of the estimators were willing to use compensation, they were many times not successful in its use. In his study, Brame (1986) concluded that estimators of limited ability used rounding but not always consistently or according to the standard rounding rules. Other commonly used strategies in the study of Brame (1986) were “averaging, using compatible” or “easier numbers” and using “the largest number” to eliminate choices. The students in the study were most successful on percent problems when they thought of percents as part of one hundred or in terms of an easier percent. The students in Brame’s (1986) research study, performed better than expected on division problems. Possibly this was because of the use of estimation in the traditional algorithm. A major difficulty encountered by the estimators of limited ability was the large number syndrome. This problem was connected to the power of ten error. Similarly, Sowder and Schappella (1994) stated that the ability to multiply and divide mentally by powers of ten is an important skill. According to many researchers, mental calculation and development of number sense could be taught to aid in the development of computational estimation strategies (Berry, 1998; Sowder, 1992; Dolma, 2002; Reys et al., 1982).

In another study reported by Levine (1982), it was determined the computational estimation ability of 89 undergraduates - none of whom were mathematics majors - and the strategies used by them which were classified into various categories. The categories had been predetermined through a literature search, pilot testing, and an examination of the computational estimation process. The categories (with an illustrative example, where necessary) were:

1) Fraction (0.76 becomes $\frac{3}{4}$)
2) Exponents (0.047 becomes 5 x 10^{-2})
3) Rounding both numbers
4) Rounding one number
5) Powers of 10 (76 x 89 becomes 100 x 100)
6) Known numbers (27.2 x 4.63 becomes 25 x 4)
7) Incomplete Partial Products(Quotients) (689 x 34 becomes 600 x 30 + 90 x 4)
8) Proceeding Algorithmically

In her study, Levine (1982) identified eight common estimation strategies used to estimate solutions to numerical problems as listed above, most of them could be served under Reys' (1991) defined strategies, which were classified in three main titles.

In a related study, using the same problems but a more mathematically wise population, Dowker (1992) identified seven strategies, four of which were also identified by Levine (1982). Both Levine (1992) and Dowker (1991) identified strategies which were use of fractions, rounding and use of algorithms as processes, commonly used to estimate solutions. Seven of the strategies specified by Dowker (1992) could be found in Levine's (1982) eight strategies fit under the reformulation and translation categories, which were identified by Reys et al. (1982).

In her study, Dowker (1992) interviewed 44 pure mathematicians to learn the computation estimation strategies used by mathematicians. They were accurate estimators and they used great variety of strategies. Dowker (1992) concluded that people often develop their own non-school based techniques for computational estimation in her mathematically wise sample.

Furthermore, Reys (1986) identified that five types self-developed strategies in her study. These were front-end, clustering, rounding, compatible numbers, special numbers in her study. She stated that like the problem solving techniques, estimation strategies are developed through instruction.

In a younger sample, Berry (1998) investigated to 8th grade students' computational estimation ability and the strategies they used. The researcher interviewed ten students using the interview format of the Accessing Computational Estimation (ACE) Test, which was developed by Reys, Rybolt, Bestgen, and Wyatt (1982). The interviews were divided in 4 segments, which were computation, application, calculator, and concept segments. In the computation segment the subjects were presented with 5 problems and were asked to "think out loud" as they estimated solution to the problems for identifying the students' strategies. In the application segment, the subjects were presented with 10 problems and then asked to answer the interviewer's probes. In the calculator segment, calculators were programmed to make systematic errors and the subjects were tested to see if they questioned the calculators' output. The last segment was the attitude/concepts segment. The questions in this segment were designed to learn about the subject's concept of estimation and to find out what factors, such as home, school, community activities, and jobs, appeared to contribute to the development of estimation strategies. In application and computation segments, Berry (1998) identified many strategies such as front-end strategies, rounding, compatible numbers, truncation, and averaging in many forms and in different situations. As a result, the researcher concluded that rounding was the most frequently observed strategy among the subjects.

Like Berry (1998), Levine (1982) concluded that the most frequently used strategy was “rounding both numbers” in the problems. The other frequently used strategy was “proceeding algorithmically,” where a form of a standard algorithm was used to calculate at first then estimate and finally combine partial products or quotients. According to Levine (1982), students of lower quantities ability used an algorithmic procedure for estimation more likely to use a variety of different estimation strategies. According to her, the compatible numbers strategy was especially useful in working percent problems.

In another perspective, Smith (1993) investigated the preservice elementary teachers’ conceptual understanding of computational estimation strategies. In Smith’s (1993) study, the results of the dialogues indicated that rounding was the only strategy that many of the preservice elementary teachers knew. Some other subjects thought about the compatible numbers strategy, as using set of numbers that could be used when doing estimation. The subjects asserted that compatible numbers could easily be manipulated mentally. A few of the subjects stated that the front-end strategies which focuses on the left-most digit of a number to provide an initial estimate followed by mental adjustment to determine a better estimate. Smith’s (1993) subjects did not much use averaging or clustering strategies, which means grouping the numbers about a particular value.

Crites’s (1992) study about the discrete quantities relied on the possession of spatial visualization, measurement, mental computation, and number sense skills. In his study, he identified two main strategies multiple benchmark and decomposition/recomposition, which were more sophisticated strategies than the others.

Benchmark defined as the comparison of a known standard to the to-be-estimated item (Crites, 1992). The comparison is made by regular decomposition, recomposition where to-be-estimated item is grouped into terms small enough to compare with a benchmark. This strategy involves dividing the item to be estimated into smaller parts until a benchmark can be applied and then recombining the parts based on a comparison with a known benchmark. If the item cannot be easily divided into parts or the parts are of different sizes, then irregular decomposition occurs. Benchmarks contribute to students' number sense by allowing them to understand the relative magnitude of fractions and to develop an intuitive feel for them (Crites, 1992).

Students can use these benchmarks to compare fractions by mentally about their relative size (Crites, 1992). Initial attention should be given to knowing which fractions are close to zero, equal to one whole, less than one whole, and greater than one whole where Reys (1986) called this strategy as "special numbers strategy." Then students should examine which fractions are equal to one-half, less than one-half, and greater than one-half (Crites, 1992; Reys, 1986).

Like Crites (1992) and Siegel et al., (1982), Heinrich (1998) concluded that the superior calculation ability developed from additional experience and maturity. The students in grades 6, 7 and 8 demonstrated that they were capable of learning to perform computational estimation tasks in a short period of time. It was found that the easiest strategy was translation and the most difficult one was the compensation among the sixth, fifth and eighth graders. He concluded that the major problem experienced by students not estimation ability skills that were lack of computational skills. The choice and use of these strategies developed flexibility in thinking about and using numbers that fit a particular situation. Students generally did poorly estimating percents, square roots and product of mixed numbers (Heinrich, 1998; Reys et al., 1991). Similar to Berry (1998) and Heinrich (1998), Sowder (1984) found that errors on estimation problems could be attributed to a lack of understanding of number size, which led students to make poor approximations.

Lemaire, Lecacheur and Farioli (2000) carried out a study to understand which strategies children use to do computational estimation, on which problem do they use each strategy, and how they choose and execute computational estimation strategies. In the study design, interview sessions were conducted with twenty-three fifth graders (ten-year-olds) to identify the computational estimation strategy on three digits addition (e.g., give an approximate answer like 400 to an arithmetic problem like 224+213). As a result, the researchers confirmed that there are four strategies (rounding with decomposition, rounding without decomposition, truncation, and compensation) were used while doing computational estimation, the fastest strategy was truncation, and the slowest was compensation. Additionally, Lemaire et al. (2000) and LeFevre et al. (1982) are considered that the children's improvement in estimation between around 9 and 12 years is due not only better coordination ability but also to increased flexibility of strategy are confirmed.

To understand why students have consistently performed so poorly when estimating solutions to problems, it is important to review research that has been conducted on assessing computational estimation skills, computational estimation strategies and the variables that affect estimation ability. In three separate studies conducted in the United States, Japan, and Mexico showed that there are some universal reason for poor ability on computational estimation (Reys, Rybolt, Bestgen, & Wyatt, 1982; Reys, Reys, & Penafiel, 1991; Reys, Reys, Nohda, Ishida, & Shimizu, 1991). In these studies, problems from the ACE (The Assessing Computational Estimation Test) with changes to reflect appropriate cultures were used to assess computational estimation ability of students in grades five through twelve. According to research studies, computational estimation was affected from the generally three constructs, number sense, mental computation, and affective constructions of the students. The studies on factors, which are affecting the computational estimation strategies gathered under the title of "components of computational estimation," are given in the next sections.

2.1.4 Components of Computational Estimation

It must emphasize that estimation itself is not a single unitary process, but it is integrated numerous components (Case & Sowder, 1990; Sowder, 1992; Reys et al., 1982, Reys, Reys, Nohda, Ishida, & Shimizu, 1991; Rubenstein, 1985). The research literature showed that a wide variety of variables, which appeared to be related to computational estimation performance (e.g., Reys et al, 1982; Rubenstein, 1982). For instance, Case and Sowder (1990) identified that estimation is a complex construct and when a child accomplished that he/she perform more than one tasks.

Moreover, Rubenstein (1985) conducted a study with three hundred eight graders aimed that to explore the relationships among the computational estimation tasks and the mathematical skills. As a result, she served that there were eight mathematical skills that were related with the computational estimation; selection of operation, making comparison, number facts, operating with tens, operating with multiple of tens, place value, rounding and judging relative size. Sowder (1992, p375) suggested that Rubenstein's “good predictors” might be too closely associated with “place value” and “basic number understanding” for all to be significant in a stepwise regression.

It was identified in Rey et al. (1982) study; the characteristics of good estimators had three distinct dimensions: number skills, cognitive processes and affective attributes, and each of these accumulated in two concepts cognitive and affective components of computational estimation. In the following section, the review of the studies is given about these components.

2.1.4.1 Cognitive Components of Computational Estimation

There are some frameworks for the cognitive components of the computational estimation (Reys, Rybolt, Bestgen, Wyatt & Wendell, 1982; Sowder, 1988; Sowder & Wheeler, 1989). These frameworks mostly similar to each others with small differences. For instance, Reys et al. (1982), Sowder and Wheeler (1989) specified four main components, which were related to the computational estimation:

1) Conceptual components,
2) Skill components,
3) Related concepts and skills,
4) Affective components.

The Conceptual Components recognized the role of approximate numbers, the potential for multiple of possible techniques and outcomes, and finally, the appropriateness of an estimate was subject to context and desired accuracy. In the Skill Components, process and outcomes of the estimation were explained, which was deeply discussed in the strategies section through Rey et al. (1982) findings. Under the title of the Related Component and Skill, the basic mathematical facts were listed. Affective Components included recognition of the usefulness of estimation, tolerance of error and confidence in mathematical ability and ability to estimate are discussed through the research literature in the next section.

Sowder and Wheeler (1989) were particularly interested in the first two components in the list above. Sowder (1988) delineated and related to various components of computational estimation, which is presented in Figure 2.1. In the Figure 2.1, Sowder (1988) specified the cognitive components of the estimation performance with the based of four cognitive structures of computational estimation. These are:

1) prerequisite skills and components,

2) primary skills required during computational estimation,
3) specific estimation process,
4) specific estimation concepts.

In the framework, first component contains some of themes such as understanding of place value of whole numbers, decimals, and factions, ability to work with multiple/powers of ten, knowledge of basic facts and ability to use properties of operations. Then the second component involved in ability to compare numbers by size and ability to mentally compute with whole numbers. The third component, which is called “specific estimation process” contain three strategies (reformulation, translation, and compensation) identified by Reys et al. (1982). The last component of the framework is “specific estimation concept” which included in approximation, appropriateness, and multiple answers of estimations. The relationships of these components are presented in the Figure 2.1 as follows.

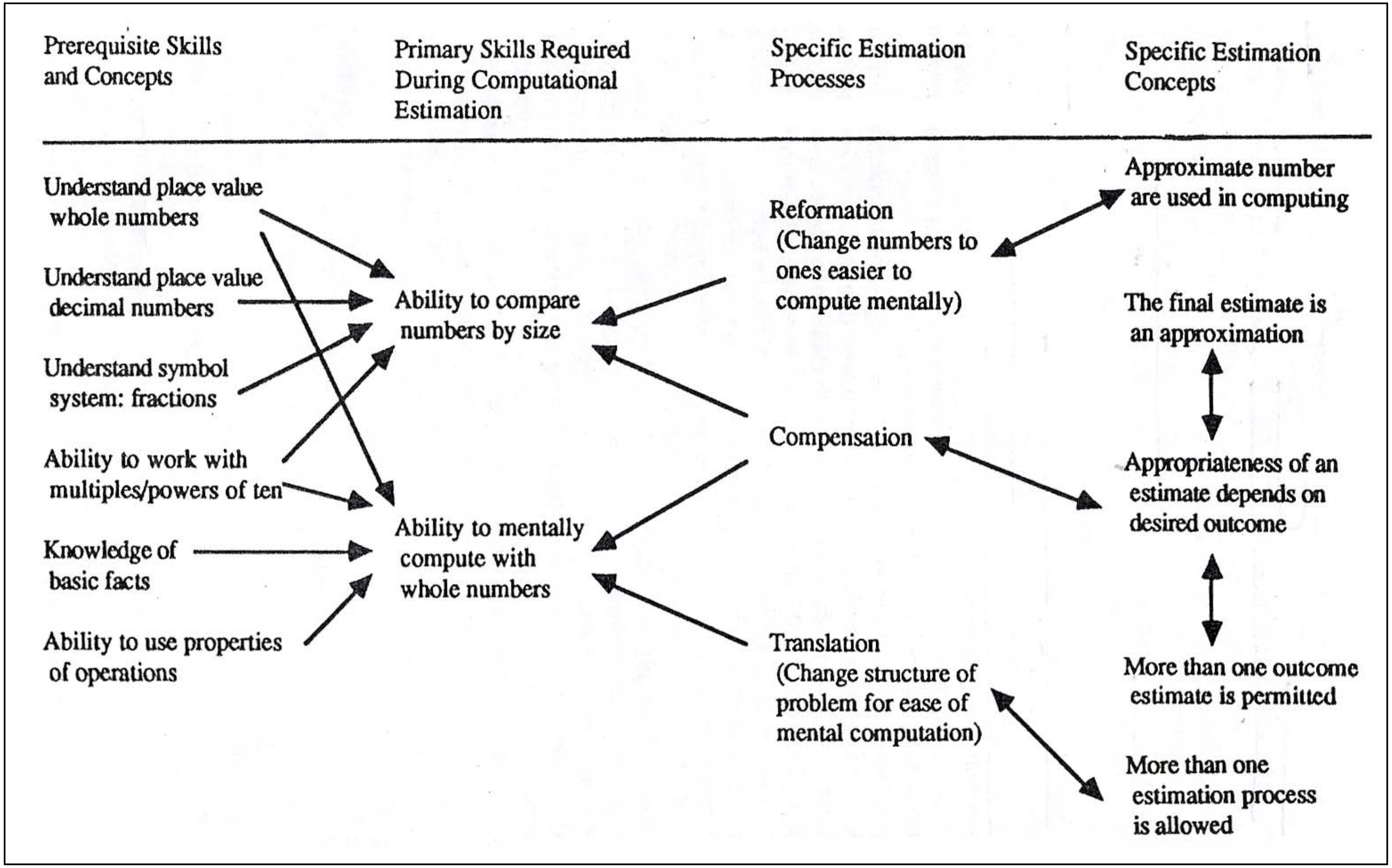

Figure 2.1 Sowder- Threadgill's (1988, p. 192) Framework

Similarly, Hogan and Brezinski (2003) suggested that computational estimation might be listed under more general, well-established mathematical abilities specifically a combination of numerical facility and reasoning quantity. The quantitative or numerical abilities have been correlated with estimation ability through the results of many studies. For instance, Levine (1982) found that quantitative ability in mathematics was positively correlated with estimation skill in college students. In addition to Levine's (1982) findings, Dowker (1997) concluded that estimation proficiency increased with arithmetical competence and decreased with the problem difficulty. Paull (1971) observed that estimation of numerical computation was significantly correlated with problem solving, mathematical ability, and verbal ability, and that the ability to compute rapidly was related to the ability to estimate numerically. Contradicting with many research findings of the studies such as Boz (2004), Dowker (2003) and Levine (1982); Gliner (1997) concluded that mathematics achievement and estimation performance were negatively correlated. This contradictory assumption might be related with exact computation dependency of higher achievers among the subjects of Gliner's (1997) study. According to him someone who was more dependent on exact computation might had lower success on computational estimation.

As reviewed above, conducted studies mostly stressed cognitive factors of computational estimation ability. In the studies indicated that good estimators had good knowledge of number operations and that the ability to estimate was positively related to quantitative ability (Bestgen et al., 1980; Bobis, 1991; Levine, 1982; Paull, 1971; Reys et al., 1982). Reys et al. (1982) identified the following numerical variables correlated with the computational estimation; basic facts of mathematics, place value, mental computation, and arithmetical processes, which contained commutativity, associative and distributive properties. In addition, Blair (2001) established strong relationship among basic facts mastery and better performance in basic computation, estimation, and numerical pattern recognition. Similar to Blair (2001), Rubenstein (1982) conducted a regression analysis to find out the factors explaining the computational estimation. She found that operating with tens, making comparisons, and getting to know the problems were the three factors confirmed by the regression analysis. On the other hand, Sowder (1989) claimed that estimation comprised of only two distinct tasks, which were approximation and mental computation.

As a summary, the findings of research studies indicated that the following variables appear to be related to ability to estimate solutions to computational problems, mathematical ability, requirements of estimation process like approximateness and compensation (Dowker, 1992; Gliner, 1991; Levine, 1982; Reys et al., 1982). Although the studies showed that estimation ability mainly correlated with mathematical variables, some research studies claimed that there were some other factors rather than cognitive, that is affective factors (Reys et al., 1982; Sowder, 1992).

2.1.4.2 Affective Components of Computational Estimation

Researchers in both psychology and education have long investigated relationship between affective and cognitive variables in mathematics (Aiken 1976; Ma & Kishor, 1997). The researchers asserted that none of the cognitive process could be performed by isolated from feelings and beliefs. In addition to considering the skills and abilities directly related to estimation tasks, it is necessary to take into account, other distinguishing factors, rather than cognitive factors, among individuals.

Sowder and Wheeler (1989) identified in their framework, there were affective components for the computational estimation. In the affective component there were four sub categories related to success in computational estimation. These were presented as follows:

1) Confidence in ability to do mathematics

2) Confidence in ability to estimate

3) Tolerance for error

4) Recognition of estimation as useful.

The first and second components, confidence in doing mathematics and doing estimation were discussed in many research studies. For instance, Bestgen et al. (1980) found that self-perception as a mathematics student was most highly related to success in computational estimation in preservice teachers. In his study, Gliner (1991) tried to answer what personal attributes may be related to success in mathematical estimation. He found the answer to "Are you a good at math?" was positively correlated with estimation scores.

Moreover, Glasgow and Rey's (1998) study, which was a replication of the study of Bestgen, Reys, Rybolt, and Wyatt (1980), tried to understand students' confidence to their estimated answers with regard to exact computation. In the study, calculators were defected in a range of 10% to 50% distance from the exact answers. Researchers asked first to find the solution by estimation then find the same question's answer by using calculator (Glasgow & Reys, 1998). Interestingly students in both studies, although subjects produced reasonable estimates solutions for the computational items by estimation, confined the results of malfunction calculators rather than trusting their own approximate answers. Where majority of preservice teachers produced reasonable estimates, only seven of twenty-five subjects questioned the accuracy of the answers produced by calculator (Glasgow & Reys, 1998). This showed that students were not confident in their estimated solution and estimation ability. However, it is not known whether this result was due to their notion of estimation as a "less trustworthy" answer. If students had been asked to produce exact answer (rather than estimates), using paper and pencil techniques prior to using the calculator, would they have been more likely to challenge the calculator result? Berry (1998) obtained a similar conclusion. In the study, it was aimed to identify the students' confidence level in estimation. He conducted four segments in the study and one of which was a calculator segment. In this segment calculators, which were designed to produce systematic errors during the calculations were given to subjects. It was observed that students were confident to calculators more than their self-produced estimated answers.

Similarly Mottram (1995) examined students' performance on estimating solutions to real life tasks, compared it performance on estimating to numerical, and word problems and relationships among performance on estimation and confidence in doing mathematics. One of the results of Mottram's (1995) study was that students who were good estimators are confident in their mathematical ability and their ability to estimate. The findings both Mottram (1995) and Reys et al. (1982) were consistent in order to good estimators had high confidence in estimation and mathematical ability.

LeFevre, Greenham, and Waheed (1993) conducted a study, which aimed to provide information about students' procedural and conceptual knowledge in solving estimation problems in order to develop a model of the estimation process. According to conducted correlation analysis of the adult questionnaire data revealed that adults who considered themselves skilled in estimation also tended to report high skill in arithmetic, high levels of confidence in their estimations, and frequent use of estimation. High self-reported estimation skill also correlated with higher math marks in high school and with the belief that estimation is useful in everyday situations. These subjects also reported more math experience in high school, recognition of math as useful in university courses development of estimation procedures outside formal schooling and less avoidance and nervousness in mathematical situations. Adults who perceived themselves as skilled estimators were less anxious when calculating tips in restaurants when completing an income tax form, and when adding up the cost of purchases. Thus adults who report high estimation skill can be described as being skilled in arithmetic with positive high school math experiences and high math marks. They are not math anxious but appear confident in their abilities in these areas, recognizing the usefulness of estimation, and using it frequently in their daily lives (Reys et al., 1982). These results support Sowder and Wheeler's (1989) contention that affective components are related to computational estimation skills. However, these correlations do not indicate of causality between estimation skills and affect about estimation.

Among the affective factors, the most interesting one was "tolerance for error," which included in a few studies. In few studies, tolerance for error explained as being comfortable with estimated solutions (Rubenstein, 1985; Sowder, 1992; LeFevre, et al., 1993). In mathematics, particularly with respect to estimations skills, a high tolerance for error has been identified as a feature of good estimators (Reys, et al., 1982).

Dowker (1992) offered two explanations; one was that people who preferred precision (those with low tolerance for error) see little point in estimating and thus they were negatively affected by their lack of experience in this skill. Another was that good estimators rely on their ability to adjust their errors and can effort to estimate broadly. This concept is difficult to identify and examine. There is a short research literature where it can be accessed according to explain how this structure can be integrated into estimation (Hogan, Laurie, Wyckoff, Krebs, Jones & Fitzgerald, 2004; LeFevre et al., 1993; Reys et al., 1982; Sowder, 1992).

Reys et al. (1982) claimed that a kind of error tolerance knowledge about estimation diffused good estimators' thoughts. This thought makes them comfortable with some computational errors. The subjects of the study of Reys et al. (1982) steadily underlined the importance of efficient, reasonably accurate computational tool and that their ability to estimate filled this need. Similarly, LeFevre et al. (1993) claimed that adults had less tolerance for error than children did in their study. According to researchers for children, calculating exact answer was difficult on even easiest problems; they preferred to use estimation to exact computation. On the other hand, adults who were calculating easily did not prefer to use estimation and had low tolerance for error.

One of the studies, which identified the tolerance for error concept, was Sowder's (1992) reviews. According to Sowder (1992), students who had high tolerance for error did not see them as "wrong" when estimating and were comfortable with error. They were comfortable with the idea that their estimates where it might be different from others and it were acceptable for them.

Reys et al. (1982) study was given as a reference in many computational estimation research studies in order to affective domains of estimation performance (Hogan, Wyckoff, Krebs, Jones, & Fitzgerald, 2004; Munakata, 2002; Rubenstein, 1985; Sowder, 1992; Sowder & Wheeler, 1989; Volkova, 2005). On the other hand, there were some criticisms on this component of computational estimation as claiming that especially the relationship of tolerance for error and estimation performance was a big assumption. According to Hogan et al. (2004) the factor tolerance for error, which was influencing students' estimation ability should be tested by psychological tests since it was a psychological construct.

According to Hogan et al. (2004), the study of Reys et al. (1982) could not infer this kind of relationship since there were no any statistical measurements about the identification of the "tolerance for error." Hogan et al. (2004) stated that although this factor contained psychological requirements, the researchers did not explain how they get that result without any psychological measurements.

To fill up this gap, Hogan and his colleagues (2004) performed a study by using very detail statistical methods. The researchers used a psychological test to identify the tolerance for error of subjects. They argued that tolerance for error as described by Reys et al (1982), Rubenstein (1985) and Sowder (1989) appeared to be specific application of “tolerance for ambiguity” one of the main descriptor of the “Openness” dimension in the Five Factors Personality Test (Hogan et al., 2004). According to researchers, “Openness” should correlate significantly with the computational estimation performance. McCrae and Costa (cited in Hogan et al., 2004) explained the relationship of “Openness” as follows:

> Need for variety, tolerance for ambiguity and preference for complexity all represent motivational aspects of openness. In addition open people can be characterized by their nontraditional attitudes, their rich and complex emotional lives and their behavioral flexibility (p. 832).

As a result, the researchers claimed that there was no statistically significant correlation between “Openness” and computational estimation performance (Hogan et al., 2004). However, they found a very small correlation between “Agreeableness” which was another factors of the “Five Factors Personality Test” and computational estimation performance. Hogan and colleagues (2004) confirmed this finding with another study and with another personality testing procedure (Hogan & Parlapiano, 2008).

The found correlation between “Agreeableness” and computational estimation score was underlined relation between the negative pole of “Agreeableness” and computational estimation score. According to Hogan et al. (2004) the negative pole of “Agreeableness” contained the descriptors as “an antagonistic, competitive, and skeptical disposition.” Although, the researchers gave statistical results about both the correlation between “Agreeableness” and computational estimation performance and no correlation between “Openness” and computational estimation performance, those findings should be triangulated by some other techniques, like conducting some interviews, or observations with their subjects to strengthen their findings. Since tolerance for error is a kind of psychological construct, it should be also conducted some interview by an expert clinical interviewer (Hogan, et al., 2004). Additionally, Piaget (cited in Ginsburg, 1981) stressed those mathematical structures especially students thoughts should not only be tested rather should be conducted a clinical interview.

Several investigators have suggested that a personality variable labeled tolerance for error partially undergirds successful performance in completing computational estimation problems. Rubenstein (1985), Sowder and Wheeler (1989), Sowder (1992) and Lefevre, Greenham and Waheed (1993) cited tolerance for error as an important or potentially important variable related to computational estimation skill from elementary school through college.

Besides affective factors, among the cognitive factors that are affecting the students' computational estimation strategies, two important topics emphasized which are "number sense and mental computation." These are reviewed through the research studies in the next sections.

2.2 Number Sense

Sowder (1992, p. 387) claims that "computational estimation is closely related to number sense and that number sense is difficult to define and therefore difficult to assess." The reviewers of The National Assessment of Educational Progress (NAEP) results concluded that students should develop a quantitative intuition before to be good estimators (Carpenter et al., 1976). In more recent years, this quantitative intuition occurred to be referred to as number sense (Sowder, 1992).

In order to understand the relationship between these terms the meaning of number sense should be specified in order to estimation context. For Sowder (1988) number sense is a well-organized network of concepts that makes it possible to relate numbers and properties of operations, that provides skill in working with numeric magnitudes. Moreover, number sense refers to a person's general understanding of numbers and operations and the ability to handle with daily-life situations that include numbers (Yang, Li, & Lin, 2008). This ability is used to develop flexible and efficient strategies (including mental computation and estimation) to handle numerical problems (McIntosh, Reys, & Reys, 1992; Reys, 1994).

Number sense is a complex process involving many different components of numbers, operations, and their relationships. It has been the focus of research and discussions among mathematics educators, educational psychologists, researchers and curricula developers. As a result, in many studies, different psychological perspectives have been provided; theoretical frameworks of number sense proposed; characteristic of number sense described and essential components of number sense have been enumerated (Case & Sowder, 1990; Greeno, 1991; McIntosh, Reys & Reys, 1992; Reys, 1994; Sowder, 1992; Tsao, 2004; Yang, Hsu & Huang, 2004).

The development of number sense in students is an important aim of mathematical instruction (NCTM, 2000). Berch (2005) asserted that possessing number sense permitted one to achieve everything from understanding the meaning of numbers to developing strategies for solving complex mathematics problems. NCTM (2000, p. 32) noted that number sense is one of the foundational ideas in mathematics. Therefore, students should:

(1) Understand number, ways of representing numbers relationships among numbers and number system,
(2) Understand meanings of operations and how they related to one another,
(3) Compute fluently and make reasonable estimates

McChesney and Biddulph (1994) gave a smart metaphor that was number sense was roads of a big city. The researchers explained this metaphor as a person with good road sense has an integrated mental picture how various roads are connected, where they lead, what they were like, how traffic behaved on them, and how they might be negotiated.

Although number sense has not a universal definition, the National Council of Teachers of Mathematics (1989) defines number sense as an intuition about numbers that is drawn from all the varied meaning of numbers. More specifically, Sowder (1992) referred it as the ability to decompose numbers naturally, use benchmarks, and use relationships among arithmetic operations to solve problems, understand base-ten number system, estimate, make sense of numbers, and recognize the relative and absolute magnitude of numbers.

Furthermore, Hatano (1988) described two types of number sense experts: Routine and adaptive. Routine experts are able to solve familiar problems quickly and accurately but are not able to invent new procedures because they lack the rich conceptual knowledge of an adaptive expert. Adaptive experts can discover rules, invent algorithms, and develop flexible uses of numbers. Number sense is not broader domain than either estimation or mental computation (Greeno 1991; McIntosh, Reys & Reys 1992; Sowder & Schappelle 1989).

Greeno (1991) stated the relationship of number sense and estimation as pointed that number sense involves several capabilities including flexible mental computation and numerical estimation and quantitative judgment. In other words, number sense includes both mental computation and computational estimation. Carroll (1996) stated that good mental computation and estimation ability is evidence of number sense and also develops number sense. According to Sowder and Schappelle (1994) number sense refers to an intuitive feeling for numbers and their various uses and interpretations; the ability to detect arithmetical errors and a common-sense approach to using numbers.

According to Sowder's (2001) study 26 middle school students representing a variety of backgrounds and achievement levels, were individually asked to estimate answers to 12 computational problems and explained how they obtained their answers. They were allowed to use writing materials. The results indicated that estimation skills were highly dependent on a student's "number sense." Yang (1995, p. 6) stated, "Computational estimation plays an important role in the development of number sense. Weakness in performance of computational estimation may reveal a lack of number sense."

Jordan, Kaplan, Olah and Locuniak (2006) classified that there are five elements of numbers sense, counting, number knowledge, number transformation, estimation and number pattern. *Counting* including grasping one to one correspondence, knowing stable order and cardinality principles, and knowing the count sequence. *Number knowledge* means that discriminating and coordinating quantities and making numerical magnitude comparisons. *Number transforming* included in calculating the nonverbal and verbal context and also transforming the sets through addition and subtraction. *Estimation* takes into account only approximating or estimating set sizes and using reference points. The last element of number sense is the recognizing the *number patterns*. It includes that coping number patterns, extending these patterns, and discerning the numerical relationships. Jordan and colleagues (2006) examined the development of number sense in 411 kindergartens. The findings suggested that gender difference in math emerge as early as kindergarten. There were small, but statistically reliable, gender effects on kindergarten level performance on overall number sense, nonverbal calculation, and estimation. In each case, boys showed an advantage over girls and the findings held above and beyond income level, age and reading ability.

According to Reys, Reys, McIntosh, Emanuelsson, Johansson, and Yang (1999) and Reys and Yang (1998), number sense meant that general understanding of number and operations, along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful and efficient strategies for managing numerical situations. It results in a view of numbers as meaningful entities and the expectation that mathematical manipulations and outcomes should make sense. Number sense involves the development of multiple relationships between mathematical concepts, facts, and skills and therefore provides multiple accesses to them when needed (Yang, 1999).

Ell (2001) served a review of the international literature on number knowledge, number strategies, and frameworks for classifying children's learning of numbers. Ell (2001) stated that a framework, which presented number sense as the backbone of the number domain, was proposed by McIntosh, Reys and Reys (1992). They proposed three strands to number sense:

1. A knowledge of and facility with numbers,
2. Knowledge of and facility with operations and
3. Applying knowledge of and facility with numbers and operations to computational settings.

Whitacre (2007) stated that students who rely heavily on standard methods evidenced to poor number sense. These students understanding of an operation seems to be tied to symbol manipulation so that they lack of flexibility. According to his research at the other end of the spectrum, students who readily employ nonstandard method, exhibit good number sense. Their understanding of the operations was independent from any particular algorithm, so that they had good flexibility.

Markovits and Sowder (1994) designed an intervention program to develop the number sense of seventh grade students. The program focused on number magnitude, mental computation, and computational estimation. These researchers concluded that number sense included using numbers flexibly when mentally computing, estimating, judging number magnitude, and judging reasonableness of results, moving between number representations, and relating numbers, symbols and operations all stemming from a disposition to make sense of numerical situations. In the results of the study, it could be said that the brief instructional unit appeared to bring about positive changes in number sense and estimation ability.

Sowder and Schappalle (1994) asserted that sense making of computation could be focus of instructional activity. They claimed that place value is one basis for the flexible decomposition and recomposition of numbers, a key element in other skills related to number sense- mental computation and estimation. Although Rubenstein (1985) did not count the place value as a predicted factor of computational estimation, she emphasized the importance of it while conducting estimation questions, especially with decimals. According to regression analysis, was conducted in her research study, among the factors of "selection of operation, making comparison, number facts, operating with tens, operating with multiple of tens, place value, rounding and judging relative size" only the three of them predicted estimation performance. These were "operating with tens, making comparison, and judging relative size" factors.

The NCTM Standards document (1989) stated that children with good number sense have well-understood number meanings, have multiple interpretations and representations of numbers, can recognize the relative and absolute magnitudes of numbers appreciate the operating on numbers and have developed a system of numerical benchmark. The proficiency of these topics improved to estimation ability.

Reys and Yang (1998) cited in their study the characteristics of the number sense as understanding of number magnitude, the use of benchmarks, the relative effects of operations, decomposition and recomposition and application of the knowledge of numbers and operations to computational situations. Sowder (1992) stated that the effective use of benchmarks has been associated with estimation ability and number sense. Similarly, Tsao (2005) conducted a study to answer the question "what cognitive processes do preservice elementary school teachers' use when they asked to solve problems involving number sense". The researcher observed five cognitive processes that are run about the number sense. The characteristics of number sense included the following processes:

1. well-understood number and symbol meaning,
2. the ability to decompose/recompose numbers
3. recognition of the relative and absolute magnitude of numbers
4. having the ability to use benchmark
5. flexibility while applying knowledge of numbers and operations to computational situation (including mental computation and computational estimation)

In the explanations of numbers sense conducted so far underlined the relations among computational estimation and mental computation. Moreover, Sowder and Schappelle (1994, p. 343) state that "knowledge of relative and absolute number size essential to judging the reasonableness of computation" which is the basics of computational estimation. Sowder- Threadgill (1984) conceived that good estimators have a good understanding of basic facts, place value, and arithmetic properties, are skilled at mental computation, demonstrate tolerance for error, can flexibly use a variety of strategies, and display self-confidence. Improving the teaching of computational estimation is related to encourage the development of number sense.

Tsao (2005) stated that high ability students were more successful on each type of number sense item than the low ability students. In the study, he concluded that the low ability students tended to use the rule-based method more frequently when answering interview items than high ability students. The low ability students also preferred the use of standard written computation algorithms rather than the use of number sense based strategies. The high ability students tended to use of benchmarks, to apply knowledge of the relative operations on numbers and decomposition/recomposition of operations on number, to reflect knowledge of number magnitude and to response flexibility with number operations when answering interview items. Tsao (2005) obtained results from the interviews indicated that items including fractions were more difficult than whole number and decimal items.

Rubenstein (1985) similarly found that eight grade students had more difficulties with decimals more than with whole numbers. During the interview, subjects conducted many comments that they could “do better” if they had pencil-paper or a calculator. Reys et al. (1991) and Yang (1997) found that many students in both Japan and Taiwan respectively were more comfortable solving computational problems exactly than estimating a solution. These researchers indicated that students resisted for giving estimates because they either did not understand the meaning of the estimation or were reluctant to accept error (Reys et al., 1991; Yang, 1997).

Since many characteristics of number sense are reflected in mental computation and estimation, number sense investigations were undertaken of students’ thinking when asked to estimate and mentally calculate (Case & Sowder, 1990; Hope, 1986; Levine, 1982; Reys, Rybolt, Bestgen & Wyatt, 1982; Rubenstein, 1983; Sowder & Wheeler, 1989). Although there is currently, a great deal of interest in number sense has not been a focus in instruction. Sowder (1992) stated that it was difficult to define and asses number sense like higher-order thinking. Assessing number sense, mental computation and all three kinds of estimation presents many difficulties. For example, with estimation the multiple correct answers are a problem for students. In a study by Sowder and Wheeler (1989), twelve students at each of grades 3, 5, 7, and 9 were individually given computational estimation tasks. The researchers found that students were willing to accept that there could be multiple strategies for finding an estimate, each producing a different answer, but students were reluctant to accept more than one “right answer.” Students at all grade levels preferred computing-then-rounding to estimate rather than the rounding-then-computing, because they believed that computing-then-rounding method was a good way to get a correct answer to the estimation task. Sowder and Wheeler (1989) claimed that perhaps this is the accumulative effect of school instruction on rounding and emphasis on unique answers.

Whitacre and Nickerson (2006) discussed their model for developing the number sense of the students according to their learning model and get impression results from it. In their study, they began to answer the question of how an instructor can support preservice teachers' development of number sense with regard to mental math. They concluded that the subjects of the study developed significantly greater number sense as a result of "hypothetical learning trajectory". They concluded that with smart instruction number sense could be improved. In addition, Markovits and Sowder (1994) examined the effect of an intervention in the instruction of 12 seventh grade students, purposed that developing number sense. The students were taught experimental units on number magnitude, mental computation and computational estimation. From the interviews and written measures, it was discovered that the students reorganized and used existing strategies rather than acquiring new knowledge structure. Markovits and Sowder (1994) stated that a brief instructional unit appeared to bring about positive changes in understanding most of aspects of number sense. Sowder (1995) also stated the same conclusion with developmental capacity of the number sense. She connected estimation and number sense in her research that instruction on estimation and mental computation could provide an avenue for developing number sense. Students who were good at estimation and mental computation could easily link symbols to concepts that contributed to development of number sense. According to Markovits and Sowder (1994) if students understood the relationship between number sense and mental computation, they could develop effective strategies to solve and estimate problems mentally.

Boz and Bulut (2002) searched the preservice mathematics, science, and childhood teachers' computational estimation abilities. The Estimation Ability Test was conducted to participant to understand their performance on computational estimation ability. The researchers concluded that the preservice teachers' computational estimation abilities were moderately low. The subjects of the study struggled with mostly on fraction number questions. The other number related categories of the estimation test were also difficult for the participant. As a result, researchers concluded the preservice teachers' quantitative intuition, number sense, was very poor.

The researchers interested in studying number sense, computational estimation and mental computation agreed on the importance of these topics, but did not necessary agree on which were the most important research issues to pursue, how research should proceed, or how these topics should be incorporated into the curriculum. Sowder (1992) presented a reason for the lack of agreement that is primarily due to the different epistemological viewpoints of the investigators. All do agree, however, that number sense should permeate the curriculum and that computational estimation and mental computational should be incorporated into all instruction on computation. Therefore, the other important concept for computational estimation is mental computation and the research studies are reviewed in the following section.

2.3 Mental Computation

Computation can be accomplished by various methods; mental, written, approximate and calculator, each appropriate given a particular problem context. In general, if it is possible to solve the problem mentally, then mental computation will be the natural tool to choice. Often the mental strategy is an invented one and is based on conceptual understanding. However, if the numbers are too complex for mental computation but estimation provides a solution that addresses the problem context then computational estimation is an appropriate tool. Again, the estimation strategy employed is generally based on conceptual understanding although some standard techniques for estimating are also practical. If however, the result from estimation are inconclusive or if more precise results are needed then exact computation is needed. On the other hand, computational estimation and mental computation are confused each other most of the time. Although mental computation needs exact results, an approximate answers enough for computational estimation. While both mental computation and computational estimation can be done mentally, the process of estimation produces a response that is close to the exact answer, which would be the result of the process of mental computation. But, either a calculator or a standard written technique is a natural tool of choice for tedious computation requiring an exact answer (Reys & Reys, 1998). Anghileri (1999) stated that mental computation was calculating *with* the head, instead of *in* the head, which means that mental computation was calculating using strategies with understanding. Thus, proficiency in mental computation was not confined to accuracy, but also included flexibility of strategy choice. Therefore, the factors that influence mental computation consist of those that affect flexibility as well as accuracy.

Computational estimation and mental computation are frequently combined together as one topic in the research studies (Bestgen et al., 1980; LeFevre, Greenham, & Waheed, 1993; Markovits & Sowder, 1994; Munakata, 2002; Reys, Reys, Nohda, Ishida, & Shimizu, 1991; Reys, Reys & Penafiel, 1991; Sowder, 2001). The research literature has shown that mental computation may be viewed as a subset of number sense, as students who exhibit proficiency in mental computation also display number sense (e.g., McIntosh, 1996; McIntosh, Reys, & Reys, 1992; Sowder, 1990; Sowder, 1992). Research on mental computation has proposed specific connections among mental computation and aspects of number sense, in particular, number facts knowledge and estimation (e.g., Heirdsfield, 1996; Sowder, 1992). Some research studies relating to computation (in particular, children's natural strategies) had reported connections with number and operation (the effects of operation on number) and numeration, for example, place value (e.g., Kamii, Lewis, & Jones, 1991).

Estimation requires competence in mental computation. Hanson and Hogan (2000) studied on level of the computational estimation ability and the number of computational estimation strategies with respect to different type of numbers on 45 college students. They prepared the three phases study to identify the students' ability. In the first phase the students tested by 20-item estimation test on the overhead projector. In the second phase students were tested individually to estimate their answers and to think aloud as they arrived their answers. The last phase of the study was containing again a testing with the sufficient time to compute the answers. The researchers concluded that the subjects did fairly well on the integer part of the test but the fraction and decimal part did relatively worse.

Sowder and Wheeler (1989) stated that the abilities to compute mentally and estimate proficiently were related skills. Researchers indicated that good estimators possessed a variety of skills and were flexible in the way they think about numbers (Bestgen, Reys, Rybolt, & Wyatt, 1980; Reys, 1986; Reys, et al., 1991). However, poor estimators had little understanding of what estimation meant. They usually tried to calculate exact answers, and then give an estimate from that answer. They also applied rigid algorithms that had been taught in the classroom, with little understanding of the appropriateness of the strategy (Reys, 1992).

Hope and Sherrill (1987) found that skilled mental computers when compared with unskilled ones used a variety of strategies. Computational estimation was a factor in proficient mental computation. The researchers emphasized that "getting the right answer," was not a unique aim for understanding the students' skills, context and appropriateness of strategy were also involved in proficiency of the good computers. An understanding of the effects of operation on number appears to be essential for flexible mental computation, as some of the strategies that good mental computers employ include decomposing and recomposing number to best suit the operations (Sowder, 1988). For example, during the calculation of 136+199 it could be perform as (136+200)-1. Sowder (1988) suggested that this kind of strategy would be both efficient and reduce demands on working memory, compared with a pen and paper algorithm performed mentally.

The study of estimation performance of middle school students, Sowder- Threadgill (1984) revealed through interviews a number of correct answers with incorrect explanations. In the study, 26 middle school students representing a variety of backgrounds and achievement levels were individually asked to estimate answers to 12 computational problems and explained how they obtained their answers. Results indicated that estimation skills were highly dependent on students' number sense and mental computation performance. This means that students could find the correct answers but they did not explain the reasons of them. Thus, interviews can give better results than written tests. At best, tests might identify students who are good estimators (i.e., score high on tests), and might identify related skills (Rubenstein, 1985); they cannot identify strategies or thinking processes. There was also a possibility that students did not estimate, that is, they computed mentally, or first computed mentally and then rounded to produce an estimate (Levine, 1982; Reys, Bestgen, Rybolt, & Wyatt, 1982; Sowder-Threadgill, 1984; Sowder & Wheeler, 1989).

Brame (1986) stated that quantitative ability is most closely related to estimation ability. Similarly, Rubenstein (1982) found that the mathematical skills, which contributed most to the prediction of estimation performance, were operating with tens, making comparisons, and judging relative size. Moreover, there is evidence found by Yang (1995, p.38) that "skill in computational estimation is associated with the flexibility of using and understanding the structure of number system and operations." Same as Yang's (1995) findings, Paull (1971) found that the ability to estimate answers to arithmetic problem was positively correlated with mathematical and verbal ability and with the ability to solve problems by trial and error. This finding also consistent with Boz (2004) results; she identified that the ninth graders computational estimation ability was positively correlated to their literature score.

According to Carpenter et al. (1976), instruction on estimation and mental computation could provide a possibility for developing number sense, or quantitative intuition. Students who were good at estimation and mental computation were easily able to link symbols to concepts. The researchers stated that estimation and mental computation were not only useful tools in everyday life but they could also lead to better number sense. NCTM (1989) also stated that mental computation and computational estimation require number sense.

According to Rubenstein (1982), estimation appears to have relationships to many goals of mathematics instruction. For example, several mathematics educators have noted its important relationship to problem solving. O'Daffer (1979) and Polya (cited in Rubenstein, 1982) thought that estimating an answer before attempting a solution would motive a pupil to pursue an exact solution. The reviewers of NAEP agreed that estimation requires genuine understanding of basic mathematical concepts and encompasses a variety of mathematical skills (Carpenter et. al., 1976).

With regard to older children, Sowder and Wheeler (1989) emphasized that arithmetical skills, components and attitudes were all important in influencing estimation ability. The concepts that the researchers proposed to be most relevant to estimation ability were:

1. understanding of the role of approximate numbers in estimation
2. understanding that estimation can involve multiple processes and have multiple answers
3. understanding that context can influence the appropriateness of an estimate

Estimation has been seen as a prerequisite to mental computation, a method of arriving at an exact answer without using paper-pencil or technology (Hope, 1986). Hope (1986) reported computational estimation should be increased in schools because of its importance in relation to mental computation. Many research studies agreed that computational estimation is related with the mental computation. In addition, they said that mental computation and computational estimation are to be accomplished without the use of paper and pencil or other tools (Reys, Rybolt, Bestgen, & Wyatt, 1982; Dowker, 1992).

According to Reys (1984), mental computation is important for estimation since it provides the cornerstone necessary for the diverse numeric processes used in the computational estimation. According to him, mental computation had two distinct characteristics. First, it produces an exact answer and second, it is performed mentally without the aid of external devices such as pencil and paper. She found that a person could be competent at mental computation but very poor at computational estimation simultaneously. However, converse is not true, that is, people who can good at computational estimation are not also good at mental computation. Reys and Yang (1998) supported to this result by the findings of their study. The researchers tried to understand sixth and eighth grade Taiwanese students' number sense through the mental and written computation procedures. They asserted that "being able to compute exact answers do not automatically lead to an ability to estimate or judge the reasonableness of answers" (p.231). Thus, it was important that not only to develop students' ability to compute fluently, but also their ability to estimate.

Hope (1986, p. 49) described the close relationship between estimation and mental calculation as stating, "Estimation is a less precise mental calculation." Mental computation is an important skill in its own right but computational estimation greatly increases its potential. Hope (1986) asserted that while students all admire the purity of Mathematics, they should learn to appreciate its flexibility, its adaptability to whatever may be the special purpose in mind. Bennet (cited from Murphy, 1989) concluded that common sense should be developed in stating and interpreting problems and should apply the appropriate tools in finding a quantitative answer to suitable and reasonable approximativeness.

Mental computation enhances a student's understanding of numbers, number properties, and operations on those numbers that is; it improved the number sense of the students. According to Gay (1990), mental computation also promotes flexible thinking and problem solving. Siegel, Goldsmith, and Madson (1982) emphasized the role of mental computation in bringing about a better understanding of the number system and estimation. Mental computation is also useful in its own right. In order to Hope (1986) in everyday world of the consumer and worker there was more need for exact or a reasonably accurate mental calculation than for a pencil-and-paper calculation. Like computational estimation, skill in mental computation is also associated with understanding the structure of the number system. Individuals skilled at mental computation use this understanding to their advantage while those poor at mental calculation tend to try to use mental analogues of paper-pencil algorithm. Because of that, these students could not cover the usefulness of mental computation (Sowder, 2001).

Mental computation and estimation play a valuable role in everyday life. Reys and Reys (1986) stated that surveys show mental computation and estimation are used in more than 80% of all real-world problem solving situations outside the classroom. In daily life people sometimes do not have calculator, paper-pencil or any other devices to make computation that's why they need their brains as stated by Maier (cited in Hope, 1986) "Other computation tools may not always be available, but people always carry their brains with them" (p.47). On the other hand, Reys and Bestgen (1981) stated that it has often been found that students are more successful when computing an exact answer with paper and pencil than when estimating an answer.

Although students more successful on mental computation than computational estimation questions (Boz, 2004), in some topics students had difficulties while conducting the mental computation questions. Students had difficulty on fraction and decimal related questions in both estimation and mental computation formats (Goodman, 1991; Hanson & Hogan, 2000; Rubenstein, 1982; Reys et al., 1995).

Children may learn isolated techniques for dealing with fractions and decimals but not make sense of these. Understanding about fractions and decimals became disconnected from their place in the number system, within the child's mind (Reys & Yang, 1998). Irwin (2001) claimed that evidence that children working on contextualized decimal problems improved their understanding more than children who worked on decontextualised problems. Irwin (2001) suggested that connection and sense making are key elements in the development of strategies for decimals.

Jurdak and Shahin (1999) conducted a study with a group of young street vendors in Beirut. The researchers aimed that to examine the computational strategies of ten young street vendors by describing, comparing, and analyzing the computational strategies used in solving three types of problems in two settings: transactions in the workplace word problems, and computation exercises in a school like settings. The result of the study showed that the school-type algorithms for performing addition, subtraction, and multiplication, which were originally memorized by the subjects without understanding were transformed by the subjects into pieces of rules remembered and combined in personal ways. A significant characteristic of these incorrect rules was that they preserved the form of the correct rules but violated their conceptual base or related understanding. Another striking result of the study was related with the word problems. The word problems were comparable to transactions in the frequency of occurrence of semantically based mental computational strategies and in the high success rate associated with them. Jurdak and Shahin (1999) concluded that the word problems, if meaningfully structured and used, they could provide a pedagogically feasible option to develop the connection between formal computational algorithms and contextual situations of real life.

Underlying children's strategies in approaching problems are their conceptions about number and their understanding of the number system. Place value and number sense are thus essential elements of children's strategic thinking. Ball (1990) and Nik Pa

(1989) found out some common difficulties associated with fractions. These are presented as follow:

1. Students have difficulty with the meaning of operations. In whole number arithmetic, division is understood to mean making smaller, and multiplication is understood to mean making larger. The rules for addition, subtraction, multiplication and division of fractions are easily confused and misapplied (Ball 1990).
2. Students have difficulty interpreting mixed numerals correctly. They may not interpret the number 4 ($\frac{1}{2}$) to mean 4+$\frac{1}{2}$ but rather as $\frac{2}{5}$ (Ball, 1990).
3. Students have difficulty associating meaning with rational numbers expressed as fractions. Students may not be able to recognize that units compared must be same size, and that the numerator and denominator do not refer to distinct regions of a closed figure (Nik Pa, 1989).

According to Johnson (1998), students have difficulty translating one rational number model, whether verbal, pictorial or symbolic into another. Fractions are symbols representing rational numbers may not be associated with real world situations and therefore may not have practical implications than merely computation.

Several studies compare children in different countries to see if there are differences in their approach to computational problems (Anghileri, 1999; McIntosh et al., 1995; Reys & Yang, 1998). These studies suggest that there are differences brought about by instructional focus. Children who were struggling with mathematics tend to continue to rely on counting strategies and additive thinking, while successful children use the more abstract and powerful ways of thinking (Anghileri, 1999).

Callingham (2005) presented a report to initial findings of a model for developing mental computation, which was applied to whole school from kindergarten to grade six. In the model, there were three themes, which are Intellectual quality, Quality learning environment and Significance. It was observed that considerable discussion and interaction between teachers and students throughout all lessons. Hence, all students were confident about taking risks and suggesting alternative approaches, suggesting the model, which was well established. Similarly, Yang (2002) presented a process-oriented activity through a class discussion, which aimed that the sixth grade students learning on fractions. He asserted that through this activity, cooperative learning, and class discussions could reduce students' difficulties on fraction. Thus, according to Yang (2002) number sense develops through communication and debates.

However, Heirdsfield (2000) stated that computational estimation did not support mental computation, which was a contradictory result with many research studies (e.g. Reys, Bestgen, Rybolt, & Wyatt, 1982). According to Heirdsfield (2000) findings, proficient mental computers did not exhibit proficiency in computational estimation. She stated that one reason could be the students were too young to have developed estimation strategies.

On the other hand, many searches asserted that there is a significant relationship between estimation abilities and skill with arithmetic operations, which requires an exact knowledge of numbers (Dowker, 1997; Rubenstein, 1985). According to McChesney and Biddulph (1994), when doing mental computation the strategies that are given below may be used;

- Using derived facts from memorized basic facts,
- Decomposing numbers, often using the nearest ten value as a benchmark
- Grouping numbers in different ways, for instance 100 as four groups of twenty-five

- Subconsciously using properties of the number system such as “the distributive property”
- Employing invented strategies
- Using counting strategies
- Using front-end strategies
- Visualizing the written form of the “standard” algorithms for the “four operations”
- Using pattern in the number sense

It appears that number sense in terms of mental computation could mean having access to a variety of strategies based on an understanding of the number system and how it works.

Teaching basic facts has always has been a part of any successful mathematics program and is very important in developing mental math skills and flexibility applying estimation skills (Leutzinger, 1999). Leutzinger (1999) addressed that too much time spent on repetitive practice instead of exploratory experiences, which gives the students the opportunity to developed thinking strategies on number sense.

2.4 Clinical Interview

Although clinical interview in mathematics education is not a new research tool, it has not been known and applied in research studies much. Therefore, in this section, clinical interview is introduced and explained through a tool for research studies and an assessment tool for mathematics education.

With the development of qualitative methodologies, interviewing has become one of the main tools in mathematics education research. Clinical interview, which is a type of tool for the qualitative methodologies get growing appreciation by researchers; since it is seen as a kind of tool for entering the child mind (Ginsburg, 1981; Groth, 2005; Heirdsfield, 2002; Hunting, 1997; Huntley, Marcus, Kahan & Miller, 2007; Karatas & Guven, 2003).

Clinical interview was originally developed by Piaget (cited in Ginsburg, 1981) for psychological research studies. According to Ginsburg (1981), Piaget intended to explore the wealthy of the children's thought to understand their fundamental activities and to establish the child's cognitive capabilities. Ginsburg (1981) claimed that Piaget produced this method to alternative of the standardized test. Since his basic research goal was to explicate the nature of thought, he realized that it could be performed by helping clinical interview not a standardized test.

The clinical method is generally a diagnostic tool applied to reasoning in children (Opper, 1977). The researchers agreed that clinical interview is not a group testing, it is a dialogue or conversation held in an individual session between an adult, the interviewer, and the child, the subject of the study (Heirdsfield, 2002; Hunting, 1997; Opper, 1977). Although most of the researchers agreed on the individual application of clinical interview, Evens and Houssart (2007) examined the paired interviews in mathematics education. In their study, the researchers were not specified the procedure conducted as clinical interview but they argued that paired interviews could possible tell more about the children approaches on questions. Even and Houssart (2007) discussed how could be form the pairs of clinical interview and how could be identified the factors appear during the interactions.

Similar to Zazkis and Hazzan (1999), Opper (1977) identified that the verbal explanations were particularly valuable for inferring the underlying mental processes of the students. Opper (1977) asserted that the interviewer consequently should make every effort to encourage the child to elaborate on and support students' statements or judgments about the different items presented. On the other hand, she added the child's verbalizations are not only information on his/her thinking and may be supplemented or even at times replaced by observations of the child's actions and manipulations of the objects.

Unfortunately, a technology that makes it possible to observe a learner's mind has not been invented yet. Therefore, only means to speculate about students' thinking or understanding is by analyzing their words and actions (Zazkis, & Hazzan, 1999). In his article, Ginsburg (1981) tried to identify three aims of the mathematical mind activities that should be investigated by clinical interview since it is the most appropriate method for these aims. Although this method could be assisted some other procedures (naturalistic observation and standardized tests) involved many different techniques. He identified that the division of clinical interview's aims through the mathematical thinking activities in threefold. These are based on Piaget's discriminations; the *discovery of cognitive activities, (structures, processes, thought pattern), the identification of cognitive activities, and the evaluation of levels of competence.*

Ginsburg (1981) aimed in his review to achieve better understanding of the clinical method as it can be used in research into mathematical thinking. In order to do that he gave detail information on clinical interview through the three kinds of aims. According to him when it was aimed that *to discover* the cognitive processes actually used by children in a variety of context, the clinical interview was planned as involving an open-ended task and further questions in a contingent manner. The main point of this issue was that the contingency of the questions are the essence of the clinical interview and could not set all of them before the interview. When the exploration was involved, the method was open-ended and employed a kind of naturalistic observation. Besides discovery, when it was aimed that *to identify* the intellectual phenomena which cognitive processes underlying them, the method was relatively focused and may employ elements of experimental procedure.

Some of the mathematics education researchers who are interested in the mathematical thinking try to describe precisely the thought process involved in mathematical tasks. Certainly, it is a complex process to identify the cognitive structure of a child, often it begins with the extensive amount of observations, which may have been produced, by any one of several different underlying structures. According to Ginsburg (1981), the third research purpose to use clinical interview that Piaget identified was *evaluation of competence*, which was involved in given mathematical tasks. In this kind of clinical interviews, the intellectual competence is detecting and identifying. Evaluating the competence contains three components the assessment of motivation, subjective equivalence and strength of belief. As a result, Ginsburg (1981) reviewed Piaget's explanations on clinical interview through the three aims of mathematics education research studies, which were *Discovery, Identification,* and *Competence*.

Golding (2000, p. 520) noted that such interviews allow researchers to "focus research attention more directly on the subjects' processes of addressing mathematical tasks, rather than just on the patterns of correct and incorrect answers in the results they produce." Nowadays, clinical interview is not only a scientific tool for the researchers but also an assessment tool for teachers. Heirdsfield (2002) discussed changing of clinical interview the form of a research tool to assessment tool in her study. She asserted that the conventional way of assessments limited to teachers about the students' thoughts processes, misconceptions and learning difficulties so they need more sophisticated tool, the clinical interview. Similarly, Hunting (1997) claimed that this method should be used as an assessment tool additionally to conventional way of assessment by teachers to plan effective teaching strategies.

Huntley, Marcus, Kahan and Miller (2007) were developed two task-based interview protocols with semi structure probes. Although Opper (1977) defined the clinical interview as a diagnostic tool and should be performed in an individual sessions, Huntley et al. (2007) preferred interviewing students with couples since they aimed to identify the students' thinking on algebraic problems. They observed in the pilot study that the students in pairs felt more at ease. Evens and Houssart (2007) mentioned about the paired interview in mathematics education and claimed that the children were interviewed in pairs provide opportunities for interaction and discussion as well as putting the children at ease. On the other hand, there is no specific explanation for the constructing the pairs that are join the interview together. This couples' interaction obviously changed the direction of the interviews and this is an obstacle for an interview.

Goldin (1998) explained task-based interview in detail description including nature and amount of intervention by interviewer, the extent to which participants are asked to verbalize their thoughts as they work through the tasks, the tools and materials available to them, the interview context, and the equipment used to make records of the interviews. The researcher claimed that it was obtained deeper information beyond students' scores compared with the assessment administered to students in paper-pencil format (Goldin, 2000).

Zazkis and Hazzan (1999) discussed about the selection of the questions of the interviews on mathematics education research studies. They were tied to some outlines of how the researchers select their questions by investigated interviews from kindergarten to university. The researchers identified the article interview questions which were categorized in six titles; *performance questions*, *unexpected why questions*, *twist questions*, *construction questions*, *give an example tasks* and *the reflection questions*. The researchers investigated the interviews were semi-structured, clinical with cognitive orientation on the subject matter. The interviews were clinical one since they all consisted extensive observation and conducted in a clinic, which were usually an office or classrooms. According to Zazkis and Hazzan, (1999), designing (good) questions was only one step in “art and science” of clinical interviewing. Different from Huntley et al. (1997) study, the investigated clinical interviews performed in individually.

Semi-structured means that planned interview with contingent questions following the interviewee’s response. Huntley et al. (1997) meant that “Orientation on the subject matter" as focusing on interviews about mathematical content, where the aim is to reveal students' understanding of mathematical concepts. This is a special case of Piaget's basic research goal, namely, to explicate the nature of thought” (Ginsburg, 1981). Although Zazkis and Hazzan (1999) tried to identify the questions that are asked in the clinical interview they gave a contradictory explanation of one of a researcher’s perspectives; she/he has pointed out that did not believe in pre-planned interview as the best or even adequate way of helping researchers understand mathematical understanding of students. She/he believed in integrating teaching space and research space, where there was no room for pre-selected questions. Acknowledging the complexity of human cognition, that researcher saw any deliberate effort to excavate what is going on in people's head as unsatisfactory. Zazkis and Hazzan (1999) agreed with this contradictory explanation by Ginsburg’s (1981) calling which was the moving beyond the standardized instruments.

Groth (2005) performed a task-based clinical interview on statistical problems, which were presented two different contexts to understand patterns of thinking of the fifteen students. Additionally to interview records the researcher took some field notes and keep students’ written works. In the educators’ interview sessions, questions were asked to identify and describe the patterns of the statistical thinking.

Some other researchers examined the clinical interview method as an assessment tool for evaluating students’ problem solving behaviors (Baki, Karatas, & Guven, 2002; Karatas & Guven, 2003). In their study, they discussed some assessment methods, which were standard test, performance assessment, and essay type questions in the evaluating procedure of problem solving. The researchers agreed that the most usable method for the evaluating the students' problem solving procedures was the clinical interview method.

Opper (1977) emphasized that building a rapport with the child was the one of the important component of clinical interview. To accomplish this rapport issue she suggested that starting with more personal questions to the interview like, his name; age whether he has sister or brother, etc. These more personal questions could then be followed by questions about the interview context.

According to Goldin (2000), the concept of reliability includes measuring the consistency with which a task-based interview is conducted, observations are taken, and inferences are made from the observations using defined criteria. It also includes measures of consistency among different observations intended to permit the same or similar inferences.

CHAPTER 3

METHOD OF THE STUDY

The purpose of the study is to describe the computational estimation strategies used by seventh grade students and to identify the factors, which influence students' estimation ability on numerical questions. In this chapter, research design, subjects of the study, the procedure, the pilot study, measurement of the instruments are discussed.

3.1 Research Design

Qualitative research is the approach of inquiry used in this study. According to research questions, the present study tries to answer "what" questions rather than how or why. Answers to these questions were sought through investigation of five students' perspectives. Therefore, multiple case study was designed for gathering information to address the research questions. According to Yin's case study design typology, embedded-multiple case study design was used for the current study (Yin, 2003; Yıldırım & Şimşek, 2003). In the embedded -multiple case studies, firstly every situation is considered by itself and later compared to each other. In the current research study, each student was concerned as a single case and then compared to each others. On the basis of Yin's (2003) explanation, unit of analysis is relevant to the fundamental problem of defining what the 'case' is; the unit of analysis of the present study is five interviewees in themselves. According to Miles and Huberman (1994), multiple-case sampling adds confidence to findings. It can strengthen the precision, the validity, and stability of the findings (Miles & Huberman, 1994).

3.2 Participants of the Study

In this section, the sampling procedure is described in detail. First, the two steps sampling process is explained and it is followed by brief descriptions of the interviewees.

The first step for the sampling started with the determination of the elementary school. Therefore, an elementary school from a moderately big city located in Aegean region was randomly selected among the forty-six schools. The middle socio economic families' children were attending this selected school. There were a hundred and twenty seventh graders (aged in 12 years old) who were enrolled in the four-classes of the selected elementary school. Three male mathematics teachers were teaching these four classes. One of them was very experienced with 30 year-service in teaching and the others were in their twentieth year of their profession. The necessary permissions for conducting a research study were obtained from the provincial directorate of National Education. The seventh graders constituted the population of the study.

As a second step for sampling process, the researcher picked up seven students according to testing results of four classes of the seventh graders. The scores were listed and seven high scores among others selected for the reaming part of the study. Therefore, purposive sampling was conducted. Purposive sampling is very common sampling procedure for qualitative research studies where Merriam (1998, p. 61) explained it as "purposive sampling emphasizes a criterion based selection of information rich cases from which a researcher can discover, understand and gain more insight on issues crucial for the study." In the current study, the criterion was "the high score" from the tests. Additionally, Miles and Huberman (1994) claimed that qualitative samples tend to be purposive rather than random, which is very important with small number of cases.

Seven students who get higher scores from the tests (word test and numerical test) were selected among a hundred sixteen students. A hundred twenty students reduced a hundred sixteen since one student did not attend testing procedure and three of them uncompleted the tests. The distribution of a hundred sixteen students according to four classes and gender is presented in the Table 3.1 below:

Table 3.1 The distribution of the students in four seventh grade classes

	Male	Female	Total
Class A	16	15	31
Class B	12	18	30
Class C	15	15	30
Class D	18	7	25
Total	61	55	116

Since, these selected seven students' scores were very similar (S1: 9 points out of 15 points, S2: 9 points, S3: 9 points, S4:9 points, S5: 9 points, S6: 8 points, S7: 8 points) to each other and there was a big gap between the scores of the first seven students and eight one (S8: 4 points out of 15), only these seven students got involved in the study. However, during the data gathering procedure two students (S5 and S7) were eliminated from the study because of their lack of communication abilities. As a result, four male and one female student formed the interview group. After elimination of the two students, the participants were enrolled in the same class, which was called as Class A. Brief descriptions of the five students are presented in the next section.

3.2.1 Personal Information about Participants

I collected some information from the three types of teachers who are related with the subjects. First one is the mathematics teacher, identified their mathematics achievements and attitudes in mathematics lessons. Other one is the guidance counselor who advises students about their personal problems. He is collecting information and keeping records about students' family, academic achievements, and some health problems. The last one is the classroom teacher who is responsible for the classroom. He collected information about each student in his classroom, like academic achievement, parental information, personal problems, or family problems. In order to get to know each student deeply, I conducted interviews with these three teachers. The mathematics teacher of Class A has been teaching for two years; therefore, he could explain students' previous semester mathematics achievement, their improvement, or thoughts on mathematics. The classroom teacher takes notes about the students' last semester and this semester grades from all courses. With the aid of these two teachers, students' last semester academic grades (mathematics and language courses only) were gathered and presented in the following Table 3.2. This table is important for deducing some conclusions about students' computational estimation since researchers claimed that subjects' mathematics and literature achievements positively correlate with their estimation performance (e.g., Boz, 2004; Cilingir & Turnuklu, 2009; Montague & van Garderen, 2003). Therefore, students' previous semester grades on mathematics and language courses are reported.

Table 3.2 Students Academic Achievement Scores from Mathematics and Turkish Courses

Interviewees	Language Grade (out of five)	Mathematics Grade (out of five)
Mert	5	5
Ayşe	5	5

Table 3.2 continued

Interviewees	Language Grade (out of five)	Mathematics Grade (out of five)
Deniz	5	4
Nevzat	5	5
Sergen	4	4

In Turkish school system, the highest grade is five for each course at elementary schools. As can be seen in the Table 3.2, except Sergen and Deniz, the others (Ayşe, Mert, and Nevzat) had five out of five for the courses.

The remaining of the section contains specific information of each participant. The presented information about the participants was gathered through teacher interviews, observations, and student interviews.

Mert

Mert's father is a doctor in a clinic and his mother is a nurse in a hospital. He has a sister who is three years older than him, and studying in an Anatolian High School. Mert was an outstanding student among others since he had a great expectation from the future. He was the only person who stated that he want to be a student of Robert College (which is the most selective independent private high school in Turkey. The 146-year-old institution is the oldest American school, still in existence in its original location outside the United States). According to his mathematics teacher, Mert is a very successful student. In the mathematics class, he finds solutions by using unordinary methods for the problems and most of the time insists that his way is better than the teacher's method. The mathematics teacher claimed that Mert has high self-confidence in mathematics lessons. According to classroom teacher, Mert sometimes exhibits aggressive behaviors towards his teachers and classmates. His classroom teacher explained his aggressiveness by stating that these behaviors occur when someone rejects his solutions to a problem. The classroom teacher confirmed Mert has a great self-confidence in general. The guidance counselor pointed out his family. According to guidance counselor' records, Mert's parents are very concerned about their children. Mert sister was also a student of that elementary school three years ago. According to the guidance counselor, Mert's parents got information about their children's success or problems regularly from the teachers. According to guidance counselor, Mert has outstanding ideas and perspectives compared to other students.

Ayşe

Ayşe's father is a manager in a government office, and mother is an elementary school teacher. According to her mathematics teacher, Ayşe is very successful student since she is a hard-working student. He added that in the classroom she listens to the mathematics lesson very mindfully. The mathematics teacher specified Ayşe as a researcher since she does her homework very properly by doing extra studies about her homework. The teacher observed an interesting point about her that is according to mathematics teacher she solves the questions always by following methods that are taught at classroom, not using a practical or shortcut way. The classroom teacher regards Ayşe as a smart, quiet, and compliant person.

Deniz

Deniz's father is a dentist and mother is an official in a government office. The mathematics teacher of Deniz stated that he is very silent in the classroom. He stated that Deniz talks in the mathematics lesson if and only if a question is asked to him. According to the mathematics teacher, his mathematics achievement is moderate and in the mathematics lessons, he is very silent and respectful. The classroom teacher defined Deniz as a good football player and social student. The classroom teacher stated that although he is a silent person in the lessons, he is very active out of class.

The guidance counselor stated that Deniz is a smart student but does not work systematically.

Nevzat

Nevzat's parents are elementary school teachers. The mathematics teacher found Nevzat poor about mathematics. Although his mathematics grade is good enough (five out of five), the mathematics teacher explained this success as saying that "Nevzat is working hard only before the exam otherwise he is not a hard-working student". According to mathematics teacher in the mathematics lessons, he raises his hand neither to answer any questions nor to ask any questions to understand any problems. The classroom teacher stated that Nevzat is a polite and respectful person. According to classroom teacher, since he cannot express himself in a precise way, he may be unsuccessful on his lessons. This observation may explain the mathematics teacher's interpretation about Nevzat's poor mathematics success in classroom contrary to exam results. The guidance counselor made a similar observation stating that Nevzat has some difficulties in expressing himself but he is a smart student in nature.

Sergen

Sergen's parents are both officials in a government office. According to his mathematics teacher, he was more successful last year. He got the highest score among the sixth graders last year. However, the mathematics teachers identified that Sergen lost his ambition about mathematics, so that his achievement decreased relative to last year. According to classroom teacher, Sergen is a skeptic person in his personal life. The classroom teacher explained Sergen's skepticism, as "he wants an explanation for every event in the classroom and wants to be convinced about the situation." According to the teacher, Sergen is a person who sticks strictly to the rules however; rules should be explained to him. The guidance counselor described Sergen as a social student and active in groups. He is a football player in the team and according to the guidance counselor; Sergen has a high level of self-confidence when compared to his peers. The guidance counselor pointed that Sergen's father is very ambitious about Sergen's school achievement and sometimes, he puts some pressure on Sergen.

3.3 Procedure

In this part of the chapter, the procedure of the study is explained. The procedure of the study consists of two main parts; pilot study and main study. In the following Table 3.3, stages of both pilot and the main research are presented:

Table 3.3 The Steps of the Research Study

The pilot study	• An elementary school was selected • Two six-grade classes were selected • One week observation was carried out • Computational Estimation tests were conducted • Six interviewees were selected • Clinical interviews were conducted in three sessions
After the pilot study	• Clinical interview protocols were redesigned • The grade level of the participants was changed • The Computational Estimation Test was redesigned

Table 3.3 Continued

Main study	• An elementary school was selected for the main study • Four classes of seventh graders were selected • The four classes of the seventh graders' mathematics classes were observed for two weeks • Computational Estimation Tests in word and numerical formats were administered to all the classes • Seven interviewees were selected according to test results • Two-session clinical interviews were conducted with five students

3.3.1 Pilot study

The pilot study was conducted in spring semester of 2007-2008 academic year with sixth graders. The aims of the pilot study were first, testing questions in the Computational Estimation Test in both numerical and word formats and the second, piloting the interview protocols in three sessions. Additionally, researcher aimed to gain experience about the clinical interviews.

First by random sampling, an elementary school was selected among the forty-six elementary schools located in the central area of the city. The selected elementary school has two sixth grade classes with forty students. The Computational Estimation Test with 20 items was applied to these two sixth grade classes. The test involved estimation questions on 5 whole numbers, 5 decimals, 5 fractions, and 5 percentages. Researcher produced the test with the help of some other instruments used by other researchers (Berry, 1998; Goodman, 1991; Heinrich, 1998; Reys, Rybolt, Bestgen & Wyatt, 1980). The items all are in numeric form and required to apply only computation. After analyzing the test results, the most successful seven students were selected. According to answers, communication skills, arithmetical and estimation ability levels, two students were eliminated from the study. Therefore, the interview group consisted of five students with one female and four males. There were three interview sessions conducted for each student, so that totally eighteen interviews were recorded by audio-tape and transcribed by the researcher.

The data gathering process had to be separated into three sessions because of some internal and external reasons. Firstly, the time was the main obstacle for the research. The elementary school conducted educational schedule in the mornings with sixth, seventh and eight graders, in the afternoon the first five grades of the classes were present in the school. Therefore, the researcher could apply the interview sessions at the end of the day for the morning group of students who were selected as interviewees.

Because the interviews should be conducted in a restricted time, which was in midday break, the interview sessions must be divided into three parts. The students participated in the study attended in the morning class, they finished the school day at the midday. Interviews could be done in this midday time as half an hour sessions. The students had to take the interviews in three weeks at the same day to finish the prepared questions in the interview schedule. The second reason is that the age constrain of the interviewees. All students were at 6^{th} grade and their mean age is 10-11 years old. As the students were hungry or tired in the midday period, they could not give full attention to the whole part of the questions, so that questions had to be divided into three separate parts. These reasons for dividing the interview sessions are external reasons; on the other hand, there are some internal ones for this separation. When we look into the sessions, we see that all of them are similar to each other within the context of the questions; on the other hand, they are different from each other within the context of the interview questions. That is, all questions required estimation but in some sessions they were presented in numeric form, in some as word problems. Additionally, some of them contained percents and only some of them had options; some of them required self-produced solutions. The examples and explanations of the questions in the three sessions are given in tables below.

In the Table 3.4, whole number and percent related questions are presented as the session 1 question examples. In session 1, eight word problems were asked to the students. The problems contained four types of numbers (whole number, decimal, percent, and fraction) and requested self-produced solutions from the students in the pilot study. Among the eight problems, three were eliminated in the main study since they were percent estimation questions like in the Table 3.4 below.

Table 3.4 Example Questions from Session 1

Questions	**Type**
There are five big cities in a region and according to last census, the population of cities are: first one is 87 419, second one is 92 765, the third one is 90 045, fourth one is 81 974 and the last one is 98 102. What is the average population of this region?	Whole number
Yusuf spent 10.83 YTL in supermarket and he saw that 15 % of this shopping was spent in haphazard manner. According to bill, approximately how much money had been spent in haphazard manner?	Percent

The following table shows, the example of questions in Session 2. The three examples given are among the 12 numerical estimation questions in fraction, decimal and whole number. These questions were used in the main study without any changes because it was observed that there was no problem while producing estimated answers for them. Some of the examples are given in the following Table 3.5.

Table 3.5 Example Questions from Session 2

Questions	Type
31 x 68 x 296	Whole number
0.7 + 0.002 + 0.81	Decimal
$3\frac{1}{2} + 19\ \frac{1}{2} + 12\frac{1}{4}$	Fraction

For each type of the numbers (fraction, decimal and whole numbers), the students were asked to estimate by explaining how they got their answers in the Session 2.

In the third session, only percent-related four questions were asked and the students were wanted to choose one of the given options. The Table 3.6 shows some examples from this session. These questions were not involved in the main study because it was observed that students had some problems while producing the estimated answers for them. For instance, they tried to contextualize the numerical questions at first then tried to estimate. Therefore, finding acceptable estimation might be related with rather than students' ability to conduct estimation but with the ability to change numerical problems to word formats.

Table 3.6 Example Questions from Session 3

Percent Questions	
What is the 25 % of 572?	Less then 200 More then 200 Have to compute
What is the 35% of 37.50?	Less then 12 More then 12 Have to compute

In this session, the students did not produce the estimated answers, they only chose appropriate option among the three options by notifying the given reference numbers. Additionally, researcher wanted students to explain how they selected the answer among the other options, so that it was checked whether students used estimation or not.

According to transcriptions for each session, themes and codes were constructed. These are presented in the Appendix F. As can be seen, these codes and themes are more complicated and amateurish than the codes of main study. An example part of results of pilot study is given in the Appendix G. According to pilot study, the process of coding transcribes could be performed easily. Pilot study produced many contributions to the main study because the pilot study was a detailed long term-process. It took nearly four months to conduct the Computational Estimation Test, interviewee-selecting procedure and perform the three session interviews with participants. The piloting made many changes to the present study. These contributions are discussed below.

After the pilot study, the researcher reexamined most of the critical points for the main qualitative study. First, the subjects' grade level was a critical point for study. After reconsidering capability of the students, seventh graders were chosen in order to sixth graders. There were some reasons to pick up the seventh graders; the first one was the sixth graders' poor performance on estimation and mathematical facts. Another reason is that the selected sixth graders had poor communication skills; hence, the clinical interview sessions were difficult to carry out with them. Additionally, some changes on estimation test were made. The test items included whole numbers, decimals, fractions, and percents. The percent-related questions were eliminated from the test since those questions were requiring different mathematical applications such as context-based requirement for subjects and strong construct of division or multiplication with hundred relationships. Every student in the pilot study correlated the percent questions in daily life money problems and this kind of connection may produce extra variables for the study where it may be difficult to control them.

Moreover, the second session of the interview in the main study was modified after pilot study. In the former one, the affective domain questions that formed the second session of the interview were embedded in the numerical format interviews in the pilot study. If the questions had not been divided into three interview sessions during the pilot study, the interview might have taken a very long time for sixth graders. Therefore, the questions were separated in three types and three interview sessions each of which were applied in thirty minutes were carried out. Although there were seven students, 21 interviews were conducted in pilot study totally but 18 were taken into consideration. However, one student who took a session could be taking the other session after three weeks, sometimes four weeks later. This kind of time interval made students away from the estimation concepts and forgot the topic, which were mentioned in the previous interview. During the pilot study, there was a striking point observed after these interviews. A student, who involved in the first session of interview, until he/she took the second or third sessions, could be more experienced on estimation or sometimes forgot about our talking on estimation. In addition, it was found that students' gaining experience affected the gathered data. This finding was also confirmed by the results of Montague and Garderen (2003) who claimed that estimation ability is likely developmental in mature and as students mature their estimation strategies, they become more sophisticated and their estimation abilities improve. As a result, affective domain questions put together and used in a separate interview domain differently from the numerical format of estimation questions, so that the number of sessions arranged was two not three. Moreover, the time interval between the interviews taken by each different student was one day.

Besides the changing of grade level of the participants, the number of interviews that was conducted with them and elimination of percent-related questions from the pilot test, some other changes were made during the data gathering processes. For example, in the pilot study, an audio recorder was used for the recording the interviews. However, during the transcription of the interviews, face and body movements during the some parts of the conversations were desired to be remembered or reexamined. Therefore, in the main study, a video camera was used and during the transcriptions, some extra observations could be done on body movements through the video recordings. The researcher trained herself to apply the interviews, particularly, to ask follow-up questions after answers of the students.

As a result, the pilot study showed some critical points to be considered by the researcher. These are: grade level is quite important for the clinical interviews, the percent related estimation questions should be separated from the other types of numbers (whole, decimal and fractions), and interview sessions' time should be closer to each other, for instance, it may be in the same week, or may be at most two days between them.

Pilot study produced great contribution to the main study. As discussed above, both the way of conducting the case study, and used tools for data collecting were revised. The revised data collecting tools and procedures are going to be explained in the following section.

3.3.2 Data Collection Tools

This section explains the tools for gathering the data. There are mainly four types of tools, which are classroom observations with taken fieldnotes, the Computational Estimation Test with two parallel forms, clinical interviews, which were conducted twice with participants, and interviews with teachers to get detailed information about the participants, are listed in the Table 3.7. In the following table, the aims of the tools are given briefly and detailed explanations are also presented in the next section under the title of each tool. The tools are presented in orders that are used during the procedure.

Table 3.7 Data Collection Tools

The data collection tools	The aims of the tools
1. Classroom Observations	Researcher conducted two-week observations of four classrooms at the very beginning of the study and took some fieldnotes. The aims of the observations were to get rapport with students, and to understand the teachers and students' interactions, students' perspectives of the "fraction" topic of the mathematics lesson.
2. The Computational Estimation Test (CET)	The test was used in word and numerical formats as parallel forms during the whole class application for identifying the higher estimation achievers. The aim of using CET is to select high scorers for interview sessions.
3. Clinical Interviews	Two sessions of clinical interviews were conducted. In the first session, CET numerical form was asked to students who were also requested to explain their solution procedures. In the second session, semi-structured questions were asked to understand participants' thoughts and feelings about computational estimation.
4. Teachers Interviews	There were five interviews conducted with the three types of teachers who were mathematics teacher, classroom teacher, and guidance counselor. The aim of the interviews with teachers was to get more information about the participants.

The data collecting procedure was started with the classrooms' observation and then following with the others, which are presented in the Table 3.7 above. The details of the steps are explained in the following sections.

3.3.2.1 Classroom Observation

Marshall and Rossman (1999) defined triangulation as an act of bringing more than one sources of data relevant to a single point. To get multiple sources of data, that is, to make the triangulation, the researcher conducted many procedures to confirm the data. In the current study, the researcher conducted two weeks' observation to take fieldnotes. The researcher observed the participants in the four classes before the whole class applications were carried out, and took some field notes during the observations.

The time of observation was planned according to "fraction" topic of mathematic curriculum in seventh graders. Therefore, observations were made while "fraction" topic of the math classes was being taught. The three mathematics teachers' four seventh grade classes were observed in two lesson periods. The focus of the observation was teachers' approaches to estimating fractions, the students' answers, and teachers' reactions to the students' estimated answers. A striking field note observed from class A is given below in Figure 3.1;

Class A: 23 December 2008 8.30 am to 9.10 am Tuesday

…….

Teacher A wrote a question ($\frac{14}{17} + \frac{6}{7}$) on blackboard and asked to the students "before conducting a computation what would you say about the answer? May be bigger than 2 or smaller than 2?" Four of the students raised their hands, and teacher gave them a hearing. Most of them gave unreasonable answers, or gave reasonable answers without any explanation. However, one student (Mert) gave his answer with correct explanation. He used "nearness of fraction to one" concept to explain his answer. Additionally, he checked his answer by conducting exact computation on the board. Then the teacher explained the estimation strategies of fractions by saying "you should control the fraction's numbers both in the denominator and numerator, so that fraction may be bigger or smaller than one, bigger or smaller than a half, or near to zero. These help you to decide where the answers should be."

Figure 3.1 An Example to the Fieldnotes from a Classroom Observation

The observation of the above-mentioned event made me aware of Mert's talent at the mathematics classes. In the other classes, there were a few students who reacted as Mert did.

Additionally, observation gave some evidence about the teachers' views on estimation. Although the Teacher A, who was the teacher of class A, tried to use of estimation in the fraction lesson, the other teachers, Teacher B (teacher of class B and class C) and Teacher C (teacher of class D) did not ask any questions related to estimation.

The other aim of the observations was to build rapport with the students. Since clinical interview requests a relationship, which is trustworthy, warm, honest and close, the rapport with students should be built by the researcher. After the lesson time, the students and researcher made some conversations, as the researcher answered the students' questions like "Why are you sitting at the end of the classroom?, Why are you making a research?, What does it mean being a researcher?" etc.

As a result, the observation, which was conducted even in a shorter period of time, yielded many clues gathered from the students and teachers. The researcher built close relationship with all interviewees by helping and interacting with them during the observation period.

3.3.2.2 Computational Estimation Test

At the beginning of the study, first word format of CET was conducted and then the numeric format of CET was applied to four classes of the seventh graders. The Computational Estimation Test consisted of 15 open-ended questions, which were obtained from different research studies and kept in their original formats (Berry, 1998; Goodman, 1991; Heinrich, 1998; Reys, Rybolt, Bestgen & Wyatt, 1980). However, the questions adapted from the studies were in numerical form and they were converted into word form for the current study to construct the application format of the CET (Berry, 1998; Goodman, 1991; Heinrich, 1998; Reys et al., 1980). Both CET tests (word and numeric) were presented to whole class by using overhead projector and giving 15-20 seconds for each question in the word format CET test and 10-15 seconds for each question in the numeric format CET. A standard answer sheet was given to the students, so that the students would not loose time for rewriting the questions. The answer sheet is given in the Appendix D. The questions in CET (word and numeric) requested self-produced solutions.

In the whole class application, the Computational Estimation test was conducted in two different forms, first in word format and then in numeric format. The researcher at the very beginning of the study prepared the word format of the test according to feedback from the pilot study. For the content validity, more than one procedure was conducted. First, researcher reviewed the questions with a selected seventh grade student who was not involved in the study group and classified as successful one by the student's teachers. Then the questions were discussed with this seventh grader's mathematics teacher. The mathematics teacher gave some insight into the wording of the questions and content. Additionally, a candidate of doctorate student reviewed the questions as an expert. After all the recommendations, some modifications were made on the wording of the test. For example, according to feedbacks, the second question presented in the Table 3.6 below was changed. The first form was "*Burak thought approximately one of every eight marbles was lost during the moving. He had 713 marbles. Approximately how many of his marbles were lost*?"

The examples of the word and numerical formats of the tests were given in the table below. The items were designed to require the same operations to control the students' problem solving abilities.

Table 3.8 Example of Word and Numeric Formats Questions

Example of word format of CET	Example of numeric format of CET
A tailor buys 835.67 m ribbon for dresses. However 0.526 m dirty part should be cut. Approximately, how long is the rest of the part?	835.67— 0.526
Burak should give his 713 marbles to his 8 friends. Approximately, how many marbles does each friend take?	$713 \div 8$

The reason for conducting the word format of the CET is to control variables that are problem-solving abilities, which were discussed in some other studies (Mottram, 1995; Reys et al., 1980; Rubenstein, 1982). These studies reported that the effect of problem solving abilities on word application of estimation questions differs from that on numerical format of estimation questions. According to literature, children might not understand the words and syntactic structure of a problem and/or may have trouble accessing mental representations of quantities when physical referents are not provided. Levine, Jordan and Huttenlocher (1992) developed a nonverbal calculation tasks that eliminated these sources of difficulties. In their study, the task requires a child to reach an exact solution to a calculation problem rather than to make a judgment about the effects of the addition or subtraction transformation. In the current study, researcher tried to control the students' problem solving ability by conducting word format and numeric format of CET, which requested to the same operations. While selecting the students, the researcher matched the questions in both word and numeric format tests of CET. After matching items in two tests, students who got the highest score in both matched items were selected. According to analysis of correlation between the word and numerical formats of the tests, there is a statistically significant correlation ($r=0.42$, $p\langle 0.001$) between the tests. Since the tests were highly correlated to each other, the students who got higher scores for both of them at the same time were chosen for the interview group.

The fifteen open-ended items were formed in word and numerical formats that consisted of three number types; whole numbers, fractions, and decimals, which appeared in four operations. All estimation questions requested self-produced answers. The table below shows which questions contain which operation and which type of number. Additionally in the Table 3.7 it is showed the aims the questions and the ability desired to be observed.

Table 3.9 Distribution of the Questions of CET

	Addition	**Subtraction**	**Multiplication**	**Division**
Whole number	Question 15	Question 3	Question 1	Question 8 Question 11
Aim	to be able to add big numbers	to be able to subtract big numbers	to be able to use ten and multiple of ten in multiplication	o to be able to divide big number by small one o to be able to divide small number by big number
Fraction	Question 10	Question 4	Question 13	Question 6 Question 9
Aim	to be able to add to mixed numbers	to be able to subtract mixed numbers	to be able to multiply mixed numbers	o to be able to divide proper fractions o to be able to divide mixed number by a proper fraction
Decimal	Question 5	Question 2	Question 12 Question 14	Question 7
Aim	to be able to add decimals	to be able to subtract decimals	o to be able to multiply too small decimals o to be able to multiply fraction by a decimal	divide a decimal by a bigger whole number

In the current study, the responses of the interviewees, which were falling within the range of 10% above or below the exact answer, were considered to be acceptable. Therefore, the answers given by students were taken as right answers and coded as one point in data sheet if they fall into the acceptable intervals (the intervals are presented in Appendix B) otherwise the answers were coded as zero. This accepted interval changes according to the subject' age. In most of the studies, the accepted interval used for identifying the correct answers changes. There is no generally accepted interval for estimation related tests. For example, in some research "acceptable interval" was considered to be within 50% of the exact answer (Baroody & Gatzke, 1991; Boz, 2004; Siegel, Goldsmith & Madson, 1982; Crites, 1992; Cilingir & Turnuklu, 2009; Rubenstein, 1982; Montague & Garderen, 2003). On the other hand, Gatzke (1989) took the acceptable estimates within 25% of the actual answer and Mottram (1995), Hanson and Hogan (2000) and Hogan and Brezinski (2003) set this interval as between 10% and 20% of the exact answer. These intervals are changing according to subjects' ages, grade levels, achievement, or estimations background.

Various question formats could be used for assessing the estimation ability. However, these various question formats –open ended, intervals, multiple choices, order of magnitude, and reference number- all have advantages as well as disadvantages (Reys, 1986). In the current study, the Computational Estimation test consists of 15 items designed in the open-ended format; that is, questions requested self-produced answers. Although the open-ended format is recommended by most of the researchers as it is consistent with the notion that there are several good estimates, establishing acceptable intervals for each response and hand scoring the test takes time (Reys, 1986; Rubenstein, 1982).

The tests were administered to 116 students and the test answers were coded in 0-1 coding system. That is if an answer is in the acceptable interval, it is coded as 1 and if not coded as 0. Therefore, 15 points is the highest possible score to get from the test. The reliability of the word and numeric format of CET was 0.61 and 0.63 respectively. Although the reliabilities of the tests are not very high, CET tests are used for selecting the five interviewees for the study.

In the answer sheet given in the Appendix D, students should also answer the two qualitative questions, which are;

- In your opinion, what does estimation mean?
- Where do you use the computational estimation in your daily life?

In the answer sheet, there are two questions related with the students' self-rating of their estimation abilities, and mathematical abilities. They rated themselves according to four-point scale (4=very good, 3=good, 2=moderate 1=poor). Although, these questions were also asked them during the interview sessions, researcher wanted to record the thoughts of students before the interviews with students about estimation. It was observed that students were consistent in themselves.

The tests were administered to four seventh grade classes after completing the observation sessions. In the observation sessions, "fraction related concepts" were taught the students in math lessons. In the mathematics curriculum of the seventh grade, estimation concept is taught only during the topic of fraction. Therefore, the testing procedure started after fraction topic was completed in mathematics lesson.

3.3.2.3 Clinical Interviews

Two weeks after the whole class application procedure was completed, the interview sessions started. The interviews were clinical, semi-structured with cognitive orientation for first session, and then affective orientation for the second session

(Ginsburg, 1997). All interviews were video-recorded and transcribed verbatim by the assistant of the researcher who was a graduate student.

The first interview was semi-structured, and interview questions, which were contingent to the numerical format of CET, were not pre-designed by the researcher. That is the estimation questions were asked to students but the other follow up questions were changed according to students' answers. The interview provided the means of learning what strategies and processes the subjects used in solving different estimation problems. They were asked to explain the thinking process they used to arrive at their estimate. The CET questions were asked one by one and the students were asked to explain how he/she found the answers. Each problem was presented on a card, which was held by the student during the answering process, and all students viewed the problems in the same order. No time limit was imposed for completion of the questions, but the students were instructed to estimate the answers not to compute the exact results. Students gave detailed explanations while solving the estimation questions. Therefore, time was not restricted in the interview sessions. To ensure that the students are mentally active while solving the questions, no paper pencil was provided for students during the interviews.

In the second session, interview was semi-structured and the questions were prepared through a literature review and pilot study observations and results (e.g., Reys et al., 1980; Reys, Reys, Nohda, Ishida, & Shimizu, 1991; Reehm, 1992). The affective domain questions (see Appendix C) were asked in the second session and the aim of these questions was to understand students' perceptions, thinking and feelings on estimation questions both at mathematics class and outside the class. Some examples of the questions of the second interview session are presented in the following Table 3.10.

The second session interview questions were designed regarding some studies' (Berry, 1998; Heinrich, 1998; Rubenstein, 1985) instruments and the pilot study results. Additionally, the mathematics teacher and the guidance counselor's feedbacks on the questions were taken into consideration. An expert on qualitative research also examined the interview questions and made some corrections on them.

Table 3.10 Example Questions from Second Session of the Interview

Questions	Aim of the question
1. What do you think about applying an approximate calculation to a mathematics question? Why?	The purpose of this question was to understand students' viewpoints about mathematics and using estimation in mathematics. The responses to this question constructed the theme of "perception on mathematics" and "perception on estimation." In "perception on mathematics" theme, some beliefs about mathematics like it should give exact result were explained. Similarly, in perception on estimation theme, beliefs about estimation such as it makes or does not make you feel like disregardful about mathematics were explained.
2. What do you think about your achievement on computational estimation ability? Why?	The purpose of this question was to understand the self-confidence of the students when doing estimation and to identify the reasons why they were feeling successful or unsuccessful on estimation. The answer to this question constructed theme of "confidence in ability to do estimation."

Each interview was coded according to a particular classification system reflecting the various strategies and hypothesizes, which influence estimation ability. According to Strauss and Corbin (1998) and Yıldırım and Simsek (2003) there are three types of coding procedure; the coding according to pre determined concepts, the coding the concepts which are gathered from the data and the coding in a general frame (Yıldırım & Simsek, 2003). The researcher conducted the third coding type, which is a combination of first two types. That is, pre-determined concepts were constructed with the help of literature (LeFevre, Greenham, & Waheed, 1993; Reys et al., 1982), and new concepts gathered from pilot study were integrated into pre-determined concepts. There were some modifications on themes and codes during data gathering procedure. As a summary, I constructed the codes with the help of literature on estimation, and the results of pilot study, which was conducted before the main study (See Appendix F) (LeFevre, Greenham, & Waheed, 1993; Reys et al., 1982).

3.3.2.4 Interviews with Teachers

The other triangulation method is the data collected from the teachers' interviews. Three types of teachers could give information about the interviewees of the study.

The researcher made some unstructured interviews with the teachers who were the mathematics teacher of the interviewees, the guidance counselor of the school, and the class-teacher who is the responsible teacher for the class A. The teachers' interviews were conducted in their spare time at teacher's room and counseling room. I met mathematics teacher three times, two times with guidance counselor and one time with classroom teacher. The meetings with guidance counselor and classroom teacher were held after the students' interview sessions in the guidance room. Nevertheless, the mathematics teacher's interview was conducted during the students' data gathering procedure in the teacher rooms.

These interviews were about the interviewees' parental situations, general achievements on school specifically on mathematics, the behaviors in classroom and school, and the relationships with their peers and teachers. Some questions from the interviews of the teachers and the reasons of the questions are exemplified in Table 3.11 below:

Table 3.11 The Examples of Questions Asked to Teachers

	Question asked each teachers	The Aim of the questions
Mathematics teacher	*What do you think about Mert's success in mathematics? Why?*	To learn mathematics teacher's perspectives of the student's success and the reasons for student's success according to him.
Guidance counselor	*Do Mert's parents visit school regularly?*	To understand the interviewees' parental situations.
Classroom teacher	*How does Mert's relationship with peers and his teachers?*	To learn student's behaviors in classroom and school, his relationships with his peers and teachers.

The unstructured interviews generally include open-ended questions that are few in number and intend to elicit views and opinions from participants (Creswell, 2003). Teachers were asked what they know about the interviewees and follow-up questions were asked depending on the teachers' answers. There was no contradictory explanation about the students among the three teachers. Although, mathematics teachers were mostly interested in students' behavior in math class or students' mathematics achievement, the classroom teacher and the guidance counselor of the school could give information different from the achievement of the students. These teachers have observed the students for two years and collected information about them during this time. Therefore, the teachers' opinions on interviewees of current study are essential and important.

3.3.3 Main Study

The data collection session took two and a half months in the fall semester of 2008-2009 academic year in the elementary school. The study started in December 2008 by selecting the elementary school and observing the mathematics classes of each seventh grade classes for two- week period. The researcher took fieldnotes for four classes in two-week period in two mathematics lessons of the seventh graders. Since the researcher was not a teacher in that school, the observation sessions were planned to understand the interactions between the mathematics teacher and students, to understand the communication of the students and teacher during the topic of "fraction" and to build rapport with the seventh grade students. According to Marshall and Rossman (1999), observation is a fundamental and very important method in all qualitative studies; basic reason to use it is to discover complex interactions in natural social settings. Even in the in-depth interview studies, observation plays an important role as the researcher notes the interviewee's body language and emotions in addition to her words (Marshall & Rossman, 1999, p. 107).

After observations of the classroom, the Computational Estimation Test (CET) was administered to four seventh grade classes to identify the students' estimation performance. The CET (see word form in Appendix A and numerical form in Appendix B) was administered by using overhead projector in the four classes in the same way. The researcher applied all testing procedure with her assistant who was a graduate student at Statistics Department. The assistant's task was to control the time for each item with the help of a chronometer and to observe the students' reactions (whether they were writing the questions or computing with / without writing, etc.) during the testing period. During word format of CET, each question took 15 to 20 seconds and 3-second interval was allocated between two questions. Before starting both tests, students were warned about the time and were told, "Not to copy the problem but do the work in their heads." They were provided with answer sheets, which should be used to record the estimated answers for all students. A week after the word form of CET was administered; the numeric form was administered to four classes in the same way by using overhead projector.

The interview sessions were conducted in a clinic, which is the guidance counselor's room in a separate and silent area. The room was allocated for the interview sessions to us for 10 days in two-week period from 12 pm to 13.30 pm. Students were involved in the interview sessions individually in their one and half-hour break time after completing their meal. In each midday break, one interviewee participated in the interview session. The assistance videotaped all interviews by a handy cam and observed the interview sessions for feedback to the researcher in each session. These sessions took about 30 or 45 minutes.

Ginsburg's (1981) reviewed that on the basis of clinical interview definition of Piaget, the aim of mathematics research is to discover cognitive activities such as structures, processes, and thought patterns. It should be achieved by conducting clinical interviews involving an open-ended task and further questions in a contingent manner (Ginsburg, 1981). Researchers agreed that clinical interview is used as a tool for understanding the students' thinking process (Heirdsfield; 2002; Hunting, 1997).

One purpose of the current research was to make students verbalize the process. It was accomplished through the use of a clinical interview protocol (Ginsburg, 1981; Hunting, 1997; Opper, 1977). The clinical interview part was divided into two separate sessions and completed in two weeks' period. In the first part of the interview, students were asked the 15-item CET numerical questions one-by-one and they were asked for their explanations while solving the problems. The questions were written on cards separately; students read them aloud and then started to explain solutions and how they got the answers. While they were giving their answers, researchers asked how and why questions to get more information from the students, for instance, "how do you get that answer, why did you use that number, would you explain by giving some detail for your work, would you give an answer in a different way, etc." This first session took approximately forty minutes for each student. When a student completed the part one interview session the next day, second part of the interview was conducted with that student. So, in two days, one student' interview-sessions were all completed. This part one interviews revealed several important trends for the first question of the study.

In the second interview session, each student was asked the semi-structured questions on computational estimation. The questions were presented in Appendix C. Students explained their thoughts or feelings about the estimation while responding to the asked questions.

3.4 Data Analysis

The following strategies were used for analyzing the qualitative data;

- The matrices were constructed by using interviews
- Fieldnotes were recorded through classroom observations
- Fieldnotes were recorded through teachers interviews
- Fieldnotes were recorded through the interview

The themes and codes were placed into the matrices to display and to analyze the cases. According to Miles and Huberman (1994), matrices essentially involve the crossing of two or more dimensions or variables to see how they interact. Therefore, to identify the interactions of cases (interviewees) and themes, matrices were constructed. These matrices lend me first, to conduct a theme-oriented analysis and then to give a chance to expand to a more holistic case oriented analysis.

Before starting the construct the data analysis matrix, all interviews were transcribed and printed out. I read transcribes in three times, first reading was for understanding the students' answers and remembering the interview session. Second reading was performed with the watching the records of video camera. This time, it was aimed to see students' gestures and body movements and also check the transcribes. In the last time, I read the transcribes for coding the selected ideas and words (see an example of coding transcribe in Appendix I). Each of the five students' transcribes were read three times. After coding all transcribes of interviewees, codes were listed. Listed codes were combined with each other and the researcher tried to find a link among them. Themes, which were general concepts that were representing all codes, were constructed. The produced themes and codes listed in Appendix E are generally based on results of the pilot study and literature review that are related with the estimation ability.

The data analysis matrices involved students in the columns and questions in the row. In the intersections of the cells, there are some comments, and codes are written. The comments are very helpful for producing the connection among the themes and also students. An example of these matrices, which was worked on it by researcher, is given in the Appendix J.

The students' estimation strategies and factors that associated with computational estimation strategies, were analyzed in two dimensions "cognitive and affective" which were divided into two and five themes, respectively (see Appendix E).

Besides the matrices, some fieldnotes were collected from classroom observations, interviews with the teachers and even interviews with the students. These fieldnotes contain sentences and paragraphs that are reflecting a more personal account of the course of the inquiry (Bogdan & Biklen, 1998). I collected fieldnotes as memos. Miles and Huberman (1994) stated that memos are primarily conceptual in intent. Therefore, my aim was to write memos not just to report the data but also to tie different pieces of data into the defined themes. Miles and Huberman (1994) emphasized that memos are one of the most useful and powerful sense making tools at hand. I wrote memos about classroom observations, the teachers' interviews, and about the interviewees after each interview with them. Some comments are written in the exceptions of interviewees in result chapter under "researcher note" title to make situation more understandable and visible.

A double coding procedure was used to identify and categorize students' responses to each item (Miles & Huberman, 1994). A measure of reliability was obtained by randomly choosing a subject and having another coder independently code the responses from the transcribed interviews. There was 95 % of agreement between the coders. Although, this is a high level of inter-rater agreement, there were some disagreed points, which were later discussed and full agreement was reached on codes.

3.5 Validity of the Study

Lincoln and Guba (1985) proposed four criteria for judging the soundness of qualitative research and explicitly offered these as alternatives to more traditional quantitatively oriented criteria. These are credibility (internal validity), transferability (external validity), dependability (reliability), and confirmability (objectivity).

The credibility criterion requires demonstrating that the results of qualitative research are credible or believable from the perspective of the participant in the research (Lincoln & Guba, 1985). I presented the analysis' results through the defined themes in an understandable and clear way. The inferences and interpretations are supported with the interviewees' quotations.

Transferability refers to the degree to which the results of the study can be generalized or transferred to other contexts or settings (Lincoln & Guba, 1985). To "transfer" the results to a different context is related with the judgment of how sensible the transfer is. The qualitative researcher can improve transferability by doing a careful and accurate description of the research context and the assumptions that are central to the research. The steps of the procedure, sampling processes and properties of interviewees, the instrumentations of the study, the interview details all were explained in detail in the current chapter so that in some extent the results of the study could be transferrable to other contexts.

Dependability is analogous to reliability, that is, the consistency of observing the same finding under similar circumstances (Lincoln & Guba, 1985). In the present study, detailed explanations of how the data collected and analyzed were provided in the study. Lodico, Spaulding, and Voegtle (2006) stated that the dependability is often the difference between an experiential report that simply summarizes a researcher's conclusions and research-based qualitative study that includes a systematic explanation of methods. Although in the study no raw data were served, certain explanations about the procedures are given.

Confirmability refers to the extent that the research findings can be confirmed or corroborated by others. It is analogous to objectivity, that is, the extent to which a researcher is aware of or accounts for individual subjectivity or bias (Lincoln & Guba, 1985). Lincoln and Guba (1985, p.320) referred to the degree to which the researcher can demonstrate the neutrality of the research interpretations through a "confirmability audit." This means providing an audit trail consisting of 1) raw data; 2) analysis notes; 3) reconstruction and synthesis products; 4) process notes; 5) personal notes; and 6) preliminary developmental information. I wrote fieldnotes, memos, and personal notes to enhance the confirmability of the study. Some crucial examples of transcripts are embedded in the report. Naturally, if the reader examines these examples of transcripts, then the interpretations and results are maximally confirmable.

CHAPTER 4

RESULTS

This chapter includes the results of the analyses through the explained methods. The chapter is divided into two sections according to research questions. In the first section, the strategies of the students are discussed in three subtitles, which are called reformulation, translation, and compensation. The second section presents factors associated with the students' computational estimation strategies. These factors are firstly separated into two categories, cognitive and affective then; each category is explained by the themes according to coded data. The cognitive factors examine in two main themes; number sense and mental computation. Affective factors are explained in two themes, mathematics related affective factors, and estimation related affective factors.

4.1 Computational Estimation Strategies

The first question of the study is "Which strategies do 7th grade students use in computational estimation tasks?" According to data gathered from interviews a number of strategies identified used by students. These strategies can be collected under three main titles, which are called as reformulation, translation, and compensation. This kind of grouping of the strategies confirmed many other researches and this is helpful for understanding the underlying construct of the strategies and properties of them (e.g., Reys, Reys & Penafiel, 1991; Reys, Reys, Nohda, Ishida, & Shimizu, 1991). In the following Table 4.1, these three strategies are exemplified from the answers of the interviewees:

Table 4.1 Strategy Table

Strategy	Subcategories of strategies	Example
Reformulation	• rule based rounding ❖ nearness of 0, ½ or 1	• 835.67-0.526→836-1 since after decimal point number should be round upper. • $14\frac{3}{4} \div \frac{5}{8} \rightarrow 15 \div 1/2$
	• situation based rounding	835.67-0.526→835-0.5
	• compatible numbers	835.67-0.526→835.5-0.5 16. 272 $\div$36→ 16$\div$36 → 16 is almost half of 36 so answer is 0,5
	• truncation ❖ ignoring fraction part of mixed number ❖ ignoring decimal part ❖ ignoring too small decimal	$3\frac{1}{2}$ x $10\frac{1}{8}$→3 x 10 16.272$\div$36→ 16$\div$36 0.7 + 0.002 + 0.81→ 0.7+0.81→1+1
Translation	convert addition to multiplication	87 419 + 92 765 + 90 045 + 81 974 + 98 102→ 90 000 x 5
	convert division to fraction	713$\div$8→720/8
Compensation	intermediate compensation	835.67-0.526→835.6-0.5
	final compensation	835.67-0.526→836-0.5=835.5 a little bit less than 835.5 ; the result may be between 830-830.5

According to Table 4.1, it cab be seen that reformulation was observed in four forms in the answers of interviewees. These are *rule based rounding*, *situation based rounding*, *compatible numbers* and *truncations*. In the Table 4.1, *rule based rounding* is exemplified with a whole number application and fraction application, where the strategy on fraction called as *nearness of 0, ½ or 1*. Rule based rounding is depending on taught rules, which is defined as a number should be rounded upwards if it ended with five or more; and the number should be rounded downward if it ended less than five. Other kind of rounding is *situation based rounding,* which is exemplified by a decimal application. In the questions, the decimal parts were managed in order to interviewees' expectance from the result. The other reformulation strategy is *compatible numbers,* which require combining complementary couples of numbers to each others. Two decimal number applications are given as exemplification of the *compatible numbers* strategy. The last reformulation strategy identified from interviews is *truncation*. It was used in decimal and fraction applications, which contains codes as *ignoring the fraction part of mixed numbers, ignoring decimal part,* and *ignoring the too small decimals*.

Translation and compensation strategies were exemplified by interviewees' answers in Table 4.1. Translation can be conducted in two ways, by *converting addition to multiplication* and *converting division to fraction*. Compensation strategy can be performed as *intermediate* and *final compensation*. The intermediate compensation requires revising numbers according to each others, where one of them is rounded upper, then other one should be rounded downwards to compensate the pay off. Different from intermediate compensation, final compensation modify the result. That is if the numbers were rounded upwards to get more reasonable result, at the end of the operation result should be downwards to compensate the pay off. The example question from decimal number is given for each compensation types in the above Table 4.1.

The interview questions consisted of three types of numbers; whole numbers, decimal and fractions with four operations (addition, multiplication, division, and subtraction). In the following sections, it is discussed students' strategies according to these three types of numbers.

In the next sections, the results are given according to reformulation in whole numbers, decimals, and fractions with the summary tables after each sections. Similarly, the findings on the translation strategy in whole numbers, and decimals are discussed in the sections together with the summary tables. The last strategy which is compensation is discussed in whole numbers, and decimals attached with the summary tables at the end of the strategies section.

4.1.1 Reformulation

Reformulation means changing the numbers into numbers that are more manageable by using rounding, truncation or compatible numbers processes. In the theme list, reformulation is investigated under four codes; *rule based rounding, situation based rounding, compatible numbers and truncation.* According to these codes, results are given and discussed under subtitle of each number types. Next section presents the results of the data on reformulation strategy in the whole number questions.

4.1.1.1 Reformulation in Whole Numbers

There are five whole number questions (Q1, Q3, Q8, Q11, and Q15) in the Computational Estimation Test (See Appendix A). These are given in the form of addition, subtraction, multiplication, and division. All subjects started the operation by reformulating, which are rounding upwards or downwards of the numbers. However, it could be performed as rule stated or according to requirements of questions. According to interviews, students mostly (almost 70% of the used strategy was rule based among the reformulation strategies) preferred *rule based rounding* process, which is taught at school and textbooks where the numbers are rounded up when they end with five or more and they are rounded down when they end with less than five. According to Reys (1993), school based learned strategy, which is the rule based rounding is the most preferred strategy among the all age groups and known as the unique strategy for estimation. Reys' (1993) finding confirmed in the study, where the rule rounding was one of the most used strategies. Among the used strategies which were a hundred thirty six, thirty one of them were rounding strategy, that is 23 % of the strategies were labeled as rounding from the transcribes of the interviews.

One of the interviewee, Sergen, explained how he used the rule based rounding strategy in Q3 (7465—572) in the following excerpt;

> ***Sergen:*** In the first number's "four hundred sixty five" should be rounded "five hundred" (7465 →7500), since sixty-five is more then fifty, and second number's seventy two should be rounded up (547 → 600) because of the same reason.
>
> (EX1)

Sergen preferred to use rule based rounding in all whole number questions, except Q8 (713 ÷8), which showed that he could not think any other rounding because the only learned strategy for him is this kind of rounding.

On the other hand, there are some exceptional situations among the students, for example, Nevzat preferred the *situation based rounding* for the Q3 (7465—572) since the numbers could be matched to each other according to him. Nevzat explained his answer as follows;

> ***Nevzat:*** The answer is 7000.
>
> **Researcher:** How did you find?
>
> ***Nevzat:*** I round the first number to 7500, hımm.
>
> **Researcher:** Why did you round like this?

> ***Nevzat:*** Because the second number is fifty thousand something. When I round the second one to 500, the operation becomes easier.
>
> (EX2)

In this question, Nevzat figured out the matching numbers and performed the operation easily. Q8 ($713 \div 8$) is an exceptional question, in which *rule based rounding* is not used by interviewees, except from Nevzat. Other interviewees preferred *situation based rounding* and *multiplication table* for controlling the multiplicands of eight. Although the rounding rule states that 713 should be truncated into 700, four of the interviewees rounded the number 713 to 720. Then they tired to remember the multiplication table. The following excerpt is an explanation of perspective of Ayşe;

> ***Ayşe:*** Seven hundred thirteen divided by eight, hımm, I should remember the multiplication table of eights.
>
> ***Researcher:*** Why do you think so?
>
> ***Ayşe:*** Because there should be a number approximately seventy in the multiplication table. Eight times eight….hımm… sixty-four. Eight times nine... yeap… seventy-two!
>
> ***Researcher:*** What is the answer?
>
> ***Ayşe:*** When I round the first one seven hundred twenty, the answer is ninety.
>
> (EX3)

In this kind of rounding, interviewees tried to check their divisions by using multiplication operation. Besides considering the division operation, they wanted to use the multiplication table for eights and found that 72 was multiplication result of 9 by 8. The students used reversed operation that is in order to divide the numbers they check multiplication of the divisor.

However, Nevzat performed rule based rounding in this question. He rounded the first number to 700 and divided by 7 instead of 8. Then he found “a hundred” as an answer, which was in predefined acceptable interval for the question.

Similar to previous division question, in Q11 (474 257÷8 127) all of the students used the *rule based rounding* and *multiplication table.* Sergen, Nevzat and Ayşe performed rounding to the nearest thousand that is they rounded 474 257 to 470 000. But since 470 000 could not be divided by 8 000 easily, they changed their mind after a while. For example, Sergen explained his solution in the following excerpt;

> ***Sergen:*** The first number is rounded 470 000.
>
> **Researcher:** Why?
>
> ***Sergen:*** Because two hundred fifty seven (257) can be ignored and think as there are zeros. Similar with this rounding I can substitute a hundred twenty seven (127) by zeros so that the second one became 8 000.
>
> **Researcher:** Next?
>
> ***Sergen:*** Hımm, this division is a little bit difficult. I want to change the 470 into 472. Does it work? Hımmm. I think no.
>
> **Researcher:** Ok. Let us think about another way. You erased the zeros?
>
> ***Sergen:*** yeap. Since both numbers have three zeros, I can remove them.
>
> **Researcher:** Ok. Then why did you change the 470 to 472?
>
> ***Sergen:*** Because in one of the previous questions, eight times nine was seventy two. So that I want to get 72. But I think 472 could not divide by 8.
>
> **Researcher:** Yeap. But you can change the numbers into more manageable forms. Do you?
>
> ***Sergen:*** I think so. Yes, the first one should be 480 so that it could be divided by 8 and it should be sixty…
>
> (EX4)

On the other hand, Deniz chose to round the 474 257 to nearest ten thousands, that is 500 000. He stated his solution in the following excerpt;

> ***Deniz:*** The first number must be 500 000.
>
> **Researcher:** Why do you think so?

Deniz: Because 474 is close to 500.

Researcher: Ok. Then?

Deniz: Hımm. I must divide five hundred by eight. Hımm…

(Researcher's Note: From his eye movements it can be understood that he was trying to perform a standard division algorithm in his head)

Researcher: Deniz you do not have to give exact answer. Give me an approximate solution.

Deniz: Ok. Then, when I round 8 to 10, the operation became five hundred divided by ten. It is easier. Therefore, the answer is around a fifty.

(EX5)

These two questions (Q8: 713 ÷8 and Q11: 474 257÷8 127) showed that in division questions students prefer to use multiplication table procedure which is revised operation of division. Most of them (four of the five interviewees) rounded the numbers 474 257 according to multiplication table of eights to find the solution.

Even rule based rounding strategy used almost all question, some of the interviewees could prefer different strategies from rule based rounding. For example, in Q15 (87 419 + 92 765 + 90 045 + 81 974 + 98 102), two of the interviewees chose the strategy except from the rule based rounding. These interviewees were Nevzat and Sergen. Nevzat chose the *situation based rounding*, whereas Sergen preferred to use the *compatible numbers*. In the following excerpt, Sergen explains how he performed the strategy;

Sergen: I round the first number 87 419 to 87 000. Then the second one can be rounded to 93 000.

Researcher: Why did you round these numbers like that?

Sergen: Because, I want to get ten by adding seven and three. Seven comes from at the end of the first number and three comes from at the end of the second number.

(EX6)

This type of reformulation, called *compatible numbers*, requests a kind of ability to convey the mental computation, which demands exact computation. Therefore, it can be said that Sergen might have good mental computation because of his ability to conduct compatible numbers strategy with giving an acceptable answers.

On the other hand, Nevzat performed situation based rounding for Q15 (87 419 + 92 765 + 90 045 + 81 974 + 98 102). He stated that "*the all numbers are very close to 90 000 so that I can round all five numbers to 90 000.*" Although he was not started the question with using the situation based rounding, after he couldn't rounded each numbers separately and after realizing that rounding each number separately was very complex process, he changed his mind and rounded all numbers to 90 000 without concerning one by one.

Ayşe was the person who preferred the rule based rounding by rounding all numbers to nearest a hundred, i.e., 87 419 to 90 000 and 81 972 to 80 000. She thought that 81 972 should be round upwards since 81 was close to 80 not 90. Among the five interviewees, only one (Ayşe) preferred rule based rounding, other one (Sergen) chose compatible numbers and the rest of them used situation based rounding.

Generally, all interviewees tended to use exact computation but specifically two of them were more eager to conduct mental computation than others. One of them was Ayşe. Because of her tendency to exact computation, she used rule based rounding and compatible numbers strategy most frequently in the questions. These strategies are depending on exact computation procedures. Ayşe used compatible numbers in Q3 (7465—572) and in following excerpt shows her perspective;

> ***Ayşe:*** The first number could be round to 7500. hımmm. No, no. It should be 7470.

Researcher: why did you change your mind?

Ayşe: Because if it is so, there is too much difference. The second one is more appropriate. The result is six thousand nine hundred.

Researcher: The six thousand nine hundred?

Ayşe: The second one is 570. Therefore, the result is six thousand nine hundred.

Researcher: How did you find this result so fast?

Ayşe: Because, the subtraction became very easy after I rounded the numbers to similar numbers. I mean that since the ends of the two numbers (7470 and 570) are same, the subtraction is easy.

(EX7)

Four of the students used reformulation in once that is, when students used rounding in a question, then they did not use it for another time in the same question. On the other hand, Mert used reformulation in many times to make operation easier for that question. This kind of flexible usage of the strategies may be seen a characteristic of the good estimators (Lemaire et al., 2000; Reys et al., 1980; Sowder, 1992). As an example, in the Q1 (31 x 68 x 296), Mert rounded the numbers as rule stated at first and then got the first multiplication result as 2100 (result of 30x70). After that, he stated his solution as follows,

Mert: Thirty multiplied by seventy is two thousand and a hundred. Then the multiplication of two thousand times three is approximately six hundred thousand.

Researcher: How did you get this result?

Mert: When we remove the two zeros of two thousand and a hundred, it became 21. Then in order to multiply twenty-one (21) by three hundred, I can multiply twenty (20) by three in which the result is sixty. The four zeros, which two of them, come from two thousand a hundred and the other two of them come from three hundred, should put at the end of sixty. Therefore, the answer is six hundred thousand.

(EX8)

According to excerpt, Mert conducted rounding procedure twice in Q1 (31 x 68 x 296). As can be identified that he was a flexible strategy user. This flexibility could be an evidence for his high tolerance for error.

In the Table 4.2, as can be seen easily, all five students preferred to use *rule based rounding* strategy for Q1 (31 x 68 x 296) and Q11 (474 257 ÷8 127). In the other questions, Q3 (7465 - 572) and Q15 (87 419 + 92 765 + 90 045 + 81 974 + 98 102), one student used *situation based rounding*, and another student among the five of them used *compatible numbers* strategies. Among the whole number questions, Q8 (713 ÷ 8) is an exceptional question since *situation based rounding* strategy was used by four of the five interviewees in this question. For the current interview group, it can not be seen a pattern for conducting the strategies in whole number questions. There are two division questions, which are Q8 (713 ÷ 8) and Q11 (474 257 ÷8 127), in the first one students preferred to use rule based rounding reformulation strategy where, in the second one most of them (four of the five students) preferred to use the situation based rounding. Therefore, a pattern could not be found in students' strategy using even in the question, which was requesting the same operations. The solution process of the two division problems (Q8: 713 ÷ 8 and Q11: 474 257 ÷8 127) are two examples for the previous claim.

Table 4.2 Reformulation Strategies Used In Whole Number Questions Based on Codes

Questions	Interviewees' Strategies				
	Deniz	Mert	Sergen	Nevzat	Ayşe
Q1 31 x 68 x 296	• rule based rounding	• rule based rounding	• rule based rounding	• rule based rounding	• rule based rounding
Q3 7465—572	• rule based rounding	• rule based rounding	• rule based rounding	• situation based rounding	• compatible numbers
Q8 713 ÷ 8	• situation based rounding	• situation based rounding	• situation based rounding	• rule based rounding	• situation based rounding
Q11 474 257 ÷ 8 127	• rule based rounding	• rule based rounding	• rule based rounding	• rule based rounding	• rule based rounding • situation based rounding
Q15 87 419 92 765 90 045 81 974 + 98 102	• rule based rounding	• rule based rounding	• compatible numbers	• situation based rounding	• rule based rounding

4.1.1.2 Reformulation in Decimals

In the decimal related questions (Q2, Q5, Q7, Q12 and Q14 see in Appendix A) as can be seen in Table 4.3, the interviewees mostly used the *rule based rounding strategy*.

In fact students had difficulties to apply estimation strategies to decimals since they were not used use it. Therefore their first reaction to do decimal questions was conducting standard paper pencil procedures. For instance, in Q2 (835.67— 0.526), although two of the students tried to perform estimate without standard algorithm, the others prefer making the numbers of decimal places equal in decimal questions. These were Nevzat and Sergen, who preferred to make numbers equal in decimal places for the Q2 (835.67— 0.526). Sergen explained her solution presented as follow;

> ***Sergen:*** One zero could be added to first number's end. So, the number becomes 835.670. The second number rounds to 0.500 (*He read the number as zero point five thousand)*
>
> **Researcher:** Why did you need to add the zero?
>
> ***Sergen:*** Because when I am doing the subtraction with decimals, the numbers of digits should be the same for both numbers.
>
> **Researcher:** Ok. How do you subtract these numbers?
>
> ***Sergen:*** I round the first number 835.700 then I subtract five hundred from the seven hundred. Therefore, the answer is eight hundred thirty five point two hundred.
>
> (EX9)

If the researcher did not force Sergen and Nevzat to do estimate the question, they had wanted to perform exact computation for Q2 (835.67— 0.526). Even this warning did not stop them to conduct estimate after exact computation procedure. There are findings of some research studies, which confirmed the result obtained from the excerpts of interviews with Sergen and Nevzat (Carpenter, Coburn, Reys., & Wilson, 1976; Carpenter, Kepner, Corbitt, Lindquist, & Reys, 1980; Sowder-Threadgill, 1984). In the studies, it was identified that students first computed exactly and then rounded the obtained results in some estimation questions. As a similar situation, in the current study, Sergen performed this kind of estimation procedure. From this kind of reactions, it may be said that students couldn't understand the usage of estimation in mathematics.

In the current study, students' preference of exact computation rather than estimation may occur because of the interviewees' confidence in their mathematical performance or giving less importance to estimation. Because, especially Sergen classified himself poor estimator and clearly stated that he did not give importance to estimation. He thought he could compute perfectly in his head but he couldn't perform estimation.

On the other hand, according to interview transcribes, even good estimators as Mert used similar procedure with Sergen. However, the reason for Mert's preference was different than Sergen. Mert used exact computation process after researcher question, which was requested closer answer for the Q2 (835.67— 0.526). It was asked such a question since, Mert found his first answer in an acceptably large interval. Then he performed as below:

> ***Mert:*** The first number's decimal part could be ignored. Then the number becomes 835 and the second number is rounded as 1. Therefore, the result of subtraction is 834.
>
> **Researcher:** Ok. Nevertheless, this answer is far away from the exact answer. Would you find a result closer to exact answer?
>
> ***Mert:*** Ok. I can round the first number as 835.650 and the second as 0.500. The operation became 835.650 minus 0.500, the result is around 835.150, and this may be rounded to 835.
>
> (EX10).

The second solution of Mert was more precise. He obtained the exact answer in his second solving procedure. This might be evidence to his high mental computation performance. In Mert's situation, it may be concluded that giving an answer in a large interval is a kind of evidence for high tolerance for error, but in Deniz's situation, it was not related with the tolerance for error.

Deniz was the other interviewee who found the answer in large interval like Mert's first solution in Q2 (835.67— 0.526). Nevertheless, the explanations of Mert and Deniz were different from each other. Since Mert did not feel uncomfortable with some pay off, he chose to perform the operation in large interval but Deniz estimated in large interval because of preference of rounding rule. Deniz explained his solution as below:

> ***Deniz:*** hım. The first number could be 836 and the other is 1.
>
> **Researcher:** Deniz why did you round the number like these?
>
> ***Deniz:*** According to the rule of rounding if the decimal part is bigger than five I should round number to upper whole number. The first number's decimal part is bigger than fifty I mean five…They are same. So that, I rounded the first decimal to 836. The second one also is bigger then five and then it becomes 1.
>
> (EX11)

In decimal related questions, students conducted another reformulation strategy, which was not used in whole numbers. This was the truncation strategy. It means rounding the numbers only backwards. According to data, interviewees used this strategy in fraction and decimal related questions and gathered data coded as *ignoring too small decimal, ignoring decimal parts,* and *ignoring fraction parts of mixed numbers*.

In Q7 ($16.272 \div 36$) all interviewees used the truncation by *ignoring the decimal parts*. Although ignoring the decimal part is seen as the same procedure as the rule based rounding strategy, in fact they are different. Students truncated the first number to 16, not because the decimal part is smaller then 500 (where rounding rule asserted in this procedure) but also they want to use a whole number instead of the decimal one in this question. For example, the reason can be seen in following statement from interview with Deniz clearly;

> ***Deniz:*** To make the decimal a whole number I can erase the numbers after the coma.
>
> (EX12)

As seen from the excerpt, in his explanation Deniz clarified that he did not use rule based rounding he used truncation strategy. If the decimal part were bigger than five, he ignored the parts again, because his aim was to get rid of the decimal parts of the number.

Other reformulation strategy was *compatible number*s. In the previous question (Q7: $16.272 \div 36$) Mert and Ayşe used *compatible numbers* strategy that built a relationship between 16 and 36. Mert explained this relationship as follows;

> ***Mert:*** 16 and 36 have half-twice relationship between them.
>
> **Researcher*:*** What does it mean?
>
> ***Mert:*** It means half of the 36 is almost 16 or twice of 16 is almost 36.
>
> **Researcher:** Ok. So what is the answer?
>
> ***Mert:*** It should be 0.5
>
> (EX13)

The truncation strategy was observed decimal related questions easily. Mert and Deniz preferred to use truncation strategy as the form of *ignoring too small decimal* *number* for in Q5 (0.7 + 0.002 + 0.81). Two of the five students solved this question by using the strategy where the second decimal is ignored and not involved in the addition process. The following is an excerpt from interview with Mert;

> ***Mert:*** The first number is seen as 1 and also the third number is seen as 1.
>
> **Researcher:** Why do you think so?
>
> ***Mert:*** Since they are very close to 1.Therefore, the addition is almost two.
>
> **Researcher:** Ok. But what about the second decimal?
>
> ***Mert:*** Oh yes, it doesn't need to use in this addition.
>
> **Researcher:** Why?
>
> ***Mert:*** It is so small, almost zero. It does not put much on the result.
>
> (EX14)

However, other students (Sergen, Nevzat and Ayşe) could not use any estimation strategies in the Q5 (0.7 + 0.002 + 0.81); they tried to add the decimals by *standard addition procedure* which is the mental computation process. Nevzat and Sergen were confused about the *decimal places of the numbers* and gave unacceptable results. However, conducting same mental computation procedure, Ayşe obtained the exact result of the question. They all performed adding zeros after the numbers on the decimal parts *to make same number of digits* of the decimal parts. This shows that these three students (Ayşe, Nevzat and Sergen) were insisting on using exact computation procedure rather than estimation procedure. They addicted to conduct exact computation even they felt that the question was difficult to solve by mentally. They might not give much importance to estimation while solving the mathematics questions. This is discussed through the following chapter.

Conducting the r*ule based rounding* for decimals may cause some problems for interviewees. As it can be seen in Table 4.3 presented at the end of this section, Q12 (98.6 x 0.041) could not be completed by any of the students. The most important reason was insisting on to apply the rule based rounding. Two of the interviewees, Deniz and Mert, confessed that according to rule 0.041 must be rounded to zero. Therefore, they found the answer as zero, which was not an acceptable answer.

On the other hand, other three students (Sergen, Nevzat and Ayşe) discriminated that the answer could not be zero, but they couldn't find answer, either. The reason why they could not perform the solution was related with the lack of knowledge about multiplication of a hundred by a decimal. The following excerpt is from interview with Sergen;

> ***Sergen:*** The result should be smaller than the current one. Since the first decimal, multiply by zero point something. Also it should not be zero, since there are forty one at the end of the zeros.
>
> ***Researcher:*** Ok. So what should be?
>
> ***Sergen:*** I took a hundred instead of first decimal and forty one for the second one.
>
> ***Researcher:*** hımm. But you said that the result should be smaller than the first number, didn't you?
>
> ***Sergen:*** Yes. I did. But hımm I couldn't remember. There should be a rule, something related with the multiplication with a hundred. I think we erase zeros from the end of the number. No, we should add I think. Or I guess we should remove the coma. Hımm. I forget that
>
> (EX15).

As can be seen from the excerpt above, student tried to remember a rule about multiplication a decimal by a hundred. This lack of ability with multiplying a power of ten with a decimal is taken into consideration as "number sense ability." The researchers stated that the ability to multiply and divide mentally by powers of ten is an important skill for the number sense (Sowder & Schappella, 1994). This question is the most problematic question among the fifteen questions; none of students obtained acceptable answer for this question.

Among the decimal related questions, Q14 ($1\frac{1}{2}$ x 1.67) was a combination of decimal and fraction numbers. This question could be solved in two ways; first, two of numbers might be converted into decimal, second, two of them might be concerned as fractions. Although these conversions are discussed in the translation strategy, they preferred to use rule based rounding procedure. Except Mert, other students rounded both numbers to two, and multiplied with each other. Nevertheless, Mert converted fraction into decimal and conducted the multiplication of 1.5x 1.5 mentally and get the exact result. He performed multiplication by using the *standard multiplication procedure* and found the exact result. This computation ability may be an evidence for Mert's number sense. The following is an excerpt from his interview;

> ***Mert:*** The fraction is also read as one point five, the second decimal also read one point five. Therefore, it became the multiplication of 1.5 by 1.5
>
> **Researcher:** How do you multiply these?
>
> ***Mert:*** In my head.
>
> **Researcher:** Ok but would you explain how do you conduct the multiplication?
>
> ***Mert:*** Well, I put these numbers under each other and multiply five by five and five by one, and so on. The answer is 2.25
>
> (EX 16).

The previous excerpt is also evidence that the interviewee Mert has high mental computation, which were approved by his mathematics teacher. In the following excerpt from his mathematics teacher's interview, shows the teachers' opinion in mathematics performance of Mert;

> **Researcher:** How is Mert computation ability in math class?
>
> ***Math Teacher A:*** Mert doesn't like using paper-pencil. He doesn't take note. Mostly he answers the questions by doing mental computation. His computational ability is very good.
>
> (EX17)

Among the reformulation strategies (Rule based rounding, situation based rounding, compatible numbers, truncation), rule based rounding was the most used one. This can be observed from the Table 4.3 in the following. During the interviews, it was observed that the interviewees of the current study had difficulty in decimals. Especially, Q12 (98.6 x 0.041) could not perform any of them and they generally used standard addition, subtraction and multiplication procedures for the questions. This may be related with the students' high performance on mathematics lessons, and their ambitions about being a most successful mathematics achiever. In the following sections it is discussed the students' perceptions of mathematics and estimation, they felt that they were successful on mathematics only if they found the exact answers for the questions. Since there is only one right answer, which is the exact one, the other estimated ones are wrong according to them.

The reformulation strategies, which were discussed above, summarized in the following Table 4.3. As can be seen from the table, rule based rounding, situation based rounding and truncation are the strategies of the students. Q12 (98.6 x 0. 041) is a remarkable questions among the five of them since nobody could produced reasonable answer for it. Because of dependency on rule based rounding and lack of ability to work with power of ten.

Table 4.3 Reformulation Strategies Used In Decimal Questions Based on Codes

Questions	Interviewees' Strategies				
	Deniz	Mert	Sergen	Nevzat	Ayşe
Q2 835.67— 0.526	• rule based rounding	• made same number decimal places • situation based rounding	• made same number decimal places • rule based rounding	• made same number decimal places *(could not performed the question)*	• situation based rounding
Q5 0.7 + 0.002 + 0.81	• rule based rounding • ignore too small decimal	• ignore too small decimal	• made same number decimal places	• made same number decimal places *(could not performed the question)*	• made same number decimal places *(could not performed the question)*
Q7 16.272 ÷36	• ignore decimal parts • rule based rounding	• ignore decimal parts • compatible numbers	• ignore decimal parts • rule based rounding	• ignore decimal parts	• ignore decimal parts • compatible numbers
Q12 98.6 x 0.041	• rule based rounding *(could not performed the question)*	• rule based rounding *(could not performed the question)*	• rule based rounding *(could not performed the question)*	• rule based rounding *(could not performed the question)*	• Situation based rounding *could not performed the question)*
Q14 1 ½ x 1.67	• rule based rounding	-	• rule based rounding	• rule based rounding *(could not performed the question)*	• rule based rounding

4.1.1.3 Reformulation in Fractions

According to interview data, there are some specific reformulation estimation strategies for fractions and decimals. Decimal related reformulation strategies are discussed in the previous section. In this section fraction related reformulation strategies are going to be examined. These are *nearness to 1, ½ and zero*, which is coded under rule based themes and *ignoring too small fractions* which is coded truncation theme

As it can be seen in Table 4.4, there are five fraction questions (Q4, Q6, Q9, Q10, and Q13 where can be seen at Appendix B) in four types of operations (addition, subtraction, multiplication, and division). The division operation is the most difficult question type for the interviewees. Two kinds of division questions were asked them and almost all of them tended to conduct the standard division algorithm. These were division of two proper fractions and division of mixed number by proper fraction. In each type of questions, students had different difficulties. These difficulties and interviewees' solutions are discussed in the following.

Only two of the students conducted a reformulation strategy that is *nearness to 1, ½ and zero* for Q6 ($\frac{13}{16} \div \frac{7}{8}$).The other interviewees preferred to conduct exact division algorithm. Strategy used interviewees, rounded the fractions according to nearness to 1, ½ or zero strategy and then found the result easily. For example, Deniz suggested a solution as follows;

> ***Deniz:*** The first fraction is so close to one.
>
> **Researcher:** Why do you think so?
>
> ***Deniz:*** Because when I check the denominator and numerator distance, it is only three. This means the fraction is three units far from to sixteen over sixteen, which is one.

Researcher: Then, what do you think about the second one?

Deniz: This is also similar to the first one. I mean the fraction is almost one. Therefore, the question became 1 divided by 1. The result is one.

(EX18)

However, except Deniz and Mert, other interviewees tried to conduct standard fraction division algorithm, which means multiplying after reversing the second fraction. This kind of application may be a result of poor estimation skills in fraction, and highly dependency on computational procedures or not giving importance to estimation. These reasons are discussed in the following chapter.

Similar to Q6 ($\frac{13}{16} \div \frac{7}{8}$), in Q9 ($14\frac{3}{4} \div \frac{5}{8}$) interviewees conducted *nearness to 1, ½ or zero strategy*. The first mixed number is rounded to 15 and the second fraction rounded to ½. However, except Ayşe, all other interviewees produced wrong results for this division question. Since they might have misconceptions on division with a fractions, that is they thought that the division result might be smaller than the dividend. For example, an excerpt is given below from Nevzat's interview;

Nevzat: The first number is nearest to 15, and the second one is nearest to $\frac{1}{2}$. The division 15 by $\frac{1}{2}$ is 7.5.

Researcher: hımm. Nevzat, could you explain the rounding procedure, and how did you find 7.5?

Nevzat: Ok. The first fraction is 14 and 3 over 4 where the three over four is so close to one. So that the first one could be round the fifteen. Am I right?

(Researcher note: he looks at me for continuing the explanations. I approve him and then he continued)

Nevzat: Then the second fraction is near to half, since it is far from one only one over eight point.

(Researcher Note: He is thinking the number line relations in his head when giving the distance among the numbers)

After that, fifteen divided by half, it is seven and half!

(EX19)

On the other hand, Ayşe could give the acceptable result for this question without any hesitation. She stated that;

Ayşe: The division of fifteen and half is thirty.

Researcher: Could you explain how did you find this result?

Ayşe: There are two halves in one whole. Then if I have fifteen whole, there should be two fifteen halves in a whole thirty.

(EX20)

The other operations (addition, subtraction, and multiplication) of fractions are easier than division for the interviewees. The easiest one was Q10 ($1\frac{7}{16} + 3\frac{5}{12} + 8\frac{1}{2}$) for all students. They rounded the fractions as regarding to *nearness to 1, ½ and zero* strategy and found the similar rounded solutions.

As a different perspectives, Mert and Ayşe preferred to use converting fractions into decimals after the rounding procedure where they used the multiple representations of the numbers. The multiple representations of the numbers are discussed in the "number sense" section.

The other reformulation strategy i*gnoring the fraction part of the mixed numbers* was used mostly in the Q13 ($3\frac{1}{2}$ x $10\frac{1}{8}$). Three of the five students (Ayşe, Nevzat and Sergen) ignored the fraction parts of the both mixed numbers and solved the questions without fractions. Ayşe explained her solution as follows;

> ***Ayşe:*** I could omit the fraction part of the numbers and the operation becomes 3 times 10. Therefore, the result is 30.
>
> **Researcher:** Why did you omit the fraction part Ayşe?
>
> ***Ayşe:*** Since they are very small factions, especially $\frac{1}{8}$. If I don't omit the first fraction ½ I can round it to four and the result is forty now.
>
> (EX21)

The rest of the interviewees only ignored the second mixed numbers' fraction part. Although Mert converted first mixed number into a decimal version as three and half, Deniz preferred to round the first mixed number to upper integer, which is four, and then applied the multiplication procedure to the numbers. By converting fraction into decimal version, Mert is the one who got the closest result among others in this question. This is an evidence for the researcher about Mert's ability of number sense.

Different from other interviewees, Mert and Ayşe were the students who solved the question in a large interval by *ignoring fraction parts of the mixed numbers* in Q4 ($7\frac{1}{6}$ - $4\frac{1}{3}$). An excerpt is given below from Mert's interview;

> ***Mert:*** The result is three.
>
> **Researcher:** You found the result so fast. How did you do?
>
> ***Mert:*** I omitted the fraction parts of the mixed numbers so that the result became three.
>
> **Researcher:** Why did you omit them?
>
> ***Mert:*** They are very small and it does not affect the result too much.
>
> (EX22)

This excerpt is an evidence for his feelings on estimation. According to him, small number does not change the answers very much, and this small pay off did not disturb him. therefore, it may be concluded that Mert has high tolerance for errors.

The reformulation strategies discussed above were listed in the following Table 4.4. As can be clearly seen that students Mert and Ayşe are two interviewees used estimation strategies most frequently. Others generally tended to compute the fractions rather than to estimate the solution. They tried to find “common denominators” of the fractions and “decomposed of mixed numbers” which are specified in “mental computation” title in hereafter of the next sections. In the Table 4.4, the used computational estimation strategies in fraction related questions were specified as “nearness of 0, ½ or 1” which is a kind of rule based rounding, was the most popular strategy since it was used seven times in the five fraction questions. Ignoring the fraction part was another strategy that was used by students and it was applied to questions in six times by interviewees.

Table 4.4 Reformulation Strategies and Used In Fraction Questions Based on Codes

Questions	Interviewees' Strategies				
	Deniz	Mert	Sergen	Nevzat	Ayşe
Q4 7 1/6 – 4 1/3	-	• ignore fraction part of mixed numbers	-	-	• ignore fraction part of mixed numbers
Q6 13/ 16 ÷ 7/8	• nearness of 0, ½ or 1	• nearness of 0, ½ or 1	-	-	-
Q9 14 ¾ ÷ 5/8	• nearness of 0, ½ or 1	• nearness of 0, ½ or 1	-	• nearness of 0, ½ or 1 *(could not performed the question)*	• nearness of 0, ½ or 1
Q10 1 7/16+3 5/12 + 8 ½	-	-	-	-	• rule based rounding
Q13 3 ½ x 10 1/8	• nearness of 0, ½ or 1 • ignore fraction part of mixed numbers	• situation based rounding	• rule based rounding • ignore fraction part of mixed numbers	• ignore fraction part of mixed numbers	• ignore fraction part of mixed numbers

4.1.2 Translation

Translation means changing the operations in the questions into more manageable form. According to data, two clusters were constructed, *changing addition into multiplication* and *changing division into fraction*. These concepts are discussed according to whole number questions and decimal questions of the Computational Estimation Test. This strategy couldn't be discussed in fraction related questions since it could not be found any applications of it in fraction related questions. The reasons are going to be discussed in the "Summary of the Computational Estimation Strategies" section. It is tried to answer why students did not use any translation strategy for the fractions.

In the following sections, results on whole numbers and decimals where translation strategy used are discussed. Each section is ended with a table in which explained strategies are summarized.

4.1.2.1 Translation in Whole Numbers

As can be seen in Table 4.5, in the whole number questions, translation strategy was used only in Q15 (87 419 + 92 765 + 90 045 + 81 974 + 98 102) and Q11 (474 257 ÷ 8 127) by the interviewees. In Q15 (87 419 + 92 765 + 90 045 + 81 974 + 98 102), all of the students preferred to change the addition process with multiplication operation in some extent. Except from Nevzat and Mert the other three interviewees preferred to add the numbers partially, that is they added the first three numbers by using the translation strategy then the rest of the numbers were added on the found addition one by one.

The following excerpt from the interview with Nevzat is an illustration of using the translation strategy in Q15 (87 419 + 92 765 + 90 045 + 81 974 + 98 102);

Nevzat: The last one could be a hundred thousand, hımm. The others…

(Researcher Note: Nevzat is thinking about rounding of each number separately but I think he can not have in mind each of the rounded numbers. Therefore, he changed his mind.)

Researcher: What do you think about Nevzat?

Nevzat: I thought that another way. I want to round all numbers to 90 000 then the result is five times 90 000 that is four hundred fifty thousand.

(EX23)

Only he could cluster the all numbers into 90 000 and then conducted the addition operation as multiplication. Similar to Nevzat, Mert used translation by rounding all numbers to a hundred thousand and stated that the result should be 500 000 which is in the acceptable range. His result showed his confidence in his estimation ability. He wasn't uncomfortable with this big result, which was given in a large interval. In the following excerpt, Mert is explaining his operation;

Mert: The answer is five hundred thousand.

Researcher: How did you find that?

Mert: All numbers could be rounded to a hundred thousand there are five of them. So that, the result is five hundred thousand (500 000).

Researcher: Could you find closer answer?

Mert: yeap. I can round first three of them 90 000 and multiply by three, and get 270 000. Then fourth and fifth one become 80 and 100 which makes180. The addition of 270 and 180 hımm, should be 450.

Researcher: So the answer?

Mert: Four hundred fifty thousand.

(EX24)

The only person who used translation strategy in another question was Nevzat. He preferred to use it in Q11 (474 257 ÷ 8 127). The following excerpt from Nevzat's interview;

> ***Nevzat:*** The first number rounded to 470 000 and the second one is to 8 000. hımm. How can I divide these?
>
> **Researcher:** Why did you round the first number like this?
>
> ***Nevzat:*** Because it is closer to 470 000. But yeah.. It can also be rounded to 480 000. and it is more useful.
>
> **Researcher:** What does useful mean?
>
> ***Nevzat:*** When I removed the three zeros, it became 480 over 8. Then I could simplify this fraction.
>
> (EX25)

As seen in the excerpt, Nevzat changed the division procedure into the fraction operation and conducted simplification on it.

The translation strategy did not used any other whole number question, except from Q11 (474 257 ÷ 8 127), in this question translation was conducted by Nevzat. In the following Table 4.5 this rare usage of the translation strategy can be seen. The reason might be related with operations' restrictions or lack of knowledge about the different strategies of computational estimation by the interviewees.

Table 4.5 Translation Strategies Used In Whole Number Questions Based on Codes

Questions	Interviewees' Strategies				
	Deniz	Mert	Sergen	Nevzat	Ayşe
Q1 31 x 68 x 296	-	-	-	-	-
Q3 7465—572	-	-	-	-	-
Q8 713 ÷ 8	-	-	-	-	-
Q11 474 257 ÷ 8 127	-	-	-	convert division to fraction	-
Q15 87 419 92 765 90 045 81 974 + 98 102	convert addition to multiplication	convert addition to multiplication	convert addition to multiplication	convert addition to multiplication	convert addition to multiplication

4.1.2.2 Translation in Decimals

The interviewees used translation in decimals for only one question, Q7 ($16.272 \div 36$). All students preferred to use estimate instead of conducting the standard division algorithm for Q7 ($16.272 \div 36$). They produced a fraction and then conducted simplification procedures. For instance, in the following excerpt, Sergen tries to explain his solution;

> ***Sergen:*** I throw the decimal part of the first number at first. Hım.. I though that this operation as a fraction.
>
> **Researcher:** What do you mean as a fraction?
>
> ***Sergen*:** I mean that I saw this as sixteen over thirty-six. Then, I think I must conduct simplification. Four can divide both of the numbers so after the simplification it becomes four over nine.
>
> **Researcher:** Where do you use the computational estimation?
>
> ***Sergen:*** Yes. I can also use it. Let us think about like this, I can round at first before the simplification. The fraction sixteen over thirty-six can be said that twenty over forty so the result easily seen as one over two which means a half. (EX26)

The reason for why the students used the fraction conversion of the division operation may be related with the dividend and divisor relations. They used to divide big number by small number but in this question situation is reverse, which means a small number divided by a big number. Because of this, the division operation is more likely to be a fraction than a division operation. Therefore, they chose to convert this division into a fraction form rather than conducting the division algorithm.

In the following Table 4.6 it can be seen that in decimals, only in one question the strategy was applied by all interviews.

Table 4.6 Translation Strategies Used In Decimal Questions Based on Codes

Questions	Interviewees' Strategies				
	Deniz	Mert	Sergen	Nevzat	Ayşe
Q2 835.67— 0.526	-	-	-	-	-
Q5 0.7 + 0.002 + 0.81	-	-	-	-	-
Q7 16.272 ÷36	changed division to fraction	• changed division to fraction	changed division to fraction	changed division to fraction	• changed division to fraction
Q12 98.6 x 0.041	-	-	-	-	-
Q14 1 ½ x 1.67	-	-	-	-	-

4.1.3 Compensation

Compensation means that rethinking the result of the question and making some changes to get closer answer to the estimation. It could be done while the operation conducting or at the end of the operation. For example, one can round the first number and to compensate it, approximately same amount of truncating is performed to the other number; this type of compensation is called as *intermediate compensation*. The other type is called the *final compensation* since the round or truncating the number is done at the end of the procedure. By doing this, the result could be closer to the exact answer.

According to research study (Reys, et al., 1982), the good estimators could use this strategy more often than others. Moreover, this strategy is more sophisticated than the other strategies.

In the interviews, the researcher waited for the interviewees conducting the compensation strategy themselves, but if they did not prefer to conduct it, she asked them "*Is your answer above or below the exact result?*" The reasons for asking them such a question is to identify both their mental computation performance and to understand whether they could see the reasonableness of their answers. In general, students did not perform compensation without asking them. It may be related with lack of conceptual knowledge on computational estimation of the students.

According to research study (Reys, et al., 1982), the good estimators could use this strategy more often than others. Moreover, this strategy is more sophisticated than the other strategies.

Since compensation strategy could not identify within the fraction related questions, next sections present compensation in whole numbers and decimals. Each section accompanied with strategy specification tables in which each students and questions are presented.

4.1.3.1 Compensation in Whole Numbers

According to Table 4.7 presented in the end of this section, in the whole number questions, among the interviewees, Mert, Ayşe and Nevzat performed the compensation strategy in some questions (Q1, Q3, Q8, and Q11). Among these three interviewees, Mert was the only person conducted the strategy in three different whole number questions. He used this strategy in Q1 (31 x 68 x 296), Q8 (713÷ 8) and Q11 (474 257÷8 127), as the forms of intermediate and final compensation. For example, in the following excerpt, Mert confidently applied the intermediate compensation strategy in Q1 (31 x 68 x 296);

> **Researcher:** Mert, you are saying, "I get 2100 but then multiply 20 by 3." How did you get those numbers?
>
> ***Mert:*** Since I rounded 68 to 70 and 296 to 300 to compensate the result I chose the 20x3 to place with 21x3.
>
> **Researcher*:*** Why did you need to compensate the result?
>
> ***Mert:*** Actually, it is not so big deal, but I do not want to round all number upper, therefore to balance this I truncate one of them.
>
> (EX27)

Similar with Mert, Nevzat conducted intermediate compensation strategy only in Q3 (7475- 572). In the following excerpt from interview with Nevzat;

> ***Nevzat:*** The result is approximately 7000.
>
> **Researcher:** How did you find the answer?
>
> ***Nevzat:*** I rounded the first one to 7500. Since it is rounded upper, the second one should be subtracted some for the

> balance. Therefore, the second one became 500. As a result, 7500 minus 500 is 7000.
>
> (EX28)

Among the three interviewees who used to compensation strategy, Mert could be identified easily in order to perform final compensation. For instance, in the following excerpt is from interview with Mert for Q8 (713÷ 8), he is explaining his solution;

> ***Mert:*** The first number could be 720. hımm. The multiplication of 8 there should be 72… Well, it is 9… yeah 90 but no, it should be 89.
>
> **Researcher:** Why not 90?
>
> ***Mert:*** since the first number rounded up so the result should be a bit smaller than 90, it may 89.
>
> (EX29)

As the researchers (Reys et al, 1982; Sowder, 1992) findings show, good estimators could use variety of strategies and they could perform compensation easily. Mert is one of the good estimators among the interviewees, in fact best estimator. Like Mert, Ayşe followed similar final compensation strategy in Q11 (474 257÷8 127). The following excerpt shows Ayşe's perspective;

> ***Ayşe:*** The first number could be rounded to 470 000 and the second one to 8 000.
>
> *(Researcher note: She is trying to divide mentally. It can be understood by the eye movement and head movements.)*
>
> **Researcher:** Ayşe, you don't have to find exact result. Could you tell me approximate one?
>
> ***Ayşe:*** Ok. Well, the first number should be rounded 480 000 then the result is sixty, something.
>
> **Researcher:** Why did you say something?

> ***Ayşe:*** Because, the exact answer not sixty… a little bit less than sixty.
>
> **Researcher:** Why do you think so?
>
> ***Ayşe:*** Because I added to first number some amount of numbers more than the second number. The division of the previous numbers should be a bit less than sixty.
>
> (EX 30)

By conducting successfully the compensation strategy in those questions, in fact Mert had some mistakes during the use of the compensation strategy in some questions. This is because of his high self-confidence. For example in Q3 (7465-572), he insisted on his compensation was true, but it was not.

> ***Mert:*** The result is approximately 6900.
>
> **Researcher:** You found the result very fast. You should explain how you found the answers to me.
>
> ***Mert:*** I round first one to 7500 and the second one 600. Then the answer is 6900. But it may a bit more.
>
> **Researcher:** How could you give this decision? A bit more?
>
> ***Mert:*** It is very easy. The first number is rounded to 35 forward and the second number is rounded 28 forward. Therefore, the result should be a bit more than six thousand nine hundred.
>
> (EX 31)

The next table, Table 4.7, shows which student used compensation strategy on which questions. As can be seen there is only three person used the compensation in three questions and Mert used it more frequently than other two interviewees. It is remarkable that Nevzat is the third person which could use this strategy. Since according to interviews' results, Ayşe and Mert are competent estimators and mental computers among others, but Nevzat is not. On the other hand, Nevzat can use the compensation strategy which is specified as "good estimator's strategy" according to some researchers (Reys et al., 1980; Reys et al., 1991). This might be related with Nevzat's out of school learning and self-training on estimation. However, it was observed that Nevzat thought that out of school application was not true or should not be used in school applications, since these learned concepts did not follow rules but mathematical applications should have formulas or rules. Therefore, according to him, the researcher’s questions should be answered in schooling process that is, followed by a rule sequences.

Table 4.7 Compensation Strategies Used In Whole Number Questions Based on Codes

Questions	Interviewees' Strategies				
	Deniz	Mert	Sergen	Nevzat	Ayşe
Q1 31 x 68 x 296	-	intermediate compensation	-	-	-
Q3 7465—572	-	-	-	intermediate compensation	-
Q8 713 ÷ 8	-	final compensation	-	-	-
Q11 474 257 ÷ 8 127	-	final compensation	-	-	final compensation
Q15 87 419 92 765 90 045 81 974 + 98 102	-	-	-	-	-

4.1.3.2 Compensation in Decimals

In the decimal part of the Computational Estimation Test, only in Q14 ($1\frac{1}{2}$ x 1.67), compensation strategy was used only two of the interviewees who are Mert and Ayşe. As can be seen from Table 4.8, except from Mert and Ayşe, nobody could use the compensation strategy in decimal questions. Both interviewees preferred to use the intermediate compensation, in which students controlled their solutions "during the procedure" to make the estimation more precise. The following excerpt is the rest of ideas of Mert for Q14 ($1\frac{1}{2}$ x 1.67), which was presented in "4.1.1.2 Reformulation of Decimals" in EX16;

> **Researcher**: Mert you found the exact answer. But I want you give me an approximate answer. If you don't have any time and have to give reasonable answer for this question, what would you do?
>
> ***Mert:*** OK. I round both of them to two and get four. But, hımm. It is a bit more. I think I only round the second one, and multiply 1.5 by 2. So that, the result could be reasonable. I mean 3 could be the result.
>
> (EX 32)

Similar with Mert's solution, Ayşe explained her answer as following;

> ***Ayşe:*** I round both of them but the result is more. Hımm. I only round one of them, the second one. Then three could be answer that is more appropriate.
>
> **Researcher:** Why does it bother you Ayşe? I mean why you tried to find another solution.
>
> ***Ayşe:*** Since four is very big result for this question, it should be smaller than four. I can find closer answer, for example 3.
>
> (EX 33)

In the following Table 4.8 shows the compensation strategies, which are used by the interviewees in decimals. There are only two students, who used this strategy.

Table 4.8 Compensation Strategies Used In Decimal Questions Based on Codes

Questions	Interviewees' Strategies				
	Deniz	Mert	Sergen	Nevzat	Ayşe
Q2 835.67— 0.526	-	-	-	-	-
Q5 0.7 + 0.002 + 0.81	-	-	-	-	-
Q7 16.272 ÷36	-	-	-	-	-
Q12 98.6 x 0.041	-	-	-	-	-
Q14 1 ½ x 1.67	-	intermediate compensation	-	-	intermediate compensation

4.2 Summary of the Computational Estimation Strategies

In this section it is discussed that the findings on computational estimation strategies which are used by the interviewees. Students were used twelve different computational estimation strategies during the interview sessions. According to similarities of concepts and aim of the used strategies, they were assembled in three different types of estimation strategies, which are reformulation, translation and compensation.

Although more than three strategies were defined in some research studies (e.g., Rubenstein, 1985; Dowker, 1992), generally these were clustered in the three main strategies (Mottram, 1995; Reys et al., 1980; Reys et al., 1991; Rubenstein, 1985; Sowder, 1992). In the current section, a brief summary and discussion related with these strategies are given. The investigation of strategies and students' perspectives on these strategies could give evidences about the second research questions which is "Which factors are associated with computational estimation strategies of seventh grade students?"

The first question is explained according to each number type under a title in the pervious sections. The strategies were presented at the end of each section separately. Additionally, all three strategies with codes are given in Appendix H, as three tables, which are Table 1, Table 2, and Table 3. These tables are summarizing the strategies of each student for each question of the CET.

Rule based rounding, situation based rounding, compatible numbers and truncation strategies are gathered under the reformulation strategy in the study. In the whole numbers, students used reformulation strategies except truncation. In Table 1 (see Appendix H), each student used "rule based rounding" at least three times for five whole number questions. Although "rule based rounding" is a strategy for computational estimation, according to other strategies it is not a precious one, since in my opinion preferring this strategy may be an evidence for dependency of the rules of mathematics and not being enough competency of estimation. For example, Ayşe is a rule-follower student, that is, she answered questions in mathematics classes according to taught rules. Therefore, it may be the reason for her the preference of rule-based rounding. Her mathematics teacher confirmed her rule dependency in mathematics lessons in the following excerpt;

> **Math Teacher A:** Ayşe is a hard-working student. She listens to the lesson very carefully, and does her homework properly. However, she could not produce an alternative solving methods for any mathematics problems. She prefers to use the standard method that she learnt in the classroom.
>
> (EX 35)

However, Deniz used rule based rounding not because of exact computation dependency but he knew only strategy as estimation strategy. He confessed that he knew rounding very well and performed it successfully, and then he used it for every question. When it was asked "How successful are you at computational estimation?", Deniz gave 10 points to his computational estimation ability; he answered the researcher's "why" question in a different way from the other interviewees as "*I can round the numbers so that I am a good estimator*."

Two interviewees preferred to use "compatible numbers" in one whole number questions, and all other interviewees used "situation based rounding" at most twice in whole number questions. The remarkable point is that reformulation strategies especially in whole numbers might not distinguish the good estimators from not good ones. This distinguish is important since the second question of this research study interested in the factors associated with students' computational estimation strategies. That is, the difference between the good estimator and poor ones serve the relating factors that one group of students have but the others not. Therefore, reformulation strategy could not say much about the good estimators since all students used this strategy. However, one significant observation can be make on "rule based rounding" that is rule dependency is important for interviewees in mathematics questions. This may lead to students' flexibility while using strategy.

Besides the whole number questions, in decimal questions it was produced some distinguishing points by students in order to used strategies. In these questions, truncation was used as reformulation strategy by two forms, which were "Ignoring too small decimal and Ignore decimal parts." As a good estimator, Mert, was more competent in using truncation in decimals.

In the fractions, students had problems on estimating the questions more than other type of question. Because rather than estimating the solutions, they attempted to conduct standard fraction operation which was seen as finding "common denominator" in the study. Except from Ayşe and Mert, others started to operation in fraction by finding the common denominator of given fractions. This is an obvious evident for giving importance to exact computation more than estimation. Students could not performed variety of estimation strategies in factions. Rule based rounding which was coded as "nearness to 1, ½ or 0", and truncation strategy coded as "ignore fraction part of mixed numbers", and situation based rounding strategies were used in fraction related computational estimation questions. In fraction questions, students rounded the fractions according to nearness to one, half or zero strategy, which was taught in schools in the fraction topic of mathematics classes. However, among the interviewees, Deniz, Sergen, Nevzat, and Ayşe mostly prefer to use exact computation procedures like finding common denominators or standard division algorithm. Although the exact computation process was more difficult than estimation strategy, students wanted to conduct the exact computation. In schools, when a new operation is introducing at first, students are taught exact computation firstly and then may be some estimation strategies are taught. This order of teaching makes students prefer to use exact computation as the first step. On the other hand, estimation might be preventing students from having lack of conceptual knowledge about the fraction. Trafton (1986) believes that fractions should be taught with early emphasis on the meaning of the topic using estimation and then mental computation as a vehicle for developing the concept of it. In his study (Trafton, 1986) it was observed that students were more component in factions than decimals among the interviewees. In contrast to research literature, in the current study, students were more successful on fractions than decimals related questions (e.g., Carpenter et al., 1976; Goodman, 1991; Hanson & Hogan, 2000). This might be related with the data gathering time, which was right after the application of the fraction topic in the math class.

The other strategy, which was used not as frequent as reformulation, was the translation strategy. Translation is converting the operations into more applicable situations. In the current study, two version of translation strategy was observed, these were "converting addition to multiplication and converting division to fraction." This strategy requests good number sense, since the relationships of the operations should be known well so that they can be converted to each other. In whole numbers questions this strategy used only once by all students in Q15 (87 419 + 92 765 +90 045 + 81 974 + 98 102). However, there were some distinguishing points among the interviewees by using the strategy. That is only Nevzat preferred to conduct translation strategy to whole numbers in the addition procedure. Other interviewees used it partially. In other words, the other interviewees grouped the numbers in three and two and added within the groups at first than two groups of results were added.

The remarkable person in this strategy was Nevzat, since he used translation in two questions, Q15 (87 419 + 92 765 +90 045 + 81 974 + 98 102) and Q11 (474 257 ÷ 8 127). In Q11 he performed rounding procedure for both of the numbers and then converted the division algorithm into the fraction. That is, he produced $\frac{480}{80}$ and conducted the simplification of fraction. Although Nevzat was the only person who used this strategy in two questions (the other interviewees used it only for Q15), Nevzat's mental computation ability is not very good. He is a relatively good estimator but he is not a good computer. Reys (1984) claim a kind of confirmation to Nevzat's case. Reys (1984) observed similar situation on people who can be good at computational estimation, should not be necessarily good at mental computation. That is a person who is a good estimator should not have to be also being good computer.

None of the interviewees could use compensation strategies in the fraction related estimation questions. The reason why the students did not use the compensation strategy might be related with the strong dependency on exact computation and difficulty of fraction questions. It was identified that the most used strategy while solving the fraction questions were "nearness of 1, 1/2 or zero," as rule based rounding and the "common denominator" as mental computation procedure. These two procedures based on the exact computation or rule dependency perspectives.

The last strategy is compensation strategy, which is used for making the estimated result closer to the actual answer. There are two ways for this adjustment of the estimated answers; these are "intermediate compensation" which requires adjustments during the operations, and the "final compensation," which requires the adjustment of result according the rounding amount. Three students could use compensation in three questions. These students (Mert, Nevzat and Ayşe) are classified as good computational estimators according to be able to use this strategy. According to the studies compensation is the most sophisticated strategy and least used one (Reys, et al, 1982; Reys et al, 1991; Sowder, 1992). This strategy is one of the most obvious evident for identifying the good estimator. Reys et al. (1982) recommended not using the compensation strategy among the middle and high school students because of the lack of conceptual understanding of what constitutes reasonable compensation. Therefore, being able to use this strategy also may be an evidence for having the conceptual knowledge and understanding of the computational estimation. Among the interviewees, Mert has not any problems about the conceptual understanding of compensation on estimation, so that he used this strategy more often than others. Ayşe and Nevzat are the other two interviewees who used the compensation in two questions. On the other hand, Mert used compensation strategies in both version, intermediate and final compensation in four questions of whole number and decimals. This is a very strong evidence for concluding that Mert is a good computational estimator. As many researchers confirmed the relationship between the uses of compensation strategy and being a good computational estimator (Reys, et al., 1982; Reys et al., 1991; Sowder, 1992; Dowker, 2003). According to Reys and his colleagues (1982), good estimators used compensation frequently and identified it as essential to successful estimation. Students who are good estimators used more strategies than students who are not good estimators (Mottram, 1995). In line with this finding, in the current study, Mert used more strategies than other four interviewees did and he used more sophisticated strategies (that is compensation strategies) than others.

There are some differences, which are observed according to strategy choices, strategy uses of the good and poor estimators. According to research studies, to develop and use estimation strategies, students must understand the power of the strategies in producing useful answers for reasoning and making mathematical judgments and be eager to use estimation rather than exact computation (Dowker, 1992; Reys et al., 1982). These constructs (mathematical judgments, eager to use estimation, understand the power of estimation, use of numbers and operations, etc.) are concerned in the next sections to understand factors that are related with them.

4.3 Factors Associated with Computational Estimation Strategies

The second research question of the current study was "*Which factors are associated with computational estimation strategies of 7th grade students?*" and discussed according to interview data in this section. There are two factors, which associated with computational estimation strategies of students; these are cognitive factors and affective factors.

4.3.1 Cognitive Factors Associated with Computational Estimation Strategies

According to interview data, cognitive factors were divided into two sub-categories, number sense, and mental computation. The number sense and mental computation ability of interviews are discussed in the next sections under the titles, Number Sense and Mental Computation.

4.3.1.1 Number Sense as Cognitive Factor

As defined in Chapter II, number sense is closely related with the computational estimation ability. According to data obtained from the interviews, number sense is investigated based on two titles; "*ability to work with powers of ten"* and *"multiple representation"* of the numbers.

In the analysis, the theme of "ability to work with powers of ten" contains, *removing zeros* while doing addition, subtracting, or multiplication; *simplification of zeros* while conducting division, and *multiplication by powers of ten.*

In her study, Rubenstein (1982) stated that estimation ability could be explained by operating with tens. Because of this, the ability to work power of ten is an important component of number sense and estimation ability. Additionally, the multiple representation of numbers means that make connections among the number types and conducting transitions if necessary.

In the following sections, the data gathered from interview are analyzed based on number sense and each type of numbers, which are whole numbers, decimals, and fractions. The codes of numbers sense can be seen in Appendix E and tables contains these codes are also presented at the end of the "4.3.1 Cognitive Factors Associated with Computational Estimation Strategies" section.

4.3.1.1.1 Number Sense on Whole Number

In the study, since high achiever students participated in interviews they mostly had good number sense. Especially "working with zeros" which is a good evidence for number sense in whole number questions, students could conducted the questions successfully. Rarely, there were some problems on students' responses.

In Q1 (31 x 68 x 296), except Nevzat and Deniz, the other three interviewees used the *removing zero* of numbers during the multiplication process and they conducted operation successfully. Following excerpt from interview with Deniz, is an illustration of removing zeros during the multiplication;

> ***Deniz:*** The first one could be 30, and the second one 70. Then thirty times seventy is two hundred ten (210). The third number rounds three hundred and the result is thirty six thousand.
>
> **Researcher:** how did you find the result? Could you explain it step by step.
>
> **Deniz:** Ok. First I rounded the numbers. Then for the first multiplication I removed the zero and three times seven is twenty one. I put back zero so that the result is two hundred ten. After that I multiply this result by the third rounded number three hundred. While doing this, I removed the zeros, and multiplied twenty one by three.
>
> **Researcher:** How did you multiply these numbers?

(Researcher note: Deniz smiled at me. Since he thought, what kind of question is this? When I asked is this multiplication easy for him, he approved by his head.)

Deniz: Ok**.** I multiply three by one and then three by two.

Researcher: What is the result?

Deniz: Thirty six thousand.

(EX 37)

In the excerpt, as can be identified, that Deniz forgot a zero after multiplication procedure while putting back zeros at the end of the multiplication result so that the solution conveyed wrong result. The reason for this misinterpretation of the questions may be due to Deniz's short-term memory. He could not keep the sequence of actions of the multiplication. Therefore, his ability to work with zeros might be poor relatively for Q1 (31 x 68 x 296).

Another question in whole number, Q15 (87 419 + 92 765 + 90 045 + 81 974 + 98 102), Deniz performed it without removing zeros correctly different from others.

Deniz: I round the first three of them ninety thousand and multiply by three. I get two hundred seventy thousand. The next number could be rounded eighty thousand and the last one a hundred thousand.

Researcher: The result?

Deniz: The result could be found by adding two hundred seventy thousand and a thousand first, then adding the eighty thousand… hımmm. So it should be firstly three hundred seventy thousand, then hımm add eighty thousand.. it is four hundred fifty thousand.

(EX 38)

Removing zeros in Q1 and Q15 is performed successfully by Mert, Ayşe and Sergen. On the other hand, simplification of the zeros during the division algorithm Sergen was confused in Q11 (474 257 ÷ 8 127). The following excerpt is the continuing part of the excerpt given in the title "4.1.1.1 Reformulation in Whole Numbers" of Sergen;

> ***Sergen:*** I think so. Yes, the first one should be 480 so that it could be divided by 8 and it should be sixty….
>
> ***Researcher:*** Your answer?
>
> ***Sergen:*** I must put the removed zeros back.
>
> ***Researcher:*** Which removed zeros?
>
> ***Sergen:*** At the beginning, I removed the three zeros from the first two numbers. Therefore, I should put them back.
>
> ***Researcher:*** hımmm. Ok. What should the answer be?
>
> ***Sergen:*** It is sixty thousand.
>
> (EX 39)

In the excerpt, it could be understood that Sergen confused the removing zeros and simplification of zeros. This may be an evidence for the poor number sense of Sergen. Ayşe and Mert chose to solve the Q11 (474 257÷8 127) by without removing zeros. These students conducted the division of 480 000÷80 000, and not being removed zeros, found an acceptable answers. It may be considered as a confirmation for good number sense of these students.

4.3.1.1.2 Number Sense on Decimals

Decimal is a problematic topic for the interviewees. As seen in Table 4.3, there are some questions, Q12 (98.6 x 0.041) and Q5 (0.7 + 0.002 + 0.81), which could not be answered by interviewees. Especially Q12 (98.6 x 0.041) was the most difficult one for all of them. There are some reasons for not answering this question. The first and most powerful reason is that the sticking with the rule based rounding. The three interviewees (Mert, Deniz, and Sergen) confused with the rules since the second decimal should be rounded to zero according to the rules. On the other hand, this multiplication result should not be zero. In the following excerpt, Mert is trying to explain his confusion;

> ***Mert:*** The second decimal could be round zero.
>
> *(Researcher's note: He is hesitating about saying zero. He was quite for a minute)*
>
> **Researcher:** So the multiplication result is zero?
>
> ***Mert:*** No… should not. But… well, it could be round zero point five.
>
> **Researcher:** Do you think that 0.041 is close to 0.5?
>
> ***Mert:*** No…I think…hımmm..
>
> (EX 40)

Even Mert had difficulties in rule based rounding in the Q12 (98.6 x 0.041), Deniz and Sergen could not get rid of this confusion. The other matter according to this question is that the multiplication of a decimal with powers of ten. Nevzat and Ayşe were struggle with this concept. The next excerpt shows Ayşe's comments on multiplication a hundred by a decimal;

> ***Ayşe:*** The first decimal could be rounded a hundred…hımmm. The second one…
>
> *(Researcher's Note: She thinks for a long time… I think she is trying to conduct exact computation but she was stuck)*
>
> **Researcher:** Ayşe would think aloud. I want to hear what are you thinking…
>
> ***Ayşe:*** Ok. I thought about the multiplication process with ten. There should be a rule about zeros and coma.
>
> **Researcher:** What kind of rule?
>
> ***Ayşe*:** We are doing something with coma and zeros…hımm.. I think we move the coma according to the number of zeros or hımm. I don't know. I could not remember the rule.
>
> (EX 41)

This question is a difficult one for the whole group. It may be related with the groups' poor learning on decimals. However, three of the interviewees had problems on rule based rounding procedure but the other had problems on multiplication of powers of tens. The interviewees' problems on this question may be related with lack of the *place value fact* or *multiplication with powers of ten* by a decimal, which is examined in next sections in the current chapter.

Morgan's (1990) findings are consistent with the situation that is multiplication by a number less than one was the most difficult one for the interviewees on his study. Hanson and Hogan's (2000) identified the subjects who were struggle with some specific operations and number types; among them, there was multiplication of decimals by power of ten.

Multiplication of decimal by ten was a difficult question also for Nevzat. In Q13 ($3\frac{1}{2}$ x $10\frac{1}{8}$), Nevzat changed his solution process because he couldn't perform the multiplication of a decimal by 10 and produced another way which was simpler than decimal multiplication procedure. The next excerpt indicated his difficulty;

> ***Nevzat:*** The first number is three and half. The second one could be seen as 10. Therefore, I should multiply three and half by 10. hımmm. I think... ok… lets say the first one is three. Then the answer is thirty.
>
> (EX 42)

"*Multiple representations of the numbers*" are other subcategory of number sense. The representation is conducting by converting decimal to fraction and fraction to decimal. Students who can easily translate from one representation form to another are able to use the representations as tools to approach problems from several different perspectives. The use of this kind of multiple representations depends on students' ability to work with either fraction or decimal. For instance, in Q14 ($1\frac{1}{2}$ x 1.67), Mert was the person used both numbers as decimals. He conducted the conversion of fraction to decimal and multiplied two decimals mentally. The excerpt (EX16) from his interview was given in the section "4.1.1.2 Reformulation in Decimals," showed his solving procedure.

In the next section, fraction related questions are examined according to students' number sense. Although questions on decimals and fractions were solved by converting each other, there were critical points on some questions and some different application through student to student

4.3.1.1.3 Number Sense on Fraction

In the fraction questions, especially in Q10 ($1\frac{7}{16} + 3\frac{5}{12} + 8\frac{1}{2}$), the interviewees Mert, Nevzat and Ayşe spelt the fractions as decimals while reading the questions. They converted the fractions into the decimals at the very beginning of the solution so that this might be an evidence for their good number sense. The following excerpt showed Mert's performance on converting fraction into decimal smoothly,

> ***Mert:*** One and half plus, three and half, and eight and half… hımm. Three and half and one and half make five. Then eight and half three and half.
>
> **Researcher:** You're so fast. Please explain each step to me.
>
> ***Mert:*** Ok. The first and second fractions could be rounded half since they are close to one over two.
>
> *(Researcher note: He is pointing out the first and second mixed number's fraction parts)*
>
> (EX 43).

The changing of the fractions to decimals can help students perform the addition procedure easily. Since, without thinking equivalence denominators of fractions they performed addition by halves and whole numbers.

Similarly, in Q13 ($3\frac{1}{2}$ x $10\frac{1}{8}$), only Mert preferred to convert first fraction to decimal and then multiply by ten. Nevzat also converted the fraction to decimal but he could not conduct the multiplication decimal by ten, then he changed his mind and ignore the first mixed number's fraction part. So that, the operation became two whole numbers multiplication and could be performed easily for Nevzat.

While Mert is very competent with mental computation and computational estimation, Nevzat is a poor computational estimator. Mert obtained the closest answer by doing the conversion of fractions to decimals and performed the multiplication successfully in the Q13 ($3\frac{1}{2}$ x $10\frac{1}{8}$) and Q10 ($1\frac{7}{16} + 3\frac{5}{12} + 8\frac{1}{2}$). Therefore, it might be said that Mert has better number sense than the others. The other interviewee, who has a good number sense, is Ayşe. She is the only person who can obtain acceptable answer for Q9 ($14\frac{3}{4} \div \frac{5}{8}$). The following excerpt shows her comments on the question;

> ***Ayşe:*** The first number rounded to 15. The second one is close to half.
>
> **Researcher:** So…?
>
> ***Ayşe:*** So, the result is 30.
>
> **Researcher**: Can you make it clear? How can you find 30?
>
> ***Ayşe:*** I should divide 15 by 0.5. So, I asked myself, how many half are there in fifteen. The answer is 30 halves.
>
> (EX 44)

To be able to use multiple representations of decimal and fraction help students produce acceptable answers in the computational estimation questions. This converting process might help Ayşe to find the answer of Q9 ($14\frac{3}{4} \div \frac{5}{8}$), since nobody used this converting process for the question and nobody could find the answer. The other reason might be related with the conceptual knowledge of division. While answering the question, Ayşe asked herself, "*how many halves are there in fifteen*?" as can be seen in the Excerpt 44 above. After asking this question, she could say the reasonable answer since her questions was a right question to get the right answer. By asking herself such a question, she made herself think about the meaning of the division process, not the standard division algorithm. Therefore, she could count how many halves in the fifteen as separate pieces. Nobody, except from her, could think to ask this kind of question to them.

Sowder and Markovits (1989) believe that meaningful understanding of the size of fraction and decimals can help students in developing number sense in general. Therefore, ability to work with fraction and decimal questions may show that students' good number sense.

Contradictory with some research result, in the current study students had difficulties on decimals more than on fractions. However, Ling (2005) asserted that in his study the interview results indicated that items including fractions were more difficult than whole number and decimal items. Different from these studies, in the current study, the interviewees performed fraction questions better than decimal questions. This may be because of the data gathering procedure was quite after the "fraction topic" in their math classes. They had been practicing in classroom for a week before the interview sessions of the current study.

Students' capability of using strategies of computational estimation is not depending on number sense but also depending on their performance of mental computation.

This performance is examined in the next section through the each type of numbers and computational estimation questions.

4.3.1.2 Mental Computation as Cognitive Factor

There are three categories of mental computation; *mathematical facts on whole number, mathematical facts on decimal and mathematical facts on fraction* are specified from the data. Each of these is discussed in the following sections.

4.3.1.2.1 Mathematical Basic Facts on Whole Number

Whole number category contains; (a) *decomposition of whole numbers* during addition, multiplication, and subtraction; (b) *standard operation algorithm,* which means that paper-pencil work and (c) usage of *multiplication table.* Dimensions of basic facts of whole numbers that were identified from the interviews are examined in the following.

Except Nevzat, all interviewees used *decomposition of the whole numbers* in Q15 (87 419 + 92 765 + 90 045 + 81 974 + 98 102) after reformulating to each numbers. Because, Nevzat chose translation strategy for this question and with rounded all numbers to 90 000, then convert addition into the multiplication procedure. However, other interviewees preferred to add numbers partially. Therefore, they should manipulate numbers for easy addition that is they used decomposition of the numbers. For example, following excerpt from Deniz's interview shows using of decomposition of the numbers while addition;

> ***Deniz:*** The first three numbers rounded to 90 000 and get 270 000 when I multiply by three. The third number could be

rounded 80 000 and the last number is 100 000. Adding 100 000 to 270 000 is 370 000. Moreover, the result is 450 000.

Researcher: how did you find the result? Tell me your addition.

Deniz: I break 80 as 30 and 50. Then add 30 to 370 we could get 400. After that, 50 could be added to 400. So that the result is 450.

(EX 45)

Decomposition of the numbers helps students to handle with addition, subtraction, and multiplication questions easily. One can separate the numbers in order to usage and easiness.

Mert conducted a similar version of this addition with decomposition, but without zeros. In the following excerpt, he stated his solution;

Mert: I get 270. Then by adding 80, I can get 350. After adding a hundred, the result is 450 000.

Researcher: How did you perform the addition?

Mert: To add 27 and 8, first I added the 27 with three and get 30 and then the other part of eight… that is five can be added 30. So that 35 is obtained. Since the last number is a hundred, the result is 450 000.

(EX 46)

Similar to addition operation, in the subtraction question decomposition was used, for example in Q3 (7465—572). Except from Nevzat and Ayşe, the other interviewees (Mert, Sergen and Deniz) rounded the numbers and used decomposition of the numbers to conduct the subtraction. The following excerpt shows Sergen's solution of the question and he explained the decomposition of the numbers during subtraction;

Sergen: The first number is 7500 and the second is six hundred. The result should be six thousand nine hundred.

Researcher: Well, Sergen would you explain your operation, please?

Sergen: Ok. I break the six hundred in a hundred and five hundred. So, seven thousand and five hundred minus five hundred is seven thousand. The remaining a hundred could be easily separated from seven thousand that is six thousand nine hundred.

(EX 47)

As a remark, Nevzat and Ayşe preferred to use compatible numbers strategy for this question, they did not conduct decomposition of the numbers process.

In mental computation themes, "standard operation algorithm" was most popular computation way among the students. Almost all of them tried at least once to conduct the standard operation for the question, but researchers convenience them to give estimated solution. For example, Ayşe, Sergen, Nevzat, and Deniz conducted the *standard operation algorithm* in Q1 (31x 68 x 296) for the multiplication of rounded numbers. The excerpt given is from Nevzat's interview shows an example below.

Researcher: How did you find the 63 000?

Nevzat: After I get 21, I multiply 21 by three.

Researcher: How did you multiply these?

Nevzat: In a normal way…

(Researcher Note: Nevzat was surprised at the question. he found the result but someone asked him how he found. Also, the question of "tell me how you multiplied them" is a weird question according to him.)

Nevzat: Ok. I put the three under twenty-one. Then multiply one by three, after that multiply two by three.

(EX 48)

Similar to Nevzat's explanations, Sergen, Ayşe and Deniz conducted the multiplication procedure by using paper-pencil algorithm. Students were confused when researchers asked them, how they multiply the numbers with each other, since this question was weird, numbers are written one under other and multiplied.

In the division problems, *the standard operation algorithm* could be easily identified during the interviews. The movement of the interviewees' heads gave clues that students were trying to draw a division cross in their head to compute the problem by paper and pencil algorithm. Moreover, when it was asked that "how did you find the answer" interviewees explained drawing the division line and trying to explain the operation of the division algorithm. For instance, Deniz conducted a standard division algorithm in his head for Q7 ($16.272 \div 36$). The following excerpt explained the way of his solution,

> **Deniz:** I want to ignore decimal part of the first number. Then it should be seen zeros. Only we have 16.
>
> **Researcher:** Why?
>
> **Deniz**: Since sixteen is smaller than thirty-six when we divide two of them, we should add zero after sixteen. Hımm, I guess it is a bit hard to divide them.
>
> **Researcher:** do you use division cross?
>
> **Deniz:** Yes. But I could not do the division.
>
> Researcher: Well, let's try something easier.
>
> **Deniz:** OK. We can round thirty-six to thirty-two.
>
> **Researcher:** Why?
>
> **Deniz:** Or we can round sixteen to twenty and thirty six to forty, so that the result should be one over two.
>
> (EX 49)

Although, students pretended to estimate the answers, it could be observed from the excerpt above, they tried to imagine a blackboard on their eyes, raise their hand as though writing in the air in front of them and tried to produce written algorithms, which was heavily emphasized in classrooms. This observation, which was conducted during the interviews, consistent with Volkova's (2006) findings. Although Volkova (2006) studied on preservice teachers, they also tended to apply standard algorithms to the problem before they could think of a possible estimation strategy.

Although Ayşe was a good estimator and good mental computer, she more often preferred to use mental computation than other interviewees did. The reason was that both she could do the computation mentally and she wanted to use it or she did not want to use estimation. There are many examples for her related with using mental computation insistently on the interviews questions. For instance, in Q3 (7465—572), she tried to compute at first; with the warning of the researcher she used estimation strategy and rounded the numbers. The continuing part of the Excerpt 7, which was given in "4.1.1.1 Reformulation in Whole Numbers," is presented below to show Ayşe's reactions;

> **Researcher:** How did you find the six thousand nine hundred?
>
> ***Ayşe:*** I subtracted the 570 from 7470.
>
> **Researcher:** How did you subtract?
>
> ***Ayşe:*** I put the 570 under 7470. Since the seventy parts removed each other, the end of the result should be zero. It should be 9, after subtracting five from four.
>
> (EX 50)

Standard operation algorithm, which known paper-pencil work, is mostly concerned as a strategy for computational estimation in most studies (Levine, 1982; Dowker, 1992). Especially in the mental computation ability, paper-pencil algorithm took an important place. In the current study, the interview group insistently used standard operation procedure in many questions, but standard operation algorithm is not concern a computational estimation strategy for the current study.

The other factor related with the performance on mental computation on whole number is "using the *multiplication table"* for whole number question. The interviewees used the multiplication table procedure in two division questions of the Computational Estimation Test during the interviews. The first one is Q8 ($713 \div 8$) and the other one is Q11 ($474\ 257 \div 8\ 127$). The following excerpt presents one of the example from students' interview how the interviewee explained the solution of Q8 ($713 \div 8$);

> **Researcher:** What are doing Sergen?
>
> ***Sergen:*** (*He is counting something silently*). I am checking the multiplication table of eighths.
>
> **Researcher:** Why?
>
> ***Sergen:*** Because in the question 71 divided by 8.
>
> **Researcher:** 71?
>
> ***Sergen:*** I removed three or I can say that 72 since I think 8 times 9 is 72. Then one zero should be put back to 9 and the result is approximately 90.
>
> (EX 51)

Similar with Sergen, the other three interviewees (except Nevzat) checked multiplication table of eights to find the division of 713 by 8. However, Nevzat preferred to round the first number to nearest hundred rather than nearest ten. Therefore, his division question became 700 divided by 8.

As a remarkable observation, standard algorithm in the division questions requires students to ignore place value. For example, in dividing 713 by 8, the interviewees ask themselves, "how many times does 8 go into 71?" rather than "how many 8s can I get out of 713?" for these reasons the standard algorithm in these division questions works against students' number sense.

In the following Table 4.9 mental computation and number sense codes of whole numbers are given. As seen from the table, except from Nevzat, all of them used various amount of strategies of mental computation. As explained previous sections, Nevzat was poor computer but this did not lead being poor estimator. On the contrary, he was one of the good estimators among the interviewees.

Table 4.9 Number Sense and Mental Computation Codes Used In Whole Number Questions

Questions	Deniz	Mert	Sergen	Nevzat	Ayşe
Q1 31 x 68 x 296	• removing zero • standard operation algorithm	• removing zero	• removing zero	• removing zero • standard operation algorithm	• removing zero
Q3 7465—572	• decomposition of the numbers	• decomposition of the numbers	• decomposition of the numbers	--	• standard operation algorithm
Q8 713 ÷ 8	• removing zero • multiplication table	• multiplication table	• multiplication table	--	• multiplication table
Q11 474 257 ÷ 8 127	• standard operation algorithm	• multiplication table	--	--	--
Q15 87 419 92 765 90 045 81 974 + 98 102	• decomposition of the numbers	• removing zero • decomposition of the numbers	• decomposition of the numbers • removing zero	--	• decomposition of the numbers • removing zero

4.3.1.2.2 Mathematical Basic Facts on Decimal

Decimal questions in the Computational Estimation Test were difficult to estimate for the interviewees. They mostly wanted to solve the questions by using the standard decimal procedure that is *making the same number of decimal places.* The data are investigated according to three sub categories of mathematical facts on decimal; these are *making the same number of decimal places, place value of decimal* and *decomposition of decimal.*

In Q2 (835.67— 0.526), Sergen and Nevzat tried to solve the question by the standard decimal procedure, they thought at first to make the same number of decimal places and then computed. However, it can be identified that this kind of solutions was having poor number sense since in order to ignore decimal parts or rounded the numbers, students chose standard decimal procedure where it was more difficult. Additionally they could not conduct this operation properly, so that this could be concerned as having poor mental computation ability. An example, the excerpt, which was also given in previous section "4.1.1.2 Reformulation in Decimals" from interview with Sergen, shows this kind of poor mental computation and poor number sense.

> ***Sergen:*** One zero could be added to first number's end. So, the number becomes 835.670. The second number is rounded to 0.500.
>
> (*Researcher Note: He read the number as zero point five thousand)*
>
> **Researcher:** Why did you need to add the zero?
>
> ***Sergen:*** Because when I am doing the subtraction with decimals, the numbers of digits should be same for both numbers.
>
> **Researcher**: Ok. How do you subtract these numbers?

Sergen: I round the first number as 835.700 then I subtract five hundred from the seven hundred. Therefore, the answer is eight hundred thirty five point two hundred (835.200).

(EX9)

In the excerpt, Sergen could not identify 0.500 and 0.5 were the same numbers. Moreover, he did not remove the founded answer's zeros because of the same reason. Therefore, these could be concern as poor number sense of Sergen. The opinion of his mathematics teacher supported the researcher's inference. In the following excerpt is from interview with Sergen's mathematics teacher, gives another perspective for performance of Sergen in mathematics classes.

Researcher: How successful is Sergen in the mathematics classes?

Math Teacher A: He has not been interested in mathematics this year. Last year he was the most successful student among the sixth graders. However, this year he is poor at computations.

(EX 51)

The other person in the interview group is Nevzat who had difficulties on decimals. In the following excerpt, it is shown that Nevzat has trouble with Q2 (835.67—0.526);

Nevzat: The second decimal could be rounded as 0.50. I mean to make the same decimal places I dropped the third digit from 0.526. So that…hımm…835.67 minus 0.50. I think…835 and sixty six and a half?

Researcher: Sorry. I missed you. How did you conduct subtraction?

Nevzat: I subtracted whole parts between each other that is 835 minus zero is 835 then in the decimal parts, sixty seven minus half is sixty-six and a half?

Researcher: hımm. Would you tell me the result?

Nevzat: Eight hundred thirty five point sixty six and a half…hımm??

(EX 52)

Nevzat had some misinterpretations on decimals' *place value* that is he thought that there were separate numbers of the two sides of coma of decimals and tried to conduct the subtraction both side of the coma separately. In her study, Sowder (1984) observed that students lacked of number sense on decimals could perform unacceptable result for computational estimation. This kind of misconception may be an evidence for Nevzat's poor number sense and poor computational ability on decimals.

Making the same number of decimal parts to be equal was also conducted in Q5 (0.7 + 0.002 + 0.81) by Sergen, Nevzat and Ayşe. Different from others, Ayşe could perform the standard paper-pencil algorithm and could get an exact result; however, the others could not. The following excerpt shows Ayşe's solution for Q5 (0.7 + 0.002 + 0.81);

Ayşe: The first number could be 0.700 the second is ok. Then the third could also be 0.810.

Researcher: Why did you put zeros?

Ayşe: When adding the numbers the zeros help to add easily.

Researcher: How?

Ayşe: When I put the numbers under each other, the first and third number is ended with zero but the second is ended with two. So the solution of addition ended with 2…hımm..then…

(Researcher Note: She put an imaginary blackboard in front of her and conducted the addition on board with her close eyes and moving finger)

> ***Ayşe:*** After then two, next comes fifteen. Hımmm. One could be written left part of coma…so the result should be one point, fifty twelve…
>
> (EX 53)

According to excerpt, Ayşe used standard paper pencil procedure for the addition of the decimals where it was difficult to conduct mentally, she could find out the exact result. Therefore, it can be said that she had good mental computation.

Although Q10 ($1\frac{7}{16} + 3\frac{5}{12} + 8\frac{1}{2}$) was labeled as a fraction related question, students mostly used the *decomposition of decimals* in this question. Mert, Deniz, and Ayşe decomposed the decimals in similar way, Nevzat and Sergen used another different way of decomposition of the numbers. That is, the first group of students (Mert, Deniz and Ayşe) conducted addition of 1.5+3.5 as 1.5+3→4.5 and after then 4.5+0.5→5. The second group of students(Nevzat and Sergen) conducted the same addition of 1.5+3.5 at first by adding decimal parts 0.5+0.5→1, and then adding the whole parts 1+3→4 and 4+1→5. The first application is an example of counting on procedure; the second one is the property of commutativity of addition. These two kinds of decompositions are not discriminated to each other in order to importance but if the steps of process are counted, the first decomposition has fewer steps than second one. It may not be very important, unless the speed of the operations would not be considered.

In the following Table 4.10 shows that students most of the time used standard operation algorithm perspective by making the same number of decimals of the numbers. Since decimals were the most difficult type of questions among others, students more were depending on standard operation rather than used estimation strategies. However, as can be seen from the table, Deniz was not used any mental computation procedure and this may lead us to conclude that he used mostly estimation strategies for decimals.

Table 4.10 Number Sense and Mental Computation Codes Used In Decimal Questions

Questions	Deniz	Mert	Sergen	Nevzat	Ayşe
Q2 835.67—0.526	--	• made same number decimal places	• made same number decimal places	• made same number decimal places *(could not performed the question)*	--
Q5 0.7 + 0.002 + 0.81	--	--	made same number decimal places	• made same number decimal places *(could not performed the question)*	• made same number decimal places *(could not performed the question)*
Q7 16.272 ÷36	--	--	--	--	--
Q12 98.6 x 0.041	*could not performed the question)*	*could not performed the question)*	*could not performed the question)*	*could not performed the question)*	• convert decimal to fraction *could not performed the question)*
Q14 1 ½ x 1.67	--	--	--	--	--

4.3.1.2.3 Mathematical Basic Facts on Fraction

According to interview data, mathematical facts on fraction is divided into four sub categories, *common denominator, division algorithm, decomposition of mixed numbers* and *misconception on fraction.*

Finding the common denominator was most preferred application for the fraction related questions. Since interviewees used exact computation on fractions more than estimation questions, they heavily rely on exact computation procedure for fraction. Among the five of them, three of interviewees (Sergen, Nevzat and Ayşe) tried to find common denominator for the Q6 ($\frac{13}{16} \div \frac{7}{8}$). The following excerpt is an example of their perspective from the interview with Nevzat;

> ***Nevzat:*** In the division of fraction I must conduct a procedure…hımm.. I must remember that. I think before that I must find the common denominator of them.
>
> **Researcher:** Ok. But let's estimate the problem rather than compute exact result.
>
> ***Nevzat:*** yes. Then I could round the first fraction as 10 over 20. I remember the rule. I should reverse the second fraction as eight over seven, and multiply by ten over twenty. That's it. The result is 80 over 140.
>
> (EX 54)

This excerpt is an evidence for inferring poor conceptual understanding of Nevzat on fraction. He concerned fraction as two separate numbers rather than a whole concept. This perspective takes into account a sub category in the *misconception on fraction* division of the current title. The misconception was observed during the interviews was that treating the numerator and denominator of a fraction as separate integers which was observed previous excerpt above. Ayşe conducted this type of misconception in Q6 ($\frac{13}{16} \div \frac{7}{8}$). Although she was the only person, who obtained the acceptable answer for Q9 ($14\frac{3}{4} \div \frac{5}{8}$), she produced a misconception in Q6 ($\frac{13}{16} \div \frac{7}{8}$). In the following excerpt (EX 55), Ayşe explains her solution for Q6 but she treated the first fraction as two separate integers. She began with finding of common denominator of the fractions but then she confused enlargement of fractions. In the following it is given that her explanations for Q6 ($\frac{13}{16} \div \frac{7}{8}$);

> ***Ayşe:*** I can make the denominator similar.
>
> *(Researcher Note: She smiled and stopped a while. I think controlling the result by doing exact computation in her mind.)*
>
> **Researcher:** ok. What are you thinking about Ayşe? Please think aloud.
>
> ***Ayşe:*** hımm, I think the answer is two.
>
> ***Researcher.*** How do you get the answer?
>
> ***Ayşe:*** I changed my mind and round the first fraction numerator as fourteen. So that, I get fourteen over sixteen where it is two times of the second fraction, seven over eight. Therefore, the result is two.
>
> (EX 55)

Different from Ayşe, Mert and Deniz used rounding strategy for the fractions in the same question. They rounded each fraction to 1 by using *nearness to 0, ½ and 1* strategy. They gave 1 as a result of this division question.

Another application for the Q6 ($\frac{13}{16} \div \frac{7}{8}$) was to make the similar denominator of the fractions. However, reason of using same denominators procedure for these fractions may be related with if the denominators of the fractions were twice each other. This may lead to students to conduct the standard fraction operation. For example, in Q4

($7\frac{1}{6}$ - $4\frac{1}{3}$), four of the five interviewees tried to made denominators similar. Among the interviewees, Mert gave an answer without using standard fraction operation, since he just subtracted whole parts. However, Deniz found an exact answer by doing making the denominators same and conducting the standard fraction algorithm mentally.

Addition to take two separate numbers of denominators and numerators of fractions, "multiplication makes bigger, division makes smaller" is another *misconception on fraction.* For example, in Q9 ($14\frac{3}{4} \div \frac{5}{8}$) students performed this popular misconception which was known as "multiplication makes bigger, division makes small." Students could easily round the first mixed number to 15. However, when dividing 15 by almost a half, was complicated process for four of the interviewees (except from Ayşe). These four interviewees could not realize that the result was more than 14 or 15. Similar to findings of Markovits and Sowder (1994) and Volkova (2006), the interviewees of the current study thought that when dividing a number by another, the division should be smaller than the divided numbers. Nevertheless, one of the students could realize that this fact is not true for all time, especially on operations with fractions and decimals. All interviewees except Ayşe, asserted that the result should be seven point five (7.5) for the Q9 ($14\frac{3}{4} \div \frac{5}{8}$). Four of the interviewees explained this division as "*when fifteen is divided by half result should be seven and half.*" When the researcher made situation clearer with asking, "How many halves are there in a whole?", then students could say correct reasoning for the question but most of them stopped a while before saying that "the answer is 30 but how could it be?" Since they thought this is a division operation and the result could not be more than 15. On the other hand, there was only one interviewee, Ayşe, who gave acceptable answer for Q9 ($14\frac{3}{4} \div \frac{5}{8}$). She followed the division algorithm properly. Additionally she asked herself some conceptual questions like, "how many halves are there in fifteen?” and then she figured out the operation in her way as saying *“fifteen should be divided by a half so that the answer should be thirty.”*

In the following Table 4.11 consisted of the codes of mental computation and number sense on fractions. It can be seen that students generally tended to use common denominator and decomposition of the mixed numbers. Converting fraction to decimal or vice versa relationship coded under number sense and Nevzat, Mert and Ayşe, were the persons used these conversion in the fraction related question more often than others. These are all an evidences of good number sense and high mental computation of these students.

Table 4.11 Number Sense and Mental Computation Codes Used in Fraction Questions

Questions	Deniz	Mert	Sergen	Nevzat	Ayşe
Q4 7 1/6 – 4 1/3	• common denominator	----	• common denominator • decomposition of mixed numbers	• common denominator • decomposition of mixed numbers	--
Q6 13/ 16 ÷ 7/8	--	--	• division algorithm • common denominators	• division algorithm • common denominators	• Misconception on fraction
Q9 14 ¾ ÷ 5/8	• division algorithm • misconception on fraction	• misconception on fraction	• common denominators • misconception on fraction	--	--
Q10 1 7/16+3 5/12 + 8 ½	• decomposition of mixed numbers	• convert fraction to decimal • decomposition of decimal	• decomposition of mixed numbers	• convert fraction to decimal • decomposition of decimal	• convert fraction to decimal • decomposition of decimal
Q13 3 ½ x 10 1/8	--	• convert fraction to decimal	--	--	--

4.3.2 Affective Factors Associated with Computational Estimation Strategies

According to the data gathered from the interviews, there are some affective factors related with the students' computational estimation strategies. The second session of the interview was designed to understand the students' thoughts on using computational estimation in daily life applications, in mathematical applications, feelings of students on using estimated solution, whether they gave importance to estimation or not.

Although, the data examined in the following section was gathered from the second interview session, and so that the second interview was designed for understand the students thought and feelings, the first interview session and observations gave considerable amount of data for answering the second question of the study.

According to interviews and observations of the students in the interview sessions, two main themes are defined which are associated with the students' computational estimation for the affective factors. These are mathematics related affective factors and estimation related affective factors. The codes, which are listed under these two themes are given below:

Mathematics Related Affective Factors

a. Confidence in ability to do mathematics
b. Perception of mathematics

Estimation Related Affective Factors

a. Confidence in ability to do estimation
b. Tolerance for error
c. Perception of estimation

Each factor is discussed below according to students' answers for related interview questions (see Appendix C). In the following sections, students' answers are examined through these themes and codes.

4.3.2.1 Mathematics Related Affective Factors

According to interviews, students answers could be collected under mathematics related affective factor, since students' perception of mathematics and confidence in their mathematical ability influencing their computational estimation strategies. In the following sections, students' answers of each question are examined.

In order to identify the students' confidence in ability to do mathematics, in the second interview session some obvious questions were asked them. Students' confidence in their mathematics performance was affecting their answers and perspectives on estimation. Similarly, the perception of mathematics was concern another mathematics related affective factors. It mainly concerns the beliefs about mathematics, such as mathematics gives exact results or mathematics means exactness. According to interview sessions, students' reactions were observed based on "exactness" concept. Through the answers, test anxiety was identified as a factor that influence students' thoughts on exactness of mathematics.

Each factor is examined through the answers of interviews and data were presented from observations and fieldnotes, which were taken from the first interview and second interview sessions. Although, second interview questions were designed for identifying these factors, some observations of first interview and classroom observations gave huge amount of data related with affective factors.

4.3.2.1.1 Confidence in Ability to do Mathematics

In the current study, three types questions were asked to identify students' confidence in mathematics performance. These were;

1. Rate your mathematics achievement,

2. Give a point out of 10 to mathematics achievement , and

3. How successful are you at mathematics?

As a first question, during conducting the Computational Estimation Test procedure, in the answer sheets (see in Appendix D) a four scale self-rating question was asked to whole class. The question is "*Rate your mathematics achievement: ( ) very good, ( ) good, ( ) moderate, ( ) poor."* The students marked a cross in the brackets to specify their achievement levels. According to results, among the five of the interviewees, Mert and Ayşe marked "very good" option and the others marked "good."

In the second interview session, there were two questions related with students' confidence in their mathematics. These were;

1. What points would you give yourself out of 10 on your mathematics achievement? Why?
2. How successful are you at mathematics? Why?

Although the first question was very similar to self-rating question, these were asked for identifying students' consistency during the current study. The self-rating question was asked at the very beginning of the study, but the other self -rating question (What points would you give yourself out of 10 on your mathematics achievement? Why) was asked in the second interview session and there was at least one mouth between them.

The answers of the first question listed above were answered in consistency with previous answers, which were given on the CET answer sheet. Although all interviewees were well-enough at mathematics lessons according to their score of math exams and their mathematics teacher's comments, the students gave points in various ranges, from five points to ten points out of ten. Among the interviewees, Ayşe and Mert gave themselves 10 points out of 10 for the first questions. The interviewees, Deniz and Nevzat gave seven points, and Sergen gave himself five points.

The students (Deniz, Sergen and Nevzat) who gave themselves lower points are generally poor at computational estimation questions. On the other hand, Ayşe and Mert who gave themselves very high points for their mathematics success differed from each other in the answer of second question.

Where Ayşe and Mert gave themselves 10 points out of 10 in the first question, their answers were very different from each other in the second question, which was "*How successful are you at mathematics? Why*?" Ayşe answered this question as "enough" but Mert answered it as "not enough." This is a very important data to understand the difference between these two students. Although they are both successful at mathematics according to exams' score and math teacher's classifications, they performed computational estimation questions differently. Ayşe was eager to conduct exact computation but Mert felt comfortable with estimated results and could give estimated results. According to Ayşe, estimation was nonsense in the mathematics, but Mert preferred to use estimation and he was good at computational estimation.

Ayşe has high confidence in to do mathematics, because of this; she might want to show her ability to find out exact results for almost all the questions in the interview.

For instance, in the Q4 ($7\frac{1}{6}$ - $4\frac{1}{3}$), the following excerpt is an example of her reaction to do estimation;

> ***Ayşe:*** May I compute the problem or give rounded answer.
>
> **Researcher:** Please, give an estimated answer.
>
> ***Ayşe:*** I can do this computation in my head without writing if you want.
>
> **Researcher:** I know you can do it. But I want you to estimate the solution.
>
> (EX 56)

As a summary, although the interviewees were all successful students according to questions that were asked them, Ayşe and Mert had high confidence in ability to do mathematics but other three interviewees did not as much as Ayşe and Mert. There should be examined another affective factor, which was called as "perception of mathematics" and in the following section there are some other data is investigated.

4.3.2.1.2 Perception of Mathematics

According to data analysis, perception of mathematics associated with students' strategy using and choosing process. Especially, exactness of the mathematics and test anxiety are two main codes of this theme. Test anxiety is affecting students' perspectives of estimation and mathematics. Since this code is very important, it is going to be presented in following section under "Test Anxiety" title. In this section, students' thought on "mathematics should be included exact results" is examined.

In the interviews, Ayşe and Sergen asserted very powerful assumptions on "mathematics needs exactness." The following excerpt from the interviews with Ayşe and Sergen show their ideas, respectively.

> **Researcher:** What do you think about to use of estimation in mathematics lesson?
>
> ***Ayşe:*** In mathematics class, I don't prefer to use it.
>
> **Researcher:** Why don't you prefer?
>
> ***Ayşe:*** Because mathematics needs exactness. Estimation contains ambiguity. In math lesson, you should find exact answers. Therefore, I don't prefer to use it in math classes.
>
> (EX 57)
>
> ………………………
>
> **Researcher:** What do you think about to use of estimation in mathematics lesson?
>
> ***Sergen:*** hımm. Estimation gives approximate solutions. It doesn't give exact result. Hımm I don't want to use it in math classes. But if I solve test questions…then estimation may help me in tests…may be…
>
> (EX 58)

As can be seen from these dialogues, students (Sergen and Ayşe) believe that mathematics does not contain the approximate answers and estimated solutions. In addition to thought on exactness, Ayşe stated an interesting explanation in the following excerpt;

> **Researcher:** Don't you ever comment on a mathematics problem without exact result?
>
> ***Ayşe:*** Hımm, Sometimes I do. Then I find the exact solution. I feel disregardful to the math question if I do not find the exact result.
>
> (EX 59)

Ayşe believe that she could show her respect to mathematics by computing the question and not giving the estimated result. These show that a general perception of mathematics is present among interviewees. This kind of thinking mostly comes from directions provided for student by teachers. The researcher observed an interesting scene about how a teacher can influence students' view on the nature of mathematics. The following figure, the fieldnotes explains this situation.

Class A: 24 December 2008 8.30 am to 9.10 am Wednesday

……........................
Teacher B starts to talk about mixed numbers. He writes a title on board "Addition with mixed numbers" then explains two ways through which we can add these numbers. First, by converting them to compound fraction, second by adding whole parts and adding fraction parts separately. Then he writes a question ($2\frac{5}{9} + 1\frac{3}{5}$) on blackboard. Students note it on their notebooks. One of the students raises her hand and says her solution in the following way;

Student: Teacher, I think result is more than four.

Teacher A: no, no... You should first convert this number (*show first fraction*) to compound fraction."

Then he warns all students about conducting the compound fractions.

Teacher A: Before conducting the operation, you should convert the mixed number to compound fraction....

The student who gave approximate answer to the question on board, is conducting teacher's way of solving question…

Figure 4.1 An Example to the Fieldnotes from a Classroom Observation

The fieldnotes was taken from the class A, where a student found an approximate answer for a fraction question and teachers gave a reaction to him. Although the student in class A gave the reasonable answer for the question, the math teacher said that it was wrong. The teacher made students gave the exact answers only, not approximate one. Since teacher gave a rule of mathematics, students did not think about the range of the answer for that question not ever for any questions.

The following excerpt from interview with Mert may shed light on the above-mentioned view. He is explaining why he does not want to use estimation in math classes.

> **Researcher:** What do you think about finding approximate solutions in math class?
>
> ***Mert:*** I use sometimes, for controlling the answers. But most of the time mathematics teacher wants me to give exact answers. Therefore, I generally conduct exact solution in math class. Sometimes, this makes me angry.
>
> **Researcher:** What makes you angry?
>
> ***Mert:*** When I say the approximate result to teacher, he thinks it is a wrong answer. But I only say approximate one, not exact one. When I explain this is an exact answer, he says "find exact one". Therefore, I don't want to use estimation in math classes.
>
> (EX 60)

As a result, teachers' perspectives and the way of teaching mathematics influenced students' perception of mathematics. These concepts are going to be discussed in the next chapter. However, before that, the other factors, which was affecting students' point of view according to computational estimation usage and strategies is the "test anxiety" which is examined in the following.

Test Anxiety was identified that from the interview results, there was a powerful factor that influencing the students2 perception of mathematics that is the tests students took. However, this test was not a teacher-applied test it was applied in nationwide. This factor appeared while answering one of the question of "confidence in ability to do mathematics". This was "How successful are you at mathematics? Why?" and students pointed out an interesting point, which was coded in the coding list as "dersane".

They confessed that their achievement levels in mathematics were changing according to place where they took the mathematics lessons. The following excerpt from second session of interview with Deniz;

> **Researcher:** How successful are you at mathematics?
>
> ***Deniz:*** I am good enough at "dersane" but not good at school.
>
> **Researcher:** why is that? What is the difference between "dersane" and school?
>
> (EX 61)

From this excerpt, it can be seen that Deniz thought his mathematics achievement was different from place to place. In "dersane," mathematics was teaching based on multiple choices testing with underlying some practical ways of the topics without showing reasons.

The reason for going to "dersane" is related with the Level Determination Exam (SBS), which is a nationwide exam for elementary students. Therefore, test anxiety is concerned as an affective factor, which influencing the computational estimation strategies of the interviewees. SBS anxiety is a cultural variable for Turkish society since in elementary schools in Turkish educational system students take this exam.

The interviewees are very seriously concerned about the SBS. The high score obtained from these exams is a kind of sign for the students and their teachers, families and friends about their achievement especially in mathematics. Because of this, Level Determination Exam is very important for all. All the interviewees are going to a "dersane" for getting high scores from the exam. "Dersane" is a kind of school but students have to pay money to attend this school and lessons are given more depending on exams and on multiple choice test. The interviewees all took lessons more seriously at "dersane" than at school. In the following excerpt, Sergen is explaining why he has to go to "dersane";

> **Researcher:** How successful are you in math classes?
>
> ***Sergen:*** In school, I am not good. But in the "dersane" I am pretty good at mathematics.
>
> **Researcher:** Sergen why do you go to "dersane"?
>
> ***Sergen:*** Because in SBS, I want to get a high score.
>
> **Researcher:** Do you have to go there? Without dersane can't you get a high score?
>
> ***Sergen:*** No, I can't. Everybody goes to dersane. In school, I cannot motivate myself. But in dersane, you have to attend the lectures, and get high score from the trial exams.
>
> **Researcher:** Why do you have to do? What if you get low score in trial?
>
> ***Sergen:*** your classroom is changed. I am in the class A, which is the class where high achievers are enrolled. If my scores decrease, then I will be transferred to lower class, which is class B.
>
> (EX 62)

The five interviewees go to same "dersane" and all in the top classes. They asserted that they should get higher scores from the tests that were conducted time to time in "dersane" to protect their achievement level and class degree. Periodic exams are taken at "dersane" to identify the students' levels, if anyone has poor performance; his/her classroom is changed and decreased the class degree. These exams make students give more importance to "dersane" than school lessons. Mert explained this situation in the following excerpt;

> ***Mert:*** I am good at both in school and dersane. However, in dersane I must be good at tests. And I should not let my achievement decrease.
>
> **Researcher:** Why?

> ***Mert:*** Because, If get lower scores, the administration changes my class. I don't want to attend lower classes. Therefore, I am trying to improve my testing skills. Estimation sometimes helps me during the test. I could eliminate some options among the four of them.
>
> (EX 63)

Although Mert made use of estimation in the tests, the other interviewees thought that in test they should found the exact solution. Moreover, except Mert, the others thought that estimation might lead them to wrong answers. Ayşe explained her view about estimation using during the tests in the following excerpt;

> **Researcher:** You do not want to use estimation in math classes. Ok. At dersane? During testing?
>
> ***Ayşe:*** I don't think so… this may mislead me.
>
> **Researcher:** Why do you think so?
>
> ***Ayşe:*** When I estimate an answer to a test question, I find approximate result. This may be more or less than the exact one. What if there are options close to each of them. No, no… Estimation should not be used in math classes in school and at tests in dersane.
>
> (EX 64)

Because of this exam anxiety, according to interviewees, the success in multiple-choice tests of mathematics is very important. However, multiple choice testing makes students more specific on their mathematical thinking and makes them think that mathematics requires a single correct answer, which is given among the options. Therefore, students' perception of mathematics and computational estimation was affecting by the perspective of "dersane" and teaching mathematics based on multiple choice testing.

Besides, mathematics based affective factors, there are some factors based on estimation nature in it. Since some students had specific thoughts and feelings on estimation. According to interview data, these thoughts and feelings are clustered in some themes and codes, which are examined in the following section.

4.3.2.2 Estimation Related Affective Factors

Estimation related affective factors influencing the students' estimation performance are investigated in three titles, *confidence in ability to do estimation, perception of estimation* and *tolerance for error.* In confidence in ability to do estimation section, the researcher discuss the students' confidence in making computational estimation through mathematics questions. Under the title of the perception of estimation, students' thoughts and feelings are discussed.

In this section, doing estimation makes students feel good or bad, what students are thinking about estimation, do they thought that is it useful or not, do they give importance to estimation in their daily life or academic life, are discussed. Under the title of tolerance for error the questions on whether estimation is an ambiguous situation or not, and may large interval-solution cause any trouble for interviewees, are examined.

4.3.2.2.1 Confidence in Ability to do Estimation

In the present study, three questions were asked to students to identify their confidence in doing estimation. One of them was asked during the testing procedure of CET and it asked them to rank their estimation ability achievement. The question is "*Rate your computational estimation ability level: (   ) very good, (   ) good, (   ) moderate, (   ) poor.*"

The other two questions were asked to the students in the second interview session. These were;

(1) What points would you give out of 10 on your computational estimation? Why?

(2) How successful are you at computational estimation? Why?

Moreover, some observations and fieldnotes, which were collected from the first interviews helped to infer students' confidence in estimation.

According to gathered data, among the five interviewees, only Mert rated himself as a "very good" estimator in the answer sheet of CET. Deniz, Sergen, and Ayşe selected the "good" options, and Nevzat thought that his estimation ability was "moderate" in the first self-rating questions.

To confirm interviewees' thoughts, in the second interview session, two questions were asked related with the confidence of the interviewees in computational estimation. the answer of the question "What points would you give out of 10 on your computational estimation? Why?" was varied fro six to ten points among the interviewees. Although, Deniz gave 10 points on his estimation ability, Ayşe gave 9 points, Mert gave 8 points, and Nevzat and Sergen gave 6 points on their computational estimation ability. The follow up questions revealed the reasons for the students' assigning these scores to themselves. In the following excerpt, Deniz and Mert explained why they gave 10 points and 8 points, respectively, to themselves;

> **Researcher:** Why did you give 10 points on your estimation ability?
>
> ***Deniz:*** Because I can round the numbers easily.
>
> (EX 65)
>
> ……………….

> **Researcher:** Why did you give 8 points on your estimation ability, Mert?
>
> ***Mert:*** I am not good enough at computational estimation. I can not find the answer in a few seconds, it takes a time. But I am a good estimator.
>
> (EX 66)

If Mert and Deniz's performances on estimation are compared, Mert is found to be more successful than Deniz in the first interview session according to computational estimation. Reehm (1992) reported that students in low ability range tended to overestimate their ability to estimate, and students in the high ability range tended to underestimate their ability. Mert and Deniz's self-rating situation could be explained by Reehm's (1992) finding. Although Mert gave lower score than Deniz and Ayşe, this did not mean Mert had low confidence in his computational estimation ability. This is related with Mert's high expectations of his achievement. Therefore, he thought his success was not enough for his high expectations.

The confidence in to do estimation was varied in interviewees and this might be affecting their preferences of computational estimation strategies. However, it was not the only factor affecting to interviewees' computational estimation, also perception of estimation affected to their strategies of computational estimation. This factor is examined in the following section.

4.3.2.2.2 Perception of Estimation

According to the interview questions, it was aimed that to understand that what students thought about computational estimation, whether they used in daily life applications or not, or whether they want to use it or not. Accomplishing to this aim, it was asked some specific questions, for example, "*Where do you use computational estimation in your daily life?*" was asked them and they answered generally "at markets or during shopping" (apart from Ayşe).

Although Ayşe gave very high point on herself about the computational estimation ability, she stated that computational estimation should not be used in daily life. The following excerpt is from the interview with Ayşe;

> **Researcher:** Where do you use computational estimation in your daily life?
>
> ***Ayşe:*** I think, I use estimation, for example while computing how far away the school from home is.
>
> **Researcher:** It is not a computational estimation procedure. It is measurement estimation. Do you use computational estimation in your daily life?
>
> ***Ayşe:*** Hımmm. I think, no.
>
> **Researcher:** During shopping at the supermarket?
>
> ***Ayşe:*** No. There is a cashier at markets. Therefore, I do not have to compute. I only use estimation when I have to measure something like length or weight.
>
> (EX 67)

In the excerpt is given above, she stated that she did not use computational estimation in daily life. In excerpt 57, she asserted that mathematics is exactness; therefore, in math classes, estimation should not be used according to her. Sergen expressed similar thoughts with Ayşe, in the following excerpt Sergen explained why estimation should not be used in math classes;

> ***Sergen*****:** In mathematics classroom you should find exact answers, estimation does not give you exact results.
>
> (EX 68)

When it was asked "what does she feel about when doing estimation question" Ayşe gave an interesting answer. While explaining her feelings, she claimed that estimation makes her uncomfortable. In the following excerpt, Ayşe explained her reasons;

> **Researcher:** What do you feel when you doing computational estimation requested questions?
>
> ***Ayşe:*** Actually, I do not prefer to use computational estimation. I can compute mentally the given question, or use pencil to compute exactly. Therefore, I don't need to use estimation.
>
> **Researcher:** ok. What if you have to do estimation…? What do you feel?
>
> ***Ayşe:*** It makes me uncomfortable. The estimated answer is not an exact one. If I could find the exact result then I should find it.
>
> **Researcher:** Why do you feel uncomfortable?
>
> ***Ayşe:*** Because, the ambiguity makes me uncomfortable. By doing estimation, I can only find approximate solutions. I don't feel good with estimation.
>
> (EX 69)

The excerpts show that Ayşe and Sergen felt estimation was not much useful. They were liked-minded in terms of using estimation in math classes and in daily life; according to their statements, "estimation should not be used in mathematics classroom." These beliefs could trigger the exact computation procedure for them so that Ayşe and Sergen were eager to find exact solution to the computational estimation question.

According to interview with mathematics teacher of Ayşe, she was not produced her own methods for problems. In the following excerpt from interview with mathematics teacher A, presented the perspectives of Ayşe in mathematics lessons through improving new ideas on problems;

> **Math Teacher A:** Ayşe prefers to conduct taught methods. She doesn't like use shortcuts or untaught ways.

Researcher: Do you promote her use another way for solving the questions?

Math Teacher A: yes. I promote every student in the class. For example, Mert could produce another ways, but Ayşe doesn't prefer. If she finds the answer, she does not need to try another way.

(EX 70)

In the current study, Ayşe frequently asked whether to compute or to give estimated solution. She wanted to exact computation of the questions for interview questions, since one of the reason for that her negative feelings on estimation. Similar with Ayşe, Sergen also insisted on using exact computation procedure rather than using estimation procedure. This is also an evidence for these students' not giving importance to computational estimation.

As a specific example, in Q5 (0.7 + 0.002 + 0.81), Ayşe solved the question by standard addition procedure. Her explanation was given in section "4.3.1.2.2 Mathematical Facts on Decimal" in EX53. She put the numbers under each other and then added as in the standard addition similar to a paper-pencil computation procedure.

In the first interview session, in Q7 (16.272 ÷36), Sergen could not conduct an operation at the beginning and he asked whether he had to use estimation with a gestured showing he got bored. This situation, which had to estimate, was annoyed him. In the following excerpt from the first interview with Sergen, it can be seen his perspective for estimation;

***Sergen**:* I have to use rounding.

Researcher: Do you feel discontent, as you use rounding?

Sergen: Yes. I little.

Researcher: Why do you feel discontent?

Sergen: Since, I usually compute math questions mentally..

Researcher: Why don't you prefer to use estimation?

> ***Sergen:*** I do not prefer, since computing questions mentally is usually very easy for me and I mostly use mental computation. “To estimate” is different for me. I rarely conduct estimation for a math question. I usually prefer directly to solve it.
>
> (EX 71)

Ayşe is the other interviewee who did not want to use estimation unless researcher made her use of it. She continuously stated that she could compute the problem in her mind if researcher wanted her to do so. For example, in Q4 ($7\frac{1}{6}$ - $4\frac{1}{3}$) was very easy for her, so that she wanted to compute mentally and find exact answer. However, when the researcher warned her to find estimated answer, she gave an answer in a big range by subtracting only whole parts of the mixed numbers not because of estimation only because of get rid of the question. She wanted to conduct mental computation for all the questions in the first interview.

The perception of estimation is influencing the students’ reactions to computational estimation questions. The specific examples were Sergen and Ayşe who were unwilling to use estimation, since they thought that estimation could not give exact computation. Exact computation is more important than estimation for these students. However, Mert, who has positive thoughts on estimation, preferred using estimation in the interview questions. He attached more importance to estimation than the other interviewees did.

4.3.2.2.3 Tolerance for Error

In the current study, tolerance for error is defined as feeling comfortable with inexact results and pay off computations. This kind of feeling is not easy to identify by exact questioning but there could be some evidences embedded in the interviews of the students. For example, in Q3 (7465—572), the following excerpt from interview with Ayşe, she explained her feelings about rounding,

> ***Ayşe:*** I could round the first one seven thousand five hundred. No, it is going to be too much.
>
> **Researcher:** What is going to more?
>
> ***Ayşe:*** If I rounded the first one to seven thousand five hundred, there will be more difference between my answer and exact one. Therefore, I should round them as seven thousand four hundred seventy and five hundred seventy.
>
> (EX 72)

Among the interviewees, Ayşe is the one who explained that the approximate computations made her uncomfortable and this kind of computation makes her feel ambiguous.

> ***Ayşe*****:** When I compute approximate solutions, this make me uncomfortable.
>
> **Researcher:** Why do you feel that?
>
> ***Ayşe:*** It is not exact solution. I mean it is not clear, it is uncertain.
>
> (EX 73)

Ayşe tried to find solutions in a very narrow interval, like in the Q4 ($7\frac{1}{6}$ - $4\frac{1}{3}$)**.** The following excerpt is continuing part of EX 56, which is given in the section "4.3.2.1.1 Confidence in ability to Do Mathematics"

> ***Ayşe:*** Ok. Then the answer is three. But I can say that less then three.
>
> **Researcher:** how can you say that?
>
> ***Ayşe:*** If I subtract 1/3 from 1/6, I should take a whole from three, since 1/3 is bigger than 1/6. Therefore, the answer is 2 and five over six.
>
> (EX 74)

She stated in almost all numerical questions that she could find the exact result by computing and tried to find answer in a small range of estimation pay off. This is a powerful evidence for concluding that Ayşe did not feel comfortable with estimated answers. therefore, this may be an evidence to understand Ayşe's low tolerance for error and why she was not comfortable with some pay off. Opposite to her, Mert is the one who has high error tolerance since he can easily make rounding in a big range and states given result being in a big range does not matter for him. For instance, the following excerpt is from the first interview with Mert while solving a decimal related question which was Q2 (835. 67— 0.526);

> ***Mert:*** I round 835.67 to 836. Then the second one can be 1. Therefore, the result is 834.
>
> **Researcher:** Would you give a closer answer?
>
> ***Mert:*** The closer answer should be eight hundred thirty five point a hundred fifty but the other one is also OK for me.
>
> (EX 75)

There is another evidence for recognizing Mert's high tolerance for error. In the question Q1 (31 x 68 x 296), he conducted rounding procedure more than once. Although the multiplication of first rounded numbers (2100 x 300) could be done easily by Mert, he preferred to conduct second rounding procedure (2000 x 300) on purpose to get the result. This shows that Mert's ignorance of pay off, which was cut from numbers while operating the multiplication.

In Q4 ($7\frac{1}{6}$ - $4\frac{1}{3}$), Mert gave an estimated answers in a big range and stated that he did not uncomfortable with this kind of pay offs. In the following excerpt, he explains his perspective;

> ***Mert:*** The result is three.
>
> **Researcher:** That's it?

Mert: Actually, a bit less than three but does not matter. Since it's too small.

(EX 76)

Additionally, Mert confessed his disappointment with the mathematics teacher's reaction when he found a solution by estimation. It was explained in the section "4.3.2.1.2 Perception on Mathematics," in the excerpt EX60, previously. Mert said that he was nervous because when he found a rounded solution, his mathematics teacher did not accept his answer and said the answer was wrong. Therefore, Mert does not conduct estimated solutions for the math questions in the math class.

Other interviewee, Sergen was the person who had low error tolerance. Similar to Ayşe, Sergen thought that mathematics contains exact solutions. Therefore, in Q15 (87 419 + 92 765 + 90 045 + 81 974 + 98 102) he prefers to use the compatible numbers strategy for reformulating the numbers. The reason to choose this strategy was finding a very specific result, which was the closest to the exact answer. In the excerpt given below, Sergen is explaining his reasons,

Sergen: the first one could be rounded to 87 000 and the second one to 93 000.

Researcher: Why would you round the numbers like this?

Sergen: I don't want to exaggerate the result.

Researcher: Exaggerate?

Sergen: I mean that I want to find an answer closer to exact one. I don't want to go far away from the exact answer. This is not true.

Researcher: Why do you think so?

Sergen: Actually, I am more comfortable with exact computation. But you want to estimate solution then I perform the addition as specific as I can. This kind of rounding is reasonable for that.

(EX 77)

The second coder of the study conducted the other significant observation. While coding the transcripts of Ayşe, the second coder identified that how low tolerance error that Ayşe had. The coder stated “Ayşe was reluctant to take risk on questions” to the researcher and confessed that how impressed from Ayşe’s results. According the second coder, Ayşe did not want to take risks on computing, so that she wanted to find exact result.

As a last word, factors, which are associated with the students’ strategies of computational estimation tried to examine in the sections presented above. In the following section, a general overview and brief results are presented for these factors.

4.4 Overall Results of the Factors Associated with Computational Estimation Strategies

Previous sections it is presented to factors associated with the students’ computational estimation according to two main themes. In this section, the given results are summarized for clear understanding. In the following sections divided into two, cognitive factors and affective factors that associated with computational estimation strategies.

4.4.1 Overall Results of Cognitive Factors

Data collected from the first interview session and observations lead us to divide the analysis according to two titles, these are “number sense” and “mental computation.” In the current study, some concepts are taken into consideration as evidence for the number sense. These are “ability to work with power of ten, and multiple representations of the numbers.”

According to strategy uses and strategy choices for the first interview questions, it was observed that Sergen and Deniz are poor computational estimators. When their number sense was investigated, it was concluded that they both have poor number sense, either. They both had difficulties on "ability to work with power of ten." In Q12 (98.4 x 0.041), Sergen and Nevzat could not multiply the decimal by a hundred. Where Sowder and Schappella (1994) emphasized that lack of ability in multiplying a power of ten with a decimal should be taken into consideration while explaining "number sense ability" so that it can be concluded that Sergen and Nevzat had poor number sense.

Another evidence for Sergen to concluded that his poor number sense is confusion of the "removing zeros" and "simplification of zeros" in the questions. The strategy-removing zero" is used when dealing with the big numbers addition and subtraction process. The zeros are removed while operation is conducting after that zeros should be added to result. On the other hand, simplification procedure is conducted in only division of the numbers. Sergen confused these procedures with each other in Q11 (474 257 $\div$ 8 127). This kind of mistake is a powerful indication of the poor number sense. Rubenstein (1982), who conducted a regression analysis and obtained a result showing the factors accounting for the estimation ability, confirms it. The most powerful factor was "operating with tens" which explained the computational estimation.

Nevzat is the other person who had poor number sense. He could not work with the power of tens in three questions; Q1 (31 x 68 x 296), Q12 (98.4 x 0.041), and Q13 ($3\frac{1}{2}$ x $10\frac{1}{8}$). For example, in Q13 ($3\frac{1}{2}$ x $10\frac{1}{8}$), he wanted to convert first fraction to decimal but since he could not perform multiplication a decimal with ten (3.5 x10). Since he did not remember "the rule" which is the multiplication of a decimal by a ten. Therefore, Nevzat changed his procedure and ignored both fractions of mixed numbers he only conducted 3x10. This shows both his poor number sense and also his flexibility of preferences on numbers and operations. He did not insist on computing the operation, which was initially produced. He changed his mind and preferred easer operation. Nevzat generally has difficulties on decimal related problems.

In Q1 (31 x 68 x 296), only Nevzat and Deniz missed zeros, which they removed at the beginning of the operation. Nevzat and Deniz removed the zeros of rounded numbers for conducting the multiplication easily and then they forget to replace the zeros to the end product. Working with powers of ten is a powerful evidence for concluding that Nevzat, Deniz and Sergen have poor number sense.

Other interviewees, Mert and Ayşe have not any problems with the ability to work with zeros. However, this does not make them having good number sense. The other evidence for good "number sense" was the skill to use "multiple representations of the numbers," which means that converting fraction to decimal and vice versa. In general, the interviewees preferred to convert fractions to decimals. Mert, Ayşe and Nevzat conducted this conversion process in all fraction related questions. Although Nevzat has poor number sense in general, he could conduct conversion of fraction to decimal successfully. Yet, this is not enough for him to have a good number sense. Mert could use decimals instead of fractions easily and smoothly, without any hesitation. He reads the fraction related questions in decimal forms at the very beginning of the solution and conducted the operation in decimal version. Similarly, Ayşe is the other person who has good number sense since she could convert fractions to decimals whenever she needs. According to Yang, Li and Lin (2007) "the multiple representations of numbers and operations" as one of the components of number sense. The researchers (Yang, Li & Lin, 2007) revealed that there is a moderate correlation among the multiple representations of numbers and operations and mathematical achievement.

All the examples presented above confirmed that poor estimators (Sergen, Deniz and Nevzat) have poor number sense and being good estimators (Mert and Ayşe) lead having good number sense. There is just one exception to this conclusion. Nevzat has poor number sense and poor mental computation but he could use "translation and compensation" strategies, which are the strategies used mostly by good estimators. Therefore, it is a bit complicated to conclude that Nevzat is a poor computational estimator based on these indications. It is clear that Nevzat is not as good estimator as Mert, but to conclude that he is a poor estimator, some other factors affecting his estimation ability should be investigated. These factors are feelings, thoughts, and tolerance for error.

The other cognitive factor, which is affecting the students' estimation performance, is "mental computation." The mental computational performance of the interviewees was checked according to codes, which are mathematical "basic facts of whole numbers, basic facts on decimals, and basic facts fractions".

Interviewees are generally good at mental computation on whole numbers. Especially Mert and Ayşe are remarkably good at mental computing among the interviewees. The basic facts on whole numbers discussed according to "decomposition of numbers, standard operation algorithm, and multiplication table" codes.

Decomposition of whole numbers was used effectively by all interviewees during the addition and subtraction of the numbers in Q15 (87 419 + 92 765 + 90 045 + 81 974 + 98 102) and in Q3 (7465—572). Apart from Ayşe and Nevzat, other interviewees subtracted the numbers by using decomposition of the numbers in Q3 (7465—572). Ayşe tried to conduct standard subtraction algorithm, where she put the second number under the first one. Attempting to exact computation procedure may be concern as an evidence of being a good mental computer. However, one should not only intend to compute exactly, he/she could perform it successfully, too. Sergen intended to compute exactly but could not perform it successfully but Ayşe both wanted to compute exactly and also performed it successfully. Therefore, Ayşe is a better mental computer than Sergen.

Ayşe and Sergen are two person insisting on exact computation for almost all estimation questions of the first interview. When the researcher did not want to exact computation, they pretended to estimate the question but they still conducting exact computation. Sowder-Threadgill (1984) confirmed in her study in this kind of case. That is, she confessed that some students went so far as to use a combination of finger writing and visual imagery to perform the computation although they asked to estimate the question.

During the first interview sessions, decimals were found to be the most challenging topic for all the interviewees in the current study. The basic facts on decimals are divided into three sub categories; "made same number decimal places, place value of decimal and decomposition of decimal." Among the interviewees, the poor computers, Sergen, Nevzat and Deniz had more difficulties than others on decimals. Sergen and Nevzat preferred to conduct standard algorithm by using the same number of digits in the decimal place of the decimals in two decimal question but they could not found the answer. Levine (1982) stated that an understanding of place value is essential to be able to estimate decimals. However, since these three interviewees (Sergen, Nevzat and Deniz) have poor number sense and poor mental computation on decimal, they could not answer estimation problems successfully. Additionally as Whitacre (2007) asserted that depending heavily on paper-pencil procedure is concerned as evidence for poor number sense, where it likes the cases of Sergen and Nevzat in decimal questions. According to Tsao (2005) students with low ability on number sense preferred the use of standard written computation algorithms rather than the use of number sense based strategies.

Except from other interviewees, Mert preferred to use rounding strategy with decimal and found an answer in a large interval. This kind of result may concern as an evidence for the high tolerance for error.

The basic facts on fractions are investigated in four categories; "common denominator, division algorithm, decomposition of mixed number and misconceptions on faction (that is, taking two independent integer of fraction, and during multiplication making denominator same)." Finding the common denominator is used three of the five students (Deniz, Sergen, and Nevzat). It is the first choice of these three students for solving the fraction questions. In their study, Yang, Reys, and Reys (2009) showed that, even preservice teachers depend on the written algorithm to find the common denominator. They also underlined that less than one third of the preservice teachers utilized components of number sense (benchmarks, estimations, and reasonableness) in explaining their answers. Interestingly, over 60% of all the responses used rule-based approaches. According to research studies (Boz, 2004; Goodman, 1991; Hanson & Hogan, 2000), fraction related estimation questions are most difficult questions among questions. However, in the current study it was not because students preferred to conduct exact computation procedure on these problems. The interviewees mostly tried to find common denominator of the fractions to find the solution.

The division algorithm of fraction is another struggle point for the interviewees. Although it is difficult to conduct mentally, the students tended to conduct standard division algorithm for the fractions. Students confused the "division of a number by a half " in one of the question since most of them (n=4) had not thought about the meaning of the division. Ayşe was the only person who could obtain an acceptable estimation in the division question. Other students did not figure out the meaning of the division operation. When the students were helped in their thinking procedures, they could find the answer but they were surprised with the result since they thought that the answer should be smaller than the dividend. Ball (1990) specified that students suppose that like in whole number arithmetic, division is understood to mean making smaller and multiplication is understood to mean making larger.

As a result, Sergen, Nevzat, and Deniz have poor number sense and poor mental computation. Consequently, they are poor computational estimators. Poor number sense and mental computation are not the only reasons for poor computational estimation but also negative emotions and thoughts about estimation are affecting the computational estimation performance. According to research results (Brame, 1986; Boz, 2004; Levine, 1982; Reys et al., 1982) quantitative ability is in a highly correlation with computational estimation. Therefore, high performance on computation either written or mental might affect the performance on computational estimation ability. According to many studies, it is observed that number sense and mental computation are affecting the students' computational estimation performance (McIntosh, De Nardi, & Swan, 1994; McIntosh, Reys, & Reys, 1992; Reys et al., 1982; Rubenstein, 1982; Sowder, 1992).

Reys (1984) said that a person could be competent at mental computation but very poor at computational estimation simultaneously. This claim explains Ayşe's situation. She is a competent computer but she has some difficulties in estimation. Actually, this may be related with her perception of estimation.

4.4.2 Overall Results of Affective Factors

Affective factors associated with students' computational estimation strategies are divided into mathematics related factors and estimation related factors according to data gathered from the interviews. Therefore, the results are given under these titles. In the first title there are two codes, "confidence in ability to do mathematics" and perception of mathematics."

According to interview results, Mert and Ayşe classified themselves as "very good" in mathematical ability. Additionally, they gave 10 credits on themselves out of ten points. This means that Ayşe and Mert are very confident in their mathematical abilities. The conducted observations it could be confirmed that is these students had high confidence in to do mathematics. The following fieldnotes presents Mert and Ayşe's confidence in to do mathematics in interview sessions, respectively.

According guidance of councilor notes, Mert was planning to be a student of Robert College in Istanbul. This ambition shows his high confidence in himself. Similarly, Ayşe specified that she was good at mathematics during the interviews. When their computational estimation is investigated, there is an interesting distinction between them. Although Mert is a good computational estimator, Ayşe is not. Therefore, the high self-rating on mathematical achievement is not enough to explain the students' good or poor computational estimator and their preferences of computational estimation strategies.

Interview Session 1 with Mert	**12th January 2009 Monday**
	Mert read the every question confidently and thought a little bit on each of them. Then he gave the answers and explained the ways of solution gradually. While doing his explanations, he was very confident in himself. This could be understood from the voice tone, the movement of his hands and sitting position on the chair. He was confidently looking straight ahead in my eyes to persuade me that the answer was right and the reason was acceptable. His voice was clear. I did not fell any hesitation in his actions while explaining the reasons of the answers.
Interview Session 1 with Ayşe	**15th January 2009 Thursday**
	Ayşe was smiling when each question was asked her. I think she thought that the questions were very easy for her, and she could not understand why the researcher was asking these easy questions. In several times, she wanted to compute the questions mentally, since the questions were very easy for her. Sometimes, she was very upset about the easiness of the questions, since she wanted to show her computational power.

Figure 4.2 Examples to the Fieldnotes from Observations of Interviews

To double check the students' thoughts on their mathematical ability, it was asked them to rate themselves according to success on mathematical ability in the second interview session. Although the interviewees, Deniz, Sergen and Nevzat thought that they were good at mathematics since they crossed "good" option in the testing session while CET conducting, they gave lower score in self-rating question in second interview session. Sergen gave himself five points and other interviewees gave seven points. Sergen was not much confident in his mathematics ability and his computational ability is also poor.

The perception of mathematics is affecting computational estimation of interviewees. Especially, Sergen and Ayşe specified that "mathematics needs exactness" and estimation should not use mathematics related concepts since estimation gave approximate answers. The students believed that mathematics questions should have exact answers rather than approximate. Although Ayşe has good number sense, and mental computation, and even had high self-confidence in her mathematics ability, she is not a good computational estimator. The belief that mathematical calculations must produce a single correct answer may contribute to a preference for exact solutions (Baroody, 1987). Ayşe tried to find exact solutions for the questions asked her in the first interview session.

According to observations and interviews with student showed that teachers could affect students' thoughts on computational estimation and mathematics that is "mathematics could be performed only for exact solutions." Mathematics teachers and mathematics lesson might make students think about mathematics as a pile of rules and exactness is the main point of it. There are some evidences about this assumption. The mathematics teacher A, made students found the solution by exact computation in a math lesson during the observation session. Additionally, Mert confessed that when he found approximate solutions, his mathematics teacher warned him to find the exact result. Again, Mert explained that in "dersane" his teachers said that "you should conduct your operations by paper-pencil, not in your head since you may probably miscompute the operations" which means that “dersane” teachers made students do not use mental computation, they systematically made students dependent on paper-pencil calculations.

The “dersane” was identified from the data and labeled in perception of mathematics, since it was observed that this variable influenced students’ perspective towards mathematics. Since the interviewees suffered from the SBS exam, all of them were attending a “dersane” for being successful in the exam. They want to get high score from the exam, so students must be successful on multiple-choice testing procedure. This kind of assessment makes students find exact answers for mathematics questions. Different from others, Mert stated that he could use estimation in test to eliminate inappropriate options; the other interviewees do not prefer to use estimation in the multiple-choice tests.

Estimation based affective factors are divided into three sub titles; “confidence in ability to do estimation, perception of estimation and tolerance for error.” In the interviews, students were asked to rate themselves in order to their computational estimation performance. Among the five interviewees, only Mert thought that he was a “very good” estimator. After him, Ayşe, Nevzat, and Deniz thought that they were “good” estimators and Sergen thought that he was “moderate” estimator. Although, in the interview session Mert gave himself eight points out of ten for his computational estimation performance, Deniz gave himself 10 points. This situation is an interesting one, since Mert was a good estimator but he thought that he could do better. However, Deniz thought since he could round the number, he was a competent estimator. This situation could be reexamined through Reehm’s (1992) assumption. The researcher (Reehm, 1992) reported that subjects in low ability range tended to overestimate their ability to estimate, and students in the high ability range tended to underestimate their ability. Although Mert gave lower score than Deniz and Ayşe, this did not mean Mert did not have confidence in his estimation ability. This was related with Mert’s high expectations of achievement. Therefore, he thought his success was not enough for his high expectations. LeFevre et al. (1993) identified that high self-reported estimation skill correlated with higher math marks in high school and with the belief that estimation is useful in everyday situation. Where Mert, explained that estimation should be used in daily life since one can deceive you.

The other affective factor based on estimation is perception of estimation. The most interesting examples are Sergen, Ayşe and Mert. The first two students, Sergen and Ayşe were two defenders of not using estimation in both math classes and daily life computation situations. The researcher asked Ayşe, while shopping whether estimation could be used, Ayşe claimed should not. She gave an extraordinary perspective that is she claimed that "when she was buying something she gave exact money for the price of that thing not approximate one."

On the other hand, Mert positively reacted to the questions about the usage place of estimation. He could easily explain that in math classes and in daily life, he could use estimation. Among these three students, Mert was a flexible computational estimator but the other two were not. Ayşe was a good mental computer but not a good computational estimator. According to Reys (1984), a person can be competent at mental computation but very poor at computational estimation simultaneously, like Ayşe in the current study.

Ayşe confessed that estimation made her uncomfortable. When she explained that she stated that ambiguity of estimated results made her feel like the question had not been completed yet. Similar to Ayşe, Sergen felt uncomfortable with estimated solutions and intended to find exact results. Sergen underlined that estimation was useless for him. He stated, "*Since I can find the exact solution, why do I try to find estimated solution?*" Students who did not prefer to use computational estimation believed that estimation was useless and needless.

The other affective factor is tolerance for error, which was identified through the first and second interview sessions. There were some evidences of high or low tolerance for error of the interviewees. These could be specified in order to understand students' perspectives on "doing estimation in a large interval" and "being not uncomfortable with approximate solutions." Students who had tolerance for error could perform estimate in a large range, which was good enough for its purposes. Sowder (1992) and Reys, Rybolt, Bestgen and Wyatt (1982) emphasized that tolerance for error is a feature of good estimators. A person who was obsessively concerned with obtaining exact answers was unlikely to see much point in estimation, which involved acceptance of the possibility and usefulness of inexact answers. With low tolerance for error, Ayşe wanted to find exact answers and conducted the exact computation enthusiastically. The good estimators studied by Reys et al. (1982) showed had high tolerance for error and it was suggesting they had greater conceptual understanding of the role of approximate numbers in estimation. Among the interviewees, Mert was the one who had high tolerance for error. He was very comfortable with approximate solutions and he could give results in large intervals. On the other hand, Ayşe and Sergen who were obsessively dependent on exact computation had low tolerance for error.

The interviewers of the current study were successful students compared to their peers in the class. However, since some of them (Sergen and Ayşe) did not give importance or appreciation to estimation, they did not want to use it in the questions. Therefore, being good in mathematics does not imply that being good at estimation.

Consequently, it can be said that confidence in to do mathematics, and estimation, positive feelings on estimation like accepting estimation as useful in mathematics classes and daily life, and high tolerance for error are all evidences of good estimators or affective factors of computational estimation.

CHAPTER 5

DISCUSSIONS, CONCLUSIONS AND RECOMMENDATIONS

This chapter consists of discussions, conclusions, and interpretations of the results with some recommendations for further studies and also a section related with a short discussion of the present study's contribution to me., In the first section, particularly the results of the study are discussed. Then, the next section consists of the restatement of results and interpretations of these results. In the third section, some suggestions for teachers, students, curriculum developers, teacher educators, and researchers are made. At the end of the chapter, the things learned from this study by the researcher are presented.

5.1 Discussions

The present study aimed to answer the question "Which strategies do 7th grade students use in computational estimation tasks, and which factors are associated with computational estimation strategies of 7^{th} grade students?" 15-items Computational Estimation Test (CET) was administered to 116 seventh grade students. Among the 116 students, five of them were selected according to their CET scores. Two sessions of clinical interviews were conducted with these five students. In the first session of the clinical interview, the strategies used by the students were identified. The questions of Computational Estimation Test were asked to students one by one without time restriction and they students also gave explanations of procedure used while solving the questions. In the second session of the clinical interview, the other research question, which was aimed to identify the associated factors of computational estimation strategies was discussed. In order to identify the factors, students were asked their thoughts and feelings about estimation. The results of the current study revealed some issues of critical importance that are worth being discussed. Indentified three computational estimation strategies and related factors are discussed below.

Reformulation strategy is one of the observed strategies among the students. It is divided into four sub-strategies according to analysis of transcribed interviews. These are rule based rounding, situation based rounding, compatible numbers and truncation. The preference of reformulation strategies is depending on some affective factors. As stated by Schoenfeld (1983) “purely cognitive behavior is extremely rare” that is cognitive and affective aspects are intertwined. Therefore, factors that are associated with reformulation strategies are investigated into two perspectives; affective factors and cognitive factors.

Rule dependent interviewees preferred rule-based rounding mostly as the reformulation strategy. Among the interviewees, Deniz and Sergen are two students who used this strategy more often than the others. Besides the rule dependency, these students think that mathematics needs exactness therefore even working with estimation, the mathematical rules are very important. Because of this, they used rule based rounding strategy. Apart from Deniz and Sergen, the other interviewees also used rule based rounding during the interviews. However, they had different reasons for using this strategy. Ayşe, Mert and Nevzat were very confident in themselves about their mathematical competence. They believed in their ability to compute mentally so that they could use rule based rounding, which had rules for rounding. When students performed the rule based rounding they might have thought that they were doing mathematics, but otherwise, they would not have felt as if that they were solving a mathematical question. Yang and Reys (1998) pointed out similar findings, which are the subjects of the study could not give importance to their solutions since they thought as the rule-based solutions and exact computations are more precise than they produced.

When students were forced to estimate, they chose the rule based rounding rather than other reformulation strategies and this might be due to the desire to both fulfill the researchers' demands of estimation and satisfy his/her thoughts on exact computation. Yang and Reys (1998) presented similar findings and stated that although high ability students were more likely to breakaway from rule-based methods, these breakaways were observed only when motivated by such questions as "can you do it another way?" An evidence for this claim might be the following statement of Ayşe from an interview with her: "*I feel disregardful to the math question if I do not find the exact result*." The factor associated with the preference of this strategy might be perception of estimation. Since Ayşe confessed that estimation made her uncomfortable and not finding the exact answers made her felt careless about mathematics, she used rule based rounding to get rid of this kind of disturbing feelings. To compute estimated answers based on rules of mathematics made her feel good.

An alternative strategy for rule based rounding is situation based rounding in the current study. Tolerance for error, perception of estimation and recognition of estimation as useful might have played an important role in choosing situation based rounding as a reformulation strategy for the questions. Mert is the student used situation based rounding more often than others (3 times in all types of the numbers). A high tolerance for error of Mert might be a strong reason for choosing this strategy. Since he had high tolerance for pay off, he could find estimated answers in an acceptably broad interval. Differently from others, Mert asserted that he recognized estimation as a useful application both in his daily life and academic life. He claimed that estimation is important in mathematics classes also since by using it, he can check the exact results' reasonableness.

Compatible numbers is another reformulation strategy associated with confidence in ability to do mathematics and coded among the affective factors in the coding list. A student who has high confidence in ability to do mathematics is more dependent on exact computation than producing estimated answers.

When the procedure of compatible number is investigated, it can be seen that it requires finding matched pairs and combining these pairs to add or subtract. Therefore, identification of these matching pairs might be related to computation ability and number sense of the students. Among the interviewees, Ayşe used this strategy more often than others. It can be said that Ayşe is the person highly dependent on exact computation since she has high confidence in ability to do mathematics and this dependency affects her strategy selection and use. However, being an addicted to use mental computation may prevent her producing estimated answers. According to Usiskin (1986), obsession with exact answers leads children to make unnecessary calculations and this kind of obsession keeps them from gaining experience and confidence in estimation judgments. Although Ayşe did not use any unnecessary computations but she thought that mathematics questions must be solve only using exact computation procedures. Therefore, such an idea may also kill intuition and reinforce the false notion that exactness is always to be preferred to estimation.

Although, in whole numbers and decimals sections, all of the interviewees could use the three strategies, they had some problems with fraction related questions because only used strategy was the reformulation strategy in these sections. In the fraction related questions, almost all of them (Deniz, Sergen, Ayşe and Nevzat) tried to compute operations by standard paper and pencil computation rather than using estimation strategies. Students relied on the exact computation procedures rather than estimation. This result might convey that if fraction related concepts might be taught first with estimation and then with exact computation procedures, students might more rely on estimation related processes rather than exact computation. Additionally, students were not capable of producing their own strategies for the estimation questions in fraction and decimal problems. This may be related with the students' good mental computation and confidence in doing mathematics. Many studies (Levine, 1982; Reys et al.,1982) asserted that mathematical achievement is significantly correlated with the estimation success, and also it was observed that self-rating of the students according to their mathematical achievement correlated with estimation scores (Bestgen, et al., 1980; Gliner, 1991; Mottram, 1995; Reys et al., 1991).

In the current study, according to the researcher's observation and interviews with teachers, interviewees had high confidence in their mental calculation ability, so that they thought that if they were mentally able to get the results, then there would no need to estimate the questions. Therefore, teachers should make students aware of the use of estimation in daily life applications and mathematical questions in the mathematics classes.

In decimal and fraction related questions, truncation was used more often than it was in whole number questions. Although students frequently chose the truncation strategy for decimals and fractions, still they wanted to conduct exact computation. The reason for that may be related with the difficulties of decimal and fraction concepts. In the current study, decimal questions were considered as difficult questions than questions on fractions by the interviewees. These kinds of thoughts were observed in many research studies (e.g., Bobis, 1991; Goodman, 1991; Hanson & Hogan, 2000; Rubenstein, 1982). Since they had difficulties on these topics, students wanted to be sure about the results of these questions, since fraction and decimal conceptual knowledge might not be matured within them. Since students could not make judgment on estimated results, they could conduct rote-learning rules of fractions, and decimals. Therefore, they generally preferred exact computation procedure in both types of topics. Mert and Ayşe are two students who used exact computation procedure fewer than other interviewees, and preferred to use truncation for decimals and fractions. According to data analysis, there are some affective factors associated with the use of truncation strategy. These are listed as tolerance for error, perception of estimation and confidence in ability to mathematics. Mert having high tolerance for error could ignore fraction part of the mixed numbers and could produce estimated answers in a large interval. He generally chose producing whole numbers from the decimals and fractions by ignoring the decimal and fraction parts of these numbers (i.e. for the question four, $7\frac{1}{6} - 4\frac{1}{3}$, Mert ignored fractions of the mixed numbers and get three as an answer). Therefore, he was a competent user of truncation strategy. The interviewees generally produced the standard operation procedure for decimals and fractions by making the same number of decimals places for decimals, and finding the common denominators for fractions. These kind of application might be related with the confidence in ability to do mathematics since they thought that they could mentally find the exact result. In addition to this, another reason for Ayşe does not prefer to conduct truncation for decimals and fractions but using the standard operation procedure might be seeing estimation as not useful for mathematical applications. Ayşe confessed in the one of the interview session that estimation should not be used in situations requiring mathematical problems .

Besides reformulation strategy, translation strategy might differentiate students among each others since it is more complicated strategy than reformulation. Reformulation strategy was generally used without any difficulties; it has a limited capacity for discriminating the interviewees. According to result of the data, most preferred second strategy was translation, which means that reconstructing the problem to a more manageable form. Especially, in whole number and decimals, three interviewees actively used this strategy. However, translation strategy provided some evidence for students' perspectives on estimation and mathematics. The identified constructs that are associated with translation strategy are discussed below right after that compensation related constructs are discussed.

The interviewees rarely used translation strategy in the interview sessions. It might be because of the heavy dependence on confidence in ability to do mathematics, since translation consists of being able to use multiple representations of numbers and being competent in computation mentally. Among the five interviewees, only Nevzat used this strategy in three times for the three questions, others used it twice. Nevzat is an exceptional interviewee since in overall perspective, he is not a good mental computer but he is a good estimator because he could use translation and compensation strategies, which are used by competent estimators. Reys and his colleagues (1982) identified that translation is more flexible than reformulation and may require an advanced level of conceptual knowledge of estimation.

Three students (Nevzat, Ayşe and Mert) could use compensation strategy, and this might be give huge amount of knowledge about students' perspectives on computational estimation. Compensation is a kind of higher level of strategy. According to researchers (e.g., Reys et al., 1991; Sowder, 1992), more competent estimator could use compensation strategy since it includes more complex constructs than other strategies. In the current study, this claim was confirmed by the finding. Related factors of compensation strategy are explained below.

Compensation strategy was chosen by good estimators and used less frequently than the other estimation strategies. It was observed that in whole number and decimal number questions, three out of five interviewees used the compensation strategy. When we look at the strategies used by the students, we can see that compensation strategy is used relatively less (for example, compensation was used seven times, and reformulation strategies were used more than a hundred times).

Although compensation was used in few times, this frequency of using and process of using the strategy explain many important underlying constructs of the students. For example, researchers (Reys et al, 1980; Reys, Reys, Nohda, Ishida, & Shimizu 1991; Sowder, 1992) agreed that compensation strategy discriminates good computational estimators. Findings of the study suggest that interviewees rarely preferred to use compensation strategy. Only three interviewees performed either intermediate or final compensation in five questions. Among three of these interviewees, only one of them is more competent strategy user and used compensation strategy more often than the others. Similar findings were reported by Sowder and Wheeler (1989) who stated that most fifth graders recognize the value of compensation but do not use it when generating computational estimates. In the current study, the reason why some students are more competent strategy user than the others might be related to some factors, which are associated with the computational estimation.

According to observations, there are some affective factors determining the use of compensation strategies by these three interviewees. These are listed as perception of mathematics and tolerance for error. Perception of mathematics is identified among interviewees as the belief that mathematics needs exactness. Among the compensation strategy users, one of the student underlined that mathematics should produce exact results. Because of this, the student prefers to use compensation strategy to make the estimated result closer to exact answers. Moreover, the other reason for the student to use the compensation strategy is low tolerance for error. It may be related with the belief that mathematics needs exactness. The student thought that mathematics should produce exact answers because of this she could not tolerate pay off for the estimated answers. Although her use of compensation strategy in the questions should be evidence for her being a good estimator, Ayşe who believes that mathematics needs exactness, and who has less tolerance for error, uses this strategy for producing closer estimation in her answers. Since she is a good mental computer she could use compensation strategy properly, but her first choice is to find the exact answer rather than estimated one.

On the other hand, the other compensation strategy user is Mert who has positive feelings on estimation. According to his interview transcribes, he is using estimation during both daily life and school time. Therefore, his reason for choosing compensation strategy is related with producing acceptable estimations rather than getting exact answers. Mert who is a good estimator, confirmed that estimation is a useful tool and he himself uses it. He stated that he used estimation to check his results' correctness. Therefore, Mert's positive feelings on estimation affect his strategy preferences and usage.

The differences and similarities between these two students may reveal that the affective factors are associated with the computational estimation. Ayşe and Mert might be specified as good mental computers but they are not both good estimators. Though Ayşe has negative feelings on estimation, Mert stated that it is useful in many areas. Ayşe's negative feelings may be due to her belief that mathematics requires exactness and estimation could not be a mathematical application. The findings of Dowker (1992) explained that people who prefer precision (those with low tolerance for error) see estimation as useless and thus they are negatively affected by their lack of experience in estimating. Since Ayşe had negative belief about estimation, she could not be a good estimator. Although, she was a good computer, she could not be an estimator, because of her beliefs about mathematics and estimation.

Heirdsfield (2000) identified that the belief about nature of mathematics could be revealed in a student's performance orientation. Such beliefs can be as mathematics should make sense, one can often conduct different ways of solving problems, and he/she may think there should be more than one answer. However, where mathematics is viewed as set of rules to be learned and need not make sense, then one only follow rules and thinks that there should be only one possible answer for the question. Similarly, Schoenfeld (1987, p.34) claimed that "Beliefs have to do with your mathematical worldview. The idea is that your sense of what mathematics is all about will determine how you approach mathematical problems."

As a result, there are some affective factors associated with compensation strategy of computational estimation. This was observed through interviewees' conversations. Belief that mathematics means exactness and tolerance for error play a role in the usage and preference of compensation strategy by the interviewees. These affective factors which were students possess might be produced by helping teachers.

In interviews with Mert, he pointed out mathematics teachers' demanding exact computations. He claimed that his mathematics teacher made him to find exact answer rather than estimated results. Therefore, a reason why students have so much trouble with estimation may be related with teachers' orientations. According to Lambert (1990) mathematics is commonly associated with certainty and being able to get the right answer quickly and teachers tell students whether their answers are right or wrong, but rarely do they encourage students to explore the assumptions, which led them to their answers. As a result, children learn that, there is only one correct answer and they become afraid to offer alternative ones. Sowder and Wheeler (1989) also emphases that schooling factors such as emphasis on unique answers and instruction on rounding and computational procedures seemed to influence students' reactions to estimate requested questions and feelings on estimation.

As a last word, interviewees used very few computational estimation strategies. There are some factors identified as associated with these strategies. These factors are discussed in relation to each strategy above. Mainly two specific factors, cognitive and affective, are influencing to students' strategy preference and use. How these are affecting are explained and discussed above.

5.2 Conclusions

The data gathered from the two interview sessions and observation fieldnotes revealed the following results;

1. Students mostly preferred the reformulation among the three types of strategies in all types of numbers, whole numbers, decimals, and fractions.
2. Rule-based rounding is the most used reformulation strategy especially used by those believing that mathematics .needs exactness.
3. More competent computational estimation user could use translation, and compensation more often than less competent one.

4. Number sense might be a construct that is associated with the students' computational estimation strategies.
    a. In particular, ability to work with power of ten may be an evidence of good number sense ability and also good computational estimator.
    b. Ability to use multiple representations of the numbers may be an evidence for good number sense and also good computational estimator.
5. Mental computation might be a construct that is associated with the students' computational estimation strategies.
    a. Skills in managing the basic facts on whole numbers may be an element of good mental computers and good computational estimators.
    b. Skills in managing the basic facts on decimals may be an element of good mental computers and good computational estimators.
    c. Skills in managing the basic facts on fraction may be an element of good mental computers and good computational estimators.
6. Mathematics-based affective factors, which are confidence in ability to do mathematics and perception of mathematics, may be affecting the students' computational estimation strategies.
    a. Confidence in mathematics ability may be affecting the students' computational estimation strategies. .
    b. Perception of mathematics may be affecting students' computational estimation strategies.
    c. Test anxiety may be an undeniable factor which is related with the perception of mathematics and which it also associated with computational estimation strategies.

7. Estimation-based affective factors, which are confidence in ability to do estimation, perception of estimation, and tolerance for error, may be affecting the students' computational estimation strategies.
    a. Confidence in estimation ability may be affecting the students' computational estimation strategies.
    b. Perception of estimation may be affecting students' computational estimation strategies.
        i. Negative feelings on estimation may affect the students' computational estimation strategies.
        ii. Recognition of estimation as useful may affect the students' computational estimation strategies.
    c. Tolerance for error may be affecting the students' computational estimation findings.

5.3 Recommendations

In this section, some recommendations are made for teachers, teacher educators, and researchers. According to result of the study, reformulation is generally known and used only strategy, particularly; rule based rounding is most preferred one in computational estimation applications. This limited exposure to computational estimation strategies might be removed the students of the opportunity to learn methods. Because of this, students should be taught variety of computational estimation strategies rather than reformulation. Since students addicted to rule based strategies, teachers should produce estimation activities through out the mathematics classes to teach students mathematics did not only base on exact results. Teachers should use and appreciate the language of estimation in arithmetic classes. While doing this, they should not require too much precision. They should emphasize the multiple answers of computational estimation where all could be acceptable. At the same time, they should emphasize and give feedbacks about the reasonableness of the estimated answers. So that student could understand that estimation could participate in mathematical applications.

Especially, compensation strategies should be emphasized in the classroom applications, since by using the strategies either final or intermediate compensation, students could identify that estimated answers could be arranged according to requirement of the precision of the questions. Based on the results, compensation strategy was observed among the good estimators; therefore improving compensation strategy might help to students being good computational estimators. More time should be spent on teaching the concept of compensation, which is the most sophisticated strategy in order to facilitate mastery of this concept.

Students could use computational estimation strategies more effectively when they had high mental computation ability and high number sense ability. Students should improve their mental computation and they should use it as a tool for using computational estimation. For using estimation strategies more effectively teachers should improve students' number sense and mental computation, which were associated factors of computational estimation. First, students should be taught how precise result is enough for the purpose at hand, and then they should decide exactness of the results. So that students should decide where they could use estimation or exact computation. Students should give importance to estimation, and believe that estimation is useful in their mathematical applications and daily life situations. Therefore, teachers solve daily life applications of computational estimation questions in mathematics classes.

Since the current research results, show that giving importance to estimation is affecting to use computational estimation strategies, teachers should help students to develop a respect for approximate answers. Therefore, teachers should give opportunities to students to find approximate answer at first, then exact answers should be produced. By conducting this kind of instruction, teachers might help students to develop positive feelings on estimated answers. As a result of this kind of instruction, teachers might change students' opinion on mathematics, which was usually regarded as mathematics needs exactness. Since this kind of opinion may affecting computational estimation strategies of students, changing students' opinion on mathematics may improve students' performance. Additionally teachers should emphasize and use real-world examples where only estimation is required so that students can look for an estimated answer almost naturally. So that students may want to use and appreciate estimation in daily life. Moreover, by solving questions, teachers may influence students' thought on the usage of estimation in mathematical problems. Since test-based exams were given more importance than written exams because of the national exam system, it should be emphasized by teachers, that students could use estimation in test exams also.

Teachers should emphasize that the exact results not always essential for making decision; on the contrary, estimated results may be more helpful than exact one for making decisions. Therefore, teachers should convince students to use computational estimation in mathematics classes.

For further research studies, researchers should pay more attention to the number sense and related abilities to make the basic rules of the mathematics because these are factors, which are associated with the computational estimation strategies. They should conduct more researches on the computational estimation and factors that related with computational estimation and strategies.

Moreover, researchers should give special emphasis on the task of improving students' own strategies of estimation and additional time should be spent on teaching methods. Not only quantitative, but also qualitative research studies should be conducted to identify the strategies of different age groups of subjects. As a critical element of mathematical problem solving, students' estimation ability and in particular that of computational estimation should be investigated. Researchers should emphasize that the sooner students are exposed to estimation and related skills, the more they give value on estimation and understand when and how to apply it effectively.

5.4 What I Have Learned From The Current Research Study?

There is no doubt I gained and improved many skills and knowledge while conducting this research. When I ask myself what I have learned from this study, the answers could be listed as follows:

1. I have learned that the research question should interest you or meaningful at least for you. While I had troubles with procedures of the study, I did not give up. Since I was wondering the answers of the questions, which were made me to continue on working. The topic that I had been studying on was make me curious about results so that I continued to find the answers.
2. I have learned that the preparations for doing a case study include prior skills such as communication skills, prevision about the possible obstacles, being unbiased . While conducting pilot study I have realized that if I could not have good communication skills I would not have rapport with children. This would prevent deep understanding of the data collected from them.
3. Since there is no strict prescribe for conducting case study, I have learned that a researcher should be flexible so that newly encountered situations can be seen as opportunities or threads. The study plan before started to conduct it was changed in many times. This changing made me uncomfortable first, but then I could adjust study plan through changed situations.
4. I have learned that a researcher who is conducting interview should able to ask good questions and interpret the answers. In the beginning of the I could realize that some probes should be prepared for the questions since during the interviews students might had short answers to the questions. to get depth understanding of the situation, researcher should asked every detail and should get a clear answers for the questions.

5. As an interviewer, I have learned that an investigator should be a good listener and not direct interviewees by his or her own ideologies or preconceptions. During the interviews, sometimes students stopped talking to answer the questions and only they were staring to the questions. In those times if researchers should cleverly use communication skills to make them answers the questions without directing them. If it could not be achieved, the data gathered from the interviewees could not be reflecting the true perspectives of the students.
6. I have learned that pilot study is very important for a research study to check the instruments, to train himself/herself, to understand the concept deeply. During the pilot study, I had learned many things, such as, to communicate with students who were shy, communicate with teachers who were not willing to participate in the study, manage with technical tools, which were audio recording and video recording, and so on. Therefore, pilot study taught how I could conduct a case study and what kinds of awkwardness could be come across during the procedure, and how I could overcome.
7. I have learned that qualitative data analysis needs a full concentration without any interruption. During data analysis, I interrupted for a week and I saw that a week time was a very long time to lose the concentration of analysis procedure. I work hard to build the links among the themes and codes again. A full concentration is very important during the analysis of a qualitative data.
8. I have learned that the recommendations of advisor are vital and they must be followed. During the research I realized again an advisor was a vital person of you since I could have lost in data so the only person who could understand and help was your advisor.
9. I have learned that writings should be read by someone else and this person's comment should be paid attention. Since I had been center of the data pile, someone as an outsider should read your writings and should give feedback for important revisions.
10. I have learned that you should not give up believing in what you are doing.

REFERENCES

Aiken, L. R. (1976). Update on attitudes and other affective variables in learning mathematics. *Review of Educational Research, 46,* 293-311.

Alajmi, A.H. (2009). Addressing computational estimation in the Kuwaiti curriculum: teachers' views. *Journal of Mathematics Teacher Education*, *12*, 263-283.

Anghileri, J. (1999). Issues in teaching multiplication and division. In I. Thompson (Ed.), *Issues in teaching numeracy in primary school* (pp. 184-194). Buckingham: Open University Press.

Ball, D. L. (1990). Prospective elementary and Secondary Teachers' Understanding of Division. *Journal for Research in Mathematics Education*, *21*(2), 132-144.

Bana, J., & Dolma, P. (2004). The relationship between the estimation ability and computational abilities of year 7 students MERGA 27: Mathematics Education for the Third Millennium: Towards 2010, 1, 63-70. Retrieved 23 June 2006 from http://www.merga.net.au/documents/RP52004.pdf

Baki, A.,Karatas, I., & Guven, B. (2002). *Klinik Mülakat yöntemi ile problem çözme becerilerinin değerlendirilmesi*. 5. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi, ODTÜ Kültür Kongre Merkezi, Ankara.

Baroody, A.J., & Gatzke, M.R. (1991). The Estimation of Set Size by Potentially Gifted Kindergarten-age Children. *Journal for Research in Mathematics Education, 22,* 59-68.

Beishuizen, M. (1993). Mental strategies and materials or models for addition and subtraction up to 100 in Dutch second graders. *Journal for Research in Mathematics Education. 24*(4), 294-323.

Benton, S.E. (1986). A summary of research on teaching and learning estimation. In H.L. Schoen & M.J. Zweng (Eds.) *Estimation and Mental Computation*: *1986 yearbook* (pp.239-248). Reston VA: National Council of Teachers of Mathematics

Berch, D. B. (2005). Making Sense of number sense –Implications for children with mathematical disabilities. *Journal of Learning Disabilities*. *38* (4), 333-339.

Berry, R. Q. (1998). *Computational Estimation Skills of eight grade students*. Unpublished master's thesis, Christopher Newport University.

Bestgen, B. J., Reys, R. E., Rybolt, J. F., & Wyatt, J. W. (1980). Effectiveness of Systematic Instruction on Attitudes and Computational Estimation Skills of Preservice Elementary Teachers. *Journal for Research in Mathematics Education*, 124-136

Blair, S.L. (2001). *The Importance of Basic Facts in Mathematics*. Unpublished doctoral dissertation, University of Kansas.

Bobis, J. (1991). The effect of instruction on the development of computational estimation strategies. *Mathematics Education Research Journal, 3* (1), 17-29.

Bogdan R. C. & Biklen, S. K. (1992). *Qualitative research for education: An introduction to theory and methods*. Boston: Allyn and Bacon.

Boz, B., & Bulut, S. (2002) *İlköğretim Matematik, Fen Bilgisi ve Okul Öncesi Öğretmen Adaylarının Tahmin Becerilerinin İncelenmesi,* 5. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi,ODTÜ Kültür Kongre Merkezi, Ankara

Boz, B. (2004), *Investigation of Estimation Ability of High School Students*, Unpublished master's thesis, Middle East Technical University, Ankara.

Callingham, R. (2005). *A whole school approach to developing mental computation strategies*. Proceedings of the 29th conference of international group fort he psychology of mathematics education. 2, 201-208, Melbourne: PME

Carpenter, T. P., Coburn, T. G., Reys, R. E., & Wilson, J. W. (1976) Notes from National Assessment: Estimation. *Arithmetic Teacher, 23*, 296–302

Carpenter, T.P., Kepner, H., Corbitt M. K., Lindquist M. M.,& Reys, R.E.(1980). Results and Implications of the Second NAEP Mathematics Assessments: Elementary School. *Arithmetic Teacher,* 27, 44-47.

Carroll, W. (1996). Mental computation of students in a reform-based mathematics curriculum. *School Science and Mathematics, 96* (6), 305-311.

Case, R. & Sowder, J.T. (1990). The Development of Computational Estimation: A Neo-Piagetian Analysis. *Cognition and Instruction*, *7(2),* 79-104

Chien, Y. C. (1990). *The Use of Self-study Material for Improving the Computational Estimation Skills of Preservice Teachers*. Unpublished doctorial dissertation, Columbia University Teachers College

Cilingir, D. & Turnuklu, E. B. (2009).Estimation Ability and Strategies of the 6th - 8th grades elementary school students. *Elementary Education Online, 8* (3), 637-650. Retrieved 01 September 2009 from http://www.ilkogretim-online.org.tr/vol8say3/v8s3m2.pfd

Clement, J. (2000). Analysis of clinical interviews: foundations and model viability. In Kelly, A.E., & Lesh, R. A. (Eds.) *Research design in mathematics and science education*. (547-590) Manwah, New Jersey: Lawrence Erlbaum Associates

Creswell, J. W. (2003). *Research Design: Quantitative, Qualitative, and Mixed Methods Approaches*. SAGE, Thousand Oaks, USA.

Crites, T. (1992). Skilled and Less-Skilled Estimators' Strategies for Estimating Discrete Quantities. *The Elementary School Journal, 92*, 601-615.

Damarin, S.K., Dziak, N. J., Stull, L., & Whiteman, F. (1988). Computer Instruction in Estimation: Improvement in High School Mathematics Students. *School Science and Mathematics, 88*, (6), 488-492.

Dolma, P. (2002). *The relationship Between Estimation Skill and computational ability of students in years 5, 7 and 9 for whole and rational numbers.* Unpublished master's thesis, Edith Cowan University, Perth

Dowker, A. (1992). Computational Estimation Strategies of Professional Mathematicians. *Journal for Research in Mathematics Education, 23,* 45-55.

Dowker, A., Flood, A., Griffiths, H., Harris, L. & Hook, L. (1996).Estimation Strategies of four groups. *Mathematical Cognition, 2* (2), 113-135.

Dowker, A. (1997).Young children's addition estimation. *Mathematical Cognition*, *3* (2), 141-154

Dowker, A. (2005). *Individual Differences in arithmetic: Implications for psychology, neuroscience and education.* New York: Psychology Press

Ell, F. (2001). *Strategies and thinking about number in children aged 9-11*. (University of Auckland Technical Report No:17). Retrieved 15 October, 2007, from http://www.tki.org.nz/r/asttle/pdf/tecnical-reports/techreport17.pdf

Evens, H and Houssart, J. (2007). Paired interviews in mathematics education. In D. Küchemann (Ed.) *Proceedings of British Society for Research into Learning Mathematics, 27* (2), 19-24.

Forrester, M. A., & Pike, C. D. (1998). Learning to estimate in the mathematics classroom: a conversion-analytic approach. *Journal for Research in Mathematics Education*, *29* (3), 334-356

Forrester, M. A., & Shire, B. (1994). The influence of object size dimension and prior context on children's estimation ability. *Educational Psychology, 14* (4), 451-465

Gay, A.S (1990). *A study of middle school students' understanding of number sense related to percent*. Unpublished doctoral dissertation, Oklahoma State University, Stillwater, Oklahoma

Gatzke, M.R. (1989), *Kindergarten Children's Estimates of Numerosity*. Unpublished doctoral dissertation, University of Illinois, Urbana-Champaign.

Geary, D.C. & Widaman, K. F.(1987). Individual Differences in Cognitive Arithmetic. *Journal of Experimental Psychology: General*. *116* (2), 154-171.

Ginsburg, H.P. (1997) *Entering the child's mind: the clinical interview in psychological research and practice.* UK: Cambridge University Press

Ginsburg, H. P. (1981). The clinical interview in Psychological research on mathematical thinking: Aim, Rationales, Techniques. *For the Learning of Mathematics,* 1, 3-11.

Ginsburg, H. P., & Seo, K. (1999). Mathematics in Children's Thinking. *Mathematical Thinking and learning,* 2, 113-129.

Glasgow, B., & Reys, B. J. (1998).The authority of calculator in the minds of college students *School Science and Mathematics, 98*(7), 383-388.

Gliner, G. S. (1991). Factors contributing to success in mathematical estimation in preservice teachers: types of problems and previous mathematical experience. *Educational Studies in Mathematics, 22,* 595-606

Goodman, T. (1991). Computational estimation skills of preservice elementary teachers. *International Journal of Mathematical Education in Science and Technology, 22* (2) 259-272.

Goldin, G. A. (1998). Observing mathematical problem solving through task-based interviews. In A.R.Teppo (Ed.), *Qualitative Research Methods in Mathematics Education* (p. 40-62). NCTM

Goldin, G. A. (2000). A scientific perspective on structured, task-based interviews in mathematics education research. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 517–546). Mahwah, NJ: Erlbaum.

Groth, R.E. (2005). An investigation of statistical thinking in two different contexts: detecting a signal in a noisy process and determining a typical value. *Journal of Mathematical Behavior, 24*, 109-124.

Hanson, S.A. & Hogan P.T. (2000). Computational Estimation Skill of College Students. *Journal for Research in Mathematics Education*, *31* (4), 483-499.

Hatano, G. (1988), Social and motivational bases for Mathematical Understanding. In Sowder, J. (1992), Estimation and Number Sense. In D.A. Grouws (Ed.), *Handbook of research in mathematics teaching and learning* (pp.371-389). New York : Macmillan

Heinrich, E.J. (1998). *Characteristics and skills exhibited by middle school students in performing the tasks of computational estimation*. Unpublished doctoral dissertations, Fordham University, New York.

Heirdsfield, A. M. (2002). The interview in mathematics education: the case of mental computation. Understanding and knowledge. *In: Annual Conference of the Australian Association for Research in Education,* Brisbane, Australia.

Heirdsfield, A. M., & Cooper, T. J. (2002). Flexibility and inflexibility in accurate mental addition and subtraction: two case study. *Journal of Mathematical Behavior*, *21*, 57-74.

Heirdsfield, A. M (2003). "Spontaneous" Mental Computation Strategies. *In International Group for the Psychology of Mathematics Education, Paper presented at the 27th International Group for the Psychology of Mathematics Education Conference Held Jointly with the 25th PME-NA Conference* (pp.55-62), Honolulu, HI, Jul 13-18

Hiebert, J., & Behr, M. (1991). *Number Concepts and Operations in the middle Grades*. The United States of America: Lawrence Erlbaum Associates

Hogan, T.P., & Brezinski, K.L. (2003). Quantitative estimation: one, two, or three abilities? *Mathematical Thinking and Learning, 5*(4), 259-280.

Hogan, T.P., Wyckoff, L. A., Krebs, P., Jones, W., & Fitzgerald, M.P. (2004). Tolerance for error and computational estimation ability. *Psychological Reports, 94,* 1393-1403.

Hogan, T.P. & Parlapiano, C.A. (2008). Personality factors related to quantitative estimation skill: confirmation and extension. *Psychological Reports*, 103, 189-198.

Hope, J.A. (1986). Mental calculation: Anachronism or basic skill? In H.L. Schoen & M.J. Zweng (Eds.) *Estimation and Mental Computation: 1986 yearbook* (pp.45-54). Reston VA: National Council of Teachers of Mathematics

Hunting, R. P. (1997). Clinical interview methods in mathematics education research and practice. *Journal of Mathematical Behavior*, *16*(2), 145-165.

Huntley, M. A., Marcus, R., Kahan, J., & Miller J. L. (2007). Investigating high-school students' reasoning strategies when they solve linear equations. *Journal of Mathematical Behavior*, *26*, 115-139.

Irwin, K. (2001). Using everyday knowledge of decimals to enhance understanding. *Journal for Research in Mathematics Education, 32*(4), 399– 420.

Jurdak, M. & Shahin, I. (1999). An ethnographic study of the computational strategies of a group of young street vendors in Beirut. *Educational Studies in Mathematics, 40,* 155-172.

Kamii, C., Lewis, B., & Jones, S. (1991). Reform in primary education: A constructivist view. *Educational Horizons. 70(1),* 19-26.

Karataş, İ., & Güven, B. (2003). Problem çözme davranışlarının değerelendirilmesinde kullanılan yöntemler: klinik mülakatın potansiyeli. İlköğretim Online, 2, Article 2. Retrieved 17 July 2008, from http://www.ilkogretim-online.org.tr/vol2say2/v02s02c.pdf

LeFevre, J., Greenham, S. L., & Waheed, N. (1993). The development of procedural and conceptual knowledge in computational estimation. *Cognition and Instruction, 11* (1), 95-132.

Lemaire, P., & Siegler, R. S. (1995). Four aspects of strategic change: contributions to children's learning of multiplication. *Journal of Experimental Psychology: General, 124* (1), 83-97.

Lemaire, P., Lecacheur & Farioli, F. (2000). Children's strategy use in computational estimation. *Canadian Journal of Experimental Psychology, 54*(2), 141-148.

Lemaire, P., & Lecacheur, M. (2002). Children's strategies in computational estimation. *Journal of Experimental Child Psychology, 82,* 281-304.

Lembke, L. O., & Reys, B. J. (1994). The development of and interaction between intuitive and school-taught ideas about percent. *Journal for Research in Mathematics Education*, *25*(3), 237-259.

Leutzinger, L. P. (1999).Developing Thinking Strategies for Addition Facts Teaching *Children Mathematics, 6,* 14-18.

Levine, D.R. (1982). Strategy use and estimation ability of college students. *Journal for Research in Mathematics Education, 15*(5), 350-359.

Levine, S.C., Jordan, N. C., & Huttenlocher, J. (1992). Development of calculation abilities in young children. *Journal of Experimental Child Psychology, 53,* 72-103.

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Beverly Hills, CA: Sage Publications, Inc.

Lodico, M. G., Spaulding, D.T., & Voegtle, K.H. (2006). *Methods in educational research from theory to practice*. San Francisco, CA: Jossey Bass.

Ma, X., & N. Kishor. (1997). Assessing the Relationship between Attitude towards Mathematics and Achievement in Mathematics: A Meta-Analysis. *Journal for Research in Mathematics Education, 28*(1), 26-47.

Markovits, Z. & Sowder, J. (1994). Developing number sense: An intervention study in grade 7. *Journal for Research in Mathematics Education, 25* (1), 4-29.

Marshall, C., & Rossman, G. B. (1999). *Designing Qualitative Research* (3rd Ed.). Thousand Oaks, CA: Sage.

McChesney, J. & Biddulph, F. (1994). Number sense. In J. Neyland (Ed.), *Mathematics Education Volume 1* (pp.7-17). Wellington: Wellington College of Education.

McIntosh, A., Reys, B., & Reys, R. (1992). A proposed framework for examining basic number sense. *For the Learning of Mathematics, 12,* 2-8.

McIntosh, A., De Nardi, E., & Swan, P. (1994). *Think Mathematically! How to teach mental maths in the primary classroom*. Melbourne: Longman Cheshire

Merriam, S.B. (1998). *Qualitative Research and Case Studies Applications in Education*. San Francisco: Jossey-Bass Publications.

Micklo, S. J. (1999). Estimation; it's more than a guess. *Childhood Education*, *5* (3), 142-145.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data Analysis: An Expanded sourcebook (3rd ed.)*.Newbury Park, CA: Sage Publications.

Milli Eğitim Bakanlığı, (2005). *Talim Terbiye Kurulu Başkanlığı, İlköğretim Matematik Dersi Öğretim Programı ve Kılavuzu 6-8. sınıflar*. Milli Eğitim Basımevi-Ankara.

Mitchell, J. H., Hawkins, E. F., Stancavage, F. B., & Dossey, J. A. (1999). Estimation skills, mathematics-in-context, and advanced skills in mathematics. U.S Department of Education, Office of Educational Research and Improvement, National Center of Educational Statistics (No ED 1.310/2:436419). Washington, D. C.: Education Publications Center

Montague, M. (1997). Cognitive strategy instruction in mathematics for students with learning disabilities. *Journal of Learning Disabilities*, *30* (2), 164-177.

Montague, M. & Van Garderen, D. (2003). A cross-sectional study of mathematics achievement, estimation skills, and academic self-perception in students of varying ability. *Journal of Learning Disabilities*, *36* (5), 437-448.

Morgan, C. (1990). Factors affecting children's strategies and success in estimation. In. G. Booker, P. Cobb & T.N. de Medicuti (Eds.), *Proceeding of the Fourteenth PME Conference with the North American Chapter Twelfth PME-NA Conference* (vol.3, pp.265-272). Oaxtepec, Mexico

Mottram, R.D. (1995). *A Comparative Study of Computational Estimation Ability and Strategies Used in Estimation Problems*, Unpublished doctoral dissertations, University of Colorado, Boulder.

Murphy, M.R. (1989). *Teaching and Applying Computational Estimation Skills in Grade 8.* Unpublished master's thesis, Simon Fraser University

Munakata, M. (2002), *Relationships Among Estimation Ability, Attitudes Toward Estimation, Category Width And Gender in Students of Grades 5-11*. Unpublished doctoral dissertations, Columbia University

National Council of Mathematics Teachers (2000). *Principles and Standard for school mathematics*. Reston, VA: National Council of Mathematics Teachers

Nik Pa, N. A. (1989). Research on children's conceptions of fractions. *Focus on Learning Problems in Mathematics, 11*(3), 3–24.

Opper, S. (1977). Piaget's clinical method. *Journal of Children's Mathematical Behavior*, *1* (4), 90-107.

Paull, D. R. (1971). *The Ability to Estimate in Mathematics,* Unpublished doctorial dissertations, Columbia University

Parmar, R. S. (2003).Understanding the concept of "Division": Assessment considerations. *Exceptionality, 11* (3), 177-189.

Pike, C. D. & Forrester M. A. (1997).The influence of number sense on children's ability to estimate measures. *Educational Psychology*, *17* (4), 483-500.

Ramachary A. & Kumar, S. (2007).Estimation and general estimating ability of secondary students-an overview. *International forum of Teaching and Studies*, *3* (1), 5-12.

Reehm, S.P. (1992). *A Comparison of Estimation Process used on Numeric and Contextual problems*. Unpublished doctorial dissertation, University of Missouri –Columbia

Reys, R. E., Bestgen, B. J., Rybolt, J. F., & Wyatt, J.W. (1980). Identification and characterization of computational estimation process used in school pupils and out-of-school adults. (No ED 197963) Washington, D.C.: National Institute of Education

Reys, R., & Bestgen, B. (1981). Teaching and assessing computational estimation skills. *The Elementary School Journal, 82* (2), 116 – 127.

Reys, R. E., Rybolt, J. F., Bestgen, B. J., & Wyatt, W. J. (1982). Process used by good computational estimators. *Journal for Research in Mathematics Education*, *13* (3), 183-201.

Reys B. J. (1986). *Teaching Computational Estimation: Concepts and Strategies*. In H. L. Schoen & M. J. Zweng (Eds.), *Estimation and mental computation: 1986 yearbook* (pp. 31-45). Reston, VA: National Council of Teachers of Mathematics.

Reys R. E. (1986). *Evaluating Computational Estimation.* In H. L. Schoen & M. J. Zweng (Eds.), *Estimation and mental computation: 1986 yearbook* (pp. 225-238). Reston, VA: National Council of Teachers of Mathematics.

Reys, B. J., Reys, R. E., & Penafiel, A. F. (1991). Estimation Performance and Strategy Use of Mexican 5th and 8th Grade Student Sample. *Educational Studies in Mathematics*, *22* (4), 353- 375.

Reys, R. E., Reys, B. J., Nohda, N., Ishida, J., & Shimizu, K. (1991). Computational estimation performance and strategies used by fifth and eight grade Japanese students. *Journal for Research in Mathematics Education*, *22* (1), 39-58.

Reys, B. (1992). Estimation. In T. Post (Ed.), *Teaching mathematics in grades K–8: Research-based methods* (pp. 279–301). Boston: Allyn & Bacon.

Reys, R. E. (1993). Research on computational estimation: What it tells us and some questions that need to be addressed. *Hiroshima Journal of Mathematics Education*, 1, 105-112.

Reys, R. E., Reys, B.J., Nohda, N., & Emori, H. (1995). Mental computation performance and strategy use of Japanese students in grades 2, 4, 6, and 8. *Journal for Research in Mathematics Education, 26* (4), 304-326.

Reys; R. E. (1998). Computation versus number sense. *Mathematics Teaching in the Middle School*, *4* (2), 110-112.

Reys, R. E., & Yang, D. (1998). Relationship between computational performance and number sense among sixth-and eighth grade students in Taiwan. *Journal for Research in Mathematics Education, 29* (2), 225-237.

Reys, R. E., Reys, B. J., McIntosh, A., Emanuelsson, G., Johansson, B., & Yang, D. (1999). Assessing Number Sense of students in Australia, Sweden, Taiwan and the United States. *School Science and Mathematics*, *99* (2), 61-70.

Rubenstein, R. (1985).Computational estimation and related mathematical skills. *Journal for Research in Mathematics Education,* 16, 109-116.

Schoen, H. L., & M.J. Zweng (Eds.) (1986). *Estimation and Mental Computation: 1986 yearbook*. Reston VA: National Council of Teachers of Mathematics

Schoen, H. L., Friesen, C. D., Jarrett, J.A., & Urbatsch, T. D. (1981). Instruction in estimating solutions of whole number computations. *Journal for Research on Mathematics Education*, *12* (3), 165-178.

Schoenfeld, A. H. (1983). Beyond the purely cognitive: Belief systems, social cognitions, and metacognitions as driving forces in intellectual performance. *Cognitive Science*, 7, 329-363

Schoenfeld, A. H. (1987).Confessions of an accidental theorist. *For the Learning of Mathematics*, *7*(1), 30-38.

Segovia, I. & Castro, E. (2009). Computational and measurement estimation: curriculum foundations and research carried out at the University of Granada, Mathematics Didactics Department. *Electronic Journal of Research in Educational Psychology, 17*, (1), 499-536.

Seethaler, P. M., & Fuchs, L. S. (2006).The cognitive correlates of computational estimation skill among third-grade students. *Learning Disabilities Research and Practice, 21* (4), 233-243.

Siegler, R. S., & Booth, J.L. (2005*). Development of Numerical Estimation.* In Jamie I. D. Campbell (Ed.) Handbook of Mathematical Cognition. New York: Psychology Press.

Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child Development, 75* (2), 428-444.

Smart, J. R. (1982). Estimation skills in mathematics. *School Science and Mathematics*, *82* (8), 642-649.

Star, J. R. & Rittle-Johnson, B. (2009). It pays to compare: an experimental study on computational estimation. *Journal of Experimental Child Psychology,* 102 (4), 408-426.

Sowder-Threadgill, J. (1984). Computational estimation procedures of school children. *Journal of Educational Research*, *77* (6), 332-336

Sowder, L. (1988). Children's solutions of story problems. *Journal of Mathematical Behavior, 7* (3), 227-238.

Sowder-Threadgill, J. (1988). Mental computation and number comparison: their roles in the development of number sense and computational estimation. In J.Hiebert & M. Behr, (Eds). *Number concepts and Operations in the middle grades*.(pp.182-197) Hillsdale:NJ:LawrenceErlbaumAssociates

Sowder, J., & Sowder, L. (1989). Research into practice: developing understanding of computational Estimation. *Arithmetic Teacher*, *36* (5), 25-27.

Sowder-Threadgill J., & Wheeler, M. M. (1989).The development of concepts and strategies used in computational estimation. *Journal for Research in Mathematics Education*, *20* (2), 130-146.

Sowder, J. (1989). Research into practice: developing understanding of computational estimation. *Arithmetic Teacher, 36* (5), 25-27.

Sowder, J. (1992), Estimation and Number Sense. In D.A. Grouws (Ed.), *Handbook of Research in Mathematics Teaching and Learnin*g (pp.371-389). New York: Macmillan.

Sowder, J. (2001). Connecting mathematics education research to practice. (Key note address). In J. Bobis, B. Perry ,& M. Mitchelmore (Eds.), *Proceedings of the 24th Annual Conference of the Mathematics Education Research Association of Australasia* (pp. 1–8). Sydney.

Sowder, J., & Schappella, B. (1994).Number sense-making. *The Arithmetic Teacher, 41*(6), 342-345.

Star, J. R., & Rittle-Johnson, B. (2009). It Pays to Compare: An Experimental Study on Computational Estimation. *Journal of Experimental Child Psychology*, 102 (4), 408-426.

Strauss, A., & Corbin, J. (1990). *Basics of qualitative research, grounded theory procedures and techniques*. California: Sage Publications, Inc.

Taylor, P. M., Simms, K., Kim, O., & Reys, R. E., (2001). Do your Students Measure Up?. *Teaching Children Mathematics*, *7*(4), 282-287.

Thompson, A.G. (1979). Estimating and Approximating. In Sowder, J. (1992), Estimation and Number Sense. In D.A. Grouws (Ed.), *Handbook of research in mathematics teaching and learning* (pp.371-389). New York: Macmillan.

Trafton, P. R. (1986). Teaching computational estimation: Establishing an estimation mind-set. In H.L. Schoen & M.J. Zweng (Eds.), *Estimation and Mental Computation.* (pp, 16-30). Reston, VA: NCTM

Tsao, Y. (2004). Exploring the connections among number sense, mental computation performance, and the written computation performance of elementary preservice school teachers. *Journal of College Teaching and Learning*, 1 (12), 71-90.

Tsao, Y. (2005). The number sense of preservice elementary school teachers. *College student Journal, 39*(4), 647-679.

Usiskin, Z. (1986). Reasons for Estimation. In H.L. Schoen & M.J. Zweng (Eds.) *Estimation and Mental Computation: 1986 yearbook* (pp.1-15). Reston VA: National Council of Teachers of Mathematics

Volkova, T. N. (2005). Characterizing middle school students' thinking in estimation. Proceedings of the 29th conference of international group for the psychology of mathematics education, 4, 289-296 Melbourne: PME. Retrieved 18 May 2005 from hhttp://www.emis.de/proceedings/PME29/PME29RRPapers/PME29Vol4Volkova.pdf

Volkova, T. N. (2006). *Characterizing preservice teachers' thinking in computational estimation with regard to whole numbers, fractions, decimals, and percents*. Unpublished doctoral dissertation, Illinois State University.

Whitacre, I., & Nickerson, S. (2006, November). Pedagogy that Makes (Number) Sense: A Classroom Teaching Experiment Around Mental Math. Paper presented at the meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Yucatán, Mexico.

Whitacre, I. (2007). *Pre-service Teachers' Number Sensible Mental Computation Strategies.* Proceedings of the Research in Undergraduate Mathematics Education. Retrieved 28 April 2007 from http://www.rume.org/crume2007/papers/whitacre.pfd

Whiteman, F. C. (1989). *The Role of Computer-Based Instruction in the Development of Strategies for Computational Estimation with Middle School Children*. Unpublished doctoral dissertation, Ohio State University.

Yang, D. C. (1995). *Number Sense performance and strategies possessed by sixth and eight grade students in Taiwan*. Unpublished doctorial dissertation, University of Missouri.

Yang, D. C., Hsu, C. J., & Huang, M. C. (2004). A Study of teaching and learning number sense for sixth grade students in Taiwan. *International Journal of Science and Mathematics Education, 2* (3), 407-430.

Yang, D. C., Li, M., & Lin (2008). A study of the performance of 5th graders in number sense and its relationship to achievement in mathematics. *International Journal of Science and Mathematics Education,* 6 (4), 789-807.

Yang, D. C., Reys, R. E., & Reys, B. J. (2009). Number sense strategies used by pre-service teachers in Taiwan. *International Journal of Science and Mathematics Education*, *7* (2), 383-403.

Yazgan, Y., Bintas, J., Altun, M. (2002), *İlköğretim 5. Sınıf Öğrencilerinin Zihinden Hesap ve Tahmin Becerilerinin Geliştirilmesi*, 5. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi, ODTÜ Kültür Kongre Merkezi, Ankara

Yıldırım A. & Şimşek H. (2003). *Sosyal Bilimlerde Nitel Araştırma Yöntemler.* Ankara: Seçkin Yayıncılık

Yin, R. K. (2003). *Case study research, design, and methods, 3rd ed.* Newbury Park: Sage Publications.

Zazkis, R., & Hazzan, O. (1999). Interviewing in the mathematics education research: choosing the questions. *Journal of Mathematical Behavior*. *17*(4), 429-439.

APPENDIX A

THE COMPUTATIONAL ESTIMATION TEST
(Word format)

	Sorular	Kabul edilen aralık	Net Cevap
1.	Şehrin su deposu dikdörtgenler prizması şeklinde olup boyutları 31 m, 68 m ve 296 m dir. Bu depo yaklaşık olarak kaç m^3 su alacaktır?	500 000-700 00	623 928
2.	Bir terzi yapacak olduğu elbiseler için toplamda 835.67 metrelik şerit almıştır. Ancak bu şeridin ucunda kirli olduğunu düşündüğü 0.526 metrelik yerini kesmek zorunda kalmıştır. Sizce geriye yaklaşık kaç metre şerit kalmıştır?	829-835	835.67
3.	Bir çiftçi bahçesine 7465 ağaç fidesi dikmiştir ancak bunların 572 si tutmamıştır. Geriye yaklaşık kaç tane fide kalmıştır?	6000-9000	6893
4.	Masa örtüsü için aldığı $7\frac{1}{6}$ metrelik kumaşın $4\frac{1}{3}$ metresini kullanarak bir masa örtüsü diken Fidan Hanımın yaklaşık olarak kaç metre kumaşı artmıştır?	2-3	17/6~2.83
5.	Özgür bir önceki gün bakkaldan yaptığı alışverişlerde ödemediği 0.7 kuruş, 0.002 kuruş ve 0.81 kuruş tutan kusüratları ertesi gün hepsini ödemek istedi. Ahmet yaklaşık olarak bakkala ne kadar ödeme yapacaktır?	1-2	0.928
6.	Babası Umut'a her hafta aynı miktarda harçlık vermektedir. Geçen haftaki harçlığının $\frac{13}{16}$ ü ile bir düzine kalem alan Umut, bu haftaki harçlığının $\frac{7}{8}$ si ile de bir roman alıyor. Umut'un aldığı romanın fiyatı, kalemlerin fiyatından yaklaşık olarak kaç kat fazladır?	1	0.928

	Sorular	Kabul Edilen Aralık	Net Cevap
7.	16.272 litrelik toprak 36 saksıya paylaştırırsam her bir saksıya yaklaşık olarak kaç litre toprak konulmuş olur?	0.5-2/3	0.452
8.	Burak, 713 bilyesini taşınırken yanında götürmek istemediği için 8 arkadaşına hediye etmek istedi. Burak her bir arkadaşına yaklaşık ne kadar bilye vermelidir?	70-100	89.125
9.	Alanı $14\frac{3}{4}$ cm^2 olan kare şeklindeki bir duvara, alanı $\frac{5}{8}cm^2$ olan küçük fayanslardan yaklaşık olarak kaç tane döşenebilir?	20-30	23.6
10.	Fatoş sağlıklı yaşam için koşu yapmaya karar vererek her hafta bir kere mahallerinde bulunan parktaki koşu yolunda koşmaya başladı. Ilk defa gittiğinde $1\frac{7}{16}$ tur sonraki gün $3\frac{5}{12}$ tur ve ondan sonrakinde de $8\frac{1}{2}$ tur attı. Fatoş ilk koşuya başladığı andan itibaren bu koşu yolunda toplamda yaklaşık olarak kaç tur atmıştır?	12-13.5	13.354
11.	Bir tekstil şirketi 474 257 paket bornozu tırlarla yurtdışına ihraç etmektedir. Herbir tır 8 127 paket bornoz taşıdığına göre yaklaşık olarak kaç tır yola çıkmıştır?	50-60	58
12.	Bir metrekarelik tarlaya 0.041 kg gübre dökülmektedir. 98.6 metrekarelik tarlanız için kaç kilogram gübre alınmalıdır?	3-5	4.04
13.	Nevzat'nın bahçeye dikdiği ağaç her yıl $10\frac{1}{8}$ cm uzamaktadır. $3\frac{1}{2}$ yıl sonra bu ağaç yaklaşık olarak kaç santimetre olacaktır?	30-40	35.48

	Sorular	Kabul Edilen Aralık	Net Cevap
14.	Efe bisikletiyle saatte 1,67 kilometre yapabilmektedir. $1\frac{1}{2}$ saat boyunca bisikletiyle giderse yaklaşık kaç kilometre gitmiş olacaktır?	1.5-3	2.505
15.	Bir şehrin beş büyük ilçesi vardır ve son nüfus sayımındaki veriler şöyledir: Birinci ilçesinin nüfusu 87 419, ikinci ilçesinin ki 92 765, üçüncü ilçesinin nüfusu 90 045, dördüncü ilçesinin ise 81 974 ve son olarak beşinci ilçesinin nufüsü 98 102 dir. Buna göre bu ilin toplam nüfusu yaklaşık ne kadardır?	400 000 - 500 000	450 305

APPENDIX B

THE COMPUTATIONAL ESTIMATION TEST
(Numerical format)

	Questions	Accepted Interval	Exact Answer
1.	31 x 68 x 296	500 000- 700 00	623 928
2.	835.67— 0.526	829-835	835.67
3.	7465—572	6000-9000	6893
4.	$7\frac{1}{6} - 4\frac{1}{3}$	2-3	17/6=2.83
5.	0.7 + 0.002 + 0.81	1-2	1.512
6.	$\frac{13}{16} \div \frac{7}{8}$	1	0.928
7.	$16.272 \div 36$	0.5-2/3	0.452
8.	$713 \div 8$	70-100	89.125
9.	$14\frac{3}{4} \div \frac{5}{8}$	20-30	23.6
10.	$1\frac{7}{16} + 3\frac{5}{12} + 8\frac{1}{2}$	12-13.5	13.354
11.	$474\,257 \div 8\,127$	50-60	58
12.	98.6 x 0.041	3-5	4.04
13.	$3\frac{1}{2} \times 10\frac{1}{8}$	30-40	35.48
14.	$1\frac{1}{2} \times 1.67$	1.5-3	2.505
15.	87 419 + 92 765 + 90 045 + 81 974 + 98 102	400 000-500 000	450 305

APPENDIX C

SECOND INTERVIEW SESSION'S QUESTIONS

1. In your opinion, what does estimation mean?
2. Where did you learn to compute by estimating?
3. Where do you use the computational estimation in your daily life?
4. What do you think about to use computational estimation the mathematics classes? Why?
5. What do you think about to use computational estimation in your daily life? Why?
6. What do you feel when you doing computational estimation requested questions? Why?
7. What points would you give out of 10 on your mathematics achievement? Why?
8. How successful are you at computational estimation? Why?
9. What points would you give out of 10 on your computational estimation? Why?
10. What do you think about to apply an approximate calculation to a mathematics question? Why?
11. How successful are you at mathematics? Why?
12. In which topic you are more successful in mathematics lesson? Why?

APPENDIX D

THE ANSWER SHEET OF COMPUTATIONAL ESTIMATION TEST

TAHMİN BECERİ TESTİ

Adınız Soyadınız:......................Sınıfınız:...........Cinsiyetiniz: ( ) Kız ( ) Erkek

Matematik başarınızı derecelendiriniz: ( )Çok İyi ( ) İyi ( ) Orta ( ) Zayıf

Tahmin ile ilgili yeteneğinizi derecelendiriniz: ( )Çok İyi ( ) İyi ( ) Orta ( ) Zayıf

YÖNERGE: Bu test işlemsel tahmin konusunda sorular içermektedir. Soruları en kısa sürede tahmin ederek cevaplayınız. Bazı sorularda cevap şıkları yoktur sizin bulduğunuz cevap şıklarını yazmanız gerekmektedir. Hiçbir soruyu boş bırakmadan ve yalnız bir şıkkı işaretleyerek cevaplayınız. Bu test yalnızca araştırma amaçlı kullanılacaktır ve verdiğimiz bilgiler <u>kesinlikle gizli tutulacaktır</u>. Sadece siz ve araştırmacı tarafından bilinecektir. Teşekkür. Burçak BOZ /OFMAE Bölümü

1.
2.
3.
4.
5.
6.
7.
8.
9.
10.
11.
12.
13.
14.
15.

1. ***Tahmini hesaplama ne demektir sence?***

2. ***Günlük yaşamda nerelerde tahmini hesaplamayı kullanırsın?***

APPENDIX E

MAIN STUDY CODING LIST

Interview Session I Themes and Codes

Computational Estimation Strategies

1. Reformulation
    1.1 Rule based rounding
        1.1.1 Nearness to 1, ½ or 0
    1. 2 situation based rounding
    1. 3 compatible numbers
    1. 4 truncation
        1. 4.1 Ignore fraction parts of mixed numbers
        1. 4.2 Ignore too small decimal
        1. 4.3 Ignore decimal parts
2. Translation
    2.1 convert addition to multiplication
    2.2 convert division to fraction
3. Compensation
    3.1 intermediate compensation
    3.2 final compensation

Interview Session II Themes and Codes

The Factors Related with Computational Estimation Ability

1. Cognitive Factors

1.1 Number Sense

1.1.1 Ability to work with powers of ten

1.1.1.1 Removing zeros

1.1.1.2 Simplification of zeros

1.1.1.3 Multiplication by powers of ten

1.1.2 Multiple representations

1.1.2.1 Convert decimal to fraction

1.1.2.2 Convert fraction to decimal

1.2 Mental computation

1.2.1 Mathematical facts on whole number

1.2.1.1 Decomposition of numbers

1.2.1.2 Standard operation algorithm (paper-pencil algorithm)

1.2.1.3 Multiplication table

1.2.2 Mathematical facts on decimal

1.2.2.1 Made same number decimal places

1.2.2.2 Place value of decimal

1.2.2.3 Decomposition of decimal

1.2.3 Mathematical facts on fraction

1.2.3.1 Common denominator

1.2.3.2 Division algorithm

1.2.3.3 Decomposition of mixed numbers

1.2.3.4 Misconception on fraction

2. Affective Factors

2.1 Mathematics Related Factors

2.1.1 Confidence in ability to do mathematics

2.1.1.1 Feel successful / not successful

2.1.1.2 Give high/ low point

2.1.1.3 Mental computation ability

2.1.2 Perception of mathematics

2.1.2.1 Exactness

2.1.2.2 Math teachers

2.1.2.3 Test Anxiety

2.1.2.3.1 Dersane

2.1.2.3.2 Class A

2.2 Estimation Related Factors

2.2.1 Confidence in ability to do estimation

2.2.1.1 Feel successful / not successful

2.2.1.2 Give high/ low point

2.2.1.3 Easy questions

2.2.2 Perception of estimation

2.2.2.1 Feelings on estimation

2.2.2.1.1 Like / not like

2.2.2.1.2 Disturbed / not disturbed

2.2.2.1.3 Feel good

2.2.2.1.4 makes feel disregardful about mathematics

2.2.2.2 Recognition of estimation as useful

2.2.2.2.1 Use / not use in daily life

2.2.2.2.2 Use /not use in math class

2.2.2.2.3 should not use in math

2.2.2.2.4 Important /not important

2.2.2.2.5 Prefer /not prefer

2.2.2.2.6 insist on computing

2.2.3 Tolerance for error

2.2.3.1 Ambiguity

2.2.3.2 Comfortable/ not comfortable with pay off

2.2.3.3 Give answer in a small/big range

APPENDIX F

PILOT STUDY CODING LIST

Phase I Themes and Codes

1. Strategies
   1.1. Reformulation
      1.1.1. Rounding
      1.1.2. Truncation
   1.2. Compensation
      1.2.1. Intermediate compensation
      1.2.2. Final compensation
   1.3. Translation
   1.4. Compatible numbers
   1.5. Leading digit

2. Cognitive factors
   2.1. Mental computation
      2.1.1. Computation strategies
         2.1.1.1.distribution law
         2.1.1.2. transformation to appropriate use
            2.1.1.2.1. On operation (percent to division, fraction to division)
            2.1.1.2.2. On numbers (percent to fraction, fraction to decimal)
   2.2. Standard algorithm
      2.2.1. Knowledge of basic facts
         2.2.1.1.decimals
         2.2.1.2.fractions
         2.2.1.3.percent

Phase II Themes and Codes

1. Rounding
    1.1. Exaggerated rounding (upper-lower)
    1.2. Round one, two/all numbers
2. Decimal
    2.1. Number of digits
        2.1.1. More digit bigger number
    2.2. converting decimals
        2.2.1. to fractions
        2.2.2. to percent
    2.3. base ten information
    2.4. think as a two number both sides of comma
    2.5. Computational rules
        2.5.1. Making same number of digit
3. Fraction
    3.1. Round fractions as decimals
    3.2. Round fractions as percent
    3.3. Rounding partially (numerator-denominator- both)
    3.4. Computational rules
        3.4.1. Denominator similarity
        3.4.2. Multiplicity rule (reverse and multiply)
        3.4.3. Simplifying
    3.5. Take fraction as division
    3.6. Used fraction and whole part separately
4. Multiplication
    4.1. Shortcut multiplication with 10 and 100
    4.2. Two fraction multiplication
    4.3. Taking off zeros
    4.4. Multiplication table
5. Division
    5.1. With big numbers

5.2. With small numbers

5.3. Taking off zeros

5.4. Relationship between fraction

5.5. Standard rules

5.6. Make numbers less digits

6. Psychological factors

6.1. Tolerance for giving big range

6.2. Ignore too small numbers

6.3. Insisting to compute exact result

6.4. Believe estimation gives wrong results

7. Cognitive Factors

7.1. Number sense

7.2. Used math rules

7.2.1. Commutativity rule

7.2.2. Distribution rule

7.3. Couldn't give a rule

7.4. Reading big numbers

Phase III Themes and Codes

1. Percent

1.1. Proportional thinking

1.2. Converting percent to fraction

1.3. Used good percents

1.4. Standard percent algorithm

1.5. Percent comparatives

1.5.1. Two percents

1.5.2. Fraction and percent

2. Computational strategies

2.1. taking off/ adding zeros

2.1.1. dividing numbers by 10 and 100

2.1.2. multiplication with 10 and 100

2.2. subtraction by adding strategy

2.3. 6x25= (4x25)+(2x25)=(2x25)x3

2.4. some mathematical rules

2.4.1. least common divisor

2.4.2. division rule (small number and big number, comma)

3. Options

3.1. Used as feedback

3.2. Let give direction to the answer

4. Strategies

4.1. Rounding

4.1.1. Standard rounding

4.1.2. Percent rounding

4.2. Compensation

4.2.1. Final compensation

4.2.2. Intermediate compensation

5. Psychological factors

5.1. Eager to find result

5.2. Intuitive knowledge

5.3. Try to find another way (don't give up)

5.4. Run away (by saying: we haven't been learnt it yet.)

APPENDIX G

EXAMPLE OF RESULTS FROM PILOT STUDY

In this section, students' answers are given to understand how they behave and how they solve the asked estimation questions. By helping of the answers, some codes and themes are constructed.

Q1: *Bir şehrin beş büyük ilçesi vardır ve son nüfus sayımındaki veriler şöyledir: 1. ilçesinin nüfusu 87 419, ikinci ilçesinin ki 92 765, üçüncü ilçesinin nüfusu 90 045, dördüncü ilçesinin ise 81 974 ve son olarak beş ilçesinin nufüsü 98 102 dir. Buna gore bu ilin toplam nüfusu yaklaşık ne kadardır? (C: 450 305)*

Zeynep : She worked with each number specifically, whether rounded or truncated. Then she added each of them with standard algorithm, which is added column by column. On the other hand when it was asked to another way to add these all numbers, she stated that there are three 90 000 so it was 270. She used to translation strategy which is the reconstructed to operation from addition to multiplication. Then she added to 100 000 to 270 and the last part 80 000 addition is completed by standard algorithm rules where she stated that "after it was added to it 80 000. When we are adding firstly we should add 8 with 7." However, she added these two digits by distribution law where she said that "when we get 2 from 7 for 8, there is 10 and 7 is now turn to 5. When we added 10 by 5 is should be 15." On the other hand after this distribution property she continue to the standard addition procedure by saying that there should be 1 from 15 so it should be added to 3 so that there is 450 000. She couldn't identified that there are almost all the numbers so close to 90 000 so there were 5 of them and we can multiply by 5 and 90 000 and get 450 000. I think she tried to use the algorithm procedures because she is the most successful students among her friends. When the researchers asked to the exact solution of the question where should be? She answered correctly by saying that her solution was smaller than the exact one. The reason to saying this was almost all of the numbers were truncated to smaller number she asserted that.

Ferhat : He started with the leading digit strategy that is he began with adding the numbers started with 9 and 8. Then he wants to change his strategy with compatible numbers during the adding numbers started with 9. On the other hand, during partial addition he used standard algorithm with column by column. He tried to find so close to exact answer by adding tens part separately. He rounded or truncated also these numbers and added two big parts to each others. He also used leading digit strategy in here the ten parts. Deciding the compensation of the result he couldn't give the correct command also the reason to saying the result was smaller the exact result give us some prejudge of estimation. Because he said that "”*my result is smaller than the exact result since I computed to appropriate numbers and also compute mentally fast.*

Metehan : He rounded or truncated more specifically to the numbers for example 92 765 to 92 800. On the other hand, he used some difficulties during the addition to these big numbers then he changed the strategies and rounded or truncated to the numbers to more manageable form. After reformulated the numbers he added the numbers with the standard algorithm approach. Also he can use one of the mental computation strategies that are distribution law to add easily. He asserted that the result he founded was smaller than the exact one since he "rounded" all the numbers. Although he tried to explain that, he truncated all of them. There is some words meaning difficulties in Turkish since there is only one word for truncation and rounding. He said that all numbers were truncated so that his result was the smaller than the exact one. He is eager to find the closest answer.

Özgür : He used rounding and truncation professionally since in the question he could easily said that each number close to 90 000 so that we can multiply the 5 by 90 000. By doing this, he also used the translation strategy. However, he made a mistake when doing multiplication he said that 5x 90 000 = 270 000. Additionally he added the tens part numbers at the end to compute closer result to the exact answer. He asserted that his answer is smaller than the exact result but he found the wrong one.

Semih : Firstly, he tried to rounded and truncated to learn form, which is the numbers, rounded/truncated to close numbers separately without thinking totally. However, when it is asked to add all of them he revise his estimation to more manageable form which are 90 000 and 80 000s. Then when adding these sums he used distribution law, which is a mental computation strategy for adding easily. It is asked that where can be the exact solution according to your result, he answered that it was over his results since he "rounded" all numbers. He gave wrong answer firstly but after partly talking on the problem he found that the acceptable solution but did not accept the making mistake. In the following Table 1, the codes of question 1 are presented. These codes are identified by the students' transcribes

Table G 1 The codes of Q1 with respect to each student

Question1	Codes
Zeynep	reformulation (part by part), standard algorithm, distribution law
Ferhat	Standard algorithm, Leading digit strategy, compatible numbers strategy, wrong final compensation, reason to compensation is using appropriate results
Metehan	Standard algorithm, distribution law, eager to find exact answer
Özgür	Translation, professionally used reformulation
Semih	distribution law, reformulation (part by part)

Q2: *Bir terzi yapacak olduğu elbiseler için toplamda 835.67 metrelik şerit almıştır. Ancak ucunda kirli olduğunu düşündüğü 0.526 metrelik yerini kesmek zorunda kalmıştır. Sizce geriye yaklaşık kaç metre şerit kalmıştır? (C: 835.144)*

Zeynep : She made exaggerated rounding to the numbers and couldn't made reasonable compensation to the results.

Ferhat : He had some difficulties on decimals as he stated that. Also he couldn't subtract the decimal part as correct. Because he asserted that the operation 835.67-0.500 answer should be 834. When the decimal part subtraction he took one digit from the ones part and said that the result is 834. Also he confesses that his result was less than the exact one.

Metehan : He truncated the second decimal and called it as half so he could use the appropriate transformation between the numbers. On the other hand, he couldn't understand the sensibility of the questions because he found 835.5 which is the same as the beginning number. Also he couldn't give reasonable compensation answer because he said that he gave the smaller answer in order to exact solution however the exact solution is less than he gave.

Özgür : He truncated both number as 835 and 0.500 and also used appropriate transformation for 0.500 as saying half. However, he said that the result was 834 because he competed to half as full. He made exaggerated rounding and found the result as 834.

Semih : He has lack of knowledge about decimal procedure. He firstly conducted exaggerated truncation both decimals, and applied the subtraction. However, he found that 700. He could not manage the decimals. In the following Table 2, the codes of question 1 are presented. These codes are identified by the students' transcribes

Table G 2 The codes of Q2 with respect to each student

Question2	Codes
Zeynep	Exaggerated rounding
Ferhat	lack of arithmetical basic facts about decimals
Metehan	Reformulation, transformation to appropriateness
Özgür	Reformulation, transformation to appropriateness, Exaggerated rounding

Table G 2 Continued

Question2	Codes
Semih	Exaggerated truncation, lack of arithmetical basic facts about decimals

Q3: *Masa örtüsü için aldığı* $2\frac{1}{2}$ *metrelik kumaşın* $1\frac{1}{4}$ *metresini kullanarak bir masa örtüsü diken Fidan Hanımın yaklaşık olarak kaç metre kumaşı artmıştır? (C:* $1\frac{1}{4}$*)*

Zeynep : She can use professionally transformation between fraction and decimal. Also she is confident to say large range of estimation results. Even she found that the exact result truncated it and said a smooth result.

Ferhat : He used standard fraction algorithm. He tried to make similar denominators for subtract the two fractions. Therefore, he accomplished and found the exact solution. When it was asked to if you would estimate the answer how would you do, he answered as I computed directly.

Metehan : He exactly found the solution with similar denominator procedure. He conducted the mental computation as paper pencil format.

Özgür : He gave very large result by doing only integer subtraction. When it was asked what happen to the proper fraction part, he said that they were not as important as in this question. The result was only one. When the researcher asked the how about the exact result he asserted that it was bigger than his result.

Semih : He exactly found the solution with similar denominator procedure. He conducted the mental computation as paper pencil format. In the following Table 3, the codes of question 1 are presented. These codes are identified by the students' transcribes

Table G. 3 The codes of Q3 with respect to each student

Question3	Codes
Zeynep	transformation to appropriateness, reformulation
Ferhat	Standard Algorithm for fraction (denominator process)
Metehan	Standard Algorithm for fraction (denominator process), mental computation
Özgür	Exaggerated rounding (fraction)
Semih	Standard Algorithm for fraction (denominator process),

Q4: *Ömer'in marketten yaptığı 10.83 YTL alışverişin yüzde 15 ini abur cubura verdiğini gördü. Buna göre Ömer abur cubur için kaç YTL vermiş oldu? (C: 1.6245)*

Zeynep : She easily rounded the first decimal to 11 and percent to 20. However, when she concluded that she would take the 1 part from 5, changed the rounded number with 10. After this revision she answered 2 as result. Besides this, she asserted that the exact number is less than the on she found.

Ferhat : He has some problems with decimals and fraction. He confused the percent standard algorithm and made some subtractions. Then he tried to take whole part percent and decimal part separately. After some guidance he could identified the 15 % as 20 % even one over 5. Before that he rounded 10.83 as 11. When it was asked what is the fifth of one of 11. He could say 2.5. He answered correctly the compensation question, as his result was bigger than the exact one.

Metehan : He consciously truncated the first number to 10 because he stated that there was a relationship between ten and percent. He didn't change the 15% and applied the mental computation strategies to it. He completed the 10 as 100 and take 15 from 100 after that divide 10. Although he gave the reasonable result to the question he couldn't explain what he was do. The researcher asked to how can find the result since he couldn't explain his way he produce a new strategy which is the transformation strategy. In this way he used 15 % as 20 % and also a fraction which is five over 1. Then he found 2 as a result and think of he rounded the percent he should off the result so he could say one and half.

Özgür : He rounded the decimal since closeness of the upper integer. Then take the percent as six over 1. He could say easily 15 is 6^{th} over one of 100. Then he tried to divide 11 to six. Also he could divide and said that approximately 1.83. After that researcher directed him to dividable number which is 12. He easily said that the result was 2. He also asserted that his result is a bit bigger than the exact number.

Semih : He firstly rounded the decimal as his previous learning but then he identified the percent relationship between 10 and 100. Then he changed his mind and truncated to 10. After that he produced mentally the percent procedure but like a paper pencil application. In the following Table 4, the codes of question 1 are presented. These codes are identified by the students' transcribes

Table G 4 The codes of Q4 with respect to each student

Question4	**Codes**
Zeynep	Reformulation, , transformation to appropriateness (percent to fraction), final compensation
Ferhat	Lack of basic facts (percent and decimal),
Metehan	Standard algorithm (percent), Mental computation, reformulation
Özgür	Reformulation, transformation to appropriateness (percent to fraction), mental computation, final compensation
Semih	Standard algorithm (percent), mental computation

Result of Phase 1

In phase 1; I have tried to identify estimation strategies that students use when dealing with the questions which are required to estimated solutions for the words problems in fraction, decimal, whole number and percent. This phase consisted of four questions and in those questions students mostly behave similar, used similar strategies which are preliminary learnt estimation strategies; for example, rounded up when the digit is 5 and more than 5. Also they mostly preferred paper pencil computing to mental computation or estimation. It is observed that most of them have problems on arithmetical facts, specifically on decimal and percent problems.

Question 1: there are five big cities in a region and according to last census the population of cities are first one 87 419, second one 92 765, the third one 90 045, fourth one 81 974 and the last one 98 102. What is the approximate population of this region?

In question one; translation and reformulation are most used estimation strategies. However; almost all students prefer standard addition algorithm by imagining the numbers in their head and adding digit by digit with drawing an addition line. The confirmation about the final compensation were differs. Most used strategy was the reformulation of the numbers, which was the rewrite the numbers rounded or truncated forms. Except from Özgür; the other students rounded or truncated the numbers one by one that is how they learnt in class. Even one of the students rounded the numbers' hundreds and tens separately. *"We could round 92765 (ıgg) 92 thousand (ıgg) 800."* Özgür identified that there were 5 of 90 000 and the answer was near the 5x 90 000. He said that: *"There are (ıgg) approximately 90 000 people live in a city. How you get this result? Because some of them above the 90 some of them under."* Almost all the student tried to add the sum of the numbers by standard addition algorithm. Only two students (Zeynep and Semih) were used commutativity rules while adding 270 and 80. When adding 8 and 7, they used (8+2) +5. Differently from others, Ferhat chose the leading digits strategy which is the adding the leading digits of the numbers. However, he could not handle the numbers by using this strategy because he wanted to compute exact result. Then changed the strategy with other one; compatible numbers strategy. Besides using these strategies, he always aimed to get the exact result. Naturally, this exact number finding ambition exist almost all of the students. Because they believed that, they could find the exact answer and also they should find it. When the researcher asked comparisons of exact and estimated answers, half of them gave the correct answers. It can be easily identified that they hadn't any arithmetical facts problems in whole numbers; they could use the taking off zeros professionally.

Question 2: A tailor buys 835.67 m ribbon for dresses. However 0.526 m dirty part should be cut. Approximately how long the rest of the part?

Although whole number problems almost done all students, the second question which was involved in decimal forced to students because of the inadequacy of knowledge on decimal numbers arithmetical facts. Ferhat asserted that:

I could round 835 to (ıgg) 830. It became 830 point and the percent part could be rounded 0 percent. I could round (ıgg) 52 to (think a while) 100. The result is (ıgg) 0 point 0100 (say long, and stress), sorry is that 526?".....OK. (ıgg) 10 billion (surprise); I could round 10 thousand to 1000. Then subtracted with (think) the result is approximately (ıgg, think) 700 thousand (think) 7 thousand 39 hundred like this.

Three of them exaggerated their rounding the numbers just like the rounding rules learnt in school, 5 and over 5 should be rounded above. On the other hand, the question has a great sensibility because if it was rounded the decimals (like taught in school) the subtraction gave an unreasonable result which was bigger than the starting point.

Question3: Mrs. Fidan bought $2\frac{1}{2}$ *m texture for making a tablecloth and used* $1\frac{1}{4}$ *m. Approximately how long texture has she got now?*

In the fraction question, almost everyone tried to conduct standard fraction denominator procedure. Although the fractions were "good ones", they didn't choose to use them. Only two of them produced a strategy about fractions. Zeynep called the fractions as “half” and “quarter” and then truncated to nearest integers. *“This* $\frac{1}{2}$ *can be half, and this is two and a half in real. And it can be rounded (think) two. This* $\frac{1}{4}$ *is quarter. That is one and a quarter.”* Similarly Özgür truncated and get rid of the fractions so that found a result in a big range. Others applied standard algorithm, as showed in the following:

…. the rest is approximately 1 (ıgg) 1 exact (think) (ıgg) that is (ıgg) (think) there is 1 exact the rest.” “How did you do?” “I subtracted whole parts. 2 minus 1 is 1” “where is the fraction part?” “I left them.” “Why did you leave them?” “(ıgg) for doing easier operation in this case (ıgg) the real answer I mean it is the less than when we do it in real (think, ıgg).” “How did you do subtraction?” “it was said that 15 percent or it is more like 17, so could divide 10 and 83 to 17” “why did you divide them?” Because it is asked to 15 percent of 10.83.

Question4: Ömer spent 10.83 YTL in supermarket and he saw that 15 % of this shopping spent on haphazard manner. According to bill approximately how much money had been spent on haphazard manner?

In the last question, everyone produce different strategies for themselves. For example Zeynep started with the rounded/truncated both numbers; decimal is truncated to nearest integer and percent is nearest manageable for, that is 20 %. Then she transformed percent to faction for taking one fifth of 10. Besides all these, she asserted that the exact answer is less then that she founded. Metehan and Semih had same mental computation strategy that is 10 was completed to 100 and take 15 from 100 then divide 15 to 10. On the other hand Özgür could see 15% is sixth of 100 so he divided six to 11 which was rounded but when he was forced to divide it he changed his mind and take it as 12 for divide 6 easily. Besides those, Ferhat has problems on percent and decimal application; he had lack of basic arithmetical facs. In the following explanation is presented from interview with Ferhat:

15 percent is approximately (ıgg) 85 (stop) not I think (ıgg), (ugg)" "what are you doing?" "(ıgg) I am doing subtractions, subtraction." "What subtraction are you doing?" He confused percent problems which part he was going to calculate, 15 or 85.

Although there could be used lots of strategies, the students had used only reformulation strategy as estimation strategy in this phase. It can be identified that they also had some arithmetical problems especially in decimal and percent problems.

APPENDIX H

TABLES OF STRATEGIES

In the following pages, three tables are presented according to each interviewee answers. The tables are including in computational estimation strategies, number sense, and mental computation codes.

Table H 1 Strategies Used In Whole Number Questions

Questions	Interviewees' Strategies				
	Deniz	Mert	Sergen	Nevzat	Ayşe
Q1 31 x 68 x 296	• rule based rounding • removing zero • standard operation algorithm	• rule based rounding • removing zero • intermediate compensation	• rule based rounding • removing zero	• rule based rounding • removing zero • standard operation algorithm	• rule based rounding • removing zero
Q3 7465—572	• rule based rounding • decomposition of the numbers	• rule based rounding • decomposition of the numbers	• rule based rounding • decomposition of the numbers	• situation based rounding • intermediate compensation	• compatible numbers • standard operation
Q8 713 ÷ 8	• situation based rounding • removing zero • multiplication table	• situation based rounding • multiplication table • final compensation	• situation based rounding • multiplication table	• rule based rounding	• situation based rounding • multiplication table
Q11 474 257 ÷ 8 127	• rule based rounding • removing zero • standard operation algorithm	• rule based rounding • multiplication table • final compensation	• rule based rounding	• rule based rounding • convert division to fraction	• rule based rounding • situation based final compensation
Q15 87 419 92 765 90 045 81 974 + 98 102	• rule based rounding • decomposition of the numbers • convert addition to multiplication	• rule based rounding • removing zero • decomposition of the numbers • convert addition to multiplication	• compatible numbers • decomposition of the numbers • removing zero • convert addition to multiplication	• situation based rounding • convert addition to multiplication	• rule based rounding • decomposition of the numbers • removing zero • convert addition to multiplication

Table H 2 Strategies Used In Decimal Questions

Questions	Interviewees' Strategies				
	Deniz	Mert	Sergen	Nevzat	Ayşe
Q2 835.67— 0.526	• rule based rounding	• made same number decimal places • situation based rounding	• made same number decimal places • rule based rounding	• made same number decimal places *(could not performed the question)*	• situation based rounding
Q5 0.7 + 0.002 + 0.81	• rule based rounding • ignore too small decimal	• ignore too small decimal	made same number decimal places	• made same number decimal places *(could not performed the question)*	• made same number decimal places *(could not performed the question)*
Q7 16.272 ÷36	• ignore decimal parts • rule based rounding • convert division to fraction	• ignore decimal parts • compatible numbers • convert division to fraction	• ignore decimal parts • rule based rounding • convert division to fraction	• ignore decimal parts • convert division to fraction	• ignore decimal parts • compatible numbers • convert division to fraction
Q12 98.6 x 0.041	• rule based rounding *(could not performed the question)*	• rule based rounding *(could not performed the question)*	• rule based rounding *(could not performed the question)*	• rule based rounding *(could not performed the question)*	• Situation based rounding • convert decimal to fraction *could not performed the question)*
Q14 1 ½ x 1.67	• rule based rounding	• convert fraction to decimal • intermediate compensation	• rule based rounding	• rule based rounding *(could not performed the question)*	• rule based rounding • intermediate compensation

Table H 3 Strategies Used In Fraction Questions

Questions	Interviewees' Strategies				
	Deniz	Mert	Sergen	Nevzat	Ayşe
Q4 7 1/6 – 4 1/3	• common denominator	• ignore fraction part of mixed numbers	• common denominator decomposition of mixed numbers	• common denominator • decomposition of mixed numbers	• ignore fraction part of mixed numbers
Q6 13/ 16 ÷ 7/8	• nearness of 0, ½ or 1	• nearness of 0, ½ or 1	• division algorithm • common denominators	• division algorithm • common denominators	• Misconception on fraction
Q9 14 ¾ ÷ 5/8	• nearness of 0, ½ or 1 • division algorithm • misconception on fraction	• nearness of 0, ½ or 1 • misconception on fraction	• common denominators • misconception on fraction	• nearness of 0, ½ or 1 *(could not performed the question)*	• nearness of 0, ½ or 1
Q10 1 7/16+3 5/12 + 8 ½	• decomposition of mixed numbers	• convert fraction to decimal • decomposition of decimal	• decomposition of mixed numbers	• convert fraction to decimal • decomposition of decimal	• convert fraction to decimal • rule based rounding • decomposition of decimal
Q13 3 ½ x 10 1/8	• nearness of 0, ½ or 1 • ignore fraction part of mixed numbers	• convert fraction to decimal • situation based rounding	• rule based rounding • ignore fraction part of mixed numbers	• ignore fraction part of mixed numbers	• ignore fraction part of mixed numbers

APPENDIX I

EXAMPLES OF TRANSCRIBE CODING

Konuşmalar	Kodlar
A: Tahmini hesaplama ne demektir sence?	
B: Yuvarlayarak yani sayıları tam bir miktara yuvarlayarak	→ rounding
A: Tam bir miktara yuvarlayarak, ne demek tam bir miktar?	
B: Tam bir sayıya yuvarlayabilirsin mesela sekiz bin yüz doksan beş onu sekiz bine yuvarlayabilirsin.	
A: Anladım. Nereden öğrendin tahmini hesap yapmayı?	
B: İlkokul beşinci sınıfta derste öğrendim.	→ at 5th grade at school
A: İlkokulda derste öğrendin. Öğretmenin mi öğretti yoksa sen böyle olursa böyle yaparım mı dedin?	teacher taught
B: Tahmin ederken öğretmen yuvarlayarak yapacaksınız dedi.	→ rounding
A: Bunu söyledi. Nasıl yuvarlama yapman gerektiğini söyledi mi?	
B: Şimdi sıfır nokta beş biz onu nereye yuvarlayabiliriz. Beş ve beşin üstündekileri bir tama yuvarlayabiliriz.	→ rule of rounding
A: Anladım, beş ve üzerini yukarıya; altını aşağıya yuvarlarız.	
A: Sence günlük yaşamda nerelerde tahmini hesaplamayı kullanıyoruz ve sen nerelerde kullanıyorsun?	in daily life
B: Hesaplama değil de yani mesela buradan şuraya kaç km veya işte ölçülerde kullanıyoruz.	she used → measurement estimation
A: Sen kullanıyor musun peki bunları?	
B: Kullanıyorum.	
A: Mesela belirli bir örnek versen şurada kullandım.	
A: Evde olabilir yolda giderken olabilir.	
B: Şuan aklımda yok ki.	
A: Uzaklık tahminlerinde kullanıyorum dedin.	
B: Onlarda kullanıyorum işte başka…	
A: Ev ile okul arasını mı tahmin ediyorsun?	
B: Evet	
A: Nasıl tahmin ediyorsun?	
B: Evle okul arası kaç metre…	
A: Kaç metre kaç km! Peki tahmini hesaplama açısından bakınca günlük yaşamda	
B: Günlük yaşamda kullanırım bazen.	
A: Nerelerde kullanırsın? Düşün bakalım kullanırsın büyük ihtimalle.	
A: Hiç aklına gelmedi mi? Peki gelince söyle o zaman.	
A: Peki matematik dersinde tahmini hesaplama kullanma konusunda ne düşünüyorsun?	
B: Ne düşünüyorum yani?	
A: Yani matematik dersinde kullanılsa ne düşünürsün, neler düşünürsün.	not prefer
B: Sayı olarak düşündüğümüzde daha pratik olur ama tam sonucu bulamıyorsun kullanılması o kadar şey değil.	→ prefer exact solution
A: Gerekli değil! Diyorsun. Tam sonucun bulunması mı gerekir sence matematikte	→ exactness
B: Bence evet.	
A: Günlük yaşamda tahmini hesaplama kullanma konusunda ne düşünürsün?	
B: Günlük yaşamda kullanılabilir yani.	→ use in daily life
A: Neden?	
B: Hayatı kolaylaştırmak için	→ est. makes life easier
A: Hayatı kolaylaştırmak için! Ne yapıyorsun da hayatın kolaylaşıyor.	
A: Küçük bir örnek ver şunu yaparak hayatımı kolaylaştırıyorum.	
B: Ben kullanmam, genelde tam hesaplama yaparım.	→ I don't use, I prefer exact computation
A: Kağıt kalem mi kullanıyorsun hesap makinesi mi kullanıyorsun?	

not use in daily life
prefer exact

1

A: Parçaların içine düşen miktar B: Daha az olacak, sekize bölünce daha az çıkacak, daha büyük sayıyı sekize böldüğümüzde daha fazla çıkacak A: Evet güzel dokuzuncu soru	
9. B: On dört tam üç bölü dört bölü beş bölü sekiz $14\frac{3}{4} \div \frac{5}{8}$ B: Beş bölü sekize sıfır nokta beş desem $5/8 \rightarrow 1/2$ A: Güzel beş bölü sekize sıfır nokta beş dedin B: Buna da on beş desem $14\,3/4 \rightarrow 15$ A: On beşi yarıma bölüyorsun? B: Otuz çıkar A: Otuz falan çıkar nasıl böldün bölme işlemini anlat B: On dört tam üç bölü dördü on beşe yuvarladım, beş bölü sekizi sıfır nokta beşe yuvarladım A: Tamam B: Böldüğümüzde otuz çıkıyor A: İşte nasıl böldün B: On beşin içinde kaç tane yarım var; yarısı olduğu içinde ikiyle çarpıcam; o yüzden otuz çıkıyor A; İşte budur! A: Net cevap otuzdan fazla mıdır az mıdır? B: Otuzdan azdır. Çünkü burada böldüğümüz sayı daha küçük bölen sayı daha fazla A: O yüzden otuzdan daha az çıkar A: Onuncu soru	0,5 round fraction directly a decimal ✓ round fraction to nearest integer ✓ the person that gave the result at the first correct time ✓ gave reasonable support to her answer ✓ true compensation ✓
10. B: Bir tam yedi bölü on altı artı üç tam beş bölü on iki artı sekiz tam bir bölü iki $1\,7/16 + 3\,5/12 + 8\,1/2$ B: Sekiz tam bir bölü ikiye dokuz desek; buna da üç buçuk desek; buna da bir buçuk desek B: Üç buçuk bir buçuk beş, dokuz demiştik on dört oluyor A: Üç buçukla bir buçuğu nasıl topladın B: Önce üç buçukla biri topladım sonra buçuk ekledim $3,5 + 1,5 = 3,5 + 1 + 0,5$ A: Beşle dokuzu nasıl topladın B: Normal topladım ☺ A: Nasıl işte normal toplama işlemi B: …………………. ☺ A: Çok basit geldiği için mi anlatamıyorsun ? B: Şimdi dokuz ekleyince ona tamamlıyorum bir çıkarıyorum; ondan beşten bir çıkardığımda dört oluyor on ekliyorum on dört A: Net cevap nerededir, on dördün üstünde midir altında mıdır? B: Altında A: Neden? B: Çünkü daha çok şeylere yuvarladık azları daha çok yuvarladık A: Hep ileriye doğru yuvarladığın için on dörtten azdır B: Azdır	round fractions to decimals ✓ round 8 1/2 as 9 ✓ adding by decomposition $9 + 5 = 9 + 1 + 4$ ✓ adding by decomposition true compensation ✓ good supporting ✓
11. A: On birinci soru $474\,257 \div 8127$ B: Dört yüz yetmiş dört bin iki yüz elli yedi bölü sekiz bin yüz yirmi yedi A: Duymuyorum 470000 8000 B: Şimdi buna dört yüz yetmiş bin desek buna da sekiz bin desem B: Yok bunu dört yüz seksen bine yuvarlasam 480000 A: Tamam B: Sekiz bine böldüğümüzde altılı bir şey çıkıyor	round firstly as rule based ✓ then round number to make convenient to each other ✓

$480000 \div 8000$

judge the result before actually conducted it. ✓

5

APPENDIX J

AN EXAMPLE OF DATA ANALYSIS MATRIX

	whole number	***fraction***	***decimal***
Deniz	• In all questions used rule based rounding(Q8-not applicable) • Q15 used translation	• Q6, Q9, Q10, Q13 Rounding • Q13, Ignore too small fraction(rounding by situation)	• Q2, Q7, Q12, Q14 used rule based rounding • Q5 ignore too small decimal (situation based rounding)
Mert	• In all questions used rule based rounding (Q8-not applicable) • Q1 intermediate compensation • Q8 final compensation • Q15 used translation	• Q4 and Q13 Ignore fraction (rounding by situation) • Q6, Q9, Q10 rounding	• Q2, used truncation • Q2 and Q12 rounding in a large interval • Q5 ignore too small decimal • Q7, Ignore decimal part of number • Q14 intermediate compensation • Q7 translation
Sergen	• In all questions used rule based rounding (Q8-not applicable), • Q15 used compatible numbers	• Q6 round fraction as two separate numbers • Q10, Q9 rounding • Q13, Ignore too small fraction	• Q7, Ignore decimal part of number • Q14 rounding, intermediate compensation • Q7 translation
Nevzat	• except Q3 and Q15, used rule based rounding • Q11 used translation-changed division to fraction form • Q15 used translation	• Q10, Q9 rounding • Q6 round fraction as two separate numbers • Q13 Ignore fractions	• Q7, Ignore decimal part of number, round integer to another integer • Q12 and Q14 rule based rounding • Q7 translation
Ayşe	• used rule based rounding (Q8-not applicable) • Q15 translation	• Q4 and Q13 Ignore fraction • Q9, Q10 rounding	• Q2 rule based rounding • Q7, Ignore decimal part of number • Q14 rounding, intermediate compensation • Q7 translation

VITA

PERSONAL INFORMATION
Surname, Name: Boz, Burçak
Nationality: Turkish
Date and Place of Birth: 17.04.1979, Denizli
Marital Status: Single
Phone: +90 258 212 90 36
Email: burcakboz@gmail.com

EDUCATION

Degree	**Institution**	**Year of Graduation**
MS	METU SSME.	2004
BS	METU SSME	2002
High School	Nazilli Anatolian Teacher Training High School, Aydın	1997

WORK EXPERIENCE

Year	**Place**	**Enrollment**
2006-Present	Muğla University, MUĞLA	Lecturer
2005-2006	Board of Education and Discipline, ANKARA	Mathematics Teacher
2002-2005	Tavas High School, DENİZLİ	Mathematics Teacher

ACADEMIC STUDIES

1. Boz, B., & Bulut, S. (2001, May). The Area and Volume of Solids. Poster session presented at the Mathematics Activities Symposium and Exhibition, ANKARA.

2. Boz, B., & Bulut, S. (2002, September). Estimation Ability of Preservice Elementary Mathematics, Elementary Science, and Early Childhood Teachers. Paper presented at the meeting of the V. National Science and Mathematics Education Symposium. METU, ANKARA.

3. Boz, B., & Bulut, S. (2004, September). Measuring and Evaluating the 9th Grade Students Estimation Ability. Paper presented at the meeting of the VI. National

Science and Mathematics Education Symposium, Marmara University ISTANBUL.

4. The new mathematics elementary curriculum of MoNE was introduced to TED Ankara College Foundation Private High School teachers at ANKARA; 3-5 September, 2005

5. Boz, B., & Bulut, S. (2006, August). Computational Estimation Ability. Paper presented at the meeting of the Third YERME Summer School, University of Jyväskylä , FINLAND.

EXTRACURRICULAR ACTIVITIES

1. At the Association of Scouts; being a voluntary teachers for children who are living at streets, February 1998- May 1998

2. At Middle East Technical University Communication Club; being a voluntary teacher for orphans at Keçiören Atatürk Orphanage; September 2000- May 2001

3. Participated in a course of Adult Literacy which was conducted by MoNE; November 2001- December 2001

4. A course “Pedagogical Use of Internet and Multimedia Tools (Eurodidaweb)” was completed as a Comenius-2 inservice tranning course at 16- 20 October 2006 in Rome, Italy