THE IMPACT OF INTERNATIONAL CAPITAL FLOWS IN A THREE-SECTOR OPEN ECONOMY: A DYNAMIC GENERAL EQUILIBRIUM ANALYSIS

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ABSTRACT

THE IMPACT OF INTERNATIONAL CAPITAL FLOWS IN A THREE-SECTOR OPEN ECONOMY: A DYNAMIC GENERAL EQUILIBRIUM ANALYSIS

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This thesis examines the effects of international capital flows on economic growth by using a dynamic general equilibrium framework based on a three-sector Ramsey Model. In order to detect the impact of financial integration on production, allocation of resources across three sectors and consumption, two different economic environments are modelled. While the first model represents a closed economy with financial autarky, the second model examplifies a financially integrated open economy with partial capital mobility. Each of the models is calibrated to Turkish economy based on the data of the year 2006. The simulation results demonstrate that the presence of international capital flows, despite being limited by a borrowing constraint, reverses the impact of economic growth on production and resource allocation. It is found that even though the importance of production in tradable-goods sector diminishes in the absence of international capital flows, it increases in the open economy model. Moreover, the findings show that while production in the closed economy model simply adjusts to domestic demand, that of the open economy model is not constrained by it. This can be explained by the augmentative effect of partial capital flows on the impact of foreign demand on domestic production.

Keywords: International Capital Flows, Human Capital, Tradable-Goods Sector, Home-Services Sector, Borrowing Constraint

ÖZ

ULUSLARARASI SERMAYE HAREKETLERİNİN ÜÇ SEKTÖRLÜ AÇIK BİR EKONOMİDEKİ ETKİSİ: BİR DİNAMİK GENEL DENGE ANALİZİ

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Bu tez, üç sektörlü bir Ramsey Modeli'ne dayanan dinamik genel denge analizi çerçevesinde, uluslararası sermaye hareketlerinin iktisadi büyümeye olan etkisini incelemektedir. Finansal birleşmenin üretime, tüketime ve kaynakların üç sektör arasındaki dağılımına olan etkisini ortaya çıkarmak amacıyla iki farklı ortam modellenmiştir. İlk model finansal özerkliğe sahip kapalı bir ekonomiyi temsil ederken, ikinci model kısmi sermaye hareketine sahip finansal olarak birleşmiş açık bir ekonomiyi örneklemektedir. Her iki model de Türk ekonomisinin 2006 yılı verilerine dayanarak kalibre edilmiştir. Simülasyon sonuçları, uluslararası sermaye hareketlerinin varlığının, borç kısıtı ile sınırlandırılmış olmasına rağmen, iktisadi büyümenin üretim ve kaynak dağılımına olan etkisini, tersine çevirdiğini göstermiştir. Uluslararası sermaye noksanlığında, ticari ürünler sektöründeki üretimin önemi azalırken, açık ekonomide bu önemin arttığı bulunmuştur. Buna ek olarak, bulgular göstermektedir ki; kapalı ekonomi modelinde üretim, yurtiçi talebe göre ayarlanırken, açık ekonomi modelinde üretim, yurtiçi taleple sınırlı değildir. Bu durum kısmi sermaye hareketlerinin, yurtiçi üretime olan yabancı talep üzerindeki artırıcı etkisi ile açıklanabilir.

Anahtar Kelimeler: Uluslararası Sermaye Hareketleri, Beşeri Sermaye, Ticari Mallar Sektörü, Yurtiçi Hizmet Sektörü, Borç Kısıtı

To my beloved sister, Ayşe Akgül

and

To my parents, Ayten and Hikmet Akgül

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CHAPTER 1

INTRODUCTION

International capital flows have become an indispensable part of global economies of the 21^{st} century. Economic growth used to be constrained by the boundaries of domestic markets since countries had to rely on their own saving and investment capabilities. However, the presence of international capital flows proposes a remedy to this problem by providing the countries with the opportunity to make use of the global resources and share the risks associated with investment. Thus, it would seem that the limits of domestic markets for economic growth cease to be a constraint for the global economies of the 21^{st} century. This line of thinking encourages the countries with closed capital accounts to engage in financial integration by liberating their capital accounts. Nonetheless, many emerging economies experienced financial crises after capital account liberalization which casts a suspicion on the prospects offered by the presence of a global financial market. Thus, the arguments raised in favor of financial integration started to lose ground leading to a re-evaluation of the benefits and costs of international capital flows. In that sense, much of the questioning is directed towards the validity of capital account liberalization and whether it exerts a positive influence on economic growth for the country in question, or not.

There are many surveys in the literature concerned with the growth impact of international capital flows and each of them reaches different conclusions. Some of the surveys argue in favor of capital account liberalization based on theoretical explanations and empirical findings. Among them, Obstfeld (1994) suggests that financial integration benefits many countries through its positive effect on steady state welfare. According to Levine (2001) financial integration enhances economic growth by improving the domestic financial system. Similarly, Klein and Olivei (2008) conclude that financial integration contributes to economic growth through financial deepening. Yet, they also add that the exhilaratory impact of financial integration is only observed in industrialized OECD countries. On the other hand, Mishkin (2007) and Edwards (2001) claim that if the countries improve their macroeconomic environments and attain a degree of economic development, financial integration would lead to significant economic growth.

In contrast to the positive image of financial integration pictured above, many authors argue against it and pose empirical evidence showing the adverse effects of international capital flows. For instance, Krugman (1993) argues that neither economic theory nor the past evidence confirms the acceleratory role of financial integration on economic growth since the access to foreign capital is not an engine for growth by itself, even though it leads to capital accumulation in the country. Similarly, according to Rodrik (1998) and Edison et al. (2002), financial integration is not associated with economic growth even when per capita income levels, institutional qualities and school enrollment rates are controlled for. Even though Arteta et al. (2001) come up with indications of a vague positive relationship between capital account liberalization and growth, the findings show that it is conditional on time, measurement and estimation.

The theoretical discussions and empirical findings on the benefits and adverse effects of capital account liberalization provide a limited analysis of international capital flows. In order to observe their practical implications on economic growth and convergence, many surveys utilize neoclassical growth models. By calibrating neoclassical models of real economy, the overlooked details in theoretical explanations and empirical surveys would be filled in. The benchmark framework in convergence and growth literature is to use the Ramsey (1928) model of optimal consumption and saving. Within the bounds of a closed economy, the model has been used to show that there is conditional convergence among economies.

Even though Ramsey model performs well in closed economy frameworks, it encounters several difficulties when international capital flows are introduced. As is discussed in Barro et al. (1992, 1995) and Barro and Sala-i-Martin (2004), there are three possible problems related with the open-economy version of the Ramsey Model. The first problem is the infinite rate of convergence observed in the economy when international capital flows are allowed eliminating any transitional behavior of endogenous variables. The second problem associated with the model is that when there is a difference in the time preference rates, the most patient country will have a high consumption level, while the consumption per head in an impatient country will approach zero. The third problem faced by the model is the wealth effect which claims that impatient countries end up with negative wealth.

The aforementioned problems related with the open economy version of the Ramsey Model have been scrutinized in the growth literature. There are three main solution methods offered by these studies. One of the methods is introducing adjustment costs in investment into the model in order to slow down the adjustment of capital stock to its steady state level.[1] Another way to deal with the problems of the open economy Ramsey Model is endogenizing the time preference parameter such that it is different for every country. This would reduce the gap between the interest rate and the time preference parameter.[2] Still another method is allowing for non-tradable investment goods in the economy which leads to imperfect capital mobility in world financial markets.

According to Gourinchas and Jeanne (2002), the economies that experience infinite convergence rates after opening their capital accounts, are, in fact, close to their conditional steady states. As is discussed in Mankiw et al. (1992), this closeness might result from a low capital share in production which increases the speed of diminishing returns to capital. In order to deal with this problem, they introduce human capital into the model so that the share of capital in production is increased. As a result, diminising returns to capital slows down. Inspired by the idea, Barro et al. (1992, 1995) and Barro and Sala-i-Martin (2004) introduce two types of capital stocks into the model, namely physical and human capital. They assume that human capital stock is non-tradable, while physical capital stock is tradable. Thus, even though foreign borrowing can be used in the accumulation of physical capital, it cannot be

[1] Cohen and Sachs (1985), Barro and Sala-i-Martin (2004) and Brock (2009) provide specific applications of this method in their respective models.

[2] The inspiration comes from the Uzawa (1968) framework and it is further questioned and analyzed by Francis and Kompas (2001) and Barro and Sala-i-Martin (2004).

used in the accumulation of human capital. The non-tradable nature of the human capital creates a distinction between the capital stocks which is reflected in their relative rates of returns. This distinction, in turn, imposes a borrowing constraint into the economy which solves the infinite convergence rate problem.

This thesis will examine the effects of international capital flows on economic growth by focusing on production, consumption and the allocation of resources across three sectors based on a dynamic general equilibrium framework. The focus on three sectors provides a more informative and detailed analysis on the effects of financial liberalization. As is discussed in Aykut and Sayek (2007), in models of real economy, not only the presence of free international capital flows, but also the sectoral composition of them are decisive in economic growth. In fact, they suggest that if international capital flows are channeled to manufacturing sector, economic growth will be improved. On the other hand, if they are channeled to primary goods or services sectors, then economic growth might be affected negatively. Therefore, if financial liberalization is considered in the context of three sectors, the analysis will include the sector specific factors of international capital flows and present more realistic conclusions.

Based on this perspective, in this thesis production will be assumed to occur in three sectors. The first sector is the home-goods sector which produces agricultural and manufacturing output only for the domestic market. For this sector, the market clears within the economy so that the price of its goods is endogenized. The second sector is the tradable-goods sector which produces all the internationally tradable goods in the economy at world prices. The output of this sector is not only for domestic consumption but also for foreign consumption. The third sector is home-services sector and it only serves to the domestic market; therefore it is also closed to international trade. As a result, the price of this sector's good is endogenously determined within the domestic market, as well. In all these sectors output is produced with three inputs: raw labor, physical capital and human capital. Production sectors are different from each other on the basis of their relative factor intensities and the type of their output. Moreover, the accumulation of physical and human capital takes place in different sectors as the economy is openned to international capital flows.

The procedure to be followed is to compare and contrast the factors of production movements across sectors in two different environments. The first environment represents an economy with financial autarky which will serve as a benchmark framework. The economy in this environment will be open to international trade in goods and services, but close to international capital flows. In other words, the model presents a closed economy with respect to financial flows. Having developed the closed economy version of the Ramsey Model, financial integration will be introduced into the framework. Thus, the second environment will extend the benchmark model by allowing for international capital flows into the economy. In short, the frameworks are differetiated from each other by the presence of international capital flows.

The difference in these models is demonstrated in the determination of the interest rate. In the closed economy model, since international capital flows are not allowed, foreign debt is zero and the interest rate is endogenously determined in the domestic market. However, in the open economy model international capital flows are liberated which means that the interest rate is pegged at the world interest rate. The constancy of the interest rate causes the open economy Ramsey model to predict problematic outcomes as already discussed above. In this thesis, the problems of the open economy framework will be overcome by introducing non-tradable investment goods into the economy. Consistent with the literature and specifically following Barro et al. (1992, 1995) this thesis will assume an imperfect capital mobility by imposing a borrowing constraint on the domestic economy. In that fashion, two types of capital stocks will be employed in production such that human capital is assumed to have a non-tradable nature. Thus, even though physical capital can be used as collateral against foreign debt, human capital cannot. Based on this assumption, investment in physical capital will be conducted by using foreign borrowing, while investment in human capital will be in the responsibility of domestic savings. Consequently, the amount of foreign borrowing of domestic agents can simply be as much as the available physical capital stock in the country.

Setting the framework, both models will be calibrated to Turkish economy for the year 2006. This thesis will examine the changes in production, consumption and reallocation of resources across sectors as the economy approaches from the initial state to its long-run growth. In that pursuit, first the initial and the steady state values of the variables in the closed economy model will be analyzed so that the evolution of the variables in the economy will be traced. In this framework, in addition to the steady state the transitional period behaviors of the control and state variables will be covered. Showing the changes occured in the absence of international capital flows, a second comparison between the inital and the steady state values of the variables will be conducted in the open economy framework. The transitional behaviors of the variables in the open economy model will be covered in a further study. Lastly, the outcome of these two models will be compared so that a conclusion about the impact of international capital flows on real economy will be obtained.

In contrast to Barro et al. (1992, 1995) and Barro and Sala-i-Martin (2004), this thesis incorporates three different sectors. It is informative to scrutinize the changes taking place in production and resource allocation recognized in various sectors. In the previous setting of the Ramsey Model with only one sector, capital accumulation is afforded by lower consumption and higher saving of the domestic agents. On the other hand, it will be shown in this analysis that the required investment is not necessarily financed by decreased consumption and increased saving in a multi-sector environment. In the closed economy model, economic growth is financed by leaving consumption behavior rather smooth and re-allocating the investment opportunities of sectors within themselves. In particular, it will be shown that as economic growth takes place, capital and labor are pulled out of the tradable-goods sector and employed in the home-goods and home-services sectors when the economy is closed to capital flows. However, there will be no change in the consumption shares of goods in total expenditure. In contrast, in the open economy environment, investment is not only financed by accessing other sector's resources, but by also utilizing the international sources through financial integration. As a result, as capital deepening takes place, the capital and labor employed in home-services sector are channeled to the tradable-goods sector in this framework.

A comparison between these two models starting from the same initial shares of sectoral production in total GDP, is expected to show different transitional changes in variables leading to different steady state outcomes. The impact of introducing imperfect capital flows into the economy will be shown to affect sectoral production and resource allocation in an opposite manner compared to the closed economy environment. In particular, calibration analysis will show that when the economy is closed to international capital flows, capital accumulation will lead to an increase in the production share of the home-goods and home-services sectors while causing a decline in that of the tradable-goods sector. Thus, the absence of international capital flows have an adverse effect on the tradable-goods sector with respect to the steady state outcomes. On the other hand, when the economy is opened to international capital flows, as the economy approaches to its long-run growth, the importance of production in tradable-goods sector will increase while that of the home-services sector declines. The reversal in the outcome can be explained by the difference in the determination of production in these frameworks. While production in the closed economy model is simply determined by domestic demand, production in the open economy is not constrained by it. In fact, opening the economy to international capital flows will be shown to increase the sensitivity of production to foreign demand. Thus, examining the changes taking place in these models by giving the shock of capital account liberalization in a three-sector environment will contribute to the understanding of open economies.

The thesis is organized as follows: In Chapter 2, the concepts of capital account liberalization, in general, and foreign direct investment, in particular, are discussed. Chapter 3 gives a brief overview of the literature on the open economy version of the Ramsey Model. Chapter 4 describes the basic properties of the closed economy model and gives the analytical solutions. Chapter 5 presents the open economy model and describes its basic properties together with the analytical solutions. Chapter 6 does the calibration analysis and compares the simulation results of the models. Chapter 7 concludes the thesis.

CHAPTER 2

THE CONCEPT OF INTERNATIONAL CAPITAL FLOWS

2.1 Issues in Capital Account Liberalization

Providing many opportunities to achieve economic growth, international capital flows have become an integral part of global economies of the 21^{st} century. The presence of capital flows is claimed to propose a remedy to the shortcomings of domestic markets. No longer constrained by domestic limitations, countries make use of global opportunities of financial markets. Thus, in order to benefit from the merits of financial integration, countries are encouraged to liberate their capital accounts. However, the experiences of many countries demonstrate that switching from financial autarky to financial integration is not always welfare enhancing in different environments. In other words, even though capital account liberalization enhances economic growth in many countries, it fails to do so in others. The optimism associated with financial integration has been shadowed by an increasing number of countries that suffer from financial crises following capital account liberalization. Since the prospects offered by the presence of a global financial market are not realized in many countries, the arguments raised in favor of financial integration started to lose ground. Consequently, the benefits and costs of international capital flows are questioned in the literature leading to a re-evaluation on the validity of capital account liberalization. The basic focus of attention is directed towards the impact of international capital flows on economic growth.

Before proceeding with the aforementioned discussions taking place in the literature, it is beneficial to define financial integration and describe the content of international financial flows. Financial openness is defined as "the ability of financial investors to choose between domestic financial assets and foreign financial assets." (Blanchard, 2003:373) In other words, when the capital accounts of the countries are liberated, capital controls eventually disappear leading to an integrated financial market where investors gain access to many international assets so that they hold not only domestic assets but also foreign assets. Thus, they have the opportunity to speculate on the fluctuations in interest rates and exchange rates of the countries which are included in the global financial system.

Broadly, there are three types of international capital flows according to the Economic Report of the President Federal Reserve Archival System for Economic Research (FRASER) (2004).[3] They are known as Foreign Direct Investment (FDI), portfolio investment and bank and other investments. The first type of capital flows, namely FDI, is described as the right to the ownership and operation of an enterprise in a different country. The investors should hold at least 10 per cent of the shares of the firm or the asset in question. The second type of capital flows, which is known as portfolio investment, includes the purchase of foreign corporate or government bonds, short-term securities and notes and the like. Comparative to the market for FDI assets, the market for portfolio investment assets is more liquid and therefore much more volatile to speculation. Thus, based on the economic and political environment of the countries in question, capital inflows can change course and turn into capital outflows. The third type of flows is bank investment which is composed of deposit holdings by foreign investors and loans to various economic agents. Among these three types, portfolio investments constitute the largest portion of international capital flows.

2.1.1 A Brief History of Global Financial Markets

Financial integration is not a new concept and it has been influencing the world economies for a long time. Mishkin (2007) examines globalization of international trade and capital flows in two phases. The first phase is comprised of the periods between 1870 and 1914. Accoring to the estimation of Obstfeld and Taylor (2002),

[3] Economic Report of the President is one of the publications of the Federal Reserve Archival System for Economic Research (FRASER). FRASER is a project of the Federal Reserve Bank of St. Louis which develops an archive of economic statistical publications and data.

the share of global foreign assets in World GDP increased from 7% in 1870 to 20% in 1914.[4] According to Obstfeld (1998) the free operation of the global financial market was disrupted by World War I.[5] The war and then the Great Depression during the 1930s weakened the global financial channels. The following World War II only made the governments impose tighter restrictions on international capital flows. Thus, domestic financial markets were controlled extensively by the governments in the 1950s with an inward-looking focus. The capital controls imposed during that era did not prevent the existence of international financial crises. Yet, it had a positive influence in terms of the fact that even though international financial crises took place, their impacts were limited within the domestic economy.

The second phase begins with the end of WWII. In fact, according to Obstfeld (1998) financial integration revitalized in the 1960s and continued to increase from then on. Nonetheless, as long as countries continue to be committed to the fixed exchange rate system of Bretton Woods, the reconstruction of free international flows remained to be under control. Specifically, according to Bosworth et al. (1999), financial inflows apart from direct investment were restricted in order for the governments to conduct monetary policy in favor of domestic markets. From the 1970s onwards, a new international system that enhances globalization in trade and capital flows has been created. Thus, the International Monetary Fund (IMF) and the World Bank (WB) came into being and accelerated the financial integration since then. In fact, international capital flows returned to and then surpassed their previous volume of 1914. In fact, according to the findings of Obstfeld and Taylor (2002), the foreign investment to World GDP ratio rose to 25% in 1980 and further increased to 62% in 1985.

International private capital movements gained a new momentum with the worldwide capital account liberalization trend in the 1990s.[6] Since then financial markets

[4] In the original paper, Obstfeld and Taylor (ibid.) examine the amount of world international capital flows. They try to come up with a consistent and reliable measure of global foreign investment. The data on foreign investment is such that certain countries enter as creditors with their asset holdings in foreign markets and some countries enter as debtors with their liabilities to foreign countries. Thus, the international capital flow measurement is given by the ratio of foreign investments to World GDP.

[5] Specifically, Obstfeld (ibid.) states that Europe, Oceania, Africa and the Far East were tied together by an international capital market before WWI.

[6] As noted by Bosworth et al. (1999), until 1990s, the majority of financial inflows to industrializing countries have been composed of official organization loans to governments and loans of commercial banks. It was only

provide the backbone of economies. Nonetheless, such strong global ties in financial markets have repercussions in the domestic markets which are reflected heavily during economic crises.

2.1.2 The Impact of Capital Account Liberalization

2.1.2.1 Positive Impacts

Financial integration has long been claimed to bring various benefits to the countries in question. There are basically two types of benefits of financial integration according to Gourinchas and Jeanne (2002). The first one is the international allocative efficiency. As noted by Fischer (1998), financial integration utilizes international capital in its most productive use so that it provides the most efficient allocation of global savings. This efficiency in allocation leads to higher gains in terms of economic growth compared to the case when financial liberalization was restricted. Moreover, international capital flows lead to a wider portfolio diversification which decreases the risks for investors both in the industrialized countries and in the industrializing countries. (Obstfeld, 1998; Fischer, 1998) Thus, financial integration provides the opportunities for a more subtle risk sharing which further improves the international allocative efficiency.

As discussed by Obstfeld (1998) and Edison et al. (2002), the efficiency in allocation of world savings also enables the capital poor countries to obtain the necessary capital through international flows and provides an inter-temporal consumption smoothing in times of output shocks. Thus, the industrializing countries, which are short of capital and undergoing recessions, would be able to access international capital and thereby finance their investments by foreign borrowing.

While the industrializing countries have more investable funds thanks to free international capital flows, the international economy profits by the widened and better-financed trading system. (Fischer, 1998) Klein and Olivei (2008) examine the effect of capital account liberalization on financial depth and economic growth in a

after the 1990s that private portfolio investment achieved an active role in global financial markets.

cross-section framework.[7] According to the findings, financial depth increased more in countries with open capital accounts compared to the countries with restrictions in financial flows. In fact, Klein and Olivei[8] claim that financial integration enlarges the size and scope of the local and international financial systems by incorporating the previously ignored parts of the market into the system. Thus, international capital flows improves financial depth of global and domestic financial markets.

The second type of benefits is the domestic allocative efficiency. As is discussed in Gourinchas and Jeanne (2002), financial integration encourages FDI in the domestic country which would improve the domestic technology and lead to knowledge spillovers. The empirical findings in Noy and Vu (2007) suggest that capital account openness has a positive impact on FDI inflows when macroeconomic and institutional environments are taken into consideration. Provided that the country in question has a stable political environment with less corruption, the positive relationship continues to prevail. However, in countries with a corrupt political system and loose regulation, the impact of capital account liberalization on FDI ceases to be beneficial.

Capital account liberalization also improves the financial system of the country. According to Eichengreen (2001) capital controls diminish the discipline on policymakers, extend the power of bureaucrats and increase the risk of rent-seeking activities through access to international capital. Since liberalization brings about competition with foreign firms, the standards in the local financial markets would improve as capital accounts are opened. (Obstfeld, 1998; Klein and Olivei, 2008) In fact, foreign competition threatens the domestic market with capital outflows which creates the incentive to reform domestic policies. Through these channels, financial integration brings discipline and regulation into the domestic markets leading to a reduction in corruption and exploitation of the financial system. (Edison et al., 2002)

Furthermore, Gourinchas and Jeanne (2002) argue that the presence of more efficient foreign banks in the domestic market leads to a better allocation of domestic savings. Thus, financial inflows stimulate domestic savings such that a higher rate of capital accumulation can be achieved which improves the domestic allocative efficiency.

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[7] The empirical survey is conducted for the periods between 1976-1995 and 1986-1995.

[8] ibid.

2.1.2.2 Negative Consequences

Even though financial liberalization has many benefits, it also imposes certain constraints on domestic economies. First of all, financial liberalization has adverse effects on employment and income equality. According to Mundell (cited in Obstfeld, 1998) international capital flows might affect wages as commodity imports would do. Since international capital can be invested anywhere in the world, it would be placed in its most productive use with the least cost possible. Thus, commodities will be produced in the countries with low labor costs. This brings about two effects. Since lower costs lead to lower prices, importing the commodities in question would be cheaper than producing them which causes unemployment in high-cost countries. Moreover, since capital markets are open, factor prices are equalized all over the world. This equalization, in turn, causes a decline in the wage rates.[9] Either effect is a distortion to the well-being of a country.

Another negative consequence of financial liberalization is its impact on fiscal policy. As is recognized in Obstfeld,[10] an open capital account makes it hard to tax capital. Therefore, the tax income of the government decreases. This might either lead to higher taxes imposed on labor or diminished social services that are provided by the government according to Rodrik. (cited in Obstfeld, 1998) Moreover, financial openness restricts the ability to implement monetary policy for domestic objectives. Obstfeld[11] argues that it is not possible for a country to simultaneously utilize a fixed exchange rate target and domestic monetary policy when its capital account is open.[12]

The global debt crises that have been taking place in the industrializing countries since the 1990s are also among the hazardous consequences of financial integration. Capital account liberalization encourages the investors to engage in risky investments

[9] For a more detailed discussion on the impact of financial integration on wages and employment, see Obstfeld (ibid.).

[10] ibid.

[11] ibid.

[12] A fixed exchange rate regime followed by a country would eliminate the risk of exchange rate instability in the economy. Exempt from this risk, "arbitrageurs would borrow in the currency with the lower interest rate and lend in the currency with the high interest rate" such that the gap between the nominal interest rates across countries would be closed eventually. (Obstfeld, 1998:7) This would restraint the ability to use monetary policies for economic objectives in the domestic market since equal interest rates requires the implementation of harmonious monetary policies in different countries.

and the banks to engage in risky loans. The confidence of the investors and domestic banks in the stability of exchange rates is the root reason in their engaging in risky investments. A fixed exchange rate regime or the assurance of stability would induce banks and investors to borrow without carefully analyzing the consequences of a devaluation of the domestic currency. (Obstfeld, 1998; Bosworth et al., 1999) Thus, their liabilities exceed their assets which damages the reliability of the banks in question. Eventually, as also noted by Dornbusch (1998), the situation leads to the bank insolvency problems experienced in the Asian crisis and in many financial crises occurred thereafter. The capital otflows and associated bank insolvencies create a signal in global financial markets which leads to a chain reaction affecting the markets all over the world. (Fischer, 1998) The contagious nature of financial failures poses the greatest disadvantage of capital account liberalization.

2.1.3 The Role of Capital Account Liberalization in Economic Growth

Most of the arguments raised in favor of capital account liberalization focus on the stimulative effect of international capital flows on economic growth. In order to assess this claim, many empirical surveys have been conducted in the literature. Even though some of them obtained results that confirm the positive relationship between financial integration and economic growth, some came up with contradictory conclusions. There is no clear-cut relationship between capital account liberalization and economic growth since every empirical survey offers different conclusions. As is discussed in Edison et al. (2004), the ambiguity derives from the divergence in the estimation technique, the data period under analysis, the set of sample countries used in the model and the dataset. For example, while some studies focus on the experiences of industrialized countries, the others examine that of industrializing countries. Moreover, according to Edison et al.[13] there are variations in the dataset used, namely cross-section, panel or time series, which lead to different conclusions. Even though each empirical survey has its own concerns and limitations, it is informative to compare the different results in terms of examining the growth effects of financial integration.

[13] ibid. The original paper gives a detailed list of the empirical surveys concerning the relationship between capital account liberalization and growth and it also shows the different approaches taken by each survey.

According to Krugman (1993) neither economic theory nor the past evidence confirms the acceleratory role of financial integration in economic development. The arguments in favor of capital account liberalization generally claim that international capital flows stimulate capital accumulation and thereby economic growth. However, capital accumulation per se is not seen as an engine for economic development in the recent growth literature. For instance, Levine (2001) acknowledges the fact that physical capital does not account for the main cross-country differences in economic growth. As long as capital inflows are low, there is no guarantee that it will spur economic growth. Thus, if financial integration is encouraged on the basis of access to international capital, it does not make much sense unless capital inflows are huge. In fact, Krugman (1993) stresses that during the postwar period, the capital flows from capital-rich to capital-poor countries that were materialized were very low contrary to the expectations.

Obstfeld (1994) examines the impact of international financial flows on economic growth through its influence on portfolio diversification. He analyzes a dynamic continuous time model in which capital account liberalization would increase the global risk-sharing and channel the investment towards more risky and high-capital yielding types. The underlying assumption is that economic growth requires "an increasing array of specialized inputs" which can be provided by engaging in risky investments. (Obstfeld, 1994:1310) This kind of investment would increase the expected consumption and leads to significant welfare gains according to the empirical findings. In fact, the calibration exercise in Obstfeld[14] imply that financial integration benefits many countries through its positive effects on steady-state welfare.

Rodrik (1998) examines the relationship between capital account liberalization and per capita growth, investment as a share of GDP and inflation during the period between 1975-1989. He uses data from industrialized as well as industrializing countries by controlling for initial per capita income, school enrollment rate, quality of institutions and regional differences. The empirical findings in Rodrik[15] show that capital account liberalization is not associated with economic growth provided that

[14] ibid.

[15] ibid.

the aforementioned determinants are included in the analysis. That is, the countries that eliminated capital controls did not grow faster or invested more compared to the countries with capital controls.

The financial crises that hampered the economies of many industrializing countries pointed to the importance of the maturity of the institutional and financial system of the countries in question prior to capital account liberalization. Even though this argument suggests that capital account liberalization is beneficial for economic growth when the country has strong institutions, Rodrik[16] finds no evidence supporting this claim. The empirical findings in his analysis imply that the quality of public institutions does not enhance the impact of capital account liberalization.

Levine (2001) examines the relationship between financial liberalization and economic growth by focusing on its development effect on the domestic financial market. The empirical findings suggest that financial liberalization improves the functioning of the domestic market. According to the analysis of Levine,[17] when the restrictions on international capital flows are reduced, the country gains access to an enhanced stock market liquidity which leads to an improvement in productivity. Moreover, the inclusion of more developed foreign banks into the domestic financial system would put the domestic banking system in order. Thus, financial integration stimulates economic growth by improving the domestic financial system which would provide a better allocation of resources leading to increased total factor productivity.

Arteta et al. (2001) assess the relationship between capital account liberalization and economic growth by taking the effects of differences in measurement and other economic factors into consideration. The empirical findings in Arteta et al.[18] imply that there is not a strong positive relationship between capital account liberalization and economic growth. Even though they find a vague positive impact of financial integration on growth, it is subject to the effect of time and measurement changes. The results argue against the augmentative influence of financial depth, development and high income on the relationship between capital account liberalization and growth.

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[16] ibid.

[17] ibid.

[18] ibid.

That is, even though strong institutions, like the rule of law, enhance the positive impact of financial liberalization on the economy, the evidence is weak to reach the same conclusion with respect to financial depth, development and income according to Arteta et al.[19]

Edison et al. (2002) analyze the effects of international financial integration on economic growth by including certain indicators into their analysis in order to elaborate on the impact of economic, financial, legal and policy conditions of the countries. The analysis shows that international financial integration has a positive relationship with banking and stock market development, legal system, real per capita GDP and educational attainment. However, international financial integration is not found to be stimulating economic growth even though economic development, financial development, legal system development, government corruption, educational attainment and macroeconomic policies are taken into consideration in the analysis. Thus, they conclude that financial integration does not exert a positive impact on economic growth.

Klein and Olivei (2008) also examine whether capital account liberalization leads to economic growth through its financial deepening effect. The empirical findings conclude that financial integration contributes to economic growth crucially through its effect on financial deepening. However, based on the study, this result is realized only in industrialized countries, in this case the OECD countries. Thus, according to Klein and Olivei (2008),[20] capital account liberalization might not lead to economic growth in industrializing countries provided that the required link is financial depth. In the end, they suggest that stabilization of macroeceonomic environment with a solid institutional setting should precede capital account liberalization in industrializing countries.

The relationship between financial development and economic growth leads to the idea that countries should follow a sequence in liberalization. The countries should, first of all, stabilize their macroeceonomic environment, improve their financial, legal and economic institutions and open their current accounts in order to benefit from

[19] ibid.

[20] ibid.

the positive growth effects of capital account liberalization. According to Bosworth et al. (1999) sequencing in financial integration is paramount. In fact, they think that domestic market functioning and regulation should be strengthened before financial liberalization takes place.[21] In the same vein, Fischer (1998) argues that in countries with weak financial systems, liberalization should not take place in an instant; instead it should be done step by step controlling for the macroeconomic stability. According to Arteta et al. (2001) opening up to trade is less effective compared to the reduction of macroeconomic imbalances. They found that capital account liberalization has a positive effect on economic growth provided that macroeconomic imbalances are eliminated which would prevent any contradictory policy that might distort the stable environment. Similarly, Mishkin (2007) states that if the countries improve their macroeconomic environments and choose right policies, financial integration would lead to significant economic growth. Edwards (2001) also claims that beneficial effects of capital account liberalization on economic growth are achieved when the country in question attained a degree of economic development.

The experiences of high income countries and low income countries might lead to different conclusions on the relationship between capital account liberalization and economic growth. However, according to Arteta et al. (2001), the findings are claimed to be dependent on the estimation and specification. In fact, Arteta et al.[22] argue that the empirical findings offer little evidence to support the view that capital account liberalization is more conducive to growth in high income countries compared to low income countries. The empirical findings in Edison et al. (2004) imply that capital account liberalization promotes economic growth in middle income countries compared to low-income or high-income countries.

In order to utilize the benefits of capital account liberalization and avoid the distortions ending up with crises certain requirements are in order. First of all, the domestic economy should follow a sound macroeconomic policy which leads to low inflation, reduces the budget deficit and controls the current account balance. Secondly, as stressed by Fischer (1998), improvements in financial system are

[21] Moreover, Bosworth et al. (1999) claim that among all the forms of capital flows, FDI is the most beneficial one for economic growth and full capital convertibility is not required to encourage FDI. Thus, capital account liberalization could be postponed until a more sound institutional environment is achieved.

[22] ibid.

necessary to eliminate banking insolvency. Similarly, according to Obstfeld (1998), in order for the benefits of capital account liberalization to outweigh its costs, a careful regulation and monitoring of the domestic financial system should take place. In that sense, a sound regulatory framework should be established to audit the banks properly so that they do not engage in too risky lending. Thirdly, certain capital control methods could also be implemented with the condition that they will be eliminated eventually. Fourthly, Fischer (1998) suggests that information about central bank reserves and operations should be published in time so that better and timely informed markets would function more efficiently. Lastly, as Fischer[23] argues, a better surveillance system would reduce the frequency of crises, even though it cannot eliminate them all.

2.2 Issues in Foreign Direct Investment

As mentioned above, international capital transactions include three types of capital flows which are foreign direct investment (FDI), portfolio investment and bank and other investments. According to the IMF definition, FDI is considered to be the foreign investment in return of which the investor is given the right of management interest, namely 10% or more of voting stock. (cited in Noy and Vu, 2007) As noted by Noy and Vu,[24] even though FDI is mainly depicted as a nonfinancial flow, it includes many financial capitals such as equity capital, reinvestment of earnings and other long and short term capital. FDI has been an increasing component of total investment in countries all over the world since 1980s regardless of their economic status.[25] Particularly for industrializing countries, the pattern of financial inflows took the form of FDI. (Bosworth et al., 1999) Moreover, since 2000, FDI inflows have reached a level that is four or five times that of 1980s. (Noy and Vu, 2007)

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[23] ibid.

[24] ibid.

[25] According to the Economic Report of the President Federal Reserve Archival System for Economic Research (FRASER) (2004), about one quarter of world capital inflows were in the form of FDI in 2002. According to the same report, the major industrial countries, namely the United States, the United Kingdom, Canada, Japan and the countries in the Euro area, obtained 40% of these flows.

Given the changing pattern of international capital flows in favor of FDI, it is possible to evaluate financial integration on the basis of FDI flows. There is an increasing literature that agrees on the beneficial effects of FDI on the economy. In fact, according to Noy and Vu,[26] FDI is considered to be a better choice for economic growth compared to other components of financial flows, namely portfolio investment or bank loans. Moreover, as indicated by Arteta et al. (2001), analyzing the effects of FDI on economic growth is less complicated than analyzing the effects of portfolio investment.[27]

2.2.1 The Impact of Foreign Direct Investment

FDI has a positive influence on the economy through two channels. The first one is a direct effect of FDI. Foreign investment is claimed to be more efficient and productive than domestic investment due to advanced technology and management skills it entails. (Borensztein et al., 1995) Thus, FDI inflows provide the potential to channel these advanced technologies to host economies. Through learning by doing the technology transfer leads to productivity spillovers in the host economy. Consequently, FDI enables productivity gains, technology transfers, which include new production processes and managerial skills, employee training and access to markets. (Alfaro et al., 2003; Bosworth et al., 1999) According to Borensztein et al. (1995), this contribution of foreign investment on the economy is accelerated when the level of human capital stock is high in the host economy.

The second effect of FDI on the economy is an indirect one. FDI contributes to capital accumulation by enhancing domestic investment. Borensztein et al.[28] give two opposing relationships between foreign investment and domestic investment. Since FDI would bring higher competition with it, there is a risk of domination of foreign capital in product and financial markets. This might cause foreign investment to

[26] ibid.

[27] Arteta et al. (ibid.) mentions one of the reasons why FDI is more conducive to empirical analysis compared to other capital flows. Financial markets suffer from information asymmetries which are hard to characterize. The information asymmetry problem distorts the picture such that the impacts and consequences of financial inflows are ambiguous. This complicates the analysis of financial flows. In contrast, empirical studies concerning the effect of FDI in models of real economy provide more reliable and definitive results which facilitates the analysis.

[28] ibid.

replace domestic investment. On the other hand, instead of a substitutive relationship, FDI might have a complementary role in domestic investment in terms of technology transfers and knowledge spillovers. This might stimulate domestic investment resulting in an indirect positive influence of FDI on the host country. As is analyzed in Borensztein et al.[29], the empirical studies show that a net FDI inflow leads to a higher increase in total investment in the domestic country leading to a crowding-in effect. In fact, there is nearly a "one-for-one relationship" between FDI and domestic investment. (Bosworth et al., 1999:164) It implies that domestic investment is stimulated by FDI leading to an acceleration in the rate and amount of domestic capital accumulation.

2.2.2 The Role of Foreign Direct Investment in Economic Growth

The impact of FDI on economic growth has been debated constantly in the literature and every survey has come up with different answers and different suggestions. As noted by Carkovic and Levine (2002), there are three general arguments associated with the growth effect of FDI. One way to look at the issue at hand is that FDI promotes economic growth since it stimulates technological externality which improves the productivity of all the firms in the economy including the ones without foreign capital. A contrary perspective is that there is no positive link between FDI and economic growth. Still another belief is that FDI accelerates economic growth only when other economic determinants are controlled for.

According to Alfaro et al. (2004), even though FDI is argued to provide spillover effects into the economy and leads to growth, the domestic country might not be able to exploit the benefits due to local conditions. There might be many distortions in the local environment that would reduce and even reverse the possible positive impact of FDI. As noted by Alfaro et al.,[30] these can take the form of limited absorptive capacity or unimproved institutional setting. They suggest that the domestic firms should change the structure according to the requirements of FDI by buying different machines, hiring new employees and the like in order to be able to take advantage of

[29] ibid.

[30] ibid.

the benefits provided by FDI. A well-functioning financial market and the presence of an efficient absorptive capacity would enable the new production system and technology brought about by FDI to be implemented in the domestic economy. Moreover, Alfaro et al. (2003) stress that mature financial markets facilitate the channeling of global capital to its most productive use since it reduces the cost of transactions.

Borensztein et al. (1995) analyze the influence of FDI on economic growth by utilizing a cross-country regression method. They examine an endogenous growth model with technological progress playing the major part in economic growth. Foreign direct investment which is carried out by multinational corporations is the basic source of technological transmission to industrializing countries. Nonetheless, it is also assumed that without a certain level of human capital[31] in the domestic country, the transferred technology cannot be implemented properly. Therefore, it is not only the access to advanced technology, but also the availability of domestic absorptive capacity that exerts an influence to economic growth. The empirical findings show that foreign direct investment accelerates economic growth by enhancing technological transfer. Moreover, the complementary relationship between human capital and FDI accelerates economic growth even more. Borensztein et al.[32] even suggest that FDI is more conducive to economic growth than domestic investment provided that a minimum threshold level of human capital is maintained in the domestic country.[33] However, their analysis also shows that if the country in question does not obtain the required threshold level of human capital, FDI exerts a negative impact on the economic growth prospects. Similar to the conclusion of Borensztein et al.,[34] Alfaro et al. (2006) argue that the availability and level of human capital is crucial in order for the positive impact of FDI on economic development to be realized.

[31] As a proxy for human capital stock Borensztein et al. (1995) utilized the initial level of average years of the male secondary schooling constructed by Barro and Lee (1993).

[32] ibid.

[33] According to the empirical findings, the threshold level of education in order for the influence of FDI on economic growth to be positive is 0,83. The analysis shows that 32 out of 69 industrializing countries are found to satisfy this requirement. See Borensztein et al. (1995) for details.

[34] ibid

Alfaro et al. (2003, 2006) examine the contribution of local financial markets in benefiting from the knowledge spillovers created by FDI. According to their findings, the maturity of financial markets has a significant impact on the relationship between FDI and economic development. In fact, Alfaro et al. (2006) note that the countries with well-functioning financial markets obtain growth rates nearly as twice as that of countries with poor financial markets. Higher FDI in a financially developed country leads to higher economic growth through the effect of backward linkages. In fact, developed financial markets enhance the absorptive capacity of the domestic agents leading to a stronger effect of FDI. Moreover, higher efficiency in the financial sector eliminates the additional costs of transaction leading to a higher social marginal product of human capital. (Alfaro et al., 2003) Nonetheless, according to their findings, even though FDI is positively associated with growth when financial markets are developed, the relationship is ambiguous when FDI is examined alone.

Carkovic and Levine (2002) suggest that the macroeconomic evidence on a positive relationship between FDI and economic growth should be examined with caution since the risk associated with simultaneity-bias, the use of lagged dependent variables and the country specific differences prevail in most of the empirical analyses. By controlling for the aforementioned factors, Carkovic and Levine[35] investigate the effect of FDI on economic growth. Their cross-country empirical findings do not support a positive influence of FDI per se on economic growth. Thus, their analysis concludes that FDI does not independently accelerate economic growth.

As can be seen, each empirical survey reaches a different conclusion about the growth effects of financial integration, in general, and FDI, in particular. The diversity in empirical findings stems from the fact that each analysis has different measurement techniques, different assumptions and different concerns. Given that, the theoretical discussions and empirical findings associated with the benefits and adverse effects of international capital flows do not provide a clear picture of the implications for economic growth. In order to understand whether the aforementioned impacts are realized in practice, it is informative to work on a model of real economy. In this way, it would be possible to trace the evolution of production taking place in various

[35] ibid.

sectors, consumption on different goods and the allocation of resources across sectors. Thus, a more concise and detailed analysis would be provided in models calibrated to real economies. In that sense, the next section will scrutinize the growth literature to obtain a pattern in modeling economies with open capital accounts.

CHAPTER 3

A REVIEW OF THE OPEN ECONOMY RAMSEY MODEL

The benefits and hazardous effects of capital account liberalization on economic growth have been discussed in theory and supported by empirical findings; yet, in practice, the implications can be seen more accurately in a model of real economy. The effort in the growth literature is to develop optimizing intertemporal models of open economies so that a comparison between financial autarky and liberalization can take place. The tendency is to use neoclassical growth models in an open economy framework.

The most frequently utilized model in the literature has been the Ramsey (1928) model of optimal consumption and saving. The Ramsey Model is a neoclassical model of economic growth in which infinitely lived households choose consumption and saving at each point in time in order to maximize their dynastic utility subject to an intertemporal budget constraint. The applications of this model into real data provides the opportunity to elaborate on consumption and saving decisions of households, the capital accumulation taking place in the country and the economic growth in the short-run and in the long-run. In order to apply these analyses to different environments, many authors extended the Ramsey Model such that it incorporates various dynamics of the economy which are not part of the original framework. [36]

[36] For instance, Cass (1965) uses a closed economy version of the Ramsey Model with one sector assuming constant returns to scale and diminishing marginal rate of substitution. The model shows that there is an optimal growth path which is unique. See Goodwin (1961) and Koopmans (1963) for further details in the initial extensions of the Ramsey model.

The Ramsey Model sets the framework to compare countries with different levels of economic development with respect to their relative convergence rates to their respective steady states. Within the bounds of a closed economy, the model has been used to show that there is conditional convergence among economies. According to the work of Barro et al. (1992, 1995), when the economy is closed to international capital flows and when the capital share is large enough, the model is consistent with the empirical evidence on convergence. Nonetheless, if capital flows are allowed, then the model encounters several difficulties.

3.1 Problems Related with the Open Economy Ramsey Model

As is recognized in Barro et al.[37] and Barro and Sala-i-Martin (2004), opening the standard Ramsey Model into international capital flows generates certain problems. There are three possible problems related with the open-economy version of the Ramsey Model.

First of all, when international capital flows are allowed in the economy, domestic saving ceases to determine capital formation all by itself since the availability of foreign saving also affects domestic investment. The access to foreign financial markets enable capital to move quickly between countries resulting in the equalization of marginal products of international capital. Thus, the return on capital will be set equal to the world interest rate which is constant based on the assumption that the world is in a steady state.[38] As is discussed in Barro and Sala-i-Martin,[39] if the initial level of the capital stock is lower than the steady state level of capital per head, then the model predicts a massive capital inflow at an infinite rate so that the gap is closed at once. Similarly, if the initial level of capital stock is higher than the steady state level, then there will be a huge capital outflow, again, leaving no gap. Therefore, the variables will jump to their steady-state levels at once. This implies that the level of capital per worker is equal to a constant since it will converge to its steady state

[37] ibid.

[38] The domestic economy is assumed to be smaller with respect to the world economy such that capital accumulation taking place in the domestic economy cannot influence the world interest rate. This is why the domestic country faces a constant world interest rate as it allows for international capital flows.

[39] ibid.

level immediately. Accordingly, the constancy in capital per worker will bring about constancy in other variables such as output per capita and the wage rate per unit of labor.

The findings of Barro and Sala-i-Martin,[40] imply that these results contradict with the gradual convergence discussions in the literature. According to Barro et al.[41] the closed economy version of the neoclassical growth model supports the empirical evidence on unconditional convergence for a homogenous group of economies provided that the capital share is large enough to slow down the diminishing returns to capital. However, when the economy is opened to financial flows, the convergence rate becomes infinite. This outcome is inconsistet with the empirical evidence. Consequently, the infinite convergence rate and the associated constancy in the variables of the model are the first problems encountered in the literature utilizing the standard Ramsey Model in an open economy framework.

The second problem associated with the model is that when there is a difference in the time preference rates of countries, the most patient country will obtain a high level of consumption per head, while that of an impatient country will approach zero. According to Barro and Sala-i-Martin,[42] when international financial flows are allowed in an impatient country with a high discount rate, the agents would use foreign borrowing in order to satisfy their current consumption needs in addition to financing their capital investments. Therefore, the country will end up having a low consumption growth in the future. In contrast, the countries with a lower discount rate tend to be more patient and obtain a higher consumption growth in the future which will be equal to the growth rate of world output. Similarly, Bhattarai (2005) discusses that economies with a low discount rate have a higher incentive to postpone their consumption so that they have higher steady state levels of output and consumption compared to economies that have higher discount rates. Thus, except for the most patient economy, all the open economies with high discount rates will experience low levels of consumption per head which eventually approaches zero.

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[40] ibid.

[41] ibid.

[42] ibid.

The third problem an open economy Ramsey Model faces is the fact that the net wealth of an impatient country will approach negative values over time. This stems from the fact that the country with a high discount rate would not only finance its capital stock by foreign debt, but also borrow to finance the present value of its wage income. Therefore, an impatient country loses all its capital and its labor income against foreign borrowing and becomes a net debtor. In contrast, the most patient country would become a net creditor and own all the wealth in the world with respect to claims on capital and present value of wage income as is discussed in Barro and Sala-i-Martin.[43] Therefore, the open economy Ramsey Model predicts an impatient country which suffers from eventual asset depletion.

The aforementioned problems limit the use of Ramsey Model in open economy frameworks since they obscure the results and lead to wrong conclusions about the impacts of free capital flows. There are many surveys in the growth literature which deal with the problems of open economy models. They not only investigate the reasons behind the failure of the Ramsey Model to incorporate the effects of capital flows, but also try to come up with remedies that will overcome those problems.

3.2 Remedies to the Problems of Open Economy Ramsey Model

As is argued in Barro and Sala-i-Martin (2004), the aforementioned 'counterfactual results' stem from the gap between the interest rate and the time-preference term. One of the basic differences between a closed economy and an open economy framework is the nature of the interest rate. In a closed economy Ramsey Model, the interest rate is determined in the domestic financial markets such that it is equal to the time-preference parameter at the steady state. However, in the open economy, the interest rate is exogenously taken as the world interest rate. This constancy of the interest rate creates a gap between the time-preference parameter and the interest rate which leads to the aforementioned problems of open economy models. According to Barro and Sala-i-Martin,[44] there are two solutions to this problem: either it is solved through an interest rate channel or it is solved through the time-preference channel.

[43] ibid.

[44] ibid.

They state that

> To avoid this result, we need some mechanism to eliminate the gap between r_i and $\rho_i+\theta_i x$ for all countries, not just for the most patient country. That is, either r_i has to differ from r, or else the effective rate of time preference, $\rho_i+\theta_i x$, has to be variable. (Barro and Sala-i-Martin, 2004:165)

In order to prevent the 'counterfactual results' in the open economy Ramsey Model three approaches have been adopted in the literature. These approaches mostly concentrate on eliminating the problem of infinite convergence speed. As the convergence speed slows down, the problems related with zero consumption and negative net wealth will also be solved accordingly. The first method to prevent the infinite speed of convergence is allowing for adjustment costs in investment. The second method is to endogenize the time preference parameter. The third approach is to find a way to endogenize the interest rate by making use of the assumption of non-tradable goods. When these solutions are implemented into the model, the capital stock will cease to be constant. Therefore, the adjustment of the capital stock to its steady state level will slow down which will cure the infinite speed of convergence problem.

3.2.1 Adjustment Costs in Investment

The presence of adjustment costs in the open economy version of the Ramsey Model slows down the instantaneous adjustment of the capital stock. As is discussed in Mulligan and Sala-i Martin (1992) the presence of adjustment costs in investment implies that the production possibility set is strictly convex.[45] Under the assumption of convex investment costs, an increase in the rate of investment raises the cost of investment, as well.

This approach has gained popularity in open economy Ramsey Models one of which is developed by Cohen and Sachs (1985). They examine an open economy version of the Ramsey Model with convex investment costs and collateral constraint to

[45] Mulligan and Sala-i Martin (ibid.) state that the relationship between strictly convex production possibility set and adjustment costs is rooted in the firm behavior of smoothing investment over time.

borrowing.[46] Solving the model by dynamic programming, they find that the borrower economy grows in two stages. In the first stage, the capital mobility is perfect so that there is no constraint on borrowing which causes the debt to grow more rapidly than the economy. However, in the second stage, a borrowing constraint is introduced; therefore, the debt and the economy grows at the same rate which is lower than the first stage. The problem related with this open economy Ramsey Model is the fact that it only provides a closed-form solution to linear production functions and the analytical solution is a bit complicated.

According to the model of Barro and Sala-i-Martin (2004) when there are two kinds of capital as factors of production, namely the physical and human capital, accumulation of human capital would be subject to adjustment costs. This is due to the fact that education process requires time and it cannot be replaced once installed. Therefore, learning process is associated with rapid diminishing returns. Their model predicts a saddle-path stable system which cures the infinite convergence speed. That is, if the initial human capital stock is below its steady state level, the accumulation cannot be instantaneous since an infinite investment in human capital would require a huge adjustment cost at once which is not optimal. Therefore, convergence to the steady state is slow and gradual.

A much more recent model has been developed by Brock (2009). Similar to Cohen and Sachs' framework, his model is an open economy Ramsey Model which assumes convex investment costs and imperfect mobility of foreign capital. Due to adjustment costs, capital is fixed in the short run which prevents lenders to claim the capital as repayment. The difference of this model is the fact that the amount of present and future capital income can be utilized as collateral; whereas, present and future labor income cannot be.[47] The analysis shows that if the collateral constraint is binding, then all the capital stock will be financed by foreign debt which is completely

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[46] Since perfect capital mobility would put the borrower in a position where the country is indifferent between debt repayment and repudiation, the model incorporates a borrowing constraint. According to the model, the country is assumed to use one kind of capital, but it can use only a fraction of this capital as collateral against foreign borrowing. The constraint on borrowing will be $D_{t+1} \leq h(K_{t+1})$, where D_{t+1} denoting the principal repayments due at time t+1, $h \in [0, 1]$ is the lending limit which keeps the country from defaulting and K_{t+1} denoting the installed capital at time t+1.

[47] The imperfect mobility of international capital is of the form $b_t \leq a\, q_t\, k_t$ where b is debt, a is a fraction, $0 \leq a \leq 1$, q_k is the value of the capital stock at time t. For details, see Brock (2009).

collateralized by future capital income. However, if the borrowing constraint is not binding, consumption will be equal to permanent income. The findings of the model show that the dynamics of the model are the same as in a standard Ramsey Model; however, the fraction of the capital stock that can be used as collateral changes the speed of adjustment of the economy.

3.2.2 Variable Time Preference Parameter

Apart from the aforementioned models, there is another method to solve for the problems of open economy version of the Ramsey model. Instead of working with convex investment costs, the model can be transformed in a way so that the rate of time preference ceases to be exogenous to the system. If the preference parameters are endogenized to the system and allowed to vary, then the gap between the interest rate and the time preference parameters is closed.

Uzawa (cited in Barro and Sala-i-Martin, 2004:177)[48] utilizes the idea of variable time preference parameters leading to an inspiration to other authors.[49] However, Francis and Kompas (2001) claim that Uzawa's model does not consider the non-linearities resulting from the time variable so that it can only be applicable in an autonomous system.

Barro and Sala-i-Martin (2004) do not agree with the outcome of the Uzawa model as well. The problem in open economy version of the Ramsey model with constant time preference parameters is the fact that countries with high rates of time preference eventually end up with zero consumption and negative assets per capita. In order to solve this problem the time preference rate should be allowed to decline as consumption and assets decline. Uzawa's time preference rate is a positive function of current and future consumption, therefore, it ensures the required result. However, according to Barro and Sala-i-Martin[50] it is not likely for people to become impatient

[48] For details of the framework, see Uzawa (1968)

[49] He builds a model under the assumption that the rate of time preference is a function of current and future consumption. However, the introduction of time into the framework requires a method to deal with two state variables. Thus, Uzawa suggests reducing the dimension of the problem such that the rate of time preference can be treated as a constant. (Francis and Kompas, 2001:3)

[50] ibid.

when their consumption levels rise. Therefore, the utilization of variable time preference does not offer a valid solution to the aforementioned problems. There are many other models that make use of Uzawa's framework, but they are beyond the scope of this work.

3.2.3 Non-Tradable Investment Goods

Another method to deal with the open economy Ramsey model is to increase the number of sectors in the economy and make the investment goods sector closed so that the return on capital in the closed sector is endogenized. The pioneering model using more than one sector is Uzawa's model (1961)[51] which inspired many economists working with the Ramsey Model.

Bade (1972) analyzes the optimal capital accumulation of an economy allowing for international capital mobility. The model first assumes perfect capital mobility, then it puts a constraint on the capital flows and compares the implications of an imperfect capital mobility to a perfect one. The financial markets are such that the country can borrow with a constraint, but it cannot lend abroad.[52] Moreover, the interest rate is assumed to be a function of the amount that has been borrowed. The results show that when there is no restriction on capital mobility, the optimal path is achieved with the instantaneous adjustment of the foreign capital stock. On the contrary, if foreign borrowing is constrained, the adjustment of foreign capital ceases to be decisive on its own. This comparison shows that optimal policy requirement and optimal paths change when international capital mobility is restricted.

Barro et al. (1992, 1995) examine an open economy version of the Ramsey Model which provides an explanation for the empirical evidence on convergence. In order

[51] Instead of a Ramsey Model, Uzawa (1963) worked on the Solow Model and assumed two sectors in the economy. These sectors produce investment goods and consumption goods respectively. Under the neoclassical hypotheses, the model shows that a steady state growth exists in a two-sector framework and it is globally stable. Thus, if the economy starts from an initial capital and labor allocation that is below the steady state, it will achieve a steady state growth. Uzawa (1963) then changes the assumption of saving and extends the model in which the propensity to save is a function of the interest rate and the current gross income per capita. The model shows that short-run equilibrium exists and the long-run growth is stable.

[52] The economy uses two factors of production and produces one good which can be either consumed or invested. In the model it is assumed that the repayment of the debt by physical capital is not possible. Moreover, the lending country has a limited supply of capital goods and the transportation facilities are also limited.

to confirm the gradual convergence argument, they analyze the Ramsey Model, first in a closed economy framework, then in an open economy framework with perfect and partial capital mobility. The model utilizes two types of capital, namely physical capital and human capital both of which are owned by the households. There is also a net stock of debt which is zero when the economy is closed and can be positive or negative when the economy is open.

If the economy is closed to capital flows, the Ramsey model provides consistent results with the convergence literature provided that the broad capital share is sufficiently high. When the economy is opened to international capital flows, there are two possibilities that should be considered. Either, the financial markets allow for perfect capital mobility or they allow for imperfect capital mobility. In the case when international capital is perfectly mobile, under the assumption of a small country, the interest rate in the economy is equal to the world interest rate which is taken as a constant when the world is in a steady state condition. The net returns on the two kinds of capital are equal to the world interest rate in this case. However, the constancy of the interest rate causes the values of physical capital, human capital and output per capita to be constant. Thus, as already mentioned above, perfect capital mobility leads to the variables to jump to their steady state levels instantaneously. The infinite convergence rate is not consistent with what the convergence literature claims.

Consequently, Barro et al.[53] modify the open economy version of the Ramsey model by assuming that the capital is partially mobile. In this framework, physical capital is allowed to be financed by foreign borrowing; however, human capital can only be financed by domestic assets. Thus, the amount of foreign debt should be equal to or less than the quantity of physical capital. As is argued by the authors, this assumption implies an asymmetry between physical and human capital. In other words, human capital cannot be used as collateral against foreign borrowing so that foreign agents cannot own human capital.[54] Under these assumptions, the return on physical capital is equal to the world interest rate since it is used as collateral against foreign debt.

[53] ibid.

[54] At this point the authors remind the reader of the difference between human and physical capital and state that the distinction is made on the basis that one is used as collateral and the other is not.

However, the return on human capital is determined in the domestic market hence it is endogenous to the system. The aim of the authors to find an open economy Ramsey model that is consistent with the empirical literature is achieved since the partial capital mobility model offers a finite speed of convergence.

Gourinchas and Jeanne (2002) utilize an open economy version of the Ramsey model with two types of capital stocks in order to show that countries with little capital do not benefit from capital account liberalization contrary to the traditional line of thinking suggests. In order to benefit from international capital flows in terms of higher welfare gains, the countries in question should be away from their steady states. This can be achieved by higher capital shares in production. Thus, Gourinchas and Jeanne,[55] following Mankiw et al. (1992)[56] introduce human capital into the model. As in Barro et al. (1992, 1995), human capital cannot serve as collateral against foreign borrowing since it should be accumulated domestically. Similar to the results of previous studies, this model predicts that the convergence to the steady state is finite, even though many countries are still close to their respective steady states.

Having discussed these three different methods, this thesis will adopt the third approach and introduce non-tradable investment goods into the framework in the fashion of Barro et al.[57] in order to overcome the problematic features of the open economy version of the Ramsey model. The previous two methods are not followed since they do not match well with the framework and analysis of this thesis. In particular, the introduction of adjustment costs into the model complicates the analytical solution, while allowing for variable time preference parameters provide an analysis for comparing multiple countries. Since in this thesis the focus is on the changes taking place within the country with respect to production, consumption and resource allocation across sectors, following the third method and introducing non-tradable investment goods into the economy will be more suitable for the solution.

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[55] ibid.

[56] The idea of adding human capital into the model in order to raise the capital share in the economy comes from Mankiw et al. (1992). They argue that including human capital leads to a change in the theoretical modelling or the empirical results of economic growth. They use human capital in the Solow model and conclude that the inclusion of human capital solves the problem of high coefficients on investment and population growth caused by a low share of capital. In fact, the augmented model, according to their work, fits the cross-country data contrary to the case where only physical capital enters into the model.

[57] ibid.

CHAPTER 4

THE CLOSED ECONOMY MODEL

This thesis will examine the effects of international capital flows on the economy by focusing on the allocation of resources between three sectors. It is informative to work with a multi-sector growth model since a one sector environment offers a very limited analysis on the effects of financial integration. In models of real economy, not only the presence of free international capital flows, but also the composition of them are decisive in economic growth.

As is discussed in Aykut and Sayek (2007) the impact of financial liberalization is different in each sector. Aykut and Sayek[58] take financial liberalization as foreign direct investment (FDI) and elaborate on the sectoral composition of FDI flows. Their analysis shows that not only the level of FDI but also the sectoral composition of it has significant influence on economic growth. The empirical findings show that FDI inflows which are in favor of the primary sector have an insignificant and a negative effect on economic growth.[59] Similarly, if FDI is channeled to the services sector, then economic growth would be negatively affected.[60] On the other hand, when the FDI inflows accumulate in the manufacturing sector, they would exert a significant and positive impact on economic growth.[61]

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[58] ibid

[59] Generally, the capital flows to the primary sector includes big projects which causes the domestic country to lose competitiveness in other sectors. This crowding-out effect might cause alterations in the domestic market structure which might result in institutional breakdown through rent-seeking activities. Moreover, the FDI attracting projects do not have strong backward and forward linkages which reduces the gains from FDI. For details, see Aykut and Sayek (ibid).

[60] But, Aykut and Sayek (ibid) also add that the subsector that receives investment might change the results.

[61] Investment in manufacturing sector usually leads to an improvement in domestic technology and leads to knowledge spillovers. Moreover, foreign firms use domestic intermediate goods which creates strong backward

In short, sector specific factors would influence the growth effect of capital flows. Therefore, if financial liberalization is considered in the context of multiple sectors, the analysis will pose more realistic conclusions. In order to conduct a thorough analysis, the effects of financial integration will be discussed in a three-sector Ramsey growth model. In this framework, production takes place in three sectors. The first sector is the home-goods sector which produces agricultural and manufacturing output only for the domestic market. The sector is assumed to provide pure consumption goods which are available for domestic trade. For this sector, the market clears within the economy so that the price of its goods is endogenized. The second sector is the tradable-goods sector which produces all the goods and services that are available to international trade at world prices. The output of the sector includes not only exportable goods and services but also importable goods and services at a given price level. The third sector is home-services sector and it only serves for the domestic market. The output of this sector is not used in international trade; therefore, the relative prices of the goods are determined within the domestic market.

The procedure to be followed is to compare and contrast the movements in factors of production between sectors. In fact, it is informative to scrutinize the changes in the factors of production recognized in various sectors. In a closed economy framework with only one sector, changes in the capital stock are recognized by investment. The required investment is afforded by lower consumption and higher saving of the domestic agents. On the other hand, in an open economy framework with more than one sector, the required investment is not necessarily financed by decreased consumption and increased saving. It might, as well, leave consumption smooth and use international capital or use the investment opportunities of the other sectors. Thus, as Mulligan and Sala-i Martin (1992) stated as well;

> It would therefore be interesting to know under what conditions agents rebuild their stock of physical capital by substituting away from other capital goods rather than by substituting away from consumption. (Mulligan and Sala-i Martin, 1992:2)

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linkages. This also brings about employment opportunities. Apart from the positive efficiency effects, FDI inflows to the manufacturing sector also improve exports which relaxes the balance of payments constraints. Thus, according to Aykut and Sayek (ibid), if manufacturing shares of FDI is increased, a significant and positive effect on economic growth would take place

The analysis will take place, first in a closed economy environment, then in an open economy framework. In the first environment, a small economy which is open to international trade but closed to international capital flows will be assumed. This closed economy will serve as a benchmark model to acknowledge the allocation of resources when there is financial autarky. In that framework, the transition and steady state behaviors of the variables will be analyzed. Then, in the second environment, free international capital flows will be allowed and a small open economy version of the Ramsey model will be built. The open economy will include both international trade and capital flows. In this framework, only the comparison between the inital period and the steady state will be provided. The transition behaviors of the variables will be covered in a further study. A comparison between these two environments will offer many conclusions about the impacts of international capital flows on different sectors, on the allocation of resources and finally on economic growth.

4.1 The Theory

This part of the thesis will describe a small closed economy with three sectors.[62] The framework will be set by introducing the assumptions related with the sectors in the economy, production technologies and household pereferences. The environment is such that it allows for international trade in goods and services but not for international capital flows. Thus, the economy in question is closed to international capital flows, not to trade in goods and services. There are three sectors in the economy which have already been introduced above. The home-goods sector produces agricultural and manufacturing goods for the domestic market and it does not allow for international trade or capital flows. Thus, the market clears within the economy with an endogenous price level. The tradable-goods sector produces all the internationally tradable goods in the economy at world prices. The outputs of this sector are not only for domestic and foreign consumption purposes, but they are also used in capital accumulation. On the other hand, the home-services sector serves only for domestic consumption and clears within the economy. In each sector, firms are perfectly competitive both in goods markets and in factors markets.

[62] In solving the three-sector Ramsey Model, the procedure of Roe et al. (forthcoming) will be followed.

In all these sectors, output is produced with the use of three kinds of factors of production at each period. Following Barro et al. (1992, 1995) and Barro and Sala-i-Martin (2004), the production function includes raw labor, physical capital and human capital in all sectors. The two kinds of capital are assumed to be perfectly reversible with no adjustment costs. All factors of production are perfectly mobile between sectors. However, the mobility of raw labor or financial capital between countries is not allowed. There is no government, technological change or population growth in the economy.

4.1.1 Households

The model assumes identical infinitely-lived households each of which has the same preference parameters, same amount of assets and obtains the same wage rate. Given these assumptions, this model will use a representative household analysis. The household supplies raw labor, physical capital and human capital to all sectors in the economy in exchange for wage compensation and interest income on assets. The income will be spent on accumulation of broad capital and consumption in three types of goods: home-goods, tradable-goods and home-services. The representative household faces two types of problems: an intertemporal problem and an intra-temporal problem.

In the intertemporal problem, the household chooses the amount of consumption and saving at each point in time in order to maximize the present value of discounted intertemporal utility subject to his/her budget constraint. The utility function of the household has a CIES [63] form and it is given as

$$U = \int_0^\infty \frac{c(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} \, dt \tag{4.1}$$

where $c(t)$ represents the composite intra-temporal consumption per capita, $\frac{1}{\theta}$ is the inter-temporal elasticity of substitution and ρ is the time preference parameter.

[63] CIES is the abbreviation for Constant Intertemporal Elasticity of Substitution

Since the economy is closed to international capital flows, households are not allowed to borrow from abroad to finance consumption or saving. Thus, foreign debt is zero which can be denoted as $d = 0$. The budget constraint of the household is as follows[64]

$$\dot{z}(t) = w(t) + r(t)\, z(t) - E(c(t), p(t)) \tag{4.2}$$

where $z(t)$ is the broad capital per head, $w(t)$ is the wage rate, $r(t)$ is the domestic return to a unit of broad capital, $E(c(t), p(t))$ is the aggregate household expenditure on three consumption goods and $p(t)$ is a vector of output prices in three sectors.

In the intra-temporal problem, the household chooses the allocation of his/her consumption between three consumption goods. Thus, the amount of expenditure on each consumption good is considered intra-temporally. The representative household faces an instantaneous composite consumption function which has a Cobb-Douglas form as follows

$$c = \beta\, c_1^{\gamma_1}\, c_2^{\gamma_2}\, c_3^{\gamma_3} \tag{4.3}$$

where c is the composite consumption per capita, $\beta > 0$ is a scaling constant,[65] $c_1 > 0$ is the home-good consumption per capita, $c_2 > 0$ is the tradable-good consumption per capita, $c_3 > 0$ is the home-services consumption per capita and $\gamma_1 + \gamma_2 + \gamma_3 = 1$. Based on this composite consumption function, the representative household minimizes his/her expenditure at each period in time by choosing a consumption bundle composed of three types of goods.

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[64] The original budget constraint used in Barro et al. (1992, 1995) is

$$\dot{h} + \dot{k} - \dot{d} = w + (R_k - n - g - \delta)k - (r - n - g)d + (R_h - n - g - \delta)h - c$$

where $\dot{h}$, $\dot{k}$ and $\dot{d}$ are changes in human capital, physical capital and foreign debt per capita respectively, r is the real interest rate, R_k and R_h are the gross returns to physical and human capital respectively, n is population growth rate, g is technological growth rate and δ is the depreciation rate in both capital stocks. When the conditions for the closed economy environment, namely $d = 0$ and $k + h = z$, are substituted into this equation, the resulting budget constraint will give the budget constraint used in this thesis.

[65] In order to avoid algebraic complexity, β is set at $\beta \equiv \gamma_1^{-\gamma_1}\, \gamma_2^{-\gamma_2}\, \gamma_3^{-\gamma_3}$.

4.1.2 Production Technologies

4.1.2.1 Home-Goods Sector:

The production function in each sector are assumed to be of Cobb-Douglas type functions fulfilling the constant-returns-to-scale property. In particular, the production function in home-goods sector is given by:

$$Y_1 = A_1 L_1^{\alpha_1} K_1^{\alpha_2} H_1^{1-\alpha_1-\alpha_2} \tag{4.4}$$

where Y_1 is the aggregate home-good, L_1, K_1 and H_1 are the aggregate raw labor, physical capital and human capital inputs respectively, $\alpha_1, \alpha_2, 1-\alpha_1-\alpha_2 \in (0,1)$ are raw labor, physical capital and human capital shares in production and $A_1 > 0$.

In the closed economy environment, agents can borrow and lend at the real interest rate in the domestic market. Since international capital flows are not allowed in this framework, foreign debt is not available and the interest rate is determined domestically. Thus, the returns on both kinds of capital should be equal to the domestic interest rate at the equilibrium. As a result of this equality in rates of return, the ratio of physical capital to human capital is constant at all points in time which is given as

$$\frac{K_1}{H_1} = \frac{\alpha_2}{1-\alpha_1-\alpha_2} \tag{4.5}$$

Since the rates of return on both kinds of capital are the same and there are no adjustment costs or irreversibility constraints, it is possible to define a broad capital stock instead of two kinds of capital in a closed economy framework following Barro et al. (1992, 1995). Rearranging the production function accordingly, the home-goods sector production function in a closed economy environment is obtained as

$$Y_1 = \tilde{A}_1 L_1^{\alpha_1} Z_1^{1-\alpha_1} \tag{4.6}$$

where Z_1 is the broad capital stock which includes both physical and human capital and $\tilde{A}_1 > 0$ is assumed to be set at $\tilde{A}_1 \equiv A_1 (1-\alpha_1)^{\alpha_1-1} \alpha_2^{\alpha_2} (1-\alpha_1-\alpha_2)^{1-\alpha_1-\alpha_2}$ for notational and algebraic simplicity.[66]

[66] For details in derivation, see Appendix (A.1.1).

4.1.2.2 Tradable-Goods Sector:

The tradable good in the second sector is produced using a Cobb-Douglas production function with constant-returns-to-scale property. The same approach used in home-goods sector will be adopted in this sector, as well, in order to obtain the production function. Even though, the actual production technology includes raw labor, physical capital and human capital stocks separately, the equivalance in the rate of returns on both kinds of capital stock makes it available to define them as a composite capital stock[67]. Thus, the production function of tradable-goods sector is as follows:

$$Y_2 = \tilde{A}_2 \, L_2^{\beta_1} \, Z_2^{1-\beta_1} \tag{4.7}$$

where L_2 is aggregate raw labor, Z_2 is the broad capital stock which includes both physical and human capital and $\tilde{A}_2 > 0$ is a scaling factor.[68]

4.1.2.3 Home-Services Sector:

The home-services good in the third sector is produced using a Cobb-Douglas production function with constant-returns-to-scale property. The same approach used in the previously mentioned sectors will be adopted in this sector, as well, in order to obtain the production function. Even though, the actual production technology includes raw labor, physical capital and human capital stocks separately, the equivalance in the rate of returns on both kinds of capital stock makes it available to define them as a composite capital stock [69]. Thus, the production function of home-services sector is as follows:

$$Y_3 = \tilde{A}_3 \, L_3^{\delta_1} \, Z_3^{1-\delta_1} \tag{4.8}$$

where L_3 is aggregate raw labor, Z_3 is the broad capital stock which includes both physical and human capital and $\tilde{A}_3 > 0$ is a scaling factor.[70]

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[67] For details in derivation, see Appendix (A.1.2).

[68] $\tilde{A}_2$ is assumed to be set at $\tilde{A}_2 = A_2 (1-\beta_1)^{\beta_1 - 1} \beta_2^{\beta_2} (1-\beta_1-\beta_2)^{1-\beta_1-\beta_2}$ in order to eliminate notational complexity.

[69] For details in derivation, see Appendix (A.1.3).

[70] $\tilde{A}_3$ is assumed to be set at $\tilde{A}_3 = A_3 (1-\delta_1)^{\delta_1 - 1} \delta_2^{\delta_2} (1-\delta_1-\delta_2)^{1-\delta_1-\delta_2}$ in order to eliminate notational complexity.

4.2 Competitive Equilibrium

This part will describe and list the conditions and requirements of a competitive equilibrium in the closed economy model.

Definition 4.2.1 *An equilibrium in this model is a list of sequences of household consumption plans* $\{c_1(t), c_2(t), c_3(t)\}_{t=0}^{\infty}$, *for each consumption item, production plans* $\{y_1(t), y_2(t), y_3(t)\}_{t=0}^{\infty}$, *for each production sector, factor plans* $\{z_j(t), l_j(t)\}_{t=0}^{\infty}$, *for each sector j, where* $j = 1, 2, 3$, *output prices* $\{p_1(t), p_2(t), p_3(t)\}_{t=0}^{\infty}$, *for each consumption good, wage rates* $\{w(t)\}_{t=0}^{\infty}$, *broad capital rental rates* $\{r(t)\}_{t=0}^{\infty}$, *and an initial condition* $z(0) \leq z_0$ *such that*

i) given z_0, $p_1(t), p_2(t), p_3(t)$, $w(t)$ *and* $r(t)$, *the sequence* $\{c_1(t), c_2(t), c_3(t)\}_{t=0}^{\infty}$, *minimizes the representative household's expenditure and maximizes the present value of his/her discounted intertemporal utility;*

ii) given z_0, $p_1(t), p_2(t), p_3(t)$, $w(t)$ *and* $r(t)$, *the sequence* $\{y_j(t), z_j(t), l_j(t)\}_{t=0}^{\infty}$ *maximize profits for each sector j,* $j = 1, 2, 3$*;*

iii) broad capital market clears;

iv) raw labor market clears;

v) the market for home-goods clears;

vi) the market for home-services clears;

vii) Walras' Law holds: $p_2 x - p_2 m = 0$, *where x is the export per capita of tradable-goods, m is the import per capita of tradable-goods such that* $x = y_2$, *and* $m = c_2 + \dot{z}$.

The model above assumes that the markets for home-goods and home-services clear within the domestic economy so that there are no international trade taking place associated with these goods or services. On the other hand, the tradable-goods sector is open to international trade; therefore, the sector incorporates both export goods and import goods. This condition requires the trade balance to take place in tradable-goods sector. Consequently, any excess supply or demand in export goods must be matched by an excess demand or supply in import goods within the tradable goods sector. In the closed economy model, it is assumed that all domestic production in tradable goods sector is exported. The sector also encompasses the imported goods which are used in household consumption and in the accumulation of broad capital. Thus, exports are equal to imports at each point in time eliminating any trade imbalances.

4.2.1 Household's Intra-Temporal Problem

Households choose bundles of consumption goods that minimize their expenditures subject to their composite consumption at each point in time. Given the output prices, the representative household faces the following minimization problem

$$\begin{aligned} \min \quad & p_1\, c_1 + p_2\, c_2 + p_3\, c_3 \\ \text{subject to} \quad & c = \beta\, c_1^{\gamma_1}\, c_2^{\gamma_2}\, c_3^{\gamma_3} \\ & c_1 > 0 \\ & c_2 > 0 \\ & c_3 > 0 \end{aligned}$$

The solution to the above problem yields the conditional consumption demands of each good in the economy as follows:[71]

$$c_1 = \gamma_1\, c\, p_1^{\gamma_1 - 1}\, p_2^{\gamma_2}\, p_3^{\gamma_3} \tag{4.9}$$

$$c_2 = \gamma_2\, c\, p_1^{\gamma_1}\, p_2^{\gamma_2 - 1}\, p_3^{\gamma_3} \tag{4.10}$$

$$c_3 = \gamma_3\, c\, p_1^{\gamma_1}\, p_2^{\gamma_2}\, p_3^{\gamma_3 - 1} \tag{4.11}$$

Substituting these consumption demands into the expenditure function, the minimized total expenditure of the representative household becomes

$$E\,(c, p) = c\, p_1^{\gamma_1}\, p_2^{\gamma_2}\, p_3^{\gamma_3} \tag{4.12}$$

4.2.2 Household's Intertemporal Problem

Households choose the amount of saving and consumption that maximizes their inter-temporal utility subject to their budget constraint, initial conditions and the

[71] For details in derivation, see Appendix (A.2).

transversality condition. The maximization problem of the representative household is as follows:

$$\begin{aligned} \max \quad & \int_0^\infty \frac{c^{1-\theta}-1}{1-\theta}\, e^{-\rho t}\, dt \\ \text{subject to} \quad & \dot{z} = w + r\,z - E\,(c,p) \\ & z\,(0) \le z_0, \quad c\,(t) \ge 0 \\ & \lim_{t\to\infty} z(t)\, e^{-\int_0^t r(v)\,dv} = 0 \end{aligned}$$

The present-value Hamiltonian is

$$H = \frac{c^{1-\theta}-1}{1-\theta}\, e^{-\rho t} + \nu\Big(w + r\,z - c\; p_1^{\gamma_1}\; p_2^{\gamma_2}\; p_3^{\gamma_3}\Big)$$

where the expression in paranthesis is equal to $\dot{z}$ and $E\,(c,p) = c\; p_1^{\gamma_1}\; p_2^{\gamma_2}\; p_3^{\gamma_3}$ is the minimized expenditure function of (4.12). The first order conditions yield

$$\frac{\partial H}{\partial c} = c^{-\theta}\, e^{-\rho t} - \nu\; p_1^{\gamma_1}\; p_2^{\gamma_2}\; p_3^{\gamma_3} = 0$$

$$\Rightarrow \qquad \nu = \frac{c^{-\theta}\; e^{-\rho t}}{p_1^{\gamma_1}\; p_2^{\gamma_2}\; p_3^{\gamma_3}} \tag{4.13}$$

$$\frac{\partial H}{\partial z} + \dot{\nu} = \nu\, r + \dot{\nu} = 0$$

$$\Rightarrow \qquad -\frac{\dot{\nu}}{\nu} = r \tag{4.14}$$

Taking logarithms on both sides of (4.13) and taking time derivatives yield

$$-\frac{\dot{\nu}}{\nu} = \theta\,\frac{\dot{c}}{c} + \rho + \gamma_1\,\frac{\dot{p}_1}{p_1} + \gamma_2\,\frac{\dot{p}_2}{p_2} + \gamma_3\,\frac{\dot{p}_3}{p_3} \tag{4.15}$$

Substituting (4.14) into (4.15), the condition becomes

$$r = \theta\,\frac{\dot{c}}{c} + \rho + \gamma_1\,\frac{\dot{p}_1}{p_1} + \gamma_2\,\frac{\dot{p}_2}{p_2} + \gamma_3\,\frac{\dot{p}_3}{p_3} \tag{4.16}$$

Rearranging, the Euler Equation can be expressed as

$$\frac{\dot{c}}{c} = \frac{1}{\theta}\left(r - \rho - \gamma_1\,\frac{\dot{p}_1}{p_1} - \gamma_2\,\frac{\dot{p}_2}{p_2} - \gamma_3\,\frac{\dot{p}_3}{p_3}\right) \tag{4.17}$$

Since the tradable-goods sector is open to international trade, the price of tradable-goods is taken as constant at world prices. Therefore, there is no change in p_2 in time which means that $\frac{\dot{p}_2}{p_2} = 0$. Including this condition into (4.17), the Euler Equation in the closed economy model becomes

$$\frac{\dot{c}}{c} = \frac{1}{\theta}\left(r - \rho - \gamma_1\,\frac{\dot{p}_1}{p_1} - \gamma_3\,\frac{\dot{p}_3}{p_3}\right) \tag{4.18}$$

4.2.3 Firms' Optimization Problems

The firms in each sector face the problem of minimizing their costs and maximizing their profits at each point in time given output and input prices. When the cost minimizing values of factors of production are obtained, they will be used in the profit maximization problem so as to find the profit maximizing level of output in each sector.

4.2.3.1 Home-Goods Sector:

Firms in home-goods sector choose factors of production, namely Z_1 and L_1, that minimize their costs subject to their production functions given the output and input prices. The cost minimization problem they face is as follows:

$$\begin{aligned} \min \quad & w L_1 + r Z_1 \\ \text{subject to} \quad & Y_1 \leq \tilde{A}_1 \, L_1^{\alpha_1} \, Z_1^{1-\alpha_1} \\ & L_1 \geq 0 \\ & Z_1 \geq 0 \end{aligned}$$

The solution to this problem gives the cost minimizing values of Z_1 and L_1. Using this information in the total cost function, the minimized total cost is obtained as[72]

$$TC_1^* = Y_1 \, \frac{w^{\alpha_1} \, r^{1-\alpha_1}}{a_1} \tag{4.19}$$

Having obtained the minimized total cost, the firms face the problem of profit maximization. Given the domestic price of home-goods, p_1 and the cost minimizing values of Z_1 and L_1, the firms choose Y_1 to maximize their profits at all points in time. The profit maximization problem is as follows:

$$\max_{Y_1} p_1 \, Y_1 - TC_1^* \tag{4.20}$$

[72] For a detailed derivation, see Appendix (A.3.1).

Substituting (4.19) into (4.20), the profit maximization condition for the home-goods sector is generated as

$$p_1 = \frac{w^{\alpha_1}\, r^{1-\alpha_1}}{a_1} \tag{4.21}$$

which is the well-known condition

$$MC_1(w, r) = p_1 \tag{4.22}$$

where MC_1 is the marginal cost of production in home-goods sector and p_1 is the relative price of home-goods.

4.2.3.2 Tradable-Goods Sector:

Firms in tradable-goods sector choose factors of production, namely Z_2 and L_2, that minimize their costs subject to their production functions given the output and input prices. The cost minimization problem they face is as follows:

$$\begin{aligned} \min \quad & w\,L_2 + r\,Z_2 \\ \text{subject to} \quad & Y_2 \leq \tilde{A}_2\, L_2^{\beta_1}\, Z_2^{1-\beta_1} \\ & L_2 \geq 0 \\ & Z_2 \geq 0 \end{aligned}$$

The solution to this problem gives the cost minimizing values of Z_2 and L_2. Using this information in the total cost function, the minimized total cost is obtained as[73]

$$TC_2^* = Y_2\, \frac{w^{\beta_1}\, r^{1-\beta_1}}{a_2} \tag{4.23}$$

Having obtained the minimized total cost, the firms face the problem of profit maximization. Given the domestic price of home-goods, p_2 and the cost minimizing values of Z_2 and L_2, the firms choose Y_2 to maximize their profits at all points in time. The profit maximization problem is as follows:

$$\max_{Y_2}\, p_2\, Y_2 - TC_2^* \tag{4.24}$$

[73] For a detailed derivation, see Appendix (A.3.2).

Substituting (4.23) into (4.24), the profit maximization condition for the home-goods sector is generated as

$$p_2 = \frac{w^{\beta_1}\ r^{1-\beta_1}}{a_2} \tag{4.25}$$

which is the well-known condition

$$MC_2(w, r) = p_2 \tag{4.26}$$

where MC_2 is the marginal cost of production in tradable-goods sector and p_2 is the relative price of tradable-goods.

4.2.3.3 Home-Services Sector:

Firms in home-services sector choose factors of production, namely Z_3 and L_3, that minimize their costs subject to their production functions given the output and input prices. The cost minimization problem they face is as follows:

$$\begin{aligned} \min \quad & w\,L_3 + r\,Z_3 \\ \text{subject to} \quad & Y_3 \leq \tilde{A}_3\ L_3^{\delta_1}\ Z_3^{1-\delta_1} \\ & L_3 \ \geq\ 0 \\ & Z_3 \ \geq\ 0 \end{aligned}$$

The solution to this problem gives the cost minimizing values of Z_3 and L_3. Using this information in the total cost function, the minimized total cost is obtained as[74]

$$TC_3^* = Y_3\ \frac{w^{\delta_1}\ r^{1-\delta_1}}{a_3} \tag{4.27}$$

Having obtained the minimized total cost, the firms face the second problem of profit maximization. Given the domestic price of home-goods, p_3 and the cost minimizing values of Z_3 and L_3, the firms choose Y_3 to maximize their profits at all points in time. The profit maximization problem is as follows:

$$\max_{Y_3} p_3\ Y_3 - TC_3^* \tag{4.28}$$

[74] For a detailed derivation, see Appendix (A.3.3).

Substituting (4.27) into (4.28), the profit maximization condition for the home-goods sector is generated as

$$p_3 = \frac{w^{\delta_1}\ r^{1-\delta_1}}{a_3} \tag{4.29}$$

which is the well-known condition

$$MC_3(w, r) = p_3 \tag{4.30}$$

where MC_3 is the marginal cost of production in home-services sector and p_3 is the relative price of home-services.

4.2.4 Market Clearing Conditions

The profit maximization conditions for the three sectors yield the following equalities

$$p_1 = \frac{w^{\alpha_1}\ r^{1-\alpha_1}}{a_1}$$
$$p_2 = \frac{w^{\beta_1}\ r^{1-\beta_1}}{a_2}$$
$$p_3 = \frac{w^{\delta_1}\ r^{1-\delta_1}}{a_3}$$

The output of tradable-goods sector will be taken as the numeriare; therefore, p_2 will be set at $p_2 \equiv 1$. Solving these conditions together, the intra-temporal equilibrium values of the endogenous variables in the model will be obtained as functions of the relative price of home-services sector.

$$\mathbf{w}\ (p_3) = \frac{(p_3\, a_3)^{\frac{1-\beta_1}{\delta_1-\beta_1}}}{(a_2)^{\frac{1-\delta_1}{\delta_1-\beta_1}}} \tag{4.31}$$

$$\mathbf{r}\ (p_3) = \frac{(a_2)^{\frac{\delta_1}{\delta_1-\beta_1}}}{(p_3\, a_3)^{\frac{\beta_1}{\delta_1-\beta_1}}} \tag{4.32}$$

$$\mathbf{p_1}\ (p_3) = \frac{(p_3\, a_3)^{\frac{\alpha_1-\beta_1}{\delta_1-\beta_1}}\ (a_2)^{\frac{\delta_1-\alpha_1}{\delta_1-\beta_1}}}{a_1} \tag{4.33}$$

At the competitive equilibrium, the markets for the sectors which are close to international trade in goods and services clear within the domestic economy. As a result, the home-goods sector and the home-services sector should clear in the domestic market. The market clearing condition for the home-goods sector is

$$\frac{\partial E\,(c, p)}{\partial p_1} = c_1 = y_1$$

Substituting (4.9) for c_1 , the market clearing condition becomes

$$\gamma_1 \, c \, p_1^{\gamma_1} \, p_2^{\gamma_2} \, p_3^{\gamma_3} = y_1 \, p_1 \tag{4.34}$$

The market clearing condition for home-services sector is as follows

$$\frac{\partial E \,(c, p)}{\partial p_3} = c_3 = y_3$$

Substituting (4.11) for c_3 , the market clearing condition becomes

$$\gamma_3 \, c \, p_1^{\gamma_1} \, p_2^{\gamma_2} \, p_3^{\gamma_3} = y_3 \, p_3 \tag{4.35}$$

The ratio of (4.34) to (4.35) gives us

$$\frac{\gamma_1 \, c \, p_1^{\gamma_1} \, p_2^{\gamma_2} \, p_3^{\gamma_3}}{\gamma_3 \, c \, p_1^{\gamma_1} \, p_2^{\gamma_2} \, p_3^{\gamma_3}} = \frac{y_1 \, p_1}{y_3 \, p_3}$$

Rearranging, the market clearing conditions can be combined as follows

$$y_3 = y_1 \frac{p_1}{p_3} \frac{\gamma_3}{\gamma_1} \tag{4.36}$$

Apart from the aforementioned conditions, the competitive equilibrium conditions also require the factor markets to clear. There are two factor markets in the closed economy framework, namely the raw labor market and the broad capital market. The raw labor market clearing condition is given by

$$\frac{\partial MC_1}{\partial w} y_1 + \frac{\partial MC_2}{\partial w} y_2 + \frac{\partial MC_3}{\partial w} y_3 = 1 \tag{4.37}$$

If the partial derivatives are substituted into (4.37), the equation becomes

$$\frac{1}{a_1}\alpha_1 \left(\frac{w}{r}\right)^{\alpha_1 - 1} y_1 + \frac{1}{a_2}\beta_1 \left(\frac{w}{r}\right)^{\beta_1 - 1} y_2 + \frac{1}{a_3}\delta_1 \left(\frac{w}{r}\right)^{\delta_1 - 1} y_3 = 1 \tag{4.38}$$

Plugging (4.36) into (4.38), the raw labor market clearing condition becomes

$$\left[\frac{1}{a_1}\alpha_1 \left(\frac{w}{r}\right)^{\alpha_1 - 1} + \frac{1}{a_3}\delta_1 \left(\frac{w}{r}\right)^{\delta_1 - 1} \frac{p_1}{p_3} \frac{\gamma_3}{\gamma_1}\right] y_1 + \frac{1}{a_2}\beta_1 \left(\frac{w}{r}\right)^{\beta_1 - 1} y_2 = 1 \tag{4.39}$$

The broad capital market clearing condition is given by

$$\frac{\partial MC_1}{\partial r} y_1 + \frac{\partial MC_2}{\partial r} y_2 + \frac{\partial MC_3}{\partial r} y_3 = z \tag{4.40}$$

The same steps should be followed in order to obtain the clearing condition as a function of per capita output levels of two sectors. Plugging the partial derivatives and (4.36) into (4.40), the broad capital market clearing condition becomes

$$\left[\frac{1}{a_1}(1 - \alpha_1)\left(\frac{w}{r}\right)^{\alpha_1} + \frac{1}{a_3}(1 - \delta_1)\left(\frac{w}{r}\right)^{\delta_1} \frac{p_1}{p_3} \frac{\gamma_3}{\gamma_1}\right] y_1 + \frac{1}{a_2}(1 - \beta_1)\left(\frac{w}{r}\right)^{\beta_1} y_2 = z \tag{4.41}$$

Using these two factor market clearing conditions, the functions of per capita output levels in home-goods sector and tradable-goods sector can be solved for. When the equilibrium wage rate, broad capital rental rate and the relative price of home-goods sector, namely (4.31), (4.32) and (4.33) respectively, are substituted into the factor market clearing conditions, they will give the per capita output functions as

$$y_1 = \mathbf{y}_1(p_3, z) \tag{4.42}$$

$$y_2 = \mathbf{y}_2(p_3, z) \tag{4.43}$$

Using (4.36), the per capita output function for the home-services good can be found as

$$y_3 = \mathbf{y}_3(p_3, z) \tag{4.44}$$

In order to see the transitional behaviors of the endogenous variables, further derivations are required. From the Euler Equation it is known that

$$\frac{\dot{c}}{c} = \frac{1}{\theta}\left(r - \rho - \gamma_1 \frac{\dot{p}_1}{p_1} - \gamma_3 \frac{\dot{p}_3}{p_3}\right)$$

Rearranging the Euler Equation

$$\frac{\dot{c}}{c} + \gamma_1 \frac{\dot{p}_1}{p_1} + \gamma_3 \frac{\dot{p}_3}{p_3} = \frac{1}{\theta}\left(r - \rho - \gamma_1 \frac{\dot{p}_1}{p_1} - \gamma_3 \frac{\dot{p}_3}{p_3}\right) + \gamma_1 \frac{\dot{p}_1}{p_1} + \gamma_3 \frac{\dot{p}_3}{p_3}$$

$$\Rightarrow \quad \frac{\dot{c}}{c} + \gamma_1 \frac{\dot{p}_1}{p_1} + \gamma_3 \frac{\dot{p}_3}{p_3} = \frac{1}{\theta}(r - \rho) + \left(\frac{\theta - 1}{\theta}\right)\gamma_1 \frac{\dot{p}_1}{p_1} + \left(\frac{\theta - 1}{\theta}\right)\gamma_3 \frac{\dot{p}_3}{p_3} \tag{4.45}$$

As has already been found above, the market clearing condition for the home-services sector is

$$c_3 = y_3$$

$$\gamma_3 \, c \, p_1^{\gamma_1} p_2^{\gamma_2} p_3^{\gamma_3} = y_3 p_3$$

Total differentiating it on both sides with respect to time, it is found that

$$\gamma_3 \, \dot{c} \, p_1^{\gamma_1} p_2^{\gamma_2} p_3^{\gamma_3} + \gamma_3 \, c \, p_1^{\gamma_1 - 1} \dot{p}_1 \gamma_1 p_2^{\gamma_2} p_3^{\gamma_3} + \gamma_3 \, c \, p_1^{\gamma_1} p_2^{\gamma_2} p_3^{\gamma_3 - 1} \dot{p}_3 \gamma_3$$
$$= \dot{p}_3 \, \mathbf{y}_3(p_3, z) + p_3 \left(\frac{\partial \mathbf{y}_3}{\partial p_3}\dot{p}_3 + \frac{\partial \mathbf{y}_3}{\partial z}\dot{z}\right)$$

Rearranging the equation,

$$\gamma_3 \, c \, p_1^{\gamma_1} p_2^{\gamma_2} p_3^{\gamma_3} \frac{\dot{c}}{c} + \gamma_3 \, c \, p_1^{\gamma_1} p_2^{\gamma_2} p_3^{\gamma_3} \frac{\dot{p}_1}{p_1}\gamma_1 + \gamma_3 \, c \, p_1^{\gamma_1} p_2^{\gamma_2} p_3^{\gamma_3} \frac{\dot{p}_3}{p_3}\gamma_3$$
$$= \dot{p}_3 \, \mathbf{y}_3(p_3, z) + p_3 \left(\frac{\partial \mathbf{y}_3}{\partial p_3}\dot{p}_3 + \frac{\partial \mathbf{y}_3}{\partial z}\dot{z}\right)$$

Taking te expression $\gamma_3\, c\, p_1^{\gamma_1} p_2^{\gamma_2} p_3^{\gamma_3}$ into paranthesis,

$$\gamma_3\, c\, p_1^{\gamma_1} p_2^{\gamma_2} p_3^{\gamma_3} \left(\frac{\dot{c}}{c} + \frac{\dot{p}_1}{p_1}\gamma_1 + \frac{\dot{p}_3}{p_3}\gamma_3 \right) = \dot{p}_3\ \mathbf{y}_3\left(p_3, z\right) + p_3 \left(\frac{\partial \mathbf{y}_3}{\partial p_3}\dot{p}_3 + \frac{\partial \mathbf{y}_3}{\partial z}\dot{z} \right) \tag{4.46}$$

Plugging (4.35) and (4.18) into (4.46) it becomes

$$p_3 \mathbf{y}_3\left(p_3, z\right) \left[\frac{1}{\theta}\left(r - \rho\right) + \left(\frac{\theta - 1}{\theta}\right)\gamma_1 \frac{\dot{p}_1}{p_1} + \left(\frac{\theta - 1}{\theta}\right)\gamma_3 \frac{\dot{p}_3}{p_3} \right] \tag{4.47}$$

$$= \dot{p}_3\ \mathbf{y}_3\left(p_3, z\right) + p_3 \left(\frac{\partial \mathbf{y}_3}{\partial p_3}\dot{p}_3 + \frac{\partial \mathbf{y}_3}{\partial z}\dot{z} \right) \tag{4.48}$$

From the market clearing conditions of home-goods sector and home-services sector, it is known that

$$\mathbf{y}_3\left(p_3, z\right) = \mathbf{y}_1\left(p_3, z\right) \frac{p_1\left(p_3\right)}{p_3} \frac{\gamma_3}{\gamma_1} \tag{4.49}$$

Finding $\frac{\partial \mathbf{y}_3}{\partial p_3}$, $\frac{\partial \mathbf{y}_3}{\partial z}$ and $\frac{\partial p_1}{\partial p_3}$ from (4.49) and plugging them into (4.48) we have

$$\begin{aligned}
& p_3\ \mathbf{y}_1\left(p_3, z\right) \frac{p_1\left(p_3\right)}{p_3}\frac{\gamma_3}{\gamma_1} \left[\frac{1}{\theta}\left(r\left(p_3\right) - \rho\right) + \left(\frac{\theta - 1}{\theta}\right)\gamma_1 \frac{\partial p_1}{\partial p_3}\frac{\dot{p}_3}{p_1\left(p_3\right)} + \left(\frac{\theta - 1}{\theta}\right)\gamma_3 \frac{\dot{p}_3}{p_3} \right] \\
=& \dot{p}_3\ \mathbf{y}_1\left(p_3, z\right) \frac{p_1\left(p_3\right)}{p_3}\frac{\gamma_3}{\gamma_1} + \\
& p_3 \left\{ \frac{\gamma_3}{\gamma_1}\frac{1}{p_3} \left[\frac{\partial \mathbf{y}_1}{\partial p_3} p_1\left(p_3\right) + \mathbf{y}_1\left(p_3, z\right)\frac{\partial p_1}{\partial p_3} - \mathbf{y}_1\left(p_3, z\right)\frac{p_1\left(p_3\right)}{p_3} \right] \dot{p}_3 + \frac{\partial \mathbf{y}_1}{\partial z}\frac{p_1\left(p_3\right)}{p_3}\frac{\gamma_3}{\gamma_1}\dot{z} \right\}
\end{aligned} \tag{4.50}$$

Rearranging (4.50) and solving it for $\dot{p}_3$ we obtain

$$\dot{p}_3 = \frac{\frac{\partial \mathbf{y}_1}{\partial z} p_1\left(p_3\right)\dot{z} - \frac{1}{\theta}\mathbf{y}_1\left(p_3, z\right) p_1\left(p_3\right)\left[r\left(p_3\right) - \rho\right]}{\left(\frac{\theta-1}{\theta}\right)\gamma_1 \mathbf{y}_1\left(p_3, z\right)\frac{\partial p_1}{\partial p_3} + \left(\frac{\theta-1}{\theta}\right)\gamma_3 \mathbf{y}_1\left(p_3, z\right)\frac{p_1\left(p_3\right)}{p_3} - \frac{\partial \mathbf{y}_1}{\partial p_3} p_1\left(p_3\right) - \mathbf{y}_1\left(p_3, z\right)\frac{\partial p_1}{\partial p_3}} \tag{4.51}$$

Thus, the differential equation for the relative price of home-services is found to be

$$\dot{p}_3 \ = \ \mathbf{P_3}(p_3, z) \tag{4.52}$$

In order to obtain the law of motion for broad capital during the transition period, the intertemporal budget constraint of households should be modified accordingly. The budget constraint is known as

$$\dot{z} = w + r\,z - E\left(c, p\right)$$

The equilibrium values of wage rate, w, and capital rental rate, r, have already been obtained above. The only remaining function is the expenditure fuction which is equal to

$$E\left(c, p\right) = c\ p_1^{\gamma_1}\ p_2^{\gamma_2}\ p_3^{\gamma_3}$$

From the home-goods market clearing condition it is found that

$$\begin{aligned} c_1 &= \mathbf{y}_1 \\ \gamma_1 \; c \; p_1^{\gamma_1} \; p_2^{\gamma_2} \; p_3^{\gamma_3} &= \mathbf{y}_1 \; p_1 \\ c \; p_1^{\gamma_1} \; p_2^{\gamma_2} \; p_3^{\gamma_3} &= \frac{\mathbf{y}_1 p_1}{\gamma_1} \end{aligned} \tag{4.53}$$

As can be seen, the left-hand side of (4.53) is equal to the expenditure function. Using this information, the equation becomes

$$E(c, p) = \frac{y_1 p_1}{\gamma_1} \tag{4.54}$$

Plugging (4.31), (4.32), (4.33) and (4.54) into the budget constraint and solving for $\dot{z}$ yields

$$\dot{z} = \frac{(p_3 a_3)^{\frac{1-\beta_1}{\delta_1-\beta_1}}}{a_2^{\frac{1-\beta_1}{\delta_1-\beta_1}}} + \frac{(p_3 a_3)^{\frac{-\beta_1}{\delta_1-\beta_1}}}{a_2^{\frac{-\delta_1}{\delta_1-\beta_1}}} z - \frac{(p_3 a_3)^{\frac{\alpha_1-\beta_1}{\delta_1-\beta_1}} \; a_2^{\frac{\delta_1-\alpha_1}{\delta_1-\beta_1}}}{a_1} \frac{\mathbf{y}_1 \, (p_3, z)}{\gamma_1} \tag{4.55}$$

where $p_2 = 1$. Thus, the differential equation for the broad capital stock is found to be

$$\dot{z} = \mathbf{Z}(p_3, z) \tag{4.56}$$

Equations (4.51) and (4.55), the initial condition for the broad capital stock, z_0, and the transversality condition fully describe the transitional dynamics of the model.

4.3 Steady-State Equilibrium Solutions

The steady-state is defined as the equilibrium where all the variables in the economy grow at their long run growth rates. Thus, in the closed economy model, the long run equilibrium requires the endogenous variables to be constant for all t. This condition can be expressed as follows:

$$\begin{aligned} \dot{z} &= 0 \\ \dot{c} &= 0 \\ \dot{p}_1 &= 0 \\ \dot{p}_3 &= 0 \end{aligned}$$

The Euler Equation is known to be

$$\frac{\dot{c}}{c} = \frac{1}{\theta}\left(r - \rho - \gamma_1 \frac{\dot{p}_1}{p_1} - \gamma_3 \frac{\dot{p}_3}{p_3}\right)$$

Since the steady state requires $\dot{p}_3 = 0$, $\dot{p}_1 = 0$ and $\dot{c} = 0$ to hold, the Euler Equation becomes

$$\frac{1}{\theta}(r_{ss} - \rho) = 0$$
$$r_{ss} = \rho \tag{4.57}$$

where r_{ss} represents the steady-state value of the rate of return to broad capital. Using (4.57) the steady state values of wage rate, w_{ss}, relative prices of home-goods, $(p_1)_{ss}$, and home-services, $(p_3)_{ss}$ can be calculated. Plugging these steady state values into the factor market clearing conditions (4.39) and (4.41), the raw labor and broad capital market clearing conditions can be obtained at the steady state as follows

$$L\,;\quad \left[\frac{1}{a_1}\alpha_1\left(\frac{w_{ss}}{r_{ss}}\right)^{\alpha_1-1} + \frac{1}{a_3}\delta_1\left(\frac{w_{ss}}{r_{ss}}\right)^{\delta_1-1}\frac{(p_1)_{ss}}{(p_3)_{ss}}\frac{\gamma_3}{\gamma_1}\right](y_1)_{ss} + \frac{1}{a_2}\beta_1\left(\frac{w_{ss}}{r_{ss}}\right)^{\beta_1-1}(y_2)_{ss} = 1 \tag{4.58}$$

$$Z\,;\quad \left[\frac{1}{a_1}(1-\alpha_1)\left(\frac{w_{ss}}{r_{ss}}\right)^{\alpha_1} + \frac{1}{a_3}(1-\delta_1)\left(\frac{w_{ss}}{r_{ss}}\right)^{\delta_1}\frac{(p_1)_{ss}}{(p_3)_{ss}}\frac{\gamma_3}{\gamma_1}\right](y_1)_{ss} + \frac{1}{a_2}(1-\beta_1)\left(\frac{w_{ss}}{r_{ss}}\right)^{\beta_1}(y_2)_{ss} = z_{ss} \tag{4.59}$$

Solving (4.58) and (4.59) first for $(y_1)_{ss}$ and then for $(y_2)_{ss}$ would give us the steady state values of per capita output for home-goods and tradable-goods sectors, respectively. In order to find the per capita output value of home-services sector at the steady state, the value of $(y_1)_{ss}$ should be substituted into (4.36). The resulting functions will be

$$(y_1)_{ss} = \mathbf{y}_1(z_{ss})$$
$$(y_2)_{ss} = \mathbf{y}_2(z_{ss})$$
$$(y_3)_{ss} = \mathbf{y}_3(z_{ss})$$

The budget constraint of the representative household is also modified at the steady state. Using the information of r_{ss} and $\dot{z} = 0$, the intertemporal budget constraint becomes

$$0 = w_{ss} + r_{ss}\,z_{ss} - E\,(c_{ss}, p_{ss}) \tag{4.60}$$

where z_{ss} is the steady state value of the broad capital stock per head and c_{ss} is the steady state value of consuption per head. Plugging the steady state values of the endogenous variables into (4.60) and solving for c_{ss}, we obtain

$$c_{ss} = \mathbf{c}(z_{ss}) \tag{4.61}$$

All the endogenous variables are found to be functions of z_{ss} at the steady state. Thus, finding the steady state solutions boils down to obtaining the value of z_{ss}. To this end, either the home-goods market clearing condition or the home-services market clearing condition can be utilized. Both of them would lead to the same value of z_{ss}. At the steady state, the home-goods market clearing condition becomes

$$\mathbf{c}_1(z_{ss}) = \mathbf{y}_1(z_{ss})$$

$$\gamma_1\, c_{ss}\, (p_1)_{ss}^{\gamma_1}\, p_2^{\gamma_2}\, (p_3)_{ss}^{\gamma_3} = \mathbf{y}_1(z_{ss})\, (p_1)_{ss} \tag{4.62}$$

Setting $p_2 = 1$ and plugging the relevant steady state values into (4.62), one can solve for the steady state value of broad capital stock per head. Knowing z_{ss}, the remaining endogenous variables, namely $(y_1)_{ss}, (y_2)_{ss}, (y_3)_{ss}$ and c_{ss} can be calculated, as well.

4.4 Transition Path Solutions

Even though the steady state behaviors provide us with significant information about the economy, it is insufficient for understanding the real dynamics at hand. As is recognized by Mulligan and Sala-i Martin (1992) it is insightful to study transitional dynamics of multi-sector models. If one drops the assumption that all the variables in the model grow at their long run growth rates, one should examine the behavior of the variables which are not at their steady state levels.

Thus, this thesis will also examine the three-sector growth model outside of the steady state. Following Mulligan and Sala-i Martin[75] and Roe, Saracoğlu, and Smith (forthcoming) the analysis will be based on the Time-Elimination Method which will serve practical in numerical solution of the dynamic optimization problems. The practicality of the Time Elimination Method comes from the fact that it transforms the boundary value type problem into an initial value problem.

In the dynamic optimization problems such as the Ramsey growth model, the optimal path for the state and control variables are obtained through the solution of the

[75] ibid.

differential equations subject to the boundary conditions, namely initial conditions of the state variables and the transversality condition. Thus, as stated in Mulligan and Sala-i Martin (1992) and Roe et al. (forthcoming), this system displays a boundary value-type problem. Numerical solutions to such boundary value problems are difficult to compute. If there is only one state-variable in the system, the Time Elimination Method offers an efficient and easy way to compute for the solution by converting it into an initial value problem. Since the closed economy model entails one state variable and one control variable, it will be possible to follow this method.

The differential equations in the closed economy model are found to be

$$\dot{z}(t) = g_z(p_3(t), z(t)) \tag{4.63}$$

$$\dot{p}_3(t) = g_{p_3}(p_3(t), z(t)) \tag{4.64}$$

where $z(t)$ is the state variable and $p_3(t)$ is the control variable. In the first step of the time elimination method the control variable will be expressed as a function of the state variable.

$$p_3 = \mathbf{P}(z) \tag{4.65}$$

The reparameterized control variable in (4.65) is a policy function which exhibits the path of equilibrium solutions for the state and control variables. Given the policy function, one can express the law of motion for the state variable as a function of only the state variable and thereby solve the system. Thus, the transitional dynamics' problem boils down to finding the policy function. One of the methods of deriving the policy function is to make use of the slope of the policy function and integrate it backwardly. Following Mulligan and Sala-i Martin (1992) and Roe et al. (forthcoming), it can be seen that the ratio of the time derivative of the control variable to the time derivative of the state variable is the first order derivative of the policy function. In order to observe it in the closed economy model, differentiate both sides of (4.65) with respect to time

$$\dot{p}_3 = \frac{\partial \mathbf{P}(z)}{\partial z}\dot{z} \tag{4.66}$$

Rearranging,

$$\frac{\dot{p}_3}{\dot{z}} = \frac{\partial \mathbf{P}(z)}{\partial z} \tag{4.67}$$

This derivative is, indeed, the slope of the policy function indepedent of time which can be used to derive the policy function itself. Extraction of the policy function from its associated slope is performed by integrating the slope backwards in the state variable domain from the steady state to the initial value such that $z \in [z_{0,} z_{ss})$. However, since the time derivatives of the variables at the steady state are zero, the slope of the policy function is undetermined at the steady state. This problem is solved by linearizing the differential equations around the steady state so that the saddle path stability is identified. This method is called the Eigenvalues-Eigenvectors Approach as also discussed in Roe et al.[76] The linearized differential equations evaluated at the steady state is given by

$$\begin{pmatrix} \dot{z} \\ \dot{p}_3 \end{pmatrix} = \begin{pmatrix} \frac{\partial g_z((p_3)_{ss}, z_{ss})}{\partial z} & \frac{\partial g_z((p_3)_{ss}, z_{ss})}{\partial p_3} \\ \frac{\partial g_{p_3}((p_3)_{ss}, z_{ss})}{\partial z} & \frac{\partial g_{p_3}((p_3)_{ss}, z_{ss})}{\partial p_3} \end{pmatrix} \begin{pmatrix} z - z_{ss} \\ p_3 - (p_3)_{ss} \end{pmatrix} \tag{4.68}$$

From this equation, it can be seen that the Jacobian with the steady state values is as follows:

$$J = \begin{pmatrix} \frac{\partial g_z((p_3)_{ss}, z_{ss})}{\partial z} & \frac{\partial g_z((p_3)_{ss}, z_{ss})}{\partial p_3} \\ \frac{\partial g_{p_3}((p_3)_{ss}, z_{ss})}{\partial z} & \frac{\partial g_{p_3}((p_3)_{ss}, z_{ss})}{\partial p_3} \end{pmatrix} \tag{4.69}$$

Since the eigenvectors are tangent to the nonlinear arms at the steady state, it is possible to distinguish stable and unstable arms. As known, in order to identify the stable arm, one should work with the eigenvector associated with the negative eigenvalue. Then, using the stable arm one can compute the general solution to the time derivatives of the state and control variables. By taking the ratio of them, the slope of the policy function at the steady state can be calculated. The slope of the policy function at the neighbourhood of the steady state is used as an initial point to integrate the differential equations backward to obtain the policy function itself. Then using the policy function, the optimal transition path of the state variable and thereby the path of the control variable can be found as follows:

$$z(t) = \mathbf{Z}(t) \tag{4.70}$$

$$p_3(t) = \mathbf{P}(\mathbf{Z}(t)) \tag{4.71}$$

Thus, using the time paths of z and p_3, it is possible to find the time paths of other endogenous variables in the closed economy model.

[76] ibid.

CHAPTER 5

THE OPEN ECONOMY MODEL

5.1 The Theory

This part of the thesis will describe a small open economy with three sectors. The framework will be set by introducing the assumptions related with the sectors in the economy, production technologies and household preferences. In this model, not only international trade in goods and services but also international capital flows are allowed for. Thus, the economy in question is open to all kinds of capital transactions. However, in this model, the focus of attention will be on FDI type of international capital flows since it is more conducive to work with in models of real economy. It is assumed that the country is small compared to the rest of the world and the world is in steady state. In such an environment the financial integration can take two forms. It can either allow for perfect capital mobility or it can restrict the capital mobility between countries.

As has already been discussed above in chapter 3, allowing for perfect capital mobility leads to certain 'counterfactual results' in a Ramsey-type growth model.[77] Obstfeld (1998) notes that open capital accounts enable the financial investors to borrow from the economies with low interest rates and lend in the economies with high interest rates. These types of transactions leads to the equalization of the interest rates in financial markets. Thus, a common world interest rate is obtained eventually. Allowing for perfect capital mobility, the domestic country in question would face a constant world interest rate. Moreover, the returns on both kinds of capital would

[77] These counterfactual results are discussed in Barro and Sala-i-Martin (2004).

also be equal to this constant world interest rate. As is discussed in Barro and Sala-i-Martin (2004), this would cause the values of physical capital per capita, k, human capital per capita, h, and the aggregate output per capita, y to be constant during the transition. Thus, in the open economy version of the Ramsey model, the endogenous variables jump to their steady state values at once. The counterfactual results stem from the infinite convergence rates of the variables.

According to Gourinchas and Jeanne (2002), the economies that experience instantaneous convergence by financial integration are, in fact, close to their conditional steady states. As discussed in Mankiw et al. (1992), this closeness might result from a low capital share in production which causes the diminishing returns to capital to set in more quickly. Introducing human capital into the model, they are able to slow down the diminishing retuns to capital. In fact, Barro et al. (1995) argue that the inclusion of human capital makes the production function less concave compared to the case when there is only one type of capital stock in the economy. However, in the framework of perfect capital mobility, even though the capital share is increased by introducing human capital, it is not sufficient to eliminate the aforementioned problems of open economy Ramsey Model.

One of the remedies of infinite convergence is to allow for constrained capital mobility instead of perfect capital mobility. In this model, the constraint in international capital mobility is introduced by a non-tradable investment good. Consistent with the literature and specifically following Barro et al. (1992, 1995) and Barro and Sala-i-Martin (2004), this thesis will use human capital as the non-tradable investment good and impose a difference between the two types of capital stocks. By introducing a borrowing constraint into the domestic economy, human capital and physical capital would be diversified. In this model, it is assumed that the domestic agents can borrow from the world financial market as long as the foreign debt is used in the investment of physical capital. Thus, the amount of foreign debt is restricted to the quantity of physical capital. Contrary to physical capital, human capital is not a suitable asset for international trade. Since it is easier to repossess and channel physical capital, the creditors accept to lend for the funding of physical capital. As noted by Barro and Sala-i-Martin (2004), provided that a default occurs, creditors would be able to acquire the physical capital stock, whereas, they cannot take the possession of the human capital stock. Therefore, human capital stock does not provide the required security for foreign investors to channel their resources to the country in question. This leads to a difference between the capital stocks such that even though physical capital can be used as collateral against foreign debt, human capital cannot serve as collateral.

Since human capital does not provide security on foreign loans, foreign creditors would not finance its investment. Therefore, it is in the responsibility of the domestic agents to provide the required investment for human capital. In that sense, domestic savings and assets should be utilized in human capital investment, while foreign savings and assets should be used in physical capital investment. Consequently, it is assumed that the quantity of human capital cannot surpass domestic saving and foreign debt cannot exceed the quantity of pysical capital. This asymmetry between the two kinds of capital stocks is also reflected in their respective rates of return. Since physical capital can be financed by foreign debt, its rate of return, r_k, is determined in the international financial market. Thus, the net return on physical capital is equal to the constant world interest rate at all points in time.[78] On the other hand, since the funding for human capital investment is carried out by domestic agents, the rate of return to human capital, r_h, is determined within the domestic market. This difference in rates of returns will become crucial in the initial period. At the steady state both rates of returns will be equalized.

Similar to the closed economy model, there are three sectors in the economy. The home-goods sector produces agricultural and manufacturing goods for the domestic market and it does not allow for international trade or capital flows. Thus, the market clears within the economy with an endogenous price level. The tradable-goods sector produces all the internationally tradable goods in the economy at world prices and it is open to international capital flows. The output of this sector is used in domestic and foreign consumption purposes. The home-services sector serves only for domestic

[78] The world interest rate, r^{ω}, is assumed to be constant at its steady state value. Following Barro et al. (1992, 1995) and Barro and Sala-i-Martin (2004), the steady state value of the world interest rate is assumed to be equal to the steady state interest rate of the closed economy case. In their own model, this equality is expressed as $r^{\omega} = \rho + \theta g$. This equality implies that the domestic country has the same patience level as the world as a whole. They note that if $r^{\omega} > \rho + \theta g$ holds, then the domestic economy ceases to be a small economy. In that case, the assumption of an exogenously taken interest rate would be violated and the interest rate would not be pegged at the world rate anymore. If, however, $r^{\omega} < \rho + \theta g$ holds, the domestic economy continues to be a small economy. The only difference is that it takes more time for the borrowing constraint to be realized in the economy.

consumption and clears within the economy. The otput of home-services sector is not only used in domestic consumption, but it is also used in the investment of human capital. Thus, the accumulation of human capital is provided by the domestic economy, as has already been discussed above. In each sector, firms are perfectly competitive both in goods markets and in factors markets.

In all these sectors, output is produced with the use of three kinds of factors of production at each period. The production function includes raw labor, physical capital and human capital in all sectors. The two kinds of capital are assumed to be perfectly reversible with no adjustment costs. All factors of production are perfectly mobile between sectors. Even though the mobility of physical capital is allowed between countries, the mobility of raw labor or human capital is not allowed. There is no government, technological change or population growth in the economy. Setting the preliminary framework, this part will concentrate on the steady state solutions of the open eceonomy model.[79] The transition period will be covered in a further study.

5.1.1 Households

Similar to the closed economy case, the open economy model assumes identical infinitely-lived households each of which has the same preference parameters, same amount of assets and obtains the same wage rate. Given these assumptions, this model will use a representative household analysis. The household supplies raw labor, physical capital and human capital to all sectors in the economy in exchange for wage compensation, return on physical and human capitals. The income will be used to accumulate physical and human capital, to pay the interest on foreign debt and to consume three types of goods. Similar to the closed economy model, the representative household faces two kinds of problem: an intertemporal problem and an intra-temporal problem.

[79] In solving the three-sector Ramsey Model, the procedure of Roe et al. (forthcoming) will be followed.

In the intertemporal problem, the household chooses the amount of consumption and saving at each point in time in order to maximize the present value of discounted intertemporal utility subject to his/her budget constraint. The utility function of the household has a CIES form and it is given as

$$U = \int_0^\infty \frac{c(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} \, dt \tag{5.1}$$

where $c(t)$ represents the composite intra-temporal consumption per capita, $\frac{1}{\theta}$ is the inverse of the elasticity of substitution across time and ρ is the time preference parameter.

Since the economy is open to international capital flows, households are allowed to borrow from abroad to finance consumption or saving. However, the borrowing constraint imposes certain restrictions on the amount of foreign debt available to the domestic economy. Since only physical capital can serve as collateral against foreign debt, the amount of borrowing, d, cannot exceed the quantity of physical capital, k. Since it is assumed that this constraint is binding, the condition $d = k$ should hold at each point in time. Following Barro et al. (1992, 1995) and Barro and Sala-i-Martin (2004), it will be assumed that the borrowing constraint is binding.[80] Thus, foreign debt per capita will be equal to physical capital per capita which can be denoted as $d = k$. Since only physical capital can be used as collateral, a change in foreign debt can only be brought about by a change in the physical capital stock. Therefore, the model also requires $\dot{d} = \dot{k}$ to hold at each point in time, as well. Setting the

[80] In the original paper, Barro et al. (ibid.) discuss the role of initial asset values in determining whether the constraint should be binding or not. They note that if the constraint is not binding, then the economy will continue to suffer from infinite convergence speed. The condition for non-binding borrowing constraint is given by

$$k(0) + h(0) - d(0) \geq h^*$$

where $k(0), h(0)$ and $d(0)$ are the initial values of physical capital, human capital and foreign debt per effective worker, respectively, and h^* is the steady state amount of human capital per capita. On the other hand, if

$$k(0) + h(0) - d(0) < h^*$$

holds, then the borrowing constraint will be binding and $d = k$ would apply.

framework, the budget constraint of the representative household is given by:[81]

$$\dot{h}(t) = w(t) + r_h(t)\ h(t) - E(c(t), p(t)) \tag{5.2}$$

where $h(t)$ is the human capital per head, $w(t)$ is the wage rate, $r_h(t)$ is the domestic return to a unit of human capital, $E(c(t), p(t))$ is the aggregate household expenditure on three consumption goods and $p(t)$ is a vector of output prices in three sectors. In this framework, all the return to physical capital serve as collateral against foreign debt. Since physical capital stock is just enough to make the interest payments on foreign borrowing, neither the return to physical capital, nor the foreign debt can have an impact on the budget constraint.

In the intra-temporal problem, the household chooses the allocation of his/her consumption between three consumption goods. Thus, the amount of expenditure on each consumption good is considered intra-temporally. The representative household faces the same instantaneous composite consumption function as in the closed economy model which is given by

$$c = \beta\, c_1^{\gamma_1}\, c_2^{\gamma_2}\, c_3^{\gamma_3} \tag{5.3}$$

where c is the composite consumption per capita, $\beta > 0$ is a scaling constant,[82] $c_1 > 0$ is the home-good consumption per capita, $c_2 > 0$ is the tradable-good consumtion per capita, $c_3 > 0$ is the home-services consumption per capita and $\gamma_1 + \gamma_2 + \gamma_3 = 1$. Based on this composite consumption function, the representative household minimizes his/her expenditure at each period in time by choosing a consumption bundle composed of three types of goods.

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[81] The original budget constraint used in Barro et al. (1992, 1995) (ibid.) is

$$\dot{h} + \dot{k} - \dot{d} = w + (R_k - n - g - \delta)k - (r - n - g)d + (R_h - n - g - \delta)h - c$$

where $\dot{h}$, $\dot{k}$ and $\dot{d}$ are changes in human capital, physical capital and foreign debt per capita respectively, r is the real interest rate, R_k and R_h are the gross returns to physical and human capital respectively, n is population growth rate, g is technological growth rate and δ is the depreciation rate in both capital stocks. The conditions for the open economy environment requires that $d = k$ and $\dot{d} = \dot{k}$ to hold. Thus, when these conditions are substituted into this equation, the resulting budget constraint will give the budget constraint used in the open economy model.

[82] In order to avoid algebraic complexity, β is set at $\beta \equiv \gamma_1^{-\gamma_1}\, \gamma_2^{-\gamma_2}\, \gamma_3^{-\gamma_3}$.

5.1.2 Production Technologies

5.1.2.1 Home-Goods Sector:

The production functions in each sector are assumed to be of Cobb-Douglas type functions fulfilling the constant-returns-to-scale property. In particular, the production function in home-goods sector is given by:

$$Y_1 = A_1 L_1^{\alpha_1} K_1^{\alpha_2} H_1^{1-\alpha_1-\alpha_2} \tag{5.4}$$

where Y_1 is the aggregate tradable-good, L_1, K_1 and H_1 are the aggregate raw labor, physical capital and human capital inputs respectively, $\alpha_1, \alpha_2, 1-\alpha_1-\alpha_2 \in (0,1)$ are raw labor, physical capital and human capital shares in production and $A_1 > 0$ is a scaling constant.

5.1.2.2 Tradable-Goods Sector:

The tradable good in the second sector is produced using a Cobb-Douglas production function with constant-returns-to-scale property. In particular, the production function of tradable-goods sector is given by

$$Y_2 = A_2 L_2^{\beta_1} K_2^{\beta_2} H_2^{1-\beta_1-\beta_2} \tag{5.5}$$

where Y_2 is the aggregate home-good, L_2, K_2 and H_2 are the aggregate raw labor, physical capital and human capital inputs respectively, $\beta_1, \beta_2, 1-\beta_1-\beta_2 \in (0,1)$ are raw labor, physical capital and human capital shares in production and $A_2 > 0$ is a scaling constant.

5.1.2.3 Home-Services Sector:

The home-service in the third sector is produced using a Cobb-Douglas production function with constant-returns-to-scale property. In particular, the production function is given by

$$Y_3 = A_3 L_3^{\delta_1} K_3^{\delta_2} H_3^{1-\delta_1-\delta_2} \tag{5.6}$$

where Y_3 is the aggregate home-service, L_3, K_3 and H_3 are the aggregate raw labor, physical capital and human capital inputs respectively, δ_1, δ_2, $1 - \delta_1 - \delta_2 \in (0, 1)$ are raw labor, physical capital and human capital shares in production and $A_3 > 0$ is a scaling constant.

5.2 Competitive Equilibrium

This part will describe and list the conditions and requirements of a competitive equilibrium in the open economy model.

Definition 5.2.1 *An equilibrium in this model is a list of sequences of household consumption plans* $\{c_1(t), c_2(t), c_3(t)\}_{t=0}^{\infty}$*, for each consumption item, production plans* $\{y_1(t), y_2(t), y_3(t)\}_{t=0}^{\infty}$*, for each production sector, factor plans* $\{l_j(t), k_j(t), h_j(t)\}_{t=0}^{\infty}$*, for each sector j, where* $j = 1, 2, 3$*, output prices* $\{p_1(t), p_2(t), p_3(t)\}_{t=0}^{\infty}$*, for each consumption good, wage rates* $\{w(t)\}_{t=0}^{\infty}$*, physical capital rental rate,* r_k*, human capital rental rate* $\{r_h(t)\}_{t=0}^{\infty}$*, and an initial condition* $h(0) < h^*$ *such that*

i) given $h(0)$*,* $p_1(t), p_2(t), p_3(t)$*,* $w(t)$*,* $r_k(t)$ *and* $r_h(t)$*, the sequence* $\{c_1(t), c_2(t), c_3(t)\}_{t=0}^{\infty}$*, minimizes the representative household's expenditure and maximizes the present value of his/her discounted intertemporal utility;*

ii) given $h(0)$*,* $p_1(t), p_2(t), p_3(t)$*,* $w(t)$*,* $r_k(t)$ *and* $r_h(t)$*, the sequence* $\{y_j(t), l_j(t), k_j(t), h_j(t)\}_{t=0}^{\infty}$ *maximize profits for each sector j,* $j = 1, 2, 3$*;*

iii) raw labor market clears;

iv) physical capital market clears;

v) human capital market clears;

vi) the market for home-goods clears;

vii) the market for home-services clears;

viii) Walras' Law holds: $p_2 x - p_2 m + r^w d = 0$*, where x is the export per capita of tradable-goods, m is the import per capita of tradable-goods,* r^w *is the constant world interest rate and d is the foreign debt per capita.*

The model above assumes that the markets for home-goods and home-services clear within the domestic economy so that there are no international trade taking place associated with these goods or services. On the other hand, the tradable-goods sector is open to international trade and international capital flows; therefore, the trade balance takes place within this sector. Since the amount of foreign debt is equal to the quantity of physical capital at each point in time, the change in capital stock will be matched by a change in foreign debt. Consequently, there will not be a problem of excess demand or supply associated with foreign borrowing.

5.2.1 Household's Intra-Temporal Problem

Households choose bundles of consumption goods that minimize their expenditures subject to their composite consumption at each point in time. Given the output prices, the representative household faces the following minimization problem

$$\begin{aligned} \min \quad & p_1\, c_1 + p_2\, c_2 + p_3\, c_3 \\ \text{subject to} \quad & c = \beta\, c_1^{\gamma_1}\, c_2^{\gamma_2}\, c_3^{\gamma_3} \\ & c_1 > 0 \\ & c_2 > 0 \\ & c_3 > 0 \end{aligned}$$

The solution to the above problem yields the same conditional consumption demands as in the closed economy model which are given by[83]

$$c_1 = \gamma_1\, c\, p_1^{\gamma_1 - 1}\, p_2^{\gamma_2}\, p_3^{\gamma_3} \tag{5.7}$$

$$c_2 = \gamma_2\, c\, p_1^{\gamma_1}\, p_2^{\gamma_2 - 1}\, p_3^{\gamma_3} \tag{5.8}$$

$$c_3 = \gamma_3\, c\, p_1^{\gamma_1}\, p_2^{\gamma_2}\, p_3^{\gamma_3 - 1} \tag{5.9}$$

Substituting these consumption demands into the expenditure function, the minimized total expenditure of the representative household becomes

$$E\,(c, p) = c\, p_1^{\gamma_1}\, p_2^{\gamma_2}\, p_3^{\gamma_3} \tag{5.10}$$

[83] Since it is the same minimization problem, the intra-temporal problem of the open economy model yields the same results as in the closed economy model. Therefore, the derivation in Appendix (A.2) still applies.

5.2.2 Household's Intertemporal Problem

Households choose the amount of saving and consumption that maximizes their inter-temporal utility subject to their budget constraint, initial conditions and the transversality condition. The maximization problem is as follows:

$$\begin{aligned} \max \quad & \int_0^\infty \frac{c^{1-\theta}-1}{1-\theta}\, e^{-\rho t}\, dt \\ \text{subject to} \quad & \dot{h} = w + r_h\, h - E\,(c,p) \\ & h\,(0) < h^*, \quad c\,(t) \geq 0 \\ & \lim_{t\to\infty} h(t)\, e^{-\int_0^t r_h(v)\,dv} = 0 \end{aligned}$$

The present-value Hamiltonian is

$$H = \frac{c^{1-\theta}-1}{1-\theta}\, e^{-\rho t} + \nu\left(w + r_h\, h - c\; p_1^{\gamma_1}\; p_2^{\gamma_2}\; p_3^{\gamma_3}\right)$$

where the expression in paranthesis is equal to $\dot{h}$ and $E\,(c,p) = c\; p_1^{\gamma_1}\; p_2^{\gamma_2}\; p_3^{\gamma_3}$ is the minimized expenditure function given by (5.10). The first order conditions yield

$$\begin{aligned} & \frac{\partial H}{\partial c} = c^{-\theta}\, e^{-\rho t} - \nu\; p_1^{\gamma_1}\; p_2^{\gamma_2}\; p_3^{\gamma_3} = 0 \\ \Rightarrow \quad & \nu = \frac{c^{-\theta}\; e^{-\rho t}}{p_1^{\gamma_1}\; p_2^{\gamma_2}\; p_3^{\gamma_3}} \end{aligned} \tag{5.11}$$

$$\begin{aligned} & \frac{\partial H}{\partial h} + \dot{\nu} = \nu\, r_h + \dot{\nu} = 0 \\ \Rightarrow \quad & -\frac{\dot{\nu}}{\nu} = r_h \end{aligned} \tag{5.12}$$

Taking logarithms on both sides of (5.11) and taking time derivatives yield

$$-\frac{\dot{\nu}}{\nu} = \theta\,\frac{\dot{c}}{c} + \rho + \gamma_1\,\frac{\dot{p}_1}{p_1} + \gamma_2\,\frac{\dot{p}_2}{p_2} + \gamma_3\,\frac{\dot{p}_3}{p_3} \tag{5.13}$$

Substituting (5.12) into (5.13), the condition becomes

$$r_h = \theta\,\frac{\dot{c}}{c} + \rho + \gamma_1\,\frac{\dot{p}_1}{p_1} + \gamma_2\,\frac{\dot{p}_2}{p_2} + \gamma_3\,\frac{\dot{p}_3}{p_3} \tag{5.14}$$

Rearranging, the Euler Equation can be expressed as

$$\frac{\dot{c}}{c} = \frac{1}{\theta}\left(r_h - \rho - \gamma_1\,\frac{\dot{p}_1}{p_1} - \gamma_2\,\frac{\dot{p}_2}{p_2} - \gamma_3\,\frac{\dot{p}_3}{p_3}\right) \tag{5.15}$$

Since the tradable-goods sector is open to international trade, the price of tradable-goods is equal to the world price level at each point in time. Therefore, there is no change in p_2 in time which means that $\dot{p}_2 = 0$. Including this condition into (5.15), the Euler Equation in the open economy model becomes

$$\frac{\dot{c}}{c} = \frac{1}{\theta}\left(r_h - \rho - \gamma_1 \frac{\dot{p}_1}{p_1} - \gamma_3 \frac{\dot{p}_3}{p_3}\right) \tag{5.16}$$

5.2.3 Firms' Optimization Problems

The firms in each sector face the problem of minimizing their costs and maximizing their profits at each point in time given output and input prices. When the cost minimizing values of factors of production are obtained, they will be used in the profit maximization problem so as to find the profit maximizing level of output in each sector.

5.2.3.1 Home-Goods Sector:

Firms in home-goods sector choose factors of production, namely L_1, K_1 and H_1, that minimize their costs subject to their production functions given the output and input prices. The cost minimization problem they face is as follows:

$$\begin{aligned} \min \quad & w\, L_1 + r_k\, K_1 + r_h\, H_1 \\ \text{subject to} \quad & Y_1 \leq A_1\, L_1^{\alpha_1}\, K_1^{\alpha_2}\, H_1^{1-\alpha_1-\alpha_2} \\ & L_1 \geq 0 \\ & K_1 \geq 0 \\ & H_1 \geq 0 \end{aligned}$$

The solution to this problem gives the cost minimizing values of L_1, K_1 and H_1. Using this information in the total cost function, the minimized total cost is obtained as[84]

$$TC_1^* = Y_1 \frac{w^{\alpha_1}\, r_k^{\alpha_2}\, r_h^{1-\alpha_1-\alpha_2}}{a_1} \tag{5.17}$$

[84] For a detailed derivation, see Appendix (B.1.1).

where a_1 is a scaling constant.[85] Having obtained the minimized total cost, the firms face the problem of profit maximization. Given the domestic price of home-goods, p_1 and the cost minimizing values of L_1, K_1 and H_1, the firms choose Y_1 to maximize their profits at all points in time. The profit maximization problem is as follows:

$$\max_{Y_1} p_1 \; Y_1 - TC_1^* \tag{5.18}$$

Substituting (5.17) into (5.18), the profit maximization condition for the home-goods sector is generated as

$$p_1 = \frac{w^{\alpha_1} \; r_k^{\alpha_2} \; r_h^{1-\alpha_1-\alpha_2}}{a_1} \tag{5.19}$$

which is the well-known condition

$$MC_1(w, r_h) = p_1 \tag{5.20}$$

where MC_1 is the marginal cost of production in home-goods sector and p_1 is the relative price of home-goods.[86]

5.2.3.2 Tradable-Goods Sector:

Firms in tradable-goods sector choose factors of production, namely L_2, K_2 and H_2, that minimize their costs subject to their production functions given the output and input prices. The cost minimization problem they face is as follows:

$$\begin{aligned} \min \quad & w\, L_2 + r_k\, K_2 + r_h\, H_2 \\ \text{subject to} \quad & Y_2 \leq A_2\, L_2^{\beta_1}\, K_2^{\beta_2}\, H_2^{1-\beta_1-\beta_2} \\ & L_2 \geq 0 \\ & K_2 \geq 0 \\ & H_2 \geq 0 \end{aligned}$$

The solution to this problem gives the cost minimizing values of L_2, K_2 and H_2. Using this information in the total cost function, the minimized total cost is obtained as

$$TC_2^* = Y_2 \; \frac{w^{\beta_1} \; r_k^{\beta_2} \; r_h^{1-\beta_1-\beta_2}}{a_2} \tag{5.21}$$

[85] In order to avoid algebraic complexity, A_1 is set equal to $A_1 = a_1\, \alpha_1^{-\alpha_1}\, \alpha_2^{-\alpha_2}\, (1-\alpha_1-\alpha_2)^{\alpha_1+\alpha_2-1}$. For details, see Appendix (B.1.1).

[86] Since r_k is taken as a constant in the model, MC_1 is not expressed as a function of r_k.

where a_2 is a scaling constant.[87] Having obtained the minimized total cost, the firms face the problem of profit maximization. Given the domestic price of home-goods, p_2 and the cost minimizing values of L_2, K_2 and H_2, the firms choose Y_2 to maximize their profits at all points in time. The profit maximization problem is as follows:

$$\max_{Y_2} p_2 \, Y_2 - TC_2^* \tag{5.22}$$

Substituting (5.21) into (5.22), the profit maximization condition for the home-goods sector is generated as

$$p_2 = \frac{w^{\beta_1} \, r_k^{\beta_2} \, r_h^{1-\beta_1-\beta_2}}{a_2} \tag{5.23}$$

which is the well-known condition

$$MC_2(w, r_h) = p_2 \tag{5.24}$$

where MC_2 is the marginal cost of production in tradable-goods sector and p_2 is the relative price of tradable-goods.

5.2.3.3 Home-Services Sector:

Firms in home-services sector choose factors of production, namely L_3, K_3 and H_3, that minimize their costs subject to their production functions given the output and input prices. The cost minimization problem they face is as follows:

$$\begin{aligned} \min \quad & w\,L_3 + r_k\,K_3 + r_h\,H_3 \\ \text{subject to} \quad & Y_3 \leq A_3 \, L_3^{\delta_1} \, K_3^{\delta_2} \, H_3^{1-\delta_1-\delta_2} \\ & L_3 \geq 0 \\ & K_3 \geq 0 \\ & H_3 \geq 0 \end{aligned}$$

The solution to this problem gives the cost minimizing values of L_3, K_3 and H_3. Using this information in the total cost function, the minimized total cost is obtained as

$$TC_3^* = Y_3 \, \frac{w^{\delta_1} \, r_k^{\delta_2} \, r_h^{1-\delta_1-\delta_2}}{a_3} \tag{5.25}$$

[87] In order to avoid algebraic complexity, A_1 is set equal to $A_2 = a_2 \beta_1^{-\beta_1} \beta_2^{-\beta_2} (1 - \beta_1 - \beta_2)^{\beta_1+\beta_2-1}$. For details, see Appendix (B.1.2).

where a_3 is a scaling constant.[88] Having obtained the minimized total cost, the firms face the second problem of profit maximization. Given the domestic price of home-goods, p_3 and the cost minimizing values of L_3, K_3 and H_3, the firms choose Y_3 to maximize their profits at all points in time. The profit maximization problem is as follows:

$$\max_{Y_3} p_3\, Y_3 - TC_3^* \tag{5.26}$$

Substituting (5.25) into (5.26), the profit maximization condition for the home-goods sector is generated as

$$p_3 = \frac{w^{\delta_1}\, r_k^{\delta_2}\, r_h^{1-\delta_1-\delta_2}}{a_3} \tag{5.27}$$

which is the well-known condition

$$MC_3(w, r_h) = p_3 \tag{5.28}$$

where MC_3 is the marginal cost of production in home-services sector and p_3 is the relative price of home-services.

5.2.4 Market Clearing Conditions

The profit maximization conditions for the three sectors yield the following equalities

$$p_1 = \frac{w^{\alpha_1}\, r_k^{\alpha_2}\, r_h^{1-\alpha_1-\alpha_2}}{a_1}$$
$$p_2 = \frac{w^{\beta_1}\, r_k^{\beta_2}\, r_h^{1-\beta_1-\beta_2}}{a_2}$$
$$p_3 = \frac{w^{\delta_1}\, r_k^{\delta_2}\, r_h^{1-\delta_1-\delta_2}}{a_3}$$

Similar to the closed economy framework, the output of tradable-goods sector will be taken as the numeriare; therefore, p_2 will be set at $p_2 \equiv 1$.

[88] In order to avoid algebraic complexity, A_3 is set equal to $A_3 = a_3\, \delta_1^{-\delta_1}\, \delta_2^{-\delta_2}\, (1 - \delta_1 - \delta_2)^{\delta_1+\delta_2-1}$. For details, see Appendix (B.1.3).

Solving these conditions together, the intra-temporal equilibrium values of the endogenous variables in the model will be obtained as functions of the relative price of home-services sector.

$$\mathbf{w}\,(p_3) = \left(\frac{(p_3\, a_3)^{1-\beta_1-\beta_2}\ (a_2)^{\delta_1+\delta_2-1}}{(r_k)^{\delta_2-\delta_2\beta_1-\beta_2+\delta_1\beta_2}} \right)^{\frac{1}{\delta_1-\delta_1\beta_2-\beta_1+\delta_2\beta_1}} \tag{5.29}$$

$$\mathbf{r_h}\,(p_3) = \left(\frac{(a_2)^{\delta_1}}{(p_3\, a_3)^{\beta_1}\ (r_k)^{\delta_1\beta_2-\delta_2\beta_1}} \right)^{\frac{1}{\delta_1-\delta_1\beta_2-\beta_1+\delta_2\beta_1}} \tag{5.30}$$

$$\mathbf{p_1}\,(p_3) = \left(\frac{(p_3\, a_3)^{\alpha_1-\alpha_1\beta_2-\beta_1+\alpha_2\beta_1}\ (a_2)^{\delta_1-\delta_1\alpha_2-\alpha_1+\delta_2\alpha_1}}{(r_k)^{\delta_1\beta_2-\delta_1\alpha_2+\delta_2\alpha_1-\delta_2\beta_1-\beta_2\alpha_1+\beta_1\alpha_2}\ a_1} \right)^{\frac{1}{\delta_1-\delta_1\beta_2-\beta_1+\delta_2\beta_1}} \tag{5.31}$$

At the competitive equilibrium, the markets for the sectors which are close to international trade in goods and services clear within the domestic economy. As a result, the home-goods sector and the home-services sector should clear in the domestic market. The market clearing condition for the home-goods sector is

$$\frac{\partial E\,(c,p)}{\partial p_1} = c_1 = y_1$$

Substituting (5.7) for c_1 , the market clearing condition becomes

$$\gamma_1\, c\, p_1^{\gamma_1}\, p_2^{\gamma_2}\, p_3^{\gamma_3} = y_1\, p_1 \tag{5.32}$$

The output of the home-services sector in the open economy model is used both in domestic consumption and in the accumulation of human capital. Therefore, the market clearing condition for home-services sector is given by

$$\frac{\partial E\,(c,p)}{\partial p_3} + \dot{h} = y_3$$

$$c_3 + \dot{h} = y_3$$

Substituting (5.9) for c_3 , the market clearing condition becomes

$$\gamma_3\, c\, p_1^{\gamma_1}\, p_2^{\gamma_2}\, p_3^{\gamma_3} + \dot{h}\, p_3 = y_3\, p_3 \tag{5.33}$$

The ratio of (5.32) to (5.33) gives us

$$\frac{\gamma_1\, c\, p_1^{\gamma_1}\, p_2^{\gamma_2}\, p_3^{\gamma_3}}{\gamma_3\, c\, p_1^{\gamma_1}\, p_2^{\gamma_2}\, p_3^{\gamma_3} + \dot{h}} = \frac{y_1\, p_1}{y_3\, p_3}$$

Rearranging, the market clearing conditions can be combined as follows

$$y_3 = y_1 \frac{p_1}{p_3} \frac{\gamma_3}{\gamma_1} + \dot{h} \tag{5.34}$$

Apart from the aforementioned conditions, the competitive equilibrium conditions also require the factor markets to clear. There are three factor markets in the open economy framework, namely the raw labor market, the physical capital market and the human capital market. The raw labor market clearing condition is given by

$$\frac{\partial MC_1}{\partial w} y_1 + \frac{\partial MC_2}{\partial w} y_2 + \frac{\partial MC_3}{\partial w} y_3 = 1 \tag{5.35}$$

If the partial derivatives are substituted into (5.35), the equation becomes

$$\frac{1}{a_1} \alpha_1 \, w^{\alpha_1 - 1} \, r_k^{\alpha_2} \, r_h^{1-\alpha_1-\alpha_2} \, y_1 + \frac{1}{a_2} \beta_1 \, w^{\beta_1 - 1} \, r_k^{\beta_2} \, r_h^{1-\beta_1-\beta_2} \, y_2 + \frac{1}{a_3} \delta_1 \, w^{\delta_1 - 1} \, r_k^{\delta_2} \, r_h^{1-\delta_1-\delta_2} \, y_3 = 1 \tag{5.36}$$

Plugging (5.34) into (5.36), the raw labor market clearing condition becomes

$$\left[\frac{1}{a_1} \alpha_1 \, w^{\alpha_1 - 1} \, r_k^{\alpha_2} \, r_h^{1-\alpha_1-\alpha_2} + \frac{1}{a_3} \delta_1 \, w^{\delta_1 - 1} \, r_k^{\delta_2} \, r_h^{1-\delta_1-\delta_2} \frac{p_1}{p_3} \frac{\gamma_3}{\gamma_1} \right] y_1 + \frac{1}{a_2} \beta_1 \, w^{\beta_1 - 1} \, r_k^{\beta_2} \, r_h^{1-\beta_1-\beta_2} \, y_2 + \frac{1}{a_3} \delta_1 \, w^{\delta_1 - 1} \, r_k^{\delta_2} \, r_h^{1-\delta_1-\delta_2} \, \dot{h} = 1 \tag{5.37}$$

The human capital market clearing condition is given by

$$\frac{\partial MC_1}{\partial r_h} y_1 + \frac{\partial MC_2}{\partial r_h} y_2 + \frac{\partial MC_3}{\partial r_h} y_3 = h \tag{5.38}$$

The same steps should be followed in order to obtain the clearing condition as a function of per capita output levels of two sectors. Plugging the partial derivatives and (5.34) into (5.38), the human capital market clearing condition becomes

$$\left[\frac{1}{a_1} (1 - \alpha_1 - \alpha_2) \, w^{\alpha_1} \, r_k^{\alpha_2} \, r_h^{-\alpha_1-\alpha_2} + \frac{1}{a_3} (1 - \delta_1 - \delta_2) \, w^{\delta_1} \, r_k^{\delta_2} \, r_h^{-\delta_1-\delta_2} \frac{p_1}{p_3} \frac{\gamma_3}{\gamma_1} \right] y_1 + \frac{1}{a_2} (1 - \beta_1 - \beta_2) \, w^{\beta_1} \, r_k^{\beta_2} \, r_h^{-\beta_1-\beta_2} \, y_2 + \frac{1}{a_3} (1 - \delta_1 - \delta_2) \, w^{\delta_1} \, r_k^{\delta_2} \, r_h^{-\delta_1-\delta_2} \, \dot{h} = h \tag{5.39}$$

There is also the market clearing condition for physical capital market. As has already been discussed, in the open economy framework, the amount of foreign debt cannot exceed the quantity of physical capital stock since only physical capital can be used as collateral against foreign debt.[89] Therefore, the following condition should hold

$$d \leq k$$

[89] For details, see Barro et al. (1992, 1995) and Barro and Sala-i-Martin (2004).

Since the inital conditions are assumed to satisfy $k(0)+h(0)-d(0) < h^*$, the constraint is binding[90] and the physical capital market clearing condition becomes

$$d = k \tag{5.40}$$

Using these three factor market clearing conditions, the functions of per capita output levels in home-goods sector and tradable-goods sector can be solved for. When the equilibrium wage rate, human capital rental rate and the relative price of home-goods sector, namely (5.29), (5.30) and (5.31) respectively, are substituted into the factor market clearing conditions, they will give the per capita output functions as

$$y_1 = \mathbf{y}_1 (p_3, h) \tag{5.41}$$

$$y_2 = \mathbf{y}_2 (p_3, h) \tag{5.42}$$

Using (5.34), the per capita output function for the home-services good can be found as

$$y_3 = \mathbf{y}_3 (p_3, h) \tag{5.43}$$

Knowing the equations for y_1, y_2 and y_3, the steady state solutions of the open economy model will be derived in the following part.

5.3 Steady-State Equilibrium Solutions

The steady-state is defined as the equilibrium where all the variables in the economy grow at their long run growth rates. Thus, in the open economy model, the long run equilibrium requires the endogenous variables to be constant for all t. This condition can be expressed as follows:

$$\begin{aligned} \dot{h} &= 0 \\ \dot{c} &= 0 \\ \dot{p}_1 &= 0 \\ \dot{p}_3 &= 0 \end{aligned}$$

[90] As noted by Barro et al. (1992, 1995) and Barro and Sala-i-Martin (2004), if initial conditions are such that

$$k(0) + h(0) - d(0) \geq h^*$$

then the borrowing constraint will be non-binding and the economy will continue to suffer from infinite convergence speed.

The Euler Equation is known to be

$$\frac{\dot{c}}{c} = \frac{1}{\theta}\left(r_h - \rho - \gamma_1 \frac{\dot{p}_1}{p_1} - \gamma_3 \frac{\dot{p}_3}{p_3}\right)$$

Since the steady state requires $\dot{p}_3 = 0$, $\dot{p}_1 = 0$ and $\dot{c} = 0$ to hold, the Euler Equation becomes

$$\frac{1}{\theta}\left[(r_h)_{ss} - \rho\right] = 0$$

$$(r_h)_{ss} = \rho \tag{5.44}$$

where $(r_h)_{ss}$ represents the steady-state value of the rate of return to human capital. Using (5.44) the steady state values of wage rate, w_{ss}, relative prices of home-goods, $(p_1)_{ss}$, and home-services, $(p_3)_{ss}$ can be calculated. Plugging these steady state values into the factor market clearing conditions (5.37) and (5.39) the raw labor and human capital market clearing conditions can be obtained at the steady state. The raw labor market clearing condition is given by

$$\left[\frac{1}{a_1}\alpha_1\, w_{ss}^{\alpha_1-1}\, r_k^{\alpha_2}\, (r_h)_{ss}^{1-\alpha_1-\alpha_2} + \frac{1}{a_3}\delta_1\, w_{ss}^{\delta_1-1}\, r_k^{\delta_2}\, (r_h)_{ss}^{1-\delta_1-\delta_2}\frac{(p_1)_{ss}}{(p_3)_{ss}}\frac{\gamma_3}{\gamma_1}\right](y_1)_{ss}$$
$$+ \frac{1}{a_2}\beta_1\, w_{ss}^{\beta_1-1}\, r_k^{\beta_2}\, (r_h)_{ss}^{1-\beta_1-\beta_2}\, (y_2)_{ss} = 1 \tag{5.45}$$

where $\dot{h}$ disappears in the equation since $\dot{h} = 0$ at the steady state. The human capital market clearing condition is given by

$$\left[\frac{1}{a_1}(1-\alpha_1-\alpha_2)\, w_{ss}^{\alpha_1}\, r_k^{\alpha_2}\, (r_h)_{ss}^{-\alpha_1-\alpha_2} + \frac{1}{a_3}(1-\delta_1-\delta_2)\, w_{ss}^{\delta_1}\, r_k^{\delta_2}\, (r_h)_{ss}^{-\delta_1-\delta_2}\frac{(p_1)_{ss}}{(p_3)_{ss}}\frac{\gamma_3}{\gamma_1}\right](y_1)_{ss}$$
$$+ \frac{1}{a_2}(1-\beta_1-\beta_2)\, w_{ss}^{\beta_1}\, r_k^{\beta_2}\, (r_h)_{ss}^{-\beta_1-\beta_2}\, (y_2)_{ss} = h_{ss} \tag{5.46}$$

where $\dot{h}$ disappears in the equation since $\dot{h} = 0$ at the steady state. Solving (5.45) and (5.46) first for $(y_1)_{ss}$ and then for $(y_2)_{ss}$ would give us the steady state values of per capita output for home-goods and tradable-goods sectors, respectively. In order to find the per capita output value of home-services sector at the steady state, the value of $(y_1)_{ss}$ should be substituted into (5.34). The resulting functions will be

$$\begin{aligned}(y_1)_{ss} &= \mathbf{y}_1(h_{ss})\\ (y_2)_{ss} &= \mathbf{y}_2(h_{ss})\\ (y_3)_{ss} &= \mathbf{y}_3(h_{ss})\end{aligned}$$

The budget constraint of the representative household is also modified at the steady state. Using the information of $(r_h)_{ss}$ and $\dot{h} = 0$, the intertemporal budget constraint becomes

$$0 = w_{ss} + (r_h)_{ss}\, h_{ss} - E\,(c_{ss}, p_{ss}) \tag{5.47}$$

where h_{ss} is the steady state value of the human capital stock per head and c_{ss} is the steady state value of consuption per head. Plugging the steady state values of the endogenous variables into (5.47) and solving for c_{ss}, we obtain

$$c_{ss} \;=\; \mathbf{c}(h_{ss}) \tag{5.48}$$

All the endogenous variables are found to be functions of h_{ss} at the steady state. Thus, finding the steady state solutions boils down to obtaining the value of h_{ss}. To this end, either the home-goods market clearing condition or the home-services market clearing condition can be utilized. Both of them would lead to the same value of h_{ss}. At the steady state, the home-goods market clearing condition becomes

$$\begin{aligned} \mathbf{c}_1(h_{ss}) &= \mathbf{y}_1(h_{ss}) \\ \gamma_1\, c_{ss}\, (p_1)_{ss}^{\gamma_1}\, p_2^{\gamma_2}\, (p_3)_{ss}^{\gamma_3} &= \mathbf{y}_1(h_{ss})\, (p_1)_{ss} \end{aligned} \tag{5.49}$$

Setting $p_2 = 1$ and plugging the relevant steady state values into (5.49), one can solve for the steady state value of human capital stock per head. Knowing h_{ss}, the remaining endogenous variables, namely $(y_1)_{ss}, (y_2)_{ss}, (y_3)_{ss}$ and c_{ss} can be calculated, as well.

CHAPTER 6

MODEL CALIBRATION AND SIMULATION RESULTS

6.1 Model Calibration

6.1.1 The Closed Economy Model

This part of the thesis describes the data used in the calibration analysis. The calibration of the closed economy model is carried out for the Turkish economy. Using the data of the year 2006,[91] a simple aggregated Social Accounting Matrix (SAM) has been constructed. The data are based on the statistics taken and generated from the National Accounts and employment information of the Turkish Statistical Institute (TURKSTAT).

The SAM of the closed economy model for the year 2006 is composed of three production sectors, a representative household and three consumption goods. There is no government assumed in the economy. There are no international capital flows taking place in the closed economy framework; however, international trade in goods and services is allowed. Thus the SAM includes the rest of the world as an economic agent with respect to international trade in goods and services.

The production sectors included in the SAM are home-goods, tradable-goods and home-services sectors. Table 6.1 summarizes the relative production in each sector and the shares of sectoral production in total output.

[91] The year 2006 is chosen for calibration based on the availability of a consistent data set.

Table 6.1: The Sectoral Composition of GDP in Turkey (2006)

	Total (thousand YTL)	**Share** (% in total)
Home-Goods Sector	243,822,751	32.1
Tradable-Goods Sector	171,926,483	22.7
Home-Services Sector	342,641,551	45.2
GDP	758,390,785	100

Sources: TURKSTAT (2006) and author's own calculations.

According to Table 6.1, among the three sectors, home-services sector dominates the aggregate production in the economy by providing about 45.2% of the total output. Following that home-goods sector produces about 32.1% of the total output, while the output of the tradable-goods sector is 22.7% of the total GDP. The output supply of the tradable-goods sector is composed of all the tradable goods in the economy including not only the exportable goods, but also the importable goods. But, the domestic production taking place within the economy is equal to all the exports of the economy in goods and services. Thus, the production statistic of the tradable-goods sector is taken from the aggregate export data of Turkish Statistical Institute (2006).

Table 6.2: The Sectoral Allocation of Labor Force in Turkey (2006)

	Total (number)	**Share** (% in total)
Home-Goods Sector	9,357,372	41.9
Tradable-Goods Sector	4,123,846	18.5
Home-Services Sector	8,848,782	39.6
Labor Force	22,330,000	100

Sources: TURKSTAT (2006) and author's own calculations.

There are two factors of production used in each sector, namely raw labor and broad capital. Table 6.2 summarizes the sectoral allocation of the labor force.[92] It is found that in 2006, the majority of the labor force, about 42%, is employed in the home-goods sector. It is followed by home-services sector which employs nearly 40% of

[92] The total work force statistic is taken from the TURKSTAT data on employment for the year 2006. In calculating the sectoral allocation of the labor force, the relative compensation ratios are utilized.

the labor force. Compared to these two sectors, the labor requirement of the tradable-goods sector seems to be very low with an 18.5%.

The production technology in each sector is given by the relative factor elasticities of raw labor and broad capital. The broad capital input consists of two types of capital stocks which are physical capital and human capital. Since in the closed economy framework, rate of return to both kinds of capital are the same, they are aggregated to compose a broad capital stock. Thus, the factor elasticity of capital is greater compared to the case when there is only one type of capital used in the production. The factor share parameters in each sector are presented in Table 6.3.[93]

Table 6.3: Factor Elasticities in the Closed Economy Model

	Share	
	Raw Labor	**Broad Capital**
Home-Goods Sector	0.40	0.60
Tradable-Goods Sector	0.25	0.75
Home-Services Sector	0.27	0.73

Source: Derived from TURKSTAT (2006) statistics.

The relative elasticities presented at the table are calculated by making use of the profit maximization conditions. The production function in each sector is of Cobb-Douglas form and it exhibits constant retuns to scale property. Based on these characteristics, profit maximization conditions demonstrate that factor elasticities are given by the initial shares of factor returns on the value of production in each sector. For instance, the elasticity of raw labor in home-goods sector is calculated by dividing the return to raw labor in that sector by the value of that sector's production. Similarly, the elasticity of broad capital in home-goods sector is found by dividing the return to

[93] The factor elasticities in home-goods sector and tradable-goods sector are taken exogenously based on the relative characteristics and requirements of the output produced in each sector. Given the factor shares in these two sectors, the factor elasticity of home-services sector is calculated accordingly.

broad capital in that sector by the value of production.[94] The same method applies to the remaining sectors, as well.

As is indicated in Table 6.3, all types of goods in the economy are, in fact, broad capital intensive. But, the degree of this intensiveness changes between sectors. The relative elasticities point to the fact that home-goods sector produces the most labor-intensive good since it has the highest labor intensity. On the other hand, the tradable goods sector produces the most broad capital intensive good since it has the highest capital intensity. Compared to these two sectors, home-services sector has a middle position with relatively more labor intensiveness than the tradable-goods sector and relatively more capital intensiveness than the home-goods sector.

The representative household in the closed economy framework spends his/her income on three types of consumption goods. The allocation of total expenditure on consumption goods is given by Table 6.4. Since the output of home-goods and home-services sectors are assumed to be consumed only by the domestic households, the consumption expenditures in each good is equal to the value of the production taking place in the relevant sector. Thus, the level of consumption expenditure in these sectors which are presented at the table are based on the production data.

[94] The elasticity of raw labor in each sector at $t = 0$ is obtained as follows:

$$\frac{w\,l_1}{p_1\,y_1} = 0.40$$
$$\frac{w\,l_2}{p_2\,y_2} = 0.25$$
$$\frac{w\,l_3}{p_3\,y_3} = 0.27$$

The elasticity of broad capital in each sector at $t = 0$ is obtained as follows:

$$\frac{r\,z_1}{p_1\,y_1} = 0.60$$
$$\frac{r\,z_2}{p_2\,y_2} = 0.75$$
$$\frac{r\,z_3}{p_3\,y_3} = 0.73$$

Table 6.4: Household Consumption in the Closed Economy Model

	Total (thousand YTL)	**Share** (% in total)
Home-Goods Sector	243,822,751	38.52
Tradable-Goods Sector	46,437,291	7.34
Home-Services Sector	342,641,551	54.14
Total Expenditure	632,901,593	100

Sources: TURKSTAT (2006) and author's own calculations.

As is indicated in Table 6.4, consumption spending on the product of home-services sector surpasses the other sectors by a significant amount. In fact, more than half of total expenditures, 54.14%, is devoted to the output of home-services sector. It is followed by the consumption on home-goods which is about 38.5% of total expenditures. The least spending is done on the output of tradable-goods sector. Since all the domestic production taking place in the tradable-goods sector is exported to the world, the consumption of the representative household is composed of the imported goods which are included in the supply of tradable-goods sector. Yet, within the bounds of a closed economy framework, the representative household devotes only 7.34% of total expenditures to imported goods which are assumed to be broad capital intensive.

Combining the information given above, the baseline specification of all the parameter values of the closed economy model are summarized in Table 6.5. The table contains the relative elasticities of raw labor and broad capital in each sector. Moreover, it gives the fractions of consumption on three types of goods produced in the economy. These values have already been analyzed in tables 6.3 and 6.4. Apart from them, the assumptions on the discount factor and the preference parameter are presented. The elasticity of intertemporal substitution is assumed to be equal to 1.

Table 6.5: The Closed Economy Model Parameter Values

	Symbol	Value
Elasticity of Raw Labor		
Home-Goods Sector	α_1	0.40
Tradable-Goods Sector	β_1	0.25
Home-Services Sector	δ_1	0.27
Elasticity of Broad Capital		
Home-Goods Sector	$1-\alpha_1$	0.60
Tradable-Goods Sector	$1-\beta_1$	0.75
Home-Services Sector	$1-\delta_1$	0.73
Fraction of Consumption on		
Home-Goods Sector Good	γ_1	0.39
Tradable-Goods Sector Good	γ_2	0.07
Home-Services Sector Good	γ_3	0.54
Preference Parameters		
Elasticity of Intertemporal Substitution	$\frac{1}{\theta}$	1
Rate of Time Preference	ρ	0.042

Sources: Derived from TURKSTAT (2006) statistics

According to Barro et al. (1995), high values attributed to θ imply low saving rates which cannot be realized based on the model specification. Specifically, they give the treshold level of 2 for the parameter value of θ and used it as such in their own calibration analysis. Thus, assuming that θ is set equal to $\theta = 1$ is consistent with the literature. The time preference rate of the representative household is assumed to be equal to 0.042.

6.1.2 The Open Economy Model

The calibration of the open economy model is also carried out for the Turkish economy using the data of the year 2006 so that a comparison between the two models can be made. Using the data, taken and generated from the National Accounts of the Turkish Statistical Institute (TURKSTAT), another simple aggregated Social Accounting Matrix (SAM) has been constructed based on the open economy model framework.

The same assumptions concerning the production sectors and consumption goods of the closed economy apply in this model, as well. The SAM of the open economy is composed of three production sectors, a representative household and three types of consumption goods. Similar to the model before, there is no government in the economy. The only difference in this framework is that the economy is open to international capital flows as well as international trade in goods and services. This has implications on the interest rate prevailing in the domestic market and also on the allocation of factors between sectors.

The production sectors included in the SAM of the open economy model are the same as in the closed economy framework, namely home-goods, tradable-goods and home-services sectors. There is no change in the production values or the relative shares of each sector in total GDP compared to the closed economy case. Table 6.6 summarizes the relative production in each sector and the shares of sectoral production in total output which give the same values on Table 6.1.

Table 6.6: The Sectoral Composition of GDP in Turkey (2006)

	Total (thousand YTL)	**Share** (% in total)
Home-Goods Sector	243,822,751	32.1
Tradable-Goods Sector	171,926,483	22.7
Home-Services Sector	342,641,551	45.2
GDP	758,390,785	100

Sources: TURKSTAT (2006) and author's own calculations.

In this framework there are three types of inputs used in each sector's production which are raw labor, physical capital and human capital. The sectoral allocation of the labor force is still the same as in the closed economy case since there is no change in the labor requirement of each sector. Table 6.7 summarizes the sectoral allocation of the labor force. As can be inferred from the table, the amount of labor force devoted to each sector as a share of total labor force continues to be the same as in Table 6.2 of the closed economy model.

Table 6.7: The Sectoral Allocation of Labor Force in Turkey (2006)

	Total (number)	**Share** (% in total)
Home-Goods Sector	9,357,372	41.9
Tradable-Goods Sector	4,123,846	18.5
Home-Services Sector	8,848,782	39.6
Labor Force	22,330,000	100

Sources: TURKSTAT (2006) and author's own calculations.

Unlike the closed economy model, the two types of capital stocks cannot be combined as a broad capital stock since their relative rates of returns are different in this framework. The presence of international capital flows and the borrowing constraint placed on the human capital stock generates a distinction between the two types of capital stocks. Thus, the impact of physical capital and human capital to the production technologies in each sector is different from each other. This characteristic of the open economy model is captured by the relative factor elasticity of each input which is presented in Table 6.8 below.

Table 6.8: Factor Elasticities in the Open Economy Model

	Share		
	Raw Labor	**Physical Capital**	**Human Capital**
Home-Goods Sector	0.40	0.33	0.27
Tradable-Goods Sector	0.25	0.33	0.42
Home-Services Sector	0.27	0.33	0.40

Source: Derived from TURKSTAT (2006) statistics.

Similar to the closed economy case, the relative elasticities presented at the table are calculated by making use of the profit maximization conditions. The production function in each sector is still of Cobb-Douglas form and it exhibits constant retuns to scale property. Based on these characteristics, profit maximization conditions demonstrate that factor elasticities are given by the initial shares of factor returns on the value of production in each sector.[95] The same method applies to the remaining sectors, as well.

As is indicated in Table 6.8, the elasticity of raw labor in each sector is the same as in the closed economy model. Thus, there is no difference in the allocation of raw labor between sectors, even though the economy is no longer closed to international capital. This indifference stems from the fact that raw labor is assumed to be immobile between countries. Thus, financial integration does not affect the share of raw labor in each sector within the framework of the open economy. On the other hand, the distinction between physical and human capital is manifested by the difference in relative factor shares in each sector. In fact, the factor allocation will be realized within the broad capital stock.

According to Table 6.8, the elasticity of physical capital is the same in each sector with a value of 0.33. This equality is the implication of the borrowing constraint assumption that has been made to slow down the convergence speed in the open economy framework. Since only physical capital can serve as collateral, the rate of return on this type of capital is equal to the constant world interest rate, while the rate of return on human capital is endogenously determined in the domestic market.

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[95] The elasticity of raw labor in each sector at $t = 0$ is obtained as follows:

$$\frac{w\,l_1}{p_1\,y_1} = 0.40$$
$$\frac{w\,l_2}{p_2\,y_2} = 0.25$$
$$\frac{w\,l_3}{p_3\,y_3} = 0.27$$

The elasticity of physical capital in each sector at $t = 0$ is obtained as follows:

$$\frac{r_k\,k_1}{p_1\,y_1} = 0.33$$
$$\frac{r_k\,k_2}{p_2\,y_2} = 0.33$$
$$\frac{r_k\,k_3}{p_3\,y_3} = 0.33$$

The elasticity of human capital in each sector at $t = 0$ is obtained as follows:

$$\frac{r_h\,h_1}{p_1\,y_1} = 0.27$$
$$\frac{r_h\,h_2}{p_2\,y_2} = 0.42$$
$$\frac{r_h\,h_3}{p_3\,y_3} = 0.40$$

The constancy in the physical capital's rate of return ensures the ratio of physical capital stock per head to output per head in each sector to be constant during the transition period to the steady state.[96] This is also reflected in each sector's relative k/y ratio. Thus, given the borrowing constraint assumption, the following condition should hold, throughout the transition period in the open economy:

$$\frac{k_1}{y_1} = \frac{k_2}{y_2} = \frac{k_3}{y_3} \tag{6.1}$$

Given (6.1), each sector should have the same elasticity of physical capital. Based on the aforementioned assumptions, home-goods sector has the highest raw labor intensity and produces the most labor-intensive good in the open economy framework, while the tradable-goods sector produces the highest human capital intensive good. Compared to these sectors, home-services sector still has a middle position with a relatively more labor intensive product than the product of tradable-goods sector and a relatively more human capital intensive product than the product of home-goods sector. The factor shares in the home-goods and home-services sectors are such that the raw labor intensiveness of the home-goods sector is equal to the human capital intensiveness of the home-services sector.

The consumption preferences of the representative household is also different in the open economy framework. The allocation of total expenditure on each consumption good is given by Table 6.9. Since the output of the home-goods sector is produced only for domestic consumption, the consumption expenditure on home-goods should be equal to the production value of this sector. Even though all the production in home-services sector is also devoted to domestic market, its use is allocated between consumption and investment in human capital stock so that the consumption expenditure on home-services is no longer equal to the output of the sector.

[96] The constancy in k/y ratio is also discussed in Barro et al. (1992, 1995) and Barro and Sala-i-Martin (2004).

Table 6.9: Household Consumption in the Open Economy Model

	Total (thousand YTL)	**Share** (% in total)
Home-Goods Sector	243,822,751	48.1
Tradable-Goods Sector	46,437,291	9.1
Home-Services Sector	217,152,359	42.8
Total Expenditure	507,412,401	100

Sources: TURKSTAT (2006) and author's own calculations.

As is indicated in Table 6.9, consumption spending on home-goods constitutes a relatively larger portion of total expenditure on consumption with a share of 48.1%. It is followed by the consumption spending on home-services which is about 42.8% of total expenditures. Thus, the share of expenditures devoted to consumption of home-goods and home-services are very close compared to the closed economy case. Yet, the least spending is done, again, on tradable-goods which constitutes 9.1% of total expenditures. Compared to the closed economy case, the share of consumption spending on home-goods has risen from 38.52% to 48.1% of total expenditure in the open economy framework, whereas the share on home-services has diminished to 42.8% from 54.14%. Thus, the consumption preference of the representative household has shifted from human capital intensive goods to raw labor intensive goods.

Combining the information given above, the baseline specification of all the parameter values of the open economy model are summarized in Table 6.10. The table lists the relative elasticities of raw labor, physical capital and human capital in each sector together with the fractions of consumption on three types of goods produced in the economy. All the parameter values given in Table 6.10 are consistent with the information given in Table 6.8 and 6.9. In addition to them, the assumptions on the discount factor and the rate of time preference are provided. The elasticity of intertemporal substitution is taken as 1 and the rate of time preference is taken as 0.042 as in the closed economy case so that a comparison can be made.

Table 6.10: The Open Economy Model Parameter Values

	Symbol	Value
Elasticity of Raw Labor		
Home-Goods Sector	α_1	0.40
Tradable-Goods Sector	β_1	0.25
Home-Services Sector	δ_1	0.27
Elasticity of Physical Capital		
Home-Goods Sector	α_2	0.33
Tradable-Goods Sector	β_2	0.33
Home-Services Sector	δ_2	0.33
Elasticity of Human Capital		
Home-Goods Sector	$1-\alpha_1-\alpha_2$	0.27
Tradable-Goods Sector	$1-\beta_1-\beta_2$	0.42
Home-Services Sector	$1-\delta_1-\delta_2$	0.40
Fraction of Consumption on		
Home-Goods Sector Good	γ_1	0.48
Tradable-Goods Sector Good	γ_2	0.09
Home-Services Sector Good	γ_3	0.43
Preference Parameters		
Elasticity of Intertemporal Substitution	$\frac{1}{\theta}$	1
Rate of Time Preference	ρ	0.042

Sources: Derived from TURKSTAT (2006) statistics

6.2 Simulation Results

6.2.1 The Closed Economy Model

The simulation results of the closed economy model are summarized in Table 6.11. A comparison between the initial and the steady state values demonstrate that there has been a change in the sectoral production and the allocation of resources between the sectors. According to the table, while production in home-goods and home-services sectors increased, production in the tradable-goods sector diminished.

Table 6.11: Model Calibration Results

	Initial Values	Steady State
Production Shares in GDP (%)		
Home-Goods Sector Output	32.1	38.5
Tradable-Goods Sector Output	22.7	7.3
Home-Services Sector Output	45.2	54.2
Sectoral Allocation of Raw Labor (%)		
Home-Goods Sector	41.9	48.4
Tradable-Goods Sector	18.5	5.8
Home-Services Sector	39.6	45.8
Sectoral Allocation of Broad Capital (%)		
Home-Goods Sector	27.8	33.9
Tradable-Goods Sector	24.5	8.1
Home-Services Sector	47.7	58.0
Consumption Shares in Expenditure (%)		
Home-Goods	38.5	38.5
Tradable-Goods	7.3	7.3
Home-Services	54.2	54.2

In value terms, Figure 6.1 displays the evolution of sectoral production. Even though the value of production in the tradable-goods sector does not seem to have declined too much, the increase in the value of production in the other sectors causes a sharp difference in sectoral shares in total production.[97] In fact, given the results in Table 6.11, it can be seen that the share of home-good sector's output in total production rises from 32.1% to 38.5% and that of the home-services sector rises from 45.2% to 54.2%. On the other hand, the share of tradable-good sector's output falls from 22.7% to 7.3%. The comparison is also represented by Figure 6.2.

[97] In the initial periods, tradable-goods sector experiences an increase in production before a sharp drop. This is reflected in Figure 6.1 by a smooth and seemingly insignificant decline until $t = 100$. The rise in tradable-good production is counterintuitive given the decline experienced afterwards. However, the increase in other sectors' production necessitates an increase in tradable-goods sector's output since it supplies the economy with capital. In fact, capital accumulation takes place only in the tradable-goods sector. Thus, in order to meet the increasing capital demand of other sectors, production in tradable-goods sector should also rise. The increase in tradable-good production continues until production in home-goods and home-services sectors adjust to demand.

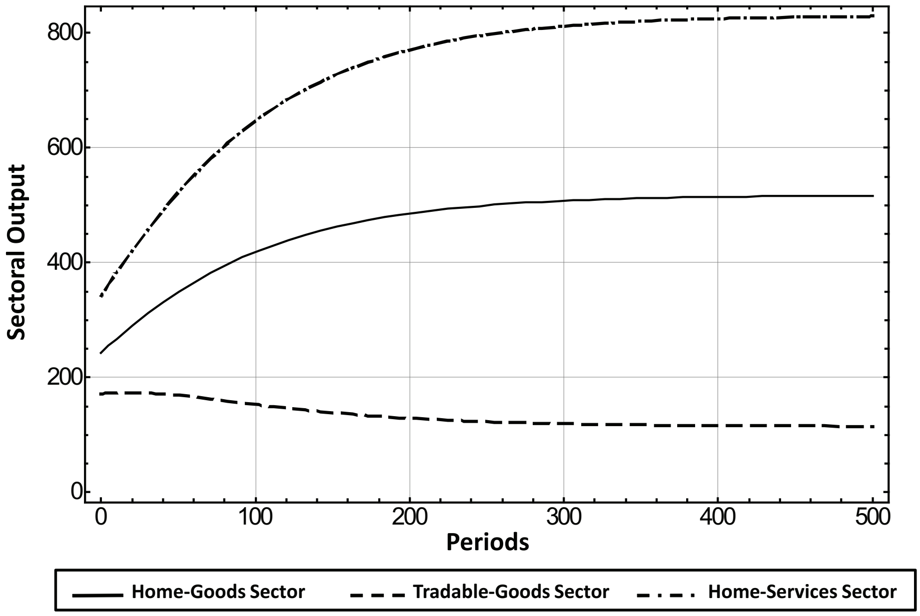

Figure 6.1: Sectoral Allocation of Output (billion YTL)

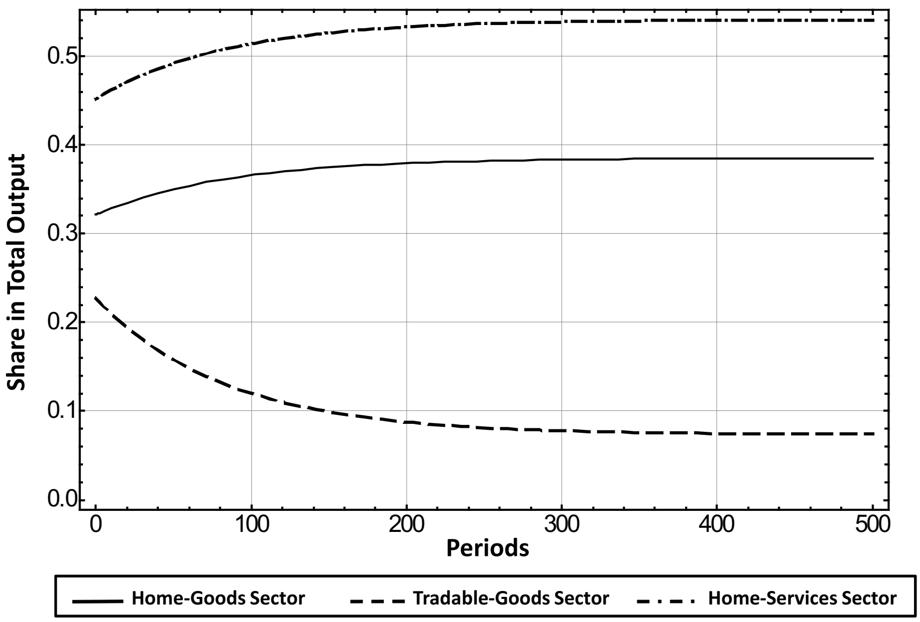

Figure 6.2: Sectoral Shares of Production in Total Output

While the economy converges to its steady state, broad capital is accumulated throughout the transition period. This capital accumulation ensures that each worker is equipped with more capital in each sector which leads to a rise in labor productivity. As a result of higher productivity, labor wages are increased which is directly reflected in household income. A higher income level induces the households to consume more of each sector's good. Thus, as depicted in Figure 6.3, the level of consumption expenditure on each sector's good increases.

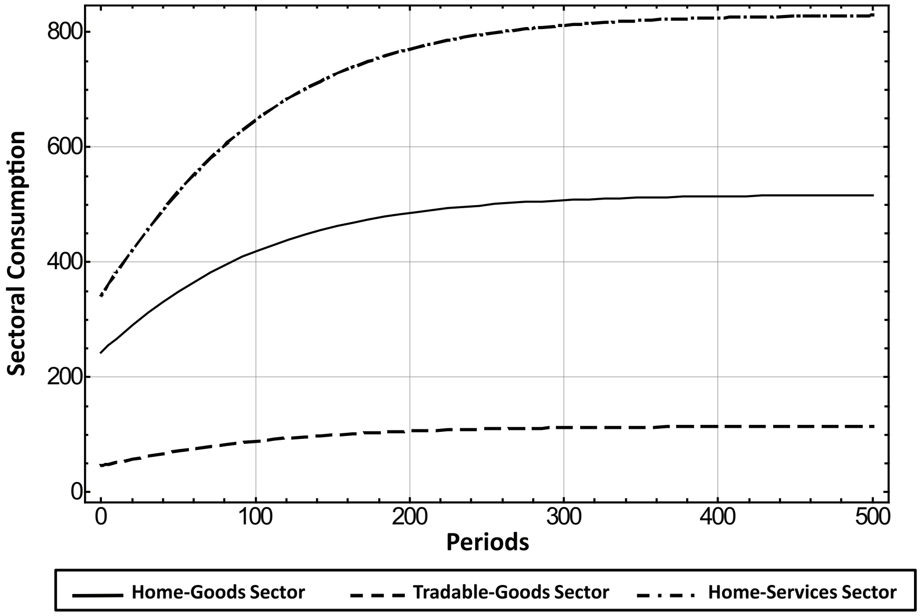

Figure 6.3: Consumption Expenditure on each Sector's Good (billion YTL)

Even though the level of consumption on each type of good increases over time, their share in total expenditures remains the same during the transition period. According to Table 6.11, the share of expenditures on home-goods does not change from its initial share of 38.5 %. Similarly, the share of consumption on tradable-goods and home-services in total expenditures stay constant at their initial shares of 7.3% and 54.2%, respectively. Since home-goods and home-services sectors produce only for domestic consumption, the steady state consumption on these goods should be equal to the steady state production. This equality is demonstrated in Table 6.11. As is indicated by the percentage changes, the share of each sector's production in total GDP is equal to the share of each type of good's consumption in total expenditures in the steady state. For instance, the share of home-goods sector's production is 38.5% at the steady state which is exactly the same as the share of expenditure on home-good consumption in total expenditure. The same applies to the remaining sectors, as well. Thus, it is implied that production in the closed economy framework adjusts to consumption demand.[98] Therefore, the increase in domestic demand for home-goods and home-services stimulates an increase in production in home-goods and home-services sectors, while a decline in domestic demand for tradable-goods reinforces a decline in production of the corresponding sector. The results show that the closed economy model provides a demand-driven environment for production.

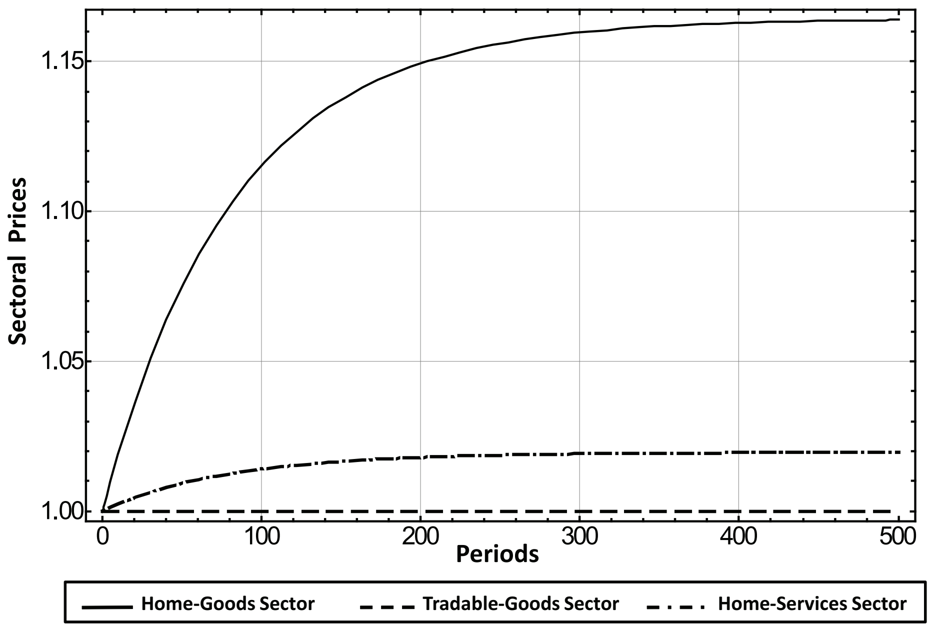

Figure 6.4: The Evolution of Prices for Each Sector's Output

[98] Consumption shares in total expenditure for home-goods and home-services are equal to their initial rates. This equality in two sectors automatically necessitates the share of expenditures spent on tradable-goods to be the same as its initial rate.

As domestic demand increases over time, prices of relative sector's goods also experience changes. The evolution of each good's price is depicted in Figure 6.4. As is indicated by the figure, the prices of the products of home-goods and home-services sectors rise, while that of tradable-goods sector stays constant troughout the transition period. Thus, the price of tradable-goods declines relative to the other two sectors which makes production in tradable-goods sector less profitable compared to the other sectors. On the other hand, production becomes more profitable in home-goods and home-services sectors so that production in these sectors increase and the share of each sector's output in GDP starts to rise until the steady state as has already been presented in Table 6.11.

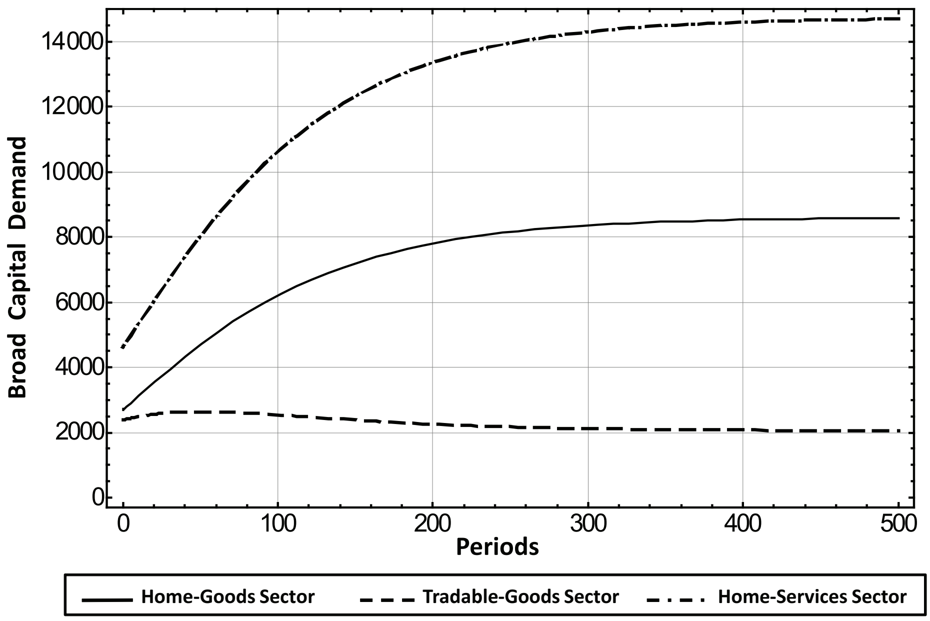

Figure 6.5: Demand for Broad Capital in Each Sector (million YTL)

As capital deepening takes place, given the production changes in each sector, labor and capital are pulled out of tradable-goods sector and employed in the home-goods and home-services sectors. As is depicted in Figure 6.5, while the demand for capital in home-goods and home-services sectors increase, that of tradable-goods sector drops over time.[99] In fact, the tradable-goods sector loses labor and capital to home-goods and home services sectors since it cannot alter production in response to capital deepening and growth due to the constant price of its goods. The other two sectors compete for the labor and capital that leave the tradable-goods sector. The new allocation of factors of production is such that the use of raw labor in home-goods sector as a share of total labor increases to a relatively higher percentage than that of the home-services sector since home-goods sector produces the most labor intensive products. The labor switch between sectors can be seen more accurately on Figure 6.6. In contrast, the use of broad capital in home-services sector as a share of total capital increases to a much higher percentage compared to that of home-goods sector which is depicted on Figure 6.7. Consequently, the allocation of raw labor and broad capital changes in favor of home-goods and home-services sectors and against the tradable-goods sector when the economy is closed to international capital flows.

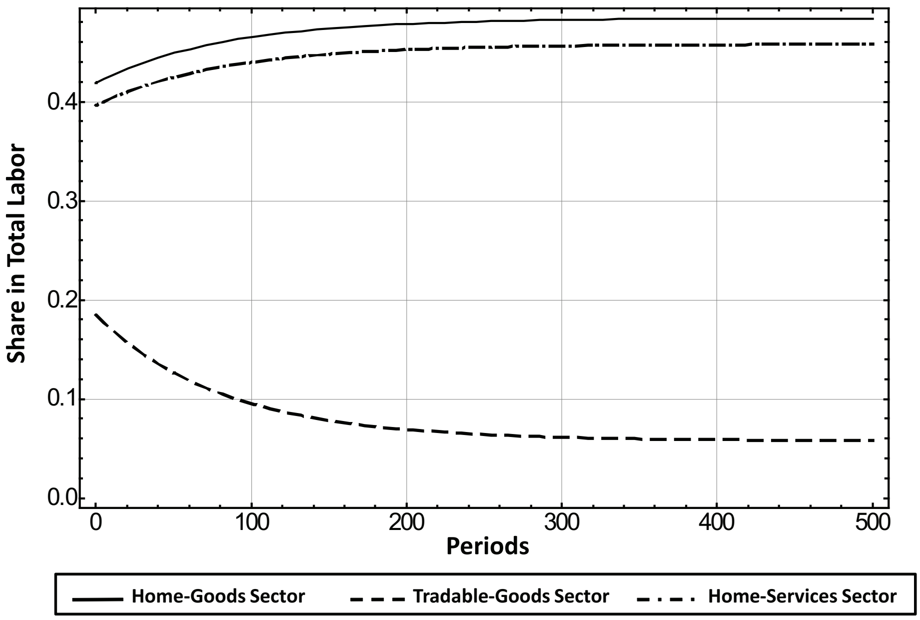

Figure 6.6: Sectoral Shares of Raw Labor in Total Labor

[99] Similar to the case in sectoral production, capital demand in tradable-goods sector experiences a sharp rise in the initial periods and then a sharp decline afterwards. This stems from the fact that even though, production in tradable-goods sector diminishes over time, production in the remaining sectors increases. Accordingly, their demand for broad capital also increase which necessitates more production in tradable-goods sector since it is the only sector that provides the required capital accumulation to the economy.

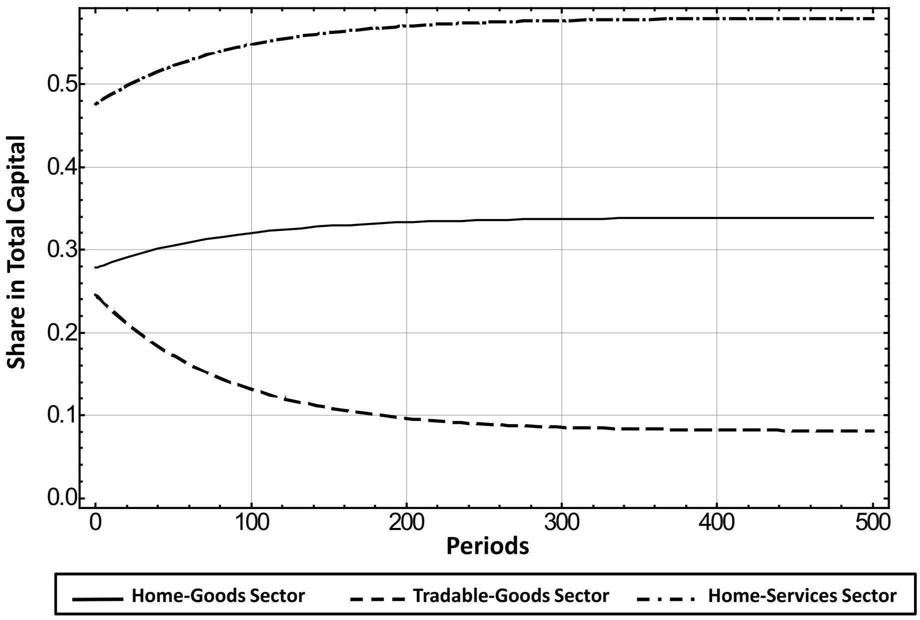

Figure 6.7: Sectoral Shares of Broad Capital in Total Capital

As a result of these changes, the share of labor income in total income rises since production moves to relatively more labor-intensive sectors. In contrast, the share of capital income diminishes but by a very small amount since the sectors that dominate production over time are also capital intensive sectors. Consequently, the transition to the steady state in a closed economy framework features an increase in accumulated capital which reinforces production in home-goods and home-services sectors, but cannot prevent the fall in production in the tradable-goods sector. Thus, the allocation of factors of production switches to the more profitable sectors which produce only for domestic consumption and away from the sector which produces for international trade purposes. In the making of this outcome, the impact of consumer demand is paramount. As has already been discussed, in the absence of international capital flows, production in each sector adjusts to domestic demand for consumption. The mechanism that brings it about is the response of each sector to the relative price changes of their good caused by demand alterations. Therefore, the closed economy model demonstrates a demand-driven economy which does not engage in international capital flows.

6.2.2 The Open Economy Model

The simulation results of the open economy model are summarized in Table 6.12. Comparing the initial period to steady state, it can be observed that there have been many changes taking place in sectoral production and the allocation of resources between sectors.

Table 6.12: Model Calibration Results

	Initial Values	**Steady State**
Production Shares in GDP (%)		
Home-Goods Sector Output	32.1	32.2
Tradable-Goods Sector Output	22.7	39.2
Home-Services Sector Output	45.2	28.6
Sectoral Allocation of Raw Labor (%)		
Home-Goods Sector	41.9	42.3
Tradable-Goods Sector	18.5	32.3
Home-Services Sector	39.6	25.4
Sectoral Allocation of Physical Capital (%)		
Home-Goods Sector	32.1	30.1
Tradable-Goods Sector	22.7	40.6
Home-Services Sector	45.2	29.3
Sectoral Allocation of Human Capital (%)		
Home-Goods Sector	23.9	23.7
Tradable-Goods Sector	26.2	45.0
Home-Services Sector	49.9	31.3
Consumption Shares in Expenditure (%)		
Home-Goods	48.1	48.1
Tradable-Goods	9.1	9.1
Home-Services	42.8	42.8
Ratio of Human Capital to Physical Capital		
Home-Goods	0.63	0.9
Tradable-Goods	0.98	1.27
Home-Services	0.94	1.22

According to the table, when the economy is partially open to international capital flows, during the transtion period, production in tradable goods sector is stimulated to increase, whereas that of home-services declines. In fact, the share of tradable-good sector's output in total production rises from 22.7% to 39.2%. On the other hand, the share of home-service sector's output falls from 45.2% to 28.6%. The increase in one sector's share nearly matches the decline in the other sector's share since the share of home-good sector's output in total production does not seem to vary much from its initial rate of 32.1%. In fact, its share rises to 32.2% which is a negligible increase in percentage points.

In the open economy framework, as the economy converges to its steady state, human capital per capita experiences an increase from its initial value to its steady state value. The human capital accumulation taking place in the economy equips each worker with more human capital so that labor productivity in each sector increases. In addition to human capital accumulation, the transition period also witnesses a rise in physical capital per capita. Given the assumption that only physical capital can be used as collateral on the world financial market against foreign debt, it is counterintuitive to observe an increase in physical capital stock over time. As is recognized in Barro et al. (1992, 1995) and Barro and Sala-i-Martin (2004), there are two factors materializing this increase. One of them is the fact that the ratio of physical capital per capita, k, to output per capita, y, in each sector stays constant throughout the transition period. Since the rate of return on physical capital is pegged at the world interest rate, the condition

$$\frac{k_1}{y_1} = \frac{k_2}{y_2} = \frac{k_3}{y_3} \tag{6.2}$$

holds not only in the initial period, but also at the steady state in the open economy framework which is also approved by the simulation results. This constancy requires that in each sector, physical capital per worker is growing at the same rate as output per worker. Thus, an increase in total income automatically brings about the accumulation in physical capital.

The second reason behind the gradual increase in physical capital stock is related with the requirements of the production functions in each sector. According to Barro et al.

(1995), since the accumulation of human capital is restricted by domestic saving and the production function necessitates a complementary relationship between human and physical capital in production, the occurence of a gradually increasing physical capital stock is not that counterintuitive. The non-tradable nature of the human capital stock limits its accumulation so that it cannot jump to its steady state amount instantaneously. Hence low values of human capital stock is observable in the initial periods. Since the production function includes both k and h, the low amount of human capital stock causes the marginal product of physical capital to be low, as well. Thus, k is lower than its steady state amount. However, based on the assumption that physical capital is financed by foreign debt, k should have converged to its steady state amount immediately instead of being low as is discussed in Barro et al.[100] Nonetheless, the low amount of human capital intervenes with the instantenous convergence of physical capital. As human capital stock increases over time to its steady state, marginal product of physical capital also rises relative to its initial value which brings about an increase in k. Consequently, the borrowing constraint on the economy leads to a gradual increase in physical capital per capita.

Given the aforementioned arguments, the open economy model experiences capital accumulation with respect to both physical and human capital stocks leading to improvements in labor productivity in each sector. As a result of higher productivity, labor wages are increased. When the simulation results of closed and open economy frameworks are compared, it can be seen that the increase in labor wages is higher in the closed economy since the initial period wages are the same in both environments. Even though the rise is lower when there are international capital flows in the economy, higher wages benefit households by increasing income levels. As a result of relatively lower wage increase, the improvement in total income is also more modest in the open economy framework. Nonetheless, a higher income level still induces the households to consume more of each good. In fact, the amount of expenditure in each sector's good increases over time. However, the share of expenditures spent on each type of good does not vary compared to their respective initial shares. This is recognized in Table 6.12 which shows that the share of expenditures spent on home-goods stays constant at its rate of 48.1%, that of tradable-goods stays constant at 9.1%

[100] ibid.

and that of home-services stays constant at 42.8%. The mechanism that brings about this constancy in consumption shares emanates from the nature of the production sectors in the economy. Since home-goods and home-services sectors are close to international trade in goods and services, all their production should be consumed by the domestic households. It means that consumption expenditure on home-goods and home-services are equal to the production taking place in respective sectors, not only in the initial period, but also at the steady state. Thus, the steady state share of consumption expenditure on each type of good is the same as its inital rate.[101]

Despite the constant shares of consumption expenditure, the increase in the consumption levels of households indicates a rising domestic demand. The improvement in domestic demand brings about changes in relative prices of goods and services. According to the results, while the prices of the products of home-goods and home-services sectors rise, that of tradable-goods sector stays constant at a given level. Thus, in comparison, the price level of the tradable-goods sector diminishes relative to the other sectors. This outcome is similar to the outcome of the closed economy model. Yet, the impact of a relatively lower price level of tradable-goods is not hampering the profitability of tradable-goods sector as in the closed economy framework. In fact, the existence of international capital flows in the economy, even though it is limited by a borrowing constraint, changed the domestic demand-determined nature of the economy. As is indicated by Table 6.12, the share of tradable-goods sector's output in total GDP, 39.2%, is no longer equal to the share of consumption expenditure on tradable-goods, 9.1%, at the steady state. The same pattern can be observed in the remaining sectors, as well. Consequently, production in the open economy is not restricted or determined by domestic demand only. The partial liberation of international capital flows enabled production to be more sensitive to foreign demand.

As capital deepening takes place in the economy, a reallocation in resources takes place. Specifically, labor and capital are pulled out of the home-services sector and employed in the tradable-goods sector contrary to the closed economy case. The results show that the use of raw labor in the tradable-goods sector as a share of total

[101] The constancy in the shares of consumption on home-goods and home-services automatically brings about the constnacy in the share of consumption expenditure on tradable-goods.

labor increases, while that of home-services sector experiences a decline. As for the home-goods sector, its use of raw labor as a share of total labor increases by a very negligible amount. As has already been stated above, the home-goods sector experiences a rather negligible increase in its production as a share of total GDP. Thus, in order to bring about that increase, the home-goods sector employs more raw labor which is the intensively used factor of production in home-goods.

When the sectoral allocation of physical capital is concerned, one can see that the use of physical capital as a share of total physical capital rises in the tradable-goods sector, while that of home-goods and home-services sectors decline. The decline in home-goods sector's share of pysical capital usage is not significant as much as that of the home-services sector. The allocation of human capital resembles the allocation of physical capital such that the use of human capital in the tradable-goods sector as a share of total human capital increased, while that of home-goods and home-services sectors declined. Again, the decline in home-goods share of usage is smaller than that of home-services sector. The results show that the physical and human capital stocks employed in the home-goods and home-services sectors are channeled to the tradable-goods sector. Thus, human capital deepening benefits the sector which produces the most human capital intensive product in the economy. The home-services sector loses raw labor to both home-goods and tradable-goods sectors even though the price of its products increased. The reason why home-goods sector is relatively less affected from this reallocation of resources is because the price of its products increased relatively more than the other sectors. Therefore, even though it is not capital intensive at all, it can compete with the other sectors for resources based on its capability of responding to relative price changes.

As a result of these changes, the share of human capital income in total income rises by a small amount since production moves to the tradable-goods sector which is the most human capital intensive sector in the economy. In contrast, the share of labor income and physical capital income diminishes over time due to the new allocation of production. The tradable-goods sector dominates production at the steady state; however, it produces the least labor intensive products in the economy. Therefore, the importance of raw labor fades away, while that of human capital grows. As for the significance of physical capital, all production functions require the same share of physical capital and none of the sectors are physical capital intensive. Moreover, the availability of financial borrowing provides easy access to physical capital. Consequently, the importance of physical capital return in household income diminishes. When a comparison is made between the respective outcomes of two models, it seems that while in the closed economy model the share of raw labor in total income increases and that of capital decreases, the open economy model presents the opposite results such that the share of human capital income increases and that of raw labor income diminishes. Thus, allowing for international capital flows in the economy not only reverses the direction of factors of production flows among sectors, but it also affects their relative significance in income.

Another impact of international capital flows can be observed by analyzing the different trends in physical and human capital stocks in closed and open economy environments. The transitional behavior of h/k ratio gives much information about the different outcomes of the respective models. In the closed economy, the rates of return to both types of capital stocks are equal; therefore, the capital stocks are considered to be the same throughout the analysis. Similar to the conclusion of Barro et al. (1992, 1995) and Barro and Sala-i-Martin (2004), in the closed economy model, the ratio of h/k in each sector is constant throughout the transition period until the steady state. Since both types of capital stocks face the same rate of return, the ratio of capital stocks is determined only by their relative elasticities which do not change over time. Hence, a constant ratio of h/k in each sector is observed. However, in the open economy framework, the rates of return are no longer the same in the initial period so that the h/k ratio changes during transition. In fact, it increases over time as can be observed from Table 6.12. Yet, the rates of return to physical capital, r_k, and human capital, r_h, are equalized at the steady state.

Even though in the initial period, the ratio in each sector is less than 1, at the steady state it increases such that certain sector's h/k ratio exceeds 1.[102] For instance, while initially h/k is equal to 0.98 in the tradable-goods sector, it increases to 1.27 at the steady state which means that the stock of human capial surpasses the stock of physical capital. Similarly, the h/k ratio in the home-services sector rises from 0.94 to

[102] According to the analysis of Erk et al. (1998), the ratio of k/h is about 4.27 in Turkey based on the time series data for the period 1960-1990.

1.22 at the steady state. Thus, in addition to the tradable-goods sector, home-services sector also employs more human capital than physical capital at the steady state. This outcome can be traced back to the relative capital intensities of each sector. In fact, the relative human capital intensities of tradable-goods and home-services sectors are very close, 0.4 and 0.42, respectively. Recall that the physical capital elasticities of each sector's production is the same. The highest h/k ratio at the steady state is recognized in the tradable-goods sector which might also contributed to the increased importance of production in tradable-goods sector relative to the other sectors.

The relatively higher physical capital stock in the initial period might stem from the availability of foreign funding in the world financial market, as also discussed by Barro et al. (1992, 1995). Thus, physical capital can be obtained much more easily and quickly than human capital in the initial period. However, towards the steady state, the accumulation of human capital exceeds that of physical capital in two sectors. Consequently, similar to the conclusions of Barro et al.,[103] the rise in h/k ratio brings about a higher impact of diminishing returns. That is, the increase in human capital leads to a faster realization of diminishing returns in the economy which also raises the speed of convergence such that it has a finite pace. When a comparison between the closed and open economy frameworks are made, one can expect that the convergence speed in the open economy is higher than that of the closed economy.

Analyzing these two frameworks, it is observed that the presence of international capital flows in the economy reverses the sectoral allocation of production and resources. Compared to the closed economy case, the sectoral allocation of production follows an opposite path in the open economy framework. In particular, when the economy does not allow international capital flows, the transition period brings about an increase in the share of home-goods and home-services sectors' production in total GDP and a drop in that of the tradable-goods sector. On the contrary, when the economy allows international capital flows with a constraint, economic development stimulates the share of tradable-good sector's production and brings about a decline in the share of home-services sector leaving the share of home-

[103] ibid.

goods sector relatively unaltered. The reversal of the outcome might be explained by two different impacts. From the demand side, the determination of production not only depends on domestic demand, but it also depends on foreign demand. Affected by the increasing foreign demand, production in the tradable-goods sector rose in the open economy model. From the supply side, the individual presence of human capital stock in the production function with an endogenous rate of return affects the production path of each sector in the open economy model. Since human capital accumulation is higher in the tradable-goods sector, production of tradable-goods increases as capital deepening takes place.

CHAPTER 7

CONCLUSION

This thesis mainly focuses on the impact of international capital flows on economic growth based on a multi-sector dynamic general equilibrium analysis. Using a three-sector Ramsey Model, a comparative analysis is conducted between the cases of financial autarky and financial integration in order to detect the movements of factors of production between sectors following a capital account liberalization. The comparative analysis is conducted by building first a closed economy model as a benchmark to represent the financial autarky case. In that sense, closed economy environment is such that international trade in goods and services is allowed but international capital flows are not. The transitional behaviours of the variables together with their steady state conditions pose as the basis for the comparative analysis. Then, in order to scrutinize the changes taking place in the economy with respect to production, consumption and reallocation of resources, international capital flows are allowed in the country. However, in order to eliminate the problems encountered in open economy versions of the Ramsey Model, a borrowing constraint assumption is imposed to the framework following the growth literature. Based on this borrowing constraint, two types of capital stocks are employed in production one of which is assumed to be non-tradable. In that sense, while physical capital can serve as collateral against foreign debt, human capital is required to be financed by domestic saving opportunities.

In both frameworks, it is assumed that there are three sectors in the economy. The home-goods sector produces agricultural and manufacturing goods for the domestic market and it does not allow for international trade or capital flows. Thus, the market clears within the economy with an endogenous price level. The tradable-goods sector produces all the internationally tradable goods in the economy at world prices. When the economy is open to international capital flows, the transactions take place witin this sector. On the other hand, the home-services sector serves only for domestic consumption and clears within the economy.

Based on these assumptions, the calibration of the closed and open economy models to Turkish economy for the year 2006 offers many conclusions about the movements of factors of production between three sectors. The transition to the steady state in a closed economy framework features an increase in the broad capital stock which reinforces production in home-goods and home-services sectors. While the respective shares of production in home-goods and home-services sectors in total GDP rise during the transition period, that of the tradable-goods sector diminishes. The change in production brings about a re-allocation of resources between sectors. Thus, raw labor and broad capital stocks are channeled from the tradable-goods sector to home-goods and home-services sectors as production in those sectors become more profitable over time. The new allocation of factors of production is such that while production in domestic consumption goods increase, production in tradable consumption goods decline. This outcome implies that the absence of international capital flows hampers the international trade prospects of the domestic country. One of the paramount outcomes of this model is the impact of consumer demand in shaping domestic production. The simulation results show that production in each sector adjusts to domestic demand for consumption until the steady state. In fact, at the steady state the share of each sector's production in total GDP is equal to the share of consumption expenditure on the corresponding good in total expenditure. The mechanism that brings it about is the response of each sector to the relative price changes of their respective goods caused by demand alterations. Therefore, the closed economy model demonstrates a domestic demand-driven environment which does not engage in international capital flows.

As the economy liberates the flow of international capital with a borrowing constraint, the simulation results demonstrate striking differences from that of the closed economy model. In the open economy framework, the transition period has not been examined; however, the comparison between the initial and the steady state values of the endogenous variables pose certain conclusions about the transitional behaviour of production, resource allocation, domestic consumption and capital accumulation. In this model, two types of capital has been analyzed. The distinction between human capital and physical capital is ensured by the borrowing constraint. Economic growth in the economy brings about capital accumulation in both types of capital. However, contrary to the closed economy model, human capital accumulates more than physical capital as can be derived from the difference between the steady state values of h/k ratios in both environments. While the transitional period in the closed economy leaves the h/k ratio constant, the ratio increases in the open economy framework.

Even though one type of capital is accumulated more than the other, the total capital accumulation brings about economic growth and a corresponding change in production. Compared to the closed economy case, the sectoral allocation of production follows an opposite path in the open economy. In particular, when the economy allows international capital flows with a constraint, the evolution to the steady state results in a higher share of tradable-goods sector's production and brings about a decline in the share of home-services sector leaving that of home-goods sector relatively unaltered. Thus, the presence of international capital flows increases the importance of production in tradable-goods sector relative to the other sectors unlike the closed economy case. The change in production also brings about a re-allocation of resources in favor of the tradable-goods sector and away from the home-goods and home-services sectors. As capital accumulation takes place, labor and both types of capital stocks are channeled to the tradable-goods sector which is the most human capital intensive sector in the economy.

In the end, starting with the same initial shares of sectoral production in total GDP, the closed economy and open economy frameworks experience different transitional changes leading to different steady state outcomes. Thus, the patterns in production, consumption and allocation of resources between sectors alter as international capital flows are liberated in the economy. When the economy is close to such flows, transitional dynamics result in capital accumulation that raises the importance of the home-goods and home-services sectors, but causes the importance of the tradable-goods sector to diminish. On the contrary, the existence of international capital flows, though restricted by a borrowing constraint, increases the importance of production in the tradable-goods sector, while it causes a decline in that of the home-services sector. As a result, opening the capital account brings about a competence between tradable-goods and home-services sectors with respect to obtaining the resources that are to be reallocated. Even though the home-goods sector is also affected by the existence of international capital flows, the changes it experienced is relatively less obvious compared to the other sectors.

Another significant feature of capital account liberalization is the fact that domestic demand ceases to be decisive in the share of sectoral production in total GDP when international capital flows are allowed in the economy. Even though in the closed economy model, the steady state shares of each sector's production is determined by domestic consumption, it is no longer the case in the open economy framework. In other words, domestic production does not have to adjust to consumption demand of domestic agents anymore. The presence of partial international capital flows raises the impact of foreign demand on domestic production reducing the significance and limitation of domestic demand.

These findings are consistent with the general literature and in line with the common wisdom concerning the growth enhancing effect of capital account liberalization. The opposite outcomes of closed and open economy models might give insight on the impact of international capital flows which can be used in policy recommendations to enhance the growth-inducing effects of capital account liberalization.

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APPENDIX A

SOLUTIONS OF THE CLOSED ECONOMY MODEL

A.1 Production Functions

A.1.1 Home-Goods Sector

The production function in home-goods sector is given by

$$Y_1 = A_1 L_1^{\alpha_1} K_1^{\alpha_2} H_1^{1-\alpha_1-\alpha_2} \tag{A.1}$$

In a closed economy framework, the net returns on both kinds of capital should be equal to the interest rate, r. The rate of return to physical capital is

$$MP_K = \alpha_2 \frac{Y_1}{K_1} = r \tag{A.2}$$

The rate of return to human capital is

$$MP_H = (1 - \alpha_1 - \alpha_2) \frac{Y_1}{H_1} = r \tag{A.3}$$

Equating (A.2) and (A.3), we have the following condition:

$$\frac{K_1}{H_1} = \frac{\alpha_2}{1 - \alpha_1 - \alpha_2} \tag{A.4}$$

Since the returns on both kinds of capital are the same and there are no adjustment costs or irreversibility constraints, we can define a broad capital stock in the production function.

If we rearrange (A.4) we can express the physical capital and human capital as functions of Z_1. The physical capital function in sector-1 is

$$K_1 = \frac{\alpha_2}{1 - \alpha_1} Z_1 \tag{A.5}$$

The human capital function in sector-1 is

$$H_1 = \frac{1-\alpha_1-\alpha_2}{1-\alpha_1} Z_1 \tag{A.6}$$

Plugging (A.5) and (A.6) into (A.1), we obtain the production function at home-goods sector for the closed economy.

$$Y_1 = A_1\, L_1^{\alpha_1} \left(\frac{\alpha_2}{1-\alpha_1} Z_1\right)^{\alpha_2} \left(\frac{1-\alpha_1-\alpha_2}{1-\alpha_1} Z_1\right)^{1-\alpha_1-\alpha_2} \tag{A.7}$$

Rearranging, we have

$$Y_1 = \tilde{A}_1\, L_1^{\alpha_1}\, Z_1^{1-\alpha_1} \tag{A.8}$$

where $\tilde{A}_1 = A_1\,(1-\alpha_1)^{\alpha_1-1}\,\alpha_2^{\alpha_2}\,(1-\alpha_1-\alpha_2)^{1-\alpha_1-\alpha_2}$.

A.1.2 Tradable-Goods Sector

We will follow the same approach in this section as well. The production function in tradable-goods sector is

$$Y_2 = A_2\, L_2^{\beta_1}\, K_2^{\beta_2}\, H_2^{1-\beta_1-\beta_2} \tag{A.9}$$

Equating the net returns on both kinds of capital yields the following condition

$$\frac{K_2}{H_2} = \frac{\beta_2}{1-\beta_1-\beta_2} \tag{A.10}$$

Using (A.10) in (A.9), the production function of tradable-goods sector is found to be

$$Y_2 = \tilde{A}_2\, L_2^{\beta_1}\, Z_2^{1-\beta_1} \tag{A.11}$$

where $\tilde{A}_2 = A_2\,(1-\beta_1)^{\beta_1-1}\,\beta_2^{\beta_2}\,(1-\beta_1-\beta_2)^{1-\beta_1-\beta_2}$.

A.1.3 Home-Services Sector

We follow the similar procedure again. The production function in the home-service goods sector is

$$Y_3 = A_3\, L_3^{\delta_1}\, K_3^{\delta_2}\, H_3^{1-\delta_1-\delta_2} \tag{A.12}$$

Equating the net returns on both kinds of capital yields

$$\frac{K_3}{H_3} = \frac{\delta_2}{1-\delta_1-\delta_2} \tag{A.13}$$

Using (A.13) in (A.12), the production function of home-services sector is found to be

$$Y_3 = \tilde{A}_3 \, L_3^{\delta_1} \, Z_3^{1-\delta_1} \tag{A.14}$$

where $\tilde{A}_3 = A_3 \, (1-\delta_1)^{\delta_1 - 1} \, \delta_2^{\delta_2} \, (1-\delta_1-\delta_2)^{1-\delta_1-\delta_2}$.

A.2 Household's Intra-Temporal Problem

Households choose bundles of consumption goods that minimize their expenditures subject to their composite consumption at each point in time. The minimization problem is as follows:

$$\begin{aligned} \min \quad & p_1 \, c_1 + p_2 \, c_2 + p_3 \, c_3 \\ \text{subject to} \quad & c = \beta \, c_1^{\gamma_1} \, c_2^{\gamma_2} \, c_3^{\gamma_3} \\ & c_1 > 0 \\ & c_2 > 0 \\ & c_3 > 0 \end{aligned}$$

The first order conditions yield

$$\frac{p_1 \, c_1}{p_2 \, c_2} = \frac{\gamma_1}{\gamma_2}$$

$$\frac{p_1 \, c_1}{p_3 \, c_3} = \frac{\gamma_1}{\gamma_3}$$

Rearranging we have

$$c_2 = \frac{\gamma_2}{\gamma_1} \, \frac{p_1}{p_2} \, c_1 \tag{A.15}$$

$$c_3 = \frac{\gamma_3}{\gamma_1} \, \frac{p_1}{p_3} \, c_1 \tag{A.16}$$

Plug (A.15) and (A.16) into the composite consumption function

$$c = \frac{1}{\beta} \, c_1^{\gamma_1} \left(\frac{\gamma_2}{\gamma_1} \, \frac{p_1}{p_2} \, c_1 \right)^{\gamma_2} \left(\frac{\gamma_3}{\gamma_1} \, \frac{p_1}{p_3} \, c_1 \right)^{\gamma_3}$$

Solving for c_1 we have

$$c_1 = \frac{1}{\beta} \, \gamma_1^{-\gamma_1} \, \gamma_2^{-\gamma_2} \, \gamma_3^{-\gamma_3} \, \gamma_1 \, c \, p_1^{\gamma_1 - 1} \, p_2^{\gamma_2} \, p_3^{\gamma_3}$$

If we set $\beta = \gamma_1^{-\gamma_1}\ \gamma_2^{-\gamma_2}\ \gamma_3^{-\gamma_3}$;

$$c_1 = \gamma_1\ c\ p_1^{\gamma_1 - 1}\ p_2^{\gamma_2}\ p_3^{\gamma_3} \tag{A.17}$$

Plug (A.17) into (A.15) and (A.16)

$$c_2 = \gamma_2\ c\ p_1^{\gamma_1}\ p_2^{\gamma_2 - 1}\ p_3^{\gamma_3} \tag{A.18}$$

$$c_3 = \gamma_3\ c\ p_1^{\gamma_1}\ p_2^{\gamma_2}\ p_3^{\gamma_3 - 1} \tag{A.19}$$

Substituting (A.17), (A.18) and (A.19) into the expenditure function, we obtain the minimum expenditure of the household

$$E\,(c, p) = p_1\ \gamma_1\ c\ p_1^{\gamma_1 - 1}\ p_2^{\gamma_2}\ p_3^{\gamma_3} + p_2\ \gamma_2\ c\ p_1^{\gamma_1}\ p_2^{\gamma_2 - 1}\ p_3^{\gamma_3} + p_3\ \gamma_3\ c\ p_1^{\gamma_1}\ p_2^{\gamma_2}\ p_3^{\gamma_3 - 1}$$

$$E\,(c, p) = c\ p_1^{\gamma_1}\ p_2^{\gamma_2}\ p_3^{\gamma_3} \tag{A.20}$$

A.3 Firms' Cost Minimization Problems

A.3.1 Home-Goods Sector

At each period in time, firms in home-goods sector choose factors of production that minimizes their costs subject to the production function of the sector as follows:

$$\begin{aligned} \min \quad & w\,L_1 + r\,Z_1 \\ \text{subject to} \quad & Y_1 \leq \tilde{A}_1\ L_1^{\alpha_1}\ Z_1^{1-\alpha_1} \\ & L_1 \ \geq\ 0 \\ & Z_1 \ \geq\ 0 \end{aligned}$$

The Lagrangian of this minimization problem is

$$\mathcal{L} = w\,L_1 + r\,Z_1 + \lambda_1 \left(Y_1 - \tilde{A}_1\ L_1^{\alpha_1}\ Z_1^{1-\alpha_1} \right)$$

The first order conditions yield

$$\frac{\partial \mathcal{L}}{\partial L_1} = w - \lambda_1\ \tilde{A}_1\ \alpha_1\ L_1^{\alpha_1 - 1}\ Z_1^{1-\alpha_1} = 0 \tag{A.21}$$

$$\frac{\partial \mathcal{L}}{\partial Z_1} = r - \lambda_1\ \tilde{A}_1\ (1 - \alpha_1)\ L_1^{\alpha_1}\ Z_1^{-\alpha_1} = 0 \tag{A.22}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = Y_1 - \tilde{A}_1\ L_1^{\alpha_1}\ Z_1^{1-\alpha_1} = 0 \tag{A.23}$$

Equating (A.21) and (A.22) we obtain

$$L_1 = \frac{\alpha_1}{1-\alpha_1} \frac{r}{w} Z_1 \tag{A.24}$$

Plug (A.24) into (A.23) to obtain Z_1

$$Z_1 = \frac{Y_1}{\tilde{A}_1} \left(\frac{1-\alpha_1}{\alpha_1}\right)^{\alpha_1} \left(\frac{w}{r}\right)^{\alpha_1} \tag{A.25}$$

Substituting (A.24) into (A.25) yields L_1 as

$$L_1 = \frac{Y_1}{\tilde{A}_1} \left(\frac{1-\alpha_1}{\alpha_1}\right)^{\alpha_1-1} \left(\frac{w}{r}\right)^{\alpha_1-1} \tag{A.26}$$

Plugging (A.25) and (A.26) into the total cost function, we obtain

$$TC_1^* = w \left[\frac{Y_1}{\tilde{A}_1} \left(\frac{1-\alpha_1}{\alpha_1}\right)^{\alpha_1-1} \left(\frac{w}{r}\right)^{\alpha_1-1}\right] + r \left[\frac{Y_1}{\tilde{A}_1} \left(\frac{1-\alpha_1}{\alpha_1}\right)^{\alpha_1} \left(\frac{w}{r}\right)^{\alpha_1}\right]$$
$$TC_1^* = \frac{Y_1}{\tilde{A}_1} w^{\alpha_1} r^{1-\alpha_1} (1-\alpha_1)^{\alpha_1-1} \alpha_1^{-\alpha_1}$$

If we set $\tilde{A}_1 = a_1 \alpha_1^{-\alpha_1} (1-\alpha_1)^{\alpha_1-1}$, the minimized total cost will be

$$TC_1^* = Y_1 \frac{w^{\alpha_1} r^{1-\alpha_1}}{a_1} \tag{A.27}$$

A.3.2 Tradable-Goods Sector

The cost minimization problem of tradable-goods sector is as follows

$$\begin{aligned} \min \quad & w L_2 + r Z_2 \\ \text{subject to} \quad & Y_2 \le \tilde{A}_2 L_2^{\beta_1} Z_2^{1-\beta_1} \\ & L_2 \ge 0 \\ & Z_2 \ge 0 \end{aligned}$$

Using the first order conditions, we obtain the following equations

$$L_2 = \frac{Y_2}{\tilde{A}_2} \left(\frac{1-\beta_1}{\beta_1}\right)^{\beta_1-1} \left(\frac{w}{r}\right)^{\beta_1-1} \tag{A.28}$$

$$Z_2 = \frac{Y_2}{\tilde{A}_2} \left(\frac{1-\beta_1}{\beta_1}\right)^{\beta_1} \left(\frac{w}{r}\right)^{\beta_1} \tag{A.29}$$

Plugging (A.28) and (A.29) into the total cost function, we obtain

$$TC_2^* = w \left[\frac{Y_2}{\tilde{A}_2} \left(\frac{1-\beta_1}{\beta_1} \right)^{\beta_1 - 1} \left(\frac{w}{r} \right)^{\beta_1 - 1} \right] + r \left[\frac{Y_2}{\tilde{A}_2} \left(\frac{1-\beta_1}{\beta_1} \right)^{\beta_1} \left(\frac{w}{r} \right)^{\beta_1} \right]$$

$$TC_2^* = \frac{Y_2}{\tilde{A}_2} \, w^{\beta_1} \, r^{1-\beta_1} \, (1-\beta_1)^{\beta_1 - 1} \, \beta_1^{-\beta_1}$$

If we set $\tilde{A}_2 = a_2 \, \beta_1^{-\beta_1} \, (1-\beta_1)^{\beta_1 - 1}$, the minimized total cost will be

$$TC_2^* = Y_2 \, \frac{w^{\beta_1} \, r^{1-\beta_1}}{a_2} \tag{A.30}$$

A.3.3 Home-Services Sector

The cost minimization problem of the home-services sector is as follows

$$\begin{aligned} \min \quad & w \, L_3 + r \, Z_3 \\ \text{subject to} \quad & Y_3 \le \tilde{A}_3 \, L_3^{\delta_1} \, Z_3^{1-\delta_1} \\ & L_3 \ge 0 \\ & Z_3 \ge 0 \end{aligned}$$

Using the first order conditions, we obtain the following equations

$$L_3 = \frac{Y_3}{\tilde{A}_3} \left(\frac{1-\delta_1}{\delta_1} \right)^{\delta_1 - 1} \left(\frac{w}{r} \right)^{\delta_1 - 1} \tag{A.31}$$

$$Z_3 = \frac{Y_3}{\tilde{A}_3} \left(\frac{1-\delta_1}{\delta_1} \right)^{\delta_1} \left(\frac{w}{r} \right)^{\delta_1} \tag{A.32}$$

Plugging (A.31) and (A.32) into the total cost function, we obtain

$$TC_3^* = w \left[\frac{Y_3}{\tilde{A}_3} \left(\frac{1-\delta_1}{\delta_1} \right)^{\delta_1 - 1} \left(\frac{w}{r} \right)^{\delta_1 - 1} \right] + r \left[\frac{Y_3}{\tilde{A}_3} \left(\frac{1-\delta_1}{\delta_1} \right)^{\delta_1} \left(\frac{w}{r} \right)^{\delta_1} \right]$$

$$TC_3^* = \frac{Y_3}{\tilde{A}_3} \, w^{\delta_1} \, r^{1-\delta_1} \, (1-\delta_1)^{\delta_1 - 1} \, \delta_1^{-\delta_1}$$

If we set $\tilde{A}_3 = a_3 \, \delta_1^{-\delta_1} \, (1-\delta_1)^{\delta_1 - 1}$, the minimized total cost will be

$$TC_3^* = Y_3 \, \frac{w^{\delta_1} \, r^{1-\delta_1}}{a_3} \tag{A.33}$$

APPENDIX B

SOLUTIONS OF THE OPEN ECONOMY MODEL

B.1 Firms' Cost Minimization Problems

B.1.1 Home-Goods Sector

At each period in time, firms in home-goods sector choose factors of production that minimizes their costs subject to the production function of the sector as follows:

$$\begin{aligned} \min \quad & w\,L_1 + r_k\,K_1 + r_h\,H_1 \\ \text{subject to} \quad & Y_1 \leq A_1\; L_1^{\alpha_1}\; K_1^{\alpha_2}\; H_1^{1-\alpha_1-\alpha_2} \\ & L_1 \;\geq\; 0 \\ & K_1 \;\geq\; 0 \\ & H_1 \;\geq\; 0 \end{aligned}$$

The Lagrangian of this minimization problem is

$$\mathcal{L} = wL_1 + r_kK_1 + r_hH_1 + \lambda_1\left(Y_1 - A_1L_1^{\alpha_1}K_1^{\alpha_2}H_1^{1-\alpha_1-\alpha_2}\right)$$

The first order conditions yield

$$\frac{\partial \mathcal{L}}{\partial L_1} = w - \lambda_1\alpha_1\;A_1\;L_1^{\alpha_1-1}\;K_1^{\alpha_2}\;H_1^{1-\alpha_1-\alpha_2} = 0 \tag{B.1}$$

$$\frac{\partial \mathcal{L}}{\partial K_1} = r_k - \lambda_1\alpha_2\;A_1\;L_1^{\alpha_1}\;K_1^{\alpha_2-1}\;H_1^{1-\alpha_1-\alpha_2} = 0 \tag{B.2}$$

$$\frac{\partial \mathcal{L}}{\partial H_1} = r_h - \lambda_1\left(1-\alpha_1-\alpha_2\right)\;A_1\;L_1^{\alpha_1}\;K_1^{\alpha_2}\;H_1^{-\alpha_1-\alpha_2} = 0 \tag{B.3}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = Y_1 - A_1\;L_1^{\alpha_1}\;K_1^{\alpha_2}\;H_1^{1-\alpha_1-\alpha_2} = 0 \tag{B.4}$$

Equating (B.1) and (B.2) we obtain

$$L_1 = \frac{\alpha_1}{\alpha_2} \frac{r_k}{w} K_1 \tag{B.5}$$

Equating (B.2) and (B.3) we obtain

$$H_1 = \frac{1-\alpha_1-\alpha_2}{\alpha_2} \frac{r_k}{r_h} K_1 \tag{B.6}$$

Plug (B.5) and (B.6) into (B.4)

$$Y_1 = A_1 \left(\frac{\alpha_1}{\alpha_2} \frac{r_k}{w} K_1\right)^{\alpha_1} K_1^{\alpha_2} \left(\frac{1-\alpha_1-\alpha_2}{\alpha_2} \frac{r_k}{r_h} K_1\right)^{1-\alpha_1-\alpha_2} \tag{B.7}$$

Solving for K_1 we have

$$K_1 = \frac{Y_1}{A_1} \alpha_2 \, w^{\alpha_1} \, {r_k}^{\alpha_2-1} \, {r_h}^{1-\alpha_1-\alpha_2} \, {\alpha_1}^{-\alpha_1} \, {\alpha_2}^{-\alpha_2} (1-\alpha_1-\alpha_2)^{\alpha_1+\alpha_2-1} \tag{B.8}$$

Plug (B.8) into (B.5) and (B.6) to get

$$\begin{aligned} L_1 &= \frac{Y_1}{A_1} \alpha_1 \, w^{\alpha_1-1} \, r_k^{\alpha_2} \, r_h^{1-\alpha_1-\alpha_2} \, \alpha_1^{-\alpha_1} \, \alpha_2^{-\alpha_2} \, (1-\alpha_1-\alpha_2)^{\alpha_1+\alpha_2-1} \\ H_1 &= \frac{Y_1}{A_1} (1-\alpha_1-\alpha_2) \, w^{\alpha_1} \, r_k^{\alpha_2} \, r_h^{-\alpha_1-\alpha_2} \, \alpha_1^{-\alpha_1} \, \alpha_2^{-\alpha_2} \, (1-\alpha_1-\alpha_2)^{\alpha_1+\alpha_2-1} \end{aligned}$$

If we set $A_1 = a_1 \, \alpha_1^{-\alpha_1} \, \alpha_2^{-\alpha_2} \, (1-\alpha_1-\alpha_2)^{\alpha_1+\alpha_2-1}$, the equations will become

$$K_1 = \frac{Y_1}{a_1} \alpha_2 \, w^{\alpha_1} \, {r_k}^{\alpha_2-1} \, {r_h}^{1-\alpha_1-\alpha_2} \tag{B.9}$$

$$L_1 = \frac{Y_1}{a_1} \alpha_1 \, w^{\alpha_1-1} \, r_k^{\alpha_2} \, r_h^{1-\alpha_1-\alpha_2} \tag{B.10}$$

$$H_1 = \frac{Y_1}{a_1} (1-\alpha_1-\alpha_2) \, w^{\alpha_1} \, r_k^{\alpha_2} \, r_h^{-\alpha_1-\alpha_2} \tag{B.11}$$

The total cost function is known to be

$$TC_1 = w \, L_1 + r_k \, K_1 + r_h \, H_1$$

Substituting (B.9), (B.10) and (B.11) into the total cost function

$$\begin{aligned} TC_1 = {} & w \left(\frac{Y_1}{a_1} \alpha_1 \, w^{\alpha_1-1} \, r_k^{\alpha_2} \, r_h^{1-\alpha_1-\alpha_2}\right) + r_k \left(\frac{Y_1}{a_1} \alpha_2 \, w^{\alpha_1} \, {r_k}^{\alpha_2-1} \, {r_h}^{1-\alpha_1-\alpha_2}\right) \\ & + r_h \left(\frac{Y_1}{a_1} (1-\alpha_1-\alpha_2) \, w^{\alpha_1} \, r_k^{\alpha_2} \, r_h^{-\alpha_1-\alpha_2}\right) \end{aligned} \tag{B.12}$$

Rearranging, we obtain the minimized total cost of the home-goods sector as follows:

$$TC_1 = Y_1 \frac{w^{\alpha_1} r_k^{\alpha_2} r_h^{1-\alpha_1-\alpha_2}}{a_1} \tag{B.13}$$

B.1.2 Tradable-Goods Sector

The cost minimization problem of tradable-goods sector is as follows

$$
\begin{aligned}
\min \quad & w\, L_2 + r_k\, K_2 + r_h\, H_2 \\
\text{subject to} \quad & Y_2 \leq A_2\, L_2^{\beta_1}\; K_2^{\beta_2}\; H_2^{1-\beta_1-\beta_2} \\
& L_2 \;\geq\; 0 \\
& K_2 \;\geq\; 0 \\
& H_2 \;\geq\; 0
\end{aligned}
$$

The Lagrangian of this minimization problem is

$$\mathcal{L} = wL_2 + r_k K_2 + r_h H_2 + \lambda_2 \left(Y_2 - A_2 L_2^{\beta_1} K_2^{\beta_2} H_2^{1-\beta_1-\beta_2} \right)$$

The first order conditions yield

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial L_2} &= w - \lambda_2 \beta_1\; A_2\; L_2^{\beta_1 - 1}\; K_2^{\beta_2}\; H_2^{1-\beta_1-\beta_2} = 0 \\
\frac{\partial \mathcal{L}}{\partial K_2} &= r_k - \lambda_2 \beta_2\; A_2\; L_2^{\beta_1}\; K_2^{\beta_2 - 1}\; H_2^{1-\beta_1-\beta_2} = 0 \\
\frac{\partial \mathcal{L}}{\partial H_2} &= r_h - \lambda_2\, (1 - \beta_1 - \beta_2)\; A_2\; L_2^{\beta_1}\; K_2^{\beta_2}\; H_2^{-\beta_1-\beta_2} = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda_2} &= Y_2 - A_2\; L_2^{\beta_1}\; K_2^{\beta_2}\; H_2^{1-\beta_1-\beta_2} = 0
\end{aligned}
$$

Using the first order conditions, we obtain the following equations

$$L_2 = \frac{Y_2}{a_2}\, \beta_1\; w^{\beta_1 - 1}\; r_k^{\beta_2}\; r_h^{1-\beta_1-\beta_2} \tag{B.14}$$

$$K_2 = \frac{Y_2}{a_2}\, \beta_2\; w^{\beta_1}\; r_k^{\beta_2 - 1}\; r_h^{1-\beta_1-\beta_2} \tag{B.15}$$

$$H_2 = \frac{Y_2}{a_2}\, (1 - \beta_1 - \beta_2)\; w^{\beta_1}\; r_k^{\beta_2}\; r_h^{-\beta_1-\beta_2} \tag{B.16}$$

where $a_2 = \frac{A_2}{\beta_1^{-\beta_1}\; \beta_2^{-\beta_2}\; (1-\beta_1-\beta_2)^{\beta_1+\beta_2-1}}$.

Substituting (B.14), (B.15) and (B.16) into the total cost function

$$
\begin{aligned}
TC_2 = w\, &\left(\frac{Y_2}{a_2}\, \beta_1\; w^{\beta_1 - 1}\; r_k^{\beta_2}\; r_h^{1-\beta_1-\beta_2} \right) + r_k \left(\frac{Y_2}{a_2}\, \beta_2\; w^{\beta_1}\; r_k^{\beta_2 - 1}\; r_h^{1-\beta_1-\beta_2} \right) \\
&+ r_h \left(\frac{Y_2}{a_2}\, (1 - \beta_1 - \beta_2)\; w^{\beta_1}\; r_k^{\beta_2}\; r_h^{-\beta_1-\beta_2} \right)
\end{aligned} \tag{B.17}
$$

Rearranging, we obtain the minimized total cost of the home-goods sector as follows:

$$TC_2 = Y_2\; \frac{w^{\beta_1} r_k^{\beta_2} r_h^{1-\beta_1-\beta_2}}{a_2} \tag{B.18}$$

B.1.3 Home-Services Sector

The cost minimization problem of home-services sector is as follows

$$\begin{aligned} \min \quad & w\,L_3 + r_k\,K_3 + r_h\,H_3 \\ \text{subject to} \quad & Y_3 \leq A_3\,L_3^{\delta_1}\,K_3^{\delta_2}\,H_3^{1-\delta_1-\delta_2} \\ & L_3 \geq 0 \\ & K_3 \geq 0 \\ & H_3 \geq 0 \end{aligned}$$

The Lagrangian of this minimization problem is

$$\mathcal{L}_3 = wL_3 + r_k K_3 + r_h H_3 + \lambda_3 \left(Y_3 - A_3 L_3^{\delta_1} K_3^{\delta_2} H_3^{1-\delta_1-\delta_2} \right)$$

The first order conditions yield

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial L_3} &= w - \lambda_3\,\delta_1\,A_3\,L_3^{\delta_1-1}\,K_3^{\delta_2}\,H_3^{1-\delta_1-\delta_2} = 0 \\ \frac{\partial \mathcal{L}}{\partial K_3} &= r_k - \lambda_3\,\delta_2\,A_3\,L_3^{\delta_1}\,K_3^{\delta_2-1}\,H_3^{1-\delta_1-\delta_2} = 0 \\ \frac{\partial \mathcal{L}}{\partial H_3} &= r_h - \lambda_3\,(1-\delta_1-\delta_2)\,A_3\,L_3^{\delta_1}\,K_3^{\delta_2}\,H_3^{-\delta_1-\delta_2} = 0 \\ \frac{\partial \mathcal{L}}{\partial \lambda_3} &= Y_3 - A_3\,L_3^{\delta_1}\,K_3^{\delta_2}\,H_3^{1-\delta_1-\delta_2} = 0 \end{aligned}$$

Using the first order conditions, we obtain the following equations

$$L_3 = \frac{Y_3}{a_3}\,\delta_1\,w^{\delta_1-1}\,r_k^{\delta_2}\,r_h^{1-\delta_1-\delta_2} \tag{B.19}$$

$$K_3 = \frac{Y_3}{a_3}\,\delta_2\,w^{\delta_1}\,r_k^{\delta_2-1}\,r_h^{1-\delta_1-\delta_2} \tag{B.20}$$

$$H_3 = \frac{Y_3}{a_3}\,(1-\delta_1-\delta_2)\,w^{\delta_1}\,r_k^{\delta_2}\,r_h^{-\delta_1-\delta_2} \tag{B.21}$$

where $a_3 = \frac{A_3}{\delta_1^{-\delta_1}\,\delta_2^{-\delta_2}\,(1-\delta_1-\delta_2)^{\delta_1+\delta_2-1}}$.

Substituting (B.19), (B.20) and (B.21) into the total cost function

$$\begin{aligned} TC_3 = {} & w\left(\frac{Y_3}{a_3}\,\delta_1\,w^{\delta_1-1}\,r_k^{\delta_2}\,r_h^{1-\delta_1-\delta_2}\right) + r_k\left(\frac{Y_3}{a_3}\,\delta_2\,w^{\delta_1}\,r_k^{\delta_2-1}\,r_h^{1-\delta_1-\delta_2}\right) \\ & + r_h\left(\frac{Y_3}{a_3}\,(1-\delta_1-\delta_2)\,w^{\delta_1}\,r_k^{\delta_2}\,r_h^{-\delta_1-\delta_2}\right) \end{aligned} \tag{B.22}$$

Rearranging we obtain the minimized total cost of the home-goods sector as follows:

$$TC_3 = Y_3\,\frac{w^{\delta_1} r_k^{\delta_2} r_h^{1-\delta_1-\delta_2}}{a_3} \tag{B.23}$$