ACOUSTIC WAVE ANALYSIS USING DIFFERENT WAVE PROPAGATION MODELS

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ABSTRACT

ACOUSTIC WAVE ANALYSIS USING DIFFERENT WAVE PROPAGATION MODELS

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In this study in order to simulate the acoustic waves, Ray Theory and Normal Mode models are used. These methods are analyzed using MATLAB simulation tool; differences between two models are examined and a region with a known bottom profile and sound velocity profiles is investigated. The Ray Theory is used in acoustic systems which is one of the applications of wave modeling. Ray theory is solved with standard Ordinary Differential Equation solvers and normal mode with finite element method. Different bottom profiles and sound velocity profiles previously taken are interpolated to form an environment and examined in the case study.

Keywords: Ray Theory, Normal Mode, Sonar Systems, Runge Kutta, Finite Differences
ÖZ

AKUSTİK DALGA DENKLEMLERİNIN DEĞİŞİK YAYILMA METODLARI KULLANILARAK ANALİZİ

Yıldırım, Baran
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Bu çalışmada akustik dalga modelleme yöntemlerinden ışın teoremi ve normal mod yöntemi kullanılmıştır. Bu metodlar MATLAB aracılığıyla çözülmüş farkları incelenmiş ve dip profilleri belli olan bir bölümün akustik incelemesi yapılmıştır. ışın teoremi, akustik uygulama alanlarından biri olan sonar sistemlerinde kullanılmıştır. ışın teoremi standart difransiyel denklem çözücüleri ile (Runga Kutta), Normal Mod sonlu elemanlar yöntemleri ile çözülmüştür. Daha önceden alınmış dip profileri ve ses hız profilleri interpolasyon yöntemi ile şekillendirilip değişik analiz ortamları oluşturulmuş ve durum çalışmasında incelenmiştir.

Anahtar Kelimeler: ışın teoremi, Normal Mod, Sonar Sistemleri, Runga Kutta, Sonlu Elemanlar
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$\theta$  Grazing Angle

$k$  Wavenumber

SVP  Sound Velocity Profile

DT  Detection Threshold

PD  Probability of Detection

PFA  Probability of False Alarm

PDF  Probability Density Functions

SNR  Signal to Noise Ratio

TL  Transmission Loss

SL  Source Level

N  Noise

AG  Array Gain

SE  Signal Excess

FOM  Figure of Merit

RL  Reverberation Level

TS  Target Strength

MFP  Match Field Processing
CHAPTER 1

INTRODUCTION

1.1 HISTORICAL BACKGROUND

Acoustics is primarily a matter of communication. The phenomenon of speaking is based on acoustic waves and it has been the base of early human communication. Recently, the introduction of Electromagnetic (EM) Waves has been a great evolution in human communication systems. However, EM waves cannot propagate much through the sea medium, and acoustic waves is still the most efficient way to transmit information through this medium. Therefore, underwater acoustics should be studied in detail.

The first acoustic studies started with Pythagoras[3]. He developed the theory of the musical scale in terms of a device called a monochord. He recognized that the lengths of these strings were inversely proportional to the frequency of sound generated when plucked. Galileo, an Italian astronomer and physicist, was the greatest contributor to our understanding of sound. He demonstrated that the frequency of sound waves determined the pitch. This was done by scraping a chisel across a brass plate producing a screech. Galileo then related the spacing of the grooves induced by the chisel to the pitch of the screech. Leonardo DaVinci discovered that sound travels in waves. In 17th century, Marin Mersenne was the first measured the speed of sound in air, Robert Boyle discovered that sound waves must travel in a medium and Sir Isaac Newton formulated a relationship between the speed of sound in a medium and the density and compressibility in a medium.
Impressive lists of physicists and mathematicians from Galileo, Newton to Rayleigh, and beyond have made major contributions to the theory of acoustics in gases, liquids, and solids. For example, Euler’s and Lagrange’s ideas on sound propagation were studied by D’Alembert[3]. He was the first to write down the partial differential equation describing the motion of a vibrating string which is now referred to as the wave equation. He developed a method to solve this type of equation: separation of variables. Newton is normally credited with the first theoretical attempt to describe sound propagation in a fluid. The development of acoustics theory has followed a different historical path than optics and electromagnetic theory. While there has been a long running battle in optics between ray and wave theory, acoustics originally developed with wave and vibration concepts as far back as ancient Greek times. For example, musical sound is the result of air motion generated by a vibrating musical instrument. This sound is propagated in an analogous manner to water surface waves which propagate disturbances (signals), but does not propagate matter to long distances. The bending of sound around corners is then easily understood in the context of diffraction in the water surface wave analogy.

Analytical investigations of sound propagation in an inhomogeneous medium are usually based on normal mode theory, the parabolic approximation, or ray theory. Each theory has its strengths and weaknesses. In normal mode theory the acoustic wave equation is solved explicitly. A complete solution in normal modes is often difficult. Nevertheless, valuable results have been obtained (Chunchuzov (1985), Zorumski and Willshire (1986)) but, the application of normal mode theory to atmospheric noise propagation problems depends on being in the far field, so it cannot be used to study data near the source. Although the parabolic approximation is used in ocean acoustics with good results, its use in atmospheric acoustics is relatively unknown. However, White and Gilbert (1986), using the parabolic approximation, also obtained good agreement between theory and
Willshire’s 1985 data at long ranges. Again, however, predictions for near-source ranges were in question. As the normal mode theory is very complicated and difficult to interpret, ray theory is often employed as an alternative or as a first approximation to describing the sound field. Ray theory has the advantages of being easy to use and of providing a simple visualization of the sound field. As a result, in underwater acoustics, in which the medium is inhomogeneous and bounded by complicated interfaces, ray theory has historically proved to be an indispensable tool for understanding and studying sound propagation. However, because it is a high frequency approximation of the wave equation, ray theory also has limits of applicability.

1.2 SCOPE OF THE THESIS

The thesis is about sound in underwater, 2-D Ray Theory and Normal Mode theory are examined. After some comparisons, Ray Theory is used during modeling. Different types of sound speed profiles and bottom profiles are discussed during the case study.

This is an introductory study for acoustics, the conventional models are used. The acoustic wave modeling of a specific region is required before the settlements of acoustic sensors, to find the right places of this settlement the modeling of this region should be done.

1.3 SUMMARY OF THE THESIS

The thesis is composed of 4 chapters.

In Chapter 1, an introduction is given. Starting from the necessity of acoustic studies, the summary of acoustic studies is explained also the development of the
wave models is introduced.

Chapter 2 is started with some basic definitions for acoustic waves, then a brief explanation of acoustic wave models and mathematical formulations of the ray theory are given. Ray theory section is finalized with ray theory simulations. The same flowing is followed for the Normal Mode. Thus; mathematical formulations and simulations are given for the Normal Mode. This chapter is finalized with discussion of the two models.

In Chapter 3 a case study is analyzed. Starting with sonar systems explanation the assumptions and simulations are shown, tied up with results.

In the last chapter the study is finalized by summarizing the basic conclusions of the work and the recommendations for future studies.
CHAPTER 2

DEFINITIONS AND SIMULATIONS

2.1 PHYSICS BEHIND THE ACOUSTICS

The variables; displacement, density and the pressure should be identified to formulate the theory of the propagation of acoustic waves:

![Diagram](image)

Fig.2.1 One-dimensional geometry for wave motion. [3]

The acoustic waves are longitudinal waves. In order to drive the acoustic wave displacement the pressure term should be defined which is actually a function of density P, such as f(ρ).
In Fig. 2.1, x is the distance in the direction of wave propagation, \( P(x) \) is the pressure \( \Delta x, t \) is the change in the volume. In this study linear acoustics is considered, illustrated in the figure volume is linear and motion along the x axis is considered. Starting from the conservation of mass and using the Newton 2\(^{nd} \) Law the acoustic wave equation in terms of displacement is found as

\[
\frac{\partial^2 \xi}{\partial t^2} - \left( \frac{B}{\rho_0} \right) \frac{\partial^2 \xi}{\partial x^2} = 0
\]  

(2.1)

where \( \xi \) is the change in the volume, \( t \) is the time and \( B \equiv \rho \frac{\partial P}{\partial \rho} \) is the bulk modulus which is the complement of the medium compressibility. From Eqn. 2.1 the sound speed is

\[
c = \sqrt{\frac{B}{\rho_0}} = \left( \frac{\frac{\partial P}{\partial \rho}}{\sqrt{\rho_0}} \right)_{0}
\]  

(2.2)

This means, the sound speed or wave speed is a property of the medium, is independent of the strength or amplitude of the acoustic wave.

The actual value of sound speed emerges not only arises from Newton’s mechanics, but it also requires the specific properties of the material, i.e., its compressibility. Newton theory explains sound speed but some thermodynamic is required to define the velocity of sound. When the thermodynamics is considered the sound velocity is found as [1]:

\[
c = \sqrt{\frac{\gamma R T_k}{M_{\text{mol}}}} = \sqrt{\frac{\gamma R T_k}{M_{\text{mol}}}}
\]  

(2.3)

Where R is the molar gas constant, \( T_k \) is the absolute temperature, \( M_{\text{mol}} \) is the molar mass and \( \gamma \) is the ratio of the specific heats, \( \gamma = \frac{C_p}{C_v} \) [1]

This equation says that Newton approach needs some corrections. It defines the speed of waves but some more additional terms are required. After further considerations, the sound speed in the ocean found as a function of temperature,
salinity and ambient pressure. Since the ambient pressure is a function of depth, it is customary to express the sound speed \( c \) as an empirical function of temperature \( T \) in degrees centigrade, salinity \( S \) in parts per thousand and depth \( z \). In general, the velocity of sound in water is given by the following equation [6]

\[
c = 1449.2 + 4.6T - 0.055T^2 + 0.00029T^3 + (1.34 - 0.01T)(S - 35) + 0.016z
\]  

(2.4)

### 2.2 SOUND PROPAGATION MODELS

All acoustic models start solving Helmholtz equation in an azimuthally symmetric environment [3]. Starting from Eqn.2.1 Helmholtz equation is stated for acoustic medium as:

\[
\nabla^2 p + \frac{\omega^2}{c^2(r,z)} p = \frac{-\delta(r-r_s)(z-z_s)}{r}
\]

(2.5)

where \( c(r,z) \) is the ocean sound speed as a function of range and depth, \( r_s \) and \( z_s \) states the point the where the source is located in cartesian coordinates. The Eqn.2.4. is approximated as Eqn.2.20 that will be discussed in Section 2.3. In addition, \( \omega \) is the angular frequency of the source which is located at the range depth coordinate \( (r_s, z_s) \).

There are mainly four types of models to describe sound propagation in the sea: ray theory, the spectral method or fast field program (FFP), normal mode (NM) and parabolic equation (PE). All of these models states that the water environment varies with depth. The two of four types will be examined during the study; Ray Theory and Normal mode which are the most frequent used among others. Spectral integral and normal mode models are closely related.
2.3 RAY THEORY

Optical ray theory can all be translated to acoustics. Snell’s Law should be known to provide first order knowledge on where rays go.

2.3.1 Snell’s Law and Rays

Snell’s Law in terms of emerging angle $\theta$ with respect to the horizontal can be rewritten as

$$\frac{\cos \theta_s}{c(z_s)} = \frac{\cos \theta}{c(z)} \Rightarrow \theta = \cos^{-1} \left[ \frac{c(z_s)}{c(z)} \cos \theta_s \right]$$

(2.6)

where Eq. 2.6 is interpreted to be the angle dependence of a ray as a function of depth as specified through the sound speed profile. After differentiating Eq. 2.6 with respect to $z$:

$$-c(z_s)\sin \theta \frac{d\theta}{dz} = \frac{dc(z)}{dz} \cos \theta_s$$

(2.7)

Figure 2.2: Schematic of two dimensional geometry.[3]
2.3.2 Ray Theory

The starting point is the Helmholtz equation in the Cartesian Coordinates; $x=(x,y,z)$,

$$\nabla^2 P + \frac{\omega^2}{c^2(x)} P = -\delta(x-x_0) \quad (2.8)$$

Here the sound speed $c(x)$ and the angular frequency $\omega$ belong to the source located at $x_0$. Assuming a high frequency solution by a series expansion in inverse frequency, each term thereby getting smaller. The series expansion of Pressure can be written as the following equation [2]

$$P(x) = e^{i\omega \tau(x)} \sum_{j=0}^{\infty} \frac{A_j(x)}{(i\omega)^j} \quad (2.9)$$

where $\tau(x)$ denotes the wavefronts, $A_j$ terms of series. Substituting this result into the Helmholtz equation, one obtains the infinite sequence of equations for the functions $\tau(x)$ and $A_j(x)$,

$$O(\omega^2): \quad |\nabla \tau|^2 = 1/c^2(x) \quad (2.10)$$

$$O(\omega): \quad 2\nabla \tau \cdot \nabla A_0 + (\nabla^2 \tau) A_0 = 0 \quad (2.11)$$

$$O(\omega^{1-j}): \quad 2\nabla \tau \cdot \nabla A_j + (\nabla^2 \tau) A_j = -\nabla^2 A_{j-1} \quad j = 1, 2, \ldots \quad (2.12)$$

The $O(\omega^2)$ equation (Eqn.2.10.) for $\tau(x)$ is known as the eikonal equation. The remaining equations (Eqn.2.11, Eqn.2.12) for $A_j(x)$ are stated as the transport equations which will not be studied in this study. Ray paths or pictures of where sound travels are inferred from the eikonal equation, the transport equations are related to the amplitude of the sound.
A single linear partial differential equation (PDE) is converted into a nonlinear PDE (the eikonal equation) plus an infinite series of linear PDEs (the transport equations) which will not be solved here. The eikonal equation:

$$|\nabla \tau|^2 = \frac{1}{c^2(x)}$$  \hspace{1cm} (2.13)

is a first-order nonlinear PDE which is solved by the method of characteristics. Constant $\tau$ defines a wavefront and the normals to that ever changing (because of varying sound speed) wavefront in space are rays. Basically, it is introduced a family of curves (rays) which are perpendicular to the level curves (wavefronts) of $\tau(x)$ as seen in Fig. 2.3. This family of rays defines a new coordinate system, and in ray coordinates the eikonal equation reduces to a linear, ordinary differential equation.

$$\frac{dx}{ds} = c\nabla \tau$$  \hspace{1cm} (2.14)

$\nabla \tau$ is a vector perpendicular to the wavefronts, $c$ is introduced so that the tangent vector $dx/ds$ has unit length. The rays can also be parameterized with respect to travel time or any other quantity which increases monotonically along the ray.
The definition for the rays is based on their being perpendicular to the level curves of \( r(x) \), a function which for the moment is an unknown. However, with some manipulations the ray equations in a form involving only \( c(x) \) can be written,

\[
\frac{d}{ds} \left( \frac{1}{c} \frac{dx}{ds} \right) = -\frac{1}{c^2} (\nabla c) \tag{2.15}
\]

The ray equations may be written in the first-order form[12]

\[
\begin{align*}
\frac{dr}{ds} &= c \eta(s), \\
\frac{d\eta}{ds} &= -\frac{1}{c^2} \frac{dc}{dr}, \\
\frac{dz}{ds} &= c \zeta(s), \\
\frac{d\zeta}{ds} &= -\frac{1}{c^2} \frac{dc}{dz} 
\end{align*} \tag{2.16, 2.17}
\]

\([r(s), z(s)]\) is the trajectory of the ray in the two dimensional range-depth plane. The auxiliary variables \( \eta(s) \) and \( \zeta(s) \) are introduced in order to write the equations in first-order form. The tangent vector to a curve \([r(s), z(s)]\) is given by \([dr/ds, dz/ds]\). Thus from the above equations the tangent vector to the ray is \( c [\eta(s), \zeta(s)] \).

This set of ordinary differential equations is solved numerically. However, to complete the specification of the rays initial conditions are needed. As indicated in Fig. 2.2, the initial conditions are that the ray starts at the source position \((r_s, z_s)\) with a specified angle \( \theta \). Thus,

\[
\begin{align*}
r &= r_s, & \xi &= \frac{\cos \theta}{c(0)} \tag{2.18} \\
z &= z_s, & \zeta &= \frac{\sin \theta}{c(0)} \tag{2.19}
\end{align*}
\]

MATLAB is used to solve these systems of equations. The sound speed profile is approximated as:

\[
c(z) = 1500.0 \left[ 1.0 + e(\hat{z} - 1 + e^{-\hat{z}}) \right] \tag{2.20}
\]
The quantity $\epsilon$ is taken as $\epsilon = 0.00737$. The scaled depth $\bar{z}$ is taken as: \[ \bar{z} = \frac{2(z - 1300)}{1300} \] (2.21)

2.3.3 Ray Theory Simulations

According to the scope defined above, the codes tried to be realized step by step. When the basic picture of Ray Theory is considered, the source is located at 1000 m depth and the range is 100 km, the graph is obtained as follows:

Figure 2.5: Ray figure (depth=1000m)
MATLAB default ODE45 and ODE23 functions are used in order to figure this graph. The equations stated in Eqn.2.16 and Eqn.2.17 are solved simultaneously with the initial conditions defined in Eqn.2.18 and Eqn.2.19. Sound speed profile is taken as Eqn.2.20. Propagation of rays perpendicular to wavefronts with different grazing angles are seen in the graph. In the case study the shallow water will be considered. When we decrease the depth, we see that the rays that penetrate do not highly effected from the sound speed profile since the sound in lower depths are nearly constant in generic sound profile (Fig.2.4.). This plot is only the skeleton of the acoustic field; to obtain the associated pressure field it should be further developed. However, this ray trace is often the most important product of a ray model. Other techniques can give more accurate transmission loss figures; however, they do not readily provide this simple graphical ray picture showing the important energy paths.

The main issue in shallow water is the interferences. The reflected waves from boundaries will interfere which will be seen in the Fig.2.6.
In Fig.2.6, the source is at 25 meters the depth is 100m and the range is 5km. Boundary conditions for this case is adapted as a full reflection at the boundaries, which means the bottom and the sea level behaves like a mirror. It is seen that reflections from the sea level and bottom make the graph more complex. When we examine much simpler case, depth is 500 meters and range is 20 km:

![Figure 2.7: Ray figure (depth=500 m)](image)

In Fig.2.7, the source is at 100 meters. It can be easily observed that, whatever the arriving angle of the rays, they are collected at nearly 7 km, thus if one puts a receiver at nearly 7 km source can be easily detected.

This is a basic illustration, thus the reflections from the bottom are considered as perfect, means full reflection from the bottom. However the reflections are not perfect and the bottom is not smooth and also sound speed profile is taken as generic. The sound speed profile depends on many factors like salinity, temperature, mostly temperature which changes even the daytime.

The basic boundary conditions (i.e. full reflection at z=0 and z=500) are easily to be handled but more complex boundaries will be applied further, thus ODE solver
which is used up to this point, should be coded. In order to accomplish this, 3\textsuperscript{rd} order Runge Kutta method is used which is assumed to be one of the effective solvers for differential equations. In the case study, 3\textsuperscript{rd} order Runge Kutta method will be used.

2.4 NORMAL MODE PROPAGATION

The sea medium can be thought as an acoustical waveguide bounded above by air and below by the sea bottom which may be a combination of sand, mud, layered rocks, etc. This waveguide is considered to be range-independent, meaning that it is a horizontally stratified medium. The two most obvious properties of a waveguide are:

1. It is infinite in the horizontal dimension (r)
2. It is finite in the vertical direction (z)

The assumption of range-independence implies that the wave equation can be solved by “separation of variables” which means that the solution is a product of solutions of one dimensional wave equations (r and z equations) which are related by a separation constant. The solution of the waveguide equation will be a product of traveling waves in the horizontal direction and standing waves in the vertical direction. The standing waves in the z coordinate are called normal modes. Waves refer to many paths that are described by many waves. In effect, the solutions are in terms of these normal modes, each of which is distributed in depth differently. Each of these modes are then the amplitude of a particular plane wave (through the separation constant) traveling in the horizontal and the total solution is the sum of these terms.
2.4.1 The picture of normal modes

A waveguide is considered which is bounded above by the air/water interface. Hence having perfect reflection with a 180 degree phase change at the surface and for paths more horizontal than the bottom critical angle, there will also be perfect reflection with an angle dependent phase change.

![Figure 2.8: Shallow water waveguide propagation](image)

As seen in Fig. 2.8a ray paths within a cone of $2\theta_c$ will propagated unattenuated down the waveguide. Since the up and down going rays have equal amplitudes, preferred angles will exist such that perfect constructive interference can occur. These particular angles can be associated with the normal modes of the waveguide. Figure 2.8b is a schematic of a ray reflected from the bottom and then the surface of a “Pekeris” waveguide (an environment with constant sound speeds and densities in the water column and fluid bottom, respectively). When a ray along the path ACDF is considered and its wavefront which is perpendicular to
the ray, the two downgoing rays, AC and DF will constructively interfere if points B and E have a phase difference of an integral number of 360 degrees (and similarly for upgoing rays).

Figure 2.9: Schematic of normal mode propagation

a) long range propagation in 2θc cone

b) propagation of waves with different wave numbers[3]

Referring to Fig. 2.9 it is seen that the more vertical the ray the more horizontal the wavefront. Hence the most vertical ray corresponds to the maximum phase velocity of c2; that is, rays more vertical than the critical angle do not propagate down the waveguide. On the other hand, a horizontal ray has a vertical wavefront and so the phase is constant in the vertical. The superposition of discrete up and downgoing waves results in a vertical amplitude distribution in the waveguide of the form,

\[ \varphi_n \approx e^{i(k_n r - \omega t)}u_n(z); \quad u_n(z) = \sin \left( \sqrt{k_1^2 - k_n^2} z \right) \] (2.22)

[3] The \( u_n \)'s are called the normal modes of the waveguide each propagate with phase velocity \( c_n \). The total field in the waveguide is a sum of all the normal mode
terms of the form of Eq. 2.22; the vertical distribution can be thought of as a superposition of up and down going plane waves at discrete propagation angles within the cone $\pm \theta_c$. Rays and modes are shown in Fig. 2.9a.

### 2.4.2 Normal mode solution from the wave equation

Beginning with the Helmholtz equation in cylindrical coordinates with sound speed depending only on depth $z$,

$$
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right) + \frac{\partial^2 p}{\partial z^2} + \frac{\omega^2}{c^2(z)} p = -\frac{\delta(r)\delta(z-z_s)}{2\pi r}
$$

(2.23)

where the right hand side is the delta function in cylindrical coordinates representing a point source at $(r, z) = (0, z_s)$. Using the technique of separation of variables, the solution is in the form $p(r, z) = \Phi(r) \Psi(z)$. The modal equation in the $z$ direction is,

$$
\frac{\partial^2 \Psi_m(z)}{\partial z^2} + \left[ \frac{\omega^2}{c^2(z)} - k_m^2 \right] \Psi_m(z) = 0
$$

(2.24)

the boundary conditions for $\Psi_m(z)$ at the surface and bottom,

$$
\Psi_m(0) = 0, \quad \frac{d \Psi_m}{dz}(D) = 0
$$

(2.25)

The modal equation is Sturm–Liouville eigenvalue problem. The function $\Psi_m(z)$ is an eigenfunction and $k_m$ is an eigenvalue. The $m^{th}$ mode has $m$ zeroes in the interval $[0,D]$ and the corresponding eigenvalues $k_m^2$ are all real and are ordered as $k_1^2 > k_2^2 > \cdots$. In addition, the modes of Sturm–Liouville problems are orthogonal,

$$
\int_0^D \frac{\Psi_m(z)\Psi_n(z)}{\rho(z)} \, dz = 0 \quad \text{for } m \neq n
$$

(2.26)

The modes are normalized so that
Finally, the modes form a complete set, which means one can represent an arbitrary function as a sum of the normal modes. Solution for the travelling wave in the $r$ direction is

$$
\frac{1}{r} \frac{d}{dr} \left[ r \frac{d\Phi_n(r)}{dr} \right] + k_m^2 \Phi_n(r) = - \frac{\delta(r) \Psi_m(z_s)}{2\pi \rho(z_s)}
$$

(2.28)

This is a standard equation whose solution is given in terms of a Hankel function as

$$
\Phi_n(r) = \frac{i}{4\rho(z_s)} \Psi_m(z_s) H_0^{(1)}(k_n r)
$$

(2.29)

The choice of $H^{(1)}_0$ or $H^{(2)}_0$ is determined by the radiation condition stating that energy should be radiating outward as $r \to \infty$. Since a time dependence of the form $\exp(-i\omega t)$ is suppressed, Hankel function of the first kind is used. Putting these all together,

$$
p(r, z) = \frac{i}{4\rho(z_s)} \sum_{m=1}^{\infty} \Psi_m(z_s) H_0^{(1)}(k_m r)
$$

(2.30)

or, using the asymptotic approximation to the Hankel function,[8]

$$
p(r, z) \approx \frac{i}{p(z_s)\sqrt{8\pi r}} e^{-ir/4} \sum_{m=1}^{\infty} \Psi_m(z_s) \Psi_m(z) \frac{e^{ik_m r}}{\sqrt{k_m}}
$$

(2.31)

Hence the acoustic field in the waveguide can be thought of as a sum of plane waves travelling horizontally with wavenumbers $k_m$. The amplitude of the $m$-th wave as function of depth is a product of the $m$-th mode as a function of depth and the value of that (normalized) mode at the source depth.
2.4.3 Normal Mode Simulations

Finite difference approximation for modal equation (Eqn 2.24) can be written as follows;

$$\Psi''_j = \frac{\Psi_{j-1} - 2\Psi'_j + \Psi_{j+1}}{h^2} + O(h^2) \quad (2.32)$$

$$\frac{\Psi_{j-1}}{h^2} + \left\{ \frac{-2}{h^2} + \frac{\omega^2}{z_j} - k^2 \right\} \Psi_j + \frac{\Psi_{j+1}}{h^2} = 0 \quad j = 1, \ldots, N - 1 \quad (2.33)$$

$$\Psi_0 = 0 \quad \Psi_{N+1} = 0 \quad (2.34)$$

Matrix form representation:

$$[A - k^2 I] \Psi = 0 \quad (2.35)$$

This equation has N eigenvalues $k^{(m)}$, and corresponding eigenvectors $\Psi^{(m)}$, $\Psi^m$ is the vector with components $\Psi_1^m, \Psi_2^m, \ldots, \Psi_N^m$

$$A = \begin{bmatrix}
  d_1 & e_2 & & & \\
  e_2 & d_3 & e_3 & e_4 & \\
  & e_3 & d_2 & e_4 & \\
  & & \ddots & \ddots & \ddots \\
  & & & e_{N-2} & d_{N-2} & e_{N-1} \\
  & & & & e_{N-1} & d_{N-1} & e_N \\
  & & & & & e_N & d_N \\
\end{bmatrix} \quad (2.36)$$

$$d_j = \frac{-2}{h^2} + \frac{\omega^2}{c^2(z_j)} \quad e_j = \frac{1}{h^2} \quad (2.37)$$

Numerical solution steps:

- Eigenvalue problem is solved $A \Psi = \lambda \Psi$
- $k_m = \sqrt{\lambda}$
- $k_m$ are sorted, largest $k$ is the most effective in solution.
- Eigenfunctions are normalized
\[ \psi^{(m)} = \frac{\psi^{(m)}}{\|\psi^{(m)}\|} \]  

(2.38)

- Eigenvectors are assembled into matrix (isd=index of source depth)

\[ \psi = [\psi^{(1)} \psi^{(2)} \ldots \psi^{(M)}] \]

(2.39)

\[ C = \begin{bmatrix} 
\psi_{isd,1} \\
\psi_{isd,2} \\
\vdots \\
\psi_{isd,M} 
\end{bmatrix} \]

(2.40)

- A mode matrix \( \tilde{\psi} \) is scaled by the mode excitation

\[ \tilde{\psi} = \psi \begin{bmatrix} 
1 \\
C_1 \\
\vdots \\
1/C_M 
\end{bmatrix} \]

(2.41)

- \( \Phi \) a phase matrix (Eqn 2.29) is formed

\[ \Phi = \begin{bmatrix} 
1/\sqrt{k_1} & e^{ik_1r_1} & e^{ik_1r_2} & \ldots & e^{ik_1r_{NR}} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1/\sqrt{k_M} & e^{ik_Mr_1} & e^{ik_Mr_2} & \ldots & e^{ik_Mr_{NR}} 
\end{bmatrix} \]

(2.42)

- Finally

\[ p = \tilde{\psi} \Phi \]

(2.43)
In order to observe the similarities and differences of two modes the situation is taken as same as the situation in Fig. 2.4. Source is at 1000 meters range is 100 km. In the Fig.2.10 it can be seen the wave propagation due to Normal mode theory with pressure distribution. The solution of Eqn.2.31 with 102 number of modes and 5000 number of finite points for $\Psi$ in the z direction. Frequency is 50 Hz which is a very small frequency for acoustics. The run time for high frequency (i.e.5000Hz) is very much compared to low frequency (i.e.50Hz). The run time for $f=50$Hz (number of modes=100) is less than 1 min, for $f=500$Hz (number of modes=1000) approximately 3 minutes and for $f=5000$Hz (number of modes=10000) run time is approximately 25 min with %50 CPU usage. This datas are taken with computer configuration: INTEL Core 2 Duo T7300 2GHz processor with 1 GB RAM.

In the graph, brighter areas show that the pressure is high at this point and areas which turn to black show that the effect of pressure is diminished at these points.
It is easily seen that the white points turn to gray while the wave penetrates. The sound speed profile is used as defined in Eqn 2.20, the graph of generic sound speed profile is as in Fig 2.4:

Figure 2.11: Normal Mode figure (depth=500m)

The depth is 500 meters and the range is 20 km. When compared with Figure 2.6 this graph is more complicated. Although number of modes is 10 the waves reflected from sea level and from the bottom are hardly distinguished.

2.5 DIFFERENCES BETWEEN TWO MODELS

The need for these various types of models stems from the diversity of applications. While one model may be capable of treating all the problems one encounters, usually at least some of the problems are more efficiently treated by another model. After simulations it is also observed that, high-frequency problems are easily treated with ray models, range-dependent problems with normal mode model.
For high frequencies (a few kilohertz or above), ray theory is the most practical. Since the solution is frequency independent the ray theory is applicable for all frequencies. This feature of ray models is that they can be adapted to broadband problems. As many modes occur for normal mode theory, number of modes increase with increasing frequency and the complexity of the code become high.

However, the normal mode is more applicable and useable at lower frequencies (below a kilohertz). A model that also takes into account horizontal variations in the environment is termed range dependent. Normal mode model assume a stratified ocean (no range dependence) and take advantage of this for speed.

When the Figures 2.4. and 2.9 are considered, ray picture and normal mode graphs are same in formal comparison. The ray mode analogy suggests that each mode corresponds to a kind of ray with an incident angle. Every time the ray bounces off the bottom, it will undergo a reflection loss because of the bottom attenuation. Bottom loss is very important in shallow water propagation that is bottom loss increases with increasing grazing angle and the number of reflections per unit distance increases with grazing angle. This combination causes the exponential damping factor to grow with mode number.

In the view of the above advantages of ray theory, another advantage arises compared to normal mode model; its numerical efficiency. Typically, the normal mode model requires about 10 nodes per wavelength. Long range problems are often thousands of wavelengths in range and tens of wavelengths in depth. The resulting linear systems of equations for the acoustic field at each node involve millions of unknowns [19]. Thus, such technique is not computationally very practical for long range problems but for short ranges Normal Mode works.

Especially mode based approaches require a large amount of computing time since the number of modes increase with frequency. In addition to the time factor, modeling of the transmission and reflection of sound at rough wave guide
boundaries is comparatively simple in the ray tracing method.

The models are two dimensional models since the index of refraction has much stronger dependence on depth than on horizontal distance. Nevertheless, bottom topography and strong sea features can cause horizontal refraction. Ray models are most easily extendable to include this added complexity. Three dimensional wave models are extremely computationally very complex.
3.1 SONAR EQUATION

There are two kinds of sonar (SOund NAvigation and Ranging): passive and active as shown in Fig. 3.1. In active sonar, the system emits a pulse of sound and then the operator listens for echoes. In passive sonar, the operator listens to sounds emitted by the object which is trying to be located.

A major application of underwater acoustics is sonar system technology. The performance of sonar is often approximately described simply in terms of the
sonar equation. The methodology of the sonar equation is analogous to an accounting procedure involving acoustic signal, interference and system characteristics. The latter involves the ability of the combination of the system and operator to discern a target in the noise clutter.

One of the way that prediction tools are used is match field processing (MFP). In MFP, the first data input is measured acoustic data from a sonar set of hydrophones, the second data input is a predicted data set, or replica data set, against which the measured data are compared. Multiple replicas are compared to the measured data and the closest match is retained. The closest match of the replica is presumed to characterize the data in some important way (e.g., a source location). More specifically, the output of a sensor of the towed array is translated to the frequency domain by applying a Discrete Fourier Transform (DFT) or a Fast Fourier Transform (FFT) to a set of contiguous time samples. A replica vector is the frequency domain N x 1 vector representing the predicted or expected values at each sensor of the sensor array for a specific frequency. The corresponding output of the method is an ambiguity surface. The ambiguity surface is a set of numbers ranging between zero and one with each number corresponding to a specific location in the sea medium. The highest values on the ambiguity surface indicate the most likely position of an acoustic source. The matched-field response is generalized by averaging the response over multiple frequencies. A response for an array may be computed by forming beams and then combining them by multiplying each by an eigenray factor before summing.

### 3.2 DETECTION THRESHOLD

The detection threshold DT is a decibel number that essentially incorporates the Sonar systems (which includes operator) ability to decide that a detection is made or not made. The detection process includes the following probabilities:
• PD (probability of detection): Probability of a signal is detected which is detected if it is present.
• 1-PD: The probability of a the signal which will not be detected if it is present;
• PFA (probability of false alarm): Probability of a signal is detected that is detected when it is not present.
• 1-PFA: The probability of a signal which will not be detected when it is not present.

Figure 3.2: Probability density functions (PDF’s) for signal plus noise and noise alone.

In practical terms, since the signal and noise are fluctuating, the detection is made (over a time interval) when the fluctuating sum of the signal and noise exceeds a threshold that is determined from empirically derived probability density functions (PDFs) of noise and signal plus noise. For example, the case that the noise alone rises above the threshold contributes to the PFA. Therefore, the process for determining a detection threshold level depends on PD and PFA. Typically numbers might be a PD of 0.5 and PFA of .0001.
3.3 PASSIVE SONAR EQUATION

A passive sonar system uses the radiated sound from a target to detect and locate the target. A radiating object of source level SL is received at a hydrophone of a sonar system at a lower signal level S, because of the transmission loss “TL”.

\[ S = SL - TL \]  
\[ (3.1) \]

The noise, N at a single hydrophone is subtracted from Eq. 3.1. to obtain the signal-to-noise ratio at a single hydrophone,

\[ SNR = SL - TL - N \]  
\[ (3.2) \]

Typically a sonar system consists of an array or antenna of hydrophones which provides signal to noise enhancement through a beam forming process. This process is quantified in decibels by an array gain AG that is added to the single hydrophone SNR to give the SNR$_{BF}$ at the output of the beam former,

\[ SNR_{BF} = SL - TL - N + AG \]  
\[ (3.3) \]

Because detection involves additional factors including sonar operator ability, it is necessary to specify a detection threshold, DT level above the SNR$_{BF}$ at which there is a 50% (by convention) probability of detection. The difference between these two quantities is called signal excess (SE),

\[ SE = SL - TL - N + AG - DT \]  
\[ (3.4) \]

This decibel bookkeeping leads to an important sonar engineering descriptor called the figure of merit, FOM, which is the transmission loss that gives a zero signal excess,

\[ FOM = SL - N + AG - DT \]  
\[ (3.5) \]

FOM encompasses the various parameters one must deal with: expected source level, the noise environment, array gain and the detection threshold. Conversely since FOM is a transmission loss, one can use the output of a propagation model
to estimate the minimum range at which a 50% probability of detection can be expected. This range changes with conditions, [13].

3.4 ACTIVE SONAR EQUATION

A monostatic active sonar transmits a pulse to a target and its echo is detected at a receiver collected with the transmitter. A bistatic active sonar has the receiver in a different location than the transmitter. The main differences between the passive and active cases are the addition of a target strength term, TS; reverberation and hence reverberation level, RL, is usually the dominant source of interference as opposed noise; and the transmission loss is over two paths: transmitter to target and target to receiver. In the monostatic case, the transmission loss is 2TL where TL is the one way transmission loss, and in the bistatic case, the transmission loss is the sum (in dB) over paths from the transmitter to the target and the target to the receiver, TL1 + TL2. The concept of the detection threshold is useful for both passive and active sonars. Hence, for signal excess,

$$SE = SL - TL_1 + TS - TL_2 - (RL + N) + AG - DT$$ (3.6)

The corresponding FOM for an active system is defined for the maximum allowable two-way transmission loss with TS = 0 dB, [13].

3.5 PERFORMANCE PREDICTIONS USING RAY THEORY

Passive sonars work in the frequency (f) range 0.01-3.5 kHz the noises and losses are approximated as; in case of the acoustic wave has an approach of $\lambda/4$ to the receiver the wave would be detected.
In Fig. 3.3 the source is at 85 meters and the receiver is at 90 meters, the depth of the water is 110 meters. Simple rays are chosen for illustration; 3 of the rays are caught by the receiver one is reflected from the sea level and two from the bottom.

Figure 3.3 represents not only passive sonars but also bi-static case of active sonars. The figure may be thought as a transmitter of an active sonar working in 3500Hz. Actually the active sonars work in the range 10-35 kHz but wideband solutions of lower frequencies is also acceptable (i.e 3500Hz). The transmitter frequency is chosen as 3500Hz which covers the both gaps for passive and active cases. The main problem for passive case is whether there is an actual target transmitting in 3500 kHz.
Figure 3.4: Time diagram transmitter receiver (f=3500 Hz)

The time of arrivals of rays are shown in the figure which are sampled at a frequency 10*f, f is equal to 3500 Hz in this figure.

Figure 3.5: Transmitter Receiver (f=50Hz)
When lower frequencies are considered the Probability of Detection (PoD) increases for passive sonars in this approximated case. In Fig 3.5, the rays, reached to the receiver are much more considered to $f=3500\text{Hz}$ case, however in $f=50\text{Hz}$ case there is no such sensitive receiver and it should be considered that active sonars are only sensitive to transmitting frequency but passive sonars are sensitive to band of frequency. To illustrate such case instead of $\lambda/4$ approximation $\lambda/40$ approximation is considered in Fig 3.6.

Figure 3.6: Transmitter Receiver ($f=50\text{ Hz}$, less sensitive case)
In Fig. 3.7., the source is at 10 meters and the receiver is at 100 meters. The bottom is 110 meters.

Figure 3.8: Time diagram (source is at 10 meters receiver is at 100 meters)
The time diagram is as in Fig3.8. The receiver catches the signals between 5.555 and 5.650 seconds. One pulse in the transmitter results four signals in the receiver.

3.6 SIMULATIONS

3.6.1 Environment

In case study a specific place whose sound velocity profiles and bottom profiles are known is investigated. The place is separated into two zones; passive sonars are placed in the outer zone for longe range detection, active sonars are placed in the inner zone to get high accuracy. The maximum efficient settlement shall be considered with the use of this code.

3.6.1.1 Sound Velocity Profiles

A case study is conducted in order to compare a summer sound velocity (Fig.3.9.) with a warm surface layer and a winter sound velocity profile with nearly constant temperature (Fig.3.10.).

![Sound Velocity Profile in Summer](image)

Figure 3.9: Sound speed velocity profile in summer
3.6.1.2 Bottom profiles

The bottom decreases strongly in the first several 1000 m to values of 90 m. In further ranges the sea become shallow. The profile is like a hollow from one coast to other coast as seen in the Fig.3.11. The sea bottom is assumed to be mostly sandy and full reflection is assumed.

Figure 3.11: Bottom profile for active sensors
For Passive sensors the bottom is degreasing continuously as seen in Fig. 3.12. For the active case Fig 3.11 is considered and for the passive case Fig. 3.12 is considered.

### 3.6.2 Active detection simulation

For active case detection the scenario is;

- There is a transmitter and an array of receivers which are separated from each other means bistatic case,
- The transmitter is in the one coast and the transmitter is in the other coast of the hollow like shaped bay.
- The receiver is an array of hydrophone, consists of 3 elements which are arranged in every 10 meters.
The different colors indicate the receiving arrays to a certain hydrophone. The rays received by the upper hydrophone represented in green, red for the middle and the rays caught by the lower one are cyan in color. As seen in the Fig 3.13 there is only one ray colored in green indicates there is only one ray reaches to the hydrophone settled near the sea surface. It can be inferred that no need for such a near surface settlement for active case. Also, there are some anomalies in the rays seen in the figure due to the bottom profile.
When the time histogram of the rays is examined in Fig. 3.14., the rays reach to the middle hydrophone is much dense as compared with the upper and lower ones. The time of arriving wave is not same in all hydrophones. In real time applications this figure is stored and by the use of MFP the same figure is expected for each wave. If there is an anomaly in the figure it should be understood that there is an obstacle between the transmitter and the receiver.

Examining the SVP of the winter in Fig. 3.10 it is seen that the velocity is nearly constant for all depths results much smooth shapes of the rays. Therefore, Fig. 3.15. is less crowded compared to Fig 3.13 as the waves are slower; the crests are much discrete. They penetrate in an inclined angle results less probability of detection by the sensors.
Further by examining time histogram (Fig 3.16.) it is observed that no ray is caught by the upper hydrophone as similar to summer case. Thus, we can infer a hydrophone near the sea level is useless.

3.6.3 Passive detection simulation

Figure 3.17: Passive Case (summer SVP)
The scenario is much more similar to active case, however in active case the intruder is understood by the change in the figure. In this case situation is little different thus, one can reverse the Fig 3.17 means the intruder is 8 km away from the receivers and try to analyze the acoustic waves spread out from the source. The receivers catch the signals coming 8 km away from the coast. Source is located at 30 meters. Again different colors indicate waves caught by different receivers.

![Figure 3.18: Time Histogram Passive Case (summer SVP)](image)

The abnormal rays are not seen in the time histogram (Figure 3.18.) since these are filtered by the program. This is also done for MFP. As seen in the figure the number of hydrophone is 4 in this case. The received rays are at the second hydrophone when winter SVP is considered as seen in Fig.3.19 and Fig. 3.20
Figure 3.19: Passive Case (winter SVP)

Figure 3.20: Time Histogram Passive Case (winter SVP)
Fig 3.21 is active case when higher frequency is considered. The figure is similar to low frequency since the rays are not affected from the frequency only difference is the assumed sensitivity of the sensors, sensors became less sensitive however the number of hydrophones increases since there is an interaction between wavelength and spacing of the hydrophone thus, a typical sonar spacing, d is half a wavelength because the angles at which destructive and constructive interference occur are most advantageous. For simplicity d is chosen as wavelength in the code. The number of hydrophones increases for active case when we consider the time histogram, it is seen as follows in Fig.3.22.
3.6.4 Results

As considered for the active and passive cases the most suitable numerical scheme seemed to be ray tracing. Ray method is applied to this case, since the model is independent of frequencies the code is applicable for Active or Passive sensors. The ray equations 2.16 and 2.17 are solved simultaneously with Runga-Kutta method. The bottom profile differs in range, so the boundaries. There was only 6 data for the bottom profile. The bottom profiles are formed with the interpolation of these datas.

Although Ray Theory is fast, run times for this application are between 25-30 minutes.

Another key issue affecting acoustic models is their direct use in the signal processing algorithms. As it is illustrated in Fig.2.4, the bending of sound and the numerous bottom and surface echoes can seriously degrade standard schemes.
based on plane-wave beam forming. Matched-field processing problems that use
acoustic models to correct for these effects are challenging for many reasons.
They involve repeated field calculations as a trial source is swept over the domain
of possible source positions to generate waveguide replica vectors. Multipath
waves cause problems in MFP [17] best way to see these multipaths which are
reflected from bottom and sea level is Ray Model. Normal Mode does not solve
this problem.
CHAPTER 4

CONCLUSION

In this study, the acoustic waves in sea medium is simulated. It is an introductory work to understand the behavior of acoustics in water. There are some methods for analyzing, Ray Theory and Normal Mode methods are used in this study. Ray Model is solved using Runga-Kutta method and the normal mode equation is solved using finite-difference methods.

When simulations are compared, it has been observed that Ray method is more applicable to high frequencies. Environmental conditions such as bottom profile and sound velocity profile are easy to adapt in Ray Model. Moreover when the run times of models are compared Ray Model is certainly better. Although the ray figures seem simpler, multipath solution can easily be obtained in Ray Model which is an important parameter in signal processing.

After comparison of Ray and Normal Mode methods, ray theory approach is chosen for the case study. In order to accomplish covering the scope of this work the codes have been enhanced with the feedbacks of previous runs. Starting from basic simulations, requirement for the next step tried to be analyzed then the next step is fulfilled. There may be further methods introduced in order to get better results, however these should be supported with experimental results. Such an infrastructure might be used for further studies. Also, this is an introductory study for sonar systems. Although our country is a peninsula these kinds of studies have not been done yet. There are a plenty of commercially available software in the world regarding acoustic wave modeling. In order to analyze anything in water prediction software should run with sensor systems. This kind of software are
integrated in acoustic systems, works as a subsystem of those systems and should well communicate with them, so one should have all know-how of this software in order to use the system. From another point of view such software works with a SVP database specific for region. Therefore, this kind of software should be nationally developed. This study may be improved to develop such national software. 3D analysis may be considered for a further study.

MATLAB is used for simulation however it has some overheads such as speed. Software development tools such as C or FORTRAN may provide better solutions.
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