# COMBINATORIAL AUCTION PROBLEMS

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ŞAFAK BAYKAL

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submitted by **ŞAFAK BAYKAL** in partial fulfillment of the requirement for the degree of **Master of Science in Industrial Engineering Department, Middle East Technical University** by,

Prof. Dr. Canan Özgen
Dean, Graduate School of **Natural and Applied Sciences** _______________

Prof. Dr. Çağlar Güven _______________
Head of Department, **Industrial Engineering**

Prof. Dr. Murat Köksalan _______________
Supervisor, **Industrial Engineering Dept., METU**

**Examining Committee Members**

Assoc. Prof. Dr. Yasemin Serin _______________
Industrial Engineering Dept., METU

Prof. Dr. Murat Köksalan _______________
Industrial Engineering Dept., METU

Assist. Prof. Dr. Seçil Savaşaneril _______________
Industrial Engineering Dept., METU

Assist. Prof. Dr. Banu Soylu _______________
Industrial Engineering Dept., Erciyes University

Dr. Banu Yüksel Özkaya _______________
Industrial Engineering Dept., HU

**Date:** **28.08.2007**

**I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.**

**Name, Last name :** Şafak BAYKAL

**Signature :**

# ABSTRACT

## COMBINATORIAL AUCTION PROBLEMS

BAYKAL, Şafak

M.S., Department of Industrial Engineering
Supervisor: Prof. Dr. Murat Köksalan

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Electronic commerce is becoming more important day by day. Many transactions and business are done electronically and many people do not want paper work anymore. When a firm wants to buy raw materials or components, it announces its need to related websites or in the newspapers. Similar demands and announcements can be seen almost everywhere nowadays. In this way, it needs to perform fast and reliable auctions as much as possible. On the other hand, buyers not only consider cost but also consider a lot of different aspects like quality, warranty period, lead time etc when they want to purchase something. This situation leads to more complex problems in the purchasing process.

As a consequence, some researchers started to consider auction mechanisms that support bids characterized by several attributes in addition to the price (quality of the product, quantity, terms of delivery, quality of the supplier etc.). These are referred to as multi-attribute combinatorial auctions.

In this thesis, Combinatorial Auctions are analyzed. Single-attribute multi-unit, multi-attribute multi-unit combinatorial auction models are studied and an interactive method is applied for solving the multi-attribute multi-unit combinatorial auction problem.

Keywords: Combinatorial Auctions, Multi-attribute, Interactive Method

# ÖZ

## KOMBİNATORYAL AÇIK ARTTIRMA PROBLEMLERİ

BAYKAL, Şafak

Yüksek Lisans, Endüstri Mühendisliği Bölümü
Tez Yöneticisi: Prof. Dr. Murat Köksalan

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Elektronik ticaret günden güne önem kazanmaktadır. Çoğu işlem ve ticaret elektronik olarak yapılmakta olup birçok insan artık evrak işleri istememektedir. Bir firma hammadde veya gerekli aksamı almak istediğinde, bu işle ilgili web sitelerine veya gazetelere ilan vermektedir. Benzeri ihtiyaçları ve duyuruları günümüzde birçok yerde görmekteyiz. Bu sebeple hızlı ve güvenilir açık arttırma (ihale) süreçlerine ihtiyaç vardır. Diğer taraftan alıcılar bir şey almak istediklerinde artık sadece maliyete değil, kalite, garanti dönemi, tedarik zamanı gibi başka niteliklere de önem vermektedirler. Bu durum da satın alma sürecinde daha karmaşık problemlere sebep olmaktadır.

Bu nedenle, bazı araştırmacılar maliyet dışında diğer faktörlerin (ürünün kalitesi, teslimat zamanı, tedarikçinin kalitesi vb.) de önemli olduğu açık arttırma (ihale) sürecini incelemeye başlamışlardır. Bu tip açık arttırmalara çoğul-nitelikli kombinatoryal açık arttırmalar da denir.

Bu tezde kombinatoryal açık arttırmalar incelenmiştir. Tekil-nitelikli çoğul-birimli ve çoğul-nitelikli çoğul-birimli kombinatoryal açık arttırmalar ve modelleri üzerinde çalışılmış ve etkileşimli bir metodun çoğul- nitelikli çoğul-birimli kombinatoryal açık arttırmalar üzerinde uygulaması yapılmıştır.

Anahtar Kelimeler: Kombinatoryal Açık Arttırmalar, Çoğul-nitelik, İnteraktif Metod

*To Hakan, Mom, Dad,*

*Badiş and Yiğitoş*

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# CHAPTER 1

# INTRODUCTION

Because people have used auctions to make trade since ancient times, auctions are very important for us. Auctions have many advantages when one wants to buy or sell something. For example, if he is not sure of the value of an item, auctions are one of the alternative ways to determine this item's value. Moreover, this item's value may change depending on the buyer. This situation is called valuation that is explained in section 2.3.

After Internet has become popular, online auctions were introduced. Online auctions have created a large marketplace where people can buy or sell whatever and whenever they want. One advantage is that one can easily sell or buy immediately whatever they want. He does not need to walk around a lot of places because it is enough to visit different websites by using a computer.

Auctions are also used by ministries around the world. US Federal Communications Commission (FCC) made an auction (spectrum auctions) in 1994 (Cramton, 1997) then many governments made spectrum auctions like third-generation mobile phone (UMTS) licenses. The Dutch government, for example, has auctioned spectrum for both DCS1800 and UMTS networks but also has considered using auctions for the allocation of slots for highway gas stations, wireless local loop spectrum and telephone numbers (Goeree *et al.*, 2006).

In this thesis, a multi objective mixed integer model for combinatorial auctions is proposed. The structure and rules of this auction are determined with respect to the application. Auctions are conducted with singleton bids and bundled bids. The assumptions are inspired by the real cases. The solutions and performances are analyzed with respect to some indicators. In addition, an interactive method is proposed for solving the multi objective combinatorial auction. The method is applied with implicit utility functions and the solutions are compared with the explicit case.

The content of the thesis is as follows: Related literature and background of auctions are given in detail and discussed in Chapter 2. In Chapter 3, some models for combinatorial auctions are presented from the literature. The proposed mixed integer models for combinatorial auctions are given in Chapter 4. The performance of these models is analyzed and evaluated. In Chapter 5, an interactive method for multi objective combinatorial auctions is proposed. Finally, conclusive remarks are provided and future research directions are discussed in Chapter 6.

# CHAPTER 2

# BACKGROUND OF AUCTION THEORY

## 2.1 AUCTIONS

### 2.1.1 Why Auctions?

World is becoming crowded, business is becoming more important and there are many ways to conduct business. Auctions are one of the popular ways of buying and selling. Auctions' popularity can be attributed to several reasons. For example, sellers and buyers discuss face to face and carry on business in a win-win strategy. Moreover, all sellers have equal chance in an auction.

Auctions also have some extra advantages. According to Wolfstetter (1996) auctions are used for three reasons:

- Speed of sale,
- To reveal information about buyers' valuations,
- To prevent dishonest dealing between the seller's agent and the buyer.

In short, auctions are advantageous and used in several kinds of situations. Auctions' structure is mainly based on equal chance of each bidder. According to Rothkopf *et al.* (2001), the most important advantages are fairness and the appearance of fairness. They claim that auctions give advantages when negotiation and posted price actions do not work properly. In the auctions the seller does not have to wait or negotiate until someone accepts these prices. In the auctions, units usually do not have a posted price. Auctioneer keeps announcing new prices until a single bidder accepts to sell/buy the units at the last announced price. Moreover, Internet based auctions reduce transaction costs because there are no overhead costs, staff costs or investments costs.

### 2.1.2 Auctioned units

The first decision in an auction is what is to be sold or purchased. There are many possible units to sell or buy in an auction. You can sell a new unit or a second hand unit or both at the same time. Rothkopf *et al.* (2001) said that you have to decide on the specifications of the unit. For example, is the product sold as is or with a warranty period? Or if you want to sell a territory, you have to decide to sell in one-piece or multi-piece. These specifications affect the auction and buyers private values.

### 2.1.3 Who may be the Bidder?

Another decision in the auction is who may be a bidder? Depending on the type of auctions explained in section 2.2, some additional requirements may be needed. For example, in the Internet, if you want to sell something at an auction, you have to be a member of a website like Gittigidiyor.com. Only website member sell or buy from this website. If a website member wants to sell or buy something from a different website, one has to join this website too. On the other hand, if a big and important thing is auctioned like airport construction, the bidders may have to be approved or controlled by government. Alternatively, government may invite only approved bidders.

## 2.2 COMMON AUCTION FORMS

Auctions have become more important recently and we can see easily many applications in the world. For example one differentiation is between oral and written auctions. In oral auctions, for example antiques auctions, bidders hear each other's bids, and can make counteroffers; each bidder knows his rivals. In a written or closed seal bid, bidders submit their bids simultaneously without revealing them to others. For example airport construction bidding, bidders submit their bids at the same time in a closed seal and they do not know others' bids. After that, auctioneer announces sometimes either only winner bids and bidder, or all bids and bidders.

The best-known and most frequently used auctions are shown in Table 2.1. They are English, Dutch, First Price Sealed Bid and Second Price Sealed Bid Auctions.

Table 2.1- The best-known and most frequently used auctions

| **Oral** | **Written (sealed-bid)** |
|---|---|
| Ascending Price (English) | Second Price |
| Descending Price (Dutch) | First Price |

Moreover Teich *et al.* (2003) listed 18 important characteristics of practical auction situations in Table 2.2.

Table 2.2 - Classification of Auction Situations Based on Their Characteristics (Teich *et al. 2003)*

| Characteristic | Range |
|---|---|
| 1. Number of Units of a Certain Good | One to Many |
| 2. Number of Goods Auctioned | One to Many |
| 3. Nature of Goods | Homogeneous to Heterogeneous |
| 4. Attributes | One to Many |
| 5. Type of Auction | Reverse vs. Forward |
| 6. Nature of Auction | One-Round vs. Progressive |
| 7. English Vs. Dutch Auction | Ascending, Descending Price |
| 8. Participation | By Invitation vs. Open |
| 9. Use of Agents | Agent Mediated Vs. Manual Mode |
| 10. Price Paid by Winner | First Price vs. Second Price vs. $n^{th}$ Price |
| 11. Price Discrimination | Yes, No |
| 12. Constraints Exist | Implicitly, Explicitly |
| 13. Follow-up Negotiation | Yes, No |
| 14. Value Function Elicitation | Yes, No |
| 15. Nature of Bids | Open-Cry vs. Semi-Sealed vs. Sealed |
| 16. Bid Vector | 1, 2, or n-dimensional |
| 17. Bids divisible | Yes, No |
| 18. Bundle Bids Allowed | Yes, No |

In Table 2.2, characteristic 1 and 2 are related with the number of goods to be auctioned. Characteristic 3 shows the nature of the goods. This means that the auctioned units can be one type of unit (like 3 μΩ resistant) or multi-type units (3 μΩ, 5 μΩ, 10 μΩ resistant). Teich *et al.* (2003) say that characteristics 5-14 concern the auction rules and format. Characteristics 15-18 concern the nature and composition of bids. For example characteristic 16 shows that a bid can be more than one dimensional such that it has dimensions like cost, quality, warranty period etc.

In the auctions, usually an agent is used (characteristic 9). For example, in antique auctions there is an auctioneer to increase the price with the rise of hands. On the other hand, in commercial sites a primitive type of agent is used. This type of agent follows an auto bidding procedure or only control who bids and decide who the winner is.

According to Teich *et al.* (2003) first price auctions are more preferential than second price. First price and second price auctions are explained in sections 2.2.3 and 2.2.4. For both auction types, neutrality trust is an important factor. Many auction sites collect the reliability votes of bidders and sellers. After a buyer buys a unit from a seller, the buyer evaluates the seller according to reliability. From the buyer's side, if a buyer bids and wins the auction but does not pay , then the auctioneer warns the buyer for this behavior and decreases the reliability credit of the buyer. Thus each side, buyer and seller, easily judge whether to trust each other to sell or buy.

Valuation or price discrimination means that everyone appreciates different values for the same unit because this unit does not have the same value for every one. Someone may overvalue the unit because it is really needed, someone may undervalue because it may not be so critical to buy it or not. Teich *et al.* claim that “price discrimination increases the monopoly power, leading to potentially higher profits than uniform pricing”. Valuation phenomenon is explained in section 2.3.

When we talk about characteristic 15, it gives us the nature of bids. “Open-cry” means that when a bidder states his bid, all rivals hear or know what he bids and if anyone wants to increase that bid, one says his increased bid and again all rivals hear/know this increment. “Sealed” bids mean bidders give their bids in a seal and rivals do not know anything about other bids. At the end of the auction, only the winner is announced. “Semi-sealed” bids are between “Open-cry” and “Sealed”. For example, when a bidder gives a bid, he only knows his rank among other bidders. Same situations hold true for all bidders. They only know the current rank of their own bid in the bid-stream.

When we put together characteristics 5 and 7, four main auction types can be seen: 1) Forward Dutch (or just Dutch), 2) Forward English (just English), 3) Reverse Dutch and 4) Reverse English (or just Reverse). According to Teich *et al.*, Reverse Dutch is apparently the least common of the four. Also they claim that, with respect to characteristics 6 and 13, auctions may consist of different stages like progressive multi-stage auction and they conclude in two-party negotiations.

Up to now, we talked about some characteristics of auctions. Hereafter, four main auction types will be discussed.

### 2.2.1 English Auction

English auction is the oldest version of auction types. Its other name is open ascending price auction. We can see many applications of the English auction. For example, an antique sale is made usually by this type of auction. Auctioneer starts auction with a beginning price (usually low price) and raises the price in small increments. The auction stops when only one bidder is interested and the unit is sold to this bidder according to the latest price.

Krishna (2002) illustrated another application of English auction. Auctioneer proposes a price and rises continuously until only one hand of bidder is raised up, because, bidders show their interest at that price in purchasing by raising a hand in this application. If the bidder finds the price too high, he lowers his hand and withdraws from the auction. Last bidder who still raises his hand wins the auction and pays the price at which the second-last bidder stopped-out.

### 2.2.2 Dutch Auction

This type of auction's name comes from Netherlands because Dutch auctions originally were used in Netherlands for flower sales. In this type, the auctioneer announces the prices and decreases it continuously until a bidder accepts the price. Krishna (2002) says that Dutch auctions are very economical for time and effort and that is why a version of Dutch auction is still used in Netherlands.

### 2.2.3 The Sealed-bid First Price

This type of auction can be seen easily in everyday life. Most of the governmental issues are auctioned using this type of auction process. In the process, bidders submit sealed bids and any bidders do not know others' bids. The bidder, who submitted the highest bid wins the auction and pays what he bids if auctioneer does not want to negotiate. If he wants to negotiate, it means that he wants to decrease (increase) the price if he is purchasing (selling). The auctioneer announces the winner price and invokes bidders once more to re-submit their new bids. If at the second iteration, the winner price does not change, the winner and the price will be declared. If it changes, the announcer will renew his announcement and new bid submission will be done again until the winner price does not change any more.

### 2.2.4 The Sealed-bid Second Price

This type of auction is very similar to the First Price Auctions. As its name suggests, bidders submit their bids in seals. But, different from First Price Auctions, the winner who submits the highest bid pays the second-highest bid value. Because Vickrey (1961) proved that under certain circumstances the second price auctions result in the same average revenue for the auctioneer as in a first price sealed-bid auction, this auction type in the literature is also known as Vickrey Auction .

## 2.3 VALUATION

Valuation phenomena come from the nature of the auctions that there are one or more sellers and buyers and both of the sides who want to maximize their gains. However none of them knows what the unit is valued for each other. If one of them knew the value of this unit for each bidder, he could have forced the other bidders to pay more.

Krishna (2002) describes three different valuation situations which change depending on the bidders. They are private value, common value, and interdependent value auctions.

If every bidder knows the value of the auctioned unit to himself in the auction, the situation is called private values. Put another way, none of the bidders know private values of other bidders and their private values would not affect the worth of the auctioned units to a bidder.

In many situations, the bidder does not know exactly how much the object is worth at the time of the auction. The bidder only realizes the value of the object from expert's estimate or some test results. Actually, certain bidders may have some extra information about the value of the object and they may attach a value to the object. Thus values are unknown at the time of the auction and may be affected by information available to other bidders. Such a specification is called one of the interdependent values and Krishna (2002) says interdependent values are suited for situations in which the object being sold that can possibly be resold after the auction.

Common value auction is the last type of valuation. It is not seen frequently because in this situation, the value of the object is the same for all bidders despite they do not know their private values for this object. Krishna (2002) gives an example for a common value model such that the value of the object can be evaluated from market price when in the auction its value is unknown.

## 2.4 SOME TYPES OF AUCTIONS

**Single or Double Auctions**: In single auctions, the bidders are either "buyers" or "sellers". If an auction is related with procurement, the auctioneer is usually the buyer, if it is related with sale, the auctioneer is usually the seller. English, Dutch, first-price sealed-bid, and second-price sealed-bid auctions are typically single auctions. On the other hand, double auctions are double-sided. They allow multiple buyers and sellers at once. Figure 2.1 shows us the single or double auction mechanisms. Continuous double auctions (CDA) are the general model for commodity and stock markets. Mostly, in the market there are many sellers and buyers. They want to maximize/minimize their profit/cost. These CDAs provide more negotiation and more competition. A buyer can bargain effectively when there exist more than one seller or vice versa.

**Sealed-bid or Out-cry Auctions:** In sealed-bid auctions the bids submitted by the bidders are not known until the auction ends. On the other hand, in the out-cry (or open-cry) auctions the bids are made public and every bidder knows what is the current winner bid during the auction.

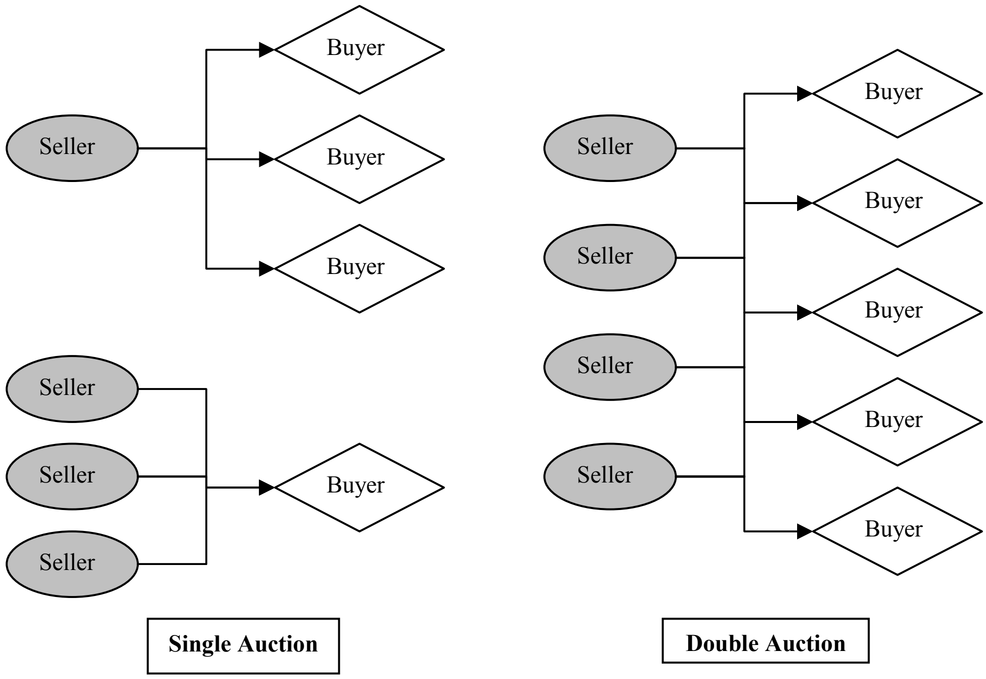


Figure 2.1 Single and Double Auction Mechanism

**Sequential or Parallel Auctions**: In sequential auctions, bidders give their bids sequentially because units are auctioned one after the other. Sandholm (2002) says that there is an easy way to determine the winner such as selecting the highest bid for each unit. However, if the bidders have same preferences over bundles, the auctioneer cannot easily select the winning bids because the auctioneer has to consider the later auctions in order to maximize his gain. Owing to the fact that bidder preferences depend on the auctioneer selections, speculations affect the future bids. This situation causes more computational cost and overhead cost. When someone bids rationally, one should consider the trade off of the computational cost against the gain, but of course it depends on the other bids. Thus bidding having preferences with respect to other bids is not meaningful in the sequential auction.

In parallel auctions, units are auctioned simultaneously. Bidders give their bids in a certain period and after this time all bids are made public. This application gives a chance to bidders to re-evaluate their bids. This specification of parallel auction decreases the uncertainty of the auction compared to sequential auctions, however similar problems arise as in sequential auctions. For example each bidder would like to wait until the end of the auction in order to see what the price is and give a bid to maximize his gain. Every bidder wants to be the last to bid and hence win the auction. According to Sandholm (2002), in order to handle this situation, some auction rules have been introduced. For example, each bidder has to bid at least a certain volume or increase the bid at a certain level.

**Reverse or Forward Auctions:** In a reverse auction or descending auction, the rule is that the bids go down. For example the seller (or the auctioneer) begins with a higher price and lowers it continuously until someone bids on it like Dutch auctions. In a forward auction the bid price goes up, that is, the bids begin low and keep increasing until a deal is made; here the bids are made by the buyer's agents. Online reverse auctions are very popular in e-business and e-sourcing (electronic sourcing) area. In online reverse auctions, multiple suppliers (i.e. sellers in the industry) bid for a contract from a buyer (the buyer submits a request for purchase (RFP)) for selling goods and/or services. Reverse and forward auctions generally include multiple rounds of bidding.

## 2.5 APPLICATIONS IN THE WORLD MARKET

Many auction applications can be seen easily all over the world. People always want to buy something and sell something. Moreover, increasing demand as a result of increasing population creates greater amount of work. Fortunately the changing world and improving technology give a lot of opportunities to use speedy computers, complex and intelligent electronic devices. Thus, people try to do these works fast, easily and optimally. Below some examples are given about auction applications over the world.

In March 2001, Motorola began a program called MINT (Motorola Internet Negotiation Tool) to implement negotiation software platform. As all known, Motorola is a big company that has a very large variety of products. For that reason, they use a large number of products. However according to Metty *et al.* (2005), they use negotiation process by meeting all potential suppliers for a particular commodity group. After meeting, commodity managers try to generate scenarios for different selections. After Motorola used MINT, it provided quick work flow, multiple online negotiations and optimization-based bid analysis. Metty *et al.* (2005) say that they started this program in 2001 within one sector and they obtained to 15 to 20 percent saving in indirect material cost and 25 to 50 percent saving in direct material cost. Furthermore, this program and Motorola were rewarded and they were the winner of the 2004 Edelman Award for Management Science Achievement is Motorola, Inc. for the project, "Reinventing the Supplier Negotiation Process at Motorola."

Another successful application is REV, used by Procter&Gamble (P&G) one of the biggest marketing companies that produce fast-moving consumer goods. P&G consists of over 135,000 employees working in over 80 countries worldwide and distributes its products in 140 countries. P&G managers gave much importance to supply chain and they wanted to create a consumer-driven supply chain. However, their wide product variety and huge supply chain had not allowed the firm to react quickly and made true decisions considering all conditions until they implemented an optimization-based software called REV which supplies a fast solution about supply chain problems (Sandholm *et al.* 2006).

REV enables companies to create their own bids. For example, sellers can express their bid as bundled offers, or conditional volume discount or unit specifications in the bid. After bidding period, REV provides possible solutions by taking into consideration buyer preferences and cost rules. Between 2001 and 2005, not only REV helped P&G save $1 billion but also strong evidences show that the suppliers of P&G also benefited in a win-win strategy.

Mars Company is another big company to use optimization software in their supply chain management. Mars-IBM team made an auction web site that provides procurement process with complex bid structures. Hohner *et al.* (2003) say that Mars firstly wanted to not only have long-term and reliable suppliers but also obtain cost-effective procurement without adversely affecting its long-term relationships. However their procurement process was based on negotiations and single sealed-bid tenders. Also they have some problems related with negotiations. At the end, Mars started a project to support strategic purchases such as small supply pools, long-term suppliers and business integration.

Hohner *et al.* (2003) designed an auction process to provide a fair negotiation process and establish long-term relationships. The iterative auction design used by Mars is illustrated in Figure 2.2. By using this auction process, they provided fairness and optimality which are the main desirable properties. Because their auction is iterative, they gave a time stamp for each bid. In this way, they optimized two objectives such as minimizing total procurement cost and minimizing the sum of the time stamp for the selected bids.

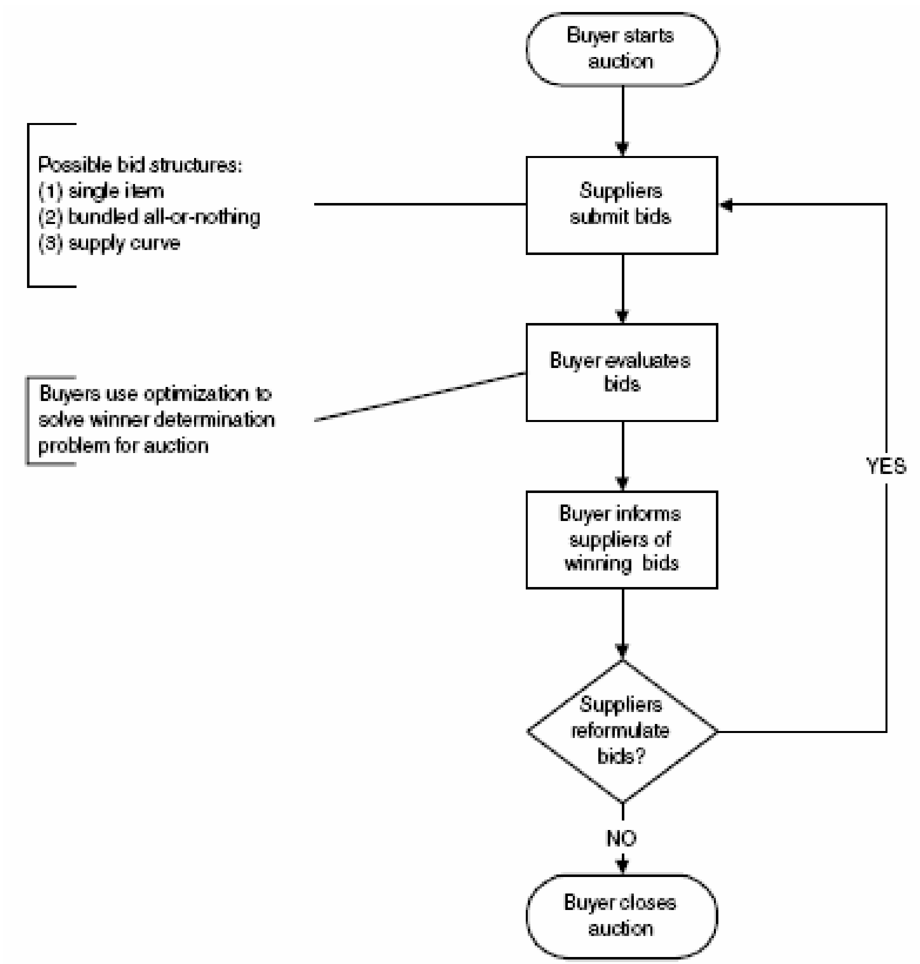


Figure 2.2 Process flow for iterative procurement auction (Hohner *et al.*, 2003)

# CHAPTER 3

## MODELS FOR COMBINATORIAL AUCTION TYPES

The literature about auctions began to grow after the paper written by Vickrey (1961). In recent years, the academic literature has begun to pay attention to design of auctions for selling large number of units with interrelated values. Due to the increase in Business-to-Business electronic commerce (B2B eCommerce) and low transaction costs on the Internet, an interest in the design of new auction mechanisms has arisen. Recently many researchers in computer science, economics, business, and game theory have presented many valuable studies on the subject of online auctions, and auction theory. From the computational perspective, combinatorial auctions are perhaps the most challenging ones. In combinatorial auctions, bidders desire to sell or buy bundles of goods rather than a single unit. (Rothkopf *et al.*, 1998). Since determination of an optimal winner combination in combinatorial auctions is an NP-hard problem (Rothkopf *et al.*, 1998), such combinatorial auctions have been of considerable interest to researchers. More information about combinatorial auctions will be given in Section 3.1.

When we consider the auction problem, its similarity to “Winner Determination Problem (WDP)” is seen easily. As is known, all auctions aim to have the best allocation and WDP is based on finding the winner with the best allocation. Research like Lehman *et al.* (2006), Sandholm (2002) or Sandholm *et al.* (2003) show computational challenges in solving the WDP and how the WDP can be applied to combinatorial auctions.

In this chapter, a brief explanation and introduction of the basic algorithm of the combinatorial auction mechanism is given. In the following chapter, some mixed integer programming models about combinatorial auctions are given.

## 3.1 COMBINATORIAL AUCTIONS

There have been several extensions to the traditional auctions in recent years (especially through growing Internet usage). One of the main active research areas has been multiple unit auctions. In a simple traditional auction, a seller sells goods to several potential buyers. Determining the auction's winner and its payment is trivial in single-unit auctions. The problem is also computationally tractable in multi-unit (homogeneous or heterogeneous) auctions when agents' valuations for 2 different units are additive, meaning that total valuation can be determined in an additive manner by their valuations for single units. In the more realistic case, the problem starts when bidder (buyer) agents have preferences over bundles, i.e. a bidder's valuation for the bundle need not be equal to the sum of his valuations of the individual units in the bundle, that is valuation need not to be additive. This problem is referred to as the combinatorial auction problem. In a combinatorial auction, a seller is faced with a set of price offers for various bundles of goods, and his aim is to allocate the goods in a way that maximizes his revenue. A combinatorial auction is desirable since it allows the bidders to express their true preferences, and thus may lead to better allocations. However, the exponential number of possible combinations usually results in computational intractability of dealing with such auctions. Due to these difficulties, only a small number of combinatorial auctions have been implemented to date. Moreover, today bidders set their bundles' quality values, delivery time or warranty terms that all need to be incorporated through the auction mechanism. According to Andersson *et al.* (2000), combinatorial auction problem can be formulated as a mixed integer problem and may include many features like

- The model can be solved by standard commercial optimization software like CPLEX.
- There may be many items for each type of units
- Bids can be constructed mutually exclusively.
- Sellers may not want preserve-prices
- One can buy and sell at the same time
- Complex bids can be expressed.

Sandholm (2002) proposes an algorithm for determining the winner and the final allocation in a combinatorial auction mechanism. He uses two main bidding languages by which bidders can express general preferences. In the first of the bidding languages, bidder offers a set of bids and bid prices can be additive when any combination is selected. For example the total price is basically sum of the selected bid prices. For the other bidding language, the set of bids is not divisible and price of the set of bids are additive.

Andersson *et al.* (2000) compare the traditional algorithms (partitioning) for computationally identical problems like set packing, propose and discuss a mixed integer program and the significance of the probability distribution of the test sets used for benchmarking in their research.

In the following section, three types of combinatorial auction models, single unit auction, single-attribute multi-unit auction and multi-attribute multi-unit auction, are given. Single unit auction is the most basic one and previous works are mostly related with this type of auction. (Sandholm 2002, Vries *et al.* 2003) Other two types, single-attribute multi-unit auction and multi-attribute multi-unit auction, are most known and applicable combinatorial auction types. Their structures and objectives are more complicated than the single unit auction.

### 3.1.1 Single-Unit Auction

This type of auction can be given as an example of a version of WDP. In this auction, like WDP, only one is sold and the aim is to maximize the auctioneer's revenue. The auctioneer has one set of units to sell and bidders submit a bid for each unit.

***Parameters:***

- N is the total number of units to be auctioned,
- M is the total number of bidders
- $p_{ij}$ is the price of unit $j$ given from bidder $i$

***Variables:***

- $x_{ij}$ is the decision variable which shows that if unit $j$ is assigned to bidder $i$, its value is 1; otherwise 0.

Max $\sum_{i=1}^{M}\sum_{j=1}^{N} p_{ij}x_{ij}$

Subject to

$$\sum_{i=1}^{M} x_{ij} \le 1 \qquad \forall\ j=1,2,...,N \tag{3.1}$$

$x_{ij} \in \{0,1\}$

According to Sandholm *et al.* (2001) if it is a free disposal auction (auctioneer does not want to keep any unit and bidders do not want to take extra units), the solution is trivial such that selecting maximum price for each unit.

### 3.1.2 Single-Attribute Multi-Unit Auction

This type of auction is the basic version of the multi-unit combinatorial auctions. The auctioneer has a set of units to sell, and the bidders submit a bundled bid that includes which units they wish to buy. The price they assign to their bid ($p_i$) is the attribute of this set of units .These types of bids contain at least one unit and the buyer cannot buy these units individually. This type of bid is called *bundled bid*.

The single-attribute multi-unit auction is to label the bids as winner or loser so as to maximize the auctioneer's revenue under the constraint that each unit can be allocated to at most one bidder:

***Parameters:***

- M is the total number of bids
- N is the total number of units to be auctioned,
- $p_i$ is the price of bundled bid $i$
- $bund_{ij}$ is the parameter that if its value is 1, bid $i$ includes unit $j$; if its value is 0, bid $i$ does not include unit $j$

***Variables:***

- $x_i$ is the decision variable which shows that if bid $i$ is assigned, its value is 1; otherwise 0.

$$\text{Max } \sum_{i=1}^{M} p_i x_i$$

*Subject to*

$$\sum_{i=1}^{M} bund_{ij} x_i \leq 1 \qquad \forall\ j=1, 2,\dots N \tag{3.2}$$

$$x_i \in \{0,1\}$$

When there are multiple indistinguishable goods for sale, Sandholm *et al.* (2002) suggest to represent these goods as multiple items of a single unit, rather than as multiple units. Different units can have multiple items in the auction. Items of one unit are indistinguishable but items of different units are distinguishable. This representation allows a bidder to place a single bid requesting the amount of each unit that he wants.

In addition, the auctioneer may have multiple items of each unit type available $U = \{u_1, u_2 ... u_N\}$ and the bidder's bid may consist of the number of items of unit k that the bid requests ( $\lambda_{ij} \geq 0$ ) and price ( $p_i$ ). The auction can be modeled in order to label the bids as winning or losing so as to maximize the auctioneer's revenue under the constraint that each unit of a unit can be allocated to at most one bidder:

***Parameters:***

- M is the total number of bids
- N is the total number of units to be auctioned,
- $p_i$ is the price of bundled bid $i$
- $\lambda_{ij}$ is the number of items of unit $j$ that the bundled bid $i$ requests
- $u_j$ is the demand of unit $j$

***Variables:***

- $x_i$ is the decision variable which shows that if bid $i$ is assigned, its value is 1; otherwise 0.

$$\text{Max } \sum_{i=1}^{M} p_i x_i$$

*subject to*

$$\sum_{i=1}^{M} \lambda_{ij} x_i \leq u_j \qquad j=1,2,...,N \tag{3.3}$$

$$x_i \in \{0,1\}$$

One extension for combinatorial auctions is multidimensional auction (sometimes it is called as multi-attribute auction). Many researchers (Che, 1993; De Smet, 2005; Teich *et al.*, 2003) review and analyze the multi-attribute auctions in their research. In multi-attribute auctions, there are multi dimensions to a transaction, such as cost, quality, delivery time and warranty period that all need to be incorporated simultaneously in the auction mechanism.

### 3.1.3 Multi-Attribute Multi-Unit Auctions

This type of auction is the most challenging auction type. Again the auctioneer has a set of units to sell, and the bidders submit their bundled bids for these units. In this auction, bundled bids have more than one attribute. For example attributes can be defined as price, quality, lead time, warranty period etc. Each bundle can have different attribute values and the auctioneer chooses the "best" according to these attribute values.

***Parameters:***

- M is the total number of bid
- N is the total number of units to be auctioned,
- $att_i^k$ is the k$^{th}$ attribute of bundled bid $i$
- $bund_{ij}$ is the parameter that if its value is 1, bid $i$ includes unit $j$; if its value is 0, bid $i$ does not include unit $j$

***Variables:***

- $x_i$ is the decision variable which shows that if bid $i$ is assigned, its value is 1; otherwise 0.

Max $\sum_{k=1}^{p}\sum_{i=1}^{M} w^k att_i^k x_i$

*Subject to*

$$\sum_{i=1}^{M} bund_{ij} x_i \le 1 \qquad \forall\ j=1, 2,..., N \tag{3.4}$$

$$\sum_{k=1}^{p} w^k = 1 \tag{3.5}$$

$x_i \in \{0,1\}$

In the real case, DM choices are not clear. A major difficulty associated with the analysis of multi-objective problems is the identification and selection of the most preferred option from the set of non-dominated solutions. In the MCDM literature, the idea of solving multi-objective optimization problem is understood as helping DM in considering the multi criteria simultaneously and in finding a nondominated solution that the DM prefers considering his/her preferences.

Below we give some definitions related with MCDM literature. For these definitions, we assumed that we have $p$ objectives and without loss of generality all objectives ( $f_i(x)$ ) are maximization type.

**Definition 1:** A vector $f(x)$, $x \in X$, is said to *dominate* another vector $f(y)$, $y \in X$, if $f_i(x) \ge f_i(y)$ for all $i$=1,2,...,p where the inequality is strict for at least one $i$. If $f(x)$ is dominated then $x$ is *inefficient.*

**Definiton 2:** A vector $f(x')$, $x' \in X$, is *nondominated* if there does not exist another $x \in X$ such that $f(x)$ dominates $f(x')$. If $f(x)$ is nondominated then $x$ is *efficient.*

A nondominated solution is supported if it is not convex dominated by other solutions. If a nondominated solution is dominated by convex combination of other solutions then it is said to be unsupported. Any supported efficient solution can be found by a weighted linear combination of objectives, choosing the weights suitably. This is not true for unsupported solutions. We can find unsupported solutions by minimizing a weighted Tchebycheff distance from the ideal point, where the ideal point is a vector obtained by optimizing each objective individually.

Most of the time, we do not know the DM's utility function. Due to the potentially large size of the non-dominated set and the undesirability of many of its elements, interactive methods have been developed which enable the DM to guide the search to more promising regions of the non-dominated set. In the MCDM literature, interactive methods have been developed since 1970's (see for example Geoffrion *et al.* 1972, Zionts and Wallenius 1976, Steuer 1986). In chapter 5, we will give an interactive method and its application to the multi-attribute multi-unit combinatorial auction process. But firstly, we will discuss the mixed integer programming models for combinatorial auctions in Chapter 4.

# CHAPTER 4

# PROPOSED MIXED INTEGER PROGRAMMING MODELS FOR COMBINATORIAL AUCTIONS

In this chapter, first, the proposed multi objective mixed integer models as an example of reverse auction for singleton bids and bundled bids are discussed. Next, input data generation algorithm is given. This data is used in the proposed mixed integer program models for combinatorial auctions. Finally, the computational results of these models are analyzed.

## 4.1 MODELS FOR SINGLE-ATTRIBUTE MULTI-UNIT AUCTION

The proposed model for single-attribute multi-unit auction structure is mainly based on the model given in section 3.1.2. Below Model-1 is a discount based model. The bidder has a chance for discount according to its account. In this model, below parameters are used:

***Parameters:***

- M is the total number of bidders,
- N is the total number of units to be auctioned,
- $c_{ij}$ is the cost of unit j of bidder $i$
- {D0, D1, D2} are discount amounts,
- {M0, M1, M2, M3} are discount zone limit points.

***Variables:***

- $x_{ij}$ is the decision variable which shows that if unit $j$ is assigned to bidder $i$, its value is 1; otherwise 0.
- $d_i$ shows the discount ratio of bidder $i$
- $y_i$ shows the number of units assigned to bidder $i$
- $z_{ik}$ shows the discount range $k$ of bidder $i$
- $t_{in}$ controls the discount range $n$ of bidder $i$

**Model-1:**

Min $\sum_{j=1}^{N}\sum_{i=1}^{M} c_{ij}x_{ij}(1-d_i)$

subject to

$$\sum_{i=1}^{M} x_{ij} = 1 \quad \forall j \tag{4.1}$$

$$\sum_{j=1}^{N} x_{ij} - y_i = 0 \quad \forall i \tag{4.2}$$

$$d_i = d_0 \cdot z_{i0} + d_1 \cdot z_{i1} + d_2 \cdot z_{i2} \quad \forall i \tag{4.3}$$

$$y_i = t_{i0} \cdot m_0 + t_{i1} \cdot m_1 + t_{i2} \cdot m_2 + t_{i3} \cdot m_3 \quad \forall i \tag{4.4}$$

$$t_{i0} \le z_{i0} \quad \forall i \tag{4.5}$$

$$t_{i1} \le z_{i0} + z_{i1} \quad \forall i \tag{4.6}$$

$$t_{i2} \le z_{i1} + z_{i2} \quad \forall i \tag{4.7}$$

$$t_{i3} \le z_{i2} \quad \forall i \tag{4.8}$$

$$\sum_{n=0}^{3} t_{in} = 1 \quad \forall i \tag{4.9}$$

$$\sum_{k=0}^{2} z_{ik} = 1 \quad \forall i \tag{4.10}$$

$x_{ij}, z_{ik} \in \{0,1\}$ , $d_i, y_i, t_{in}, c_{ij} \geq 0$

In the model, first constraint (4.1) aims that every unit should be assigned. Other constraints are related with determination of discount ratio applied to each bidder. Model firstly counts the assigned units on every bidder (4.2) then determines the discount ratio $d_i$ (4.3) of each bidder. For example, if r number of items are assigned (M1<r<M2) to a bidder, the bidder will discount D1% on the total amount. Rest of the constraints (4.4, 4.5, 4.6, 4.7, 4.8, 4.9, 4.10) determine the discount range of each bidder and control the discount ratio applied to bidders.

The objective function of the this model aims to calculate the total cost with respect to the assigned items' cost and determined bidder's discount ratio and this objective is nonlinear because two variables ($x_{ij}$ and $d_i$) are multiplied with each other. In order to eliminate this non-linearity, replace this multiplication with a new variable $o_{ij}$ such that $x_{ij} * d_i = o_{ij}$. This linear model Model-2 is given below. Model-1 and Model-2 are similar models with respect to properties of the auction:

**Model-2:**

Min $\sum_{j=1}^{N}\sum_{i=1}^{M} c_{ij}.x_{ij} - \sum_{j=1}^{N}\sum_{i=1}^{M} c_{ij}.o_{ij}$

subject to

$$\sum_{i=1}^{M} x_{ij} = 1 \quad \forall j \qquad (4.11)$$

$$\sum_{j=1}^{N} x_{ij} - y_i = 0 \qquad \forall i \tag{4.12}$$

$$d_i = d_0 \cdot z_{i0} + d_1 \cdot z_{i1} + d_2 \cdot z_{i2} \tag{4.13}$$

$$y_i = t_{i0} \cdot m_0 + t_{i1} \cdot m_1 + t_{i2} \cdot m_2 + t_{i3} \cdot m_3 \tag{4.14}$$

$$t_{i0} \le z_{i0} \qquad \forall i \tag{4.15}$$

$$t_{i1} \le z_{i0} + z_{i1} \qquad \forall i \tag{4.16}$$

$$t_{i2} \le z_{i1} + z_{i2} \qquad \forall i \tag{4.17}$$

$$t_{i3} \le z_{i2} \qquad \forall i \tag{4.18}$$

$$\sum_{n=0}^{3} t_{in} = 1 \qquad \forall i \tag{4.19}$$

$$\sum_{k=0}^{2} z_{ik} = 1 \qquad \forall i \tag{4.20}$$

$$o_{ij} \le x_{ij} \cdot L \qquad \forall i, j \tag{4.21}$$

$$o_{ij} \le d_i \qquad \forall i, j \tag{4.22}$$

$$x_{ij}, z_{ik} \in \{0,1\} \;, \quad t_{in}, y_i, c_{ij}, o_{ij} \ge 0$$

In this model, variables are the same except $o_{ij}, x_{ij}$ and $d_i$. In order to limit $o_{ij}$, we can add two constraints like 4.21 and 4.22. These constraints control the value of $o_{ij}$ according to our variables $x_{ij}$ and $d_i$ (L is very large number). When one of the two variables' ($x_{ij}$ and $d_i$) value is zero, $o_{ij}$'s value becomes zero.

This model can also be constructed with some additional side constraints like maximum/minimum trade constraints. The constraints on the minimum and maximum number of winning bidders limit the number of winning bidders. To achieve this, we can introduce a new variable $wb_i$ for each bidder $i$ which takes the value 1 if the bidder has any winning bids and 0 otherwise (Hohner *et al*., 2003). These constraints (4.23 and 4.24) can be written (L is a very large number) such that:

$$\sum_{j=0}^{N} x_{ij} \leq wb_i * L \qquad \forall i \tag{4.23}$$

$$W_{\min} < \sum_{i=0}^{M} wb_i < W_{\max} \qquad \forall i \tag{4.24}$$

Moreover Hohner *et al.* (2003) propose to decide the range of winning bidders' revenues. For example if any units are assigned to bidder $i$, then the amount of revenue of bidder $i$ can be restricted in the decided range $[L_i, U_i]$. $L_i$ is the lower limit of revenue of bidder $i$ and $U_i$ is the upper limit of revenue of bidder $i$. This limitation provides economy of scale because much more bidders mean that much more transaction costs and time. This constraint (4.25) can be expressed in the following way:

$$wb_i * L_i \leq (\sum_{j=0}^{N} c_{ij} * x_{ij}) \leq wb_i * U_i \qquad \forall i \tag{4.25}$$

Constraints 4.23, 4.24 and 4.25 can be used altogether depending on the firm's strategy.

## 4.2 MODELS FOR MULTI-ATTRIBUTE MULTI-UNIT AUCTION

Multi-attribute multi-unit auctions can be considered as the most challenging auction types. A bidder can define any attribute for the proposed unit. For example a bidder can define a quality attribute for his unit because he trusts his unit with respect to quality however he may deliver this unit very late. Because of similar reasons, the auctioneer needs to define attributes for different aspects in the auction. On the other hand, the auctioneer should define which attributes are important for him because more attributes mean more complex problem. This causes assignment and generation of different scenarios hard. Therefore auctioneer should avoid defining much more attributes.

In this part of the thesis, two-attribute multi-unit auction programs (Model-3 and Model-4) are written and solved. The two attributes are defined as cost and lead time. The generation of the value of the attributes is explained in section 4.3.

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Two-attribute multi-unit auction model (Model -3) and its parameters and variables are given below:

***Parameters:***

- M is the number of bundled bids,
- N is the number of units to be auctioned,
- $w$ is the weight of cost attribute
- $c_i$ is the cost of bundled bid $i$
- $lt_i$ is the lead time of bundled bid $i$
- $bund_{ij}$ is the parameter that if its value is 1, bid $i$ includes unit $j$; if its value is 0, bid $i$ does not include unit $j$

***Variables:***

- $y_i$ is the decision variable which shows that if bundle bid $i$ is assigned, its value is 1; otherwise 0.
- $cost$ is the total cost of auction
- $leadtime$ is the total lead time of auction

**Model-3**

Min $w * cost + (1-w) * leadtime$

Subject to

$$cost = \sum_{i}^{M} c_i * y_i \tag{4.26}$$

$$leadtime = \sum_{i}^{M} lt_i * y_i \tag{4.27}$$

$$\sum_{i}^{M} bund_{ij} * y_i \geq 1 \qquad \forall j \in N \tag{4.28}$$

In this model, constraint 4.26 and 4.27 calculate the total cost and lead time of auction according to assignment of bids respectively. Constraint 4.28 guarantees that at least one unit for each unit is bought. The objective function aims to minimize cost and lead time according to bid assignment.

With Model-3, we can find the supported efficient solutions. Furthermore, if we want to find unsupported efficient solutions, we can apply the weighted Tchebycheff method (Steuer 1986, 419-435). The model for augmented weighted Tchebycheff method (Model-4) and its parameters and variables are given below:

***Parameters:***

- M is the number of bundled bids,
- N is the number of units to be auctioned,
- $w$ is the weight of cost attribute
- $zc$ is the ideal point for cost
- $zl$ is the ideal point for lead time
- $c_i$ is the cost of bundled bid $i$
- $lt_i$ is the lead time of bundled bid $i$

- $bund_{ij}$ is the parameter that if its value is 1, bid $i$ includes unit $j$; if its value is 0, bid $i$ does not include unit $j$

***Variables:***

- $y_i$ is the decision variable which shows that if bundle bid $i$ is assigned, its value is 1; otherwise 0.
- $cost$ is the total cost of auction
- $leadtime$ is the total lead time of auction

**<u>Model-4</u>**

Min $\alpha + \varepsilon * ((cost - zc) + (leadtime - zl))$

Subject to

$$\alpha \geq w * (cost - zc) \tag{4.29}$$

$$\alpha \geq (1 - w) * (leadtime - zl) \tag{4.30}$$

$$cost = \sum_{i}^{M} c_i * y_i \tag{4.31}$$

$$leadtime = \sum_{i}^{M} lt_i * y_i \tag{4.32}$$

$$\sum_{i}^{M} bund_{ij} * y_i \geq 1 \qquad \forall j \in N \tag{4.33}$$

This model is very similar to Model-3 with respect to variables and parameters. However the main aim for Model-4 is minimizing the weighted Tchebycheff distance (i.e. minimizing the maximum weighted distance from ideal point) between ideal point and the solution. Constraints 4.29 and 4.30 help minimizing distance between cost and lead time objective values and their respective ideal points. We found these ideal points for cost and lead time by solving the each objective independently. In the objective function part, the differences of solutions and ideal points are multiplied with a small positive constant ($\varepsilon$). This part prevents obtaining weakly efficient but inefficient solutions.

As a matter of fact, auction process may need to contain multiple attributes for decision making. Multiple attributes allow auctioneer to decide accurately which bidders should win but it should be also considered that much more attributes cause more complex problem. Because of this, auctioneer firstly should decide which attributes are used. For multiple attributes, we can write a multi objective programming model given below.

***Parameters:***

- M is the number of bundled bids,
- N is the number of units to be auctioned,
- $w_k$ is the weight of $k$ th attribute
- $att_{ik}$ is the value of attribute $k$ of bundled bid $i$
- $bund_{ij}$ is the parameter that if its value is 1, bid $i$ includes unit $j$; if its value is 0, bid $i$ does not include unit $j$

***Variables:***

- $y_i$ is the decision variable which shows that if bundle bid $i$ is assigned, its value is 1; otherwise 0.
- $totatt_k$ is the total value of attribute $k$ of the auction

$$\text{Min } \sum_{k=1}^{p} w_k * totatt_k$$

Subject to

$$totatt_k = \sum_{i=1}^{M} attr_{ik} * y_i \qquad \forall k \qquad (4.34)$$

$$\sum_{i=1}^{M} bund_{ij} * y_i \geq 1 \qquad \forall j \in N \qquad (4.35)$$

$$\sum_{k=1}^{p} w_k = 1 \qquad (4.36)$$

There are $p$ attributes in the auction and $w_k$'s are the weights of the attributes. In order to write the weights of each attribute, the auctioneer should decide the value of the weights firstly, it means that the relative importance of each attribute should be decided then the solution can be found according to these attributes. Without loss of generality, all attributes are to be minimized. The constraint 4.34 calculates the total value of the each attribute of the current bid assignment. All bundled bids should be assigned (Constraint 4.35). Constraint 4.36 maintains normalization of weights.

More attributes provide more accurate decision among these bids; however in order to decide which bids should be assigned, either we should search the whole solution space or know auctioneer (buyer) preferences. In practice, the auctioneer preferences are implicit and in order to elicit preferences their utility functions are required. Basic approach is coming from decision analysis techniques such as MAUT (Keeny *et al.*, 1993), SMART (Edwards, 1977) and AHP (Saaty, 1980). These techniques mostly use linear, weighted value functions. In the auction process, weights of the attributes are very important because the solution depends on these weights, so deciding on the weights are the key point of the auction. Several techniques have been proposed to help users assign reasonable weights. One approach is called pricing out used in MAUT. This approach is based on determining the value of the objective in terms of another. For example, one may say that ten days faster delivery time is worth $600. Another approach for weight determination is used by AHP. This approach is based on pair-wise comparisons of attributes. A consistent matrix is formed from this comparison and it generates the weights from eigenvectors' of this matrix.

Many researches and studies investigated these approaches. For example Bichler *et al.*(2005) studied a bid evaluation technique in multi-attribute auction with an additive utility function and concept of multi-attribute winner determination with multi sourcing and configurable offers. Teich *et al.* (1999) proposed an algorithm for multiple unit auctions. In their algorithm, bidders want to buy units and submit their bid including quantity and cost for desired unit. When a bidder outbid, other bidders are informed and invited to resubmit. Another approach for combinatorial auctions is Park *et al.* (2005) study. In their approach, bidders are allowed to determine combination of units and they proposed that using bidder-combination bids provides fairness and realization of economically-important combinations.

In the next section, we will give an example of an interactive method (Achievement Scalarizing Function) and its application to the multi-attribute multi-unit combinatorial auctions.

## 4.3 INPUT DATA GENERATION

Bids' structure and specifications are important issues in the auction process. These issues are based upon the nature of the auction. For example if an auction is related with antique sales, it is a single unit auction and each bidder can only bid for each commodity. On the other hand, if the auction is related with a procurement process, most probably the auction type is multi-attribute multi-unit auction and bidders offer bundled bids.

In this thesis, singleton and bundled bids are used. Singleton bids consist of attributes for each unit individually and when a buyer wants to buy more than one unit, the attributes' properties stay the same, without converting them into something else. For example cost attribute is an additive attribute and the total amount of auction is basically the summation of cost of assigned bids. However, bundled bids have different structures. The bundled bid is not basically equal to the sum of attributes' values of the bid, because the bidder can have preferences over units. It means that the bidder may have preference over bundle of units. For example a seller can prefer to sell units a and b only. Moreover, the price of bundled bid {a,b} can be less than the sum of prices of a and b for this seller. Bundled bid structure is more realistic than other bid structures, because in the real world, bidders have own "private prices" for each unit and valuation can be different when two units are sold together or separately and need not be additive. Valuation is explained in detail in Section 2.3.

In order to use as input data in the models, singleton and bundled bids are generated. The structure of bundled bids consist of proposed units, cost and lead time. This structure is shown below in Table 4.1 where {a1,a2,…,a50} are auctioned units. If any unit is proposed by bidder, the value is 1, otherwise 0. In the below example, bidder 7 gives a bid including units 9 and 47, these two units' cost 16.98 and bidder 7 will deliver these two units in 9 unit-time. We called this bid as bundled bid with sized 2. Detailed input data and singleton and bundled bids generation algorithm is explained in Appendix A.

Table 4.1 - Bundled Bid structure

| **Bidder No.** | **a1** | **a2** | **a3** | **…..** | **a9** | **a10** | **…..** | **a46** | **a47** | **a48** | **a49** | **a50** | **Cost** | **Lead time** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | 0 | 0 | 0 | …. | 1 | 0 | ….. | 0 | 1 | 0 | 0 | 0 | 16.98 | 9 |

## 4.4 EXAMPLE SOLUTIONS OF MODEL-3 AND MODEL-4

In the proposed Model-3 and Model-4, there are 900 bundled bids (150 bundled bids with size 5, 450 bundled bids with size 6 and 300 bundled bids with size 7) and each bid has two attributes such as cost and lead time. Model-3 is solved in GAMS 22.2 and by changing $w$ systematically between 0 and 1, we obtained the solutions in Figures 4.1 and 4.2. When we solved the Model-4 again by changing $w$ systematically between 0 and 1, we obtained many unsupported efficient solutions. The resulting supported and unsupported solutions are compared in Figure 4.3.

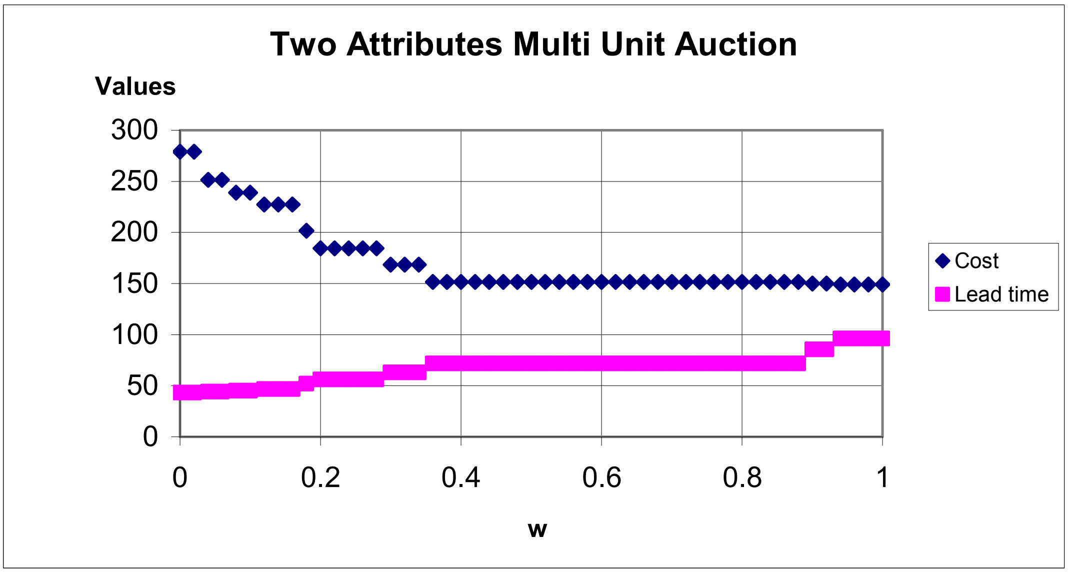


Figure 4.1 - Chart of Two Attributes Multi Unit Auction

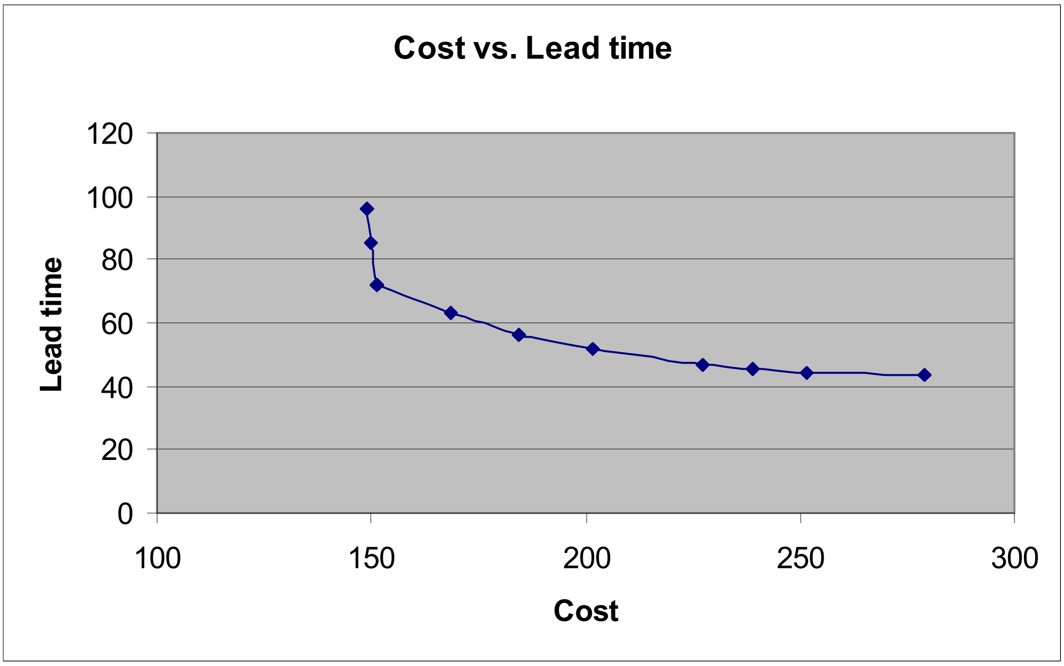


Figure 4.2 – Relation Between Two Attributes

According to Figure 4.1 and Figure 4.2, cost and lead time attributes conflict with each other. As we expect that when cost gets smaller, lead time increases.

Up to now, we solved four different models for multi-attribute multi-unit auctions where the input data included bundles of size 5, 6 and 7. For the bundle size 5, the bundled bid consists of 5 different units and the bid's cost and lead time are related to these five units.

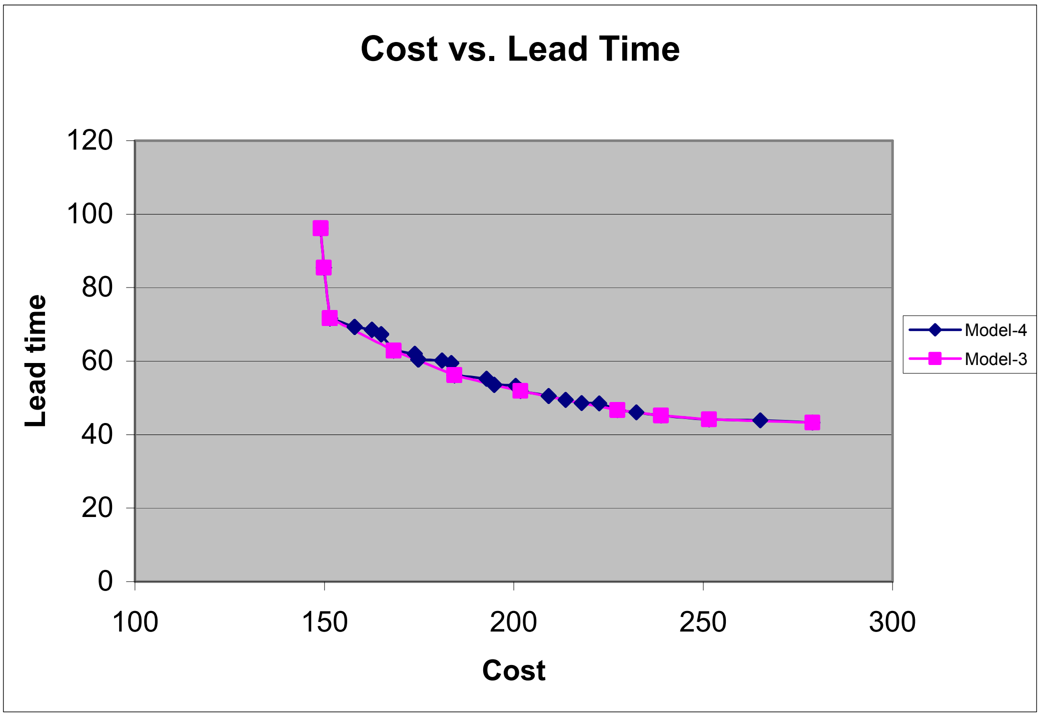


Figure 4.3 Comparative Chart

Firstly we run Model-3 with 900 bundled bids (150 bundled bids with size 5, 450 bundled bids with size 6 and 300 bundled bids with size 7) by changing $w$ (the weight of cost) systematically between 0 and 1 with increments of 0.01. For each $w$ value, approximately 10 bids are assigned (i.e., there are about 10 winning bids). Out of the 100 runs with different $w$ values, the frequency of assignment of each of the 900 bids is given in Figure 4.4. According to the figure, over 40 different bundled bids are assigned more than three times. This means that the auctioneer shall have many alternatives corresponding to different $w$ values. On the other hand, when we change the bundle size and solve Model-3 (with 150 bundled bids of size 7, 450 bundled bids of size 9 and 300 bundled bids of size 10) again by changing $w$ systematically between 0 and 1, Only 18 different bundled bids are assigned more than three times (see Figure 4.5). These two figures (Figure 4.4 and Figure 4.5) show us how bundle size affects the solution space. When the bundle sizes get larger, the number of efficient solutions decreases. This can also be observed from Figure 4.6.

Moreover high frequencies lead to dependency of the auctioneer to bidders. When we look at the Figures 4.4 and 4.5, we can easily detect the popular bidders from among other bidders. In order to avoid this situation, most auction processes over the Internet (Sandholm *et al.* 2006, Hohner *et al.*, 2003) prevent supplier dependency. According to Hohner *et al.* (2003), they control the assignment of the bids with respect to previously determined minimum and maximum winning suppliers in the solution. Moreover, they can limit the quantity of units supplied from a bidder in a range of minimum and maximum quantity level. In the winning allocation, if a bidder has any allocation, it must lie in this range. We have already given an example for this issue in section 4.2.

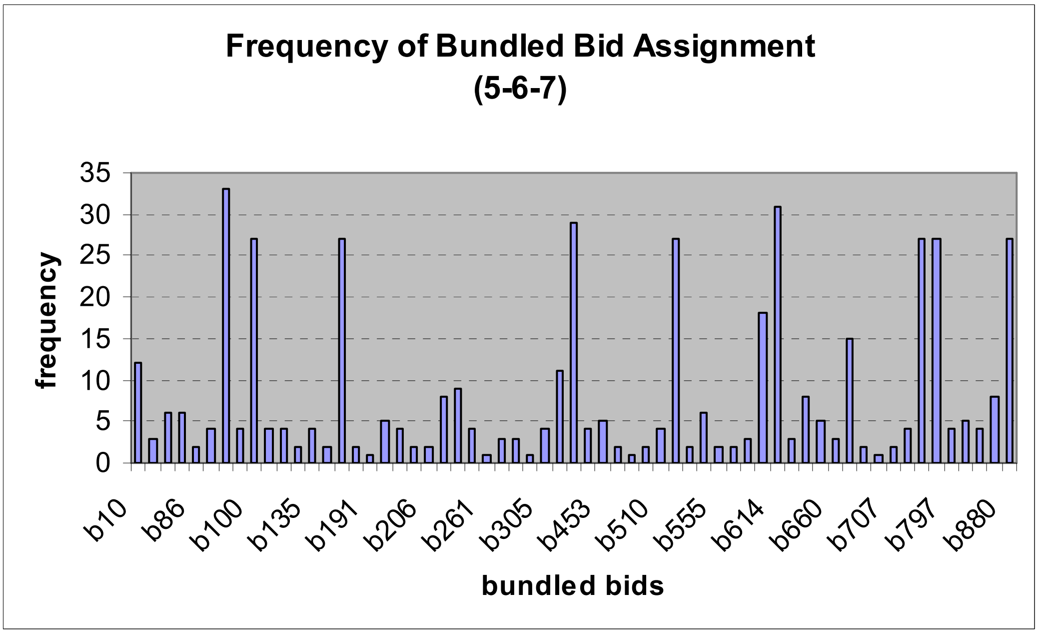


Figure 4.4 - Frequency of 5-6-7 sized Bundled Bids

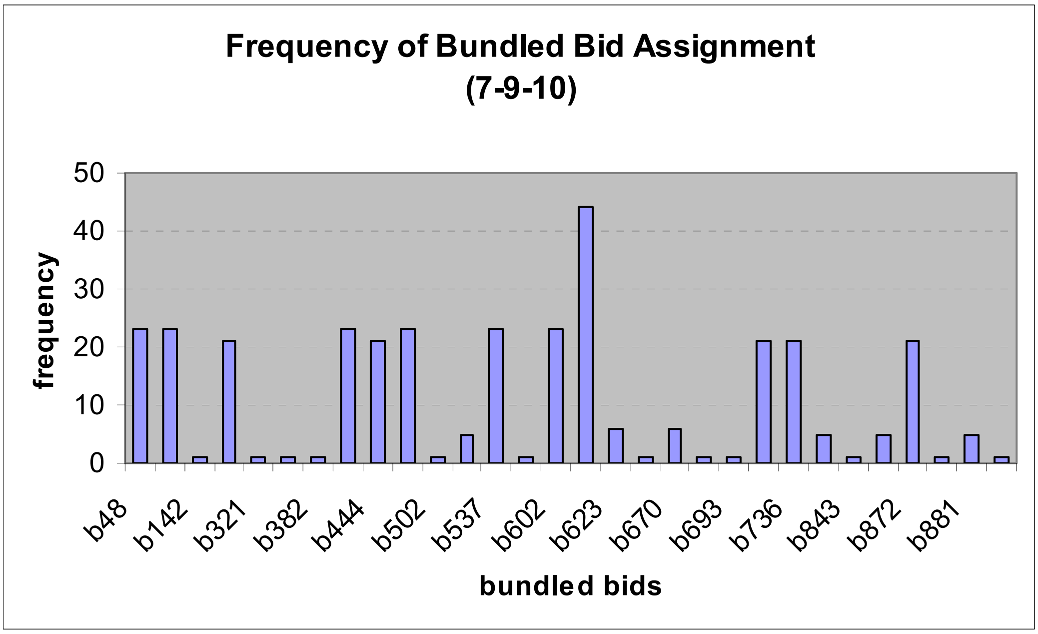


Figure 4.5 - Frequency of 7-9-10 sized Bundled Bids

When we compared these two different bundled sized models, we recognize easily the difference of objective values and how many efficient solutions exist. In our example, when we use 7-9-10 sized bundled bids, only 5 different efficient solutions can be found. On the other hand, when 5-6-7 sized bundled bids are used, efficient solutions go as high as 10 different ones. The comparison of these two models is given in Figure 4.6. According to this figure, efficient solutions of models that used large sized bundles dominates small sized bundles, partially because cost objective coefficients of models that use large sized bundles are in general better than small ones due to cost discount when bundles are generated (see Appendix A). On the other hand, the DM has only five different efficient solutions to select the best when large bundle sizes are used. This situation is one of the trade-offs between large and small sized bundles.

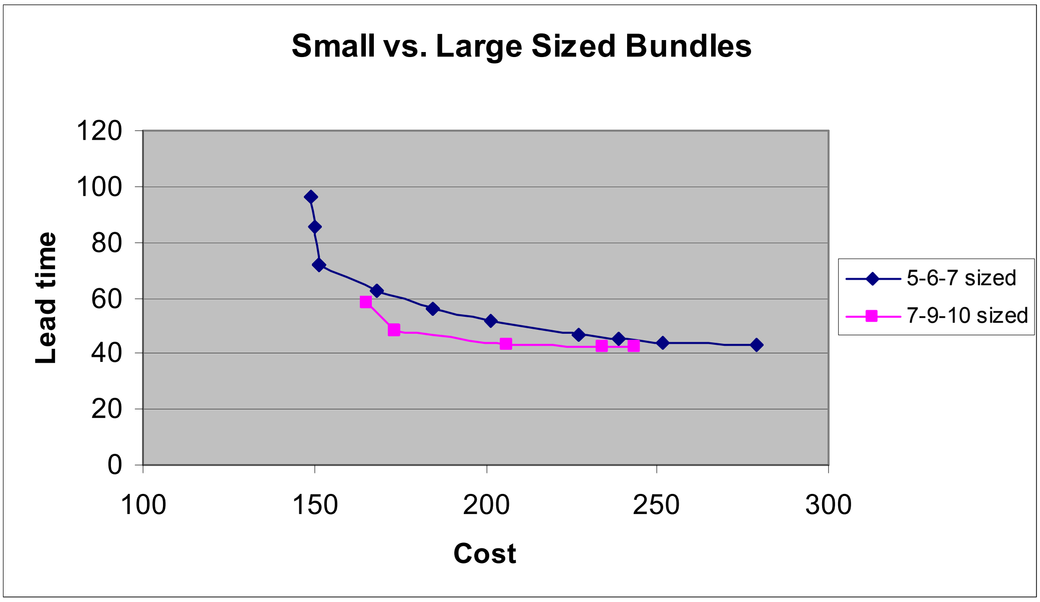


Figure 4.6 – Objective Values of Two Different Sized Bundled Bids

Other than the bundle size, many factors can affect results. For example if the number of bundles increase, we expect the efficient solutions to improve in all criteria because the solution space is expanded. Moreover, solution time naturally depends on the number of bundles (since we have a binary variable for each bundle) and increases as the number of bundles increases.

# CHAPTER 5

# INTERACTIVE METHODS FOR COMBINATORIAL AUCTIONS

## 5.1 ACHIEVEMENT SCALARIZING FUNCTION (ASF)

What can we do if we do not know the DM's utility function? Korhonen and Laakso (1986) proposed a method for multi objective programs when DM's utility function is not known. The method is not based on an explicitly known utility function; only assumes that the utility function is pseudoconcave. This method desires to help the DM to find improved solutions and enables to evaluate a subset of the efficient frontier.

The method assumes that the DM specifies reasonable or desirable aspiration levels forming a reference point. An achievement scalarizing function (ASF) projects any given point (reference point) onto the efficient frontier. It means that the model searches on efficient frontier. The reference point consists of aspiration levels reflecting desirable values for the objective functions. It reflects the DM's preference information and affects the next set of solution generated.

ASF has been introduced by Wierzbicki in 1980. Also general framework is similar to GDF model (Geoffrion *et al.*, 1972). But according to Korhonen and Laakso (1986), method of Wierzbicki does not help DM find improved solutions, only considers discrete set of alternatives and does not check the optimality of the current solution.

### 5.1.1 The Algorithm

Korhonen and Laakso (1986) summarize their algorithm as follows:

*Step 0:* Find an arbitrary point $q^0$ in the criterion space. Let k=1 ($q^0$ does not need to be efficient)

*Step 1:* Specify a vector $g^k$ and take vector $d^k = g^k - q^{k-1}$ as the new reference vector.

*Step 2:* Find the set $Q^k$ of efficient vectors $q$ that solve the Achievement Scalarizing Program (ASP)

Min $s(q, w, z)$

Subject to

$$z = q^{k-1} + \theta d^k \,, \quad q \in Q \tag{5.1}$$

$\theta$ is increased from zero to infinity, $s$ is the achievement scalarizing function (ASF) and $w$ is a weighing vector. We can write $s$ as follows:

$$s(q, w, z) = \max\left\{w_i(q_i - z_i)\right\} - \varepsilon \sum_{i=1}^{p} z_i \tag{5.2}$$

The last term is included to avoid weakly efficient but inefficient points. In order to project the point, we simply minimize the function $s$ in the solution space.

*Step 3:* Find the most preferred solution $q^k$ in $Q^k$.

*Step 4:* If $q^{k-1} \neq q^k$, let k=k+1 and return back to Step 1. Otherwise, check the optimality condition. If the conditions are satisfied, stop. $q^k$ is an optimal solution. If the conditions are not satisfied, let k=k+1 and $d^k$ is a new direction identified by the procedure.

If anyone wants to check whether $q*$ is an optimal solution, we have to assume the utility function of the DM is pseudoconcave on the set of all alternatives. Thus if $q^{k-1} = q^k$ condition is satisfied, the projection of $d^k$ vector is not an improvement direction and if any other improving directions are not found, then $q^k$ is an optimal solution. The method can also be used to search the efficient solution space without explicitly trying to prove optimality.

### 5.1.2 Examples

In this section, we demonstrate an implementation of Korhonen and Laakso's (1986) method on an auction process. Firstly, index sets, parameters, and decision variables are given. Then, the mixed integer programming model for the auction problem is given.

***Parameters:***

- M is the number of bundled bids,
- N is the number of units to be auctioned,
- $c_i$ is the cost of bid $i$,
- $q_i$ is the quality of bid $i$,
- $lt_i$ is the lead time of bid $i$,
- $q_1, q_2, q_3$ are the reference points of cost, quality and lead time respectively.
- $d_1, d_2, d_3$ are the directions of reference points respectively.
- $bund_{ij}$ is the parameter that if its value is 1, bid $i$ includes unit $j$; if its value is 0, bid $i$ does not include unit $j$.

***Variables:***

- $y_i$ is the decision variable that takes a value of 1 if bundle bid $i$ is assigned, and 0 otherwise.

- *cost* is the total cost of the auction
- *quality* is the total quality of the auction
- *leadtime* is the total lead time of the auction

The mathematical model is given as follows.

Min $\alpha + \varepsilon * (cost - quality + leadtime)$

Subject to

$$\alpha \geq cost - (q_1 + (\theta * d_1)) \tag{5.3}$$

$$\alpha \geq (q_2 + (\theta * d_2)) - quality \tag{5.4}$$

$$\alpha \geq leadtime - (q_3 + (\theta * d_3)) \tag{5.5}$$

$$cost = \sum_{i=0}^{M} \mathrm{c}_i * y_i \tag{5.6}$$

$$quality = \sum_{i=0}^{M} q_i * y_i \tag{5.7}$$

$$leadtime = \sum_{i=0}^{M} lt_i * y_i \tag{5.8}$$

$$\sum_{i=0}^{M} bund_{ij} * y_i = 1 \qquad \forall j \in N \tag{5.9}$$

According to the objective function, we want to minimize the total cost and lead time, maximize the total quality. The values of total cost, lead time and quality are calculated from the assigned (winning) bids. The cost, quality and lead time values of each unit are randomly generated and the rule which is explained in Appendix A is applied when the bundles are composed.

In our example, there are 950 bundled bids which consist of 50 bundled bids having 1-unit, 150 bundled bids having 5-units, 450 bundled bids having 6-units and 200 bundled bids having 7-units. Totally we want to assign 950 units optimally with respect to cost, quality and lead time attributes of bundled bids. Model evaluated total cost, quality and lead time in constraints 5.6, 5.7 and 5.8

respectively. In order to minimize $\alpha$ , we try to minimize total cost and lead time, maximize total quality by the constraints 5.3, 5.4 and 5.5. Note that we do not differentiate between the relative importance of criteria in the ASP (i.e. we use equal weights). With the last constraint 5.9, we guarantee that every unit is assigned only once.

In the first example, to simulate the responses of a DM, we assumed that the DM has the following implicit linear utility function:

$$u = 4 * cost - 3.5 * quality + 10 * leadtime \quad (5.10)$$

While applying the method, the most preferred solution on the efficient curve is determined according to the utility function in 5.10. It means that the minimum utility function value of the solutions is the most preferred solution among the efficient points. These efficient points are found by changing $\theta$ while we search the efficient frontier. But for higher $\theta$ values, the same objective values are found. This situation is related with line search in the method and reaching a border of the efficient solution space. With maximum $\theta$ value, reference vector shoots the last extreme point (last searching point in the piecewise line). Because of this, with the larger $\theta$ values, the objective values get the same value (last supported efficient solution). A graphical representation of the search process on a hypothetical efficient frontier is shown in Figure 5.1.

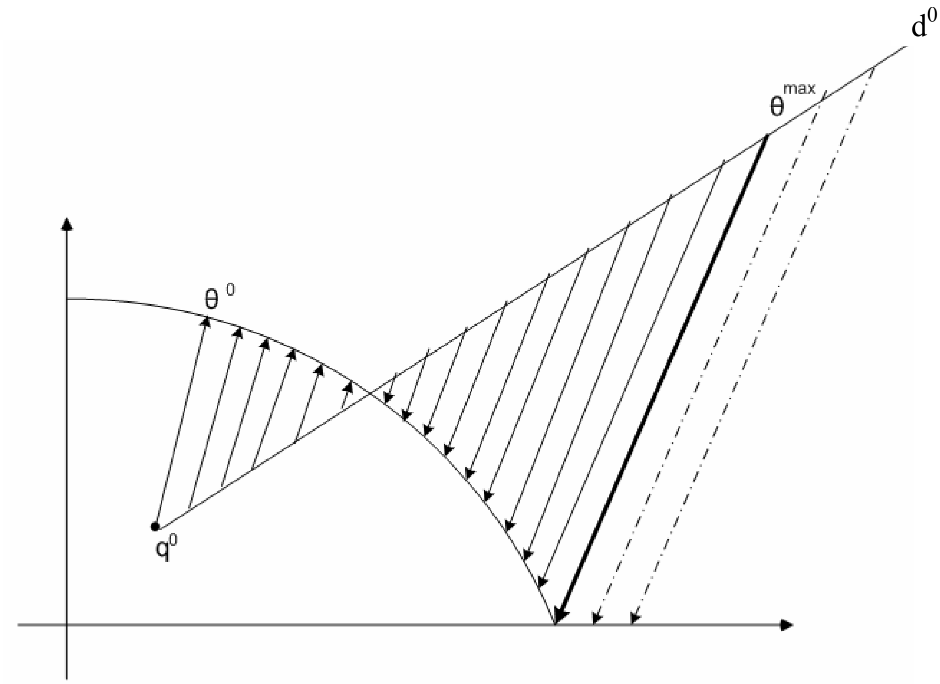


Figure 5.1 A graphical representation of search process

Let us define $\alpha_1, \alpha_2, \alpha_3$ as the (RHS) values of constraints 5.3, 5.4 and 5.5 respectively. In order to find $\theta_{\max}$, at least two constraints (among constraint 5.3, 5.4 and 5.5) are binding and the RHS values of two among three constraints are equal. For example if $\alpha_1 = \alpha_2$, then $\alpha_3$ should be less than both of them because the equal valued constraints should be binding.

Cost vs. Quality ($\alpha_1 = \alpha_2$): $\theta_{\max} = \dfrac{cost + quality - q^1 - q^2}{d^1 + d^2}$ (5.11)

Cost vs. Lead Time ($\alpha_1 = \alpha_3$): $\theta_{\max} = \dfrac{cost - leadtime - q^1 + q^3}{d^1 - d^3}$ (5.12)

Lead Time vs. Quality ($\alpha_3 = \alpha_2$): $\theta_{\max} = \dfrac{leadtime + quality - q^2 - q^3}{d^2 + d^3}$ (5.13)

To find $\theta_{\max}$, firstly we run the model with a very high $\theta$ and find the extreme point values of each objective in that reference direction. Then we substitute the objective values in each equation (5.11, 5.12 and 5.13) and calculate $\theta_{\max}$.

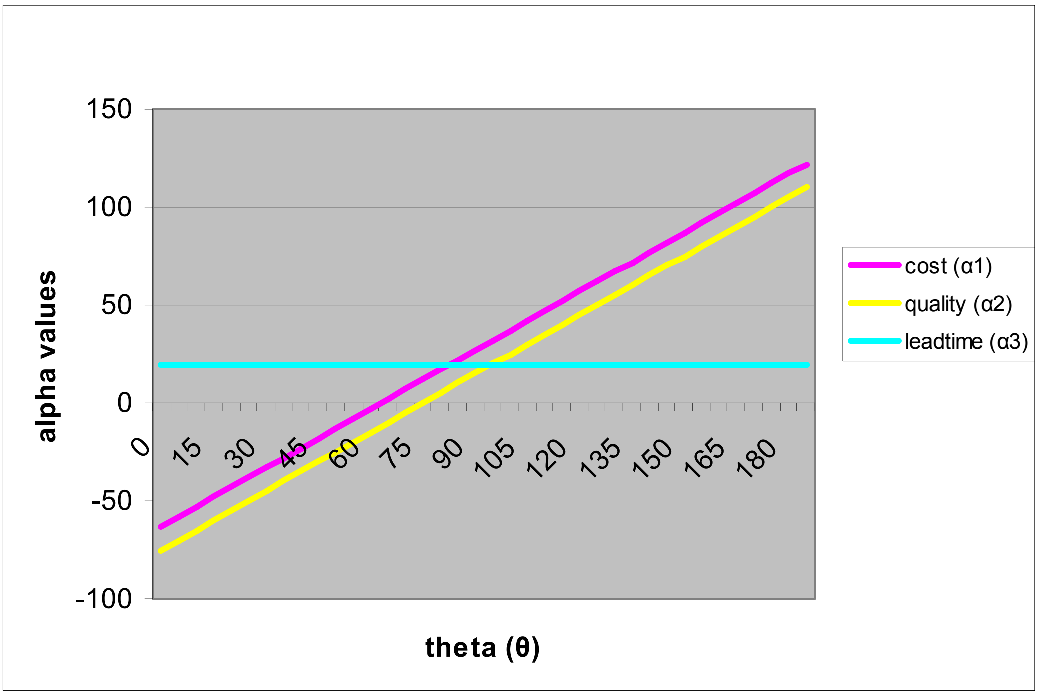


Figure 5.2 Graphical representation of changing $\alpha_1, \alpha_2, \alpha_3$ values with respect to $\theta$

For example in Figure 5.2, there are two intersection points between three $\alpha$ values. First intersection point is seen from equality of $\alpha_1$ and $\alpha_3$ at around $\theta$=85 and at that $\theta$ value, $\alpha_2$ value is less than $\alpha_1$ and $\alpha_3$. Thus this $\theta$ value is our $\theta_{\max}$. On the contrary, second intersection point is seen from equality of $\alpha_2$ and $\alpha_3$ at around $\theta$=95 but $\alpha_1$ value is higher than other two at that $\theta$ value. So we keep $\theta_{\max}$ =85.

Let us come back to our example and define the initial reference point as $q^0$ = (200,170,100). Suppose that the DM's reference vector is $d^0$=(-1, 1,0). It means that the DM wants to decrease total cost and increase the total quality values. The method is applied according to these two reference vectors. Firstly $\theta_{max}$ is found by the method explained above and we can find the efficient points on piecewise linear curve by changing $\theta$ values. (The step size of changing $\theta$ values depends on the $\theta_{max}$ because we aim to search the efficient frontier piece with minimum 10 steps until we reach to $\theta_{max}$ .)

In iteration 1, from the decided initial reference point and directions, we can find $\theta_{max}$ =82.21 and we solved the model by changing $\theta$ with the step size increments of 10. Table 5.1 shows us the solutions on the efficient frontier. Rightmost column shows the (implicit) utility function values of the related solutions. So we can decide to select new reference point as $q^1$=(163.24, 231, 68.98) by considering the utility function values.

Table 5.1 Efficient Points ( $d^0$=(-1, 1,0))

| **Theta (θ)** | **cost** | **quality** | **leadtime** | **DM's utility function value** |
|---|---|---|---|---|
| **0** | ***163.24*** | ***231*** | ***68.98*** | ***534*** |
| 10 | 161.24 | 230 | 72.84 | 568 |
| 20 | 151.00 | 227 | 73.57 | 545 |
| 30 | 150.28 | 224 | 80.67 | 624 |
| 40 | 150.25 | 228 | 87.99 | 683 |
| 50 | 140.01 | 229 | 91.02 | 669 |
| 60 | 140.01 | 229 | 91.02 | 669 |
| 70 | 139.80 | 250 | 102.63 | 711 |

Table 5.1 (continued)

| **Theta (θ)** | **cost** | **quality** | **leadtime** | **DM's utility function value** |
|---|---|---|---|---|
| 80 | 136.86 | 245 | 119.07 | 881 |
| 90 | 136.86 | 245 | 119.07 | 881 |

Continuing with the second iteration, let us assume that the DM defines the new reference direction (-1,0,-3) since he wants to decrease total cost and lead time values. $\theta_{\max}$ is found as 16.82. That is, at $\theta_{\max}$ =16.82 the last efficient solution is reached in the diagonal direction of the contours defined by the ASF. Since this is a discrete problem, the same solution can be reached for some smaller $\theta$ values and for all larger $\theta$ values. Once more the model is solved with a step size of 2 and the resulting solutions are given in Table 5.2.

Table 5.2 Efficient Points ( $d^1$ =(-1, 0,-3))

| **Theta (θ)** | **cost** | **quality** | **leadtime** | **DM's utility function value** |
|---|---|---|---|---|
| 0 | 163.24 | 231 | 68.98 | 534 |
| 2 | 163.24 | 231 | 68.98 | 534 |
| 4 | 170.85 | 244 | 64.75 | 477 |
| 6 | 170.85 | 244 | 64.75 | 477 |
| **8** | ***173.01*** | ***261*** | ***61.99*** | ***398*** |
| 10 | 173.01 | 261 | 61.99 | 398 |
| 12 | 173.01 | 261 | 61.99 | 398 |
| 14 | 184.07 | 253 | 56.17 | 412 |

Table 5.2 (continued)

| Theta (θ) | cost | quality | leadtime | DM's utility function value |
|---|---|---|---|---|
| 16 | 184.07 | 253 | 56.17 | 412 |
| 18 | 184.07 | 253 | 56.17 | 412 |

If you look at the last three objective values for different $\theta$, the objective values are seen to be fixed after $\theta$=14. It means that last supported efficient solution is (184.07, 253, 56.17). Figure 5.3 shows us the procedure of finding the efficient points during searching.

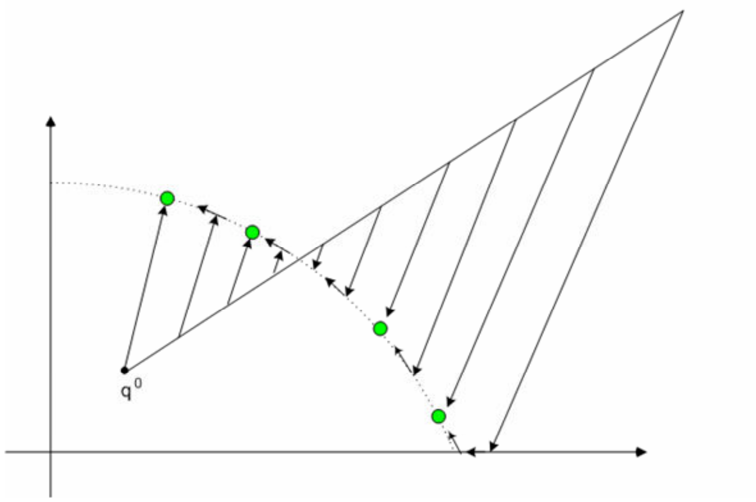


Figure 5.3 Procedure of finding the efficient points during the searching

In the second step, the DM selects point (173.01, 261, 61.99) as the reference point with respect to the underlying utility function. The new reference point becomes $q^2$ = (173.01, 261, 61.99). Let us assume that the DM wants to improve the quality objective and decides the new direction vector to be $d^2$ = (-2, 3,-2). With the new reference point and direction, $\theta_{max}$ is found as 138.16. The model is solved once more by changing the $\theta$ values with the step size of 13 and the resulting efficient point values are given in Table 5.3.

Among the values in Table 5.3, the DM selects point (180.41, 277, 63.5) based on his (implicit) utility function values. At this step, because the DM wants to improve the cost objective and the quality objective, assume that the new reference direction is $d^3$ = (-3, 2,-1) and the new reference point is $q^3$ = (180.41, 277, 63.5).

After $\theta_{max}$ is found as 75.55, the model is solved once more with the current reference point $q^3$ = (180.41, 277, 63.5), the current direction vector $d^3$ =(-3, 2,-1) and a step size of 8. The efficient points from this reference point with the determined reference vector can be seen in Table 5.4. However, the DM cannot find an improved solution among these values given in the table according to his utility function. He prefers the point (180.41, 277, 63.5) again.

Table 5.3 Efficient Points ($d^2$ = (-2, 3,-2))

| **Theta (θ)** | **cost** | **quality** | **leadtime** | **DM's utility function value** |
|---|---|---|---|---|
| 0 | 173.01 | 261 | 61.99 | 398 |
| **13** | ***180.41*** | ***277*** | ***63.5*** | ***387*** |
| 26 | 180.41 | 277 | 63.5 | 387 |
| 39 | 174.24 | 290 | 76.31 | 445 |
| 52 | 189.3 | 303 | 76.59 | 463 |

Table 5.3 (continued)

| Theta (θ) | cost | quality | leadtime | DM's utility function value |
|---|---|---|---|---|
| 65 | 191.17 | 305 | 80.22 | 499 |
| 78 | 192.79 | 315 | 85.84 | 527 |
| 91 | 184.45 | 324 | 95.96 | 563 |
| 104 | 207.89 | 328 | 99.92 | 683 |
| 117 | 217.1 | 330 | 100.51 | 719 |
| 130 | 233.62 | 332 | 96.59 | 738 |
| 138 | 237.17 | 335 | 117.18 | 948 |

Table 5.4 Efficient Points ( $d^3$ =(-3, 2,-1))

| Theta (θ) | cost | quality | leadtime | DM's utility function value |
|---|---|---|---|---|
| **0** | ***180.41*** | ***277*** | ***63.5*** | ***387*** |
| 8 | 173.41 | 277 | 73.2 | 456 |
| 16 | 166.12 | 279 | 79.68 | 485 |
| 24 | 159.71 | 277 | 83.69 | 506 |
| 32 | 150.81 | 273 | 91.24 | 560 |
| 40 | 145.46 | 276 | 102.38 | 640 |
| 48 | 145.46 | 276 | 102.38 | 640 |
| 56 | 143.22 | 261 | 101.94 | 679 |
| 64 | 141.08 | 253 | 128.75 | 966 |
| 72 | 138.76 | 250 | 121.62 | 896 |
| 80 | 136.86 | 245 | 119.07 | 881 |

Now it is time to check whether the current point $q'$=(180.41, 277, 63.5) is optimal or not. According to Korhonen and Laakso (1986), if all the constraints and objective functions are linear, a cone can always be expressed in terms of feasible directions. At least one of these directions has to be improvement direction if the current solution is not optimal. Thus, we have to check all directions for improvement. The previous step, cost was tried to be improved but we did not see any improvement. Two more directions should be tried for quality and lead time. For quality, we can use $d^q$=(0,1,0) and for lead time, we can use $d^l$=(0,0,-1).

With directions $d^q$=(0,1,0) and $d^l$=(0,0,-1), the current solution does not improve with respect to the DM's utility function. Thus the current point $q^*$=(180.41, 277, 63.5) is the optimal solution for our problem. The directions and the best solutions identified at each iteration are depicted in Figures 5.4 and 5.5. Moreover, if the DM's utility function was explicitly known, the optimal solution would be the same ($q^*$=(180.41, 277, 63.5)) as the solution found above.

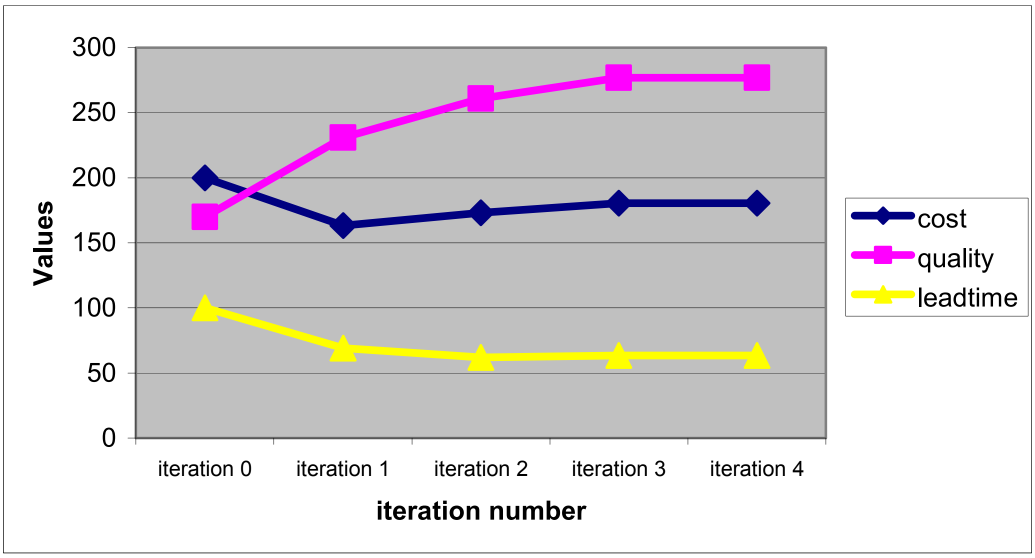


Figure 5.4 The graph of how objective functions change in each iteration

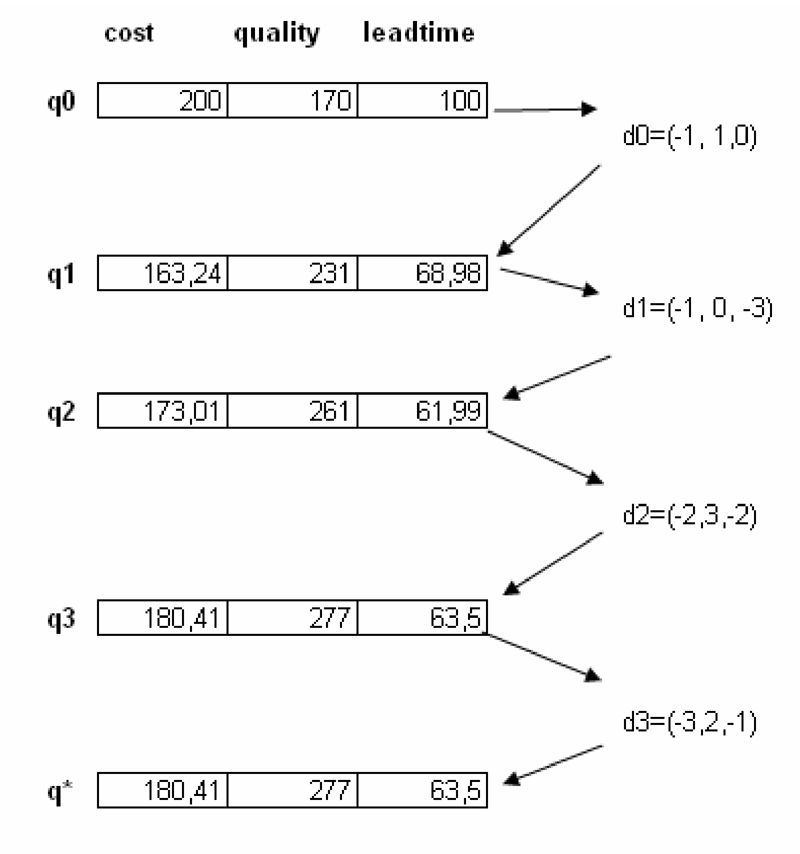


Figure 5.5 Iterations for the first example

Another example will be applied based on an implicit utility function that aims to minimize the Tchebycheff distance from ideal points of each objective. To simulate the DM's decisions, we assumed that the implicit utility function is

$$u = 5(cost - c^*) + 3(q^* - quality) + 4(leadtime - lt^*) \tag{5.14}$$

In this example we want to show how the DM's utility function changes the progress of the method. Let us start with the previous initial point $q^0$ = (200,170,100) and previous initial direction $d^0$ =(-1, 1,0). Note that for all future iterations, we will take the same directions as previous application and again we aim to search the efficient frontier in that direction with about 10 $\theta$ values between 0 and $\theta_{\max}$ . After finding $\theta_{\max}$ =82.21 and solving the model by changing $\theta$ with the step size of 10, we find the efficient solutions shown in Table 5.5.

Table 5.5 Efficient Points ( $d^0$ =(-1, 1,0))

| Theta (θ) | cost | quality | Lead time | DM's utility function value |
|---|---|---|---|---|
| 0 | 163.24 | 231 | 68.98 | 495.15 |
| 10 | 161.24 | 230 | 72.84 | 503.59 |
| 20 | 151 | 227 | 73.57 | 464.31 |
| 30 | 150.28 | 224 | 80.67 | 498.11 |
| 40 | 150.25 | 228 | 87.99 | 515.24 |
| 50 | 140.01 | 229 | 91.02 | 473.16 |
| 60 | 140.01 | 229 | 91.02 | 473.16 |
| **70** | ***139.8*** | ***250*** | ***102.63*** | ***455.55*** |
| 80 | 136.86 | 245 | 119.07 | 521.61 |
| 90 | 136.86 | 245 | 119.07 | 521.61 |

From Table 5.5, the DM selects a new reference point $q^1$ =(139.8, 250, 102.63) according to his utility function. With our previous $d^1$ =(-1, 0, -3) and $q^1$ , we find $\theta_{\max}$ =45.37. Our model is solved with a step size of 5 and the resulting solutions are given in Table 5.6.

Table 5.6 Efficient Points ($d^1$=(-1, 0,-3))

| Theta (θ) | cost | quality | Lead time | DM's utility function value |
|---|---|---|---|---|
| 0 | 139.8 | 250 | 102.63 | 455.55 |
| 5 | 145.44 | 252 | 91.19 | 431.99 |
| 10 | 145.44 | 252 | 91.19 | 431.99 |
| 15 | 150.51 | 242 | 82.41 | 452.22 |
| 20 | 151 | 227 | 73.57 | 464.31 |
| 25 | 159.74 | 223 | 72.97 | 517.61 |
| 30 | 163.24 | 231 | 68.98 | 495.15 |
| 35 | 170.85 | 244 | 64.75 | 477.28 |
| **40** | ***173.01*** | ***261*** | ***61.99*** | ***426.04*** |
| 45 | 184.07 | 253 | 56.17 | 482.06 |
| 50 | 184.07 | 253 | 56.17 | 482.06 |

According to the DM's utility function, the point $q^2$=(173.01, 261, 61.99) is selected. With the new reference point and the second iteration direction $d^2$=(-2, 3,-2), $\theta_{max}$ is found as 138.16. The model is solved by changing $\theta$ values with a step size of 10 and the efficient solutions are given in Table 5.7.

Table 5.7 Efficient Points ($d^2$=(-2, 3,-2))

| Theta (θ) | cost | quality | Lead time | DM's utility function value |
|---|---|---|---|---|
| 0 | 173.01 | 261 | 61.99 | 426.04 |
| 10 | 178.31 | 265 | 65.18 | 453.30 |

Table 5.7 (continued)

| **Theta (θ)** | **cost** | **Quality** | **Lead time** | **DM's utility function value** |
|---|---|---|---|---|
| 20 | 180.41 | 277 | 63.5 | 421.08 |
| 30 | 184.05 | 281 | 71.93 | 461.00 |
| **40** | ***174.24*** | ***290*** | ***76.31*** | ***402.47*** |
| 50 | 189.3 | 303 | 76.59 | 439.89 |
| 60 | 189.3 | 303 | 76.59 | 439.89 |
| 70 | 192.79 | 315 | 85.84 | 458.34 |
| 80 | 192.79 | 315 | 85.84 | 458.34 |
| 90 | 204.4 | 318 | 88.18 | 516.75 |
| 100 | 184.45 | 324 | 95.96 | 430.12 |
| 110 | 207.89 | 328 | 99.92 | 551.16 |
| 120 | 217.1 | 330 | 100.51 | 593.57 |
| 130 | 233.62 | 332 | 96.59 | 654.49 |
| 140 | 237.17 | 335 | 117.18 | 745.60 |

Among the values in Table 5.6, the DM selects the point $q^3$=(174.24, 290, 76.31) based on his (implicit) utility function. In this iteration, our new direction is $d^3$=(-3, 2,-1). After $\theta_{max}$ is found as 82.38, the model is solved once more with a step size of 8. The new efficient solutions are given in Table 5.8.

Table 5.8 Efficient Points ( $d^3$ =(-3, 2,-1))

| Theta (θ) | cost | quality | Lead time | DM's utility function value |
|---|---|---|---|---|
| 0 | 174.24 | 290 | 76.31 | 402.47 |
| **8** | ***168.12*** | ***301*** | ***79.99*** | ***353.59*** |
| 16 | 162.54 | 288 | 87.67 | 395.41 |
| 24 | 157.86 | 293 | 105.19 | 427.09 |
| 32 | 153.83 | 292 | 113.67 | 443.86 |
| 40 | 147.12 | 284 | 121.22 | 464.51 |
| 48 | 145.3 | 273 | 130.89 | 527.09 |
| 56 | 145.3 | 273 | 130.89 | 527.09 |
| 64 | 143.22 | 261 | 101.94 | 436.89 |
| 72 | 141.08 | 253 | 128.75 | 557.43 |
| 80 | 138.76 | 250 | 121.62 | 526.31 |
| 88 | 136.86 | 245 | 119.07 | 521.61 |

The new reference point is selected as $q^4$ = (168.12, 301, 79.99). Up to now, we have used the same directions we used for the underlying linear utility function case. Hereafter we will use new directions to search the efficient frontier. Let us assume the DM decides the new direction as $d^4$ = (-4, 1, -1). With the new direction and the reference point we find $\theta_{\max}$ =29.09. We solve the model once more with a step size 3 and the efficient solutions found are given in Table 5.9.

Table 5.9 Efficient Points ( $d^3$ =(-4, 1,-1))

| Theta (θ) | cost | quality | Lead time | DM's utility function value |
|---|---|---|---|---|
| **0** | ***168.12*** | ***301*** | ***79.99*** | ***353.59*** |
| 3 | 168.12 | 301 | 79.99 | 353.59 |
| 6 | 162.54 | 288 | 87.67 | 395.41 |
| 9 | 158.16 | 283 | 99.71 | 436.67 |
| 12 | 155.2 | 278 | 101.28 | 443.15 |
| 15 | 145.46 | 276 | 102.38 | 404.85 |
| 18 | 145.46 | 276 | 102.38 | 404.85 |
| 21 | 143.22 | 261 | 101.94 | 436.89 |
| 24 | 142.05 | 257 | 126.6 | 541.68 |
| 27 | 138.76 | 250 | 121.62 | 526.31 |
| 30 | 136.86 | 245 | 119.07 | 521.61 |

At this time, the DM cannot select a different solution because the last reference point is still the best solution for the DM according to his (implicit) utility function. Hereafter we have to check the optimality by conditions proposed by Korhonen and Laakso (1986). In the last step, the DM wanted to improve cost. Thus we have to check any improvement in quality and lead time. For quality, we can use $d^q$=(0,1,0) and for lead time, we can use $d^l$=(0,0,-1). After checking any improvement for each objective, we see that we keep our last efficient solution which is $q^4$=(168.12, 301, 79.99). When the solution is compared with the optimal solution which is $q^*$=(162.253, 288, 87.67) (found by solving the model with the explicitly known utility function), we see some differences between optimal solution and selected solution. This is probably due to the discretized step size which does not cover all possible step sizes. The iterations for the second example are given in Figure 5.6.

Although we tried to use the same directions in the two examples, in practice we will not do so. The DM will indicate the direction in terms of the objectives he wishes to improve given the current reference solution. Furthermore, we used somewhat different number of step sizes in different iterations. It might be reasonable to fix the number of steps in different iterations or to use more steps at parts that are desired to be searched in more detail.

Furthermore, when we discuss about the solution time, the execution time is reasonable to apply this method. For each $\theta$ value, the model is solved in a few minutes. Moreover, we had alternative formulations to the above model that excluded the singleton bids. We defined the objectives in a slightly different way. However, we decided not to pursue those models since the solution times were excessive. We give the details of those models Appendix B.

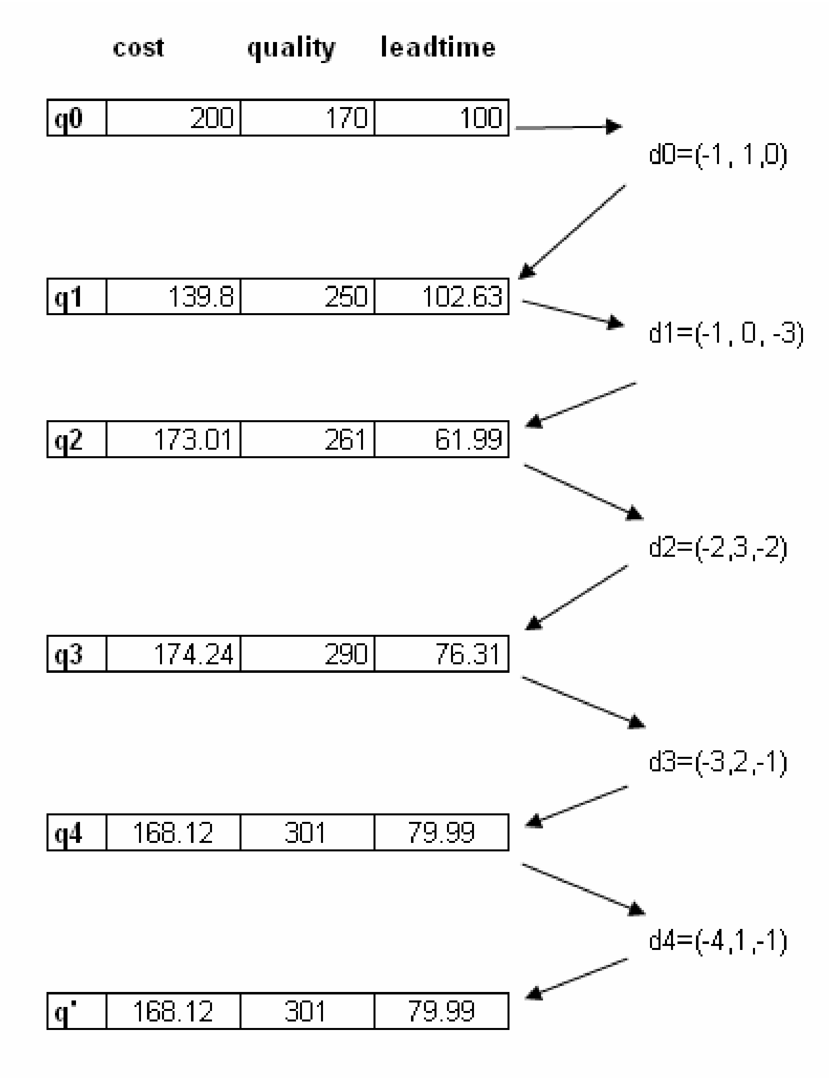


Figure 5.6 Iterations for the second example

# CHAPTER 6

# CONCLUSIONS

## 6.1 CONCLUDING REMARKS

In this thesis, the combinatorial auctions were analyzed. Firstly we explained the auctions' history and why people need auctions. In Chapter 2, common forms of auctions, their advantages and disadvantages are discussed. Also we give some examples of auctions in the world market. In Chapter 3, models for combinatorial auctions are analyzed. Single-unit, single-attribute multi-unit, multi-attribute multi-unit auction models are given and discussed how they differ from each other. At the end, we apply a variation of Korhonen and Laakso's (1986) method to solve a multi-attribute multi-unit auction using two different implicit utility functions.

## 6.2 FUTURE WORK

Based on the models proposed in Chapter 4 and implementation of Korhonen and Laakso (1986) method in Chapter 5, we can say that there are many future research issues in that area. With respect to the increasing Internet usage, auctions will be used in many areas. People want to make their transactions in a transparent environment and they want to trust the reliability of the system.

E-commerce and its applications will probably be more popular in future. In Chapter 4, some mixed integer programming models are proposed. These types of models can be developed and implemented in real time applications. DMs can add different constraints to the models and the models can be implemented in parallel or sequential auctions.

Moreover, interactive multi objective decision making (MCDM) models can further be applied on auctions. We give an example of Korhonen and Laakso (1986) method to investigate the best solution on the efficient frontier. As a future work, different interactive MCDM methods can be applied to auction process and a new method for auctions can be developed for comparison among bidders.

## REFERENCES

Andersson A., Tenhunen M, and Ygge F., 2000, “ Integer programming for combinatorial auction winner determination”, ICMAS, pp. 39-46

Bertsekas, D. P.,1979, “A Distributed Algorithm for the Assignment Problem,” Lab. for Information and Decision Systems Working Paper, M.I.T., March.

Bichler M., Kalagnanam J., 2005, Configurable offers and winner determination in multi attribute auctions, European Journal Of Operational Research, Vol. 160, pp. 380-394

Che, Y, 1993, “Design Competition through multidimensional auctions”, Rand Journal of Economics 24, pp. 668-679

De Smet Yves, 2005, “Multi-criteria auctions without full comparability of bids”, European Journal of Operational Research, Volume 177, Issue 3, 16 March 2007, pp. 1433-1452

Edwards W., 1997, How to use multiattribute utility measurement for social decisionmaking, IEEE Transactions on Systems, Man and Cybernetics SMC 7, pp. 326-340

Fujishima Y. Leyton-Brown K. and Shoham Y., 1999, “Taming the Computational Complexity of Combinatorial Auctions: Optimal and Approximate Approaches”, In Proceeding of the Sixteenth International Joint Conference on Artificial Intelligence, IJCAI’99, August, pp. 548-553

Geoffrion A.M., Dyer J.S, Feinberg A., 1972, “An interactive approach for multi-criterion optimization with an application to the operation of an academic department”, Management Science, Vol. 19, pp. 357-368

Hohner G., Rich J, Ng E., Reid G., Davenport A.J., Kalagnanam J.R, Lee H.S and An C., 2003, Interfaces Vol. 33, No. 1, pp. 23-35

Keeny R.L., Raiffa H., 1993, Decision Making With Multiple Objectives:Preferences and Value Tradeoffs, Cambridge University Press, Cambridge

Korhonen P.J, Laakso J, 1986, A visual interactive method for solving the multiple criteri problem, European Journal Of Operational Research Vol 24, pp. 277-287

Krishna, V, 2002, “Auction Theory”, Academic Press, 0-12-426297-X

Lehmann, D., Mueller, R., and Sandholm, T., 2006, “The Winner Determination Problem” Chapter 12, Combinatorial Auctions, Cramton, Shoham, and Steinberg, eds., MIT Press.

Metty T., Harlan R., Samelson Q., Moore T., Morris T., Sorensen R., Schneur A., Raskina O., Schneur R., Kanner J., Potts K., Robbins J., 2005, "Reinventing The Supplier Negotiation Process at Motorola", Vol. 35, No. 1, pp. 7-23

Milgrom, P., 2000, "Putting Auction Theory to Work: The Simultaneous Ascending Auction", April, Journal of Political Economy

Nisan N., 2000, "Bidding and allocation in combinatorial auctions.", ACM Ecommerce, October 17-20, pp. 1-12

Park S., Rothkopf M., 2005, "Auctions with bidder-determined allowable combinations", European Journal of Operational Research, Vol. 161, pp 399-415

Rothkopf M.H., Pekec A. and Harstad M., (1998), "Computationally Manageable Combinatorial Auctions", Management Science, Vol. 44(8) pp.1131-1147

Rothkopf, M.H., Sunju P., 2001, "An Elementary Introduction to Auctions", INTERFACES, Vol. 31: 6 November–December, pp. 83–97

Saaty T.L., 1980, The Analytic Hierarchy Process, McGraw Hill, New York

Sandholm T., Suri S., Gilpin A. and Levine L., 2001, "Winner Determination in Combinatorial Auction Generalizations"

Sandholm T., 2002, "Algorithm for optimal winner determination in combinatorial auctions", Artificial Intelligence 135, pp.1-54

Sandholm, Tuomas, and Subhash Suri (2003), “BOB: Improved Winner Determination in Combinatorial Auctions and Generalizations," Artificial Intelligence, 145, 33{58. Early version: AAAI-00).

Sandholm T., Suri S., Gilpin A. and Levine L., 2005, “CABOB: A Fast Optimal Algorithm for Winner Determination in Combinatorial Auctions”, Management Science, Vol. 51, No. 3, March 2005, pp. 374–390

Sandholm T., Levine D., Concordia M., Martyn P., Hughes R., Jacobs J., Begg D., 2006, “Changing the Game in Strategic Sourcing at Procter&Gamble: Expressive Competition Enabled by Optimization”, Vol. 36, No. 1,pp. 55-68

Steuer R.,1986, “Multiple Criteria Optimization: Theory, Computation and Application”, Wiley, New York.

Teich J., Wallenuis H., Wallenuis J and Zaitsev A.,1999, “A Multiple Unit Auction Algorithm:Some Theory and a Web Implementation”, Electronic Markets, Vol. 9, No. 3, pp 199-205

Teich J.E., Wallenius H., Wallenius J., Koppius O. R., 2003, “Emerging Multiple Issue e-auctions”, ERIM Report Series Reference No. ERS-2003-058-LIS

Vickrey, W., 1961, “Counterspeculation, Auctions and Competitive Sealed Tenders”, Journal of Finance, Vol.16 No.1, pp. 8-37

Vries S., Vohra R., 2003, “Combinatorial auctions: A survey”, INFORMS Journal on Computing, Vol. 15, No. 3, pp. 284–309

Wierzbicki AP, 1980, "The Use of Reference Objectives in Multi Objective Optimization", Multi criteria decision making theory and applications, pp 468-486, Springer, Berlin Heidelberg New York

Wolfstetter,E., 1996, "Auctions: An Introduction," Journal of Economic Surveys, Vol. 10

Zionts S., Wallenius J., 1976, "An Interactive Programming Method for Solving the Multiple Criteria Problem", Management Science, Vol. 22, No. 6, pp. 652-663.

# APPENDIX A

## INPUT DATA GENERATION

In order to simulate and find a solution for auctions, we can generate singleton and bundled bids randomly. A singleton bid generation code is written in C. In the code, a cost range is defined randomly for all units because in the real world, all things have a cost range. This range can be large or small depending on the type of unit. For example, a basic type of electronic board (PCB) has a price range and people decide to buy whether the price of this PCB is suitable or not in relation to its properties (number of resistances, microprocessors, memory unit, capacitors …etc.). So according to the units' properties, we can decide a price (cost) range for the units. Program firstly decides the price range of all units that some of units are more expensive than others, it means more important than others. Moreover, some units' costs are generated to be "-". This means that bidders do not want to sell these units; the singleton bids do not include these units. For example, in the Table A.1, bidder 7 does not sell unit a2, thus its cost value is zero.

Table A.1 - Singleton bid of bidder 7

| Unit / Bidder | **a0** | **a1** | **a2** | **a3** | **a4** | **a5** |
|---|---|---|---|---|---|---|
| **Bidder7** | 11 | 6 | - | 33 | 12 | 25 |

After the cost attribute is generated, there exist some scaling problems between cost and other attributes (quality and lead time) because quality and lead time vary between 0 and 10 but cost values vary in a larger range. To avoid this scaling issue, the cost attributes for each item is normalized to suit the scaling range of 0 to 10.

Bundled bids generation code is also written in C. Bundled bids are composed from singleton bids' cost, quality and lead times. However, bundled bid attributes' values are not simply the summation of the related attributes. For example cost of the bundle is the 'private' value of bidder on the bundle of the units, not the same as the summation of cost of units. For example when 5-unit, 6-unit and 7-unit bundled bids are generated, bundled bids' attributes are calculated according to the specified rules. Moreover bundled bids' size can be changed if one wants to create different sized bids. Small and large bundled bids effects are analyzed in Section 4.3.

In the program, the main way of the bundled bid generation is selecting bidders from the bidder list and forming bundled bid according to their bid items' attributes. For example, in the program we decide to form 5-units, 6- units and 7-units bundled bids according to following rules:

***Parameters:***

- $ucost_{jk}$ is the cost of selected unit $j$ of bidder $k$
- $uquality_{jk}$ is the quality of selected unit $j$ of bidder $k$
- $uleadtime_{jk}$ is the lead time of selected unit $j$ of bidder $k$
- $c_{ik}$ is the cost of bundled bid $i$ of bidder $k$
- $q_{ik}$ is the quality of bundled bid $i$ of bidder $k$
- $lt_{ik}$ is the lead time of bundled bid $i$ of bidder $k$

***Specifications:***

- Bundle size: 5-units, 6-units, 7-units

- Discount 3% per unit in the bundle

$$c_{ik} : \sum_{j=1}^{Bundle\_size} ucost_{jk} * (1 - (Bundle\_size * 0.03))$$

- Sum up the quality attribute

$$q_{ik} : \sum_{j=1}^{Bundle\_size} uquality_{jk}$$

- Take maximum lead time throughout bundled units

$$lt_{ik} : \underset{j=1,2,..bundle\_size}{Max} \{leadtime_{jk}\}$$

Attributes are chosen randomly and calculated in a specific way. These can naturally be used in different ways suiting different requirements of DMs. For example, quality can be measured by the minimum quality among the items' qualities in the bundle or other measures can be developed depending on the DM's wishes.

When a random number is generated, firstly bidder is selected from this number, then units are started to select. In order to select unit from the list, a linked list is created. (N is the total number of units)

1 2 3 ……… N-1 N

Total unit number is N in the linked list

For unit selection, first random number is generated then 'pick' the unit from the list. For example unit 3 is selected.

1 2 4 ……… N-1 N

Total unit number is N-1 in the linked list

For second unit, let the random number is 10, then 10th unit is selected from the list, it means that unit 11 is selected.

1 2 4 5 6 7 8 9 10 12 … N-1 N

Total unit number is N-2 in the linked list

Bundled bids generation procedure continues until all bundled bids are generated. This procedure allows us to use all random numbers generated as exactly as generated.

# APPENDIX B

## SOME EXTENSIONS

When we applied Korhonen and Laakso (1986) method for auction process, firstly we generated only the multi-unit bundled bids (5-units, 6-units and 7-units) for auction. However, the model cannot assign units only once because a feasible solution does not exist. If we allow assigning more than once for each unit, quality constraint cannot be a binding constraint because model assigns all bundled bid in order to maximize total quality and buyer has to buy all assigned units. Thus we add a new variable ($totitem$) to control the assignment and modify the model slightly.

***Parameters:***

- M is the number of bundled bids,
- N is the number of units to be auctioned,
- $c_i$ is the cost of bid $i$,
- $q_i$ is the quality of bid $i$,
- $lt_i$ is the lead time of bid $i$,
- $q_1, q_2, q_3$ are the reference points of cost, quality and lead time respectively.
- $d_1, d_2, d_3$ are the directions of reference points respectively.
- $bund_{ij}$ is the parameter that if its value is 1, bid $i$ includes unit $j$; if its value is 0, bid $i$ does not include unit $j$.

***Variables:***

- $y_i$ is the decision variable that takes a value of 1 if bundle bid $i$ is assigned, and 0 otherwise.
- $cost$ is the total cost of the auction
- $quality$ is the total quality of the auction
- $leadtime$ is the total lead time of the auction
- $totitem$ is the total number of assigned items in the auction

Min $\alpha + \varepsilon * (cost + leadtime) - \varepsilon * (\varepsilon * quality - totitem)$

Subject to

$$\alpha \geq cost - (q^1 + \theta * d^1) \tag{B.1}$$

$$\alpha \geq leadtime - (q^3 + \theta * d^3) \tag{B.2}$$

$$\alpha \geq q^2 + \theta * d^2 - (\varepsilon * quality - totitem) \tag{B.3}$$

$$cost = \sum_{i=1}^{M} \mathrm{c}_i * y_i \tag{B.4}$$

$$quality = \sum_{i=1}^{M} q_i * y_i \tag{B.5}$$

$$leadtime = \sum_{i=1}^{M} lt_i * y_i \tag{B.6}$$

$$\sum_{i=1}^{M} bund_{ij} * y_i = 1 \qquad \forall j \in N \tag{B.7}$$

$$totitem = \sum_{j=1}^{N} \sum_{i=1}^{M} bund_{ij} * y_i \tag{B.8}$$

The term "$\varepsilon * quality - totitem$" provides an improvement in quality objective without increasing total number of bids uncontrolled. In constraint B.8, we find the total assigned units in the auction process then add this variable to the objective in order to control the assignment.

But when we run the model in GAMS 22.2, the model cannot find a feasible integer solution with 900 bundles (150 bids (5-units), 450 bids (6-units), 300 bids (7-units)). Because of this, we change the constraint B.7. Instead of "=", we can use "≥" to find a feasible solution. However, in this time the memory of GAMS is not enough to solve this problem and we cannot find an optimum solution.

Another attempt to find a solution from same problem set is about controlling the extra assigned units. For example, we have 50 units in our set and we should assign all kind of unit at least one. (Of course our aim is to assign all units only once). The modified model can be written like this:

Min $e + \varepsilon * (\alpha + \varepsilon * (cost + leadtime - quality))$

Subject to

$$\alpha \geq cost - (q^1 + \theta * d^1) \quad \text{(B.9)}$$

$$\alpha \geq leadtime - (q^3 + \theta * d^3) \quad \text{(B.10)}$$

$$\alpha \geq q^2 + \theta * d^2 - quality \quad \text{(B.11)}$$

$$cost = \sum_{i=0}^{M} \mathrm{c}_i * y_i \quad \text{(B.12)}$$

$$quality = \sum_{i=0}^{M} q_i * y_i \quad \text{(B.13)}$$

$$leadtime = \sum_{i=0}^{M} lt_i * y_i \quad \text{(B.14)}$$

$$\sum_{i=0}^{M} bund_{ij} * y_i \geq 1 \qquad \forall j \in N \quad \text{(B.15)}$$

$$\sum_{j=0}^{N} \sum_{i=0}^{M} bund_{ij} - e = 50 \quad \text{(B.16)}$$

For this model, the variable $e$ is the deviation variable calculated in constraint 5.29. In order to minimize the deviation from 50 units, we can add the variable $e$ into the objective and provide minimum extra assignment. When we solve the model, the execution time is much more than we want. For example, 900-

bundled-bid model is solved in 1 hour and half. Execution time is important for us because we want to suggest at least 10 alternative solutions in each iteration to DM in order to select his “best” solution among these solutions and the procedure may consist of 5 to 20 iteration.

Thus we can propose an alternative way in response to explained two models and find a set of solution to DM in order to select the “best” solution among them in a suitable time interval. We can generate 1-unit bids for all units (for 50 units, generated 50 bids). In this way, all units will be assigned only once (like constraint 5.9) and we can propose efficient solutions to DM to select his “best” solution. The model given in section 4.3 can be given as an example for multi-unit and single-unit concurrent bundled-bid model.