HYBRID RANKING APPROACHES BASED ON DATA ENVELOPMENT ANALYSIS AND OUTRANKING RELATIONS

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ABSTRACT

HYBRID RANKING APPROACHES BASED ON
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In this study two different hybrid ranking approaches based on data envelopment analysis and outranking relations.for ranking alternatives are proposed Outranking relations are widely used in Multicriteria Decision Making (MCDM) for ranking the alternatives and appropriate in situations when we have limited information on the preference structure of the decision maker. Yet to apply these methods DM should provide exact values for method parameters (weights, thresholds etc.) as well as basic information such as alternative scores. DEA is used for classification of decision making units according to their efficiency scores in a non-parameteric way. The proposed hybrid approaches utilize PROMETHEE (a well known method based on outranking relations) to construct outranking relations by pairwise comparisons and a technique similar to DEA cross-efficiency ranking for aggregating comparisons. While first of the proposed approaches can deal with imprecise specification of criterion weights, second approach can utilize imprecise weights and thresholds.

Keywords: Data envelopment analysis, Outranking relations, PROMETHEE, Cross-efficiency, Ranking

ÖZ

VERİ ZARFLAMA ANALİZİ VE BASKINLIK İLİŞKİLERİNİ
TEMEL ALAN HİBRİT SIRALAMA YAKLAŞIMLARI

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Bu çalışmada veri zarflama analizi ve baskınlık ilişkileri metodlarına dayanan iki ayrı hibrit sıralama yaklaşımı önerilmektedir. Baskınlık ilişkileri metodları çok kriterli karar verme (ÇKKV) problemlerinde sıralama için yaygın kullanımı olan ve karar vericinin tercih yapısı hakkında sınırlı bilginin olduğu durumlarda kullanımı uygun metodlardır. Bu metodların kullanımı için karar vericinin kriter skorları gibi temel bilgilerle beraber birçok parametreyi (kriter ağırlıkları, eşik değerleri vb.) tam olarak belirlemesi gerekir. Veri zarflama analizi (VZA) ise karar verme birimlerinin verimliliklerine göre parametrik olmayan bir şekilde sınıflandırılmasında kullanılır. Önerilen yaklaşımlar PROMETHEE (baskınlık ilişkileri metodu) yoluyla baskınlık ilişkilerinin ikili karşılaştırmalarla oluşturulmasında ve VZA çapraz verimlilik sıralaması benzeri bir yöntemi ise baskınlık ilişkilerinin birleştirilmesinde kullanmaktadır. Birinci yaklaşım kriter ağırlıklarının eksik bir şekilde belirtildiği durumda, ikinci yaklaşım ise hem kriter ağırlıklarının hem de tercih fonksiyonu eşik değerlerinin net olmadığı durumda kullanılabilmektedir.

Anahtar Kelimeler: Veri Zarflama Analizi, Baskınlık İlişkileri, PROMETHEE, Çapraz Verimlilik, Sıralama.

To My Family

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LIST OF ABBREVIATIONS

AR: Assurance Region

DEA: Data Envelopment Analysis

DM: Decision Maker

MBA: Master of Business Administration

MCDM: Multicriteria Decision Making

S: Standard Error

PROMETHEE: Preference Ranking Organisation Method for Enrichment Evaluations

SBM: Slack Based Measure

SE: Super-efficiency Method

SMART: Simple Multi-Attribute Rating Technique

SXE: Simple Cross-efficiency Method

CHAPTER 1

INTRODUCTION

In multi-criteria decision making problems where there is a finite number of alternatives, the decision maker (DM) may aim to select the best alternative, rank the alternatives or sort/classify the alternatives. For ranking the alternatives from most desired to least in the presence of multiple criteria, there are several different methods. In the MCDM framework two distinct viewpoints exist, multi-attribute utility based methods and outranking relations.

In contrast to multi-attribute utility based methods, outranking methods avoid making strong assumptions about the preference structure of the decision maker. These methods are based on collecting evidence about the preferences of the decision maker by building outranking relations from pairwise comparisons. Most known outranking methods are ELECTRE [62] and PROMETHEE [10] both of which also have many variants.

PROMETHEE method forces DM to specify weights and thresholds precisely at the beginning of the decision process. To minimize the number of parameters the decision maker has to provide precisely in the beginning of the solution process, in this study we propose two different approaches. In the first approach the DM determines his preference structure (preference functions and preference function parameters) for each criterion in the same way described by the PROMETHEE method, except the criterion weights. Aggregation is done for each alternative by maximizing its own score relative to average of the others and overall ranking is done by aggregation of these evaluations. Two different ideas borrowed from DEA approach is used in the development of the methods. First is that the aggregation of outranking information among criteria is done based on most favorable weights for each alternative similar to the main idea of DEA, and secondly final ranking is done based on both self evaluation and other alternatives' evaluations in the same way as DEA cross-efficiency method does. Constraints similar to assurance region constraints used in DEA based methods and global constraints on the criterion weights can be added. In the second method the DM may also specify lower and upper bounds for the preference function parameters. For each criterion pairwise outranking relations are built such that the preference of the alternative under consideration is maximized.

The organization of the thesis is as follows:

In Chapter 2, we review the literature on DEA models, relation between DEA and MCDM, ranking methods based on DEA, application of DEA ranking in MBA programs, PROMETHEE method and alternative approaches for determining parameters in PROMETHEE. In Chapter 3, we present the background information on PROMETHEE method, DEA, cross-efficiency ranking approach, and types of constraints imposed in DEA models. In Chapter 4, we introduce our approaches and in Chapter 5 we describe the decision aid we develop to implement our methods. In the conclusion chapter, we discuss the significance of this study and indicate direction for further research.

CHAPTER 2

LITERATURE REVIEW

In this chapter, we review the literature on DEA and PROMETHEE. In Section 2.1 we present a review of DEA methods, relation between DEA and MCDM, DEA based ranking approaches, and application of DEA in MBA programs ranking. In Section 2.2, we present PROMETHEE and review studies that aim to deal with the DM's problem of determining parameters in PROMETHEE and incorporating imprecise information.

2.1. Data Envelopment Analysis

There has been extensive research and literature on DEA. Seiford [67] provides exhaustive list of DEA journal articles. Tavares [71] also supplies various statistics based on publication type, author, keywords, journal, university and country in addition to exhaustive listing of publications. Examining the keyword statistics, it can be concluded that main application domains for DEA are bank evaluation and educational institution evaluation. Gattoufi et al. [31] provides an exhaustive list containing refereed journal publications, books, conference proceedings and technical reports. They also analyzed the content of the DEA publications between 1996 and 2001 and classified them according to their nature (theory, application or both) and research strategy used [30].

In Section 2.1.1 general DEA models are briefly presented. Section 2.1.2 review the studies that aim to investigate the relationship between MCDM and DEA approaches and Section 2.1.3 presents DEA based ranking approaches. Finally studies that deal with application of DEA in MBA programs ranking are presented.

2.1.1 Models

The first explicit introduction of DEA method was by the classic paper of Charnes et. al. [15]. In this paper an efficiency concept based on multiple inputs and outputs is developed and a method for determining an efficient set of units based on observational data is provided. The model they provide solely based on the idea of Pareto efficiency principle which is "a unit is fully efficient if and only if none of its input or outputs can be improved without worsening some of its other inputs and outputs." Former study for measuring the efficiency of units was by Farell [26] which deals with single output case.

Primary distinguishing factor of DEA efficiency analysis is that each Decision Making Unit (DMU) is scored to increase its own advantage –an input or output of which a DMU ranks better is weighted higher when calculating the score of the DMU- with minimum required input from the analyst [15]. For the simplest DEA model the only data needed for DEA analysis is the levels of inputs and outputs for each DMU.

Later the DEA formulation of Charnes et al. [15] which is called the CCR model was modified and alternative formulations were introduced. The assumption of CCR model was that the inputs and outputs can be scaled such that the ratio of inputs and outputs stays constant. The BCC formulation proposed by Banker et al. [6] adds another constraint to the CCR model for variable returns to scale production frontier which limits the scaling of the DMUs. DMU that may not be efficient compared to set of DMUs based on constant returns to scale DEA model (CCR model), may be efficient if variable returns to scale model is used (BCC Model).

Both of these models require solving two linear programming (LP) problems, first to maximize efficiency and to find the efficiency score, second to find slacks and discriminate weakly efficient DMUs. Alternatively objectives of the two LPs can be combined using a very small archimedean constant (ε) as a multiplier for the second objective.

In CCR or BCC models solution is found either by projection in the input space by taking the outputs constant or in the output space by taking the inputs constant. The set of DMUs classified as efficient will be the same while the efficiency scores of the inefficient DMUs will differ for input and output oriented models.

To decrease computational burden, additive formulation was developed to enable finding efficient DMUs by solving the LP once for each DMU. In this model both input and output slacks are varied simultaneously [13]. As the efficiency of the unit is concluded by the sum of slacks, the calculated efficiency score is not intuitive as in CCR or BCC. In addition to decreased computational burden, the additive model has advantage of translation invariance (i.e. translation of input or output measures do not effect efficiency score) but also has some drawbacks as direct inefficiency score is based on slacks and hard to interpret [19].

CCR, BCC and additive model are equivalent in the fact that they project inefficient DMUs radially to efficient frontier meaning that the efficiency scores are found based on a radial measure from the frontier [19]. Yet the efficient set of DMUs may change depending on the return to scale characteristics assumed (i.e. CCR has CRS assumption, BCC has VRS assumption, additive model may have CRS or VRS assumption). Moreover, as the efficient set and frontier is affected, therefore the scores of the inefficient DMUs based on the radial distance to the frontier may also differ.

A more recent measure of efficiency, slack based measure (SBM), is suggested by Tone [74]. Tone [74] states that commonly accepted desirable attributes of a DEA based efficiency measure are units invariance (the efficiency score should not depend on criteria measure as long as it is applied to all criteria equally), being monotonic (shall increase as any of the outputs increase, decrease as any of the inputs increase and vice vs.) and translation invariance (no change with translation of criteria), concluding that such a measure shall only depend on the reference efficient DMU's and develops a slack based measure for efficiency. A method for dealing with zero inputs and outputs and comparison with Russell efficiency measure which is more commonly used in economics literature is presented.

2.1.2 MCDM and DEA Relationship

The interactive approach given by Belton and Vickers [7] was one of the first attempts to use DEA as an MCDM tool. Aggregated inputs and outputs are used and both input weights and output weights add up to unity. A unit or alternative is efficient if it can obtain a combination of input and output score that is higher than those of other units based on goal programming formulation. Also for a user controllable set of inputs and outputs the aggregated input and output is displayed. This study also states the coincidence between DEA and MCDM where alternatives in MCDM terms are regarded as DMUs and criteria are regarded as inputs and outputs. Stewart also analyzes the correspondence between the ratio form efficiency definition and distance to the pareto frontier in linear form value function model [70].

Li [51] proposed a multi criteria approach to DEA problem by formulating the first objective maximizing self efficiency, second minimizing maximum of deviation of the input or outputs from the efficient point (slacks) and third minimizing sum of the deviations (slacks).

Joro et al. [46] showed the connection between DEA and MOLP approaches formulating DEA as a reference point model and reference point model as a DEA model. Later these results are used by Halme et. al. [34] to arrive at different measures of efficiency that incorporates DMs preferences called value efficiency. An interactive way for improvement of the calculation of value efficiency is presented later by Joro et. al. [47].

Besides these approaches there are attempts to use several multiple criteria approaches with DEA simultaneously. A short review of such studies on use DEA with AHP is given by Ramanathan [60]. Another approach that incorporates DEA and MAVT is given in Mavrotas and Trifillis [54].

2.1.3 DEA Based Ranking Approaches

At first DEA was developed for differentiating between efficient and inefficient DMUs. Later DEA researchers aim to rank both efficient and inefficient DMUs. Adler et al. [1] made a survey of DEA ranking approaches. Sarkis et. al. [66] compares some of these methods with other MCDM methods.

The first approach for full-ranking using DEA was cross-efficiency approach [24]. It was based on former work that utilizes DEA to rank units based on votes. Aiming at consensus it may be utilized in cases where the DMUs are also part of the decision process. Each DMU evaluates itself and also other DMUs, and final evaluation is based on average of these evaluations. The problem is that during self evaluation the set of weights that a DMU is classified as efficient is not unique. To arrive unique weights two different approaches are formulated namely aggressive and benevolent cross-efficiency. Both solve for the weights formulating a second LP model either by suppressing other DMUs efficiency scores (aggressive model) or favoring them while holding the efficiency score of the DMU under consideration constant at the level found in first LP.

Benchmarking approach which may not result with a full-rank use the number of times the efficient unit is in the reference set for inefficient DMUs as a basis for ranking efficient units [1]. Being a simple measure and can be obtained easily, the model is the most used approach in available DEA software tools [20].

Another ranking approach is the super-efficiency approach of Andersen and Peterson [4] also called AP method. In this method, the units having an efficiency value of one are marked as efficient and are scored by running DEA again by excluding corresponding constraint that prevents them having an efficiency value greater than one. It has weak points such that the dual formulation ends up with infeasible solutions, and very small input values may cause very high variability in DEA scores.

Tone [75] proposes to use SBM for ranking both efficient and inefficient units. The inefficient units are ranked as in AP method (by excluding the constraint for the efficient unit under consideration) by using input or output oriented SBM. The zero values in input or output levels are handled by the model.

Other variants that use DEA and aim to rank all the DMUs exist in the literature [5][36][37][39][41][57].

Bouyysou [9] presents some weaknesses of DEA use in the MCDM problems. First the simple DEA models -which aim only ranking inefficient units- are criticized for not being able produce a consistent rank for inefficient units as different models produce different rankings and all the efficient units are ranked better than inefficient units. Secondly, super-efficiency approach [4] has additional problems such as rank reversal occurs for efficient unit when a similar inefficient unit is introduced. Third different cross-efficiency methods [24] produce different rankings with different formulations and an alternative may rank better when its score on one of the criteria is decreased.

Main application areas of DEA are bank evaluation and educational institution evaluation [29]. Cooper et.al. [20] presents other application areas of DEA, such as engineering applications, benchmarking in sports, retailing applications, health care applications. DEA is firstly used as a method for evaluating non-profit organizations; recently it is used for performance appraisal of firms from various sectors.

2.1.4 Ranking of MBA Programs

There are a large number of studies that evaluate performance of universities, university departments etc. using DEA. We present a survey of studies that evaluate particularly MBA programs. Also some of these studies introduce extensions to original DEA method.

The first study [65] utilizing DEA for MBA program evaluation introduces different perspectives and focuses on the applicants perspective. In that sense it is the first paper that uses DEA to evaluate a university department from the applicant's perspective. The applicant can select the relevant criteria among The Times league table, categorize them as less important, important, very important that are modeled as weight restrictions enabling a nearly complete rank. The second study [56] utilizes five criteria, where the performances are gathered from "Peterson's Guide to MBA Programs" and puts weight restrictions on each pair of outputs and inputs. Negative perturbations to performances of the efficient DMUs are done to examine whether their efficiency status changes or not. The third study [17] also accommodates different viewpoints of students and recruiters and adjusts outputs accordingly. Also outputs and inputs are combined for some of the trials resulting smaller number of efficient units as expected. More recent study [61] uses CCR, BBC and Russell efficiency measure to rank the business schools based on business week data.

2.2. Outranking Methods

Outranking methods are developed as an alternative to utility based approaches which aim to model the underlying preference structure of the decision maker based on strong assumptions. The outranking information is collected by pairwise comparisons and formulating statements whether an alternative is inferior, indifferent or preferred to another alternative.

Also the degree of uncertainty in criteria can be modeled by pseudocriteria concept and incorporated in the decision process. According to the pseudoucriteria concept, we cannot state that evidence of an alternative outranking another for a given criterion for which the performance advantage do not pass a certain threshold called the indifference threshold. The indifference value can be interpreted as the minimum margin of error acceptable to the DM or the minimum value of difference resulting a perception of difference by the DM. The preference threshold value can be interpreted as maximum margin of error acceptable or the minimum value that indicates a certain preference of the DM for the given criterion.

The weight concept used in outranking relations is distinct from the weight concept of multi attribute utility based methods as they represent the relative importance of the criteria [29].

PROMETHEE method is introduced by Brans et al. [10][11]. Unlike Electre's concept of concordance and discordance, positive and negative outranking flow concepts are used for gathering evidence about preference of alternatives and building the outranking relation. After aggregation of outranking relations criterion wise ranking is done by using the information on the level of evidence that shows how much the alternative outranks other alternatives (positive outranking net flow) and how much the alternative is outranked by others (negative outranking net flow). PROMETHEE I relies on an ordinal aggregation of these evidence and produces a partial rank where a better ranked alternative has both a higher positive outranking net flow and a lower negative outranking net flow. PROMETHEE II aggregates these evidences cardinally and produces a complete rank [8].

As obtaining exact values of parameters from the DM is a problem, so fuzzy PROMETHEE (F-PROMETHEE) method [32] and Monte Carlo Simulation with PROMETHEE [35] are proposed to cope with imprecise information. In F-PROMETHEE the alternative performance values are regarded as fuzzy parameters and ranking is done accordingly. First limitation of the F-PROMETHEE is that the criterion weights are taken as crisp with special difficulty of incorporating fuzzy weights as they add up to one. Also the studies do not provide an example of specifying fuzzy preference parameters and only the alternative performances are modeled by fuzzy variables. In the second approach, the distribution functions are assigned to parameters based choices of group of decision makers, which are later used to generate data and calculate rankings. In general it will be hard to provide enough number of estimations for these parameters by the DMs to formulate a distribution function for the PROMETHEE parameters.

There is another study that aims to compute the credibility indices of Electre method given partial information on pseudocriteria, namely preference, indifference and veto parameters and criterion weights [22]. Under partial information about the parameters, robustness of outranking among two actions is examined. The study does not deal with ranking of the alternatives or robustness of ranking if more then two actions are present. Another study based on Electre method aims to find the weights that makes the certain alternative best when weights are imprecise [58]. The study does not provide the solution exactly and an interactive search procedure is proposed.

Özerol and Karasakal [59] develop a PROMETHEE based interactive approach for selecting best alternative and ranking the alternatives when the criterion weights and preference function parameters are imprecise. However, this method do not guarantee complete ranking.

The review of the relevant literature demonstrates that there seems to be no method aiming complete ranking when there is no or partial information on criterion weights or preference parameters and this study proposes two methods for arriving at a ranking under such circumstances.

CHAPTER 3

PROMETHEE and DEA

In this chapter PROMETHEE and DEA are presented. These methods are the building blocks of the hybrid approaches proposed in Chapter 4.

3.1. PROMETHEE Method

In this section we will explain PROMETHEE method in five steps which are initialization, determination of method parameters, calculation of preferences, calculation of outranking flows, and ranking the alternatives based on the netflow.

Step 1: Initialization

DM formulates the alternatives, selects the criteria and assesses the performance of the alternatives for the criteria. After this step, we will have a performance matrix (S) for n alternatives and m criteria. S_{ij} represents ith alternative's performance in criterion j.

$$S = \begin{bmatrix} S_{11} & S_{12} & \dots & \dots & S_{1m} \\ S_{21} & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ S_{n1} & \dots & \dots & \dots & S_{nm} \end{bmatrix}$$

Step 2: Determine Method Parameters

There are two types of parameters in PROMETHEE methods, intra-criteria and inter-criteria information [29].

The intra-criteria information is used to model the preference structure of the DM for a criterion. Each criterion selected in step 1, is modeled by the appropriate preference function (P_j) and parameters. Originally there are 6 types of functions and three different threshold parameters, indifference (q_j), preference (p_j) and Gaussian thresholds (s_j) as shown in Figure 1 [10]. If the difference between performance values of the alternatives ($\Delta_{ik}^{j} = S_{ij} - S_{kj}$) is smaller than the indifference threshold then there is no evidence that one alternative is preferred to other in that criterion. Whereas any difference bigger than the preference threshold is a clear indication of preference of the better performing alternative. If the difference is between indifference and preference the preference changes linearly for "type 3" or "type 5" functions and takes discrete value of 1/2 for "type 4" function. Gaussian threshold on the other hand has an intermediate meaning and the preference changes continuously, approaching one in the limit as the difference approaches infinity.

As the inter-criteria information the weights of the criteria shall be provided to the model. The criterion weights are determined so that sum of the weights is one. The weights are used for aggregating the outranking information and are not meaningful for scaling alternative performance values.

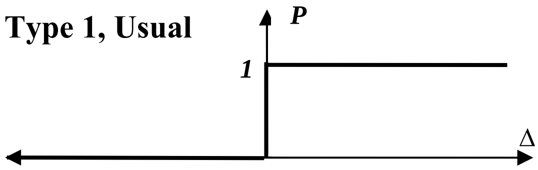
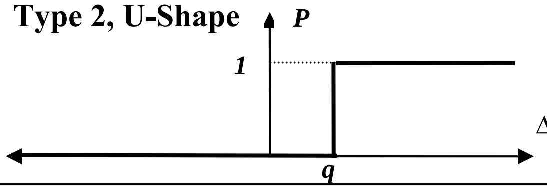
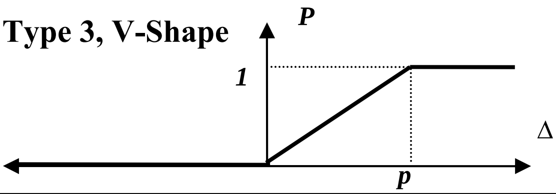
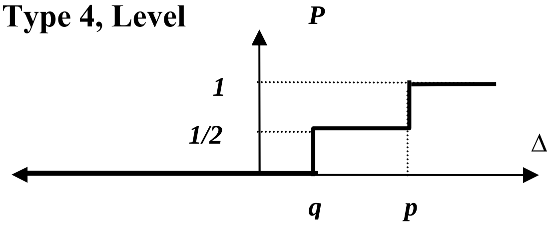
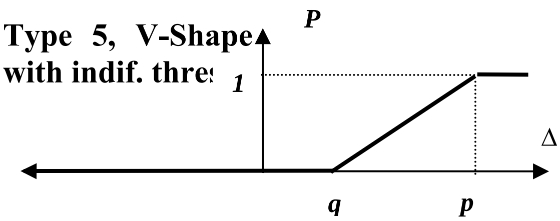
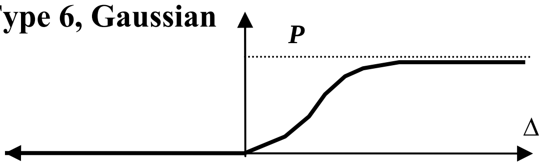
Type, Graphical Illustration	Preference Formula
Type 1, Usual 	$P(\Delta)=\begin{cases}0 & \Delta\le 0\\ 1 & \Delta>0\end{cases}$
Type 2, U-Shape 	$P(\Delta)=\begin{cases}0 & \Delta\le q\\ 1 & \Delta>q\end{cases}$
Type 3, V-Shape 	$P(\Delta)=\begin{cases}0 & \Delta\le 0\\ \Delta/p & p\ge\Delta>0\\ 1 & \Delta>p\end{cases}$
Type 4, Level 	$P(\Delta)=\begin{cases}0 & \Delta\le q\\ 1/2 & p\ge\Delta>q\\ 1 & \Delta>p\end{cases}$
Type 5, V-Shape with indif. thres 	$P(\Delta)=\begin{cases}0 & \Delta\le q\\ (\Delta-q)/(p-q) & p\ge\Delta>q\\ 1 & \Delta>p\end{cases}$
Type 6, Gaussian 	$P(\Delta)=\begin{cases}0 & \Delta\le 0\\ 1-e^{-\Delta^2/2s^2} & \Delta>0\end{cases}$

Figure 1. PROMETHEE Preference Functions

Step 3: Calculate Preferences

The preference between the alternatives for each criterion is calculated by using preference functions (P_j) and parameters of the preference function.

$$P_{ik}^{j} = P_j\,(i,\,k) = P_j\,(\Delta_j^{ik})$$

The preference of alternative *i* to alternative *k,* denoted as $\prod(i,k)$, is calculated by aggregating preference values using criterion weights.

$$\prod(i,k) = \sum_j w_j P_{ik}^{j}$$

Likewise, the preference of alternative *k* to alternative *i,* denoted as $\prod(k,i)$, is calculated by aggregating preference values using criterion weights.

$$\prod(k,i) = \sum_j w_j P_{ki}^{j}$$

Step 4: Calculate Outranking Flows

For each alternative the positive outranking flow is found by evaluating the average preference value of the alternative.

$$\Phi^{+}{}_{i} = \frac{1}{n-1}\sum_k \prod(i,k)$$

For each alternative the negative outranking flow is found by evaluating the average preference value of other alternatives preference values compared to alternative *i*.

$$\Phi^{-}{}_{i} = \frac{1}{n-1}\sum_k \prod(k,i)$$

The positive outranking flow of an alternative is interpreted as the quantitative level of evidence that the given alternative outranks other alternatives and negative outranking flow is interpreted as the level of evidence that the given alternative is outranked by other alternatives.

Step 5: Rank the alternatives

PROMETHEE I exploits these values conservatively for ranking such that an alternative is ranked better than another only if it has both a larger positive outranking flow and a smaller negative outranking flow than the other, so some of the alternatives may not be ranked.

PROMETHEE II aiming full ranking of the alternatives, acts more liberally and aggregates two types of evidence and uses the net outranking flows ($\Phi^{net}{}_i$), difference between positive and negative outranking flows.

$$\Phi^{net}{}_i = \Phi^{+}{}_i - \Phi^{-}{}_i$$

3.2. DEA

In this section DEA models of particular interest are presented. Across the many DEA models we will present the basic CCR model which is used for classification of units, super-efficiency approach, and cross-efficiency approach, and how value judgments are incorporated in DEA models.

3.2.1 DEA CCR Model

DEA CCR model is the first model proposed for classification of the units The fractional form of the model presented by Charnes et. al. [15] is as follows:

(CCR)

$$\max h_k = \frac{\sum_{j=1}^{m} v_j y_{kj}}{\sum_{s=1}^{l} u_s x_{ks}}$$

$$subject \quad to:$$

$$\frac{\sum_{j=1}^{m} v_j y_{ij}}{\sum_{s=1}^{l} u_s x_{is}} \leq 1; \quad i = 1,\ldots,.n$$

$$v_j, u_s \geq 0; \quad k = 1,\ldots,m, \quad s = 1,\ldots,l$$

This fractional program can be converted to a linear program equating the denominator to 1 by adding the constraint (C1).

$$\text{(C1)} \sum_{s=1}^{l} u_s x_{ks} = 1$$

If there are only outputs in the model, the denominators can be equated to unity. Generally rather than the multiplier model presented above, the dual problem called envelopment model is solved.

In the end some of the units can obtain an efficiency value of 1 and classified as efficient. Inefficient units obtain efficiency scores between 0 and 1 based on radial distance to the frontier and can be ranked based on these scores.

3.2.2 Super-efficiency Approach

In order to rank the efficient units under evaluation all of which have efficiency score of 1 which are ranked as efficient, super-efficiency approach is proposed [4]. The constraint stating that all efficiency scores should be smaller than 1 is excluded only for the unit under evaluation (*k)*, so a new score for the unit that may be greater than 1 is obtained. The score is based on the radial distance of the alternative to this new frontier. For the marginal alternatives the exclusion will affect the frontier radically causing high super-efficiency scores.

(Super efficieny)

$$\max h_k = \frac{\sum_{j=1}^{m} v_j y_{kj}}{\sum_{s=1}^{l} u_s x_{ks}}$$

$$subject \quad to:$$

$$\frac{\sum_{j=1}^{m} v_j y_{ij}}{\sum_{s=1}^{l} u_s x_{is}} \le 1\,; \quad i \ne k, \quad i = 1,\ldots,.n$$

$$v_j, u_s \ge 0\,; \quad k = 1,\ldots,m, \quad s = 1,\ldots,l$$

3.2.3 Cross-efficiency Approach

Originally the DEA ranking is based on the alternative efficiency scores (h_k). To obtain a full ranking, cross-efficiency approach introduces self evaluation and peer evaluation concepts [24]. Self evaluation is the efficiency value determined from the solution of the LP. Peer evaluation of alternative *i* by alternative *k* is calculated by using optimal weights for *k*.

$$E_{ki} = \frac{\sum_{j=1}^{m} v_{kj} y_{ij}}{\sum_{s=1}^{l} u_{ks} x_{is}}$$

The output oriented CCR model in terms of cross efficiencies is given as:

(CCR-O)

$$\max E_{kk} = \sum_{j=1}^{m} v_{kj} y_{kj}$$
$$subject \quad to:$$
$$\sum_{k=1}^{l} u_{ks} x_{ks} = 1$$
$$E_{ki} \leq 1; \quad i = 1, \ldots, .n$$
$$v_{kj}, u_{ks} \geq 0; \quad k = 1, \ldots, m, \quad s = 1, \ldots, l$$

In fact $E_{ki} \leq 1$ is also a linear constraint so above LP can be solved for finding self efficiency. Usually the optimal input and output weights may not be unique. So after optimizing self efficiency a second LP is solved either to minimize or maximize the sum of other alternatives' efficiency while preserving the efficiency score of the alternative under evaluation (E_{kk}) found in step 1. Cross-efficiency values (E_{ki}) values are calculated by using weights determined in the second stage.

At last final evaluation score is found by averaging self and peer evaluations.

$$h_i = \frac{\sum_{k=1}^{n} E_{ki}}{n}$$

3.2.4 Incorporating Value Judgments

Value judgments of the decision maker can be imposed on input and output weights or on virtual weights. Normalization of data is needed if weight restrictions are added to the model. Two types of weight restrictions exist in the thesis, absolute weight restriction and assurance region restrictions.

Absolute Weight Restrictions

These restrictions can be specified on input weights (u_s) or output weights (v_j).

$$U_s \geq u_s \geq L_s$$
$$U_j \geq v_j \geq L_{js}$$

Assurance Region Restrictions

These restrictions can be specified for ratio of two input weights (u_s / u_l) or ratio of two output weights (v_j / v_t).

$$U_{sl} \geq u_s / u_l \geq L_{sl}$$
$$U_{jt} \geq v_j / v_t \geq L_{jt}$$

There are other types of weight restrictions that is not mentioned here, assurance region restrictions between inputs and outputs [72] and restricting weights by cone ratio method [16].

3.3. An Example Ranking Problem

To illustrate problems of PROMETHEE and DEA based ranking, a small example problem with 5 alternatives and 2 outputs (criteria) is provided below. Input values of all alternatives are assumed to be 1. Both criteria are increasing as they are outputs. Such examples should be treated with caution but can be helpful for illustrative purposes.

Table 1. An Example Problem with Two Criteria

	Alt. 1	Alt. 2	Alt. 3	Alt. 4	Alt. 5
Criterion 1	8	6	8	7	2
Criterion 2	2	3	1	4	8

When the criteria scores are plotted in the criterion plane (See Figure 2), we can observe that four alternatives lie in lower right region of the criterion plane while alternative 5 lies in the higher left portion alone.

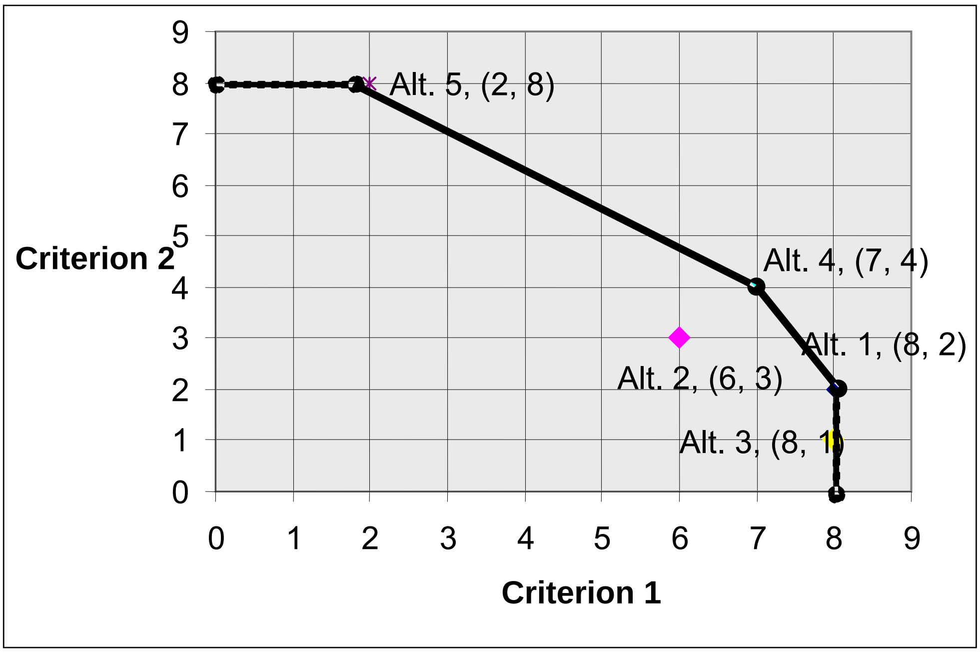


Figure 2. Alternatives in Criterion Space and Efficiency Frontier

Our aim is to rank the alternatives. We try different techniques starting with DEA, super-efficiency, cross-efficiency and PROMETHEE.

3.3.1 DEA Ranking

First we calculate the DEA CCR scores for the alternatives using basic model and scores and slacks are given in Table 2. There are more than one alternative in the efficiency frontier so complete ranking is not possible with technical efficiency score of basic CCR model. Except alternative 2 all the remaining alternatives will get technical efficiency score of 1. However Alternative 3 is weakly efficient alternative so the slack of output two is positive, so it is ranked inferior to efficient alternatives according to a variation of CCR model.

Table 2. Ranking Using CCR-O Model

	Alt. 1	**Alt. 2**	**Alt. 3**	**Alt. 4**	**Alt. 5**
Rank	1	5	4	1	1
DEA Score	1	0.83	1	1	1
Slack 1	0	1.2	0	0	0
Slack 2	0	0.6	1	0	0

From the ranking we obtained using basic DEA model, we can conclude that a complete rank for the DMUs may not be obtained.

3.3.2 DEA Super-efficiency Ranking

In this case for the efficient alternatives we exclude the corresponding constraint so the alternative under evaluation can attain efficiency score greater than 1. The super-efficiency scores of the alternatives is given in Table 3.

Table 3. Ranking Using DEA Super-efficiency Method

	Alt. 1	**Alt. 2**	**Alt. 3**	**Alt. 4**	**Alt. 5**
Rank	3	5	4	2	1
DEA Score	1.04	0.83	1	1.09	2
Slack 1	0	1.2	0	0	0
Slack 2	0	0.6	1	0	0

As seen from the Table 3, alternative 5 obtains a very extreme score and ranked first. Alternative 1 whose values are just the reverse of the alternative 5 is ranked third getting a much less score. Super-efficiency method favors the marginal alternatives even if there is no information about the importance of the criteria.

3.3.3 DEA Cross-efficiency Ranking

By using the weights calculated by CCR-O, other alternatives are evaluated and final scores are obtained by taking averages of the evaluations.

Table 4. Ranking Using DEA Cross-efficiency Method

	Alt. 1	**Alt. 2**	**Alt. 3**	**Alt. 4**	**Alt. 5**
Rank	2	4	3	1	5
Score	0.87	0.75	0.80	0.90	0.69

Cross-efficiency method favors the alternatives in the crowded region. Alternative 5 which is an efficient alternative ranks worst while inefficient alternatives 2 and 3 rank better.

3.3.4 PROMETHEE Ranking

For PROMETHE ranking we need to determine preference functions (type and preference parameters) and weights. The preference functions are assumed as given in Table 5.

Table 5. PROMETHEE Preference Functions and Parameters for Ranking Example

Criteria	Weight	Preference Function Type	Indifference Threshold	Preference Threshold
Criterion 1	0.5	Type 5	0	3
Criterion 2	0.5	Type 5	0	3

For the sample the results of a PROMETHEE II ranking are shown in Table 6.

Table 6. Ranking Using PROMETHEE II Method

	Alt. 1	Alt. 2	Alt. 3	Alt. 4	Alt. 5
Rank	2	4	5	1	3
PROMETHEE II Score	0.041	-0.125	-0.125	0.203	0

PROMETHEE II method produces a reasonable rank compared to previous approaches. Alternative 4 is ranked as in the first place and alternative 1 and alternative 5 are ranked second and third with a very little score difference. Alternative 2 and alternative 3 are ranked 4 and 5 respectively. The ranking is also reasonable compared to DEA ranking, which ranks all the weakly efficient DMUs better than inefficient ones. To arrive such a ranking we had to specify both weights and preference structure precisely.

CHAPTER 4

HYBRID METHODS BASED ON PROMETHEE AND DEA

4.1. General Outline

The proposed methods explained in this section extend PROMETHEE so that it can be used when there is uncertainty in criterion weights, indifference and preference thresholds. The general flow of operations can be seen as show in Figure 3.

In the first method, PROMETHEE is used for building outranking relations based on pairwise comparisons. Instead of aggregating preference values among criteria to arrive at a general preference of an alternative to another (Section 3.1, step 3), netflows for alternatives for each criterion are evaluated. Then outranking netflows are aggregated using a procedure similar to DEA cross-efficiency ranking. If DM desires, s/he can define constraints on weights either as specifying absolute upper and lower bounds for individual weights or upper and lower bounds on ratios of two criterion weights.

In the second method, the outranking relations are built based on the partial information of preference structure (preference function type and parameters) such that netflow for each alternative and criterion is maximized. After finding preference and indifference thresholds for each alternative and criterion the outranking netflows are calculated. After this step, the netflows of outranking may be aggregated using exact weights specified by the DM a priori or by using the first method if uncertainties in criterion weights exist.

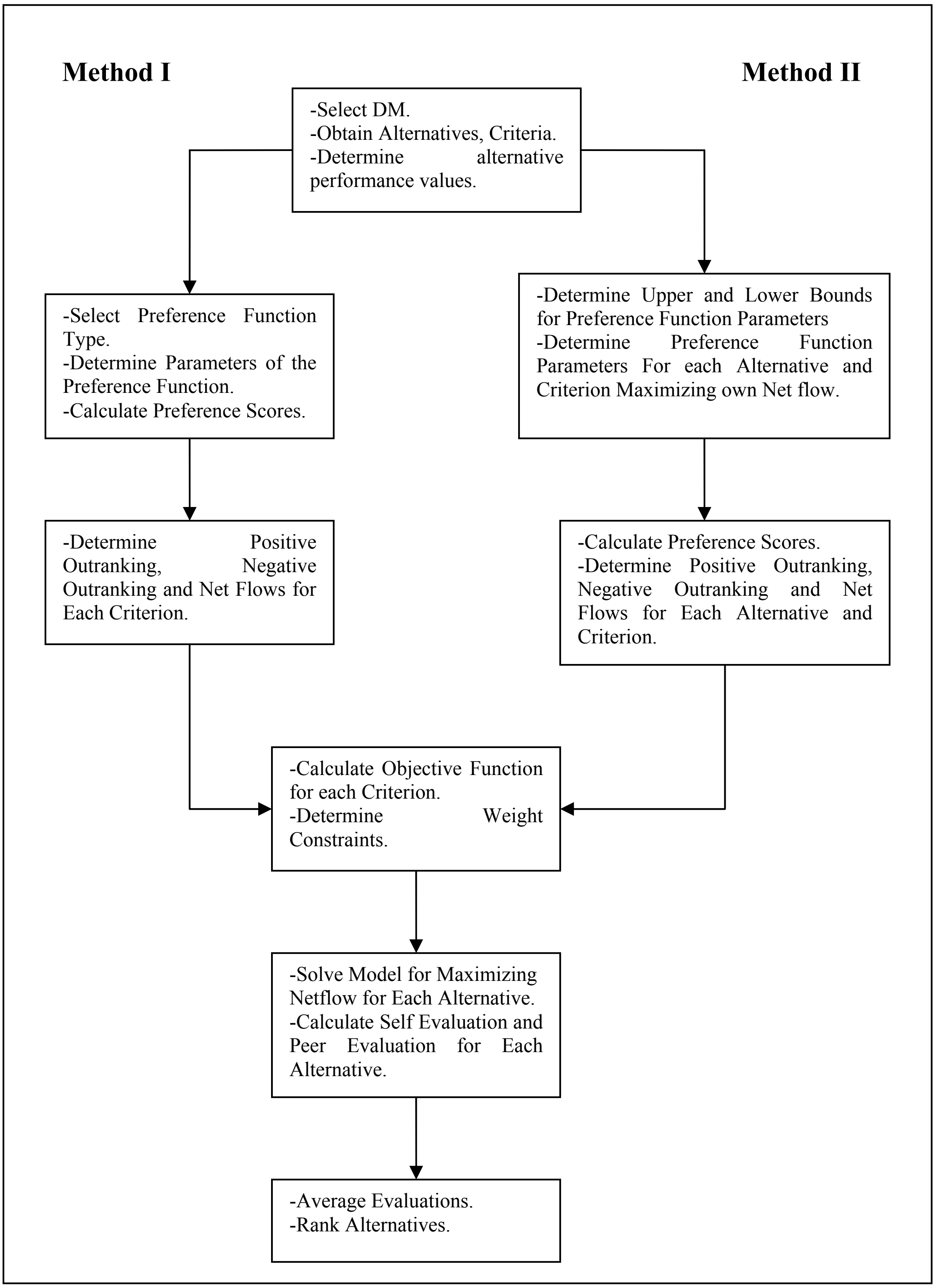


Figure 3. General Flow of the Methods

4.2. The First Approach: Ranking When Weights are not Precisely Specified

PROMETHEE method does not address a specific way to determine the weights for the criteria for aggregating the preferences. In our case the weights are determined for each alternative separately by using DEA. Assurance region constraints on weights can be specified by the DM. We present the steps of the algorithm below.

Step 1: Initialization

Ask the DM to determine criteria and alternatives and to evaluate all alternatives $(i = 1,..,n)$ in all criteria $(j = 1,..,m)$.

Matrix of alternative performance values (S) with n rows and m columns is prepared. S_{ij} represents the ith alternative's performance in criterion j.

$$S = \begin{bmatrix} S_{11} & S_{12} & \dots & \dots & S_{1m} \\ S_{21} & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ S_{n1} & \dots & \dots & \dots & S_{nm} \end{bmatrix}$$

Step 2: Calculate Preference Values

Ask the DM to determine preference function type for each criteria j. Then for each pair of alternatives (i, k) using preference function (P_j) the preference score of an alternative i with respect to alternative k in criteria j, P_{ik}^{j} is calculated.

$$P_{ik}^{j} = P_j\,(i,\,k)$$

The entry of P_{ik}^{j} refers to the preference value for ith alternative compared to kth alternative. The preference function (P_j) may be any of the functions used in PROMETHEE.

$$P^j = \begin{bmatrix} P_{11}^j & P_{12}^j & \dots & \dots & P_{1n}^j \\ P_{21}^j & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ P_{n1}^j & \dots & \dots & \dots & P_{nn}^j \end{bmatrix}$$

Step 3: Calculate Outranking Flows

The positive outranking flow of alternative *i* in criterion *j* is calculated by aggregating the preference values.

$$\Phi^{+}{}_{ij} = \frac{1}{n-1} \sum_k P_{ik}^j$$

For the negative outranking flow similar approach is utilized.

$$\Phi^{-}{}_{ij} = \frac{1}{n-1} \sum_k P_{ki}^j$$

Net flow is calculated by taking difference of the positive outranking flow and negative outranking score:

$$\Phi^{net}{}_{kj} = \Phi^{+}{}_{kj} - \Phi^{-}{}_{kj}$$

Unlike PROMETHEE approach, the preferences between alternatives are aggregated for each alternative and criterion. Thus Φ^{+} and Φ^{-} are not aggregated flows over all the criteria but intermediate measures.

Let Φ^{+} be the matrix of positive outranking flows and Φ_{ij}^{+} representing the positive outranking flow of alternative *i* for criterion *j,* calculated by the equation above.

$$\Phi^+ = \begin{bmatrix} \Phi^+_{11} & \Phi^+_{12} & .. & .. & \Phi^+_{1m} \\ \Phi^+_{21} & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ \Phi^+_{n1} & .. & .. & .. & \Phi^+_{nm} \end{bmatrix}$$

Likewise Φ^- be matrix of negative outranking flows.

$$\Phi^- = \begin{bmatrix} \Phi^-_{11} & \Phi^-_{12} & .. & .. & \Phi^-_{1m} \\ \Phi^-_{21} & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ \Phi^-_{n1} & .. & .. & .. & \Phi^-_{nm} \end{bmatrix}$$

Step 4: Add Constraints on Weights

At this step DM may provide information on the weights (w_j) of the criteria. Any Linear constraint can be specified by DM.

Upper bound (U_j) and lower bound (L_j) on criterion weights can be specified. Specification of lower bound is critical if every criterion is desired to contribute overall score by the DM. The DEA counterparts of these constraints are called absolute assurance region constraints.

$$L_j \leq w_j \leq U_j$$

Weight constraints can be added based on the importance of the criteria between weights or ratio on weights. In DEA models these are called relative assurance region constraints.

$$Aw_j \leq w_l$$

Additional linear constraints on weights may be added to the model.

Finally as the PROMETHEE method suggests we add the constraint that asserts sum of the weights is one. This constraint is also meaningful as an alternative that is scoring well in a criterion will force that weight to infinity in the absence of such a constraint.

$$\sum_{j=1}^{m} w_j \quad = 1$$

Finally we can formulate feasible weight set for W_j.

$$W_j = \left\{ w_j \in R \;\middle|\; L_j \le w_j \le U_j, A w_j \le w_l, \sum_{j=1}^{m} w_j \quad = 1, w_j \ge 0 \right\}$$

The objective is to maximize self net flow relative to the average of net flows of other alternatives. The decision variables (v_{kj}) are the weights assigned to criteria j by alternative k.

$$\underset{v_{kj} \in W_j}{Max} \sum_{j=1}^{m} v_{kj} (\Phi^+{}_{kj} - \Phi^-{}_{kj}) - \frac{\sum_{i=1, i \ne k}^{n} \sum_{j=1}^{m} v_{kj} (\Phi^+{}_{ij} - \Phi^-{}_{ij})}{n-1}$$

Terms in parentheses are constant and were calculated in Step 3.
(M1)

$$\underset{v_{kj} \in W_j}{Max} \sum_{j=1}^{m} v_{kj} \left[(\Phi^+{}_{kj} - \Phi^-{}_{kj}) - \frac{\sum_{i=1, i \ne k}^{n} (\Phi^+{}_{ij} - \Phi^-{}_{ij})}{n-1} \right]$$

Step 5: Construct Cross-efficiency Matrix

For each alternative, LP with the objective function and constraints given in step 4 is solved. The optimal objective function value is recorded as the self score (E_{kk}) of the given alternative.

$$E_{kk} = \sum_{j=1}^{m} v^*_{kj} \left[(\Phi^+_{kj} - \Phi^-_{kj}) - \frac{\sum_{l=1, l \neq i}^{n} (\Phi^+_{lj} - \Phi^-_{lj})}{n-1} \right]$$

Using the optimal criterion weights other alternatives are evaluated. E_{ki} is the evaluation of the *ith* alternative by using weights of alternative *k*.

$$E_{ki} = \sum_{j=1}^{m} v^*_{kj} \left[(\Phi^+_{ij} - \Phi^-_{ij}) - \frac{\sum_{l=1, l \neq i}^{n} (\Phi^+_{lj} - \Phi^-_{lj})}{n-1} \right]$$

Now we can form a matrix of self (E_{kk}) and peer evaluations (E_{ki}), like cross-efficiency matrix of DEA.

$$E = \begin{bmatrix} E_{11} & E_{12} & \cdots & E_{1n} \\ E_{21} & E_{22} & \cdots & E_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ E_{n1} & E_{n2} & \cdots & E_{nn} \end{bmatrix}$$

Step 6: Rank the Alternatives

To arrive at a final score for ranking the self and peer evaluations are averaged. The ranking of the alternatives is done based on this average (h_i).

$$h_i = \frac{\sum_{k=1}^{n} E_{ki}}{n}$$

4.2.1 Example Ranking by the First Proposed Approach

We apply proposed method to illustrative problem described in Section 3.3. The scores of alternatives and rank is given in Table 7. For the weights no constraints are added.

Table 7. Ranking Using First Proposed Approach

	Alt. 1	Alt. 2	Alt. 3	Alt. 4	Alt. 5
Rank	3	4	5	1	2
Method 1 Score	-0.063	-0.146	-0.312	0.271	0.250

In the ranking, alternative 5 manages to be positioned before alternative 1. Alternative 1 is ranked higher than alternative 2 and 3. Proposed method can rank distinct alternatives which has a high outranking value and yet able to discriminate between other alternatives. So alternative 5 can be ranked higher but just not the best because it is distinct from the others like in the super-efficiency method. Proposed method does not favor alternatives in the crowded region. Also we assumed that no information on weights is available and did not define any weight constraints for this example which will refine the ranking.

4.3. The Second Approach: Ranking When Weights and Preference Function Parameters are not Precisely Specified

In the second approach, we assume that DM can provide partial information about preference function parameters. DM may not exactly state indifference thresholds and/or preference thresholds so the additional assumption of the method is that only upper and lower bounds for the preference and indifference thresholds can be specified by the decision maker for some of the criteria.

The preference of the DM for each criterion is assumed to be one of the type 1, type 2, type 3 and type 5 (Section 3.1, Figure 1). A generalized form of above mentioned four types of preference is defined shown as Figure 4.

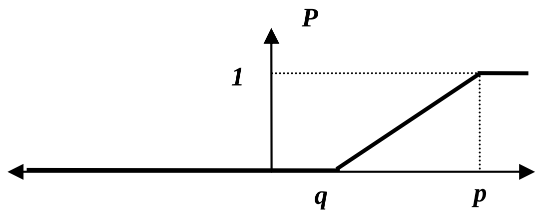


Figure 4. Generic Preference Function

We call this preference function type g, standing for generic. The type g which is similar to type 5 “linear preference function with indifference threshold” has two parameters q and p. The feasible set of (*q, p*) pairs, can be defined as:

$$G = \{(q, p) \in R^2 | q \geq 0, p \geq q\}$$

For any given real number pair $g \in G$, there exists a PROMETHEE preference function of the types 1, 2, 3 or 5. So by constraining the parameters of the preference function type g; type 1, type 2, type 3 and type 5 preference functions can be obtained as shown in Figure 5.

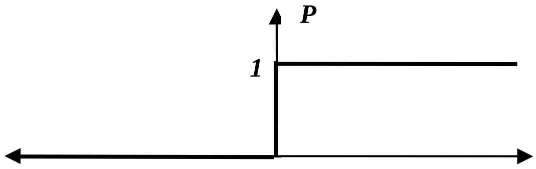
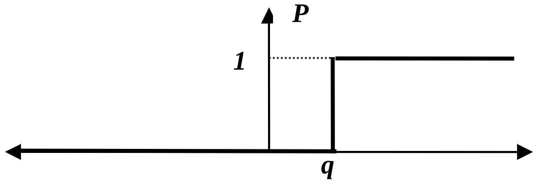
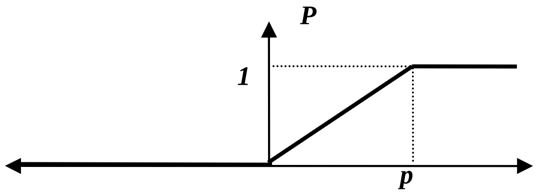
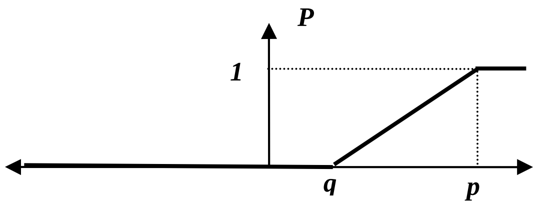
<table>
<tr><th>Graphical Illustration</th><th>Type, Parameter</th></tr>
<tr><td>
</td><td><u>Type 1</u>
q = p = 0</td></tr>
<tr><td>
</td><td><u>Type 2</u>
q > 0, p = q</td></tr>
<tr><td>
</td><td><u>Type 3</u>
q = 0, p > 0</td></tr>
<tr><td>
</td><td><u>Type 5</u>
p > 0 , q > 0, p > q</td></tr>
</table>

Figure 5. PROMETHEE Preference Functions as Special Cases of a Generic Preference Function

The preference between two alternatives for the generic function can be defined as:

$$P_j\,(\,i,\,l) = \begin{cases} 1 & S_{ij} - S_{lj} > p \\ 1 & S_{ij} - S_{lj} = p, p = q \\ (S_{ij} - S_{lj} - q)/(p-q) & q \le S_{ij} - S_{lj} \le p,\ p \ne q \\ 0 & S_{ij} - S_{lj} \le q \end{cases}$$

The steps of the proposed algorithm is presented below.

Step 1: Initialization

Ask the DM to to determine criteria and alternatives and to evaluate all alternatives $(i = 1,..,n)$ 1 in all criteria $(j = 1,..,m)$.

Matrix of alternative performance values (*S*) with *n* rows and *m* columns is prepared. S_{ij} represents the *i*th alternatives performance in criterion *j*.

$$S = \begin{bmatrix} S_{11} & S_{12} & \cdots & \cdots & S_{1m} \\ S_{21} & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ S_{n1} & \cdots & \cdots & \cdots & S_{nm} \end{bmatrix}$$

Step 2: Calculate Preference Function Parameters

In this approach the DM does not provide preference function completely. In fact s/he only provides some upper and lower bounds for the parameters of the type g function. So there are no explicit preference values. By determining lower (Lq_j, Lp_j) and upper (Uq_j, Up_j) bounds for indifference and preference thresholds respectively, the set of feasible values for criterion *j*, G_j is defined as:

$$G_j = \left\{ (q_j, p_j) \in R^2 \middle| q_j, p_j \in R, q_j \ge 0, p_j \ge q_j, Lq_j \le q \le Uq_j, Lp_j \le p \le Up_j \right\}$$

If the DM can not specify a lower bound it may be taken as 0. The upper bound is also specified by the DM; if not it may be taken as the difference between the best performing score and the worst performing alternative's score for the criterion under consideration.

As in the first proposed method, we want to solve the model M2, weights (v_{kj}) and thresholds (q_{kj}, p_{kj}) are the decision variables.

(M2)

$$\max_{v_{kj}\in W_j,(q_{kj},p_{kj})\in G_j} \sum_{j=1}^{m} v_{kj}\left[(\Phi^{+}{}_{kj}-\Phi^{-}{}_{kj})-\frac{\sum_{i=1,i\neq k}^{n}(\Phi^{+}{}_{ij}-\Phi^{-}{}_{ij})}{n-1}\right]$$

M2 is a nonlinear model and can be split into two terms; weights (v_{kj}) and the term that only depends on thresholds (F) by Theorem 1.

$$F=\left[(\Phi^{+}{}_{kj}-\Phi^{-}{}_{kj})-\frac{\sum_{i=1,i\neq k}^{n}(\Phi^{+}{}_{ij}-\Phi^{-}{}_{ij})}{n-1}\right]$$

Note that the term (F) is independent of the weights and thresholds are the only decision variables in the second part. Theorems 1 and 2 show how optimal solution to model M2 can be found.

Theorem 1:

Let $\Phi^{+}{}_{kj}$, $\Phi^{-}{}_{kj}$ be the positive and negative outranking flows of alternative k for criterion j respectively.

$$G_j=\left\{(q_j,p_j)\in R^2 \middle| q_j,p_j\in R, q_j\geq 0, p_j\geq q_j, Lq_j\leq q\leq Uq_j, Lp_j\leq p\leq Up_j\right\}$$

$$\max_{v_{kj}\in W_j,(q_{kj},p_{kj})\in G_j} \sum_{j=1}^{m} v_{kj}\left[(\Phi^{+}{}_{kj}-\Phi^{-}{}_{kj})-\frac{\sum_{i=1,i\neq k}^{n}(\Phi^{+}{}_{ij}-\Phi^{-}{}_{ij})}{n-1}\right]=\max_{v_{kj}\in W_j}\Big(\sum_{j=1}^{m} v_{kj}\max_{(q_{kj},p_{kj})\in G_j}\Big(\left[(\Phi^{+}{}_{kj}-\Phi^{-}{}_{kj})-\frac{\sum_{i=1,i\neq k}^{n}(\Phi^{+}{}_{ij}-\Phi^{-}{}_{ij})}{n-1}\right]\Big)\Big)$$

Proof: See Appendix A.

To find maximum of M2, given linear constraints on weights and upper and lower bounds on preference function parameters, first the maximization of individual flow of an alternative and criterion can be done, then, using these parameters corresponding optimal criterion weights can be found.

Theorem 2:

Let $\Phi^{+}{}_{kj}$, $\Phi^{-}{}_{kj}$ be the positive and negative outranking flows of alternative k for criterion j respectively.

$$GD_j = \left\{(x, y) \middle| x \in \Delta \cup \left\{Lq_j, Uq_j\right\}, y \in \Delta \cup \left\{Lp_j, Up_j\right\}, x \le y\right\}$$

$$\max_{(q_{kj}, p_{kj}) \in G_j} \left(\left[(\Phi^{+}{}_{kj} - \Phi^{-}{}_{kj}) - \frac{\sum_{i=1, i \ne k}^{n} (\Phi^{+}{}_{ij} - \Phi^{-}{}_{ij})}{n-1} \right] \right) = \max_{(q_{kj}, p_{kj},) \in GD_j} \left(\left[(\Phi^{+}{}_{kj} - \Phi^{-}{}_{kj}) - \frac{\sum_{i=1, i \ne k}^{n} (\Phi^{+}{}_{ij} - \Phi^{-}{}_{ij})}{n-1} \right] \right)$$

Proof: See Appendix B.

So the q, p values that maximize the unweighted outranking flow can be determined by trying a discrete set of values (members of the set GD_j) for q and p. The values of q and p that maximize objective function (i.e. optimal solution to M2) are denoted by $\left(q^{*}_{kj}, p^{*}_{kj}\right)$ where k stands for the alternative under evaluation and j is the criterion under consideration

Step 3: Calculate Outranking Flows

Now we have determined the values of the preference function parameters for each alternative and criterion. Using $\left(q^{*}_{kj}, p^{*}_{kj}\right)$ found in step 2, that specify P_j preference score of alternative i with respect to alternative l is calculated.

$$P^{kj}_{il} = P_j(i, l) \quad where \quad (q, p) = \left(q^{*}_{kj}, p^{*}_{kj}\right)$$

We can form the matrix of preference scores for each alternative and criterion (P^{kj}).

$$P^{kj} = \begin{bmatrix} P_{11}^{kj} & P_{12}^{kj} & \dots & \dots & P_{1n}^{kj} \\ P_{21}^{kj} & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ P_{n1}^{kj} & \dots & \dots & \dots & P_{nn}^{kj} \end{bmatrix}$$

We next calculate the second part of the objective function of M2 which will be used as objective function coefficient of LP in step 4.

The positive outranking netflows (${\Phi^{+k}}_{ij}$) can be calculated by using pairwise preference matrix (P^{kj}) of alternative under evaluation (k).

$${\Phi^{+k}}_{ij} = \frac{1}{n-1}\sum_{l} P_{il}^{kj}$$

The negative outranking netflows (${\Phi^{-k}}_{ij}$)can be calculated by using pairwise preference matrix (P^{kj}) of alternative under evaluation (k).

$${\Phi^{-k}}_{ij} = \frac{1}{n-1}\sum_{l} P_{li}^{kj}$$

The net outranking flow (${\Phi^{net(k)}}_{ij}$) can then be evaluated.

$${\Phi^{net(k)}}_{ij} = {\Phi^{+k}}_{ij} - {\Phi^{-k}}_{ij}$$

Step 4: Calculate Criterion weights

The nonlinear program (M2) is transformed into an LP and solved just as M1. For the objective function coefficients the values found in step 3 are used. For each alternative, LP with objective function coefficients and constraints given in previous step is solved. The objective function value is recorded as the self score. Different types of weight constraints that form feasible weight set (W_j) explained in Section 4.2, can be used.

$$\max_{v_{kj}\in W_j}\sum_{j=1}^{m} v_{kj}\left[(\Phi^{+k}{}_{kj}-\Phi^{-k}{}_{kj})-\frac{\sum_{i=1,i\neq k}^{n}(\Phi^{+k}{}_{ij}-\Phi^{-k}{}_{ij})}{n-1}\right]$$

The optimal criterion weights $v^*{}_{kj}$ are calculated for each alternative and criterion.

Step 5: Construct Cross-efficiency Matrix

The cross evaluations are done similar to first method. During cross evaluations, negative, positive and net outranking flows found in step 2 are used first to calculate net flow for each alternative and criterion (see step 3).

Then the calculated weights for alternative that is under evaluation are used for aggregating these. Unlike first method, for different alternatives, different preference functions exist, resulting in different preference values. So the unweighted positive, negative and net flows are not unique for each alternative and criterion but also depend on the alternative under evaluation.

$$E_{kk}=\sum_{j=1}^{m} v^*{}_{kj}\left[(\Phi^{+k}{}_{kj}-\Phi^{-k}{}_{kj})-\frac{\sum_{l=1,l\neq i}^{n}(\Phi^{+k}{}_{lj}-\Phi^{-k}{}_{lj})}{n-1}\right]$$

Using the optimal criterion weights and flows calculated in step 3 other alternatives are evaluated.

$$E_{ki}=\sum_{j=1}^{m} v^*{}_{kj}\left[(\Phi^{+k}{}_{ij}-\Phi^{-k}{}_{ij})-\frac{\sum_{l=1,l\neq i}^{n}(\Phi^{+k}{}_{lj}-\Phi^{-k}{}_{lj})}{n-1}\right]$$

Now we can form a matrix of self and peer evaluations, like cross-efficiency matrix of DEA.

$$E = \begin{bmatrix} E_{11} & E_{12} & \cdots & E_{1n} \\ E_{21} & E_{22} & \cdots & E_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ E_{n1} & E_{n2} & \cdots & E_{nn} \end{bmatrix}$$

Step 6: Rank the Alternatives

To arrive at a final score for ranking the self and peer evaluations are averaged similar to first method. The ranking of the alternatives is done based on this score.

$$h_i = \frac{\sum_{k=1}^{n} E_{ki}}{n}$$

4.3.1 Example Ranking by the Second Proposed Approach

We apply proposed method to illustrative problem described in Section 3.3. The scores of alternatives and rank is given in Table 8. For the weights no constraints are added. Instead of setting indifference threshold to 0 and preference threshold to 3 as in method 1, we set lower and upper bounds on the thresholds as follows:

$$0 \le q_j \le 0.3 \quad j = 1,2$$
$$2.7 \le p_j \le 3.3 \quad j = 1,2$$

Then we calculate the resultant scores and ranking for the illustrative example. The resultant scores and ranks are shown in Table 8.

Table 8. Ranking Using Second Proposed Approach

	Alt. 1	**Alt. 2**	**Alt. 3**	**Alt. 4**	**Alt. 5**
Rank	3	4	5	1	2
Method 2 Score	-0.056	-0.173	-0.313	0.277	0.269

Although scores of alternatives slightly changed, overall ranking is the same as ranking of proposed method 1. We should also state that overall ranking may change if the upper and lower bounds of thresholds change.

CHAPTER 5

SOFTWARE

Proposed methods are implemented as a decision aid tool. For data input, a user form and excel spreadsheets are utilized. The user form and spreadsheets are explained in Section 5.1 and Section 5.2 respectively.

5.1. User Form

Five input windows exist in the main user form, general parameter window includes basic information, criteria information window includes the criteria information, thresholds window includes preference type and parameters inputs, auto generate window includes inputs to generate symmetric bounds for preference parameters, and weights and output window includes inputs for customizing output information.

5.1.1 General Parameters Window

Number of alternatives (n) and number of criteria (m) is specified, also the user can define the area that alternative score information is placed (See Figure 6.). By default score information resides in "Scores" worksheets but another area may be defined by the user using scores textbox.

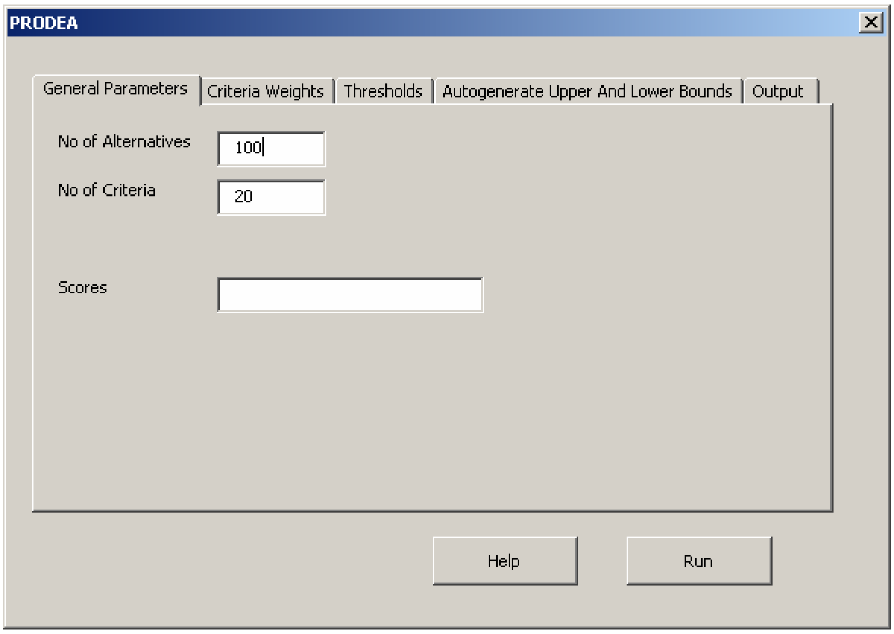


Figure 6. Screenshot of General Parameters Window

5.1.2 Criteria Information Window

The criterion information is entered in this tab. The user specifies whether exact weights or constraints on weights are used. If the user prefers using constraints on weights, then s/he may select type of constraints on the criterion weights; i.e. absolute lower bounds and upper bounds, lower and upper bounds on ratios, or other linear constraints. If the user does not want to use default sheet ("Criteria") s/he may specify the location of the criteria input data (See Figure 7). If constraints on weights is selected either the first proposed approach or the second will be used depending on the selected options on thresholds.

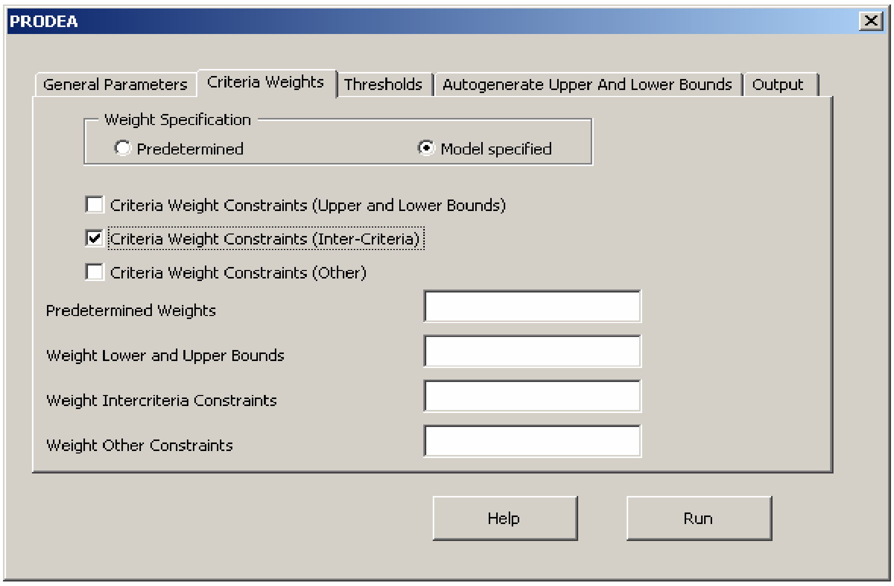


Figure 7. Screenshot of Criterion weights Window

5.1.3 Threshold Information Window

This tab is used to enter information about the criteria. The user either selects to specify the thresholds (predetermined option) or let the model (model specified option) determine the thresholds (See Figure 8). The second approach described in Section 4.3 will be used to determine the thresholds if the second option is selected.

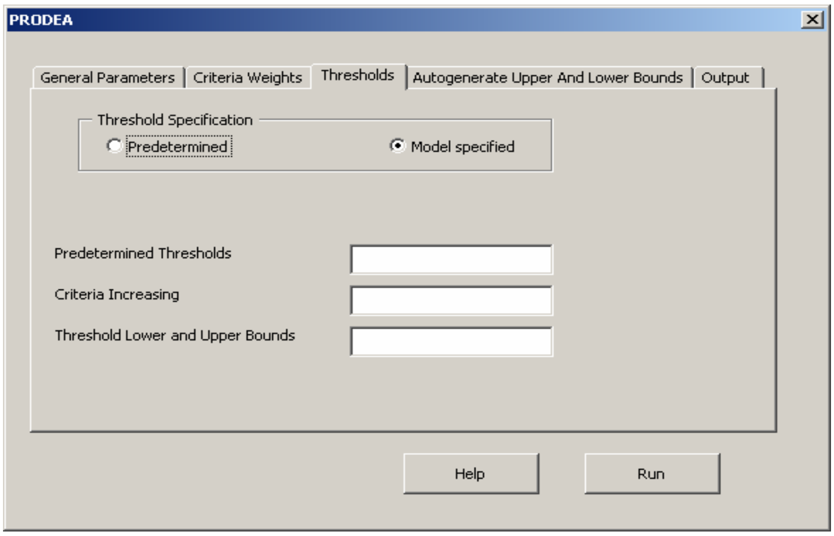


Figure 8. Screenshot of Thresholds Window

5.1.4 Auto Generate Upper and Lower Bounds Window

Rather than creating bounds on weights by manually entering, the user can use this tab to generate upper and lower bounds for the weights and thresholds automatically. The user may input the percentage of relaxation on the predetermined values (See Figure 9). If auto generate weights option is used, then the approach described in Section 4.2 is used if both options are specified, the approach described in Section 4.3 will be used.

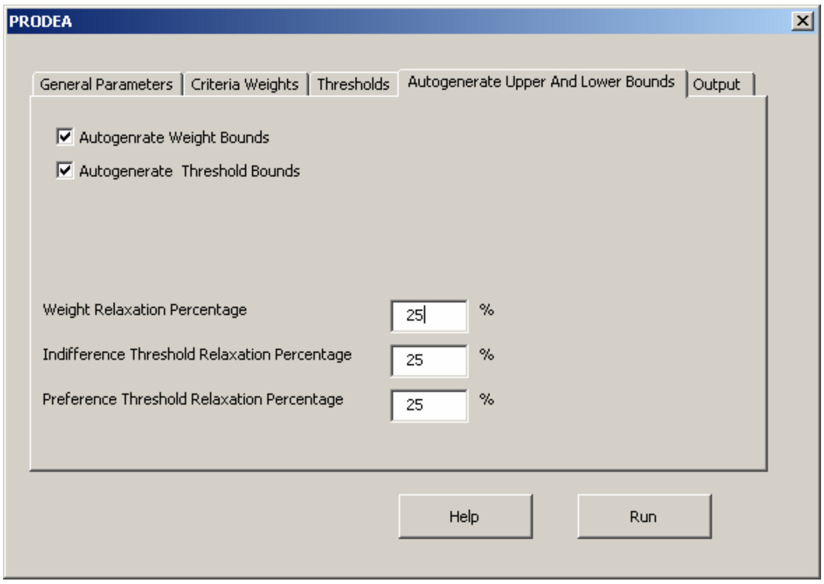


Figure 9. Screenshot of Auto generate Upper and Lower Bounds Window

5.1.5 Output Window

The place where output of the model will be displayed is determined in the output column (See Figure 10). Also any other ranking of the same alternatives that may be placed in output column to compare with produced ranking are specified. The user can select S or absolute difference metric to be calculated based on two rankings. Additionally two rankings may be graphed in a chart.

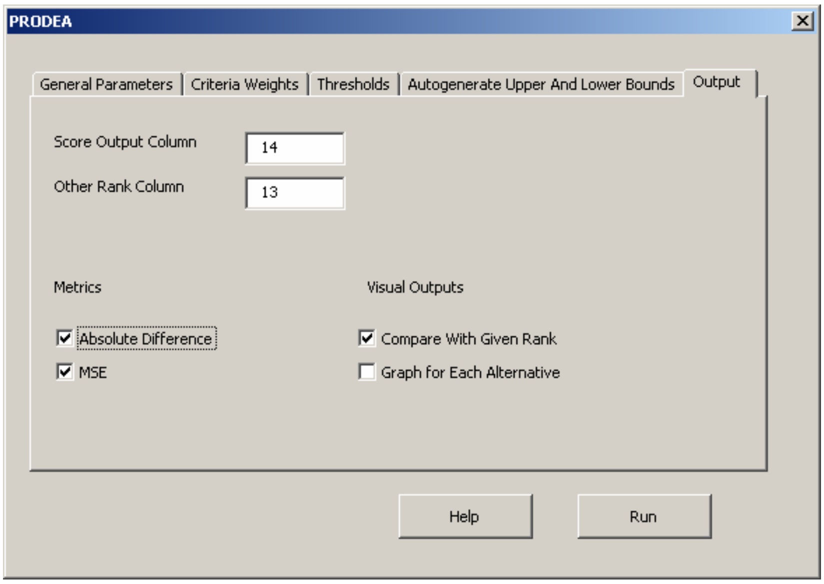


Figure 10. Screenshot of Output Window

5.1.6 Run Button

After successfully determining model parameters run button is used to execute the model.

5.1.7 Help Button

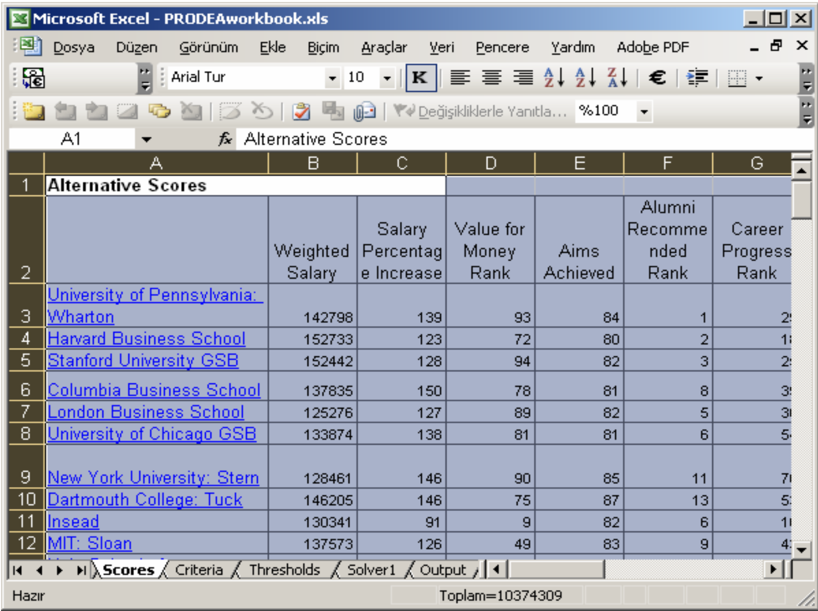
Help for the software is displayed,

5.2. Worksheets

For the inputs by default "Scores" and "Criteria" worksheets are used. The outputs are provided in the "Output" worksheet.

5.2.1 Scores Worksheet

The alternatives are listed in rows and criteria in columns (See Figure 11). The alternative score information is read from this worksheet considering the number of alternatives and criteria provided in general information window.



	A	B	C	D	E	F	G
1	Alternative Scores						
2		Weighted Salary	Salary Percentage Increase	Value for Money Rank	Aims Achieved	Alumni Recommended Rank	Career Progress Rank
3	University of Pennsylvania: Wharton	142798	139	93	84	1	2
4	Harvard Business School	152733	123	72	80	2	1
5	Stanford University GSB	152442	128	94	82	3	2
6	Columbia Business School	137835	150	78	81	8	3
7	London Business School	125276	127	89	82	5	3
8	University of Chicago GSB	133874	138	81	81	6	5
9	New York University: Stern	128461	146	90	85	11	7
10	Dartmouth College: Tuck	146205	146	75	87	13	5
11	Insead	130341	91	9	82	6	1
12	MIT: Sloan	137573	126	49	83	9	4

Figure 11. Screenshot of Scores Worksheet

5.2.2 Criteria Worksheet

The criteria worksheet includes information on preference functions, preference parameter bounds, weights, intra-criterion weight constraints (absolute bounds on weights), inter-criterion weight constraints (bounds on ratio of weights) (See Fıgure 12).

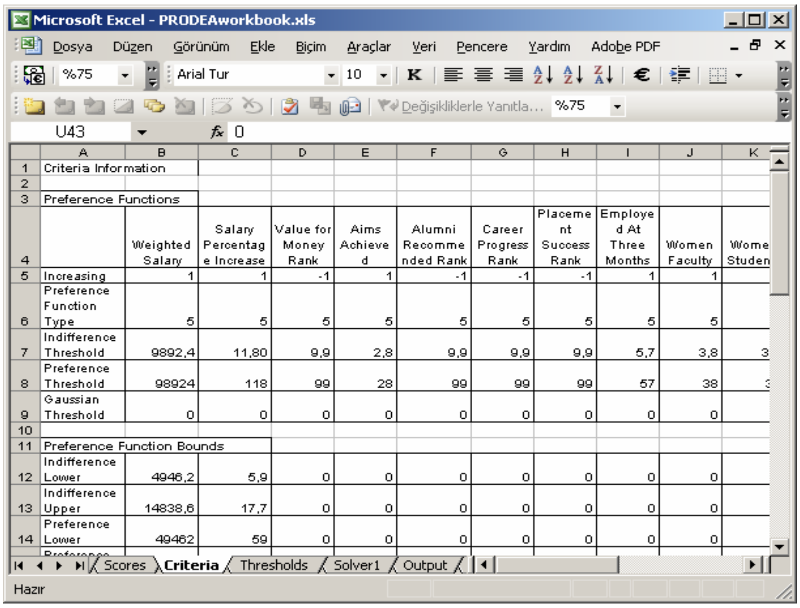


Figure 12. Screenshot of Criteria Worksheet

5.2.3 Output Worksheet

In the output worksheet the net outranking flows, calculated objective function parameters foe M1, optimal weights, cross-evaluation scores, final scores and metrics are displayed (See Figure 13).

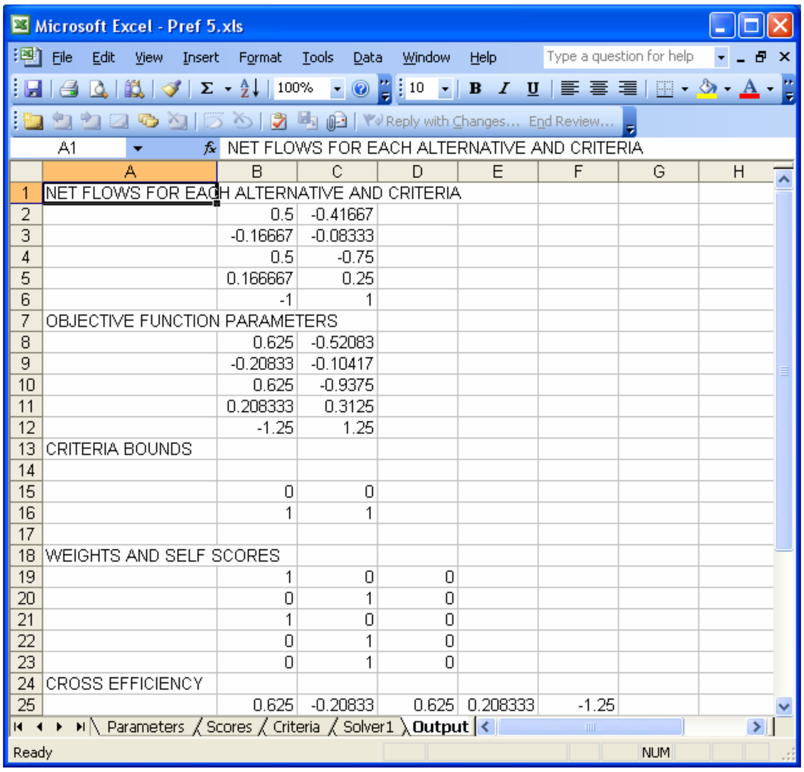


Figure 13. Screenshot of Outputs Worksheet

CHAPTER 6

CASE STUDY

We will use the MBA program ranking problem for testing and comparing proposed methods and other ranking approaches. In Section 6.1, information on MBA ranking problem and used data are presented. In Section 6.2, various MCDM approaches are applied and evaluated. We compare the methods in Section 6.3. Finally in the last section, the results of the case study are presented.

6.1. MBA Program Ranking Data

Various sources provide data sets of performance of MBA programs and their rankings. Main reasons for choosing the FT data for the case study are:

a. It is the most comprehensive data, according to the number of criteria.
b. Both international and US programs are ranked and the number of programs for which performances are given for each criterion is higher than other rankings.
c. The school performances are provided free of charge in FT website.

Financial Times (FT) 2006 data is used for the case study. The data set includes performances of 100 MBA programs in twenty criteria. Exact performance values are provided for twelve criteria and ranks of the graduate programs are provided for the rest.

6.2. Ranking Methods

To compare different ranking approaches and examine the weak and strong points of our methods numerous ranking approaches are applied to the FT data. Following rankings are considered:

a. Ranking of Financial Times (FT).
b. Ranking using SMART method (SMART) using FT weights.
c. Ranking by PROMETHEE method (PROM) using FT weights.
d. Rankings by using different mixtures (MIX), for cases with unconstrained weights and with AR constraints.
e. Ranking based on basic DEA CCR model, for cases with unconstrained weights and with AR constraints.
f. Ranking based on super-efficiency approach (SE) approach explained in Subsection 3.2.2, for cases with unconstrained weights and with AR constraints.
g. Ranking based on simple cross-efficiency (SXE) approach explained in Subsection 3.2.3 , for cases with unconstrained weights and with AR constraints.
h. Ranking based on proposed method 1 (M1), for cases with unconstrained weights and with AR constraints.
i. Ranking based on proposed method 2 (M2).

Ranking of FT, SMART (with FT weights) and PROMETHEE (with exact values for preference parameters and FT weights) methods used for comparison of different ranking approaches. Main reason for the comparison with these rankings is to show the difference of various DEA based methods with the case when exact parameters are available.

Ranking using different mixtures is used to understand whether different set of weights provide substantially different rankings or not for the data set.

SE, SXE, M1 methods are used for two kinds of weight set, unrestricted weights and imprecise weights. For the first case it is assumed that no information on weights exist and for the second case the weights are assumed to be not known exactly and constrained by relaxing the weights provided by FT (F_j), by a fixed percentage p (1%, %25, %50, %75, 100%). So the following assurance region (AR) constraint is added for each criterion j.

$$(1-p)F_j \leq w_j \leq F_j(1+p)$$

The alternatives still classified as efficient by CCR for various AR constraints are also determined for comparison of approaches. Also for different cases of AR constraints, mixtures of weights are used to examine the impreciseness of the ranking.

Rankings of each method are compared to FT rank using various measures. The sum of absolute value of rank difference over all programs (ABS), the standard error (S), and number of programs that change at least 10 rank places is used as metrics for measuring the difference of the ranking from FT ranking. Kendall's Tau-b measure is also used when comparing trial rankings by MIX method and different rankings. We do not provide Kendall's Tau-b statistics for comparing rankings by the same method with different relaxations of weights as they are extremely high. Finally the ranking methods are compared and strengths and weaknesses of DEA based methods and proposed methods are presented. Ranking results for different methods are provided in Appendix E.

6.2.1 FT Ranking

FT 2006 ranking is produced by normalizing the performance values and calculating z scores and linearly aggregating these scores based on criterion weights. Final scores of the schools are not provided and only ranks of the schools are given.

6.2.2 Ranking Using SMART Approach

We intend to examine if the ranking will vary for different weights. For this purpose we assume a linear utility function and normalized the given FT data to use the same weights as FT ranking. Normalization is also needed since AR constraints on criterion weights are defined in various DEA models (DEA CCR, SE and SXE). For the criterion (*j*) whose raw performance values (S_{ij}) are given, the normalization is done by using the formula:

$$SN_{ij} = \frac{S_{ij} - \min_i(S_{ij})}{\max_i(S_{ij}) - \min_i(S_{ij})}$$

For the criterion (*j*) whose ranks (R_{ij}) are given a slightly different version of the above formula is used:

$$SN_{ij} = 1 - \frac{R_{ij} - 1}{100} = \frac{101 - R_{ij}}{100}$$

Then for each program final score is calculated using normalized score (SN_{ij}) and weights provided by FT. FT rank and SMART is compared in Table 9. Differences of these two rankings can be attributed to the availability of limited information for some of the criteria (ranks are provided for eight of the criteria) and normalization method used.

Table 9. Comparison of FT Ranking and Ranking Using Normalized Scores

Kendall's Tau-b	**ABS**	**Average ABS**	**S**	**Rank difference ≥10**
0.959	516	5.16	83.07	10

6.2.3 PROMETHEE Ranking

For PROMETHEE ranking, DM has to provide preference function type, parameters and bounds on weights. Type 5 preference function is used for all the criteria, the preference threshold *(p)* is set to difference of performance values of best performing program and worst performing program and indifference threshold *(q)* is set to 10% of that value. FT weights are used for aggregating the flows of various criteria.

Table 10. Comparison of FT Ranking and PROMETHEE Ranking

Kendall's Tau-b	ABS	Average ABS	S	Rank difference ≥10
0.945	628	6.28	95.88	22

As seen form Table 9 and Table 10, compared to SMART, PROMETHEE ranking is more different from FT as PROMETHEE ranking uses preference information.

6.2.4 Ranking Using Different Mixtures of Weights

Before comparing various ranking approaches, it will be beneficial to examine if different approaches can produce different rankings from the data set for the cases where no AR is defined or AR is defined by relaxing the FT weights as explained in Section 6.2. Most comprehensible method is to check whether various feasible weight combinations produce different rankings using a linear model for aggregation (similar to SMART). For the case where no AR constraints defined, we use a simplex lattice design, and various weight mixtures are used for ranking the programs. Secondly, AR is defined as 1%, 25%, 50%, 75%, 100% around FT weights. This time D-optimal design is used to find extreme weight mixtures and examine the variation of the ranking. The summary information about mixture design that are used to produce trial weight sets are shown in Table 11. A more detailed explanation on mixture designs used is provided in Appendix D.

Table 11. Properties of Mixture Design Experiments

	No of Mixtures	Constraints	Explanations
Simplex Lattice	231	-	Degree of Lattice = 2, Includes Augmented Points
D-Optimal (For Dif. AR Cases)	100	Weight Lower and Upper Bounds (Relaxations of FT weights)	Linear

100 mixtures are selected from the first design and pairwise Kendall's Tau-b statistics are calculated for both cases. The summary of the analysis are given in Table 12.

Table 12. Correlation of Mixture Experiments

	No of Mixtures	No of Pairwise Comparisons	No of Correlated Observations (0.05 Level) *	No of Correlated Observations (0.01 Level)
Simplex Lattice	100	4950	536	2368
D-Optimal (100% Relaxation)	100	4950	-	4950

*Observations that are not correlated for 0.01 confidence level but correlated for 0.05 level.

We now can conclude that for the given set of weights, correlation exist for a fraction of the experiments (~60%). For the unconstrained problem we can say that different weight sets will not necessarily produce correlated rankings.

For the constrained problem we observe that all the rankings are correlated even for the 100% relaxation case. For this case, we further analyze if the rankings observed are far different from rankings provided by FT. As shown in Table 13, the rankings do not agree with FT and a high number of programs ranked significantly different. Since 41.87 observations have on the average more than 10 rank difference compared to FT and variability of the measures are high, the need for a ranking approach still exists for the constrained case.

Table 13. Comparison Constrained Mixture experiments with FT

	Kendall's Tau-b	**ABS**	**Average ABS**	**S**	**Alt. with Rank Difference ≥10**
Minimum	0.465	572	5.72	84.33	18
Maximum	0.827	1542	15.42	248.60	71
Average	0.688	1078	10.78	145.9	41.87

We then use D-optimal designs for the cases where FT weights are relaxed by 1%, 25%, 50%, 75% and compare the variability of each individual program among different ARs. For each case, 100 mixtures are obtained and lowest and highest ranking of the programs are determined The difference of these two values are calculated to show rank impreciseness of each program (See Figure 14 and Appendix D.3). It is observed that as AR constraints get tighter, the highest and lowest ranks an alternative obtains become closer and difference decreases.

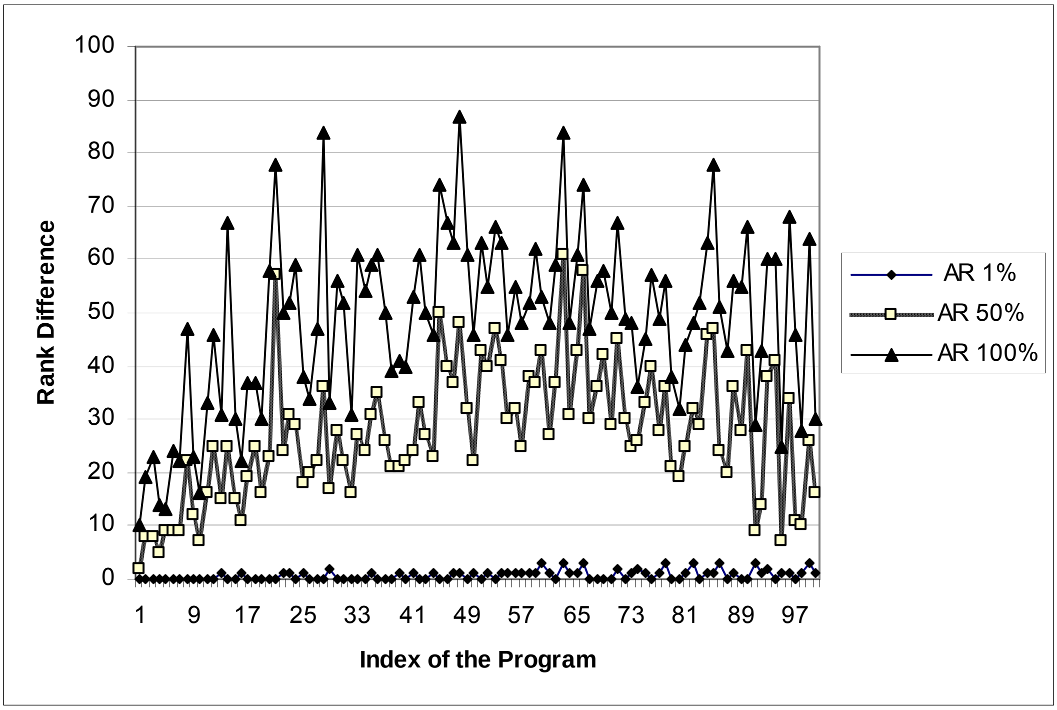


Figure 14. Rank Impreciseness of MBA Programs for Mixture Designed Set of Weights

For the high ranking alternatives (Top 15) the rank difference is limited to 30-45 rank places, while for the middle ranking alternatives rank differences are very high at least 40 for the AR defined by 100% relaxation of FT weights. For the very low ranking programs rank difference exists but is lower than middle ranking programs. Average rank impreciseness of top 15, bottom 16 (there are two 85th ranked program in FT list and no 86th) and all the alternatives for different AR is provided in Figure 15. So we can conclude that the ranks of the programs vary considerably for AR defined as 50% relaxation of FT weights. Even for 25% relaxation variation exists but there is absolutely very little variation when the AR is limited to 1% relaxation of FT weights.

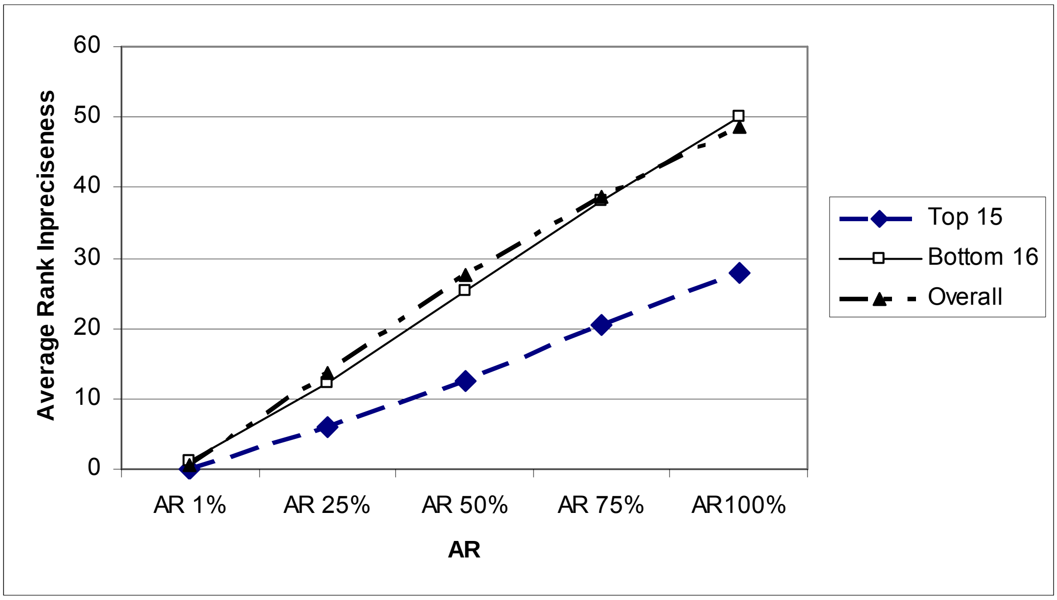


Figure 15. Average Rank Impreciseness of Ranks for Top 15, Bottom 16 and All Programs

6.2.5 DEA CCR Ranking

We ranked the programs using DEA CCR method. There are 70 programs which are classified as efficient and cannot be ranked. Generally programs that rank highest in subset of criterion maximize its score by equating the sum of corresponding criterion weights to unity to be efficient. If the subset has only one criterion the weighting is similar to vertices of the simplex of unconstrained mixture design, so at least we can say that for different vertices of the simplex different programs are ranked first. This result is in agreement with the result of the mixture experiments presented in previous section which shows the variability of the ranking for different weights.

Then we find the efficient alternatives by using the assurance region defined by upper and lower bounds around FT weights. CCR efficient programs are listed in Table 14 except for the unconstrained case for which set of efficient programs is too large.

Table 14. CCR Efficient Programs

	Number of CCR Efficient Prog.	Set of CCR Efficient Programs
Unconstrained	70	*
1% Relaxation	0	{}
25% Relaxation	0	{}
50% Relaxation	3	{1, 5, 9}
75% Relaxation	9	{1, 2, 3, 4, 5, 8, 9, 12, 14a**}
100% Relaxation	14	{1, 2, 3, 4, 5, 7, 8, 9, 11, 12, 13, 14a**, 18, 21}

*Set of efficient programs is large and not presented in the table.
**The first of the programs ranked in the 14^{th}, which is IMD program.

For the given data set the number of efficient programs is very high if no AR is defined. We observe that incorporating AR constraints drop the number of CCR efficient programs radically down. As we introduce the AR constraints by relaxing around the FT weights certain programs that perform very high in low weighted criteria is no more classified as efficient.

6.2.6 DEA Super-efficiency Ranking (SE)

Unlike DEA CCR model, SE approach produces a full ranking. First the programs are ranked without AR constraints. Then lower bounds and upper bounds are set as fixed relaxation percentage of FT weights. The rankings are compared with FT ranking and the summary of the comparison is given in Table 15.

Table 15. DEA Super-efficiency Ranking Compared to FT Ranking

	Kendall's Tau-b	**ABS**	**Average ABS**	**S**	**Alt. with Rank Difference ≥10**
Unconstrained	0.294	2418	24.18	310.10	71
1% Relaxation	0.852	516	5.16	83.07	13
25% Relaxation	0.852	516	5.16	83.07	13
50% Relaxation	0.857	502	5.02	80.05	13
75% Relaxation	0.815	666	6.66	91.69	18
100% Relaxation	0.761	866	8.66	116.14	28

For the constrained case for small relaxations (1%-25% interval), the solution is not affected by percentage of relaxation as there is no program having an efficiency value greater than one and all the scores increase proportionally as the constraints are relaxed. Finally higher relaxation results in rank changes for significant number of programs as programs with marginal characteristics are able to rank better by SE.

Also it shall be noted that the ranking varies significantly for the SE method. For the case with no AR constraints, Hong Kong UST business school ranks the 1st, that ranks in 30-40 range when constraints are introduced. That is because this school has very promising scores in various criteria; "women students", "international faculty", "international board", "research rank", "placement success", and "women board" which are weighted heavily in SE ranking of the program. Various other examples of this pattern (ranking high for the unconstrained case) exist (such as Ashridge, George Washington University, Birmingham Business School), and the reverse of this pattern (such as Stanford University GSB, Dartmouth College and University Oxford) also exist. Finally we could conclude that the ranking of SE method is very variable if no AR constraints are provided. SE rank depends on the AR heavily and ranking changes abruptly as the constraints become tighter. If more than one program has similar performance, they will not be classified as marginal and ranked high by SE.

6.2.7 DEA Cross-efficiency Ranking (SXE)

DEA SXE is used for ranking the programs using same AR constraints given in the SE ranking. The rankings are compared with FT and results are provided in Table 16.

Table 16. DEA Cross-efficiency Ranking Compared to FT Ranking

	Kendall's Tau-b	**ABS**	**Average ABS**	**S**	**Alt. with Rank Difference ≥10**
Unconstrained	0.580	1408	14.08	181.9	60
1% Relaxation	0.852	468	4.68	77.54	13
25% Relaxation	0.854	470	4.70	77.74	13
50% Relaxation	0.882	424	4.24	71.71	10
75% Relaxation	0.876	434	4.34	74.97	11
100% Relaxation	0.847	554	5.54	86.41	18

SXE method produces results closer to FT in all the measures compared SE approach. However this method ranks some inefficient programs better than efficient programs. This weakness will be discussed in section 6.3.

Ceibs which ranks high among financial criteria (weighted salary, salary percentage increase and value for money), is not ranked high by SXE method. Another example is Yale Business School which ranks 48th without any constraints but ranks the 11th when AR constraints are introduced. SXE approach favors schools which are good at in diversity criteria (9th to 16th explained in Appendix C.2) whereas the schools that are better in financial and career related criteria (1st to 8th in Appendix C.2) are not favored.

6.2.8 Ranking by Proposed Method 1 (M1)

For the proposed method 1, DM has to provide preference function type, parameters and bounds on weights. Type 5 preference function is used for all the criteria, the preference threshold *(p)* is set to the difference of performance values of best performing program and worst performing program and indifference threshold *(q)* is set to 10% of that value. The AR constraints are incorporated by relaxing FT weights and rankings are compared with FT and results are provided in Table 17.

Table 17. Ranking by Proposed Method 1 Compared to FT Ranking

	Kendall's Tau-b	**ABS**	**Average ABS**	**S**	**Alt. with Rank Difference ≥10**
Unconstrained	0.459	1864	18.64	235.32	67
1% Relaxation	0.817	632	6.32	92.52	22
25% Relaxation	0.831	602	6.02	91.72	22
50% Relaxation	0.827	618	6.18	95.35	21
75% Relaxation	0.821	676	6.76	93.81	23
100% Relaxation	0.806	724	7.24	99.26	27

Some of the criteria both SE and SXE ignored are taken into account in proposed method 1. Generally MBA programs have low performance in language scores because only a few of them have language education. MBA programs of schools such as Insead, University of Michigan: Ross, Esade Bussiness School, ECSP rank higher in overall score. This shows proposed method 1 is better at discriminating programs that only a minor number of programs are better than the average but perform similar to each other. SE does not provide very promising scores for such schools as they are not radically different. SXE on the other hand undervalues language criterion as most of the programs underperformed in this criterion. Other criteria that are undervalued by SXE are "women board" and "international board". SE may also fail to discriminate the few good performing programs in these criteria if a few programs perform equally well. Proposed method 1 will provide a better ranking for these as the difference causes a high net flow and high net flow will cause a higher weight for the corresponding criterion.

To analyze sensitivity of M1 to preference function parameters and examine the effects of preference functions and compared the change of preference function type on the ranking produced by proposed method 1. For different types of preference functions the rankings are compared with each other for AR 25% around FT weights.

Table 18. Average ABS for Difference for Different Preference Functions

	Pref. Func.1	**Pref. Func. 2**	**Pref. Func. 3**	**Pref. Func. 5**
Pref. Func. 1	-	2.94	5.26	7.2
Pref. Func. 2	2.94	-	4.52	6.36
Pref. Func. 3	5.26	4.52	-	2.34
Pref. Func. 5	7.2	6.36	2.34	-

The preference threshold (p) and indifference threshold (q) is set as explained in the beginning of this subsection. For preference function 1 no preference parameter is needed, for the preference function 2 only indifference threshold is used, for the preference function 3 only preference threshold is used and for the preference function 5 both of the parameters are used. Average ABS is given for different pairwise comparison of ranks in Table 18. For this case more rank difference exists between for functions 1 and 5 as expected. As we use very high value for preference threshold the difference of rankings using preference function 1 or 2 is considerably different than preference function 3 and 5.

Table 19. Kendall's Tau-b Correlation for Different Preference Functions

	Pref. Func. 1	Pref. Func. 2	Pref. Func. 3	Pref. Func. 5
Pref. Func. 1	1	0.926	0.858	0.808
Pref. Func. 2	0.926	1	0.877	0.828
Pref. Func. 3	0.858	0.877	1	0.938
Pref. Func. 5	0.808	0.828	0.938	1

The correlations of ranks for different preference functions are also substantially high (See Table 19) and all ranks are correlated in 0.01 confidence level (See Table 19). We can conclude that average ABS is low and rank correlation is high for different preference functions.

6.2.9 Ranking by Proposed Method 2 (M2)

For proposed method 2 we introduce two test cases, weights are unconstrained and weights are constrained by 1% around FT weights. For both of the cases, feasible region of thresholds are formed by relaxing the preference threshold 5% and relaxing the indifference threshold 50% around the values used for proposed method 1. The comparisons with method 1 rankings are given in Table 20.

Table 20. Ranking by Proposed Method 2 Compared to Proposed Method 1

	Kendall's Tau-b	ABS	Average ABS	S	Alt. with Rank Difference ≥10
Unconstrained	0.974	110	1.10	16	-
1% Relaxation	0.990	46	0.46	6.78	-

We observe from the table that the ranking by method 2 is very similar to method 1. For the unconstrained weight case, introducing uncertainty for thresholds effects ranking more compared the case where weights are tightly constrained because in the second case weights are so tightly bounded so that rank change is limited. In this study we analyze the proposed method 2 to measure the sensitivity of result to uncertainty in thresholds in a limited way. Detailed analysis is left out for further study.

6.3. Comparison of Rankings

In this section we aim to compare different ranking methods. In the first subsection we examine the correlations of rankings using Kendall's Tau-b statistics, in the second we compare rankings of all programs in general; in the third subsection we examine ranking of top and bottom programs, and in the last subsection we examine some programs whose rankings change much.

6.3.1 Correlations of Rankings

In the first case, we compare FT, SMART and PROMETHEE rankings with other methods where weights are unconstrained. Kendall's Tau-b statistics are listed in Appendix F.

We observe that SE, SXE and M1 methods correlate with FT ranking. The SXE method correlates more with FT ranking then other approaches (See Figure 16). Knowing that SXE ranking is based on average weights of individual DEA calculations, an agreed weight set exist that makes it more similar to a linear aggregation method. The peak observed for correlation of SXE 50% AR case with FT (See Figure 16) stems from the fact that FT rank and SXE does not use the same performance values and normally the SXE correlation will increase by restricting the weights more. We can observe this fact by examining the correlations of SXE and SMART for different AR constraints shown in the Appendix E.

For very tight AR constraints (25 % for the case study) SXE and SE converges to a similar ranking and correlation of SE with FT is equal to the correlation SXE with FT. Proposed method 1 also depends on cross-evaluation principle like SXE but outranking information is used rather than normalized scores so lower correlation can be justified.

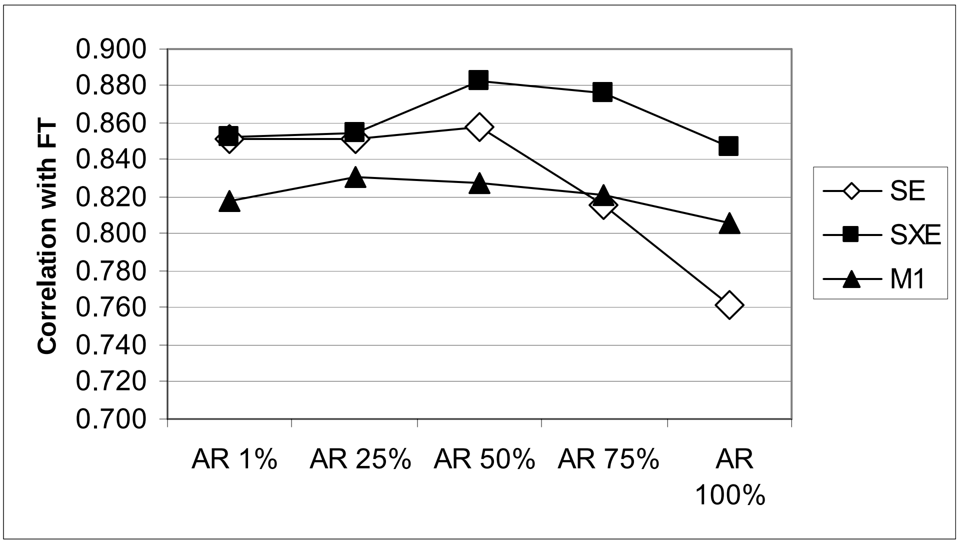


Figure 16. Correlation of rankings with FT

Proposed method produces a ranking more different than FT ranking compared to SE and SXE when AR constraints are incorporated. Next we examine the correlation of methods (SE, SXE, and M1) with PROM ranking. We observe that SXE has higher correlation with FT and PROM when constraints are not tight. As the constraints get tighter the correlation of proposed method with PROM approach becomes the highest of three rankings. Main reason for this fact is that small differences are eliminated preliminary by using preference functions and flow calculations are similar to PROM. Secondly, our objective function also has term for other units' appraisal and even for small weight set each program focuses on this relative measure.

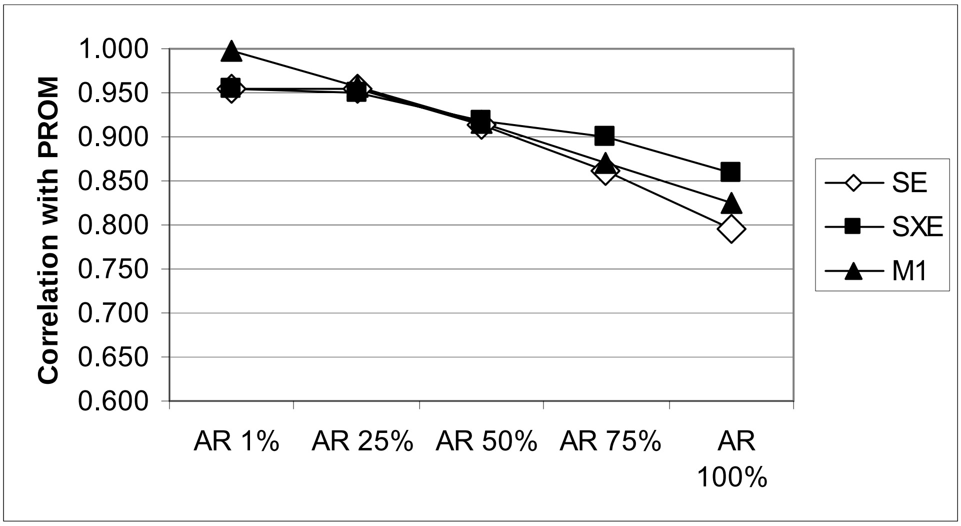


Figure 17. Correlation of Methods with PROM

In general the Kendall's Tau-b measures are high and all the rankings are found to be significant according to this measure. This does not mean that the differences in rankings are insignificant as we observed impreciseness of rankings in the mixture design experiments in which AR constraints are introduced. In the next subsection we will examine these differences.

6.3.2 General Ranking Differences

In this subsection we aim to compare the ranking of SE, SXE and M1 methods generally. For this purpose we illustrate the rankings of MBA programs by graphs where the horizontal axis is programs' index and vertical axis is the rank place this program is ranked by a particular approach. If FT ranks are plotted in such a graph the result is nearly the diagonal as program index is taken from FT list (See Figure 18).

First we compare the rankings of SE, SXE, and M1 when no AR constraints are defined in Figure 18 and 19. For SE the ranking of alternatives are much more dispersed compared to SXE and M1. In SXE and M1 for top ranked alternatives are similar to those of FT ranking and particularly SXE produces more similar ranks to

FT. As SE ranks alternatives based on their marginality, the programs are free to set DEA weights favoring their marginality when no AR is defined. As different programs are marginal for different set of criteria, programs that are ranked in the bottom by FT can be ranked in the top and vice versa.

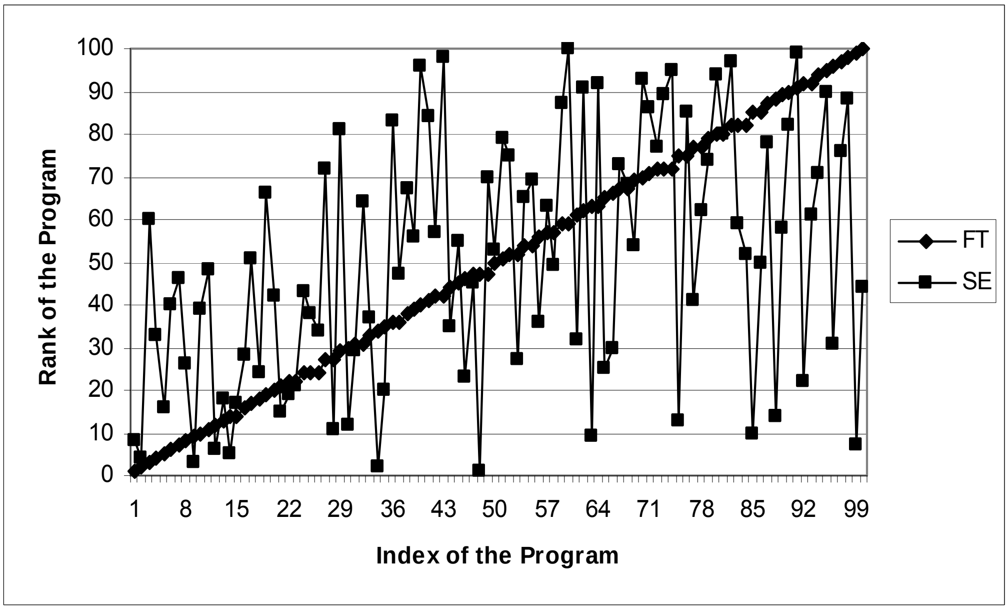


Figure 18. FT and SE Rankings (No AR Constraints)

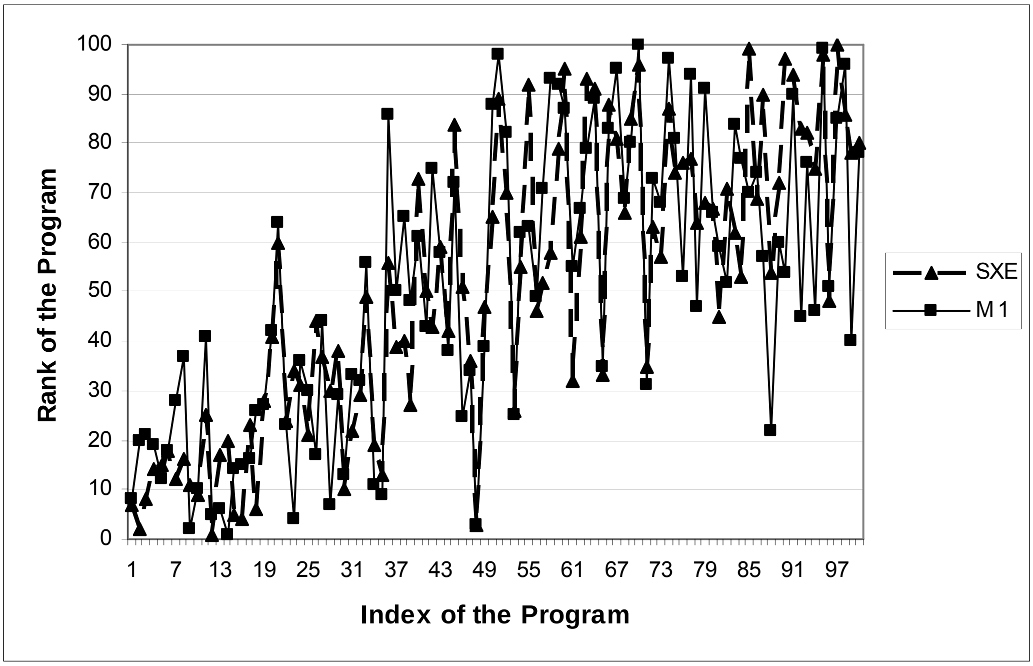


Figure 19. SXE and M1 Rankings (No AR constraints)

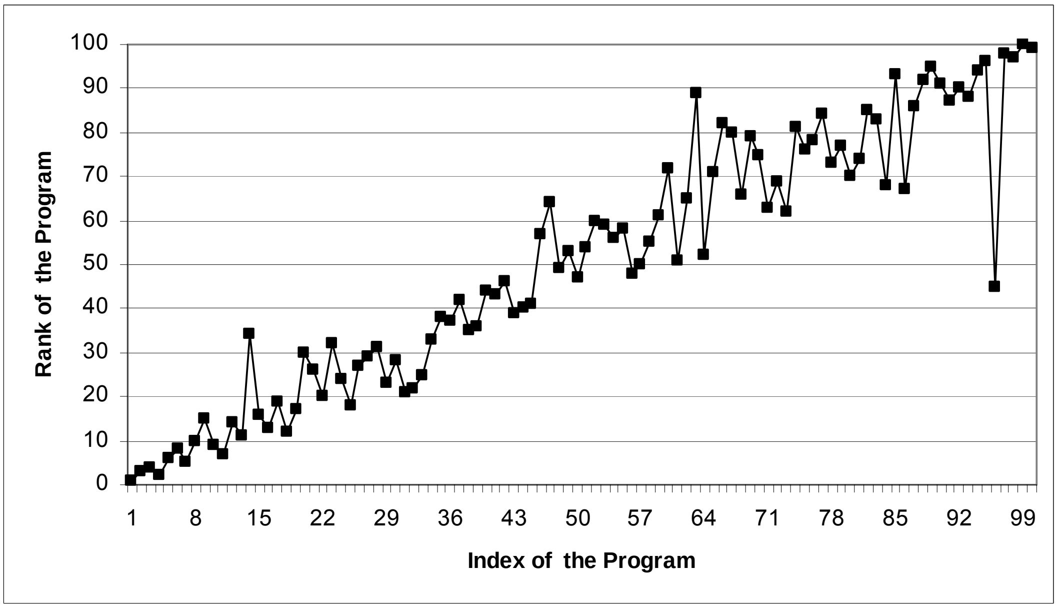


Figure 20. SXE (AR 100%) Ranking

When AR constraints around 100 % weights of FT are defined in the case SXE (See Figure 20), the ranking difference with FT decreases considerably and the difference is observed more on the programs approximately after the 45^{th} in FT ranking. SXE method favors programs that are better in criteria where most of the alternatives have high scores. The significant weight difference between the FT weights and implied weights of SXE method if no AR constraints are defined causes rank difference. For 100% relaxed case some criteria are partially free but the weights of no AR case cannot be obtained. When AR is defined the implied weights obtain values more similar to FT weights.

For M1 imposing AR constraints have a similar effect to that observed in SXE as shown in Figure 21. While the programs ranked by FT in the upper and lower portions are very similar, some alternatives are ranked differently in the middle portion. The difference of ranking of M1 is considerably more significant than SXE even if AR constraints are defined.

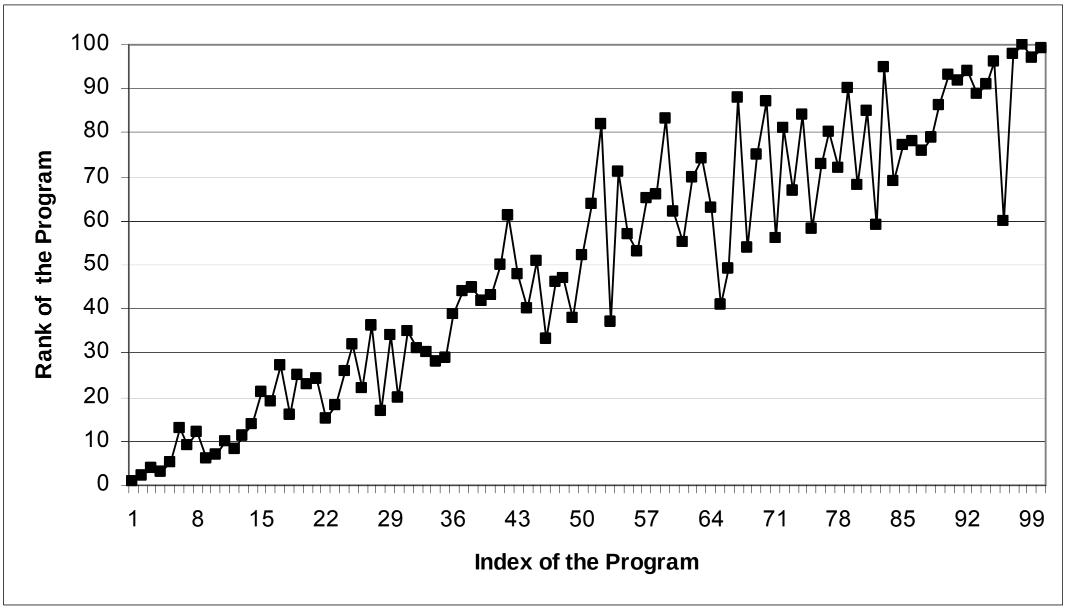


Figure 21. M1 (AR 100%) Ranking

Further tightening the constraint results in more similar rankings for lower and upper extreme programs for M1 as shown in Figure 22**.**

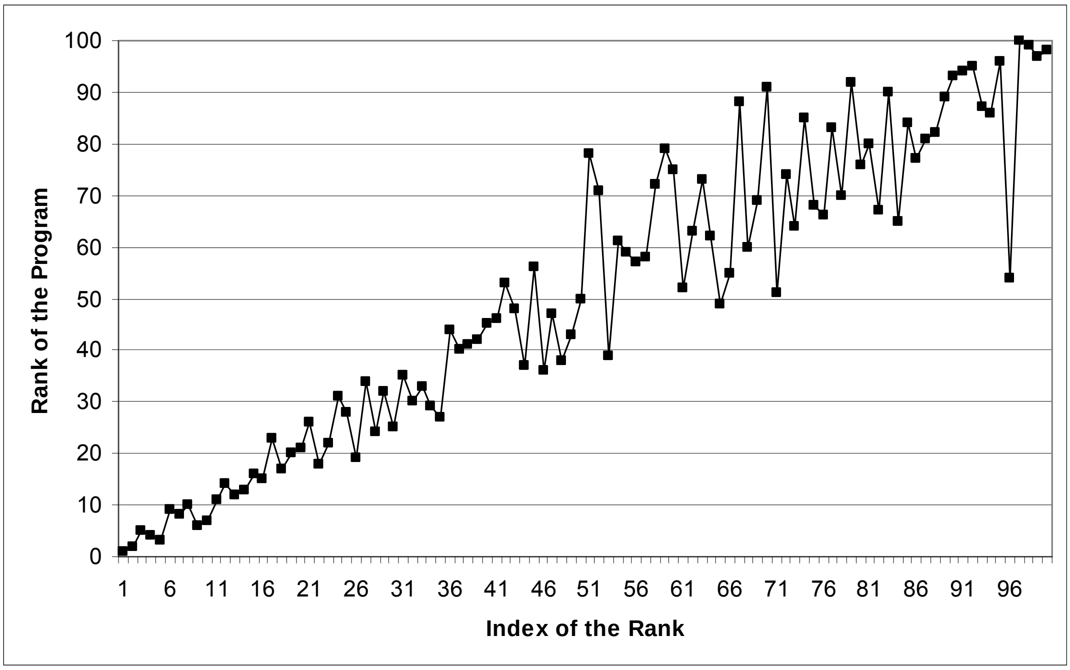


Figure 22. M1 (AR 50%) Ranking

6.3.3 Ranking of Extreme Programs

In this subsection we examine the ranking of top and bottom portion of FT ranking and ranking of DEA efficient alternatives.

For different weight relaxations, we provide a measure which is the number of inefficient programs that rank higher than at least one efficient program. While SE ranks efficient units (given in Table 14) always higher than inefficient units SXE and M1 do not. We provide the number of inefficient units ranked better than at least one efficient program for SXE and M1 for different AR constraints in Table 21.

Table 21. Number of Inefficient Programs Ranking Better than Efficient Programs for SXE and M1

AR	SXE	M1
1%	0	0
25%	0	0
50%	8	3
75%	15	5
100%	20	10

For the 50 % relaxation 8 inefficient units rank better than efficient units in case of SXE and for M1 this number is 3 much less than that. For 75% and 100% relaxations, number of inefficient units ranked better than efficient are 15 and 20 respectively for SXE, while for M1 it is much less, 5 and 10 respectively. So for these assurance regions, M1 is better at ranking efficient units higher than inefficient units. In a problem where number of efficient units is limited for the original CCR model when no AR constraints exist, M1 method may be better at ranking of efficient units higher than inefficient ones.

We now can examine the rankings of different methods for top and bottom ranking alternatives in FT. We select the first 15 programs and last 16 (there are two 85th ranked program in FT list and no 86th) programs. If there are no AR constraints almost any program can achieve the best rank or the worst by different weight combinations. We observe that methods using fixed weights (such as FT, SMART and PROMETHEE) ends up with similar rankings for top 15 programs (See Table 22). However we could observe that DEA based methods ranks these top 15 problems differently from the case where exact weights are available. For the case of SE, only 5 programs are still ranked in top 15 and 5 programs are ranked marginally different (i.e., new ranks range between 39th and 60th). And even the first program is no longer in top 15. For SE rankings of top 15 changes drastically. For example alternative ranked the first (48th in FT ranking) bases its SE score on criteria, “women students”, “international faculty”, “international board”, “research rank", "placement success", and "women board". On the other hand the 2nd program's score is based on criteria "women board", "languages", "women faculty" and "research rank". SE scores of programs are based on only self evaluations which depend on the distance to the efficient frontier and differ from scores of additive approaches. In the case of SXE, top 15 programs in FT ranking also ranked higher. Only one program is ranked lower than 20th and it is ranked 25th. M1 ranking of top 15 programs is more different from SXE but compared to SE more similar to FT. Half of the alternatives in top 15 of FT ranking are still ranked in top 15, and only one alternative is ranked higher than 40th.

Table 22. Ranks of Top 15 and Bottom 16 Alternatives (No AR Const.)

FT	SMART	PROM	SE	SXE	M1
1	1	1	8	7	8
2	3	3	4	2	20
3	5	5	60	8	21
4	4	6	33	14	19
5	2	2	16	15	12
6	8	8	40	18	18
7	9	9	46	12	28
8	10	10	26	16	37
9	6	4	3	11	2
10	7	7	39	9	10
11	11	11	48	25	41
12	16	17	6	1	5
13	13	14	18	17	6
14a	15	12	5	20	1
14b	14	15	17	5	14

FT	SMART	PROM	SE	SXE	M1
85a	84	87	10	99	70
85b	76	72	50	69	74
87	86	86	78	90	57
88	90	83	14	54	22
89	92	90	58	72	60
90	93	93	82	97	54
91	96	94	99	94	90
92a	94	97	22	83	45
92b	89	84	61	82	76
94	85	82	71	75	46
95	99	96	90	98	99
96	52	53	31	48	51
97	100	100	76	100	85
98	97	98	88	86	96
99	95	99	7	78	40
100	98	95	44	80	78

Secondly we inspect ranks of top 15 alternatives ranks when AR constraints are appended to SE, SXE and M1 (See Table 23). Looking at the mixture design results, we observe that only three of the program ranks (1st, 4th and 5th in the FT ranking) remain in top 15 for different weight combinations. But SE, SXE and M1 rank at least 13 of these programs (i.e. top 15 of FT) in top 15. The rankings of these programs however depend on the method and the programs that are ranked 2^{nd}-10^{th} in FT list change places. Introducing AR for SE makes the rank more similar to fixed weight approaches such as FT, SMART and PROMETHEE. SXE ranks some of the programs that are marginal much worse than SE. M1 produces an intermediate result, it does not assign very high ranks for programs that are marginally different in a set of criteria as SE, but does not rank them as low as SXE. In order to be ranked high by M1, programs should have clear difference from the rest of the alternatives in most of the criteria and should not have very low performance in criteria where some other programs perform well.

Table 23. Ranks of Upper 15 Programs (With AR Constraints)

					SE					SXE					M1				
FT	**MIX-MIN***	**MIX-MAX****	**SMART**	**PROM**	**1%**	**25%**	**50%**	**75%**	**100%**	**1%**	**25%**	**50%**	**75%**	**100%**	**1%**	**25%**	**50%**	**75%**	**100%**
1	1	11	1	1	1	1	1	2	4	1	1	1	1	1	1	1	1	1	1
2	1	20	3	3	3	3	4	4	6	2	2	3	3	3	3	3	2	2	2
3	2	25	5	5	5	5	6	9	12	4	4	4	4	4	5	4	5	5	4
4	1	15	4	6	4	4	5	7	8	3	3	2	2	2	6	5	4	3	3
5	1	14	2	2	2	2	3	3	2	5	5	5	5	6	2	2	3	4	5
6	3	27	8	8	8	8	9	13	18	8	8	8	7	8	8	8	9	9	13
7	3	25	9	9	9	9	8	11	13	9	9	6	6	5	9	9	8	8	9
8	1	48	10	10	10	10	7	8	7	10	10	9	9	10	10	10	10	10	12
9	1	24	6	4	6	6	2	1	1	6	6	11	11	15	4	6	6	6	6
10	4	20	7	7	7	7	13	16	20	7	7	7	8	9	7	7	7	7	7
11	4	37	11	11	11	11	11	12	14	11	11	10	10	7	11	11	11	12	10
12	1	47	16	17	16	16	10	6	3	18	17	16	15	14	16	15	14	11	8
13	1	32	13	14	13	13	14	10	9	14	14	14	14	11	14	12	12	13	11
14a	1	68	15	12	15	15	12	5	5	17	18	19	24	34	12	13	13	14	14
14b	8	38	14	15	14	14	17	21	26	13	13	13	13	16	15	16	16	18	21

*Minimum rank obtained from different mixtures when AR constraints around 100% FT weights are incorporated.

**Maximum rank obtained from different mixtures when AR constraints around 100% FT weights are incorporated.

For the case of lower 16 programs of FT by examining Table 24, we observe that at least 2 and at most 5 of the programs are no more ranked in lower 16 by SE, SXE and M1 and the change of ranking is higher compared to top 15 programs in AR case. The rank changes of the programs which are in the 85th-88th places and the 96th place in FT ranking are observable.

Table 24. Ranks of Lower 16 Programs (with AR Constraints)

					SE					SXE					M1				
FT	**MIX-MIN***	**MIX-MAX****	**SMART**	**PROM**	**1%**	**25%**	**50%**	**75%**	**100%**	**1%**	**25%**	**50%**	**75%**	**100%**	**1%**	**25%**	**50%**	**75%**	**100%**
85a	22	100	84	87	84	84	80	72	63	87	87	88	88	93	87	85	84	81	77
85b	44	95	76	72	76	76	78	79	76	72	72	71	68	67	72	75	77	76	78
87	51	94	86	86	86	86	91	92	95	88	88	87	87	86	86	82	81	79	76
88	41	97	90	83	90	90	85	81	75	91	91	92	90	92	83	81	82	82	79
89	43	98	92	90	92	92	92	93	94	89	89	89	93	95	90	90	89	86	86
90	34	100	93	93	93	93	93	80	64	94	95	95	95	91	93	93	93	93	93
91	71	100	96	94	96	96	96	97	96	95	94	93	91	87	94	94	94	94	92
92a	57	100	94	97	94	94	95	94	89	93	93	94	92	90	97	95	95	95	94
92b	38	98	89	84	89	89	90	84	82	92	92	91	89	88	84	87	87	88	89
94	37	97	85	82	85	85	87	85	85	90	90	90	94	94	82	84	86	90	91
95	75	100	99	96	99	99	97	96	98	96	96	96	96	96	96	96	96	96	96
96	23	91	52	53	52	52	50	50	44	48	48	46	46	45	53	54	54	57	60
97	54	100	100	100	100	100	100	100	97	98	98	98	98	98	100	100	100	100	98
98	72	100	97	98	97	97	98	98	100	97	97	97	97	97	98	99	99	99	100
99	36	100	95	99	95	95	94	95	92	99	100	100	100	100	99	98	97	97	97
100	70	100	98	95	98	98	99	99	99	100	99	99	99	99	95	97	98	98	99

* Minimum rank obtained among different mixtures when AR constraints around 100% FT weights are incorporated.

** Maximum rank obtained among different mixtures when AR constraints around 100% FT weights are incorporated.

6.3.4 Illustrative Examples

In this section we provide some rankings that illustrate the specific properties of M1. We present some programs that are ranked differently by M1 compared to SE and SXE rankings and explain the causes of differences in ranks. We select some of the programs that are ranked differently by M1 which are shown in Figure 23. Programs that are better ranked are marked with empty rectangle and that are ranked worse are marked with empty triangle. Although for 50% AR restricted case the changes in the rankings of the programs are between 10 and 30, these changes are considerably higher for the unconstrained case. The scores of programs that are ranked better by M1 for both the unconstrained case and AR case are presented in Table 25 and programs that are ranked worse are presented in Table 26. In the table the bold entries stand for the criterion that a program has score that is in the first quartile and gray background stands for programs that have scores in the lowest quartile. This is based on descriptive statistics given in Appendix C.5.

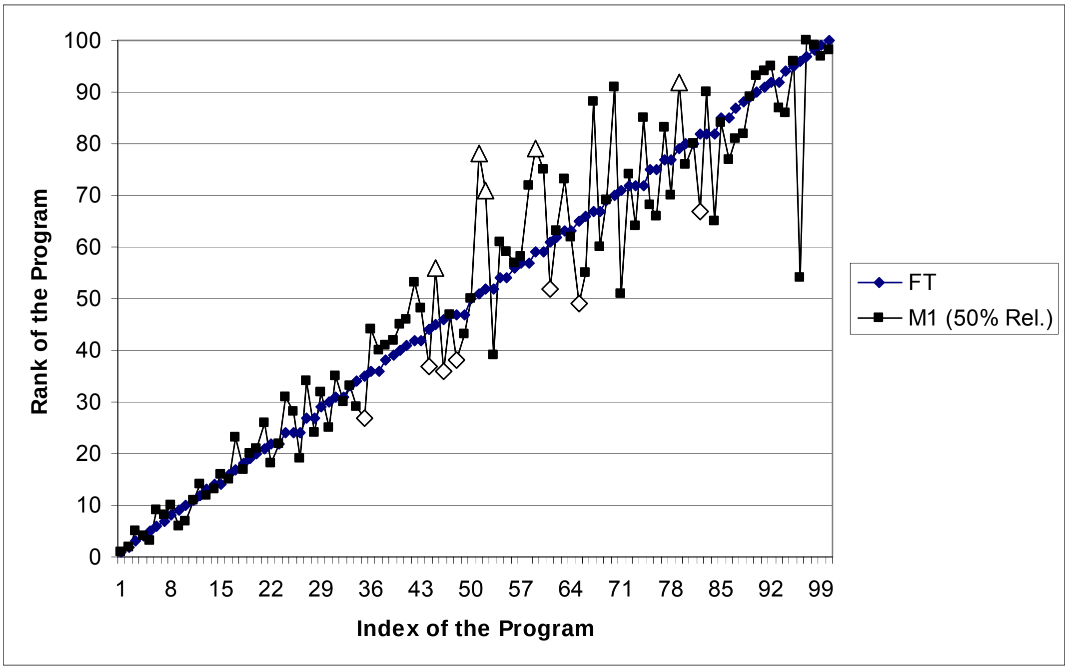


Figure 23. Some of the Alternatives Ranked Higher By M1

We can observe that the programs ranked high by M1 have very promising scores in a high number of the criteria while not performing very poor in most of the others. The programs with high score in criteria "career progress rank", "international faculty", "international students", "international board", "international mobility", “international experience rank”, “languages” and “FT doctoral rank” is more emphasized by M1. In order to better rank by M1 programs shall have high performances in these criteria and shall have not very low performance in others.

Table 25. Alternatives Ranked Better by M1

Criterion	University of Cambridge: Judge	McGill University	Cranfield School of Management	Hong Kong UST Business School	University of Washington Business School	University of Bradford/ Universiteit Nimbas	Edinburgh University Management School
Weighted Salary	0.5453	0.3701	**0.6519**	0.2078	0.3555	0.3434	0.3434
Salary Percentage Increase	0.3644	0.5763	0.4576	0.3644	0.5339	0.4831	0.4407
Value for Money Rank	**0.8200**	**0.9700**	**0.8500**	0.5300	0.3400	**0.9500**	**0.8100**
Aims Achieved	**0.8571**	**0.7857**	**0.7857**	0.6071	0.6429	0.6429	0.6071
Alumni Recommended Rank	0.5600	0.7200	**0.7600**	0.1800	0.2600	0.1100	0.2300
Career Progress Rank	0.7000	0.5300	**0.9100**	0.5700	0.5000	0.5500	**0.7900**
Placement Success Rank	0.4900	0.2500	0.3400	0.7400	0.5400	0.2200	0.1500
Employed At Three Months	0.7368	0.5614	**0.8070**	**0.9649**	**0.9298**	0.8772	**0.7895**
Women Faculty	0.5000	**0.5789**	0.4737	0.2368	**0.5789**	**0.8684**	0.4737
Women Students	0.2895	0.5000	0.1053	**1.0000**	**0.8158**	0.4211	0.0263
Women Board	0.5000	0.3167	**0.7000**	**0.6500**	0.1833	0.3333	**0.4167**
International Faculty	0.6020	**0.4592**	0.2143	**0.8980**	0.0816	**0.4490**	0.3878
International Students	**0.9326**	0.5056	0.6517	**0.8989**	0.4494	**0.9101**	**0.7528**
International Board	**0.4787**	0.2979	0.2660	**1.0000**	0.0319	0.3511	0.3511
International Mobility	**0.8000**	**0.9100**	0.6600	0.6700	0.4900	**0.8300**	0.6900
International Experience Rank	**0.8700**	0.3800	0.2500	**0.9400**	0.4100	0.5100	0.5800
Languages	0.0000	0.0000	**0.5000**	**0.5000**	0.0000	0.0000	**0.5000**
Faculty with Doctorates	**0.9615**	0.7308	0.6154	**1.0000**	0.8846	0.6538	0.8077
FT Doctoral Rank	**0.9200**	0.4100	0.6700	0.6000	**0.7700**	0.7300	0.3900
FT Research Rank	0.4900	0.6100	0.1300	0.6500	0.6900	0.0900	0.1700
FT Rank	35	44	46	47b	61	65	82a
M1 Rank (AR %50)	27	37	36	38	52	49	67
Best Quartile*	8	5	8	9	4	5	6
Worst Quartile**	3	4	4	4	3	4	5

* Number of criterion for which program is in best quartile
** Number of criterion for which program is in worst quartile

Table 26. Alternatives Ranked Worse by M1

Criterion	Brigham Young University: Marriott	College of William and Mary	Washington University: Olin	University of Notre Dame: Mendoza	University of California: Davis
Weighted Salary	0.3595	0.4008	0.4347	0.4630	0.4168
Salary Percentage Increase	**0.9746**	**0.8814**	0.5847	0.6950	0.5763
Value for Money Rank	**0.8300**	0.7000	0.1300	0.2400	0.4000
Aims Achieved	**0.8571**	0.6786	0.6786	**0.7860**	0.7143
Alumni Recommended Rank	0.4600	0.1700	0.4800	0.4500	0.2400
Career Progress Rank	0.4600	0.1400	0.2100	0.1600	0.4500
Placement Success Rank	**0.7900**	0.2900	0.6600	0.5800	0.5300
Employed At Three Months	0.8596	0.6842	0.8596	**0.8950**	0.7544
Women Faculty	0.0263	0.3158	0.2368	0.4470	**0.5000**
Women Students	0.0000	**0.5526**	0.4474	0.2370	**0.5526**
Women Board	0.1500	0.1833	0.2167	0.2170	0.2333
International Faculty	0.0000	0.1224	**0.4694**	0.0920	0.3776
International Students	0.0000	0.2584	0.2921	0.1910	0.0899
International Board	0.1064	0.0000	0.0000	0.0320	0.0000
International Mobility	0.1900	0.1200	0.2000	0.1400	0.0800
International Experience Rank	0.0800	0.2600	0.3400	0.6300	0.3200
Languages	0.0000	0.0000	0.0000	0	0.0000
Faculty with Doctorates	0.8974	**0.9744**	**1.0000**	0.8970	**1.0000**
FT Doctoral Rank	0.0100	0.0100	0.2900	0.0100	0.0100
FT Research Rank	0.4300	0.3000	**0.8700**	0.6900	0.4900
FT Rank	45	51	52a	59a	79
M1 Rank (AR 50%)	56	78	71	79	92
Best Quartile*	4	3	3	2	2
Worst Quartile**	11	8	6	8	5

* Number of criterion for which program is in best quartile
** Number of criterion for which program is in worst quartile

The programs ranked by FT in the middle (44th, 46th, 47th, 61st and 65th as shown in the Table 25) are ranked upper by M1. Programs which are also ranked in the middle by FT (45th, 51st, 52nd and 59th as shown Table 26) are ranked lower by M1. the effect of the change of the ranking . Because the first set of programs have considerably better performance than others compared to second set. This contributes to their net flow and they rank better than the second set.

We analyze the rankings of "Stanford University: GSB", "Purdue University: Krannert" and "Washington University: Olin" to see the effects of different ranking approaches.

Rankings of "Stanford University: GSB" is listed in Table 27. "Stanford University: GSB" which is a high ranking program considering the FT Rank also having high average scores for most of the criteria. We observe an equally good performance with the SMART ranking. But it can not assume a good ranking position in SE unconstrained case. In the financial (first to third in Appendix C.2) and career related (fourth to eight in Appendix C.2) criteria, it performs well but is not marginally different from other high ranking alternatives. Both SXE and M1 produces a better ranking for Stanford University GSB as it has high score in a quite large number of criteria. Still the ranking is worse for no AR case as criteria including financial and career related ones are more uniformly weighted in these approaches.

Table 27. Ranks of Stanford University GSB

	Rank		
Ranking Method	**Constant**	**No AR**	**AR 100%**
FT Rank	3	N/A	N/A
Normalized Rank	5	N/A	N/A
PROMETHEE Rank	5	N/A	N/A
SE Rank	N/A	60	12
SXE Rank	N/A	8	4
Method 1 Rank	N/A	21	4

In the second case we examine the rankings of "Purdue University: Krannert" (See Table 28). M1 produces a better ranking for the program than others in no AR case. The program does not perform very poor in most of the criteria, and performance is outstanding for the criteria such as "placement success rank" and "FT doctoral rank". So we had this school far better ranked by M1 then SXE. This program do not have very low negative net flows for most of the criteria but have sufficiently larger net flow for the others so it is ranked better by M1.

Table 28. Ranks of Purdue University: Krannert

	Rank		
Ranking Method	**Constant**	**No AR**	**AR 100%**
FT Rank	77b	N/A	N/A
SMART Rank	70	N/A	N/A
PROMETHEE Rank	66	N/A	N/A
SE Rank	N/A	62	81
SXE Rank	N/A	64	73
Method 1 Rank	N/A	47	72

"Washington University: Olin" ranks lower then FT, SMART and PROMETHEE in all cases as shown in Table 29. The reason for the lower ranking obtained by SXE and M1 is that it only ranks better than others in criteria "Faculty with Doctorates" and the difference is not notable (the average is already so high). The programs do not have high average performance for the other criteria this program performs well so the evaluation will not be advantageous in the case of SXE. An examination of the score shows that this program ranks in the lowest quartile for 6 of the criteria and in second for most of the others. In the case of SE, it has a low rank as it is not marginally different in any of the criteria from the other programs.

Table 29. Ranks of Washington University: Olin

	Rank		
Ranking Method	**Constant**	**No AR**	**AR 100%**
FT Rank	52a	N/A	N/A
Normalized Rank	56	N/A	N/A
PROMETHEE Rank	58	N/A	N/A
SE Rank	N/A	75	68
SXE Rank	N/A	70	60
Method 1 Rank	N/A	82	82

6.4. Results

Finally we can conclude that each DEA based ranking methodology has different characteristics. When no assurance region constraints are provided these characteristics are more apparent but the introduction of assurance region constraints forces methods to arrive a similar ranking very quickly. Basic DEA CCR model could not rank the programs and even the classification of efficient and inefficient set is very poor in case of large number of criteria. SE tends to rank marginal programs very high but the marginality is defined locally and a few programs having similar scores but very different from large set of programs may not be ranked higher. We saw such a case for language criterion in the case study. SXE on the other hand favors programs that score similar to most programs. In our case most of the schools get similar scores from gender and international diversity criterion so they are heavily weighted while programs that have high performance in financial and career related criteria are ranked lower. The advantage of SXE is that it produces a robust ranking for a fixed set of programs. Proposed method 1 can rank programs higher that are significantly better than most of the programs in a criterion or a set of criteria and also robust compared to SE method. Also note that the selection of indifference and preference values by the DM has an effect on the ranks of the programs. Proposed Method 1 increases its correlation with PROMETHEE as AR constraints are imposed and becomes the most correlated approach. It is also shown that introducing moderate amount of uncertainty in the thresholds does not affect the ranking of proposed method 1 and proposed methods 1 and 2 produce similar results.

CHAPTER 7

CONCLUSION

In this thesis two methods are proposed based on PROMETHEE and outranking methods. In these methods the outranking relations are aggregated using a method similar to DEA cross-efficiency ranking. These methods can be used when:

i. When outranking relations can be built but the weight information is not available so the aggregation of these relations is a problem.
ii. When there is only partial information about criterion weights, absolute bounds or relative bounds on criterion weights can be specified.
iii. When both information about the preference structure and weights are not precise. The bounds on indifference and preference parameters and constraints are used as inputs.

Also the effects of change of parameters of preference functions on net flow are analyzed, which will provide benefit for future studies that examine the robustness of PROMETHEE methods.

In a case study, the proposed methods together with some other ranking approaches are applied to MBA program ranking problem. In this problem the proposed method 1, produced a robust ranking when there is imprecise information on weights. Differences of ranking exist between the proposed method 1 and other DEA based methods because proposed method 1 uses preference information taken from the DM.

As a future work, the approach can be applied for other real-life MCDM problems. In our case study proposed method 2 is compared with proposed method 1 in order to show that impreciseness introduced in thresholds has minor effects in ranking. More detailed analysis of proposed method 2 will provide sensitivity of ranks to thresholds. The preference information is taken to be a proportion of performance range for the case study. If this information can be obtained from the DM the advantages of the method will become more evident.

Also in the case study in the proposed method 2, lower bounds or upper bounds for thresholds are used in calculating alternative scores. A more efficient algorithm for the selection of thresholds is a problem to be addressed. Similarly the Gaussian preference (type 6) function is not studied and used in our analysis and introducing it will provide an approach that has a larger scope.

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APPENDIX A

PROOF OF THEOREM 1

Remark:

Let $a_1 \le b_1,\ a_2 \le b_2,\ a_3 \le b_3,\ \ldots, a_N \le b_N$ for $a_1,\ldots a_N, b_1,\ldots b_N \in R$,

Then for $v_k \ge 0$,

$$\underset{v_k \in V}{Max} \sum_{i=1}^{N} a_i v_k \le \underset{v_k \in V}{Max} \sum_{i=1}^{N} b_i v_k$$

Proof:

Let us define *K(p,q)* which is independent of weights (v_{kj}) and defined for values$(p_{kj}\ ,q_{kj}\ ) \in G_j$

$$K(p_{kj},q_{kj}) = \left[(\Phi^{+}{}_{kj} - \Phi^{-}{}_{kj}) - \frac{\sum_{i=1,i\ne k}^{n} (\Phi^{+}{}_{ij} - \Phi^{-}{}_{ij})}{n-1} \right]$$

$K(p_{kj},q_{kj})$ has an upper bound as the preference function definitions are limited to one and number of alternatives is finite. There is a pair of values, let us call $(p_{kj}^{\ *},q_{kj}^{\ *}) \in G_j$ such that

$$K(p_{kj},q_{kj}) \le K(p_{kj}^{\ *},q_{kj}^{\ *}) \quad \forall k, j$$

Then by using the remark and preceeding result

$$\max_{v_{kj}\in W_j,(q_{kj},p_{kj})\in G_j} \sum_{j=1}^{m} v_{kj}K(p_{kj},q_{kj}) \le \max_{v_{kj}\in W_j} \sum_{j=1}^{m} v_{kj}K(p_{kj}{}^{*},q_{kj}{}^{*})$$

This is equivalent to:

$$\max_{v_{kj}\in W_j,(q_{kj},p_{kj})\in G_j} \sum_{j=1}^{m} v_{kj}\left[(\Phi^{+}{}_{kj}-\Phi^{-}{}_{kj})-\frac{\sum_{i=1,i\neq k}^{n}(\Phi^{+}{}_{ij}-\Phi^{-}{}_{ij})}{n-1}\right] \le \max_{v_{kj}\in W_j} (\sum_{j=1}^{m} v_{kj} \max_{(q_{kj},p_{kj},)\in G_j}(\left[(\Phi^{+}{}_{kj}-\Phi^{-}{}_{kj})-\frac{\sum_{i=1,i\neq k}^{n}(\Phi^{+}{}_{ij}-\Phi^{-}{}_{ij})}{n-1}\right]))$$

Also for any value of (v_{kj},p_{kj},q_{kj}) used in the right hand side just replacing these values will provide the same result in the left hand side, so it is impossible for right hand side to be greater. Then two sides should be equal and proof is completed.

$$\max_{v_{kj}\in W_j,(q_{kj},p_{kj})\in G_j} \sum_{j=1}^{m} v_{kj}\left[(\Phi^{+}{}_{kj}-\Phi^{-}{}_{kj})-\frac{\sum_{i=1,i\neq k}^{n}(\Phi^{+}{}_{ij}-\Phi^{-}{}_{ij})}{n-1}\right] = \max_{v_{kj}\in W_j} (\sum_{j=1}^{m} v_{kj} \max_{(q_{kj},p_{kj},)\in G_j}(\left[(\Phi^{+}{}_{kj}-\Phi^{-}{}_{kj})-\frac{\sum_{i=1,i\neq k}^{n}(\Phi^{+}{}_{ij}-\Phi^{-}{}_{ij})}{n-1}\right]))$$

APPENDIX B

PROOF OF THEOREM 2

To simplify the proof let us take an instance problem for a fixed j and k. Dropping the corresponding indices, k stands for the index of the alternative that calculations are done for and is fixed.

$$\underset{(q\ ,p\ )\in G2}{Max}\left[(\Phi_k^{\ +}-\Phi_k^{\ -})-\frac{\sum_{i=1,i\neq k}^{n}(\Phi^+{}_i-\Phi^-{}_i)}{n-1}\right]$$

Where flows are calculated by using the generic preference function for fixed k and j.

$$\Phi^+{}_k=\sum_{i=1(S_{kj}-S_{ij}\geq q\ ,S_{kj}-S_{ij}\leq p\ ,p\neq q)}^{n}\frac{S_{kj}-S_{ij}-q}{p\ -q}+\sum_{i=1,(S_{kj}-S_{ij}>p\ )}^{n}1+\sum_{i=1,S_{kj}-S_{ij}=p,p=q}^{n}1$$

$$\Phi^-{}_k=\sum_{i=1(S_{ij}-S_{kj}\geq q\ ,S_{ij}-S_{kj}\leq p\ ,p\neq q)}^{n}\frac{S_{ij}-S_{kj}-q}{p\ -q}+\sum_{i=1,(S_{ij}-S_{kj}>p)}^{n}1+\sum_{i=1,(S_{ij}-S_{kj}=p\ ,p=q)}^{n}1$$

$$\Phi^+{}_i=\sum_{l=1(S_{ij}-S_{lj}\geq q\ ,S_{ij}-S_{lj}\leq p\ ,p\neq q)}^{n}\frac{S_{ij}-S_{lj}-q}{p\ -q}+\sum_{l=1,(S_{ij}-S_{lj}>p\ )}^{n}1+\sum_{l=1,(S_{ij}-S_{lj}=p\ ,p=q)}^{n}1$$

$$\Phi^{-}{}_i = \sum_{l=1(S_{lj}-S_i \geq q \;\; ,S_l - S_i \leq p \;\; ,p \neq q)}^{n} \frac{S_{lj} - S_{ij} - q}{p \;\; - q} + \sum_{l=1,(S_l - S_i > p \;\;)}^{n} 1 + \sum_{l=1,(S_i - S_l > p \;\; ,p=q)}^{n} 1$$

There are n*n $\Delta^j{}_{ki}$ value where a is the number of alternatives. Some of these values may be same as different alternatives may have same value in that criterion. Now let us form a set from these $|\Delta^j{}_{ki}|$ values. The set will have less then $n(n-1)/2+1$ members[1]. Then order these values such that:

$\Delta^j{}_{(1)} = \min(\Delta^j)$

$\vdots$

$\Delta^j{}_{(k)} = \min(\Delta^j \text{ - } \{|\Delta_{(1)}|,..,|\Delta_{(k-1)}|\})$

$\vdots$

$\Delta^j{}_{(n')} = \max(\Delta^j)$

where $n' \leq n(n-1)/2 \; + 1$

---

[1] The equivalence may be a result of negative differences. $\Delta^j{}_{ki} = S_k - S_i = -\Delta_{ik} \quad \forall k,i$. So $|\Delta^j{}_{ki}| = |-\Delta^j{}_{ik}| \quad \forall k,i$ resulting $n(n+1)/2$ distinct $|\Delta^j{}_{ki}|$ values at most. Diagonals are equal to zero resulting $n(n-1)/2+1$ values at most ($\Delta^j{}_{kk} = 0 \quad \forall k$). Other then these, two alternatives may have same scores resulting 0 differences as in case b. Two alternatives may have same scores resulting equalities. If alternative k and l have the same scores than for every other alternative ($|\Delta^j{}_{ik}| = |S_{ij} - S_{kj}| = |S_{ij} - S_{lj}| = |\Delta^j{}_{il}| \quad \forall i$ other then *k, l*). Even the alternative scores may result in different scores. There may be alternatives i, k, l, m such that no two of them have same scores but resulting differences will be same ($|\Delta_{ik}| = |S_i - S_k| = |S_l - S_m| = |\Delta_{lm}|$)

We have $n^{'}$ intervals such that $\left[\Delta^{j}{}_{(1)},\Delta^{j}{}_{(2)}\right]$, $\left[\Delta^{j}{}_{(2)},\Delta^{j}{}_{(3)}\right]$, …, $\left[\Delta^{j}{}_{(n'-1)},\Delta^{j}{}_{(n')}\right]$ covering the $\left[0,\Delta^{j}{}_{(n')}\right]$ interval. The q and p values which are positive may be valued between one of these intervals or may be greater then $|\Delta^{j}{}_{(n')}|$.

Then we can rewrite the set G_j as an intersection of sets based on the above formulation as in Table 30. The gray area is discarded as only one point for the entire union is feasible (*p*=*q*) which is also included in the diagonal set in that row where q and p take upper bound values.

Table 30. Decomposition of Feasible Set into Intervals for q and p

(q,p) bounds for subsets	**q in the 1st interval**	**….**	**q in the kth interval**	**….**	**q in the n'th interval**
p in the 1st interval	$(\lfloor\Delta_{(1)},\Delta_{(2)}\rfloor, \lfloor\Delta_{(1)},\Delta_{(2)}\rfloor)$	(…………, $\lfloor\Delta_{(1)},\Delta_{(2)}\rfloor$)	$(\lfloor\Delta_{(k)},\Delta_{(k+1)}\rfloor, \lfloor\Delta_{(1)},\Delta_{(2)}\rfloor)$	(…………, $\lfloor\Delta_{(1)},\Delta_{(2)}\rfloor$)	$(\lfloor\Delta_{(n'-1)},\Delta_{(n')}\rfloor, \lfloor\Delta_{(1)},\Delta_{(2)}\rfloor)$
….	($\lfloor\Delta_{(1)},\Delta_{(2)}\rfloor$, …………)	(…………, …………)	($\lfloor\Delta_{(k)},\Delta_{(k+1)}\rfloor$, ……………)	(…………, …………)	($\lfloor\Delta_{(n'-1)},\Delta_{(n')}\rfloor$, ……………)
p in the kth interval	$(\lfloor\Delta_{(1)},\Delta_{(2)}\rfloor, \lfloor\Delta_{(k)},\Delta_{(k+1)}\rfloor)$	…………, $\lfloor\Delta_{(k)},\Delta_{(k+1)}\rfloor$	$\lfloor\Delta_{(k)},\Delta_{(k+1)}\rfloor$, $\lfloor\Delta_{(k)},\Delta_{(k+1)}\rfloor$	………...., $\lfloor\Delta_{(k)},\Delta_{(k+1)}\rfloor$	$(\lfloor\Delta_{(n'-1)},\Delta_{(n')}\rfloor, \lfloor\Delta_{(k)},\Delta_{(k+1)}\rfloor)$
….	$\lfloor\Delta_{(1)},\Delta_{(2)}\rfloor$, …………	…………, …………	$\lfloor\Delta_{(k)},\Delta_{(k+1)}\rfloor$, ……………	…………, …………	$\lfloor\Delta_{(n'-1)},\Delta_{(n')}\rfloor$ …………..
p in the n'th interval	$(\lfloor\Delta_{(1)},\Delta_{(2)}\rfloor, \lfloor\Delta_{(n'-1)},\Delta_{(n')}\rfloor)$	…………, $\lfloor\Delta_{(n'-1)},\Delta_{(n')}\rfloor$	$\lfloor\Delta_{(k)},\Delta_{(k+1)}\rfloor$, ……………	…………, $\lfloor\Delta_{(n'-1)},\Delta_{(n')}\rfloor$	$(\lfloor\Delta_{(n'-1)},\Delta_{(n')}\rfloor, \lfloor\Delta_{(n'-1)},\Delta_{(n')}\rfloor)$

To maximize the objective, q and p will take values that will be in one of the intervals given in white cells of Table 30.

i) For subspaces that are feasible and *p* is definitely greater then *q* (elements that are not neighbor to diagonal elements).
ii) Neighborhood of diagonal elements *p*=*q* is possible only when *p* takes the lowest and *q* takes the highest value.
iii) For the diagonal intervals where *p*=*q* may be true for the all the set values in the interval.

Case i:

Given $\Delta^j{}_{(i')} \le q \le \Delta^j{}_{(i'+1)}$, $\Delta^j{}_{(k')} \le p \le \Delta^j{}_{(k'+1)}$, then q is valued in the *i*th interval ($q \in \left[\Delta^j{}_{(i')}, \Delta^j{}_{(i'+1)}\right]$) and p in the *k*th interval($p \in \left[\Delta^j{}_{(k')}, \Delta^j{}_{(k'+1)}\right]$) where $i+2 \le k$ and formulate the problem for this specific case. $N(\Delta^j{}_{ki})$ is used for denoting the $\Delta^j{}_{ki}$ values in the corresponding interval as explained.

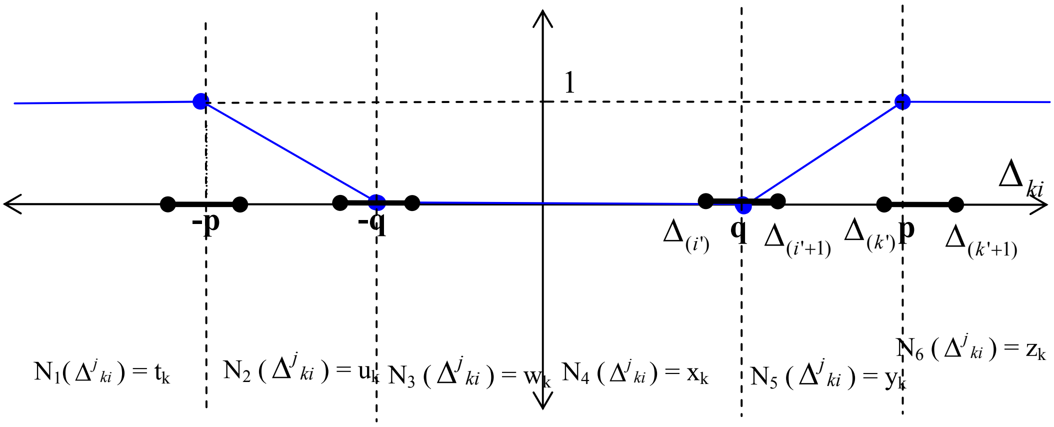

Figure 24. Illustration of the Feasible Subset Region for p Strictly Greater than q

For any alternative k, $\Delta^j{}_{ki}$ will either positive or negative.

If $\Delta^j{}_{ki} \ge 0$, then one of the below conditions hold for $\Delta^j{}_{ki}$;

i) $\Delta^j{}_{ki} > \Delta^j{}_{(k'+1)}$ so it will contribute directly to positive outranking flow. Assume there are z_k such values.

ii) $\Delta^j{}_{(i'+1)} \le \Delta^j{}_{ki} \le \Delta^j{}_{(k')}$ so it will have proportional contribution to positive outranking flow. Assume there are y_k such values.

iii) $\Delta^j{}_{ki} < \Delta^j{}_{(i')}$ so it will not contribute to positive outranking flow. Assume there are x_k such values.

If $\Delta^j{}_{ki} < 0$, then one of the below conditions hold for $\Delta^j{}_{ki}$;

i) $-\Delta^j{}_{ki} > \Delta^j{}_{(k'+1)}$ so it will contribute directly to negative outranking flow. Assume there are t_k such values.

ii) $\Delta^j{}_{(i'+1)} \le -\Delta^j{}_{ki} \le \Delta^j{}_{(k')}$ so it will have proportional contribution to negative outranking flow. Assume there are u_k such values.

iii) $-\Delta^j{}_{ki} < \Delta^j{}_{(i')}$ so it will not contribute to negative outranking flow. Assume there are w_k such values.

For any alternative k, $x_k + y_k + z_k + t_k + u_k + w_k = n$ will hold. Assuming $p \neq q$ and replacing equation we get:

$$\Phi^+{}_k = \sum_{i=1(S_{kj}-S_{ij}\ge q\ ,S_{kj}-S_{ij}\le p\ ,p\neq q)}^{n} \frac{S_{kj}-S_{ij}-q}{p\ -q} + \sum_{i=1(S_{kj}-S_{ij}>p\ )}^{n} 1 = \sum_{i=1(\Delta^j{}_{ki}\in[\Delta^j{}_{(i'+1)},\Delta^j{}_{(k')}])}^{n} \frac{\Delta^j{}_{ki}-q}{p\ -q} + z_k$$

$$\Phi^-{}_k = \sum_{i=1(S_{ij}-S_{kj}\ge q\ ,S_{ij}-S_{kj}\le p\ ,p\neq q)}^{n} \frac{S_{ij}-S_{kj}-q}{p\ -q} + \sum_{i=1,(S_{ij}-S_{kj}>p)}^{n} 1 = \sum_{i=1(\Delta^j{}_{ki}\in[-\Delta^j{}_{(k')},-\Delta^j{}_{(i'+1)}])}^{n} \frac{-\Delta^j{}_{ki}-q}{p\ -q} + t_k$$

$$\Phi^+{}_i = \sum_{l=1(S_{ij}-S_{lj}\ge q\ ,S_{ij}-S_{lj}\le p\ ,p\neq q)}^{n} \frac{S_{ij}-S_{lj}-q}{p\ -q} + \sum_{l=1,(S_{ij}-S_{lj}>p\ )}^{n} 1 = \sum_{l=1(\Delta^j{}_{il}\in[\Delta^j{}_{(i'+1)},\Delta^j{}_{(k')}])}^{n} \frac{\Delta^j{}_{il}-q}{p\ -q} + z_i$$

$$\Phi^-{}_i = \sum_{l=1(S_{lj}-S_{ij}\ge q\ ,S_{lj}-S_{ij}\le p\ ,p\neq q)}^{n} \frac{S_{lj}-S_{ij}-q}{p\ -q} + \sum_{l=1,(S_{lj}-S_{ij}>p\ )}^{n} 1 = \sum_{l=1(\Delta^j{}_{li}\in[-\Delta^j{}_{(k')},-\Delta^j{}_{(i'+1)}])}^{n} \frac{-\Delta^j{}_{il}-q}{p\ -q} + t_i$$

By replacing the result in the equation we arrive at the function:

$$Max\quad z_k + \sum_{i=1(\Delta^j{}_{ki}\in[\Delta^j{}_{(i'+1)},\Delta^j{}_{(k')}])}^{n} \left(\frac{\Delta^j{}_{ki}-q}{p-q}\right) - \sum_{i=1(\Delta^j{}_{ki}\in[-\Delta^j{}_{(k')},-\Delta^j{}_{(i'+1)}])}^{n} \left(\frac{-q-\Delta^j{}_{ki}}{p-q}\right) - t_k$$
$$- \frac{\sum_{i=1,i\neq k}^{n} \left(z_i + \sum_{l=1(\Delta^j{}_{il}\in[\Delta^j{}_{(i'+1)},\Delta^j{}_{(k')}])}^{n} \left(\frac{\Delta^j{}_{il}-q}{p-q}\right) - \sum_{l=1(\Delta^j{}_{il}\in[-\Delta^j{}_{(k')},-\Delta^j{}_{(i'+1)}])}^{n} \left(\frac{-q-\Delta^j{}_{il}}{p-q}\right) - t_i\right)}{n-1}$$

The z and t terms will not effect the function as long as (p, q) pair do not pass the limits causing x, y, z, t, u, w to change.

$$Max \quad z_k + \sum_{i=1(\Delta^j{}_{ki} \in [\Delta^j{}_{(i'+1)}, \Delta^j{}_{(k')}])}^{n} \left(\frac{\Delta^j{}_{ki}}{p-q} - \frac{q}{p-q}\right) - \sum_{i=1(\Delta^j{}_{ki} \in [-\Delta^j{}_{(k')}, -\Delta^j{}_{(i'+1)}])}^{n} \left(-\frac{q}{p-q} - \frac{\Delta^j{}_{ki}}{p-q}\right) - t_k$$

$$- \frac{\sum_{i=1, i\neq k}^{n} \left(z_i + \sum_{l=1(\Delta^j{}_{il} \in [\Delta^j{}_{(i'+1)}, \Delta^j{}_{(k')}])} \left(\frac{\Delta^j{}_{il}}{p-q} - \frac{q}{p-q}\right) - \sum_{l=1(\Delta^j{}_{il} \in [-\Delta^j{}_{(k')}, -\Delta^j{}_{(i'+1)}])} \left(-\frac{q}{p-q} - \frac{\Delta^j{}_{il}}{p-q}\right) - t_i\right)}{n-1}$$

Grouping similar terms we get:

$$Max \quad z_k + \left(\frac{1}{p-q}\left(\sum_{i=1(\Delta^j{}_{ki} \in [\Delta^j{}_{(i'+1)}, \Delta^j{}_{(k')}])}^{n} (\Delta^j{}_{ki}) - \sum_{i=1(\Delta^j{}_{ki} \in [-\Delta^j{}_{(k')}, -\Delta^j{}_{(i'+1)}])}^{n} (\Delta^j{}_{ki})\right)\right) - \left(\frac{q}{p-q}(y_k - u_k)\right) - t_k$$

$$- \frac{\sum_{i=1, i\neq k}^{n} \left(z_i + \left(\frac{1}{p-q}\left(\sum_{l=1(\Delta^j{}_{il} \in [\Delta^j{}_{(i'+1)}, \Delta^j{}_{(k')}])}^{n} (\Delta^j{}_{il}) - \sum_{l=1(\Delta_{il} \in [-\Delta^j{}_{(k')}, -\Delta^j{}_{(i'+1)}])}^{n} (\Delta^j{}_{il})\right)\right) - \left(\frac{q}{p-q}(y_i - u_i)\right) - t_i\right)}{n-1}$$

By rearranging:

$$Max \quad (z_k - t_k) - \frac{\sum_{i=1, i\neq k}^{n} (z_i - t_i)}{n-1}$$

$$+ \frac{1}{p-q}\left(\left(\sum_{i=1(\Delta^j{}_{ki} \in [\Delta^j{}_{(i'+1)}, \Delta^j{}_{(k')}])}^{n} \Delta^j{}_{ki} - \sum_{i=1(\Delta^j{}_{ki} \in [-\Delta^j{}_{(k')}, -\Delta^j{}_{(i'+1)}])}^{n} \Delta^j{}_{ki}\right) - \frac{\sum_{i=1, i\neq k}^{n} \left(\sum_{l=1(\Delta^j{}_{il} \in [\Delta^j{}_{(i'+1)}, \Delta^j{}_{(k')}])}^{n} \Delta^j{}_{il} - \sum_{l=1(\Delta^j{}_{ji} \in [-\Delta^j{}_{(k')}, -\Delta^j{}_{(i'+1)}])}^{n} \Delta^j{}_{il}\right)}{n-1}\right)$$

$$+ \left(\frac{q}{p-q}\right)\left((u_k - y_k) - \frac{\sum_{i=1, i\neq k}^{n} (u_i - y_i)}{n-1}\right)$$

Then let us replace the terms that can be calculated by available information in order to simplify the formulation:

$$K = \left(\left(\sum_{i=1(\Delta^j{}_{ki} \in [\Delta^j{}_{(i'+1)}, \Delta^j{}_{(k')}])}^{n} \Delta^j{}_{ki} - \sum_{i=1(\Delta^j{}_{ki} \in [-\Delta^j{}_{(k')}, -\Delta^j{}_{(i'+1)}])}^{n} \Delta^j{}_{ki}\right) - \frac{\sum_{i=1, j\neq k}^{n} \left(\sum_{l=1(\Delta^j{}_{il} \in [\Delta^j{}_{(i'+1)}, \Delta^j{}_{(k')}])}^{n} \Delta^j{}_{il} - \sum_{l=1(\Delta^j{}_{il} \in [-\Delta^j{}_{(k')}, -\Delta^j{}_{(i'+1)}])}^{n} \Delta^j{}_{il}\right)}{n-1}\right)$$

$$L = \left((u_k - y_k) - \frac{\sum_{i=1, j\neq k}^{n} (u_i - y_i)}{n-1}\right)$$

$A = \Delta^j{}_{(i')}$, $B = \Delta^j{}_{(i'+1)}$, $C = \Delta_{(k')}$ $D = \Delta_{(k'+1)}$

This problem is a fractional programming problem.

$$Max \quad \frac{K}{p-q} + (\frac{qL}{p-q})$$

Subject to

$A \le q$

$q \le B$

$C \le p$

$p \le D$

$p, q \ge 0$

Solution can be found by first transforming it to an LP based on method of Charnes et al.[14]. Noting than *p-q* always greater than 0 as *p>q*. Let $q' = q/(p-q)$, $p' = p/(p-q)$, $z = 1/(p-q)$ are the variables for the corresponding LP and S_i are the slack variables. The LP in standard form is given below:

$Max \quad Kz + q'L$

Subject to

$p' - q' - 1 = 0 \qquad (1)$

$Az - q' + S_1 = 0 \qquad (2)$

$q' - Bz + S_2 = 0 \qquad (3)$

$Cz - p' + S_3 = 0 \qquad (4)$

$p' - Dz + S_4 = 0 \qquad (5)$

$p', q', z \ge 0 \qquad (6)$

Where $p = p'/z$ and $q = q'/z$ are the optimal solution of the original fractional problem.

The maximum objective will be on one of the basic feasible solution. As there are 7 variables 5 constraints, there will be 21 basic solutions. However all these solution will not be feasible, the feasibility of these solutions is shown in Table 31.

Table 31. Basic Solutions of the LP (Subproblem)

Basic Variables							**Solution**			
p'	**q'**	**z**	**S1**	**S2**	**S3**	**S4**	**Feasible or Not, Constraints Causing Inf.**	**p'**	**q'**	**Objective Fun. Value**
-	-	√	√	√	√	√	No (1),(6)	-	-	-
-	√	-	√	√	√	√	No (1),(6)	-	-	-
-	√	√	-	√	√	√	No (1),(6)	-	-	-
-	√	√	√	-	√	√	No (1),(6)	-	-	-
-	√	√	√	√	-	√	No (1),(6)	-	-	-
-	√	√	√	√	√	-	No (1),(6)	-	-	-
√	-	-	√	√	√	√	No (1),(5),(6)	-	-	-
√	-	√	-	√	√	√	No (1),(2),(5),(6)	-	-	-
√	-	√	√	-	√	√	No (1),(3),(5),(6)	-	-	-
√	-	√	√	√	-	√	Yes or No*	1	0	K
√	-	√	√	√	√	-	Yes or No*	1	0	K
√	√	-	-	√	√	√	No (1),(5),(6)	-	-	-
√	√	-	√	-	√	√	No (1),(5),(6)	-	-	-
√	√	-	√	√	-	√	No (1),(5),(6)	-	-	-
√	√	-	√	√	√	-	No (1),(5),(6)	-	-	-
√	√	√	-	-	√	√	No (2),(3),(6)	-	-	-
√	√	√	-	√	-	√	Yes	C/(C-A)	A/(C-A)	(K+L *A)/(C-A)
√	√	√	-	√	√	-	Yes	D/(D-A)	A/(D-A)	(K+L *A)/(D-A)
√	√	√	√	-	-	√	Yes	C/(C-B)	B/(C-B)	(K+L *B)/(C-B)
√	√	√	√	-	√	-	Yes	D/(D-B)	B/(D-B)	(K+L *B)/(D-B)
√	√	√	√	√	-	-	No (4),(5),(6)	-	-	-

*These cases are only feasible when lower bound for *q* is equal to zero (A = 0), which will make the solution equivalent with *q*=A.

Therefore (p,q) pairs for the basic feasible solutions are (C,A), (D,A), (C,B) and (D, B) which means the optimal point of the problem will occur these boundary points depending on value of K and L.

So we can conclude that:

$$\underset{(q\ ,p)\in G2\ ,(p>q)}{Max}\left[(\Phi_k^{\ +}-\Phi_k^{\ -})-\frac{\sum_{i=1,i\neq k}^{n}(\Phi^+_i-\Phi^-_i)}{n-1}\right]=\underset{(q,p)\in G3,(p>q)}{Max}\left[(\Phi_k^{\ +}-\Phi_k^{\ -})-\frac{\sum_{i=1,i\neq k}^{n}(\Phi^+_i-\Phi^-_i)}{n-1}\right]$$

Case ii:

Given $\Delta^j{}_{(i')}\le q\le\Delta^j{}_{(i'+1)}$, $\Delta^j{}_{(k')}\le p\le\Delta^j{}_{(k'+1)}$, then q is valued in the *i*th interval ($q\in\left[\Delta^j{}_{(i')},\Delta^j{}_{(i'+1)}\right]$) and p in the *k*th interval($p\in\left[\Delta^j{}_{(k')},\Delta^j{}_{(k'+1)}\right]$), as these intervals are neighboorhood intervals we equate $i+1=k$ and replace *i*, we have $q\in\left[\Delta^j{}_{(k')},\Delta^j{}_{(k'-1)}\right]$ and $p\in\left[\Delta^j{}_{(k')},\Delta^j{}_{(k'+1)}\right]$ as shown in Figure 25.

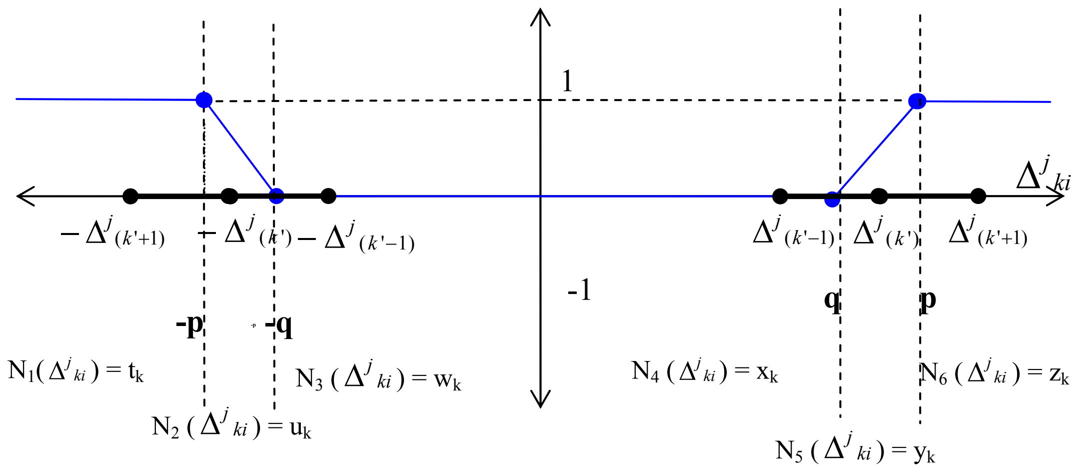

Figure 25. Illustration of the Feasible Subset Region for p and q in Neighborhood Intervals

If $p\neq q$ the function to be maximized is:

$$Max \quad (z_k - t_k) - \frac{\sum_{i=1, i\neq k}^{n} (z_i - t_i)}{n-1}$$

$$+ \frac{1}{p-q}((\sum_{i=1(\Delta^j{}_{ki}=\Delta^j{}_{(k')})}^{n} \Delta^j{}_{ki} - \sum_{i=1(\Delta^j{}_{ki}=-\Delta^j{}_{(k')}])}^{n} \Delta^j{}_{ki}) - \frac{\sum_{i=1,i\neq k}^{n} (\sum_{l=1(\Delta^j{}_{il}=\Delta^j{}_{(k')})}^{n} \Delta^j{}_{il} - \sum_{l=1(\Delta^j{}_{il}=-\Delta^j{}_{(k')})}^{n} \Delta^j{}_{il})}{n-1})$$

$$+ (\frac{q}{p-q})((u_k - y_k) - \frac{\sum_{i=1,i\neq k}^{n} (u_i - y_i)}{n-1})$$

$$Max \quad (z_k - t_k) - \frac{\sum_{i=1,i\neq k}^{n} (z_i - t_i)}{n-1} + \frac{1}{p-q}(\Delta^j{}_{ki}(y_k - u_k) - \frac{\sum_{i=1,i\neq k}^{n} \Delta^j{}_{il}(y_i - u_i)}{n-1})$$

$$+ \frac{q}{p-q}((u_k - y_k) - \frac{\sum_{i=1,i\neq k}^{n} (u_i - y_i)}{n-1})$$

$$= Max \quad (z_k - t_k) - \frac{\sum_{i=1,i\neq k}^{n} (z_i - t_i)}{n-1} + (y_k - u_k)\frac{\Delta^j{}_{ki} - q}{p-q} - \frac{\sum_{i=1,i\neq k}^{n} (y_i - u_i)(\Delta^j{}_{il} - q)}{(n-1)(p-q)}$$

We can call $\Delta = \Delta^j{}_{(k')}$, and $\Delta = \Delta^j{}_{ki} = \Delta^j{}_{il}$.

$$= Max \quad (z_k - t_k) - \frac{\sum_{i=1,i\neq k}^{n} (z_i - t_i)}{n-1} + \frac{(\Delta - q)}{(p-q)}((y_k - u_k) - \frac{1}{(n-1)} \sum_{i=1,i\neq k}^{n} (y_i - u_i))$$

$$M = ((y_k - u_k) + \frac{1}{(n-1)} \sum_{i=1,i\neq k}^{n} (y_i - u_i))$$

Then if $M < 0$, to maximize objective, it is better to decrease the contribution of second term to zero and equate $q = \Delta = \Delta^j{}_{(k')}$ and $p \in \left[\Delta^j{}_{(k')}, \Delta^j{}_{(k'+1)}\right]$. As all p values except $p = \Delta^j{}_{(k')}$ is feasible then the upper bound value for p may be selected which is $\Delta^j{}_{(k'+1)}$.

If M > 0, then $\dfrac{(\Delta - q)}{(p-q)}$ is to be maximized. The maximum will occur when p is equated to Δ ($p = \Delta = \Delta^j{}_{(k')}$). In this case again q can be selected any feasible value for instance $q = \Delta^j{}_{(k'-1)}$.

If $p=q$, then using generic function the objective value is equal to:

$$(z_k + y_k - u_k - t_k) - \frac{\sum_{i=1, i\neq k}^{n} (z_i + y_i - u_i - t_i)}{n-1}$$

This value is also same with the M > 0 case. So we can conclude that the maximum occurs one of the points $(p,q) = (\Delta^j{}_{(k'+1)}, \Delta^j{}_{(k')}),(\Delta^j{}_{(k')}, \Delta^j{}_{(k'-1)})$ or $(\Delta^j{}_{(k')}, \Delta^j{}_{(k')})$. This is a subset of combinations of boundary points of the region ($q \in [\Delta^j{}_{(i')}, \Delta^j{}_{(i'+1)}]$), ($p \in [\Delta^j{}_{(k')}, \Delta^j{}_{(k'+1)}]$) where $k=i+1$, so by iterating the maximum of the four distinct boundary combinations, we can get the maximum for the continuous subspace.

Case iii:

Lastly let us examine the situation where q and p are in the same interval illustrated in Figure 26.

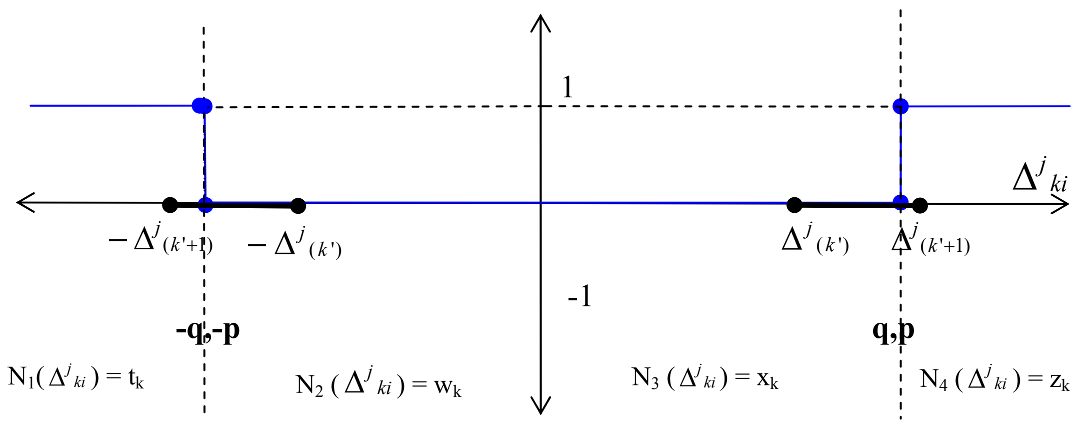

Figure 26. Illustration of the Feasible Subset Region for p and q in the Same Interval

By definition $\left|\Delta^{j}{}_{ki}\right| \notin (\ \Delta^{j}{}_{(k')}, \Delta^{j}{}_{(k'+1)})$, so the function does not change value when p and q is varied in the interval except boundary points. There will be two distinct values function takes.

When $\Delta^{j}{}_{(k'+1)} > q = p \geq \Delta^{j}{}_{(k')}$

$$\underset{q\ \in[\Delta_{(k')},\Delta_{(k'+1)}],(p=q)}{Max} \left[(\Phi_{k}{}^{+} - \Phi_{k}{}^{-}) - \frac{\sum_{i=1,i\neq k}^{n} (\Phi^{+}{}_{i} - \Phi^{-}{}_{i})}{n-1} \right] = (z_{k} - t_{k}) - \frac{\sum_{i=1,i\neq k}^{n} (z_{i} - t_{i})}{n-1}$$

When $q = p = \Delta^{j}{}_{(k'+1)}$ the function takes another distinct value. As one of these values is simply greater we can state that:

$$\underset{(q\ ,p)\in G_{j},(p=q)}{Max} \left[(\Phi_{k}{}^{+} - \Phi_{k}{}^{-}) - \frac{\sum_{i=1,i\neq k}^{n} (\Phi^{+}{}_{i} - \Phi^{-}{}_{i})}{n-1} \right] = \underset{(p=q=\Delta^{j}{}_{(k')},p=q=\Delta^{j}{}_{(k'+1)})}{Max} \left[(\Phi_{k}{}^{+} - \Phi_{k}{}^{-}) - \frac{\sum_{i=1,i\neq k}^{n} (\Phi^{+}{}_{i} - \Phi^{-}{}_{i})}{n-1} \right]$$

For all three cases, the maximum occurs in the boundary points. So we can conclude that

$$\max_{(q_{kj},p_{kj})\in G_{j}} (\left[(\Phi^{+}{}_{kj} - \Phi^{-}{}_{kj}) - \frac{\sum_{i=1,i\neq k}^{n} (\Phi^{+}{}_{ij} - \Phi^{-}{}_{ij})}{n-1} \right]) = \max_{(q_{kj},p_{kj},)\in GD_{j}} (\left[(\Phi^{+}{}_{kj} - \Phi^{-}{}_{kj}) - \frac{\sum_{i=1,i\neq k}^{n} (\Phi^{+}{}_{ij} - \Phi^{-}{}_{ij})}{n-1} \right])$$

APPENDIX C

MBA PROGRAM RANKING DATA

Various institutions prepare league tables of full-time MBA programs for the MBA applicants. The tables are formed by collecting information from schools and alumni and generally smoothed over several years to prevent abrupt rank changes. Also these tables do not provide the reader with the raw data and even for some categories only rankings for individual criteria are provided. In Table 32 some of these rankings are listed.

Table 32. MBA Program Rankings

Source	No of Programs Ranked	No of Criteria	Scope	Website	Free/Not Free
Bussiness Week	30, 10	N/A	US, international	http://www.businessweek.com/bschools/05/geographic.htm	Not Free
Financial Times	100	20	Global	http://media.ft.com/cms/c51a4c7c-8f2d-11da-b430-0000779e2340.pdf	Free
Forbes	67, 18	7 (Category)	US, non-US	http://www.forbes.com/2003/09/24/bschooland.html	Free
U.S News	50	10	Global	http://www.usnews.com/usnews/edu/grad/rankings/mba/brief/mbarank_brief.php	Not Free
WSJ	10, 10, 10	5	Natiomal, regional, international	http://www.careerjournal.com/reports/bschool/	Free

C.1 Financial Times MBA Ranking 2006 Data

Financial Times Data is reproduced from http://media.ft.com/cms/c51a4c7c-8f2d-11da-b430-0000779e2340.pdf .

C.2 MBA Criteria Key

Key for Financial Times 2006 MBA rankings is reproduced from http://www.ft.com/CareerAdvisor/MBARankings/pdf/2006_key_mba.pdf. The number in parenthesis is the FT weight of the criterion.

0) Salary Today: An average of salaries – three years after graduation – from the 2004, 2005 and 2006 surveys. The figure is in US dollars and is not used in the ranking.

1) Weighted Salary (20): The average 'salary today' with adjustment for salary variations between industry sectors. The figure is a weighted average of salaries three years after graduation from the 2004, 2005 and 2006 surveys and is in US dollars.

2) Salary percentage increase (20): The percentage increase in salary from the beginning of the MBA to three years after graduation. The figure is a weighted average of the increases from the 2004, 2005 and 2006 surveys.

3) Value for money (3): The value for money criterion is a short-term indicator calculated using the salary earned by alumni three years after graduation and course costs, including the opportunity cost of not working for the duration of the course.

4) Career progress (3): The degree to which alumni have moved up the career ladder three years after graduating. Progression is measured through changes in level of seniority and the size of company in which they are employed. The data in this field has been combined with career progress results from the MBA 2005 and MBA 2004 surveys.

5) Aims achieved (3): The extent to which alumni fulfilled their goals or reasons for doing an MBA. This is measured as a percentage of total returns for a school.

6) Placement success (2): The percentage of alumni, who graduated in 2002, that gained employment with the help of career advice. The data is presented as a rank.

The figure behind the rank is a weighted average of the placement success results from MBA 2004, 2005 and 2006.

7) Alumni recommendation (2): Alumni of 2002 were asked to name three business schools from which they would recruit MBA graduates. The figure represents the number of votes received by each school. The data is a weighted average from the 2004, 2005 and 2006 surveys and is presented as a rank.

8) Employed at three months (2): The percentage of the most recent graduating class that had gained employment within three months. The figure in brackets is the percentage of the class on which the school was able to provide employment data.

9) Women faculty (2): Percentage of female faculty.

10) Women students (2): Percentage of female students.

11) Women board (1): Female members of the advisory board, as a percentage.

12) International faculty (4): The percentage of faculty whose nationality differs from their country of employment.

13) International students (4): The percentage of international students.

14) International board (2): The percentage of the board whose nationality differs from the country in which the business school is situated.

15) International mobility (6): A rating system that measures the degree of international mobility based on the employment movements of alumni between graduation and today.

16) International experience (2): Weighted average of four criteria that measure international exposure during the course.

17) Languages (2): Number of additional languages required on completion of the MBA. Where a proportion of students require a further language due to an additional diploma, that figure is included in the calculations but not presented in the final table.

18) Faculty with doctorates (5): Percentage of faculty with a doctoral degree.

19) FT Doctoral rating (5): Number of doctoral graduates from the last three academic years with additional weighting for those graduates taking up a faculty position at one of the top 50 schools in MBA 2005.

20) FT Research rating (10): A rating of faculty publications in 40 international academic and practitioner journals. Points are accrued by the business school at which the author is presently employed. Adjustment is made for faculty size.

C.3 Correlation Analysis

The correlation among each pair of criteria is presented in Table 33. Only for the first criterion is significantly correlated with the second which is not used by FT in ranking and in the case study.

	Weighted Salary (US$)	Salary Percentage Increase	Value for Money Rank	Aims Achieved (%)	Alumni Recommended Rank	Career Progress Rank	Placement Success Rank	Employed At Three Months (%)	Women Faculty (%)	Women Students (%)	Women Board (%)	International Faculty (%)	International Students (%)	International Board (%)	International Mobility	International Experience Rank	Languages	Faculty with Doctorates (%)	FT Doctoral Rank	FT Research Rank
Weighted Salary (US$)	1.0000																			
Salary Percentage Increase (US$)	0.2234	1.0000																		
Value for Money Rank	0.3490	0.3049	1.0000																	
Aims Achieved (%)	0.4492	0.2935	0.2321	1.0000																
Recommended Rank	-0.6750	-0.3404	-0.3967	-0.4539	1.0000															
Career Progress Rank	-0.2242	0.0966	0.2036	0.0511	0.1365	1.0000														
Placement Success Rank	-0.4497	-0.4497	-0.4506	-0.5392	0.5226	-0.2338	1.0000													
Employed At Three Months (%)	0.2297	0.2172	0.2361	0.1784	-0.2381	-0.0361	-0.2424	1.0000												
Women Faculty (%)	-0.1965	-0.2492	-0.1755	-0.2045	0.2169	0.0569	0.2762	-0.2085	1.0000											
Women Students (%)	-0.1182	-0.1062	0.1015	-0.0632	0.3146	0.0195	0.0373	-0.1430	0.1226	1.0000										
Women Board (%)	0.1067	-0.2195	-0.2684	-0.1413	0.0806	-0.2141	0.1634	-0.0686	0.3170	0.0614	1.0000									
International Faculty (%)	0.1237	-0.3240	-0.1534	0.0120	-0.1872	-0.2400	0.1085	0.0497	-0.1929	-0.0392	0.1696	1.0000								
International Students (%)	-0.1728	-0.4128	-0.4472	-0.2175	0.1017	-0.4345	0.4564	-0.1016	0.1234	0.0447	0.2234	0.5543	1.0000							
International Board (%)	0.0251	-0.1592	-0.2442	-0.0277	-0.0579	-0.4321	0.1614	-0.0159	-0.0993	0.0224	0.1910	0.6139	0.6458	1.0000						
International Mobility	-0.0047	0.3374	0.3951	0.1007	0.0555	0.4896	-0.4295	0.1797	-0.0319	0.0104	-0.1709	-0.4722	-0.7985	-0.6098	1.0000					
International Experience Rank	-0.0578	0.1232	0.0480	0.1120	0.1404	0.3618	-0.2248	0.0313	0.0045	0.0728	-0.1260	-0.3474	-0.4582	-0.4293	0.4580	1.0000				
Languages	-0.0540	-0.0688	-0.2355	-0.1173	-0.0346	-0.4028	0.1405	-0.0884	-0.0453	-0.2322	0.0714	0.3097	0.4242	0.5583	-0.3783	-0.4706	1.0000			
Faculty with Doctorates (%)	0.1374	0.1624	0.2210	0.1667	-0.2867	0.2082	-0.3675	0.1452	-0.1183	0.0403	-0.2570	0.1938	-0.0752	-0.0512	0.0999	0.0858	-0.1743	1.0000		
FT Doctoral Rank	-0.1641	0.0099	0.0414	-0.0068	0.2863	0.0355	0.2137	-0.2732	-0.0174	0.0081	-0.0979	-0.0610	-0.0871	0.0294	0.1713	0.0539	0.0488	-0.2232	1.0000	
FT Research Rank	-0.4470	-0.2330	-0.4622	-0.3655	0.5955	-0.2489	0.6123	-0.4283	0.2770	0.0372	0.1484	-0.1531	0.2903	0.1974	-0.2418	-0.0839	0.3068	-0.5289	0.4131	1.0000

Table 33. Correlations Between Criterion for FT 2006 MBA Rankings Data

C.4 Normalized Data

Table 34. Normalized Performance Values for MBA Programs

2006 FT Rank	School Name	Weighted Salary (US$)	Salary Percentage Increase (US$)	Value for Money Rank	Aims Achieved (%)	Alumni Recommended Rank	Career Progress Rank	Placement Success Rank	Employed At Three Months (%)	Women Faculty (%)	Women Students (%)	Women Board (%)	International Faculty (%)	International Students (%)	International Board (%)	International Mobility	International Experience Rank	Languages	Faculty with Doctorates (%)	FT Doctoral Rank	FT Research Rank
1	University of Pennsylvania: Wharton	0.960	0.822	0.080	0.857	1.000	0.720	0.820	0.877	0.289	0.474	0.067	0.296	0.371	0.798	0.550	0.650	0.000	0.962	1.000	0.990
2	Harvard Business School	1.000	0.686	0.290	0.714	0.990	0.830	0.650	0.912	0.421	0.632	0.817	0.337	0.315	0.160	0.590	0.470	0.000	0.949	0.820	1.000
3	Stanford University GSB	0.997	0.729	0.070	0.786	0.980	0.760	0.810	0.930	0.289	0.474	0.267	0.378	0.337	0.213	0.410	0.610	0.000	0.987	0.950	0.980
4	Columbia Business School	0.849	0.915	0.230	0.750	0.930	0.620	0.840	0.877	0.184	0.447	0.133	0.571	0.270	0.383	0.520	0.360	0.000	0.974	0.810	0.960
5	London Business School	0.722	0.720	0.120	0.786	0.960	0.710	0.420	0.947	0.184	0.211	0.133	0.908	0.921	0.638	0.970	0.910	0.500	0.987	0.710	0.920
6	University of Chicago GSB	0.809	0.814	0.200	0.750	0.950	0.470	0.970	0.895	0.184	0.368	0.367	0.357	0.258	0.191	0.380	0.690	0.000	0.974	0.830	0.970
7	New York University: Stern	0.755	0.881	0.110	0.893	0.900	0.250	0.900	0.860	0.342	0.500	0.217	0.408	0.202	0.032	0.480	0.480	0.000	1.000	0.970	0.890
8	Dartmouth College: Tuck	0.934	0.881	0.260	0.964	0.880	0.480	0.990	0.895	0.474	0.447	0.350	0.133	0.247	0.106	0.360	0.570	0.000	0.949	0.010	0.870
9	Insead	0.774	0.415	0.920	0.786	0.950	0.850	0.330	0.807	0.158	0.079	0.217	0.867	0.865	0.798	0.950	0.920	1.000	0.962	0.730	0.940
10	MIT: Sloan	0.847	0.712	0.520	0.821	0.920	0.580	0.850	0.930	0.158	0.421	0.183	0.296	0.270	0.255	0.560	0.710	0.000	1.000	0.940	0.840
11	Yale School of Management	0.798	0.949	0.250	0.964	0.770	0.410	0.860	0.825	0.237	0.526	0.300	0.296	0.202	0.074	0.530	0.330	0.000	0.974	0.350	0.750
12	Instituto de Empresa	0.524	0.932	0.900	0.750	0.590	1.000	0.680	0.807	0.737	0.579	0.367	0.408	0.742	0.894	0.770	0.680	0.500	0.821	0.010	0.170
13	Iese Business School	0.454	0.890	0.190	0.714	0.820	0.840	0.720	0.982	0.184	0.237	0.167	0.327	0.742	0.926	0.940	1.000	0.500	0.974	0.530	0.530
14a	IMD	0.818	0.322	0.960	0.893	0.840	0.930	0.780	0.912	0.079	0.211	0.167	1.000	1.000	0.809	1.000	0.450	1.000	0.949	0.010	0.430
14b	University of Michigan: Ross	0.714	0.661	0.500	0.893	0.910	0.220	0.950	0.772	0.526	0.447	0.683	0.296	0.281	0.117	0.440	0.440	0.000	0.936	0.890	0.940
16	UC Berkeley: Haas	0.691	0.619	0.310	0.786	0.850	0.740	0.870	0.877	0.421	0.474	0.300	0.347	0.315	0.085	0.600	0.820	0.000	1.000	0.860	0.950
17	Northwestern University: Kellogg	0.728	0.593	0.350	0.786	0.970	0.520	0.980	0.877	0.289	0.447	0.150	0.245	0.191	0.064	0.260	0.620	0.000	0.949	0.930	0.920
18	York University: Schulich	0.350	0.881	0.930	0.714	0.690	0.870	0.160	0.842	0.500	0.632	0.333	0.592	0.685	0.564	0.810	0.780	0.000	0.987	0.290	0.530
19	UCLA: Anderson	0.769	0.636	0.370	0.750	0.860	0.750	0.830	0.912	0.079	0.500	0.167	0.214	0.191	0.106	0.320	0.700	0.000	1.000	0.710	0.890
20	University of Oxford: Said	0.733	0.500	0.740	0.750	0.640	0.980	0.240	0.719	0.342	0.237	0.233	0.469	0.933	0.266	0.890	0.550	0.000	1.000	0.310	0.560
21	Ceibs	0.826	0.949	0.010	0.607	0.380	0.360	0.600	0.965	0.079	0.579	0.167	0.633	0.112	0.532	0.020	0.880	0.500	0.923	0.010	0.300
22a	Manchester Business School	0.499	0.746	0.550	0.857	0.700	0.820	0.410	0.895	0.605	0.289	0.000	0.214	0.865	0.000	0.920	0.950	0.000	0.795	0.920	0.270
22b	HEC Paris	0.502	0.729	0.630	0.750	0.510	0.970	0.700	0.368	0.368	0.395	0.200	0.286	0.831	0.340	0.930	0.900	1.000	0.846	0.500	0.130
24a	University of Virginia: Darden	0.713	0.822	0.330	0.893	0.870	0.730	0.960	0.912	0.447	0.263	0.350	0.010	0.191	0.064	0.300	0.200	0.000	0.936	0.390	0.390
24b	University of Toronto: Rotman	0.435	0.746	0.620	0.786	0.810	0.310	0.500	0.895	0.421	0.289	0.633	0.592	0.404	0.489	0.460	0.160	0.000	0.962	0.760	0.840
24c	RSM Erasmus University	0.523	0.653	0.670	0.464	0.620	0.920	0.170	0.667	0.158	0.368	0.333	0.194	0.944	0.319	0.880	0.990	0.500	0.910	0.870	0.490
27a	Duke University: Fuqua	0.643	0.669	0.020	0.786	0.890	0.320	0.940	0.842	0.421	0.211	0.267	0.337	0.303	0.085	0.220	0.560	0.000	0.936	0.620	0.920
27b	Esade Business School	0.387	0.873	0.470	0.571	0.630	0.940	0.450	0.912	0.579	0.158	0.367	0.153	0.753	0.830	0.860	0.960	1.000	0.667	0.460	0.090
29	University of North Carolina: Kenan-Flagler	0.585	0.737	0.450	0.786	0.830	0.180	0.880	0.772	0.289	0.316	0.083	0.265	0.213	0.053	0.350	0.590	0.000	0.910	0.710	0.870
30	Lancaster University Management School	0.461	0.703	0.990	0.607	0.170	0.900	0.350	0.895	0.500	0.421	0.733	0.276	0.742	0.468	0.720	0.860	0.000	0.821	0.920	0.170
31a	Michigan State University: Broad	0.435	0.881	0.610	1.000	0.470	0.030	1.000	0.930	0.395	0.500	0.533	0.061	0.360	0.000	0.180	0.290	0.000	0.936	0.500	0.750
31b	University of Western Ontario: Ivey	0.441	0.703	0.710	0.786	0.800	0.540	0.390	0.895	0.368	0.316	0.217	0.449	0.629	0.479	0.740	0.600	0.000	0.962	0.350	0.760
33	University of Iowa: Tippie	0.415	1.000	0.760	0.607	0.030	0.370	0.570	0.737	0.368	0.474	0.283	0.153	0.506	0.074	0.500	0.050	0.000	0.910	0.470	0.390
34	SDA Bocconi	0.391	0.729	0.650	0.179	0.540	0.600	0.190	0.632	1.000	0.237	0.783	0.245	0.461	0.255	0.730	0.930	1.000	0.808	0.990	0.390
35	University of Cambridge: Judge	0.545	0.364	0.820	0.857	0.560	0.700	0.490	0.737	0.500	0.289	0.500	0.602	0.933	0.479	0.800	0.870	0.000	0.962	0.920	0.490
36a	Georgetown University: McDonough	0.578	0.864	0.270	0.786	0.660	0.420	0.320	0.807	0.579	0.395	0.267	0.235	0.247	0.117	0.390	0.520	0.000	0.897	0.010	0.390
36b	Cornell University: Johnson	0.663	0.593	0.060	0.714	0.780	0.260	0.930	0.807	0.553	0.342	0.250	0.388	0.281	0.468	0.270	0.280	0.000	0.936	0.390	0.750
38	University of Maryland: Smith	0.482	0.695	0.510	0.786	0.580	0.060	0.520	0.877	0.447	0.447	0.100	0.255	0.326	0.160	0.420	0.040	0.000	1.000	0.760	0.790
39	University of Illinois at Urbana-Champaign	0.368	0.729	0.490	0.679	0.300	0.390	0.670	0.947	0.447	0.579	0.283	0.276	0.506	0.000	0.280	0.240	0.000	1.000	0.850	0.660
40	University of Rochester: Simon	0.537	0.746	0.100	0.679	0.390	0.200	0.760	0.807	0.184	0.500	0.233	0.224	0.461	0.479	0.580	0.210	0.000	0.859	0.410	0.530
41	Carnegie Mellon: Tepper	0.507	0.619	0.140	0.643	0.750	0.120	0.910	0.825	0.289	0.211	0.250	0.378	0.236	0.085	0.290	0.370	0.000	0.949	0.890	0.800
42a	Emory University: Goizueta	0.583	0.602	0.050	0.714	0.680	0.670	0.800	0.895	0.579	0.474	0.250	0.204	0.315	0.074	0.150	0.400	0.000	0.910	0.010	0.840
42b	Pennsylvania State: Smeal	0.446	0.805	0.540	0.643	0.450	0.130	0.690	0.789	0.342	0.289	0.283	0.163	0.270	0.021	0.240	0.430	0.000	0.846	0.540	0.690
44	McGill University	0.370	0.576	0.970	0.786	0.720	0.530	0.250	0.561	0.579	0.500	0.317	0.459	0.506	0.298	0.910	0.380	0.000	0.731	0.410	0.610
45	Brigham Young University: Marriott	0.359	0.975	0.830	0.857	0.460	0.460	0.790	0.860	0.026	0.000	0.150	0.000	0.000	0.106	0.190	0.080	0.000	0.897	0.010	0.430
46	Cranfield School of Management	0.652	0.458	0.850	0.786	0.760	0.910	0.340	0.807	0.474	0.105	0.700	0.214	0.652	0.266	0.660	0.250	0.500	0.615	0.670	0.130
47a	Imperial College London: Tanaka	0.552	0.356	0.780	0.607	0.430	0.770	0.300	0.754	0.395	0.526	0.667	0.735	0.573	0.532	0.620	0.130	0.000	0.936	0.660	0.300
47b	Hong Kong UST Business School	0.208	0.364	0.530	0.607	0.180	0.570	0.740	0.965	0.237	1.000	0.650	0.898	0.899	1.000	0.670	0.940	0.500	1.000	0.600	0.650
47c	City University: Cass	0.488	0.508	0.840	0.250	0.330	0.890	0.210	0.895	0.395	0.368	0.267	0.429	0.685	0.309	0.780	0.420	0.000	0.885	0.800	0.190
50	Boston University School of Management	0.504	0.653	0.040	0.357	0.430	0.960	0.100	0.877	0.500	0.684	0.267	0.245	0.348	0.149	0.470	0.540	0.000	0.833	0.460	0.560

Table 34. Normalized Performance Values for MBA Programs (continued)

2006 FT Rank	School Name	Weighted Salary (US$)	Salary Percentage Increase(US$)	Value for Money Rank	Aims Achieved (%)	Alumni Recommended Rank	Career Progress Rank	Placement Success Rank	Employed At Three Months (%)	Women Faculty (%)	Women Students (%)	Women Board (%)	International Faculty (%)	International Students (%)	International Board (%)	International Mobility	International Experience Rank	Languages	Faculty with Doctorates (%)	FT Doctoral Rank	FT Research Rank
51	College of William and Mary	0.401	0.881	0.700	0.679	0.170	0.140	0.290	0.684	0.316	0.553	0.183	0.122	0.258	0.000	0.120	0.260	0.000	0.974	0.010	0.300
52a	Washington University: Olin	0.435	0.585	0.130	0.679	0.480	0.210	0.660	0.860	0.237	0.447	0.217	0.469	0.292	0.000	0.200	0.340	0.000	1.000	0.290	0.870
52b	Warwick Business School	0.388	0.407	0.800	0.536	0.730	0.560	0.070	0.842	0.816	0.158	0.533	0.388	0.764	0.340	0.710	0.830	0.500	0.897	0.980	0.270
54a	University of Southern California: Marshall	0.509	0.559	0.030	0.893	0.610	0.230	0.890	0.737	0.553	0.421	0.167	0.245	0.146	0.085	0.160	0.530	0.000	0.744	0.530	0.840
54b	Thunderbird: Garvin	0.436	0.619	0.420	0.536	0.710	0.650	0.430	0.105	0.368	0.421	0.150	0.255	0.371	0.245	0.840	0.770	0.000	1.000	0.010	0.430
56	University of Georgia: Terry	0.352	0.703	0.860	0.500	0.290	0.510	0.560	0.877	0.395	0.684	0.000	0.092	0.292	0.000	0.130	0.140	0.000	0.923	0.850	0.390
57a	University of Minnesota: Carlson	0.382	0.610	0.220	0.643	0.410	0.040	0.750	0.965	0.421	0.342	0.250	0.184	0.348	0.021	0.450	0.190	0.000	0.923	0.660	0.790
57b	Boston College: Carroll	0.514	0.754	0.380	0.571	0.090	0.340	0.480	0.947	0.711	0.526	0.183	0.143	0.236	0.021	0.050	0.180	0.000	0.821	0.250	0.490
59a	University of Notre Dame: Mendoza	0.463	0.695	0.240	0.786	0.450	0.160	0.580	0.895	0.447	0.237	0.217	0.092	0.191	0.032	0.140	0.630	0.000	0.897	0.010	0.690
59b	Vanderbilt University: Owen	0.497	0.771	0.210	0.750	0.670	0.240	0.550	0.684	0.368	0.289	0.133	0.122	0.315	0.085	0.170	0.100	0.000	0.821	0.250	0.270
61	University of Washington Business School	0.355	0.534	0.340	0.643	0.260	0.500	0.540	0.930	0.579	0.816	0.183	0.082	0.449	0.032	0.490	0.410	0.000	0.885	0.770	0.690
62	University of Texas at Austin: McCombs	0.511	0.508	0.390	0.536	0.790	0.170	0.590	0.842	0.500	0.289	0.233	0.173	0.146	0.021	0.230	0.270	0.000	0.910	0.740	0.750
63a	Ashridge	0.880	0.203	0.590	0.714	0.080	0.810	0.130	0.579	0.526	0.211	1.000	0.592	0.461	0.426	0.540	0.850	0.000	0.449	0.010	0.270
63b	Case Western Reserve: Weatherhead	0.360	0.712	0.360	0.464	0.270	0.290	0.260	0.596	0.421	0.316	0.000	0.102	0.281	0.000	0.640	0.310	0.000	0.949	0.560	0.610
65	University of Bradford/Universiteit Nimbas	0.343	0.483	0.950	0.643	0.110	0.550	0.220	0.877	0.868	0.421	0.333	0.449	0.910	0.351	0.830	0.510	0.000	0.654	0.730	0.090
66	University of Cape Town	0.865	0.381	0.680	0.429	0.400	0.490	0.020	1.000	0.526	0.316	0.417	0.112	0.337	0.223	0.700	0.740	0.000	0.551	0.250	0.050
67a	Rice University: Jones	0.519	0.576	0.150	0.714	0.650	0.150	0.620	0.877	0.579	0.447	0.167	0.265	0.258	0.021	0.090	0.060	0.000	0.949	0.010	0.490
67b	Temple University: Fox	0.257	0.593	0.690	0.607	0.050	0.380	0.090	0.754	0.263	0.447	0.450	0.367	0.382	0.415	0.850	0.750	0.000	0.910	0.630	0.270
69	Melbourne Business School	0.497	0.373	0.580	0.107	0.550	0.400	0.050	0.491	0.447	0.289	0.450	0.643	0.854	0.096	0.900	0.670	0.000	1.000	0.220	0.490
70	Wake Forest University: Babcock	0.402	0.788	0.320	0.750	0.250	0.300	0.400	0.772	0.237	0.158	0.267	0.031	0.202	0.043	0.060	0.070	0.000	0.936	0.010	0.430
71	University of British Columbia: Sauder	0.247	0.407	0.640	0.679	0.350	0.690	0.460	0.860	0.289	0.526	0.333	0.694	0.607	0.117	0.820	0.730	0.000	0.833	0.640	0.610
72a	Ohio State University: Fisher	0.356	0.602	0.460	0.571	0.500	0.190	0.610	0.930	0.474	0.316	0.167	0.163	0.348	0.021	0.110	0.390	0.000	0.872	0.550	0.750
72b	Arizona State University: Carey	0.366	0.576	0.430	0.500	0.530	0.070	0.470	0.877	0.474	0.395	0.150	0.153	0.180	0.000	0.400	0.660	0.000	0.885	0.800	0.650
72c	SMU: Cox	0.481	0.644	0.170	0.500	0.520	0.350	0.370	0.842	0.474	0.579	0.217	0.122	0.315	0.011	0.250	0.510	0.000	0.872	0.010	0.270
75a	George Washington University	0.439	0.475	0.180	0.679	0.200	0.860	0.200	0.561	0.526	0.974	0.150	0.071	0.360	0.053	0.650	0.110	0.000	0.872	0.800	0.270
75b	Australian Graduate School of Man.	0.493	0.339	0.600	0.393	0.490	0.020	0.440	0.509	0.447	0.474	0.183	0.551	0.764	0.160	0.750	0.840	0.000	0.936	0.430	0.560
77a	Babson College: Olin	0.484	0.492	0.090	0.643	0.570	0.950	0.180	0.825	0.579	0.395	0.350	0.194	0.416	0.074	0.330	0.970	0.000	0.910	0.010	0.270
77b	Purdue University: Krannert	0.414	0.441	0.280	0.607	0.600	0.100	0.920	0.877	0.237	0.211	0.167	0.194	0.225	0.032	0.570	0.100	0.000	0.962	0.960	0.610
79	University of California: Davis	0.417	0.576	0.400	0.714	0.240	0.450	0.530	0.754	0.500	0.553	0.233	0.378	0.090	0.000	0.080	0.320	0.000	1.000	0.010	0.490
80a	Georgia Institute of Technology	0.352	0.636	0.660	0.607	0.320	0.590	0.630	0.842	0.289	0.263	0.217	0.316	0.146	0.000	0.370	0.230	0.000	0.923	0.320	0.390
80b	University of Wisconsin-Madison	0.405	0.449	0.410	0.571	0.370	0.660	0.770	0.877	0.395	0.553	0.433	0.173	0.146	0.021	0.210	0.040	0.000	0.974	0.570	0.650
82a	Edinburgh University Management School	0.343	0.441	0.810	0.607	0.230	0.790	0.150	0.789	0.474	0.026	0.417	0.388	0.753	0.351	0.690	0.580	0.500	0.808	0.390	0.170
82b	University of California at Irvine: Merage	0.375	0.466	0.300	0.607	0.280	0.090	0.640	0.842	0.658	0.474	0.183	0.337	0.348	0.138	0.070	0.170	0.000	0.923	0.420	0.790
82c	Leeds University Business School	0.205	0.517	0.870	0.571	0.060	0.330	0.060	0.947	0.447	0.553	0.150	0.296	0.787	0.234	0.790	0.300	0.000	0.897	0.590	0.610
85a	Trinity College Dublin	0.648	0.373	0.980	0.750	0.210	0.440	0.120	0.386	0.342	0.842	0.500	0.122	0.618	0.266	0.990	0.150	0.000	0.244	0.210	0.050
85b	Texas A & M University: Mays	0.332	0.610	0.890	0.500	0.320	0.080	0.360	0.965	0.474	0.105	0.100	0.051	0.124	0.000	0.340	0.640	0.000	0.859	0.620	0.620
87	Queen's School of Business	0.395	0.441	0.790	0.714	0.740	0.630	0.380	0.316	0.632	0.105	0.150	0.439	0.438	0.319	0.510	0.040	0.000	0.846	0.390	0.190
88	Birmingham Business School	0.278	0.415	0.940	0.071	0.010	0.640	0.510	0.298	0.737	0.737	0.750	0.296	0.899	0.777	0.760	0.220	0.000	0.782	0.440	0.100
89	University College Dublin: Smurfit	0.402	0.246	0.770	0.393	0.120	0.780	0.080	0.965	0.474	0.211	0.200	0.306	0.472	0.596	0.870	0.460	0.500	0.885	0.350	0.170
90	Incae	0.000	0.737	0.480	0.357	0.340	0.280	0.270	0.456	0.079	0.526	0.000	0.592	0.787	0.862	0.630	0.810	0.500	0.846	0.010	0.050
91	University of Tennessee at Knoxville	0.251	0.695	0.750	0.607	0.110	0.050	0.280	0.737	0.447	0.553	0.283	0.020	0.090	0.000	0.100	0.120	0.000	0.795	0.590	0.130
92a	Coppead	0.331	0.576	1.000	0.679	0.020	0.010	0.730	0.000	0.763	0.447	0.283	0.071	0.000	0.000	0.030	0.790	0.500	1.000	0.260	0.050
92b	National University of Singapore	0.156	0.364	0.720	0.571	0.190	0.270	0.030	0.789	0.395	0.316	0.250	0.439	0.955	0.404	0.960	0.890	0.000	0.936	0.270	0.390
94	University of Durham Business School	0.248	0.297	0.880	0.000	0.080	0.880	0.040	0.895	0.368	0.342	0.217	0.520	0.820	0.415	0.680	0.760	0.500	0.833	0.500	0.390
95	Pepperdine University: Graziadio	0.344	0.525	0.160	0.607	0.220	0.610	0.140	0.404	0.474	0.763	0.183	0.020	0.270	0.000	0.610	0.350	0.000	0.910	0.010	0.090
96	University of Florida	0.424	0.636	0.910	0.643	0.130	0.800	0.710	0.842	0.184	0.395	0.100	0.122	0.090	0.000	0.010	0.040	0.000	1.000	0.710	0.750
97	Ipade	0.175	0.890	0.440	0.429	0.370	0.990	0.310	0.807	0.000	0.132	0.283	0.071	0.011	0.000	0.040	0.720	0.500	0.000	0.010	0.050
98	University of Alberta	0.120	0.339	0.570	0.750	0.140	0.110	0.110	0.719	0.421	0.421	0.267	0.592	0.427	0.245	0.430	0.490	0.000	0.949	0.310	0.530
99	ESCP-EAP	0.259	0.000	0.560	1.000	0.040	0.680	0.230	0.614	0.500	0.868	0.117	0.347	0.899	0.777	0.980	0.980	0.500	0.679	0.200	0.090
100	Nottingham University Business School	0.145	0.305	0.730	0.107	0.150	0.430	0.010	0.754	0.605	0.816	0.633	0.255	0.865	0.138	0.310	0.800	0.000	0.782	0.530	0.390

C.5 Descriptive Statistics For the Data Set

Table 35. Descriptive Statistics for the Data Set

Criteria	Mean	St.dev.	Minimum	Q1	Median	Q3	Maximum
Weighted Salary	0.4958	0.2060	0.0000	0.3614	0.4576	0.6287	1.0000
Salary Percentage Increase	0.6165	0.1956	0.0000	0.4682	0.6186	0.7436	1.0000
Value for Money Rank	0.5050	0.2901	0.0100	0.2525	0.5050	0.7575	1.0000
Aims Achieved	0.6514	0.1940	0.0000	0.5714	0.6786	0.7857	1.0000
Alumni Recommended Rank	0.5058	0.2897	0.0100	0.2525	0.5050	0.7575	1.0000
Career Progress Rank	0.5050	0.2901	0.0100	0.2525	0.5050	0.7575	1.0000
Placement Success Rank	0.5050	0.2901	0.0100	0.2525	0.5050	0.7575	1.0000
Employed At Three Months	0.7858	0.1850	0.0000	0.7412	0.8421	0.8947	1.0000
Women Faculty	0.4134	0.1826	0.0000	0.2895	0.4211	0.5000	1.0000
Women Students	0.4153	0.1965	0.0000	0.2895	0.4211	0.5263	1.0000
Women Board	0.2920	0.1926	0.0000	0.1667	0.2500	0.3500	1.0000
International Faculty	0.3154	0.2107	0.0000	0.1556	0.2806	0.4235	1.0000
International Students	0.4463	0.2731	0.0000	0.2472	0.3483	0.7275	1.0000
International Board	0.2448	0.2638	0.0000	0.0319	0.1383	0.3989	1.0000
International Mobility	0.5050	0.2901	0.0100	0.2525	0.5050	0.7575	1.0000
International Experience Rank	0.5058	0.2890	0.0400	0.2525	0.5100	0.7575	1.0000
Languages	0.1250	0.2694	0.0000	0.0000	0.0000	0.0000	1.0000
Faculty with Doctorates	0.8812	0.1492	0.0000	0.8462	0.9231	0.9615	1.0000
FT Doctoral Rank	0.4920	0.3153	0.0100	0.2525	0.5150	0.7600	1.0000
FT Research Rank	0.5221	0.2848	0.0500	0.2700	0.5300	0.7575	1.0000

APPENDIX D

INFORMATION ON MIXTURE DESIGN

In this section we will provide details of the mixture design method that is used to evaluate the variability of rankings based on linear aggregation of performance values. Basic limitation of such an approach is the number factors which is high.

D.1 Simplex Lattice Design

First we will provide information on the mixture design based on simplex formed by the constraint that sum of the criteria equals to unity:

$$\sum_{j=1}^{m} w_j = 1$$

For this problem we can use the simplex lattice design approach and MINITAB is used for design of mixture. The degree of lattice is chosen as 2, so points other than vertices of the simplex can be included. Also using augmentation 20 points and center point of the simplex are added. In table we can summarize the points of the design and their types:

Table 36. Number of Mixture Design Points for Each Type

Point Type	**No.**
Vertex	20
Double Blend	190
Center Point	1
Axial Point	20

Finally in the above table we selected a sample of 100 points from above design points in order to ease calculation of Kendall’s Tau-b correlations.

D.2 D-Optimal Design

Now we add constraints on mixture weights such that the mixture proportion is limited by imposing lower and upper bounds (AR) relaxing the FT weight of the criterion $j\ (F_j)$ (given in Appendix 0) by 1%, 25%, 50%, 75%, 100%.

Design Expert is utilized and D-Optimal design is used to produce 100 points for each relaxation. Linear model and coordinate exchange method is used to find the points. Design-Expert uses the CONVERT algorithm to find vertices. (see Piepel, *Journal of Quality Technology*, pp125-133, April, 1988.)

D.3 Rank Impreciseness for Different AR

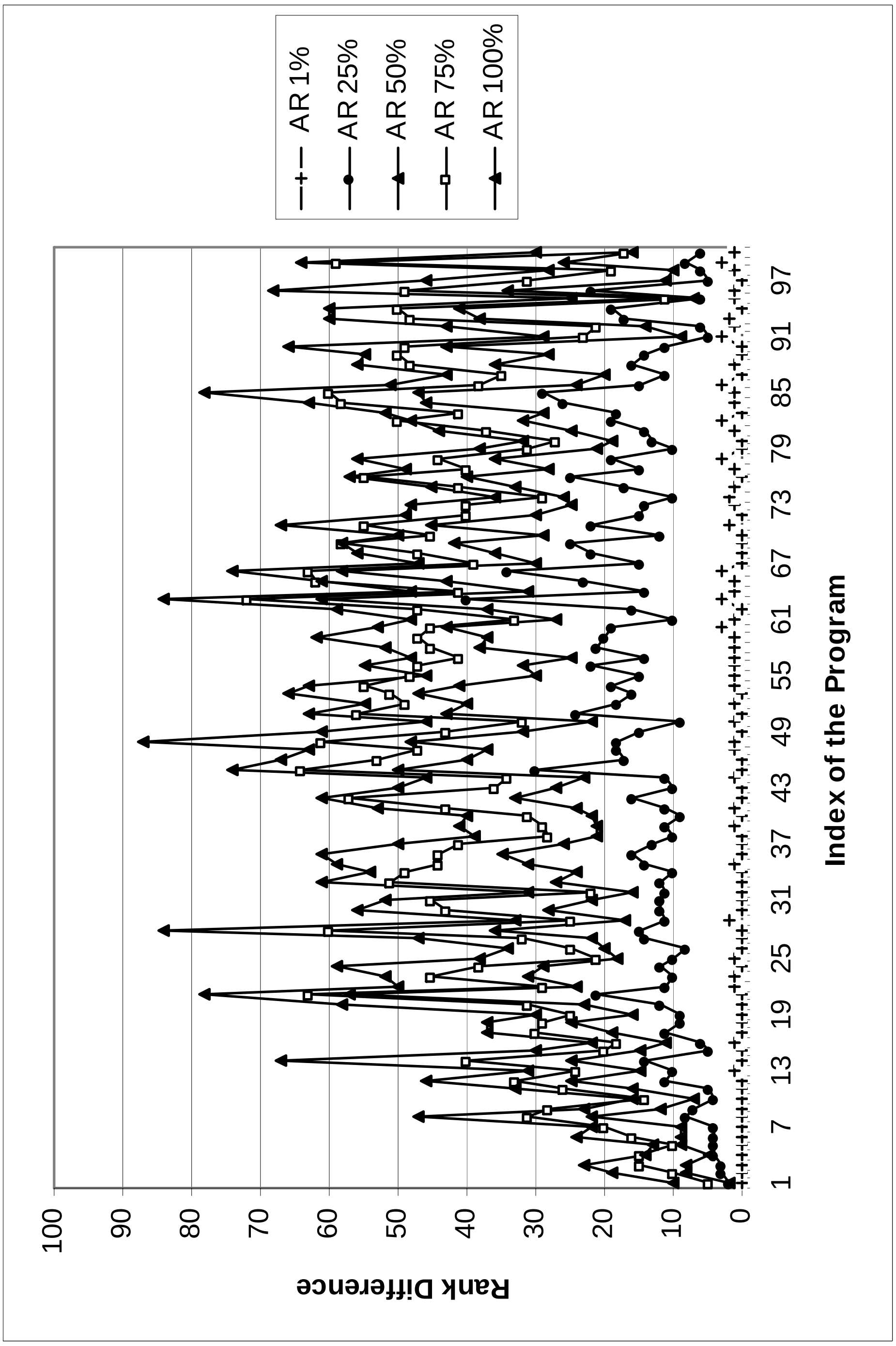

Figure 27. Rank Impreciseness of Programs for All AR Cases

APPENDIX E

RANKING RESULTS

Table 37. Ranking Results

			SE						SXE						Proposed Method 1						PM 2	
FT	SMART	PROM	SE (No AR)	SE (1% Rel.)	SE (25% Rel.)	SE (50% Rel.)	SE (75% Rel.)	SE (100% Rel.)	SXE (No AR)	SXE(1% Rel.)	SXE (25% Rel.)	SXE (50% Rel.)	SXE (75% Rel.)	SXE (100% Rel.)	M 1 (No AR)	M1 (1% Rel.)	M1 (25% Rel.)	M1 (50% Rel.)	M1 (75% Rel.)	M1 (100% Rel.)	M2 (No AR)	M2 (1% Rel.)
1	1	1	8	1	1	1	2	4	7	1	1	1	1	1	8	1	1	1	1	1	8	1
2	3	3	4	3	3	4	4	6	2	2	2	3	3	3	20	3	3	2	2	2	19	3
3	5	5	60	5	5	6	9	12	8	4	4	4	4	4	21	5	4	5	5	4	21	5
4	4	6	33	4	4	5	7	8	14	3	3	2	2	2	19	6	5	4	3	3	18	6
5	2	2	16	2	2	3	3	2	15	5	5	5	5	6	12	2	2	3	4	5	13	2
6	8	8	40	8	8	9	13	18	18	8	8	8	7	8	18	8	8	9	9	13	17	8
7	9	9	46	9	9	8	11	13	12	9	9	6	6	5	28	9	9	8	8	9	28	9
8	10	10	26	10	10	7	8	7	16	10	10	9	9	10	37	10	10	10	10	12	39	10
9	6	4	3	6	6	2	1	1	11	6	6	11	11	15	2	4	6	6	6	6	2	4
10	7	7	39	7	7	13	16	20	9	7	7	7	8	9	10	7	7	7	7	7	10	7
11	11	11	48	11	11	11	12	14	25	11	11	10	10	7	41	11	11	11	12	10	41	11
12	16	17	6	16	16	10	6	3	1	18	17	16	15	14	5	16	15	14	11	8	4	17
13	13	14	18	13	13	14	10	9	17	14	14	14	14	11	6	14	12	12	13	11	6	12
14a	15	12	5	15	15	12	5	5	20	17	18	19	24	34	1	12	13	13	14	14	1	14
14b	14	15	17	14	14	17	21	26	5	13	13	13	13	16	14	15	16	16	18	21	14	15
16	12	13	28	12	12	15	18	23	4	12	12	12	12	13	15	13	14	15	17	19	15	13
17	19	19	51	19	19	24	31	34	23	16	16	17	18	19	16	19	21	23	25	27	20	19
18	18	18	24	18	18	16	14	11	6	19	19	18	16	12	26	18	17	17	16	16	22	18
19	17	16	66	17	17	20	28	33	28	15	15	15	17	17	27	17	18	20	24	25	27	16
20	20	20	42	20	20	19	22	22	41	22	22	23	27	30	42	20	20	21	22	23	42	20
21	29	32	15	29	29	21	15	10	60	23	23	21	20	26	64	32	30	26	26	24	58	32
22a	22	22	19	22	22	18	17	15	24	27	27	27	22	20	23	22	22	18	15	15	25	22
22b	25	26	21	25	25	22	20	17	34	29	30	30	32	32	4	26	23	22	20	18	5	26
24a	32	33	43	32	32	30	27	29	31	28	28	24	25	24	36	33	32	31	27	26	38	33
24b	23	23	38	23	23	25	24	24	21	20	20	20	19	18	30	23	25	28	31	32	29	23
24c	21	21	34	21	21	23	23	21	44	24	26	28	28	27	17	21	19	19	21	22	16	21
27a	28	30	72	28	28	32	36	37	37	25	24	26	29	29	44	30	33	34	34	36	44	29
27b	30	28	11	30	30	26	19	16	30	32	34	36	33	31	7	28	24	24	19	17	7	28
29	26	27	81	26	26	31	33	35	38	21	21	22	21	23	29	27	31	32	33	34	30	27
30	31	31	12	31	31	28	25	19	10	34	32	31	30	28	13	31	28	25	23	20	12	31
31a	34	34	29	34	34	34	32	27	22	33	31	29	26	21	33	35	34	35	35	35	35	34
31b	24	24	64	24	24	27	26	28	29	26	25	25	23	22	32	24	27	30	30	31	31	24

Table 37. Ranking Results (continued)

			SE						SXE						Proposed Method 1						PM 2	
FT	SMART	PROM	SE (No AR)	SE (1% Rel.)	SE (25% Rel.)	SE (50% Rel.)	SE (75% Rel.)	SE (100% Rel.)	SXE (No AR)	SXE(1% Rel.)	SXE (25% Rel.)	SXE (50% Rel.)	SXE (75% Rel.)	SXE (100% Rel.)	M 1 (No AR)	M1 (1% Rel.)	M1 (25% Rel.)	M1 (50% Rel.)	M1 (75% Rel.)	M1 (100% Rel.)	M2 (No AR)	M2 (1% Rel.)
71	54	50	86	54	54	52	53	55	35	59	59	63	63	63	31	49	50	51	54	56	32	50
72a	72	68	77	72	72	72	73	71	63	69	69	67	67	69	73	68	70	74	78	81	73	68
72b	66	62	89	66	66	70	71	72	57	61	61	64	65	62	68	62	62	64	64	67	68	63
72c	88	91	95	88	88	89	91	93	87	86	86	85	82	81	97	91	89	85	85	84	97	91
75a	74	76	13	74	74	76	76	78	74	78	78	79	78	76	81	76	72	68	62	58	82	76
75b	59	59	85	59	59	65	70	74	76	66	66	69	74	78	53	59	65	66	70	73	54	59
77a	83	85	41	83	83	81	82	83	77	84	83	84	85	84	94	85	83	83	83	80	93	84
77b	70	66	62	70	70	74	78	81	64	65	65	68	73	73	47	66	69	70	71	72	50	66
79	87	89	74	87	87	86	90	90	68	82	82	81	79	77	91	89	91	92	91	90	90	89
80a	80	78	94	80	80	83	87	87	67	76	76	74	69	70	66	78	76	76	74	68	67	78
80b	78	77	80	78	78	82	89	91	45	75	75	78	76	74	59	77	79	80	84	85	61	77
82a	77	74	97	77	77	75	77	79	71	83	84	86	86	85	52	73	71	67	61	59	51	75
82b	81	80	59	81	81	84	86	86	62	81	81	82	83	83	84	80	86	90	92	95	84	79
82c	64	60	52	64	64	62	56	48	53	74	74	76	70	68	77	60	63	65	67	69	75	60
85a	84	87	10	84	84	80	72	63	99	87	87	88	88	93	70	87	85	84	81	77	70	88
85b	76	72	50	76	76	78	79	76	69	72	72	71	68	67	74	72	75	77	76	78	74	71
87	86	86	78	86	86	91	92	95	90	88	88	87	87	86	57	86	82	81	79	76	60	86
88	90	83	14	90	90	85	81	75	54	91	91	92	90	92	22	83	81	82	82	79	26	85
89	92	90	58	92	92	92	93	94	72	89	89	89	93	95	60	90	90	89	86	86	56	90
90	93	93	82	93	93	93	80	64	97	94	95	95	95	91	54	93	93	93	93	93	53	93
91	96	94	99	96	96	96	97	96	94	95	94	93	91	87	90	94	94	94	94	92	89	94
92a	94	97	22	94	94	95	94	89	83	93	93	94	92	90	45	97	95	95	95	94	48	98
92b	89	84	61	89	89	90	84	82	82	92	92	91	89	88	76	84	87	87	88	89	76	83
94	85	82	71	85	85	87	85	85	75	90	90	90	94	94	46	82	84	86	90	91	43	82
95	99	96	90	99	99	97	96	98	98	96	96	96	96	96	99	96	96	96	96	96	99	97
96	52	53	31	52	52	50	50	44	48	48	48	46	46	45	51	53	54	54	57	60	52	53
97	100	100	76	100	100	100	100	97	100	98	98	98	98	98	85	100	100	100	100	98	85	100
98	97	98	88	97	97	98	98	100	86	97	97	97	97	97	96	98	99	99	99	100	95	96
99	95	99	7	95	95	94	95	92	78	99	100	100	100	100	40	99	98	97	97	97	40	99
100	98	95	44	98	98	99	99	99	80	100	99	99	99	99	78	95	97	98	98	99	78	95

Table 37. Ranking Results (continued.)

			SE						SXE						Proposed Method 1						PM 2	
FT	SMART	PROM	SE (No AR)	SE (1% Rel.)	SE (25% Rel.)	SE (50% Rel.)	SE (75% Rel.)	SE (100% Rel.)	SXE (No AR)	SXE(1% Rel.)	SXE (25% Rel.)	SXE (50% Rel.)	SXE (75% Rel.)	SXE (100% Rel.)	M 1 (No AR)	M1 (1% Rel.)	M1 (25% Rel.)	M1 (50% Rel.)	M1 (75% Rel.)	M1 (100% Rel.)	M2 (No AR)	M2 (1% Rel.)
33	37	36	37	37	37	35	35	31	49	37	37	35	31	25	56	36	35	33	32	30	57	36
34	33	29	2	33	33	33	29	25	19	31	33	34	34	33	11	29	29	29	29	28	11	30
35	27	25	20	27	27	29	30	30	13	30	29	32	36	38	9	25	26	27	28	29	9	25
36a	43	46	83	43	43	40	39	39	56	39	39	38	37	37	86	46	45	44	40	39	86	46
36b	36	37	47	36	36	38	44	57	39	36	36	37	39	42	50	37	38	40	41	44	49	37
38	35	39	67	35	35	37	37	38	40	35	35	33	35	35	65	39	39	41	43	45	64	38
39	39	41	56	39	39	41	38	36	27	40	40	40	38	36	48	41	41	42	42	42	46	40
40	42	44	96	42	42	45	48	54	73	42	42	42	43	44	61	44	44	45	45	43	62	44
41	41	40	84	41	41	43	43	52	50	38	38	39	42	43	43	40	43	46	48	50	45	41
42a	46	49	57	46	46	44	46	53	43	44	44	44	45	46	75	50	51	53	59	61	77	49
42b	45	45	98	45	45	48	47	46	59	43	43	41	40	39	58	45	47	48	49	48	59	45
44	40	38	35	40	40	42	42	43	42	41	41	43	41	40	38	38	37	37	38	40	37	39
45	63	67	55	63	63	53	51	42	84	57	57	48	44	41	72	67	60	56	52	51	71	67
46	44	42	23	44	44	39	40	40	51	46	47	47	49	57	24	42	40	36	36	33	24	43
47a	49	48	45	49	49	49	49	47	36	50	50	51	53	64	34	48	48	47	46	46	33	48
47b	38	35	1	38	38	36	34	32	3	45	45	50	48	49	3	34	36	38	44	47	3	35
47c	48	47	70	48	48	46	41	41	47	51	51	52	50	53	39	47	46	43	39	38	36	47
50	50	51	53	50	50	51	60	66	65	47	46	45	47	47	88	51	49	50	50	52	87	51
51	79	81	79	79	79	73	69	62	89	79	79	65	62	54	98	81	80	78	73	64	98	81
52a	56	58	75	56	56	61	61	68	70	56	56	54	56	60	82	58	66	71	77	82	83	58
52b	47	43	27	47	47	47	45	45	26	52	52	58	60	59	25	43	42	39	37	37	23	42
54a	51	55	65	51	51	59	67	73	55	49	49	49	52	56	62	55	56	61	65	71	63	55
54b	57	61	69	57	57	56	59	59	92	58	58	57	57	58	63	61	57	59	58	57	65	62
56	65	65	36	65	65	69	63	60	46	62	62	61	55	48	49	65	59	57	53	53	47	65
57a	55	54	63	55	55	57	58	56	52	54	54	53	51	50	71	54	55	58	60	65	72	54
57b	67	73	49	67	67	64	66	70	58	63	63	59	58	55	93	74	73	72	68	66	92	72
59a	71	75	87	71	71	66	68	69	79	64	64	62	64	61	92	75	77	79	80	83	94	73
59b	75	79	100	75	75	77	75	77	95	77	77	72	72	72	87	79	78	75	69	62	88	80
61	53	52	32	53	53	55	57	58	32	55	55	56	54	51	55	52	52	52	55	55	55	52
62	58	56	91	58	58	67	74	80	61	53	53	55	61	65	67	57	58	63	66	70	69	56
63a	69	71	9	69	69	63	55	51	93	70	70	75	84	89	79	71	74	73	75	74	79	74
63b	62	63	92	62	62	68	65	65	91	60	60	60	59	52	89	63	61	62	63	63	91	61
65	61	57	25	61	61	54	52	49	33	73	73	77	71	71	35	56	53	49	47	41	34	57
66	68	70	30	68	68	60	54	50	88	67	67	66	77	82	83	70	64	55	51	49	81	70
67a	82	88	73	82	82	79	83	88	81	80	80	80	80	80	95	88	88	88	87	88	96	87
67b	73	69	68	73	73	71	64	61	66	71	71	73	66	66	69	69	67	60	56	54	66	69
69	60	64	54	60	60	58	62	67	85	68	68	70	75	79	80	64	68	69	72	75	80	64
70	91	92	93	91	91	88	88	84	96	85	85	83	81	75	100	92	92	91	89	87	100	92

APPENDIX F

RANK CORRELATIONS

Kendall's Tau-b rank correlations for various rankings are provided in the following tables.

Table 38. Correlation of Methods (No AR)

	FT	SMART	PROM	SE	SXE	M1
FT	1.000	0.852	0.819	0.294	0.580	0.459
SMART	0.852	1.000	0.954	0.324	0.666	0.554
PROM	0.819	0.954	1.000	0.333	0.692	0.583
SE	0.294	0.324	0.333	1.000	0.439	0.450
SXE	0.580	0.666	0.692	0.439	1.000	0.676
M1	0.459	0.554	0.583	0.450	0.676	1.000

Table 39. Correlation of Methods (AR with 100% Relaxation around FT Weights)

	FT	SMART	PROM	SE	SXE	M1
FT	1.000	0.852	0.819	0.761	0.847	0.806
SMART	0.852	1.000	0.954	0.798	0.875	0.824
PROM	0.819	0.954	1.000	0.796	0.860	0.825
SE	0.761	0.798	0.796	1.000	0.805	0.820
SXE	0.847	0.875	0.860	0.805	1.000	0.825
M1	0.806	0.824	0.825	0.820	0.825	1.000

Table 40. Correlation of Methods (AR with 75% Relaxation around FT Weights)

	FT	SMART	PROM	SE	SXE	M1
FT	1.000	0.852	0.819	0.815	0.876	0.821
SMART	0.852	1.000	0.954	0.870	0.923	0.867
PROM	0.819	0.954	1.000	0.861	0.901	0.870
SE	0.815	0.870	0.861	1.000	0.877	0.871
SXE	0.876	0.923	0.901	0.877	1.000	0.874
M1	0.821	0.867	0.870	0.871	0.874	1.000

Table 41. Correlation of Methods (AR with 50% Relaxation around FT Weights)

	FT	SMART	PROM	SE	SXE	M1
FT	1.000	0.852	0.819	0.857	0.883	0.827
SMART	0.852	1.000	0.954	0.932	0.955	0.907
PROM	0.819	0.954	1.000	0.914	0.918	0.917
SE	0.857	0.932	0.914	1.000	0.924	0.914
SXE	0.883	0.955	0.918	0.924	1.000	0.890
M1	0.827	0.907	0.917	0.914	0.890	1.000

Table 42. Correlation of Methods (AR with 25% Relaxation around FT Weights)

	FT	SMART	PROM	SE	SXE	M1
FT	1.000	0.852	0.819	0.852	0.855	0.831
SMART	0.852	1.000	0.954	1.000	0.992	0.938
PROM	0.819	0.954	1.000	0.954	0.950	0.956
SE	0.852	1.000	0.954	1.000	0.992	0.938
SXE	0.855	0.992	0.950	0.992	1.000	0.935
M1	0.831	0.938	0.956	0.938	0.935	1.000

Table 43. Correlation of Methods (AR with 1% Relaxation around FT Weights)

	FT	SMART	PROM	SE	SXE	M1
FT	1.000	0.852	0.819	0.852	0.852	0.818
SMART	0.852	1.000	0.954	1.000	1.000	0.952
PROM	0.819	0.954	1.000	0.954	0.954	0.998
SE	0.852	1.000	0.954	1.000	1.000	0.952
SXE	0.852	1.000	0.954	1.000	1.000	0.953
M1	0.818	0.952	0.998	0.952	0.953	1.000