AN ANALYSIS OF GEOMETRY PROBLEMS IN 6 - 8 GRADES TURKISH MATHEMATICS TEXTBOOKS

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Approval of the Graduate School of Natural and Applied Sciences

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This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

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ABSTRACT

AN ANALYSIS OF GEOMETRY PROBLEMS IN 6 - 8 GRADES TURKISH MATHEMATICS TEXTBOOKS

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The purpose of this study was to analyze geometry problems in a widely used sixth-, seventh-, and eighth-grade Turkish elementary mathematics textbooks series based on the cognitive assessment frameworks of the most recent TIMSS, the Trends in International Mathematics and Science Study 2003. To compare geometry problems in the textbooks and TIMSS 1999, in which Turkish students poorly performed, the cognitive behaviors that the problems required were determined and categorized. After the analysis, it was found that the two most frequent behaviors that the problems require are computing and applying with a total percentage of 72, in case of the textbooks. In TIMSS 1999 geometry problems case, however, applying and analyzing are the most frequent cognitive behaviors with a total percentage of 47. It was also found that a great majority of 22 behaviors of the framework were either not represented or underrepresented by the textbooks geometry problems. When we consider the four major categories of behaviors, 86 percent of the

textbooks geometry problems required behaviors belong to two cognitive domains: Knowing Facts and Procedures or Solving Routine Problems. TIMSS 1999 geometry problems, however, mostly belong to Solving Routine Problems or Reasoning with a percentage of 65. In both the textbooks and TIMSS 1999 cases, a relatively small part of the problems required behaviors belong to Using Concepts. The results are discussed in the light of Turkey's performance in TIMSS 1999 and some suggestions related to the textbook problems were given.

Keywords: Behavior, Cognitive Domain, Problem, Textbook, Geometry, The Trends in International Mathematics and Science Study

ÖZ

TÜRK 6 – 8. SINIF MATEMATİK DERS KİTAPLARINDAKİ
GEOMETRİ PROBLEMLERİNİN ANALİZİ

DELİL, Hüseyin

Yüksek Lisans, Ortaöğretim Fen ve Matematik Alanları Eğitimi Bölümü

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Bu çalışmanın amacı, Türkiye'de en çok kullanılan 6., 7., ve 8. sınıf matematik ders kitaplarındaki geometri problemlerini son yapılan Uluslararası Matematik ve Fen Bilgisi Çalışmasındaki Eğilimler 2003'ün değerlendirme çatısındaki zihinsel davranışlar temelinde incelemektir. Türk öğrencilerinin başarısız sonuçlar aldığı Üçüncü Uluslararası Matematik ve Fen Bilgisi Çalışmasının Tekrarı (TIMSS 1999)'ndaki problemler ile ders kitabındaki problemleri karşılaştırmak amacıyla, bu problemlerin gerektirdiği zihinsel davranışlar belirlenmiş ve sınıflandırılmıştır. İnceleme sonucunda, ders kitaplarındaki problemlerin gerektirdiği davranışlar içinde en sık karşılaşılan iki davranışın, toplam yüzde 72'lik oranla hesaplamak ve uygulamak olduğu, buna karşılık TIMSS 1999 geometri problemlerinde en sık karşılaşılan iki davranışın toplam yüzde 47'lik oranla uygulamak ve çözümlemek olduğu görülmüştür. Bu incelemede ayrıca, ders kitaplarındaki problemlerin, değerlendirme çatısındaki 22 davranışın büyük

çoğunluğunu ya hiç temsil etmediği ya da çok az temsil ettiği görülmüştür. Davranışların ait olduğu dört temel beceriyi dikkate aldığımızda, Türkiye'deki ders kitaplarındaki geometri problemlerinin gerektirdiği davranışların % 86'lık bir oranla ya Matematiksel Gerçek ve İşlemleri Bilme ya da Alışıldık Problemleri Çözme becerilerine ait olduğu, buna karşılık TIMSS 1999 geometri problemlerin gerektirdiği davranışların % 65'nin ya Alışıldık Problemleri Çözme ya da Muhakeme Etme becerilerine ait olduğu ortaya çıkmıştır. Her iki durumda da Kavramları Kullanma becerisine ait davranışları gerektiren problemlerinin oranının göreceli olarak düşük olduğu gözlenmiştir. Sonuçlar, Türkiye'nin TIMSS 1999'daki başarı durumu açısından değerlendirilmiş ve ders kitaplarındaki problemler ile ilgili önerilerde bulunulmuştur.

Anahtar Kelimeler: Davranış, Beceri, Problem, Ders Kitabı, Geometri, Uluslararası Matematik ve Fen Bilgisi Çalışmasındaki Eğilimler

To Maşallah, beloved brother

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Last and the most important, I wish to express my gratitude to Maşallah DELİL, my brother to whom I owed my life and therefore all my successes. To him I dedicated this thesis.

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LIST OF ABBREVIATIONS

EARGED : Eğitimi Araştırma ve Geliştirme Dairesi (Educational Research and Development Directorate).

IEA : International Association for the Evaluation of Educational Achievement.

ISC : International Study Center (of IEA)

MEB : Milli Eğitim Bakanlığı (Ministry of National Education).

OECD : Organisation for Economic Cooperation and Development.

OKS : Ortaöğretim Kurumları Sınavı (The Secondary Schools Entrance Exam)

ÖSS : Öğrenci Seçme Sınavı (The University Entrance Exam)

PISA 2000 : Programme for International Student Assessment 2000.

PISA 2003 : Programme for International Student Assessment 2003.

TD : Milli Eğitim Bakanlığı Tebliğler Dergisi (Journal for Announcements of Ministry of National Education)

TIMSS : Third International Mathematics and Science Study.

TIMSS 1995 : Third International Mathematics and Science Study.

TIMSS 1999 : Repeat of the Third International Mathematics and Science Study.

TIMSS 2003 : The Trends in International Mathematics and Science Study 2003.

TIMSS 2007 : The Trends in International Mathematics and Science Study 2007

CHAPTER 1

INTRODUCTION

Geometry, the branch of mathematics concerned with the measurement and relationships between points, lines, curves, and surfaces (Webster, n.d.), is frequently used to model that we call the "real world" and has many applications in solving practical problems (The Royal Society, 2001 July). Geometry education has proven as one powerful means of improving spatial abilities, which is an important component of human intelligence (Kaufmann, Steinbügl, Dünser, & Glück, n.d.). A thorough understanding of the principles of geometry is important to a wide range of scientific and technological fields of study such as engineering, physics, and cartography. Also the study of geometry sharpens thinking skills by teaching the process of deductive reasoning (Joyce, n.d.). For that reason, geometry is a very important subject in school curriculum. So, many international and national standardized achievement tests such as The Third International Mathematics and Science Study (TIMSS 1995), The Repeat of the Third International Mathematics and Science Study (TIMSS 1999), The Trends in International Mathematics and Science Study 2003 (TIMSS 2003), the university entrance exams, ÖSS, and secondary schools entrance exams, OKS, in Turkey include a large number of geometry problems. In TIMSS 1995, 1999, and 2003 about 15 % of the mathematics items were in content area of geometry (Martin & Kelly, 1996; Mullis, Martin, Gonzalez, Gregory, Garden, O'Connor, Chrostowski & Smith, 2000; Martin, Mullis & Chrostowski, 2004). ÖSS and OKS have about 35% of their mathematics problems in content area of geometry (OKS, 2005; ÖSS, 2005).

Considerable research, including the findings from TIMSS 1999, has shown that Turkish students have great difficulty in geometry (Olkun & Aydoğdu, 2003;

Ulusal Rapor, 2003). In TIMSS 1999, Turkish 8th-grade students' performance in geometry was the poorest among all the content areas of mathematics i.e., geometry, number, algebra, measurement, and data (Mullis et al., 2000). Understanding factors affecting performance differences among countries has long been concerned (Husen, 1967; Robitaille & Garden, 1989; Beaton, Mullis, Martin, Gonzales, Kelly, & Smith, 1996). Many studies exploring possible contributing factors in cross-national differences in students' mathematical achievement have consistently showed that curriculum materials are the key factors (Schmidt et al., 1997; Schmidt, McKnight, Houang, Wang, Wiley, & Cogan, et al., 2001). Textbook is a very important part of curriculum materials (Törnroos, 2005; Johansson, 2005). Textbook is seen as an authoritative part of curriculum and also seen as a mediator between the intent of curricular policy and the instruction that occurs in classrooms (Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002). Many researchers have analyzed textbooks to understand their potential effect on students' mathematical achievement (Schmidt et al., 1997; Li, 1999; Cai, Lo, & Watanabe, 2002; Zhu & Fan, 2004). TIMSS 1995 included an analysis of hundreds of textbooks with other curricular materials from about 50 countries (Schmidt et al., 1997). However, existing textbook studies have focused on content analysis (Cai et al., 2002; Carter, Li, & Ferucci, 1997; Fan, 1999), including content-topic coverage and page space devoted to each topic (Schmidt et al, 1997; Törnroos, 2005). Less attention has been placed on the analysis of problems presented in textbook (Li, 1999; Li, 2000; Zhu & Fan, 2004). The studies analyzing textbook problems either focused on content areas other than geometry (Li, 1999; Li, 2000) or on the representation of problem types (Zhu & Fan, 2004). On the other hand, in Turkey, studies related to TIMSS are either examining few TIMSS items (e.g., Olkun & Aydoğdu, 2003; Toluk, 2003), or using Linear Structural Modeling (e.g., Berberoğlu, Çelebi, Özdemir, Uysal & Yayan, 2003) or assessing Turkish students' performance based on TIMSS 1999 reports (e.g., Özgün-Koca & Şen, 2003)

Research Question

What are the similarities and differences of to-be-solved geometry problems in lower secondary Turkish mathematics textbooks and TIMSS 1999 geometry items with respect to their cognitive requirements?

Sub-questions

1- What are the required behaviors of to-be-solved geometry problems in 6th-grade Turkish mathematics textbooks?
2- What are the required behaviors of to-be-solved geometry problems in 7th-grade Turkish mathematics textbooks?
3- What are the required behaviors of to-be-solved geometry problems in 8th-grade Turkish mathematics textbooks?
4- What are the similarities and differences of to-be-solved geometry problems in 6th-, 7th-, and 8th-grade Turkish mathematics textbooks with respect to their required behaviors?
5- What are the required behaviors of TIMSS 1999 geometry items?
6- What are the similarities and differences between to-be-solved geometry problems in lower secondary Turkish mathematics textbooks and TIMSS 1999 geometry items with respect to their required behaviors?
7- What are the similarities and differences between to-be-solved geometry problems in 6th-, 7th-, and 8th-grade Turkish mathematics textbooks with respect to their cognitive domains?
8- What are the similarities and differences between to-be-solved geometry problems in lower secondary Turkish mathematics textbooks and TIMSS 1999 geometry items with respect to their cognitive domains?

Significance of the Study

The significance of this study is two-fold. At one level, the study provides information about cognitive requirements of geometry problems in Turkish elementary textbooks and this information can be used to improve mathematics curriculum and teaching in Turkey. At another level, it explores new ways of gaining

information from international comparative assessments of mathematics achievement and, possibly, limitations of detailed inference from such testing. The grade levels that this study applied are 6, 7 and 8 grade levels in Turkish educational system and the popularity of these grade levels comes from that Turkey participated TIMSS 1999 with 8th-grade students' (Mullis et al., 2000). Efforts to understand students' performance differences are meaningful and analysis of textbook problems in terms of their cognitive requirements at lower secondary level may contribute to having a better understanding on this issue. Many researchers indicated that the instruction of problem solving was more effective at lower secondary level than other grade levels (Hembree, 1992; Pace, 1986).

In a standardized test or a textbook case, having an even distribution of many required behaviors of problems will possibly serve improving students' thinking skills better. In both cases, analysis of items in terms of their cognitive domains requires a framework and possibly the method used in this study may contribute the efforts for those who analyzes test items.

Definition of Terms

Textbooks are curriculum materials that directly relate to teacher's teaching and students' learning activities in classroom (Li, 1999). In Turkey, textbooks are approved by the Turkish Ministry of National Education (MEB).

Worked examples are problems which appear in the content presentation part of topics and have accompanying complete solutions or answers (Li, 1999). They can also be called worked-out examples (Li, 1999).

To-be-solved problem are problem which are presented for student practice and have no accompanying solutions or answers (Li, 1999). These problems appear mainly under headings "exercises" and "test" in the lower secondary Turkish textbooks.

Cognitive domains involve knowledge and the development of intellectual skills in terms of major categories. They include “Knowing Facts and Procedures”, Using Concepts”, “Solving Routine Problems”, or “Reasoning” (Mullis et al., 2003).

Required behaviors are underlying behaviors of each major cognitive domain categories. For example, “Recall”, “Recognize”, “Compute”, and “Use tools” are behaviors underlying “Knowing Facts and Procedures” (Mullis et al., 2003).

CHAPTER 2

LITERATURE REVIEW

In this chapter, cognitive domains of TIMSS 2003 Mathematics Assessment Framework (Mullis et al., 2003), the importance textbooks analysis, and Turkish students' difficulties with solving geometry problems will be reviewed to have a framework to compare cognitive requirements of geometry problems of Turkish textbooks and TIMSS 1999 geometry items.

Cognitive Domains of Student's Learning

Efforts to understand the dimensions of human learning have led to the conclusion that every learning incident is in terms of the three types of domains: Cognitive (thinking), Affective (feeling), and Psychomotor (doing) (Bloom, Englehart, Furst, Hill, & Krathwohl, 1956; Krathwohl, Bloom, & Masia, 1964; Simpson, 1972). Among these domains, there is an increased emphasis on Cognitive Domain (Stahl, R. J., & Murphy, G. T., 1981; Anderson & Krathwohl, 2001) and, perhaps the most effectual work on cognitive domain is the so called Bloom's Taxonomy (or "Taxonomy of Educational Objectives") (Bloom et al., 1956) arranges the cognitive domains of learning in ascending order of complexity as: *Knowledge* (recalling and recognizing information), *Comprehension* (interpreting, translating or summarizing given information), *Application* (using information in a situation different from original learning context), *Analysis* (separating wholes into parts until relationships are clear), *Synthesis* (combining elements to form new entity from the original one), and *Evaluation* (making a decision based on a given criteria or rationale).

Mathematics Cognitive Domains and Cross-national Studies

Assessing cognitive domains of students' learning is an important part of many cross-national mathematics studies such as TIMSS 1995, TIMSS 1999, TIMSS 2003 (Martin, Mullis, & Chrostowski, 2004), PISA 2000 (OECD, 1999), or PISA 2003 (OECD, 2003). Each cross-national study has its own goals and therefore, its own framework. From TIMSS point of view, mathematics cognitive domains are "sets of behaviors expected of students as they engage with mathematics content" (Mullis et al., 2003, p. 9). These cognitive domains (e.g. "Reasoning" for the TIMSS 2003 framework) include some behaviors (e.g. "make suitable conjectures while investigating patterns" (p. 32)) that students need to perform in the process of responding items appropriately. On the other hand, PISA used the term "mathematical competencies" that ".. have to be activated in order to connect the real world, in which the problems are generated, with mathematics, and thus to solve the problems" to describe cognitive dimension of student performance (OECD, 2003, p. 30). However, many similarities can be seen between the assessment frameworks of these studies in terms of their cognitive domains and underlying behaviors. Table 2.1 presents an overview of the assessment frameworks of these studies.

Table 2.1

Behaviors under the cognitive domains of TIMSS 1995-1999, TIMSS 2003, TIMSS 2007, PISA 2000, PISA 2003 Mathematics frameworks.

CROSS-NATIONAL STUDY	COGNITIVE DOMAINS (TIMSS) / CLUSTERS (PISA)	BEHAVIOURS (TIMSS) / COMPETENCIES (PISA)
TIMSS 1995- TIMSS1999 (Robitaille et al., 1993)	▪ Knowing	Representing, Recognizing equivalents, Recalling mathematical objects and properties
	▪ Using Routine Procedures	Using equipment, Performing routine procedures, Using complex procedures
	▪ Investigating and Problem Solving	Formulating and classifying problems and situations, developing strategy, Solving, Predicting, Verifying
	▪ Mathematical Reasoning	Developing notation and vocabulary, Developing algorithm, Generalizing, Conjecturing, Justifying and Proving, Axiomatizing
	▪ Communicating	Using vocabulary and notation, Relating representations, Describing, Discussing, Critiquing
TIMSS 2003 (Mullis et al., 2003)	▪ Knowing Facts and Procedures	Recall, Recognize /Identify, Compute, Use tools
	▪ Using Concepts	Know, Classify, Represent, Formulate, Distinguish
	▪ Solving Routine Problems	Select, Model, Interpret, Apply, Verify/Check
	▪ Reasoning	Hypothesize /Conjecture /Predict, Analyze, Evaluate, Generalize, Connect, Synthesize/Integrate, Solve non-routine problems, Justify /Prove
TIMSS 2007 (Mullis et al., 2005)	▪ Knowing	Recall, Recognize, Compute, Retrieve, Measure, Classify / Order
	▪ Applying	Select, Represent, Model, Implement, Solve routine problems
	▪ Reasoning	Analyze, Generalize, Synthesize / Integrate, Justify, Solve non-routine problems
PISA 2000 (OECD, 1999)	▪ Carrying out procedures ▪ Making connections ▪ Mathematical thinking and generalizations	Mathematical thinking skill, Mathematical argumentation skill, Modeling skill, Problem posing and solving skill, Representation skill, Symbolic, formal and technical skill, Communication skill, Aids and tools skill
PISA 2003 (OECD, 2003)	1. Reproduction 2. Connections 3. Reflection	Thinking and reasoning, Argumentation, Communication, Modeling, Problem posing and solving, Representation, Using symbolic, formal and technical language and operations, Use of aids and tools

Table 2.1 shows that, the frameworks of IEA's studies, i.e. TIMSS 1995 framework, TIMSS 2003 framework, and TIMSS 2007 framework) are getting simpler in terms of number of cognitive domains and their underlying behaviors. However, for OECD's PISA frameworks, the underlying competencies, which can be analogous to behaviors in TIMSS terms, did not change much. Also, the clusters, which can be analogous to cognitive domains in TIMSS terms, did not change much. On the other hand, for both TIMSS and PISA, the reflection of many cognitive domains of Bloom's work (Bloom et al., 1956) can be seen (e.g. knowing cognitive domain of TIMSS 2007 framework and reproduction cluster of PISA 2003 framework with comprehension cognitive domain of Bloom's Taxonomy (Bloom et al., 1956)). These show some connections among TIMSS, PISA frameworks, and Bloom's taxonomy of cognitive domains.

An Overview of TIMSS 2003 Mathematics Assessment Framework

Besides having different philosophies from OECD's PISA studies (OECD, 2003), TIMSS improved its scope and qualities as IEA's studies take place for over 40 years (Mullis at al., 2003). This is possibly the reason for why about 50 countries participated in the last cycle of TIMSS, TIMSS 2003. TIMSS 2003 Mathematics Assessment Framework updated the common framework of TIMSS 1995 and TIMSS 1999 is to serve as "the basis of TIMSS 2003 and beyond" (Mullis et al., 2003, p. 3). By TIMSS 2003 framework, not only content areas and their underlying topics but also cognitive domains and their underlying behaviors of TIMSS 1999 framework were revised to reflect changes in the curricula of the participant countries (Mullis et al., 2003). A detailed explanation for the cognitive domains of TIMSS 2003 Mathematics Assessment Framework for 8th graders is given below (Martin, Mullis, & Chrostowski, 2004).

> Knowing Facts and Procedures: Facts encompass the factual knowledge that provides the basic language of mathematics and the essential mathematical facts and properties that form the foundation for mathematical thought. Procedures form a bridge between more basic knowledge and the use of mathematics for solving routine problems, especially those encountered by people in their daily lives. Students need to be efficient and accurate in using a variety of computational procedures and tools.

> Using Concepts: Familiarity with mathematical concepts is essential for the effective use of mathematics for problem solving, for reasoning, and thus for developing mathematical understanding. Knowledge of concepts enables students to make connections between elements of knowledge, make extensions beyond their existing knowledge, and create mathematical representations.
>
> Solving Routine Problems: Problem solving is a central aim of teaching school mathematics and features prominently in school mathematics textbooks. Routine problems may be standard in classroom exercises designed to provide practice in particular methods or techniques. Some of these problems may be set in a quasi-real context, and may involve extended knowledge of mathematical properties (e.g., solving equations). Though they range in difficulty, routine problems are expected to be sufficiently familiar to students that they essentially involve selecting and applying learned procedures.
>
> Reasoning: Mathematical reasoning involves the capacity for logical, systematic thinking. It includes intuitive and inductive reasoning based on patterns and regularities that can be used to arrive at solutions to non-routine problems, i.e., problems very likely to be unfamiliar to students. Such problems may be purely mathematical or may have real-life settings, and involve application of knowledge and skills to new situations, with interactions among reasoning skills usually a feature (p. 30-31).

Analysis of TIMSS Items according to TIMSS Frameworks

Similar to PISA, TIMSS reports its results in many contexts i.e., achievement, school, classroom, home, and national contexts but the achievement context is the most interesting to public (AFT, 1999). Although TIMSS keep some conducted items secure for beyond TIMSS studies to get trend data of participating countries (Mullis et al., 2000), it releases more than half of the conducted items accompanying their content and cognitive dimensions (World Wide Web b; World Wide Web c). TIMSS also reports students' performances on some of these items (Mullis et al., 2000) (See Appendix B). This data on released items gives very helpful information for academics, researchers, curriculum specialists, or policy makers to see stronger or weaker areas of their students.

Up to now, TIMSS gathered a three-level information on the content dimension of TIMSS items i.e., content domain, topic area, and objective, and only one- level information on the cognitive dimension of the items i.e., cognitive domain (Mullis et al., 2000; Martin, Mullis, and Chrostowski, 2004). In fact, analysis of TIMSS items according to their behaviors under cognitive domains is a very ambitious effort and it requires work of expert panels (E. Erberber, personal communication, April 11). TIMSS is going to report underlying behaviors of

cognitive domains of TIMSS 2007 items (E. Erberber, personal communication, April 11). This shows that determining underlying behaviors of cognitive domains of problems is meaningful and provides useful data.

Curricular Influence on Students' Performance in Cross-national Studies

Cross-national mathematics studies have been conducted both to indicate students' mathematics performances and uncover factors that contribute to the observed differences in achievement (Schmidt, McKnight, Valverde, Houang, & Wiley, 1997). Among several factors, educational researchers reached a general consensus that curriculum is the key factor to explain performance differences (Li, 1999). TIMSS 1995 and TIMSS 1999 had a three-level of curriculum model to describe educational systems: *intended*, i.e. documented national goals and objectives –"what is actually taught", *implemented*, i.e. teaching practices –"how it is taught", and *attained*, i.e., results of education including knowledge, skills and attitudes -"what it is that students have learned" (Mullis et al., 2003; Robitaille et. al., 1993). In this model, it was assumed that relative success or failure of a country's student in an international achievement study may result from one or a combination of all. It should also be noted that, in this model, textbook is a part of intended curriculum, which has an influence on the implemented curriculum (Robitaille et al., 1993).

The Importance of Textbook

Textbook is being used in mathematics classrooms very frequently all over the world (Haggarty & Pepin, 2002; Johansson, 2005). However, the importance of textbook was not realized as much as today. To get information on participant countries, TIMSS 1995 gathered a huge number of textbooks and other curriculum materials (Li, 2000). Schmidt, McKnight, Houang, Wang, Wiley, & Cogan, et al., (2001) carefully analyzed this data to see the correlation between students' achievement scores on TIMSS 1995 item and that learning opportunities provided for corresponding items in those students' textbooks. For each item of the test, the amount of material given in textbooks to answer that item was calculated. They

found very clear positive correlations between textbooks opportunity to learn a specific topic and students' success on that topic (Schmidt et al., 2001).

To emphasize that textbooks both reflect national curriculum and influence the teaching practices critically, it has become natural to put a fourth level as "potentially implemented curriculum" between intended curriculum and implemented curriculum (Valverde et al., 2002; Törnroos, 2004; Johansson, 2005). Figure 2.1 shows the model that Valverde et al. (2002) used to describe textbook with other curricular materials.

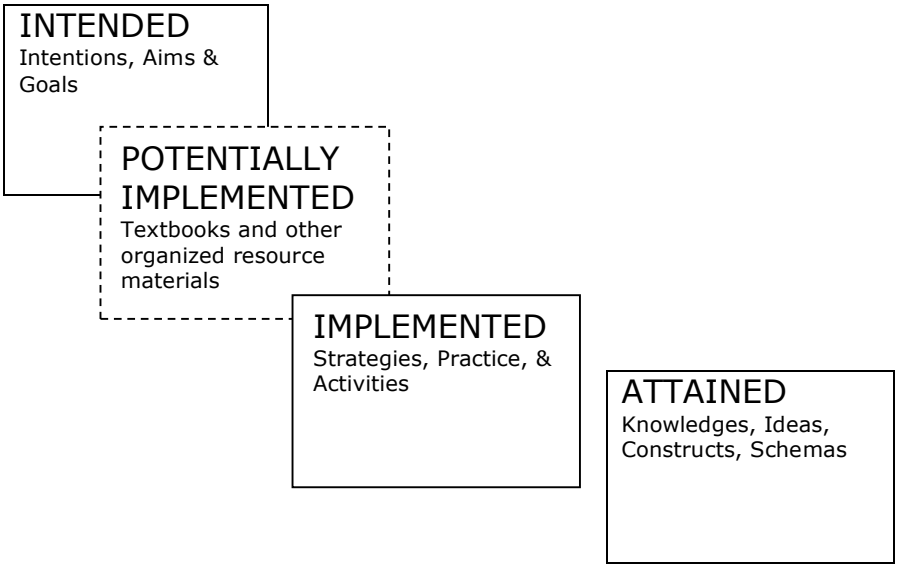


Figure 2.1. Textbooks and the tripartite model (Valverde et. al., 2002, p.13).

To be put as a fourth level into the model, the importance of textbook is stressed. The critical role of textbooks is also expressed by Johansson (2005) as "the textbooks identify the topics and order them in a way students should explore them" (p. 119).

Textbook and Opportunity To Learn Mathematics

Opportunity to learn refers to equitable conditions or circumstances within the school or classroom that promote learning for all students (NCREL, n.d.).

Opportunity to learn is a very important factor to explain students' performance difference (Schmidt et al. 2001) showed. Although opportunity to learn is not a newly introduced variable (e.g., Husén, 1967), measuring opportunity to learn is a difficult issue (Törnroos, 2005). To find the best way of measuring opportunity to learn, Törnroos (2005) approached Finnish students' opportunity to learn TIMSS 1999 mathematics topics in three ways, to find the one which is best correlated with Finnish students' TIMSS 1999 performances. For each TIMSS 1999 item, he had TIMSS 1999 teacher questionnaire data on the adequateness of teaching of the topic the item related to, he had analyzed textbook coverage in proportion of the material for the topic which the item belong to, and as a third way the coverage of that item in the textbook (Törnroos, 2005). He found that, among the three, item based textbook analysis is the best method to measure opportunity to learn of TIMSS 1999 mathematics items (Törnroos, 2005).

Analysis of Textbook Problems

Besides content analysis of textbooks (Törnroos, 2005), analyzing textbook problems as a window through which to view students' mathematical experiences is another useful idea in educational research (Li, 2000). For example, Nicely (1985) analyzed mathematical problems in content sections on complex numbers in secondary school mathematics textbooks and found that a majority of these problems had low-level cognitive requirements, such as recall and reproduction. Nicely, Fiber, and Bobango (1986) made a similar analysis on 3rd-, 4th-, 5th-, and 6th-grade mathematics textbooks on decimals and found that a majority of these problems had low-level cognitive requirements.

Stigler, Fuson, Ham, & Kim (1986) carried out textbook-problem analysis in a cross-national study to compare US and Soviet grade 1, grade 2 and grade 3 elementary mathematics word problems and found that Soviet text have much larger variability on problem types and also on their difficulty levels. There are also other cross-national studies to analyze elementary mathematics textbooks in terms of their problem types (e.g. Zhu & Fan, 2004) or cognitive requirements (Li, 1999; 2000). Zhu & Fan (2004) formulated a seven main classification of problem types, i.e.

routine versus non-routine, open-ended versus close-ended, etc., to analyze Chinese and US lower secondary mathematics textbooks and he found that US texts contain more non-traditional problem types i.e., non-routine, open-ended, etc. after the curriculum reform in US.

Li (1999) in his dissertation compared content presentations and cognitive requirements of to-be-solved problems of algebra chapters of eighth-grade mathematics textbooks from Hong Kong, Mainland China, Singapore, and US. To differentiate mathematical content i.e., algebra or non-algebra, he adapted TIMSS 1995 Mathematics Assessment Framework (Robitaille et al., 1993). He analyzed problems in three dimensions as: mathematical feature dimension i.e., the number of steps as single or multiple, contextual feature dimension i.e., purely mathematical context or illustrative context, and the third dimension as performance requirements. In performance requirements dimension he analyzed both the *response type* i.e., explanation/solution required or no explanation/solution required, and cognitive requirement. The cognitive requirements are "conceptual understanding", "performing routine procedures", "using complex procedures", "problem solving", and "other" adapted from TIMSS 1995 framework (Robitaille, et al., 1993) as he states "…the categories given in the TIMSS curriculum frameworks have the advantage of identifying comparable content in algebra across textbooks from different educational systems" (Li, 1999, p. 96). He found that US and Asian texts have different expectations for and approaches to the teaching and learning of Algebra. Li (2000) compared US and Chinese grade 5 to grade 8 mathematics textbooks to compare to-be-solved problems of addition and subtraction problems. He found that US textbooks included more variety in problem requirements than the Chinese textbooks.

Olkun & Toluk (2002) analyzed addition and subtraction word problems of 3rd- and 4th-grade Turkish elementary textbooks according to standard types to get their frequency, and also surveyed 3rd- and 4th-grade students' success on a test composed of those standard problem types. They found that the textbooks do not represent all problem types. They also found that the students were less successful on

the problem types that were underrepresented in the textbooks. As it is shown above (e.g. Li, 2000; Zhu & Fan, 2004; Olkun & Toluk, 2002) analyzing textbook problems gives very much information and this can be used to improve students' problem solving abilities.

Student Difficulties in Solving Problems

Textbook problem analysis will be more meaningful if the results can be connected with students' problem solving difficulties. On the other hand, students' problem solving difficulties are closely related to at students' problem solving processes (Montague, 2005, April 19). Charles, Lester, O'Daffer (1987) indicated that the process of solving problems involves seven important thinking skills:

1. Understand/formulate the question in a problem.
2. Understand the conditions and variables in the problem.
3. Select or find the data needed to solve the problem.
4. Formulate sub-problems and select appropriate solution strategies to pursue.
5. Correctly implement the solution strategy or strategies and solve sub-problems.
6. Give an answer in terms of the data in the problem.
7. Evaluate the reasonableness of the answer.

The first three skills concern the solvers ability to understand the problem. The preceding three involves procedures to solve the problem. The last is to check the solution of the main problem. It should be noted that if student misses a step in solving a problem then she is unlikely to get the solution for the problem. She must understand the given problem, grasp the context with its variables, find and take the data needed, formulate sub-problems and get the more specific data needed, implement strategy, reach a result, evaluate the reasonableness of the answer. It is needless to say that this methodology is applicable to all kinds of problems. According to Jonassen (2000) " a problem is an unknown quantity in some situation (where between a goal state and a current state)".

Caldwell & Goldin (1987) compared the difficulties of lower and upper level secondary school students in solving different types of mathematics problems in two dimension: (C)concreteness versus (A)abstractness and (F)factuality versus (H)hypotheticality. They denoted the concrete and factual problems ("concrete factual") by CF and similarly denoted "concrete hypothetical", "abstract factual", and for "abstract hypothetical" problem types by CH, AF, and AH respectively. They found that for lower grades i.e., grades 5 to 8, the difficulty level in ascending order is CF → CH → AH → AF but for upper grades i.e., grades 9 to 12, the difficulty level in ascending order is CF → AF → CH→ AH. This result show that, although concrete and factual problems are the easiest for both level, for six, seven, and eight graders, abstract and factual problem are the hardest type in contrast to nine to twelfth grade students who finds abstract and hypothetical ones the hardest (Caldwell & Goldin, 1987). We see that difficulty levels of the problem types change with grade levels of the students.

Turkish Students' Difficulties with Solving Geometry Problems

There is considerable research about Turkish students' difficulties in geometry content area of mathematics (Olkun & Aydoğdu, 2003; Olkun & Toluk, 2002). TIMSS 1999 reports of achievement context also show the poor geometry performance of Turkish students among all the content areas of mathematics i.e., geometry, algebra, number, data, and measurement (Mullis et al., 2000; EARGED, 2003). Instead of content dimension of the items when we consider cognitive dimension of TIMSS 1999, however, Turkish students' international standing is better on the items that require low level cognitive behaviors or moderate level cognitive behaviors but on the items that require high level cognitive behaviors the relative success of the Turkish students' performance on that item decline dramatically (Mullis et al., 2000). Table 2.2 shows Turkish students' standing at the achievement on TIMSS reported items and cognitive requirements of these items proposed by the researcher.

Table 2.2

Some released TIMSS items and Turkish students' achievement on them (SOURCE: Mullis et al., 2000 p. 64-99) (See Appendix B).

TIMSS 1999 ITEM CODE	CONTENT AREA OF ITEM	COGNITIVE DOMAIN OF ITEM*	BEHAVIOR OF ITEM*	ITEM DIFFICULTY **		TURKEY'S STANDING (OVER 38 COUNTRIES)
				INT	TURK	
J10	Measurement (M)	Solving Routine Problems (SRP)	Apply	42	25	35
V02	Data (D)	Reasoning (R)	Connect	24	10	31
P10	Geometry (G)	SRP	Apply	37	22	38
V04	Algebra (A)	R	Generalize	30	11	32
N19	Number (N)	Using Concepts (UC)	Represent	49	26	33
R15	N	SRP	Apply	44	26	33
T03	M	SRP	Apply	43	20	33
R11	G	R	Analyze	46	30	31
L17	A	Knowing Facts and Procedures	Compute	44	32	24
P13	N	SRP	Apply	65	50	31
J16	G	SRP	Interpret	58	32	32
B12	A	UC	Represent	65	41	34
H09	N	KFP	Compute	80	74	28
R07	N	KFP	Compute	77	71	30
R13	N	KFP	Compute	74	69	26
P16	D	R	Connect	79	38	38

* *Determined according to TIMSS 2003 Mathematics Assessment Framework.*

** *INT is for percentage of correct answers over all students; TURK for percentage of correct answers over all Turkish students.*

From Table 2.2, we see that Turkish students' performance on the items that require computation is relatively better as Turkish students' standings for those items are 24, 28, 30, 26 compared to 31, Turkish students' performance in mathematics TIMSS 1999 (Mullis et al., 2000; EARGED, 2003). Nevertheless, this is not surprising as Olkun and Toluk (2002) indicated that Turkish curriculum overemphasized procedures. On the other hand, Turkish students' standings for the items require behaviors in Reasoning (e.g. connect, generalize, analyze) are 31, 32, 31, 38 which are clearly lower than the ones require behaviors in Knowing Facts and Procedures mentioned before (e.g. compute). When we consider the average correct answers we see that the average correct answer for the items that require low-level cognitive behaviors i.e., computation, is 0.615 but it is 0.223 for the items that requires high-level cognitive behaviors. The average standings of the Turkish students on the items that require low-level behaviors and high level behaviors are 27 and 33, respectively over 38 countries.

Similarly, Olkun and Aydoğdu (2003) analyzed the cognitive requirements of two released TIMSS 1999 geometry items on which Turkish students performed very poorly. They discussed on some possible reasons such as lack of some skills and suggest some activities to improve those skills. On the limitations of lower elementary Turkish textbook problems, Olkun and Toluk (2002) suggested teachers and prospective teachers be provided recent research on children's learning. It is seen from the literature that Turkish students have difficulties in solving problems partly resulted from limitations of the textbooks.

The Importance of Geometry

The two most important standard national tests (the university entrance exams, ÖSS and secondary schools entrance exams, OKS) in Turkey include large number of geometry problems, like many standardized international mathematics achievement tests (e.g. TIMSS, PISA). In TIMSS 1995, 1999, and 2003 about 15 % of the mathematics items were in content area of geometry (Martin & Kelly, 1996; Mullis et al., 2000; Martin, Mullis & Chrostowski, 2004). This proportion will be 20 % in TIMSS 2007 (Mullis, Martin, Ruddok, O'Sullivan, Arora, & Erberber, 2005).

ÖSS and OKS, however have about 35% of their mathematics problems in geometry (ÖSS, 2005; OKS, 2005). We see that geometry is emphasized much in Turkey; the reasons for the poor performance of Turkish students should be searched.

When we consider curricula of countries all around the world we also see that geometry is very important in school mathematics curricula (US National Council of Teachers of Education –NCTM-, 1989; NCTM 2000). Although, national achievement tests (ÖSS, OKS) in Turkey devote a very large portion of items to geometry, the degree of emphasis that geometry topics receive in Turkish textbooks or the amount of time that devoted to geometry topics during instruction are not definite. On the other hand, geometry chapters are usually the last chapters of Turkish elementary textbooks, which may result in not providing enough of this material to children (Avşar, 2006). These are possible factors affecting students' opportunities to learn geometry. Table 2.3 shows geometry topics that are included in TIMSS 1999 Assessment Framework. (Mullis et al., 2000).

Table 2.3
Geometry topics in TIMSS 1999 Framework and the inclusion of them in Turkish curriculum (Mullis et al., 2000 p. 171).

GEOMETRY TOPICS	TURKEY'S CURRICULUM**
Cartesian Coordinates of Points in a Plane*	All or almost all students (= at least 90 % students)
Coordinates of points on a given straight line*	All or almost all students
Simple Two Dimensional Geometry* (Angles on a straight line, Parallel lines, Triangles, and Quadrilaterals)	All or almost all students
Congruence and Similarity*	All or almost all students
Angles (acute, right, supplementary, etc.)	All or almost all students
Pythagorean Theorem –without proof-	All or almost all students
Symmetry and Transformations* (Reflection, and Rotation)	All or almost all students
Visualization of Three-Dimensional Shapes*	Not included in the curriculum
Geometric Constructions with Straight-Edge and Compass	All or almost all students
Regular Polygons and their Properties-Names (e.g. Hexagon, and Octagon) and Sum of Angles, etc.)	All or almost all students
Proofs (Formal deductive demonstrations of geometric relationships)	Not included in the curriculum
Sine, Cosine, and Tangent in Right-angled Triangles	All or almost all students
Nets of Solids	All or almost all students

** The topics were included in TIMSS 1999.*

*** Yurdanur Atlıoğlu, Educational Research and Development Directorate (EARGED) of MEB provided the background data.*

Table 2.3, based on EARGED directorate, shows that the five of six TIMSS 1999 topics are included, just the topic "Visualization of Three- Dimensional Objects" is not covered in Turkish geometry curriculum for 8th graders. Although the national research coordinate reported inclusion of "Nets of Solids", it was not found in the textbooks in the analysis, which is consistent with TIMSS 1999 –Turkish teacher questionnaire that reports 37 % (Mullis et al., 2000 p. 180) as the percentage of students taught on this topic. In addition to this, "Rotational Symmetry", which is included in TIMSS 1999 is also not covered by Turkish curriculum. It is also seen from the table that seven of thirteen geometry topics were not included in TIMSS 1999. Non-inclusion of other geometry topics perhaps resulted from consensus among participating countries (Mullis et al., 2000).

Geometry in Turkish Elementary Mathematics Textbooks

Ministry of National Education (MEB) of Turkey prepares the curriculum and also approves curriculum materials for the schools (MEB, 2004). Textbooks are very frequently used in Turkish mathematics classrooms. Besides MEB series, there are many private textbook series. However, MEB series are the dominating ones (M. Demiroğlu, personal communication, January 27, 2006). The MEB series textbooks have been in use since 1999, after the revision of Turkish curriculum. Mathematics is a compulsory subject in grades 1 to 9 and an elective subject in grades 10 to 12 for some school types in Turkey, (TD, 2005 a; TD, 2005 b). The mathematics curriculum for lower secondary level, grades 6 to 8, had been used since 1991 and was revised in 1999 (Mullis et al, 2000). However, the total hour for mathematics (and for other courses) per week did not change (World Wide Web, 2006 d). In this curriculum, although the notion of "student-centered learning" introduced and aimed, the revised textbooks did not reflect this claim and found having traditional view, for example, on problem solving (Toluk and Olkun, 2003). The revision mainly affect total year of compulsory schooling as it was enlarged to 8. Although "student-centered education" resulted to more colorful textbooks it did not affect teaching practices much. Also, these textbooks overemphasized computation (Olkun and Toluk, 2002). Table 2.4 shows the major characteristics of Turkish intended

curriculum based on TIMSS 1999 Turkish national research coordinator (Yurdanur Atlıoğlu)'s data.

This table shows that Turkish curriculum does not support real-life applications of mathematics, working on mathematics projects, thematic approach, or multicultural approach. Also, as indicated in the table, solving non-routine problems, and deriving formal proofs are not emphasized very much. Instead, for example mastering basic skills is emphasized. On the other hand, a new curriculum of mathematics was introduced in 2004 and some these limitations were concerned such as working on mathematics projects or thematic approach (MEB, 2004). The new curricula textbooks for grades 1 to 5 are ready but they have not been ready for 6 to 8 grades yet. They will be in use next school year.

Table 2.4
Major Characteristics of Turkish Intended Curriculum

APPROACHES AND PROCESSES OF TURKISH INTENDED CURRICULUM	TURKISH INTENDED CURRICULUM EMPHASIS
Mastering Basic Skills	Major Emphasis
Understanding Mathematics Concepts	Major Emphasis
Real-life Applications of Mathematics	Minor / No Emphasis
Communicating Mathematically	Moderate Emphasis
Solving Non-Routine Problems	Moderate Emphasis
Deriving Formal Proofs	Moderate Emphasis
Working on Mathematics Projects	Minor / No Emphasis
Integration of Mathematics with Other School Subjects	Moderate Emphasis
Thematic Approach	Minor / No Emphasis
Multicultural Approach	Minor / No Emphasis
Assessing Student Learning	Major Emphasis

SOURCE: Mullis et al., (2000). Background data provided by Yurdanur Atlıoğlu (EARGED Directorate of MEB)

Summary of the Literature Review

Many national and international researches, studies, projects, websites and papers were reviewed and as a result, several important points were investigated.

1. TIMSS is one of the large-scale cross-national studies to assess students' knowledge and cognitive domains (Mullis, et al., 2003). There are 22 behaviors underlying four cognitive domains of TIMSS 2003 Mathematics Assessment Framework.

2. Problem solving is a very important skill and Turkish students have many difficulties, especially in geometry (Mullis et al., 2000). Turkish curriculum does not fully reflect some topics of TIMSS 1999 in which Turkey participated and had very poor results in geometry (Mullis et al., 2000).
3. Analyzing textbooks is found a good way of explaining cross-national differences (Schmidt et al., 2001). Many studies analyzing textbooks (Törnroos, 2005) and textbooks problems (Nicely, 1985; Stigler et al., 1986; Li, 1999, 2000; Zhu and Fan, 2004) have been found in the literature. Li (1999) analyzed to-be-solved problems of Asian texts and US text by adapting cognitive domains of TIMSS 1995 framework.
4. However, analyzing Turkish textbooks to look for some explanation of the poor performance was not found.

CHAPTER 3

METHODOLOGY

This is a qualitative research study that used content analysis techniques (Weber, 1990) to examine geometry problems of three lower secondary level textbooks. In order to get information about cognitive domains and underlying behaviors of geometry problems of the textbooks, the behaviors of TIMSS 2003 Mathematics Assessment Framework (Mullis et al., 2003) were used in this study. The methodology will be explained in the following order. First, the framework that was used in this study will be introduced. Then, the characteristics of the textbooks that were selected for the analysis will be explained. After giving the explanation for the coding procedure, the reliability and validity issues will be discussed. At the end, some examples of coding of problem types will be given.

TIMSS 2003 Mathematics Assessment Framework

In this study, TIMSS 2003 Mathematics Assessment Framework was used in the analysis of data. The reason for using this framework instead TIMSS 1999 framework is that TIMSS 2003 provides "the vision necessary to take the TIMSS cycles beyond 2003 assessments" (Mullis et al., 2001 p. i, 3). As mentioned before, to assess cognitive domains of the students' learning, TIMSS 2003 framework has four cognitive domains composed of a total of 22 distinct behaviors. The detailed explanations of cognitive domains each of which combine some specific behaviors are given below (Martin, Mullis, & Chrostowski, 2004).

> Knowing Facts and Procedures: Facts encompass the factual knowledge that provides the basic language of mathematics and the essential mathematical facts and properties that form the foundation for mathematical thought. Procedures form a bridge between more basic knowledge and the use of mathematics for solving routine problems,

especially those encountered by people in their daily lives. Students need to be efficient and accurate in using a variety of computational procedures and tools.

Using Concepts: Familiarity with mathematical concepts is essential for the effective use of mathematics for problem solving, for reasoning, and thus for developing mathematical understanding. Knowledge of concepts enables students to make connections between elements of knowledge, make extensions beyond their existing knowledge, and create mathematical representations.

Solving Routine Problems: Problem solving is a central aim of teaching school mathematics and features prominently in school mathematics textbooks. Routine problems may be standard in classroom exercises designed to provide practice in particular methods or techniques. Some of these problems may be set in a quasi-real context, and may involve extended knowledge of mathematical properties (e.g., solving equations). Though they range in difficulty, routine problems are expected to be sufficiently familiar to students that they essentially involve selecting and applying learned procedures.

Reasoning: Mathematical reasoning involves the capacity for logical, systematic thinking. It includes intuitive and inductive reasoning based on patterns and regularities that can be used to arrive at solutions to non-routine problems, i.e., problems very likely to be unfamiliar to students. Such problems may be purely mathematical or may have real-life settings, and involve application of knowledge and skills to new situations, with interactions among reasoning skills usually a feature (p. 30-31).

For each cognitive domain, there are underlying behaviors: Knowing Facts and Procedures includes four behaviors, Using Concepts contains five behaviors, the number of the behaviors under Solving Routine Problems the cognitive domains is five, and the last domain Reasoning has eight distinct behaviors. These all twenty-two behaviors are shown in Table 3.1.

According to Table 3.1 there are 22 behaviors underlying four cognitive domains of TIMSS 2003 framework. Mullis et al. (2003) gives characteristic features of these behaviors as follow:

".. [R]ecall .. Recall definitions; vocabulary units; number facts; number properties of plane figures; mathematical conventions of plane figures; mathematical conventions. ..

Recognize/Identify .. Recognize/Identify mathematical entities that are mathematically equivalent, i.e., areas of parts of figures to represent fractions, equivalent familiar fractions, decimals, and percents; simplified algebraic expressions; differently oriented simple geometric figures. ..

Compute .. Know algorithmic procedures for +, –, ×, ÷, or a combination of these; know procedures for approximating numbers, estimating measures, solving equations, evaluating expressions and formulas, dividing a quantity in a given ratio, increasing or decreasing a quantity by a given percent. Simplify, factor, expand algebraic and numerical expressions; collect like terms. ..

Use tools .. Use mathematics and measuring instruments; read scales; draw lines, angles, or shapes to given specifications. Use straightedge and compass to construct the perpendicular bisector of a line, angle bisector, triangles, and quadrilaterals, given necessary measures. ..

Know .. Know that length, area, and volume are conserved under certain conditions; have an appreciation of concepts such as inclusion and exclusion, generality, equal likelihood, representation, proof, cardinality and ordinality, mathematical relationships, place value. ..

Classify .. Classify/group objects, shapes, numbers, expressions, and ideas according to common properties; make correct decisions about class membership; order numbers and objects by attributes. ..

Represent .. Represent numbers using models; display mathematical information or data in diagrams, tables, charts, graphs; generate equivalent representations for a given mathematical entity or relationship. ..

Formulate .. Formulate problems or situations that could be modeled by given equations or expressions. ..

Distinguish .. Distinguish questions that can be addressed by given information, such as a data set, from those that cannot. ..

Select .. Select/use an efficient method or strategy for solving problems where there is a known algorithm or method of solution, i.e., an algorithm or method students at the target level could be expected to be familiar with. Select appropriate algorithms, formulas, or units. ..

Model Generate an appropriate model, such as an equation or diagram, for solving a routine problem. ..

Interpret .. Interpret given mathematical models (equations, diagrams, etc.); follow and execute a set of mathematical instructions. ..

Apply ..Apply knowledge of facts, procedures, and concepts to solve routine mathematical (including real-life) problems, i.e., problems similar to those target students are likely to have encountered in class. ..

Verify/Check .. Verify/check the correctness of the solution to a problem; evaluate the reasonableness of the solution to a problem. ..

Hypothesize/Conjecture/Predict .. Make suitable conjectures while investigating patterns, discussing ideas, proposing models, examining data sets; specify an outcome (number, pattern, amount, transformation, etc.) that will result from some operation or experiment before it is performed. ..

Analyze ..Determine and describe or use relationships between variables or objects in mathematical situations; analyze univariate statistical data; decompose geometric figures to simplify solving a problem; draw the net of a given unfamiliar solid; make valid inferences from given information. ..

Evaluate .. Discuss and critically evaluate a mathematical idea, conjecture, problem solving strategy, method, proof, etc. ..

Generalize .. Extend the domain to which the result of mathematical thinking and problem solving is applicable by restating results in more general and more widely applicable terms. ..

Connect .. Connect new knowledge to existing knowledge; make connections between different elements of knowledge and related representations; make linkages between related mathematical ideas or objects. ..

Synthesize/Integrate .. Combine (disparate) mathematical procedures to establish results; combine results to produce a further result. ..

Solve non-routine problems .. Solve problems set in mathematical or real-life contexts where target students are very unlikely to have encountered closely similar items; apply mathematical procedures in unfamiliar contexts. ..

Justify/Prove .. Provide evidence for the validity of an action or the truth of a statement by reference to mathematical results or properties; develop mathematical arguments to prove or disprove statements, given relevant information. .. (p. 27-33).

Table 3.1
An overview of TIMSS 2003 cognitive domains and their underlying behaviors (Mullis et al., 2000, p. 25-33).

	COGNITIVE DOMAINS			
	KNOWING FACTS & PROCEDURES	USING CONCEPTS	SOLVING ROUTINE PROBLEMS	REASONING
BEHAVIOURS	▪ Recall ▪ Recognize /Identify ▪ Compute ▪ Use tools	▪ Know ▪ Classify ▪ Represent ▪ Formulate ▪ Distinguish	▪ Select ▪ Model ▪ Interpret ▪ Apply ▪ Verify /Check	▪ Hypothesize /Conjecture /Predict ▪ Analyze ▪ Evaluate ▪ Generalize ▪ Connect ▪ Synthesize /Integrate ▪ Solve non-routine problems ▪ Justify/Prove

The coding scheme used in this study was derived from behaviors of TIMSS 2003 Mathematics Assessment Framework (Mullis et al., 2003) since cognitive domains of students were decided to be searched according to this framework in the study. The characteristics of behaviors of this framework were summarized in Table 3.2 to be used as the coding scheme of the study.

Table 3.2:
The coding scheme of the study.

KNOWING FACTS & PROCEDURES	USING CONCEPTS	SOLVING ROUTINE PROBLEMS	REASONING
Recall a definition/word/unit. a fact on numbers. a property of a number /plane figure a convention (e.g. $ab = a \times b$, $3a = a + a + a$). **Recognize /Identify** a familiar equivalent representation of a fraction/decimal /percent. a simplified algebraic expression. a differently oriented simple geometric figure. **Compute** a combination of $+,-,\times,\div,\%$ for rationales. approximation of a number. estimation of measures. an unknown in an equation. simplification/ factorization/expansion of algebraic/numerical expressions. the conformity of an expression / formula. **Use (tools)** a scale to measure length. measuring instrument to construct lines angles shapes. straightedge and compass to construct the perpendicular bisector of a line, angle angle bisector triangle quadrilaterals to given specifications.	**Know /Appreciate** conservation of length /area /volume with certain conditions. concepts: place value /inclusion/exclusion/ relation /generality/ representation /equally likelihood/proof /cardinality/ordinality. **Classify** objects / shapes/ numbers / expressions / ideas according to common properties members to classes. **Represent** a number by a model. data by a diagram/chart /graph. a mathematical entity /relationship by its other representations. **Formulate** problems / situations (that could be modeled by given equations or expressions). **Distinguish** questions (that can be addressed by given information) from questions (those cannot be addressed by given information).	**Select** an appropriate known strategy /method /algorithm/formula /unit for a routine problem. **Model** an equation / diagram for a routine problem. **Interpret** a given model (an equation / diagram) a set of mathematical instructions. **Apply** knowledge of facts/ procedures/ concepts for solving a routine problem. **Verify /Check** the correctness / reasonableness of a given solution to a problem.	**Hypothesize /Conjecture /Predict** about patterns/ ideas/data sets /models correctly. about the result of an operation/experiment. **Analyze** the relations between variables /objects in a mathematical situation. univariate statistical data. a geometric object (decomposing). given information to make valid inferences. **Evaluate** a mathematical idea (conjecture/strategy /method/proof) critically. **Generalize** a domain to which a result is applicable a result by restating it in more general terms. **Connect** elements of knowledge. representations. mathematical ideas/objects. **Synthesize /Integrate** mathematical procedures to establish other results. results to produce other results. **Solve non-routine problems** by applying mathematical procedures. **Justify /Prove** a statement by providing evidence from mathematical results or developing mathematical arguments.

As Table 3.2 indicates, the scheme categorizes a problem according to the behaviors required to solve it. The coding scheme identifies 22 types of behaviors of TIMSS 2003 framework (Mullis et al., 2003). These behaviors are grouped into four major categories as cognitive domains: Knowing Facts and Procedures, Using Concepts, Solving Routine Problems, and Reasoning. The behaviors under "Knowing Facts and Procedures" were coded as 11, 12, 13, and 14; the behaviors under "Using Concepts" were coded as 21, 22, 23, 24, an 25; and so on.

Selection of Textbooks

Elementary mathematics textbook series at the sixth (Bilgi, Ekmen & Gürsoy, 1999), seventh (Tortumlu & Kılıç 1999), and eighth (Polatoğlu, Çamlı & Çalıkoğlu 1999)-grade levels were selected for the analysis for two reasons. First, Turkey participated TIMSS 1999 with eighth-grade students (Mullis et al., 2000). Second, lower secondary grades are important in improving students' problem solving skills (Hembree, 1992; Pace, 1986)These textbooks have been the most widely selected and used standard textbooks series among about 25 private textbook series for 6th-, 7th-, and 8th-grades in Turkey (M. Demiroğlu, personal communication, December 25, 2005). It also should be noted that each textbook either private or not is approved by MEB for a maximum of 5 years. That is, any approved textbook may be regarded as invalid for the next years. MEB series dominates the style of content presentation and problem variability just because all series should be approved by Turkish Ministry of National education and the ministry has specified the content and skills for all grades (TD, 1995; TD, 2004). For this reason, there are very small differences about content presentation and problem variability among different publications. So, the selected textbooks represent the population of Turkish textbooks.

Description of Textbooks

The sixth grade textbook, *Elementary School Textbook Mathematics 6* (Bilgi, Ekmen & Gürsoy, 1999) has 234 pages with paper size 19.0 cm x 27.0 cm. It consists of 9 chapters. The seventh grade textbook, *Elementary School Textbook Mathematics 7* (Tortumlu & Kılıç, 1999) has 194 pages with paper size 19.0 cm x 27.0 cm. It contains 7 chapters. The eighth grade textbook, *Elementary School*

Textbook Mathematics 8 (Polatoğlu, Çamlı, & Çalıkoğlu, 1999) has 202 pages with paper size 19.0 cm x 27.0 cm. It contains 6 chapters. Each page of the textbooks was counted to the nearest half of a page as "non-geometry" and "geometry". The total number of geometry pages was found 60, 89, and 75 for sixth-, seventh-, and eighth-grade textbooks respectively. As a result, total number of pages of the three textbooks is 630 and totally 224 of this is geometry. That is, 35.56 % of the pages of the textbooks are devoted to geometry. The amount of pages devoted to geometry for content presentation and to-be-solved problems sections for sixth-, seventh- and eighth-grade textbooks are shown in Table 3.3.

Table 3.3
Pages devoted to content presentation and to-be-solved problem sections of the analyzed geometry topics of the textbooks.

TEXTBOOKS	PAGES (devoted to geometry sections of the textbooks)		
	Content presentation with worked examples	"Exercises" sections of to-be-solved problems	"Test" sections of to-be-solved problems
6th grade textbook (60 pages)	38.5 (64.17 %)	15 (25.00 %)	6.5 (10.83 %)
7th grade textbook (89 pages)	69,5 (78.09 %)	10.5 (11.80 %)	9 (10.11 %)
8th grade textbook (75 pages)	53 (70.67 %)	16 (21.33 %)	6 (8.00 %)
6,7, and 8th grade textbooks total (percentages) for each section	161 (71.88 %)	41.5 (18.53 %)	21.5 (9.60 %)

Table 3.3 shows that about 72 % of pages of geometry parts of the textbooks are devoted to content presentation and about 28 % of the pages are devoted to to-be-solved problems. The “Exercises” sections of the to-be-solved problems took about two times pages of that “Test” parts do. For the sixth grade textbook, total number of pages devoted to geometry is 60 (25.64 % of the total page of the textbook). 64 % of that page is devoted to content presentation with worked examples, 25 % is devoted to “Exercises” sections and 11 % is devoted to “Test” sections. For the seventh grade textbook, total number of pages devoted to geometry is 89 (45.88 % of the total page of the textbook). 78 % of that page is devoted to content presentation with worked examples, 13 % is devoted to “Exercises” sections and 12 % is devoted to “Test” sections. For the eighth grade textbook, total number of pages devoted to geometry is 75 (37.13 % of the total page of the textbook). 71 % of that page is devoted to content presentation with worked examples, 21 % is devoted to “Exercises” sections and 8 % is devoted to “Test” sections. The distribution of problems in the content presentation i.e., worked problems, and to-be-solved problems sections of geometry chapters of the textbooks are given in Table 3.4. “Problems” sections of to-be-solved problems were not included in the analysis, as they have very small numbers and they did not appear in most of the chapters.

Table 3.4
The distribution of worked-examples and to-be-solved problems of the analyzed geometry topics of the textbooks.

TEXTBOOKS	PROBLEMS (in geometry chapters)		
	worked examples (of the content presentation)	"Exercises" sections of the to-be-solved problems	"Test" sections of the to-be-solved problems
6th grade textbook (153 problems)	33 (21.57 %)	75 (49.02 %)	45 (29.41 %)
7th grade textbook (250 problems)	71 (28.40 %)	112 (44.80 %)	67 (26.80 %)
8th grade textbook (208 problems)	79 (37.98 %)	93 (44.71 %)	36 (17.31 %)
TOTAL (percentages) for each section	183 (29.95 %)	280 (45.83 %)	148 (24.22 %)

Table 3.4 shows that the content presentation sections have about 30 % of problems of the geometry chapters of the textbooks and to-be-solved problems sections have and about 70 % of the problems. Seventh-grade textbook contains the most number of the problems. As grade level increases the ratio of the number of to-be-solved problems over worked examples decreases. However, in all cases a great majority of the problems are in to-be-solved problems sections.

Selection of Content

TIMSS 1999 included only 6 geometry topics of 13 geometry topics of its framework because of having a consensus between participating countries curricula (Mullis et al., 2000). These six topics of TIMSS 1999 are given in Table 3.5.

Table 3.5
Geometry topics included in the TIMSS 1999. (Mullis et al., 2000 p. 171).

TOPICS INCLUDED IN TIMSS 1999
Cartesian Coordinates of Points in a Plane
Coordinates of points on a given straight line
Simple Two Dimensional Geometry (*Angles on a straight line, Parallel lines, Triangles, and Quadrilaterals*)
Congruence and Similarity
Symmetry and Transformations (*Reflection, and Rotation*)
Visualization of Three-Dimensional Shapes

Any chapter of the textbooks containing one of the topics that included in the analysis was specified as a geometry chapter and was selected to be used in this study. The textbooks did not cover some of the topics of TIMSS 1999 as Table 2.3 indicates. Therefore, these TIMSS 1999 geometry topics (e.g. visualization of three-dimensional shapes, rotational symmetry) were not included in the analysis. On the other hand, some of the topics that are not included in TIMSS 1999 were included in the analysis. Table 3.6 shows all topics included in the analysis and the rationale for their inclusion.

Table 3.6
The topics included in this study and the reason for the inclusion.

GEOMETRY TOPICS THAT INCLUDED IN THE ANALYSIS	LOCATION	REASON FOR INCLUSION
Symmetry, Coordinates of a point in a plane, Line graphs (pages 78-95, 97, 98)	Textbook 7 / chapter 3	Matches to the TIMSS 1999 topics. (see Table 3.5)
Angles and Polygons (123-130, 134-165)	Textbook 7 / chapter 5	Matches to the TIMSS 1999 topics. (see Table 3.5)
Proportional Line Segments and Similar Triangles (68-84,86-98, 112-132)	Textbook 8 / chapter 3	Matches to the TIMSS 1999 topics. (see Table 3.5)
Geometric Constructions with Straight-Edge and Compass (130-134)	Textbook 7 / chapter 5	Concluded as focused on "Simple Two Dimensional Geometry" topic of TIMSS 1999 curriculum
Surface Area and Volumes (149-182)	Textbook 8 / chapter 5	Concluded as comparable to "Visualization of Three-Dimensional Shapes" topic of TIMSS 1999 curriculum which was not covered by the texts.
Pythagorean Theorem, and Euclides' Relations (91-98)	Textbook 8 / chapter 3	Concluded as a part of "Congruence and Similarity" topic of TIMSS 1999 curriculum
Coordinates of Points on a Given Straight Line	NOT COVERED	-
Visualization of Three-Dimensional Shapes	NOT COVERED	-
Point, Line, Plane, Space, Line Segment, Ray (145-158)	Textbook 6 / chapter 6	Concluded as a part of "Simple Two D. Geometry" topic of TIMSS 1999 curriculum.
Perimeter of Triangle, Square, Quadrilateral (189-192)	Textbook 6 / chapter 8	Concluded as focused on "Simple Two D. Geo." topic of TIMSS 1999 curriculum.
Area of Square, Quadrilateral, Right Triangle (198-201)	Textbook 6 / chapter 8	Concluded as focused on "Simple Two Dimensional Geometry" topic of TIMSS 1999 curriculum
Volume of Cube, Rectangular Prism (205-208)	Textbook 6 / chapter 8	Concluded as comparable to "Visualization of Three-Dimensional Shapes" topic of TIMSS 1999 curriculum which was not covered by the texts.
Circle, Circular Region, Cylinder (166-186)	Textbook 7 / chapter 6	Concluded as very related to "Simple Two Dimensional Geometry" topic of TIMSS 1999 curriculum and "Visualization of Three-Dimensional Shapes" topics of TIMSS 1999 curriculum.

As it is seen from Table 3.6, three topics of the textbooks directly match to TIMSS 1999 geometry topics. There are some topics, though not directly matched to TIMSS topics, contain elements of those topics (e.g. "Angle, Triangle, and Their Classifications" topic of seventh chapter of the 6th–grade textbook). On the other hand, since some TIMSS topics are not covered by the textbooks (e.g. "Visualization of the Three-Dimensional Objects"), the possible corresponding geometry topics of the textbook (e.g. "Volume of Cube, Rectangular Prism") were selected for the analysis. The chapters, which contain any of the geometry topics of the study, were specified as a geometry chapter. These chapters are "Point, Line, Plane, Space, Line Segment, Ray", "Angle, Triangle", "Measures" for 6th-grade textbook. For the 7th-grade textbook, these chapters are "Equations, Line Graphs"," Angles, and Polygons", and "Circle, Cylinder". There are two geometry chapters of 8th-grade textbook: "Proportional Line Segments, Similar Triangles", and "Surface Areas, Volume". "Measurement of liquids, mass and time topics" of Measurement chapter of 6th-grade textbook which was concluded as a "non-geometry topic" and therefore not included to the analysis. For the 7th-grade textbook, "Solving first degree linear equations" topic of "Equations and Line Graphs" chapter was not included as it was regarded as a topic related to algebra i.e., non-geometry. For the 8th-grade textbook, two topics "Trigonometric relations in right triangles" and "Inequalities with two variables" of "Proportional Line Segments, Similar Triangles" chapter were not included to the analysis as they were not geometry topics of TIMSS 1999 framework (Mullis et al., 2000). Nevertheless, almost all geometry topics in the textbooks were included in the study and this made a better comparison of geometry chapters of the textbooks possible.

Selection of Problems

This study analyzes to-be-solved geometry problems of textbooks i.e., the problems that are presented for students to practice. To-be-solved problems, in contrast to worked examples, refer to the problems that do not have accompanying solutions or answers (except for possible answers provided at the end of a textbook (Li, 1999). In Turkish textbooks, these problems often appear under headings

“exercises” or “test”. The worked examples, the problems given for instructional purposes in the content presentation parts of the textbooks, were not included in the analysis. The to-be-solved problems were found mainly under titles “Alıştırmalar”-Exercises or “Test” but very few of problems were found under “Problemler”-Problems. Since they were very few and their sections were non-regular i.e., not appeared in most of the geometry chapters, these problems were not included to the analysis except for one of these problems, a unique example for the behavior of “Solving non-routine problems” (given as a coding example) included to the analysis to have coding variability. The exception problem is under “Problems” title of Measurement chapter of the 6^{th-} grade textbooks (will be given as Figure 3.14). Some of the to-be-solved problems (the ones under “Test”) were in multiple-choice format i.e., the problems requiring students select the correct one from the given four alternatives.

Selection of TIMSS 1999 Geometry Items

Since two of the sub-problems of the research are directly related to TIMSS 1999 geometry items, TIMSS 1999 geometry items were searched via Internet. 9 of the 21 items were found in the Released Set of TIMSS 1999 Mathematics Items (World Wide Web b, 2005). The item code of these geometry items are B11, D07, J11, J15, J16, L16, N12, P10, and R11 in this set (See from Figure A.1 to Figure A.9). On the other hand, 8 of TIMSS 1999 geometry which were not released in the Released Set of TIMSS 1999 Mathematics Items were found in the Relesead Set of TIMSS 2003 Mathematics Items (World Wide Web c, 2005). The item code of these items are M012005, M012015, M012026, M012039, M022202, M022142, M022154, and M022016 (See from Figure A.10 to Figure A.17). As a result, 17 of 21 TIMSS 1999 geometry items were gathered for the analysis. In fact, the rest four geometry items of TIMSS 1999 will be used in beyond TIMSS to get the trend data of participant countries (Mullis et al., 2000). That is, these items will be kept secure until TIMSS 2007 conducted.

Procedure

At the beginning of the study, to prepare the coding scheme, TIMSS 2003 Mathematics Assessment Framework (Mullis et al., 2003) was examined. The behaviors under each cognitive domain of the framework were specified as a distinct behavior such as "Recall", "Recognize", "Compute" etc. and put into a table to form the coding scheme. For each behavior, its characteristic features were derived from the assessment framework (Mullis et al., p. 27-33) and put into Table 3.2, under their behavior titles to be able to compare behaviors more easily. As a result, a coding scheme of 22 behaviors was formed. Behaviors were put in columns in the scheme as the behaviors in the columns show distinct cognitive domains. The names of the columns i.e., cognitive domains, were written on the top of the column. All to-be-solved geometry problems were coded according to the scheme. The problems which are composed of sub-problems and for which the same procedures or methods or strategies were required to answer them correctly were assumed as one problem. On the other hand, some of the problems required more than one behavior. These problems, as a principle, were coded by the number of behavior which has the highest number.

The worked examples were not included in the study for two reasons: firstly, as for each worked example, it has an accompanying solution (i.e. it has the solution strategy), one of the two critical attributes of a problem does not exist. For Jonassen (2000) "the difference between a goal state and a current state" is one of two critical attributes of a problem with "having social, cultural, or intellectual value for finding the unknown" (p. 4). For the worked examples we see that there is no gap between the problem situation and the goal state, since the solution is given. So, one of the critical attributes of a problem does not exist. Second, and more important, reason for not including them in the analysis is that worked examples are presented in textbooks for instructional purposes (Li, 1999) not for exercising their thinking which we have focused on this study. For those reasons, worked examples were not included in the analysis.

To get a comparison of content sections and to-be-solved-problems sections of the textbooks the procedure is given: the length of text area, which is found constant among pages for each textbook, was calculated for all textbooks. Then the amount of material given for that specific content (either belong to content presentation or to-be solved problems) page-by-page calculated and the partial pages were rounded to nearest half. "Problems" sections of geometry chapters were not included in the analysis since they had very limited problems and pages. This procedure was done for each geometry topic in each textbook. These findings were tabulated.

To get a comparison of worked examples and to-be-solved-problems of the textbooks the problems were classified under titles "worked examples" and "to-be-solved problem" and this procedure was done for all topics. The findings were tabulated. To-be-solved problems also categorized into chapters to see variability among chapters of the textbooks. These findings were tabulated. For each one of 15 problem types that were appeared in the textbooks, an example will be given as a figure at the end of this chapter.

As a result, total of 428 to-be-solved problems of selected topics in textbooks were selected for the analysis. These problems, with their underlying cognitive domains were tabulated (and will be given at the end of the result chapter in Table 4.3).

The released TIMSS 1999 geometry items were also analyzed according to the same coding scheme to get cognitive domains and the underlying behavior for each geometry item according to TIMSS 2003 Mathematics Assessment Framework (Mullis et al., 2003).

Reliability

To work out the coding procedure, two coders working independently coded 90 problems of the "Angles and Polygons" chapter in the seventh-grade textbook (Tortumlu & Kılıç, 1999) as it has the largest number of problems. One of the coders

is the researcher and other one is a mathematics educator. The two coders read Mathematics Cognitive Domains section of TIMSS 2003 Mathematics Assessment Framework (Mullis et al., 2003, p.25-33) independently. Then, the coders met and read the framework together. The coders were trained on some sample questions of the largest geometry chapter of textbooks and also some sample TIMSS 1999 and 2003 geometry problems. The disagreements were discussed until an agreement was reached. Then 90 other problems of the chapter were coded independently. After the initial coding, the *inter-rater reliability* was calculated as 96% (86 out of 90 problems) of the problems in the chapter. Each problem for which the coders did not agree was then discussed until an agreement was reached on how the problem would be coded. Since the inter-rater reliability score (96 %) is large enough (Li, 1999), the researcher coded all the problems of the textbooks.

Validity

Validity refers to, according to Fraenkel & Wallen (1990), "the appropriateness, meaningfulness, and the usefulness of the specific *inferences* researchers make based on the data they collect" (p. 153). To be able to draw valid conclusions, we need to have validity of the study. The researcher looked into these three aspects of inferences i.e., appropriateness, meaningfulness, and the usefulness, to understand the extend to which the study satisfies these criteria. For the "appropriateness" aspect, the researcher checked the goal of the study i.e., determining cognitive requirements of geometry problems of Turkish 6th-, 7th-, and 8th-grade textbooks and then to compare cognitive requirements of TIMSS 1999 geometry items first aspect. Then, the researcher checked TIMSS 2003 Mathematics Assessment Framework (Mullis et al., 2003) to get data on. Mullis et al. (2001) say

> "[T]he following factors were considered in finalizing the content domains and the topics and objectives of the assessment frameworks: Inclusion of the content in the curricula of a significant number of participating countries. Alignment of the content domains with the reporting categories of TIMSS 1995 and TIMSS 1999. The likely importance of the content to future developments in mathematics and science education. Appropriateness for the populations of students being assessed. Suitability for being assessed in a large-scale international study. Contribution to overall test balance and coverage of content and cognitive domains" (p. 5)

This clearly shows that this assessment framework is appropriate for the goal of the research i.e., determining cognitive domains and most importantly, it has an "alignment of the content domains with the reporting categories of TIMSS 1995 and TIMSS 1999" (p.5) and it has an "overall .. coverage of .. cognitive domains" (p. 5). This framework is appropriate for lower secondary level, since 8th graders are the second population of TIMSS 2003 (Mullis et al., 2003). So it is concluded that this framework is appropriate for the research purpose.

How meaningful will be the results? This question is directly related to the "meaningfulness" of the study. Determining cognitive requirements in terms cognitive domains and underlying behaviors explained above, very important to understand the impact of textbooks on students' performance. For example, it may give some explanation of Turkish students' very poor performance in TIMSS 1999 (Mullis et al., 2000). So, it was concluded that the research is meaningful.

How useful will be the results? This is another question related to validity of this study and should be carefully examined. Such an analysis (i.e. determining behaviors that the problems require to respond appropriately) would give invaluable information about Turkish texts. More importantly, test developers may obtain very useful information about such an analysis and this will probably influence the variability of textbook problems in terms of their cognitive domains. Those are found enough evidence for the usefulness of the study. As a result, the aspects of the study found enough to make valid inferences on the research question.

Data Analysis

The coding results were analyzed and compared among the textbooks in terms of the percentages of the behaviors and cognitive domains that these problems required. The same analysis was done for TIMSS 1999 geometry items. The results were interpreted in the context of the textbooks and TIMSS 1999 geometry items. This interpretation is descriptive in nature and serves as a means to identifying the nature of similarities and differences across the textbooks from Turkey. This

interpretation has also been done between the textboks and TIMSS 1999 in terms of their geometry problems.

To answer the research question, the summarized results of the sub-questions were used. For the first three sub-questions about the required behaviors of the textbook were determined. A similar analysis was also done for the fifth sub-question. For the fourth and sixth sub-questions, distributions of the problems among different textbooks and TIMSS 1999 items were compared to examine qualitative and quantitative differences. For the last two sub-questions, the distribution of the cognitive domains of the textbooks and TIMSS items were determined and compared.

Coding Examples

In the next pages of this chapter, there will be figures, from Figure 3.1 to Figure 3.15, to indicate examples for 15 distinct behaviors that appeared in the geometry chapters of the textbooks. As seven of 22 behaviors did not appear in the textbooks, not any example for these could be given.

Problem 3 in "Test 6" of 6th-grade textbook (p. 156)**:**
How many planes can pass through three non-linear points?

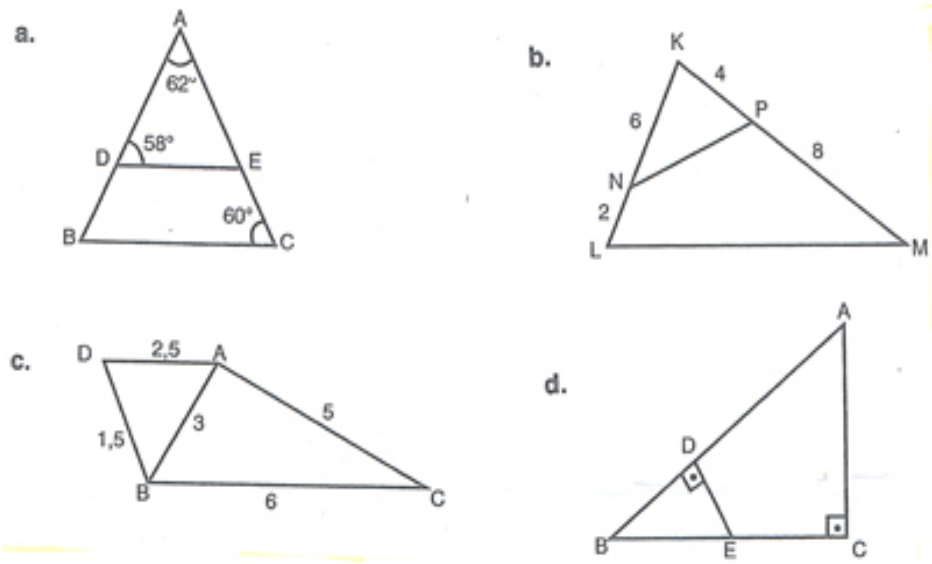
a) 1 b) 2 c) 3 d) 4

Coding: Recall (11)

Figure 3.1. Sample textbook problem cognitive requirement of which is "Recall".

In the 6th-grade textbook, after giving some properties of planes including "three distinct points determine a plane" (p. 148), this question 3 was posed. Therefore, the required behavior for this question is recalling this property.

Problem 1 in Exercises" of 8th-grade textbook (p. 82):
In each of the following figures, there are two triangles given. Search for their similarity.



Coding: Recognize (12)

Figure 3.2. Sample textbook problem cognitive requirement of which is "Recognize".

Similarity of figures is given in the 8th-grade textbook as "the figures in the same format but not necessarily in the same measure" (p. 75). The similar triangles in the figure can be seen as a rotated form of each other. So, the figures can be seen as "differently oriented simple geometric figures" (Table 3.2). The required behavior can be seen as recognizing or identifying differently oriented simple geometric figures. Thus, this problem was coded as "recognize"

Problem 25 in "Test 5" of 7th-grade textbook (p. 164):

How many sides does a polygon whose the sum of the measure of interrior angles is 1980° have?
a) 11 b) 12 c) 13 d) 14

Coding: Compute (13)

Figure 3.3. Sample textbook problem cognitive requirement of which is "Compute".

The formula for the sum of interior angles of a polygon in terms of the number of its sides is given in the textbook (p. 145) and then the problem is posed. The required behaviors are recalling this formula and computing the number of the sides or "conform of the formula" (in Table 3.2). So Racall and Compute are two required behaviors for this problem. However, Compute (13) has a higher coding number than Recall (11) has. For this reason, this problem was coded as "Compute".

Problem 5 in "Exercises" of 6th-grade textbook (p. 149):
On your notebook locate five points three of which are non-linear. Draw all lines that pass through those points.

Coding: Use tools (14)

Figure 3.4. Sample textbook problem cognitive requirement of which is "Use tools".

Problem 5 requires locating points and the drawing lines passing through that points. To do this, the student needs to use a straight edge. So the problem was coded as "Use tools" with the coding number of 13.

Problem 1 in "Exercises" of 7th-grade textbook (p. 143):

Which of the following given three side lengths form a triangle? Write your answer down.

a) a = 4 cm, b = 3 cm, c = 1 cm
b) a = 7 cm, b = 9 cm, c = 12 cm
c) a = 5 cm, b = 8 cm, c = 3 cm
d) a = 6 cm, b = 5 cm, c = 7 cm

Coding: Know (21)

Figure 3.5. Sample textbook problem cognitive requirement of which is "Know".

This problem requires mainly knowing the triangle inequality on the lengths of the sides of a triangle. That is, if student know this property of a triangle then it is expected for her to solve this problem. Therefore, this problem was coded as "Know".

Problem 7 in "Exercises" of 7th-grade textbook (p. 163):

According to the figure:

a) Which angles are acute?

b) Which angles are right?

c) Which angles are obtuse?

Coding: Classify (22)

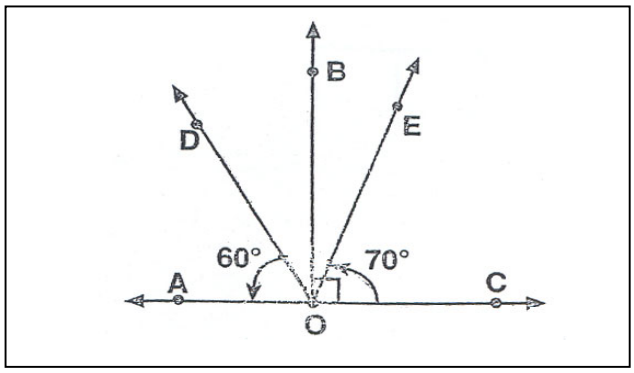


Figure 3.6. Sample textbook problem cognitive requirement of which is "Classify".

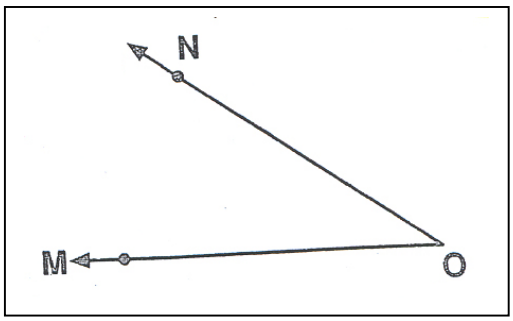
For this problem, the expected behavior is recalling the vocabulary about some concepts and classifying given figures according to their common properties. Since Classify (22) has a higher coding number than Recall (11) this problem was coded as “Classify”.

Problem 1 in “Test 7” of 6th-grade textbook (p. 178)**:**

Which of the following does not indicate the angle in the figure?

a) <NOM b) <MON

c) <O d) <MNO



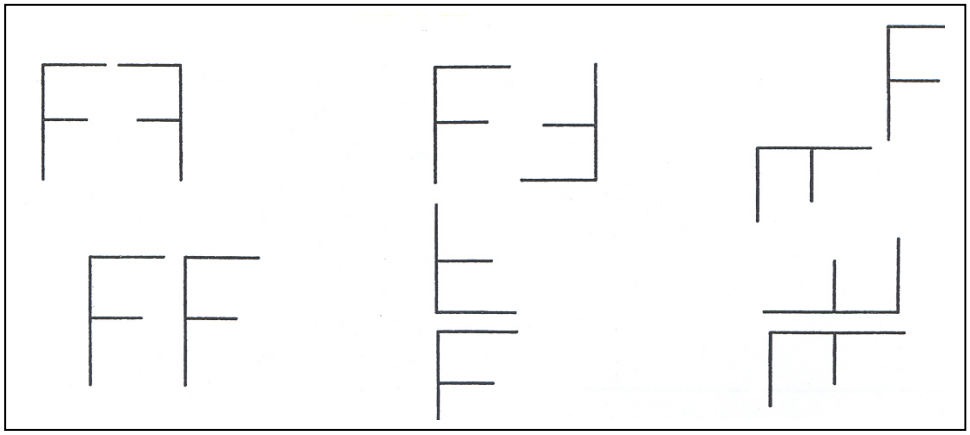
Coding: Represent (23)

Figure 3.7. Sample textbook problem cognitive requirement of which is “Represent”.

This problem requires student to “generate equivalent representations of a given mathematical quantity” (in Table 3.2). Therefore, this problem was coded as “Represent” with the coding number as 23.

Problem 6 in "Exercises" of 7th-grade textbook (p. 83):

In which of the followings the symmetry axes can be drawn?
Draw the symmetry axes of which that can be drawn?

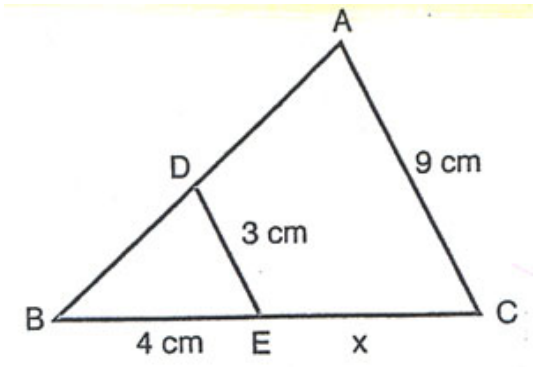


Coding: Distinguish (25)

Figure 3.8. Sample textbook problem cognitive requirement of which is "Distinguish".

This problem can be seen as a combination of six problems for which the same question is asked. Student is expected to "distinguish the questions that can be addressed by given information from those that cannot" and this is given in the fifth box of the second column of the box.

Problem 2 in "Exercises" of 8th-grade textbook (p. 82):



ABC triangle; **[DE]∥[AC]**, **|DE|** = 3 cm, **|AC|** = 9 cm, **|BE|**= 4 cm, **|EC|** = x

According to the data above, what cm is the **|EC|** = **x** ?

Coding: Apply (35)

Figure 3.9. Sample textbook problem cognitive requirement of which is "Apply"

In this problem student is expected first to recognize the similarity of the triangles and then apply this "knowledge of .. concept to solve routine mathematical .problems" (in Table 3.2). This behavior is appeared in the fourth box of the third column of the coding scheme i.e., "Apply".

Problem 10 in "Test 5" of 8th-grade textbook (p. 181):

A sphere of 13 cm radius is intersected by a plane that is 5 cm away from the center.

What cm is the perimeter of the intersected circular are? (Take $\pi = 3$)

a) 64 b) 72 c) 78 d) 86

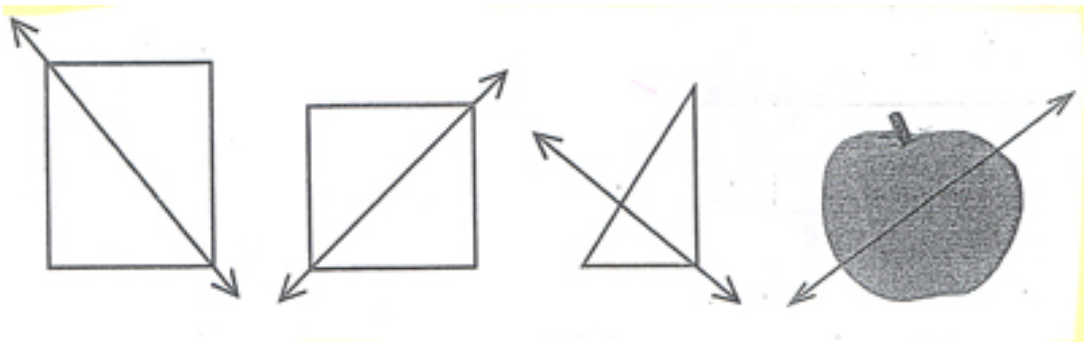
Coding: Analyze (42)

Figure 3.10. Sample textbook problem cognitive requirement of which is "Analyze".

This problem requires student to "determine and describe .. relationships between objects in mathematical situations" (in Table 3.2). The relation between a sphere and a circle is expected to be determined for this problem. Therefore, this problem was coded as "Analyze".

Problem 8 in "Exercises" of 7th-grade textbook (p. 84):

Determine whether the lines drawn on the figures below are the symmetry axes or not. Explain the reason why for the ones that are not symmetry axes.



Coding: Evaluate (43)

Figure 3.11. Sample textbook problem cognitive requirement of which is "Evaluate".

The problem can be seen of the evaluation of the statement conjecturing that the lines intersecting the figures are their symmetry lines. In this problem, student is expected to "discuss and critically evaluate a conjecture" (in Table 3.2). Therefore, this problem was coded as "Evaluate".

Problem 10 in "Exercises" of 6th-grade textbook (p. 201)**:**

The longer side of a rectangular field, whose area is 40 *a* (1 *a* = 100 meter-square), is 80 m. What meter-square is the area of a square field whose perimeter is equal to the perimeter of the rectangular field?

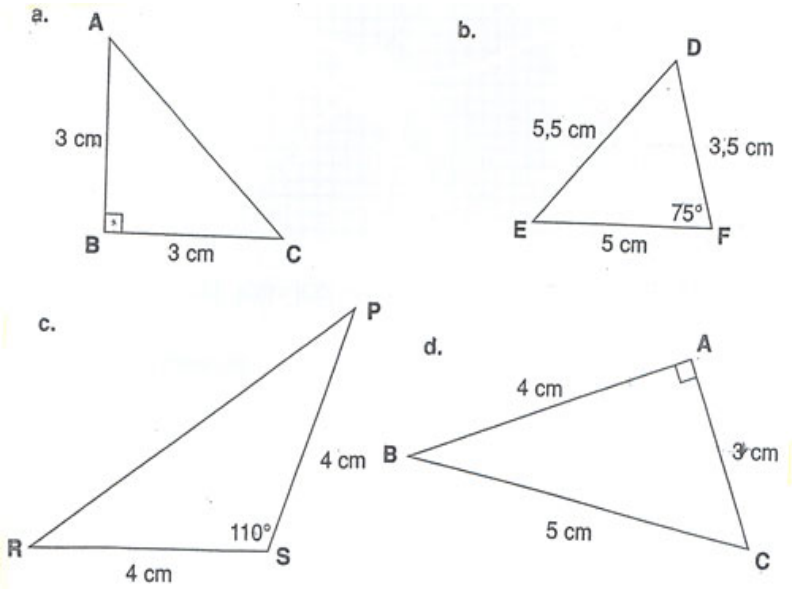
Coding: Connect (45)

Figure 3.12. Sample textbook problem cognitive requirement of which is "Connect".

The area and perimeter of the rectangle are different elements of the knowledge and the connection between them is expected to be established for this problem. Therefore, the problem was coded as "Connect".

Problem 6 in "Exercises" of 6th-grade textbook (p. 177)**:**

Label the following triangles according to their angles and sides.



Coding: Synthesize (46)

Figure 3.13. Sample textbook problem cognitive requirement of which is "Synthesize".

In this problem some elements of the problems i.e., side lengths and an angle of the triangles were given. Student is first expected to recognize the type of triangle according to its length. Then the student is expected to decide or at least make a conjecture on the measures of the other angles of the triangles based on the given side lengths of the triangle. After this, the student will be able to label the triangle according to both their angles and side length. So this example requires student to "synthesize .. mathematical procedures". This example helps us make the difference between cognitive complexity and item difficulty. Although this example requires

higher order thinking such as synthesizing, it can be expected that the most of the students will solve the problem.

Problem 12 in "Problems" of 6th-grade textbook (p. 229)**:**

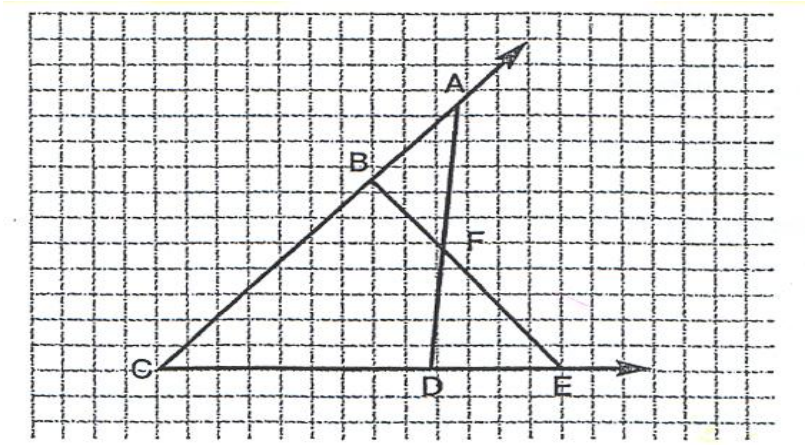
The wool needed to weave a carpet of 1.3 m width and 220 cm length, is used to weave another carpet whose width is 1.1 meter. What cm will the length of this carpet be?

Coding: Solve non-routine problems (47)

Figure 3.14. Sample textbook problem cognitive requirement of which is "Solve non-routine problems".

This problem was found as "non-routine" for two reasons. First, the context is different. In most cases the problems are either in a familiar real context such as "field" or a mathematical context such as parallelogram but here it is a real object. The second reason is that area of a carpet and the wool needed to weave that carpet was connected. Such a connection was not found in the textbooks.

Problem 4 in "Exercises" of 8th-grade textbook (p. 73):



In the figure **|CA|** = **|CE|**; **|CB|** = **|CD|**. Show that $\Delta BCE \cong \Delta DCA$

Coding: Justify (48)

Figure 3.15. Sample textbook problem cognitive requirement of which is "Justify".

In this problem, student is expected to provide evidence for the truth of the statement about congruence of the given triangles by referencing mathematical results or properties.

Summary of the Methodology

This is a qualitative study that used content analysis techniques to examine the cognitive requirements, cognitive domains and underlying behaviors, of geometry problems in Turkish mathematics textbooks to compare them with cognitive requirements of TIMSS 1999 geometry problems. Three widely used and widely sold lower secondary Turkish mathematics textbooks were selected for the analysis. The geometry content was based on TIMSS 1999 geometry topics (Mullis et al., 2000). However, since some of TIMSS 1999 geometry topics are not directly covered by the textbooks, some other topics in the textbooks which are concluded as

related to those TIMSS 1999 topics were selected for the analysis, in addition to the ones directly matches TIMSS 1999 topics. In this study, the problems that do not have accompanying solutions, to-be-solved problems, were selected for the analysis. Nonetheless, content presentations and to-be-solved problems sections were compared in both number of pages devoted to and the number of problems to have a better picture of the selected contents. 17 of 21 TIMSS 1999 geometry items had been released and they were also selected for the analysis to compare textbooks and TIMSS 1999 in terms of cognitive requirements of their geometry problems. For the analysis of cognitive requirements of the problems, TIMSS 2003 Mathematics Assessment Framework was used (Mullis et al., 2003). Both cognitive domains and their underlying behaviors were determined according to the selected framework. SPSS for Windows (Released 10.0.7 2000) was used to analyze the data on the behaviors of the problems of textbooks and TIMSS 1999, and the results were given in tables and graphs.

CHAPTER 4

RESULTS

In this chapter the results of data analysis are reported to answer the research question "What are the similarities and differences of to-be-solved geometry problems in lower secondary Turkish mathematics textbooks and TIMSS 1999 geometry items with respect to their cognitive requirements?" and its sub-questions.

Required Behaviors of To-be-solved Geometry Problems in the Textbooks

The required behaviors of the to-be-solved geometry problems of the textbooks are displayed in Figure 4.1, was used. In this figure, required behaviors of 6th-, 7th- and 8th-grade textbooks are given comparatively. As the textbooks have different numbers of to-be-solved geometry problems (see Table 3.4) the distributions of the problems over behaviors are given in percentages to compare required behaviors of the textbooks.

The to-be-solved geometry problems in 6th-grade textbook which require behaviors such as Apply, Compute, Recall, Analyze, Use tool, Represent, Classify constitute a total percentage of 97.50. Each group of the textbook problems that represent each one of these behaviors has constitute at least 1 percent of the textbook problems. The rest 2.50 % of the to-be-solved geometry problems of the 6th-grade textbook represent 3 behaviors of 22. Therefore, 12 of the behaviors of the framework i.e., Recognize, Know, Formulate, Distinguish, Select, Model, Interpret, Verify, Conjecture, Evaluate, Generalize, and Justify, are not represented in the textbook and this is very striking. In case of the 7th-grade textbook, the to-be-solved

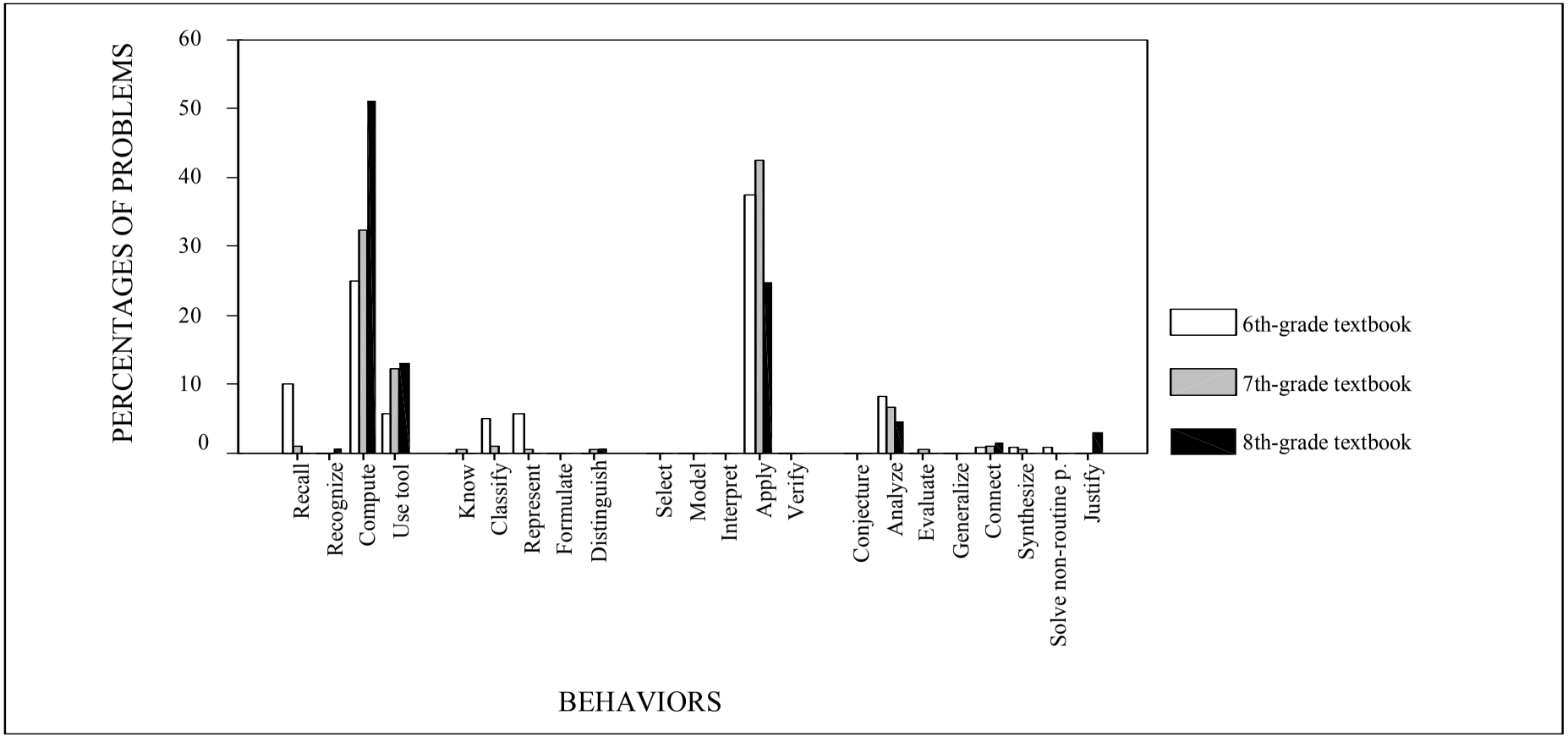


Figure 4.1. Percentages of geometry problems in 6th-, 7th-, and 8th-grade Turkish mathematics textbooks according to their required behaviors.

geometry problems which require behaviors such as Apply, Compute, Use tool, Analyze, Recall, Classify, and Connect constitute 97.21 % of the textbook problems. which is striking, too. Recognize, Formulate, Select, Model, Interpret, Verify, Conjecture, Generalize, Solve non-routine problems, and Justify are not represented by any of the to-be-solved geometry problems in the 7th-grade textbook. In case of 8th-grade textbook, the most frequent six required behaviors are Compute, Apply, Use tool, Analyze, Justify, and Connect with a total percentage of 98.45. Recognize, Distinguish are the other behaviors represented in the 8th-grade textbook. In this case, 14 of the behaviors of the framework are not reresented.

Those all above show two basic results: first, the distributions of the problems over behaviors are odd for each case of the textbook. Some behaviors have not been represented in the textbooks: Formulate, Select, Model, Interpret, Verify, Conjecture, and Generalize. Second, a great majority of the 22 behaviors, if any, have very small percentages compared to the percentage of the perfect even distribution of all behaviors i.e., 100 / 22 or about 4.55. When we search for the similarities and differences between the textbooks we see that similarities are larger than the differences among the textbooks: 6th-, and 7th-grade textbooks resemble to each other in terms of the leading behaviors. In both cases, Apply is the most frequent behavior and Compute is the second. For the 6th-grade textbook, Apply and Compute have percentages 37.50 and 25.00, respectively. For the 7th-grade textbook, the percentages for these behaviors are 42.46 and 32.40, respectively. This order is changed for 8th-grade textbook for which Compute is the most frequent and Apply is the second. For this case, Compute has a percentage of 51.16 and Apply has a percentage of 24.81. For the other frequent behaviors we can say that there is also a similarity. The third most frequent behavior for 6th-, 7th-, and 8th-grade textbooks are Recall, Use tool, and Use tool respectively and thses all are in the same cognitive domain i.e., Knowing Facts and Procedures. The fourth frequent behavior is the same for all those: Analyze with percentages of 8.33, 6.70, and 4.65. It can be said that the percentage for the problems requiring Analyze decreases as the grade level increases.

For the fourth sub-question related to required behaviors of the overall to-be-solved geometry problems in the textbooks, we see that the most frequent behaviors that have percentages greater than or equal to 1 are Compute (35.98 %), Apply (35.75 %), Use tool (10.75 %), Analyze (6.54 %), Recall (3.27 %), Classify (1.87 %), Represent (1.87 %), and Connect (1.17 %) with a total percentage of 97.20. The rest 14 of 22 behaviors have a total percentage of 2.80. The second important result of this figure is that the two most frequent behaviors of the to-be-solved geometry problems in all textbooks are Compute and Apply with an average percentage greater than 71. This shows that a great majority of the geometry problems of the textbooks require computing and applying.

Required Behaviors of the Geometry Problems of the Textbooks and TIMSS 1999

The required behaviors of TIMSS 1999 geometry problems and the textbooks are given comparatively in Figure 4.2. As the numbers in the cases are different (17 problems in TIMSS 1999 and 428 to-be-solved geometry problems in the textbooks) the distributions of the problems over behaviors are given in percentages. Figure 4.2 immediately indicates that there is a more even distribution of behaviors in TIMSS 1999 case. The two most frequent behaviors are Apply and Analyze with a percentage of 29.41 and 17.65. When we compare the textbooks and TIMSS 1999 in terms of the required behaviors of their geometry problems we see some similarities and differences: The immediate difference is that Compute is the most frequent behavior for the textbooks with a percentage of 35.98 but it has only 5.88 percentage in TIMSS 1999 case. Another difference is that the percentages of the most frequent three behaviors constitute 58.82 and 82.45 percent cases for TIMSS and the textbooks, respectively. On the other hand, for both cases Apply is a very frequent behavior for both cases with percentages of 35.75 and 29.41 for the textbooks and TIMSS 1999, respectively. Another similarity is that: for both cases, 3 of the most frequent 5 behaviors coincide: Apply, Analyze, and Recall. The problems requiring these constitute 58.82 and 45.56 percent of the problems in TIMSS 1999 and the textbooks cases, respectively.

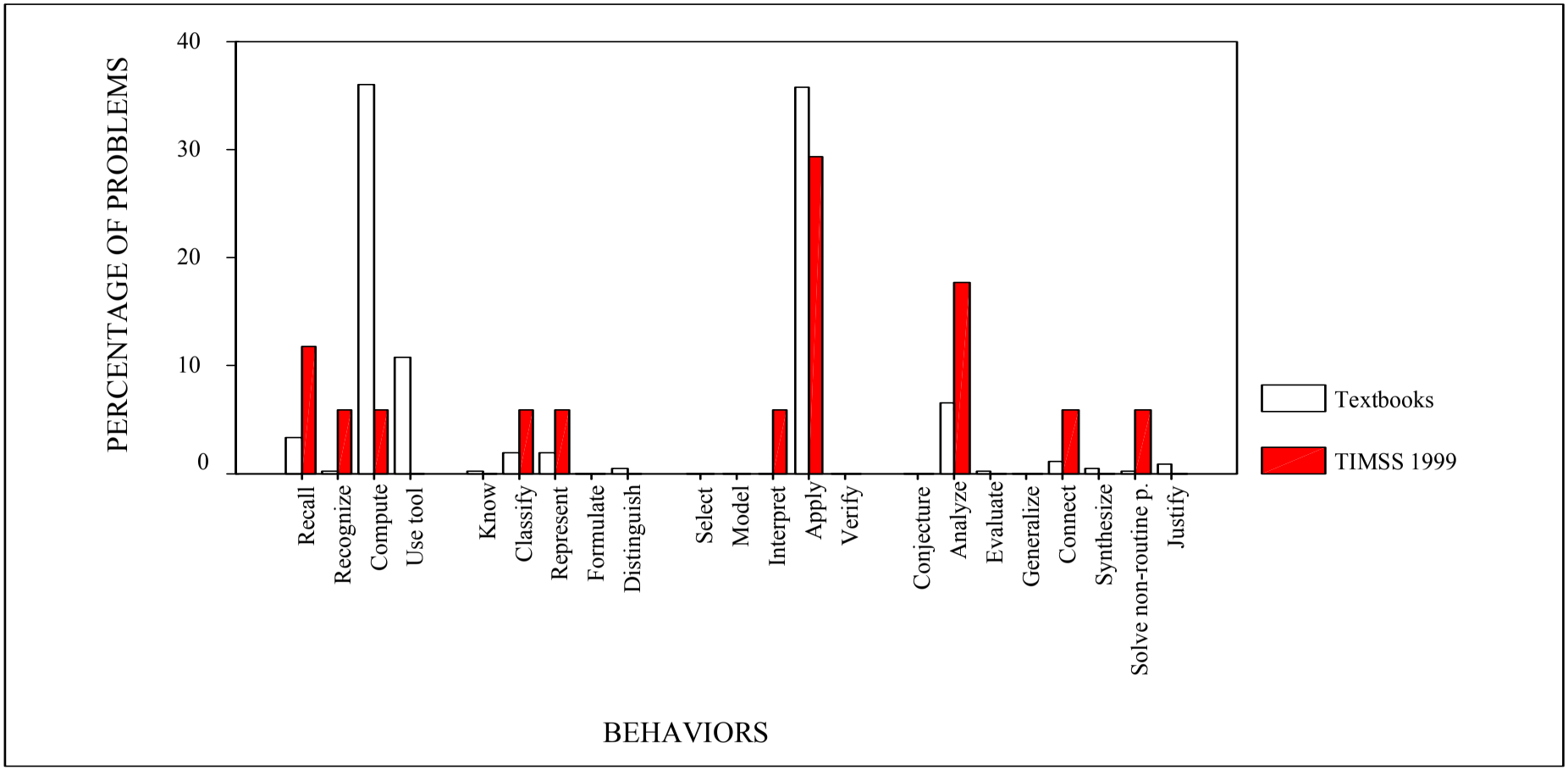


Figure 4.2. Percentages of geometry problems in Turkish lower secondary mathematics textbooks and TIMSS 1999 according to their required behaviors

Cognitive Domains of the Geometry Problems of the Textbooks and TIMSS 1999

Table 4.1 presents the overall results of the study. This table reveals that 6th- and 7th-grade textbooks resemble each other in terms of the percentages of their problems in "Knowing Facts and Procedures" and "Solving Routine Problems" cognitive domains. However, 65% of geometry problems of 8th-grade textbook are in "Knowing Facts and Procedures" cognitive domains. For the three textbooks, however, the percentages of items that require "Using Concepts" and "Reasoning" are very rare. This table also shows that there are clear differences between lower secondary Turkish textbooks and TIMSS 1999 in terms of the cognitive domains of their geometry problems. Despite the similarity between their percentages of problems in "Solving Routine Problems" the percentages of problems in other cognitive domains are different. Turkish textbooks contain problems mainly in the cognitive domains "Knowing Facts and Procedures" and "Solving Routine Problems". Only 9 % of geometry problems of Turkish textbooks require "Reasoning" but 29 % of geometry problems of TIMSS 1999 require "Reasoning". Another important result is that the problems that require "Using Concepts" are very rare in Turkish textbooks.

Summary of the Results

As Figure 4.1 indicates, the distribution of the required behaviors of to-be-solved geometry problems across all three grade levels resemble to each other. In these textbooks, "Compute" and "Apply" are the most frequent ones among 22 behaviors. The percentage of problems that require one of the two most frequent behaviors i.e., Compute or Apply, took 62.50 %, 74.86 and 75.97 % for 6th-, 7th-, and 8th-grade textbooks respectively. This shows that majority of the problems of all the textbooks requires only two behavior. The next most frequent behaviors are "Use tools" and "Analyze". The most four frequent behaviors of geometry problems of the textbooks took 89.02 percent of the overall geometry problems of the textbooks. This can be seen as a clear evidence of not even distribution of the behaviors. There is no, or in some cases just one, problem requiring behaviors "Recognize", "Know",

”Formulate”, “Select”, “Generate”, “Interpret”, “Follow”, “Check”, “Conjecture”, “Evaluate”, “Generalize”, “Solve non-routine problems”. It can be said that more than half of the behaviors are not represented in Turkish curriculum.

The required behaviors of the released TIMSS 1999 geometry items has much more variability than the required behaviors of geometry problems of the Turkish textbooks as Figure 4.2 shows. Not only in required behaviors but also in cognitive domains, TIMSS geometry items are more balanced than the textbooks as the comparative Figure 4.15 shows,. For TIMSS 1999, the percentage of items in cognitive domains “Knowing Facts and Procedures”, “Using Concepts”, “Solving Routine Problems”, and “Reasoning” are 23.53 %, 11.76 %, 35.29 %, and 29.41 % respectively. On the other hand, for the textbook geometry problems these percentages are 50.23 %, 4.44 %, 35.75 %, and 9.58 % respectively. This is a clear evidence for that the Turkish texts do not reflect cognitive domains and their underlying behaviors of TIMSS 2003 Mathematics Assessment Framework as much as TIMSS 1999 geometry items do. Although the distribution of geometry items of TIMSS 1999 across cognitive domains looks balanced, the distribution of them in required behaviors is not so balanced. For example the first two most frequent performance expectations, namely “Apply” and “Analyze” together took 47.06 % of TIMSS 1999 geometry items.

In general, it can be concluded that Turkish students are exposed to a less variability of geometry problems in terms of their cognitive dimension (i.e. cognitive domains and underlying behaviors) comparing to TIMSS 1999 geometry items. For each grade textbook, the two most frequent cognitive domains are “Knowing Facts and Procedures” and “Solving Routine Problems”. However, in case of the TIMSS 1999 geometry items, they are “Solving Routine Problems” and “Reasoning”.

Table 4.1

The cognitive requirements of geometry problems in lower secondary Turkish mathematics textbooks.

TEXTBOOKS (GEOMETRY CHAPTERS) → / REQUIRED BEHAVIORS ↓	6TH-GRADE TEXTBOOK				7TH-GRADE TEXTBOOK				8TH-GRADE TEXTBOOK		
	Point, Line, Plane, Space, Line Segment, Ray	Angle, Triangle	Measures	6th grade total	Equations, Line Graphs	Angles, and Polygons	Circle, Cylinder	7th grade total	Proportional Line Segments, Similar Triangles	Surface Areas, Volume	8th grade total
11 Recall	10	2	0	12 (10 %)	2	0	0	2 (1.12 %)	0	0	0 (0 %)
12 Recognize / Identify	0	0	0	0 (0 %)	0	0	0	0 (0 %)	1	0	1 (0.76 %)
13 Compute	0	12	18	30 (25 %)	8	33	17	58 (32.40 %)	24	42	66 (51.16 %)
14 Use tools	4	3	0	7 (5.83 %)	14	8	0	22 (12.29 %)	17	0	17 (13.19 %)
Knowing Facts and Proc. (11 - 14)	14	17	18	49	24	41	17	82	42	42	84
21 Know	0	0	0	0 (0 %)	0	1	0	1 (0.56 %)	0	0	0 (0 %)
22 Classify	1	5	0	6 (5 %)	2	0	0	2 (1.12 %)	0	0	0 (0 %)
23 Represent	2	5	0	7 (5.83 %)	1	0	0	1 (0.56 %)	0	0	0 (0 %)
24 Formulate	0	0	0	0 (0 %)	0	0	0	0 (0 %)	0	0	0 (0 %)
25 Distinguish	0	0	0	0 (0 %)	1	0	0	1 (0.56 %)	1	0	1 (1.76 %)
Using Concepts (21 - 25)	3	10	0	13	4	1	0	5	1	0	1
31 Select	0	0	0	0 (0 %)	0	0	0	0 (0 %)	0	0	0 (0 %)
32 Model	0	0	0	0 (0 %)	0	0	0	0 (0 %)	0	0	0 (0 %)
33 Interpret	0	0	0	0 (0 %)	0	0	0	0 (0 %)	0	0	0 (0 %)
34 Apply	11	22	12	45 (37.5 %)	0	55	21	76 (42.46 %)	24	8	32 (24.81 %)
35 Verify / Check	0	0	0	0 (0 %)	0	0	0	0 (0 %)	0	0	0 (0 %)
Solving Routine Prob. (31 - 35)	11	22	12	45	0	55	21	76	24	8	32
41 Hypothesize / Conjecture/	0	0	0	0 (0 %)	0	0	0	0 (0 %)	0	0	0 (0 %)
42 Analyze	6	2	2	10 (8.33 %)	1	0	11	12 (6.70 %)	1	5	6 (4.65 %)
43 Evaluate	0	0	0	0 (0 %)	1	0	0	1 (0.56 %)	0	0	0 (0 %)
44 Generalize	0	0	0	0 (0 %)	0	0	0	0 (0 %)	0	0	0 (0 %)
45 Connect	0	0	1	1 (0.83 %)	0	1	1	2 (1.12 %)	0	2	2 (1.55 %)
46 Synthesize / Integrate	0	1	0	1 (0.83 %)	0	1	0	1 (0.56 %)	0	0	0 (0 %)
47 Solve non-routine problems	0	0	1	1 (0.83 %)	0	0	0	0 (0 %)	0	0	0 (0 %)
48 Justify / Prove	0	0	0	0 (0 %)	0	0	0	0 (0 %)	4	0	4 (3.10 %)
Reasoning (41 - 48)	6	3	2	13	2	2	12	16	5	7	12
TOTAL of PROBLEMS	34	52	34	120 (100 %)	30	99	50	179 (100 %)	72	57	129 (100 %)

CHAPTER 5

DISCUSSION

The goal of this research is to determine the cognitive requirements of geometry problems in Turkish lower secondary textbooks, according to the TIMSS 2003 Mathematics Assessment Framework. The most important finding of this study is that over 70 % of geometry problems of Turkish textbooks require computation and application. This finding is consistent with the literature such as Olkun & Toluk (2002) who found that "…overemphasis on procedures in the Turkish Curriculum…" (p. 5). It is also consisted with Toluk & Olkun (2002) too as they found that Turkish textbooks have traditional view of "…application of learnt facts…" (p. 579) for lower elementary grades case.

It is clear from Figure 4.1 that there is no harmony between the required behaviors of to-be-solved geometry problems of Turkish textbooks and the framework we have used in this study. It seems that, the designers of the textbook series did not attempt to provide an even distribution of geometry problems across the various types of behaviors. Besides, they did not concern to rules that suggests a variation, as well as repetition, which is crucial for learning. Here, we can only conjecture that these weaknesses are very important in developing geometry problem solving skills among elementary school children in Turkey. Perhaps this was caused by the mathematics curricula of Turkey. As it was mentioned earlier, there is no variation among the textbooks analyzed in this study, in terms of the problem types. Another important reason for this result is that the curriculum overemphasizes the objectives of the course. In the mathematics curriculum of lower secondary schools we see that "goals" and "objectives" are emphasized (Babadoğan, & Olkun, 2006). Providing knowledge and procedures related to a topic with some application is probably found enough by the authors of the textbooks. That is to say, this odd

distribution of the problems according to their required behaviors most probably resulted from the curriculum of the subject.

To understand Turkish students' performances in TIMSS 1999 better, five geometry-related items of the released set of TIMSS 1999, for which international standings were given, were examined. These items were given as Figures C.1., C.3., C.7., C.8., and C.11. For these items, the average standings of Turkish students on theese items is about 33 and very close to 34, the standings of Turkish students' performance on overall geometry items (Mullis et al., 2000). The items with the codes J10, T03, and R11 are related to Measures chapter of the 6th-grade textbook. There are 34 to-be-solved problems are exposed in this chapter related to these items. One of the the other two items is the item with the code J16, which is related to Equations and Line Graphs chapter of 7th-grade textbook. There are 30 problems are exposed in the textbook related to this item. The remained item has the code P10 and related to Proportional Line Segments and Similar Triangle chapter of the eighth-grade textbook. There are 72 problems exposed for these items. It could be concluded that there is enough material in the textbooks to answer these items. However, average of standings of Turkish students is 36 in this case. This lead us understand that a content based analysis of the items will not be enough to understand the student performances. In the chapter related to the items, about 33 percent of the problem requiring application (see Table 4.1). From this point, it could be concluded that a large enough amount of opportunity to solve a particular type of problem in terms of cognitive behaviors were given in the textbook. However, for these three requiring application, the student performances are relatively worse compared to overall performance on mathematics items. This leads us to conclude that exposing problems that require a particular type behavior in the textbook will not guarantee the students to solve them. Of course, teaching practices are very important in this point. Although it was shown that textbook influence what to teach in the class, how to teach that material is primarily depends on teachers in Turkey. It is known that traditional ways of teaching (e.g. lecturing, or drill and exercise) is common in Turkish textbooks (Toluk, & Olkun, 2002) and therefore among Turkish teachers. This automatically results in a teacher-centered education in which students

are passive learners and there are strict opportunities of improving their thinking skills.

As it was mentioned before, opportunity to learn is an important factor for explaining performance differences. Among the other components of opportunity to learn, opportunity to solve a particular type of problem is an important one. For example, solving a particular type of problem may lead students to be familiar with the problem type and positively affect the performances (Jonassen, 2000). When we concentrate on the behaviors of the item, we see that Turkish students' performance on the items that requires computation are relatively better (see Table 2.2). As it is found for geometry case, about 36 percent of the textbook problems require computation and this is the highest percetage among all 22 behaviors. From this point of view, it can be concluded that providing a particular type of problem affected the students' performances. However, determining the degree of exposure of a particular type of problem does not provide understanding the performance related to these type of problems. As it is clear from Figure 4.2, the textbooks emphasized computation procedures and application of the learnt facts. As opportunity to solve a particular type of a problem is a component of opportunity to learn given before, the emphasis of the problems requiring computation and application behaviors could affect the relative success of Turkish students on the TIMSS items these behaviors. For this issue, the results of Table 2.2 given will be discussed here. When we examine the table, we see that on the items requiring computation, Turkish students performance is relatively better compared to Turkish students standing on overall mathematics items (see Appendix D). From the table we see that Turkish students' average performance on the items requiring computation is 26 is lower than 32, performance of the overall TIMSS mathematics items (see Appendix D). However, on the case of the items requiring application Turkish students performed poorly. Although the problems requiring application was emphasized in the textbooks, this situation is quite surprising. When we examine the items more closely we see that the average difficulties of these items is 0.46 and this is lower than 0.69, the average difficulty of the items requiring computation. So, we may conjecture that, generally,

as the item gets more difficult, Turkish students' performance gets poorer. So this result led us to understand the importance of determining the difficulty level of test.

In Turkey, determining difficulty level of tests used in the schools is usually a subjective work of teachers. In some cases, not surprisingly however, students get very low scores on such tests. It is not easy to understand why our expectation and the reality do not match. Perhaps this situation results from the fact that "any problem is easy if you know the solution" as teachers always say. In Turkey, teachers write items or select them from a variety of books for their instructional purposes.

Up to now, two dimensions of problems (content and cognitive requirements) were introduced. Another dimension is, possibly, the context. The context, from the point of view of PISA frameworks (e.g. OECD, 1999), is that "knowledge and skills are applied or drawn on" (p. 12). Familiarity with the content (e.g. topic area, or objective) and required skills makes the problem "routine" and easy for a student. Mullis et al. (2000) state that "[T]he routine problems will have been standard in classroom exercises designed to provide practice in particular methods or techniques" (p. 30). Generally, non-routine problems are regarded as more difficult than the routine ones. So, the more non-routine problems in a test the more the test difficult will be.

Problem difficulty is not a disjoint characteristic other than content, cognitive requirements, or context but in most, it is a function of those variables. However, if we think on problem solving case, individual differences is an important factor (Jonassen, 2000) that affects problem solving and therefore problem difficulty. There are many variables (e.g. the motivational variable, abstractness, (Caldwell, & Goldin, 1987)) were also variables affecting problem solving.

Many times teachers aim to prepare a test of "moderate" difficulty level. Preparing a test of items having very good variability among distinct cognitive requirements could be claimed as a good way of preparing a "moderate" difficult test. However, this even, does not guarantee such a test with "moderate" difficulty. On this issue Mullis et al. (2003) say:

> "[N]evertheless, cognitive complexity should not be confused with item difficulty. For nearly all of the cognitive skills listed, it is possible to create relatively easy items as well as very challenging items" (p. 25)

So, as Mullis et al. (2003) determined, having an even distribution among all cognitive domains (i.e. Knowing Facts and Procedures, Using Concepts etc.) when preparing a test does not guarantee the test difficulty. Letting students practice on the problems requiring different cognitive domains will probably improve students' thinking skills. However, the items that require complex behaviors (e.g. evaluate, prove etc.) can be easy ones so one should be careful in case of adjusting overall test difficulty.

Improving thinking skills is a very ambitious aim of mathematics curricula, especially at the elementary school level (Olkun & Toluk, 2002). From this point of view, this study may help other teachers be able to classify the problems that they use in their class and this will help them improve students' thinking skills. Such a work of teachers will probably help to narrow the gap resulted from the mathematics curricula. However the best way is always putting this demand into mathematics curricula. This will affect the textbooks and then practice in classrooms.

Perhaps, the most benefit of this study is for the test developers (e.g. exercise book authors, or standard test developers, etc.). The variability of the test items should be yielded not only in content dimension but also in cognitive requirement dimension to improve thinking skills of students.

Publishing companies that prepare textbooks can also be benefited from this study. As it is known, Turkish elementary mathematics curriculum was revised (MEB, 2004; TD, 2005 c; TD, 2005 d; TD, 2005 e) and the new curriculum requires many skills such as problem posing, or communicating mathematically, that were not demanded by the previous ones. To reflect these demands, especially in problem level, textbooks should reflect those skills. This study will probably be useful for that effort.

Understanding cognitive requirements of textbook problems solely is not enough to understand students' problem solving difficulties. Although, by exposing more distinct problem types, it is possible to improve students' problem solving skills and this will improve individual differences as a variable which will affect problem solving in a positive way. However, improving students' cognitive skills will, in most cases, not be enough to improve their problem solving abilities, as there are many variables that exist on this product. Opportunity to learn is one of these variables. Törnroos (2005) showed that opportunity to learn data should be taken for long periods of time. However, this study is limited as it analyzes only the last three grades of TIMSS 1999 population i.e., 6th-, 7th-, 8th-grades. On the other hand, an analysis of textbooks is not a sufficient condition to understand what really happen in the classroom teaching. The quality of teaching is an important factor in learning environment. Not having enough knowledge about the textbooks used, leads teachers may not be able to use them in an effective way. In addition to having opportunity to learn (e.g. specific types of problems), quality of teaching, and students' characteristics also affect actual learning (Floden, 2002).

Further Study

The method used in this study (content analysis of textbook problems) can be applied to other school level mathematics textbook problems in any content area (e.g. number, data, etc.) Such a work will give valuable information about textbooks. However, the immediate work is needed to assess the problem types of the new curricula mathematics textbooks. Such a work will help us in determining the degree to which the reform textbooks reflect the new curricula.

For analyzing textbook problem types, in terms of their cognitive requirements, other assessment frameworks may be used. For example, using TIMSS 2007 Mathematics Assessment Framework (Mullis et al., 2005) will give information on both the usability of the framework and consistency between the findings of this study.

Another possible analysis can be made to gather cognitive demands, the skills that are emphasized, of the new Turkish curriculum and a standard achievement tests' framework (e.g. TIMSS 2007 framework (Mullis et al., 2005)). Such an analysis, especially for the curriculum part, may require other content analysis techniques (e.g. Weber, 1990).

In conclusion, I hope that I have demonstrated that analysis of the instructional environment is important and must be considered with task analysis in theories of how children learn. Children's difficulty with solving geometry problems probably results from developmental limitations and from restricted opportunities for learning and practice.

CHAPTER 6

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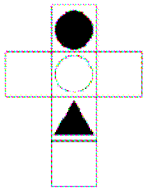
APPENDIX A
Released Geometry Items of TIMSS 1999

Cube from folded 2-dimensional net					B11
Content Category	Performance Expectation	Item Key	Score Points	International Average Percentage of 8th Grade Students Responding Correctly	Used in 1995
Geometry	Using Complex Procedures	C	1	59	Y



Which of these cubes could be made by folding the figure above?

A. 
B. 
C. 
D. 

Figure A.1. Released geometry items of TIMSS 1999 with item code B11. (From ISC, n.d. a).

Angles in symmetric polygon					D07
Content Category	Performance Expectation	Item Key	Score Points	International Average Percentage of 8th Grade Students Responding Correctly	Used in 1995
Geometry	Investigating and Solving Problems	C	1	62	Y

The line m is a line of symmetry for figure $ABCDE$.

The measure of angle BCD is

A. 30°

B. 50°

C. 60°

D. 70°

E. 110°

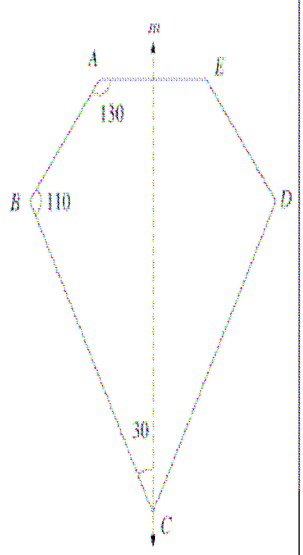


Figure A.2. Released geometry items of TIMSS 1999 with item code D07 (From ISC, n.d. a).

NOT true for all rectangles					J11
Content Category	Performance Expectation	Item Key	Score Points	International Average Percentage of 8th Grade Students Responding Correctly	Used in 1995
Geometry	Knowing	E	1	54	N

Of the following, which is NOT true for all rectangles?

A. The opposite sides are parallel.

B. The opposite sides are equal.

C. All angles are right angles.

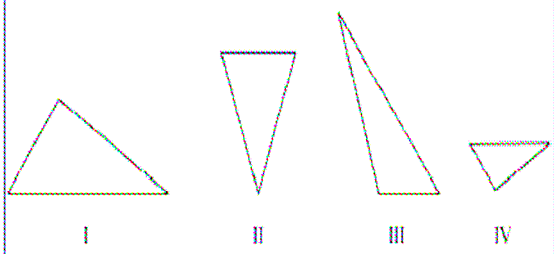
D. The diagonals are equal.

E. The diagonals are perpendicular.

Figure A.3. Released geometry items of TIMSS 1999 with item code J11 (From ISC, n.d. a).

Two similar triangles					J15
Content Category	Performance Expectation	Item Key	Score Points	International Average Percentage of 8th Grade Students Responding Correctly	Used in 1995
Geometry	Knowing	C	1	62	N

Two of the triangles below are similar.



Which two triangles are similar?

A. I and II

B. I and III

C. I and IV

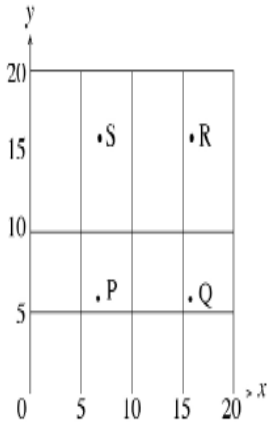
D. II and IV

E. III and IV

Figure A.4. Released geometry items of TIMSS 1999 with item code J15 (From ISC, n.d. a).

Point on graph from coordinates					J16
Content Category	Performance Expectation	Item Key	Score Points	International Average Percentage of 8th Grade Students Responding Correctly	Used in 1995
Geometry	Using Routine Procedures	D	1	58	N

Which point on the graph could have coordinates (7,16)?



A. Point P

B. Point Q

C. Point R

D. Point S

Figure A.5. Released geometry items of TIMSS 1999 with item code J16 (From ISC, n.d. a).

Measure of angle in quadrilateral					L16
Content Category	Performance Expectation	Item Key	Score Points	International Average Percentage of 8th Grade Students Responding Correctly	Used in 1995
Geometry	Investigating and Solving Problems	A	1	40	N

In a quadrilateral, each of two angles has a measure of 115°. If the measure of a third angle is 70°, what is the measure of the remaining angle?

A. 60°

B. 70°

C. 130°

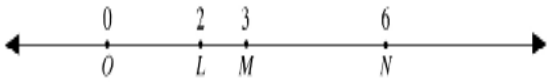
D. 140°

E. None of the above

Figure A.6. Released geometry items of TIMSS 1999 with item code L16 (From ISC, n.d. a).

Position of point on number line					N12
Content Category	Performance Expectation	Item Key	Score Points	International Average Percentage of 8th Grade Students Responding Correctly	Used in 1995
Geometry	Using Routine Procedures	A	1	42	N

Point P (not shown) on the number line is 5 units from point N and 2 units from point M.



Where is point P located?

A. Between O and L

B. Between L and M

C. Between M and N

D. To the right of N

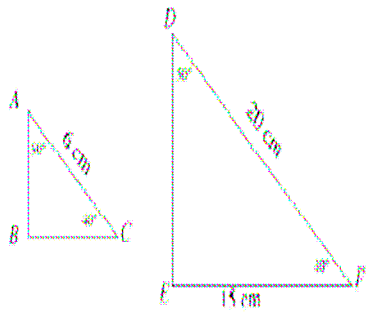
Figure A.7. Released geometry items of TIMSS 1999 with item code N12 (From ISC, n.d. a).

Length of side from similar triangle

P10

Content Category	Performance Expectation	Item Key	Score Points	International Average Percentage of 8th Grade Students Responding Correctly	Used in 1995
Geometry	Using Routine Procedures	B	1	37	N

The figure represents two similar triangles. The triangles are not drawn to scale.



In the actual triangle *ABC*, what is the length of side *BC*?

A. 3.5 cm

B. 4.5 cm

C. 5 cm

D. 5.5 cm

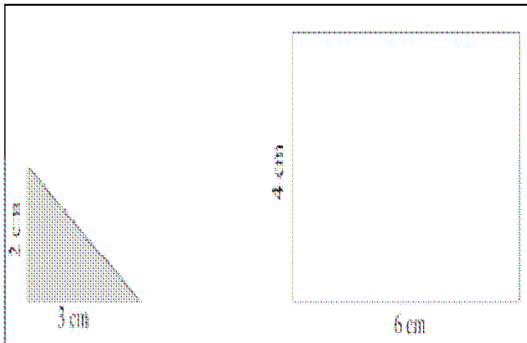
E. 8 cm

Figure A.8. Released geometry items of TIMSS 1999 with item code P10 (From ISC, n.d. a).

Right triangles to cover rectangle

R11

Content Category	Performance Expectation	Item Key	Score Points	International Average Percentage of 8th Grade Students Responding Correctly	Used in 1995
Geometry	Using Complex Procedures	C	1	46	N



How many of the shaded right triangles shown above are needed to exactly cover the surface of the rectangle?

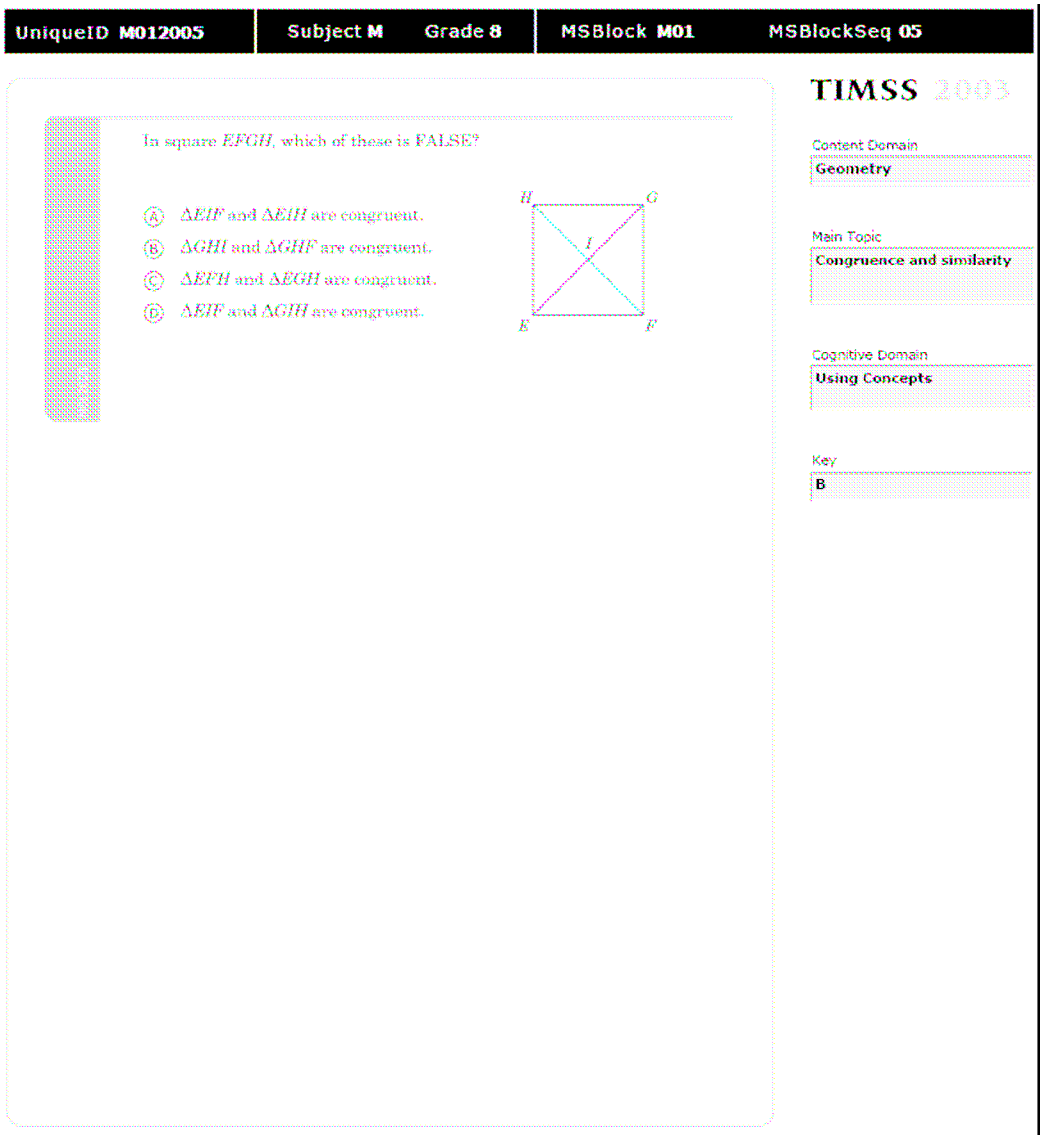
A. Four

B. Six

C. Eight

D. Ten

Figure A.9. Released geometry items of TIMSS 1999 with item code R11 (From ISC, n.d. a).



UniqueID M012005 | Subject M | Grade 8 | MSBlock M01 | MSBlockSeq 05

TIMSS 2003

In square *EFGH*, which of these is FALSE?

(A) ΔEIF and ΔEIH are congruent.
(B) ΔGHI and ΔGHF are congruent.
(C) ΔEFH and ΔEGH are congruent.
(D) ΔEIF and ΔGIH are congruent.

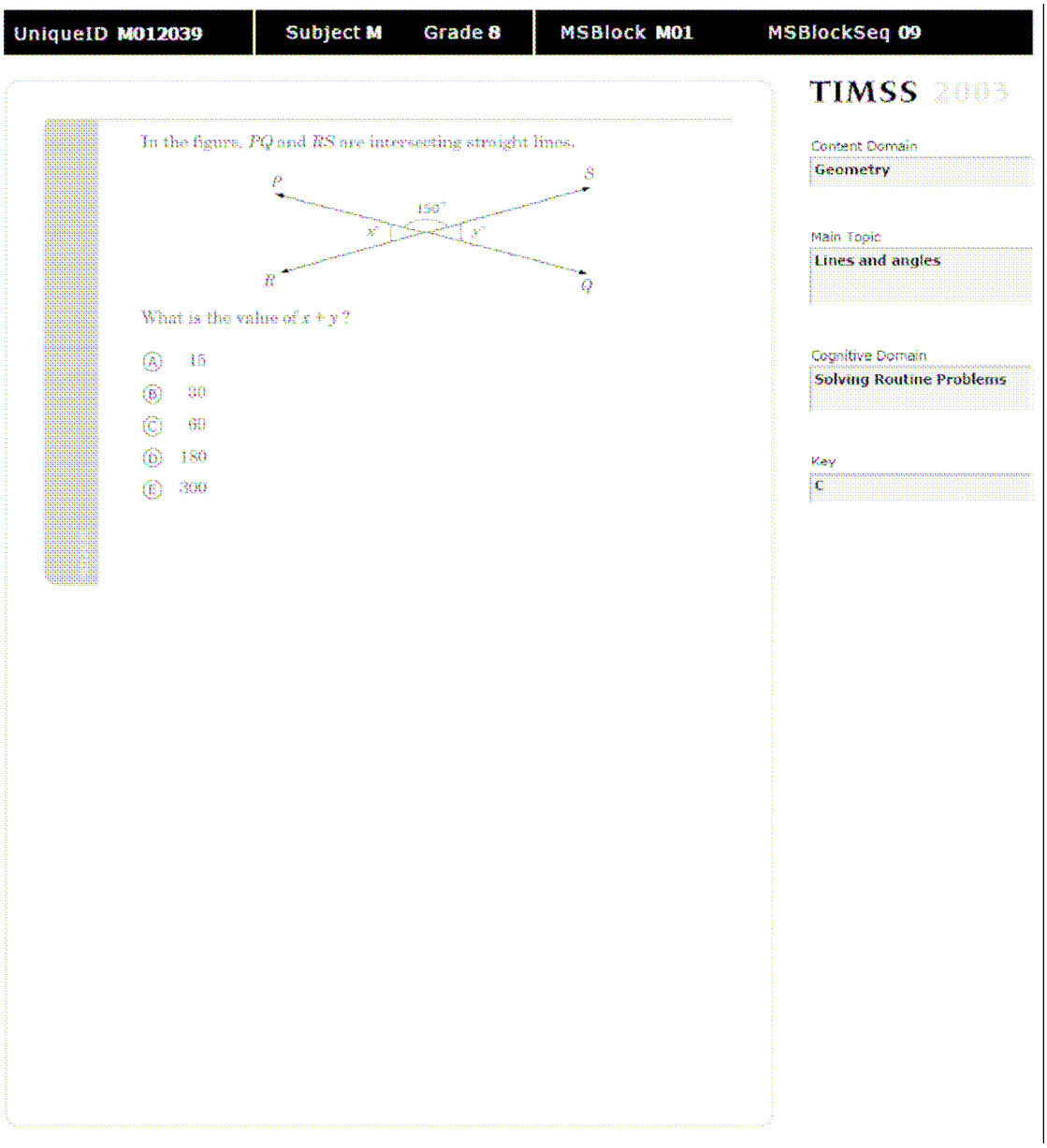
Content Domain
Geometry

Main Topic
Congruence and similarity

Cognitive Domain
Using Concepts

Key
B

Figure A.10. Released geometry items of TIMSS 1999 with item code M012005 (From ISC, n.d. b).



UniqueID M012039 | Subject M | Grade 8 | MSBlock M01 | MSBlockSeq 09

TIMSS 2003

In the figure, PQ and RS are intersecting straight lines.

What is the value of $x + y$?

(A) 15
(B) 30
(C) 60
(D) 180
(E) 300

Content Domain
Geometry

Main Topic
Lines and angles

Cognitive Domain
Solving Routine Problems

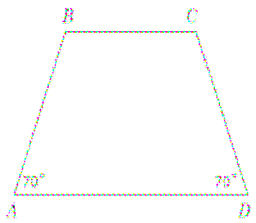
Key
C

Figure A.11. Released geometry items of TIMSS 1999 with item code M012039 (From ISC, n.d. b).

UniqueID M012015	Subject M	Grade 8	MSBlock M02	MSBlockSeq 03

TIMSS 2003

ABCD is a trapezoid.



Another trapezoid, *GHIJ* (not shown), is congruent (the same size and shape) to *ABCD*. Angles *G* and *J* each measure 70°. Which of these could be true?

(A) *GH* = *AB*

(B) Angle *H* is a right angle.

(C) All sides of *GHIJ* are the same length.

(D) The perimeter of *GHIJ* is 3 times the perimeter of *ABCD*.

(E) The area of *GHIJ* is less than the area of *ABCD*.

Content Domain
Geometry

Main Topic
Congruence and similarity

Cognitive Domain
Reasoning

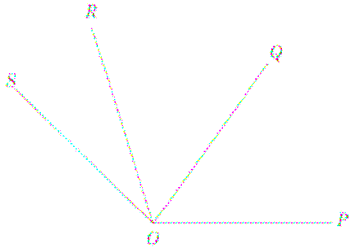
Key
A

Figure A.12. Released geometry items of TIMSS 1999 with item code M012015 (From ISC, n.d. b).

UniqueID M022202	Subject M Grade 8	MSBlock M02	MSBlockSeq 15

TIMSS 2003

In the figure, the measure of $\angle POR$ is 110°, the measure of $\angle QOS$ is 90°, and the measure of $\angle POS$ is 140°.



What is the measure of $\angle QOR$?

Answer: _____

Content Domain
Geometry

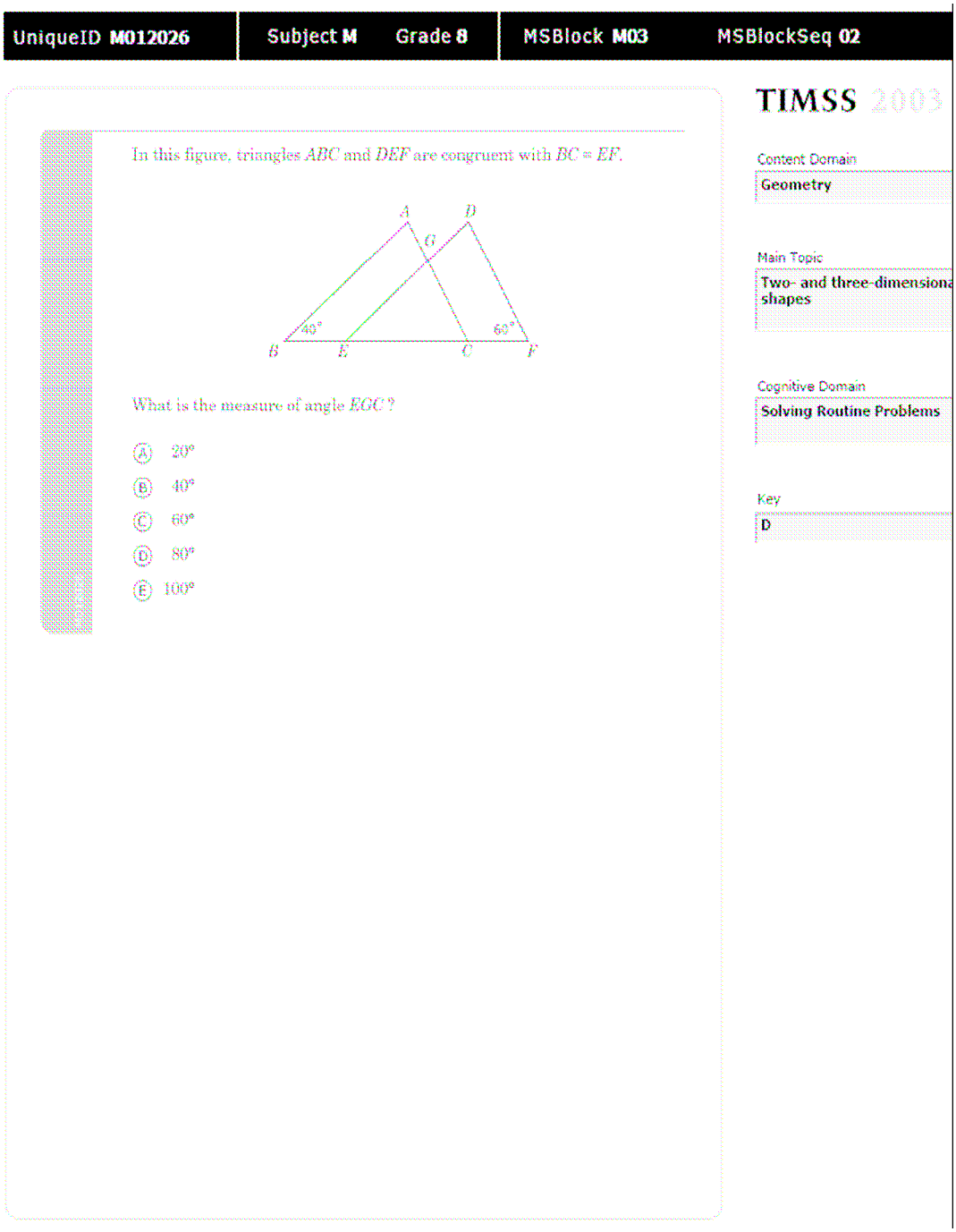
Main Topic
Lines and angles

Cognitive Domain
Reasoning

Key
See scoring guide

Figure A.13. Released geometry items of TIMSS 1999 with item code M022202 (From ISC, n.d. b)

.



UniqueID M012026 | Subject M | Grade 8 | MSBlock M03 | MSBlockSeq 02

TIMSS 2003

In this figure, triangles ABC and DEF are congruent with $BC = EF$.

What is the measure of angle EGC?

(A) 20°
(B) 40°
(C) 60°
(D) 80°
(E) 100°

Content Domain
Geometry

Main Topic
Two- and three-dimensiona shapes

Cognitive Domain
Solving Routine Problems

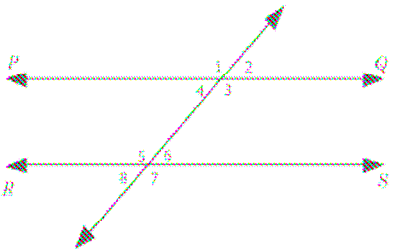
Key
D

Figure A.14. Released geometry items of TIMSS 1999 with item code M012026 (From ISC, n.d. b).

UniqueID M022142	Subject M Grade 8	MSBlock M03	MSBlockSeq 09

TIMSS 2003

In this figure, PQ and RS are parallel.



Of the following, which pair of angles has the sum of 180°?

(A) ∠5 and ∠7

(B) ∠8 and ∠6

(C) ∠1 and ∠5

(D) ∠1 and ∠7

(E) ∠2 and ∠8

Content Domain

Geometry

Main Topic

Lines and angles

Cognitive Domain

Knowing Facts and Procedures

Key

B

Figure A.15. Released geometry items of TIMSS 1999 with item code M022142 (From ISC, n.d. b).

UniqueID M022154	Subject M Grade 8	MSBlock M03	MSBlockSeq 14

TIMSS 2003

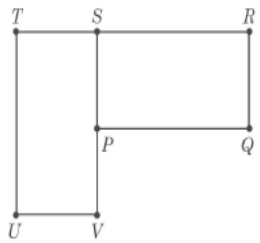
Content Domain
Geometry

Main Topic
Symmetry and transformations

Cognitive Domain
Reasoning

Key
C

Rectangle *PQRS* can be rotated (turned) onto rectangle *UVST*.



What point is the center of rotation?

Ⓐ *P*
Ⓑ *R*
Ⓒ *S*
Ⓓ *T*
Ⓔ *V*

M022154

Figure A.16. Released geometry items of TIMSS 1999 with item code M022154 (From ISC, n.d. b).

UniqueID M022016	Subject M	Grade 8	MSBlock M04	MSBlockSeq 08

TIMSS 2003

A straight line passes through the points (2,3) and (4,7). Which of these points is also on the line?

(A) (0,2)
(B) (1,2)
(C) (2,4)
(D) (3,5)
(E) (4,5)

Content Domain
Geometry

Main Topic
Locations and spatial relationships

Cognitive Domain
Knowing Facts and Procedures

Key
D

Figure A.17. Released geometry items of TIMSS 1999 with item code M022016 (From ISC, n.d. b).

APPENDIX B
Standings on Geometry Items of TIMSS 1999

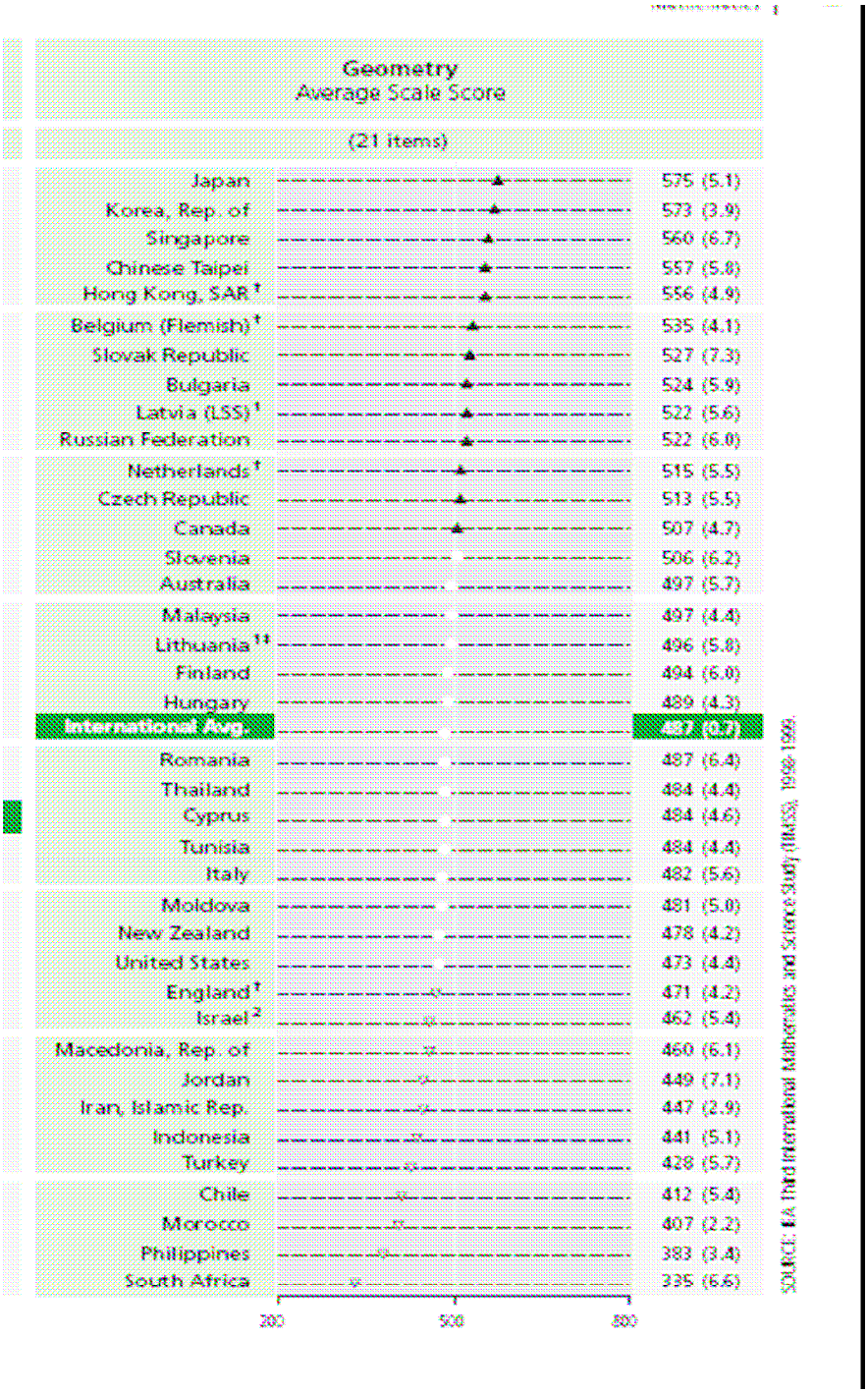


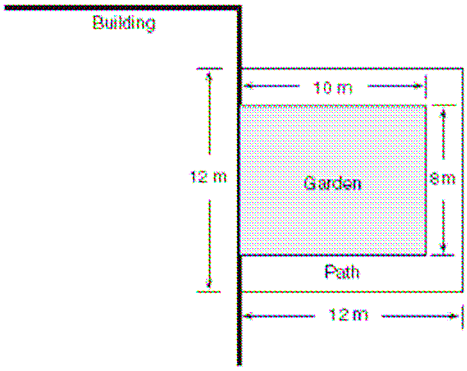
Figure B.1. International standings on geometry items of TIMSS 1999. (From Mullis et al., 2000, p. 97).

APPENDIX C
Item Examples of TIMSS 1999 and Performances

Content Area: Measurement

Description: Finds the area between two rectangles when one is inside the other and their sides are parallel.

A rectangular garden that is next to a building has a path around the other three sides, as shown.



What is the area of the path?

A. 144 m^2

B. 64 m^2

C. 44 m^2

D. 16 m^2

	Overall Percent Correct	
Hong Kong, SAR [1]	79 (2.0)	▲
Singapore	78 (2.6)	▲
Japan	74 (1.9)	▲
Chinese Taipei	73 (2.1)	▲
Korea, Rep. of	67 (1.7)	▲
Netherlands [1]	57 (4.4)	▲
Australia	52 (2.6)	▲
Malaysia	52 (2.1)	▲
Slovak Republic	51 (3.3)	
Canada	51 (3.0)	
Belgium (Flemish) [1]	51 (2.2)	▲
Finland	46 (3.0)	
Hungary	46 (2.7)	
Slovenia	46 (3.2)	
Cyprus	45 (3.0)	
Italy	45 (2.7)	
Bulgaria	42 (3.4)	
International Avg.	**42 (0.4)**	
Czech Republic	40 (3.5)	
England [1]	40 (3.3)	
New Zealand	40 (2.6)	
Tunisia	38 (2.0)	
Russian Federation	38 (3.2)	
Thailand	35 (2.1)	
Moldova	34 (2.7)	
United States	33 (1.6)	▼
Morocco	31 (2.1)	▼
Lithuania [1*]	31 (3.0)	▼
Macedonia, Rep. of	30 (2.5)	▼
Romania	29 (2.6)	▼
Jordan	29 (2.3)	▼
Israel [2]	28 (2.1)	▼
Latvia (LSS) [1]	28 (2.5)	▼
Iran, Islamic Rep.	26 (2.1)	▼
Indonesia	25 (2.0)	▼
Turkey	22 (1.6)	▼
Chile	18 (1.6)	▼
Philippines	15 (1.2)	▼
South Africa	15 (1.2)	▼

▲ Country average significantly higher than international average

No statistically significant difference between country average and international average

▼ Country average significantly lower than international average

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

Figure C.1. Item example of TIMSS 1999 with code J10 and student performances (From Mullis et al., 2000, 64).

Content Area: Data Representation, Analysis and Probability

Description: Selects relevant information from two advertisements to solve a complex word problem involving decimals.

Chris plans to order 24 issues of a magazine. He reads the following advertisements for two magazines. *Ceds* are the units of currency in Chris' country.

Teen Life Magazine	**Teen News Magazine**
24 issues	24 issues
First four issues FREE	First six issues FREE
The rest	The rest
3 *ceds* each.	3.5 *ceds* each.

Which magazine is the least expensive for 24 issues? How much less expensive? Show your work.

Teen Life = 20
× 3
60 ceds
24 = 60 ceds

Teen News = 18
× 3.5
90
540
63.0
ceds
24 = 63 ceds

Teen Life is less expensive by 3 ceds.

The answer shown illustrates the type of student response that was given full credit.

	Overall Percent Correct	
Singapore	57 (2.1)	▲
Korea, Rep. of	52 (1.5)	▲
Chinese Taipei	50 (1.8)	▲
Belgium (Flemish) [1]	42 (1.7)	▲
Japan	39 (1.5)	▲
Slovak Republic	36 (2.3)	▲
Slovenia	36 (2.1)	▲
Hungary	35 (2.1)	▲
Latvia (LSS) [1]	35 (2.1)	▲
Hong Kong, SAR [1]	34 (1.8)	▲
Czech Republic	34 (2.5)	▲
Canada	32 (1.8)	▲
Russian Federation	30 (2.4)	
Australia	29 (2.0)	
Finland	28 (2.0)	
Italy	27 (1.7)	
United States	26 (1.4)	
Netherlands [1]	25 (2.7)	
Lithuania [4]	25 (2.0)	
International Avg.	**24 (0.3)**	
Bulgaria	22 (2.6)	
Thailand	21 (1.8)	
Cyprus	21 (1.8)	
Romania	20 (2.2)	
Malaysia	19 (1.4)	▼
Israel [3]	19 (1.5)	▼
New Zealand	18 (1.7)	▼
Macedonia, Rep. of	17 (1.3)	▼
England [1]	17 (1.8)	▼
Moldova	16 (1.8)	▼
Jordan	12 (1.1)	▼
Turkey	10 (1.3)	▼
Tunisia	9 (0.8)	▼
Iran, Islamic Rep.	9 (0.7)	▼
Chile	5 (1.0)	▼
Indonesia	5 (0.5)	▼
Philippines	3 (0.7)	▼
Morocco	2 (0.4)	▼
South Africa	1 (0.3)	▼

▲ Country average significantly higher than international average

No statistically significant difference between country average and international average

▼ Country average significantly lower than international average

Significance tests adjusted for multiple comparisons

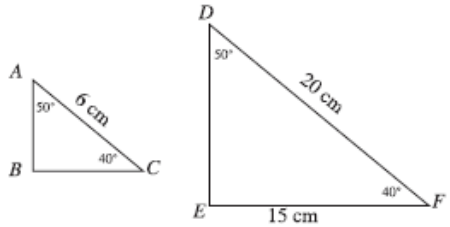
SOURCE: IEA, 1999 International Mathematics and Science Study (TIMSS), 1999-2000.

Figure C.2. Item example of TIMSS 1999 with code V02 and student performances (From Mullis et al., 2000, p. 65).

Content Area: Geometry

Description: Uses properties of similar triangles to find the length of a corresponding side.

The figure represents two similar triangles. The triangles are not drawn to scale.



In the actual triangle *ABC*, what is the length of side *BC*?

A 3.5 cm

B. 4.5 cm

C. 5 cm

D. 5.5 cm

E. 8 cm

	Overall Percent Correct	
Korea, Rep. of	70 (1.9)	▲
Japan	68 (1.9)	▲
Singapore	64 (2.7)	▲
Hong Kong, SAR [1]	56 (2.2)	▲
Chinese Taipei	52 (2.3)	▲
Belgium (Flemish) [1]	50 (3.2)	▲
Netherlands [1]	44 (3.1)	●
Hungary	43 (2.9)	●
Russian Federation	41 (2.7)	●
Finland	39 (2.9)	●
Australia	39 (2.8)	●
Romania	38 (2.9)	●
Slovak Republic	38 (3.0)	●
International Avg.	**37 (0.4)**	
United States	36 (1.6)	●
Moldova	36 (2.4)	●
Canada	35 (2.2)	●
New Zealand	34 (2.7)	●
Slovenia	34 (2.4)	●
England [1]	34 (2.7)	●
Bulgaria	33 (3.8)	●
Czech Republic	32 (2.5)	●
Malaysia	32 (1.9)	●
Jordan	32 (2.1)	●
Lithuania [1]	31 (2.6)	●
Cyprus	31 (2.1)	●
Latvia (LSS) [1]	30 (2.8)	●
Thailand	30 (1.9)	▼
Italy	29 (2.4)	▼
Israel [2]	29 (2.4)	●
Macedonia, Rep. of	27 (2.5)	▼
Philippines	27 (1.4)	▼
Indonesia	26 (2.0)	▼
Iran, Islamic Rep.	26 (2.1)	▼
Tunisia	24 (1.9)	▼
Chile	23 (1.7)	▼
South Africa	23 (1.3)	▼
Turkey	22 (1.4)	▼

▲ Country average significantly higher than international average

● No statistically significant difference between country average and international average

▼ Country average significantly lower than international average

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

Figure C.3. Item example of TIMSS 1999 with code P10 and student performances (From Mullis et al., 2000, p. 66).

Content Area: Algebra

Description: Given the initial terms in a sequence and, for example, the 50th term of that sequence, generalizes to find the next term.

The figures show four sets consisting of circles.

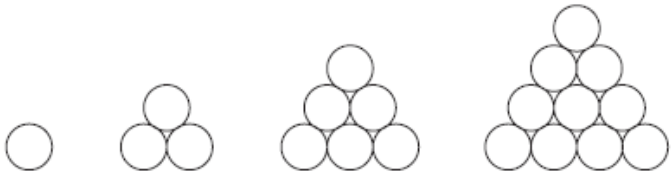


Figure 1 Figure 2 Figure 3 Figure 4

a) Complete the table below. First, fill in how many circles make up Figure 4. Then, find the number of circles that would be needed for the 5th figure if the sequence of figures is extended.

Figure	Number of circles
1	1
2	3
3	6
4	10
5	15

b) The sequence of figures is extended to the 7th figure. How many circles would be needed for Figure 7?

Answer: 28

c) The 50th figure in the sequence contains 1275 circles. Determine the number of circles in the 51st figure. Without drawing the 51st figure, explain or show how you arrived at your answer.

Because it is the 51st figure you have to add 51 to the base of the figure before it

1275
+ 51
1326

The answer shown illustrates the type of student response that was given full credit.

	Overall Percent Correct	
Korea, Rep. of	70 (1.2)	▲
Chinese Taipei	68 (1.5)	▲
Japan	66 (1.6)	▲
Singapore	65 (2.4)	▲
Hong Kong, SAR †	57 (2.0)	▲
Netherlands †	48 (3.0)	▲
Belgium (Flemish) †	44 (1.7)	▲
Canada	43 (2.2)	▲
Australia	39 (2.3)	▲
Hungary	38 (1.9)	▲
Malaysia	37 (1.7)	▲
Slovenia	37 (2.3)	●
England †	35 (2.5)	●
United States	34 (1.3)	▲
Czech Republic	34 (2.5)	●
Slovak Republic	31 (2.5)	●
New Zealand	31 (2.0)	●
International Avg.	**30 (0.3)**	
Finland	30 (2.2)	●
Israel ²	27 (1.6)	●
Russian Federation	27 (2.0)	●
Moldova	26 (2.3)	●
Bulgaria	26 (2.2)	●
Thailand	25 (2.0)	●
Italy	24 (1.8)	▼
Indonesia	24 (1.6)	▼
Latvia (LSS) ¹	22 (2.1)	▼
Romania	19 (2.0)	▼
Lithuania ³	19 (1.9)	▼
Cyprus	15 (1.5)	▼
Macedonia, Rep. of	13 (1.3)	▼
Jordan	13 (1.3)	▼
Turkey	11 (1.2)	▼
Philippines	9 (0.9)	▼
Chile	8 (1.0)	▼
Tunisia	8 (0.9)	▼
Iran, Islamic Rep.	8 (0.9)	▼
South Africa	3 (0.6)	▼
Morocco	3 (0.5)	▼

▲ Country average significantly higher than international average

● No statistically significant difference between country average and international average

▼ Country average significantly lower than international average

Significance tests adjusted for multiple comparisons

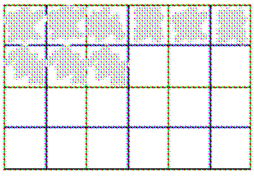
SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

Figure C.4. Item example of TIMSS 1999 with code V04 and student performances (From Mullis et al., 2000, p. 67).

Content Area: Fractions and Number Sense

Description: Shades squares in a rectangular grid to represent a given fraction.

Shade in $\frac{3}{8}$ of the unit squares in the grid.



$8\overline{)24}$

$3 \times 3 = 9$

The answer shown illustrates the type of student response that was given credit.

	Overall Percent Correct	
Singapore	89 (1.7)	▲
Hong Kong, SAR [1]	87 (1.7)	▲
Belgium (Flemish) [1]	87 (1.8)	▲
Korea, Rep. of	81 (1.4)	▲
Chinese Taipei	80 (1.9)	▲
Japan	78 (1.9)	▲
Malaysia	73 (2.1)	▲
Canada	68 (2.6)	▲
Finland	65 (2.5)	▲
Hungary	63 (2.5)	▲
Netherlands [1]	61 (4.7)	
Australia	60 (2.9)	▲
Slovenia	55 (2.7)	
Bulgaria	54 (4.3)	
Cyprus	54 (2.6)	
England [1]	52 (2.9)	
Slovak Republic	52 (3.3)	
Russian Federation	52 (3.2)	
United States	49 (1.9)	
International Avg.	**49 (0.4)**	
Thailand	49 (2.9)	
New Zealand	46 (2.9)	
Italy	46 (2.6)	
Latvia (LSS) [1]	46 (2.8)	
Moldova	44 (3.2)	
Czech Republic	42 (3.2)	
Israel [2]	40 (2.4)	▼
Romania	39 (2.9)	▼
Macedonia, Rep. of	32 (2.4)	▼
Jordan	31 (2.3)	▼
Iran, Islamic Rep.	31 (2.1)	▼
Tunisia	28 (1.8)	▼
Turkey	26 (2.2)	▼
Lithuania [1]	26 (2.8)	▼
Indonesia	21 (2.0)	▼
Chile	13 (1.7)	▼
Philippines	11 (1.3)	▼
Morocco	8 (1.1)	▼
South Africa	7 (1.4)	▼

▲ Country average significantly higher than international average

No statistically significant difference between country average and international average

▼ Country average significantly lower than international average

Significance tests adjusted for multiple comparisons

SOURCE: IEA, Third International Mathematics and Science Study (TIMSS), 1998-1999.

Figure C.5. Item example of TIMSS 1999 with code N19 and student performances (From Mullis et al., 2000, p. 72).

Content Area: Fractions and Number Sense

Description: Solves a multi-step word problem that involves dividing a quantity in a given ratio.

John sold 60 magazines and Mark sold 80 magazines. The magazines were all sold for the same price. The total amount of money received for the magazines was $700. How much money did Mark receive?

Answer: $400

60
+80
140 TOTAL

MARK = 80/140 = 8/14 = 4/7

100
7 | 700

100
×4
$400

The answer shown illustrates the type of student response that was given credit.

	Overall Percent Correct	
Singapore	84 (2.0)	▲
Chinese Taipei	75 (1.8)	▲
Hong Kong, SAR [1]	72 (2.1)	▲
Korea, Rep. of	69 (1.4)	▲
Japan	67 (2.0)	▲
Malaysia	65 (2.0)	▲
Slovenia	60 (2.7)	▲
Belgium (Flemish) [1]	60 (3.7)	▲
Hungary	58 (2.5)	▲
Moldova	54 (3.1)	▲
Czech Republic	54 (3.8)	
Slovak Republic	54 (3.3)	
Lithuania [1]	54 (2.9)	▲
Netherlands [1]	53 (4.5)	
Russian Federation	52 (3.1)	
Bulgaria	50 (3.9)	
Latvia (LSS) [1]	48 (3.4)	
Finland	47 (3.2)	
Canada	46 (2.4)	
International Avg	**41 (0.4)**	
Australia	44 (3.2)	
Romania	43 (3.1)	
United States	41 (2.0)	
Cyprus	40 (2.5)	
Tunisia	39 (2.0)	
Thailand	38 (2.3)	
Italy	36 (2.6)	
New Zealand	33 (2.7)	▼
England [1]	31 (2.6)	▼
Israel [2]	30 (2.5)	▼
Macedonia, Rep. of	30 (2.6)	▼
Iran, Islamic Rep.	28 (2.1)	▼
Indonesia	27 (1.8)	▼
Turkey	26 (1.9)	▼
Jordan	23 (2.0)	▼
Chile	22 (1.7)	▼
Philippines	12 (1.3)	▼
South Africa	9 (1.3)	▼
Morocco	3 (0.6)	▼

▲ Country average significantly higher than international average

No statistically significant difference between country average and international average

▼ Country average significantly lower than international average

Significance tests adjusted for multiple comparisons

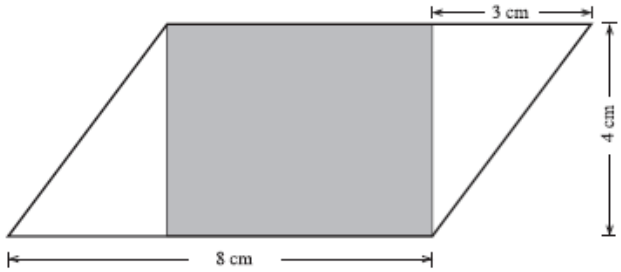
SOURCE: IEA, Third International Mathematics and Science Study (TIMSS), 1998-1999.

Figure C.6. Item example of TIMSS 1999 with code R15 and student performances (From Mullis et al., 2000, p. 73).

Content Area: Measurement

Description: Finds the area of a rectangle contained in a parallelogram of given dimensions.

The figure shows a shaded rectangle inside a parallelogram.



What is the area of the shaded rectangle?

Answer: 20

8-3=5

$$\begin{array}{r} 5 \\ \times\ 4 \\ \hline 20 \end{array}$$

The answer shown illustrates the type of student response that was given credit.

	Overall Percent Correct	
Singapore	83 (1.5)	▲
Japan	80 (1.2)	▲
Hong Kong, SAR †	78 (1.6)	▲
Korea, Rep. of	78 (1.3)	▲
Chinese Taipei	75 (1.4)	▲
Belgium (Flemish) †	65 (2.0)	▲
Canada	58 (1.6)	▲
Slovak Republic	57 (2.5)	▲
Finland	57 (2.3)	▲
Malaysia	56 (1.9)	▲
Netherlands †	55 (4.7)	●
Australia	55 (1.8)	▲
Bulgaria	52 (3.2)	●
Slovenia	49 (2.1)	●
Russian Federation	49 (2.8)	●
Italy	48 (2.1)	●
England †	48 (2.3)	●
Czech Republic	46 (2.9)	●
Hungary	45 (2.0)	●
Latvia (LSS) 1	44 (2.5)	●
International Avg.	**43 (0.3)**	
Romania	43 (2.7)	●
New Zealand	41 (2.3)	●
Cyprus	41 (1.9)	●
Moldova	38 (2.6)	●
Tunisia	38 (1.6)	▼
Lithuania **	35 (2.4)	▼
United States	34 (1.4)	▼
Thailand	33 (2.1)	▼
Israel 2	28 (1.8)	▼
Jordan	26 (1.5)	▼
Iran, Islamic Rep.	25 (2.0)	▼
Macedonia, Rep. of	25 (1.9)	▼
Turkey	20 (1.7)	▼
Indonesia	20 (1.4)	▼
Morocco	8 (0.9)	▼
Chile	7 (1.2)	▼
Philippines	6 (1.0)	▼
South Africa	3 (0.7)	▼

▲ Country average significantly higher than international average

● No statistically significant difference between country average and international average

▼ Country average significantly lower than international average

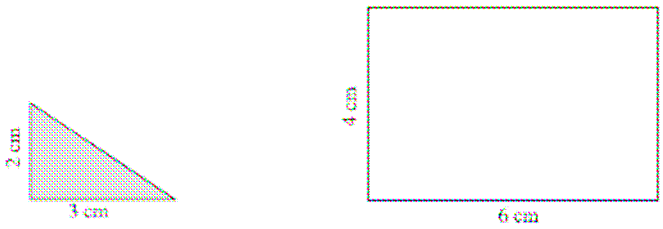
Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

Figure C.7. Item example of TIMSS 1999 with code T03 and student performances (From Mullis et al., 2000,p. 74).

Content Area: Geometry

Description: Determines the number of triangles of given dimensions needed to cover a given rectangle.



How many of the shaded right triangles shown above are needed to exactly cover the surface of the rectangle?

A. Four

B. Six

C. Eight

D. Ten

	Overall Percent Correct
Japan	80 (1.8) ▲
Korea, Rep. of	76 (1.7) ▲
Hong Kong, SAR [1]	75 (2.0) ▲
Singapore	72 (2.2) ▲
Belgium (Flemish) [1]	68 (2.7) ▲
Netherlands [1]	66 (3.8) ▲
Malaysia	60 (2.2) ▲
Chinese Taipei	60 (1.8) ▲
Hungary	59 (2.4) ▲
Slovenia	57 (2.6) ▲
Slovak Republic	57 (3.1) ▲
Australia	56 (2.7) ▲
Czech Republic	55 (3.6)
New Zealand	55 (2.4) ▲
Canada	50 (2.4)
Finland	49 (2.8)
Italy	49 (2.7)
England [1]	48 (2.6)
Latvia (LSS) [1]	48 (2.9)
United States	47 (2.0)
International Avg.	**46 (0.4)**
Russian Federation	44 (2.8)
Lithuania [1*]	43 (3.2)
Iran, Islamic Rep.	42 (2.1)
Israel [2]	41 (2.1)
Thailand	40 (2.0)
Cyprus	37 (3.1)
Moldova	37 (2.8) ▼
Romania	35 (2.7) ▼
Bulgaria	34 (3.8) ▼
Tunisia	33 (1.9) ▼
Turkey	30 (1.7) ▼
Macedonia, Rep. of	30 (2.6) ▼
Indonesia	29 (1.9) ▼
Chile	27 (1.8) ▼
Jordan	26 (2.0) ▼
Morocco	21 (1.7) ▼
Philippines	15 (1.4) ▼
South Africa	12 (1.5) ▼

▲ Country average significantly higher than international average

No statistically significant difference between country average and international average

▼ Country average significantly lower than international average

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

Figure C.8. Item example of TIMSS 1999 with code R11 and student performances (From Mullis et al., 2000, p. 75).

Content Area: Algebra

Description: Solves a linear equation involving transposing.

Find the value of x if $12x - 10 = 6x + 32$

Answer: 7

$12x - 6x - 10 = 32$

$\Rightarrow 6x = 42$

$\Rightarrow \frac{6x}{6} = \frac{42}{6}$

$\Rightarrow x = 7.$

The answer shown illustrates the type of student response that was given credit.

	Overall Percent Correct	
Japan	85 (1.4)	▲
Hong Kong, SAR †	80 (1.9)	▲
Korea, Rep. of	80 (1.5)	▲
Slovak Republic	78 (2.6)	▲
Russian Federation	77 (3.1)	▲
Slovenia	76 (2.8)	▲
Singapore	75 (2.8)	▲
Hungary	74 (2.6)	▲
Chinese Taipei	73 (2.0)	▲
Romania	70 (3.2)	▲
Czech Republic	66 (2.8)	▲
Lithuania **	62 (3.4)	▲
Latvia (LSS) †	58 (2.9)	▲
Belgium (Flemish) †	58 (1.9)	▲
Moldova	56 (3.0)	▲
Macedonia, Rep. of	54 (3.1)	●
Cyprus	51 (3.4)	●
Israel 2	51 (3.1)	●
Italy	46 (2.8)	●
International Avg.	**44 (0.4)**	
Malaysia	43 (2.7)	●
Bulgaria	34 (3.1)	▼
United States	34 (1.8)	▼
Canada	33 (3.1)	▼
Turkey	32 (2.6)	▼
Australia	31 (3.0)	▼
Thailand	29 (2.8)	▼
England †	26 (2.7)	▼
Finland	24 (2.9)	▼
Iran, Islamic Rep.	23 (1.8)	▼
Netherlands †	19 (2.9)	▼
New Zealand	19 (2.0)	▼
Jordan	18 (1.9)	▼
Indonesia	18 (2.0)	▼
Chile	12 (1.9)	▼
Morocco	7 (1.0)	▼
Philippines	6 (1.4)	▼
Tunisia	6 (1.0)	▼
South Africa	5 (0.9)	▼

▲ Country average significantly higher than international average

● No statistically significant difference between country average and international average

▼ Country average significantly lower than international average

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

Figure C.9 .: Item example of TIMSS 1999 with code L17 and student performances (From Mullis et al., 2000, p. 76).

Content Area: Fractions and Number Sense

Description: In a word problem, uses rounding to identify the number sentence that gives the best estimate for the product.

There are 68 rows of cars in a parking lot. Each row has 92 cars. Which of these would give the closest estimate of the total number of cars in the parking lot?

A. $60 \times 90 = 5400$

B. $60 \times 100 = 6000$

C. $70 \times 90 = 6300$

D. $70 \times 100 = 7000$

	Overall Percent Correct	
Singapore	94 (1.0)	▲
Hong Kong, SAR [1]	85 (1.7)	▲
Belgium (Flemish) [1]	83 (3.0)	▲
Japan	82 (1.4)	▲
Korea, Rep. of	82 (1.2)	▲
Chinese Taipei	81 (1.5)	▲
Netherlands [1]	81 (3.1)	▲
Finland	79 (2.5)	▲
United States	79 (1.8)	▲
Slovak Republic	78 (2.4)	▲
Hungary	78 (2.1)	▲
Canada	78 (2.1)	▲
Czech Republic	78 (2.3)	▲
Malaysia	78 (1.6)	▲
Australia	77 (2.3)	▲
Slovenia	76 (2.5)	▲
England [1]	74 (2.8)	▲
New Zealand	67 (2.6)	
Russian Federation	65 (2.7)	
International Avg	**65 (0.4)**	
Israel [2]	63 (2.4)	
Latvia (LSS) [1]	62 (2.6)	
Cyprus	60 (2.7)	
Bulgaria	60 (4.7)	
Thailand	58 (2.3)	
Jordan	58 (2.3)	
Lithuania [1]	57 (3.5)	
Romania	55 (3.0)	
Macedonia, Rep. of	53 (2.8)	▼
Italy	52 (2.5)	▼
Moldova	52 (2.7)	▼
Turkey	50 (2.0)	▼
Chile	48 (2.4)	▼
Tunisia	48 (2.1)	▼
Iran, Islamic Rep.	48 (2.0)	▼
Indonesia	44 (2.1)	▼
Philippines	42 (1.9)	▼
South Africa	30 (1.8)	▼
Morocco	17 (1.3)	▼

▲ Country average significantly higher than international average

No statistically significant difference between country average and international average

▼ Country average significantly lower than international average

Significance tests adjusted for multiple comparisons

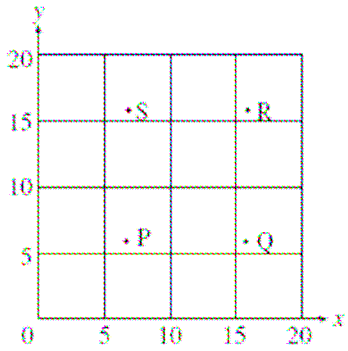
SOURCE: IEA, Third International Mathematics and Science Study (TIMSS), 1998-1999.

Figure C.10. Item example of TIMSS 1999 with code P13 and student performances (From Mullis et al., 2000, p. 80).

Content Area: Geometry

Description: Locates the point on a grid with 5-unit divisions when the point lies between the grid lines

Which point on the graph could have coordinates (7,16)?



A. Point P

B. Point Q

C. Point R

D. Point S

	Overall Percent Correct	
Japan	84 (1.7)	▲
Korea, Rep. of	84 (1.4)	▲
Chinese Taipei	83 (1.5)	▲
Hong Kong, SAR [1]	81 (1.7)	▲
Singapore	80 (2.3)	▲
Netherlands [1]	78 (2.5)	▲
Malaysia	78 (1.7)	▲
Slovenia	76 (2.4)	▲
Slovak Republic	76 (2.5)	▲
England [1]	75 (3.2)	▲
Australia	74 (2.3)	▲
Finland	72 (2.7)	▲
New Zealand	72 (2.6)	▲
Hungary	71 (2.5)	▲
Russian Federation	71 (2.2)	▲
Belgium (Flemish) [1]	71 (2.5)	▲
Canada	67 (2.6)	▲
United States	67 (1.6)	▲
Lithuania [3]	63 (2.9)	
Italy	62 (2.2)	
Czech Republic	58 (3.2)	
International Avg.	**58 (0.4)**	
Jordan	57 (2.6)	
Iran, Islamic Rep.	55 (2.2)	
Bulgaria	53 (2.8)	
Israel [2]	51 (2.7)	
Indonesia	50 (2.1)	▼
Moldova	48 (2.9)	▼
Romania	47 (2.7)	▼
Latvia (LSS) [1]	46 (2.9)	▼
Macedonia, Rep. of	44 (2.7)	▼
Thailand	37 (2.2)	▼
Turkey	32 (1.9)	▼
Morocco	26 (2.1)	▼
Cyprus	24 (2.1)	▼
Philippines	23 (1.7)	▼
Chile	23 (1.6)	▼
South Africa	20 (1.7)	▼
Tunisia	10 (1.2)	▼

▲ Country average significantly higher than international average

No statistically significant difference between country average and international average

▼ Country average significantly lower than international average

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1995-1999.

Figure C.11. Item example of TIMSS 1999 with code J16 and student performances (From Mullis et al., 2000, p. 81).

Content Area: Algebra

Description: Identifies the linear equation corresponding to a given verbal statement involving a variable.

n is a number. When n is multiplied by 7, and 6 is then added, the result is 41. Which of these equations represents this relation?

A. $7n + 6 = 41$

B. $7n - 6 = 41$

C. $7n \times 6 = 41$

D. $7(n + 6) = 41$

	Overall Percent Correct	
Hong Kong, SAR [1]	93 (0.9)	▲
Singapore	89 (1.7)	▲
Japan	86 (0.8)	▲
Korea, Rep. of	85 (0.7)	▲
Chinese Taipei	84 (1.1)	▲
Slovenia	83 (1.1)	▲
Canada	82 (1.0)	▲
Russian Federation	82 (1.6)	▲
Slovak Republic	81 (1.5)	▲
Belgium (Flemish) [1]	81 (1.2)	▲
Netherlands [1]	80 (2.5)	▲
Hungary	80 (1.3)	▲
United States	77 (1.3)	▲
Bulgaria	76 (2.0)	▲
Australia	72 (1.9)	▲
Czech Republic	72 (1.7)	▲
Latvia (LSS) [1]	71 (1.6)	▲
Lithuania [1]	71 (1.8)	
Finland	68 (1.5)	
Israel [2]	68 (1.7)	
Thailand	67 (1.5)	
Romania	67 (2.1)	
Cyprus	66 (1.3)	
International Avg.	**65 (0.3)**	
Moldova	65 (1.6)	
Macedonia, Rep. of	63 (1.9)	
England [1]	62 (2.1)	
Italy	58 (1.6)	▼
New Zealand	58 (2.2)	▼
Tunisia	58 (1.4)	▼
Malaysia	57 (1.8)	▼
Jordan	46 (1.4)	▼
Iran, Islamic Rep.	46 (1.5)	▼
Turkey	41 (1.6)	▼
Chile	38 (1.6)	▼
Indonesia	37 (1.4)	▼
Morocco	35 (1.1)	▼
South Africa	21 (1.3)	▼
Philippines	19 (1.6)	▼

▲ Country average significantly higher than international average

No statistically significant difference between country average and international average

▼ Country average significantly lower than international average

Significance tests adjusted for multiple comparisons

SOURCE: IEA, Third International Mathematics and Science Study (TIMSS), 1998-1999.

Figure C.12. Item example of TIMSS 1999 with code B12 and student performances (From Mullis et al., 2000, p. 82).

Content Area: Fractions and Number Sense

Description: Rounds to estimate the sum of two three-digit numbers.

The sum 691 + 208 is closest to the sum

A. 600 + 200

B. 700 + 200

C. 700 + 300

D. 900 + 200

	Overall Percent Correct	
Singapore	97 (0.5)	▲
Belgium (Flemish) [1]	96 (0.7)	▲
Japan	95 (0.5)	▲
Netherlands [1]	95 (0.8)	▲
Hong Kong, SAR [1]	93 (0.7)	▲
Canada	93 (0.7)	▲
United States	93 (0.7)	▲
Hungary	93 (0.9)	▲
Korea, Rep. of	93 (0.6)	▲
Slovenia	92 (0.8)	▲
England [1]	92 (1.0)	▲
Czech Republic	91 (1.0)	▲
Australia	91 (0.8)	▲
Finland	91 (1.0)	▲
Slovak Republic	90 (1.1)	▲
Chinese Taipei	89 (0.7)	▲
New Zealand	88 (1.0)	▲
Malaysia	88 (0.8)	▲
Latvia (LSS) [1]	87 (1.4)	▲
Bulgaria	86 (1.6)	▲
Cyprus	85 (1.1)	▲
Lithuania [1]	84 (1.5)	●
Russian Federation	83 (1.9)	●
Israel [2]	83 (1.6)	●
International Avg.	**80 (0.2)**	
Macedonia, Rep. of	79 (1.4)	●
Italy	77 (1.9)	●
Thailand	77 (1.5)	●
Turkey	74 (1.3)	▼
Romania	73 (1.8)	▼
Tunisia	67 (1.3)	▼
Jordan	66 (1.5)	▼
Moldova	66 (1.6)	▼
Chile	65 (1.3)	▼
Iran, Islamic Rep.	58 (1.5)	▼
Indonesia	54 (1.6)	▼
Philippines	53 (1.6)	▼
Morocco	43 (1.2)	▼
South Africa	37 (1.6)	▼

▲ Country average significantly higher than international average

● No statistically significant difference between country average and international average

▼ Country average significantly lower than international average

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

Figure C.13. Item example of TIMSS 1999 with code H09 and student performances (From Mullis et al., 2000, p. 86).

Content Area: Fractions and Number Sense

Description: Subtracts a three-decimal-place number from another with multiple regrouping.

Subtract: 4.722 − 1.935 =

A. 2.787

B. 2.797

C. 2.887

D. 2.897

	Overall Percent Correct	
Malaysia	92 (1.1)	▲
Singapore	90 (1.4)	▲
Hungary	90 (1.7)	▲
Slovenia	90 (1.6)	▲
Korea, Rep. of	88 (1.2)	▲
Russian Federation	88 (1.9)	▲
Slovak Republic	87 (2.1)	▲
Japan	86 (1.3)	▲
Lithuania [1]	86 (2.1)	▲
Czech Republic	85 (2.8)	
Chinese Taipei	84 (1.5)	▲
Hong Kong, SAR [1]	83 (1.8)	▲
Thailand	83 (1.6)	▲
Tunisia	82 (1.6)	
Bulgaria	81 (2.6)	
Moldova	80 (2.3)	
Canada	80 (1.8)	
Latvia (LSS) [1]	79 (2.4)	
Indonesia	78 (1.9)	
Romania	77 (2.5)	
United States	77 (1.7)	
Italy	77 (2.3)	
International Avg.	**77 (0.4)**	
Chile	75 (1.7)	
Australia	74 (2.7)	
Belgium (Flemish) [1]	73 (2.0)	
Finland	72 (3.0)	
Cyprus	71 (2.2)	
Macedonia, Rep. of	71 (2.4)	
Iran, Islamic Rep.	71 (2.3)	
Turkey	71 (1.9)	
Netherlands [1]	69 (4.3)	
Philippines	69 (1.8)	▼
Jordan	65 (2.4)	▼
Israel [1]	63 (2.5)	▼
Morocco	62 (2.5)	▼
New Zealand	61 (2.5)	▼
England [1]	59 (2.7)	▼
South Africa	42 (1.8)	▼

▲ Country average significantly higher than international average

No statistically significant difference between country average and international average

▼ Country average significantly lower than international average

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

Figure C.14. Item example of TIMSS 1999 with code R07 and student performances (From Mullis et al., 2000, p. 87).

Content Area: Fractions and Number Sense

Description: Subtracts a four-digit number from another involving zeroes.

Subtract:

$$\begin{array}{r} 7003 \\ -\ 4078 \\ \hline \end{array}$$

A. 2035

B. 2925

C. 3005

D. 3925

	Overall Percent Correct	
Malaysia	94 (0.9)	▲
Singapore	92 (1.3)	▲
Chinese Taipei	90 (1.2)	▲
Hong Kong, SAR [1]	90 (1.3)	▲
Korea, Rep. of	88 (1.2)	▲
Hungary	87 (1.8)	▲
Slovak Republic	86 (1.9)	▲
Japan	86 (1.4)	▲
Belgium (Flemish) [1]	85 (2.1)	▲
Slovenia	83 (2.2)	▲
Canada	83 (1.4)	▲
Czech Republic	82 (2.4)	
United States	81 (1.6)	▲
Lithuania [1]	80 (2.7)	
Tunisia	80 (1.7)	▲
Russian Federation	79 (2.2)	
Moldova	79 (2.2)	
Netherlands [1]	79 (3.4)	
Australia	77 (2.5)	
Thailand	77 (1.8)	
Finland	76 (2.4)	
Bulgaria	76 (2.9)	
International Avg.	**74 (0.4)**	
Latvia (LSS) [1]	74 (3.1)	
Iran, Islamic Rep.	73 (1.9)	
Cyprus	70 (2.2)	
Turkey	69 (1.9)	
Jordan	69 (2.1)	
Romania	68 (2.9)	
Israel [2]	67 (2.4)	
Italy	67 (2.7)	
Macedonia, Rep. of	65 (2.7)	▼
Chile	59 (2.0)	▼
Philippines	58 (1.9)	▼
New Zealand	58 (2.4)	▼
Indonesia	55 (2.6)	▼
Morocco	54 (2.1)	▼
England [1]	51 (3.1)	▼
South Africa	37 (2.0)	▼

▲ Country average significantly higher than international average

No statistically significant difference between country average and international average

▼ Country average significantly lower than international average

Significance tests adjusted for multiple comparisons

SOURCE: IEA, Third International Mathematics and Science Study (TIMSS), 1996–1999.

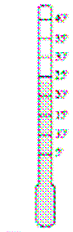
Figure C.15. Item example of TIMSS 1999 with code R13 and student performances (From Mullis et al., 2000, p. 88).

Content Area: Data Representation, Analysis and Probability

Description: Reads a thermometer and locates the reading in a table.

This table shows temperatures at various times on four days.

TEMPERATURE					
	6 a.m.	9 a.m.	Noon	3 p.m.	6 p.m.
Monday	15°	17°	24°	21°	18°
Tuesday	20°	16°	15°	10°	9°
Wednesday	8°	14°	16°	19°	15°
Thursday	8°	11°	19°	26°	20°



Thermometer

On which day and at what time was the temperature shown in the table the same as that shown on the thermometer.

A. Monday, Noon

B. Tuesday, 6 a.m.

C. Wednesday, 3 p.m.

D. Thursday, 3 p.m.

	Overall Percent Correct	
Japan	96 (0.8)	▲
Singapore	95 (0.9)	▲
Belgium (Flemish) [1]	95 (1.5)	▲
Finland	93 (1.4)	▲
Korea, Rep. of	92 (0.9)	▲
England [1]	92 (2.2)	▲
Chinese Taipei	91 (1.2)	▲
Slovenia	91 (1.7)	▲
Czech Republic	91 (1.9)	▲
Australia	91 (2.2)	▲
Slovak Republic	91 (1.5)	▲
Hong Kong, SAR [1]	90 (1.5)	▲
Netherlands [1]	90 (2.6)	▲
Canada	89 (2.6)	▲
United States	89 (1.2)	▲
New Zealand	88 (1.9)	▲
Hungary	87 (2.0)	▲
Cyprus	86 (1.4)	▲
Russian Federation	85 (2.6)	
Malaysia	85 (1.4)	▲
Lithuania [1]	84 (2.4)	
Latvia (LSS) [1]	83 (2.3)	
Italy	81 (2.0)	
International Avg.	**78 (0.3)**	
Israel [2]	74 (2.9)	
Bulgaria	72 (2.8)	
Chile	67 (1.9)	▼
Moldova	66 (2.8)	▼
Romania	65 (2.8)	▼
Jordan	65 (1.9)	▼
Macedonia, Rep. of	65 (2.9)	▼
Iran, Islamic Rep.	59 (2.5)	▼
Philippines	54 (2.0)	▼
Indonesia	50 (2.3)	▼
South Africa	43 (2.1)	▼
Turkey	38 (1.9)	▼

▲ Country average significantly higher than international average

No statistically significant difference between country average and international average

▼ Country average significantly lower than international average

Significance tests adjusted for multiple comparisons

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

Figure C.16. Item example of TIMSS 1999 with code P16 and student performances (From Mullis et al., 2000, p. 89).

APPENDIX D
Standings on Mathematics Items of TIMSS 1999

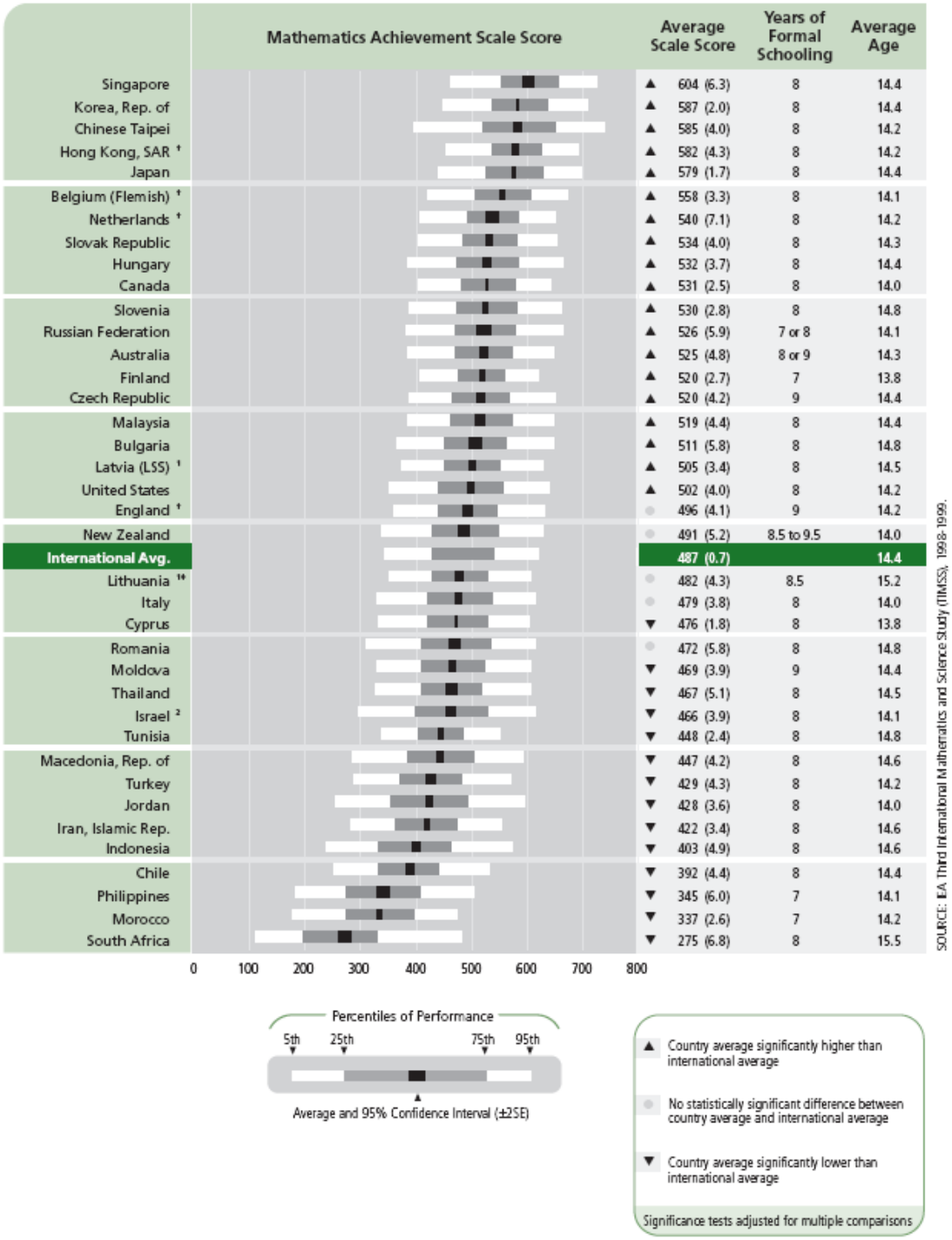


Figure D.1. International standings on overall mathematics items of TIMSS 1999 (From Mullis et al., 2000, p. 32).