AN APPROXIMATE MODEL FOR KANBAN CONTROLLED ASSEMBLY SYSTEMS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

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IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
INDUSTRIAL ENGINEERING

AUGUST 2005

Approval of the Graduate School of Natural and Applied Sciences

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I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

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This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

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ABSTRACT

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August 2005, 160 Pages

In this thesis, an approximation is proposed to evaluate the steady-state performance of kanban controlled assembly systems. The approximation is developed for the systems with two components making up an assembly. Then, it is extended to systems with more than two components. A continuous-time Markov model is aggregated keeping the model exact, and this aggregate model is approximated replacing some state-dependent transition rates with constant rates. Decomposition of the approximate aggregate model into submodels guarantees product-form steady-state distribution for each subsystem. Finally, submodels are combined in such a way that the size of the problem becomes independent of the number of kanbans. This brings about the computational advantage in solving the combined model using numerical matrix-geometric solution algorithms. Based on the numerical comparisons with simulation, the exact model, an approximate aggregate model and another approximation in a previous study in the literature, the approximation is observed to be good in terms of accuracy with respect to computational burden and has the potential to be a building block for the analysis of systems that are more complex but closer to real-life applications.

Keywords: Assembly Systems, Approximation, Performance Evaluation, Matrix Geometric Solution, Kanban Control, Steady-State Behavior.

ÖZ

KANBAN DENETİMİNDEKİ MONTAJ SİSTEMLERİ İÇİN BİR YAKLAŞIK MODEL

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Ağustos 2005, 160 Sayfa

Bu tezde kanban denetimli montaj sistemlerinin kararlı-durum performansını değerlenirmek üzere bir yaklaşıklama önerilmektedir. Yaklaşıklama, iki alt ürünün montajlandığı sistemler için geliştirilmiştir. Daha sonra, bu ikiden fazla alt ürünlü sistemlere genişletilmiştir. Bir sürekli Markov modeli, modelin kesinliğini koruyarak kümelenmiştir. Bu kümelenmiş model, duruma bağımlı geçiş oranları sabit geçiş oranları ile değiştirerek yaklaşıklanmıştır. Yaklaşık kümelenmiş modelin alt modellere ayrışımı her bir alt model için çarpım halinde kararlı-durum dağılımlarını garanti etmektedir. Son olarak, alt modeller problemin boyutu kanban sayısından bağımsız olacak biçimde birleştirilmiştir. Bu matris-geometrik çözümleme algoritmalarını kullanarak birleşik modelin çözülmesindeki hesaplama avantajını getirir. Benzetim, kesin model, yaklaşık kümelenmiş model ve daha önceki bir çalışmadaki başka bir yaklaşıklama ile olan sayısal karşılaştırmalar baz alındığında hesaplama sıkıntısına kıyasla doğruluk bakımından yaklaşıklamanın başarılı olduğu gözlemlenmiştir ve daha kompleks fakat gerçek hayat uygulamalarına daha yakın sistemlerin analizlerinde yapıtaşı olma potansiyeline sahip olduğu görülmektedir.

Anahtar Kelimeler: Montaj Sistemleri, Yaklaştırım, Performans Değerlendirmesi, Matris Geometrik Çözümleme, Kanban-Stok Denetimi, Kararlı Durum.

ACKNOWLEDGEMENT

I would like to thank my thesis supervisor Assist. Prof. Dr. Z. Müge AVŞAR for her support, guidance and especially for her patience throughout my thesis.

I am thankful to my chairman Prof. Dr. Fetih YILDIRIM, for his tolerance and support for my graduate studies.

I would also like to thank Prof. Dr. Refik GÜLLÜ for being patient while replying my questions that are urgent for my study, and also I owe a great debt to Dr. Andrea MATTA for sending me the results of his study which I have utilized.

I present my deepest thanks to my mother and father, Gülsiye and Hüseyin TOPAN and special thanks to my brother M. Ersin TOPAN.

I thank my friends Çağrı ÇOŞKUN, Aybeniz YİĞİT, Bora KAT, Halil ÖZBEY, Ender YILDIRIM, İlter ÖNDER and especially İpek SEYRAN and her family for their support during my study. Thanks to everyone I could not mention here.

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LIST OF ABBREVATIONS

BSCS: Base-Stock Control System

CONWIP: Constant Work-in-Process

EB: Expected Backorder

EKCS: Extended Kanban Control System

FP: Expected Number of Assemblies in Stock

FR: Fill Rate

GKCS: Generalized Kanban Control System

IKCS: Independent Kanban Control System

KCS: Kanban Control System

PAC: Production Authorization Card

QBD: Quasi Birth and Death Process

SKCS: Simultaneous Kanban Control System

SP: Stockout Probability

WIP: Work-in-Process

CHAPTER 1

1. INTRODUCTION

With the rise of just-in-time (JIT) philosophy that is introduced to eliminate the unnecessary inventory, pull-type control systems started playing a more important role. The most popular one of these is the kanban control system (KCS). In this study, an approximation is proposed in order to evaluate the steady-state performance measures of kanban controlled assembly systems modeled as Markovian queuing networks. Two-stage systems are under consideration. Upper stage is for the manufacture and synchronization of the components making up an assembly and the lower stage is for the assembly of the components to satisfy the customer demand. The latter is called as the assembly stage. The simplest system structure is considered in this thesis for the development of the approximation proposed. The manufacturing facilities for the components and the assembly facility are all single exponential servers, there is a manufacturing facility for each component type. Two-component systems are analyzed in the first place, and the immediate extensions are considered for systems with more than two components. Two versions of the approximation proposed are for the simultaneous and independent release of kanbans attached to the components making up an assembly.

Even for the two-component, rather simple, systems characterized above, it is difficult to numerically solve the corresponding Markov model for the steady-state probabilities. Especially for large-scale problems with large number of kanbans and many components, the computational burden is prohibitive. There are not many studies on the kanban controlled assembly systems in the literature, and the existing ones are mostly on the approximation methods. In fact, to the best of our knowledge, there isn't any study to obtain the exact numerical solution, even to see what the levels of computational burden would be to go for the exact solution for problems of different sizes. In Di Mascolo and Dallery (1996) and Matta et al.

(2005), exactly the same type of systems as the ones in this thesis are studied using multi-class approximation and product-form approximation techniques developed in a series of studies, Baynat and Dallery (1993a), (1993b), (1995), (1996), Baynat et al. (2001), and Di Mascolo et al. (1996), on some other types of systems.

As for the approximation proposed in this thesis, the previous work to be referred to are Avşar et al. (2004) and Spanjers et al. (2005). These studies are both on the same approximation approach, the former is for the base-stock controlled assembly systems and the latter is for the closed loop two-echelon repairable item systems. The approximation in these studies gives product-form steady-state distributions, and then the relevant performance measures can be approximated accurately in terms of these probability distributions. The same approximation approach is considered in this thesis for kanban controlled assembly systems relying on the good approximation performance observed in Avşar et al. (2004) and Spanjers et al. (2005) for similar systems, especially for the assembly systems in Avşar et al. (2004). We proceed with the same system structure as the one in Avşar et al. (2004), the only difference is that the material coordination is by kanban control policy instead of base-stock policy. Then, the queueing network in our case turns out to be a semi-open network, unlike the open network in Avşar et al. (2004) and the closed loop in Spanjers et al.(2005). As a result, with the expectation of a satisfactory accuracy, we aim to present an alternative approximation for the kanban controlled assembly systems on which the work in the literature is very limited.

This study is organized as follows: In Chapter 2, related studies in the literature are reviewed, concentrating on the assembly systems under different pull control policies and on the solution techniques for the analysis of continuous-time Markov chains. In Chapter 3, the kanban controlled two-component assembly systems; the corresponding models and the approximation approach are introduced. Numerical results are reported in Chapter 4 in order to evaluate the performance of the method as compared to simulation and to the approximation in Matta et al. (2005), and also the approximation is extended to cases with more than two components. Finally, in Chapter 5 we wrap up with concluding remarks.

CHAPTER 2

2. RELATED STUDIES

Kanban control systems (KCS) considered in this study are pull-type production control systems coordinating the release of parts into each stage of the system with the arrival of customer demands for final products. Such control mechanisms are to resolve the trade-off between customer service levels and cost of work-in-process inventories. Due to the stochastic nature of the input parameters identifying the demand and service processes, the studies on pull-type control systems are restricted to easily implementable, rather simple, policies instead of trying to resolve the trade-off over the class of all policies as noted in Liberopoulos and Dallery (2000) . Basic pull control policies are kanban (KCS) and base-stock control policies (BSCS), many others in the literature being derivatives of these basic policies. Constant Work-in-Process (CONWIP) control policy is a special WIP limiting type of kanban control mechanism. Generalized Kanban Control Systems (GKCS) and Extended Kanban Control Systems (EKCS), on the other hand, are to combine KCS and BSCS with the respective advantages of keeping work-in-process inventories at low levels and quick response to customer needs. An overview of the pull control mechanisms is given in Bonvik et al. (1997) and Liberopoulos and Dallery (2000).

As for the assembly systems under pull control mechanisms, policies considered in the literature are as follows: In Di Mascolo and Dallery (1996) and Matta et al. (2005), kanban control systems are analyzed. Chaouiya et al. (2000) and Sbiti et al. (2005) are on the use of EKCS and GKCS, respectively. In Hazra et al. (1999), assembly problems are considered under CONWIP control systems. Avşar et al. (working paper) and Sbiti et al. (2002), on the other hand, are for BSCS. These studies are reviewed in detail in this chapter. Other related references can be found in Schömig and Kahnt (1995).

Most of the models in the literature used for analysis of the systems under pull control mechanisms are stochastic continuous-time formulations as in this thesis, in addition to some deterministic or discrete-time formulations. For the latter, some references are given in Aktürk and Erhun (1999) and Di Mascolo et al. (1996). There are also studies using stochastic Petri nets as in Chaouiya and Dallery (1997), Moore and Gupta (1999). For more on the use of Petri nets for control strategies, see the references in Chaouiya and Dallery (1997). Since our analysis is based on the use of classical queuing theory techniques, the review in this chapter includes only the studies in this category. Since the continuous-time Markov models of manufacturing systems are in general difficult to solve for the steady-state probabilities (structure of the models is not simple to find an analytical solution and multi-dimensional state space is large to numerically solve it), exact numerical solutions can only be found for small systems, e.g., when the number of kanbans is small and there are only a few stations. Logarithmic reduction algorithm, for example, is an iterative procedure that would serve this purpose for Quasi Birth and Death (QBD) processes based on the use of matrix-geometric approach. Among other iterative methods, we particularly refer to this logarithmic reduction algorithm in Latouche and Ramaswami (1993) and (1999) because we employ it in this study. For large systems, there are still implementation difficulties regarding the use of such iterative procedures, which makes approximation unavoidable. Decomposition is the most common approximation approach as revealed to us by the review below, but obviously not the only one for analyzing the assembly systems which we focus on for the review. Before proceeding with the review on assembly systems, we summarize the non-iterative approximation proposed in Albright and Soni (1988) for finding the steady-state distribution of birth and death type continuous-time Markov chains with multi-dimensional state descriptions, the basic of which we employ in this thesis.

In Albright and Soni (1988), continuous-time Markov chains with multi-dimensional state descriptions like $(x_1, x_2, x_3, \ldots x_n)$ are considered. The idea is to fix one of x_i, say $X_1 = x_1$, and ignoring the transitions that change x_1 to solve the problem for conditional probabilities $P\left(X_2 = x_2, X_3 = x_3, \ldots\ldots X_n = x_n \mid X_1 = x_1\right)$. This is repeated for each value of the random variable X_1. Then, using these conditional probabilities to find the expected transition rates that change x_1, one-dimensional birth and death processes are obtained for state description to be solved x_1 for the approximate marginal probabilities $P(x_1)$. Finally, multiplication of these marginal and conditional probabilities gives the steady-state joint distribution. What is proposed in Albright and Soni (1988) is to repeat all these for each of $i \in \{1,......,n\}$ and then to approximate the joint distribution as a weighted average of n different approximate joint distributions. As for the choice of the weights, the authors give some guidelines. The proposed approximation is similar to the aggregation-disaggregation methods in the literature the related references are given in Albright and Soni (1988).

Among many studies on various system structures under different pull control policies other than assembly systems under kanban control policy, we refer to Di Mascolo et al. (1996) and a series of studies by Baynat and Dallery ((1993a), (1993b), (1995), (1996)) and Baynat et al. (2001) because these lead to the analysis in Di Mascolo and Dallery (1996) and Matta et al. (2005) which are for kanban controlled assembly systems. Apart from the preference to present the development in the literature that gives rise to a limited number of studies on kanban controlled assembly systems based on the work in Baynat and Dallery (1993a), (1993b), (1995), (1996), Baynat et al. (2001), and Di Mascolo et al. (1996), conditional throughputs and load-dependent service rates proposed in these studies may show alternative ways of computing the state-independent (expected) conditional probabilities introduced for the approximation in this thesis. Critical importance of working with the right state-independent conditional probabilities will be clear in Chapter 5. Also Buzacott and Shanthikumar (1993) develop an approximation for Production Authorization Card (PAC) systems using conditional q probabilities similar to the ones used in this thesis. Buzacott and Shanthikumar (1993) investigate also serial KCS as one of the special cases of PAC systems. Not only approximation approach, but also their numerical results and interpretations for cases with large errors are parallel to ours. Similarly, although Sbiti et al. (2002) is not on kanban controlled assembly systems we considered in this thesis but under base-stock controlled assembly systems, it is reviewed in this section. This is because the way the decomposed submodels are combined in our study is analogous to the one in Sbiti et al. (2002). Avşar et al. (2004) and Spanjers et al. (2005), on the other hand, are cited because our work is, in fact, an extension of the ones in these references.

Di Mascolo et al. (1996), develop a general purpose analytical method for single-item serial multi-stage kanban controlled systems. This method can handle the cases with any number of machines (stations) at a stage. Di Mascolo et al. propose to decompose the whole system into subsystems, each corresponding to a single stage, and to analyze each subsystem independently of the others using the product-form approximation technique in Baynat and Dallery (1993b). This technique is based on considering each subsystem as a closed queueing network with a constant number of customers in the network (the number of kanbans associated with the stage under consideration) and with all the stations at this stage in addition to two synchronization stations. Defining stations of this closed network as load-dependent service stations, the network becomes of product-form type. The load-dependent rates of the stations are determined approximating conditional throughputs. Later, Baynat et al. (2001) present another way of coming up with the method in Di Mascolo et al. (1996) making further extensions possible to handle the kanban systems with multiple customers and suppliers, kanban controlled assembly systems and generalized kanban systems. As a matter of fact, Frein et al. (1995) and Di Mascolo and Dallery (1996) on the performances of GKCS and kanban controlled assembly systems, respectively, appear as two extensions. The approach in Baynat et al. (2001) is to regard the same system as the one in Di Mascolo et al. (1996) as a multi-class queueing network where each kanban loop corresponds to a customer class. Then, for the analysis of multi-class closed queueing networks, Baynat et al. (2001) employ the technique in Baynat and Dallery (1996) which allows class-dependent general service time distributions, priorities, resource, sharing, fork/join mechanisms.

In Di Mascolo and Dallery (1996) and Matta et al. (2005), kanban controlled assembly systems are studied under both simultaneous and independent release mechanisms. As noted above, multi-class approximation technique introduced in Baynat et al. (2001) allows Di Mascolo and Dallery (1996) to proceed with kanban controlled assembly systems for both of the release mechanisms. In Matta et al. (2005), on the other hand, the work in Di Mascolo and Dallery (1996) is extended to propose a new method for independent release of kanbans based on product-form approximation in Baynat and Dallery (1993a), (1993b) and multi-class aggregation techniques in Baynat and Dallery (1995). They call the analysis based on the exact solution of the assembly synchronization stations as exact SS, and which can be used for smaller systems. But in case of larger systems with simultaneous release of kanbans, assembly synchronization station is represented by a collection of smaller synchronization stations with two queues, one of the queues to represent the behaviour of one class (one component) and the other to represent behaviour of the aggregate class consisting of all the other classes. Then, the number of smaller synchronization stations is equal to the number of components. This approximation is referred to as 2qSS. An alternative one is 3qSS with three queues for each class, two of them are as the ones in 2qSS and a third one is to represent the class related with demand. Also, simultaneous and independent release mechanisms are compared numerically in Matta et al. (2005) as to which one to choose depending on the system parameters. Independent release mechanism performs better when the system throughput is close to the demand rate.

As noted before, the approximation used for the base-stock controlled assembly systems in Sbiti et al. (2002) is similar to the one in this thesis. Two-component system is studied first, and then extensions are considered for systems with more than two components and more than one server in series at the assembly stage. The approximation proposed by Sbiti et al. is to decompose the system into two, one subsystem (upper stage) manufacturing and storing the components to be simultaneously picked up later and sent to the assembly stage and the other subsystem (assembly stage) assembling the components and storing the finished assemblies. State space of the first subsystem is truncated and solved for the steady-state distribution using matrix-geometric method (Neuts (1981)). Then, in the first subsystem, four aggregate states are identified for each state of the second subsystem. This model is solved again using matrix-geometric method, solution of the first subsystem being the input for the model with aggregate states. As compared to this approximation, advantage of the one in this thesis for kanban controlled assembly systems is that approximate closed-form distributions are obtained in the first step making it possible to avoid matrix-geometric method with truncation.

The problem considered by Sbiti et al.(2002) is also investigated in Avşar et al. (2004). Based on the analysis of two-component case; extensions with more than two components and with component commonality are considered by Avşar et al. (2004). In their study, a partially aggregated but exact model is approximated assuming that the state-dependent transition rates arising as a result of the partial aggregation are state-independent. This approximation leads to the derivation of a closed-form steady-state probability distribution, which is of product-form. The approximation proposed for kanban controlled assembly systems in this thesis is an extension of the one by Avşar et al. (2004). As a matter of fact, the closed-form probability distribution mentioned at the end of the previous paragraph is guaranteed proceeding as in Avşar et al. (2004) for the decomposed models. At this point, Spanjers et al.(2005) on the closed loop two-echelon repairable item systems should also be referred to because both of the approximations in Avşar et al. (2004) and Spanjers et al. (2005) are built upon the same approach. The former in Avşar et al. (2004) being accurate for the open assembly systems and the latter in Spanjers et al. (2005) for the closed loop systems, the mentioned approach is considered in this thesis also for the kanban controlled assembly systems falling into the class of semi-open queueing networks (refer to Coenen et al. (2003) for such queueing networks).

Research on kanban controlled assembly systems is rather new and restricted to only the studies by Di Mascolo and Dallery (1996), Matta et al. (2005) and Hazra et al. (1999). Unlike these, the work by Chaouiya et al. (2000) is just to introduce a new combined policy called EKCS for the assembly systems without any performance evaluation using simulation or approximation. Sbiti et al. (2005), on the other hand, is to see how GKCS can be applied for the assembly systems and what properties the corresponding models have. Assembly systems under pull-type control mechanisms, particularly under kanban control mechanism, are, in fact, in one of the application areas of fork/join queues. It is possible to consider applications in queueing models of not only manufacturing but also computer systems. In order to place the type of models we work on in this thesis among all such applications, the reader is referred to Krishnamurthy et al. (2003).

CHAPTER 3

3. TWO-COMPONENT ASSEMBLY SYSTEMS

In this chapter, a simplified kanban controlled assembly system is studied for the case of only two different types of components to be manufactured and assembled at single-server facilities. This system is considered as a building block for the analysis of more complex systems. The system under consideration is two-staged; stage 1 for the manufacture and synchronization of the components and stage 2 for the assembly operations. There are two manufacturing facilities, each dedicated to one of two components, and an assembly facility. The system works as follows: when there is an available kanban for a component, say a component of type 1, raw material is sent to the queue of orders to be processed at the manufacturing facility dedicated to components of type 1 and then to be stored as a ready-for-use component at the corresponding stock point. It is assumed that there are always available raw materials for both types of components. A free kanban for the assembly and one of each type of ready-for-use components in stock are merged at a synchronization station to send component pairs to the assembly facility. Customer demand occurs for the assemblies to be withdrawn from the assembly stock point. There are two different kanban release mechanisms for the synchronization of the components: simultaneous release and independent release. Accordingly, a system is called as Simultaneous Kanban Control System (SKCS) or Independent Kanban Control System (IKCS). In this study, the model and the proposed approach are explained in detail for SKCS. The changes in this model and the analysis are mentioned later to handle IKCS.

An approximate queuing model of the system is given working on an aggregated model, which is exact in spite of two partial aggregations. Even the approximate model is still difficult to analyze, i.e., to solve this approximate model for the steady–state probabilities. Therefore, first the model is decomposed into subproblems, each with a closed-form solution, and then using results of the decomposition a simpler model is solved by logarithmic reduction algorithm.

3.1 Modeling for SKCS

The system outlined above is depicted in Figure 3.1. There are three sets of kanbans circulating. K_i is the number of kanbans for component i, i = 1, 2 and K_3 for the assemblies in order to keep the sum of items (components or assemblies) in stock and item requests awaiting to be processed at the facilities constant. $\overline{n}_1$ ($\overline{n}_2$) is the number of ready-for-use components of type 1 (2) at the respective stock point. $\overline{n}_3$ is defined similarly for the assemblies in stock. n_i is the number of kanbans (attached to the raw materials to be processed) to replenish the stock of component i, i = 1, 2. n_3, on the other hand, is the number of kanbans attached to the component pairs awaiting assembly operation or being assembled. d_1 is the number of backordered requests for component pairs (free kanbans of the assembly stage), and d_2 is the backorders for external demand. Demand arrivals occur according to a Poisson process with rate λ, and service time for station i is exponentially distributed with rate μ_i. It is assumed that $\mu_i > \lambda$ for i = 1, 2, 3; otherwise, the system would explode (d_2 would go to infinity). Queueing discipline is first-come-first-serve at each station.

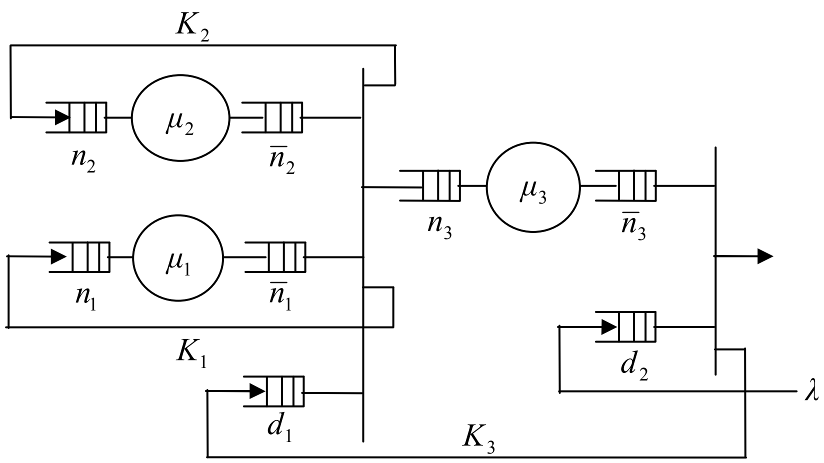


Figure 3.1 Simultaneous Kanban Control System.

When an external demand arrives at stage 2, it is immediately satisfied if there is ready-for-use assembly in stock. Otherwise, the demand is backordered. In the first case, kanban is detached from an available assembly in stock, assembly is sent to the customer and the kanban is sent to stage 1 (the queue of size d_1) as a request of a pair of components to be assembled next to replenish the assembly stock. On the other hand, while transferring manufactured components of type 1 and 2 to stage 2, the kanbans are detached from the components and a free kanban of the assembly stage awaiting as a request is attached to the component pair. Kanbans detached from the components become available to pick up raw materials to replenish the component stocks. The following equations are due to the use of kanban control policy and synchronizations:

$$n_i + \overline{n}_i = K_i \text{ for } i = 1, 2, \quad (3.1)$$

$$n_3 + \overline{n}_3 + d_1 = K_3, \quad (3.2)$$

$$\overline{n}_1 \cdot \overline{n}_2 \cdot d_1 = 0 \text{ and } \overline{n}_3 \cdot d_2 = 0. \quad (3.3)$$

As a result, this kanban controlled assembly system can be modeled as a continuous-time Markov chain with state description $(n_1, n_2, n_3, d_1, d_2)$ (see the relations in Appendix A implied by the equations).

For the purpose of extending the approximation approach proposed for base-stock controlled assembly systems in Avşar et al. (2004) to kanban-controlled assembly systems, an alternative model given in Figure 3.2 is considered. Otherwise it would not be possible to easily employ the mentioned approach, and even if we employ it for two-component systems, extensions to cases with more than two components would not be immediate. The reason for switching to the alternative model will be clear in section 3.3. In the alternative model, components are picked up sequentially. That is, a request for the components can be merged with an available component of type 1 regardless of the availability of component 2. The request (free kanban of the assembly stage in the queue of d_{11}) that is merged with component 1 proceeds and becomes a request for component 2. Therefore, d_1 in Figure 3.1 is split into two as d_{11} and d_{12}, representing the backordered requests for component 1 and 2, respectively.

Mechanics of the models in Figures 3.1 and 3.2 are the same regardless of the sequence in which the components are picked up. This is due to the fact that in both models a request cannot be sent to the assembly stage without picking up one of each component. Equivalence of the two models can be proved as in Avşar et al. (2004), noting that d_1 and $\overline{n}_1$ in Figure 3.1, are equal to $d_{11} + d_{12}$ and $\overline{n}_1 + d_{12}$ in Figure 3.2, respectively. Then, for the alternative model, inventory balance equations implied by the use of kanban control policy and synchronizations take the following form:

$$n_1 + \overline{n}_1 + d_{12} = K_1, \tag{3.4}$$

$$n_2 + \overline{n}_2 = K_2, \tag{3.5}$$

$$n_3 + \overline{n}_3 + d_{11} + d_{12} = K_3, \tag{3.6}$$

$$\overline{n}_1 \cdot d_{11} = 0, \ \ \overline{n}_2 \cdot d_{12} = 0 \ \text{ and } \ \overline{n}_3 \cdot d_2 = 0. \tag{3.7}$$

This is as if the number of kanbans for component 1 is variable and equal to $K_1 - d_{12}$, compare equations (3.1) and (3.4).

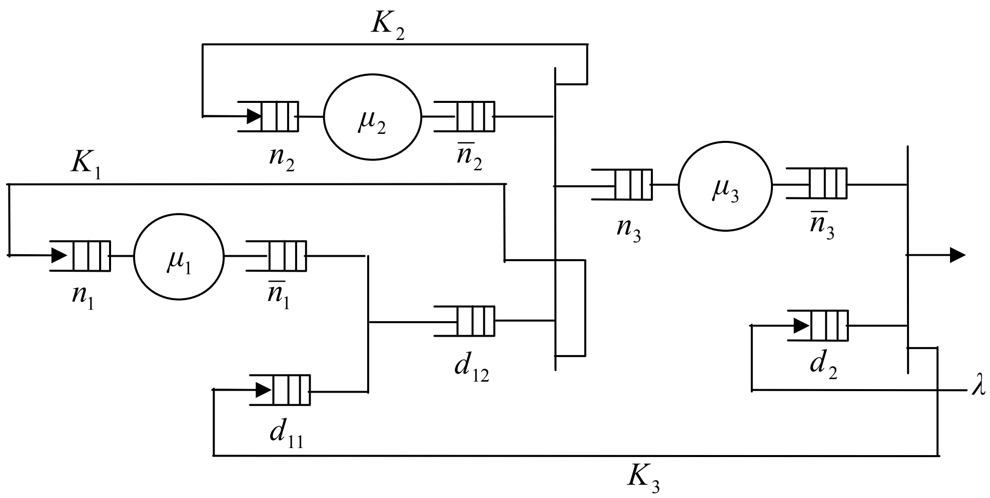


Figure 3.2 Alternative Model for the Simultaneous Kanban Control System.

Let $m_i = n_i + d_{1i}$, $i = 1$, 2, and $m_3 = n_3 + d_2$. Now, it is shown that d_{11}, d_{12} and d_2 values can be determined in terms of m_1, m_2 and m_3. By making use of the definition of m_2, equation (3.5) can be rewritten as $d_{12} - \overline{n}_2 = m_2 - K_2$. Then, referring to the synchronization equation (3.7) for d_{12} and $\overline{n}_2$, we obtain

$$d_{12} = \max\{ m_2 - K_2, 0 \}. \tag{3.8}$$

Similarly, equation (3.4) can be written as $d_{11} - \overline{n}_1 = m_1 - K_1 + d_{12}$ by definition of m_1; and then, using the synchronization equation (3.7) for d_{11} and $\overline{n}_1$,

$$d_{11} = \max\{ m_1 - K_1 + d_{12}, 0 \} \tag{3.9}$$

where d_{12} is given by (3.8). Finally, rewriting equation (3.6) in terms of m_3 as $d_2 - \overline{n}_3 = m_3 - K_3 + d_{11} + d_{12}$ and referring to the synchronization for d_2 and $\overline{n}_3$, we come up with

$$d_2 = \max\{ m_3 - K_3 + d_{11} + d_{12}, 0 \} \tag{3.10}$$

where d_{12} and d_{11} are as given by (3.8) and (3.9), respectively. Thus, (m_1, m_2, m_3) completely determines the system state (details for each possible state are given in Appendix B).

Cross-sections of the three dimensional state-transition diagram representing the system behavior is given in Figure 3.3. (+) and (-) signs next to some transition rates in the figure denote an increase and a decrease, respectively, in m_3. μ_3(-2) denotes two units of decrease in m_3 with rate μ_3. Recall that m_3 is the sum of n_3 and d_2. When $n_3 > 0$ and $d_2 > 0$, the number of entities in both of the queues of size n_3 and d_2 decrease by one upon completion of an assembly operation with rate μ_3. Also, the transitions that are denoted by μ_1' represent two units of decrease in m_1 and at the same time one unit of increase in m_2 with rate μ_1. Since m_1 is the sum of n_1 and d_{11}, when both n_1 and d_{11} are positive, the number of entities in both of these queues decrease by one upon completion of a manufacturing process for component 1. As it is seen in Figure 3.3, there are four different cases; namely, $m_3 = 0$, $m_3 < K_3$, $m_3 = K_3$, $m_3 > K_3$. Each of these cases is detailed next in Figures 3.4(a), (b), (c) and (d) where the regions with different d_{11}, d_{12} and d_2 values are identified referring to (3.9), (3.8) and (3.10), respectively. Note that the figures are drawn for the cases where K_1 is greater than or equal to K_3. When $K_1 < K_3$, the boundaries of the state space will change and so are the Figures 3.4(a), (b), (c) and (d), but still there will be the same regions with different d_{11}, d_{12} and d_2 values and the same transitions in and between these regions. Therefore, the analytical approach in this study can be considered for any of the cases $K_1 \geq K_3$ and $K_1 < K_3$ as long as the state space and the regions mentioned above are clearly identified. Keeping this in mind, we will proceed with $K_1 \geq K_3$ just for the demonstration of the state space of SKCS.

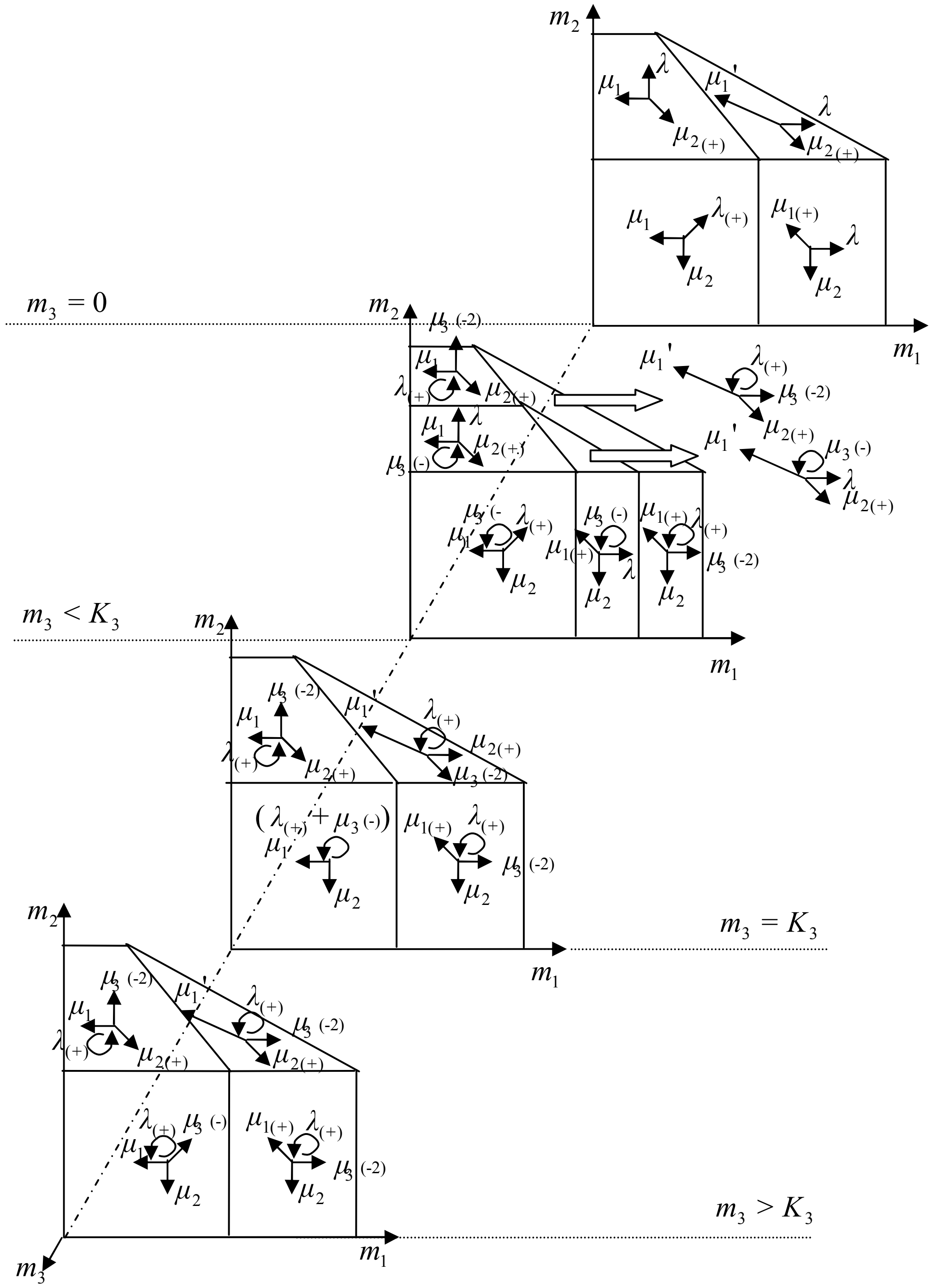


Figure 3.3 Cross-Sections of the State-Transition Diagram for the Model with State Description (m_1, m_2, m_3).

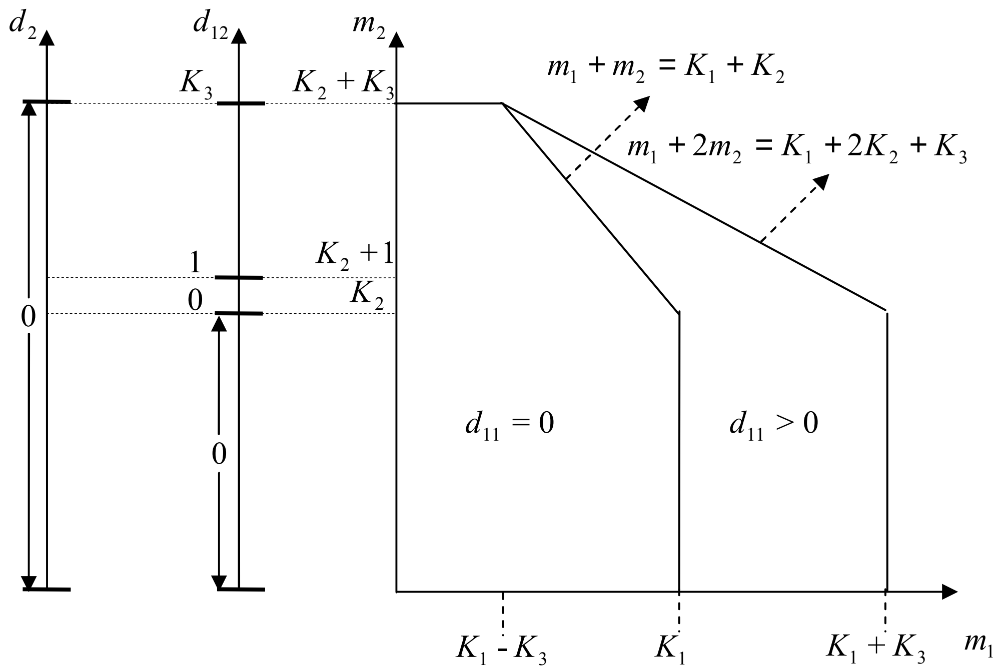


Figure 3.4(a) Cross-Section of the State Space for $m_3 = 0$.

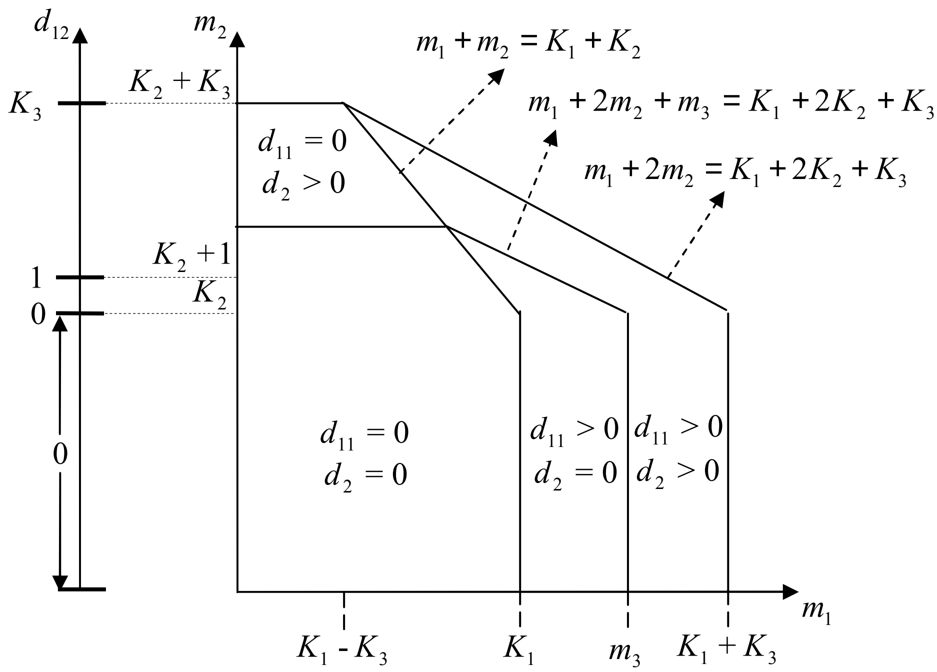


Figure 3.4(b) Cross-Section of the State Space for $m_3 < K_3$.

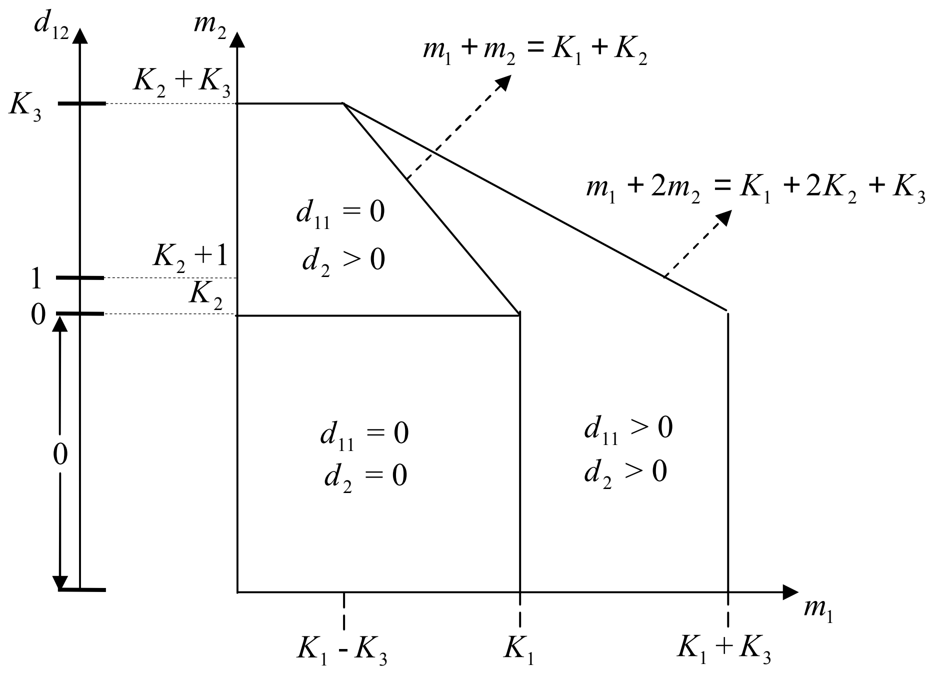


Figure 3.4(c) Cross-Section of the State Space for $m_3 = K_3$.

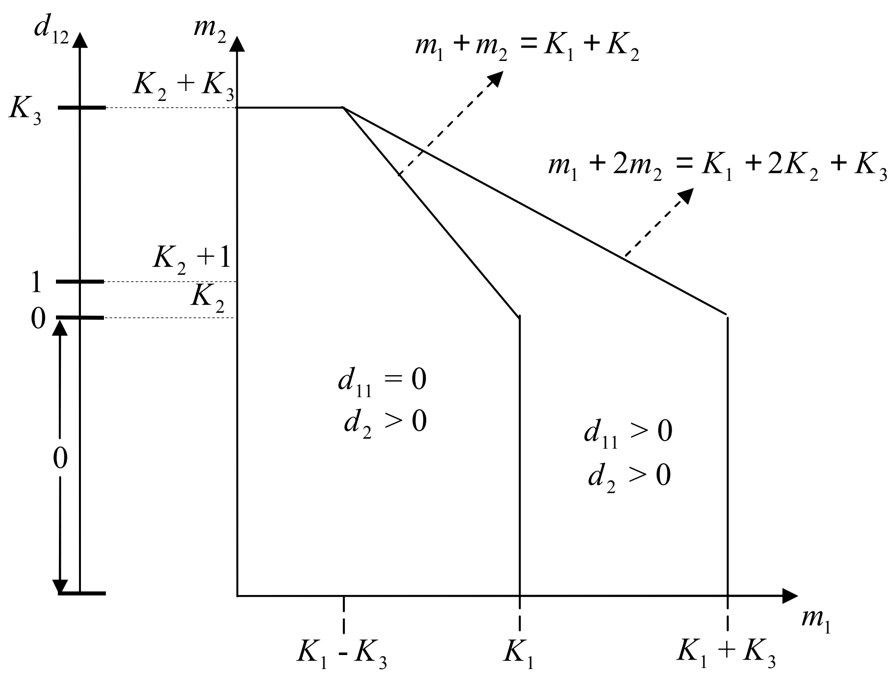


Figure 3.4(d) Cross-Section of the State Space for $m_3 > K_3$.

State-transition diagram of the model with (m_1, m_2, m_3) is seen in Figure 3.5 for different values of d_2. There is a three-dimensional structure for $d_2 = 0$, i.e., every (m_1, m_2, m_3) triple in or on this volume gives $d_2 = 0$. This volume-depth structure is shown by bold lines. The outer part of this volume is wrapped up (covered) by parallel surfaces each representing an additional d_2 level for $d_2 > 0$. Two of them are shown by dashed lines. As d_2 can take infinitely large values, this structure repeats itself in the direction of m_3 for m_3 =0, 1, 2, ... Note that the state space is finite for any given d_2 because $n_3 \le K_3$, $m_1 \le K_1 + K_3$, and $m_2 \le K_2 + K_3$.

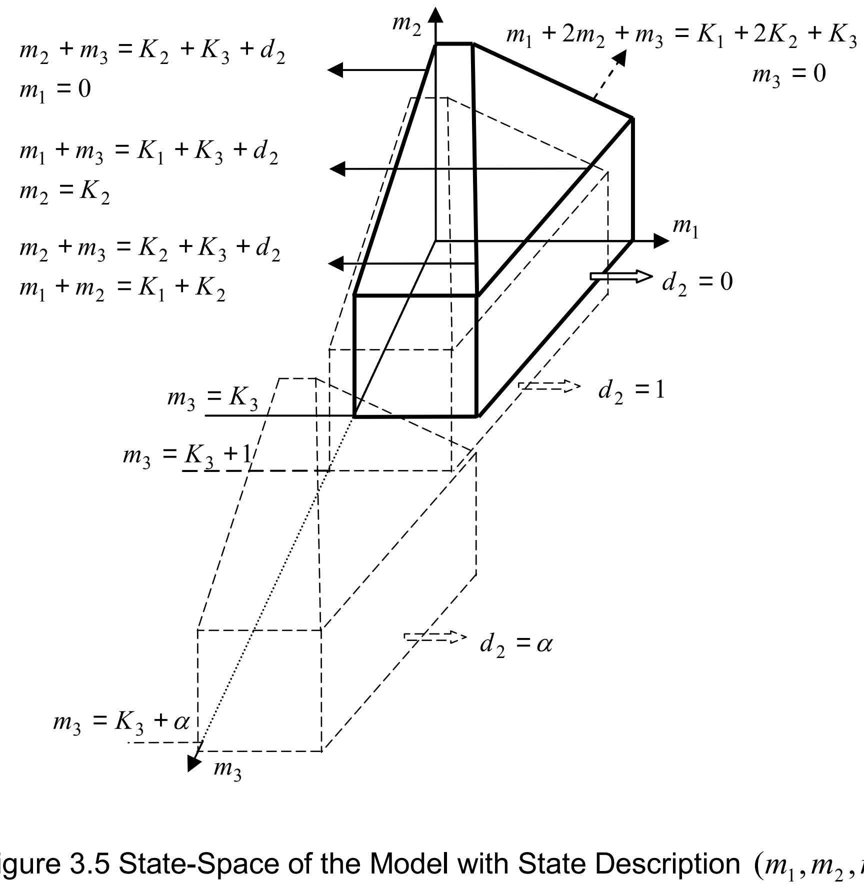


Figure 3.5 State-Space of the Model with State Description (m_1, m_2, m_3).

Since $m_3 = n_3 + d_2$, it is possible to consider also the state description (d_2, m_1, m_2, n_3). Switching from (m_1, m_2, m_3) to (d_2, m_1, m_2, n_3) is preferred for the development of the approximation as it will be clear later in section 3.4. The state-transition diagram of (d_2, m_1, m_2, n_3) is shown in Figures 3.6 (a), (b), (c), (d) for $d_2 = 0$ and in Figure 3.7 for $d_2 > 0$. Unit changes in d_2, m_1, m_2 and n_3 are demonstrated in parentheses next to the transition rates in the order of d_2, m_1, m_2, n_3. For each $d_2 > 0$, the transitions are the same (i.e., structure of the diagram does not change depending on the value d_2 as long as $d_2 > 0$).

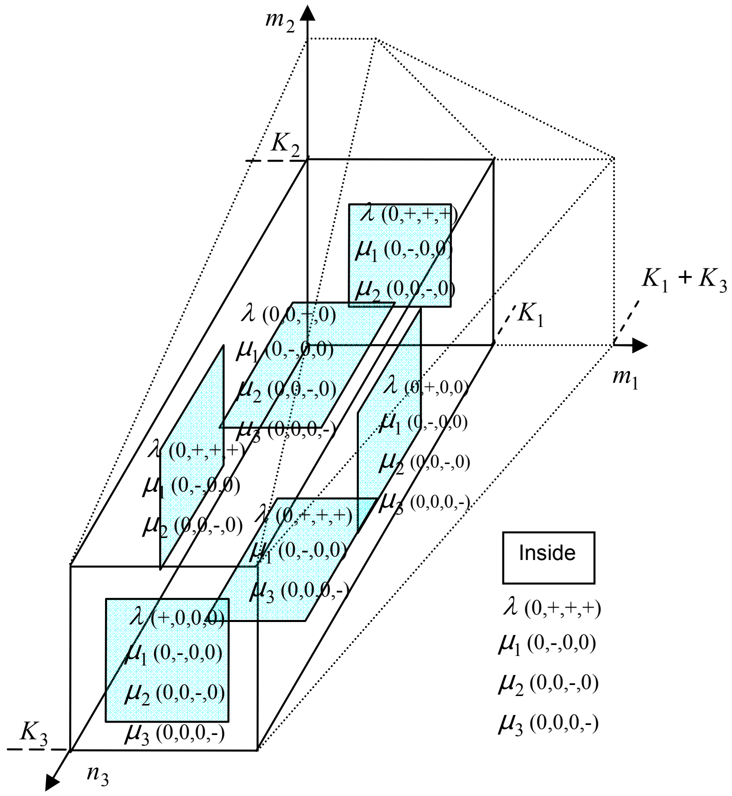


Figure 3.6(a) State-Transition Diagram for $d_2 = 0$, $d_{11} = 0$, $d_{12} = 0$.

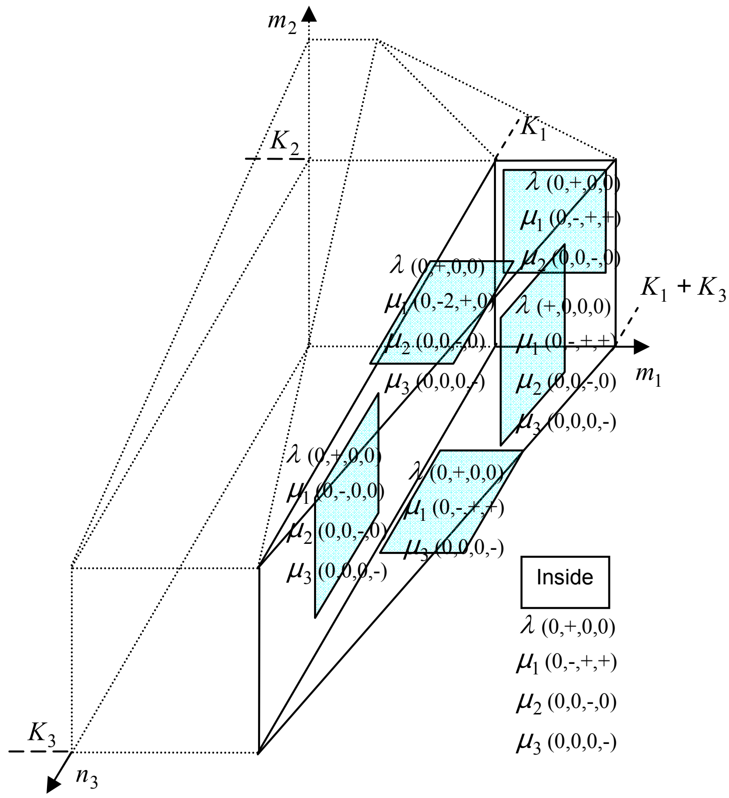


Figure 3.6(b) State-Transition Diagram for $d_2 = 0$, $d_{11} > 0$, $d_{12} = 0$.

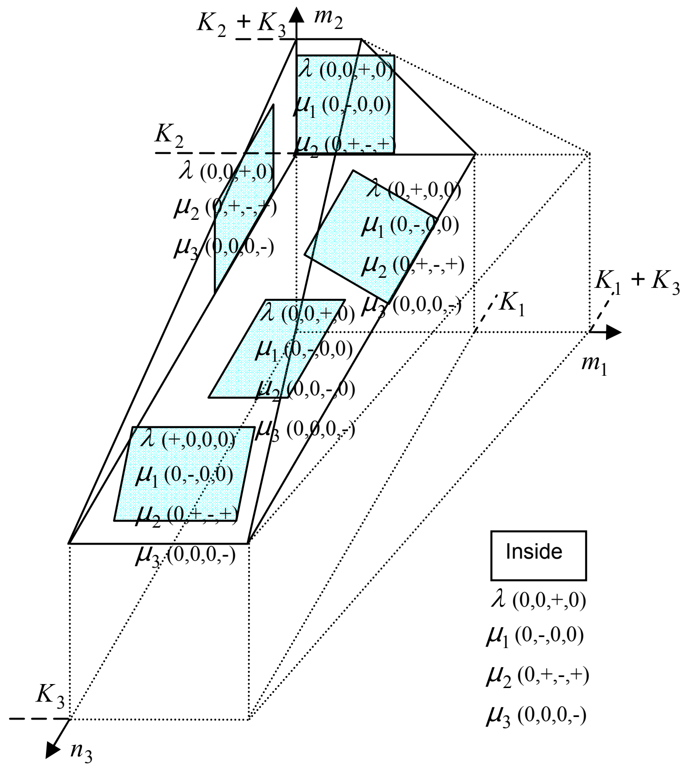


Figure 3.6(c) State-Transition Diagram for $d_2 = 0$, $d_{11} = 0$, $d_{12} > 0$.

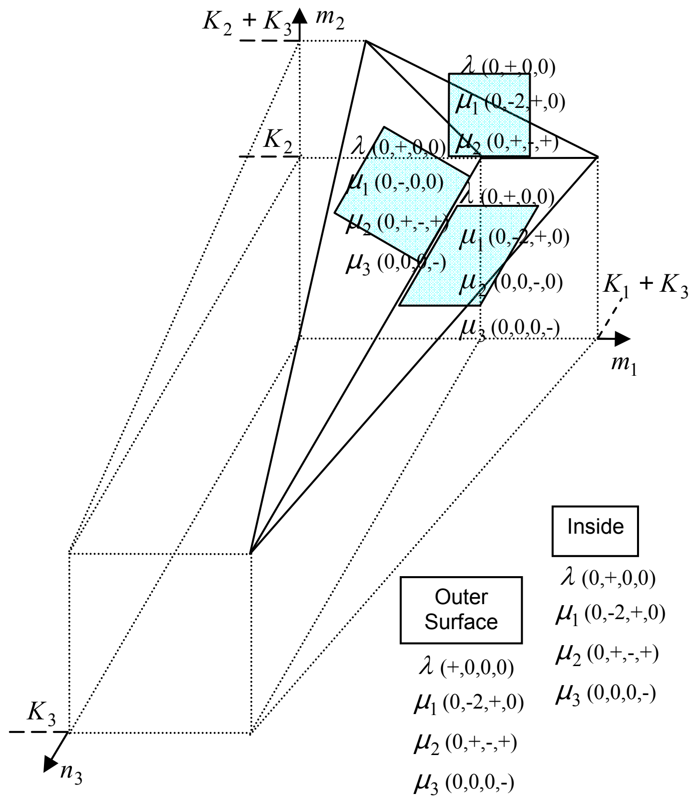


Figure 3.6(d) State-Transition Diagram for $d_2 = 0$, $d_{11} > 0$, $d_{12} > 0$.

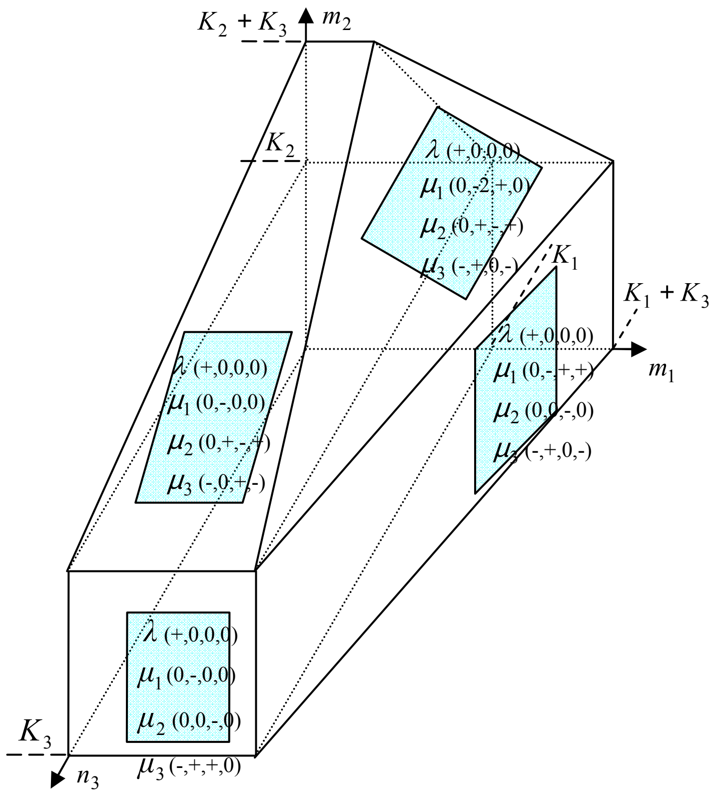


Figure 3.7 State-Transition Diagram for $d_2 > 0$.

3.2 Solution of the Exact Model for SKCS

In this section, a numerical solution procedure is given to find the steady-state distribution of the model with (d_2, m_1, m_2, n_3) using matrix-geometric method (Neuts (1981)). Since the same system structure is repeated along d_2 for all $d_2 > 0$, using the terminology in the matrix-geometric theory d_2 is called as the level of the process. Let $P_{d_2 d_{11} d_{12} n_3}$ be the steady-state probability of being in state $(d_2, d_{11}, d_{12}, n_3)$, then the steady-state distribution at level d_2, is

$$\boldsymbol{\pi}_{d_2} = [\ P_{d_2 00K_3}\ \ P_{d_2 00,K_3-1}\ \ P_{d_2 10,K_3-2}\ \ldots\ P_{d_2,K_3-1,01}\ \ P_{d_2 K_3 00}\ ],$$

and the balance equations and the normalization equation to be solved are

$$[\ \boldsymbol{\pi}_0\ \ \boldsymbol{\pi}_1\ \ \boldsymbol{\pi}_2\ \ \boldsymbol{\pi}_3\ \ldots\ ]\begin{bmatrix} A_{00} & A_{01} & 0 & 0 & \cdots \\ A_{10} & A_{11} & A_{12} & 0 & \cdots \\ 0 & A_{21} & A_{22} & A_{23} & \cdots \\ 0 & 0 & A_{32} & A_{33} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} = \mathbf{0}, \quad \sum_{d_2=0}^{\infty} \left|\boldsymbol{\pi}_{d_2}\right| = 1,$$

where A_{ij} is the rate matrix for the transitions from level i to j, and $\left|\boldsymbol{\pi}_{d_2}\right|$ shows cardinality of $\boldsymbol{\pi}_{d_2}$. From the transition diagrams in Figures 3.6(a), (b), (c), (d) and 3.7, it can be seen that

$$A_{21} = A_{32} = A_{43} = \ldots = A_{d_2+1,d_2}$$

$$A_{11} = A_{22} = A_{33} = \ldots = A_{d_2 d_2}$$

$$A_{12} = A_{23} = A_{34} = \ldots = A_{d_2,d_2+1}$$

for all $d_2 = 1, 2, \ldots$, (see Appendix C for the rate matrix of the example model with $K_1 = K_2 = K_3 = 1$). So, letting $A_{d_2+1,d_2} = A_2$, $A_{d_2,d_2} = A_1$ and $A_{d_2,d_2+1} = A_0$ for $d_2 > 0$, the balance equations can be rewritten as

$$
\begin{bmatrix} \boldsymbol{\pi}_0 & \boldsymbol{\pi}_1 & \boldsymbol{\pi}_2 & \boldsymbol{\pi}_3 & \ldots\ldots \end{bmatrix}
\begin{bmatrix}
A_{00} & A_{01} & 0 & 0 & \cdots \\
A_{10} & A_1 & A_0 & 0 & \cdots \\
0 & A_2 & A_1 & A_0 & \cdots \\
0 & 0 & A_2 & A_1 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots
\end{bmatrix} = \mathbf{0}
$$

That is,

$$\boldsymbol{\pi}_0 A_{00} + \boldsymbol{\pi}_1 A_{10} = \mathbf{0}, \tag{3.11}$$

$$\boldsymbol{\pi}_0 A_{01} + \boldsymbol{\pi}_1 A_1 + \boldsymbol{\pi}_2 A_2 = \mathbf{0}, \tag{3.12}$$

$$\boldsymbol{\pi}_{d_2} A_0 + \boldsymbol{\pi}_{d_2+1} A_1 + \boldsymbol{\pi}_{d_2+2} A_2 = \mathbf{0} \quad \text{for } d_2 > 0. \tag{3.13}$$

Structure of the generator matrix shows that the model with state description (d_2, m_1, m_2, n_3) is an infinite homogenous QBD process. There are efficient algorithms to solve such models for the steady-state distributions, one of them is the logarithmic reduction algorithm (see Latouche and Ramaswami (1993) for the details). It is known to be one of the advantageous algorithms in the literature (see Bini et al. (2002), Tran and Do (2000) for a comprehensive comparison of these algorithms), so it is used here for solving the exact SKCS model with state description (d_2, m_1, m_2, n_3). But, it should be noted that the solution would then be expressed in terms of bulky and huge matrices which are difficult to solve, especially for systems with large kanban sizes. The difficulties to solve the exact model will be discussed later in Chapter 4. Now, we proceed to show how the exact solution can be obtained.

Using the logarithmic reduction algorithm, matrix R is obtained as the minimal nonnegative solution of the following matrix-quadratic equation:

$$A_0 + R A_1 + R^2 A_2 = \mathbf{0}. \tag{3.14}$$

When this equation is multiplied by $\boldsymbol{\pi}_{d_2}$, we obtain

$\boldsymbol{\pi}_{d_2} A_0 + \boldsymbol{\pi}_{d_2} R\, A_1 + \boldsymbol{\pi}_{d_2} R^2 A_2 = \mathbf{0}$ for all $d_2 > 0$,

which gives

$\boldsymbol{\pi}_{d_2+1} = \boldsymbol{\pi}_{d_2} R$ and $\boldsymbol{\pi}_{d_2+2} = \boldsymbol{\pi}_{d_2} R^2$ referring to (3.13).

Proceeding recursively, $\boldsymbol{\pi}_{d_2} = \boldsymbol{\pi}_1 R^{d_2 -1}$ for any $d_2 > 0$. Then, using this relation in (3.12),

$$\boldsymbol{\pi}_1 = -\boldsymbol{\pi}_0 A_{01} (A_1 + RA_2)^{-1} \tag{3.15}$$

when ($A_1 + RA_2$) is invertible. Finally, equation (3.11), i.e.,

$$\boldsymbol{\pi}_0 (A_{00} - A_{01} (A_1 + RA_2)^{-1} A_{10}) = \mathbf{0} \tag{3.16}$$

needs to be solved for $\boldsymbol{\pi}_0$. Since $\boldsymbol{\pi}_{d_2}$ is expressed in terms of $\boldsymbol{\pi}_0$ for all $d_2 > 0$, the normalization equation takes the following form:

$$|\boldsymbol{\pi}_0| - | \boldsymbol{\pi}_0 A_{01} (A_1 + RA_2)^{-1} (\mathrm{I} + R + R^2 + \ldots) | = 1. \tag{3.17}$$

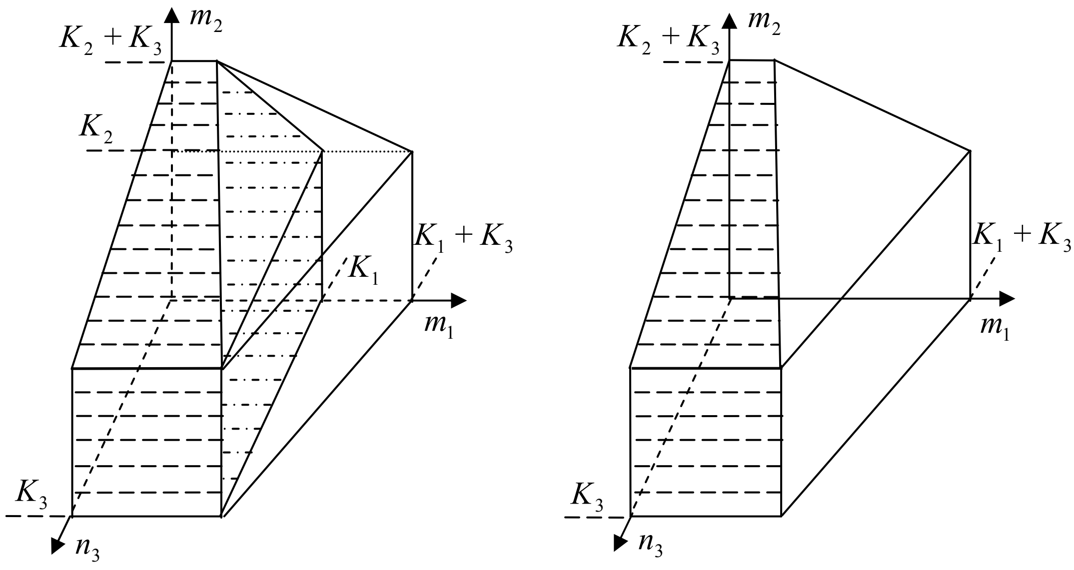
By using logarithmic reduction algorithm in Latouche and Ramaswami (1993) with inputs A_0, A_1, and A_2, matrix R is found and equations (3.16), (3.17) are solved. Note that in order to obtain the exact solution, ($A_1 + RA_2$) should be invertible. Here, we do not examine the nonsingularity of this matrix. Rather, we employ the algorithm for SKCS system with various parameters. The experimental results have shown that there is not any singularity problem except for the exploding systems. Therefore, we do not investigate the conditions for the systems to reach steady-state in the long run. Readers who are interested in the necessary stability conditions for the matrices used in the logarithmic reduction algorithm are referred to Latouche and Ramaswami (1999). The numerical results for the exact (sequential) model are discussed in Chapter 4.

3.3 Aggregation for SKCS

A two-step aggregation is considered in this section to switch from state description (d_2, m_1, m_2, n_3) to (d_2, d_{11}, m_2, n_3) and then to $(d_2, d_{11}, d_{12}, n_3)$. These aggregations are natural because the second (first) one is due to observing that the shaded three-dimensional structure for $d_2 = 0$ and the shaded (two) surface(s) for $d_2 > 0$ in Figure 3.10 (3.8) correspond to $d_{12} = 0$ ($d_{11} = 0$), and in the remaining regions not shaded d_{12} (d_{11}) increases along m_2-axis (m_1-axis). As a result of the first aggregation, $\hat{q}(d_2, m_2, n_3)$ and $\bar{\hat{q}}(d_2, m_2, n_3)$ appear in the transition diagram as seen in Figure 3.9 to adjust the transition rates for $d_{11} = 0$. Note that $\bar{\hat{q}} = 1 - \hat{q}$. $\hat{q}(d_2, m_2, n_3)$ is the steady-state conditional probability that an arriving request of component 1 has to wait (it is backordered), given that it finds no other waiting requests in front of it, when the current state is (d_2, m_2, n_3), i.e.,

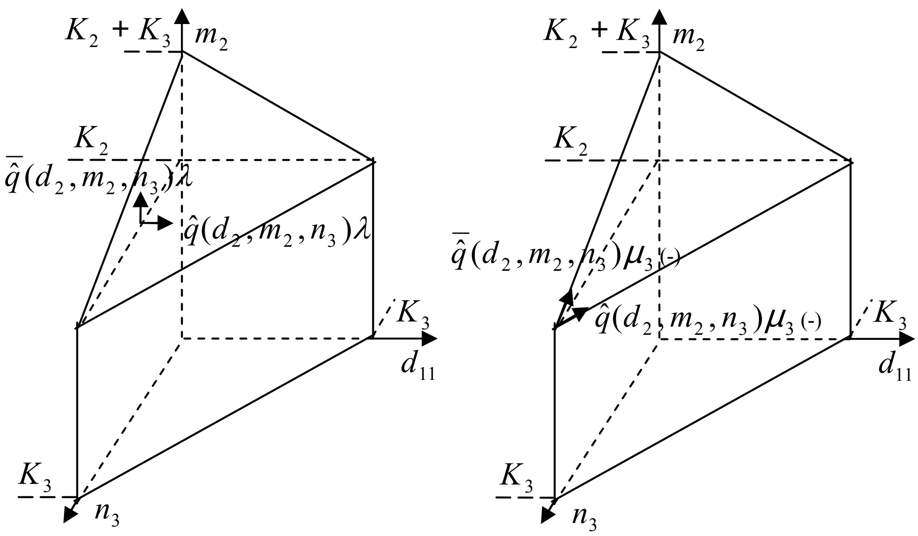
$$\begin{aligned}\hat{q}(d_2, m_2, n_3) &= Pr(\overline{N}_1 = 0 \mid D_2 = d_2, D_{11} = 0, M_2 = m_2, N_3 = n_3) \\ &= Pr(M_1 = K_1 - \max\{M_2 - K_2, 0\} \mid D_2 = d_2, D_{11} = 0, M_2 = m_2, N_3 = n_3)\end{aligned} \tag{3.18}$$

where the second equation results from (3.8) and (3.9).



a) $d_2 = 0$ b) $d_2 > 0$

Figure 3.8 First Aggregation.



a) $d_2 = 0$ b) $d_2 > 0$

Figure 3.9 State Space After the First Aggregation.

The second aggregation leads to $q'(d_2, d_{11}, n_3)$ terms to adjust the transition rates for $d_{12} = 0$, in addition to q and q'' terms. $q'(d_2, d_{11}, n_3)$ is the conditional steady-state probability that an arriving request of component 2 has to wait (it is backordered), given that it finds no other waiting requests in front of it, when the current state is (d_2, d_{11}, n_3). $q''(d_2, n_3)$, on the other hand, is the conditional steady-state probability that an arriving request of component 1 is merged with an available component but this merged entity that passes to backorder queue for component 2 has to wait, given that it finds no other waiting requests in front of it, in state (d_2, n_3). Finally, $q(d_2, d_{12}, n_3)$ is the steady-state conditional probability that an arriving request of component 1 has to wait, given that it finds no other waiting requests in front of it, in state (d_2, d_{12}, n_3), i.e.,

$$q(d_2, d_{12}, n_3) = \sum_{m_2=0}^{K_2} \hat{q}(d_2, m_2, n_3) \cdot Pr(M_2 = m_2 \mid D_2 = d_2, D_{11} = 0, M_2 \leq K_2, N_3 = n_3) \qquad \text{for } d_{12} = 0,$$

$$q(d_2, d_{12}, n_3) = \hat{q}(d_2, m_2 - K_2, n_3) \qquad \text{for } d_{12} > 0.$$

These steady-state conditional probabilities are given below.

$$\begin{aligned} q(d_2, d_{12}, n_3) &= Pr(\overline{N}_1 = 0 \mid D_2 = d_2, D_{11} = 0, D_{12} = d_{12}, N_3 = n_3) \\ &= Pr(M_1 = K_1 - D_{12} \mid D_2 = d_2, D_{11} = 0, D_{12} = d_{12}, N_3 = n_3), \end{aligned} \qquad (3.19)$$

$$\begin{aligned} q'(d_2, d_{11}, n_3) &= Pr(\overline{N}_2 = 0 \mid D_2 = d_2, D_{11} = d_{11}, D_{12} = 0, N_3 = n_3) \\ &= Pr(M_2 = K_2 \mid D_2 = d_2, D_{11} = d_{11}, D_{12} = 0, N_3 = n_3), \end{aligned} \qquad (3.20)$$

$$\begin{aligned} q''(d_2, n_3) &= Pr(\overline{N}_1 > 0, \overline{N}_2 = 0 \mid D_2 = d_2, D_{11} = 0, D_{12} = 0, N_3 = n_3) \\ &= Pr(M_1 < K_1 - D_{12}, M_2 = K_2 \mid D_2 = d_2, D_{11} = 0, D_{12} = 0, N_3 = n_3). \end{aligned} \qquad (3.21)$$

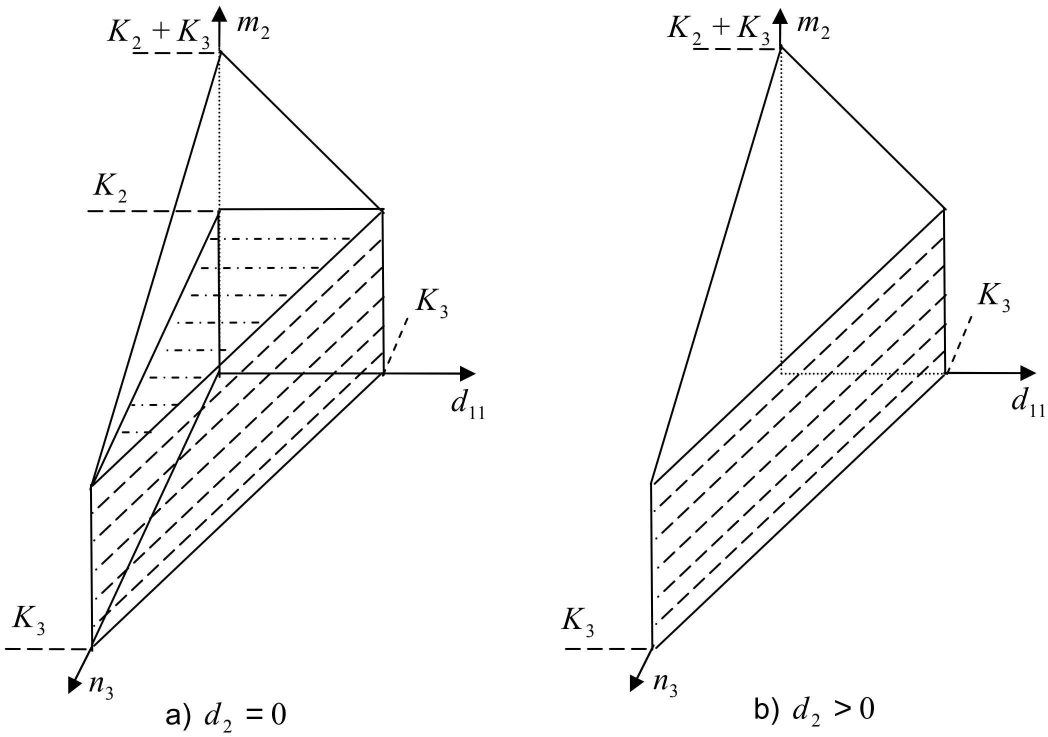


Figure 3.10 Second Aggregation.

The resulting aggregate model is seen in Figures 3.11(a) and (b) without showing the dependence of q, q', and q'' on d_{11}, d_{12}, n_3, d_2 as in equations (3.19), (3.20) and (3.21), respectively, in order to keep the drawing simpler. Aggregated state space is a (volume-depth) pyramid for the case $d_2 = 0$, whereas it is a triangle surface for each $d_2 > 0$ with corners at (d_{11}, d_{12}, n_3) equals $(K_3, 0, 0)$ and $(0, K_3, 0)$ and $(0, 0, K_3)$. For $d_2 > 0$, the state space does not include the points such that $d_{11} + d_{12} + n_3 < K_3$ but only $d_{11} + d_{12} + n_3 = K_3$. That is, due to synchronization when d_2 is greater than zero, $\overline{n}_3$ is equal to zero which results in $d_{11} + d_{12} + n_3 = K_3$. Plus and minus signs in parentheses in Figures 3.11(a) and (b) represent the increase and decrease in d_2, respectively.

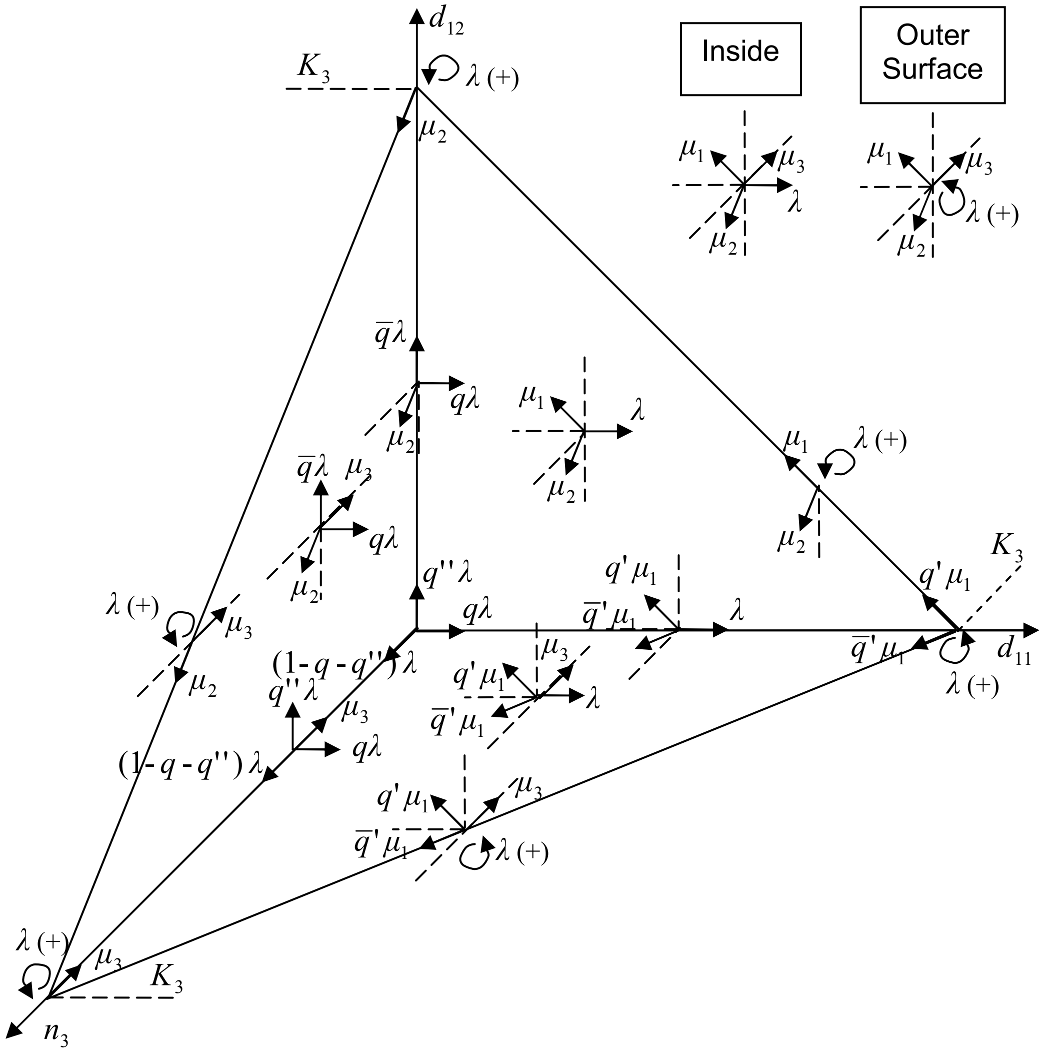


Figure 3.11(a) State-Transition Diagram of the Aggregate Model for $d_2 = 0$.

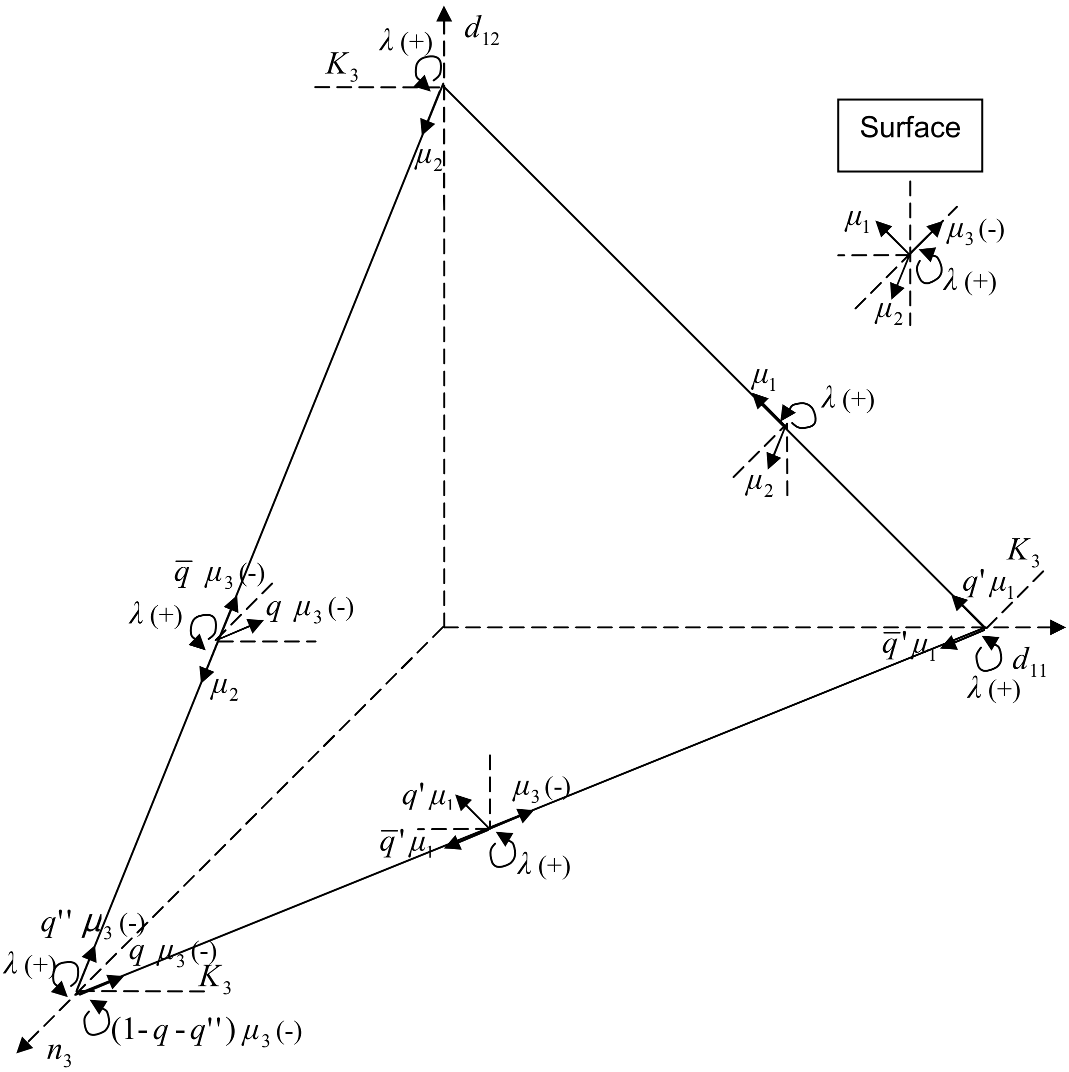


Figure 3.11(b) State-Transition Diagram of the Aggregate Model for $d_2 > 0$.

The two-step aggregation enables to represent the behaviour of the station 1 (station 2) in terms of d_{11} (d_{12}) and state-dependent conditional probability q (q' and q''). This can be achieved working with the alternative model where components are picked up sequentially. Otherwise, i.e., if the same approximation approach is to be applied to the original model, then it would not be possible to define such conditional probabilities, each handle the behaviour of one station where the respective type of components are processed. But even after the two-step aggregation, the model in Figures 3.11(a) and (b) is still exact but can not be solved as is, note that the conditional probabilities q, q', q'' are all expressed in terms of unknown steady-state probabilities. Thus, proceeding with an approximation is unavoidable. The solution method proposed in this study is based on approximating the aggregate model by assuming that q, q', q'' values are not state-dependent, i.e., they are assumed to be state-independent. As it is mentioned in Chapter 1, the method proposed in this thesis is based on the approximation approach used previously in Avşar et al. (2004) and Spanjers et al. (2005). In both studies, i.e., after the approximation of the partially aggregated model of the systems analyzed, the authors come up with tandem networks with product-form steady-state distributions. The model in Spanjers et al. (2005) is a typical-server closed queuing network, whereas the model in Avşar et al. (2004) is an open network (although they do not call the system as that). However, the model that we come across in this thesis is neither an open nor a closed one, and a product-form solution is not observed for the approximate aggregate model whose transition diagram is given in Figures 3.11(a) and (b), but for the submodels obtained decomposing the approximate aggregate model. Therefore one can utilize product-form solutions for sub-problems obtained decomposing the resultant approximate aggregate model.

To sum up encountering difficulties that can not be handled to find the steady-state probability distribution even for the approximate version of the aggregate model, we also propose to decompose the approximate aggregate model into submodels (a submodel for each d_2 value), and solve them independently, and then combine these submodels introducing the dependence relations among them in terms of the product-form probability distributions of the independently solved submodels. Unfortunately, we can not avoid solving the combined model using matrix-geometric method. Doing all these, the two important issues considered are the following: the way we decompose the system so that the steady-state solution for each submodel turns out to be of product-form and the choice or computation of the estimates used for q, q', q'' in order for the approximate results to be accurate. Regarding presentation and discussion are in the following sections.

The same approximation is considered in this thesis for kanban controlled assembly systems relying on the good performance in Avşar et al. (2004) and Spanjers et al. (2005) for similar systems, especially the assembly systems in Avşar et al. (2004). We proceed with the same system structure as the one in Avşar et al. (2004); the only difference is that the material coordination is by kanban control policy instead of base-stock policy. Then, the queueing network in our case turns out to be a semi-open network, unlike the open network in Avşar et al. (2004) and the closed loop in Spanjers et al. (2005). As a result, with the expectation of a satisfactory accuracy, we aim to present an alternative approximation for the kanban controlled assembly systems on which the work in the literature is very limited.

Before proceeding with decomposition, we now consider the direct solution of the approximate aggregate model (with state-independent q, q', q'' values) as is, i.e., without decomposing it but using the solution procedure to the one analogous presented in section 3.2. This is to have a comparison basis to evaluate the performance of the decomposition for given q, q', q'' values. From the transition diagram in Figure 3.11(a) and (b) (see also Appendix D), it can be observed that, as in the case of exact model in section 3.2, the approximate aggregate model with state description $(d_2, d_{11}, d_{12}, n_3)$ is an infinite homogenous QBD process. The rate matrices which would be the input for logarithmic reduction algorithm are given in Appendix D for the example case $K_1 = K_2 = K_3 = 2$. The discussion on the numerical results for the approximate aggregate models is deferred to Chapter 4.

3.4 Decomposition for SKCS

The aggregate model given in Figures 3.11(a) and (b) is decomposed into submodels in such a way that d_2 is fixed for each submodel. To be able to investigate the submodels independently, the link between them is kept out of consideration by replacing the transitions from one to a neighbouring d_2 level with the transitions staying at the same d_2 level as the one they emanate from. That is, (+) or (-) signs, denoting unit changes in d_2 for some of the transitions in Figures 3.11(a) and (b), are all dropped in the submodels as seen in Figures 3.12(a) and (b). The idea behind the decomposition is similar to the one proposed by Albright and Soni (1988) to obtain the approximate steady-state distribution of multidimensional birth and death processes. As in Albright and Soni (1988), the steady-state distribution of each of these submodels in Figures 3.12(a) and (b) obtained by fixing one of the random variables (in this case D_2) is used as an approximation for the steady-state distributions conditioned on $D_2 = d_2$, i.e., $\Pr(D_{11} = d_{11}, D_{12} = d_{12}, N_3 = n_3 \mid D_2 = d_2)$. Apart from this, the manner the decomposed models are analyzed and combined is almost as proposed by Sbiti et al. (2002) for base-stock controlled assembly systems.

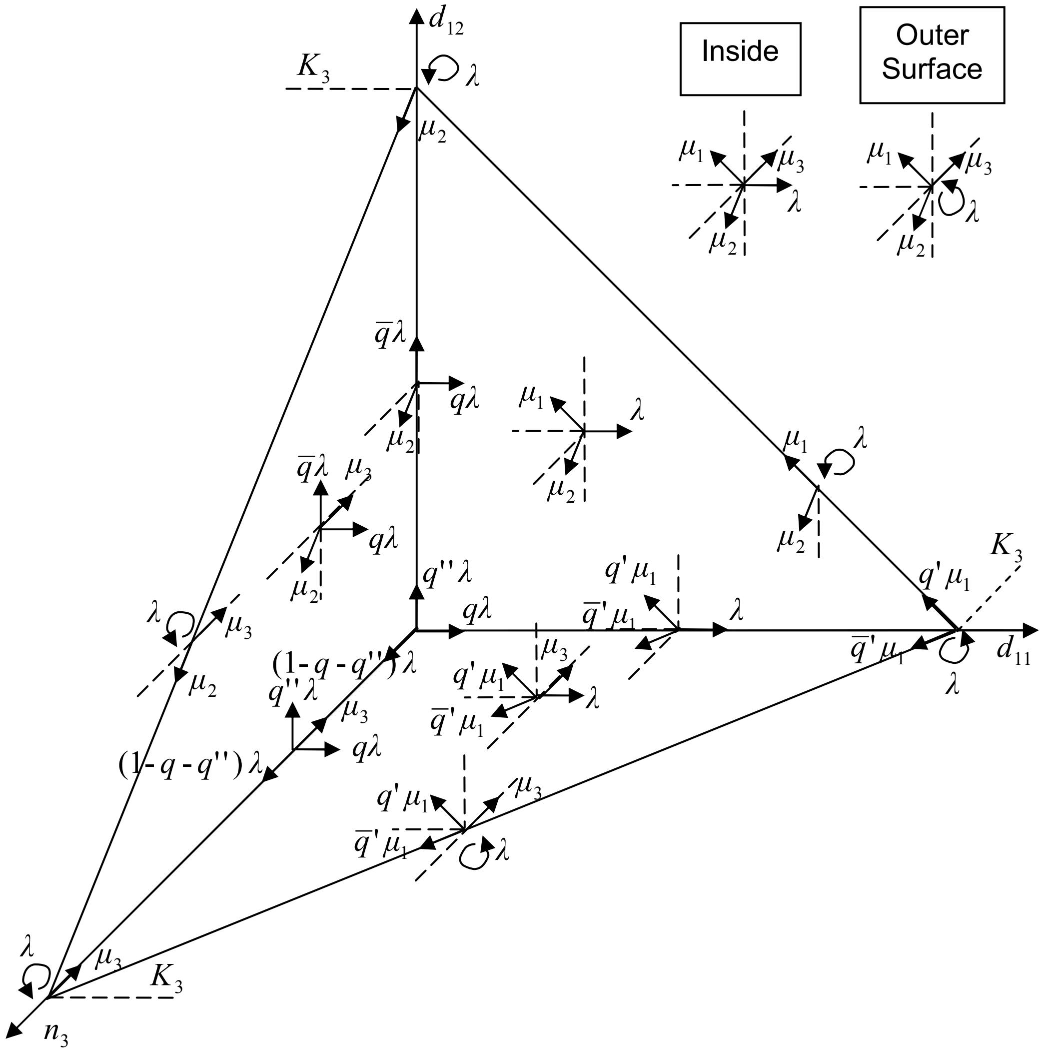


Figure 3.12(a) State-Transition Diagram of the Submodel for $d_2 = 0$.

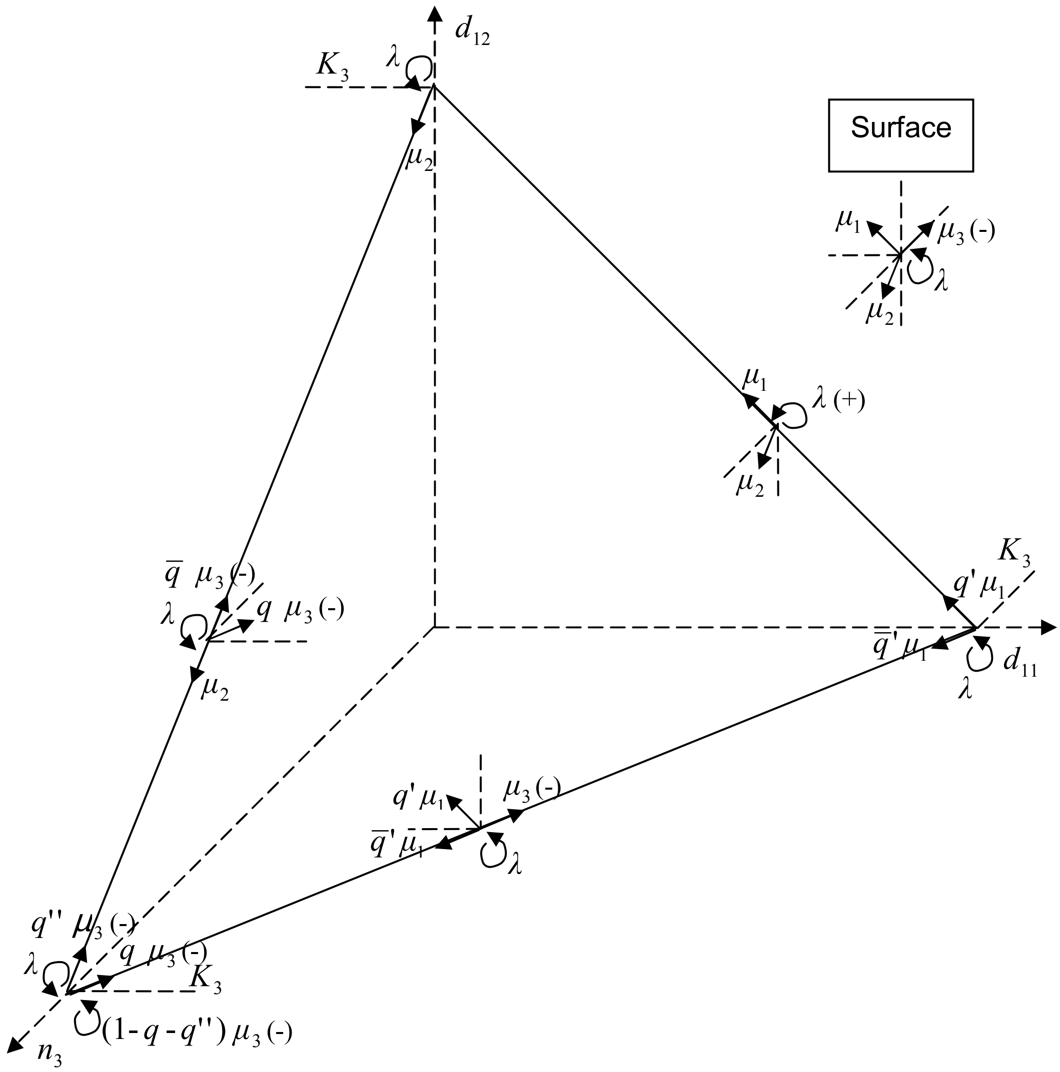


Figure 3.12(b) State-Transition Diagram of the Submodel for $d_2 > 0$.

When d_2 is equal to zero, the system with the transition diagram in Figure 3.12(a) turns out to be an open network with arrivals are blocked when $d_{11} + d_{12} + n_3 = K_3$. In fact, this is equivalent to the system in Figure 3.13(a), which is a closed network with product-form solution. Effective service rates μ_1^* and μ_2^* to process the requests in queues of size d_{11} and d_{12}, respectively, are given as follows:

$$\mu_1^* = \begin{cases} \mu_1 & \text{for } D_{11} > 0, \\ \mu_1 \text{ with probability } q & \\ \infty \text{ with probability } 1\text{-}q & \text{for } D_{11} = 0, \end{cases}$$

$$\mu_2^* = \begin{cases} \mu_2 & \text{for } D_{12} > 0, \\ \mu_2 \text{ with probability } q' & \\ \infty \text{ with probability } 1\text{-}q' & \text{for } D_{12} = 0. \end{cases}$$

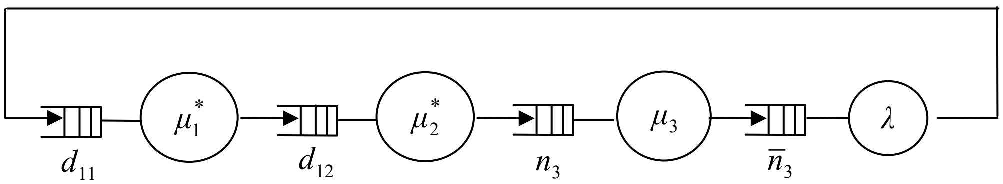


Figure 3.13(a) Equivalent Closed Network of the Submodel for $d_2 = 0$

The model in Figure 3.13 (a) is a kind of typical-server closed queueing network (TCQN) which has a product-form solution (see Spanjers et al. (2005) for TCQN). The product-form steady-state distribution of this closed network is given in (3.22) defining ρ_i as $\frac{\lambda}{\mu_i}$ for $i = 1, 2, 3$.

$$\Pr(D_{11} = d_{11}, D_{12} = d_{12}, N_3 = n_3 | D_2 = 0) = \begin{cases} K\ \rho_1^{d_{11}} \rho_2^{d_{12}} \rho_3^{n_3} & d_{11} > 0, d_{12} > 0, \\ \dfrac{K}{q} \rho_1^{d_{11}} \rho_2^{d_{12}} \rho_3^{n_3} & d_{11} = 0, d_{12} > 0, \\ \dfrac{K}{q'} \rho_1^{d_{11}} \rho_2^{d_{12}} \rho_3^{n_3} & d_{11} > 0, d_{12} = 0, \\ \dfrac{K}{qq'} \rho_1^{d_{11}} \rho_2^{d_{12}} \rho_3^{n_3} & d_{11} = 0, d_{12} = 0. \end{cases}$$

(3.22)

Similarly, for any d_2 greater than zero, we have $\bar{n}_3 = 0$ and the closed network equivalent to the system for which the transition diagram is given in Figure 3.12(b) is as in Figure 3.13(b). Then, letting $\tau_i = \dfrac{\mu_3}{\mu_i}$ for $i = 1, 2$,

$$\Pr(D_{11} = d_{11}, D_{12} = d_{12}, N_3 = n_3 | D_2 = d_2) = \begin{cases} G\ \tau_1^{d_{11}} \tau_2^{d_{12}} & d_{11} > 0,\ d_{12} > 0, \\ \dfrac{G}{q} \tau_1^{d_{11}} \tau_2^{d_{12}} & d_{11} = 0,\ d_{12} > 0, \\ \dfrac{G}{q'} \tau_1^{d_{11}} \tau_2^{d_{12}} & d_{11} > 0,\ d_{12} = 0, \\ \dfrac{G}{qq'} \tau_1^{d_{11}} \tau_2^{d_{12}} & d_{11} = 0,\ d_{12} = 0. \end{cases}$$

(3.23)

for all $d_2 > 0$.

Since the submodels are all the same for $d_2 > 0$, the steady-state distribution for these submodels is not dependent on the value of d_2. The proofs for (3.22) and (3.23) would follow by substituting these probabilities into the balance equations of the transition diagrams in Figures 3.12(a) and 3.12(b), respectively. For further explanation, see Appendix E.

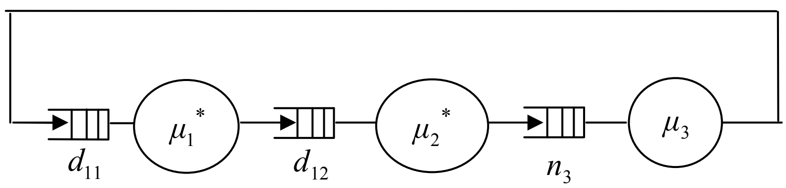


Figure 3.13(b) Equivalent Closed Network of the Submodel for $d_2 > 0$.

To sum up decomposing the approximate aggregate model obtained in the previous section by fixing d_2, we come up with typical-server closed networks for the submodels and the corresponding solutions are of product-form. In the next section, the submodels are combined to obtain the steady-state probabilities of the overall system.

3.5 Combining Submodels of SKCS

The submodels are combined by identifying and aggregating the states with common transition at each d_2 level. The rates of some common transitions are expressed in terms of the (conditional) steady-state probabilities of the submodels. Then, the compact model in Figure 3.14 results in with four aggregate states for $d_2 = 0$ and three aggregate for each of $d_2 > 0$. Introducing an additional random variable Y, (d_2, y) is defined to represent aggregate states of the combined model. The manner in which the decomposed submodels are combined to reflect the links between d_2 levels is in the spirit of the one in Sbiti et al. (2002). Recall some more details given in Chapter 2.

For all d_2, we have

$(d_2,0)$: $\{ (d_{11}, d_{12}, n_3) | \, d_{11} + d_{12} + n_3 = K_3, \, d_{12} = 0, \, n_3 = 0 \}$,

$(d_2,1)$: $\{ (d_{11}, d_{12}, n_3) | \, d_{11} + d_{12} + n_3 = K_3, \, d_{12} > 0, \, n_3 = 0 \}$,

$(d_2,2)$: $\{\ (d_{11},d_{12},n_3) \mid d_{11}+d_{12}+n_3=K_3,\ n_3>0\ \}$,

and for $d_2=0$,

$(d_2,3)$: $\{\ (d_{11},d_{12},n_3) \mid d_{11}+d_{12}+n_3<K_3\ \}$.

Note that $(d_2,0)$ is singleton for any given d_2, it corresponds to $(d_2,d_{11},d_{12},n_3)=(d_2,K_3,0,0)$.

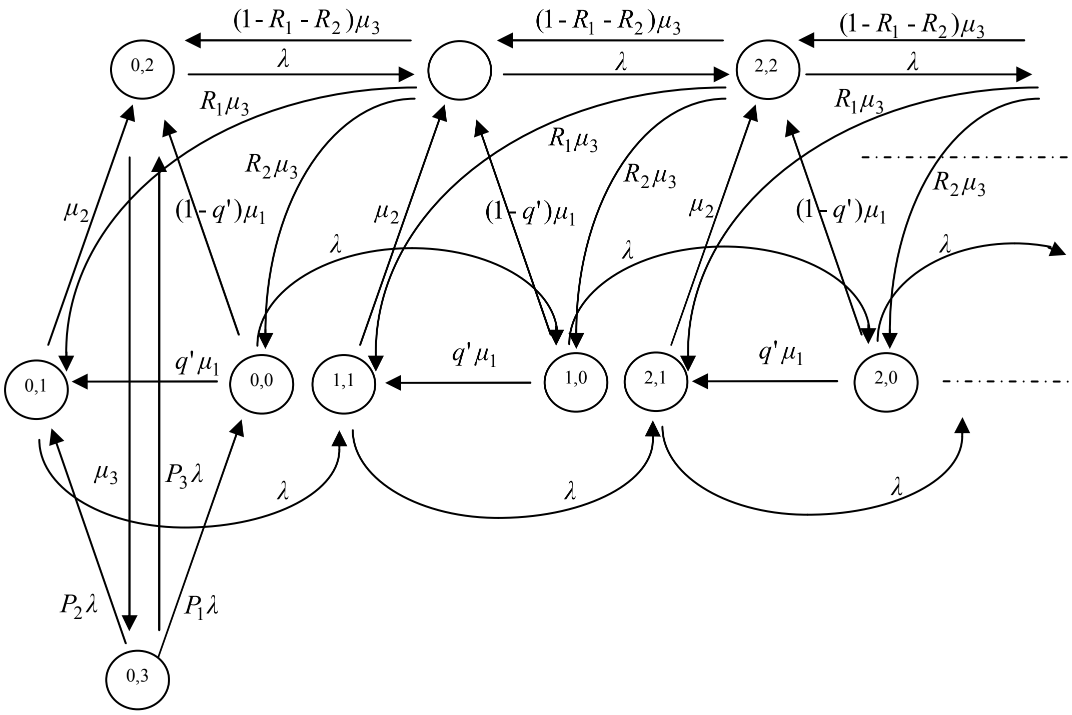


Figure 3.14 Transition Diagram of the Combined Model.

The probabilities to adjust the transition rates between the aggregate states are

$$P_1 = Pr(D_{11}+D_{12}+N_3=K_3-1, D_{12}=0, N_3=0 \mid D_2=0, D_{11}+D_{12}+N_3<K_3),$$

$$P_2 = Pr(D_{11}+D_{12}+N_3=K_3-1, D_{12}>0, N_3=0 \mid D_2=0, D_{11}+D_{12}+N_3<K_3),$$

$P_3 = Pr(D_{11} + D_{12} + N_3 = K_3 - 1, N_3 > 0 \mid D_2 = 0, D_{11} + D_{12} + N_3 < K_3)$,

$R_1(d_2) = Pr(N_3 = 1, D_{12} > 0 \mid D_2 = d_2, N_3 > 0)$,

$R_2(d_2) = Pr(N_3 = 1, D_{12} = 0 \mid D_2 = d_2, N_3 > 0)$.

P_1, P_2 and P_3 are defined for $(d_2, y) = (0,3)$ and obtained using the (conditional) steady-state distribution of the submodel for $d_2 = 0$ as follows.

$$P_1 = \frac{\Pr(D_{11} = K_3 - 1, D_{12} = 0, N_3 = 0 \mid D_2 = 0)}{\sum_{(d_{11}, d_{12}, n_3) \ni d_{11} + d_{12} + n_3 \le K_3 - 1} \Pr(D_{11} = d_{11}, D_{12} = d_{12}, N_3 = n_3 \mid D_2 = 0)},$$

$$P_2 = \frac{\sum_{(d_{11}, d_{12}) \ni d_{11} + d_{12} = K_3 - 1, d_{12} > 0} \Pr(D_{11} = d_{11}, D_{12} = d_{12}, N_3 = 0 \mid D_2 = 0)}{\sum_{(d_{11}, d_{12}, n_3) \ni d_{11} + d_{12} + n_3 \le K_3 - 1} \Pr(D_{11} = d_{11}, D_{12} = d_{12}, N_3 = n_3 \mid D_2 = 0)},$$

$$P_3 = \frac{\sum_{(d_{11}, d_{12}, n_3) \ni d_{11} + d_{12} + n_3 = K_3 - 1, n_3 > 0} \Pr(D_{11} = d_{11}, D_{12} = d_{12}, N_3 = n_3 \mid D_2 = 0)}{\sum_{(d_{11}, d_{12}, n_3) \ni d_{11} + d_{12} + n_3 \le K_3 - 1} \Pr(D_{11} = d_{11}, D_{12} = d_{12}, N_3 = n_3 \mid D_2 = 0)},$$

R_1 and R_2, on the other hand, are defined for $(d_2, y) = (d_2, 2)$ for all $d_2 > 0$. In terms of the (conditional) steady-state distributions of the submodels of $d_2 > 0$, we have

$$R_1 = \frac{\sum_{d_{12} > 0} \Pr(D_{11} = K_3 - D_{12} - 1, D_{12} = d_{12}, N_3 = 1 \mid D_2 = d_2)}{\sum_{n_3 > 0} \Pr(D_{11} = K_3 - D_{12} - n_3, D_{12} = d_{12}, N_3 = n_3 \mid D_2 = d_2)},$$

$$R_2 = \frac{\Pr(D_{11} = K_3 - 1, D_{12} = 0, N_3 = 1 \mid D_2 = d_2)}{\sum_{n_3 > 0} \Pr(D_{11} = K_3 - D_{12} - n_3, D_{12} = d_{12}, N_3 = n_3 \mid D_2 = d_2)},$$

regardless of the value of d_2 because the submodels are all the same for $d_2 > 0$. Recall Figure 3.12(b) and equation (3.23).

As it is seen in Figure 3.14 (see also Appendix F), the combined model with state description (d_2, y) is an infinite homogeneous QBD. The steady-state probabilities can be calculated by the matrix-geometric method, i.e., using logarithmic reduction algorithm, as in section 3.2. Then, the steady-state probability distribution of the aggregate model, i.e., $\Pr(D_2 = d_2, D_{11} = d_{11}, D_{12} = d_{12}, N_3 = n_3)$, can be approximated as $\Pr(D_{11} = d_{11}, D_{12} = d_{12}, N_3 = n_3 \mid D_2 = d_2) \sum_y \Pr(D_2 = d_2, Y = y)$ for all d_{11}, d_{12}, n_3, d_2 where the first (conditional) probability is the steady-state solution to the submodel either in Figure 3.12(a) or (b) (or equivalently Figure 3.13(a) or (b)) depending on the value of d_2 and $\Pr(D_2 = d_2, Y = y)$ is the steady-state solution of the combined model in Figure 3.14. Then, the summation over y gives the marginal steady-state probability for the combined model. Recall that the first probability above is calculated using the product-form distributions in (3.22) or (3.23) and the second term is obtained by employing logarithmic reduction algorithm.

To sum up, the proposed approximate solution to be obtained in two steps, finding product-form solutions of the submodels and then solving the combined model using logarithmic reduction algorithm, needs to be compared to the solutions of the exact model and the approximate aggregate model calculated by the logarithmic reduction algorithm in terms of the efforts required and computational limitations and accuracy of the results. Towards that end, extensive numerical experiments are considered in Chapter 4. Now, we are to propose the state-independent q, q' and q'' values to be used for all the models above except the exact one.

3.6 State-Independent Conditional Probabilities for SKCS

As the dependencies of $q(d_2, d_{12}, n_3)$, $q'(d_2, d_{11}, n_3)$ and $q''(d_2, n_3)$ on d_2, d_{11}, d_{12} and n_3 are ignored in the approximate aggregate model, distributions conditioned on d_2 (solutions to submodels) turn out to be of product-form. Recall the expressions (3.22) and (3.23) which are valid for any choice of state-independent probabilities q, q' and q''. But, then, the question to be addressed is how to compute q, q' and q''. The state-independent q, q' and q'' values used in this study are derived in this section. As noted in Avşar et al. (2004) where the same approximation approach is employed in base-stock controlled assembly systems, avoiding state-dependencies through the use of weighted averages (expected values of $q(D_2, D_{12}, N_3)$, $q'(D_2, D_{11}, N_3)$ and $q''(D_2, N_3)$) would make sense. However, since the kanban system under consideration in this work is more complicated compared to the case in Avşar et al. (2004), there is no claim of the state-independent q, q' and q'' derived here being weighted averages. It should be pointed out that performance of the approximation would differ with the use of some other alternative state-independent q, q' and q'' values.

Referring to (3.19) and ignoring the dependence on d_2, d_{12} and n_3, q is formulated as follows: $q = Pr(M_1 = K_1 - D_{12} \mid D_{11} = 0)$. Now, recall (3.9); then, q is written in terms of M_1 as

$$
\begin{aligned}
q &= Pr(M_1 = K_1 - D_{12} \mid M_1 - K_1 + D_{12} \leq 0) \\
&= \frac{\Pr(M_1 + \max\{M_2 - K_2, 0\} = K_1)}{\Pr(M_1 + \max\{M_2 - K_2, 0\} \leq K_1)} \text{ from (3.8).}
\end{aligned}
\tag{3.24}
$$

Here, we may also use $E[D_{12}]$ instead of working with random variable $D_{12} = \max\{M_2 - K_2, 0\}$, i.e.

$$
q = \frac{\Pr(M_1 = K_1 - E[D_{12}])}{\Pr(M_1 \leq K_1 - E[D_{12}])} \tag{3.25}
$$

where $E[D_{12}] = E[\max\{M_2 - K_2, 0\}]$ from (3.8). As noted before in section 3.1, this is like considering the number of kanbans for component 1 being equal to $K_1 - D_{12}$. Then, $K_1 - E[D_{12}]$ comes up as a further approximation to somehow reflect the average behaviour, which is analogous to the approximations of this sort in Avşar et al. (2004).

Similarly, ignoring the dependence on d_2, d_{11} and n_3, q' is formulated as follows:

$$q' = Pr(M_2 = K_2 \mid D_{12} = 0)$$
$$= Pr(M_2 = K_2 \mid M_2 - K_2 \le 0) \text{ from (3.8)}$$
$$= \frac{\Pr(M_2 = K_2)}{\Pr(M_2 \le K_2)}. \qquad (3.26)$$

Approximate distributions of M_1 and M_2 to be used in (3.25) and (3.26), are obtained analyzing the respective parts of the alternative model in Figure 3.2, namely Figures 3.15 and 3.16, where the arrivals of requests for component 1 and 2 are assumed Poisson with rate λ (the rate can really be λ on the average but we do not question this here). The model in Figure 3.16 turns out to be an $M/M/1$ queue with limited queue size $K_2 + K_3$ when the state description is m_2, note that $m_2 \le K_2 + K_3$. Note that the upper bound on d_2 is K_3 although it is not seen in Figure 3.16. The corresponding transition diagram is given in Figure 3.17. Relation (3.8) implies the following: $d_{12} = 0$ for $m_2 \le K_2$ and $d_{12} = m_2 - K_2$, $\overline{n}_2 = 0$, $n_2 = K_2$ for $m_2 > K_2$. Note that it is not possible to have $d_2 = m_2$, $n_2 = 0$ at some $m_2 > 0$ because this would mean $K_2 = 0$. So, the departures in Figure 3.17 are always with rate μ_2. Arrival rates are, in fact, state-dependent but, we assume they are state-independent as noted above.

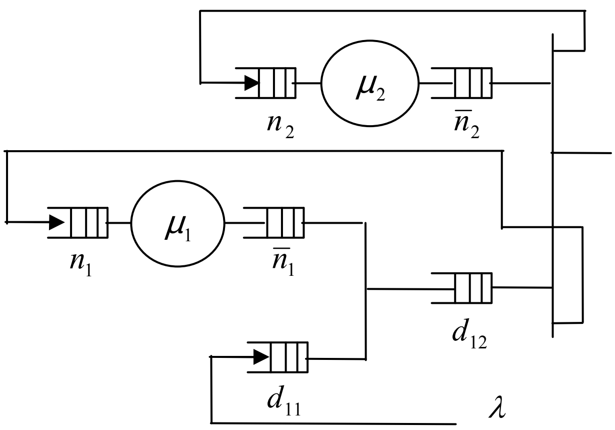


Figure 3.15 Model to compute q.

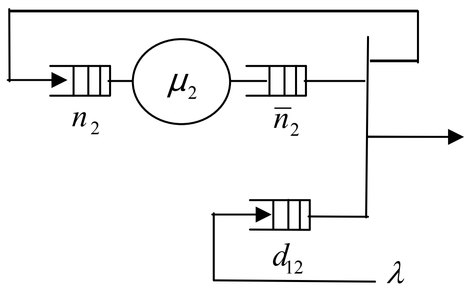


Figure 3.16 Model to compute q'.

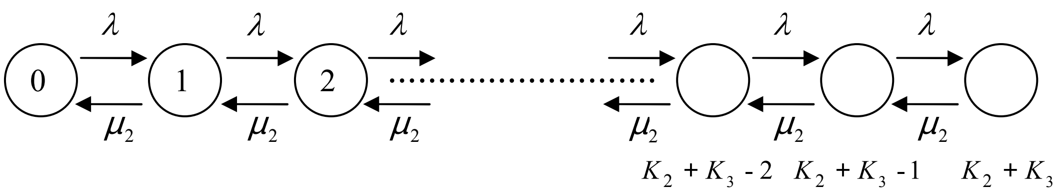


Figure 3.17 State-Transition Diagram of the Model in Figure 3.16 with State Description m_2.

Steady-state solution of the model in Figure 3.17 gives the following expression for q':

$$q' = \frac{(1-\rho_2)\rho_2^{K_2}}{(1-\rho_2^{K_2+1})} \tag{3.27}$$

where $\rho_2 = \dfrac{\lambda}{\mu_2}$.

We can think of three alternative ways of computing q. First is based on considering the distribution of random variable $(M_1 + D_{12})$ as that of $M/M/1$ with limited queue size $K_2 + K_3$ and $\rho_1 = \lambda/\mu_1$. This is equivalent to treating the model in Figure 3.18 independently of the rest of the system (recall Figure 3.15 or the original model in Figure 3.1).

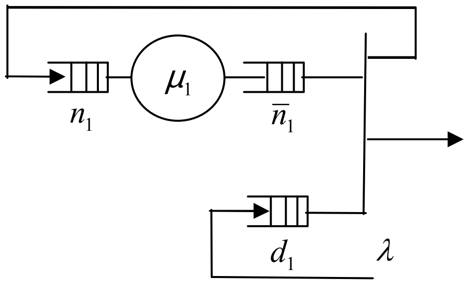


Figure 3.18 Model to compute q_1.

In this case, $q_1 = \frac{(1-\rho_1)\rho_1^{K_1}}{(1-\rho_1^{K_1+1})}$, which may be expected to be the worst as an estimate for q. Second estimate, q_2, is computed considering random variable M_i as the state description of $M/M/1$ with limited queue size $K_2 + K_3$ and $\rho_i = \lambda/\mu_i$ for $i = 1$, 2. Then, treating M_1 and M_2 independently, q_2 is found as in (3.24). Finally, proceeding with the distributions of M_i used for the second estimate, $E[D_{12}]$ appearing in (3.25) is calculated and the third estimate becomes $q_3 = \frac{(1-\rho_1)\rho_1^{K_1-E[D_{12}]}}{(1-\rho_1^{K_1-E[D_{12}]+1})}$. Note that one can formulate $E[D_{12}]$ using (3.8) and the distribution of M_2 as in the following expression:

$$E[D_{12}] = \frac{\rho_2^{K_2}(1+K_3\rho_2^{K_3+1}-(K_3+1)\rho_2^{K_3})}{\rho_2^{K_3+K_2+1}(1-\rho_2)}. \tag{3.28}$$

Finally, $q''(d_2, n_3)$ can be written as the multiplication of $\Pr(M_2 = K_2 \mid M_1 < K_1 - D_{12}, D_2 = d_2, D_{11} = 0, D_{12} = 0, N_3 = n_3)$ and

$\Pr(M_1 < K_1 - D_{12} \mid D_2 = d_2, D_{11} = 0, D_{12} = 0, N_3 = n_3)$. The first probability is replaced with $\Pr(M_2 = K_2 \mid D_{11} = 0, D_{12} = 0, N_3 = n_3)$, which is further approximation. Then, referring to (3.19) and (3.20), $q''(d_2, n_3)$ is assumed to be equal to $q'(d_2, 0, n_3)\,(1 - q(d_2, 0, n_3))$. Ignoring the dependence on d_2 and n_3, we come up with $q'' = (1-q)\,q'$.

3.7 Two-Component Assembly Systems With Independent Release

The independent kanban control system (IKCS) is depicted in Figure 3.19. As compared to the SKCS in Figure 3.1, the kanbans attached for manufacture of components are released independently. This is why there are separate queues of size d_{11} and d_{21} for backordered requests of components 1 and 2, respectively. Upon satisfaction of a customer's assembly request from the corresponding stock point, the kanban that is detached from the assembly sent to the customer is split into two parts as requests of the two components. When a ready-for-use component, say of type 1 (2), and a request for it are merged to proceed to the secondary queue of size d_{12} (d_{22}), the kanban detached from the component is immediately released to manufacture a component and replenish the respective stock. Another synchronization is to merge one of each type of components, each with a part of the assembly stage kanban, to be assembled next at the assembly facility. As noted in Di Mascolo and Dallery (1996), the component pairs are sent to the assembly stage at the same time in both SKCS and IKCS, but the difference is that kanbans attached for manufacture of components are released earlier in IKCS. Therefore, IKCS reacts faster than SKCS. Unlike the case of SKCS, there is no need to proceed with an alternative model of two-component IKCS where components are picked up sequentially. But, this is not the case for more than two components, see Appendix M. Also, the development for the proposed approximation is not dependent on any relation between K_1 (or K_2) and K_3.

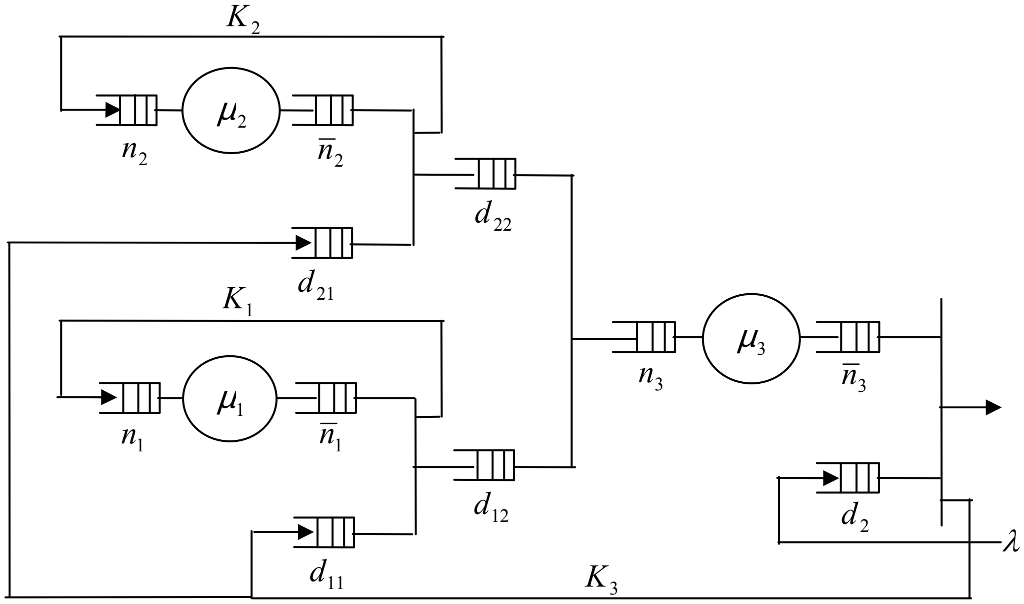


Figure 3.19 Independent Kanban Control System.

The following equations are implied by the use of kanban control policy and synchronizations:

$n_i + \bar{n}_i = K_i$ for $i = 1, 2$, (3.29)

$n_3 + \bar{n}_3 + d_{11} + d_{12} = K_3$, (3.30)

$d_{11} + d_{12} = d_{21} + d_{22}$, (3.31)

$\bar{n}_i \cdot d_{i1} = 0$ for $i = 1, 2$, $d_{12} \cdot d_{22} = 0$ and $\bar{n}_3 \cdot d_2 = 0$. (3.32)

Define $m_i = n_i + d_{i1}$ for $i = 1, 2$ and $m_3 = n_3 + d_2$. From (3.29), $d_{i1} - \bar{n}_i = m_i - K_i$ for $i = 1, 2$. Then, the synchronization equations in (3.32) for d_{i1} and $\bar{n}_i$ lead to

$d_{i1} = \max\{ m_i - K_i, 0 \}$ for $i = 1, 2$. (3.33)

Also, (3.31) can be rewritten as $d_{12} - d_{22} = \max\{m_2 - K_2, 0\} - \max\{m_1 - K_1, 0\}$ using (3.33). Then,

$$d_{12} = \max\{\max\{m_2 - K_2, 0\} - \max\{m_1 - K_1, 0\}, 0\} \quad (3.34)$$

$$d_{22} = \max\{\max\{m_1 - K_1, 0\} - \max\{m_2 - K_2, 0\}, 0\} \quad (3.35)$$

follow from the synchronization constraint $d_{12} \cdot d_{22} = 0$ in (3.32).

The approach introduced for SKCS in sections 3.1 to 3.6 is used here also for the analysis of IKCS. The system as a whole can be modeled as an infinite homogeneous QBD process with state description (d_2, m_1, m_2, n_3). Leaving details of the development to Appendix G, the steady-state distribution of the exact model is again found by logarithmic reduction algorithm (refer to Appendix H for the input matrices).

The aggregate model is constructed defining the conditional probabilities q, q' and q'' similar to the ones in the case of SKCS. $q(d_2, d_{12}, n_3)$ is defined as before, but the definitions of $q'(d_2, d_{11}, n_3)$ and $q''(d_2, n_3)$ are updated according to the operating mechanism of IKCS. $q'(d_2, d_{11}, n_3)$ is the conditional steady-state probability that a ready-for-use component of type 1 has to wait in the secondary queue, given that it finds no other manufactured component of type 1 in front of it, in state (d_2, d_{11}, n_3). $q''(d_2, n_3)$ is the conditional steady-state probability that an arriving request for component 1 is satisfied with an available component in stock but this component that passes to the secondary queue of type 1 has to wait, given that it finds no other waiting requests in front of it, in state (d_2, n_3). That is,

$$\begin{aligned} q(d_2, d_{12}, n_3) &= Pr(\overline{N}_1 = 0 \mid D_2 = d_2, D_{11} = 0, D_{12} = d_{12}, N_3 = n_3) \\ &= Pr(M_1 = K_1 \mid D_2 = d_2, D_{11} = 0, D_{12} = d_{12}, N_3 = n_3), \end{aligned} \quad (3.36)$$

$$\begin{aligned} q'(d_2, d_{11}, n_3) &= Pr(D_{22} = 0 \mid D_2 = d_2, D_{11} = d_{11}, D_{12} = 0, N_3 = n_3) \\ &= Pr(M_2 = K_2 \mid D_2 = d_2, D_{11} = d_{11}, D_{12} = 0, N_3 = n_3), \end{aligned} \quad (3.37)$$

$$\begin{aligned} q''(d_2, n_3) &= Pr(\overline{N}_1 > 0, D_{22} = 0, \mid D_2 = d_2, D_{11} = 0, D_{12} = 0, N_3 = n_3) \\ &= Pr(M_2 = K_2 \mid D_2 = d_2, D_{11} = 0, D_{12} = 0, N_3 = n_3). \end{aligned} \quad (3.38)$$

Proceeding exactly as in the case of SKCS, $q''(d_2, n_3)$ is approximated as $q'(d_2,0,n_3)\ (1-q(d_2,0,n_3))$.

Although the definitions of these conditional probabilities are slightly different than the ones in the cases of SKCS, as these conditional probabilities are assumed state-independent the aggregate model (in terms of q and q') for IKCS is the same as in Figures 3.11(a) and (b). Because of the fact that the aggregate models are identical for both types of release mechanisms, exactly the same results in sections 3.3 to 3.5 follow also for the IKCS. The difference between SKCS and IKCS results from the values of q, q'.

Assuming that q in (3.36) is state-independent,

$$
\begin{aligned}
q &= Pr(\overline{N}_1 = 0 \mid D_{11} = 0) \\
&= Pr(M_1 = K_1 \mid M_1 - K_1 \le 0) \text{ from (3.33) for } i = 1 \\
&= \frac{Pr(M_1 = K_1)}{Pr(M_1 \le K_1)}. \qquad (3.39)
\end{aligned}
$$

Three alternative state-independent q' values are considered. The first one, q_1', in a natural way follows from (3.37) assuming that there is no dependence on d_2, d_{11} and n_3. Then,

$$
\begin{aligned}
q_1' &= Pr(D_{22} = 0 \mid D_{12} = 0) \\
&= \frac{Pr(\max\{M_1 - K_1, 0\} = \max\{M_2 - K_2, 0\})}{Pr(\max\{M_2 - K_2, 0\} \le \max\{M_1 - K_1, 0\})} \qquad (3.40)
\end{aligned}
$$

from (3.34) and (3.35). As for the second alternative,

$$
\begin{aligned}
q_2' &= Pr(D_{22} = 0 \mid D_{12} = 0), \\
&= Pr(D_{21} = D_{11} \mid D_{21} \le D_{11})
\end{aligned}
$$

because $d_{12} - d_{22} = d_{21} - d_{11}$ from (3.31) and $d_{12} \cdot d_{22} = 0$ from (3.32).

Assuming that $d_{21} = m_2 - K_2$ which is obtained by slightly modifying the expression in (3.33),

$$q_2' = Pr(M_2 = K_2 + D_{11} \mid M_2 \leq K_2 + D_{11})$$

where $D_{11} = \max\{M_1 - K_1, 0\}$.

Instead, we may proceed with

$$q_2' = Pr(M_2 = K_2 + E[D_{11}] \mid M_2 \leq K_2 + E[D_{11}]) \tag{3.41}$$

where $E[D_{11}] = E[\max\{M_1 - K_1, 0\}]$.

Finally, similar to the derivations in section 3.6 state-independent q'' is chosen as $q'' = (1 - q)q'$.

Approximate distribution of the random variable M_i is calculated treating the model in Figure 3.20 independently of the remaining part of the system and assuming that arrivals are Poisson with constant rate λ. Note that, in fact, the arrival rate is state-dependent. Then, the model in Figure 3.20 is an $M/M/1$ queue with a limited queue size of $K_i + K_3$ for i = 1, 2, and (3.39) becomes

$$q = \frac{(1 - \rho_1)\rho_1^{K_1}}{(1 - \rho_1^{K_1+1})}. \tag{3.42}$$

q_1' and q_2' are also obtained in terms of these approximate probability distributions of M_1 and M_2. Using the approximate distribution of M_1,

$$E[D_{11}] = \frac{\rho_1^{K_1}(1 + K_3\rho_1^{K_3+1} - (K_3 + 1)\rho_1^{K_3})}{\rho_1^{K_3+K_1+1}(1 - \rho_1)}. \tag{3.43}$$

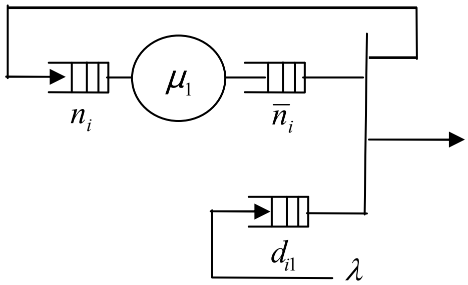


Figure 3.20 Model to compute q and q'.

Note that, all the estimates for state-independent conditional probabilities, i.e., q and q' in sections 3.6 and 3.7, are obtained by assuming that the arrival rates are state-independent and ignoring the correlation between M_1 and M_2. The proposed estimates could be improved by working with the state-dependent arrival rates as in Matta et al. (2005) and maybe this way reflecting the correlation between M_1 and M_2.

CHAPTER 4

4. PERFORMANCE OF THE APPROXIMATION

In this chapter, performance of the proposed approximation is investigated in terms of both accuracy and the required efforts to come up with the approximate solutions for different problem sizes with various traffic intensities. The performance evaluation is not only based on comparing the proposed approximate performance measures to the simulation results but also to the solutions of the exact model and the approximate aggregate model based on the matrix-geometric method. Recall that the approximate aggregate model is the one in Figures 3.11 (a) and (b) with state-independent q, q', q'' values for both SKCS and IKCS. Exact model, on the other hand, is the one given in section 3.1 for SKCS with its matrix-geometric solution derived in section 3.2 and the one in section 3.7 for IKCS.

As noted before, the steady-state distributions for the exact model can be obtained only for small problems while the distribution of the approximate aggregate model can be obtained for some large-scale problems. For both of these models, huge and bulky matrices need to be solved even for small problems. Even decomposing the approximate aggregate model, we can not avoid employing matrix-geometric method to solve the combined model. However, sizes of the matrices become reasonable because of using the product-form distributions of the decomposed submodels in constructing the combined model.

4.1 Two-Component Systems

Let Z denote the term $A_{00} - A_{01}(A_1 + RA_2)^{-1}A_{10}$ appearing in equations (3.16) and (3.17), and recall that R is the matrix obtained solving logarithmic reduction algorithm. The matrix sizes are functions of the number of kanbans for the exact and approximate aggregate models, but constant for the combined model. R is a 3×3

matrix and Z is a 4×4 matrix for the combined model of the two-component systems. Sizes of the matrices R and Z for some example cases are given in Table 4.1 and CPU times required are in Table 4.2, showing the advantage of the approximate aggregate model over the exact one and of the combined model over the approximate aggregate model in this respect. Note that each CPU time given in Table 4.2 is for 27 different ρ_i, $i=1$, 2, 3, combinations for a given (K_1, K_2, K_3). Exact, aggregate and combined in the tables and figures refer to the exact model, approximate aggregate model and the combined model, respectively. The highest $K_i = K$, $i=1$, 2, 3 value for which the exact model can be solved is 10, this number increases up to 25 for the approximate aggregate model. However, for the combined model practically there is no such limitation on the number of kanbans (see Figure 4.1).

Table 4.1 Sizes of the matrices R and Z.

Kanbans			Aggregate		Exact	
K1	K2	K3	Z	R	Z	R
3	3	3	10×10	10×10	106×106	37×37
5	5	5	35×35	21×21	391×391	91×91
7	7	7	84×84	36×36	960×960	169×169
10	10	10	220×220	66×66	2556×2556	331×331
20	20	20	1540×1540	231×231	18211×18211	1261×1261

Table 4.2 CPU Times for different kanban sizes.

			CPU Time (sec)					
Kanbans			SKCS			IKCS		
K1	K2	K3	Combined	Aggregate	Exact	Combined	Aggregate	Exact
3	3	3	0.922	1.188	11.250	0.719	0.906	19.532
5	5	5	0.656	1.312	57.156	0.547	1.188	124.937
7	7	7	0.609	3.609	324.340	0.531	3.657	696.469

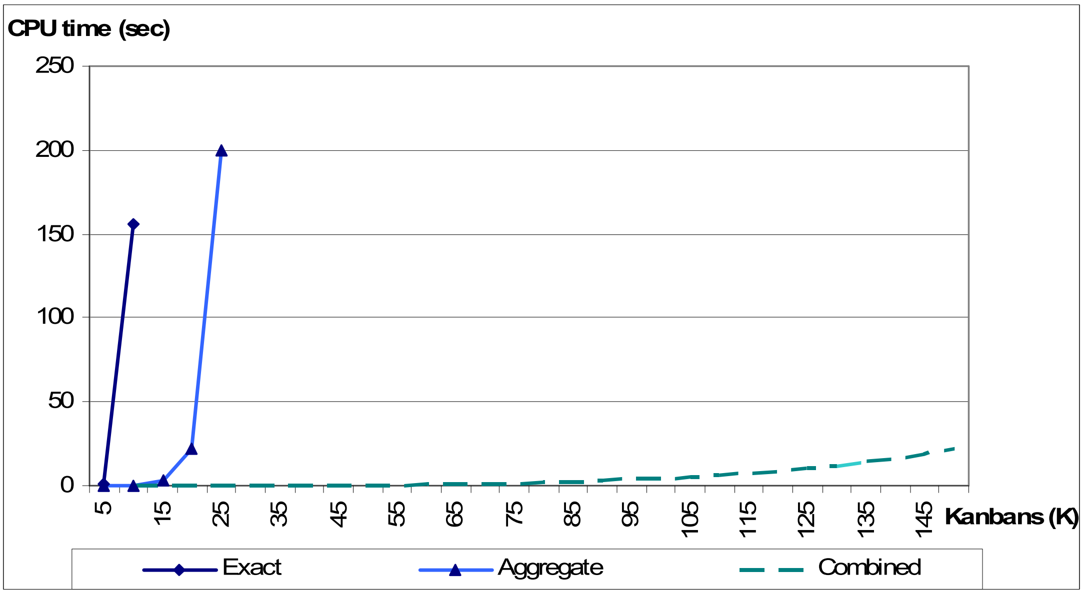


Figure 4.1 Comparisons for SKCS with $K_i = K$ for $i = 1, 2, 3$.

Although we have a solution procedure for the exact model, we prefer to use simulation results for testing the approximation performance. The main reason is that size of the problems that can be solved by the exact model is limited. Also, simulation is not only used for testing accuracy of the approximate aggregate and the combined models, but also to verify the exact solution. Considering these, we choose cases such that solutions of the exact model can be obtained and, for the sake of completeness, all the errors are calculated relative to the simulation results. Later in this section, we also consider the cases beyond these limitations. The full test bed consists of the cases with $\lambda = 1$ entity/unit time and μ_i taking values of 1.25, 1.61 and 2 in order to consider different combinations of the traffic intensities ($\rho_i = \dfrac{\lambda}{\mu_i}$) 0.5, 0.65 and 0.8. K_i takes the values of 3, 5 and 7. As for the comparison of the performances of the exact model and the approximate aggregate model and the combined model, the following performance measures are considered: stockout probability SP, $\Pr(D_2 > 0)$, fill rate FR, $\Pr(\overline{N}_3 > 0)$, expected backorders EB for the assembly, $E[D_2]$, and expected number of assemblies in stock FP, $E[\overline{N}_3]$. The approximate performance measures are calculated using Matlab 6.5. For simulation, Arena 4.0 software is used. Given a parameter set, 15 replications are considered, each with a simulation length of 1.000.000 time units and a warm-up period of 200.000 time units. The graphs to observe the warm-up period for a system with high traffic intensity are in Appendix I. The simulation time is taken as 5 times the warm-up period, and half widths for a confidence level 95% are tabulated in Appendix O for an example case. CPU times on an Intel Centrino 1.7 processor are given in Table 4.3 for 729 parameter sets. The object-oriented visualization of the code can be seen in Appendix J.

Table 4.3 CPU Times for the whole test bed.

Model	SKCS	IKCS
Simulation	103 hr. 16 min. 29 sec.	122 hr. 16 min. 5 sec.
Exact	1 hr. 8 min. 19.651 sec.	2 hr. 26 min. 51.921 sec.
Aggregate	51.787 sec.	52.781 sec.
Combined	13.594 sec.	13.156 sec.

Recall that, in section 3.6, we have proposed three alternative estimates of q for SKCS. Similarly, in section 3.7, two alternative estimates of q' are proposed for IKCS. According to the preliminary numerical results, q_3 and q_2' are the best. Therefore, we proceed with q_3 for SKCS and q_2' for IKCS for more extensive numerical experiments. The preliminary results for SKCS are summarized in Table 4.4. For IKCS, q_2' is much better than q_1', that's why we don't need to tabulate the results in this case. Absolute (relative percent) errors are considered for FR and SP (EB and FP).

Table 4.4 Comparison of the performance of the alternative estimates for q in SKCS.

Performance Measure	Approximation Error					
	q_1		q_2		q_3	
	Average	**Max**	**Average**	**Max**	**Average**	**Max**
EB	20.39	68.56	18.73	64.24	18.79	62.55
FP	4.28	65.95	3.62	36.87	3.38	42.47
FR	0.0212	0.1602	0.0179	0.1593	0.0175	0.1240
SP	0.0238	0.1967	0.0201	0.1666	0.0199	0.1576

Before going through the outcomes of the numerical analysis, it should be noted that the observations reported in the following paragraphs are almost the same for each performance measure because error patterns are similar for given kanban combinations as seen in Figures 4.2 (a), (b), (c), (d), (e) and (f) for 27 example cases with $K_i = 5$, $i = 1$, 2, 3. The numerical results for SKCS and IKCS are tabulated for the same cases in Tables 4.5 (a) and (b), respectively. For both the tables and the graphs of some other parameter sets, see Appendix K. The different parameter sets in Table 4.3 appear as cases in the same order along the horizontal axis in Figure 4.2. The highest approximation errors are observed for EB, but it should be noted that in almost all cases with high EB errors the expected backorders and the corresponding approximations are below 1 or even very close to zero. That is, although the EB approximations are good, a small deviation from the exact value which is below 1 causes the relative error to turn out to be high. In addition to these with very small EB values, there are also a few with EB values above 1 but the relative error being large when the traffic intensity of the system is high.

As expected, results of the exact model are almost the same as the simulation results. Almost all of the results obtained by solving the exact model fall into 95% confidence intervals of the performance measures constructed using simulation results for the whole test bed. However, with the results obtained by the approximate aggregate model 15.23%, 8.37% and 8.57% of the results fall into 95% confidence intervals for EB, FR and SP, respectively. As for the results of the combined model, 7.27%, 6.04% and 5.28% of the results fall into 95% confidence intervals for EB, FR and SP, respectively. Note that simulation is used in order to test our approximation performance for the approximate aggregate and the combined models, whereas it is used as a verification tool for the exact model. As an example, EB and FR results of the case with $K_1 = 5, K_2 = 5, K_3 = 5$ are given in Appendix L. As it is mentioned at the beginning of this section, we also consider large-scale models for which the exact solution cannot be obtained. As an example, we give the results of the one with $K_i = 15$, $i = 1, 2, 3$ in Appendix K.

One of the immediate observations is that approximation errors are larger for cases with $\rho_3 < \rho_1$ or $\rho_3 < \rho_2$ ($\mu_3 > \mu_1$ or $\mu_3 > \mu_2$) as compared to other cases for both the approximate aggregate model and combined model. In fact, in such cases approximation seems to overestimate (underestimate) FR and FP (EB and SP) as seen in Tables 4.5(a) and (b) and in Appendix K. This is parallel to the findings of Buzacott and Shanthikumar (1993) who investigate serial KCS using conditional probabilities similar to the q values in this thesis. In order to see whether such large errors could be eliminated by employing the same approximation approach with some other state-independent q and q' values, fill rate errors are plotted over all possible q and q' values for the combined model of SKCS with two different ρ_i combinations in Figures 4.3 (a) and (b). This is to question how good the state-independent q and q' values proposed in Chapter 3 are when $\rho_3 < \rho_1$ or $\rho_3 < \rho_2$ as compared to the other ρ_i combinations with low errors. Figures 4.3 (a) and (b) show that there exist q and q' values with approximation errors around zero for both of the given parameter sets. In these figures, the arrows represent the (q, q') pairs proposed in Chapter 3. The proposed (q, q') pair falls into the region where errors are almost zero for the case with $\rho_1 = \rho_2 = \rho_3$ (which is a special case of $\rho_1 \le \rho_3$ and $\rho_2 \le \rho_3$) in Figure 4.3 (a), unlike the case with $\rho_3 < \rho_1$ or $\rho_3 < \rho_2$. Although the drawings in Figures 4.3 (a) and (b) are only for two different ρ combinations of only fill rate errors in SKCS, our observations for many other ρ combinations of also other performance measures in not only SKCS but also IKCS are in accordance with the one pointed out above.

An explanation for large approximation errors of cases $\rho_3 < \rho_1$ or $\rho_3 < \rho_2$ could be that dependence of state-independent q and q' values (expected q and q' values we approximate in Chapter 3) on μ_3 (or ρ_3) can not be incorporated into the derivations of the approximate q and q' values. Recall that, for these derivations, it is assumed that arrivals at upper stage(s) are Poisson with rate λ although this rate is in fact, state-dependent. This arrival rate is equal to μ_3 when there are backordered requests at the assembly stage, i.e., $d_2 > 0$. If large errors for $\rho_3 < \rho_1$ or $\rho_3 < \rho_2$ cases can really be attributed to ignoring this dependence on μ_3, apparently this does not affect the approximation performance much as long as $\rho_3 < \rho_1$ and $\rho_3 < \rho_2$. When $\rho_3 > \rho_1$ or $\rho_3 > \rho_2$, upper stages can not feed the assembly stage as fast as the component pairs are processed by the assembly facility and it might be more likely to be in stockout at the component stockpoints, the impact of this cannot be reflected to the approximate state-independent q and q' through derivations in Chapter 3. Note that in Figure 4.3(b) where the condition $\rho_3 < \rho_1$ or $\rho_3 < \rho_2$ is satisfied, q and q' values giving errors around zero are larger than our respective estimates. This observation is intuitively apparent based on the explanation above. As it is mentioned before Buzacott and Shanthikumar (1993) have similar numerical observations but for the approximation of serial KCS, and their explanation for the problematic cases with large errors is that the variability of the arrivals at the upper station in the real system cannot be captured with the use of exponential interarrival time distribution in calculating similar q values. We should note that both the approximation approach in Buzacott and Shanthikumar (1993) and the problematic cases are directly comparable to ours. In other words, the reason for the variability mentioned by Buzacott and Shanthikumar (1993) is nothing but the ignorance of the state dependence while estimating the conditional q probabilities. On the other hand, there is no such observation as for problematic combinations of kanban cards. It seems relative values of ρ_i are dominant in determining the approximation errors, at least for the K_i ranges tried in the experiments.

Another observation based on numerical results is the following: As the number of kanban cards and/or service rates decrease for a given customer arrival rate, approximation errors get larger. Otherwise, approximation accuracy gets better even for the cases with $\rho_3 < \rho_1$ or $\rho_3 < \rho_2$.

Noting all the observations above for both IKCS and SKCS, it is observed that the approximation gives better results for IKCS than for SKCS. This may be due to the fact that the independent release of kanbans at the upper stage is not contradictory, maybe only to a certain extent, to the independence assumptions used in estimating q and q'.

One may expect the approximate aggregate model to perform better than the combined model because the latter is obtained as a further approximation of the former. In fact, the numerical results are in accordance with such an expectation. Accuracy of the approximate aggregate model is almost the same as the combined model when $\rho_3 > \rho_1$ and $\rho_3 > \rho_2$, but better when $\rho_3 < \rho_1$ or $\rho_3 < \rho_2$. The improvement in accuracy is more apparent for IKCS. Note that all these improvements in accuracy are at the cost of extra computational efforts required to solve the approximate aggregate model instead of the combined model. Also, there is a limitation on the size of the systems (number of kanbans) to be solved by the approximate aggregate model as noted at the beginning of this section. Accordingly, one can choose which model to solve for given ρ_i and K_i combinations.

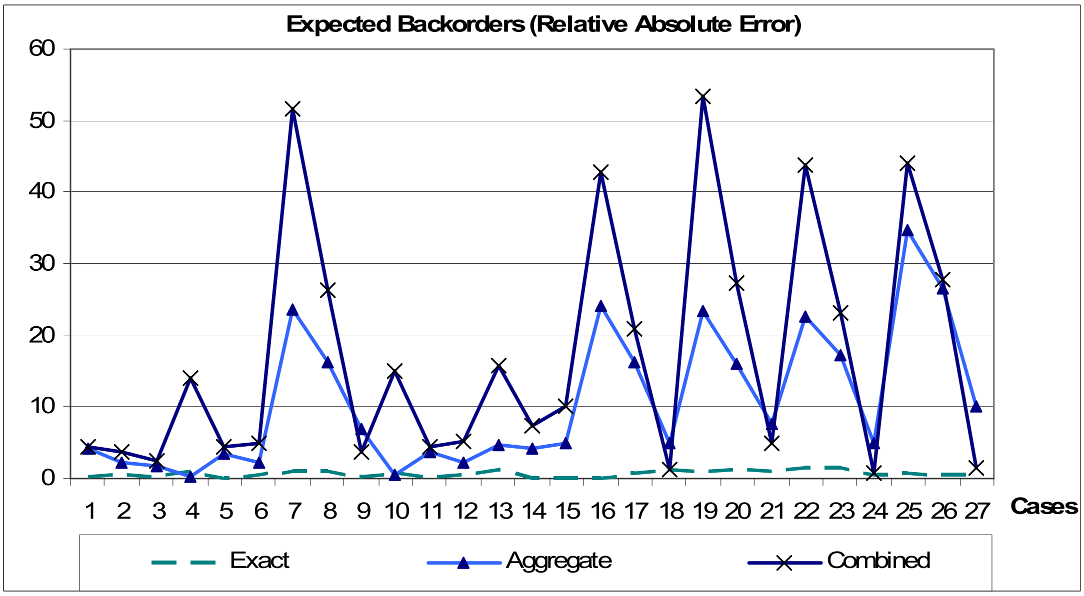


Figure 4.2 (a) Errors in Expected Backorders for SKCS with $K_i = 5$, $i = 1, 2, 3$.

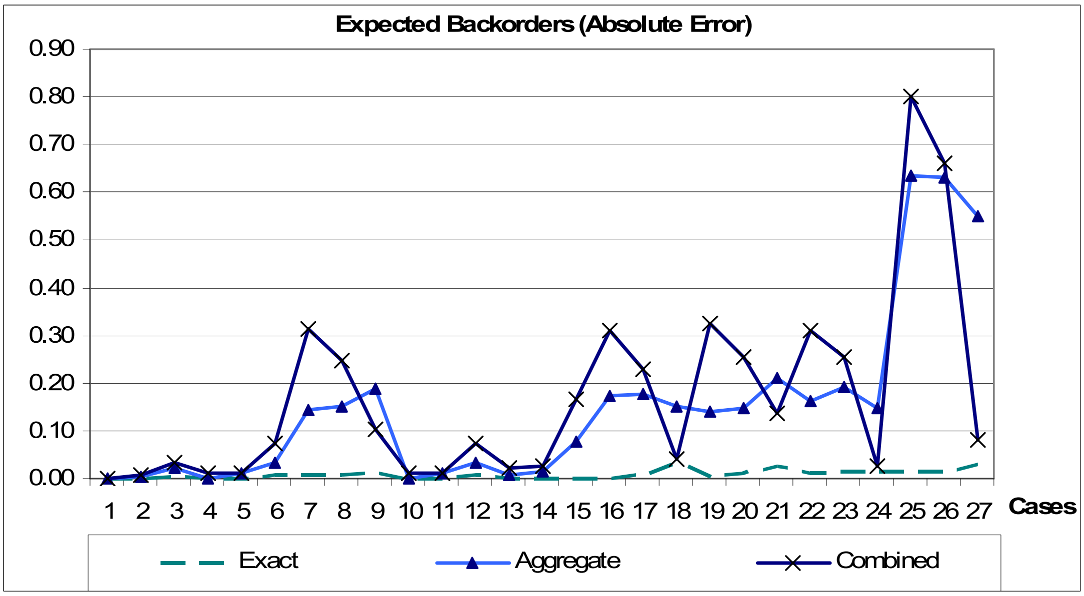


Figure 4.2 (b) Errors in Expected Backorders for SKCS with $K_i = 5$, $i = 1, 2, 3$.

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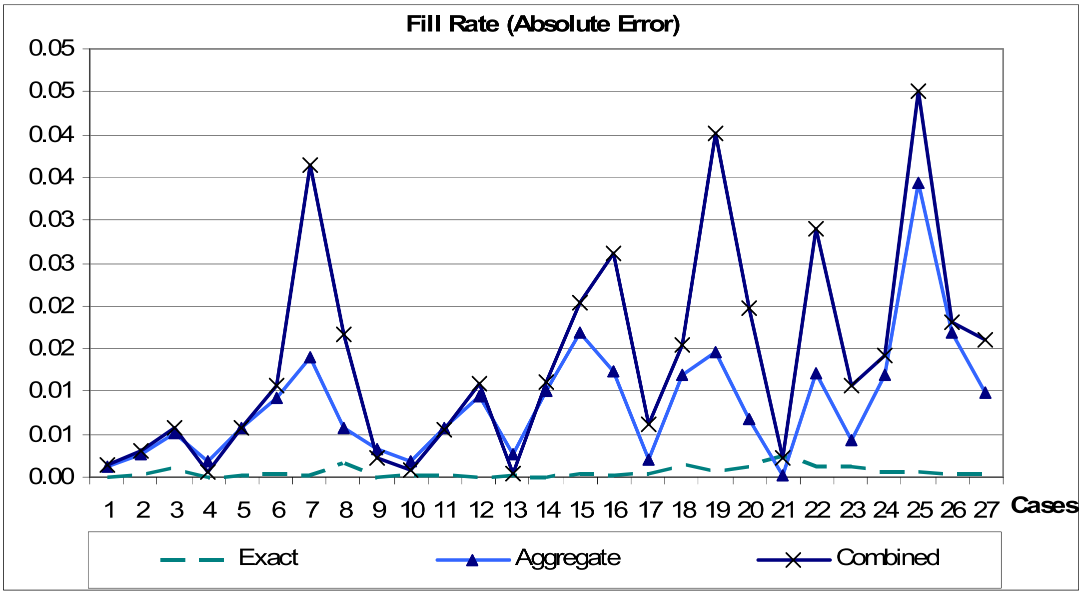

Figure 4.2 (c) Errors in Fill rates for SKCS with $K_i = 5$, $i = 1, 2, 3$.

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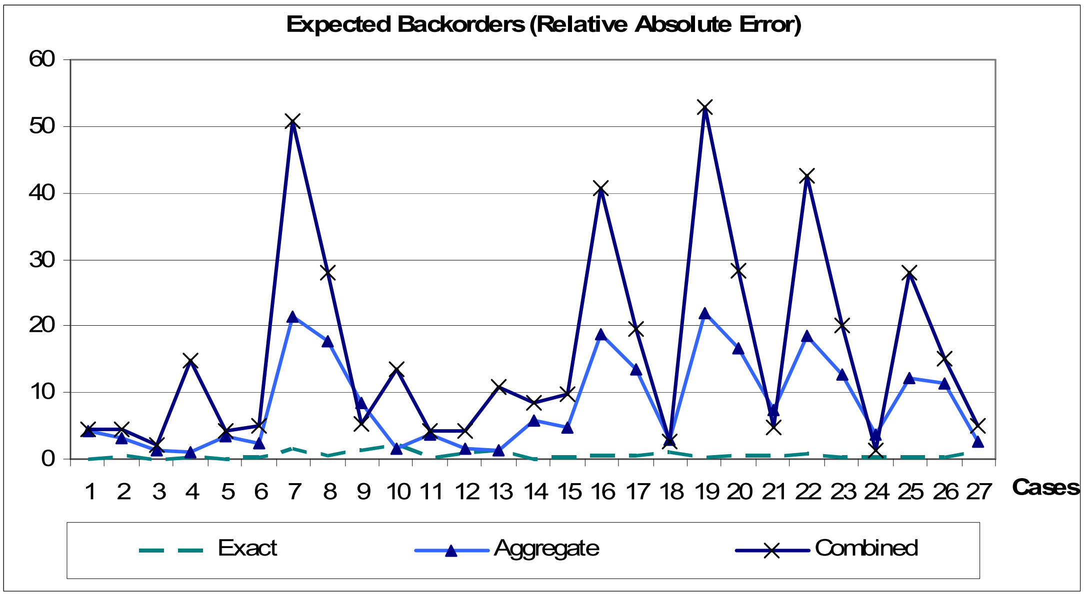

Figure 4.2 (d) Errors in Expected Backorders for IKCS with $K_i = 5$, $i = 1, 2, 3$.

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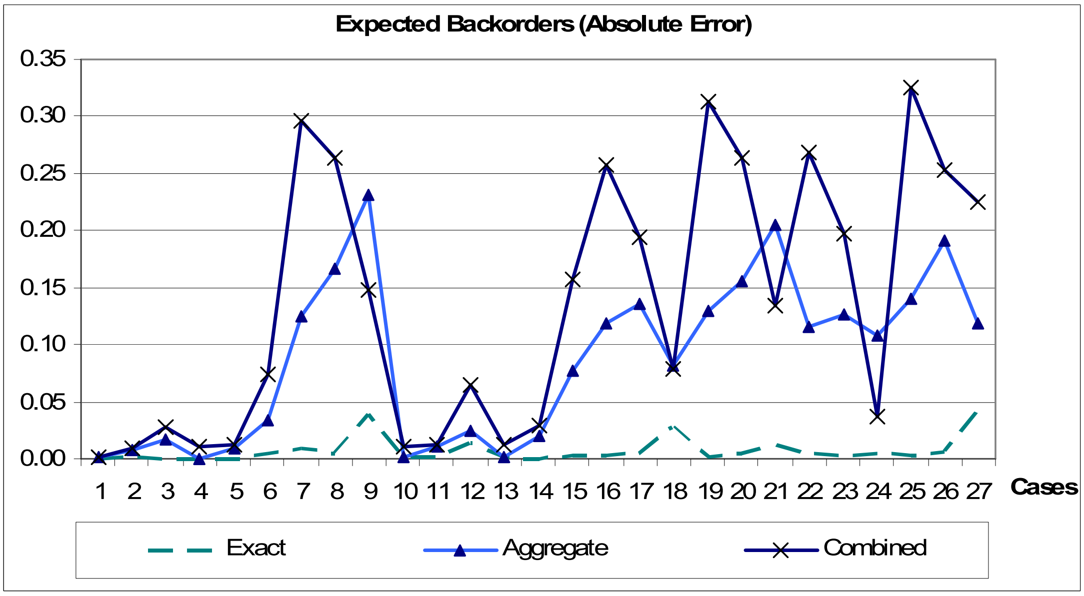

Figure 4.2 (e) Errors in Expected Backorders for IKCS with $K_i = 5$, $i = 1, 2, 3$.

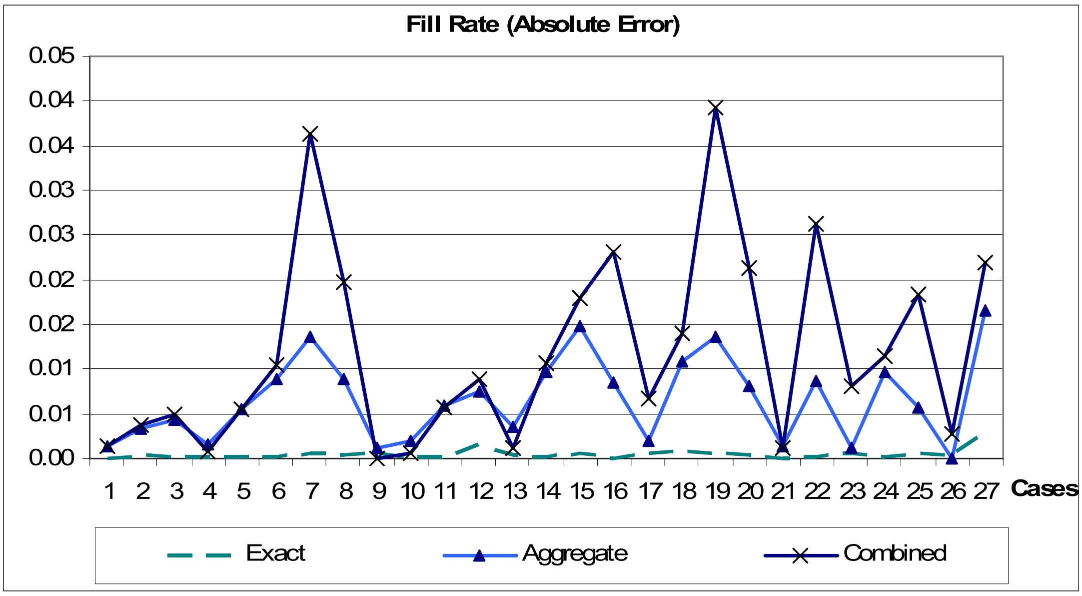

Figure 4.2 (f) Errors in Fill rates for IKCS with $K_i = 5$, $i = 1, 2, 3$.

Table 4.5 (a) Numerical results for SKCS with $K_1 = 5, K_2 = 5, K_3 = 5$.

SKCS			Expected Backorder							Expected Number of Assemblies in Stock						
K1	K2	K3	Sim.	Exact	Abs. Rel. Error (%)	Agg.	Abs. Rel. Error (%)	Com.	Abs. Rel. Error (%)	Sim.	Exact	Abs. Rel. Error (%)	Agg.	Abs. Rel. Error (%)	Com.	Abs. Rel. Error (%)
5	5	5														
ρ1	ρ2	ρ3														
0.5	0.5	0.5	0.037	0.037	0.25	0.038	4.16	0.038	4.51	3.996	3.996	0.02	3.975	0.51	3.975	0.51
0.5	0.5	0.65	0.226	0.225	0.41	0.231	2.31	0.234	3.60	3.332	3.335	0.08	3.307	0.75	3.306	0.78
0.5	0.5	0.8	1.330	1.334	0.29	1.352	1.62	1.363	2.44	2.298	2.294	0.17	2.269	1.26	2.268	1.33
0.5	0.65	0.5	0.079	0.080	1.10	0.079	0.26	0.068	13.98	3.872	3.871	0.01	3.837	0.91	3.844	0.71
0.5	0.65	0.65	0.287	0.287	0.06	0.297	3.39	0.300	4.41	3.235	3.236	0.01	3.182	1.64	3.183	1.63
0.5	0.65	0.8	1.483	1.475	0.53	1.515	2.18	1.557	5.01	2.218	2.218	0.00	2.163	2.50	2.159	2.67
0.5	0.8	0.5	0.609	0.603	1.03	0.466	23.57	0.295	51.65	3.296	3.298	0.05	3.319	0.68	3.393	2.94
0.5	0.8	0.65	0.936	0.946	0.98	0.785	16.17	0.689	26.39	2.753	2.745	0.27	2.731	0.80	2.764	0.40
0.5	0.8	0.8	2.754	2.745	0.34	2.564	6.89	2.652	3.72	1.816	1.817	0.04	1.773	2.38	1.777	2.15
0.65	0.5	0.5	0.079	0.080	0.84	0.079	0.43	0.067	14.91	3.873	3.871	0.04	3.838	0.91	3.846	0.69
0.65	0.5	0.65	0.286	0.287	0.31	0.297	3.63	0.299	4.41	3.235	3.236	0.03	3.183	1.60	3.184	1.56
0.65	0.5	0.8	1.481	1.475	0.40	1.515	2.29	1.556	5.07	2.218	2.218	0.00	2.163	2.48	2.160	2.62
0.65	0.65	0.5	0.133	0.131	1.23	0.126	4.64	0.112	15.68	3.751	3.751	0.01	3.695	1.49	3.703	1.29
0.65	0.65	0.65	0.358	0.358	0.00	0.373	4.30	0.384	7.36	3.138	3.139	0.02	3.054	2.68	3.051	2.78
0.65	0.65	0.8	1.626	1.627	0.06	1.704	4.81	1.791	10.13	2.146	2.144	0.07	2.053	4.31	2.044	4.72
0.65	0.8	0.5	0.721	0.721	0.03	0.548	24.00	0.413	42.79	3.182	3.179	0.07	3.172	0.31	3.216	1.08
0.65	0.8	0.65	1.093	1.086	0.63	0.915	16.31	0.864	20.99	2.648	2.650	0.07	2.595	1.98	2.608	1.51
0.65	0.8	0.8	3.052	3.018	1.11	2.903	4.90	3.091	1.28	1.738	1.743	0.31	1.655	4.76	1.647	5.25
0.8	0.5	0.5	0.608	0.603	0.89	0.467	23.30	0.284	53.36	3.296	3.298	0.06	3.324	0.85	3.408	3.41
0.8	0.5	0.65	0.933	0.946	1.30	0.784	15.97	0.678	27.39	2.750	2.745	0.18	2.736	0.53	2.776	0.94
0.8	0.5	0.8	2.771	2.745	0.95	2.560	7.61	2.635	4.92	1.807	1.817	0.52	1.777	1.70	1.785	1.23
0.8	0.65	0.5	0.710	0.721	1.50	0.549	22.68	0.399	43.80	3.184	3.179	0.15	3.182	0.06	3.237	1.65
0.8	0.65	0.65	1.102	1.086	1.40	0.912	17.25	0.847	23.16	2.645	2.650	0.19	2.606	1.48	2.625	0.75
0.8	0.65	0.8	3.034	3.018	0.51	2.887	4.82	3.058	0.79	1.746	1.743	0.12	1.664	4.67	1.659	4.95
0.8	0.8	0.5	1.827	1.841	0.79	1.193	34.70	1.025	43.90	2.599	2.595	0.13	2.660	2.35	2.691	3.56
0.8	0.8	0.65	2.384	2.397	0.57	1.753	26.48	1.722	27.75	2.152	2.150	0.08	2.141	0.47	2.145	0.32
0.8	0.8	0.8	5.509	5.539	0.55	4.959	9.98	5.426	1.50	1.338	1.336	0.14	1.258	5.95	1.243	7.12

SKCS			Fill Rate							Stockout Probability						
K1	K2	K3	Sim.	Exact	Abs. Error	Agg.	Abs. Error	Com.	Abs. Error	Sim.	Exact	Abs. Error	Agg.	Abs. Error	Com.	Abs. Error
5	5	5														
ρ1	ρ2	ρ3														
0.5	0.5	0.5	0.965	0.965	0.000	0.964	0.001	0.964	0.001	0.018	0.018	0.000	0.019	0.001	0.019	0.001
0.5	0.5	0.65	0.880	0.880	0.000	0.877	0.003	0.877	0.003	0.078	0.078	0.000	0.080	0.002	0.081	0.002
0.5	0.5	0.8	0.670	0.669	0.001	0.665	0.005	0.664	0.006	0.264	0.265	0.001	0.269	0.004	0.270	0.005
0.5	0.65	0.5	0.947	0.947	0.000	0.945	0.002	0.948	0.001	0.031	0.031	0.000	0.032	0.001	0.029	0.002
0.5	0.65	0.65	0.862	0.862	0.000	0.856	0.006	0.856	0.006	0.093	0.093	0.000	0.097	0.004	0.097	0.004
0.5	0.65	0.8	0.650	0.651	0.000	0.641	0.009	0.640	0.011	0.283	0.283	0.001	0.291	0.007	0.293	0.009
0.5	0.8	0.5	0.839	0.839	0.000	0.853	0.014	0.875	0.037	0.125	0.124	0.000	0.110	0.015	0.087	0.038
0.5	0.8	0.65	0.755	0.754	0.002	0.761	0.006	0.772	0.017	0.191	0.192	0.001	0.182	0.009	0.170	0.021
0.5	0.8	0.8	0.545	0.545	0.000	0.542	0.003	0.543	0.002	0.390	0.390	0.000	0.390	0.000	0.390	0.000
0.65	0.5	0.5	0.947	0.947	0.000	0.945	0.002	0.948	0.001	0.031	0.031	0.000	0.032	0.001	0.029	0.002
0.65	0.5	0.65	0.862	0.862	0.000	0.856	0.006	0.856	0.005	0.093	0.093	0.000	0.097	0.004	0.097	0.004
0.65	0.5	0.8	0.651	0.651	0.000	0.641	0.009	0.640	0.011	0.283	0.283	0.000	0.291	0.008	0.293	0.010
0.65	0.65	0.5	0.928	0.929	0.000	0.926	0.003	0.928	0.000	0.046	0.045	0.000	0.046	0.001	0.044	0.002
0.65	0.65	0.65	0.844	0.844	0.000	0.834	0.010	0.832	0.011	0.109	0.109	0.000	0.115	0.007	0.117	0.008
0.65	0.65	0.8	0.633	0.633	0.000	0.616	0.017	0.613	0.020	0.300	0.300	0.000	0.314	0.014	0.319	0.019
0.65	0.8	0.5	0.817	0.817	0.000	0.829	0.012	0.843	0.026	0.143	0.144	0.000	0.128	0.015	0.113	0.030
0.65	0.8	0.65	0.733	0.733	0.000	0.735	0.002	0.739	0.006	0.212	0.212	0.000	0.205	0.007	0.200	0.012
0.65	0.8	0.8	0.524	0.525	0.001	0.512	0.012	0.509	0.015	0.411	0.410	0.001	0.419	0.008	0.424	0.013
0.8	0.5	0.5	0.838	0.839	0.001	0.853	0.015	0.878	0.040	0.125	0.124	0.001	0.109	0.016	0.084	0.041
0.8	0.5	0.65	0.755	0.754	0.001	0.762	0.007	0.775	0.020	0.191	0.192	0.001	0.181	0.010	0.168	0.023
0.8	0.5	0.8	0.542	0.545	0.002	0.542	0.000	0.545	0.002	0.393	0.390	0.002	0.389	0.003	0.388	0.005
0.8	0.65	0.5	0.818	0.817	0.001	0.830	0.012	0.847	0.029	0.142	0.144	0.001	0.128	0.015	0.110	0.032
0.8	0.65	0.65	0.732	0.733	0.001	0.736	0.004	0.743	0.011	0.213	0.212	0.001	0.204	0.009	0.197	0.016
0.8	0.65	0.8	0.526	0.525	0.001	0.514	0.012	0.512	0.014	0.409	0.410	0.000	0.417	0.008	0.421	0.011
0.8	0.8	0.5	0.689	0.689	0.001	0.724	0.034	0.734	0.045	0.263	0.264	0.001	0.223	0.040	0.211	0.053
0.8	0.8	0.65	0.611	0.610	0.000	0.628	0.017	0.629	0.018	0.332	0.332	0.000	0.307	0.025	0.305	0.027
0.8	0.8	0.8	0.411	0.411	0.000	0.401	0.010	0.395	0.016	0.532	0.532	0.000	0.536	0.004	0.545	0.012

Table 4.5 (b) Numerical results for IKCS with $K_1 = 5,\ K_2 = 5,\ K_3 = 5$.

IKCS			Expected Backorder							Expected Number of Assemblies in Stock						
K1	K2	K3	Sim.	Exact	Abs. Rel. Error (%)	Agg.	Abs. Rel. Error (%)	Com.	Abs. Rel. Error (%)	Sim.	Exact	Abs. Rel. Error (%)	Agg.	Abs. Rel. Error (%)	Com.	Abs. Rel. Error (%)
5	5	5														
ρ1	ρ2	ρ3														
0.5	0.5	0.5	0.037	0.037	0.09	0.038	4.21	0.038	4.55	3.997	3.997	0.01	3.976	0.53	3.976	0.53
0.5	0.5	0.65	0.223	0.224	0.41	0.230	3.13	0.233	4.40	3.337	3.335	0.04	3.308	0.85	3.307	0.89
0.5	0.5	0.8	1.334	1.334	0.02	1.351	1.29	1.362	2.09	2.296	2.295	0.04	2.270	1.11	2.269	1.18
0.5	0.65	0.5	0.078	0.078	0.24	0.078	0.95	0.067	14.73	3.875	3.875	0.00	3.842	0.86	3.849	0.66
0.5	0.65	0.65	0.285	0.285	0.05	0.295	3.32	0.297	4.25	3.237	3.238	0.03	3.187	1.55	3.187	1.53
0.5	0.65	0.8	1.476	1.472	0.31	1.510	2.29	1.551	5.05	2.219	2.220	0.04	2.166	2.36	2.163	2.53
0.5	0.8	0.5	0.585	0.594	1.54	0.460	21.36	0.288	50.72	3.307	3.305	0.05	3.333	0.78	3.408	3.05
0.5	0.8	0.65	0.942	0.937	0.55	0.775	17.69	0.678	28.01	2.749	2.750	0.06	2.744	0.17	2.778	1.06
0.5	0.8	0.8	2.770	2.731	1.40	2.539	8.33	2.621	5.37	1.818	1.820	0.12	1.784	1.87	1.789	1.62
0.65	0.5	0.5	0.077	0.078	2.07	0.078	1.51	0.066	13.41	3.876	3.875	0.03	3.842	0.87	3.851	0.64
0.65	0.5	0.65	0.285	0.285	0.33	0.295	3.63	0.297	4.32	3.240	3.238	0.06	3.187	1.61	3.188	1.58
0.65	0.5	0.8	1.485	1.472	0.92	1.510	1.66	1.550	4.35	2.213	2.220	0.29	2.167	2.10	2.164	2.24
0.65	0.65	0.5	0.118	0.117	1.43	0.120	1.41	0.106	10.77	3.770	3.771	0.04	3.716	1.43	3.724	1.23
0.65	0.65	0.65	0.343	0.342	0.08	0.362	5.75	0.372	8.56	3.152	3.153	0.04	3.073	2.50	3.070	2.59
0.65	0.65	0.8	1.599	1.602	0.17	1.677	4.86	1.757	9.87	2.152	2.154	0.07	2.069	3.84	2.061	4.23
0.65	0.8	0.5	0.632	0.628	0.56	0.513	18.89	0.374	40.83	3.242	3.240	0.06	3.230	0.37	3.277	1.08
0.65	0.8	0.65	0.996	0.991	0.46	0.860	13.61	0.802	19.52	2.692	2.694	0.08	2.649	1.61	2.663	1.07
0.65	0.8	0.8	2.844	2.872	1.00	2.762	2.86	2.922	2.75	1.775	1.774	0.09	1.701	4.15	1.694	4.56
0.8	0.5	0.5	0.592	0.594	0.38	0.462	21.99	0.278	52.97	3.308	3.305	0.07	3.335	0.83	3.420	3.41
0.8	0.5	0.65	0.932	0.937	0.49	0.776	16.74	0.668	28.31	2.752	2.750	0.05	2.746	0.21	2.787	1.28
0.8	0.5	0.8	2.743	2.731	0.45	2.539	7.45	2.609	4.89	1.821	1.820	0.03	1.786	1.93	1.794	1.45
0.8	0.65	0.5	0.633	0.628	0.68	0.516	18.40	0.364	42.43	3.239	3.240	0.05	3.235	0.13	3.291	1.63
0.8	0.65	0.65	0.988	0.991	0.30	0.862	12.78	0.791	20.00	2.695	2.694	0.05	2.654	1.55	2.675	0.77
0.8	0.65	0.8	2.868	2.872	0.15	2.761	3.74	2.905	1.31	1.775	1.774	0.08	1.706	3.91	1.702	4.11
0.8	0.8	0.5	1.158	1.161	0.27	1.017	12.16	0.833	28.03	2.838	2.835	0.11	2.808	1.06	2.847	0.32
0.8	0.8	0.65	1.694	1.700	0.39	1.502	11.31	1.440	14.96	2.331	2.331	0.01	2.277	2.32	2.285	1.99
0.8	0.8	0.8	4.390	4.433	0.97	4.272	2.70	4.615	5.12	1.473	1.462	0.73	1.377	6.48	1.364	7.41

IKCS			Fill Rate							Stockout Probability						
K1	K2	K3	Sim.	Exact	Abs. Error	Agg.	Abs. Error	Com.	Abs. Error	Sim.	Exact	Abs. Error	Agg.	Abs. Error	Com.	Abs. Error
5	5	5														
ρ1	ρ2	ρ3														
0.5	0.5	0.5	0.965	0.965	0.000	0.964	0.001	0.964	0.001	0.018	0.018	0.000	0.019	0.001	0.019	0.001
0.5	0.5	0.65	0.881	0.880	0.000	0.877	0.003	0.877	0.004	0.078	0.078	0.000	0.080	0.002	0.081	0.003
0.5	0.5	0.8	0.669	0.669	0.000	0.665	0.004	0.664	0.005	0.265	0.265	0.000	0.269	0.004	0.269	0.004
0.5	0.65	0.5	0.948	0.948	0.000	0.946	0.002	0.948	0.001	0.031	0.031	0.000	0.031	0.001	0.029	0.002
0.5	0.65	0.65	0.862	0.863	0.000	0.857	0.006	0.857	0.005	0.093	0.093	0.000	0.097	0.004	0.097	0.004
0.5	0.65	0.8	0.651	0.651	0.000	0.642	0.009	0.640	0.010	0.283	0.282	0.000	0.290	0.007	0.292	0.009
0.5	0.8	0.5	0.841	0.840	0.001	0.854	0.014	0.877	0.036	0.122	0.123	0.001	0.108	0.014	0.085	0.037
0.5	0.8	0.65	0.755	0.755	0.000	0.764	0.009	0.775	0.020	0.192	0.191	0.000	0.180	0.012	0.168	0.023
0.5	0.8	0.8	0.545	0.546	0.001	0.544	0.001	0.545	0.000	0.390	0.389	0.001	0.388	0.002	0.387	0.003
0.65	0.5	0.5	0.948	0.948	0.000	0.946	0.002	0.949	0.001	0.030	0.031	0.000	0.031	0.001	0.029	0.002
0.65	0.5	0.65	0.863	0.863	0.000	0.857	0.006	0.857	0.006	0.093	0.093	0.000	0.097	0.004	0.096	0.004
0.65	0.5	0.8	0.649	0.651	0.002	0.642	0.008	0.641	0.009	0.284	0.282	0.002	0.290	0.006	0.292	0.008
0.65	0.65	0.5	0.932	0.932	0.000	0.928	0.004	0.931	0.001	0.042	0.042	0.000	0.044	0.002	0.042	0.001
0.65	0.65	0.65	0.846	0.847	0.000	0.837	0.010	0.836	0.011	0.106	0.106	0.000	0.113	0.007	0.114	0.008
0.65	0.65	0.8	0.634	0.635	0.001	0.620	0.015	0.616	0.018	0.298	0.298	0.001	0.311	0.013	0.315	0.017
0.65	0.8	0.5	0.830	0.830	0.000	0.839	0.008	0.853	0.023	0.131	0.131	0.000	0.121	0.011	0.105	0.026
0.65	0.8	0.65	0.743	0.744	0.001	0.745	0.002	0.750	0.007	0.201	0.201	0.000	0.196	0.006	0.190	0.011
0.65	0.8	0.8	0.535	0.534	0.001	0.524	0.011	0.521	0.014	0.400	0.401	0.001	0.407	0.007	0.412	0.012
0.8	0.5	0.5	0.841	0.840	0.001	0.854	0.014	0.880	0.039	0.122	0.123	0.001	0.108	0.014	0.082	0.040
0.8	0.5	0.65	0.756	0.755	0.000	0.764	0.008	0.777	0.021	0.191	0.191	0.000	0.180	0.011	0.166	0.025
0.8	0.5	0.8	0.546	0.546	0.000	0.544	0.001	0.547	0.001	0.389	0.389	0.000	0.387	0.002	0.386	0.004
0.8	0.65	0.5	0.830	0.830	0.000	0.839	0.009	0.856	0.026	0.131	0.131	0.000	0.121	0.011	0.102	0.029
0.8	0.65	0.65	0.745	0.744	0.001	0.746	0.001	0.753	0.008	0.200	0.201	0.001	0.195	0.005	0.188	0.012
0.8	0.65	0.8	0.534	0.534	0.000	0.525	0.010	0.523	0.011	0.401	0.401	0.000	0.407	0.006	0.410	0.009
0.8	0.8	0.5	0.747	0.747	0.001	0.753	0.006	0.765	0.018	0.206	0.207	0.000	0.197	0.009	0.183	0.024
0.8	0.8	0.65	0.659	0.658	0.000	0.659	0.000	0.662	0.003	0.282	0.283	0.001	0.277	0.005	0.274	0.008
0.8	0.8	0.8	0.451	0.448	0.003	0.434	0.017	0.429	0.022	0.488	0.492	0.003	0.501	0.013	0.508	0.020

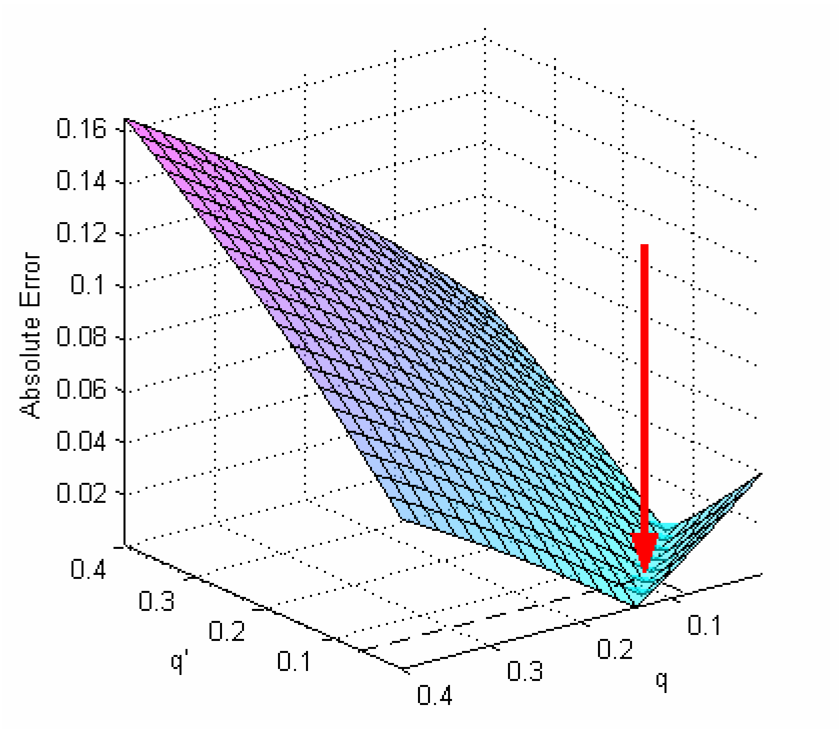

Figure 4.3 (a) Absolute errors for FR in the combined model of SKCS with $\rho_i = 0.5$, $K_i = 3$, $i = 1, 2, 3$.

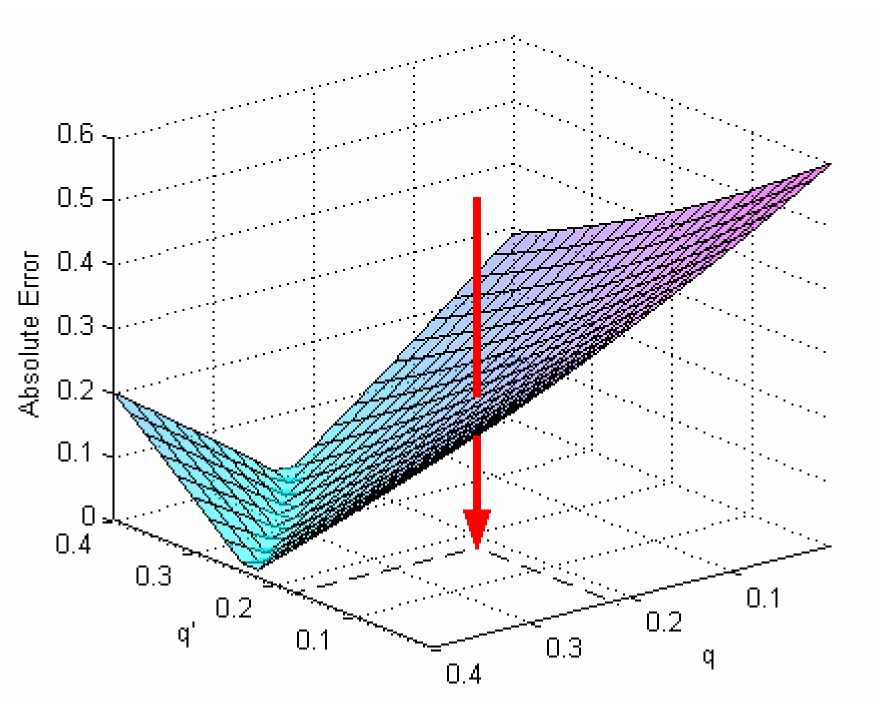

Figure 4.3 (b) Absolute errors for FR in the combined model of SKCS with $\rho_1 = \rho_2 = 0.8$, $\rho_3 = 0.5$, $K_i = 3$, $i = 1, 2, 3$.

As mentioned in Chapter 2, the kanban controlled assembly systems studied by Matta et al. (2005) are the same as the systems considered in this thesis. Among the three (two) approximations in Matta et al. (2005) for SKCS (IKCS), what Matta et al. call exact SS is based on an exact analysis of the assembly synchronization station. As for the other two approximations, 2qSS and 3qSS, Matta et al. (2005) introduce alternative (approximate) representations of the assembly synchronization station. In other words, an isolated part of the system, i.e., the assembly synchronization station, is analyzed exactly in exact SS, while it is approximated in 2qSS and 3qSS. Note that exact SS is an approximation rather than an exact solution method. Matta et al. (2005) test performance of the approximations they propose for the single case with $K_1 = K_2 = 3, K_3 = 5$ while taking $\mu_i = 1$ for all $i = 1, 2, 3$, and considering different λ values in order to cover different levels of traffic intensities. Approximations proposed in this thesis are compared with the ones in Matta et al. (2005) in Tables 4.6(a) and (b), and Figures 4.7(a), (b) and (c), (d) for SKCS and IKCS, respectively. Note that the values in Tables 4.6(a) and (b) for Exacts SS, 2qSS, 3qSS are from a spreadsheet provided by Andrea Matta as details of Matta et al. (2005). Unfilled values are represented by *. Both the approximate aggregate model and the combined model presented in Chapter 3 give better results than 3qss and 2qss for SKCS, these results are even as good as the exact SS. However, as noted in Matta et al. (2005), the disadvantage for exact SS is that it can be used only for very small problems, whereas the approximations proposed in this thesis (especially the combined model) can be used to solve large-scale problems. In the case of IKCS, our approximate results are better than 3qSS, and almost the same as the results of exact SS, but this time exact SS seems to perform better than our approximations for high traffic intensity. As a result, noting that exact SS can be used only for very small problems, it can be concluded that the approximation proposed in this thesis with or without decomposition is more advantageous than the multi-class aggregation techniques used by Matta et al. (2005). The comparison of the performance of our approximation with the one in Matta et al. (2005) is based on only the accuracy; CPU times can not be compared because these values are not reported in Matta et al. (2005).

Table 4.6 (a) Numerical results for SKCS with $K_1 = 3, K_2 = 3, K_3 = 5$.

Demand Rate	Expected Backorders						
	Sim.	Exact	Agg.	Com.	Exact SS	2qSS	3qSS
0.20	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.30	0.0012	0.0013	0.0013	0.0013	0.0013	0.0021	0.0016
0.40	0.0100	0.0100	0.0106	0.0107	0.0105	0.0190	0.0136
0.50	0.0592	0.0588	0.0622	0.0643	0.0626	0.1182	0.0817
0.60	0.2962	0.3036	0.3093	0.3327	0.3213	0.6185	0.4217
0.70	1.6489	1.6776	1.5392	1.7409	1.7127	3.6312	2.3493
0.75	4.7950	4.8043	3.9562	4.5770	4.7269	13.0908	7.1958
0.80	30.5084	29.7179	16.7337	19.6902	27.5040	*	190.3730
0.81	76.0924	76.1275	28.3374	33.4267	69.2286	*	*
Demand Rate	**Fill Rate**						
	Sim.	**Exact**	**Agg.**	**Com.**	**Exact SS**	**2qSS**	**3qSS**
0.20	0.9997	0.9997	0.9997	0.9997	0.9997	0.9996	0.9996
0.30	0.9972	0.9972	0.9971	0.9971	0.9971	0.9960	0.9967
0.40	0.9865	0.9865	0.9856	0.9856	0.9858	0.9792	0.9833
0.50	0.9517	0.9514	0.9476	0.9471	0.9485	0.9242	0.9398
0.60	0.8569	0.8558	0.8455	0.8427	0.8477	0.7836	0.8249
0.70	0.6266	0.6270	0.6164	0.6079	0.6147	0.4819	0.5647
0.75	0.4261	0.4240	0.4277	0.4171	0.4146	0.2378	0.3425
0.80	0.1330	0.1301	0.1734	0.1663	0.1286	*	0.0235
0.81	0.0588	0.0576	0.1135	0.1085	0.0573	*	*
Demand Rate	**Expected Number of Assemblies in Stock**						
	Sim.	**Exact**	**Agg.**	**Com.**	**Exact SS**	**2qSS**	**3qSS**
0.20	4.7476	4.7477	4.7461	4.7461	4.7463	4.7428	4.7448
0.30	4.5586	4.5580	4.5494	4.5494	4.5510	4.5320	4.5438
0.40	4.2855	4.2847	4.2569	4.2567	4.2629	4.2013	4.2418
0.50	3.8678	3.8667	3.8005	3.7991	3.8170	3.6695	3.7700
0.60	3.2042	3.2031	3.0868	3.0793	3.1163	2.8314	3.0246
0.70	2.1269	2.1323	2.0143	1.9932	2.0287	1.5627	1.8636
0.75	1.3751	1.3694	1.3167	1.2918	1.2870	0.7261	1.0654
0.80	0.4075	0.3984	0.5021	0.4863	0.3721	*	0.0682
0.81	0.1785	0.1745	0.3248	0.3136	0.1632	*	*

Table 4.6 (b) Comparison of the results for IKCS with $K_1 = 3, K_2 = 3, K_3 = 5$.

Demand Rate	Expected Backorders					
	Sim.	Exact	Agg.	Com.	Exact SS	3qSS
0.20	0.0001	0.0001	0.0001	0.0001	0.0001	0.9997
0.30	0.0012	0.0012	0.0013	0.0013	0.0013	0.9972
0.40	0.0097	0.0097	0.0104	0.0105	0.0103	0.9867
0.50	0.0557	0.0542	0.0598	0.0615	0.0590	0.9531
0.60	0.2507	0.2556	0.2824	0.3005	0.2841	0.8696
0.70	1.2294	1.1867	1.2809	1.4170	1.3322	0.6732
0.75	2.7861	2.8164	2.9739	3.3484	3.1472	0.5196
0.80	8.6936	8.7026	8.9077	10.1704	9.5841	0.2953
0.81	11.7434	11.8602	12.0360	13.7725	13.0030	0.2451
0.82	16.8628	17.2732	17.3420	19.8832	18.8415	0.1903
0.83	27.8087	28.4777	28.1377	32.3172	30.8829	0.1263
0.84	53.0444	64.4123	61.3176	70.5293	69.3981	0.0632
Demand Rate	Fill Rate					
	Sim.	Exact	Agg.	Com.	Exact SS	3qSS
0.20	0.9997	0.9997	0.9997	0.9997	0.9997	0.9996
0.30	0.9972	0.9972	0.9971	0.9971	0.9971	0.9967
0.40	0.9867	0.9867	0.9858	0.9858	0.9861	0.9841
0.50	0.9531	0.9535	0.9492	0.9488	0.9505	0.9448
0.60	0.8696	0.8679	0.8553	0.8531	0.8588	0.8468
0.70	0.6732	0.6773	0.6544	0.6479	0.6595	0.6394
0.75	0.5196	0.5175	0.4935	0.4849	0.4978	0.4727
0.80	0.2953	0.2961	0.2788	0.2710	0.2807	0.2496
0.81	0.2451	0.2428	0.2284	0.2214	0.2295	0.1970
0.82	0.1903	0.1862	0.1753	0.1694	0.1755	0.1414
0.83	0.1263	0.1260	0.1193	0.1149	0.1184	0.0826
0.84	0.0632	0.0621	0.0604	0.0580	0.0582	0.0205
Demand Rate	Expected Number of Assemblies in Stock					
	Sim.	Exact	Agg.	Com.	Exact SS	3qSS
0.20	4.7482	4.7477	4.7461	4.7461	4.7463	4.7449
0.30	4.5587	4.5583	4.5497	4.5497	4.5514	4.5451
0.40	4.2904	4.2873	4.2599	4.2598	4.2663	4.2501
0.50	3.8773	3.8812	3.8176	3.8164	3.8339	3.8050
0.60	3.2680	3.2622	3.1505	3.1444	3.1779	3.1343
0.70	2.3129	2.3237	2.1833	2.1666	2.2106	2.1502
0.75	1.7008	1.6912	1.5632	1.5422	1.5823	1.5099
0.80	0.9148	0.9204	0.8380	0.8196	0.8439	0.7555
0.81	0.7547	0.7472	0.6793	0.6629	0.6821	0.5896
0.82	0.5785	0.5672	0.5158	0.5022	0.5154	0.4183
0.83	0.3815	0.3800	0.3474	0.3375	0.3437	0.2416
0.84	0.1896	0.1854	0.1740	0.1686	0.1669	0.0594

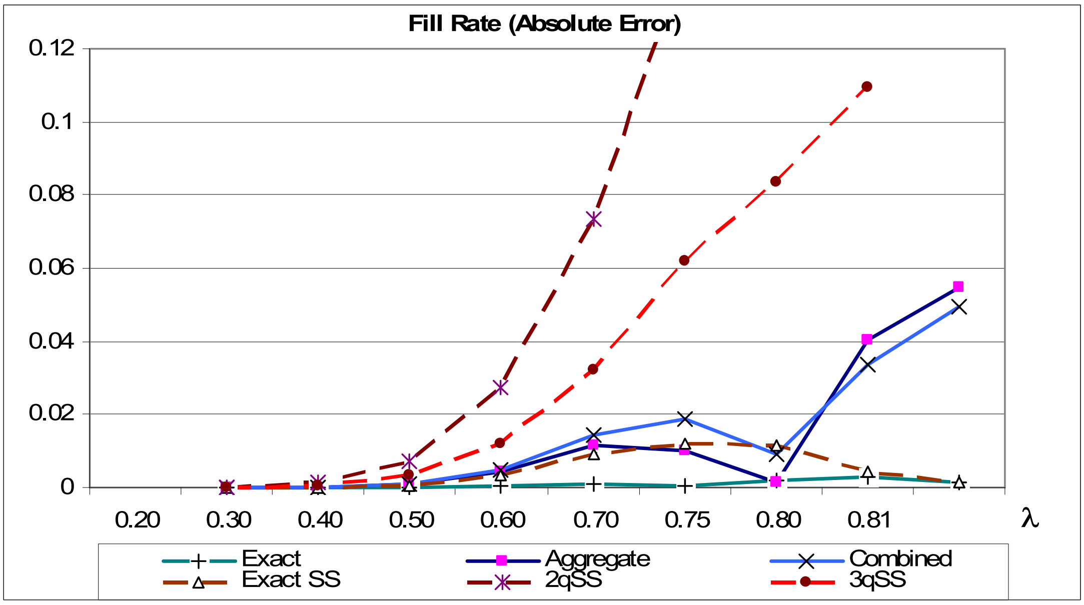

Figure 4.4 (a) Fill rate errors for SKCS with $K_1 = 3,\ K_2 = 3,\ K_3 = 5$.

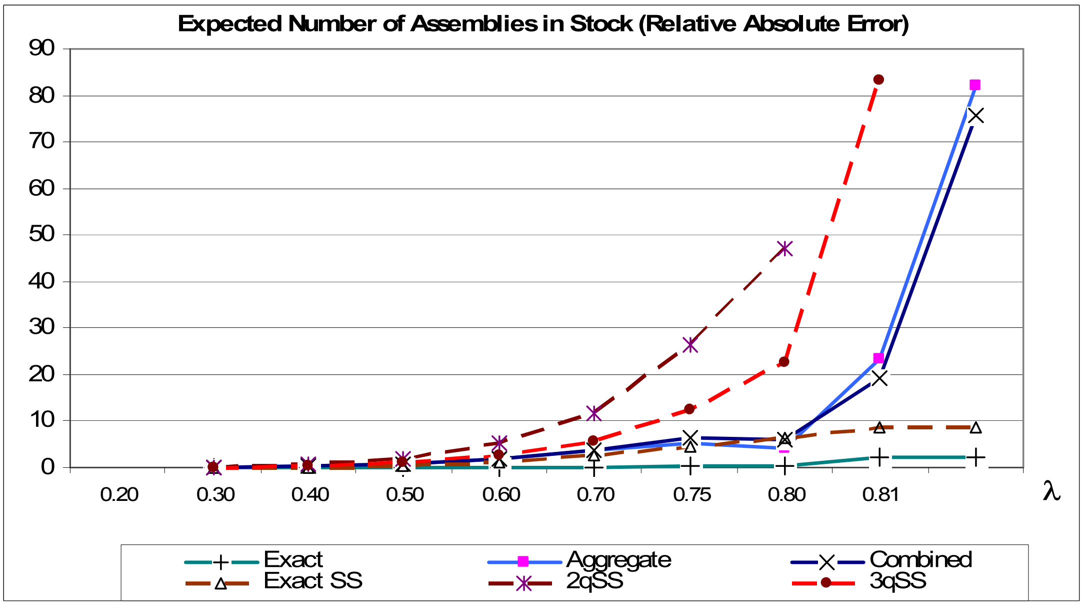

Figure 4.4 (b) Errors for expected number of assemblies in stock for SKCS with $K_1 = 3,\ K_2 = 3,\ K_3 = 5$.

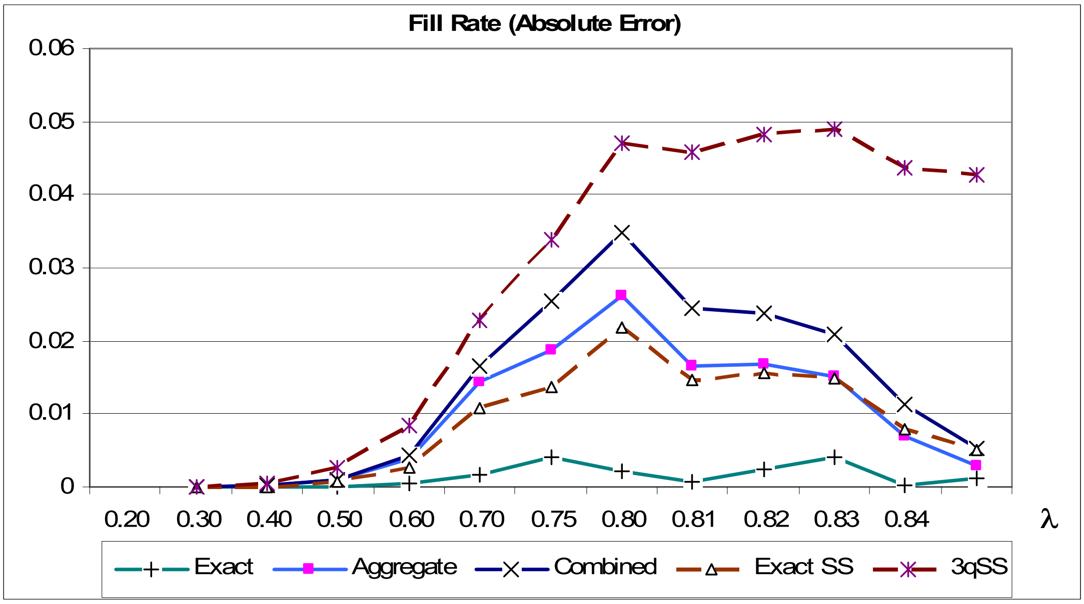

Figure 4.4 (c) Fill rate errors for IKCS with $K_1 = 3, K_2 = 3, K_3 = 5$.

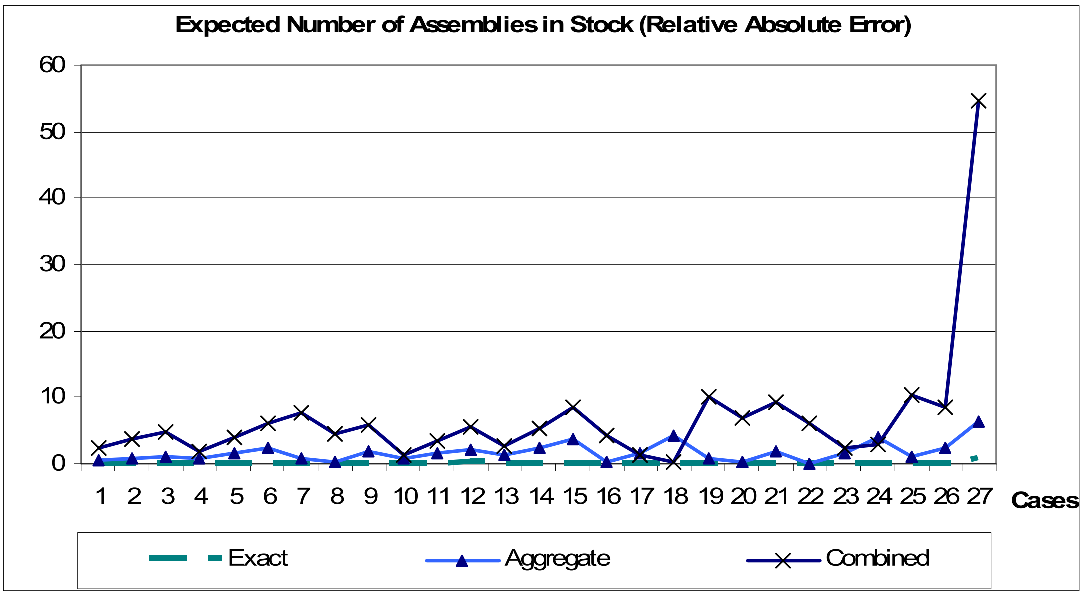

Figure 4.4 (d) Errors for expected number of assemblies in stock for IKCS with $K_1 = 3, K_2 = 3, K_3 = 5$.

4.2 Systems With More Than Two Components

Extension of the approximation in Chapter 3 to systems with more than two components is immediate when we consider the alternative model to pick up components sequentially, i.e., to take advantage of the (sequential) partial aggregations. A conditional probability is introduced for each component in addition to some other conditional probabilities that are expressed in terms of the conditional probabilities introduced for each component as in the case of q'' in the two-component systems. Then, as before, the approximate aggregate model with state-independent conditional probabilities is decomposed into independent submodels, each with a product-form steady-state distribution. The model to combine these submodels by aggregating states with common transition rates gets larger as the number of components increases. For example, state space of the combined model for the three-component (two-component) system consists of four (three) aggregate states when $d_2 = 0$ and three (two) aggregate states when $d_2 > 0$ introducing conditional probabilities P_1 to P_4 (P_1 to P_3) and R_1 to R_3 (R_1 and R_2) to adjust transition rates between aggregate states. Accordingly, there would be increase in efforts required for the matrix-geometric solution of the combined model. Details of the development for the approximation of three-component systems for SKCS and IKCS are given in Appendix M. In this section, we give numerical results for three- and four-component systems.

In order to test performance of the approximation for systems with three (four) components, number of kanbans is kept the same for all stages at either 5 or 8 or 10 and for each of these cases, 16 (32) different ρ combinations of 0.5 and 0.8 are considered. Numerical results for three- (four-) component systems with $K_i = 8$ $i = 1, 2, 3, 4$ ($i = 1, 2, 3, 4, 5$) are in Tables 4.7(a) and (b) (Tables 4.8(a) and(b)). Numerical results for some other parameter sets are in Appendix N. Approximation (app) column in all the tables referred to above is for the solution of the combined model, matrix-geometric solution of the approximate aggregate model is not considered in this section.

As in the case of two-component systems, it is observed that the approximation gives better results for IKCS than for SKCS. Also, as the number of kanban cards and/or service rates decrease for a given customer arrival rate, approximation errors get larger. Unlike the two-component case, we can not differentiate certain parameter sets from the others as the ones with larger errors. But, according to Figures 4.5(a) and (b), similar to section 4.1, approximation seems to overestimate (underestimate) FR and FP (EB and SP) for cases with $\rho_4 < \rho_1$ or $\rho_4 < \rho_2$ or $\rho_4 < \rho_3$. But, this time, for the cases with $\rho_4 > \rho_1$ and $\rho_4 > \rho_2$ and $\rho_4 > \rho_3$, FR and FP (EB and SP) are underestimated (overestimated) (this is observed also for some of the two-component cases, but not almost always as in the three-component systems). The reason for that can again be thought as the ignorance of the dependence of arrival stream on downstream station in the derivations of conditional q probabilities. Trying different state-independent q values, we observe that the relative values of the performance measures for the parameter sets with $\rho_4 < \rho_1$ or $\rho_4 < \rho_2$ or $\rho_4 < \rho_3$ and with $\rho_4 > \rho_1$ and $\rho_4 > \rho_2$ and $\rho_4 > \rho_3$ stay almost the same but overestimations may turn to underestimations or underestimations may turn to overestimations. Any comment that is made in this section is also valid for four-component systems.

Table 4.7 (a) Numerical results for SKCS with $K_i = 8$, $i = 1, 2, 3, 4$.

K1=8, K2=8, K3=8, K4=8 (SKCS)															
Traffic Intensity				EB			FR			SP			FG		
ρ1	ρ2	ρ3	ρ4	Sim.	App.	Abs. Rel. Error (%)	Sim.	App.	Abs. Error	Sim.	App.	Abs. Error	Sim.	App.	Abs. Rel. Error (%)
0.5	0.5	0.5	0.5	0.004	0.004	2.17	0.996	0.996	0.000	0.002	0.002	0.000	6.998	6.992	0.07
0.5	0.5	0.5	0.8	0.667	0.674	1.12	0.833	0.832	0.001	0.134	0.135	0.001	4.672	4.662	0.21
0.5	0.5	0.8	0.5	0.153	0.091	40.44	0.960	0.967	0.007	0.031	0.024	0.007	6.504	6.521	0.27
0.5	0.5	0.8	0.8	0.976	1.016	4.11	0.791	0.783	0.008	0.172	0.179	0.006	4.359	4.268	2.08
0.5	0.8	0.5	0.5	0.152	0.091	39.65	0.960	0.967	0.007	0.031	0.024	0.007	6.503	6.523	0.31
0.5	0.8	0.5	0.8	0.987	1.016	2.96	0.789	0.784	0.006	0.174	0.178	0.005	4.349	4.270	1.81
0.5	0.8	0.8	0.5	0.391	0.266	31.94	0.917	0.926	0.009	0.068	0.058	0.010	6.032	5.999	0.54
0.5	0.8	0.8	0.8	1.382	1.492	7.97	0.748	0.729	0.019	0.213	0.230	0.016	4.058	3.855	5.01
0.8	0.5	0.5	0.5	0.150	0.091	38.84	0.959	0.967	0.007	0.032	0.024	0.007	6.494	6.523	0.44
0.8	0.5	0.5	0.8	0.981	1.016	3.58	0.791	0.784	0.008	0.172	0.178	0.006	4.361	4.270	2.09
0.8	0.5	0.8	0.5	0.394	0.266	32.43	0.916	0.926	0.010	0.069	0.058	0.011	6.026	5.999	0.45
0.8	0.5	0.8	0.8	1.388	1.492	7.55	0.748	0.729	0.019	0.213	0.230	0.016	4.053	3.855	4.90
0.8	0.8	0.5	0.5	0.382	0.252	34.16	0.918	0.929	0.012	0.068	0.055	0.012	6.033	6.039	0.10
0.8	0.8	0.5	0.8	1.396	1.453	4.10	0.747	0.733	0.014	0.214	0.226	0.011	4.049	3.886	4.02
0.8	0.8	0.8	0.5	0.692	0.510	26.22	0.874	0.881	0.007	0.106	0.097	0.010	5.596	5.491	1.88
0.8	0.8	0.8	0.8	1.884	2.109	11.97	0.702	0.672	0.031	0.257	0.284	0.027	3.754	3.452	8.04

Table 4.7 (b) Numerical results for IKCS with $K_i = 8$, $i = 1, 2, 3, 4$.

K1=8, K2=8, K3=8, K4=8 (IKCS)															
Traffic Intensity				EB			FR			SP			FP		
ρ1	ρ2	ρ3	ρ4	Sim.	App.	Abs. Rel. Error (%)	Sim.	App.	Abs. Error	Sim	App.	Abs. Error	Sim	App.	Abs. Rel. Error (%)
0.5	0.5	0.5	0.5	0.004	0.004	4.47	0.996	0.996	0.000	0.002	0.002	0.000	3.981	3.949	0.81
0.5	0.5	0.5	0.8	0.674	0.674	0.08	0.832	0.832	0.000	0.135	0.135	0.000	2.286	2.248	1.65
0.5	0.5	0.8	0.5	0.151	0.091	39.44	0.960	0.967	0.007	0.031	0.024	0.007	3.302	3.355	1.58
0.5	0.5	0.8	0.8	0.999	1.015	1.58	0.790	0.784	0.007	0.173	0.178	0.005	1.817	1.758	3.20
0.5	0.8	0.5	0.5	0.156	0.091	41.57	0.960	0.967	0.007	0.032	0.024	0.007	3.305	3.354	1.49
0.5	0.8	0.5	0.8	0.981	1.015	3.44	0.791	0.784	0.007	0.172	0.178	0.006	1.817	1.758	3.24
0.5	0.8	0.8	0.5	0.293	0.247	15.61	0.929	0.930	0.001	0.057	0.054	0.002	2.831	2.774	2.00
0.5	0.8	0.8	0.8	1.301	1.440	10.69	0.755	0.734	0.021	0.206	0.224	0.018	1.462	1.307	10.65
0.8	0.5	0.5	0.5	0.156	0.091	41.61	0.959	0.967	0.007	0.032	0.024	0.007	3.300	3.363	1.89
0.8	0.5	0.5	0.8	0.998	1.015	1.63	0.790	0.784	0.006	0.173	0.178	0.005	1.817	1.764	2.88
0.8	0.5	0.8	0.5	0.289	0.235	18.79	0.930	0.933	0.003	0.056	0.052	0.004	2.829	2.833	0.14
0.8	0.5	0.8	0.8	1.285	1.405	9.33	0.755	0.738	0.017	0.206	0.221	0.015	1.469	1.352	7.94
0.8	0.8	0.5	0.5	0.288	0.235	18.69	0.930	0.933	0.003	0.056	0.052	0.004	2.826	2.833	0.22
0.8	0.8	0.5	0.8	1.276	1.405	10.11	0.756	0.738	0.018	0.205	0.221	0.016	1.456	1.352	7.15
0.8	0.8	0.8	0.5	0.420	0.423	0.64	0.905	0.896	0.008	0.078	0.083	0.005	2.477	2.338	5.58
0.8	0.8	0.8	0.8	1.573	1.880	19.50	0.725	0.692	0.033	0.235	0.265	0.030	1.182	0.970	17.97

Table 4.8 (a) Numerical results for SKCS with $K_i = 8$, i = 1, 2, 3, 4, 5.

K1=8, K2=8, K3=8, K4=8, K5=8 (SKCS)																
Traffic Intensity					EB			FR			SP			FG		
ρ1	ρ2	ρ3	ρ4	ρ5	Sim.	App.	Abs. Rel. Error (%)	Sim.	App.	Abs. Error	Sim.	App.	Abs. Error	Sim.	App.	Abs. Rel. Error (%)
0.5	0.5	0.5	0.5	0.5	0.004	0.004	1.60	0.996	0.996	0.000	0.002	0.002	0.000	6.996	6.989	0.10
0.5	0.5	0.5	0.5	0.8	0.663	0.675	1.81	0.832	0.831	0.001	0.134	0.135	0.001	4.671	4.658	0.27
0.5	0.5	0.5	0.8	0.5	0.154	0.091	40.71	0.959	0.966	0.007	0.032	0.024	0.007	6.496	6.517	0.32
0.5	0.5	0.5	0.8	0.8	1.000	1.018	1.80	0.790	0.783	0.007	0.173	0.179	0.005	4.354	4.264	2.05
0.5	0.5	0.8	0.5	0.5	0.151	0.092	39.21	0.960	0.966	0.007	0.031	0.024	0.007	6.502	6.519	0.26
0.5	0.5	0.8	0.5	0.8	0.983	1.017	3.45	0.790	0.783	0.007	0.173	0.179	0.006	4.354	4.267	2.01
0.5	0.5	0.8	0.8	0.5	0.388	0.266	31.39	0.917	0.926	0.009	0.068	0.058	0.010	6.031	5.995	0.60
0.5	0.5	0.8	0.8	0.8	1.402	1.495	6.61	0.747	0.728	0.018	0.214	0.230	0.015	4.050	3.851	4.92
0.5	0.8	0.5	0.5	0.5	0.161	0.092	43.07	0.959	0.966	0.008	0.032	0.024	0.008	6.498	6.519	0.33
0.5	0.8	0.5	0.5	0.8	0.980	1.017	3.82	0.791	0.783	0.008	0.172	0.179	0.007	4.364	4.267	2.23
0.5	0.8	0.5	0.8	0.5	0.383	0.266	30.54	0.918	0.926	0.008	0.067	0.058	0.009	6.034	5.995	0.65
0.5	0.8	0.5	0.8	0.8	1.414	1.495	5.71	0.746	0.728	0.017	0.215	0.230	0.015	4.040	3.851	4.67
0.5	0.8	0.8	0.5	0.5	0.377	0.252	33.24	0.918	0.929	0.011	0.067	0.055	0.012	6.037	6.035	0.03
0.5	0.8	0.8	0.5	0.8	1.394	1.455	4.35	0.746	0.733	0.013	0.215	0.226	0.011	4.044	3.883	3.97
0.5	0.8	0.8	0.8	0.5	0.712	0.510	28.31	0.872	0.881	0.009	0.108	0.097	0.012	5.583	5.487	1.72
0.5	0.8	0.8	0.8	0.8	1.891	2.113	11.72	0.703	0.671	0.031	0.257	0.285	0.028	3.754	3.448	8.15
0.8	0.5	0.5	0.5	0.5	0.155	0.092	41.08	0.959	0.966	0.007	0.032	0.024	0.007	6.501	6.519	0.28
0.8	0.5	0.5	0.5	0.8	0.982	1.017	3.62	0.792	0.783	0.008	0.172	0.179	0.007	4.365	4.267	2.26
0.8	0.5	0.5	0.8	0.5	0.382	0.266	30.26	0.917	0.926	0.009	0.068	0.058	0.010	6.028	5.995	0.55
0.8	0.5	0.5	0.8	0.8	1.428	1.495	4.71	0.745	0.728	0.017	0.216	0.230	0.014	4.042	3.851	4.74
0.8	0.5	0.8	0.5	0.5	0.382	0.252	34.11	0.918	0.929	0.011	0.067	0.055	0.012	6.032	6.035	0.05
0.8	0.5	0.8	0.5	0.8	1.392	1.455	4.52	0.748	0.733	0.016	0.213	0.226	0.013	4.060	3.883	4.35
0.8	0.5	0.8	0.8	0.5	0.702	0.511	27.31	0.872	0.881	0.008	0.108	0.097	0.011	5.593	5.486	1.90
0.8	0.5	0.8	0.8	0.8	1.912	2.113	10.54	0.701	0.671	0.029	0.259	0.285	0.026	3.742	3.448	7.84
0.8	0.8	0.5	0.5	0.5	0.386	0.252	34.70	0.918	0.929	0.011	0.068	0.055	0.012	6.032	6.035	0.06
0.8	0.8	0.5	0.5	0.8	1.412	1.455	3.04	0.746	0.733	0.013	0.215	0.226	0.010	4.045	3.883	3.99
0.8	0.8	0.5	0.8	0.5	0.721	0.511	29.15	0.871	0.881	0.009	0.109	0.097	0.012	5.580	5.486	1.67
0.8	0.8	0.5	0.8	0.8	1.922	2.113	9.95	0.703	0.671	0.032	0.257	0.285	0.028	3.757	3.448	8.22
0.8	0.8	0.8	0.5	0.5	0.686	0.472	31.18	0.874	0.887	0.014	0.106	0.091	0.015	5.597	5.563	0.61
0.8	0.8	0.8	0.5	0.8	1.922	2.011	4.66	0.701	0.680	0.021	0.258	0.276	0.018	3.747	3.510	6.33
0.8	0.8	0.8	0.8	0.5	1.118	0.831	25.65	0.825	0.832	0.007	0.151	0.140	0.011	5.169	4.996	3.35
0.8	0.8	0.8	0.8	0.8	2.481	2.916	17.54	0.659	0.611	0.048	0.300	0.343	0.043	3.478	3.057	12.11

Table 4.8 (b) Numerical results for IKCS with $K_i = 8$, i = 1, 2, 3, 4, 5.

K1=8, K2=8, K3=8, K4=8, K5=8 (IKCS)																	
Traffic Intensity					EB			FR			SP			FG			
					Sim.	App.	Abs. Rel. Error (%)	Sim.	App.	Abs. Error	Sim.	App.	Abs. Error	Sim.	App.	Abs. Rel. Error (%)
ρ1	ρ2	ρ3	ρ4	ρ5												
0.5	0.5	0.5	0.5	0.5	0.004	0.004	0.12	0.996	0.996	0.000	0.002	0.002	0.000	6.996	6.989	0.10
0.5	0.5	0.5	0.5	0.8	0.661	0.675	2.10	0.833	0.831	0.002	0.133	0.135	0.002	4.679	4.659	0.44
0.5	0.5	0.5	0.8	0.5	0.145	0.091	37.27	0.961	0.967	0.006	0.030	0.024	0.006	6.516	6.521	0.07
0.5	0.5	0.5	0.8	0.8	0.979	1.016	3.81	0.791	0.783	0.008	0.172	0.179	0.007	4.363	4.268	2.18
0.5	0.5	0.8	0.5	0.5	0.159	0.091	42.74	0.959	0.967	0.008	0.032	0.024	0.008	6.499	6.521	0.34
0.5	0.5	0.8	0.5	0.8	1.001	1.016	1.53	0.789	0.783	0.006	0.174	0.179	0.005	4.348	4.268	1.84
0.5	0.5	0.8	0.8	0.5	0.292	0.247	15.31	0.930	0.930	0.000	0.056	0.054	0.002	6.137	6.049	1.43
0.5	0.5	0.8	0.8	0.8	1.261	1.442	14.32	0.757	0.734	0.023	0.204	0.224	0.021	4.114	3.894	5.35
0.5	0.8	0.5	0.5	0.5	0.148	0.091	38.26	0.960	0.967	0.006	0.031	0.024	0.007	6.507	6.521	0.22
0.5	0.8	0.5	0.5	0.8	0.981	1.016	3.54	0.791	0.783	0.007	0.173	0.179	0.006	4.356	4.268	2.03
0.5	0.8	0.5	0.8	0.5	0.282	0.247	12.49	0.931	0.930	0.001	0.055	0.054	0.001	6.152	6.049	1.68
0.5	0.8	0.5	0.8	0.8	1.294	1.442	11.42	0.755	0.734	0.021	0.206	0.224	0.019	4.105	3.894	5.15
0.5	0.8	0.8	0.5	0.5	0.283	0.247	12.51	0.931	0.930	0.001	0.055	0.054	0.001	6.150	6.049	1.65
0.5	0.8	0.8	0.5	0.8	1.277	1.442	12.91	0.756	0.734	0.022	0.205	0.224	0.019	4.105	3.894	5.15
0.5	0.8	0.8	0.8	0.5	0.412	0.455	10.48	0.905	0.890	0.015	0.077	0.088	0.011	5.856	5.597	4.43
0.5	0.8	0.8	0.8	0.8	1.548	1.966	27.00	0.728	0.684	0.044	0.232	0.272	0.040	3.911	3.537	9.57
0.8	0.5	0.5	0.5	0.5	0.155	0.091	41.08	0.959	0.967	0.007	0.032	0.024	0.007	6.501	6.522	0.32
0.8	0.5	0.5	0.5	0.8	0.986	1.016	2.97	0.790	0.784	0.006	0.174	0.178	0.005	4.353	4.269	1.91
0.8	0.5	0.5	0.8	0.5	0.284	0.234	17.55	0.930	0.933	0.003	0.056	0.052	0.004	6.139	6.085	0.88
0.8	0.5	0.5	0.8	0.8	1.289	1.407	9.09	0.757	0.738	0.019	0.204	0.221	0.016	4.111	3.923	4.59
0.8	0.5	0.8	0.5	0.5	0.281	0.235	16.56	0.930	0.933	0.002	0.056	0.052	0.003	6.143	6.085	0.94
0.8	0.5	0.8	0.5	0.8	1.304	1.407	7.85	0.755	0.738	0.017	0.206	0.221	0.015	4.098	3.923	4.27
0.8	0.5	0.8	0.8	0.5	0.408	0.423	3.77	0.906	0.896	0.009	0.077	0.083	0.007	5.858	5.664	3.31
0.8	0.5	0.8	0.8	0.8	1.561	1.882	20.53	0.727	0.691	0.035	0.233	0.265	0.032	3.897	3.591	7.86
0.8	0.8	0.5	0.5	0.5	0.287	0.235	18.38	0.930	0.933	0.002	0.056	0.052	0.004	6.144	6.085	0.96
0.8	0.8	0.5	0.5	0.8	1.284	1.407	9.57	0.756	0.738	0.018	0.205	0.221	0.016	4.109	3.923	4.52
0.8	0.8	0.5	0.8	0.5	0.421	0.423	0.37	0.904	0.896	0.008	0.078	0.083	0.005	5.845	5.664	3.10
0.8	0.8	0.5	0.8	0.8	1.571	1.882	19.79	0.726	0.691	0.034	0.234	0.265	0.031	3.895	3.591	7.81
0.8	0.8	0.8	0.5	0.5	0.420	0.423	0.80	0.904	0.896	0.008	0.078	0.083	0.005	5.844	5.664	3.07
0.8	0.8	0.8	0.5	0.8	1.557	1.882	20.87	0.725	0.691	0.033	0.234	0.265	0.031	3.887	3.591	7.61
0.8	0.8	0.8	0.8	0.5	0.539	0.656	21.60	0.883	0.858	0.024	0.097	0.117	0.020	5.611	5.263	6.21
0.8	0.8	0.8	0.8	0.8	1.836	2.456	33.80	0.699	0.644	0.055	0.259	0.311	0.052	3.715	3.273	11.91

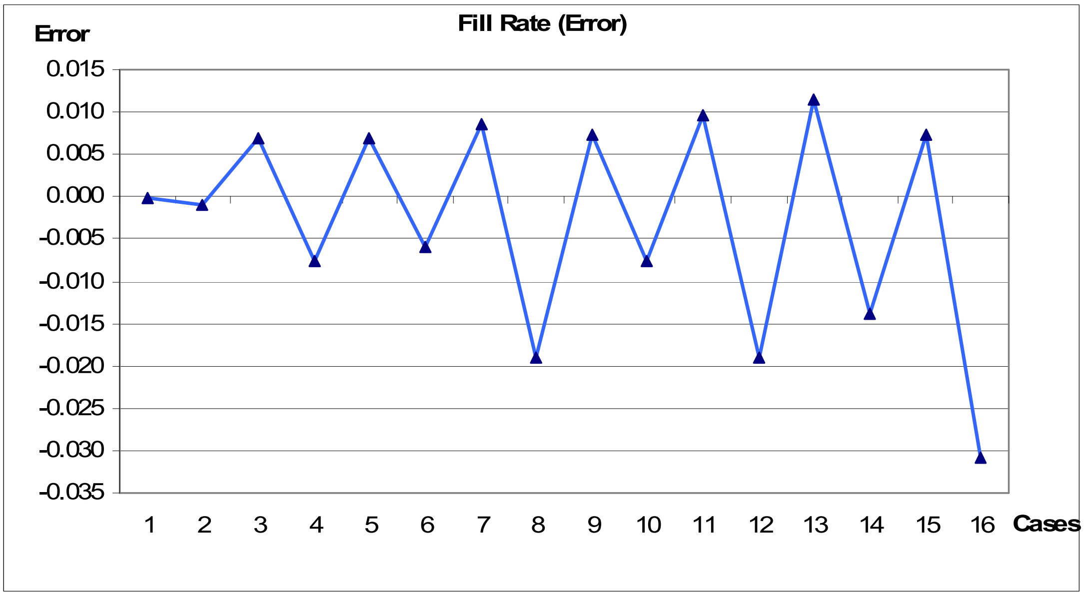

Figure 4.5 (a) Fill rate errors for SKCS with $K_i = 8$, $i = 1, 2, 3, 4$.

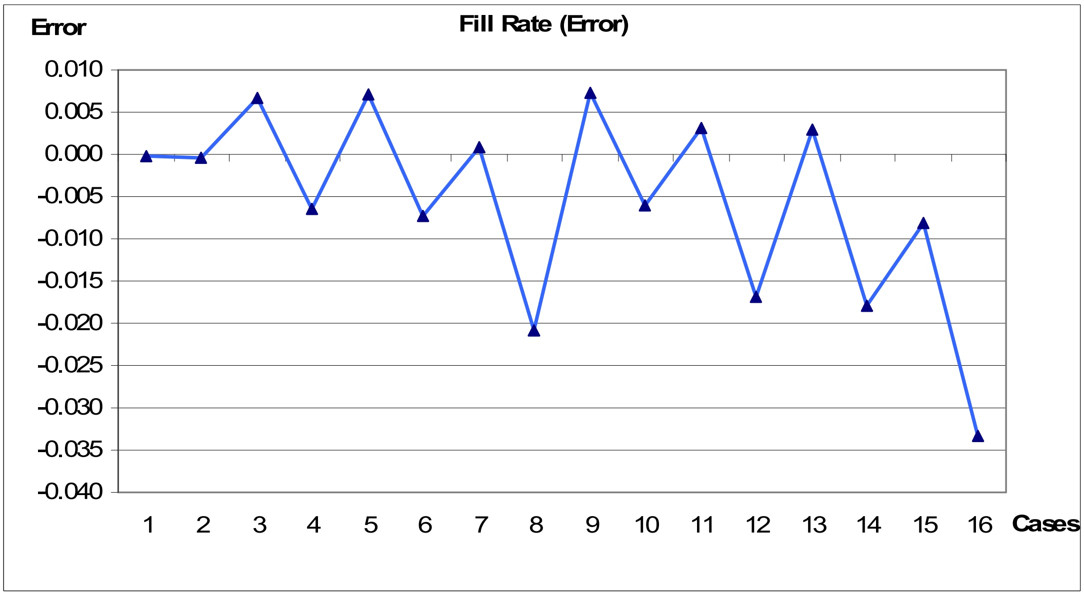

Figure 4.5 (b) Fill rate errors for IKCS with $K_i = 8$, $i = 1, 2, 3, 4$.

The numerical results given in Matta et al. (2005) for three-component systems are compared with the approximate performance measures we propose in Tables 4.9 (a) and (b) for SKCS and IKCS, respectively, and it is observed that our approximation seems to give better results also in the case of more than two components. Note that Matta et al. (2005) test the performance of their methods only for three-component systems. Therefore, the comparison with Matta et al. (2005) is only for the cases with three components.

Table 4.9 (a) Comparison of the results with the ones in Matta et al. (2005) for SKCS.

EB (Simultaneous)										
Kanban Size				**Sim**	**Decomposition**		**3qss**		**2qss**	
K1	**K2**	**K3**	**K4**		**Approx.**	**Abs. Rel.Er.(%)**	**Approx.**	**Abs. Rel.Er.(%)**	**Approx.**	**Abs. Rel.Er.(%)**
10	10	10	10	0.2557	0.2857	11.75	0.4866	90.31	0.4984	94.92
8	8	8	10	0.3593	0.4028	12.12	0.5481	52.55	0.9229	156.86
10	10	10	8	0.4471	0.4779	6.89	0.5510	23.24	0.7298	63.23
FG (Simultaneous)										
Kanban Size				**Sim**	**Decomposition**		**3qss**		**2qss**	
K1	**K2**	**K3**	**K4**		**Approx.**	**Abs. Rel.Er.(%)**	**Approx.**	**Abs. Rel.Er.(%)**	**Approx.**	**Abs. Rel.Er.(%)**
10	10	10	10	6.9126	6.7508	2.34	6.7171	2.83	6.4621	6.52
8	8	8	10	6.6509	6.4104	3.62	6.2904	5.42	5.8707	11.73
10	10	10	8	5.0587	4.9364	2.42	4.8863	3.41	4.7036	7.02
FR (Simultaneous)										
Kanban Size				**Sim**	**Decomposition**		**3qss**		**2qss**	
K1	**K2**	**K3**	**K4**		**Approx.**	**Abs.Er.**	**Approx.**	**Abs.Er.**	**Approx.**	**Abs.Er.**
10	10	10	10	0.9258	0.9180	0.0078	0.9058	0.0200	0.8865	0.0393
8	8	8	10	0.9059	0.8952	0.0107	0.8765	0.0294	0.8323	0.0736
10	10	10	8	0.8724	0.8637	0.0087	0.8537	0.0187	0.8284	0.0440

Table 4.9 (b) Comparison of the results with the ones in Matta et al. (2005) for IKCS.

EB (Independent)								
Kanban Size				Sim	Decomposition		3qss	
K1	K2	K3	K4		Approx.	Abs. Rel.Er.(%)	Approx.	Abs. Rel.Er.(%)
10	10	10	10	0.2510	0.2786	11.02	0.3253	29.60
8	8	8	10	0.3159	0.3764	19.15	0.4465	41.34
10	10	10	8	0.4274	0.4684	9.60	0.5255	22.95
FG (Independent)								
Kanban Size				Sim	Decomposition		3qss	
K1	K2	K3	K4		Approx.	Abs. Rel.Er.(%)	Approx.	Abs. Rel.Er.(%)
10	10	10	10	6.9218	6.7735	2.14	6.7294	2.78
8	8	8	10	6.7151	6.4822	3.47	6.4403	4.09
10	10	10	8	5.0768	4.9537	2.42	4.9161	3.17
FR (Independent)								
Kanban Size				Sim	Decomposition		3qss	
K1	K2	K3	K4		Approx.	Abs.Er.	Approx.	Abs.Er.
10	10	10	10	0.9266	0.9194	0.0072	0.9124	0.0142
8	8	8	10	0.9122	0.9001	0.0121	0.8910	0.0212
10	10	10	8	0.8750	0.8655	0.0095	0.8576	0.0174

Performance of the proposed approximation for increasing number of components is seen in Table 4.10 in terms of both the average accuracy and the average CPU times. It is observed that the CPU time incases and the accuracy gets worse as the number of components increases.

A final investigation is on how the sequence the components are picked up in the alternative model affect the approximation accuracy. Recall that our claim is that the original and the alternative models are equivalent regardless of the sequence the components are picked up in the alternative model. Although a formal proof is not given for this equivalence, the alternative model is constructed to preserve the operating mechanism in the original model. Also, the aggregated version of the alternative model is exact as long as the conditional probabilities (one for each of the components and some others expressed in terms of these) are kept state-dependent. However, when these conditional probabilities are assumed state-independent, the approximate aggregate models obtained with different sequences of the components would be different. This is what we observe with the numerical experiments, see Appendix P. K_i values are all the same but the ρ_i combinations identify different sequences of the components. However, based on these numerical results, we can not claim advantages of a sequence over the other sequence.

Table 4.10 Effect of the number of components, $K_i = 5$ for all i.

SKCS					
Number of Components	**EB (Abs. Rel. Error %)**	**FR (Abs. Error)**	**SP (Abs. Error)**	**FP (Abs. Rel. Error %)**	**Average CPU time**
2	11.53	0.012	0.013	1.91	0.041
3	21.34	0.025	0.027	4.87	0.167
4	31.98	0.034	0.038	8.97	0.673
IKCS					
Number of Components	**EB (Abs. Rel. Error %)**	**FR (Abs. Error)**	**SP (Abs. Error)**	**FP (Abs. Rel. Error %)**	**Average CPU time**
2	10.60	0.010	0.011	2.05	0.033
3	17.48	0.017	0.018	3.60	0.160
4	17.17	0.020	0.021	5.51	0.684

CHAPTER 5

5. CONCLUSION

In this thesis, the steady-state performance of the kanban controlled assembly systems is approximated by analyzing the corresponding Markovian queuing network models. Basic steps of our analysis are directly comparable to the ones in Avşar et al. (2004). When the aggregate model in Avşar et al. (2004) is approximated assuming that the conditional probabilities (q , q' and q'' in the case of two-component systems), that result from the aggregation to adjust transition rates for the aggregate states, are state-independent, the steady-state probability distribution for this approximate aggregate model turns out to be of product-form. However, due to the nature of the kanban controlled systems, in our case we can not avoid decomposing the approximate aggregate model into submodels. The product-form distributions can only be guaranteed for the independently treated submodels. The submodels are, then, combined identifying a very small number of aggregate states for each submodel. Although the combined QBD process becomes manageable with four and three aggregate states at the initial and the following levels of the process, respectively, we can not find a closed-form solution for this model and have to proceed by using iterative numerical algorithms based on matrix-geometric method. But, for a given number of components, the computational efforts required for the use of these algorithms are determined by the constant number of aggregate states defined for each submodel regardless of the number of kanbans at each stages. This comes up as an advantage of our approach in spite of not being able to avoid the use of matrix-geometric method. For the use of the same algorithms to solve the exact or the approximate aggregate model, the computational burden is a function of the number of kanbans, which makes these algorithms prohibitive for such models.

Extensive numerical experiments reveal the performance of the proposed approximation in terms of the accuracy for the relevant performance measures and the computational efforts to be spent. The approximation in this thesis turns out to be more accurate than the ones in Di Mascolo and Dallery (1996) and Matta et al. (2005). Since the required computational efforts are not reported in Di Mascolo and Dallery (1996) and Matta et al. (2005), in this respect our comparisons are with the exact and the approximate aggregate models. The advantages over these two models result from the reduction of the state space in the combined model as already mentioned above.

Two versions of the approximation arise with the use of different state-independent conditional probabilities for simultaneous and independent kanban release mechanisms. In other words, structures of the (approximate) aggregate model are the same for both mechanisms but the definitions of the (state-independent) conditional probabilities differ. The proposed approximation is also considered for the systems with more than two components employing the same arguments repetitively in order to handle each component. The numerical results show that the performance of the approximation decreases as the number of components increases. This is something one may expect because including one more component brings about the requirement to estimate one more state-independent conditional probability at the expense of some additional independence assumptions to approximate the correlated behaviour of the queues. But, the accuracy still puts forward this approximation as the best in the literature.

Numerical results show that the approximation errors increase when the assembly facility is faster than at least one of the manufacturing facilities feeding the lower stage. Based on numerical observations, this is attributed to estimating expected values of the conditional probabilities of the assembly stage, i.e., to (q, q' and q'' values) independently working with constant arrival rate λ. On the other hand, we also observe that the approximation errors may drop even to zero for some q, q' and q'' values. But, note that, then, these best q, q' and q'' would be different for different performance measures. Thus, all these point out a further research direct to see if the approximation performance can be improved estimating state-independent conditional probabilities , i.e., q, q', q'', using state-dependent arrival rates like the ones computed in Matta et al. (2005) by iterative procedures as a further research direction.

An immediate extension of the proposed approximation would be for similar system structures under different kanban control policies. For such extensions, how the state-independent conditional probabilities that result from partial aggregation are estimated is central for good performance of the approximation. Also, as in the case of assembly systems, the approximation approach under consideration can not be employed for the original model, but for some alternative (preferably equivalent) models. Thus, in case it is inevitable to proceed with an alternative model, structure of this model would determine the way it is aggregated and the conditional probabilities are defined and introduced accordingly. For example, in the case of the assembly systems in this thesis and in Avşar et al. (2004), the alternative model is to sequentially pick up the components, and this raises questions about the approximation accuracy with respect to the sequence the components are picked up. Even more challenging extensions can be considered to analyze more complex system structures possibly closer to real-life applications for which the systems like the ones in this thesis would serve as building blocks. Immediate examples of such extensions are for more realistic service facilities instead of single exponential servers or systems to produce more than one type of assembly with common components.

REFERENCES

1.Aktürk, M. S. and Erhun, F., (1999). "An Overview of Design and Operational Issues of Kanban Systems", International Journal of Production Research, Vol. 37 (17), pp. 3859-3881.

2.Albright, S.C., and Soni, A., (1988). "An approximation to the Stationary Distribution of a Multidimensional Markov Process", *IEE Transactions*, Vol. 20(1), pp. 111-118.

3.Avşar, Z.M., Zijm, W.H.M., and Rodoplu, U., (2004). "An Approximate Model for Base-Stock Controlled Assembly Systems", working paper.

4.Baynat, B. and Dallery, Y., (1993a). "Approximate Techniques for General Closed Queueing Networks with Subnetworks Having Population Constraints", *European Journal of Operational Research*, Vol. 69, pp. 250-264.

5.Baynat, B. and Dallery, Y., (1993b). "A Unified View of Product-form Approximation Techniques for General Closed Queueing Networks", *Performance Evaluation*, Vol. 18(3), pp. 205-224.

6.Baynat, B. and Dallery, Y., (1995). "Approximate Analysis of Multi-class Synchronized Closed Queueing Networks", MASCOTS'95, *Proceedings of the Third International Workshop on Modeling, Analysis, and Simulation of Computer and Telecommunication Systems*, 18-20 Jan., pp. 23-27.

7.Baynat, B. and Dallery, Y., (1996). "A Product-form Approximation Method for General Closed Queuing Networks with Several Classes of Customers" , *Performance Evaluation*, Vol. 24 (3), pp. 165-188.

8.Baynat, B., Dallery, Y., Di Mascolo, M. and Frein, Y., (2001). "A Multi-Class Approximation Technique for the Analysis of Kanban-Like Control Systems", *International Journal of Production Research, Special Issue on Modeling, Specification and Analysis of Manufacturing Systems*, Vol. 39 (2), pp. 307-328.

9.Bini, D.A., Lataouche, G., Meini, B., (2002). "Solving Matrix Polynomial Equations Arising in Queueing Problems", *Linear Algebra and Its Applications*, Vol. 340, pp. 225-244.

10.Bonvik, A.M., Couch, C., Gershwin, S.B., (1997). “A Comparison of Production Line Control Mechanisms”, *International Journal of Production Research*, Vol. 35, pp. 789-804.

11.Buzacott, J.A. and Shanthikumar, J.G., (1993). *Stochastic Models of Manufacturing Systems*, Prentice Hall, New Jersey.

12.Chaouiya, C. and Dallery, Y., (1997). “Petri Net models of Pull Control Systems for Assembly Manufacturing Systems", *Proceedings of the 2nd International Workshop on Manufacturing and Petri Nets, International Conference on Application and Theory of Petri Nets*, edited by F. DiCesare, M. Silva and R. Valette. CNRS/LAAS, Toulouse, France, 23 June, pp. 85-103.

13.Chaouiya, C., Liberopoulos, G., Dallery, Y., (2000). “The Extended Kanban Control System for Production Coordination of Assembly Manufacturing Systems”, *IIE Transactions*, Vol. 32, pp.999-1012.

14.Coenen, T., Heragu, S.S., and Zijm, W.H.M., (2003). “Analysis of a Multi-Class Manufacturing Systems via Semi-Open Queuing Networks”, DSES Technical Report No. 38-03-503 (under revision for re-submission to *Manufacturing and Service Operations Management*).

15. Di Mascolo, M., Frein, Y., Dallery, Y., (1996). “An Analytical Method for Performance Evaluation of Kanban Controlled Production Systems”, *Operations Research*, Vol. 44(1), pp. 50-64.

16.Di Mascolo, M. and Dallery, Y., (1996). “Performance Evaluation of Kanban Controlled Assembly Systems”, *Proceedings of the Symposium on Discrete Events and Manufacturing Systems of the Multiconference on Computational Engineering in Systems Applications (CESA'96, IMACS)*, Lille, France.

17.Frein, Y., Di Mascolo, M. and Dallery, Y., (1995). “On the Design of Generalized Kanban Control Systems”, *International Journal of Operations and Production Management*, Vol. 15, pp. 158-184.

18.Hazra, J., Schwetzer, P.J. and Seidmann, A., (1999). "Analyzing Closed Kanban-Controlled Assembly Systems by Iterative Aggregation-Disaggregation", *Computers and Operations Research*, Vol. 26, pp. 1015-1039.

19.Krishnamurthy, A., Suri, R., and Vernon, M., (2003). “Two Moment Approximations for Throughput and Mean Queue Length of a Fork/Join Station with General Input Processes”, *Stochastic Modeling and Optimization of Manufacturing Systems and Supply Chains*, Shanthikumar, J.G., Yao, D.D., and Zijm, W.H.M.

(Eds.), Kluwer International Series in Operations Research and Management Science, pp.87-126.

20.Latouche, G. and Ramaswami, V., (1993). “A Logarithmic Reduction Algorithm for Quasi-Birth-Death Processes”, *Journal of Applied Probability*, Vol. 30, pp. 650-674.

21.Latouche, G. and Ramaswami, V., (1999). Introduction to Matrix Analytic Methods in Stochastic Modeling, ASA-SIAM Series on Statistics and Applied Probability.

22.Liberopoulos, G., and Dallery, Y., (2000). “A Unified Framework for Pull Control Mechanisms in Multi-Stage Manufacturing Systems”, *Annals of Operations Research*, Vol. 93, pp. 325-355.

23.Matta, A., Dallery, Y., Di Mascolo, M., (2005). "Analysis of Assembly Systems Controlled with Kanbans", *European Journal of Operational Research*, Vol. 166(2), pp. 310-336.

24.Moore, K.E., and Gupta, S.M., (1999). “Stochastic Coloured Petri Net (SCPN) Models of Traditional and Flexible Kanban Systems", *International Journal of Production Research*, Vol. 37(9), pp.2135-2158.

25.Neuts, M.F., (1981). *Matrix-Geometric Solutions in Stochastic Models. An Algorithmic Approach*, The John Hopkins University Press, Baltimore MD.

26.Sbiti, N., Di Mascolo, M., Bennani, T., Amghar, M., (2002). “Modeling and Performance Evaluation of Base-Stock Controlled Assembly Systems”, S.B. Gershwin, Y. Dallery, C.T. Papadopoulos, J. McGregor-Smith (eds.): *Kluwer Academic Publishers Special Issue on Analysis and Modeling of Manufacturing Systems*, Chapter 13.

27.Sbiti, N., Di Mascolo, M. and Amghar, M., (2005). “Modeling and Properties of Generalized Kanban Controlled Assembly Systems”, *5th International Conference on Analysis of Manufacturing Systems-Production Management*, Zakynthos Island, Greece, pp. 235-242.

28.Schömig, A.K. and Kahnt, M., (1995). “Performance Modeling of Pull manufacturing Systems with Batch Servers and Assembly-Like Structure”, *Universität Würzburg Institut für Informatik Research Report Series*, Report No. 108, March.

29.Spanjers, L., Van Ommeren, J.C.W. and Zijm, W.H.M, (2005). “Closed Loop Two-Echelon Repairable Item Systems", *OR Spektrum*, Vol. 27, pp. 369-398.

30.Tran, H.T., Do, T.V., (2000). “Computational Aspects for Steady State Analysis of QBD Processes ", *Periodica Polytechnica Ser. El. Eng.*, Vol. 44(2), pp. 179-200.

APPENDIX A

RELATIONS IMPLIED BY THE INVENTORY BALANCE EQUATIONS IN THE ORIGINAL MODEL OF SKCS WITH STATE DESCRIPTION $(n_1, n_2, n_3, d_1, d_2)$

If $n_1 < K_1$, $n_2 < K_2$ and $n_3 < K_3$,
then $\overline{n}_1 = K_1 - n_1$, $\overline{n}_2 = K_2 - n_2$, $\overline{n}_3 = K_3 - n_3$, $d_1 = 0$ and $d_2 = 0$.

If $n_1 = K_1$, $n_2 < K_2$, $n_3 < K_3$, $d_1 = K_3 - n_3$ *and* $d_2 = D_2$,
then $\overline{n}_1 = 0$, $\overline{n}_2 = K_2 - n_2$, $\overline{n}_3 = 0$, $d_1 = K_3 - n_3$ *and* $d_2 = D_2$.

If $n_1 = K_1$, $n_2 < K_2$, $n_3 < K_3$ *and* $d_1 = D_1 < K_3 - n_3$,
then $\overline{n}_1 = 0$, $\overline{n}_2 = K_2 - n_2$, $\overline{n}_3 = K_3 - d_1 - n_3$, $d_1 = D_1$ *and* $d_2 = 0$.

If $n_1 < K_1$, $n_2 = K_2$, $n_3 < K_3$, $d_1 = K_3 - n_3$ *and* $d_2 = D_2$,
then $\overline{n}_1 = K_1 - n_1$, $\overline{n}_2 = 0$, $\overline{n}_3 = 0$, $d_1 = K_3 - n_3$ *and* $d_2 = D_2$.

If $n_1 < K_1$, $n_2 = K_2$, $n_3 < K_3$, $d_1 = D_1 < K_3 - n_3$,
then $\overline{n}_1 = K_1 - n_1$, $\overline{n}_2 = 0$, $\overline{n}_3 = K_3 - d_1 - n_3$, $d_1 = 0$ *and* $d_2 = 0$.

If $n_1 < K_1$, $n_2 < K_2$ and $n_3 = K_3$, $d_2 = D_2$,
then $\overline{n}_1 = K_1 - n_1$, $\overline{n}_2 = K_2 - n_2$, $\overline{n}_3 = 0$, $d_1 = 0$ and $d_2 = D_2$.

If $n_1 = K_1$, $n_2 < K_2$ and $n_3 = K_3$, $d_2 = D_2$,
then $\overline{n}_1 = 0$, $\overline{n}_2 = K_2 - n_2$, $\overline{n}_3 = 0$, $d_1 = 0$ and $d_2 = D_2$.

If $n_1 < K_1$, $n_2 = K_2$ and $n_3 = K_3$, $d_2 = D_2$,

then $\overline{n}_1 = K_1 - N_1$, $\overline{n}_2 = 0$, $\overline{n}_3 = 0$, $d_1 = 0$ and $d_2 = D_2$.

If $n_1 = K_1$, $n_2 = K_2$ and $n_3 < K_3$, $d_1 = D_1 < K_3$, $d_2 = D_2$,

then $\overline{n}_1 = 0$, $\overline{n}_2 = 0$, $\overline{n}_3 = K_3 - d_1 - n_3$, $d_1 = D_1$ and $d_2 = 0$.

If $n_1 = K_1$, $n_2 = K_2$ and $n_3 = K_3$, $d_2 = D_2$,

then $\overline{n}_1 = 0$, $\overline{n}_2 = 0$, $\overline{n}_3 = 0$, $d_1 = D_1$ and $d_2 = D_2$.

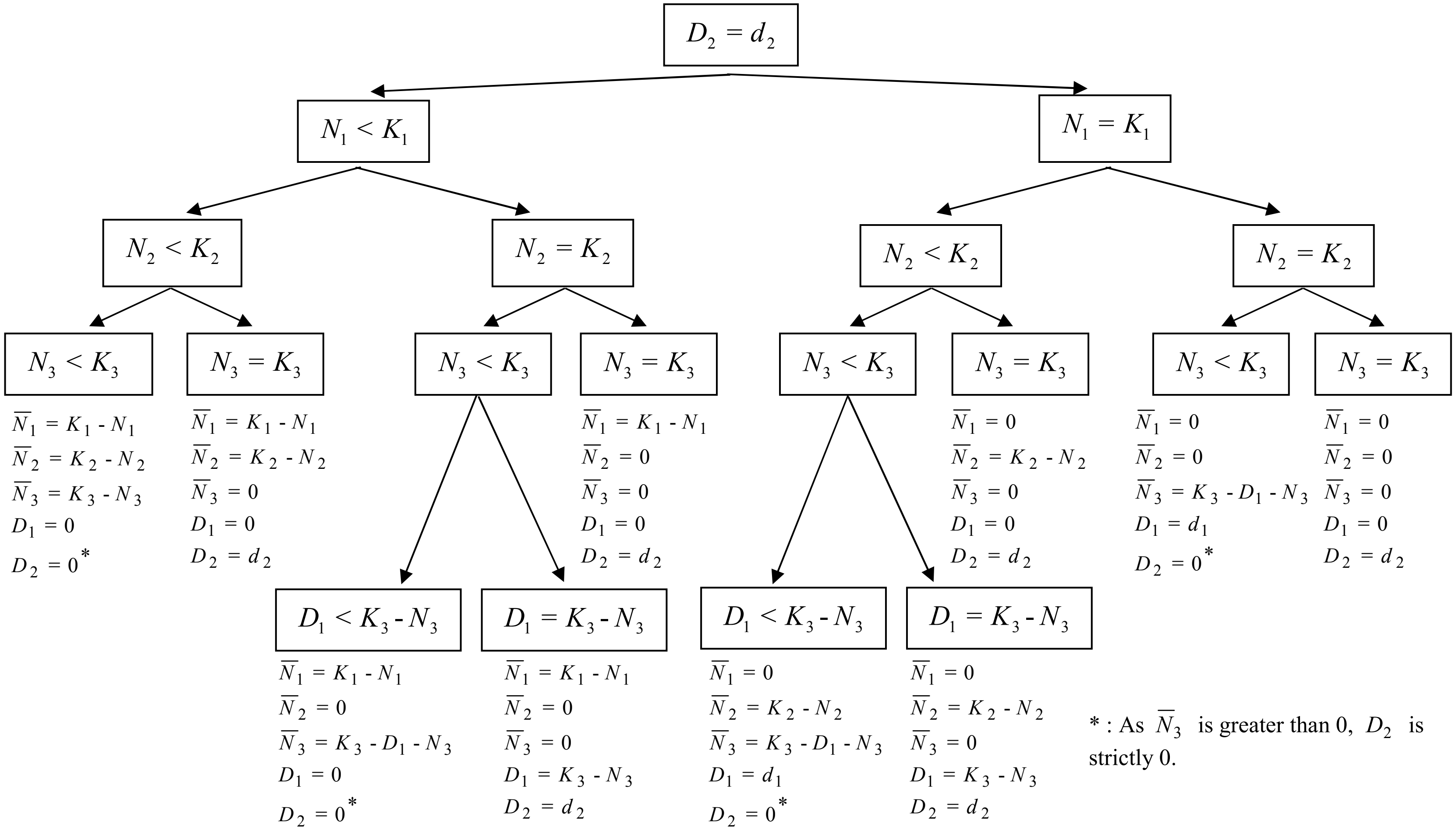

Figure A.1 Tree Structure of the Relations in the Original Model of SKCS with state description $(n_1, n_2, n_3, d_1, d_2)$

APPENDIX B

RELATIONS IMPLIED BY THE INVENTORY BALANCE EQUATIONS IN THE ALTERNATIVE MODEL OF SKCS WITH STATE DESCRIPTION (m_1, m_2, m_3)

Using (3.8), (3.9) and (3.10) and the feasibility conditions (for every m_i, n_i, $\overline{n}_i$, and $d_{1i} \geq 0$ and $d_2 \geq 0$), one can obtain the values of each random variable in terms of m_i, $i = 1, 2, 3$.

If $0 \leq m_1 < K_1$, $0 \leq m_2 < K_2$ and $0 \leq m_3 < K_3$,

then $n_1 = m_1$, $\overline{n}_1 = K_1 - m_1$, $d_{11} = 0$,

$n_2 = m_2$, $\overline{n}_2 = K_2 - m_2$, $d_{12} = 0$,

$n_3 = m_3$, $\overline{n}_3 = K_3 - m_3$, $d_2 = 0$.

If $m_1 \geq K_1$, $0 \leq m_2 < K_2$ and $0 \leq m_3 < K_3 - (m_1 - K_1)$,

then $n_1 = K_1$, $\overline{n}_1 = 0$, $d_{11} = m_1 - K_1$,

$n_2 = m_2$, $\overline{n}_2 = K_2 - m_2$, $d_{11} = 0$,

$n_3 = m_3$, $\overline{n}_3 = K_3 - (m_1 - K_1) - m_3$, $d_2 = 0$.

If $0 \leq m_1 < K_1 - (m_2 - K_2)$, $m_2 \geq K_2$ and $0 \leq m_3 < K_3 - (m_2 - K_2)$,

then $n_1 = m_1$, $\overline{n}_1 = K_1 - (m_2 - K_2) - m_1$, $d_{11} = 0$,

$n_2 = K_2$, $\overline{n}_2 = 0$, $d_{12} = m_2 - K_2$,

$n_3 = m_3$, $\overline{n}_3 = K_3 - (m_2 - K_2) - m_3$, $d_2 = 0$.

If $0 \leq m_1 < K_1$, $0 \leq m_2 < K_2$ and $m_3 \geq K_3$,

then $n_1 = m_1$, $\overline{n}_1 = K_1 - m_1$, $d_{11} = 0$,

$n_2 = m_2$, $\overline{n}_2 = K_2 - m_2$, $d_{12} = 0$,

$n_3 = K_3$, $\overline{n}_3 = 0$, $d_2 = m_3 - K_3$.

If $m_1 \geq K_1 - (m_2 - K_2) \geq 0$, $m_2 \geq K_2$, $0 \leq m_3 < K_3 - (m_2 - K_2) - (m_1 - (K_1 - (m_2 - K_2)))$,

then $n_1 = K_1 - (m_2 - K_2)$, $\overline{n}_1 = 0$, $d_{11} = m_1 - (K_1 - (m_2 - K_2))$,

$n_2 = K_2$, $\overline{n}_2 = 0$, $d_{12} = m_2 - K_2$,

$n_3 = m_3$, $\overline{n}_3 = K_3 - (m_1 - K_1) - 2(m_2 - K_2) - m_3$, $d_2 = 0$.

If $m_1 \geq K_1$, $0 \leq m_2 < K_2$ and $m_3 \geq K_3 - (m_1 - K_1) \geq 0$,

then $n_1 = K_1$, $\overline{n}_1 = 0$, $d_{11} = m_1 - K_1$,

$n_2 = m_2$, $\overline{n}_2 = K_2 - m_2$, $d_{12} = 0$,

$n_3 = K_3 - (m_1 - K_1)$, $\overline{n}_3 = 0$, $d_2 = m_3 - (K_3 - (m_1 - K_1))$.

If $0 \leq m_1 < K_1 - (m_2 - K_2)$, $m_2 \geq K_2$ and $m_3 \geq K_3 - (m_2 - K_2) \geq 0$,

then $n_1 = m_1$, $\overline{n}_1 = K_1 - (m_2 - K_2) - m_1$, $d_{11} = 0$,

$n_2 = K_2$, $\overline{n}_2 = 0$, $d_{12} = M_2 - K_2$,

$n_3 = K_3 - (m_2 - K_2)$, $\overline{n}_3 = 0$, $d_2 = m_3 - (K_3 - (m_2 - K_2))$.

If $m_1 \geq K_1 - (m_2 - K_2) \geq 0$, $m_2 \geq K_2$, $m_3 \geq K_3 - (m_2 - K_2) - (m_1 - (K_1 - (m_2 - K_2))) \geq 0$,

then $n_1 = K_1 - (m_2 - K_2)$, $\overline{n}_1 = 0$, $d_{11} = m_1 - (K_1 - (m_2 - K_2))$,

$n_2 = K_2$, $\overline{n}_2 = 0$, $d_{12} = m_2 - K_2$,

$n_3 = K_3 - 2(m_2 - K_2) - (m_1 - K_1)$, $\overline{n}_3 = 0$,

$d_2 = m_3 - (K_3 - 2(m_2 - K_2) - (m_1 - K_1))$.

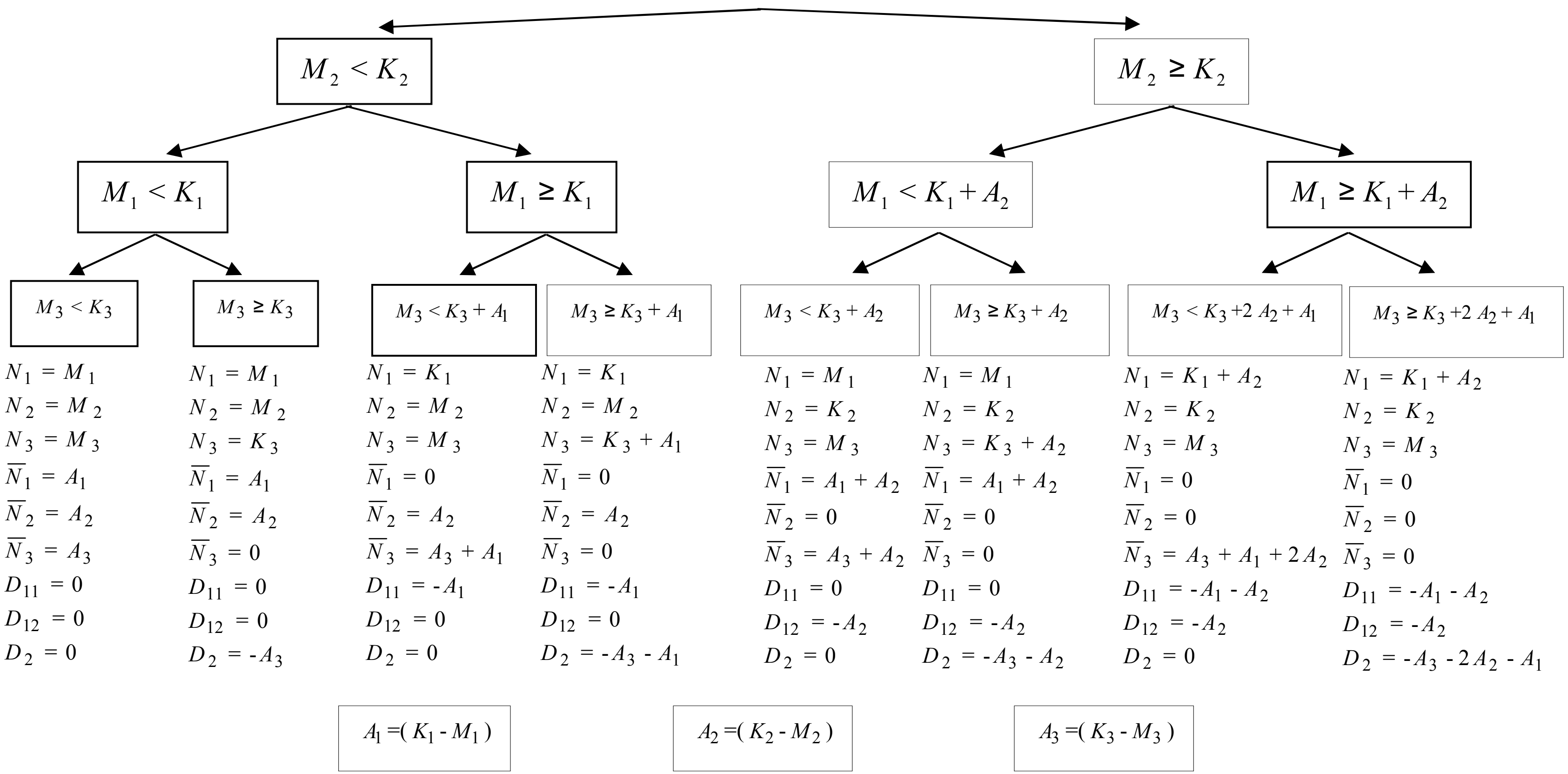

Figure B.1 Tree Structure of the Relations in the Alternative Model of SKCS with state description (m_1, m_2, m_3).

APPENDIX C

THE MATRICES (A_{00}, A_{01}, A_{10}, A_0, A_1 and A_2) FOR THE EXACT MODEL OF SKCS WITH $K_i = 1$, $i = 1, 2, 3$

Since the state space of the exact model gets larger as a function of the K_1, K_2 and K_3 values, the simplest case, with $K_1 = K_2 = K_3 = 1$ is considered for show the rate matrices. In the exact model of SKCS with $K_1 = K_2 = K_3 = 1$ the states (m_1, m_2, m_3) for level are ordered as; $(0,0,0)$, $(0,0,1)$, $(0,1,0)$, $(0,1,1)$, $(0,2,0)$, $(1,0,0)$, $(1,0,1)$, $(1,1,0)$, $(1,1,1)$, $(1,2,0)$, $(2,0,0)$, $(2,1,0)$ and $(2,2,0)$ for $d_2 = 0$, and as $(0,0,1)$, $(0,1,1)$, $(0,2,0)$, $(1,0,1)$, $(1,1,1)$, $(1,2,0)$, $(2,0,0)$, $(2,1,0)$ and $(2,2,0)$ for $d_2 > 0$. Then, the matrices used in the logarithmic reduction algorithm are the following:

$$A_{01} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix},$$

$$A_{00} = \begin{bmatrix} \theta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 \\ \mu_3 & \theta_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mu_2 & 0 & \theta_3 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & \mu_2 & \mu_3 & \theta_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu_2 & \theta_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mu_1 & 0 & 0 & 0 & 0 & \theta_2 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 \\ 0 & \mu_1 & 0 & 0 & 0 & \mu_3 & \theta_6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu_1 & 0 & 0 & \mu_2 & 0 & \theta_5 & 0 & 0 & 0 & 0 & \lambda \\ 0 & 0 & 0 & \mu_1 & 0 & 0 & \mu_2 & \mu_3 & \theta_8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu_1 & 0 & 0 & 0 & \mu_2 & \theta_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mu_1 & 0 & 0 & 0 & \theta_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_1 & 0 & \mu_2 & \theta_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_1 & 0 & \mu_2 & \theta_5 \end{bmatrix},$$

$$A_{10} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_{0} = \begin{bmatrix} \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix},$$

$$A_1 = \begin{bmatrix} \theta_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mu_2 & \theta_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu_2 & \theta_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mu_1 & 0 & 0 & \theta_6 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu_1 & 0 & \mu_2 & \theta_8 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu_1 & 0 & \mu_2 & \theta_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu_1 & 0 & 0 & \theta_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu_1 & 0 & \mu_2 & \theta_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu_1 & 0 & \mu_2 & \theta_5 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & \mu_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

where $\theta_1 = -\lambda$, $\theta_2 = -(\lambda + \mu_1)$, $\theta_3 = -(\lambda + \mu_2)$, $\theta_4 = -(\lambda + \mu_3)$, $\theta_5 = -(\lambda + \mu_1 + \mu_2)$, $\theta_6 = -(\lambda + \mu_1 + \mu_3)$, $\theta_7 = -(\lambda + \mu_2 + \mu_3)$ and $\theta_8 = -(\lambda + \mu_1 + \mu_2 + \mu_3)$.

APPENDIX D

THE MATRICES (A_{00} , A_{01} , A_{10} , A_0 , A_1 and A_2) FOR THE APPROXIMATE AGGREGATE MODEL OF SKCS WITH $K_i = 2$, i = 1, 2, 3

The states (d_{11}, d_{12}, n_3) are ordered as $(0,0,0)$, $(0,0,1)$, $(0,0,2)$, $(0,1,0)$, $(0,1,1)$, $(0,2,0)$, $(1,0,0)$, $(1,0,1)$, $(1,1,0)$ and $(2,0,0)$ for $d_2 = 0$ and as $(0,0,2)$, $(0,1,1)$, $(0,2,0)$, $(1,0,1)$, $(1,1,0)$ and $(2,0,0)$ for $d_2 > 0$. Then, the corresponding matrices are the following:

$$A_{01} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix},$$

$$A_{00} = \begin{bmatrix} \theta_1 & \overline{q}\overline{q}'\lambda & 0 & \overline{q}q'\lambda & 0 & 0 & q\lambda & 0 & 0 & 0 \\ \mu_3 & \theta_4 & \overline{q}\overline{q}'\lambda & 0 & \overline{q}q'\lambda & 0 & 0 & q\lambda & 0 & 0 \\ 0 & \mu_3 & \theta_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu_2 & 0 & \theta_3 & 0 & \overline{q}\lambda & 0 & 0 & q\lambda & 0 \\ 0 & 0 & \mu_2 & \mu_3 & \theta_7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu_2 & \theta_3 & 0 & 0 & 0 & 0 \\ 0 & \overline{q}'\mu_1 & 0 & q'\mu_1 & 0 & 0 & \theta_2 & 0 & 0 & \lambda \\ 0 & 0 & \overline{q}'\mu_1 & 0 & q'\mu_1 & 0 & \mu_3 & \theta_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu_1 & 0 & \mu_2 & \theta_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \overline{q}'\mu_1 & q'\mu_1 & \theta_2 \end{bmatrix},$$

$$A_{10} = \begin{bmatrix} 0 & 0 & \bar{q}\bar{q}'\mu_3 & 0 & \bar{q}q'\mu_3 & 0 & 0 & q\mu_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{q}\mu_3 & 0 & 0 & q\mu_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_0 = \begin{bmatrix} \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix},$$

$$A_1 = \begin{bmatrix} \theta_4 & 0 & 0 & 0 & 0 & 0 \\ \mu_2 & \theta_7 & 0 & 0 & 0 & 0 \\ 0 & \mu_2 & \theta_3 & 0 & 0 & 0 \\ \bar{q}'\mu_1 & q'\mu_1 & 0 & \theta_6 & 0 & 0 \\ 0 & 0 & \mu_1 & \mu_2 & \theta_5 & 0 \\ 0 & 0 & 0 & \bar{q}'\mu_1 & q'\mu_1 & \theta_2 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} \bar{q}\bar{q}'\mu_3 & \bar{q}q'\mu_3 & 0 & q\mu_3 & 0 & 0 \\ 0 & 0 & \bar{q}\mu_3 & 0 & q\mu_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu_3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

where $\theta_1 = -\lambda$, $\theta_2 = -(\lambda + \mu_1)$, $\theta_3 = -(\lambda + \mu_2)$, $\theta_4 = -(\lambda + \mu_3)$, $\theta_5 = -(\lambda + \mu_1 + \mu_2)$, $\theta_6 = -(\lambda + \mu_1 + \mu_3)$, $\theta_7 = -(\lambda + \mu_2 + \mu_3)$ and $\theta_8 = -(\lambda + \mu_1 + \mu_2 + \mu_3)$.

Because the approximate aggregate models for SKCS and IKCS are identical in terms of q and q', the rate matrices are the same also for IKCS.

APPENDIX E

PRODUCT-FORM DISTRIBUTIONS FOR THE SUBMODELS

As in Spanjers et al. [27] the steady-state distribution of the closed queuing network in Figure 3.13 (a) is given as

$$\Pr(d_{11}, d_{12}, n_3, \bar{n}_3) = \begin{cases} K' \dfrac{1}{\mu_1^{d_{11}}} \dfrac{1}{\mu_2^{d_{12}}} \dfrac{1}{\mu_3^{n_3}} \dfrac{1}{\lambda^{\bar{n}_3}} & \text{for } d_{11} > 0, d_{12} > 0, \\ \dfrac{K'}{q} \dfrac{1}{\mu_1^{d_{11}}} \dfrac{1}{\mu_2^{d_{12}}} \dfrac{1}{\mu_3^{n_3}} \dfrac{1}{\lambda^{\bar{n}_3}} & \text{for } d_{11} = 0, d_{12} > 0, \\ \dfrac{K'}{q'} \dfrac{1}{\mu_1^{d_{11}}} \dfrac{1}{\mu_2^{d_{12}}} \dfrac{1}{\mu_3^{n_3}} \dfrac{1}{\lambda^{\bar{n}_3}} & \text{for } d_{11} > 0, d_{12} = 0, \\ \dfrac{K'}{qq'} \dfrac{1}{\mu_1^{d_{11}}} \dfrac{1}{\mu_2^{d_{12}}} \dfrac{1}{\mu_3^{n_3}} \dfrac{1}{\lambda^{\bar{n}_3}} & \text{for } d_{11} = 0, d_{12} = 0, \end{cases}$$

where K' is the normalization constant. Since the system is a closed queueing network, i.e., $d_{11} + d_{12} + n_3 + \bar{n}_3 = K_3$, the term $\frac{1}{\mu_1^{d_{11}}} \frac{1}{\mu_2^{d_{12}}} \frac{1}{\mu_3^{n_3}} \frac{1}{\lambda^{\bar{n}_3}}$ can be rewritten as $\frac{1}{\mu_1^{d_{11}}} \frac{1}{\mu_2^{d_{12}}} \frac{1}{\mu_3^{n_3}} \frac{1}{\lambda^{K_3 - d_{11} - d_{12} - n_3}}$, which turns out to be $K \frac{\lambda}{\mu_1^{d_{11}}} \frac{\lambda}{\mu_2^{d_{12}}} \frac{\lambda}{\mu_3^{n_3}} \frac{1}{\lambda^{K_3}}$. Letting $K = K' \frac{1}{\lambda^{K_3}}$. (3.23) follows similarly.

APPENDIX F

THE MATRICES (A_{00} , A_{01} , A_{10} , A_0 , A_1 and A_2) FOR THE COMBINED MODEL OF SKCS

The combined models for the two-component cases are the same for given q', R_1, R_2, P_1, P_2 and P_3. For each d_2, the states are considered in the increasing order of y. The rate matrices are given below.

$$A_{01} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \\ 0 & 0 & 0 \end{bmatrix}, \quad A_{00} = \begin{bmatrix} \theta_1 & q'\mu_1 & \bar{q}'\mu_1 & 0 \\ 0 & \theta_2 & \mu_2 & 0 \\ 0 & 0 & \theta_3 & \mu_3 \\ P_1\lambda & P_2\lambda & P_3\lambda & P\lambda \end{bmatrix},$$

$$A_{10} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ R_2\mu_3 & R_1\mu_3 & R\mu_3 & 0 \end{bmatrix}, \quad A_0 = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix},$$

$$A_1 = \begin{bmatrix} \theta_1 & q'\mu_1 & \bar{q}'\mu_1 \\ 0 & \theta_2 & \mu_2 \\ 0 & 0 & \theta_3 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ R_2\mu_3 & R_1\mu_3 & R\mu_3 \end{bmatrix},$$

where $R = (1 - R_1 - R_2)$, $P = -(P_1 + P_2 + P_3)$, $\theta_1 = -(\lambda + \mu_1)$, $\theta_2 = -(\lambda + \mu_2)$ and $\theta_3 = -(\lambda + \mu_3)$.

APPENDIX G

STATE SPACE AND THE TRANSITION DIAGRAM OF IKCS

Let $m_i = n_i + d_{i1}$ for $i = 1$, 2 and $m_3 = n_3 + d_2$, then the state-transition diagram of the exact model with the state description (m_1, m_2, m_3) can be analyzed for four different cases in Figures G.1(a), (b), (c) and (d) where the regions with different d_{11} and d_{21} values are identified referring to (3.33) (whereas for d_2 refer to (3.10), since d_2 is the same in both SKCS and IKCS).

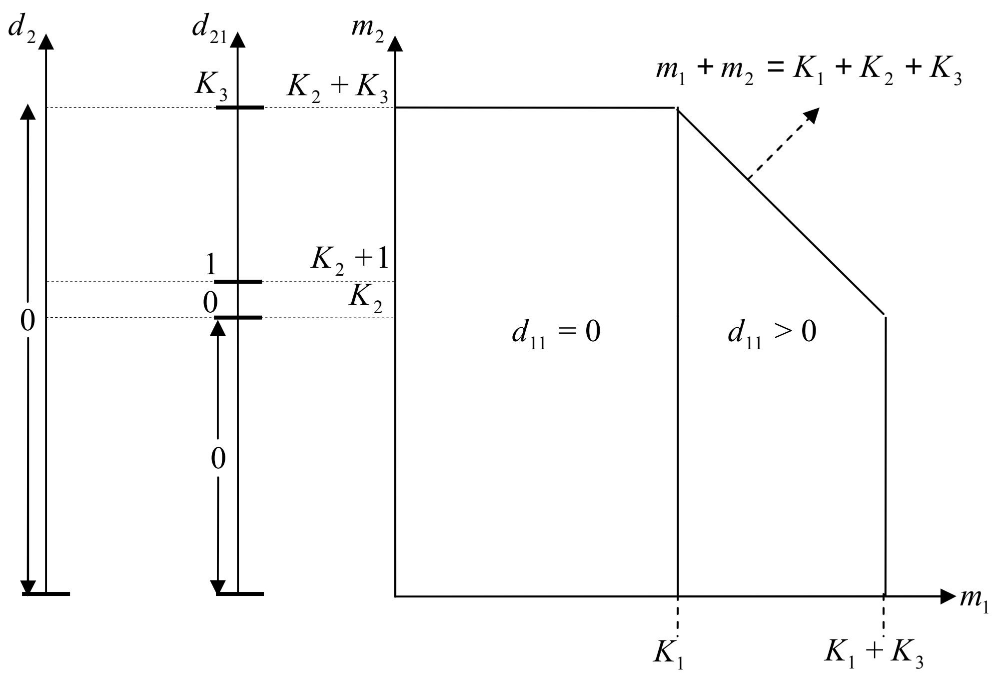

Figure G.1(a) Cross-Section of the State Space for $m_3 = 0$.

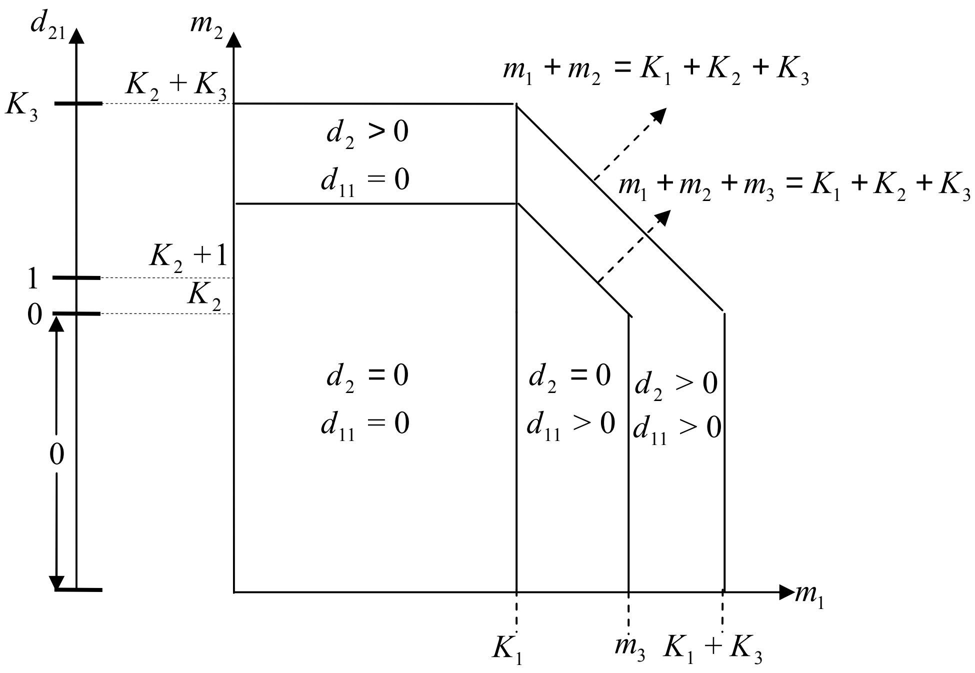

Figure G.1(b) Cross-Section of the State Space for $m_3 < K_3$.

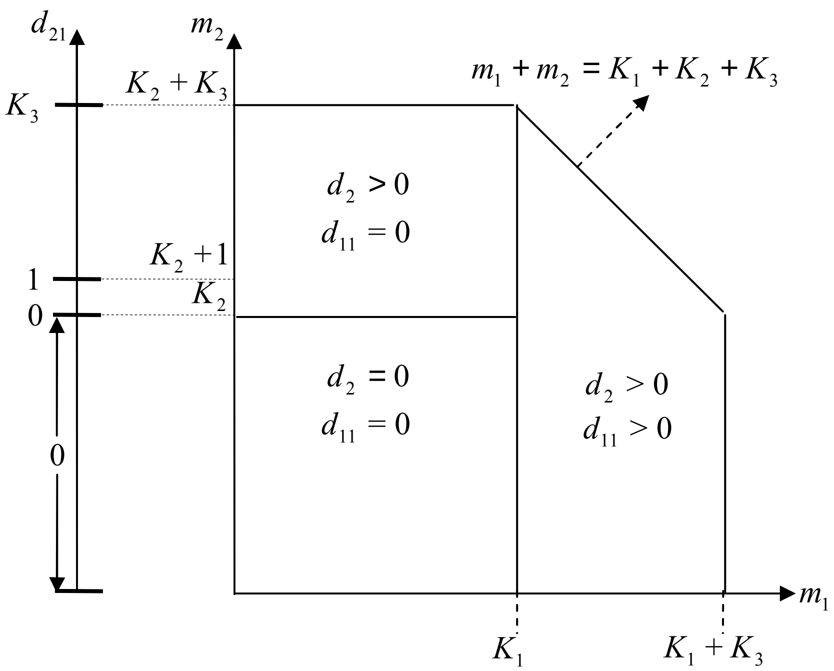

Figure G.1(c) Cross-Section of the State Space for $m_3 = K_3$.

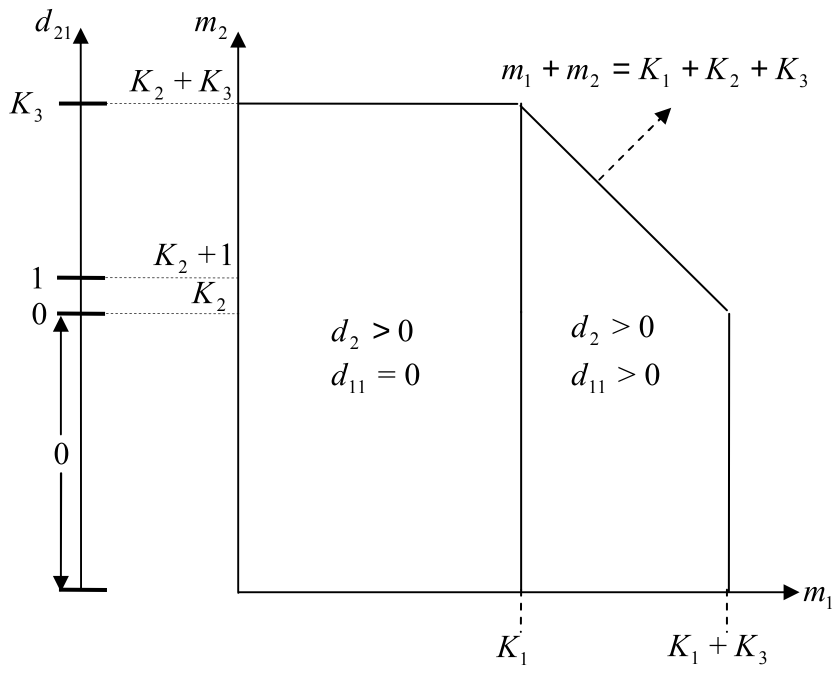

Figure G.1(d) Cross-Section of the State Space for $m_3 > K_3$.

State-transition diagram of the model with (m_1, m_2, m_3) is in Figure G.2. As in SKCS, there is a three-dimensional structure for $d_2 = 0$ and parallel surfaces for each $d_2 > 0$.

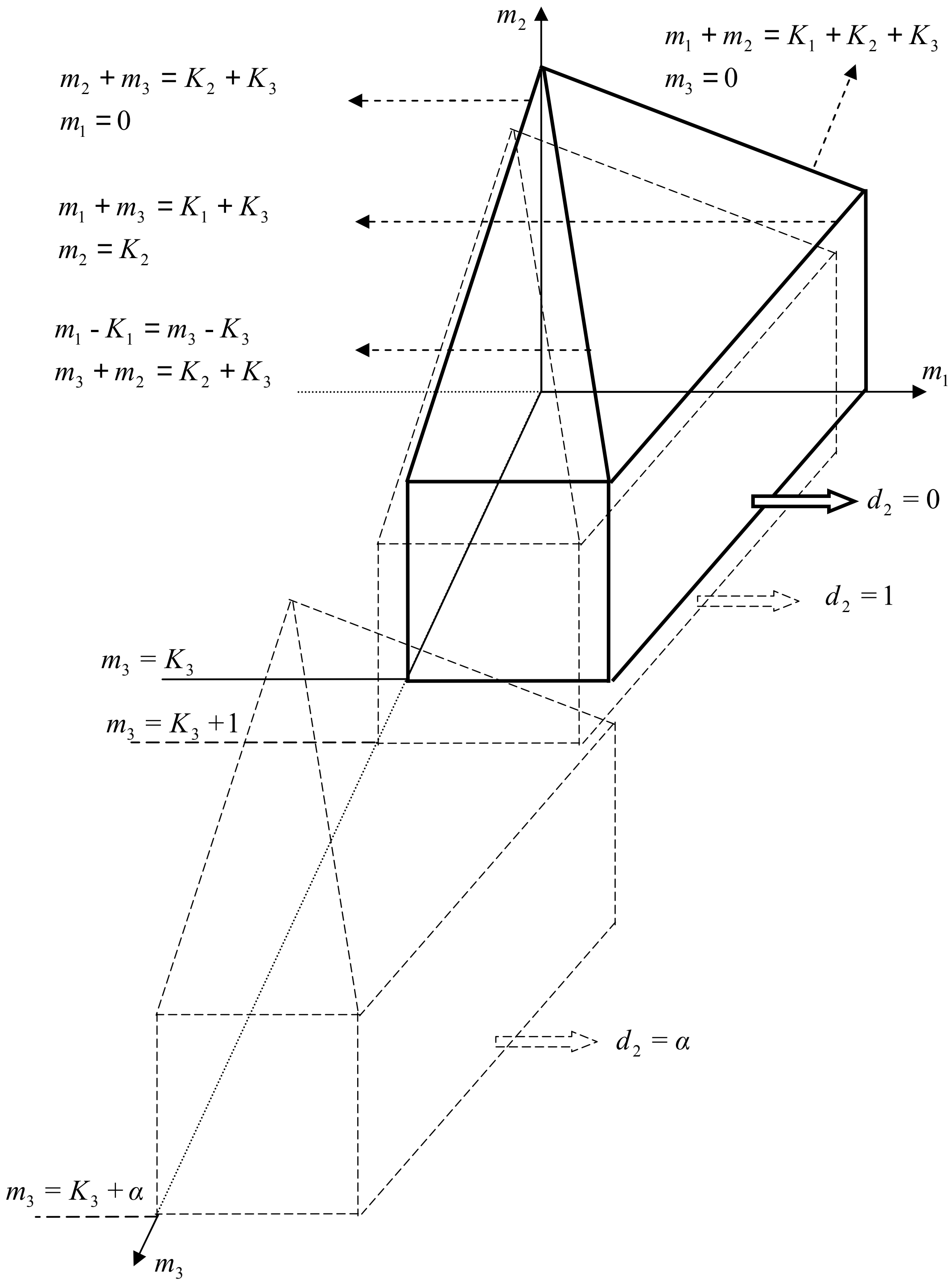

Figure G.2 State-Space of the Model with State Description (m_1, m_2, m_3).

Switching from state description (m_1, m_2, m_3) to (d_2, m_1, m_2, n_3) results in the state-transition diagram shown in Figures G.3(a), (b), (c), (d) for $d_2 = 0$ and Figure G.4 being the same for all $d_2 > 0$. Unit changes in d_2, m_1, m_2 and n_3 are demonstrated in parentheses as before next to the transition rates in the order of d_2, m_1, m_2, n_3.

$$\mu_1^* = \begin{cases} \mu_1(0,-,0,+) & \text{when } m_1 - K_1 > m_2 - K_2, \\ \mu_1(0,-,0,0) & \text{otherwise,} \end{cases}$$

$$\mu_2^* = \begin{cases} \mu_2(0,0,-,+) & \text{when } m_2 - K_2 > m_1 - K_1, \\ \mu_2(0,0,-,0) & \text{otherwise,} \end{cases}$$

in Figure G.3(d) and

$$\mu_1^* = \begin{cases} \mu_1(0,-,0,+) & \text{when } m_1 - K_1 > m_2 - K_2, \\ \mu_1(0,-,0,0) & \text{otherwise,} \end{cases}$$

$$\mu_2^* = \begin{cases} \mu_2(0,0,-,+) & \text{when } m_2 - K_2 > m_1 - K_1, \\ \mu_2(0,0,-,0) & \text{otherwise,} \end{cases}$$

in Figure G.4.

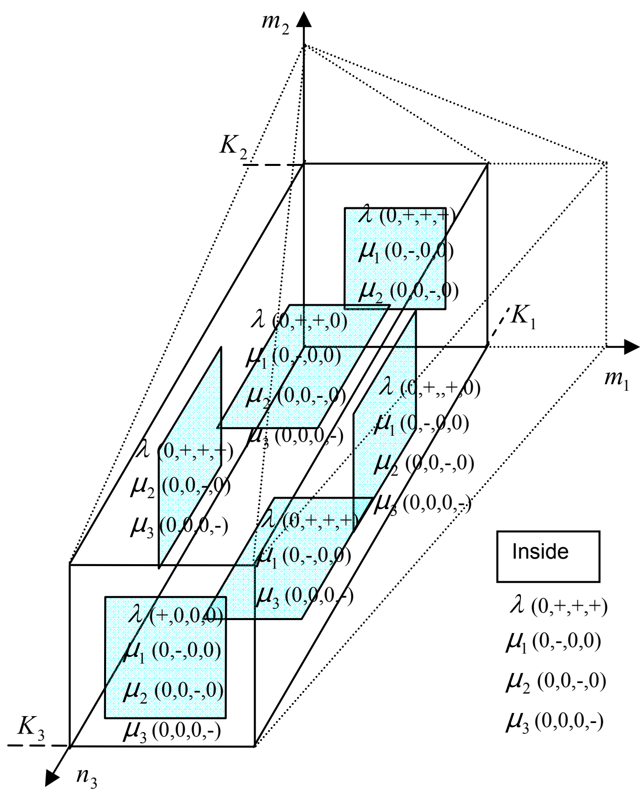

Figure G.3(a) State-Transition Diagram for $d_2 = 0,\ d_{11} = 0,\ d_{21} = 0$.

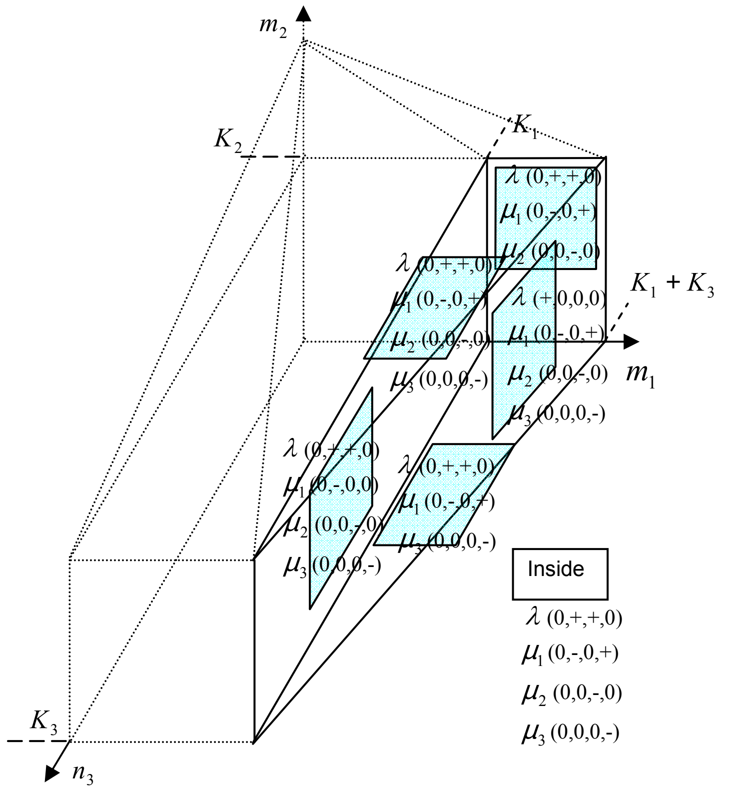

Figure G.3(b) State-Transition Diagram for $d_2 = 0,\ d_{11} > 0,\ d_{21} = 0$.

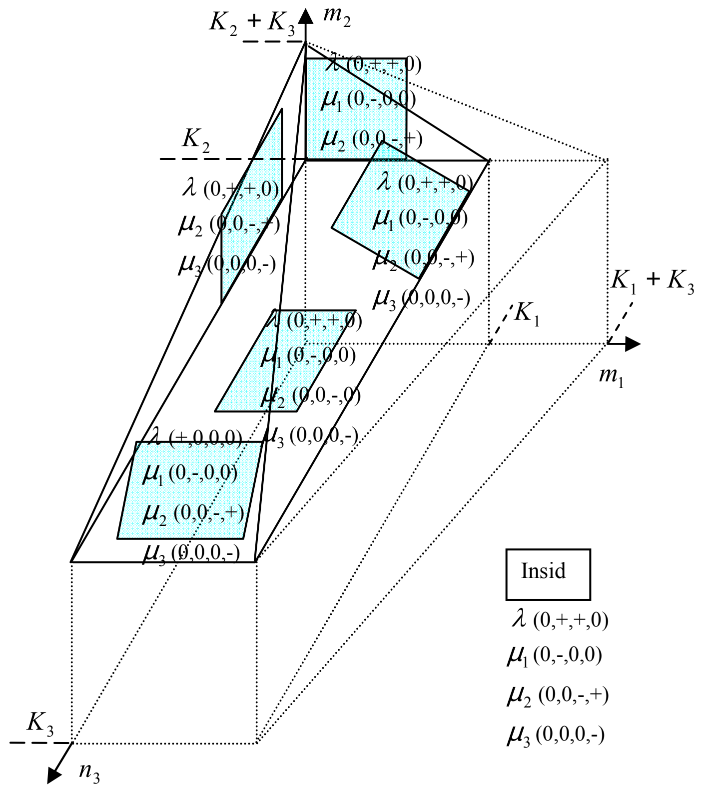

Figure G.3(c) State-Transition Diagram for $d_2 = 0,\ d_{11} = 0,\ d_{21} > 0$.

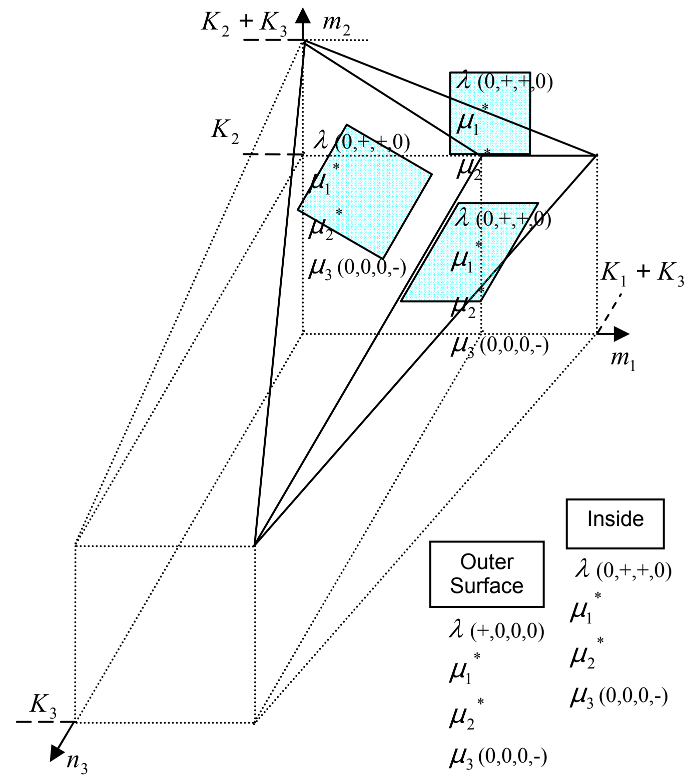

Figure G.3(d) State-Transition Diagram for $d_2 = 0,\ d_{11} > 0,\ d_{21} > 0$.

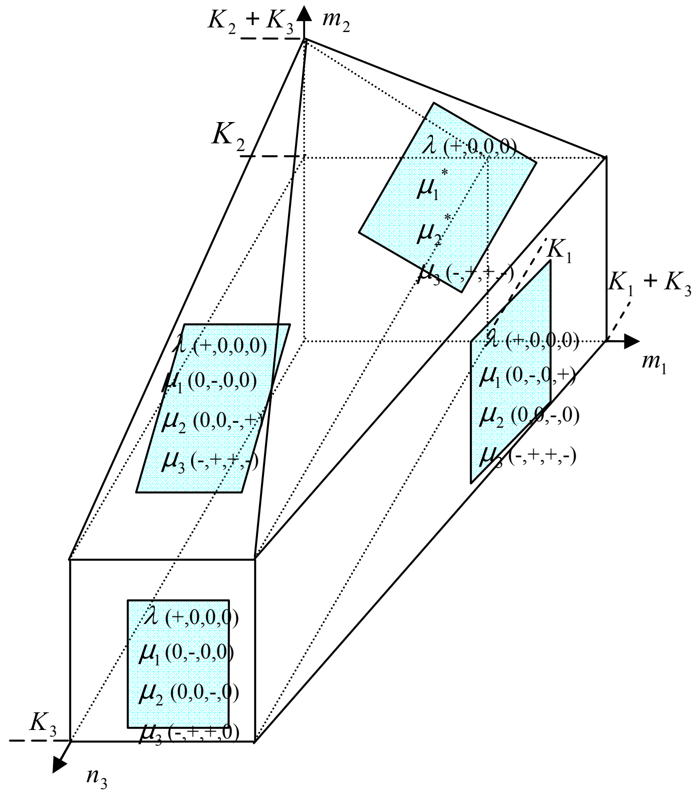

Figure G.4 State-Transition Diagram for $d_2 > 0$.

APPENDIX H

THE MATRICES (A_{00} , A_{01} , A_{10} , A_0 , A_1 and A_2) FOR THE EXACT MODEL OF IKCS WITH $K_i = 1$, $i = 1, 2, 3$

From Figures G.3 (a), (b), (c), (d) and G.4, one can easily obtain the rate matrices for the exact model of IKCS. Since the approximate aggregate models for SKCS and IKCS are identical in terms of q and q', the matrices for the corresponding systems are the same. But, we should note that the definition of q' changes according to the mechanics of IKCS.

The states (m_1, m_2, m_3) are ordered as $(0,0,0)$, $(0,0,1)$, $(0,1,0)$, $(0,1,1)$, $(0,2,0)$, $(1,0,0)$, $(1,0,1)$, $(1,1,0)$, $(1,1,1)$, $(2,0,0)$ and $(2,1,0)$ for $d_2 = 0$, and as $(0,0,1)$, $(0,1,1)$, $(0,2,0)$, $(1,0,1)$, $(1,1,1)$, $(2,0,0)$ and $(2,1,0)$ for $d_2 > 0$. The rate matrices for IKCS with $K_1 = K_2 = K_3 = 1$ are as follows:

$$A_{01} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix},$$

$$A_{00} = \begin{bmatrix} \theta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 \\ \mu_3 & \theta_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mu_2 & 0 & \theta_3 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu_2 & \mu_3 & \theta_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \theta_3 & 0 & 0 & 0 & \mu_2 & 0 & 0 \\ \mu_1 & 0 & 0 & 0 & 0 & \theta_2 & 0 & 0 & 0 & \lambda & 0 \\ 0 & \mu_1 & 0 & 0 & 0 & \mu_3 & \theta_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu_1 & 0 & 0 & \mu_2 & 0 & \theta_5 & 0 & 0 & \lambda \\ 0 & 0 & 0 & \mu_1 & 0 & 0 & \mu_2 & \mu_3 & \theta_8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_1 & \theta_2 & 0 \\ 0 & 0 & 0 & 0 & \mu_1 & 0 & 0 & 0 & 0 & \mu_2 & \theta_5 \end{bmatrix},$$

$$A_{10} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_3 & 0 \\ 0 & 0 & 0 & 0 & \mu_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_{0} = \begin{bmatrix} \lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix},$$

$$A_1 = \begin{bmatrix} \theta_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mu_2 & \theta_7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \theta_3 & 0 & \mu_2 & 0 & 0 \\ \mu_1 & 0 & 0 & \theta_6 & 0 & 0 & 0 \\ 0 & \mu_1 & 0 & \mu_2 & \theta_8 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu_1 & \theta_2 & 0 \\ 0 & 0 & \mu_1 & 0 & 0 & \mu_2 & \theta_6 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & \mu_3 & 0 & 0 \\ 0 & 0 & \mu_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mu_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

where $\theta_1 = -\lambda$, $\theta_2 = -(\lambda + \mu_1)$, $\theta_3 = -(\lambda + \mu_2)$, $\theta_4 = -(\lambda + \mu_3)$, $\theta_5 = -(\lambda + \mu_1 + \mu_2)$, $\theta_6 = -(\lambda + \mu_1 + \mu_3)$, $\theta_7 = -(\lambda + \mu_2 + \mu_3)$ and $\theta_8 = -(\lambda + \mu_1 + \mu_2 + \mu_3)$.

APPENDIX I

WARM-UP PERIODS FOR THE SIMULATION RUNS

$K_1 = K_2 = K_3 = 3$, $\rho_1 = \rho_2 = 0.8$, $\rho_3 = 0.65$.

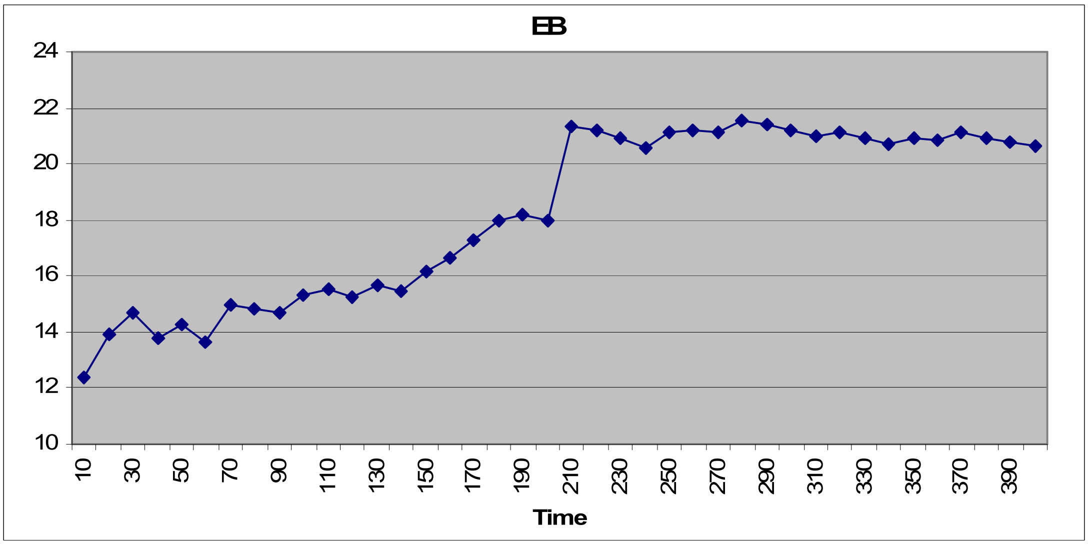

Figure I.1 (a) Expected Backorder Versus Simulation Time.

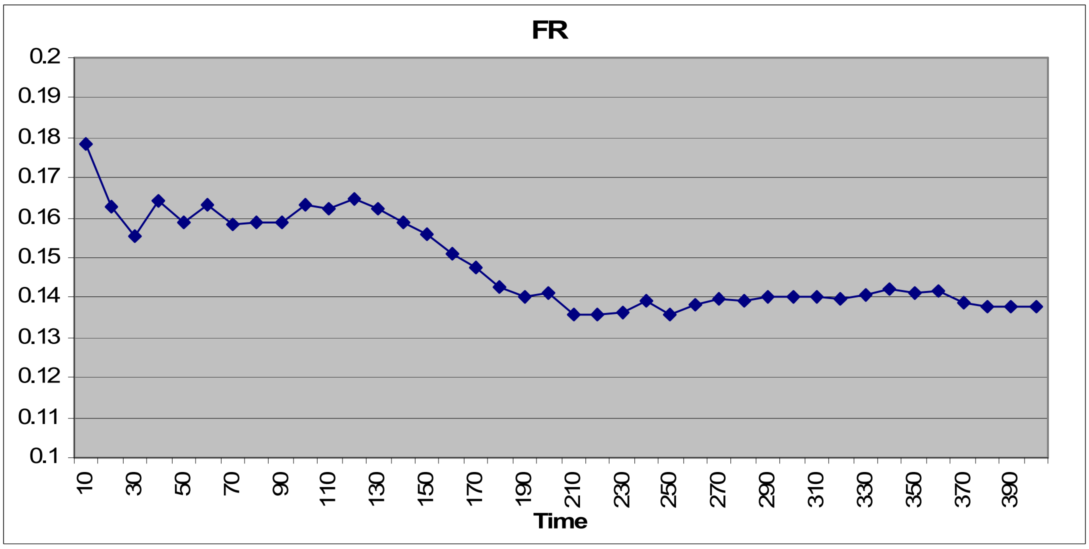

Figure I.1 (b) Fill Rate Versus Simulation Time.

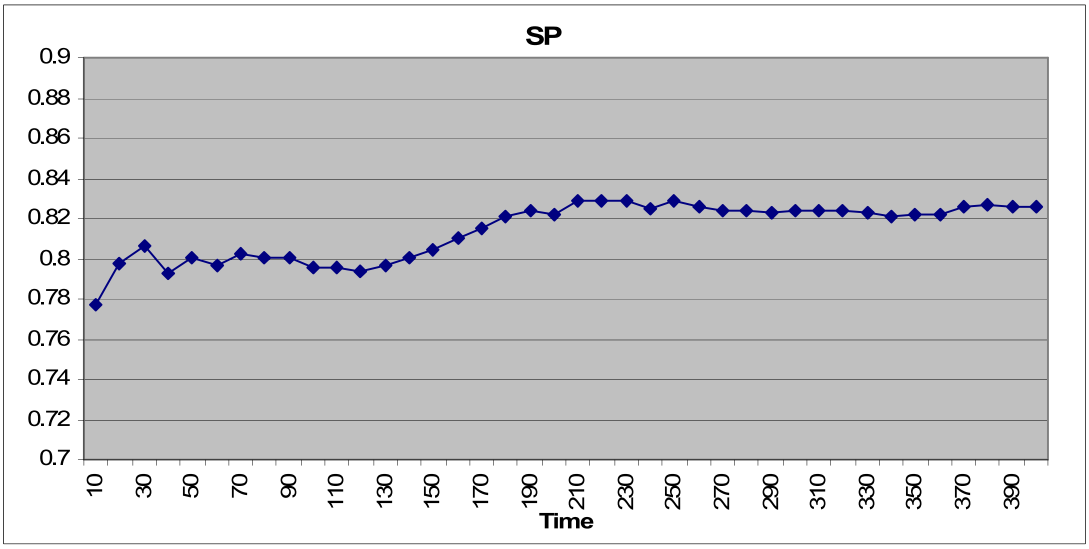

Figure I.1 (c) Stockout Probability Versus Simulation Time.

APPENDIX J

SIMAN CODE FOR THE SIMULATION

Code for simulation model for SKCS:

```
BEGIN,       Yes,No;
PROJECT,                                                          "Thesis","Engin
Topan",02/10/20003,Yes,Yes,Yes,Yes,Yes,Yes,Yes,Yes;
ATTRIBUTES:   TimeIn;
QUEUES:      1,Queue of Process 1,FirstInFirstOut:
             2,Queue of Process 2,FirstInFirstOut:
             3,Queue of Process 3,FirstInFirstOut:
             4,Output Queue of Process 1,FirstInFirstOut:
             5,Output Queue of Process 2,FirstInFirstOut:
             6,Output Queue of Process 3,FirstInFirstOut:
             7,Demand of Region 1,FirstInFirstOut:
             8,Demand of Region 2,FirstInFirstOut:
             9,Output Queue of Subassembly in Process 1,FirstInFirstOut;
RESOURCES:    1,Machine 1,Capacity(1),,Stationary,COST(0.0,0.0,0.0):
             2,Machine 2,Capacity(1),,Stationary,COST(0.0,0.0,0.0):
             3,Machine 3,Capacity(1),,Stationary,COST(0.0,0.0,0.0);
COUNTERS:     Number of Backorders,,Replicate:
             Number of demand arrivals,,Replicate;
TALLIES:      Flowtime;
DSTATS:       1,NR(Machine 1),Machine 1 Utilization:
             2,NR(Machine 2),Machine 2 Utilization:
             3,NR(Machine 3),Machine 3 Utilization:
             4,NC(Number of Backorders),Backorders:
             5,NQ(Demand of Region 2),A2,"eb.dat":
             6,(NQ(Output Queue of Process 3)>0),fillrate,"fill.dat":
             7,(NQ(Demand of Region 2)>0),stockout,"stock.dat":
```

```
            8,NQ(Output Queue of Process 1),FP11:
            9,NQ(Output Queue of Process 2),FP12:
            10,NQ(Output Queue of Process 3),FP2;
OUTPUTS:    1,DAVG(5):
            2,DAVG(6),,fill:
            3,DAVG(7),,stock:
            4,DAVG(8),,FP_11:
            5,DAVG(9),,FP_12:
            6,DAVG(10),,FP_2:
            7,TAVG(1),,Time Delay;
REPLICATE,  15,0.0,1000000,Yes,Yes,200000,,,24.0,Hours;


3$          CREATE,     1:EXPO(1):MARK(TimeIn):NEXT(32$);
32$         COUNT:      Number of demand arrivals,1;
30$         BRANCH,     1:
                        If,NQ(Output Queue of Process 3)>0,create4,Yes:
                        Else,31$,Yes;
create4     QUEUE,      Demand of Region 2;
18$         SCAN:       (NQ(Output Queue of Process 3)>0) .AND. (NQ(Demand of
Region 2)>0);
26$         SIGNAL:     2;
14$         DUPLICATE:  1,create3:NEXT(21$);
21$         TALLY:      Flowtime,INT(TimeIn),1;
20$         DISPOSE:    No;
create3     QUEUE,      Demand of Region 1;
15$         SCAN:
                        (NQ(Output Queue of Process 2)>0 ).AND.(NQ(Output Queue of
Process 1)>0 ).AND.(NQ(Demand of Region 1)>0);
22$         SIGNAL:     1;
16$         DISPOSE:    No;
31$         COUNT:      Number of Backorders,1:NEXT(create4);
27$         CREATE,     3,0::NEXT(7$);
7$          QUEUE,      Output Queue of Process 2;
23$         WAIT:       1,1;
```

```
28$         DUPLICATE:    1,create2:NEXT(17$);
17$         COMBINE:      2,Product;
9$          QUEUE,        Queue of Process 3;
10$         SEIZE,        1,Other:
                          Machine 3,1:NEXT(11$);
11$         DELAY:        EXPO(0.5),,Other:NEXT(12$);
12$         RELEASE:      Machine 3,1;
13$         QUEUE,        Output Queue of Process 3;
25$         WAIT:         2,1;
19$         DISPOSE:      No;
create2     QUEUE,        Queue of Process 2;
4$          SEIZE,        1,Other:
                          Machine 2,1:NEXT(5$);
5$          DELAY:        EXPO(0.5),,Other:NEXT(6$);
6$          RELEASE:      Machine 2,1:NEXT(7$);
33$         CREATE,       3::NEXT(8$);
8$          QUEUE,        Output Queue of Process 1;
24$         WAIT:         1,1;
29$         DUPLICATE:    1,create1:NEXT(17$);
create1     QUEUE,        Queue of Process 1;
0$          SEIZE,        1,Other:
                          Machine 1,1:NEXT(1$);
1$          DELAY:        EXPO(0.5),,Other:NEXT(2$);
2$          RELEASE:      Machine 1,1:NEXT(8$);
34$         CREATE,       3::NEXT(13$);
```

Code for simulation model for IKCS:

```
BEGIN,       Yes,No;
PROJECT,                                                            "Thesis","Engin
Topan",02/10/20003,Yes,Yes,Yes,Yes,Yes,Yes,Yes,Yes;
ATTRIBUTES:  TimeIn;
QUEUES:      1,Queue of Process 1,FirstInFirstOut:
```

```
            2,Queue of Process 2,FirstInFirstOut:
            3,Queue of Process 3,FirstInFirstOut:
            4,Output Queue of Process 1,FirstInFirstOut:
            5,Output Queue of Process 2,FirstInFirstOut:
            6,Output Queue of Process 3,FirstInFirstOut:
            7,Demand of Region 11,FirstInFirstOut:
            8,Demand of Region 12,FirstInFirstOut:
            9,Demand of Region 2,FirstInFirstOut:
            10,Output Queue of Subassembly in Process 1,FirstInFirstOut:
            11,Output Queue of Subassembly in Process 2,FirstInFirstOut;
RESOURCES:  1,Machine 1,Capacity(1),,Stationary,COST(0.0,0.0,0.0):
            2,Machine 2,Capacity(1),,Stationary,COST(0.0,0.0,0.0):
            3,Machine 3,Capacity(1),,Stationary,COST(0.0,0.0,0.0);

COUNTERS:   Number of Backorders,,Replicate:
            Number of demand arrivals,,Replicate;
TALLIES:    Flowtime;
DSTATS:     1,NR(Machine 1),Machine 1 Utilization:
            2,NR(Machine 2),Machine 2 Utilization:
            3,NR(Machine 3),Machine 3 Utilization:
            4,NC(Number of Backorders),Backorders:
            5,NQ(Demand of Region 2),A3,"eb.dat":
            6,(NQ(Output Queue of Process 3)>0),fillrate,"fill.dat":
            7,(NQ(Demand of Region 2)>0),stockout,"stock.dat":
            8,NQ(Output Queue of Process 1),FP11:
            9,NQ(Output Queue of Process 2),FP12:
            10,NQ(Output Queue of Process 3),FP2:
            11,NQ(Output Queue of Subassembly in Process 1),FP112:
            12,NQ(Output Queue of Subassembly in Process 2),FP122;
OUTPUTS:    1,DAVG(5):
            2,DAVG(6),,fill:
            3,DAVG(7),,stock:
            4,DAVG(8),,FP_11:
            5,DAVG(9),,FP_12:
            6,DAVG(10),,FP_2:
```

```
            7,TAVG(1),,Time Delay:
            8,DAVG(11),,FP_112:
            9,DAVG(12),,FP_122;
REPLICATE,    15,0.0,1000000,Yes,Yes,200000,,,24.0,Hours;


3$           CREATE,       1:EXPO(1):MARK(TimeIn):NEXT(34$);

34$          COUNT:        Number of demand arrivals,1;
32$          BRANCH,       1:
                           If,NQ(Output Queue of Process 3)>0,create5,Yes:
                           Else,33$,Yes;
create5      QUEUE,        Demand of Region 2;
18$          SCAN:         (NQ(Output Queue of Process 3)>0) .AND. (NQ(Demand of
Region 2)>0);
26$          SIGNAL:       4;
14$          DUPLICATE:    1,create3:NEXT(41$);
41$          DUPLICATE:    1,create4:NEXT(21$);
21$          TALLY:        Flowtime,INT(TimeIn),1;
20$          DISPOSE:      No;
create4      QUEUE,        Demand of Region 12;
36$          SCAN:         (NQ(Output Queue of Process 2)>0 ).AND.(NQ(Demand of
Region 12)>0);
38$          SIGNAL:       2;
37$          DISPOSE:      No;
create3      QUEUE,        Demand of Region 11;
15$          SCAN:         (NQ(Output Queue of Process 1)>0 ).AND.(NQ(Demand of
Region 11)>0);
22$          SIGNAL:       1;
16$          DISPOSE:      No;
33$          COUNT:        Number of Backorders,1:NEXT(create5);
27$          CREATE,       3,0::NEXT(7$);
7$           QUEUE,        Output Queue of Process 2;
23$          WAIT:         2,1;
28$          DUPLICATE:    1,create2:NEXT(39$);
```

```
39$        QUEUE,       Output Queue of Subassembly in Process 2;
40$        WAIT:        3,1;
17$        COMBINE:     2,Product;
9$         QUEUE,       Queue of Process 3;
10$        SEIZE,       1,Other:
                        Machine 3,1:NEXT(11$);
11$        DELAY:       EXPO(0.5),,Other:NEXT(12$);
12$        RELEASE:     Machine 3,1;
13$        QUEUE,       Output Queue of Process 3;
25$        WAIT:        4,1;
19$        DISPOSE:     No;
create2    QUEUE,       Queue of Process 2;
4$         SEIZE,       1,Other:
                        Machine 2,1:NEXT(5$);
5$         DELAY:       EXPO(0.5),,Other:NEXT(6$);
6$         RELEASE:     Machine 2,1:NEXT(7$);
42$        CREATE,      3::NEXT(8$);
8$         QUEUE,       Output Queue of Process 1;
24$        WAIT:        1,1;
35$        DUPLICATE:   1,create1:NEXT(29$);
29$        QUEUE,       Output Queue of Subassembly in Process 1;
30$        SCAN:        (NQ(Output Queue of Subassembly in Process 1)>0
).AND.(NQ(Output Queue of Subassembly in Process 2)>0);
31$        SIGNAL:      3:NEXT(17$);
create1    QUEUE,       Queue of Process 1;
0$         SEIZE,       1,Other:
                        Machine 1,1:NEXT(1$);
1$         DELAY:       EXPO(0.5),,Other:NEXT(2$);
2$         RELEASE:     Machine 1,1:NEXT(8$);
43$        CREATE,      3::NEXT(13$);
```

APPENDIX K

NUMERICAL RESULTS FOR TWO-COMPONENT SYSTEMS

Because there is a huge number of kanban combinations selected for the test runs, here we only present the results of a part of the test bed. The system with traffic intensities $\rho_i = 0.8$ and $K_i = 3$, i = 1, 2, 3, explodes. The algorithm for the models considered does not converge in this case.

Table K.1 (a) Numerical results for SKCS with $K_i = 3$, $i = 1, 2, 3$.

SKCS			Expected Backorders							Expected Number of Assemblies in Stock						
K1	K2	K3	Sim.	Exact	Abs. Rel. Error (%)	Agg.	Abs. Rel. Error (%)	Com.	Abs. Rel. Error (%)	Sim.	Exact	Abs. Rel. Error (%)	Agg.	Abs. Rel. Error (%)	Com.	Abs. Rel. Error (%)
3	3	3														
ρ1	ρ2	ρ3														
0.5	0.5	0.5	0.206	0.206	0.20	0.215	3.96	0.220	6.59	1.993	1.994	0.07	1.947	2.31	1.945	2.38
0.5	0.5	0.65	0.652	0.651	0.08	0.685	5.13	0.717	9.95	1.545	1.546	0.07	1.492	3.47	1.488	3.71
0.5	0.5	0.8	2.473	2.469	0.17	2.560	3.51	2.661	7.58	0.953	0.954	0.10	0.909	4.67	0.905	5.07
0.5	0.65	0.5	0.439	0.436	0.77	0.396	9.90	0.382	12.94	1.801	1.803	0.12	1.762	2.16	1.766	1.90
0.5	0.65	0.65	1.010	1.009	0.11	1.004	0.55	1.052	4.19	1.382	1.382	0.00	1.322	4.30	1.320	4.46
0.5	0.65	0.8	3.414	3.447	0.98	3.520	3.09	3.740	9.56	0.812	0.809	0.44	0.756	6.95	0.752	7.47
0.5	0.8	0.5	1.770	1.773	0.12	1.114	37.07	0.959	45.83	1.326	1.325	0.03	1.395	5.25	1.425	7.48
0.5	0.8	0.65	3.041	3.008	1.07	2.214	27.18	2.215	27.17	0.962	0.963	0.08	0.989	2.86	1.000	3.95
0.5	0.8	0.8	9.708	9.924	2.22	7.910	18.52	8.445	13.01	0.439	0.436	0.76	0.452	2.85	0.452	3.02
0.65	0.5	0.5	0.440	0.436	0.93	0.395	10.28	0.372	15.46	1.800	1.803	0.16	1.766	1.91	1.776	1.34
0.65	0.5	0.65	1.005	1.009	0.33	1.001	0.44	1.038	3.22	1.381	1.382	0.04	1.326	4.01	1.328	3.88
0.65	0.5	0.8	3.441	3.447	0.19	3.507	1.90	3.712	7.88	0.809	0.809	0.06	0.759	6.24	0.757	6.48
0.65	0.65	0.5	0.771	0.770	0.17	0.638	17.30	0.629	18.41	1.608	1.609	0.09	1.577	1.94	1.578	1.86
0.65	0.65	0.65	1.518	1.513	0.33	1.441	5.06	1.539	1.43	1.214	1.216	0.16	1.152	5.08	1.146	5.59
0.65	0.65	0.8	4.899	4.923	0.48	5.026	2.59	5.465	11.55	0.663	0.663	0.00	0.601	9.34	0.595	10.34
0.65	0.8	0.5	2.694	2.704	0.37	1.582	41.26	1.507	44.06	1.120	1.121	0.10	1.206	7.75	1.212	8.28
0.65	0.8	0.65	4.479	4.461	0.40	3.156	29.53	3.319	25.88	0.787	0.789	0.16	0.814	3.44	0.810	2.88
0.65	0.8	0.8	17.349	17.348	0.01	13.671	21.20	15.002	13.53	0.284	0.285	0.41	0.291	2.66	0.287	1.14
0.8	0.5	0.5	1.781	1.773	0.47	1.106	37.89	0.893	49.84	1.323	1.325	0.17	1.408	6.40	1.460	10.37
0.8	0.5	0.65	3.003	3.008	0.18	2.187	27.16	2.125	29.24	0.964	0.963	0.14	1.001	3.84	1.027	6.47
0.8	0.5	0.8	9.841	9.924	0.85	7.742	21.32	8.186	16.82	0.436	0.436	0.02	0.461	5.91	0.469	7.57
0.8	0.65	0.5	2.717	2.704	0.49	1.564	42.43	1.435	47.18	1.120	1.121	0.10	1.220	8.93	1.241	10.87
0.8	0.65	0.65	4.464	4.461	0.07	3.101	30.54	3.202	28.26	0.789	0.789	0.03	0.827	4.85	0.832	5.51
0.8	0.65	0.8	17.328	17.348	0.11	13.149	24.12	14.340	17.24	0.285	0.285	0.11	0.302	6.04	0.301	5.63
0.8	0.8	0.5	8.934	9.046	1.25	3.413	61.80	3.346	62.55	0.608	0.603	0.85	0.850	39.86	0.853	40.29
0.8	0.8	0.65	16.867	16.819	0.28	7.299	56.72	7.765	53.97	0.339	0.338	0.29	0.488	43.79	0.483	42.47
0.8	0.8	0.8	*	0.000	*	0.000	*	0.000	*	0.000	0.000	*	0.000	*	0.000	*

SKCS			Fill Rate							Stockout Probability						
K1	K2	K3	Sim.	Exact	Abs. Error	Agg.	Abs. Error	Com.	Abs. Error	Sim.	Exact	Abs. Error	Agg.	Abs. Error	Com.	Abs. Error
3	3	3														
ρ1	ρ2	ρ3														
0.5	0.5	0.5	0.835	0.835	0.000	0.825	0.010	0.824	0.011	0.092	0.091	0.000	0.097	0.005	0.098	0.006
0.5	0.5	0.65	0.687	0.687	0.000	0.671	0.016	0.669	0.018	0.212	0.212	0.000	0.223	0.011	0.227	0.015
0.5	0.5	0.8	0.448	0.449	0.000	0.433	0.016	0.430	0.018	0.452	0.451	0.000	0.465	0.014	0.470	0.018
0.5	0.65	0.5	0.767	0.768	0.001	0.762	0.005	0.765	0.002	0.150	0.149	0.001	0.148	0.002	0.145	0.005
0.5	0.65	0.65	0.622	0.622	0.000	0.606	0.016	0.604	0.018	0.275	0.275	0.000	0.284	0.009	0.288	0.013
0.5	0.65	0.8	0.386	0.384	0.002	0.366	0.020	0.362	0.023	0.522	0.523	0.002	0.539	0.018	0.545	0.024
0.5	0.8	0.5	0.580	0.580	0.000	0.623	0.042	0.639	0.059	0.333	0.333	0.000	0.279	0.054	0.260	0.073
0.5	0.8	0.65	0.443	0.443	0.000	0.466	0.023	0.471	0.028	0.467	0.466	0.000	0.431	0.036	0.426	0.041
0.5	0.8	0.8	0.212	0.211	0.001	0.224	0.012	0.223	0.011	0.729	0.731	0.002	0.709	0.020	0.711	0.018
0.65	0.5	0.5	0.767	0.768	0.001	0.763	0.003	0.769	0.002	0.150	0.149	0.001	0.147	0.003	0.142	0.008
0.65	0.5	0.65	0.622	0.622	0.000	0.607	0.015	0.607	0.014	0.275	0.275	0.000	0.284	0.009	0.285	0.010
0.65	0.5	0.8	0.384	0.384	0.000	0.367	0.017	0.365	0.020	0.523	0.523	0.000	0.538	0.015	0.543	0.019
0.65	0.65	0.5	0.695	0.696	0.001	0.696	0.001	0.697	0.002	0.216	0.215	0.000	0.206	0.010	0.204	0.011
0.65	0.65	0.65	0.553	0.554	0.001	0.538	0.016	0.533	0.020	0.345	0.344	0.001	0.352	0.008	0.359	0.014
0.65	0.65	0.8	0.318	0.318	0.000	0.295	0.023	0.290	0.027	0.600	0.600	0.000	0.621	0.021	0.629	0.029
0.65	0.8	0.5	0.497	0.497	0.000	0.549	0.052	0.553	0.056	0.419	0.419	0.000	0.350	0.069	0.344	0.075
0.65	0.8	0.65	0.366	0.367	0.001	0.390	0.024	0.387	0.021	0.552	0.551	0.001	0.513	0.040	0.518	0.035
0.65	0.8	0.8	0.138	0.139	0.001	0.147	0.008	0.143	0.005	0.821	0.820	0.001	0.805	0.016	0.811	0.010
0.8	0.5	0.5	0.579	0.580	0.001	0.626	0.047	0.655	0.076	0.334	0.333	0.001	0.276	0.058	0.246	0.089
0.8	0.5	0.65	0.444	0.443	0.001	0.470	0.026	0.484	0.040	0.466	0.466	0.001	0.427	0.039	0.413	0.052
0.8	0.5	0.8	0.211	0.211	0.000	0.228	0.017	0.231	0.021	0.731	0.731	0.000	0.704	0.027	0.702	0.029
0.8	0.65	0.5	0.497	0.497	0.000	0.554	0.057	0.566	0.069	0.419	0.419	0.000	0.346	0.074	0.332	0.088
0.8	0.65	0.65	0.367	0.367	0.000	0.396	0.029	0.398	0.031	0.551	0.551	0.000	0.507	0.044	0.506	0.045
0.8	0.65	0.8	0.139	0.139	0.000	0.152	0.013	0.150	0.011	0.820	0.820	0.000	0.799	0.022	0.802	0.018
0.8	0.8	0.5	0.276	0.274	0.002	0.398	0.122	0.400	0.124	0.666	0.669	0.003	0.512	0.154	0.508	0.158
0.8	0.8	0.65	0.161	0.161	0.000	0.240	0.079	0.237	0.076	0.797	0.798	0.001	0.690	0.107	0.695	0.102
0.8	0.8	0.8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	0.000	1.000	0.000	1.000	0.000

Table K.1 (b) Numerical results for SKCS with $K_i = 7$, $i = 1, 2, 3$.

SKCS			Expected Backorders							Expected Number of Assemblies in Stock						
K1	K2	K3	Sim.	Exact	Abs. Rel. Error (%)	Agg.	Abs. Rel. Error (%)	Com.	Abs. Rel. Error (%)	Sim.	Exact	Abs. Rel. Error (%)	Agg.	Abs. Rel. Error (%)	Com.	Abs. Rel. Error (%)
7	7	7														
ρ1	ρ2	ρ3														
0.5	0.5	0.5	0.008	0.008	0.96	0.008	2.53	0.008	2.56	5.999	5.999	0.01	5.993	0.10	5.993	0.10
0.5	0.5	0.65	0.092	0.092	0.19	0.093	1.11	0.093	1.35	5.228	5.229	0.01	5.220	0.16	5.219	0.17
0.5	0.5	0.8	0.842	0.840	0.17	0.844	0.23	0.845	0.34	3.838	3.836	0.03	3.827	0.27	3.827	0.28
0.5	0.65	0.5	0.016	0.017	2.50	0.017	3.44	0.014	13.71	5.937	5.936	0.02	5.918	0.31	5.922	0.25
0.5	0.65	0.65	0.104	0.105	0.25	0.107	3.07	0.107	3.02	5.179	5.181	0.03	5.151	0.55	5.151	0.55
0.5	0.65	0.8	0.861	0.867	0.72	0.882	2.47	0.891	3.56	3.807	3.805	0.06	3.769	1.01	3.767	1.06
0.5	0.8	0.5	0.243	0.240	0.88	0.216	11.10	0.119	51.04	5.418	5.426	0.15	5.417	0.02	5.486	1.25
0.5	0.8	0.65	0.386	0.384	0.62	0.344	11.07	0.273	29.44	4.740	4.742	0.04	4.708	0.68	4.746	0.12
0.5	0.8	0.8	1.360	1.344	1.11	1.327	2.37	1.350	0.73	3.465	3.468	0.09	3.394	2.05	3.398	1.92
0.65	0.5	0.5	0.016	0.017	1.99	0.017	3.00	0.014	14.32	5.935	5.936	0.00	5.919	0.28	5.922	0.23
0.65	0.5	0.65	0.105	0.105	0.42	0.107	2.38	0.107	2.29	5.180	5.181	0.03	5.151	0.56	5.151	0.55
0.65	0.5	0.8	0.863	0.867	0.45	0.882	2.20	0.891	3.27	3.807	3.805	0.05	3.769	1.00	3.767	1.04
0.65	0.65	0.5	0.026	0.026	2.97	0.026	1.69	0.022	17.42	5.874	5.876	0.03	5.843	0.53	5.847	0.46
0.65	0.65	0.65	0.118	0.118	0.50	0.123	4.31	0.125	5.21	5.132	5.135	0.06	5.080	1.00	5.080	1.02
0.65	0.65	0.8	0.892	0.894	0.19	0.923	3.45	0.943	5.69	3.776	3.774	0.05	3.709	1.77	3.705	1.89
0.65	0.8	0.5	0.265	0.260	1.83	0.231	12.62	0.148	44.19	5.373	5.372	0.02	5.336	0.69	5.387	0.26
0.65	0.8	0.65	0.410	0.407	0.78	0.369	10.04	0.311	24.21	4.700	4.699	0.01	4.632	1.45	4.657	0.91
0.65	0.8	0.8	1.378	1.382	0.28	1.388	0.75	1.436	4.23	3.443	3.439	0.11	3.328	3.32	3.324	3.46
0.8	0.5	0.5	0.238	0.240	1.03	0.216	9.30	0.116	51.19	5.429	5.426	0.05	5.418	0.19	5.491	1.15
0.8	0.5	0.65	0.388	0.384	1.07	0.344	11.46	0.270	30.32	4.739	4.742	0.06	4.709	0.62	4.750	0.23
0.8	0.5	0.8	1.330	1.344	1.11	1.327	0.20	1.347	1.29	3.475	3.468	0.20	3.395	2.30	3.401	2.13
0.8	0.65	0.5	0.253	0.260	2.85	0.232	8.04	0.145	42.53	5.379	5.372	0.13	5.341	0.70	5.396	0.32
0.8	0.65	0.65	0.401	0.407	1.38	0.369	8.02	0.308	23.31	4.702	4.699	0.06	4.637	1.38	4.665	0.79
0.8	0.65	0.8	1.356	1.382	1.91	1.387	2.27	1.431	5.52	3.443	3.439	0.11	3.333	3.19	3.330	3.27
0.8	0.8	0.5	0.635	0.628	1.20	0.512	19.39	0.411	35.27	4.874	4.877	0.06	4.828	0.95	4.869	0.11
0.8	0.8	0.65	0.822	0.831	1.10	0.702	14.59	0.628	23.57	4.276	4.270	0.13	4.178	2.28	4.199	1.80
0.8	0.8	0.8	2.045	2.042	0.15	2.022	1.15	2.154	5.32	3.105	3.106	0.05	2.940	5.30	2.925	5.79

SKCS			Fill Rate							Stockout Probability						
K1	K2	K3	Sim.	Exact	Abs. Error	Agg.	Abs. Error	Com.	Abs. Error	Sim.	Exact	Abs. Error	Agg.	Abs. Error	Com.	Abs. Error
7	7	7														
ρ1	ρ2	ρ3														
0.5	0.5	0.5	0.992	0.992	0.000	0.992	0.000	0.992	0.000	0.004	0.004	0.000	0.004	0.000	0.004	0.000
0.5	0.5	0.65	0.951	0.951	0.000	0.950	0.000	0.950	0.000	0.032	0.032	0.000	0.032	0.000	0.032	0.000
0.5	0.5	0.8	0.790	0.790	0.000	0.789	0.001	0.789	0.001	0.168	0.168	0.000	0.169	0.001	0.169	0.001
0.5	0.65	0.5	0.988	0.988	0.000	0.988	0.000	0.988	0.000	0.007	0.007	0.000	0.007	0.000	0.006	0.000
0.5	0.65	0.65	0.946	0.946	0.000	0.945	0.002	0.945	0.001	0.036	0.035	0.000	0.037	0.001	0.036	0.001
0.5	0.65	0.8	0.786	0.786	0.000	0.782	0.004	0.781	0.004	0.172	0.172	0.000	0.175	0.003	0.175	0.004
0.5	0.8	0.5	0.935	0.936	0.001	0.939	0.004	0.952	0.017	0.050	0.050	0.001	0.047	0.004	0.033	0.017
0.5	0.8	0.65	0.891	0.891	0.000	0.892	0.002	0.901	0.010	0.084	0.084	0.000	0.081	0.003	0.072	0.011
0.5	0.8	0.8	0.729	0.730	0.000	0.724	0.006	0.724	0.005	0.226	0.225	0.001	0.229	0.004	0.229	0.003
0.65	0.5	0.5	0.988	0.988	0.000	0.988	0.000	0.988	0.000	0.007	0.007	0.000	0.007	0.000	0.006	0.000
0.65	0.5	0.65	0.946	0.946	0.000	0.945	0.001	0.945	0.001	0.036	0.035	0.000	0.037	0.001	0.036	0.001
0.65	0.5	0.8	0.786	0.786	0.000	0.782	0.004	0.782	0.004	0.172	0.172	0.000	0.175	0.003	0.175	0.003
0.65	0.65	0.5	0.984	0.984	0.000	0.983	0.000	0.984	0.000	0.010	0.010	0.000	0.010	0.000	0.009	0.001
0.65	0.65	0.65	0.942	0.942	0.000	0.939	0.003	0.939	0.003	0.039	0.039	0.000	0.041	0.002	0.041	0.002
0.65	0.65	0.8	0.781	0.781	0.000	0.774	0.007	0.773	0.008	0.176	0.176	0.000	0.181	0.005	0.183	0.007
0.65	0.8	0.5	0.931	0.931	0.000	0.933	0.002	0.943	0.012	0.054	0.053	0.000	0.051	0.003	0.041	0.013
0.65	0.8	0.65	0.886	0.886	0.000	0.885	0.000	0.891	0.005	0.088	0.088	0.000	0.086	0.001	0.081	0.007
0.65	0.8	0.8	0.726	0.725	0.001	0.715	0.011	0.713	0.012	0.229	0.229	0.001	0.237	0.009	0.239	0.010
0.8	0.5	0.5	0.936	0.936	0.000	0.939	0.003	0.953	0.017	0.049	0.050	0.000	0.047	0.003	0.033	0.017
0.8	0.5	0.65	0.890	0.891	0.001	0.892	0.003	0.901	0.011	0.084	0.084	0.001	0.081	0.003	0.072	0.012
0.8	0.5	0.8	0.731	0.730	0.001	0.724	0.007	0.725	0.006	0.224	0.225	0.001	0.229	0.005	0.228	0.004
0.8	0.65	0.5	0.932	0.931	0.001	0.934	0.001	0.944	0.012	0.052	0.053	0.001	0.051	0.002	0.040	0.012
0.8	0.65	0.65	0.886	0.886	0.000	0.886	0.001	0.892	0.005	0.087	0.088	0.000	0.086	0.001	0.080	0.007
0.8	0.65	0.8	0.726	0.725	0.001	0.715	0.011	0.714	0.012	0.228	0.229	0.001	0.237	0.009	0.238	0.010
0.8	0.8	0.5	0.871	0.871	0.001	0.878	0.007	0.886	0.015	0.106	0.106	0.001	0.098	0.008	0.089	0.017
0.8	0.8	0.65	0.825	0.825	0.001	0.826	0.000	0.831	0.005	0.142	0.143	0.001	0.139	0.003	0.133	0.009
0.8	0.8	0.8	0.666	0.666	0.000	0.649	0.017	0.645	0.021	0.287	0.287	0.001	0.300	0.013	0.305	0.018

Table K.1 (c) Numerical results for SKCS with $K_i = 15$, $i = 1, 2, 3$.

SKCS			Expected Backorders					Expected Number of Assemblies in Stock				
K1	K2	K3	Sim.	App. Agg.	Abs. Rel. Error (%)	App. Com.	Abs. Rel. Error (%)	Sim.	App. Agg.	Abs. Rel. Error (%)	App. Com.	Abs. Rel. Error (%)
15	15	15										
ρ1	ρ2	ρ3										
0.5	0.5	0.5	0.000	0.000	19.22	0.000	19.22	13.999	14.000	0.01	14.000	0.01
0.5	0.5	0.65	0.003	0.003	5.19	0.003	5.19	13.147	13.146	0.01	13.146	0.01
0.5	0.5	0.8	0.136	0.141	3.71	0.141	3.71	11.149	11.141	0.07	11.141	0.07
0.5	0.65	0.5	0.000	0.000	9.70	0.000	17.70	13.997	13.997	0.00	13.997	0.00
0.5	0.65	0.65	0.003	0.003	3.64	0.003	3.66	13.141	13.143	0.01	13.143	0.01
0.5	0.65	0.8	0.139	0.141	1.55	0.141	1.56	11.142	11.138	0.04	11.138	0.04
0.5	0.8	0.5	0.007	0.007	4.27	0.005	32.53	13.870	13.866	0.03	13.870	0.00
0.5	0.8	0.65	0.012	0.012	2.43	0.008	29.59	13.035	13.014	0.16	13.020	0.11
0.5	0.8	0.8	0.158	0.161	1.71	0.160	1.61	11.057	11.020	0.34	11.020	0.33
0.65	0.5	0.5	0.000	0.000	44.28	0.000	31.49	13.996	13.997	0.01	13.997	0.01
0.65	0.5	0.65	0.003	0.003	0.67	0.003	0.65	13.144	13.143	0.01	13.143	0.01
0.65	0.5	0.8	0.143	0.141	1.42	0.141	1.41	11.126	11.138	0.11	11.138	0.11
0.65	0.65	0.5	0.000	0.000	6.11	0.000	17.31	13.996	13.994	0.01	13.994	0.01
0.65	0.65	0.65	0.003	0.003	11.18	0.003	11.18	13.143	13.140	0.02	13.140	0.02
0.65	0.65	0.8	0.139	0.141	1.35	0.141	1.38	11.138	11.135	0.02	11.135	0.02
0.65	0.8	0.5	0.007	0.007	0.40	0.005	22.98	13.874	13.863	0.08	13.866	0.06
0.65	0.8	0.65	0.011	0.012	9.71	0.008	22.58	13.029	13.011	0.13	13.017	0.10
0.65	0.8	0.8	0.157	0.161	2.29	0.161	2.36	11.073	11.017	0.51	11.017	0.51
0.8	0.5	0.5	0.007	0.007	6.43	0.005	34.32	13.867	13.866	0.01	13.870	0.02
0.8	0.5	0.65	0.012	0.012	1.38	0.008	30.38	13.033	13.014	0.14	13.020	0.10
0.8	0.5	0.8	0.156	0.161	2.66	0.160	2.56	11.063	11.020	0.39	11.020	0.39
0.8	0.65	0.5	0.006	0.007	7.03	0.005	17.50	13.877	13.863	0.10	13.866	0.08
0.8	0.65	0.65	0.012	0.012	4.77	0.008	32.87	13.028	13.012	0.13	13.017	0.09
0.8	0.65	0.8	0.158	0.161	1.97	0.161	2.04	11.059	11.017	0.38	11.017	0.38
0.8	0.8	0.5	0.013	0.014	10.26	0.013	1.54	13.753	13.729	0.18	13.730	0.16
0.8	0.8	0.65	0.021	0.022	4.98	0.016	21.36	12.926	12.880	0.35	12.887	0.30
0.8	0.8	0.8	0.171	0.182	6.50	0.183	7.13	10.999	10.896	0.94	10.895	0.94

SKCS			Fill Rate					Stockout Probability				
K1	K2	K3	Sim.	App. Agg.	Abs. Error	App. Com.	Abs. Error	Sim.	App. Agg.	Abs. Error	App. Com.	Abs. Error
15	15	15										
ρ1	ρ2	ρ3										
0.5	0.5	0.5	1.000	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.5	0.65	0.998	0.998	0.000	0.998	0.000	0.001	0.001	0.000	0.001	0.000
0.5	0.5	0.8	0.966	0.965	0.001	0.965	0.001	0.027	0.028	0.001	0.028	0.001
0.5	0.65	0.5	1.000	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.65	0.65	0.998	0.998	0.000	0.998	0.000	0.001	0.001	0.000	0.001	0.000
0.5	0.65	0.8	0.965	0.965	0.000	0.965	0.000	0.028	0.028	0.000	0.028	0.000
0.5	0.8	0.5	0.998	0.998	0.000	0.999	0.000	0.001	0.001	0.000	0.001	0.000
0.5	0.8	0.65	0.996	0.996	0.000	0.997	0.000	0.003	0.003	0.000	0.002	0.000
0.5	0.8	0.8	0.962	0.961	0.001	0.961	0.001	0.031	0.031	0.001	0.031	0.001
0.65	0.5	0.5	1.000	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
0.65	0.5	0.65	0.998	0.998	0.000	0.998	0.000	0.001	0.001	0.000	0.001	0.000
0.65	0.5	0.8	0.964	0.965	0.001	0.965	0.001	0.029	0.028	0.001	0.028	0.001
0.65	0.65	0.5	1.000	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
0.65	0.65	0.65	0.998	0.998	0.000	0.998	0.000	0.001	0.001	0.000	0.001	0.000
0.65	0.65	0.8	0.965	0.965	0.000	0.965	0.000	0.028	0.028	0.000	0.028	0.000
0.65	0.8	0.5	0.998	0.998	0.000	0.999	0.000	0.001	0.001	0.000	0.001	0.000
0.65	0.8	0.65	0.996	0.996	0.000	0.997	0.000	0.003	0.003	0.000	0.002	0.000
0.65	0.8	0.8	0.962	0.961	0.001	0.961	0.001	0.031	0.031	0.001	0.031	0.001
0.8	0.5	0.5	0.998	0.998	0.000	0.999	0.000	0.001	0.001	0.000	0.001	0.000
0.8	0.5	0.65	0.996	0.996	0.000	0.997	0.000	0.003	0.003	0.000	0.002	0.000
0.8	0.5	0.8	0.962	0.961	0.001	0.961	0.001	0.030	0.031	0.001	0.031	0.001
0.8	0.65	0.5	0.998	0.998	0.000	0.999	0.000	0.001	0.001	0.000	0.001	0.000
0.8	0.65	0.65	0.996	0.996	0.000	0.997	0.000	0.003	0.003	0.000	0.002	0.000
0.8	0.65	0.8	0.962	0.961	0.001	0.961	0.001	0.031	0.031	0.001	0.031	0.001
0.8	0.8	0.5	0.997	0.997	0.000	0.997	0.000	0.003	0.003	0.000	0.003	0.000
0.8	0.8	0.65	0.994	0.994	0.000	0.994	0.000	0.005	0.005	0.000	0.004	0.001
0.8	0.8	0.8	0.960	0.957	0.002	0.957	0.003	0.033	0.035	0.002	0.035	0.002

Table K.2 (a) Numerical results for IKCS with $K_i = 3$, $i = 1, 2, 3$.

IKCS			Expected Backorders							Expected Number of Assemblies in Stock						
K1	K2	K3	Sim.	Exact	Abs. Rel. Error (%)	Agg.	Abs. Rel. Error (%)	Com.	Abs. Rel. Error (%)	Sim.	Exact	Abs. Rel. Error (%)	Agg.	Abs. Rel. Error (%)	Com.	Abs. Rel. Error (%)
3	3	3														
ρ_1	ρ_2	ρ_3														
0.5	0.5	0.5	0.197	0.197	0.13	0.209	5.71	0.214	8.15	2.002	2.003	0.02	1.957	2.26	1.956	2.32
0.5	0.5	0.65	0.635	0.640	0.66	0.674	6.06	0.703	10.64	1.554	1.552	0.07	1.501	3.38	1.498	3.61
0.5	0.5	0.8	2.440	2.440	0.00	2.526	3.52	2.620	7.36	0.959	0.959	0.01	0.917	4.41	0.913	4.79
0.5	0.65	0.5	0.397	0.400	0.79	0.375	5.50	0.359	9.51	1.826	1.825	0.07	1.787	2.16	1.792	1.85
0.5	0.65	0.65	0.957	0.963	0.64	0.964	0.78	1.005	5.00	1.400	1.398	0.16	1.345	3.93	1.344	4.04
0.5	0.65	0.8	3.313	3.337	0.72	3.388	2.26	3.585	8.20	0.823	0.822	0.20	0.776	5.74	0.772	6.19
0.5	0.8	0.5	1.596	1.609	0.85	1.042	34.66	0.875	45.18	1.367	1.366	0.09	1.439	5.32	1.473	7.75
0.5	0.8	0.65	2.764	2.793	1.07	2.066	25.24	2.040	26.20	0.997	0.993	0.46	1.031	3.34	1.043	4.62
0.5	0.8	0.8	9.107	9.176	0.76	7.187	21.08	7.615	16.38	0.462	0.459	0.71	0.488	5.55	0.490	5.92
0.65	0.5	0.5	0.401	0.400	0.30	0.375	6.45	0.351	12.62	1.823	1.825	0.09	1.789	1.88	1.799	1.30
0.65	0.5	0.65	0.955	0.963	0.80	0.964	0.87	0.994	4.04	1.399	1.398	0.08	1.347	3.71	1.349	3.55
0.65	0.5	0.8	3.351	3.337	0.42	3.384	0.99	3.568	6.48	0.823	0.822	0.12	0.777	5.49	0.776	5.68
0.65	0.65	0.5	0.603	0.606	0.54	0.570	5.44	0.555	8.00	1.678	1.677	0.08	1.632	2.74	1.634	2.60
0.65	0.65	0.65	1.303	1.306	0.22	1.311	0.58	1.388	6.52	1.266	1.266	0.02	1.203	4.97	1.198	5.39
0.65	0.65	0.8	4.406	4.403	0.06	4.536	2.96	4.895	11.09	0.700	0.703	0.35	0.647	7.63	0.640	8.54
0.65	0.8	0.5	1.896	1.897	0.06	1.343	29.17	1.238	34.72	1.261	1.259	0.13	1.303	3.36	1.314	4.17
0.65	0.8	0.65	3.339	3.355	0.48	2.649	20.65	2.736	18.04	0.892	0.894	0.25	0.905	1.45	0.903	1.20
0.65	0.8	0.8	12.297	12.231	0.54	10.175	17.26	11.033	10.28	0.368	0.367	0.31	0.373	1.34	0.369	0.15
0.8	0.5	0.5	1.612	1.609	0.16	1.045	35.15	0.824	48.86	1.364	1.366	0.12	1.444	5.89	1.500	9.96
0.8	0.5	0.65	2.786	2.793	0.26	2.063	25.95	1.981	28.89	0.995	0.993	0.20	1.035	4.08	1.063	6.83
0.8	0.5	0.8	9.174	9.176	0.03	7.150	22.06	7.510	18.14	0.457	0.459	0.40	0.492	7.53	0.500	9.34
0.8	0.65	0.5	1.926	1.897	1.51	1.344	30.23	1.192	38.10	1.256	1.259	0.27	1.309	4.20	1.334	6.24
0.8	0.65	0.65	3.366	3.355	0.34	2.640	21.56	2.678	20.43	0.895	0.894	0.12	0.910	1.65	0.917	2.49
0.8	0.65	0.8	12.387	12.231	1.26	10.075	18.67	10.861	12.32	0.366	0.367	0.28	0.377	3.11	0.377	2.94
0.8	0.8	0.5	3.852	3.841	0.27	2.560	33.54	2.409	37.45	0.920	0.920	0.05	1.008	9.61	1.016	10.45
0.8	0.8	0.65	7.130	7.135	0.07	5.062	29.01	5.259	26.24	0.583	0.583	0.02	0.636	9.03	0.632	8.49
0.8	0.8	0.8	69.993	71.382	1.98	37.198	46.86	40.689	41.87	0.080	0.080	0.84	0.125	57.21	0.123	54.60

IKCS			Fill Rate							Stockout Probability						
K1	K2	K3	Sim.	Exact	Abs. Error	Agg.	Abs. Error	Com.	Abs. Error	Sim.	Exact	Abs. Error	Agg.	Abs. Error	Com.	Abs. Error
3	3	3														
ρ_1	ρ_2	ρ_3														
0.5	0.5	0.5	0.838	0.838	0.000	0.828	0.010	0.827	0.011	0.089	0.089	0.000	0.095	0.006	0.096	0.007
0.5	0.5	0.65	0.690	0.689	0.000	0.675	0.015	0.672	0.018	0.209	0.210	0.000	0.220	0.011	0.224	0.015
0.5	0.5	0.8	0.451	0.451	0.000	0.436	0.015	0.433	0.017	0.449	0.449	0.000	0.462	0.013	0.466	0.017
0.5	0.65	0.5	0.776	0.776	0.000	0.770	0.006	0.773	0.003	0.141	0.142	0.000	0.142	0.001	0.139	0.003
0.5	0.65	0.65	0.630	0.629	0.001	0.614	0.015	0.613	0.017	0.267	0.268	0.001	0.277	0.010	0.280	0.013
0.5	0.65	0.8	0.391	0.390	0.001	0.374	0.016	0.371	0.019	0.516	0.517	0.001	0.530	0.014	0.535	0.019
0.5	0.8	0.5	0.597	0.597	0.001	0.638	0.041	0.657	0.060	0.316	0.316	0.001	0.265	0.050	0.244	0.071
0.5	0.8	0.65	0.458	0.456	0.002	0.482	0.024	0.489	0.031	0.450	0.452	0.002	0.414	0.035	0.408	0.042
0.5	0.8	0.8	0.223	0.222	0.002	0.241	0.017	0.241	0.017	0.715	0.717	0.002	0.689	0.027	0.690	0.025
0.65	0.5	0.5	0.775	0.776	0.001	0.771	0.005	0.777	0.001	0.142	0.142	0.001	0.142	0.000	0.136	0.006
0.65	0.5	0.65	0.629	0.629	0.001	0.615	0.014	0.616	0.014	0.267	0.268	0.001	0.276	0.009	0.277	0.010
0.65	0.5	0.8	0.390	0.390	0.000	0.375	0.016	0.373	0.017	0.516	0.517	0.001	0.530	0.013	0.533	0.017
0.65	0.65	0.5	0.723	0.722	0.001	0.715	0.008	0.717	0.006	0.188	0.189	0.001	0.189	0.001	0.187	0.001
0.65	0.65	0.65	0.575	0.575	0.000	0.558	0.018	0.554	0.021	0.321	0.321	0.000	0.333	0.012	0.338	0.017
0.65	0.65	0.8	0.335	0.336	0.001	0.316	0.019	0.311	0.024	0.580	0.578	0.001	0.597	0.017	0.604	0.025
0.65	0.8	0.5	0.557	0.556	0.001	0.586	0.030	0.593	0.036	0.355	0.355	0.001	0.314	0.040	0.306	0.049
0.65	0.8	0.65	0.413	0.415	0.001	0.429	0.016	0.427	0.014	0.498	0.496	0.001	0.471	0.027	0.474	0.024
0.65	0.8	0.8	0.179	0.179	0.001	0.186	0.007	0.183	0.004	0.769	0.770	0.001	0.756	0.014	0.761	0.008
0.8	0.5	0.5	0.596	0.597	0.001	0.639	0.043	0.670	0.074	0.317	0.316	0.001	0.265	0.052	0.232	0.084
0.8	0.5	0.65	0.457	0.456	0.001	0.484	0.026	0.498	0.041	0.450	0.452	0.001	0.413	0.037	0.398	0.052
0.8	0.5	0.8	0.221	0.222	0.001	0.242	0.021	0.246	0.025	0.718	0.717	0.001	0.687	0.031	0.684	0.034
0.8	0.65	0.5	0.554	0.556	0.002	0.588	0.033	0.602	0.048	0.357	0.355	0.002	0.313	0.043	0.297	0.060
0.8	0.65	0.65	0.415	0.415	0.001	0.431	0.016	0.434	0.019	0.496	0.496	0.001	0.469	0.026	0.466	0.029
0.8	0.65	0.8	0.178	0.179	0.001	0.188	0.010	0.187	0.009	0.771	0.770	0.001	0.753	0.017	0.757	0.014
0.8	0.8	0.5	0.414	0.415	0.000	0.464	0.049	0.469	0.055	0.506	0.506	0.000	0.442	0.064	0.435	0.071
0.8	0.8	0.65	0.275	0.275	0.000	0.307	0.032	0.305	0.030	0.658	0.657	0.000	0.611	0.047	0.614	0.044
0.8	0.8	0.8	0.039	0.040	0.000	0.063	0.024	0.062	0.023	0.948	0.948	0.000	0.915	0.034	0.917	0.031

Table K.2 (b) Numerical results for IKCS with $K_i = 7$, $i = 1, 2, 3$.

IKCS			Expected Backorders							Expected Number of Assemblies in Stock						
K1	K2	K3	Sim.	Exact	Abs. Rel. Error (%)	Agg.	Abs. Rel. Error (%)	Com.	Abs. Rel. Error (%)	Sim.	Exact	Abs. Rel. Error (%)	Agg.	Abs. Rel. Error (%)	Com.	Abs. Rel. Error (%)
7	7	7														
ρ1	ρ2	ρ3														
0.5	0.5	0.5	0.008	0.008	3.04	0.008	4.68	0.008	4.71	5.997	5.999	0.03	5.993	0.08	5.993	0.08
0.5	0.5	0.65	0.091	0.092	0.92	0.093	1.84	0.093	2.09	5.230	5.229	0.02	5.220	0.19	5.220	0.20
0.5	0.5	0.8	0.841	0.840	0.11	0.844	0.28	0.845	0.40	3.839	3.836	0.06	3.827	0.30	3.827	0.31
0.5	0.65	0.5	0.016	0.017	0.57	0.017	1.65	0.014	15.21	5.936	5.936	0.00	5.919	0.28	5.922	0.22
0.5	0.65	0.65	0.104	0.104	0.08	0.107	2.86	0.107	2.81	5.181	5.181	0.01	5.151	0.57	5.152	0.56
0.5	0.65	0.8	0.874	0.867	0.81	0.882	0.90	0.891	1.96	3.801	3.805	0.11	3.769	0.83	3.767	0.88
0.5	0.8	0.5	0.234	0.240	2.56	0.215	8.00	0.118	49.40	5.430	5.427	0.06	5.420	0.18	5.489	1.08
0.5	0.8	0.65	0.378	0.384	1.41	0.343	9.36	0.272	28.13	4.746	4.743	0.07	4.711	0.73	4.749	0.07
0.5	0.8	0.8	1.344	1.344	0.00	1.325	1.38	1.347	0.25	3.473	3.468	0.13	3.396	2.19	3.401	2.07
0.65	0.5	0.5	0.017	0.017	1.82	0.017	0.70	0.014	17.41	5.938	5.936	0.03	5.919	0.31	5.923	0.25
0.65	0.5	0.65	0.105	0.104	0.33	0.107	2.45	0.107	2.35	5.182	5.181	0.01	5.151	0.59	5.152	0.58
0.65	0.5	0.8	0.865	0.867	0.26	0.882	1.98	0.891	3.04	3.810	3.805	0.12	3.769	1.06	3.768	1.11
0.65	0.65	0.5	0.024	0.024	3.19	0.025	8.02	0.021	9.32	5.883	5.880	0.04	5.848	0.59	5.852	0.52
0.65	0.65	0.65	0.117	0.116	0.50	0.122	4.74	0.123	5.60	5.139	5.138	0.01	5.085	1.04	5.085	1.05
0.65	0.65	0.8	0.902	0.892	1.18	0.920	1.97	0.939	4.11	3.772	3.776	0.10	3.713	1.57	3.709	1.68
0.65	0.8	0.5	0.250	0.246	1.61	0.225	10.02	0.141	43.47	5.388	5.393	0.09	5.361	0.50	5.412	0.45
0.65	0.8	0.65	0.392	0.393	0.07	0.360	8.38	0.301	23.33	4.713	4.715	0.03	4.655	1.23	4.681	0.68
0.65	0.8	0.8	1.372	1.364	0.51	1.368	0.28	1.411	2.89	3.450	3.448	0.06	3.348	2.96	3.344	3.07
0.8	0.5	0.5	0.240	0.240	0.16	0.215	10.35	0.116	51.83	5.429	5.427	0.03	5.421	0.14	5.494	1.20
0.8	0.5	0.65	0.384	0.384	0.07	0.343	10.64	0.270	29.73	4.747	4.743	0.10	4.712	0.74	4.752	0.11
0.8	0.5	0.8	1.322	1.344	1.63	1.325	0.23	1.345	1.70	3.470	3.468	0.05	3.397	2.10	3.403	1.93
0.8	0.65	0.5	0.250	0.246	1.65	0.226	9.47	0.139	44.31	5.390	5.393	0.05	5.364	0.48	5.419	0.54
0.8	0.65	0.65	0.391	0.393	0.39	0.361	7.84	0.299	23.63	4.717	4.715	0.05	4.658	1.25	4.687	0.64
0.8	0.65	0.8	1.382	1.364	1.27	1.368	1.03	1.408	1.88	3.446	3.448	0.07	3.351	2.75	3.349	2.82
0.8	0.8	0.5	0.452	0.453	0.25	0.457	1.09	0.354	21.75	5.023	5.019	0.09	4.935	1.77	4.979	0.88
0.8	0.8	0.65	0.660	0.656	0.57	0.633	4.05	0.555	15.78	4.379	4.382	0.05	4.276	2.36	4.299	1.83
0.8	0.8	0.8	1.805	1.834	1.66	1.882	4.28	1.989	10.22	3.189	3.179	0.33	3.025	5.16	3.011	5.58

IKCS			Fill Rate							Stockout Probability						
K1	K2	K3	Sim.	Exact	Abs. Error	Agg.	Abs. Error	Com.	Abs. Error	Sim.	Exact	Abs. Error	Agg.	Abs. Error	Com.	Abs. Error
7	7	7														
ρ1	ρ2	ρ3														
0.5	0.5	0.5	0.992	0.992	0.000	0.992	0.000	0.992	0.000	0.004	0.004	0.000	0.004	0.000	0.004	0.000
0.5	0.5	0.65	0.951	0.951	0.000	0.950	0.000	0.950	0.001	0.032	0.032	0.000	0.032	0.000	0.032	0.000
0.5	0.5	0.8	0.790	0.790	0.000	0.789	0.001	0.789	0.001	0.168	0.168	0.000	0.169	0.001	0.169	0.001
0.5	0.65	0.5	0.988	0.988	0.000	0.988	0.000	0.988	0.000	0.007	0.007	0.000	0.007	0.000	0.006	0.000
0.5	0.65	0.65	0.946	0.946	0.000	0.945	0.002	0.945	0.001	0.035	0.035	0.000	0.037	0.001	0.036	0.001
0.5	0.65	0.8	0.785	0.786	0.001	0.782	0.003	0.782	0.003	0.173	0.172	0.001	0.175	0.002	0.175	0.003
0.5	0.8	0.5	0.937	0.936	0.001	0.939	0.002	0.953	0.016	0.049	0.049	0.001	0.046	0.002	0.033	0.016
0.5	0.8	0.65	0.891	0.891	0.001	0.893	0.001	0.901	0.010	0.083	0.083	0.001	0.081	0.002	0.072	0.011
0.5	0.8	0.8	0.730	0.730	0.001	0.724	0.006	0.725	0.006	0.224	0.225	0.000	0.229	0.004	0.228	0.004
0.65	0.5	0.5	0.988	0.988	0.000	0.988	0.000	0.988	0.000	0.007	0.007	0.000	0.007	0.000	0.006	0.000
0.65	0.5	0.65	0.946	0.946	0.000	0.945	0.002	0.945	0.001	0.036	0.035	0.000	0.037	0.001	0.036	0.001
0.65	0.5	0.8	0.786	0.786	0.001	0.782	0.004	0.782	0.005	0.171	0.172	0.001	0.175	0.003	0.175	0.004
0.65	0.65	0.5	0.985	0.984	0.000	0.984	0.001	0.984	0.000	0.009	0.009	0.000	0.010	0.001	0.009	0.000
0.65	0.65	0.65	0.942	0.942	0.000	0.939	0.003	0.939	0.003	0.038	0.039	0.000	0.041	0.002	0.041	0.002
0.65	0.65	0.8	0.781	0.781	0.001	0.775	0.006	0.774	0.007	0.176	0.176	0.001	0.181	0.005	0.182	0.006
0.65	0.8	0.5	0.933	0.934	0.001	0.935	0.002	0.945	0.012	0.052	0.051	0.001	0.049	0.003	0.039	0.013
0.65	0.8	0.65	0.888	0.888	0.000	0.888	0.000	0.893	0.005	0.086	0.086	0.000	0.085	0.001	0.079	0.007
0.65	0.8	0.8	0.727	0.727	0.000	0.717	0.009	0.716	0.010	0.228	0.228	0.000	0.235	0.007	0.236	0.009
0.8	0.5	0.5	0.936	0.936	0.000	0.939	0.003	0.953	0.017	0.049	0.049	0.000	0.047	0.003	0.033	0.017
0.8	0.5	0.65	0.891	0.891	0.000	0.893	0.002	0.901	0.010	0.083	0.083	0.000	0.081	0.003	0.072	0.011
0.8	0.5	0.8	0.730	0.730	0.001	0.724	0.006	0.725	0.005	0.224	0.225	0.001	0.229	0.005	0.228	0.004
0.8	0.65	0.5	0.933	0.934	0.000	0.935	0.002	0.946	0.013	0.052	0.051	0.000	0.049	0.002	0.039	0.013
0.8	0.65	0.65	0.888	0.888	0.000	0.888	0.001	0.894	0.006	0.085	0.086	0.000	0.085	0.001	0.078	0.007
0.8	0.65	0.8	0.726	0.727	0.000	0.718	0.009	0.717	0.009	0.228	0.228	0.001	0.234	0.006	0.236	0.007
0.8	0.8	0.5	0.893	0.892	0.000	0.889	0.004	0.898	0.005	0.086	0.087	0.000	0.089	0.003	0.079	0.007
0.8	0.8	0.65	0.843	0.843	0.000	0.838	0.004	0.844	0.001	0.126	0.126	0.000	0.128	0.002	0.122	0.004
0.8	0.8	0.8	0.682	0.680	0.002	0.663	0.019	0.660	0.023	0.270	0.273	0.002	0.286	0.016	0.291	0.020

Table K.2 (c) Numerical results for IKCS with $K_i = 15$, $i = 1, 2, 3$.

SKCS			Expected Backorders					Expected Number of Assemblies in Stock				
K1	K2	K3	Sim.	App. Agg.	Abs. Rel. Error (%)	App. Com.	Abs. Rel. Error (%)	Sim.	App. Agg.	Abs. Rel. Error (%)	App. Com.	Abs. Rel. Error (%)
15	15	15										
ρ1	ρ2	ρ3										
0.5	0.5	0.5	0.000	0.000	19.22	0.000	19.22	13.999	14.000	0.01	14.000	0.01
0.5	0.5	0.65	0.003	0.003	5.19	0.003	5.19	13.147	13.146	0.01	13.146	0.01
0.5	0.5	0.8	0.136	0.141	3.71	0.141	3.71	11.149	11.141	0.07	11.141	0.07
0.5	0.65	0.5	0.000	0.000	9.70	0.000	17.70	13.997	13.997	0.00	13.997	0.00
0.5	0.65	0.65	0.003	0.003	3.64	0.003	3.66	13.141	13.143	0.01	13.143	0.01
0.5	0.65	0.8	0.139	0.141	1.55	0.141	1.56	11.142	11.138	0.04	11.138	0.04
0.5	0.8	0.5	0.007	0.007	4.27	0.005	32.53	13.870	13.866	0.03	13.870	0.00
0.5	0.8	0.65	0.012	0.012	2.43	0.008	29.59	13.035	13.014	0.16	13.020	0.11
0.5	0.8	0.8	0.158	0.161	1.71	0.160	1.61	11.057	11.020	0.34	11.020	0.33
0.65	0.5	0.5	0.000	0.000	44.28	0.000	31.49	13.996	13.997	0.01	13.997	0.01
0.65	0.5	0.65	0.003	0.003	0.67	0.003	0.65	13.144	13.143	0.01	13.143	0.01
0.65	0.5	0.8	0.143	0.141	1.42	0.141	1.41	11.126	11.138	0.11	11.138	0.11
0.65	0.65	0.5	0.000	0.000	6.11	0.000	17.31	13.996	13.994	0.01	13.994	0.01
0.65	0.65	0.65	0.003	0.003	11.18	0.003	11.18	13.143	13.140	0.02	13.140	0.02
0.65	0.65	0.8	0.139	0.141	1.35	0.141	1.38	11.138	11.135	0.02	11.135	0.02
0.65	0.8	0.5	0.007	0.007	0.40	0.005	22.98	13.874	13.863	0.08	13.866	0.06
0.65	0.8	0.65	0.011	0.012	9.71	0.008	22.58	13.029	13.011	0.13	13.017	0.10
0.65	0.8	0.8	0.157	0.161	2.29	0.161	2.36	11.073	11.017	0.51	11.017	0.51
0.8	0.5	0.5	0.007	0.007	6.43	0.005	34.32	13.867	13.866	0.01	13.870	0.02
0.8	0.5	0.65	0.012	0.012	1.38	0.008	30.38	13.033	13.014	0.14	13.020	0.10
0.8	0.5	0.8	0.156	0.161	2.66	0.160	2.56	11.063	11.020	0.39	11.020	0.39
0.8	0.65	0.5	0.006	0.007	7.03	0.005	17.50	13.877	13.863	0.10	13.866	0.08
0.8	0.65	0.65	0.012	0.012	4.77	0.008	32.87	13.028	13.012	0.13	13.017	0.09
0.8	0.65	0.8	0.158	0.161	1.97	0.161	2.04	11.059	11.017	0.38	11.017	0.38
0.8	0.8	0.5	0.013	0.014	10.26	0.013	1.54	13.753	13.729	0.18	13.730	0.16
0.8	0.8	0.65	0.021	0.022	4.98	0.016	21.36	12.926	12.880	0.35	12.887	0.30
0.8	0.8	0.8	0.171	0.182	6.50	0.183	7.13	10.999	10.896	0.94	10.895	0.94

SKCS			Fill Rate					Stockout Probability				
K1	K2	K3	Sim.	App. Agg.	Abs. Error	App. Com.	Abs. Error	Sim.	App. Agg.	Abs. Error	App. Com.	Abs. Error
15	15	15										
ρ1	ρ2	ρ3										
0.5	0.5	0.5	1.000	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.5	0.65	0.998	0.998	0.000	0.998	0.000	0.001	0.001	0.000	0.001	0.000
0.5	0.5	0.8	0.966	0.965	0.001	0.965	0.001	0.027	0.028	0.001	0.028	0.001
0.5	0.65	0.5	1.000	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.65	0.65	0.998	0.998	0.000	0.998	0.000	0.001	0.001	0.000	0.001	0.000
0.5	0.65	0.8	0.965	0.965	0.000	0.965	0.000	0.028	0.028	0.000	0.028	0.000
0.5	0.8	0.5	0.998	0.998	0.000	0.999	0.000	0.001	0.001	0.000	0.001	0.000
0.5	0.8	0.65	0.996	0.996	0.000	0.997	0.000	0.003	0.003	0.000	0.002	0.000
0.5	0.8	0.8	0.962	0.961	0.001	0.961	0.001	0.031	0.031	0.001	0.031	0.001
0.65	0.5	0.5	1.000	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
0.65	0.5	0.65	0.998	0.998	0.000	0.998	0.000	0.001	0.001	0.000	0.001	0.000
0.65	0.5	0.8	0.964	0.965	0.001	0.965	0.001	0.029	0.028	0.001	0.028	0.001
0.65	0.65	0.5	1.000	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
0.65	0.65	0.65	0.998	0.998	0.000	0.998	0.000	0.001	0.001	0.000	0.001	0.000
0.65	0.65	0.8	0.965	0.965	0.000	0.965	0.000	0.028	0.028	0.000	0.028	0.000
0.65	0.8	0.5	0.998	0.998	0.000	0.999	0.000	0.001	0.001	0.000	0.001	0.000
0.65	0.8	0.65	0.996	0.996	0.000	0.997	0.000	0.003	0.003	0.000	0.002	0.000
0.65	0.8	0.8	0.962	0.961	0.001	0.961	0.001	0.031	0.031	0.001	0.031	0.001
0.8	0.5	0.5	0.998	0.998	0.000	0.999	0.000	0.001	0.001	0.000	0.001	0.000
0.8	0.5	0.65	0.996	0.996	0.000	0.997	0.000	0.003	0.003	0.000	0.002	0.000
0.8	0.5	0.8	0.962	0.961	0.001	0.961	0.001	0.030	0.031	0.001	0.031	0.001
0.8	0.65	0.5	0.998	0.998	0.000	0.999	0.000	0.001	0.001	0.000	0.001	0.000
0.8	0.65	0.65	0.996	0.996	0.000	0.997	0.000	0.003	0.003	0.000	0.002	0.000
0.8	0.65	0.8	0.962	0.961	0.001	0.961	0.001	0.031	0.031	0.001	0.031	0.001
0.8	0.8	0.5	0.997	0.997	0.000	0.997	0.000	0.003	0.003	0.000	0.003	0.000
0.8	0.8	0.65	0.994	0.994	0.000	0.994	0.000	0.005	0.005	0.000	0.004	0.001
0.8	0.8	0.8	0.960	0.957	0.002	0.957	0.003	0.033	0.035	0.002	0.035	0.002

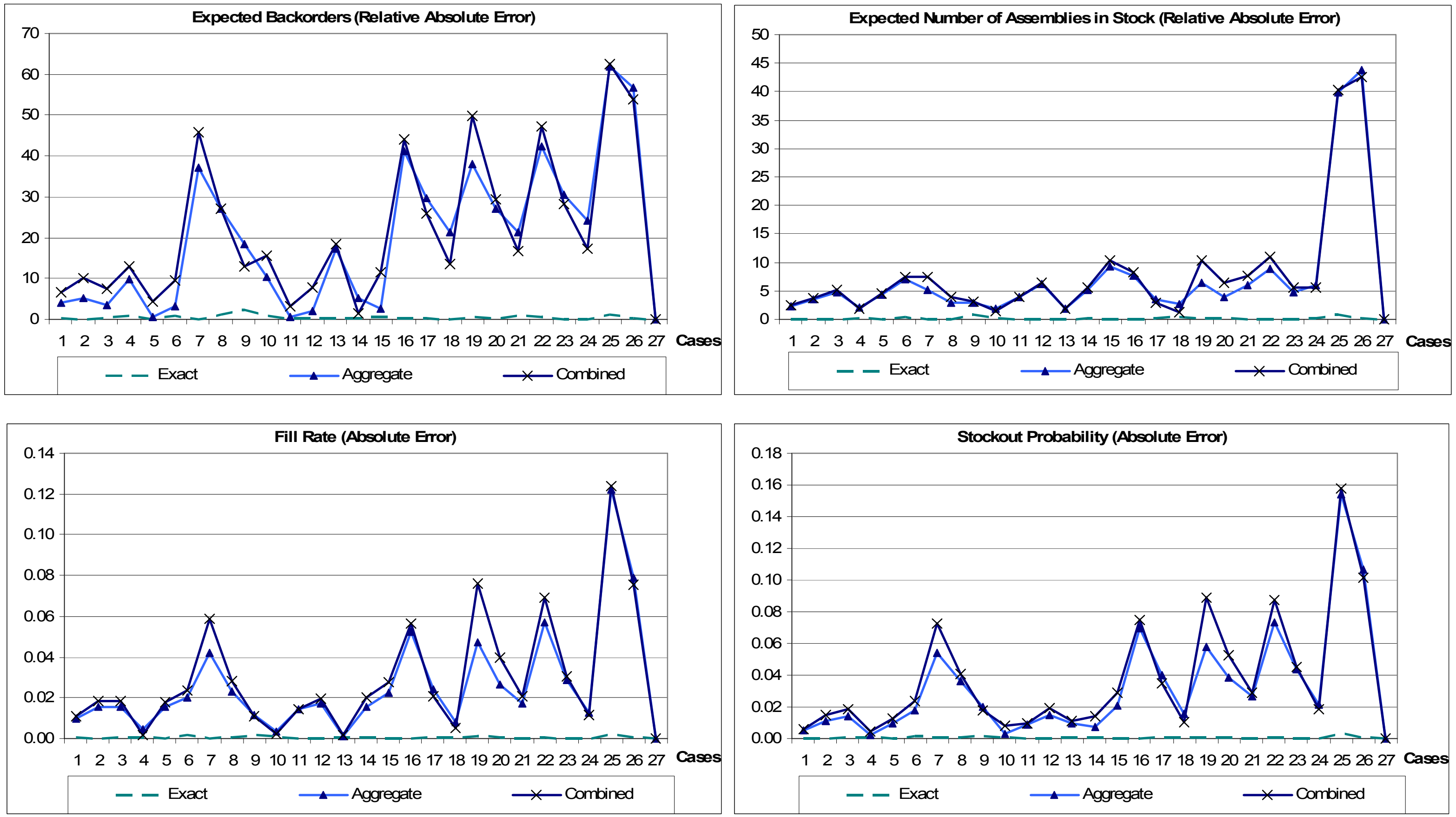

Figure K.1(a) Numerical Results for SKCS with K_i=3, i=1, 2, 3.

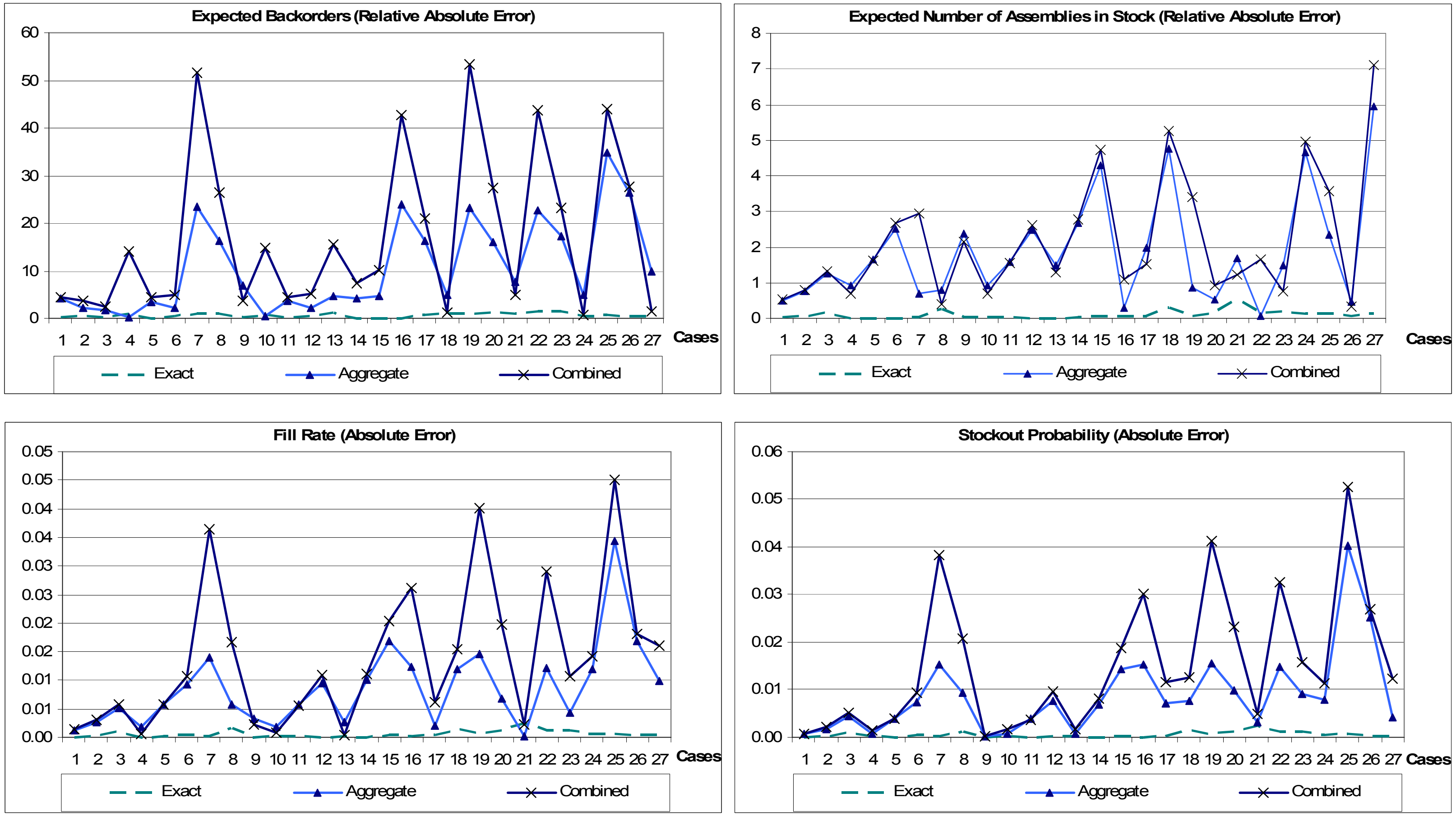

Figure K.1(b) Numerical Results for SKCS with K_i=5, i=1, 2, 3.

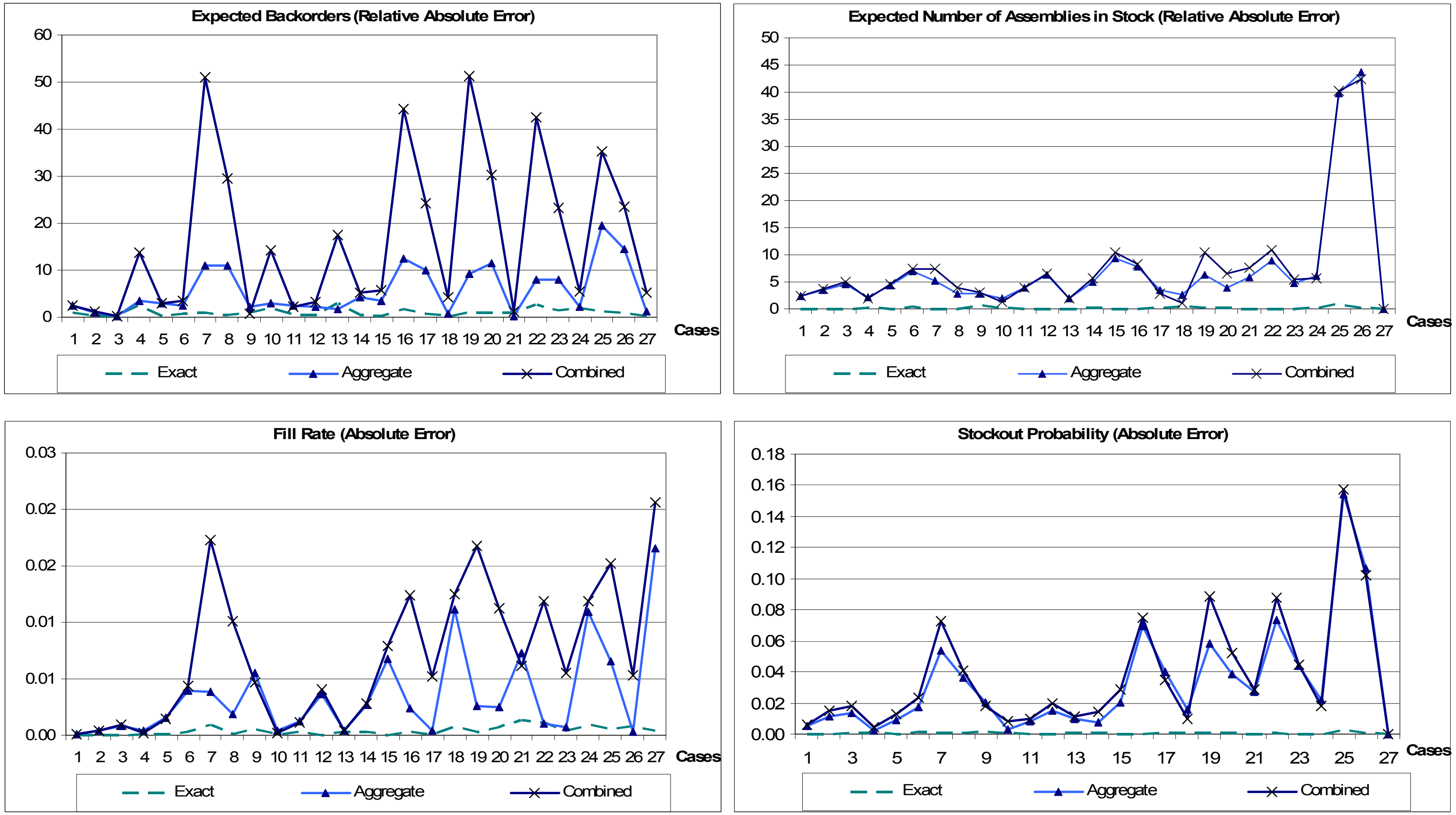

Figure K.1(c) Numerical Results for SKCS with K_i=7, i=1, 2, 3.

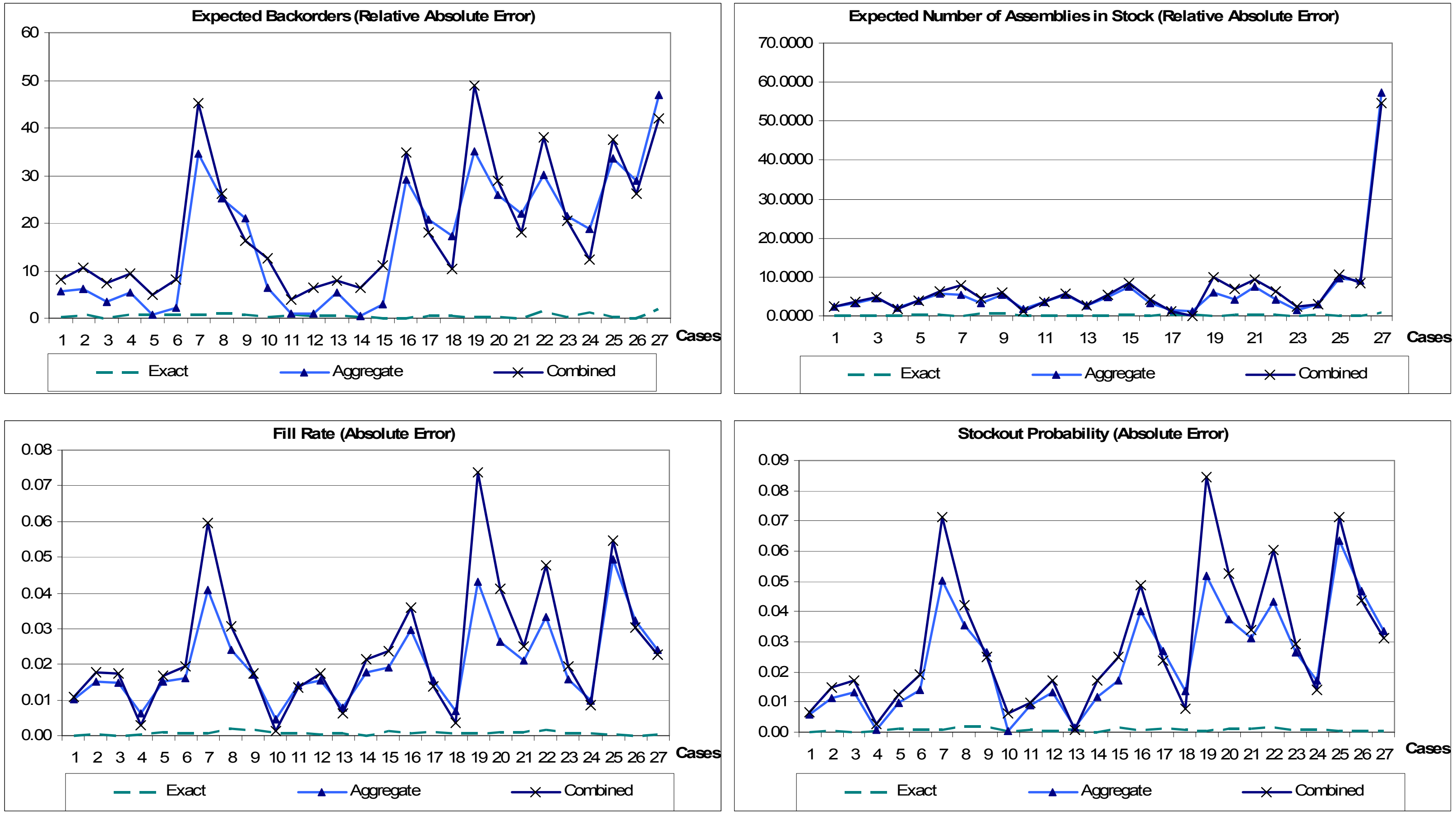

Figure K.2(a) Numerical Results for IKCS with K_i=3, i=1, 2, 3.

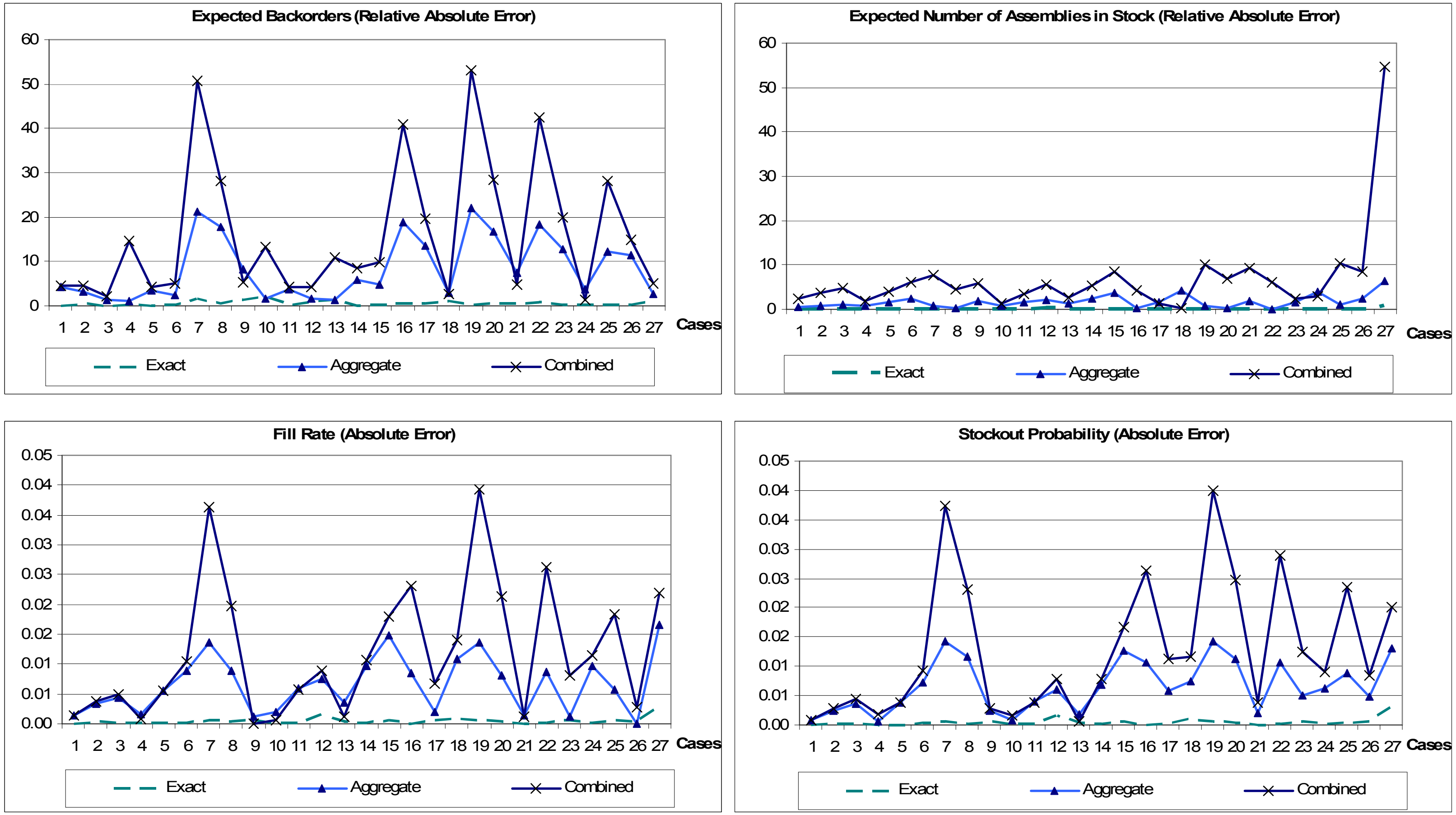

Figure K.2(b) Numerical Results for IKCS with K_i=5, i=1, 2, 3.

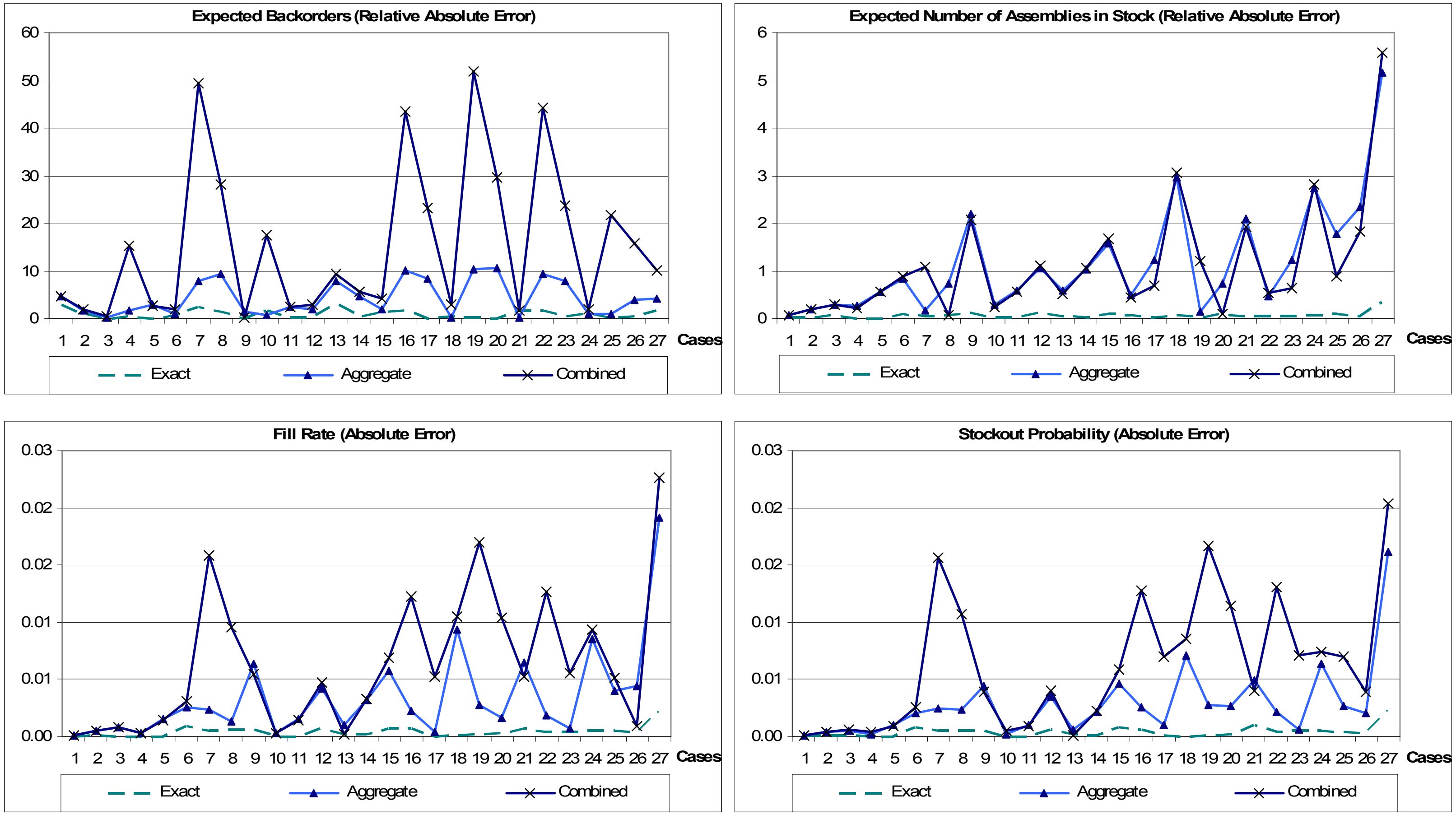

Figure K.2(c) Numerical Results for IKCS with K_i=7, i=1, 2, 3.

APPENDIX L

CONFIDENCE INTERVALS FOR PERFORMANCE MEASURES

Table L.1 95% Confidence Intervals for EB, SKCS with $K_i = 5$, $i = 1, 2, 3$.

Traffic Intensities			Mean	Half Width	Lower Limit	Upper Limit	Exact	Agg.	Com.	Exact	App. Agg.	App. Com.
ρ1	ρ2	ρ3										
0.5	0.5	0.5	0.0368	0.0004	0.0363	0.0372	0.0369	0.0383	0.0384	1	0	0
0.5	0.5	0.65	0.2255	0.0038	0.2217	0.2293	0.2246	0.2307	0.2336	1	0	0
0.5	0.5	0.8	1.3303	0.0140	1.3163	1.3443	1.3342	1.3519	1.3628	1	0	0
0.5	0.65	0.5	0.0789	0.0017	0.0772	0.0806	0.0798	0.0787	0.0679	1	1	0
0.5	0.65	0.65	0.2869	0.0039	0.2831	0.2908	0.2871	0.2966	0.2996	1	0	0
0.5	0.65	0.8	1.4825	0.0157	1.4668	1.4982	1.4746	1.5148	1.5568	1	0	0
0.5	0.8	0.5	0.6092	0.0137	0.5954	0.6229	0.6029	0.4656	0.2945	1	0	0
0.5	0.8	0.65	0.9364	0.0230	0.9134	0.9594	0.9456	0.7850	0.6893	1	0	0
0.5	0.8	0.8	2.7539	0.0466	2.7073	2.8005	2.7446	2.5641	2.6515	1	0	0
0.65	0.5	0.5	0.0791	0.0012	0.0779	0.0803	0.0798	0.0788	0.0673	1	1	0
0.65	0.5	0.65	0.2862	0.0051	0.2811	0.2913	0.2871	0.2966	0.2988	1	0	0
0.65	0.5	0.8	1.4806	0.0267	1.4539	1.5073	1.4746	1.5145	1.5557	1	0	0
0.65	0.65	0.5	0.1326	0.0025	0.1302	0.1351	0.1310	0.1265	0.1118	1	0	0
0.65	0.65	0.65	0.3577	0.0056	0.3521	0.3633	0.3577	0.3730	0.3840	1	0	0
0.65	0.65	0.8	1.6259	0.0183	1.6076	1.6442	1.6269	1.7042	1.7905	1	0	0
0.65	0.8	0.5	0.7212	0.0206	0.7006	0.7419	0.7210	0.5482	0.4126	1	0	0
0.65	0.8	0.65	1.0932	0.0186	1.0746	1.1118	1.0864	0.9149	0.8637	1	0	0
0.65	0.8	0.8	3.0521	0.0517	3.0004	3.1038	3.0183	2.9026	3.0912	1	0	1
0.8	0.5	0.5	0.6083	0.0086	0.5997	0.6169	0.6029	0.4665	0.2837	1	0	0
0.8	0.5	0.65	0.9334	0.0103	0.9231	0.9437	0.9456	0.7844	0.6777	0	0	0
0.8	0.5	0.8	2.7709	0.0560	2.7149	2.8269	2.7446	2.5600	2.6346	1	0	0
0.8	0.65	0.5	0.7104	0.0135	0.6968	0.7239	0.7210	0.5492	0.3992	1	0	0
0.8	0.65	0.65	1.1018	0.0218	1.0800	1.1236	1.0864	0.9118	0.8467	1	0	0
0.8	0.65	0.8	3.0336	0.0611	2.9726	3.0947	3.0183	2.8874	3.0577	1	0	1
0.8	0.8	0.5	1.8268	0.0504	1.7764	1.8772	1.8412	1.1928	1.0249	1	0	0
0.8	0.8	0.65	2.3839	0.0435	2.3404	2.4274	2.3974	1.7527	1.7223	1	0	0
0.8	0.8	0.8	5.5085	0.0995	5.4090	5.6080	5.5390	4.9588	5.4261	1	0	1

Number of results that fall in to intervals=	26	2	3

Table L.2 95% Confidence Intervals for EB, IKCS with $K_i = 5$, $i = 1, 2, 3$.

Traffic Intensities			Mean	Half Width	Lower Limit	Upper Limit	Exact	Agg.	Com.	Exact	App. Agg.	App. Com.
ρ1	ρ2	ρ3										
0.5	0.5	0.5	0.0366	0.0004	0.0363	0.0370	0.0367	0.0382	0.0383	1	0	0
0.5	0.5	0.65	0.2235	0.0037	0.2197	0.2272	0.2244	0.2304	0.2333	1	0	0
0.5	0.5	0.8	1.3339	0.0191	1.3148	1.3530	1.3337	1.3511	1.3618	1	1	0
0.5	0.65	0.5	0.0784	0.0019	0.0766	0.0803	0.0782	0.0777	0.0669	1	1	0
0.5	0.65	0.65	0.2853	0.0038	0.2815	0.2891	0.2854	0.2948	0.2974	1	0	0
0.5	0.65	0.8	1.4762	0.0147	1.4615	1.4909	1.4716	1.5099	1.5508	1	0	0
0.5	0.8	0.5	0.5851	0.0104	0.5747	0.5955	0.5941	0.4601	0.2883	1	0	0
0.5	0.8	0.65	0.9421	0.0164	0.9257	0.9585	0.9369	0.7755	0.6782	1	0	0
0.5	0.8	0.8	2.7696	0.0370	2.7326	2.8066	2.7309	2.5388	2.6209	0	0	0
0.65	0.5	0.5	0.0767	0.0018	0.0748	0.0785	0.0782	0.0778	0.0664	1	1	0
0.65	0.5	0.65	0.2845	0.0033	0.2812	0.2878	0.2854	0.2948	0.2968	1	0	0
0.65	0.5	0.8	1.4853	0.0195	1.4658	1.5048	1.4716	1.5099	1.5499	1	0	0
0.65	0.65	0.5	0.1183	0.0021	0.1163	0.1204	0.1166	0.1200	0.1056	1	1	0
0.65	0.65	0.65	0.3426	0.0052	0.3374	0.3479	0.3424	0.3623	0.3720	1	0	0
0.65	0.65	0.8	1.5992	0.0246	1.5746	1.6238	1.6019	1.6770	1.7570	1	0	0
0.65	0.8	0.5	0.6318	0.0120	0.6198	0.6439	0.6283	0.5125	0.3739	1	0	0
0.65	0.8	0.65	0.9959	0.0155	0.9805	1.0114	0.9913	0.8604	0.8015	1	0	0
0.65	0.8	0.8	2.8437	0.0534	2.7903	2.8971	2.8720	2.7624	2.9219	1	0	0
0.8	0.5	0.5	0.5919	0.0162	0.5756	0.6081	0.5941	0.4617	0.2784	1	0	0
0.8	0.5	0.65	0.9324	0.0165	0.9159	0.9488	0.9369	0.7763	0.6684	1	0	0
0.8	0.5	0.8	2.7432	0.0544	2.6888	2.7976	2.7309	2.5389	2.6092	1	0	0
0.8	0.65	0.5	0.6326	0.0129	0.6197	0.6455	0.6283	0.5162	0.3642	1	0	0
0.8	0.65	0.65	0.9883	0.0199	0.9684	1.0083	0.9913	0.8620	0.7907	1	0	0
0.8	0.65	0.8	2.8678	0.0563	2.8115	2.9241	2.8720	2.7606	2.9054	1	0	1
0.8	0.8	0.5	1.1580	0.0281	1.1299	1.1861	1.1611	1.0172	0.8334	1	0	0
0.8	0.8	0.65	1.6936	0.0354	1.6582	1.7290	1.7002	1.5021	1.4403	1	0	0
0.8	0.8	0.8	4.3903	0.1066	4.2837	4.4969	4.4330	4.2717	4.6150	1	0	0

Number of results that fall in to intervals =	26	4	1

Table L.3 95% Confidence Intervals for FR, SKCS with $K_i = 5$, $i = 1, 2, 3$.

Traffic Intensities			Mean	Half Width	Lower Limit	Upper Limit	Exact	App. Agg.	App. Com.	Exact	App. Agg.	App. Com.
ρ1	ρ2	ρ3										
0.5	0.5	0.5	0.9651	0.0003	0.9648	0.9653	0.9651	0.9638	0.9637	1	0	0
0.5	0.5	0.65	0.8799	0.0010	0.8790	0.8809	0.8804	0.8773	0.8769	1	0	0
0.5	0.5	0.8	0.6698	0.0016	0.6682	0.6714	0.6688	0.6646	0.6641	1	0	0
0.5	0.65	0.5	0.9473	0.0006	0.9466	0.9479	0.9472	0.9455	0.9479	1	0	1
0.5	0.65	0.65	0.8620	0.0010	0.8610	0.8629	0.8621	0.8561	0.8562	1	0	0
0.5	0.65	0.8	0.6502	0.0016	0.6487	0.6518	0.6506	0.6410	0.6395	1	0	0
0.5	0.8	0.5	0.8385	0.0015	0.8370	0.8400	0.8388	0.8525	0.8750	1	0	0
0.5	0.8	0.65	0.7554	0.0022	0.7533	0.7576	0.7539	0.7613	0.7721	1	0	0
0.5	0.8	0.8	0.5447	0.0027	0.5420	0.5475	0.5448	0.5415	0.5425	1	0	1
0.65	0.5	0.5	0.9474	0.0003	0.9470	0.9477	0.9472	0.9455	0.9481	1	0	0
0.65	0.5	0.65	0.8620	0.0012	0.8608	0.8632	0.8621	0.8562	0.8565	1	0	0
0.65	0.5	0.8	0.6506	0.0024	0.6482	0.6530	0.6506	0.6411	0.6397	1	0	0
0.65	0.65	0.5	0.9285	0.0007	0.9278	0.9291	0.9287	0.9257	0.9281	1	0	1
0.65	0.65	0.65	0.8436	0.0012	0.8424	0.8449	0.8436	0.8335	0.8324	1	0	0
0.65	0.65	0.8	0.6329	0.0018	0.6311	0.6347	0.6325	0.6160	0.6125	1	0	0
0.65	0.8	0.5	0.8169	0.0022	0.8147	0.8191	0.8167	0.8291	0.8429	1	0	0
0.65	0.8	0.65	0.7326	0.0020	0.7306	0.7346	0.7330	0.7347	0.7388	1	0	0
0.65	0.8	0.8	0.5241	0.0029	0.5212	0.5271	0.5255	0.5123	0.5087	1	0	0
0.8	0.5	0.5	0.8382	0.0011	0.8370	0.8393	0.8388	0.8528	0.8783	1	0	0
0.8	0.5	0.65	0.7551	0.0013	0.7538	0.7563	0.7539	0.7619	0.7749	1	0	0
0.8	0.5	0.8	0.5424	0.0026	0.5398	0.5450	0.5448	0.5422	0.5446	1	1	1
0.8	0.65	0.5	0.8178	0.0012	0.8166	0.8191	0.8167	0.8301	0.8468	1	0	0
0.8	0.65	0.65	0.7318	0.0020	0.7298	0.7338	0.7330	0.7362	0.7425	1	0	0
0.8	0.65	0.8	0.5261	0.0031	0.5230	0.5291	0.5255	0.5141	0.5119	1	0	0
0.8	0.8	0.5	0.6894	0.0027	0.6867	0.6921	0.6887	0.7238	0.7344	1	0	0
0.8	0.8	0.65	0.6109	0.0028	0.6081	0.6136	0.6105	0.6278	0.6290	1	0	0
0.8	0.8	0.8	0.4110	0.0031	0.4079	0.4142	0.4107	0.4012	0.3950	1	0	0

Number of results that fall in to intervals =	27	1	4

Table L.4 95% Confidence Intervals for FR, IKCS with $K_i = 5$, $i = 1, 2, 3$.

Traffic Intensities			Mean	Half Width	Lower Limit	Upper Limit	Exact	App. Agg.	App. Com.	Exact	App. Agg.	App. Com.
ρ1	ρ2	ρ3										
0.5	0.5	0.5	0.9652	0.0002	0.9650	0.9654	0.9652	0.9639	0.9638	1	0	0
0.5	0.5	0.65	0.8808	0.0008	0.8800	0.8816	0.8805	0.8775	0.8771	1	0	0
0.5	0.5	0.8	0.6691	0.0016	0.6675	0.6707	0.6689	0.6648	0.6642	1	0	0
0.5	0.65	0.5	0.9476	0.0004	0.9472	0.9481	0.9477	0.9460	0.9484	1	0	0
0.5	0.65	0.65	0.8624	0.0006	0.8617	0.8630	0.8626	0.8568	0.8569	1	0	0
0.5	0.65	0.8	0.6507	0.0012	0.6495	0.6519	0.6510	0.6418	0.6403	1	0	0
0.5	0.8	0.5	0.8408	0.0012	0.8397	0.8420	0.8403	0.8544	0.8771	1	0	0
0.5	0.8	0.65	0.7547	0.0014	0.7533	0.7561	0.7551	0.7635	0.7745	1	0	0
0.5	0.8	0.8	0.5451	0.0019	0.5432	0.5470	0.5457	0.5440	0.5452	1	1	1
0.65	0.5	0.5	0.9480	0.0005	0.9475	0.9485	0.9477	0.9460	0.9486	1	0	0
0.65	0.5	0.65	0.8628	0.0009	0.8620	0.8637	0.8626	0.8568	0.8572	1	0	0
0.65	0.5	0.8	0.6494	0.0016	0.6478	0.6510	0.6510	0.6418	0.6405	1	0	0
0.65	0.65	0.5	0.9321	0.0005	0.9316	0.9326	0.9325	0.9285	0.9308	1	0	0
0.65	0.65	0.65	0.8465	0.0011	0.8454	0.8475	0.8467	0.8368	0.8358	1	0	0
0.65	0.65	0.8	0.6344	0.0022	0.6322	0.6366	0.6350	0.6196	0.6163	1	0	0
0.65	0.8	0.5	0.8300	0.0011	0.8289	0.8311	0.8300	0.8385	0.8531	1	0	0
0.65	0.8	0.65	0.7434	0.0012	0.7421	0.7446	0.7439	0.7452	0.7501	1	0	0
0.65	0.8	0.8	0.5348	0.0022	0.5326	0.5369	0.5339	0.5238	0.5208	1	0	0
0.8	0.5	0.5	0.8408	0.0019	0.8389	0.8427	0.8403	0.8543	0.8800	1	0	0
0.8	0.5	0.65	0.7555	0.0018	0.7537	0.7573	0.7551	0.7637	0.7769	1	0	0
0.8	0.5	0.8	0.5457	0.0028	0.5429	0.5485	0.5457	0.5443	0.5468	1	1	1
0.8	0.65	0.5	0.8298	0.0016	0.8283	0.8314	0.8300	0.8386	0.8560	1	0	0
0.8	0.65	0.65	0.7445	0.0013	0.7433	0.7458	0.7439	0.7457	0.7526	1	1	0
0.8	0.65	0.8	0.5342	0.0027	0.5315	0.5370	0.5339	0.5245	0.5228	1	0	0
0.8	0.8	0.5	0.7471	0.0021	0.7450	0.7492	0.7465	0.7527	0.7655	1	0	0
0.8	0.8	0.65	0.6588	0.0023	0.6565	0.6610	0.6584	0.6588	0.6616	1	1	0
0.8	0.8	0.8	0.4508	0.0034	0.4474	0.4542	0.4478	0.4342	0.4288	1	0	0

Number of results that fall in to intervals =	27	4	2

APPENDIX M

APPROXIMATION FOR THREE-COMPONENT SYSTEMS

Three-component kanban controlled assembly systems are depicted in Figures M.1(a) and (b) with two different kanban release mechanisms for the components, and the alternative sequential models are given in Figures M.2(a) and (b). Sequential partial aggregations of the alternative models, then, give the aggregate model with state description $(d_2, d_{11}, d_{12}, d_{13}, n_4)$.

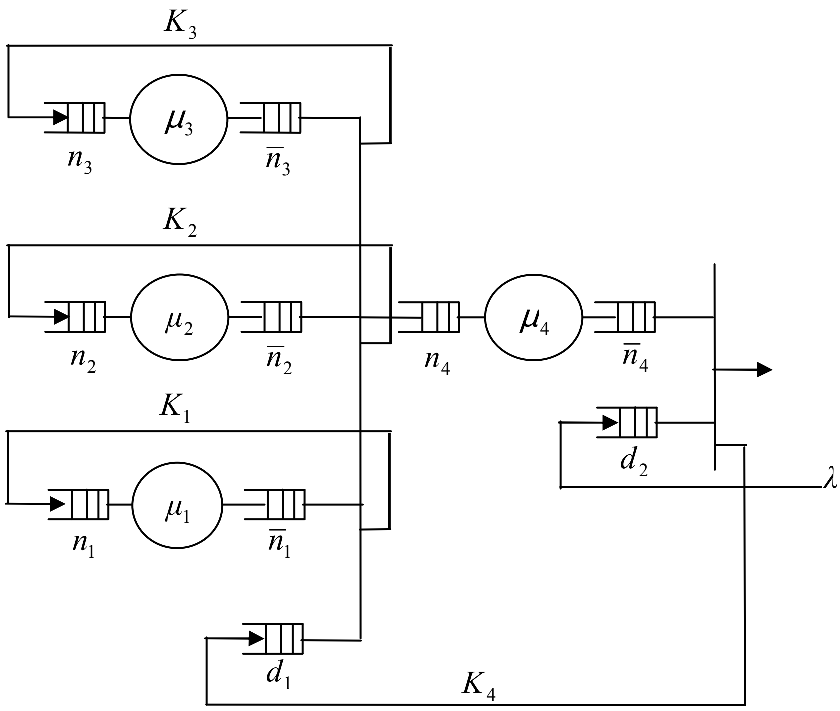

Figure M.1(a) Three-Component Assembly System with Simultaneous Release of Kanbans.

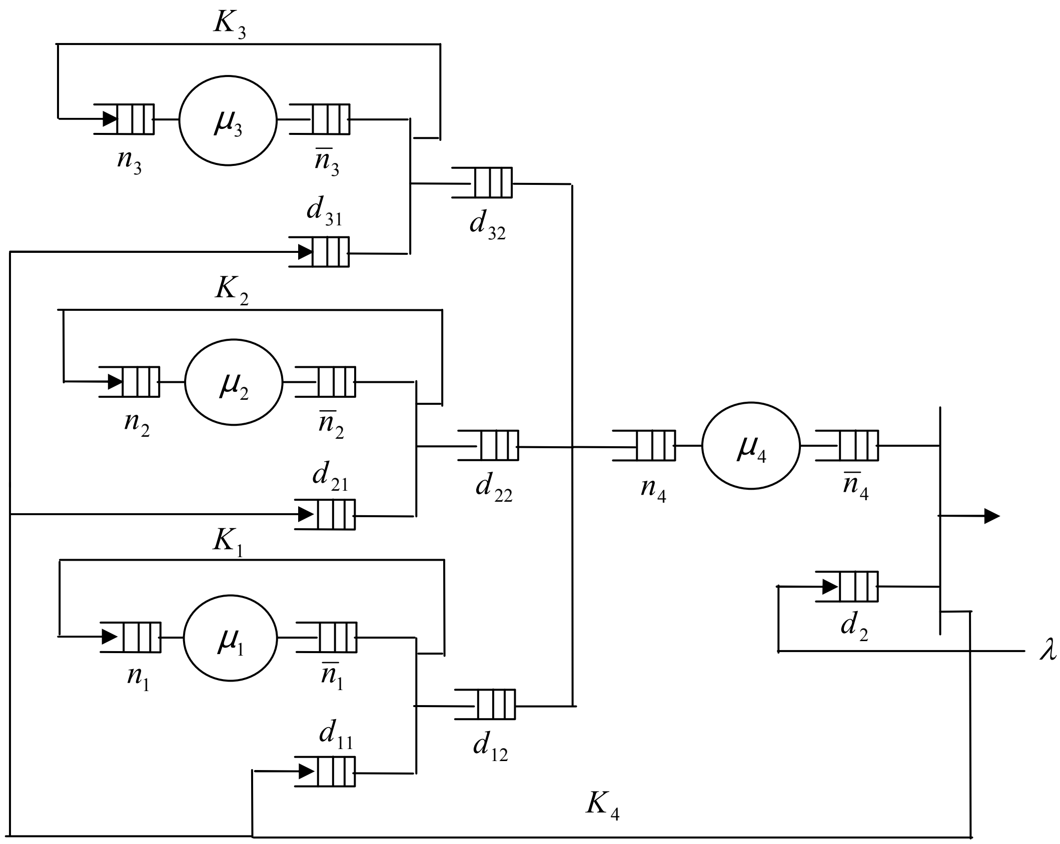

Figure M.1(b) Three-Component Assembly System with Independent Release of Kanbans.

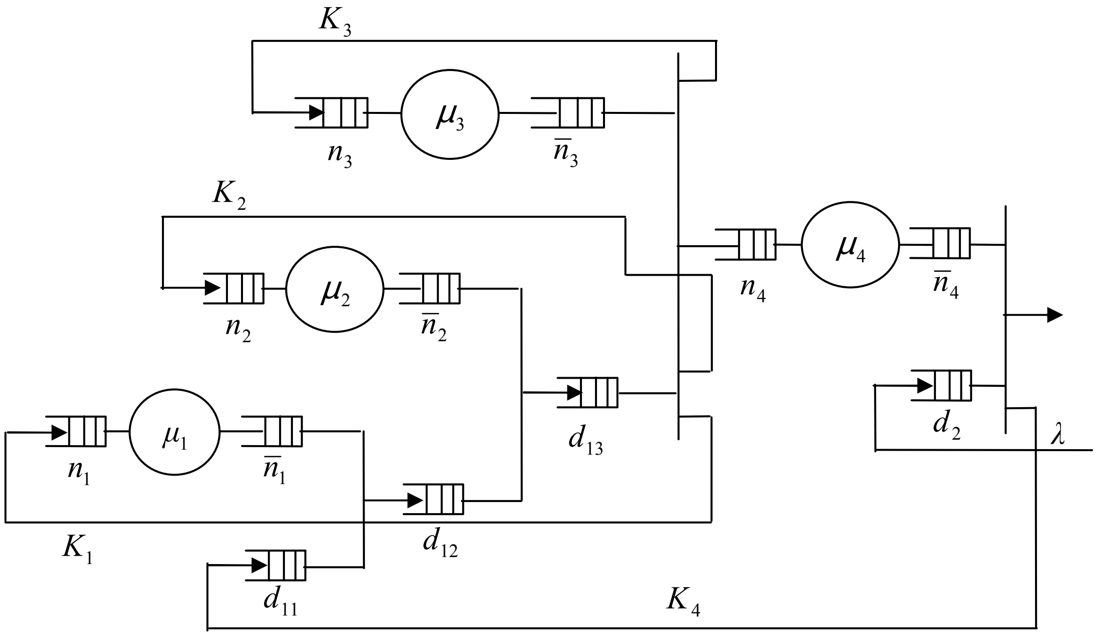

Figure M.2(a) Alternative Model for the Simultaneous Assembly Kanban System.

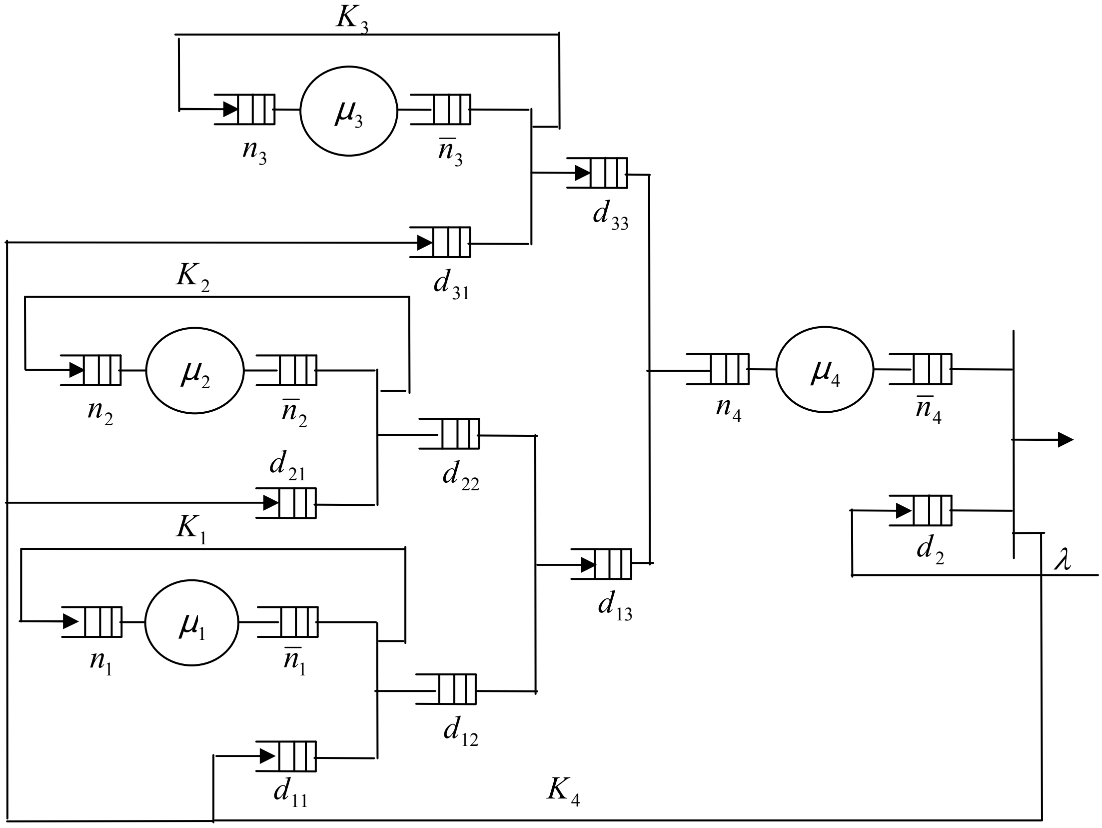

Figure M.2(b) Alternative Model for the Independent Assembly Kanban System.

For the case of SKCS, conditional probabilities that result from the aggregations are the following:

$$q(d_2, d_{12}, d_{13}, n_4) = Pr(\overline{N}_1 = 0 \mid D_2 = d_2, D_{11} = 0, D_{12} = d_{12}, D_{13} = d_{13}, N_4 = n_4),$$

$$q'(d_2, d_{11}, d_{13}, n_4) = Pr(\overline{N}_2 = 0 \mid D_2 = d_2, D_{12} = 0, D_{11} = d_{11}, D_{13} = d_{13}, N_4 = n_4),$$

$$q''(d_2, d_{11}, d_{12}, n_4) = Pr(\overline{N}_3 = 0 \mid D_2 = d_2, D_{11} = d_{11}, D_{12} = d_{12}, D_{13} = 0, N_4 = n_4),$$

$$q^{(3)}(d_2, d_{13}, n_4) = Pr(\overline{N}_1 > 0, \overline{N}_2 = 0 \mid D_2 = d_2, D_{11} = 0, D_{12} = 0, D_{13} = d_{13}, N_4 = n_4),$$

$$q^{(4)}(d_2, d_{13}, n_4) = Pr(\overline{N}_2 > 0, \overline{N}_3 = 0 \mid D_2 = d_2, D_{11} = d_{11}, D_{12} = 0, D_{13} = 0, N_4 = n_4),$$

$$q^{(5)}(d_2, n_4) = Pr(\overline{N}_1 > 0, \overline{N}_2 > 0, \overline{N}_3 = 0 \mid D_2 = d_2, D_{11} = 0, D_{12} = 0, D_{13} = 0, N_4 = n_4).$$

(M.1)

For IKCS, the definition of $q(d_2, d_{12}, d_{13}, n_4)$ is same as the one above and the other conditional probabilities are given as follows:

$$q'(d_2, d_{11}, d_{13}, n_4) = Pr(D_{22} = 0 \mid D_2 = d_2, D_{12} = 0, D_{11} = d_{11}, D_{13} = d_{13}, N_4 = n_4),$$
$$q''(d_2, d_{11}, d_{12}, n_4) = Pr(D_{33} = 0 \mid D_2 = d_2, D_{13} = 0, D_{11} = d_{11}, D_{12} = d_{12}, N_4 = n_4),$$
$$q^{(3)}(d_2, d_{13}, n_4) = Pr(\overline{N}_1 > 0, D_{22} = 0 \mid D_2 = d_2, D_{11} = 0, D_{12} = 0, D_{13} = d_{13}, N_4 = n_4),$$
$$q^{(4)}(d_2, d_{13}, n_4) = Pr(D_{22} > 0, D_{33} = 0 \mid D_2 = d_2, D_{11} = d_{11}, D_{12} = 0, D_{13} = 0, N_4 = n_4),$$
$$q^{(5)}(d_2, n_4) = Pr(\overline{N}_1 > 0, D_{22} > 0, D_{33} = 0 \mid D_2 = d_2, D_{11} = 0, D_{12} = 0, D_{13} = 0, N_4 = n_4).$$
(M.2)

The aggregate model is approximated by assuming that these conditional probabilities are state-independent as seen below. The state-independent conditional probabilities are computed analogously to the ones in (3.25), (3.26) and (3.39), (3.41) for SKCS and IKCS, respectively. In the simultaneous release case,

$$q = Pr(\overline{N}_1 = 0 \mid D_{11} = 0),$$
$$q' = Pr(\overline{N}_2 = 0 \mid D_{12} = 0),$$
$$q'' = Pr(\overline{N}_3 = 0 \mid D_{13} = 0),$$
$$q^{(3)} = (1-q)\cdot q',$$
$$q^{(4)} = (1-q')\cdot q'',$$
$$q^{(5)} = (1-q)\cdot(1-q')\cdot q'', \qquad \text{(M.3)}$$

whereas for the independent release case,

$$q = Pr(\overline{N}_1 = 0 \mid D_{11} = 0),$$
$$q' = Pr(D_{22} = 0 \mid D_{12} = 0),$$
$$q'' = Pr(D_{33} = 0 \mid D_{13} = 0),$$
$$q^{(3)} = (1-q)\cdot q',$$
$$q^{(4)} = (1-q')\cdot q'',$$

$$q^{(5)} = (1-q)\cdot(1-q')\cdot q''. \tag{M.4}$$

Next step is to decompose the approximate aggregate model. When d_2 is equal to zero, the system turns out to be equivalent to the closed network in Figure M.3 (a).

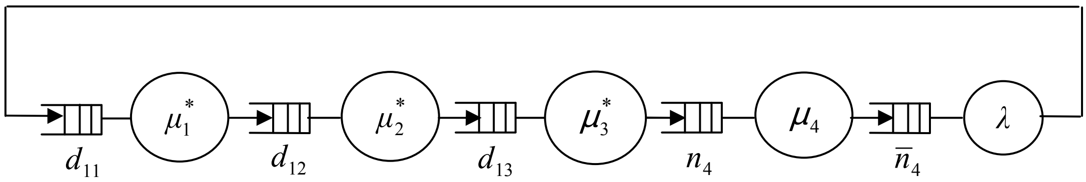

Figure M.3(a) Equivalent Closed Network of the Submodel for $d_2 = 0$.

Effective service rates μ_i^* to process the requests in queues of size d_{1i}, $i = 1, 2, 3$, are given as follows:

$$\mu_i^* = \begin{cases} \mu_i & \text{for } d_{1i} > 0, \\ \mu_i \text{ with probability } \alpha \\ \infty \text{ with probability } 1\text{-}\alpha & \text{for } d_{1i} = 0, \end{cases}$$

α being q and q' and q'' for $i = 1, 2, 3$, respectively. The product-form steady-state distribution for this closed network, i.e., $\Pr(D_{11} = d_{11}, D_{12} = d_{12}, D_{13} = d_{13}, N_4 = n_4 | D_2 = 0)$, is

$$
\begin{cases}
G\rho_1^{D_{11}}\rho_2^{D_{12}}\rho_3^{D_{13}}\rho_4^{N_4} & \text{for } D_{11} > 0,\ D_{12} > 0,\ D_{13} > 0, \\
\frac{G}{q}\rho_1^{D_{11}}\rho_2^{D_{12}}\rho_3^{D_{13}}\rho_4^{N_4} & \text{for } D_{11} = 0,\ D_{12} > 0,\ D_{13} > 0, \\
\frac{G}{q'}\rho_1^{D_{11}}\rho_2^{D_{12}}\rho_3^{D_{13}}\rho_4^{N_4} & \text{for } D_{11} > 0,\ D_{12} = 0,\ D_{13} > 0, \\
\frac{G}{q''}\rho_1^{D_{11}}\rho_2^{D_{12}}\rho_3^{D_{13}}\rho_4^{N_4} & \text{for } D_{11} > 0,\ D_{12} > 0,\ D_{13} = 0, \\
\frac{G}{qq'}\rho_1^{D_{11}}\rho_2^{D_{12}}\rho_3^{D_{13}}\rho_4^{N_4} & \text{for } D_{11} = 0,\ D_{12} = 0,\ D_{13} > 0, \\
\frac{G}{qq''}\rho_1^{D_{11}}\rho_2^{D_{12}}\rho_3^{D_{13}}\rho_4^{N_4} & \text{for } D_{11} = 0,\ D_{12} > 0,\ D_{13} = 0, \\
\frac{G}{q'q''}\rho_1^{D_{11}}\rho_2^{D_{12}}\rho_3^{D_{13}}\rho_4^{N_4} & \text{for } D_{11} > 0,\ D_{12} = 0,\ D_{13} = 0, \\
\frac{G}{qq'q''}\rho_1^{D_{11}}\rho_2^{D_{12}}\rho_3^{D_{13}}\rho_4^{N_4} & \text{for } D_{11} = 0,\ D_{12} = 0,\ D_{13} = 0,
\end{cases}
\quad \text{(M.5)}
$$

where $\rho_i = \frac{\lambda}{\mu_i}$ for $i = 1, 2, 3, 4$ and G is the normalization constant.

Similarly, for any d_2 greater than zero, the closed network is as in Figure M.3 (b).

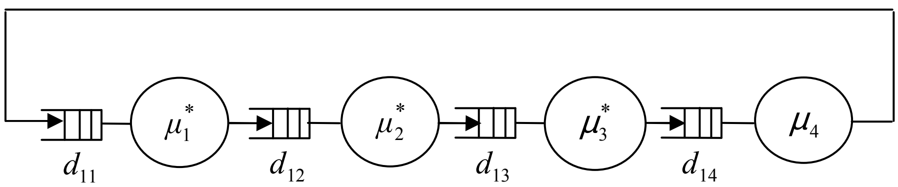

Figure M.3(b) Equivalent Closed Network of the Submodel for $d_2 > 0$.

Then, letting $\tau_i = \dfrac{\mu_4}{\mu_i}$ for $i = 1, 2, 3,$ the product-form steady-state distribution $\Pr(D_{11} = d_{11}, D_{12} = d_{12}, D_{13} = d_{13}, N_4 = n_4 | D_2 = d_2)$ is

$$
\begin{cases}
G\tau_1^{D_{11}}\tau_2^{D_{12}}\tau_3^{D_{13}} & D_{11} > 0, D_{12} > 0, D_{13} > 0, \\
\dfrac{G}{q}\tau_1^{D_{11}}\tau_2^{D_{12}}\tau_3^{D_{13}} & D_{11} = 0, D_{12} > 0, D_{13} > 0, \\
\dfrac{G}{q'}\tau_1^{D_{11}}\tau_2^{D_{12}}\tau_3^{D_{13}} & D_{11} > 0, D_{12} = 0, D_{13} > 0, \\
\dfrac{G}{q''}\tau_1^{D_{11}}\tau_2^{D_{12}}\tau_3^{D_{13}} & D_{11} > 0, D_{12} > 0, D_{13} = 0, \\
\dfrac{G}{qq'}\tau_1^{D_{11}}\tau_2^{D_{12}}\tau_3^{D_{13}} & D_{11} = 0, D_{12} = 0, D_{13} > 0, \\
\dfrac{G}{qq''}\tau_1^{D_{11}}\tau_2^{D_{12}}\tau_3^{D_{13}} & D_{11} = 0, D_{12} > 0, D_{13} = 0, \\
\dfrac{G}{q'q''}\tau_1^{D_{11}}\tau_2^{D_{12}}\tau_3^{D_{13}} & D_{11} > 0, D_{12} = 0, D_{13} = 0, \\
\dfrac{G}{qq'q''}\tau_1^{D_{11}}\tau_2^{D_{12}}\tau_3^{D_{13}} & D_{11} = 0, D_{12} = 0, D_{13} = 0.
\end{cases}
\tag{M.6}
$$

for $d_2 > 0$.

Combining submodels which are investigated independently of the others, we come up with the aggregate states (d_2, y) for all d_2 and $y =$0, 1, 2, 3 and $(d_2, 4)$ for $d_2 = 0$ given below.

$$(d_2,0) : \{\ (d_{11}, d_{12}, d_{13}, n_3) |\ d_{11} + d_{12} + n_3 + n_4 = K_4,\ n_4 = 0,\ d_{13} = 0, d_{12} = 0\ \},$$
$$(d_2,1) : \{\ (d_{11}, d_{12}, d_{13}, n_3) |\ d_{11} + d_{12} + n_3 + n_4 = K_4,\ n_4 = 0,\ d_{13} = 0, d_{12} > 0\ \},$$
$$(d_2,2) : \{\ (d_{11}, d_{12}, d_{13}, n_3) |\ d_{11} + d_{12} + n_3 + n_4 = K_4,\ n_4 = 0,\ d_{13} > 0\ \},$$
$$(d_2,3) : \{\ (d_{11}, d_{12}, d_{13}, n_3) |\ d_{11} + d_{12} + n_3 + n_4 = K_4,\ n_4 > 0\ \},$$

$$(d_2,4):\{\ (d_{11},d_{12},d_{13},n_3)|\ d_{11}+d_{12}+n_3+n_4<K_4\ \}.$$

In order to adjust the transition rates between aggregate states in the combined model, the following probabilities are defined:

$$P_1 = Pr(D_{11}+D_{12}+D_{13}+N_4 = K_4 -1, D_{12}=0, D_{13}=0, N_4=0 \mid D_2=0, D_{11}+D_{12}+D_{13}+N_4<K_4),$$

$$P_2 = Pr(D_{11}+D_{12}+D_{13}+N_4 = K_4 -1, D_{12}>0, D_{13}=0, N_4=0 \mid D_2=0, D_{11}+D_{12}+D_{13}+N_4<K_4),$$

$$P_3 = Pr(D_{11}+D_{12}+D_{13}+N_4 = K_4 -1, N_4=0, D_{13}>0 \mid D_2=0, D_{11}+D_{12}+D_{13}+N_4<K_4),$$

$$P_3 = Pr(D_{11}+D_{12}+D_{13}+N_4 = K_4 -1, N_4>0 \mid D_2=0, D_{11}+D_{12}+D_{13}+N_4<K_4),$$

$$R_1(d_2) = Pr(N_4=1, D_{12}>0, D_{13}>0 \mid D_2=d_2, N_4>0),$$

$$R_2(d_2) = Pr(N_4=1, D_{12}>0, D_{13}=0 \mid D_2=d_2, N_4>0),$$

$$R_3(d_2) = Pr(N_4=1, D_{12}=0, D_{13}=0 \mid D_2=d_2, N_4>0).$$

The (conditional) steady-state distributions of the submodels are used to determine these probabilities as seen below:

$$P_1 = \frac{\Pr(D_{11}=K_4-1, D_{12}=0, D_{13}=0, N_4=0\ |D_2=0)}{\sum_{d_{11}+d_{12}+n_3\le K_4\text{-}1}\Pr(D_{11}=d_{11}, D_{12}=d_{12}, D_{13}=d_{13}, N_4=n_4\ |D_2=0)},$$

$$P_2 = \frac{\sum_{d_{11}+d_{12}+d_{13}=K_4\text{-}1, d_{12}>0}\Pr(D_{11}=d_{11}, D_{12}=d_{12}, D_{13}=0, N_4=0\ |D_2=0)}{\sum_{d_{11}+d_{12}+n_3\le K_4\text{-}1}\Pr(D_{11}=d_{11}, D_{12}=d_{12}, D_{13}=d_{13}, N_4=n_4\ |D_2=0)},$$

$$P_3 = \frac{\sum_{d_{11}+d_{12}+d_{13}=K_4\text{-}1, d_{13}>0}\Pr(D_{11}=d_{11}, D_{12}=d_{12}, D_{13}=d_{13}, N_4=0\ |D_2=0)}{\sum_{d_{11}+d_{12}+n_3\le K_4\text{-}1}\Pr(D_{11}=d_{11}, D_{12}=d_{12}, D_{13}=d_{13}, N_4=n_4\ |D_2=0)},$$

$$P_4 = \frac{\sum\limits_{d_{11}+d_{12}+d_{13}=K_4-1, n_4>0} \Pr(D_{11} = d_{11}, D_{12} = d_{12}, D_{13} = d_{13}, N_4 = n_4 \ | D_2 = 0)}{\sum\limits_{d_{11}+d_{12}+n_3 \le K_4-1} \Pr(D_{11} = d_{11}, D_{12} = d_{12}, D_{13} = d_{13}, N_4 = n_4 \ | D_2 = 0)},$$

$$R_1(d_2) = \frac{\sum\limits_{d_{12}>0, d_{13}>0} \Pr(D_{11} = K_4 - D_{12} - D_{13} - 1, D_{12} = d_{12}, D_{13} = d_{13}, N_4 = 1 \ | D_2 = d_2)}{\sum\limits_{n_4>0} \Pr(D_{11} = K_4 - D_{12} - D_{13} - n_4, D_{12} = d_{12}, N_4 = n_4 \ | D_2 = d_2)},$$

$$R_2(d_2) = \frac{\sum\limits_{d_{12}>0} \Pr(D_{11} = K_4 - D_{12} - 1, D_{12} = d_{12}, D_{13} = 0, N_4 = 1 \ | D_2 = d_2)}{\sum\limits_{n_4>0} \Pr(D_{11} = K_4 - D_{12} - D_{13} - n_4, D_{12} = d_{12}, N_4 = n_4 \ | D_2 = d_2)},$$

$$R_3(d_2) = \frac{\Pr(D_{11} = K_4 - 1, D_{12} = 0, D_{13} = 0, N_4 = 1 \ | D_2 = d_2)}{\sum\limits_{n_4>0} \Pr(D_{11} = K_4 - d_{12} - d_{13} - n_4, D_{12} = d_{12}, N_4 = n_4 \ | D_2 = d_2)}.$$

Recall that R_1, R_2 and R_3 values are not dependent on d_2 when $d_2 > 0$. By the adjustments of the transition rates, the combined model which is a QBD process with infinite number of levels is given in Figure M.4. Note that $\bar{q}' = (1-q')$, $\bar{q}'' = (1-q'')$.

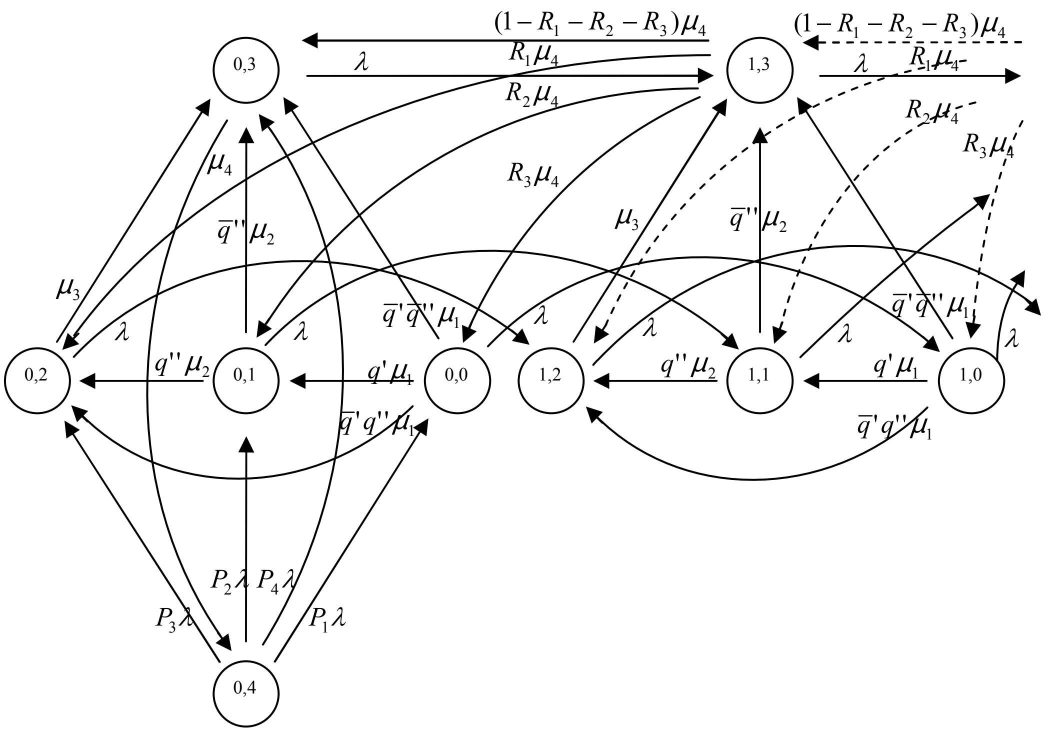

Figure M.4 Transition Diagram of the Combined Model for Three-Component System.

As in the case of two-component systems, the combined models of the three-component systems are the same for both simultaneous and independent release of kanbans given q, q', q'', R_1, R_2, R_3, P_1, P_2, P_3 and P_4 and so are the matrices A_{00}, A_{01}, A_{10}, A_0, A_1 and A_2. But, note that the conditional probabilities q, q', q'', R_1, R_2, R_3, P_1, P_2, P_3 and P_4 are different for simultaneous and independent release cases. For level $d_2 = 0$, the states are ordered as 0, 1, 2, 3 and 4 for y, whereas the states for level $d_2 > 0$ are ordered as 0, 1, 2 and 3 for y. Then, the corresponding rate matrices are given as

$$A_{01} = \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad A_{00} = \begin{bmatrix} \theta_1 & q'\mu_1 & \bar{q}'q''\mu_2 & \bar{q}'\bar{q}'' & 0 \\ 0 & \theta_2 & q''\mu_2 & \bar{q}''\mu_2 & 0 \\ 0 & 0 & \theta_3 & \mu_3 & 0 \\ 0 & 0 & 0 & \theta_4 & \mu_4 \\ P_1\lambda & P_2\lambda & P_3\lambda & P_4\lambda & P\lambda \end{bmatrix},$$

$$A_{10} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ R_3\mu_4 & R_2\mu_4 & R_1\mu_4 & R\mu_4 & 0 \end{bmatrix}, \quad A_0 = \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix},$$

$$A_1 = \begin{bmatrix} \theta_1 & q'\mu_1 & \bar{q}'q''\mu_1 & \bar{q}'\bar{q}''\mu_1 \\ 0 & \theta_2 & q''\mu_2 & \bar{q}''\mu_2 \\ 0 & 0 & \theta_3 & \mu_3 \\ 0 & 0 & 0 & \theta_4 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ R_3\mu_4 & R_2\mu_4 & R_1\mu_4 & R\mu_4 \end{bmatrix},$$

where $R = (1 - R_1 - R_2 - R_3)$, $P = (P_1 + P_2 + P_3 + P_4)$, $\theta_1 = -(\lambda + \mu_1)$, $\theta_2 = -(\lambda + \mu_2)$, $\theta_3 = -(\lambda + \mu_3)$ and $\theta_4 = -(\lambda + \mu_4)$.

Using these matrices with matrix-geometric approach (logarithmic reduction algorithm) explained in section 3.2, steady-state distribution of the model can be calculated. Up to this point, the development applies to both simultaneous and independent kanban systems. The difference between them results from the approximation of q, q' and q'' ($q^{(3)}$, $q^{(4)}$, $q^{(5)}$ are expressed in terms of former ones).

The model in Figure M.2 (a) leads to the following equations:

$$n_1 + \bar{n}_1 + d_{12} + d_{13} = K_1,$$

$$n_2 + \bar{n}_2 + d_{13} = K_2,$$

$$n_3 + \bar{n}_3 = K_3,$$

$$n_4 + \bar{n}_4 + d_{11} + d_{12} = d_{13} = K_4, \tag{M.7}$$

$$\bar{n}_1 \cdot d_{11} = 0,\ \bar{n}_2 \cdot d_{12} = 0,\ \bar{n}_3 \cdot d_{13} = 0 \text{ and } \bar{n}_4 \cdot d_2 = 0. \tag{M.8}$$

Defining M_i, as $M_i = N_i + D_{1i}$, each station with service rate μ_i is analyzed as an independent $M/M/1$ with limited queue size $K_i + K_4$ as in section 3.6 for i = 1, 2, 3. That is, the state-dependent arrival rates are assumed equal to λ. Referring to the definitions of the conditional probabilities q, q' and q'' and the balance equations above, we obtain the following state-independent conditional probabilities:

$$q = Pr(M_1 = K_1 - D_{12} - D_{13} \mid M_1 \le K_1 - D_{12} - D_{13}),$$

$$q' = Pr(M_2 = K_2 - D_{13} \mid M_2 \le K_2 - D_{13}),$$

$$q'' = Pr(M_3 = K_3 \mid M_3 \le K_3),$$

We again prefer to use the expected values of the random variables D_{11}, D_{12}, D_{13} whenever needed as in section 3.6, and come up with the following expressions:

$$q = \frac{(1-\rho_1)\rho_1^{(K_1 - E[D_{12}] - E[D_{13}])}}{\left(1-\rho_1^{K_1+1-E[D_{12}]-E[D_{13}]}\right)} \tag{M.9}$$

$$q' = \frac{(1-\rho_2)\rho_2^{(K_2 - E[D_{13}])}}{\left(1-\rho_2^{K_2+1-E[D_{13}]}\right)}, \tag{M.10}$$

$$q'' = \frac{(1-\rho_3)\rho_3^{K_3}}{\left(1-\rho_3^{K_3+1}\right)}, \tag{M.11}$$

where $E[D_{12}] = E[\max\{M_2 - K_2 + D_{13}, 0\}] = E[\max\{M_2 - K_2 + \max\{M_3 - K_3, 0\}, 0\}]$, $E[D_{13}] = E[\max\{M_3 - K_3, 0\}]$, and $\rho_i = \dfrac{\lambda}{\mu_i}$ for i = 1, ...,3.

Working with the (approximate) M/M/1 distribution of M_3,

$$E[D_{13}] = \frac{\rho_3^{K_3}(1 + K_4\rho_3^{K_4+1} - (K_4+1)\rho_3^{K_4})}{\rho_3^{K_4+K_3+1}(1-\rho_3)}. \qquad \text{(M.12)}$$

This is completely analogous to in (3.28).

In stead of proceeding with $E[D_{12}]$ expressed in terms of random variables M_2 and M_3, $E[D_{12}]$ is calculated directly using the distribution of D_{12} whose state-transition diagram is given in Figure M.5. This is nothing but evaluating the partially aggregated but exact version of the system in Figure 3.16 with its state-transition diagram in Figure 3.17.

Figure M.5 State-Transition Diagram of the Partially Aggregated Model in Figure 3.16 with State Description d_{12}.

Therefore, one can obtain the following expression for $E[D_{12}]$:

$$E[D_{12}] = \frac{\rho_2}{\frac{1}{(1-q')} + \frac{(1-\rho_2^{K_4+1})}{1-\rho_2}} \cdot \frac{(1 + K_4\,\rho_2^{K_4+1} - (K_4+1)\rho_1^{K_4})}{(1-\rho_2)^2} \qquad \text{(M.13)}$$

For three-component systems, the state-independent conditional probabilities are calculated in the following order: q'', $E[D_{13}]$, q', $E[D_{12}]$ and q in (M.11), (M.12), (M.10), (M.13) and (M.19), respectively.

As for the independent kanban assembly, the equations to be satisfied in the model in Figure M.2 (b) are as follows:

$n_i + \overline{n}_i = K_i$ for $i = 1, 2, 3$,

$$n_4 + \overline{n}_4 + d_{11} + d_{12} = d_{13} = K_4 , \tag{M.14}$$

$d_{11} + d_{12} = d_{21} + d_{22}$,

$$d_{11} + d_{12} + d_{13} = d_{31} + d_{33} , \tag{M.15}$$

$\overline{n}_1 \cdot d_{11} = 0$, $d_{22} \cdot d_{12} = 0$, $d_{33} \cdot d_{13} = 0$ $\overline{n}_2 \cdot d_{21} = 0$, $\overline{n}_3 \cdot d_{31} = 0$

and $\overline{n}_4 \cdot d_2 = 0$. (M.16)

Then, letting $M_i = N_i + D_{i1}$, each being the state description of an $M/M/1$ with a limited queue size of $K_i + K_4$ treated independently, we come up with the following state-independent q, q' and q'' analogous to the ones in section 3.7:

$$q = Pr(M_1 = K_1 \mid M_1 \le K_1) ,$$

$$q' = Pr(M_2 = K_2 + D_{11} \mid M_2 \le K_2 + D_{11}) ,$$

$$q'' = Pr(D_{31} = D_{11} + D_{12} \mid D_{31} \le D_{11} + D_{12})$$
$$= Pr(M_3 = K_3 + D_{11} + D_{12} \mid M_3 \le K_3 + D_{11} + D_{12})$$

As before, working with the expected values of D_{1i} whenever D_{1i} appears,

$$q = \frac{(1-\rho_1)\rho_1^{K_1}}{(1-\rho_1^{K_1+1})} , \tag{M.17}$$

$$q' = \frac{(1-\rho_2)\rho_2^{K_2+E[D_{11}]}}{(1-\rho_2^{K_2+1+E[D_{11}]})} , \tag{M.18}$$

$$q'' = \frac{(1-\rho_3)\rho_3^{K_3+E[D_{11}]+E[D_{12}]}}{(1-\rho_3^{K_3+1+E[D_{11}]+E[D_{12}]})}, \tag{M.19}$$

where $E[D_{11}] = E[\max\{M_1 - K_1, 0\}]$ and

$E[D_{12}] = E[\max\{\max\{M_2 - K_2, 0\} - \max\{M_1 - K_1, 0\}, 0]$, and $\rho_i = \dfrac{\lambda}{\mu_i}$ for $i = 1,...,3$.

Using the same approach used for (3.43),

$$E[D_{11}] = \frac{\rho_1^{K_1}(1 + K_4\rho_1^{K_4+1} - (K_4+1)\rho_1^{K_4})}{\rho_1^{K_4+K_1+1}(1-\rho_1)}. \tag{M.20}$$

Referring to the state-transition diagram in Figure M.5, the equation in (M.13) for $E[D_{12}]$ is valid also for IKCS .

Hence, the order of computation is as follows: q, $E[D_{11}]$, q', $E[D_{12}]$ and q'' in (M.17), (M.20), (M.18), (M.13) and (M.19), respectively.

For four-component SKCS, similarly the following state-independent conditional probabilities are obtained:

$$q = Pr(M_1 = K_1 - D_{12} - D_{13} - D_{14} \mid M_1 \le K_1 - D_{12} - D_{13} - D_{14}),$$

$$q' = Pr(M_2 = K_2 - D_{13} - D_{14} \mid M_2 \le K_2 - D_{13} - D_{14}),$$

$$q'' = Pr(M_3 = K_3 - D_{14} \mid M_3 \le K_3 - D_{14}),$$

$$q^{(3)} = Pr(M_4 = K_4 \mid M_4 \le K_4),$$

to be calculated as

$$q = \frac{(1 - \rho_1)\rho_1^{\left(K_1 - E[D_{12}] - E[D_{13}] - E[D_{14}]\right)}}{\left(1 - \rho_1^{K_1 + 1 - E[D_{12}] - E[D_{13}] - E[D_{14}]}\right)}, \qquad \text{(M.21)}$$

$$q' = \frac{(1 - \rho_2)\rho_2^{\left(K_2 - E[D_{13}] - E[D_{14}]\right)}}{\left(1 - \rho_2^{K_2 + 1 - E[D_{13}] - E[D_{14}]}\right)}, \qquad \text{(M.22)}$$

$$q'' = \frac{(1 - \rho_3)\rho_3^{\left(K_3 - E[D_{14}]\right)}}{\left(1 - \rho_3^{K_3 + 1 - E[D_{14}]}\right)}, \qquad \text{(M.23)}$$

$$q^{(3)} = \frac{(1 - \rho_4)\rho_4^{K_4}}{\left(1 - \rho_4^{K_4 + 1}\right)}. \qquad \text{(M.24)}$$

$E[D_{12}]$ and $E[D_{13}]$ are calculated as in (M.13), whereas $E[D_{14}]$ is calculated as in (M.12).

Finally, the state-independent conditional probabilities for four-component IKCS are given as follows:

$$q = Pr(M_1 = K_1 \mid M_1 \le K_1),$$

$$q' = Pr(M_2 = K_2 - D_{11} \mid M_2 \le K_2 - D_{11}),$$

$$q'' = Pr(M_3 = K_3 - D_{11} - D_{12} \mid M_3 \le K_3 - D_{11} - D_{12}),$$

$$q^{(3)} = Pr(M_4 = K_4 - D_{11} - D_{12} - D_{13} \mid M_4 \le K_4 - D_{11} - D_{12} - D_{13}),$$

being replaced by

$$q = \frac{(1 - \rho_1)\rho_1^{K_1}}{\left(1 - \rho_1^{K_1 + 1}\right)} \qquad \text{(M.25)}$$

$$q' = \frac{(1 - \rho_2)\rho_2^{(K_2 - E[D_{11}])}}{\left(1 - \rho_2^{K_2 + 1 - E[D_{11}]}\right)}, \tag{M.26}$$

$$q'' = \frac{(1 - \rho_3)\rho_3^{(K_3 - E[D_{11}] - E[D_{12}])}}{\left(1 - \rho_3^{K_3 + 1 - E[D_{11}] - E[D_{12}]}\right)}, \tag{M.27}$$

$$q^{(3)} = \frac{(1 - \rho_4)\rho_4^{(K_4 - E[D_{11}] - E[D_{12}] - E[D_{13}])}}{\left(1 - \rho_4^{K_4 + 1 - E[D_{11}] - E[D_{12}] - E[D_{13}]}\right)}. \tag{M.28}$$

$E[D_{12}]$ and $E[D_{13}]$ are calculated as in (M.13), whereas $E[D_{11}]$ is calculated as in (M.20).

APPENDIX N

RESULTS OF THE APPROXIMATE COMBINED MODEL FOR THREE- AND FOUR-COMPONENT SYSTEMS

Table N.1 (a) Numerical results for SKCS with $K_i = 5$, $i = 1, 2, 3, 4$.

K1=5, K2=5, K3=5, K4=5 (SKCS)															
Traffic Intensity				EB			FR			SP			FG		
				Sim.	App.	Abs. Rel. Error (%)	Sim.	App.	Abs. Error	Sim.	App.	Abs. Error	Sim.	App.	Abs. Rel. Error (%)
ρ1	ρ2	ρ3	ρ4												
0.5	0.5	0.5	0.5	0.040	0.042	7.10	0.963	0.961	0.002	0.019	0.020	0.001	3.981	3.947	0.86
0.5	0.5	0.5	0.8	1.351	1.390	2.90	0.667	0.660	0.007	0.267	0.273	0.006	2.286	2.246	1.74
0.5	0.5	0.8	0.5	0.626	0.339	45.75	0.835	0.862	0.026	0.127	0.098	0.029	3.279	3.327	1.48
0.5	0.5	0.8	0.8	2.779	2.752	0.96	0.542	0.532	0.010	0.393	0.400	0.007	1.806	1.738	3.81
0.5	0.8	0.5	0.5	0.627	0.336	46.36	0.836	0.863	0.028	0.127	0.097	0.031	3.281	3.342	1.84
0.5	0.8	0.5	0.8	2.822	2.730	3.27	0.540	0.535	0.005	0.395	0.398	0.002	1.799	1.748	2.83
0.5	0.8	0.8	0.5	1.868	1.043	44.19	0.685	0.730	0.045	0.267	0.214	0.053	2.576	2.661	3.31
0.5	0.8	0.8	0.8	5.581	5.571	0.17	0.408	0.389	0.020	0.535	0.551	0.016	1.328	1.218	8.28
0.8	0.5	0.5	0.5	0.611	0.337	44.81	0.837	0.863	0.026	0.126	0.097	0.029	3.286	3.341	1.68
0.8	0.5	0.5	0.8	2.751	2.731	0.71	0.544	0.534	0.009	0.391	0.398	0.006	1.812	1.748	3.51
0.8	0.5	0.8	0.5	1.879	1.045	44.41	0.685	0.730	0.045	0.268	0.214	0.053	2.577	2.661	3.24
0.8	0.5	0.8	0.8	5.612	5.576	0.65	0.408	0.389	0.020	0.535	0.551	0.016	1.328	1.218	8.27
0.8	0.8	0.5	0.5	1.916	0.957	50.03	0.683	0.744	0.061	0.270	0.202	0.068	2.573	2.733	6.22
0.8	0.8	0.5	0.8	5.535	5.174	6.52	0.410	0.404	0.006	0.533	0.534	0.002	1.334	1.274	4.50
0.8	0.8	0.8	0.5	4.497	2.345	47.85	0.522	0.587	0.065	0.430	0.351	0.080	1.898	2.032	7.08
0.8	0.8	0.8	0.8	12.147	12.273	1.04	0.268	0.241	0.026	0.690	0.715	0.024	0.851	0.728	14.47

Table N.1(b) Numerical results for SKCS with $K_i = 10$, $i = 1, 2, 3, 4$.

K1=10, K2=10, K3=10, K4=10 (SKCS)															
Traffic Intensity				EB			FR			SP			FG		
				Sim.	App.	Abs. Rel. Error (%)	Sim.	App.	Abs. Error	Sim.	App.	Abs. Error	Sim.	App.	Abs. Rel. Error (%)
ρ1	ρ2	ρ3	ρ4												
0.5	0.5	0.5	0.5	0.001	0.001	6.30	0.999	0.999	0.000	0.000	0.000	0.000	9.000	8.998	0.02
0.5	0.5	0.5	0.8	0.420	0.430	2.45	0.893	0.893	0.001	0.085	0.086	0.001	6.434	6.427	0.11
0.5	0.5	0.8	0.5	0.058	0.042	28.30	0.984	0.986	0.002	0.012	0.010	0.002	8.653	8.651	0.02
0.5	0.5	0.8	0.8	0.569	0.581	2.06	0.872	0.869	0.003	0.105	0.107	0.002	6.199	6.130	1.11
0.5	0.8	0.5	0.5	0.063	0.042	33.64	0.984	0.986	0.003	0.013	0.010	0.003	8.643	8.652	0.10
0.5	0.8	0.5	0.8	0.565	0.580	2.67	0.873	0.869	0.004	0.104	0.107	0.003	6.210	6.131	1.28
0.5	0.8	0.8	0.5	0.158	0.113	28.45	0.966	0.969	0.003	0.028	0.024	0.004	8.311	8.274	0.45
0.5	0.8	0.8	0.8	0.710	0.769	8.29	0.853	0.842	0.011	0.122	0.131	0.009	5.998	5.820	2.96
0.8	0.5	0.5	0.5	0.063	0.042	34.13	0.984	0.986	0.002	0.013	0.010	0.002	8.650	8.652	0.01
0.8	0.5	0.5	0.8	0.560	0.581	3.66	0.873	0.869	0.004	0.103	0.107	0.004	6.209	6.131	1.25
0.8	0.5	0.8	0.5	0.147	0.113	23.44	0.966	0.969	0.003	0.027	0.024	0.003	8.312	8.274	0.46
0.8	0.5	0.8	0.8	0.705	0.769	9.01	0.853	0.842	0.011	0.122	0.131	0.010	6.000	5.820	2.99
0.8	0.8	0.5	0.5	0.144	0.109	24.44	0.967	0.970	0.003	0.026	0.023	0.003	8.328	8.296	0.39
0.8	0.8	0.5	0.8	0.708	0.757	7.02	0.853	0.844	0.010	0.121	0.130	0.008	6.004	5.838	2.77
0.8	0.8	0.8	0.5	0.247	0.203	17.87	0.950	0.950	0.001	0.042	0.040	0.002	8.029	7.902	1.59
0.8	0.8	0.8	0.8	0.895	0.990	10.65	0.833	0.814	0.019	0.140	0.157	0.016	5.803	5.514	4.99

Table N.2(a) Numerical results for IKCS with $K_i = 5$, $i = 1, 2, 3, 4$.

K1=5, K2=5, K3=5, K4=5 (IKCS)															
Traffic Intensity				EB			FR			SP			FG		
ρ1	ρ2	ρ3	ρ4	Sim.	App.	Abs. Rel. Error (%)	Sim.	App.	Abs. Error	Sim.	App.	Abs. Error	Sim.	App.	Abs. Rel. Error (%)
0.5	0.5	0.5	0.5	0.040	0.042	5.93	0.964	0.961	0.002	0.019	0.020	0.001	3.981	3.949	0.81
0.5	0.5	0.5	0.8	1.347	1.388	3.03	0.667	0.660	0.007	0.267	0.273	0.006	2.286	2.248	1.65
0.5	0.5	0.8	0.5	0.592	0.329	44.52	0.840	0.865	0.025	0.123	0.095	0.028	3.302	3.355	1.58
0.5	0.5	0.8	0.8	2.754	2.696	2.09	0.545	0.537	0.008	0.390	0.395	0.005	1.817	1.758	3.20
0.5	0.8	0.5	0.5	0.587	0.329	43.93	0.841	0.865	0.024	0.122	0.095	0.027	3.305	3.354	1.49
0.5	0.8	0.5	0.8	2.746	2.698	1.74	0.545	0.537	0.008	0.390	0.395	0.005	1.817	1.758	3.24
0.5	0.8	0.8	0.5	1.162	0.906	22.02	0.746	0.752	0.006	0.207	0.194	0.013	2.831	2.774	2.00
0.5	0.8	0.8	0.8	4.386	4.954	12.94	0.448	0.413	0.035	0.491	0.525	0.034	1.462	1.307	10.65
0.8	0.5	0.5	0.5	0.593	0.328	44.69	0.840	0.866	0.026	0.123	0.095	0.028	3.300	3.363	1.89
0.8	0.5	0.5	0.8	2.728	2.687	1.50	0.545	0.538	0.007	0.390	0.394	0.004	1.817	1.764	2.88
0.8	0.5	0.8	0.5	1.172	0.839	28.36	0.745	0.764	0.018	0.208	0.184	0.024	2.829	2.833	0.14
0.8	0.5	0.8	0.8	4.387	4.666	6.37	0.450	0.426	0.024	0.489	0.511	0.022	1.469	1.352	7.94
0.8	0.8	0.5	0.5	1.169	0.840	28.14	0.745	0.764	0.018	0.208	0.184	0.024	2.826	2.833	0.22
0.8	0.8	0.5	0.8	4.505	4.668	3.61	0.446	0.426	0.020	0.493	0.511	0.018	1.456	1.352	7.15
0.8	0.8	0.8	0.5	1.758	1.649	6.20	0.670	0.657	0.013	0.277	0.284	0.006	2.477	2.338	5.58
0.8	0.8	0.8	0.8	6.482	8.176	26.13	0.368	0.315	0.054	0.577	0.633	0.056	1.182	0.970	17.97

Table N.2(b) Numerical results for IKCS with $K_i = 10$, $i = 1, 2, 3, 4$.

K1=10, K2=10, K3=10, K4=10 (IKCS)															
Traffic Intensity				EB			FR			SP			FG		
ρ1	ρ2	ρ3	ρ4	Sim.	App.	Abs. Rel. Error (%)	Sim.	App.	Abs. Error	Sim.	App.	Abs. Error	Sim.	App.	Abs. Rel. Error (%)
0.5	0.5	0.5	0.5	0.001	0.001	0.01	0.999	0.999	0.000	0.000	0.000	0.000	3.981	3.949	0.81
0.5	0.5	0.5	0.8	0.424	0.430	1.35	0.894	0.893	0.001	0.085	0.086	0.001	2.286	2.248	1.65
0.5	0.5	0.8	0.5	0.058	0.042	28.65	0.984	0.986	0.002	0.012	0.010	0.002	3.302	3.355	1.58
0.5	0.5	0.8	0.8	0.570	0.580	1.88	0.872	0.869	0.004	0.104	0.107	0.003	1.817	1.758	3.20
0.5	0.8	0.5	0.5	0.061	0.042	32.15	0.984	0.986	0.002	0.013	0.010	0.002	3.305	3.354	1.49
0.5	0.8	0.5	0.8	0.574	0.580	1.04	0.872	0.869	0.003	0.104	0.107	0.003	1.817	1.758	3.24
0.5	0.8	0.8	0.5	0.124	0.107	13.22	0.971	0.970	0.000	0.024	0.023	0.000	2.831	2.774	2.00
0.5	0.8	0.8	0.8	0.703	0.754	7.27	0.854	0.844	0.010	0.121	0.129	0.008	1.462	1.307	10.65
0.8	0.5	0.5	0.5	0.062	0.042	32.59	0.984	0.986	0.002	0.013	0.010	0.002	3.300	3.363	1.89
0.8	0.5	0.5	0.8	0.564	0.580	2.99	0.872	0.869	0.003	0.104	0.107	0.003	1.817	1.764	2.88
0.8	0.5	0.8	0.5	0.119	0.103	13.28	0.971	0.971	0.001	0.024	0.022	0.001	2.829	2.833	0.14
0.8	0.5	0.8	0.8	0.684	0.744	8.71	0.856	0.845	0.010	0.119	0.128	0.009	1.469	1.352	7.94
0.8	0.8	0.5	0.5	0.115	0.103	10.20	0.971	0.971	0.000	0.023	0.022	0.001	2.826	2.833	0.22
0.8	0.8	0.5	0.8	0.685	0.744	8.62	0.855	0.845	0.010	0.120	0.128	0.008	1.456	1.352	7.15
0.8	0.8	0.8	0.5	0.174	0.179	2.89	0.959	0.955	0.004	0.033	0.036	0.003	2.477	2.338	5.58
0.8	0.8	0.8	0.8	0.801	0.929	15.99	0.839	0.821	0.018	0.134	0.150	0.016	1.182	0.970	17.97

Table N.3 (a) Numerical results for SKCS with $K_i = 5$, $i = 1, 2, 3, 4, 5$.

K1=5, K2=5, K3=5, K4=5, K5=5 (SKCS)																
Traffic Intensity					EB			FR			SP			FG		
					Sim.	App.	Abs. Rel. Error (%)	Sim.	App.	Abs. Error	Sim.	App.	Abs. Error	Sim.	App.	Abs. Rel. Error (%)
ρ1	ρ2	ρ3	ρ4	ρ5												
0.5	0.5	0.5	0.5	0.5	0.043	0.046	8.49	0.962	0.958	0.003	0.020	0.022	0.002	3.966	3.918	1.20
0.5	0.5	0.5	0.5	0.8	1.364	1.419	4.04	0.665	0.655	0.009	0.269	0.277	0.008	2.278	2.224	2.36
0.5	0.5	0.5	0.8	0.5	0.636	0.352	44.68	0.833	0.857	0.024	0.129	0.101	0.028	3.264	3.292	0.85
0.5	0.5	0.5	0.8	0.8	2.797	2.822	0.89	0.541	0.526	0.015	0.394	0.406	0.012	1.802	1.710	5.09
0.5	0.5	0.8	0.5	0.5	0.629	0.346	45.11	0.834	0.860	0.026	0.129	0.099	0.029	3.270	3.315	1.38
0.5	0.5	0.8	0.5	0.8	2.774	2.781	0.27	0.543	0.530	0.013	0.392	0.402	0.010	1.810	1.728	4.55
0.5	0.5	0.8	0.8	0.5	1.871	1.064	43.11	0.684	0.725	0.041	0.268	0.218	0.050	2.572	2.629	2.23
0.5	0.5	0.8	0.8	0.8	5.670	5.726	0.99	0.408	0.382	0.026	0.535	0.558	0.023	1.328	1.193	10.20
0.5	0.8	0.5	0.5	0.5	0.634	0.346	45.43	0.834	0.860	0.026	0.129	0.099	0.029	3.270	3.314	1.37
0.5	0.8	0.5	0.5	0.8	2.793	2.783	0.36	0.541	0.530	0.011	0.394	0.402	0.008	1.803	1.728	4.19
0.5	0.8	0.5	0.8	0.5	1.911	1.066	44.22	0.681	0.725	0.044	0.271	0.218	0.053	2.562	2.629	2.62
0.5	0.8	0.5	0.8	0.8	5.600	5.731	2.35	0.408	0.382	0.026	0.535	0.558	0.023	1.328	1.192	10.19
0.5	0.8	0.8	0.5	0.5	1.880	0.972	48.31	0.684	0.740	0.056	0.268	0.205	0.064	2.573	2.709	5.29
0.5	0.8	0.8	0.5	0.8	5.695	5.279	7.30	0.406	0.399	0.006	0.538	0.539	0.002	1.319	1.255	4.87
0.5	0.8	0.8	0.8	0.5	4.608	2.386	48.23	0.515	0.582	0.067	0.437	0.355	0.082	1.870	2.004	7.21
0.5	0.8	0.8	0.8	0.8	11.912	12.753	7.06	0.268	0.234	0.033	0.690	0.723	0.033	0.850	0.704	17.22
0.8	0.5	0.5	0.5	0.5	0.619	0.347	44.00	0.834	0.860	0.025	0.128	0.100	0.029	3.271	3.314	1.32
0.8	0.5	0.5	0.5	0.8	2.796	2.785	0.41	0.542	0.530	0.012	0.393	0.402	0.009	1.806	1.727	4.35
0.8	0.5	0.5	0.8	0.5	1.899	1.068	43.76	0.686	0.725	0.039	0.267	0.219	0.048	2.580	2.629	1.87
0.8	0.5	0.5	0.8	0.8	5.604	5.737	2.37	0.407	0.382	0.025	0.537	0.558	0.022	1.322	1.192	9.85
0.8	0.5	0.8	0.5	0.5	1.892	0.973	48.56	0.684	0.740	0.057	0.269	0.205	0.064	2.570	2.708	5.39
0.8	0.5	0.8	0.5	0.8	5.719	5.283	7.64	0.405	0.399	0.006	0.538	0.540	0.001	1.317	1.255	4.69
0.8	0.5	0.8	0.8	0.5	4.468	2.389	46.52	0.520	0.582	0.062	0.432	0.356	0.076	1.890	2.004	6.02
0.8	0.5	0.8	0.8	0.8	12.568	12.767	1.58	0.263	0.234	0.029	0.695	0.723	0.028	0.837	0.704	15.90
0.8	0.8	0.5	0.5	0.5	1.878	0.974	48.13	0.685	0.740	0.055	0.267	0.205	0.063	2.576	2.708	5.12
0.8	0.8	0.5	0.5	0.8	5.632	5.286	6.15	0.408	0.399	0.008	0.536	0.540	0.004	1.326	1.255	5.35
0.8	0.8	0.5	0.8	0.5	4.561	2.393	47.54	0.518	0.582	0.064	0.435	0.356	0.079	1.879	2.003	6.61
0.8	0.8	0.5	0.8	0.8	12.248	12.778	4.33	0.264	0.234	0.030	0.695	0.723	0.028	0.841	0.703	16.32
0.8	0.8	0.8	0.5	0.5	4.487	2.073	53.80	0.519	0.611	0.092	0.433	0.327	0.106	1.885	2.133	13.15
0.8	0.8	0.8	0.5	0.8	12.434	10.622	14.57	0.264	0.266	0.001	0.694	0.687	0.007	0.841	0.807	4.01
0.8	0.8	0.8	0.8	0.5	11.174	4.894	56.20	0.337	0.433	0.096	0.625	0.508	0.117	1.188	1.423	19.74
0.8	0.8	0.8	0.8	0.8	39.203	46.297	18.10	0.116	0.084	0.032	0.864	0.899	0.035	0.362	0.243	32.90

Table N.3 (b) Numerical results for SKCS with $K_i = 10$, $i = 1, 2, 3, 4, 5$.

K1=10, K2=10, K3=10, K4=10, K5=10 (SKCS)																
Traffic Intensity					EB			FR			SP			FG		
					Sim.	App.	Abs. Rel. Error (%)	Sim.	App.	Abs. Error	Sim.	App.	Abs. Error	Sim.	App.	Abs. Rel. Error (%)
ρ1	ρ2	ρ3	ρ4	ρ5												
0.5	0.5	0.5	0.5	0.5	0.001	0.001	12.15	0.999	0.999	0.000	0.000	0.000	0.000	8.999	8.997	0.02
0.5	0.5	0.5	0.5	0.8	0.428	0.430	0.38	0.893	0.892	0.000	0.086	0.086	0.000	6.432	6.426	0.09
0.5	0.5	0.5	0.8	0.5	0.064	0.042	35.37	0.983	0.986	0.003	0.013	0.010	0.003	8.644	8.650	0.07
0.5	0.5	0.5	0.8	0.8	0.563	0.581	3.13	0.873	0.869	0.004	0.104	0.107	0.003	6.209	6.129	1.28
0.5	0.5	0.8	0.5	0.5	0.061	0.042	32.22	0.984	0.986	0.002	0.013	0.010	0.002	8.649	8.651	0.02
0.5	0.5	0.8	0.5	0.8	0.559	0.581	3.79	0.873	0.869	0.004	0.104	0.107	0.003	6.216	6.130	1.39
0.5	0.5	0.8	0.8	0.5	0.148	0.113	23.60	0.967	0.969	0.003	0.027	0.024	0.003	8.314	8.273	0.50
0.5	0.5	0.8	0.8	0.8	0.715	0.769	7.51	0.853	0.842	0.011	0.122	0.131	0.009	6.001	5.819	3.04
0.5	0.8	0.5	0.5	0.5	0.059	0.042	29.52	0.984	0.986	0.002	0.012	0.010	0.002	8.653	8.651	0.02
0.5	0.8	0.5	0.5	0.8	0.561	0.581	3.46	0.872	0.869	0.004	0.104	0.107	0.003	6.202	6.130	1.16
0.5	0.8	0.5	0.8	0.5	0.144	0.113	21.94	0.967	0.969	0.002	0.027	0.024	0.003	8.321	8.273	0.57
0.5	0.8	0.5	0.8	0.8	0.715	0.769	7.56	0.853	0.842	0.011	0.122	0.131	0.009	5.997	5.819	2.97
0.5	0.8	0.8	0.5	0.5	0.148	0.109	26.78	0.967	0.970	0.003	0.027	0.023	0.004	8.319	8.295	0.30
0.5	0.8	0.8	0.5	0.8	0.718	0.758	5.59	0.853	0.843	0.009	0.122	0.130	0.007	5.999	5.837	2.71
0.5	0.8	0.8	0.8	0.5	0.255	0.203	20.46	0.947	0.950	0.003	0.043	0.040	0.004	8.000	7.901	1.24
0.5	0.8	0.8	0.8	0.8	0.904	0.990	9.61	0.831	0.814	0.017	0.142	0.157	0.015	5.786	5.513	4.72
0.8	0.5	0.5	0.5	0.5	0.063	0.042	33.73	0.984	0.986	0.002	0.013	0.010	0.002	8.649	8.651	0.02
0.8	0.5	0.5	0.5	0.8	0.570	0.581	1.89	0.872	0.869	0.004	0.104	0.107	0.003	6.209	6.130	1.27
0.8	0.5	0.5	0.8	0.5	0.154	0.113	26.62	0.966	0.969	0.003	0.028	0.024	0.003	8.313	8.273	0.48
0.8	0.5	0.5	0.8	0.8	0.714	0.769	7.76	0.853	0.842	0.011	0.122	0.131	0.009	6.000	5.819	3.01
0.8	0.5	0.8	0.5	0.5	0.144	0.109	24.81	0.967	0.970	0.003	0.026	0.023	0.003	8.328	8.295	0.40
0.8	0.5	0.8	0.5	0.8	0.717	0.758	5.73	0.854	0.843	0.010	0.121	0.130	0.008	6.011	5.837	2.90
0.8	0.5	0.8	0.8	0.5	0.258	0.203	21.28	0.948	0.950	0.002	0.043	0.040	0.003	8.009	7.901	1.36
0.8	0.5	0.8	0.8	0.8	0.903	0.990	9.64	0.832	0.814	0.018	0.141	0.157	0.015	5.794	5.513	4.86
0.8	0.8	0.5	0.5	0.5	0.151	0.109	28.08	0.966	0.970	0.004	0.027	0.023	0.004	8.321	8.295	0.32
0.8	0.8	0.5	0.5	0.8	0.715	0.758	5.97	0.852	0.843	0.009	0.123	0.130	0.007	5.997	5.837	2.67
0.8	0.8	0.5	0.8	0.5	0.240	0.203	15.36	0.950	0.950	0.000	0.041	0.040	0.002	8.031	7.901	1.62
0.8	0.8	0.5	0.8	0.8	0.878	0.990	12.75	0.833	0.814	0.019	0.140	0.157	0.016	5.798	5.513	4.93
0.8	0.8	0.8	0.5	0.5	0.238	0.193	19.12	0.950	0.952	0.003	0.042	0.038	0.003	8.025	7.943	1.02
0.8	0.8	0.8	0.5	0.8	0.875	0.964	10.19	0.833	0.817	0.016	0.140	0.154	0.014	5.805	5.548	4.44
0.8	0.8	0.8	0.8	0.5	0.373	0.313	16.23	0.929	0.930	0.001	0.060	0.057	0.003	7.717	7.535	2.37
0.8	0.8	0.8	0.8	0.8	1.092	1.249	14.38	0.810	0.785	0.025	0.162	0.184	0.022	5.588	5.210	6.75

Table N.4 (a) Numerical results for IKCS with $K_i = 5$, $i = 1, 2, 3, 4, 5$.

K1=5, K2=5, K3=5, K4=5, K5=5 (IKCS)																
Traffic Intensity					EB			FR			SP			FG		
					Sim.	App.	Abs. Rel. Error (%)	Sim.	App.	Abs. Error	Sim.	App.	Abs. Error	Sim.	App.	Abs. Rel. Error (%)
ρ1	ρ2	ρ3	ρ4	ρ5												
0.5	0.5	0.5	0.5	0.5	0.041	0.046	10.54	0.962	0.959	0.003	0.020	0.022	0.002	3.968	3.922	1.15
0.5	0.5	0.5	0.5	0.8	1.353	1.414	4.53	0.666	0.656	0.010	0.268	0.277	0.009	2.283	2.228	2.43
0.5	0.5	0.5	0.8	0.5	0.601	0.336	44.18	0.838	0.863	0.024	0.125	0.097	0.028	3.290	3.332	1.27
0.5	0.5	0.5	0.8	0.8	2.780	2.738	1.52	0.542	0.533	0.009	0.393	0.399	0.006	1.807	1.741	3.65
0.5	0.5	0.8	0.5	0.5	0.588	0.336	42.81	0.839	0.862	0.023	0.123	0.097	0.026	3.296	3.332	1.10
0.5	0.5	0.8	0.5	0.8	2.766	2.740	0.94	0.543	0.533	0.010	0.392	0.399	0.007	1.809	1.741	3.77
0.5	0.5	0.8	0.8	0.5	1.163	0.915	21.36	0.746	0.750	0.003	0.207	0.196	0.011	2.831	2.756	2.65
0.5	0.5	0.8	0.8	0.8	4.428	5.025	13.49	0.447	0.410	0.037	0.492	0.528	0.036	1.460	1.292	11.48
0.5	0.8	0.5	0.5	0.5	0.593	0.337	43.18	0.839	0.862	0.023	0.124	0.097	0.026	3.294	3.331	1.14
0.5	0.8	0.5	0.5	0.8	2.722	2.741	0.70	0.544	0.533	0.011	0.390	0.399	0.009	1.814	1.741	4.05
0.5	0.8	0.5	0.8	0.5	1.171	0.916	21.82	0.745	0.750	0.005	0.208	0.196	0.012	2.825	2.756	2.46
0.5	0.8	0.5	0.8	0.8	4.445	5.029	13.13	0.447	0.410	0.037	0.493	0.528	0.036	1.459	1.292	11.47
0.5	0.8	0.8	0.5	0.5	1.153	0.917	20.46	0.747	0.750	0.003	0.206	0.196	0.010	2.831	2.755	2.68
0.5	0.8	0.8	0.5	0.8	4.511	5.031	11.53	0.446	0.410	0.036	0.494	0.529	0.035	1.455	1.291	11.22
0.5	0.8	0.8	0.8	0.5	1.768	1.869	5.72	0.669	0.632	0.037	0.279	0.307	0.029	2.472	2.222	10.10
0.5	0.8	0.8	0.8	0.8	6.573	9.428	43.43	0.368	0.287	0.080	0.578	0.663	0.085	1.180	0.878	25.60
0.8	0.5	0.5	0.5	0.5	0.597	0.334	44.08	0.838	0.864	0.025	0.125	0.096	0.028	3.291	3.345	1.64
0.8	0.5	0.5	0.5	0.8	2.758	2.719	1.41	0.544	0.535	0.008	0.391	0.397	0.006	1.813	1.751	3.39
0.8	0.5	0.5	0.8	0.5	1.154	0.846	26.63	0.746	0.762	0.015	0.206	0.186	0.021	2.831	2.819	0.44
0.8	0.5	0.5	0.8	0.8	4.417	4.719	6.83	0.448	0.423	0.024	0.492	0.514	0.023	1.460	1.341	8.15
0.8	0.5	0.8	0.5	0.5	1.158	0.847	26.85	0.746	0.762	0.015	0.207	0.186	0.021	2.832	2.818	0.48
0.8	0.5	0.8	0.5	0.8	4.451	4.721	6.06	0.447	0.423	0.024	0.492	0.514	0.022	1.459	1.341	8.09
0.8	0.5	0.8	0.8	0.5	1.752	1.658	5.36	0.669	0.655	0.014	0.278	0.285	0.007	2.472	2.327	5.87
0.8	0.5	0.8	0.8	0.8	6.653	8.274	24.37	0.363	0.312	0.051	0.582	0.635	0.053	1.167	0.960	17.74
0.8	0.8	0.5	0.5	0.5	1.172	0.848	27.68	0.746	0.762	0.016	0.207	0.186	0.022	2.827	2.818	0.32
0.8	0.8	0.5	0.5	0.8	4.442	4.723	6.32	0.447	0.423	0.023	0.493	0.514	0.021	1.458	1.341	8.02
0.8	0.8	0.5	0.8	0.5	1.752	1.659	5.31	0.670	0.655	0.015	0.277	0.286	0.008	2.476	2.326	6.05
0.8	0.8	0.5	0.8	0.8	6.672	8.277	24.07	0.365	0.312	0.053	0.580	0.635	0.055	1.171	0.960	18.00
0.8	0.8	0.8	0.5	0.5	1.745	1.660	4.86	0.670	0.655	0.016	0.277	0.286	0.008	2.477	2.326	6.09
0.8	0.8	0.8	0.5	0.8	6.609	8.281	25.29	0.366	0.312	0.054	0.580	0.636	0.056	1.173	0.960	18.15
0.8	0.8	0.8	0.8	0.5	2.363	2.899	22.69	0.607	0.547	0.060	0.338	0.392	0.054	2.194	1.873	14.61
0.8	0.8	0.8	0.8	0.8	9.419	15.817	67.93	0.297	0.202	0.095	0.655	0.760	0.104	0.940	0.606	35.53

Table N.4(b) Numerical results for IKCS with $K_i = 10$, $i = 1, 2, 3, 4, 5$.

K1=10, K2=10, K3=10, K4=10, K5=10 (IKCS)																
Traffic Intensity					EB			FR			SP			FG		
ρ1	ρ2	ρ3	ρ4	ρ5	Sim.	App.	Abs. Rel. Error (%)	Sim.	App.	Abs. Error	Sim.	App.	Abs. Error	Sim.	App.	Abs. Rel. Error (%)
0.5	0.5	0.5	0.5	0.5	0.001	0.001	2.96	0.999	0.999	0.000	0.001	0.000	0.000	9.000	8.997	0.03
0.5	0.5	0.5	0.5	0.8	0.436	0.430	1.46	0.892	0.892	0.000	0.087	0.086	0.000	6.422	6.426	0.06
0.5	0.5	0.5	0.8	0.5	0.065	0.042	35.64	0.983	0.986	0.003	0.013	0.010	0.003	8.643	8.651	0.10
0.5	0.5	0.5	0.8	0.8	0.565	0.581	2.80	0.873	0.869	0.004	0.104	0.107	0.003	6.205	6.130	1.21
0.5	0.5	0.8	0.5	0.5	0.064	0.042	35.13	0.984	0.986	0.003	0.013	0.010	0.003	8.645	8.651	0.07
0.5	0.5	0.8	0.5	0.8	0.544	0.581	6.78	0.875	0.869	0.007	0.101	0.107	0.006	6.227	6.130	1.56
0.5	0.5	0.8	0.8	0.5	0.116	0.107	7.60	0.971	0.970	0.001	0.023	0.023	0.000	8.378	8.301	0.92
0.5	0.5	0.8	0.8	0.8	0.673	0.754	12.03	0.856	0.844	0.012	0.119	0.129	0.011	6.033	5.842	3.16
0.5	0.8	0.5	0.5	0.5	0.059	0.042	29.02	0.984	0.986	0.002	0.012	0.010	0.002	8.657	8.651	0.07
0.5	0.8	0.5	0.5	0.8	0.557	0.581	4.26	0.873	0.869	0.004	0.103	0.107	0.004	6.213	6.130	1.33
0.5	0.8	0.5	0.8	0.5	0.116	0.107	7.69	0.971	0.970	0.001	0.023	0.023	0.000	8.382	8.301	0.97
0.5	0.8	0.5	0.8	0.8	0.702	0.754	7.47	0.854	0.844	0.010	0.121	0.129	0.008	6.009	5.842	2.77
0.5	0.8	0.8	0.5	0.5	0.121	0.107	11.02	0.971	0.970	0.001	0.023	0.023	0.000	8.380	8.301	0.94
0.5	0.8	0.8	0.5	0.8	0.674	0.754	11.95	0.856	0.844	0.012	0.119	0.129	0.011	6.029	5.842	3.11
0.5	0.8	0.8	0.8	0.5	0.173	0.188	8.98	0.959	0.953	0.006	0.033	0.037	0.004	8.151	7.960	2.34
0.5	0.8	0.8	0.8	0.8	0.811	0.953	17.49	0.839	0.819	0.021	0.134	0.152	0.018	5.857	5.562	5.04
0.8	0.5	0.5	0.5	0.5	0.062	0.042	33.02	0.984	0.986	0.002	0.013	0.010	0.002	8.647	8.651	0.05
0.8	0.5	0.5	0.5	0.8	0.561	0.581	3.52	0.873	0.869	0.004	0.104	0.107	0.003	6.205	6.130	1.20
0.8	0.5	0.5	0.8	0.5	0.119	0.103	12.81	0.971	0.971	0.000	0.023	0.022	0.001	8.380	8.321	0.70
0.8	0.5	0.5	0.8	0.8	0.689	0.744	8.02	0.855	0.845	0.010	0.120	0.128	0.008	6.021	5.859	2.70
0.8	0.5	0.8	0.5	0.5	0.123	0.103	15.64	0.970	0.971	0.001	0.024	0.022	0.001	8.368	8.321	0.56
0.8	0.5	0.8	0.5	0.8	0.683	0.744	8.95	0.856	0.845	0.011	0.119	0.128	0.009	6.033	5.859	2.89
0.8	0.5	0.8	0.8	0.5	0.171	0.179	4.60	0.959	0.955	0.004	0.033	0.036	0.003	8.144	7.999	1.79
0.8	0.5	0.8	0.8	0.8	0.803	0.930	15.83	0.840	0.821	0.019	0.134	0.150	0.016	5.859	5.594	4.53
0.8	0.8	0.5	0.5	0.5	0.113	0.103	8.22	0.972	0.971	0.000	0.023	0.022	0.000	8.383	8.321	0.73
0.8	0.8	0.5	0.5	0.8	0.674	0.744	10.37	0.856	0.845	0.011	0.118	0.128	0.010	6.032	5.859	2.87
0.8	0.8	0.5	0.8	0.5	0.175	0.179	2.56	0.959	0.955	0.004	0.033	0.036	0.003	8.146	7.999	1.80
0.8	0.8	0.5	0.8	0.8	0.800	0.930	16.14	0.839	0.821	0.018	0.134	0.150	0.016	5.860	5.594	4.54
0.8	0.8	0.8	0.5	0.5	0.176	0.179	1.70	0.958	0.955	0.003	0.034	0.036	0.002	8.142	7.999	1.76
0.8	0.8	0.8	0.5	0.8	0.812	0.930	14.55	0.840	0.821	0.019	0.133	0.150	0.016	5.862	5.594	4.58
0.8	0.8	0.8	0.8	0.5	0.220	0.267	21.33	0.949	0.938	0.011	0.042	0.050	0.009	7.957	7.686	3.41
0.8	0.8	0.8	0.8	0.8	0.914	1.139	24.58	0.826	0.797	0.029	0.146	0.172	0.026	5.715	5.336	6.62

APPENDIX O

SIMULATION RESULTS WITH $K_i = 5$, $i = 1, 2, 3$.

Table O.1 Mean and the half width with 95% confidence level for SKCS.

Traffic Intensity			EB		FR		SP		FP	
ρ1	ρ2	ρ3	Mean	Half width	Mean	Half width	Mean	Half width	Mean	Half width
0.5	0.5	0.5	0.0368	0.0004	0.9651	0.0003	0.0179	0.0002	3.9955	0.0021
0.5	0.5	0.65	0.2255	0.0038	0.8799	0.0010	0.0784	0.0008	3.3322	0.0048
0.5	0.5	0.8	1.3303	0.0140	0.6698	0.0016	0.2644	0.0017	2.2983	0.0059
0.5	0.65	0.5	0.0789	0.0017	0.9473	0.0006	0.0310	0.0005	3.8718	0.0035
0.5	0.65	0.65	0.2869	0.0039	0.8620	0.0010	0.0932	0.0008	3.2353	0.0046
0.5	0.65	0.8	1.4825	0.0157	0.6502	0.0016	0.2833	0.0016	2.2183	0.0063
0.5	0.8	0.5	0.6092	0.0137	0.8385	0.0015	0.1248	0.0014	3.2963	0.0071
0.5	0.8	0.65	0.9364	0.0230	0.7554	0.0022	0.1911	0.0021	2.7527	0.0095
0.5	0.8	0.8	2.7539	0.0466	0.5447	0.0027	0.3901	0.0029	1.8161	0.0096
0.65	0.5	0.5	0.0791	0.0012	0.9474	0.0003	0.0309	0.0003	3.8729	0.0027
0.65	0.5	0.65	0.2862	0.0051	0.8620	0.0012	0.0932	0.0010	3.2347	0.0057
0.65	0.5	0.8	1.4806	0.0267	0.6506	0.0024	0.2829	0.0024	2.2183	0.0094
0.65	0.65	0.5	0.1326	0.0025	0.9285	0.0007	0.0455	0.0006	3.7513	0.0037
0.65	0.65	0.65	0.3577	0.0056	0.8436	0.0012	0.1086	0.0011	3.1384	0.0059
0.65	0.65	0.8	1.6259	0.0183	0.6329	0.0018	0.3000	0.0019	2.1457	0.0065
0.65	0.8	0.5	0.7212	0.0206	0.8169	0.0022	0.1435	0.0021	3.1817	0.0098
0.65	0.8	0.65	1.0932	0.0186	0.7326	0.0020	0.2119	0.0020	2.6477	0.0083
0.65	0.8	0.8	3.0521	0.0517	0.5241	0.0029	0.4114	0.0032	1.7380	0.0107
0.8	0.5	0.5	0.6083	0.0086	0.8382	0.0011	0.1251	0.0010	3.2959	0.0058
0.8	0.5	0.65	0.9334	0.0103	0.7551	0.0013	0.1912	0.0012	2.7500	0.0059
0.8	0.5	0.8	2.7709	0.0560	0.5424	0.0026	0.3926	0.0027	1.8074	0.0100
0.8	0.65	0.5	0.7104	0.0135	0.8178	0.0012	0.1424	0.0012	3.1842	0.0052
0.8	0.65	0.65	1.1018	0.0218	0.7318	0.0020	0.2129	0.0020	2.6447	0.0084
0.8	0.65	0.8	3.0336	0.0611	0.5261	0.0031	0.4095	0.0032	1.7456	0.0109
0.8	0.8	0.5	1.8268	0.0504	0.6894	0.0027	0.2634	0.0027	2.5985	0.0112
0.8	0.8	0.65	2.3839	0.0435	0.6109	0.0028	0.3320	0.0029	2.1516	0.0108
0.8	0.8	0.8	5.5085	0.0995	0.4110	0.0031	0.5321	0.0036	1.3380	0.0101

Table O.2 Mean and the half width with 95% confidence level for IKCS.

Traffic Intensity			EB		FR		SP		FP	
ρ1	ρ2	ρ3	Mean	Half width	Mean	Half width	Mean	Half width	Mean	Half width
0.5	0.5	0.5	0.0366	0.0004	0.9652	0.0002	0.0178	0.0002	3.9973	0.0020
0.5	0.5	0.65	0.2235	0.0037	0.8808	0.0008	0.0778	0.0007	3.3367	0.0043
0.5	0.5	0.8	1.3339	0.0191	0.6691	0.0016	0.2650	0.0016	2.2957	0.0063
0.5	0.65	0.5	0.0784	0.0019	0.9476	0.0004	0.0308	0.0004	3.8748	0.0024
0.5	0.65	0.65	0.2853	0.0038	0.8624	0.0006	0.0928	0.0006	3.2368	0.0033
0.5	0.65	0.8	1.4762	0.0147	0.6507	0.0012	0.2828	0.0012	2.2189	0.0052
0.5	0.8	0.5	0.5851	0.0104	0.8408	0.0012	0.1224	0.0011	3.3071	0.0060
0.5	0.8	0.65	0.9421	0.0164	0.7547	0.0014	0.1915	0.0014	2.7485	0.0063
0.5	0.8	0.8	2.7696	0.0370	0.5451	0.0019	0.3900	0.0021	1.8179	0.0065
0.65	0.5	0.5	0.0767	0.0018	0.9480	0.0005	0.0305	0.0004	3.8758	0.0030
0.65	0.5	0.65	0.2845	0.0033	0.8628	0.0009	0.0926	0.0007	3.2395	0.0042
0.65	0.5	0.8	1.4853	0.0195	0.6494	0.0016	0.2840	0.0017	2.2133	0.0063
0.65	0.65	0.5	0.1183	0.0021	0.9321	0.0005	0.0424	0.0004	3.7700	0.0027
0.65	0.65	0.65	0.3426	0.0052	0.8465	0.0011	0.1060	0.0009	3.1520	0.0053
0.65	0.65	0.8	1.5992	0.0246	0.6344	0.0022	0.2983	0.0023	2.1519	0.0075
0.65	0.8	0.5	0.6318	0.0120	0.8300	0.0011	0.1312	0.0010	3.2421	0.0055
0.65	0.8	0.65	0.9959	0.0155	0.7434	0.0012	0.2014	0.0012	2.6919	0.0054
0.65	0.8	0.8	2.8437	0.0534	0.5348	0.0022	0.4001	0.0024	1.7752	0.0081
0.8	0.5	0.5	0.5919	0.0162	0.8408	0.0019	0.1225	0.0018	3.3076	0.0087
0.8	0.5	0.65	0.9324	0.0165	0.7555	0.0018	0.1909	0.0017	2.7517	0.0071
0.8	0.5	0.8	2.7432	0.0544	0.5457	0.0028	0.3893	0.0030	1.8207	0.0107
0.8	0.65	0.5	0.6326	0.0129	0.8298	0.0016	0.1314	0.0015	3.2388	0.0073
0.8	0.65	0.65	0.9883	0.0199	0.7445	0.0013	0.2004	0.0013	2.6953	0.0049
0.8	0.65	0.8	2.8678	0.0563	0.5342	0.0027	0.4007	0.0029	1.7750	0.0099
0.8	0.8	0.5	1.1580	0.0281	0.7471	0.0021	0.2062	0.0021	2.8376	0.0092
0.8	0.8	0.65	1.6936	0.0354	0.6588	0.0023	0.2823	0.0023	2.3310	0.0084
0.8	0.8	0.8	4.3903	0.1066	0.4508	0.0034	0.4883	0.0038	1.4727	0.0114

APPENDIX P

EFFECT OF THE SEQUENCE THE COMPONENTS ARE PICKED UP

Table P.1 Effect of the sequence the components are picked up.

	Release Mechanism	Kanban Size			Traffic Intensity			EB		FG		FR		SP	
		K1	K2	K3	ρ1	ρ2	ρ3	Com.	Abs. Rel. Error (%)	Com.	Abs. Rel. Error (%)	Com.	Abs. Error	Com.	Abs. Error
1	SKCS	3	3	3	0.5	0.65	0.8	3.740	9.56	0.752	7.47	0.362	0.023	0.545	0.024
	SKCS	3	3	3	0.65	0.5	0.8	3.712	7.88	0.757	6.48	0.365	0.020	0.543	0.019
2	SKCS	3	3	3	0.5	0.8	0.5	0.959	45.83	1.425	7.48	0.639	0.059	0.260	0.073
	SKCS	3	3	3	0.8	0.5	0.5	0.893	49.84	1.500	9.96	0.655	0.076	0.246	0.089
3	SKCS	7	7	7	0.5	0.65	0.8	0.891	3.56	3.767	1.06	0.781	0.004	0.175	0.004
	SKCS	7	7	7	0.65	0.5	0.8	0.891	3.27	3.767	1.04	0.782	0.004	0.175	0.003
4	SKCS	7	7	7	0.5	0.8	0.5	0.116	51.19	5.486	1.25	0.952	0.017	0.033	0.017
	SKCS	7	7	7	0.8	0.5	0.5	0.116	51.83	5.491	1.15	0.953	0.017	0.033	0.017
5	IKCS	3	3	3	0.5	0.65	0.8	3.585	8.20	0.772	6.19	0.371	0.019	0.535	0.019
	IKCS	3	3	3	0.65	0.5	0.8	3.568	6.48	0.776	5.68	0.373	0.017	0.533	0.017
6	IKCS	3	3	3	0.5	0.8	0.5	0.875	45.18	1.473	7.75	0.657	0.060	0.244	0.071
	IKCS	3	3	3	0.8	0.5	0.5	0.824	48.86	1.500	9.96	0.670	0.074	0.232	0.084
7	IKCS	7	7	7	0.5	0.65	0.8	0.891	1.96	3.767	0.88	0.782	0.003	0.175	0.003
	IKCS	7	7	7	0.65	0.5	0.8	0.891	3.04	3.768	1.11	0.782	0.005	0.175	0.004
8	IKCS	7	7	7	0.5	0.8	0.5	0.118	49.40	5.489	1.08	0.953	0.016	0.033	0.016
	IKCS	7	7	7	0.8	0.5	0.5	0.116	51.83	5.494	1.20	0.953	0.017	0.033	0.017