DYNAMIC PERFORMANCES OF KINEMATICALLY AND DYNAMICALLY ADJUSTABLE PLANAR MECHANISMS

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ABSTRACT

DYNAMIC PERFORMANCES OF KINEMATICALLY AND DYNAMICALLY ADJUSTABLE PLANAR MECHANISMS

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In this thesis, the dynamic performances of kinematically and dynamically adjustable planar mechanisms have been investigated. An adjustable mechanism is here defined to be a mechanism where some of the kinematic and/or dynamic parameters are changed in a controlled manner in order to optimize the dynamic behaviour of the mechanism in spite of variable operating conditions. Here, variable operating conditions refer to variable load(s) on the mechanism and/or variable desired input motion. The dynamic behaviour of the mechanism may be optimized via minimization of the actuator torque/force fluctuations, minimization of energy consumed by the actuators etc.

According to the type of the adjustable parameter, the adjustable mechanisms are classified into two groups namely, dynamically adjustable mechanisms and kinematically adjustable mechanisms. Mechanisms, where the main concern is to change a dynamic parameter(s) are called dynamically adjustable mechanisms. In the kinematically adjustable mechanisms, on the other hand , the main concern is to change a kinematic parameter(s).

The main objective of this study is to investigate the benefits of adjustable planar mechanisms, regarding different dynamic behaviours under variable operating conditions. To achieve this objective , various simulations have been performed on the computer. In these simulations, practical constraints that will exist in a real application have been taken into account as much as possible. The results reveal that, in many cases, the dynamic behaviour of a planar mechanism may be improved quite extensively via adjustable mechanisms which are obtained from the original mechanisms with slight modifications.

Keywords : Optimization of dynamic behaviour, adjustable mechanisms, dynamic performance .

ÖZ

KİNEMATİK VE DİNAMİK OLARAK AYARLANABİLİR DÜZLEMSEL MEKANİZMALARIN DİNAMİK PERFORMANSI

İyiay, Erdinç
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Bu tezde, kinematik ve dinamik olarak ayarlanabilir düzlemsel mekanizmaların dinamik performansları araştırılmıştır. Burada ayarlanabilir mekanizma , değişken çalışma şartlarında mekanizmanın dinamik davranışını optimize etmek için üzerinde kinematik ve/veya dinamik parametrelerin kontrollü bir biçimde değiştirildiği bir mekanizma olarak tanımlanmaktadır. Burada değişken çalışma şartları mekanizma üzerindeki değişken yükleri ve/veya istenen değişken giriş hareketini simgelemektedir. Mekanizmanın dinamik davranışı tahrik torkundaki/kuvvetindeki dalgalanmaların minimizasyonu, tahrik motorları tarafından tüketilen enerjinin minimizasyonu vs. ile optimize edilebilmektedir.

Ayarlanabilir mekanizmalar ayarlanabilir parametrenin türüne göre dinamik olarak ve kinematik olarak ayarlanabilir mekanizmalar olmak üzere iki gruba ayrılırlar. Esas amacın bir dinamik parametre veya parametreleri değiştirmek olduğu mekanizmalar dinamik olarak ayarlanabilir mekanizmalar olarak isimlendirilirler. Diğer taraftan, kinematik olarak ayarlanabilir mekanizmalarda esas amaç kinematik bir parametreyi veya parametreleri değiştirmektir.

Bu çalışmanın temel amacı, değişken çalışma şartlarında, farklı dinamik davranışları dikkate alarak ayarlanabilir mekanizmaların faydalarını araştırmaktır. Bu amaçla bilgisayarda değişik simülasyonlar gerçekleştirilmiştir. Bu simülasyonlarda, gerçek uygulamalarda ortaya çıkacak pratik kısıtlamalar mümkün olduğunca dikkate alınmıştır. Sonuçlar göstermektedir ki , bir çok durumda mekanizmanın dinamik davranışı , küçük değişikliklerle orijinal mekanizmalardan elde edilen ayarlanabilir mekanizmalar ile oldukça etkin bir biçimde düzelmektedir.

Anahtar Kelimeler : Dinamik davranışın optimizasyonu, ayarlanabilir mekanizmalar, dinamik performans .

ACKNOWLEDGMENTS

I would like to express my gratitude to my thesis supervisor Prof. Dr. Reşit Soylu for his continuous help , encouragement and his unbelievable effort even in the times when I felt hopeless and weak in solving some problems.

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$O_1X_1Y_1$	Fixed coordinate system
$O_iX_iY_i$	Body fixed coordinate system for link i
θ_{1i}	Angular position of link i measured from the X_1 axis to the X_i axis in a right hand sense around the Z_1 axis.
s_{ij}	Relative linear displacement between links i and j
N	Degree of freedom of the adjustable mechanism
N_m	Degree of freedom of the original mechanism
$N_{a,i}$	Degree of freedom of the i'th adjuster
n_a	Number of adjusters on the mechanism
$\vec{q}$	N component generalized coordinates vector
$\vec{q}_s$	Vector of specified generalized coordinates
$\vec{q}_u$	Vector of unspecified generalized coordinates
$\vec{U}$	Unknown position variables vector
NR(i)	Number of revolute joints on link i
NP(i)	Number of prismatic joints on link i
NE(i)	Number of external forces acting on link i
NFR(i)	Number of friction forces (viscous & coulomb friction) acting on link i
$\vec{F}^x_{r,i}$	x component of the r'th revolute joint reaction
$\vec{F}^x_{p,i}$	x component of the r'th prismatic joint reaction
$\vec{F}^x_{e,i}$	x component of the e'th external force
$\vec{F}^x_{w,i}$	x component of the weight of link i
$\vec{F}^x_{in,i}$	x component of the inertia force acting on link i

$\vec{F}^{x}_{fr,i}$	x component of the fr'th friction force
$\vec{F}^{y}_{r,i}$	y component of the r'th revolute joint reaction
$\vec{F}^{y}_{p,i}$	y component of the r'th prismatic joint reaction
$\vec{F}^{y}_{e,i}$	y component of the e'th external force
$\vec{F}^{y}_{w,i}$	y component of the weight of link i
$\vec{F}^{y}_{in,i}$	y component of the inertia force acting on link i
$\vec{F}^{y}_{fr,i}$	y component of the fr'th friction force
NM(i)	Number of external moments acting on link i
$\vec{M}_{r,i}$	Moment of the r'th revolute joint reaction
$\vec{M}_{p,i}$	Moment of the p'th prismatic joint reaction
$\vec{M}_{e,i}$	Moment of the e'th external force
$\vec{M}_{m,i}$	m'th external moment acting on link i
$\vec{M}_{w,i}$	moment of the weight of link i
$\vec{M}_{fr,i}$	moment of the fr'th friction force
$(\vec{T}_{in})_{O_i}$	inertia torque of link i plus moments of inertia forces about O_i
$\vec{F}^{x}_{j,i}, \vec{F}^{y}_{j,i}$	x and y components of the reaction force (in $O_iX_1Y_1$ system) exerted by link j on link i acting at joint k , respectively
P_{ji}, γ_{ji}	polar coordinates of joint k in the $O_iX_iY_i$ system
β_{ji}	angle measured from $\overrightarrow{O_iQ_{j,i}}$ to $\overrightarrow{Q_{j,i}R_{j,i}}$ in a right hand sense around Z_1
$\vec{a}^{x}_{CG_i}, \vec{a}^{y}_{CG_i}$	x and y components of the absolute acceleration of CG_i in the X_1Y_1 system
$\bar{I}_i$	Centroidal moment of inertia of link i
$\vec{\alpha}_{1i}$	Absolute angular acceleration of link i (positive if in the Z_1 direction)
$\vec{a}_{O_i}$	Absolute acceleration of O_i in the X_1Y_1 system

m_i	mass of link i
MX_i	mass times x coordinate of center of mass of link i in a convenient body fixed coordinate system
MY_i	mass times y coordinate of center of mass of link i in a convenient body fixed coordinate system
I_i	Moment of inertia of link i with respect to the origin of its body fixed coordinate system
$\vec{a}_n$	Mass center acceleration of the nonmoving actuator in the $O_iX_1Y_1$ (fixed) system
$\vec{a}_b$	Mass center acceleration of the moving block in the $O_iX_1Y_1$ (fixed) system
s_n	constant position of the non-moving actuator
$s_b(t)$	Position of the block with respect to the origin of the link on which it moves
J	The performance measure
G_i	The i'th Lagrangian function corresponding to the i'th desired behaviour of the mechanism
t_0 , t_f	Initial and final values of the time space over which J is defined
w_i	Weighting coefficient associated with G_i
n_i	Number of G_i's that appear in the definition of J
$\vec{I}_m$	Vector of specified inertial parameters of the original mechanism
$\vec{I}_n$	Vector of specified inertial parameters of the nonmoving actuator
$\vec{I}_s$	Vector of specified inertial parameters of the adjustable mechanism
$\vec{d}_c$	Vector of constant design parameters
$\vec{d}_u$	Vector of design parameters related to the unspecified generalized coordinates.
$\vec{d}_J$	Vector of design parameters that affect J
$L_{ij}(t)$	Total variable length of the combined link consisting of links i and j which are connected by a prismatic joint.
W_{CL}	Energy dissipated in the armature circuit due to copper losses

J_{CL}	Performance measure related to the minimization of copper losses
J_{EC}	Performance measure related to the minimization of energy consumed
J_{SF}	Performance measure related to the minimization of the shaking force fluctuations
F_s^x	x component of the shaking force
F_{sd}^x	desired value of F_s^x
F_s^y	y component of the shaking force
F_{sd}^y	desired value of F_s^y

CHAPTER I

INTRODUCTION

Mechanisms during their practical operation are often exposed to different dynamical effects, which cause some important problems. One of these problems is the existence of variable shaking forces and shaking moments. Recall that resultant of all the forces transmitted to the frame from the machine due to inertial effects only are known as shaking forces, moment of forces transmitted to the frame due to inertial effects only is known as shaking moment which occur especially in the mechanisms working at higher speeds. If the speed and/or the total mass in the linkage is increased, the vibrations, noise, unneccessary wear, and fatique also increase. The shaking forces and moments cause problems that affect the life of the machine. Therefore balancing of the shaking forces and shaking moments has been a challenging problem for the designers of the mechanisms for a long time. Besides the balancing of the shaking forces and shaking moments, minimization of the generalized actuator force (input torque or force) fluctuations, minimization of energy consumed, minimization of joint reaction fluctuations are also important, so that a machine works efficiently and smoothly. Various methods may be used in order to realize these objectives. In this thesis, the kinematic and/or dynamic properties of the mechanism will be adjusted in order to realize these objectives.

An adjustable mechanism is a mechanism where some of the kinematic and/or dynamic parameters are changed in a controlled manner in order to optimize the various dynamic behaviours of the mechanism in spite of variable operating conditions. There exist many methods in the literature for adjusting the mechanisms.

Vukobratovic and Potkonjak ([1] and [2]), considered the modelling and control of active systems with variable geometry. Every active system during its practical operation is subject to more or less intense static, kinematic and dynamic requirements. To meet all these requirements is a diffucult problem. To overcome this problem they suggested system reconfiguration by changing its geometry, so that it is possible to achieve better system performance. In [1] the general approach is derived and the applications are discussed. In [2] case studies and numerical results are presented.

Chen, Modi and Silva ([3], and [4]) presented a relatively general formulation for studying dynamics and control of a novel multi-module mobile manipulator, with slewing as well as deployable links. The deployment character leads to several advantages including reduced coupling, fewer singular configurations and ease of obstacle avoidance.

Furuya and Higashiyama [5] present dynamical characteristics of a variable geometry truss (VGT) manipulator system which consists of a two-dimensional statically determinate truss for space applications. Formulation takes into account geometrical effects of the closed-link constraints , variable length mechanisms, rotational degrees of freedom at the joints and internal control forces and is developed by using Kane's equations. In this study, the effects of the internal control forces on the attitude of the manipulator system in a space environment and the characteristics of the inverse kinetics of the manipulator are discussed.

Boutin and Arun [6] considered dynamics formulation and vibration control of variable geometry truss structures which may be regarded as kinematically adjustable mechanisms. The truss system is modelled as a collection of sub-structures consisting of truss booms, prismatic actuator elements and in some cases a manipulator at the end. For vibration control, the singular perturbation method is employed to construct two reduced-order models, for quasi-static motion and for modal coordinates, respectively.

The major objective of this study is to determine the optimal design parameters that optimize the dynamic behaviour of adjustable planar mechanisms. Here, optimization of the dynamic behaviour refers to the minimization of actuator torque/force fluctuations, minimization of energy consumed in the mechanism, minimization of shaking force/moment fluctuations etc. The design parameters depend on the type of the adjustment, variable operating conditions etc.

For each type of adjustable mechanism an algorithm has been developed to derive the equations of motion of the mechanism. These algorithms have been implemented using the software package MATHEMATICA. The dynamic behaviour of the mechanism has been optimized via appropiate performance measures (PM). Again MATHEMATICA has been used to minimize the PM's.

The outline of the thesis is as follows:

In Chapter II, the methods used for derivation of the equations of motion of the adjustable mechanisms are presented.

In Chapter III, the performance measures used are introduced. The chapter also discusses the methods used to compute and optimize PM's.

Case studies related to the optimization of the dynamic behaviour(s) of different types of adjustable mechanisms are given in Chapter IV.

Finally, Chapter V is devoted to the conclusions.

CHAPTER II

KINEMATIC AND FORCE ANALYSIS OF ADJUSTABLE MECHANISMS

2.1 Introduction to Adjustable Mechanisms

An adjustable mechanism is a mechanism where some of the kinematic or dynamic parameters are changed in a controlled manner in order to optimize the dynamic behaviour of the mechanism in spite of variable operating conditions. Here, load(s) on the mechanism and/or the desired motion may be considered as variable operating conditions. The dynamic behaviour of the mechanism may be optimized via

1) Minimization of actuator torque / force fluctuations.
2) Minimization of energy consumed in the actuators.
3) Minimization of the shaking force and shaking moment fluctuations from their desired values
4) Minimization of the joint reaction fluctuations from their desired values.

or, a weighted combination of the above tasks.

According to the type of the adjustable parameter, the adjustable mechanisms may be classified into two groups, namely :

1) Dynamically Adjustable Mechanisms
2) Kinematically Adjustable Mechanisms

Mechanisms where the main concern is to change a dynamic parameter(s) are called dynamically adjustable mechanisms. Figure 2.1 shows the i'th link of such a mechanism.

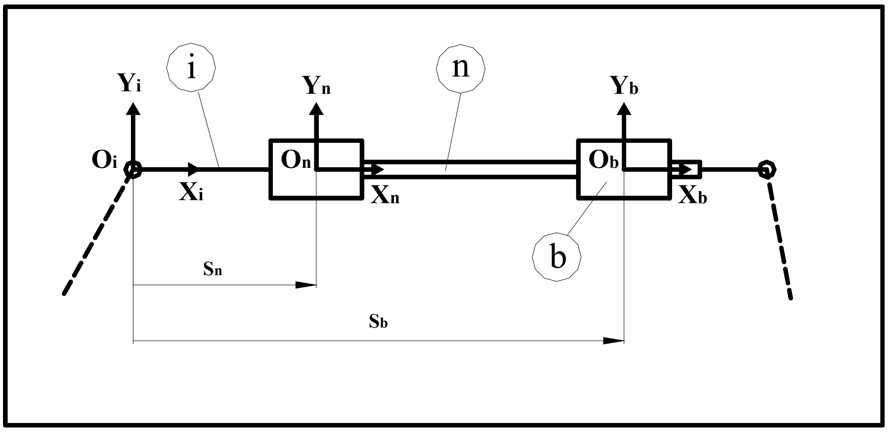

Figure 2.1 Link i of a Dynamically Adjustable Mechanism

As seen in Figure 2.1, the idea of dynamic adjustment consists of a moving block (denoted by b) which is located on a link (denoted by i) of the mechanism. The block moves on the link by means of a non-moving linear actuator (denoted by n). The actuator-block system can also be called as the dynamic adjuster of the mechanism. The major objective in this type of adjustment is to determine the optimal values of the design parameters that optimize the dynamic behaviour of the mechanism. Here, the design parameters include the constant position s_n of the non-moving actuator, the adjustable position $s_b(t)$ and the inertial parameters of the moving block.

Another type of adjustable mechanisms is the kinematically adjustable mechanisms where the main concern is to change a kinematic parameter(s). Figure 2.2 shows an example of this type.

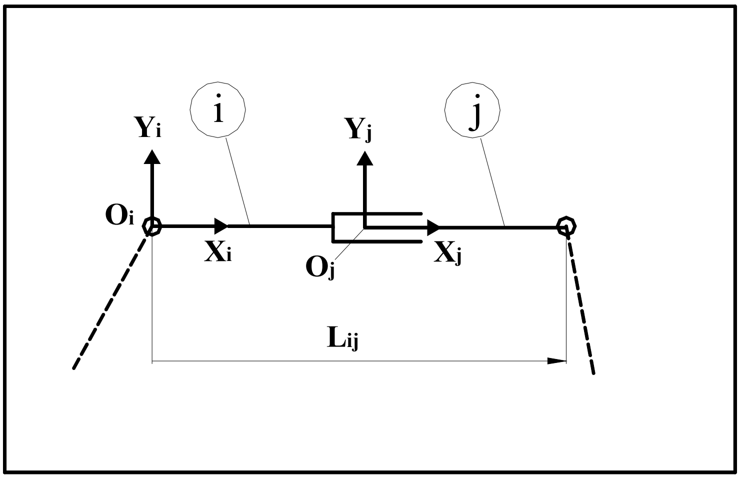

Figure 2.2 Link i of a Kinematically Adjustable Mechanism

As seen above , kinematic adjustment on the mechanism is based on the concept of variable geometry. By designing one or more than one of the links with variable length, the mechanism may be adjusted for various tasks. In this type of adjustment, the major objective is to determine the optimal values of the design parameters, that optimize the dynamic behaviour of the mechanism . Here, the only design parameter (actually, design function) is the variable length $L_{ij}(t)$ of the combined link consisting of links i and j. This combined link system may also be called as the kinematic adjuster. A hydraulic actuator may be regarded as an example of kinematic adjuster.

If desired, one can design a mechanism which is both kinematically and dynamically adjustable. The two type of adjustments may be on the same link, or on different links of the mechanism.

Whether kinematic or dynamic, the adjustment on the mechanism can be made before, or during the regular motion. This affects the degree of freedom (DOF) of the mechanism. The DOF of an adjustable mechanism is given by

$$N = N_m + \sum_{i=1}^{n_a} N_{a,i} \tag{2.1}$$

where

N : DOF of the adjustable mechanism

N_m : DOF of the original mechanism (The case of no adjustment)

$N_{a,i}$: DOF of the i'th adjuster

n_a : Number of adjusters on the mechanism

The degree of freedom of an adjuster is 0 if the adjustment is made before the regular motion, whereas it is 1 if the adjustment is made during the regular motion of the mechanism.

2.2 Kinematic Analysis

For the kinematic and dynamic analysis of adjustable planar mechanisms, a MATHEMATICA package developed by Tursun [7] has been used together with the algorithm explained in section 2.4 . This package performs the complete kinematic and dynamic analysis of a planar mechanism independently. The material covered in this section has been taken from [7] and appropiate modifications have been made for the case of adjustable mechanisms.

The first step in performing the kinematic & force analysis of a planar mechanism is to define a body fixed coordinate system, $O_iX_iY_iZ_i$, for each link, where i denotes the link number (see Figure 2.3). Since link 1 is considered to be the fixed link in general, the inertial or fixed coordinate system is taken to be the $O_1X_1Y_1Z_1$ system. The angular position of link i is given by θ_{1i} which is measured from the X_1 axis to the X_i axis in a right hand sense around the Z_1 axis. The relative linear displacement of two links, say, i and j , connected by a prismatic joint, on the other hand, is designated by $s_{i,j}$ (See Figure 2.7) .

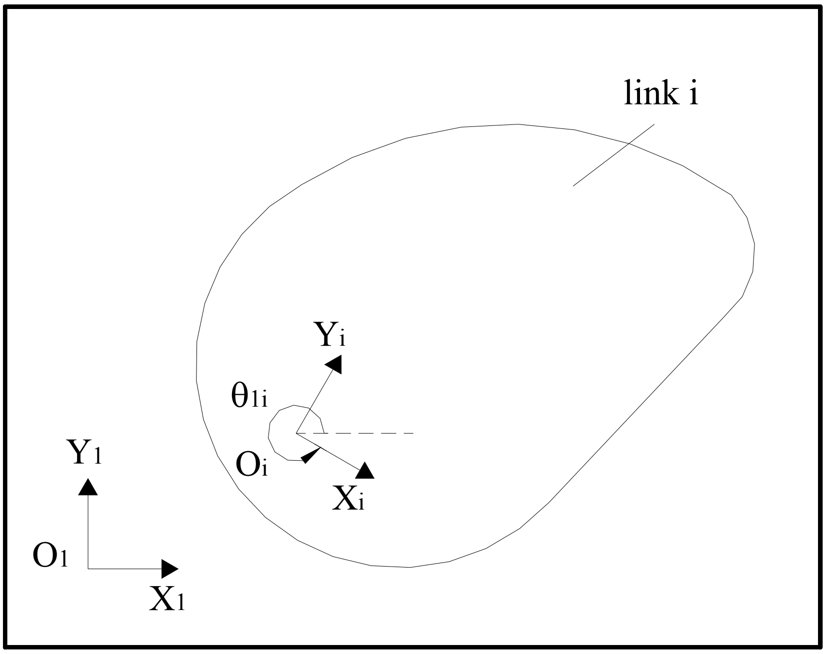

Figure 2.3 Body fixed and inertial coordinate systems

The θ_{1i}'s and $s_{i,j}$'s constitute the so-called position variables vector $\vec{P}$, of the mechanism. For an N degree of freedom planar mechanism, the position variables vector is given by

$$\vec{P} \stackrel{\Delta}{=} \left[\vec{q} \;\; \vec{U} \right] \tag{2.2}$$

where $\vec{q}$ is the N-component generalized coordinates vector and $\vec{U}$ is the unknown position variables vector.

2.2.1 Position Analysis

Position analysis is the determination of the $\vec{U}$ vector when the $\vec{q}$ vector and the dimensions of the mechanism are given. To perform position analysis , firstly the loop closure equations (LCE) which are supplied to the package (developed by Tursun [7]) as input , are transformed into an algebraic equation system (in terms of the s_i, c_i, and $s_{i,j}$ type variables) by the addition of the equations

$$s_i^2 + c_i^2 = 1 \tag{2.3}$$

for each θ_{1i}, where $s_i = \sin\theta_{1i}$ and $c_i = \cos\theta_{1i}$. Then this algebraic equation system is solved by using the nonlinear algebraic equation solver, NSOLVE, of the MATHEMATICA package. Using this solver, it is possible to obtain all solutions of the equation system corressponding to all closures (or, assembly configurations) of the mechanism. Since it is possible to display the solutions in a convenient manner, the user of the package can identify the different closures of the mechanism. Following this identification, the rest of the kinematic analysis and the force analysis can be carried out by using any desired closure of the mechanism.

2.2.2 Velocity Analysis

Velocity analysis is the determination of the $\dot{\vec{U}}$ vector when the $\dot{\vec{q}}$ vector, $\vec{q}$ vector, $\vec{U}$ vector and the dimensions of the mechanism are given. To achieve this task, firstly the LCE's are differentiated with respect to time. The scalar components of the resulting equations yield a linear equation system (in the components of $\dot{\vec{U}}$) which is solved by using the LINEARSOLVE package of MATHEMATICA.

2.2.3 Acceleration Analysis

Acceleration analysis is the determination of the $\ddot{\vec{U}}$ vector when the $\vec{q}$, $\dot{\vec{q}}$, $\ddot{\vec{q}}$, $\vec{U}$, $\dot{\vec{U}}$ vectors and the dimensions of the mechanism are given. To achieve this task, firstly the second time derivative of the LCE's are determined. The scalar components of the resulting equations yield a linear equation system

(in the components of $\ddot{U}$) which is solved by using the LINEARSOLVE package of MATHEMATICA.

2.3 Force Analysis

For the force analysis D'Alembert's principle is used and the dynamic equilibrium of each of the moving links is considered. Again, the material covered in this section has been taken from Tursun [7] and has been modified for the adjustable mechanisms (see section 2.4). The three dynamic equilibrium equations (in the $O_iX_1Y_1$ system) for link i are given by the following three equations.

Force equilibrium in x direction:

$$\sum_{r=1}^{r=NR(i)} \vec{F}_{r,i}^{x} + \sum_{p=1}^{p=NP(i)} \vec{F}_{p,i}^{x} + \sum_{e=1}^{e=NE(i)} \vec{F}_{e,i}^{x} + \vec{F}_{w,i}^{x} + \vec{F}_{in,i}^{x} + \sum_{fr=1}^{fr=NFR(i)} \vec{F}_{fr,i}^{x} = \vec{0} \tag{2.4}$$

where

$NR(i)$: # of revolute joints on link i

$NP(i)$: # of prismatic joints on link i

$NE(i)$: # of external forces acting on link i

$NFR(i)$: # of friction forces (viscous & coulomb friction) acting on link i

$\vec{F}_{r,i}^{x}$: x component of the r'th revolute joint reaction.

$\vec{F}_{p,i}^{x}$: x component of the p'th prismatic joint reaction.

$\vec{F}_{e,i}^{x}$: x component of the e'th external force

$\vec{F}_{w,i}^{x}$: x component of the weight of link i

$\vec{F}_{in,i}^{x}$: x component of the inertia force acting on link i

$\vec{F}_{fr,i}^{x}$: x component of the fr'th friction force

Force equilibrium in y direction:

$$\sum_{r=1}^{r=NR(i)} \vec{F}_{r,i}^{y} + \sum_{p=1}^{p=NP(i)} \vec{F}_{p,i}^{y} + \sum_{e=1}^{e=NE(i)} \vec{F}_{e,i}^{y} + \vec{F}_{w,i}^{y} + \vec{F}_{in,i}^{y} + \sum_{fr=1}^{fr=NFR(i)} \vec{F}_{fr,i}^{y} = \vec{0} \tag{2.5}$$

where

$\vec{F}_{r,i}^{y}$	: y component of the r'th revolute joint reaction.
$\vec{F}_{p,i}^{y}$	: y component of the p'th prismatic joint reaction.
$\vec{F}_{e,i}^{y}$	: y component of the e'th external force
$\vec{F}_{w,i}^{y}$	: y component of the weight of link i
$\vec{F}_{in,i}^{y}$	: y component of the inertia force acting on link i
$\vec{F}_{fr,i}^{y}$	: y component of the fr'th friction force

Moment equilibrium about O_i :

$$\sum_{r=1}^{r=NR(i)} \vec{M}_{r,i} + \sum_{p=1}^{p=NP(i)} \vec{M}_{p,i} + \sum_{f=1}^{f=NE(i)} \vec{M}_{e,i} + \sum_{m=1}^{m=NM(i)} \vec{M}_{m,i} + \vec{M}_{w,i} + \sum_{fr=1}^{fr=NFR(i)} \vec{M}_{fr,i} + \left(\vec{T}_{in}\right)_{O_i} = \vec{0} \tag{2.6}$$

where

$NM(i)$	: # of external moments acting on link i
$\vec{M}_{r,i}$	: Moment of the r'th revolute joint reaction
$\vec{M}_{p,i}$	: Moment of the p'th prismatic joint reaction
$\vec{M}_{e,i}$	: Moment of the e'th external force
$\vec{M}_{m,i}$	: m'th external moment acting on link i
$\vec{M}_{w,i}$	: Moment of the weight of link i
$\vec{M}_{fr,i}$	: Moment of the fr'th friction force

$\left(\vec{T}_{in}\right)_{Oi}$: Inertia torque of link i plus moments of inertia forces about O_i

The contribution of the revolute joint reaction forces to the equilibrium equations can be explained by referring to Figure 2.4. Here, $F_{j,i}^{x}$ and $F_{j,i}^{y}$ denote the x and y components of the reaction force (in $O_iX_1Y_1$ system) exerted by link j on link i acting at joint k respectively. Furthermore, $P_{j,i}$ and $\gamma_{j,i}$ denote the polar coordinates of joint k in the $O_iX_iY_i$ system. The contribution of the k'th revolute joint to the first summation in equations (2.4)-(2.6) can be expressed as $F_{j,i}^{x}\,\vec{i}_1$, $F_{j,i}^{y}\,\vec{j}_1$ and $\left[P_{j,i}\,F_{j,i}^{x}\,Sin\left(-\gamma_{j,i}-\theta_{1,i}\right)+P_{j,i}\;F_{j,i}^{y}\;Sin\left(\frac{\pi}{2}-\gamma_{j,i}-\theta_{1,i}\right)\right]\vec{k}_1$ respectively, where $\vec{i}_1$, $\vec{j}_1$, and $\vec{k}_1$ denote the unit vectors along the X_1, Y_1 and Z_1 axes.

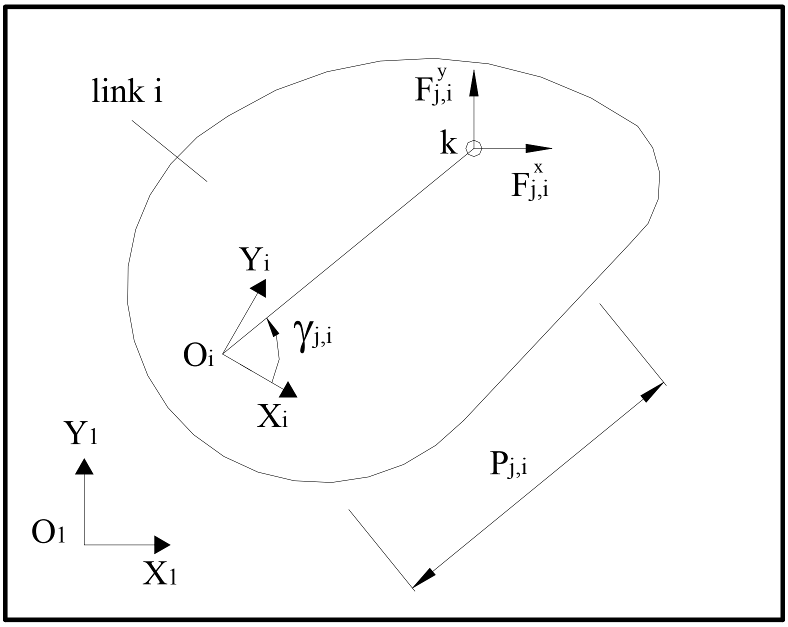

Figure 2.4 Revolute joint reaction on link i

For prismatic joints, the two kinematic elements that make up the joint are labelled as slider and slot. The slider and slot parts of prismatic joint j on link i are shown in figures 2.5 and 2.6.

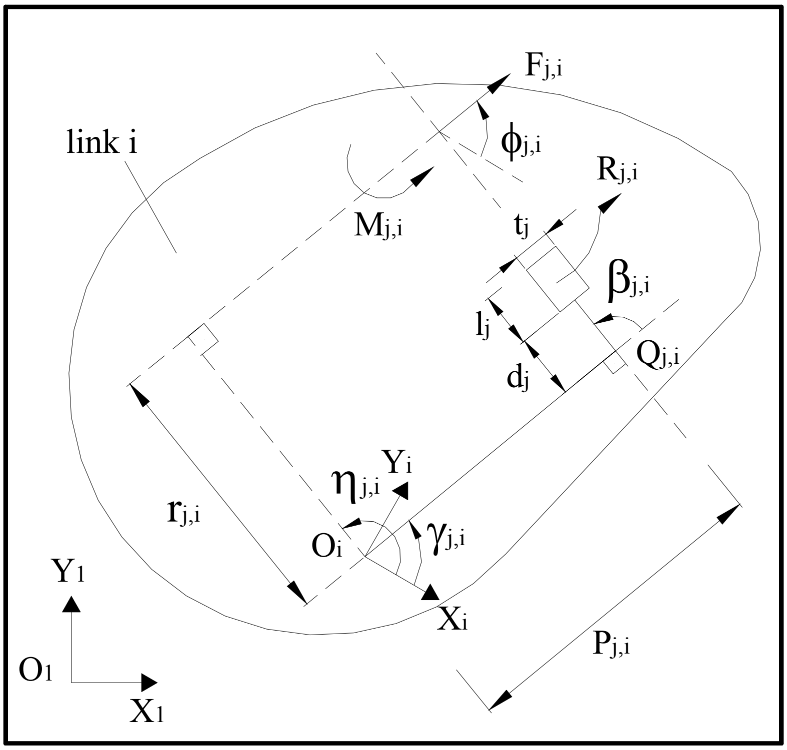

Figure 2.5 Slider (numbered j) on link i

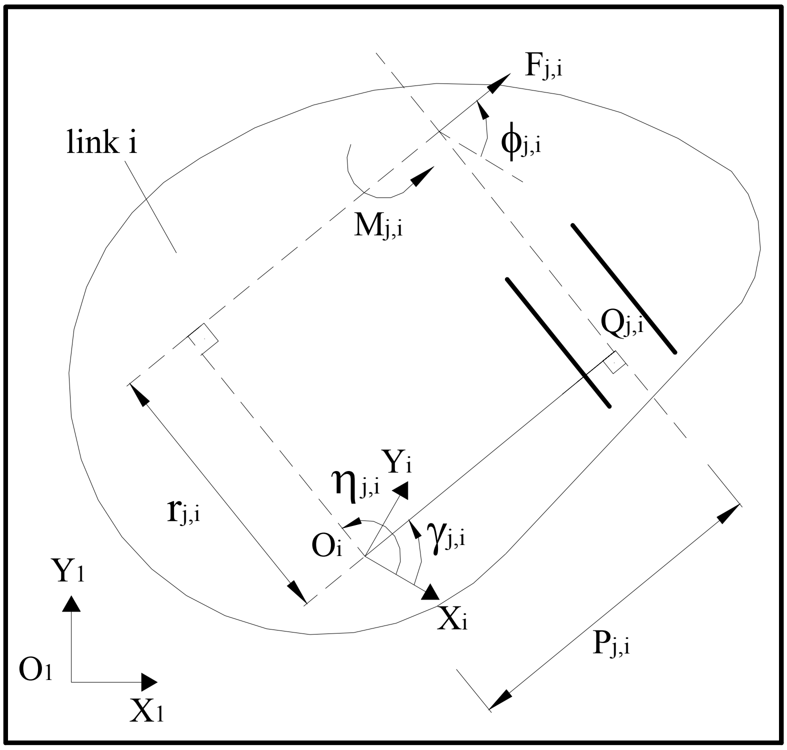

Figure 2.6 Slot (numbered j) on link i

In these figures $F_{j,i}$ and $M_{j,i}$ denote the reaction force and reaction moment due to the j'th prismatic joint acting on link i respectively . Also, the polar coordinates of point $Q_{j,i}$ in the $O_iX_iY_i$ system are designated by $P_{j,i}$ and $\gamma_{j,i}$. The angle $\beta_{j,i}$ is either (+) or (-) $\pi/2$, and it is measured from $\overrightarrow{O_iQ_{j,i}}$ to $\overrightarrow{Q_{j,i}R_{j,i}}$ in a right hand sense around Z_1.

The contributions of the j'th prismatic joint to the second summation in equations (2.4)-(2.6) will then be $F_{j,i}\, Cos(\phi_{j,i} + \theta_{1i})\,\vec{i}_1$, $F_{j,i}\, Sin(\phi_{j,i} + \theta_{1i})\,\vec{j}_1$ and $\left[M_{j,i} + r_{j,i}\, F_{j,i}\, Sin\left(\phi_{j,i} - \eta_{j,i}\right)\right]\vec{k}_1$.

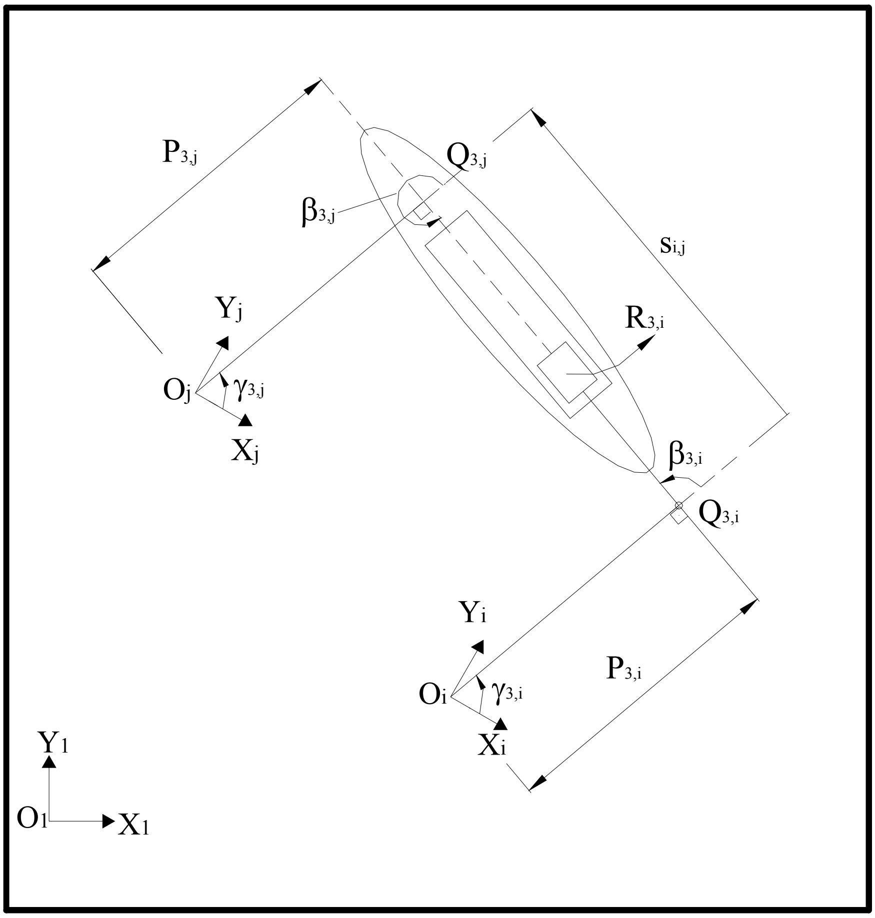

Figure 2.7 Prismatic joint number 3 connecting links i and j

Figure 2.7 illustrates the prismatic joint, say 3 connecting links i and j. As can be seen from the figure, the position variable $s_{i,j}$ is measured from $Q_{3,i}$, to $Q_{3,j}$, the positive sense being the $\overrightarrow{Q_{3,i}R_{3,i}}$ direction.

Figure 2.8 shows f'th externally applied force acting on link i. The contribution of this force to the third summation in equations (2.4)-(2.6) are given by $F_{f,i}\, Cos\,(\psi_{f,i} + \theta_{1i})\,\vec{i}_1$, $F_{f,i}\, Sin\,(\psi_{f,i} + \theta_{1i})\,\vec{j}_1$ and $\left[q_{f,i}\, F_{f,i}\, Sin\left(\psi_{f,i} - \delta_{f,i}\right)\right]\vec{k}_1$ respectively.

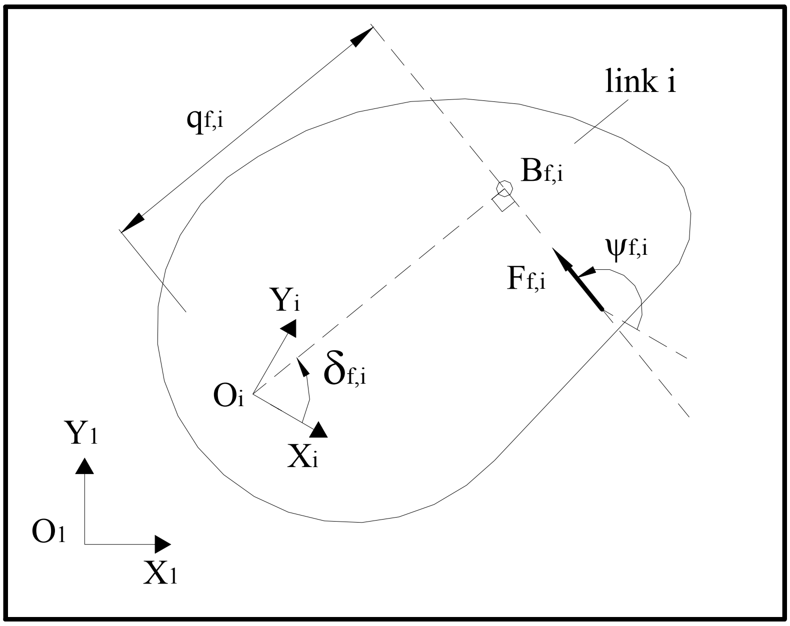

Figure 2.8 f'th externally applied force acting on link i

Figure 2.9 shows the m'th externally applied moment acting on link i, which is considered to be positive if in the Z_1 direction. Clearly, the contribution of this moment to the fourth summation in (2.6) is $M_{m,i}\,\vec{k}_1$.

Figure 2.10 shows link i with mass m_i and center of gravity CG_i . Here, $\bar{x}$ and $\bar{y}$ denote the coordinates of CG_i in the $O_iX_iY_i$ system, and $\vec{g} = g\langle\xi$ denotes the gravitational acceleration vector in the $O_1X_1Y_1$ system. The contribution of the weight of link i to the fourth summation in (2.4) , (2.5) and the fifth summation in (2.6) are given by $\left(m_i\, g\, Cos(\xi)\right)\vec{i}_1$, $\left(m_i\, g\, Sin(\xi)\right)\vec{j}_1$, and $MX_i\, g\, Sin(\xi - \theta_{1i}) - MY_i\, g\, Cos(\xi - \theta_{1i})$ respectively.

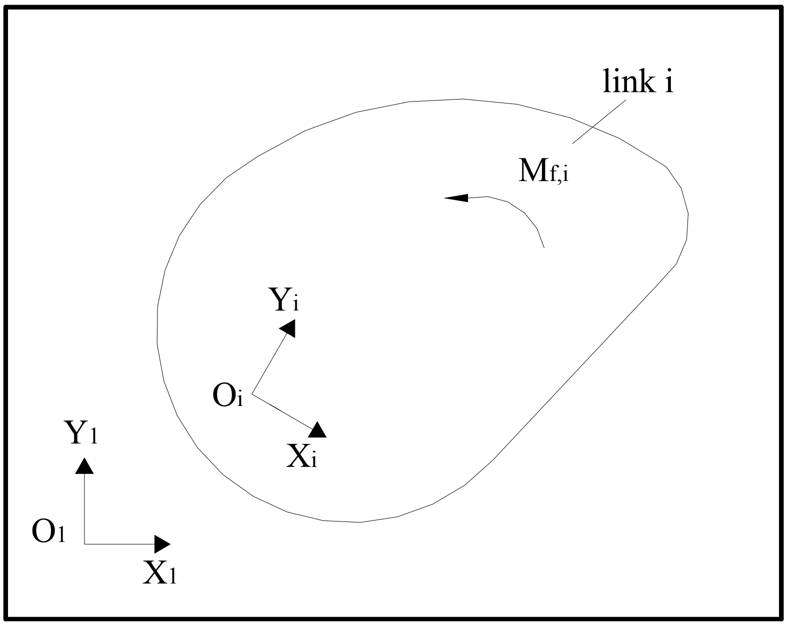

Figure 2.9 m'th externally applied moment acting on link i

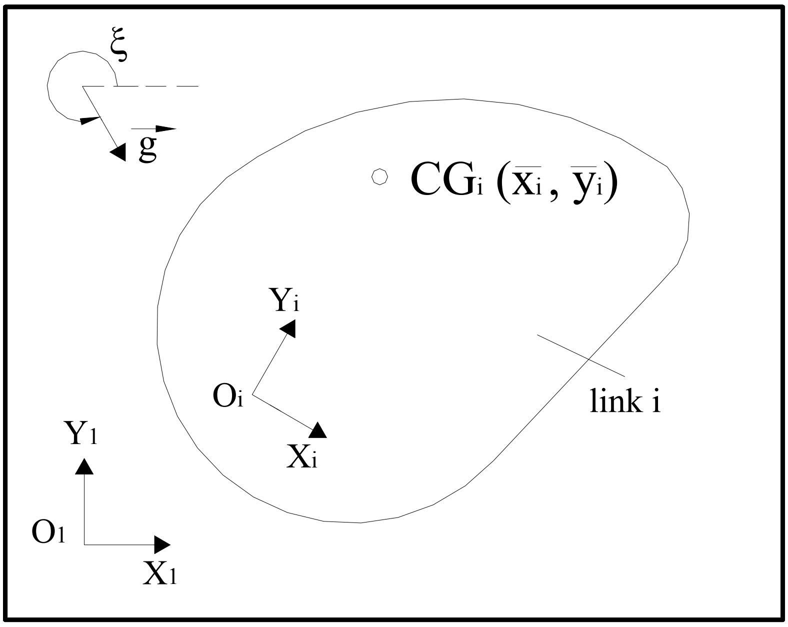

Figure 2.10 Coordinates of the center of gravity of link i

2.3.1 Equivalent Inertia Force System

Consider the inertia force system shown in Figure 2.11 . The inertia forces in the X_1, Y_1 directions and the inertia torque in the Z_1 direction are given by

$$\vec{F}^{x}_{in,i} = - m_i \, \vec{a}^{x}_{CG_i} \tag{2.7}$$

$$\vec{F}^{y}_{in,i} = - m_i \, \vec{a}^{y}_{CG_i} \tag{2.8}$$

$$\vec{T}_{in,i} = - \bar{I}_i \, \vec{\alpha}_{1i} \tag{2.9}$$

respectively, where

$\vec{a}^{x}_{CG_i}$, $\vec{a}^{y}_{CG_i}$: x and y components of the absolute acceleration of CG_i in the $O_1X_1Y_1$ system.

$\bar{I}_i$: Centrodial moment of inertia of link i.

$\vec{\alpha}_{1i}$: Absolute angular acceleration of link i (positive if in the Z_1 direction)

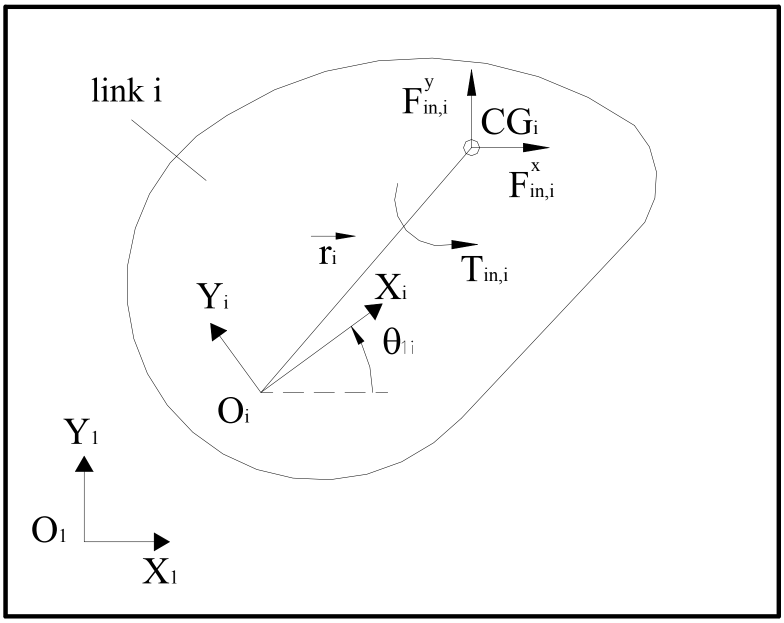

Figure 2.11 Inertia force system acting at CG_i

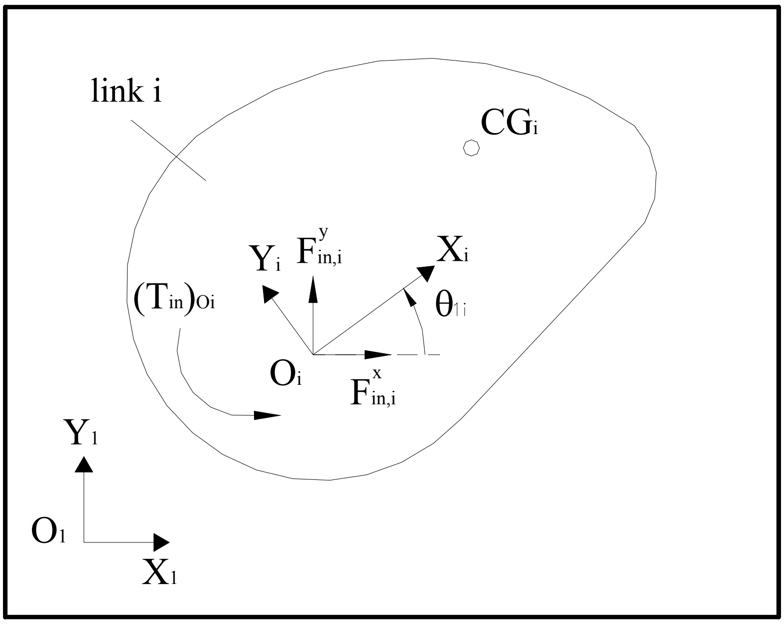

Figure 2.12 Inertia force system acting at O_i

Now, let the inertia force system in Figure 2.11 be equivalent to the inertia force system shown in Figure 2.12 . Clearly these two systems will be equivalent if and only if the moments about O_i are the same for both systems, i.e. ,

$$\vec{T}_{in,i} + \vec{r}_i \times \left(\vec{F}^{x}_{in,i} + \vec{F}^{y}_{in,i}\right) = \left(\vec{T}_{in}\right)_{O_i} \tag{2.10}$$

where

$$\vec{r}_i = \left(\bar{x}_i\, c_i - \bar{y}_i\, s_i\right)\vec{i}_1 + \left(\bar{x}_i\, s_i + \bar{y}_i\, c_i\right)\vec{j}_1 \tag{2.11}$$

and

$$c_i = Cos\left(\theta_{1i}\right)$$

$$s_i = Sin\left(\theta_{1i}\right)$$

Using simple kinematic relations one may write

$$\vec{a}_{CG_i} = \vec{a}_{O_i} + \vec{\omega}_{1i} \times \left(\vec{\omega}_{1i} \times \vec{r}_i\right) + \vec{\alpha}_{1i} \times \vec{r}_i \tag{2.12}$$

where

$\vec{a}_{O_i} = \left[a_{O_i}^x \quad a_{O_i}^y \right]$: Absolute acceleration of O_i in the X_1Y_1 system.

$\vec{\omega}_{1i}$: Absolute angular velocity of link i (positive if in the Z_1 direction).

Furthermore the parallel axis theorem yields

$$\bar{I}_i = I_i - m_i \, \bar{x}_i^2 - m_i \, \bar{y}_i^2 \tag{2.13}$$

Using equations (2.7), (2.8), (2.9), (2.10), (2.11), (2.12) and (2.13) and making some algebraic manipulations one obtains

$$\vec{F}_{in,i}^{x} = \left[m_i \left(-a_{O_i}^x \right) + MX_i \left(\omega_{1i}^2 \, c_i + \alpha_{1i} \, s_i \right) + MY_i \left(-\omega_{1i}^2 \, s_i + \alpha_{1i} \, c_i \right) \right] \vec{i}_1 \tag{2.14}$$

$$\vec{F}_{in,i}^{y} = \left[m_i \left(-a_{O_i}^y \right) + MX_i \left(\omega_{1i}^2 \, s_i - \alpha_{1i} \, c_i \right) + MY_i \left(\omega_{1i}^2 \, c_i + \alpha_{1i} \, s_i \right) \right] \vec{j}_1 \tag{2.15}$$

$$\left(\vec{T}_{in} \right)_{O_i} = \left[I_i \left(-\alpha_{1i} \right) + MX_i \left(a_{O_i}^x \, s_i - a_{O_i}^y \, c_i \right) + MY_i \left(a_{O_i}^x \, c_i + a_{O_i}^y \, s_i \right) \right] \tag{2.16}$$

Now one can substitute (2.14), (2.15) and (2.16) into equations (2.4)-(2.6) respectively.

2.4 Kinematic and Force Analysis of Adjustable Planar Mechanisms

Upto now, the methods used in Tursun's [7] program for kinematic and dynamic analysis of a planar mechanism, have been discussed. Note that the package developed by Tursun [7] is restricted to 1 DOF planar mechanisms only. However, the dynamically adjustable mechanisms may have more than one DOF. In this section the methods used in kinematic and dynamic analysis of adjustable mechanisms will be discussed.

Consider an adjustable mechanism with the generalized coordinate vector

$$\vec{q}(t) = \left[q_1(t)\ \ q_2(t) \ldots\ldots\ldots q_N(t)\right]^T = \begin{bmatrix} \vec{q}_s(t) \\ \vec{q}_u(t) \end{bmatrix} \tag{2.17}$$

where

$\vec{q}_s(t)$: Vector of specified generalized coordinates (i.e. the desired input motion) .

$\vec{q}_u(t)$: Vector of unspecified generalized coordinates (adjustable design parameters to be designed).

Note that $\vec{q}_s(t)$ denotes the vector of generalized coordinates in case of no adjustment (original mechanism). In other words, $\vec{q}(t) = \vec{q}_s(t)$ in the case of no adjustment. In this thesis, the study will be restricted to the mechanisms where the original (non-adjusted) mechanism has 1 degree of freedom (i.e., $\vec{q}_s(t) = q_s(t)$) . The methods used in kinematic and force analysis of such adjustable mechanisms depend on whether the adjustment is kinematic and/or dynamic.

2.4.1 Kinematic and Force Analysis of Dynamically Adjustable Mechanisms

Consider a 1 DOF mechanism , which is to be dynamically adjusted by means of attaching an actuator-block system on its i'th link as shown in Figure 2.13. Note that when the mechanism is adjusted dynamically by means of an actuator-block system (dynamic adjuster) , there exists another generalized coordinate which is unspecified . This generalized coordinate is the adjustable position $s_b(t)$ of the moving block and is expressed as $\vec{q}_u(t) = s_b(t)$. However, since it has no effect on the dimensions of the original mechanism, the kinematic analysis of the adjustable mechanism is the same as the kinematic analysis of the original mechanism. Therefore, one can use the program developed by Tursun [7] for the kinematic analysis of the adjustable mechanism.

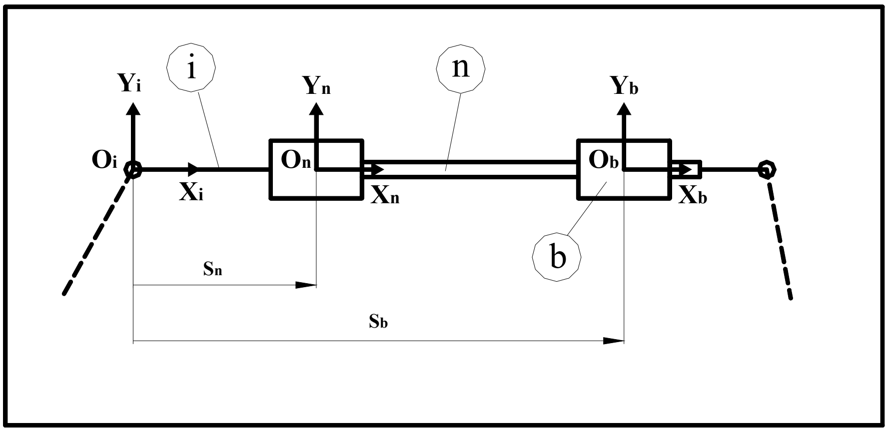

Figure 2.13 Link i of a Dynamically Adjustable Mechanism

The following algorithm may be used in conjunction with the program developed by Tursun [7] for deriving the equations of motion of the dynamically adjustable mechanisms where the original (non-adjusted) mechanism has 1 DOF.

1) Let $q_s(t)$ be the specified input motion of the original mechanism. Using the program developed by Tursun [7], perform velocity, acceleration and force analysis, and determine the actuator torque or force T_m and the reaction forces/moments vector $\vec{R}$ in terms of $\vec{U}$, $\dot{q}_s$, $\ddot{q}_s$ and $\vec{I}_m$. Here, $\vec{U}$ denotes the vector of unknown position variables, $\dot{q}_s$ denotes the input generalized velocity, $\ddot{q}_s$ denotes the input generalized acceleration and $\vec{I}_m$ denotes the vector of inertial parameters of the mechanism. One can express $\vec{I}_m$ as

$$\vec{I}_m = \{ m_2 , MX_2 , MY_2 , I_2 ; m_3 , MX_3 , MY_3 , I_3 ; .... m_l , MX_l , MY_l , I_l \} \tag{2.18}$$

where

m_i : mass of link i .

$MX_i \stackrel{\Delta}{=} m_i \, \bar{x}_i$

$MY_i \stackrel{\Delta}{=} m_i \, \bar{y}_i$

I_i : Moment of inertia of link i with respect to O_i .

While finding T_m and $\vec{R}$ put an external force F^j_{ext} and an external moment M^j_{ext} on each link (where j denotes the link number) so that the program displays T_m and $\vec{R}$ in terms of the external force and moment that act on each link symbolically. Note that the angles and the moment arms of the external forces will also be displayed symbolically as ψ^j_{ext}, δ^j_{ext} and q^j_{ext} .

2) Determine the accelerations of all origins of the links of the original mechanism in terms of $\vec{U}$, $\dot{q}_s$, and $\ddot{q}_s$.

3) Now let's put a moving block and a non-moving actuator on link i as shown in Figure 2.13. Note that in this figure , $O_iX_iY_i$ is the body fixed coordinate system (attached to link i on which the moving block will be working), $O_nX_nY_n$ is the body fixed coordinate system attached to the non-moving actuator n and $O_bX_bY_b$ is the body fixed coordinate system attached to the moving block. Now consider the free body diagram of link i shown in Figure 2.14.

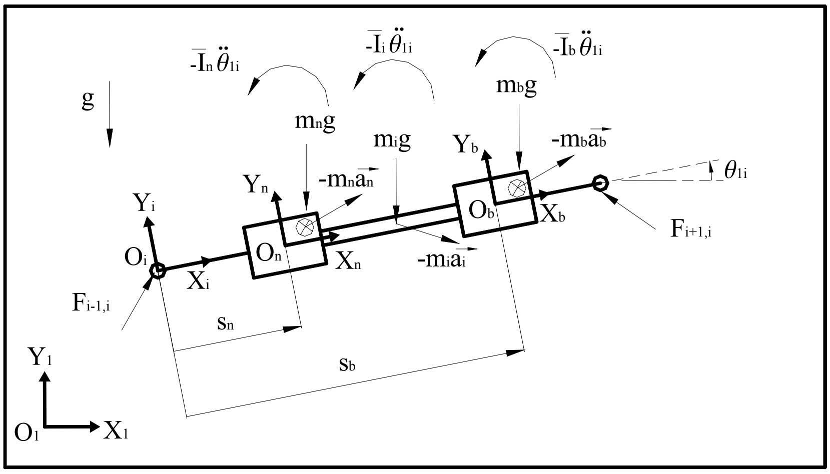

Figure 2.14 Free body diagram of link i

In this figure $\bar{I}_n$ and $\bar{I}_b$ denote the centroidal moment of inertias of the non-moving actuator and the moving block. Also $\vec{a}_n$ and $\vec{a}_b$ denote the mass center accelerations of the non-moving actuator and the moving block in the $O_iX_1Y_1$ (fixed) system. Using the original the origin acceleration of link i (determined in step 2), one can determine $\vec{a}_n$ and $\vec{a}_b$ by

$$\vec{a}_n = \vec{a}_{O_i} + \vec{a}_{n/O_i} \tag{2.19}$$

$$\vec{a}_b = \vec{a}_{O_i} + \vec{a}_{b/O_i} \tag{2.20}$$

where

$$\begin{aligned}\vec{a}_{n/O_i} = &\{Cos(\theta_{1i}(t))\,[(-\bar{x}_n - s_n)\dot{\theta}_{1i}^{\,2}(t) \ - \bar{y}_n\,\ddot{\theta}_{1i}(t)] \\ &+ Sin(\theta_{1i}(t))\,[\bar{y}_n\,\dot{\theta}_{1i}^{\,2}(t) - (\bar{x}_n + s_n)\ddot{\theta}_{1i}(t)\}\,\vec{i}_1 \\ &\{Sin(\theta_{1i}(t))\,[(-\bar{x}_n - s_n)\dot{\theta}_{1i}^{\,2}(t) \ - \bar{y}_n\,\ddot{\theta}_{1i}(t)] \\ &+ Cos(\theta_{1i}(t))\,[\,-\bar{y}_n\,\dot{\theta}_{1i}^{\,2}(t) + (\bar{x}_n + s_n)\ddot{\theta}_{1i}(t)\}\,\vec{j}_1\end{aligned} \tag{2.21}$$

$$\begin{aligned}\vec{a}_{b/O_i} = &\{Cos(\theta_{1i}(t))\,[(-\bar{x}_b - s_b)\dot{\theta}_{1i}^{\,2}(t) \ + \ \ddot{s}_b(t) \ - \bar{y}_b\,\ddot{\theta}_{1i}(t)] \\ &+ Sin(\theta_{1i}(t))\,[-2\dot{s}_b(t)\,\dot{\theta}_{1i}(t) + \bar{y}_b\,\dot{\theta}_{1i}^{\,2}(t) - (\bar{x}_b + s_b)\ddot{\theta}_{1i}(t)\}\,\vec{i}_1 \\ &\{Sin(\theta_{1i}(t))\,[(-\bar{x}_b - s_b)\dot{\theta}_{1i}^{\,2}(t) \ + \ \ddot{s}_b(t) \ - \bar{y}_b\,\ddot{\theta}_{1i}(t)] \\ &+ Cos(\theta_{1i}(t))\,[\,2\dot{s}_b(t)\,\dot{\theta}_{1i}(t) - \bar{y}_b\,\dot{\theta}_{1i}^{\,2}(t) + (\bar{x}_b + s_b)\ddot{\theta}_{1i}(t)\}\,\vec{j}_1\end{aligned} \tag{2.22}$$

Note that $\vec{a}_{b/O_i}$ and $\vec{a}_{n/O_i}$ can be determined in the inertial (fixed) coordinate system $O_1X_1Y_1$, in terms of θ_{1i} , $\dot{\theta}_{1i}$, $\ddot{\theta}_{1i}$, $s_b, \dot{s}_b$, $\ddot{s}_b$, s_n and the center of mass coordinates of the non-moving actuator n and the moving block b.

4) Note that the inertia forces , inertia moments and weights of the non-moving actuator and the moving block shown in Figure 2.14 are now available in terms of θ_{1i} , $\dot{\theta}_{1i}$, $\ddot{\theta}_{1i}$, $s_b, \dot{s}_b$, $\ddot{s}_b$, s_n ,center of mass coordinates of the non-moving actuator and the moving block $(\bar{x}_n, \bar{y}_n, \bar{x}_b \ and \ \bar{y}_b)$, $\bar{I}_n$, $\bar{I}_b$, m_n and m_b . By making some algebraic manipulations one can easily obtain these forces and

moments in terms of θ_{1i} , $\dot{\theta}_{1i}$, $\ddot{\theta}_{1i}$, $s_b, \dot{s}_b$, $\ddot{s}_b$, s_n , MX_n , MY_n , MX_b , MY_b , I_n , I_b, m_n and m_b where

m_n : mass of the non-moving actuator n .

$MX_n \stackrel{\Delta}{=} m_n \, \bar{x}_n$

$MY_n \stackrel{\Delta}{=} m_n \, \bar{y}_n$

I_n : Moment of inertia of the non-moving actuator with respect to O_n which can simply be obtained by $I_n = \bar{I}_n + m_n \, (\bar{x}_n^2 + \bar{y}_n^2)$.

m_b : mass of the non-moving actuator n .

$MX_b \stackrel{\Delta}{=} m_b \, \bar{x}_b$

$MY_b \stackrel{\Delta}{=} m_b \, \bar{y}_b$

I_b : Moment of inertia of the moving block with respect to O_b which can simply be obtained by $I_b = \bar{I}_b + m_b \, (\bar{x}_b^2 + \bar{y}_b^2)$.

Note that all these forces and moments can be reduced to a single force and single moment which can be treated to be an “external” force and moment acting on link i of the original mechanism .

5) Find the contribution of the external force and moment found in (3) to T_m and $\vec{R}$ by applying the method of superposition. If there are any other external forces and/or moments acting on the other moving links, find also their contributions to T_m and $\vec{R}$.

6) At the end of step 5 , one has T_m and $\vec{R}$ in terms of $\vec{d}_c$, $\vec{q}_u(t) = s_b(t)$(and its first two time derivatives), $\vec{I}_n$, $\vec{I}_m$, $\vec{U}$, $\dot{q}_s$ and $\ddot{q}_s$ where $\dot{q}_s$ and $\ddot{q}_s$ are known. Here $\vec{I}_n$ denotes the vector of inertial parameters of the non-moving

actuator n and $\vec{d}_c$ denotes the vector of constant design parameters including the inertial parameters of the moving block and the constant position s_n of the non-moving actuator n . The vectors $\vec{I}_m$ and $\vec{I}_n$ can be combined in a vector $\vec{I}_s$ which is the vector of specified inertial parameters of the mechanism. Therefore T_m and $\vec{R}$ can be obtained in closed form in terms of time, $\vec{I}_s$, $\vec{d}_c$, $\vec{q}_u(t)$ and $\vec{U}$.

7) If the adjustment is made during the regular motion, one should also determine the actuator force F_b that actuates the block, and the reaction force and reaction moment due to prismatic joint between the actuator and the block. To determine these 3 unknowns, one should draw the free body diagram of the block seperately as shown in Figure 2.15. In this figure, F_b denotes the actuator force that is required to move the block, M_p and F_p denote the reaction moment and reaction force due to prismatic joint between the block and the actuator. One can easily determine F_b, M_p and F_p in terms of time $\vec{I}_s$, $\vec{d}_c$, $\vec{q}_u(t)$ and $\vec{U}$ by solving the three equilibrium equations for the moving block.

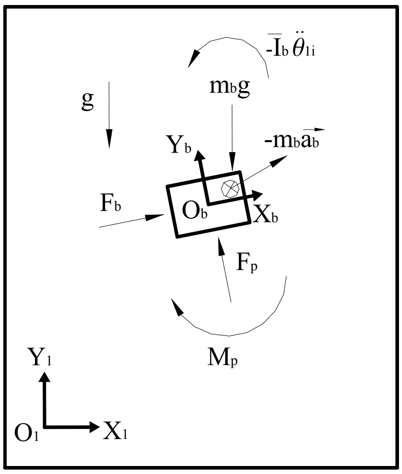

Figure 2.15 Free body diagram of the moving block

8) At this stage, the terms T_m , $\vec{R}$, F_b, M_p and F_p have been determined in closed form in terms of time, $\vec{I}_s$, $\vec{d}_c$, $\vec{q}_u(t)$ and $\vec{U}$. Note that for the specified input motion $q_s(t)$ one can obtain $\vec{U}$ by performing the position analysis of the mechanism. Since the position analysis of the mechanism , in general, does not yield a closed form solution, one can solve position analysis numerically using the package developed by Tursun[7] . Solving position analysis numerically, T_m , $\vec{R}$, F_b , M_p and F_p can be determined corresponding to a given time value, in terms of $\vec{I}_s$, $\vec{d}_c$ and $\vec{q}_u(t_i)$ only, where $\vec{q}_u(t_i)$ denotes the value of $\vec{q}_u(t)$ at the time value t_i . Here t_i denotes the i'th time value of the operating time interval $[t_0 , t_f]$ which is divided into a certain number of subintervals for numerical solution. Note that $\vec{q}_u(t_i)$ is displayed symbolically in the expressions of T_m , $\vec{R}$, F_b, M_p and F_p for each t_i .

Note that the algorithm above may easily be applied for the mechanisms where there are more than one adjusters on different moving links of the mechanism. In this case, contribution of each adjuster to the actuator forces/torques and the joint reactions can be determined as explained in steps 3 and 4. However, in order to distinguish the parameters of the adjusters one should replace the subscripts b and n by bk and nk where k denotes the k'th dynamic adjuster (i.e s_{nk}, s_{bk}, m_{bk} , m_{nk} etc).

2.4.2 Kinematic and Force Analysis of Kinematically Adjustable Mechanisms

The kinematic and force analysis of kinematically adjustable mechanisms is more diffucult than that of the dynamically adjustable mechanisms. As discussed in section 2.4.1, in the case of dynamic adjustment, the unspecified generalized coordinates vector $\vec{q}_u(t)$ has no effect on the kinematic analysis of the mechanism. However, in case of kinematic adjustment, $\vec{q}_u(t)$ is the variable link length $L_{ij}(t)$ consisting of links i and j of the mechanism shown in Figure 2.2. Therefore it affects the kinematic and force analysis of the mechanism directly. In case of kinematic adjustment, one can't use the program developed by Tursun [7] for the kinematic and dynamic analysis. One should analyze the mechanism starting from strach by considering the free body diagrams of each link. The following algorithm may be implemented in MATHEMATICA for the kinematic and force analysis of kinematically adjustable mechanisms. Note that the study in this thesis is restricted to kinematically adjustable mechanisms where the original (non-adjusted) mechanism has 1 DOF.

1) Perform velocity, acceleration and force analysis of the mechanism and determine the actuator torques/forces vector $\vec{T}_m$ and the reaction forces vector $\vec{R}$ in terms of time, $\dot{\vec{q}}_s$, $\ddot{\vec{q}}_s$, $\vec{U}$ and $\vec{q}_u(t)$ (and its first two time derivatives) and $\vec{I}_s$. Here $\vec{I}_s$ denotes the vector of inertial parameters which is specified.

2) Perform the position analysis of the mechanism and solve $\vec{U}$ in terms of $\vec{q}_u(t)$, for a given $\vec{q}_s(t)$.

3) Substitute $\vec{U}$ (determined in (2)) into $\vec{T}_m$ and $\vec{R}$ expressions and obtain them in terms of time , $\vec{I}_s$ and $\vec{q}_u(t)$.

Again , note that if $\vec{U}$ can't be obtained in closed form in terms of $\vec{q}_u(t)$, perform position analysis numerically by dividing the operating time [t_0, t_f] into a number of subintervals and determine $\vec{U}$ numerically as $\vec{U}(t_i)$ in terms of $\vec{q}_u(t_i)$ where t_i denotes the i'th time value in the interval [t_0, t_f]. Using these results determine $\vec{T}_m$ and $\vec{R}$ numerically as $\vec{T}_m(t_i)$ and $\vec{R}(t_i)$ for each t_i.

CHAPTER III

THE PERFORMANCE MEASURE

3.1 Computation of the Performance Measure

In this study, the dynamic behaviour of adjustable planar mechanisms will be quantified using the concept of performance measure (PM). The performance measure is the objective function to be minimized, or maximized, in order to obtain the optimal control for the desired behaviour of the system in optimal control problems. Depending upon the type of the optimal control problem, the performance measure may be expressed in many different forms (see Kirk [9]).

Consider an adjustable planar mechanism with the generalized coordinates vector

$$\vec{q}\,(t) = \left[q_1(t)\ \ q_2(t)\ldots\ldots\ldots\ldots q_N(t)\right]^T = \begin{bmatrix} \vec{q}_s(t) \\ \vec{q}(t) \end{bmatrix} \tag{3.1}$$

where

$\vec{q}_s(t)$: Vector of specified generalized coordinates (i.e. the desired input motion) .

$\vec{q}_u(t)$: Vector of unspecified generalized coordinates (adjustable design parameters to be determined).

Now let the performance measure be defined by

$$J = \int_{t_0}^{t_f} \left\{ \sum_{i=1}^{n_i} w_i\, G_i \right\} dt \tag{3.2}$$

where

t_0, t_f : Initial and final values of the time-space over which J is defined.

G_i : The i'th Lagrangian function corresponding to the i'th desired behaviour of the mechanism

w_i : Weighting coefificient associated with G_i (user specified).

n_i : Number of G_i's that appear in the definition of J .

As seen in equation (3.2), by selecting the definition of the G_i's appropiately, and combining them in a weighted manner, one can obtain different performance measures representing different dynamic behaviours of the mechanism. Two possible definitions of J are shown below for achieving various dynamic requirements.

1) Consider a 1 DOF mechanism that is actuated by an armature controlled DC motor. Let the design task be the minimization of the copper losses *(i^2R)* of this motor, where i and R denote the armature current and resistance respectively. Note that for this type of motor, the generalized actuator force, T_m, (it may be a force or torque depending upon the type of the actuator) and the current are related by

$$T_m = K\, i \tag{3.3}$$

Energy dissipated in the armature circuit due to copper losses is then given by

$$W_{CL} = \int_{t_0}^{t_f} (i^2 R)\, dt = \int_{t_0}^{t_f} (\frac{T_m^2}{K^2} R)\, dt = \frac{R}{K^2} \int_{t_0}^{t_f} T_m^2\, dt \; = K_m \int_{t_0}^{t_f} T_m^2\, dt \tag{3.4}$$

where $K_m = \dfrac{R}{K^2}$ is the motor constant. To minimize W_{CL}, one has to minimize

$$J_{CL} = \int_{t_0}^{t_f} T_m^2\, dt \tag{3.5}$$

where the subscript CL denotes the copper losses.

Note that equation (3.5) is obtained from equation (3.2) by setting $n_i = 1$, $G_i = T_m^2$ and $w_i = 1$.

2) One may also consider minimization of the energy consumed (EC) in the mechanism.In this case, if the motor(s) are not regenerative, the performance measure is formulated to be

$$J_{EC} = \int_{t_0}^{t_f} \left(\sum_{i=1}^{N} \left| T_{mi} \, \dot{q}_i \, (t) \right| \right) \, dt \tag{3.6}$$

where T_{mi} and $\dot{q}_i(t)$ denote the i'th actuator torque/force and the associated generalized velocity respectively.

Similar expressions may be obtained for the minimization of the fluctuations of shaking forces, shaking moments and joint reactions. Note that if there are external forces and/or moments due to the loads, the shaking force or moment will include the effects of the external forces and/or moments as well. The classical shaking force/moment concept on the other hand, takes into account only the inertia forces and moments. Suppose that the desired objective is the minimization of the fluctuations of the components of shaking forces (SF) from their desired values. Then the performance measure to be minimized can be expressed as

$$J_{SF} = \int_{t_0}^{t_f} \left\{ w_1 \, (F_s^x - F_{sd}^x)^2 + w_2 \, (F_s^y - F_{sd}^y)^2 \right\} dt \tag{3.7}$$

where

w_1, w_2 : weighting factors

F_s^x , F_s^y : x and y components of the shaking force.

F_{sd}^x , F_{sd}^y : desired values of F_s^x and F_s^y . Each of these values could be any specified constant (including zero).

In order to compute the performance measure J, one should obtain the G_i*'s* that appear in the definition of J. This is realized by using the algorithms developed in section 2.4. By implementing these algorithms, one may obtain any G_i in closed form in terms of time, $\vec{U}$, $\vec{I}_s$, $\vec{d}_c$, $\vec{q}_u(t)$, $\dot{\vec{q}}_u(t)$, and $\ddot{\vec{q}}_u(t)$. Here, $\vec{U}$ denotes the unknown position variables vector, $\vec{I}_s$ denotes the vector of specified inertial parameters, $\vec{d}_c$ denotes the vector of constant design parameters, and $\vec{q}_u(t)$ denotes the vector of unspecified generalized coordinates (consists of design functions) of the adjustable mechanism.

Note that the elements of the design parameter vector $\vec{d}_c$, and $\vec{q}_u(t)$, depend on the type of the adjustable mechanism. Two possible cases are given below.

1) If the mechanism is a dynamically adjustable one (Figure 2.1) , then

$$\vec{d}_c = \{m_b, MX_b, MY_b, I_b, s_n\} \tag{3.8}$$

$$\vec{q}_u(t) = \{s_b(t)\} \tag{3.9}$$

2) If the mechanism is a kinematically adjustable one (Figure 2.2), then there are no constant design parameters. The only function to be designed is the unspecified generalized coordinate $\vec{q}_u(t)$ which is given by

$$\vec{q}_u(t) = \{L_{ij}(t)\} \tag{3.10}$$

where

$L_{ij}(t)$: Variable link length of the combined link consisting of links i and j .

Note that the vectors $\vec{q}_u(t)$, $\dot{\vec{q}}_u(t)$, $\ddot{\vec{q}}_u(t)$ and $\vec{U}$ are in implicit form in the definition of the G_i's. Therefore, it is impossible to obtain J by evaluating the integral given by (3.2) . In order for the G_i's to be integrable over the time interval [t_0 , t_f], one should express the vectors $\vec{q}_u(t)$ and $\vec{U}$ explicitly.

The expression of $\vec{q}_u(t)$ depends on whether the adjustment is made before or during the regular motion. If the adjustment is made before the regular motion, then $\vec{q}_u(t)$ is fixed during the motion. Since it is a constant, it is already in explicit form and creates no problem in the integration of G_i's. Note that in this the performance measure J is a function of $\vec{q}_u(t)$. Therefore the optimization problem is a static one which can be solved easily.

However, in the case of adjustment during the regular motion, the unspecified generalized coordinates vector $\vec{q}_u(t)$ is a function of time. Therefore the performance measure J becomes a *functional* (function of a function) of $\vec{q}_u(t)$. In this case, one has a dynamic optimization problem which is more diffucult to solve. Depending on the type of the problem, there are different solution methods (See Kirk [9]). In this study, a suboptimal solution method using Piecewise Continuous Polynomial Parameterization has been used (see Yılmaz [8]). The main idea of the method is to assign piecewise continuous polynomials for the elements of $\vec{q}_u(t)$, so that one obtains J in terms of some independent polynomial coefficients using equation (3.2) . Note that by this method, the dynamic optimization problem is converted to a static one, therefore the minimization of J is much easier as in the case of adjustment before the regular motion. A detailed information about the Piecewise Continuous Polynomial Parameterization is given in section 3.2 .

In order to perform the integration given by equation (3.2), one should also express the unknown position variables vector $\vec{U}$ explicitly in the definitions of the G_i's . This is realized by performing the position analysis of the mechanism. Note that if the position analysis yields a closed form solution , it is possible to obtain J by analytical integration, using equation (3.2). However, in general the position analysis of a mechanism does not yield a closed form solution , therefore in most cases it is impossible to obtain J analytically using the equation (3.2). On the other hand, if the time is specified numerically, one obtains any G_i corresponding to the given time value (since the position analysis may be solved numerically) . Using the values of G_i's at the given time values, the performance measure is then obtained by symbo-numeric integration. Here, symbo-numeric integration refers to the implementation of a numerical integration algorithm using a symbolic manipulation package. The numerical integration algorithm used in this study is the Gaussian Quadrature method, where the number of Gaussian Quadrature points may be selected by the user.

When the weighted combination of the G_i's are integrated, one obtains the performance measure J in terms of $\vec{I}_s$, $\vec{d}_c$, and $\vec{d}_u$. Since $\vec{I}_s$ is specified, the resulting expression of J will be only in terms of $\vec{d}_c$, and $\vec{d}_u$. Here $\vec{d}_u$ denotes the vector of design parameters related to the unspecified generalized coordinates $\vec{q}_u(t)$. As an example, consider that an adjustable mechanism has a single unspecified generalized coordinate given by $\vec{q}_u(t) = f(t)$. If the adjustment is made before the regular motion $\left(i.e, f(t) = f = cst \ , \ \dot{f}(t) = \ddot{f}(t) = 0 \right)$, then $\vec{d}_u$ is defined by $\vec{d}_u = \{f\}$. On the other hand, if the adjustment is made during the regular motion, then the elements of $\vec{d}_u$ are the independent coefficients of the piecewise continuous polynomial which approximates $f(t)$. The vectors $\vec{d}_c$, and $\vec{d}_u$, when combined, constitute the design parameters vector $\vec{d}_J$ that affect J . Now J is ready to be minimized with respect to the elements of $\vec{d}_J$. Note that due to the geometry of the mechanism and its variable operating conditions, some elements of the vector $\vec{d}_c$ may not appear in the expression for J .

3.2 Piecewise Continuous Polynomial Parameterization

In this section, the method of approximating the unspecified generalized coordinates via piecewise continuous polynomials is explained. As stated in the previous section for the case of adjustment during the regular motion, piecewise continuous polynomials are used to approximate the unspecified genaralized coordinates, so that the optimization problem becomes easier. This is a very effective method especially in systems which involve highly nonlinear and complicated equations.

A MATHEMATICA program called PiecewisePolyGenerate has been developed for generating the piecewise continuous polynomials. Consider that a function $\hat{g}(x)$ is to be approximated as a piecewise continuous polynomial g(x) on the interval $[x_i , x_f]$ where x denotes the independent variable, x_i denotes the initial value of x, and x_f denotes the final value of x . For this purpose, the interval $[x_i , x_f]$ is divided into n_p number of subintervals (not neccessarily of equal length) and a polynomial $g_j(x)$ (j = 1, 2 , ...n_p) is defined for each subinterval, as seen in figure 3.1 such that $g(x) = g_j(x)$ if $x_j \leq x \leq, x_{j+1}$.

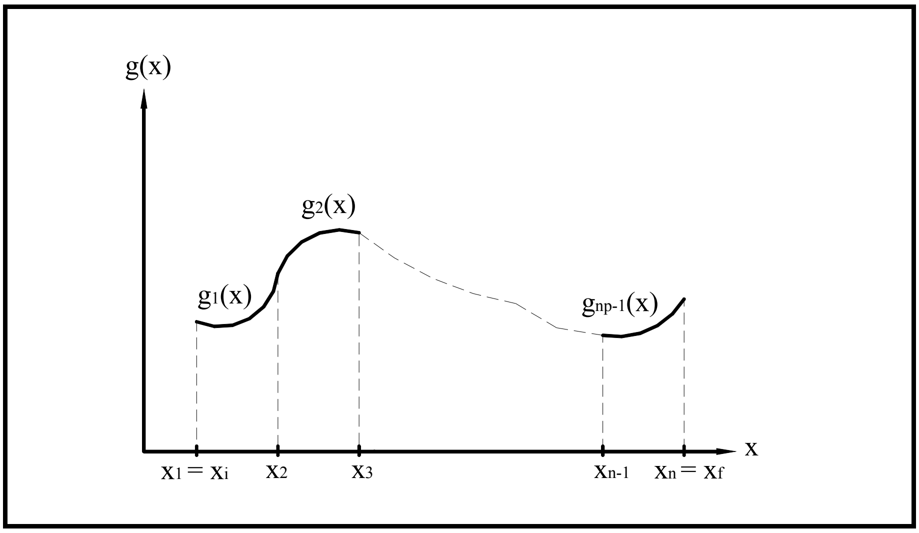

Figure 3.1 Piecewise Continuous Polynomials

The program displays the piecewise continuous polynomial g(x) in terms of the design coefficients, which are the values of the $g_j(x)$'s and their derivatives at the knot points (x_j's). Now lets introduce the notation used in expressing these design coefficients . As an example, consider a polynomial $g_j(x)$ ($j = 1,2,......,n-1$) on the subinterval $[x_j , x_{j+1}]$. The initial value of this polynomial is its value at $x = x_j$ and it is denoted by the coefficient $g_{jI,\,0P}$. Here I denotes the initial value and 0P denotes the 0'th prime (derivative) of $g_j(x)$. Similarly the final value of $g_j(x)$ is denoted by $g_{jF,\,0P} = g_j(x_{j+1})$. A few examples are given below for this notation.

$$
\begin{aligned}
&g_{jI,0P} = g_j(x_j)\ ,\ g_{jF,0P} = g_j(x_{j+1}) \\
&g_{jI,1P} = \dot{g}_j(x_j),\ g_{jF,1P} = \dot{g}_j(x_{j+1}) \\
&g_{jI,2P} = \ddot{g}_j(x_j),\ g_{jF,2P} = \ddot{g}_j(x_{j+1}) \quad \text{etc .}
\end{aligned}
\tag{3.11}
$$

The neccessary inputs to run the program are n , $\overrightarrow{xv}$, $\vec{S}$, $\vec{C}$, bv and nd .

1) n is the number of knot points on the interval $[x_i , x_f]$. Note that $x_i = x_1$ and $x_f = x_n$.

2) $\overrightarrow{xv}$ is the n dimensional vector of knot points on the interval $[x_1 , x_n]$, given by $\overrightarrow{xv} = \{x_1 , x_2 ,............x_n\}$.

3) $\vec{S}$ is the n dimensional vector of number of specified g values at the knots . This vector can be expressed as $\vec{S} = \{s_1 , s_2 ,............s_n\}$. For instance, s_1 denotes the number of specified g values at x_1 . If $s_1 = 3$, this means $g_1(x_1)$, $\dot{g}_1(x_1)$ and $\ddot{g}_1(x_1)$ are specified. Note that some of these values may not be specified numerically and left as free variables. The elements of the vector $\vec{S}$ also gives us information about the degree of the polynomials . One can obtain the degree of the j'th polynomial , $g_j(x)$, by the equation $\deg(g_j(x)) = s_j + s_{j+1} - 1$.

4) $\vec{C}$ is the (n-2) dimensional vector of degree of continuity at the knot points, given by $\vec{C} = \{c_2 , c_3 ,........... c_{n-1}\}$. For instance if $c_2 = 3$, then at the point $x = x_2$

one has 3 contiunity equations which are $g_1(x_2)=g_2(x_2)$, $\dot{g}_1(x_2)=\dot{g}_2(x_2)$ and $\ddot{g}_1(x_2)=\ddot{g}_2(x_2)$.

5) *bv* is a string which contains all specified (numerical or symbolic) values at $x = x_1$ and $x = x_n$.

6) nd is the # of derivatives of g (x) to be displayed as output . For instance when one enters nd = 2, it is possible to obtain the polynomial and its first two derivatives with respect to the independent variable x.

The program simply obtains the $g_j(x)$'s in terms of the parameters $g_{jI,kP}$ and $g_{jF,kP}$ where j = 1,2,....np and k = 1,2,...nd by simply solving a set of linear equations. It should be noted that if there are any continuity requirements and/or initial/final value requirements, then only some of the parameters $g_{jI,kP}$ and $g_{jF,kP}$ will be independent. When the program is executed, the piecewise continious polynomial g(x) which satisfies all the boundary conditions and the contiunity requirements is displayed as

$$g(x) = g_1(x)\,[u(x-x_1)-u(x-x_2)] + g_2(x)\,[u(x-x_2)-u(x-x_3)] + \ldots\ldots\ldots\ldots$$

$$+\, g_{n-1}(x)\,[u(x-x_{n-1})-u(x-x_n)] \tag{3.12}$$

where u(x) denotes the unitstep function. If needed, the derivatives of g(x) can also be displayed in the same way.

3.3 Minimization of the Performance Measure

In order to optimize the dynamic behaviour of the adjustable mechanism, the performance measure given by equation (3.2) should be minimized with respect to the design parameters . In order for the optimal design parameters to be phsically meaningful, they must satisfy certain constraints. These constraints depend on the type of the adjustable mechanism.

If the mechanism is a dynamically adjustable one (Figure 2.1), then one should determine the optimal $\vec{q}_u(t)$ and $\vec{d}_c$ where $\vec{q}_u(t) = \{s_b(t)\}$, and $\vec{d}_c = \{m_b, MX_b, MY_b, I_b, s_n\}$. In order for the block to be physically realizable, the inertial parameters of the block , namely m_b, MX_b, MY_b and I_b should satisfy certain constraints. Two such constraints which are quite obvious are given by

$$m_b > 0 \tag{3.13}$$

$$I_b > 0 \tag{3.14}$$

The third constraint for the block to be phsically realizable is given by

$$\bar{I}_b > 0 \tag{3.15}$$

where $\bar{I}_b$ denotes the moment of inertia of the block with respect to its mass center. If $\bar{I}_b$ is substituted from the parallel axis theorem, the inequality (3.15) becomes

$$I_b - m_b\left(\bar{x}_b^2 + \bar{y}_b^2\right) > 0 \tag{3.16}$$

which can be written as

$$\frac{m_b\, I_b - (MX_b)^2 - (MY_b)^2}{m_b} > 0 \tag{3.17}$$

The three constraints given by (3.13), (3.14), and (3.17) can be easily shown to be equivalent to the following two inequalities.

$$m_b > 0 \tag{3.18}$$

$$m_b\, I_b - (MX_b)^2 - (MY_b)^2 > 0 \tag{3.19}$$

Note that the constraint given by (3.18) may not be satisfactory for practical purposes. In this case, one may replace (3.18) by

$$m_{bl} \le m_b \le m_{bu} \tag{3.20}$$

where m_{bl} and m_{bu} are user specified lower and upper bounds for m_b . Again due to practical reasons one may replace (3.14) by

$$I_{bl} \le I_b \le I_{bu} \tag{3.21}$$

where I_{bl} and I_{bu} are the user specified lower and upper bounds for I_b .

Note that due to the restrictions in available space, , one may also consider additional constraints on the center of mass coordinates of the moving block. These constraints may be in the form

$$\bar{x}_{bl} \le \bar{x}_b \le \bar{x}_{bu} \tag{3.22}$$

$$\bar{y}_{bl} \le \bar{y}_b \le \bar{y}_{bu} \tag{3.23}$$

where $\bar{x}_{bl}$, $\bar{x}_{bu}$, $\bar{y}_{bl}$ and $\bar{y}_{bu}$ are user specified lower and upper bounds for $\bar{x}_b$ and $\bar{y}_b$.

Multiplying each term by m_b in (3.22) and (3.23), one obtains

$$MX_{bl} \le MX_b \le MX_{bu} \tag{3.24}$$

$$MY_{bl} \le MY_b \le MY_{bu} \tag{3.25}$$

Due to practical reasons , one may impose constraints on the design parameter s_n in the form

$$s_{nl} \le s_n \le s_{nu} \tag{3.26}$$

where s_{nl} and s_{nu} are user specified lower and upper bounds for s_n .

The final constraint for the optimization of the dynamically adjustable mechanisms is the constraint on the position $s_b(t)$ of the moving block , in other words the unspecified generalized coordinate. The constraint for $s_b(t)$ may be in the form

$$s_{bl} \leq s_b(t) \leq s_{bu} \tag{3.27}$$

where s_{bl} and s_{bu} are user specified lower and upper bounds for $s_b(t)$.

For kinematically adjustable mechanisms the only design function is the variable length $L_{ij}(t)$ of the combined link consisting of links i and j . The constraint for this function may be in the form

$$L_{ij,l} \leq L_{ij}(t) \leq L_{ij,u} \tag{3.28}$$

where $L_{ij,l}$ and $L_{ij,u}$ are user specified lower and upper bounds for $L_{ij}(t)$.

In this study, the minimization of the performance measure has been performed using the *NMinimize* command of the *NumericalMath* package of MATHEMATICA . This command may be used to search for the global minimum value of any multivariable function numerically, subject to any type of constraints. Although the command selects the best of the 4 numerical minimization methods automatically during its execution, the method to be used may also be specified by the user .

3.3.1 The Minimization Algorithm

In this study, the following algorithm will be used to minimize the performance measure J , which optimizes the desired behaviour of the adjustable mechanism.

1) Obtain the G_i's that appear in the definition of J in closed form in terms of time , $\vec{I}_s$, $\vec{d}_c$, and $\vec{d}_u$, or numerically in terms of $\vec{I}_s$, $\vec{d}_c$, and $\vec{d}_u$. Also specify the weighting factors for each G_i .

2) Compute the performance measure J analytically or by symbonumeric integration (depending upon whether G_i's are determined in closed form or numerically) in terms of $\vec{I}_s$, $\vec{d}_c$, and $\vec{d}_u$ only. Note that since $\vec{I}_s$ (specified inertial parameters vector) is specified numerically, substitute it into J and obtain J in terms of $\vec{d}_c$, and $\vec{d}_u$ only.

3) Form the design parameters vector (that affect J) $\vec{d}_J$ by combining $\vec{d}_c$, and $\vec{d}_u$. Also decide upon the neccessary constraints to be taken into account on the elements of $\vec{d}_J$.

4) Finally, using the NMinimize command , determine the minimum value of J (denoted by J^{opt}) and the corresponding optimal design parameters vector $\vec{d}_J^{opt}$.

3.3.2 Conversion of Dynamic Constraints to Static Constraints

The definition of the constraints in NMinimize command of MATHEMATICA depends on the structure of the constraint. As an example consider a function $J = J(\vec{d}_J)$ which is to be minimized with respect to the elements of the design parameters vector $\vec{d}_J$, subject to the following constraints

$$f_1(\vec{d}_J) \geq 0 \tag{3.29}$$

$$f_2(\vec{d}_J, t) \geq 0 \quad \text{for all } t \in [t_0, t_f] \tag{3.30}$$

As seen above the first constraint involves the design parameters vector $\vec{d}_J$, only. Therefore, one can directly enter this inequality constraint in the NMinimize command (i.e. it is a static constraint). However, the constraint given by (3.30) is a function of the independent variable t (i.e., it is a dynamic constraint) . Note that $f_2(\vec{d}_J,t) \geq 0$ will be satisfied on the interval $[t_0, t_f]$ if and only if the minimum of $f_2(\vec{d}_J,t)$ on $[t_0, t_f]$ is greater than 0 . Therefore, a command ConsMin has been developed in MATHEMATICA. This command evaluates the function $f_2(\vec{d}_J,t)$ at certain points on $[t_0, t_f]$ and selects the minimum of these values as the minimum (with respect to time) of the function on $[t_0, t_f]$. The general form of the command is expressed as $ConsMin\,[F, t_0, t_f, \Delta t]$, where F is the function whose minimum value is required, t_0 is the initial value of the independent variable , t_f is the final value of the independent variable , and Δt is the increment used for the variable t.

Turning back to our example , the neccessary command to minimize $J(\vec{d}_J)$ subject to the constraints (3.29) and (3.30), can be expressed as

$$NMinimize\left[\left\{J(\vec{d}_J),\ \left\{f_1(\vec{d}_J) \geq 0\ ,\ ConsMin\ [\ f_2(\vec{d}_J,t),\ t_0\ ,\ t_f\ ,\Delta t] \geq 0\right\}\right\},\ \vec{d}_J\right]$$

Note that in some problems the dynamic constraints may be of the form

$$h_l \leq h(\vec{d}_J,t) \leq h_u \tag{3.31}$$

where h_l and h_u are the user specified lower and upper bounds for $h(\vec{d}_J,t)$. One may also express this inequality constraint as

$$[h_u - h(\vec{d}_J,t)] \geq 0 \tag{3.32}$$

and

$$[h(\vec{d}_J,t) - h_l] \geq 0 \tag{3.33}$$

As seen above both of the constraints given by (3.32) and (3.33) are in the form given by equation (3.30). Note that one may also express the inequalities (3.32) and (3.33) in a combined form as

$$[h_u - h(\vec{d}_J, t)][h(\vec{d}_J, t) - h_l] \geq 0 \tag{3.34}$$

CHAPTER IV

CASE STUDIES

4.1 The Dynamically Adjustable Oscillating Elliptic Trammel

Consider the adjustable oscillating elliptic trammel (OET) shown in Figure 4.1.

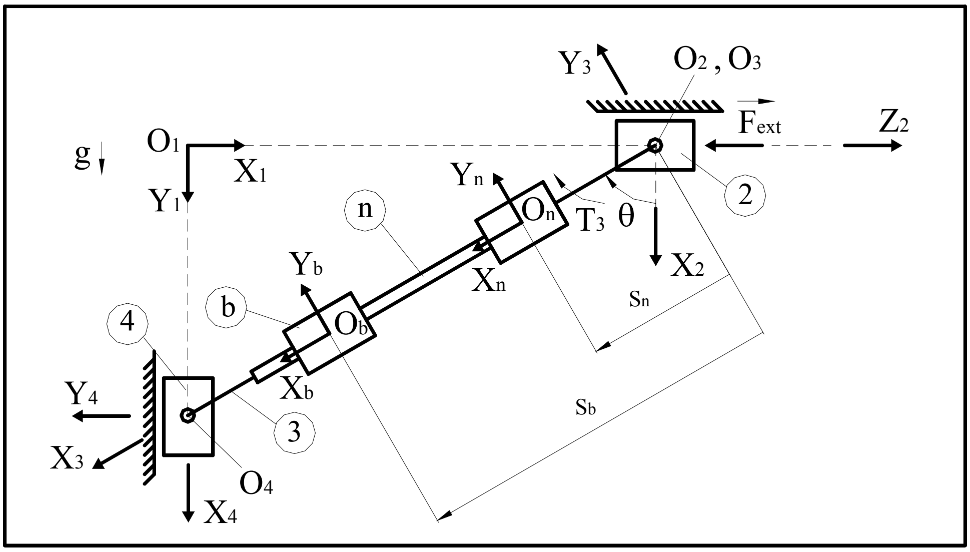

Figure 4.1 The Dynamically Adjustable Oscillating Elliptic Trammel

As seen in the figure, this is a dynamically adjustable mechanism where b denotes the moving block and n denotes the non-moving actuator that actuates this block. The dimensions, inertial parameters and some other neccessary data for this mechanism are given as follows.

m_2 = 7 kg, MX_2 = 0, MY_2 = 0, I_2 = 0.1 kg.m^2
m_3 = 15 kg , MX_3 = 6 kg.m, MY_3 = 0, I_3 = 0.9 kg.m^2
m_4 = 8 kg, MX_4 = 0, MY_4 = 0, I_4 = 0.12 kg.m^2
L = 1 m, g = 9.81 m/s^2 (4.1)

Also assume that the external force F_{ext} is applied to link 2 (the horizontal slider) in the negative Z_2 direction for $0 \le \theta \le \pi/2$ and $3\pi/2 \le \theta \le 2\pi$, and has a magnitude of 300 N. One can express this external force as

$$F_{ext} = 300\left(1 - UnitStep\left(\theta - \frac{\pi}{2}\right) + UnitStep\left(\theta - \frac{3\pi}{2}\right)\right) \; N \tag{4.2}$$

The free body diagrams of the links are shown in Figure 4.2.

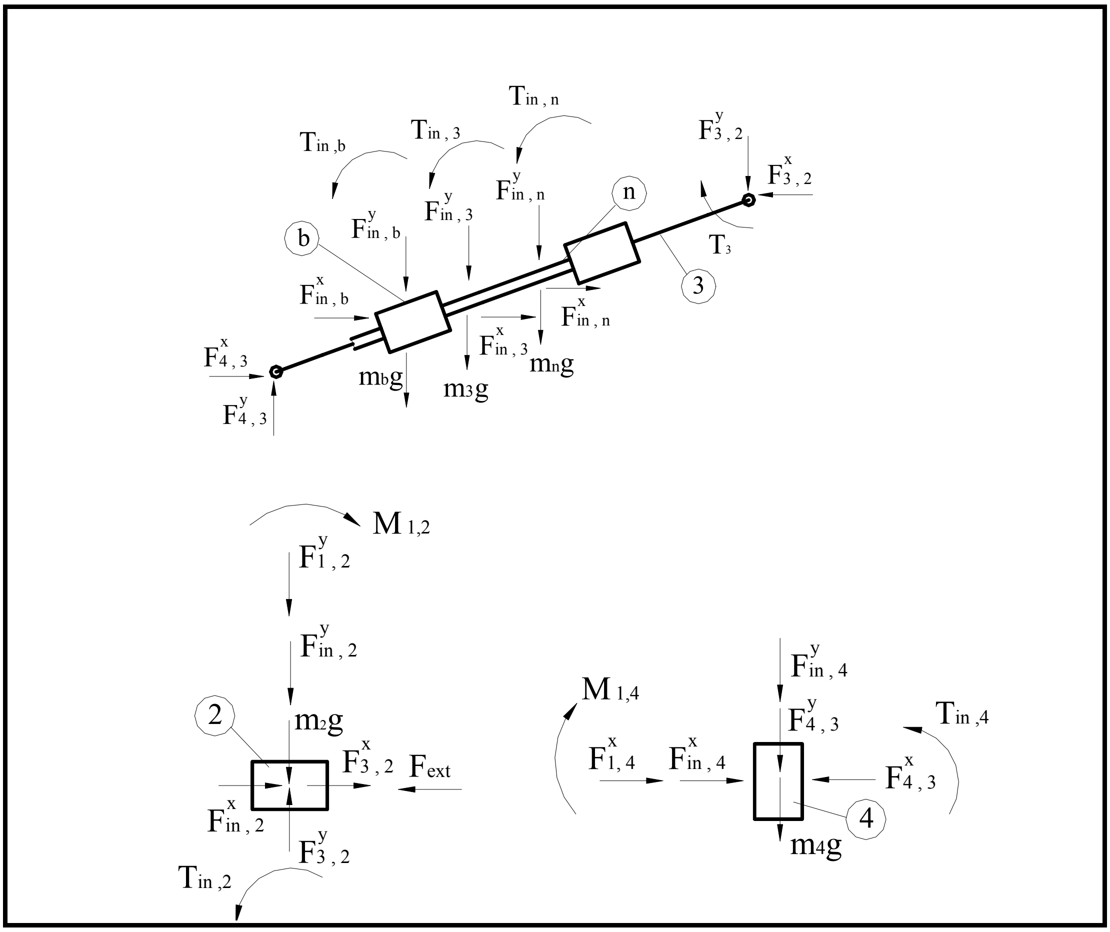

Figure 4.2 Free-body Diagrams of the Dynamically Adjustable OET

If the position $s_b(t)$ of the moving block is adjusted during the regular motion, one should also consider the free-body diagram of the moving block as shown in Figure 4.3 .

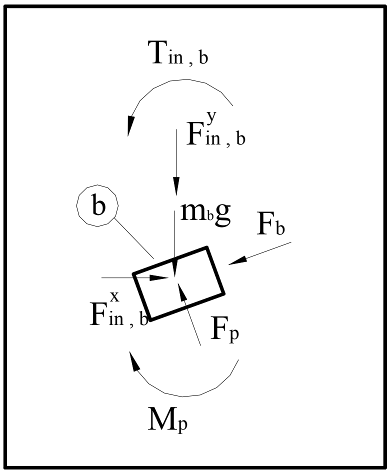

Figure 4.3 Free-body diagram of the moving block

Note that in Figure 4.3, M_p and F_p denote the reaction force and moment due to the prismatic joint between link 3 and the block and F_b denotes the actuator force applied to the block by the non-moving actuator n .

The equations of motion for this mechanism depend on whether the adjustment is made before or during the regular motion. Since, the mechanism is a simple one, it is possible to obtain the equations of motion in closed form using the algorithm in section 2.4.1. If the adjustment is made *during the regular motion*, then taking $q_s(t) = \theta(t)$ and $q_u(t) = s_b(t)$ as the generalized coordinates, the equation of motion of the mechanism can be expressed as

$$H_1(\theta, s_b)\ddot{\theta} + H_2(\theta)\ddot{s}_b + C_1(\theta, s_b)\dot{\theta}^2 + C_2(\theta, s_b)\dot{\theta}\dot{s}_b + G_1(\theta) - T_e = T_3 \tag{4.3}$$

$$H_3(\theta)\ddot{\theta} + H_4\ddot{s}_b + +C_4(\theta)\dot{\theta}^2 + G_2(\theta) = F_b \tag{4.4}$$

where

$$H_1(\theta, s_b) = Sin(2\theta)\, L\, (MY_3 + MY_n + MY_b) - L^2\, Cos^2\,(\theta) \left(2\frac{MX_3}{L} + 2\,\frac{MX_n}{L} + 2\frac{MX_b}{L} \right.$$
$$\left. + 2\frac{m_n\, s_n}{L} + 2\,\frac{m_b\, s_b}{L} + m_4 - m_3 - m_2 - m_n - m_b \right) + m_4\, L^2 + I_b + I_n + I_3 + 2\, MX_n s_n$$
$$+ m_n s_n^2 + 2\, MX_b s_b + m_b s_b^2 \tag{4.5}$$

$$H_2(\theta) = \left(-MY_b - m_b\, L\, Sin\,(\theta)\, Cos\,(\theta)\right) \tag{4.6}$$

$$H_3(\theta) = \left(-MY_b - m_b\, L\, Sin\,(\theta)\, Cos\,(\theta)\right) \tag{4.7}$$

$$H_4 = m_b \tag{4.8}$$

$$C_1(\theta, s_b) = L\left[Sin(\theta)\, Cos(\theta) L\left(m_4 - m_3 - m_2 - m_n - m_b\right) + Sin(2\theta)\left(MX_3 + MX_n + MX_b\right.\right.$$
$$\left.\left. + m_n s_n + m_b s_b\right) + Cos(2\theta)\left(MY_3 + MY_n + MY_b\right)\right] \tag{4.9}$$

$$C_2(\theta, s_b) = 2\ (MX_b + m_b s_b - m_b\ L\ Cos^2\,(\theta)) \tag{4.10}$$

$$C_4(\theta) = (m_b\, L\, Sin^2\,(\theta) - m_b\, s_b - MX_b) \tag{4.11}$$

$$G_1(\theta) = g\left[Sin(\theta)\left(m_4 L + MX_3 + MX_n + MX_b + m_n s_n + m_b s_b\right) + Cos(\theta)\ \left(MY_3 + MY_n\right.\right.$$
$$\left.\left. + MY_b\right)\right] \tag{4.12}$$

$$G_2(\theta) = -\, m_b\, g\, Cos(\theta) \tag{4.13}$$

$$T_e = -\, F_{ext}\ L\ Cos\,(\theta) \tag{4.14}$$

Also note that

T_3 : generalized actuator force (input torque) associated with θ

F_b : generalized actuator force associated with s_b

T_e	: generalized force due to the externally applied force F_{ext}
L	: length of the connecting rod ($L = \|O_3O_4\|$)
MX_i, MY_i	: mass times x and y coordinates of the links in their own body fixed coordinate systems (i = 2,3,4,n,b)
s_n	: constant position of the non-moving actuator n, with respect to $O_3\ X_3\ Y_3\ Z_3$ system
s_b	: position of the moving block with respect to the $O_3\ X_3\ Y_3\ Z_3$ system
I_3, I_n, I_b	: moment of inertias of the rod, non-moving actuator n, and the moving block b with respect to their body fixed Z axes
g	: gravitational acceleration (vertically downward)

However , if the adjustment is made *before the regular motion*, the DOF of the mechanism reduces to 1. In this case, the position $s_b(t)$ is constant throughout the motion leading to $\dot{s}_b(t) = \ddot{s}_b(t) = 0$. Therefore, the only equation of motion is obtained by setting $\dot{s}_b(t) = \ddot{s}_b(t) = 0$ in equation (4.3) , whereas F_b, M_p and F_p become internal forces.

In this case study, the following dynamic behaviours of the mechanism have been optimized.

1) Minimization of the copper losses (of the armature controlled DC motors that actuate the mechanism and the block).
2) Minimization of energy consumed in the actuators which are assumed to be nonregenerative.

4.1.1 Minimization of the Copper Losses of the Armature Controlled DC Motors of the Mechanism

Let the design task be the minimization of the copper losses of the armature controlled DC motors that actuate the mechanism and the moving block. As discussed in Chapter III , energy dissipated in the armature circuit due to copper losses is given by

$$W_{CL} = \int_0^{t_f} \left(K_1 T_3^2 + K_2 F_b^2 \right) dt \tag{4.15}$$

where K_1 and K_2 are motor related constants. Suppose that $K_1 = K_2$. In this case to minimize W_{CL} , one has to minimize the performance measure given by

$$J_{CL} = \int_0^{t_f} \left(T_3^2 + F_b^2 \right) dt \tag{4.16}$$

Let the specified input motion of the mechanism be given by

$$q_s(t) = \theta(t) = \omega t \tag{4.17}$$

where ω is constant. Note that for the given input motion $t_f = 2\pi / \omega$ for one cycle of the mechanism. Due to changing operating conditions it is known that $\omega \in R_\omega$ where

$$R_\omega = \{\omega : \omega_l \le \omega \le \omega_u\} \tag{4.18}$$

Here, ω_l and ω_u denote the specified lower and upper bounds for ω .

4.1.1.1 Adjustment Before the Regular Motion

Assume that the position $s_b(t)$ is adjusted before the regular motion , i.e. $\dot{s}_b(t) = \ddot{s}_b(t) = 0$ during the motion. In this case, the only generalized actuator force is the actuator torque T_3 that actuates the mechanism. Since the adjustment is made before the regular motion, assume that the position of the block is adjusted manually, i.e there is not an actuator ($m_n = MX_n = MY_n = I_n = s_n = 0$). Then the performance measure given by (4.16) reduces to

$$J_{CLbefore} = \int_{0}^{t_f=\frac{2\pi}{\omega}} T_3^2 \, dt \tag{4.19}$$

where the subscript CLbefore denotes the copper losses for the case of adjustment before the regular motion.

The problem is to determine the optimum values of the inertial parameters I_b , m_b, MX_b, MY_b of the moving block and the optimum value of the position $s_b(t)$ (which is fixed for adjustment before the regular motion) of the moving block. As a special case, consider that, the block to be designed is a symmetric body , therefore two of the inertial parameters are given by $MX_b = MY_b = 0$. Evaluating the integral and making the neccessary substitutions, one obtains

$$\begin{aligned}J_{CLbefore} &= \frac{1}{\omega}\,(202283 + 8465.37\, m_b s_b + 302.335\, m_b^2 s_b^2\,) + \omega\,(0.00566149 \\ &+ 0.00283075\, m_b - 0.0052571\, m_b s_b + 0.000202196\, m_b^2 s_b - 0.000404392\, m_b^2\, s_b^2) \\ &+ \omega^3 (3.14441 + 3.14441 m_b + 0.786101 m_b^2 - 6.28881 m_b s_b - 3.14441\, m_b^2 s_b \\ &+\ 3.14441\, m_b^2\, s_b^2\end{aligned} \tag{4.20}$$

The design parameters vectors $\vec{d}_c$ and $\vec{d}_u$ that affect $J_{CLbefore}$ are given by

$$\vec{d}_c = \{m_b\} \tag{4.21}$$

$$\vec{d}_u = \{s_b\} \tag{4.22}$$

The constraints on the design parameters are selected to be

$$1 \le m_b \le 10 \quad kg \tag{4.23}$$

$$0.45 \le s_b \le 0.85 \quad m \tag{4.24}$$

Depending upon the operating conditions , there are two methods to minimize the performance measure $J_{CLbefore}$ for this mechanism. Method 1 is to determine the optimum values of $\vec{d}_c$ and $\vec{d}_u$ for a specified angular velocity ω by using the algorithm given in section 3.3.1. This method is suggested if the mechanism is to operate at only a few ω values, because for each ω, one has to design a new $\vec{d}_c$ and $\vec{d}_u$. In this method, in order to obtain the best dynamic performance from the mechanism, one manufactures a distinct block for each distinct operating condition (i.e. , for each distinct ω value in this example) . If the mechanism is known to operate at a certain ω, then the optimal block corresponding to that ω value is installed on the mechanism at the optimal position given by s_b^{opt} = constant . The mechanism is then allowed to operate. When the ω value is changed, the block should be changed with the new optimal block before the operation starts. Indeed, this method will be practical if and only if the number of different operating conditions is not too many.

Method 2 on the other hand, is based on the minimization of $J_{CLbefore}$ for a given ω range $[\omega_l , \omega_u]$. In this method one determines a single optimal $\vec{d}_c$ value for a given range of ω. The aim of the method is to design a single block whose position s_b is to be adjusted depending upon the operating condition ω in the range $[\omega_l , \omega_u]$. This method is suggested if the number of different operating conditions is too many. The algorithm for the method is as follows.

1) Obtain the performance measure J in terms of $\vec{d}_c$, $\vec{d}_u$, and ω, according to the desired behaviour .
2) Discretize each element of $\vec{d}_c$ between their lower and upper bounds with specified increments. (For instance discretize m_b between 1 and 10 kg with an increment of 1 kg)
3) Obtain a list L_c containing all possible combinations of the discretized values of each element of $\vec{d}_c$.
4) Generate a list of ω values (denoted by L_ω) by discretizing the range $[\omega_l , \omega_u]$ with a specified increment .
5) For a given element of L_c, determine the optimum values of $\vec{d}_u$ and J numerically for each element of L_ω , using the NMinimize command of Mathematica, taking into acccount the constraints on elements of $\vec{d}_u$.

Then, calculate the avarage value of J over the range [ω_l , ω_u] by numerical integration.

6) Repeat 5 for all elements of L_c .
7) Select the minimum of the average J values obtained in steps 5 and 6. The optimal value of $\vec{d}_c$ is the element of L_c , corresponding to this minimum value. The value of $\vec{d}_u$ (determined in step 5 for this element), which is a function of ω, is the optimal value of $\vec{d}_u$.

Both of these methods have been applied to the mechanism for the case of adjustment before the regular motion.

Optimization Results for a Specified ω Value (Method 1)

Consider that the mechanism is to operate only at the constant ω values of 6, 25 and 40 rad /s . Applying Method 1, minimization of $J_{CLbefore}$, subject to the constraints (4.23) and (4.24) , yields the optimal values (of the design parameters m_b and s_b) given in table 4.1 .

Table 4.1 Optimization results of $J_{CLbefore}$ in case of adjustment before the regular motion using Method 1

ω (rad/s)	$J_{CLbefore}$	J_{CLno}	$m_b^{opt}\,(kg)$	$s_b^{opt}\,(m)$
6	35107.7	34393	1	0.45
25	8980.25	57222.8	2.82913	0.85
40	5615.16	206299	2.85286	0.85

As seen in table 4.1, for ω = 6 rad/s , adjusting the mechanism is not preferrable, since the performance measure $J_{CLbefore}$ is larger than the performance measure in the case of no adjustment J_{Clno} (i.e. J_{CL} when $m_b = MX_b = MY_b = I_b = 0$ and $m_n = MX_n = MY_n = I_n = 0$) . This means that the copper losses increases, when the mechanism is adjusted for ω = 6 rad/s. However, for ω = 25 rad/s and ω = 40 rad/s, notice that $J_{CLbefore}$ is much less than J_{Clno} . Actually for ω = 25 rad/s and ω = 40 rad/s , the copper losses of the adjustable mechanism reduce to approximately 21 % and 6 % of the copper losses in the case of the original mechanism. This means that there is a saving of 79 % and 94 % in the copper losses for the cases of ω = 25 rad/s and ω = 40 rad/s. This shows that, if the mechanism is to operate at these ω values, the adjustment on the mechanism is a must for the minimization of the copper losses. Also note that , as the angular velocity increases, the savings in copper losses increases as well in a nonlinear manner. Recall from Chapter III that, the copper losses in the motor is directly related to the actuator torque T_3 that actuates the mechanism. Figure 4.4 shows the variation of T_3 with respect to time for ω = 40 rad/s . Note the reduction in the amount of the actuator torque in the case of adjustment (undashed curve) compared to the csae of no adjustment (dashed curve) .

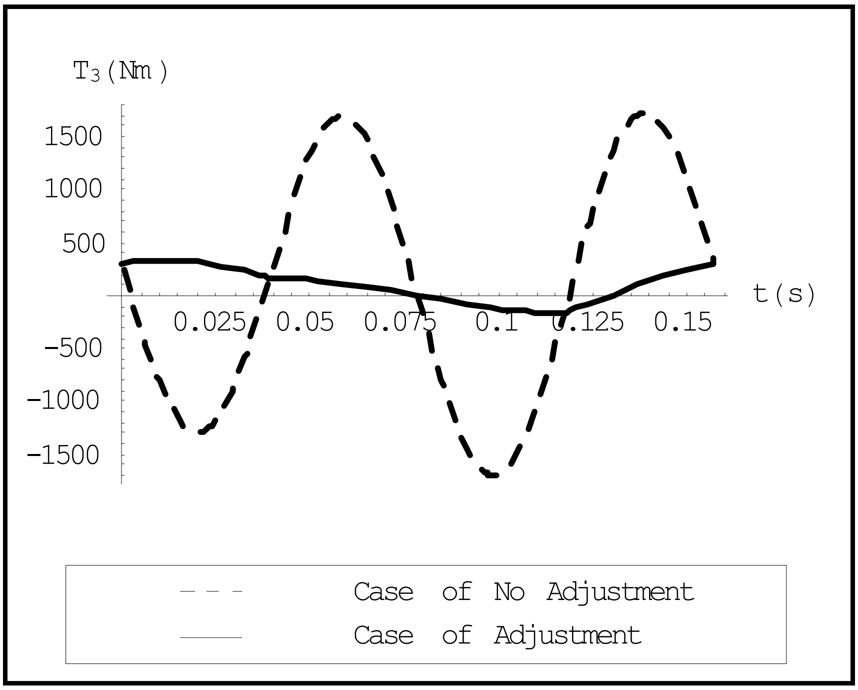

Figure 4.4 Variation of T_3 for $\omega = 40$ rad/s

Optimization Results for a Specified ω Range (Method 2)

Now consider that the mechanism is to operate, with a single block, at many different ω values on an operating range given by $[\omega_l , \omega_u] = [5 , 40]$ rad/s . For this case, Method 2 will be applied to the problem. Let ω be discretized from 5 rad/s to 40 rad/s with an increment of 1 rad/s . Also let m_b be discretized from 1 kg to 10 kg with an increment of 1kg . Using the developed algorithm, one obtains the optimal value of m_b to be

$$m_b^{opt} = 3 \ kg \tag{4.25}$$

The variations of $J_{CLbefore}^{opt}$ and s_b^{opt} (which is a constant at a given operating speed ω) with respect to ω are shown in Figures 4.5 and 4.6 . In Figure 4.5, the dotted curve indicates the case of no adjustment whereas the other one indicates the case of adjustment.

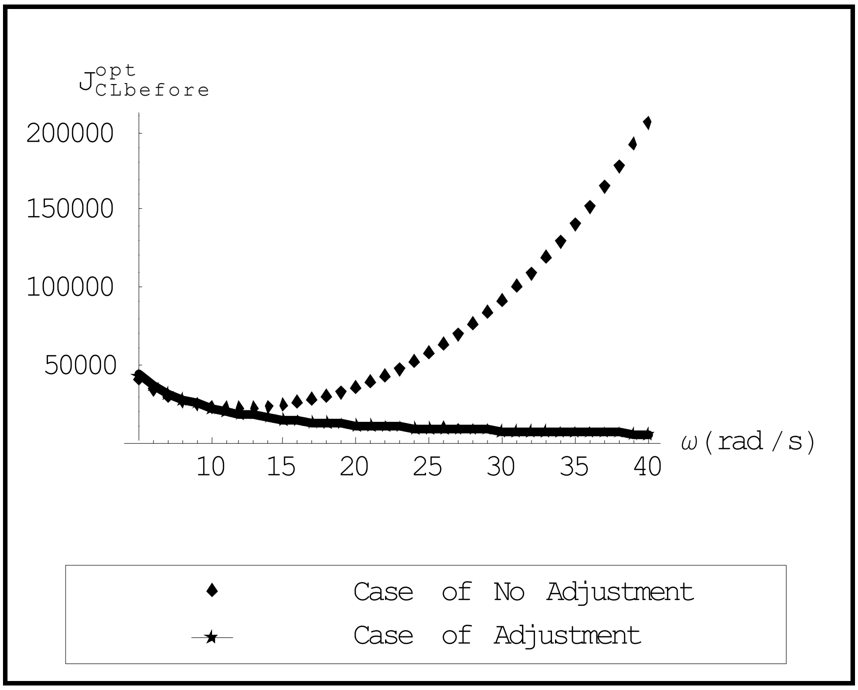

Figure 4.5 Variation of $J_{CLbefore}^{opt}$ with respect to ω

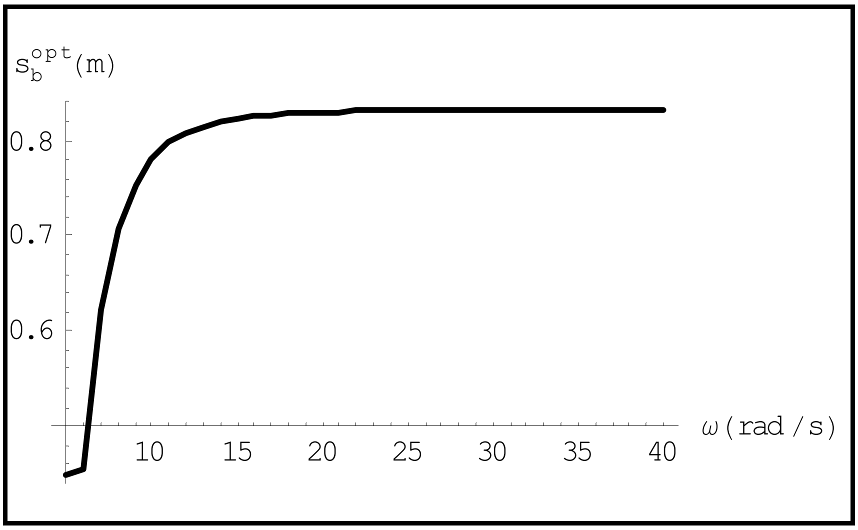

Figure 4.6 Variation of s_b^{opt} with respect to ω

As seen in Figure 4.5, for small angular velocities, the value of the performance measure in case of adjustment (undotted curve) is close to that of the case of no adjustment (dotted curve). This implies that the adjustment is not useful for small angular velocities. However, as ω gets larger, the performance measure in the case of adjustment becomes less than that of the case of no adjustment. This means that , adjusting the mechanism at higher angular velocities, is extremely useful for minimizing the copper losses.

Note that, once the operating speed ω is known, the block position will be set to the optimal one (given by Figure 4.6) before the regular motion. The position of the block will, indeed, be fixed during the regular motion. Also note that for about $\omega \geq 20$ rad/s , one doesn't have to change s_b for every change in ω, since the curve is almost constant.

The variations of the actuator torque T_3 with respect to time for $\omega = 6$, 25 and 40 rad/s are shown in Figures 4.7, 4.8 and 4.9. In these curves, the undashed curve indicates the case of adjustment, and the dashed one indicates the case of no adjustment.

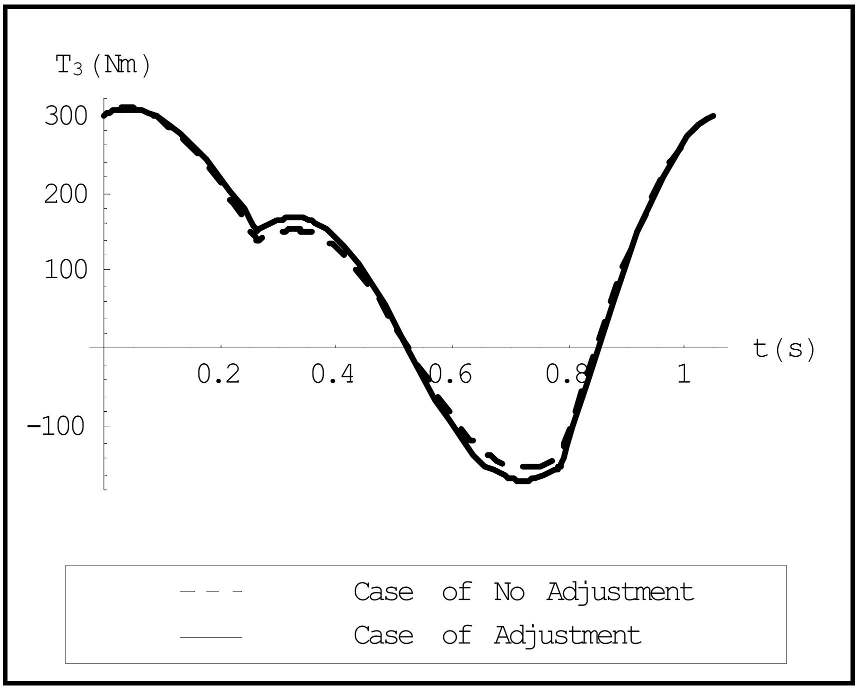

Figure 4.7 Variation of T_3 for $\omega = 6$ rad/s

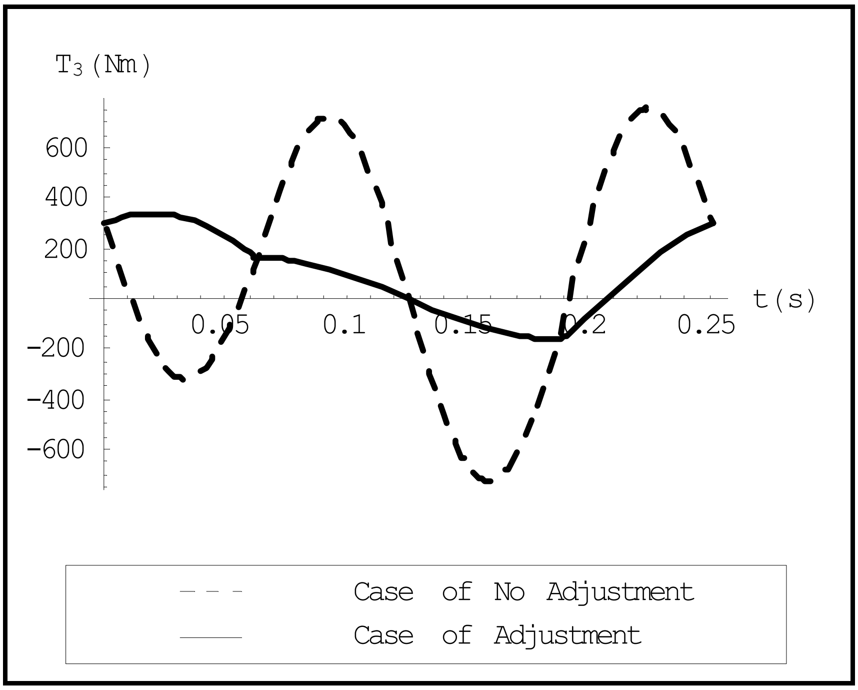

Figure 4.8 Variation of T_3 for $\omega = 25$ rad/s

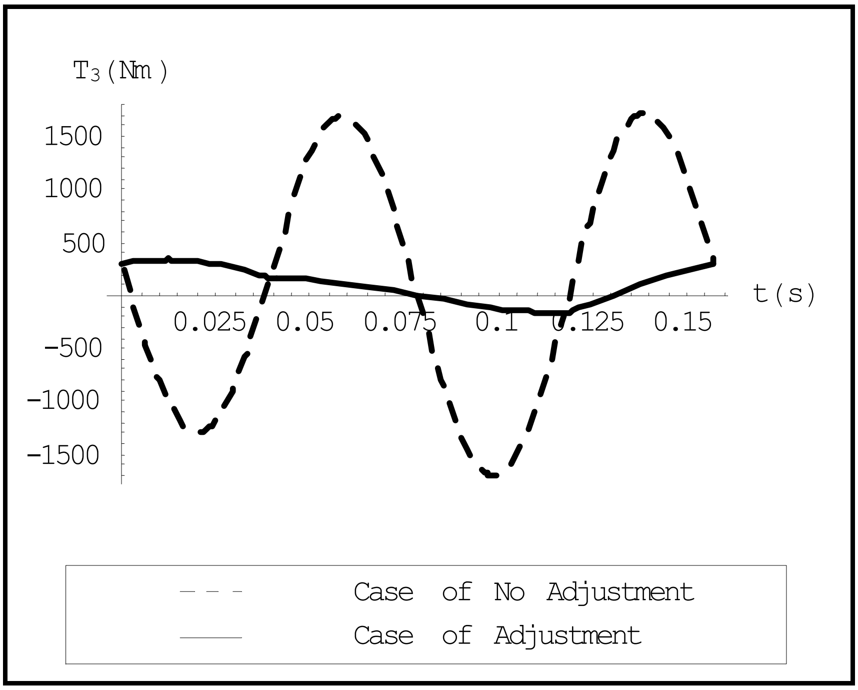

Figure 4.9 Variation of T_3 for $\omega = 40$ rad/s

As seen in the figures above, for $\omega = 6$ rad/s , adjusting the mechanism, in general, increases the amount of the actuator torque , whereas for $\omega = 25$ rad/s and $\omega = 40$ rad/s, it decreases the actuator torque (therefore the copper losses), compared to the case of no adjustment.

Note that the external force F_{ext} (given by equation (4.2)), which is applied to the horizontal slider, is discontinuous at $\theta = \pi / 2$ and $\theta = 3\pi / 2$. Therefore the actuator torque T_3 will also be discontinuous at these points. These discontiunities can easily be seen in Figures (4.7) - (4.9) especially on the undashed curves. They can certainly be visualized better if the plot ranges of the horizontal and vertical axes are adjusted in a convenient manner.

4.1.1.2 Adjustment During the Regular Motion

Now consider the case where the position $s_b(t)$ of the moving block is adjusted *during the regular motion*, in other words it is a function of time. Since the adjustment is made during the regular motion, one should use a non-moving actuator (denoted by n in Figure 4.1) in order to actuate the block. Let the inertial parameters of this actuator be given by

$$\begin{aligned} m_n &= 5\ kg \\ I_n &= 0.1\ kg\, m^2 \\ MX_n &= 1.2\ kg\, m \\ MY_n &= 0 \end{aligned} \tag{4.26}$$

In this case, for the minimization of the copper losses, one should also consider the actuator that actuates the block. Assume that the motor constants K_1 and K_2 are equal to each other in equation (4.15). Then to minimize the copper losses , one has to minimize the performance measure given by

$$J_{CLduring} = \int_0^{t_f=\frac{2\pi}{\omega}} \left(T_3^2 + F_b^2\right) dt \tag{4.27}$$

Note that in addition to the inertial parameters of the moving block, there is an additional design parameter s_n which is the constant position of the non-moving actuator. Again assuming the block to be symmetric ($MX_b = MY_b = 0$), the constant design parameters vector is given by

$$\vec{d}_c = \{m_b, s_n\} \tag{4.28}$$

Since the adjustment is made during the regular motion, piecewise continuous polynomial parameterization will be used. Let $s_b(t)$ be represented by a single 5'th order polynomial $g(t) = g_1(t)$ on the interval $[0\ ,\ t_f\,]$, satisfying the following boundary conditions :

$$\begin{aligned} s_b(0) &= s_b(t_f) = Free \\ \dot{s}_b(0) &= \dot{s}_b(t_f) = Free \\ \ddot{s}_b(0) &= \ddot{s}_b(t_f) = Free \end{aligned} \tag{4.29}$$

where $t_f = 2\pi / \omega$ for one cycle for the given input motion $\theta(t) = \omega t$. As seen from equation (4.29), due to the periodical motion , the position , velocity, and acceleration of the block at the beginning and at the end of the cycle are desired be equal to each other. Using the developed code, the polynomial for $s_b(t)$, can be generated as

$$s_b(t) = (g_{1F,0P} + g_{1F,1P}\ t + \frac{1}{2}\ g_{1F,2P}\ t^2 - \frac{1}{2\pi}\ g_{1F,2P}\ \omega\ t^3 - \frac{5}{2\pi^2}\ g_{1F,1P}\ \omega^2 t^3$$
$$+ \frac{1}{8\pi^2}\ g_{1F,2P}\ \omega^2\ t^4 + \frac{15}{8\pi^3}\ g_{1F,1P}\ \omega^3 t^4 - \frac{3}{8\pi^4}\ g_{1F,1P}\ \omega^4 t^5\ )\ (\ u(t) - u(t - t_f))$$

(4.30)

where u(t) denotes the unitstep function. Also note that we have 3 independent polynomial coefficients $g_{1F,0P}$, $g_{1F,1P}$, and $g_{1F,2P}$. Substituting equation (4.30) into (4.27), evaluating the integral and making the neccessary substitutions one obtains $J_{CLduring}$ in terms of ω , $\vec{d}_c$, and $\vec{d}_u$, where

$$\vec{d}_u = \{g_{1F,0P}, g_{1F,1P}, g_{1F,2P}\} \tag{4.31}$$

The constraints on the elements of $\vec{d}_c$ and $\vec{d}_u$ are selected to be

$$1 \le m_b \le 10 \quad kg \tag{4.32}$$
$$0.2 \le s_n \le 0.3 \ \ m \tag{4.33}$$
$$0.45 \le s_b(t) \le 0.85 \tag{4.34}$$

As seen above, the constraint given by (4.34) is a dynamic constraint, which can be converted to a static constraint by using the method discussed in section 3.3.2.

Note that the vector $\vec{d}_c$ contains the position s_n of the actuator as a design parameter. If Method 1 is applied to the problem, one has to change the position of the actuator for every change in the operating condition ω. This is not practical. Therefore, only Method 2 will be used to minimize $J_{CLduring}$. As in section 4.1.1.2, consider that the mechanism is to operate in a range of ω values given by $[\omega_l, \omega_u] = [5, 40]$ rad/s. Let this range be discretized by 1 rad/s. Also let m_b be discretized from 1 kg to 10 kg with an increment of 1 kg, and let s_n be discretized from 0.2 m to 0.3 m with an increment of 0.05 m. Applying the algorithm in section 4.1.1, one obtains one obtains the optimal values of the constant design parameters as

$$\begin{aligned} m_b^{opt} &= 1\ kg \\ s_n^{opt} &= 0.3\ m \end{aligned} \tag{4.35}$$

Figure 4.10 shows the variation of $J_{CLduring}^{opt}$ with respect to ω compared to the case of no adjustment. Note that the savings in the copper losses increase nonlinearly as the operating speed ω increases.

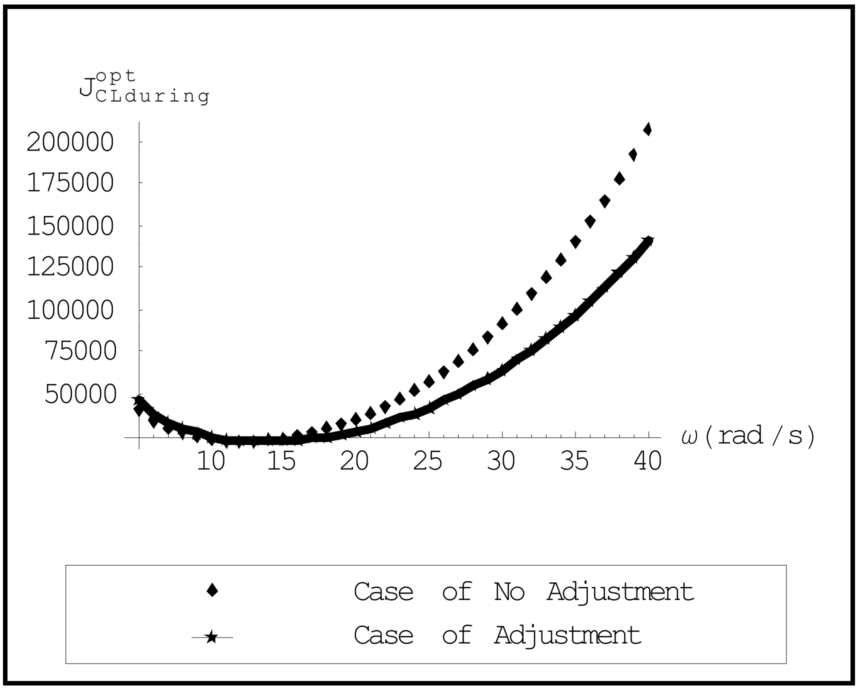

Figure 4.10 Variation of $J_{CLduring}^{opt}$ with respect to ω

The variations of the actuator torque T_3 and $s_b^{opt}(t)$ with respect to time for $\omega = 6$, 25 and 40 rad/s are shown in figures 4.11-4.16.

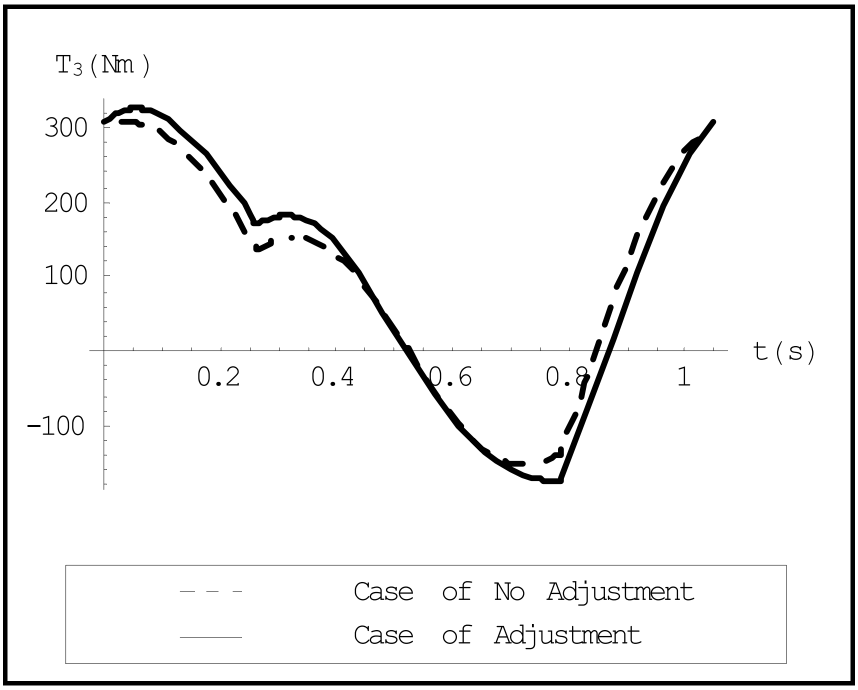

Figure 4.11 Variation of T_3 for ω =6 rad/s

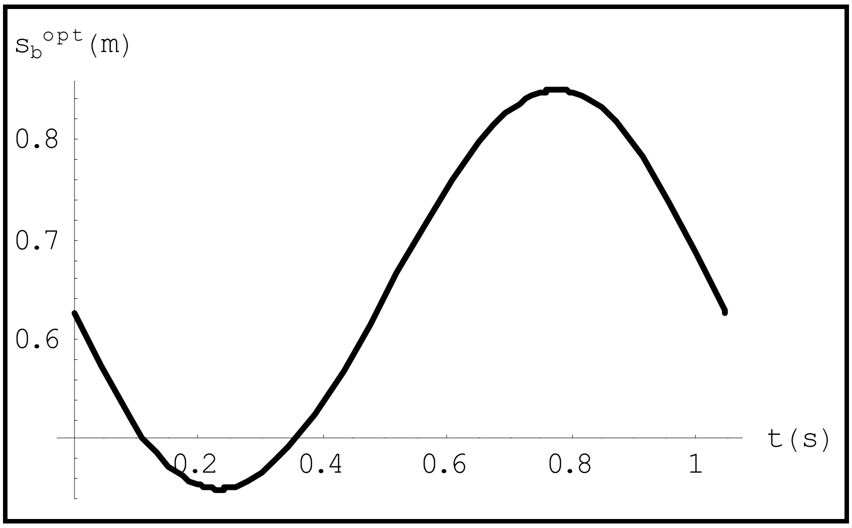

Figure 4.12 Variation of s_b^{opt} for ω =6 rad/s

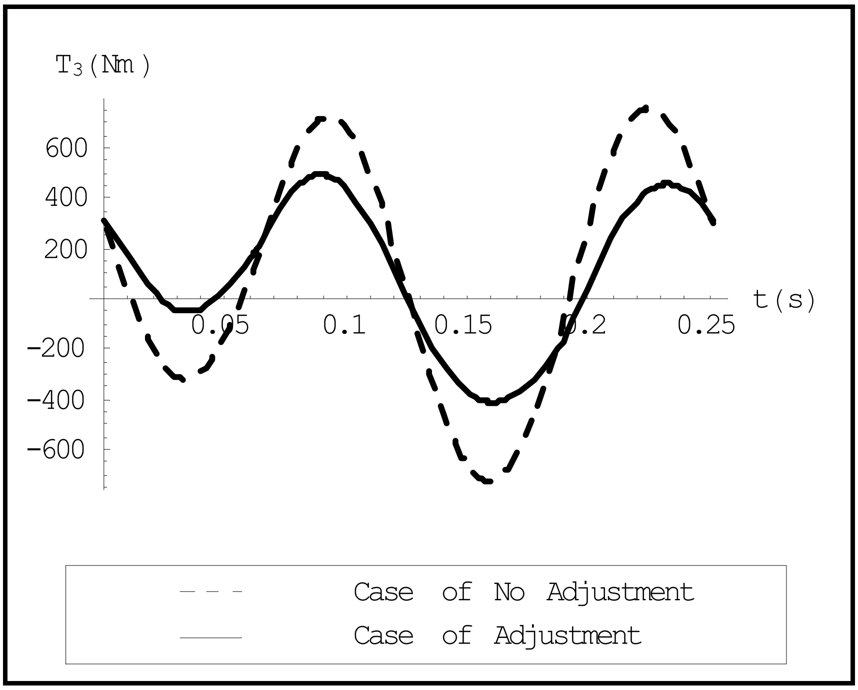

Figure 4.13 Variation of T_3 for $\omega = 25$ rad/s

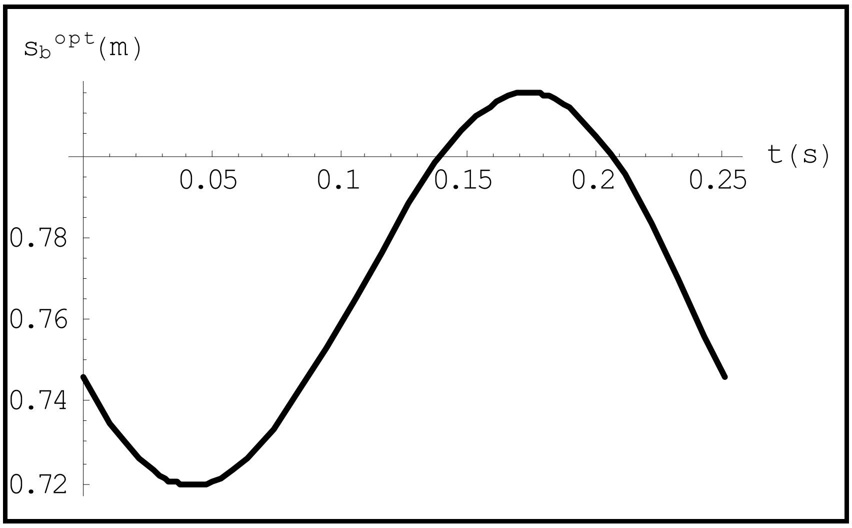

Figure 4.14 Variation of s_b^{opt} for $\omega = 25$ rad/s

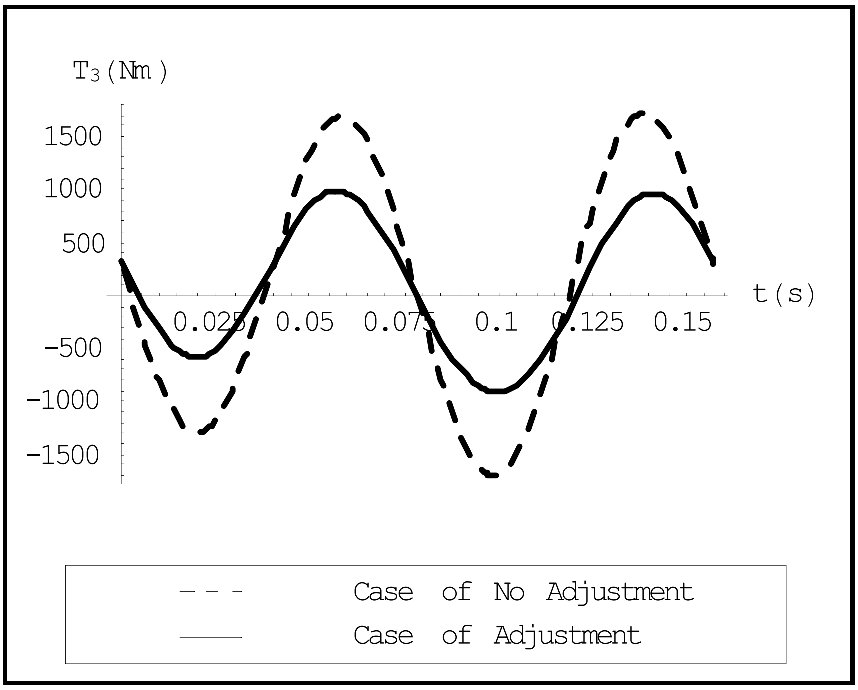

Figure 4.15 Variation of T_3 for $\omega = 40$ rad/s

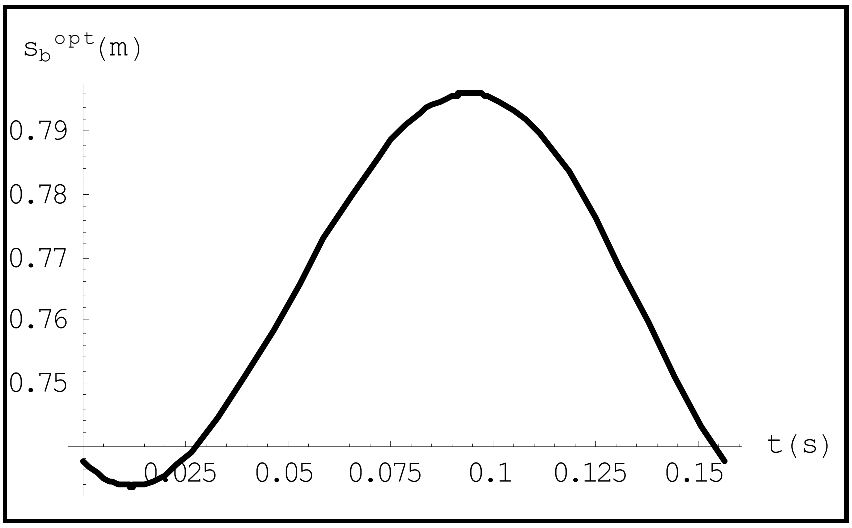

Figure 4.16 Variation of s_b^{opt} for $\omega = 40$ rad/s

As seen from the plots, the actuator torque T_3 decreases in the case of adjustment during the regular motion at high angular velocities. Therefore, the adjustment during the regular motion is neccessary to minimize the copper losses, if the mechanism is to run at high ω values.

4.1.2 Minimization of the Energy Consumed

Now , let the design task be the minimization of the energy consumed (EC) For this purpose, if the motors are not regenerative, the performance measure to be minimized is given by

$$J_{EC} = \int_0^{t_f=\frac{2\pi}{w}} \left(\left| T_3\,\dot{\theta} \right| + \left| F_b\,\dot{s}_b \right| \right) dt \tag{4.36}$$

Note that the above performance measure illustrates the most general case, i.e . the adjustment during the regular motion (where the block is actuated by a force F_b) .

4.1.2.1 Adjustment Before the Regular Motion

Consider that the position $s_b(t)$ of the moving block is adjusted before the regular motion, i.e. $(\dot{s}_b(t) = \ddot{s}_b(t) = 0)$. In this case , the performance measure related to energy consumed is given by

$$J_{ECbefore} = \int_0^{t_f=\frac{2\pi}{w}} \left(\left| T_3\,\dot{\theta} \right| \right) dt \tag{4.37}$$

Again as in the previous sections, assume that the input motion is given by $\theta = \omega t$, where ω is constant. Also assume that, the block to be designed is symmetric, leading to $MX_b = MY_b = 0$ and the position of the block is adjusted manually (i.e. there is not an actuator) . Then the constant design parameters vector $\vec{d}_c$, and the unspecified generalized coordinate design parameters vector $\vec{d}_u$ are given by

$$\vec{d}_c = \{m_b\} \tag{4.38}$$

$$\vec{d}_u = \{s_b\} \tag{4.39}$$

The constraints on these parameters are the same as the previous ones given by equations (4.23) and (4.24) .

Optimization Results for a Specified ω Value (Method 1)

Consider that the mechanism is to operate at ω = 6 , 25 and 40 rad/s . Using the developed algorithm discussed in section 3.3.1, the optimal values of $\vec{d}_c$ and $\vec{d}_u$ and the minimum value of $J_{ECbefore}$ compared to the case of no adjustment (J_{ECno}) are given in Table 4.2 .

Table 4.2 Optimization results of $J_{ECbefore}$ in case of adjustment before the regular motion using Method 1

ω (rad/s)	$J_{ECbefore}$ (J)	J_{ECno} (J)	$m_b^{opt}\,(kg)$	$s_b^{opt}\,(m)$
6	995.533	1023.47	4.1617	0.85
25	897.466	2835.58	3.27893	0.84999
40	905.843	6861.83	4.0026	0.764476

As seen in the table, for all three of the operating conditions, the energy consumed in the case of adjustment is less than that of the case of no adjustment. Therefore adjusting the mechanism is preferrable for these ω values. Also note that as ω increases, the energy consumed in case of adjustment becomes much less than that of the case of no adjustment. For instance for ω = 6 rad/s , the energy consumed for the case of adjustment is about 97 % of the case of no adjustment, whereas for ω = 40 rad/s, this percentage becomes about 13 % . Note that, by minimizing the energy consumed, the power requirement of the actuator decreases. Figure 4.17 shows the variation of power required for ω = 40 rad/s . Here, the dashed curve indicates the case of no adjustment whereas the undashed one indicates the case of adjustment.

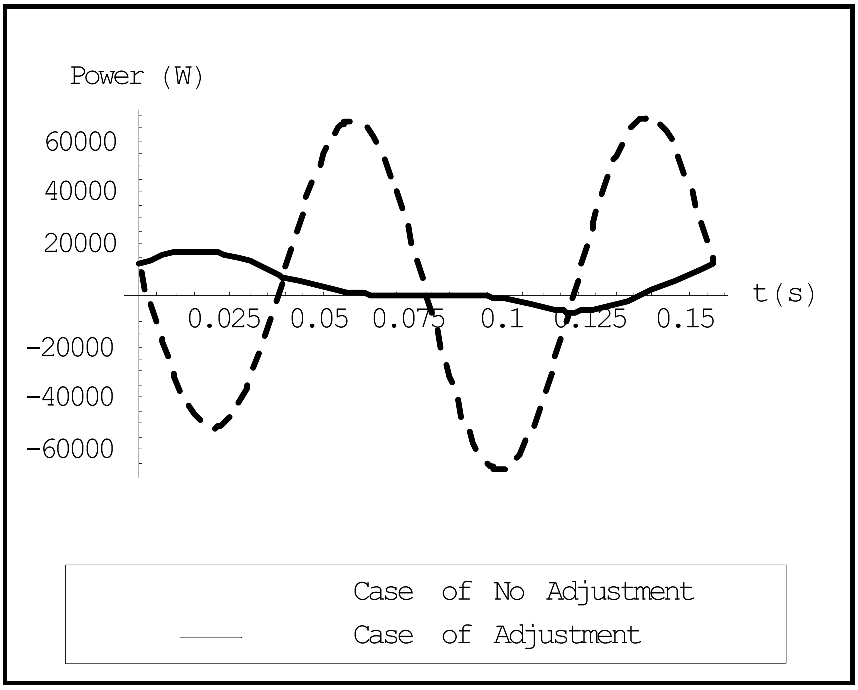

Figure 4.17 Variation of power for $\omega = 40$ rad/s

Optimization Results for a Specified ω Range (Method 2)

Consider that the mechanism is to operate at many ω values in the range $[\omega_l , \omega_u] = [5, 40]$ rad/s . As in the case of copper losses minimization, lets apply the second method , which gives us a single optimal $\vec{d}_c = \{m_b\}$ value for the range [5, 40] rad/s . Using the same discretized values for m_b and ω, the optimum value of m_b can be obtained as

$$m_b^{opt} = 4 \ kg \tag{4.40}$$

The variation of the $J_{ECbefore}^{opt}$ with respect to ω compared to the case of no adjustment is shown in Figure 4.18. Here the case of adjustment has been illustrated by the undotted curve. As seen below, it increases very slowly compared to the case of no adjustment (dotted curve) as ω increases.

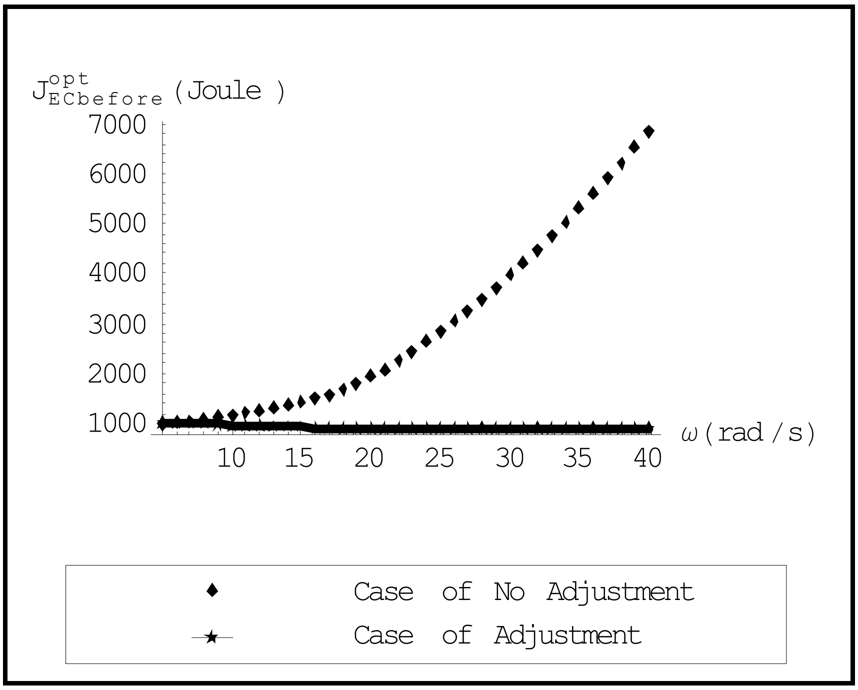

Figure 4.18 Variation of $J_{ECbefore}^{opt}$ with respect to ω

Figure 4.18 shows that, the energy consumed for the adjusted OET is much less than that of the unadjusted OET. Furthermore the savings in energy consumed increase nonlinearly as ω is increased. For instance the savings in energy consumed for ω = 10, 20 and 30 rad/s are about 13 %, 49 % and 76 %. The variation of optimal s_b with respect to ω is shown in Figure 4.19.

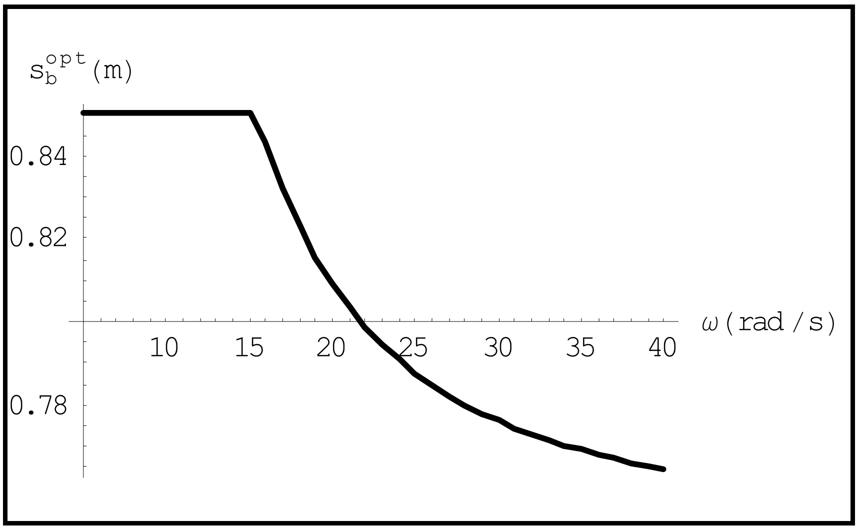

Figure 4.19 Variation of s_b^{opt} with respect to ω

The variations of the power wrt time for $\omega = 6$, 25 , and 40 rad/s are shown in figures 4.20 - 4.22 .

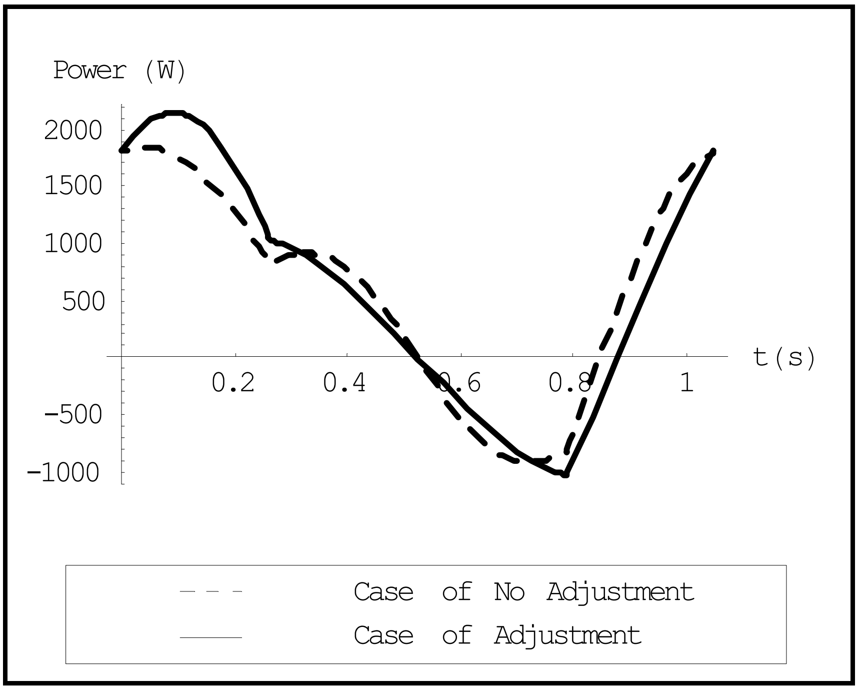

Figure 4.20 Variation of power for $\omega = 6$ rad/s

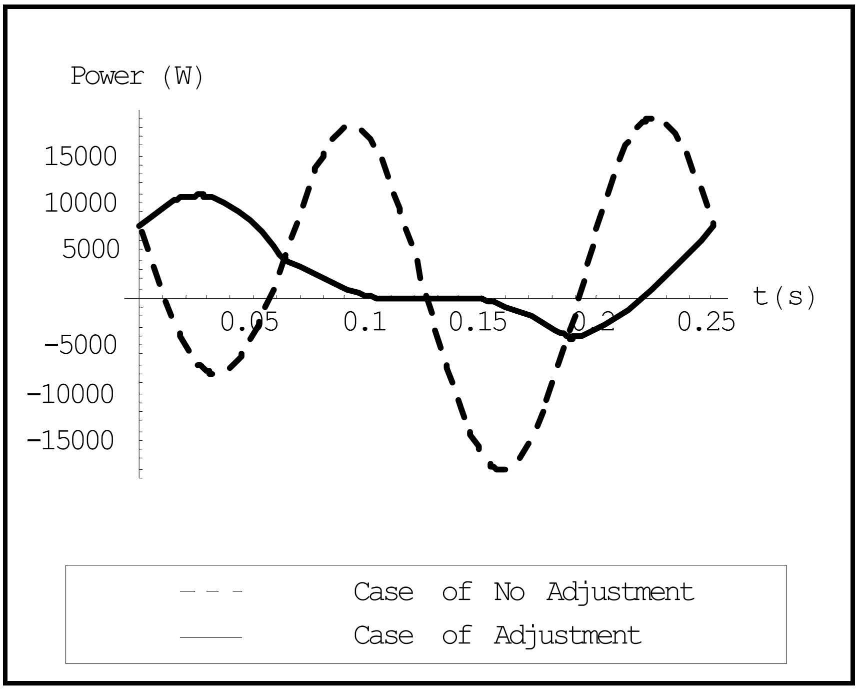

Figure 4.21 Variation of power for $\omega = 25$ rad/s

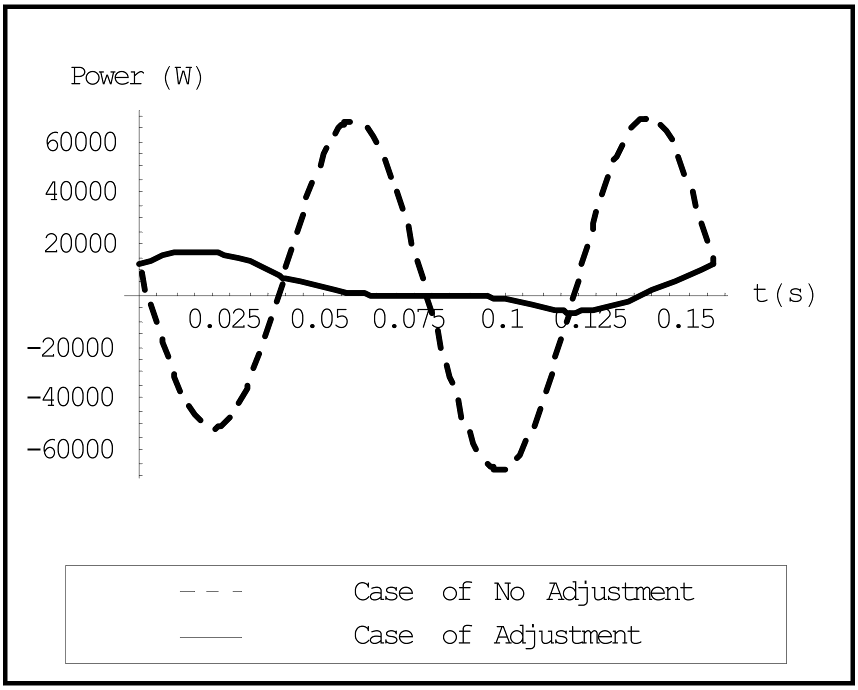

Figure 4.22 Variation of power for $\omega = 40$ rad/s

Note that for ω = 25 and 40 rad/s, the power required in the case of adjustment (undashed curve), is, in general, less than that of the case of no adjustment (dashed curve). However, for ω = 6 rad/s the situation is reversed. The results indicate that by minimizing energy consumed, one also, simultaneously, reduces the power requirements of the actuator which is another desirable feature.

4.1.2.2 Adjustment During the Regular Motion

As in section 4.1.1.2, again consider that the position $s_b(t)$ of the moving block is adjusted during the regular motion. In this case , the performance measure related to energy consumed is given by

$$J_{ECduring} = \int_0^{t_f=\frac{2\pi}{w}} \left(\left| T_3\,\dot{\theta} \right| + \left| F_b\,\dot{s}_b \right| \right) dt \tag{4.41}$$

Note that since the adjustment is made during the regular motion, the energy consumed in the actuator of the block is also taken into account. The inertial parameters of the actuator are the same as given in equation (4.26). Again $s_b(t)$ is represented by a 5'th order polynomial (given by (4.30)), subject to the BC's given by (4.29) . The design parameter vectors $\vec{d}_c$ and $\vec{d}_u$ are the same as given in equations (4.28) and (4.31) . Also the constraints on these parameters are the same as given by equations (4.32)-(4.34). Again, as in section 4.1.1.2, the second method will be used to minimize $J_{ECduring}$, since designing the position s_n of the actuator for each given ω is not practical. Let ω be discretized from 5 rad/s to 40 rad/s with an increment of 1 rad/s. Also let m_b be discretized from 1 kg to 10 kg with an increment of 1 kg and let s_n be discretized from 0.2 m to 0.3 m with an increment of 0.05 m . Using the developed algorithm in section 4.1.1.1, one obtains the optimal values of m_b and s_n as

$$\begin{aligned} m_b^{opt} &= 3\ kg \\ s_n^{opt} &= 0.3\, m \end{aligned} \tag{4.42}$$

Note that due to the assumption of symmetric block, the inertial parameters MX_b and MY_b of the block have been taken to be 0 at the beginning of the problem. Also note that the moment of inertia I_b (which should be strictly positive) of the block is free to be determined. Figure 4.23 shows the variation of $J^{opt}_{ECduring}$ in case of adjustment during the regular motion, compared to the case of no adjustment. Here the undotted curve indicates the case of adjustment whereas the dotted one indicates the case of no adjustment.

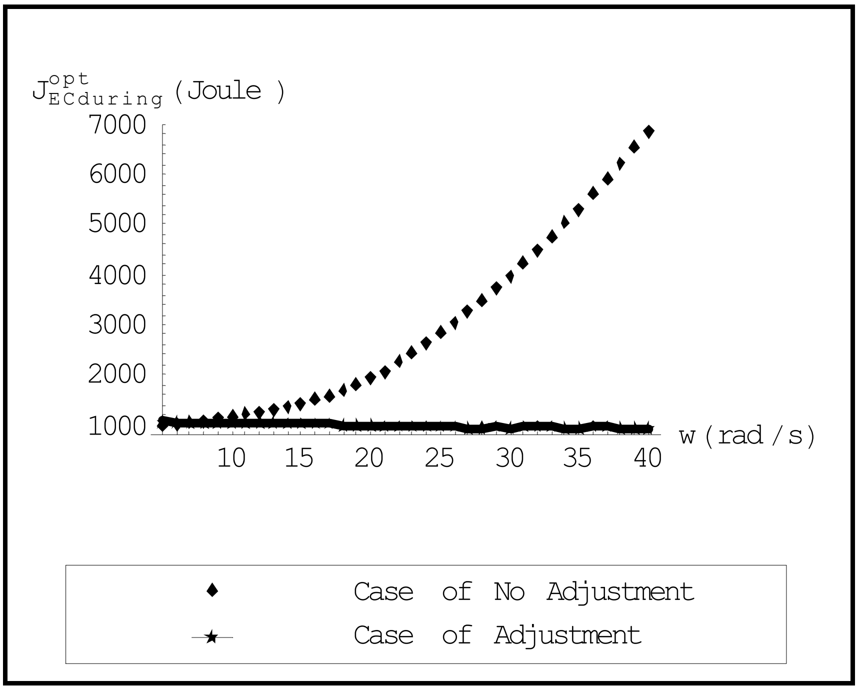

Figure 4.23 Variation of $J^{opt}_{ECduring}$ with respect to ω

As seen in the figure above, adjusting the mechanism is useful at high ω values. Note that as ω increases, the energy consumed in the case of adjustment becomes much less than the case of no adjustment.

As stated before, minimization of the energy consumed decreases the total power required for the mechanism. The variations of the total power required and the optimal $s_b(t)$ with respect to time for ω = 6 ,25 and 40 rad/s are shown in figures 4.24-4.29. In the power variation curves , the dashed curves indicate the case of no adjustment, whereas the undashed ones indicate the case of adjustment.

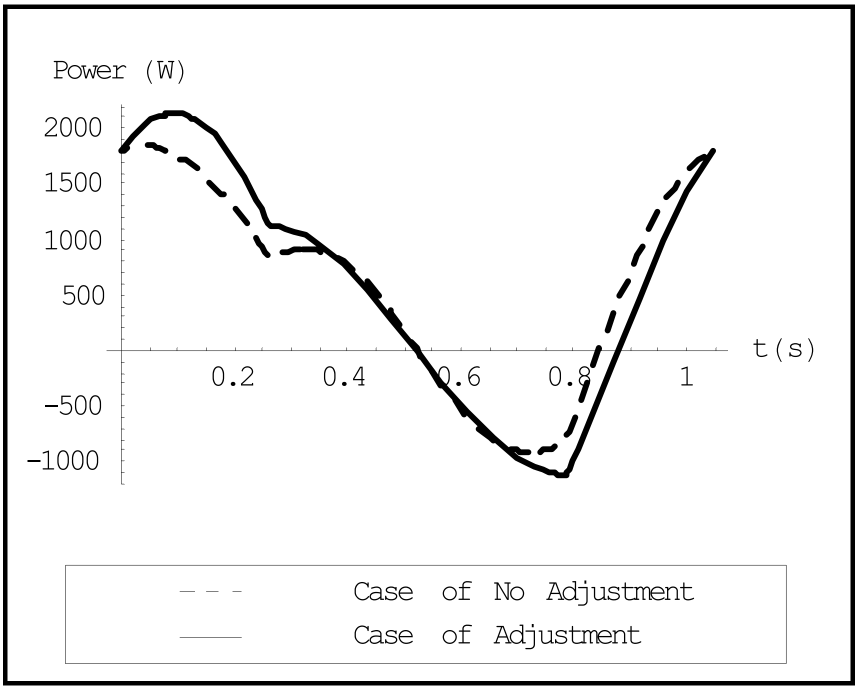

Figure 4.24 Variation of power for $\omega = 6$ rad/s

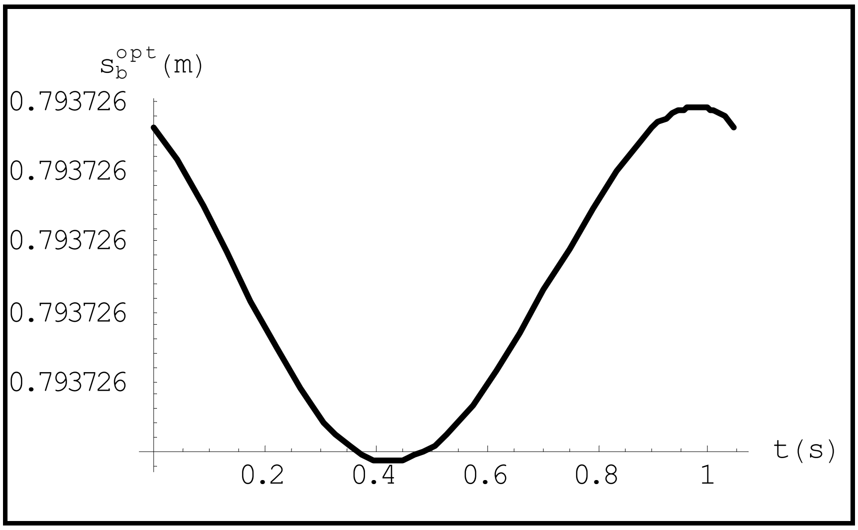

Figure 4.25 Variation of s_b^{opt} for $\omega = 6$ rad/s

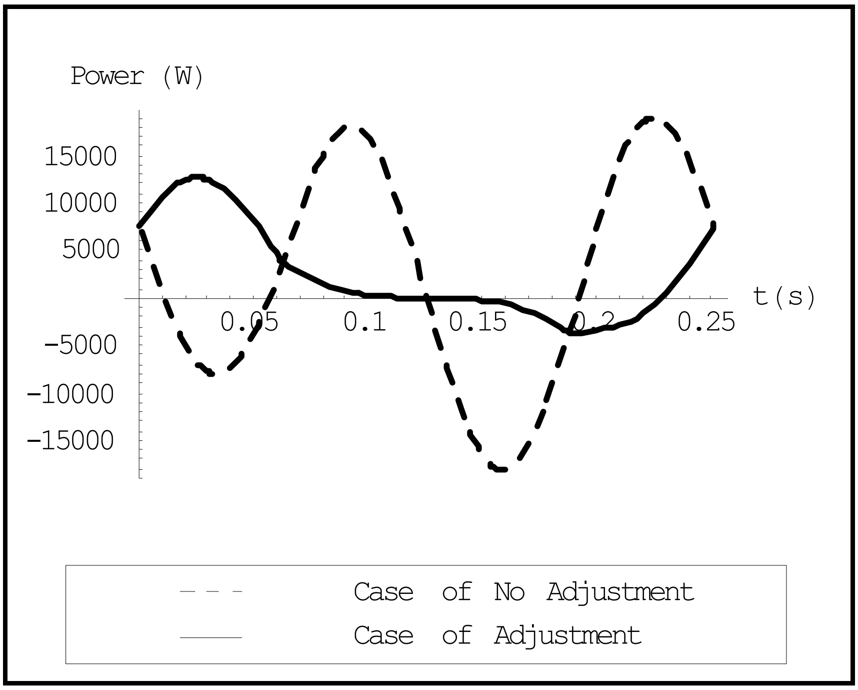

Figure 4.26 Variation of power for $\omega = 25$ rad/s

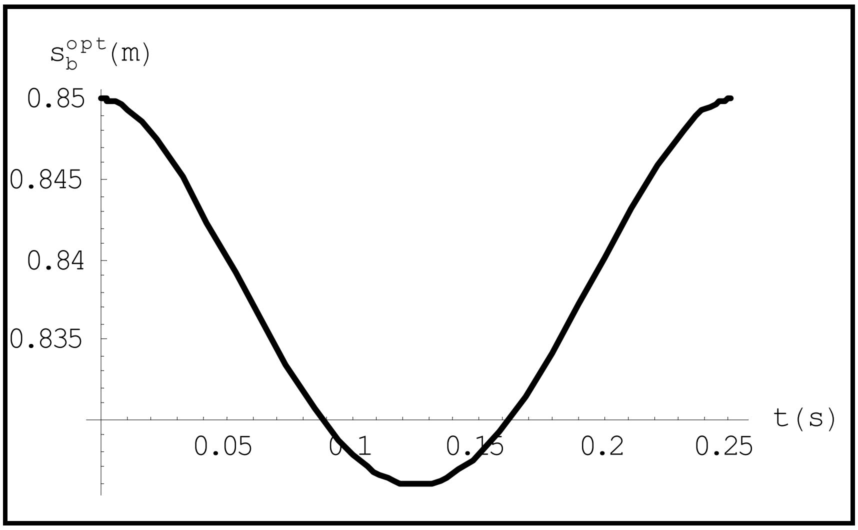

Figure 4.27 Variation of s_b^{opt} for $\omega = 25$ rad/s

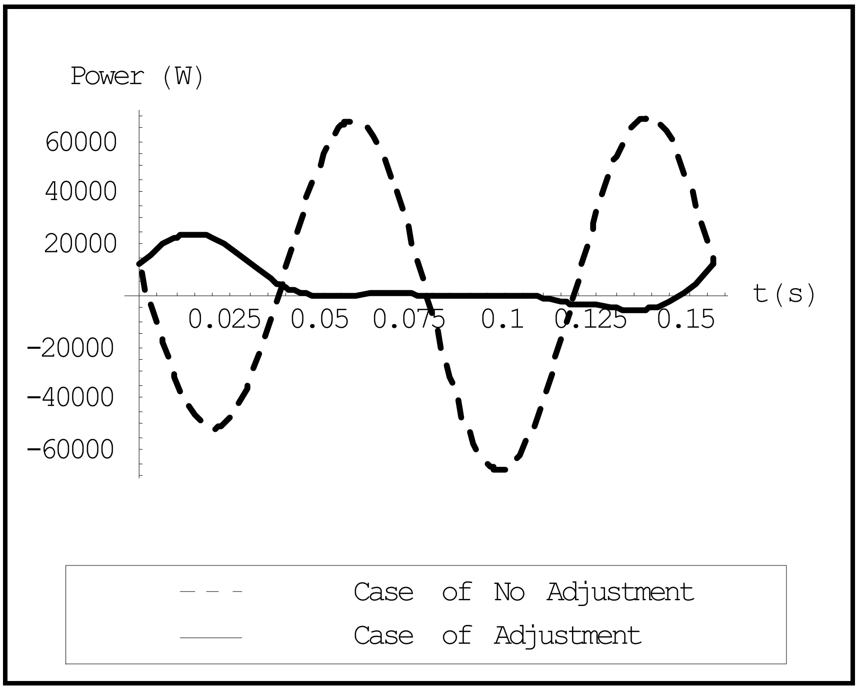

Figure 4.28 Variation of power for ω = 40 rad/s

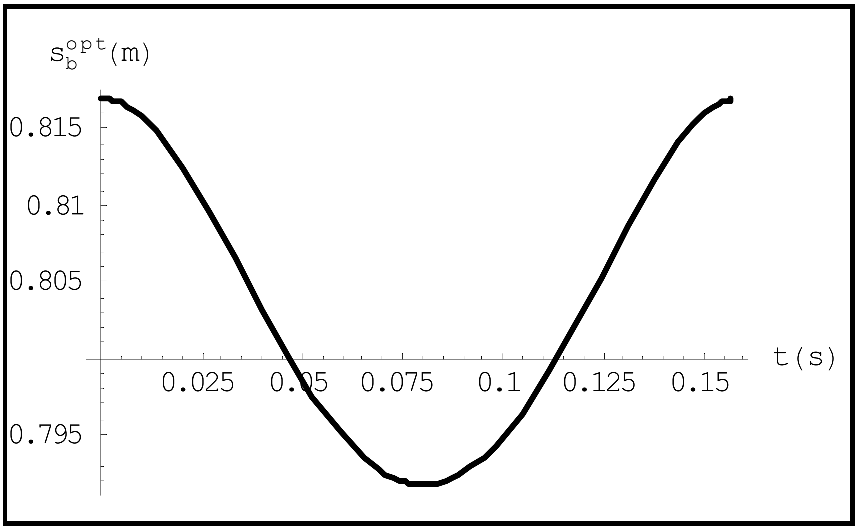

Figure 4.29 Variation of s_b^{opt} for ω = 40 rad/s

As seen in the figures above, for ω = 25 and ω = 40 rad/s, the total power required for the case of adjustment during the regular motion is less than that of the case of no adjustment. However, for ω = 6 rad/s, the situation is reversed as in section 4.1.2.1. This means that adjusting the mechanism during the regular motion for minimization of the energy consumed, is useful at high angular velocities. Also note that for ω = 6 rad/s , the chnages in $s_b(t)$ are very small. Therefore one should better fix the block at s_b = 0.79 m , rather than moving it.

4.2 The Kinematically Adjustable Oscillating Elliptic Trammel

Consider the kinematically adjustable oscillating elliptic trammel shown in Figure 4.30.

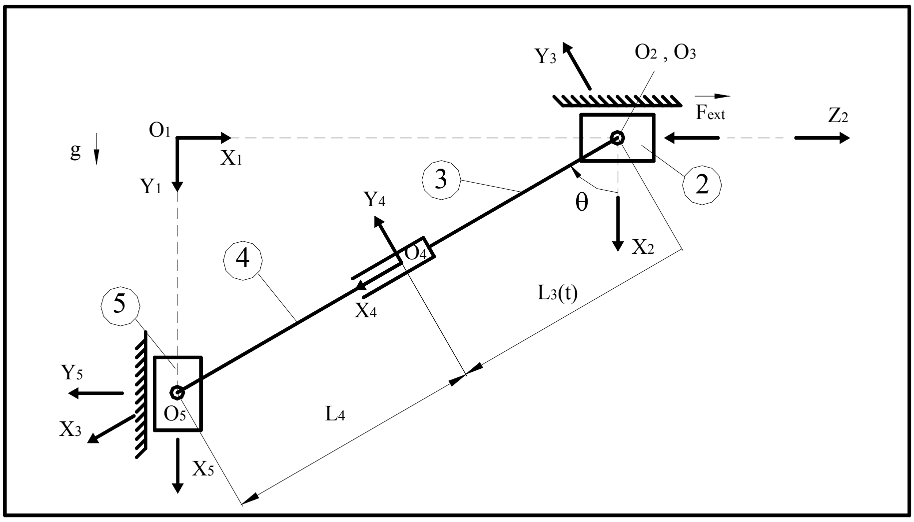

Figure 4.30 The Kinematically Adjustable Oscillating Elliptic Trammel

As seen in the figure this is a kinematically adjustable mechanism, since the length $L_{34}(t) = L_3(t) + L_4$ of the combined link associated with links 3 and 4 is an adjustable kinematic parameter. The dimensions, inertial parameters, and some other neccessary data for this mechanism are given as follows.

$$
\begin{aligned}
&m_2 = 7 \text{ kg} , MX_2 = 0, MY_2 = 0, I_2 = 0.1 \text{ kg.m}^2 \\
&m_3 = 6 \text{ kg} , MX_3 = 0.6 \text{ kg.m} , MY_3 = 0, I_3 = 0.08 \text{ kg.m}^2 \\
&m_4 = 9 \text{ kg} , MX_4 = 1.8 \text{ kg.m}, MY_4 = 0 , I_4 = 0.15 \text{ kg.m}^2 \\
&m_5 = 8 \text{ kg} , MX_5 = 0, MY_5 = 0 , I_5 = 0.13 \text{ kg.m}^2 \\
&L_4 = 0.4 \text{ m} , g = 9.81 \text{ m / s}^2
\end{aligned}
\tag{4.43}
$$

Also assume that the external force F_{ext} is applied to link 2 (the horizontal slider) in the negative Z_2 direction for $0 \le \theta \le \pi/2$ and $3\pi/2 \le \theta \le 2\pi$, and has a magnitude of 300 N. One can express this external force as

$$F_{ext} = 300\left(1 - UnitStep\left(\theta - \frac{\pi}{2}\right) + UnitStep\left(\theta - \frac{3\pi}{2}\right)\right)\ N \tag{4.44}$$

The free body diagrams of the links are shown in Figure 4.31.

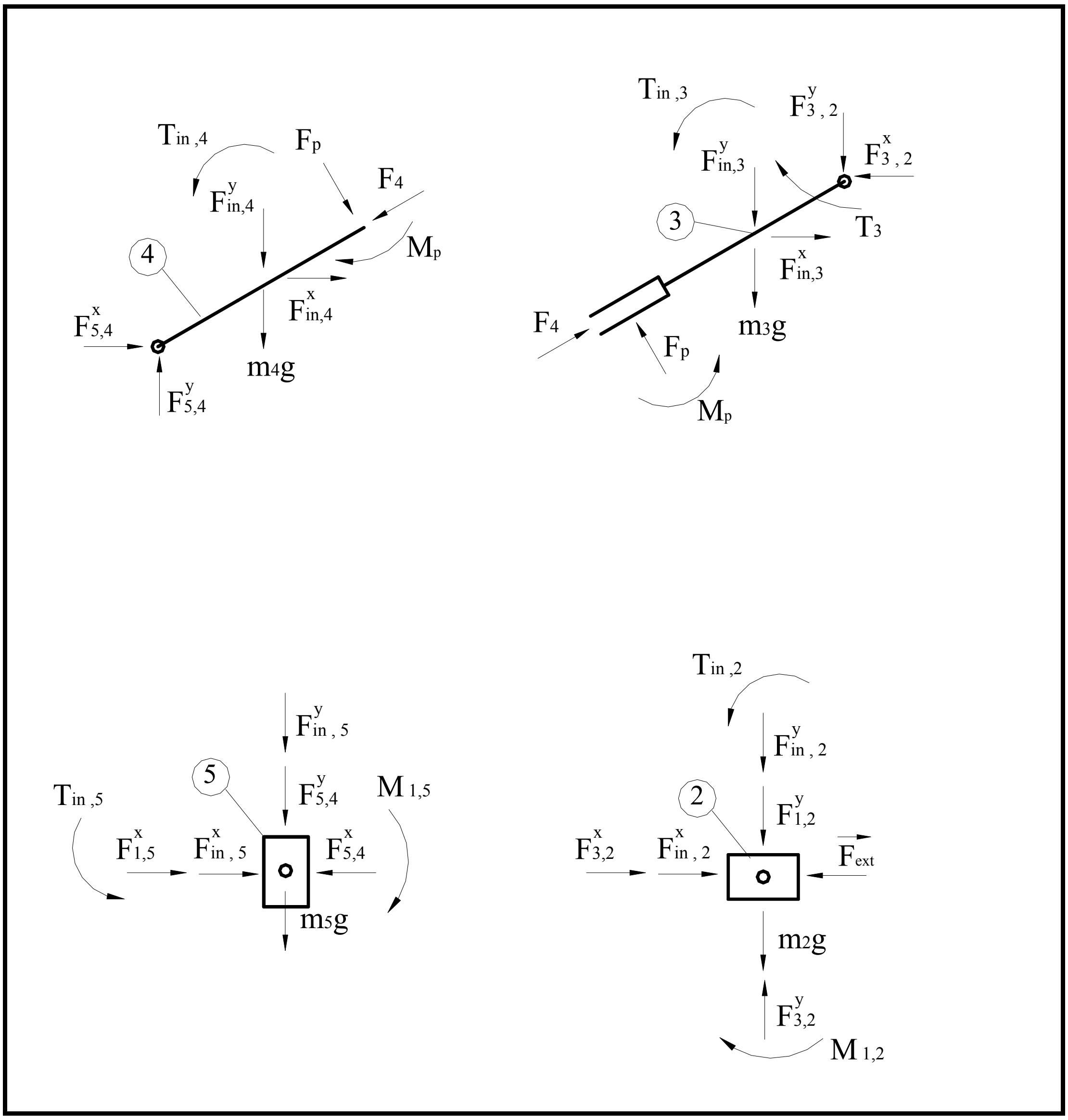

Figure 4.31 Free body diagrams of the Kinematically Adjustable OET

Note that in Figure 4.31 M_p and F_p denote the reaction force and moment due to the prismatic joint between link 3 and link 4 and F_4 denotes the actuator force applied to link 4 in order to adjust the length $L_3(t)$.

The equations of motion for this mechanism depend on whether the adjustment is made before or during the regular motion. Since, the mechanism is a simple one, it is possible to obtain the equations of motion in closed form using the algorithm in section 2.4.2. If the adjustment is made *during the regular motion*, then taking $q_s(t) = \theta(t)$ and $q_u(t) = L_3(t)$ as the generalized coordinates, the equation of motion of the mechanism can be expressed as

$$H_1(\theta, L_3)\ddot{\theta} + H_2(\theta)\ddot{L}_3 + C_1(\theta, L_3)\dot{\theta}^2 + C_2(\theta, L_3)\,\dot{\theta}\dot{L}_3 + G_1(\theta) - T_e = T_3 \tag{4.45}$$

$$H_3(\theta, L_3)\,\ddot{\theta} + H_4(\theta)\,\ddot{L}_3 + C_3(\theta, L_3)\,\dot{\theta}^2 + C_4(\theta)\dot{\theta}\dot{L}_3 + G_2(\theta) + F_e = F_4 \tag{4.46}$$

where

$$\begin{aligned} H_1(\theta, L_3) = {} & Sin(2\theta)(L_3 + L_4)(MY_3 + MY_4) - (L_3 + L_4)^2\,Cos^2(\theta)\left(2\frac{MX_3}{L_3 + L_4}\right. \\ & \left. + 2\,\frac{MX_4}{L_3 + L_4} + m_5 - m_3 - m_2\right) + m_5\,(L_3 + L_4)^2 + I_3 + I_4 + m_4 L_3^2 + m_4\,(L_4^2 - L_3^2)\,Cos^2(\theta) \\ & + 2\,MX_4 L_3(t) \end{aligned} \tag{4.47}$$

$$\begin{aligned} H_2(\theta, L_3) = {} & Cos(\theta)\,Sin(\theta)\,\{(L_3 + L_4)(m_2 + m_3 - m_5) - MX_3 - MX_4 - m_4 L_3\} \\ & - MY_4\,Cos^2(\theta) + MY_3\,Sin^2(\theta) \end{aligned} \tag{4.48}$$

$$\begin{aligned} C_1(\theta, L_3) = {} & Sin(2\theta)\,\{0.5\,(L_3 + L_4)^2\,(m_5 - m_2 - m_3) + 0.5(L_3^2 - L_4^2)\,m_4 \\ & + (L_3 + L_4)\,(MX_3 + MX_4)\} + Cos(2\theta)(L_3 + L_4)\{MY_3 + MY_4\} \end{aligned} \tag{4.49}$$

$$C_2(\theta, L_3) = Cos^2(\theta)\ \{2(L_3 + L_4)(m_2 + m_3) - 2\,MX_3\} + Sin^2(\theta)\{2\,m_5\,(L_3 + L_4) + 2m_4 L_3\ + 2\,MX_4\} + Sin(2\theta)[MY_3 + MY_4] \tag{4.50}$$

$$G_1(\theta, L_3) = g\,[\,Sin(\theta)\ \{m_5\,(L_3 + L_4) + MX_3 + MX_4 + m_4 L_3\} + Cos(\theta)\{MY_3 + MY_4\}] \tag{4.51}$$

$$T_e = -F_{ext}\,(L_3 + L_4)\ Cos\,(\theta) \tag{4.52}$$

$$H_3(\theta, L_3) = 0.5\ [\ MY_3 - MY_4 - (MY_3 + MY_4)\,Cos(2\theta) + (m_2 L_4 + m_3 L_4 - m_5 L_4 - MX_3 - MX_4 + (\,m_2 + m_3 - m_4 - m_5\,)\,L_3\,)\,Sin(2\theta)] \tag{4.53}$$

$$H_4(\theta) = m_2 + m_3 + (-m_2 - m_3 + m_4 + m_5\,)\,Cos^2(\theta) \tag{4.54}$$

$$C_3(\theta, L_3) = -\,m_2 L_4 - m_3 L_4 + MX_3 + MY_3\,Cot\,(\theta) + Cos^2(\theta)[L_4(m_2 + m_3 - m_5) - MX_3 - MX_4 - MY_3\,Cot\,(\,\theta\,)\,] + [\,-m_2 - m_3 + (\,m_2 + m_3 - m_4 - m_5\,)\,Cos^2(\theta)]\,L_3(t) + MY_4\,Cos(\theta)\,Sin(\theta) \tag{4.55}$$

$$C_4(\theta) = (m_2 + m_3 - m_4 - m_5\,)\ Sin(\,2\,\theta\,) \tag{4.56}$$

$$G_2(\theta) = (-m_4 - m_5\,)\,g\,Cos(\theta) \tag{4.57}$$

$$F_e = F_{ext}\,Sin(\theta) \tag{4.58}$$

Also note that

T_3 : generalized actuator force (input torque) associated with θ

F_4 : generalized actuator force associated with L_3

T_e : generalized force due to the externally applied force F_{ext}

F_e : generalized force due to the externally applied force F_{ext}

L_4	: length of link 4 ($L_4 = \|O_4O_5\|$)
MX_i, MY_i	: mass times x and y coordinates of the links in their own body fixed coordinate systems (i = 2,3,4,n,b)
L_3	: The adjustable generalized coordinate
I_3, I_4	: moment of inertias of the links 3 and 4 with respect to their body fixed Z axes
g	: gravitational acceleration (vertically downward)

However , if the adjustment is made *before the regular motion*, the DOF of the system reduces to 1. In this case, the length $L_3(t)$ is constant throughout the motion, leading to $\dot{L}_3(t) = \ddot{L}_3(t) = 0$. Therefore, the only equation of motion is obtained by setting $\dot{L}_3(t) = \ddot{L}_3(t) = 0$ in equation (4.45) , whereas F_4, M_p and F_p become internal forces. Note that the unspecified generalized coordinate is taken to be $q_u(t) = L_3(t)$ rather than $L_{34}(t)$. This does not make any difference, since once $L_3(t)$ is obtained, one can determine $L_{34}(t)$ by $L_{34}(t) = L_3(t) + L_4$. In this study the minimization of the energy consumed in the actuator(s) of the mechanism will be discussed only.

4.2.1 Minimization of the Energy Consumed

Let the design task be the minimization of the energy consumed in the actuator(s) of the mechanism which are assumed to be nonregenerative. As in section 4.1.2, the energy consumed in the actuators depends on whether the adjustment is made before or during the regular motion.

4.2.1.1 Adjustment Before the Regular Motion

Assume that the adjustable length $L_3(t) = |O_3O_4|$, is constant throughout the motion, i.e. $\dot{L}_3(t) = \ddot{L}_3(t) = 0$ during the motion. In this case, one considers the energy consumed in the actuator of the mechanism only. Therefore the performance measure to be minimized is given by

$$J_{ECbefore} = \int_{0}^{t_f=\frac{2\pi}{\omega}} \left| T_3\, \dot{\theta} \right| \, dt \tag{4.59}$$

where the subscript ECbefore denotes the energy consumed for the adjustment before the regular motion. Furthermore, let the input motion of the mechanism (the specified generalized coordinate) be given by

$$q_s(t) = \theta(t) = \omega t \tag{4.60}$$

where ω is constant. Note that for the given input motion $t_f = 2\pi / \omega$ for one cycle of the mechanism. Due to changing operating conditions it is known that $\omega \in R_\omega$ where

$$R_\omega = \{\omega : \omega_l \le \omega \le \omega_u\} \tag{4.61}$$

Here, ω_l and ω_u denote the specified lower and upper bounds for ω .

Note that when the integral given by (4.59) is evaluated, one obtains the performance measure $J_{ECbefore}$ in terms of $L_3(t)$ and ω only. The problem is to determine the optimal length $L_3(t)$ (which is fixed during the regular motion) that minimizes the energy consumed given by (4.59) for a specified ω . The constraint on $L_3(t) = L_3$ = constant , is selected to be

$$0.2 \le L_3 \le 0.5 \ m \tag{4.62}$$

Now, consider that the mechanism is to operate at different angular velocities in an operating range of $[\omega_l , \omega_u] = [5 , 40]$ rad/s . Dividing this range with an increment of 1 rad/s, the performance measure $J_{ECbefore}$ has been minimized with respect to L_3 for each ω. The variations of $J_{ECbefore}^{opt}$ and L_3^{opt} with respect to ω are shown in Figures 4.32 and 4.33. Note that in Figure 4.32, the variation of $J_{ECbefore}^{opt}$ (undotted curve) has been shown compared to a nonoptimal case (dotted curve). Here the nonoptimal case is the value of $J_{ECbefore}$ at $L_3 = 0.2$ m, which is the minimum of $J_{ECbefore}$ values at $L_3 = 0.2$ m and $L_3 = 0.5$ m.

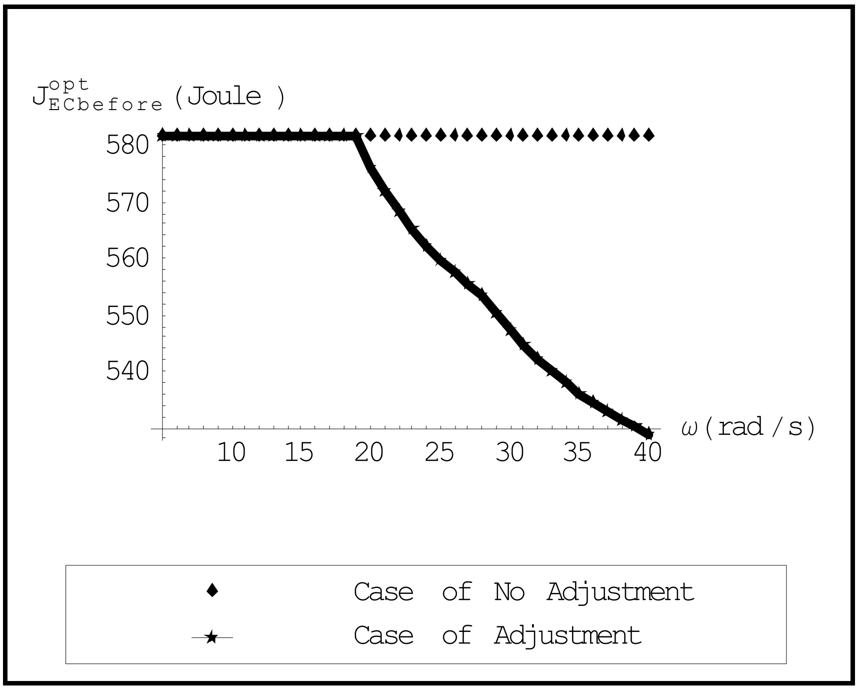

Figure 4.32 Variation of $J_{ECbefore}^{opt}$ with respect to ω

As seen from Figure 4.31, for the given operating range, the energy consumed for the case of adjustment (i.e., when $L_3 = L_3^{opt}$) is less than that of the nonoptimal case for about after ω = 20 rad/s. Note that, as the operating speed ω incrases, the adjustment on the mechanism becomes more and more useful, since the savings in the energy increase in a nonlinear manner. Also note that the variation of the optimal link length L_{34}^{opt} of the combined link associated with links

3 and 4 can be obtained by simply shifting the plot in Figure 4.33 upward with an amount of $L_4 = 0.4$ m , since $L_{34}(t) = L_3(t) + L_4$.

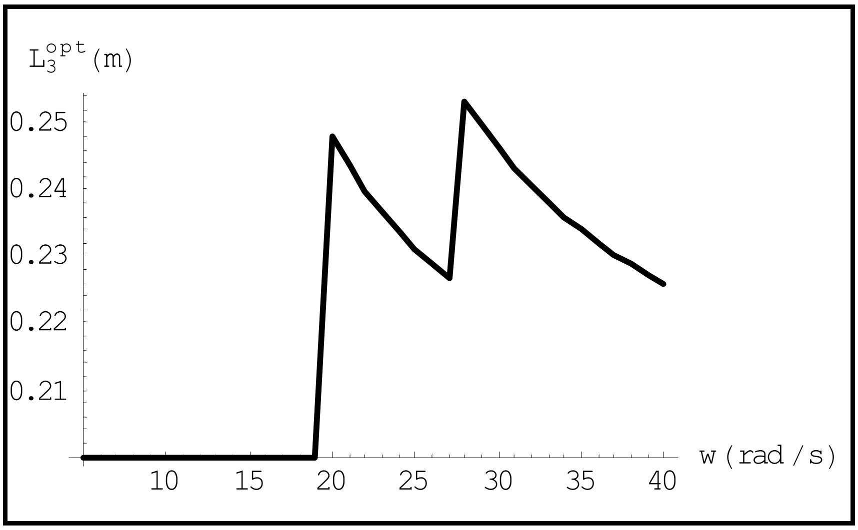

Figure 4.33 Variation of L_3^{opt} with respect to ω

The energy consumed in the actuators is directly related to the power requirements of the actuators. Figures 4.34 and 4.35 show the variation of total power with respect to time for $\omega = 15$ rad/s and $\omega = 35$ rad/s compared to the nonoptimal case(dashed curve) in which $L_3 = L_3^{\min} = 0.2\,\text{m}$.

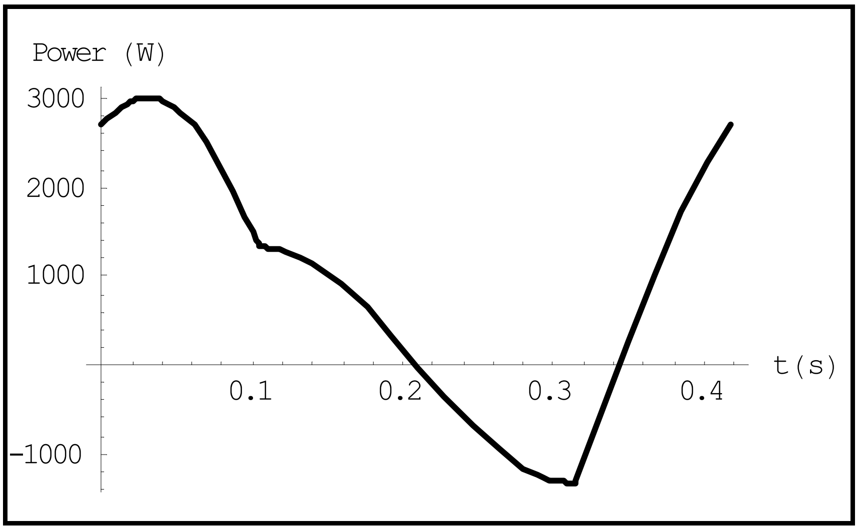

Figure 4.34 Variation of power with respect to time for $\omega = 15$ rad/s

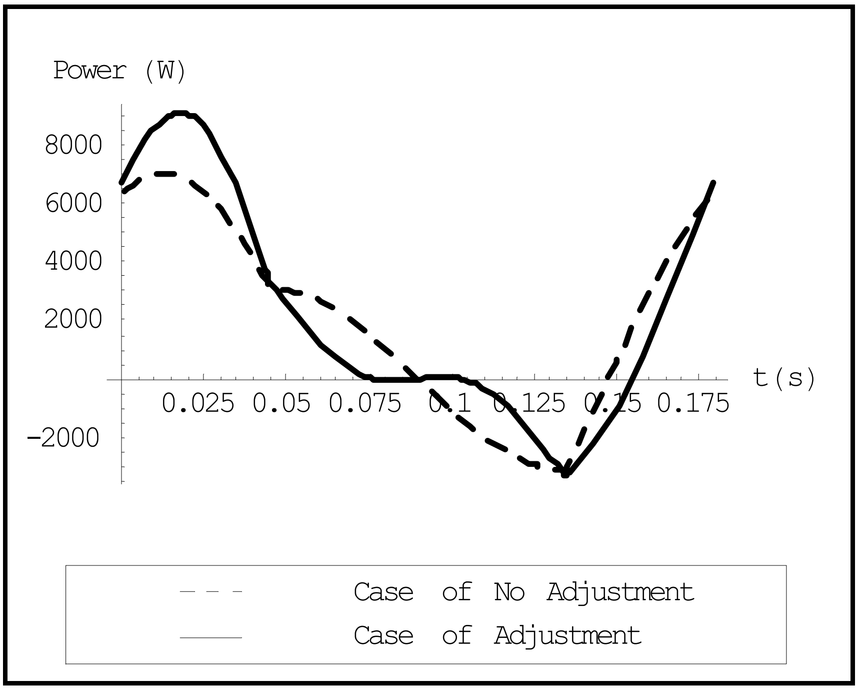

Figure 4.35 Variation of power with respect to time for ω = 35 rad/s

As seen from the plots, for ω = 15 rad/s the optimal and non-optimal cases are coincident, whereas for ω = 35 rad/s one can easily notice that the adjustment decreases the power requirement of the actuator during the whole cycle.

4.2.1.2 Adjustment During the Regular Motion

Now consider the case when the length $L_3(t)$ is adjusted during the regular motion by means of a linear actuator. In this case, for the minimization of the energy consumed one should also take this linear actuator into account. In other words, the performance measure to be minimized is given by

$$J_{ECduring} = \int_0^{t_f=\frac{2\pi}{\omega}} \left\{ \left| T_3\,\dot{\theta}\right| + \left|F_4\,\dot{L}_3\right| \right\} dt \qquad (4.63)$$

Since the adjustment is made during the regular motion, the optimization problem is a dynamic one. Therefore, piecewise continuous polynomial parameterization will be used. Using the developed code, define $L_3(t)$ as two piecewise continuous polynomials satisfying the following boundary conditions:

$$
\begin{aligned}
&L_3(0) = L_3(t_f) = Free \\
&\dot{L}_3(0) = \dot{L}_3(t_f) = Free \qquad (4.64) \\
&\ddot{L}_3(0) = \ddot{L}_3(t_f) = Free
\end{aligned}
$$

where $t_f = 2\pi / \omega$ for one cycle for the given input motion $\theta(t) = \omega t$. As seen from equation (4.64), due to the periodical motion, $L_3(t)$ and its first two time derivatives at the beginning and at the end of the cycle are desired to be equal to each other. When the developed code runs, the polynomial for $L_3(t)$, can be generated in the form

$$L_3(t) = g(t) = g_1(t)[u(t) - u(t - t_1)] + g_2(t)[u(t - t_1) - u(t_1 - t_f)] \qquad (4.65)$$

where $g_1(t)$ and $g_2(t)$ are the polynomial functions in terms of the elements of the design parameters vector (independent polynomial coefficients) given by

$$\vec{d}_J = \{g_{1F,0P},\ g_{1F,1P},\ g_{1F,2P},\ g_{2F,0P},\ g_{2F,1P},\ g_{2F,2P},\ g_{2I,2P}\} \qquad (4.66)$$

When equation (4.63) is integrated, one obtains $J_{ECduring}$ in terms of $\vec{d}_J$ and ω only. For a given ω, one can easily determine the optimal $\vec{d}_J$ and the optimal $J_{ECduring}$ by applying the algorithm discussed in section 3.3.1. Figure 4.36 shows the variation of $J_{ECduring}^{opt}$ with respect to ω compared to the nonoptimal case (which is the value of the energy consumed when L_3 is fixed at $L_3 = 0.2$ m). As seen from this figure , for the given operating range, the energy consumed in case of adjustment (undashed curve) is less than that of the nonoptimal case (dashed curve).

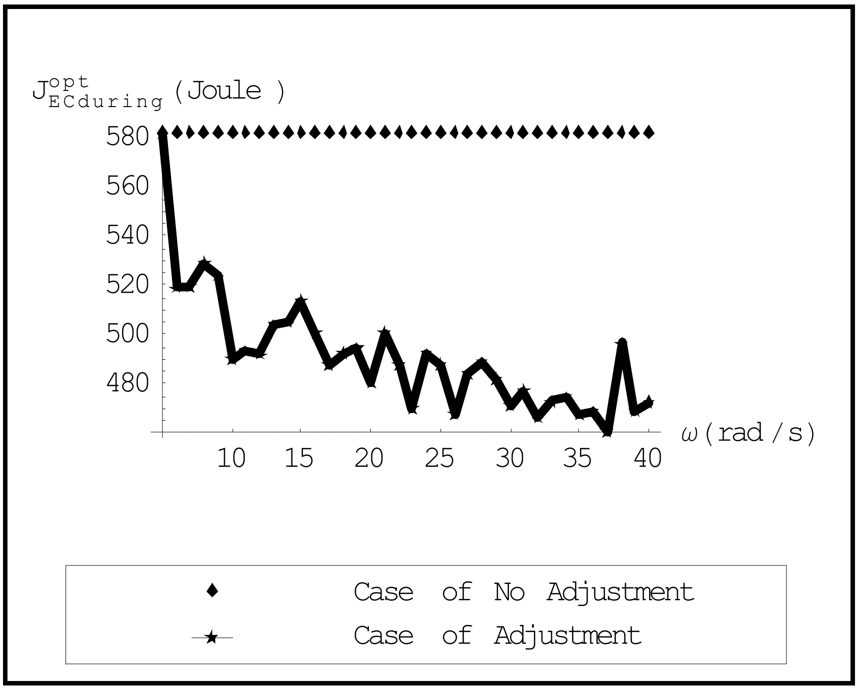

Figure 4.36 Variation of $J^{opt}_{ECduring}$ with respect to ω

The variations of the power and L_3^{opt} with respect to time for $\omega = 15$ rad/s and $\omega = 35$ rad/s are shown in Figures 4.37-4.40.

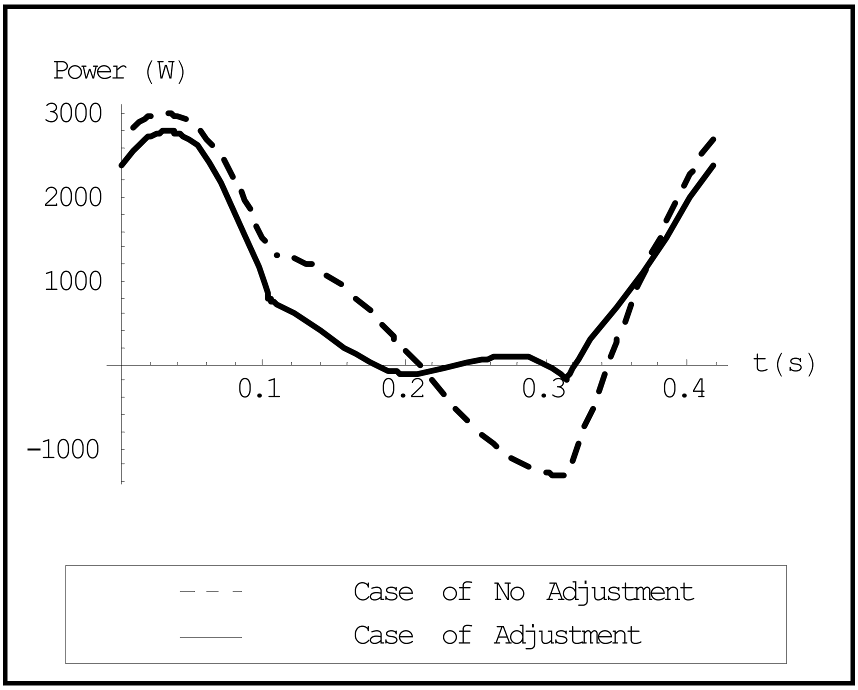

Figure 4.37 Variation of power for $\omega = 15$ rad/s

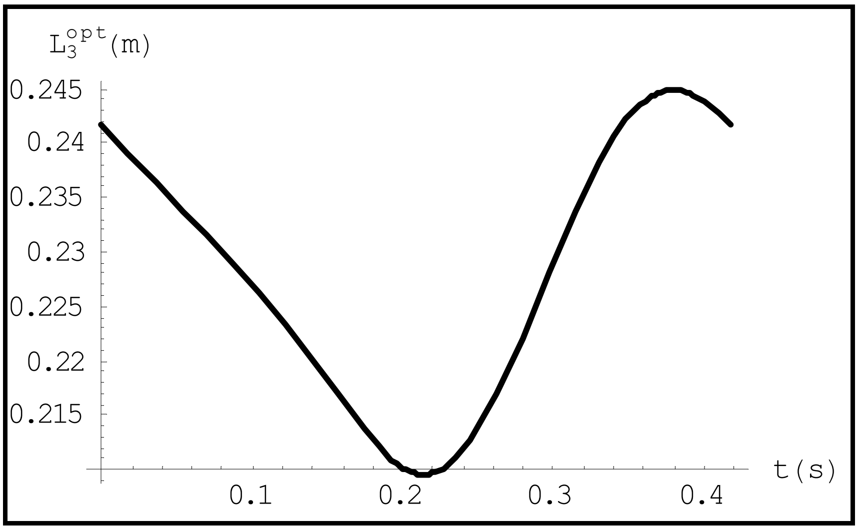

Figure 4.38 Variation of L_3^{opt} for ω = 15 rad/s

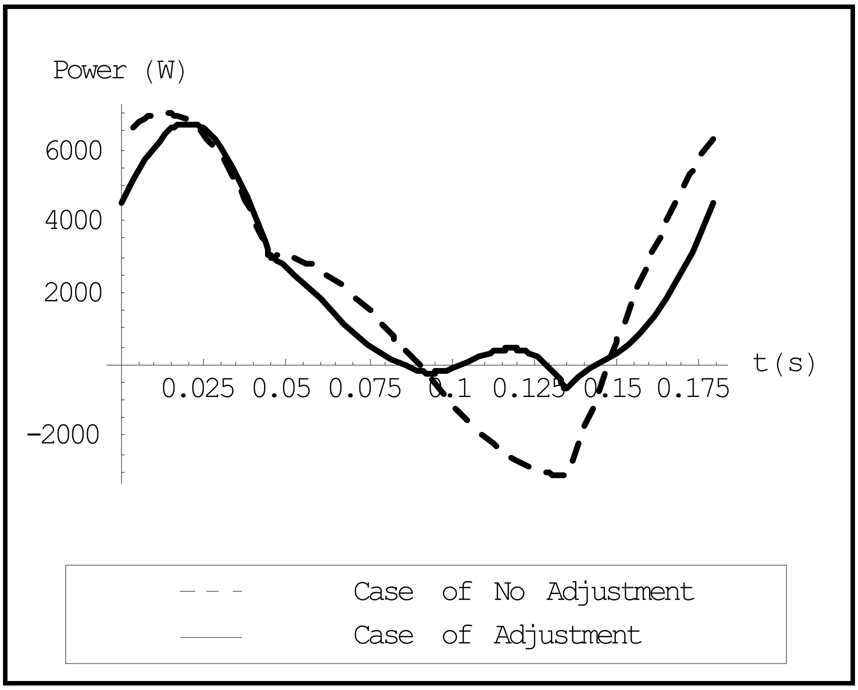

Figure 4.39 Variation of power for ω = 35 rad/s

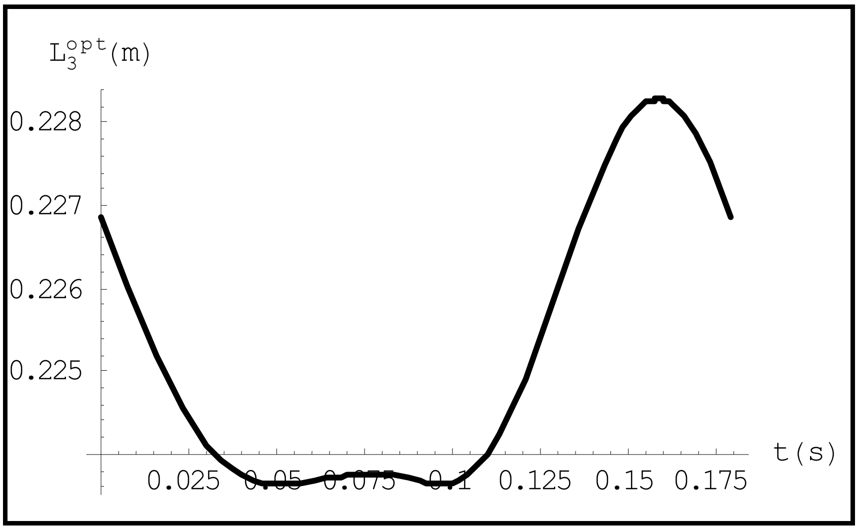

Figure 4.40 Variation of L_3^{opt} for $\omega = 35$ rad/s

As seen from the plots above, the adjustment on the mechanism decreases the power requirements of the actuators for both $\omega = 15$ rad/s and $\omega = 35$ rad/s , therefore it is useful.

4.3 The Dynamically Adjustable Fourbar Mechanism

Consider the adjustable fourbar mechanism shown in Figure 4.41. As seen in this figure , this is a dynamically adjustable mechanism where b denotes the moving block and n denotes the nonmoving actuator . The dimensions of this mechanism are given by

$r_1 = |O_2 O_4| = 0.9$ m (meters)
$r_2 = |O_2 O_3| = 0.3$ m
$r_3 = |O_3 B_1| = 0.7$ m
$r_4 = |B_1 O_4| = 0.6$ m (4.67)

Assume that links 2 , 3 and 4 are made of steel (which has a density of $\rho_s = 7769$ kg / m^3) and have cylindirical cross sections with a radius of 0.015 m. Also assume that mass center and geometric center of each link are coincident . Using this data, the mass and moment of inertia and other inertial parameters of each link with respect to its body fixed Z axis can be determined . The results are as follows.

$$\begin{aligned}
m_2 &= 1.65 \text{ kg} \\
MX_2 &= 0.25 \text{ kg.m} \\
MY_2 &= 0 \\
I_2 &= 0.05 \text{ kg.m}^2 \\
m_3 &= 3.84 \text{ kg} \\
MX_3 &= 1.34 \text{ kg.m} \\
MY_3 &= 0 \\
I_3 &= 0.63 \text{ kg.m}^2 \\
m_4 &= 3.3 \text{ kg} \\
MX_4 &= 0.99 \text{ kg.m} \\
MY_4 &= 0 \\
I_4 &= 0.4 \text{ kg.m}^2
\end{aligned} \tag{4.68}$$

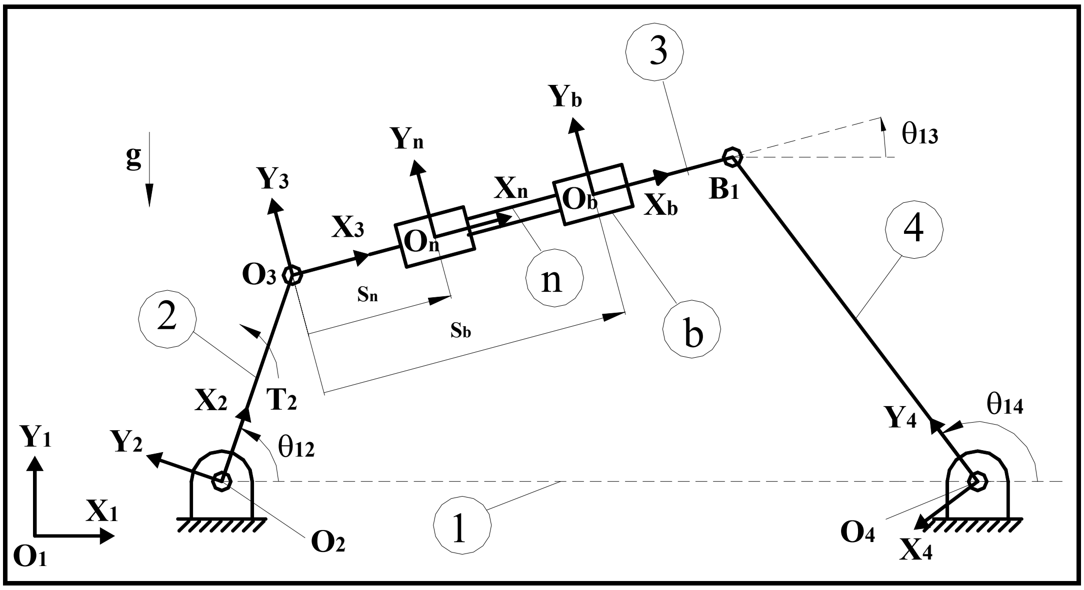

Figure 4.41 The Dynamically Adjustable Fourbar Mechanism

The free body diagrams of the links are shown in Figure 4.42.

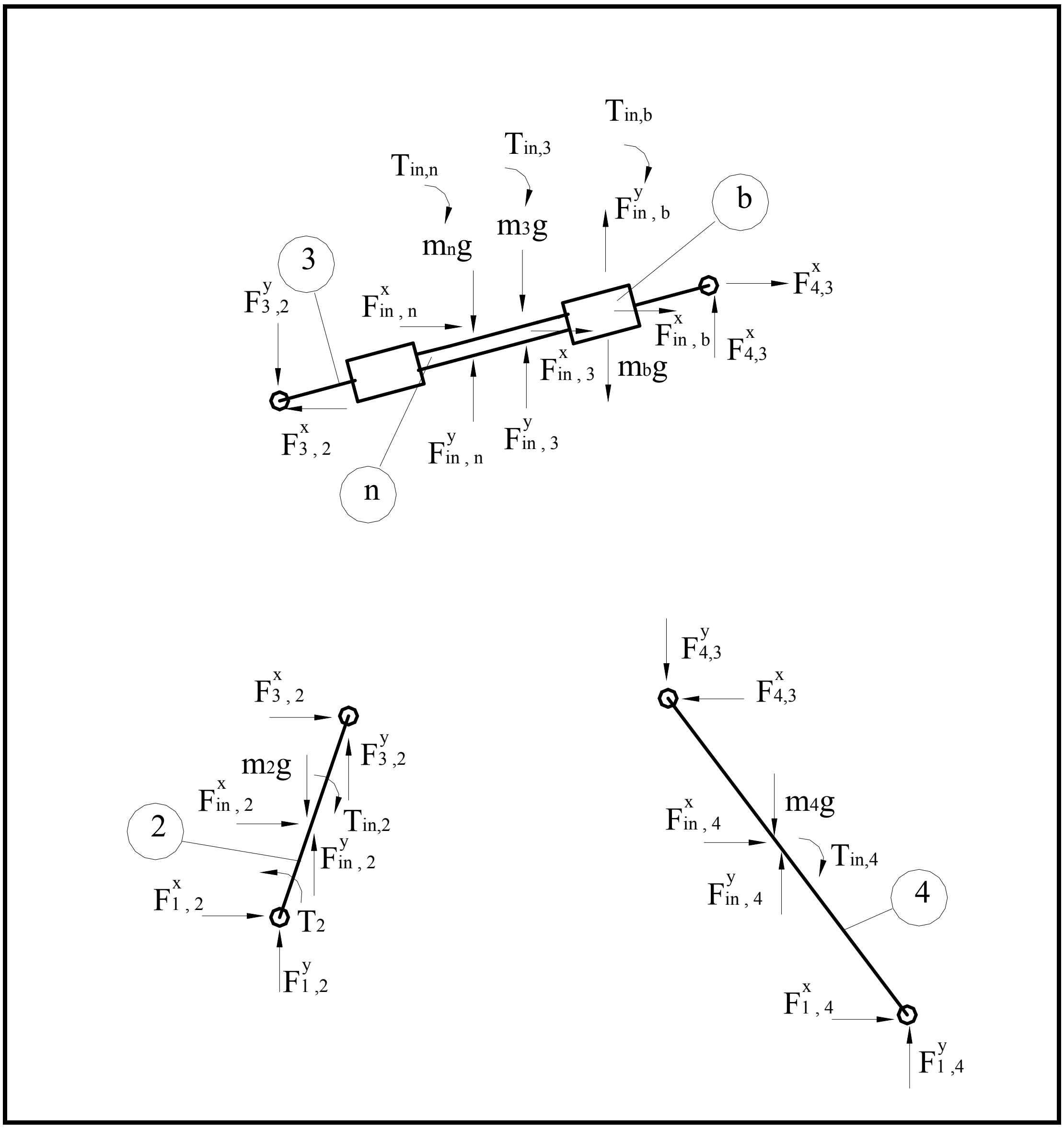

Figure 4.42 Free body diagrams of the Adjustable Fourbar Mechanism

For the case of adjustment during the regular motion, one should also consider the free body diagram of the moving block as shown in Figure 4.43. Note that in Figure 4.42 M_p and F_p denote the reaction force and moment due to the prismatic joint between link 3 and the block and F_b denotes the actuator force applied to the block by the non-moving actuator n .

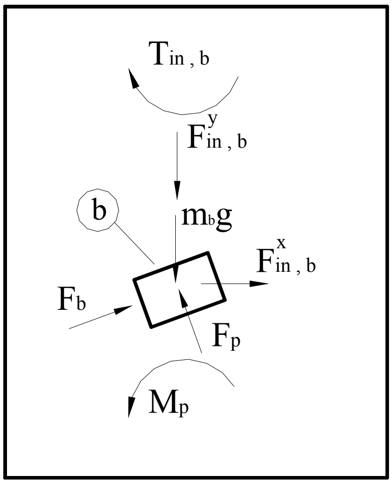

Figure 4.43 Free body diagram of the moving block

Let the input motion of the mechanism be given by

$$q_s(t) = \theta_{12}(t) = \omega\, t \tag{4.69}$$

where ω is constant. Note that for the given input motion $t_f = 2\pi / \omega$ for one cycle of the mechanism. Also as in sections 4.1 and 4.2 , due to changing operating conditions it is known that $\omega \in R_\omega$ where

$$R_\omega = \{\omega : \omega_l \le \omega \le \omega_u\} \tag{4.70}$$

The equations of motion for this mechanism depend on whether the adjustment is made before or during the regular motion. If the adjustment is made before the regular motion, the DOF of the system is 1, therefore the only generalized actuator torque is the actuator torque T_2 (see figure 4.41) associated with the specified generalized coordinate θ_{12} (t). However, if the adjustment is made during the regular motion, DOF of the system is 2. In this case, one has two equations of motion .

The algorithm discussed in section 2.4.1, and the program developed by Tursun [7] has been used in order to determine the equations of motion of the mechanism. Note that there are two closures of the mechanism. The kinematic and force analysis have been performed numerically by dividing the time interval [0, t_f] into 120 points using Tursun's program and the algorithm discussed in section 2.4.1. In this case study the minimization of the energy consumed for the case of adjustment before the regular motion has been analyzed.

4.3.1 Minimization of the Energy Consumed

Let the design task be the minimization of the energy consumed . As stated in the previous sections, the performance measure related to the energy consumed depends on whether the adjustment is made before or during the regular motion. Note that the solutions obtained in this section belong to the first closure of the mechanism (the closure in which θ_{13} = 36.48 deg and θ_{14} = 99.06 deg when θ_{12} = 36 deg).

4.3.3.1 Adjustment Before the Regular Motion

Consider the case where the position $s_b(t)$ is adjusted before the regular motion , i.e. $\dot{s}_b(t) = \ddot{s}_b(t) = 0$ during the motion. Since the adjustment is made before the regular motion, assume that the position of the block is adjusted manually, i.e. there is not an actuator ($m_n = MX_n = MY_n = I_n = s_n = 0$). The performance measure to be minimized is given by

$$J_{ECbefore} = \int_0^{t_f=\frac{2\pi}{\omega}} \left| T_2\,\dot{\theta}_{12} \right| dt \tag{4.71}$$

The constant design parameters vector $\vec{d}_c$ and the design parameters vector related to the unspecified generalized coordinate $(\vec{q}_u(t) = s_b)$, $\vec{d}_u$ are given by

$$\vec{d}_c = \{m_b, MX_b, MY_b, I_b\} \tag{4.72}$$

$$\vec{d}_u = \{s_b\} \tag{4.73}$$

Assume that the moving block is a symmetric body, therefore $MX_b = MY_b = 0$. Then the design parameters to be determined are m_b, I_b and s_b only. The constraints on these parameters are selected to be

$$\begin{aligned} &1 \le m_b \le 10 \;\; kg \\ &0.2 \le s_b \le 0.7 \;\; m \\ &0.1 \le I_b \le 1 \; kg\, m^2 \end{aligned} \tag{4.74}$$

Now, consider that the mechanism is to operate at many different ω values in a range $[\omega_l, \omega_u] = [15, 45]$ rad/s. Using the algorithm given in section 3.1.1, minimizing the performance measure given by (4.71) subject to the constraints (4.74), one obtains the optimal m_b, I_b and s_b for each ω in the range $[\omega_l, \omega_u] = [15, 45]$ rad/s. Figure 4.44 shows the variation of $J^{opt}_{ECbefore}$ with respect to ω. Here, the undotted curve indicates the case of adjustment whereas the dotted one indicates the case of no adjustment.

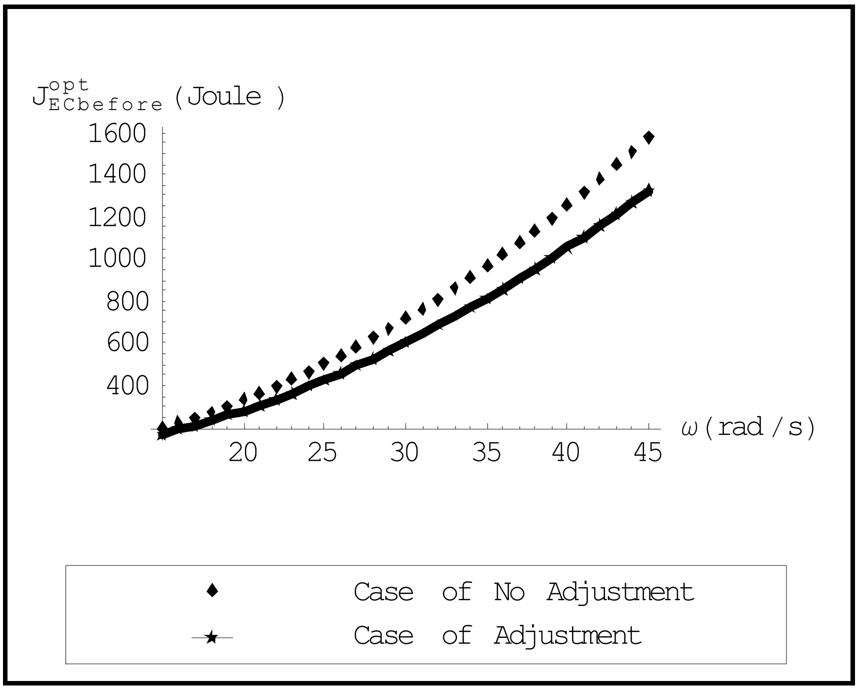

Figure 4.44 Variation of $J^{opt}_{ECbefore}$ with respect to ω

The variations of the optimal m_b, I_b and s_b with respect to ω are shown in Figures 4.45 – 4.47.

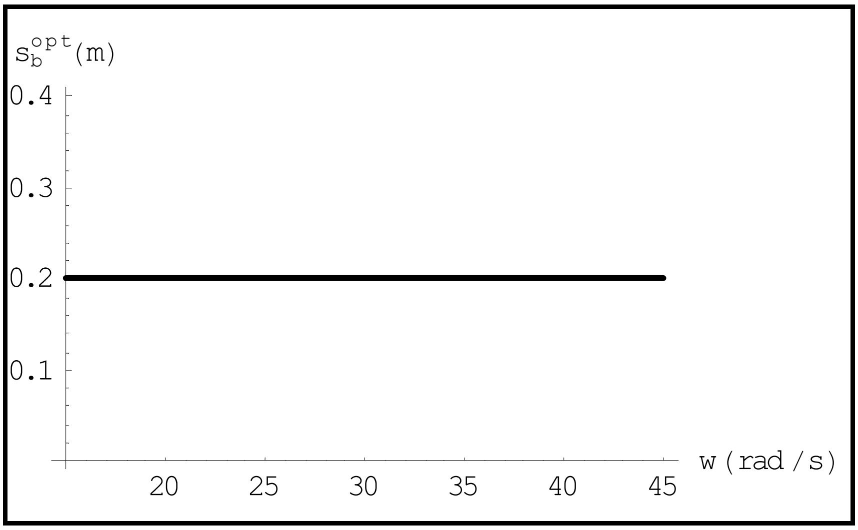

Figure 4.45 Variation of s_b^{opt} with respect to ω

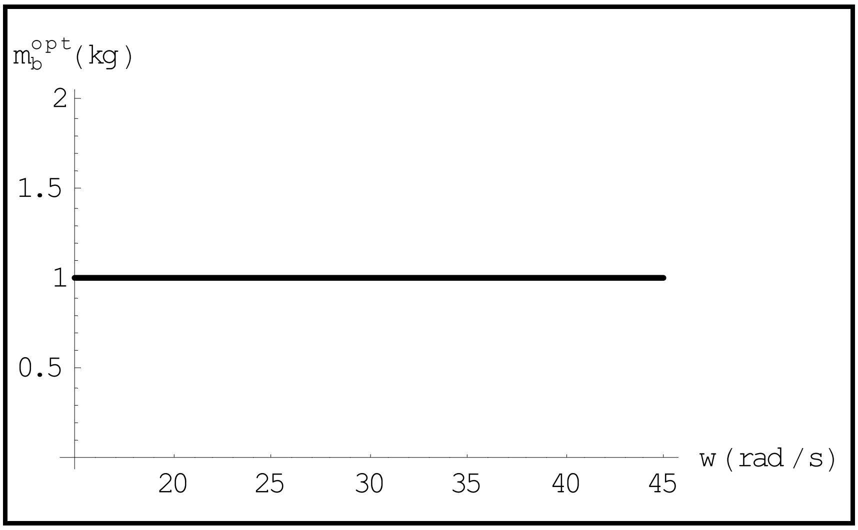

Figure 4.46 Variation of m_b^{opt} with respect to ω

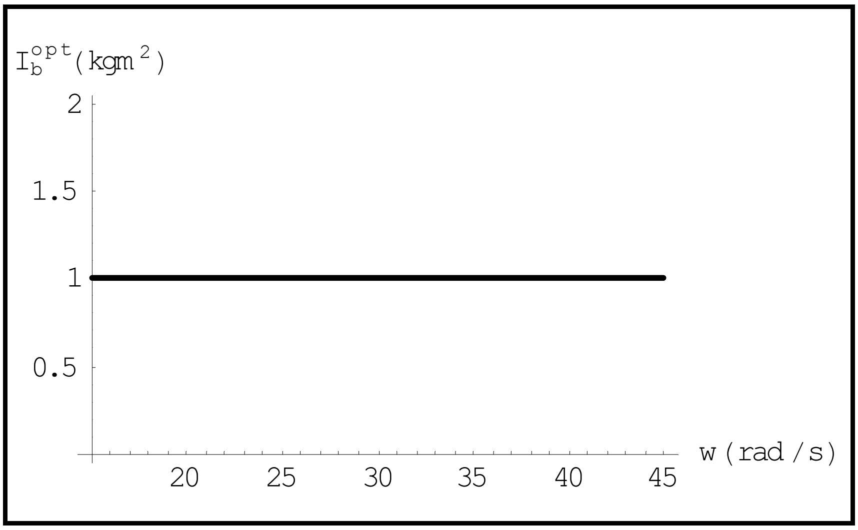

Figure 4.47 Variation of I_b^{opt} with respect to ω

As seen from Figure 4.43, the dynamic adjustment on the fourbar mechanism is useful at high angular velocities, since it reduces the energy consumed compared to the case of no adjustment. Indeed, relaxing the constraints (4.74), will lead to better results. Also note that the optimal m_b and I_b are constant for the given ω range. This is an interesting result, since it was obtained by using the algorithm discussed in section 3.3.1 (Method 1) from which we expect different optimal m_b ,I_b and s_b values for different ω values. Of course, this result is valid for only the data given by (4.67) , (4.68) and for the constraints given by (4.74). For the given operating range the optimal results are given by

$$
\begin{aligned}
m_b^{opt} &= 1\ kg \\
I_b^{opt} &= 1\ kgm^2 \\
s_b^{opt} &= 0.2\ m
\end{aligned}
\tag{4.75}
$$

Notice that the optimal values of m_b and s_b are the lower bounds of their constraints whereas the optimal value of I_b is the upper bound of its constraint. This means that better results could be obtained, locally, if one uses a block with a smaller mass and higher mass moment of inertia and if one positions this block nearer to the origin O_3 of the body fixed frame of the coupler (Link 3).

CHAPTER V

CONCLUSIONS

In this study, the benefits of adjustable planar mechanisms, regarding different dynamic behaviours under variable operating conditions, have been investigated.

Different methods have been applied to derive the equations of motion for kinematically and dynamically adjustable mechanisms. For the dynamically adjustable mechanisms, an algorithm has been developed and has been used in conjunction with the package developed by Tursun [7] for the kinematic and force analysis of adjustable planar mechanisms. On the other hand , the kinematic and force analysis of kinematically adjustable mechanisms have been determined by using another developed algorithm discussed in Chapter 2.

In order to optimize the dynamic behaviour of the mechanism, the concept performance measure has been used. By defining the performance measure appropiately, it has been possible to optimize various dynamic behaviours of the mechanism in a weighted manner. Throughout the thesis, the minimization of the performance measure has been performed using the NMinimize command of MATHEMATICA. This command may be used to find the global minimum, hopefully, of any multivariable function subject to any type of constraints.

For the dynamic optimization problems, the method of Piecewise Continuous Polynomial Parameterization has been used to convert dynamic optimization problems to static optimization problems. The transformed optimization problem has been solved via the NMinimize command of MATHEMATICA.

The developed algorithms have been applied to different types of mechanisms in Chapter 4. All case studies have been made as realistic as possible. The results reveal that, in many cases, the dynamic behaviour of a planar mechanism may be improved quite extensively via adjustable mechanisms. It should be noted that the adjustment mechanisms suggested in this study , which convert an unadjustable mechanism to an adjustable one , are rather easily implementable to an existing planar mechanism.

The studies performed in this section is restricted to planar mechanisms only. In the future a similar study can be realized for spatial mechanisms as well.

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