THE EFFECTS OF $\rho - \omega$ MIXING IN RADIATIVE VECTOR MESON DECAYS

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I certify that this thesis satisfies all the requirements as a thesis for the degree of Doctor of Philosophy.

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This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Doctor of Philosophy.

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ABSTRACT

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The radiative $\omega \to \pi^0\pi^0\gamma$, $\rho \to \pi^0\pi^0\gamma$, $\omega \to \pi^+\pi^-\gamma$ and $\rho \to \pi^+\pi^-\gamma$ decays are studied by adding the effect of vector meson mixing to the amplitude of these decays. For the above decays we consider only $\rho - \omega$ mixing. In addition to the $\rho - \omega$ mixing, we also analyse the contributions coming from different intermediate states to examine the decay mechanism of these decays in a phenomenological framework. For $\omega \to \pi^0\pi^0\gamma$ decay, we consider the contributions of the ρ-meson and σ-meson intermediate states and of the kaon-loop, and for the $\rho \to \pi^0\pi^0\gamma$ decay we calculate the amplitude using the contributions of the ω-meson and σ-meson intermediate states and pion-loop. Moreover, the radiative $\omega \to \pi^+\pi^-\gamma$ decay is studied by considering the contributions of σ-meson and ρ-meson intermediate states and the decay $\rho \to \pi^+\pi^-\gamma$ is investigated by taking into account the contributions of bremmsstrahlung, pion-loop and σ-meson intermediate state amplitude. We also estimate the coupling constant $g_{\omega\sigma\gamma}$ utilizing the latest experimental value of the branching ratio $\omega \to \pi^0\pi^0\gamma$.

Keywords: $\rho - \omega$ Mixing, Radiative Decay, Vector Meson, Bremsstrahlung,

Kaon-loop, Pion-loop, Branching Ratio, Coupling Constant

ÖZ

IŞINSAL VEKTÖR MEZON BOZUNMALARINDA $\rho - \omega$ KARIŞIMININ ETKİLERİ

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Genliğe vektör mezon karışımın etkisi eklenerek ışınsal $\omega \to \pi^0\pi^0\gamma$, $\rho \to \pi^0\pi^0\gamma$, $\omega \to \pi^+\pi^-\gamma$ ve $\rho \to \pi^+\pi^-\gamma$ bozunmaları çalışıldı. Bu bozunmalarda sadece $\rho - \omega$ karışımı gözönüne alındı. $\rho - \omega$ karışımına ek olarak, bu bozunmaların bozunma mekanizmalarına farklı katkıları fenomenolojik bir çerçevede analiz edildi. $\omega \to \pi^0\pi^0\gamma$ bozunması için ρ-mezon ve σ-mezon ara durumları ve kaon-döngüsü katkıları düşünüldü, $\rho \to \pi^0\pi^0\gamma$ bozunması için ise ω-mezon ve σ-mezon ara durumları ve pion-döngüsü katkıları kullanılarak genlik hesabı yapıldı. Bundan başka, $\omega \to \pi^+\pi^-\gamma$ bozunması σ-mezon ve ρ-mezon ara durumları katkıları düşünülerek çalışıldı ve $\rho \to \pi^+\pi^-\gamma$ bozulması bremsstrahlung, pion-döngüsü ve σ-mezon ara durumu katkıları dikkate alınarak çalışıldı. Ayrıca, $\omega \to \pi^0\pi^0\gamma$ bozunmasının en son deneysel dallanma oranı kullanılarak $g_{\omega\sigma\gamma}$ çiftlenim sabiti hesaplandı.

Anahtar Sözcükler: $\rho - \omega$ Karışımı, Bremsstrahlung, Işınsal Bozunma, Vektor Mezon, Kaon-döngü, Pion-döngü, Dallanma Oranı, çiftlenim Sabiti

..to mystery...

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CHAPTER 1

INTRODUCTION

The SND Collaboration measured the branching ratio of the radiative $\omega \rightarrow \pi^0\pi^0\gamma$ and $\rho \rightarrow \pi^0\pi^0\gamma$ decays [1] and they obtained the value $BR(\omega \rightarrow \pi^0\pi^0\gamma) = (6.6^{+1.4}_{-0.8} \pm 0.6) \times 10^{-5}$ which is in good agreement with GAMS Collaboration measurement of the branching ratio, $BR(\omega \rightarrow \pi^0\pi^0\gamma) = (7.2 \pm 2.5) \times 10^{-5}$ [2] but it has a higher accuracy, and for the $\rho \rightarrow \pi^0\pi^0\gamma$ decay, they obtained $BR(\rho \rightarrow \pi^0\pi^0\gamma) = (4.1^{+1.0}_{-0.9} \pm 0.3) \times 10^{-5}$ and with this value they improved their previous preliminary report of the value $BR(\rho \rightarrow \pi^0\pi^0\gamma) = (4.8^{+3.4}_{-1.8} \pm 0.3) \times 10^{-5}$ [3].This last result can be explained by means of a significant contribution of the $\sigma\gamma$ intermediate state together with the well-known $\omega\pi$ contribution. For the charged mode of $\omega \rightarrow \pi\pi\gamma$ decay only upper limit exists and its branching ratio has been measured as $BR(\omega \rightarrow \pi^+\pi^-\gamma) < 3.6 \times 10^{-3}$ [4]. The experimental study of $\rho \rightarrow \pi^+\pi^-\gamma$ decay was reported earlier by Novosibirsk group who measured the branching ratio as $BR(\rho \rightarrow \pi^+\pi^-\gamma) = (9.9 \pm 1.6) \times 10^{-3}$ [5, 6] and for this decay they observed that the main mechanism is the pion bremsstrahlung with the structural radiation proceeding through the intermediate scalar resonance [5].

The theoretical studies of radiative ρ-meson and ω-meson decays were initiated by Singer [7, 8] in 1960's. For the $\omega \to \pi\pi\gamma$ and $\rho \to \pi\pi\gamma$ decays, he assumed that they proceed through $\omega \to (\rho)\pi \to \pi\pi\gamma$ and $\rho \to (\omega)\pi \to \pi\pi\gamma$ mechanism, respectively. Moreover, by considering the bremsstrahlung mechanism, he calculated the amplitude for the $\rho^0 \to \pi^+\pi^-\gamma$ decay. Renard considered the decay modes $V \to PP^{'}\gamma$ where V and P denote vector meson and pseudoscalar meson respectively, in a gauge invariant way using current algebra, the hard-pion and Ward-identities technique and also considering intermediate meson states [9]. Furthermore, he discussed the angular distributions, photon spectra and decay rate in terms of the coupling constants and of the intermediate state meson parameters, mass and coupling constants. He observed that the intermediate meson term modifies the shape of the photon spectrum for high momenta differently in the case of the different mass of the meson considered. In particular, he concluded that the σ-meson term could be analysed in $V \to PP^{'}\gamma$ decays in terms of its mass and of the $g_{\sigma\pi\pi}$ and $g_{V\sigma\gamma}$ coupling constants, and the σ-meson intermediate state makes the largest contribution to the radiative amplitude. Then the radiative $V \to P^0P^0\gamma$ decays were studied by Fajfer and Oakes [10] using a low energy effective Lagrangian approach with gauged Wess-Zumino terms, there being no bremsstrahlung contributions. Scalar meson contributions were neglected and the branching ratios for the $\omega \to \pi^0\pi^0\gamma$ and $\rho \to \pi^0\pi^0\gamma$ decays were obtained as $BR(\omega \to \pi^0\pi^0\gamma) = 8.21 \times 10^{-5}$ and $BR(\rho \to \pi^0\pi^0\gamma) = 2.89 \times 10^{-5}$. Bramon et al. [11] also studied the contribution of intermediate vector meson (VMD) to the vector meson decay into two pseudoscalars and a single photon, $V \rightarrow PP^{'}\gamma$, using standard Lagrangians obeying the SU(3)-symmetry. Moreover, they also considered the $V \rightarrow PP^{'}\gamma$ decays within the framework of chiral effective Lagrangians using chiral perturbation theory [12]. In particular for the branching ratio of the decays $\omega \rightarrow \pi^0\pi^0\gamma$ and $\rho \rightarrow \pi^0\pi^0\gamma$ they found the results $BR(\omega \rightarrow \pi^0\pi^0\gamma) = 2.8 \times 10^{-5}$ and $BR(\rho \rightarrow \pi^0\pi^0\gamma) = 1.1 \times 10^{-5}$ [11]. However, they also observed that final state interactions could lead to a larger value for the branching ratio $BR(\rho \rightarrow \pi^0\pi^0\gamma)$ through the mechanism $\rho \rightarrow (\pi^+\pi^-)\gamma \rightarrow (\pi^0\pi^0)\gamma$, but that was very unlikely for the branching ratio $BR(\omega \rightarrow \pi^0\pi^0\gamma)$. If chiral perturbation theory Lagrangians are used there is no three-level contribution to the amplitudes for the decay processes $V \rightarrow PP^{'}\gamma$, and moreover the one-loop contributions including both $\pi\pi$ and $K\overline{K}$ intermediate loops, are finite and to this order no counterterms are required. For the $\omega \rightarrow \pi^0\pi^0\gamma$ amplitude, π-loop contributions vanish in the good isospin limit and the contribution of K-loops, resulting in the decay rate $\Gamma(\omega \rightarrow \pi^0\pi^0\gamma)_K = 1.8$ eV, is two orders of magnitude smaller than the contribution of VMD amplitude. Therefore, the contribution of the VMD amplitude essentially accounts for the decay rate of the $\omega \rightarrow \pi^0\pi^0\gamma$ decay. Bramon et al. [11] also considered the decay $\rho \rightarrow \pi^0\pi^0\gamma$ in this approach where the decay proceeds mainly through the charged pion loops, obtaining the contribution of pion-loops to the decay rate as $\Gamma(\rho^0 \rightarrow \pi^0\pi^0\gamma)_\pi = 1.42 \times 10^3$ eV which was three orders of magnitude larger than that due to kaon-loops. As the result show the pion-loop contribution is of the same order of magnitude as the VMD contribution which is $\Gamma(\rho^0 \to \pi^0\pi^0\gamma)_{VMD} = 1.62 \times 10^3$ eV. Therefore, the global $\rho^0 \to \pi^0\pi^0\gamma$ decay width was given by the sum of the pion-loop contribution and the VMD amplitude as $\Gamma(\rho^0 \to \pi^0\pi^0\gamma)_{VMD+\pi} = 3.88 \times 10^3$ eV and the branching ratio as $BR(\rho^0 \to \pi^0\pi^0\gamma)_{VMD+\pi} = 2.6 \times 10^{-6}$ which, however is smaller than the latest experimental result [1]. Hubert and Neufeld [13] investigated the process $\rho \to \pi^+\pi^-\gamma$, using the close relationship between the low-energy constants of chiral perturbation theory and the chiral invariant interactions of the vector meson resonances with the pseudoscalar mesons. They observed that for small photon energies, the decay rate is dominated by bremsstrahlung, however, near the endpoint of the photon energy spectrum, the solution favoured by chiral vector meson dominance shows a sizable enhancement comparable with the contribution from the pure bremsstrahlung mechanism. Such a particular shape of the differential decay rate has indeed been observed experimentally and turns out to be an important confirmation of the theoretical concept of chiral vector dominance. They also compared the theoretical and the experimental branching ratio and they found the branching ratio of the $\rho \to \pi^+\pi^-\gamma$ decay as $BR(\rho \to \pi^+\pi^-\gamma) = 1.1 \times 10^{-2}$ for $E_\gamma > 50$ MeV, while the measured value is given by Dolinsky et al [5] as $BR(\rho \to \pi^+\pi^-\gamma) = (0.99 \pm 1.6) \times 10^{-2}$ for $E_\gamma > 50$ MeV.

Guetta and Singer [14] in a recent work reexamined the theoretical value for the branching ratio $BR(\omega \to \pi^0\pi^0\gamma)$ of the decay $\omega \to \pi^0\pi^0\gamma$ which is

$BR(\omega \to \pi^0\pi^0\gamma) = (4.1 \pm 1.1) \times 10^{-5}$. Determining the Born amplitude for VMD mechanism of the $\omega \to \pi^0\pi^0\gamma$ decay, they calculated the decay width of $\omega \to \pi^0\pi^0\gamma$ which is proportional to the coupling constants $g^2_{\omega\rho\gamma}$ and $g^2_{\rho\pi\gamma}$. They also assumed that the decay $\omega \to 3\pi$ proceeds with the same mechanism as $\omega \to \pi^0\pi^0\gamma$ decay, that is as the sequential transition $\omega \to (\rho)\pi \to \pi\pi\gamma$. Then they used the experimental inputs for the decay rates $\Gamma(\omega \to 3\pi)$, $\Gamma(\rho^0 \to \pi^0\gamma)$ and $\Gamma(\rho \to \pi\pi)$ and the Born amplitude for $\omega \to \pi^0\pi^0\gamma$ decay. Furthermore they employed a momentum dependent width for ρ-meson and obtained the value $BR(\omega \to \pi^0\pi^0\gamma) = (4.1 \pm 1.0) \times 10^{-5}$. If a constant ρ-meson width is used, for the branching ratio of the decay $\omega \to \pi^0\pi^0\gamma$ they obtained the value $BR(\omega \to \pi^0\pi^0\gamma) = (3.6 \pm 0.9) \times 10^{-5}$. Therefore, there appears to be a serious discrepancy between the theoretical result and the experimental value for the branching ratio of the $\omega \to \pi^0\pi^0\gamma$ decay.

Guetta and Singer [14] noted that in the theoretical framework based on chiral perturbation theory and vector meson dominance one feature has been neglected. This is the possibility of ρ–ω mixing, one consequence of which is the isospin violating $\omega \to \pi^+\pi^-$ decay width the branching ratio $BR(\omega \to \pi^+\pi^-) = (2.21 \pm 0.30)^0/_0$ [6]. The phenomenon of $\rho - \omega$ mixing has been observed in the electromagnetic form factor of pion improving the standard VMD model result involving vector-meson intermediate state. This phenomena is explained in the following chapter in some detail. Guetta and Singer [14] calculated the effect of ρ–ω mixing using the Born amplitude for VMD mechanism of $\omega \to \pi^0\pi^0\gamma$ decay, and they showed that it increases the $\omega \to \pi^0\pi^0\gamma$ decay width by $5^0/_0$ which is less than $12^0/_0$ increase provided by using a momentum dependent width for ρ-meson in the calculation using the VMD amplitude. They then combined all the improvements on the simple Born term of VMD mechanism, that is ρ–ω mixing, momentum dependence of ρ-meson width and the inclusion of the chiral loop amplitude as given by Bramon et al. [12], and using the resulting amplitude for the decay rate they obtained the theoretical result $\Gamma(\omega \to \pi^0\pi^0\gamma) = (390 \pm 96)$ eV and $BR(\omega \to \pi^0\pi^0\gamma) = (4.6 \pm 1.1) \times 10^{-5}$ for the branching ratio of the $\omega \to \pi^0\pi^0\gamma$ decay. Their values are still smaller though barely consistent with the existing experimental value of this decay. Besides Guetta and Singer, Palomar et al. [15] also analysed the radiative $V \to PP^{'}\gamma$ decays using the sequential vector meson decay mechanism in addition to $\rho - \omega$ mixing and chiral loops obtained using unitarized chiral perturbation theory. For the sequential mechanism (VMD) they followed the approach of Bramon et al.[11], but for the loop contributions they adopted the approach of Marco et al. [16]. They obtained the branching ratio of the $\omega \to \pi^0\pi^0\gamma$ decay as $BR(\omega \to \pi^0\pi^0\gamma) = (4.7 \pm 0.9) \times 10^{-5}$. These theoretical results are still seriously less than the latest experimental result, $BR(\omega \to \pi^0\pi^0\gamma) = (6.6^{+1.4}_{-0.8} \pm 0.6) \times 10^{-5}$. For the $\rho \to \pi^0\pi^0\gamma$ decay, Palomar et al. [15] found the branching ratio from the sum of the sequential and loop mechanism as $BR(\rho \to \pi^0\pi^0\gamma) = 4.2 \times 10^{-5}$ which is about three times larger than with either mechanism alone, and this value is comparable with the present experimental value, $BR(\rho \to \pi^0\pi^0\gamma) = (4.8^{+3.4}_{-1.8} \pm 0.3) \times 10^{-5}$.

From these results it follows that the possibility of additional contributions to the mechanism of $\omega \to \pi^0\pi^0\gamma$ and $\rho \to \pi^0\pi^0\gamma$ and also $\omega \to \pi^+\pi^-\gamma$ and $\rho \to \pi^+\pi^-\gamma$ decays should be investigated. For these decays the amplitude involving scalar-isoscalar σ-meson as an intermediate state, may provide one such additional contribution.

About the existence of an isospin zero, broad scalar resonance in $\pi\pi$ scattering there is a long standing argument in the literature. Lately, the possible existence of the sigma resonance was discussed along the whole conference and at this meeting all speakers took the light σ-meson for granted and many mass and width estimations near 500 MeV were presented. Törnqvist summarized the most important results presented at the conference in his report [17]. He emphasized that if the light and broad σ is accepted as a true resonance it explains many basic problems of low energy hadronic physics in a simple way, especially if the linear sigma model ($L\sigma M$) is used as an approximate effective low energy theory.

The sigma meson has been difficult to find in the data due to the very large width of the resonance. In the Fermilab E791 experiment it was found that the σ-meson manifests itself in the D-meson decay $D \to \pi^+\pi^+\pi^-$, being responsible for approximately half of the decays through the resonant sequence $D \to \pi^+\sigma \to \pi^+\pi^+\pi^-$ [18]. A $\pi^+\pi^-$ pair in the final state appears through the formation of an intermediate σ-meson resonance. In a coherent amplitude analysis of the 3π Dalitz plot the scalar resonance is determined with $M_\sigma = (438 \pm 31)$ MeV and the total width $\Gamma_\sigma = (338 \pm 48)$ MeV, where statistical and systematic errors have been added in quadrature [19].

Gökalp and Yılmaz [20] studied the $\omega \to \pi\pi\gamma$ decays by adding to the amplitude calculated within the framework of chiral perturbation theory and vector meson dominance the amplitude of σ-meson intermediate state in a phenomenological approach. To calculate the decay rate for the $\omega \to \pi\pi\gamma$ decay, they considered ρ-pole vector meson dominance amplitude as well as the σ-pole amplitude. Then by employing the available experimental value for this branching ratio $BR(\omega \to \pi\pi\gamma) = (7.2 \pm 2.5) \times 10^{-5}$ [2] which is somewhat less accurate than the present new value $BR(\omega \to \pi\pi\gamma) = (6.6^{+1.4}_{-0.8} \pm 0.6) \times 10^{-5}$ [1], they obtained for the coupling constant $g_{\omega\sigma\gamma}$ the values $g_{\omega\sigma\gamma} = 0.13$ and $g_{\omega\sigma\gamma} = -0.27$ using the set of values $M_\sigma = 478$ MeV and $\Gamma_\sigma = 374$ MeV. Besides this, they determined the coupling constant $g_{\omega\sigma\gamma}$ from the $\omega \to \pi^+\pi^-\gamma$ decay using experimental upper limit for its decay rate, $BR(\omega \to \pi^+\pi^-\gamma) < 3.6 \times 10^{-3}$ [4]. They concluded that σ-meson intermediate state amplitude makes an important contribution by itself and by its interference with the VMD amplitude. Later the same authors investigated the $\omega\sigma\gamma$-vertex and again estimated the coupling constant $g_{\omega\sigma\gamma}$ in the framework of the light cone QCD sum rules methods [21] and its value was deduced as $|g_{\omega\sigma\gamma}| = (0.72 \pm 0.08)$. This result is in reasonable agreement with the results obtained from the phenomenological analysis of the $\omega \to \pi\pi\gamma$ decays [20]. Aliev et al. [22] also calculated the coupling constant $g_{\rho\sigma\gamma}$ using the light cone QCD sum rules techniques, and they obtained the value

$g_{\rho\sigma\gamma} = (2.2 \pm 0.4)$ from which by using SU(3)-symmetry it follows that the coupling constant $g_{\omega\sigma\gamma}$ should have the value $g_{\omega\sigma\gamma} = 0.73$. Thus, there seems to be a serious discrepancy between the values obtained for the coupling constant $g_{\omega\sigma\gamma}$ using the light cone QCD sum rules method and the phenomenological analysis of $\omega \to \pi\pi\gamma$ decays.

To include the effect of σ-meson in the decay mechanism of the ρ^0-meson, it is considered as a σ-pole intermediate state. Thus an amplitude characterizing the contribution of σ-meson to the $\rho^0 \to \pi\pi\gamma$ decays result from the sequential $\rho^0 \to (\sigma)\gamma \to \pi\pi\gamma$ interaction. To calculate the branching ratio $BR(\rho^0 \to \pi^+\pi^-\gamma)$ in a phenomenological approach, Gökalp and Yılmaz [23] used pion bremsstrahlung amplitude and σ-meson pole amplitude, and determined the coupling constant $g_{\rho\sigma\gamma}$ by using the experimental value of the branching ratio $BR(\rho^0 \to \pi^+\pi^-\gamma)$. Then, same authors used this value of the coupling constant $g_{\rho\sigma\gamma}$ in their calculation of the branching ratio of the $\rho^0 \to \pi^0\pi^0\gamma$ decay in their following work [24]. They considered the contribution of σ-meson and ω-meson intermediate states, and the pion-loop amplitude in their work. However, including the contribution of the mechanism $\rho^0 \to (\sigma)\gamma \to \pi^0\pi^0\gamma$ the value they obtained for the branching ratio $BR(\rho^0 \to \pi^0\pi^0\gamma) = 4.7 \times 10^{-5}$ was much larger than the experimental result $BR(\rho^0 \to \pi^0\pi^0\gamma) = (1.9^{+0.9}_{-0.8} \pm 0.4) \times 10^{-5}$. This unrealistic result was due to the constant $\rho^0 \to \sigma\gamma$ amplitude employed and consequently the large coupling constant $g_{\rho\sigma\gamma}$ that was deduced using the experimental branching ratio of the $\rho^0 \to \pi^+\pi^-\gamma$ decay. In conclusion, the

σ-meson should be considered as dynamically generated from pion loops and therefore it should not be included in the mechanism of vector meson decays as an intermediate σ-pole state. Moreover, Marco et al. [16] also studied the radiative $V \rightarrow PP'\gamma$ decays using the techniques of chiral unitary approach, which was developed earlier by Oller et al. [25], to deal with the final state interaction of the two pion system which was included by unitary resummation of the pion-loops through the Bethe-Salpeter equation. Marco et al. noted that the energies of two meson system are too large in some decays to be treated with standard chiral perturbation theory. The novelty in their work is that the strong interaction vertex is evaluated using the unitary chiral amplitudes instead of the lowest order amplitudes used in [12]. Considering the total contribution to the $\rho^0 \rightarrow \pi^+\pi^-\gamma$ decay, they obtained the branching ratio of the decay as $BR(\rho^0 \rightarrow \pi^+\pi^-\gamma) = 1.18 \times 10^{-2}$ for $E_\gamma > 50$ MeV. The branching ratio for $\rho^0 \rightarrow \pi^0\pi^0\gamma$ that they obtained was $BR(\rho^0 \rightarrow \pi^0\pi^0\gamma) = 1.4 \times 10^{-5}$ which could be interpreted as resulting from the $\rho^0 \rightarrow (\sigma)\gamma \rightarrow \pi^0\pi^0\gamma$ mechanism. These values are consistent with experimental results. Therefore, it seems that a natural way to include the effect of σ-meson in the mechanism of radiative ρ^0-meson decays is to assume that σ-meson couples to ρ-meson through the pion-loop.

Bramon et al. [26] studied the scalar σ-meson effects in vector meson decays in a recent work using chiral loop, Linear sigma Model (LσM), vector meson dominance (VMD) as well as $\rho - \omega$ mixing which was first analysed by Guetta and Singer [14]. They observed that there is a sizeable VMD contribution to the $\rho^0 \to \pi^0\pi^0\gamma$ decay, besides this there also exists a large contribution coming from pion-loop which couple strongly to the low mass σ-meson. For the $\omega \to \pi^0\pi^0\gamma$ decay, the parallel analysis is more involved because $\rho-\omega$ mixing plays a crucial role [14]. Moreover, for this decay the main contribution comes from a less well fixed VMD amplitude and the effects of scalar meson exchange are much more difficult to disentangle. Then, the complementarity between chiral perturbation theory and the Linear sigma Model model was used to study scalar meson exchange in $V \to P^0P^0\gamma$ decays by Escribano [27]. Experimental data on $\rho^0 \to \pi\pi\gamma$ decays seem to prefer a low mass and moderately narrow $\sigma(500)$. For the reference values $M_\sigma = 478$ MeV and $\Gamma_\sigma = 324$ MeV, the branching ratio $BR(\rho^0 \to \pi^0\pi^0\gamma)$ was obtained as $BR(\rho^0 \to \pi^0\pi^0\gamma) = 3.8 \times 10^{-5}$, in agreement with the experimental result. Therefore, taking the different values M_σ and Γ_σ, as required by the LσM, one finds different value of the branching ratio of the $\rho^0 \to \pi^0\pi^0\gamma$ decay. So, the smallness of the former value disfavours a broad σ-meson while the smallness of the chiral loop contribution confirms the need for the effects of a moderately narrow σ-meson.

Recently, there are two valuable studies of the σ-meson effects in the radiative vector meson decays. First, Gökalp, Solmaz and Yılmaz [28] reexamined the approach used in the references [24, 26]. They studied the radiative ρ^0-meson decays in a phenomenological framework in which the contribution of vector meson dominance, chiral-loop and σ-meson intermediate state amplitudes was considered, and assumed that the σ-meson couples to the ρ^0-meson through a pion-loop. They observed that their results for the branching ratios of the ρ^0-meson decay was in good agreement with the experimental values, and the contribution coming from the σ-meson intermediate state amplitude should be included in the analysis of radiative ρ^0-meson decays and moreover σ-meson should be considered to couple to the ρ^0-meson through a pion-loop. Second, Bramon and Escribano [29] suggested a consistent description of $\sigma(500)$-meson effects in $\rho^o \to \pi\pi\gamma$ decays in terms of reasonably simple amplitudes which reproduced the expected chiral-loop behaviour for large M_σ values. In their study, for the $\rho^o \to \pi^0\pi^0\gamma$ decay, in addition to the well known ω-meson exchange, there is an important contribution from the $\sigma(500)$-meson and for the $\rho^o \to \pi^+\pi^-\gamma$ decay, the dominant contribution comes from bremsstrahlung, the effects of the $\sigma(500)$-meson are relevant only at high values of the photon energy. In their conclusion, the $\rho^o \to \pi\pi\gamma$ decays have been shown to be an important source of information on the low-mass $\pi\pi$ spectrum in the s-channel. A global analysis of both processes, with a common amplitude interfering with markedly different but well established backgrounds, should contribute to clarify the σ-meson status. According to their analysis, present data already suggest the existence of such a low-mass state.

Singer [7] also discussed the electromagnetic decays of the ω-meson in the first order of the fine structure constant α with special emphasis on the $2\pi + \gamma$ mode. This decay provides the opportunity for investigating the dynamics of

$\pi - \pi$ interaction in even angular momentum states because of the particular final-state configuration of the pion pair, as a result of the charge-conjugation invariance of the electromagnetic interactions. This way he obtained the relation between the decay rates of the two possible final charge states, that is $\Gamma(\omega \to \pi^+\pi^-\gamma) = 2\Gamma(\omega \to \pi^0\pi^0\gamma)$. Then, Levy and Singer [30] presented a detailed study of the first-order electromagnetic $\omega \to \pi\pi\gamma$ decays. Apart from providing a theoretical understanding of the radiative transitions among vector and pseudoscalar mesons, there is an additional feature making this process an interesting object for experimental and theoretical study. In their study, they used the dispersion-theoretical approach and they showed that final-state interactions resulting in a decay rate of the same order of magnitude as the one calculated from the Born term can be parametrized with the effective pole approximation.

In this thesis, we mainly examine the effects of $\rho - \omega$ mixing in the radiative vector meson decays, $\omega \to \pi\pi\gamma$ and $\rho \to \pi\pi\gamma$ besides the σ-meson intermediate state, vector meson pole state and chiral loop amplitude. We reexamine the $\omega \to \pi^0\pi^0\gamma$ decay in a phenomenological framework in order to the assess the role of σ-meson in the mechanism of the radiative $\omega \to \pi^0\pi^0\gamma$ decay. Then, utilizing the latest experimental value of the branching ratio $BR(\omega \to \pi^0\pi^0\gamma)$, we redetermine the coupling constant $g_{\omega\sigma\gamma}$ which is essential in different studies. Furthermore, we calculate the decay rate for the $\omega \to \pi^0\pi^0\gamma$ decay by considering ρ-pole vector meson dominance amplitude, chiral loop amplitude,

σ-pole amplitude as well as the effects of $\omega - \rho$ mixing which was not taken into account by Gökalp and Yılmaz while investigating the $\omega \to \pi^0\pi^0\gamma$ decay [20]. We also examine the radiative $\rho \to \pi^0\pi^0\gamma$ decay, adding the effects of $\rho - \omega$ mixing to the amplitude calculated with the aid of ω-meson pole intermediate state, chiral loop and σ-meson pole which couples to the ρ-meson through a pion-loop, in the framework of a phenomenological approach. We compare the branching ratio calculated using different contributions with the experimental result for this decay. In order to investigate the role of σ-meson in radiative $\omega \to \pi^+\pi^-\gamma$ decay in the same approach, we consider the vector meson dominance, chiral loop and σ-meson intermediate state amplitudes and the effects of $\rho - \omega$ mixing. Using these contributions in our approach, we calculate the branching ratio of this decay and obtain the photon spectra for the branching ratio of $\omega \to \pi^+\pi^-\gamma$ decay which can be tested experimentally. If only Born term VMD amplitude is used in the calculation one has the important relation $\Gamma(\omega \to \pi^0\pi^0\gamma) = (1/2)\Gamma(\omega \to \pi^+\pi^-\gamma)$ which was noticed by Singer [7]. As mentioned above, this relation follows from charge conjugation invariance to order α which imposes pion pairs of even angular momentum. Since the 1/2 factor in the relation holds to the first order in α, our calculation is also of interest since the amplitude resulting from the assumed decay mechanism for ω-meson and ρ-meson decays in our work contains terms of order e^3. The last decay we consider in our framework is the decay $\rho \to \pi^+\pi^-\gamma$ which is dominated by the pion-bremsstrahlung amplitude. In addition to the σ-meson intermediate state amplitude, we examine the effects of $\rho{-}\omega$ mixing for this decay. However, in this case the $\rho - \omega$ mixing does not play a significant role, because the $\rho \to \pi^+\pi^-\gamma$ decay does not have a contribution coming from the VMD amplitude. In our phenomenological approach, we try to calculate the decay rates of all radiative decays by considering the contributions of different mechanisms represented by diagrams using effective Lagrangians where we employ the coupling constants that are determined from the experimental values of the relevant quantities.

CHAPTER 2

RHO-OMEGA MIXING

The isospin symmetry which is the invariance under arbitrary rotations in the isospin space is the one of the basic symmetries in nuclear physics. It is an internal symmetry that results in the relations between different particle states. The charge symmetry is a special case of isospin symmetry which is rotations through 180^0 about the number 2 axis in isospin space that converts protons into neutrons, and vice versa. These symmetries assume that the proton and neutron are identical and are distinguished only by the direction of their isospin. However, we know that the nucleons are not completely identical proton being charged and neutron being electrically neutral. Therefore, these symmetries must be broken by the electromagnetic effects and by the mass differences of the up and down quarks.

Most theoretical efforts have been devoted to understand the charge symmetry breaking (CSB) phenomena in nuclear physics. These phenomena are well explained in terms of one-boson-exchange potentials, with the electromagnetic effects, the neutron-proton mass difference, and isoscalar-isovector meson mixing included. The violation of the charge symmetry was first observed in neutron-proton scattering at 477 MeV as a difference between the neutron and proton polarization asymmetries at TRIUMF experiment [31]. Other experiment was performed by Knutson et al. [32] measuring of the spin-dependent left-right asymmetries for n-p elastic scattering at 183 MeV. In Quantum Chromodynamics (QCD) CSB is understood in terms of electromagnetic effects and the up and down quark mass difference. Important source of CSB arises through isospin mixing of vector mesons, $\rho-\omega$ mixing, in single meson-exchange models of the strong interaction in two-nucleon system

2.1 $\rho - \omega$ mixing in quark model

The wave functions of ρ and ω meson in the quark model are given schematically as

$$|\rho^0\rangle = \frac{1}{\sqrt{2}}(|u\overline{u}\rangle - |d\overline{d}\rangle) \tag{2.1}$$

$$|\omega\rangle = \frac{1}{\sqrt{2}}(|u\overline{u}\rangle + |d\overline{d}\rangle) \tag{2.2}$$

so that the mixing matrix is obtained in the form

$$\prod_{\rho\omega} = \langle\rho^0|H|\omega\rangle = \frac{1}{2}\langle u\overline{u}|H|u\overline{u}\rangle - \frac{1}{2}\langle d\overline{d}|H|d\overline{d}\rangle \quad . \tag{2.3}$$

Mixing matrix vanishes if the Hamiltonian does not include the effect of mass term that distinguish between the up and down quarks $(m_u - m_d)$. Therefore, the mixing matrix element is strongly affected by the quark mass difference as well as by the electromagnetic effects.

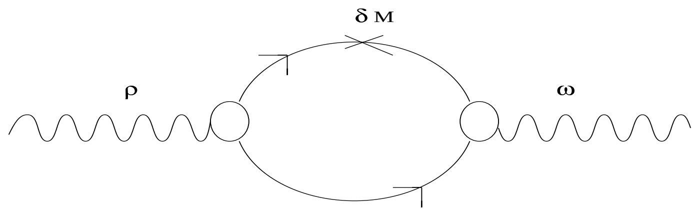

Figure 2.1: Feynman graph for $\rho - \omega$ mixing through a $q\overline{q}$ intermediate state.

In the one-boson-exchange model of the nucleon-nucleon interaction the $\rho{-}\omega$ potential is produced by $\rho - \omega$ mixing in the intermediate vector-meson propagator as shown by Coon and Barrett [33]. They showed that the charge asymmetric potential, which then depends on the electromagnetic transition matrix element between the vector mesons ρ and ω, is about $140^0/_0$ stronger than previous estimates. The input to the calculation was a new measurement of the G-parity forbidden decay $\omega \to \pi\pi$ which has extremely low statistical errors and an inherently clean interpretation. Goldman, Henderson and Thomas (GHT) [34] analyzed the problem associated with the assumed off-shell behavior of the $\rho - \omega$ mixing matrix element. They used a simple model where the vector mesons are considered as quark-antiquark $(q - \overline{q})$ composites. The $\rho - \omega$ mixing amplitude is entirely generated by an intermediate quark loop due to the small mass difference between the up and down quarks, $\delta \equiv m_u - m_d$, and the process that they calculated is illustrated in Fig. 2. 1. A ρ meson of four-momentum q^μ dissociates into a quark-antiquark pair through a vertex function $F(k^2)$, which is the form-factor, describing the meson structure where k_μ is the free momentum of the quark loop. Then, for the quarks free Dirac propagators were used, thus ignoring the question of confinement. GHT calculated the mixing amplitude and obtained the static central potential in coordinate-space by Fourier transform. Their conclusion was that the $\rho - \omega$ mixing amplitudes are strongly momentum dependent,they plotted the potential-radial distance from source, there is a node in the exact potential at about 0.7-0.9 fm, and the potential changes sign due to the node which is the most important feature.

Then Krein et al. [35] investigated the role of quark confinement and the nature of the quark propagator in a $q - \overline{q}$ based description of the $\rho - \omega$ mixing amplitude. Their study was motivated by one of the difficulties associated with the GHT calculation which did not include quark confinement, namely that an unphysical $q\overline{q}$-pair production threshold resulted in the timelike region at $q^2 = 4M_q^2$. One possible mechanism of quark confinement is that the quark propagator does not have a mass pole and one explicit quark model including this property uses the solution of a quark model Dyson-Schwinger (DS) equation. Therefore they described the momentum dependence of the $\rho - \omega$ mixing amplitude using the analytic confining, Dyson-Schwinger equation, and also for comparison they considered recent hadronic calculations [36], where an essentially parameter free calculation was made using an $N\overline{N}$-loop and the $n - p$ mass difference. They showed that the Dyson-Schwinger propagator and the hadronic models predict the opposite sign at smaller r and have nodes between 0.5 and 0.8 fm whereas the heavy free quark and confining quark cases have the same sign as the usual assumed potential. They saw that this increase in mass has removed the node in potential for the free propagator case. As a conclusion they observed that all calculations appear capable of fitting the data with typical parameters and the momentum dependence is qualitatively similar in each case, but for free and confining cases it should be noted that the results are sensitive to the choice of the average up-down quark masses used. Their obtained results varied with choice of form factors but not strongly.

One particularly important idea of hadronic physics is about the interaction between the photon and hadronic matter. This has been remarkably well described using the vector meson dominance (VMD) model. A simple example to introduce VMD is the electromagnetic pion-form factor, $F_\pi(q^2)$, which has played such a crucial role in our understanding of $\rho - \omega$ mixing. This quantity is measured experimentally in the process $e^+e^- \to \pi^+\pi^-$ in which the non-perturbative strong interaction effects produce the significant enhancement seen in the cross-section. In the $\rho - \omega$ resonance region the cross-section displays a narrow interference shoulder resulting from the superposition of narrow resonant ω and broad resonant ρ exchange amplitudes.

2.2 $\rho - \omega$ mixing in Vector Meson Dominance model

VMD supposes that the vector mesons play the dominant role in the interaction of the photon with hadronic matter. O'Connell et al. [37, 38] described how the interactions of the photon with hadronic systems can be modelled usefully using vector mesons. The traditional representation of VMD which is called VMD2 supposes that the photon couples to hadronic matter exclusively through a vector meson. For the photon-rho-pion system, the relevant part of the VMD2 Lagrangian is

$$\mathcal{L}_{VMD2} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}M_\rho^2(\rho_\mu)^2 - g_{\rho\pi\pi}\rho_\mu J_\pi^\mu$$
$$-\frac{eM_\rho^2}{g_\rho}\rho_\mu A^\mu + \frac{1}{2}\left(\frac{e}{g_\rho}\right)^2 M_\rho^2 A_\mu A^\mu, \qquad (2.4)$$

where J_π^μ is the hadronic current and $F_{\mu\nu}$ and $G_{\mu\nu}$ are the electromagnetic and ρ field strength tensors, respectively. The pion form-factor is obtained from this equation as

$$F_\pi(q^2) = -\frac{M_\rho^2}{q^2 - M_\rho^2 + iM_\rho\Gamma_\rho(q^2)}\frac{g_{\rho\pi\pi}}{g_\rho} \quad . \qquad (2.5)$$

The VMD2 Lagrangian has a fixed photon-matter coupling as well as a photon-rho coupling, but if this coupling were also generated by the kind of momentum dependent loop processes used in for $\rho-\omega$ mixing, also the photon-rho coupling would be strongly momentum dependent. For this reason, an equivalent alternative formulation of VMD, developed by Sakurai in the 1960's [39], which is called VMD1 is preferred. As discussed by O'Connell et al. [37], the alternative formulation, VMD1, is given by the following Lagrangian

$$\mathcal{L}_{VMD1} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}M_\rho^2\rho_\mu\rho^\mu - g_{\rho\pi\pi}\rho_\mu J_\pi^\mu - \frac{e}{2g_\rho}F_{\mu\nu}\rho^{\mu\nu}. \quad (2.6)$$

From this Lagrangian we can derive a pion-form factor of the form

$$F_\pi(q^2) = 1 - \frac{q^2}{[q^2 - M_\rho^2 + iM_\rho\Gamma_\rho(q^2)]}\frac{g_{\rho\pi\pi}}{g_\rho} \quad . \tag{2.7}$$

We have the constraint $F_\pi(0) = 1$ at zero momentum transfer and it reflects the fact that the photon sees only the charge of the pions. In the limit of universality, $g_\rho = g_{\rho\pi\pi}$, which is seen to be only approximate in nature [40], the two representations of VMD become equivalent and without universality only VMD1 continues to satisfy the constraint condition.

In many ways VMD1 version differs from VMD2, for instance, in VMD1 photon-hadron interactions take place exclusively through a vector meson and VMD1 does not have a photon mass term and it has a term which produces a momentum-dependent photon-rho coupling of the form

$$\mathcal{L}_{\gamma\rho} = -\frac{e}{2g_\rho}\ F_{\mu\nu}G^{\mu\nu} \rightarrow \ -\frac{e}{g_\rho}\ q^2 A_\mu \rho^\mu. \tag{2.8}$$

We reach the result that this decouples the photon from the ρ at $q^2 = 0$, for this reason keeping the photon massless in a natural way. In Fig. 2. 2 we display the difference the two representations of VMD. O'Connell et al. [38] in two recent papers also discussed the two established representations of vector meson dominance (VMD) model for photons coupling to matter, with vanishing of vector meson-meson and meson-photon mixing self energies at $q^2 = 0$ and showed that one of these representations is completely consistent with such a coupling.

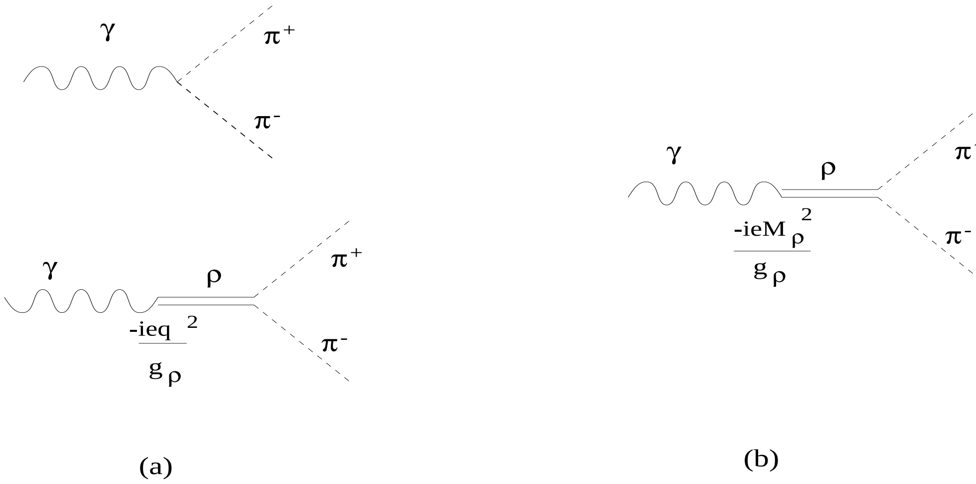

Figure 2.2: Contributions to the pion form factor in the two representations of VMD a) VMD1 b) VMD2.

2.3 $\rho - \omega$ mixing in $e^+e^- \to \pi^+\pi^-$

The observation of the interference of the ω-meson in the reaction $e^+e^- \to \pi^+\pi^-$ and the improved the resolution of the cross-section plot, revealed that there was a G symmetry violating interactions of the ω-meson, such as $\omega \to \pi^+\pi^-$, which could not be explained by electromagnetism alone [37]. This situation is shown by corresponding diagram in Fig. 2. 3.

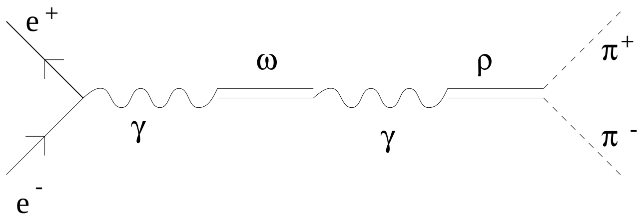

Figure 2.3: Electromagnetic contribution to the ω-resonance of $e^+e^- \to \pi^+\pi^-$.

Examination of the decay $\omega \to \pi\pi$ concluded that there was not significant statistical evidence for the direct decay. It was suggested that despite a possibly substantial direct decay rate, some process produced a cancellation giving a zero result. The strong symmetry breaking theory allowed for a mixing of the two mesons, introducing the quantity, transition matrix element between the vector mesons ρ and ω. In Fig. 2. 4. the relevant diagram for the reaction $e^+e^- \to \pi^+\pi^-$ is displayed.

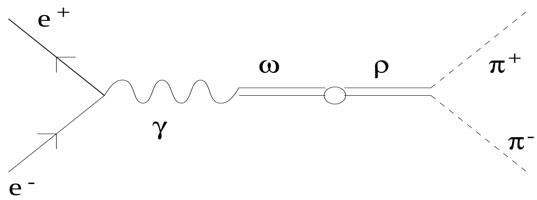

Figure 2.4: $\rho - \omega$ mixing contribution to $e^+e^- \to \pi^+\pi^-$.

2.4 $\rho - \omega$ mixing Amplitude

Writing the ρ and ω meson propagators in matrix form and generating the mixing by dressing the bare isospin pure matrix elements this interference of the ω meson can be included into the VMD picture formalism. Thus, in the pure isospin limit we have the bare matrix, $D^0_{\mu\nu} = -g_{\mu\nu}D^0$, in order that vector mesons coupled to conserved currents where D^0 is the scalar propagator whose matrix form is

$$D^0 = \begin{pmatrix} D^I_{\rho\rho} & 0 \\ 0 & D^I_{\omega\omega} \end{pmatrix}. \tag{2.9}$$

Here $D^I_{\rho\rho}$ and $D^I_{\omega\omega}$ are the scalar parts of the renormalised propagators for the isospin pure fields, and there is no $\rho - \omega$ mixing and no direct $\omega_I \to \pi\pi$ coupling. Then we take the full expression for the dressed propagator given by $D_{\mu\nu} = D^0_{\mu\nu} + D^0_{\mu\alpha} \prod^{\alpha\beta} D_{\beta\nu}$ where the polarization function $\prod_{\mu\nu}(q^2) = (g_{\mu\nu} - q_\mu q_{nu}/q^2) \prod_{\rho\omega}$ has off-diagonal elements of the isospin violating mixing self-energy $\prod_{\rho\omega}$ which generates the mixing between the isospin pure ρ_I and ω_I states and we keep terms to first order in isospin breaking [41] to obtain

$$D^0 = \begin{pmatrix} D^I_{\rho\rho} & 0 \\ 0 & D^I_{\omega\omega} \end{pmatrix} \to D^I = \begin{pmatrix} D^I_{\rho\rho} & D^I_{\rho\omega}(q^2) \\ D^I_{\rho\omega}(q^2) & D^I_{\omega\omega} \end{pmatrix} \tag{2.10}$$

$$= \begin{pmatrix} D^I_{\rho\rho} & D^I_{\rho\rho} \prod_{\rho\omega}(q^2) D^I_{\omega\omega} \\ D^I_{\rho\rho} \prod_{\rho\omega}(q^2) D^I_{\omega\omega} & D^I_{\omega\omega} \end{pmatrix}. \tag{2.11}$$

The dressed propagator $D^I_{\rho\omega}(q^2)$ contains both a broad ρ resonance and narrow ω resonance piece.

In the ρ, ω basis, the combinations of the ρ_I and ω_I, for which only the diagonal elements of the propagator matrix contain poles, define the physical ρ and ω fields as mentioned by Maltman et al. [42] and O'Connell et al. [37]. This associates the broad resonant part of the full amplitude with the ρ and narrow resonant part with the ω. One would find also narrow resonant structure in the off-diagonal element of the vector meson propagator using different linear combinations of ρ_I and ω_I in the ρ' and ω' basis. The propagator could be diagonalised by transforming to the physical basis

$$\rho = \rho_I - \varepsilon\omega_I \ , \qquad \omega = \omega_I + \varepsilon\rho_I \ . \tag{2.12}$$

The transformation matrix, C, is defined as follows [37]

$$\begin{pmatrix} \rho \\ \\ \omega \end{pmatrix} = C \begin{pmatrix} \rho_I \\ \\ \omega_I \end{pmatrix} = \begin{pmatrix} 1 & -\varepsilon \\ \\ \varepsilon & 1 \end{pmatrix} \begin{pmatrix} \rho_I \\ \\ \omega_I \end{pmatrix} \tag{2.13}$$

where ε is given by

$$\varepsilon = \frac{\prod_{\rho\omega}}{M_\omega^2 - M_\rho^2 - i(M_\omega\Gamma_\omega - M_\rho\Gamma_\rho)} \ . \tag{2.14}$$

The standard assumption is that the mixing amplitude, $\prod_{\rho\omega}$, is momentum independent with its value extracted from the experimental data on $e^+e^- \rightarrow \pi^+\pi^-$ at the ω pole, $q^2 \simeq M_\omega^2$, while the exchanged mesons have spacelike four momentum, $q^2 < 0$, and are therefore highly virtual. But, Goldman, Henderson and Thomas (GHT) [34] raise the possibility of significant q^2-dependence of the $\rho - \omega$ mixing matrix element, constructing a simple model in which $\prod_{\rho\omega}$ is generated by an intermediate quark loop as a consequence of the difference between up and down quark masses.

Then, the significant momentum dependence has been obtained using many theoretical approaches by other various authors. Two of them are Piekarewicz and Williams [36], they calculated the momentum dependence of the $\rho - \omega$ mixing amplitude in a purely hadronic model, the basic assumption of which is that the mixing amplitude is generated by $N\overline{N}$ loops and thus driven by the neutron-proton mass difference. Using standard values for parameters obtained from fits to two-nucleon data they determined a value for the $\rho - \omega$ mixing amplitude at the on-shell ω-meson point in good agreement with experiment. Then extending their results to the spacelike region, they computed the contribution from the off-shell $\rho - \omega$ mixing amplitude to the N-N potential. Their results were compared to a recent calculation of GHT of the mixing amplitude in terms of $q\overline{q}$ loops. In spite of the obvious differences between the two models, their findings agree with the main conclusion drawn from that GHT's work that the momentum dependence of the $\rho - \omega$ mixing amplitude is significant.

From the experimental point of view, $\rho - \omega$ interference has always been observed through the G-symmetry violation interaction $\omega \to \pi^+\pi^-$ of ω mesons produced in different reactions. Coon and Barrett [33] determined the mixing amplitude using the amplitude for G-parity forbidden decay $\omega \to 2\pi$ as $\prod_{\rho\omega} = (-4.52 \pm 0.60) \times 10^{-3}\ GeV^2$. The sign of the mixing amplitude is determined from the relative phase of the ω and ρ amplitudes in $e^+e^- \to \pi^+\pi^-$ near M_ρ and M_ω. They used the values $M_\rho = 775.9 \pm 1.1$ MeV and $\Gamma_\rho = 150.5 \pm 3.0$ MeV in their calculations. Then Bernicha et al. [43] determined the ρ mass and width by applying the S matrix formalism to the reaction $e^+e^- \to \pi^+\pi^-$ in the timelike region. Their obtained values are significantly smaller than the values quoted by the Particle Data Group [4]. To calculate the strength of the $\rho - \omega$ mixing, they used their obtained values, determined a small dimensionless parameter

(y) which quantifies the isospin-breaking contribution and also used the ratio $g_{\omega\pi\gamma}/g_{\rho\pi\gamma}$ obtained from the leptonic partial rate. Then, using $M_\rho = (757.5 \pm 1.5)$ MeV and $y = (-1.91 \pm 0.15) \times 10^{-3}$, the value $\prod_{\rho\omega} = (-3.735 \pm 0.300) \times 10^{-3}$ GeV^2 is obtained, whereas from the values of $M_\rho = (757.0 \pm 0.59)$ MeV and $y = (-2.16 \pm 0.35) \times 10^{-3}$ they obtaine $\prod_{\rho\omega} = (-4.225 \pm 0.684) \times 10^{-3}$ GeV^2 which agrees well the values obtained by Coon and Barrett [33] despite the fact that for the ρ mass and width quite different values are used. Also from the values $M_\rho = (757.03 \pm 0.76)$ MeV and $y = (-1.87 \pm 0.15) \times 10^{-3}$, the $\rho - \omega$ mixing strenght is derived as $\prod_{\rho\omega} = (-3.669 \pm 0.30) \times 10^{-3}$ GeV^2. As for Urech [44], he derived the $\rho - \omega$ mixing amplitude using the fourier transform of the two-point function and compared his numerical result for the on-shell expression with the calculations found in the literature. For the calculation of the on-shell amplitude, he considered the $\omega \rightarrow \rho \rightarrow \pi^+\pi^-$ decay and found following expression

$$\begin{aligned} \Gamma(\omega \rightarrow \pi^+\pi^-) &= \frac{\prod_{\rho\omega}^2 \Gamma(\rho \rightarrow \pi^+\pi^-)}{|M_\omega^2 - M_\rho^2 - i(M_\omega\Gamma_\omega - M_\rho\Gamma_\rho)|^2} \\ &\simeq \frac{\prod_{\rho\omega}^2}{4M_\rho^2} \frac{\Gamma(\rho \rightarrow \pi^+\pi^-)}{(M_\omega - M_\rho)^2 + \frac{1}{4}(\Gamma_\omega - \Gamma_\rho)^2} \ . \end{aligned} \quad (2.15)$$

Thus, Urech determined the amplitude $\prod_{\rho\omega}$ as

$$\prod_{\rho\omega} = 2M_\rho(m_u - m_d) + \frac{1}{3}\, e^2 F_V^2 \quad (2.16)$$

and found the value $\prod_{\rho\omega} = (-3.91 \pm 0.30) \times 10^{-3}$ GeV^2. If the mass difference $M_\omega - M_\rho$ and the width Γ_ω is neglected in Eq. 2.15, the mixing amplitude is $\prod_{\rho\omega} = -4.08 \times 10^{-3}$ GeV^2. The first of the results is in good agreement with the values quoted in the literature. He also discussed the off-shell mixing, for this he again considered the two-point function. In conclusion, he found that $\prod_{\omega\rho}(q^2)$ contains a zero at $q^2 = 0$ and is positive in the spacelike region. Gardner and O'Connell et al. [45] extracted the G-parity violating branching ratio $BR(\omega \rightarrow \pi^+\pi^-)$ from the effective $\rho - \omega$ mixing matrix element $\prod_{\rho\omega}$ that is determined from the $e^+e^- \rightarrow \pi^+\pi^-$ data. They obtained three different equations for branching ratios of $\omega \rightarrow \pi^+\pi^-$ where they also defined an effective, isospin-violating coupling constant $g^{eff.}_{\omega\pi\pi}$ which was used to calculate branching ratio. This way, they found a relation that is equivalent to Eq. 2. 15 determined by Urech [44]. In this study they also calculated $\prod_{\rho\omega}$ mixing matrix element which results from using following equation with $g_{\rho\pi\gamma}/g_{\omega\pi\gamma}$ ratio

$$\prod_{\rho\omega}(M_\omega^2) = \frac{1}{3}\frac{g_{\rho\pi\gamma}}{g_{\omega\pi\gamma}}(-3500\ MeV^2) \quad . \tag{2.17}$$

The average value of mixing matrix element was determined as $\prod_{\rho\omega}(M_\omega^2) = -3900 \pm 300\ MeV^2$. Through these studies it is thus possible to understand $\omega \rightarrow \pi^+\pi^-$ decay and $\rho - \omega$ mixing and also the connection between $\prod_{\rho\omega}(M_\omega^2)$ and the branching ratio of $\omega \rightarrow \pi^+\pi^-$ decay.

O'Connell et al. [37] reviewed their study of constraints on the momentum dependence of $\rho - \omega$ mixing and concluded that the $\rho - \omega$ mixing amplitude should also vanish at $q^2 = 0$ in a large class of models. A re-analysis of the pion form-factor using this formulation gave an excellent fit to the data, while careful re-analysis near the ω-pole gave a value $\prod_{\rho\omega}(M_\rho^2) = -3800 \pm 370\ MeV^2$

[38]. There are two principle sources of error in the value of $\prod_{\rho\omega}$, the first one is a statistical uncertainty of 310 MeV^2 resulting from the fit to data, and the second one (200 MeV^2) is due to the error quoted in the ratio $g_{\omega\pi\gamma}/g_{\rho\pi\gamma}$. These errors are added in quadrature. This value differs from other modern fits mainly because of using the most recent value of $g_{\omega\pi\gamma}/g_{\rho\pi\gamma}$ which is obtained by Bernicha et al. [43] utilizing the leptonic partial rate of neutral vector meson [46] $\Gamma(\rho \to e^+e^-) = (6.77 \pm 0.32)$ keV and $\Gamma(\omega \to e^+e^-) = (0.60 \pm 0.02)$ keV as

$$\frac{g_{\omega\pi\gamma}}{g_{\rho\pi\gamma}} = \sqrt{\frac{M_\omega}{M_\rho}\frac{\Gamma(\rho \to e^+e^-)}{\Gamma(\omega \to e^+e^-)}} \;=\; 3.5 \pm 0.18 \; . \qquad (2.18)$$

The ratio between $g_{\omega\pi\gamma}$ and $g_{\rho\pi\gamma}$ has long been considered to be approximately 1/3 and this value is obtained in a recent QCD- based investigation by Dillon and Morpurgo [47]. In their study, to analize the $V - \gamma$ couplings $g_{V\gamma}$ in the decays $V \to e^+e^-$, they applied the general parametrization method that is obtained from the general properties of QCD. They derived the quasi no flavor breaking theorem using the method of general parametrization. Then employing this theorem, they showed that the ratio of $\rho\gamma$ and $\omega\gamma$ coupling is almost unaffected by flavor breaking, and therefore, equal to 3. Also, they found two values for the ratio of these couplings, using different value of $\Gamma(\omega \to e^+e^-)$ and the formula in Eq. 2. 18. Their results were $|g_{\rho\pi\gamma}/g_{\omega\pi\gamma}| = 3.36 \pm 0.07$ and $|g_{\rho\pi\gamma}/g_{\omega\pi\gamma}| = 3.18 \pm 0.12$.

The underlying theory of strong interactions, QCD, is inaccessible for studies in the low energy region, therefore it was difficult to make some model-independent statement about $\rho-\omega$ mixing. However, some model-independent treatments of low energy strong interactions have later been developed and applied to examine the $\rho-\omega$ mixing. The technique of QCD sum rules (QCDSR) is one such technique which examines two-point functions of different hadronic currents, expanding them in powers of $1/q^2$. Using this technique and dispersion relations, Hatsuda et al. [48] analysed the q^2 dependence of $\rho-\omega$ mixing amplitude. They set up the problem by considering the two-point function

$$\prod_{\rho\omega}^{\mu\nu}(q^2) = i\int d^4x e^{iq\cdot x} < 0 \mid T(J_\rho^\mu(x) J_\omega^\nu(0)) \mid 0 > \quad , \tag{2.19}$$

where the vector currents are

$$J_\rho^\mu = (\overline{u}\gamma_\mu u - \overline{d}\gamma_\mu d)/2, \quad J_\omega^\mu = (\overline{u}\gamma_\mu u + \overline{d}\gamma_\mu d)/6 \ . \tag{2.20}$$

$\prod_{\rho\omega}^{\mu\nu}$ has to be of transversal structure, because the currents J_ρ^μ and J_ω^ν are conserved,

$$\prod_{\rho\omega}^{\mu\nu}(q^2) = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\prod_{\rho\omega}(q^2) \ . \tag{2.21}$$

They equated this current correlator (Eq. 2. 21) with the mixed propagator

$$\prod_{\rho\omega}^{\mu\nu}(q^2) = -(g_{\mu\nu} - q_\mu q_\nu/q^2)\frac{\prod_{\rho\omega}(q^2)}{(q^2 - M_\rho^2)(q^2 - M_\omega^2)} \tag{2.22}$$

and found a rapid variation of the mixing matrix element $\prod(q^2)$ with q^2. Going off shell the mixing decreases, changes sign for positive q^2 and is always negative in the spacelike region ($q^2 < 0$). Although the variation of $\prod(q^2)$ obtained in this work was qualitatively similar to previous results based on different models, their value of $\prod(q^2)$ has a stronger q^2-dependence than those of others. In the quark-loop model ($\rho \rightarrow q\overline{q} \rightarrow \omega$) with momentum cut-off [34], $\prod(q^2)$ changes sign at a spacelike momentum ($q^2 \sim -M_\rho^2/2$), therefore the variation of $\prod(q^2)$ is more moderate. The nucleon-loop model ($\rho \rightarrow N\overline{N} \rightarrow \omega$) with dimensional regularization [36] predicts a sign change of $\prod(q^2)$ at $q^2 = 0$, which still corresponds to a more moderate variation than result of Hatsuda et al. [48] who noticed that there is a crucial assumption with no theoretical justification in both quark-loop and nucleon-loop models that is the effect of the isospin breaking other than the QED effect is solely attributed to the mass difference between u and d constituent quarks in the quark-loop model or to the mass difference between the proton and neutron in the nucleon-loop model. There is no priori reason, however, to neglect the isospin breaking in the coupling constants of the vector mesons with the constituent quarks or the nucleons, which generates an extra effect to the $\rho - \omega$ mixing of order $O(m_d - m_u)$. They also examined the central part of the N-N potential contributed by the $\rho - \omega$ mixing in order to observe how the nuclear force is affected by the q^2 variation of $\prod(q^2)$. The long-range exponential part of the potential due to the $\rho - \omega$ mixing is strongly suppressed by the q^2 dependence of the mixing. As a result, the potential changes sign at r=0.9 fm which is the region of interest for the symmetry breaking effect, as mentioned before.

The other model-independent method for considering the strong interaction at low energies is the Chiral Perturbation Theory (ChPT). It sets up an effective model involving all the interactions of the pseudoscalar meson octet and admitting all terms allowed by the symmetry of the original QCD Lagrangian. Maltman [49] described two model-independent results on the momentum-dependence of $\rho - \omega$ mixing. He displayed an explicit choice of interpolating fields for the vector mesons for which both the mixing in the propagator and the isospin-breaking at the nucleon-vector meson vertices vanish identically at $q^2 = 0$. He also showed, using the constraints of unitarity and analyticity on the spectral function of the vector meson propagator, that there is no possible choice of interpolating fields for the ρ, ω mesons such that mixing matrix element is independent of momentum. The standard approach of charge symmetry breaking in few-body systems is physically realizable. In consequence, since the standard treatment can not be interpreted as arising from any effective meson-baryon Lagrangian it must be interpreted as being purely phenomenological in nature.

Also that the $\rho - \omega$ mixing amplitude has a zero at $q^2 = 0$ has been shown by O'Connell et al. [50] within a broad class of models in which the mixing is either zero everywhere or is necessarily momentum dependent. They argued that the mixing amplitude vanishes at $q^2 = 0$ in any effective Lagrangian model, where there are no explicit mass mixing terms in the bare Lagrangian and where the vector mesons have a local coupling to conserved currents which satisfy the usual vector current commutation relations as QCD. They concluded that the $\rho-\omega$ mixing might play a minor role in the theoretical understanding of charge-symmetry breaking in nuclear systems.

Moreover, Iqbal et al. [51] studied mesonic width effects on the momentum dependence of the $\rho - \omega$ mixing matrix element. Theoretical calculations of the off-shell variations of the $\rho - \omega$ mixing matrix element have used various models that include mixing through $q\overline{q}$ loops [34, 35], $N\overline{N}$ loops [36], QCD sum rule calculation [50]. In all these calculations, the ρ and ω mesons are treated as stable particles and their widths are neglected. Iqbal et al. [51] showed in a model independent way that the large difference in ρ and ω widths, $\Gamma_\rho = 151.5 MeV, \Gamma_\omega = 8.4 MeV$, gives rise to a new source of momentum dependence for the $\rho - \omega$ mixing matrix element. The q^2 dependence arising due to the meson widths leads to a significant alteration of the result obtained in the zero-width approximation typically discussed in the literature [34, 35, 36, 50]. They concluded in a model independent way that the inclusion of ρ and ω widths significantly alters the q^2 dependence of the $\rho - \omega$ mixing matrix element and hence of the mixed meson propagator. This behavior arises from the fact that the widths of ρ and ω are different. Any model calculation addressing the q^2-dependence of the $\rho - \omega$ mixing matrix element that does not include meson width effects is incomplete.

Recently, Ya. I. Azimov [52] reconsidered the isospin violating mixing of ρ- and ω-mesons in terms of propagators. He studied various pairs of ρ, ω-decays to the same final states. An interesting situation appears in the $\omega \rightarrow \pi^+\pi^-$ decay due to transition of ω into ρ having the near mass and large width $\rho \rightarrow \pi^+\pi^-$ decay. Nevertheless, experiments of this decay can extract only one parameter and,therefore are not sufficient to explain the isospin violation mechanisms. From this point of view, the decay such as $(\rho, \omega) \rightarrow \eta\gamma$ and $(\rho, \omega) \rightarrow e^+e^-$ has attracted much attention in recent years. Taking the unperturbed propagator for vector meson V with bare mass M_V Azimov described the propagators for mixing of vector particles in different form. Then he rewrote the amplitude through contributions of the physical states with physical propagators and physical amplitudes for the meson production and for meson decays considering a process $i \rightarrow f$ with intermediate ρ- and ω-mesons. In calculations, he made all numerical estimations taking all necessary parameters constant, which are ρ, ω complex masses, that is masses and widths and mixing parameters, and used the leading role of the $\rho - \omega$ mixing for isospin violation. In summary we can say that, the isospin violation due to $\rho - \omega$ mixing that was known for some time in the forbidden decay $\omega \rightarrow \pi^+\pi^-$ is later suggested for the radiative decay $\rho \rightarrow \pi^+\pi^-$. The mixing also affect all pairs of decays of ρ, ω to the same final state and decays of heavier particles with the production of ρ, ω. The $\rho - \omega$ mixing affects various pairs of ρ, ω-decays in a regular, correlated manner. At higher experimental sensitivity, the universal nature of the mixing parameter will allow to separate mixing isospin violation due to ρ, ω-transitions from direct isospin violation in amplitudes of bare unmixed states ρ, ω. Even present data from experiments give some proof for necessity of such direct violating effects. As noted by Azimov [52] in the near future it can be expected that the meson radiative decays with participation of ρ and/or ω may certainly be attractive and useful for studying the $\rho - \omega$ mixing and other manifestations of isospin violation.

CHAPTER 3

FORMALISM

We extend previous studies of $\omega \rightarrow \pi^0\pi^0\gamma$, $\omega \rightarrow \pi^+\pi^-\gamma$ [21], $\rho^0 \rightarrow \pi^0\pi^0\gamma$, $\rho^0 \rightarrow \pi^+\pi^-\gamma$ [28, 29] decays by considering the effect of $\omega-\rho$ mixing in addition to the vector meson exchange, σ-meson intermediate states and chiral loop amplitudes. We follow a phenomenological approach and attempt to calculate the decay rate and explain the latest experimental result about the branching ratio for these decays.

Although the contribution of chiral kaon-loop diagram shown in Fig. 3. 1b to decay rate of $\omega \rightarrow \pi^0\pi^0\gamma$ decay is small, we also include the corresponding amplitude of these diagrams in our calculation for completeness. For this radiative decay, pion-loop contributions vanish in the good isospin limit. However, since we lack any experimental information to describe the ωK^+K^--vertex and $K^+K^- - \pi^0\pi^0$ amplitudes for the contribution of this diagram we use the amplitude given by Bramon et al. [12] derived using chiral perturbation theory. This may not be entirely consistent with the philosophy of our phenomenological approach, but since their contributions are shown to be small we do not think that this way of including kaon-loop diagram into our calculation constitutes a serious inconsistency. Moreover, in Fig. 3. 2b and Fig. 3. 7b in addition to pion-loop intermediate state there is also a contribution to $\rho^0 \to \pi^0\pi^0\gamma$ and $\rho^0 \to \pi^+\pi^-\gamma$ decays coming from $K\overline{K}$ intermediate state. However, as shown by Bramon et al.[12] for these decays the kaon-loop intermediate states give a contribution which is 10^3 times smaller than the contribution coming from the charged-pion loops. Therefore, in our calculation we do not take the kaon-loop amplitude in $\rho^0 \to \pi^0\pi^0\gamma$ and $\rho^0 \to \pi^+\pi^-\gamma$ decays into account.

3.1 Radiative $\omega \to \pi^0\pi^0\gamma$ and $\rho^0 \to \pi^0\pi^0\gamma$ decays

The contributions of VMD, chiral loops, σ-meson intermediate state amplitudes and the effect of $\rho - \omega$ mixing are considered for the radiative $\omega \to \pi^0\pi^0\gamma$ decay. In the case of the radiative $\rho^0 \to \pi^0\pi^0\gamma$ decay we calculate the decay rate assuming that this decay also proceeds through the same mechanism as well, that is its amplitude is provided by VMD, chiral loops, σ-meson intermediate state amplitudes and $\rho - \omega$ mixing. In order to calculate the effect of the $\rho - \omega$ mixing in the $\omega \to \pi^0\pi^0\gamma$ decay, we need an amplitude characterizing the contribution coming from the different amplitudes to the $\rho^0 \to \pi^0\pi^0\gamma$ decay. Likewise, for evaluating the effects of the $\rho - \omega$ mixing in the $\rho^0 \to \pi^0\pi^0\gamma$ decay, we use the amplitude resulting from the different amplitudes to the $\omega \to \pi^0\pi^0\gamma$ decay.

Our phenomenological approach is based on the Feynman diagrams shown in Fig. 3. 1 for $\omega \to \pi^0\pi^0\gamma$ decay and in Fig. 3. 2 for $\rho^0 \to \pi^0\pi^0\gamma$ decay. To establish the gauge invariance, the direct terms shown in the diagrams in Fig. 3. 1b and in Fig. 3. 2b,c are required. The interaction term for two vector mesons and one pseudoscalar meson is given by the Wess-Zumino anomaly term of the chiral Lagrangian [53], therefore we describe the $\omega\rho\pi$- vertex by the effective Lagrangian [54]

$$\mathcal{L}^{eff.}_{\omega\rho\pi} = \frac{g_{\omega\rho\pi}}{M_\omega}\epsilon^{\mu\nu\alpha\beta}\partial_\mu\omega_\nu\partial_\alpha\vec{\rho}_\beta\cdot\vec{\pi} \quad , \tag{3.1}$$

which also defines the coupling constant $g_{\omega\rho\pi}$. This coupling constant was determined by Achasov et al. [55] through an experimental analysis as $g_{\omega\rho\pi} = (14.4) \pm 0.2)GeV^{-1}$ assuming that $\omega \to 3\pi$ decay proceeds with the intermediate $\rho\pi$ state as $\omega \to (\rho)\pi \to \pi\pi\pi$ and they used the experimental value of the $\omega \to 3\pi$ width. Similarly, the $V\pi\gamma$-vertices where $V = \rho, \omega$ are described by the effective Lagrangian [56]

$$\mathcal{L}^{eff.}_{V\pi\gamma} = g_{V\pi\gamma}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu V_\nu\partial_\alpha A_\beta\pi \quad . \tag{3.2}$$

To deduce the coupling constants $g_{\omega\pi\gamma}$ and $g_{\rho\pi\gamma}$ we use the experimental partial widths of the radiative $V \to \pi\gamma$ decays which are given as

$$\Gamma\left[V \to \pi\gamma\right] = \frac{\alpha}{24}\frac{(M_v^2 - M_\pi^2)^3}{M_v^5}g^2_{V\pi\gamma} \quad . \tag{3.3}$$

This way for the coupling constants $g_{\omega\pi\gamma}$ and $g_{\rho\pi\gamma}$ we obtain the values $g_{\omega\pi\gamma} = (0.706 \pm 0.021)GeV^{-1}$ and $g_{\rho\pi\gamma} = (0.274 \pm 0.035)GeV^{-1}$. The $\sigma\pi\pi$-vertex is described by the effective Lagrangian [57]

$$\mathcal{L}^{eff.}_{\sigma\pi\pi} = \frac{1}{2}g_{\sigma\pi\pi}M_\sigma\overrightarrow{\pi}\cdot\overrightarrow{\pi}\sigma \quad . \tag{3.4}$$

The decay width of the σ-meson that follows from this effective Lagrangian is given as

$$\Gamma[\sigma \to \pi\pi] = \frac{g^2_{\sigma\pi\pi}}{4\pi}\frac{3M_\sigma}{8}\left[1 - \left(\frac{2M_\pi}{M_\sigma}\right)^2\right]^{1/2} \quad . \tag{3.5}$$

Using this expression and experimentally measured values of the mass M_σ and the width Γ_σ as $M_\sigma = (483\pm31)$MeV and $\Gamma_\sigma = (338\pm48)$MeV, where statistical and systematic errors are added in quadrature [18, 19], we obtain the strong coupling constant as $g_{\sigma\pi\pi} = 5.3 \pm 0.55$. We describe the $\omega\sigma\gamma$-vertex by the effective Lagrangian [58]

$$\mathcal{L}^{eff.}_{\omega\sigma\gamma} = \frac{e}{M_\omega} g_{\omega\sigma\gamma}\partial^\alpha\omega^\beta[\partial_\alpha A_\beta - \partial_\beta A_\alpha]\sigma \quad , \tag{3.6}$$

which also defines the coupling constant $g_{\omega\sigma\gamma}$ that will be determined by our analysis. For the $\rho\pi\pi$-vertex the effective Lagrangian [59]

$$\mathcal{L}^{eff.}_{\rho\pi\pi} = g_{\rho\pi\pi}\overrightarrow{\rho}_\mu \cdot (\partial^\mu\overrightarrow{\pi} \times \overrightarrow{\pi}) \quad , \tag{3.7}$$

is used. The decay width of ρ-meson that follows from this effective Lagrangian is

$$\Gamma[\rho \to \pi\pi] = \frac{g_{\rho\pi\pi}2}{4\pi}\frac{M_\rho}{12}\left[1 - \left(\frac{2M_\pi}{M_\rho}\right)^2\right]^{\frac{3}{2}} \quad . \tag{3.8}$$

For the coupling constant $g_{\rho\pi\pi}$ using the experimental decay width of the decay $\rho \to \pi\pi$ [6] we obtain the value $g_{\rho\pi\pi} = (6.03 \pm 0.02)$. The effective Lagrangians $\mathcal{L}^{eff.}_{\sigma\pi\pi}$ and $\mathcal{L}^{eff.}_{\rho\pi\pi}$ are obtained from an extension of the σ model where the isovector ρ is included through a Yang-Mills local gauge theory based on isospin with the vector meson mass generated through the Higgs mechanism [60]. In order to describe the π^4-vertex again we consider the σ-model with spontaneous symmetry breaking [61] and we describe the π^4-vertex by the effective Lagrangian

$$\mathcal{L}^{eff.} = \frac{\lambda}{4}(\vec{\pi} \cdot \vec{\pi})^2 \quad , \tag{3.9}$$

where the coupling constant λ is given as $\lambda = -\frac{g^2_{\pi NN}}{2}\frac{M^2_\sigma - M^2_\pi}{M^2_N}$ and the value $\frac{g^2_{\pi NN}}{4\pi} = 14$ is used. We note that this effective interaction results in only isospin I=0 amplitudes. The small I=2 amplitudes were also neglected in previous calculations within the framework of chiral unitary theory [16].

In our calculation of the invariant amplitude, in ρ-meson and σ-meson propagators we make the replacement $q^2 - M^2 \rightarrow q^2 - M^2 + iM\Gamma$ in order to take into account the finite widths of these unstable particles. We use the energy dependent width for σ-meson that follows from Eq. 3. 4

$$\Gamma_\sigma(q^2) = \Gamma_\sigma \frac{M_\sigma}{q^2}\left(\frac{q^2 - 4M^2_\pi}{M^2_\sigma - 4M^2_\pi}\right)^{\frac{1}{2}} \theta(q^2 - 4M^2_\pi) \quad , \tag{3.10}$$

and for ρ-meson we use the following momentum dependent width as conventionally adopted [37]

$$\Gamma_\rho(q^2) = \Gamma_\rho \frac{M_\rho}{\sqrt{q^2}}\left(\frac{q^2 - 4M^2_\pi}{M^2_\rho - 4M^2_\pi}\right)^{\frac{3}{2}} \theta(q^2 - 4M^2_\pi) \quad . \tag{3.11}$$

In order to evaluate the loop diagrams in Fig. 3. 1 and Fig. 3. 2 we note that similar loop integrals were evaluated by Lucio and Pestiau [62] using dimensional regularization and their calculations were confirmed by Close et al. [63] We use their results and, for example, we express the contribution of the pion-loop amplitude corresponding to $\rho^0 \to (\pi^+\pi^-)\gamma \to \pi^0\pi^0\gamma$ reaction in Fig. 3. 2b as

$$\mathcal{A}_\pi = -\frac{e g_{\rho\pi\pi}\lambda}{2\pi^2 M_{pi}^2} I(a,b)[(p\cdot k)(\epsilon\cdot u) - (p\cdot\epsilon)(k\cdot u)] \ , \qquad (3.12)$$

where $a = \frac{M_\rho^2}{M_\pi^2}$, $b = \frac{(p-k)^2}{M_\pi^2}$, p,k the momenta and u,$\epsilon$ the polarization vector of ρ-meson and photon, respectively. The amplitude corresponding to $\rho^0 \to (\pi^+\pi^-)\gamma\sigma \to \pi^0\pi^0\gamma$ reaction can similarly be written. The function I(a,b) is given as

$$I(a,b) = \frac{1}{2(a-b)} - \frac{2}{(a-b)^2}\left[f(\frac{1}{b}) - f(\frac{1}{a})\right] + \frac{a}{(a-b)^2}\left[g(\frac{1}{b}) - g(\frac{1}{a})\right] \qquad (3.13)$$

where

$$f(x) = \begin{cases} -\left[\arcsin(\frac{1}{2\sqrt{x}})\right]^2 , & x > \frac{1}{4} \\ \frac{1}{4}\left[\ln(\frac{\eta_+}{\eta_-}) - i\pi\right]^2 , & x < \frac{1}{4} \end{cases}$$

$$g(x) = \begin{cases} (4x-1)^{\frac{1}{2}} \arcsin(\frac{1}{2\sqrt{x}}) , & x > \frac{1}{4} \\ \frac{1}{2}(1-4x)^{\frac{1}{2}}\left[\ln(\frac{\eta_+}{\eta_-}) - i\pi\right] , & x < \frac{1}{4} \end{cases}$$

$$\eta_\pm = \frac{1}{2x}\left[1 \pm (1-4x)^{\frac{1}{2}}\right] \ . \qquad (3.14)$$

In addition to the vector meson dominance contribution that is displayed the corresponding Feynman diagrams in Fig. 3. 1a for $\omega \to \pi^0\pi^0\gamma$ decay and Fig. 3. 2a for $\rho \to \pi^0\pi^0\gamma$ decay, the incorporation of isospin violation effects allowing the mixing of the ρ and ω resonances is readily possible. This is the $\rho-\omega$ mixing which is well known and it has been seen to be relevant in processes like the $\omega \to \pi^+\pi^-$ decay or in the pion form factor in the ω region as mentioned in the previous chapter. The $\rho-\omega$ mixing is described by an effective Lagrangian of the form

$$\mathcal{L}^{eff.}_{\rho-\omega} = \prod^2_{\rho\omega} \omega_\mu \rho^\mu \quad , \tag{3.15}$$

where ω_μ and ρ_μ denote pure isospin field combinations. Therefore, the corresponding physical states can be written as [37]

$$|\rho\rangle = |\rho, I=1\rangle - \epsilon|\omega, I=0\rangle \tag{3.16}$$

$$|\omega\rangle = |\omega, I=0\rangle + \epsilon|\rho, I=1\rangle \tag{3.17}$$

where

$$\epsilon = \frac{\prod^2_{\rho\omega}}{M^2_\omega - M^2_\rho + i(M_\omega\Gamma_\omega - M_\rho\Gamma_\rho)} \quad . \tag{3.18}$$

O'Connell et al. [37] determined values of $\prod_{\rho\omega}$ from fits the $e^+e^- \to \pi^+\pi^-$ data as $\prod_{\rho\omega} = [-3800 \pm 370]MeV^2$. Urech [44] also determined the $\prod_{\rho\omega}$ using the chiral perturbation theory, as mentioned in the previous section, and this result is in good agreement with the values quoted in the literature. Then, using the experimental values for M_V and Γ_V where $V = \rho, \omega$, the mixing parameter ϵ is obtained as $\epsilon = (-0.006 + i0.036)$. Besides the mixing of the states, there is another effect of $\rho-\omega$ mixing, which is that it modifies the ρ-propagator in diagrams in Fig. 3. 1a for calculation of $\omega \to \pi^0\pi^0\gamma$ decay amplitude and ω-propagator in diagrams in Fig. 3. 2a for the amplitude of $\rho \to \pi^0\pi^0\gamma$ decay as

$$\frac{1}{D_{V'}(s)} \to \frac{1}{D_{V'}(s)}[1 + \frac{g_{V\pi\gamma}}{g_{V'\pi\gamma}} \frac{\prod^2_{VV'}}{D_V(s)}] \quad , \tag{3.19}$$

where

$$D_V(s) = s - M_V^2 + iM_V\Gamma_V(s) \quad . \tag{3.20}$$

New contribution coming from the $\rho - \omega$ mixing makes the whole $V \to PP^{'}\gamma$ amplitude to be written as $\mathcal{A}_0(V \to PP^{'}\gamma) + \epsilon\widetilde{\mathcal{A}}(V^{'} \to PP^{'}\gamma)$, where $\mathcal{A}_0$ and $\widetilde{\mathcal{A}}$ include the contributions coming from the different terms. Therefore, the amplitude of the decay $\omega \to \pi^0\pi^0\gamma$ can then be written as $\mathcal{A} = \mathcal{A}_0 + \epsilon\widetilde{\mathcal{A}}$ where $\mathcal{A}_0$ includes the contribution coming from the diagrams shown in Fig. 3. 1 for $\omega \to \pi^0\pi^0\gamma$ and $\widetilde{\mathcal{A}}$ represents the contributions of the diagrams in Fig. 3. 2 for $\rho \to \pi^0\pi^0\gamma$.

Similarly, the amplitude of the decay $\rho \to \pi^0\pi^0\gamma$ can be written same way that is $\mathcal{A} = \mathcal{A}_0 - \varepsilon\widetilde{\mathcal{A}}$ and in this case $\mathcal{A}_0$ includes the contribution coming from the diagrams shown in Fig. 3. 2 for $\rho \to \pi^0\pi^0\gamma$ and $\widetilde{\mathcal{A}}$ represents the contribution of the diagrams in Fig. 3. 1 for $\omega \to \pi^0\pi^0\gamma$.

We calculate invariant amplitude $\mathcal{A}(E_\gamma, E_1)$ this way for the decays $\omega \to \pi^0\pi^0\gamma$ and $\rho \to \pi^0\pi^0\gamma$ from the corresponding Feynman diagrams shown in Fig. 3. 1 and Fig. 3. 2. Then, the differential decay probability of $V \to \pi^0\pi^0\gamma$ decay for an unpolarized V-meson at rest is then given as

$$\frac{d\Gamma}{dE_\gamma dE_1} = \frac{1}{(2\pi)^3}\,\frac{1}{8M_V}\ \mid \mathcal{A} \mid^2 \quad , \tag{3.21}$$

where E_γ and E_1 are the photon and pion energies respectively. We perform an average over the spin states of vector-meson and a sum over the polarization

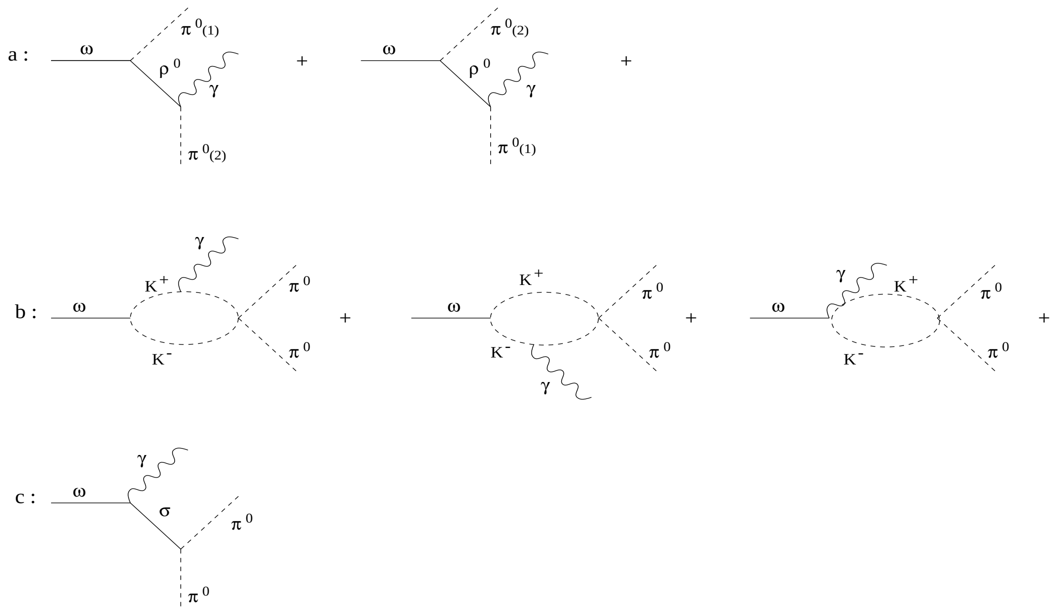

Figure 3.1: Feynman diagrams of $\omega \to \pi^0\pi^0\gamma$ decay.

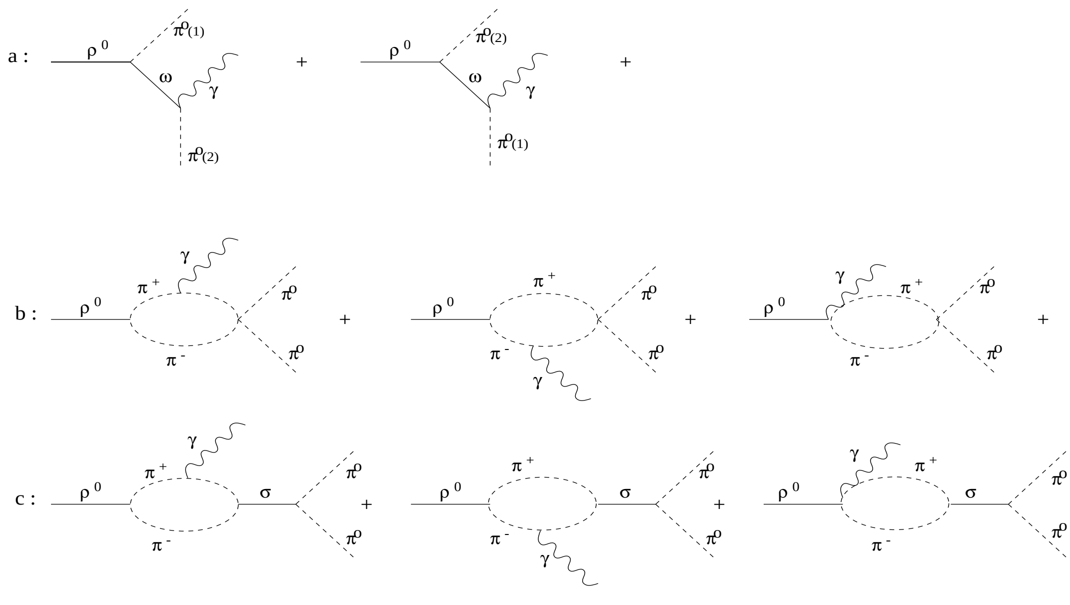

Figure 3.2: Feynman diagrams of $\rho \to \pi^0\pi^0\gamma$ decay.

states of the photon. We obtain the decay width $\Gamma(\omega \to \pi^0\pi^0\gamma)$ by integration

$$\Gamma = \frac{1}{2}\int_{E_{\gamma,min.}}^{E_{\gamma,max.}} dE_\gamma \int_{E_{1,min.}}^{E_{1,max.}} dE_1 \frac{d\Gamma}{dE_\gamma dE_1} \tag{3.22}$$

where the factor $\frac{1}{2}$ is included because of the $\pi^0\pi^0$ pair in the final state. The minimum photon energy is $E_{\gamma,min.} = 0$ and the maximum photon energy is given as $E_{\gamma,max.} = (M_V^2 - 4M_\pi^2)/2M_V$. Thus, we determine the maximum photon energy as $E_{\gamma,max.} = 341$ MeV for $\omega \to \pi^0\pi^0\gamma$ decay and $E_{\gamma,max.} = 338$ MeV for $\rho \to \pi^0\pi^0\gamma$ decay. The maximum and minimum values for pion energy E_1 are given by

$$\frac{1}{2(2E_\gamma M_V - M_V^2)}[-2E_\gamma^2 M_V + 3E_\gamma M_V^2 - M_V^3 \\ \pm E_\gamma\sqrt{(-2E_\gamma M_V + M_V^3)(-2E_\gamma M_V + M_V^2 - 4M_\pi^2)}\ ] \ . \tag{3.23}$$

3.1.1 Numerical analysis of $\omega \to \pi^0\pi^0\gamma$ decay

The contribution of different amplitudes to the branching ratio of the radiative decay $\omega \to \pi^0\pi^0\gamma$ are considerably different. We first consider the VMD amplitudes for the branching ratio and we obtain the values $BR(\omega \to \pi^0\pi^0\gamma) = 3.96 \times 10^{-5}$ and $BR(\omega \to \pi^0\pi^0\gamma) = 4.22 \times 10^{-5}$ without and with the effect of the $\rho - \omega$ mixing, respectively. These results are quite close to the values calculated in Ref. [14]. Then we use VMD amplitudes and chiral amplitudes, the resulting values for the branching ratio are $BR(\omega \to \pi^0\pi^0\gamma) = 3.98 \times 10^{-5}$ and $BR(\omega \to \pi^0\pi^0\gamma) = 4.67 \times 10^{-5}$ without and with the effect of the $\rho - \omega$ mixing included, respectively. Again these calculated values for the branching ratio seem to be quite in agreement with the previous results, in particular, with the results of Bramon et al. [12] and Palomar et al. [15]. When we consider the contribution of σ-meson intermediate state, the main difference with previous results is observed. Indeed, using the full amplitude including the contributions of VMD, chiral loop, and σ-meson intermediate state diagrams we obtain the branching ratio as $\mathrm{BR}(\omega \to \pi^0\pi^0\gamma) = 7.29 \times 10^{-5}$, and this value is reduced to the value $\mathrm{BR}(\omega \to \pi^0\pi^0\gamma) = 6.6 \times 10^{-5}$ when we add the effect of the $\rho - \omega$ mixing. We thus observe that the effect of $\rho - \omega$ mixing on the amplitude of the $\omega \to \pi^0\pi^0\gamma$ decay is reasonably pronounced and moreover the contribution of the σ-meson intermediate state is quite substantial.

The theoretical decay rate for $\omega \to \pi^0\pi^0\gamma$ decay that we calculate using Feynman diagrams in Fig. 3. 1 and Fig. 3. 2 results in a quadric equation for the coupling constant $g_{\omega\sigma\gamma}$. Using the experimental value for this decay rate [1], we obtain the values $g_{\omega\sigma\gamma} = (0.11 \pm 0.01)$ and $g_{\omega\sigma\gamma} = (-0.21 \pm 0.02)$ for the coupling constant $g_{\omega\sigma\gamma}$ [64]. These values are smaller than the values $g_{\omega\sigma\gamma} = 0.13$ and $g_{\omega\sigma\gamma} = -0.27$ that were obtained in the previous phenomenological analysis which did not include the effect of $\rho - \omega$ mixing [20]. As a result of our calculation, the $\rho - \omega$ mixing does make a reasonably substantial contribution to the $\omega \to \pi^0\pi^0\gamma$ decay amplitude when σ-meson intermediate state is taken into account and this consequently results in a reduced value for the coupling constant $g_{\omega\sigma\gamma}$.

In Fig. 3. 3 we plot the distribution $dB/dM_{\pi^0\pi^0}$ for the radiative $\omega \to \pi^0\pi^0\gamma$

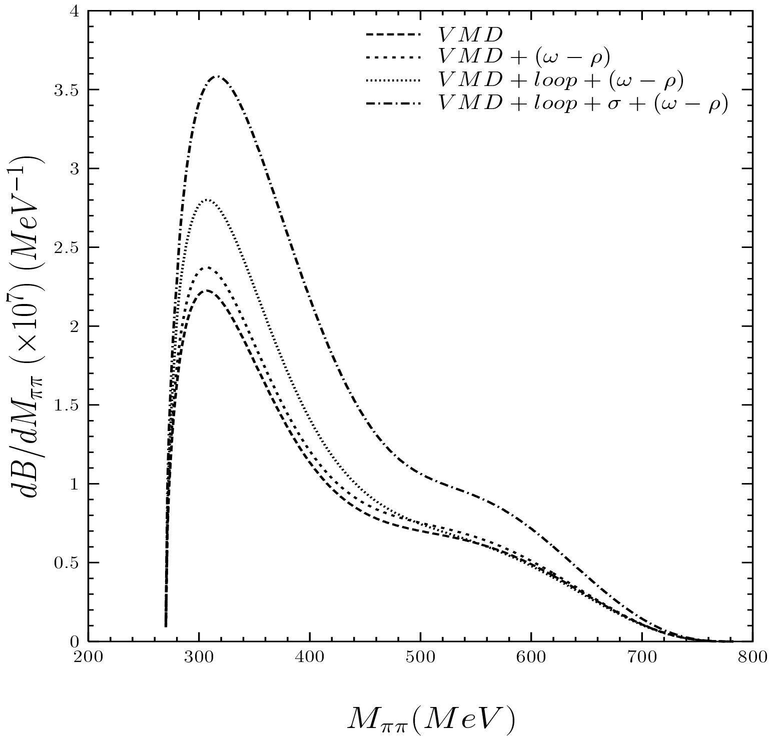

Figure 3.3: The $\pi^0\pi^0$ invariant mass spectrum of the decay $\omega \to \pi^0\pi^0\gamma$ for $g_{\omega\sigma\gamma} = 0.11$. The separate contributions resulting from the amplitudes of VMD; VMD and $\rho - \omega$ mixing; VMD, chiral loop, $\rho - \omega$ mixing; VMD, chiral loop, σ-meson intermediate state, $\rho - \omega$ mixing are shown.

decay in our phenomenological approach choosing coupling constat $g_{\omega\sigma\gamma} = 0.11$, as a function of invariant mass $M_{\pi\pi}$ of $\pi^0\pi^0$ system, where we also indicate the contributions coming from the different amplitudes. The interference term between the different amplitudes is positive over the whole region. When we take into account the effect of $\rho - \omega$ mixing by including the contribution coming from the diagrams in Fig. 3. 2a, the contribution of the VMD amplitude calculated from the diagrams in Fig. 3. 1a does not change appreciably. Then the situation changes somewhat if we consider VMD and chiral loop amplitudes and $\rho - \omega$ mixing as well. However, the significant alteration is obtained when we include VMD, chiral loop, and σ-meson intermediate state amplitudes with $\rho-\omega$ mixing. In Fig. 3. 4, we plot the resulting $\pi^0\pi^0$-invariant mass distribution for the decay which using the coupling constant $g_{\omega\sigma\gamma} = -0.21$. These figures clearly show the importance of the σ-meson intermediate state amplitude, and again it makes a very significant contribution. The interference term between the σ-meson and VMD and chiral loop amplitudes is negative some regions of the spectrum. Moreover, the overall shape of the spectrum is quite different from the previous case.

For the mechanism of $\rho \to \pi^0\pi^0\gamma$ decay, we consider a new approach in our work. Neglecting a direct $\rho\sigma\gamma$-vertex, we assume that the $\rho \to \pi^0\pi^0\gamma$ decay proceeds by a two-step mechanism with σ coupling to ρ^0-meson with $\pi^+\pi^-$ intermediate loop. We show the corresponding Feynman diagrams in Fig. 3. 2 for this decay. Scalar σ-meson effects in radiative ρ^0-meson decays are studied in recent work by Gökalp and Yılmaz in detail [20]. In that work they use the standard $\pi^0\pi^0 \to \pi^+\pi^-$ amplitude of chiral perturbation theory in the loop diagrams. Using the same $\pi\pi \to \pi\pi$ amplitude in our work, we then obtain the value of the branching ratios as BR$(\omega \to \pi^0\pi^0\gamma) = 6.12 \times 10^{-5}$ from the full amplitude including the contributions of VMD, chiral loop, and σ-meson intermediate state diagram as well as the effects of $\rho-\omega$ mixing. This evaluated value is not very different from BR$(\omega \to \pi^0\pi^0\gamma) = 6.6 \times 10^{-5}$ that is obtained employig the effective Lagrangian given in Eq. 3. 9 to described the π^4-vertex.

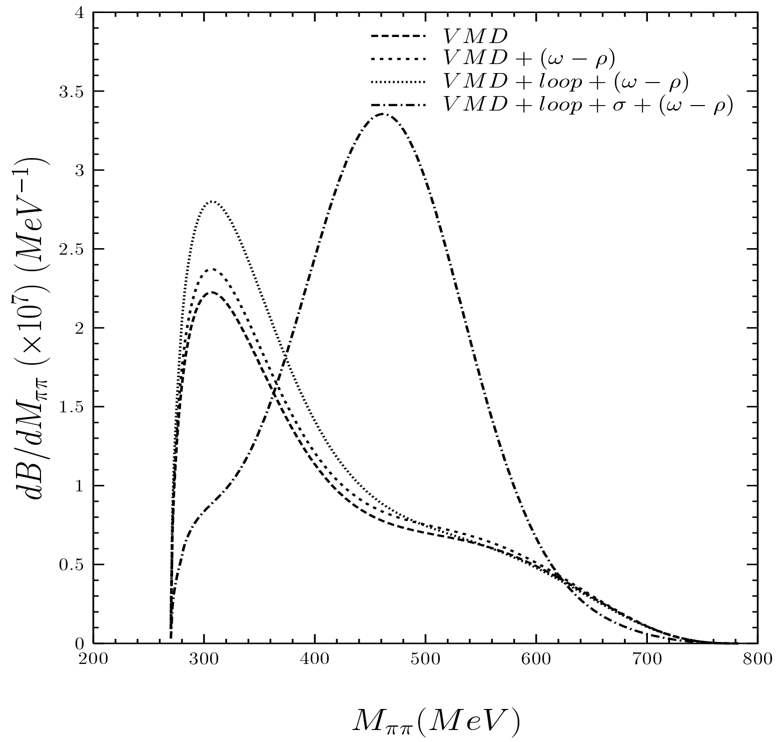

Figure 3.4: The $\pi^0\pi^0$ invariant mass spectrum of the decay $\omega \to \pi^0\pi^0\gamma$ for $g_{\omega\sigma\gamma} = -0.21$. The separate contributions resulting from the amplitudes of VMD; VMD and $\rho - \omega$ mixing; VMD, chiral loop, $\rho - \omega$ mixing; VMD, chiral loop, σ-meson intermediate state, $\rho - \omega$ mixing are shown.

An essential assumption of this decay calculation is that there is no SU(3) vector meson-sigma-gamma vertex. Thus, the $\omega\sigma\gamma$-vertex cannot be related to the $\rho\sigma\gamma$-vertex. The $\omega\sigma\gamma$-vertex that we use may be considered as representing the effective final state interactions in the $\pi\pi$-channel. The small value of the coupling constant $g_{\omega\sigma\gamma}$ that we obtain leads to a change in the Born amplitude of the $\omega \to \pi^0\pi^0\gamma$ decay which is of the same of the magnitude, as it is typical of final state interactions [64]. Finally, we noted that our analysis suggests that the coupling constant $g_{\omega\sigma\gamma}$ has actually a much smaller value than obtained by light cone QCD sum rules calculations.

3.1.2 Numerical analysis of $\rho \to \pi^0\pi^0\gamma$ decay

In the case of $\rho \to \pi^0\pi^0\gamma$ decay, the photon spectra for the decay rate is plotted in Fig. 3. 5 as a function of photon energy E_γ. The contributions of VMD amplitude, ρ–ω mixing amplitude, loop amplitude, σ-meson intermediate state amplitude and the total interference term are indicated. As it is clearly seen that there is no effect of ρ–ω mixing for the decay of $\rho \to \pi^0\pi^0\gamma$. So, the ρ–ω mixing gives the contribution as well as the contribution of VMD amplitude. The total branching ratio, including $\rho - \omega$ mixing, is obtained as $\mathrm{BR}(\rho \to \pi^0\pi^0\gamma)) = (4.90 \pm 0.82) \times 10^{-5}$ which is in good agreement with the experimental result $\mathrm{BR}(\rho \to \pi^0\pi^0\gamma)) = (4.1^{+1.0}_{-0.9} \pm 0.3) \times 10^{-5}$ [1]. The contribution of VMD amplitude that we obtain $\mathrm{BR}(\rho \to \pi^0\pi^0\gamma)) = (1.03 \pm 0.02) \times 10^{-5}$ is the same the $\rho - \omega$ mixing contribution and it's also in good agreement with the previous calculations [11, 12, 28, 29]. On the other hand the contributions coming from the pion-loop amplitude and σ-meson intermediate state amplitude including the effect of $\rho - \omega$ mixing are $\mathrm{BR}(\rho \to \pi^0\pi^0\gamma)) = (1.07 \pm 0.02) \times 10^{-5}$ and $\mathrm{BR}(\rho \to \pi^0\pi^0\gamma)) = (4.92 \pm 0.16) \times 10^{-5}$, respectively. As one compares the contributions of VMD, $\rho - \omega$ mixing and pion-loop amplitudes, it is seen that the σ-meson intermediate state makes an important contribution. Chiral loop and VMD in the radiative vector meson decays are also studied by Palomar et al. [15]. Their study includes the mechanisms of sequential vector meson decay,

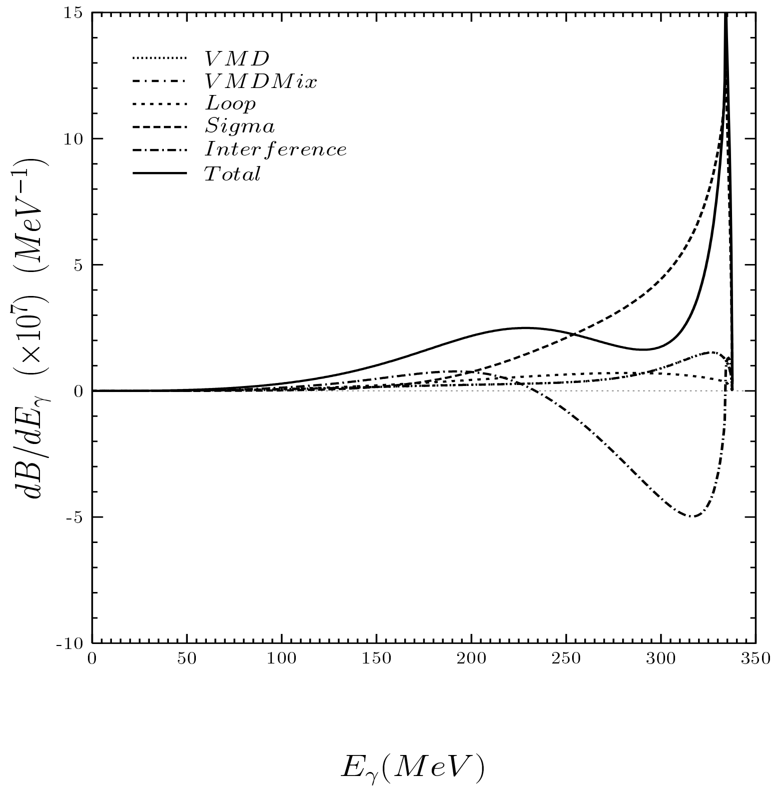

Figure 3.5: The photon spectra for the branching ratio of the decay $\rho \to \pi^0\pi^0\gamma$. The separate contributions resulting from the amplitudes of VMD; VMD and $\rho-\omega$ mixing; chiral loop; σ-meson intermediate state; and from the full amplitude obtained using the diagrams in Fig. 3. 2 and in Fig. 3. 1 as well as the total interference are shown.

chiral loops that is obtained using a chiral unitary approach to deal with the final state interaction of the meson meson system and the effect of $\rho-\omega$ mixing. For the radiative $\rho \to \pi^0\pi^0\gamma$ decay, they demonstrated that the loop contribution is very important and the branching ratio obtained as BR$(\rho \to \pi^0\pi^0\gamma) = 4.2\times10^{-5}$ with the sum of the sequential and loop mechanisms is about three times larger than with either mechanism alone (which is approximately the same value) and then this value is compatible with present experimental value. They also found that the $\rho - \omega$ mixing effects were negligible in the case of the $\rho \to \pi^0\pi^0\gamma$ decay as we reach the same conclusion for this decay. However, the mixing is relevant in the rest of the decays and it has important interferences with the sequential contribution and in addition modifies the resonance propagator involved even though the mixing contribution is by itself small.

For the radiative $\rho \to \pi^0\pi^0\gamma$ decay σ-meson effects are also investigated by Gokalp et al. [28] and Bramon et al. [29]. Gokalp et al. consider the contribution coming from the σ intermediate state as well as VMD and chiral pion-loop contributions in a phenomenological approach. Their study shows clearly that the contribution of the σ-meson intermediate amplitude has to be included in the analysis of radiative $\rho \to \pi^0\pi^0\gamma$ decay, moreover σ-meson has to be considered to couple to the ρ^0 meson through a pion-loop. As for Bramon et al., they propose the description of σ-meson effects in $\rho \to \pi^0\pi^0\gamma$ decay in terms of reasonably simple amplitudes reproduced the chiral-loop. They find that there is an important contribution from the σ meson, in addition to the well known ω-exchange, for the $\rho \to \pi^0\pi^0\gamma$ decay. They obtained the decay width of charged loops, VMD, as $\Gamma(\rho^0 \to \pi^0\pi^0\gamma)_\chi = 1.55$ keV and $\Gamma(\rho^0 \to \pi^0\pi^0\gamma)_\omega = 1.89$ keV, respectively. Then, for σ-meson they determined two values due to free parameter (k). The value $k = 1$ corresponds to the Linear Sigma Model (LσM) and the value $k \simeq 2.5$ matches for phenomenological context, and the results were $\Gamma(\rho^0 \to \pi^0\pi^0\gamma)_{L\sigma M} = 2.63$ keV and $\Gamma(\rho^0 \to \pi^0\pi^0\gamma)_{\sigma-phen} = 1.84$

keV. Moreover, they found the interference of VMD amplitude with the chiral-pion loops, Linear sigma Model and sigma phenomenological context amplitudes as $\Gamma(\rho^0 \to \pi^0\pi^0\gamma)_{\chi+\omega} = 4.40$ keV, $\Gamma(\rho^0 \to \pi^0\pi^0\gamma)_{L\sigma M+\omega} = 6.29$ keV, and $\Gamma(\rho^0 \to \pi^0\pi^0\gamma)_{\sigma-phen+\omega} = 5.10$ keV. Our results are coming from the VMD, chiral loop and σ-meson intermediate state and also interference of VMD with the pion-loop and σ-intermediate state amplitude, $\Gamma(\rho^0 \to \pi^0\pi^0\gamma)_{VMD} = 1.54$ keV, $\Gamma(\rho^0 \to \pi^0\pi^0\gamma)_{\pi} = 1.60$ keV, $\Gamma(\rho^0 \to \pi^0\pi^0\gamma)_{\sigma} = 7.44$ keV and $\Gamma(\rho^0 \to \pi^0\pi^0\gamma)_{\pi+VMD} = 2.49$ keV, $\Gamma(\rho^0 \to \pi^0\pi^0\gamma)_{\sigma+VMD} = 8.25$ keV, respectively. The σ-meson effects are quite different from our results because of using different approaches. We add the σ-meson effects to chiral-loop contribution, however, Bramon and Escribano proposed a consistent description of σ meson effects in terms of simple amplitudes which reproduced the expected chiral-loop behaviour for large M_σ values.

3.2 Radiative $\omega \to \pi^+\pi^-\gamma$ and $\rho^0 \to \pi^+\pi^-\gamma$ decays

We consider the contributions of vector meson dominance model, σ-meson intermediate state amplitudes and $\omega - \rho$ mixing in radiative $\omega \to \pi^+\pi^-\gamma$ decay. In the case of the radiative $\rho \to \pi^+\pi^-\gamma$ decay, the contributions of the pion-bremsstrahlung amplitude, pion-loop, σ-meson intermediate state amplitude and $\rho - \omega$ mixing are considered. To calculate the effects of the $\rho - \omega$ mixing in the $\omega \to \pi^+\pi^-\gamma$ decay, we use the amplitude resulting from the different amplitudes to the $\rho \to \pi^+\pi^-\gamma$ decay. Similarly, for evaluating the effects of the

$\rho - \omega$ mixing in the $\rho \to \pi^+\pi^-\gamma$ decay, we need an amplitude coming from the different amplitudes to the $\omega \to \pi^+\pi^-\gamma$ decay.

Our calculation is based on the Feynman diagrams shown in Fig. 3. 6 for $\omega \to \pi^+\pi^-\gamma$ decay and in Fig. 3. 7 for $\rho \to \pi^+\pi^-\gamma$ decay. The direct terms shown in the diagrams in Fig. 3. 7a, b, c are required to establish the gauge invariance. For the $\omega\rho\pi$, $\rho\pi\gamma$, $\omega\sigma\gamma$ and $\rho\pi\pi$ vertices, we use the effective Lagrangians described in Eq. 3. 1, 2, 6, 4 and 7 and also related coupling constants we take the same values except the coupling constant $g_{\omega\sigma\gamma}$. For this coupling constant, we use the values of $g_{\omega\sigma\gamma} = 0.11$ and $g_{\omega\sigma\gamma} = -0.23$ that are estimated by Gökalp et al. [64] in their analysis of $\omega \to \pi^0\pi^0\gamma$ decay.

Oller and Oset [65] studied the meson-meson interactions in the scalar sector using the standard chiral Lagrangian in lowest order of chiral perturbation theory. Their model predicts the mass and partial decay widths of the scalar resonances, as well as the different scattering amplitudes, in good agreement with experimental results and requires the use of only one parameter which is the cut off parameter around 1.2 GeV, in the loop integrations. We use their result for the isospin $I = 0$ four pseudoscalar amplitude $\pi^+\pi^- \to \pi^+\pi^-$ that we need in the loop diagrams in Fig. 1b, thus the small $I = 2$ amplitude is neglected.

Oller [66] noted that the important point in the argumentation is that the off-shell part of the meson-meson amplitude, which should be kept inside the loop integration, do not contribute, and consequently the amplitude $\mathcal{A}_{\chi}(\pi^+\pi^- \to$

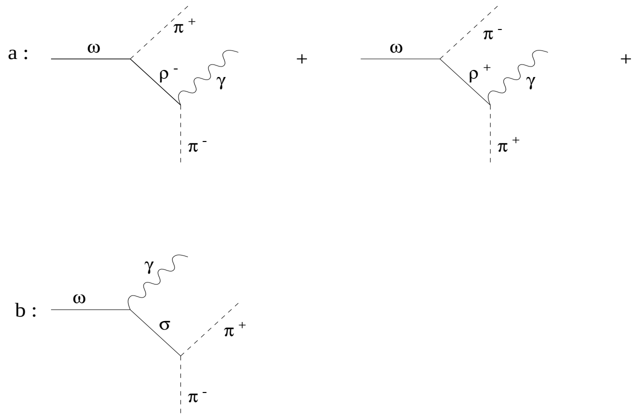

Figure 3.6: Feynman diagrams of $\omega \to \pi^+\pi^-\gamma$ decay.

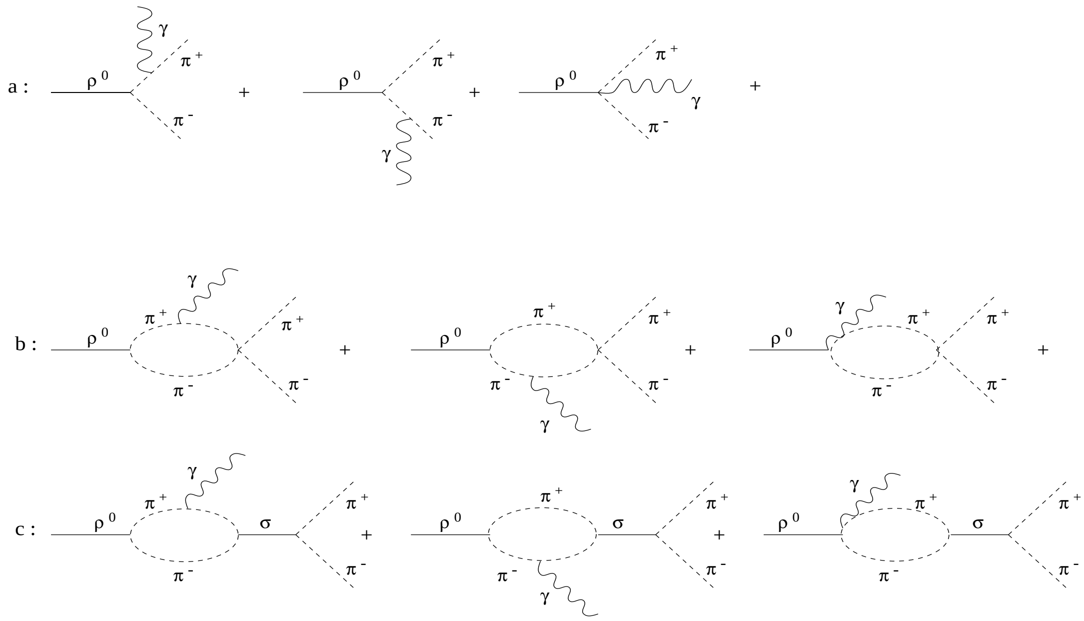

Figure 3.7: Feynman diagrams of $\rho \to \pi^+\pi^-\gamma$ decay.

$\pi^+\pi^-$) factorizes in the expression for the loop diagrams.

For ρ and σ mesons, we make the replacement in the propagator as calculation of $\omega \to \pi^0\pi^0\gamma$ and $\rho \to \pi^0\pi^0\gamma$ decays, and use the energy dependent widths given in Eq. 3. 10 and 3. 11 in invariant amplitude calculation of $\omega \to \pi^+\pi^-\gamma$ and $\rho \to \pi^+\pi^-\gamma$ decays. Moreover, we use the loop integrals appearing in Fig. 6 and 7 evaluated by Lucio and Pestiau [62] as mentioned before. The contribution of the pion loop amplitude corresponding to $\rho^0 \to (\pi^+\pi^-)\gamma \to\to \pi^+\pi^-\gamma$ reaction in Fig. 3. 7b can be written as

$$\mathcal{A}_\pi = -\frac{e g_{\rho\pi\pi} \mathcal{A}(\pi^+\pi^- \to \pi^+\pi^-)}{2\pi^2 M_\pi^2} I(a,b) \left[(p.k)(\varepsilon.u) - (p.\varepsilon)(k.u)\right] \quad , \qquad (3.24)$$

where $a = M_\rho^2/M_\pi^2$, $b = (p-k)^2/M_\pi^2$, $\mathcal{A}_\chi = -(2/f_\pi^2)(s + M_\pi^2/6)$, $s = (p-k)^2$, $f_\pi = 92.4$ MeV, which is the pion decay constant, p(u) and k(ε) being the momentum (polarization vector) of ρ-meson and photon, respectively. A similar amplitude corresponding to $\rho^0 \to (\pi^+\pi^-)\gamma\sigma \to \pi^+\pi^-\gamma$ reaction can also be written as follows

$$\mathcal{A} = -\frac{e g_{\rho\pi\pi} (g_{\sigma\pi\pi} M_\sigma)^2}{2\pi^2 M_\pi^2} D_\sigma(p-k) I(a,b) \left[(\varepsilon.u)(k.p) - (\varepsilon.p)(k.u)\right] \quad , \qquad (3.25)$$

where $D_\sigma(p-k)$ is the propagator of the σ-meson. For the function I(a,b), we again use the definition in Eq. 13 and 14. To calculate $\rho - \omega$ mixing part for $\omega \to \pi^+\pi^-\gamma$ and $\rho \to \pi^+\pi^-\gamma$ decays, we follow the formalism that developed for the decays of $\omega \to \pi^0\pi^0\gamma$ and $\rho \to \pi^0\pi^0\gamma$.

The mixing allows the transition $V \to V'$ in the process $V \to \pi^+\pi^-\gamma$, thus the amplitude of the decay $\omega \to \pi^+\pi^-\gamma$ can be written as $\mathcal{A} = \mathcal{A}_0 + \varepsilon\widetilde{\mathcal{A}}$ where

$\mathcal{A}_0$ includes the contributions coming from the diagrams shown in Fig. 3. 6 for $\omega \to \pi^+\pi^-\gamma$ and $\widetilde{\mathcal{A}}$ represents the contributions of the diagrams in Fig. 3. 7 for $\rho \to \pi^+\pi^-\gamma$. Then for the radiative $\rho \to \pi^+\pi^-\gamma$ decay the amplitude is written as $\mathcal{A} = \mathcal{A}_0 - \varepsilon\widetilde{\mathcal{A}}$ where $\mathcal{A}_0$ contains the contributions coming from the diagrams shown in Fig. 3. 7 for $\rho \to \pi^+\pi^-\gamma$ and $\widetilde{\mathcal{A}}$ includes the contributions of the diagrams in Fig. 3. 6 for $\omega \to \pi^+\pi^-\gamma$.

Another effect of $\rho - \omega$ mixing is to replace the ρ propagator in first part of amplitude of $\omega \to \pi^+\pi^-\gamma$ decay by the propagator given in Eq. 3. 19. Since according to SU(3) relation $g_{\omega\pi\gamma}/g_{\rho\pi\gamma} = 3$ this effect is relevant and it makes a sizeable contribution. However, since there is no VMD diagram for the $\rho \to \pi^+\pi^-\gamma$ decay, the first part of the effects of $\rho - \omega$ mixing will not exist.

The invariant amplitude $\mathcal{A}(E_{\gamma,E_1})$ is calculated this way for the radiative $\omega \to \pi^+\pi^-\gamma$ and $\rho \to \pi^+\pi^-\gamma$ decays from the corresponding Feynman diagrams are shown in Fig. 3. 6 and Fig. 3. 7. In this part of calculation we use Eq. 3. 21 and 3. 22, but we do not take the factor 1/2 in Eq. 3. 22 since final state particles are not identical. In our calculations the minimum photon energy $E_{\gamma,min.} = 0$ for the $\omega \to \pi^+\pi^-\gamma$ decay in $\rho - \omega$ mixing calculation of the $\omega \to \pi^+\pi^-\gamma$ decay it is taken as $E_{\gamma,min.} = 30$ MeV because of the presence of bremsstrahlung amplitude coming from the $\rho \to \pi^+\pi^-\gamma$ decay. The same situation also exist in calculation of $\rho - \omega$ mixing of $\rho \to \pi^+\pi^-\gamma$ decay. The maximum photon energy is given as $E_{\gamma,max.} = (M_V^2 - 4M_\pi^2)/2M_V$. Eq. 23 is used for the maximum and minimum values for pion energy E_1.

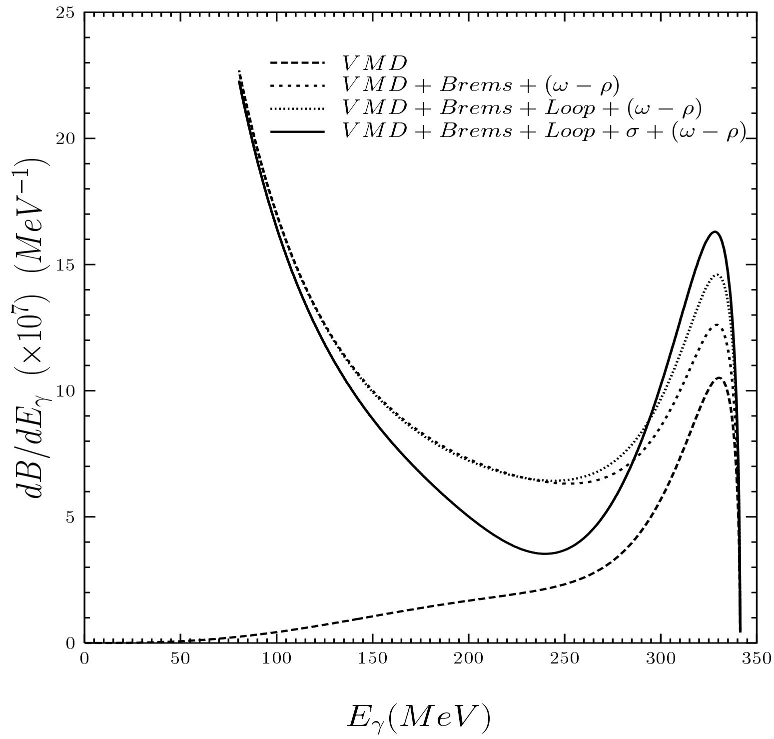

Figure 3.8: The photon spectra for the branching ratio of the decay $\omega \to \pi^+\pi^-\gamma$ for $g_{\omega\sigma\gamma} = 0.11$. The separate contributions resulting from the amplitudes of VMD; VMD and bremsstrahlung with $\rho - \omega$ mixing; VMD, bremsstrahlung, chiral loop with $\rho - \omega$ mixing; and from the full amplitude obtained using the diagrams in Fig. 3. 6 and in Fig. 3. 7 including σ-meson intermediate state with $\rho - \omega$ mixing.

3.2.1 Numerical analysis of $\omega \to \pi^+\pi^-\gamma$ decay

For the branching ratio of the decay $\omega \to \pi^+\pi^-\gamma$, the photon spectra are plotted in Fig. 3. 8 for $g_{\omega\sigma\gamma} = 0.11$ and in Fig. 3. 9 for $g_{\omega\sigma\gamma} = -0.21$ as a function of photon energy E_γ and also the contributions of the different amplitudes are indicated. The general shape of the spectrum as well as the relative contributions of different terms for positive and negative values of $g_{\omega\sigma\gamma}$ are quite different. These figures clearly show that the bremsstrahlung amplitude

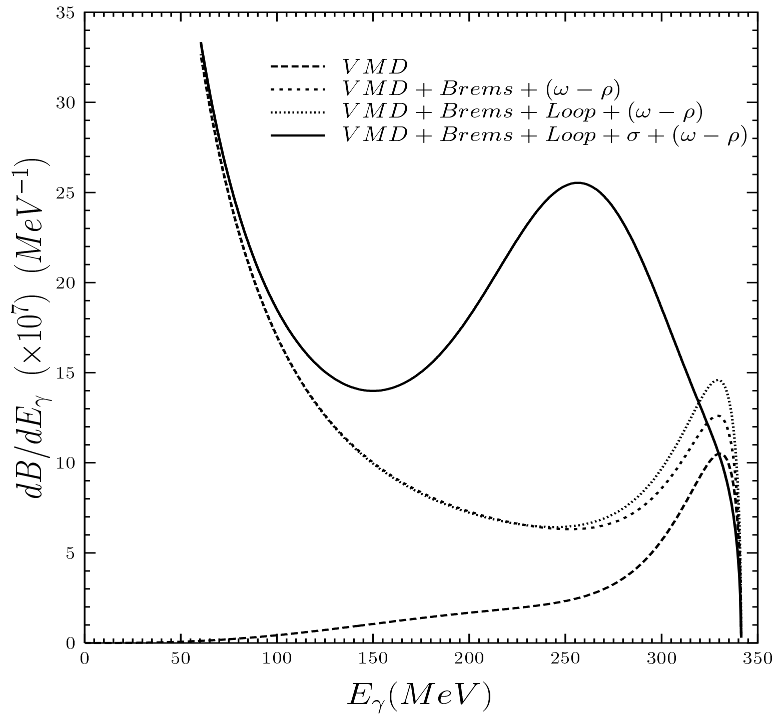

Figure 3.9: The photon spectra for the branching ratio of the decay $\omega \to \pi^+\pi^-\gamma$ for $g_{\omega\sigma\gamma} = -0.21$. The separate contributions resulting from the amplitudes of VMD; VMD and bremsstrahlung with $\rho - \omega$ mixing; VMD, bremsstrahlung, chiral loop with $\rho - \omega$ mixing; and from the full amplitude obtained using the diagrams in Fig. 3. 6 and in Fig. 3. 7 including σ-meson intermediate state with $\rho - \omega$ mixing.

of $\rho \to \pi^+\pi^-\gamma$ decay as the result of $\rho - \omega$ mixing affects mostly the lower part of the photon spectra, changing it drastically, but becomes practically negligible toward the higher photon energy part of the spectrum.

In our calculation from the contribution of VMD amplitude for the $\omega \to \pi^+\pi^-\gamma$ decay, we obtain the branching ratio $BR(\omega \to \pi^+\pi^-\gamma) = 7.2 \times 10^{-5}$. If we consider the VMD amplitude for $\omega \to \pi^+\pi^-\gamma$ decay and the bremsstrahlung amplitude for $\rho \to \pi^+\pi^-\gamma$ decay, as a result of $\rho - \omega$ mixing, we obtain the branching ratio $BR(\omega \to \pi^+\pi^-\gamma) = 0.46 \times 10^{-3}$. On the other hand, if we consider VMD and σ-meson intermediate state amplitudes for $\omega \to \pi^+\pi^-\gamma$ decay and do not consider $\rho - \omega$ mixing effect, the obtaining branching ratio is $BR(\omega \to \pi^+\pi^-\gamma) = 0.13 \times 10^{-3}$ for $g_{\omega\sigma\gamma} = 0.11$ and $BR(\omega \to \pi^+\pi^-\gamma) = 0.12 \times 10^{-3}$ for $g_{\omega\sigma\gamma} = -0.21$. These values show the importance of $\rho - \omega$ mixing and the σ-meson intermediate state amplitude in $\omega \to \pi^+\pi^-\gamma$ decay.

Finally, if we consider the full amplitudes resulting from the Feynman diagrams in Fig. 3. 6 and in Fig. 3. 7, we obtain for the branching of $\omega \to \pi^+\pi^-\gamma$ decay the value $BR(\omega \to \pi^+\pi^-\gamma) = 0.43 \times 10^{-3}$ using the coupling constant $g_{\omega\sigma\gamma} = 0.11$ and $BR(\omega \to \pi^+\pi^-\gamma) = 0.67 \times 10^{-3}$ if we use the coupling constant $g_{\omega\sigma\gamma} = -0.21$ [67]. These values are consistent with the experimental upper limit $BR(\omega \to \pi^+\pi^-\gamma) < 3.6 \times 10^{-3}$ [6].

3.2.2 Numerical analysis of $\rho \to \pi^+\pi^-\gamma$ decay

For the $\rho \to \pi^+\pi^-\gamma$ decay, the contributions of the bremsstrahlung amplitude, pion-loop amplitude and σ-meson intermediate state amplitude to the branching ratio of the decay are $BR(\rho \to \pi^+\pi^-\gamma)_\gamma = (1.14 \pm 0.01) \times 10^{-2}$, $BR(\rho \to \pi^+\pi^-\gamma)_\pi = (0.45 \pm 0.08) \times 10^{-5}$ and $BR(\rho \to \pi^+\pi^-\gamma)_\sigma = (0.83 \pm 0.16) \times 10^{-4}$, respectively. Gokalp et al. [28] also obtained the same results in their analysis for this decay. For the total branching ratio including effects of $\rho - \omega$ mixing we obtain $BR(\rho \to \pi^+\pi^-\gamma) = (1.22 \pm 0.02) \times 10^{-2}$ for $E_\gamma > 50$ MeV which is in reasonably good agreement with the experimental

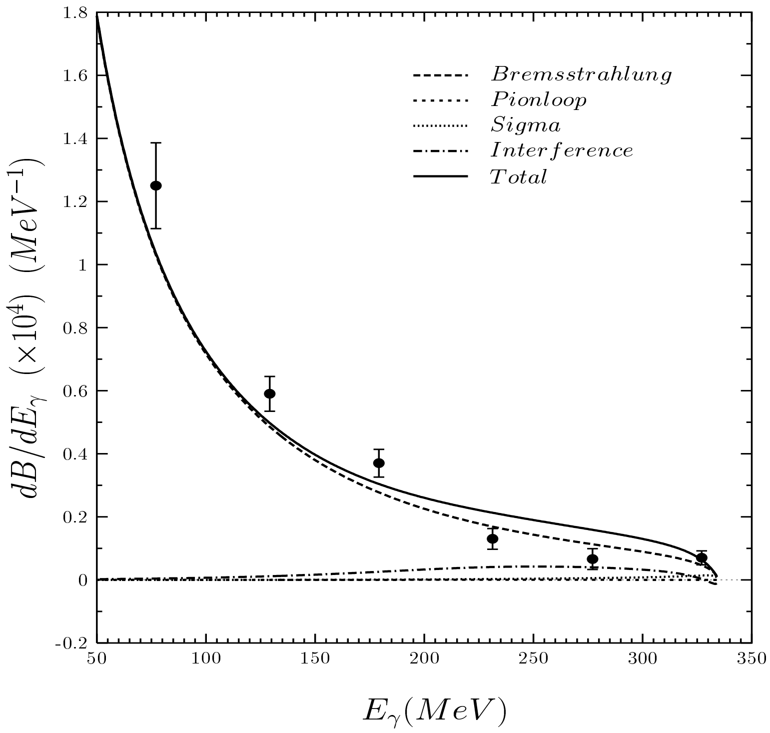

Figure 3.10: The photon spectra for the branching ratio of the decay $\rho^0 \rightarrow \pi^+\pi^-\gamma$. The separate contributions resulting from the amplitudes of VMD; bremsstrahlung; pion loop; σ-meson intermediate state; and from the full amplitude obtained using the diagrams in Fig. 3. 7 and in Fig. 3. 6 as well as the total interference are shown.

result $BR(\rho \rightarrow \pi^+\pi^-\gamma) = (0.99 \pm 0.16) \times 10^{-2}$ [5]. Also this value was obtained without the effects of $\rho - \omega$ mixing [28] and as the result we can say that the $\rho - \omega$ mixing is not important for $\rho \rightarrow \pi^+\pi^-\gamma$ decay. Therefore the dominant contribution comes from bremsstrahlung and the contribution of σ-meson intermediate state amplitude should be included in the analysis of the decay but the $\rho - \omega$ mixing effects should not be considered in this decay since there is no contribution coming from the VMD mechanism.

In Fig. 3. 10 we show the photon spectra for the branching ratio of the decay

$\rho \to \pi^+\pi^-\gamma$ as a function of photon energy E_γ and also the experimental data points [5]. The contribution of the pion-bremsstrahlung amplitude is the main contribution, pion-loop and σ-meson intermediate states become noticeable only in the region of high photon energies.

Bramon and Escribano [29] also analysed this decay and determined the decay width contributed by bremsstrahlung, chiral loop and σ-meson amplitudes. Similar to the $\rho \to \pi^0\pi^0\gamma$ decay there are two values of σ-meson amplitudes coming from the free parameter k. They obtained the values $\Gamma(\rho \to \pi^+\pi^-\gamma)_{brems} = 1.706$ MeV (for $E_\gamma > 50$ MeV), $\Gamma(\rho \to \pi^+\pi^-\gamma)_{\chi} = 0.93$ keV and $\Gamma(\rho \to \pi^+\pi^-\gamma)_{L\sigma M} = 5.21$ keV, $\Gamma(\rho \to \pi^+\pi^-\gamma)_{\sigma-phen} = 3.84$ keV, respectively. For the contribution of different amplitudes to the decay width, we find $\Gamma(\rho \to \pi^+\pi^-\gamma)_\gamma = 1.718$ MeV from the bremsstrahlung amplitude, $\Gamma(\rho \to \pi^+\pi^-\gamma)_\pi = 0.67$ keV from the pion-loop amplitude and $\Gamma(\rho \to \pi^+\pi^-\gamma)_\sigma = 0.0125$ MeV from σ-meson intermediate state amplitude. Again the differences between these results can be attributed to the different approaches followed.

CHAPTER 4

CONCLUSIONS

We have studied the radiative decays of the ρ and ω mesons into pseudoscalar mesons including the mechanism of vector meson dominance model (VMD), chiral loop, σ-meson intermediate state amplitude and $\omega - \rho$ mixing in a phenomenological approach.

Results of our study can be briefly summarized as follows

- The effects of $\rho - \omega$ mixing is the efficient mechanism for having the dominant contribution if to the vector meson dominance amplitude makes a contribution to the decays considered.

- In the analysis of the $\omega \to \pi^0\pi^0\gamma$ decay in order to explain the latest experimental result, the σ-meson intermediate state and $\rho - \omega$ mixing should be considered.

- The $\rho - \omega$ mixing is a small effect for the $\omega \to \pi^0\pi^0\gamma$ decay whose width increases by $5^0/_0$ only due to $\rho - \omega$ mixing, even less than the $12^0/_0$ increase provided by using a momentum dependent width for ρ meson in the calculation amplitude of VMD.

- In the full amplitude, if the σ meson intermediate state is considered with the $\rho-\omega$ mixing the value of the branching ratio of $\omega \to \pi^0\pi^0\gamma$ decay is reduced. This conclusion affects the value of the coupling constant $g_{\omega\sigma\gamma}$. Therefore, the value of the coupling constant $g_{\omega\sigma\gamma}$ that we obtain is smaller than the values obtained in the previous studies which was not considered the effects of $\rho-\omega$ mixing.

- The shape of the $\pi\pi$ invariant mass distribution for $\omega \to \pi^0\pi^0\gamma$ decay is found to depend on the mechanism considered, especially σ-meson intermediate state changes the shape considerably.

- There is very slight effect of $\rho-\omega$ mixing for the $\rho \to \pi^0\pi^0\gamma$ decay, so this effect is negligible in mechanism of this decay.

- ρ and ω meson widths, which are rather different from each other, significantly changes the results of the $\rho-\omega$ mixing mechanism [51], thus, this may be a reason of why the $\rho \to \pi^0\pi^0\gamma$ decay has not been effected by the $\rho-\omega$ mixing.

- In the case of the $\rho \to \pi^0\pi^0\gamma$ decay, the loop and σ-meson contributions are very important and the branching ratio is obtained with the sum of the VMD, pion-loop amplitude and σ-meson intermediate state amplitude.

- For the $\omega \to \pi^+\pi^-\gamma$ decay, the σ-meson intermediate state amplitude and $\rho-\omega$ mixing make important contributions to the branching ratio.

- Since the bremsstrahlung that comes from the effects of $\rho-\omega$ mixing is the main contribution to the decay it affects the lower part of the photon spectrum in $\omega \to \pi^+\pi^-\gamma$ decay but for the higher part it is unimportant.

- The bremsstrahlung contribution provides the dominant contribution of the $\rho \to \pi^+\pi^-\gamma$ decay, however σ-meson intermediate state amplitude also makes an appreciable contribution for this decay.

- The $\rho \to \pi^+\pi^-\gamma$ decay does not have a contribution coming from the VMD mechanism, therefore the $\rho - \omega$ mixing is not pronounced in this decay and thus its contribution is negligible.

- Our predicted branching ratios of the $\omega \to \pi\pi\gamma$ and $\rho \to \pi\pi\gamma$ decays are in good agreement with the latest experimental results.

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APPENDIX A

TWO BODY DECAY RATES

The probability of transition for a particular transition from initial state ($| i >$) to a final state ($| f >$) is given by $|S_{fi}|^2 = |< f|S \mid i > |^2$ where the element of the scattering matrix S is defined as

$$S_{fi} = \delta_{fi} + i(2\pi)^4\delta^4(p_f - p_i)T_{fi} \tag{A.1}$$

where T is the transition matrix. A particle of mass M and energy E decays into any number of particles 1,2,..... , if the invariant matrix element for the process is $\mathcal{A}_{fi}$, the decay rate is obtained by multiplying the transition probability per unit time by number of final states as follows

$$d\Gamma = (2\pi)^4\delta^4(p_f - p_i)|\mathcal{A}_{fi}|^2\frac{1}{2E}\prod_i \frac{d^3p_i}{(2\pi)^3(2E_i)} \quad . \tag{A.2}$$

If we consider that the decay produces two particles, then in the rest frame of the decaying particle $\vec{p_1} = -\vec{p_2} \equiv \vec{p}$, $E_1 + E_2 = M$, thus

$$d\Gamma = \frac{1}{(2\pi)^2}|\mathcal{A}_{fi}|^2\frac{1}{2M}\frac{1}{4E_1E_2}\delta(\vec{p_1} + \vec{p_2})\delta(E_1 + E_2 - M)d^3p_1d^3p_2 \quad . \tag{A.3}$$

The integration over d^3p_2 eliminates the first delta function, the differential d^3p_1 is written as

$$d^3p = p^2d|\vec{p}|d\Omega = |\vec{p}|d\Omega\frac{E_1E_2d(E_1 + E_2)}{E_1 + E_2} \tag{A.4}$$

since $E_1^2 - M_1^2 = E_2^2 - M_2^2 = \vec{p}^2$. The second delta function is eliminated by integration over $(E_1 + E_2)$ and the result is

$$d\Gamma = \frac{1}{32\pi^2 M^2} |\mathcal{A}_{fi}|^2 |\vec{p}| d\Omega \quad . \tag{A.5}$$

In the rest frame of the decaying particle $|\vec{p}|$ is given as

$$|\vec{p}| = \frac{1}{2} \sqrt{\frac{[M^2 - (M_1 + M_2)^2][M^2 - (M_1 - M_2)^2]}{M^2}} \quad . \tag{A.6}$$

Therefore, for the decay $M \to M_1 + M_2$ where $M_1 = M_2$

$$|\vec{p}| = \frac{1}{2} \sqrt{1 - \left(\frac{2M_1}{M}\right)^2} \quad , \tag{A.7}$$

and for the decay $M \to M_1 + \gamma$

$$|\vec{p}| = \frac{1}{2} M \left[1 - \left(\frac{M_1}{M}\right)^2\right] \quad . \tag{A.8}$$

For the decay $\sigma \to \pi\pi$, the invariant matrix element that follows from the effective Lagrangian

$$\mathcal{L}_{\sigma\pi\pi}^{eff.} = \frac{1}{2} g_{\sigma\pi\pi} M_\sigma \vec{\pi} \cdot \vec{\pi} \sigma \tag{A.9}$$

is given by $g_{\sigma\pi\pi} M_\sigma$, therefore

$$\Gamma_\sigma \equiv \Gamma(\sigma \to \pi\pi) = \frac{g_{\sigma\pi\pi}^2}{4\pi} \frac{3M_\sigma}{8} \left[1 - \left(\frac{2M_\pi}{M_\sigma}\right)^2\right]^{1/2} \quad . \tag{A.10}$$

For the decay $\rho^0 \to \pi\pi$ which is described by the effective Lagrangian

$$\mathcal{L}_{\rho\pi\pi}^{eff.} = g_{\rho\pi\pi} \vec{\rho}_\mu \cdot (\partial^\mu \vec{\pi} \times \vec{\pi}) \tag{A.11}$$

the invariant matrix element is $\mathcal{A} = ig_{\rho\pi\pi}(2q_1 - p)_\mu U^\mu$, where q_1 is the momentum of one of the pions and p(U) is the momentum (polarization) of the decaying ρ-meson. Thus the decay rate can be obtained as

$$\Gamma(\rho \to \pi\pi) = \frac{g_{\rho\pi\pi}^2}{4\pi}\frac{M_\rho}{12}\left[1 - \left(\frac{2M_\pi}{M_\rho}\right)^2\right]^{3/2} \quad . \tag{A.12}$$

The radiative decay $V^0 \to \varphi^0\gamma$ is described by the effective Lagrangian

$$\mathcal{L}^{eff.}_{V\varphi\gamma} = \frac{e}{M_V} g_{V\varphi\gamma}\epsilon^{\mu\nu\alpha\beta}\partial_\mu V^0_\nu \partial_\alpha A_\beta \varphi^0 \quad , \tag{A.13}$$

where V denotes the decaying vector meson and φ denotes the pseudoscalar meson. The invariant amplitude can be obtained as $\mathcal{A} = i\frac{e}{M_V} g_{V\varphi\gamma}\epsilon^{\mu\nu\alpha\beta} p_\mu U_\nu k_\alpha \epsilon_\beta$, where p(U) and k($\epsilon$) are the momenta (polarization) of vector meson and photon respectively, and the decay rate is

$$\Gamma(V^0 \to \varphi^0\gamma) = \frac{\alpha}{24}\frac{(M_V^2 - M_\varphi^2)^3}{M_V^5}\; g^2_{V\varphi\gamma} \quad . \tag{A.14}$$

Utilizing the experimental values of different two body decay rates, we determine various coupling constants. Using the decay rates of $\rho^0 \to \pi\pi$ and $\rho^0 \to \pi^0\gamma$ decays [6] we find the coupling constants $g_{\rho\pi\pi}$ and $g_{\rho\pi\gamma}$ as $g_{\rho\pi\pi} = 6.047$ and $g_{\rho\pi\gamma} = 0.696$, respectively where the mass and width for ρ is taken as $M_\rho = 770$ MeV and $\Gamma_\rho = 150.2$ MeV in our calculation. We also determine the coupling constant $g_{\omega\pi\gamma}$ as $g_{\omega\pi\gamma} = 1.821$ from the experimental value of the $\omega \to \pi^0\gamma$ decay rate [6] using the value of $M_\omega = 782$ MeV and $\Gamma_\omega = 8.44$ MeV. Furthermore, to calculate the coupling constant $g_{\sigma\pi\pi}$, we use the mass and width of sigma as $M_\sigma = 478$ MeV, $\Gamma_\sigma = 324$ MeV and finally find this coupling constant as $g_{\sigma\pi\pi} = 5.290$.

APPENDIX B

THREE BODY DECAY AND THE BOUNDARY OF DALITZ PLOT

For the three particle decay $M(p) \to M_1(q_1) + M_2(q_2) + \gamma(k)$, the differential decay rate is given by

$$d\Gamma = (2\pi)^4 \delta^4(p-k-q_1-q_2) \frac{1}{2E_p} \frac{d^3k}{(2\pi)^3(2E_\gamma)} \frac{d^3q_1}{(2\pi)^3(2E_1)} \frac{d^3q_2}{(2\pi)^3(2E_2)} \overline{|\mathcal{A}_{fi}|^2} \quad \text{(B.1)}$$

where $\overline{|\mathcal{A}_{fi}|^2}$ is the average over spin states of the absolute square of the decay invariant matrix element. Therefore, due to spin average, we can write $\overline{|\mathcal{A}_{fi}|^2} = F(E_1, E_2)$. In the rest frame of the decaying particle $\delta^4(p - k - q_1 - q_2) = \delta(M - E_\gamma - E_1 - E_2)\delta^3(\vec{k} + \vec{q_1} + \vec{q_2})$, and the momentum delta function can be eliminated by performing the integral over d^3q_2. Since

$$\frac{d^3k}{2E_\gamma} = \frac{1}{2} E_\gamma dE_\gamma d\Omega_\gamma \quad , \quad \text{(B.2)}$$

and

$$\frac{d^3q_1}{2E_1} = \frac{1}{2} |\vec{q_1}| dE_1 d\Omega_1 \quad , \quad \text{(B.3)}$$

we obtain

$$\frac{d\Gamma}{dE_\gamma dE_1} = \frac{|\vec{q_1}| E_\gamma \overline{|\mathcal{A}_{fi}|^2}}{16M(2\pi)^5} \int d\Omega_\gamma d\Omega_1 \frac{\delta(M - E_\gamma - E_1 + \sqrt{(\vec{k} + \vec{q_1})^2 + M_2^2})}{\sqrt{(\vec{k} + \vec{q_1})^2 + M_2^2}} \quad \text{(B.4)}$$

If we consider the integral defined by

$$I = |\vec{q_1}|E_\gamma \int d\Omega_\gamma d\Omega_1 \frac{\delta(M - E_\gamma - E_1 - \sqrt{(\vec{k}+\vec{q_1})^2 + M_2^2})}{\sqrt{(\vec{k}+\vec{q_1})^2 + M_2^2}} \tag{B.5}$$

we perform the angular integrals and obtain

$$I = 8\pi^2 \int_{-1}^{1} d(\cos\theta)|\vec{q_1}|E_\gamma \frac{\delta(M - E_\gamma - E_1 - \sqrt{E_\gamma^2 + E_1^2 - M_1^2 + 2E_\gamma|\vec{q_1}|\cos\theta + M_2^2})}{\sqrt{E_\gamma^2 + E_1^2 - M_1^2 + 2E_\gamma|\vec{q_1}|\cos\theta + M_2^2}} \tag{B.6}$$

where θ is defined by $\vec{k}\cdot\vec{q_1} = |\vec{k}||\vec{q_1}|\cos\theta$. A change of variable

$$\xi = \sqrt{E_\gamma^2 + E_1^2 - M_1^2 + 2E_\gamma|\vec{q_1}|\cos\theta + M_2^2} \tag{B.7}$$

gives

$$I = 8\pi^2 \int d\xi \delta(M - E_\gamma - E_1 - \xi) = 8\pi^2 \tag{B.8}$$

subject to the condition $M - E_\gamma - E_1 - \xi = 0$. Therefore the double differential decay rate can be obtain as

$$\frac{d\Gamma}{dE_\gamma dE_1} = \frac{1}{(2\pi)^3}\frac{1}{8M}\overline{|\mathcal{A}_{fi}|^2} \tag{B.9}$$

The limits of integral are defined by the condition $M - E_\gamma - E_1 - \xi = 0$ as

$$M - E_\gamma - E_1 = \sqrt{E_\gamma^2 + E_1^2 - M_1^2 + 2E_\gamma|\vec{q_1}|\cos\theta + M_2^2} \quad , \tag{B.10}$$

or

$$-1 \leq \frac{(M - E_\gamma - E_1)^2 - E_\gamma^2 - E_1^2 + M_1^2 - M_2^2}{2E_\gamma\sqrt{E_1^2 - M_1^2}} \leq 1 \quad . \tag{B.11}$$

APPENDIX C

INVARIANT AMPLITUDES FOR THE $V \to \pi^0\pi^0\gamma$ DECAYS

The invariant amplitude of the radiative decay $V(p) \to \pi^0(q_1)\pi^0(q_2)\gamma(k)$ including the contribution of the $\rho-\omega$ mixing is $\mathcal{A}[V \to \pi^0\pi^0\gamma] = \mathcal{A}^0[V \to \pi^0\pi^0\gamma] + \epsilon\widetilde{\mathcal{A}}[V' \to \pi^0\pi^0\gamma]$ where $\mathcal{A}^0$ and $\widetilde{\mathcal{A}}$ include the contributions coming from the vector meson dominance amplitude, chiral loop and σ-meson intermediate state amplitude. In case $V,\ V' = \omega$ the amplitude is calculated using the Feynman diagrams shown in Fig. 3. 1 (a), (b) and (c), and for $V,\ V' = \rho$ the Feynman diagrams shown Fig. 3. 2 (a), (b) and (c) are used. Moreover, in the calculation of the amplitude A^0 the modified vector meson propagator is used as given by Eqs. (3.19) and (3.20). Therefore, we can write the invariant amplitude including the effects of $\rho-\omega$ mixing for $V \to \pi^0\pi^0\gamma$ decay as

$$\mathcal{A} = \mathcal{A}^0_{V,VMD} + \mathcal{A}^0_{V,loop} + \mathcal{A}^0_{V,\sigma} + \varepsilon[\widetilde{\mathcal{A}}_{V',VMD} + \widetilde{\mathcal{A}}_{V',loop} + \widetilde{\mathcal{A}}_{V',\sigma}] \qquad \text{(C.1)}$$

where $\varepsilon \equiv c + id$. The numerical values of c and d are c=-0.006 and d=0.036 [38].

In the rest frame for the $V(p) \to P(q_1)P'(q_2)\gamma(k)$ decay, when P and P' denote any one of the pseudoscalar mesons π^0, π^+ and π^-

$$k \cdot p \;=\; M_V E_k$$

$$
\begin{aligned}
k \cdot q_1 &= \frac{1}{2}(M_V^2 - 2M_V E_2) \\
k \cdot q_2 &= \frac{1}{2}(M_V^2 - 2M_V E_1) \\
p \cdot p &= p^2 = M_V^2 \\
p \cdot q_1 &= M_V^2 E_1 \\
p \cdot q_2 &= M_V^2 E_2 \\
q_1 \cdot q_1 &= q_2 \cdot q_2 = M_\pi^2 \\
q_1 \cdot q_2 &= \frac{1}{2}(M_V^2 - 2M_V E_k - 2M_\pi^2) \quad . \qquad \text{(C.2)}
\end{aligned}
$$

We organize this Appendix as follows. First, the amplitudes contributing to the $\omega \to \pi^0\pi^0\gamma$ and $\rho \to \pi^0\pi^0\gamma$ decays are introduced. Then, starting with the VMD contribution we add the contributions of loops and σ-meson intermediate state. Finally, we give the square of the invariant amplitudes as we discuss in the text.

C.1 $\omega \to \pi^0\pi^0\gamma$ decay

For the $\omega \to \pi^0\pi^0\gamma$ decay we use the contributions coming from the VMD, kaon-loop, and σ-meson intermediate state amplitudes. As the result of the $\rho - \omega$ mixing, we change the propagator of the ρ-meson that is shown in Fig. 3. 1 (a). Therefore, we write the amplitudes that includes the change in the propagator in the vector meson dominance amplitude as

$$
\mathcal{A}^0_{\omega,VMD} = ReA^0_\omega(p - q_1) * F_\omega(p - q_1) + ReA^0_\omega(p - q_2) * F_\omega(p - q_2)
$$

$$+i[ImA^0_\omega(p-q_1)*F_\omega(p-q_1)+ImA^0_\omega(p-q_2)*F_\omega(p-q_2)] \tag{C.3}$$

$$\mathcal{A}^0_{\omega,K} = ReA^0_{\omega,K}*F_{\omega,K}+iImA^0_{\omega,K}*F_{\omega,K} \tag{C.4}$$

$$\mathcal{A}^0_{\omega,\sigma} = ReA^0_{\omega,\sigma}*F_{\omega,\sigma}+iImA^0_{\omega,\sigma}*F_{\omega,\sigma} \tag{C.5}$$

where

$$\begin{aligned} ReA^0_\omega(n) &= (\Gamma_\rho M_\rho)D_\omega(n)+\frac{g_{\omega\pi\gamma}}{g_{\rho\pi\gamma}}M^2_{\rho\omega} \\ &\times\{(n^2-M^2_\omega)(\Gamma_\rho M_\rho)-(n^2-M^2_\rho)(\Gamma_\omega M_\omega)\}G_\omega(n) \end{aligned} \tag{C.6}$$

$$\begin{aligned} ImA^0_\omega(n) &= (n^2-M^2_\rho)D_\omega(n)+\frac{g_{\omega\pi\gamma}}{g_{\rho\pi\gamma}}M^2_{\rho\omega} \\ &\times\{(n^2-M^2_\omega)(n^2-M^2_\rho)-(\Gamma_\rho M_\rho)(\Gamma_\omega M_\omega)\}G_\omega(n) \end{aligned} \tag{C.7}$$

$$\begin{aligned} F_\omega(n) &= -\left(\frac{e}{M_\rho}g_{\rho\pi\gamma}\right)\left(\frac{g_{\omega\rho\pi}}{M_\omega}\right)\epsilon^{\mu\nu\alpha\beta}p_\alpha U_\beta n_\mu \epsilon^{\mu'\nu'\alpha'\beta'}n_{\mu'}k_{\alpha'}\epsilon_{\beta'} \\ &\times\left[-g_{\nu\nu'}+\frac{n_\nu n_{\nu'}}{M^2_\rho}\right] \end{aligned} \tag{C.8}$$

$$D_\omega(n) = \frac{1}{(n^2-M^2_\rho)^2+(\Gamma_\rho M_\rho)^2} \tag{C.9}$$

$$G_\omega(n) = \frac{1}{[(n^2-M^2_\rho)^2+(\Gamma_\rho M_\rho)^2][(n^2-M^2_\omega)^2+(\Gamma_\omega M_\omega)^2]} \tag{C.10}$$

$$ReA^0_{\omega,K} = ReI(a,b) \tag{C.11}$$

$$ImA^0_{\omega,K} = ImI(a,b) \tag{C.12}$$

$$F_{\omega,K} = -\left(\frac{eg}{4\sqrt{2}\pi^2 f_\pi^2 M_k^2}\right)(p^2 - 2k\cdot p)p_\alpha U_\beta(k_\alpha\epsilon_\beta - k_\beta\epsilon_\alpha) \quad \text{(C.13)}$$

$$ReA^0_{\omega,\sigma} = (\Gamma_\sigma M_\sigma)D_\sigma(p-k) \quad \text{(C.14)}$$

$$ImA^0_{\omega,\sigma} = [(p-k)^2 - M_\sigma^2]D_\sigma(p-k) \quad \text{(C.15)}$$

$$F_{\omega,\sigma} = -\left(\frac{e}{M_\omega}g_{\omega\sigma\gamma}\right)(g_{\sigma\pi\pi}M_\sigma)p_\alpha U_\beta(k_\alpha\epsilon_\beta - k_\beta\epsilon_\alpha) \quad . \quad \text{(C.16)}$$

ImI and ReI are given in Eq. 3.13 and Eq. 3.14, respectively.

C.2 $\rho \to \pi^0\pi^0\gamma$ decay

For the $\rho \to \pi^0\pi^0\gamma$ decay we use the contribution of the VMD, pion-loop, and σ-meson intermediate state. As the result of the $\rho-\omega$ mixing, we change the propagator of the ω-meson that is shown in Fig. 3. 2 (a). Therefore, we write the amplitudes that includes the change in the propagator in the vector meson dominance amplitude as

$$\begin{aligned}\mathcal{A}^0_{\rho,VMD} &= ReA^0_\rho(p-q_1) * F_\rho(p-q_1) + ReA^0_\rho(p-q_2) * F_\rho(p-q_2) \\ &\quad + i[ImA^0_\rho(p-q_1) * F_\rho(p-q_1) + ImA^0_\rho(p-q_2) * F_\rho(p-q_2)]\end{aligned} \quad \text{(C.17)}$$

$$\mathcal{A}^0_{\rho,\pi} = ReA^0_{\rho,\pi} * F_{\rho,\pi} + iImA^0_{\rho,\pi} * F_{\rho,\pi} \quad \text{(C.18)}$$

$$\mathcal{A}^0_{\rho,\sigma} = ReA^0_{\rho,\sigma} * F_{\rho,\sigma} + iImA^0_{\rho,\sigma} * F_{\rho,\sigma} \quad \text{(C.19)}$$

where

$$
\begin{aligned}
ReA^0_\rho(n) &= (\Gamma_\omega M_\omega)D_\rho(n) + \frac{g_{\rho\pi\gamma}}{g_{\omega\pi\gamma}}M^2_{\rho\omega} \\
&\quad \times\{(n^2 - M^2_\omega)(\Gamma_\rho M_\rho) - (n^2 - M^2_\rho)(\Gamma_\omega M_\omega)\}G_\rho(n) \qquad \text{(C.20)}
\end{aligned}
$$

$$
\begin{aligned}
ImA^0_\rho(n) &= (n^2 - M^2_\omega)D_\rho(n) + \frac{g_{\rho\pi\gamma}}{g_{\omega\pi\gamma}}M^2_{\rho\omega} \\
&\quad \times\{(n^2 - M^2_\omega)(n^2 - M^2_\rho) - (\Gamma_\rho M_\rho)(\Gamma_\omega M_\omega)\}G_\rho(n) \qquad \text{(C.21)}
\end{aligned}
$$

$$
\begin{aligned}
F_\rho(n) &= -\left(\frac{e}{M_\omega}g_{\omega\pi\gamma}\right)\left(\frac{g_{\rho\omega\pi}}{M_\rho}\right)\epsilon^{\mu\nu\alpha\beta}p_\alpha U_\beta n_\mu \epsilon^{\mu'\nu'\alpha'\beta'}n_{\mu'}k_{\alpha'}\epsilon_{\beta'} \\
&\quad \times\left[-g_{\nu\nu'} + \frac{n_\nu n_{\nu'}}{M^2_\omega}\right] \qquad \text{(C.22)}
\end{aligned}
$$

$$
D_\rho(n) = \frac{1}{(n^2 - M^2_\omega)^2 + (\Gamma_\omega M_\omega)^2} \qquad \text{(C.23)}
$$

$$
G_\rho(n) = \frac{1}{[(n^2 - M^2_\rho)^2 + (\Gamma_\rho M_\rho)^2][(n^2 - M^2_\omega)^2 + (\Gamma_\omega M_\omega)^2]} \qquad \text{(C.24)}
$$

$$
ReA^0_{\rho,\pi} = ReI(a,b) \qquad \text{(C.25)}
$$

$$
ImA^0_{\rho,\pi} = ImI(a,b) \qquad \text{(C.26)}
$$

$$
\begin{aligned}
F_{\rho,\pi} &= 2g_{\rho\pi\pi}(e\lambda)U_\mu\epsilon_\nu \\
&\quad \times\int\frac{d^4q}{(2\pi)^4}\left\{\frac{-4q_\mu q_\nu + g_{\mu\nu}(q^2 - M^2_\pi)}{(q^2 - M^2_\pi)[(q-k)^2 - M^2_\pi][(p-q)^2 - M^2_\pi]}\right\}
\end{aligned}
$$

$$= -\left(\frac{e\ g_{\rho\pi\pi}}{2\pi^2 M_\pi^2}\right)\ \lambda\ p_\alpha U_\beta (k_\alpha \epsilon_\beta - k_\beta \epsilon_\alpha) \tag{C.27}$$

where

$$\lambda = -\frac{g_{\pi NN}^2}{2} \frac{M_{\sigma - M_\pi^2}^2}{M_N^2} \tag{C.28}$$

$$ReA_{\rho,\sigma}^0 = \{[((p-k)^2 - M_\sigma^2) ReI(a,b) + (\Gamma_\sigma M_\sigma) ImI(a,b)]\} D_\sigma (p-k) \tag{C.29}$$

$$ImA_{\rho,\sigma}^0 = [((p-k)^2 - M_\sigma^2) ImI(a,b) - (\Gamma_\sigma M_\sigma) ReI(a,b)]\} D_\sigma (p-k) \tag{C.30}$$

$$F_{\rho,\sigma} = -\left(\frac{e g_{\rho\pi\pi}}{2\pi^2 M_\pi^2}\right) (g_{\sigma\pi\pi} M_\sigma)^2\, p_\alpha U_\beta (k_\alpha \epsilon_\beta - k_\beta \epsilon_\alpha)\ . \tag{C.31}$$

C.3 The full amplitude of the $\omega \to \pi^0\pi^0\gamma$ decay including the $\rho - \omega$ mixing

The invariant amplitude of the radiative decay $\omega(p) \to \pi^0(q_1)\pi^0(q_2)\gamma(k)$ including the contribution of the $\rho - \omega$ mixing is $\mathcal{A}[\omega \to \pi^0\pi^0\gamma] = \mathcal{A}^0[\omega \to \pi^0\pi^0\gamma] + \varepsilon\widetilde{\mathcal{A}}[\rho \to \pi^0\pi^0\gamma]$ where in the $\rho \to \pi^0\pi^0\gamma$ decay amplitude we do not include the change in the vector meson propagator. The contribution coming from the VMD is

$$\begin{aligned}\mathcal{A}_{VMD} = {} & ReA_{\omega1}^0 * F_{\omega1} + ReA_{\omega2}^0 * F_{\omega2} + c(Re\widetilde{A}_{\rho1} * F_{\rho1} + Re\widetilde{A}_{\rho2} * F_{\rho2}) \\ & -d(Im\widetilde{A}_{\rho1} * F_{\rho1} + Im\widetilde{A}_{\rho2} * F_{\rho2}) + i[ImA_{\omega1}^0 * F_{\omega1} + ImA_{\omega2}^0 * F_{\omega2} \\ & +c(Im\widetilde{A}_{\rho1} * F_{\rho1} + Im\widetilde{A}_{\rho2} * F_{\rho2}) + d(Re\widetilde{A}_{\rho1} * F_{\rho1} + Re\widetilde{A}_{\rho2} * F_{\rho2})]\ . \end{aligned} \tag{C.32}$$

The square of the invariant VMD amplitude is then obtained as

$$|\mathcal{A}_{VMD}|^2 = [(ReA_{\omega1}^0)^2 + (ImA_{\omega1}^0)^2](F_{\omega1})^2 + [(ReA_{\omega2}^0)^2 + (ImA_{\omega2}^0)^2](F_{\omega2})^2$$

$$+(c^2+d^2)\{[(Re\widetilde{A}_{\rho 1})^2+(Im\widetilde{A}_{\rho 1})^2](F_{\rho 1})^2+[(Re\widetilde{A}_{\rho 2})^2$$

$$+(Im\widetilde{A}_{\rho 2})^2](F_{\rho 2})^2+2(Re\widetilde{A}_{\rho 1}Re\widetilde{A}_{\rho 2}+Im\widetilde{A}_{\rho 1}Im\widetilde{A}_{\rho 2})(F_{\rho 1}F_{\rho 2})\}$$

$$+2[(ReA^0_{\omega 1}ReA^0_{\omega 2}+ImA^0_{\omega 1}ImA^0_{\omega 2})(F_{\omega 1}F_{\omega 2})+(ReA^0_{\omega 1}*F_{\omega 1}$$

$$+ReA^0_{\omega 2}*F_{\omega 2})(cRe\widetilde{A}_{\rho 1}*F_{\rho 1}-dIm\widetilde{A}_{\rho 1}*F_{\rho 1})+(ImA^0_{\omega 1}*F_{\omega 1}$$

$$+ImA^0_{\omega 2}*F_{\omega 2})(cIm\widetilde{A}_{\rho 1}*F_{\rho 1}+dRe\widetilde{A}_{\rho 1}*F_{\rho 1})+(ReA^0_{\omega 1}*F_{\omega 1}$$

$$+ReA^0_{\omega 2}*F_{\omega 2})(cRe\widetilde{A}_{\rho 2}*F_{\rho 2}-dIm\widetilde{A}_{\rho 2}*F_{\rho 2})+(ImA^0_{\omega 1}*F_{\omega 1}$$

$$+ImA^0_{\omega 2}*F_{\omega 2})(cIm\widetilde{A}_{\rho 2}*F_{\rho 2}+dRe\widetilde{A}_{\rho 2}*F_{\rho 2})] \tag{C.33}$$

where

$$\begin{aligned}(F_{\omega 1})^2 &= \left(\frac{e}{M_\rho}g_{\rho\pi\gamma}\right)^2\left(\frac{g_{\omega\rho\pi}}{M_\omega}\right)^2\frac{1}{3}\{-2k\cdot p\ k\cdot q_1\\ &\times[p^2(p\cdot q_1-2q_1^2)+p\cdot q_1\ q_1^2]+(k\cdot p)^2\\ &\times[2(p\cdot q_1)^2-p^2q_1^2-2p\cdot q_1\ q_1^2+q_1^4]+(k\cdot q_1)^2\\ &\times[p^4+2(p\cdot q_1)^2-p^2(2p\cdot\ q_1+q_1^2)]\}\end{aligned} \tag{C.34}$$

$$(F_{\omega 2})^2 = (F_{\omega 1})^2(q_1\to q_2) \tag{C.35}$$

$$(F_{\rho 1})^2 = (F_{\omega 1})^2(interchange\ \omega,\rho) \tag{C.36}$$

$$(F_{\rho 2})^2 = (F_{\omega 2})^2(interchange\ \omega,\rho) \tag{C.37}$$

$$\begin{aligned}(F_{\omega 1}F_{\omega 2}) &= \left(\frac{e}{M_\rho}g_{\rho\pi\gamma}\right)^2\left(\frac{g_{\omega\rho\pi}}{M_\omega}\right)^2\frac{1}{3}\{(k\cdot q_2)^2[(p\cdot q_1)^2-p^2\ q_1^2]\\ &+k\cdot p\ k\cdot q_2[p\cdot q_2\ q_1^2-2p\cdot q_1\ q_1\cdot q_2+p^2(-p\cdot q_1+q_1^2\end{aligned}$$

$$+ q_1 \cdot q_2)] + (k \cdot p)^2[-p \cdot q_2\ q_1^2 + q_1 \cdot q_2(-p^2 + q_1 \cdot q_2)$$

$$+ p \cdot q_1(2p \cdot q_2 - q_2)] + (k \cdot q_1)^2[(p \cdot q_2)^2 - p^2\ q_2^2] + k \cdot q_1$$

$$\times [k \cdot q_2\ p^2(p^2 - p \cdot q_1 - p \cdot q_2 + q_1 \cdot q_2) + k \cdot p(-2p \cdot q_2$$

$$\times q_1 \cdot q_2 + p \cdot q_1\ q_2^2 + p^2(-p \cdot q_2 + q_1 \cdot q_2 + q_2^2))]\} \quad (C.38)$$

$$(F_{\rho 1}F_{\rho 2}) = (F_{\omega 1}F_{\omega 2})(interchange\ \omega, \rho) \quad (C.39)$$

$$(F_{\omega 1}F_{\rho 1}) = \left(\frac{e}{M_\rho}g_{\rho\pi\gamma}\right)\left(\frac{g_{\omega\rho\pi}}{M_\omega}\right)\left(\frac{e}{M_\omega}g_{\omega\pi\gamma}\right)\left(\frac{g_{\rho\omega\pi}}{M_\rho}\right)$$

$$\times\frac{1}{3}\{-2k \cdot p\ k \cdot q_1[p^2(p \cdot q_1 - 2q_1^2) + p \cdot q_1$$

$$\times q_1^2] + (k \cdot p)^2[2(p \cdot q_1)^2 - p^2 q_1^2 - 2p \cdot q_1$$

$$\times q_1^2 + q_1^4] + (k \cdot q_1)^2[p^4 + 2(p \cdot q_1)^2$$

$$-p^2(2p \cdot q_1 + q_1^2)]\} \quad (C.40)$$

$$(F_{\omega 1}F_{\rho 2}) = \left(\frac{e}{M_\rho}g_{\rho\pi\gamma}\right)\left(\frac{g_{\omega\rho\pi}}{M_\omega}\right)\left(\frac{e}{M_\omega}g_{\omega\pi\gamma}\right)\left(\frac{g_{\rho\omega\pi}}{M_\rho}\right)$$

$$\times\frac{1}{3}\{(k \cdot q_2)^2[(p \cdot q_1)^2 - p^2\ q_1^2] + k \cdot p\ k \cdot q_2[p \cdot q_2\ q_1^2$$

$$- 2p \cdot q_1\ q_1 \cdot q_2 + p^2(-p \cdot q_1 + q_1^2 + q_1 \cdot q_2)] + (k \cdot p)^2$$

$$\times [-p \cdot q_2\ q_1^2 + q_1 \cdot q_2(-p^2 + q_1 \cdot q_2) + p \cdot q_1(2p \cdot q_2$$

$$- q_2)] + (k \cdot q_1)^2[(p \cdot q_2)^2 - p^2\ q_2^2] + k \cdot q_1[k \cdot q_2\ p^2$$

$$\times (p^2 - p \cdot q_1 - p \cdot q_2 + q_1 \cdot q_2) + k \cdot p(-2p \cdot q_2 q_1 \cdot q_2$$

$$+ p \cdot q_1\ q_2^2 + p^2(-p \cdot q_2 + q_1 \cdot q_2 + q_2^2))]\} \quad (C.41)$$

$$(F_{\omega 2}F_{\rho 2}) \;=\; (F_{\omega 1}F_{\rho 1})(q_1 \to q_2) \quad . \tag{C.42}$$

After adding the chiral loop amplitude to the above amplitude the resulting invariant amplitude is $\mathcal{A}[\omega \to \pi^0\pi^0\gamma] = \mathcal{A}[\omega \to \pi^0\pi^0\gamma]_{VMD} + \mathcal{A}[\omega \to \pi^0\pi^0\gamma]_{loops}$ where $\mathcal{A}[\omega \to \pi^0\pi^0\gamma]_{loops} = \mathcal{A}^0_{\omega,K} + \varepsilon\, \widetilde{\mathcal{A}}_{\rho,\pi}$. Therefore, we can write the invariant amplitude as

$$\begin{aligned}
\mathcal{A} &= \mathcal{A}_{VMD} + \mathcal{A}_{loop} \\
&= \mathcal{A}_{VMD} + ReA^0_{\omega,K} * F_{\omega,K} + (cRe\widetilde{A}_{\rho,\pi} - dIm\widetilde{A}_{\rho,\pi}) * F_{\rho,\pi} \\
&\quad +i[ImA^0_{\omega,K} * F_{\omega,K} + (dRe\widetilde{A}_{\rho,\pi} + cIm\widetilde{A}_{\rho,\pi}) * F_{\rho,\pi}] \quad .
\end{aligned} \tag{C.43}$$

The square of the invariant amplitude is then obtained as

$$\begin{aligned}
|\mathcal{A}|^2 &= |\mathcal{A}_{VMD} + \mathcal{A}_{loop}|^2 \\
&= |\mathcal{A}_{VMD}|^2 + [(ReA^0_{\omega,K})^2 + (ImA^0_{\omega,K})^2] * (F_{\omega,K})^2 + (c^2 + d^2) \\
&\quad \times[(Re\widetilde{A}_{\rho,\pi})^2 + (Im\widetilde{A}_{\rho,\pi})^2] * (F_{\rho,\pi})^2 + 2[(ReA^0_{\omega 1}ReA^0_{\omega,K} + ImA^0_{\omega 1} \\
&\quad \times ImA^0_{\omega,K})(F_{\omega 1}F_{\omega,K}) + (ReA^0_{\omega 2}ReA^0_{\omega,K} + ImA^0_{\omega 2}ImA^0_{\omega,K})(F_{\omega 2}F_{\omega,K}) \\
&\quad +ReA^0_{\omega,K}(cRe\widetilde{A}_{\rho,1} - dIm\widetilde{A}_{\rho,1})(F_{\omega,K}F_{\rho 1}) + ImA^0_{\omega,K}(cIm\widetilde{A}_{\rho,1} \\
&\quad +dRe\widetilde{A}_{\rho,1})(F_{\omega,K}F_{\rho 1}) + ReA^0_{\omega,K}(cRe\widetilde{A}_{\rho,2} - dIm\widetilde{A}_{\rho,2})(F_{\omega,K}F_{\rho 2}) \\
&\quad +ImA^0_{\omega,K}(cIm\widetilde{A}_{\rho,2} + dRe\widetilde{A}_{\rho,2})(F_{\omega,K}F_{\rho 2}) + ReA^0_{\omega 1}(cRe\widetilde{A}_{\rho,\pi} \\
&\quad -dIm\widetilde{A}_{\rho,\pi})(F_{\omega 1}F_{\rho,\pi}) + ReA^0_{\omega 2}(cRe\widetilde{A}_{\rho,\pi} - dIm\widetilde{A}_{\rho,\pi})(F_{\omega 2}F_{\rho,\pi}) \\
&\quad +ReA^0_{\omega,K}(cRe\widetilde{A}_{\rho,\pi} - dIm\widetilde{A}_{\rho,\pi})(F_{\omega,K}F_{\rho,\pi}) + ImA^0_{\omega 1}(cIm\widetilde{A}_{\rho,\pi} \\
&\quad +dRe\widetilde{A}_{\rho,\pi})(F_{\omega 1}F_{\rho,\pi}) + ImA^0_{\omega 2}(cIm\widetilde{A}_{\rho,\pi} + dRe\widetilde{A}_{\rho,\pi})(F_{\omega 2}F_{\rho,\pi}) \\
&\quad +ImA^0_{\omega,K}(cIm\widetilde{A}_{\rho,\pi} + dRe\widetilde{A}_{\rho,\pi})(F_{\omega,K}F_{\rho,\pi}) + (cRe\widetilde{A}_{\rho,1} - dIm\widetilde{A}_{\rho,1})
\end{aligned}$$

$$\times(cRe\widetilde{A}_{\rho,\pi} - dIm\widetilde{A}_{\rho,\pi})(F_{\rho 1}F_{\rho,\pi}) + (cIm\widetilde{A}_{\rho,1} + dRe\widetilde{A}_{\rho,1})(cIm\widetilde{A}_{\rho,\pi}$$

$$+bdRe\widetilde{A}_{\rho,\pi})(F_{\rho 1}F_{\rho,\pi}) + (cRe\widetilde{A}_{\rho,2} - dIm\widetilde{A}_{\rho,2})(cRe\widetilde{A}_{\rho,\pi} - dIm\widetilde{A}_{\rho,\pi})$$

$$\times(F_{\rho 2}F_{\rho,\pi}) + (cIm\widetilde{A}_{\rho,2} + dRe\widetilde{A}_{\rho,2})(cIm\widetilde{A}_{\rho,\pi} + dRe\widetilde{A}_{\rho,\pi})(F_{\rho 2}F_{\rho,\pi})]$$

$$\text{(C.44)}$$

where

$$(F_{\omega,K})^2 = \left(\frac{eg}{4\sqrt{2}\pi^2 f_\pi^2 M_k^2}\right)^2 (p^2 - 2k\cdot p)^2 \left[\frac{2}{3}(k\cdot p)^2\right] \tag{C.45}$$

$$(F_{\rho,\pi})^2 = \left(\frac{e\ g_{\rho\pi\pi}}{2\pi^2 M_\pi^2}\right)^2 \ \lambda^2 \left[\frac{2}{3}(k\cdot p)^2\right] \tag{C.46}$$

$$\begin{aligned}(F_{\omega 1}F_{\omega,K}) &= \left(\frac{e}{M_\rho} g_{\rho\pi\gamma}\right)\left(\frac{g_{\omega\rho\pi}}{M_\omega}\right)\left(\frac{eg}{4\sqrt{2}\pi^2 f_\pi^2 M_K^2}\right)(p^2 - 2k\cdot p) \\ &\quad \times\frac{1}{3}\{k\cdot p(2k\cdot q_1 p^2 - p^2(k\cdot q_1)^2/k\cdot p + k\cdot p(-2k\cdot q_1 + q_1^2))\}\end{aligned} \tag{C.47}$$

$$(F_{\omega 2}F_{\omega,K}) = (F_{\omega 1}F_{\omega,K})(q_1 \to q_2) \tag{C.48}$$

$$(F_{\omega,K}F_{\rho 1}) = (F_{\omega,1}F_{\omega,K})(interchange\ \omega,\rho) \tag{C.49}$$

$$(F_{\omega,K}F_{\rho 2}) = (F_{\omega,K}F_{\rho 1})(q_1 \to q_2) \tag{C.50}$$

$$\begin{aligned}(F_{\omega 1}F_{\rho,\pi}) &= \left(\frac{e}{M_\rho} g_{\rho\pi\gamma}\right)\left(\frac{g_{\omega\rho\pi}}{M_\omega}\right)\left(\frac{e\ g_{\rho\pi\pi}}{2\pi^2 M_\pi^2}\right)\ \lambda \\ &\quad \times\frac{1}{3}\{k\cdot p[2k\cdot q_1 p^2 - p^2(k\cdot q_1)^2/k\cdot p + k\cdot p(-2k\cdot q_1 + q_1^2)]\}\end{aligned} \tag{C.51}$$

$$(F_{\omega 2}F_{\rho,\pi}) \;=\; (F_{\omega 1}F_{\rho,\pi})(q_1 \to q_2) \tag{C.52}$$

$$(F_{\omega,K}F_{\rho,\pi}) \;=\; \left(\frac{eg}{4\sqrt{2}\pi^2 f_\pi^2 M_k^2}\right)\left(\frac{e\ g_{\rho\pi\pi}}{2\pi^2 M_\pi^2}\right)\ \lambda\ [p^2 - 2k\cdot p]\left[\frac{2}{3}(k\cdot p)^2\right] \tag{C.53}$$

$$\begin{aligned}(F_{\rho 1}F_{\rho,\pi}) \;=\; & \left(\frac{e}{M_\omega}g_{\omega\pi\gamma}\right)\left(\frac{g_{\rho\omega\pi}}{M_\rho}\right)\left(\frac{e\ g_{\rho\pi\pi}}{2\pi^2 M_\pi^2}\right)\ \lambda \\ & \times\frac{1}{3}\{k\cdot p[2k\cdot q_1 p^2 - p^2(k\cdot q_1)^2/k\cdot p + k\cdot p(-2k\cdot q_1 + q_1^2)]\}\end{aligned} \tag{C.54}$$

$$(F_{\rho 2}F_{\rho,\pi}) \;=\; (F_{\rho 1}F_{\rho,\pi})(q_1 \to q_2)\quad . \tag{C.55}$$

The full amplitude including the contribution of VMD, chiral loop and σ-meson intermediate state is $\mathcal{A}[\omega \to \pi^0\pi^0\gamma] = \mathcal{A}[\omega \to \pi^0\pi^0\gamma]_{VMD} + \mathcal{A}[\omega \to \pi^0\pi^0\gamma]_{loops} + \mathcal{A}[\omega \to \pi^0\pi^0\gamma]_\sigma$ where $\mathcal{A}[\omega \to \pi^0\pi^0\gamma]_\sigma = \mathcal{A}^0_{\omega,\sigma} + \varepsilon\ \widetilde{\mathcal{A}}_{\rho,\sigma}$. Therefore, the invariant amplitude can be written as

$$\begin{aligned}\mathcal{A} &= \mathcal{A}_{VMD} + \mathcal{A}_{loop} + \mathcal{A}_\sigma \\ &= \mathcal{A}_{VMD} + \mathcal{A}_{loops} + ReA^0_{\omega,\sigma} * F_{\omega,\sigma} + (cRe\widetilde{A}_{\rho,\sigma} - bIm\widetilde{A}_{\rho,\sigma}) * F_{\rho,\sigma} \\ &\quad + i(ImA^0_{\omega,\sigma} * F_{\omega,\sigma} + (cIm\widetilde{A}_{\rho,\sigma} + dRe\widetilde{A}_{\rho,\sigma}) * F_{\rho,\sigma})\quad .\end{aligned} \tag{C.56}$$

The square of the invariant amplitude is obtained in the form

$$\begin{aligned}|\mathcal{A}|^2 &= |\mathcal{A}_{VMD} + \mathcal{A}_{loop} + \mathcal{A}_\sigma|^2 \\ &= |\mathcal{A}_{VMD}|^2 + |\mathcal{A}_{loops}|^2 + [(ReA^0_{\omega,\sigma})^2 + (ImA^0_{\omega,\sigma})^2](F_{\omega,\sigma})^2 + (c^2+d^2) \\ &\quad \times((Re\widetilde{A}_{\rho,\sigma})^2 + (Im\widetilde{A}_{\rho,\sigma})^2)(F_{\rho,\sigma})^2 + 2[(ReA^0_{\omega 1}ReA^0_{\omega,\sigma} \\ &\quad + ImA^0_{\omega 1}ImA^0_{\omega,\sigma})(F_{\omega 1}F_{\omega,\sigma}) + (ReA^0_{\omega 2}ReA^0_{\omega,\sigma} + ImA^0_{\omega 2}ImA^0_{\omega,\sigma})\end{aligned}$$

$$
\begin{aligned}
&\times(F_{\omega 2}F_{\omega,\sigma}) + (ReA^0_{\omega,K}ReA^0_{\omega,\sigma} + ImA^0_{\omega,K}ImA^0_{\omega,\sigma})(F_{\omega,K}F_{\omega,\sigma}) \\
&+ReA^0_{\omega,\sigma}(cRe\widetilde{A}_{\rho,1} - dIm\widetilde{A}_{\rho,1})(F_{\omega,\sigma}F_{\rho 1}) + ImA^0_{\omega,\sigma}(cIm\widetilde{A}_{\rho,1} \\
&+dRe\widetilde{A}_{\rho,1})(F_{\omega,\sigma}F_{\rho 1}) + ReA^0_{\omega,\sigma}(cRe\widetilde{A}_{\rho,2} - dIm\widetilde{A}_{\rho,2})(F_{\omega,\sigma}F_{\rho 2}) \\
&+ImA^0_{\omega,\sigma}(cIm\widetilde{A}_{\rho,2} + dRe\widetilde{A}_{\rho,2})(F_{\omega,\sigma}F_{\rho 2}) + ReA^0_{\omega,\sigma}(cRe\widetilde{A}_{\rho,\pi} \\
&-dIm\widetilde{A}_{\rho,\pi})(F_{\omega,\sigma}F_{\rho,\pi}) + ImA^0_{\omega,\sigma}(cIm\widetilde{A}_{\rho,\pi} + dRe\widetilde{A}_{\rho,\pi})(F_{\omega,\sigma}F_{\rho,\pi}) \\
&+ReA^0_{\omega 1}(cRe\widetilde{A}_{\rho,\sigma} - dIm\widetilde{A}_{\rho,\sigma})(F_{\omega 1}F_{\rho,\sigma}) + ReA^0_{\omega 2}(cRe\widetilde{A}_{\rho,\sigma} \\
&-dIm\widetilde{A}_{\rho,\sigma})(F_{\omega 2}F_{\rho,\sigma}) + ReA^0_{\omega,K}(cRe\widetilde{A}_{\rho,\sigma} - dIm\widetilde{A}_{\rho,\sigma})(F_{\omega,K}F_{\rho,\sigma}) \\
&+ReA^0_{\omega,\sigma}(cRe\widetilde{A}_{\rho,\sigma} - dIm\widetilde{A}_{\rho,\sigma})(F_{\omega,\sigma}F_{\rho,\sigma}) + ImA^0_{\omega 1}(cIm\widetilde{A}_{\rho,\sigma} \\
&+dRe\widetilde{A}_{\rho,\sigma})(F_{\omega 1}F_{\rho,\sigma}) + ImA^0_{\omega 2}(cIm\widetilde{A}_{\rho,\sigma} + dRe\widetilde{A}_{\rho,\sigma})(F_{\omega 2}F_{\rho,\sigma}) \\
&+ImA^0_{\omega,K}(cIm\widetilde{A}_{\rho,\sigma} + dRe\widetilde{A}_{\rho,\sigma})(F_{\omega 2}F_{\rho,\sigma}) + ImA^0_{\omega,\sigma}(cIm\widetilde{A}_{\rho,\sigma} \\
&+dRe\widetilde{A}_{\rho,\sigma})(F_{\omega,\sigma}F_{\rho,\sigma}) + (cRe\widetilde{A}_{\rho,1} - dIm\widetilde{A}_{\rho,1})(cRe\widetilde{A}_{\rho,\sigma} - dIm\widetilde{A}_{\rho,\sigma}) \\
&\times(F_{\rho 1}F_{\rho,\sigma}) + (cIm\widetilde{A}_{\rho,1} + dRe\widetilde{A}_{\rho,1})(cIm\widetilde{A}_{\rho,\sigma} + dRe\widetilde{A}_{\rho,\sigma})(F_{\rho 1}F_{\rho,\sigma}) \\
&+(cRe\widetilde{A}_{\rho,2} - dIm\widetilde{A}_{\rho,2})(cRe\widetilde{A}_{\rho,\sigma} - dIm\widetilde{A}_{\rho,\sigma})(F_{\rho 2}F_{\rho,\sigma}) + (cIm\widetilde{A}_{\rho,2} \\
&+dRe\widetilde{A}_{\rho,2})(cIm\widetilde{A}_{\rho,\sigma} + dRe\widetilde{A}_{\rho,\sigma})(F_{\rho 2}F_{\rho,\sigma}) + (cRe\widetilde{A}_{\rho,\pi} - dIm\widetilde{A}_{\rho,\pi}) \\
&\times(cRe\widetilde{A}_{\rho,\sigma} - dIm\widetilde{A}_{\rho,\sigma})(F_{\rho,\pi}F_{\rho,\sigma}) + (cIm\widetilde{A}_{\rho,2} + dRe\widetilde{A}_{\rho,2})(cIm\widetilde{A}_{\rho,\sigma} \\
&+dRe\widetilde{A}_{\rho,\sigma})(F_{\rho,\pi}F_{\rho,\sigma})]
\end{aligned}
\tag{C.57}
$$

where

$$
(F_{\omega,\sigma})^2 = \left(\frac{e}{M_\omega}g_{\omega\sigma\gamma}\right)^2 (g_{\sigma\pi\pi}M_\sigma)^2 \left[\frac{2}{3}(k\cdot p)^2\right] \tag{C.58}
$$

$$(F_{\rho,\sigma})^2 = \left(\frac{eg_{\rho\pi\pi}}{2\pi^2 M_\pi^2}\right)^2 (g_{\sigma\pi\pi}M_\sigma)^4 \left[\frac{2}{3}(k\cdot p)^2\right] \tag{C.59}$$

$$\begin{aligned}(F_{\omega 1}F_{\omega,\sigma}) &= \left(\frac{e}{M_\rho}g_{\rho\pi\gamma}\right)\left(\frac{g_{\omega\rho\pi}}{M_\omega}\right)\left(\frac{e}{M_\omega}g_{\omega\sigma\gamma}\right)(g_{\sigma\pi\pi}M_\sigma) \\ &\times\frac{1}{3}\{k\cdot p[2k\cdot q_1 p^2 - p^2(k\cdot q_1)^2/k\cdot p + k\cdot p(-2k\cdot q_1 + q_1^2)]\}\end{aligned} \tag{C.60}$$

$$(F_{\omega 2}F_{\omega,\sigma}) = (F_{\omega 1}F_{\omega,\sigma})(q_1 \to q_2) \tag{C.61}$$

$$(F_{\omega,K}F_{\omega,\sigma}) = \left(\frac{eg}{4\sqrt{2}\pi^2 f_\pi^2 M_k^2}\right)\left(\frac{e}{M_\omega}g_{\omega\sigma\gamma}\right)(g_{\sigma\pi\pi}M_\sigma)\left[\frac{2}{3}(k\cdot p)^2\right] \tag{C.62}$$

$$\begin{aligned}(F_{\omega,\sigma}F_{\rho 1}) &= \left(\frac{e}{M_\omega}g_{\omega\sigma\gamma}\right)(g_{\sigma\pi\pi}M_\sigma)\left(\frac{e}{M_\omega}g_{\omega\pi\gamma}\right)\left(\frac{g_{\rho\omega\pi}}{M_\rho}\right) \\ &\times\frac{1}{3}\{k\cdot p[2k\cdot q_1 p^2 - p^2(k\cdot q_1)^2/k\cdot p + k\cdot p(-2k\cdot q_1 + q_1^2)]\}\end{aligned} \tag{C.63}$$

$$(F_{\omega,\sigma}F_{\rho 2}) = (F_{\omega,\sigma}F_{\rho 1})(q_1 \to q_2) \tag{C.64}$$

$$(F_{\omega,\sigma}F_{\rho,\pi}) = \left(\frac{e}{M_\omega}g_{\omega\sigma\gamma}\right)(g_{\sigma\pi\pi}M_\sigma)\left(\frac{e\ g_{\rho\pi\pi}\ \lambda}{2\pi^2 M_\pi^2}\right)\left[\frac{2}{3}(k\cdot p)^2\right] \tag{C.65}$$

$$\begin{aligned}(F_{\omega 1}F_{\rho,\sigma}) &= \left(\frac{e}{M_\rho}g_{\rho\pi\gamma}\right)\left(\frac{g_{\omega\rho\pi}}{M_\omega}\right)\left(\frac{eg_{\rho\pi\pi}}{2\pi^2 M_\pi^2}\right)(g_{\sigma\pi\pi}M_\sigma)^2 \\ &\times\frac{1}{3}\{k\cdot p[2k\cdot q_1 p^2 - p^2(k\cdot q_1)^2/k\cdot p + k\cdot p(-2k\cdot q_1 + q_1^2)]\}\end{aligned} \tag{C.66}$$

$$(F_{\omega 2}F_{\rho,\sigma}) = (F_{\omega 1}F_{\rho,\sigma})(q_1 \to q_2) \tag{C.67}$$

$$(F_{\omega,K}F_{\rho,\sigma}) = \left(\frac{eg}{4\sqrt{2}\pi^2 f_\pi^2 M_k^2}\right)\left(\frac{eg_{\rho\pi\pi}}{2\pi^2 M_\pi^2}\right)(g_{\sigma\pi\pi}M_\sigma)^2\left[\frac{2}{3}(k\cdot p)^2\right] \tag{C.68}$$

$$(F_{\omega,\sigma}F_{\rho,\sigma}) = \left(\frac{e}{M_\omega}g_{\omega\sigma\gamma}\right)(g_{\sigma\pi\pi}M_\sigma)^3\left(\frac{eg_{\rho\pi\pi}}{2\pi^2 M_\pi^2}\right)\left[\frac{2}{3}(k\cdot p)^2\right] \tag{C.69}$$

$$\begin{aligned}(F_{\rho 1}F_{\rho,\sigma}) = & \left(\frac{e}{M_\omega}g_{\omega\pi\gamma}\right)\left(\frac{g_{\rho\omega\pi}}{M_\rho}\right)\left(\frac{eg_{\rho\pi\pi}}{2\pi^2 M_\pi^2}\right)(g_{\sigma\pi\pi}M_\sigma)^2 \\ & \times\frac{1}{3}\{k\cdot p[2k\cdot q_1 p^2 - p^2(k\cdot q_1)^2/k\cdot p + k\cdot p(-2k\cdot q_1 + q_1^2)]\}\end{aligned} \tag{C.70}$$

$$(F_{\rho 2}F_{\rho,\sigma}) = (F_{\rho 1}F_{\rho,\sigma})(q_1 \rightarrow q_2) \tag{C.71}$$

$$(F_{\rho,\pi}F_{\rho,\sigma}) = \left(\frac{e\ g_{\rho\pi\pi}\ \lambda}{2\pi^2 M_\pi^2}\right)\left(\frac{eg_{\rho\pi\pi}}{2\pi^2 M_\pi^2}\right)(g_{\sigma\pi\pi}M_\sigma)^2\left[\frac{2}{3}(k\cdot p)^2\right] \quad . \tag{C.72}$$

C.4 The full amplitude of the $\rho \rightarrow \pi^0\pi^0\gamma$ decay including the $\rho - \omega$ mixing

The invariant amplitude of the radiative decay $\rho(p) \rightarrow \pi^0(q_1)\pi^0(q_2)\gamma(k)$ including the contribution of the $\rho - \omega$ mixing is $\mathcal{A}[\rho \rightarrow \pi^0\pi^0\gamma] = \mathcal{A}^0[\rho \rightarrow \pi^0\pi^0\gamma] + \varepsilon\widetilde{\mathcal{A}}[\omega \rightarrow \pi^0\pi^0\gamma]$ where $\mathcal{A}^0$ and $\widetilde{\mathcal{A}}$ include the contributions coming from the VMD amplitude, chiral loop and σ-meson intermediate states amplitudes that result from the Feynman diagrams in Fig. 3. 2 (a), (b) and (c) for $\rho \rightarrow \pi^0\pi^0\gamma$ decay and in Fig. 3. 1 (a), (b) and (c) for $\omega \rightarrow \pi^0\pi^0\gamma$ decay. Therefore, we can write the invariant amplitude including the effects of $\rho - \omega$ mixing for

$\rho \to \pi^0\pi^0\gamma$ decay as

$$\mathcal{A} = \mathcal{A}^0_{\rho,VMD} + \mathcal{A}^0_{\rho,\pi} + \mathcal{A}^0_{\rho,\sigma} + \varepsilon[\widetilde{\mathcal{A}}_{\omega,VMD} + \widetilde{\mathcal{A}}_{\omega,K} + \widetilde{\mathcal{A}}_{\omega,\sigma}] \quad . \tag{C.73}$$

For the $\rho \to \pi^0\pi^0\gamma$ decay the amplitudes, which are $\mathcal{A}^0_{\rho,VMD}$, $\mathcal{A}^0_{\rho,\pi}$ and $\mathcal{A}^0_{\rho,\sigma}$, are described in section C. 2. For the $\omega \to \pi^0\pi^0\gamma$ decay, we use the same amplitude which is described in section C.1, but for the calculation of the VMD amplitude in $\widetilde{\mathcal{A}}(\omega \to \pi^0\pi^0\gamma)$ we use the unmodified propagator for the intermediate vector meson. Therefore, we can write the contributions coming from the different amplitudes as

$$\begin{aligned}\mathcal{A}_{VMD} \quad = \quad & ReA^0_{\rho 1} * F_{\rho 1} + ReA^0_{\rho 2} * F_{\rho 2} + c(Re\widetilde{A}_{\omega 1} * F_{\omega 1} + Re\widetilde{A}_{\omega 2} * F_{\omega 2}) \\ & -d(Im\widetilde{A}_{\omega 1} * F_{\omega 1} + Im\widetilde{A}_{\omega 2} * F_{\omega 2}) + i[ImA^0_{\rho 1} * F_{\rho 1} + ImA^0_{\rho 2} * F_{\rho 2} \\ & +c(Im\widetilde{A}_{\omega 1} * F_{\omega 1} + Im\widetilde{A}_{\omega 2} * F_{\omega 2}) + d(Re\widetilde{A}_{\omega 1} * F_{\omega 1} + Re\widetilde{A}_{\omega 2} * F_{\omega 2})]\end{aligned} \tag{C.74}$$

$$\begin{aligned}\mathcal{A} &= \mathcal{A}_{VMD} + \mathcal{A}_{loop} \\ &= \mathcal{A}_{VMD} + ReA^0_{\rho,\pi} * F_{\rho,\pi} + (cRe\widetilde{A}_{\omega,K} - dIm\widetilde{A}_{\omega,K}) * F_{\omega,K} \\ &\quad +i[ImA^0_{\rho,\pi} * F_{\rho,\pi} + (dRe\widetilde{A}_{\omega,K} + cIm\widetilde{A}_{\omega,K}) * F_{\omega,K}]\end{aligned} \tag{C.75}$$

$$\begin{aligned}\mathcal{A} &= \mathcal{A}_{VMD} + \mathcal{A}_{loop} + \mathcal{A}_{\sigma} \\ &= \mathcal{A}_{VMD} + \mathcal{A}_{loops} + ReA^0_{\rho,\sigma} * F_{\rho,\sigma} + (cRe\widetilde{A}_{\rho,\sigma} - bIm\widetilde{A}_{\omega,\sigma}) * F_{\omega,\sigma} \\ &\quad +i[ImA^0_{\rho,\sigma} * F_{\rho,\sigma} + (cIm\widetilde{A}_{\omega,\sigma} + dRe\widetilde{A}_{\omega,\sigma}) * F_{\omega,\sigma}] \quad .\end{aligned} \tag{C.76}$$

The square of the full amplitude of $\rho \to \pi^0\pi^0\gamma$ decay including VMD, chiral loop and σ-meson intermediate state amplitude is obtained as

$$
\begin{aligned}
|\mathcal{A}|^2 &= |\mathcal{A}_{VMD} + \mathcal{A}_{loop} + \mathcal{A}_{\sigma}|^2 \\
&= [(ReA^0_{\rho 1})^2 + (ImA^0_{\rho 1})^2](F_{\rho 1})^2 + [(ReA^0_{\rho 2})^2 + (ImA^0_{\rho 2})^2](F_{\rho 2})^2 \\
&\quad +(c^2+d^2)\{[(Re\widetilde{A}_{\omega 1})^2 + (Im\widetilde{A}_{\omega 1})^2](F_{\omega 1})^2 + [(Re\widetilde{A}_{\omega 2})^2 \\
&\quad +(Im\widetilde{A}_{\omega 2})^2](F_{\omega 2})^2 + 2(Re\widetilde{A}_{\omega 1}Re\widetilde{A}_{\omega 2} + Im\widetilde{A}_{\omega 1}Im\widetilde{A}_{\omega 2})(F_{\omega 1}F_{\omega 2})\} \\
&\quad +2[(ReA^0_{\rho 1}ReA^0_{\rho 2} + ImA^0_{\rho 1}ImA^0_{\rho 2})(F_{\rho 1}F_{\rho 2}) + (ReA^0_{\rho 1} * F_{\rho 1} \\
&\quad +ReA^0_{\rho 2} * F_{\rho 2})(cRe\widetilde{A}_{\omega 1} * F_{\omega 1} - dIm\widetilde{A}_{\omega 1} * F_{\omega 1}) + (ImA^0_{\rho 1} * F_{\rho 1} \\
&\quad +ImA^0_{\rho 2} * F_{\rho 2})(cIm\widetilde{A}_{\omega 1} * F_{\omega 1} + dRe\widetilde{A}_{\omega 1} * F_{\omega 1}) + (ReA^0_{\rho 1} * F_{\rho 1} \\
&\quad +ReA^0_{\rho 2} * F_{\rho 2})(cRe\widetilde{A}_{\omega 2} * F_{\omega 2} - dIm\widetilde{A}_{\omega 2} * F_{\omega 2}) + (ImA^0_{\rho 1} * F_{\rho 1} \\
&\quad +ImA^0_{\rho 2} * F_{\rho 2})(cIm\widetilde{A}_{\omega 2} * F_{\omega 2} + dRe\widetilde{A}_{\omega 2} * F_{\omega 2}) + (ReA^0_{\rho,\pi})^2 \\
&\quad +(ImA^0_{\rho,\pi})^2] * (F_{\rho,\pi})^2 + (c^2+d^2)[(Re\widetilde{A}_{\omega,K})^2 + (Im\widetilde{A}_{\omega,K})^2] \\
&\quad \times(F_{\omega,K})^2 + 2[(ReA^0_{\rho 1}ReA^0_{\rho,\pi} + ImA^0_{\rho 1}ImA^0_{\rho,\pi}) * (F_{\rho 1}F_{\rho,\pi}) \\
&\quad +(ReA^0_{\rho 2}ReA^0_{\rho,\pi} + ImA^0_{\rho 2}ImA^0_{\rho,\pi})(F_{\rho 2}F_{\rho,\pi}) + ReA^0_{\rho,\pi} \\
&\quad \times(cRe\widetilde{A}_{\omega,1} - dIm\widetilde{A}_{\omega,1})(F_{\rho,\pi}F_{\omega 1}) + ImA^0_{\rho,\pi}(cIm\widetilde{A}_{\omega,1} \\
&\quad +dRe\widetilde{A}_{\omega,1})(F_{\rho,\pi}F_{\omega 1}) + ReA^0_{\rho,\pi}(cRe\widetilde{A}_{\omega,2} - dIm\widetilde{A}_{\omega,2})(F_{\rho,\pi}F_{\omega 2}) \\
&\quad +ImA^0_{\rho,\pi}(cIm\widetilde{A}_{\omega,2} + dRe\widetilde{A}_{\omega,2})(F_{\rho,\pi}F_{\omega 2}) + ReA^0_{\rho 1}(cRe\widetilde{A}_{\omega,K} \\
&\quad -dIm\widetilde{A}_{\omega,K})(F_{\rho 1}F_{\omega,K}) + ReA^0_{\rho 2}(cRe\widetilde{A}_{\omega,K} - dIm\widetilde{A}_{\omega,K})(F_{\rho 2}F_{\omega,K}) \\
&\quad +ReA^0_{\rho,\pi}(cRe\widetilde{A}_{\omega,K} - dIm\widetilde{A}_{\omega,K})(F_{\rho,\pi}F_{\omega,K}) + ImA^0_{\rho 1}(cIm\widetilde{A}_{\omega,K} \\
&\quad +dRe\widetilde{A}_{\omega,K})(F_{\rho 1}F_{\omega,K}) + ImA^0_{\rho 2}(cIm\widetilde{A}_{\omega,K} + dRe\widetilde{A}_{\omega,K})(F_{\rho 2}F_{\omega,K})
\end{aligned}
$$

$$+ImA^0_{\rho,\pi}(cIm\widetilde{A}_{\omega,K}+dRe\widetilde{A}_{\omega,K})(F_{\rho,\pi}F_{\omega,K})+(cRe\widetilde{A}_{\omega,1}-dIm\widetilde{A}_{\omega,1})$$

$$\times(cRe\widetilde{A}_{\omega,K}-dIm\widetilde{A}_{\omega,K})(F_{\omega1}F_{\omega,K})+(cIm\widetilde{A}_{\omega,1}+dRe\widetilde{A}_{\omega,1})(cIm\widetilde{A}_{\omega,K}$$

$$+bdRe\widetilde{A}_{\omega,K})(F_{\omega1}F_{\omega,K})+(cRe\widetilde{A}_{\omega,2}-dIm\widetilde{A}_{\omega,2})(cRe\widetilde{A}_{\omega,K}-dIm\widetilde{A}_{\omega,K})$$

$$\times(F_{\omega2}F_{\omega,K})+(cIm\widetilde{A}_{\omega,2}+dRe\widetilde{A}_{\omega,2})(cIm\widetilde{A}_{\omega,K}+dRe\widetilde{A}_{\omega,K})$$

$$\times(F_{\omega2}F_{\omega,K})+(ReA^0_{\rho,\sigma})^2+(ImA^0_{\rho,\sigma})^2](F_{\rho,\sigma})^2+(c^2+d^2)$$

$$\times((Re\widetilde{A}_{\omega,\sigma})^2+(Im\widetilde{A}_{\omega,\sigma})^2)(F_{\omega,\sigma})^2+2[(ReA^0_{\rho1}ReA^0_{\rho,\sigma}$$

$$+ImA^0_{\rho1}ImA^0_{\rho,\sigma})(F_{\rho1}F_{\rho,\sigma})+(ReA^0_{\rho2}ReA^0_{\rho,\sigma}+ImA^0_{\rho2}ImA^0_{\rho,\sigma})$$

$$\times(F_{\rho2}F_{\rho,\sigma})+(ReA^0_{\rho,\pi}ReA^0_{\rho,\sigma}+ImA^0_{\rho,\pi}ImA^0_{\rho,\sigma})(F_{\rho,\pi}F_{\rho,\sigma})$$

$$+ReA^0_{\rho,\sigma}(cRe\widetilde{A}_{\omega,1}-dIm\widetilde{A}_{\omega,1})(F_{\rho,\sigma}F_{\omega1})+ImA^0_{\rho,\sigma}(cIm\widetilde{A}_{\omega,1}$$

$$+dRe\widetilde{A}_{\omega,1})(F_{\rho,\sigma}F_{\omega1})+ReA^0_{\rho,\sigma}(cRe\widetilde{A}_{\omega,2}-dIm\widetilde{A}_{\omega,2})(F_{\rho,\sigma}F_{\omega2})$$

$$+ImA^0_{\rho,\sigma}(cIm\widetilde{A}_{\omega,2}+dRe\widetilde{A}_{\omega,2})(F_{\rho,\sigma}F_{\omega2})+ReA^0_{\rho,\sigma}(cRe\widetilde{A}_{\omega,K}$$

$$-dIm\widetilde{A}_{\omega,K})(F_{\rho,\sigma}F_{\omega,K})+ImA^0_{\rho,\sigma}(cIm\widetilde{A}_{\omega,K}+dRe\widetilde{A}_{\omega,K})(F_{\rho,\sigma}F_{\omega,K})$$

$$+ReA^0_{\rho1}(cRe\widetilde{A}_{\omega,\sigma}-dIm\widetilde{A}_{\omega,\sigma})(F_{\rho1}F_{\omega,\sigma})+ReA^0_{\rho2}(cRe\widetilde{A}_{\omega,\sigma}$$

$$-dIm\widetilde{A}_{\omega,\sigma})(F_{\rho2}F_{\omega,\sigma})+ReA^0_{\rho,\pi}(cRe\widetilde{A}_{\omega,\sigma}-dIm\widetilde{A}_{\omega,\sigma})(F_{\rho,\pi}F_{\omega,\sigma})$$

$$+ReA^0_{\rho,\sigma}(cRe\widetilde{A}_{\omega,\sigma}-dIm\widetilde{A}_{\omega,\sigma})(F_{\rho,\sigma}F_{\omega,\sigma})+ImA^0_{\rho1}(cIm\widetilde{A}_{\omega,\sigma}$$

$$+dRe\widetilde{A}_{\omega,\sigma})(F_{\rho1}F_{\omega,\sigma})+ImA^0_{\rho2}(cIm\widetilde{A}_{\omega,\sigma}+dRe\widetilde{A}_{\omega,\sigma})(F_{\rho2}F_{\omega,\sigma})$$

$$+ImA^0_{\rho,\pi}(cIm\widetilde{A}_{\omega,\sigma}+dRe\widetilde{A}_{\omega,\sigma})(F_{\rho2}F_{\omega,\sigma})+ImA^0_{\rho,\sigma}(cIm\widetilde{A}_{\omega,\sigma}$$

$$+dRe\widetilde{A}_{\omega,\sigma})(F_{\rho,\sigma}F_{\omega,\sigma})+(cRe\widetilde{A}_{\omega1}-dIm\widetilde{A}_{\omega1})(cRe\widetilde{A}_{\omega,\sigma}-dIm\widetilde{A}_{\omega,\sigma})$$

$$\times(F_{\omega1}F_{\omega,\sigma})+(cIm\widetilde{A}_{\omega1}+dRe\widetilde{A}_{\omega1})(cIm\widetilde{A}_{\omega,\sigma}+dRe\widetilde{A}_{\omega,\sigma})(F_{\omega1}F_{\omega,\sigma})$$

$$+(cRe\widetilde{A}_{\omega2}-dIm\widetilde{A}_{\omega2})(cRe\widetilde{A}_{\omega,\sigma}-dIm\widetilde{A}_{\omega,\sigma})(F_{\omega2}F_{\omega,\sigma})+(cIm\widetilde{A}_{\omega2}$$

$$+dRe\widetilde{A}_{\omega 2})(cIm\widetilde{A}_{\omega,\sigma} + dRe\widetilde{A}_{\omega,\sigma})(F_{\omega 2}F_{\omega,\sigma}) + (cRe\widetilde{A}_{\omega,K} - dIm\widetilde{A}_{\omega,K})$$

$$\times(cRe\widetilde{A}_{\omega,\sigma} - dIm\widetilde{A}_{\omega,\sigma})(F_{\omega,K}F_{\omega,\sigma}) + (cIm\widetilde{A}_{\omega 2} + dRe\widetilde{A}_{\omega 2})(cIm\widetilde{A}_{\omega,\sigma}$$

$$+dRe\widetilde{A}_{\omega,\sigma})(F_{\omega,K}F_{\omega,\sigma})] \tag{C.77}$$

The full amplitude of the $\rho \to \pi^0\pi^0\gamma$ decay can then be calculated using the same expressions as for the $\omega \to \pi^0\pi^0\gamma$ decay described in detail in the previous section.

APPENDIX D

INVARIANT AMPLITUDE FOR THE $V \to \pi^+\pi^-\gamma$ DECAY

The invariant amplitude of the radiative decay $V(p) \to \pi^+(q_1)\pi^-(q_2)\gamma(k)$ including the contribution of the $\rho-\omega$ mixing is $\mathcal{A}[V \to \pi^+\pi^-\gamma] = \mathcal{A}^o[V \to \pi^+\pi^-\gamma] + \varepsilon\widetilde{\mathcal{A}}[V' \to \pi^+\pi^-\gamma]$ where $V,\ V' = \rho,\ \omega$, and $\mathcal{A}^0$ and $\widetilde{\mathcal{A}}$ include the contributions coming from the different amplitudes that follow from the Feynman diagrams in Fig. 3. 6 (a), (b) and (c) for $\omega \to \pi^+\pi^-\gamma$ decay and in Fig. 3. 7 (a), (b) and (c) for $\rho \to \pi^+\pi^-\gamma$ decay. Moreover in the calculation of VMD amplitude in A^0 modified propagator is used for the intermediate vector meson given by Eqs. (3.19) and (3.20).

D.1 $\omega \to \pi^+\pi^-\gamma$ decay including $\rho-\omega$ mixing

For the $\omega(p) \to \pi^+(q_1)\pi^-(q_2)\gamma(k)$ decay we consider the contributions of VMD and σ-meson intermediate state amplitudes, and $\rho-\omega$ mixing. The total amplitude of this decay is $\mathcal{A}[\omega \to \pi^+\pi^-\gamma] = \mathcal{A}^0[\omega \to \pi^+\pi^-\gamma] + \varepsilon\widetilde{\mathcal{A}}[\rho \to \pi^+\pi^-\gamma]$. Therefore, we can express the invariant amplitude for $\omega \to \pi^+\pi^-\gamma$ decay as

$$\mathcal{A} = \mathcal{A}^0_{\omega,VMD} + \mathcal{A}^0_{\omega,\sigma} + \varepsilon[\widetilde{\mathcal{A}}_{brems.} + \widetilde{\mathcal{A}}_{\rho,\pi} + \widetilde{\mathcal{A}}_{\rho,\sigma}] \tag{D.1}$$

where

$$\mathcal{A}^0_{\omega,VMD} = ReA^0_{\omega1} * F_{\omega1} + ReA^0_{\omega2} * F_{\omega2} + i[ImA^0_{\omega1} * F_{\omega1} + ImA^0_{\omega2} * F_{\omega2}] \quad \text{(D.2)}$$

$$\mathcal{A}^0_{\omega,\sigma} = ReA^0_{\omega,\sigma} * F_{\omega,\sigma} + iImA^0_{\omega,\sigma} * F_{\omega,\sigma} \quad \text{(D.3)}$$

$$\widetilde{\mathcal{A}}_{brems.} = i[\widetilde{A}_1 * F_{b1} + \widetilde{A}_2 * F_{b2} + \widetilde{A}_3 * F_{b3}] \quad \text{(D.4)}$$

$$\widetilde{\mathcal{A}}_{\rho,\pi} = Re\widetilde{A}_{\rho,\pi} * F_{\rho,\pi} + iIm\widetilde{A}_{\rho,\pi} * F_{\rho,\pi} \quad \text{(D.5)}$$

$$\widetilde{\mathcal{A}}_{\rho,\sigma} = Re\widetilde{A}_{\rho,\sigma} * F_{\rho,\sigma} + iIm\widetilde{A}_{\rho,\sigma} * F_{\rho,\sigma} \quad . \quad \text{(D.6)}$$

For the $\omega \to \pi^+\pi^-\gamma$ decay the terms including VMD and σ-meson intermediate state amplitude $ReA^0_{\omega1}$, $ReA^0_{\omega2}$,$F_{\omega1}$,$F_{\omega2}$, $ImA^0_{\omega1}$, $ImA^0_{\omega2}$, $ReA^0_{\omega,\sigma}$,$ImA^0_{\omega,\sigma}$,$F_{\omega,\sigma}$, are defined in Appendix C for $\omega \to \pi^0\pi^0\gamma$ decay. Also, for the $\rho \to \pi^+\pi^-\gamma$ decay the terms coming from the pion-loop and σ-meson intermediate state amplitude, $Re\widetilde{A}_{\rho,\pi}$, $Im\widetilde{A}_{\rho,\pi}$,$F_{\rho,\pi}$, $Re\widetilde{A}_{\rho,\sigma}$, $Im\widetilde{A}_{\rho,\sigma}$,$F_{\rho,\sigma}$, are given in Appendix C for the $\rho \to \pi^0\pi^0\gamma$ decay. Therefore, the contributions coming from the bremsstrahlung amplitude are

$$\widetilde{A}_1 = \frac{1}{[(p-q_2)^2 - M_\pi^2]} \quad \text{(D.7)}$$

$$\widetilde{A}_2 = \frac{1}{[(p-q_1)^2 - M_\pi^2]} \quad \text{(D.8)}$$

$$F_{b1} = -4eg_{\rho\pi\pi}U_\mu q_2^\mu q_{1\nu}\varepsilon^\nu \quad \text{(D.9)}$$

$$F_{b2} = -4eg_{\rho\pi\pi}U_\mu q_1^\mu q_{2\nu}\varepsilon^\nu \quad \text{(D.10)}$$

$$F_{b3} = -2eg_{\rho\pi\pi}U_\mu\varepsilon^\mu \quad . \quad \text{(D.11)}$$

The full amplitude including all the contributions coming from the VMD, chiral loop, σ-meson intermediate state amplitudes and $\rho - \omega$ mixing can be written as

$$\begin{aligned}
\mathcal{A} &= ReA^0_{\omega 1} * F_{\omega 1} + ReA^0_{\omega 2} * F_{\omega 1} + ReA^0_{\omega,\sigma} * F_{\omega,\sigma} \\
&\quad + c(Re\widetilde{A}_{\rho,\pi} * F_{\rho,\pi} + Re\widetilde{A}_{\rho,\sigma} * F_{\rho,\sigma}) - d(\widetilde{A}_1 * F_{b1} + \widetilde{A}_2 * F_{b2} \\
&\quad + \widetilde{A}_3 * F_{b3} + Im\widetilde{A}_{\rho,\pi} * F_{\rho,\pi} + Im\widetilde{A}_{\rho,\sigma} * F_{\rho,\sigma}) + i[ImA^0_{\omega 1} * F_{\omega 1} \\
&\quad + ImA^0_{\omega 2} * F_{\omega 2} + ImA^0_{\omega,\sigma} * F_{\omega,\sigma} + c(\widetilde{A}_1 * F_{b1} + \widetilde{A}_2 * F_{b2} \\
&\quad + \widetilde{A}_3 * F_{b3} + Im\widetilde{A}_{\rho,\pi} * F_{\rho,\pi} + Im\widetilde{A}_{\rho,\pi}) * F_{\rho,\pi} + d(Re\widetilde{A}_{\rho,\pi} * F_{\rho,\pi} \\
&\quad + Re\widetilde{A}_{\rho,\sigma} * F_{\rho,\sigma})] \quad . \qquad \text{(D.12)}
\end{aligned}$$

The square of the total amplitude is obtained as

$$\begin{aligned}
|\mathcal{A}|^2 &= [(ReA^0_{\omega 1})^2 + (ImA^0_{\omega 1})^2](F_{\omega 1})^2 + [(ReA^0_{\omega 2})^2 + (ImA^0_{\omega 2})^2](F_{\omega 2})^2 \\
&\quad +[(ReA^0_{\omega,\sigma})^2 + (ImA^0_{\omega,\sigma})^2](F_{\omega,\sigma})^2 + (c^2 + d^2)\{(\widetilde{A}_1)^2(F_{b1})^2 \\
&\quad +(\widetilde{A}_2)^2(F_{b2})^2 + (F_{b3})^2 + [(Re\widetilde{A}_{\rho,\pi})^2 + (Im\widetilde{A}_{\rho,\pi})^2](F_{\rho,\pi})^2 \\
&\quad +[(Re\widetilde{A}_{\rho,\sigma})^2 + (Im\widetilde{A}_{\rho,\sigma})^2](F_{\rho,\sigma})^2 + 2[(\widetilde{A}_1 Im\widetilde{A}_{\rho,\pi})(F_{b1}F_{\rho,\pi}) \\
&\quad +(\widetilde{A}_1 Im\widetilde{A}_{\rho,\sigma})(F_{b1}F_{\rho,\sigma}) + (\widetilde{A}_2 Im\widetilde{A}_{\rho,\pi})(F_{b2}F_{\rho,\pi}) + (\widetilde{A}_2 Im\widetilde{A}_{\rho,\sigma}) \\
&\quad \times(F_{b2}F_{\rho,\sigma}) + Im\widetilde{A}_{\rho,\pi}(F_{b3}F_{\rho,\pi}) + Im\widetilde{A}_{\rho,\sigma}(F_{b3}F_{\rho,\sigma}) + (Re\widetilde{A}_{\rho,\pi}Re\widetilde{A}_{\rho,\sigma} \\
&\quad +Im\widetilde{A}_{\rho,\pi}Im\widetilde{A}_{\rho,\sigma})(F_{\rho,\pi}F_{\rho,\sigma})]\} + 2[(ReA^0_{\omega 1}ReA^0_{\omega 2} + ImA^0_{\omega 1}ImA^0_{\omega 2}) \\
&\quad \times(F_{\omega 1}F_{\omega 2}) + (ReA^0_{\omega 1}ReA^0_{\omega,\sigma} + ImA^0_{\omega 1}ImA^0_{\omega,\sigma})(F_{\omega 1}F_{\omega,\sigma}) \\
&\quad +(ReA^0_{\omega 2}ReA^0_{\omega,\sigma} + ImA^0_{\omega 2}ImA^0_{\omega,\sigma})(F_{\omega 2}F_{\omega,\sigma}) - ReA^0_{\omega 1}(d\widetilde{A}_1) \\
&\quad \times(F_{\omega 1}F_{b1}) - ReA^0_{\omega 2}(d\widetilde{A}_1)(F_{\omega 2}F_{b1}) - ReA^0_{\omega,\sigma}(d\widetilde{A}_1)(F_{\omega,\sigma}F_{b1})
\end{aligned}$$

$$
\begin{aligned}
&+ImA^0_{\omega 1}(c\widetilde{A}_1)(F_{\omega 1}F_{b1}) + ImA^0_{\omega 2}(c\widetilde{A}_1)(F_{\omega 2}F_{b1}) + ImA^0_{\omega,\sigma}(c\widetilde{A}_1) \\
&\times(F_{\omega,\sigma}F_{b1}) - ReA^0_{\omega 1}(d\widetilde{A}_2)(F_{\omega 1}F_{b2}) - ReA^0_{\omega 2}(d\widetilde{A}_2)(F_{\omega 2}F_{b2}) \\
&-ReA^0_{\omega,\sigma}(d\widetilde{A}_2)(F_{\omega,\sigma}F_{b2}) + ImA^0_{\omega 1}(c\widetilde{A}_2)(F_{\omega 1}F_{b2}) + ImA^0_{\omega 2}(c\widetilde{A}_2) \\
&\times(F_{\omega 2}F_{b2}) + ImA^0_{\omega,\sigma}(c\widetilde{A}_1)(F_{\omega,\sigma}F_{b2}) - ReA^0_{\omega 1}d(F_{\omega 1}F_{b3}) \\
&- ReA^0_{\omega 2}d(F_{\omega 2}F_{b3}) - ReA^0_{\omega,\sigma}d(F_{\omega,\sigma}F_{b3}) + ImA^0_{\omega 1}c(F_{\omega 1}F_{b3}) \\
&+ ImA^0_{\omega 2}c(F_{\omega 2}F_{b3}) + ImA^0_{\omega,\sigma}c(F_{\omega,\sigma}F_{b3}) - ReA^0_{\omega 1}(d\widetilde{A}_{\rho,\pi}) \\
&\times(F_{\omega 1}F_{\rho,\pi}) - ReA^0_{\omega 2}(d\widetilde{A}_{\rho,\pi})(F_{\omega 2}F_{\rho,\pi}) - ReA^0_{\omega,\sigma}(d\widetilde{A}_{\rho,\pi}) \\
&\times(F_{\omega,\sigma}F_{\rho,\pi}) + ImA^0_{\omega 1}(c\widetilde{A}_{\rho,\pi})(F_{\omega 1}F_{\rho,\pi}) + ImA^0_{\omega 2}(c\widetilde{A}_{\rho,\pi}) \\
&\times(F_{\omega 2}F_{\rho,\pi}) + ReA^0_{\omega,\sigma}(c\widetilde{A}_{\rho,\pi})(F_{\omega,\sigma}F_{\rho,\pi}) - ReA^0_{\omega 1}(d\widetilde{A}_{\rho,\sigma}) \\
&\times(F_{\omega 1}F_{\rho,\sigma}) - ReA^0_{\omega 2}(d\widetilde{A}_{\rho,\sigma})(F_{\omega 2}F_{\rho,\sigma}) - ReA^0_{\omega,\sigma}(d\widetilde{A}_{\rho,\sigma}) \\
&\times(F_{\omega,\sigma}F_{\rho,\sigma}) + ImA^0_{\omega 1}(c\widetilde{A}_{\rho,\sigma})(F_{\omega 1}F_{\rho,\sigma}) + ImA^0_{\omega 2}(c\widetilde{A}_{\rho,\sigma}) \\
&\times(F_{\omega 2}F_{\rho,\sigma}) + ReA^0_{\omega,\sigma}(c\widetilde{A}_{\rho,\sigma})(F_{\omega,\sigma}F_{\rho,\sigma})] \qquad \text{(D.13)}
\end{aligned}
$$

where the terms related to the VMD and σ-meson are defined in Appendix C and the terms related to the bremsstrahlung amplitude are given as

$$F^2_{b1} = \frac{1}{3}(q_1)^2\left[q_2^2 - \frac{(p\cdot q_2)^2}{M_\rho^2}\right] \qquad \text{(D.14)}$$

$$F^2_{b2} = \frac{1}{3}(q_2)^2\left[q_1^2 - \frac{(p\cdot q_1)^2}{M_\rho^2}\right] \qquad \text{(D.15)}$$

$$F^2_{b3} = (2eg_{\rho\pi\pi})^2 \qquad \text{(D.16)}$$

$$
\begin{aligned}
(F_{\omega 1} F_{b1}) &= \left(\frac{e}{M_\rho} g_{\rho\pi\gamma}\right)\left(\frac{g_{\omega\rho\pi}}{M_\omega}\right)(4 e g_{\rho\pi\pi}) \\
&\quad \times \frac{1}{3}\{k\cdot q_2[(p\cdot q_1)^2 - p^2 q_1^2] + k\cdot q_1[(-p\cdot q_1)(p\cdot q_2) \\
&\quad + p^2(q_1\cdot q_2) + k\cdot p\ [(p\cdot q_2)q_1^2 - (p\cdot q_1)(q_1\cdot q_2)]]\} \qquad \text{(D.17)}
\end{aligned}
$$

$$
(F_{\omega 2} F_{b1}) = (F_{\omega 1} F_{b1}) \qquad \text{(D.18)}
$$

$$
\begin{aligned}
(F_{\omega,\sigma} F_{b1}) &= \left(\frac{e}{M_\omega} g_{\omega\sigma\gamma}\right)(g_{\sigma\pi\pi} M_\sigma)(4 e g_{\rho\pi\pi}) \\
&\quad \times \left\{\frac{1}{3}[k\cdot p\ q_1\cdot q_2 - p\cdot q_1\ k\cdot q_2]\right\} \qquad \text{(D.19)}
\end{aligned}
$$

$$
(F_{\omega 1} F_{b2}) = (F_{\omega 1} F_{b1}) \qquad \text{(D.20)}
$$

$$
\begin{aligned}
(F_{\omega 2} F_{b2}) &= \left(\frac{e}{M_\rho} g_{\rho\pi\gamma}\right)\left(\frac{g_{\omega\rho\pi}}{M_\omega}\right)(4 e g_{\rho\pi\pi}) \\
&\quad \times \frac{1}{3}\{k\cdot q_2[(-p\cdot q_1)(p\cdot q_2) + p^2(q_1\cdot q_2)] + k\cdot q_1 \\
&\quad [(p\cdot q_2)^2 - p^2 q_2^2] + k\cdot p\ [(-p\cdot q_2)(q_1\cdot q_2) + (p\cdot q_1)q_2^2]\} \qquad \text{(D.21)}
\end{aligned}
$$

$$
\begin{aligned}
(F_{\omega,\sigma} F_{b2}) &= \left(\frac{e}{M_\omega} g_{\omega\sigma\gamma}\right)(g_{\sigma\pi\pi} M_\sigma)(4 e g_{\rho\pi\pi}) \\
&\quad \times \left\{\frac{1}{3}[k\cdot p\ q_1\cdot q_2 - p\cdot q_2\ k\cdot q_1]\right\} \qquad \text{(D.22)}
\end{aligned}
$$

$$
\begin{aligned}
(F_{\omega 1} F_{b3}) &= \left(\frac{e}{M_\rho} g_{\rho\pi\gamma}\right)\left(\frac{g_{\omega\rho\pi}}{M_\omega}\right)(2 e g_{\rho\pi\pi}) \\
&\quad \times \left\{\frac{2}{3}\left[k\cdot q_1(p^2 - p\cdot q_1) + k\cdot p\ (-p\cdot q_1 + q_1^2)\right]\right\} \qquad \text{(D.23)}
\end{aligned}
$$

$$(F_{\omega 2}F_{b3}) = (F_{\omega 2}F_{b1})(q_1 \to q_2) \tag{D.24}$$

$$(F_{\omega,\sigma}F_{b3}) = \left(\frac{e}{M_\omega} g_{\omega\sigma\gamma}\right)(g_{\sigma\pi\pi}M_\sigma)(2eg_{\rho\pi\pi})(k\cdot p) \tag{D.25}$$

$$(F_{b1}F_{b2}) = (4eg_{\rho\pi\pi})^2\left\{\frac{1}{3}(q_1\cdot q_2)\left[q_1\cdot q_2 - \frac{(p\cdot q_1)(p._{q_2})}{M_\rho^2}\right]\right\} \tag{D.26}$$

$$(F_{b1}F_{b3}) = 8(eg_{\rho\pi\pi})^2\left\{\frac{1}{3}\left[q_1\cdot q_2 - \frac{(p\cdot q_1)(p\cdot q_2)}{M_\rho^2}\right]\right\} \tag{D.27}$$

$$(F_{b2}F_{b3}) = (F_{b1}F_{b3}) \tag{D.28}$$

$$\begin{aligned}(F_{b1}F_{\rho,\pi}) &= (4eg_{\rho\pi\pi})\left(\frac{eg_{\rho\pi\pi}}{2\pi^2M_\pi^2}\right)\ \lambda \\ &\quad\times\left\{\frac{1}{3}\left[(p\cdot k)(q_1\cdot q_2)-(p\cdot q_1)(k\cdot q_2)\right]\right\}\end{aligned} \tag{D.29}$$

$$\begin{aligned}(F_{b1}F_{\rho,\sigma}) &= 4(eg_{\rho\pi\pi})^2\left(\frac{(g_{\sigma\pi\pi}M_\sigma)^2}{2\pi^2M_\pi^2}\right) \\ &\quad\times\left\{\frac{1}{3}\left[(p\cdot k)(q_1\cdot q_2)-(p\cdot q_1)(k\cdot q_2)\right]\right\}\end{aligned} \tag{D.30}$$

$$\begin{aligned}(F_{b2}F_{\rho,\pi}) &= (4eg_{\rho\pi\pi})\left(\frac{eg_{\rho\pi\pi}}{2\pi^2M_\pi^2}\right)\ \lambda \\ &\quad\times\left\{\frac{1}{3}\left[(k\cdot p)(q_1\cdot q_2)-(p\cdot q_2)(k\cdot q_1)\right]\right\}\end{aligned} \tag{D.31}$$

$$(F_{b2}F_{\rho,\sigma}) = (F_{b1}F_{\rho,\sigma}) \tag{D.32}$$

$$(F_{b3}F_{\rho,\pi}) = \left(\frac{2(eg_{\rho\pi\pi})^2}{2\pi^2 M_\pi^2}\right) \lambda\, (p\cdot k) \tag{D.33}$$

$$(F_{b3}F_{\rho,\sigma}) = \left(\frac{2(eg_{\rho\pi\pi})^2}{2\pi^2 M_\pi^2}\right) (g_{\sigma\pi\pi}M_\sigma)^2\, (p\cdot k) \quad . \tag{D.34}$$

D.2 $\rho \to \pi^+\pi^-\gamma$ decay including $\rho-\omega$ mixing

For the $\rho(p) \to \pi^+(q_1)\pi^-(q_2)\gamma(k)$ decay we consider the contributions of pion-bremsstrahlung amplitudes, pion-loop, σ-meson intermediate state amplitude and $\rho-\omega$ mixing. Total amplitude of $\rho \to \pi^+\pi^-\gamma$ decay is $\mathcal{A}[\rho \to \pi^+\pi^-\gamma] = \mathcal{A}^0[\rho \to \pi^+\pi^-\gamma] + \varepsilon\widetilde{\mathcal{A}}[\omega \to \pi^+\pi^-\gamma]$. Therefore, we can express the invariant amplitude as

$$\mathcal{A} = \mathcal{A}^0_{brems.} + \mathcal{A}^0_{\rho,\pi} + \mathcal{A}^0_{\rho,\sigma} + \varepsilon[\widetilde{\mathcal{A}}_{\omega,VMD} + \widetilde{\mathcal{A}}_{\omega,\sigma}] \tag{D.35}$$

where

$$\mathcal{A}^0_{brems.} = i[A_1^0 * F_{b1} + A_2^0 * F_{b2} + A_3^0 * F_{b3}] \tag{D.36}$$

$$\mathcal{A}^0_{\rho,\pi} = ReA^0_{\rho,\pi} * F_{\rho,\pi} + iImA^0_{\rho,\pi} * F_{\rho,\pi} \tag{D.37}$$

$$\mathcal{A}^0_{\rho,\sigma} = ReA^0_{\rho,\sigma} * F_{\rho,\sigma} + iImA^0_{\rho,\sigma} * F_{\rho,\sigma} \tag{D.38}$$

$$\begin{aligned}\widetilde{\mathcal{A}}_{\omega,VMD} &= Re\widetilde{A}_{\omega1} * F_{\omega1} + Re\widetilde{A}_{\omega2} * F_{\omega2} \\ &\quad + i[Im\widetilde{A}_{\omega1} * F_{\omega1} + Im\widetilde{A}_{\omega2} * F_{\omega2}]\end{aligned} \tag{D.39}$$

$$\widetilde{\mathcal{A}}_{\omega,\sigma} = Re\widetilde{A}_{\omega,\sigma} * F_{\omega,\sigma} + iIm\widetilde{A}_{\omega,\sigma} * F_{\omega,\sigma} \quad . \tag{D.40}$$

For the $\rho \to \pi^+\pi^-\gamma$ decay, we use the amplitudes $\mathcal{A}^0_{\rho,\pi}$, $\mathcal{A}^0_{\rho,\sigma}$ as described in section C.2. Similarly, for the $\omega \to \pi^+\pi^-\gamma$ decay we utilize the same amplitudes for VMD and σ-meson intermediate amplitude which are described in section C.1, but in VMD amplitude in $\widetilde{\mathcal{A}}(\omega \to \pi^0\pi^0\gamma)$ we use unmodified propagator for the vector meson.

For the $\rho \to \pi^+\pi^-\gamma$ decay the square of the full amplitude including bremsstrahlung amplitudes, pion-loop, VMD and σ-meson intermediate state amplitude is obtained as

$$
\begin{aligned}
|\mathcal{A}|^2 \;=\;& (A_1^0)^2(F_{b1})^2 + (A_2^0)^2(F_{b2})^2 + (F_{b3})^2 + [(ReA^0_{\rho,\pi})^2 + (ImA^0_{\rho,\pi})^2] \\
& \times(F_{\rho,\pi})^2 + [(ReA^0_{\rho,\sigma})^2 + (ImA^0_{\rho,\sigma})^2](F_{\rho,\sigma})^2 + (c^2+d^2)[(Re\widetilde{A}_{\omega 1})^2 \\
& +(Im\widetilde{A}_{\omega 1})^2](F_{\omega 1})^2 + [(Re\widetilde{A}_{\omega 2})^2 + (Im\widetilde{A}_{\omega 2})^2](F_{\omega 2})^2 + (Re\widetilde{A}_{\omega,\sigma})^2 \\
& +(Im\widetilde{A}_{\omega,\sigma})^2](F_{\omega,\sigma})^2 + 2[(A_1^0)(A_2^0)(F_{b1})(F_{b2}) + (A_1^0)(F_{b1})(F_{b3}) \\
& +(A_1^0)(ReA^0_{\rho,\pi})(F_{b1})(F_{\rho,\pi}) + (A_1^0)(ReA^0_{\rho,\sigma})(F_{b1})(F_{\rho,\sigma}) \\
& +(A_2^0)(F_{b2})(F_{b3}) + (A_2^0)(ReA^0_{\rho,\pi})(F_{b2})(F_{\rho,\pi}) + (A_2^0)(ReA^0_{\rho,\sigma}) \\
& \times(F_{b2})(F_{\rho,\sigma}) + (ReA^0_{\rho,\pi})(F_{b3})(F_{\rho,\pi}) + (ReA^0_{\rho,\sigma})(F_{b3})(F_{\rho,\sigma}) \\
& +[(ReA^0_{\rho,\pi})(ReA^0_{\rho,\sigma}) + (ImA^0_{\rho,\pi})(ImA^0_{\rho,\sigma})](F_{\rho,\pi})(F_{\rho,\sigma}) \\
& +(ReA^0_{\rho,\pi})(Re\widetilde{A}_{\omega 1})(F_{\rho,\pi})(cF_{\omega 1}) + (ReA^0_{\rho,\sigma})(Re\widetilde{A}_{\omega 1})(F_{\rho,\sigma})(cF_{\omega 1}) \\
& -(ReA^0_{\rho,\pi})(Im\widetilde{A}_{\omega 1})(F_{\rho,\pi})(dF_{\omega 1}) - (ReA^0_{\rho,\sigma})(Im\widetilde{A}_{\omega 1})(F_{\rho,\sigma})(dF_{\omega 1}) \\
& +(A_1^0)(cIm\widetilde{A}_{\omega 1} + dRe\widetilde{A}_{\omega 1})(F_{b1})(F_{\omega 1}) + (A_2^0)(cIm\widetilde{A}_{\omega 1} + dRe\widetilde{A}_{\omega 1}) \\
& \times(F_{b2})(F_{\omega 1}) + (cIm\widetilde{A}_{\omega 1} + dRe\widetilde{A}_{\omega 1})(F_{b3})(F_{\omega 1}) + (ImA^0_{\rho,\pi})(cIm\widetilde{A}_{\omega 1} \\
& +dRe\widetilde{A}_{\omega 1})(F_{\rho,\pi})(F_{\omega 1}) + (ImA^0_{\rho,\sigma})(cIm\widetilde{A}_{\omega 1} + dRe\widetilde{A}_{\omega 1})(F_{\rho,\sigma})(F_{\omega 1})
\end{aligned}
$$

$$
\begin{aligned}
&+(ReA^0_{\rho,\pi})(Re\widetilde{A}_{\omega 2})(F_{\rho,\pi})(cF_{\omega 2}) + (ReA^0_{\rho,\sigma})(Re\widetilde{A}_{\omega 2})(F_{\rho,\sigma})(cF_{\omega 2}) \\
&-(ReA^0_{\rho,\pi})(Im\widetilde{A}_{\omega 2})(F_{\rho,\pi})(dF_{\omega 2}) - (ReA^0_{\rho,\sigma})(Im\widetilde{A}_{\omega 2})(F_{\rho,\sigma})(dF_{\omega 2}) \\
&+(A^0_1)(cIm\widetilde{A}_{\omega 2} + dRe\widetilde{A}_{\omega 2})(F_{b1})(F_{\omega 2}) + (A^0_2)(cIm\widetilde{A}_{\omega 2} + dRe\widetilde{A}_{\omega 2}) \\
&\times(F_{b2})(F_{\omega 2}) + (cIm\widetilde{A}_{\omega 2} + dRe\widetilde{A}_{\omega 2})(F_{b3})(F_{\omega 2}) + (ImA^0_{\rho,\pi})(cIm\widetilde{A}_{\omega 2} \\
&+dRe\widetilde{A}_{\omega 2})(F_{\rho,\pi})(F_{\omega 2}) + (ImA^0_{\rho,\sigma})(cIm\widetilde{A}_{\omega 2} + dRe\widetilde{A}_{\omega 2})(F_{\rho,\sigma})(F_{\omega 2}) \\
&+(ReA^0_{\rho,\pi})(Re\widetilde{A}_{\omega,\sigma})(F_{\rho,\pi})(cF_{\omega,\sigma}) + (ReA^0_{\rho,\sigma})(Re\widetilde{A}_{\omega,\sigma})(F_{\rho,\sigma})(cF_{\omega,\sigma}) \\
&-(ReA^0_{\rho,\pi})(Im\widetilde{A}_{\omega,\sigma})(F_{\rho,\pi})(dF_{\omega,\sigma}) - (ReA^0_{\rho,\sigma})(Im\widetilde{A}_{\omega,\sigma})(F_{\rho,\sigma})(dF_{\omega,\sigma}) \\
&+(A^0_1)(cIm\widetilde{A}_{\omega,\sigma} + dRe\widetilde{A}_{\omega,\sigma})(F_{b1})(F_{\omega,\sigma}) + (A^0_2)(cIm\widetilde{A}_{\omega,\sigma} + dRe\widetilde{A}_{\omega,\sigma}) \\
&\times(F_{b2})(F_{\omega,\sigma}) + (cIm\widetilde{A}_{\omega,\sigma} + dRe\widetilde{A}_{\omega,\sigma})(F_{b3})(F_{\omega,\sigma}) + (ImA^0_{\rho,\pi})(cIm\widetilde{A}_{\omega,\sigma} \\
&+dRe\widetilde{A}_{\omega,\sigma})(F_{\rho,\pi})(F_{\omega,\sigma}) + (ImA^0_{\rho,\sigma})(cIm\widetilde{A}_{\omega,\sigma} + dRe\widetilde{A}_{\omega,\sigma})(F_{\rho,\sigma})(F_{\omega,\sigma}) \\
&+(c^2 + d^2)[(Re\widetilde{A}_{\omega 1})(Re\widetilde{A}_{\omega 2})(F_{\omega 1})(F_{\omega 2}) + (Re\widetilde{A}_{\omega 1})(Re\widetilde{A}_{\omega,\sigma})(F_{\omega 1}) \\
&\times(F_{\omega,\sigma}) + (Re\widetilde{A}_{\omega 2})(Re\widetilde{A}_{\omega,\sigma})(F_{\omega 2})(F_{\omega,\sigma}) + (Im\widetilde{A}_{\omega 1})(Im\widetilde{A}_{\omega 2})(F_{\omega 1}) \\
&\times(F_{\omega 2}) + (Im\widetilde{A}_{\omega 1})(Im\widetilde{A}_{\omega,\sigma})(F_{\omega 1})(F_{\omega,\sigma}) + (Im\widetilde{A}_{\omega 2})(Im\widetilde{A}_{\omega,\sigma})(F_{\omega 2}) \\
&\times(F_{\omega,\sigma})]] \qquad \text{(D.41)}
\end{aligned}
$$

The square of the full amplitude of the $\rho \to \pi^+\pi^-\gamma$ decay can be calculated using the similar expressions which are described for the square of the invariant amplitude of the $\omega \to \pi^+\pi^-\gamma$ decay.

VITA

Ayşe Küçükarslan, born on December 28, 1970, received her B.Sc. degree in 1992 and her M.Sc. in 1996 from the Department of Physics in Black Sea Technical University. She worked as an assistant director in family firm besides her academic studies from 1992 onwards. Her main area of interest is the theoretical study of high energy nuclear physics, in particular scalar mesons and the role of $\rho - \omega$ mixing in radiative vector meson decays.

Apart from these, she has been studying ceramics and held various exhibitions on costume design.