# DYNAMIC SIMULATION AND PERFORMANCE OPTIMIZATION OF A CAR WITH CONTINUOUSLY VARIABLE TRANSMISSION

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# ABSTRACT

DYNAMIC SIMULATION AND PERFORMANCE OPTIMIZATION OF A CAR WITH CONTINUOUSLY VARIABLE TRANSMISSION

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The continuously variable transmission (CVT), which has been in use in some of the vehicles in the market today, presents the possibility of decoupling the engine speed and the vehicle speed. By this way, it is now possible to operate the engine at its maximum efficient or performance point and fix it at that operating point without losing from the vehicle speed. Instead of using gears, which are the main transmission elements of conventional transmission, CVT uses two pulleys and a belt. By changing the pulley diameters, a continuously variable transmission ratio is obtained. Besides all its advantages, it has some big drawbacks like low efficiency, torque transmission ability and limited speed range. With developing technology, however, new solutions are developed to eliminate these drawbacks.

In this study simulation models for the performance and fuel consumption of different types and arrangements of continuously variable transmission (CVT) systems are developed. Vehicles, which are equipped with two different arrangements of CVT and an automatic transmission, are modelled by using Matlab's simulation toolbox Simulink. By defining the required operating points for better acceleration performance and fuel consumption, and operating the engine at these points, performance optimization is satisfied. These transmissions are compared with each other according to their '0-100 kph' acceleration performances, maximum speeds, required time to travel 1000 m. and fuel consumptions for European driving cycles ECE and EUDC.

These comparisons show that CVT systems are superior to automatic transmission, according to their acceleration and fuel consumption performances. CVTs also provide smoother driving, while they can eliminate jerks at gear shifting points.

**Keywords:** CVT, dynamic simulation, fuel consumption, performance

# ÖZ

SÜREKLİ DEĞİŞKEN TRANSMİSYONLU BİR ARABANIN DİNAMİK SİMULASYONU VE PERFORMANS OPTİMİZASYONU

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Bugün pazarda bulunan birkaç araçta kullanılanılmakta olan, sürekli değişken transmisyon (CVT), motor hızı ile araç hızını birbirinden ayırabilme imkanı sunuyor. Bu yolla, motoru maksimum verimli veya maksimum performansa sahip olduğu noktada çalıştırmak ve araç hızından kaybetmeden o çalışma noktasında tutmak mümkün olmaktadır. Geleneksel bir transmisyonun ana transmisyon elemanları olan dişlileri kullanmak yerine, CVT, iki kasnak ve bir kayış kullanmaktadır. Kasnak çapları değiştirilerek, sürekli değişken bir transmisyon oranı sağlanmaktadır. Bütün ajantajlarının yanı sıra, düşük verim ve

tork aktarabilme yeteneği, ve sınırlı hız aralığı gibi bazı büyük dezavantajlara sahiptir. Ancak yakın gelecekte bu dezavantajları elimine edecek yeni çözümler üretilmesi beklenmektedir**.**

Bu tezde, değişik tip ve düzenlemelerden oluşan sürekli değişken transmisyon (CVT) sistemlerinin performans ve yakıt sarfiyatlarını incelemk üzere simulasyon modelleri hazırlanmıştır. İki farklı sürekli değişken transmisyon düzenlemesi ve bir otomatik transmisyon ile donatılmış araçlar, Matlab'ın simulasyon aracı Simulink kullanılarak modellenmiştir. Daha iyi ivmelenme performansı ve yakıt sarfiyatı için gerekli çalışma noktaları tanımlanıp, motor bu noktalarda çalıştırılarak, performans optimizasyonu sağlanmıştır. Bu transmisyonlar birbirleriyle, '0-100 km/s' ivmelenme performası, maksimum hız, 1000 m seyahat için gerekli  zaman ve Avrupa sürüş turları olan ECE ve EUDC için gerekli yakıt sarfiyatı açısından karşılaştırılmaktadır.

Bu karşılaştırma, CVT sistemlerinin, ivmelenme ve yakıt sarfiyatı açısından otomatik viteslerden üstün olduğunu göstermiştir. Ayrıca CVTler, vites değişimlerindeki sarsıntıları elimine edebildikleri için daha sarsıntısız bir sürüş sağlamaktadır.

**Anahtar Kelimeler:** Sürekli Değişken Transmisyon, dinamik simulasyon, yakıt sarfiyatı, performans

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| | |
|---|---|
| $a^*$, $b^*$ | Rolling resistance coefficients |
| $A_f$ | Frontal Area |
| bsfc | Basic Specific Fuel Consumption |
| $C_D$ | Air Drag Coefficient |
| $C_r$ | Torque Ratio |
| CVPST | Continuously Variable Power Split Transmission |
| CVT | Continuously Variable Transmission |
| ECE | European Urban Cycle |
| EUDC | Extra Urban Driving Cycle |
| $F_D$ | Aerodynamic Drag |
| $F_{res}$ | Resistance Force |
| $F_t$ | Tractive Force |
| G | Gradient in percentages |
| K | Capacity Factor |
| i | Speed Ratio |
| $i_{cg}$ | Speed Ratio of Control Gear |
| $i_{cvt}$ | Speed Ratio of CVT |
| $i_g$ | Speed Ratio of Counter-shaft Gear |
| $i_{step}$ | Step-up Gear Ratio |
| $i_t$ | Total Transmission Ratio |
| $J_e$ | Engine Rotational Inertia |
| $J_p$ | Primary Pulley Rotational Inertia |

| | |
|---|---|
| $J_s$ | Secondary Pulley Rotational Inertia |
| $M_{eff}$ | Effective Mass of Vehicle |
| OOL | Optimum Operating Line |
| PGT | Planetary Gear Train |
| $R_G$ | Gradient Resistance |
| $R_r$ | Rolling Resistance |
| $r_w$ | Wheel Radius |
| SR | Torque Converter Speed Ratio |
| T | Torque |
| $T_t$ | Total Resistance Torque |
| TC | Torque Converter |
| TVO | Throttle Valve Opening |
| V | Vehicle Velocity |
| $V_r$ | Relative Velocity |
| W | Weight of Vehicle |
| WOT | Wide Open Throttle |
| $\rho$ | Air Density |
| $\omega$ | Rotational Speed |
| $\eta_t$ | Transmission Efficieny |

# CHAPTER 1

# INTRODUCTION

No matter how well the engine performs, if that power is not transmitted properly, desirable results cannot be expected. In that line of thought, one of the key factors to total power output is the transmission. Transmission systems are the main and one of the most important parts of a vehicle. In these systems, torque and power, which are produced by the engine, are transmitted to the driving wheels of the vehicle. In recent years, there are so many different transmission systems are used. As a result of the demand for more powerful and efficient vehicles, most of the engineers focus on transmission system. For many years, engineers are studying on more efficient transmission systems. Today in most of the vehicles, both manual and automatic transmission systems, power and torque developed are transmitted to the driving shaft by using gear pairs. In recent years, vehicle and transmission producers spent too much money for making their transmissions perfect. But different from all other transmission systems, nowadays another system which is known for years is started to use in vehicles. This system is Continuously Variable Transmission (CVT).

A Continuously Variable Transmission (CVT) provides a continuously variable ratio between the power source of the vehicle and its wheels. A conventional CVT has an infinite (or a very large number) of gear ratios. It selects a ratio that can deliver to the wheels the power being demanded by the driver, but keeps the engine spin rate low within this constraint to improve economy and engine life. Different from the traditional manual and automatic gearbox systems, CVT uses belt or chain systems whose ratio are changing depending on hydrodynamic principles.

In fact, the idea of CVT is not a new concept. In 1490, Leonardo da Vinci made a sketch of his CVT. Different manufacturers have worked on CVT for years. One of these is General Motors. In early 1930s, they developed a CVT system but after testing procedures, they decided not to use it. In 1960s, they again studied on it but never used it. Another manufacturer which interested in CVT is Austin, a British manufacturer. Different from General Motors, they used CVT in small cars. In 1958, a Dutch firm Daf developed a CVT [1]. Although a lot of studies are done on CVTs, their popularity is not so high in those days. There are some reasons for that. First of all, CVTs have high costs. The other problems are poor reliability, inadequate torque transmission, problems with noise and rough starts.

Another company working on CVT is Audi. They have worked on CVTs nearly for 20 years. At last they can overcome most of the problems and use this new kind of CVT in Audi A6 2.8 model. After the test done by Audi, it is seen that this new model called multitronic A6 accelerates from 0-100 km/h 1.3s quicker than a geared automatic transmission and is 0.1s quicker over the same speed than an equivalent model with “optimum” use of a five speed manual gearbox [1].

After so many years, there must be some reasons for why manufactures tend to CVT. One of them is the governments’ new regulations (CAFE in USA for example) for automotive fuel economy and emissions. In dependent of vehicle speed, CVT allow the engine to work at its most efficient point without disturbing the driver with discrete shifts. Since the engine works at its optimum point, the fuel consumption decreases compared with a vehicle equipped with manual transmission. CVT provide continuously variable ratio. As a result of this, harshness of shifts can be avoided. Also engine is not forced by different gears. These yield less engine fatigue and more reliable transmission.

A transmission, which offers such advantages and solutions for most of the recent problems, can not be ignored just because of its drawbacks like low efficiency and limited torque capacity. These disadvantages lead the engineer to different types and arrangements of CVT.

In this study, some of these solutions are investigated and the comparison between different solution proposals are made and the superiorities or insufficiency between these new technologies and currently used automatic transmissions. In Chapter 2, some of the recent studies on different solution proposal, CVT control types and simulation studies are presented. In Chapter 3, two of the proposed solutions in the literature are investigated deeply and required mathematical models and CVT control strategies are developed. In Chapter 4, by using the developed models in Simulink environment, acceleration and fuel consumption performances of the cars are investigated and the simulation results and analysis of these results are presented. In Chapter 5, some conclusions are made by the help of the simulation results and analysis. Also some future studies to develop this study and obtain better and accurate results are suggested.

# CHAPTER 2

# LITERATURE REVIEW

## 2.1 Introduction

With the developments in automobiles also came the creation and development of transmission systems. Early vehicles were simply equipped with manual transmissions with manual control. As advances have been made in vehicles over the past several decades, developments in automobile transmission systems also came out. First result of this development is the automatic transmissions, which supply ease of use, especially in congestion in urban areas. It is also a very good solution to deal with the new high power engines. But in recent years, another transmission system, called continuously variable transmission (CVT), is revealed. The main advantage of this new transmission is to allow the engine to work at its most efficient point without disturbing the driver with discrete shifts. Theoretically, this leads to reduction in the fuel consumption of the vehicle. But their limited torque transmission capacity and short ratio range are the main drawbacks that put them far away from being a standard transmission for most of the vehicles.

### 2.2. Possible CVT configurations

There are some suggestions for the solution of these disadvantages. Some of them have application areas in the modern vehicles and some others are theoretical. In the following paragraphs these suggestions are presented.

#### 2.2.1. CVT with torque converter

As discussed above, the limited torque transmission capacity is one of the disadvantages of CVTs and one way to solve this problem is to use a torque converter as proposed in reference [2]. In this work, a high torque capacity belt-drive with a torque converter was developed. The aim of his study is to apply CVTs to the vehicles with high motor torque, improve the driveability and fuel consumption and give the freedom of choosing the driving mode to driver. The parts of the CVT system can be seen in 2.1.

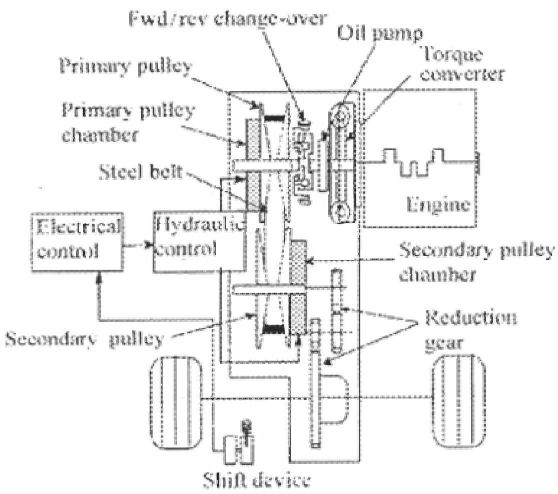


**Figure 2.1** CVT system diagram (Kurosawa, 1999)

One of the main problems in the transmission of higher torque levels is the strength of the belt. For this reason, as shown in figure 2.2, steel belts were used. Belt torque capacity is directly proportional with belt clamping force exerted on pulleys. This force is a function of hydraulic pressure and area of hydraulic pressure on back of secondary pulley, and radius of minimum belt contact on primary pulley. These are also enhanced in the study.

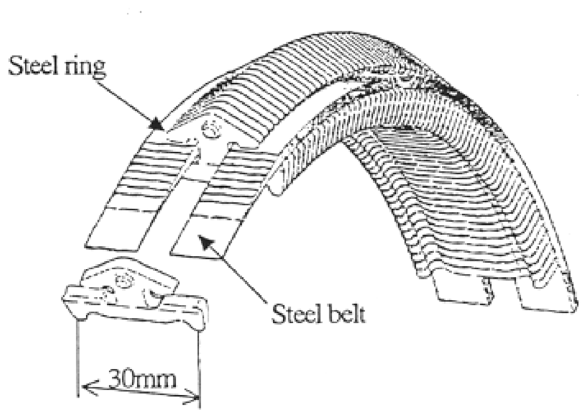


**Figure 2.2** Structure of steel belt (Kurosawa, 1999)

The torque converter also improves the start-off acceleration performance when it is compared with the traditional CVTs.

The idea of inserting torque converter was first applied by one of the big Japanese manufacturer Nissan. Nissan, accompany with its partner Subaru, is one of the leader companies in CVT technology, especially after it launched M6 Hyper-CVT in the Japanese Primera. This M6 Hyper-CVT is equipped with torque converter and it can handle 190 hp and nearly 110 N.m torque. It also provides the driver a manual mode, which simulates a 6-speed manual gearbox. After Nissan Primera, March, Cube (M6 Hyper-CVT), Fiat Punto (Speed Gear), Subaru Pleo (I-CVT) and Rover MGF (Steptronic) also use this type of CVT.

### 2.2.2. CVT with chain drive

The idea of using a torque converter with CVT gives quite good results when one compares this new CVT system with the other traditional ones. But there are still some drawbacks of this system. In reality, it is no faster or more frugal than automatic transmissions. Although the torque values that it can handle is a ‘record’ of its time, still it cannot deal with engines with great torque values. Also when the accelerator pedal was pressed, CVT immediately brings the rpm up to a high level. The engine put out its maximum performance with the corresponding level of noise but the car slowly catches up in acceleration. This gives one the feeling of a slipping clutch. This is called as rubber band effect and it is another disadvantage of this system.

For these reasons, for 20 years Audi’s research and development engineers have tried to find a better CVT system that can overcome these disadvantages and at last they came up with a CVT equipped with a chain and variator system [1].

This new developed CVT system, which is called as Multitronic by Audi is used in Audi A6 2.8. Multitronic is different from the other CVT in two aspects. One is the chain instead of belt and the second is the torque sensor. By using a variator, a new transmission element called link-plate chain can be inserted into

CVT to deal with high torque and power values like A6's engine which produces a peak torque 280 N.m. This variator enables high torque ratios and acceleration from rest is now possible without using a torque converter.

Torque sensor determines the pulley's chain clamping force and supply sufficient contact pressure but not excessive. This also proves the fuel efficiency. By ensuring the engine speed increases with increasing vehicle velocity, it eliminates rubber band effect. How this system works can be seen in figure 2.3. Also a general view of Audi Multitronic CVT can be seen in figure 2.4.

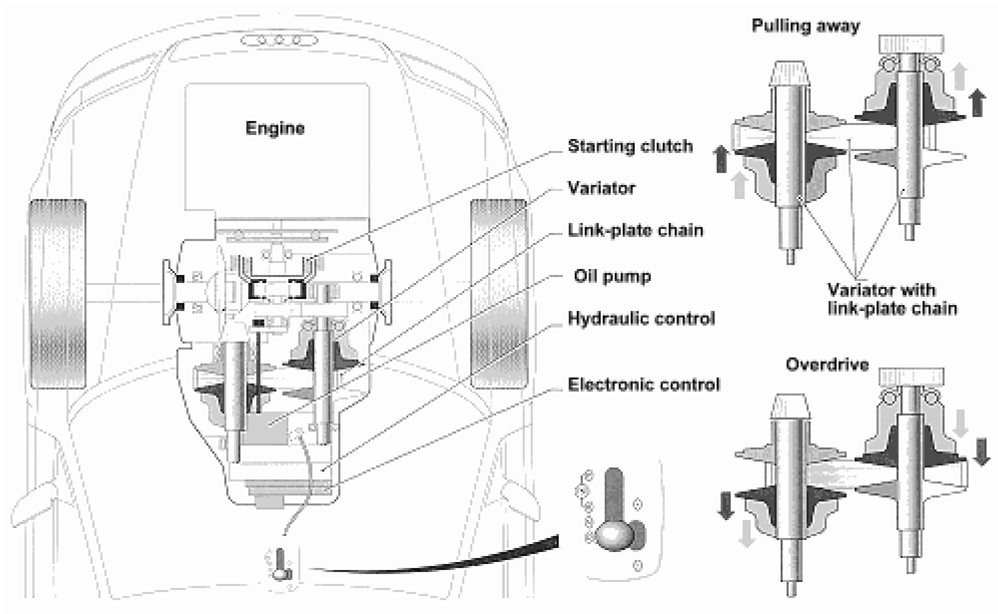


**Figure 2.3** Audi's new multitronic CVT (Birch, 2000)

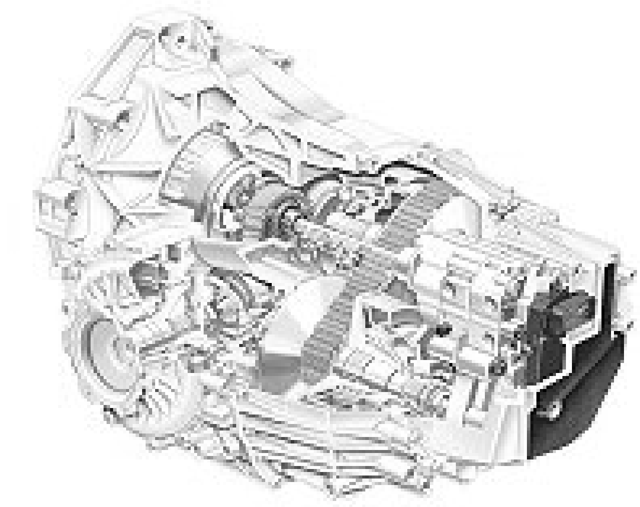

**Figure 2.4** General view of Audi's multitronic (Birch, 2000)

### 2.2.3. Continuously Variable Power Split Transmission (CVPST)

As stated in the preceding paragraphs, the biggest problem in CVTs is the limited torque and power transmission. One way to eliminate this is to use a stronger belt and increase the friction between the pulley and belt. This is done by Audi. Another possible solution is to split power off the CVT and divert it through a planetary or differential gear train [3].

At low speed ratios when the vehicle accelerates from rest or low velocities, high torque values are required. At that instant, the input power is mostly diverted to the planetary gear set and it deals with high torque instead of CVT. When the desired speed or a pre-determined speed value is achieved, the input power passes through CVT. The power split feature accomplishes two goals: first the gear sizes required to attain variable velocity ratios need not change due to the CVT connection between the gear train and the output shaft; second, the belt capacity no longer limits the maximum torque capacity of the system.

To expand the overall transmission ratio, two different step-up gears are used. By the help of this step-up gearbox, in this study, the ratio range 3.785 to 0.694 for the CVPST can be obtained. It is not a very wide range when compared with the Audi's Multitronic, which provides a ratio range from 6.01 to 1. A basic CVPST system can be seen in figure 2.5.

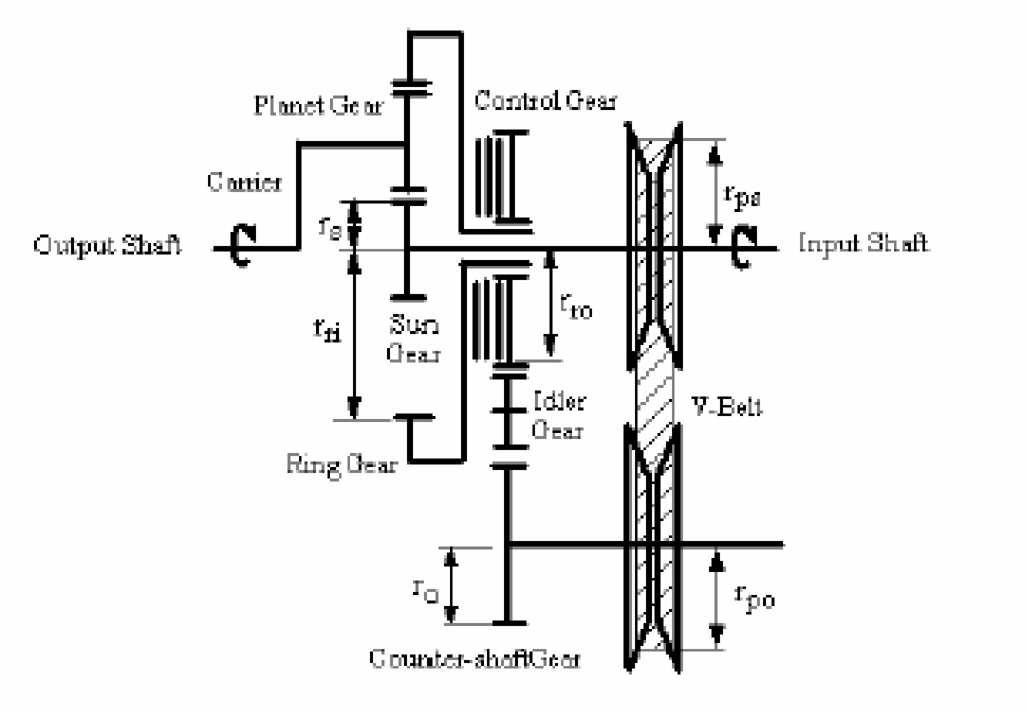


**Figure 2.5** A Basic CVPST system (Lu Z., 1999)

### 2.2.4. Extroid CVT

Extroid CVT is introduced by a Japanese manufacturer, Nissan, in 1999. The type of CVT was totally different from the other companies. They use a torodial CVT which is another type of CVT. Instead of using a belt or chain as the media for varying transmission ratio, Nissan's Extroid uses two pairs of rollers. Input is transferred via the first torus disc by roller friction and flows over a second sheave to the transmission output shaft. By changing the angle of the rollers, the ratio change can be obtained. By this way, an overall ratio of 4.41 to 1 was obtained. The fundamental advantage is a higher torque capacity than a CVT, with other advantages being greater driving comfort and faster ratio changes. Drawbacks are the manufacturing costs and limited range of ratio.

## 2.3. Survey of CVT controls

As described in previous parts, CVT provides continuously variable changing transmission ratios. During a driving cycle, there exists numerous operating scenarios and CVT ratio must be determined according the corresponding demands of the driver. For example, in most of these automated transmission systems, there exists two driving modes. One is for economical driving, whereas the other is sport mode. There are different studies on control of CVT ratios during driving cycle.

As described in reference [4], the recent efforts on CVT control can be categorized in three groups.

### 2.3.1. Single Track

The logic behind the single track control system is to hold the engine speed or torqueses at the peak efficient point by controlling the ratio of CVT. Internal combustion (IC) engines have peak efficiency curves and the fuel consumption can be reduced by setting the operating point of the engine on this efficiency curve. There exist different studies on this control strategy. As described reference [4], in one of these studies, it is tried to achieve this strategy by controlling the speed of the IC engine with an engine throttle governor and controlling the engine torque independently with the ratio variations of a generic CVT. Proportional controllers are used to fix the torque, which was the axle torque in low vehicle speeds and the engine torque at high vehicle speeds. There is also a very similar study, in which the driver input is simulated as proportional and integral control on engine speed error and the CVT ratio change is done according to a predetermined ratio, which is a smooth function of engine speed.

In another case, a hybrid vehicle is used to demonstrate another shift control strategy. The shift control strategy maintains the two point of IC engine operation and the vehicle response to drive-by-wire input signals by varying the CVT.

Another suggestion is to use a speed ratio map similar to the ones used in automatic transmissions. This speed ratio map is determined according to either the peak efficiency curve or the optimal operation line (OOL). According to the driver input, which is the amount of the driver's pressing the accelerator pedal, the system convert this into a percent throttle angle opening, the desired CVT ratio regulates the engine speed such that engine operation tracks the desired path, which is OOL or the peak efficiency curve.

Although it is a straightforward approach to implement, which is known to perform well with respect to fuel consumption, it has disadvantages for the drivability of the vehicle. It also strongly relies on the exact knowledge of peak efficiency curve or OOL and does not consider the efficiency of CVT.

### 2.3.2. Speed Envelope

This is a control strategy, which has more applications than single track solution. It can be thought as the core of the controller in most of today's CVT-equipped vehicles. In this strategy, there exists a desired operating area, which is formed by two curves, each can be approximated by a polynomial, on vehicle speed versus engine speed plane and they form a speed envelope in which the CVT regulates the powertrain responses. The design of the curves of the speed envelope strongly effects the fuel consumption and drivability of the vehicle.

Honda uses a strategy, which is a form of speed envelope philosophy, in their recent commercial gasoline engine vehicles. There are different driving modes offered to the driver and a given mode at a given vehicle speed and throttle opening will force the CVT to modify engine speed only within a particular portion of the overall speed envelope. PI controllers are used to satisfy the required operating points. Ratio change rates are also controlled and for engine braking and other high power demanded situations, the shift controller provides maximum permissible performance and ratio change rate.

One step further of Honda's strategy is the Bosch Cartronic Powertrain Structure. The improvement in this new structure is to automate the driving mode selection manually available in Hondas. The driver gas pedal input identifies the driver style and needs, and the driving modes are selected according to this identification by "dynamic driving program".

### 2.3.3. Off the Beaten Track

This strategy is similar to single track strategy, but this strategy uses two different tracks represent the economy and performance modes. According to the driver's mode selection, the shift control unit tracks different paths to reach the desired operating point. The throttle opening is also controlled by the controller. The driver's gas pedal input is correlated to the demand for engine power. This power demand is processed and the desired operating point is found. Then according to the driving mode, engine speed, accordingly CVT ratio is determined.

In the economy mode, the throttle is opened as quickly as possible to the level corresponding to the desired operating point. Then the engine operates along that throttle level and the engine speed is modulated through the CVT ratio until the desired operating point is satisfied.

In the performance mode, the engine is forced to operate on the iso-power curve corresponding to the desired power level. The throttle is opened as quickly as possible to achieve this power level. If the torque capability is not enough, than the engine will be operated at the maximum torque line until the demanded iso-power curve is reached.

These strategies vary with the system configurations, operating philosophy and design resolution. It is also sure that none of these strategies use precise optimization techniques. These are some practical and easily applicable solutions, and those, which have application areas in today’s vehicles. To achieve precise optimization and study the behavior of these presented control strategies, there are several simulation studies are done.

**2.4. Studies on Simulation and Optimization of CVTs**

There are several simulation studies on CVT performance according to fuel consumption and drivability. Some of these are done for comparison of CVT with other transmission system like automatic and manual transmission. The other ones are done for optimization and development purposes.

A computer-based dynamic model was developed in reference [5]. For control purposes, a single model, covering the transmission and the rest of the drivetrain was developed.  The dynamics of the engine and the tire slip were also taken into account in the bicycle model of the vehicle. Newtonian formulation of the equations of motion was used. The drivetrain was modeled as an interconnected multi-degree-of-freedom system, where each subelement was treated as a single lumped-mass system. In figure 2.6, driveline and vehicle model schematics can be seen.

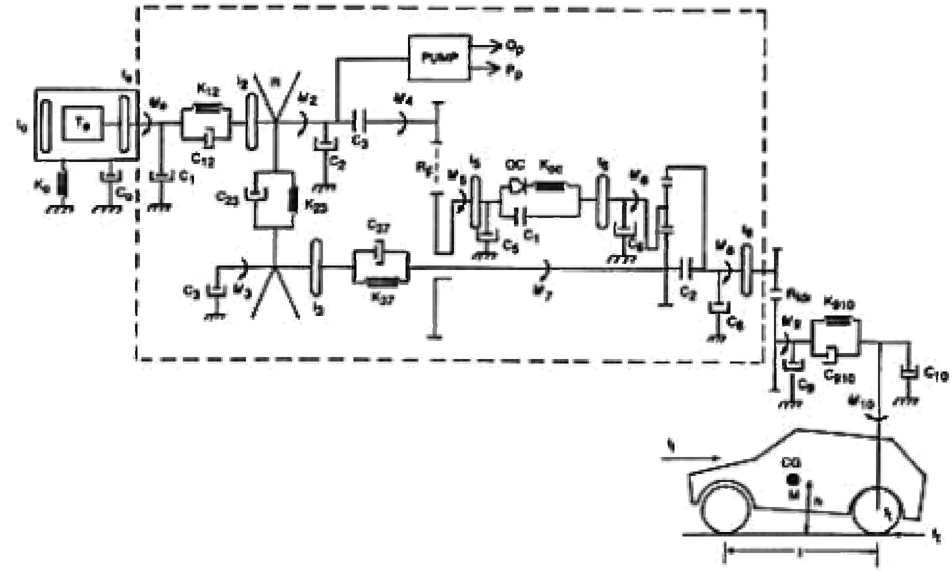


**Figure 2.6** Driveline and Vehicle Model Schematics (Vahabzadeh, 1991)

After writing necessary equations for all systems and then making necessary substitutions lead to an equation of motion for a generic CVT driveline.

$$\overset{o}{\omega}_d = \frac{T_e - R_{CVT}.(T_R + T_l)}{I_d + \frac{I_e}{R^2{}_{CVT}}} \cdot \frac{1}{R_{CVT}} + \frac{I_e \cdot \omega_e}{I_d + \frac{I_e}{R^2{}_{CVT}}} \cdot \overset{o}{R}_{CVT} \qquad (2.1)$$

where $\omega$ = Rotational speed

T = Torque

I = Rotational Inertia

d = Driveshaft

e = Engine

$R_{CVT}$ = Speed ratio of CVT

In the simulation study, throttle angle, speed ratio and change rate and clutch positions are taken as control parameters. Figure 2.7 shows the control hierarchy and structure.

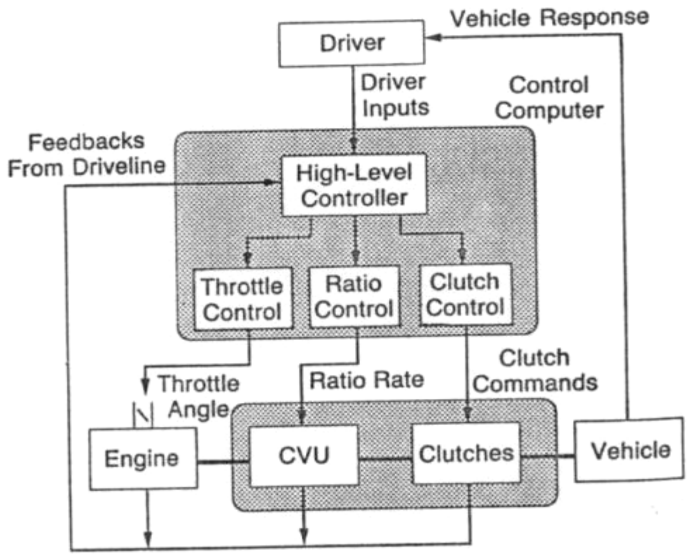


**Figure 2.7** Control Hierarchy (Vahabzadeh, 1991)

Two different driving modes exist in the simulation. One is for performance and the other is for fuel consumption. Two different optimal operation lines were calculated and an “off the beaten track” type control strategy was applied.

In reference [6], the objective is to optimize a real CVT system. There exist system nonlinearities and some unstable elements, like engine. It is tried to find a suitable set of parameters to stabilize the system and obtain a uniform response for any input signal within the range of driver control.

A block diagram, as shown in figure 2.8, is prepared for control system design. The engine is forced to operate along an OOL. "Single track" control strategy is used.

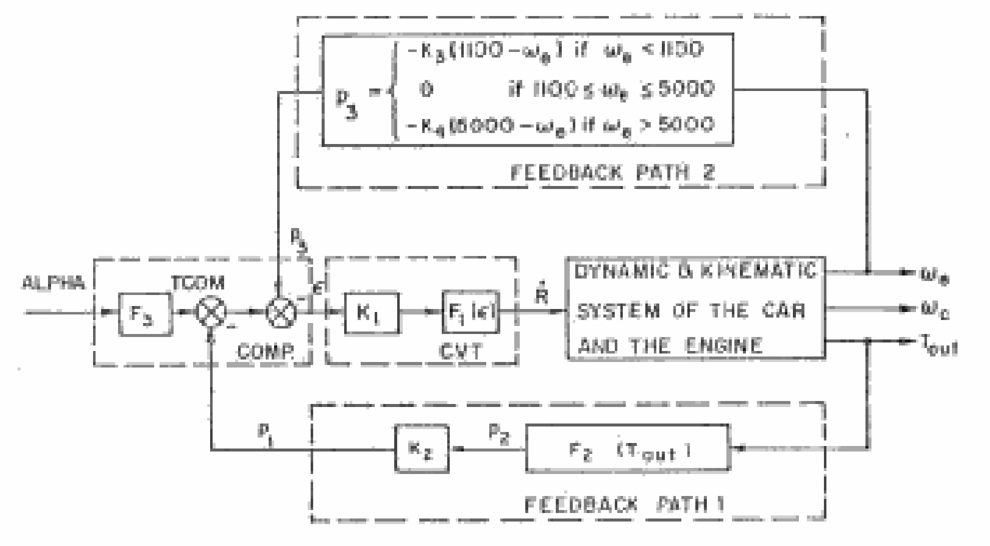


**Figure 2.8** Block Diagram of the CVT control system (Yang 1985)

Mathematical model of the transmission is same with reference [5]. As shown in figure 2.8, the input of CVT is an error signal ε which is the output of a comparing element of the system. Dynamic and kinematics system of the car and engine is presented as a block and the input for that block is the ratio rate of CVT. Outputs are the speed of the engine, speed of the car and the torque of the output shaft of the CVT. The speed of the engine and output torque is also used as feedback signals in the system.

In the mathematical model of the system, there exist twelve unknowns, four parameters of the system and a given input signal ALPHA, which is generated by the accelerator pedal. By using different cost functions and solve the equations repeatedly, system parameters are obtained and for different parameters, speed of the car and engine is simulated.

Reference [7] is a study done on a hybrid vehicle. The aim is to calculate torques for both IC engine and electric motor and the optimal CVT gear ratios for a given driving cycle of the vehicle.

A simple second order kinematic model of the driveline, which is shown in the figure 2.9, is used.

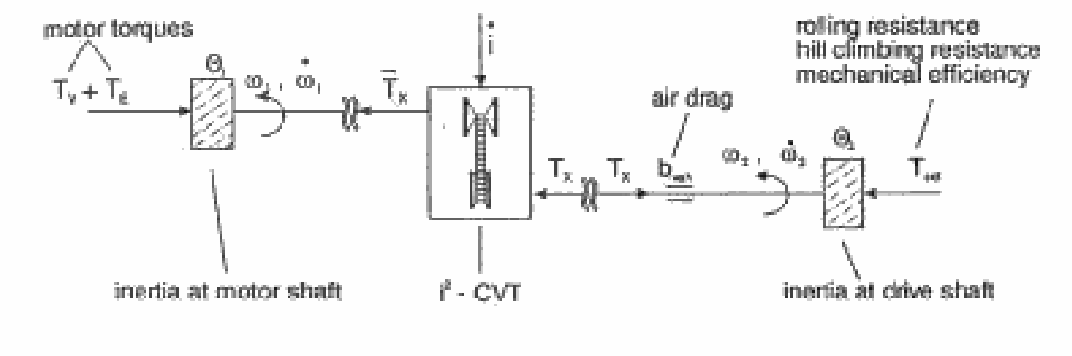


**Figure 2.9** Kinematic model of the driveline (Zoelch, 1998)

The equations of the system are as follows.

$$i = \frac{\omega_1}{\omega_2} \tag{2.2}$$

$$\frac{d}{dt}\omega_2 = \frac{\left(T_V + T_E\right) \cdot i - T_{ext} - b_{veh} \cdot \omega_2{}^2 - \theta_1 \cdot \left(\frac{d}{dt} i\right) \cdot i \cdot \omega_2}{\theta_1 \cdot i^2 + \theta_2} \tag{2.3}$$

i = gear ratio

$\omega_2$ = rational speed of the wheels

The torque at the input shaft of the gearbox can be obtained by using the following formula.

$$\overline{T_X} = T_V + T_E - \theta_1 \cdot \omega_2 \cdot \frac{d}{dt} i \tag{2.4}$$

The CVT dynamics is modeled as an integrator.

$$i = \frac{\delta_i}{T_i} \tag{2.5}$$

There also exist equations for battery model, and when the system is represented in state space, the following state and input vectors are obtained.

$$x = \begin{pmatrix} \omega_2 \\ i \\ T_V \\ I_z \\ w \end{pmatrix} \qquad u = \begin{pmatrix} T_{Vsoll} \\ T_E \\ \delta_i \end{pmatrix}$$

where $T_V$ and $T_{Vsoll}$ is electric motor torque, $I_z$ is motor parameter, w is fuel consumption.

The aim of the optimization is to minimize the fuel consumption. By defining Hamiltonian and writing costate differential equations, an optimal control problem is obtained. Then by using an optimization software and Simulink for a complete system model, simulations are done. As a result, an optimal trajectory on fuel consumption map of the vehicle is obtained.

In reference [8], it is required to find an optimal operation in transient conditions of vehicle, to satisfy an improvement of the fuel economy of the passenger cars. Firstly, the discussion of the present control strategies, which are also presented in section 2.3, is done. Then a system modeling including the engine, CVT and powertrain is done. A sketch of the system is shown in figure 2.10.

Two different type of engine is modeled in the study. One is downsized supercharged SI engine and the other is conventional SI engine.

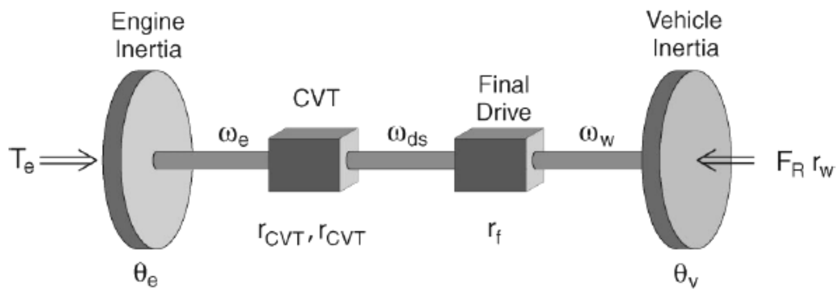


**Figure 2.10** Drive train structure (Pfiffner, 2001)

As similar with the other studies, one of the control parameter is taken as the gear ratio change rate where it is defined as;

$$\frac{d}{dt} r_{CVT}(t) = u_2(t) \tag{2.6}$$

where $u_2$ (t) is the system input. The other control input is accelerator pedal signal $u_1$ (t).

The gear ratio and state constrains are;

$$r_{CVT}(t) = \frac{\omega_{ds}(t)}{\omega_e(t)} \tag{2.7}$$

$$r_{CVTmin} \leq r_{CVT}(t) \leq r_{CVTmax}$$

Different from the other studies, a CVT efficiency concept is introduced. A CVT efficiency map, which models the losses within the CVT system, is used. It is a static function of three variables, input torque, input speed and gear ratio.

As a whole powertrain with CVT, driveline mathematical model is given as follows;

$$\frac{d}{dt}\omega_e = \frac{1}{\theta_e \cdot \eta_{CVT} + \theta_v \cdot r^2} \cdot \left(T_e \cdot \eta_{CVT} - r \cdot r_w \cdot F_R - r \cdot r_f \cdot u_2 \cdot \theta_v \cdot \omega_e\right) \tag{2.8}$$

where $r_f$, $r_w$, $\theta_v$, $\theta_e$, r are final drive ratio, wheel radius, vehicle inertia, engine inertia and overall gear ratio respectively.

By selecting a driving cycle, and defining the boundary conditions and constraints with state equations, an optimization problem is constructed. Then using a suitable software package, this problem is solved and the optimal trajectories, which are the result of simulation, are presented by graphs and the comparison of the results with the ones obtained from conventional control strategies is done.

In reference [9] and [10], the comparison of lightweight passenger cars with three different types of engine and two different type of transmission according to fuel consumption is done. One type of transmission is CVT. A sketch of the system with CVT can be seen in figure 2.11.

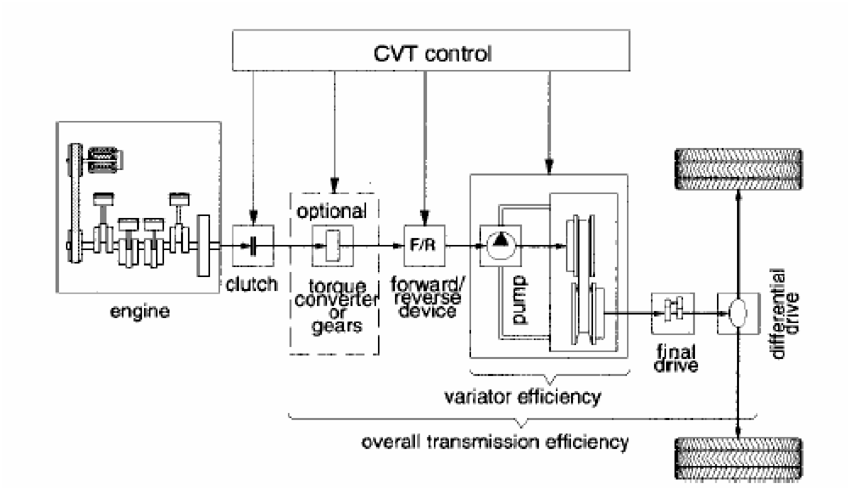


**Figure 2.11** Sketch of a CVT powertrain (Soltic 2001)

In the control of CVT system, “single track” strategy is used.  This means the engine throttle is also controlled by the system. The model is composed of three different subsystems. These systems are composed of different parts. First one is composed of engine, flywheel and primary clutch side. The second one is composed of gearbox, final drive and differential. The last one is the wheel. The necessary Newtonian equations are written for each of the subsystems and as a result, a closed-loop tracking control system can be developed. The powertrain model and block diagram of the system can be seen in figure 2.12 and 2.13.

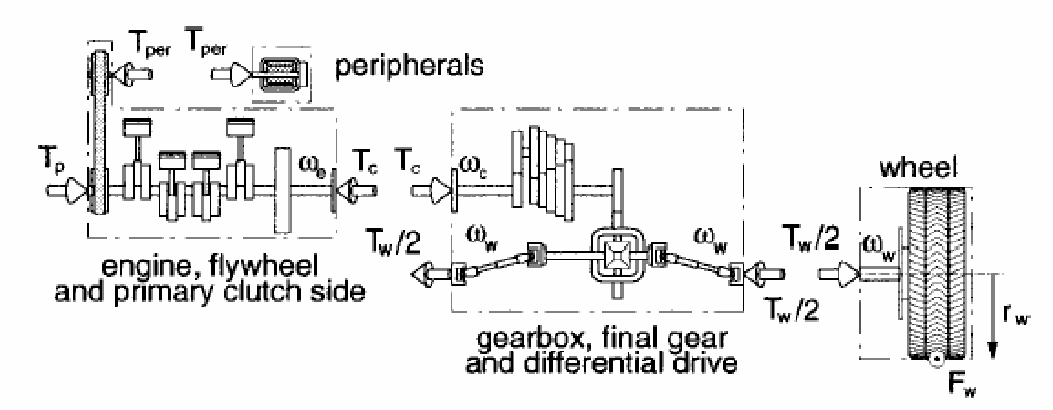


**Figure 2.12** Powertrain Model (Soltic, 2001)

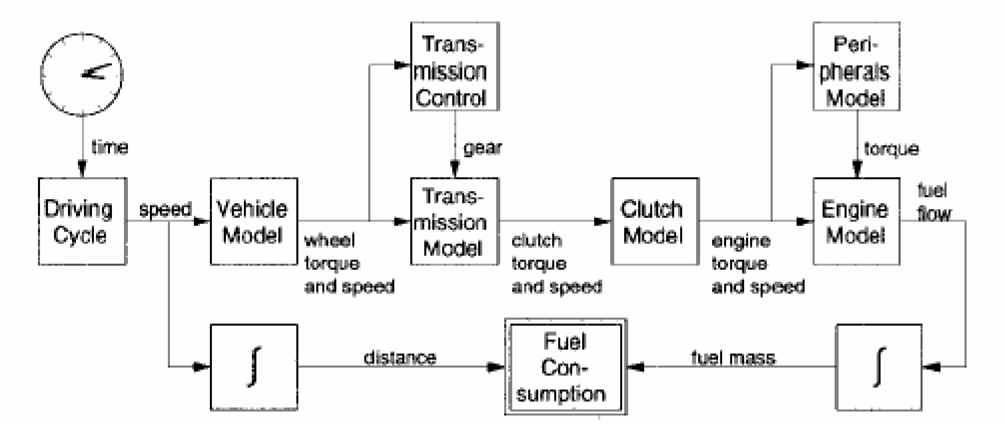


**Figure 2.13** Working scheme of driving cycle simulator (Soltic, 2001)

The wheel torque is modeled as;

$$T_w = \left( m \cdot g \cdot k_{roll} + \frac{\rho_{air}}{2} \cdot C_D \cdot A_{front} \cdot V^2 \right) \cdot r_w + \left[ (1+\delta) \cdot m \cdot r_w \right] \cdot \frac{d}{dt} V \qquad (2.9)$$

The inertia of the rotating parts is equal to a vehicle mass increase of δ in the vehicle acceleration term.

Clutch speed and torque, and the model are as follows;

$$T_c = \frac{T_w \cdot i_t}{\eta_t(i)} \qquad \omega_c = \frac{\omega_w}{i_t}$$

$$\omega_e = \begin{cases} \omega_c & \text{if } \omega_c \geq \omega_{min}(T_c) \\ \omega_{min} & \text{if } \omega_c < \omega_{min}(T_c) \end{cases} \qquad (2.10)$$

The engine torque can be calculated as follows;

$$T_p = T_c + T_{per} + J_e \cdot \frac{d}{dt} \omega_e \qquad (2.11)$$

Peripherals are fuel pump, engine electrics and electronics, instrumentation, lights, defroster, A/C, power steering, etc. The necessary torques for these is also taken into account in the engine torque equation.

After the simulations and comparisons, it is stated that because of the CVTs' poor efficiency, CVT system is not feasible to use instead of automatic transmission.

In reference [3], the idea is to find a solution for the limited torque capacity problem of CVTs. The biggest torque requirements in vehicles are at start-ups and at low speeds. Starting from this point, it is tried to transmit the torque across another transmission system like a planetary geartrain at operating points require high torque values, and at other times, it uses the benefits of CVT. Also by using two step-up gears, the overall transmission ratio span is expanded significantly. Existence of a planetary gear set makes it possible to arrange the ratio span according to the demands of the designer. In reference [11], the details of this kind of a design study is also explained.

A computer program is written to simulate this new type of CVT, manual and automatic transmission and the comparison between them is made.

**2.5. Scope of Thesis**

With developing technology, today, there are much more vehicles on roads equipped with CVT, when it is compared with the amount ten years ago. Most of the big automotive manufactures spend lots of money on research and development in CVT systems. As a result, different solutions from different companies come out. In this study, two of these solution proposals are investigated. One of them is already used in production, whereas the other is not used widely yet. The aim is to develop a vehicle model equipped with these CVT systems and make acceleration and fuel consumption simulations of these vehicles. Although the main aim is to develop simulations, by adjusting the parameters of CVT, also an optimization of these systems can be done. To compare with current systems, a model for a vehicle equipped with automatic transmission will also be developed. At last, the evaluation of the driveability and the performance of these systems will be done.

# CHAPTER 3

# VEHICLE POWERTRAIN MODELS

Vehicles are composed of different mechanical parts like engine, transmission, differential, etc. For simulation study, the whole system can be treated as a lump mass, which is concentrated at center of gravity (CG) of the vehicle. The point mass at the CG, with appropriate rotational moments of inertia, is dynamically equivalent to the vehicle itself for all motions in which it is reasonable to assume the vehicle to be a rigid body. The motion of a vehicle hence belongs to a rigid body motion. For the purpose of a drive test of a vehicle used for the comparison of different types of transmission, it is sufficient to consider only forward or longitudinal motion. In this study, the simulation of the vehicle is therefore focused on the longitudinal direction to test the characteristics of different types and configurations of continuously variable transmissions.

## 3.1. Newton's Second Law

The basis of the model for the longitudinal motion of the vehicle is mostly based on Newton's Second Law. For the translational system in longitudinal direction, the sum of the external forces acting on the vehicle in this direction is equal to the product of the vehicle mass and the acceleration in that direction.

$$\mathbf{\Sigma\ F_x = Ma_x} \tag{3.1}$$

For the rotational systems, the sum of the torques acting on a body about a given axis is equal to the product of its rotational moment of inertia and the rotational acceleration about the axis.

$$\mathbf{\Sigma\ T = J\alpha} \tag{3.2}$$

## 3.2 Tractive Force

Traction force is directly proportional to the engine torque. This engine torque passes through the transmission system and creates traction force at the wheels. Some of this torque is used for overcoming the inertial effects like the inertias of engine, transmission, etc.

The engine torque is a function of engine throttle valve opening and the engine speed. For a given throttle valve opening, the engine torque varies with engine speed. A typical engine map is shown in figure 3.1 [14].

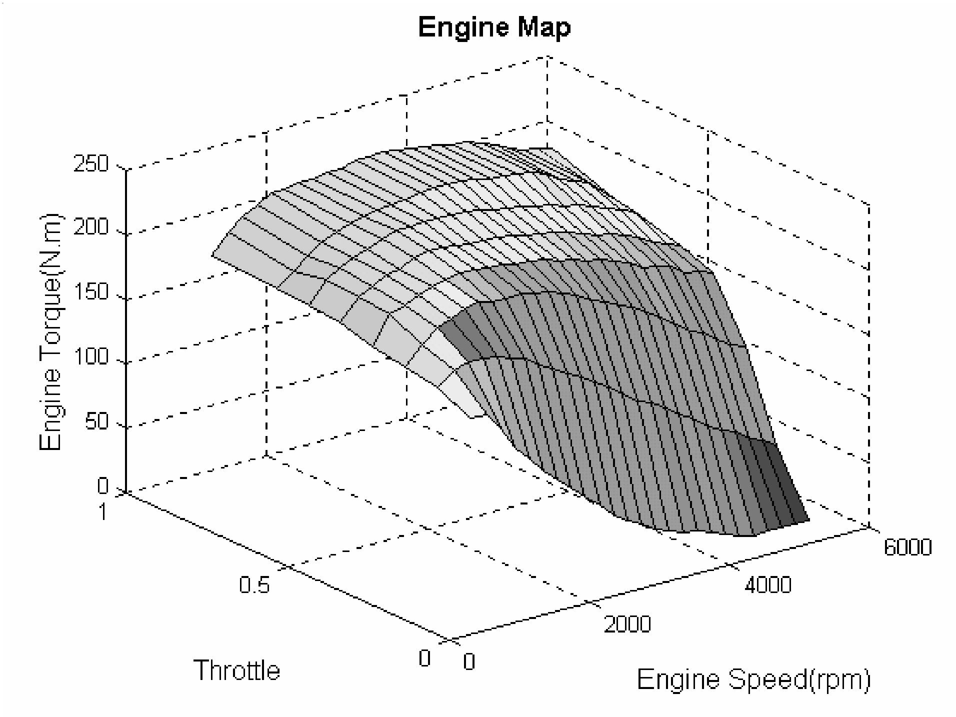


**Figure 3.1** A typical engine map (Salaani, 1998)

As it can be seen from the figure, engine torque is not a linear function of engine speed for a given throttle valve opening. So the engine torque must be checked for all the times from the engine map for all the engine speeds and throttle valve openings.

For a given engine, two curves are fixed. One is maximum torque curve, which corresponds to the wide open throttle valve position. The other is the optimum operating line, which is defined as the operating points where the vehicle consumes optimum fuel. These two curves can be seen in figure 3.2. [18]

In order to perform a vehicle acceleration simulation, the throttle valve opening must be scheduled for each time steps.

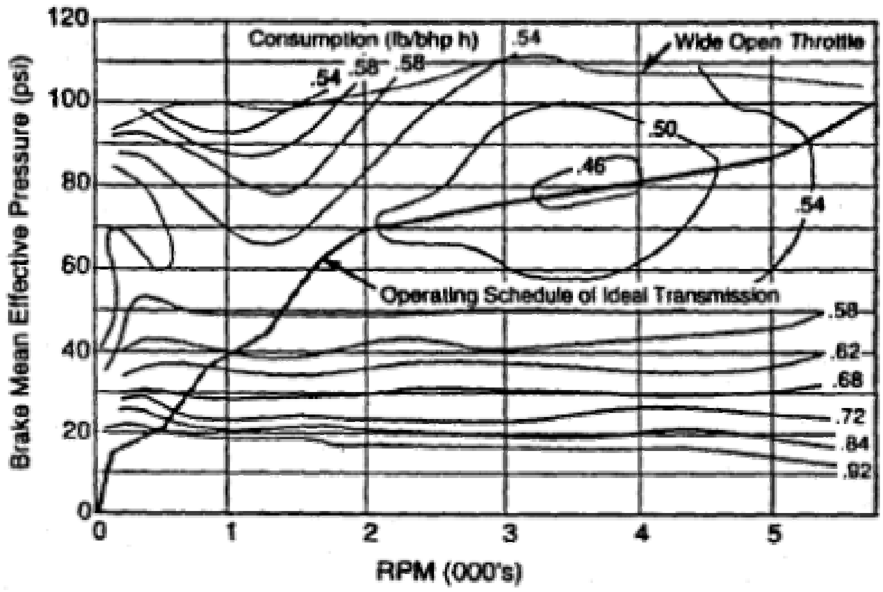


**Figure 3.2** V8 engine map (Gillespie, 1992)

### 3.3. Resistances to Motion

Dynamic loads acting on the vehicle is shown on the figure 3.3. Among all the forces shown in the figure, $F_{xf}$ and $F_{xr}$ are traction forces, which push the vehicle to move forward. The rest of the forces are resistance forces.

#### 3.3.1. Aerodynamic Drag

As a result of the air stream interacting with the vehicle body, in three coordination system (x, y, z), there occurs six components of forces and moments. But the most important one in vehicle acceleration simulation studies is the drag force which is in longitudinal direction and the most effective aerodynamic force

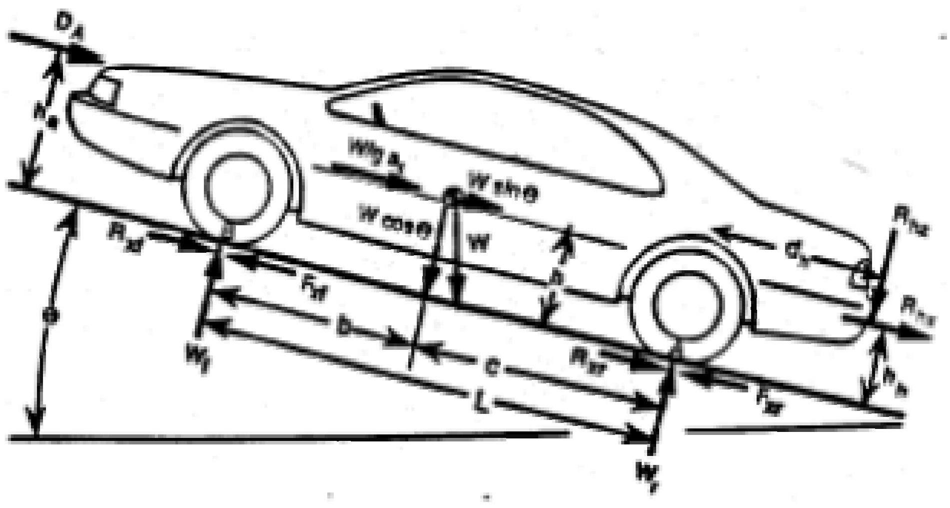

**Figure 3.3** Forces acting on a vehicle

at normal highway speeds. Aerodynamic drag is a function of vehicle relative velocity and can be calculated by the following formula.

$$F_D = \frac{1}{2} \cdot \rho \cdot V_r^2 \cdot C_D \cdot A_f \qquad (3.3)$$

where $\rho$ is air density, $V_r$ is relative velocity of the vehicle, $C_D$ is aerodynamic drag coefficient and $A_f$ is the frontal area of the vehicle.

Air density depends on the pressure and the temperature of the environment. But for this study, the environmental factors are neglected and assume to be constant. Then the air density is taken as 1.227 kg/m$^3$.

The drag coefficient varies over a wide range for different types of cars. For passenger cars, it can be taken as $C_D$ = 0.3 ~ 0.4. Frontal areas and drag coefficients of some selected cars can be seen in Table 3.1 [17].

Vehicle relative velocity can be calculated by summing up the vehicle velocity and the wind velocity. When the wind blows towards the vehicle, a headwind is present. Wind blowing in the direction of the vehicle travel is a tailwind. For headwind, the wind velocity is taken as positive, whereas for tailwind, it is negative

**Table 3.1** Frontal Areas for different types of cars [17]

| Vehicle | Year | $C_D$ | $A_f$ |
|---|---|---|---|
| BMW Coupe 850i | 1990 | 0.29 | 2.06 |
| Citroen AX GT5 | 1989 | 0.31 | 1.69 |
| Citroen XM 3.0 SEI Auto | 1990 | 0.30 | 1.98 |
| Ferrari 348tb | 1989 | 0.32 | 1.83 |
| Fiat Croma CHT | 1990 | 0.32 | 2.02 |
| Toyota Lexus LS400 | 1990 | 0.29 | 2.07 |
| Nissan Prairie | 1989 | 0.36 | 2.20 |
| Rover Sterling Catallyst | 1989 | 0.32 | 2.18 |
| Vauxhall Calibra 2.0i 16V | 1990 | 0.26 | 1.78 |

### 3.3.2. Rolling Resistance

Due to the deformation of tyre and the dissipation of energy through impact, one of the important resistances to the motion of the vehicle, which is rolling resistance, originates. Its value depends upon the vehicle speed, tyre inflation pressure, vertical load on the tyre, tyre diameter, and the road surface. As in reference [17], the rolling resistance can be calculated as;

$$\mathbf{R_r = f_r.W} \tag{3.4}$$

where

$$\mathbf{f_r = a^* + b^* V^n} \tag{3.5}$$

$R_r$ = Rolling Resistance

$f_r$ = Coefficient of rolling resistance

W = Total weight of vehicle and its load

$a^*$ , $b^*$ = Coefficient

V = Vehicle speed

n = index (commonly taken to be unity for simplicity)

Rolling resistance coefficients, $a^*$ and $b^*$, can be obtained experimentally. As a result of these experiments, $a^*$ can be calculated by using the experimental data. Inflation pressure of tires and load distribution have effect on coefficient $a^*$. The velocity coefficient $b^*$ can be taken as $1.5x10^{-5}$ for radial ply tyres.

### 3.3.3. Gradient Resistance

When the vehicle climbs up a gradient, the component of its weight parallel to the road surface acts a resistance force to its motion and it must be taken into consideration. If the vehicle goes down a hill, this component will help to accelerate.

$$R_G = \frac{W \cdot G}{100} \tag{3.6}$$

Maximum gradient (G) for state highways is 10 %, for city roads 15 % and for rural areas 22 % [17].

### 3.4. Acceleration Simulation of CVT with Torque Converter

A powertrain is a multi-input, multi-output, large scale, complex, nonlinear dynamic system. As such an all-encompassing, highly-detailed powertrain model would undoubtedly require a significant investment of time and resources to develop, and would most likely prove to be intractable for simulation and control design purposes. Thus, any attempt to model a powertrain's behavior must be preceded by a clear concept of the model's ultimate use, thereby defining the scope of the model and its level of detail and accuracy. The model described in this work will be used to develop and compare the different types and configurations of continuously variable transmission.

The models include mathematical models, which are consisting of physical laws and heuristics. In addition, assumptions and simplification, such as lumped-parameter models or static mappings, must be recognized. The system includes look-up tables and data-driven networks. The model includes the engine, torque converter, CVT and vehicle model. Figure 3.4 depicts the interactions between the subsystems.

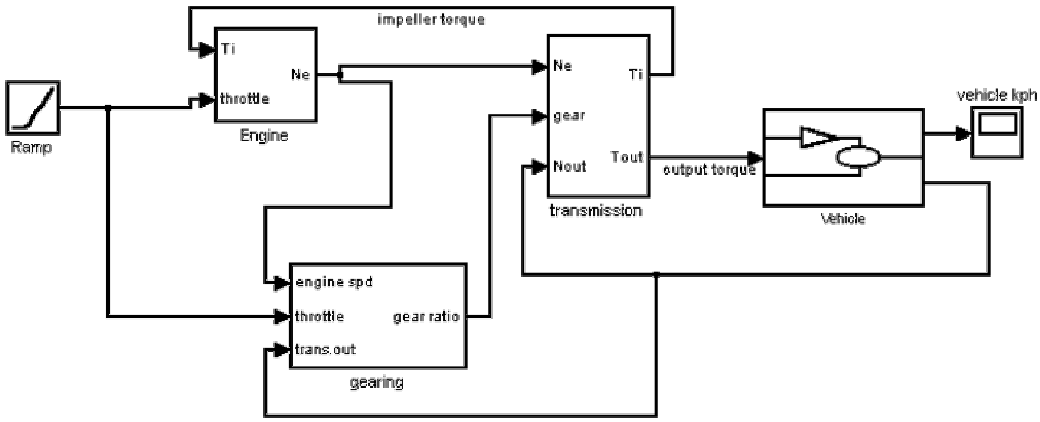


**Figure 3.4** Interactions between subsystems of the model

### 3.4.1. Engine

Engine is the main source of the vehicle for producing traction force. Engine torque develops the traction force after passing the powertrain of vehicle. As described in section 3.2.1, this torque is a function of the throttle valve opening and the engine speed. Thus engine can be modeled as a static engine map whose inputs are engine throttle and engine speed. The engine equation is as follow:

$$J_e \cdot \frac{d}{dt}\omega_e = T_e - T_{ac} - T_i \qquad (3.7)$$

The engine torque $T_e$ is obtained from the surface data (Figure 3.2) using bilinear interpolation of throttle input $\varphi_f$ and the engine speed $\omega_e$. $T_i$ is the torque absorbed by the torque converter. As described in reference [14], the accessory torque $T_{ac}$ is the torque lost due to engine friction and pumping losses, and can be estimated as a linear function of engine speed.

$$T_{ac} = a_0 + a_1 \cdot \omega_e \tag{3.8}$$

In figure 3.5, Simulink model of the engine can be seen.

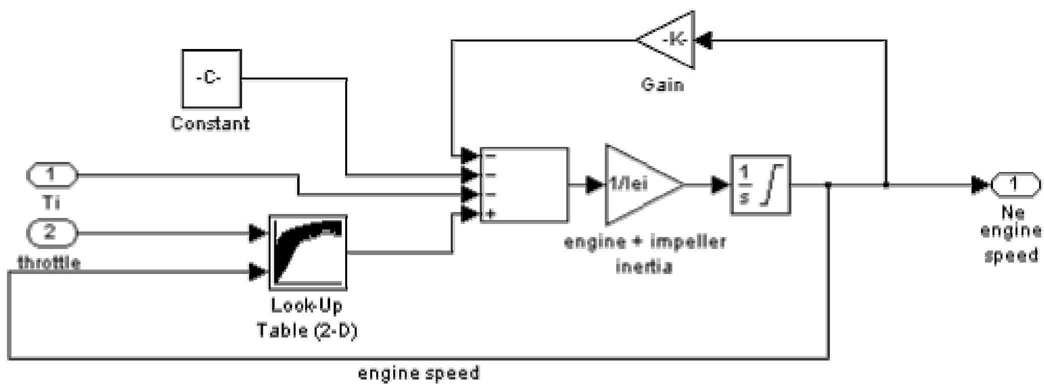


**Figure 3.5** Simulink Block of Engine Model

### 3.4.2. Torque Converter

A torque converter is used for improving start-off acceleration performance, which has been a weak point of the current CVT and improved driveability at very low speeds, such as when putting a vehicle in a garage. By using a torque converter, a maximum torque approximately 2 times greater than the engine torque can be produced.

The engine is connected to the torque converter that acts very much like a clutch under some conditions while more like a direct connection in others. While the torque converter transmits power to the transmission there is a speed reduction across the unit during low speed operation. Once higher vehicle speeds are attained, the torque converter input and output may be locked together to achieve a direct drive though the unit.

The main duty of a torque converter is to transfer the power between the engine and the mechanical parts of the transmission, and allow torque amplification during vehicle travel. Modeling and design of the mechanical and fluid dynamics of torque multiplication is an entire research topic in itself, therefore the dilemma is how to sufficiently but accurately represents its behavior without introducing considerable complexity. The torque converter is assumed to be always operating at its capacity and therefore can be adequately modeled by its steady-state performance characteristics when it is unlocked.

For steady-state modeling, a speed ratio is first defined as the ratio of primary pulley of the CVT to engine speed. Two additional variables are introduced which are experimentally-measured functions of the speed ratio. These functions, the torque ratio, $C_r$ and the capacity factor, K-factor, are plotted in figure 3.6 [14]. The governing relationship between speed and torque is:

$$T_i = \frac{\omega_e^2}{K^2} \tag{3.9}$$

The output torque is then the product of the input torque and the torque ratio.

$$T_o = C_r \cdot T_i \tag{3.10}$$

In figure 3.7, Simulink model of the torque converter is shown.

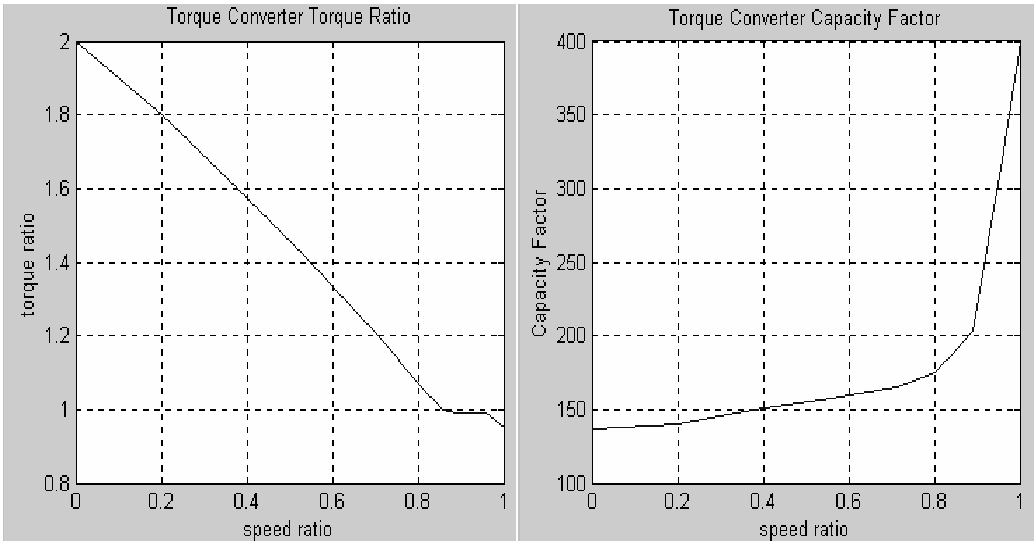


**Figure 3.6** Torque Ratio and Capacity Factor vs. Speed Ratio Graphs of Torque Converter (Salaani, 1998)

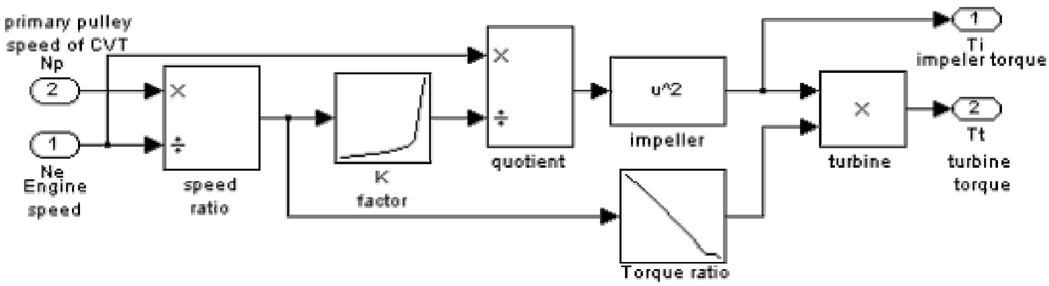


**Figure 3.7** Simulink Block of Torque Converter Model

### 3.4.3. CVT Model

The continuously variable transmission consists of two pulley connected by PIV chain. The primary pulley is driven by torque converter, while the secondary pulley connects to the intermediate drive gear assembly. As with fixed gears the CVT ratio is defined by the speed ratio, in this case primary speed divided by the secondary speed,

$$i = \frac{\omega_p}{\omega_s} \tag{3.11}$$

and is physically limited by the pulley diameters and belt length to

$$i_{min} \leq i \leq i_{max}$$

Assuming no belt slippage occurs, by conservation of energy, the secondary torque divided by the primary torque is also the same ratio

$$i = \frac{T_s}{T_p} \tag{3.12}$$

CVT ratio change is done according to the selected drive mode. Two different drive modes exist in the CVT model. The purpose of the first mode is to optimize the fuel consumption. In this mode, the engine should be operated on the optimal operating line (OOL). Optimum operating line can be obtained by combining the minimum fuel consumption points for each throttle valve opening. In figure 3.8, engine characteristic curves and OOL can be seen [21]. Single track type control strategy is used in the model. CVT ratio should be adjusted to hold the engine speed at the optimum operating point by using the driver's throttle valve opening input. CVT ratio change process can be investigated in two parts. In the first operating period, CVT ratio is fixed at maximum allowable limit. In this period, engine speed is compared with the required engine speed in gear ratio block and the ratio is fixed until the engine speed reaches the required engine speed at that throttle valve opening. After that, the second operating period starts. In this period, CVT ratio decreases while the engine speed is constant. By decreasing the CVT ratio, the speed of the secondary pulley is increased. Accordingly the vehicle velocity is also increased.

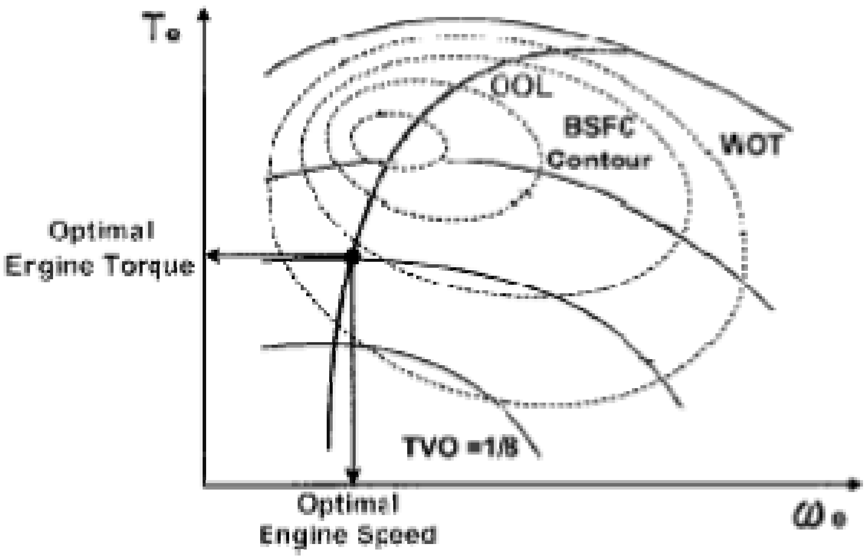


**Figure 3.8** OOL line and Engine Characteristic Curves (Kim, 2002)

The content of the gear ratio block is given in figure 3.9. As seen in the figure, this CVT ratio control is done by using switches and relational operators.

In the first stage, using relational operators, the engine speed at that instant is compared with the required engine speed which is determined from the look-up table of engine speed vs. throttle valve opening. This look-up table is developed from the engine power curve graph by combining the maximum power points at each throttle valve opening. As a result, optimum operating line for performance driving is obtained. When the required speed achieved, this means the second stage starts, and the required CVT ratio is calculated by using the required engine speed, transmission output speed and the required speed ratio (SR) for torque converter. Transmission output speed is a feed back signal of vehicle block. To find the required speed ratio, another Simulink block is developed. As mentioned before, engine equation is:

$$J_e \cdot \frac{d}{dt}\omega_e = T_e - T_{ac} - T_i \tag{3.13}$$

When the engine speed is constant at maximum power point, the inertia term at the left hand side of equation is equal to zero and also since the engine speed and throttle valve opening is fixed, then engine torque and accessory torque values are fixed. From the equation, required impeller torque can be calculated. By using capacity factor graph as look up table and impeller torque together, speed ratio requirement for torque converter is obtained.

From engine to final drive, there are two transmission steps. One is torque converter and the other is CVT. Speed ratio is;

$$SR = \frac{\omega_p}{\omega_e} \tag{3.14}$$

Combining the CVT ratio equation and speed ratio equation;

$$i = \frac{\omega_e \cdot SR}{\omega_s} \tag{3.15}$$

Secondary pulley speed is same with transmission output speed, which is obtained from vehicle block.

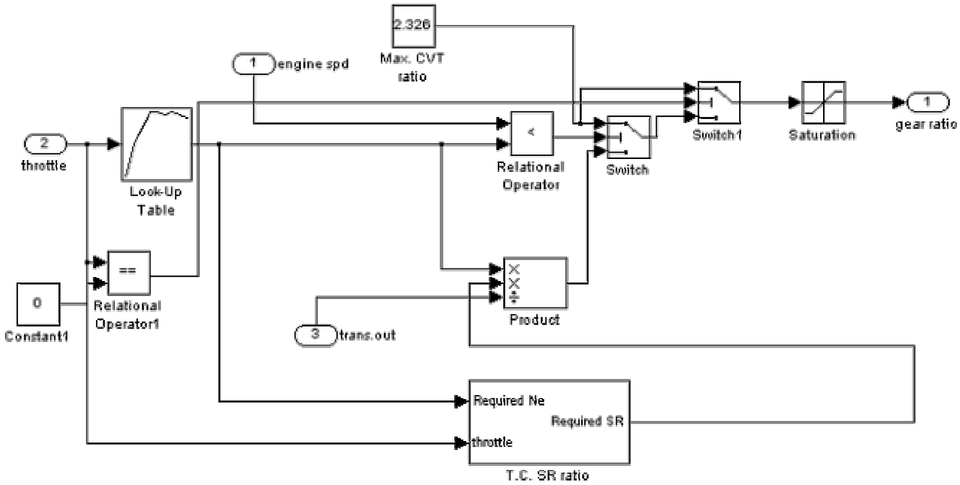


**Figure 3.9** Simulink Block of CVT model

CVT efficiency concept can also be inserted into the model. CVT efficiency is effected by speed ratio, input speed and input torque. CVT efficiency concept can be another research topic itself. Therefore, in this work the mathematical model in reference [12] is used. Efficiency of CVT is affected by the variator ratio, the input speed and torque of CVT. So the efficiency is a function of these three parameters. The efficiency function of PIV chain drive type is used. Figure 3.10 shows the effect of input speed to maximum engine speed ratio over the efficiency of the CVT at a variator ratio of 1 [12].

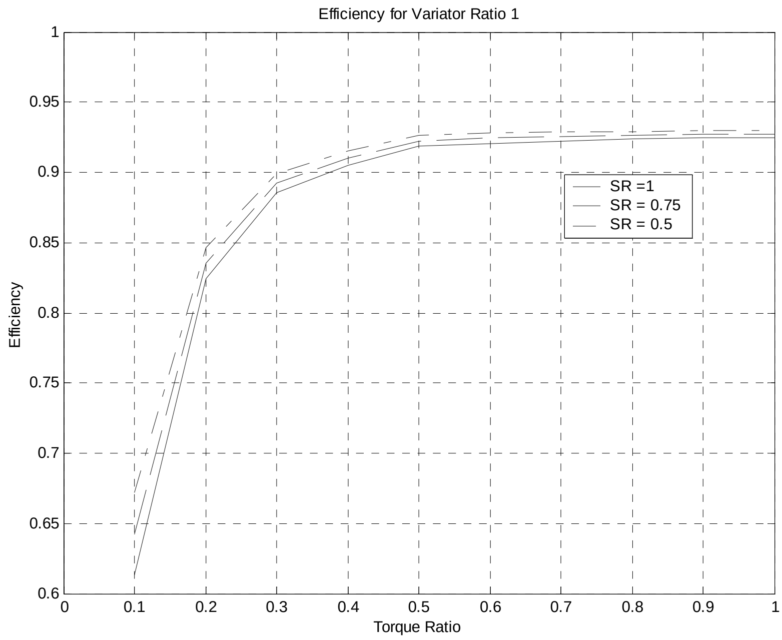


**Figure 3.10** Efficiency Graph of PIV Chain Drive type CVT (Singh, 1992)

### 3.4.4. Vehicle Model

Vehicle can be modeled using Newton's equation, with vehicle speed, V, tractive force, $F_t$ and resistant force, $F_{res}$.

$$M_{eff} \cdot \frac{d}{dt} V = F_t - F_{res} \tag{3.16}$$

$M_{eff}$ is the effective mass of vehicle, which includes inertia of the wheel, CVT, differential. These rotational inertias are addition to the vehicle mass in the order of 3.4 %. Resistant force is composed of rolling, drag and gradient resistances. To obtain the wheel speed, vehicle velocity can be converted into rotational speed by using the wheel radius. The vehicle model is shown in figure 3.11.

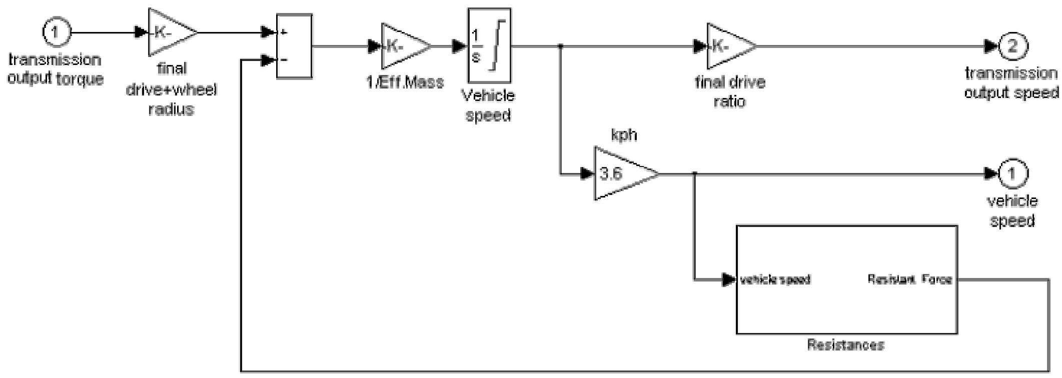


**Figure 3.11** Simulink Block of Vehicle and Resistances Model

### 3.5. Acceleration Simulation of Continuously Variable Power Split Transmission (CVPST)

Continuously Variable Power Split Transmission (CVPST) is a combination of V-belt type CVT and a planetary gear train (PGT). This combination leads to a continuously variable transmission. There exist different combinations of CVT and PGT [13]. In this study, the power split configuration is simulated. In this type, the input power is distributed to the CVT and PGT, at low speeds (high torque and power requirements) the fraction of power flowing through the CVT element is smallest. To expand the overall ratio span, an external step-up gearbox is added. The power distribution is shown in figure 3.12.

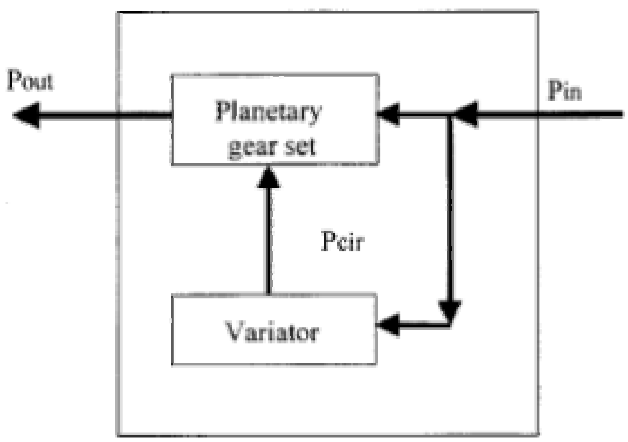


**Figure 3.12** Power Split Configuration

Figure 3.13 shows the basic elements of the vehicle equipped with CVPST from the engine to the wheels. The motive power developed by the engine is transmitted down to the driveline system and finally to the driving wheels. The engine shaft is directly connected to the primary pulley of CVT and the sun gear of the planetary gear train as shown in figure 3.14. Some of the power is transmitted through CVT and the rest is through the PGT by the help of sun gear and countershaft. In reference [11], the system is described in details.

When the equation of motion for the engine is;

$$\left(J_e + J_p\right) \cdot \frac{d}{dt}\omega_e = T_e - T_{ac} - T_{in} \qquad (3.17)$$

where $J_p$ and $T_{in}$ are the rotational inertia of primary pulley of CVT and torque delivered to the CVT and PGT, respectively. The delivered torque is distributed and transmitted to the step-up gear through CVT and PGT. Passing through the step-up gear and differential, engine torque supplied to the wheels to overcome the resistances and accelerate the vehicle.

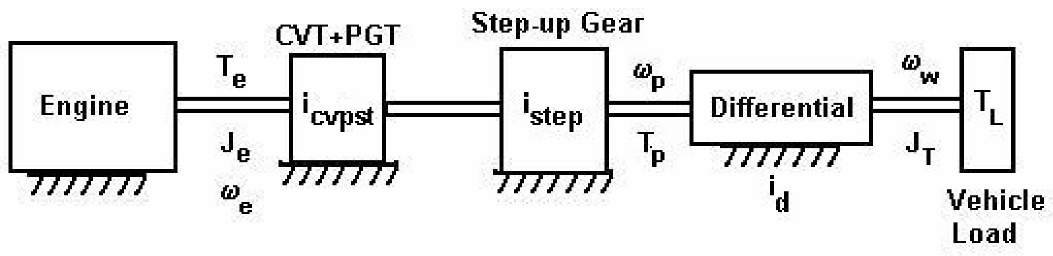


**Figure 3.13** Basic Elements of Vehicle equipped with CVPST

By using equation (3.16), the transmission output speed can be found. Since the engine speed is needed for finding the engine torque, an algebraic loop must be constructed to calculate the engine speed. The engine speed is;

$$\omega_e = i_t \cdot \omega_{out} \tag{3.18}$$

$\omega_{out}$ is the transmission output speed and $i_t$ is the transmission ratio described as follow;

$$i_t = i_{cvpst} \cdot i_{step} \tag{3.19}$$

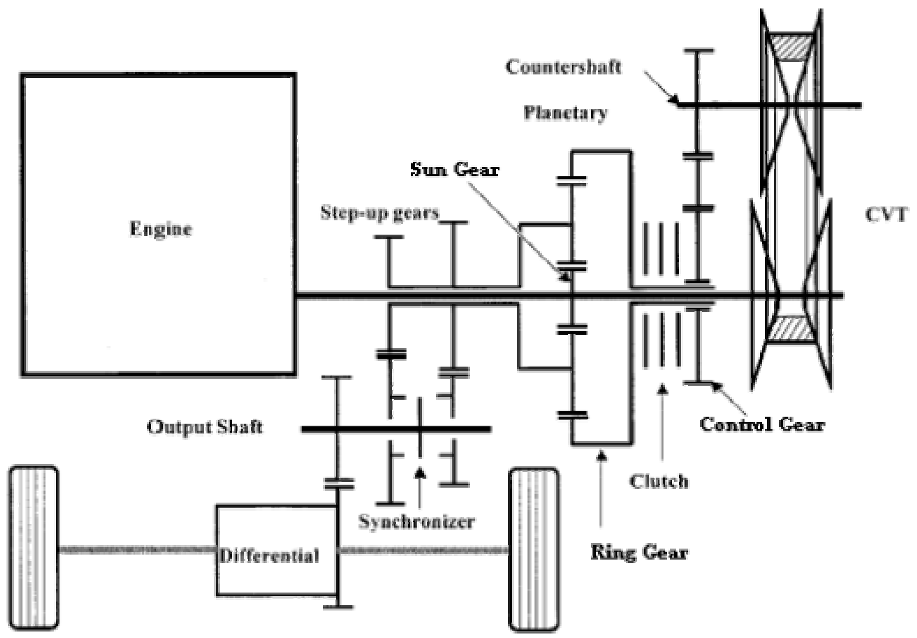


**Figure 3.14** Transmission system consisting of a basic CVPST and a two-stage step-up gearbox (Mucino, 2001)

Also engine speed is required for calculating $i_{cvt}$. So an algebraic Simulink loop is constructed by feeding back the transmission output speed as shown in figure 3.15.

### 3.5.1. CVPST Model

Since continuously variable power split transmission is a combination of CVT and PGT, it has one constant ratio and one variable ratio part. These two ratios need to be adjusted well during travel of the vehicle to force the engine to work at optimum operating points. The control strategy is similar to the previous model. Two different modes and accordingly two different optimum operating lines (OOL) are defined for control purposes. The engine is forced to track these OOLs.

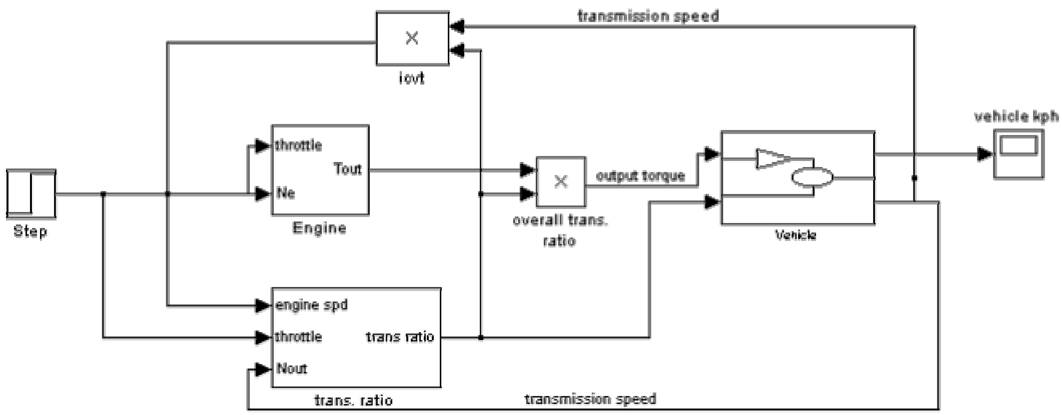


**Figure 3.15** Simulink Block of CVPST model

Since there are two different ratio parts, the overall ratio of the system must be defined as a function of CVT ratio and PGT ratios. As stated in reference [11], the overall transmission ratio of CVPST can be written as;

$$i_{cvpst} = \frac{\omega_{in}}{\omega_{out}} = \frac{i_{cvt} \cdot \left(1 + i_g\right)}{i_{cvt} \cdot i_g + i_{cg}} \tag{3.20}$$

$i_g$ and $i_{cg}$ are the speed ratios of counter-shaft gear and control gear, respectively. These two ratios are determined in the design stage of the CVPST and they are constant ratios. [11]

The transmission ratio block works very similar to the one in previous model. But in this model, there is no need to adjust the speed ratio since no torque converter is present in the system. According the driver's throttle input, the desired operating point, which defines the required engine speed, is determined by using the 2-D look-up table. This required engine speed compares with the actual engine speed and the CVPST ratio is fixed to the maximum possible value, since at low speeds, the torque requirement is usually high. When the engine speed reaches to the desired value, the CVPST ratio is calculated as;

$$i_{cvpst} = \frac{\omega_{req\_e}}{\omega_{out\_cvpst}} \tag{3.21}$$

By using the two transmission ratio, $i_{cvt}$ can be calculated as;

$$i_{cvt} = \frac{i_{cg} \cdot i_{cvpst}}{\left(1 + i_g\right) - i_{cvpst} \cdot i_g} \tag{3.22}$$

After reaching the required speed, by using a switch, the required transmission ratio is calculated as described above. Since the ratio span of the CVT is not very high, step-up gears are used for expanding the ratio span. In the first period, due to the high torque requirement, step-up gear with the high speed ratio is engaged. After switching, the CVT ratio is decreased until its minimum ratio. At this point, the second step-up gear is engaged and the CVT ratio is adjusted such that the overall ratio before second step-up gear can be obtained.

The efficiency of CVPST system show different characteristics according to the arrangement of the system. For this CVPST system, the efficiency is a function of the variator ratio and the efficiency curve can be seen from figure 3.16. [13]

This curve is curve fitted by the following quadratic equations.

$$\begin{aligned} \eta_t &= -36.091 \cdot i_{cvt}^2 + 69.84 \cdot i_{cvt} + 59.47 \qquad 0.5 \le i_{cvt} \le 1 \\ \eta_t &= -4.2592 \cdot i_{cvt}^2 + 7.9364 \cdot i_{cvt} + 89.54 \qquad 1 \le i_{cvt} \le 2 \end{aligned} \tag{3.23}$$

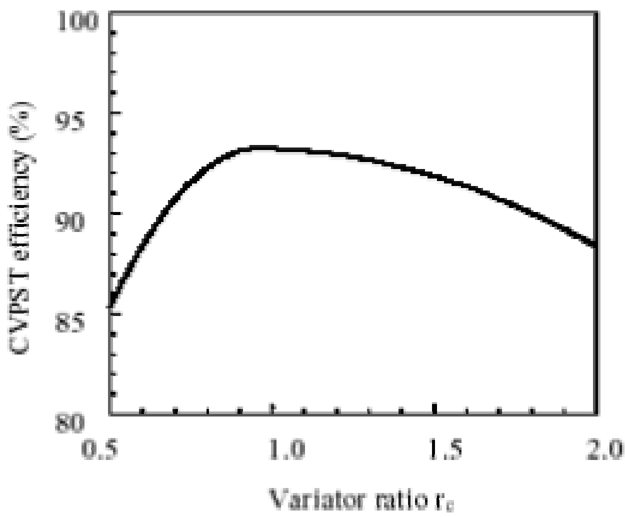


**Figure 3.16** CVPST efficiency as a function of variator ratio
(Mantriota, 2001)

### 3.6. Acceleration Simulation of Vehicle with Automatic Transmission

For comparison purposes, a vehicle equipped with automatic transmission is also modeled. The model is very similar to the CVT equipped with torque converter. Currently available in production vehicles with automatic transmission use torque converters, clutches, and planetary gear sets for the selection of different output ratios. Different speed ratios can be achieved by planetary gear sets. Required speed ratio of automatic transmission are found from the shift logic map of the transmission according to the knowledge of the current gear, throttle input and the output shaft speed. Automatic shift logic map of 1994 Ford Taurus [14] can be seen in figure 3.17.

Shift logic map is modeled by using look-up tables. The output speed is first compared with the previous output speed. As a result of this comparison, required shifting regime, upshift or downshift, is determined. After that decision, output speed is compared with the corresponding look-up tables to decide which gear will be in use. After the decision of the gear number, corresponding gear ratio is used.

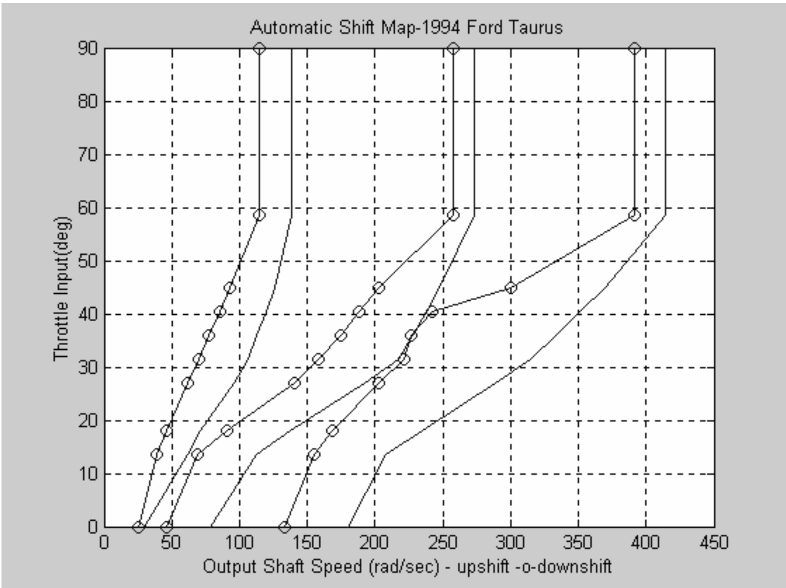


**Figure 3.17** Automatic Shift Logic Map (Salaani, 1998)

### 3.7. Fuel Consumption Models

To check the fuel consumption efficiencies of different types of CVT systems, another Simulink model must be developed. In this model, different from models for acceleration simulation studies, the input for the system is a required velocity profile. A very similar models to the acceleration models are developed, which are tracing back from the vehicle velocity to the engine speed and torque.

To calculate the fuel consumption of a system, another engine characteristic, which is specific fuel consumption map, is required. Specific fuel consumption maps are obtained experimentally, and consist of a plot of constant specific fuel consumption lines on engine torque or brake mean effective pressure (bmep). A typical specific fuel consumption map can be seen in figure 3.18.

For fuel consumption model, OOL must pass through the minimum fuel consumption points and CVT must force the engine to operate along this line. Similar to the flow diagram of the simulator of reference [9] in figure 2.13, velocity profile is an input for the vehicle resistance and axle shaft model. Using that input, firstly the required at the output side of transmission is calculated. This value is corrected by efficiency of the engine. Engine efficiency is quite low in the order of 62.5 %. By using this value, required engine power is calculated. Then using a look-up table, which consist of OOL, the required engine speed is obtained. In this model, it is assumed that the driver presses the accelerator enough to satisfy the velocity pattern requirements. Using this required engine speed and the required wheel speed, CVT ratio can be calculated. In transmission model, the differences between CVT types are obvious. One model uses torque converter with CVT, other uses CVT with planetary geartrain. Starting with the required wheel torque to satisfy the velocity profile and tracing back and passing through the transmission model, the engine torque can be obtained. Using this engine torque and speed as the inputs for the specific fuel consumption map, the specific fuel consumption at that operating point is obtained. To calculate the amount of fuel used in 100 km, the required engine power and the specific fuel consumption is used through the following equation;

$$\text{"Amount of fuel (lt/100 km)"} = \frac{bsfc \cdot P_e}{\rho_f \cdot V} \tag{3.24}$$

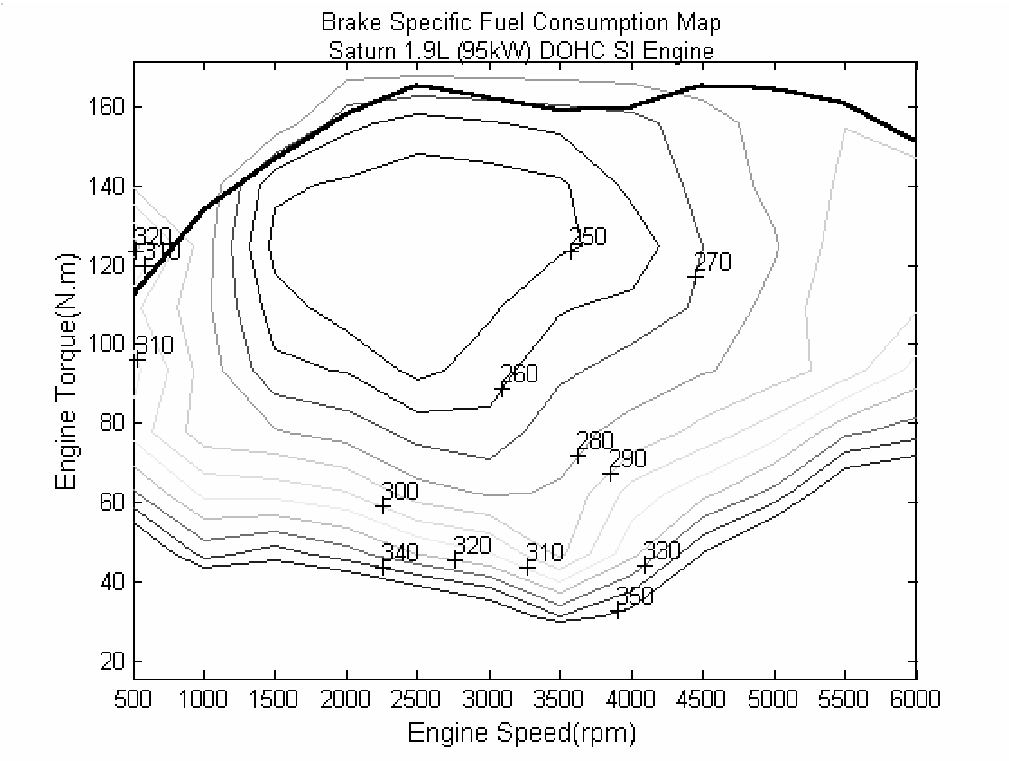


**Figure 3.18** Fuel Consumption Map of a 95 kW SI engine

### 3.7.1. Model Details of CVT with torque converter

In this model, it is important to determine the values of the CVT ratio and the speed ratio across the torque converter precisely to obtain the required engine speed. The main problem is to determine the required engine speed and the required torque at the same time since the torque converter impeller has a capacity which is determined by the capacity factor and stall speed of torque converter. For that reason, the required engine speed, which is obtained from OOL, can not be reached all the time while that speed can not satisfy the torque requirement. To find the operating point, which is closest to the optimum operating point and satisfies the torque requirement, a ‘while’ loop is constructed in the fuel consumption model of CVT with torque converter. The working principle of the loop is to check all the ratio range of CVT to find the optimum CVT ratio that satisfies the nearest engine speed and torque to required ones obtained from the OOL. It also checks the engine torque requirement at that engine speed and if the engine torque requirement is higher than the maximum torque that the engine can supply at that engine speed, system continues to check the ratio range to satisfy the torque requirement in the range of the maximum torque that the engine can supply. In figure 3.19, Simulink block of the ‘while’ loop is shown.

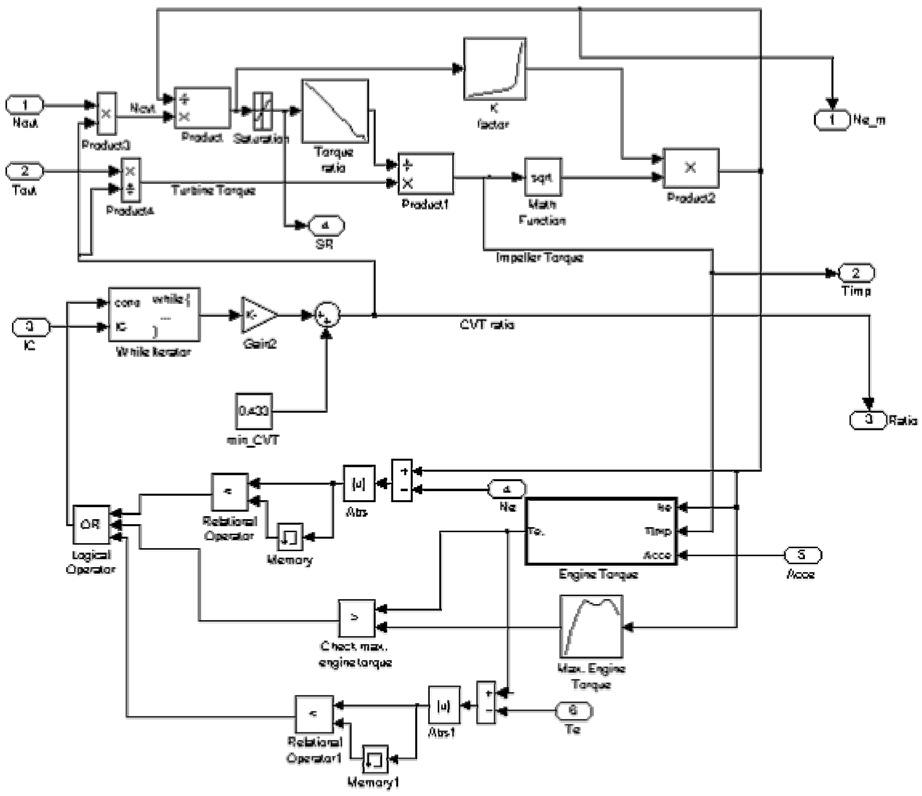

**Figure 3.19** Simulink Block of ‘while’ loop

### 3.7.2. Model Details of CVPST

This model is not as complicated as the previous one. The idea is to track the optimum operating points. The main constraint is the CVPST ratio range. The model calculates the ratio of required engine speed obtained from OOL to the required transmission output speed, which is necessary to obtain the required velocity pattern. If the ratio is out of the bounds of the CVPST ratio range, then the model uses the maximum or the minimum CVPST ratio and calculates the engine speed and torque with this CVT ratio.

### 3.7.3. Model Details of Automatic Transmission

Since no continuously variable transmission ratio is available in this model, it is impossible to determine an OOL and track these operating points for minimum fuel consumption. Instead, to obtain minimum fuel consumption, shift logic map of automatic transmission can be modified. In the model, the required torque and speed is used to calculate the corresponding engine torque and speed required. This calculation is done for each gear ratio. The required transmission output speed is used with look-up tables, which are constructed from the shift logic map, to find gear number. After finding gear number, the required engine speed and torque are also known. By using these values with the engine map look-up table, the required throttle valve opening is calculated and this value is checked if the selected gear is correct or not.

# CHAPTER 4

# SIMULATION RESULTS AND ANALYSIS

In this study, three different vehicles, which are equipped with two different arrangements of CVT systems and an automatic transmission, are modeled by using Simulink 4, which is a tool for dynamic system simulation in Matlab 6.01. By using mathematical models of dynamic systems and with the help of the Simulink Library, which contains blocks that are used for expressing the mathematical equations as signal input and output systems, vehicle models are developed, which are composed of algebraic loops and differential equations. By setting the suitable solver from the simulation parameters menu, the required outputs, such as vehicle velocity, acceleration, fuel consumption, etc. can be obtained in graphical form or in the form of data array. In the simulations, fifth order ordinary differential equation solver with fixed step size is used, while the models contain continuous states.

Totally six models are developed for two different purposes. One of them is to measure and compare the performances of these three vehicles and discuss the superiorities of these three transmissions with performance aspects. Second purpose is to find the fuel consumptions of them and compare them in that aspect.

Also setting the parameters of the CVT system appropriately, optimization of these systems is satisfied.

**4.1. Simulation Conditions**

To make comparison between transmission systems, parameters that affect the performance and fuel consumption must be comparable with respect to each other. Overall transmission ratio from differential to the engine shaft, weight, drag coefficient, etc. should be same if possible. The same engine characteristics should be used. For acceleration simulations, the engine characteristic shown in figure 4.1 is used [14].

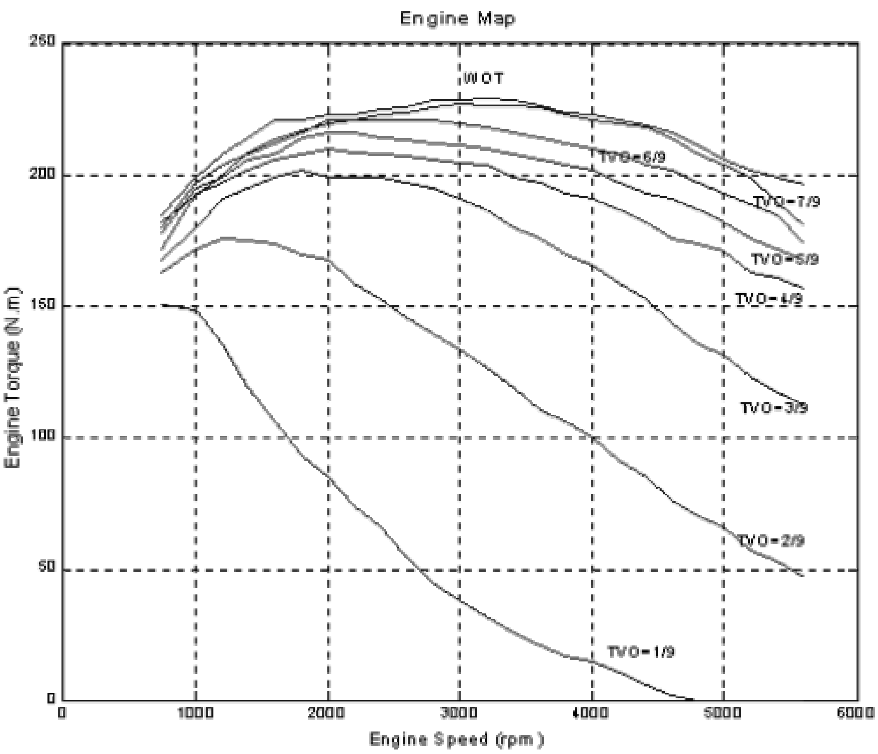


**Figure 4.1** Engine Map used for Acceleration Simulations (Salaani, 1998)

The transmission ratios of automatic transmission are fixed, and these values and also shift logic map for that transmission is obtained from reference [14]. To obtain the same overall transmission, CVT transmission system ratios must be adjusted if possible. For CVPST system, as described in reference [11], the required ratio range can be obtained by designing the system appropriately. But for CVT system with torque converter, this can not be possible, so while comparison is done, this should be taken into account. To satisfy the automatic transmission ratio, CVPST ratio range should be 2.77-0.69. With a final drive ratio, 4.125, the overall transmission ratio is 11.43-2.85. The transmission ratios of three system and the required parameters of CVPST system is given in Table 4.1.

**Table 4.1** Transmission ratios of the simulation

| **Automatic Transmission** | | **CVPST** | | **CVT with Torque Converter** |
|---|---|---|---|---|
| Gear | Gear Ratio | | Ratio | Ratio |
| 1 | 2.77 | CVT | 2.5 – 0.4 | 2.326 – 0.434 |
| 2 | 1.54 | Control/ Counter-Shaft | 0.44 | |
| 3 | 1.0 | Sun / Ring | 0.373 | |
| 4 | 0.69 | CVPST | 2.5 – 0.932 | |
| | | Step-up | 1.108 – 0.413 | |

For fuel consumption simulation, a SI engine with 95 kW maximum power at 6000 rpm and 165 N.m maximum torque at 4800 rpm is used. Specific fuel consumption data and other required engine data is obtained from reference [15]. The selected driving cycles for fuel consumption simulations are European urban cycle (ECE) and extra urban driving cycle (EUDC). The required cycle data is also obtained from reference [15]. Figure 4.2 and 4.3 show the general pattern of these two cycles and the time and speed data is also available in Appendix-B.

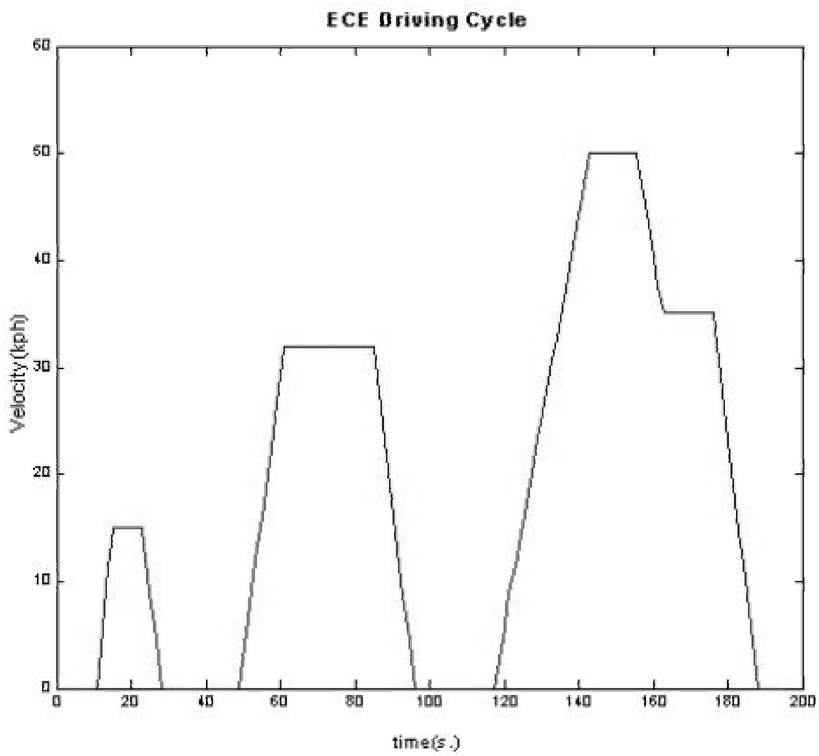


**Figure 4.2** Velocity Pattern of ECE cycle

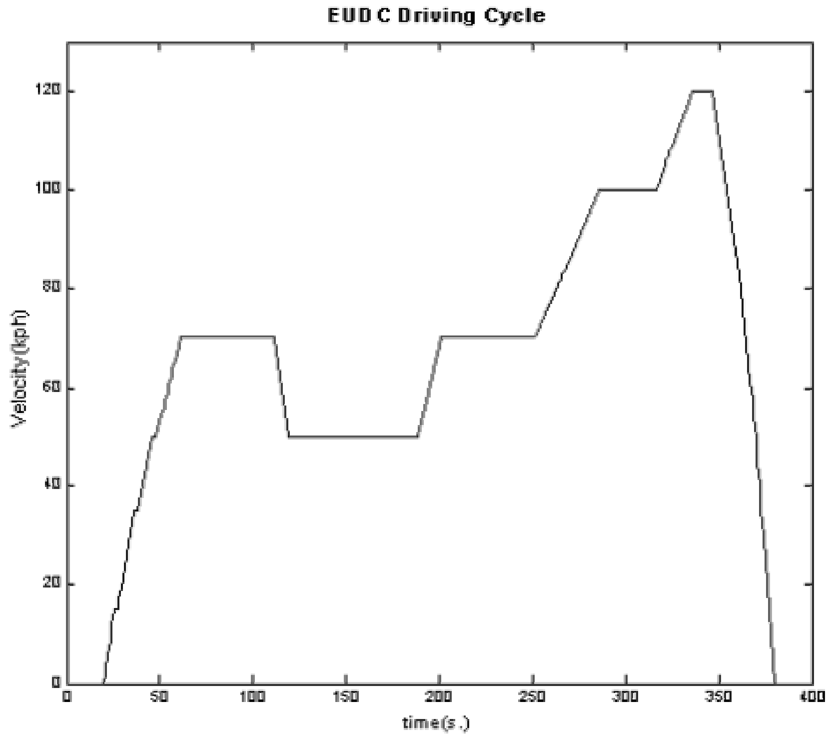


**Figure 4.3** Velocity Pattern of EUDC cycle

### 4.1.1. Simulation Assumptions

Throughout the simulation period, for acceleration or deceleration purposes, gear shifting for automatic transmission, variator ratio change for CVT systems takes place. These ratio adjustments also take time and the required time for these adjustments needs further research on this field and it is out of the scope of this thesis. So in this thesis, it is assumed that the shift is completed in an instant for all three systems.

The transmission ratio of CVT systems is determined by the variator ratio. To change the ratio, the diameters of the primary and secondary pulleys are adjusted by hydraulic cylinders. The maximum gear change rate is therefore mainly limited by the maximum hydraulic fluid flow rate. It is assumed that the change rate does not exceed the possible values.

## 4.2. Simulation of Torque-Limited Acceleration with Automatic Transmission

For torque-limited acceleration (TLA) simulation, the throttle valve opening input is taken as a step input with a final value of wide open position. It is also assumed that there is no slip between drive wheels and road. The simulation data, which is also same for other type transmissions, is shown in Table 4.2. These values are written in an M-file and can be changed from that ‘parameter.m’ file.

**Table 4.2** General Simulation Parameters

| | |
|---|---|
| Simulation Time (s.) | 20 |
| Simulation Time Step | 0.01 |
| Vehicle Mass (kg) | 1300 |
| Wheel Radius (m) | 0.3273 |
| Final Drive Ratio | 4.125 |
| Final Drive Efficiency | 0.98 |
| Frontal Area ($m^2$) | 1.9 |
| Drag Coefficient | 0.3 |
| Road Grade (%) | 0 |
| Wind Velocity (kph) | 5 (Head) |
| Tire Type | Steel-Belted Radial |

The simulation result for vehicle equipped with automatic transmission is presented throughout figures 4.4, 4.5, 4.6, 4.7 and numerical values are also available in Appendix-A. The required time for a vehicle speed 100 kph is 12.55 s.

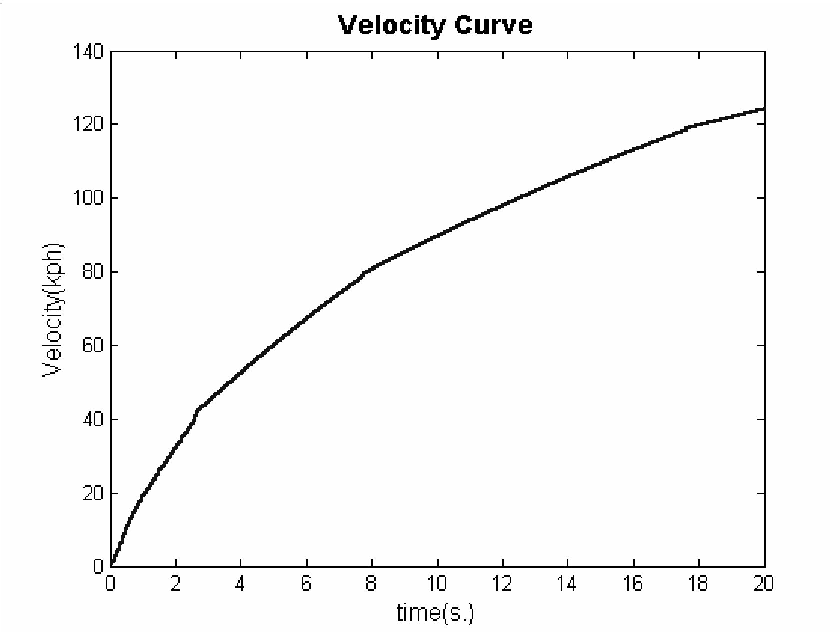


**Figure 4.4** Vehicle Velocity as a function of time (Automatic)

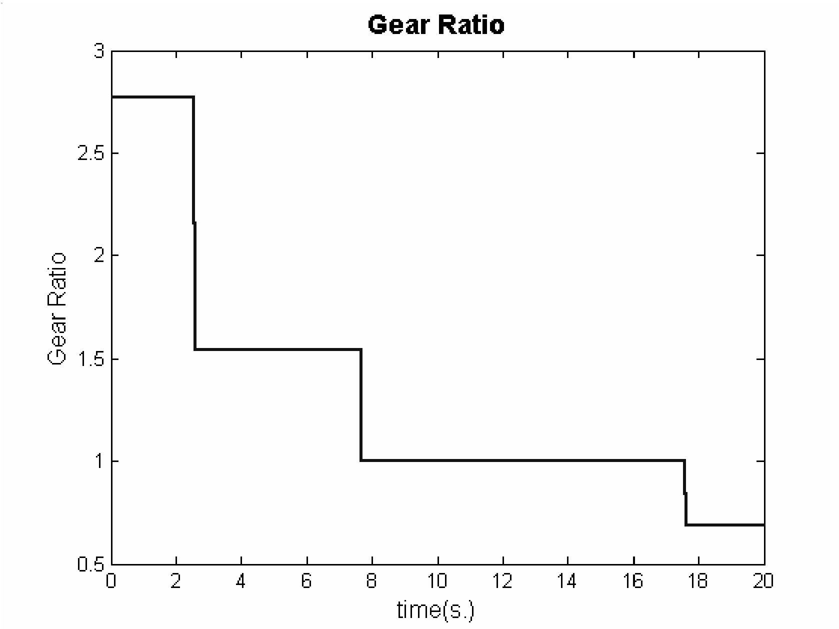


**Figure 4.5** Gear Ratio as a function of time (Automatic)

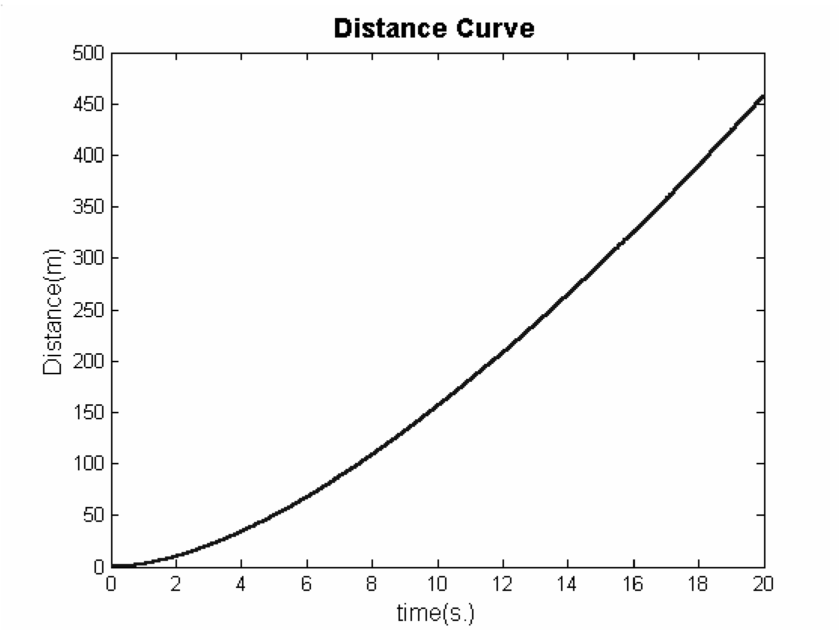


**Figure 4.6** Distance covered as a function of time (Automatic)

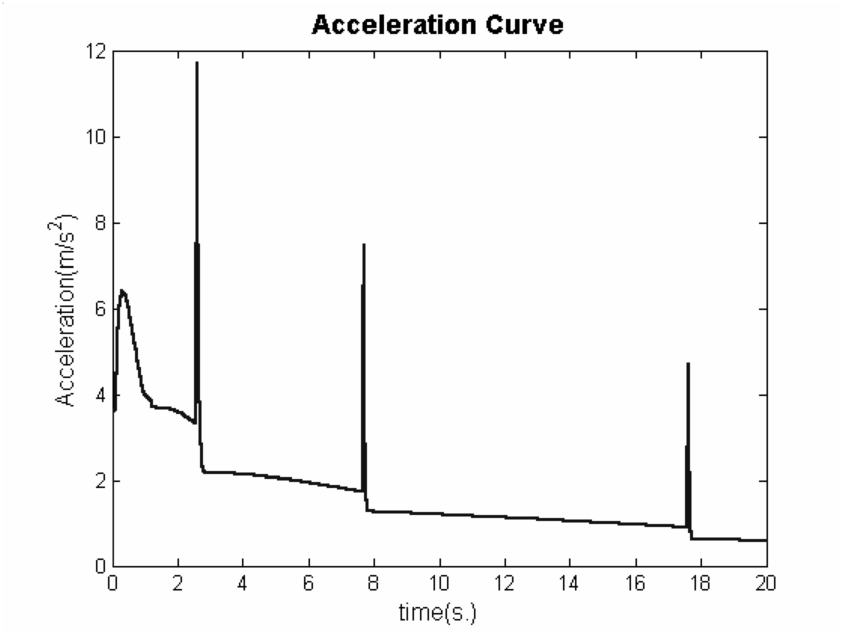


**Figure 4.7** Acceleration as a function of time (Automatic)

### 4.3. Simulation of Torque-Limited Acceleration with CVT with torque converter

The same parameters are used with this simulation also, but the transmission ratio range is not as large as automatic transmission. The CVT ratio span used is 2.326 – 0.434. Torque converter data is same with automatic transmission. The vehicle accelerates to 100 kph at 10.06 s. Simulation results are presented in figures 4.8, 4.9, 4.10, 4.11 and numerical values are also available in Appendix-A.

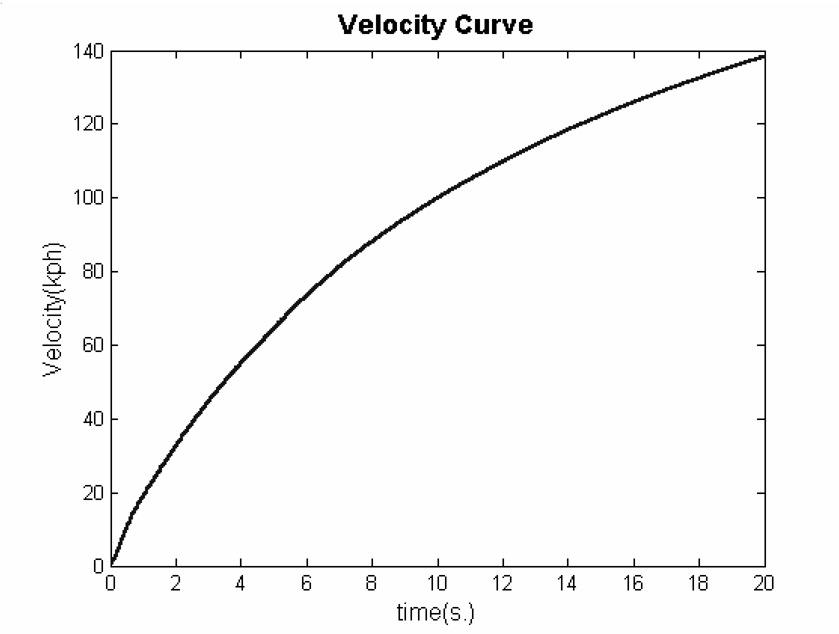


**Figure 4.8** Vehicle Velocity as a function of time (CVT with TC)

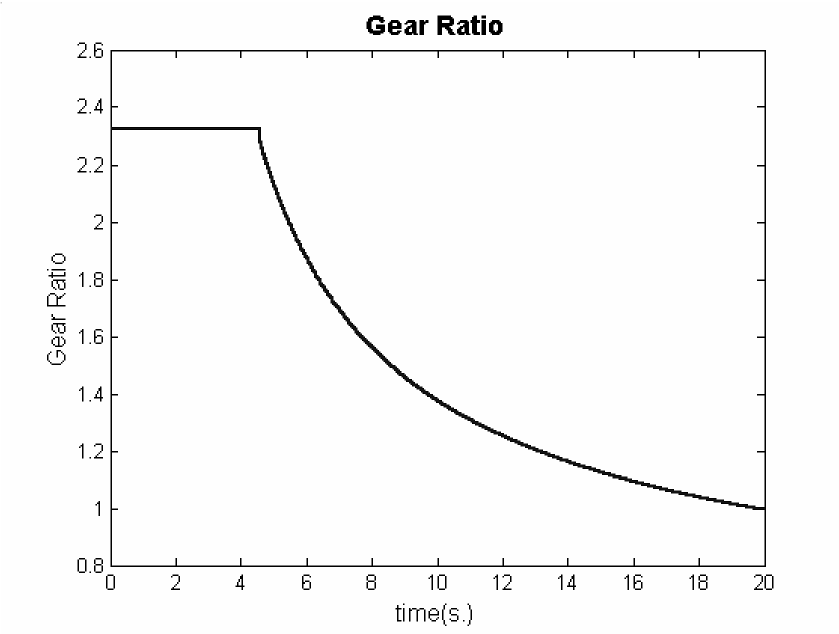


**Figure 4.9** Gear Ratio as a function of time (CVT with TC)

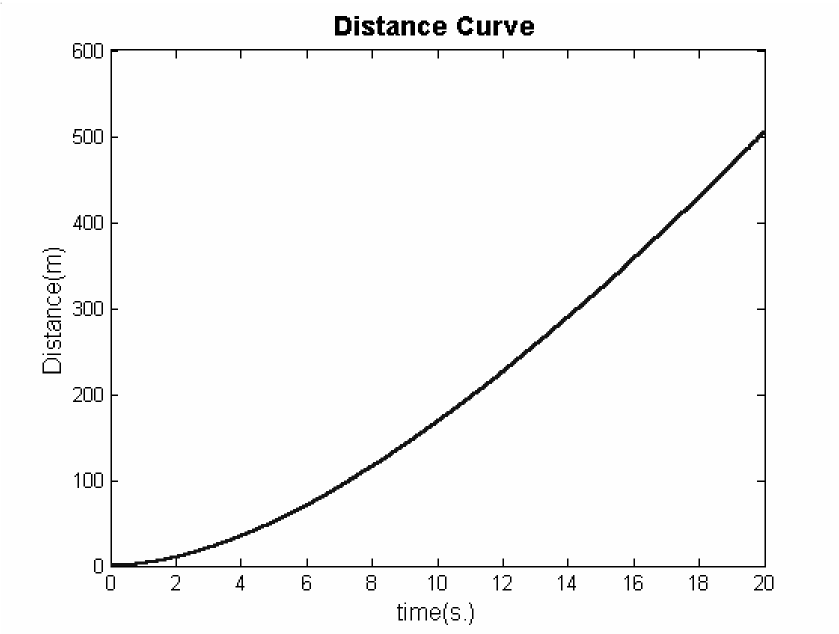


**Figure 4.10** Distance covered as a function of time (CVT with TC)

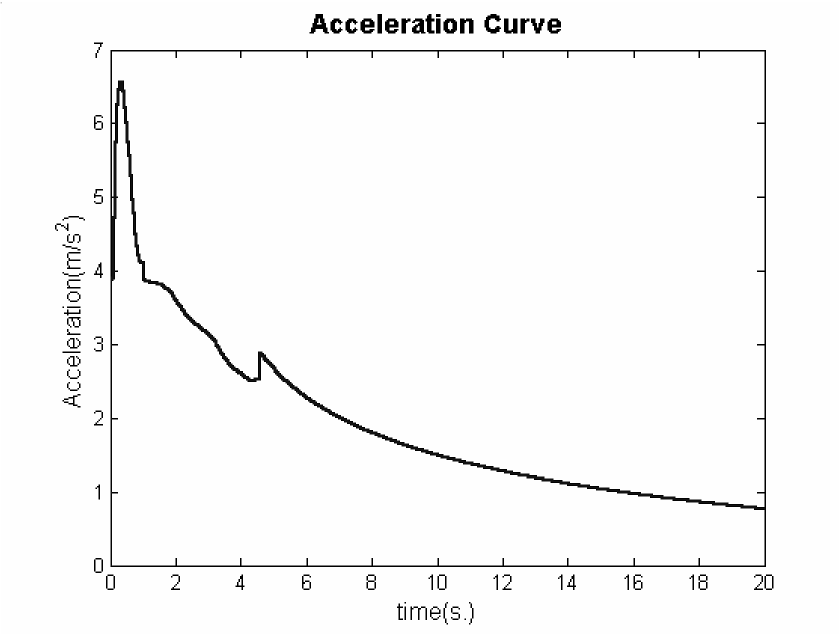


**Figure 4.11** Acceleration as a function of time (CVT with TC)

## 4.4. Simulation of Torque-Limited Acceleration with CVPST

The overall CVPST ratio span is same with automatic transmission. There exists two step up gears. At first, the first step up gear is engaged. When CVT ratio reaches the minimum possible value, the second step up gear is engaged and the CVT ratio adjusted to give the ratio before shifting. The vehicle accelerates to 100 kph at 10.96 s. Simulation results are presented in figures 4.12, 4.13, 4.14, 4.15 and numerical values are also available in Appendix-A.

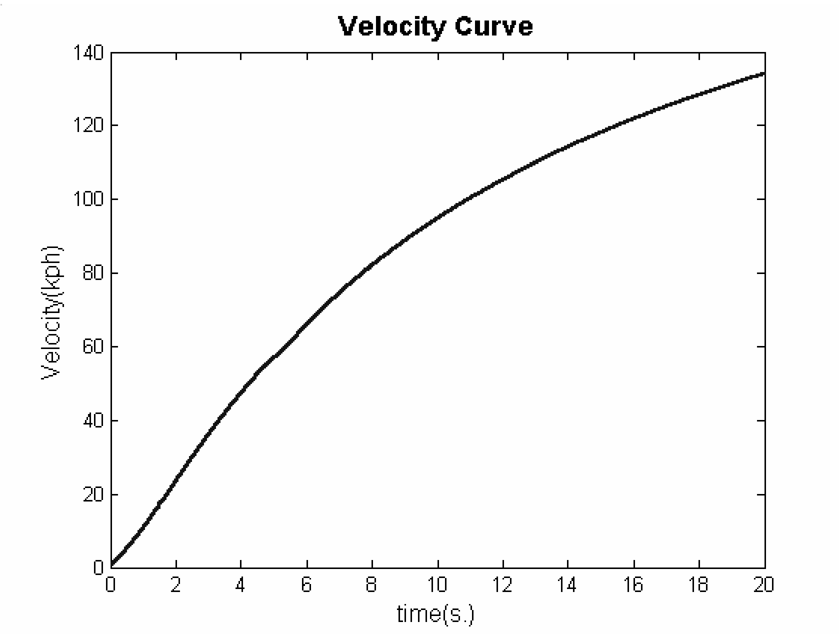


**Figure 4.12** Vehicle Velocity as a function of time (CVPST)

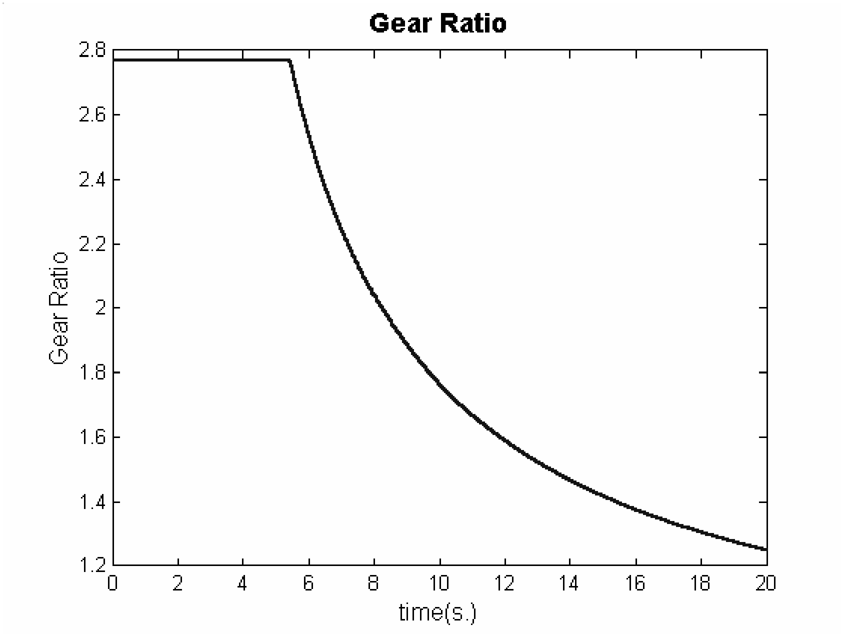


**Figure 4.13** Gear Ratio as a function of time (CVPST)

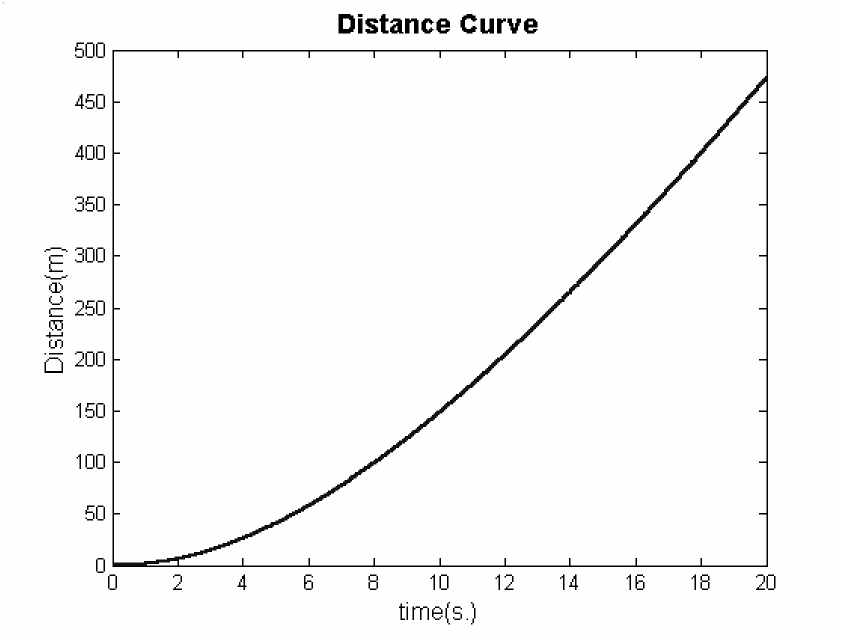


**Figure 4.14** Distance cover as a function of time (CVPST)

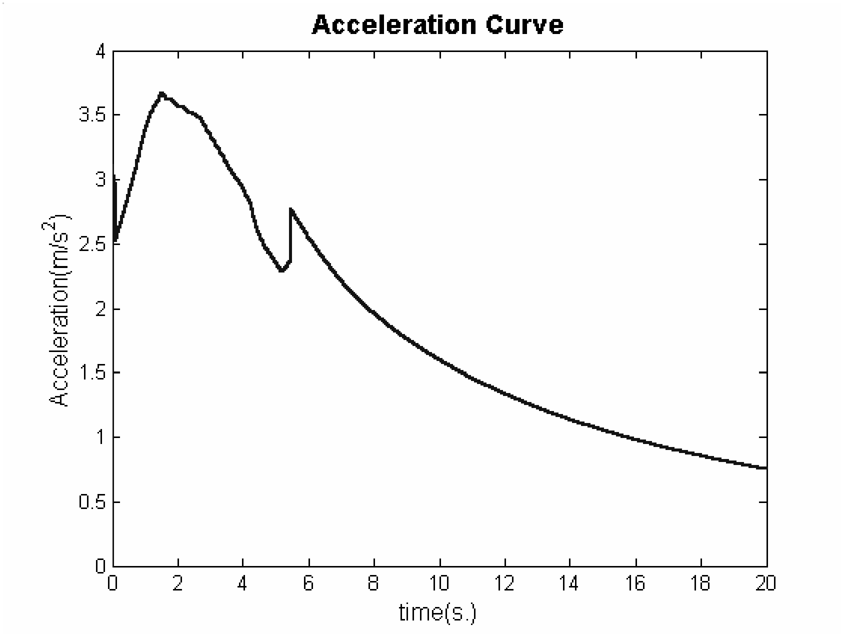


**Figure 4.15** Acceleration as a function of time (CVPST)

### 4.5. Acceleration Simulation Analysis

Simulation results for all three systems are tabulated in Table 4.3. It can be seen from results that CVT with torque converter accelerates to 100 kph quicker that the other two transmission. Automatic transmission can not cope with CVT systems from ‘0 – 100 kph acceleration’ point of view.

When the comparison of CVTs is done, it can be seen that ‘0-100 kph acceleration’ performance of CVT with torque converter is better than CVPST. This is quite a reasonable result since using of torque converter improves the start-off acceleration performance, which has been a weak point of the current CVTs. Investigation of figures 4.7, 4.11 and 4.15 shows that from maximum acceleration point of view, with a value in the order of 3.5 $m/s^2$ CVPST is the worst of all three. Its ability to maintain this value for a large period of time when it's compared with the other two is the reason for its superiority over automatic transmission. In Table 4.4, average accelerations for specified periods are tabulated. When this table is examined, it can be seen that acceleration values are approximately 9 % better than automatic transmission for the first 35 s. This is the reason for better '0–100 kph acceleration' although the maximum acceleration value of automatic transmission is better than CVPST.

When the maximum speed values are investigated, it can be said CVTs are better. But there is a handicap for CVPST. It is 6 kph faster than automatic transmission, but it takes longer to achieve that speed. When the time requirement of CVPST for 0-186 kph, which is only 65.81 s., is considered, the superiority over automatic transmission looks so obvious.

**Table 4.3** Simulation Results of Acceleration Performance

| **Transient Performance** | **Automatic** | **CVT with TC** | **CVPST** |
|---|---|---|---|
| Elapsed time (s.) for $0 \rightarrow 100$ kph | 12.55 | 10.06 | 10.98 |
| Traveling Distance (m.) for $0 \rightarrow 100$ kph | 222.34 | 168.74 | 174.2 |
| Maximum Speed (kph) | 186 | 192 | 193 |
| Elapsed time (s.) for max. speed | 145.5 | 107.92 | 172.51 |

**Table 4.3** (continued)

| | | | |
|---|---|---|---|
| Traveling Distance (m.) for max. speed | 6488 | 4906.6 | 8299.2 |
| Elapsed time for 0 – 1000 m. | 34.26 | 31.8 | 32.78 |
| Speed (kph) @ 1000 m. | 148.68 | 162.426 | 160.72 |

**Table 4.4** Average Acceleration Values

| **Transmission Type** | **0-35 s. ($m/s^2$)** | **0-120 s. ($m/s^2$)** |
|---|---|---|
| Automatic Transmission | 1.1876 | 0.4283 |
| CVT with TC | 1.3232 | 0.4451 |
| CVPST | 1.2997 | 0.4454 |

## 4.6. Fuel Consumption Simulations

The average fuel consumptions of three transmissions are given in Table 4.5. Also figures 4.16, 4.17, 4.18, 4.19, 4.20, 4.21 are showing the operating points of the transmissions and optimum operating line on fuel consumption map of SI engine with 95 kW maximum power.

**Table 4.5** Simulation Results of Fuel Consumption Models

| Type of transmission | Fuel Consumption (lt/100 km) | |
|---|---|---|
| Automatic | 11.604 (ECE) | 10.530 (EUDC) |
| CVPST | 7.292 (ECE) | 9.004 (EUDC) |
| CVT with TC | 11.295 (ECE) | 10.704 (EUDC) |
| Ideal Car(Reference) | 5.496 (ECE) | 5.765 (EUDC) |

In the figures, the maximum torque curve (thick solid line at the top) of the engine is also shown. Operating points of the engine are shown with cross (x) signs and optimum operating line is marked with ‘o’.

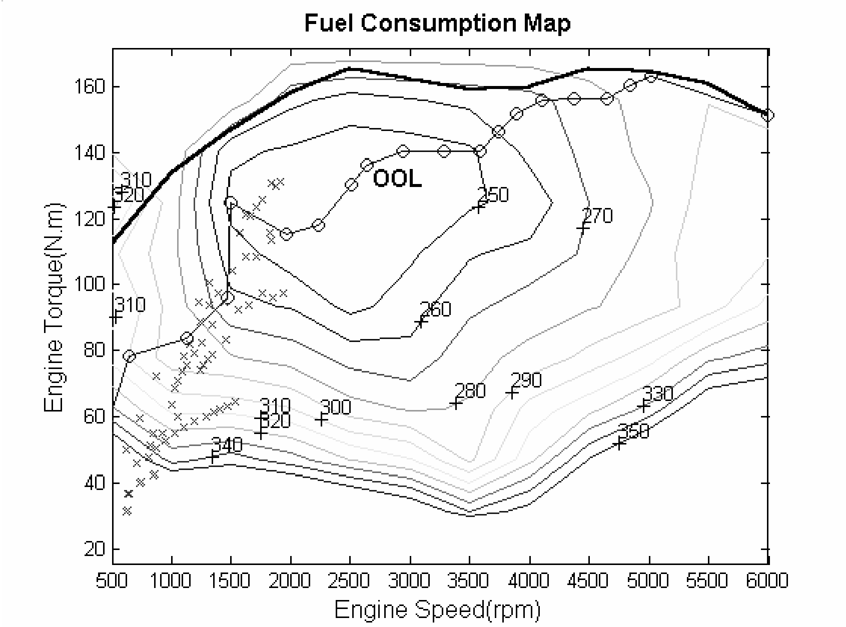


**Figure 4.16** Operating Points of Automatic Transmission (ECE)

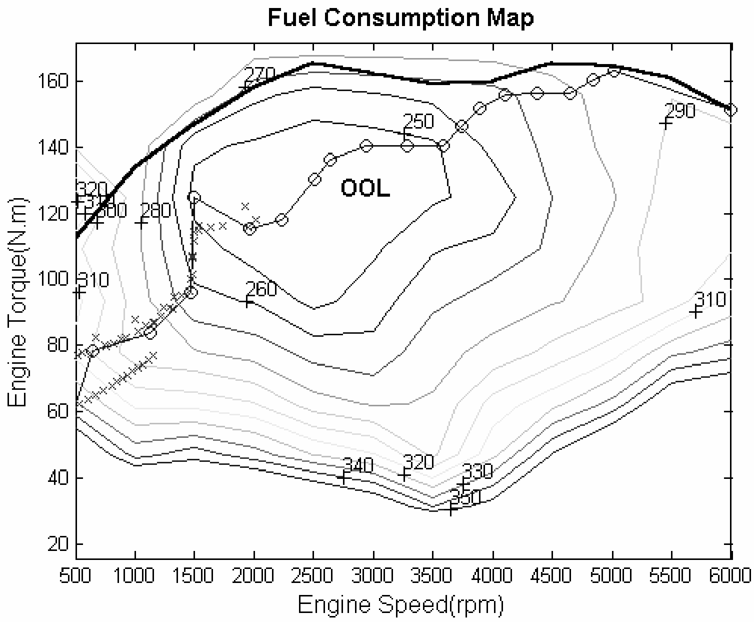


**Figure 4.17** Operating Points of CVPST (ECE)

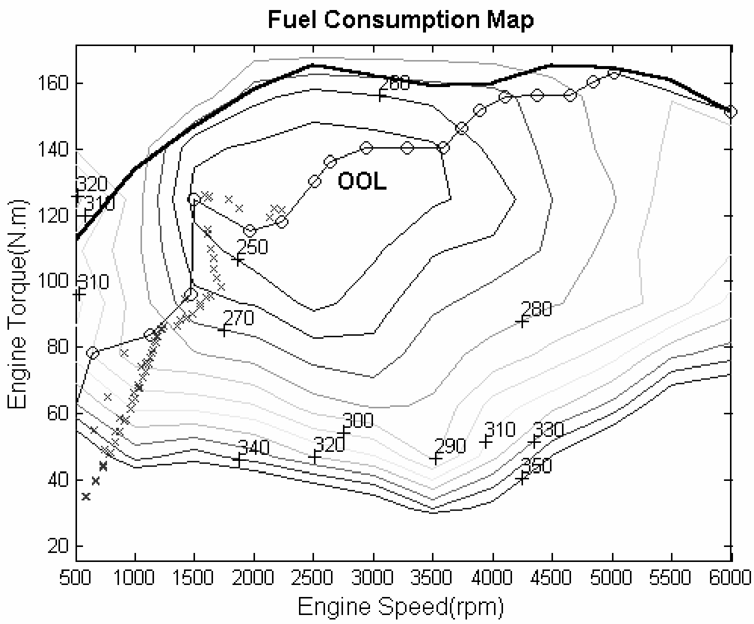


**Figure 4.18** Operating Points of CVT with TC (ECE)

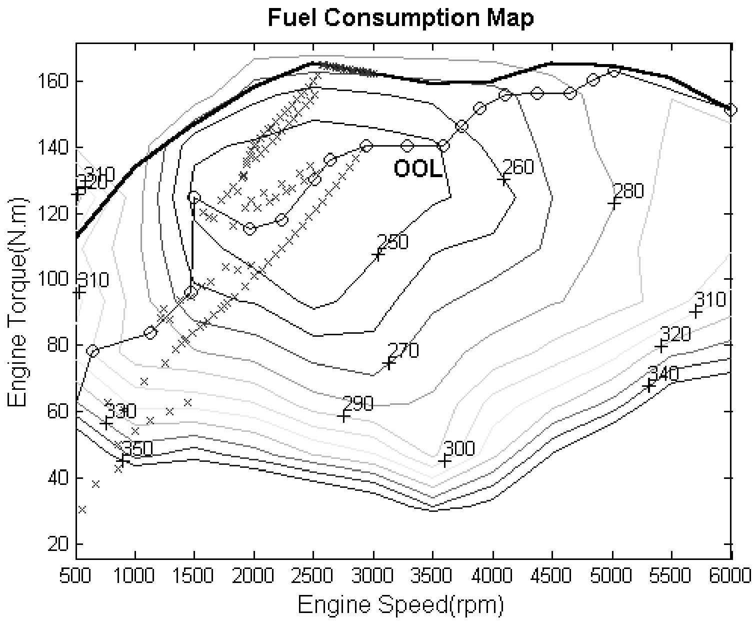


**Figure 4.19** Operating Points of Automatic Transmission (EUDC)

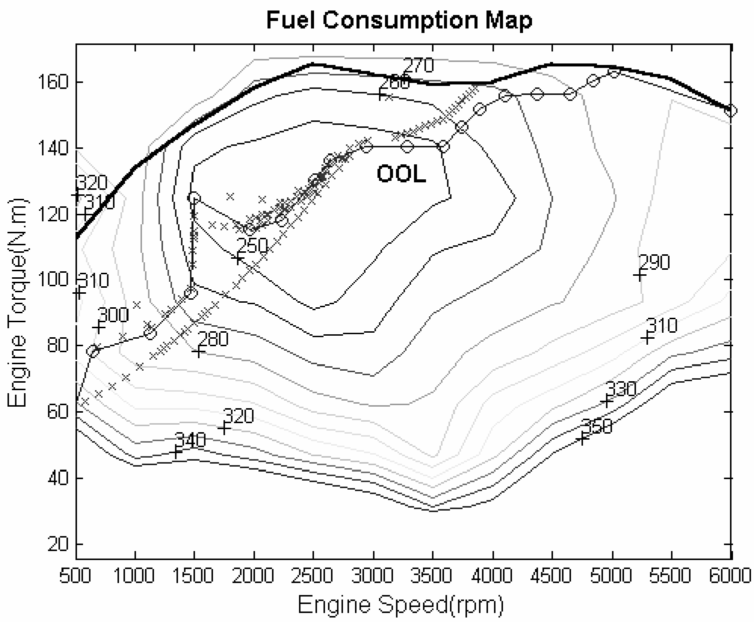


**Figure 4.20** Operating Points of CVPST (EUDC)

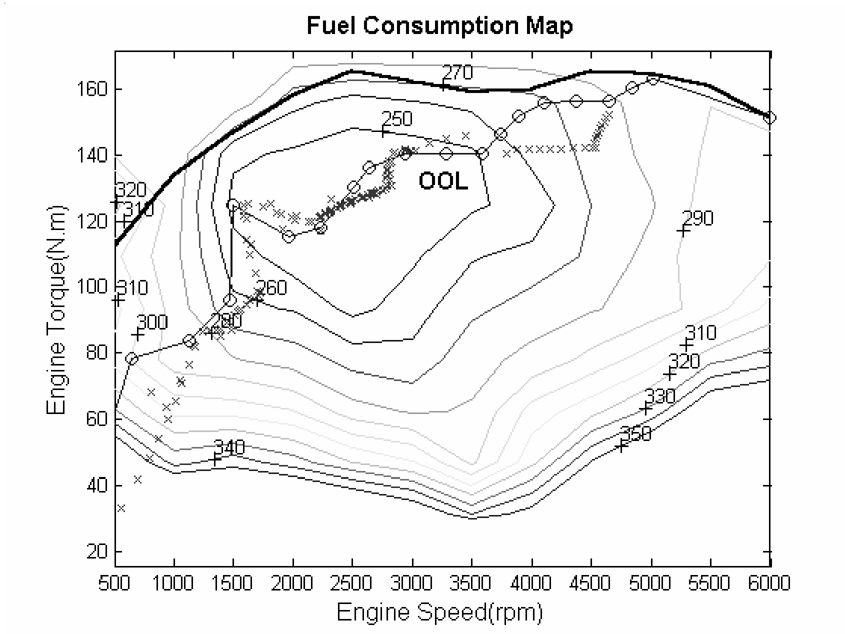


**Figure 4.21** Operating Points of CVT with TC (EUDC)

## 4.7. Fuel Consumption Simulation Analysis

When Table 4.5 is investigated, the superiority of CVT over automatic transmission can be seen easily. Especially for ECE cycle, fuel consumption improvement of CVPST is approximately 37 %. CVT with TC could satisfy 2.7 % improvement. When figures 4.16, 4.17 and 4.18 are investigated, it can be seen that CVPST could follow the OOL better than the other two type of transmission. For an ideal car, which has an infinitely large transmission ratio span and no engine losses due to inertia and accessories, fuel consumption value is about 5.5 lt/100km. This ideal car is operating at the optimum operating points. So when this value is compared with CVPST's fuel consumption, and the engine losses are taken into account, it can be said that CVPST could follow the optimum operating line satisfactorily.

When the efficiencies of the systems are examined, it can be seen that low efficiency value of CVT with torque converter system, which is approximately 78.8 %, is the main reason for smaller fuel consumption improvement value than CVPST. CVPST has an efficiency value 92.8 %. Although it is lower than the efficiency of automatic transmission, which is 96 %, CVPST's ability to track the OOL results in improved fuel consumption characteristics.

When the simulation results for EUDC cycle is examined, it can be seen that CVPST could again follow the OOL better than the other two and show an improvement in the order of 14.5 %. When Table 4.5 is investigated, it can be seen that the systems equipped with torque converter could improve their fuel consumptions for EUDC driving cycle, whereas CVPST could not show that kind of an improvement. The reason is the characteristics of the driving cycles. ECE cycle is developed for representing the city driving conditions, which are low vehicle speed and load. ECE cycle includes three start-ups from idling condition. As it can be seen from the acceleration simulations, systems equipped with torque converter could show better performance than CVPST. As a cost of this performance improvement, these systems use a little higher power than CVPST at these start-ups. As a result of that, fuel consumption values could not be as good as CVPST. For EUDC cycle, which represents the urban driving conditions, fuel consumption values of CVT with torque converter systems show improvement because of lower acceleration requirements.

From efficiency point of view, the values for CVPST and automatic transmission do not change, whereas CVT with torque converter has an improved efficiency value, which is approximately 87.7 %. But examining the fuel consumption values shows that CVT with torque converter is the worst of all in EUDC cycle. This phenomenon can be explained by its lower ratio span than the other two type of transmission.

# CHAPTER 5

# CONCLUSION AND FUTURE WORK

This thesis work aimed at simulating the acceleration performance and fuel consumption of a car equipped with a continuously variable transmission. Although CVT is an old idea, it couldn't find a wide application in automotive engineering until recent years. The main two different types, which are developed and most frequently used by the automotive engineers, are modeled and compared with each other and a 4-speed automatic transmission.

Analysis of the acceleration simulations shows that with CVT systems could provide a continuously variable ratio range and as a result of that, a smooth and continuous acceleration change can be obtained. As a result of this continuous acceleration change, jerk, which occurs in vehicle with automatic transmission and disturbs the drivers and passengers, can be eliminated. For that reason, it is more comfortable for drivers and passengers to travel in a vehicle equipped with a CVT.

A CVT with a torque converter is better than automatic transmission in acceleration performance. In spite of its low ratio span, it shows better performance than CVPST at start-ups due to the usage of torque converter. These improvements can be obtained by defining the optimum operating line for performance and adjust the CVT ratios such that the engine speed could follow that optimum operating line.

The fuel consumption simulation results show that the ability of CVTs' forcing the engine to operate at a predefined optimum operating line, which can satisfy the minimum fuel consumptions at each power requirement. The simulations show that CVPST system is the most economic one throughout the other two types of transmission.

It can be seen that from acceleration and fuel consumption point of views, CVTs are superior to automatic transmissions. It can be seen that these different two types of CVT can show improvements in different areas. During a design process of a car, the objective must be well defined and the appropriate CVT can be used.

The next phase of this study can be development of control system of CVT, since the required CVT ratios obtained are actually determined by hydraulic cylinders. Modeling of these hydraulic systems and designing of a control system which can satisfy these required CVT ratios can be a useful study.

Also the inertia variations due to the variator changes, which are non-linear, can be modeled and inserted in this study. Using developed control system and the detailed model of transmission, which includes inertia variations, an optimization function can be described, which has weight functions for performance and economy, and the solution of this optimization problem can give a control strategy, which can compromise between performance and economy according to the designer's demand. It can be better than a single track control strategy.

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# APPENDIX A

## Acceleration Simulation Results of CVT with Torque Converter

| Time (s.) | V (kph) | a ($m/s^2$) | Time (s.) | V (kph) | a ($m/s^2$) |
|---|---|---|---|---|---|
| 0.5 | 10.1761 | 6.0362 | 8.5 | 91.0955 | 1.7199 |
| 1.0 | 18.8295 | 4.1148 | 9.0 | 94.1169 | 1.6400 |
| 1.5 | 25.7831 | 3.8270 | 9.5 | 97.0013 | 1.5674 |
| 2.0 | 32.5320 | 3.6132 | 10.0 | 99.7607 | 1.5009 |
| 2.5 | 38.7509 | 3.3020 | 10.5 | 102.4053 | 1.4397 |
| 3.0 | 44.5850 | 3.1530 | 11.0 | 104.9441 | 1.3830 |
| 3.5 | 49.9909 | 2.8311 | 11.5 | 107.3845 | 1.3303 |
| 4.0 | 54.8684 | 2.6163 | 12.0 | 109.7334 | 1.2811 |
| 4.5 | 59.4507 | 2.5235 | 12.50 | 111.9966 | 1.2350 |
| 5.0 | 64.3919 | 2.6895 | 13.0 | 114.1795 | 1.1917 |
| 5.5 | 69.0279 | 2.4737 | 13.5 | 116.2868 | 1.1509 |
| 6.0 | 73.3132 | 2.2967 | 14.0 | 118.3227 | 1.1123 |
| 6.5 | 77.3069 | 2.1476 | 14.5 | 120.2911 | 1.0758 |
| 7.0 | 81.0521 | 2.0194 | 15.0 | 122.1955 | 1.0411 |
| 7.5 | 84.5822 | 1.9075 | 15.5 | 124.0390 | 1.0082 |
| 8.0 | 87.9230 | 1.8085 | 16.0 | 125.8246 | 0.9768 |

| Time (s.) | V (kph) | a ($m/s^2$) | Time (s.) | V (kph) | a ($m/s^2$) |
|---|---|---|---|---|---|
| 16.5 | 127.5551 | 0.9468 | 18.5 | 133.9718 | 0.8392 |
| 17.0 | 129.2328 | 0.9181 | 19.0 | 135.4600 | 0.8150 |
| 17.5 | 130.8601 | 0.8907 | 19.5 | 136.9056 | 0.7918 |
| 18.0 | 132.4391 | 0.8644 | 20.0 | 138.3102 | 0.7694 |

**Acceleration Simulation Results of CVPST**

| Time (s.) | V (kph) | a ($m/s^2$) | Time (s.) | V (kph) | a ($m/s^2$) |
|---|---|---|---|---|---|
| 0.5 | 5.0030 | 2.8670 | 9.5 | 91.7169 | 1.6838 |
| 1.0 | 10.6107 | 3.3670 | 10.0 | 94.6744 | 1.6049 |
| 1.5 | 16.9699 | 3.6593 | 10.5 | 97.4949 | 1.5313 |
| 2.0 | 23.4944 | 3.5758 | 11.0 | 100.1877 | 1.4629 |
| 2.5 | 29.8730 | 3.5090 | 11.5 | 102.7618 | 1.3993 |
| 3.0 | 36.0730 | 3.3477 | 12.0 | 105.2255 | 1.3400 |
| 3.5 | 41.9027 | 3.1317 | 12.50 | 107.5861 | 1.2847 |
| 4.0 | 47.3559 | 2.9371 | 13.0 | 109.8505 | 1.2329 |
| 4.5 | 52.3530 | 2.5813 | 13.5 | 112.0247 | 1.1844 |
| 5.0 | 56.7689 | 2.3539 | 14.0 | 114.1145 | 1.1389 |
| 5.5 | 61.0301 | 2.7538 | 14.5 | 116.1248 | 1.0961 |
| 6.0 | 65.8032 | 2.5563 | 15.0 | 118.0603 | 1.0557 |
| 6.5 | 70.2387 | 2.3792 | 15.5 | 119.9251 | 1.0175 |
| 7.0 | 74.3759 | 2.2242 | 16.0 | 121.7232 | 0.9814 |
| 7.5 | 78.2524 | 2.0887 | 16.5 | 123.4581 | 0.9472 |
| 8.0 | 81.9003 | 1.9694 | 17.0 | 125.1330 | 0.9147 |
| 8.5 | 85.3461 | 1.8635 | 17.5 | 126.7509 | 0.8838 |
| 9.0 | 88.6121 | 1.7689 | 18.0 | 128.3147 | 0.8544 |

| Time (s.) | V (kph) | a (m/s$^2$) | Time (s.) | V (kph) | a (m/s$^2$) |
|---|---|---|---|---|---|
| 18.5 | 129.8267 | 0.8264 | 19.5 | 132.7053 | 0.7741 |
| 19.0 | 131.2895 | 0.7997 | 20.0 | 134.0760 | 0.7496 |

**Acceleration Simulation Results of Automatic Transmission**

| Time (s.) | V (kph) | a (m/s$^2$) | Time (s.) | V (kph) | a (m/s$^2$) |
|---|---|---|---|---|---|
| 0.5 | 9.6296 | 6.0757 | 11.0 | 93.6783 | 1.1618 |
| 1.0 | 18.5855 | 4.0416 | 11.5 | 95.7530 | 1.1439 |
| 1.5 | 25.4125 | 3.6715 | 12.0 | 97.7957 | 1.1261 |
| 2.0 | 31.9806 | 3.6053 | 12.50 | 99.8065 | 1.1085 |
| 2.5 | 38.2692 | 3.3659 | 13.0 | 101.7859 | 1.0909 |
| 3.0 | 44.4962 | 2.1698 | 13.5 | 103.7284 | 1.0685 |
| 3.5 | 48.3807 | 2.1498 | 14.0 | 105.6318 | 1.0470 |
| 4.0 | 52.2348 | 2.1313 | 14.5 | 107.4970 | 1.0259 |
| 4.5 | 56.0460 | 2.0974 | 15.0 | 109.3246 | 1.0052 |
| 5.0 | 59.7769 | 2.0493 | 15.5 | 111.1152 | 0.9847 |
| 5.5 | 63.4224 | 2.0024 | 16.0 | 112.8679 | 0.9633 |
| 6.0 | 66.9787 | 1.9450 | 16.5 | 114.5824 | 0.9423 |
| 6.5 | 70.4218 | 1.8824 | 17.0 | 116.2596 | 0.9217 |
| 7.0 | 73.7534 | 1.8196 | 17.5 | 117.9002 | 0.9016 |
| 7.5 | 76.9700 | 1.7563 | 18.0 | 119.6928 | 0.6159 |
| 8.0 | 80.5465 | 1.2644 | 18.5 | 120.7952 | 0.6088 |
| 8.5 | 82.8116 | 1.2527 | 19.0 | 121.8835 | 0.6006 |
| 9.0 | 85.0515 | 1.2353 | 19.5 | 122.9572 | 0.5925 |
| 9.5 | 87.2580 | 1.2167 | 20.0 | 124.0164 | 0.5845 |
| 10.0 | 89.4311 | 1.1982 | | | |
| 10.5 | 91.5710 | 1.1799 | | | |

# APPENDIX B

**Velocity Data for ECE Cycle**

| **Time(s.)** | **Velocity (kph)** | **Time(s.)** | **Velocity (kph)** | **Time(s.)** | **Velocity (kph)** | **Time(s.)** | **Velocity (kph)** |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 18 | 14.9968 | 36 | 0 | 54 | 13.2971 |
| 1 | 0 | 19 | 14.9968 | 37 | 0 | 55 | 15.9965 |
| 2 | 0 | 20 | 14.9968 | 38 | 0 | 56 | 18.6960 |
| 3 | 0 | 21 | 14.9968 | 39 | 0 | 57 | 21.2954 |
| 4 | 0 | 22 | 14.9968 | 40 | 0 | 58 | 23.9948 |
| 5 | 0 | 23 | 14.9968 | 41 | 0 | 59 | 26.6942 |
| 6 | 0 | 24 | 11.9974 | 42 | 0 | 60 | 29.2937 |
| 7 | 0 | 25 | 8.9981 | 43 | 0 | 61 | 31.9931 |
| 8 | 0 | 26 | 5.9987 | 44 | 0 | 62 | 31.9931 |
| 9 | 0 | 27 | 2.9994 | 45 | 0 | 63 | 31.9931 |
| 10 | 0 | 28 | 0 | 46 | 0 | 64 | 31.9931 |
| 11 | 0 | 29 | 0 | 47 | 0 | 65 | 31.9931 |
| 12 | 3.7992 | 30 | 0 | 48 | 0 | 66 | 31.9931 |
| 13 | 7.4984 | 31 | 0 | 49 | 0 | 67 | 31.9931 |
| 14 | 11.2976 | 32 | 0 | 50 | 2.6994 | 68 | 31.9931 |
| 15 | 14.9968 | 33 | 0 | 51 | 5.2989 | 69 | 31.9931 |
| 16 | 14.9968 | 34 | 0 | 52 | 7.9983 | 70 | 31.9931 |
| 17 | 14.9968 | 35 | 0 | 53 | 10.6977 | 71 | 31.9931 |

| Time(s.) | Velocity (kph) | Time(s.) | Velocity (kph) | Time(s.) | Velocity (kph) | Time(s.) | Velocity (kph) |
|---|---|---|---|---|---|---|---|
| 72 | 31.9931 | 99 | 0 | 126 | 17.2963 | 153 | 49.9892 |
| 73 | 31.9931 | 100 | 0 | 127 | 19.1959 | 154 | 49.9892 |
| 74 | 31.9931 | 101 | 0 | 128 | 21.1954 | 155 | 49.9892 |
| 75 | 31.9931 | 102 | 0 | 129 | 23.3950 | 156 | 48.0896 |
| 76 | 31.9931 | 103 | 0 | 130 | 24.9946 | 157 | 46.1900 |
| 77 | 31.9931 | 104 | 0 | 131 | 26.8942 | 158 | 44.3904 |
| 78 | 31.9931 | 105 | 0 | 132 | 28.7938 | 159 | 42.4908 |
| 79 | 31.9931 | 106 | 0 | 133 | 30.7934 | 160 | 40.5912 |
| 80 | 31.9931 | 107 | 0 | 134 | 32.6929 | 161 | 38.7916 |
| 81 | 31.9931 | 108 | 0 | 135 | 34.5925 | 162 | 36.8920 |
| 82 | 31.9931 | 109 | 0 | 136 | 36.4921 | 163 | 34.9925 |
| 83 | 31.9931 | 110 | 0 | 137 | 38.4917 | 164 | 34.9925 |
| 84 | 31.9931 | 111 | 0 | 138 | 40.3913 | 165 | 34.9925 |
| 85 | 31.9931 | 112 | 0 | 139 | 42.2909 | 166 | 34.9925 |
| 86 | 29.0937 | 113 | 0 | 140 | 44.1905 | 167 | 34.9925 |
| 87 | 26.1943 | 114 | 0 | 141 | 46.1900 | 168 | 34.9925 |
| 88 | 23.2950 | 115 | 0 | 142 | 48.0896 | 169 | 34.9925 |
| 89 | 20.3956 | 116 | 0 | 143 | 49.9892 | 170 | 34.9925 |
| 90 | 17.4962 | 117 | 0 | 144 | 49.9892 | 171 | 34.9925 |
| 91 | 14.4969 | 118 | 1.8996 | 145 | 49.9892 | 172 | 34.9925 |
| 92 | 11.5975 | 119 | 3.7992 | 146 | 49.9892 | 173 | 34.9925 |
| 93 | 8.6981 | 120 | 5.7987 | 147 | 49.9892 | 174 | 34.9925 |
| 94 | 5.7987 | 121 | 7.6983 | 148 | 49.9892 | 175 | 34.9925 |
| 95 | 2.8994 | 122 | 9.5979 | 149 | 49.9892 | 176 | 34.9925 |
| 96 | 0 | 123 | 11.4975 | 150 | 49.9892 | 177 | 32.0931 |
| 97 | 0 | 124 | 13.4971 | 151 | 49.9892 | 178 | 29.1937 |
| 98 | 0 | 125 | 15.3967 | 152 | 49.9892 | 179 | 26.194 |

| Time(s.) | Velocity (kph) | Time(s.) | Velocity (kph) | Time(s.) | Velocity (kph) | Time(s.) | Velocity (kph) |
|---|---|---|---|---|---|---|---|
| 180 | 23.2950 | 184 | 11.6975 | 188 | 0 | 192 | 0 |
| 181 | 20.3956 | 185 | 8.6981 | 189 | 0 | 193 | 0 |
| 182 | 17.4962 | 186 | 5.7987 | 190 | 0 | 194 | 0 |
| 183 | 14.5969 | 187 | 2.8994 | 191 | 0 | 195 | 0 |

**Velocity Data for EUDC Cycle**

| Time(s.) | Velocity (kph) | Time(s.) | Velocity (kph) | Time(s.) | Velocity (kph) | Time(s.) | Velocity (kph) |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 18 | 0 | 36 | 34.9925 | 54 | 59.1872 |
| 1 | 0 | 19 | 0 | 37 | 34.9925 | 55 | 60.7869 |
| 2 | 0 | 20 | 0 | 38 | 34.9925 | 56 | 62.2866 |
| 3 | 0 | 21 | 2.9994 | 39 | 36.8920 | 57 | 63.7862 |
| 4 | 0 | 22 | 5.9987 | 40 | 38.7916 | 58 | 65.3859 |
| 5 | 0 | 23 | 8.9981 | 41 | 40.5912 | 59 | 66.8856 |
| 6 | 0 | 24 | 11.9974 | 42 | 42.4908 | 60 | 68.4852 |
| 7 | 0 | 25 | 14.9968 | 43 | 44.3904 | 61 | 69.9849 |
| 8 | 0 | 26 | 14.9968 | 44 | 46.2900 | 62 | 69.9849 |
| 9 | 0 | 27 | 14.9968 | 45 | 48.0896 | 63 | 69.9849 |
| 10 | 0 | 28 | 17.1963 | 46 | 49.9892 | 64 | 69.9849 |
| 11 | 0 | 29 | 19.3958 | 47 | 49.9892 | 65 | 69.9849 |
| 12 | 0 | 30 | 21.6953 | 48 | 49.9892 | 66 | 69.9849 |
| 13 | 0 | 31 | 23.8948 | 49 | 51.4889 | 67 | 69.9849 |
| 14 | 0 | 32 | 26.0944 | 50 | 53.0885 | 68 | 69.9849 |
| 15 | 0 | 33 | 28.2939 | 51 | 54.5882 | 69 | 69.9849 |
| 16 | 0 | 34 | 30.5934 | 52 | 56.1879 | 70 | 69.9849 |
| 17 | 0 | 35 | 32.7929 | 53 | 57.6876 | 71 | 69.9849 |

| Time(s.) | Velocity (kph) | Time(s.) | Velocity (kph) | Time(s.) | Velocity (kph) | Time(s.) | Velocity (kph) |
|---|---|---|---|---|---|---|---|
| 72 | 69.9849 | 99 | 69.9849 | 126 | 49.9892 | 153 | 49.9892 |
| 73 | 69.9849 | 100 | 69.9849 | 127 | 49.9892 | 154 | 49.9892 |
| 74 | 69.9849 | 101 | 69.9849 | 128 | 49.9892 | 155 | 49.9892 |
| 75 | 69.9849 | 102 | 69.9849 | 129 | 49.9892 | 156 | 49.9892 |
| 76 | 69.9849 | 103 | 69.9849 | 130 | 49.9892 | 157 | 49.9892 |
| 77 | 69.9849 | 104 | 69.9849 | 131 | 49.9892 | 158 | 49.9892 |
| 78 | 69.9849 | 105 | 69.9849 | 132 | 49.9892 | 159 | 49.9892 |
| 79 | 69.9849 | 106 | 69.9849 | 133 | 49.9892 | 160 | 49.9892 |
| 80 | 69.9849 | 107 | 69.9849 | 134 | 49.9892 | 161 | 49.9892 |
| 81 | 69.9849 | 108 | 69.9849 | 135 | 49.9892 | 162 | 49.9892 |
| 82 | 69.9849 | 109 | 69.9849 | 136 | 49.9892 | 163 | 49.9892 |
| 83 | 69.9849 | 110 | 69.9849 | 137 | 49.9892 | 164 | 49.9892 |
| 84 | 69.9849 | 111 | 69.9849 | 138 | 49.9892 | 165 | 49.9892 |
| 85 | 69.9849 | 112 | 67.4854 | 139 | 49.9892 | 166 | 49.9892 |
| 86 | 69.9849 | 113 | 64.9860 | 140 | 49.9892 | 167 | 49.9892 |
| 87 | 69.9849 | 114 | 62.4865 | 141 | 49.9892 | 168 | 49.9892 |
| 88 | 69.9849 | 115 | 59.9871 | 142 | 49.9892 | 169 | 49.9892 |
| 89 | 69.9849 | 116 | 57.4876 | 143 | 49.9892 | 170 | 49.9892 |
| 90 | 69.9849 | 117 | 54.9881 | 144 | 49.9892 | 171 | 49.9892 |
| 91 | 69.9849 | 118 | 52.4887 | 145 | 49.9892 | 172 | 49.9892 |
| 92 | 69.9849 | 119 | 49.9892 | 146 | 49.9892 | 173 | 49.9892 |
| 93 | 69.9849 | 120 | 49.9892 | 147 | 49.9892 | 174 | 49.9892 |
| 94 | 69.9849 | 121 | 49.9892 | 148 | 49.9892 | 175 | 49.9892 |
| 95 | 69.9849 | 122 | 49.9892 | 149 | 49.9892 | 176 | 49.9892 |
| 96 | 69.9849 | 123 | 49.9892 | 150 | 49.9892 | 177 | 49.9892 |
| 97 | 69.9849 | 124 | 49.9892 | 151 | 49.9892 | 178 | 49.9892 |
| 98 | 69.9849 | 125 | 49.9892 | 152 | 49.9892 | 179 | 49.9892 |

| Time(s.) | Velocity (kph) | Time(s.) | Velocity (kph) | Time(s.) | Velocity (kph) | Time(s.) | Velocity (kph) |
|---|---|---|---|---|---|---|---|
| 180 | 49.9892 | 207 | 69.9849 | 234 | 69.9849 | 261 | 78.5830 |
| 181 | 49.9892 | 208 | 69.9849 | 235 | 69.9849 | 262 | 79.3829 |
| 182 | 49.9892 | 209 | 69.9849 | 236 | 69.9849 | 263 | 80.2827 |
| 183 | 49.9892 | 210 | 69.9849 | 237 | 69.9849 | 264 | 81.0825 |
| 184 | 49.9892 | 211 | 69.9849 | 238 | 69.9849 | 265 | 81.9823 |
| 185 | 49.9892 | 212 | 69.9849 | 239 | 69.9849 | 266 | 82.8821 |
| 186 | 49.9892 | 213 | 69.9849 | 240 | 69.9849 | 267 | 83.6819 |
| 187 | 49.9892 | 214 | 69.9849 | 241 | 69.9849 | 268 | 84.5818 |
| 188 | 49.9892 | 215 | 69.9849 | 242 | 69.9849 | 269 | 85.3816 |
| 189 | 51.4889 | 216 | 69.9849 | 243 | 69.9849 | 270 | 86.2814 |
| 190 | 53.0885 | 217 | 69.9849 | 244 | 69.9849 | 271 | 87.0812 |
| 191 | 54.5882 | 218 | 69.9849 | 245 | 69.9849 | 272 | 87.9810 |
| 192 | 56.1879 | 219 | 69.9849 | 246 | 69.9849 | 273 | 88.8808 |
| 193 | 57.6876 | 220 | 69.9849 | 247 | 69.9849 | 274 | 89.6807 |
| 194 | 59.1872 | 221 | 69.9849 | 248 | 69.9849 | 275 | 90.5805 |
| 195 | 60.7869 | 222 | 69.9849 | 249 | 69.9849 | 276 | 91.3803 |
| 196 | 62.2866 | 223 | 69.9849 | 250 | 69.9849 | 277 | 92.2801 |
| 197 | 63.7862 | 224 | 69.9849 | 251 | 69.9849 | 278 | 93.0799 |
| 198 | 65.3859 | 225 | 69.9849 | 252 | 70.8847 | 279 | 93.9797 |
| 199 | 66.8856 | 226 | 69.9849 | 253 | 71.6845 | 280 | 94.8795 |
| 200 | 68.4852 | 227 | 69.9849 | 254 | 72.5843 | 281 | 95.6794 |
| 201 | 69.9849 | 228 | 69.9849 | 255 | 73.3842 | 282 | 96.5792 |
| 202 | 69.9849 | 229 | 69.9849 | 256 | 74.2840 | 283 | 97.3790 |
| 203 | 69.9849 | 230 | 69.9849 | 257 | 75.0838 | 284 | 98.2788 |
| 204 | 69.9849 | 231 | 69.9849 | 258 | 75.9836 | 285 | 99.0786 |
| 205 | 69.9849 | 232 | 69.9849 | 259 | 76.8834 | 286 | 99.9784 |
| 206 | 69.9849 | 233 | 69.9849 | 260 | 77.6832 | 287 | 99.9784 |

| Time(s.) | Velocity (kph) | Time(s.) | Velocity (kph) | Time(s.) | Velocity (kph) | Time(s.) | Velocity (kph) |
|---|---|---|---|---|---|---|---|
| 288 | 99.9784 | 315 | 99.9784 | 342 | 119.974 | 369 | 53.7884 |
| 289 | 99.9784 | 316 | 99.9784 | 343 | 119.974 | 370 | 49.9892 |
| 290 | 99.9784 | 317 | 100.978 | 344 | 119.974 | 371 | 44.9903 |
| 291 | 99.9784 | 318 | 101.978 | 345 | 119.974 | 372 | 39.9914 |
| 292 | 99.9784 | 319 | 102.978 | 346 | 119.974 | 373 | 34.9925 |
| 293 | 99.9784 | 320 | 103.978 | 347 | 117.475 | 374 | 29.9935 |
| 294 | 99.9784 | 321 | 104.977 | 348 | 114.975 | 375 | 24.9946 |
| 295 | 99.9784 | 322 | 105.977 | 349 | 112.476 | 376 | 19.9957 |
| 296 | 99.9784 | 323 | 106.977 | 350 | 109.976 | 377 | 14.9968 |
| 297 | 99.9784 | 324 | 107.977 | 351 | 107.477 | 378 | 9.9978 |
| 298 | 99.9784 | 325 | 108.977 | 352 | 104.977 | 379 | 4.9989 |
| 299 | 99.9784 | 326 | 109.976 | 353 | 102.478 | 380 | 0 |
| 300 | 99.9784 | 327 | 110.976 | 354 | 99.9784 | 381 | 0 |
| 301 | 99.9784 | 328 | 111.976 | 355 | 97.4790 | 382 | 0 |
| 302 | 99.9784 | 329 | 112.976 | 356 | 94.9795 | 383 | 0 |
| 303 | 99.9784 | 330 | 113.975 | 357 | 92.4801 | 384 | 0 |
| 304 | 99.9784 | 331 | 114.975 | 358 | 89.9806 | 385 | 0 |
| 305 | 99.9784 | 332 | 115.975 | 359 | 87.4811 | 386 | 0 |
| 306 | 99.9784 | 333 | 116.975 | 360 | 84.9817 | 387 | 0 |
| 307 | 99.9784 | 334 | 117.975 | 361 | 82.4822 | 388 | 0 |
| 308 | 99.9784 | 335 | 118.974 | 362 | 79.9827 | 389 | 0 |
| 309 | 99.9784 | 336 | 119.974 | 363 | 76.2835 | 390 | 0 |
| 310 | 99.9784 | 337 | 119.974 | 364 | 72.4844 | 391 | 0 |
| 311 | 99.9784 | 338 | 119.974 | 365 | 68.7852 | 392 | 0 |
| 312 | 99.9784 | 339 | 119.974 | 366 | 64.9860 | 393 | 0 |
| 313 | 99.9784 | 340 | 119.974 | 367 | 61.2868 | 394 | 0 |
| 314 | 99.9784 | 341 | 119.974 | 368 | 57.4876 | 395 | 0 |

| **Time(s.)** | **Velocity (kph)** | **Time(s.)** | **Velocity (kph)** | **Time(s.)** | **Velocity (kph)** | **Time(s.)** | **Velocity (kph)** |
|---|---|---|---|---|---|---|---|
| 396 | 0 | 397 | 0 | 398 | 0 | 399 | 0 |
| 400 | 0 | | | | | | |