FREE FLEXURAL (OR BENDING) VIBRATIONS ANALYSIS OF DOUBLY STIFFENED, COMPOSITE, ORTHOTROPIC AND/OR ISOTROPIC BASE PLATES AND PANELS (IN AERO-STRUCTURAL SYSTEMS)

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Approval of the Graduate School of Natural and Applied Sciences

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ABSTRACT

"FREE FLEXURAL (or BENDING) VIBRATIONS ANALYSIS OF DOUBLY STIFFENED, COMPOSITE, ORTHOTROPIC AND/OR ISOTROPIC BASE PLATES OR PANELS" (IN AERO-STRUCTURAL SYSTEMS)

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In this Thesis, the problem of the "Free Vibrations Analysis of Doubly Stiffened Composite, Orthotropic and/or Isotropic, Base Plates or Panels (with Orthotropic Stiffening Plate Strips)" is investigated. The composite plate or panel system is made of an "Orthotropic and/or Isotropic Base Plate" stiffened or reinforced by adhesively bonded "Upper and Lower Orthotropic Stiffening Plate Strips". The plates are assumed to be the "Mindlin Plates" connected by relatively very thin adhesive layers. The general problem under study is considered in terms of three problems, namely "Main PROBLEM I", "Main PROBLEM II" and "Main PROBLEM III". The theoretical formulation of the "Main PROBLEMS" is based on a "First Order Shear Deformation Plate Theory (FSDPT)" that is, in this case, the "Mindlin Plate Theory". The entire composite system is assumed to have simple supports along the two opposite edges so that the "Classical Levy's Solutions" can be

applied in that direction. Thus, the transverse shear deformations and the rotary moments of inertia of plates are included in the formulation. The very thin, yet elastic deformable adhesive layers are considered as continua with transverse normal and shear stresses. The damping effects in the plates and the adhesive layers are neglected. The sets of the systems of equations of the “Mindlin Plate Theory” are reduced to a set of the “Governing System of First Order Ordinary Differential Equations” in the “state vector” form. The sets of the “Governing System” for each “Main PROBLEM” constitute a “Two-Point Boundary Value Problem” in the y-direction which is taken along the length of the plates. Then, the system is solved by the “Modified Transfer Matrix Method (with Interpolation Polynomials and/or Chebyshev Polynomials)” which is a relatively semi-analytical and numerical technique. The numerical results and important parametric studies of the natural modes and the corresponding frequencies of the composite system are presented.

Keywords: Free Vibrations, Orthotropic Composite Plate or Panel Vibrations, Bonded Repair of Cracked Plates, Multi-Layer Plates, Bonded Plates, Stiffened Composite Plates.

ÖZ

"ÇİFT DESTEKLİ, KOMPOZİT, ORTOTROPİK VE/VEYA İZOTROPİK, TABAN LEVHA (PLAKA) VEYA PANELLERİNİN SERBEST EĞİLME TİTREŞİMLERİNİN ANALİZİ" (HAVA-ARACI YAPISAL SİSTEMLERİNDE)

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Bu tezde, "Çift Destekli, Kompozit, Ortotropik Şerit Plakalarla Desteklenmiş, Ortotropik ve/veya İzotropik Taban Levha (Plaka) ve Panellerinin Serbest Titreşimlerinin Analizi" problemi incelenmiştir."Ortotropik ve/veya İsotropik Taban Plakaları", yapıştırıcıyla birleştirilmiş "Alt ve Üst Ortotropik Destekleyici Şerit Plakalar" ile güçlendirilerek veye takviye edilerek "Kompozit Plaka ve Panel Sistemi" ni oluşturmuştur.Plakaların kendilerine göre çok ince yapıştırıcı tabakalarla birbirine bağlanmış "Mindlin Plakaları" oldukları varsayılmıştır. Ana problem "Ana PROBLEM I", "Ana PROBLEM II", "Ana PROBLEM III" olarak üçe ayrılmıştır. "Ana PROBLEMLER" in teorik formülleştirilmesi "Birinci Derece Kayma Deformasyon Plaka Teorisi"ne dayanmaktadır, bu da "Mindlin Plaka Teorisi"dir. Tüm kompozit sistemin karşılıklı iki kenarında basit mesnetli sınır durumlarına sahip olduğu kabul edilir, bu nedenle "Klasik Lévy Çözümleri" bu yönde uygulanabilir. Sonuç olarak, enine kayma deformasyonu ve plakalardaki dönme atalet momenti

denklemlerde dahil edilmiştir. Çok ince aynı zamanda elastik olarak deforme olabilen yapıştırıcı tabakaların enine dik ve kayma gerilmeleriyle süreklilik gösterdiği hesaba katılmıştır. Plakalardaki ve yapıştırıcı tabakalardaki titreşim sönümleyici özellikler ihmal edilmiştir. Sonra,"Mindlin Plaka Teorisi" denklem sistemleri takımı, "durum vektörü" halindeki "Esas Birinci Derece Basit Diferansiyel Denklemler Sistemi" ne dönüşmüştür. "Esas Sistem" in üç denklem takımı her bir problem için plakaların boyları doğrultusunda alınan y-yönünde "İki Noktalı Sınır Değer Mekanik Problemi"ni oluşturmaktadır.Bu durumda, daha önce adı geçen sistem, bazı belli plaka ve kabuk titreşim problemleri için geliştirilmiş yarı-analitik ve sayısal teknik olan, "Değiştirilmiş Transfer Matris Yöntemi (Interpolasyon Polinomları ve/veya Chebyshev Polinomlarıyla)" ile çözülmüştür. Kompozit sitemin doğal mod şekilleri ve karşılık gelen frekanslarının sayısal sonuçları ve önemli parametrik çalışmaları her üç "Ana PROBLEM" için sunulmuştur.

Anahtar Kelimeler: Serbest Titreşimler, Ortotropik Kompozit Plaka ve Panellerin Titreşimleri, Çatlamış Plakaların Yapıştırılarak Tamiri, Çok-tabakalı Plakalar, Yapıştırılmış Plakalar, Desteklenmiş Kompozit Plakalar.

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CHAPTER 1

INTRODUCTION

1.1 Introductory Remarks

For many years, aircraft designers could propose theoretical designs that could not be build because the materials needed to construct them did not exist. (The term "unobtainium" is sometimes used to identify materials that are desired but not yet available.) For instance, large spacecraft like the “Space Shuttle” would have proven extremely difficult, if not impossible, to build without heat-resistant ceramic tiles to protect them during reentry. And high-speed forward-swept-wing airplanes like Grumman's experimental X-29 or the Russian Sukhoi S-27 Berkut would not have been possible without the development of “Composite Materials” to keep their wings from bending out of shape.

Because of the significant developments in the “Advanced Composites” technology together with the “Adhesive Bonding” techniques and the epoxy-based adhesives, the multi-layer composite structural systems are being increasingly used in aerospace, marine and other light-weight, high-strength vehicles as primary and secondary structures and structural components. (Lubin [I.1], Hoskins [I.2], Marshall [I.3], ASM International Handbook[I.4], Schwartz [I.5], Baker [I.6]).

The “Advanced Composites” are the most important materials developed and adapted for aviation since the starting the use of aluminum in the 1930s. In general, Composites are materials that are combinations of two or more components or phases. One material serves as a “resin” or "matrix," which is the material that holds

everything together, while the other material serves as a “reinforcement”, in the form of fibers embedded in the matrix. Until recently, the most common matrix materials were "thermosetting" materials such as epoxy, bismaleimide, or polyimide. The reinforcing materials can be glass fiber, boron fiber, carbon fiber, Kevlar, or other more exotic mixtures.

Fiberglass is the most common composite material, and consists of glass fibers embedded in a resin matrix. Fiberglass was first used widely in the 1950s for boats and automobiles, and today most cars have fiberglass bumpers covering a steel frame. Fiberglass was first used in the Boeing 707 passenger jet in the 1950s, where it comprised about two percent of the structure. By the 1960s, other composite materials became available, in particular boron fibers and graphite fibers, embedded in epoxy resins. The U.S. Air Force and U.S. Navy began research into using these materials for aircraft control surfaces like ailerons and rudders. The first major military production use of boron fiber was in the horizontal stabilizers on the U.S.Navy's F-14 Tomcat interceptor. By 1981, the British Aerospace-McDonnell Douglas developed AV-8B Harrier aircraft with over 25 percent of its structure made of composite materials.

The manufacturing of composite aircraft structures is more complex than manufacturing most aluminum alloy structures. To make a composite structure, the composite material, in tape or fabric form, is laid out and put in a mold and cured under heat and pressure. The resin matrix material flows and when the heat is removed, it solidifies. It can be formed into various shapes. In some cases, the fibers are wound tightly to increase strength. One useful feature of composites is that they can be layered, with the fibers in each layer running in a different direction. This allows materials engineers to design structures that behave in desired ways. For instance, they can design a structure that will bend in one direction, but not in another. The designers of the Grumman X-29 Experimental Aircraft used this attribute of composite materials to design forward-swept wings that did not bend up at the tips like aluminum alloy wings of the same shape would have bent in flight.

The greatest advantage of “Advanced Composites” is that they are light-weight and high-strength which are their main characteristics The heavier an aircraft weighs, the more fuel it burns, so reducing weight is crucial to aeronautical and astronautical engineers. The weight savings of the order of 45-50% are generally considered to be achievable using current “Advanced Composites” in place of conventional light-weight metal alloys.

Despite their high-strength and low-weight, composites have not been a miracle solution for air and space vehicle structures. They have some disadvantages, too. Composites are hard to inspect for flaws. Some of them absorb moisture. And they age by passing time. Most importantly, they can be expensive, primarily because they are labor intensive and often require complex and expensive fabrication methods and equipments. Aluminum, by contrast, is easy to manufacture and repair. Anyone who has ever gotten into a minor car accident has learned that the dented metal can be hammered back into shape, but a crunched fiberglass bumper has to be completely replaced. The same is true for many composite materials used in aviation.

The modern airliners, currently in use all over the world, employ significant amounts of composites to achieve lighter weight. About 10% of the structural weight of the Boeing 777, for instance, is made of composite materials. Modern military aircraft, such as F-22 “Joint Strike Fighter”, use composites for at least 45% of their weight. Some experts have predicted that future military aircraft will be, weight-wise, more than two-thirds will be made of composite materials. But for now, military aircraft use substantially greater percentages of composite materials than commercial passenger aircraft primarily because of the different ways that commercial and military aircraft are designed and manufactured and employed.

Aluminum is a very tolerant material and can take a great deal of punishment before it fails. It can be dented or punctured and still hold together. Composites are not like this. If they are damaged, they require immediate repair, which is difficult and expensive. An airplane made entirely from aluminum can be repaired almost anywhere. This is not the case for composite materials, particularly as they use different and more exotic materials. Because of this, composites will probably

always be used more in military aircraft, which are constantly being maintained, than in commercial aircraft, which have to require less maintenance.

Composites can be classified on the basis of the form of their structural components: fibrous (composed of fibers in a matrix), laminar (composed of layers of materials), and particulate (composed of particles in a matrix). Resins can be classified as thermoplastic (capable of being repeatedly hardened and softened by increase and decrease, respectively, in temperature) or thermoset (changing into a substantially infusible and insoluble material when cured by the application of heat or by chemical means). At present, the use of thermosetting resins (polyester, poyimide and epoxy) predominates. (Lubin [I.1]).

Thermoplastics are a relatively new material that is replacing thermosets as the matrix material for composites. They hold much promise for aviation applications. One of their big advantages is that they are easy to produce. They are also more durable and tougher than thermosets, particularly for light impacts, such as when a wrench dropped on a wing accidentally. The wrench could easily crack a thermoset material but would bounce off a thermoplastic composite material.

In addition to composites, other advanced materials are under development for aviation. During the 1980s, many aircraft designers became enthusiastic about ceramics, which seemed particularly promising for lightweight jet engines, because they could tolerate hotter temperatures than conventional metals. But their brittleness and difficulty to manufacture were major drawbacks, and research on ceramics for many aviation applications decreased by the 1990s.

“Aluminum” and “Advanced Aluminum Alloys” still remain a remarkably useful material for aircraft and some spacecraft structures and metallurgists have worked hard to develop better aluminum alloys (a mixture of aluminum and other materials). In particular, aluminum-lithium is the most successful of these alloys. It is approximately ten percent lighter than standard aluminum. Beginning in the later 1990s it was used for the Space Shuttle's large external tank in order to reduce weight and enable the shuttle to carry more payload. Its adoption by commercial

aircraft manufacturers has been slower, however, due to the cost of lithium and the greater difficulty of using aluminum-lithium (in particular, it requires much care during welding). But it is likely that aluminum-lithium will eventually become a widely used material for both commercial and military aircraft.

The composite materials that are generally used for aircraft structures belong to the class known as ‘‘fiber composites’’ comprising continuous fibers embedded in a resin (or ‘‘plastic’’) matrix. Currently, graphite/epoxy and, to a lesser extend, kevlar/epoxy are seen as the most important composites for aircraft structural applications. These composite materials in the form of laminated and multi-layer shells, plates and beams are extensively used in aircraft structures. Another group of composites is the fiber reinforced metal alloys (metal matrix composites) which are stronger than the usual metals.

Some major applications of composites in aircraft structures are given in Table 1.1. (Hoskins and Baker[I.2], ASM International Handbook [I.4])

Most aero-structures consist of an assembly of a number of individual elements or components connected or joined together to form a structural system appropriate for a certain type of vehicle. These connections or joints are potentially the weakest points in the structure and may determine its viability. In general, it is desirable to minimize the number of joints in a structure in order to minimize both its weight and cost. The fiber reinforced composites have an important advantage over metals in this respect, since large one-piece components can readily be produced. They are, then, joined together to create the final structure.

The types of joints can be classified as: (a) adhesively bonded; (b) mechanical, either fastened (bolted,riveted, etc.) or wedge loaded; and (c) combinations of types (a) and (b). In the first type, loads are carried by the surface of the joint elements in shear, bending and tension through a layer of adhesive or resin. In the second type, the loads are carried by the joint elements in compression on the contacting faces. Comparing the two major joining techniques shows that adhesive bonding, where appropriate, is usually preferred over conventional fastening

techniques since it offers greater uniformity in load distribution (i.e. more structural efficiency) , smoother joints in reducing drag, better fatigue properties, sealing against corrosion, and relatively light weight. Hereafter, the name ‘‘Laminated Plates’’ will refer to the cases where similar or different layers of fibers are put in the same matrix to form the plate and ‘‘Multi-Layer Plates’’ will refer to a system in which similar or different plates are somehow bonded to construct the desired shape of the structure.

The theoretical and experimental knowledge and reliable information are needed on the free and forced vibrations of the stiffened composite plates and panels in the design of the flight vehicle structures. This kind of knowledge is extremely important in the studies of panel flutter, sonic fatigue and sound transmission etc. in panels of the complex structural systems used in aircraft structures. Consequently, these stiffened or partially stiffened plates and shells are increasingly receiving the attention of the research workers in the scientific and engineering literature. Among these studies some important ones will be mentioned and briefly reviewed in the following section.

1.2 Literature Survey and Brief Review

There are many research studies on the “Advanced Materials” such as “Composites” and their manufacturing techniques and applications in various areas of engineering. Lubin [I.1], ASM International Handbook [I.4] and Schwartz [I.5] are some of the important handbooks about ‘‘Composites’’. Also, the application of composites to the aircraft and space vehicle structures can be obtained from a book by Hoskins and Baker [I.2] and also from the text by Marshall and Devauts [I.3] and from other references included in those books.

Leissa [II.1], in this monograph, basically concentrated on the ‘‘Classical Plate Theory (CLPT)’’ and isotropic materials and presented a collection of investigations on the free vibration of plates. Ambartsumyan [II.2] presented a very extensive work on the vibration of anisotropic plates based on his improved theory.

Ambartsumyan also formulated symmetrically laminated anisotropic plates. The most recent review of the theories of dynamics and vibrations of isotropic and anisotropic plates and multi-layer composite plates is available in Kapania and Raciti [II.3,II.4]. In Noor and Burton [III.1] and in Reddy and Robbins [III.2], one can find some comparisons of various theories of multi-layer composite plates.

The "Stiffened Plates or Panels" and/or "Stepped- Thickness Plates" and their variants are used in various applications in several areas of engineering and technology. These classes of plates are primarily used in light-weight, high-speed air and spacecraft, and marine vehicle structures. They are also increasingly used in automotive vehicle structures. Our main concern and review in this thesis are limited to the base plates or panels which are "centrally, non-centrally or at supports doubly stiffened base plates or panels by the upper and lower stiffening plate strips".

Some of the relatively old studies concentrated on the free vibrations of thin plates with uniformly spaced multi-stiffeners also use some kind of approximations. A commonly used one is to approximate or replace the stiffened thin plate by an orthotropic one, that is to define the flexural rigidities in two directions such that the isotropic plate with stiffeners becomes an orthotropic plate. The free vibrations of isotropic, thin plates with two (or more) stiffeners or beams are investigated by Wah [IV.1], Lin [IV.2], Long[IV.3,IV.5], Smith [IV.4], Bath [IV.7]. Rectangular plates with identical and equally spaced stiffeners (beams with rectangular cross-section) are studied in Wah [IV.1] and Lin [IV.2]. The free vibration of rectangular plates continuous over identical and equally spaced elastic beams are investigated by Wah [IV.1]. Initially, Wah used the combination of frequencies obtained from the classical theories. Then, he used an orthotropic plate in his study to increase the accuracy of simulating the vibration modes of a stiffened plate and the results were compared to those of orthotropic plate theory. Lin [IV.2] also determined the natural frequencies and normal modes of vibration for panels with identical and equally spaced stringers and assumed simple supports on all four edges of the system. Long [IV.3] and Smith [IV.4] carried out a more detalied analysis for panels with multi stiffening beams which are simply supported along the edges normal to the direction

of stiffening plate. Later on, Long [IV.5] further simplified the analysis by neglecting inplane displacement in the direction normal to the direction of stiffening plate. Long [IV.5] used the matrix method as the solution method. Yurkovich [IV.6] studied dynamic analysis of a stiffened panel system. Bath [IV.7] used Rayleigh-Ritz method to analyze the vibration of plates with nonuniformly spaced stiffeners. Besides the methods mentioned above, the finite element method used by Olson and Hazeli [IV.8], Mukherjee and Mukhopadhyay [IV.9] and the finite strip methods employed by Srinivasan and Munaswamy [IV.10], Chen et al. [IV.11] in order to solve the stiffened plate problems. The free vibrations of thin, isotropic plates with one stiffener are considered by Kirk [IV.12], Aksu and Ali [IV.13]. Aksu [IV.14, VII.5]. Kirk [IV.12] determined the natural frequencies of the first symmetric and first antisymmetric modes of a simply supported rectangular plate reinforced by a single integral stiffener (or beam with rectangular corss-section) placed along its centre line. The free vibrations of thin, isotropic plates with a central stiffener (beam with rectangular cross-section) were studied by developing a variational technique in conjunction with finite difference formulation by Aksu and Ali [IV.13]. Aksu [IV.14] investigated the effect of the transverse shear deformation and rotatory inertia and also inplane inertia on the free vibration of plates with a single stiffener. Lee and Lee [IV.15] investigated the vibration analysis of anisotropic thin plates reinforced by a central stiffener. Applegarth, Bull, Bettess, Bond and Thompson [IV.16] developed an auto-adaptive finite element analysis software for stiffened plate and shell structures. In their studies, Barrette, Berry and Beslin [IV.17] used hierarchical trigonometric functions to investigate the vibrations of stiffened plates. Zeng and Bert [IV.18] studied on the differential quadrature analysis of vibration for rectangular stiffened plates. Wei-Chau Xie and Richard Zhi-Hua Chen [IV.19] made the vibration mode localization in rib-stiffened plates with misplaced stiffeners in one direction. Sheikh and Mukhopadhyay [IV.20] studied on linear and nonlinear transient vibration analysis of stiffened plate structures.

In multi-layer plates, the thickness of the bonding or adhesive layer is generally neglected in most of the studies in literature. That is, the bonding between the plate layers is assumed to be rigid bonding. However, the natural frequencies and

the corresponding mode shapes can be significantly affected by the relatively small thicknesses and the elastic constants of the adhesive layers. Also, the fracture and the delamination of the multi-layer plate systems usually initiate in the adhesive layer and then propagate to the final failure. There are very few studies in layered composites taking into account the relative flexibility and deformations of adhesive layers. Some of these studies on orthotropic plates are presented by Yuceoglu and Özerciyes [V.4-V.13]. In assuming a very thin but deformable adhesive layer or layers between the laminae one can significantly change the static and dynamic characteristics and responses of the entire multi-layer composite system as pointed out in Irie et al [V.1] and in Yuceoglu et al. [V.2, V.3] and in Yuceoglu and Özerciyes [V.4 to V.13].

The studies about the stepped-thickness and/or stiffened plates available in the scientific and engineering literature are based mostly on the ‘‘Classical Plate Theory’’ and on isotropic plates. Another very important point is that, in all the above mentioned papers [IV.1-IV.20], the stiffener(s) is(are) beam(s) with rectangular cross section(s). Another restriction or deficiency here is that the transverse shear deformations and the rotatory moments of inertia of plates are not taken into account in the dynamic equations.

It is a well-known fact that the natural frequencies obtained from the “Classical Plate Theory (CLPT)” and “First Order Shear Deformation Plate Theories (FSDPT)” such as Reissner [VI.1, VI.2, VI.5] and Mindlin [VI.3, VI.4] and “Higher Order Shear Deformation Plate Theories (HSDPT)” (some of which can be found in Noor and Burton [III.1] and in Reddy [VI.6]) are significantly different.

Messina and Soldatos [VI.7] studied on the vibration of completely free composite plates and cylindrical shell panels by a higher-order theory. Attia and El-Zafrany [VI.8] used a higher-order shear element in their study for nonlinear vibration analysis of composite layered plates and shells. Singh, Yadav and Iyengar [VI.9] obtained natural frequencies of composite plates with random material properties using a higher-order shear deformation theory instead of first order shear deformation theory. Kant and Swaminathan [VI.10] also investigated the free

vibration of isotropic, orthotropic, and sandwich plates based on higher order refined theories. Wittrick [VI.11] studied about analytical 3-D elasticity solutions to some plate problems and some observations on Mindlin's Plate Theory.

If the stiffening plate strip(s) is/are connected to the base plate by adhesive layer(s), the composite plate system is a two or more layer plate system in the stiffened region. Therefore, the influence of the transverse shear deformations and the rotatory moments of inertia should definitely be considered in the formulation and the solution of the problem at least in this multi-layer plate region. The importance of this point in the statics and dynamics of multi-layer plates or laminated plates are investigated by Reissner and Stavsky [VII.1], Whitney [VII.2], Whitney and Pagano [VII.3], Whitney[VII.4] and by Dawe and Craig [VII.6], Craig and Dawe [VII.7]. Aslo, Aksu [VII.5] investigated the effect of transverse shear derformation and rotatory inertia on vibrations of stepped-thickness plates. Additionally, the influence of the adhesive layer(s) on the deformations of the stiffening plate strip(s) and the base plate can be significant if the very thin adhesive layer(s) is/are deformable (even though with relatively small strains). This also affects the natural frequencies and the corresponding mode shapes of the entire partially stiffened composite plate or panel system.

From the above brief review, it can be concluded that the composite orthotropic and/or isotropic base plates or panels doubly reinforced by the upper and lower stiffening plate strips are not considered in the literature. Therefore, free dynamic response of those classes of Stiffened Plates or Panels are the subject of this thesis.

In the present study, the transverse shear deformations and the rotatory inertias of both the base plate and the stiffening plate strips will be included in the formulation in terms of the Mindlin Plate Theory for orthotropic plates. Also, the transverse shear and normal strains and stresses are to be taken into account in the very thin, yet deformable in-between adhesive layers. The reason for not considering "Higher Order Shear Deformation Theories (HSDT)" (see for instance Reddy [VI.6])) is that there is not significant difference in the natural frequencies of the Laminated plates obtained by (HSDPT) and "First Order Shear Deformation

Theories (FSDPT)". This fact is clearly demonstrated in a recent study by Khadeir and Librescu [VII.8].

Hunter [VIII.1] used the "Integrating Matrix Method" for determining the natural vibration characterics of propeller blades. The present method of solution which is used in this thesis can also be seen in Yuceoglu et al. [VIII.2-VIII.7]

Table 1.1. “Advanced Composites” and “Composite Components” in Aircraft Structures for Various Types of Aircraft (most of the data below taken from Ref. [I.2] and [I.4])”

Composite Component	F-14	F -15	F-16	F-18 Hornet	B-1	AV-8B	DC-10 Demo	L-1011	B-737 Demo	B-727	B-757	B-767	Lear Fan	JAS-39 Gripen	F-18 Super Hornet
Doors	√			√	√	√					√	√	√	√	√
Rudder		√				√	√				√	√	√	√	√
Elevator										√	√	√	√	√	√
Vertical tail		√	√	√	√	√	√	√						√	√
Horizontal tail	√	√	√	√	√	√			√				√		√
Aileron						√		√			√	√		√	√
Spoiler									√		√	√		√	√
Flap					√	√					√		√	√	√
Wing box				√		√							√	√	√
Body						√							√		
Miscellaneous	Fairings	Speed brake		Speed brake, fairings	Slats, inlets	Fairings		Fairings			Fairings	Fairings	Fairings	Fairings, speed brake, slats, canard	Fairings, speed brake, slats

Note :

(1): √ means that composite is used for that component.

(2): Some of the composite applications in the above table are given according to the information available in 1997 or so.

CHAPTER 2

MAIN PURPOSE AND SCOPE

2.1 Introductory Remarks and Motivation for Present Study

In this Section, a certain group or class of the "Free Flexural (or Bending) Vibration Problems of Stiffened Mindlin Plates" will be considered.

From the brief review of studies on stiffened plates available in the scientific and the engineering literature, and also from the practical applications point of view in aerospace vehicles structures (or aero-structures), one may conclude that the following free vibrations problems need to be considered because of,

- The "Stiffening and/or Reinforcing" of relatively slender "Advanced Composite" or "Advanced Metal Alloy" plate or panel systems in aero-structures or aero-structural systems.
- The "Bonded Repairing and/or Strengthening" of relatively slender and already cracked "Advanced Composite" or "Advanced Metal Alloy" plate or panel systems in aero-structures or aero-structural systems.
- Also, the "Free Dynamic Response" of doubly stiffened composite orthotropic and/or isotropic base plates or panels are extremely important from the point of view of sound transmission, sonic fatigue and fracture, dynamic fatigue and fracture, panel flutter, dynamic panel structural stability, etc in aero-structures.

The motivation and justification for the present Thesis and research work, therefore, are based on the above mentioned general free flexural (or bending) vibration problems encountered in aero-structures. In order to explain and also to give a general idea about the practical applications and importance of these problems, one may refer to the three basic cases presented in Figure 2.1 through 2.3. These cases will be converted to and be properly defined as and referred to as the " Main PROBLEMS" which are the main study topics of the present Thesis. Thus, the "Main PROBLEMS" are properly defined in Chapter 3 in great detail. Some general ideas, however, are given in Section 2.3.

2.2 Main Purpose and Objectives

The "main purpose" of this Thesis is to analyze the "Free Flexural (or Bending) Vibrations of Doubly Stiffened Composite Orthtropic and/or Isotropic Base Plates or Panels", particularly, in "Aero-Structural Systems".

The "main objectives" are,

- First, to determine free dynamic response characteristics in terms of their natural frequencies and corrsponding mode shapes.

- Second, to perform some important parametric studies of these composite systems in order to help the aerospace vehicle designers.

- Third, to investigate the free dynamic response, after the repairing or strengthining by upper and lower stiffening composite orthotropic plate strips, of the copmosite and/or isotropic (metal alloy) base plates or panels with through or part-through crack(s).

In the present study, the base plates are relatively thick composite orthotropic and/or isotropic Mindlin plates. The base plates are "centrally" and/or "non-centrally" and/or "at supports" doubly stiffened by dissimilar stiffening plate strips.

The "Main PROBLEMS", in general terms, involved in this work are stated or listed briefly in the following section.

2.3 Statements of "Main PROBLEMS" and General Configurations

These are, in general terms, the "Doubly Stiffened Plates" such that:

The "Main PROBLEM I" is defined as the "Free Flexural (or Bending)" Vibrations of "Centrally Doubly Stiffened, Composite Orthotropic and/or Isotropic Plates or Panels (with Upper and Lower Orthotropic Stiffening Plate Strips)".

The general configuration and its longitudinal cross section of this type of stiffened plate (i.e. "Main PROBLEM I") are shown in Figure 3.2 and Figure 3.3, respectively. The analysis will be based on Mindlin Plate Theory [VI.3,VI.4], and thus the transverse shear deformations and the rotatory inertias in the plate layers will be included in the formulation.

The "Main PROBLEM II" is defined as the "Free Flexural (or Bending)" Vibrations of "Non-Centrally Doubly Stiffened, Composite Orthotropic and/or Isotropic Plates or Panels (with Upper and Lower Orthotropic Stiffening Plate Strips)".

The general configuration and its longitudinal cross section of this type of stiffened plate (i.e. " Main PROBLEM II") are presented in Figure 3.4 and Figure 3.5, respectively. The formulation will be based on Mindlin Plate Theory [VI.3,VI.4] taking into account the transverse shear deformations and the rotatory inertias in the plate layers.

The "Main PROBLEM III" ("Special case of "Main PROBLEM II") is defined as the "Free Flexural (or Bending)" Vibrations of Doubly Stiffened-at

Support, Composite Orthotropic and/or Isotropic Plates or Panels (with Upper and Lower Orthotropic Stiffening Plate Strips)".

Again, the general configuration and its longitudinal cross section of this type of stiffened plate (i.e. " Main PROBLEM III") are given in Figure 3.6 and Figure 3.7, respectively. The analysis employs the Mindlin Plate Theory [VI.3,VI.4] taking into account the transverse shear deformations and the rotatory inertias in the plate layers.

2.4 Original Contributions

The main original contributions of the present Thesis can be summarized in the following way:

- In the "Doubly Stiffened Composite Plate or Panel System", even when the entire geometry, material and the boundary conditions are symmetric with respect to the middle vertical plane (or mid-plane) (perpendicular to the y-axis), it was found that mode shapes are not exactly symmetric, skew-symmetric. This is because of the shear deformations due to the adhesive shear modulus G_a. When the adhesive shear modulus G_a=0 (10^{-8}), the mode shapes are shown that the symmetric and subsequently skew-symmetric mode shapes develop and folllow each other in the composite plate system.

- An important additional contribution of this study is that the detailed free dynamic response of the "Bonded Repairing and/or Strengthening" of cracked plates or panels (Composite and/or Metal Alloy) is obtained which were not currently available in the open scientific and engineering literature.

- Again, another important contribution of the present work is that, in the slender "Doubly Stiffened Composite Plate or Panel Systems", the free flexural vibrations response of the "Doubly Stiffened Plate or Panel" configurations (after "Stiffening and/or Reinforcing") are obtained which might significantly affect the design of this type of aero-structures or aero-

structural components. (These systems are not yet investigated and their dynamic response is not available in the open literature).

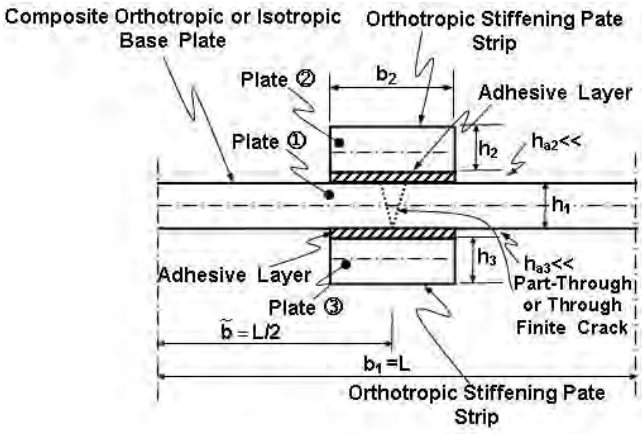

Figure 2.1 ‘‘Centrally Bonded Repair or Bonded Stiffening’’ in Composite Orthotropic and/or Isotropic Base Plate or Panel.

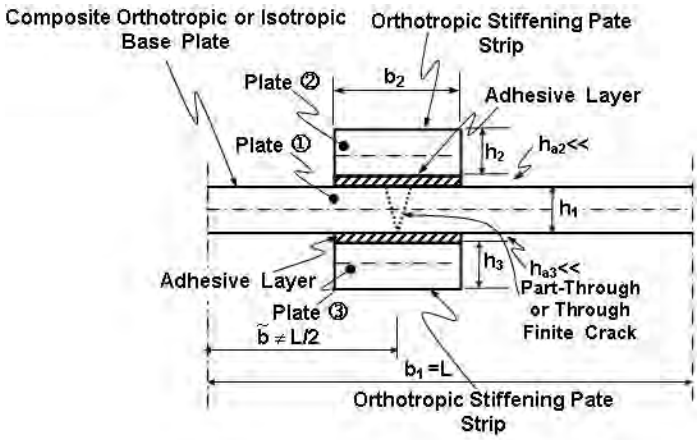

Figure 2.2 ‘‘Non-Centrally Bonded Repair or Bonded Stiffening’’ in Composite Orthotropic and/or Isotropic Base Plate or Panel.

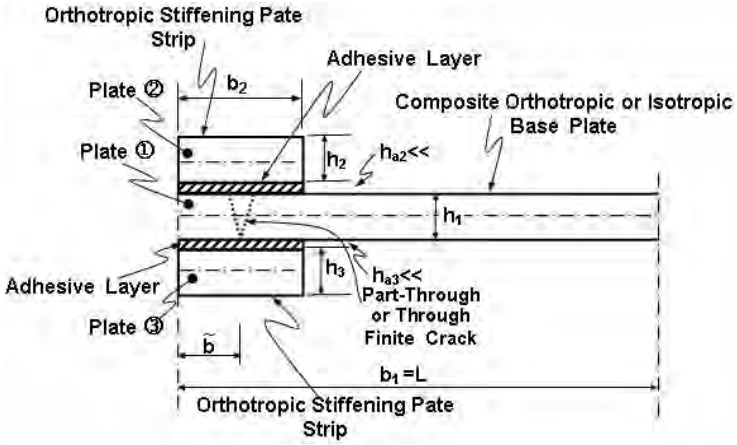

Figure 2.3 ‘‘At-Support Bonded Repair or Bonded Stiffening’’ in Composite Orthotropic and/or Isotropic Base Plate or Panel.

CHAPTER 3

MAIN PROBLEMS

3.1 Introductory Remarks

In this Chapter, the "Higher Order" and the "First Order" theories for the plate dynamic equations will be briefly reviewed. Then, proper definitions of the "Main PROBLEMS (I, II, III)" will be stated. Finally, the corresponding theoretical formulations of these problems will be developed in the next Chapter 4.

3.2 Brief Remarks on "Higher Order Shear Deformation Plate Theories (HSDPT)" used in Multi-Layer Plates

The studies carried out by research workers in the last few decades were aimed at improving the "Classical Plate Theory (CLPT)" which does not include the effect of the transverse shear deformations, the rotatory moments of inertia, and the transverse normal stress. The several studies by research workers toward new and improved theories, such as many different "First Order Shear Deformation Plate Theories (FSDPT)" which have been proposed in order to improve the "Classical Plate Theory (CLPT)" showed that the "Classical Plate Theory (CLPT)" is imperfect. The results obtained by using that theory is not accurate enough for a lot of practical problems. For example, some problems which include the vibration problems, elastic wave propagation problems, the analysis of anistropic plates, the stresses concentration due to holes or cut-outs, etc. can be mentioned.

Reissner [VI.1,VI.2] was the first researcher to produce the simplest of all the improved plate theories which incorporates the effect of shear deformations. The derivation given by Reissner results in displacements of the form;

$$\left|\begin{array}{l} u(x,y) = u_0(x,y) + z\psi_x(x,y) \\ v(x,y) = v_0(x,y) + z\psi_y(x,y) \\ w(x,y) = w_0(x,y) \end{array}\right. \tag{3.1}$$

where u_0, v_0, ψ_x, ψ_y and w_0 have a dependence upon the in-plane coordinates x and y, and ψ_x, ψ_y and w_0 are actually weighted averages, and z is the coordinate that is normal to the middle plane. The basic assumption used by Reissner [VI.1,VI.2] included consistent form for the stress distribution across the thickness, that is, the bending stresses are assumed to be distributed linearly over the thickness of the plates as in the standard theory of thin plates.

$$\left|\begin{array}{l} \sigma_x = \dfrac{M_x}{h^2/6}\dfrac{z}{h/2} \\ \sigma_y = \dfrac{M_y}{h^2/6}\dfrac{z}{h/2} \\ \tau_{xy} = \dfrac{M_{xy}}{h^2/6}\dfrac{z}{h/2} \end{array}\right. \tag{3.2}$$

A special variational theorem which consists of an application of Castigliano's theorem combined with the Lagrangian multiplier method of the calculus of variations by accounting the energy of transverse shear stresses was used to determine both equilibrium equations in terms of resultants and stress-strain relations in the form of involving resultants and the functions in (3.1).

In the same order of approximation, Mindlin [VI.3,VI.4] assumed displacements of the form in (3.1) and obtained governing equations without introducing the corresponding stress distribution assumption. The relation (3.1) predicts a uniform shear stress through the thickness of the plate, which is incorrect and would violate the surface conditions in general. Therefore, a shear correction

factor κ^2 evaluated by comparison with an exact elasticity solution was used in Mindlin's derivation.

Both Mindlin and Reissner theories can be considered as the "First Order Shear Deformation Plate Theories (FSDPT)" by considering the terms in (3.1) as the first terms in a power series in z.

The in-plane displacement functions have a non-linear dependence on z in "Higher Order Shear Deformation Plate Theories (HSDPT)" which have higher order than those of Reissner and Mindlin theories. One of them is based upon the following displacement forms:

$$\left|\begin{array}{l} u(x,y) = u_0(x,y) + z\psi_x(x,y) \\ v(x,y) = v_0(x,y) + z\psi_y(x,y) \\ w(x,y) = w_0(x,y) + z\psi_z(x,y) + z^2\xi_z(x,y) \end{array}\right. \tag{3.3}$$

The above form of displacements has been used to derive a general "Higher Order Shear Deformation Shell Theory (HSDST)" and also to derive the corresponding "Higher Order Shear Deformation Plate Theory (HSDPT)". They are also used in development of a theory of laminated cylindrical shells. (see the review by Kapania and Raciti [II.3,II.4])

The another "Higher Order Shear Deformation Plate Theory (HSDPT)" is based upon the following displacement forms:

$$\left|\begin{array}{l} u(x,y) = u_0(x,y) + z\psi_x(x,y) + z^2\xi_x(x,y) \\ v(x,y) = v_0(x,y) + z\psi_y(x,y) + z^2\xi_y(x,y) \\ w(x,y) = w_0(x,y) + z\psi_z(x,y) + z^2\xi_z(x,y) \end{array}\right. \tag{3.4}$$

From the applications of the above theory to multi-layer plates, it was seen that the second order terms in z do not provide significant advantages over the lower order theories such as Reissner and Mindlin theories [VI.1,VI.2.VI.3,VI.4].

Reissner [VI.5] has another theory for bending such that,

$$\left|\begin{array}{l} u(x,y) = z\psi_x(x,y) + z^3\phi_x(x,y) \\ v(x,y) = z\psi_y(x,y) + z^3\phi_y(x,y) \\ w(x,y) = w_0(x,y) + z^2\xi_z(x,y) \end{array}\right. \tag{3.5}$$

For pure bending of an infinite plate with a circular hole, Reissner used the above displacement field and he obtained very accurate results than the ones obtained by using the other theories. However, there is a disadvantage because it considers only out-of-plane effects. But, it does not account the contribution of in-plane modes of deformation.

More recently, Reddy [III.2] developed a "Higher Order Shear Deformation Plate Theory (HSDPT)" which has the same order of approximation as the Reissner theory. However, this theory included in-plane modes of deformation besides out-of-plane modes of deformation and it involves the displacement field of the following type:

$$\left|\begin{array}{l} u(x,y) = u_0(x,y) + z\psi_x(x,y) + z^2\xi_x(x,y) + z^3\zeta_x(x,y) \\ v(x,y) = v_0(x,y) + z\psi_y(x,y) + z^2\xi_y(x,y) + z^3\zeta_y(x,y) \\ w(x,y) = w_0(x,y) + z\psi_z(x,y) + z^2\xi_z(x,y) \end{array}\right. \tag{3.6}$$

where the "Shear Correction Factors" are not needed.

In general, the governing equations of a laminated plate system and the natural boundary conditions based upon (3.6) can be derived from the "Hamilton's Principle" such that,

$$0 = \int_0^T (\delta U + \delta V - \delta K)dt \tag{3.7}$$

where δ stands for the variations of the strain energy U, the work done by applied forces V and the kinetic energy K of the entire system.

Reddy's theory [III.2] has been used for some class of laminated plate problems. However, this thoery can also be applied to other classes of plate problems such as, cutouts, loaded holes, surface cracks, and also problems involving impact caused localized stress gradients that are essentially three-dimensional in nature, require the application of this type of higher-order theory.

In the stress analysis, "Higher Order Shear Deformation Plate Theories (HSDPT)" proved to be more accurate than "First Order Shear Deformation Plate Theories (FSDPT)". However, they are also more cumbersome and involve a large number of dependent unknowns. There are significant differences in the stresses obtained from (HSDPT) and (FSDPT) in vibration and buckling problems. However, the natural frequencies obtained from both theories were practically very close as was shown by Reddy [VI.6] and Khdeir and Librescu [VII.8] (with differences about 0.01). On the other hand, the differences in natural frequencies obtained by using the (CLPT)'s and the (HSDPT)'s are approximately 0.1. In another problem, orthotropic plates under uniformly distributed transverse loads were analyzed and it was proved that there are no differences in deflection shapes but some differences in stresses. Hence, it can be concluded that it is more practical to use a "First Order Shear Deformation Plate Theory (FSDPT)" such as "Mindlin's Plate Theory" [VI.3] instead of using a complicated "Higher Order Shear Deformation Plate Theory (HSDPT)", in order to obtain accurate natural frequencies and corresponding mode shapes for moderately thick plates and multi-layer plates.

In passing, one may mention the appearance of certain higher order moment and transverse shear force resultants is another disadvantage in the employment of higher order plate theories. These higher order resultants are due to the particular form of the proposed displacement field. They can not easily be seen to have physical meaning and they may create a complicated and, therefore, a somehow unusual and formidable image for the entire analysis of composite plate systems.

Some of the significant differences between "Classical Plate Theory (CLPT)", "First Order Shear Deformation Plate Theories (FSDPT)" and "Higher Order Shear Deformation Plate Theories (HSDPT)" are as follows:

One of the main assumptions used in "Classical Plate Theory (CLPT)" is that normals to the mid-plane before deformation remain straight and normal to the mid-plane after deformation. Also, the rotatory moments of inertia, in general, are neglected.

The corresponding assumption used in the "First Order Shear Deformation Plate Theories (FSDPT)" is that plane sections originally perpendicular to the mid-plane of the plate remain plane after deformation, but not necessarily perpendicular to the mid-plane. The "Shear Correction Factors κ_x, κ_y" are used to account for the transverse shear stresses and also the rotatory moments of inertia are included in the formulation in (FSDPT)'s.

The "Higher Order Shear Deformation Plate Theories (HSDPT)", however, allow for distorsion (of third order) of normals to the mid-plane of the undeformed plate. By this means, the need for the specification of the "Shear Correction Factors" can be eliminated from the equations. However, very complicated inertia and stress resultant terms without usual physical meanings appear in the governing equations.

3.3 "First Order Shear Deformation Theories of Plates (FSDPT)" and Mindlin's Plate Theory

In this present study, in the theoretical formulations of the "Main PROBLEMS", the "Mindlin Plate Theory" is to be employed. As mentioned in Section 3.1, according to a study by Khadeir and Librescu [VII.8], there is no difference in the natural frequencies obtained by the (FSDPT)'s and the (HSDPT)'s. Therefore, a "First Order Shear Deformation Plate Theory (FSDPT)" such as Mindlin's Plate Theory is chosen for the theoretical analysis in the present study. Therefore, the complete set of dynamic equations of the Mindlin Plate Theory are developed in this section for easy reference.

The general coordinate system and the positive sign convention for the displacements, stress resultants and external surface loads or stresses for the "Main

PROBLEMS" under consideration are given in Figure 3.1. The basic assumptions of the Mindlin Plate Theory [VI.3] are that the in-plane displacement u and v are proportional to z and transverse displacement w is independent of z. Then,

$$\left|\begin{array}{l} u = z\psi_x(x,y,t), \\ v = z\psi_y(x,y,t), \\ w = w(x,y,t) \end{array}\right. \tag{3.8}$$

where angle of rotations ψ_x and ψ_y are negative of the rotations in xz and yz-planes, respectively. (see also Figures 4.1 and 4.2 in Chapter 4)

The Hooke's law or strain-stress relations for orthotropic materials is,

$$\left|\begin{array}{l} \varepsilon_x = a_{11}\sigma_x + a_{12}\sigma_y + a_{13}\sigma_z \\ \varepsilon_y = a_{21}\sigma_x + a_{22}\sigma_y + a_{23}\sigma_z \\ \varepsilon_z = a_{31}\sigma_x + a_{32}\sigma_y + a_{33}\sigma_z \\ \gamma_{yz} = a_{44}\tau_{yz} \\ \gamma_{xz} = a_{55}\tau_{xz} \\ \gamma_{xy} = a_{66}\tau_{xy} \end{array}\right. \tag{3.9}$$

Of the above six equations, the one which is ignored, namely, the one containing the strain (ε_z) normal to the faces of the plate which are the planes at $z=\pm h/2$. The remaining equations are then solved for σ_x, σ_y, τ_{xy}, τ_{xz} and τ_{yz} in terms of ε_x, ε_y, γ_{xy}, γ_{xz}, γ_{yz} and σ_z .

$$\left|\begin{array}{l} \sigma_x = B_{11}\varepsilon_x + B_{12}\varepsilon_y - C_1\sigma_z \\ \sigma_y = B_{21}\varepsilon_x + B_{22}\varepsilon_y - C_2\sigma_z \\ \tau_{yz} = B_{44}\gamma_{yz} \\ \tau_{xz} = B_{55}\gamma_{xz} \\ \tau_{xy} = B_{66}\gamma_{xy} \end{array}\right. \tag{3.10}$$

In the case of an orthotropic material, the coefficients B_{ij} can be expressed in terms of the engineering material constants as,

$$\left|\begin{array}{ll} B_{11} = \dfrac{E_1}{1-\nu_{12}\nu_{21}}, & B_{44} = G_{23} \\ B_{22} = \dfrac{E_2}{1-\nu_{12}\nu_{21}}, & B_{55} = G_{13} \\ B_{12} = B_{21} = \nu_{21}B_{11} = \nu_{12}B_{22}, & B_{66} = G_{12} \end{array}\right. \tag{3.11}$$

where E and G are moduli of elsaticity and rigidity respectively, ν is Poisson's ratio and subscripts denote the directions parallel to the coordinate axes of the plate.

The bending and twisting moments and the transverse shear resultants, all per unit length, are written in the following integral form.

$$\left|\begin{array}{l} \left(M_x, M_y, M_{yx}\right) = \displaystyle\int_{-h/2}^{h/2} \left(\sigma_x, \sigma_y, \tau_{xy}\right) z\, dz \\ \left(Q_x, Q_y\right) = \displaystyle\int_{-h/2}^{h/2} \left(\tau_{xz}, \tau_{yz}\right) dz \end{array}\right. \tag{3.12}$$

The integrations are now performed across the thickness of the plate to obtain the stress resultants-strain relations. The results are then altered in two respects:

(1) The weighted average of σ_z is ignored, since this stress is very small compared to the bending and twisting stresses. The integrals containing σ_z are dropped. This assumption is also made in other "First Order Shear Deformation Plate Theories (FSDPT)" and the "Classical Plate Theory (CLPT)".

(2) The coefficients of the integrals containing γ_{xz} and γ_{yz} are replaced by constants whose magnitudes are to be determined later. The transverse shear deformation are taken into account in a (FSDPT). In (CLPT), however, they are neglected.

The following relations between stress resultants and strains are obtained,

$$\left|\begin{array}{l} M_x = \left[B_{11}\Gamma_x + B_{12}\Gamma_y\right] \\ M_y = \left[B_{12}\Gamma_x + B_{22}\Gamma_y\right] \\ M_{yx} = B_{66}\Gamma_{xy} \\ Q_x = \kappa_x^2 B_{55}\Gamma_{xz} \\ Q_y = \kappa_y^2 B_{44}\Gamma_{yz} \end{array}\right. \tag{3.13}$$

where

$$\left|\begin{array}{l} \left(\Gamma_x, \Gamma_y, \Gamma_{xy}\right) = \int_{-h/2}^{h/2} \left(\varepsilon_x, \varepsilon_y, \gamma_{xy}\right) z\, dz \\ \left(\Gamma_{xz}, \Gamma_{yz}\right) = \int_{-h/2}^{h/2} \left(\gamma_{xz}, \gamma_{yz}\right) dz \end{array}\right. \tag{3.14}$$

where the "Shear Correction Factor", κ^2, of the Mindlin Plate theory, is defined to account the transverse shear stresses which are not distributed uniformly through the thickness.

In the present study, κ^2, is taken as used in Mindlin's Plate Theory. In Mindlin's Plate Theory the "Shear Correction Factor" is determined by equating the exact results obtained from three dimensional elasticity equation and the result based on the "Mindlin's Plate Theory" for the case of straight-crested flexural waves in an infinite plate. These values are found to be $\pi^2/12$. [VI.3] in isotropic plates. In orthotropic plates, the "Shear Correction Factors" are defined as κ_x^2,κ_y^2, respectively and these are obtained from a study by Wittrick [VI.11].

By considering the strain-displacement equations in three dimensional elasticity;

$$\left|\begin{array}{ll} \varepsilon_x = \dfrac{\partial u}{\partial x} & \gamma_{xy} = \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \\ \varepsilon_y = \dfrac{\partial v}{\partial y} & \gamma_{xz} = \dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial x} \\ \varepsilon_z = \dfrac{\partial w}{\partial z} & \gamma_{yz} = \dfrac{\partial v}{\partial z} + \dfrac{\partial w}{\partial y} \end{array}\right| \tag{3.15}$$

Strain-displacement relations can easily be obtained by inserting the assumed form of displacement functions (3.8) into the strain-displacement equations in three dimensional elasticity (3.15):

$$\left|\begin{array}{ll} \varepsilon_x = z\dfrac{\partial \psi_x}{\partial x} & \gamma_{xy} = z\left(\dfrac{\partial \psi_x}{\partial y} + \dfrac{\partial \psi_y}{\partial x}\right) \\ \varepsilon_y = z\dfrac{\partial \psi_y}{\partial y} & \gamma_{xz} = \left(\psi_x + \dfrac{\partial w}{\partial x}\right) \\ \varepsilon_z = 0 & \gamma_{yz} = \left(\psi_y + \dfrac{\partial w}{\partial y}\right) \end{array}\right| \tag{3.16}$$

Therefore, the following relations between stress resultants and plate displacements functions are obtained:

$$\left|\begin{array}{l} M_x = \dfrac{h^3}{12}\left(B_{11}\dfrac{\partial \psi_x}{\partial x} + B_{12}\dfrac{\partial \psi_y}{\partial y}\right) \\ M_y = \dfrac{h^3}{12}\left(B_{12}\dfrac{\partial \psi_x}{\partial x} + B_{22}\dfrac{\partial \psi_y}{\partial y}\right) \\ M_{yx} = \dfrac{h^3}{12}B_{66}\left(\dfrac{\partial \psi_x}{\partial y} + \dfrac{\partial \psi_y}{\partial x}\right) \\ Q_x = \kappa_x^2 h B_{55}\left(\psi_x + \dfrac{\partial w}{\partial x}\right) \\ Q_y = \kappa_y^2 h B_{44}\left(\psi_y + \dfrac{\partial w}{\partial y}\right) \end{array}\right| \tag{3.17.a}$$

Also, in terms of the ‘‘Orthotropic Plate Stiffnesses’’,

$$
\left|\begin{array}{l}
M_x = D_{11}\dfrac{\partial \psi_x}{\partial x} + D_{12}\dfrac{\partial \psi_y}{\partial y} \\
M_y = D_{12}\dfrac{\partial \psi_x}{\partial x} + D_{22}\dfrac{\partial \psi_y}{\partial y} \\
M_{yx} = D_{66}\left(\dfrac{\partial \psi_x}{\partial y} + \dfrac{\partial \psi_y}{\partial x}\right) \\
Q_x = A_{55}\left(\psi_x + \dfrac{\partial w}{\partial x}\right) \\
Q_y = A_{44}\left(\psi_y + \dfrac{\partial w}{\partial y}\right)
\end{array}\right. \qquad (3.17.b)
$$

where the “Bending Stiffness D’s” and the “Shear Stiffness A’s” of the Orthotropic Mindlin Plate are given as,

$$
\left|\begin{array}{l}
D_{ik} = \dfrac{h^3 B_{ik}}{12}(i,k = 1,2) \\
D_{66} = \dfrac{h^3 B_{66}}{12} \\
A_{44} = \kappa_y^2 h B_{44} \\
A_{55} = \kappa_x^2 h B_{44}
\end{array}\right. \qquad (3.17.c)
$$

The across-thickness integration of the stress equations of motion of the three dimensional elasticity theory are needed. Then, these equations are,

$$
\left|\begin{array}{l}
\dfrac{\partial \sigma_x}{\partial x} + \dfrac{\partial \tau_{yx}}{\partial y} + \dfrac{\partial \tau_{zx}}{\partial z} = \rho\dfrac{\partial^2 u}{\partial t^2} \\
\dfrac{\partial \tau_{xy}}{\partial x} + \dfrac{\partial \sigma_y}{\partial y} + \dfrac{\partial \tau_{zy}}{\partial z} = \rho\dfrac{\partial^2 v}{\partial t^2} \\
\dfrac{\partial \tau_{xz}}{\partial x} + \dfrac{\partial \tau_{yz}}{\partial y} + \dfrac{\partial \sigma_z}{\partial z} = \rho\dfrac{\partial^2 w}{\partial t^2}
\end{array}\right. \qquad (3.18)
$$

The integration of the above equations results in the first two governing equations of the Mindlin Plate Theory for orthotropic plates:

$$\left|\begin{array}{l} \dfrac{\partial M_x}{\partial x}+\dfrac{\partial M_{yx}}{\partial y}-Q_x+\dfrac{h}{2}\left(q_{zx}^{(+)}+q_{zx}^{(-)}\right)=\dfrac{\rho h^3}{12}\dfrac{\partial^2\psi_x}{\partial t^2} \\ \dfrac{\partial M_{yx}}{\partial x}+\dfrac{\partial M_y}{\partial y}-Q_y+\dfrac{h}{2}\left(q_{zy}^{(+)}+q_{zy}^{(-)}\right)=\dfrac{\rho h^3}{12}\dfrac{\partial^2\psi_y}{\partial t^2} \end{array}\right. \tag{3.19}$$

where $q_{zx}^{(+)}$, $q_{zy}^{(+)}$ and $q_{zx}^{(-)}$, $q_{zy}^{(-)}$ represent the surface stresses at z=+h/2 and z=-h/2, respectively. In the right hand sides of (3.19) the coefficient terms are for the rotatory moments of inertia.

Additionally, the third equation of (3.18) is integrated over the plate thickness and, using (3.12) yields;

$$\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}+\left(q_z^{(+)}-q_z^{(-)}\right)=\rho h\frac{\partial^2 w}{\partial t^2} \tag{3.20}$$

where $q_z^{(+)}$ and $q_z^{(-)}$ represent the surface stresses at z=+h/2 and z=-h/2, respectively. Thus, the Equations (3.19) and (3.20) are the dynamic equations of the Mindlin Plate Theory.

Also, from (3.17), (3.19) and (3.20), one can obtain the following equations,

$$\left|\begin{array}{l} \dfrac{\partial\psi_y}{\partial y}=\dfrac{1}{B_{22}}\left(\dfrac{12}{h^3}M_y-B_{12}\dfrac{\partial\psi_x}{\partial x}\right) \\ \dfrac{\partial\psi_x}{\partial y}=\dfrac{12}{h^3B_{66}}M_{yx}-\dfrac{\partial\psi_y}{\partial x} \\ \dfrac{\partial w}{\partial y}=\dfrac{1}{\kappa_y^2 hB_{44}}Q_y-\psi_y \\ \dfrac{\partial M_{yx}}{\partial y}=\dfrac{\rho h^3}{12}\dfrac{\partial^2\psi_x}{\partial t^2}-\dfrac{\partial M_x}{\partial x}+Q_x-\dfrac{h}{2}\left(q_{zx}^{(+)}+q_{zx}^{(-)}\right) \\ \dfrac{\partial M_y}{\partial y}=\dfrac{\rho h^3}{12}\dfrac{\partial^2\psi_y}{\partial t^2}-\dfrac{\partial M_{yx}}{\partial x}+Q_y-\dfrac{h}{2}\left(q_{zy}^{(+)}+q_{zy}^{(-)}\right) \\ \dfrac{\partial Q_y}{\partial y}=\rho h\dfrac{\partial^2 w}{\partial t^2}-\dfrac{\partial Q_x}{\partial x}-\left(q_z^{(+)}-q_z^{(-)}\right) \end{array}\right. \tag{3.21}$$

Even though the "Classical Plate Theory (CLPT)" allows prescription of two "Boundary Conditions" along each edge, the "Mindlin Plate Theory" requires prescription of three Boundary conditions along each edge. Then, in the Mindlin Plate Theory,

The "Boundary Conditions" are,

(F) (free)	$M_{nt}=M_n=Q_n=0$	
(S) (simply supported)	$w=\psi_t=M_n=0$	(3.22)
(C) (clamped)	$w=\psi_n=\psi_t=0$	

where n and t are normal and tangential coordinates of the edges.

3.4 Definition of "Main PROBLEMS" and Scope and Limitations

In general, the three "Main PROBLEMS" considered in the present Thesis, in more detail, are as follows:

- **"<u>Main PROBLEM I</u>"**

  "Free Flexural (or Bending) Vibrations of <u>Centrally</u> Doubly Stiffened, Composite Base Plates or Panels (with Upper and Lower Orthotropic Stiffening Plate Strips Dissimilar to the Base Plate or Panel)" in which,

  I.a) <u>Orthotropic</u> Base Plate or Panel with Upper and Lower Orthotropic Stiffening Plate Strips

  I.b) <u>Isotropic</u> Base Plate or Panel with Upper and Lower Orthotropic Stiffening Plate Strips

- **"Main PROBLEM II"**

  "Free Flexural (or Bending) Vibrations of Non-Centrally Doubly Stiffened, Composite Base Plates or Panels (with Upper and Lower Orthotropic Stiffening Plate Strips Dissimilar to the Base Plate or Panel)" in which,

  II.a) Orthotropic Base Plate or Panel with Upper and Lower Orthotropic Stiffening Plate Strips

  II.b) Isotropic Base Plate or Panel with Upper and Lower Orthotropic Stiffening Plate Strips

- **"Main PROBLEM III" ("Special Case of Main PROBLEM II")**

  "Free Flexural (or Bending) Vibrations of Doubly Stiffened-at Support, Composite Base Plates or Panels (with Upper and Lower Orthotropic Stiffening Plate Strips Dissimilar to the Base Plate or Panel)" in which,

  III.a) Orthotropic Base Plate or Panel with Upper and Lower Orthotropic Stiffening Plate Strips

  III.b) Isotropic Base Plate or Panel with Upper and Lower Orthotropic Stiffening Plate Strips

The general configurations and the coordinate systems of above mentioned "Main PROBLEMS" are presented in Figures 3.2 and 3.3, 3.4 and 3.5 and 3.6 and 3.7.

In the analytical formulation of the above "Main PROBLEMS", the "Mindlin Plate Theory" is used taking into account the influences of the thickness shear deformations and the rotatory moments of inertia of plates in the dynamic equations. The basic equations of the theory are systematically derived from the fundamental

equations of three-dimensional elasticity in the preceding section. The transverse normal and shear strains and the corresponding stresses in the relatively very thin adhesive layers are included in the formulation. In all three “Main PROBLEMS”, the entire system are assumed to have simple support boundary conditions on two opposite edges while the other two opposite edges may have arbitrary support conditions in the sense of the “Mindlin Plate Theory”. This, as will be seen in later Chapters, allows to use the “Classical Lévy’s Solution” in the x-direction which facilitates the present “Method of Solution” employed in this Thesis.

In this first study, the damping effects in the plates and in the adhesive layers are neglected. In later studies, however, these effects could be included in the theoretical formulation of the “Main PROBLEMS”. Thus, their influence on the free vibration response of the entire composite system can be accounted for.

The “Method of Solution” in this work is the “Modified Transfer Matrix Method" which is very effective and accurate in handling certain class of plate and shell free vibrations problems. This method is a semi-analytical and numerical technique which combines, the ‘‘Classical Levy’s Method”, the “Integrating Matrix Method” and the “Transfer Matrix Method”. The solution method here is used in Yuceoglu, et al. [V.2,V.3] and, specially, in Yuceoglu and Özerciyes [V.4-V.13 and VIII.3-VIII.7] (who developed several versions of this method) is a considerable extension, modification and further development of the method employed in Yuceoglu et al [VIII.2].

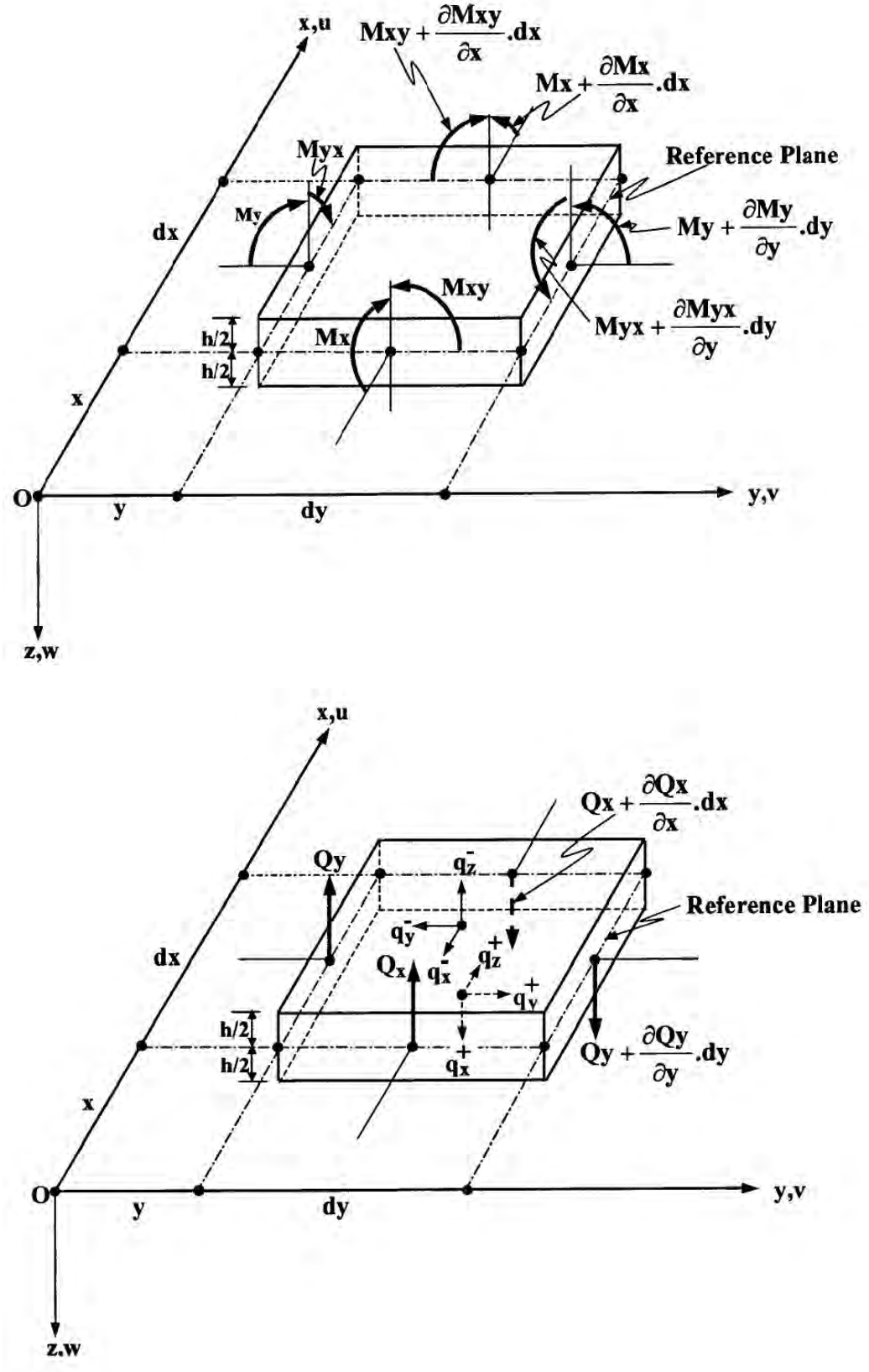

Figure 3.1 Coordinate System and Sign Convention for Displacements and Stress Resultants in Mindlin Plates in Flexure (or Bending).

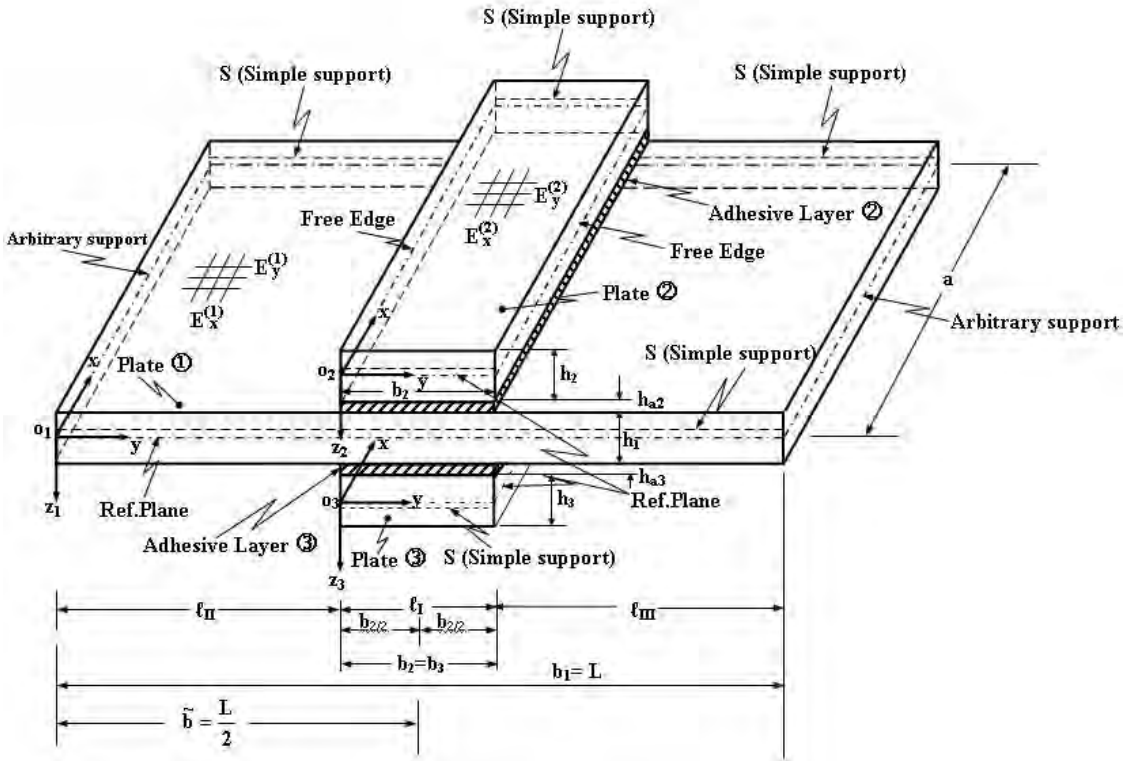

Figure 3.2 General Configuration and Coordinate System of "Centrally Doubly Stiffened, Composite Orthotropic and/or Isotropic Plates or Panels (with Upper and Lower Orthotropic Stiffening Plate Strips)" (''Main PROBLEM I'')

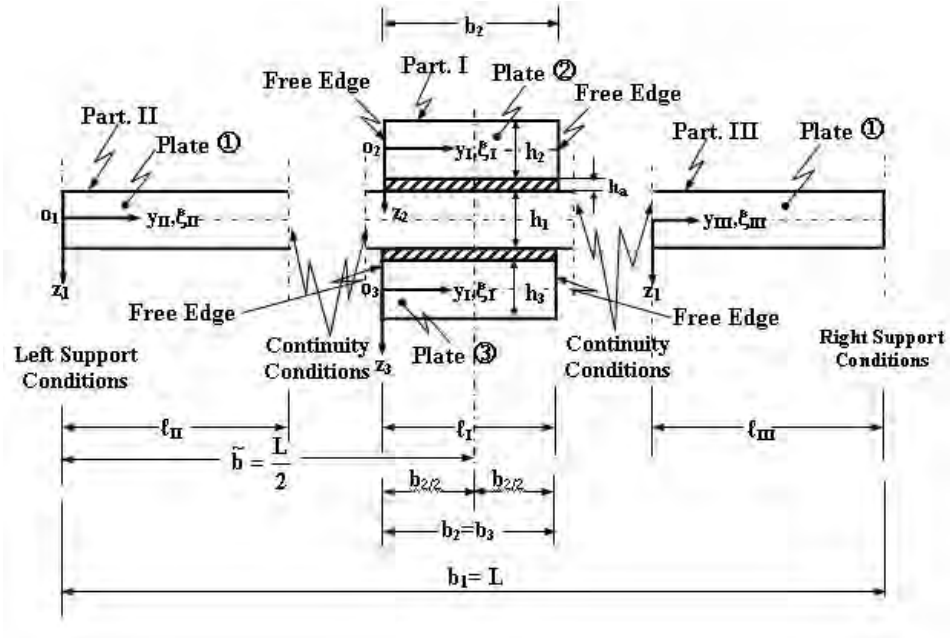

Figure 3.3 Longitudinal Cross-Section of "Centrally Doubly Stiffened, Composite Orthotropic and/or Isotropic Plates or Panels (with Upper and Lower Orthotropic Stiffening Plate Strips)" (''Main PROBLEM I'')

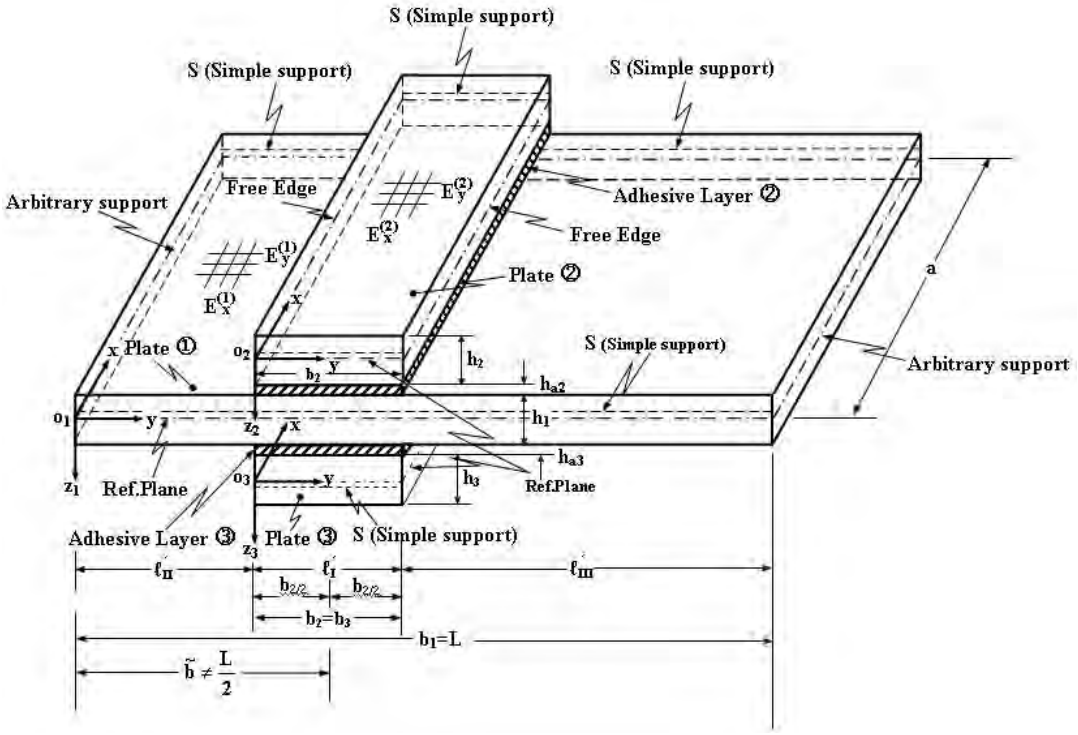

Figure 3.4 General Configuration and Coordinate System of "Non-Centrally Doubly Stiffened, Composite Orthotropic and/or Isotropic Plates or Panels (with Upper and Lower Orthotropic Stiffening Plate Strips)" (''Main PROBLEM II'')

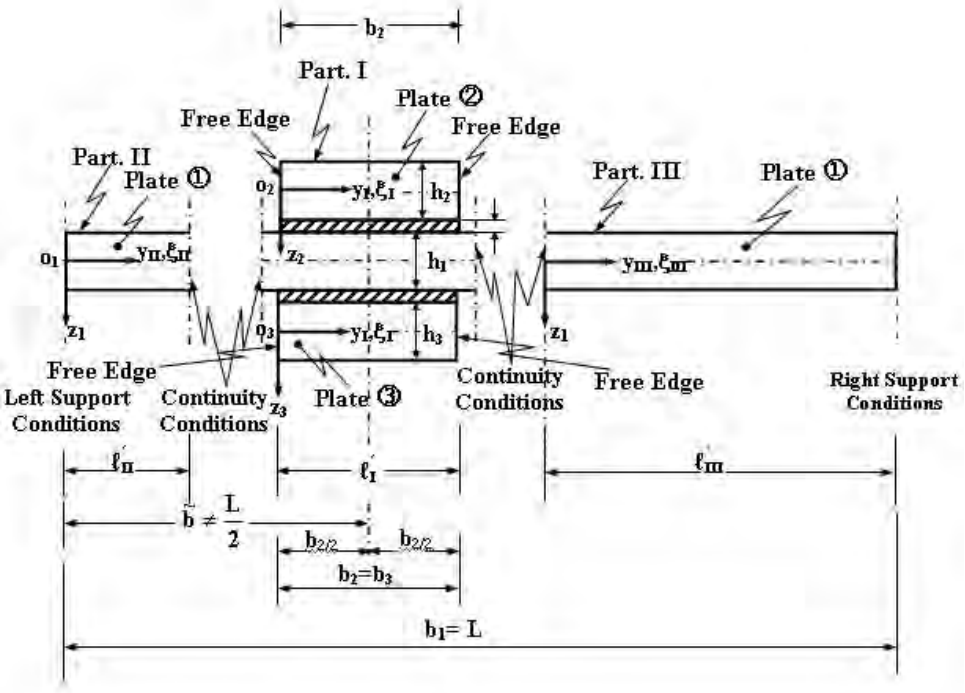

Figure 3.5 Longitudinal Cross-Section of "Non-Centrally Doubly Stiffened, Composite Orthotropic and/or Isotropic Plates or Panels (with Upper and Lower Orthotropic Stiffening Plate Strips)" (''Main PROBLEM II'')

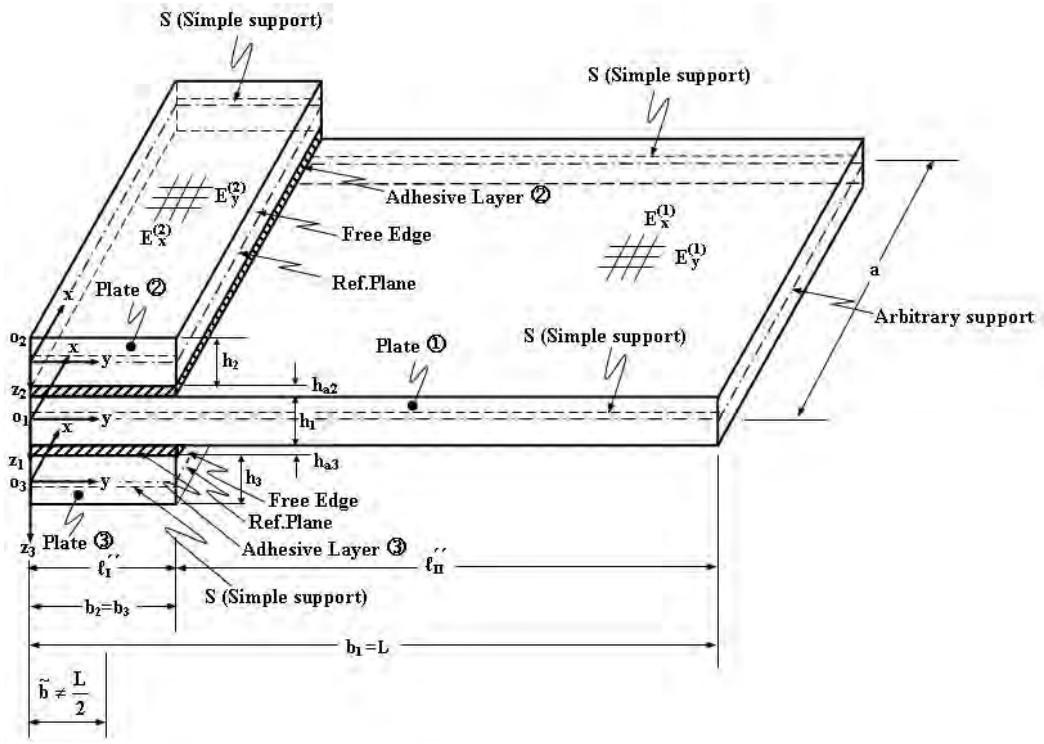

Figure 3.6 General Configuration and Coordinate System of "Doubly Stiffened-at Support, Composite Orthotropic and/or Isotropic Plates or Panels (with Upper and Lower Orthotropic Stiffening Plate Strips)" (''Main PROBLEM III'')

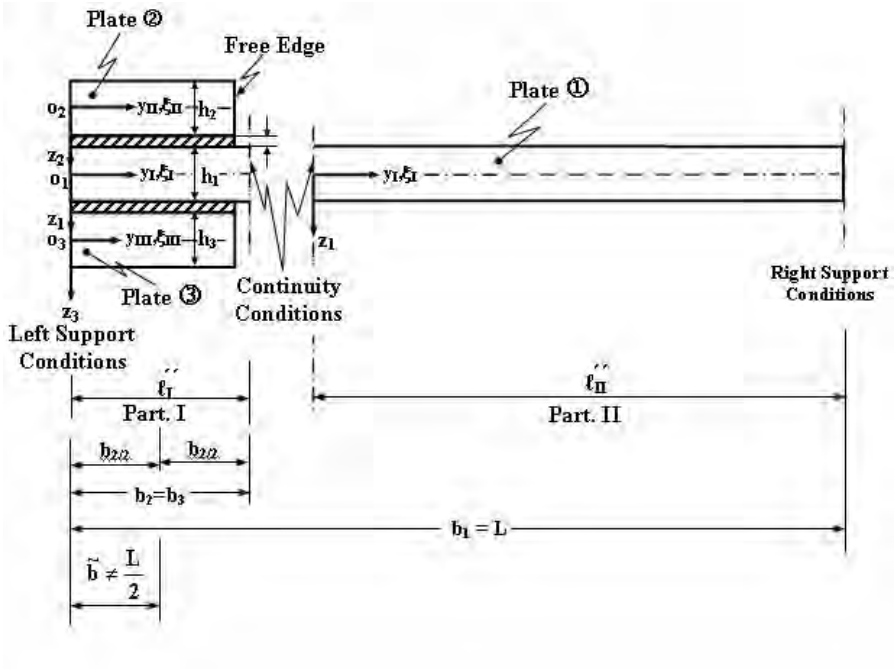

Figure 3.7 Longitudinal Cross-Section of "Doubly Stiffened-at Support, Composite Orthotropic and/or Isotropic Plates or Panels (with Upper and Lower Orthotropic Stiffening Plate Strips)" (''Main PROBLEM III'')

CHAPTER 4

("Main PROBLEM I.a'') __FREE FLEXURAL (or BENDING) VIBRATIONS of CENTRALLY DOUBLY STIFFENED, COMPOSITE ORTHOTROPIC PLATES or PANELS (Orthotropic Base Plate or Panel Case)

4.1 Main Assumptions and Analytical Modeling

Figure 3.2 shows the general configuration of the "Centrally Doubly Stiffened, Composite Plate System" which has an "Orthotropic Base Plate or Panel with Upper and Lower Stiffening Plate Strips" ("Main PROBLEM I.a") under consideration here and Figure 3.3 presents its longitudinal cross-section. In the aforementioned composite plate or panel system, the plates are connected by a relatively very thin, elastic adhesive layers.

The analytical formulation is based on the following assumptions;

(i) The analysis is carried out only for the free flexural (or bending) vibrations of the stiffened plate system. The in-plane moments of inertia are neglected, but the rotatory moments of inertia of the plates are included in the formulation.

(ii) The rotatory moments of inertia of the base plate and stiffening strips as well as their thickness shear deformations are taken into account in the sense of the "Mindlin Plate Theory".

(iii) There is no slip and separation on the interfaces of the adhesive layers and plates.

(iv) Since the thickness of the adhesive layers is very small relative to the thickness of the plates, the inertia and the mass of the adhesive layers are neglected, and both adhesive normal and shear stresses are to be constant across the its thickness.

(v) Damping effects in Mindlin Plates and in the adhesive layers are neglected.

(vi) Stiffening Plate strips and base plate are assumed to be simply supported along edges x=0 and x=a while arbitrary support conditions may be specified in the y-direction at y=0,b_1 and y=0,b_2 as shown in Figure 3.2 and Figure 3.3.

(vii) The coordinate system of each plate is attached to its medium plane or the reference plane.

(viii) The principal directions of orthotropy in plates are parallel to the edges and to the coordinates as shown in Figure 3.2.

For the general formulation of the problem, the entire "Composite Plate or Panel System" of Figure 3.2 is divided into three parts, namely, Part I, Part II and Part III in the y-direction as shown in Figure 3.3. Part I corresponds to the "overlap region" which contains all of the three plates and Part II and Part III correspond the continuation of the middle base plate as a single plate in the y-direction.

4.2 Theoretical Formulation of "Main PROBLEM I.a" (Theoretical Analysis)

4.2.1 Analysis of Adhesive Layer in the "Overlap Region"

In the previous section, basic equations for a one-layer Mindlin Plate of composite plate system have been derived. In this section, the aim is to develop a suitable mathematical model for the system shown in Figure 3.2 in order to obtain the natural frequencies and corresponding mode shapes. The system in "Overlap Region" or Part I region in Figure 3.2 is composed of three plates which are

adhesively bonded by means of very thin adhesive layers. The stresses at the upper and lower faces of the plates due to the adhesive layers are considered as external surface stresses or loads on the plates. The adhesive stresses should be related unknown displacement functions and angle of rotations of the base plate and the stiffening plate strips in the “Overlap Region”. Figures 4.1 and 4.2 show an exaggerated view of the deformation of an infinitesimal element of three-layer plate system. The positive sign convention for displacements, stress resultants and angles of rotation is also shown in those figures.

The purpose of this section is to obtain the relations between the stresses in the upper and lower adhesive layers and the plate interfaces and displacements of the upper, lower and base plates as shown in Figure 4.1 and Figure 4.2, respectively.

Then, the shear strains $\gamma_{xz}^{(j)}$ and $\gamma_{yz}^{(j)}$ at upper (j=1,2) and lower (j=1,3) interfaces can be found by considering the distortion in the right angles of the adhesive infinitesimal element as given in Figure 4.2 after deformation. It can be seen from Figure 4.2 that,

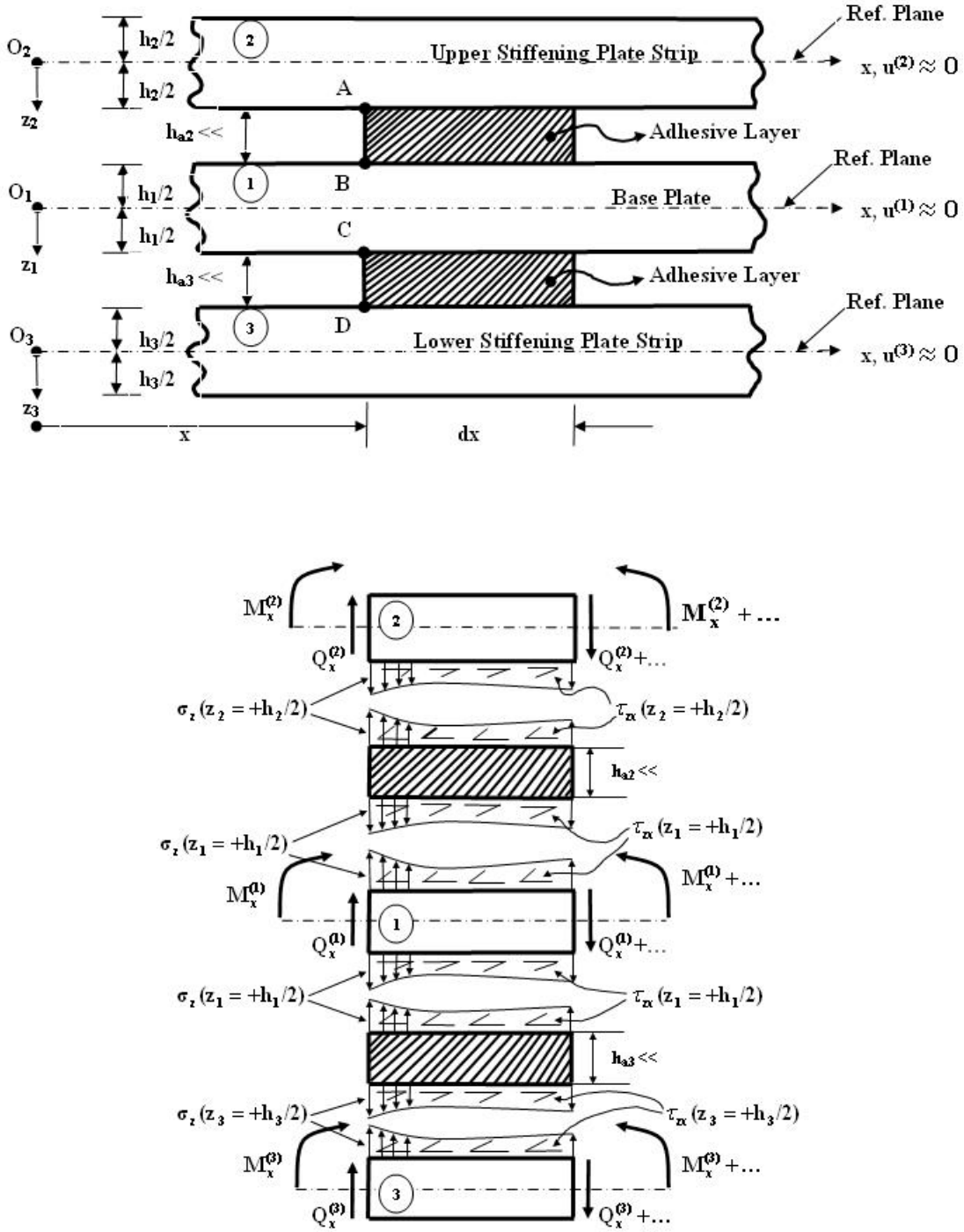

Figure 4.1 Stress Distribution at Plate Adhesive Layer Interfaces in the "Overlap Region"

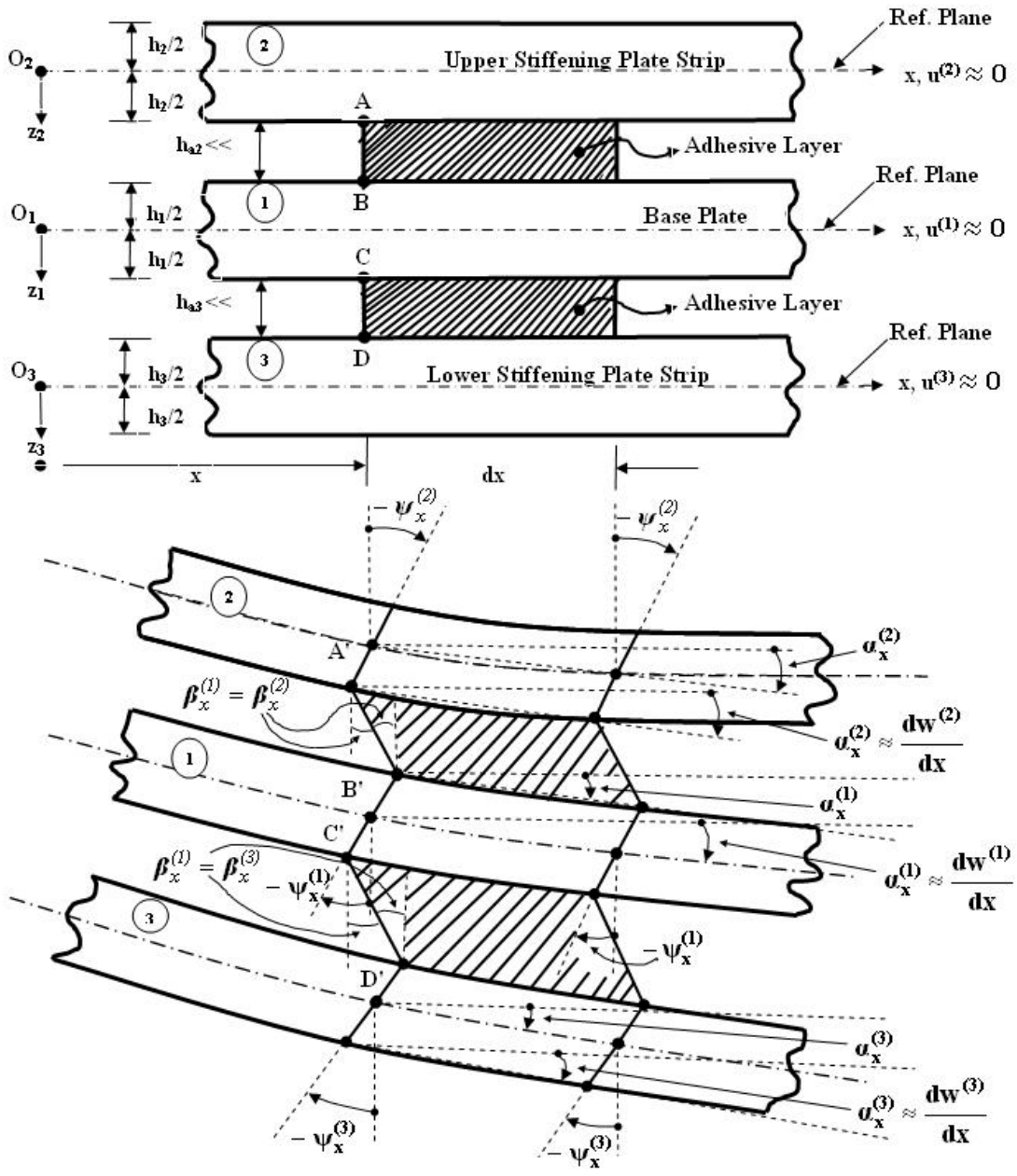

Figure 4.2 Deformations of Upper, Lower and Base Plates (Mindlin Plates) and in-between Adhesive Layers in the “Overlap Region”

$$\left|\begin{array}{l}\gamma_{xz}^{(j)}=\alpha_x^{(j)}+\beta_x^{(j)}\\ \gamma_{yz}^{(j)}=\alpha_y^{(j)}+\beta_y^{(j)}\end{array}\right. \qquad \text{(j=1,2,3)} \qquad (4.1)$$

With the assumption of small deformations, $\alpha^{(j)}$ and $\beta^{(j)}$ can be expressed as the slopes of the upper, lower and base plates and the displacements in x and y directions, respectively. That is,

Between Plates 1 and 2,

$$\left|\begin{array}{l}tan\left(\alpha_s^{(j)}\right)=\dfrac{\partial w^{(j)}}{\partial s}\approx\alpha_s^{(j)}\\ tan\left(\beta_s^{(j)}\right)=\dfrac{\left(u_B\right)_s-\left(u_A\right)_s}{h_{a2}}\approx\beta_s^{(j)}\end{array}\right. \qquad \text{(j=1,2)} \qquad (4.2.a)$$

Between Plates 1 and 3,

$$\left|\begin{array}{l}tan\left(\alpha_s^{(j)}\right)=\dfrac{\partial w^{(j)}}{\partial s}\approx\alpha_s^{(j)}\\ tan\left(\beta_s^{(j)}\right)=\dfrac{\left(u_B\right)_s-\left(u_C\right)_s}{h_{a3}}\approx\beta_s^{(j)}\end{array}\right. \qquad \text{(j=1,3)} \qquad (4.2.b)$$

where h_{a2} and h_{a3} are the thicknesses of the upper and lower adhesive layers, respectively and $(u_A)_s$, $(u_B)_s$ and $(u_C)_s$ are axial deformation of points A, B and C in s direction respectively. By assuming that the axial deformations $(u_A)_s$, $(u_B)_s$ and $(u_C)_s$ are caused only by the bending of the plates, then, it can be shown from the basic assumption on the displacement components (3.1) that,

$$\left|\begin{array}{l}\left(u_A\right)_s=+\dfrac{h_2}{2}\psi_s^{(2)}\\ \left(u_B\right)_s=-\dfrac{h_1}{2}\psi_s^{(1)}\\ \left(u_C\right)_s=+\dfrac{h_3}{2}\psi_s^{(3)}\end{array}\right. \qquad (4.3)$$

Hence, using (4.1) through (4.3), one can express the shear strains in the interfaces in terms of displacements and angles of rotation of the upper and lower plates as,

Between Plates 1 and 2 (Adhesive Strains),

$$\left|\begin{array}{l}\gamma_{xz}^{(j)} = -\dfrac{1}{2h_{a2}}\left(h_1\psi_x^{(1)} + h_2\psi_x^{(2)}\right) + \underline{\underline{\dfrac{\partial w^{(j)}}{\partial x}}} \\ \gamma_{yz}^{(j)} = -\dfrac{1}{2h_{a2}}\left(h_1\psi_y^{(1)} + h_2\psi_y^{(2)}\right) + \underline{\underline{\dfrac{\partial w^{(j)}}{\partial y}}}\end{array}\right. \qquad \text{(j=1,2)} \qquad \text{(4.4.a)}$$

Between Plates 1 and 3 (Adhesive Strains),

$$\left|\begin{array}{l}\gamma_{xz}^{(j)} = -\dfrac{1}{2h_{a3}}\left(h_1\psi_x^{(1)} + h_3\psi_x^{(3)}\right) + \underline{\underline{\dfrac{\partial w^{(j)}}{\partial x}}} \\ \gamma_{yz}^{(j)} = -\dfrac{1}{2h_{a3}}\left(h_1\psi_y^{(1)} + h_3\psi_y^{(3)}\right) + \underline{\underline{\dfrac{\partial w^{(j)}}{\partial y}}}\end{array}\right. \qquad \text{(j=1,3)} \qquad \text{(4.4.b)}$$

It should be mentioned here that in some previous studies by Yuceoglu and Özerciyes [V.4-V.13], the double underlined terms (=) are neglected. In this present study; however, they are included in the analysis and the governing equations.
Hence, the interface shear stresses in the adhesive layers can be expressed as,

Between Plates 1 and 2 (Adhesive Stresses),

$$\left|\begin{array}{l}\tau_{xz}^{(j)} = G_{a2}\gamma_{xz}^{(j)} = -\dfrac{G_{a2}}{2h_{a2}}\left(h_1\psi_x^{(1)} + h_2\psi_x^{(2)}\right) + \underline{\underline{G_{a2}\dfrac{\partial w^{(j)}}{\partial x}}} \\ \tau_{yz}^{(j)} = G_{a2}\gamma_{yz}^{(j)} = -\dfrac{G_{a2}}{2h_{a2}}\left(h_1\psi_y^{(1)} + h_2\psi_y^{(2)}\right) + \underline{\underline{G_{a2}\dfrac{\partial w^{(j)}}{\partial y}}}\end{array}\right. \qquad \text{(j=1,2)} \qquad \text{(4.5.a)}$$

Between Plates 1 and 3 (Adhesive Stresses),

$$\left|\begin{array}{l}\tau_{xz}^{(j)} = G_{a3}\gamma_{xz}^{(j)} = -\dfrac{G_{a3}}{2h_{a3}}\left(h_1\psi_x^{(1)} + h_3\psi_x^{(3)}\right) + \underline{\underline{G_{a3}\dfrac{\partial w^{(j)}}{\partial x}}} \\ \tau_{yz}^{(j)} = G_{a3}\gamma_{yz}^{(j)} = -\dfrac{G_{a3}}{2h_{a3}}\left(h_1\psi_y^{(1)} + h_3\psi_y^{(3)}\right) + \underline{\underline{G_{a3}\dfrac{\partial w^{(j)}}{\partial y}}}\end{array}\right. \qquad \text{(j=1,3)} \qquad \text{(4.5.b)}$$

where G_a is the shear modulus of adhesive layer and superscript (j) denotes middle base plate for j=1, upper plate for j=2 and lower plate for j=3. The thicknesses h_{a2} and h_{a3} of the adhesive layers are assumed to be much smaller than the thicknesses of the plates and $\psi_x^{(j)}, \psi_y^{(j)}$ have the same order of magnitudes when compared with $\frac{\partial w^{(j)}}{\partial x}$ and $\frac{\partial w^{(j)}}{\partial y}$.Therefore, one may assume that the variation of transverse displacement in the adhesive layer is to be linear that is equivalent to constant normal strain ε_z across the thickness assumption.

Therefore, normal and tangential stresses at the interface can be written as,

Between Plates 1 and 2 (Adhesive Stresses),

$$\left|\begin{array}{l} \tau_{xz}^{(j)} = G_{a2}\gamma_{xz}^{(j)} = -\dfrac{G_{a2}}{2h_{a2}}\left(h_1\psi_x^{(1)} + h_2\psi_x^{(2)}\right) + \underline{\underline{G_{a2}\dfrac{\partial w^{(j)}}{\partial x}}} \\ \tau_{yz}^{(j)} = G_{a2}\gamma_{yz}^{(j)} = -\dfrac{G_{a2}}{2h_{a2}}\left(h_1\psi_y^{(1)} + h_2\psi_y^{(2)}\right) + \underline{\underline{G_{a2}\dfrac{\partial w^{(j)}}{\partial y}}} \quad (j=1,2) \\ \sigma_z^{(j)} = E_{a2}\varepsilon_z^{(j)} = \dfrac{E_{a2}}{h_{a2}}\left(w^{(1)} - w^{(2)}\right) \end{array}\right. \qquad (4.6.a)$$

Between Plates 1 and 3 (Adhesive Stresses),

$$\left|\begin{array}{l} \tau_{xz}^{(j)} = G_{a3}\gamma_{xz}^{(j)} = -\dfrac{G_{a3}}{2h_{a3}}\left(h_1\psi_x^{(1)} + h_3\psi_x^{(3)}\right) + \underline{\underline{G_{a3}\dfrac{\partial w^{(j)}}{\partial x}}} \\ \tau_{yz}^{(j)} = G_{a3}\gamma_{yz}^{(j)} = -\dfrac{G_{a3}}{2h_{a3}}\left(h_1\psi_y^{(1)} + h_3\psi_y^{(3)}\right) + \underline{\underline{G_{a3}\dfrac{\partial w^{(j)}}{\partial y}}} \quad (j=1,3) \\ \sigma_z^{(j)} = E_{a3}\varepsilon_z^{(j)} = \dfrac{E_{a3}}{h_{a3}}\left(w^{(3)} - w^{(1)}\right) \end{array}\right. \qquad (4.6.b)$$

where E_{a2} and E_{a3} are the modulus of elasticity (or Young's modulus) and G_{a2} and G_{a3} are the modulus of rigidity (or shear modulus) of the upper and lower adhesive layers, respectively.

4.2.2 Analysis of Part I (or the "Overlap Region") of Composite Plate System

The adhesive stresses should be related to unknown displacement functions and angle of rotations of the stiffening plate strips and the base plate in the "Overlap Region" since they can be considered as surface loads or external stresses acting on the upper and lower faces of the plates as shown in Figure 4.2. The Equation (4.6) shows the normal and tangential stresses at the interface may be considered as the ''compatibility or coupling conditions'' of the three plates in the "Overlap Region" of Part I in Figure 3.3. Then, surface load terms q_{zx}'s and q_{zy}'s in the governing equations (or plate equations of motion) given in (3.19) and (3.20) can be replaced by adhesive stresses. Therefore, one can write the plate equations of motion of the three-layer coupled plate system shown in the Part I of Figure 3.3 as,

For Plates 1 and 2 (Base Plate=1, Stiffening Plate Strip=2),

$$\begin{aligned}
&\frac{\partial M_x^{(j)}}{\partial x}+\frac{\partial M_{yx}^{(j)}}{\partial y}-Q_x^{(j)}-\underline{\frac{h_j G_{a2}}{4h_{a2}}\left(h_1\psi_x^{(1)}+h_2\psi_x^{(2)}\right)}+\underline{\underline{\frac{h_j G_{a2}}{2}\frac{\partial w^{(j)}}{\partial x}}}=\frac{\rho_j h_j^3}{12}\frac{\partial^2\psi_x^{(j)}}{\partial t^2}\\
&\frac{\partial M_{yx}^{(j)}}{\partial x}+\frac{\partial M_y^{(j)}}{\partial y}-Q_y^{(j)}-\underline{\frac{h_j G_{a2}}{4h_{a2}}\left(h_1\psi_y^{(1)}+h_2\psi_y^{(2)}\right)}+\underline{\underline{\frac{h_j G_{a2}}{2}\frac{\partial w^{(j)}}{\partial y}}}=\frac{\rho_j h_j^3}{12}\frac{\partial^2\psi_y^{(j)}}{\partial t^2} \quad (j=1,2) \qquad (4.7.a)\\
&\frac{\partial Q_x^{(j)}}{\partial x}+\frac{\partial Q_y^{(j)}}{\partial y}+(-1)^{(j+1)}\underline{\frac{E_{a2}}{h_{a2}}\left(w^{(1)}-w^{(2)}\right)}=\rho_j h_j\frac{\partial^2 w^{(j)}}{\partial t^2}
\end{aligned}$$

For Plates 1 and 3 (Base Plate=1, Stiffening Plate Strip=3),

$$\begin{aligned}
&\frac{\partial M_x^{(j)}}{\partial x}+\frac{\partial M_{yx}^{(j)}}{\partial y}-Q_x^{(j)}-\underline{\frac{h_j G_{a3}}{4h_{a3}}\left(h_1\psi_x^{(1)}+h_3\psi_x^{(3)}\right)}+\underline{\underline{\frac{h_j G_{a3}}{2}\frac{\partial w^{(j)}}{\partial x}}}=\frac{\rho_j h_j^3}{12}\frac{\partial^2\psi_x^{(j)}}{\partial t^2}\\
&\frac{\partial M_{yx}^{(j)}}{\partial x}+\frac{\partial M_y^{(j)}}{\partial y}-Q_y^{(j)}-\underline{\frac{h_j G_{a3}}{4h_{a3}}\left(h_1\psi_y^{(1)}+h_3\psi_y^{(3)}\right)}+\underline{\underline{\frac{h_j G_{a3}}{2}\frac{\partial w^{(j)}}{\partial y}}}=\frac{\rho_j h_j^3}{12}\frac{\partial^2\psi_y^{(j)}}{\partial t^2} \quad (j=1,3) \qquad (4.7.b)\\
&\frac{\partial Q_x^{(j)}}{\partial x}+\frac{\partial Q_y^{(j)}}{\partial y}+(-1)^{(j+1)}\underline{\frac{E_{a3}}{h_{a3}}\left(w^{(1)}-w^{(3)}\right)}=\rho_j h_j\frac{\partial^2 w^{(j)}}{\partial t^2}
\end{aligned}$$

where superscript (j) denotes the base plate for j=1, upper stiffening platestrip for j=2 and lower stiffening plate strip for j=3 in the "Overlap Region" or the Part I region of Figure 3.3. Also, in the above equations of both types, the "underlined terms" are the

"coupling terms" between the base plate in the middle and the upper and lower stiffening plate strips.

The three-layer composite orthotropic plate system shown in the "Overlap Region" or Part I region of Figure 3.3 is simply supported along x=0,a edges. $M_x^{(j)}$ and $Q_x^{(j)}$ are taken as subsidiary variables and they are eliminated from the governing partial differential equations because they can not be prescribed in the cross-sections perpendicular to y-directions. Then, the remaining "fundamental dependent variables" $\psi_x^{(j)}, \psi_y^{(j)}, w^{(j)}, M_{yx}^{(j)}, M_y^{(j)}, Q_y^{(j)}$ (j=1,2,3) are chosen as intrinsic variables and unknowns of the problem under consideration. Six of the twentyfour equations (nine equations of motion (4.7) and three sets five stress resultants-displacement relations (3.17) one set for each plate) are algebraically manipulated to eliminate the subsidiary variables $M_x^{(j)}$ and $Q_x^{(j)}$. The remaining eighteen equations are used to obtain the following set of the coupled "Governing System of First Order Governing Partial Differential Equations" of motion in matrix form for the three plates in the "Overlap Region" or Part I. region,

$$\frac{\partial}{\partial y_I}\begin{Bmatrix} \boldsymbol{Y}^{(2)} \\ \boldsymbol{Y}^{(1)} \\ \boldsymbol{Y}^{(3)} \end{Bmatrix} = \begin{bmatrix} \boldsymbol{C}_{11} & \boldsymbol{C}_{12} & \boldsymbol{C}_{13} \\ \boldsymbol{C}_{21} & \boldsymbol{C}_{22} & \boldsymbol{C}_{23} \\ \boldsymbol{C}_{31} & \boldsymbol{C}_{32} & \boldsymbol{C}_{33} \end{bmatrix} \begin{Bmatrix} \boldsymbol{Y}^{(2)} \\ \boldsymbol{Y}^{(1)} \\ \boldsymbol{Y}^{(3)} \end{Bmatrix}, (0<y_I<\ell_I) \tag{4.8}$$

Rewriting (4.8), one can write ,

In the "Overlap Region" or Part I region,

$$\frac{\partial}{\partial y_I}\begin{Bmatrix} \boldsymbol{Y}^{(2)} \\ \boldsymbol{Y}^{(1)} \\ \boldsymbol{Y}^{(3)} \end{Bmatrix} = \left[\boldsymbol{C}\left(\frac{\partial}{\partial \boldsymbol{x}}, \frac{\partial^2}{\partial \boldsymbol{t}^2}, etc.\right)\right] \begin{Bmatrix} \boldsymbol{Y}^{(2)} \\ \boldsymbol{Y}^{(1)} \\ \boldsymbol{Y}^{(3)} \end{Bmatrix}, (0<y_I<l_I) \tag{4.8.a}$$

where the "Coefficient Matrix [$\boldsymbol{C}$]" includes, as shown above, the partial differential operators and the geometric and the material characteristics of the three-layer plates and adhesive layers.

Similarly, for the base plate,

In the single layer plate or Part II region,

$$\frac{\partial}{\partial y_I}\left\{\boldsymbol{Y}^{(1)}\right\}=\left[\boldsymbol{D}\left(\frac{\partial}{\partial \boldsymbol{x}},\frac{\partial^2}{\partial \boldsymbol{t}^2},etc.\right)\right]\left\{\boldsymbol{Y}^{(1)}\right\},(0<y_{II}<l_{II}) \qquad (4.8.b)$$

where the "Coefficient Matrix [***D***]" is composed of partial differential operators and the geometric and the material characteristics of the base plate in Part II region.

Also, one can write,

In the single layer plate or Part III region,

$$\frac{\partial}{\partial y_I}\left\{\boldsymbol{Y}^{(1)}\right\}=\left[\boldsymbol{E}\left(\frac{\partial}{\partial \boldsymbol{x}},\frac{\partial^2}{\partial \boldsymbol{t}^2},etc.\right)\right]\left\{\boldsymbol{Y}^{(1)}\right\},(0<y_{III}<l_{III}) \qquad (4.8.c)$$

where the "Coefficient Matrix [***E***]" include the partial differential operators and the geometric and the material characteristics of the base plate in Part III region.

In the above (4.8.a), (4.8.b) and (4.8.c), the "state vector" for Part I, Part II and Part III regions, respectively,

$$\left\{\boldsymbol{Y}^{(j)}\right\}=\left\{\psi_x^{(j)},\psi_y^{(j)},w^{(j)};M_{yx}^{(j)},M_y^{(j)},Q_y^{(j)}\right\}^T \qquad (j=1,2,3) \qquad (4.9)$$

where the independent space variables are y_I, y_{II} and y_{III}, respectively.

The "Column Matrix $\{\boldsymbol{Y}^{(j)}\}$" includes the "fundamental dependent variables" which are the angles of rotation, displacements and stress resultants appearing in the arbitrary cross-sections of the plates defined by y=const. in the y-direction in Part I region in Figure 3.3.

At this stage, the "Classical Levy's Method" with the trigonometric series expansions is to be considered. The "Classical Lévy's Method" is restricted to

rectangular plates with any two opposite edges simply supported. However, the other two edges may have arbitrary boundary conditions.

In the present study, the edges at x=0 and x=a are simply supported and the plate boundary conditions which have to be satisfied along these edges are as follows,

$$\text{at x=0,a} \quad \rightarrow \qquad w^{(j)} = 0,\ M_x^{(j)} = 0,\ \psi_y^{(j)} = 0 \quad \text{(j=1,2,3)} \qquad (4.10)$$

Since it is assumed that all plates in the composite system are simply supported along the edges at x=0 and x=a, the classical "Classical Levy-Type Solutions" in trigonometric series which satisfy the boundary conditions at these edges can be assumed. Then, one can expand the angles of rotations and the displacements of the upper, lower stiffening plate strips and the base plate in appropriate trigonometric series in dimensionless quantities in the x-direction. Then,

The Displacements and Angles of Rotation, are,

$$w^{(j)}(\eta,\xi_I,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} h_1 \overline{W}_{mn}^{(j)}(\xi_I)\sin(m\pi\eta)e^{i\overline{\omega}_{mn}t}$$

$$\psi_x^{(j)}(\eta,\xi_I,t) = \frac{a}{h_1}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \overline{\Psi}_{mnx}^{(j)}(\xi_I)\cos(m\pi\eta)e^{i\overline{\omega}_{mn}t} \qquad \text{(j=1,2,3)} \qquad (4.11)$$

$$\psi_y^{(j)}(\eta,\xi_I,t) = \frac{h_1}{a}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \overline{\Psi}_{mny}^{(j)}(\xi_I)\sin(m\pi\eta)e^{i\overline{\omega}_{mn}t}$$

where "m" is the number of half-waves in the x-direction. The nondimensional independent space variables η and ξ_I are defined as x/a and y_I/l_I, respectively. Note that superscript (j) denotes base plate for j=1, upper plate for j=2 and lower plate for j=3 whereas i isdefined as $\sqrt{-1}$ and the “barred” (—) quantities are the dimensionless transverse displacements and angles of rotation. And "$\overline{\omega}_{mn}$" is the dimensionless circular frequency or the natural frequency of the flexural (or bending) vibrations of the entire composite plate or panel system (The non-dimensionalization process will be explained later).

The components of stress resultants can be expressed in trigonometric series in the x-direction using plate stress resultant-displacement relations (3.17) as,

The Stress Resultants, are,

$$M_x^{(j)}(\eta,\xi_I,t)=\frac{h_1^3 B_{11}^{(1)}}{a}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\overline{M}_{mnx}^{(j)}(\xi_I)\sin(m\pi\eta)e^{i\overline{\omega}_{mn}t}$$

$$M_{yx}^{(j)}(\eta,\xi_I,t)=h_1^2 B_{11}^{(1)}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\overline{M}_{mnyx}^{(j)}(\xi_I)\cos(m\pi\eta)e^{i\overline{\omega}_{mn}t}$$

$$M_y^{(j)}(\eta,\xi_I,t)=h_1^2 B_{11}^{(1)}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\overline{M}_{mny}^{(j)}(\xi_I)\sin(m\pi\eta)e^{i\overline{\omega}_{mn}t} \qquad (j=1,2,3) \qquad (4.12)$$

$$Q_y^{(j)}(\eta,\xi_I,t)=h_1 B_{11}^{(1)}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\overline{Q}_{mny}^{(j)}(\xi_I)\sin(m\pi\eta)e^{i\overline{\omega}_{mn}t}$$

$$Q_x^{(j)}(\eta,\xi_I,t)=\frac{h_1^2 B_{11}^{(1)}}{a}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\overline{Q}_{mnx}^{(j)}(\xi_I)\cos(m\pi\eta)e^{i\overline{\omega}_{mn}t}$$

where the "barred" (¯) $\overline{w}_{mn}^{(j)}$, $\overline{\Psi}_{mnx}^{(j)}$, $\overline{\Psi}_{mny}^{(j)}$, $\overline{M}_{mnx}^{(j)}$, $\overline{M}_{mnyx}^{(j)}$, $\overline{M}_{mny}^{(j)}$, $\overline{Q}_{mny}^{(j)}$, $\overline{Q}_{mnx}^{(j)}$ are "dimensionless fundamental dependent variables".

Further nondimensionalization of the equations can be achieved in the following manner:

"$B_{11}^{(1)}$", "h_1", "ρ_1" and "a" are chosen as main or reference quantities or parameters and all other parameters are nondimensionalized with respect to these quantities.

The dimensionless coordinates or independent space variables are,

$$\eta=x/a, \qquad \xi_I=y_I/l_I, \qquad \xi_{II}=y_{II}/l_{II}, \qquad \xi_{III}=y_{III}/l_{III} \qquad (4.13)$$

The dimensionless parameters related to orthotropic elastic constant and the adhesive layers are,

$$
\left|\begin{array}{ll}
\overline{B}_{ik}^{(j)} = B_{ik}^{(j)} \Big/ B_{11}^{(1)} & \left(j = 1,2,3 \, and \, i,k = 1,2,3\right), \\
\overline{B}_{\ell\ell}^{(j)} = B_{\ell\ell}^{(j)} \Big/ B_{11}^{(1)} & \left(j = 1,2,3 \, and \, \ell = 4,5,6\right) \\
\overline{G}_{a2} = G_{a2} \Big/ B_{11}^{(1)}, \overline{G}_{a3} = G_{a3} \Big/ B_{11}^{(1)} & \\
\overline{E}_{a2} = E_{a2} \Big/ B_{11}^{(1)}, \overline{E}_{a3} = E_{a3} \Big/ B_{11}^{(1)} &
\end{array}\right. \tag{4.14}
$$

The dimensionless parameters related to the densities and the geometry of the plates and adhesive layers are,

$$
\left|\begin{array}{l}
\overline{\rho}_2 = \rho_2 \Big/ \rho_1, \overline{\rho}_3 = \rho_3 \Big/ \rho_1 \\
\overline{h}_{a2} = h_{a2} \Big/ h_1, \overline{h}_{a3} = h_{a3} \Big/ h_1 \\
\overline{L}_I = \ell_I \Big/ a, \overline{L}_{II} = \ell_{II} \Big/ a, \overline{L}_{III} = \ell_{III} \Big/ a \\
\overline{h}_2 = h_2 \Big/ h_1, \overline{h}_3 = h_3 \Big/ h_1
\end{array}\right. \tag{4.15}
$$

The dimensionless frequency parameter $\overline{\omega}_{mn}$ of the entire doubly stiffened, composite plate or panel system of the "Main PROBLEM I.a":

$$
\left|\begin{array}{ll}
\overline{\omega}_{mn} = \rho_1 a^4 \omega_{mn}^2 \Big/ h_1^2 B_{11}^{(1)} & \\
\overline{\Omega} = \overline{\omega}_{mn} & \text{(m,n=1,2,3...)}
\end{array}\right. \tag{4.16}
$$

where the dimensionless natural frequency parameter $\overline{\omega}_{mn}$ is ordered in terms of its magnitudes as $\overline{\Omega}_1 < \overline{\Omega}_2 < \overline{\Omega}_3 < ....$ with the subscript indicating the first, second, third,..... dimensionless natural frequencies depending on the given m, n values.

For the “Overlap Region” or Part I region, one can obtain the following partial differential equations after writing (3.21) with respect to the dimensionless independent variables ξ_I and η given in (4.13) in the y- and x-directions, respectively.

$$
\left|\begin{array}{l}
\dfrac{1}{l_I}\dfrac{\partial \psi_y^{(j)}}{\partial \xi_I} = \dfrac{1}{B_{22}^{(j)}}\left(\dfrac{12}{h_j^3}M_y^{(j)} - B_{12}^{(j)}\dfrac{1}{a}\dfrac{\partial \psi_x^{(j)}}{\partial \eta}\right) \\
\dfrac{1}{l_I}\dfrac{\partial \psi_x^{(j)}}{\partial \xi_I} = \dfrac{12}{h_j^{(3)}B_{66}^{(j)}}M_{yx}^{(j)} - \dfrac{1}{a}\dfrac{\partial \psi_y^{(j)}}{\partial \eta} \\
\dfrac{1}{l_I}\dfrac{\partial w^{(j)}}{\partial \xi_I} = \dfrac{1}{\kappa_y^2 h_j B_{44}^{(j)}}Q_y^{(j)} - \psi_y^{(j)} \\
\dfrac{1}{l_I}\dfrac{\partial M_{yx}^{(j)}}{\partial \xi_I} = \dfrac{\rho_j h_j^3}{12}\dfrac{\partial^2 \psi_x^{(j)}}{\partial t^2} - \dfrac{1}{a}\dfrac{\partial M_x^{(j)}}{\partial \eta} + Q_x^{(j)} - \dfrac{h_j}{2}\left(q_{zx}^{(+)} + q_{zx}^{(-)}\right) \\
\dfrac{1}{l_I}\dfrac{\partial M_y^{(j)}}{\partial \xi_I} = \dfrac{\rho_j h_j^3}{12}\dfrac{\partial^2 \psi_y^{(j)}}{\partial t^2} - \dfrac{1}{a}\dfrac{\partial M_{yx}^{(j)}}{\partial \eta} + Q_y^{(j)} - \dfrac{h_j}{2}\left(q_{zy}^{(+)} + q_{zy}^{(-)}\right) \\
\dfrac{1}{l_I}\dfrac{\partial Q_y^{(j)}}{\partial \xi_I} = \rho_j h_j \dfrac{\partial^2 w^{(j)}}{\partial t^2} - \dfrac{1}{a}\dfrac{\partial Q_x^{(j)}}{\partial \eta} - \left(q_z^{(+)} - q_z^{(-)}\right)
\end{array}\right. \tag{4.17}
$$

where q’s are surface loads and the stresses on the upper (-) and lower (+) surfaces of the plates. Also, note that superscript and subscript (j) denotes the base plate for j=1, upper plate for j=2 and lower plate for j=3.

After substituting (3.17), (4.11) and (4.12) into (4.17) and making necessary non-dimensionalizations with respect to (4.14), (4.15) and (4.16), one can obtain the “Governing System of First Order Ordinary Differential Equations” for the base plate in terms of the “dimensionless fundamental dependent variables” in the “Overlap Region” or Part I region in Figure 3.3. Then, in detailed form,

For the Orthotropic Base Plate in the "Overlap Region" or Part I region,

$$
\begin{aligned}
\frac{d\overline{\psi}^{(1)}_{mnx}}{d\xi_I} &= \frac{12\overline{L}_1}{\overline{B}^{(1)}_{66}}\overline{M}^{(1)}_{mnyx} - \overline{L}_1 m\pi\left(\frac{h_1}{a}\right)^2\overline{\psi}^{(1)}_{mny} \\
\frac{d\overline{\psi}^{(1)}_{mny}}{d\xi_I} &= \frac{12\overline{L}_1}{\overline{B}^{(1)}_{22}}\left(\frac{a}{h_1}\right)^2\overline{M}^{(1)}_{mny} + \overline{L}_1 m\pi\frac{\overline{B}^{(1)}_{12}}{\overline{B}^{(1)}_{22}}\left(\frac{a}{h_1}\right)^2\overline{\psi}^{(1)}_{mnx} \\
\frac{d\overline{W}^{(1)}_{mn}}{d\xi_I} &= \frac{\overline{L}_1}{\kappa_y^2\overline{B}^{(1)}_{44}}\left(\frac{a}{h_1}\right)\overline{Q}^{(1)}_{mny} - \overline{L}_1\overline{\psi}^{(1)}_{mny} \\
\frac{d\overline{M}^{(1)}_{mnyx}}{d\xi_I} &= \left\{\begin{array}{l}
\left\{-\frac{\overline{L}_1}{12}\left(\frac{h_1}{a}\right)^2\overline{\Omega} + \overline{L}_1\kappa_x^2\overline{B}^{(1)}_{55}\left(\frac{a}{h_1}\right)^2\right\}\psi^{(1)}_{mnx} \\
\left\{+\frac{\overline{L}_1}{12}\left(\frac{am\pi}{h_1}\right)^2\left(\overline{B}^{(1)}_{11} - \frac{\overline{B}^{(1)2}_{12}}{\overline{B}^{(1)}_{22}}\right)\right\} \\
+\left\{\kappa_x^2\overline{L}_1\overline{B}^{(1)}_{55}m\pi - \underline{\underline{\overline{L}_1\frac{m\pi}{2}\overline{G}_{a2}}} - \underline{\underline{\overline{L}_1\frac{m\pi}{2}\overline{G}_{a3}}}\right\}\overline{W}^{(1)}_{mn} \\
-\left(\frac{\overline{B}^{(1)}_{12}}{\overline{B}^{(1)}_{22}}\overline{L}_1 m\pi\right)\overline{M}^{(1)}_{mny} + \frac{\overline{G}_{a2}\overline{L}_1}{4\overline{h}_{a2}}\left(\frac{a}{h_1}\right)^2\left(\overline{\psi}^{(1)}_{mnx} + \overline{h}_2\overline{\psi}^{(2)}_{mnx}\right) \\
+\frac{\overline{G}_{a3}\overline{L}_1}{4\overline{h}_{a3}}\left(\frac{a}{h_1}\right)^2\left(\overline{\psi}^{(1)}_{mnx} + \overline{h}_3\overline{\psi}^{(3)}_{mnx}\right)
\end{array}\right\} \\
\frac{d\overline{M}^{(1)}_{mny}}{d\xi_I} &= \left\{\begin{array}{l}
\left\{-\frac{\overline{L}_1}{12}\left(\frac{h_1}{a}\right)^4\overline{\Omega} + \underline{\underline{\frac{\overline{G}_{a2}\overline{L}_1}{2}}} + \underline{\underline{\frac{\overline{G}_{a3}\overline{L}_1}{2}}}\right\}\overline{\psi}^{(1)}_{mny} + \left(\frac{\overline{L}_1 a}{h_1} - \underline{\underline{\overline{L}_1\left(\frac{a}{h_1}\right)\frac{\overline{G}_{a2}}{2\kappa^2\overline{B}^{(1)}_{44}}}} - \underline{\underline{\overline{L}_1\left(\frac{a}{h_1}\right)\frac{\overline{G}_{a3}}{2\kappa_y^2\overline{B}^{(1)}_{44}}}}\right)\overline{Q}^{(1)}_{mny} \\
+\left(\overline{L}_1 m\pi\right)\overline{M}^{(1)}_{mnyx} + \frac{\overline{G}_{a2}\overline{L}_1}{4\overline{h}_{a2}}\left(\overline{\psi}^{(1)}_{mny} + \overline{h}_2\overline{\psi}^{(2)}_{mny}\right) + \frac{\overline{G}_{a3}\overline{L}_1}{4\overline{h}_{a3}}\left(\overline{\psi}^{(1)}_{mny} + \overline{h}_3\overline{\psi}^{(3)}_{mny}\right)
\end{array}\right\} \\
\frac{d\overline{Q}^{(1)}_{mny}}{d\xi_I} &= \left\{\begin{array}{l}
\left\{-\overline{L}_1\left(\frac{h_1}{a}\right)^3\overline{\Omega} + \overline{L}_1\kappa_x^2\left(\frac{h_1}{a}\right)\left(\frac{am\pi}{h_1}\right)^2\overline{B}^{(1)}_{55}\right\}\overline{W}^{(1)}_{mn} \\
+\left\{\overline{L}_1\kappa_x^2 B^{(1)}_{55}\left(\frac{a}{h_1}\right)m\pi\right\}\psi^{(1)}_{mnx} \\
+\frac{\overline{L}_1\overline{E}_{a2}}{\overline{h}_{a2}}\left(\frac{a}{h_1}\right)\left\{\overline{W}^{(1)}_{mn} - \overline{W}^{(2)}_{mn}\right\} + \frac{\overline{L}_1\overline{E}_{a3}}{\overline{h}_{a3}}\left(\frac{a}{h_1}\right)\left\{\overline{W}^{(1)}_{mn} - \overline{W}^{(3)}_{mn}\right\}
\end{array}\right\}
\end{aligned}
\tag{4.18}
$$

where the double underlined (=) terms come from $\partial w^{(j)}/\partial x$ and $\partial w^{(j)}/\partial y$ terms in the adhesive layer equations.

Similarly, for the stiffening plate strips, one may write:

For the Upper Orthotropic Stiffening Plate Strip in the ‘‘Overlap Region’’ or Part I region,

$$
\begin{aligned}
\frac{d\overline{\psi}_{mnx}^{(2)}}{d\xi_I} &= \frac{12\overline{L}_1}{\overline{B}_{66}^{(2)}\overline{h}_2{}^3}\overline{M}_{mnyx}^{(2)} - \overline{L}_1 m\pi\left(\frac{h_1}{a}\right)^2\overline{\psi}_{mny}^{(2)} \\
\frac{d\overline{\psi}_{mny}^{(2)}}{d\xi_I} &= \frac{12\overline{L}_1}{\overline{B}_{22}^{(2)}\overline{h}_2{}^3}\left(\frac{a}{h_1}\right)^2\overline{M}_{mny}^{(2)} + \overline{L}_1 m\pi\frac{\overline{B}_{12}^{(2)}}{\overline{B}_{22}^{(2)}}\left(\frac{a}{h_1}\right)^2\overline{\psi}_{mnx}^{(2)} \\
\frac{d\overline{W}_{mn}^{(2)}}{d\xi_I} &= \frac{\overline{L}_1}{\kappa_y^2\overline{B}_{44}^{(2)}\overline{h}_2}\left(\frac{a}{h_1}\right)\overline{Q}_{mny}^{(2)} - \overline{L}_1\overline{\psi}_{mny}^{(2)} \\
\frac{d\overline{M}_{mnyx}^{(2)}}{d\xi_I} &= \left\{\begin{array}{l}
\left\{\begin{array}{l}
-\dfrac{\overline{L}_1}{12}\overline{\rho}_2\overline{h}_2{}^3\left(\dfrac{h_1}{a}\right)^2\overline{\Omega} + \overline{L}_1\kappa_x^2\overline{B}_{55}^{(2)}\overline{h}_2\left(\dfrac{a}{h_1}\right)^2 \\
+\dfrac{\overline{L}_1}{12}\left(\dfrac{am\pi}{h_1}\right)^2\overline{h}_2{}^3\left(\overline{B}_{11}^{(2)} - \dfrac{\overline{B}_{12}^{(2)^2}}{\overline{B}_{22}^{(2)}}\right)
\end{array}\right\}\psi_{mnx}^{(2)} \\
+\left\{\overline{h}_2\kappa_x^2\overline{L}_1\overline{B}_{55}^{(2)}m\pi - \underline{\underline{\overline{L}_1\dfrac{m\pi}{2}\overline{h}_2\overline{G}_{a2}}}\right\}\overline{W}_{mn}^{(2)} \\
-\left(\dfrac{\overline{B}_{12}^{(2)}}{\overline{B}_{22}^{(2)}}\overline{L}_1 m\pi\right)\overline{M}_{mny}^{(j)} + \dfrac{\overline{G}_{a2}\overline{L}_1\overline{h}_2}{4\overline{h}_{a2}}\left(\dfrac{a}{h_1}\right)^2\left(\overline{\psi}_{mnx}^{(1)} + \overline{h}_2\overline{\psi}_{mnx}^{(2)}\right)
\end{array}\right. \\
\frac{d\overline{M}_{mny}^{(2)}}{d\xi_I} &= \left\{\begin{array}{l}
\left\{-\dfrac{\overline{L}_1}{12}\overline{\rho}_2\overline{h}_2{}^3\left(\dfrac{h_1}{a}\right)^4\overline{\Omega} + \underline{\underline{\dfrac{\overline{G}_{a2}\overline{L}_1\overline{h}_2}{2}}}\right\}\overline{\psi}_{mny}^{(2)} + \left(\dfrac{\overline{L}_1 a}{h_1} - \underline{\underline{\overline{L}_1\left(\dfrac{a}{h_1}\right)\dfrac{\overline{G}_{a2}}{2\kappa_y^2\overline{B}_{44}^{(2)}}}}\right)\overline{Q}_{mny}^{(2)} \\
+\left(\overline{L}_1 m\pi\right)\overline{M}_{mnyx}^{(2)} + \dfrac{\overline{G}_{a2}\overline{L}_1\overline{h}_2}{4\overline{h}_{a2}}\left(\overline{\psi}_{mny}^{(1)} + \overline{h}_2\overline{\psi}_{mny}^{(2)}\right)
\end{array}\right. \\
\frac{d\overline{Q}_{mny}^{(2)}}{d\xi_I} &= \left\{\begin{array}{l}
\left\{-\overline{L}_1\overline{\rho}_2\overline{h}_2\left(\dfrac{h_1}{a}\right)^3\overline{\Omega} + \overline{L}_1\kappa_x^2\overline{h}_2\left(\dfrac{h_1}{a}\right)\left(\dfrac{am\pi}{h_1}\right)^2\overline{B}_{55}^{(2)}\right\}\overline{W}_{mn}^{(2)} \\
+\left\{\overline{L}_1\kappa_x^2 B_{55}^{(2)}\overline{h}_2\left(\dfrac{a}{h_1}\right)m\pi\right\}\psi_{mnx}^{(2)} \\
-\dfrac{\overline{L}_1\overline{E}_{a2}}{\overline{h}_{a2}}\left(\dfrac{a}{h_1}\right)\left\{\overline{W}_{mn}^{(1)} - \overline{W}_{mn}^{(2)}\right\}
\end{array}\right.
\end{aligned}
\tag{4.19}
$$

where the double underlined (=)terms come from $\partial w^{(j)}/\partial x$ and $\partial w^{(j)}/\partial y$ terms in the adhesive layer equations.

For the Lower Orthotropic Stiffening Plate Strip in the ‘‘Overlap Region’’ or Part I region,

$$
\begin{aligned}
\frac{d\overline{\psi}^{(3)}_{mnx}}{d\xi_I} &= \frac{12\overline{L}_1}{\overline{B}^{(3)}_{66}\overline{h}_3{}^3}\overline{M}^{(3)}_{mnyx} - \overline{L}_1 m\pi\left(\frac{h_1}{a}\right)^2\overline{\psi}^{(3)}_{mny} \\
\frac{d\overline{\psi}^{(3)}_{mny}}{d\xi_I} &= \frac{12\overline{L}_1}{\overline{B}^{(3)}_{22}\overline{h}_3{}^3}\left(\frac{a}{h_1}\right)^2\overline{M}^{(3)}_{mny} + \overline{L}_1 m\pi\frac{\overline{B}^{(3)}_{12}}{\overline{B}^{(3)}_{22}}\left(\frac{a}{h_1}\right)^2\overline{\psi}^{(3)}_{mnx} \\
\frac{d\overline{W}^{(3)}_{mn}}{d\xi_I} &= \frac{\overline{L}_1}{\kappa_y^2\overline{B}^{(3)}_{44}\overline{h}_3}\left(\frac{a}{h_1}\right)\overline{Q}^{(3)}_{mny} - \overline{L}_1\overline{\psi}^{(3)}_{mny} \\
\frac{d\overline{M}^{(3)}_{mnyx}}{d\xi_I} &= \left\{\begin{array}{l}
\left\{-\frac{\overline{L}_1}{12}\overline{\rho}_3\overline{h}_3{}^3\left(\frac{h_1}{a}\right)^2\overline{\Omega} + \overline{L}_1\kappa_x^2\overline{B}^{(3)}_{55}\overline{h}_3\left(\frac{a}{h_1}\right)^2\right\}\psi^{(3)}_{mnx} \\
+\frac{\overline{L}_1}{12}\left(\frac{am\pi}{h_1}\right)^2\overline{h}_3{}^3\left(\overline{B}^{(3)}_{11} - \frac{\overline{B}^{(3)^2}_{12}}{\overline{B}^{(3)}_{22}}\right) \\
+\left\{\overline{h}_3\kappa_x^2\overline{L}_1\overline{B}^{(3)}_{55}m\pi - \underline{\underline{\frac{\overline{L}_1}{2}\frac{m\pi}{2}\overline{h}_3\overline{G}_{a3}}}\right\}\overline{W}^{(3)}_{mn} \\
-\left(\frac{\overline{B}^{(3)}_{12}}{\overline{B}^{(3)}_{22}}\overline{L}_1 m\pi\right)\overline{M}^{(3)}_{mny} + \frac{\overline{G}_{a3}\overline{L}_1\overline{h}_3}{4\overline{h}_{a3}}\left(\frac{a}{h_1}\right)^2\left(\overline{\psi}^{(1)}_{mnx} + \overline{h}_3\overline{\psi}^{(3)}_{mnx}\right)
\end{array}\right\} \\
\frac{d\overline{M}^{(3)}_{mny}}{d\xi_I} &= \left\{\begin{array}{l}
\left\{-\frac{\overline{L}_1}{12}\overline{\rho}_3\overline{h}_3{}^3\left(\frac{h_1}{a}\right)^4\overline{\Omega} + \underline{\underline{\frac{\overline{G}_{a3}\overline{L}_1\overline{h}_3}{2}}}\right\}\overline{\psi}^{(3)}_{mny} + \left(\frac{\overline{L}_1 a}{h_1} - \underline{\underline{\overline{L}_1\left(\frac{a}{h_1}\right)\frac{\overline{G}_{a3}}{2\kappa_y^2\overline{B}^{(3)}_{44}}}}\right)\overline{Q}^{(3)}_{mny} \\
+\left(\overline{L}_1 m\pi\right)\overline{M}^{(3)}_{mnyx} + \frac{\overline{G}_{a3}\overline{L}_1\overline{h}_3}{4\overline{h}_{a3}}\left(\overline{\psi}^{(1)}_{mny} + \overline{h}_3\overline{\psi}^{(3)}_{mny}\right)
\end{array}\right\} \\
\frac{d\overline{Q}^{(3)}_{mny}}{d\xi_I} &= \left\{\begin{array}{l}
\left\{-\overline{L}_1\overline{\rho}_3\overline{h}_3\left(\frac{h_1}{a}\right)^3\overline{\Omega} + \overline{L}_1\kappa_x^2\overline{h}_3\left(\frac{h_1}{a}\right)\left(\frac{am\pi}{h_1}\right)^2\overline{B}^{(3)}_{55}\right\}\overline{W}^{(3)}_{mn} \\
+\left\{\overline{L}_1\kappa_x^2 B^{(3)}_{55}\overline{h}_3\left(\frac{a}{h_1}\right)m\pi\right\}\psi^{(3)}_{mnx} \\
-\frac{\overline{L}_1\overline{E}_{a3}}{\overline{h}_{a3}}\left(\frac{a}{h_1}\right)\left\{\overline{W}^{(1)}_{mn} - \overline{W}^{(2)}_{mn}\right\}
\end{array}\right\}
\end{aligned}
\tag{4.20}
$$

where the double underlined (=) terms come from $\partial w^{(j)}/\partial x$ and $\partial w^{(j)}/\partial y$ terms appearing in the adhesive layer equations.

One can write the "Governing System of First Order Ordinary Differential Equations" in terms of the "dimensionless fundamental dependent variables" given in (4.18), (4.19) and (4.20) for the "Overlap Region" or Part I region in Figure 3.3 in a more general form as follows,

$$\frac{d\overline{\psi}_{mnx}^{(j)}}{d\xi_I} = \frac{12\overline{L}_1}{\overline{B}_{66}^{(j)}\overline{h}_j^{\,3}}\overline{M}_{mnyx}^{(j)} - \overline{L}_1 m\pi\left(\frac{h_1}{a}\right)^2\overline{\psi}_{mny}^{(j)}$$

$$\frac{d\overline{\psi}_{mny}^{(j)}}{d\xi_I} = \frac{12\overline{L}_1}{\overline{B}_{22}^{(j)}\overline{h}_j^{\,3}}\left(\frac{a}{h_1}\right)^2\overline{M}_{mny}^{(j)} + \overline{L}_1 m\pi\frac{\overline{B}_{12}^{(j)}}{\overline{B}_{22}^{(j)}}\left(\frac{a}{h_1}\right)^2\overline{\psi}_{mnx}^{(j)}$$

$$\frac{d\overline{W}_{mn}^{(j)}}{d\xi_I} = \frac{\overline{L}_1}{\kappa_y^2\overline{B}_{44}^{(j)}\overline{h}_j}\left(\frac{a}{h_1}\right)\overline{Q}_{mny}^{(j)} - \overline{L}_1\overline{\psi}_{mny}^{(j)}$$

$$\frac{d\overline{M}_{mnyx}^{(j)}}{d\xi_I} = \left\{\begin{array}{l} \left\{\begin{array}{l} -\dfrac{\overline{L}_1}{12}\overline{\rho}_j\overline{h}_j^{\,3}\left(\dfrac{h_1}{a}\right)^2\overline{\Omega} + \overline{L}_1\kappa_x^2\overline{B}_{55}^{(j)}\overline{h}_j\left(\dfrac{a}{h_1}\right)^2 \\ +\dfrac{\overline{L}_1}{12}\left(\dfrac{am\pi}{h_1}\right)^2\overline{h}_j^{\,3}\left(\overline{B}_{11}^{(j)} - \dfrac{\overline{B}_{12}^{(j)^2}}{\overline{B}_{22}^{(j)}}\right) \end{array}\right\}\psi_{mnx}^{(j)} \\ +\left\{\overline{h}_j\kappa_x^2\overline{L}_1\overline{B}_{55}^{(j)}m\pi - \underline{\underline{\overline{L}_1\dfrac{m\pi}{2}\overline{h}_j\overline{G}_{a2}}} - \underline{\underline{\overline{L}_1\dfrac{m\pi}{2}\overline{h}_j\overline{G}_{a3}}}\right\}\overline{W}_{mn}^{(j)} \\ -\left(\dfrac{\overline{B}_{12}^{(j)}}{\overline{B}_{22}^{(j)}}\overline{L}_1 m\pi\right)\overline{M}_{mny}^{(j)} + \dfrac{\overline{G}_{a2}\overline{L}_1\overline{h}_j}{4\overline{h}_{a2}}\left(\dfrac{a}{h_1}\right)^2\left(\overline{\psi}_{mnx}^{(1)} + \overline{h}_2\overline{\psi}_{mnx}^{(2)}\right) \\ +\dfrac{\overline{G}_{a3}\overline{L}_1\overline{h}_j}{4\overline{h}_{a3}}\left(\dfrac{a}{h_1}\right)^2\left(\overline{\psi}_{mnx}^{(1)} + \overline{h}_3\overline{\psi}_{mnx}^{(3)}\right) \end{array}\right.$$

(where $\dfrac{\overline{G}_{a2}}{\overline{h}_{a2}} = 0$ ($\overline{G}_{a2} = 0$) for j=3 , $\dfrac{\overline{G}_{a3}}{\overline{h}_{a3}} = 0$ ($\overline{G}_{a3} = 0$) for j=2)

$$\frac{d\overline{M}_{mny}^{(j)}}{d\xi_I} = \left\{\begin{array}{l} \left\{-\dfrac{\overline{L}_1}{12}\overline{\rho}_j\overline{h}_j^{\,3}\left(\dfrac{h_1}{a}\right)^4\overline{\Omega} + \underline{\underline{\dfrac{\overline{G}_{a2}\overline{L}_1\overline{h}_j}{2}}} + \underline{\underline{\dfrac{\overline{G}_{a3}\overline{L}_1\overline{h}_j}{2}}}\right\}\overline{\psi}_{mny}^{(j)} \\ +\left(\dfrac{\overline{L}_1 a}{h_1} - \underline{\underline{\overline{L}_1\left(\dfrac{a}{h_1}\right)\dfrac{\overline{G}_{a2}}{2\kappa_y^2\overline{B}_{44}^{(j)}}}} - \underline{\underline{\overline{L}_1\left(\dfrac{a}{h_1}\right)\dfrac{\overline{G}_{a3}}{2\kappa_y^2\overline{B}_{44}^{(j)}}}}\right)\overline{Q}_{mny}^{(j)} \\ +\left(\overline{L}_1 m\pi\right)\overline{M}_{mnyx}^{(j)} + \dfrac{\overline{G}_{a2}\overline{L}_1\overline{h}_j}{4\overline{h}_{a2}}\left(\overline{\psi}_{mny}^{(1)} + \overline{h}_2\overline{\psi}_{mny}^{(2)}\right) + \dfrac{\overline{G}_{a3}\overline{L}_1\overline{h}_j}{4\overline{h}_{a3}}\left(\overline{\psi}_{mny}^{(1)} + \overline{h}_3\overline{\psi}_{mny}^{(3)}\right) \end{array}\right.$$

(where $\dfrac{\overline{G}_{a2}}{\overline{h}_{a2}} = 0$ ($\overline{G}_{a2} = 0$) for j=3 , $\dfrac{\overline{G}_{a3}}{\overline{h}_{a3}} = 0$ ($\overline{G}_{a3} = 0$) for j=2)

where the double underlined (=) terms come from $\partial w^{(j)}/\partial x$ and $\partial w^{(j)}/\partial y$ terms appearing in the adhesive layer equations.

$$\frac{d\overline{Q}_{mny}^{(j)}}{d\xi_I}=\left\{\begin{array}{l}\left\{-\overline{L}_1\overline{\rho}_j\overline{h}_j\left(\frac{h_1}{a}\right)^3\overline{\Omega}+\overline{L}_1\kappa_x^2\overline{h}_j\left(\frac{h_1}{a}\right)\left(\frac{am\pi}{h_1}\right)^2\overline{B}_{55}^{(j)}\right\}\overline{W}_{mn}^{(j)}\\ +\left\{\overline{L}_1\kappa_x^2B_{55}^{(j)}\overline{h}_j\left(\frac{a}{h_1}\right)m\pi\right\}\psi_{mnx}^{(j)}\\ +(-1)^{(j+1)}\frac{\overline{L}_1\overline{E}_{a2}}{\overline{h}_{a2}}\left(\frac{a}{h_1}\right)\left\{\overline{W}_{mn}^{(1)}-\overline{W}_{mn}^{(2)}\right\}-(-1)^{(j+1)}\frac{\overline{L}_1\overline{E}_{a3}}{\overline{h}_{a3}}\left(\frac{a}{h_1}\right)\left\{\overline{W}_{mn}^{(1)}-\overline{W}_{mn}^{(3)}\right\}\end{array}\right\}$$

(where $\frac{\overline{E}_{a2}}{\overline{h}_{a2}}=0$ for j=3 , $\frac{\overline{E}_{a3}}{\overline{h}_{a3}}=0$ for j=2 , $\overline{E}_{a3}=-1\times\overline{E}_{a3}$ for j=1)

$$(0<\xi_I<l_I),(j=1,2,3) \qquad (4.21)$$

The quantities having the subscript “m” are called as “dimensionless fundamental dependent variables” of this problem. As given above, the system of partial differential equations are finally reduced to a ‘‘Governing System of First Order Ordinary Differential Equations’’ in ξ_I or y_I direction. This creates a ‘‘Two-Point Boundary Value Problem in the ‘‘Overlap Region’’ or Part I region. Then, the “Governing System of First Order Ordinary Differential Equations” in the compact matrix or ‘‘state vector’’ form for the ‘‘Overlap Region’’ or Part I region can be written as,

$$\frac{d}{d\xi_I}\left\{\begin{array}{c}\overline{\boldsymbol{Y}}_{\boldsymbol{mn}}^{(2)}\\ \overline{\boldsymbol{Y}}_{\boldsymbol{mn}}^{(1)}\\ \overline{\boldsymbol{Y}}_{\boldsymbol{mn}}^{(3)}\end{array}\right\}=\left[\begin{array}{ccc}\overline{\mathsf{C}}_{1,1} & \overline{\mathsf{C}}_{1,2} & \overline{\mathsf{C}}_{1,3}\\ \overline{\mathsf{C}}_{2,1} & \overline{\mathsf{C}}_{2,2} & \overline{\mathsf{C}}_{2,3}\\ \overline{\mathsf{C}}_{3,1} & \overline{\mathsf{C}}_{3,2} & \overline{\mathsf{C}}_{3,3}\end{array}\right]\left\{\begin{array}{c}\overline{\boldsymbol{Y}}_{\boldsymbol{mn}}^{(2)}\\ \overline{\boldsymbol{Y}}_{\boldsymbol{mn}}^{(1)}\\ \overline{\boldsymbol{Y}}_{\boldsymbol{mn}}^{(3)}\end{array}\right\},\qquad (0<\xi_I<1) \qquad (4.22)$$

with the ‘‘Arbitrary Boundary Conditions’’ at ξ_I =0,1 for the stiffening strips and the ‘‘Continuity Conditions’’ at ξ_I =0,1 for the base plate.

where ξ_I is defined as y_I/l_I , and the superscripts show the related plate layer, the sub-matrices $[\overline{\mathsf{C}}_{i,j}]$ are partitioned square matrices of dimension (6x6) which explicitly include the nondimensional geometric and material characteristics of the base plate and the upper and lower stiffening plate strips and the adhesive layers and dimensionless natural frequency parameter $\overline{\omega}_{mn}$ of the entire composite system.

Also, $\overline{Y}_{mn}^{(j)}$ (j=1,2,3) is the ‘‘state vector’’ corresponding to state variables or ‘‘dimensionless fundamental dependent variables’’. Thus,

$$\left\{\overline{Y}_{mn}^{(j)}\right\}=\left\{\overline{\Psi}_{mnx}^{(j)},\overline{\Psi}_{mny}^{(j)},\overline{W}_{mn}^{(j)};\overline{M}_{mnyx}^{(j)},\overline{M}_{mny}^{(j)},\overline{Q}_{mny}^{(j)}\right\}^{T},\ (j=1,2,3) \qquad (4.23)$$

where the elements of $\left\{\overline{Y}_{mn}^{(j)}\right\}$ are the quantities related to the displacements, angles of rotation and the stress resultants which appear at ξ_I=constant segments or cross-sections of the plate system in the “Overlap Region” or Part I region (see also Figure 3.3.)

4.2.3 Analysis of Part II of Composite Plate System

One can obtain the governing system of equations for the base plate in Part II region of Figure 3.3 by using the same procedure in the previous section. Thus, one may substitute j=1, ξ_I is replaced by ξ_{II} in equations (4.21) and (4.22) for Part II. region. There is no adhesive layer in Part II. Therefore, coupling terms in (4.21) including the adhesive layer elastic constants are dropped. The “Governing System of First Order Ordinary Differential Equations” in the ‘‘state vector’’ form, for Part I region,

$$\frac{d}{d\xi_{II}}\left\{\overline{Y}_{mn}^{(1)}\right\}=\left[\overline{D}\right]\left\{\overline{Y}_{mn}^{(1)}\right\} \qquad (0<\xi_{II}<1) \qquad (4.24)$$

with the ‘‘Arbitrary Boundary Conditions’’ at ξ_{II} =0 and the ‘‘Continuity Conditions’’ at ξ_{II} =1 for the orthotropic base plate.

where ξ_{II} is defined as y_{II}/l_{II} and $[\overline{D}]$ is the “Coefficient Matrix” of dimension (6x6) which explicitly includes dimensionless geometric and material characteristics of the base plate in the Part II region as well as the dimensionless natural frequencies $\overline{\omega}_{mn}$ of the entire composite plate system. The column matrix or the “state vector” $\left\{\overline{Y}_{mn}^{(1)}\right\}$ is defined as in (4.23) by taking j=1 only, that is,

$$\left\{\overline{Y}_{mn}^{(1)}\right\}=\left\{\overline{\Psi}_{mnx}^{(1)},\overline{\Psi}_{mny}^{(1)},\overline{W}_{mn}^{(1)};\overline{M}_{mnyx}^{(1)},\overline{M}_{mny}^{(1)},\overline{Q}_{mny}^{(1)}\right\}^{T},\ (j=1) \qquad (4.25)$$

4.2.4 Analysis of Part III of Composite Plate System

In order to obtain the governing system of ordinary differential equations in a "state vector" form for the Part III region, a similar procedure as in Section 4.2.3 is followed. However, in (4.24), ξ_{II} is replaced by ξ_{III} in Part III region. Then, the "Governing System of First Order Ordinary Differential Equations" in the ''state vector'' form for the Part III region,

$$\frac{d}{d\xi_{III}}\left\{\overline{Y}_{mn}^{(1)}\right\}=\left[\overline{E}\right]\left\{\overline{Y}_{mn}^{(1)}\right\}, \qquad (0<\xi_{III}<1) \qquad (4.26)$$

with the ''Arbitrary Boundary Conditions'' at $\xi_{III}=1$ and the ''Continuity Conditions'' at $\xi_{III}=0$ for the orthotropic base plate.

where ξ_{III} is defined as y_{III}/l_{III} and $[\overline{E}]$ is the "Coefficient Matrix" of dimension (6x6) which explicitly includes dimensionless geometric and material characteristics of the base plate in the Part III region as well as the dimensionless natural frequency parameter $\overline{\omega}_{mn}$ of the entire composite plate system. The column matrix or the "state vector" $\left\{\overline{Y}_{mn}^{(1)}\right\}$ is defined as in (4.25).

4.2.5 System of Governing Differential Equations ("Main PROBLEM I.a")

In the previous sections, the governing system of coupled ordinary differential equations in the matrix or "state vector" form is obtained in (4.22) for the "Overlap Region" or Part I region, in (4.24) for Part II region and in (4.26) for Part III region. Then, one can write these equations in open form as follows,

For Part I region or the "Overlap Region" (or Three-Layer Composite Plate Region),

$$\frac{d}{d\xi_I}\begin{Bmatrix}\overline{\psi}^{(2)}_{mnx}\\ \overline{\psi}^{(2)}_{mny}\\ \overline{W}^{(2)}_{mn}\\ \overline{M}^{(2)}_{mnyx}\\ \overline{M}^{(2)}_{mny}\\ \overline{Q}^{(2)}_{mny}\\ \overline{\psi}^{(1)}_{mnx}\\ \overline{\psi}^{(1)}_{mny}\\ \overline{W}^{(1)}_{mn}\\ \overline{M}^{(1)}_{mnyx}\\ \overline{M}^{(1)}_{mny}\\ \overline{Q}^{(1)}_{mny}\\ \overline{\psi}^{(3)}_{mnx}\\ \overline{\psi}^{(3)}_{mny}\\ \overline{W}^{(3)}_{mn}\\ \overline{M}^{(3)}_{mnyx}\\ \overline{M}^{(3)}_{mny}\\ \overline{Q}^{(3)}_{mny}\end{Bmatrix}=\begin{bmatrix}
0 & c^{(2)}_{1,2} & 0 & c^{(2)}_{1,4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
c^{(2)}_{2,1} & 0 & 0 & 0 & c^{(2)}_{2,5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & c^{(2)}_{3,2} & 0 & 0 & 0 & c^{(2)}_{3,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
c^{(2)}_{4,1} & 0 & c^{(2)}_{4,3} & 0 & c^{(2)}_{4,5} & 0 & r^{(2)}_{1,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & c^{(2)}_{5,2} & 0 & c^{(2)}_{5,4} & 0 & c^{(2)}_{5,6} & 0 & r^{(2)}_{2,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
c^{(2)}_{6,1} & 0 & c^{(2)}_{6,3} & 0 & 0 & 0 & 0 & 0 & r^{(2)}_{3,3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{(1)}_{1,2} & 0 & c^{(1)}_{1,4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & c^{(1)}_{2,1} & 0 & 0 & 0 & c^{(1)}_{2,5} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{(1)}_{3,2} & 0 & 0 & 0 & c^{(1)}_{3,6} & 0 & 0 & 0 & 0 & 0 & 0\\
r^{(2)}_{1,1} & 0 & 0 & 0 & 0 & 0 & c^{(1)}_{4,1} & 0 & c^{(1)}_{4,3} & 0 & c^{(1)}_{4,5} & 0 & r^{(3)}_{1,1} & 0 & 0 & 0 & 0 & 0\\
0 & r^{(2)}_{2,2} & 0 & 0 & 0 & 0 & 0 & c^{(1)}_{5,2} & 0 & c^{(1)}_{5,4} & 0 & c^{(1)}_{5,6} & 0 & r^{(3)}_{2,2} & 0 & 0 & 0 & 0\\
0 & 0 & r^{(2)}_{3,3} & 0 & 0 & 0 & c^{(1)}_{6,1} & 0 & c^{(1)}_{6,3} & 0 & 0 & 0 & 0 & 0 & r^{(3)}_{3,3} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{1,2} & 0 & c^{(3)}_{1,4} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{2,1} & 0 & 0 & 0 & c^{(3)}_{2,5} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{3,2} & 0 & 0 & 0 & c^{(3)}_{3,6}\\
0 & 0 & 0 & 0 & 0 & 0 & r^{(3)}_{1,1} & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{4,1} & 0 & c^{(3)}_{4,3} & 0 & c^{(3)}_{4,5} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & r^{(3)}_{2,2} & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{5,2} & 0 & c^{(3)}_{5,4} & 0 & c^{(3)}_{5,6}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & r^{(3)}_{3,3} & 0 & 0 & 0 & c^{(3)}_{6,1} & 0 & c^{(3)}_{6,3} & 0 & 0 & 0
\end{bmatrix}\begin{Bmatrix}\overline{\psi}^{(2)}_{mnx}\\ \overline{\psi}^{(2)}_{mny}\\ \overline{W}^{(2)}_{mn}\\ \overline{M}^{(2)}_{mnyx}\\ \overline{M}^{(2)}_{mny}\\ \overline{Q}^{(2)}_{mny}\\ \overline{\psi}^{(1)}_{mnx}\\ \overline{\psi}^{(1)}_{mny}\\ \overline{W}^{(1)}_{mn}\\ \overline{M}^{(1)}_{mnyx}\\ \overline{M}^{(1)}_{mny}\\ \overline{Q}^{(1)}_{mny}\\ \overline{\psi}^{(3)}_{mnx}\\ \overline{\psi}^{(3)}_{mny}\\ \overline{W}^{(3)}_{mn}\\ \overline{M}^{(3)}_{mnyx}\\ \overline{M}^{(3)}_{mny}\\ \overline{Q}^{(3)}_{mny}\end{Bmatrix} \qquad (0<\xi_I<1) \qquad (4.27.a)$$

In the above, the elements of the ''Coefficient Sub-Matrix $[\Lambda]^{(j)}$'' corresponding to the plate layers are,

$$[\Lambda]^{(j)}=\begin{bmatrix}
0 & c^{(j)}_{1,2} & 0 & c^{(j)}_{1,4} & 0 & 0\\
c^{(j)}_{2,1} & 0 & 0 & 0 & c^{(j)}_{2,5} & 0\\
0 & c^{(j)}_{3,2} & 0 & 0 & 0 & c^{(j)}_{3,6}\\
c^{(j)}_{4,1} & 0 & c^{(j)}_{4,3} & 0 & c^{(j)}_{4,5} & 0\\
0 & c^{(j)}_{5,2} & 0 & c^{(j)}_{5,4} & 0 & c^{(j)}_{5,6}\\
c^{(j)}_{6,1} & 0 & c^{(j)}_{6,3} & 0 & 0 & 0
\end{bmatrix} \qquad (j=1,2,3) \qquad (4.27.b)$$

where superscript j denotes base plate for j=1, upper stiffening plate strip for j=2 and lower stiffening plate strip for j=3.

Similarly, the elements of the ''Coefficient Sub-Matrix $[\alpha]^{(k)}$'' related to the adhesive layers are,

$$[\alpha]^{(k)}=\begin{bmatrix}
r^{(k)}_{1,1} & 0 & 0\\
0 & r^{(k)}_{2,2} & 0\\
0 & 0 & r^{(k)}_{3,3}
\end{bmatrix} \qquad (k=2,3) \qquad (4.27.c)$$

where superscript k denotes the upper adhesive layer for k=2, and the lower adhesive layer for k=3 .

The elements, in the open form, of the ‘‘Coefficient Sub-Matrix $[\Lambda]^{(j)}$,’’ related to the plate layers are,

$$c_{1,2}^{(j)} = -\overline{L}_1 m\pi \left(\frac{h_1}{a}\right)^2 \qquad c_{1,4}^{(j)} = \frac{12\overline{L}_1}{\overline{B}_{66}^{(k)}\overline{h}_j^3}$$

$$c_{2,1}^{(j)} = \overline{L}_1 m\pi \frac{\overline{B}_{12}^{(j)}}{\overline{B}_{22}^{(j)}}\left(\frac{a}{h_1}\right)^2 \qquad c_{2,5}^{(j)} = \frac{12\overline{L}_1}{\overline{B}_{22}^{(j)}\overline{h}_j^{(3)}}\left(\frac{a}{h_1}\right)^2$$

$$c_{3,2}^{(j)} = -\overline{L}_1 \qquad c_{3,6}^{(j)} = \frac{\overline{L}_1}{\kappa_y^2 \overline{B}_{44}^{(j)}\overline{h}_j}\left(\frac{a}{h_1}\right)$$

$$c_{4,1}^{(j)} = -\frac{\overline{L}_1}{12}\overline{\rho}_j \overline{h}_j^3\left(\frac{h_1}{a}\right)^2 \overline{\Omega} + \overline{L}_1\kappa_x^2 \overline{B}_{55}^{(j)}\overline{h}_j\left(\frac{a}{h_1}\right)^2 + \frac{\overline{L}_1}{12}\left(\frac{am\pi}{h_1}\right)^2 \overline{h}_j^3\left(\overline{B}_{11}^{(j)} - \frac{\overline{B}_{12}^{(j)^2}}{\overline{B}_{22}^{(j)}}\right) + \frac{\overline{G}_{aj}\overline{L}_1\overline{h}_j^2}{4\overline{h}_{aj}}\left(\frac{a}{h_1}\right)^2$$

where $\dfrac{\overline{G}_{a1}}{\overline{h}_{a1}} = \dfrac{\overline{G}_{a2}}{\overline{h}_{a2}} + \dfrac{\overline{G}_{a3}}{\overline{h}_{a3}}$ for j=1

$$c_{4,3}^{(j)} = \overline{h}_j\kappa_x^2\overline{L}_1\overline{B}_{55}^{(j)} m\pi - \underline{\underline{\overline{L}_1\frac{m\pi}{2}\overline{h}_j\overline{G}_{aj}}} \qquad c_{4,5}^{(j)} = -\overline{L}_1 m\pi\frac{\overline{B}_{12}^{(j)}}{\overline{B}_{22}^{(j)}}$$

where $\overline{G}_{a1} = \overline{G}_{a2} + \overline{G}_{a3}$ for j=1

$$c_{5,2}^{(j)} = -\frac{\overline{L}_1}{12}\overline{\rho}_j\overline{h}_j^3\left(\frac{h_1}{a}\right)^4\overline{\Omega} + \frac{\overline{G}_{aj}\overline{L}_1\overline{h}_j^2}{4\overline{h}_{aj}} + \underline{\underline{\frac{\overline{G}_{aj}\overline{L}_1\overline{h}_j}{2}}} \qquad c_{5,4}^{(j)} = \overline{L}_1 m\pi$$

$$c_{5,6}^{(j)} = \overline{L}_1\left(\frac{a}{h_1}\right) - \underline{\underline{\overline{L}_1\left(\frac{a}{h_1}\right)\frac{\overline{G}_{aj}}{2\kappa_y^2\overline{B}_{44}^{(j)}}}}$$

$$c_{6,1}^{(j)} = \overline{h}_j\kappa_x^2\overline{L}_1\overline{B}_{55}^{(j)}\left(\frac{a}{h_1}\right)m\pi$$

$$c_{6,3}^{(j)} = -\overline{L}_1\overline{\rho}_j\overline{h}_j\left(\frac{h_1}{a}\right)^3\overline{\Omega} + \overline{L}_1\kappa_x^2\overline{h}_j\left(\frac{h_1}{a}\right)\left(\frac{am\pi}{h_1}\right)^2\overline{B}_{55}^{(j)} + \frac{\overline{L}_1\overline{E}_{aj}}{\overline{h}_{aj}}\left(\frac{a}{h_1}\right)$$

where $\dfrac{\overline{G}_{a1}}{\overline{h}_{a1}} = \dfrac{\overline{G}_{a2}}{\overline{h}_{a2}} + \dfrac{\overline{G}_{a3}}{\overline{h}_{a3}}$ and $\overline{G}_{a1} = \overline{G}_{a2} + \overline{G}_{a3}$ (for j=1).

and $\dfrac{\overline{E}_{a1}}{\overline{h}_{a1}} = \dfrac{\overline{E}_{a2}}{\overline{h}_{a2}} + \dfrac{\overline{E}_{a3}}{\overline{h}_{a3}}$ (for j=1) (j=1,2,3) (4.27.d)

Also, in the above, the double underlined (=) terms come from $\partial w^{(j)}/\partial x$ and $\partial w^{(j)}/\partial y$ terms appearing in the adhesive layer equations.

The elements, in the open form, of the ‘‘Coefficient Sub-Matrix $[\alpha]^{(k)}$,,’’ associated with the plate layers are,

$$r_{1,1}^{(k)}=\frac{\overline{G}_{ak}\overline{L}_1\overline{h}_k}{4\overline{h}_{ak}}\left(\frac{a}{h_1}\right)^2 \qquad r_{2,2}^{(k)}=\frac{\overline{G}_{ak}\overline{L}_1\overline{h}_k}{4\overline{h}_{ak}} \qquad r_{3,3}^{(k)}=-\frac{\overline{L}_1\overline{E}_{ak}}{\overline{h}_{ak}}\left(\frac{a}{h_1}\right)$$

(k=2,3) (4.27.e)

Similar to Part I, one may write,

For Part II region (or Single Layer Orthotropic Base Plate Region),

$$\frac{d}{d\xi_{II}}\begin{Bmatrix}\overline{\psi}_{mnx}^{(1)}\\ \overline{\psi}_{mny}^{(1)}\\ \overline{W}_{mn}^{(1)}\\ \overline{M}_{mnyx}^{(1)}\\ \overline{M}_{mny}^{(1)}\\ \overline{Q}_{mny}^{(1)}\end{Bmatrix}=\begin{bmatrix}0 & d_{1,2} & 0 & d_{1,4} & 0 & 0\\ d_{2,1} & 0 & 0 & 0 & d_{2,5} & 0\\ 0 & d_{3,2} & 0 & 0 & 0 & d_{3,6}\\ d_{4,1} & 0 & d_{4,3} & 0 & d_{4,5} & 0\\ 0 & d_{5,2} & 0 & d_{5,4} & 0 & d_{5,6}\\ d_{6,1} & 0 & d_{6,3} & 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}\overline{\psi}_{mnx}^{(1)}\\ \overline{\psi}_{mny}^{(1)}\\ \overline{W}_{mn}^{(1)}\\ \overline{M}_{mnyx}^{(1)}\\ \overline{M}_{mny}^{(1)}\\ \overline{Q}_{mny}^{(1)}\end{Bmatrix}, \quad (0<\xi_{II}<1) \qquad (4.28.a)$$

where the elements of the above ‘‘Coefficient Matrix $\left[\overline{\mathsf{D}}\right]$,,’’ are,

$$d_{1,2}=-\overline{L}_2 m\pi\left(\frac{h_1}{a}\right)^2 \qquad d_{1,4}=\frac{12\overline{L}_2}{\overline{B}_{66}^{(1)}}$$

$$d_{2,1}=\overline{L}_2 m\pi\frac{\overline{B}_{12}^{(1)}}{\overline{B}_{22}^{(1)}}\left(\frac{a}{h_1}\right)^2 \qquad d_{2,5}=\frac{12\overline{L}_2}{\overline{B}_{22}^{(1)}}\left(\frac{a}{h_1}\right)^2$$

$$d_{3,2}=-\overline{L}_2 \qquad d_{3,6}=\frac{\overline{L}_2}{\kappa_y^2\overline{B}_{44}^{(1)}}\left(\frac{a}{h_1}\right)$$

$$d_{4,1}=-\frac{\overline{L}_2}{12}\left(\frac{h_1}{a}\right)^2\overline{\Omega}+\overline{L}_2\kappa_x^2\overline{B}_{55}^{(1)}\left(\frac{a}{h_1}\right)^2+\frac{\overline{L}_2}{12}\left(\frac{am\pi}{h_1}\right)^2\left(\overline{B}_{11}^{(1)}-\frac{\overline{B}_{12}^{(1)^2}}{\overline{B}_{22}^{(1)}}\right) \qquad d_{4,3}=\kappa_x^2\overline{L}_2\overline{B}_{55}^{(1)}m\pi$$

$$d_{4,5}=-\frac{\overline{B}_{12}^{(1)}}{\overline{B}_{22}^{(1)}}\overline{L}_2 m\pi$$

$$d_{5,2}=-\frac{\overline{L}_2}{12}\overline{\Omega} \qquad d_{5,4}=\overline{L}_2 m\pi \qquad d_{5,6}=\frac{\overline{L}_2 a}{h_1}$$

$$d_{6,1}=\kappa_x^2\overline{L}_2 B_{55}^{(1)}\left(\frac{a}{h_1}\right)m\pi \qquad d_{6,3}=-\overline{L}_2\left(\frac{h_1}{a}\right)^3\overline{\Omega}+\overline{L}_2\kappa_x^2\left(\frac{h_1}{a}\right)\left(\frac{am\pi}{h_1}\right)^2\overline{B}_{55}^{(1)}$$

(4.28.b)

For Part III region (or Single Layer Orthotropic Base Plate Region),

$$\frac{d}{d\xi_{III}}\begin{Bmatrix}\overline{\psi}_{mnx}^{(1)}\\ \overline{\psi}_{mny}^{(1)}\\ \overline{W}_{mn}^{(1)}\\ \overline{M}_{mnyx}^{(1)}\\ \overline{M}_{mny}^{(1)}\\ \overline{Q}_{mny}^{(1)}\end{Bmatrix}=\begin{bmatrix}0 & e_{1,2} & 0 & e_{1,4} & 0 & 0\\ e_{2,1} & 0 & 0 & 0 & e_{2,5} & 0\\ 0 & e_{3,2} & 0 & 0 & 0 & e_{3,6}\\ e_{4,1} & 0 & e_{4,3} & 0 & e_{4,5} & 0\\ 0 & e_{5,2} & 0 & e_{5,4} & 0 & e_{5,6}\\ e_{6,1} & 0 & e_{6,3} & 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}\overline{\psi}_{mnx}^{(1)}\\ \overline{\psi}_{mny}^{(1)}\\ \overline{W}_{mn}^{(1)}\\ \overline{M}_{mnyx}^{(1)}\\ \overline{M}_{mny}^{(1)}\\ \overline{Q}_{mny}^{(1)}\end{Bmatrix},\qquad (0<\xi_{III}<1)\qquad (4.29.a)$$

where the elements of the above ‘‘Coefficient Matrix $\left[\overline{E}\right]$’’ are,

$$e_{1,2}=-\overline{L}_3 m\pi\left(\frac{h_1}{a}\right)^2 \qquad e_{1,4}=\frac{12\overline{L}_3}{\overline{B}_{66}^{(1)}}$$

$$e_{2,1}=\overline{L}_3 m\pi\,\frac{\overline{B}_{12}^{(1)}}{\overline{B}_{22}^{(1)}}\left(\frac{a}{h_1}\right)^2 \qquad e_{2,5}=\frac{12\overline{L}_3}{\overline{B}_{22}^{(1)}}\left(\frac{a}{h_1}\right)^2$$

$$e_{3,2}=-\overline{L}_3 \qquad e_{3,6}=\frac{\overline{L}_3}{\kappa_y^2\overline{B}_{44}^{(1)}}\left(\frac{a}{h_1}\right)$$

$$e_{4,1}=-\frac{\overline{L}_3}{12}\left(\frac{h_1}{a}\right)^2\overline{\Omega}+\overline{L}_3\kappa_x^2\overline{B}_{55}^{(1)}\left(\frac{a}{h_1}\right)^2+\frac{\overline{L}_3}{12}\left(\frac{am\pi}{h_1}\right)^2\left(\overline{B}_{11}^{(1)}-\frac{\overline{B}_{12}^{(1)^2}}{\overline{B}_{22}^{(1)}}\right)$$

$$e_{4,3}=\kappa_x^2\overline{L}_3\overline{B}_{55}^{(1)}m\pi \qquad e_{4,5}=-\overline{L}_3 m\pi\,\frac{\overline{B}_{12}^{(1)}}{\overline{B}_{22}^{(1)}}$$

$$e_{5,2}=-\frac{\overline{L}_3}{12}\left(\frac{h_1}{a}\right)^4\overline{\Omega} \qquad e_{5,4}=\overline{L}_3 m\pi \qquad e_{5,6}=\overline{L}_3\left(\frac{a}{h_1}\right)$$

$$e_{6,1}=\kappa_x^2\overline{L}_3 B_{55}^{(1)}\left(\frac{a}{h_1}\right)m\pi \qquad e_{6,3}=-\overline{L}_3\left(\frac{h_1}{a}\right)^3\overline{\Omega}+\overline{L}_3\kappa_x^2\left(\frac{h_1}{a}\right)\left(\frac{am\pi}{h_1}\right)^2\overline{B}_{55}^{(1)}$$

(4.29.b)

The “Boundary Conditions” at x=0 and x=a are already satisfied by trigonometric expansion in ‘‘Classical Lévy’s Type Solution’’. Then, in order to be able to solve the ‘‘Governing System of First Order Ordinary Differential

Equations'' (4.27), (4.28) and (4.29), the ''Appropriate Boundary Conditions'' and the ''Continuity Conditions'' are needed. These are,

The ''Boundary Conditions'' along the edges in the y-direction,

F (Free): $\overline{M}_{yx}^{(j)} = \overline{M}_{y}^{(j)} = \overline{Q}_{y}^{(j)} = 0$

C (Clamped): $\overline{w}^{(j)} = \overline{\psi}_{x}^{(j)} = \overline{\psi}_{y}^{(j)} = 0 \quad (j=1,2,3)$ (4.30)

S (Simply Supported): $\overline{w}^{(j)} = \overline{\psi}_{x}^{(j)} = \overline{M}_{y}^{(j)} = 0$

The ''Continuity Condition'' between Part I and Part II (see also Fig. 3.3),

$$\left\{\overline{\boldsymbol{Y}}_{\xi_{II}=1}^{(1)}\right\} = \left\{\overline{\boldsymbol{Y}}_{\xi_{I}=0}^{(1)}\right\} \tag{4.31}$$

The ''Continuity Condition'' between Part I and Part III (see also Fig.3.3),

$$\left\{\overline{\boldsymbol{Y}}_{\xi_{III}=0}^{(1)}\right\} = \left\{\overline{\boldsymbol{Y}}_{\xi_{I}=1}^{(1)}\right\} \tag{4.32}$$

Finally, the set of the "Governing System of First Order Ordinary Differential Equations" for "Main PROBLEM I.a" is as follows,

$$\frac{d}{d\xi_I}\begin{Bmatrix}\overline{\boldsymbol{Y}}_{mn}^{(2)}\\ \overline{\boldsymbol{Y}}_{mn}^{(1)}\\ \overline{\boldsymbol{Y}}_{mn}^{(3)}\end{Bmatrix} = \begin{bmatrix}\overline{\mathsf{C}}_{1,1} & \overline{\mathsf{C}}_{1,2} & \overline{\mathsf{C}}_{1,3}\\ \overline{\mathsf{C}}_{2,1} & \overline{\mathsf{C}}_{2,2} & \overline{\mathsf{C}}_{2,3}\\ \overline{\mathsf{C}}_{3,1} & \overline{\mathsf{C}}_{3,2} & \overline{\mathsf{C}}_{3,3}\end{bmatrix}\begin{Bmatrix}\overline{\boldsymbol{Y}}_{mn}^{(2)}\\ \overline{\boldsymbol{Y}}_{mn}^{(1)}\\ \overline{\boldsymbol{Y}}_{mn}^{(3)}\end{Bmatrix}, \quad (0<\xi_I<1) \quad \text{(in Part I)}$$

$$\frac{d}{d\xi_{II}}\left\{\overline{\boldsymbol{Y}}_{mn}^{(1)}\right\} = \left[\overline{\mathsf{D}}\right]\left\{\overline{\boldsymbol{Y}}_{mn}^{(1)}\right\}, \quad (0<\xi_{II}<1) \quad \text{(in Part II)} \quad \text{(4.33 a,b,c)}$$

$$\frac{d}{d\xi_{III}}\left\{\overline{\boldsymbol{Y}}_{mn}^{(1)}\right\} = \left[\overline{\mathsf{E}}\right]\left\{\overline{\boldsymbol{Y}}_{mn}^{(1)}\right\}, \quad (0<\xi_{III}<1) \quad \text{(in Part III)}$$

with the "Appropriate Boundary Conditions" and the "Continuity Conditions" for each differential equation above.

The above system of equations forms a “Two-Point Boundary Value Problem” for the “Main PROBLEM I.a”. It is obvious that, once the natural frequencies are obtained, then, the Equations (4.33.a,b,c) can be integrated numerically for a given particular geometry, materials and the support conditions.

CHAPTER 5

(‘‘Main PROBLEM I.b’’)__ FREE FLEXURAL (or BENDING) VIBRATIONS of CENTRALLY DOUBLY STIFFENED, COMPOSITE PLATES or PANELS (Isotropic Base Plate or Panel Case)

5.1 Application of Mindlin’s Plate Theory to Isotropic Base Plate and Orthotropic Stiffening Plate Strips

The coordinate system and the positive sign convention for the stress resultants and surface loads or external stresses for plates are shown in Figure 3.2. The faces of the plate are the planes at z=±h/2.

The basic assumptions of the “Mindlin Plate Theory” [VI.3] is given in Chapter 3. The in-plane displacements u and v are proportional to z and the transverse displacement w is independent of z. That is,

$$\left| \begin{array}{l} u = z\psi_x(x,y,t) \\ v = z\psi_y(x,y,t) \\ w = w(x,y,t) \end{array} \right. \qquad (5.1)$$

The positive sign convention for the angles of rotation ψ_x and ψ_y are the negatives of rotations in xz and yz-planes respectively.(see also figure 3.2)

The strain-displacement equations in three dimensional elasticity are,

$$\left|\begin{array}{ll} \varepsilon_x = \dfrac{\partial u}{\partial x} & \gamma_{xy} = \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \\ \varepsilon_y = \dfrac{\partial v}{\partial y} & \gamma_{xz} = \dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial x} \\ \varepsilon_z = \dfrac{\partial w}{\partial z} & \gamma_{yz} = \dfrac{\partial v}{\partial z} + \dfrac{\partial w}{\partial y} \end{array}\right. \tag{5.2}$$

One can obtain another form of strain-displacement relations by inserting the assumed form of displacement functions (5.1) into the strain-displacement relations in three dimensional elasticity (5.2) as the follows,

$$\left|\begin{array}{ll} \varepsilon_x = z\dfrac{\partial \psi_x}{\partial x} & \gamma_{xy} = z\left(\dfrac{\partial \psi_x}{\partial y} + \dfrac{\partial \psi_y}{\partial x}\right) \\ \varepsilon_y = z\dfrac{\partial \psi_y}{\partial y} & \gamma_{xz} = \left(\psi_x + \dfrac{\partial w}{\partial x}\right) \\ \varepsilon_z = 0 & \gamma_{yz} = \left(\psi_y + \dfrac{\partial w}{\partial y}\right) \end{array}\right. \tag{5.3}$$

From (3.9), the Hooke's law (or Strain-Stress Equations) for the orthotropic base plate is,

$$\left|\begin{array}{l} \varepsilon_x = a_{11}\sigma_x + a_{12}\sigma_y + a_{13}\sigma_z \\ \varepsilon_y = a_{21}\sigma_x + a_{22}\sigma_y + a_{23}\sigma_z \\ \varepsilon_z = a_{31}\sigma_x + a_{32}\sigma_y + a_{33}\sigma_z \\ \gamma_{yz} = a_{44}\tau_{yz} \\ \gamma_{xz} = a_{55}\tau_{xz} \\ \gamma_{xy} = a_{66}\tau_{xy} \end{array}\right. \tag{5.4}$$

Of the above six equations, the one which is ignored, namely, the one containing the strain (ε_z) normal to the faces of the plate which are the planes at

z=±h/2. The remaining equations are then solved for σ_x, σ_y, τ_{xy}, τ_{xz} and τ_{yz} in terms of ε_x, ε_y, γ_{xy}, γ_{xz}, γ_{yz} and σ_z. Then,

Stress-Strain Equations

$$\begin{aligned}
\sigma_x &= B_{11}\varepsilon_x + B_{12}\varepsilon_y - C_1\sigma_z \\
\sigma_y &= B_{21}\varepsilon_x + B_{22}\varepsilon_y - C_2\sigma_z \\
\tau_{yz} &= B_{44}\gamma_{yz} \\
\tau_{xz} &= B_{55}\gamma_{xz} \\
\tau_{xy} &= B_{66}\gamma_{xy}
\end{aligned} \tag{5.5}$$

The Hooke's law (or Strain-Stress Equations) for isotropic upper and lower plates,

$$\begin{aligned}
\varepsilon_x &= S_{11}\sigma_x + S_{12}\sigma_y + S_{12}\sigma_z \\
\varepsilon_y &= S_{12}\sigma_x + S_{11}\sigma_y + S_{12}\sigma_z \\
\varepsilon_z &= S_{12}\sigma_x + S_{12}\sigma_y + S_{11}\sigma_z \\
\gamma_{yz} &= 2(S_{11} - S_{12})\tau_{yz} \\
\gamma_{xz} &= 2(S_{11} - S_{12})\tau_{xz} \\
\gamma_{xy} &= 2(S_{11} - S_{12})\tau_{xy}
\end{aligned} \tag{5.6}$$

where, for the isotropic material of the base plate,

$$\begin{aligned}
&S_{11} = \frac{1}{E},\ S_{12} = -\frac{\nu}{E} \\
&\textit{and}\quad 2(S_{11} - S_{12}) = G
\end{aligned} \tag{5.7}$$

Of the above six equations, the one which is ignored, namely, the one containing the strain (ε_z) normal to the upper and lower surfaces of the plate at z=±h/2. The remaining equations are then solved for σ_x, σ_y, τ_{xy}, τ_{xz} and τ_{yz} in terms of ε_x, ε_y, γ_{xy}, γ_{xz}, γ_{yz} and σ_z .

$$
\begin{aligned}
\sigma_x &= B_{11}\varepsilon_x + B_{12}\varepsilon_y - D\sigma_z \\
\sigma_y &= B_{12}\varepsilon_x + B_{11}\varepsilon_y - D\sigma_z \\
\tau_{yz} &= B_{44}\gamma_{yz} \\
\tau_{xz} &= B_{44}\gamma_{xz} \\
\tau_{xy} &= B_{44}\gamma_{xy}
\end{aligned}
\tag{5.8}
$$

In the case of an orthotropic material, the coefficients B_{ij} can be expressed in terms of the engineering constants of the orthotropic material as,

$$
\begin{aligned}
&B_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}, && B_{44} = G_{23} \\
&B_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}, && B_{55} = G_{13} \\
&B_{12} = B_{21} = \nu_{21}B_{11} = \nu_{12}B_{22}, && B_{66} = G_{12}
\end{aligned}
\tag{5.9}
$$

where subscripts denote the material directions.

In the case of an isotropic material, the coefficients B_{ij} can be expressed in terms of the isotropic material engineering constants as,

$$
\begin{aligned}
&B_{11} = \frac{E}{1-\nu^2} = B_{22}, && B_{44} = B_{55} = B_{66} = G, \\
&B_{12} = \nu B_{11}
\end{aligned}
\tag{5.10}
$$

where subscripts denote the directions in which they are meant.

For the plates, the bending and twisting moments and the transverse shearing forces, all per unit length, are written in the integral form.,

$$
\begin{aligned}
\left(M_x, M_y, M_{yx}\right) &= \int_{-h/2}^{h/2} \left(\sigma_x, \sigma_y, \tau_{xy}\right) z\,dz \\
\left(Q_x, Q_y\right) &= \int_{-h/2}^{h/2} \left(\tau_{xz}, \tau_{yz}\right) dz
\end{aligned}
\tag{5.11}
$$

From Section 3.2, it is known that relations between stress-resultants and plate displacements functions for orthotropic plates are as follows:

$$
\begin{aligned}
M_x &= \frac{h^3}{12}\left(B_{11}\frac{\partial \psi_x}{\partial x} + B_{12}\frac{\partial \psi_y}{\partial y} \right) \\
M_y &= \frac{h^3}{12}\left(B_{12}\frac{\partial \psi_x}{\partial x} + B_{22}\frac{\partial \psi_y}{\partial y} \right) \\
M_{yx} &= \frac{h^3}{12} B_{66}\left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x} \right) \\
Q_x &= \kappa_x^2 h B_{55}\left(\psi_x + \frac{\partial w}{\partial x} \right) \\
Q_y &= \kappa_y^2 h B_{44}\left(\psi_y + \frac{\partial w}{\partial y} \right)
\end{aligned}
\qquad (5.12.a)
$$

where B's are defined as in (5.9).

Also, in terms of the ''Orthotropic Plate Stiffnesses'',

$$
\begin{aligned}
M_x &= D_{11}\frac{\partial \psi_x}{\partial x} + D_{12}\frac{\partial \psi_y}{\partial y} \\
M_y &= D_{12}\frac{\partial \psi_x}{\partial x} + D_{22}\frac{\partial \psi_y}{\partial y} \\
M_{yx} &= D_{66}\left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x} \right) \\
Q_x &= A_{55}\left(\psi_x + \frac{\partial w}{\partial x} \right) \\
Q_y &= A_{44}\left(\psi_y + \frac{\partial w}{\partial y} \right)
\end{aligned}
\qquad (5.12.b)
$$

where the "Bending Stiffness D's" and the "Shear Stiffness A's" are given as,

$$
\begin{aligned}
D_{ik} &= \frac{h^3 B_{ik}}{12} (i,k = 1,2) \\
D_{66} &= \frac{h^3 B_{66}}{12} \\
A_{44} &= \kappa_y^2 h B_{44} \\
A_{55} &= \kappa_x^2 h B_{44}
\end{aligned}
\qquad (5.12.c)
$$

Under similar assumptions in Section 3.2; (a) the weighted average of σ_z is ignored. τ_{xz} and τ_{yz} are written with a correction factor κ^2 of the Mindlin Plate Theory, the following relation between stress-resultants and plate displacements functions for isotpropic plate are obtained as,

$$
\begin{aligned}
M_x &= \frac{h^3}{12}\left(B_{11}\frac{\partial \psi_x}{\partial x} + B_{12}\frac{\partial \psi_y}{\partial y} \right) \\
M_y &= \frac{h^3}{12}\left(B_{12}\frac{\partial \psi_x}{\partial x} + B_{11}\frac{\partial \psi_y}{\partial y} \right) \\
M_{yx} &= \frac{h^3}{12} B_{44}\left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x} \right) \\
Q_x &= \kappa^2 h B_{44}\left(\psi_x + \frac{\partial w}{\partial x} \right) \\
Q_y &= \kappa^2 h B_{44}\left(\psi_y + \frac{\partial w}{\partial y} \right)
\end{aligned}
\qquad (5.13.a)
$$

where B's are given in (5.10).

The Stress Resultants in terms of "Isotropic Plate Stiffnesses",

$$
\begin{aligned}
M_x &= D_{11}\frac{\partial \psi_x}{\partial x} + D_{12}\frac{\partial \psi_y}{\partial y} \\
M_y &= D_{12}\frac{\partial \psi_x}{\partial x} + D_{22}\frac{\partial \psi_y}{\partial y} \\
M_{yx} &= D_{44}\left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x} \right) \\
Q_x &= A_{44}\left(\psi_x + \frac{\partial w}{\partial x} \right) \\
Q_y &= A_{44}\left(\psi_y + \frac{\partial w}{\partial y} \right)
\end{aligned}
\qquad (5.13.b)
$$

where the "Bending Stiffness D's" and the "Shear Stiffness A's " of the isotropic plate are given as,

$$
\left|\begin{array}{l}
D_{ik}=\dfrac{h^3 B_{ik}}{12}\,(i,k=1,2) \\
D_{44}=\dfrac{h^3 B_{44}}{12} \\
A_{44}=\kappa^2 h B_{44}
\end{array}\right. \qquad (5.13.c)
$$

where B's are given in (5.6).

From Section 3.2, it is known that the plate stress resultants-displacement equations or the governing equations of the "Mindlin Plate Theory" for both orthotropic and isotropic plates,

$$
\left|\begin{array}{l}
\dfrac{\partial M_x}{\partial x}+\dfrac{\partial M_{yx}}{\partial y}-Q_x+\dfrac{h}{2}\left(q_{zx}^{(+)}+q_{zx}^{(-)}\right)=\dfrac{\rho h^3}{12}\dfrac{\partial^2 \psi_x}{\partial t^2} \\
\dfrac{\partial M_{yx}}{\partial x}+\dfrac{\partial M_y}{\partial y}-Q_y+\dfrac{h}{2}\left(q_{zy}^{(+)}+q_{zy}^{(-)}\right)=\dfrac{\rho h^3}{12}\dfrac{\partial^2 \psi_y}{\partial t^2} \\
\dfrac{\partial Q_x}{\partial x}+\dfrac{\partial Q_y}{\partial y}+\left(q_z^{(+)}-q_z^{(-)}\right)=\rho h\dfrac{\partial^2 w}{\partial t^2}
\end{array}\right. \qquad (5.14)
$$

where $q_{zx}^{(+)}, q_{zy}^{(+)}, q_z^{(+)}$ and $q_{zx}^{(-)}, q_{zy}^{(-)}, q_z^{(-)}$ represent the surface stresses or surface loads at z=+h/2 and z=-h/2, respectively.

The ''Mindlin Plate Theory'' requires prescription of three ''Boundary Conditions'' along each edge. Hence,

The "Boundary Conditions" are,

(F) (free)	$M_{nt}=M_n=Q_n=0$	
(S) (simply supported)	$w=\psi_t=M_n=0$	(5.15)
(C) (clamped)	$w=\psi_n=\psi_t=0$	

where n and t are normal and tangential coordinates along the edges.

5.2 Main Assumptions and Analytical Modeling

Figure 3.2 shows the general configuration of the "Centrally Doubly Stiffened, Composite Plate System which has Isotropic Base Plate with Orthotropic

Stiffening Strips (Special Case of “Main PROBLEM I”)” under consideration and Figure 3.3 presents its longitudinal cross-section. However, in this special case, the stiffening strips are made of orthotropic materials but the base plate is made of an isotropic material(for instance, Aluminum Alloy in this study).

The composite plate or panel system of the “Main PROBLEM I.b” is composed of three rectangular plates (isotropic base plate and orthotropic stiffening plate strips) connected through relatively thin elastic adhesive layers.

The formulation for the present case is to be based on the same basic assumptions through one to eight used in Section 4.1 for the previous more general problem.

5.3 Theoretical Formulation of “Main PROBLEM I.b” (Theoretical Analysis)

5.3.1 Analysis of Part I (or the “Overlap Region”) of Composite Plate System

In this section, a suitable mathematical model will be set up as in the preceding Chapter in order to obtain mode shapes and natural frequencies of “Centrally Doubly Stiffened Composite Plate or Panel System (with Isotropic Base Plate and Orthotropic Stiffening Plate Strips)”. The system is composed of three rectangular plates (isotropic base plate and orthotropic stiffening strips) of equal width connected through the relatively thin adhesive layers.

The adhesive stresses should be related to the unknown displacement functions and angles of rotation of the base plate and the stiffening plate strips in the “Overlap Region”. They can be considered as surface loads or external stresses acting on the upper and lower surfaces of the plates as shown in Figure 4.2.

As in section (4.2.1), normal (σ_z) and tangential or shear (τ_{xz}, τ_{yz}) stresses at the interfaces can be written as,

Between Plates 1 and 2 (Adhesive Stresses),

$$\left|\begin{array}{l} \tau_{xz}^{(j)} = G_{a2}\gamma_{xz}^{(j)} = -\dfrac{G_{a2}}{2h_{a2}}\left(h_1\psi_x^{(1)} + h_2\psi_x^{(2)}\right) + \underline{\underline{G_{a2}\dfrac{\partial w^{(j)}}{\partial x}}} \\ \tau_{yz}^{(j)} = G_{a2}\gamma_{yz}^{(j)} = -\dfrac{G_{a2}}{2h_{a2}}\left(h_1\psi_y^{(1)} + h_2\psi_y^{(2)}\right) + \underline{\underline{G_{a2}\dfrac{\partial w^{(j)}}{\partial y}}} \quad \text{(j=1,2)} \\ \sigma_z^{(j)} = E_{a2}\varepsilon_z^{(j)} = \dfrac{E_{a2}}{h_{a2}}\left(w^{(1)} - w^{(2)}\right) \end{array}\right. \qquad (5.16.a)$$

Between Plates 1 and 3 (Adhesive Stresses),

$$\left|\begin{array}{l} \tau_{xz}^{(j)} = G_{a3}\gamma_{xz}^{(j)} = -\dfrac{G_{a3}}{2h_{a3}}\left(h_1\psi_x^{(1)} + h_3\psi_x^{(3)}\right) + \underline{\underline{G_{a3}\dfrac{\partial w^{(j)}}{\partial x}}} \\ \tau_{yz}^{(j)} = G_{a3}\gamma_{yz}^{(j)} = -\dfrac{G_{a3}}{2h_{a3}}\left(h_1\psi_y^{(1)} + h_3\psi_y^{(3)}\right) + \underline{\underline{G_{a3}\dfrac{\partial w^{(j)}}{\partial y}}} \quad \text{(j=1,3)} \\ \sigma_z^{(j)} = E_{a3}\varepsilon_z^{(j)} = \dfrac{E_{a3}}{h_{a3}}\left(w^{(3)} - w^{(1)}\right) \end{array}\right. \qquad (5.16.b)$$

where E_{a2} and E_{a3} are the modulus of elasticity (or Young's modulus) and G_{a2} and G_{a3} are the modulus of rigidity (or shear modulus) of the upper and lower adhesive layers, respectively

The adhesive normal and tangential stresses at the interfaces may be thought as the ''compatibility or coupling conditions'' of the three plates in the "Overlap Region" or Part I region in Figure 3.3.

One can obtain the governing system of coupled ordinary differential equations in the matrix or "state vector" form for the "Overlap Region" or Part I region by following similar steps to those of Section 4.2.2. During this process, it should be noted that $"B_{11}^{(1)}"$, $"h_1"$, $"\rho_1"$ and "a" are also chosen as main or reference magnitudes or parameters and all other parameters are again nondimensionalized with respect to these quantities.

The dimensionless coordinates or independent variables are,

$$\eta = x/a, \qquad \xi_I = y_I/l_I, \quad \xi_{II} = y_{II}/l_{II} \tag{5.17}$$

The dimensionless parameters related to orthotropic elastic constants of the plates and adhesive layer are,

$$\begin{aligned}
&\overline{B}_{ik}^{(j)} = B_{ik}^{(j)} \Big/ B_{11}^{(1)} \qquad (j = 1,2,3 \; and \; i,k = 1,2,3), \\
&\overline{B}_{\ell\ell}^{(j)} = B_{\ell\ell}^{(j)} \Big/ B_{11}^{(1)} \qquad (j = 1,2,3 \; and \; \ell = 4,5,6) \\
&\overline{G}_{a2} = G_{a2} \Big/ B_{11}^{(1)}, \overline{G}_{a3} = G_{a3} \Big/ B_{11}^{(1)} \\
&\overline{E}_{a2} = E_{a2} \Big/ B_{11}^{(1)}, \overline{E}_{a3} = E_{a3} \Big/ B_{11}^{(1)}
\end{aligned} \tag{5.18}$$

The dimensionless parameters related to the densities and to the geometry of the plates and adhesive layer are,

$$\begin{aligned}
&\overline{\rho}_2 = \rho_2 \Big/ \rho_1, \overline{\rho}_3 = \rho_3 \Big/ \rho_1 \\
&\overline{h}_{a2} = h_{a2} \Big/ h_1, \overline{h}_{a3} = h_{a3} \Big/ h_1 \\
&\overline{L}_I = \ell_I \Big/ a, \overline{L}_{II} = \ell_{II} \Big/ a, \overline{L}_{III} = \ell_{III} \Big/ a \\
&\overline{h}_2 = h_2 \Big/ h_1, \overline{h}_3 = h_3 \Big/ h_1
\end{aligned} \tag{5.19}$$

The dimensionless frequency parameter of the entire three-layer, centrally doubly stiffened, composite plate system which has isotropic base plate and orthotropic stiffening plate strips in “Main PROBLEM I.b”,

$$\begin{aligned}
&\overline{\omega}_{mn} = \rho_1 a^4 \omega_{mn}^2 \Big/ h_1^2 B_{11}^{(1)} \\
&\overline{\Omega} = \overline{\omega}_{mn}
\end{aligned} \qquad (m,n=1,2,3...) \tag{5.20}$$

where the dimensionless natural frequency parameter $\overline{\Omega} = \overline{\omega}_{mn}$ is ordered in terms of its magnitudes as $\overline{\Omega}_1 < \overline{\Omega}_2 < \overline{\Omega}_3 < ....$ with the subscript indicating the first, second, third,..... dimensionless natural frequencies depending on the given m, n values.

Finally, the "Governing System of First Order Ordinary Differential Equations" in the matrix or "state vector" form for the "Overlap Region" or Part I region can be written as,

$$\frac{d}{d\xi_I}\begin{Bmatrix} \overline{\boldsymbol{Y}}_{mn}^{(2)} \\ \overline{\boldsymbol{Y}}_{mn}^{(1)} \\ \overline{\boldsymbol{Y}}_{mn}^{(3)} \end{Bmatrix} = \begin{bmatrix} \overline{\mathsf{C}}_{1,1}' & \overline{\mathsf{C}}_{1,2}' & \overline{\mathsf{C}}_{1,3}' \\ \overline{\mathsf{C}}_{2,1}' & \overline{\mathsf{C}}_{2,2}' & \overline{\mathsf{C}}_{2,3}' \\ \overline{\mathsf{C}}_{3,1}' & \overline{\mathsf{C}}_{3,2}' & \overline{\mathsf{C}}_{3,3}' \end{bmatrix} \begin{Bmatrix} \overline{\boldsymbol{Y}}_{mn}^{(2)} \\ \overline{\boldsymbol{Y}}_{mn}^{(1)} \\ \overline{\boldsymbol{Y}}_{mn}^{(3)} \end{Bmatrix}, \qquad (0 < \xi_I < 1) \tag{5.21}$$

with the ''Arbitrary Boundary Conditions'' at ξ_I =0,1 for the stiffening plate strips and the ''Continuity Conditions'' at ξ_I =0,1 for the isotropic base plate.

where ξ_I is defined as y_I/l_I , superscripts show the particular plate layer, the "sub-matrices [$\overline{\mathsf{C}}_{i,j}'$]" are partitioned square matrices of dimension (6x6) which explicitly include the nondimensional geometric and material characteristics of the isotropic base plate and of the orthotropic stiffening strips and the adhesive layers and the dimensionless natural frequency parameter $\overline{\omega}_{mn}$ of the composite system. The "Column Matrix $\{\overline{\boldsymbol{Y}}_{\boldsymbol{m}}^{(j)}\}$" (j=1,2,3) is the "state vector" corresponding to the state variables or the "dimensionless fundamental dependent variables" of the stiffening strips and the base plate, defined as before,

$$\left\{\overline{\boldsymbol{Y}}_{\boldsymbol{mn}}^{(j)}\right\} = \left\{\overline{\Psi}_{mnx}^{(j)}, \overline{\Psi}_{mny}^{(j)}, \overline{W}_{mn}^{(j)}; \overline{M}_{mnyx}^{(j)}, \overline{M}_{mny}^{(j)}, \overline{Q}_{mny}^{(j)}\right\}^T, \ (j=1,2,3) \tag{5.22}$$

where the elements of $\left\{\overline{\boldsymbol{Y}}_{\boldsymbol{mn}}^{(j)}\right\}$ are the transverse displacement, angles of rotation and stress resultants which appear at ξ_I=constant segments or cross-sections of the plate system in the "Overlap Region" or Part I region as shown in Figure 3.3.

Similarly, one can write the governing system of equations in the matrix or "state vector" form for the Part II region (see Figure 3.3).

5.3.2 Analysis of Part II of Composite Plate System

$$\frac{d}{d\xi_{II}}\left\{\overline{Y}_{mn}^{(1)}\right\}=\left[\overline{D}'\right]\left\{\overline{Y}_{mn}^{(1)}\right\}, \qquad (0<\xi_{II}<1) \qquad (5.23)$$

with the ''arbitrary boundary conditions'' at ξ_{II} =0 and the ''continuity conditions'' at ξ_{II} =1 for the isotropic base plate.

where ξ_{II} is defined as y_{II}/l_{II} and $[\overline{D}']$ is a coefficient matrix of dimension (6x6) which explicitly includes dimensionless geometric and material characteristics of the isotropic base plate in the Part II region as well as the dimensionless natural frequencies $\overline{\omega}_{mn}$ of the entire composite plate system. The column matrix or the "state vector" as $\left\{\overline{Y}_{mn}^{(1)}\right\}$ is defined as in (5.22) by taking j=1 only, that is,

$$\left\{\overline{Y}_{mn}^{(1)}\right\}=\left\{\overline{\Psi}_{mnx}^{(1)},\overline{\Psi}_{mny}^{(1)},\overline{W}_{mn}^{(1)};\overline{M}_{mnyx}^{(1)},\overline{M}_{mny}^{(1)},\overline{Q}_{mny}^{(1)}\right\}^{T}, \ (j=1) \qquad (5.24)$$

5.3.3 Analysis of Part III of Composite Plate System

Similarly, one can write the governing system of equations in the matrix or "state vector" form for Part III region shown in Figure 3.3 as,

$$\frac{d}{d\xi_{III}}\left\{\overline{Y}_{mn}^{(1)}\right\}=\left[\overline{E}'\right]\left\{\overline{Y}_{mn}^{(1)}\right\}, \qquad (0<\xi_{III}<1) \qquad (5.25)$$

with the ''arbitrary boundary conditions'' at ξ_{III} =1 and the ''continuity conditions'' at ξ_{III} =0 for the isotropic base plate.

where ξ_{III} is defined as y_{III}/l_{III} and $\left[\overline{E}'\right]$ is a coefficient matrix of dimension (6x6) which explicitly includes dimensionless geometric and material characteristics of the

isotropic base plate in the Part III region as well as the dimensionless natural frequencies $\overline{\omega}_{mn}$ of the entire composite plate system. The column matrix or the "state vector" as $\left\{\overline{Y}_{mn}^{(1)}\right\}$ is defined as in (5.24).

5.3.4 System of Governing Differential Equations ("Main PROBLEM I.b")

In the previous sections, the governing system of coupled ordinary differential equations in the matrix or state vector form is obtained in (5.21) for the overlap or Part I region and in (5.23) for Part II region and in (5.25) for Part III region. Then, one can write these equations, in open form, as follows,

For Part I region or "Overlap Region" (or Three-Layer Composite Plate Region),

$$\frac{d}{d\xi_I}\begin{Bmatrix}\overline{\psi}^{(2)}_{mnx}\\ \overline{\psi}^{(2)}_{mny}\\ \overline{W}^{(2)}_{mn}\\ \overline{M}^{(2)}_{mnyx}\\ \overline{M}^{(2)}_{mny}\\ \overline{Q}^{(2)}_{mny}\\ \overline{\psi}^{(1)}_{mnx}\\ \overline{\psi}^{(1)}_{mny}\\ \overline{W}^{(1)}_{mn}\\ \overline{M}^{(1)}_{mnyx}\\ \overline{M}^{(1)}_{mny}\\ \overline{Q}^{(1)}_{mny}\\ \overline{\psi}^{(3)}_{mnx}\\ \overline{\psi}^{(3)}_{mny}\\ \overline{W}^{(3)}_{mn}\\ \overline{M}^{(3)}_{mnyx}\\ \overline{M}^{(3)}_{mny}\\ \overline{Q}^{(3)}_{mny}\end{Bmatrix}=\begin{bmatrix}
0 & c^{(2)}_{1,2} & 0 & c^{(2)}_{1,4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
c^{(2)}_{2,1} & 0 & 0 & 0 & c^{(2)}_{2,5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & c^{(2)}_{3,2} & 0 & 0 & 0 & c^{(2)}_{3,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
c^{(2)}_{4,1} & 0 & c^{(2)}_{4,3} & 0 & c^{(2)}_{4,5} & 0 & r^{(2)}_{1,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & c^{(2)}_{5,2} & 0 & c^{(2)}_{5,4} & 0 & c^{(2)}_{5,6} & 0 & r^{(2)}_{2,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
c^{(2)}_{6,1} & 0 & c^{(2)}_{6,3} & 0 & 0 & 0 & 0 & 0 & r^{(2)}_{3,3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{'(1)}_{1,2} & 0 & c^{'(1)}_{1,4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & c^{'(1)}_{2,1} & 0 & 0 & 0 & c^{'(1)}_{2,5} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{'(1)}_{3,2} & 0 & 0 & 0 & c^{'(1)}_{3,6} & 0 & 0 & 0 & 0 & 0 & 0\\
r^{(2)}_{1,1} & 0 & 0 & 0 & 0 & 0 & c^{'(1)}_{4,1} & 0 & c^{'(1)}_{4,3} & 0 & c^{'(1)}_{4,5} & 0 & r^{(3)}_{1,1} & 0 & 0 & 0 & 0 & 0\\
0 & r^{(2)}_{2,2} & 0 & 0 & 0 & 0 & 0 & c^{'(1)}_{5,2} & 0 & c^{'(1)}_{5,4} & 0 & c^{'(1)}_{5,6} & 0 & r^{(3)}_{2,2} & 0 & 0 & 0 & 0\\
0 & 0 & r^{(2)}_{3,3} & 0 & 0 & 0 & c^{'(1)}_{6,1} & 0 & c^{'(1)}_{6,3} & 0 & 0 & 0 & 0 & 0 & r^{(3)}_{3,3} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{1,2} & 0 & c^{(3)}_{1,4} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{2,1} & 0 & 0 & 0 & c^{(3)}_{2,5} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{3,2} & 0 & 0 & 0 & c^{(3)}_{3,6}\\
0 & 0 & 0 & 0 & 0 & 0 & r^{(3)}_{1,1} & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{4,1} & 0 & c^{(3)}_{4,3} & 0 & c^{(3)}_{4,5} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & r^{(3)}_{2,2} & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{5,2} & 0 & c^{(3)}_{5,4} & 0 & c^{(3)}_{5,6}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & r^{(3)}_{3,3} & 0 & 0 & 0 & c^{(3)}_{6,1} & 0 & c^{(3)}_{6,3} & 0 & 0 & 0
\end{bmatrix}\begin{Bmatrix}\overline{\psi}^{(2)}_{mnx}\\ \overline{\psi}^{(2)}_{mny}\\ \overline{W}^{(2)}_{mn}\\ \overline{M}^{(2)}_{mnyx}\\ \overline{M}^{(2)}_{mny}\\ \overline{Q}^{(2)}_{mny}\\ \overline{\psi}^{(1)}_{mnx}\\ \overline{\psi}^{(1)}_{mny}\\ \overline{W}^{(1)}_{mn}\\ \overline{M}^{(1)}_{mnyx}\\ \overline{M}^{(1)}_{mny}\\ \overline{Q}^{(1)}_{mny}\\ \overline{\psi}^{(3)}_{mnx}\\ \overline{\psi}^{(3)}_{mny}\\ \overline{W}^{(3)}_{mn}\\ \overline{M}^{(3)}_{mnyx}\\ \overline{M}^{(3)}_{mny}\\ \overline{Q}^{(3)}_{mny}\end{Bmatrix}$$

$$(0<\xi_I<1) \qquad (5.26.a)$$

where the "Coefficient Sub-Matrices" $[\Lambda]^{(j)}$ and $[\alpha]^{(k)}$ are,

$$[\Lambda]^{(j)}=\begin{bmatrix}0 & c^{(j)}_{1,2} & 0 & c^{(j)}_{1,4} & 0 & 0\\ c^{(j)}_{2,1} & 0 & 0 & 0 & c^{(j)}_{2,5} & 0\\ 0 & c^{(j)}_{3,2} & 0 & 0 & 0 & c^{(j)}_{3,6}\\ c^{(j)}_{4,1} & 0 & c^{(j)}_{4,3} & 0 & c^{(j)}_{4,5} & 0\\ 0 & c^{(j)}_{5,2} & 0 & c^{(j)}_{5,4} & 0 & c^{(j)}_{5,6}\\ c^{(j)}_{6,1} & 0 & c^{(j)}_{6,3} & 0 & 0 & 0\end{bmatrix} \qquad (j=1,2,3) \qquad (5.26.b)$$

$$[\alpha]^{(k)}=\begin{bmatrix}r^{(k)}_{1,1} & 0 & 0\\ 0 & r^{(k)}_{2,2} & 0\\ 0 & 0 & r^{(k)}_{3,3}\end{bmatrix} \qquad (k=2,3) \qquad (5.26.c)$$

The elements of the ‘‘Coefficient Sub-Matrix $[\Lambda]^{(j)}$’’ are,

$$c_{1,2}^{(j)} = -\overline{L}_1 m\pi \left(\frac{h_1}{a}\right)^2 \qquad c_{1,4}^{(j)} = \frac{12\overline{L}_1}{\overline{B}_{66}^{(j)} \overline{h}_j^3}$$

$$c_{2,1}^{(j)} = \overline{L}_1 m\pi \frac{\overline{B}_{12}^{(j)}}{\overline{B}_{22}^{(j)}} \left(\frac{a}{h_1}\right)^2 \qquad c_{2,5}^{(j)} = \frac{12\overline{L}_1}{\overline{B}_{22}^{(j)} \overline{h}_j^{(3)}} \left(\frac{a}{h_1}\right)^2$$

$$c_{3,2}^{(j)} = -\overline{L}_1 \qquad c_{3,6}^{(j)} = \frac{\overline{L}_1}{\kappa_y^2 \overline{B}_{44}^{(j)} \overline{h}_j} \left(\frac{a}{h_1}\right)$$

$$c_{4,1}^{(j)} = -\frac{\overline{L}_1}{12} \overline{\rho}_j \overline{h}_j^3 \left(\frac{h_1}{a}\right)^2 \overline{\Omega} + \overline{L}_1 \kappa_x^2 \overline{B}_{55}^{(j)} \overline{h}_j \left(\frac{a}{h_1}\right)^2 + \frac{\overline{L}_1}{12} \left(\frac{am\pi}{h_1}\right)^2 \overline{h}_j^3 \left(\overline{B}_{11}^{(j)} - \frac{\overline{B}_{12}^{(j)^2}}{\overline{B}_{22}^{(j)}}\right) + \frac{\overline{G}_{a\,j} \overline{L}_1 \overline{h}_j^2}{4\overline{h}_{a\,j}} \left(\frac{a}{h_1}\right)^2$$

$$c_{4,3}^{(j)} = \overline{h}_j \kappa_x^2 \overline{L}_1 \overline{B}_{55}^{(j)} m\pi - \underline{\underline{\overline{L}_1 \frac{m\pi}{2} \overline{h}_j \overline{G}_{aj}}} \qquad c_{4,5}^{(j)} = -\overline{L}_1 m\pi \frac{\overline{B}_{12}^{(j)}}{\overline{B}_{22}^{(j)}}$$

$$c_{5,2}^{(j)} = -\frac{\overline{L}_1}{12} \overline{\rho}_j \overline{h}_j^3 \left(\frac{h_1}{a}\right)^4 \overline{\Omega} + \frac{\overline{G}_{a\,j} \overline{L}_1 \overline{h}_j^2}{4\overline{h}_{a\,j}} + \underline{\underline{\frac{\overline{G}_{aj} \overline{L}_1 \overline{h}_j}{2h_1}}} \qquad c_{5,4}^{(j)} = \overline{L}_1 m\pi$$

$$c_{5,6}^{(j)} = \overline{L}_I \left(\frac{a}{h_1}\right) - \underline{\underline{\overline{L}_1 \left(\frac{a}{h_1}\right) \frac{\overline{G}_{aj}}{2\kappa_y^2 \overline{B}_{44}^{(j)}}}}$$

$$c_{6,1}^{(j)} = \overline{h}_j \kappa_x^2 \overline{L}_1 \overline{B}_{55}^{(j)} \left(\frac{a}{h_1}\right) m\pi$$

$$c_{6,3}^{(j)} = -\overline{L}_1 \overline{\rho}_j \overline{h}_j \left(\frac{h_1}{a}\right)^3 \overline{\Omega} + \overline{L}_1 \kappa_x^2 \overline{h}_j \left(\frac{h_1}{a}\right) \left(\frac{am\pi}{h_1}\right)^2 \overline{B}_{55}^{(j)} + \frac{\overline{L}_1 \overline{E}_{a\,j}}{\overline{h}_{a\,j}} \left(\frac{a}{h_1}\right)$$

(j=2,3) (5.26.d)

where (j=2) indicates the “Upper Orthotropic Stiffening Plate Strip” and (j=3) defines the “Lower Orthotropic Stiffening Plate Strip”. Thus,

The elements of the ‘‘Coefficient Sub-Matrix $[\Lambda]^{(1)}$,’’ for the Isotropic Base Plate are shown below,

$$c_{1,2}^{'(1)} = -\bar{L}_1 m\pi \left(\frac{h_1}{a}\right)^2 \qquad c_{1,4}^{'(1)} = \frac{12\bar{L}_1}{\bar{B}_{44}^{(1)}}$$

$$c_{2,1}^{'(1)} = \bar{L}_1 m\pi \bar{B}_{1,2}^{(1)} \left(\frac{a}{h_1}\right)^2 \qquad c_{2,5}^{'(1)} = 12\bar{L}_1 \left(\frac{a}{h_1}\right)^2$$

$$c_{3,2}^{'(1)} = -\bar{L}_1 \qquad c_{3,6}^{'(1)} = \frac{\bar{L}_1}{\kappa^2 \bar{B}_{44}^{(1)}} \left(\frac{a}{h_1}\right)$$

$$c_{4,1}^{'(1)} = -\frac{\bar{L}_1}{12}\left(\frac{h_1}{a}\right)^2 \bar{\Omega} + \bar{L}_1 \kappa^2 \bar{B}_{44}^{(1)} \left(\frac{a}{h_1}\right)^2 + \frac{\bar{L}_1}{12}\left(\frac{am\pi}{h_1}\right)^2 \left(1 - \bar{B}_{12}^{(1)^2}\right) + \left(\frac{\bar{G}_{a2}}{\bar{h}_{a2}} + \frac{\bar{G}_{a3}}{\bar{h}_{a3}}\right)\frac{\bar{L}_1}{4}\left(\frac{a}{h_1}\right)^2$$

$$c_{4,3}^{'(1)} = \kappa^2 \bar{L}_1 \bar{B}_{44}^{(1)} m\pi - \underline{\underline{\bar{L}_1 \frac{m\pi}{2}\left(\bar{G}_{a2} + \bar{G}_{a3}\right)}} \qquad c_{4,5}^{'(1)} = -\bar{L}_1 m\pi \bar{B}_{1,2}^{(1)}$$

$$c_{5,2}^{'(1)} = -\frac{\bar{L}_1}{12}\left(\frac{h_1}{a}\right)^4 \bar{\Omega} + \left(\frac{\bar{G}_{a2}}{\bar{h}_{a2}} + \frac{\bar{G}_{a3}}{\bar{h}_{a3}}\right)\frac{\bar{L}_1}{4} + \underline{\underline{\left(\bar{G}_{a2} + \bar{G}_{a3}\right)\frac{\bar{L}_1}{2}}} \qquad c_{5,4}^{'(1)} = \bar{L}_I m\pi$$

$$c_{5,6}^{'(1)} = \bar{L}_I \left(\frac{a}{h_1}\right) - \underline{\underline{\bar{L}_1 \left(\frac{a}{h_1}\right)\frac{1}{2\kappa^2 \bar{B}_{44}^{(1)}}\left(\bar{G}_{a2} + \bar{G}_{a3}\right)}} \qquad c_{6,1}^{'(1)} = \kappa^2 \bar{L}_1 \bar{B}_{44}^{(1)} \left(\frac{a}{h_1}\right) m\pi$$

$$c_{6,3}^{'(1)} = -\bar{L}_1 \left(\frac{h_1}{a}\right)^3 \bar{\Omega} + \bar{L}_1 \kappa^2 \left(\frac{h_1}{a}\right)\left(\frac{am\pi}{h_1}\right)^2 \bar{B}_{44}^{(1)} + \left(\frac{\bar{E}_{a2}}{\bar{h}_{a2}} + \frac{\bar{E}_{a3}}{\bar{h}_{a3}}\right)\bar{L}_1 \left(\frac{a}{h_1}\right)$$

(5.26.e)

where the double underlined (=) terms in above equations come from $\partial w^{(j)}/\partial x$ and $\partial w^{(j)}/\partial y$ terms in the adhesive layer equations.

$$r_{1,1}^{(k)} = \frac{\bar{G}_{ak}\bar{L}_1\bar{h}_k}{4\bar{h}_{ak}}\left(\frac{a}{h_1}\right)^2 \qquad r_{2,2}^{(k)} = \frac{\bar{G}_{ak}\bar{L}_1\bar{h}_k}{4\bar{h}_{ak}} \qquad r_{3,3}^{(k)} = -\frac{\bar{L}_1\bar{E}_{ak}}{\bar{h}_{ak}}\left(\frac{a}{h_1}\right)$$

(k=2,3)

(5.26.f)

For Part II region (or Single Layer Isotropic Base Plate Region),

$$\frac{d}{d\xi_{II}}\begin{Bmatrix}\overline{\psi}^{(1)}_{mnx}\\ \overline{\psi}^{(1)}_{mny}\\ \overline{W}^{(1)}_{mn}\\ \overline{M}^{(1)}_{mnyx}\\ \overline{M}^{(1)}_{mny}\\ \overline{Q}^{(1)}_{mny}\end{Bmatrix}=\begin{bmatrix}0 & d'_{1,2} & 0 & d'_{1,4} & 0 & 0\\ d'_{2,1} & 0 & 0 & 0 & d'_{2,5} & 0\\ 0 & d'_{3,2} & 0 & 0 & 0 & d'_{3,6}\\ d'_{4,1} & 0 & d'_{4,3} & 0 & d'_{4,5} & 0\\ 0 & d'_{5,2} & 0 & d'_{5,4} & 0 & d'_{5,6}\\ d'_{6,1} & 0 & d'_{6,3} & 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}\overline{\psi}^{(1)}_{mnx}\\ \overline{\psi}^{(1)}_{mny}\\ \overline{W}^{(1)}_{mn}\\ \overline{M}^{(1)}_{mnyx}\\ \overline{M}^{(1)}_{mny}\\ \overline{Q}^{(1)}_{mny}\end{Bmatrix} \qquad (0<\xi_{II}<1) \quad (5.27.a)$$

where the elements of the above ‘‘Coefficient Matrix $\left[\overline{D'}\right]$’’ are,

$$d'_{1,2}=-\overline{L}_2 m\pi\left(\frac{h_1}{a}\right)^2 \qquad d'_{1,4}=\frac{12\overline{L}_2}{\overline{B}^{(1)}_{44}}$$

$$d'_{2,1}=\overline{L}_2 m\pi\overline{B}^{(1)}_{12}\left(\frac{a}{h_1}\right)^2 \qquad d'_{2,5}=12\overline{L}_2\left(\frac{a}{h_1}\right)^2$$

$$d'_{3,2}=-\overline{L}_2 \qquad d'_{3,6}=\frac{\overline{L}_2}{\kappa^2\overline{B}^{(1)}_{44}}\left(\frac{a}{h_1}\right)$$

$$d'_{4,1}=-\frac{\overline{L}_2}{12}\left(\frac{h_1}{a}\right)^2\overline{\Omega}+\overline{L}_2\kappa^2\overline{B}^{(1)}_{44}\left(\frac{a}{h_1}\right)^2+\frac{\overline{L}_2}{12}\left(\frac{am\pi}{h_1}\right)^2\left(1-\overline{B}^{(1)^2}_{1,2}\right)$$

$$d'_{4,3}=\kappa^2\overline{L}_2\overline{B}^{(1)}_{55}m\pi \qquad d'_{4,5}=-\overline{L}_2 m\pi\overline{B}^{(1)}_{12}$$

$$d'_{5,2}=-\frac{\overline{L}_2}{12}\left(\frac{h_1}{a}\right)^4\overline{\Omega} \qquad d'_{5,4}=\overline{L}_2 m\pi \qquad d'_{5,6}=\overline{L}_2\left(\frac{a}{h_1}\right)$$

$$d'_{6,1}=\kappa^2\overline{L}_2\overline{B}^{(1)}_{55}\left(\frac{a}{h_1}\right)m\pi \qquad d'_{6,3}=-\overline{L}_2\left(\frac{h_1}{a}\right)^3\overline{\Omega}+\overline{L}_2\kappa^2\left(\frac{h_1}{a}\right)\left(\frac{am\pi}{h_1}\right)^2\overline{B}^{(1)}_{44}$$

(5.27.b)

and similarly,

For Part III region (or Single Layer Isotropic Base Plate Region),

$$\frac{d}{d\xi_{III}}\begin{Bmatrix}\overline{\psi}^{(1)}_{mnx}\\ \overline{\psi}^{(1)}_{mny}\\ \overline{W}^{(1)}_{mn}\\ \overline{M}^{(1)}_{mnyx}\\ \overline{M}^{(1)}_{mny}\\ \overline{Q}^{(1)}_{mny}\end{Bmatrix}=\begin{bmatrix}0 & e'_{1,2} & 0 & e'_{1,4} & 0 & 0\\ e'_{2,1} & 0 & 0 & 0 & e'_{2,5} & 0\\ 0 & e'_{3,2} & 0 & 0 & 0 & e'_{3,6}\\ e'_{4,1} & 0 & e'_{4,3} & 0 & e'_{4,5} & 0\\ 0 & e'_{5,2} & 0 & e'_{5,4} & 0 & e'_{5,6}\\ e'_{6,1} & 0 & e'_{6,3} & 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}\overline{\psi}^{(1)}_{mnx}\\ \overline{\psi}^{(1)}_{mny}\\ \overline{W}^{(1)}_{mn}\\ \overline{M}^{(1)}_{mnyx}\\ \overline{M}^{(1)}_{mny}\\ \overline{Q}^{(1)}_{mny}\end{Bmatrix}, \qquad (0<\xi_{III}<1) \qquad (5.28.a)$$

where the elements of the above ‘‘Coefficient Matrix $\left[\overline{E'}\right]$’’ are,

$$e'_{1,2}=-\overline{L}_3 m\pi\left(\frac{h_1}{a}\right)^2 \qquad e'_{1,4}=\frac{12\overline{L}_3}{\overline{B}^{(1)}_{44}}$$

$$e'_{2,1}=\overline{L}_3 m\pi\overline{B}^{(1)}_{12}\left(\frac{a}{h_1}\right)^2 \qquad e'_{2,5}=12\overline{L}_3\left(\frac{a}{h_1}\right)^2$$

$$e'_{3,2}=-\overline{L}_3 \qquad e'_{3,6}=\frac{\overline{L}_3}{\kappa^2\overline{B}^{(1)}_{44}}\left(\frac{a}{h_1}\right)$$

$$e'_{4,1}=-\frac{\overline{L}_3}{12}\left(\frac{h_1}{a}\right)^2\overline{\Omega}+\overline{L}_3\kappa^2\overline{B}^{(1)}_{44}\left(\frac{a}{h_1}\right)^2+\frac{\overline{L}_3}{12}\left(\frac{am\pi}{h_1}\right)^2\left(1-\overline{B}^{(1)^2}_{1,2}\right)$$

$$e'_{4,3}=\kappa^2\overline{L}_3\overline{B}^{(1)}_{55}m\pi \qquad e'_{4,5}=-\overline{L}_3 m\pi\frac{\overline{B}^{(1)}_{12}}{\overline{B}^{(1)}_{22}}$$

$$e'_{5,2}=-\frac{\overline{L}_3}{12}\left(\frac{h_1}{a}\right)^4\overline{\Omega} \qquad e'_{5,4}=\overline{L}_3 m\pi \qquad e'_{5,6}=\overline{L}_3\left(\frac{a}{h_1}\right)$$

$$e'_{6,1}=\kappa^2\overline{L}_3\overline{B}^{(1)}_{55}\left(\frac{a}{h_1}\right)m\pi$$

$$e'_{6,3}=-\overline{L}_3\left(\frac{h_1}{a}\right)^3\overline{\Omega}+\overline{L}_3\kappa^2\left(\frac{h_1}{a}\right)\left(\frac{am\pi}{h_1}\right)^2\overline{B}^{(1)}_{44}$$

(5.28.b)

It should be noted here that the “primed” quantities belong to the isotropic base plate.

It will be recalled that the “Boundary Conditions” at x=0 and x=a are already satisfied by trigonometric expansions in the “Classical Lévy’s Type Solution”. Then, the aim is it to determine $\overline{\Psi}_{mnx}^{(j)}$,$\overline{\Psi}_{mny}^{(j)}$,$\overline{W}_{mn}^{(j)}$;$\overline{M}_{mnyx}^{(j)}$,$\overline{M}_{mny}^{(j)}$,$\overline{Q}_{mny}^{(j)}$ in such a form that they satisfy the ‘‘Boundary Conditions’’ in the y-direction at the sides ξ_I=0, ξ_I=1 for the stiffening plate strips, at ξ_{II}=0, ξ_{III}=1 for the base plate and also the ‘‘continuity conditions’’ at ξ_{II}=1 (ξ_I=0) and ξ_{III}=0 (ξ_I=1) for the base plate.

The ‘‘Boundary Conditions’’ along the edges in the y-direction,

F (Free): $\overline{M}_{yx}^{(j)} = \overline{M}_{y}^{(j)} = \overline{Q}_{y}^{(j)} = 0$

C (Clamped): $\overline{w}^{(j)} = \overline{\psi}_{x}^{(j)} = \overline{\psi}_{y}^{(j)} = 0$ (j=1,2,3) (5.29)

S (Simply Supported): $\overline{w}^{(j)} = \overline{\psi}_{x}^{(j)} = \overline{M}_{y}^{(j)} = 0$

The ‘‘Continuity Condition’’ between Part I and Part II is (see also Fig. 3.3),

$$\left\{\overline{\boldsymbol{Y}}_{\xi_{II}=1}^{(1)}\right\} = \left\{\overline{\boldsymbol{Y}}_{\xi_{I}=0}^{(1)}\right\} \tag{5.30}$$

The ‘‘Continuity Condition’’ between Part I and Part III is (see also Fig. 3.3),

$$\left\{\overline{\boldsymbol{Y}}_{\xi_{III}=0}^{(1)}\right\} = \left\{\overline{\boldsymbol{Y}}_{\xi_{I}=1}^{(1)}\right\} \tag{5.31}$$

Finally, one can summerize the enitre set of the “Governing System of First Order Ordinary Differential Equations” as,

$$\frac{d}{d\xi_I}\begin{Bmatrix}\overline{Y}_{mn}^{(2)}\\ \overline{Y}_{mn}^{(1)}\\ \overline{Y}_{mn}^{(3)}\end{Bmatrix}=\begin{bmatrix}\overline{C}_{1,1}' & \overline{C}_{1,2}' & \overline{C}_{1,3}'\\ \overline{C}_{2,1}' & \overline{C}_{2,2}' & \overline{C}_{2,3}'\\ \overline{C}_{3,1}' & \overline{C}_{3,2}' & \overline{C}_{3,3}'\end{bmatrix}\begin{Bmatrix}\overline{Y}_{mn}^{(2)}\\ \overline{Y}_{mn}^{(1)}\\ \overline{Y}_{mn}^{(3)}\end{Bmatrix}, \quad (0<\xi_I<1) \quad \text{(in Part I)}$$

$$\frac{d}{d\xi_{II}}\left\{\overline{Y}_{mn}^{(1)}\right\}=\left[\overline{D}'\right]\left\{\overline{Y}_{mn}^{(1)}\right\}, \quad (0<\xi_{II}<1) \quad \text{(in Part II)} \qquad (5.32\ a,b,c)$$

$$\frac{d}{d\xi_{III}}\left\{\overline{Y}_{mn}^{(1)}\right\}=\left[\overline{E}'\right]\left\{\overline{Y}_{mn}^{(1)}\right\}, \quad (0<\xi_{III}<1) \quad \text{(in Part III)}$$

with the “Appropriate Boundary Conditions” and the “Continuity Conditions” for the above differential equations.

As mentioned in the previous section, the above set of equations creates a “Two-Point Boundary Value Problem” for the “Main PROBLEM I.b”. The above system can be integrated numerically if the natural frequencies are obtained or known for a particular geometry and materials and the boundary conditions. In Chapter 10, the “Method of Solution” and the way the natural frequencies are computed will be explained in detail.

CHAPTER 6

(“Main PROBLEM II.a’’) __FREE FLEXURAL (or BENDING) VIBRATIONS of NON-CENTRALLY DOUBLY STIFFENED, COMPOSITE ORTHOTROPIC PLATES or PANELS (Orthotropic Base Plate or Panel Case)

6.1 Main Assumptions and Analytical Modeling

The general configuration of the “Non-Centrally Doubly Stiffened, Composite Plate or Panel System (with Orthotropic Base Plate and Orthotropic Stiffening Plate Strips)” is shown in Figure 3.4. The longitudinal cross-section of the system in the y-direction is given in Figure 3.5. The coordinate systems and the orthotropic plate material directions are again indicated in Figure 3.4.

The theoretical formulation for the present case is once more based on the same basic assumptions through one to eight and the “Mindlin Plate Theory” explained in Section 4.1 for the previous ‘‘Main PROBLEM I.a’’ in Chapter 4.

6.2 Theoretical Formulation of “Main PROBLEM II.a” (Theoretical Analysis)

6.2.1 Analysis of Part I (or the “Overlap Region”) of Composite Plate System

After following the same steps in Sections (4.2.1), (4.2.2), (4.2.3) and (4.2.4), one can obtain the ‘‘Governing System of First Order Ordinary Differential

Equations”in terms of the ‘‘dimensionless fundamental dependent variables’’ for the ‘‘Overlap Region’’ or Part I region in Figure 3.5 as follows,

$$\frac{d\overline{\psi}_{mnx}^{(j)}}{d\xi_I} = \frac{12\overline{L}_1}{\overline{B}_{66}^{(j)}\overline{h}_j^{\,3}}\overline{M}_{mnyx}^{(j)} - \overline{L}_1 m\pi\left(\frac{h_1}{a}\right)^2\overline{\psi}_{mny}^{(j)}$$

$$\frac{d\overline{\psi}_{mny}^{(j)}}{d\xi_I} = \frac{12\overline{L}_1}{\overline{B}_{22}^{(j)}\overline{h}_j^{\,3}}\left(\frac{a}{h_1}\right)^2\overline{M}_{mny}^{(j)} + \overline{L}_1 m\pi\frac{\overline{B}_{12}^{(j)}}{\overline{B}_{22}^{(j)}}\left(\frac{a}{h_1}\right)^2\overline{\psi}_{mnx}^{(j)}$$

$$\frac{d\overline{W}_{mn}^{(j)}}{d\xi_I} = \frac{\overline{L}_1}{\kappa_y^2\overline{B}_{44}^{(j)}\overline{h}_j}\left(\frac{a}{h_1}\right)\overline{Q}_{mny}^{(j)} - \overline{L}_1\overline{\psi}_{mny}^{(j)}$$

$$\frac{d\overline{M}_{mnyx}^{(j)}}{d\xi_I} = \left\{\begin{array}{l} \left\{\begin{array}{l} -\dfrac{\overline{L}_1}{12}\overline{\rho}_j\overline{h}_j^{\,3}\left(\dfrac{h_1}{a}\right)^2\overline{\Omega} + \overline{L}_1\kappa_x^2\overline{B}_{55}^{(j)}\overline{h}_j\left(\dfrac{a}{h_1}\right)^2 \\ +\dfrac{\overline{L}_1}{12}\left(\dfrac{am\pi}{h_1}\right)^2\overline{h}_j^{\,3}\left(\overline{B}_{11}^{(j)} - \dfrac{\overline{B}_{12}^{(j)^2}}{\overline{B}_{22}^{(j)}}\right) \end{array}\right\}\psi_{mnx}^{(j)} \\ +\left\{\overline{h}_j\kappa_x^2\overline{L}_1\overline{B}_{55}^{(j)}m\pi - \underline{\underline{\overline{L}_1\dfrac{m\pi}{2}\overline{h}_j\overline{G}_{a2}}} - \underline{\underline{\overline{L}_1\dfrac{m\pi}{2}\overline{h}_j\overline{G}_{a3}}}\right\}\overline{W}_{mn}^{(j)} \\ -\left(\dfrac{\overline{B}_{12}^{(j)}}{\overline{B}_{22}^{(j)}}\overline{L}_1 m\pi\right)\overline{M}_{mny}^{(j)} + \dfrac{\overline{G}_{a2}\overline{L}_1\overline{h}_j}{4\overline{h}_{a2}}\left(\dfrac{a}{h_1}\right)^2\left(\overline{\psi}_{mnx}^{(1)} + \overline{h}_2\overline{\psi}_{mnx}^{(2)}\right) \\ +\dfrac{\overline{G}_{a3}\overline{L}_1\overline{h}_j}{4\overline{h}_{a3}}\left(\dfrac{a}{h_1}\right)^2\left(\overline{\psi}_{mnx}^{(1)} + \overline{h}_3\overline{\psi}_{mnx}^{(3)}\right) \end{array}\right\}$$

$$\frac{d\overline{M}_{mny}^{(j)}}{d\xi_I} = \left\{\begin{array}{l} \left\{-\dfrac{\overline{L}_1}{12}\overline{\rho}_j\overline{h}_j^{\,3}\left(\dfrac{h_1}{a}\right)^4\overline{\Omega} + \underline{\underline{\dfrac{\overline{G}_{a2}\overline{L}_1\overline{h}_j}{2}}} + \underline{\underline{\dfrac{\overline{G}_{a3}\overline{L}_1\overline{h}_j}{2}}}\right\}\overline{\psi}_{mny}^{(j)} \\ +\left(\dfrac{\overline{L}_1 a}{h_1} - \underline{\underline{\overline{L}_1\left(\dfrac{a}{h_1}\right)\dfrac{\overline{G}_{a2}}{2\kappa^2\overline{B}_{44}^{(j)}}}} - \underline{\underline{\overline{L}_1\left(\dfrac{a}{h_1}\right)\dfrac{\overline{G}_{a3}}{2\kappa_y^2\overline{B}_{44}^{(j)}}}}\right)\overline{Q}_{mny}^{(j)} \\ +\left(\overline{L}_1 m\pi\right)\overline{M}_{mnyx}^{(j)} + \dfrac{\overline{G}_{a2}\overline{L}_1\overline{h}_j}{4\overline{h}_{a2}}\left(\overline{\psi}_{mny}^{(1)} + \overline{h}_2\overline{\psi}_{mny}^{(2)}\right) + \dfrac{\overline{G}_{a3}\overline{L}_1\overline{h}_j}{4\overline{h}_{a3}}\left(\overline{\psi}_{mny}^{(1)} + \overline{h}_3\overline{\psi}_{mny}^{(3)}\right) \end{array}\right\}$$

where the double underlined (=) terms come from $\partial w^{(j)}/\partial x$ and $\partial w^{(j)}/\partial y$ terms in the adhesive layer equations.

$$\frac{d\overline{Q}_{mny}^{(j)}}{d\xi_I}=\left\{\begin{array}{l}\left\{-\overline{L}_1\overline{\rho}_j\overline{h}_j\left(\frac{h_1}{a}\right)^3\overline{\Omega}+\overline{L}_1\kappa_x^2\overline{h}_j\left(\frac{h_1}{a}\right)\left(\frac{am\pi}{h_1}\right)^2\overline{B}_{55}^{(j)}\right\}\overline{W}_{mn}^{(j)}\\ +\left\{\overline{L}_1\kappa_x^2B_{55}^{(j)}\overline{h}_j\left(\frac{a}{h_1}\right)m\pi\right\}\psi_{mnx}^{(j)}\\ +(-1)^{(j+1)}\frac{\overline{L}_1\overline{E}_{a2}}{\overline{h}_{a2}}\left(\frac{a}{h_1}\right)\left\{\overline{W}_{mn}^{(1)}-\overline{W}_{mn}^{(2)}\right\}-(-1)^{(j+1)}\frac{\overline{L}_1\overline{E}_{a3}}{\overline{h}_{a3}}\left(\frac{a}{h_1}\right)\left\{\overline{W}_{mn}^{(1)}-\overline{W}_{mn}^{(3)}\right\}\end{array}\right\}$$

where $\frac{\overline{G}_{a2}}{\overline{h}_{a2}}=0$ ($\overline{G}_{a2}=0$) for j=3 , $\frac{\overline{G}_{a3}}{\overline{h}_{a3}}=0$ ($\overline{G}_{a3}=0$) for j=2

and $\frac{\overline{E}_{a2}}{\overline{h}_{a2}}=0$ for j=3 , $\frac{\overline{E}_{a3}}{\overline{h}_{a3}}=0$ for j=2 , $\overline{E}_{a3}=-1\times\overline{E}_{a3}$ for j=1

$(0< \xi_I <l_I),(j=1,2,3)$

(6.1)

The Equations (6.1) shows that the entire of partial differential equations are reduced to a set ‘‘Governing System of First Order Ordinary Differential Equations’’ in ξ_I or y_I direction and this creates a ‘‘Two-Point Boundary Value Problem’’ in the ‘‘Overlap Region’’ or Part I region. Similarly, one can obtain the ‘‘Governing System of Equations’’ for Part II and Part III regions. The ‘‘Governing System of First Order Ordinary Differential Equations’’ in the matrix or ‘‘state vector’’ form is given below.

For Part I region ‘’Overlap Region’’) (or Three-Layer Composite Plate Region),

$$\frac{d}{d\xi_I}\left\{\begin{array}{c}\overline{Y}_{mn}^{(2)}\\ \overline{Y}_{mn}^{(1)}\\ \overline{Y}_{mn}^{(3)}\end{array}\right\}=\left[\begin{array}{ccc}\overline{C}_{1,1} & \overline{C}_{1,2} & \overline{C}_{1,3}\\ \overline{C}_{2,1} & \overline{C}_{2,2} & \overline{C}_{2,3}\\ \overline{C}_{3,1} & \overline{C}_{3,2} & \overline{C}_{3,3}\end{array}\right]\left\{\begin{array}{c}\overline{Y}_{mn}^{(2)}\\ \overline{Y}_{mn}^{(1)}\\ \overline{Y}_{mn}^{(3)}\end{array}\right\},(0<\xi_I<1) \qquad (6.2)$$

with the ‘‘Arbitrary Boundary Conditions’’ at ξ_I =0,1 for the orthotropic stiffening plate strips and the ‘‘Continuity Conditions’’ at ξ_I =0,1 for the orthotropic base plate.

where ξ_I is defined as $y_I / l_I^{'}$, the superscripts show the related plate layer, submatrices $[\overline{C}_{i,j}]$ are partitioned square matrices of dimension (6x6) which explicitly include the nondimensional geometric and material characteristics of the base plate and stiffening strips and the adhesive layer and dimensionless natural frequencies parameter $\overline{\omega}_{mn}$ of the entire composite system. The "Column Matrix $\{\overline{Y}_{mn}^{(j)}\}$" (j=1,2,3) is the ''state vector'' corresponding to the state variables or the ''dimensionless fundamental dependent variables'', defined as,

$$\{\overline{Y}_{mn}^{(j)}\} = \{\overline{\Psi}_{mnx}^{(j)}, \overline{\Psi}_{mny}^{(j)}, \overline{W}_{mn}^{(j)}; \overline{M}_{mnyx}^{(j)}, \overline{M}_{mny}^{(j)}, \overline{Q}_{mny}^{(j)}\}^T, \quad (j=1,2,3) \tag{6.3}$$

where the elements of $\{\overline{Y}_{mn}^{(j)}\}$ are the quantities related to the displacements, angles of rotation and the stress resultants which appear at ξ_I=constant segments or cross-sections of the plate system in the ''Overlap Region'' or Part I region as shown in Figure 3.5.

6.2.2 Analysis of Part II of Composite Plate System

One can obtain from Equations (6.1), the Orthotropic Base Plate Equations for Part II region. Then,

For Part II (or Single Layer Orthotropic Base Plate Region),

$$\frac{d}{d\xi_{II}}\{\overline{Y}_{mn}^{(1)}\} = [\overline{D}]\{\overline{Y}_{mn}^{(1)}\}, \qquad (0< \xi_{II} <1) \tag{6.4}$$

with the ''arbitrary boundary conditions'' at ξ_{II} =0 and the ''continuity conditions'' at ξ_{II} =1 for the orthotropic base plate.

where ξ_{II} is defined as $y_{II} / l_{II}^{'}$ and $[\overline{D}]$ is a coefficient matrix of dimension (6x6) which explicitly includes dimensionless geometric and material characteristics of the base plate in the Part II region as well as the dimensionless natural frequencies $\overline{\omega}_{mn}$

of the entire composite plate system. The column matrix or the ''state vector'' $\{\overline{Y}_{mn}^{(1)}\}$ is defined as in (6.3) by taking j=1 only, that is,

$$\{\overline{Y}_{mn}^{(1)}\} = \{\overline{\Psi}_{mnx}^{(1)}, \overline{\Psi}_{mny}^{(1)}, \overline{W}_{mn}^{(1)}; \overline{M}_{mnyx}^{(1)}, \overline{M}_{mny}^{(1)}, \overline{Q}_{mny}^{(1)}\}^T, \quad (j=1) \tag{6.5}$$

6.2.3 Analysis of Part III of Composite Plate System

For Part III (or Single Layer Orthotropic Base Plate Region),

$$\frac{d}{d\xi_{III}}\{\overline{Y}_{mn}^{(1)}\} = [\overline{E}]\{\overline{Y}_{mn}^{(1)}\}, \qquad (0< \xi_{III} <1) \tag{6.6}$$

with the ''Arbitrary Boundary Conditions'' at ξ_{III} =1 and the ''Continuity Conditions'' at ξ_{III} =0 for the base orthotropic plate.

where ξ_{III} is defined as y_{III}/l'_{III} and $[\overline{E}]$ is a coefficient matrix of dimension (6x6) which explicitly includes dimensionless geometric and material characteristics of the base plate in the Part III region as well as the dimensionless natural frequencies $\overline{\omega}_{mn}$ of the entire composite plate system. The column matrix or the state-vector $\{\overline{Y}_{mn}^{(1)}\}$ is defined as in (6.5).

6.2.4 System of Governing Differential Equations (''Main PROBLEM II.a'')

In the previous section, the sets of the ''Governing System of First Order Ordinary Differential Equations'' in the ''state vector'' form for the ''Overlap Region'' or Part I region, for Part II region and for Part III region are obtained. Then, rewriting these equations, in open form, as follows,

For Part I region or “Overlap Region” (or Three-Layer Composite Plate Region),

$$\frac{d}{d\xi_I}\begin{Bmatrix}\overline{\psi}^{(2)}_{mnx}\\ \overline{\psi}^{(2)}_{mny}\\ \overline{W}^{(2)}_{mn}\\ \overline{M}^{(2)}_{mnyx}\\ \overline{M}^{(2)}_{mny}\\ \overline{Q}^{(2)}_{mny}\\ \overline{\psi}^{(1)}_{mnx}\\ \overline{\psi}^{(1)}_{mny}\\ \overline{W}^{(1)}_{mn}\\ \overline{M}^{(1)}_{mnyx}\\ \overline{M}^{(1)}_{mny}\\ \overline{Q}^{(1)}_{mny}\\ \overline{\psi}^{(3)}_{mnx}\\ \overline{\psi}^{(3)}_{mny}\\ \overline{W}^{(3)}_{mn}\\ \overline{M}^{(3)}_{mnyx}\\ \overline{M}^{(3)}_{mny}\\ \overline{Q}^{(3)}_{mny}\end{Bmatrix}=\begin{bmatrix}
0 & c^{(2)}_{1,2} & 0 & c^{(2)}_{1,4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
c^{(2)}_{2,1} & 0 & 0 & 0 & c^{(2)}_{2,5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & c^{(2)}_{3,2} & 0 & 0 & 0 & c^{(2)}_{3,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
c^{(2)}_{4,1} & 0 & c^{(2)}_{4,3} & 0 & c^{(2)}_{4,5} & 0 & r^{(2)}_{1,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & c^{(2)}_{5,2} & 0 & c^{(2)}_{5,4} & 0 & c^{(2)}_{5,6} & 0 & r^{(2)}_{2,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
c^{(2)}_{6,1} & 0 & c^{(2)}_{6,3} & 0 & 0 & 0 & 0 & 0 & r^{(2)}_{3,3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{(1)}_{1,2} & 0 & c^{(1)}_{1,4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & c^{(1)}_{2,1} & 0 & 0 & 0 & c^{(1)}_{2,5} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{(1)}_{3,2} & 0 & 0 & 0 & c^{(1)}_{3,6} & 0 & 0 & 0 & 0 & 0 & 0\\
r^{(2)}_{1,1} & 0 & 0 & 0 & 0 & 0 & c^{(1)}_{4,1} & 0 & c^{(1)}_{4,3} & 0 & c^{(1)}_{4,5} & 0 & r^{(3)}_{1,1} & 0 & 0 & 0 & 0 & 0\\
0 & r^{(2)}_{2,2} & 0 & 0 & 0 & 0 & 0 & c^{(1)}_{5,2} & 0 & c^{(1)}_{5,4} & 0 & c^{(1)}_{5,6} & 0 & r^{(3)}_{2,2} & 0 & 0 & 0 & 0\\
0 & 0 & r^{(2)}_{3,3} & 0 & 0 & 0 & c^{(1)}_{6,1} & 0 & c^{(1)}_{6,3} & 0 & 0 & 0 & 0 & 0 & r^{(3)}_{3,3} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{1,2} & 0 & c^{(3)}_{1,4} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{2,1} & 0 & 0 & 0 & c^{(3)}_{2,5} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{3,2} & 0 & 0 & 0 & c^{(3)}_{3,6}\\
0 & 0 & 0 & 0 & 0 & 0 & r^{(3)}_{1,1} & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{4,1} & 0 & c^{(3)}_{4,3} & 0 & c^{(3)}_{4,5} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & r^{(3)}_{2,2} & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{5,2} & 0 & c^{(3)}_{5,4} & 0 & c^{(3)}_{5,6}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & r^{(3)}_{3,3} & 0 & 0 & 0 & c^{(3)}_{6,1} & 0 & c^{(3)}_{6,3} & 0 & 0 & 0
\end{bmatrix}\begin{Bmatrix}\overline{\psi}^{(2)}_{mnx}\\ \overline{\psi}^{(2)}_{mny}\\ \overline{W}^{(2)}_{mn}\\ \overline{M}^{(2)}_{mnyx}\\ \overline{M}^{(2)}_{mny}\\ \overline{Q}^{(2)}_{mny}\\ \overline{\psi}^{(1)}_{mnx}\\ \overline{\psi}^{(1)}_{mny}\\ \overline{W}^{(1)}_{mn}\\ \overline{M}^{(1)}_{mnyx}\\ \overline{M}^{(1)}_{mny}\\ \overline{Q}^{(1)}_{mny}\\ \overline{\psi}^{(3)}_{mnx}\\ \overline{\psi}^{(3)}_{mny}\\ \overline{W}^{(3)}_{mn}\\ \overline{M}^{(3)}_{mnyx}\\ \overline{M}^{(3)}_{mny}\\ \overline{Q}^{(3)}_{mny}\end{Bmatrix}$$

$$(0<\xi_I<1) \qquad (6.7)$$

where the elements of the ‘‘Coefficient Matrix $\left[\overline{C}\right]$’’, in detailed form, are the same with those of Equations (4.27) in Section (4.2.5).

For Part II (or Single Layer Orthotropic Base Plate Region),

$$\frac{d}{d\xi_{II}}\begin{Bmatrix}\overline{\psi}^{(1)}_{mnx}\\ \overline{\psi}^{(1)}_{mny}\\ \overline{W}^{(1)}_{mn}\\ \overline{M}^{(1)}_{mnyx}\\ \overline{M}^{(1)}_{mny}\\ \overline{Q}^{(1)}_{mny}\end{Bmatrix}=\begin{bmatrix}
0 & d_{1,2} & 0 & d_{1,4} & 0 & 0\\
d_{2,1} & 0 & 0 & 0 & d_{2,5} & 0\\
0 & d_{3,2} & 0 & 0 & 0 & d_{3,6}\\
d_{4,1} & 0 & d_{4,3} & 0 & d_{4,5} & 0\\
0 & d_{5,2} & 0 & d_{5,4} & 0 & d_{5,6}\\
d_{6,1} & 0 & d_{6,3} & 0 & 0 & 0
\end{bmatrix}\begin{Bmatrix}\overline{\psi}^{(1)}_{mnx}\\ \overline{\psi}^{(1)}_{mny}\\ \overline{W}^{(1)}_{mn}\\ \overline{M}^{(1)}_{mnyx}\\ \overline{M}^{(1)}_{mny}\\ \overline{Q}^{(1)}_{mny}\end{Bmatrix}, \qquad (0<\xi_{II}<1) \qquad (6.8)$$

where $d_{i,j}$, elements of the ‘‘Coefficient Matrix $\left[\overline{D}\right]$’’ in detailed form are the same with those of Equations(4.28) in Section (4.2.5).

For Part III (or Single Layer Orthotropic Base Plate Region),

$$\frac{d}{d\xi_{III}}\begin{Bmatrix}\overline{\psi}^{(1)}_{mnx}\\ \overline{\psi}^{(1)}_{mny}\\ \overline{W}^{(1)}_{mn}\\ \overline{M}^{(1)}_{mnyx}\\ \overline{M}^{(1)}_{mny}\\ \overline{Q}^{(1)}_{mny}\end{Bmatrix}=\begin{bmatrix}0 & e_{1,2} & 0 & e_{1,4} & 0 & 0\\ e_{2,1} & 0 & 0 & 0 & e_{2,5} & 0\\ 0 & e_{3,2} & 0 & 0 & 0 & e_{3,6}\\ e_{4,1} & 0 & e_{4,3} & 0 & e_{4,5} & 0\\ 0 & e_{5,2} & 0 & e_{5,4} & 0 & e_{5,6}\\ e_{6,1} & 0 & e_{6,3} & 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}\overline{\psi}^{(1)}_{mnx}\\ \overline{\psi}^{(1)}_{mny}\\ \overline{W}^{(1)}_{mn}\\ \overline{M}^{(1)}_{mnyx}\\ \overline{M}^{(1)}_{mny}\\ \overline{Q}^{(1)}_{mny}\end{Bmatrix}, \qquad (0<\xi_{III}<1) \qquad (6.9)$$

where $e_{i,j}$, elements of the ''Coefficient Matrix $\left[\overline{E}\right]$'' in detailed form are the same with those of Equations(4.29) in Section (4.2.5).

In order to be able to solve the sets of the ''Governing System of Equations'' given by (6.7), (6.8) and (6.9), the ''Boundary Conditions'' and the ''Continuity Conditions'' are needed. Hence,

The ''Boundary Conditions'' along the edges in the y-direction,

F (Free): $\overline{M}^{(j)}_{yx}=\overline{M}^{(j)}_{y}=\overline{Q}^{(j)}_{y}=0$

C (Clamped): $\overline{w}^{(j)}=\overline{\psi}^{(j)}_{x}=\overline{\psi}^{(j)}_{y}=0$ (j=1,2,3) (6.10)

S (Simply Supported): $\overline{w}^{(j)}=\overline{\psi}^{(j)}_{x}=\overline{M}^{(j)}_{y}=0$

The ''Continuity Condition'' between Part I and Part II (see also Figure 3.5),

$$\left\{\overline{\boldsymbol{Y}}^{(1)}_{\xi_{II}=1}\right\}=\left\{\overline{\boldsymbol{Y}}^{(1)}_{\xi_{I}=0}\right\} \qquad (6.11)$$

The ''Continuity Condition'' between Part I and Part III (see also Figure 3.5),

$$\left\{\overline{\boldsymbol{Y}}^{(1)}_{\xi_{III}=0}\right\}=\left\{\overline{\boldsymbol{Y}}^{(1)}_{\xi_{I}=1}\right\} \qquad (6.12)$$

Hence, the full set of the "Governing System of First Order Ordinary Differential Equations" for the "Main PROBLEM I.a." are,

$$\frac{d}{d\xi_I}\begin{Bmatrix}\overline{Y}_{mn}^{(2)}\\ \overline{Y}_{mn}^{(1)}\\ \overline{Y}_{mn}^{(3)}\end{Bmatrix}=\begin{bmatrix}\overline{C}_{1,1} & \overline{C}_{1,2} & \overline{C}_{1,3}\\ \overline{C}_{2,1} & \overline{C}_{2,2} & \overline{C}_{2,3}\\ \overline{C}_{3,1} & \overline{C}_{3,2} & \overline{C}_{3,3}\end{bmatrix}\begin{Bmatrix}\overline{Y}_{mn}^{(2)}\\ \overline{Y}_{mn}^{(1)}\\ \overline{Y}_{mn}^{(3)}\end{Bmatrix}, \qquad (0<\xi_I<1) \qquad \text{(in Part I)}$$

$$\frac{d}{d\xi_{II}}\left\{\overline{Y}_{mn}^{(1)}\right\}=\left[\overline{D}\right]\left\{\overline{Y}_{mn}^{(1)}\right\}, \qquad (0<\xi_{II}<1) \quad \text{(in Part II)} \qquad \text{(6.13 a,b,c)}$$

$$\frac{d}{d\xi_{III}}\left\{\overline{Y}_{mn}^{(1)}\right\}=\left[\overline{E}\right]\left\{\overline{Y}_{mn}^{(1)}\right\}, \qquad (0<\xi_{III}<1) \quad \text{(in Part III)}$$

with the "Appropriate Boundary Conditions" and the "Continuity Conditions" for the above system.

Here, needless to say that the above system constitutes a "Two-Point Boundary Value Problem" for the "Main PROBLEM II.a". The "Method of Solution" is similar to that of the "Main PROBLEM I". In the above equations, once the natural frequencies are obtained, then, a numerical integration can be performed for a given particular geometry, materials and the support conditions.

CHAPTER 7

(‘‘Main PROBLEM II.b’’)__ FREE FLEXURAL (or BENDING) VIBRATIONS of NON-CENTRALLY DOUBLY STIFFENED, COMPOSITE PLATES or PANELS (Isotropic Base Plate or Panel Case)

7.1 Main Assumptions and Analytical Modeling

In Figure 3.4, the general configuration and geometric and material characteristics of the “Non-Centrally Doubly Stiffened, Composite Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)’’are presented in detail. The longitudinal cross-section of the entire composite system is given in Figure 3.5. The material directions are also given in Figure 3.4.

In the above case, the theoretical analysis is again based on the main assumptions and the analytical (or mathematical) modeling developed for the preceding cases (or problems) and, in particular, the ‘‘Main PROBLEM I.b’’ in Chapter 4.

7.2 Theoretical Formulation of "Main PROBLEM II.b" (Theoretical Analysis)

7.2.1 Analysis of Part I (or the ''Overlap Region'') of Composite Plate System

The ''Overlap Region'' or Part I is composed of an "Isotropic Base Plate reinforced by the bonded "Upper and Lower Orthtropic Stiffening Plate Strips". In other words, Part I, as in previous cases, is a three-layer composite system in which the "Mindlin Plate Theory" elements are connected by relatively thin, yet elastic, deformable adhesive layers.

After going through the same steps as was done in Sections (5.3.1), one can obtain the ''Governing System of First Order Ordinary Differential Equations'' in the simpler matrix or ''state vector'' form for the ''Overlap Region'' or Part I region.

For Part I region (or ''Overlap Region'') (or Three-Layer Composite Plate Region),

$$\frac{d}{d\xi_I}\begin{Bmatrix}\overline{\boldsymbol{Y}}_{mn}^{(2)}\\ \overline{\boldsymbol{Y}}_{mn}^{(1)}\\ \overline{\boldsymbol{Y}}_{mn}^{(3)}\end{Bmatrix}=\begin{bmatrix}\overline{\mathsf{C}}_{1,1}^{\,\prime} & \overline{\mathsf{C}}_{1,2}^{\,\prime} & \overline{\mathsf{C}}_{1,3}^{\,\prime}\\ \overline{\mathsf{C}}_{2,1}^{\,\prime} & \overline{\mathsf{C}}_{2,2}^{\,\prime} & \overline{\mathsf{C}}_{2,3}^{\,\prime}\\ \overline{\mathsf{C}}_{3,1}^{\,\prime} & \overline{\mathsf{C}}_{3,2}^{\,\prime} & \overline{\mathsf{C}}_{3,3}^{\,\prime}\end{bmatrix}\begin{Bmatrix}\overline{\boldsymbol{Y}}_{mn}^{(2)}\\ \overline{\boldsymbol{Y}}_{mn}^{(1)}\\ \overline{\boldsymbol{Y}}_{mn}^{(3)}\end{Bmatrix},\quad (0<\xi_I<1) \tag{7.1}$$

with the ''Arbitrary Boundary Conditions'' at ξ_I =0,1 for the orthotropic stiffening plate strips and the ''Continuity Conditions'' at ξ_I =0,1 for the isotropic base plate.

where ξ_I is defined as $y_I/l_I^{'}$, and the superscripts show the particular plate layer, submatrices $[\overline{\mathsf{C}}_{i,j}^{\,\prime}]$ are partitioned square matrices of dimension (6x6) which explicitly include the nondimensional geometric and material characteristics of the isotropic base plate and the orthotropic stiffening plate strips, the adhesive layer and the dimensionless natural frequency parameter $\overline{\omega}_{mn}$ of the entire composite system.

The ‘‘state vector’’ $\{\overline{Y}_{m}^{(j)}\}$ (j=1,2,3) includes the state variables or the ‘‘dimensionless fundamental dependent variables’’ of the present ‘‘Main PROBLEM II.b’’.Then,

$$\left\{\overline{Y}_{mn}^{(j)}\right\}=\left\{\overline{\Psi}_{mnx}^{(j)},\overline{\Psi}_{mny}^{(j)},\overline{W}_{mn}^{(j)};\overline{M}_{mnyx}^{(j)},\overline{M}_{mny}^{(j)},\overline{Q}_{mny}^{(j)}\right\}^{T}, \quad \text{(j=1,2,3)} \tag{7.2}$$

where the elements of $\left\{\overline{Y}_{mn}^{(j)}\right\}$ are the quantities related to the displacements, angles of rotation and the stress resultants which appear at ξ_I =constant segments or cross-sections of the plate system.

7.2.2 Analysis of Part II of Composite Plate System

For Part II (or Single Layer Isotropic Base Plate Region),

$$\frac{d}{d\xi_{II}}\left\{\overline{Y}_{mn}^{(1)}\right\}=\left[\overline{D'}\right]\left\{\overline{Y}_{mn}^{(1)}\right\}, \qquad (0<\xi_{II}<1) \tag{7.3}$$

with the ‘‘Arbitrary Boundary Conditions’’ at ξ_{II} =0 and the ‘‘Continuity Conditions’’ at ξ_{II} =1 for the isotropic base plate.

where ξ_{II} is defined as y_{II}/l'_{II} and $[\overline{D'}]$ is a coefficient matrix of dimension (6x6) which explicitly includes dimensionless geometric and material characteristics of the isotropic base plate in the Part II region as well as the dimensionless natural frequencies $\overline{\omega}_{mn}$ of the system. The column matrix or the ‘‘state vector’’ $\left\{\overline{Y}_{mn}^{(1)}\right\}$ is defined as in (7.2) by taking j=1 only, that is,

$$\left\{\overline{Y}_{mn}^{(1)}\right\}=\left\{\overline{\Psi}_{mnx}^{(1)},\overline{\Psi}_{mny}^{(1)},\overline{W}_{mn}^{(1)};\overline{M}_{mnyx}^{(1)},\overline{M}_{mny}^{(1)},\overline{Q}_{mny}^{(1)}\right\}^{T}, \text{ (j=1)} \tag{7.4}$$

7.2.3 Analysis of Part III of Composite Plate System

For Part III (or Single Layer Isotropic Base Plate Region),

$$\frac{d}{d\xi_{III}}\left\{\bar{Y}_{mn}^{(1)}\right\}=\left[\bar{E}'\right]\left\{\bar{Y}_{mn}^{(1)}\right\}, \qquad (0<\xi_{III}<1) \qquad (7.5)$$

with the ‘‘Arbitrary Boundary Conditions’’ at ξ_{III} =1 and the ‘‘Continuity Conditions’’ at ξ_{III} =0 for the isotropic base plate.

where ξ_{III} is defined as y_{III}/l'_{III} and $\left[\bar{E}'\right]$ is a coefficient matrix of dimension (6x6) which explicitly includes dimensionless geometric and material characteristics of the isotropic base plate in the Part II region as well as the dimensionless natural frequencies $\bar{\omega}_{mn}$ of the system. The column matrix or the ‘‘state vector’’ $\left\{\bar{Y}_{mn}^{(1)}\right\}$ is defined as in (7.4).

7.2.4 System of Governing Differential Equations (‘‘Main PROBLEM II.b’’)

In the previous sections, the sets of the ‘‘Governing System of Ordinary Differential Equations’’ in the matrix or ‘‘state vector’’ form are presented in (7.1) for the ‘‘Overlap Region’’ or Part I region, in (7.3) for Part II region and in (7.5) for Part III region, respectively. Then, one can rewrite these equations in open form,

For Part I region or “Overlap Region” (or Three-Layer Composite Plate Region),

$$\frac{d}{d\xi_I}\begin{Bmatrix}\overline{\psi}^{(2)}_{mnx}\\ \overline{\psi}^{(2)}_{mny}\\ \overline{W}^{(2)}_{mn}\\ \overline{M}^{(2)}_{mnyx}\\ \overline{M}^{(2)}_{mny}\\ \overline{Q}^{(2)}_{mny}\\ \overline{\psi}^{(1)}_{mnx}\\ \overline{\psi}^{(1)}_{mny}\\ \overline{W}^{(1)}_{mn}\\ \overline{M}^{(1)}_{mnyx}\\ \overline{M}^{(1)}_{mny}\\ \overline{Q}^{(1)}_{mny}\\ \overline{\psi}^{(3)}_{mnx}\\ \overline{\psi}^{(3)}_{mny}\\ \overline{W}^{(3)}_{mn}\\ \overline{M}^{(3)}_{mnyx}\\ \overline{M}^{(3)}_{mny}\\ \overline{Q}^{(3)}_{mny}\end{Bmatrix}=\begin{bmatrix}0 & c^{(2)}_{1,2} & 0 & c^{(2)}_{1,4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ c^{(2)}_{2,1} & 0 & 0 & 0 & c^{(2)}_{2,5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & c^{(2)}_{3,2} & 0 & 0 & 0 & c^{(2)}_{3,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ c^{(2)}_{4,1} & 0 & c^{(2)}_{4,3} & 0 & c^{(2)}_{4,5} & 0 & r^{(2)}_{1,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & c^{(2)}_{5,2} & 0 & c^{(2)}_{5,4} & 0 & c^{(2)}_{5,6} & 0 & r^{(2)}_{2,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ c^{(2)}_{6,1} & 0 & c^{(2)}_{6,3} & 0 & 0 & 0 & 0 & 0 & r^{(2)}_{3,3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{'(1)}_{1,2} & 0 & c^{'(1)}_{1,4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & c^{'(1)}_{2,1} & 0 & 0 & 0 & c^{'(1)}_{2,5} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{'(1)}_{3,2} & 0 & 0 & 0 & c^{'(1)}_{3,6} & 0 & 0 & 0 & 0 & 0 & 0\\ r^{(2)}_{1,1} & 0 & 0 & 0 & 0 & 0 & c^{'(1)}_{4,1} & 0 & c^{'(1)}_{4,3} & 0 & c^{'(1)}_{4,5} & 0 & r^{(3)}_{1,1} & 0 & 0 & 0 & 0 & 0\\ 0 & r^{(2)}_{2,2} & 0 & 0 & 0 & 0 & 0 & c^{'(1)}_{5,2} & 0 & c^{'(1)}_{5,4} & 0 & c^{'(1)}_{5,6} & 0 & r^{(3)}_{2,2} & 0 & 0 & 0 & 0\\ 0 & 0 & r^{(2)}_{3,3} & 0 & 0 & 0 & c^{'(1)}_{6,1} & 0 & c^{'(1)}_{6,3} & 0 & 0 & 0 & 0 & 0 & r^{(3)}_{3,3} & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{1,2} & 0 & c^{(3)}_{1,4} & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{2,1} & 0 & 0 & 0 & c^{(3)}_{2,5} & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{3,2} & 0 & 0 & 0 & c^{(3)}_{3,6}\\ 0 & 0 & 0 & 0 & 0 & 0 & r^{(3)}_{1,1} & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{4,1} & 0 & c^{(3)}_{4,3} & 0 & c^{(3)}_{4,5} & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & r^{(3)}_{2,2} & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{5,2} & 0 & c^{(3)}_{5,4} & 0 & c^{(3)}_{5,6}\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & r^{(3)}_{3,3} & 0 & 0 & 0 & c^{(3)}_{6,1} & 0 & c^{(3)}_{6,3} & 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}\overline{\psi}^{(2)}_{mnx}\\ \overline{\psi}^{(2)}_{mny}\\ \overline{W}^{(2)}_{mn}\\ \overline{M}^{(2)}_{mnyx}\\ \overline{M}^{(2)}_{mny}\\ \overline{Q}^{(2)}_{mny}\\ \overline{\psi}^{(1)}_{mnx}\\ \overline{\psi}^{(1)}_{mny}\\ \overline{W}^{(1)}_{mn}\\ \overline{M}^{(1)}_{mnyx}\\ \overline{M}^{(1)}_{mny}\\ \overline{Q}^{(1)}_{mny}\\ \overline{\psi}^{(3)}_{mnx}\\ \overline{\psi}^{(3)}_{mny}\\ \overline{W}^{(3)}_{mn}\\ \overline{M}^{(3)}_{mnyx}\\ \overline{M}^{(3)}_{mny}\\ \overline{Q}^{(3)}_{mny}\end{Bmatrix}$$

$$(0< \xi_I <1) \qquad (7.6)$$

where the elements of the “Coefficient Matrix $\left[\overline{C'}\right]$”, in detailed form, are same with the ones in Equation (5.26) in Section (5.3.2).

For Part II (or Single Layer Isotropic Base Plate Region),

$$\frac{d}{d\xi_{II}}\begin{Bmatrix}\overline{\psi}^{(1)}_{mnx}\\ \overline{\psi}^{(1)}_{mny}\\ \overline{W}^{(1)}_{mn}\\ \overline{M}^{(1)}_{mnyx}\\ \overline{M}^{(1)}_{mny}\\ \overline{Q}^{(1)}_{mny}\end{Bmatrix}=\begin{bmatrix}0 & d'_{1,2} & 0 & d'_{1,4} & 0 & 0\\ d'_{2,1} & 0 & 0 & 0 & d'_{2,5} & 0\\ 0 & d'_{3,2} & 0 & 0 & 0 & d'_{3,6}\\ d'_{4,1} & 0 & d'_{4,3} & 0 & d'_{4,5} & 0\\ 0 & d'_{5,2} & 0 & d'_{5,4} & 0 & d'_{5,6}\\ d'_{6,1} & 0 & d'_{6,3} & 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}\overline{\psi}^{(1)}_{mnx}\\ \overline{\psi}^{(1)}_{mny}\\ \overline{W}^{(1)}_{mn}\\ \overline{M}^{(1)}_{mnyx}\\ \overline{M}^{(1)}_{mny}\\ \overline{Q}^{(1)}_{mny}\end{Bmatrix}, \quad (0< \xi_{II} <1) \qquad (7.7)$$

where the elements of the above ‘‘Coefficient Matrix $\left[\overline{D'}\right]$’’, in detailed form, are same with the ones in Equation (5.27) in Section (5.3.2).

For Part III (or Single Layer Isotropic Plate Region),

$$\frac{d}{d\xi_{III}}\begin{Bmatrix}\overline{\psi}_{mnx}^{(1)}\\ \overline{\psi}_{mny}^{(1)}\\ \overline{W}_{mn}^{(1)}\\ \overline{M}_{mnyx}^{(1)}\\ \overline{M}_{mny}^{(1)}\\ \overline{Q}_{mny}^{(1)}\end{Bmatrix}=\begin{bmatrix}0 & e'_{1,2} & 0 & e'_{1,4} & 0 & 0\\ e'_{2,1} & 0 & 0 & 0 & e'_{2,5} & 0\\ 0 & e'_{3,2} & 0 & 0 & 0 & e'_{3,6}\\ e'_{4,1} & 0 & e'_{4,3} & 0 & e'_{4,5} & 0\\ 0 & e'_{5,2} & 0 & e'_{5,4} & 0 & e'_{5,6}\\ e'_{6,1} & 0 & e'_{6,3} & 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}\overline{\psi}_{mnx}^{(1)}\\ \overline{\psi}_{mny}^{(1)}\\ \overline{W}_{mn}^{(1)}\\ \overline{M}_{mnyx}^{(1)}\\ \overline{M}_{mny}^{(1)}\\ \overline{Q}_{mny}^{(1)}\end{Bmatrix}, \quad (0< \xi_{III} <1) \qquad (7.8)$$

where the elements of the above ‘‘Coefficient Matrix $\left[\overline{E'}\right]$’’ are given in detail and they are identical with the ones in Equation (5.28) in Section (5.3.2).

In addition to the above set of Equations (7.6), (7.7) and (7.8), the appropriate “Boundary Conditions” and the appropriate ‘‘Continuity Conditions’’ of the entire composite system are needed to solve the problem. Therefore,

The ‘‘Boundary Conditions’’ prescribed along the edges in the y-direction,

F (Free): $\overline{M}_{yx}^{(j)} = \overline{M}_{y}^{(j)} = \overline{Q}_{y}^{(j)} = 0$

C (Clamped): $\overline{w}^{(j)} = \overline{\psi}_{x}^{(j)} = \overline{\psi}_{y}^{(j)} = 0$ (j=1,2,3) (7.9)

S (Simply Supported): $\overline{w}^{(j)} = \overline{\psi}_{x}^{(j)} = \overline{M}_{y}^{(j)} = 0$

The ‘‘Continuity Condition’’ between Part I and Part II is (see also Figure 3.5) ,

$$\left\{\overline{\boldsymbol{Y}}_{\xi_{II}=1}^{(1)}\right\}=\left\{\overline{\boldsymbol{Y}}_{\xi_{I}=0}^{(1)}\right\} \qquad (7.10)$$

The ‘‘Continuity Condition’’ between Part I and Part III is (see also Figure 3.5),

$$\left\{\overline{\boldsymbol{Y}}_{\xi_{III}=0}^{(1)}\right\}=\left\{\overline{\boldsymbol{Y}}_{\xi_{I}=1}^{(1)}\right\} \qquad (7.11)$$

Now, one may summarize the complete set of the “Governing System of First Order Ordinary Differential Equations” for the “Main PROBLEM II.b” as,

$$\frac{d}{d\xi_I}\begin{Bmatrix}\overline{Y}_{mn}^{(2)}\\ \overline{Y}_{mn}^{(1)}\\ \overline{Y}_{mn}^{(3)}\end{Bmatrix}=\begin{bmatrix}\overline{C}_{1,1}' & \overline{C}_{1,2}' & \overline{C}_{1,3}'\\ \overline{C}_{2,1}' & \overline{C}_{2,2}' & \overline{C}_{2,3}'\\ \overline{C}_{3,1}' & \overline{C}_{3,2}' & \overline{C}_{3,3}'\end{bmatrix}\begin{Bmatrix}\overline{Y}_{mn}^{(2)}\\ \overline{Y}_{mn}^{(1)}\\ \overline{Y}_{mn}^{(3)}\end{Bmatrix}, \quad (0<\xi_I<1) \quad \text{(in Part I)}$$

$$\frac{d}{d\xi_{II}}\left\{\overline{Y}_{mn}^{(1)}\right\}=\left[\overline{D}'\right]\left\{\overline{Y}_{mn}^{(1)}\right\}, \qquad (0<\xi_{II}<1) \quad \text{(in Part II)} \quad \text{(7.12 a,b,c)}$$

$$\frac{d}{d\xi_{III}}\left\{\overline{Y}_{mn}^{(1)}\right\}=\left[\overline{E}'\right]\left\{\overline{Y}_{mn}^{(1)}\right\}, \qquad (0<\xi_{III}<1) \quad \text{(in Part III)}$$

with the “Appropriate Boundary Conditions” and the “Continuity Conditions” for the above set of differential equations.

The above system creates a “Two-Point Boundary Value Problem” for the “Main PROBLEM II.b” considered here. The above system of equations can be numerically integrated once the natural frequencies are obtained for a particular geometry, materials and the boundary conditions. The “Method of Solution” (see also Chapter 10) in the above case is similar to the “Main PROBLEM I.b”.

CHAPTER 8

(“Main PROBLEM III.a”) (‘‘SPECIAL CASE of Main PROBLEM II.a’’)__ FREE FLEXURAL (or BENDING) VIBRATIONS of DOUBLY STIFFENED-at SUPPORT, COMPOSITE ORTHOTROPIC PLATES or PANELS (Orthotropic Base Plate or Panel Case)

8.1 Main Assumptions and Analytical Modeling

The problem under consideration here, is the “Doubly Stiffened-at Support, Composite, Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”(“Main PROBLEM III.a”). The general configuration and the geometric and material characteristics are shown in detail in Figure 3.6. This problem, in a way, is a ‘‘SPECIAL CASE of Main PROBLEM II.a’’. This can be seen by simple comparison of Figures 3.4 and 3.5 with Figures 3.6 and 3.7, respectively. The longitudinal cross-section in the y-direction of the entire composite system is presented in Figure 3.7. In the composite system, the base plate and the stiffening plate strips are dissimilar Orthotropic Mindlin plates joined by very thin, yet flexible adhesive layers. In the theoretical developments, upper and lower adhesive layers are considered to be not identical at first. But in numerical calculations, they are assumed to be, from the practical applications point of view, exactly the same.

The theoretical formulation, in the present case the ‘‘Main PROBLEM III.a’’, the basic assumptions and the analytical modeling are similar to those of the ‘‘Main PROBLEM II.a’’ in Chpater 6. Therefore, they will not be repeated here.

8.2 Theoretical Formulation of ‘‘Main PROBLEM III.a’’ (Theoretical Analysis)

In this section, the theoretical analysis will be based on the formulation of the ‘‘Main PROBLEM II.a’’ and the same assumptions regarding the Mindlin Plates and the adhesive layers. However, intermediate steps in the derivations will be omitted and more or less the final form of the equations will be included in the subsequent sections.

8.2.1 Analysis of Adhesive Layer in the “Overlap Region”

The adhesive stresses should be related unknown displacement functions and angles of rotation of the orthotropic base plate and the orthotropic stiffening strips in the ‘‘Overlap Region’’.

Thus, in a similar fashion, as in section (4.2.1), normal (σ_z) and tangential or shear (τ_{xz}, τ_{yz}) stresses at the interfaces can be written as,

Between the Orthotropic Base Plate 1 and the Upper Orthotropic Stiffening Plate Strip 2,

$$\left|\begin{array}{l} \tau_{xz}^{(j)} = G_{a2}\gamma_{xz}^{(j)} = -\underline{\frac{G_{a2}}{2h_{a2}}\left(h_1\psi_x^{(1)} + h_2\psi_x^{(2)}\right)} + \underline{\underline{G_{a2}\frac{\partial w^{(j)}}{\partial x}}} \\ \tau_{yz}^{(j)} = G_{a2}\gamma_{yz}^{(j)} = -\underline{\frac{G_{a2}}{2h_{a2}}\left(h_1\psi_y^{(1)} + h_2\psi_y^{(2)}\right)} + \underline{\underline{G_{a2}\frac{\partial w^{(j)}}{\partial y}}} \quad \text{(j=1,2)} \\ \sigma_z^{(j)} = E_{a2}\varepsilon_z^{(j)} = \underline{\frac{E_{a2}}{h_{a2}}\left(w^{(1)} - w^{(2)}\right)} \end{array}\right. \qquad (8.1.a)$$

Between the Orthotropic Base Plate 1 and the Lower Orthotropic Stiffening Plate Strip 3,

$$\left|\begin{array}{l} \tau_{xz}^{(j)} = G_{a3}\gamma_{xz}^{(j)} = -\underline{\frac{G_{a3}}{2h_{a3}}\left(h_1\psi_x^{(1)} + h_3\psi_x^{(3)}\right)} + \underline{\underline{G_{a3}\frac{\partial w^{(j)}}{\partial x}}} \\ \tau_{yz}^{(j)} = G_{a3}\gamma_{yz}^{(j)} = -\underline{\frac{G_{a3}}{2h_{a3}}\left(h_1\psi_y^{(1)} + h_3\psi_y^{(3)}\right)} + \underline{\underline{G_{a3}\frac{\partial w^{(j)}}{\partial y}}} \quad (j=1,3) \\ \sigma_z^{(j)} = E_{a3}\varepsilon_z^{(j)} = \underline{\frac{E_{a3}}{h_{a3}}\left(w^{(3)} - w^{(1)}\right)} \end{array}\right. \qquad (8.1.b)$$

where E_{a2} and E_{a3} are the modulus of elasticity (or Young's modulus) and G_{a2} and G_{a3} are the modulus of rigidity (or shear modulus) of the upper and lower adhesive layers, respectively. In the above Equations (8.1.a) and (8.1.b), the double underlined (=) terms are coming from $\frac{\partial w^{(j)}}{\partial x}, \frac{\partial w^{(j)}}{\partial y}$ terms in adhesive layers deformation equations.

8.2.2 Analysis of Part I (or "Overlap Region") of Composite Plate System

The Equations (8.1) which show the adhesive normal and the shear stresses at the interfaces may be thought as the ''Compatibility or Coupling Conditions'' of the three plates in the ''Overlap Region'' or Part I region in Figures 3.6 and 3.7. Then, the surface load terms q_{zx}'s and q_{zy}'s in the plate dynamic equations of motion given in (3.19) and (3.20) can be replaced by adhesive stresses. Therefore, one can write the plate equations of motion of the three layer coupled plate system shown in the Part I of Figure 3.6 as,

For Plates 1 and 2 (Base Plate=1, Upper Stiffening Plate Strip=2),

$$\begin{aligned}
&\frac{\partial M_x^{(j)}}{\partial x}+\frac{\partial M_{yx}^{(j)}}{\partial y}-Q_x^{(j)}-\underline{\frac{h_j G_{a2}}{4h_{a2}}\left(h_1\psi_x^{(1)}+h_2\psi_x^{(2)}\right)}+\underline{\underline{\frac{h_j G_{a2}}{2}\frac{\partial w^{(j)}}{\partial x}}}=\frac{\rho_j h_j^3}{12}\frac{\partial^2\psi_x^{(j)}}{\partial t^2}\\
&\frac{\partial M_{yx}^{(j)}}{\partial x}+\frac{\partial M_y^{(j)}}{\partial y}-Q_y^{(j)}-\underline{\frac{h_j G_{a2}}{4h_{a2}}\left(h_1\psi_y^{(1)}+h_2\psi_y^{(2)}\right)}+\underline{\underline{\frac{h_j G_{a2}}{2}\frac{\partial w^{(j)}}{\partial y}}}=\frac{\rho_j h_j^3}{12}\frac{\partial^2\psi_y^{(j)}}{\partial t^2}\qquad (j=1,2)\qquad (8.2.a)\\
&\frac{\partial Q_x^{(j)}}{\partial x}+\frac{\partial Q_y^{(j)}}{\partial y}+(-1)^{(j+1)}\underline{\frac{E_{a2}}{h_{a2}}\left(w^{(1)}-w^{(2)}\right)}=\rho_j h_j\frac{\partial^2 w^{(j)}}{\partial t^2}
\end{aligned}$$

For Plates 1 and 3 (Base Plate=1, Lower Stiffening Plate Strip=3),

$$\begin{aligned}
&\frac{\partial M_x^{(j)}}{\partial x}+\frac{\partial M_{yx}^{(j)}}{\partial y}-Q_x^{(j)}-\underline{\frac{h_j G_{a3}}{4h_{a3}}\left(h_1\psi_x^{(1)}+h_3\psi_x^{(3)}\right)}+\underline{\underline{\frac{h_j G_{a3}}{2}\frac{\partial w^{(j)}}{\partial x}}}=\frac{\rho_j h_j^3}{12}\frac{\partial^2\psi_x^{(j)}}{\partial t^2}\\
&\frac{\partial M_{yx}^{(j)}}{\partial x}+\frac{\partial M_y^{(j)}}{\partial y}-Q_y^{(j)}-\underline{\frac{h_j G_{a3}}{4h_{a3}}\left(h_1\psi_y^{(1)}+h_3\psi_y^{(3)}\right)}+\underline{\underline{\frac{h_j G_{a3}}{2}\frac{\partial w^{(j)}}{\partial y}}}=\frac{\rho_j h_j^3}{12}\frac{\partial^2\psi_y^{(j)}}{\partial t^2}\qquad (j=1,3)\qquad (8.2.b)\\
&\frac{\partial Q_x^{(j)}}{\partial x}+\frac{\partial Q_y^{(j)}}{\partial y}+(-1)^{(j+1)}\underline{\frac{E_{a3}}{h_{a3}}\left(w^{(1)}-w^{(3)}\right)}=\rho_j h_j\frac{\partial^2 w^{(j)}}{\partial t^2}
\end{aligned}$$

where superscript (j) denotes the orthotropic base plate for j=1, upper orthotropic stiffening plate strip for j=2 and lower orthotropic stiffening plate strip for j=3 in the "Overlap Region" or Part I region of Figures 3.6 and 3.7. Also, the underlined terms are the coupling terms between the base plate and the stiffening plate strips.

The three-layer orthotropic composite plate system shown in the ''Overlap Region'' or Part I region of Figure 3.6 is simply supported along x=0,a edges. $M_x^{(j)}$ and $Q_x^{(j)}$ are taken as ''subsidiary variables'' and they are eliminated from the governing partial differential equations because they can not be prescribed in the cross-sections perpendicular to y-directions. Then, the remaining dependent variables $\psi_x^{(j)},\psi_y^{(j)},w^{(j)},M_{yx}^{(j)},M_y^{(j)},Q_y^{(j)}$ (j=1,2,3) are chosen as ''intrinsic'' variables of the problem. Here, six of the twentyfour equations, that is, the nine equations of motion (4.7) and five stress resultants-displacement relation (3.17) for each plate, are algebraically manipulated to eliminate the subsidiary variables $M_x^{(j)}$ and $Q_x^{(j)}$. The remaining eighteen equations are used to obtain the following set of the ''Coupled

Governing First Order Partial Differential Equations" of motion in matrix form for the three plates in the ''Overlap Region" or in Part I region. After then, using non-dimensionalization and the "Classical Levy's Solution" in the x-direction, the governing system of equation are obtained.

The procedure for the non-dimensionalization is the same with the one in Section 4.2.2. Therefore,

The "dimensionless coordinates" or "independent space variables'' are,

$$\eta = x/a, \qquad \xi_I = y_I / l_I^{"}, \qquad \xi_{II} = y_{II} / l_{II}^{"} \tag{8.3}$$

The dimensionless parameters related to orthotropic elastic constants of the plates and the adhesive layers are,

$$\begin{aligned}
&\overline{B}_{ik}^{(j)} = B_{ik}^{(j)} \Big/ B_{11}^{(1)} \qquad (j = 1,2,3 \; and \; i,k = 1,2,3), \\
&\overline{B}_{\ell\ell}^{(j)} = B_{\ell\ell}^{(j)} \Big/ B_{11}^{(1)} \qquad (j = 1,2,3 \; and \; \ell = 4,5,6) \\
&\overline{G}_{a2} = G_{a2} \Big/ B_{11}^{(1)}, \overline{G}_{a3} = G_{a3} \Big/ B_{11}^{(1)} \\
&\overline{E}_{a2} = E_{a2} \Big/ B_{11}^{(1)}, \overline{E}_{a3} = E_{a3} \Big/ B_{11}^{(1)}
\end{aligned} \tag{8.4}$$

The dimensionless parameters related to the densities and the geometry of the plates and adhesive layer are,

$$\begin{aligned}
&\overline{\rho}_2 = \rho_2 \Big/ \rho_1, \overline{\rho}_3 = \rho_3 \Big/ \rho_1 \\
&\overline{h}_{a2} = h_{a2} \Big/ h_1, \overline{h}_{a3} = h_{a3} \Big/ h_1 \\
&\overline{L}_I = \ell_I^{"} \Big/ a, \overline{L}_{II} = \ell_{II}^{"} \Big/ a \\
&\overline{h}_2 = h_2 \Big/ h_1, \overline{h}_3 = h_3 \Big/ h_1
\end{aligned} \tag{8.5}$$

The dimensionless frequency parameter, $\overline{\omega}_{mn}$, of the entire composite plate system,

$$\left|\begin{array}{l} \overline{\omega}_{mn} = \rho_1 a^4 \omega_{mn}^2 \big/ h_1^2 B_{11}^{(1)} \\ \overline{\Omega} = \overline{\omega}_{mn} \end{array}\right. \qquad \text{(m,n=1,2,3...)} \qquad (8.6)$$

where the dimensionless natural frequency parameter $\overline{\Omega}$ is ordered or sequenced in terms of its magnitudes as $\overline{\Omega}_1<\overline{\Omega}_2<\overline{\Omega}_3<....$ with the subscripts indicating the first, second, third,..... dimensionless natural frequencies depending on the given m, n values.

First, all parameters are nondimensionalized with respect to the base or main parameters ($"B_{11}^{(1)}", "h_1", "\rho_1"$ and "a") and then, by means of some algebraic manipulations, the "Governing System of First Order Ordinary Differential Equations" for the "Overlap Region" or Part I region in Figures 3.6 and 3.7 is obtained in the matrix or the "state vector" form,

$$\left|\begin{array}{l} \dfrac{d}{d\xi_I}\begin{Bmatrix} \overline{\boldsymbol{Y}}_{\boldsymbol{mn}}^{(2)} \\ \overline{\boldsymbol{Y}}_{\boldsymbol{mn}}^{(1)} \\ \overline{\boldsymbol{Y}}_{\boldsymbol{mn}}^{(3)} \end{Bmatrix} = \begin{bmatrix} \overline{C}_{1,1} & \overline{C}_{1,2} & \overline{C}_{1,3} \\ \overline{C}_{2,1} & \overline{C}_{2,2} & \overline{C}_{2,3} \\ \overline{C}_{3,1} & \overline{C}_{3,2} & \overline{C}_{3,3} \end{bmatrix} \begin{Bmatrix} \overline{\boldsymbol{Y}}_{\boldsymbol{mn}}^{(2)} \\ \overline{\boldsymbol{Y}}_{\boldsymbol{mn}}^{(1)} \\ \overline{\boldsymbol{Y}}_{\boldsymbol{mn}}^{(3)} \end{Bmatrix}, \qquad (0<\xi_I<1) \qquad (8.7) \\ \text{with the ''Arbitrary Boundary Conditions'' at } \xi_I = 0,1 \text{ for the orthotropic stiffening plate strips, and the ''Continuity Conditions'' at } \xi_I = 1 \text{ for the orthotropic base plate.} \end{array}\right.$$

where ξ_I is defined as $y_I / l_I^{"}$, and the superscripts indicate the related plate layer The "sub-matrices $[\overline{C}_{i,j}]$" are partitioned square matrices of dimension (6x6) which explicitly include the nondimensional geometric and material characteristics of the orthotropic base plate and the orthotropic stiffening plate strips and the adhesive layer and also the dimensionless natural frequency parameter $\overline{\omega}_{mn}$ of the composite system. In Equation (8.7), the "Column Matrix $\{\overline{\boldsymbol{Y}}_{\boldsymbol{mn}}^{(j)}\}$" (j=1,2,3), the "state vector"

corresponding to the state variables or the dimensionless fundamental dependent variables which is given as before,

$$\left\{\overline{Y}_{mn}^{(j)}\right\}=\left\{\overline{\Psi}_{mx}^{(j)},\overline{\Psi}_{my}^{(j)},\overline{W}_{m}^{(j)};\overline{M}_{myx}^{(j)},\overline{M}_{my}^{(j)},\overline{Q}_{my}^{(j)}\right\}^{T},\ (j=1,2,3) \qquad (8.8)$$

where the elements of $\left\{\overline{Y}_{mn}^{(j)}\right\}$ are the quantities related to the displacements, angles of rotation and the stress resultants which appear at ξ_I=const segments or cross-sections of the plate system in the "Overlap Region".

8.2.3 Analysis of Part II of Composite Plate System

By following the steps in Section 4.2.3, one can obtain the governing system of ordinary differential equations in a "state vector" form for the Part II region as follows,

$$\frac{d}{d\xi_{II}}\left\{\overline{Y}_{mn}^{(1)}\right\}=\left[\overline{D}\right]\left\{\overline{Y}_{mn}^{(1)}\right\}, \qquad (0<\xi_{II}<1) \qquad (8.9)$$

with the ''Arbitrary Boundary Conditions'' at ξ_{II} =1 and the ''Continuity Conditions'' at ξ_{II} =0 for the orthotropic base plate.

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where ξ_{II} is defined as $y_{II}/l_{II}^{''}$ and [$\overline{D}$] the "Coefficient Matrix" of dimension (6x6) which explicitly includes dimensionless geometric and material characteristics of the orthotropic base plate in Part II region as well as the dimensionless natural frequencies $\overline{\omega}_{mn}$ of the entire composite plate system. The column matrix or the "state vector" $\left\{\overline{Y}_{mn}^{(1)}\right\}$ is defined as in (8.8) by,

$$\left\{\overline{Y}_{mn}^{(1)}\right\}=\left\{\overline{\Psi}_{mnx}^{(1)},\overline{\Psi}_{mny}^{(1)},\overline{W}_{mn}^{(1)};\overline{M}_{mnyx}^{(1)},\overline{M}_{mny}^{(1)},\overline{Q}_{mny}^{(1)}\right\}^{T},\ (j=1) \qquad (8.10)$$

8.2.4 System of Governing Differential Equations ("Main PROBLEM III.a")

In preceding sections, the "Governing System of First Order Ordinary Differential Equations" in "state vector" form is obtained in (8.7) for the "Overlap Region" or Part I region. Similarly, for Part II and Part III regions, the Equations (8.8) and (8.9) are developed acccordingly. Thus, one can write the sets of "Governing System of First Order Ordinary Differential Equations" in open matrix form:

For Part I region or "Overlap Region" (or Three-Layer Composite Plate Region),

$$\frac{d}{d\xi_I}\begin{Bmatrix}\overline{\psi}^{(2)}_{mnx}\\ \overline{\psi}^{(2)}_{mny}\\ \overline{W}^{(2)}_{mn}\\ \overline{M}^{(2)}_{mnyx}\\ \overline{M}^{(2)}_{mny}\\ \overline{Q}^{(2)}_{mny}\\ \overline{\psi}^{(1)}_{mnx}\\ \overline{\psi}^{(1)}_{mny}\\ \overline{W}^{(1)}_{mn}\\ \overline{M}^{(1)}_{mnyx}\\ \overline{M}^{(1)}_{mny}\\ \overline{Q}^{(1)}_{mny}\\ \overline{\psi}^{(3)}_{mnx}\\ \overline{\psi}^{(3)}_{mny}\\ \overline{W}^{(3)}_{mn}\\ \overline{M}^{(3)}_{mnyx}\\ \overline{M}^{(3)}_{mny}\\ \overline{Q}^{(3)}_{mny}\end{Bmatrix}=\begin{bmatrix}
0 & c^{(2)}_{1,2} & 0 & c^{(2)}_{1,4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
c^{(2)}_{2,1} & 0 & 0 & 0 & c^{(2)}_{2,5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & c^{(2)}_{3,2} & 0 & 0 & 0 & c^{(2)}_{3,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
c^{(2)}_{4,1} & 0 & c^{(2)}_{4,3} & 0 & c^{(2)}_{4,5} & 0 & r^{(2)}_{1,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & c^{(2)}_{5,2} & 0 & c^{(2)}_{5,4} & 0 & c^{(2)}_{5,6} & 0 & r^{(2)}_{2,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
c^{(2)}_{6,1} & 0 & c^{(2)}_{6,3} & 0 & 0 & 0 & 0 & 0 & r^{(2)}_{3,3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{(1)}_{1,2} & 0 & c^{(1)}_{1,4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & c^{(1)}_{2,1} & 0 & 0 & 0 & c^{(1)}_{2,5} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{(1)}_{3,2} & 0 & 0 & 0 & c^{(1)}_{3,6} & 0 & 0 & 0 & 0 & 0 & 0\\
r^{(2)}_{1,1} & 0 & 0 & 0 & 0 & 0 & c^{(1)}_{4,1} & 0 & c^{(1)}_{4,3} & 0 & c^{(1)}_{4,5} & 0 & r^{(3)}_{1,1} & 0 & 0 & 0 & 0 & 0\\
0 & r^{(2)}_{2,2} & 0 & 0 & 0 & 0 & 0 & c^{(1)}_{5,2} & 0 & c^{(1)}_{5,4} & 0 & c^{(1)}_{5,6} & 0 & r^{(3)}_{2,2} & 0 & 0 & 0 & 0\\
0 & 0 & r^{(2)}_{3,3} & 0 & 0 & 0 & c^{(1)}_{6,1} & 0 & c^{(1)}_{6,3} & 0 & 0 & 0 & 0 & 0 & r^{(3)}_{3,3} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{1,2} & 0 & c^{(3)}_{1,4} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{2,1} & 0 & 0 & 0 & c^{(3)}_{2,5} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{3,2} & 0 & 0 & 0 & c^{(3)}_{3,6}\\
0 & 0 & 0 & 0 & 0 & 0 & r^{(3)}_{1,1} & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{4,1} & 0 & c^{(3)}_{4,3} & 0 & c^{(3)}_{4,5} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & r^{(3)}_{2,2} & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{5,2} & 0 & c^{(3)}_{5,4} & 0 & c^{(3)}_{5,6}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & r^{(3)}_{3,3} & 0 & 0 & 0 & c^{(3)}_{6,1} & 0 & c^{(3)}_{6,3} & 0 & 0 & 0
\end{bmatrix}\begin{Bmatrix}\overline{\psi}^{(2)}_{mnx}\\ \overline{\psi}^{(2)}_{mny}\\ \overline{W}^{(2)}_{mn}\\ \overline{M}^{(2)}_{mnyx}\\ \overline{M}^{(2)}_{mny}\\ \overline{Q}^{(2)}_{mny}\\ \overline{\psi}^{(1)}_{mnx}\\ \overline{\psi}^{(1)}_{mny}\\ \overline{W}^{(1)}_{mn}\\ \overline{M}^{(1)}_{mnyx}\\ \overline{M}^{(1)}_{mny}\\ \overline{Q}^{(1)}_{mny}\\ \overline{\psi}^{(3)}_{mnx}\\ \overline{\psi}^{(3)}_{mny}\\ \overline{W}^{(3)}_{mn}\\ \overline{M}^{(3)}_{mnyx}\\ \overline{M}^{(3)}_{mny}\\ \overline{Q}^{(3)}_{mny}\end{Bmatrix}$$

$$(0<\xi_I<1) \qquad (8.11)$$

where the elements of the ''Coefficient Matrix $\left[\overline{C}\right]$''and also the "Sub-Matrix $[\Lambda]^{(j)}$" and the "Sub-Matrix $[\alpha]^{(k)}$", in detailed form, are given in Equations (4.27) in Section (4.2.5).

For Part II (or Single Layer Orthotropic Base Plate Region),

$$\frac{d}{d\xi_{II}}\begin{Bmatrix}\overline{\psi}_{mnx}^{(1)}\\ \overline{\psi}_{mny}^{(1)}\\ \overline{W}_{mn}^{(1)}\\ \overline{M}_{mnyx}^{(1)}\\ \overline{M}_{mny}^{(1)}\\ \overline{Q}_{mny}^{(1)}\end{Bmatrix}=\begin{bmatrix}0 & d_{1,2} & 0 & d_{1,4} & 0 & 0\\ d_{2,1} & 0 & 0 & 0 & d_{2,5} & 0\\ 0 & d_{3,2} & 0 & 0 & 0 & d_{3,6}\\ d_{4,1} & 0 & d_{4,3} & 0 & d_{4,5} & 0\\ 0 & d_{5,2} & 0 & d_{5,4} & 0 & d_{5,6}\\ d_{6,1} & 0 & d_{6,3} & 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}\overline{\psi}_{mnx}^{(1)}\\ \overline{\psi}_{mny}^{(1)}\\ \overline{W}_{mn}^{(1)}\\ \overline{M}_{mnyx}^{(1)}\\ \overline{M}_{mny}^{(1)}\\ \overline{Q}_{mny}^{(1)}\end{Bmatrix},\quad (0<\xi_{II}<1) \qquad (8.12)$$

where the elements of the ‘‘Coefficient Matrix $\left[\overline{\mathsf{D}}\right]$’’ are identical with the ones given in Equations (4.28) in Section (4.2.5).

As explained in several previous sections, the ”Boundary Conditions” at x=0 and x=a are already satisfied by the trigonometric expansions in “Classical Lévy’s Type Solution”. Then, the aim is it to determine $\overline{\Psi}_{mnx}^{(j)}, \overline{\Psi}_{mny}^{(j)}, \overline{W}_{mn}^{(j)}; \overline{M}_{mnyx}^{(j)}, \overline{M}_{mny}^{(j)}, \overline{Q}_{mny}^{(j)}$ in such a form that they satisfy the “Appropriate Boundary Conditions” for the base plate and the stiffening plate strips,and also the “Appropriate Continuity Condition” and also satisfy the “Governing System of Ordinary Differential Equations” in (8.11) and (8.12). Hence,

The ‘‘Boundary Conditions’’prescribed in the y-direction,

F (Free): $\overline{M}_{yx}^{(j)} = \overline{M}_{y}^{(j)} = \overline{Q}_{y}^{(j)} = 0$

C (Clamped): $\overline{w}^{(j)} = \overline{\psi}_{x}^{(j)} = \overline{\psi}_{y}^{(j)} = 0$ (j=1,2,3) (8.13)

S (Simply Supported): $\overline{w}^{(j)} = \overline{\psi}_{x}^{(j)} = \overline{M}_{y}^{(j)} = 0$

The ‘‘Continuity Condition’’ between Part I and Part II is (see also Fig. 3.7),

$$\left\{\overline{\boldsymbol{Y}}_{\xi_I=1}^{(1)}\right\}=\left\{\overline{\boldsymbol{Y}}_{\xi_{II}=0}^{(1)}\right\} \qquad (8.14)$$

Hence, one may express the complete set of the "Governing System of First Order Ordinary Differential Equations" for the "Main PROBLEM III.a" as,

$$\frac{d}{d\xi_I}\begin{Bmatrix}\overline{Y}_{mn}^{(2)}\\ \overline{Y}_{mn}^{(1)}\\ \overline{Y}_{mn}^{(3)}\end{Bmatrix}=\begin{bmatrix}\overline{C}_{1,1} & \overline{C}_{1,2} & \overline{C}_{1,3}\\ \overline{C}_{2,1} & \overline{C}_{2,2} & \overline{C}_{2,3}\\ \overline{C}_{3,1} & \overline{C}_{3,2} & \overline{C}_{3,3}\end{bmatrix}\begin{Bmatrix}\overline{Y}_{mn}^{(2)}\\ \overline{Y}_{mn}^{(1)}\\ \overline{Y}_{mn}^{(3)}\end{Bmatrix}, \qquad (0<\xi_I<1) \quad \text{(in Part I)}$$

$$\frac{d}{d\xi_{II}}\left\{\overline{Y}_{mn}^{(1)}\right\}=\left[\overline{D}\right]\left\{\overline{Y}_{mn}^{(1)}\right\}, \qquad (0<\xi_{II}<1) \quad \text{(in Part II)} \qquad (8.15\ a,b)$$

with the "Appropriate Boundary Conditions" and the "Continuity Conditions" for the above given system.

Again, the above system forms a "Two-Point Boundary Value Problem" for the "Main PROBLEM III.a" under consideration here. Once the dimensionless natural frequency parameter $\overline{\omega}_{mn}$ is known or computed, then the above set of equations can be numerically integrated (see the "Method of Solution" in Chapter 10).

CHAPTER 9

(“Main PROBLEM III.b”) (‘‘SPECIAL CASE of Main PROBLEM II.b’’)__FREE FLEXURAL (or BENDING) VIBRATIONS OF DOUBLY STIFFENED-at SUPPORT, COMPOSITE PLATES or PANELS
(Isotropic Base Plate or Panel Case)

9.1 Main Assumptions and Analytical Modeling

In this chapter, the problem of the “Free Flexural (or Bending) Vibrations of Doubly Stiffened-at Support. Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)” (“Main PROBLEM III b”) is to be analyzed. This problem may be considered as a “special case” of the “Main PROBLEM II b” which is analyzed in Chapter 7. The general configuration and also the geometric and the material characteristics and the coordinate system of the problem are shown in Figure 3.6. The longitudinal cross-section in the y-direction of the composite plate system is given in Figure 3.7.

The main assumptions and analytical modeling regarding the formulation of this problem are exactly same as was given in Chapter 7. In this particular case, the composite plate system is a combination of an isotropic base plate doubly stiffened-at support by the bonded upper and lower orthotropic stiffening plate strips. The upper and lower orthotropic stiffening plate strips and the adhesive layers connecting them to the isotropic base plate are assumed to be identical in practical applications and for numerical results in the present Thesis. In the theoretical analysis, however, the upper and lower adhesive layers as well as the upper and the lower orthotropic stiffening plate strips are assumed to be dissimilar and the problem is formulated accordingly.

9.2 Theoretical Formulation of "Main PROBLEM III.b" (Theoretical Analysis)

9.2.1 Analysis of Adhesive Layer in Part I (or "Overlap Region")

The theoretical formulation here, follows the same steps as was done in the "Main PROBLEM II.b" in chapter 7. In this case, however, in order to facilitate the theoretical analysis, the composite plate system in y-direction is divided instead into 2 parts namely Part I and Part II as shown in Figure 3.7.

In what follows, the intermediate steps of the derivatives involved will be omitted. Thus, only the final results are given.

The adhesive stresses should be related to the unknown displacement functions and angles of rotation of the isotropic base plate and the orthotropic stiffening strips in the "Overlap Region" or Part I Region. As was done in previous chapters and sections, the adhesive stresses, at the interfaces of plates and adhesive layers, are considered as the surface loads on the plate upper and lower surfaces.

As in section (4.2.1), one can write the adhesive stresses as,

Between Plates 1 and 2 (Adhesive Stresses),

$$\left|\begin{array}{l} \tau_{xz}^{(j)} = G_{a2}\gamma_{xz}^{(j)} = \underline{-\dfrac{G_{a2}}{2h_{a2}}\left(h_1\psi_x^{(1)} + h_2\psi_x^{(2)}\right)} + \underline{\underline{G_{a2}\dfrac{\partial w^{(j)}}{\partial x}}} \\ \tau_{yz}^{(j)} = G_{a2}\gamma_{yz}^{(j)} = \underline{-\dfrac{G_{a2}}{2h_{a2}}\left(h_1\psi_y^{(1)} + h_2\psi_y^{(2)}\right)} + \underline{\underline{G_{a2}\dfrac{\partial w^{(j)}}{\partial y}}} \quad (j=1,2) \\ \sigma_z^{(j)} = E_{a2}\varepsilon_z^{(j)} = \underline{\dfrac{E_{a2}}{h_{a2}}\left(w^{(1)} - w^{(2)}\right)} \end{array}\right. \qquad (9.1.a)$$

Between Plates 1 and 3 (Adhesive Stresses),

$$\tau_{xz}^{(j)} = G_{a3}\gamma_{xz}^{(j)} = -\underline{\frac{G_{a3}}{2h_{a3}}\left(h_1\psi_x^{(1)} + h_3\psi_x^{(3)}\right)} + \underline{\underline{G_{a3}\frac{\partial w^{(j)}}{\partial x}}}$$

$$\tau_{yz}^{(j)} = G_{a3}\gamma_{yz}^{(j)} = -\underline{\frac{G_{a3}}{2h_{a3}}\left(h_1\psi_y^{(1)} + h_3\psi_y^{(3)}\right)} + \underline{\underline{G_{a3}\frac{\partial w^{(j)}}{\partial y}}} \quad (j=1,3) \qquad (9.1.b)$$

$$\sigma_z^{(j)} = E_{a3}\varepsilon_z^{(j)} = \underline{\frac{E_{a3}}{h_{a3}}\left(w^{(3)} - w^{(1)}\right)}$$

where E_{a2} and E_{a3} are the moduli of elasticity (or Young's moduli) and G_{a2} and G_{a3} are the moduli of rigidity (or shear moduli) of the upper and lower adhesive layers, respectively. Also in the above equations double underlined terms (=) are coming from the adhesive layer equations.

The Equations (9.1) which shows the transverse normal and tangential stresses at the plate-adhesive layer interfaces may also be thought as the ''Compatibility or Coupling Conditions'' of the three plates in the "Overlap Region" or Part I Region in Figure 3.7.

9.2.2 Analysis of "Overlap Region"(or Part I Region) of Composite Plate System

One can obtain the "Governing System of First Order Ordinary Differential Equations" in the "state vector" form for the "Overlap Region" or Part I Region by following similar steps as in Section 4.2.2. During this process, it should be noted that $"B_{11}^{(1)}", "h_1", "\rho_1"$ and "a" are also chosen as "main or reference parameters" and all other parameters are nondimensionalized with respect to these quantities.

The dimensionless coordinates or "independent space variables" are,

$$\eta = x/a, \qquad \xi_I = y_I/l_I^{"}, \qquad \xi_{II} = y_{II}/l_{II}^{"} \qquad (9.2)$$

The dimensionless parameters related to orthotropic elastic constant and the adhesive layer are,

$$\left|\begin{array}{ll}
\overline{B}_{ik}^{(j)} = B_{ik}^{(j)} \Big/ B_{11}^{(1)} & \left(j = 1,2,3 \; and \; i,k = 1,2,3\right), \\
\overline{B}_{\ell\ell}^{(j)} = B_{\ell\ell}^{(j)} \Big/ B_{11}^{(1)} & \left(j = 1,2,3 \; and \; \ell = 4,5,6\right) \\
\overline{G}_{a2} = G_{a2} \Big/ B_{11}^{(1)}, \overline{G}_{a3} = G_{a3} \Big/ B_{11}^{(1)} & \\
\overline{E}_{a2} = E_{a2} \Big/ B_{11}^{(1)}, \overline{E}_{a3} = E_{a3} \Big/ B_{11}^{(1)} &
\end{array}\right| \tag{9.3}$$

The dimensionless parameters related to the densities and the geometry of the plates and adhesive layer are,

$$\left|\begin{array}{l}
\overline{\rho}_2 = \rho_2 \Big/ \rho_1, \overline{\rho}_3 = \rho_3 \Big/ \rho_1 \\
\overline{h}_{a2} = h_{a2} \Big/ h_1, \overline{h}_{a3} = h_{a3} \Big/ h_1 \\
\overline{L}_I = \ell''_I \Big/ a, \overline{L}_{II} = \ell''_{II} \Big/ a \\
\overline{h}_2 = h_2 \Big/ h_1, \overline{h}_3 = h_3 \Big/ h_1
\end{array}\right| \tag{9.4}$$

The dimensionless natural frequency $\overline{\omega}_{mn}$ parameter of the entire three-layer, support doubly stiffened, stepped-thickness, composite plate system which has isotropic base plate and orthotropic stiffening strips of the Special Case of Problem III is as follows,

$$\left|\begin{array}{ll}
\overline{\omega}_{mn} = \rho_1 a^4 \omega_{mn}^2 \Big/ h_1^2 B_{11}^{(1)} & \\
\overline{\Omega} = \overline{\omega}_{mn} & \text{(m,n=1,2,3...)}
\end{array}\right| \tag{9.5}$$

where the dimensionless natural frequency parameter $\overline{\Omega}$ is ordered or sequenced in terms of its magnitudes as $\overline{\Omega}_1 < \overline{\Omega}_2 < \overline{\Omega}_3 < \ldots$ with the subscript indicating

the first, second, third,..... dimensionless natural frequencies depending on the given m, n values.

Finally, the "Governing System of First Order Ordinary Differential Equations" in the matrix or "state vector" form for the "Overlap Region" or Part I Region can be written as,

$$\frac{d}{d\xi_I}\begin{Bmatrix}\overline{\boldsymbol{Y}}_{mn}^{(2)}\\ \overline{\boldsymbol{Y}}_{mn}^{(1)}\\ \overline{\boldsymbol{Y}}_{mn}^{(3)}\end{Bmatrix}=\begin{bmatrix}\overline{\boldsymbol{C}}_{1,1}' & \overline{\boldsymbol{C}}_{1,2}' & \overline{\boldsymbol{C}}_{1,3}'\\ \overline{\boldsymbol{C}}_{2,1}' & \overline{\boldsymbol{C}}_{2,2}' & \overline{\boldsymbol{C}}_{2,3}'\\ \overline{\boldsymbol{C}}_{3,1}' & \overline{\boldsymbol{C}}_{3,2}' & \overline{\boldsymbol{C}}_{3,3}'\end{bmatrix}\begin{Bmatrix}\overline{\boldsymbol{Y}}_{mn}^{(2)}\\ \overline{\boldsymbol{Y}}_{mn}^{(1)}\\ \overline{\boldsymbol{Y}}_{mn}^{(3)}\end{Bmatrix}, \qquad (0<\xi_I<1) \tag{9.6}$$

with the ''Arbitrary Boundary Conditions'' at ξ_I =0,1 for the orthotropic stiffening plate strips, and the ''Continuity Conditions'' at ξ_I =1 for the isotropic base plate.

where ξ_I is defined as y_I/l_I'' ,and the superscripts show the particular plate layer, submatrices $[\overline{\boldsymbol{C}}_{i,j}']$ are partitioned square matrices of dimension (6x6) which explicitly include the nondimensional geometric and material characteristics the isotropic base plate and the orthotropic stiffening strips, the adhesive layer and dimensionless natural frequencies parameter $\overline{\omega}_{mn}$ of the composite system,

In the above, $\{\overline{\mathbf{Y}}_{\mathbf{m}}^{(j)}\}$ (j=1,2,3) is the "state vector" corresponding to state variables or "dimensionless fundamental dependent variables" of the upper, lower stiffening plate strips and the in-between base plate, defined as,

$$\{\overline{\boldsymbol{Y}}_{\boldsymbol{mn}}^{(j)}\}=\{\overline{\Psi}_{mnx}^{(j)},\overline{\Psi}_{mny}^{(j)},\overline{W}_{mn}^{(j)};\overline{M}_{mnyx}^{(j)},\overline{M}_{mny}^{(j)},\overline{Q}_{mny}^{(j)}\}^T, \quad (j=1,2,3) \tag{9.7}$$

where the elements of $\{\overline{\boldsymbol{Y}}_{mn}^{(j)}\}$ are the quantities related to the displacement, angle of rotations and stress resultants which appear at ξ_I=const segments or cross-sections of the plate system in the "Overlap Region" or Part I region. (see also Figure 3.7.)

9.2.3 Analysis of Part II of Composite Plate System

Similarly, one can write the "Governing System of Equations" in the matrix or "state vector" form for the Part II and region shown in Figure 3.7 as,

For Part II region,

$$\frac{d}{d\xi_{II}}\left\{\overline{Y}_{mn}^{(I)}\right\}=\left[\overline{\mathcal{D}'}\right]\left\{\overline{Y}_{mn}^{(I)}\right\}, \qquad (0<\xi_{II}<1) \qquad (9.8)$$

with the ''Arbitrary Boundary Conditions'' at ξ_{II} =1 and the ''Continuity Conditions'' at ξ_{II} =0 for isotropic the base plate.

where ξ_{II} is defined as $y_{II}/l_{II}^{''}$ and is the "Coefficient Matrix $\left[\overline{D'}\right]$" of dimension (6x6) which explicitly includes dimensionless geometric and material characteristics of the isotropic base plate in the Part II region as well as the dimensionless natural frequencies $\overline{\omega}_{mn}$ of the entire composite plate system. The column matrix or the "state vector" $\left\{\overline{Y}_{mn}^{(I)}\right\}$ is defined as in (9.7), that is,

$$\left\{\overline{Y}_{mn}^{(j)}\right\}=\left\{\overline{\Psi}_{mnx}^{(j)},\overline{\Psi}_{mny}^{(j)},\overline{W}_{mn}^{(j)};\overline{M}_{mnyx}^{(j)},\overline{M}_{mny}^{(j)},\overline{Q}_{mny}^{(j)}\right\}^{T}, \ (j=1) \qquad (9.9)$$

9.2.4 System of Governing Differential Equations ("Main PROBLEM III b")

In previous Sections, the governing system of coupled ordinary differential equations in the matrix or "state vector" form is obtained in (9.6) for the "Overlap Region" or Part I region, and in (9.8) for Part II region. Then, one can write these equations, in open form, as follows,

For Part I region or "overlap region" (or Three-Layer Plate Composite Region),

$$\frac{d}{d\xi_I}\begin{Bmatrix}\overline{\psi}^{(2)}_{mnx}\\ \overline{\psi}^{(2)}_{mny}\\ \overline{W}^{(2)}_{mn}\\ \overline{M}^{(2)}_{mnyx}\\ \overline{M}^{(2)}_{mny}\\ \overline{Q}^{(2)}_{mny}\\ \overline{\psi}^{(1)}_{mnx}\\ \overline{\psi}^{(1)}_{mny}\\ \overline{W}^{(1)}_{mn}\\ \overline{M}^{(1)}_{mnyx}\\ \overline{M}^{(1)}_{mny}\\ \overline{Q}^{(1)}_{mny}\\ \overline{\psi}^{(3)}_{mnx}\\ \overline{\psi}^{(3)}_{mny}\\ \overline{W}^{(3)}_{mn}\\ \overline{M}^{(3)}_{mnyx}\\ \overline{M}^{(3)}_{mny}\\ \overline{Q}^{(3)}_{mny}\end{Bmatrix}=\begin{bmatrix}0 & c^{(2)}_{1,2} & 0 & c^{(2)}_{1,4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ c^{(2)}_{2,1} & 0 & 0 & 0 & c^{(2)}_{2,5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & c^{(2)}_{3,2} & 0 & 0 & 0 & c^{(2)}_{3,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ c^{(2)}_{4,1} & 0 & c^{(2)}_{4,3} & 0 & c^{(2)}_{4,5} & 0 & r^{(2)}_{1,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & c^{(2)}_{5,2} & 0 & c^{(2)}_{5,4} & 0 & c^{(2)}_{5,6} & 0 & r^{(2)}_{2,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ c^{(2)}_{6,1} & 0 & c^{(2)}_{6,3} & 0 & 0 & 0 & 0 & 0 & r^{(2)}_{3,3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{'(1)}_{1,2} & 0 & c^{'(1)}_{1,4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & c^{'(1)}_{2,1} & 0 & 0 & 0 & c^{'(1)}_{2,5} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{'(1)}_{3,2} & 0 & 0 & 0 & c^{'(1)}_{3,6} & 0 & 0 & 0 & 0 & 0 & 0\\ r^{(2)}_{1,1} & 0 & 0 & 0 & 0 & 0 & c^{'(1)}_{4,1} & 0 & c^{'(1)}_{4,3} & 0 & c^{'(1)}_{4,5} & 0 & r^{(3)}_{1,1} & 0 & 0 & 0 & 0 & 0\\ 0 & r^{(2)}_{2,2} & 0 & 0 & 0 & 0 & 0 & c^{'(1)}_{5,2} & 0 & c^{'(1)}_{5,4} & 0 & c^{'(1)}_{5,6} & 0 & r^{(3)}_{2,2} & 0 & 0 & 0 & 0\\ 0 & 0 & r^{(2)}_{3,3} & 0 & 0 & 0 & c^{'(1)}_{6,1} & 0 & c^{'(1)}_{6,3} & 0 & 0 & 0 & 0 & 0 & r^{(3)}_{3,3} & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{1,2} & 0 & c^{(3)}_{1,4} & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{2,1} & 0 & 0 & 0 & c^{(3)}_{2,5} & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{3,2} & 0 & 0 & 0 & c^{(3)}_{3,6}\\ 0 & 0 & 0 & 0 & 0 & 0 & r^{(3)}_{1,1} & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{4,1} & 0 & c^{(3)}_{4,3} & 0 & c^{(3)}_{4,5} & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & r^{(3)}_{2,2} & 0 & 0 & 0 & 0 & 0 & c^{(3)}_{5,2} & 0 & c^{(3)}_{5,4} & 0 & c^{(3)}_{5,6}\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & r^{(3)}_{3,3} & 0 & 0 & 0 & c^{(3)}_{6,1} & 0 & c^{(3)}_{6,3} & 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}\overline{\psi}^{(2)}_{mnx}\\ \overline{\psi}^{(2)}_{mny}\\ \overline{W}^{(2)}_{mn}\\ \overline{M}^{(2)}_{mnyx}\\ \overline{M}^{(2)}_{mny}\\ \overline{Q}^{(2)}_{mny}\\ \overline{\psi}^{(1)}_{mnx}\\ \overline{\psi}^{(1)}_{mny}\\ \overline{W}^{(1)}_{mn}\\ \overline{M}^{(1)}_{mnyx}\\ \overline{M}^{(1)}_{mny}\\ \overline{Q}^{(1)}_{mny}\\ \overline{\psi}^{(3)}_{mnx}\\ \overline{\psi}^{(3)}_{mny}\\ \overline{W}^{(3)}_{mn}\\ \overline{M}^{(3)}_{mnyx}\\ \overline{M}^{(3)}_{mny}\\ \overline{Q}^{(3)}_{mny}\end{Bmatrix}$$

$$(0<\xi_I<1) \qquad (9.10)$$

where the elements of the "Coefficient Matrix $\left[\overline{\mathcal{C}'}\right]$"in detailed are presented in Equations (5.2b) in Section (5.3.2).

For Part II (or Single Layer Isotropic Base Plate Region),

$$\frac{d}{d\xi_{II}}\begin{Bmatrix}\overline{\psi}^{(1)}_{mnx}\\ \overline{\psi}^{(1)}_{mny}\\ \overline{W}^{(1)}_{mn}\\ \overline{M}^{(1)}_{mnyx}\\ \overline{M}^{(1)}_{mny}\\ \overline{Q}^{(1)}_{mny}\end{Bmatrix}=\begin{bmatrix}0 & d'_{1,2} & 0 & d'_{1,4} & 0 & 0\\ d'_{2,1} & 0 & 0 & 0 & d'_{2,5} & 0\\ 0 & d'_{3,2} & 0 & 0 & 0 & d'_{3,6}\\ d'_{4,1} & 0 & d'_{4,3} & 0 & d'_{4,5} & 0\\ 0 & d'_{5,2} & 0 & d'_{5,4} & 0 & d'_{5,6}\\ d'_{6,1} & 0 & d'_{6,3} & 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}\overline{\psi}^{(1)}_{mnx}\\ \overline{\psi}^{(1)}_{mny}\\ \overline{W}^{(1)}_{mn}\\ \overline{M}^{(1)}_{mnyx}\\ \overline{M}^{(1)}_{mny}\\ \overline{Q}^{(1)}_{mny}\end{Bmatrix}(0<\xi_{II}<1) \qquad (9.11)$$

where the elements of the "Coefficient Matrix $\left[\overline{\mathcal{D}'}\right]$" are given in Equations (5.27) in Section (5.3.2).

The “Appropriate B.C.’s” and the “Appropriate Continuity Conditions” to be used with the sets of equations in (9.10) and (9.11) in the solution of the “Main PROBLEM III b” are given below.

The ‘‘Boundary Conditions’’ along the edges in the y-direction as,

F (Free): $\overline{M}_{yx}^{(j)} = \overline{M}_{y}^{(j)} = \overline{Q}_{y}^{(j)} = 0$

C (Clamped): $\overline{w}^{(j)} = \overline{\psi}_{x}^{(j)} = \overline{\psi}_{y}^{(j)} = 0$ (j=1,2,3) (9.12)

S (Simply Supported): $\overline{w}^{(j)} = \overline{\psi}_{x}^{(j)} = \overline{M}_{y}^{(j)} = 0$

The ‘‘Continuity Conditions’’ between Part I and Part II is (see also Fig. 3.7),

$$\left\{\overline{\boldsymbol{Y}}_{\xi_I=1}^{(1)}\right\} = \left\{\overline{\boldsymbol{Y}}_{\xi_{II}=0}^{(1)}\right\} \tag{9.13}$$

Thus, the complete set of the “Governing System of First Order Ordinary Differential Equations” for the “Main PROBLEM III.b” are,

$$\frac{d}{d\xi_I}\begin{Bmatrix}\overline{\boldsymbol{Y}}_{mn}^{(2)}\\ \overline{\boldsymbol{Y}}_{mn}^{(1)}\\ \overline{\boldsymbol{Y}}_{mn}^{(3)}\end{Bmatrix} = \begin{bmatrix}\overline{\boldsymbol{C}}_{1,1}' & \overline{\boldsymbol{C}}_{1,2}' & \overline{\boldsymbol{C}}_{1,3}'\\ \overline{\boldsymbol{C}}_{2,1}' & \overline{\boldsymbol{C}}_{2,2}' & \overline{\boldsymbol{C}}_{2,3}'\\ \overline{\boldsymbol{C}}_{3,1}' & \overline{\boldsymbol{C}}_{3,2}' & \overline{\boldsymbol{C}}_{3,3}'\end{bmatrix}\begin{Bmatrix}\overline{\boldsymbol{Y}}_{mn}^{(2)}\\ \overline{\boldsymbol{Y}}_{mn}^{(1)}\\ \overline{\boldsymbol{Y}}_{mn}^{(3)}\end{Bmatrix} \quad (0<\xi_I<1) \quad \text{(in Part I)}$$

$$\frac{d}{d\xi_{II}}\left\{\overline{\boldsymbol{Y}}_{mn}^{(1)}\right\} = \left[\overline{\boldsymbol{\mathcal{D}}}'\right]\left\{\overline{\boldsymbol{Y}}_{mn}^{(1)}\right\} \quad (0<\xi_{II}<1) \quad \text{(in Part II)} \tag{9.14 a,b}$$

with the “Appropriate Boundary Conditions” and the “Continuity Conditions” for the above given system.

Again, the above system constitutes a “Two-Point Boundary Value Problem” for the problem under consideration here. The above system can be integrated numerically if the dimensionless frequency parameter $\overline{\omega}_{mn}$ is computed. In the “Method of Solution” in Chapter 10, this procedure will be explained in detail.

CHAPTER 10

METHOD OF SOLUTION

10.1 Introduction

Several different approaches and numerical methods and also some approximate methods have been developed for the solution of plate "Initial and Boundary Value Problems". Since very few analytical and/or closed form solutions are available in the literature and in the graduate level texts, the solution of complicated plate problems[1] are, in general, attempted by means of several numerical and some approximate methods developed over the years (Leissa [II.1]).

In the present Thesis, a semi-analytical and numerical method which is the "Modified Transfer Matrix Method" is employed to solve the "Main PROBLEMS" considered here. The ''Modified Transfer Matrix Method'' [V.4-V.13 and VIII.3-VIII.7] is a combination of the "Classical Levy's Method" and the ''Integrating Matrix Method'' and the "Transfer Matrix Method".

Yuceoglu and Özerciyes [V.4-V.13 and VIII.3-VIII.7] have developed several versions of the present method of solution such that,

- "Modified Transfer Matrix Method (with Interpolation Polynomials)"
- "Modified Transfer Matrix Method (with Chebyshev Polynomials)"
- "Modified Transfer Matrix Method (with Eigenvalue Approach)"

[1] Some of these problems are multi-layer composite plates, composite multi-layer and/or single layer plates with one (or more) stiffener(s) (or holes(s) or cut-out, etc.), etc.

Aforementioned methods are essentially semi-analytical and numerical techniques.

Here in this Thesis, the first two methods mentioned above are employed.The earlier version of the present technique is in Yuceoglu et al [V.2, V.3, VIII.2].

The present method of solution will be systematically applied to the “Main PROBLEMS” defined and formulated in preceding Chapters 4-9.

10.2 Method of Solution of “Main PROBLEM I.a”

Here in this Section, the application of the present solution technique will be explained in some detail for the “Main PROBLEM I” without making any distinction for I.a and I.b part of the “Main PROBLEM I”.

The initial step in the present method of solution is to write the “Governing System of Differential Equations” in a first order “state vector” form as in (4.22), (4.24) and (4.26). This is because only a single step integration is required in order to eliminate the differential operator and obtain numerically the “fundamental dependent variables” when the equations are expressed in the first order form.

Referring to Chapters 4-9, the entire set of “Governing System of Equations” may be rewritten in compact matrix form with the “state vectors” as unknowns. Then,

The "Governing System of First Order Ordinary Differential Equations" are,

$$\frac{d}{d\xi_I}\begin{Bmatrix}\overline{\boldsymbol{Y}}_{mn}^{(2)}\\ \overline{\boldsymbol{Y}}_{mn}^{(1)}\\ \overline{\boldsymbol{Y}}_{mn}^{(3)}\end{Bmatrix}=\left[\overline{\mathsf{C}}\right]\begin{Bmatrix}\overline{\boldsymbol{Y}}_{mn}^{(2)}\\ \overline{\boldsymbol{Y}}_{mn}^{(1)}\\ \overline{\boldsymbol{Y}}_{mn}^{(3)}\end{Bmatrix}, \qquad (0<\xi_I<1) \quad \text{(in Part I)}$$

$$\frac{d}{d\xi_{II}}\left\{\overline{\boldsymbol{Y}}_{mn}^{(1)}\right\}=\left[\overline{\mathsf{D}}\right]\left\{\overline{\boldsymbol{Y}}_{mn}^{(1)}\right\}, \qquad (0<\xi_{II}<1) \quad \text{(in Part II)} \qquad (10.1.a,b,c)$$

$$\frac{d}{d\xi_{III}}\left\{\overline{\boldsymbol{Y}}_{mn}^{(1)}\right\}=\left[\overline{\mathsf{E}}\right]\left\{\overline{\boldsymbol{Y}}_{mn}^{(1)}\right\}, \qquad (0<\xi_{III}<1) \quad \text{(in Part III)}$$

with the "Appropriate Boundary Conditions" and the "Continuity Conditions" for the particular problem under consideration.

where ξ_I, ξ_{II}, ξ_{III} are defined as y_I/l_I, y_{II}/l_{II}, y_{III}/l_{III}, respectively and the "Coefficient Matrix $[\overline{\mathsf{C}}]$" is of dimension (18x18) which explicitly includes the nondimensional geometric and material characteristics of the base plate, the stiffening plate strips and the adhesive layers and also the dimensionless natural frequency parameter $\overline{\omega}_{mn}$ of the composite system. Similarly, the "Coefficient Matrix $[\overline{\mathsf{D}}]$" and "Coefficient Matrix $[\overline{\mathsf{E}}]$" both of dimension (6x6) which explicitly include dimensionless geometric and material characteristics of the base plate in Part II and Part III regions, respectively, and also the dimensionless natural frequency parameter $\overline{\omega}_{mn}$.The "Column Matrix $\left\{\overline{\boldsymbol{Y}}_{mn}^{(j)}\right\}$" (j=1,2,3) is the "state vector" including state variables or "dimensionless fundamental dependent variables" of the problem,

$$\left\{\overline{\boldsymbol{Y}}_{mn}^{(j)}\right\}=\left\{\overline{\Psi}_{mnx}^{(j)},\overline{\Psi}_{mny}^{(j)},\overline{W}_{mn}^{(j)};\overline{M}_{mnyx}^{(j)},\overline{M}_{mny}^{(j)},\overline{Q}_{mny}^{(j)}\right\}^T \qquad (j=1,2,3) \qquad (10.2)$$

The next step involves a discretization of the "fundamental dependent variables" of the problem under investigation and the "Coefficient Matrices" in (10.1) with respect to the independent space variables ξ_I, ξ_{II}, ξ_{III}, respectively. They are taken as spatial coordinates along the Part I, Part II and Part III regions, respectively. This is realized by dividing the Part I, Part II and Part III regions in Figure 3.2 into sufficient number of segments or stations along ξ_I, ξ_{II} and ξ_{III}

directions and pre-multiplying discrete version of (4.22), (4.24) and (4.26) by the appropriate "Global Integrating Matrix $[L\ \ ]$" and dropping "mn" subscript for convenience, which yields,

For the "Overlap Region" or Part I region (Three-Layer Composite Plate),

$$\begin{Bmatrix} \dot{\bar{\boldsymbol{Y}}}^{(2)} \\ \dot{\bar{\boldsymbol{Y}}}^{(1)} \\ \dot{\bar{\boldsymbol{Y}}}^{(3)} \end{Bmatrix} - \begin{Bmatrix} \dot{\bar{\boldsymbol{Y}}}_{1}^{(2)} \\ \dot{\bar{\boldsymbol{Y}}}_{1}^{(1)} \\ \dot{\bar{\boldsymbol{Y}}}_{1}^{(3)} \end{Bmatrix} = [L\ \ _{\mathrm{I}}] \begin{bmatrix} \text{—} \\ \\ \end{bmatrix} \begin{Bmatrix} \dot{\bar{\boldsymbol{Y}}}^{(2)} \\ \dot{\bar{\boldsymbol{Y}}}^{(1)} \\ \dot{\bar{\boldsymbol{Y}}}^{(3)} \end{Bmatrix}, \qquad \text{(in Part I)} \qquad (10.3)$$

For Part II region (Single Layer Orthotropic or Isotropic Base Plate),

$$\left\{\dot{\bar{\boldsymbol{Y}}}^{(1)}\right\} - \left\{\dot{\bar{\boldsymbol{Y}}}_{1}^{(1)}\right\} = [L\ \ _{\mathrm{II}}] \begin{bmatrix} \dot{\text{—}} \\ \\ \end{bmatrix} \left\{\dot{\bar{\boldsymbol{Y}}}^{(1)}\right\}, \qquad \text{(in Part II)} \qquad (10.4)$$

For Part III region (Single Layer Orthotropic or Isotropic Base Plate),

$$\left\{\dot{\bar{\boldsymbol{Y}}}^{(1)}\right\} - \left\{\dot{\bar{\boldsymbol{Y}}}_{1}^{(1)}\right\} = [L\ \ _{\mathrm{III}}] \begin{bmatrix} \dot{\bar{E}} \end{bmatrix} \left\{\dot{\bar{\boldsymbol{Y}}}^{(1)}\right\}, \qquad \text{(in Part II)} \qquad (10.5)$$

where $\left\{\dot{\bar{\boldsymbol{Y}}}^{(j)}\right\}, \begin{bmatrix} \dot{\text{—}} \\ \\ \end{bmatrix}, \begin{bmatrix} \dot{\text{—}} \\ \\ \end{bmatrix}, \begin{bmatrix} \dot{\bar{E}} \end{bmatrix}$ ("dot" or "." indicating the discretization along the ξ-direction) are the discrete versions of "state vector $\left\{\bar{\boldsymbol{Y}}_{mn}^{(j)}\right\}$" and the coefficient matrices $\begin{bmatrix} \text{—} \\ \\ \end{bmatrix}, \begin{bmatrix} \text{—} \\ \\ \end{bmatrix}, \begin{bmatrix} \bar{E} \end{bmatrix}$ in (10.1), respectively. The "state vector $\left\{\bar{\boldsymbol{Y}}_{1}^{(j)}\right\}$" is evaluated at the "initial" end point, i.e. at $\xi_{\mathrm{I}}=0$, $\xi_{\mathrm{II}}=0$, and $\xi_{\mathrm{III}}=0$.for Part I, Part II and Part III, respectively The superscripts (1,2,3) denote the base plate and stiffening plate strips, respectively. The subscripts "m" and "n" will be dropped from the matrices for convience in the following equations. In detailed form, one can write the "state vector" at a "general station" as $\left\{\dot{\bar{\boldsymbol{Y}}}^{(j)}\right\}$, and the "state vector" at the "initial end point" with the subscript "1" as $\left\{\bar{\boldsymbol{Y}}_{1}^{(j)}\right\}$ and $\begin{bmatrix} \dot{\text{—}} \\ \\ \end{bmatrix}$, $\begin{bmatrix} \dot{\text{—}} \\ \\ \end{bmatrix}$ and $\begin{bmatrix} \dot{\bar{E}} \end{bmatrix}$ explicitly as,

$$\left\{\dot{\bar{Y}}^{(1)}\right\}=\begin{Bmatrix}\bar{\Psi}^{(1)}_{x1}\\ \bar{\Psi}^{(1)}_{x2}\\ .\\ .\\ \bar{\Psi}^{(1)}_{xn}\\ \bar{\Psi}^{(1)}_{y1}\\ \bar{\Psi}^{(1)}_{y2}\\ .\\ .\\ .\\ \bar{Q}^{(1)}_{y1}\\ .\\ .\\ \bar{Q}^{(1)}_{yn}\end{Bmatrix},\left\{\dot{\bar{Y}}^{(2)}\right\}=\begin{Bmatrix}\bar{\Psi}^{(2)}_{x1}\\ \bar{\Psi}^{(2)}_{x2}\\ .\\ .\\ \bar{\Psi}^{(2)}_{xn}\\ \bar{\Psi}^{(2)}_{y1}\\ \bar{\Psi}^{(2)}_{y2}\\ .\\ .\\ .\\ \bar{Q}^{(2)}_{y1}\\ .\\ .\\ \bar{Q}^{(2)}_{yn}\end{Bmatrix},\left\{\dot{\bar{Y}}^{(3)}\right\}=\begin{Bmatrix}\bar{\Psi}^{(3)}_{x1}\\ \bar{\Psi}^{(3)}_{x2}\\ .\\ .\\ \bar{\Psi}^{(3)}_{xn}\\ \bar{\Psi}^{(3)}_{y1}\\ \bar{\Psi}^{(3)}_{y2}\\ .\\ .\\ .\\ \bar{Q}^{(3)}_{y1}\\ .\\ .\\ \bar{Q}^{(3)}_{yn}\end{Bmatrix},\left\{\dot{\bar{Y}}^{(1)}_{1}\right\}=\begin{Bmatrix}\bar{\Psi}^{(1)}_{x1}\\ \bar{\Psi}^{(1)}_{x1}\\ .\\ .\\ \bar{\Psi}^{(1)}_{x1}\\ \bar{\Psi}^{(1)}_{y1}\\ \bar{\Psi}^{(1)}_{y1}\\ .\\ .\\ .\\ \bar{Q}^{(1)}_{y1}\\ .\\ .\\ \bar{Q}^{(1)}_{y1}\end{Bmatrix},\left\{\dot{\bar{Y}}^{(2)}_{1}\right\}=\begin{Bmatrix}\bar{\Psi}^{(2)}_{x1}\\ \bar{\Psi}^{(2)}_{x1}\\ .\\ .\\ \bar{\Psi}^{(2)}_{x1}\\ \bar{\Psi}^{(2)}_{y1}\\ \bar{\Psi}^{(2)}_{y1}\\ .\\ .\\ .\\ \bar{Q}^{(2)}_{y1}\\ .\\ .\\ \bar{Q}^{(2)}_{y1}\end{Bmatrix},\left\{\dot{\bar{Y}}^{(3)}_{1}\right\}=\begin{Bmatrix}\bar{\Psi}^{(3)}_{x1}\\ \bar{\Psi}^{(3)}_{x1}\\ .\\ .\\ \bar{\Psi}^{(3)}_{x1}\\ \bar{\Psi}^{(3)}_{y1}\\ \bar{\Psi}^{(3)}_{y1}\\ .\\ .\\ .\\ \bar{Q}^{(3)}_{y1}\\ .\\ .\\ \bar{Q}^{(3)}_{y1}\end{Bmatrix} \quad (10.6)$$

and the "Coefficient Matrices" $\left[\dot{\bar{\ }}\right],\left[\dot{\bar{\ }}\right],\left[\dot{\bar{E}}\right]$ as,

$$\left[\dot{\bar{\ }}\right]=\begin{bmatrix}\left[\ _{1,1}\right]_{nxn} & \left[\ _{1,2}\right]_{nxn} & . & . & \left[\ _{1,18}\right]_{nxn}\\ . & . & . & . & \left[\ _{2,18}\right]_{nxn}\\ . & . & . & . & .\\ . & . & . & . & .\\ \left[\ _{18,1}\right]_{nxn} & . & . & . & \left[\ _{18,18}\right]_{nxn}\end{bmatrix} \quad (10.7)$$

$$\left[\dot{\bar{\ }}\right]=\begin{bmatrix}\left[\ _{1,1}\right]_{nxn} & \left[\ _{1,2}\right]_{nxn} & . & . & \left[\ _{1,6}\right]_{nxn}\\ . & . & . & . & \left[\ _{2,6}\right]_{nxn}\\ . & . & . & . & .\\ . & . & . & . & .\\ \left[\ _{6,1}\right]_{nxn} & . & . & . & \left[\ _{6,6}\right]_{nxn}\end{bmatrix} \quad (10.8)$$

$$\left[\dot{\bar{E}}\right]=\begin{bmatrix}\left[E_{1,1}\right]_{nxn} & \left[E_{1,2}\right]_{nxn} & . & . & \left[E_{1,6}\right]_{nxn}\\ . & . & . & . & \left[E_{2,6}\right]_{nxn}\\ . & . & . & . & .\\ . & . & . & . & .\\ \left[E_{6,1}\right]_{nxn} & . & . & . & \left[E_{6,6}\right]_{nxn}\end{bmatrix} \quad (10.9)$$

The second subscripts in (10.6), (10.7), (10.8) and (10.9) indicate the discretization point or the "station" with which they are associated. [$_{i,j}$], [$_{i,j}$] and [$E_{i,j}$] are the diagonal "Sub-Matrices" and $_{i,j}$, $_{i,j}$ and $E_{i,j}$ are the elements of the "Coefficient Matrices" in (4.27), (4.28) and (4.29), respectively.

In order to explain the numerical part of the solution technique the "Integrating Matrix Method" is needed. Hunter [VIII.1] gave a detailed discussion of the "Integrating Matrix Method". In this Thesis, the algorithm of the development of the "Integrating Matrix" will be briefly explained. The "Integrating Matrix" is a means by which a continuous function may be integrated with the use of a finite-difference approach. This numerical method is based upon the assumption that the function f(x) may be represented by a polynomial of degree r as follows,

Interpolation Polynomial (of degree r)

$$f(x)=a_0+a_1x+a_2x^2+.....+a_rx^r \qquad (10.10)$$

The "Integrating Matrix" is developed by expressing (10.10) in the form of "Newton's Forward Difference Interpolation" formula. The integrand may be represented conveniently by polynomials of any degree. When the "Integrating Matrix" as presented in this study is employed, any number of station "n" may be chosen (so long as n≥r) (see Hunter [VIII.1]). The "Integrating Matrix" can be developed, in general, for any value of n.

The "Global Integrating Matrix [L " works as an operator, in a similar way as the conventional integral symbol, that is,

$$\mathcal{L} \leftrightarrow \int_0^s (.....)ds \qquad (10.11)$$

The following "Integrating Matrix" is repeated from Hunter [VIII.1] However, it is worthwhile to point out that the symbol $[L_{\ I}]$ in (10.3) will be used for the "Global Integrating Matrix" for Part I, which is a square block diagonal

matrix composed of 18×18 (since the "state vector" in (10.2) is composed of 18 "dimensionless fundamental dependent variables" for "Overlap Region" or Part I region) ''basic'' square blocks of dimension n_s, (i.e. n_s=18), and n_1 is the number of discretization points along Part I region. Each basic block "Integrating Sub-Matrix [L is responsible for the integration of one single state variable or fundamental dependent variable of the problem under consideration. Therefore, the dimension of the "Global Integrating Matrix $[L_{\ I}]$ for Part I in (10.3) is ($18n_1$×$18n_1$). Similarly, $[L_{\ II}]$ in (10.4) and $[L_{\ III}]$ in (10.5) stand for the "Global Integrating Matrices" for Part II and Part III, respectively. Then, these are square block diagonal matrices composed of six (since the "state vector"" in (10.2) is composed of six "dimensionless fundamental dependent variables" for Part II and Part III) ''basic'' square blocks of dimension n_s, (i.e. n_s=6), and n_2 and n_3 are the number of discretization points along Part II and Part III regions, respectively. Therefore, the dimensions of the "Global Integrating Matrices $[L_{\ II}]$ for Part II and $[L_{\ III}]$ for Part III in (10.4) and (10.5) are ($6n_2$×$6n_2$) and ($6n_3$×$6n_3$), respectively. The explicit expression for the "Global Integrating Matrix [L]" is as follows,

$$[L\ ]=\begin{bmatrix} L & 0 & . & . & . & 0 \\ 0 & L & 0 & . & . & .0 \\ 0 & 0 & L & 0 & . & .0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & 0 & . & . & 0 & L \end{bmatrix} = \text{(Diagonal Matrix)} \qquad (10.12)$$

Both accuracy and convergence of the method used are affected by the degree of the "Approximating Polynomial or Interpolation Polynomial". However, the effect of the degree of the "Approximating Polynomial" on the accuracy of the calculated frequencies is dependent on those of the number of discretization points. A proper degree of the "Assumed Interpolation Polynomial" may result in convergence with fewer discretization points. However, it is not expected to obtain more accurate results as the degree of the "Assumed Polynomial" increases arbitrarily. This is because of the fact that the polynomial shows a more oscillating behavior as the

degree of it increases. This does not necessarily resemble the actual behavior of the plate system. Therefore one has to be careful in that regard. Also, another important point is that, one has to check if the far-end support conditions in the y-direction are satisfied properly in the numerical sense.

The "Integrating Sub-Matrix [L]" which is a component of the "Global Integrating Matrix [$\mathcal{L}$]" is normalized for integrations in the unitary interval [0, 1] in the following way,

$$[L] \quad [\boldsymbol{S}][\boldsymbol{W}_n] \tag{10.13}$$

where [S] is an ($n_1 \times n_1$) lower triangular matrix for Part I, ($n_2 \times n_2$) lower triangular matrix for Part II and ($n_3 \times n_3$) lower triangular matrix for Part III. Thus, the "Lower Triangular Matrix **[*S*]**" is given as,

$$[\boldsymbol{S}] = \begin{bmatrix} 1 & 0 & . & . & 0 \\ 1 & 1 & . & . & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ 1 & 1 & . & . & 1 \end{bmatrix} \tag{10.14}$$

where $[\boldsymbol{W}_n]$ is the "Weighting Matrix":

$$[\boldsymbol{W}_n] = \frac{\Delta n}{1440} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & \ldots & 0 \\ 475 & 1427 & -798 & 482 & -173 & 27 & 0 & \ldots & \ldots & 0 \\ -27 & 637 & 1022 & -258 & 77 & -11 & 0 & \ldots & \ldots & 0 \\ 11 & -93 & 802 & 802 & -93 & 11 & 0 & \ldots & \ldots & 0 \\ 0 & 11 & -93 & 802 & 802 & -93 & 11 & 0 & \ldots & 0 \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ 0 & \ldots & \ldots & 11 & -93 & .802 & .802 & .-93 & 11 & 0 \\ 0 & \ldots & \ldots & 0 & 11 & -93 & 802 & 802 & -93 & 11 \\ 0 & \ldots & \ldots & 0 & -11 & 77 & -258 & 1022 & 637 & -27 \\ 0 & \ldots & \ldots & 0 & 27 & -173 & 482 & -798 & 1427 & 475 \end{bmatrix} \tag{10.15}$$

where Δn is the "step size", given by,

$$\Delta n = \frac{1}{(n-1)} \tag{10.16}$$

The above expression (10.15), is based on a "5th Order Interpolation Polynomial" and n is the number of discretization points depending on Part I, Part II and Part III regions. Moreover, the "Weighting Matrix $[\boldsymbol{W_n}]$" for the "6th Order Interpolation Polynomial" which is also used in this Thesis is given in Hunter [VIII.1].

After rearranging (10.3), (10.4) and (10.5), the relations between the "state vector" at a ''general station'' along ξ_I, ξ_{II} and ξ_{III} in Part I, Part II and Part III regions, respectively and the "state vector" at the "initial end points'' ξ_I=0, ξ_{II}=0, ξ_{III}=0 along Part I, Part II and Part III regions, respectively, as shown, are obtained. Then, from Equation (10.3),

For the "Overlap Region" or Part I region,

$$\begin{Bmatrix} \dot{\bar{Y}}^{(2)} \\ \dot{\bar{Y}}^{(1)} \\ \dot{\bar{Y}}^{(3)} \end{Bmatrix} = [\dot{\mathcal{U}}] \begin{Bmatrix} \dot{\bar{Y}}_I^{(2)} \\ \dot{\bar{Y}}_I^{(1)} \\ \dot{\bar{Y}}_I^{(3)} \end{Bmatrix} \tag{10.17}$$

where $[\dot{U}]$ is the "Modified Transfer Matrix" for Part I region,

$$[\dot{\mathcal{U}}] = \left([I] - [\mathcal{L}_{\mathrm{I}}] \left[\dot{\bar{C}} \right] \right)^{-1} \tag{10.18}$$

Similarly, from Equation (10.4),

For Part II region,

$$\left\{ \dot{\bar{Y}}^{(1)} \right\} = [\dot{\mathcal{V}}] \left\{ \dot{\bar{Y}}_I^{(1)} \right\} \tag{10.19}$$

where $[\dot{V}]$ is the discretized version of "Modified Transfer Matrix" for Part II region,

$$[\dot{\mathcal{V}}] = \left([I] - [\mathcal{L}_{\mathrm{II}}] \left[\dot{\bar{\mathcal{D}}} \right] \right)^{-1} \tag{10.20}$$

Again, similarly, from Equation (10.5),

For Part III region,

$$\left\{\dot{\bar{Y}}^{(I)}\right\} = [\dot{W}]\left\{\dot{\bar{Y}}_I^{(I)}\right\} \tag{10.21}$$

where $[\dot{W}]$ is the discretized version of "Modified Transfer Matrix" for Part III region,

$$[\dot{W}] = \left([I] - [\mathcal{L}_{III}]\left[\dot{\bar{\varepsilon}}\right]\right)^{-1} \tag{10.22}$$

In the above expressions, the "Global Integrating Matrix $[\mathcal{L}_I]$, $[\mathcal{L}_{II}]$ and $[\mathcal{L}_{III}]$" are for Part I, Part II and Part III, respectively.

Also, in the above expressions, $[\dot{Z}]$, $[\dot{V}]$, $[\dot{W}]$ are the discretized versions of the "Modified Transfer Matrix" between a ''general station'' along ξ_I, ξ_{II}, ξ_{III} and the "initial end point $\xi_I=0$, $\xi_{II}=0$, $\xi_{III}=0$", respectively. Thus one can express, in open form,

For the “Overlap Region” or Part I region (Three-Layer Composite Plate),

$$\begin{Bmatrix} \overline{\Psi}_{x1}^{(2)} \\ \overline{\Psi}_{x2}^{(2)} \\ \cdot \\ \cdot \\ \overline{\Psi}_{xn}^{(2)} \\ \overline{\Psi}_{y1}^{(2)} \\ \overline{\Psi}_{y2}^{(2)} \\ \cdot \\ \cdot \\ \overline{\Psi}_{x1}^{(1)} \\ \cdot \\ \cdot \\ \overline{Q}_{y1}^{(3)} \\ \overline{Q}_{y2}^{(3)} \\ \cdot \\ \cdot \\ \overline{Q}_{yn}^{(3)} \end{Bmatrix} = \begin{bmatrix} \dot{\mathcal{I}}_{1,1} & \dot{\mathcal{I}}_{1,2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \dot{\mathcal{I}}_{1,18} \\ \dot{\mathcal{I}}_{2,1} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \dot{\mathcal{I}}_{1,18} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \dot{\mathcal{I}}_{18,18} \end{bmatrix} \begin{Bmatrix} \overline{\Psi}_{x1}^{(2)} \\ \overline{\Psi}_{x1}^{(2)} \\ \cdot \\ \cdot \\ \overline{\Psi}_{x1}^{(2)} \\ \overline{\Psi}_{y1}^{(2)} \\ \overline{\Psi}_{y1}^{(2)} \\ \cdot \\ \cdot \\ \overline{\Psi}_{x1}^{(1)} \\ \cdot \\ \cdot \\ \overline{Q}_{y1}^{(3)} \\ \overline{Q}_{y1}^{(3)} \\ \cdot \\ \cdot \\ \overline{Q}_{y1}^{(3)} \end{Bmatrix} \tag{10.23}$$

For Part II region (Single-Layer Base Plate),

$$\begin{Bmatrix} \overline{\Psi}_{x1}^{(1)} \\ \overline{\Psi}_{x2}^{(1)} \\ \cdot \\ \cdot \\ \overline{\Psi}_{xn_2}^{(1)} \\ \overline{\Psi}_{y1}^{(1)} \\ \overline{\Psi}_{y2}^{(1)} \\ \cdot \\ \cdot \\ \cdot \\ \overline{Q}_{y1}^{(1)} \\ \cdot \\ \cdot \\ \overline{Q}_{yn_2}^{(1)} \end{Bmatrix} = \begin{bmatrix} \dot{\mathcal{V}}_{1,1} & \cdot & \cdot & \cdot & \dot{\mathcal{V}}_{1,6} \\ \dot{\mathcal{V}}_{2,1} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \dot{\mathcal{V}}_{6,1} & \cdot & \cdot & \cdot & \dot{\mathcal{V}}_{6,6} \end{bmatrix} \begin{Bmatrix} \overline{\Psi}_{x1}^{(1)} \\ \overline{\Psi}_{x1}^{(1)} \\ \cdot \\ \cdot \\ \overline{\Psi}_{x1}^{(1)} \\ \overline{\Psi}_{y1}^{(1)} \\ \overline{\Psi}_{y1}^{(1)} \\ \cdot \\ \cdot \\ \cdot \\ \overline{Q}_{y1}^{(1)} \\ \cdot \\ \cdot \\ \overline{Q}_{y1}^{(1)} \end{Bmatrix} \tag{10.24}$$

For Part III region (Single-Layer Base Plate),,

$$\begin{Bmatrix} \overline{\Psi}_{x1}^{(2)} \\ \overline{\Psi}_{x2}^{(2)} \\ \cdot \\ \cdot \\ \overline{\Psi}_{xn_3}^{(2)} \\ \overline{\Psi}_{y1}^{(2)} \\ \overline{\Psi}_{y2}^{(2)} \\ \cdot \\ \cdot \\ \cdot \\ \overline{Q}_{y1}^{(2)} \\ \cdot \\ \cdot \\ \overline{Q}_{yn_3}^{(2)} \end{Bmatrix} = \begin{bmatrix} \dot{w}_{1,1} & \cdot & \cdot & \cdot & \dot{w}_{1,6} \\ \dot{w}_{2,1} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \dot{w}_{6,1} & \cdot & \cdot & \cdot & \dot{w}_{6,6} \end{bmatrix} \begin{Bmatrix} \overline{\Psi}_{x1}^{(2)} \\ \overline{\Psi}_{x1}^{(2)} \\ \cdot \\ \cdot \\ \overline{\Psi}_{x1}^{(2)} \\ \overline{\Psi}_{y1}^{(2)} \\ \overline{\Psi}_{y1}^{(2)} \\ \cdot \\ \cdot \\ \cdot \\ \overline{Q}_{y1}^{(2)} \\ \cdot \\ \cdot \\ \overline{Q}_{y1}^{(2)} \end{Bmatrix} \tag{10.25}$$

The matrix $[\dot{\mathcal{U}}]$ is composed of (18×18) square blocks of dimension n_1 and each individual block represents a relation between a state variable at any "general station ξ_I" along the Part I region and the state variable at the "initial end point ξ_I=0". The subscript n_1 is the number of discretization points in Part I region or the "Overlap Region".

The matrix $[\dot{\mathcal{V}}]$ is composed of (6×6) blocks of dimension n_2 and each individual block represents a relation between a state variable at any "general station ξ_{II}" along the Part II region and another state variable at the "initial end point ξ_{II}=0". The subscript n_2 is the number of discretization points in Part II.

The matrix $[\dot{\mathcal{W}}]$ is composed of (6×6) blocks of dimension n_3 and each individual block represents a relation between a state variable at any "general station ξ_{III}" along the Part II region and another state variable at the "initial end point ξ_{III}=0". The subscript n_3 is the number of discretization points in Part III.

Then, from (10.23), (10.24) and (10.25), one can obtain the relation between the "state vector" at the "initial end points $\xi_I=0$, $\xi_{II}=0$ and $\xi_{III}=0$" and the "final end points $\xi_I=1$, $\xi_{II}=1$ and $\xi_{III}=1$" and along Part I, Part II and Part III regions, respectively. Then, by summing the elements on the integer multiples of nth row of $\left[\dot{\mathcal{A}}\right]$, $\left[\dot{\mathcal{V}}\right]$ and $\left[\dot{\mathcal{W}}\right]$ one can obtain the following,

<u>For the "Overlap Region" or Part I region (Three-Layer Composite Plate),</u>

$$\begin{Bmatrix} \overline{\Psi}_{xn}^{(2)} \\ \overline{\Psi}_{yn}^{(2)} \\ \cdot \\ \cdot \\ \cdot \\ \overline{Q}_{yn}^{(3)} \end{Bmatrix} = \begin{bmatrix} \sum_{i=1}^{n}\left(\dot{\mathcal{A}}_{1,1}\right)_{n,i} & \sum_{i=n+1}^{2n}\left(\dot{\mathcal{A}}_{1,2}\right)_{n,i} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \sum_{i=17n+1}^{18n}\left(\dot{\mathcal{A}}_{18,18}\right)_{n,i} \end{bmatrix} \begin{Bmatrix} \overline{\Psi}_{x1}^{(2)} \\ \overline{\Psi}_{y1}^{(2)} \\ \cdot \\ \cdot \\ \cdot \\ \overline{Q}_{y1}^{(3)} \end{Bmatrix} \qquad (10.26.a)$$

or,

$$\begin{Bmatrix} \overline{\Psi}_{xn}^{(2)} \\ \overline{\Psi}_{yn}^{(2)} \\ \cdot \\ \cdot \\ \cdot \\ \overline{Q}_{yn}^{(3)} \end{Bmatrix} = \left[\tilde{\mathcal{A}}\right] \begin{Bmatrix} \overline{\Psi}_{x1}^{(2)} \\ \overline{\Psi}_{y1}^{(2)} \\ \cdot \\ \cdot \\ \cdot \\ \overline{Q}_{y1}^{(3)} \end{Bmatrix} \qquad (10.26.b)$$

<u>For Part II region (Single-Layer Base Plate),</u>

$$\begin{Bmatrix} \overline{\Psi}_{xn_2}^{(1)} \\ \overline{\Psi}_{yn_2}^{(1)} \\ \cdot \\ \cdot \\ \cdot \\ \overline{Q}_{yn_2}^{(1)} \end{Bmatrix} = \begin{bmatrix} \sum_{i=1}^{n}\left(\dot{\mathcal{V}}_{1,1}\right)_{n,i} & \sum_{i=n+1}^{2n}\left(\dot{\mathcal{V}}_{1,2}\right)_{n,i} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \sum_{i=5n+1}^{6n}\left(\dot{\mathcal{V}}_{1,6}\right)_{n,i} \end{bmatrix} \begin{Bmatrix} \overline{\Psi}_{x1}^{(1)} \\ \overline{\Psi}_{y1}^{(1)} \\ \cdot \\ \cdot \\ \cdot \\ \overline{Q}_{y1}^{(1)} \end{Bmatrix} \qquad (10.27.a)$$

or,

$$\begin{Bmatrix} \overline{\Psi}_{xn_2}^{(l)} \\ \overline{\Psi}_{yn_2}^{(l)} \\ \cdot \\ \cdot \\ \cdot \\ \overline{Q}_{yn_2}^{(l)} \end{Bmatrix} = \left[\tilde{\boldsymbol{v}}\right] \begin{Bmatrix} \overline{\Psi}_{x1}^{(l)} \\ \overline{\Psi}_{y1}^{(l)} \\ \cdot \\ \cdot \\ \cdot \\ \overline{Q}_{y1}^{(l)} \end{Bmatrix} \tag{10.27.b}$$

For Part III region (Single-Layer Base Plate),

$$\begin{Bmatrix} \overline{\Psi}_{xn_3}^{(l)} \\ \overline{\Psi}_{yn_3}^{(l)} \\ \cdot \\ \cdot \\ \cdot \\ \overline{Q}_{yn_3}^{(l)} \end{Bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} \left(\dot{\boldsymbol{w}}_{1,1} \right)_{n,i} & \sum_{i=n+1}^{2n} \left(\dot{\boldsymbol{w}}_{1,2} \right)_{n,i} & \cdot \cdot & \cdot \\ \cdot & \cdot & \cdot \cdot & \cdot \\ \cdot & \cdot & \cdot \cdot & \cdot \\ \cdot & \cdot & \cdot \cdot & \cdot \\ \cdot & \cdot & \cdot \cdot & \sum_{i=5n+1}^{6n} \left(\dot{\boldsymbol{w}}_{1,6} \right)_{n,i} \end{bmatrix} \begin{Bmatrix} \overline{\Psi}_{x1}^{(l)} \\ \overline{\Psi}_{y1}^{(l)} \\ \cdot \\ \cdot \\ \cdot \\ \overline{Q}_{y1}^{(l)} \end{Bmatrix} \tag{10.28.a}$$

or,

$$\begin{Bmatrix} \overline{\Psi}_{xn_3}^{(l)} \\ \overline{\Psi}_{yn_3}^{(l)} \\ \cdot \\ \cdot \\ \cdot \\ \overline{Q}_{yn_3}^{(l)} \end{Bmatrix} = \left[\tilde{\boldsymbol{w}}\right] \begin{Bmatrix} \overline{\Psi}_{x1}^{(l)} \\ \overline{\Psi}_{y1}^{(l)} \\ \cdot \\ \cdot \\ \cdot \\ \overline{Q}_{y1}^{(l)} \end{Bmatrix} \tag{10.28.b}$$

where in the above expressions $\left[\tilde{\boldsymbol{z}}\right]$, $\left[\tilde{\boldsymbol{v}}\right]$ and $\left[\tilde{\boldsymbol{w}}\right]$ are the "final form" of the "Modified Transfer Matrices" for Part I, Part II and Part III regions, respectively. They transfer the discretizied quantities from the "initial end point ξ_t=0" (t=Parts I, II, III) to the general end point ξ_t=1" (t=Parts I, II, III), respectively.

Therefore, one can write (10.26), (10.27) and (10.28) ,in a compact matrix form, such that,

For the "Overlap Region" or Part I region,

$$\begin{Bmatrix} \overline{Y}^{(2)}_{\xi_I=1} \\ \overline{Y}^{(1)}_{\xi_I=1} \\ \overline{Y}^{(3)}_{\xi_I=1} \end{Bmatrix} = \begin{bmatrix} \tilde{\mathcal{U}}_{1,1} & \tilde{\mathcal{U}}_{1,2} & \tilde{\mathcal{U}}_{1,3} \\ \tilde{\mathcal{U}}_{2,1} & \tilde{\mathcal{U}}_{2,2} & \tilde{\mathcal{U}}_{2,3} \\ \tilde{\mathcal{U}}_{3,1} & \tilde{\mathcal{U}}_{3,2} & \tilde{\mathcal{U}}_{3,3} \end{bmatrix}_{01} \begin{Bmatrix} \overline{Y}^{(2)}_{\xi_I=0} \\ \overline{Y}^{(1)}_{\xi_I=0} \\ \overline{Y}^{(3)}_{\xi_I=0} \end{Bmatrix} \tag{10.29}$$

For Part II region,

$$\left\{ \overline{Y}^{(1)}_{\xi_{II}=1} \right\} = \left[\tilde{\mathcal{V}}_{1,1} \right]_{01} \left\{ \overline{Y}^{(1)}_{\xi_{II}=0} \right\} \tag{10.30}$$

For Part III region,

$$\left\{ \overline{Y}^{(1)}_{\xi_{III}=1} \right\} = \left[\tilde{\mathcal{W}}_{1,1} \right]_{01} \left\{ \overline{Y}^{(1)}_{\xi_{III}=0} \right\} \tag{10.31}$$

where the subscript (01) in the above expressions means that the "final form" of the "Modified Transfer Matrix" transferring the above quantities from the "initial end point, ("0" point)", to the "final end point, ("1" point)". The matrices $\left[\tilde{\mathcal{U}}_{i,j}\right]$ are the partitioned square matrices of dimension (6x6) which implicitly includes the dimensionless natural frequencies $\overline{\omega}_{mn}$. Similarly, $\left[\tilde{\mathcal{V}}_{i,j}\right]$and $\left[\tilde{\mathcal{W}}_{i,j}\right]$are (6x6) matrices which also implicitly include the dimensionless natural frequency parameter $\overline{\omega}_{mn}$ of the entire composite plate system of the "Main PROBLEM I "

.

In equations (10.29), (10.30) and (10.31), the relations between the state-vectors at the "initial end points" and "final end points" of the Part I, Part II and Part III regions are obtained, respectively. Now, the aim is to formulate the nonclassical eigenvalue problem of the entire composite plate system of the Problem I. Therefore, the "Boundary Conditions" and the "Continuity Conditions" in the y-direction are

used to obtain the natural frequencies of the entire system shown in Figure 3.1. One can prescribe any combination of the "Boundary Conditions" at $\xi_I=0$, $\xi_I=1$ for the stiffening plate strips, and $\xi_{II}=0$, $\xi_{III}=1$ for the base plate in the y-direction as shown in Figure 3.2. The boundary conditions for the stiffening plate strips at $\xi_I=0$ and $\xi_I=1$ are "Free Edge Boundary Conditions" for practical purposes, as shown in Figure 3.2.

The ''Continuity Conditions'' between the Part I and Part II,

$$\left\{\overline{\boldsymbol{Y}}^{(1)}_{\xi_I=0}\right\}=\left\{\overline{\boldsymbol{Y}}^{(1)}_{\xi_{II}=1}\right\} \tag{10.32}$$

The ''Continuity Conditions'' between the Part I and Part III,

$$\left\{\overline{\boldsymbol{Y}}^{(1)}_{\xi_I=1}\right\}=\left\{\overline{\boldsymbol{Y}}^{(1)}_{\xi_{III}=0}\right\} \tag{10.33}$$

After inserting (10.32) into (10.30),

$$\left\{\overline{\boldsymbol{Y}}^{(1)}_{\xi_I=0}\right\}=\left[\tilde{\boldsymbol{\nu}}_{1,1}\right]_{01}\left\{\overline{\boldsymbol{Y}}^{(1)}_{\xi_{II}=0}\right\} \tag{10.34}$$

Then, one can write,

$$\begin{Bmatrix}\overline{\boldsymbol{Y}}^{(2)}_{\xi_I=0}\\ \overline{\boldsymbol{Y}}^{(1)}_{\xi_I=0}\\ \overline{\boldsymbol{Y}}^{(3)}_{\xi_I=0}\end{Bmatrix}=\begin{bmatrix}\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \tilde{\boldsymbol{\nu}}_{1,1} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}\end{bmatrix}\begin{Bmatrix}\overline{\boldsymbol{Y}}^{(2)}_{\xi_I=0}\\ \overline{\boldsymbol{Y}}^{(1)}_{\xi_{II}=0}\\ \overline{\boldsymbol{Y}}^{(3)}_{\xi_I=0}\end{Bmatrix} \tag{10.35}$$

After inserting (10.33) into (10.31),

$$\left\{\overline{\boldsymbol{Y}}^{(1)}_{\xi_I=1}\right\}=\left[\tilde{\boldsymbol{\nu}}_{1,1}\right]^{-1}\left\{\overline{\boldsymbol{Y}}^{(1)}_{\xi_{III}=1}\right\} \tag{10.36}$$

Then, one can write,

$$\begin{Bmatrix}\overline{\boldsymbol{Y}}^{(2)}_{\xi_I=1}\\ \overline{\boldsymbol{Y}}^{(1)}_{\xi_I=1}\\ \overline{\boldsymbol{Y}}^{(3)}_{\xi_I=1}\end{Bmatrix}=\begin{bmatrix}\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \tilde{\boldsymbol{\nu}}^{-1}_{1,1} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}\end{bmatrix}\begin{Bmatrix}\overline{\boldsymbol{Y}}^{(2)}_{\xi_I=1}\\ \overline{\boldsymbol{Y}}^{(1)}_{\xi_{III}=1}\\ \overline{\boldsymbol{Y}}^{(3)}_{\xi_I=1}\end{Bmatrix} \tag{10.37}$$

Again, substituting (10.35) and (10.37)into (10.29),

$$\begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \tilde{\boldsymbol{W}}_{1,1}^{-1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \begin{Bmatrix} \overline{\boldsymbol{Y}}_{\xi_I=1}^{(2)} \\ \overline{\boldsymbol{Y}}_{\xi_{III}=1}^{(1)} \\ \overline{\boldsymbol{Y}}_{\xi_I=1}^{(3)} \end{Bmatrix} = \begin{bmatrix} \tilde{\boldsymbol{U}}_{1,1} & \tilde{\boldsymbol{U}}_{1,2} & \tilde{\boldsymbol{U}}_{1,3} \\ \tilde{\boldsymbol{U}}_{2,1} & \tilde{\boldsymbol{U}}_{2,2} & \tilde{\boldsymbol{U}}_{2,3} \\ \tilde{\boldsymbol{U}}_{3,1} & \tilde{\boldsymbol{U}}_{3,2} & \tilde{\boldsymbol{U}}_{3,3} \end{bmatrix} \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \tilde{\boldsymbol{V}}_{1,1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \begin{Bmatrix} \overline{\boldsymbol{Y}}_{\xi_I=0}^{(2)} \\ \overline{\boldsymbol{Y}}_{\xi_{II}=0}^{(1)} \\ \overline{\boldsymbol{Y}}_{\xi_I=0}^{(3)} \end{Bmatrix} \quad (10.38)$$

where **[I]** is a unit matrix with dimension (6x6), $[\tilde{\boldsymbol{W}}_{1,1}]$ and $[\tilde{\boldsymbol{V}}_{1,1}]$ are also (6x6) matrices defined as before in (10.30) and (10.31) which implicitly include the dimensionless natural frequency $\overline{\omega}_{mn}$ and $[\tilde{\boldsymbol{U}}_{i,j}]$ are partitioned square matrices of dimension (6x6) defined as before in (10.29) which also implicitly includes the dimensionless natural frequencies $\overline{\omega}_{mn}$ of the plate system. All the "state vectors" in (10.38) are at stations where the boundary or support conditions of the system are prescribed as in Figure 3.2. Therefore, one can write,

$$\begin{Bmatrix} \overline{\boldsymbol{Y}}_{\xi_I=1}^{(2)} \\ \overline{\boldsymbol{Y}}_{\xi_{III}=1}^{(1)} \\ \overline{\boldsymbol{Y}}_{\xi_I=1}^{(3)} \end{Bmatrix} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \tilde{\boldsymbol{W}}_{1,1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \tilde{\boldsymbol{U}}_{1,1} & \tilde{\boldsymbol{U}}_{1,2} & \tilde{\boldsymbol{U}}_{1,3} \\ \tilde{\boldsymbol{U}}_{2,1} & \tilde{\boldsymbol{U}}_{2,2} & \tilde{\boldsymbol{U}}_{2,3} \\ \tilde{\boldsymbol{U}}_{3,1} & \tilde{\boldsymbol{U}}_{3,2} & \tilde{\boldsymbol{U}}_{3,3} \end{bmatrix} \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \tilde{\boldsymbol{V}}_{1,1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \begin{Bmatrix} \overline{\boldsymbol{Y}}_{\xi_I=0}^{(2)} \\ \overline{\boldsymbol{Y}}_{\xi_{II}=0}^{(1)} \\ \overline{\boldsymbol{Y}}_{\xi_I=0}^{(3)} \end{Bmatrix} \quad (10.39)$$

or finally, rearranging,

$$\begin{Bmatrix} \overline{\boldsymbol{Y}}_{\xi_I=1}^{(2)} \\ \overline{\boldsymbol{Y}}_{\xi_{III}=1}^{(1)} \\ \overline{\boldsymbol{Y}}_{\xi_I=1}^{(3)} \end{Bmatrix} = \begin{bmatrix} \boldsymbol{Q}_{1,1} & \boldsymbol{Q}_{1,2} & \boldsymbol{Q}_{1,3} \\ \boldsymbol{Q}_{2,1} & \boldsymbol{Q}_{2,2} & \boldsymbol{Q}_{2,3} \\ \boldsymbol{Q}_{3,1} & \boldsymbol{Q}_{3,2} & \boldsymbol{Q}_{3,3} \end{bmatrix}_{01} \begin{Bmatrix} \overline{\boldsymbol{Y}}_{\xi_I=0}^{(2)} \\ \overline{\boldsymbol{Y}}_{\xi_{II}=0}^{(1)} \\ \overline{\boldsymbol{Y}}_{\xi_I=0}^{(3)} \end{Bmatrix} \quad (10.40)$$

Six of the eighteen state variables must be prescribed at "each end point". After substitution of the "Boundary Conditions" in the y-direction at ξ_I=0, ξ_I=1 for the stiffening plate strips, and ξ_{II}=0, ξ_{III}=1 for the base plate into (10.40), the following matrix equation can be obtained,

$$[\mathbf{C}_0]_{(\mathrm{I.a})} \{\mathbf{Y}_0\}_{(\mathrm{I.a})} = \{\mathbf{0}\} \qquad \Rightarrow \qquad \left| [\mathbf{C}_0]_{(\mathrm{I.a})} \right| = \{\mathbf{0}\} \quad (10.41)$$

where $[\mathbf{C}_0]_{(\mathrm{I.a})}$ is a (6x6) "Coefficient Matrix" which implicitly includes the dimensionless natural frequency parameter $\overline{\omega}_{mn}$. The "Coefficient Matrix $[\mathbf{C}_0]_{(\mathrm{I.a})}$" is obtained from the "Coefficient Matrix" between the "final end point" and the "initial end point" in (10.40) by eliminating the rows corresponding to nonzero state variable at the "final end point'' and the columns corresponding to zero state variable at the "initial end point''. Here, $\{\mathbf{Y}_0\}$ is a vector whose elements are the nonzero components of $\left\{\boldsymbol{Y}^{(2)}_{\xi_1=0}, \boldsymbol{Y}^{(1)}_{\xi_1=0}, \boldsymbol{Y}^{(3)}_{\xi_1=0}\right\}^{\mathrm{T}}$

It is essential that the determinant of the "Coefficient Matrix $[\mathbf{C}_0]_{(\mathrm{I.a})}$" be equal to zero in order to have nontrivial solution for the system of (10.41). The values of the dimensionless natural frequency parameter $\overline{\omega}_{mn}$ which makes the determinant of the "Coefficient Matrix $[\mathbf{C}_0]_{(\mathrm{I.a})}$" equal to zero are the eigenvalues of the system under consideration. Therefore, the procedure should be repeated for a specific set of "Boundary Conditions" in order to find the roots of the determinant of the "Coefficient Matrix $[\mathbf{C}_0]_{(\mathrm{I.a})}$". The roots correspond to the nondimensionalized natural frequencies, $\overline{\omega}_{mn}$ of the entire composite plate system of the "Main PROBLEM I.a". One can easily find the corresponding mode shapes, after the natural frequencies (and eigenvectors) are obtained for a particular set of "Boundary Conditions" in the y-directions.

10.3 Method of Solution for "Main PROBLEM I.b"

Here, in this case, the intermediate steps in the solution technique will be omitted.

Then, as is done in Section (10.2), the first step is to write, from (5.21), (5.23) and (5.25), the sets of the "Governing System of First Order Ordinary Differential Equations" in a "state vector" form in Part I, Part II and Part III regions, respectively,

$$\frac{d}{d\xi_I}\begin{Bmatrix}\overline{\boldsymbol{Y}}_{mn}^{(2)}\\ \overline{\boldsymbol{Y}}_{mn}^{(1)}\\ \overline{\boldsymbol{Y}}_{mn}^{(3)}\end{Bmatrix}=\left[\overline{\boldsymbol{C}}'\right]\begin{Bmatrix}\overline{\boldsymbol{Y}}_{mn}^{(2)}\\ \overline{\boldsymbol{Y}}_{mn}^{(1)}\\ \overline{\boldsymbol{Y}}_{mn}^{(3)}\end{Bmatrix}, \qquad (0<\xi_I<1) \qquad \text{(in Part I region)} \quad (10.42)$$

$$\frac{d}{d\xi_{II}}\left\{\overline{\boldsymbol{Y}}_{mn}^{(1)}\right\}=\left[\overline{\boldsymbol{\mathcal{D}}}'\right]\left\{\overline{\boldsymbol{Y}}_{mn}^{(1)}\right\}, \qquad (0<\xi_{II}<1) \qquad \text{(in Part II region)} \quad (10.43)$$

$$\frac{\boldsymbol{d}}{\boldsymbol{d}\xi_{III}}\left\{\overline{\boldsymbol{Y}}_{mn}^{(1)}\right\}=\left[\overline{\boldsymbol{\mathcal{E}}}'\right]\left\{\overline{\boldsymbol{Y}}_{mn}^{(1)}\right\}, \qquad (0<\xi_{III}<1) \qquad \text{(in Part III region)} \quad (10.44)$$

where ξ_I, ξ_{II} and ξ_{III} are defined as y_I/l_I, y_{II}/l_{II} and y_{III}/l_{III}, respectively. The primed quantities correspond to the isotropic base plate with orthotropic stiffening plate strips (i.e. "Main PROBLEM I.b"). The "Coefficient Matrix $[\overline{\boldsymbol{C}}']$" is of dimension (18x18) which explicitly includes the nondimensional geometric and material characteristics of the base plate and stiffening strips and the adhesive layer and dimensionless natural frequencies parameter $\overline{\omega}_{mn}$ of the composite system.. $[\overline{\boldsymbol{\mathcal{D}}}']$, $[\overline{\boldsymbol{\mathcal{E}}}']$ are "Coefficient Matrices" of dimension (6x6). They include the dimensionless geometric and material characteristics of the base plate in the Part II and Part III regions, respectively, as well as the dimensionless natural frequencies $\overline{\omega}_{mn}$."The "Column Matrix" $\left\{\overline{\boldsymbol{Y}}_{\boldsymbol{m}}^{(j)}\right\}$ (j=1,2,3) is the "state vector" including the "dimensionless fundamental state variables" of the base plate and the stiffening plate strips,

$$\left\{\overline{\boldsymbol{Y}}_{\boldsymbol{m}}^{(j)}\right\}=\left\{\overline{\Psi}_{mx}^{(j)},\overline{\Psi}_{my}^{(j)},\overline{W}_{m}^{(j)};\overline{M}_{myx}^{(j)},\overline{M}_{my}^{(j)},\overline{Q}_{my}^{(j)}\right\}^{\mathrm{T}} \quad \text{(j=1,2,3)} \qquad (10.45)$$

After following the similar steps as is done in Section 10.1, one can obtain,

$$\begin{Bmatrix}\overline{\boldsymbol{Y}}_{\xi_I=1}^{(2)}\\ \overline{\boldsymbol{Y}}_{\xi_{III}=1}^{(1)}\\ \overline{\boldsymbol{Y}}_{\xi_I=1}^{(3)}\end{Bmatrix}=\begin{bmatrix}\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \tilde{\boldsymbol{W}}_{1,1}' & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}\end{bmatrix}\begin{bmatrix}\tilde{\boldsymbol{\mathcal{U}}}_{1,1}' & \tilde{\boldsymbol{\mathcal{U}}}_{1,2}' & \tilde{\boldsymbol{\mathcal{U}}}_{1,3}'\\ \tilde{\boldsymbol{\mathcal{U}}}_{2,1}' & \tilde{\boldsymbol{\mathcal{U}}}_{2,2}' & \tilde{\boldsymbol{\mathcal{U}}}_{2,3}'\\ \tilde{\boldsymbol{\mathcal{U}}}_{3,1}' & \tilde{\boldsymbol{\mathcal{U}}}_{3,2}' & \tilde{\boldsymbol{\mathcal{U}}}_{3,3}'\end{bmatrix}\begin{bmatrix}\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \tilde{\boldsymbol{V}}_{1,1}' & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}\end{bmatrix}\begin{Bmatrix}\overline{\boldsymbol{Y}}_{\xi_I=0}^{(2)}\\ \overline{\boldsymbol{Y}}_{\xi_{II}=0}^{(1)}\\ \overline{\boldsymbol{Y}}_{\xi_I=0}^{(3)}\end{Bmatrix} \quad (10.46)$$

or, simply rearranging,

$$\begin{Bmatrix} \overline{Y}^{(2)}_{\xi_I=1} \\ \overline{Y}^{(1)}_{\xi_{III}=1} \\ \overline{Y}^{(3)}_{\xi_I=1} \end{Bmatrix} = \begin{bmatrix} Q'_{1,1} & Q'_{1,2} & Q'_{1,3} \\ Q'_{2,1} & Q'_{2,2} & Q'_{2,3} \\ Q'_{3,1} & Q'_{3,2} & Q'_{3,3} \end{bmatrix}_{01} \begin{Bmatrix} \overline{Y}^{(2)}_{\xi_I=0} \\ \overline{Y}^{(1)}_{\xi_{II}=0} \\ \overline{Y}^{(3)}_{\xi_I=0} \end{Bmatrix} \qquad (10.47)$$

where the primed quantities indicate that this is the isotropic base plate case.

In the above, six of the eighteen dependent state variables must be prescribed at “each end point”. After substitution of the “Boundary Conditions” in the y-direction at ξ_I=0, ξ_I=1 for the upper and lower stiffening plates and for the base plate at ξ_{II}=0, ξ_{III}=1 into (10.47), the following matrix equation is obtained,

$$\left[\mathbf{C}'_0\right]_{(\mathrm{I.b})} \left\{\mathbf{Y}_0\right\}_{(\mathrm{I.b})} = \{\mathbf{0}\} \qquad \Rightarrow \qquad \left| \left[\mathbf{C}'_0\right]_{(\mathrm{I.b})} \right| = \{\mathbf{0}\} \qquad (10.48)$$

where $\left[\mathbf{C}'_0\right]_{(\mathrm{I.b})}$ is a (6×6) “Coefficient Matrix” which implicitly includes the dimensionless natural frequency parameter $\overline{\omega}_{mn}$. The “Coefficient Matrix $\left[\mathbf{C}'_0\right]_{(\mathrm{I.b})}$” is obtained from the “Coefficient Matrix” between the ‘‘final end point’’ and the ‘‘initial end point" in (10.47) by eliminating the rows corresponding to nonzero state variables at the “final end point’’ and the columns corresponding to zero state variables at the “initial end point’’. As in previous problems, the “Column Matrix $\{\mathbf{Y}_0\}$” includes the nonzero components of . $\left\{Y^{(2)}_{\xi_I=0}, Y^{(1)}_{\xi_I=0}, Y^{(3)}_{\xi_I=0}\right\}^{\mathrm{T}}$.

For a non-trivial solution of (10.48), the determinant of “Coefficient Matrix $\left[\mathbf{C}'_0\right]_{(\mathrm{I.b})}$” has to be equal to zero. The values of the dimensionless natural frequency parameter $\overline{\omega}_{mn}$ which makes the determinant of the “Coefficient Matrix $\left[\mathbf{C}'_0\right]_{(\mathrm{I.b})}$”equal to zero are the eigenvalues of the system under consideration. These are computed by assigning m, n values in (10.48). Therefore, the procedure should be repeated for a specific “Boundary Condition” in order to find the roots of the determinant of the “Coefficient Matrix $\left[\mathbf{C}'_0\right]_{(\mathrm{I.b})}$”. The roots are the nondimensionalized natural frequencies, $\overline{\omega}_{mn}$, in the composite plate system. After

then, the corresponding mode shapes, for a particular set of “Boundary Conditions” in the y-directions can be easily obtained.

10.4 Method of Solution for “Main PROBLEM II.a”

In this “Main PROBLEM II.a”, a procedure and algebraic operations will be performed similar to those in method of solution of the “Main PROBLEM I.a” in Chapter 4.

After following the same steps in Section 10.2, one can obtain the relation between the “state vectors” at the “initial end points” and “final end points” of the “Main PROBLEM II.a”,

$$\begin{Bmatrix} \overline{Y}^{(2)}_{\xi_I=1} \\ \overline{Y}^{(1)}_{\xi_{III}=1} \\ \overline{Y}^{(3)}_{\xi_I=1} \end{Bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & \tilde{W}_{1,1} & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \tilde{U}_{1,1} & \tilde{U}_{1,2} & \tilde{U}_{1,3} \\ \tilde{U}_{2,1} & \tilde{U}_{2,2} & \tilde{U}_{2,3} \\ \tilde{U}_{3,1} & \tilde{U}_{3,2} & \tilde{U}_{3,3} \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & \tilde{V}_{1,1} & 0 \\ 0 & 0 & I \end{bmatrix} \begin{Bmatrix} \overline{Y}^{(2)}_{\xi_I=0} \\ \overline{Y}^{(1)}_{\xi_{II}=0} \\ \overline{Y}^{(3)}_{\xi_I=0} \end{Bmatrix} \quad (10.49)$$

or rearranging,

$$\begin{Bmatrix} \overline{Y}^{(2)}_{\xi_I=1} \\ \overline{Y}^{(1)}_{\xi_{III}=1} \\ \overline{Y}^{(3)}_{\xi_I=1} \end{Bmatrix} = \begin{bmatrix} Q_{1,1} & Q_{1,2} & Q_{1,3} \\ Q_{2,1} & Q_{2,2} & Q_{2,3} \\ Q_{3,1} & Q_{3,2} & Q_{3,3} \end{bmatrix}_{01} \begin{Bmatrix} \overline{Y}^{(2)}_{\xi_I=0} \\ \overline{Y}^{(1)}_{\xi_{II}=0} \\ \overline{Y}^{(3)}_{\xi_I=0} \end{Bmatrix} \quad (10.50)$$

In the above, six of the eighteen state variables must be prescribed at “each end point”. After substitution of the “Boundary Conditions” in the y-direction at $\xi_I=0$, $\xi_I=1$ for the stiffening strips and at $\xi_{II}=0$, $\xi_{III}=1$ for the base plate into (10.50), the following matrix equation can be obtained,

$$[\mathbf{C}_0]_{(\mathrm{II.a})}\{\mathbf{Y}_0\}_{(\mathrm{II.a})} = \{\mathbf{0}\} \qquad \Rightarrow \qquad \left|[\mathbf{C}_0]_{(\mathrm{II.a})}\right| = \{\mathbf{0}\} \quad (10.51)$$

where $[\mathbf{C}_0]_{(\text{II.a})}$ is a (6x6) "Coefficient Matrix" which implicitly includes the dimensionless natural frequency parameter $\overline{\omega}_{mn}$. The "Coefficient Matrix $[\mathbf{C}_0]_{(\text{II.a})}$" is obtained from the "Coefficient Matrix" between the ''final end point'' and the ''initial end point" in (10.50) by eliminating the rows corresponding to nonzero state variables at the "final end point'' and the columns corresponding to zero state variables at the "initial end point''. Thus, $\{\mathbf{Y}_0\}$ is a "Column Matrix" whose elements are the nonzero components of $\left\{\boldsymbol{Y}^{(2)}_{\xi_1=0}, \boldsymbol{Y}^{(1)}_{\xi_1=0}, \boldsymbol{Y}^{(3)}_{\xi_1=0}\right\}^{\mathrm{T}}$.

Again, the determinant of the "Coefficient Matrix $[\mathbf{C}_0]_{(\text{II.a})}$" to be equal to zero in order to have nontrivial solution for the system in (10.51). The values of the dimensionless natural frequency parameter $\overline{\omega}_{mn}$ which makes the determinant of the "Coefficient Matrix $[\mathbf{C}_0]_{(\text{II.a})}$" equal to zero are the eigenvalues of the composite plate system under consideration. The procedure should be repeated for a specific "Boundary Conditions" in order to find the roots of the determinant of the "Coefficient Matrix $[\mathbf{C}_0]_{(\text{II.a})}$". The nondimensionalized natural frequencies $\overline{\omega}_{mn}$ of the entire plate system may be obtained by assigning m, n values. After then, one can easily find the corresponding mode shapes, (after the natural frequencies and the corresponding eigenvectors are obtained) for a particular the set of the "Boundary Conditions" in the y-direction.

10.5 Method of Solution for "Main PROBLEM II.b"

This is the "Isotropic Base Plate Case" in "Main PROBLEM II". Here too, the intermediate operations required in the solution technique will be omitted.

Taking a similar approach to those of Section 10.2 and Section 10.3, one can obtain the relation between the "state vectors" at the "initial end points" and "final end points" of the "Main PROBLEM II.b" such that,

$$\begin{Bmatrix} \overline{Y}^{(2)}_{\xi_I=1} \\ \overline{Y}^{(1)}_{\xi_{III}=1} \\ \overline{Y}^{(3)}_{\xi_I=1} \end{Bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & \tilde{W}'_{1,1} & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \tilde{\mathcal{U}}'_{1,1} & \tilde{\mathcal{U}}'_{1,2} & \tilde{\mathcal{U}}'_{1,3} \\ \tilde{\mathcal{U}}'_{2,1} & \tilde{\mathcal{U}}'_{2,2} & \tilde{\mathcal{U}}'_{2,3} \\ \tilde{\mathcal{U}}'_{3,1} & \tilde{\mathcal{U}}'_{3,2} & \tilde{\mathcal{U}}'_{3,3} \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & \tilde{V}'_{1,1} & 0 \\ 0 & 0 & I \end{bmatrix} \begin{Bmatrix} \overline{Y}^{(2)}_{\xi_I=0} \\ \overline{Y}^{(1)}_{\xi_{II}=0} \\ \overline{Y}^{(3)}_{\xi_I=0} \end{Bmatrix} \quad (10.52)$$

or, combining the matrices,

$$\begin{Bmatrix} \overline{Y}^{(2)}_{\xi_I=1} \\ \overline{Y}^{(1)}_{\xi_{III}=1} \\ \overline{Y}^{(3)}_{\xi_I=1} \end{Bmatrix} = \begin{bmatrix} \mathcal{Q}'_{1,1} & \mathcal{Q}'_{1,2} & \mathcal{Q}'_{1,3} \\ \mathcal{Q}'_{2,1} & \mathcal{Q}'_{2,2} & \mathcal{Q}'_{2,3} \\ \mathcal{Q}'_{3,1} & \mathcal{Q}'_{3,2} & \mathcal{Q}'_{3,3} \end{bmatrix}_{01} \begin{Bmatrix} \overline{Y}^{(2)}_{\xi_I=0} \\ \overline{Y}^{(1)}_{\xi_{II}=0} \\ \overline{Y}^{(3)}_{\xi_I=0} \end{Bmatrix} \quad (10.53)$$

In Equation (10.53), six of the eighteen state variables must be prescribed at "each end point". The substitution of the "Appropriate Boundary Conditions" in the y-direction at ξ_I=0, ξ_I=1 for the upper and lower stiffening plate strips and at ξ_{II}=0, ξ_{III}=1 for the base plate into (10.53), yield the following matrix equation,

$$\left[\mathbf{C}'_0\right]_{(\mathrm{II.b})} \{\mathbf{Y}_0\}_{(\mathrm{II.b})} = \{\mathbf{0}\} \qquad \Rightarrow \qquad \left| \left[\mathbf{C}'_0\right]_{(\mathrm{II.b})} \right| = \{\mathbf{0}\} \qquad (10.54)$$

where $\left[\mathbf{C}'_0\right]_{(\mathrm{II.b})}$ is a (6×6) "Coefficient Matrix" which implicitly includes the dimensionless natural frequency parameter $\overline{\omega}_{mn}$. The "Coefficient matrix $\left[\mathbf{C}'_0\right]_{(\mathrm{II.b})}$" is obtained from the "Coefficient Matrix" between the ''final end point'' and ''initial end points'' in (10.53) by eliminating the rows corresponding to nonzero state variable at the "final end point'' and the columns corresponding to zero state variable at the "initial end point''. Then, $\{\mathbf{Y}_0\}$ is a "Column Matrix" whose elements are the nonzero components of $\left\{Y^{(2)}_{\xi_I=0}, Y^{(1)}_{\xi_I=0}, Y^{(3)}_{\xi_I=0}\right\}^{\mathrm{T}}$.

The determinant of the "Coefficient Matrix $\left[\mathbf{C}'_0\right]_{(\mathrm{II.b})}$" which is equal to zero yields the nontrivial solution for the (10.54). The values of the dimensionless natural frequency parameter $\overline{\omega}_{mn}$ which makes the determinant of the "Coefficient Matrix

$\left[\mathbf{C}_0'\right]_{(\mathrm{II.b})}$" equal to zero are the eigenvalues of the problem under consideration. Therefore, the procedure should be repeated by assigning m, n values and a particular set of the "Boundary Conditions" in order to find the roots of the determinant of the "Coefficient Matrix $\left[\mathbf{C}_0'\right]_{(\mathrm{II.b})}$". The roots correspond to the nondimensionalized natural frequencies, $\overline{\omega}_{mn}$ of the entire composite plate system of the "Main PROBLEM II.b". After then, the corresponding mode shapes are obtained for a particular set of the "Boundary Conditions" in the y-directions.

10.6 Method of Solution for "Main PROBLEM III.a"

As pointed out previously, this is a "special case" of the "Main PROBLEM II.a". The doubly stiffened part of the composite system is moved to the left support. (see also Figures 3.6 and 3.7). In such a case, there are only two parts, namely Part I and Part II as shown in Figure 3.7.

Similar to the solution method of the proceeding problems, the initial step is to write the "Governing Systems of First Order Ordinary Differential Equations" in a "state vector" form as in (8.7) and (8.9). This is because only a single step integration is required in order to eliminate the differential operator and obtain the "dimensionless" fundamental dependent variables. Then rewriting the "Governing System of First Order Ordinary Differential Equations" for Part I as,

$$\frac{d}{d\xi_I}\begin{Bmatrix}\overline{\boldsymbol{Y}}_{mn}^{(2)}\\ \overline{\boldsymbol{Y}}_{mn}^{(1)}\\ \overline{\boldsymbol{Y}}_{mn}^{(3)}\end{Bmatrix}=\begin{bmatrix}\overline{\boldsymbol{C}}_{1,1} & \overline{\boldsymbol{C}}_{1,2} & \overline{\boldsymbol{C}}_{1,3}\\ \overline{\boldsymbol{C}}_{2,1} & \overline{\boldsymbol{C}}_{2,2} & \overline{\boldsymbol{C}}_{2,3}\\ \overline{\boldsymbol{C}}_{3,1} & \overline{\boldsymbol{C}}_{3,2} & \overline{\boldsymbol{C}}_{3,3}\end{bmatrix}\begin{Bmatrix}\overline{\boldsymbol{Y}}_{mn}^{(2)}\\ \overline{\boldsymbol{Y}}_{mn}^{(1)}\\ \overline{\boldsymbol{Y}}_{mn}^{(3)}\end{Bmatrix},\qquad (0<\xi_I<1)\qquad (10,0.55)$$

where ξ_I is defined as y_I/l_I''. The "Coefficient Matrix $[\overline{\boldsymbol{C}}]$" is of dimension (18x18) which explicitly includes the nondimensional geometric and material characteristics of the upper, and lower stiffening plate strips and the base plate and the adhesive layers and dimensionless natural frequency parameter $\overline{\omega}_{mn}$ of the composite system.

In the above, $\{\overline{Y}_{mn}^{(j)}\}$ (j=1,2,3) is the "state vector" including "dimensionless fundamental dependent variables", such that,

$$\{\overline{Y}_{mn}^{(j)}\}=\{\overline{\Psi}_{mnx}^{(j)},\overline{\Psi}_{mny}^{(j)},\overline{W}_{mn}^{(j)};\overline{M}_{mnyx}^{(j)},\overline{M}_{mny}^{(j)},\overline{Q}_{mny}^{(j)}\}^{T} \quad \text{(j=1,2,3)} \tag{10.56}$$

The next step involves the discretization of the "fundamental dependent variables" of the problem and the "Coefficient Matrices" in (8.7) or in (10.55) with respect to the independent variable ξ_I, along Part I region. This is realized by dividing the Part I region in Figure 3.6 into sufficient number of segments along ξ_I and pre-multiplying discrete version of (8.7) by "Global Integrating Matrix $[\mathcal{L}_I]$" which yields,

$$\begin{Bmatrix}\dot{\overline{Y}}^{(2)}\\ \dot{\overline{Y}}^{(1)}\\ \dot{\overline{Y}}^{(3)}\end{Bmatrix}-\begin{Bmatrix}\dot{\overline{Y}}_I^{(2)}\\ \dot{\overline{Y}}_I^{(1)}\\ \dot{\overline{Y}}_I^{(3)}\end{Bmatrix}=[\mathcal{L}_I]\left[\dot{\overline{C}}\right]\begin{Bmatrix}\dot{\overline{Y}}^{(2)}\\ \dot{\overline{Y}}^{(1)}\\ \dot{\overline{Y}}^{(3)}\end{Bmatrix} \tag{10.57}$$

where $\{\dot{\overline{Y}}^{(j)}\},\left[\dot{\overline{C}}\right]$ are the discrete versions of "state vector" $\{\overline{Y}_{mn}^{(j)}\}$ and the "Coefficient Matrix $[\overline{C}]$" in (8.7), respectively. Also, $\{\overline{Y}_I^{(j)}\}$ is the "state vector" evaluated at the "initial end point, ξ_I=0". The subscripts "m" and "n" in (10.57) are dropped from the matrices for convience, In (10.57), the superscripts denotes upper, lower stiffening plate strips and the base plate in the Part I region. In the detailed and open form, one can write the discretized $\{\dot{\overline{Y}}^{(j)}\}$, $\{\dot{\overline{Y}}_I^{(j)}\}$ and $\left[\dot{\overline{C}}\right]$ explicitly as,

$$\left\{\dot{\bar{Y}}^{(1)}\right\}=\begin{Bmatrix}\bar{\Psi}_{x1}^{(1)}\\ \bar{\Psi}_{x2}^{(1)}\\ .\\ .\\ \bar{\Psi}_{xn}^{(1)}\\ \bar{\Psi}_{y1}^{(1)}\\ \bar{\Psi}_{y2}^{(1)}\\ .\\ .\\ .\\ \bar{Q}_{y1}^{(1)}\\ .\\ .\\ \bar{Q}_{yn}^{(1)}\end{Bmatrix},\left\{\dot{\bar{Y}}^{(2)}\right\}=\begin{Bmatrix}\bar{\Psi}_{x1}^{(2)}\\ \bar{\Psi}_{x2}^{(2)}\\ .\\ .\\ \bar{\Psi}_{xn}^{(2)}\\ \bar{\Psi}_{y1}^{(2)}\\ \bar{\Psi}_{y2}^{(2)}\\ .\\ .\\ .\\ \bar{Q}_{y1}^{(2)}\\ .\\ .\\ \bar{Q}_{yn}^{(2)}\end{Bmatrix},\left\{\dot{\bar{Y}}^{(3)}\right\}=\begin{Bmatrix}\bar{\Psi}_{x1}^{(3)}\\ \bar{\Psi}_{x2}^{(3)}\\ .\\ .\\ \bar{\Psi}_{xn}^{(3)}\\ \bar{\Psi}_{y1}^{(3)}\\ \bar{\Psi}_{y2}^{(3)}\\ .\\ .\\ .\\ \bar{Q}_{y1}^{(3)}\\ .\\ .\\ \bar{Q}_{yn}^{(3)}\end{Bmatrix},\left\{\dot{\bar{Y}}_{1}^{(1)}\right\}=\begin{Bmatrix}\bar{\Psi}_{x1}^{(1)}\\ \bar{\Psi}_{x1}^{(1)}\\ .\\ .\\ \bar{\Psi}_{x1}^{(1)}\\ \bar{\Psi}_{y1}^{(1)}\\ \bar{\Psi}_{y1}^{(1)}\\ .\\ .\\ .\\ \bar{Q}_{y1}^{(1)}\\ .\\ .\\ \bar{Q}_{y1}^{(1)}\end{Bmatrix},\left\{\dot{\bar{Y}}^{(2)}\right\}=\begin{Bmatrix}\bar{\Psi}_{x1}^{(2)}\\ \bar{\Psi}_{x1}^{(2)}\\ .\\ .\\ \bar{\Psi}_{x1}^{(2)}\\ \bar{\Psi}_{y1}^{(2)}\\ \bar{\Psi}_{y1}^{(2)}\\ .\\ .\\ .\\ \bar{Q}_{y1}^{(2)}\\ .\\ .\\ \bar{Q}_{y1}^{(2)}\end{Bmatrix},\left\{\dot{\bar{Y}}_{1}^{(3)}\right\}=\begin{Bmatrix}\bar{\Psi}_{x1}^{(3)}\\ \bar{\Psi}_{x1}^{(3)}\\ .\\ .\\ \bar{\Psi}_{x1}^{(3)}\\ \bar{\Psi}_{y1}^{(3)}\\ \bar{\Psi}_{y1}^{(3)}\\ .\\ .\\ .\\ \bar{Q}_{y1}^{(3)}\\ .\\ .\\ \bar{Q}_{y1}^{(3)}\end{Bmatrix} \tag{10.58}$$

and,

$$\left[\dot{\bar{\mathcal{C}}}\right]=\begin{bmatrix}[\mathcal{C}_{1,1}]_{nxn} & [\mathcal{C}_{1,2}]_{nxn} & . & . & [\mathcal{C}_{1,18}]_{nxn}\\ . & . & . & . & [\mathcal{C}_{2,18}]_{nxn}\\ . & . & . & . & .\\ . & . & . & . & .\\ [\mathcal{C}_{18,1}]_{nxn} & . & . & . & [\mathcal{C}_{18,18}]_{nxn}\end{bmatrix} \tag{10.59}$$

The function of the second subscripts in (10.58) and (10.59) are to indicate the station with which they are associated or evaluated. In the above, the "Matrices. $[\mathcal{C}_{i,j}]$" (small characters) are the "Diagonal Matrices" which are the discretized elements of the "Coefficient Matrix $\left[\dot{\bar{\mathcal{C}}}\right]$" in (8.11) or in (10.55).

The detailed explanation of the "Global Integrating Matrix $[\mathcal{L}_I]$" is given in section 10.1. Therefore, it will not be repeated here. The important point is that the "Integrating Matrix" in (10.57) has a dimension of $18n_1 \times 18n_1$ and n_1 is the number of discretization points along Part I region.

After rearranging (10.57), a relation between the “state vector” at a ‘‘general station’’ ξ_I along in Part I region and the “state vector” at the “initial end point’’ $\xi_I=0$ in Part I region is obtained;

$$\left\{\dot{\bar{Y}}\right\}=[\dot{\mathcal{A}}]\left\{\dot{\bar{Y}}_I\right\} \tag{10.60}$$

where

$$[\dot{\mathcal{A}}]=\left([I]-[\mathcal{L}_I]\left[\dot{\bar{C}}\right]\right)^{-1} \tag{10.61}$$

In the above, $[\dot{\mathcal{A}}]$ is the discretized version of the “Modified Transfer Matrix” between a ‘‘general station along ξ_I” and the “initial end point $\xi_I=0$”.

In a detailed and open form, one can write,

$$\begin{Bmatrix} \bar{\Psi}_{x1}^{(2)} \\ \bar{\Psi}_{x2}^{(2)} \\ . \\ . \\ \bar{\Psi}_{xn}^{(2)} \\ \bar{\Psi}_{y1}^{(2)} \\ \bar{\Psi}_{y2}^{(2)} \\ . \\ . \\ \bar{\Psi}_{x1}^{(1)} \\ . \\ . \\ \bar{Q}_{y1}^{(3)} \\ \bar{Q}_{y2}^{(3)} \\ . \\ . \\ \bar{Q}_{yn}^{(3)} \end{Bmatrix} = \begin{bmatrix} \dot{\mathcal{A}}_{1,1} & \dot{\mathcal{A}}_{1,2} & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & \dot{\mathcal{A}}_{1,18} \\ \dot{\mathcal{A}}_{2,1} & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ \dot{\mathcal{A}}_{1,18} & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & \dot{\mathcal{A}}_{18,18} \end{bmatrix} \begin{Bmatrix} \bar{\Psi}_{x1}^{(2)} \\ \bar{\Psi}_{x1}^{(2)} \\ . \\ . \\ \bar{\Psi}_{x1}^{(2)} \\ \bar{\Psi}_{y1}^{(2)} \\ \bar{\Psi}_{y1}^{(2)} \\ . \\ . \\ \bar{\Psi}_{x1}^{(1)} \\ . \\ . \\ \bar{Q}_{y1}^{(3)} \\ \bar{Q}_{y1}^{(3)} \\ . \\ . \\ \bar{Q}_{y1}^{(3)} \end{Bmatrix} \tag{10.62}$$

The matrix $[\mathcal{Z}_1]$ is composed of (18x18) square blocks of dimension n_1 with respect to (10.61) and (10.64) and each individual block represents a relation between a state variable at "any general station ξ_I" along Part I region and another state variable at the "initial end point ξ_I=0" in Part I region.

Then, from the Equation (10.62), one can obtain the relation between the "state vector" at the "final end point ξ_I=1" and the "initial end point ξ_I=0" in Part I region by summing the elements on the integer multiples of nth row of $[\mathcal{Z}_1]$as follows,

$$\begin{Bmatrix} \overline{\Psi}^{(2)}_{xn} \\ \overline{\Psi}^{(2)}_{yn} \\ \cdot \\ \cdot \\ \cdot \\ \overline{Q}^{(3)}_{yn} \end{Bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} (\mathcal{Z}_{1,1})_{n,i} & \sum_{i=n+1}^{2n} (\mathcal{Z}_{1,2})_{n,i} & \cdot\ \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot & \sum_{i=17n+1}^{18n} (\mathcal{Z}_{18,18})_{n,i} \end{bmatrix}_{01} \begin{Bmatrix} \overline{\Psi}^{(2)}_{x1} \\ \overline{\Psi}^{(2)}_{y1} \\ \cdot \\ \cdot \\ \cdot \\ \overline{Q}^{(3)}_{y1} \end{Bmatrix} \tag{10.63}$$

Thus, rewriting (10.57),

$$\begin{Bmatrix} \overline{Y}^{(2)}_{\xi_I=1} \\ \overline{Y}^{(1)}_{\xi_I=1} \\ \overline{Y}^{(3)}_{\xi_I=1} \end{Bmatrix} = \begin{bmatrix} \tilde{\mathcal{Z}}_{1,1} & \tilde{\mathcal{Z}}_{1,2} & \tilde{\mathcal{Z}}_{1,3} \\ \tilde{\mathcal{Z}}_{2,1} & \tilde{\mathcal{Z}}_{2,2} & \tilde{\mathcal{Z}}_{2,3} \\ \tilde{\mathcal{Z}}_{3,1} & \tilde{\mathcal{Z}}_{3,2} & \tilde{\mathcal{Z}}_{3,3} \end{bmatrix}_{01} \begin{Bmatrix} \overline{Y}^{(2)}_{\xi_I=0} \\ \overline{Y}^{(1)}_{\xi_I=0} \\ \overline{Y}^{(3)}_{\xi_I=0} \end{Bmatrix} \tag{10.64}$$

where the end points are defined as ''initial end point ξ_I=0" and ''final end point ξ_I=1" along Part I region (as shown in Figures 3.6 and 3.7). The sub-matrices $[\tilde{\mathcal{Z}}_{i,j}]$ are partitioned square matrices of dimension (6x6) which implicitly includes the dimensionless natural frequencies $\overline{\omega}_{mn}$. The subscripts in "state vectors" of (10.64) denote the station with which the "state vector" is associated or evaluated along the "Overlap Region" or Part I Region.

Then, in a similar way, one can obtain a relation between the “state vectors” at the ‘‘initial end point $\xi_{II}=0$” and the “state vectors” at the ‘‘final end point $\xi_{II}=1$” for the Part II region (as shown in Figure 3.6 and 3.7) such that,

$$\begin{Bmatrix} \overline{\Psi}^{(1)}_{x1} \\ \overline{\Psi}^{(1)}_{x2} \\ \cdot \\ \cdot \\ \overline{\Psi}^{(1)}_{xn_2} \\ \overline{\Psi}^{(1)}_{y1} \\ \overline{\Psi}^{(1)}_{y2} \\ \cdot \\ \cdot \\ \cdot \\ \overline{Q}^{(1)}_{y1} \\ \cdot \\ \cdot \\ \overline{Q}^{(1)}_{yn_2} \end{Bmatrix} = \begin{bmatrix} \dot{w}_{1,1} & \cdot & \cdot & \cdot & \dot{w}_{1,6} \\ \dot{w}_{2,1} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \dot{w}_{6,1} & \cdot & \cdot & \cdot & \dot{w}_{6,6} \end{bmatrix} \begin{Bmatrix} \overline{\Psi}^{(1)}_{x1} \\ \overline{\Psi}^{(1)}_{x1} \\ \cdot \\ \cdot \\ \overline{\Psi}^{(1)}_{x1} \\ \overline{\Psi}^{(1)}_{y1} \\ \overline{\Psi}^{(1)}_{y1} \\ \cdot \\ \cdot \\ \cdot \\ \overline{Q}^{(1)}_{y1} \\ \cdot \\ \cdot \\ \overline{Q}^{(1)}_{y1} \end{Bmatrix} \tag{10.65}$$

where n_2 indicates the number of discretization points in Part II region and $[\tilde{w}_{i,j}]$ is the square block matrix of dimension n_2.

A relation between the “state vector” at the ‘‘initial end point $\xi_{II}=0$” and the “state vector” at the ‘‘final end point $\xi_{II}=1$” for Part II can be obtained from (10.65) as ,

$$\left\{ \overline{Y}^{(1)}_{\xi_{II}=1} \right\} = \left[\tilde{w}_{1,1} \right]_{01} \left\{ \overline{Y}^{(1)}_{\xi_{II}=0} \right\} \tag{10.66}$$

where $[\tilde{w}_{1,1}]$ is a (6x6) matrix which also implicitly includes the dimensionless natural frequency parameter $\overline{\omega}_{mn}$ of the composite system of the “Main PROBLEM III.a”.

In equations (10.64) and (10.66), the relations between the "state vectors" at the "initial end points" and "final end points" of the Part I and Part II regions are obtained, respectively. Therefore, the "Boundary Conditions" and the "Continuity Conditions" in the y-direction are used to obtain the natural frequencies of the plate system. One can prescribe any combination of the "Boundary Conditions" at $\xi_I=0$, $\xi_I=1$ for the stiffening plate strips and at $\xi_I=0$, $\xi_{II}=1$ for the base plate in the y-direction (as indicated in Figure 3.6). (The "Boundary Conditions" for the stiffening plate strips at $\xi_I=1$ are free edge boundary conditions, as shown in Figure 3.6). Then, writing,

The ''Continuity Conditions'' between the Part I and Part II as,

$$\left\{\overline{Y}^{(1)}_{\xi_I=1}\right\}=\left\{\overline{Y}^{(1)}_{\xi_{II}=0}\right\} \tag{10.67}$$

After inserting (10.67) into (10.66),

$$\left\{\overline{Y}^{(1)}_{\xi_I=1}\right\}=\left[\tilde{\mathcal{W}}_{1,1}\right]^{-1}\left\{\overline{Y}^{(1)}_{\xi_{II}=1}\right\} \tag{10.68}$$

Then, further substitutions give,

$$\begin{bmatrix} I & 0 & 0 \\ 0 & \tilde{\mathcal{W}}_{1,1}^{-1} & 0 \\ 0 & 0 & I \end{bmatrix}\begin{Bmatrix} \overline{Y}^{(2)}_{\xi_I=1} \\ \overline{Y}^{(1)}_{\xi_{II}=1} \\ \overline{Y}^{(3)}_{\xi_I=1} \end{Bmatrix}=\begin{bmatrix} \tilde{\mathcal{U}}_{1,1} & \tilde{\mathcal{U}}_{1,2} & \tilde{\mathcal{U}}_{1,3} \\ \tilde{\mathcal{U}}_{2,1} & \tilde{\mathcal{U}}_{2,2} & \tilde{\mathcal{U}}_{2,3} \\ \tilde{\mathcal{U}}_{3,1} & \tilde{\mathcal{U}}_{3,2} & \tilde{\mathcal{U}}_{3,3} \end{bmatrix}\begin{Bmatrix} \overline{Y}^{(2)}_{\xi_I=0} \\ \overline{Y}^{(1)}_{\xi_I=0} \\ \overline{Y}^{(3)}_{\xi_I=0} \end{Bmatrix} \tag{10.69}$$

where **[*I*]** is a "unit matrix" with dimension (6x6), $[\tilde{\mathcal{W}}_{1,1}]$ is a (6×6) matrix which implicitly includes the dimensionless natural frequency $\overline{\omega}_{mn}$ and $[\tilde{\mathcal{U}}_{i,j}]$ are partitioned square matrices of dimension (6x6) which also implicitly includes the dimensionless natural frequencies $\overline{\omega}_{mn}$ of the plate system. All the "state vectors" in (10.69) are in stations where the boundary or support conditions of the system are prescribed as in Figure 3.6. Therefore, one can obtain,

$$\begin{Bmatrix} \overline{Y}^{(2)}_{\xi_I=1} \\ \overline{Y}^{(1)}_{\xi_{II}=1} \\ \overline{Y}^{(3)}_{\xi_I=1} \end{Bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & \tilde{W}_{1,1} & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \tilde{U}_{1,1} & \tilde{U}_{1,2} & \tilde{U}_{1,3} \\ \tilde{U}_{2,1} & \tilde{U}_{2,2} & \tilde{U}_{2,3} \\ \tilde{U}_{3,1} & \tilde{U}_{3,2} & \tilde{U}_{3,3} \end{bmatrix} \begin{Bmatrix} \overline{Y}^{(2)}_{\xi_I=0} \\ \overline{Y}^{(1)}_{\xi_I=0} \\ \overline{Y}^{(3)}_{\xi_I=0} \end{Bmatrix} \quad (10.70)$$

or rearranging,

$$\begin{Bmatrix} \overline{Y}^{(2)}_{\xi_I=1} \\ \overline{Y}^{(1)}_{\xi_{II}=1} \\ \overline{Y}^{(3)}_{\xi_I=1} \end{Bmatrix} = \begin{bmatrix} Q_{1,1} & Q_{1,2} & Q_{1,3} \\ Q_{2,1} & Q_{2,2} & Q_{2,3} \\ Q_{3,1} & Q_{3,2} & Q_{3,3} \end{bmatrix}_{01} \begin{Bmatrix} \overline{Y}^{(2)}_{\xi_I=0} \\ \overline{Y}^{(1)}_{\xi_I=0} \\ \overline{Y}^{(3)}_{\xi_I=0} \end{Bmatrix} \quad (10.71)$$

In above Equation (10.71), six of the eighteen state variables must be prescribed at each end point. After inserting of the "Boundary Conditions" in the y-direction at ξ_I=0 and ξ_{II}=1 into (10.71), the matrix equation below is obtained then,

$$[\mathbf{C}_0]_{(\text{III.a})}\{\mathbf{Y}_0\}_{(\text{III.a})} = \{\mathbf{0}\} \qquad \Rightarrow \qquad \left|[\mathbf{C}_0]_{(\text{III.a})}\right| = \{\mathbf{0}\} \quad (10.72)$$

where $[\mathbf{C}_0]_{(\text{III.a})}$ is a (6x6) "Coefficient Matrix" which implicitly includes the dimensionless natural frequency parameter $\overline{\omega}_{mn}$. The "Coefficient Matrix $[\mathbf{C}_0]_{(\text{III.a})}$" is obtained from the "Coefficient Matrix" between the ''final and initial end points" in (10.71) by eliminating the rows corresponding to nonzero state variables at the "final end point'' and the columns corresponding to zero state variable at the "initial end point''. In Equation (10.72) $\{\mathbf{Y}_0\}$ is a vector whose elements are the nonzero components of $\left\{Y^{(2)}_{\xi_I=0}, Y^{(1)}_{\xi_I=0}, Y^{(3)}_{\xi_I=0}\right\}^{\mathrm{T}}$

The determinant of the "Coefficient Matrix $[\mathbf{C}_0]_{(\text{III.a})}$" be equal to zero yields the nontrivial solution for the system of (10.72) which are the dimensionless natural frequency parameter $\overline{\omega}_{mn}$ of the composite system. As in previous cases, the procedure should be repeated for a given set of the "Boundary Conditions" in order

to find the roots of the determinant of the "Coefficient Matrix $\left[\mathbf{C}_0\right]_{(\mathrm{III.a})}$". The roots are the nondimensional natural frequencies, $\overline{\omega}_{mn}$ of the plate system. One can easily find the corresponding mode shapes, (after the natural frequencies are obtained) for a particular set of "Boundary Conditions" in the y-directions.

10.7 Method of Solution for "Main PROBLEM III.b"

This problem may be considered as the "special case" of the "Main PROBLEM II.b" such that the doubly stiffened part is moved to or positioned at the left support (see Figures 3.6 and 3.7.).

As a first step, writing the set of the "Governing Systems of First Order Ordinary Differential Equations",

$$\frac{d}{d\xi_I}\begin{Bmatrix}\overline{\boldsymbol{Y}}_{mn}^{(2)}\\ \overline{\boldsymbol{Y}}_{mn}^{(1)}\\ \overline{\boldsymbol{Y}}_{mn}^{(3)}\end{Bmatrix}=\left[\overline{\overline{\boldsymbol{\alpha}}}'\right]\begin{Bmatrix}\overline{\boldsymbol{Y}}_{mn}^{(2)}\\ \overline{\boldsymbol{Y}}_{mn}^{(1)}\\ \overline{\boldsymbol{Y}}_{mn}^{(3)}\end{Bmatrix},\qquad (0<\xi_{\mathrm{I}}<1)\qquad \text{(in Part I region)}\qquad (10.73)$$

$$\frac{d}{d\xi_{II}}\left\{\overline{\boldsymbol{Y}}_{mn}^{(1)}\right\}=\left[\overline{\overline{\boldsymbol{\alpha}}}'\right]\left\{\overline{\boldsymbol{Y}}_{mn}^{(1)}\right\},\qquad (0<\xi_{\mathrm{II}}<1)\qquad \text{(in Part II region)}\qquad (10.74)$$

where ξ_{I} and ξ_{II} are defined as $y_{\mathrm{I}}/l_I^{''}$ and $y_{\mathrm{II}}/l_{II}^{''}$, respectively. The "Coefficient Matrix $\left[\overline{\overline{\boldsymbol{\alpha}}}'\right]$" is of dimension (18x18) explicitly includes the nondimensional geometric and material characteristics of the base plate and stiffening strips and the adhesive layer and also the dimensionless natural frequency parameter $\overline{\omega}_{mn}$ of the composite system. The "Coefficient Matrix $\left[\overline{\overline{\boldsymbol{\alpha}}}'\right]$" is of dimension (6x6) also explicitly includes the dimensionless geometric and material characteristics of the base plate in the Part II region as well as the dimensionless natural frequency $\overline{\omega}_{mn}$ of the plate system. $\left\{\overline{\boldsymbol{Y}}_{m}^{(j)}\right\}$ (j=1,2,3) is the "state vector" includes "dimensionless fundamental dependent variables" of the problem,

$$\left\{\overline{\boldsymbol{Y}}_{\boldsymbol{m}}^{(j)}\right\}=\left\{\overline{Y}_{mx}^{(j)},\overline{Y}_{my}^{(j)},\overline{W}_{m}^{(j)};\overline{M}_{myx}^{(j)},\overline{M}_{my}^{(j)},\overline{Q}_{my}^{(j)}\right\}^{\mathrm{T}} \quad \text{(j=1,2,3)} \tag{10.75}$$

Following the same steps as in Section 10.6, one can obtain,

$$\begin{Bmatrix} \overline{\boldsymbol{Y}}_{\xi_I=1}^{(2)} \\ \overline{\boldsymbol{Y}}_{\xi_{II}=1}^{(1)} \\ \overline{\boldsymbol{Y}}_{\xi_I=1}^{(3)} \end{Bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & \tilde{\boldsymbol{\mathcal{W}}}_{11} & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \tilde{\ } & \tilde{\ } & \tilde{\ } \\ 11 & 12 & 13 \\ \tilde{\ } & \tilde{\ } & \tilde{\ } \\ 21 & 22 & 23 \\ \tilde{\ } & \tilde{\ } & \tilde{\ } \\ 31 & 32 & 33 \end{bmatrix} \begin{Bmatrix} \overline{\boldsymbol{Y}}_{\xi_I=0}^{(2)} \\ \overline{\boldsymbol{Y}}_{\xi_I=0}^{(1)} \\ \overline{\boldsymbol{Y}}_{\xi_I=0}^{(3)} \end{Bmatrix} \tag{10.76}$$

or, simply combining,

$$\begin{Bmatrix} \overline{\boldsymbol{Y}}_{\xi_I=1}^{(2)} \\ \overline{\boldsymbol{Y}}_{\xi_{II}=1}^{(1)} \\ \overline{\boldsymbol{Y}}_{\xi_I=1}^{(3)} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{21} & Q_{22} & Q_{23} \\ Q_{31} & Q_{32} & Q_{33} \end{bmatrix}_{01} \begin{Bmatrix} \overline{\boldsymbol{Y}}_{\xi_I=0}^{(2)} \\ \overline{\boldsymbol{Y}}_{\xi_I=0}^{(1)} \\ \overline{\boldsymbol{Y}}_{\xi_I=0}^{(3)} \end{Bmatrix} \tag{10.77}$$

In Equation (10.77) six of the eighteen state variables must be prescribed at "each end point". Futher substitution of the "Boundary Conditions" in the y-direction at ξ_I=0 and ξ_{II}=1 into (10.77), one can obtain the following matrix equation,

$$\left[\mathbf{C}_0\right]_{(\mathrm{III.b})}\left\{\mathbf{Y}_0\right\}_{(\mathrm{III.b})}=\{\mathbf{0}\} \qquad \Rightarrow \qquad \left|\left[\mathbf{C}_0\right]_{(\mathrm{III.b})}\right|=\{\mathbf{0}\} \tag{10.78}$$

where $\left[\mathbf{C}_0\right]_{(\mathrm{III.b})}$ is a (6×6) "Coefficient Matrix" which implicitly includes the dimensionless natural frequency parameter $\overline{\omega}_{mn}$. The "Coefficient Matrix $\left[\mathbf{C}_0\right]_{(\mathrm{III.b})}$" is obtained from the coefficient matrix between the ''final and initial end points" in (10.77) by eliminating the rows corresponding to nonzero state variable at the "final end point'' and the columns corresponding to zero state variable at the "initial end point''. In Equation (10.78), $\{\mathbf{Y}_0\}$ includes the nonzero components of $\left\{\boldsymbol{Y}_{\xi_I=0}^{(2)},\boldsymbol{Y}_{\xi_I=0}^{(1)},\boldsymbol{Y}_{\xi_I=0}^{(3)}\right\}^{\mathrm{T}}$

The non-trivial solution of the matrix equation of (10.78) corresponds to the natural frequency parameter $\overline{\omega}_{mn}$ of the composite plate system. These natural frequencies are, then, the roots of the determinant of the "Coefficient Matrix $[\mathbf{C}_0]_{(\text{III.b})}$" as shown in Equation (10.78). After the natural frequencies are obtained, the corresponding mode shapes can easily be calculated and plotted.

CHAPTER 11

NUMERICAL RESULTS AND DISCUSSION

11.1 Some Remarks on Geometric and Material Characteristics and Boundary Conditions

In the previous Chapters, the theoretical formulations of the "Composite Plate or Panel System" in terms of "Main PROBLEM I", "Main PROBLEM II" and "Main PROBLEM III" were given. Then, recalling and rewriting,

- "Main PROBLEM I "

I-a) Free Flexural (or Bending) Vibrations of "Centrally Doubly Stiffened, Composite Orthotropic Plates or Panels (Orthotropic Base Plate or Panel Case)".

I-b) Free Flexural (or Bending) Vibrations of "Centrally Doubly Stiffened, Composite Plates or Panels (Isotropic Base Plate or Panel Case)".

- "Main PROBLEM II "

II-a) Free Flexural (or Bending) Vibrations of "Non-Centrally Doubly Stiffened, Composite Orthotropic Plates or Panels (Orthotropic Base Plate or Panel Case)".

II-b) Free Flexural (or Bending) Vibrations of "Non-Centrally Doubly Stiffened, Composite Plates or Panels (Isotropic Base Plate or Panel Case)".

- "Main PROBLEM III "

III-a) Free Flexural (or Bending) Vibrations of "Doubly Stiffened-at Support, Composite Plates or Panels (Orthotropic Base Plate or Panel Case)".

III-b) "Free Flexural (or Bending)" Vibrations of "Doubly Stiffened-at Support, Composite Plates or Panels (Isotropic Base Plate or Panel Case)".

In this Chapter, the parametric studies and the numerical results for three "Main PROBLEMS" defined in Chapter 3 are presented. In each "Main PROBLEM",

a) "Orthotropic Base Plate or Panel (with Orthotropic Stiffening Plate Strip)"
b) "Isotropic Base Plate or Panel (with Orthotropic Stiffening Plate Strip)"

are considered and the mode shapes and the parametric studies of the natural frequencies, depending upon various "Boundary Conditions" are presented in detail.

In the "Composite Plate or Panel System" (which has the "Orthotropic Base Plate (Kevlar-Epoxy) with Upper and Lower Orthotropic Stiffening Plate Strips (Graphite-Epoxy)"), the material and the geometric characteristics of the plates and the adhesive layers are displayed in Table 11.1.

Similarly, the "Composite Plate or Panel System" (which has the "Isotropic Base Plate (Aluminum Alloy) with Upper and Lower Orthotropic Stiffening Plate Strips (Graphite-Epoxy)"), the material and the geometric characteristics of the plates and the adhesive layers are displayed in Table 11.2.

In order to investigate the significant effects of the adhesive layer elastic constants on the mode shapes and on the natural frequencies and on the mode shapes, the "hard" and the relatively "soft" adhesive characteristics as shown in both Tables 11.1 and 11.2.are taken into account in the numerical studies.

For all cases, since the "Classical Lévy's Type Solutions" were used in the formulation of the problems, the "Boundary Conditions" are always assumed to be simply supported at sides x=0 and x=a since the Lévy's type solutions were used in

the formulation of problems. Hence, only the “Support Conditions” at sides y=0 and $y=b_2=L$ (i.e. supports in y-direction) will be given in Figures.

In the “Boundary Conditions” on all figures, the letter (S) is used to define ‘‘Simple Support Condition’’, (C) for ‘‘Clamped Support Condition’’ and (F) for ‘‘Free Edge Condition’’. Also, the “Boundary Conditions” are read from left to right. The first two letters show the support conditions of the upper stiffening plate strip, the second and third letters indicate the support conditions of the base plate and the last two letters indicate the boundary conditions of the lower stiffening plate strip.

For instance, (FFCCFF) means that, reading from left to right, the upper stiffening strip has ‘‘Free Edge Boundary Conditions’’ at y=0 and $y=b_2$, the base plate has ‘‘Clamped Boundary Conditions’’ at y=0 and $y=b_1=L$ and the lower stiffening strip has ‘‘Free Edge Boundary Conditions’’ at y=0 and $y=b_3=\ell_I$

Similarly, (SFSSSF) means that, reading from left to right, the upper stiffening strip has ‘‘Simple Support Boundary Conditions’’ at y=0 and ‘‘Free Edge Boundary Conditions’’ at $y=b_2=L$, the base plate has ‘‘Simple Support Boundary Conditions’’ at y=0 and $y=b_1=L$ and the lower stiffening strip has ‘‘Simple Support Boundary Conditions’’ at y=0 and ‘‘Free Edge Boundary Conditions’’ at $y=b_3=\ell_I$.

Table 11.1. Composite Plate or Panel System
(Geometric and Material Characteristics)
(Orthotropic Base Plate and Orthotropic Stiffening Plate Strips)

Kevlar - Epoxy (Base Plate) (j=1)	*Graphite- Epoxy (Stiffening Plate Strips) (j=2) (upper one) (j=3) (lower one)*	*Adhesive Layer (soft) (k=2) (upper one) (k=3) (lower one)*	*Adhesive Layer (hard) (k=2) (upper one) (k=3) (lower one)*
$Ex_1 = 5.50$ GPa $Ey_1 = 76.0$ GPa $Gxy_1 = 2.10$ GPa $Gxz_1 = 1.50$ GPa $Gyz_1 = 2.00$ GPa $\nu xy_1 = 0.024$ $\nu yx_1 = 0.34$ $\rho_1 = 1.3$ gr/cm^3 $h_1 = 0.007$ m. $a = 0.5$ m. $b_1 = L = 1.0$ m.	$Ex_j = 11.71$ GPa $Ey_j = 137.8$ GPa $Gxy_j = 5.51$ Gpa $Gxz_j = 2.50$ GPa $Gyz_j = 3.00$ GPa $\nu xy_j = 0.0213$ $\nu yx_j = 0.25$ $\rho_j = 1.6$ gr/cm^3 $h_j = 0.007$ m. $a = 0.5$ m. $b_2 = b_3 = \ell_1 = 0.3$ m. $\tilde{b} = 0.35$ m.	$Ea_k / B_{11}^{(1)} = 4.22 \times 10^{-4}$ $Ga_k / B_{11}^{(1)} = 2.11 \times 10^{-4}$ $ha_k = 0.15 \times 10^{-3}$ m. ρ_a = neglected --- $Ea_k = 2.34$ MPa $Ga_k = 1.17$ MPa	$Ea_k / B_{11}^{(1)} = 0.340$ $Ga_k / B_{11}^{(1)} = 0.119$ $ha_k = 0.15 \times 10^{-3}$ m. ρ_a = neglected --- $Ea_k = 4.0$ GPa $Ga_k = 1.4$ GPa

Table 11.2. Composite Plate or Panel System
(Geometric and Material Characteristics)
(Isotropic Base Plate and Orthotropic Stiffening Plate Strips)

Aircraft Aluminum (Base Plate) (j=1)	*Graphite- Epoxy (Stiffening Plate Stips) (j=2) (upper one) (j=3) (lower one)*	*Adhesive Layer (soft) (k=2) (upper one) (k=3) (lower one)*	*Adhesive Layer (hard) (k=2) (upper one) (k=3) (lower one)*
$E = 72.69$ GPa $G = 25.78$ GPa $\nu = 0.313$ $\rho = 2.796$gr/cm^3 $h_1 = 0.007$m. $a = 0.5$ m. $b_2 = b_3 = \ell_1 = 0.3$ m. $b_1 = L = 1.0$ m.	$Ex_j = 11.71$ GPa $Ey_j = 137.8$ GPa $Gxy_j = 5.51$ Gpa $Gxz_j = 2.50$ GPa $Gyz_j = 3.00$ GPa $\nu xy_j = 0.0213$ $\nu yx_j = 0.25$ $\rho_j = 1.6$ gr/cm^3 $h_j = 0.007$ m. $a = 0.5$ m. $b_2 = b_3 = \ell_1 = 0.3$ m. $\tilde{b} = 0.35$ m.	$Ea_k = 2.34$ MPa $Ga_k = 1.17$ MPa $ha_k = 0.15 \times 10^{-3}$ m ρ_a = neglected	$Ea_k = 4.0$ GPa $Ga_k = 1.4$ GPa $ha_k = 0.15 \times 10^{-3}$ m ρ_a = neglected

11.2 Numerical Results and Discussion for "Main PROBLEM I.a"

In this "Main PROBLEM I.a", the "Orthotropic Base Plate" is made of evlar-Epoxy and the " pper and Lower Stiffening Plate Strips" are chosen as Graphite-Epoxy plates. In Figures, the adhesive layers are chosen as the "hard" and the "soft" adhesive cases with various boundary conditions. Also, in order to see the significant effects of the adhesive layer on the mode shapes and the corresponding frequencies, the adhesive shear modulus is assumed to be $G_{a2} = G_{a3} = 0$ (10^{-8}) 0. The material and geometric characteristics of the composite plate system are shown in Table 11.1. The elastic constants of the upper and lower adhesive layers are assumed to be identical in the numerical calculations (they are assumed to be dissimilar in the theoretical formulations).

Several mode shapes corresponding to the several support conditions will be considered and discussed next.

In Figure 11.1, the first four mode shapes and the corresponding dimensionless natural frequencies of the composite plate or panel system are displayed for the "hard" adhesives case and with the (FFCCFF) support conditions.

It can be easily observed from Figure 11.1, that in spite of the symmetry of the entire system (the geometry, the material and the B.C.'s) with respect to the mid-plane perpendicular to the base plate, the mode shapes are not exactly symmetric and/or skew-symmetric. However, in Figure 11.2 for the "soft" adhesive case with the same (FFCCFF) support conditions and, in particular, in Figure 11.5 with the same B.C.'s and with the adhesive shear modulus $G_{a2} = G_{a3} = 0$ (10^{-8}) 0, it can be observed that the mode shapes become symmetric and/or skew-symmetric with respect to the previously defined mid-plane. These observations can also be correlated by comparing the side-views of the mode shapes as shown in Figures 11.1(b) and 11.5(b).

Thus, one can conclude that, in spite of the symmetric geometry, material and the B.C.'s, if the adhesive shear modulus is not sufficiently small (or zero), then the

mode shapes are not going to be symmetric and/or skew-symmetric. Also, from the aforementioned Figures, it can be said that the adhesive shear modulus significantly influence the mode shapes and the corresponding natural frequencies.

Another important result of the present study is that, in the “hard” adhesives case, the central portion of the composite system is almost stationary due to the ,overly stiffening effect of the upper and lower stiffening plate strips and also due to the “hard” adhesive. This conclusion is also very important from the point of view of fracture mechanics. (The almost stationary nature of the doubly stiffened portion of the plate system in the “hard” adhesives case, acts favorably in slowing down and/or eliminating the propagation of the existing crack(s) in the base plate or panel). It is obvious here, that similar conclusions can be drawn by comparing Figures 11.3 and 11.4 and also Figure 11.3 and 11.6. It is obvious that the “soft” adhesive, even when the base plate is doubly stiffened, is not going to prevent deflections in the stiffened portion of the plate system. Hence, the cracks, if they are any, is going to propagate relatively easily.

Hence, the mode shapes and the corresponding natural frequencies are drastically affected by the elastic constants (specially by the shear modulus) of the adhesive layers between the base plate and the stiffening plate strips.

In Figures 11.1 and 11.2, “hard” and the “soft” adhesives cases, respectively, in Figures 11.3 and 11.4 correspond with the (FFCCFF) and (FFSSFF) support conditions, respectively. Although the mode shapes exhibit similar trends, but the corresponding frequencies are considerable different due to the significant influence of the boundary conditions in the y-direction.

It can be said as a conclusion that the stiffer (or more constraining) the support conditions in the y-direction, the higher natural frequencies (provided the adhesive layers, in both cases are same).

Now, the parametric studies for “Main PROBLEM I.a” will be considered and discussed next.

In Figures 11.7, 11.8, and 11.9, 11.10 the “Aspect Ratio (b_2/a)” are considered. The dependency of the natural frequencies on the (m,n) values (and also the dependency on natural modes) is plotted in the “hard” adhesives case with the (FFCCFF) and the (FFSSFF) support conditions, respectively. Recalling “m” is the number of half waves in the x-direction and “n” is the second subscript in the natural frequency parameter $\overline{\omega}_{mn}$, the natural frequencies grow gradually up to the “Aspect Ratio=1.2” value, and, after that value, start increasing sharply, in the third and higher modes. A similar trend in the natural frequencies is observed in Figures 11.11 and 11.12 for the (FFFFFF) boundary conditions.

The natural frequencies with respect to the “Aspect Ratio” (b_2/a) from the first to the fourth are plotted in the “hard” adhesives case, for various boundary conditions in Figures 11.13 through 11.16, respectively. As the “Aspect Ratio (b_2/a)” or the wetted-area increases, so the natural frequencies. This result is expected because of the increasing stiffness of the composite system as the wetted-area increases or spreads.

In Figures 11.17 and 11.18 the natural frequency parameter versus the “Aspect Ratio (b_2/a)” are plotted in the “soft” adhesives cases and with the (FFCCFF) boundary conditions. Similarly, in Figures 11.19 and 11.20 the natural frequencies versus the “Aspect Ratio (b_2/a)” presented in the “soft” adhesives case and with the (FFSSFF) support conditions. Also, similarly, in Figures 11.19 and 11.20 the natural frequencies versus the “Aspect Ratio (b_2/a)” presented in the “soft” adhesives case and with the (FFFFFF) support conditions.

One may conclude from the aforementioned figures, the natural frequencies increase with some sort of a wavy pattern with the increasing the “Aspect Ratio (b_2/a)”.

In the parametric studies, the influence of the “ oint Length Ratio (L/b_2)” is investigated. Then, the natural frequencies versus the “ oint Length Ratio (L/b_2)” are presented in Figures 11.23, 11.24 and 11.25, 11.26 and 11.27 and 11.28, in the “hard” adhesives case for the (FFCCFF), (FFSSFF) and the (FFFFFF) boundary

conditions respectively. In all the above Figures, the frequency curves corresponding to the modes from first to fourth exhibit closely similar patterns. In other words, as the "Joint Length Ratio (L/b_2)" increases the frequencies sharply decrease as can be expected and eventually they become asymptotic to certain values.

Similar to the aforementioned figures, in Figures 11.29, 11.30,and 11.31, 11.32 and 11.33, 11.34, the natural frequencies versus the "Joint Length Ratio (L/b_2)" are plotted, in the "soft" adhesives case, with the aforementioned the (FFCCFF), (FFSSFF) and (FFFFFF) support conditions, respectively. The trend or the behavior of the natural frequencies is similar. However, the first and the second and the third, the fourth dimensionless frequencies are very close to each other in the "soft" adhesives case.

The effect of the "Bending Rigidity Ratio $\boldsymbol{D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})}$" on the natural frequencies are also investigated. Thus, Figure 11.35, 11.36 and 11.37, 11.38 and 11.39, 11.40 presents the natural frequencies versus the "Bending Rigidity Ratio $\boldsymbol{D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})}$" in the "hard" adhesives case with the (FFCCFF), (FFSSFF), and the (FFFFFF) support conditions , respectively. It is interesting to observe that the first, second, and the third, fourth dimensionless frequencies are extremely close to each other. Also, the trend is that they drop sharply for the small values of the "Bending Rigidity Ratio $\boldsymbol{D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})}$" up to 0.6, after than they decrease gradually.

It should be pointed out here, that, while the dimensionless natural frequency parameters decrease as the "Bending Rigidity Ratio $\boldsymbol{D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})}$" increases, the real dimensional natural frequency increases as excepted (see for instance, the expression (4.16) for the dimensionless natural frequency parameter).

The effect of the "Bending Rigidity Ratio $\boldsymbol{D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})}$" on the dimensionless natural frequencies in the "soft" adhesives case are computed in Figures 11.41, 11.42 and 11.43, 11.44 and 11.45, 11.46 for several modes corresponding to the (FFCCFF), (FFSSFF) and the (FFFFFF) boundary conditions

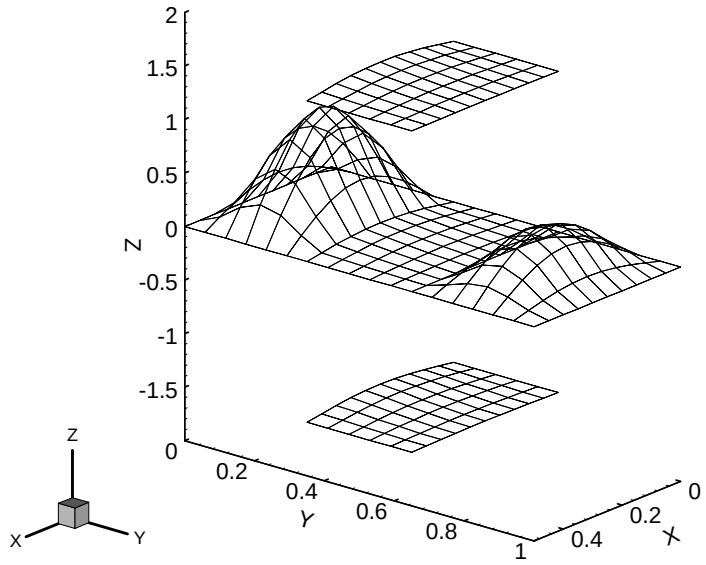
respectively. The dimensionless natural frequencies decrease relatively sharply, at first, (up to 0.6 value of the "Bending Rigidity Ratio $\boldsymbol{D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})}$") and, after that value, the decrease is rather gradual. However, there does not exist another common pattern in those figures. As in the similar "hard" adhesives case, the real dimensional natural frequencies increase while the dimensionless natural frequencies decrease (see again for example, the expression (4.16))

The influence of the "Cross Bending Rigidity Ratio $\boldsymbol{D_{22}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})}$" on the natural frequencies, in the "hard" adhesives case, are demonstrated in Figures 11.59, 11.60 and 11.61, 11.62 and 11.63, 11.64 for the support conditions (FFCCFF), (FFSSFF) and (FFFFFF), respectively. The general trend in the natural frequencies shows almost linear increases as the "Cross-Bending Rigidity Ratio" increases.

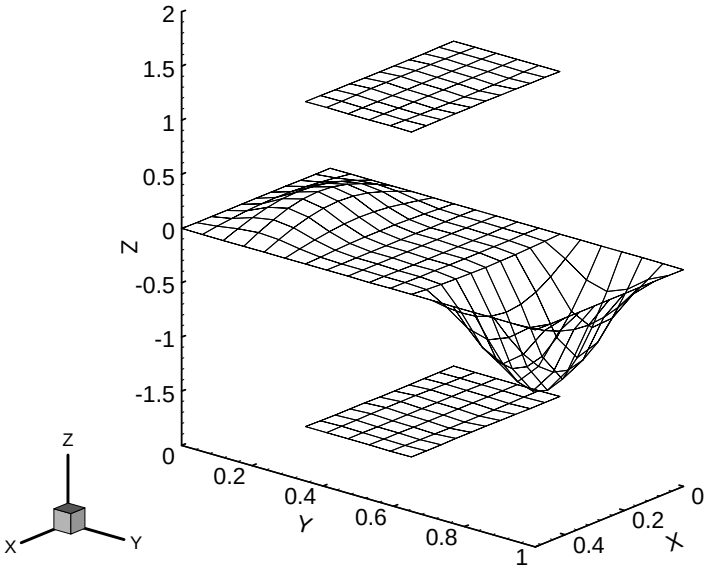
In Figures 11.65, 11.66 and 11.67, 11.68 and 11.69, 11.70, the natural frequencies versus "Cross Bending Rigidity Ratio $\boldsymbol{D_{22}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})}$" are plotted, in the "soft" adhesives case, for the boundary conditions (FFCCFF), (FFSSFF) and (FFFFFF), respectively. The common trend or pattern in all those Figures is that, for the first through up to the fourth mode, the dimensionless natural frequency increases non-linearly with the increasing "Cross Bending Stiffness Ratio". In the first mode, however, the increases in the natural frequency are again non-linear although extremely gradual.

The direct effects of the adhesive layer elastic constants E_a, G_a on the dimensionless natural frequencies are investigated for the "Main PROBLEM I.a". For this purpose, the dimensionless natural frequencies versus the "Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$" are plotted (while the other elastic constant kept constant) in Figures 11.71, 11.73 and 11.75 for the support conditions (FFCCFF), (FFSSFF) and (FFFFFF), respectively. Similarly, the dimensionless natural frequencies versus the "Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3})/B_{11}^{(1)}$" are presented in Figures 11.72, 11.74 and 11.76 for the boundary condition (FFCCFF), (FFSSFF), (FFFFFF), respectively.

It is obvious from the aforementioned Figures, the “Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3})/B_{11}^{(1)}$” has very significant effect on the natural frequencies of the composite plate systems considered in the “Main PROBLEM I.a”. This result is expected if the shear deformable character of the Mindlin Plates is taken into account in the composite system. The influence of the “Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$” on the natural frequencies is rather relatively weak depending on the support conditions on the plate or panel system.



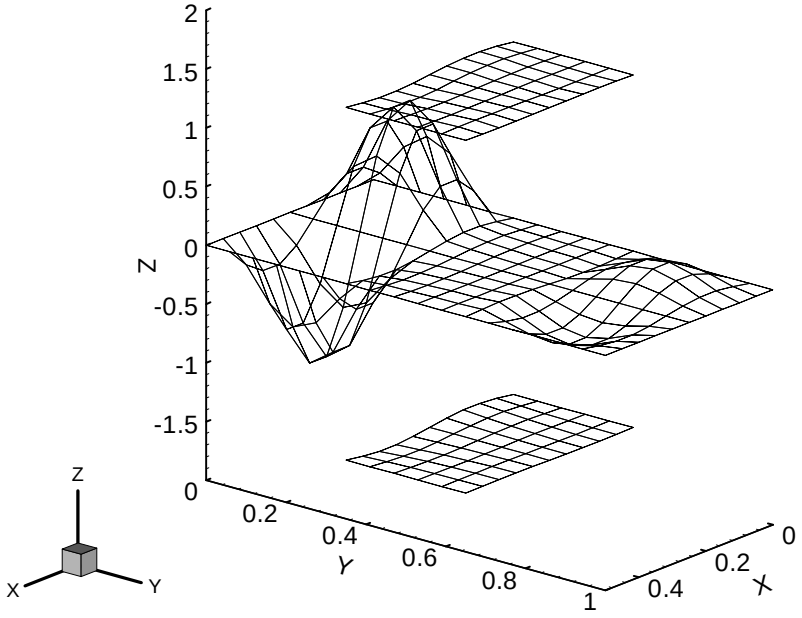
a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 1949.796$



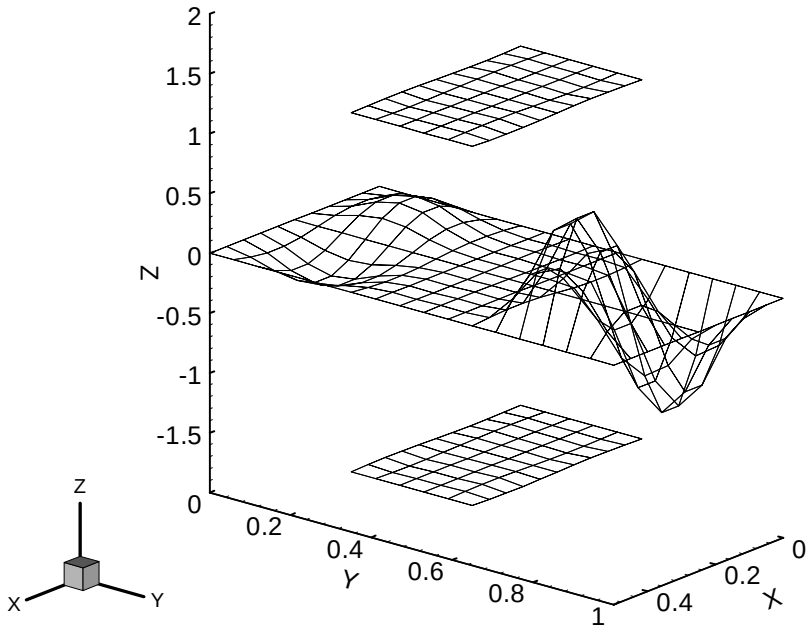
b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{12} = 2004.785$

Figure 11.1. Mode Shapes and Natural Frequencies of "Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFCCFF) B.C.'s in y-direction, "Hard" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

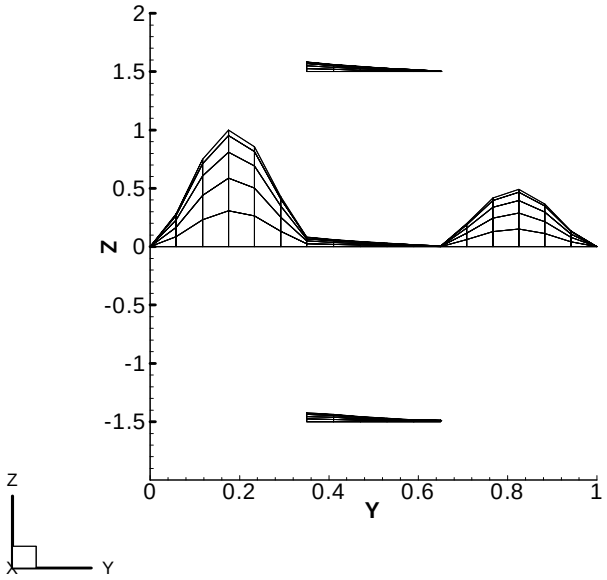


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{21} = 2233.800$

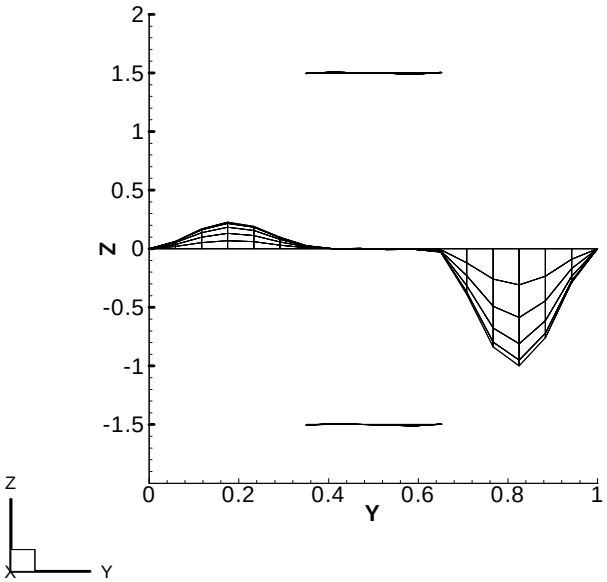


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{22} = 2245.243$

Figure 11.1. (continued)



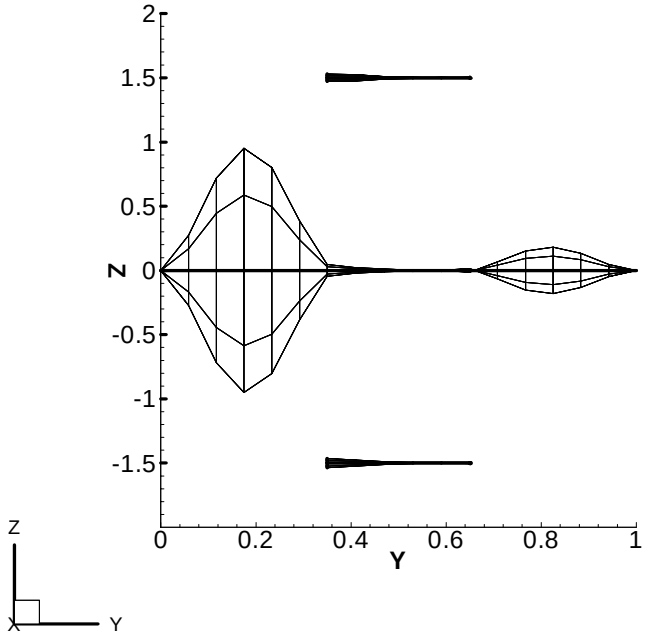
a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 1949.796$



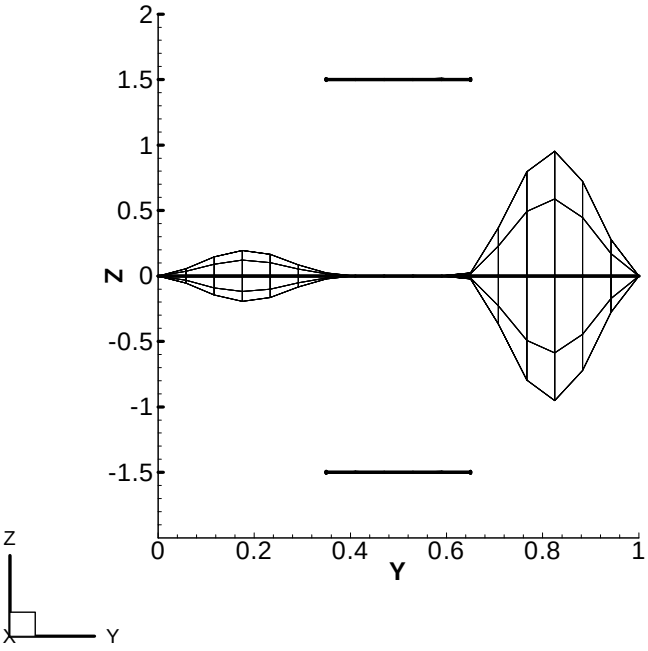
b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{12} = 2004.785$

Figure 11.1(b). "Side Views" of Mode Shapes and Natural Frequencies of "Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFCCFF) B.C.'s in y-direction, "Hard" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

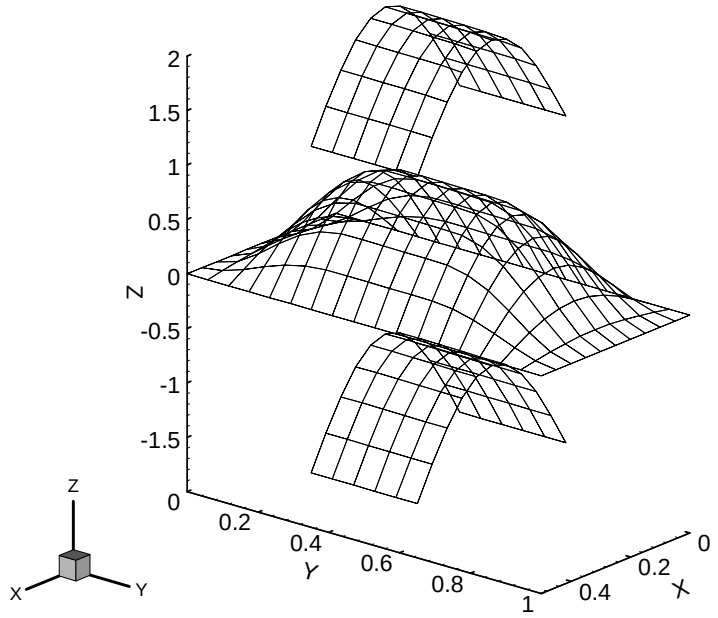


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{21} = 2233.800$

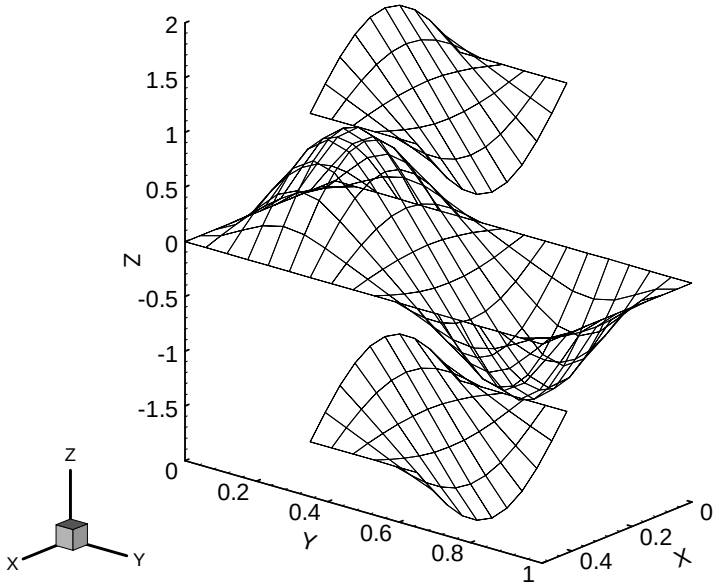


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{22} = 2245.243$

Figure 11.1(b). (continued)



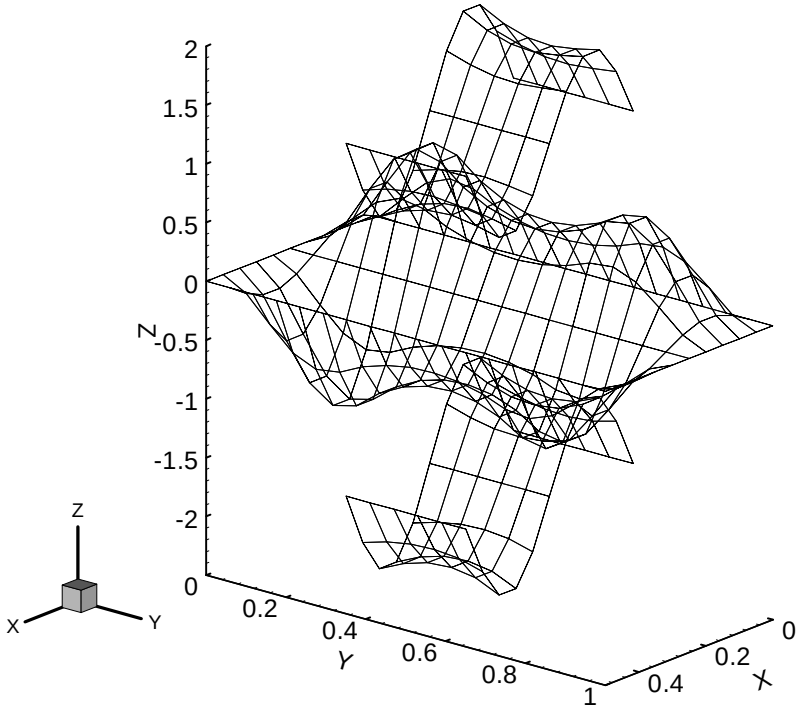
a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 262.471$



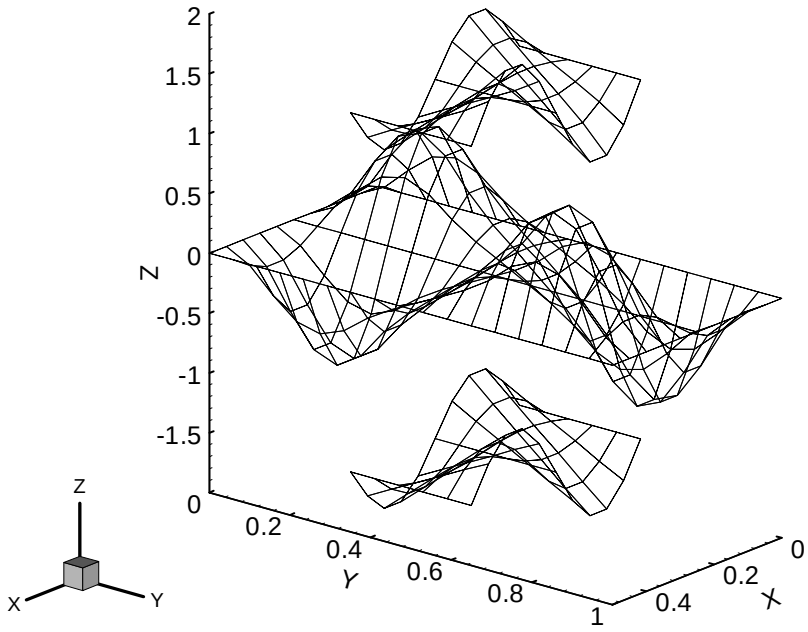
b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{12} = 710.457$

Figure 11.2. Mode Shapes and Natural Frequencies of "Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFCCFF) B.C.'s in y-direction, "Soft" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

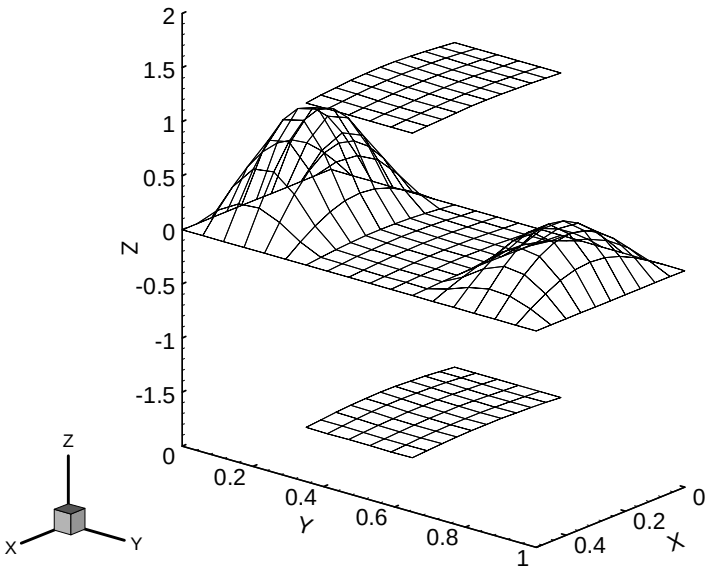


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{21} = 1003.954$

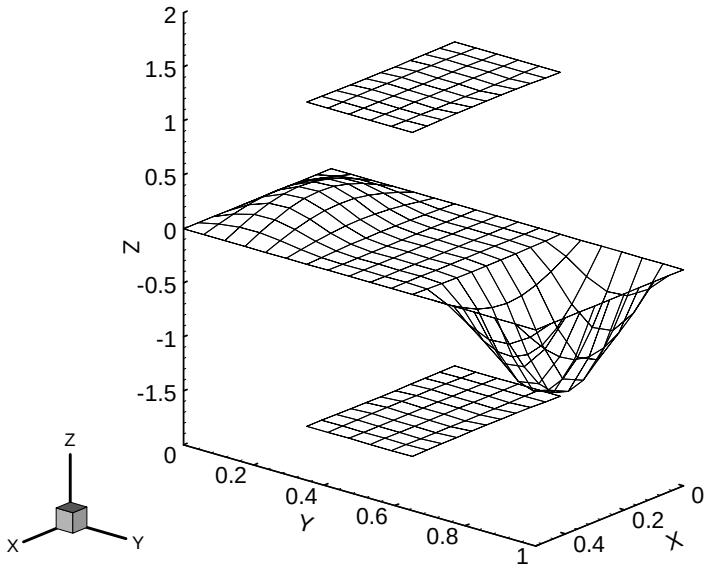


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{22} = 1152.216$

Figure 11.2. (continued)



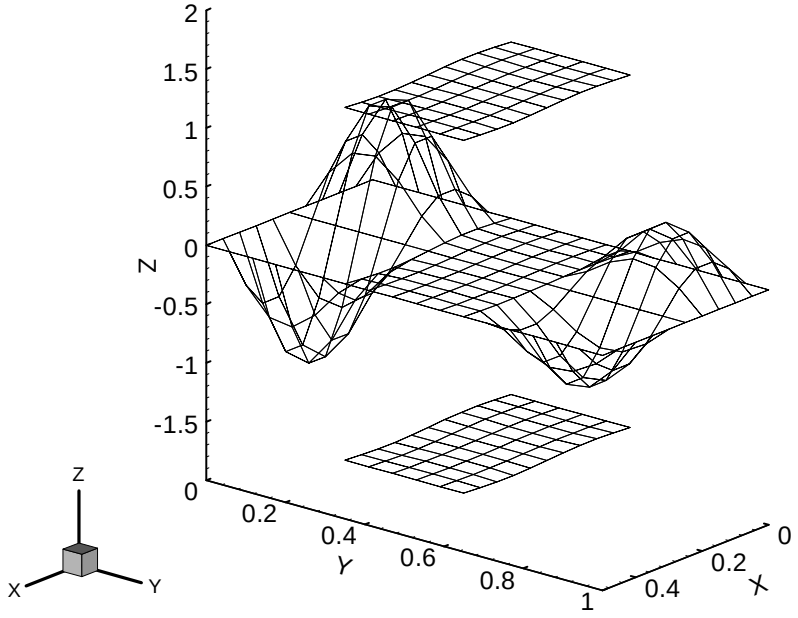
a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 983.032$



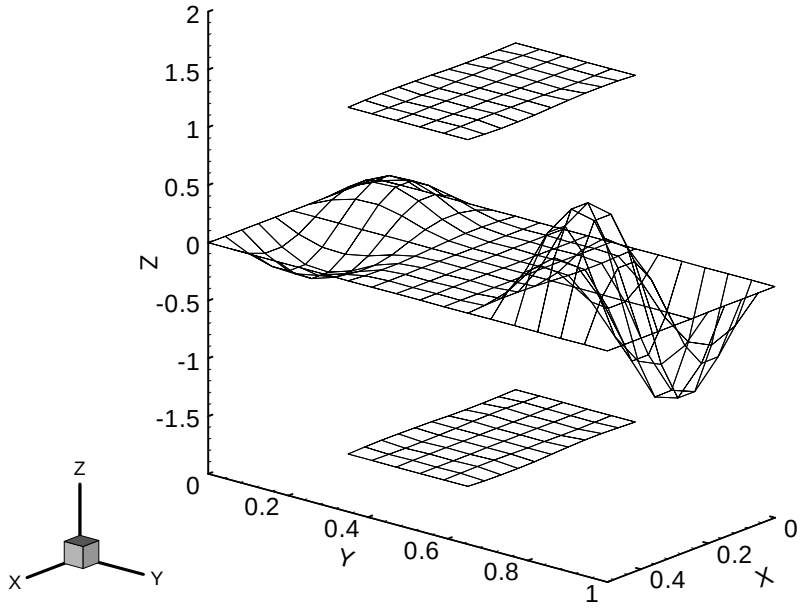
b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{12} = 1009.697$

Figure 11.3. Mode Shapes and Natural Frequencies of "Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFSSFF) B.C.'s in y-direction, "Hard" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

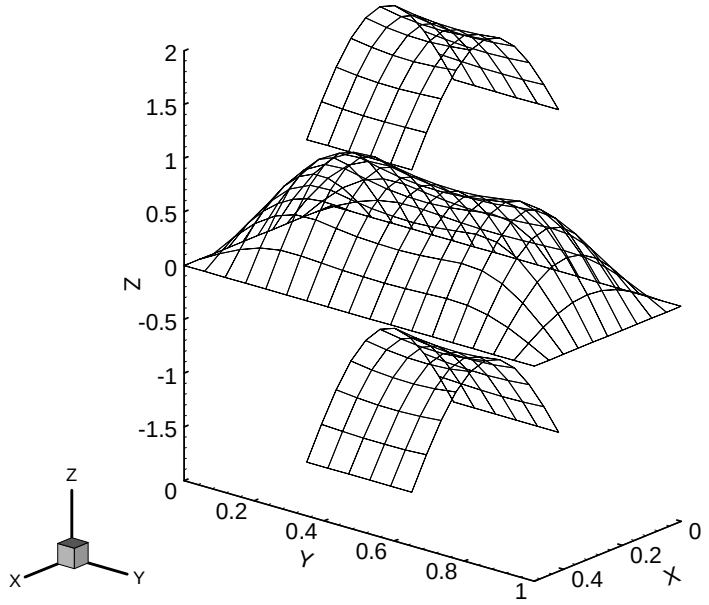


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{21} = 1232.446$

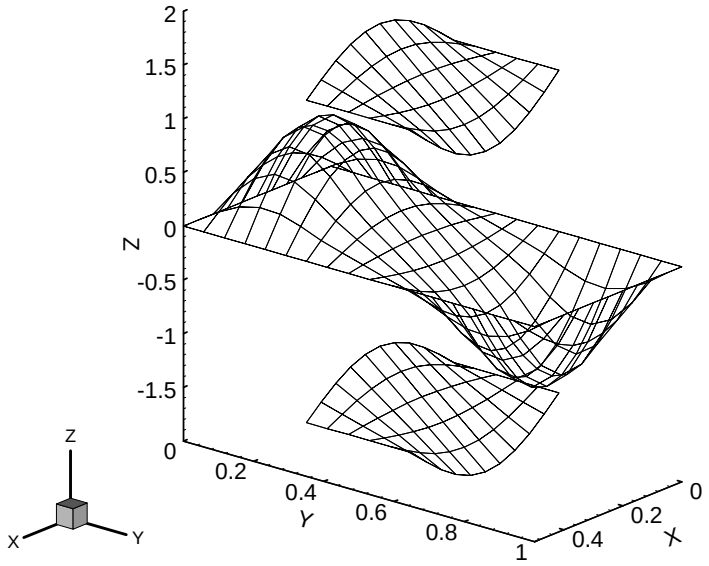


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{22} = 1244.606$

Figure 11.3. (continued)



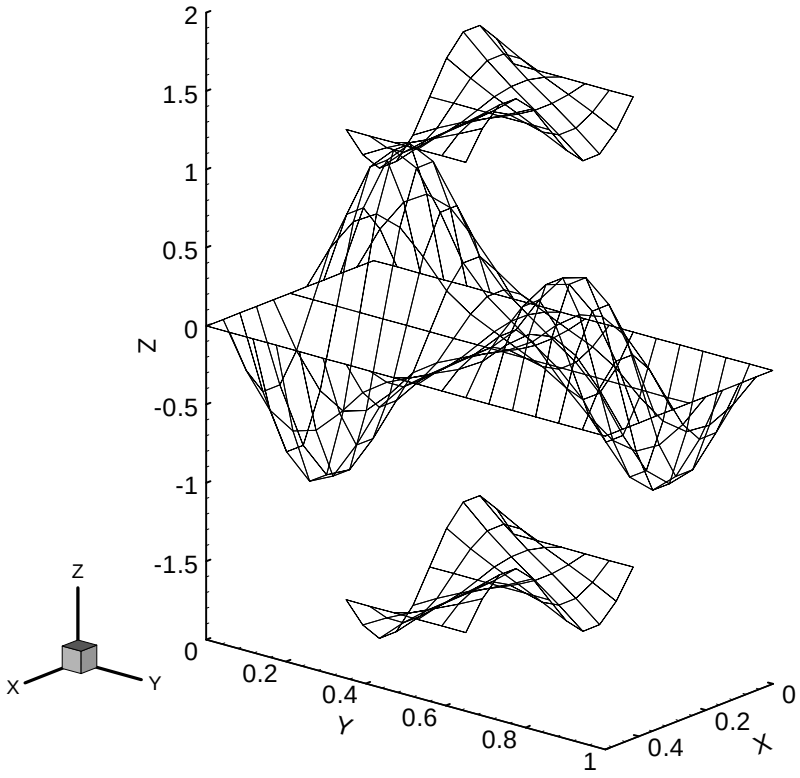
a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 221.318$



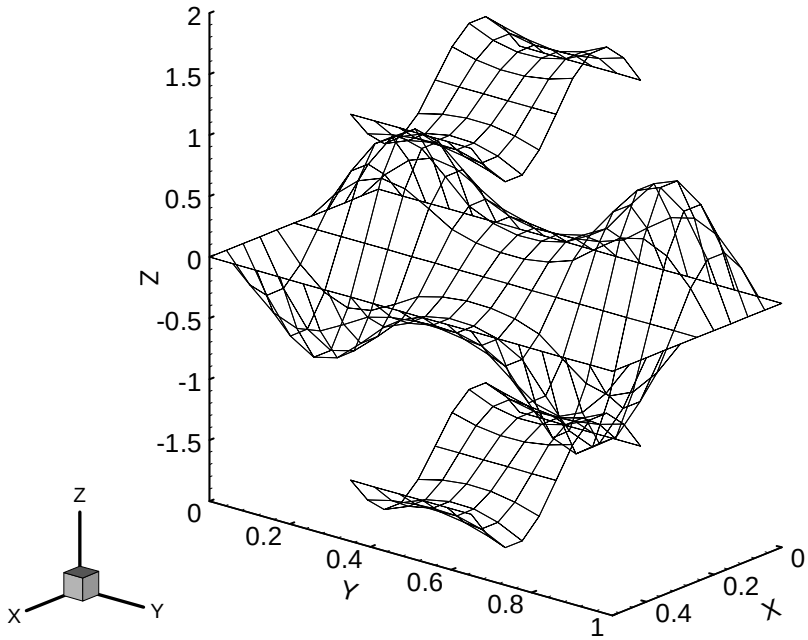
b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{12} = 434.435$

Figure 11.4. Mode Shapes and Natural Frequencies of "Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFSSFF) B.C.'s in y-direction, "Soft" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

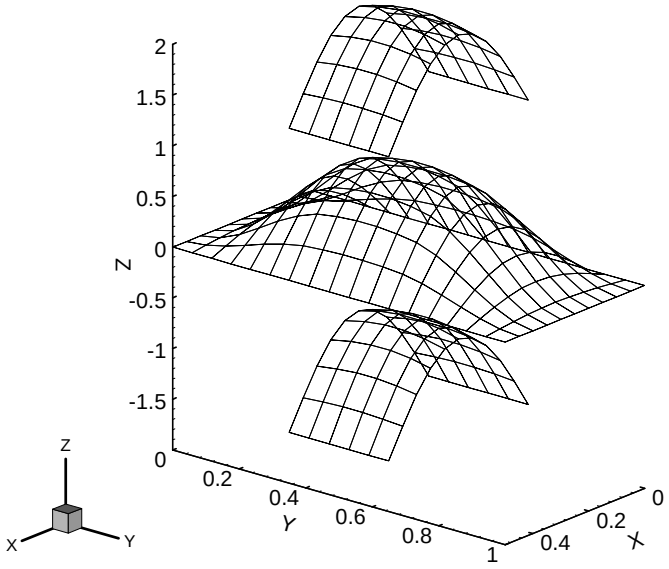


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{21} = 741.990$

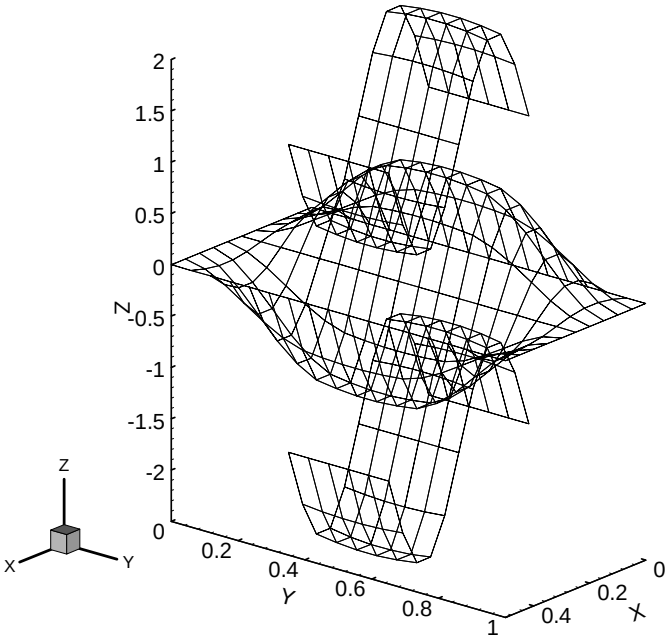


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{22} = 756.576$

Figure 11.4. (continued)



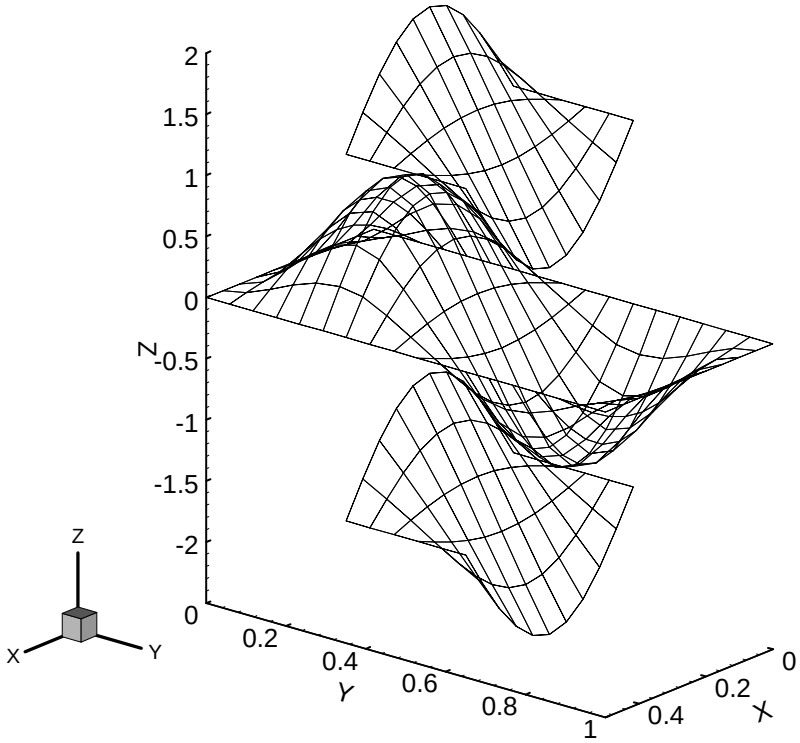
a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 35.746$



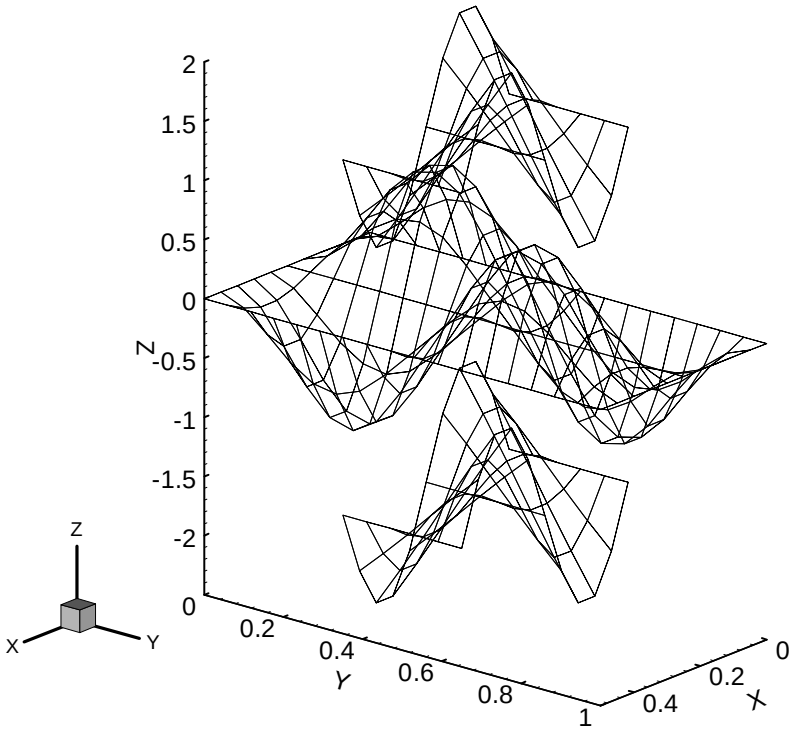
b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{21} = 216.903$

Figure 11.5. Mode Shapes and Natural Frequencies of "Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFCCFF) B.C.'s in y-direction, Adhesive $G_{a2}=G_{a3}\approx 0$, b_2=0.30 m,a=0.50 m, L=1.0 m)

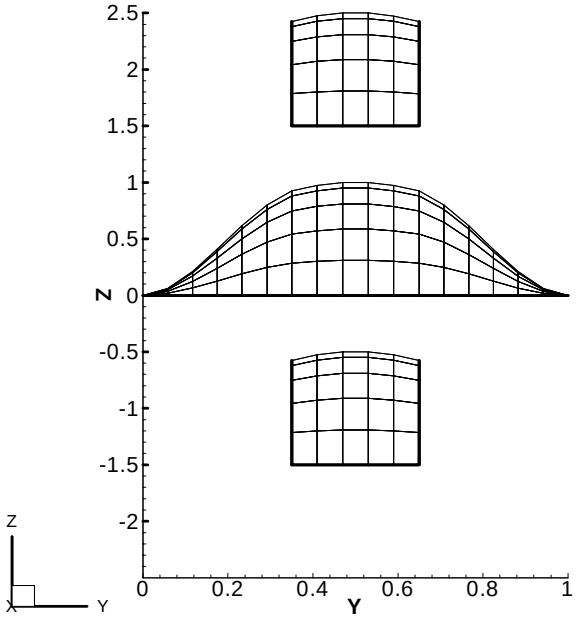


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{12} = 240.842$

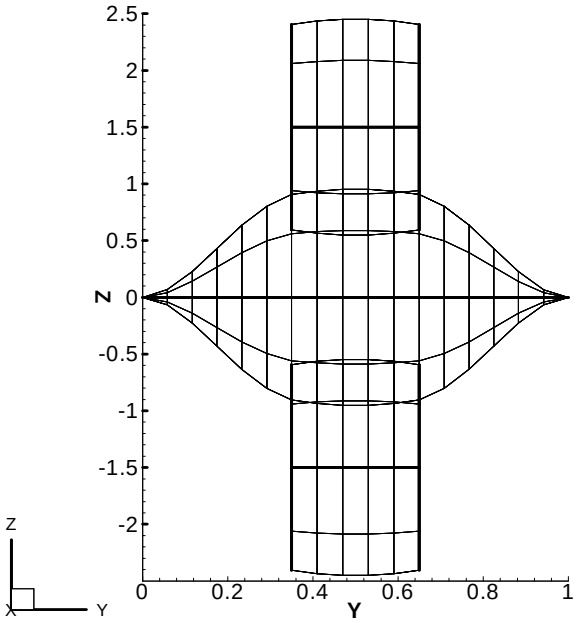


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{22} = 508.826$

Figure 11.5. (continued)



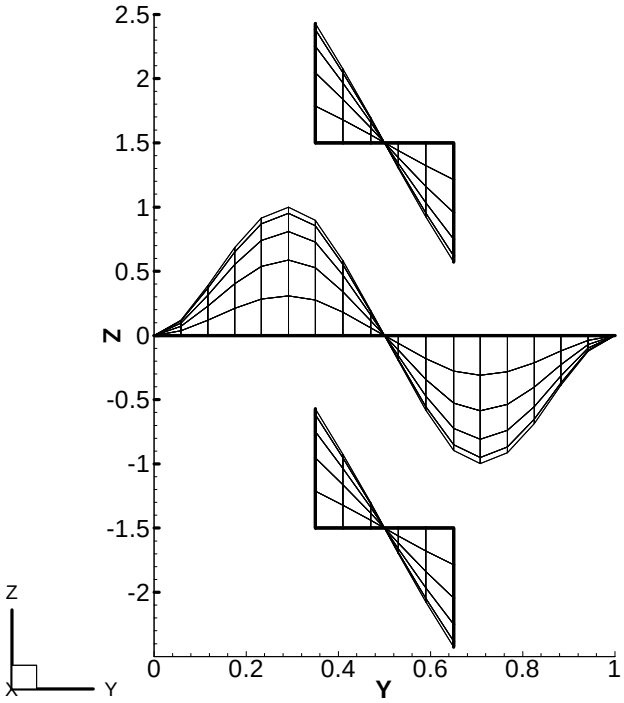
a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 35.746$



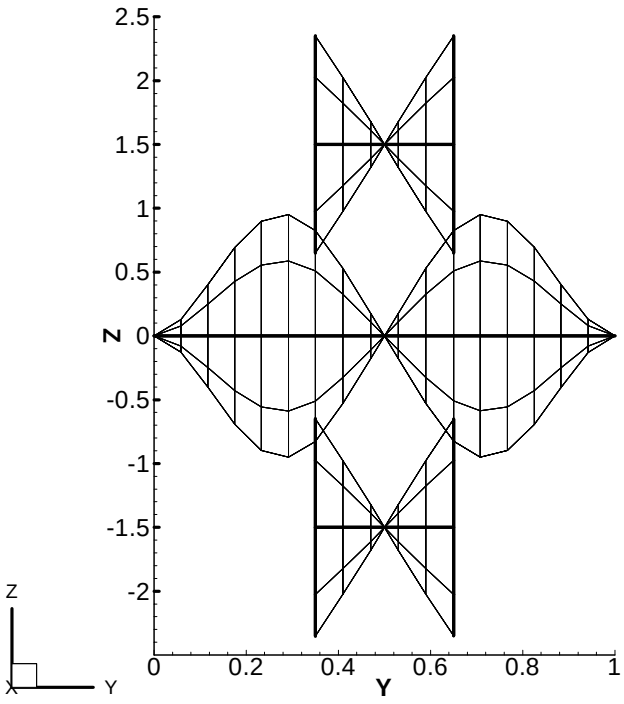
b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{21} = 216.903$

Figure 11.5(b). "Side Views" of Mode Shapes and Natural Frequencies of "Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy) ((FFCCFF) B.C.'s in y-direction, Adhesive $G_{a2}=G_{a3}\approx 0$, b_2=0.30 m, a=0.50 m, L=1.0 m)

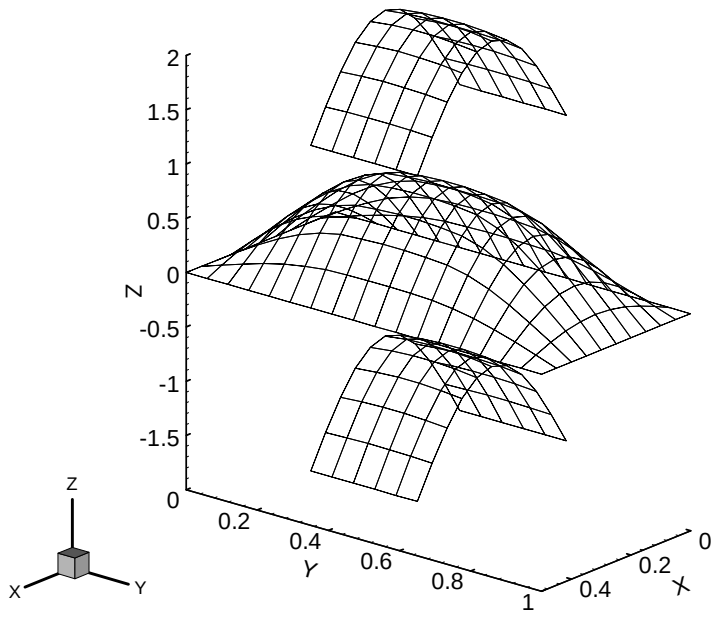


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{12} = 240.842$

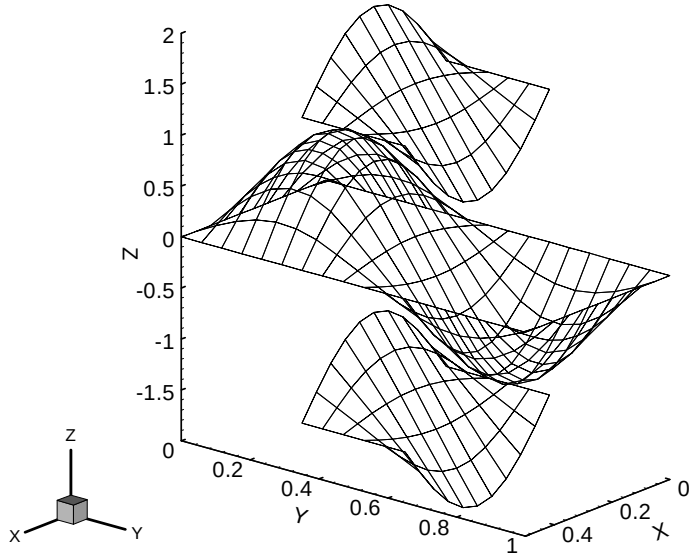


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{22} = 508.826$

Figure 11.5(b). (continued)



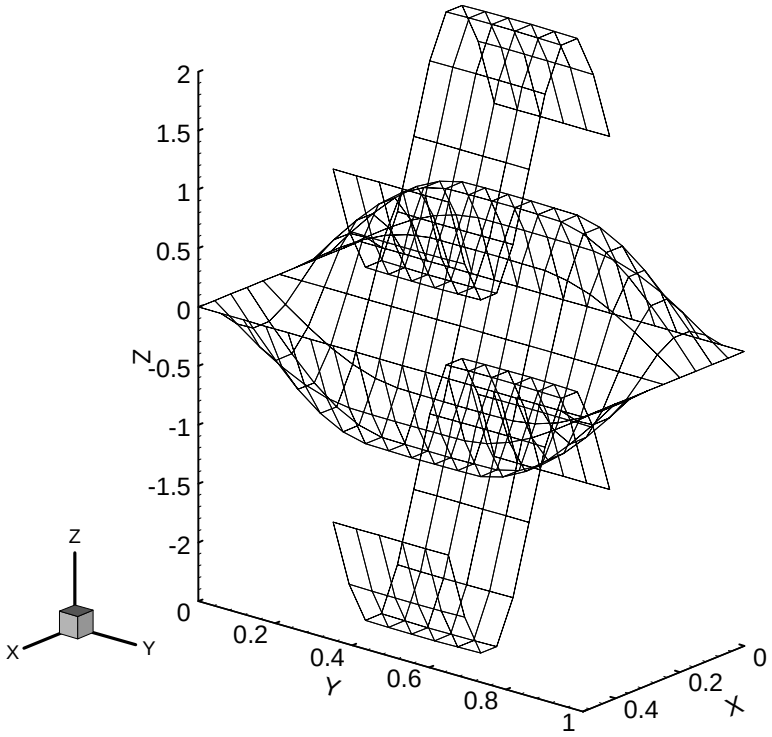
a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 18.834$



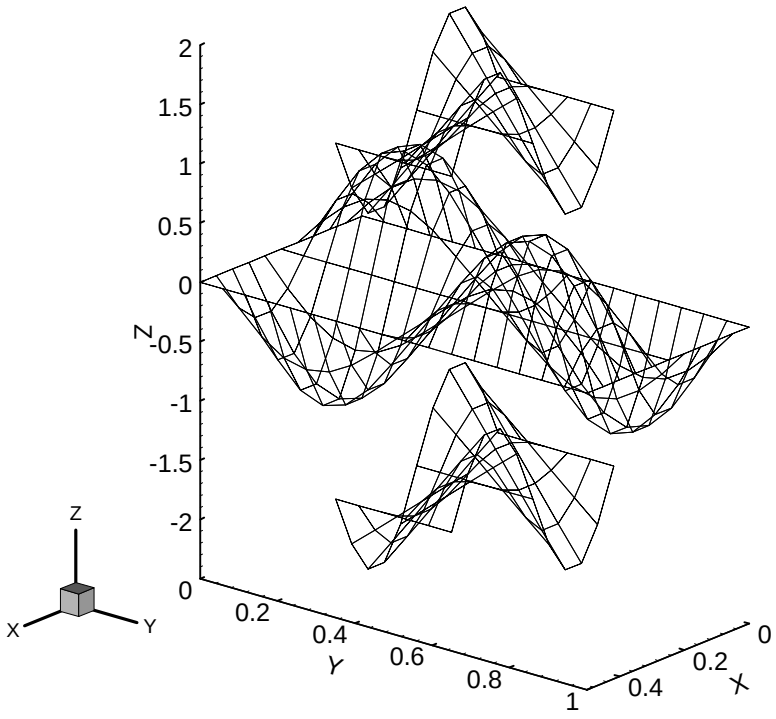
b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{12} = 130.117$

Figure 11.6. Mode Shapes and Natural Frequencies of "Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFSSFF) B.C.'s in y-direction, Adhesive $G_{a2}=G_{a3}\approx 0$, b_2=0.30 m, a=0.50 m, L=1.0 m)



c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{21} = 194.884$



d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{22} = 369.423$

Figure 11.6. (continued)

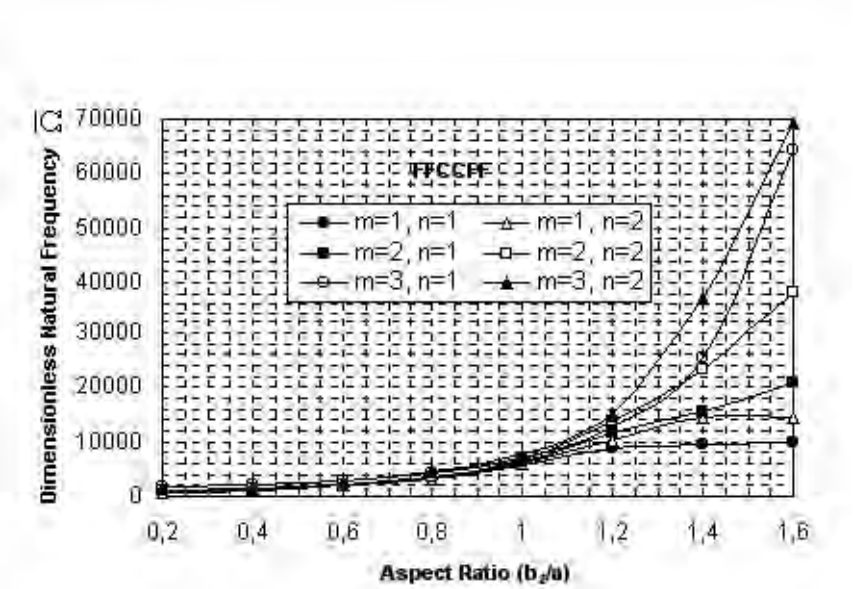


Figure 11.7. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on number of half waves in y- and x-direction)
(Base Plate=Plate 1= evlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFCCFF) B.C.’s, ‘‘Hard” Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

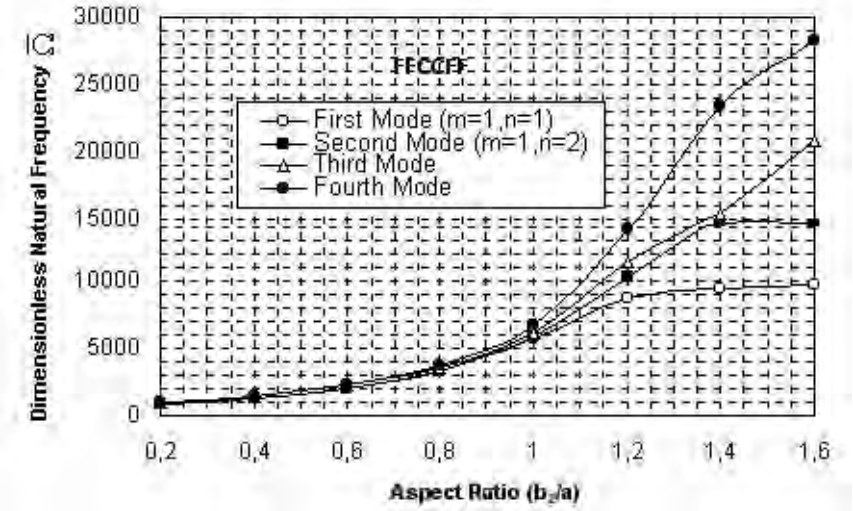


Figure 11.8. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Base Plate=Plate 1= evlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
(Various Modes with (FFCCFF) B.C.’s, ‘‘Hard” Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

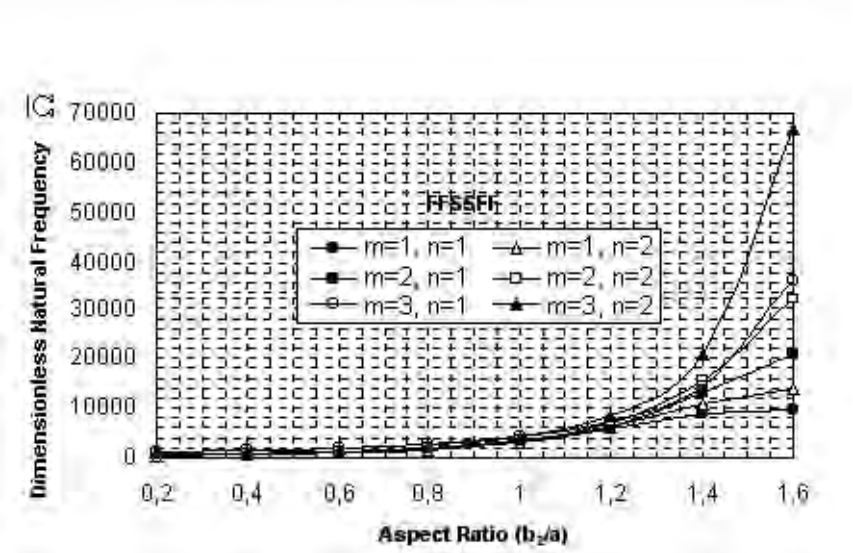


Figure 11.9. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on number of half waves in y- and x-direction)
(Base Plate=Plate 1= evlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFSSFF) B.C.’s, ‘‘Hard” Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

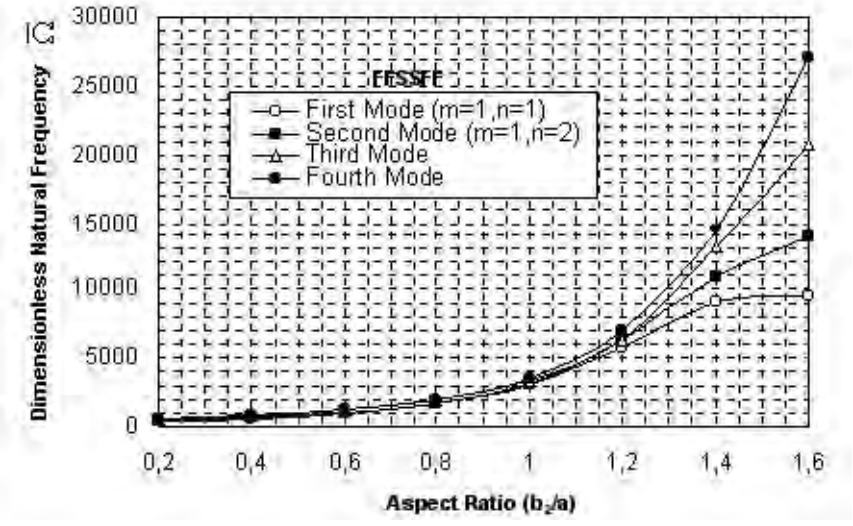


Figure 11.10. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFSSFF) B.C.’s, ‘‘Hard” Adhesive, b_2=varies a=0.50 m, L=1.0 m)

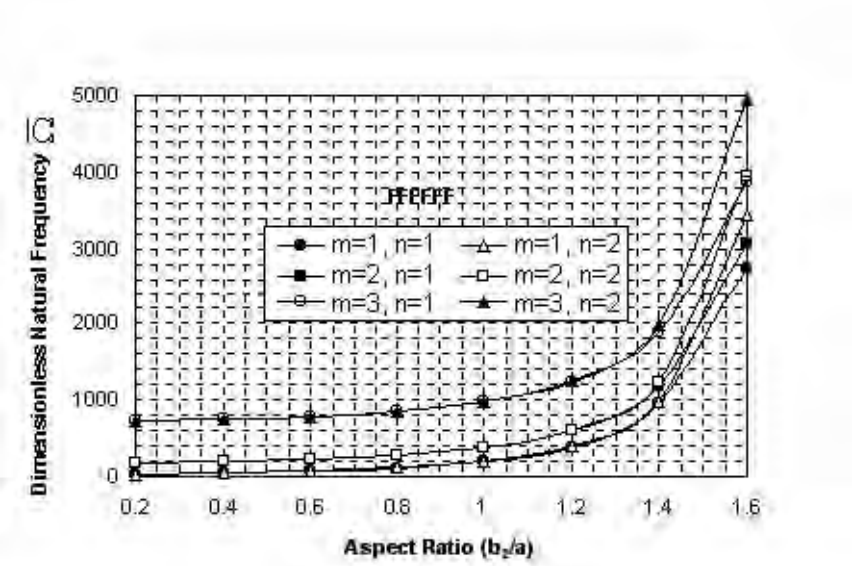


Figure 11.11. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on number of half waves in y- and x-direction)
(Base Plate=Plate 1= evlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFFFFF) B.C.’s, ‘‘Hard” Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

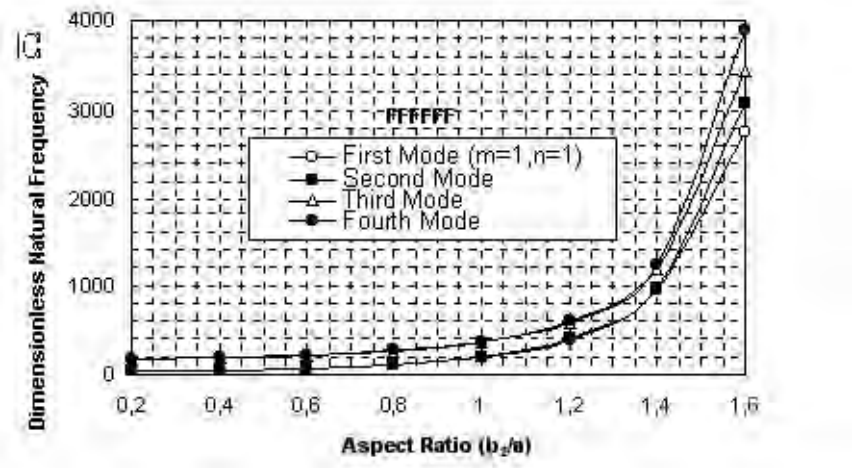


Figure 11.12. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFFFFF) B.C.’s, ‘‘Hard” Adhesive’’, b_2=varies a=0.50 m, L=1.0 m)

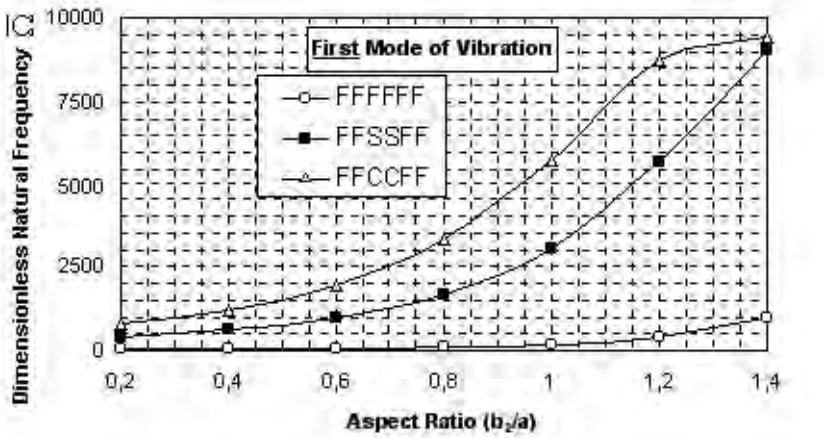


Figure 11.13. First Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Base Plate=Plate 1= evlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
(For Various B.C.’s, ‘‘Hard” Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

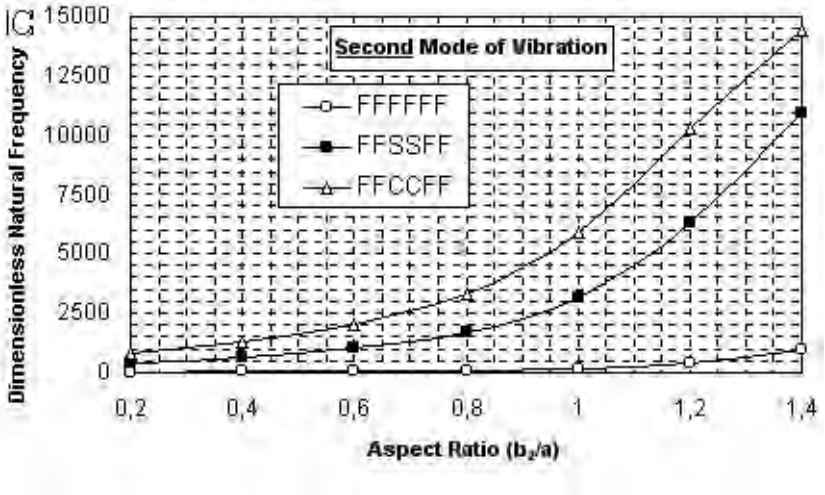


Figure 11.14. Second Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Base Plate=Plate 1= evlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
(For Various B.C.’s, ‘‘Hard” Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

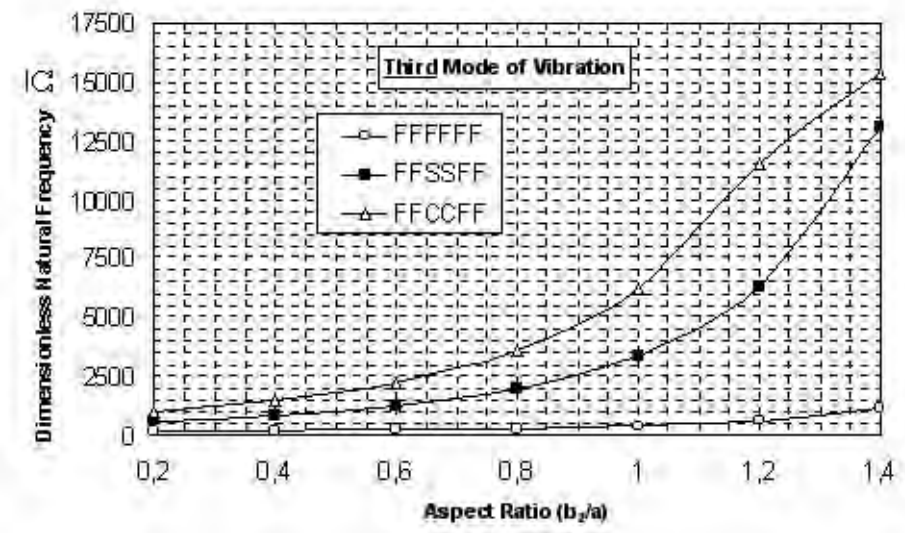


Figure 11.15. Third Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Base Plate=Plate 1= evlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
(For Various B.C.’s, ‘‘Hard” Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

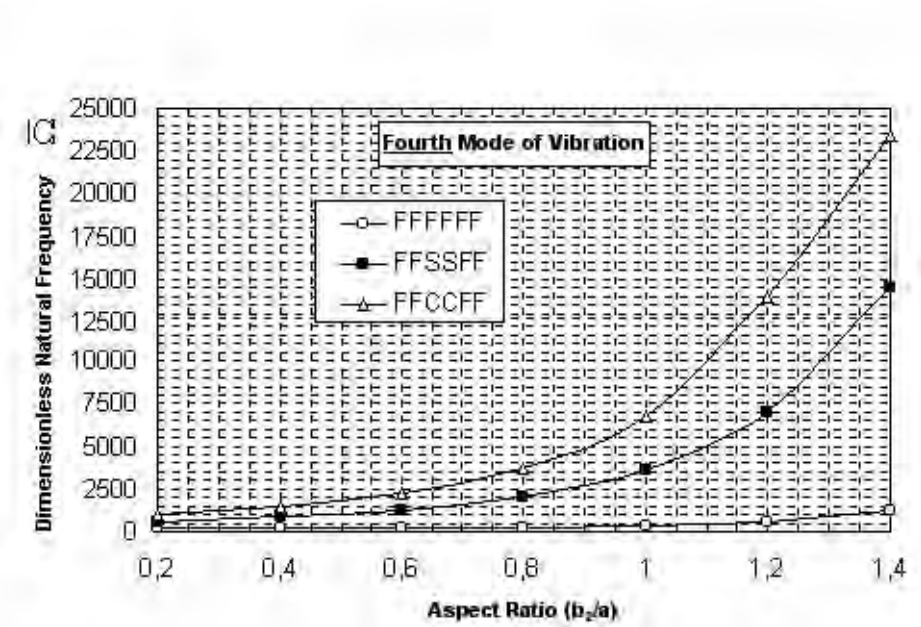


Figure 11.16. Fourth Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Base Plate=Plate 1= evlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
(For Various B.C.’s, ‘‘Hard” Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

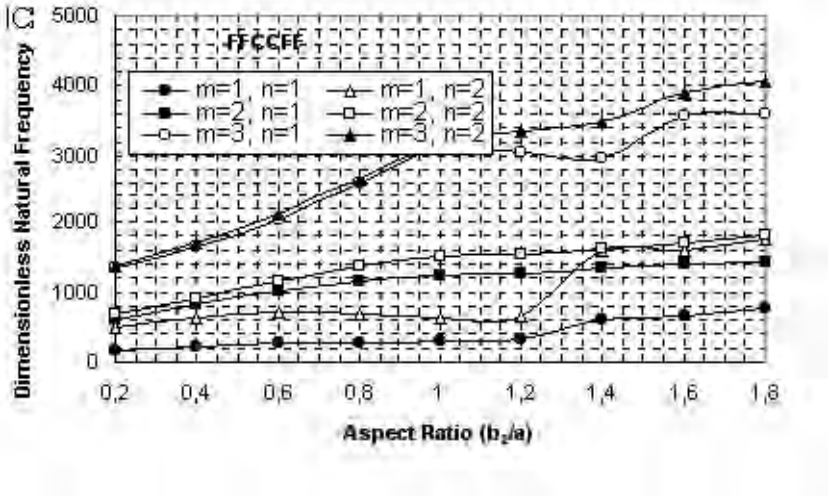


Figure 11.17. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on number of half waves in y- and x-direction)
(Base Plate=Plate 1= evlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFCCFF) B.C.’s, ‘‘Soft” Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

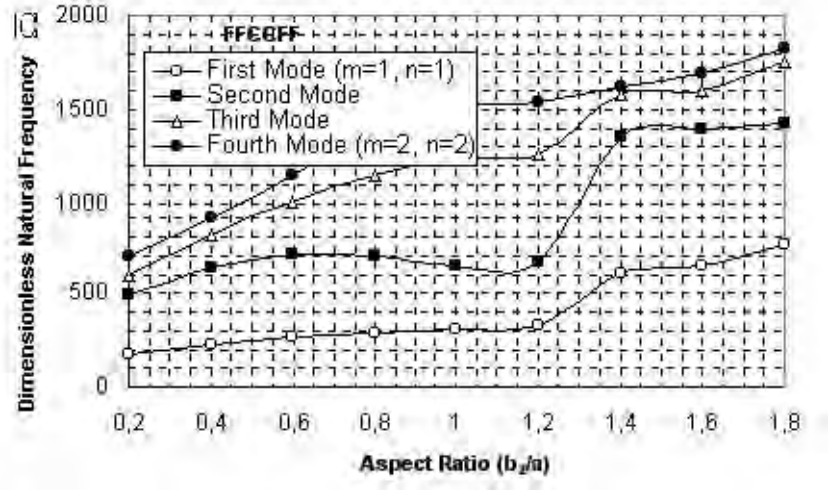


Figure 11.18. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFCCFF) B.C.’s, ‘‘Soft” Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

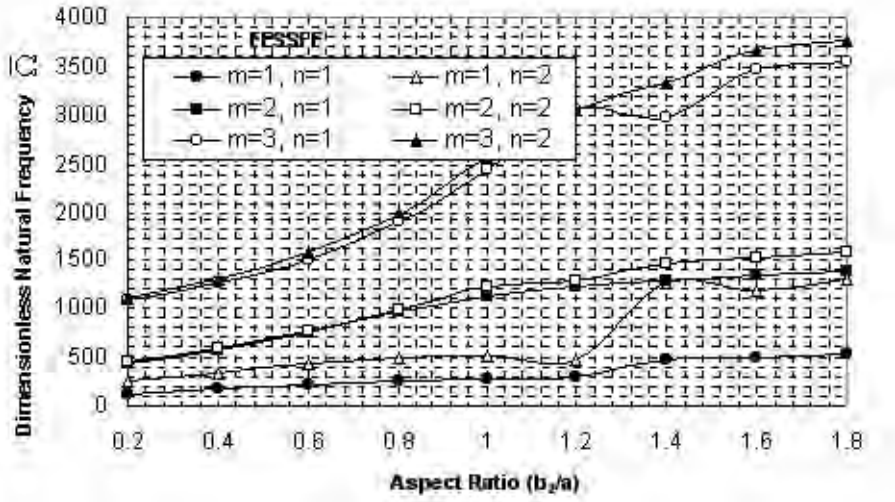


Figure 11.19. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on number of half waves in y- and x-direction)
(Base Plate=Plate 1= evlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFSSFF) B.C.’s, ‘‘Soft” Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

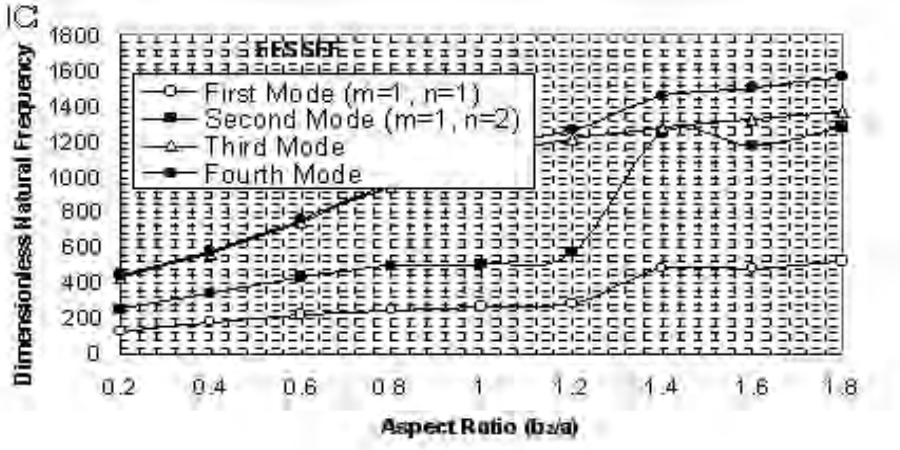


Figure 11.20. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFSSFF) B.C.’s, ‘‘Soft” Adhesive’’, b_2=varies, a=0.50 m, L=1.0 m)

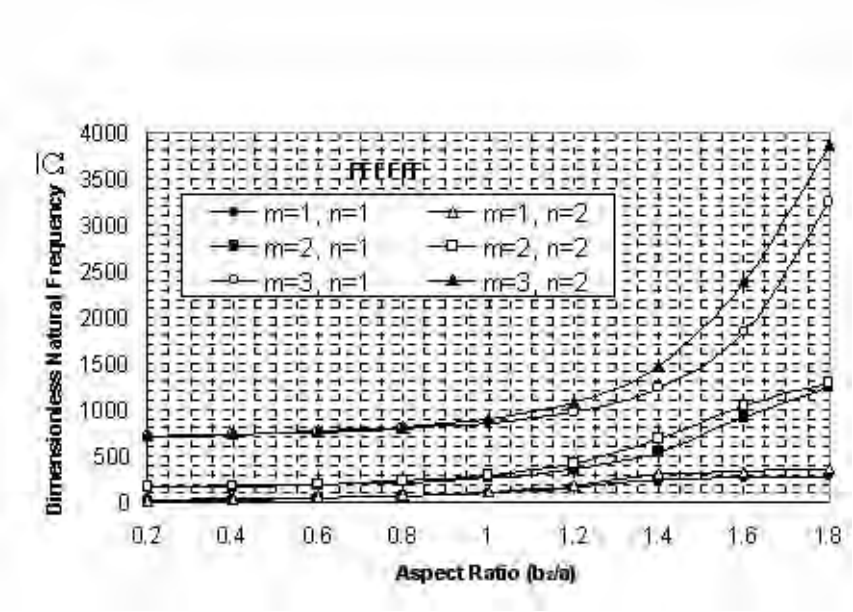


Figure 11.21. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on number of half waves in y- and x-direction)
(Base Plate=Plate 1= evlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFFFFF) B.C.’s, ‘‘Soft” Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

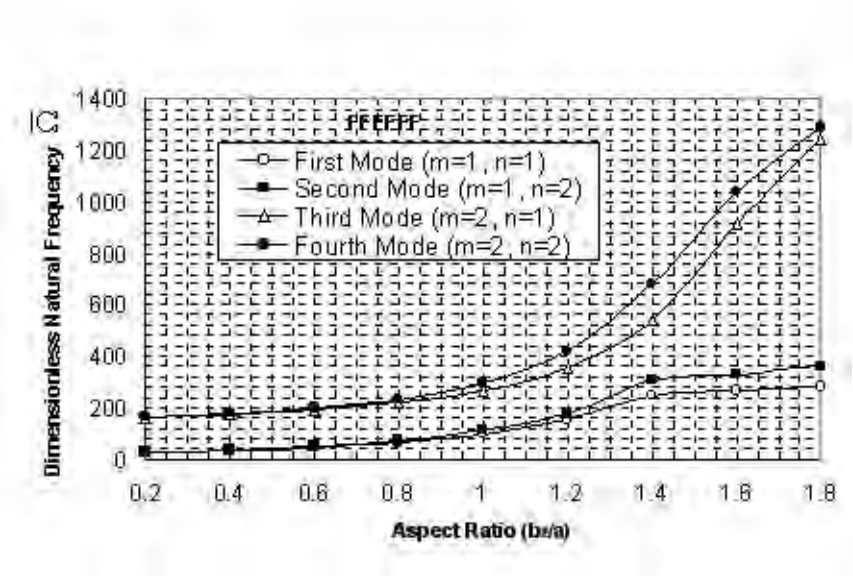


Figure 11.22. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFFFFF) B.C.’s, ‘‘Soft” Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

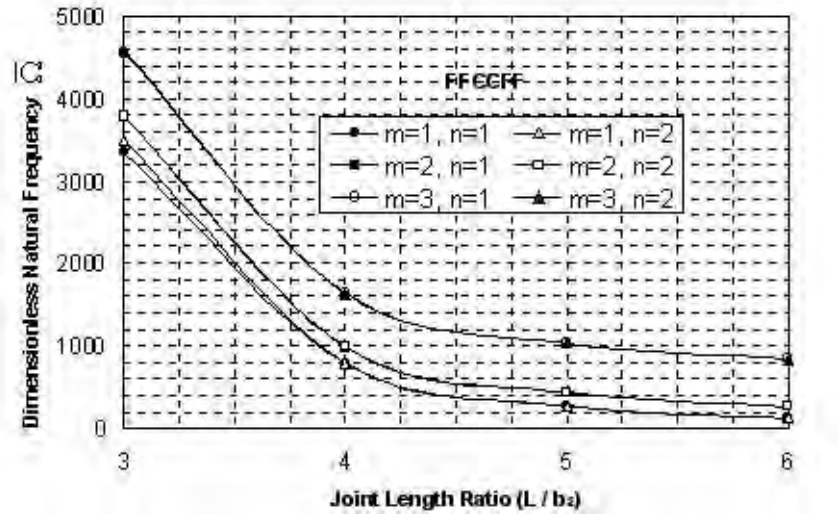


Figure 11.23. Dimensionless Natural Frequency $\bar{\Omega}$ versus ‘‘ oint Length Ratio L/b_2’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\bar{\omega}_{mn}$ on number of half waves in y- and x-direction)
(Base Plate=Plate 1= evlar-Epoxy, Plate 2= Plate 3= Graphite-Epoxy)
((FFCCFF) B.C.’s, ‘‘Hard” Adhesive, b_2=0.30m, a=0.50 m, L=varies)

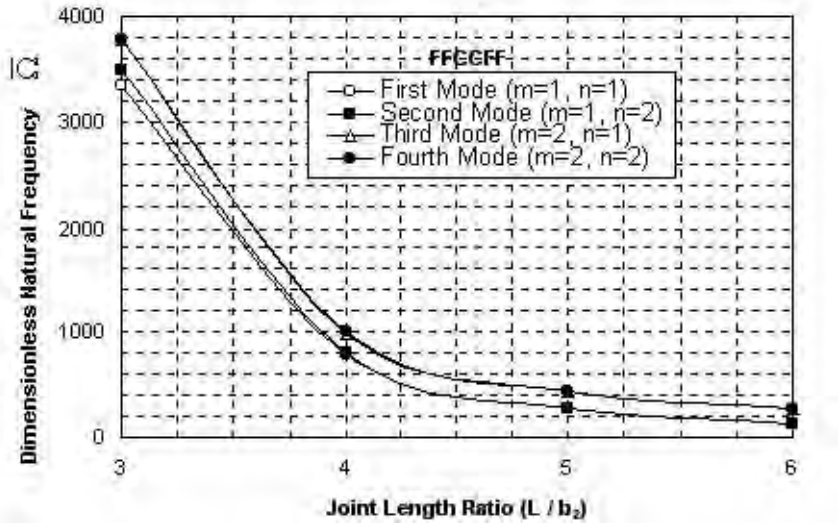


Figure 11.24. Dimensionless Natural Frequency $\bar{\Omega}$ versus ‘‘ oint Length Ratio L/b_2’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFCCFF) B.C.’s, ‘‘Hard” Adhesive, b_2=0.30m, a=0.50 m, L=varies)

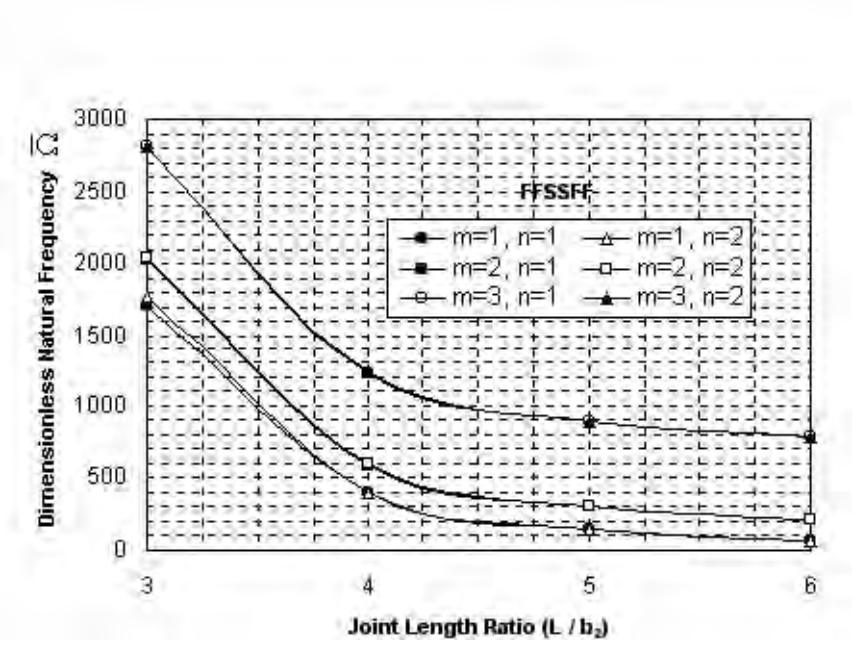


Figure 11.25. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘ oint Length Ratio L/b_2’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on number of half waves in y- and x-direction)
(Base Plate=Plate 1= evlar-Epoxy, Plate 2= Plate 3= Graphite-Epoxy)
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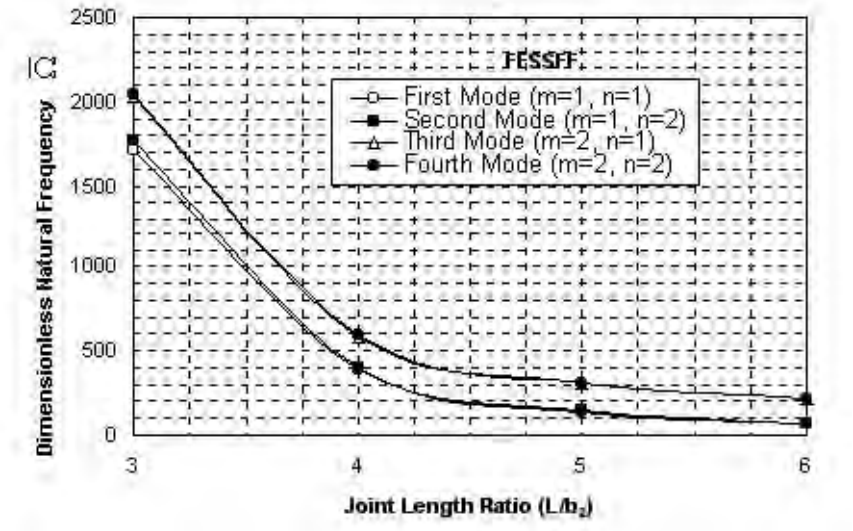

Figure 11.26. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘ oint Length Ratio L/b_2’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFSSFF) B.C.’s, ‘‘Hard” Adhesive, b_2=0.30m, a=0.50 m, L=varies)

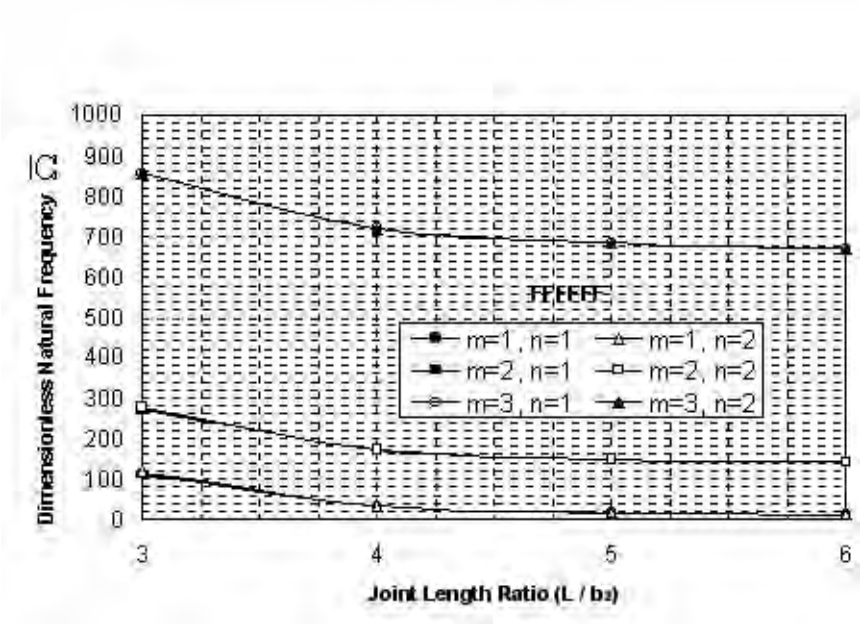

Figure 11.27. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘ oint Length Ratio L/b_2’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on number of half waves in y- and x-direction)
(Base Plate=Plate 1= evlar-Epoxy, Plate 2= Plate 3= Graphite-Epoxy)
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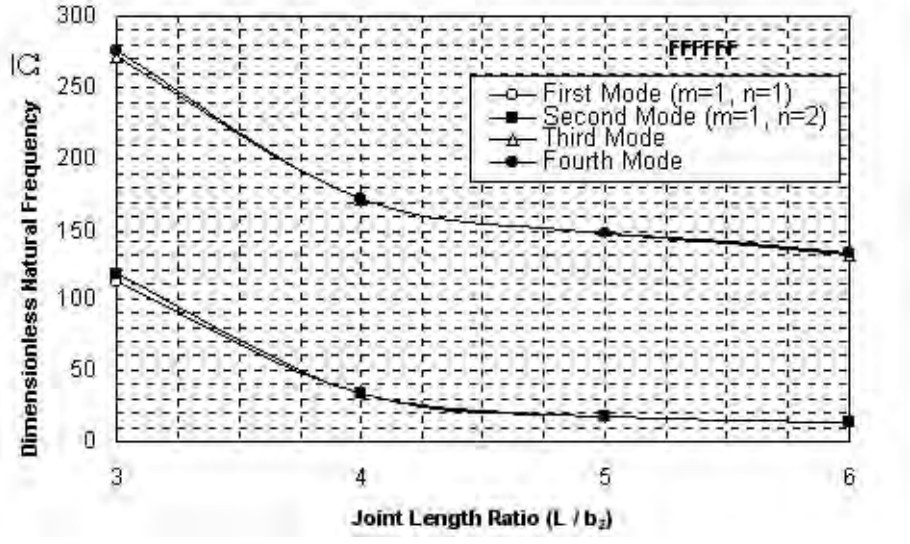

Figure 11.28. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘ oint Length Ratio L/b_2’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFFFFF) B.C.’s, ‘‘Hard” Adhesive, b_2=0.30m, a=0.50 m, L=varies)

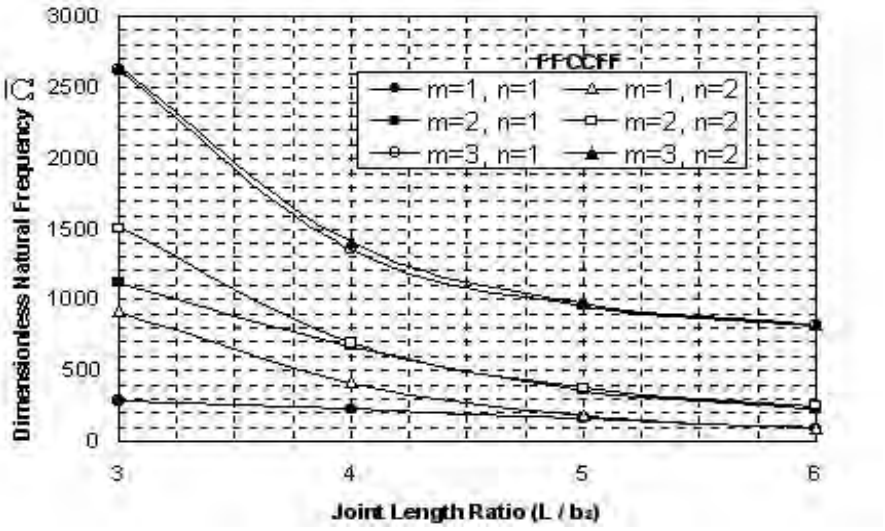

Figure 11.29. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘ oint Length Ratio L/b_2’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on number of half waves in y- and x-direction)
(Base Plate=Plate 1= evlar-Epoxy, Plate 2= Plate 3= Graphite-Epoxy)
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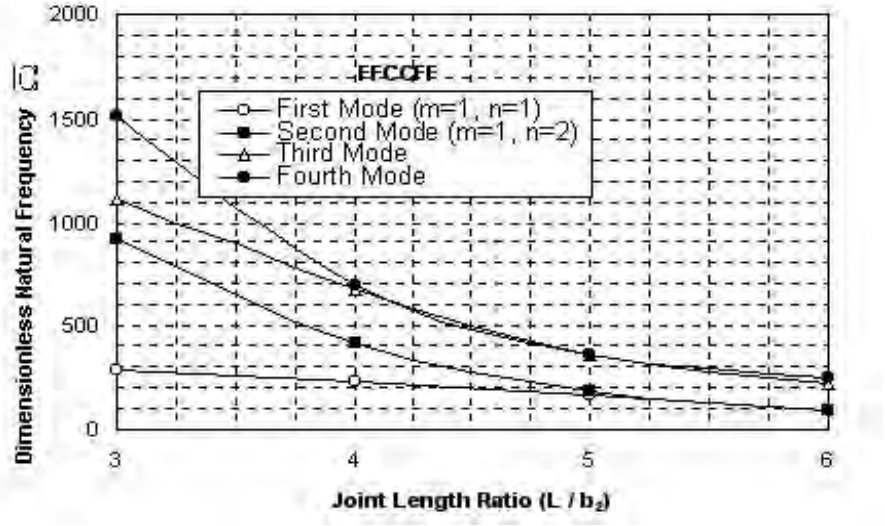

Figure 11.30. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘ oint Length Ratio L/b_2’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFCCFF) B.C.’s, ‘‘Soft” Adhesive, b_2=0.30m, a=0.50 m, L=varies)

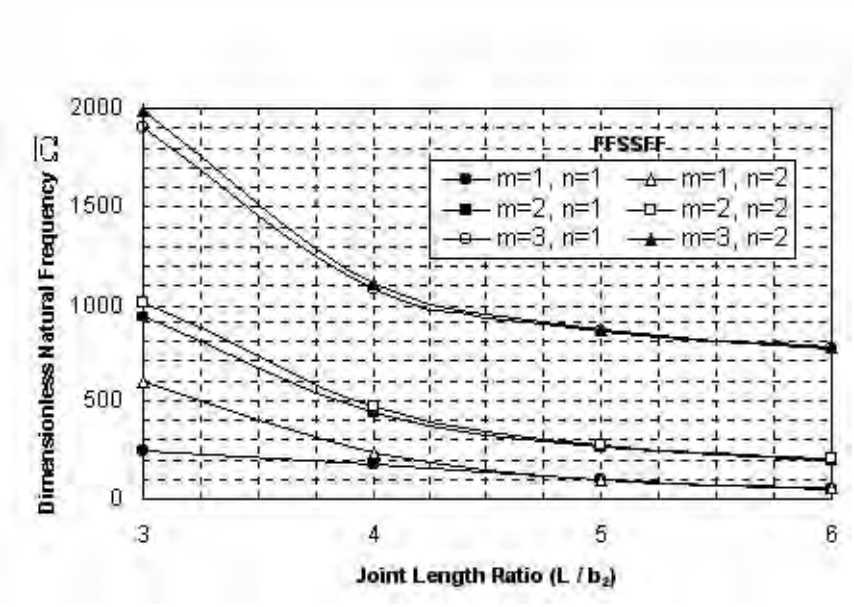

Figure 11.31. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘ oint Length Ratio L/b_2’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on number of half waves in y- and x-direction)
(Base Plate=Plate 1= evlar-Epoxy, Plate 2= Plate 3= Graphite-Epoxy)
((FFSSFF) B.C.’s, ‘‘Soft” Adhesive, b_2=0.30m, a=0.50 m, L=varies)

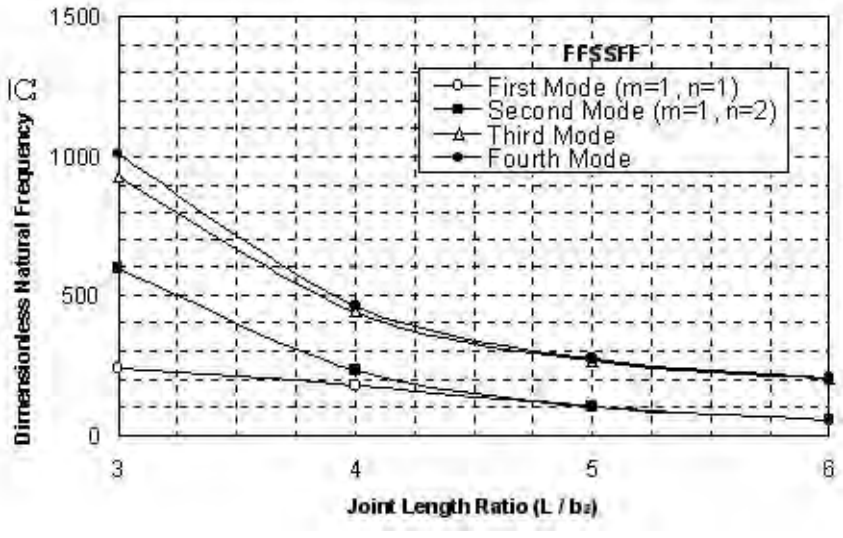

Figure 11.32. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘ oint Length Ratio L/b_2’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFSSFF) B.C.’s, ‘‘Soft” Adhesive, b_2=0.30m, a=0.50 m, L=varies)

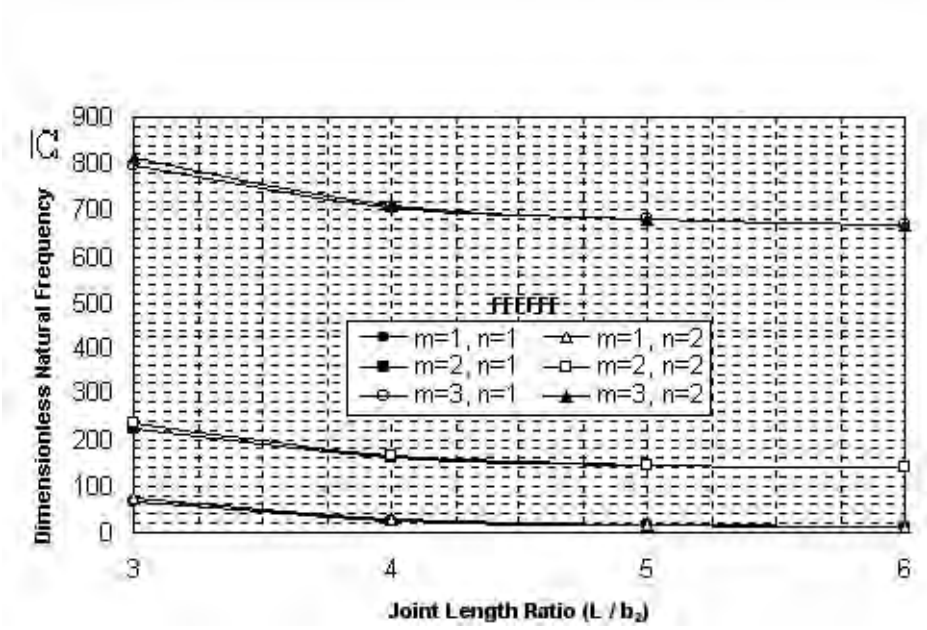

Figure 11.33. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘ oint Length Ratio L/b_2’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on number of half waves in y- and x-direction)
(Base Plate=Plate 1= evlar-Epoxy, Plate 2= Plate 3= Graphite-Epoxy)
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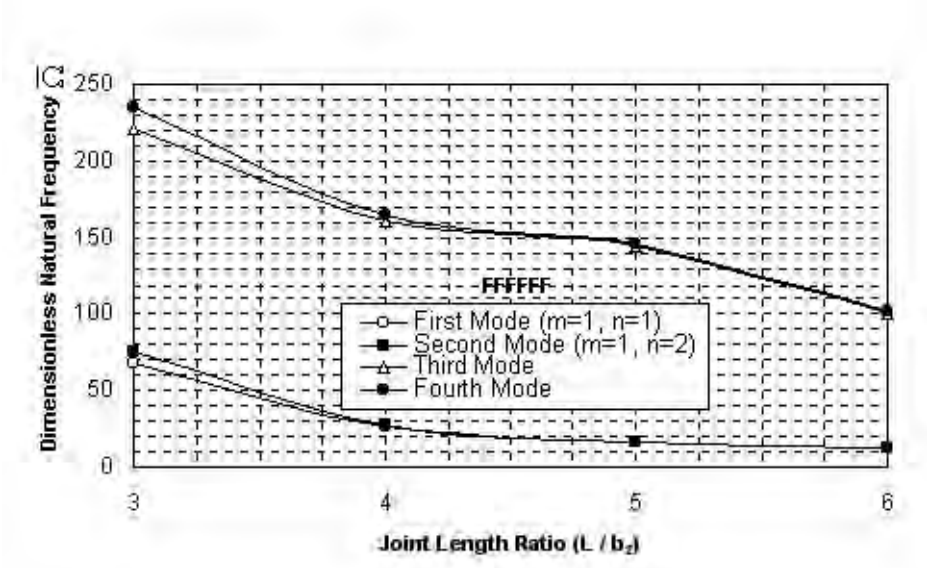

Figure 11.34. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘ oint Length Ratio L/b_2’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFFFFF) B.C.’s, ‘‘Soft” Adhesive, b_2=0.30m, a=0.50 m, L=varies)

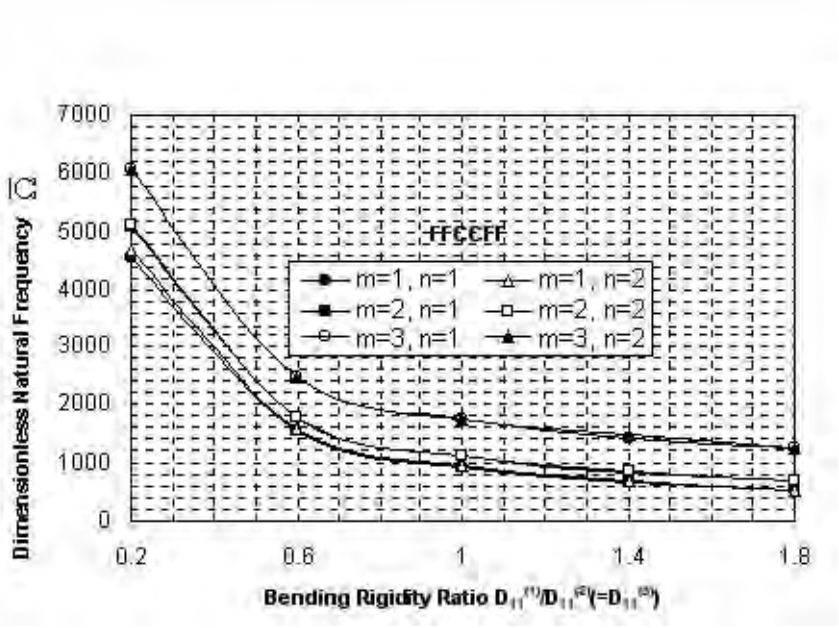

Figure 11.35. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})$ ’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)” (Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on the number of half waves in y- and x-direction)
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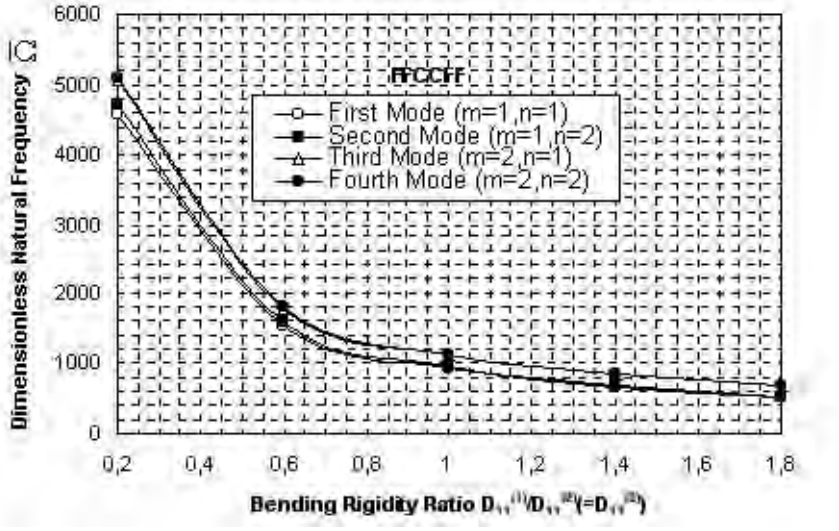

Figure 11.36. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})$ ’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)” (Various Modes with (FFCCFF) B.C.’s, ‘‘Hard’’ Adhesive b_2=0.30m, a=0.50 m, L=1.0 m, $D_{11}^{(1)}$=varies)

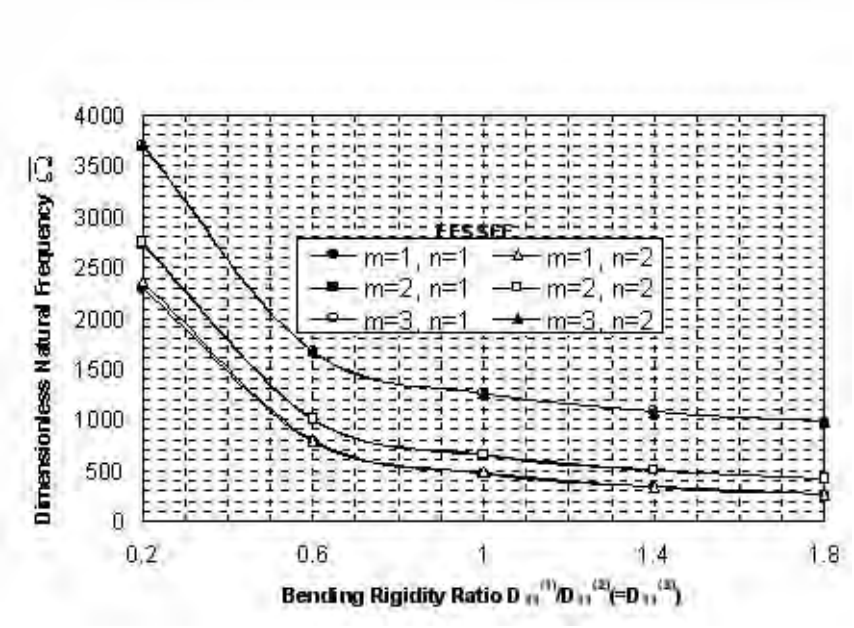

Figure 11.37. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})$ ’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)” (Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on the number of half waves in y- and x-direction)
((FFSSFF) B.C.’s, ‘‘Hard’’ Adhesive, b_2=0.30m, a=0.50 m, L=1.0 m, $D_{11}^{(1)}$=varies)

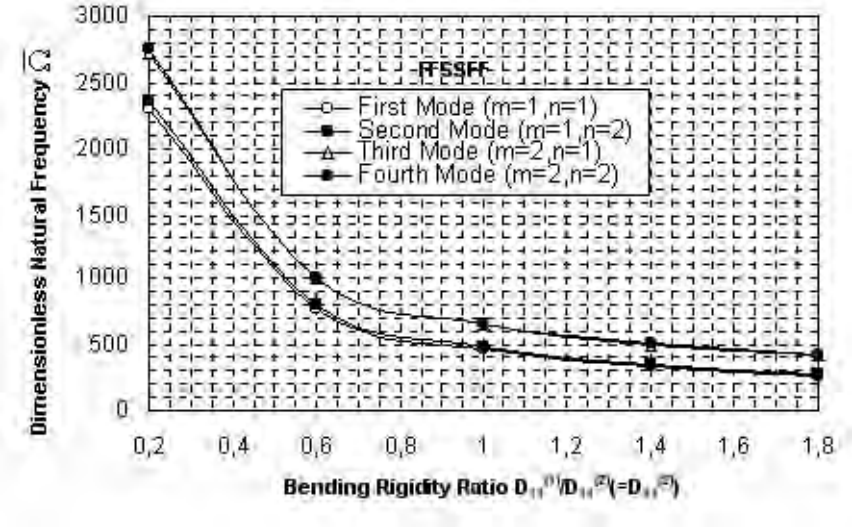

Figure 11.38. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})$ ’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)” (Various Modes with (FFSSFF) B.C.’s, ‘‘Hard’’ Adhesive b_2=0.30m, a=0.50 m, L=1.0 m, $D_{11}^{(1)}$=varies)

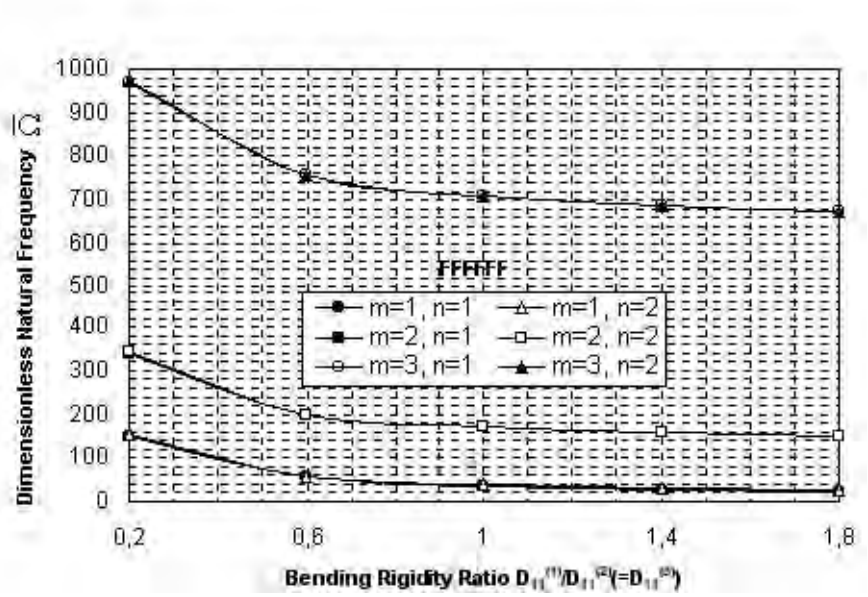

Figure 11.39. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})$ ’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on the number of half waves in y- and x-direction)
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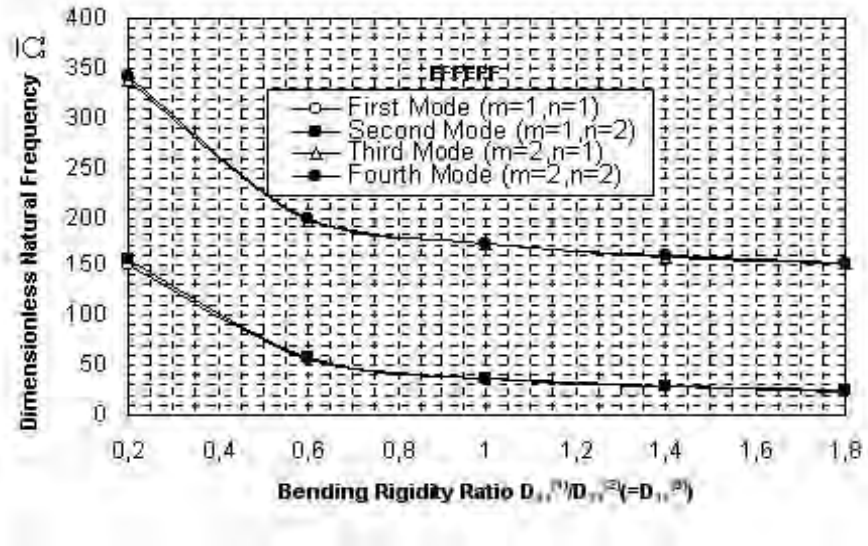

Figure 11.40. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})$ ’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFFFFF) B.C.’s, ‘‘Hard’’ Adhesive b_2=0.30m, a=0.50 m, L=1.0 m, $D_{11}^{(1)}$=varies)

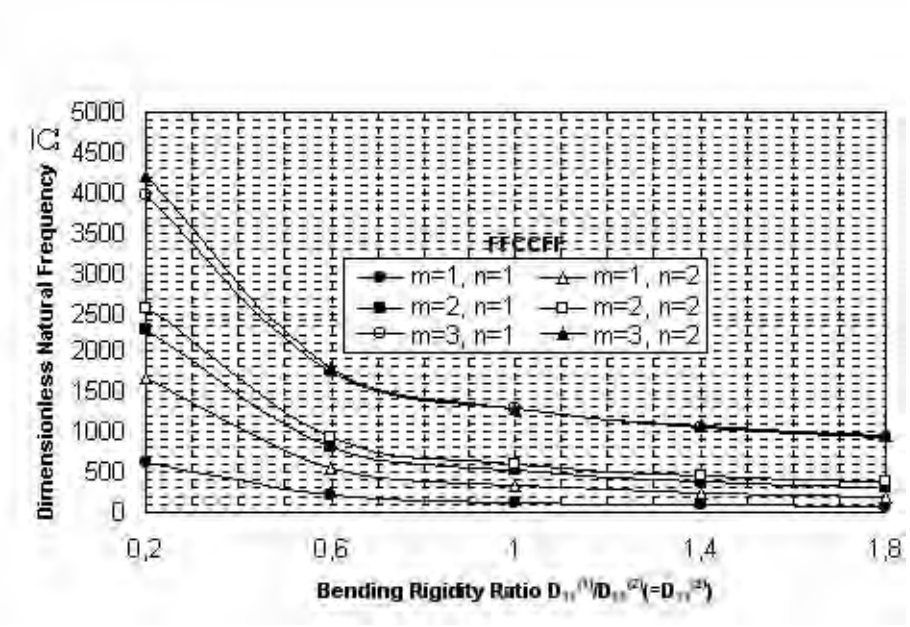

Figure 11.41. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})$’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)” (Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on the number of half waves in y- and x-direction)
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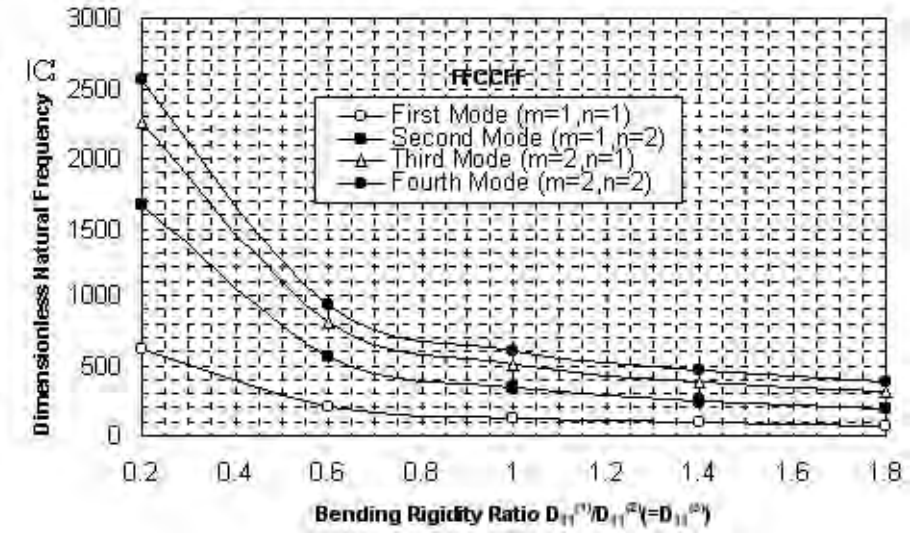

Figure 11.42. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})$’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)” (Various Modes with (FFCCFF) B.C.’s, ‘‘Soft’’ Adhesive b_2=0.30m, a=0.50 m, L=1.0 m, $D_{11}^{(1)}$=varies)

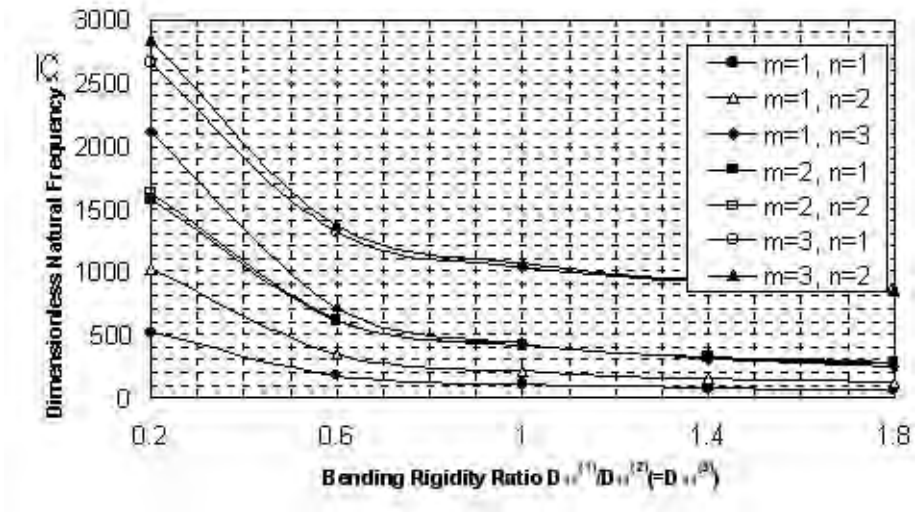

Figure 11.43. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})$’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on the number of half waves in y- and x-direction)
((FFSSFF) B.C.’s, ‘‘Soft’’ Adhesive, b_2=0.30m, a=0.50 m, L=1.0 m, $D_{11}^{(1)}$=varies)

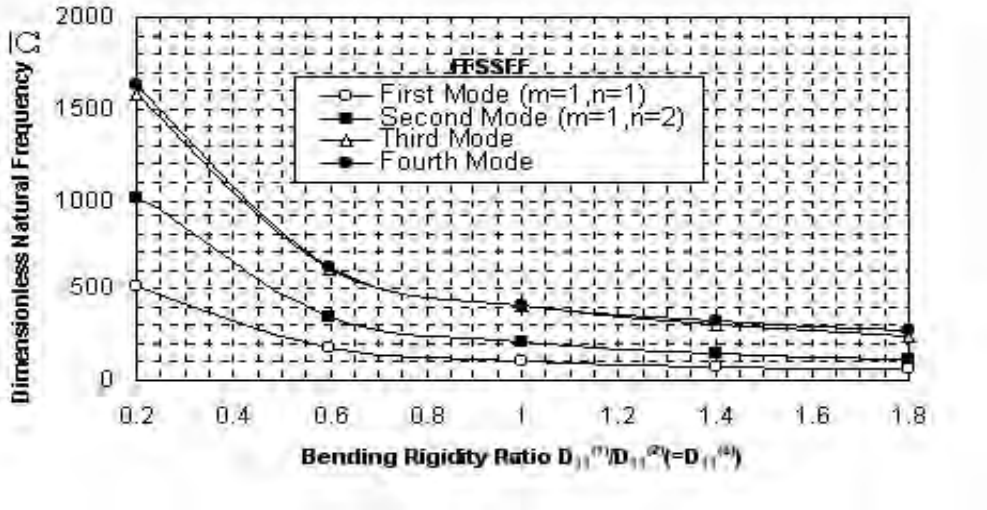

Figure 11.44. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})$’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFSSFF) B.C.’s, ‘‘Soft’’ Adhesive b_2=0.30m, a=0.50 m, L=1.0 m, $D_{11}^{(1)}$=varies)

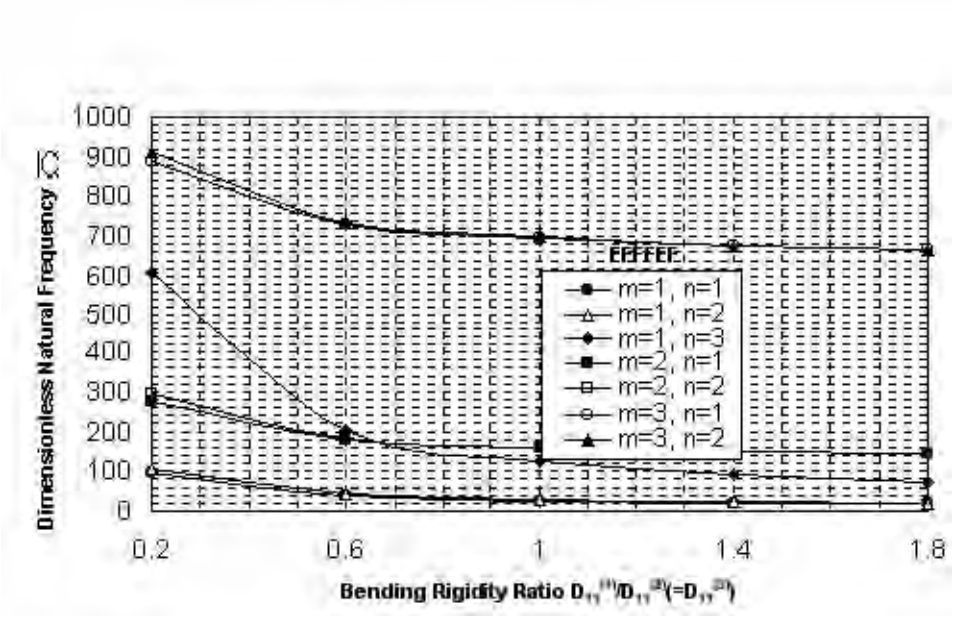

Figure 11.45. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})$’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)” (Dependency of natural frequency $\overline{\Omega}$ $\overline{\omega}_{mn}$ on the number of half waves in y- and x-direction)
((FFFFFF) B.C.’s, ‘‘Soft’’ Adhesive, b_2=0.30m, a=0.50 m, L=1.0 m, $D_{11}^{(1)}$=varies)

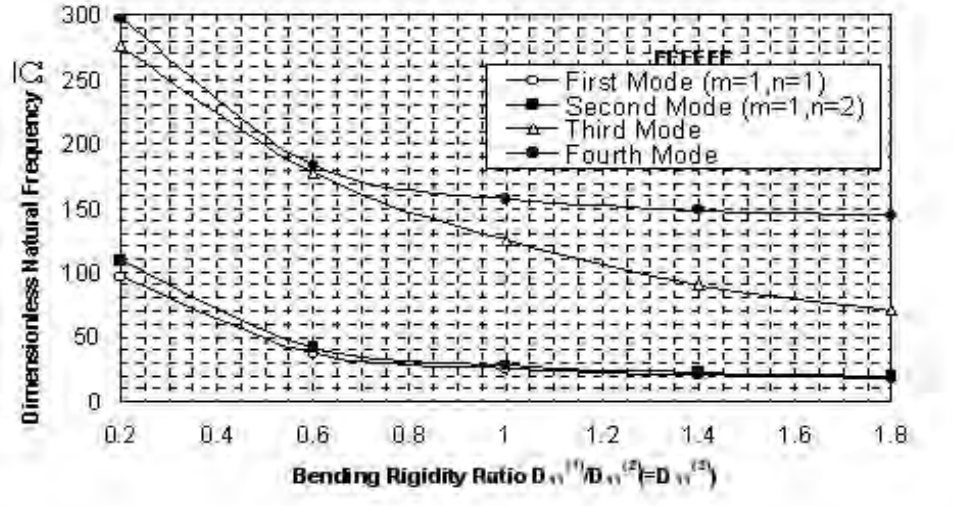

Figure 11.46. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})$’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)” (Various Modes with (FFFFFF) B.C.’s, ‘‘Soft’’ Adhesive b_2=0.30m, a=0.50 m, L=1.0 m, $D_{11}^{(1)}$=varies)

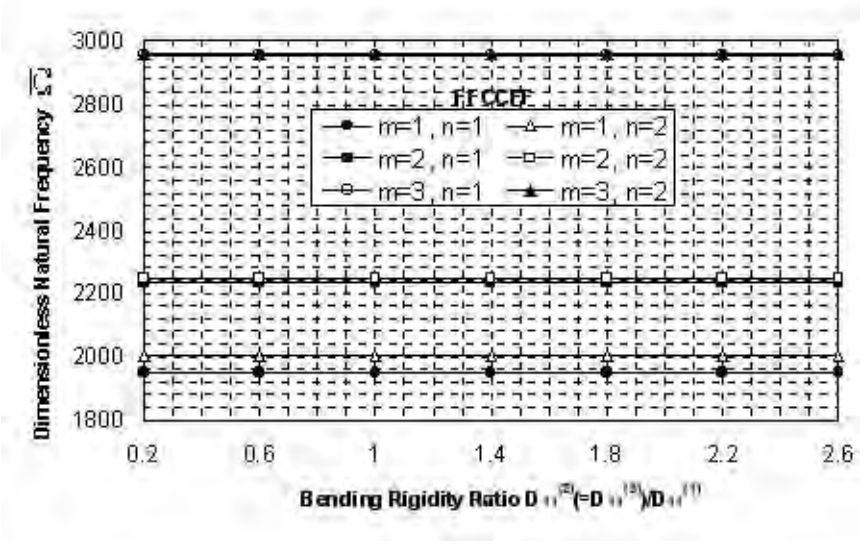

Figure 11.47. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(2)}(=D_{11}^{(3)})/D_{11}^{(1)}$ ’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega}=\overline{\omega}_{mn}$ on the number of half waves in y- and x-direction)
((FFCCFF) B.C.’s, ‘‘Hard’’ Adhesive, b_2=0.30m, a=0.50 m, L=1.0 m, $D_{11}^{(2)}(=D_{11}^{(3)})$=varies)

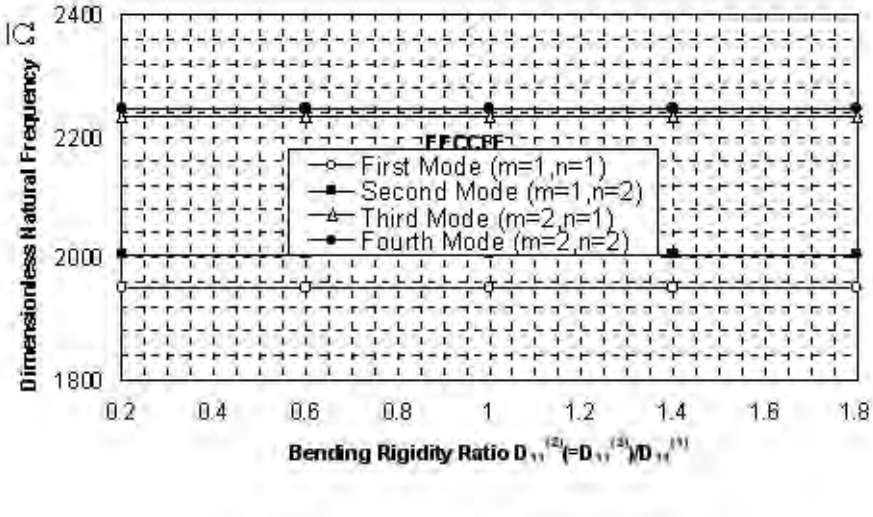

Figure 11.48. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(2)}(=D_{11}^{(3)})/D_{11}^{(1)}$ ’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFCCFF) B.C.’s, ‘‘Hard’’ Adhesive b_2=0.30m, a=0.50 m, L=1.0 m, $D_{11}^{(2)}(=D_{11}^{(3)})$=varies)

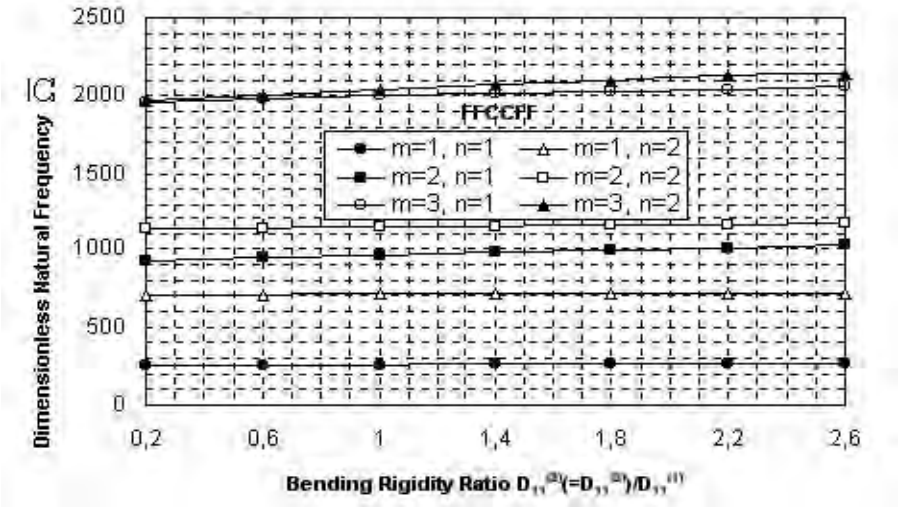

Figure 11.49. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(2)}(=D_{11}^{(3)})/D_{11}^{(1)}$ ’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega}=\overline{\omega}_{mn}$ on the number of half waves in y- and x-direction)
((FFCCFF) B.C.’s, ‘‘Soft’’ Adhesive, b_2=0.30m, a=0.50 m, L=1.0 m, $D_{11}^{(2)}(=D_{11}^{(3)})$=varies)

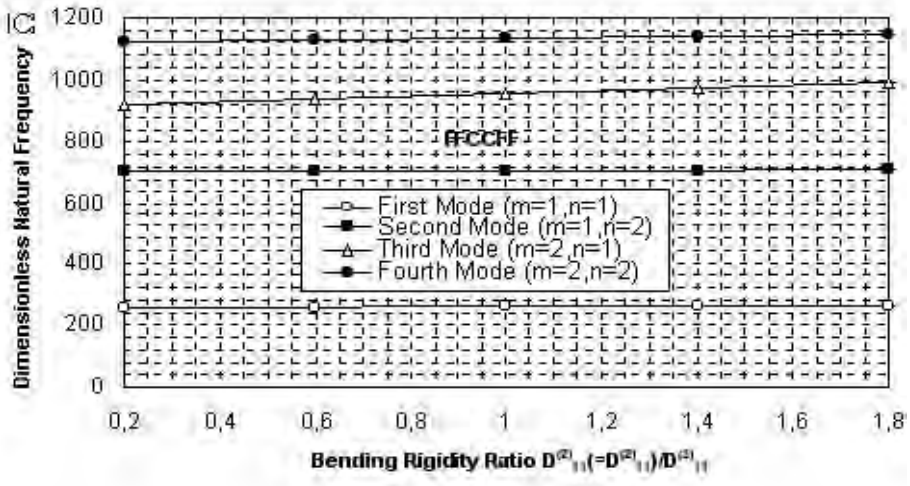

Figure 11.50. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(2)}(=D_{11}^{(3)})/D_{11}^{(1)}$ ’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFCCFF) B.C.’s, ‘‘Soft’’ Adhesive b_2=0.30m, a=0.50 m, L=1.0 m, $D_{11}^{(2)}(=D_{11}^{(3)})$=varies)

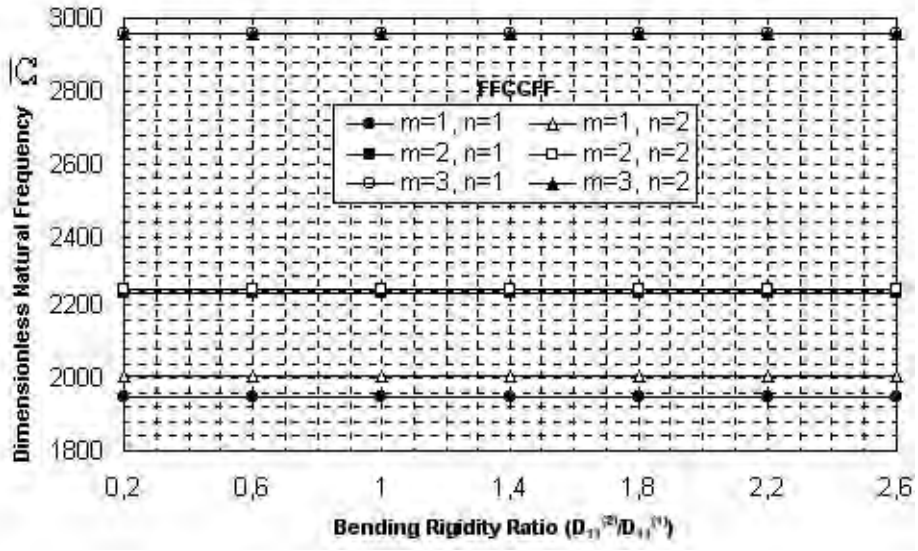

Figure 11.51. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(2)} / D_{11}^{(1)}$ ’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on the number of half waves in y- and x-direction)
((FFCCFF) B.C.’s, ‘‘Hard’’ Adhesive, b_2=0.30m, a=0.50 m, L=1.0 m, $D_{11}^{(2)}$=varies)

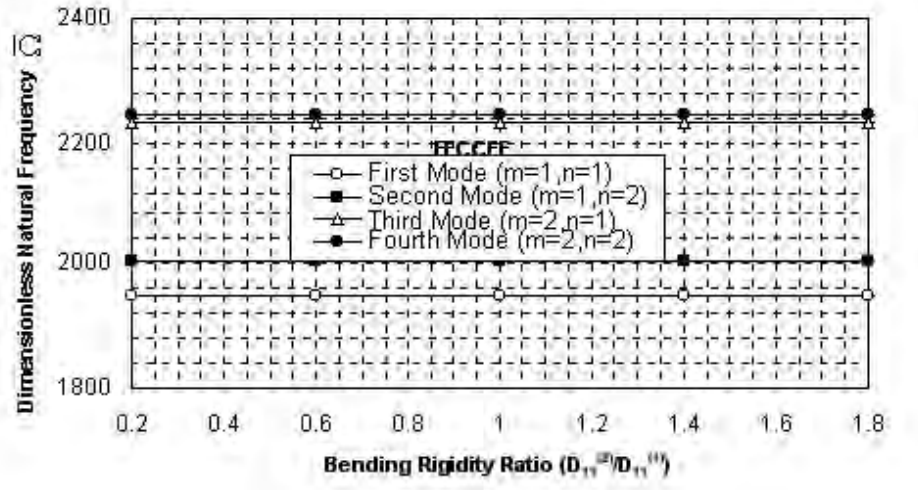

Figure 11.52. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(2)} / D_{11}^{(1)}$ ’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFCCFF) B.C.’s, ‘‘Hard’’ Adhesive b_2=0.30m, a=0.50 m, L=1.0 m, $D_{11}^{(2)}$=varies)

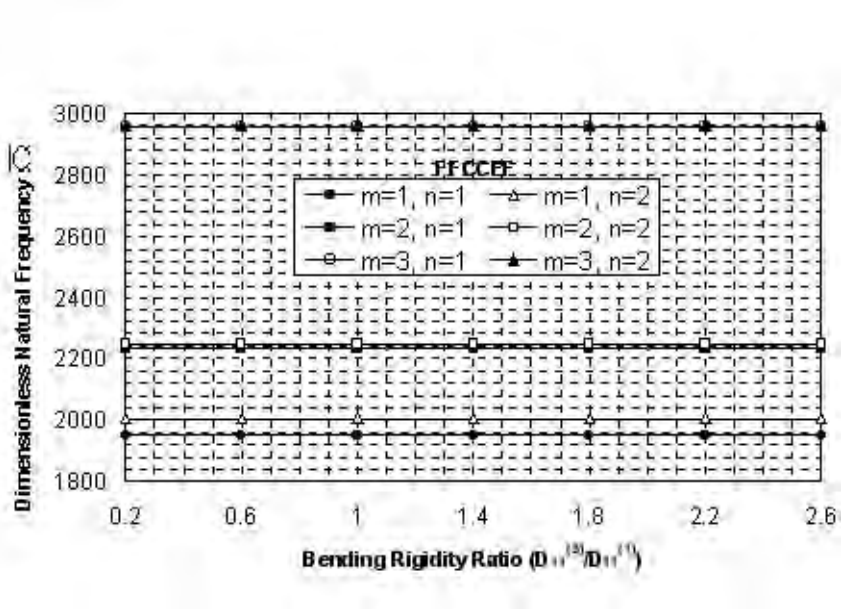

Figure 11.53. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(3)} / D_{11}^{(1)}$ ’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on the number of half waves in y- and x-direction)
((FFCCFF) B.C.’s, ‘‘Hard’’ Adhesive, b_2=0.30m, a=0.50 m, L=1.0 m, $D_{11}^{(3)}$=varies)

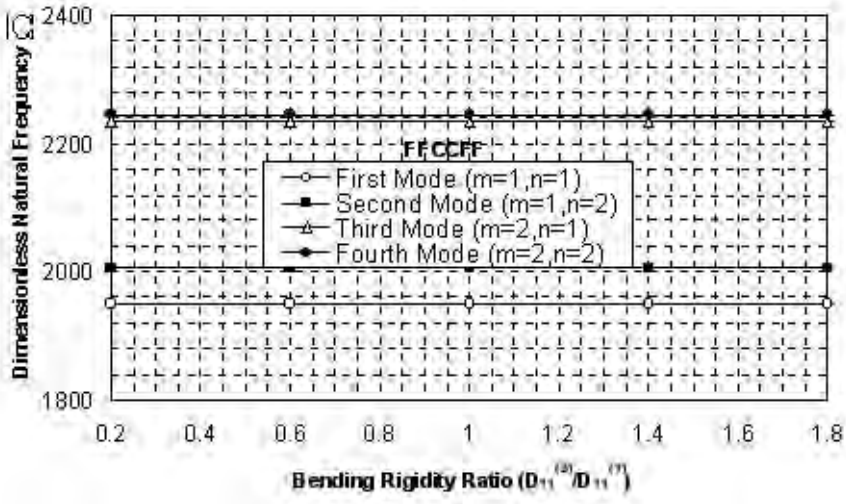

Figure 11.54. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(3)} / D_{11}^{(1)}$ ’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFCCFF) B.C.’s, ‘‘Hard’’ Adhesive b_2=0.30m, a=0.50 m, L=1.0 m, $D_{11}^{(3)}$=varies)

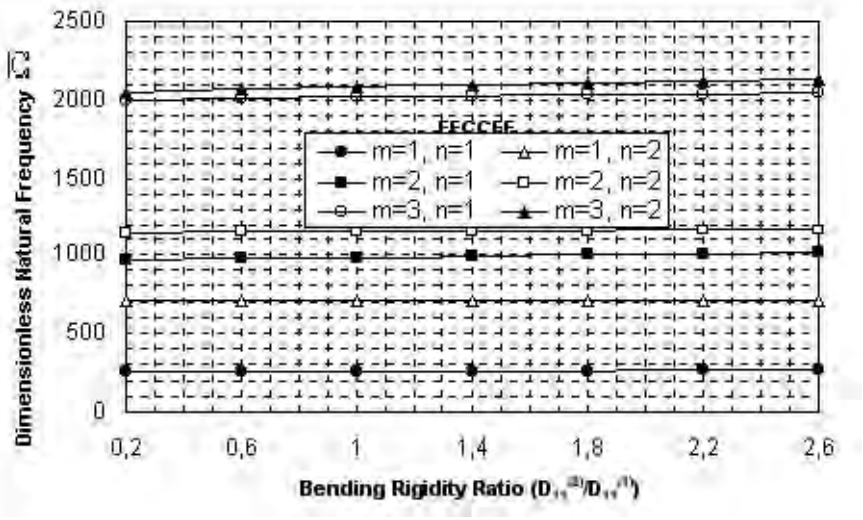

Figure 11.55. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(2)} / D_{11}^{(1)}$ ’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\ } = \overline{\ }_{mn}$ on the number of half waves in y- and x-direction)
((FFCCFF) B.C.’s, ‘‘Soft’’ Adhesive, b_2=0.30m, a=0.50 m, L=1.0 m, $D_{11}^{(2)}$=varies)

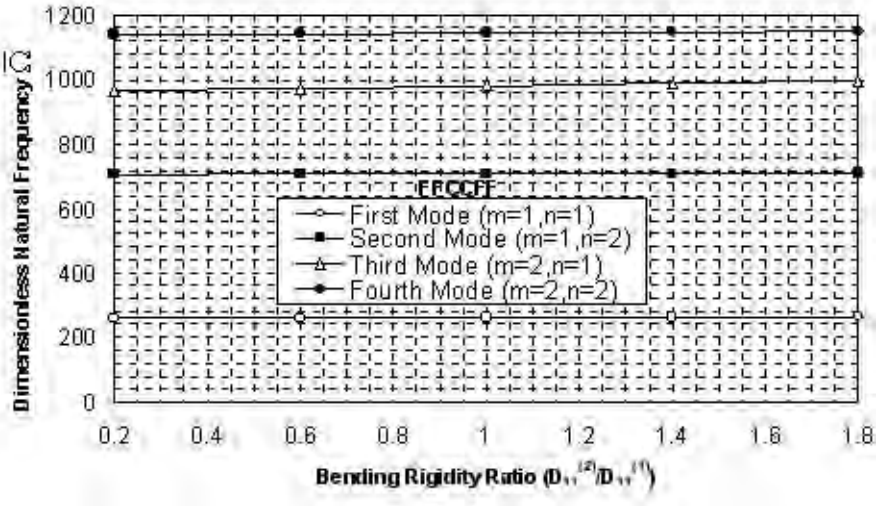

Figure 11.56. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(2)} / D_{11}^{(1)}$ ’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFCCFF) B.C.’s, ‘‘Soft’’ Adhesive b_2=0.30m, a=0.50 m, L=1.0 m, $D_{11}^{(2)}$=varies)

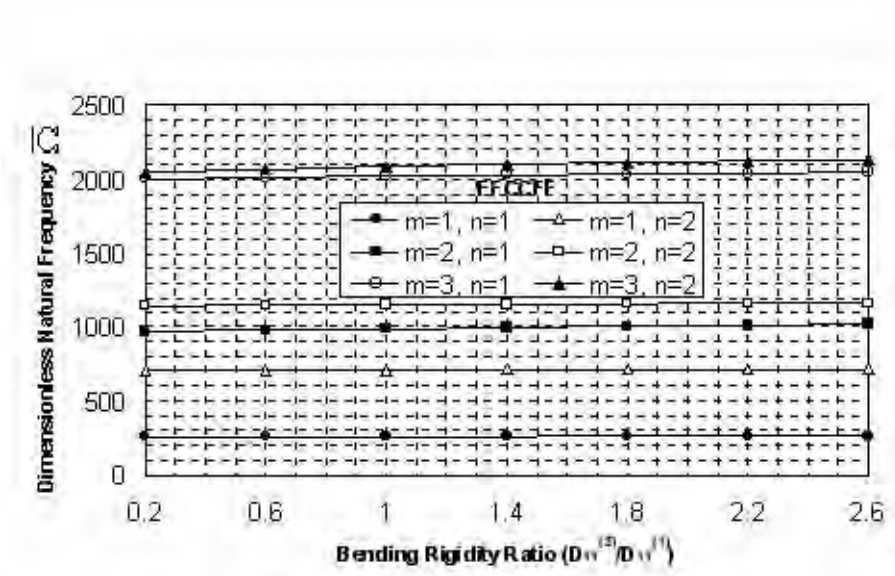

Figure 11.57. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(3)} / D_{11}^{(1)}$ ’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on the number of half waves in y- and x-direction)
((FFCCFF) B.C.’s, ‘‘Soft’’ Adhesive, b_2=0.30m, a=0.50 m, L=1.0 m, $D_{11}^{(3)}$=varies)

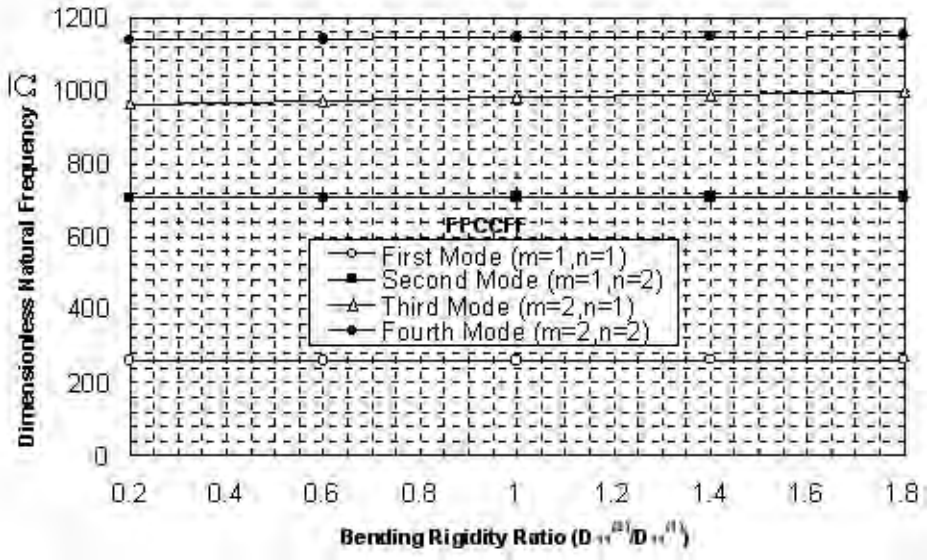

Figure 11.58. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(3)} / D_{11}^{(1)}$ ’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFCCFF) B.C.’s, ‘‘Soft’’ Adhesive b_2=0.30m, a=0.50 m, L=1.0 m, $D_{11}^{(3)}$=varies)

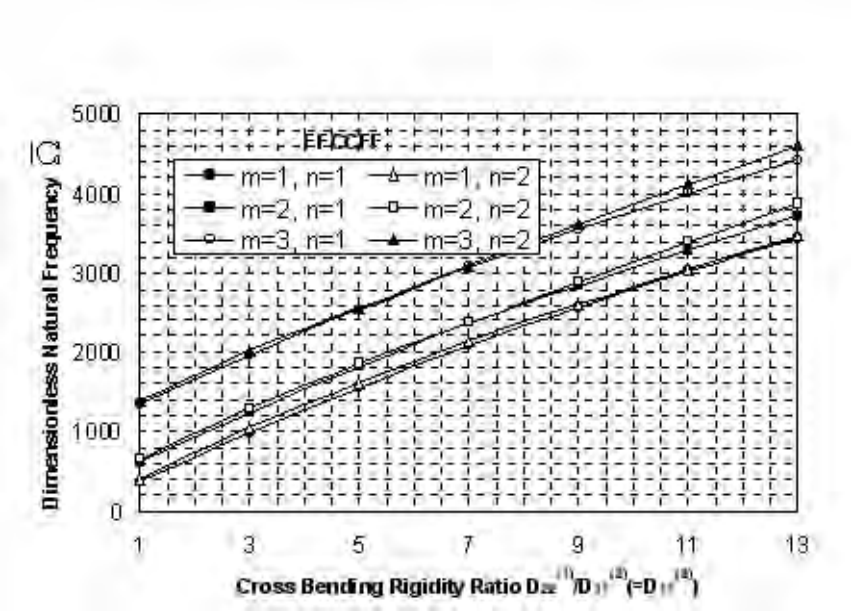

Figure 11.59. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Cross-bending Rigidity Ratio $D_{22}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})$ '' in "Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)" (Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on the number of half waves in y- and x-direction)
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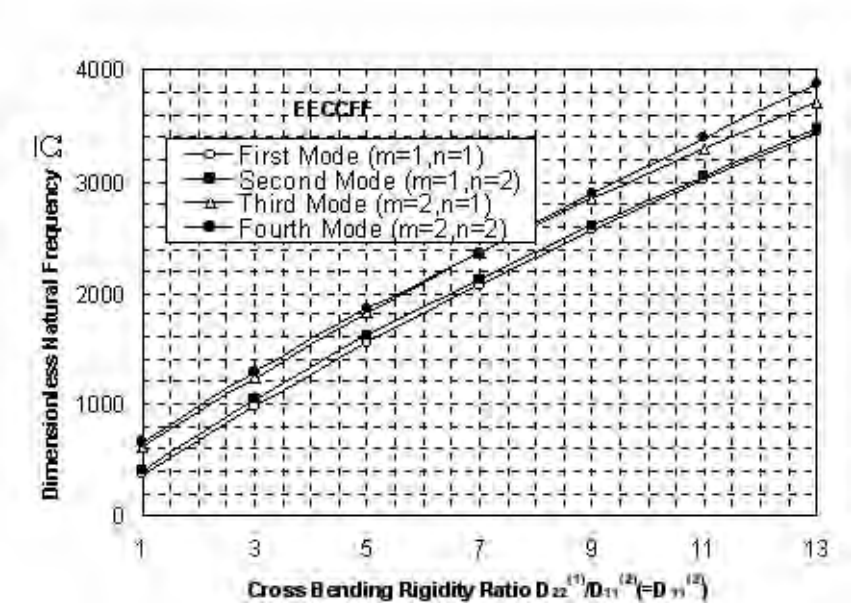

Figure 11.60. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Cross-bending Rigidity Ratio $D_{22}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})$ '' in "Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)"
((FFCCFF) B.C.'s, ''Hard'' Adhesive b_2=0.30m, a=0.50 m, L=1.0 m, $D_{22}^{(1)}$=varies)

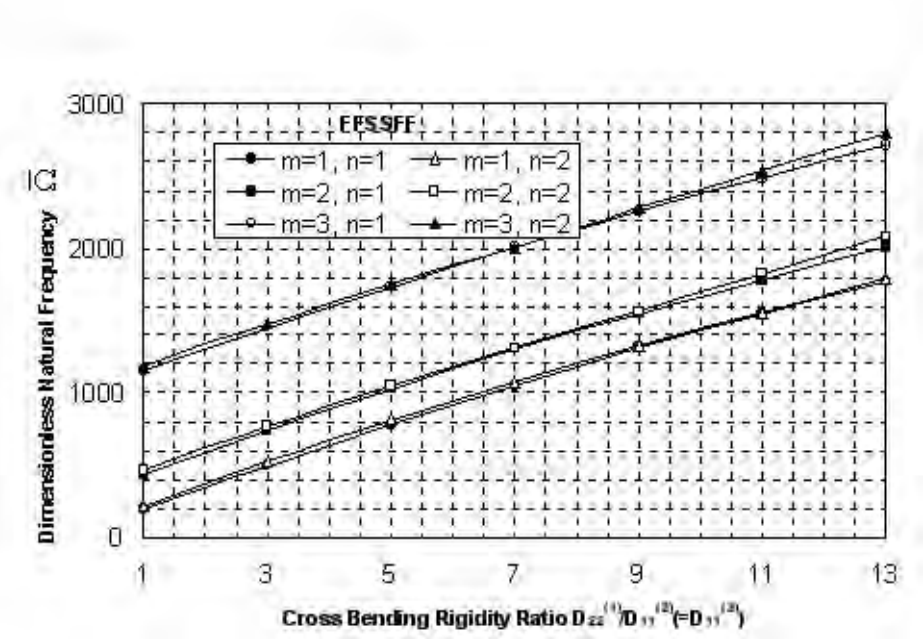

Figure 11.61. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Cross-bending Rigidity Ratio $D_{22}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})$ '' in "Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)" (Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on the number of half waves in y- and x-direction)
((FFSSFF) B.C.'s, ''Hard'' Adhesive, b_2=0.30m, a=0.50 m, L=1.0 m, $D_{22}^{(1)}$=varies)

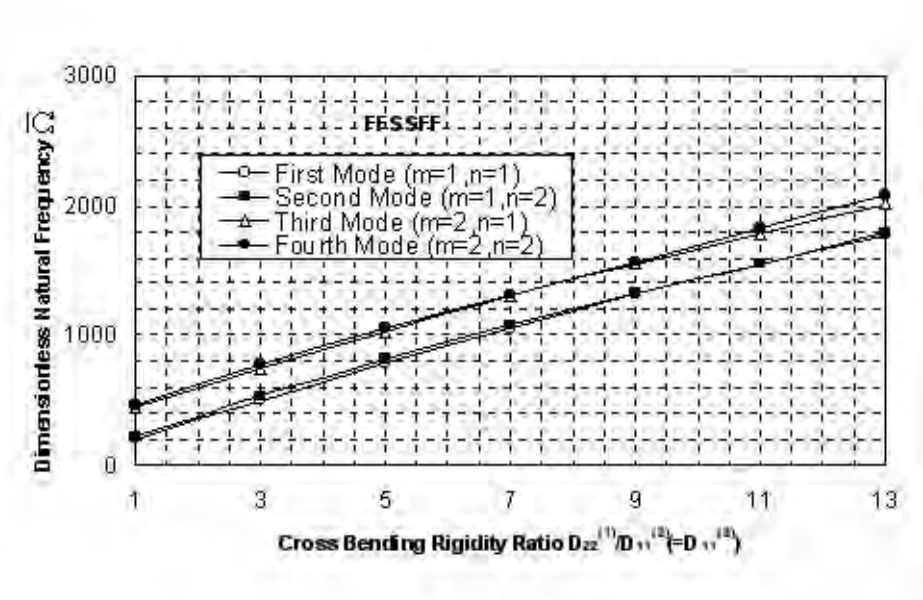

Figure 11.62. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Cross-bending Rigidity Ratio $D_{22}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})$ '' in "Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)" ((FFSSFF) B.C.'s, ''Hard'' Adhesive b_2=0.30m, a=0.50 m, L=1.0 m, $D_{22}^{(1)}$=varies)

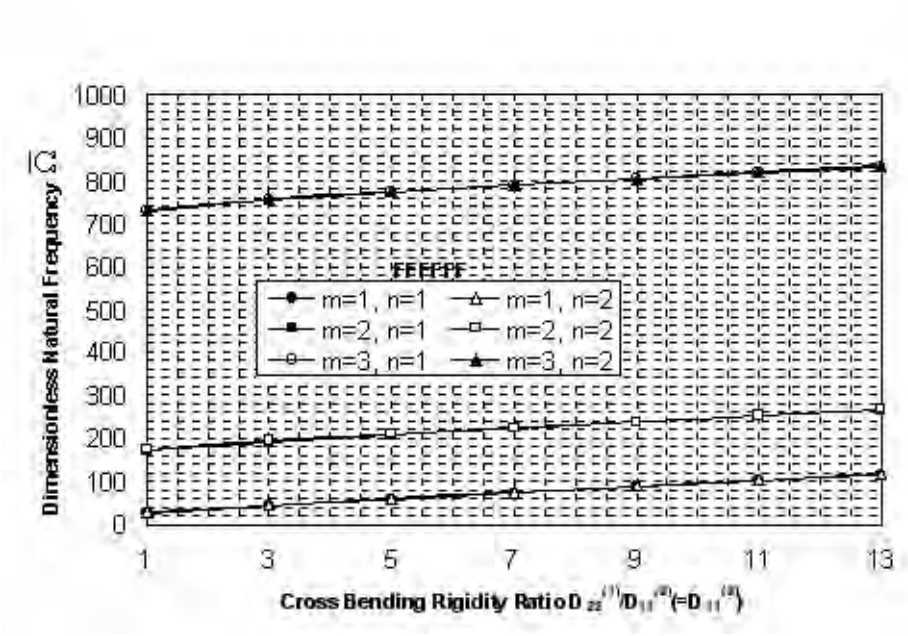

Figure 11.63. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Cross-bending Rigidity Ratio $D_{22}^{(1)}/(D_{11}^{(2)} \quad D_{11}^{(3)})$’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)” (Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on the number of half waves in y- and x-direction)
((FFFFFF) B.C.’s, ‘‘Hard’’ Adhesive, b_2=0.30m, a=0.50 m, L=1.0 m, $D_{22}^{(1)}$=varies)

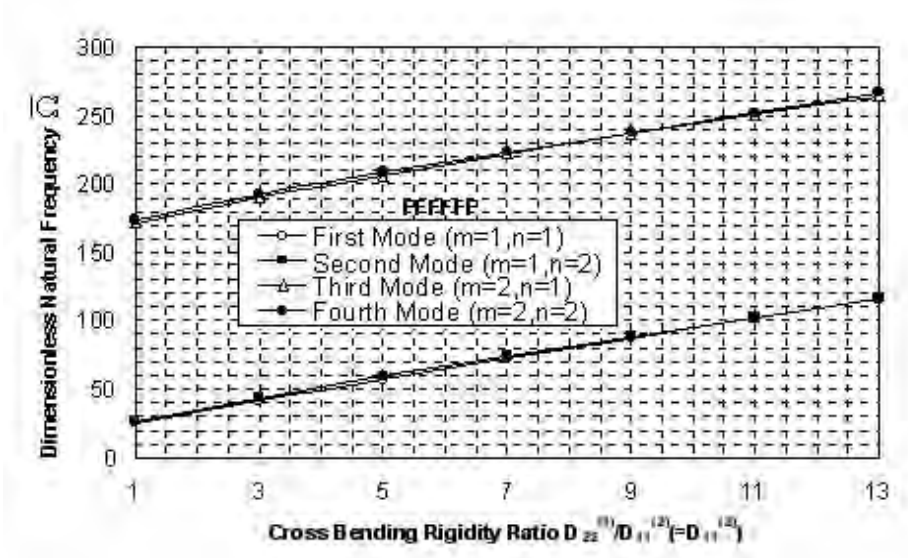

Figure 11.64. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Cross-bending Rigidity Ratio $D_{22}^{(1)}/(D_{11}^{(2)} \quad D_{11}^{(3)})$’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
((FFFFFF) B.C.’s, ‘‘Hard’’ Adhesive b_2=0.30m, a=0.50 m, L=1.0 m, $D_{22}^{(1)}$=varies)

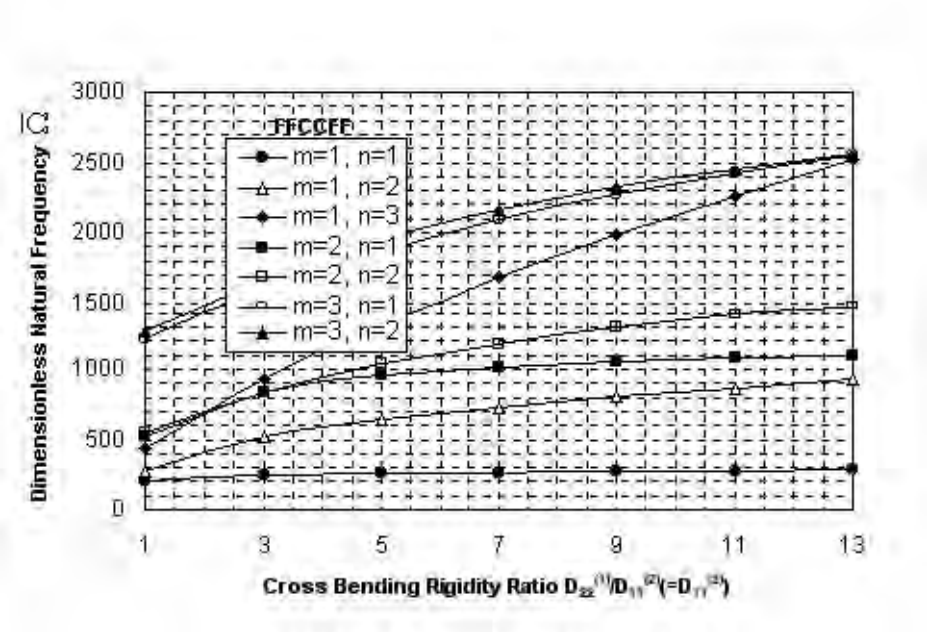

Figure 11.65. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Cross-bending Rigidity Ratio $D_{22}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})$ ’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)” (Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on the number of half waves in y- and x-direction)
((FFCCFF) B.C.’s, ‘‘Soft’’ Adhesive, b_2=0.30m, a=0.50 m, L=1.0 m, $D_{22}^{(1)}$=varies)

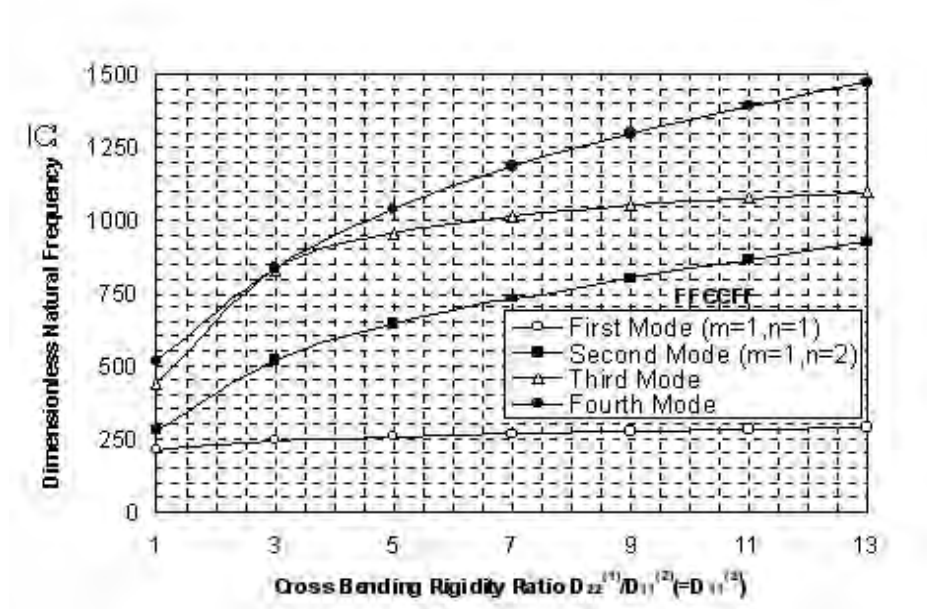

Figure 11.66. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Cross-bending Rigidity Ratio $D_{22}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})$ ’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
((FFCCFF) B.C.’s, ‘‘Soft’’ Adhesive b_2=0.30m, a=0.50 m, L=1.0 m, $D_{22}^{(1)}$=varies)

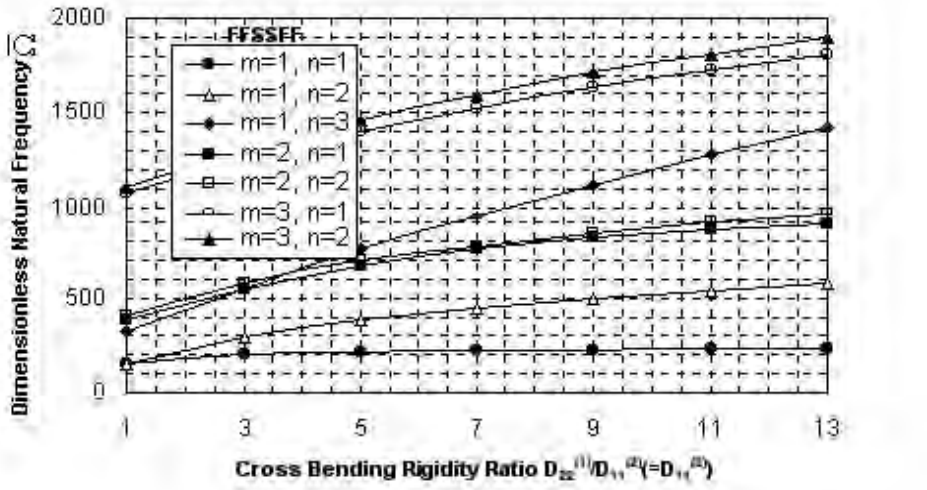

Figure 11.67. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Cross-bending Rigidity Ratio $D_{22}^{(1)}/(D_{11}^{(2)} \quad D_{11}^{(3)})$ ’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)” (Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on the number of half waves in y- and x-direction)
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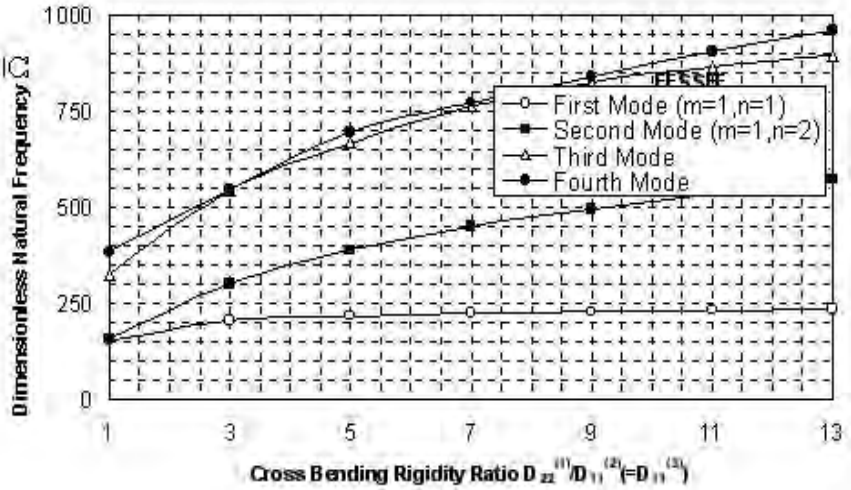


Figure 11.68. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Cross-bending Rigidity Ratio $D_{22}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})$ ’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
((FFSSFF) B.C.’s, ‘‘Soft’’ Adhesive b_2=0.30m, a=0.50 m, L=1.0 m, $D_{22}^{(1)}$=varies)

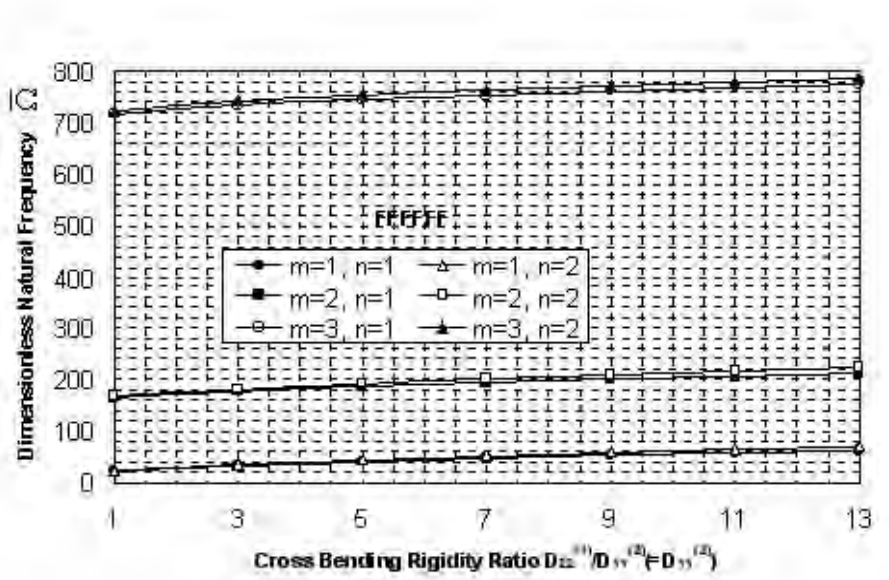


Figure 11.69. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Cross-bending Rigidity Ratio $D_{22}^{(1)}/(D_{11}^{(2)} \quad D_{11}^{(3)})$ ’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)” (Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on the number of half waves in y- and x-direction)
((FFFFFF) B.C.’s, ‘‘Soft’’ Adhesive, b_2=0.30m, a=0.50 m, L=1.0 m, $D_{22}^{(1)}$=varies)

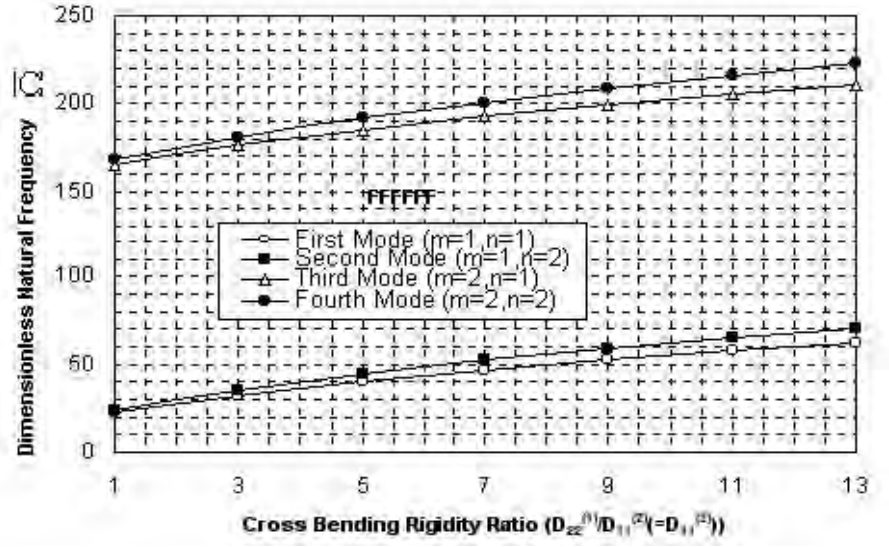


Figure 11.70. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Cross-bending Rigidity Ratio $D_{22}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$ ’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
((FFFFFF) B.C.’s, ‘‘Soft’’ Adhesive b_2=0.30m, a=0.50 m, L=1.0 m, $D_{22}^{(1)}$=varies)

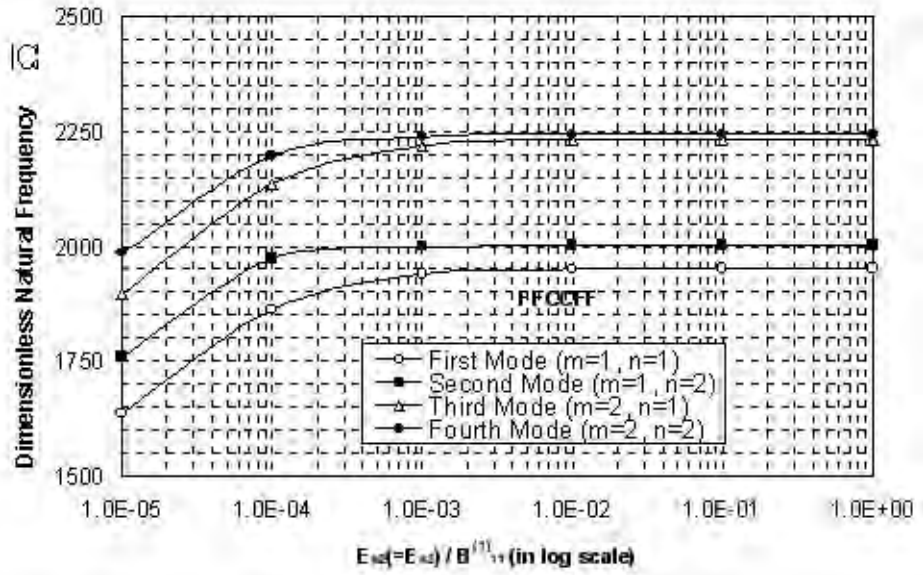


Figure 11.71. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFCCFF) B.C.’s, b_2=0.30m, a=0.50 m, L=1.0 m)

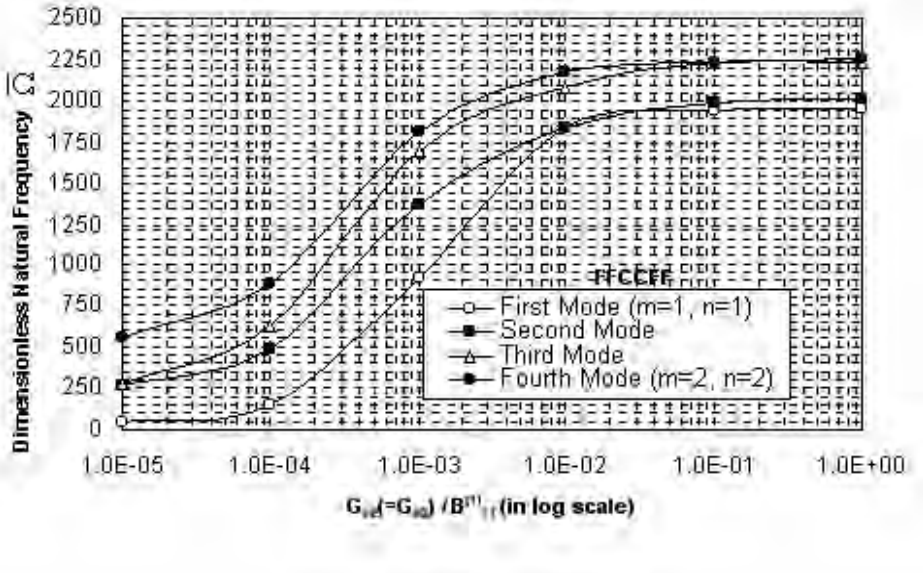


Figure 11.72. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3})/B_{11}^{(1)}$’’ in “Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFCCFF) B.C.’s, b_2=0.30m, a=0.50 m, L=1.0 m)

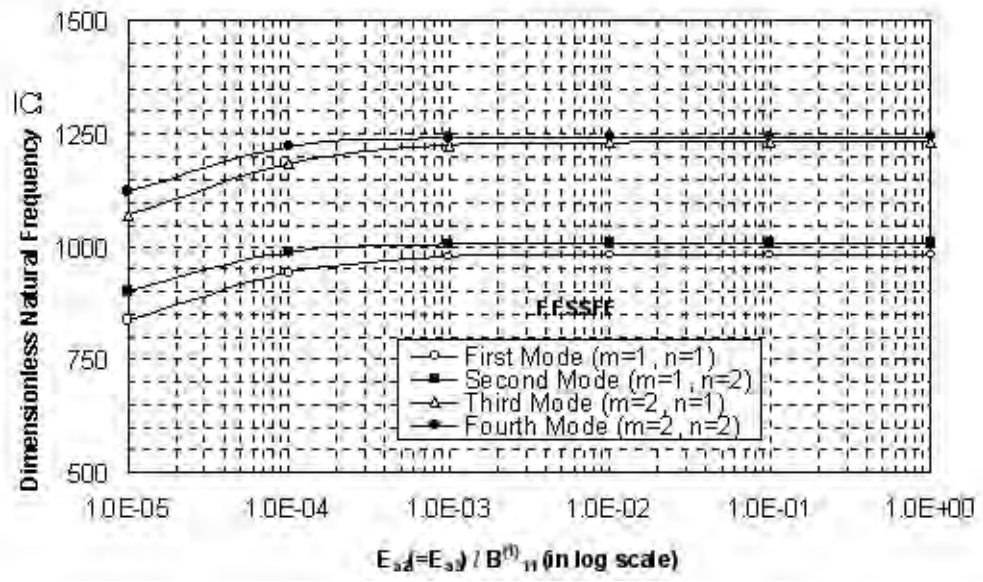


Figure 11.73. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$'' in "Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)"
(Various Modes with (FFSSFF) B.C.'s, b_2=0.30m, a=0.50 m, L=1.0 m)

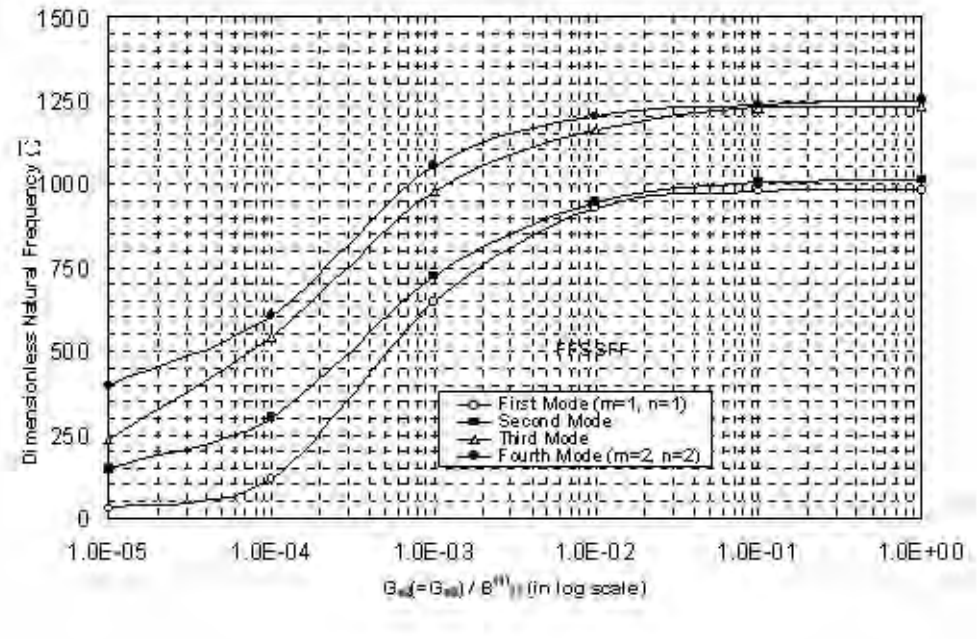


Figure 11.74. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3})/B_{11}^{(1)}$'' in "Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)"
(Various Modes with (FFSSFF) B.C.'s, b_2=0.30m, a=0.50 m, L=1.0 m)

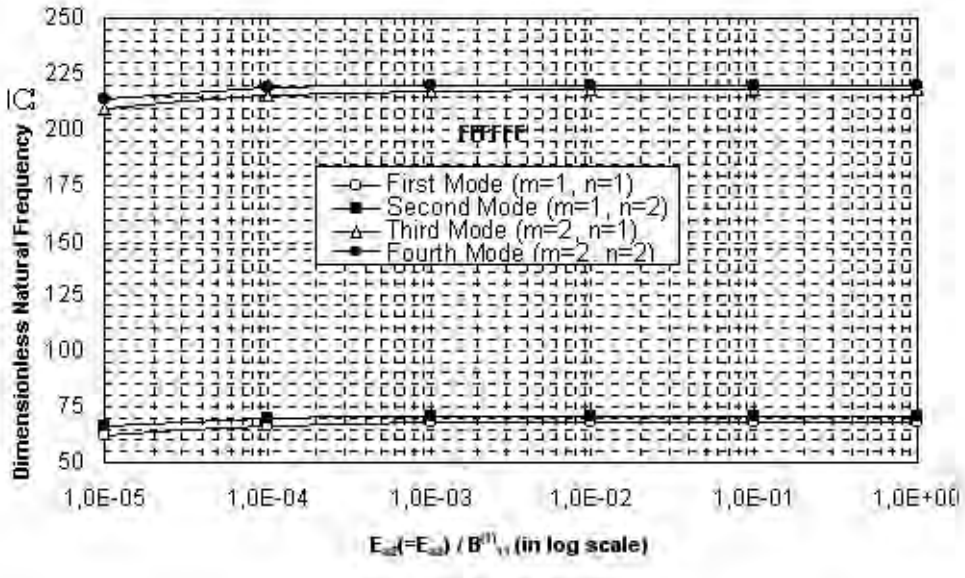


Figure 11.75. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$'' in "Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)"
(Various Modes with (FFFFFF) B.C.'s, b_2=0.30m, a=0.50 m, L=1.0 m)

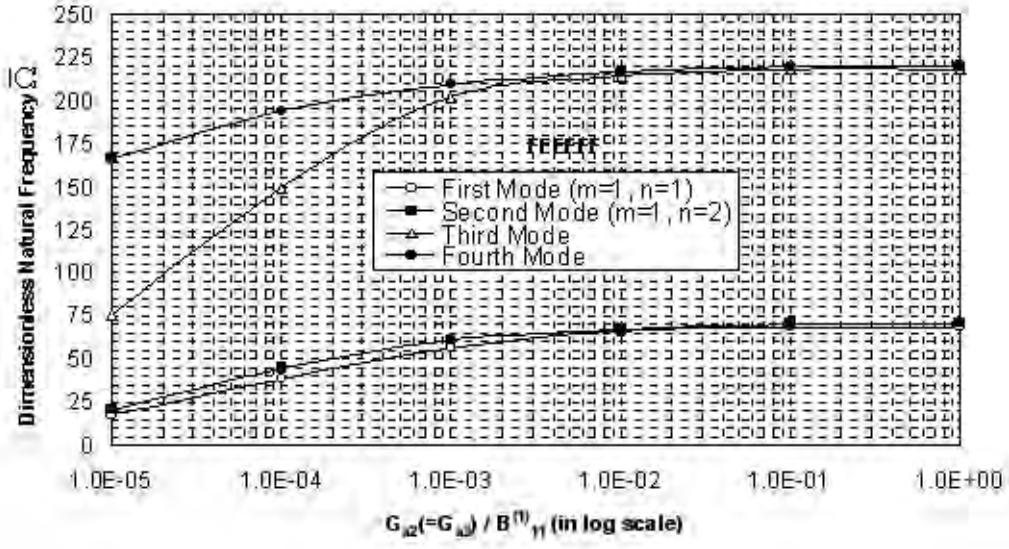


Figure 11.76. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3})/B_{11}^{(1)}$'' in "Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with pper and Lower Orthotropic Stiffening Plate Strips)"
(Various Modes with (FFFFFF) B.C.'s, b_2=0.30m, a=0.50 m, L=1.0 m)

11.3 Numerical Results and Discussion for “Main PROBLEM I.b”

In this “Main PROBLEM I.b”, the composite plate or panel system includes an “Isotropic Base Plate or Panel” (substituting for an “Aircraft Aluminum Alloy” panel) reinforced by the centrally positioned “Upper and Lower Orthotropic Stiffening Plate Strips”. Both stiffening strips are identical and made of Graphite-Epoxy plates. Again in Figures, adhesive layers are chosen as the “hard” and the “soft” adhesive cases with various support conditions. Although, they are not shown here, in order to demonstrate the significant effects of the adhesive shear modulus, $G_{a2}=G_{a3}=0$ $(10^{-8})\approx 0$ is considered as an extreme case in some Figures not shown here. It was found that the symmetry and/or skew-symmetry of mode shapes hold provided the geometry, the material and the B.C.’s are symmetric with respect to the mid-plane. The geometric, and material characteristics of the composite plate system under consideration are given in Table 11.2. In this “Main PROBLEM I.b”, (similar to the preceding “Main PROBLEM I.a”) the elastic constants of the upper and lower adhesive layers are considered to be identical (In the theoretical formulation, they are assumed to be dissimilar).

In what follows, first, the mode shapes corresponding to the several support conditions, in the “hard” and the “soft” adhesive cases are presented and discussed in some detail.

Figure 11.77, the first four mode shapes with the corresponding dimensionless natural frequencies of the composite system of “Main PROBLEM I b” are displayed, for the “hard” adhesives case, with the (FFCCFF) boundary conditions.

Similarly, the first four mode shapes and the corresponding dimensionless natural frequencies for the same system with the same boundary conditions in the ‘‘soft’’ adhesives case are presented in Figure 11.78.

As in “Main PROBLEM I.a”, it can be easily observed from Figure 11.77 that the mode shapes are not exactly symmetric and/or skew-symmetric while the entire

system (geometry, B.C.'s and material) is symmetric with respect to the previously defined mid-plane perpendicular to the base plate. However, in Figure 11.78 for the "soft" adhesives case with the same (FFCCFF) support conditions, it can be observed that the mode shapes become symmetric and/or skew-symmetric with respect to the previously defined mid-plane.

Also, by comparing, the "hard" and the "soft" adhesive cases, in the above Figures, it can be concluded that the adhesive layer elastic modulus and shear modulus significantly influence the mode shapes and the corresponding natural frequencies.

In Figures 11.79 and 11.80, the same composite system of Figures 11.77 and 11.78 are now considered with (FFSSFF) boundary conditions. From the comparison of those Figures, it is obvious that the mode shapes have essentially the same pattern in both cases of the boundary conditions.

It is again interesting to observe, in the "hard" adhesive cases, the doubly stiffened central portion of the composite plate system is almost stationary. This result is important from crack creation and/or propagation point of view in the "Isotropic Base Plate and Panel" (or the Aluminum Alloy Base Plate).

Another important observation is that the support conditions in the y-direction, greatly influenced the mode shapes and the corresponding frequencies. It can be concluded that the stiffer or more constraining the boundary conditions, the higher the natural frequencies.

The parametric studies for "Main PROBLEM I.b" will be considered next.

In Figures 11.81 through 11.96 dimensionless natural frequency $\overline{\Omega}$ versus "Aspect Ratio b_2/a" are plotted for both ''hard'' and ''soft'' adhesives cases for various boundary condition. For the "hard" adhesives case, it is obvious from these Figures that as the "Aspect Ratio (b_2/a)", or the wetted-area increases, so the natural frequencies. It can also be stated that as the support conditions get increasingly stiffer

or more constraining, the natural frequency curves go higher and with sharper slopes specially in the ‘‘hard’’ adhesives case. However, for the “soft” adhesives case, the adhesive layers effects on the natural frequencies are not dominant in comparison with other parameters. Hence, as the “Aspect Ratio b_2/a” increases, the dimensionless natural frequencies remain almost constant or change in a very small range.

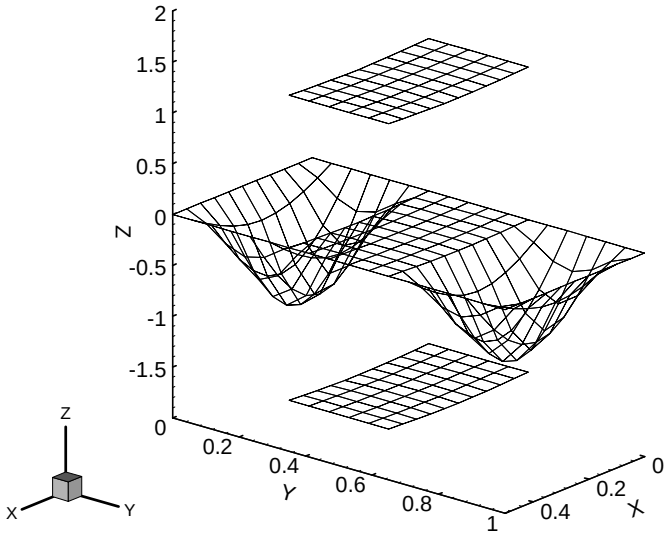
In Figures 11.97, 11.98 and 11.99, 11.100 and 11.101, 11.102, the effect of the “Bending Rigidity Ratio $\boldsymbol{D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})}$” on the natural frequencies are investigated in the “hard” adhesives case with the (FFCCFF), (FFSSFF), and the (FFFFFF) support conditions , respectively. It is interesting to observe that the first, second, and the third, fourth dimensionless frequencies are extremely close to each other. Also, the general trend is that they drop sharply for the small values of the “Bending Rigidity Ratio $\boldsymbol{D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})}$” up to 4.0, after than they decrease rather gradually.

It should be pointed out here, that, while the dimensionless natural frequency parameter decreases as the “Bending Rigidity Ratio $\boldsymbol{D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})}$” increases, the real dimensional natural frequency increases as excepted (see for instance, the expression (4.16) for the dimensionless natural frequency paramater).

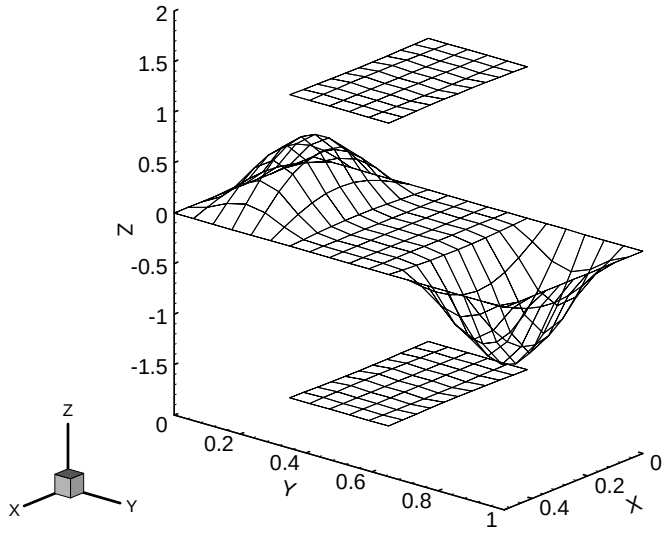
The effect of the “Bending Rigidity Ratio $\boldsymbol{D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})}$” on the dimensionless natural frequencies in the “soft” adhesives case are plotted in Figures 11.103, 11.104 and 11.105, 11.106 and 11.107, 11.108 for several modes corresponding to the (FFCCFF), (FFSSFF) and the (FFFFFF) boundary conditions, respectively. The dimensionless natural frequencies decrease relatively sharply, at first, (up to 4.0 value of the “Bending Rigidity Ratio $\boldsymbol{D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})}$”) and, after that value, the decrease is rather gradual. However, there is not another common pattern in those figures. As in the similar “hard” adhesives case, the real dimensional natural frequencies increase while the dimensionless natural frequencies decrease (see again for example, the expression. (4.16))

Lastly, the direct effects of the adhesive layer elastic constants E_a, G_a on the dimensionless natural frequencies are investigated for the "Main PROBLEM I.b". In order to show these effects, the dimensionless natural frequencies versus the "Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$" are plotted (while the other elastic constant kept constant) in Figures 11.109, 11.111 and 11.113 for the support conditions (FFCCFF), (FFSSFF) and (FFFFFF), respectively. Similarly, the dimensionless natural frequencies versus the "Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3})/B_{11}^{(1)}$" are presented in Figures 11.100, 11.112 and 11.114 for the boundary condition (FFCCFF), (FFSSFF), (FFFFFF), respectively. These curves are very much similar to those of the "Main PROBLEM I.a".

It is obvious from the aforementioned figures, "Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3})/B_{11}^{(1)}$" has very significant effect on the natural frequencies of the composite plate systems considered here in the "Main PROBLEM I.b". This result is expected if, in the composite systems, the shear deformable character of the Mindlin Plates is taken into consideration. The influence of the "Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$" on the natural frequencies is rather relatively weak and depends on the support conditions in the y-direction on the composite plate or panel system.



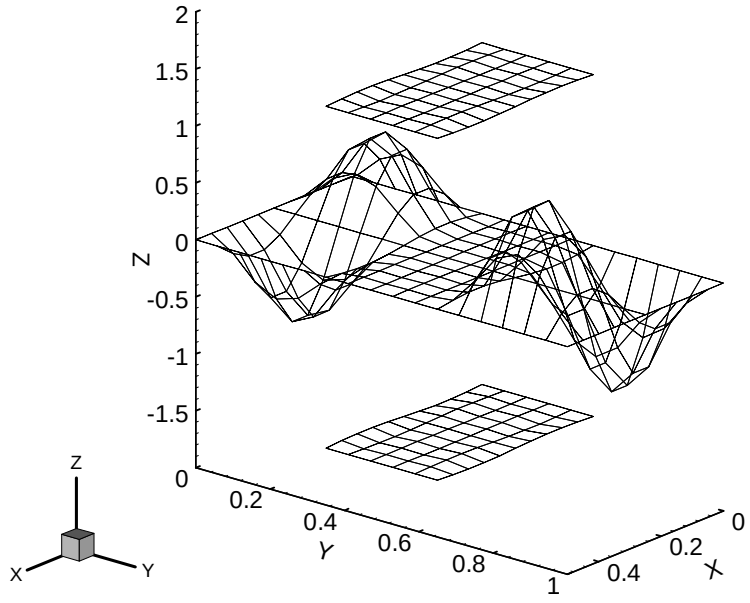
a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 193.744$



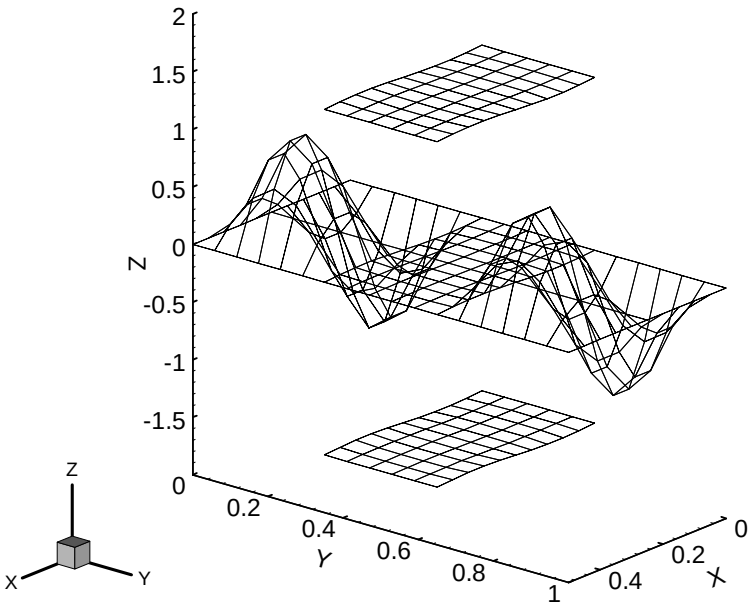
b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{12} = 195.026$

Figure 11.77. Mode Shapes and Natural Frequencies of "Centrally Doubly Stiffened, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Base Plate=Plate 1= Aluminum , Plate 2= Plate 3= Graphite-Epoxy)
((FFCCFF) B.C.'s in y-direction, "Hard" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

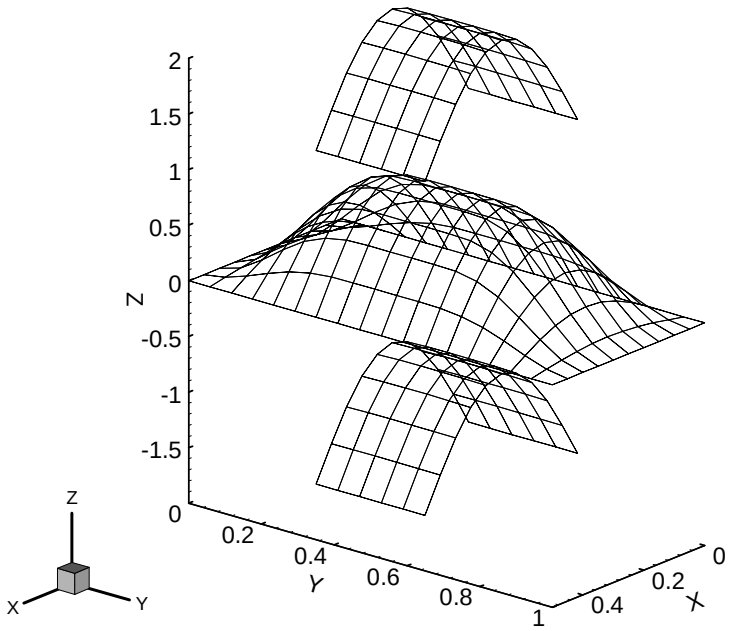


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{21} = 420.630$

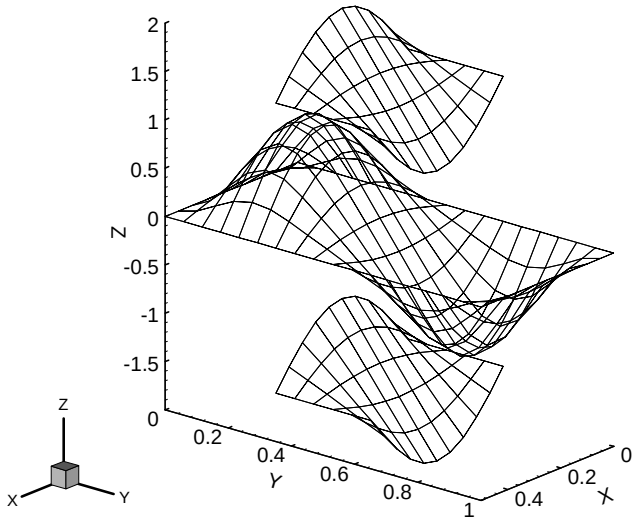


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{22} = 423.282$

Figure 11.77. (continued)



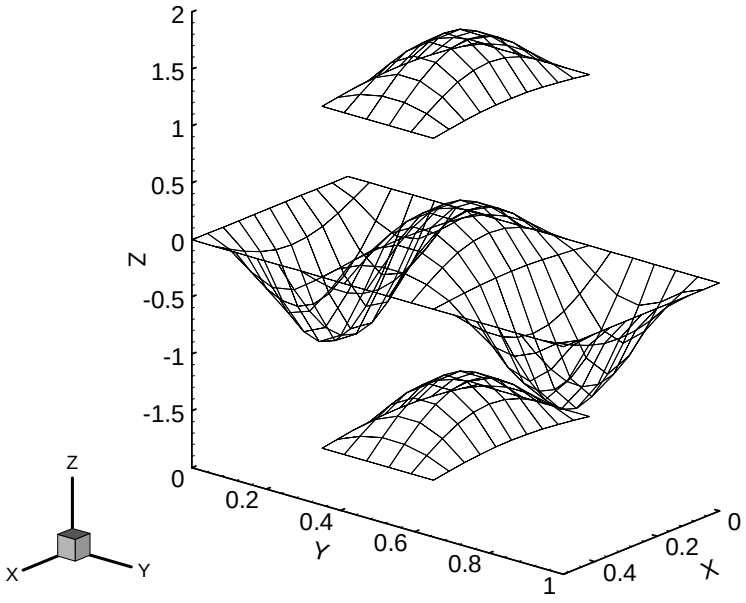
a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 33.850$



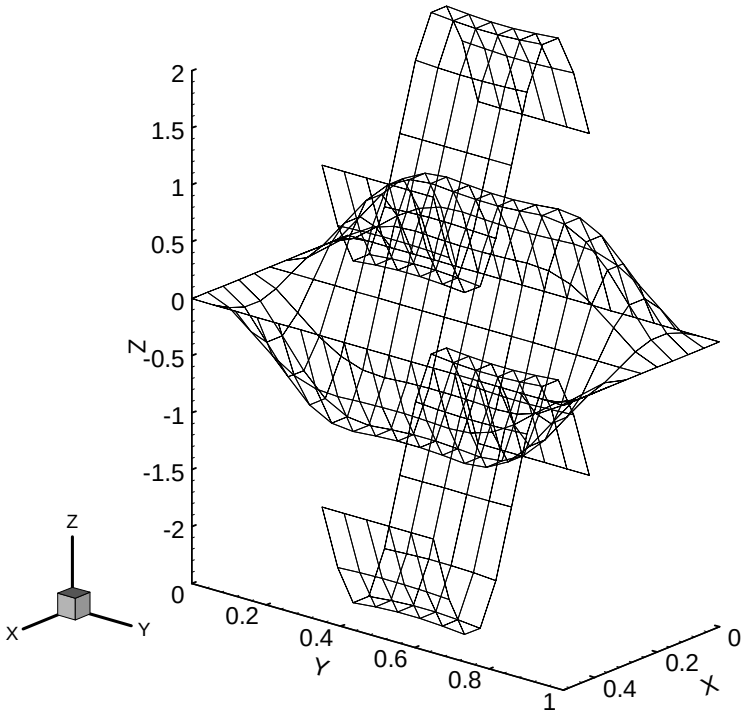
b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{12} = 78.969$

Figure 11.78. Mode Shapes and Natural Frequencies of "Centrally Doubly Stiffened, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Base Plate=Plate 1= Aluminum , Plate 2= Plate 3= Graphite-Epoxy)
((FFCCFF) B.C.'s in y-direction, "Soft" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m.)

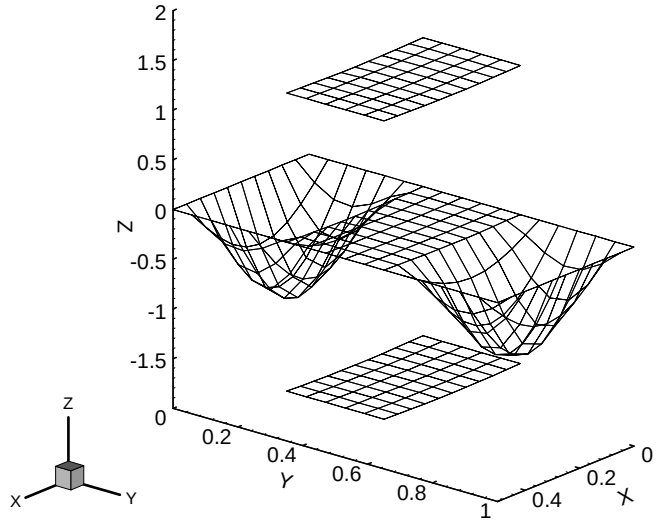


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{13} = 168.475$

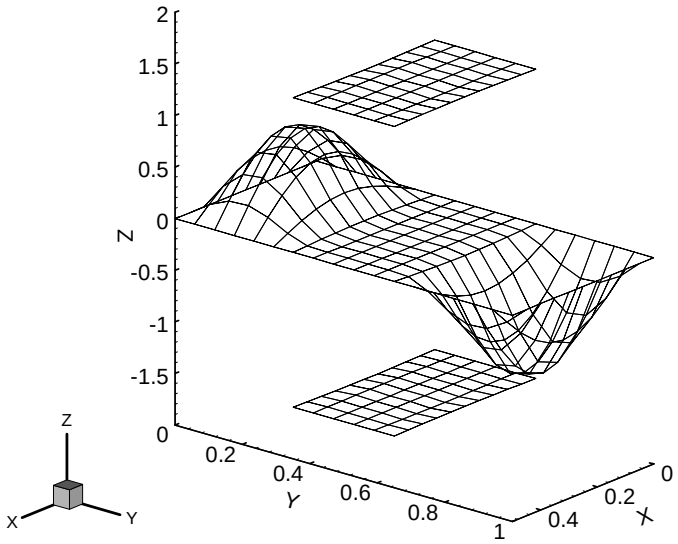


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{21} = 194.681$

Figure 11.78. (continued)



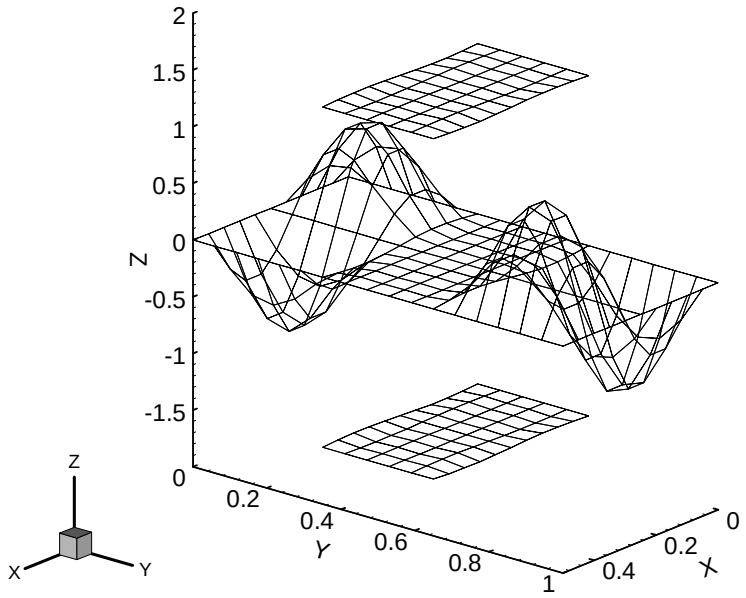
a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 113.134$



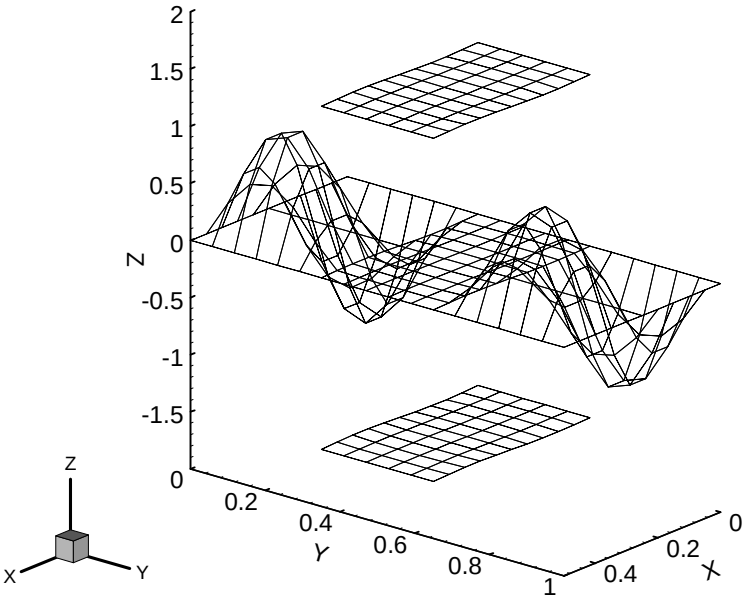
b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{12} = 114.099$

Figure 11.79. Mode Shapes and Natural Frequencies of "Centrally Doubly Stiffened, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Base Plate=Plate 1= Aluminum , Plate 2= Plate 3= Graphite-Epoxy)
((FFSSFF) B.C.'s in y-direction, "Hard" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m.)

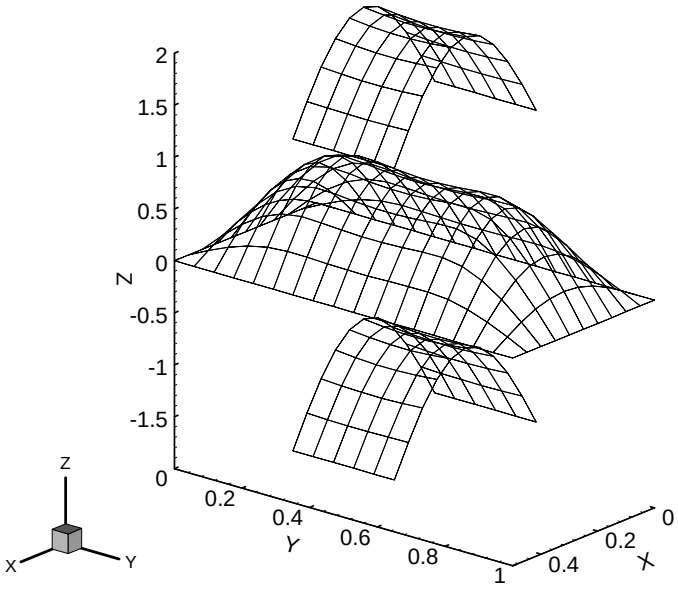


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{21} = 333.224$

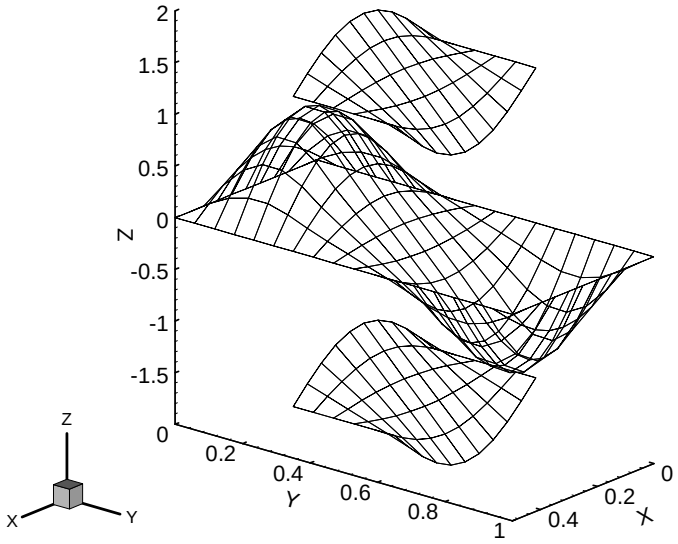


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{22} = 334.592$

Figure 11.79. (continued)



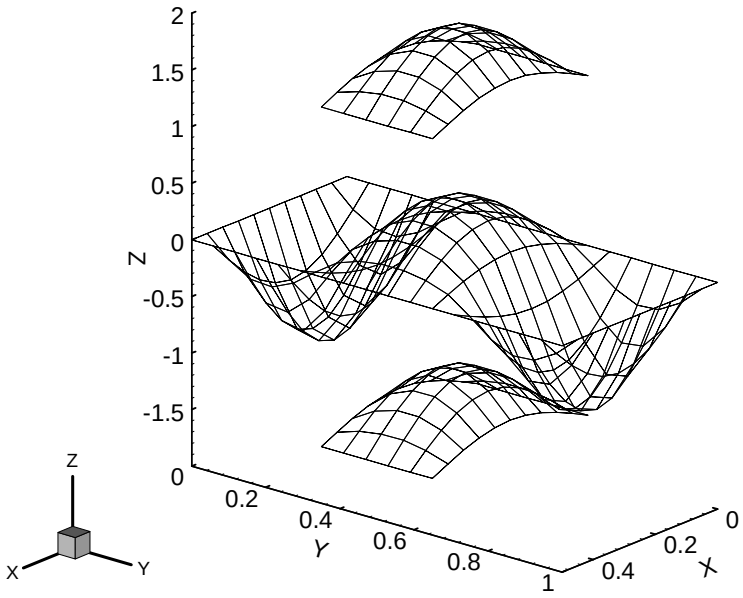
a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 29.560$



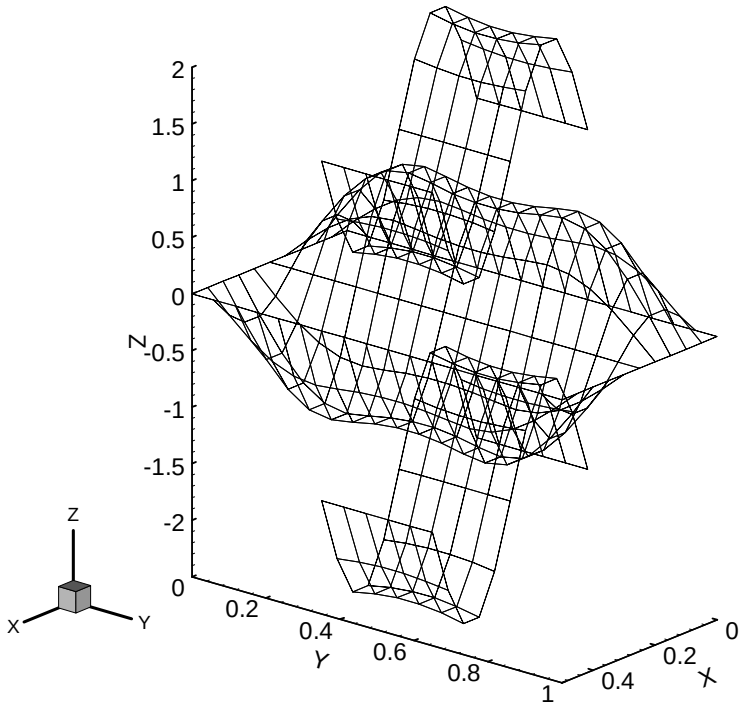
b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{12} = 55.568$

Figure 11.80. Mode Shapes and Natural Frequencies of "Centrally Doubly Stiffened, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Base Plate=Plate 1= Aluminum , Plate 2= Plate 3= Graphite-Epoxy)
((FFSSFF) B.C.'s in y-direction, "Soft" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m.)



c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{13} = 111.299$



d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{21} = 187.693$

Figure 11.80. (continued)

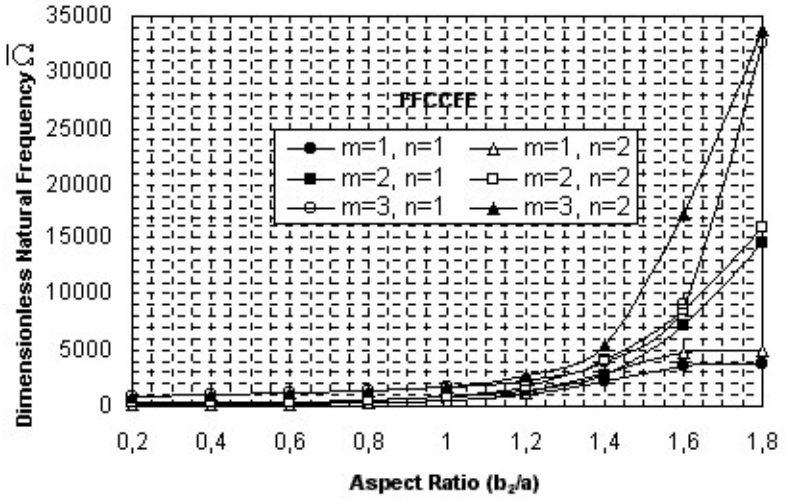


Figure 11.81. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Dependency of natural frequency parameter on the number of half waves in y- and x-directions)
(Base Plate=Plate 1= Aluminum , Plate 2= Plate 3= Graphite-Epoxy)
((FFCCFF) B.C.'s, ''Hard" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

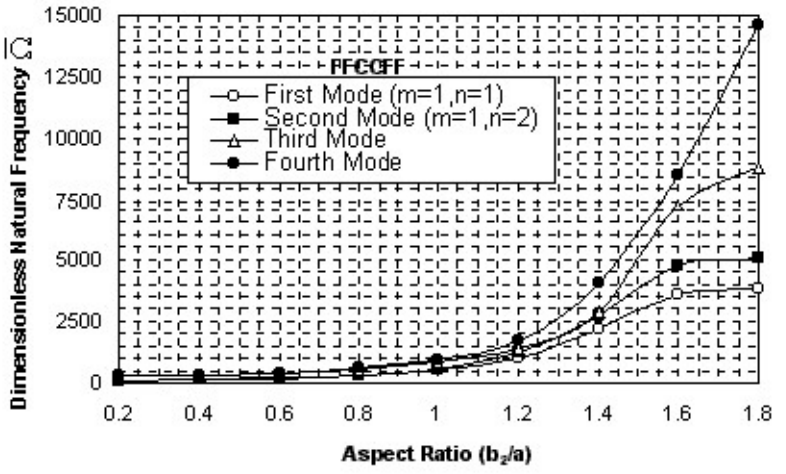


Figure 11.82. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Various Modes with (FFCCFF) B.C.'s, ''Hard'' Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m))
(Base Plate=Plate 1= Aluminum , Plate 2= Plate 3= Graphite-Epoxy)

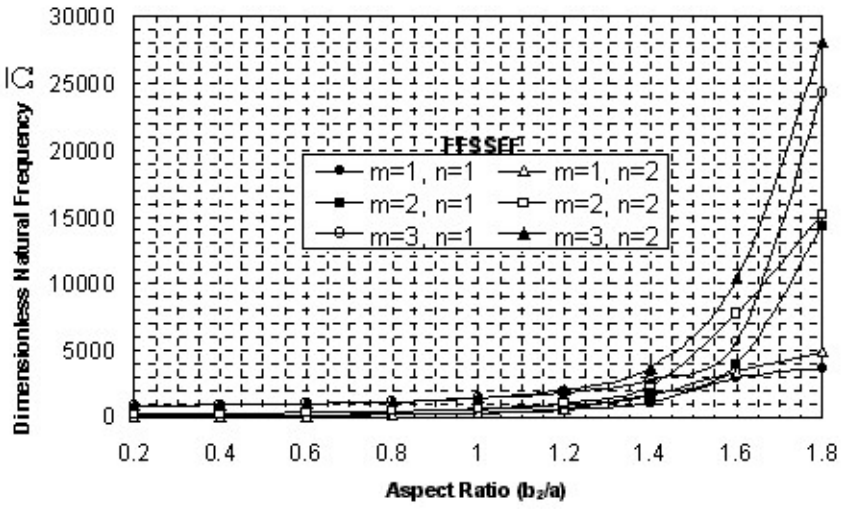


Figure 11.83. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Dependency of natural frequency parameter on the number of half waves in y- and x-directions)
(Base Plate=Plate 1= Aluminum , Plate 2= Plate 3= Graphite-Epoxy)
((FFSSFF) B.C.'s, ''Hard" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

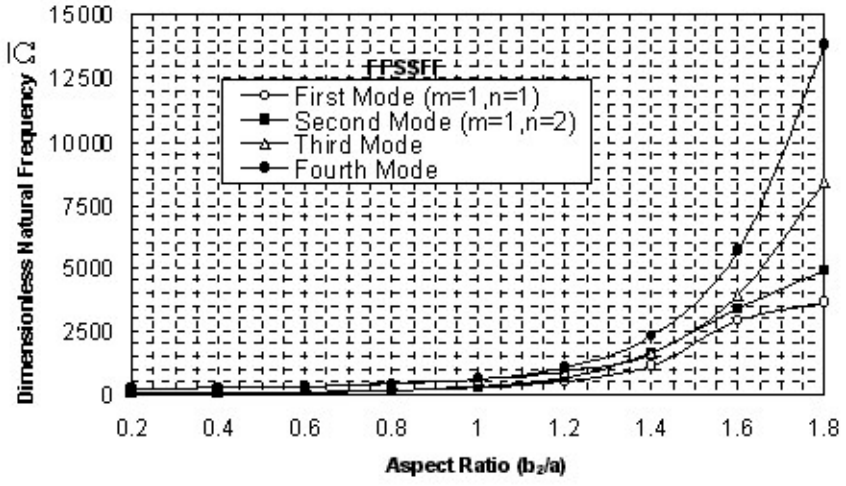


Figure 11.84. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Various Modes with (FFSSFF) B.C.'s, ''Hard'' Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)
(Base Plate=Plate 1= Aluminum, Plate 2= Plate 3= Graphite-Epoxy)

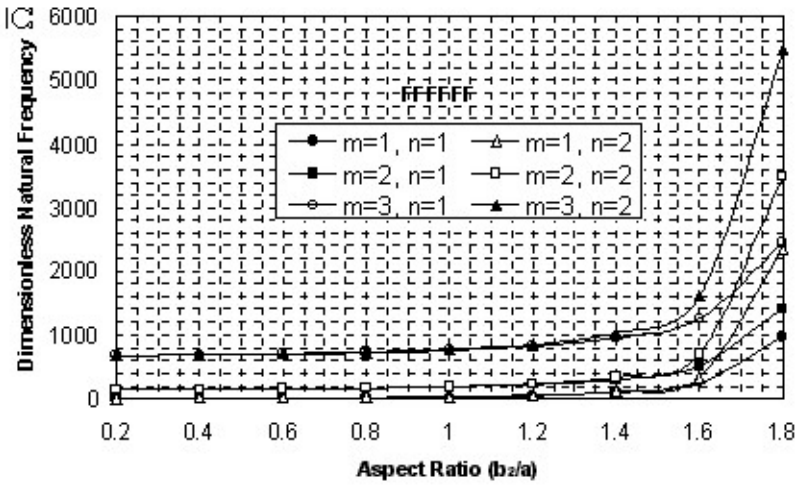


Figure 11.85. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Dependency of natural frequency parameter on the number of half waves in y- and x-directions)
(Base Plate=Plate 1= Aluminum , Plate 2= Plate 3= Graphite-Epoxy)
((FFFFFF) B.C.'s, ''Hard" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

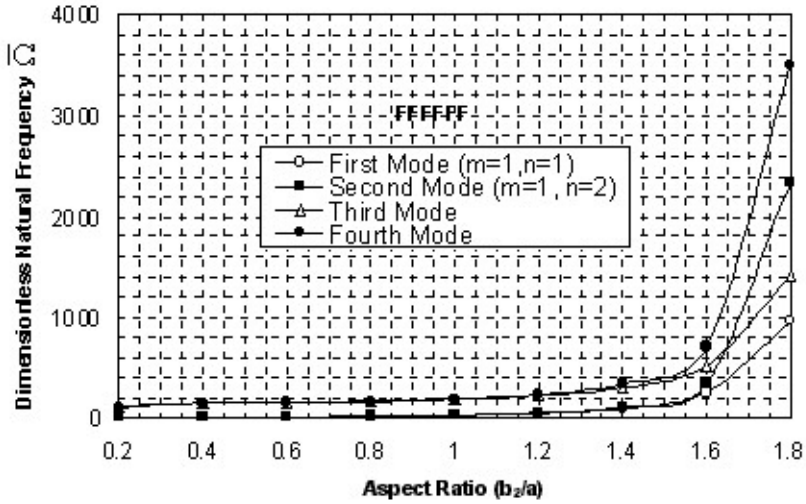


Figure 11.86. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Various Modes with (FFFFFF) B.C.'s, ''Hard'' Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)
(Base Plate=Plate 1= Aluminum , Plate 2= Graphite-Epoxy, Plate 3= Graphite)

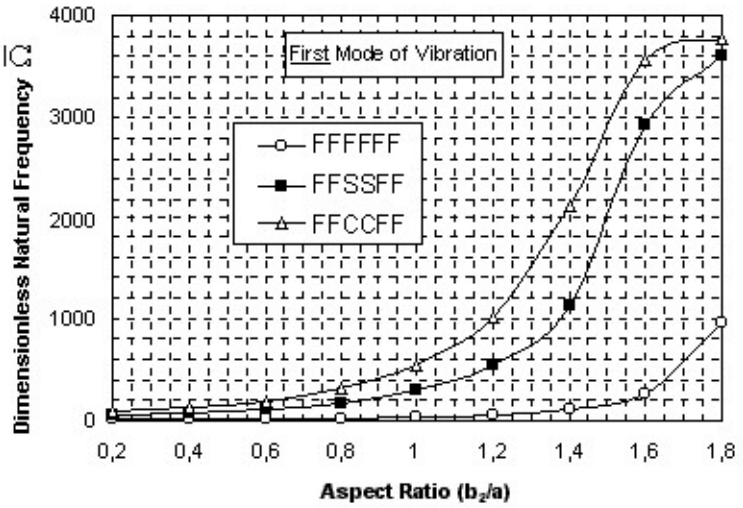


Figure 11.87. First Mode Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a’’ in “Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
(For Various B.C.’s, ‘‘Hard” Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

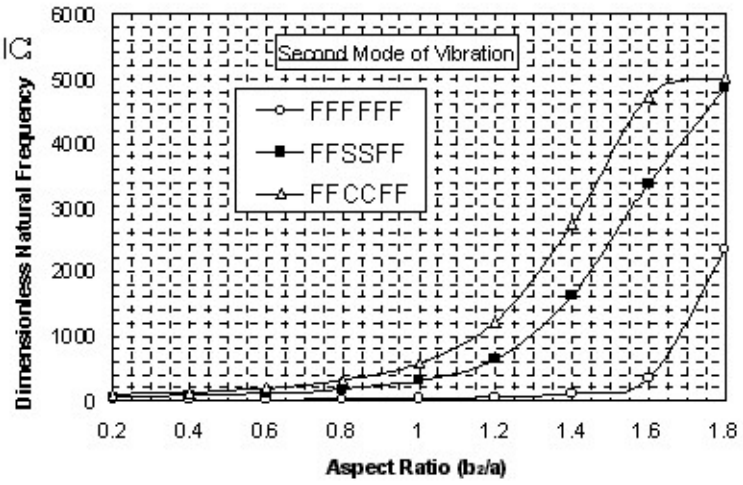


Figure 11.88. Second Mode Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a’’ in “Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
(For Various B.C.’s, ‘‘Hard” Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

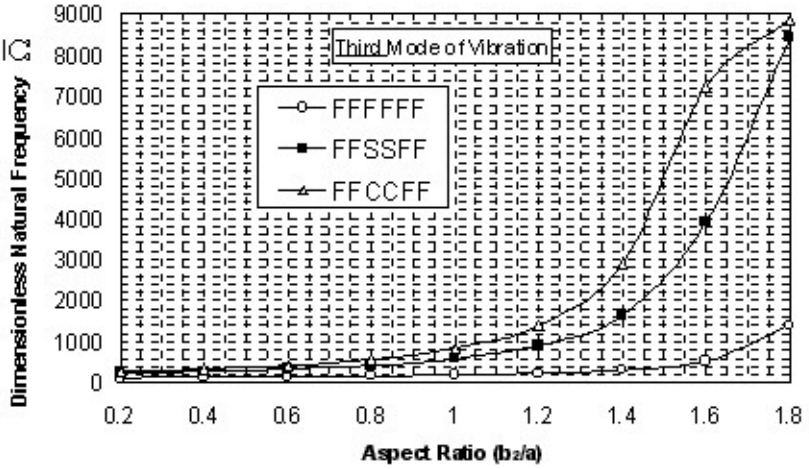


Figure 11.89. Third Mode Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a’’ in “Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
(For Various B.C.’s, ‘‘Hard” Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

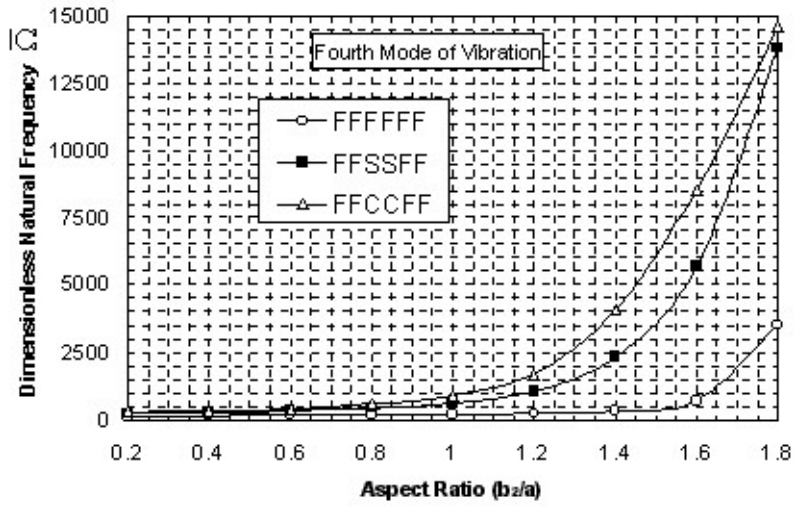


Figure 11.90. Fourth Mode Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a’’ in “Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
(For Various B.C.’s, ‘‘Hard” Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

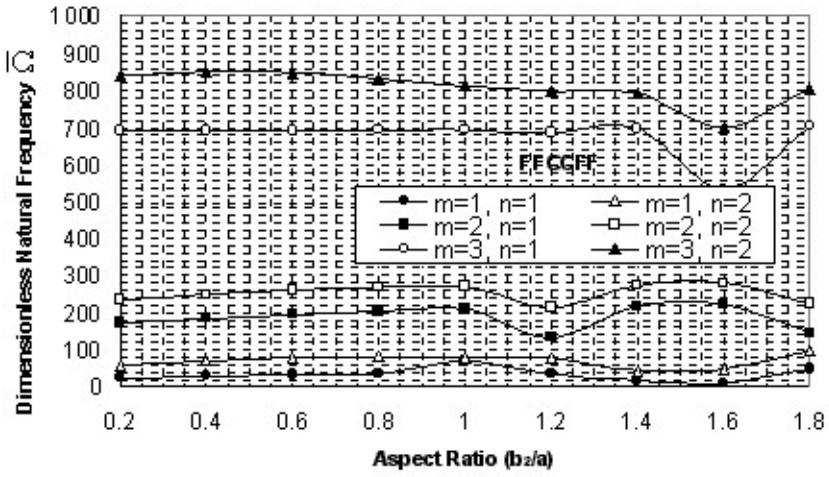


Figure 11.91. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on the number of half waves in y- and x-direction)

((FFCCFF) B.C.'s, ''Soft'' Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

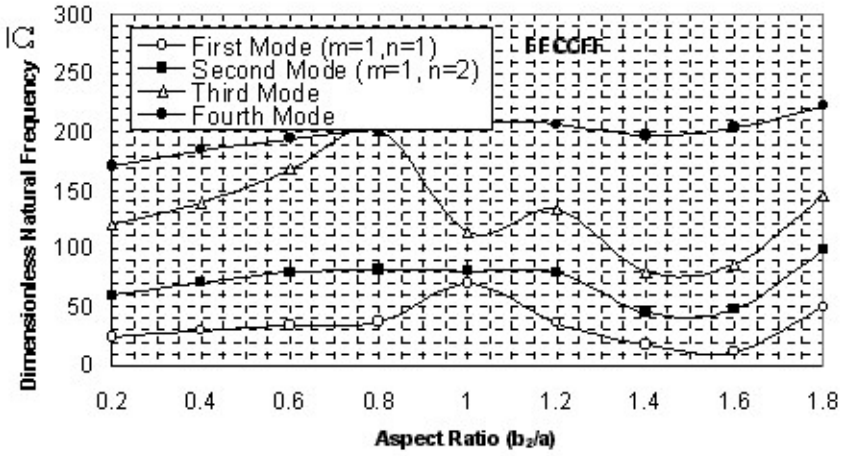


Figure 11.92. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Various Modes with (FFCCFF) B.C.'s, ''Soft'' Adhesive, b_2=varies, a=0.50 m,

L=1.0 m)

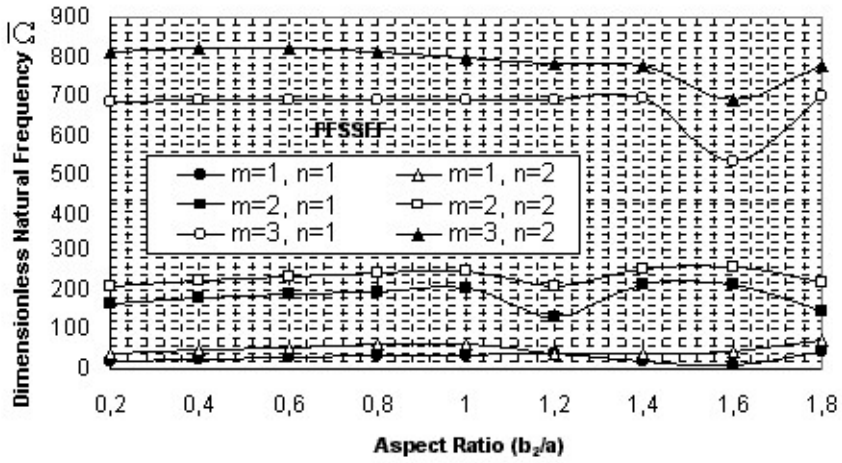


Figure 11.93. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on the number of half waves in y- and x-direction)
((FFSSFF) B.C.'s, ''Soft'' Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

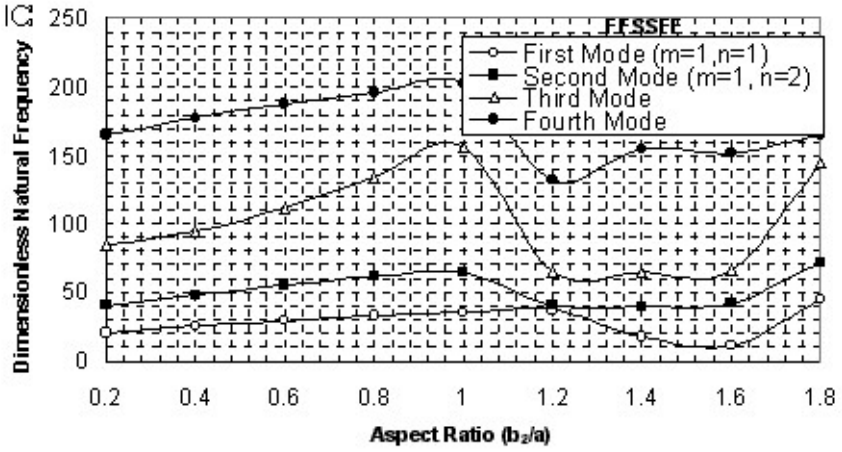


Figure 11.94. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Various Modes with (FFSSFF) B.C.'s, ''Soft'' Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

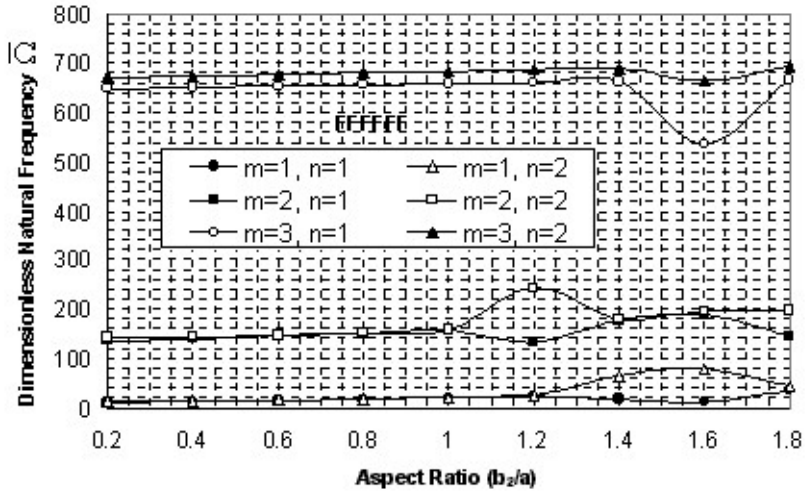


Figure 11.95. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on the number of half waves in y- and x-direction)

((FFFFFF) B.C.'s, ''Soft'' Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

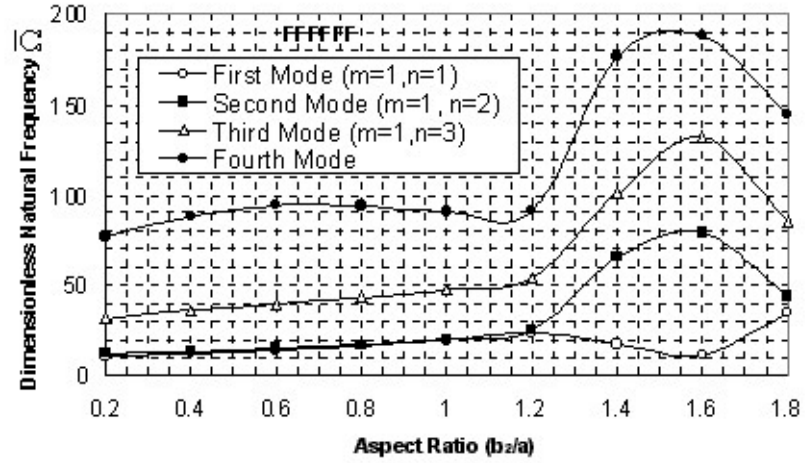


Figure 11.96. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Various Modes with (FFFFFF) B.C.'s, ''Soft'' Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

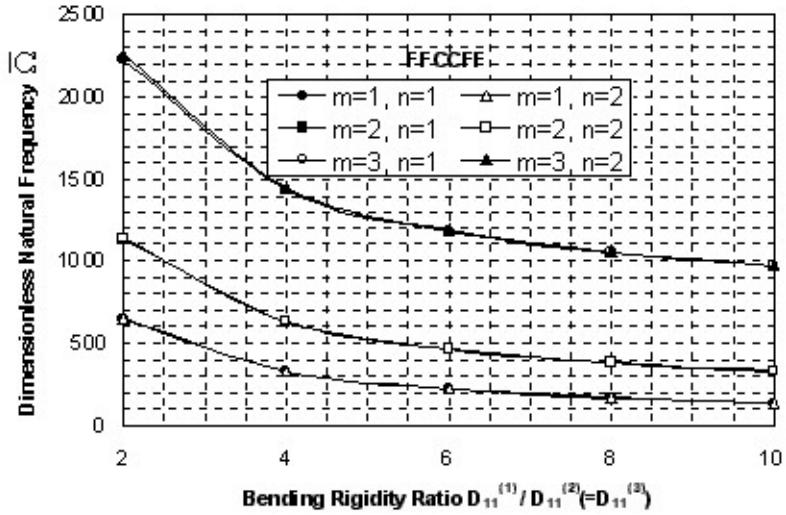


Figure 11.97. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$” in “Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega}=\overline{\omega}_{mn}$ on the number of half waves in y-and x-directions)
((FFCCFF) B.C.’s, ‘‘Hard’’ Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

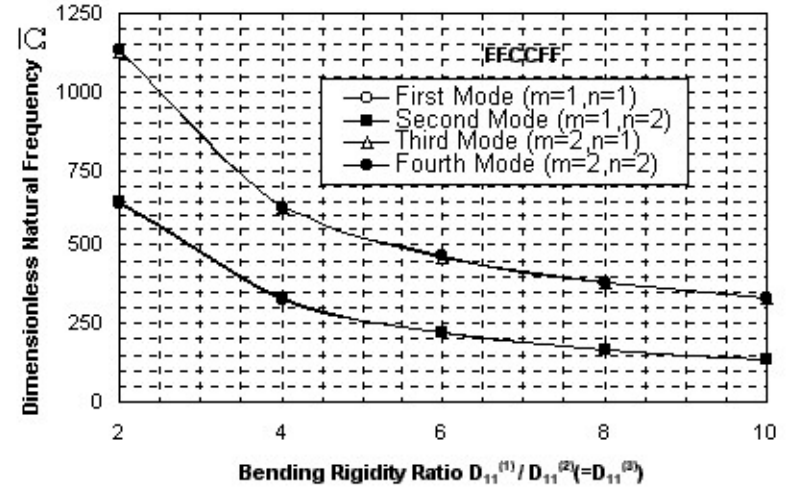


Figure 11.98. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$’’ in “Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFCCFF) B.C.’s, ‘‘Hard’’ Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

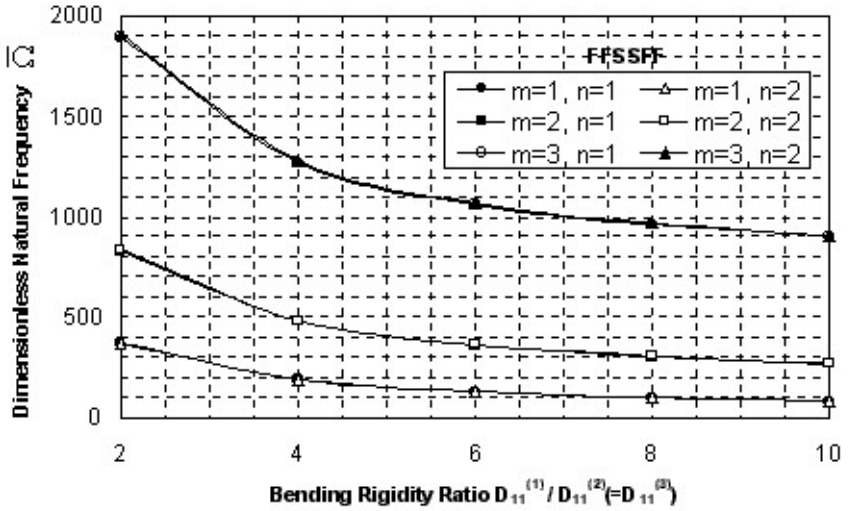


Figure 11.99. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$” in “Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega}=\overline{\omega}_{mn}$ on the number of half waves in y-and x-directions)
((FFSSFF) B.C.’s, ‘‘Hard’’ Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

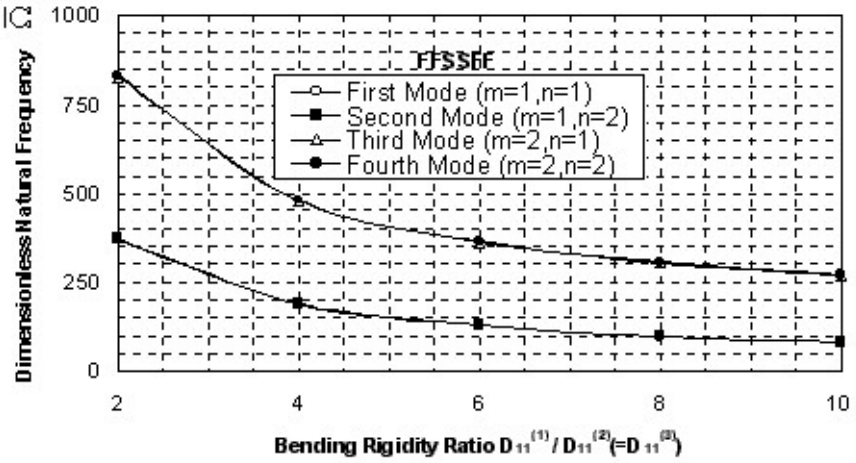


Figure 11.100. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$’’ in “Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFSSFF) B.C.’s, ‘‘Hard’’ Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

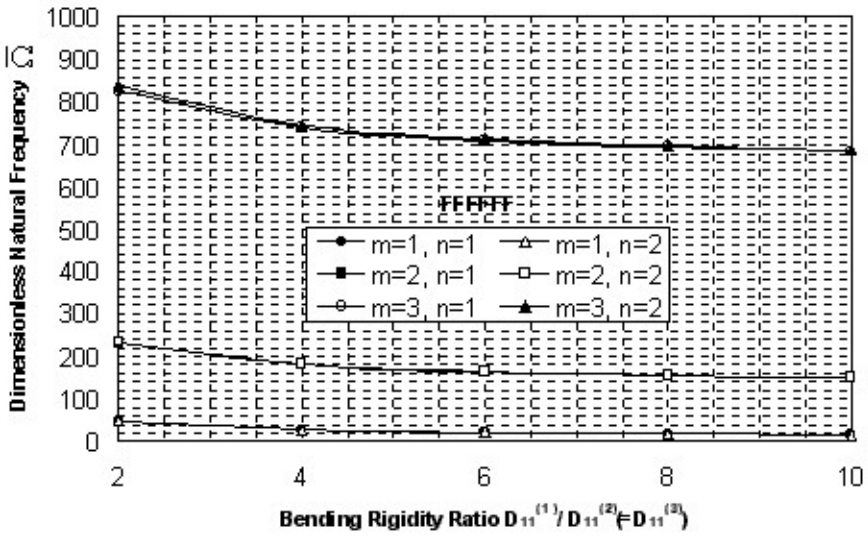


Figure 11.101. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$’’ in “Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega}=\overline{\omega}_{mn}$ on the number of half waves in y-and x-directions)
((FFFFFF) B.C.’s, ‘‘Hard’’ Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

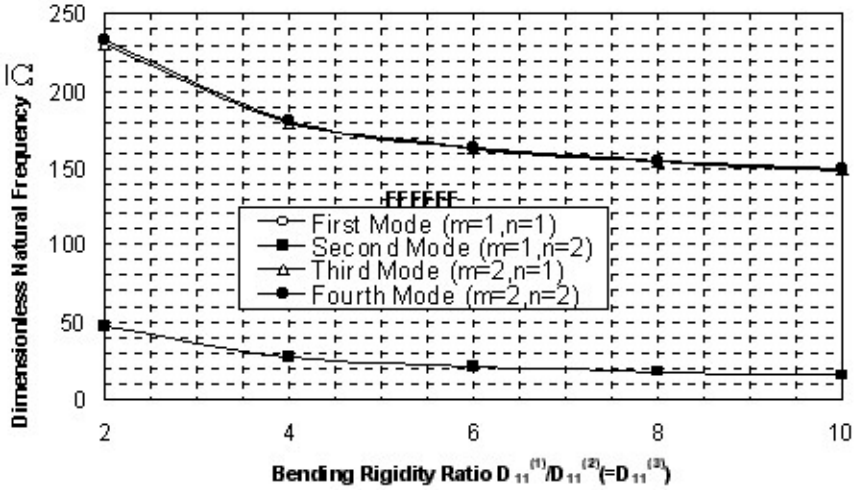


Figure 11.102. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$’’ in “Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFFFFF) B.C.’s, ‘‘Hard’’ Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

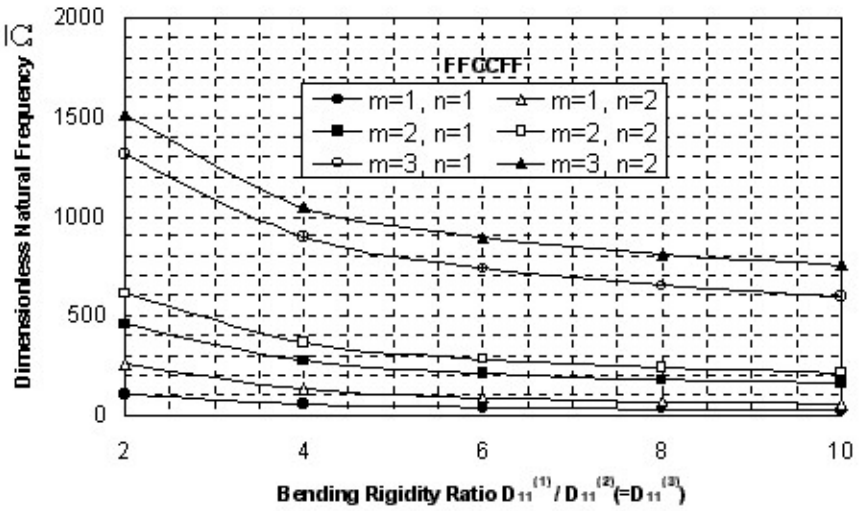


Figure 11.103. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$'' in "Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Dependency of natural frequency $\overline{\Omega}=\overline{\omega}_{mn}$ on the number of half waves in y-and x-directions)
((FFCCFF) B.C.'s, ''Soft'' Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

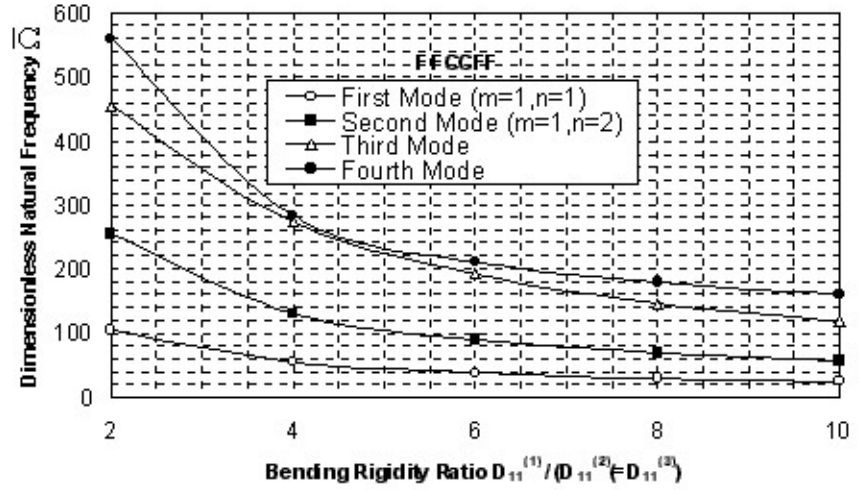


Figure 11.104. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$'' in "Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Various Modes with (FFCCFF) B.C.'s, ''Soft'' Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

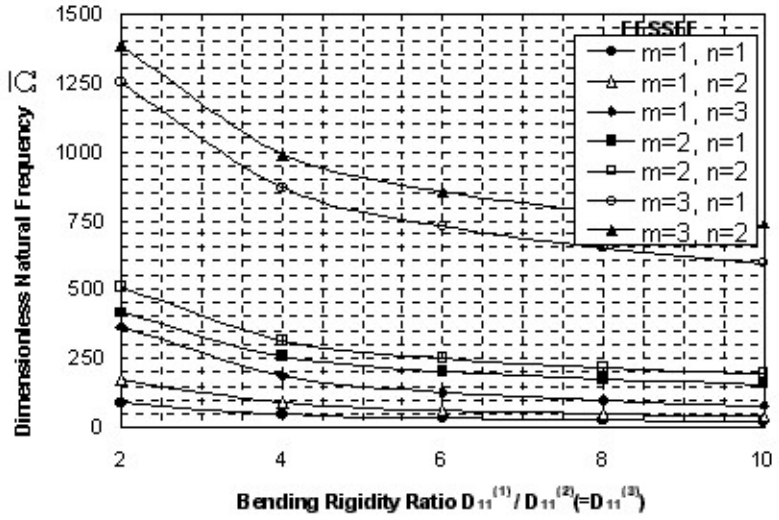


Figure 11.105. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$ ” in “Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega}=\overline{\omega}_{mn}$ on the number of half waves in y-and x-directions)
((FFSSFF) B.C.’s, ‘‘Soft’’ Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

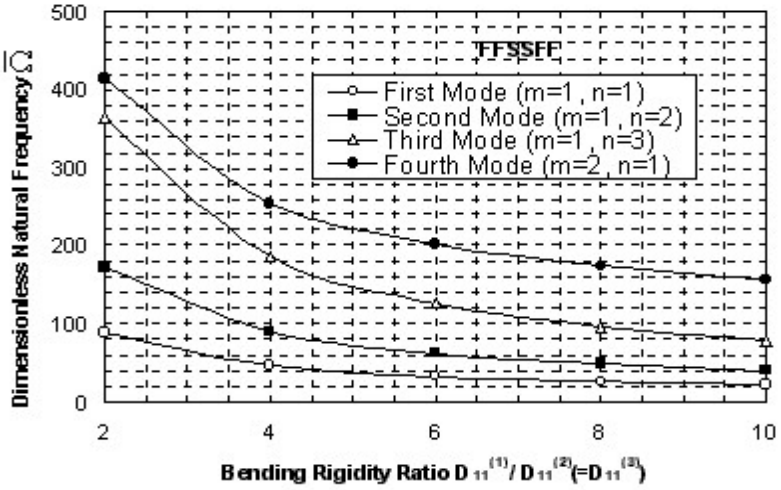


Figure 11.106. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$ ’’ in “Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFSSFF) B.C.’s, ‘‘Soft’’ Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

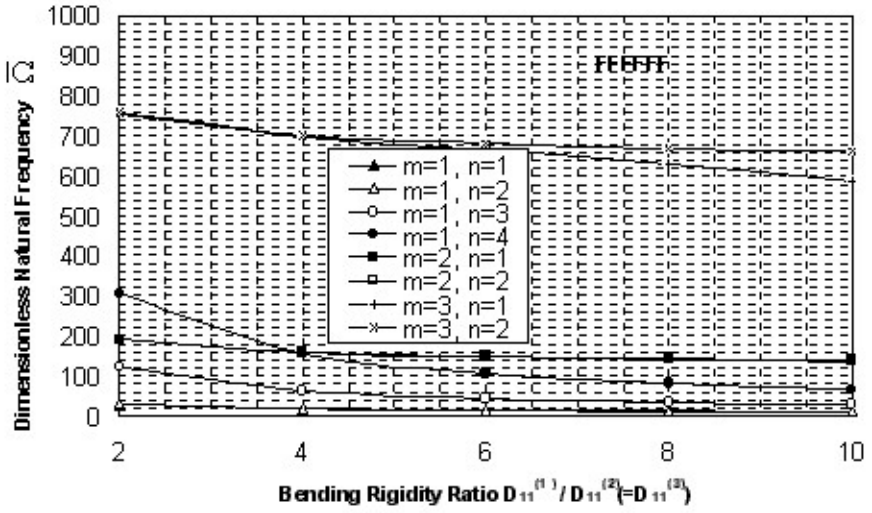


Figure 11.107. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$” in “Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”

(Dependency of natural frequency $\overline{\Omega}=\overline{\omega}_{mn}$ on the number of half waves in y-and x-directions)

((FFFFFF) B.C.’s, ‘‘Soft’’ Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

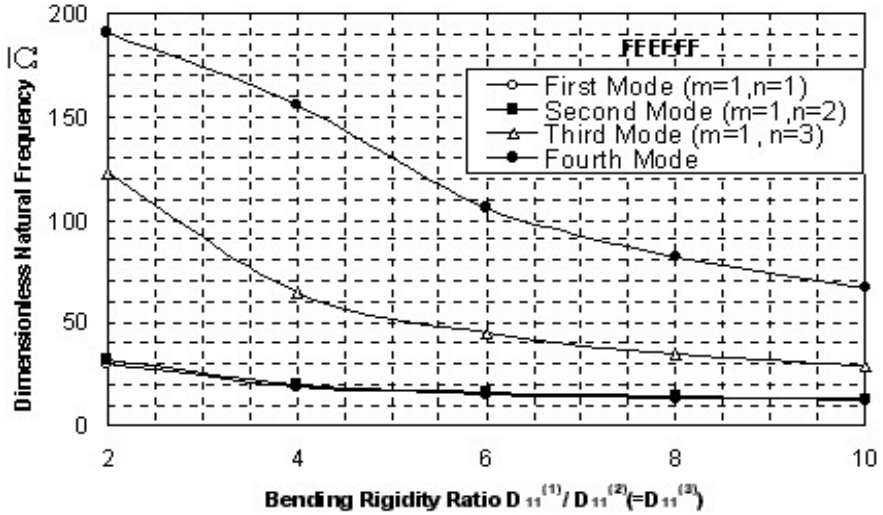


Figure 11.108. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$’’ in “Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”

(Various Modes with (FFFFFF) B.C.’s, ‘‘Soft’’ Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

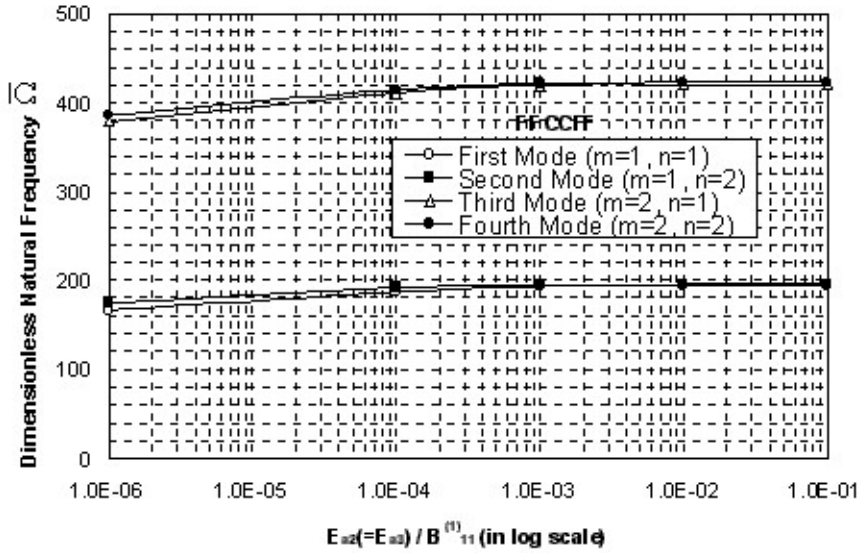


Figure 11.109. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$'' in "Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)" (Various Modes with (FFCCFF) B.C.'s, b_2=0.30 m, a=0.50 m, L=1.0 m)

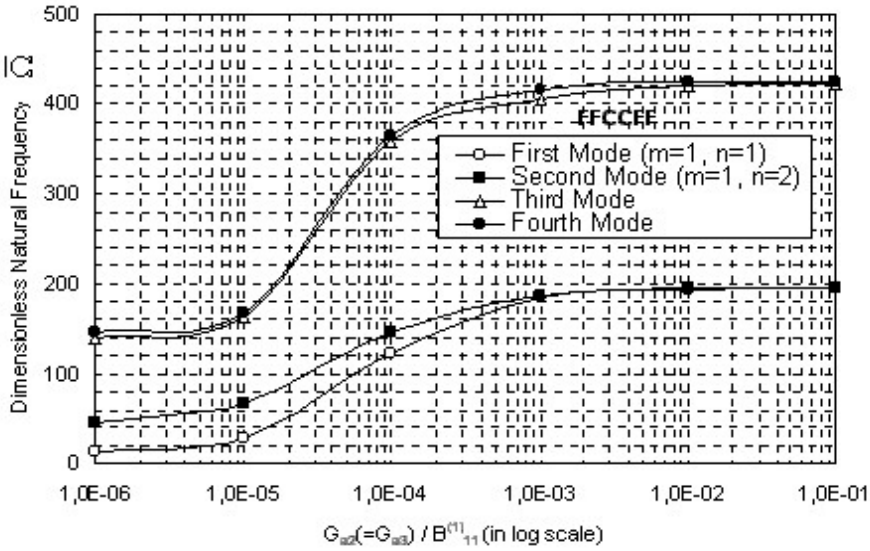


Figure 11.110. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3})/B_{11}^{(1)}$'' in "Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)" (Various Modes with (FFCCFF) B.C.'s, b_2=0.30 m, a=0.50 m, L=1.0 m)

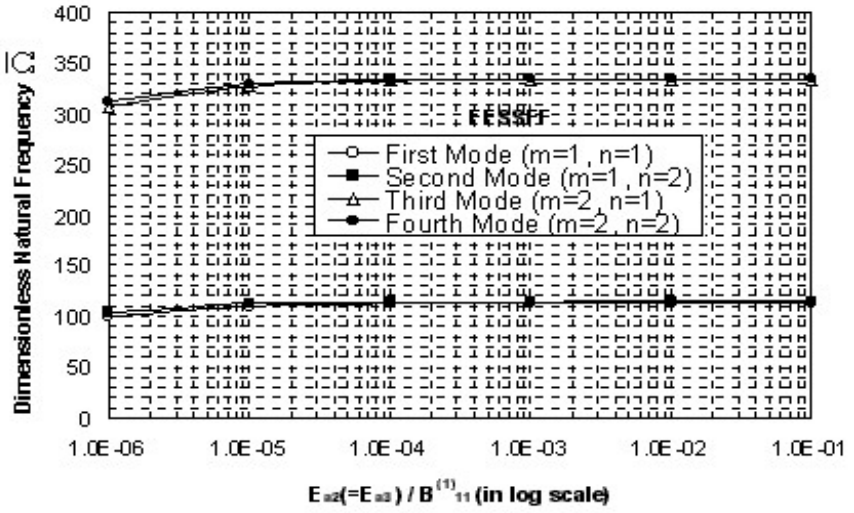


Figure 11.111. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$'' in "Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)" (Various Modes with (FFSSFF) B.C.'s, b_2=0.30 m, a=0.50 m, L=1.0 m)

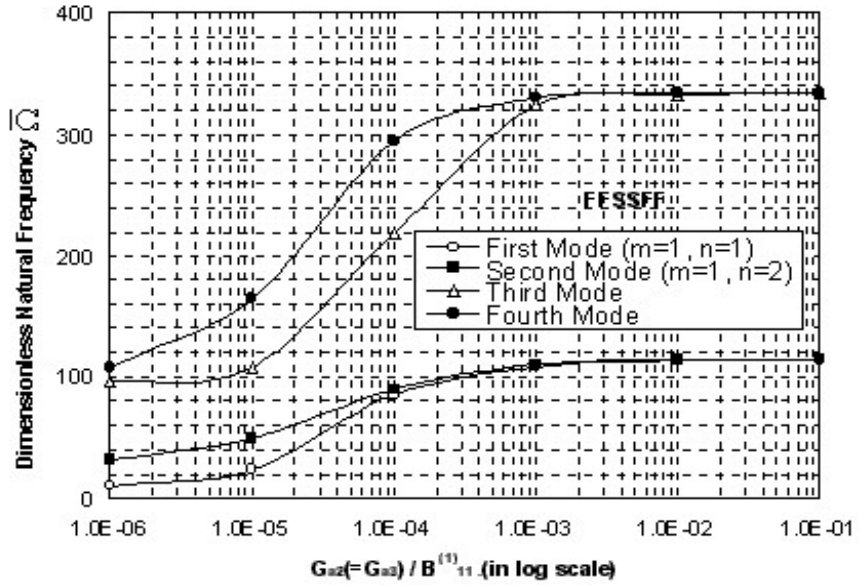


Figure 11.112. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3})/B_{11}^{(1)}$'' in "Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)" (Various Modes with (FFSSFF) B.C.'s, b_2=0.30 m, a=0.50 m, L=1.0 m)

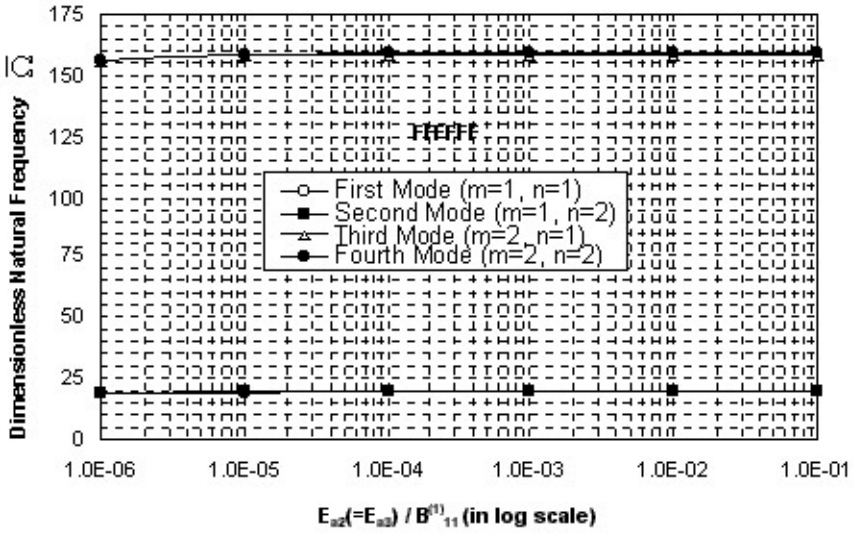


Figure 11.113. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B^{(1)}_{11}$ '' in "Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)" (Various Modes with (FFFFFF) B.C.'s, b_2=0.30 m, a=0.50 m, L=1.0 m)

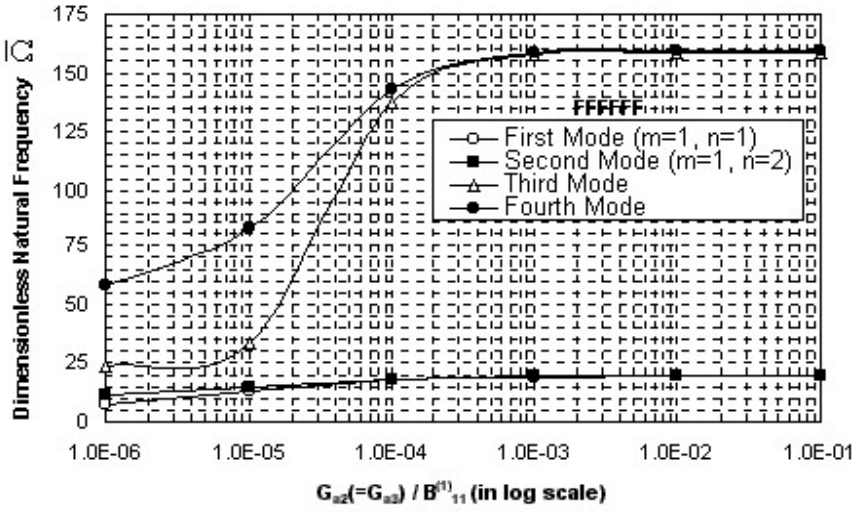


Figure 11.114. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3})/B^{(1)}_{11}$ '' in "Centrally Doubly Stiffened, Isotropic Base, Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)" (Various Modes with (FFFFFF) B.C.'s, b_2=0.30 m, a=0.50 m, L=1.0 m)

11.4 Numerical Results and Discussion for “Main PROBLEM II.a”

In this “Main PROBLEM II.a”, the “Orhotropic Base Plate is made of Kevlar-Epoxy and “Upper and Lower Stiffening Plate Strips” are chosen as Graphite-Epoxy.

In Figure 11.115, the first four mode shapes and the corresponding dimensionless natural frequencies of the composite plate or panel system are plotted for the “hard” adhesives case and with the (FFCCFF) support conditions.Also, Figure 11.116 is plotted the “soft” adhesives case with the same (FFCCFF) support conditions and, in particular, Figure 11.119 is plotted with the same B.C.’s and with the adhesive shear modulus $G_{a2} = G_{a3} = 0$ $(10^{-8}) \approx 0$. By comparing those Figures, it can be said that the adhesive shear modulus significantly influence the mode shapes and the corresponding natural frequencies. It is obvious, here that similar conclusions can be obtianed by comparing Figures 11.117 and 11.118 and also Figure 11.120.

By comparing Figures 11.117 and 11.118 with Figures 11.115 and 11.116, it is obvious that the mode shapes are essentially the same in both cases of the boundary conditions in the y-direction. However, the natural frequencies are drastically affected by the boundary conditions in y-direction. Also, by comparing Figures 11.115 through 11.118 with Figures 11.1 through 11.4, it can be concluded that both mode shapes and natural frequencies are essentially affected by the positions of the stiffening plate strips.

The parametric studies for “Main PROBLEM II.a” will be considered next.

In Figure 11.121, the dimensionless natural frequency parameter $\overline{\Omega}$ versus the “Aspect Ratio (b_2/a)” is considered for “Main PROBLEM II.a”. In this Figure, the dependency of the natural frequencies on the (m,n) values (and also the dependency on natural modes) is shown in the “hard” adhesives case with the (FFCCFF) support conditions. The natural frequencies with respect to the “Aspect Ratio” (b_2/a) from the first to the fourth are plotted in the “hard” adhesives case, for

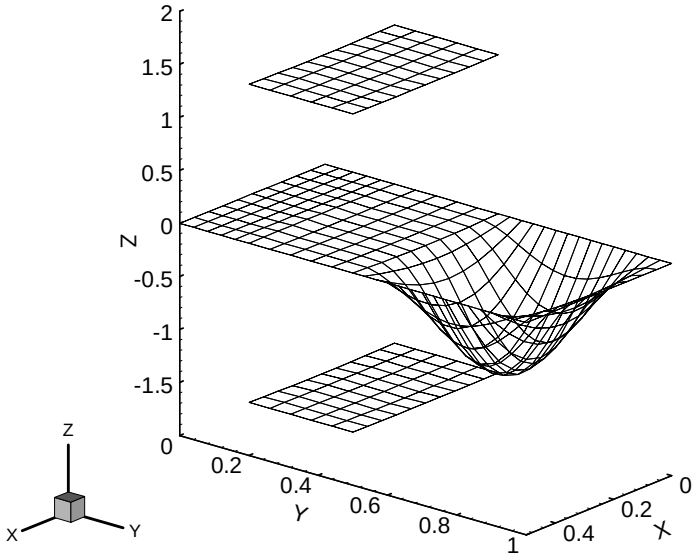
various boundary conditions in Figures 11.122 through 11.125, respectively. As the "Aspect Ratio (b_2/a)" or the wetted-area increases, so the natural frequencies.

Similarly, in Figure 11.126, the "Aspect Ratio (b_2/a)" are considered. However, in this Figure, the dependency of the natural frequencies on the (m,n) values (and also the dependency on natural modes) is shown in the "soft" adhesives case with the (FFCCFF) support conditions. Also, the natural frequencies with respect to the "Aspect Ratio" (b_2/a) from the first to the fourth are plotted in the "soft" adhesives case, for various boundary conditions in Figures 11.127 through 11.130, respectively. As the "Aspect Ratio (b_2/a)" or the wetted-area increase, so generally the natural frequencies .

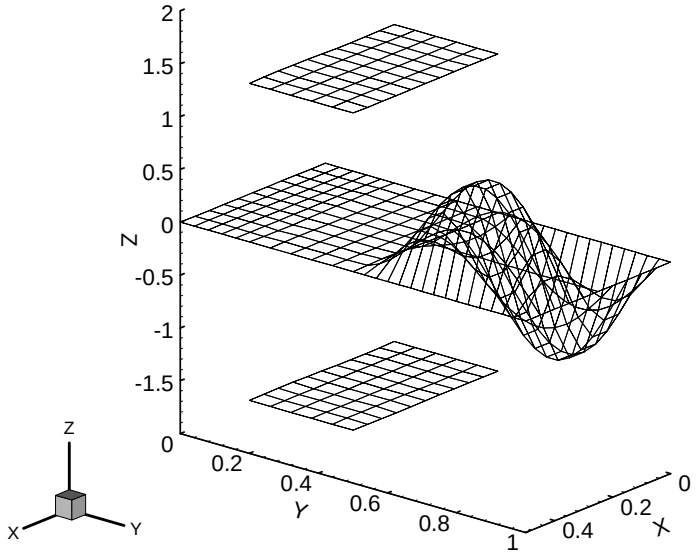
In Figures 11.131, 11.132 and 11.133, the effect of the "Bending Rigidity Ratio $\boldsymbol{D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})}$" on the natural frequencies are investigated in the "hard" adhesives case with the (FFCCFF), (FFSSFF), and the (FFFFFF) support conditions , respectively. Also, the effect of the "Bending Rigidity Ratio $\boldsymbol{D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})}$" on the dimensionless natural frequencies in the "soft" adhesives case are plotted in Figures 11.134, 11.135 and 11.136 for several modes corresponding to the (FFCCFF), (FFSSFF) and the (FFFFFF) boundary conditions, respectively. It is obvious from these Figures, the trend of the dimensionless natural frequency curves is that they drop sharply for the small values of the "Bending Rigidity Ratio $\boldsymbol{D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})}$" up to 0.6 as in "Main PROBLEM I.a", after than they decrease gradually for both "hard" and "soft" adhesive cases.

Lastly, the direct effects of the adhesive layer elastic constants E_a, G_a on the dimensionless natural frequencies are investigated for the "Main PROBLEM II.a". For this purpose, the dimensionless natural frequencies versus the "Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$" are plotted (while the other elastic constants kept constant) in Figures 11.137, 11.139 and 11.141 for the support conditions (FFCCFF), (FFSSFF) and (FFFFFF), respectively. Similarly, the dimensionless natural frequencies versus the "Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3})/B_{11}^{(1)}$" are presented in Figures 11.138, 11.140 and 11.142 for the boundary condition (FFCCFF), (FFSSFF), (FFFFFF), respectively.

It is obvious from the aforementioned figures, “Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3})/B_{11}^{(1)}$” has very significant effect on the natural frequencies of the composite plate systems considered in the “Main PROBLEM II.a”. The influence of the “Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$” on the natural frequencies is rather relatively weak depending on the support conditions on the plate or panel system.

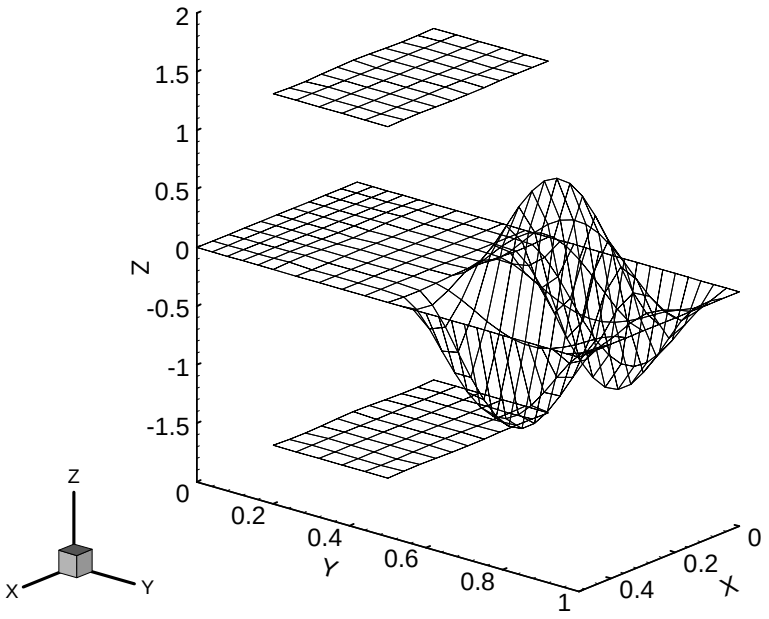


a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 532.617$

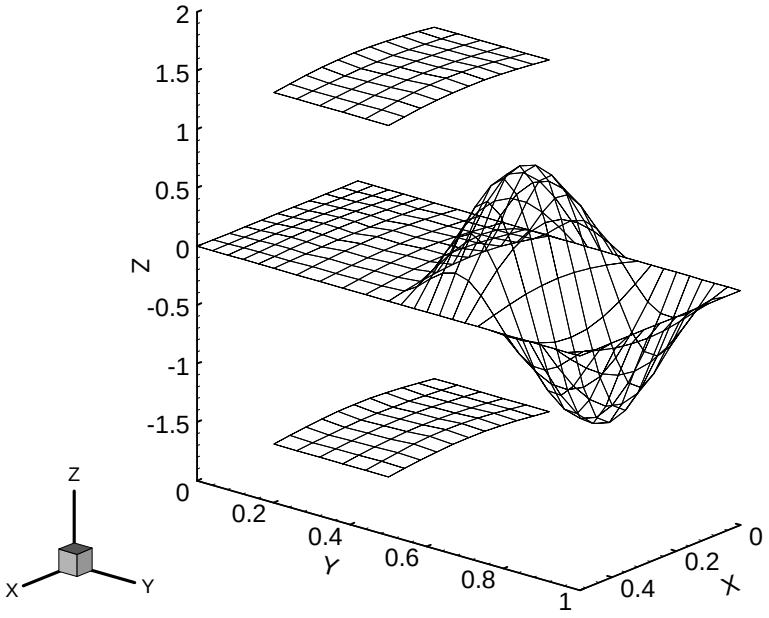


b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{21} = 715.903$

Figure 11.115. Mode Shapes and Natural Frequencies of "Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFCCFF) B.C.'s in y-direction, "Hard" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

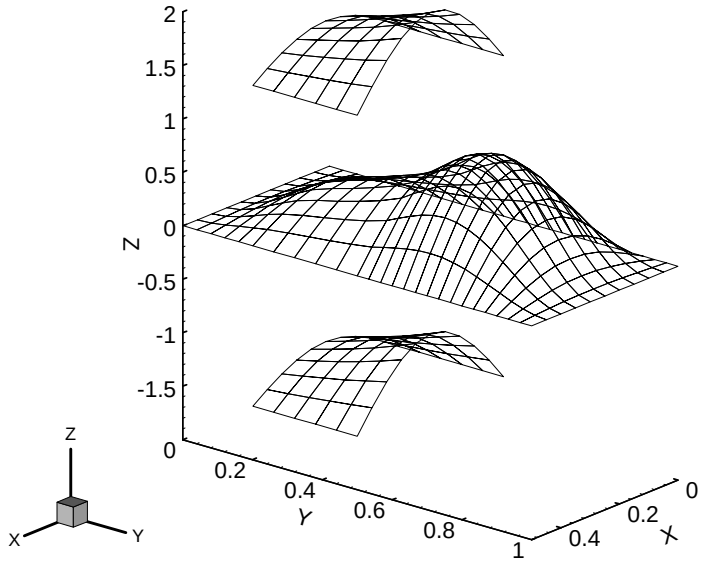


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{31} = 1337.453$

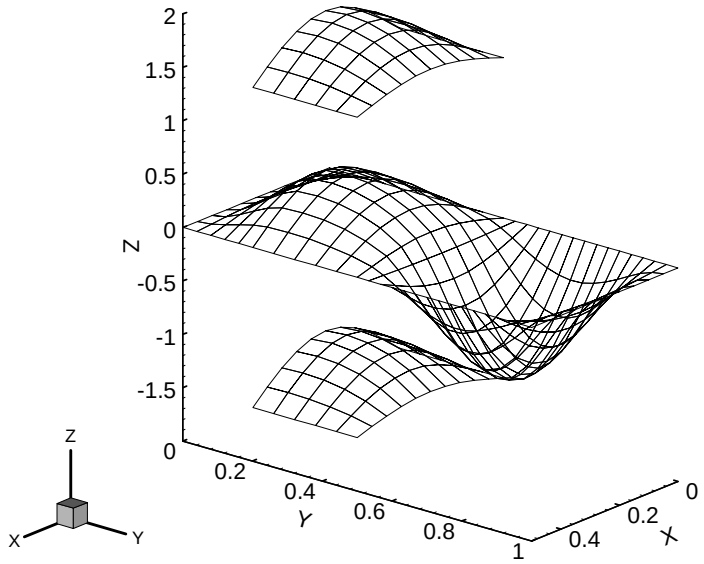


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{12} = 3690.309$

Figure 11.115. (continued)

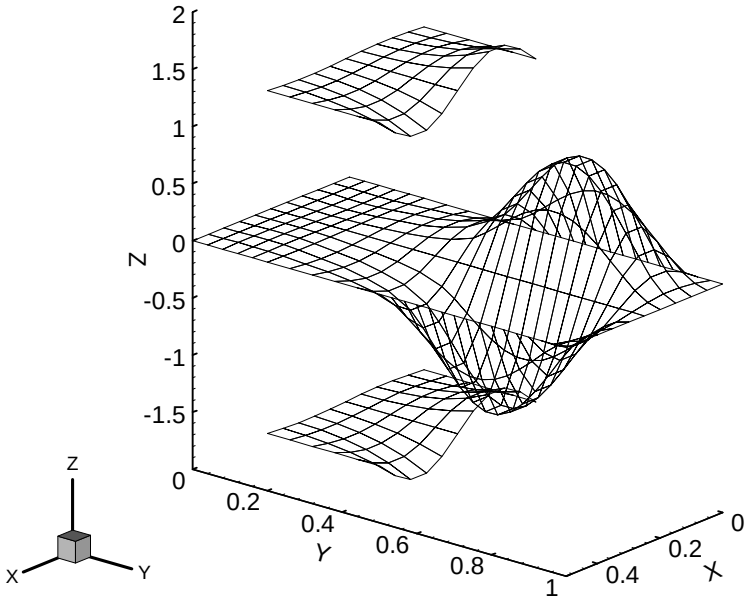


a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 237.941$

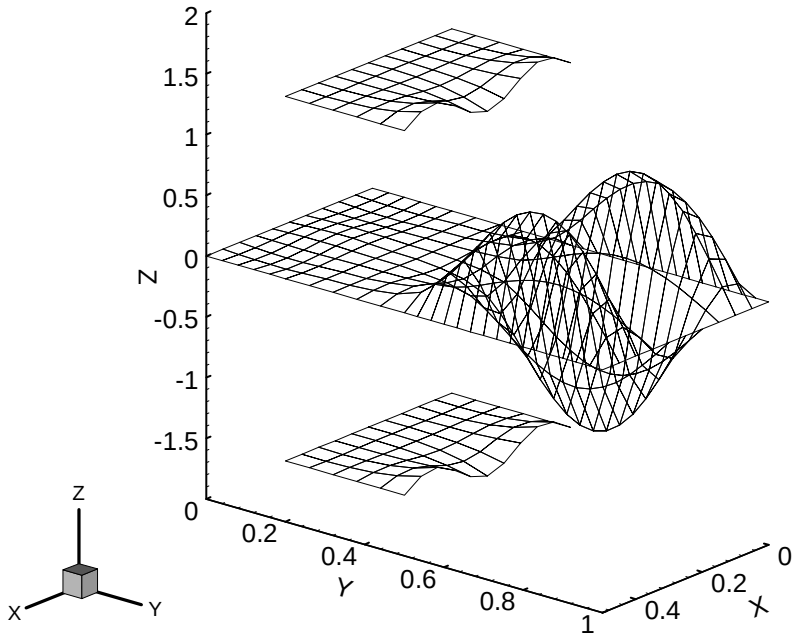


b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{12} = 486.309$

Figure 11.116. Mode Shapes and Natural Frequencies of "Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFCCFF) B.C.'s in y-direction, "Soft" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

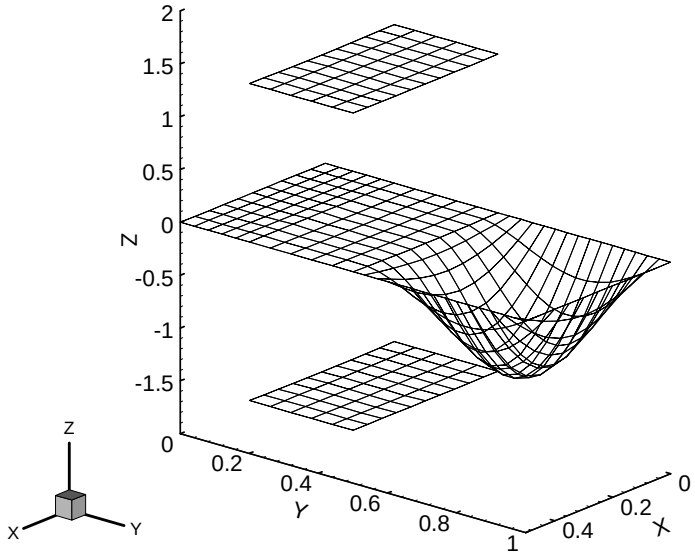


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{21} = 540.771$

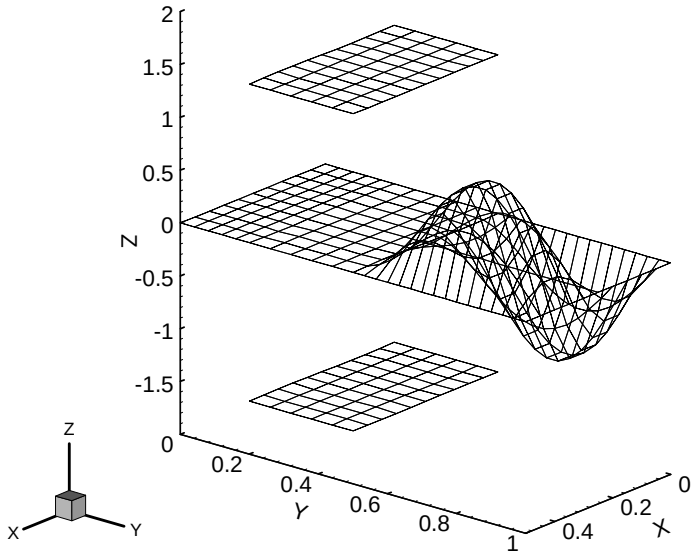


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{31} = 1183.419$

Figure 11.116. (continued)

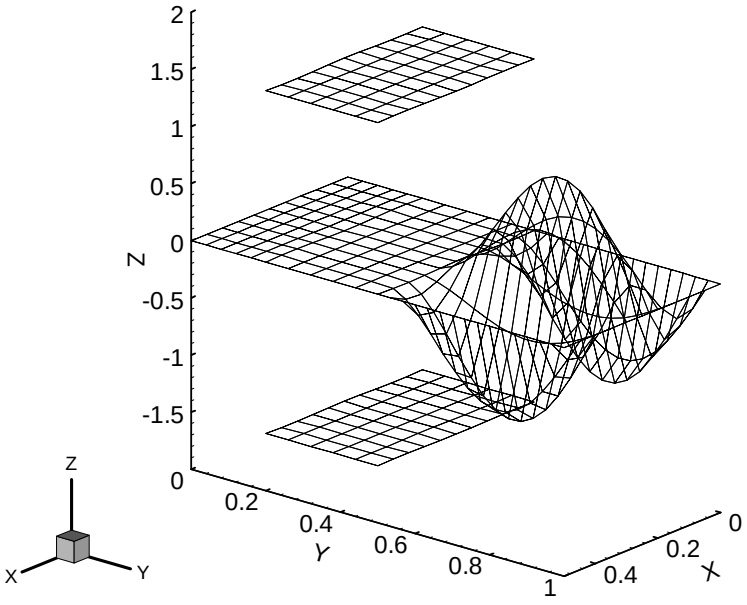


a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 271.094$

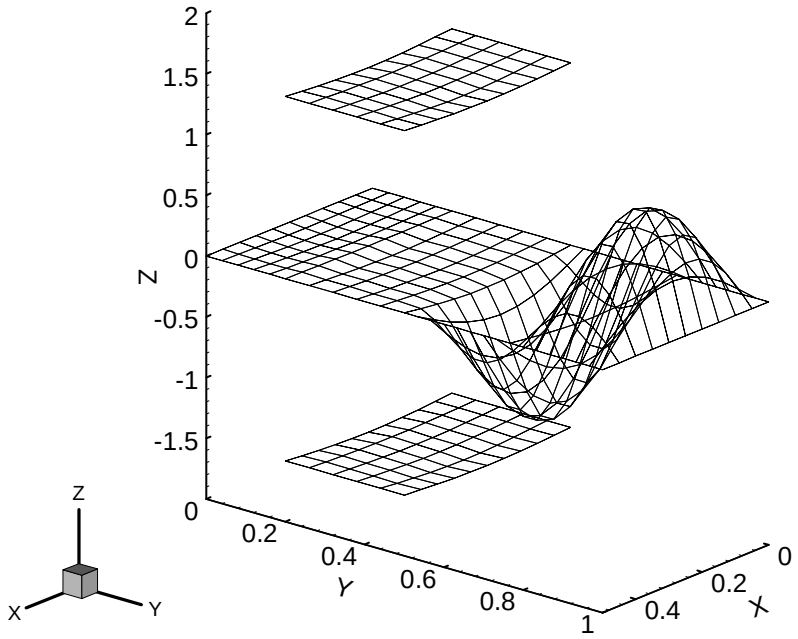


b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{21} = 450.488$

Figure 11.117. Mode Shapes and Natural Frequencies of "Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFSSFF) B.C.'s in y-direction, "Hard" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

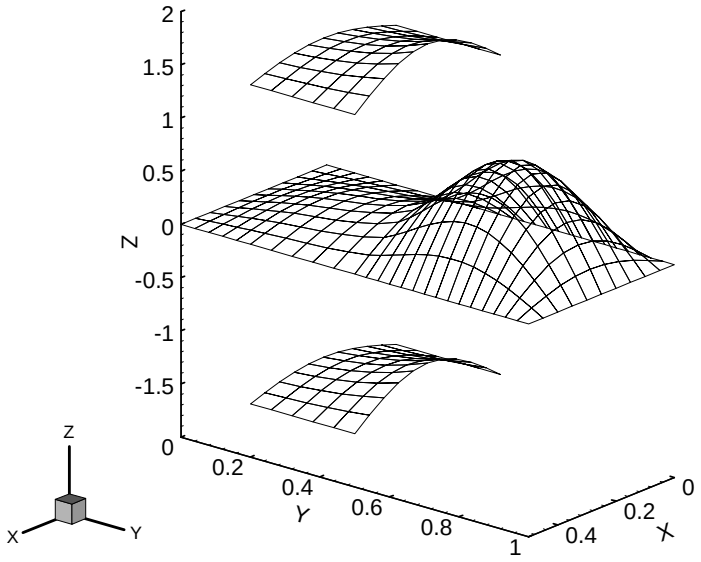


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{31} = 1067.370$

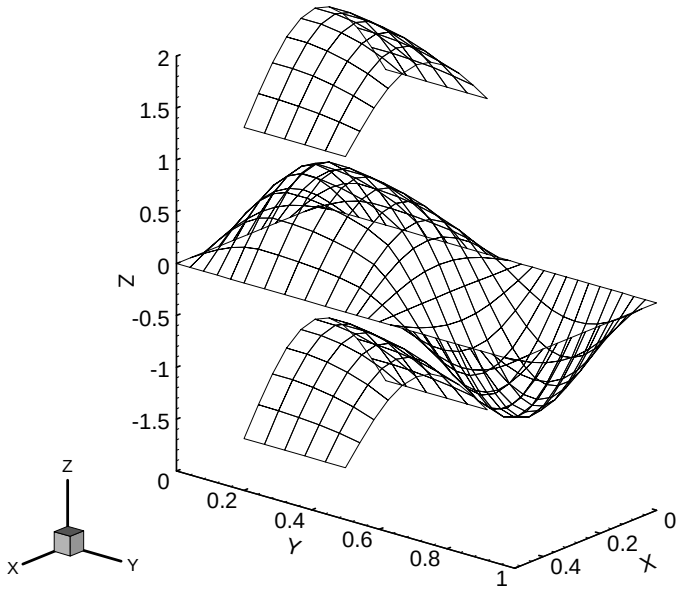


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{12} = 2526.580$

Figure 11.117. (continued)

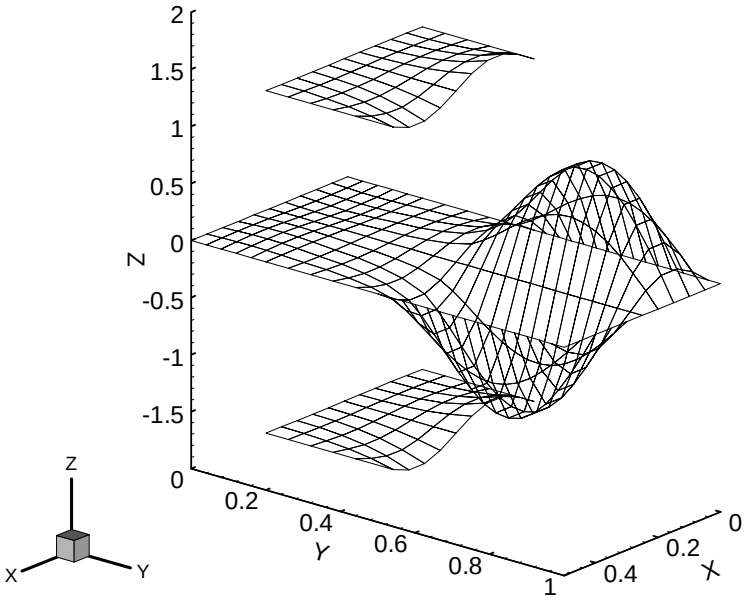


a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 159.323$

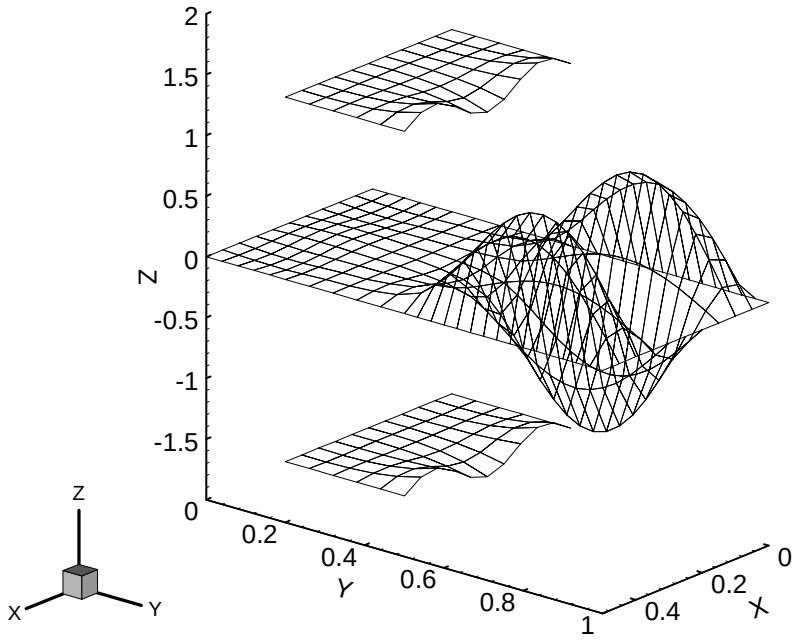


b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{12} = 327.689$

Figure 11.118. Mode Shapes and Natural Frequencies of "Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFSSFF) B.C.'s in y-direction, "Soft" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

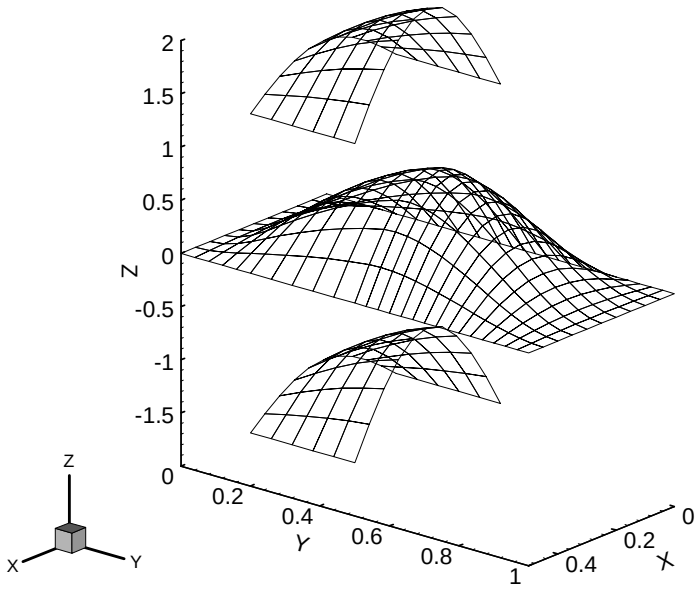


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{21} = 367.277$

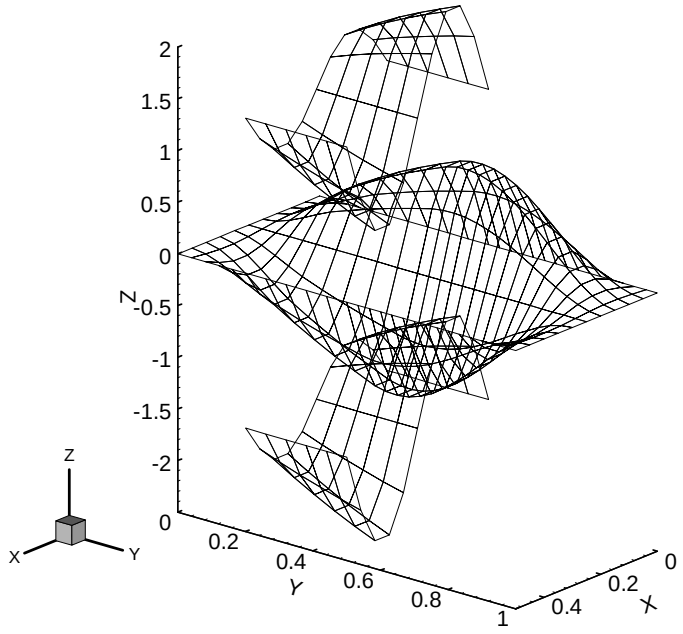


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{31} = 984.610$

Figure 11.118. (continued)

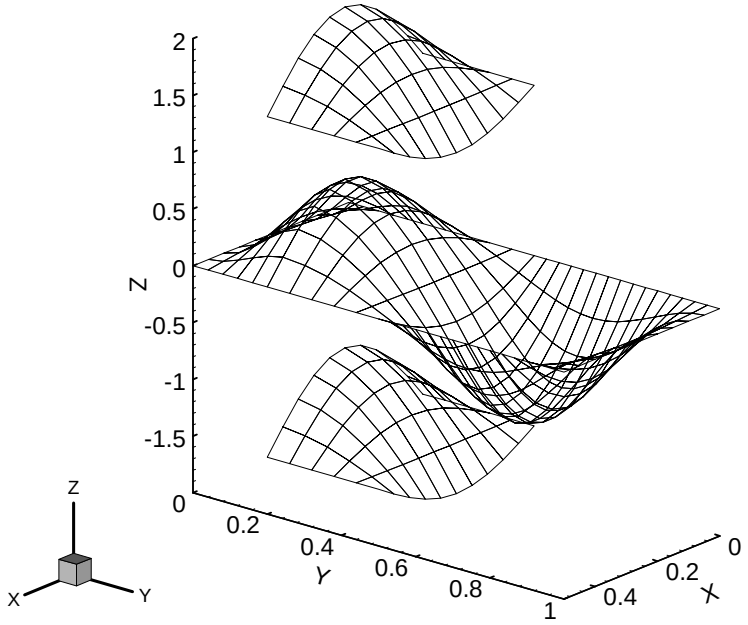


a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 37.729$

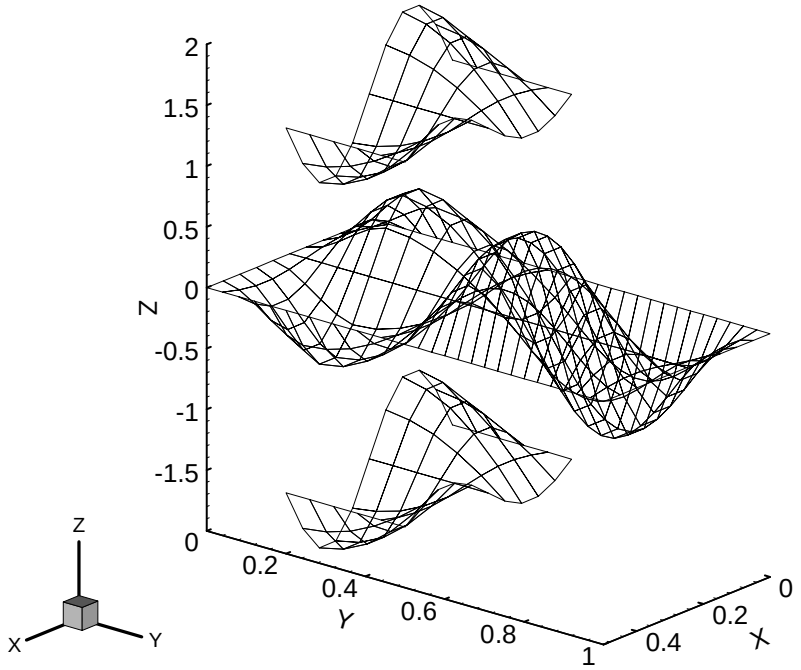


b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{21} = 218.051$

Figure 11.119. Mode Shapes and Natural Frequencies of "Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFCCFF) B.C.'s in y-direction, $G_{a2}=G_{a3}\approx 0$, $b_2=0.30$ m, a=0.50 m, L=1.0 m)

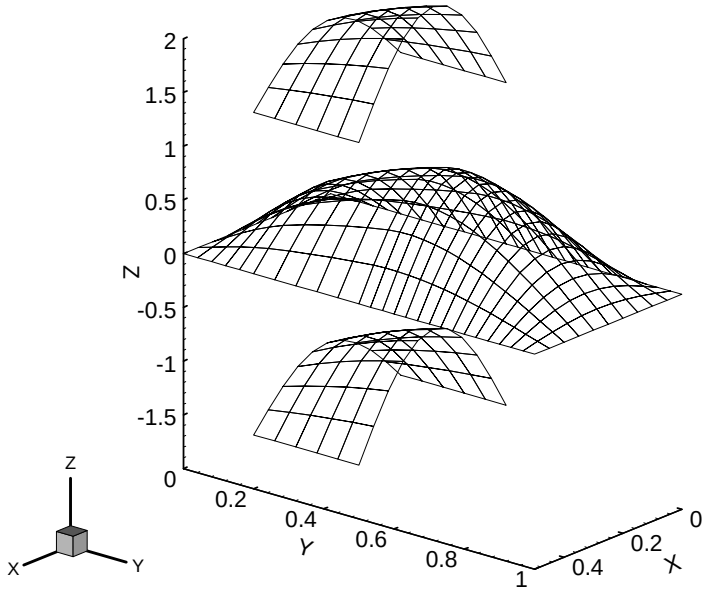


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{12} = 283.347$

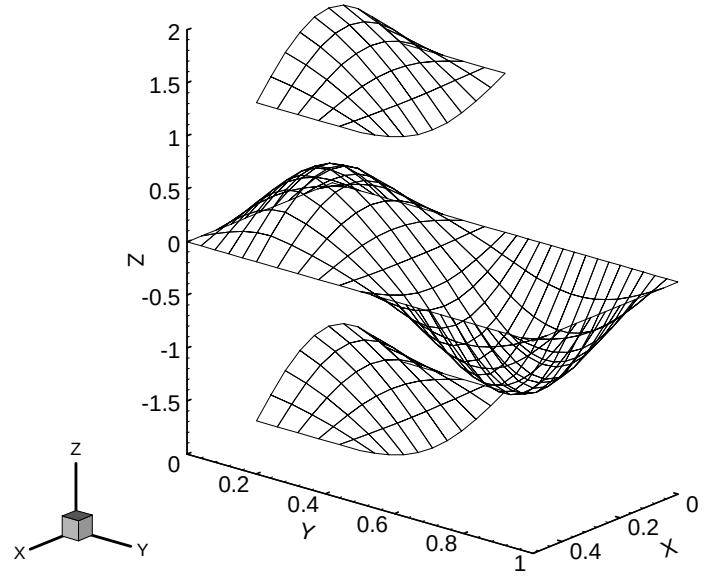


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{22} = 506.120$

Figure 11.119. (continued)

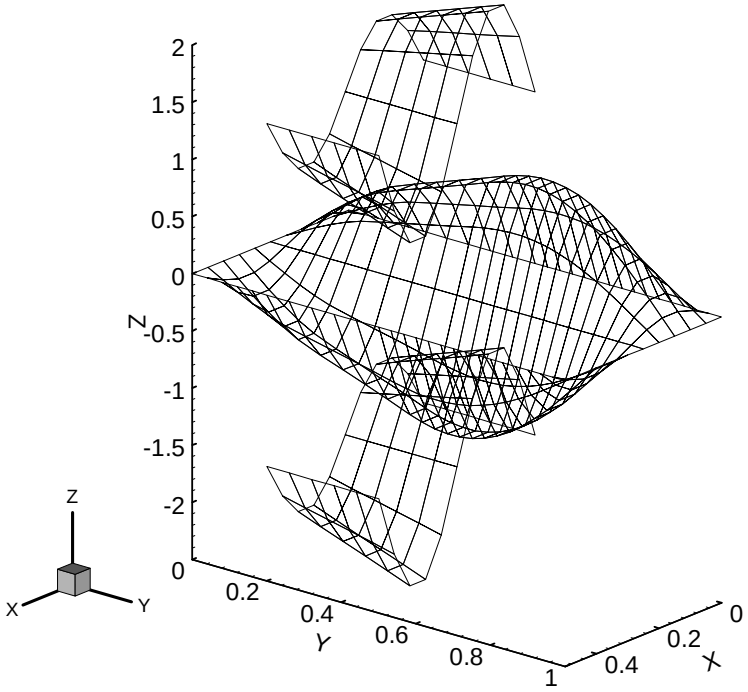


a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 23.754$

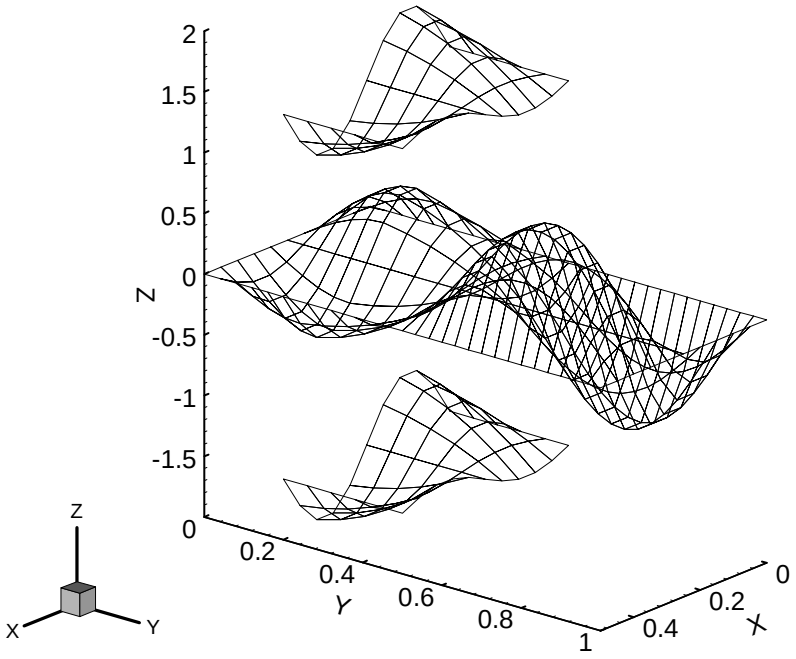


b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{12} = 131.030$

Figure 11.120. Mode Shapes and Natural Frequencies of "Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFSSFF) B.C.'s in y-direction, $G_{a2}=G_{a3}\approx 0$, $b_2=0.30$ m, a=0.50 m, L=1.0 m)



c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{21} = 181.696$



d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{22} = 336.571$

Figure 11.120. (continued)

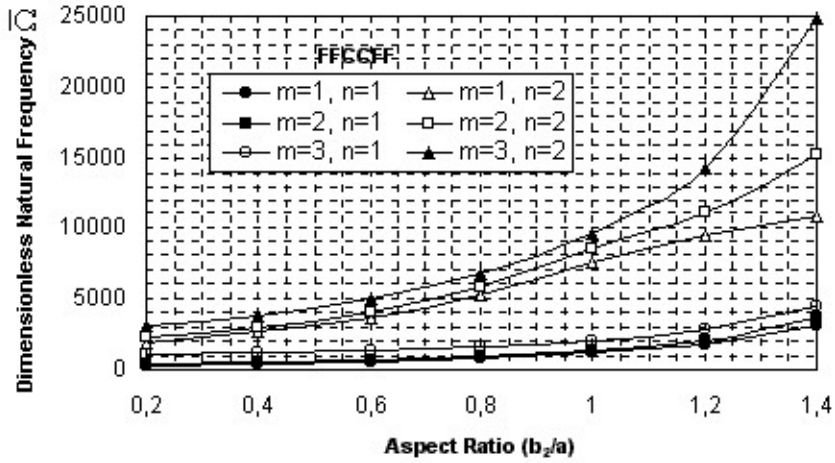


Figure 11.121. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on the number of half waves in y- and x-directions)
((FFCCFF) B.C.'s, ''Hard'' Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

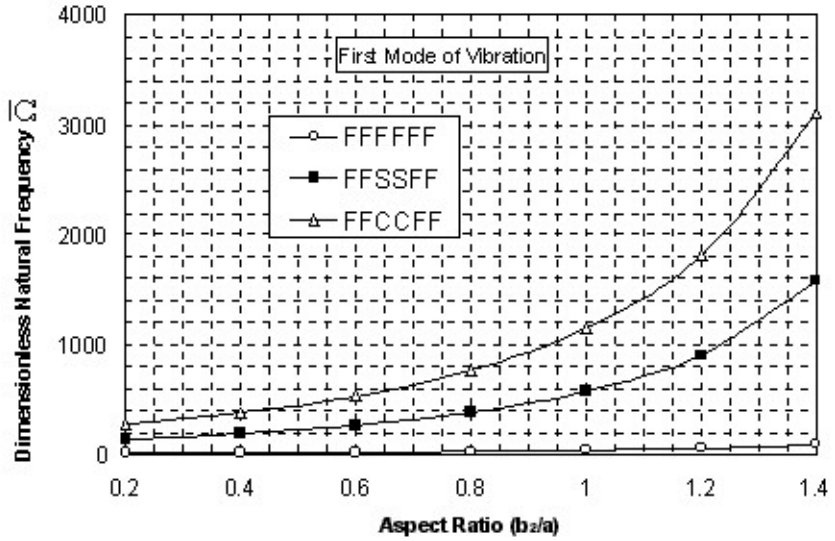


Figure 11.122. First Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(For Various B.C.'s, ''Hard'' Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

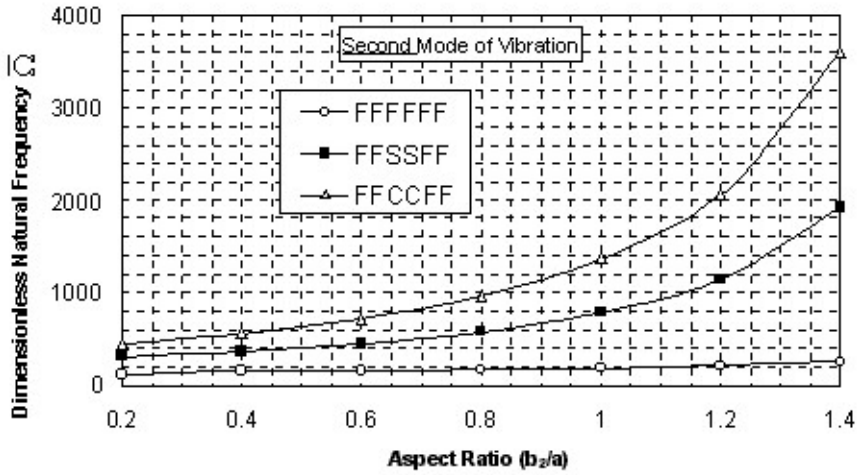


Figure 11.123. Second Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)" (For Various B.C.'s, ''Hard'' Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

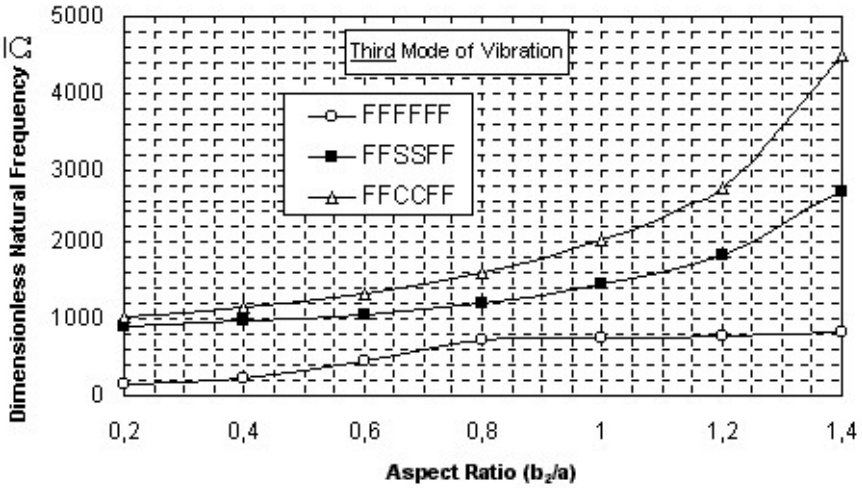


Figure 11.124. Third Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)" (For Various B.C.'s, ''Hard'' Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

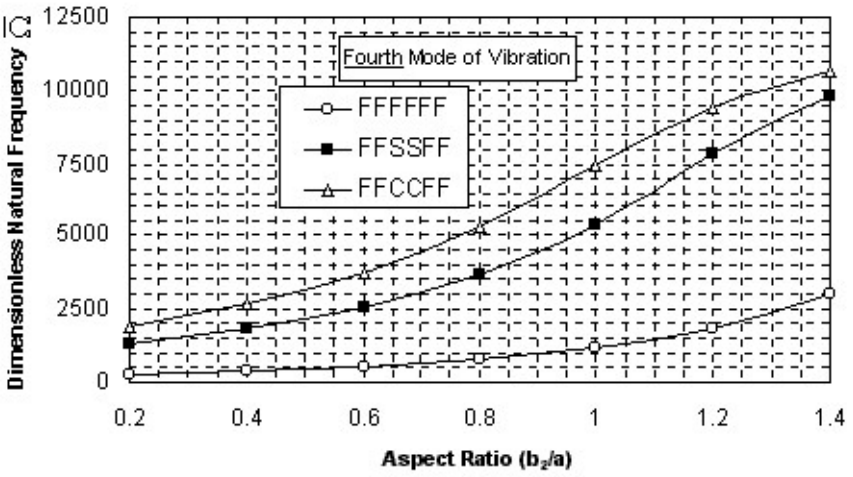


Figure 11.125. Fourth Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(For Various B.C.'s, ''Hard'' Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

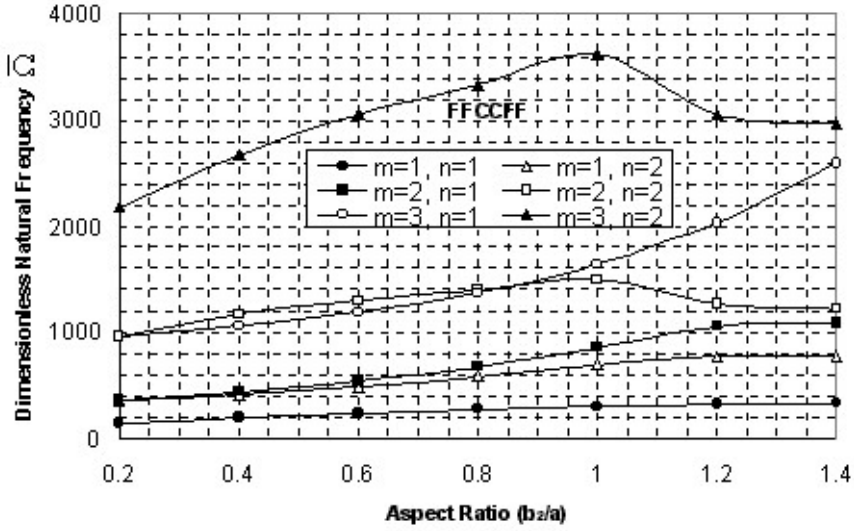


Figure 11.126. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Dependency of natural frequency parameter on the number of half waves in y- and x-directions)
((FFCCFF) B.C.'s, ''Soft'' Adhesive b_2=varies, a=0.50 m, L=1.0 m)

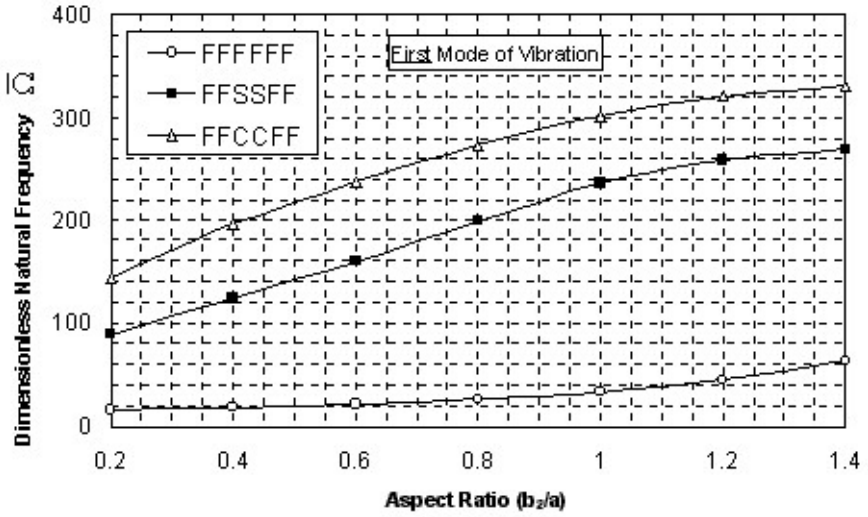
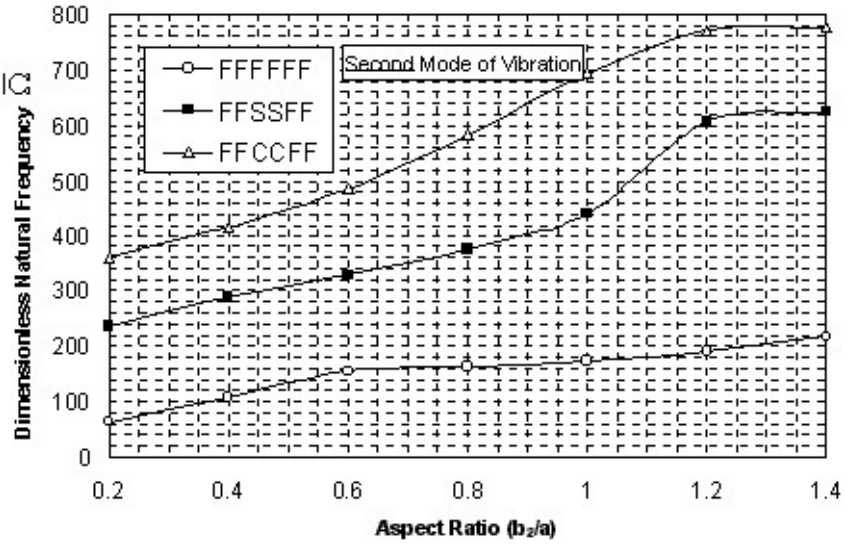
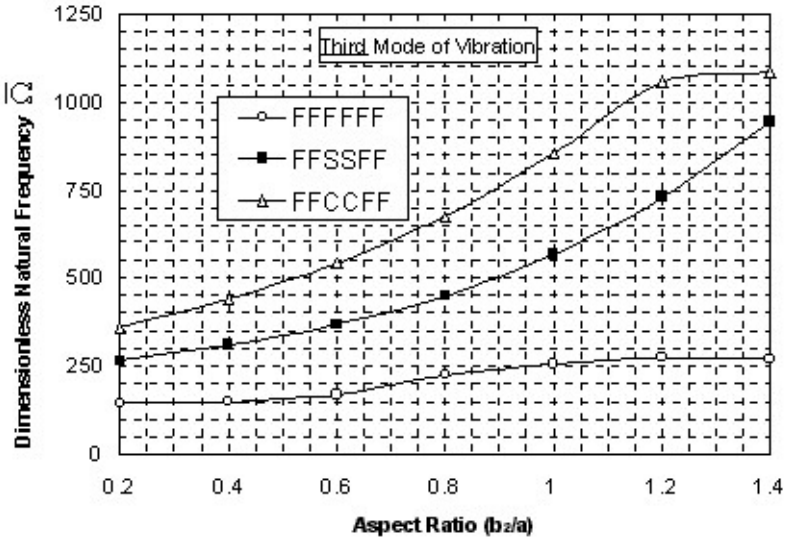


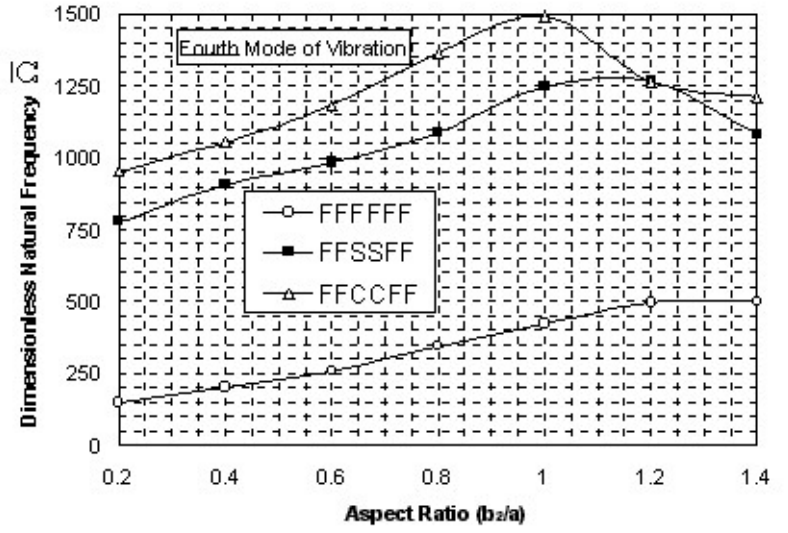
Figure 11.127. First Dimensionless Natural Frequency $\bar{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(For Various B.C.'s, ''Soft'' Adhesive, b_2=varies, a=0.50 m, L=1.0 m)



11.128. Second Dimensionless Natural Frequency $\bar{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(For Various B.C.'s, ''Soft'' Adhesive, b_2=varies, a=0.50 m, L=1.0 m)



11.129. Third Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(For Various B.C.'s, ''Soft'' Adhesive, b_2=varies, a=0.50 m, L=1.0 m)



11.130. Fourth Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(For Various B.C.'s, ''Soft'' Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

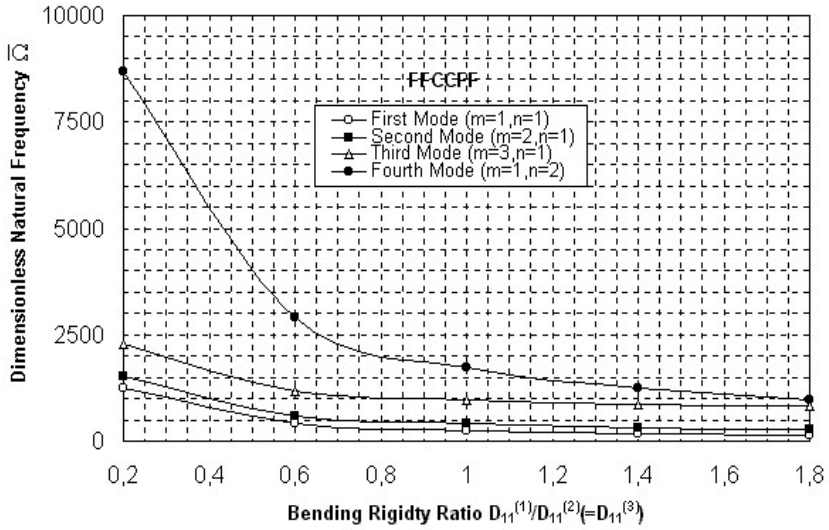


Figure 11.131. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$’’ in “Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)” (Various Modes with (FFCCFF) B.C.’s, ‘‘Hard” Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

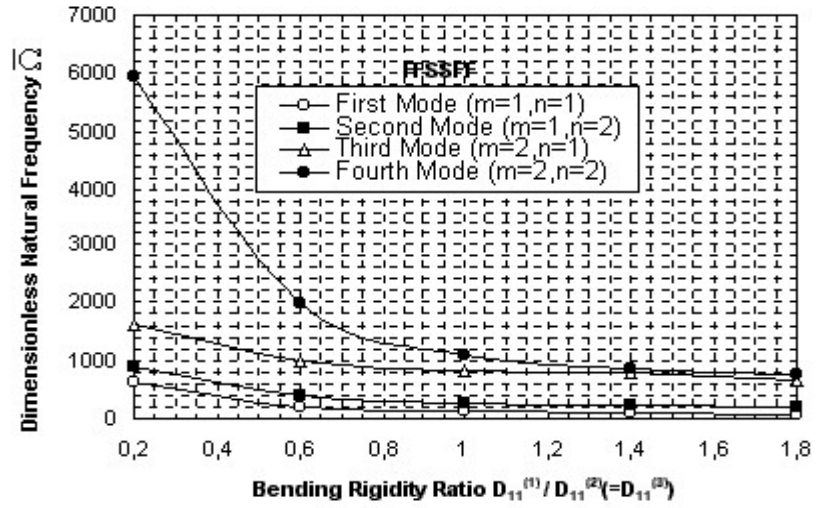


Figure 11.132. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$’’ in “Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)” (Various Modes with (FFSSFF) B.C.’s, ‘‘Hard” Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

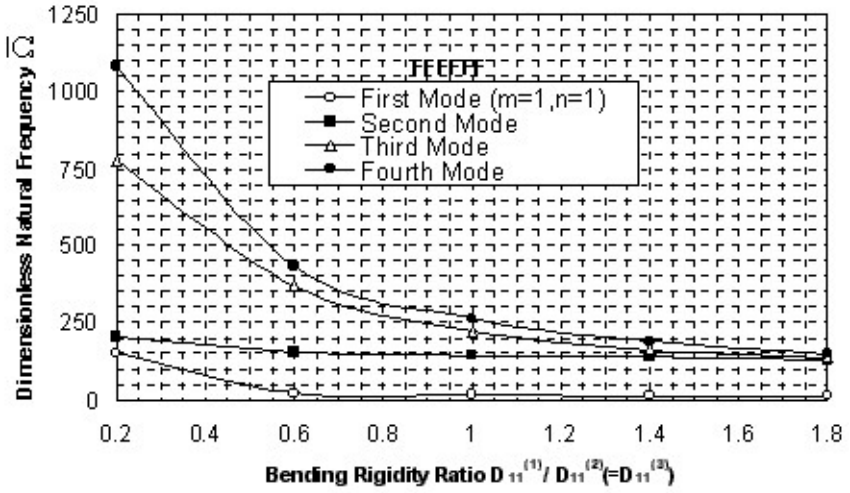


Figure 11.133. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$’’ in “Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)” (Various Modes with (FFFFFF) B.C.’s, ‘‘Hard” Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

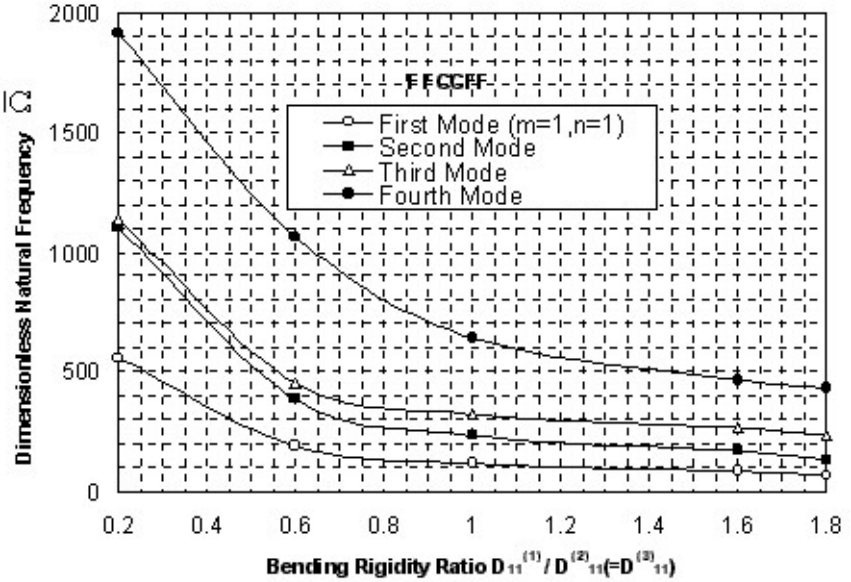


Figure 11.134. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$’’ in “Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)” (Various Modes with (FFCCFF) B.C.’s, ‘‘Soft” Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

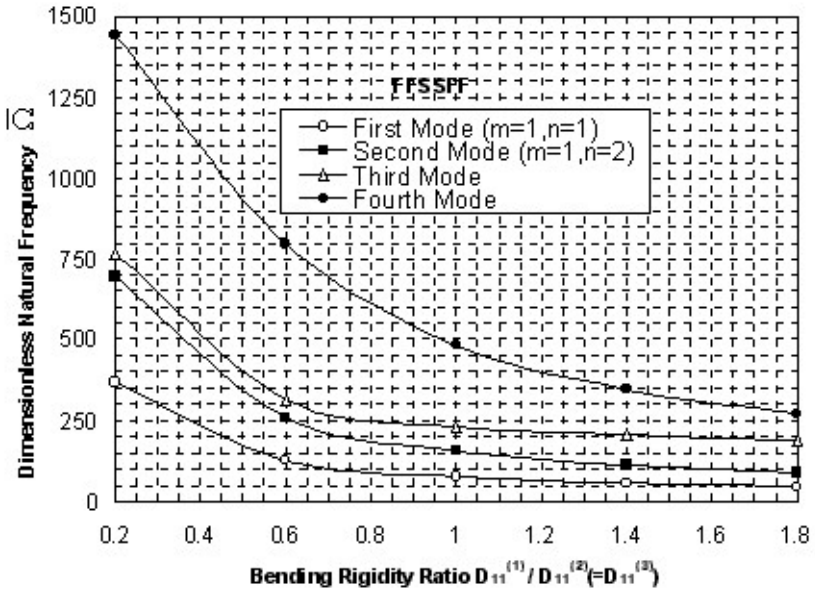


Figure 11.135. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$’’ in “Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)” (Various Modes with (FFSSFF) B.C.’s, ‘‘Soft” Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

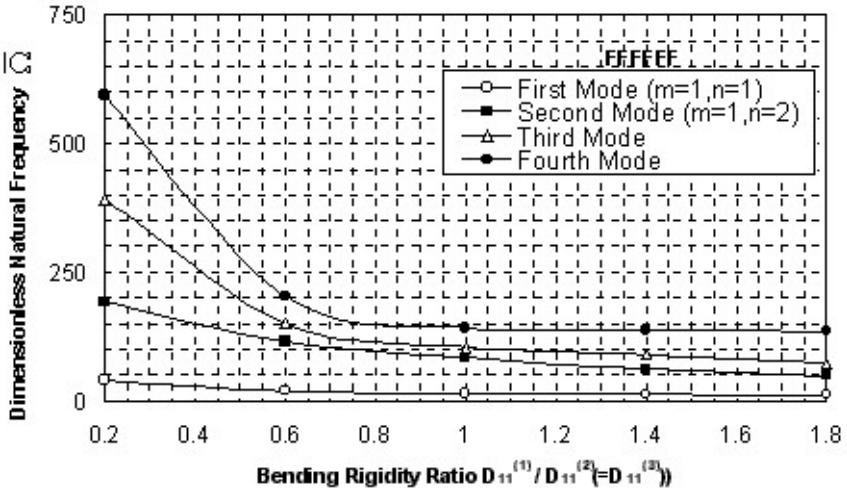


Figure 11.136. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$’’ in “Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)” (Various Modes with (FFFFFF) B.C.’s, ‘‘Soft” Adhesive, b_2=varies, a=0.50 m, L=1.0 m)

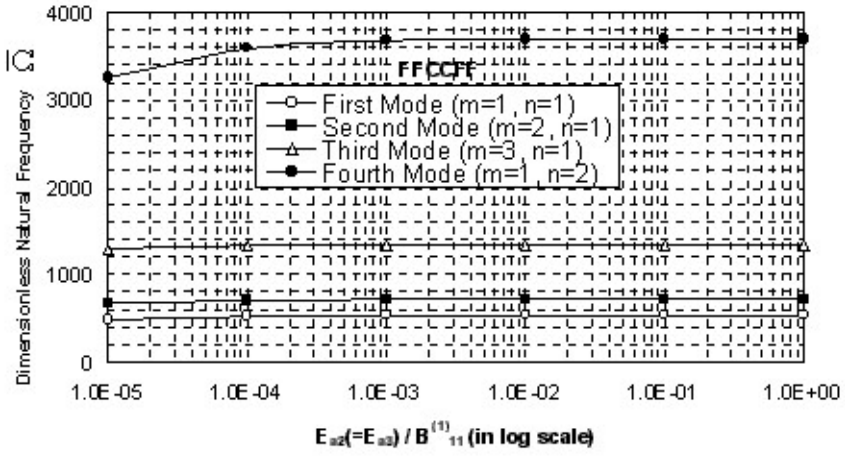


Figure 11.137. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$ ’’ in “Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFCCFF) B.C.’s, b_2=varies, a=0.50 m, L=1.0 m)

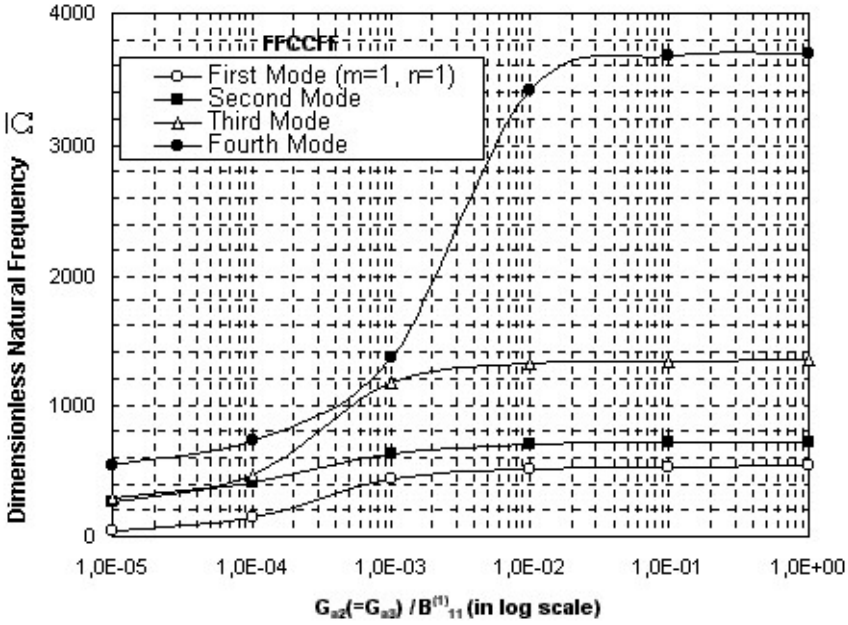


Figure 11.138. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3})/B_{11}^{(1)}$ ’’ in “Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFCCFF) B.C.’s, b_2=varies, a=0.50 m, L=1.0 m)

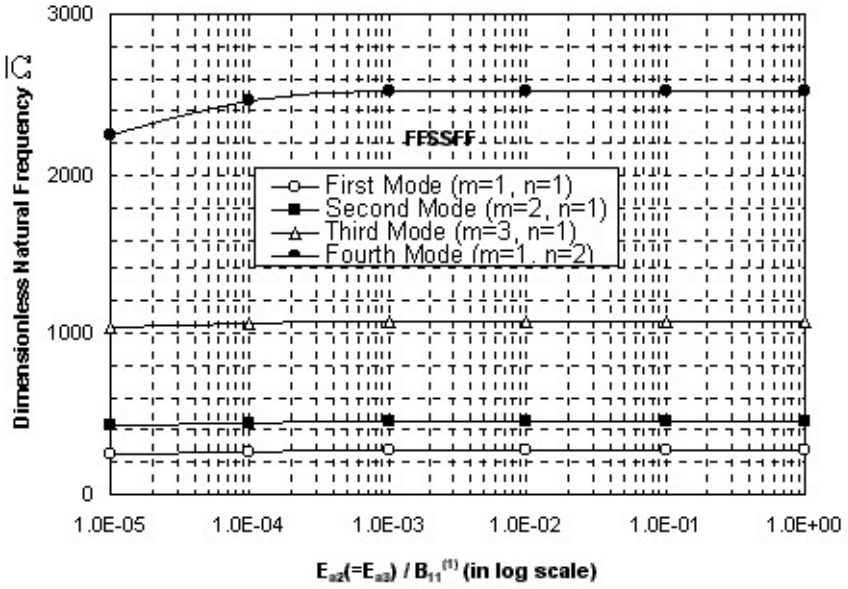


Figure 11.139. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$ ’’ in “Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFSSFF) B.C.’s, b_2=varies, a=0.50 m, L=1.0 m)

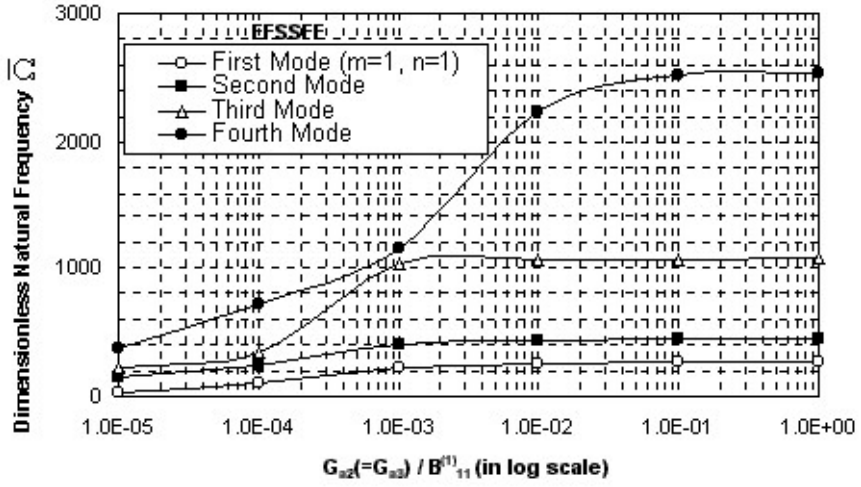


Figure 11.140. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3})/B_{11}^{(1)}$ ’’ in “Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (FFCCFF) B.C.’s, b_2=varies, a=0.50 m, L=1.0 m)

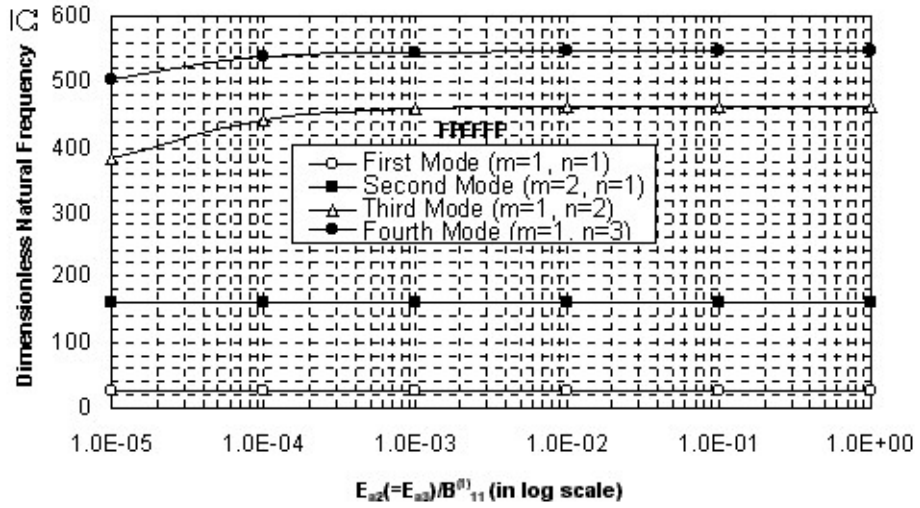


Figure 11.141. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$'' in "Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Various Modes with (FFFFFF) B.C.'s, b_2=varies, a=0.50 m, L=1.0 m)

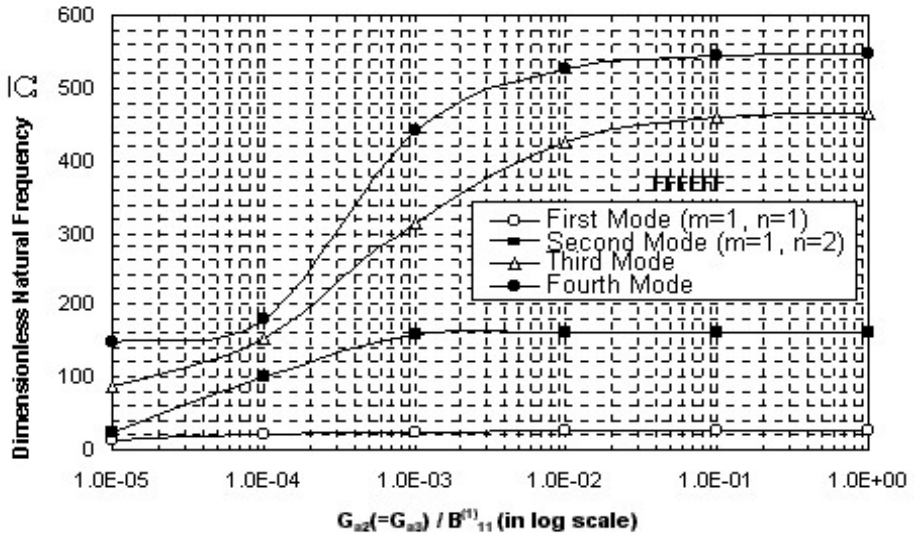


Figure 11.142. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3})/B_{11}^{(1)}$'' in "Non-Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Various Modes with (FFFFFF) B.C.'s, b_2=varies, a=0.50 m, L=1.0 m)

11.5 Numerical Results and Discussion for “Main PROBLEM II.b”

In this “Main PROBLEM II.b”, the “Isotropic Base Plate is made of Aluminum and “Upper and Lower Stiffening Plate Strips” are chosen as Graphite-Epoxy. In Figures, the adhesive layer are chosen as the “hard” and the “soft” adhesive cases with various boundary conditions.

Figure 11.143 represents the first four mode shapes with the corresponding dimensionless natural frequencies of the entire composite system of “Main PROBLEM II.b” for the “hard” adhesives case with (FFCCFF) boundary conditions in the y-direction. Similarly, the first four mode shapes and the corresponding dimensionless natural frequencies for the same system with the same boundary conditions for the ‘‘soft’’ adhesivs case are presented in Figure 11.144.

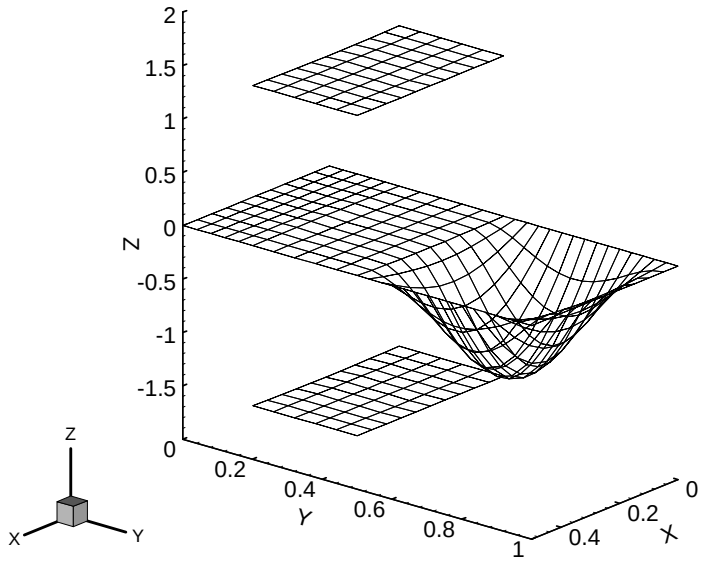
In Figures 11.145 and 11.146, the same composite system of Figures 11.143 and 11.144 are now considered with (FFSSFF) boundary conditions in the y-direction. From the comparison of these figures, it is obvious that the mode shapes are essentially the same in both cases of the boundary conditions in the y-direction. However, boundary conditions change the natural frequencies drastically.

The parametric studies for “Main PROBLEM II.a” will be considered next.

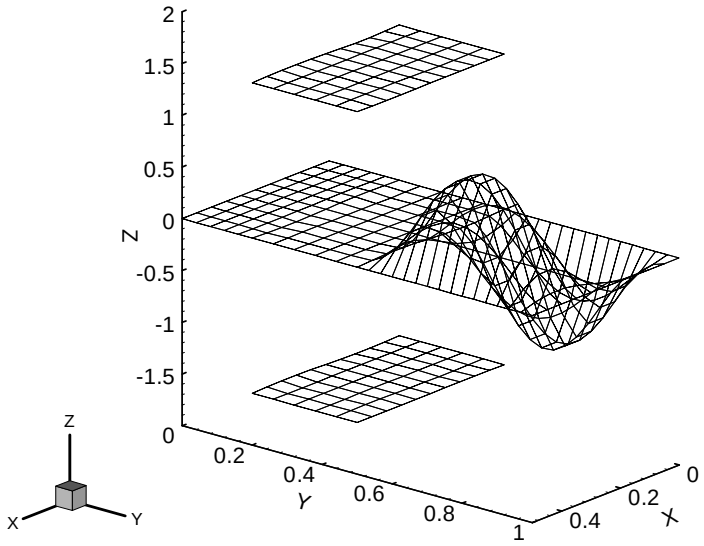
In Figures 11.147 through 11.151, the dimensionless natural frequency $\overline{\Omega}$ versus “Aspect Ratio b_2/a” are given for only ‘‘hard’’ adhesive case for different support conditions. It is clear from these Figures that the natural frequencies increase as the “Aspect Ratio (b_2/a)” or the wetted-area increases.

In Figures 11.152, 11.153 and 11.154, the effect of the ‘‘Bending Rigidity Ratio $\boldsymbol{D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})}$ ’’ on the natural frequencies $\overline{\Omega}$ for the ‘‘hard’’ adhesives case with the (FFCCFF), (FFSSFF), and the (FFFFFF) support conditions is plotted, respectively. It can be stated that as the “Bending Rigidity or Stiffness Ratio $\boldsymbol{D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})}$ ” increases, the dimensionless natural frequency curves of each mode decrease sharply for the small values of the “Bending Rigidity Ratio $\boldsymbol{D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})}$ ” up to 4.0, after than they decrease gradually.

In Figures 11.155, 11.157 and 11.159, the dimensionless natural frequencies versus the "Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$" are plotted (while the other elastic constants kept constant) for the support conditions (FFCCFF), (FFSSFF) and (FFFFFF), respectively. Similarly, the dimensionless natural frequencies versus the "Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3})/B_{11}^{(1)}$" are presented in Figures 11.156, 11.158 and 11.160 for the boundary condition (FFCCFF), (FFSSFF), (FFFFFF), respectively. Again, "Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$" has relatively weak influence on the natural frequencies of the composite plate systems considered in the "Main PROBLEM II.b" while the "Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3})/B_{11}^{(1)}$" has very significant effect on the natural frequencies.

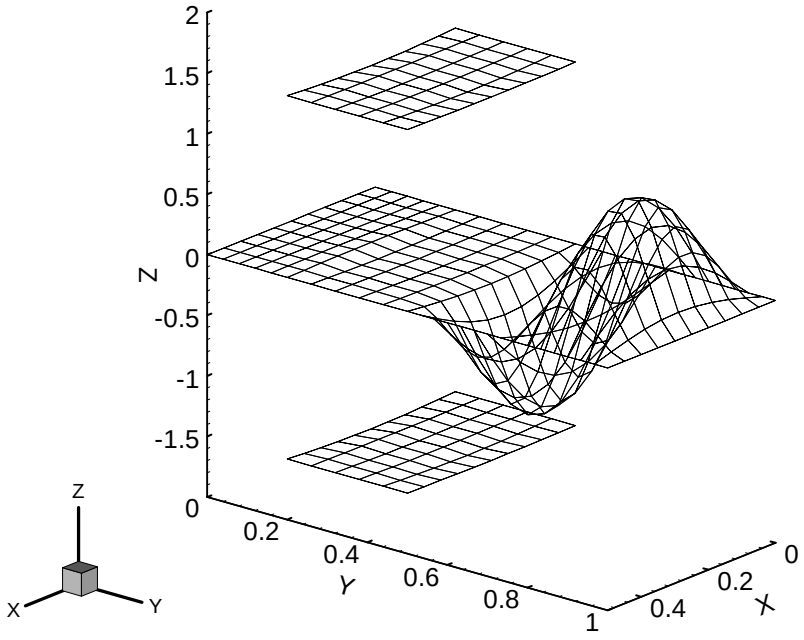


a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 63.959$

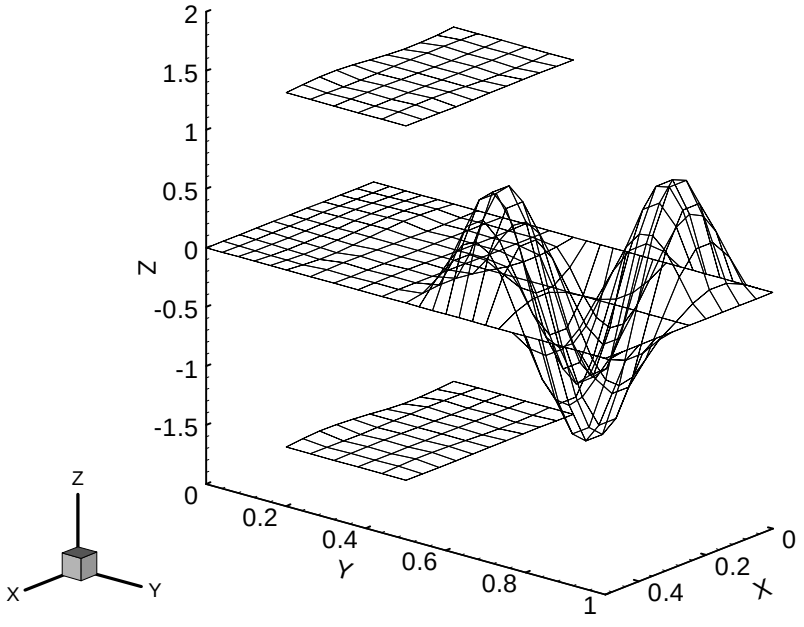


b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{21} = 237.470$

Figure 11.143. Mode Shapes and Natural Frequencies of "Non-Centrally Doubly Stiffened, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Base Plate=Plate 1= Aluminum , Plate 2= Plate 3= Graphite-Epoxy)
((FFCCFF) B.C.'s in y-direction, "Hard" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m.)

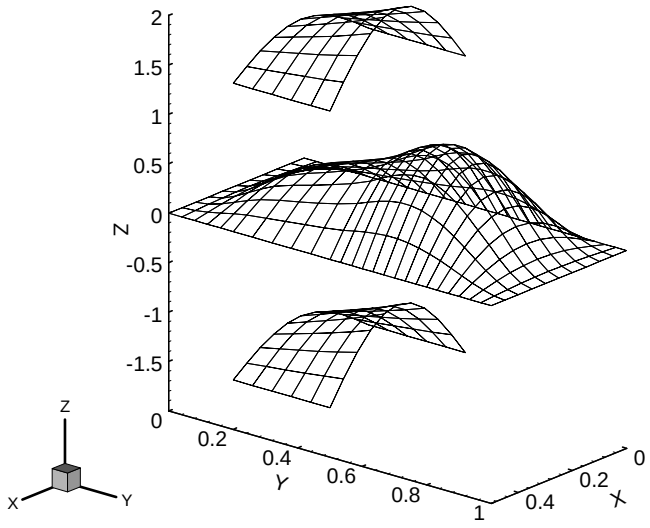


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{12} = 359.796$

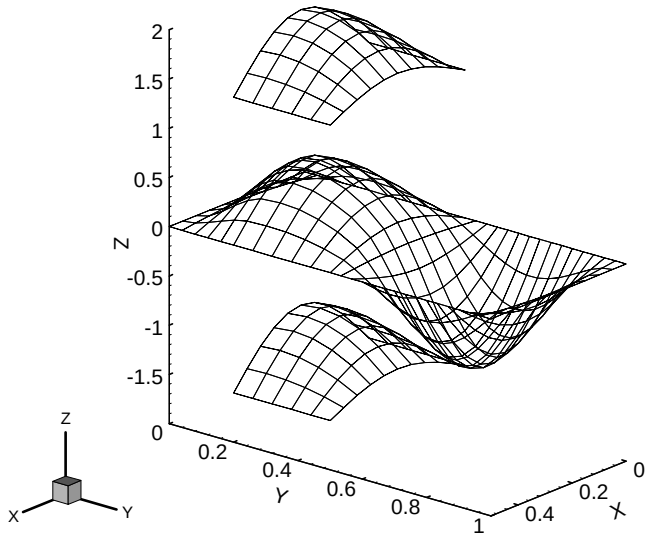


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{22} = 681.194$

Figure 11.143. (continued)

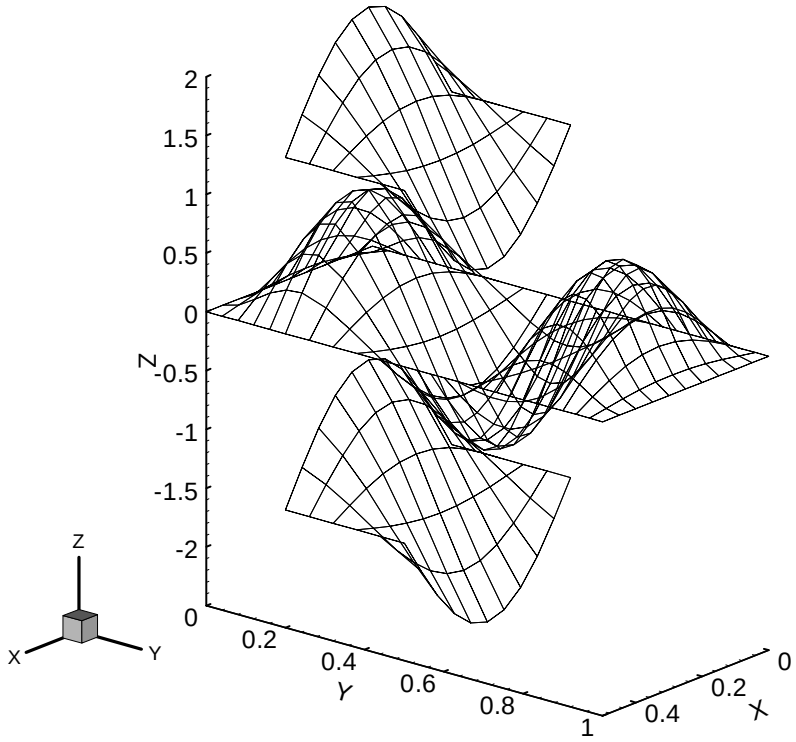


a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 31.143$

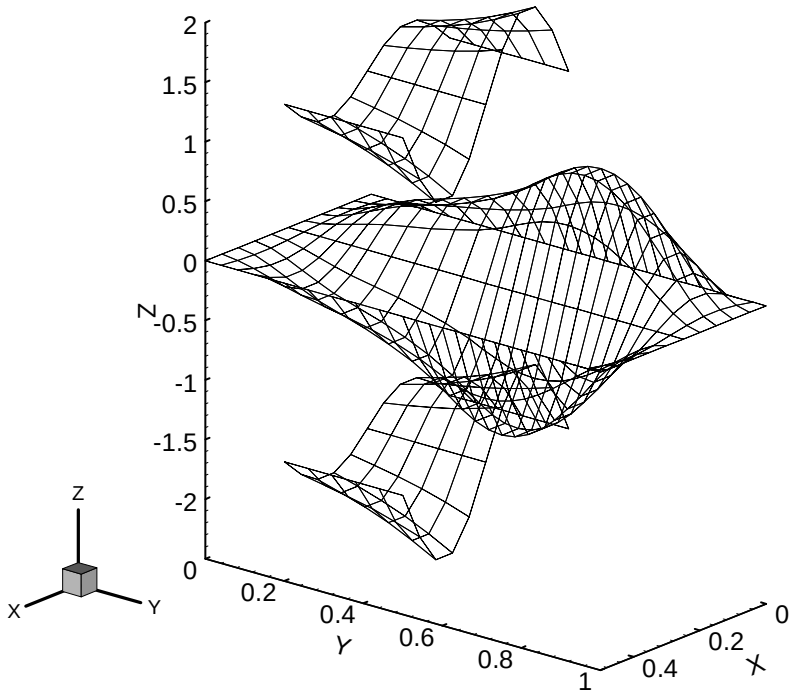


b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{12} = 63.439$

Figure 11.144. Mode Shapes and Natural Frequencies of "Non-Centrally Doubly Stiffened, Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Base Plate=Plate 1= Aluminum , Plate 2= Plate 3= Graphite-Epoxy)
((FFCCFF) B.C.'s in y-direction, "Soft" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m.)

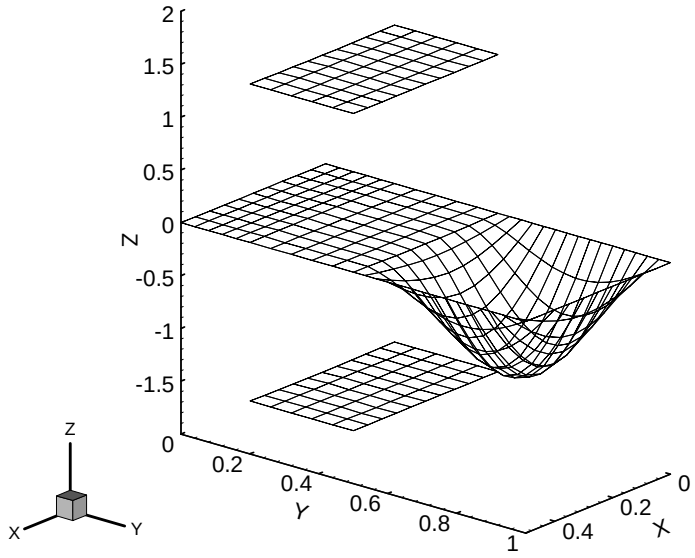


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{13} = 159.432$

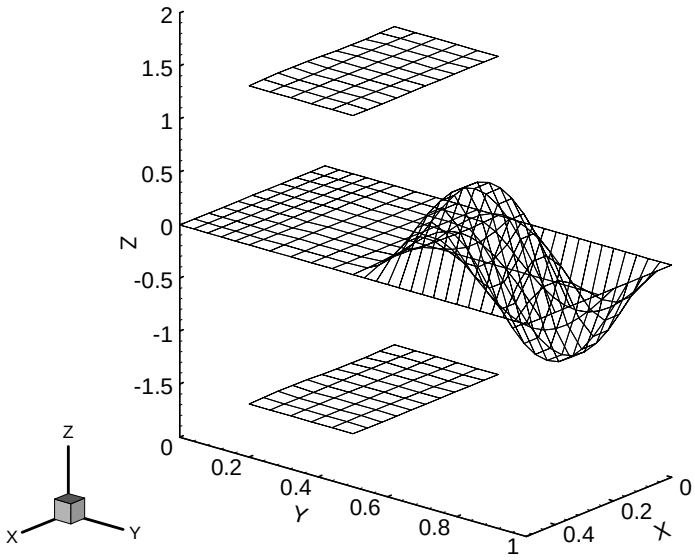


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{21} = 183.622$

Figure 11.144. (continued)

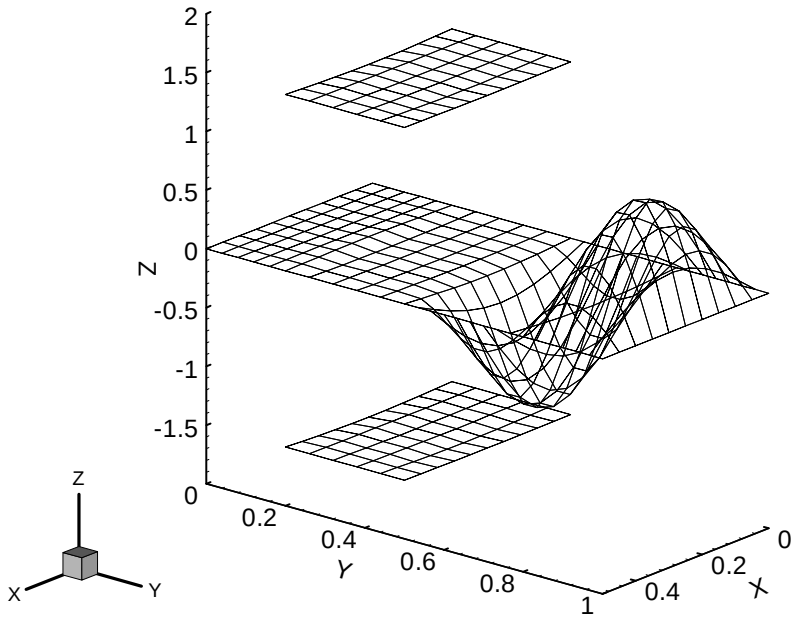


a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 42.989$

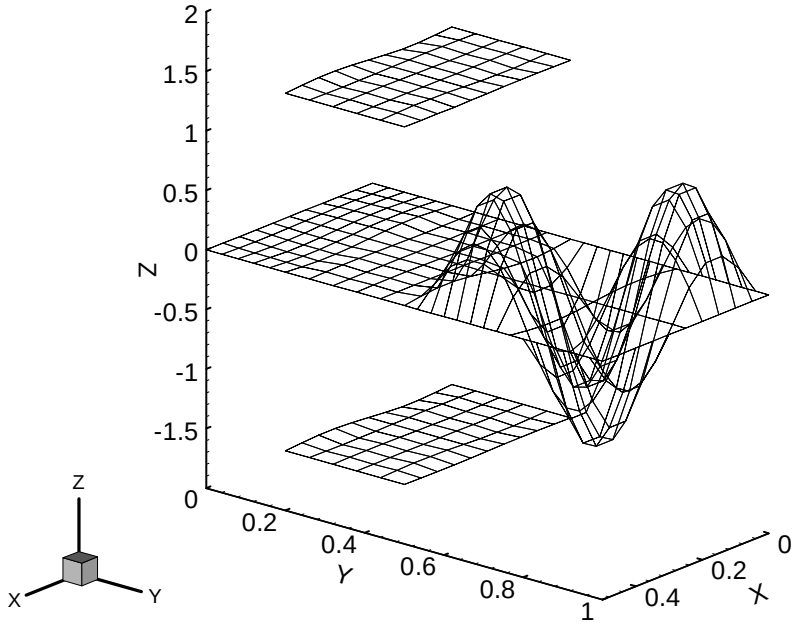


b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{21} = 213.083$

Figure 11.145. Mode Shapes and Natural Frequencies of "Non-Centrally Doubly Stiffened, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Base Plate=Plate 1= Aluminum , Plate 2= Plate 3= Graphite-Epoxy)
((FFSSFF) B.C.'s in y-direction, "Hard" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m.)

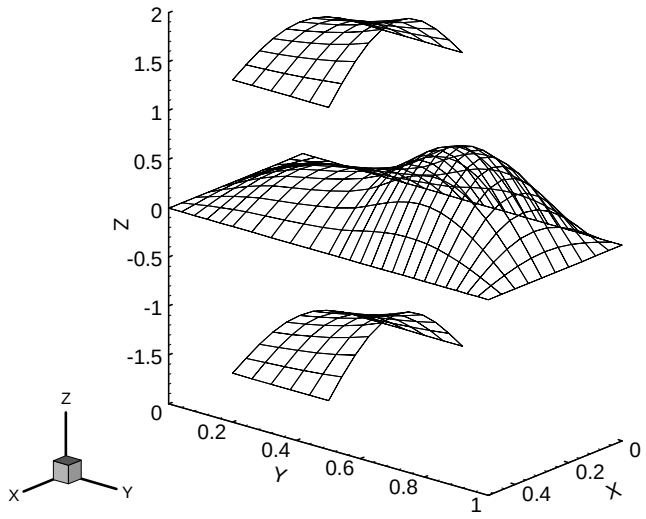


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{12} = 258.701$

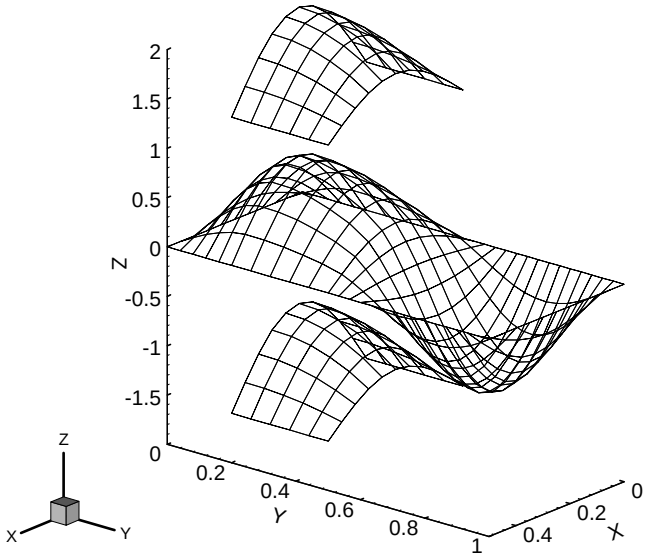


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{22} = 568.376$

Figure 11.145. (continued)

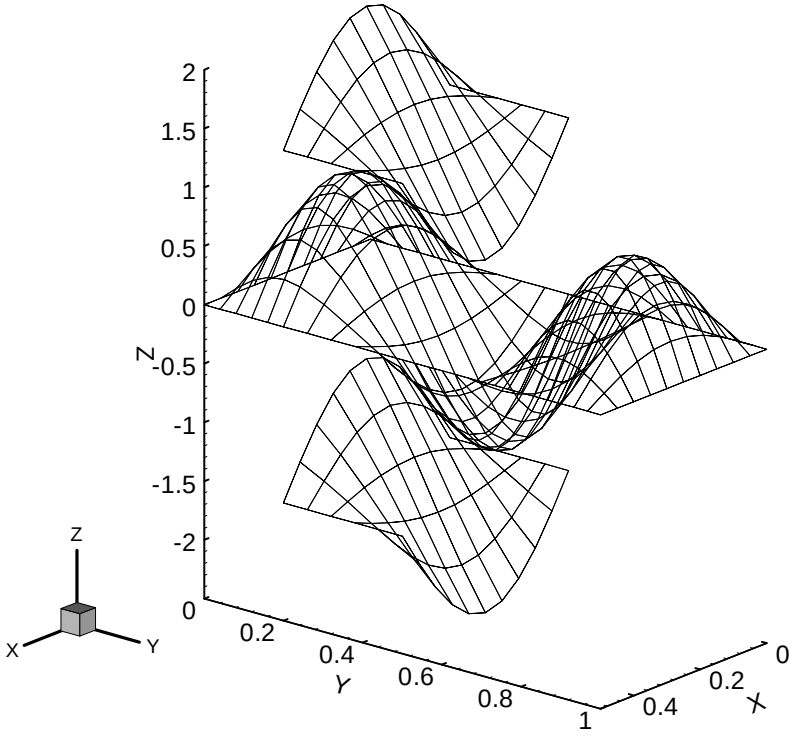


a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 25.653$

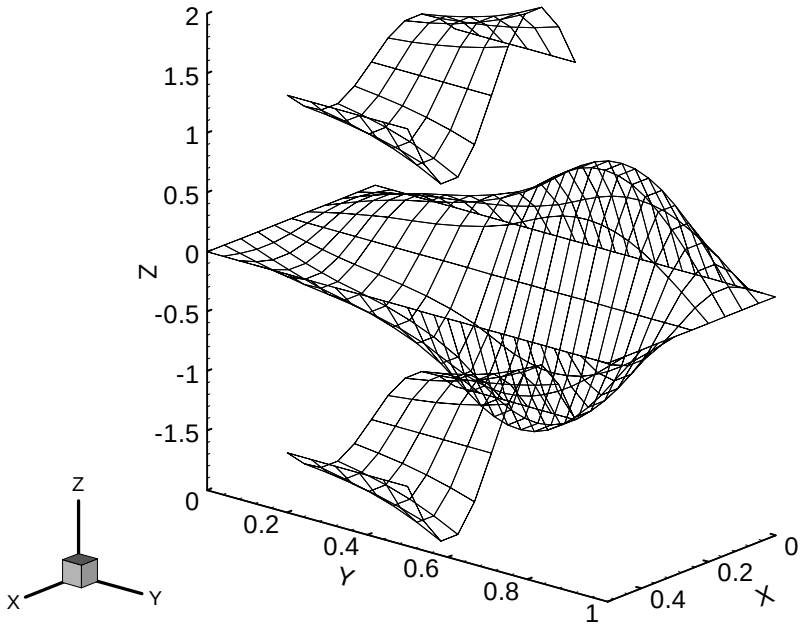


b) Second Mode with $\overline{\Omega}_{\mathbf{2}} = \overline{\omega}_{\mathbf{12}} = 47.714$

Figure 11.146. Mode Shapes and Natural Frequencies of "Non-Centrally Doubly Stiffened, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Base Plate=Plate 1= Aluminum , Plate 2= Plate 3= Graphite-Epoxy)
((FFSSFF) B.C.'s in y-direction, "Soft" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m.)



c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{13} = 121.099$



d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{21} = 175.951$

Figure 11.146. (continued)

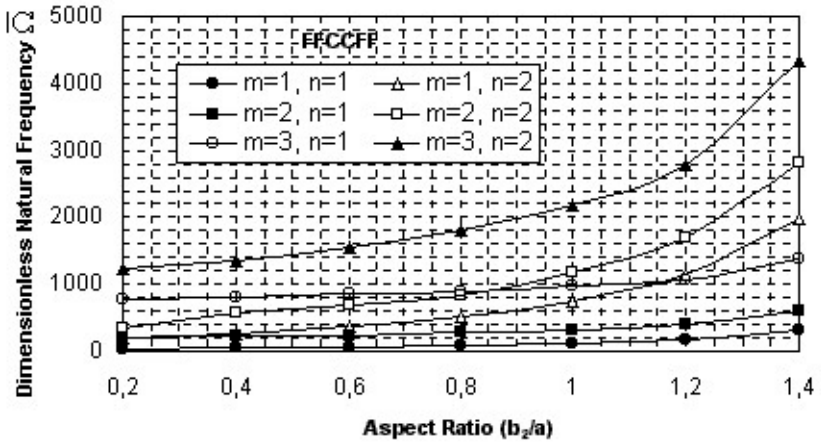


Figure 11.147. Dimensionless Natural Frequency. $\overline{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Non-Centrally Doubly Stiffened, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on the number of half waves in y- and x-directions)

((FFCCFF) B.C.'s in y-direction, ''Hard" Adhesive, b_2=0.30 m, $\tilde{b}$ =0.35 m, a=0.50 m, L=1.0 m)

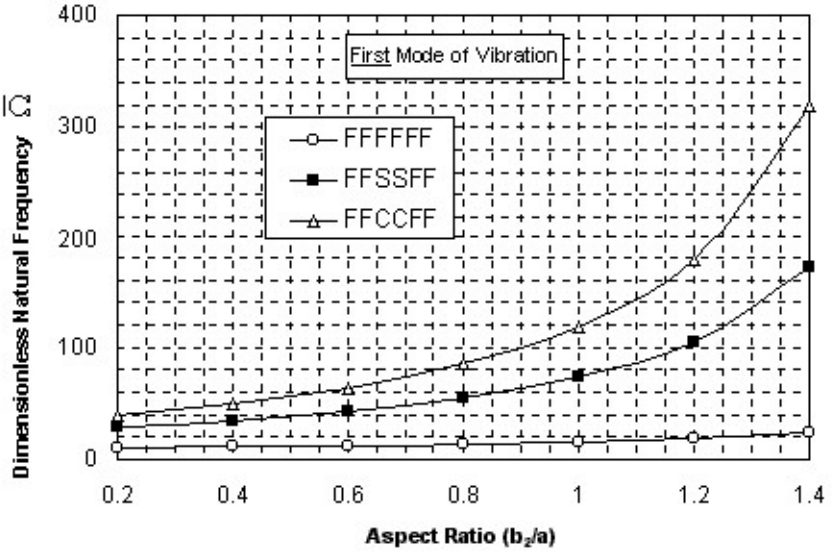


Figure 11.148. First Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a" in "Non-Centrally Doubly Stiffened, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(For Various B.C.'s, ''Hard" Adhesive, b_2=0.30 m, $\tilde{b}$ =0.35 m, a=0.50 m, L=1.0 m)

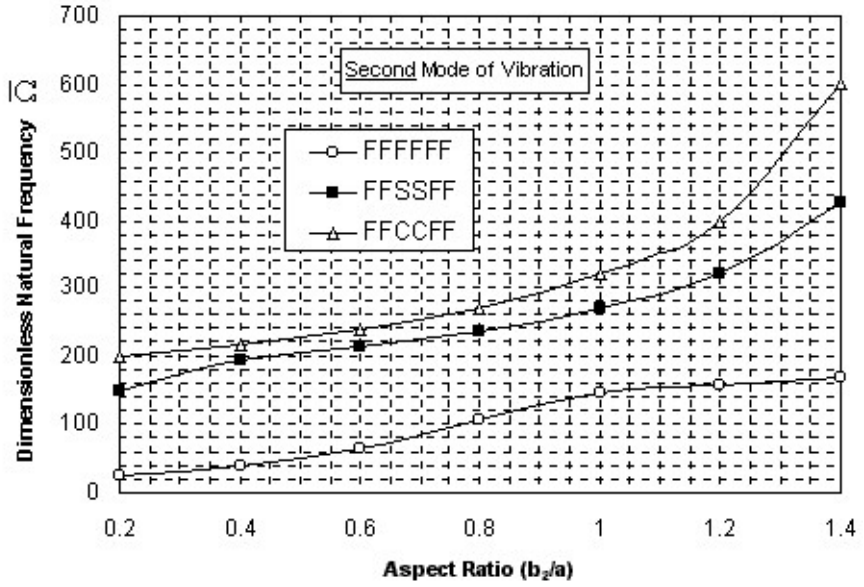


Figure 11.149. Second Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a" in "Non-Centrally Doubly Stiffened, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(For Various B.C.'s, ''Hard" Adhesive, b_2=0.30 m, $\tilde{b}$ =0.35 m, a=0.50 m, L=1.0 m)

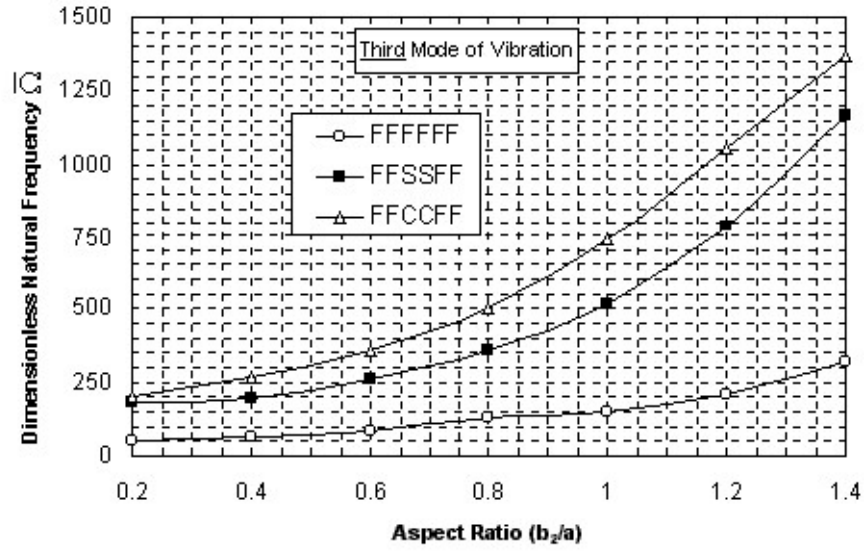


Figure 11.150. Third Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a" in "Non-Centrally Doubly Stiffened, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(For Various B.C.'s, ''Hard" Adhesive, b_2=0.30 m, $\tilde{b}$ =0.35 m, a=0.50 m, L=1.0 m)

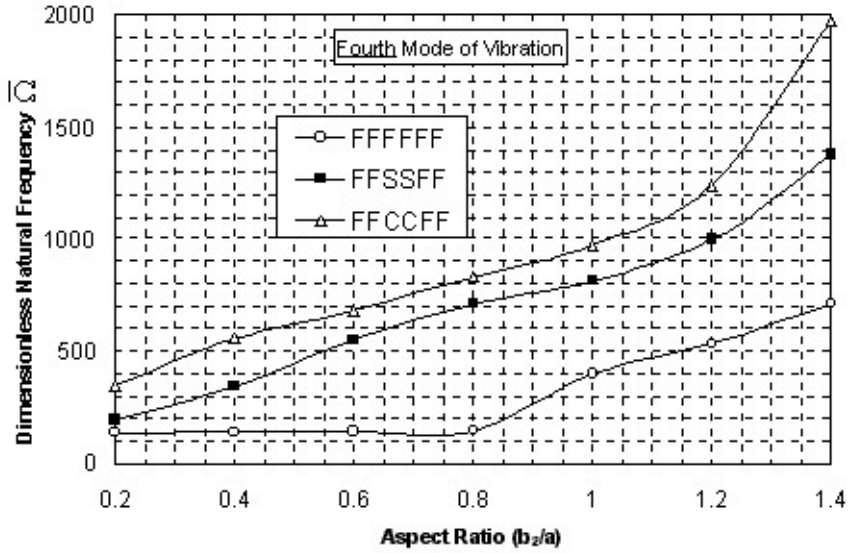


Figure 11.151. Fourth Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a" in "Non-Centrally Doubly Stiffened, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(For Various B.C.'s, ''Hard" Adhesive, b_2=0.30 m, $\tilde{b}$ =0.35 m, a=0.50 m,L=1.0 m)

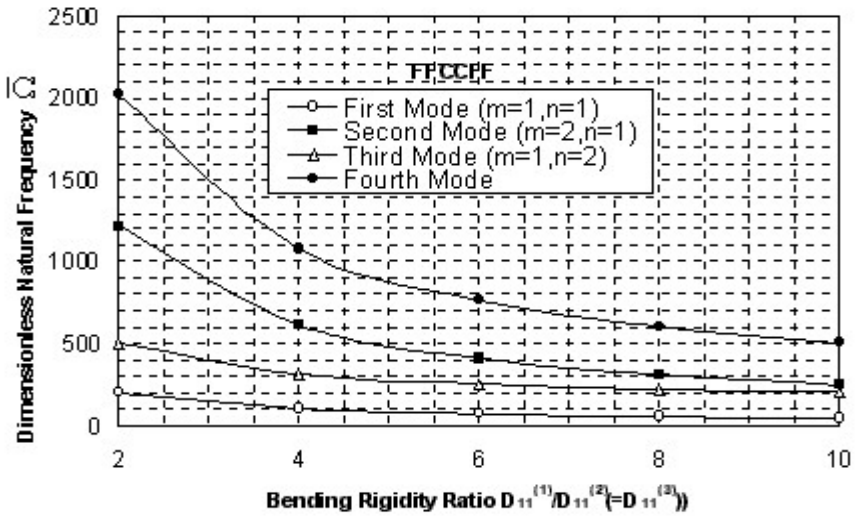


Figure 11.152. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$'' in "Non-Centrally Doubly Stiffened, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Several Modes with (FFCCFF) B.C.'s, ''Hard" Adhesive,
b_2=0.30 m, $\tilde{b}$ =0.35 m, a=0.50 m, L=1.0 m)

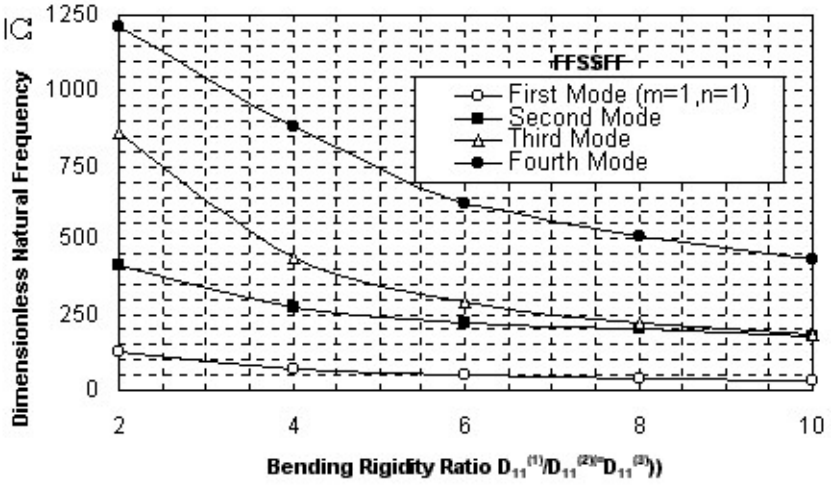


Figure 11.153. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$’’ in “Non-Centrally Doubly Stiffened, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Several Modes with (FFSSFF) B.C.’s, ‘‘Hard” Adhesive,
b_2=0.30 m, $\tilde{b}$ =0.35 m, a=0.50 m, L=1.0 m)

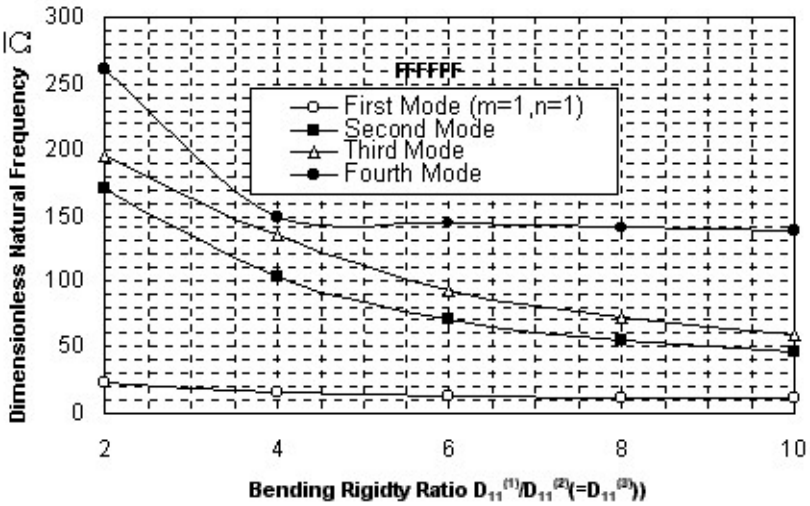


Figure 11.154. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$’’ in “Non-Centrally Doubly Stiffened, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Several Modes with (FFFFFF) B.C.’s, ‘‘Hard” Adhesive,
b_2=0.30 m, $\tilde{b}$ =0.35 m, a=0.50 m, L=1.0 m)

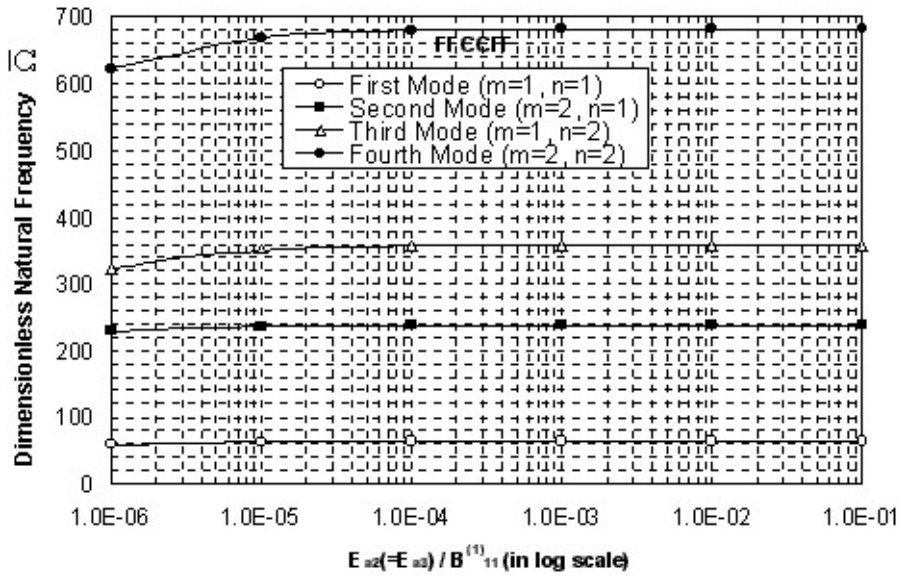


Figure 11.155. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$ ’’ in “Non-Centrally Doubly Stiffened, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)” (Several Modes with (FFCCFF) B.C.’s, b_2=0.30 m, $\tilde{b}$ =0.35 m, a=0.50 m, L=1.0 m)

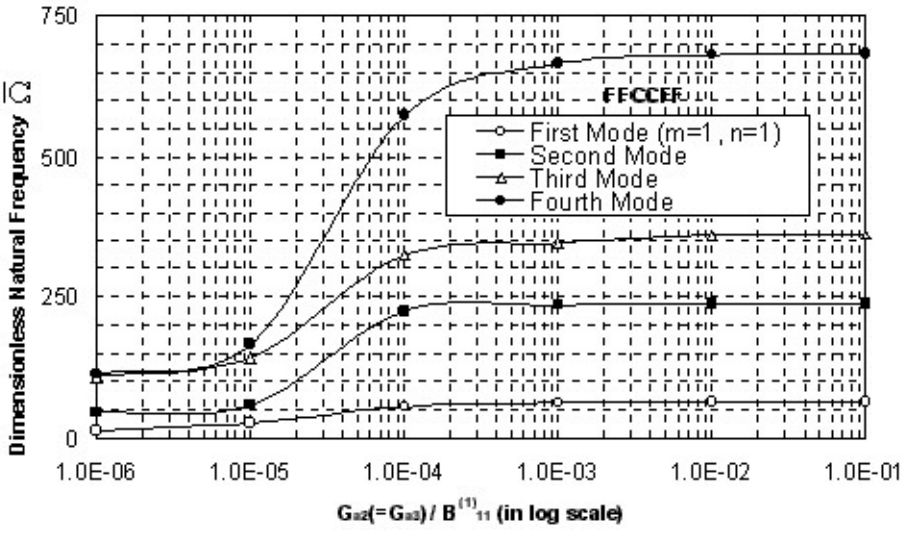


Figure 11.156. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3})/B_{11}^{(1)}$ ’’ in “Non-Centrally Doubly Stiffened, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)” (Several Modes with (FFCCFF) B.C.’s, b_2=0.30 m, $\tilde{b}$ =0.35 m, a=0.50 m, L=1.0 m)

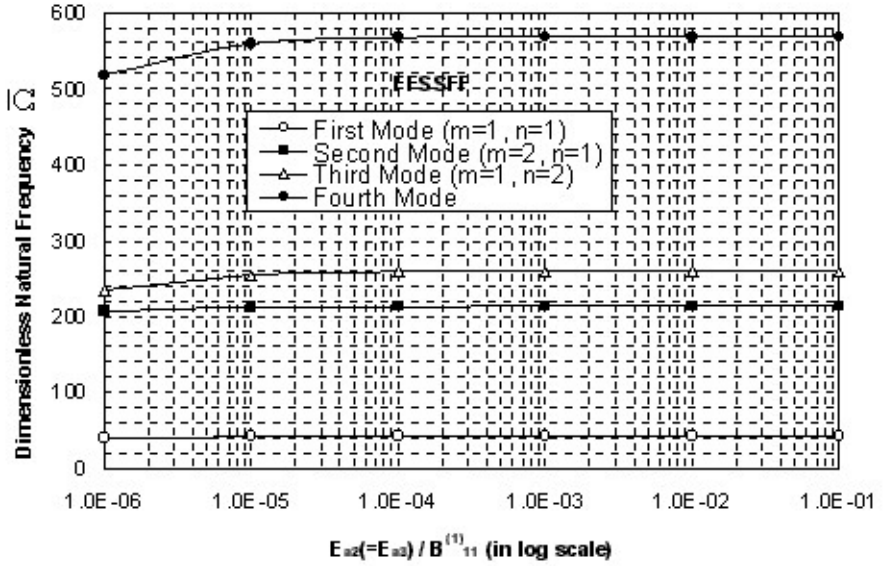


Figure 11.157. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$ ’’ in “Non-Centrally Doubly Stiffened, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)” (Several Modes with (FFSSFF) B.C.’s, b_2=0.30 m, $\tilde{b}$ =0.35 m, a=0.50 m, L=1.0 m)

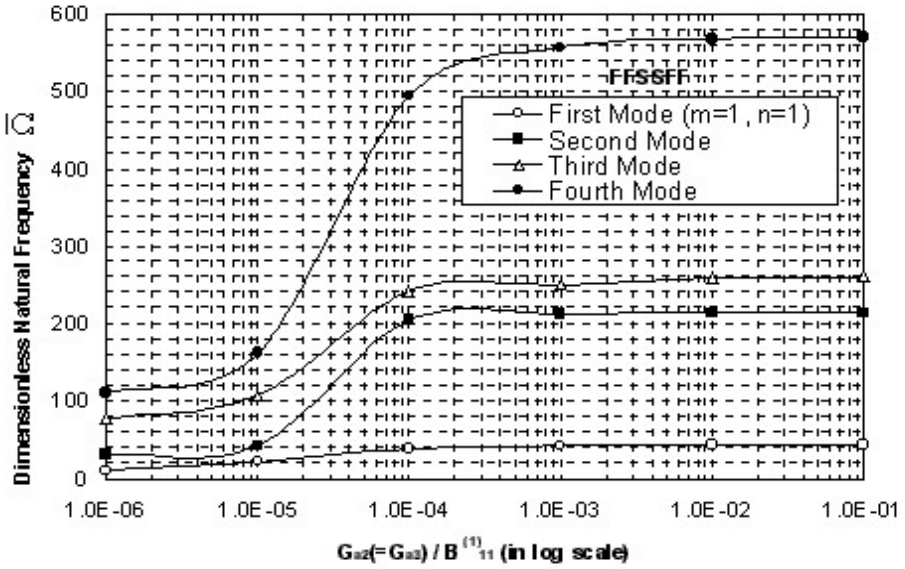


Figure 11.158. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3})/B_{11}^{(1)}$ ’’ in “Non-Centrally Doubly Stiffened, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)” (Several Modes with (FFSSFF) B.C.’s, b_2=0.30 m, $\tilde{b}$ =0.35 m, a=0.50 m, L=1.0 m)

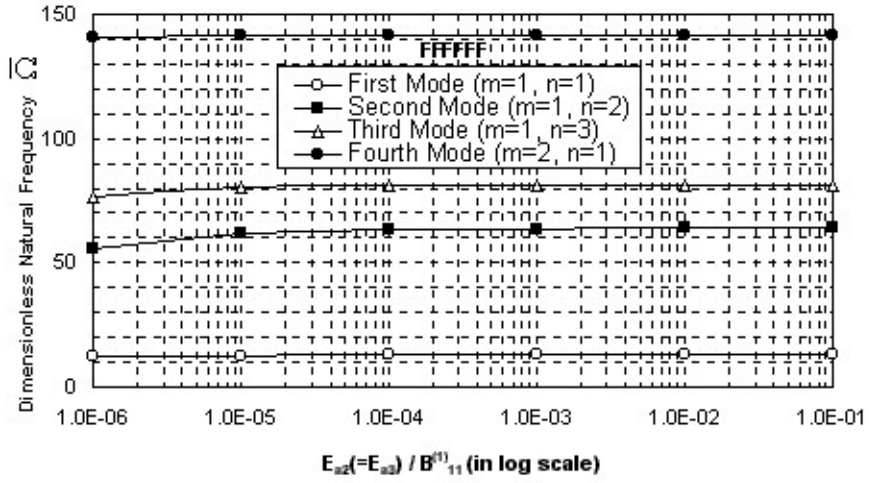


Figure 11.159. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$ ’’ in “Non-Centrally Doubly Stiffened, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)” (Several Modes with (FFFFFF) B.C.’s, b_2=0.30 m, $\tilde{b}$ =0.35 m, a=0.50 m, L=1.0 m)

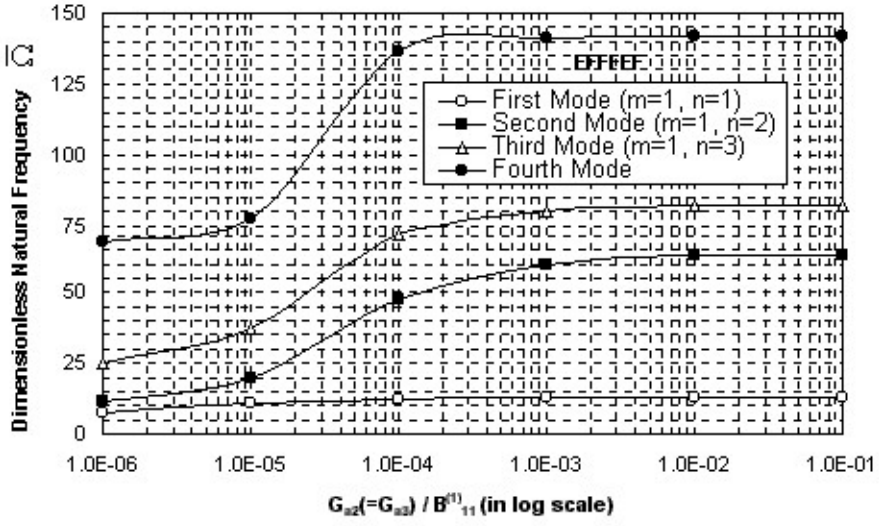


Figure 11.160. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3})/B_{11}^{(1)}$ ’’ in “Non-Centrally Doubly Stiffened, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)” (Several Modes with (FFFFFF) B.C.’s, b_2=0.30 m, $\tilde{b}$ =0.35 m, a=0.50 m, L=1.0 m)

11.6 Numerical Results and Discussion for “Main PROBLEM III.a”

In this “Main PROBLEM III.a”, the “Orthotropic Base Plate is made of Kevlar-Epoxy and “Upper and Lower Stiffening Plate Strips” are chosen as Graphite-Epoxy. In Figures, the adhesive layer are chosen as the “hard” and the “soft” adhesive cases with various boundary conditions.

In Figures 11.161 and 11.162, the first four mode shapes and corresponding natural frequencies of the composite plate or panel system of “Main PROBLEM III.a” with the (CFCCCF) support conditions are displayed for the “hard” and “soft” adhesive cases, respectively. From the above mentioned Figures, it can be observed that the in-between adhesive layers seriously influence or rather change the mode shapes as well as the natural frequencies of the composite plate or panel system.

In a similar manner, in Figures 11.163 and 11.164, the first four mode shapes and the associated natural frequencies of the composite plate or panel system of the present problem are plotted for the “hard” and “soft” adhesive cases, respectively. This time, in both cases, the support conditions in the y-directions are shown as (SFSSSF). By comparing these Figures with Figures 11.161 and 11.162, it can be stated that mode shapes are almost the same but natural frequencies are drastically affected by the support conditions in y-direction.

Figure 11.165 represents the same composite system of Figure 11.161 again with the (CFCCCF) boundary conditions but for the adhesive shear modulus $G_{a2} = G_{a3} = 0$ $(10^{-8}) \approx 0$. From the comparison of those Figures, the effect of the adhesive shear modulus on both mode shapes and the corresponding natural frequencies can easily be seen.

Similarly, the first four mode shapes of the same composite plate or panel system for “hard” and “soft” adhesive cases, but with the (FFFFFF) boundary conditions are presented in Figures 11.166. and 11.167. In these Figures, due to less restraining support conditions in the y-direction, very interesting mode shapes and

the corresponding natural frequencies are observed. This can be seen by comparing the mode shapes of these Figures with the previous ones.

The parametric studies for “Main PROBLEM III.a” will be considered next.

In Figure 11.168, the dimensionless natural frequencies $\overline{\Omega}$ versus the “Aspect Ratio b_2/a” are plotted for the(CFCCCF) boundary conditions and for the “hard” adhesives case. In this Figure, “m” is the number of half waves in the x-direction and “n” is the second subscript of the natural frequencies $\overline{\omega}_{mn}$.

In Figures 11.169, 11.170, 11.171, 11,172 and 11.173, the dimensionless natural frequency $\overline{\Omega}$ versus the “Aspect Ratio b_2/a” for the first four modes and for the “hard” adhesives case are investigated. The boundary conditions in the y-directions are the (CFCCCF), (SFSSSF) and (FFFFFF), respectively. It can be concluded that as the “Aspect Ratio b_2/a” increases, the natural frequencies get relatively higher. Also, it is interesting when the ‘‘Aspect Ratio b_2/a” takes 2.0 value (i.e. the base plate and the stiffening strips are of the same length of 1.0 value), the natural frequencies for the same modes but with different boundary conditions converge to almost same values regardless of the boundary conditions.

In Figure 11.174, the dimensionless natural frequencies $\overline{\Omega}$ versus “the Aspect Ratio b_2/a” are plotted for the “soft” adhesives case with the (CFCCCF) boundary conditions. It is wortwhile to compare 11.174 with the previous Figure 11.168.

In Figure 11.175, 11.176 and 11.177, in a similar manner to the preceding Figures 11.169, 11.171 and 11.173, the dimensionless natural frequencies $\overline{\Omega}$ versus the “Aspect Ratio b_2/a” are plotted for the “soft” adhesives case with the (CFCCCF), (SFSSSF) and (FFFFFF) support conditions, respectively. It can be concluded from these three Figures that, in contrast with the “hard” adhesives case, the natural frequency curves corresponding to each mode are very much separate from each other. Also, one may conclude from the aforementioned figures, the natural frequencies increase with a wavy pattern with the increasing the “Aspect Ratio b_2/a”.

This is, of course, due to the "soft"ness of adhesive layers and consequently to the relatively loose coupling (because of the "soft" adhesives) between the base plate and the stiffening strips.

In Figures 11.178 through 11.184, the effect of the "Bending Rigidity Ratio $\boldsymbol{D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})}$" on the natural frequencies are investigated in the "hard" and "soft" adhesive cases with the (CFCCCF), (SFSSSF), and the (FFFFFF) support conditions , respectively. It is obvious from these Figures that the dimensionless natural frequencies decrease relatively sharply, at first, (up to 0.6 value of the "Bending Rigidity Ratio $\boldsymbol{D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})}$") and, after that value, the decrease is rather gradual.

The direct effects of the adhesive layer elastic constants E_a, G_a on the dimensionless natural frequencies are investigated for the "Main PROBLEM I.a". For this purpose, the dimensionless natural frequencies versus the "Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$" are plotted (while the other elastic constants kept constant) in Figures 11.185, 11.187 and 11.189 for the support conditions (FFCCFF), (FFSSFF) and (FFFFFF), respectively. Similarly, the dimensionless natural frequencies versus the "Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3})/B_{11}^{(1)}$" are presented in Figures 11.186, 11.188 and 11.190 for the boundary condition (FFCCFF), (FFSSFF), (FFFFFF), respectively.

It is obvious from the aforementioned figures, "Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3})/B_{11}^{(1)}$" has very significant effect on the natural frequencies of the composite plate systems considered in the "Main PROBLEM III.a". This result is expected if taking into account the shear deformable character of the Mindlin Plates in the composite systems. The influence of the "Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$" on the natural frequencies is rather relatively weak depending on the support conditions on the plate or panel system.

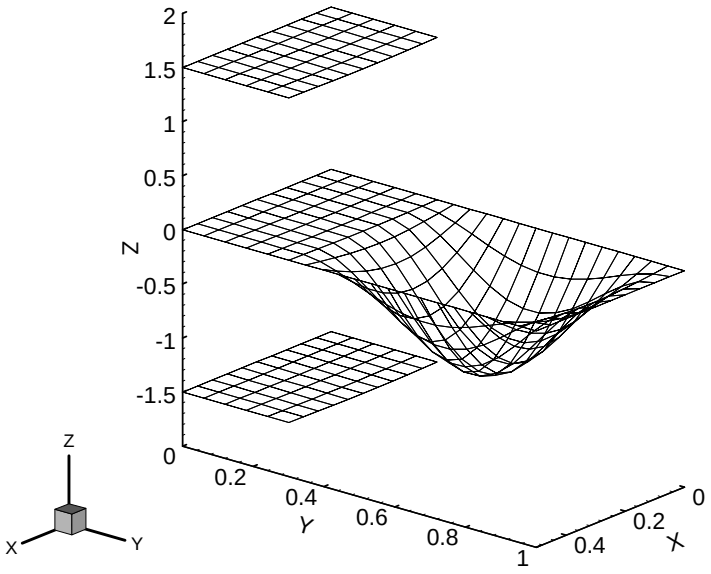
The position of the upper and lower stiffening plate strips will obviously have some effects on the natural frequencies for the "Main PROBLEM III.a". In order to observe this effect, the dimensionless natural frequencies versus the "Stiffeners

Position Ratio ($\tilde{b}/L$)" are plotted in the "hard" adhesives case in Figures 11.191, 11.192 and 11.193, 11.194 and 11.195, 11.196 taking in to account several boundary conditions.

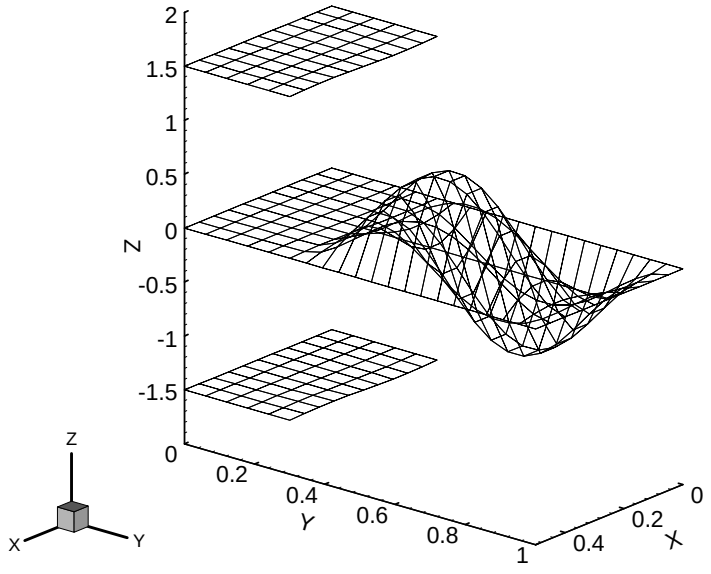
It is interesting to observe that, in Figure 11.192 corresponding to the (FFCCFF) boundary conditions, the first three frequencies gradually increase as the "Stiffeners Position Ratio ($\tilde{b}/L$)" increases up to the 0.5 value (which means that the stiffening plate strips reach the central position in the plate system). In the case of the fourth natural frequency the natural frequencies increase relatively sharply and reach a maximum value corresponding to the "Position Ratio ($\tilde{b}/L$)=0.4". After this point, the fourth natural frequency decreases rather fast and finally reaches the fourth natural frequency in the case of the central position of the stiffening plate strips in the composite system.

Similar conclusions can be reached in the case of the (FFSSFF) support conditions as can be seen from Figure 11.194. However, in the case of the (FFFFFF) support conditions, there are some similarities in the fourth natural frequencies, but for the first three frequencies, there are no similarities with those Figures 11.192 and 11.194 corresponding to the (FFCCFF) and (FFSSFF) support conditions, respectively. This is the consequence very loose (FFFFFF) boundary conditions with very little constraining effects on the entire composite system.

In Figures 11.197 through 11.202, the effect of the "Stiffeners Position Ratio ($\tilde{b}/L$)" on the natural frequencies are investigated in the "soft" adhesives case with the (FFCCFF), (FFSSFF), and the (FFFFFF) support conditions, respectively. In the case of the (FFCCFF) and the (FFSSFF) support conditions, similar results to those of the "hard" adhesives case are obtained. However, in the case of the (FFFFFF) support conditions, the general trend in the first , the second and the third dimensionless natural frequencies show increases or decreases in a wavy pattern while the first natural frequency increases gradually as the "Stiffeners Position Ratio ($\tilde{b}/L$)" increases.



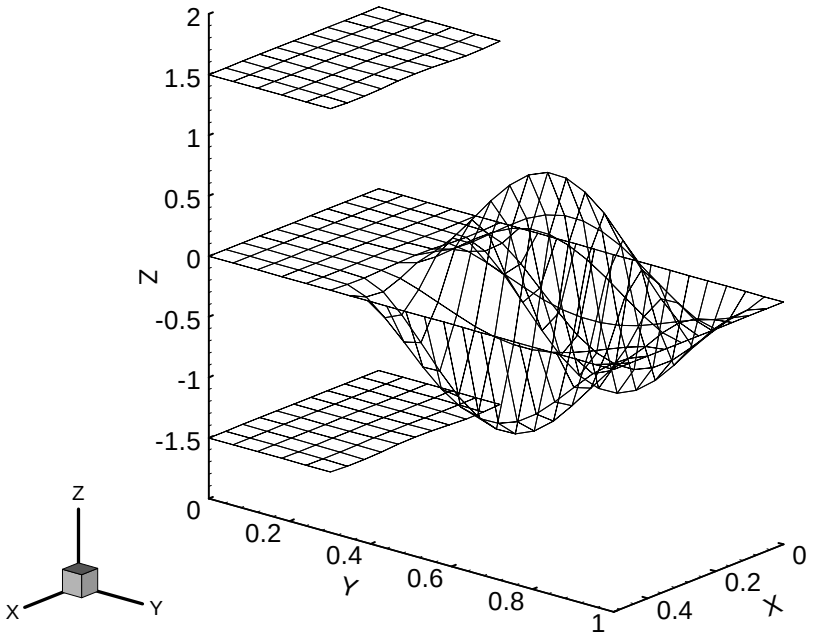
a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 162.678$



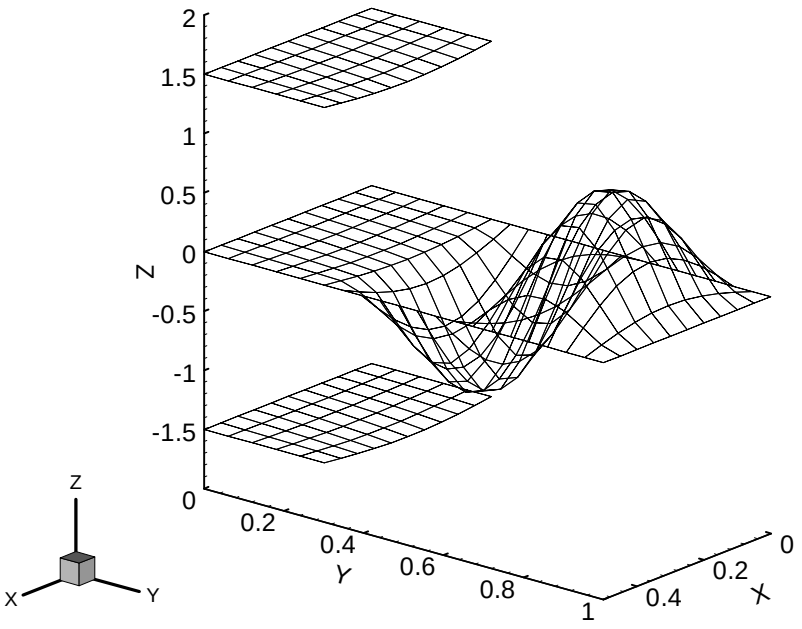
b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{21} = 316.390$

Figure 11.161. Mode Shapes and Natural Frequencies of "Doubly Stiffened-at Support, Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((CFCCCF) B.C.'s in y-direction, "Hard" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m.)

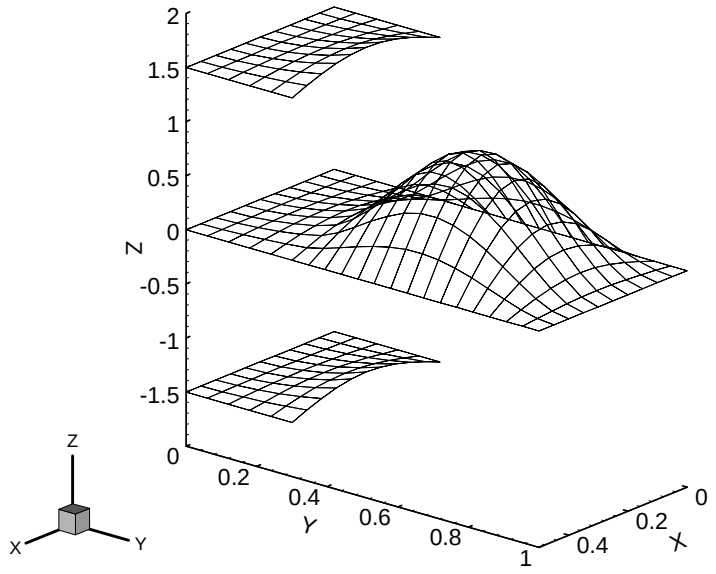


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{31} = 891.845$

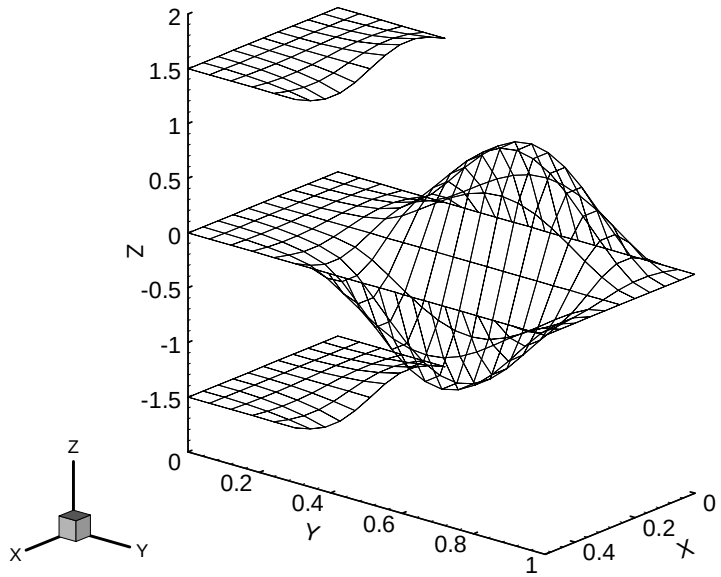


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{12} = 1112.105$

Figure 11.161. (continued)



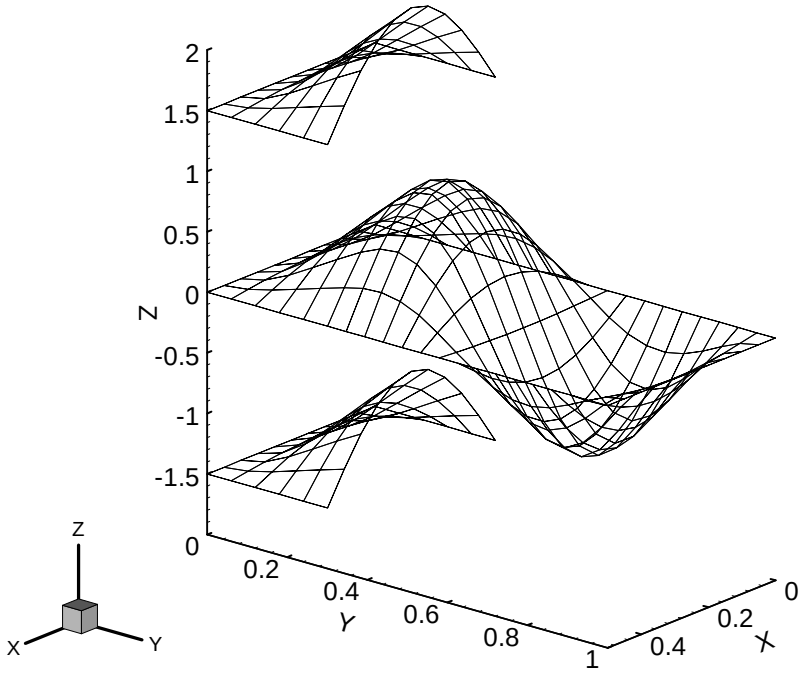
a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 119.588$



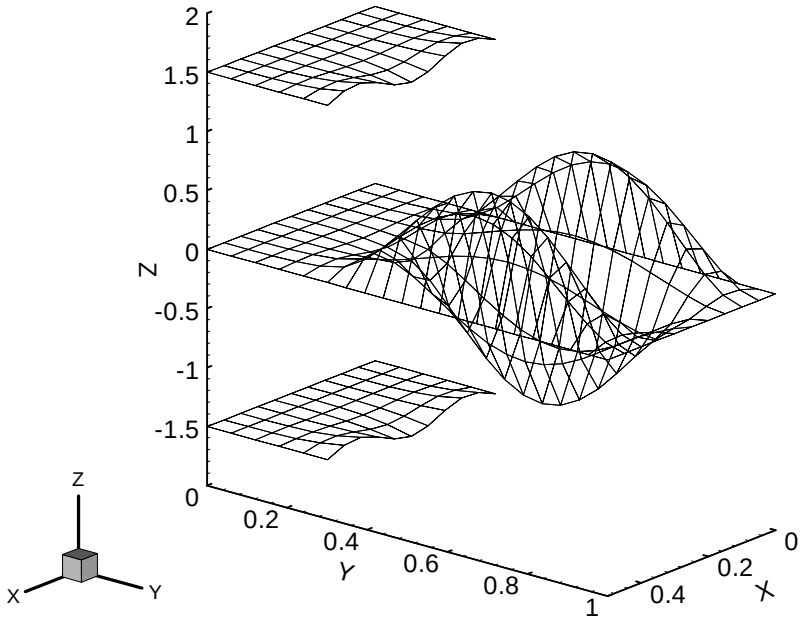
b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{21} = 277.881$

Figure 11.162. Mode Shapes and Natural Frequencies of "Doubly Stiffened-at Support, Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((CFCCCF) B.C.'s in y-direction, "Soft" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m.)

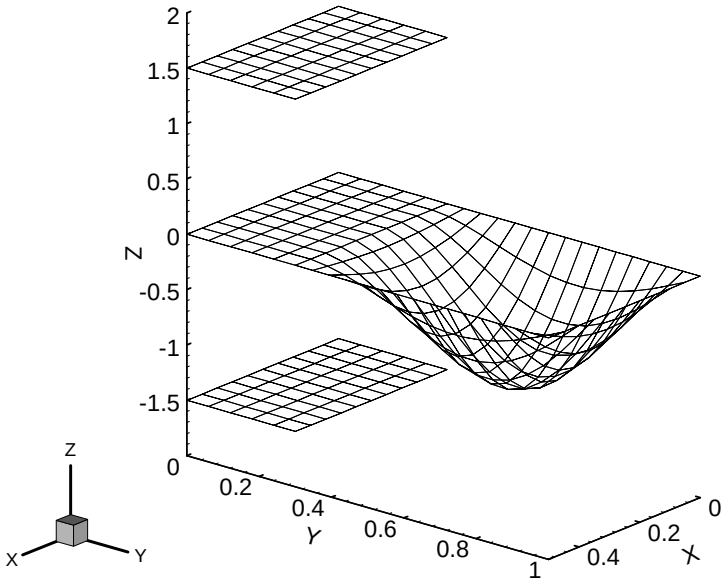


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{12} = 543.492$

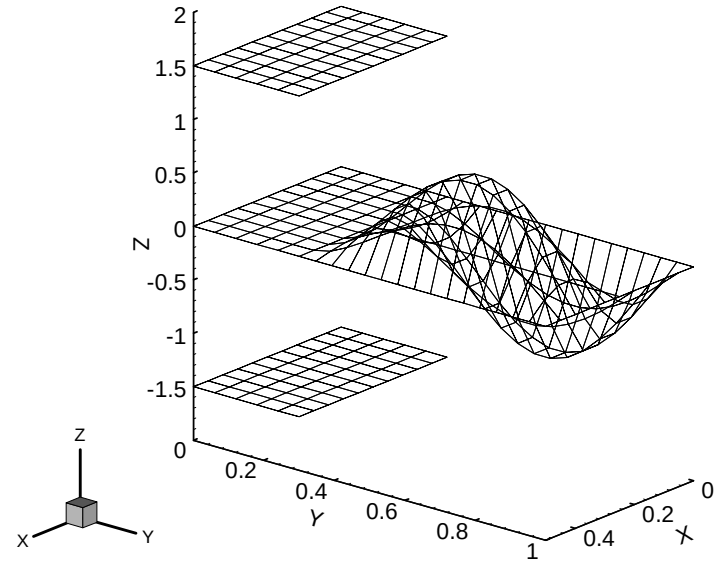


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{31} = 852.537$

Figure 11.162. (continued)



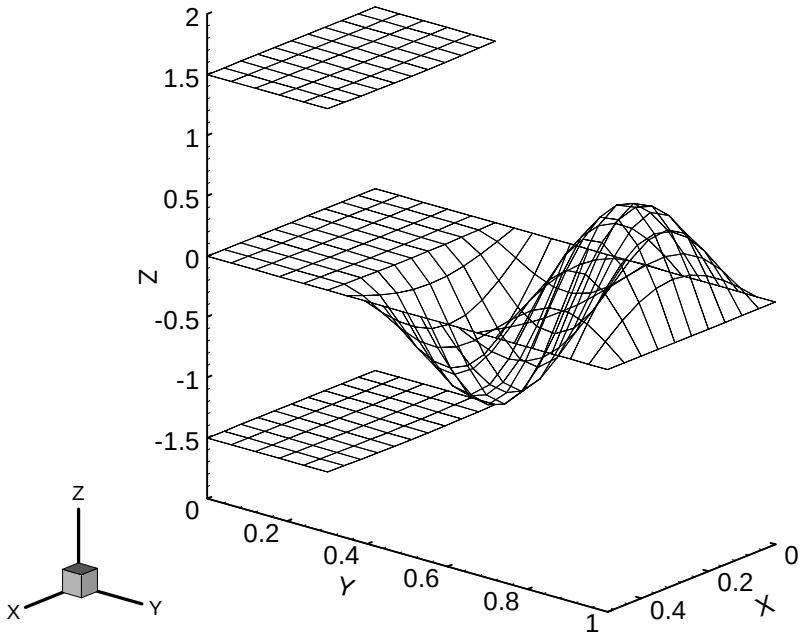
a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 85.443$



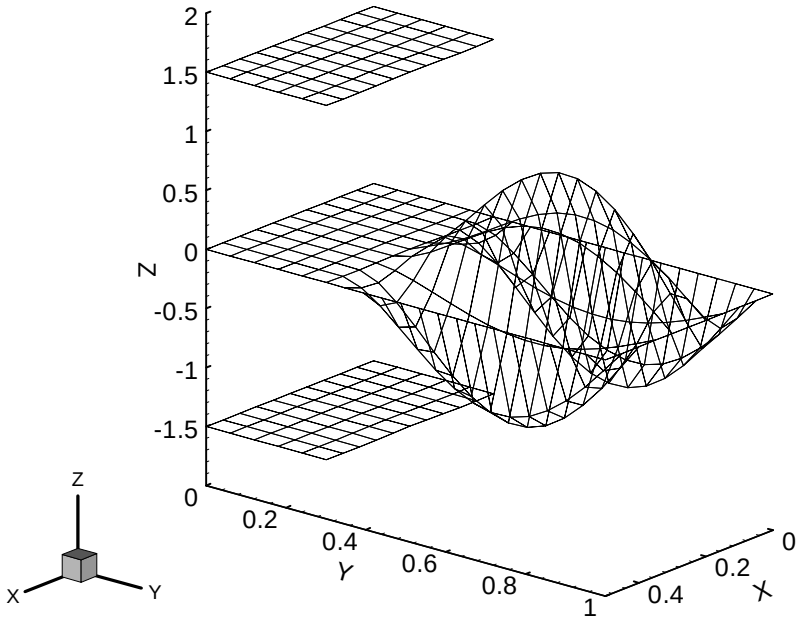
b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{21} = 236.960$

Figure 11.163. Mode Shapes and Natural Frequencies of "Doubly Stiffened-at Support, Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((SFSSSF) B.C.'s in y-direction, "Hard" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m.)

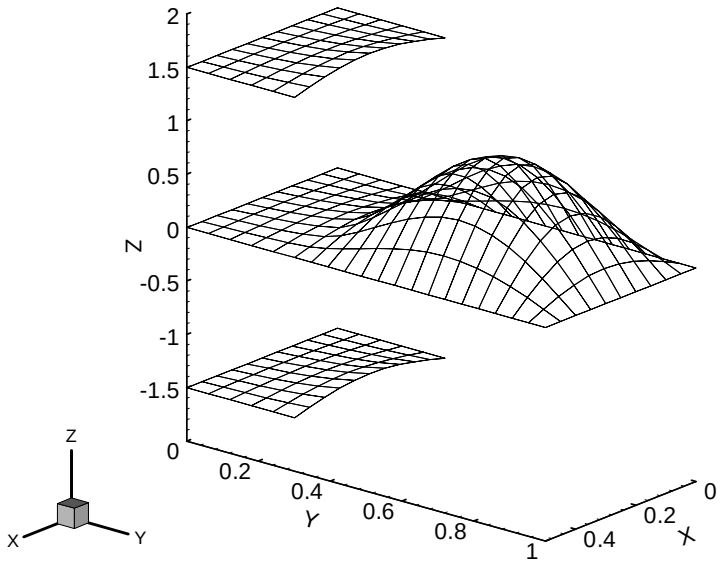


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{12} = 738.391$

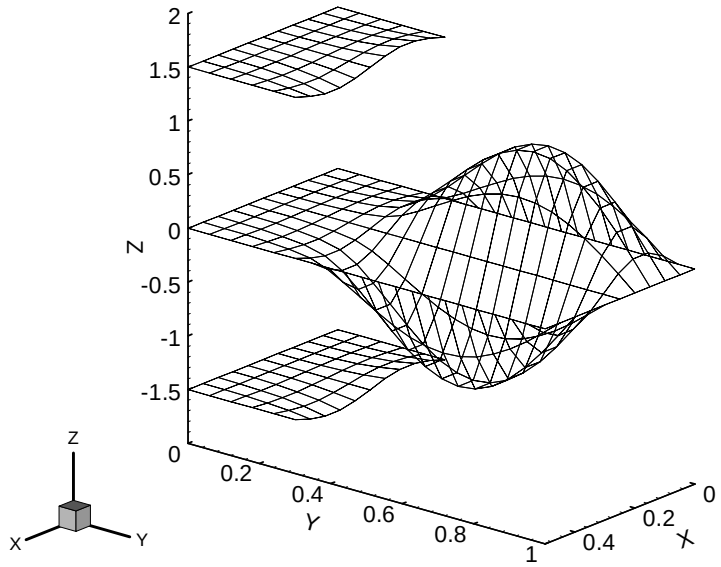


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{31} = 808.898$

Figure 11.163. (continued)



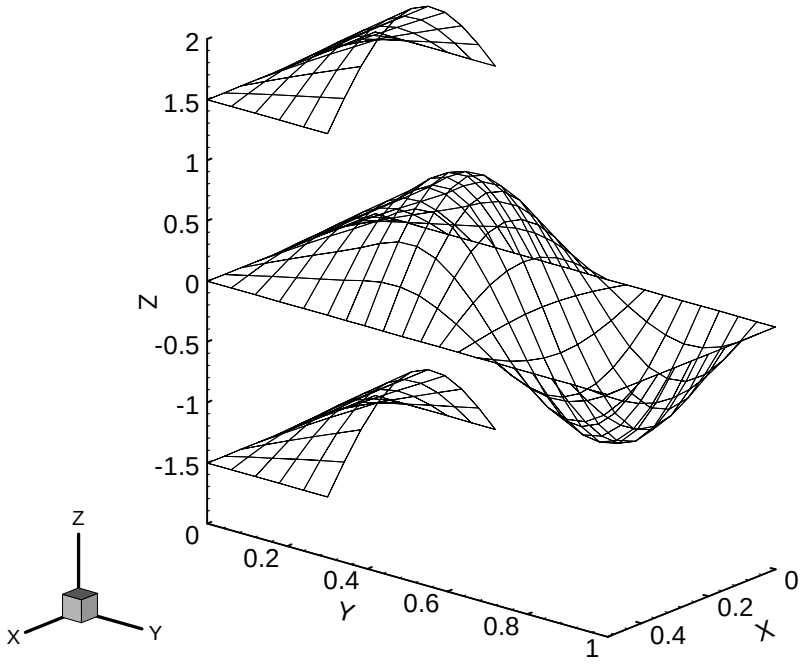
a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 66.807$



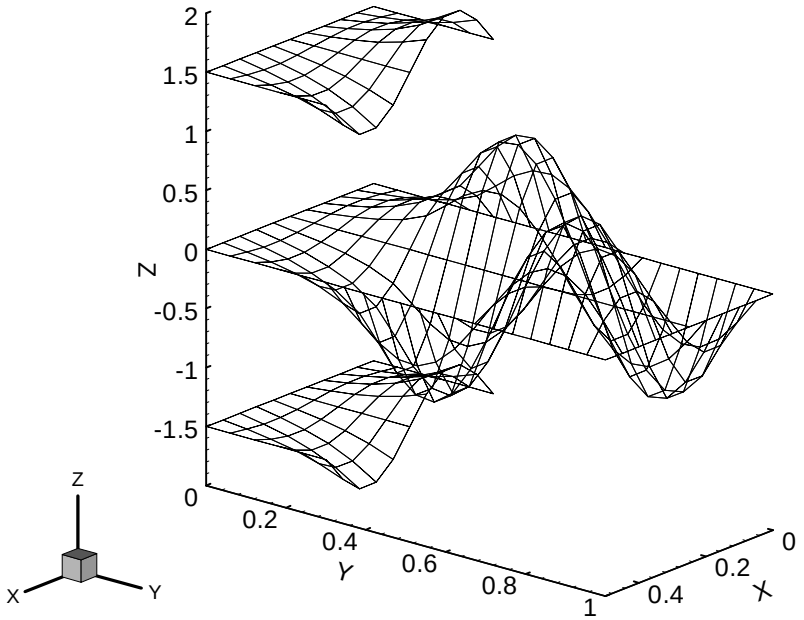
b) Second Mode with $\overline{\Omega}_{\mathbf{2}} = \overline{\omega}_{\mathbf{21}} = 218.943$

Figure 11.164. Mode Shapes and Natural Frequencies of "Doubly Stiffened-at Support, Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((SFSSSF) B.C.'s in y-direction, "Soft" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m.)

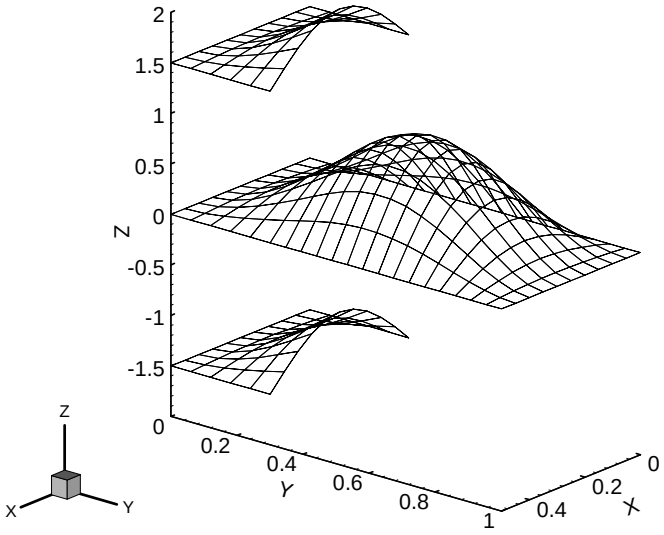


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{12} = 367.731$

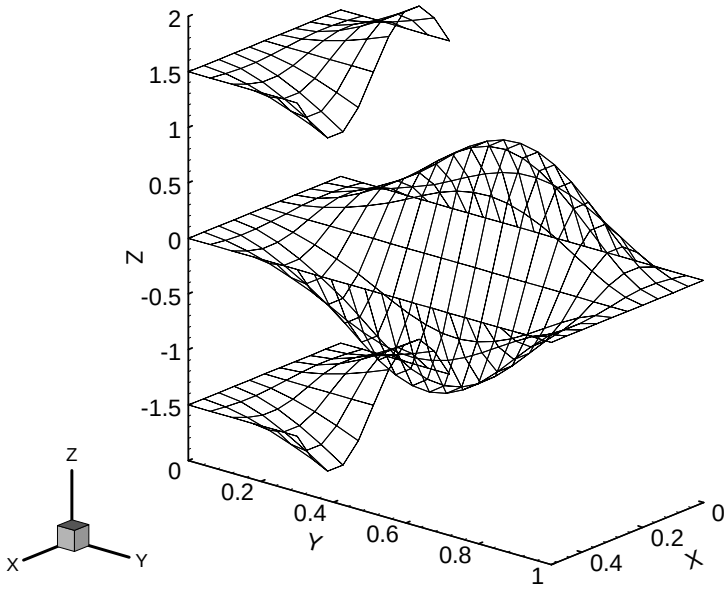


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{22} = 746.381$

Figure 11.164. (continued)



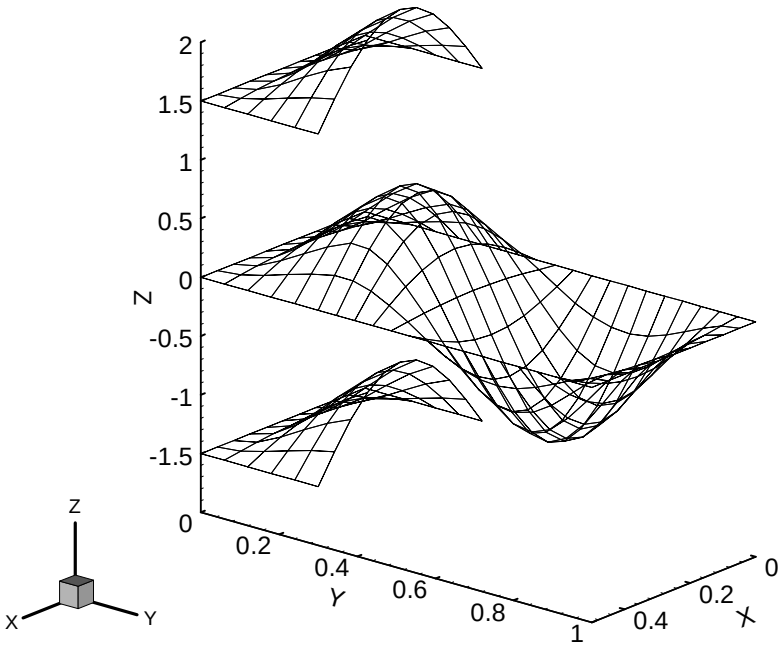
a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 72.286$



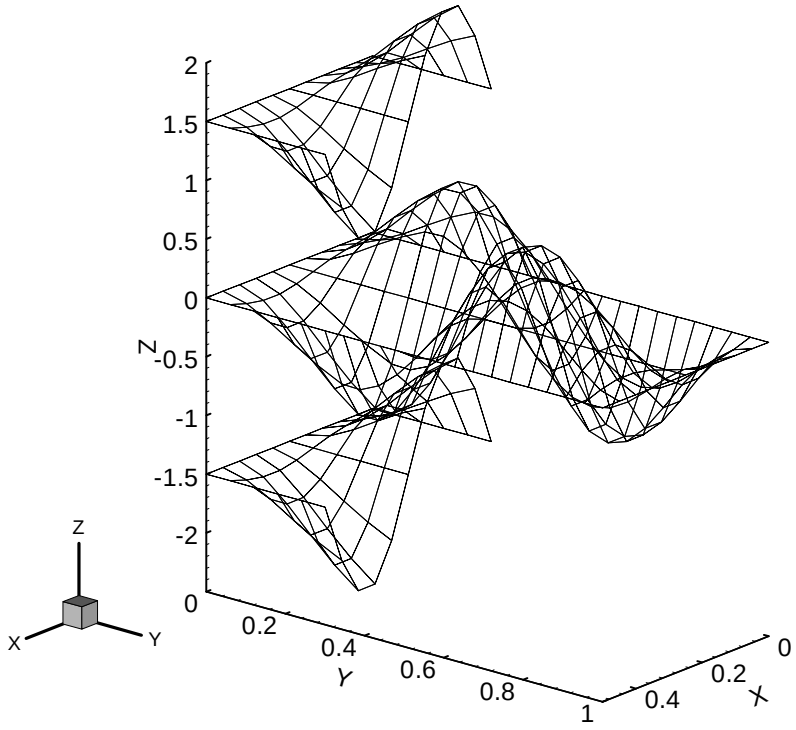
b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{21} = 228.450$

Figure 11.165. Mode Shapes and Natural Frequencies of "Doubly Stiffened-at Support, Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((CFCCCF) B.C.'s in y-direction, $G_{a2}=G_{a3}\approx 0$, $b_2=0.30$ m, a=0.50 m, L=1.0 m)

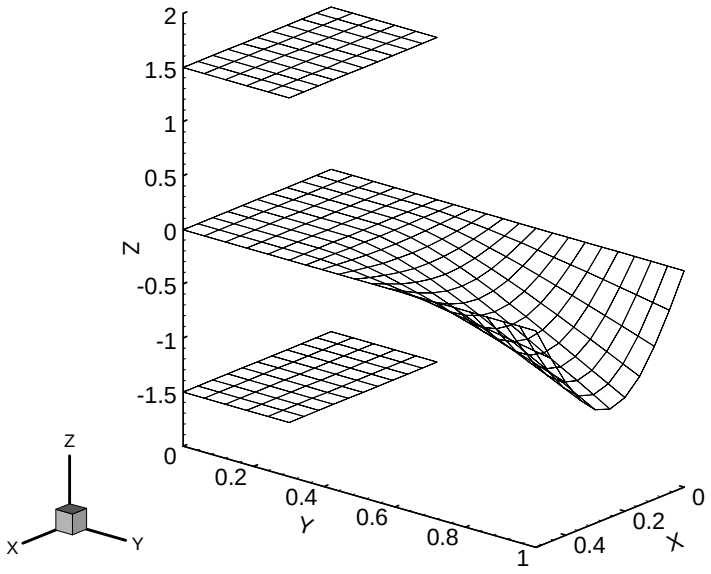


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{12} = 301.589$

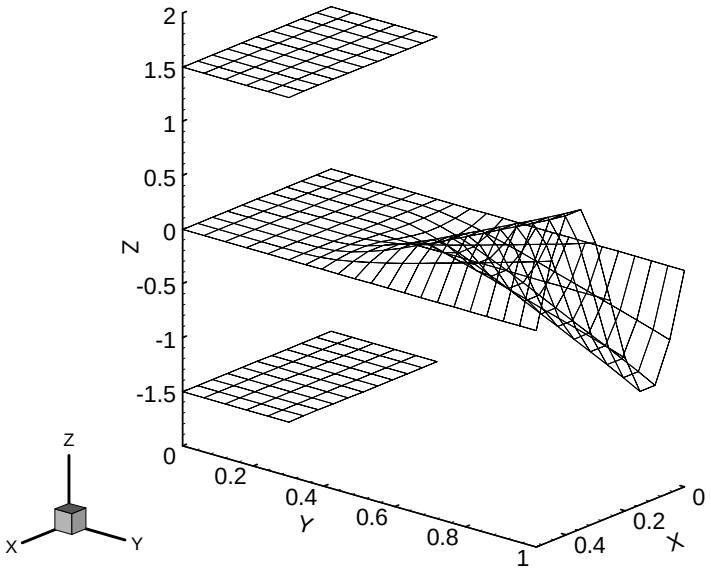


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{22} = 517.814$

Figure 11.165. (continued)



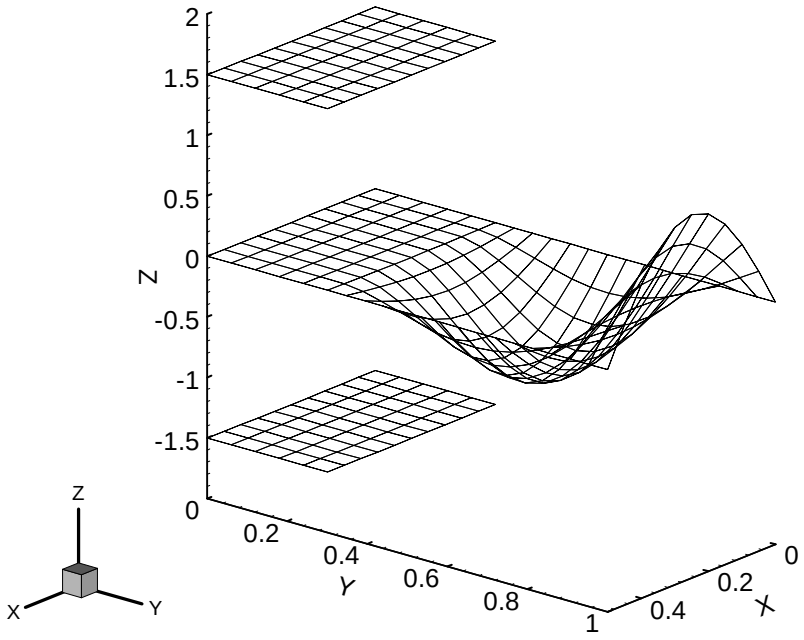
a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 14.074$



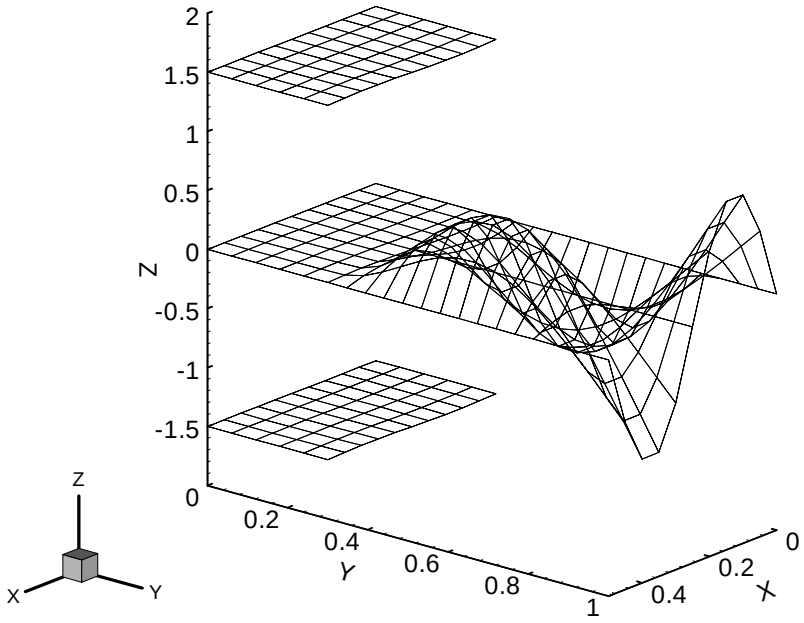
b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{21} = 142.166$

Figure 11.166. Mode Shapes and Natural Frequencies of "Doubly Stiffened-at Support, Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFFFFF) B.C.'s in y-direction, "Hard" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m.)

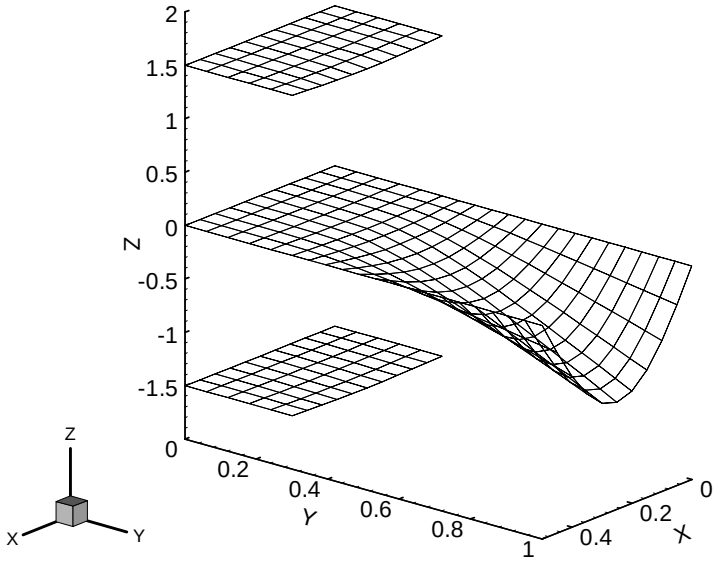


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{12} = 165.185$

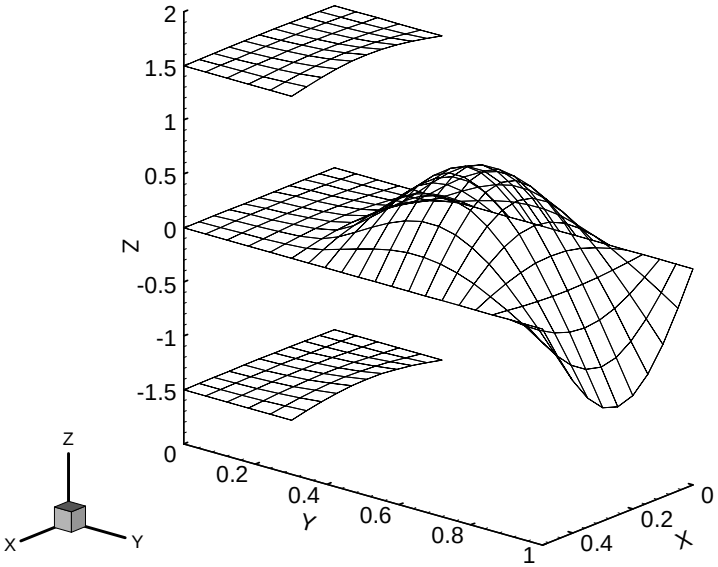


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{22} = 353.822$

Figure 11.166. (continued)



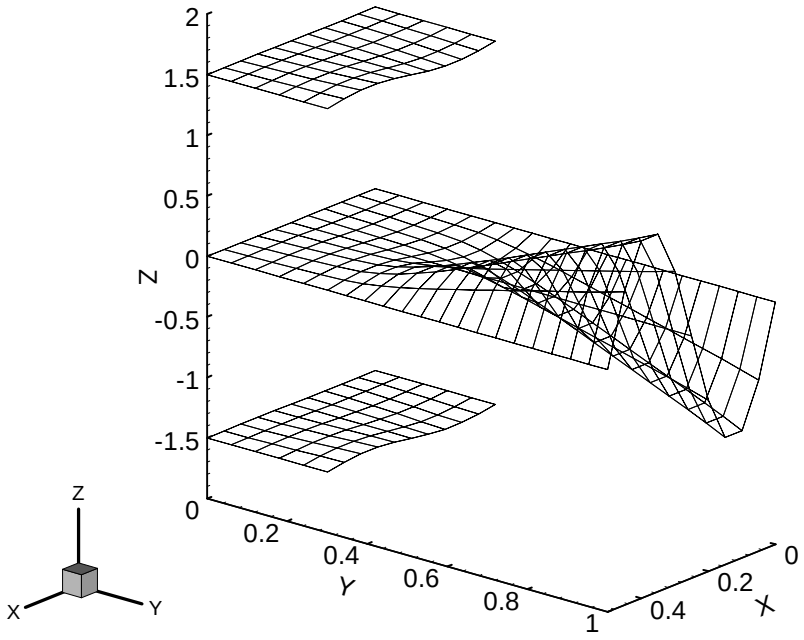
a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 13.127$



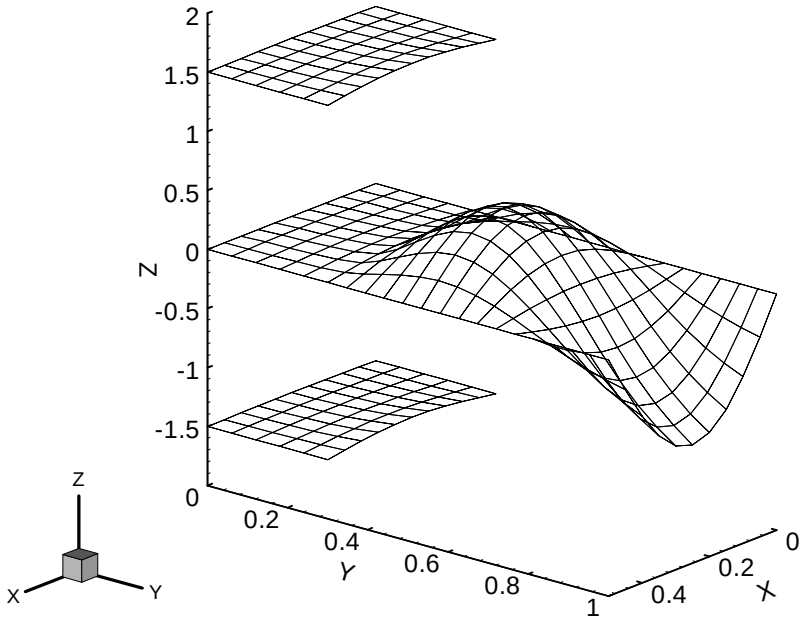
b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{12} = 120.654$

Figure 11.167. Mode Shapes and Natural Frequencies of "Doubly Stiffened-at Support, Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFFFFF) B.C.'s in y-direction, "Soft" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m.)



c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{21} = 140.580$



d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{13} = 289.911$

Figure 11.167. (continued)

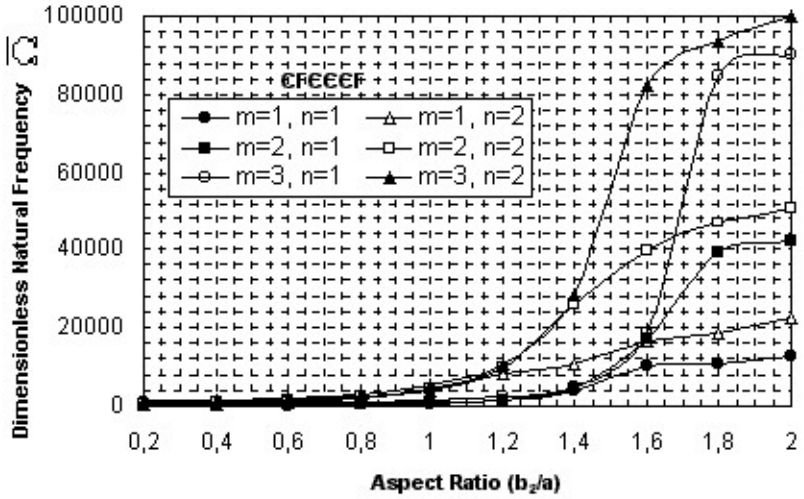


Figure 11.168. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a (“Aspect Ratio up to 2.0”)’’ in “Doubly Stiffened-at Support Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on the number of half waves in y- and x-directions)
((CFCCCF) B.C.’s, in y-direction, “Hard” Adhesive, b_2 =varies, $\tilde{b}$ =varies, a=0.5 m, L=1.0 m)

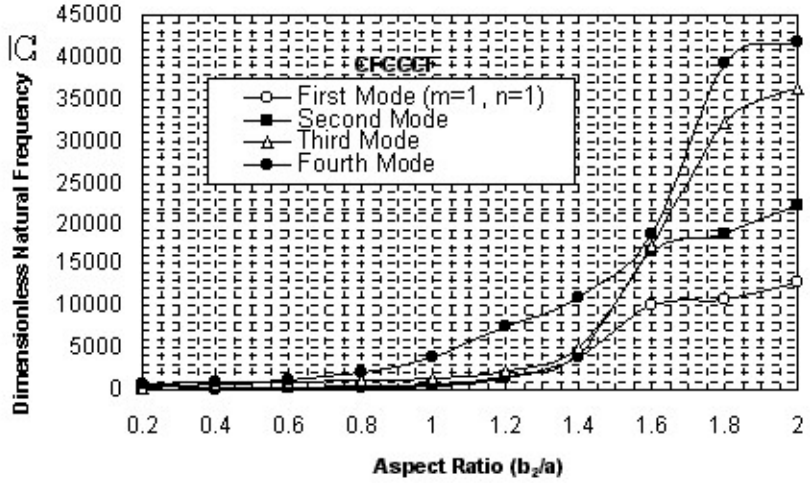


Figure 11.169. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a (“Aspect Ratio up to 2.0”)’’ in “Doubly Stiffened-at Support Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (CFCCCF) B. C.’s, ‘‘Hard” Adhesive,
b_2 =varies, $\tilde{b}$ =varies, a=0.5 m, L=1.0 m)

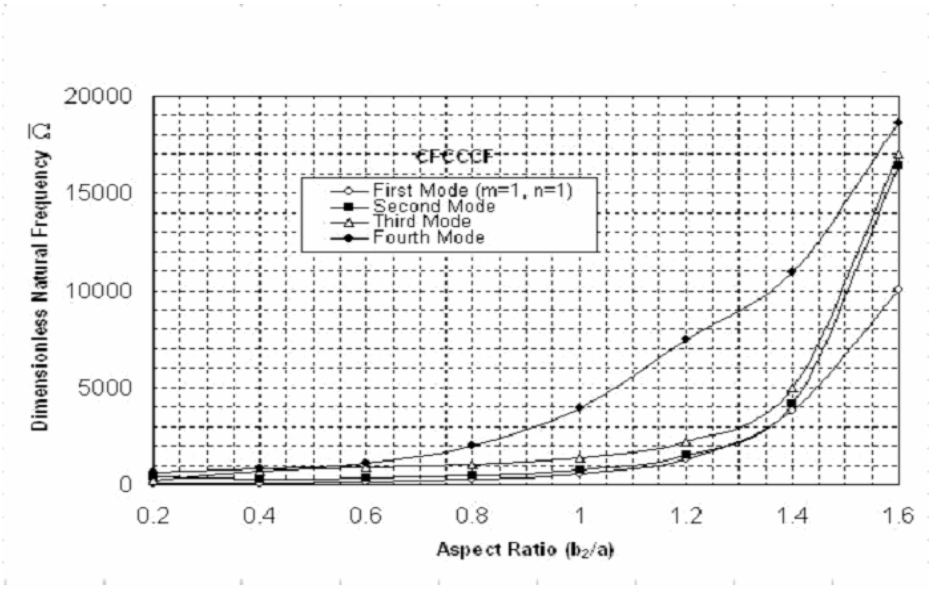


Figure 11.170. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a (“Aspect Ratio up to 1.6”)’’ in “Doubly Stiffened-at Support Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)” (Various Modes with (CFCCCF) B. C.’s, ‘‘Hard” Adhesive, b_2 =varies, $\tilde{b}$ =varies, a=0.5 m, L=1.0 m)

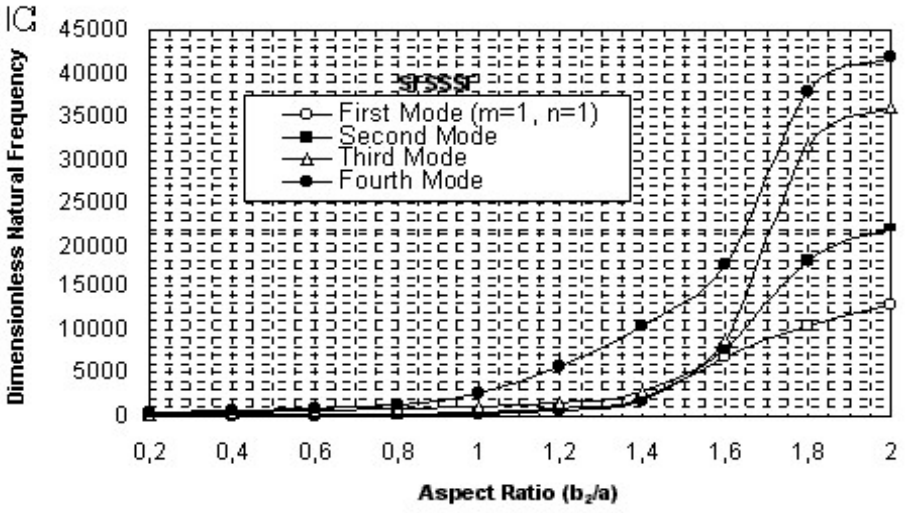


Figure 11.171. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a (“Aspect Ratio up to 2.0”)’’ in “Doubly Stiffened-at Support Composite Orthotropic Base Plate or
Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (SFSSSF) B. C.’s, ‘‘Hard” Adhesive,
b_2 =varies, $\tilde{b}$ =varies, a=0.5 m, L=1.0 m)

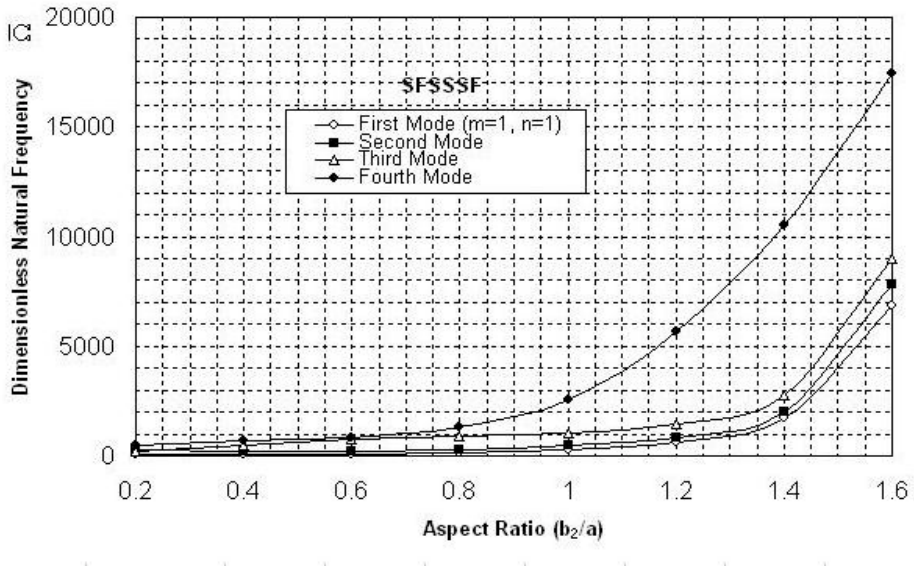


Figure 11.172. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a ("Aspect Ratio up to 1.6")'' in "Doubly Stiffened-at Support Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)" (Various Modes with (SFSSSF) B. C.'s, ''Hard" Adhesive, b_2 =varies, $\tilde{b}$ =varies, a=0.5 m, L=1.0 m)

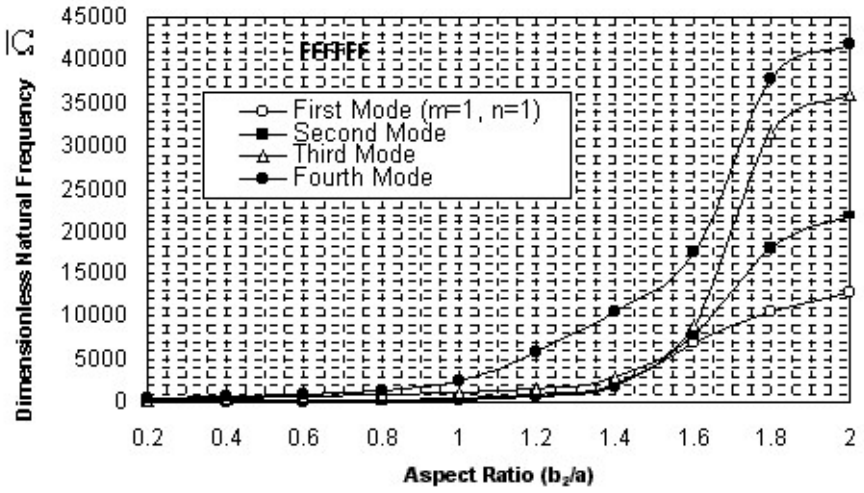


Figure 11.173. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a ("Aspect Ratio up to 2.0")'' in "Doubly Stiffened-at Support Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)" (Various Modes with (FFFFFF) B. C.'s, ''Hard" Adhesive, b_2 =varies, $\tilde{b}$ =varies, a=0.5 m, L=1.0 m)

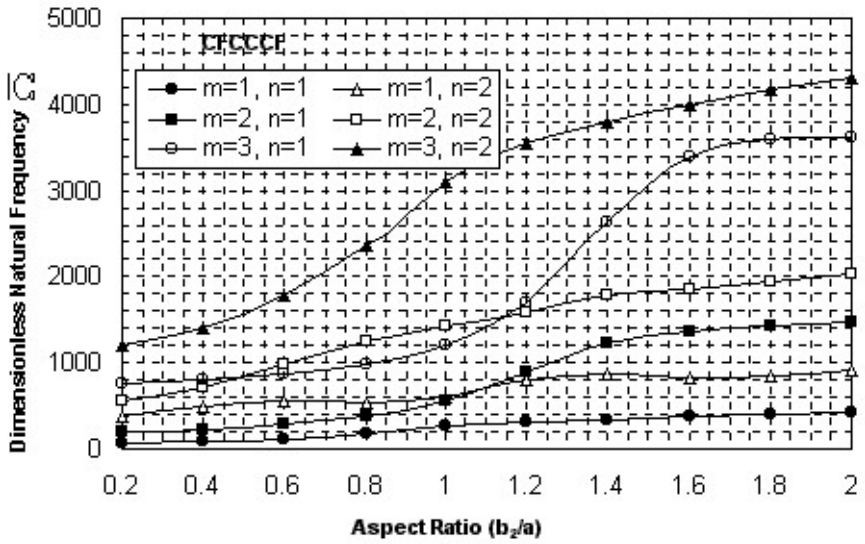


Figure 11.174. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a (“Aspect Ratio up to 2.0”)’’ in “Doubly Stiffened-at Support Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on the number of half waves in y- and x-directions)
((CFCCCF) B.C.’s, in y-direction, “Soft” Adhesive, b_2 =varies, $\tilde{b}$ =varies, a=0.5 m, L=1.0 m)

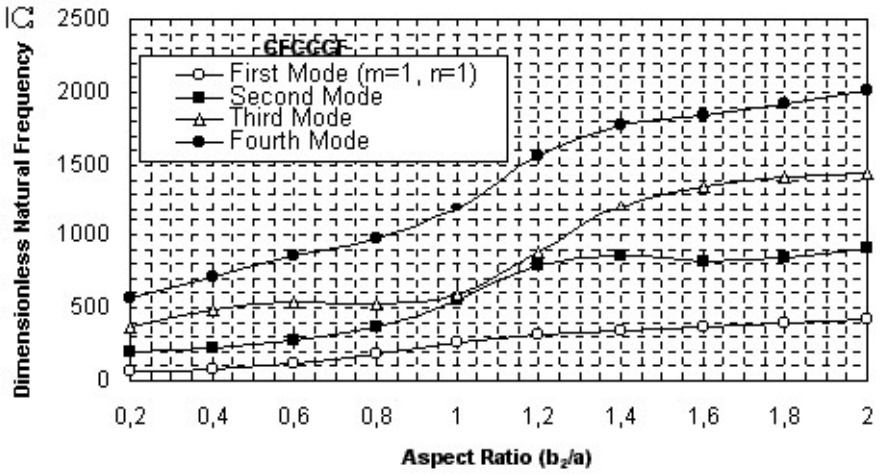


Figure 11.175. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a (“Aspect Ratio up to 2.0”)’’ in “Doubly Stiffened-at Support Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Various Modes with (CFCCCF) B. C.’s, ‘‘Soft” Adhesive,
b_2 =varies, $\tilde{b}$ =varies, a=0.5 m, L=1.0 m)

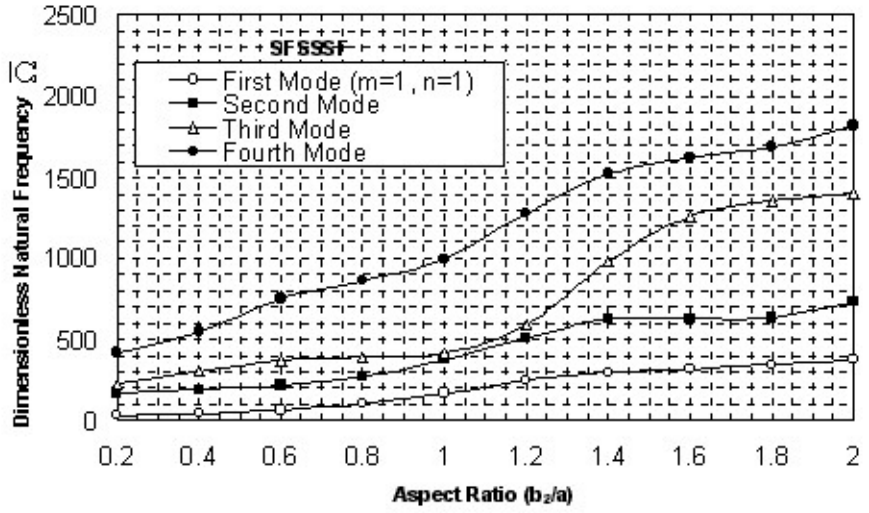


Figure 11.176. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a (“Aspect Ratio up to 2.0”)’’ in “Doubly Stiffened-at Support Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)” (Various Modes with (SFSSSF) B. C.’s, ‘‘Soft” Adhesive, b_2 =varies, $\tilde{b}$ =varies, a=0.5 m, L=1.0 m)

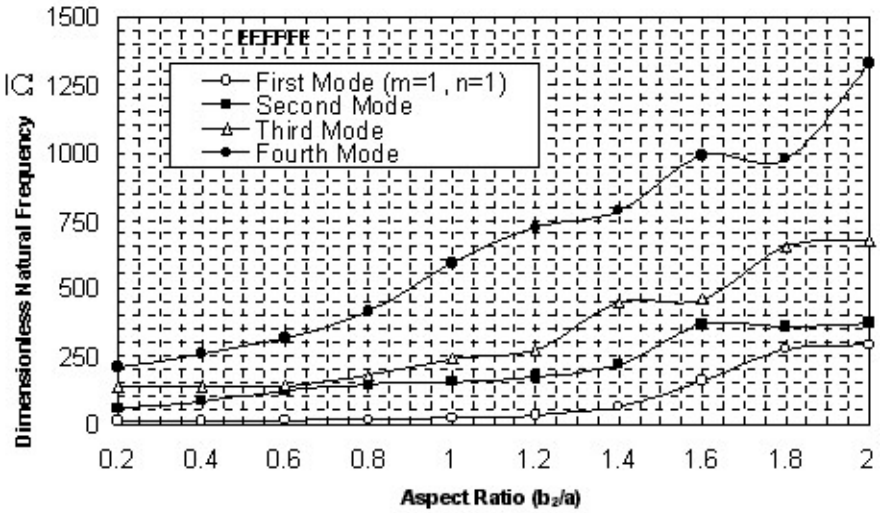


Figure 11.177. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Aspect Ratio b_2/a (“Aspect Ratio up to 2.0”)’’ in “Doubly Stiffened-at Support Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)” (Various Modes with (FFFFFF) B. C.’s, ‘‘Soft” Adhesive, b_2 =varies, $\tilde{b}$ =varies, a=0.5 m, L=1.0 m)

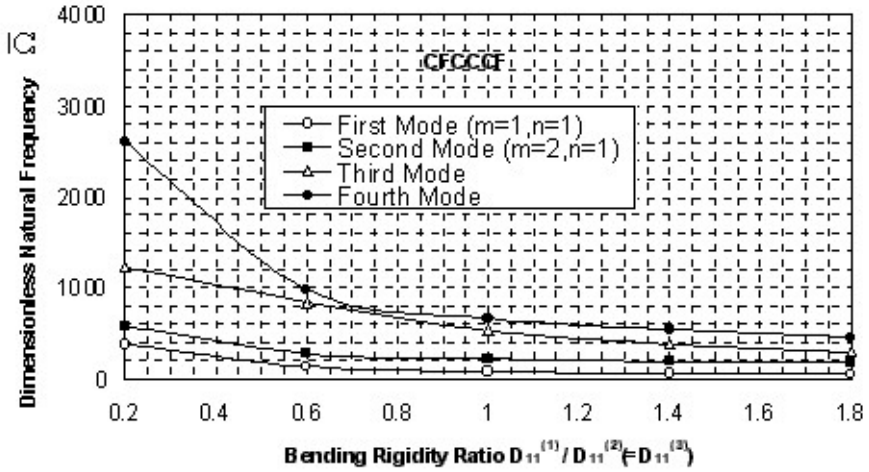


Figure 11.178. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})$’’ in “Doubly Stiffened-at Support Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)” (Various Modes with (CFCCCF) B. C.’s, ‘‘Hard” Adhesive, b_2 =0.30 m, $\tilde{b}$ =0.15 m, a=0.5 m, L=1.0 m)

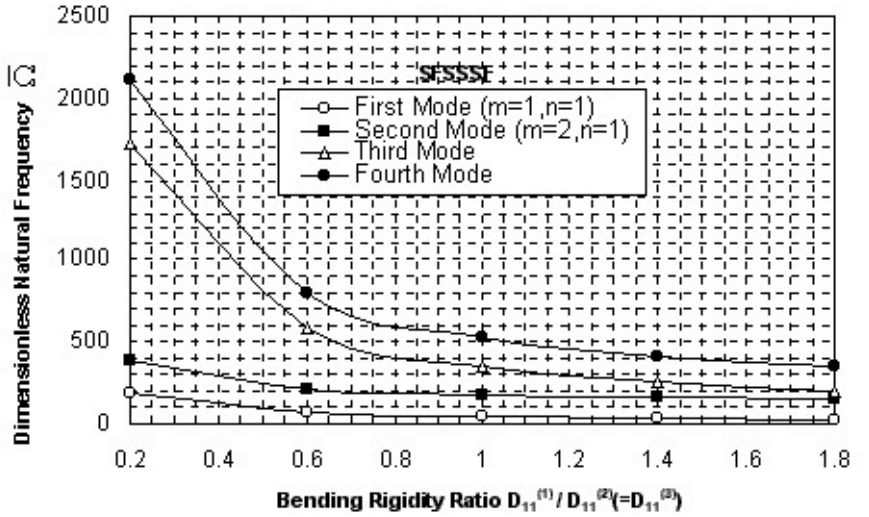


Figure 11.179. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})$’’ in “Doubly Stiffened-at Support Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)” (Various Modes with (SFSSSF) B. C.’s, ‘‘Hard” Adhesive, b_2 =0.30 m, $\tilde{b}$ =0.15 m, a=0.5 m, L=1.0 m)

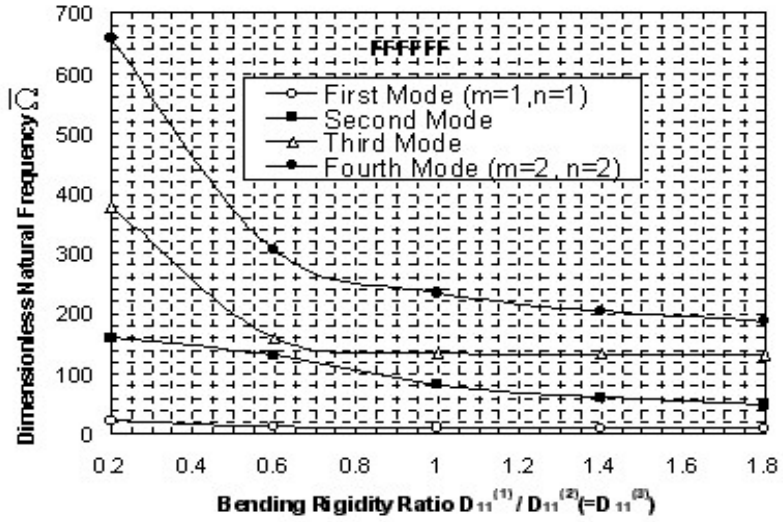
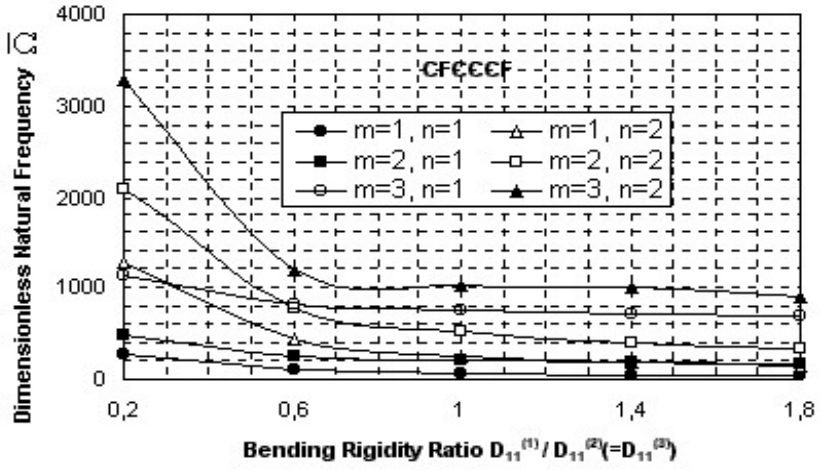


Figure 11.180. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$’’ in “Doubly Stiffened-at Support Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)” (Various Modes with (FFFFFF) B. C.’s, ‘‘Soft” Adhesive, b_2=0.30 m, $\tilde{b}$ =0.15 m, a=0.5 m, L=1.0 m)



11.181. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$’’ in “Doubly Stiffened-at Support Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)” (Dependency of natural frequency $\overline{\Omega}=\overline{\omega}_{mn}$ on the number of half waves in y- and x-directions)
((CFCCCF) B.C.’s, in y-direction, “Soft” Adhesive, b_2 =varies, $\tilde{b}$ =varies, a=0.5 m, L=1.0 m)

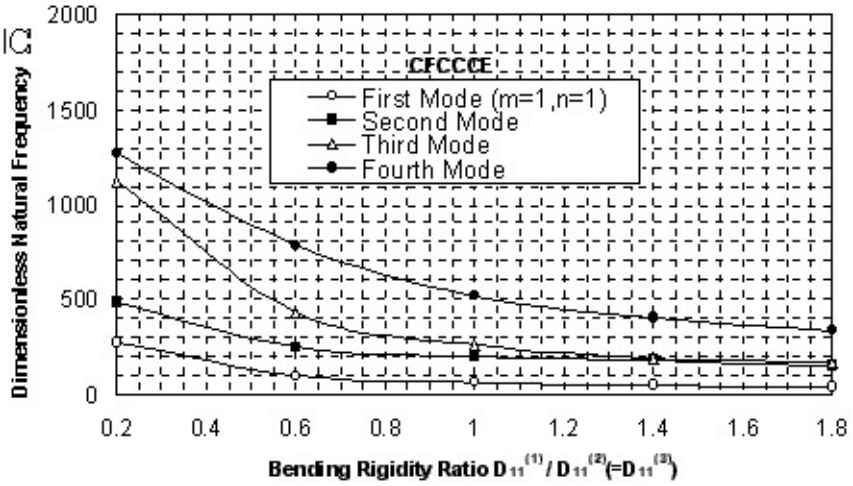


Figure 11.182. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$'' in "Doubly Stiffened-at Support Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)" (Various Modes with (CFCCCF) B.C.'s, ''Soft" Adhesive, b_2=0.30 m, $\tilde{b}$ =0.15 m, a=0.5 m, L=1.0 m)

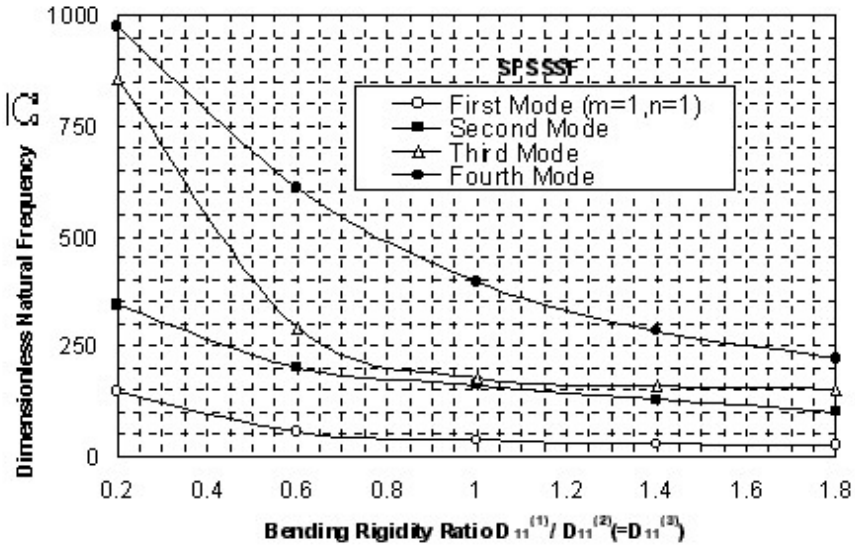


Figure 11.183. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$'' in "Doubly Stiffened-at Support Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)" (Various Modes with (SFSSSF) B.C.'s, ''Soft" Adhesive, b_2=0.30 m, $\tilde{b}$ =0.15 m, a=0.5 m, L=1.0 m)

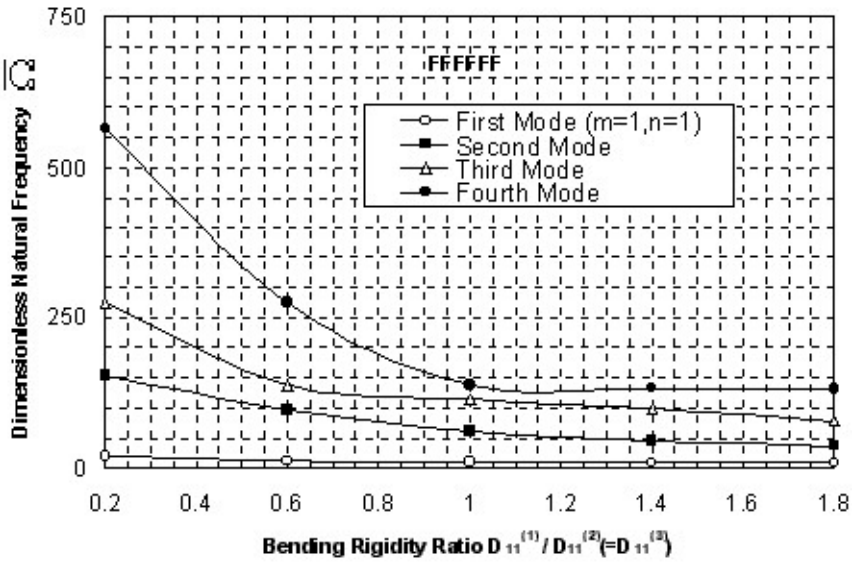


Figure 11.184. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$’’ in “Doubly Stiffened-at Support Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)” (Various Modes with (FFFFFF) B.C.’s, ‘‘Soft” Adhesive, b_2=0.30 m, $\tilde{b}$ =0.15 m, a=0.5 m, L=1.0 m)

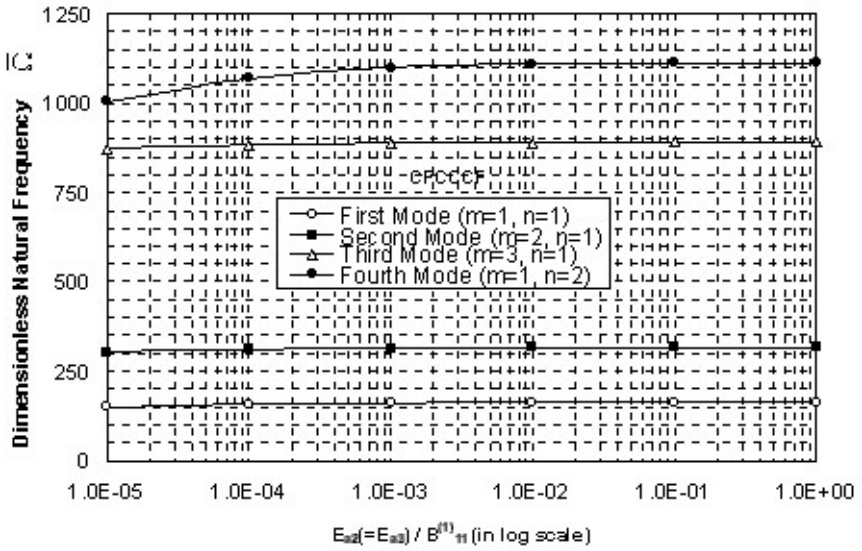


Figure 11.185. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$'' in "Doubly Stiffened-at Support Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Various Modes with (CFCCCF) B.C.'s, b_2 =0.30 m, $\tilde{b}$ =0.15 m, a=0.5 m, L=1.0 m)

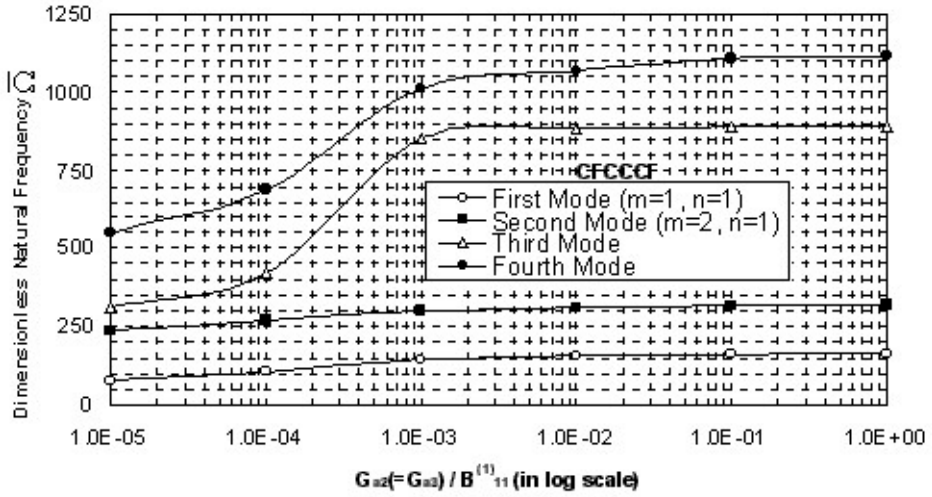


Figure 11.186. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3})/B_{11}^{(1)}$'' in "Doubly Stiffened-at Support Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Various Modes with (CFCCCF) B.C.'s, b_2 =0.30 m, $\tilde{b}$ =0.15 m, a=0.5 m, L=1.0 m)

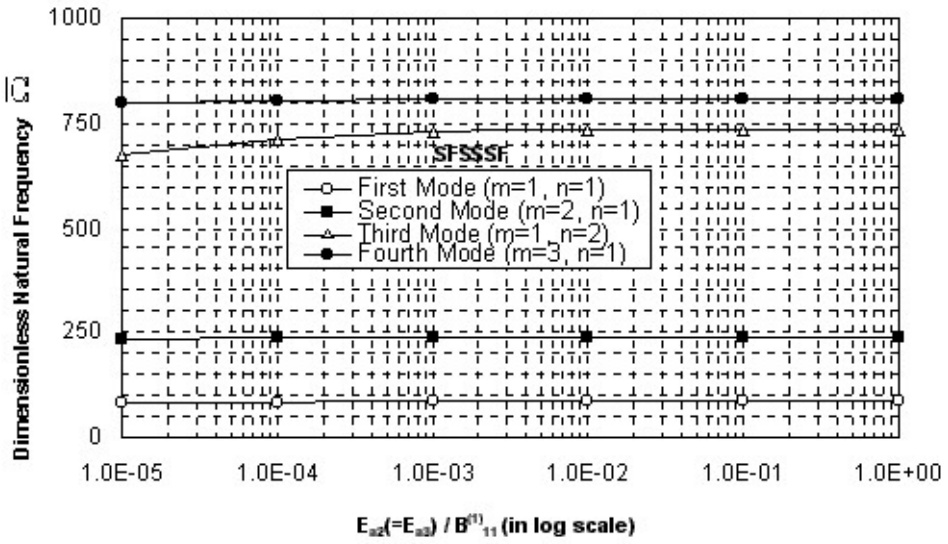


Figure 11.187. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$'' in "Doubly Stiffened-at Support Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Various Modes with (SFSSSF) B.C.'s, b_2 =0.30 m, $\tilde{b}$ =0.15 m, a=0.5 m, L=1.0 m)

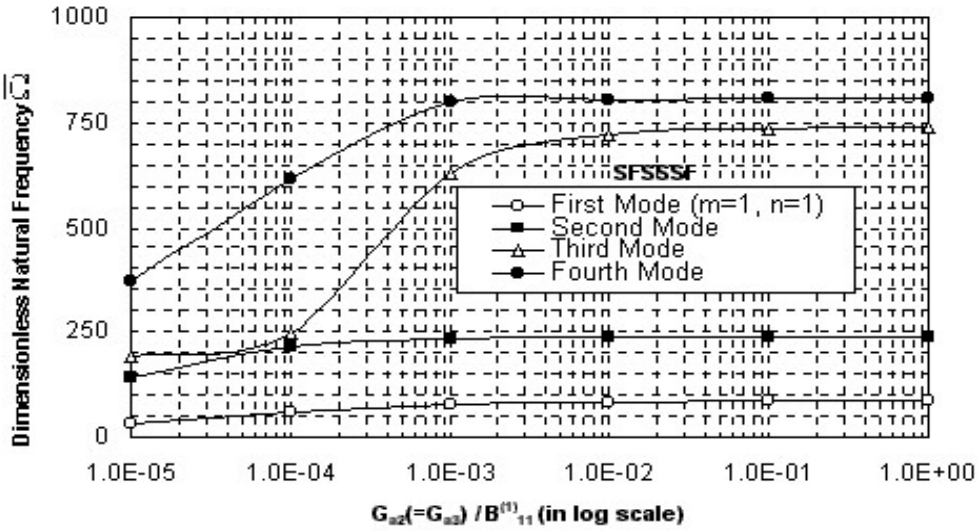


Figure 11.188. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3})/B_{11}^{(1)}$'' in "Doubly Stiffened-at Support Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Various Modes with (SFSSSF) B.C.'s, b_2 =0.30 m, $\tilde{b}$ =0.15 m, a=0.5 m, L=1.0 m)

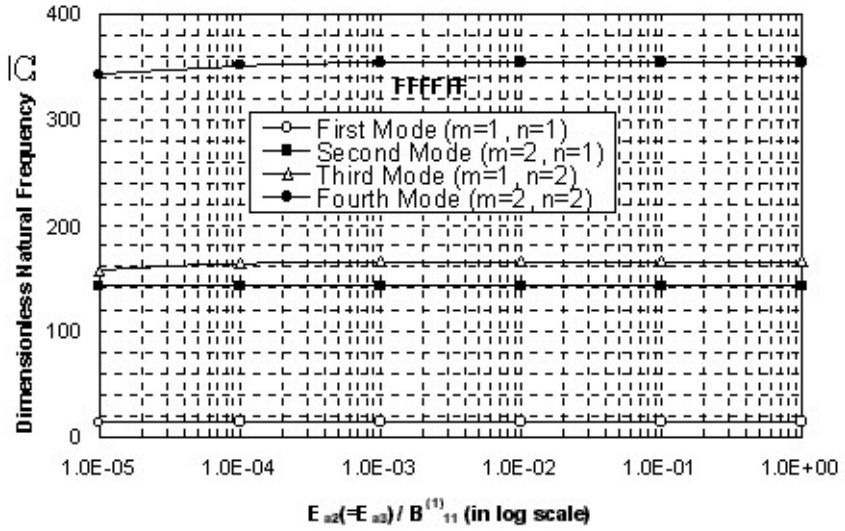


Figure 11.189. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$'' in "Doubly Stiffened-at Support Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Various Modes with (FFFFFF) B.C.'s, b_2 =0.30 m, $\tilde{b}$ =0.15 m, a=0.5 m, L=1.0 m)

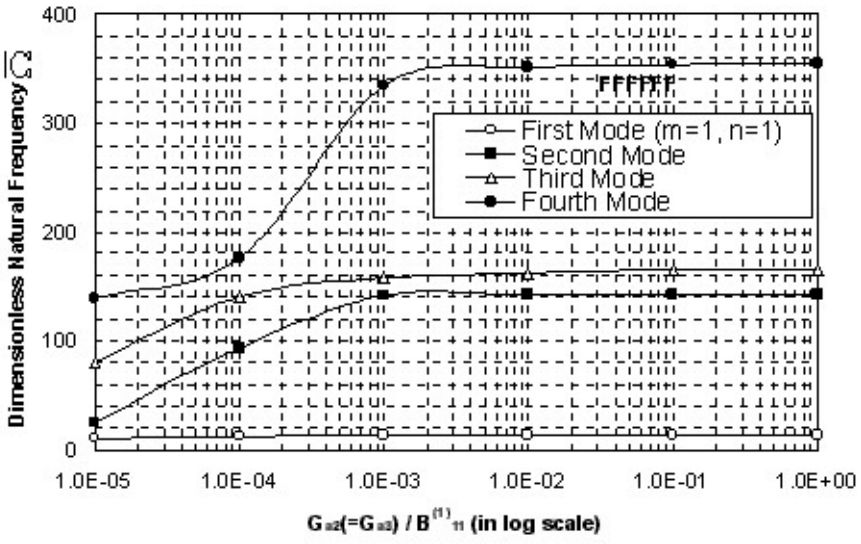


Figure 11.190. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3})/B_{11}^{(1)}$'' in "Doubly Stiffened-at Support Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Various Modes with (FFFFFF) B.C.'s, b_2 =0.30 m, $\tilde{b}$ =0.15 m, a=0.5 m, L=1.0 m)

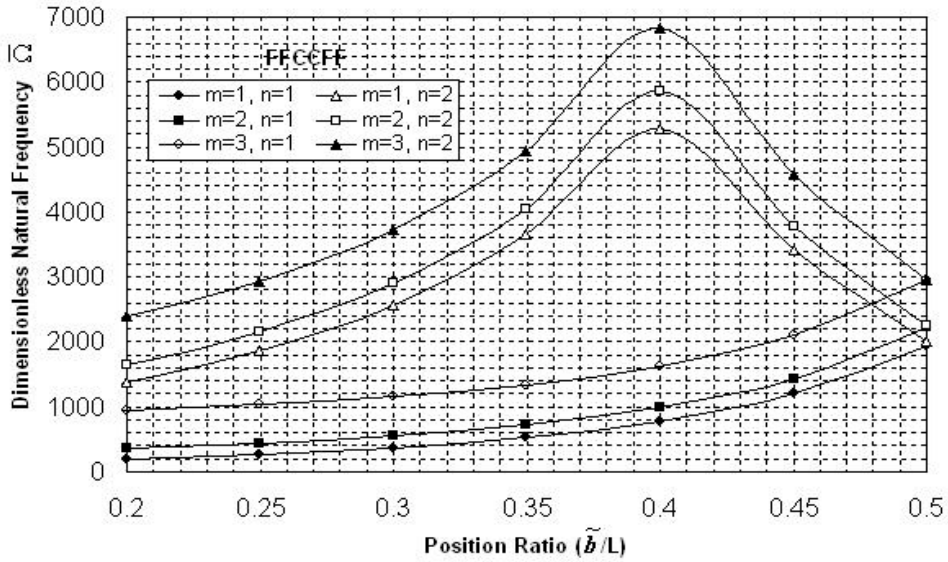


Figure 11.191. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Position Ratio of Stiffeners $\tilde{b}$ /L'' in "Doubly Stiffened Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on number of half waves in y- and x-direction)
(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFCCFF) B.C.'s in y-direction, ''Hard" Adhesive, $\tilde{b}$ =varies, a=0.50 m, L=1.0 m)

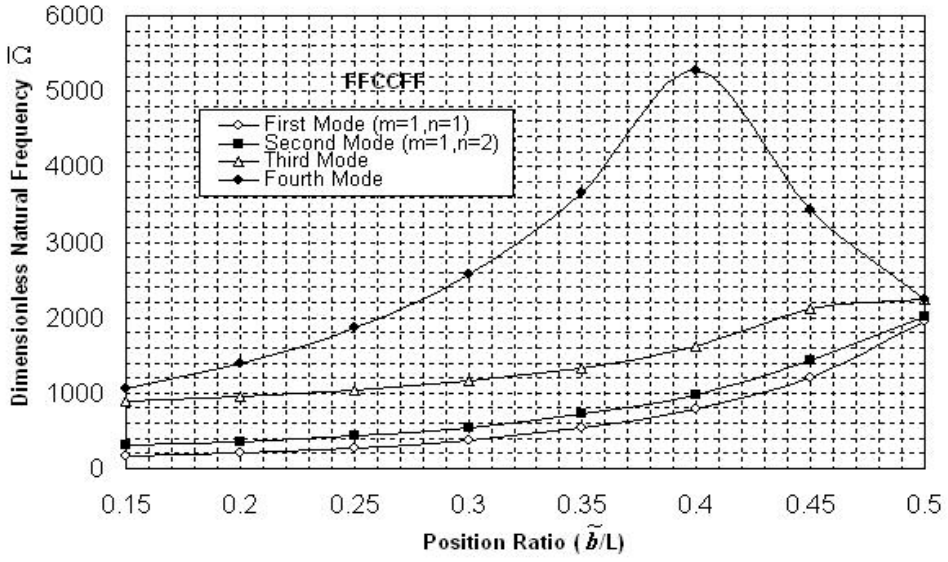


Figure 11.192. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Position Ratio of Stiffeners $\tilde{b}$ /L'' in "Doubly Stiffened Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
(Various Modes with (FFCCFF) B.C.'s in y-direction, ''Hard" Adhesive, $\tilde{b}$ =varies, a=0.50 m, L=1.0 m)

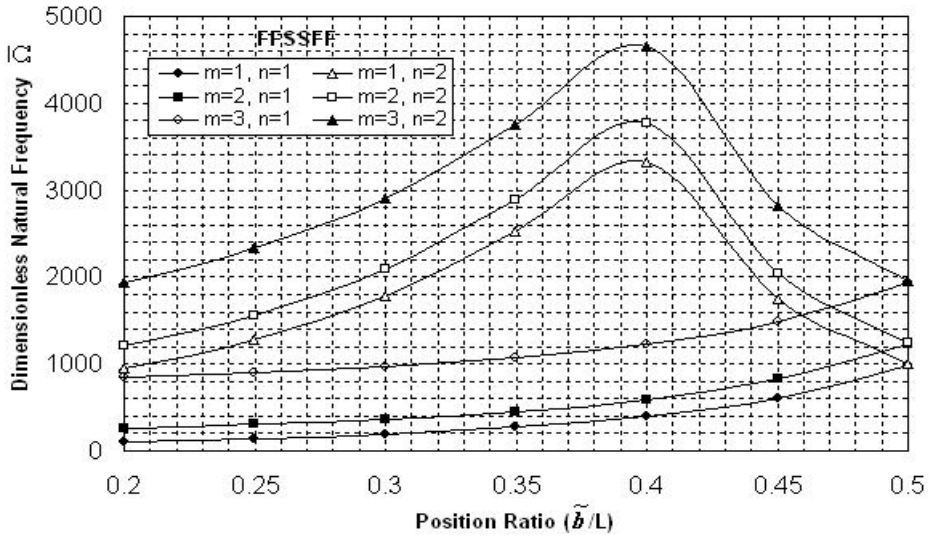


Figure 11.193. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Position Ratio of Stiffeners $\tilde{b}$ /L’’ in “Doubly Stiffened Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on number of half waves in y- and x-direction)
(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFSSFF) B.C.’s in y-direction, ‘‘Hard” Adhesive, $\tilde{b}$ =varies, a=0.50 m, L=1.0 m)

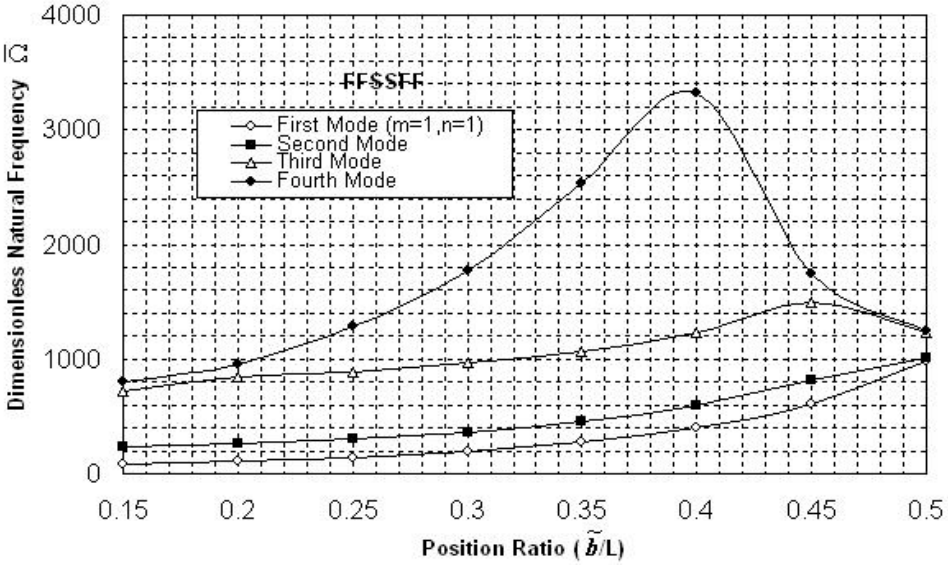


Figure 11.194. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Position Ratio of Stiffeners $\tilde{b}$ /L’’ in “Doubly Stiffened Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
(Various Modes with (FFSSFF) B.C.’s in y-direction, ‘‘Hard” Adhesive, $\tilde{b}$ =varies, a=0.50 m, L=1.0 m)

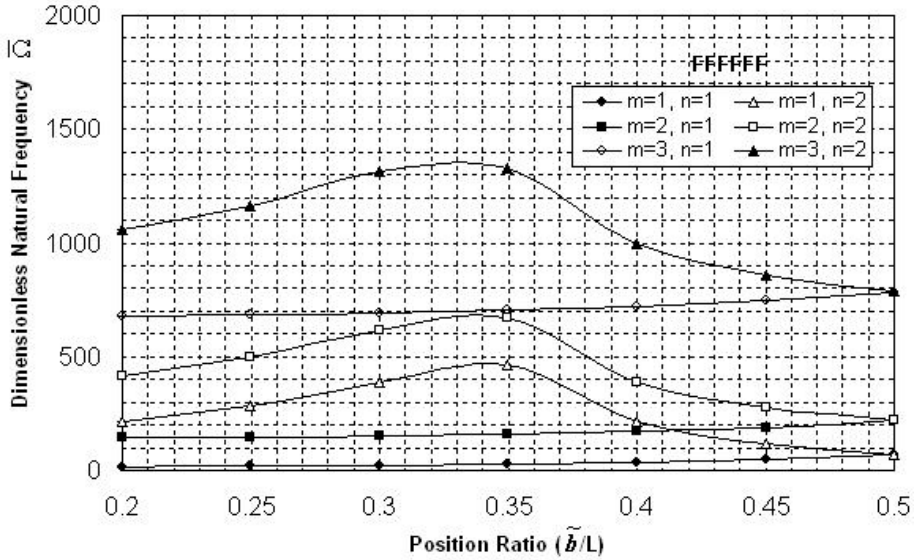


Figure 11.195. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Position Ratio of Stiffeners $\tilde{b}$ /L’’ in “Doubly Stiffened Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on number of half waves in y- and x-direction)
(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFFFFF) B.C.’s in y-direction, ‘‘Hard” Adhesive, $\tilde{b}$ =varies, a=0.50 m, L=1.0 m)

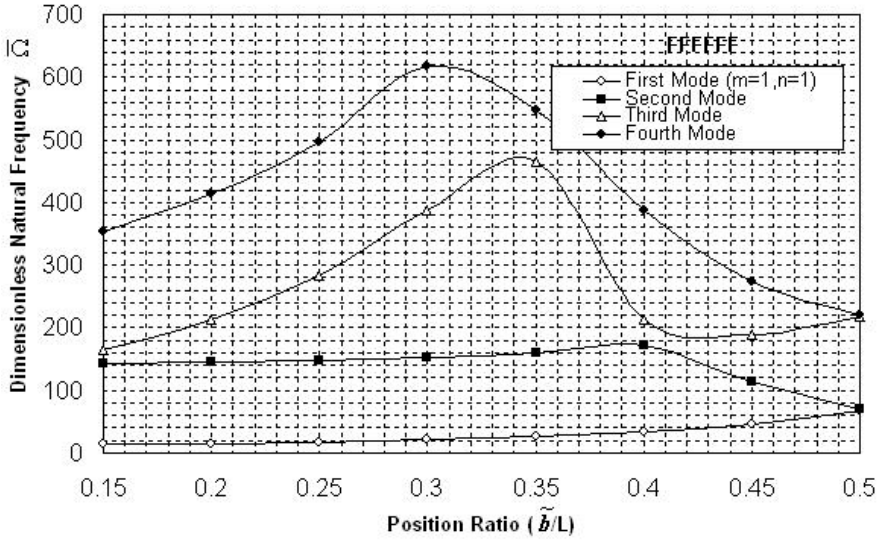


Figure 11.196. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Position Ratio of Stiffeners $\tilde{b}$ /L’’ in “Doubly Stiffened Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
(Various Modes with (FFFFFF) B.C.’s in y-direction, ‘‘Hard” Adhesive, $\tilde{b}$ =varies, a=0.50 m, L=1.0 m)

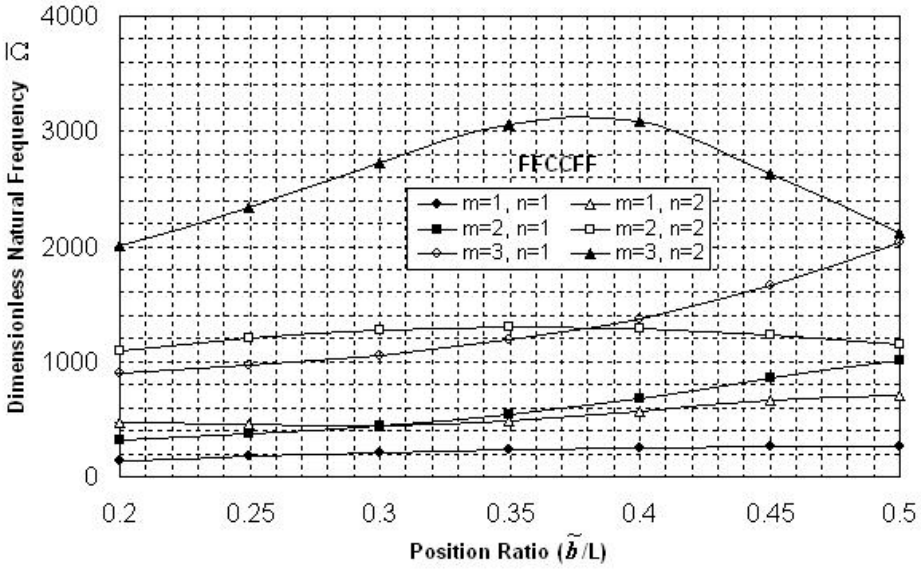


Figure 11.197. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Position Ratio of Stiffeners $\tilde{b}$ /L’’ in “Doubly Stiffened Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on number of half waves in y- and x-direction)
(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFCCFF) B.C.’s in y-direction, ‘‘Soft” Adhesive, $\tilde{b}$ =varies, a=0.50 m, L=1.0 m)

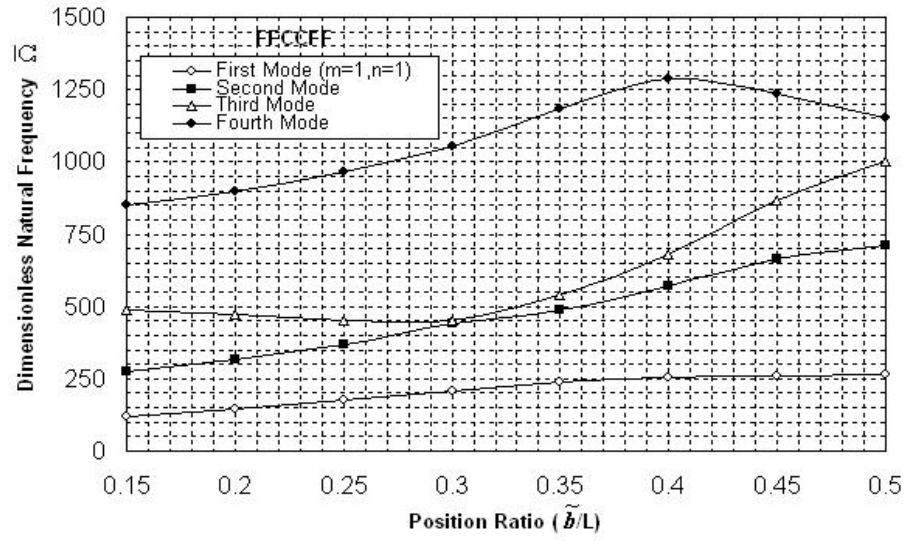


Figure 11.198. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Position Ratio of Stiffeners $\tilde{b}$ /L’’ in “Doubly Stiffened Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
(Various Modes with (FFCCFF) B.C.’s in y-direction, ‘‘Soft” Adhesive, $\tilde{b}$ =varies, a=0.50 m, L=1.0 m)

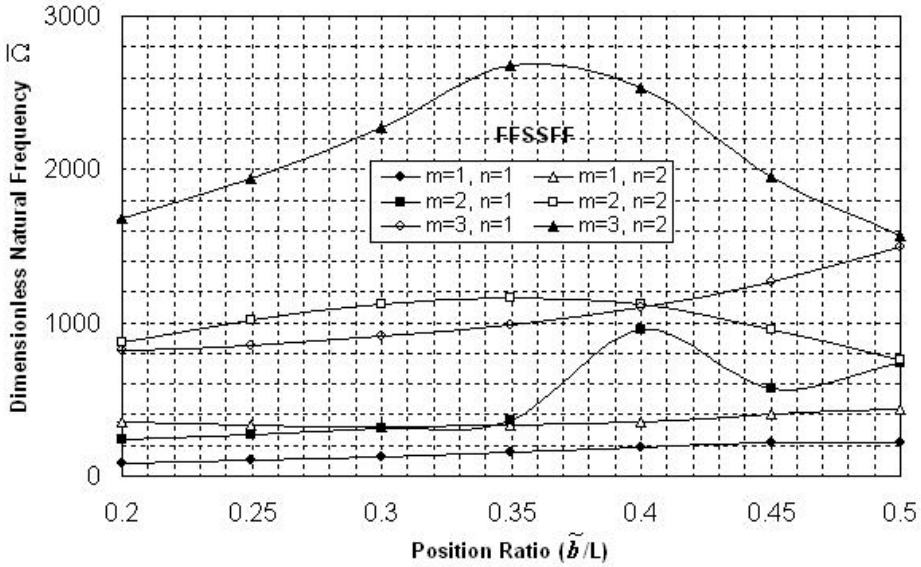


Figure 11.199. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Position Ratio of Stiffeners $\tilde{b}$/L’’ in “Doubly Stiffened Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on number of half waves in y- and x-direction)
(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFSSFF) B.C.’s in y-direction, ‘‘Soft” Adhesive, $\tilde{b}$ =varies, a=0.50 m, L=1.0 m)

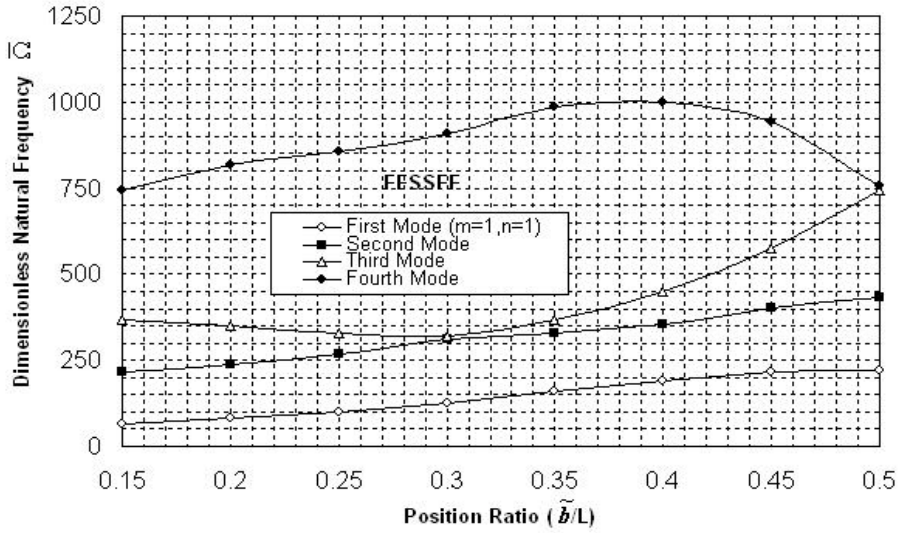


Figure 11.200. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Position Ratio of Stiffeners $\tilde{b}$/L’’ in “Doubly Stiffened Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
(Various Modes with (FFSSFF) B.C.’s in y-direction, ‘‘Soft” Adhesive, $\tilde{b}$ =varies, a=0.50 m, L=1.0 m)

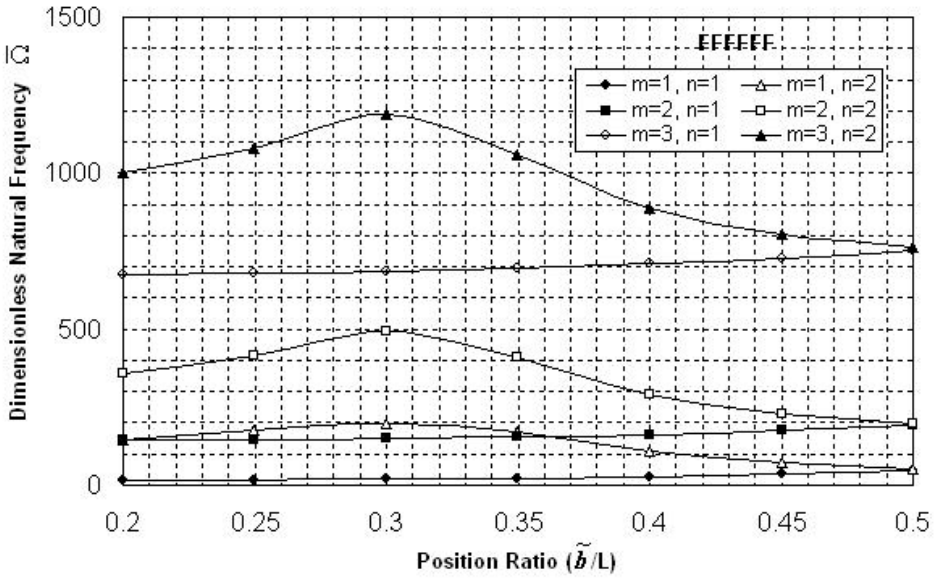


Figure 11.201. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Position Ratio of Stiffeners $\tilde{b}$/L’’ in “Doubly Stiffened Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on number of half waves in y- and x-direction)
(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFFFFF) B.C.’s in y-direction, ‘‘Soft” Adhesive, $\tilde{b}$ =varies, a=0.50 m, L=1.0 m)

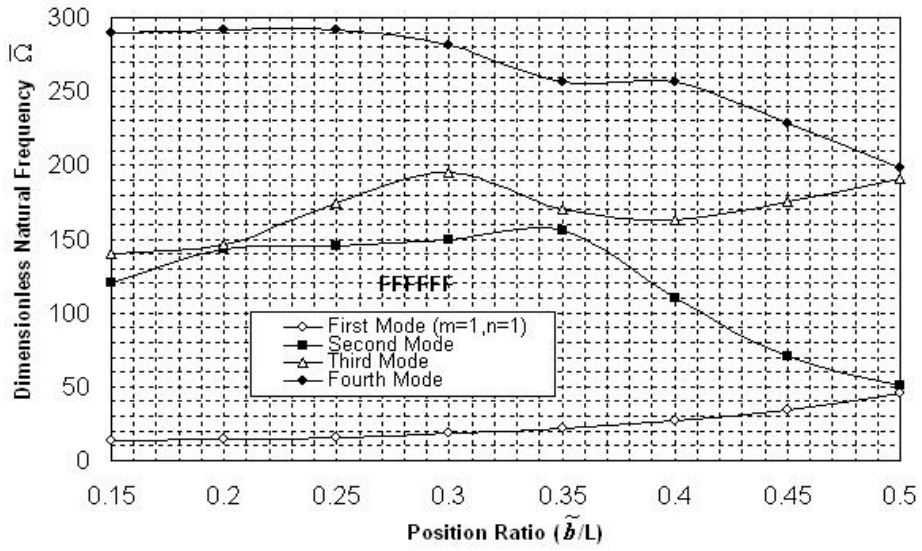


Figure 11.202. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Position Ratio of Stiffeners $\tilde{b}$/L’’ in “Doubly Stiffened Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)”
(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
(Various Modes with (FFFFFF) B.C.’s in y-direction, ‘‘Soft” Adhesive, $\tilde{b}$ =varies, a=0.50 m, L=1.0 m)

11.7 Numerical Results and Discussion for “Main PROBLEM III.b”

The “Isotropic Base Plate is made of Aluminum and “Upper and Lower Stiffening Plate Strips” are chosen as Graphite-Epoxy in this “Main PROBLEM III.b”.

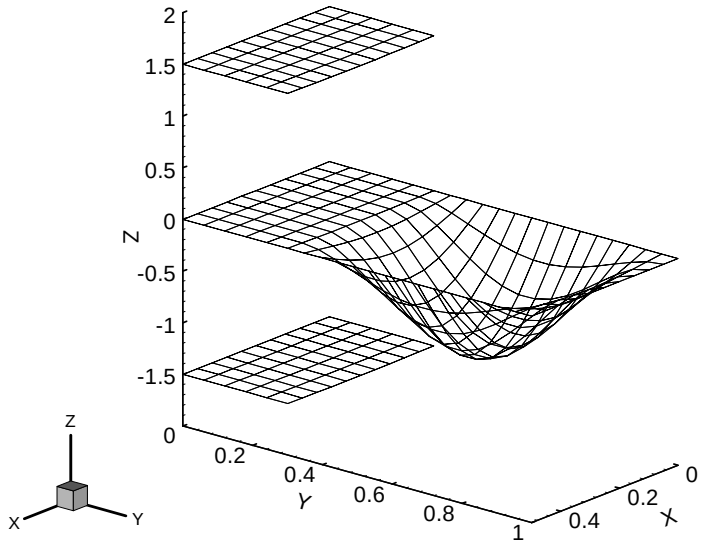
In Figures 11.203 and 11.204, the first four mode shapes with the corresponding dimensionless natural frequencies of the composite system of “Main PROBLEM III.b” for the “hard” and the ‘‘soft’’ adhesive layer cases with the (CFCCCF) boundary conditions are plotted

Similarly, in Figures 11.205 and 11.206, the same composite system of Figures 11.203 and 11.204 are now considered with (SFSSSF) support conditions. From the comparison of these figures, it is obvious that the mode shapes are essentially the same in both cases of the boundary conditions. However, boundary conditions change the natural frequencies drastically.

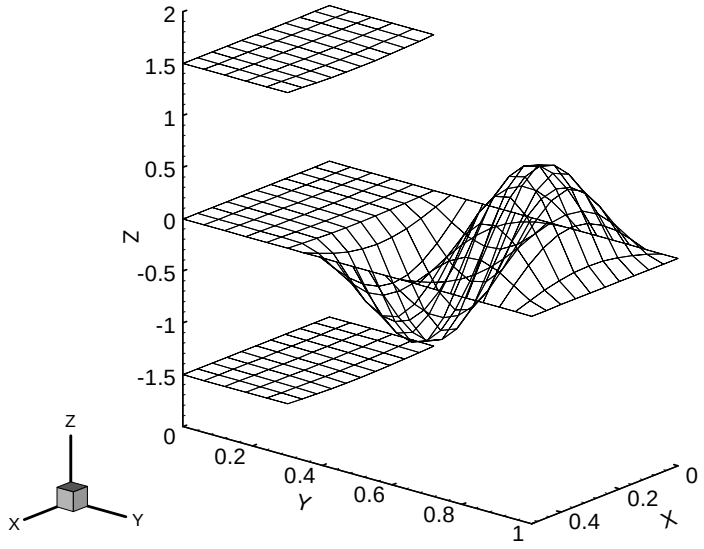
The parametric studies for “Main PROBLEM III.b” will be considered next.

In Figures 11.207 through 11.211 dimensionless natural frequency $\overline{\Omega}$ versus “Aspect Ratio b_2/a” are displayed for the ‘‘hard’’ adhesives case for different boundary conditions in y-direction. From the comparison of those Figures, it can be stated that dimensionless natural frequencies increase as the “Aspect Ratio b_2/a” increases regardless of the support conditions.

In Figures 11.212 through 11.217, the natural frequency parameter versus both “Adhesive Elastic Modulus and Adhesive Shear Modulus Ratios” are plotted in a logarithmic scale for various modes and boundary conditions. These curves are similar to those of the orthotropic case in the previous section (i.e. “Main PROBLEM III.a”). However, the natural frequencies are smaller than those for “Main PROBLEM III.a” as expected.



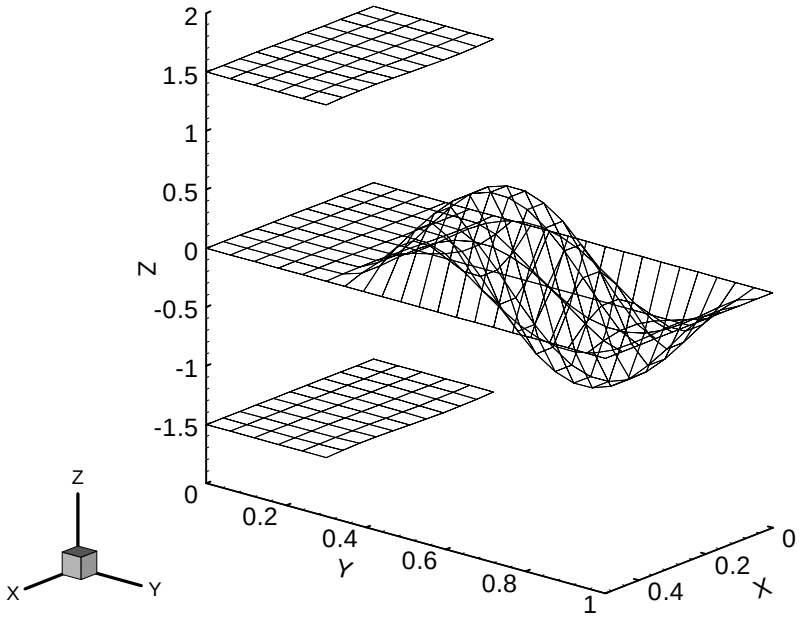
a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 28.285$



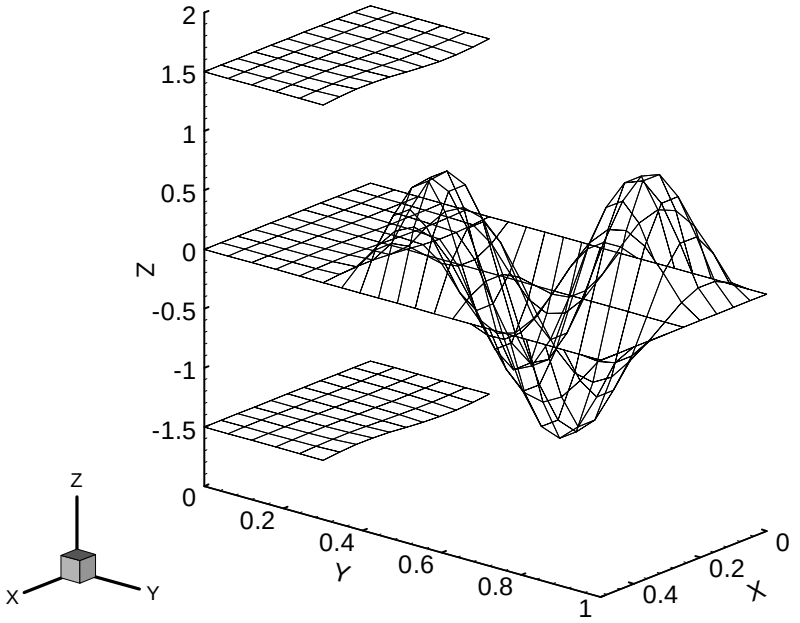
b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{12} = 124.607$

Figure 11.203. Mode Shapes and Natural Frequencies of "Doubly Stiffened-at Support, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Base Plate=Plate 1= Aluminum , Plate 2= Plate 3= Graphite-Epoxy)
((CFCCCF) B.C.'s in y-direction, "Hard" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m.)

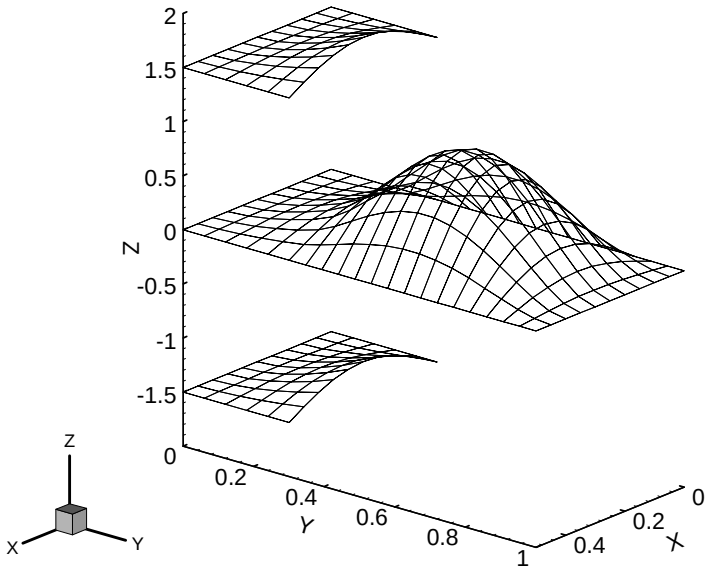


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{21} = 176.728$

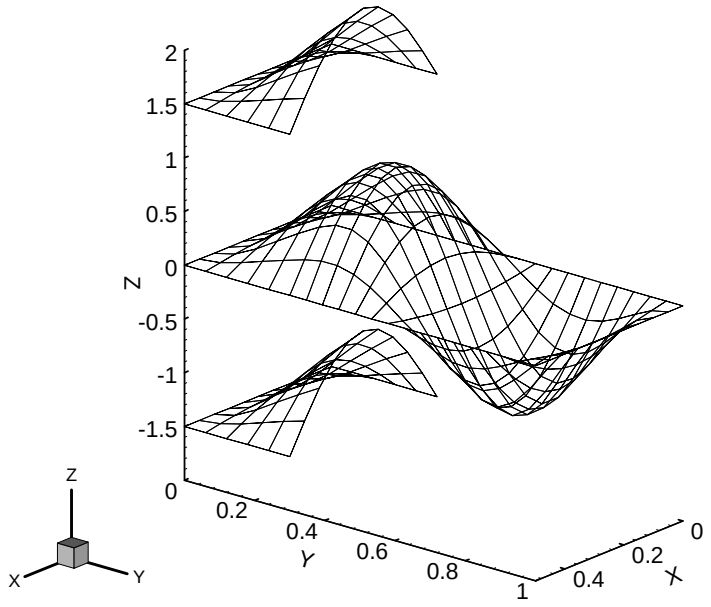


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{22} = 350.858$

Figure 11.203. (continued)



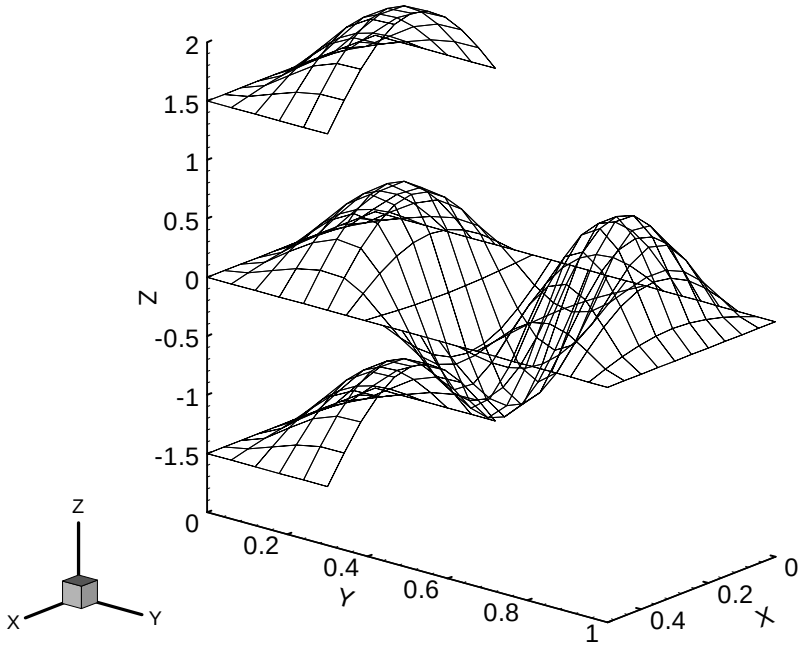
a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 22.181$



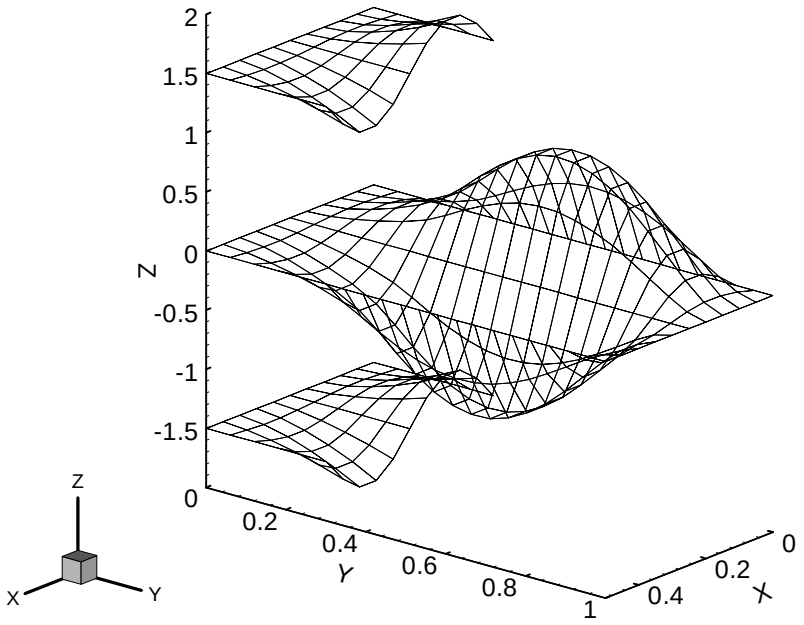
b) Second Mode with $\overline{\Omega}_{\mathbf{2}} = \overline{\omega}_{\mathbf{12}} = 67.383$

Figure 11.204 Mode Shapes and Natural Frequencies of "Doubly Stiffened-at Support, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Base Plate=Plate 1= Aluminum , Plate 2= Plate 3= Graphite-Epoxy)
((CFCCCF) B.C.'s in y-direction, "Soft" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m.)

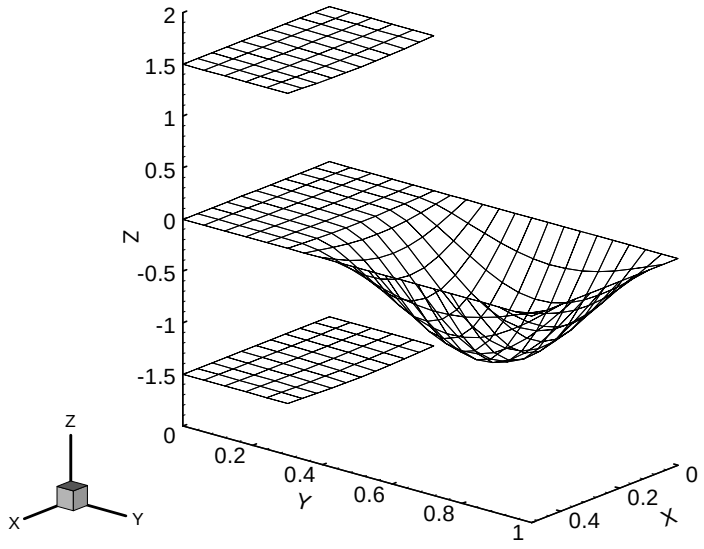


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{13} = 148.567$

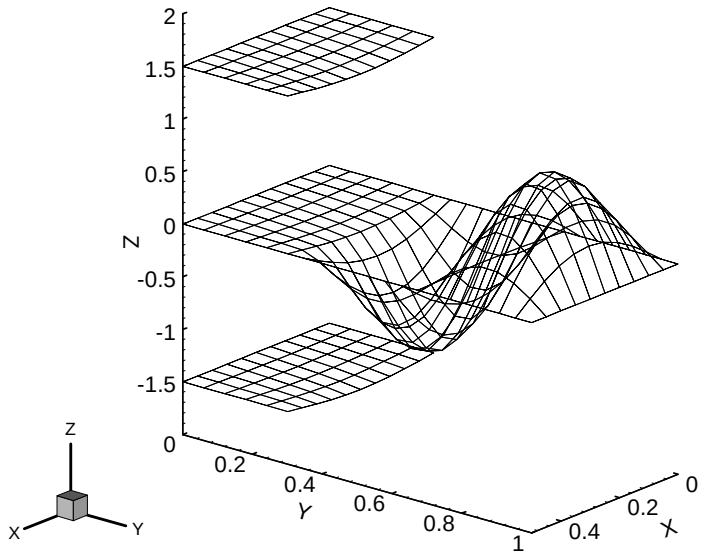


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{21} = 162.729$

Figure 11.204. (continued)



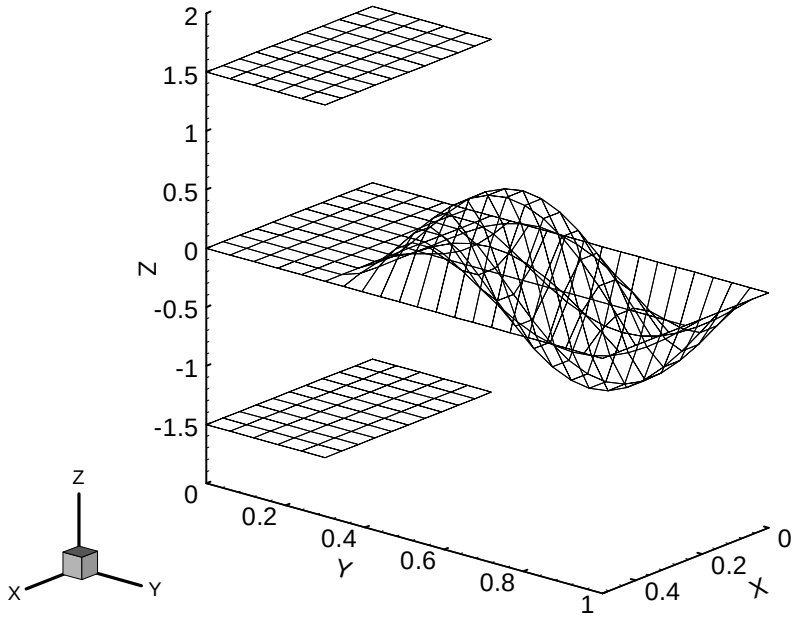
a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 21.812$



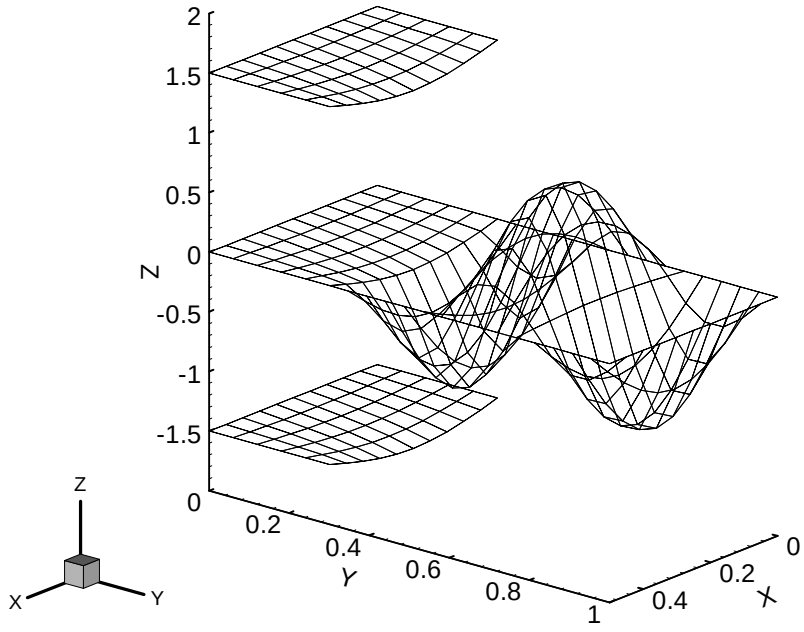
b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{12} = 93.036$

Figure 11.205. Mode Shapes and Natural Frequencies of "Doubly Stiffened-at Support, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Base Plate=Plate 1= Aluminum , Plate 2= Plate 3= Graphite-Epoxy)
((SFSSSF) B.C.'s in y-direction, "Hard" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m.)

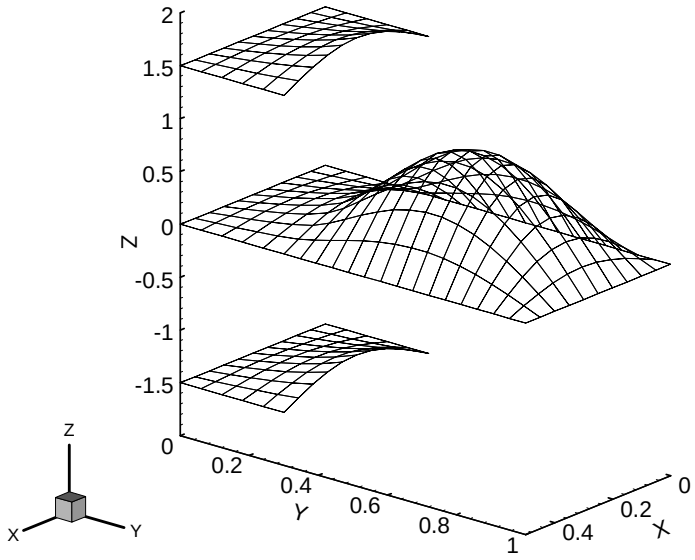


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{21} = 168.272$

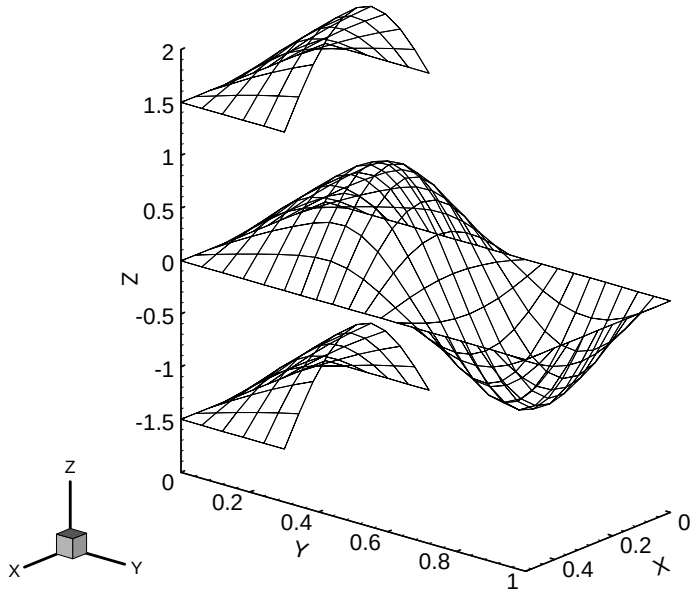


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{13} = 304.879$

Figure 11.205. (continued)



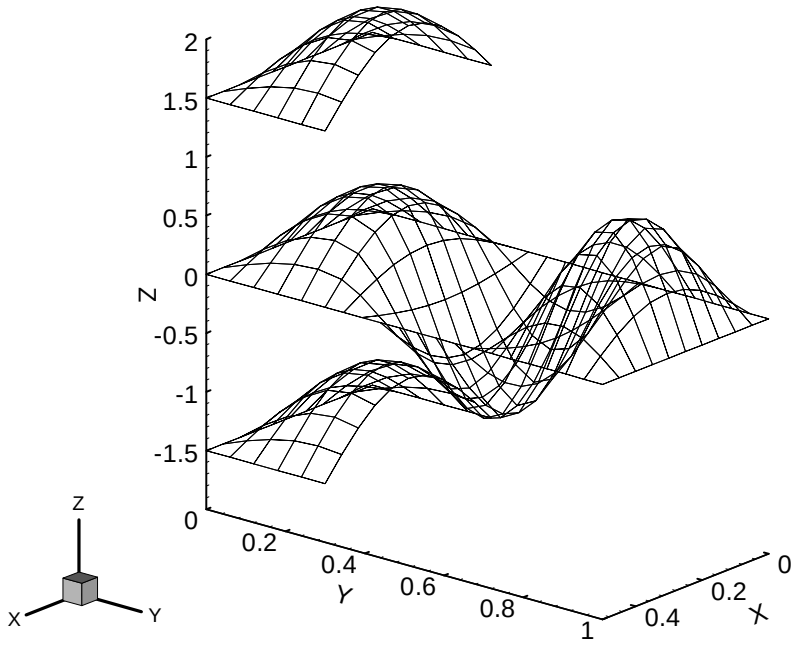
a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 18.057$



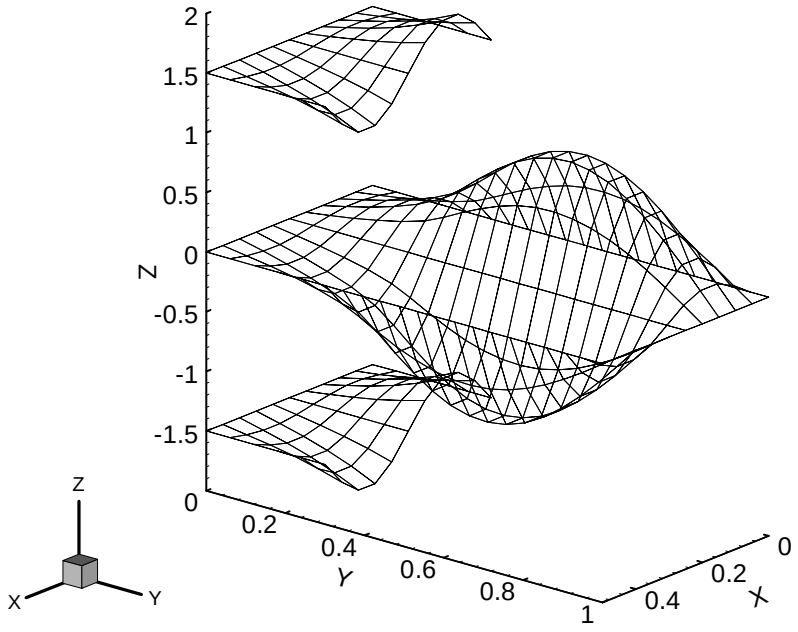
b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{12} = 50.414$

Figure 11.206 Mode Shapes and Natural Frequencies of "Doubly Stiffened-at Support, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Base Plate=Plate 1= Aluminum , Plate 2= Plate 3= Graphite-Epoxy)
((SFSSSF) B.C.'s in y-direction, "Soft" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m.)



c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{13} = 107.135$



d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{21} = 157.500$

Figure 11.206. (continued)

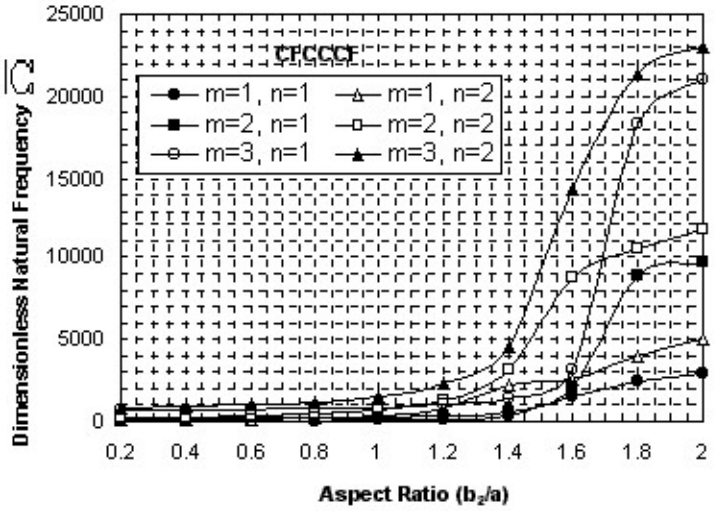


Figure 11.207. Dimensionless Natural Frequency. $\overline{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Doubly Stiffened-at Support, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on the number of half waves in y- and x-directions)

((CFCCCF) B.C.'s in y-direction, ''Hard" Adhesive, b_2= varies, $\tilde{b}$ =varies, a=0.50 m, L=1.0 m)

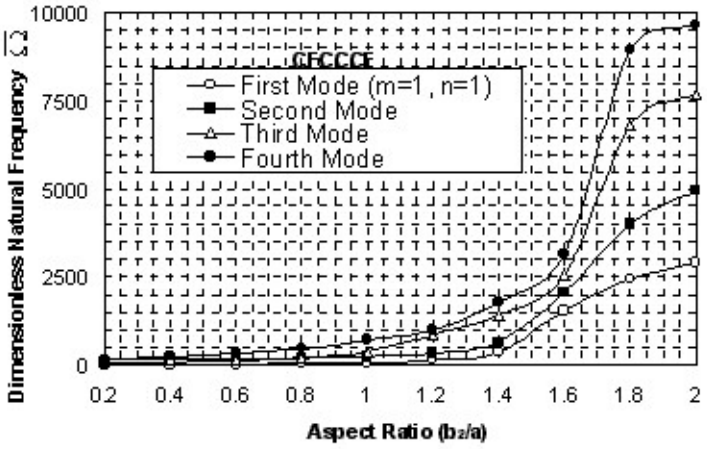


Figure 11.208. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Doubly Stiffened-at Support, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Various Modes with (CFCCCF) B.C.'s, ''Hard" Adhesive,

b_2= varies, $\tilde{b}$ =varies, a=0.50 m, L=1.0 m)

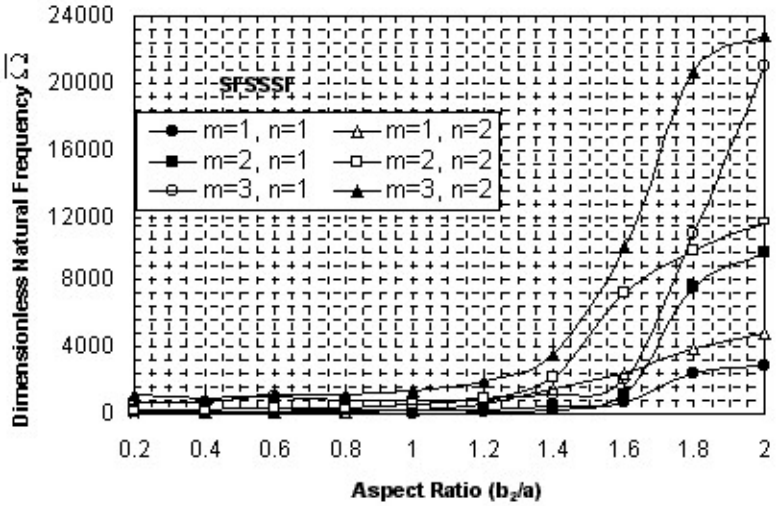


Figure 11.209. Dimensionless Natural Frequency. $\overline{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Doubly Stiffened-at Support, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Dependency of natural frequency $\overline{\Omega} = \overline{\omega}_{mn}$ on the number of half waves in y- and x-directions)

((SFSSSF) B.C.'s in y-direction, ''Hard" Adhesive, b_2=varies, $\tilde{b}$ =varies, a=0.50 m, L=1.0 m)

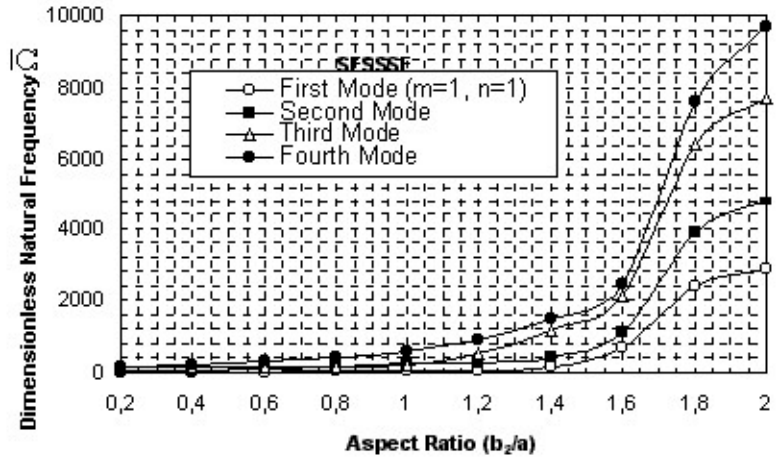


Figure 11.210. Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Doubly Stiffened-at Support, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Various Modes with (SFSSSF) B.C.'s, ''Hard" Adhesive, b_2=varies, $\tilde{b}$ =varies, a=0.50 m, L=1.0 m)

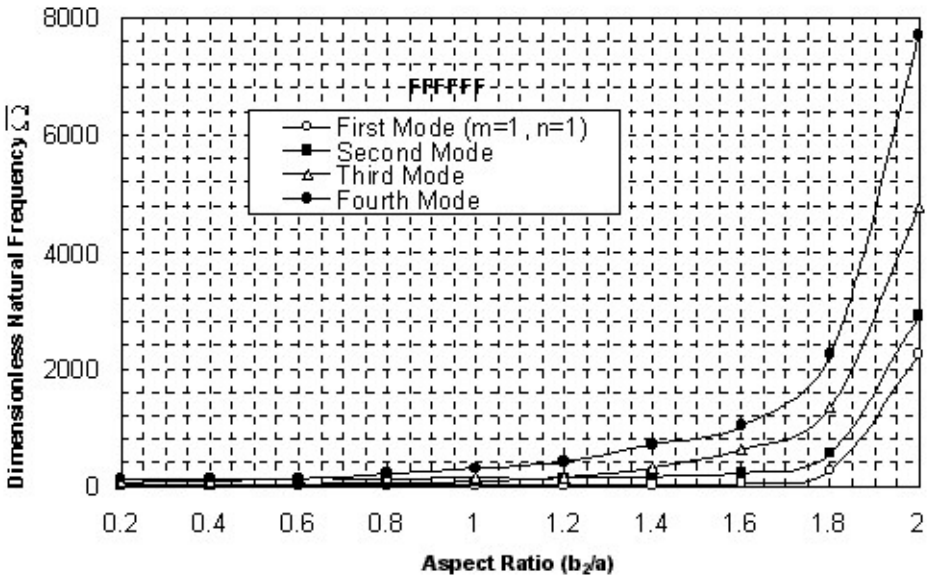


Figure 11.211 Dimensionless Natural Frequency $\overline{\Omega}$ versus ''Aspect Ratio b_2/a'' in "Doubly Stiffened-at Support, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Various Modes with (FFFFFF) B.C.'s, ''Hard" Adhesive,
b_2=varies, $\tilde{b}$ =varies, a=0.50 m, L=1.0 m)

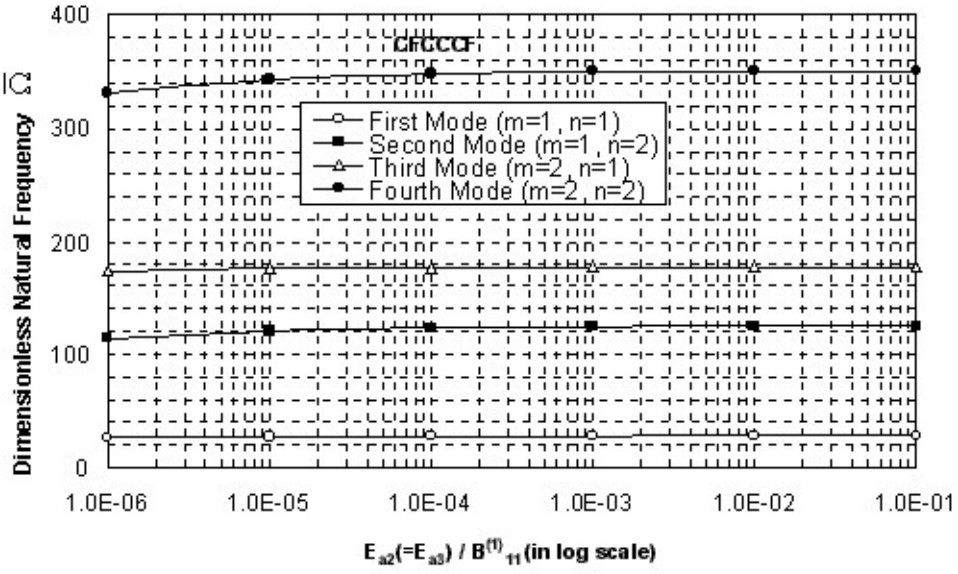


Figure 11.212. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$’’ in “Doubly Stiffened-at Support, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)” (Various Modes with (CFCCCF) B.C.’s, b_2=0.30 m, $\tilde{b}$ =0.15 m, a=0.50 m, L=1.0 m)

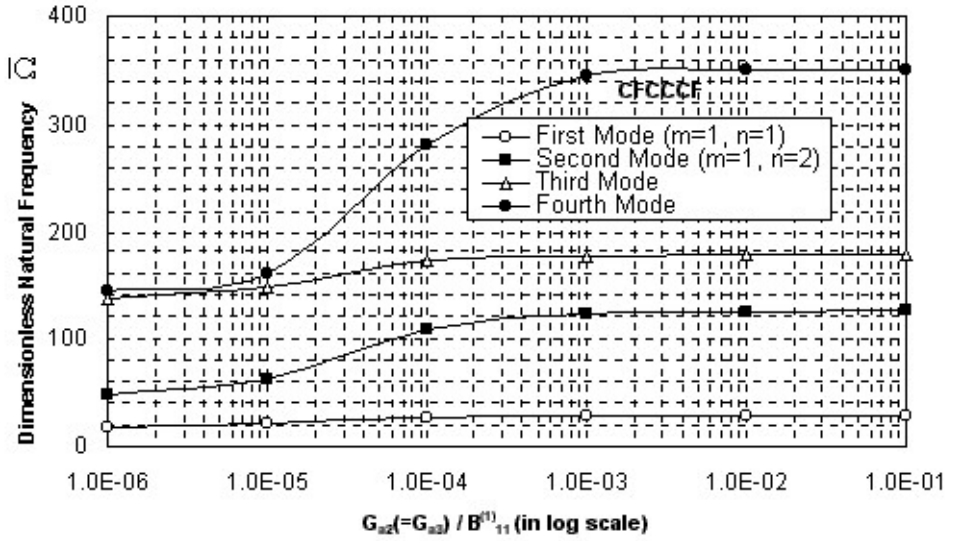


Figure 11.213. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Adhesive Shear Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$’’ in “Doubly Stiffened-at Support, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)” (Various Modes with (CFCCCF) B.C.’s, b_2=0.30 m, $\tilde{b}$ =0.15 m, a=0.50 m, L=1.0 m)

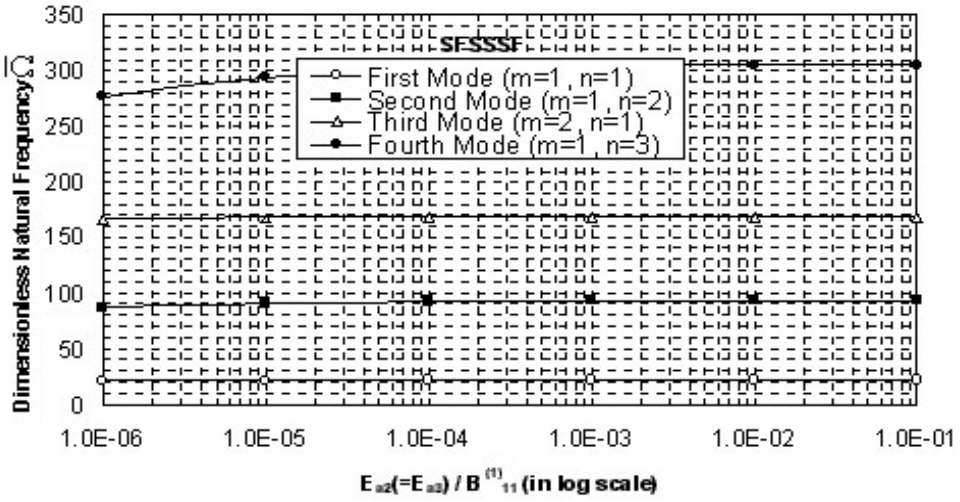


Figure 11.214. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$’’ in “Doubly Stiffened-at Support, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)” (Various Modes with (SFSSSF) B.C.’s, b_2=0.30 m, $\tilde{b}$ =0.15 m, a=0.50 m, L=1.0 m)

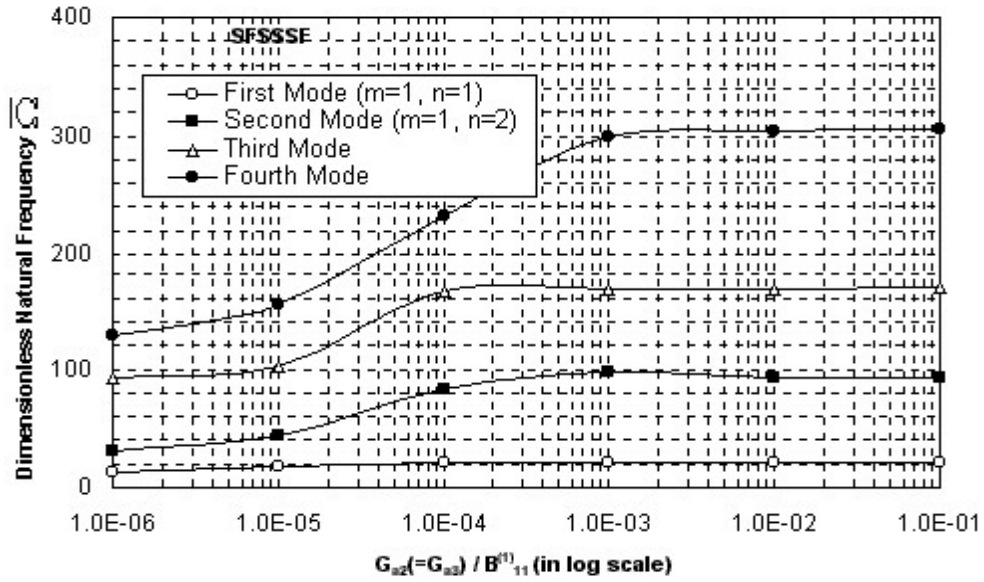


Figure 11.215. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Adhesive Shear Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$’’ in “Doubly Stiffened-at Support, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)” (Various Modes with (SFSSSF) B.C.’s, b_2=0.30 m, $\tilde{b}$ =0.15 m, a=0.50 m, L=1.0 m)

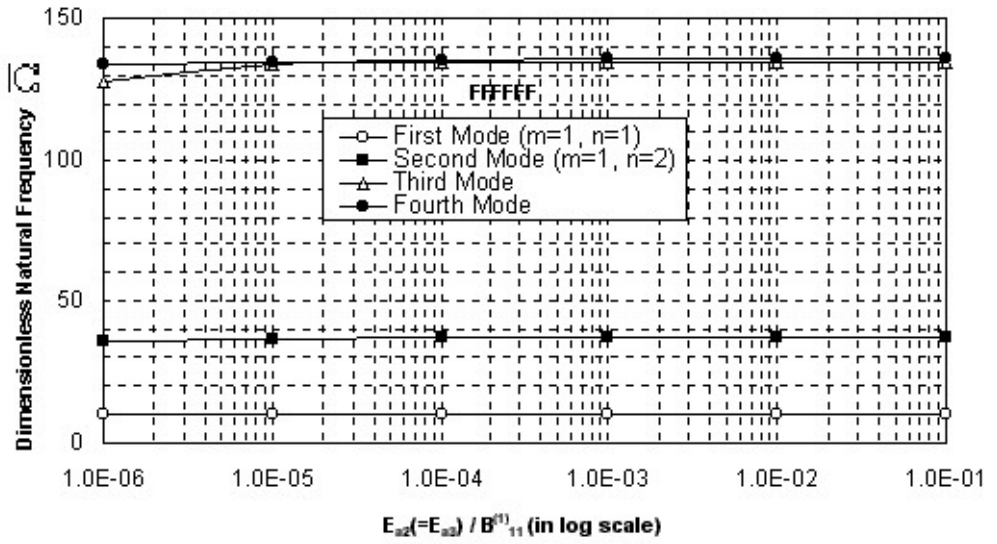


Figure 11.216. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$’’ in “Doubly Stiffened-at Support, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)” (Various Modes with (FFFFFF) B.C.’s, b_2=0.30 m, $\tilde{b}$ =0.15 m, a=0.50 m, L=1.0 m)

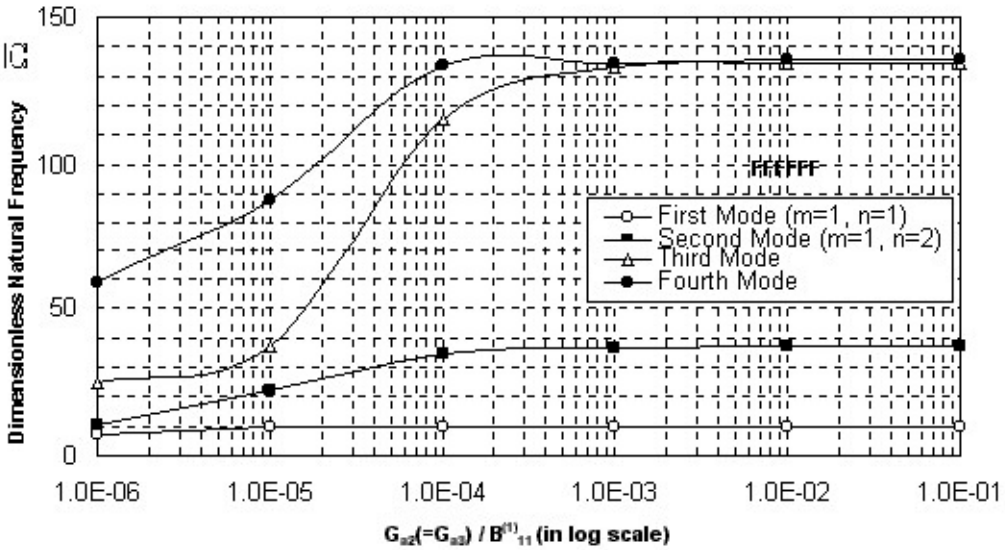


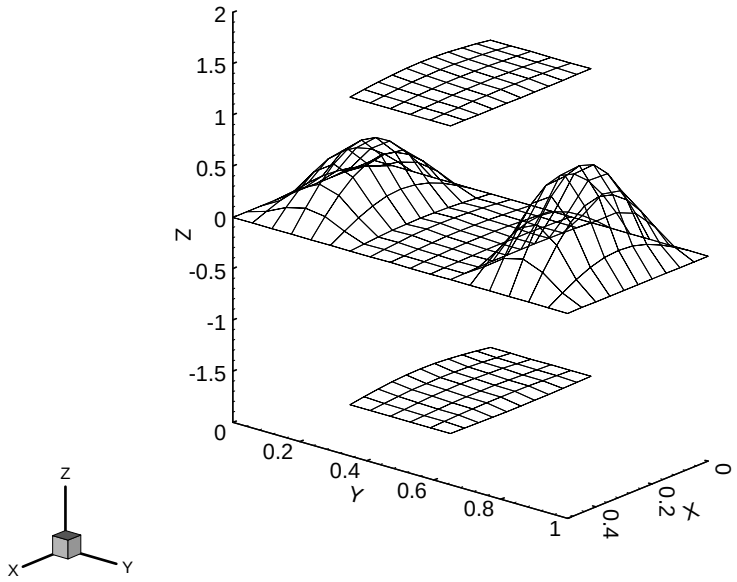
Figure 11.217. Dimensionless Natural Frequency $\overline{\Omega}$ versus ‘‘Adhesive Shear Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$’’ in “Doubly Stiffened-at Support, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)” (Various Modes with (FFFFFF) B.C.’s, b_2=0.30 m, $\tilde{b}$ =0.15 m, a=0.50 m, L=1.0 m)

11.8 Influence of Additional Partial Derivative Terms in Adhesive Layer Equations

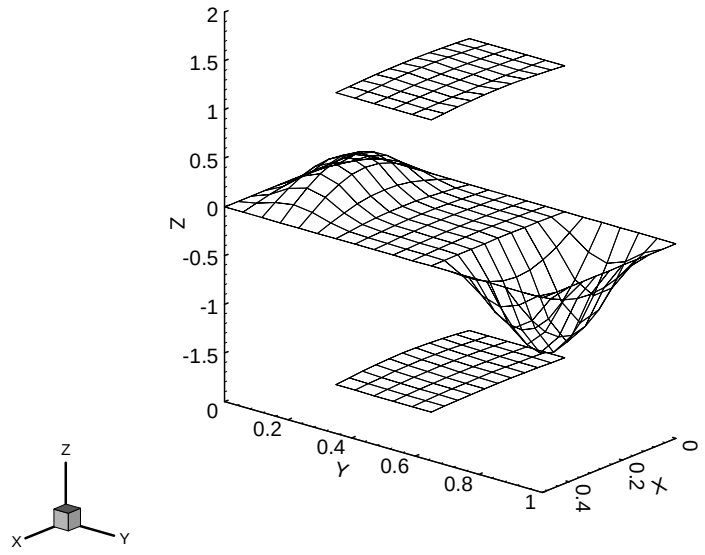
For this purpose, two cases, namely, the “Orthotropic Base Plate or Panel with Orthotropic Stiffening Plate Strips” and also the “Isotropic Base Plate or Panel with Orthotropic Stiffening Plate Strips” are investigated. For both cases, the “hard” and the “soft” adhesive layers are considered. The free flexural vibrations response in terms of their modes and the natural frequencies of these aforementioned two cases are presented in Figures 218 and 219 and in Figures 220 and 221, respectively.

From the comparison of these Figures with the Figures 11.1 and 11.2 and Figures 11.77 and 11.78, it can be observed that the mode shapes exhibit similar trends whether the additional partial derivative terms $\frac{\partial \omega}{\partial x}$ *and* $\frac{\partial \omega}{\partial y}$ (which are “double underlined” terms in the adhesive layer equations) (see Equations (4.4), (4.5), (4.6) and (4.7)) are taken into account or not. However, as expected, there are some increases in the natural frequencies if those additional terms are considered. Their effect on the first natural frequency is relatively small, however, in higher frequencies, their influence starts becoming significant (as can be seen from the calculated natural frequencies corresponding to the higher modes). These conclusions are valid for the “hard” adhesives case.

In the “soft” adhesives case, however, the effect of those $\frac{\partial \omega}{\partial x}$ *and* $\frac{\partial \omega}{\partial y}$ terms seems rather negligible.



a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 1955.486$

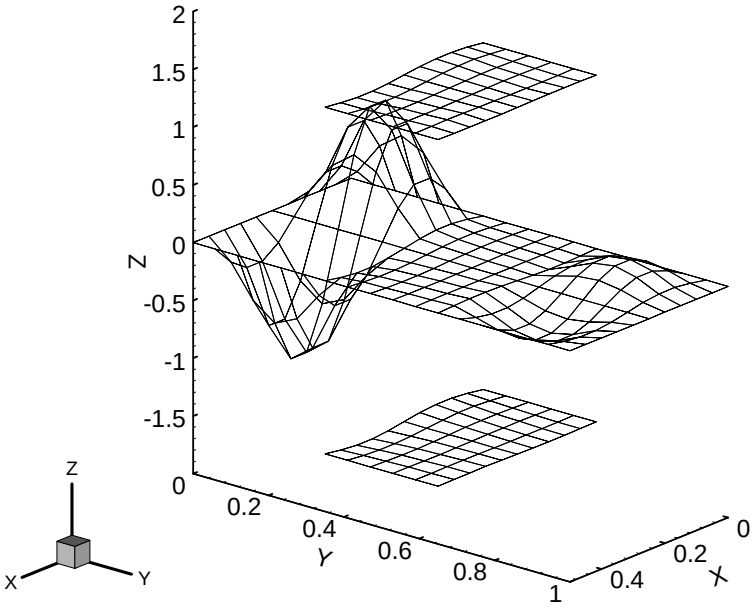


b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{12} = 2135.842$

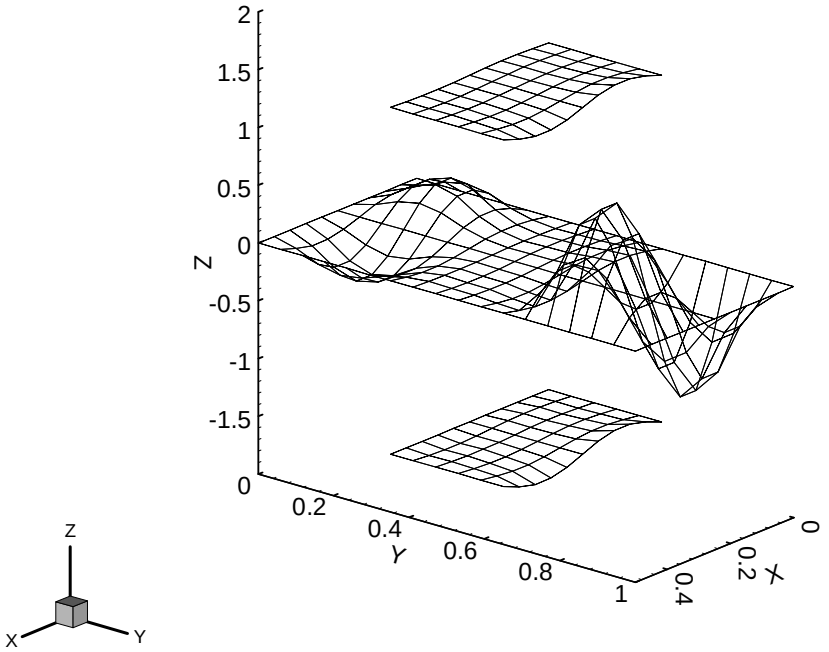
Figure 11.218. Mode Shapes and Natural Frequencies of "Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Adhesive Layer Equations with $\frac{\partial \omega}{\partial x}$ *and* $\frac{\partial \omega}{\partial y}$ terms)

(Base Plate= Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFCCFF) B.C.'s in y-direction, "Hard" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m.)

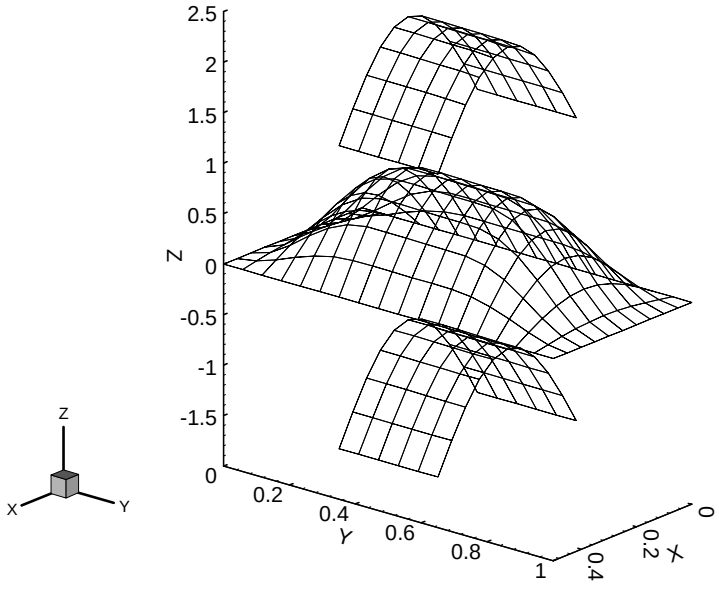


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{21} = 2253.297$

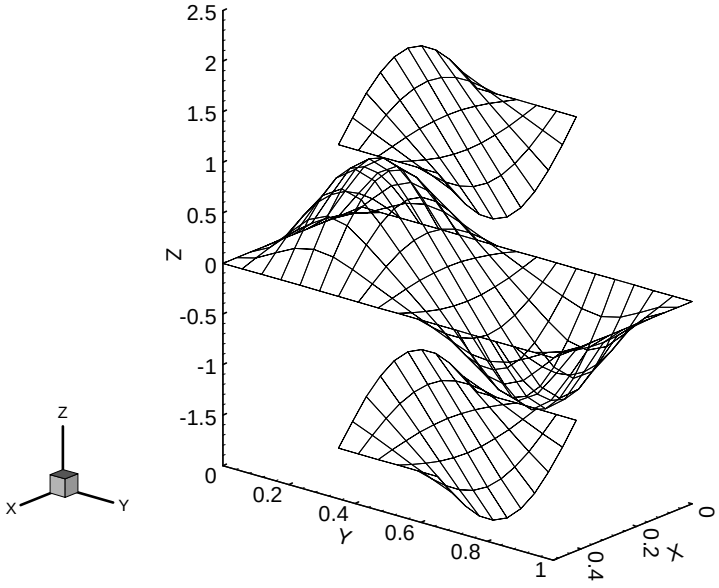


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{22} = 2397.774$

Figure 11.218. (continued)



a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 272.172$

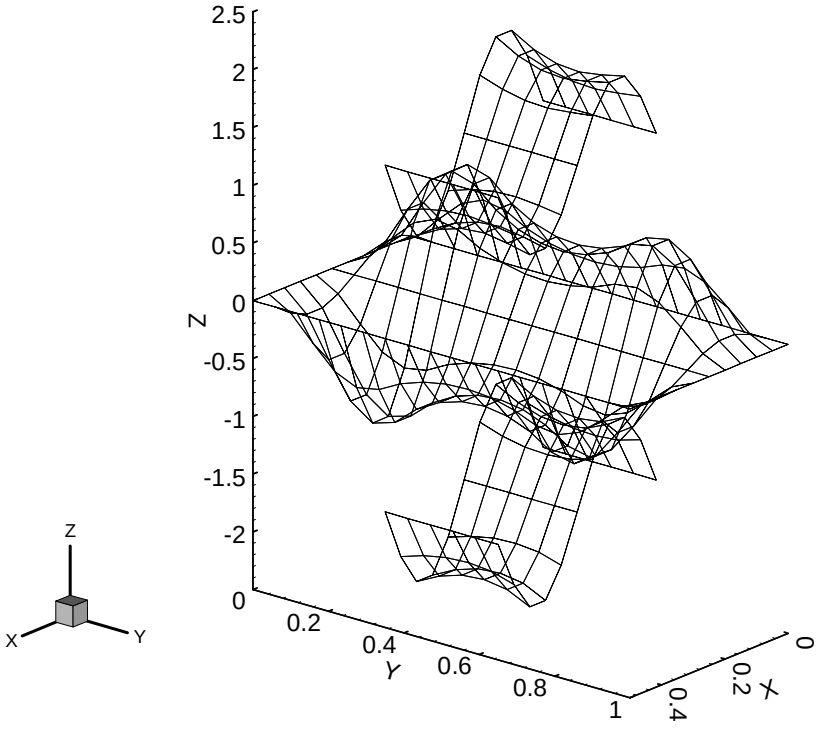


b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{12} = 722.625$

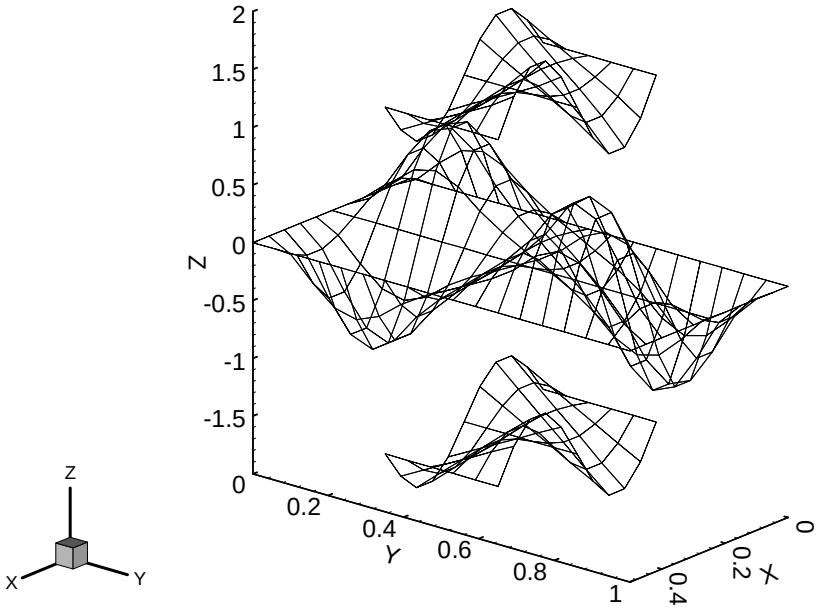
Figure 11.219 Mode Shapes and Natural Frequencies of "Centrally Doubly Stiffened, Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Adhesive Layer Equations with $\frac{\partial \omega}{\partial x}$ *and* $\frac{\partial \omega}{\partial y}$ terms)

(Base Plate= Plate 1= Kevlar-Epoxy, Plate 2= Plate 3= Graphite-Epoxy)
((FFCCFF) B.C.'s in y-direction, "Soft" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m.)

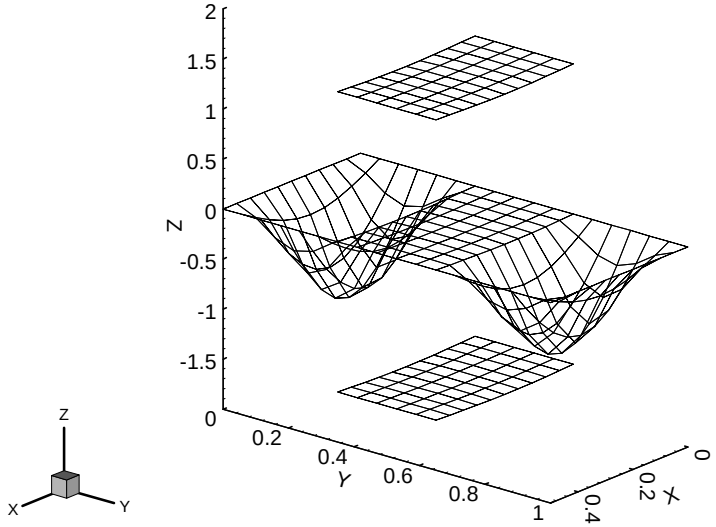


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{21} = 1033.557$

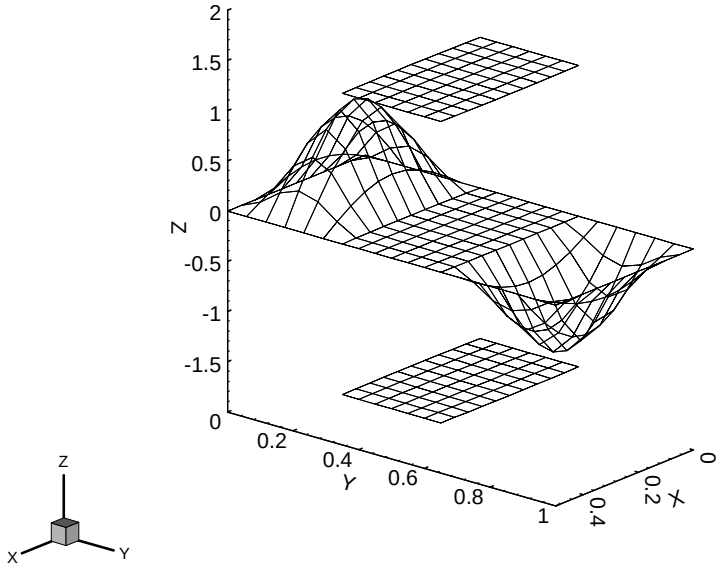


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{22} = 1175.582$

Figure 11.219. (continued)



a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 193.815$

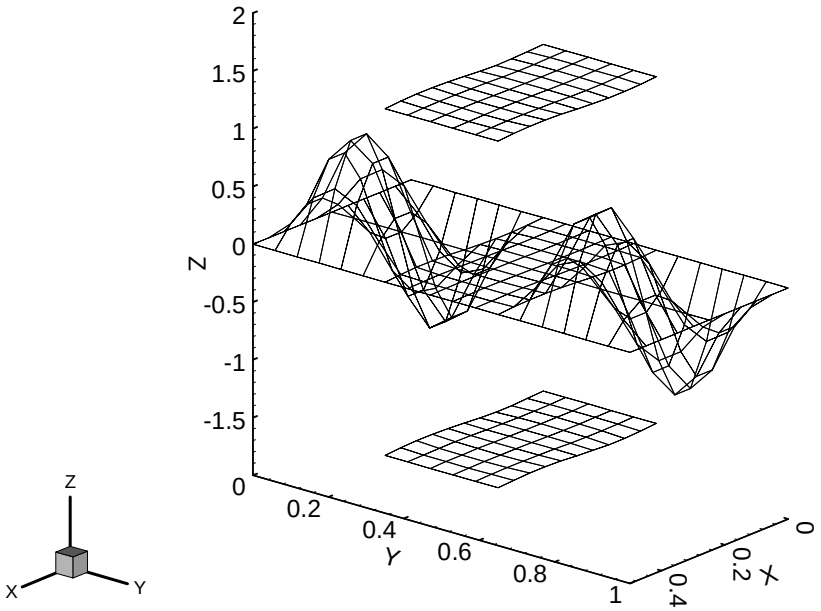


b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{12} = 205.096$

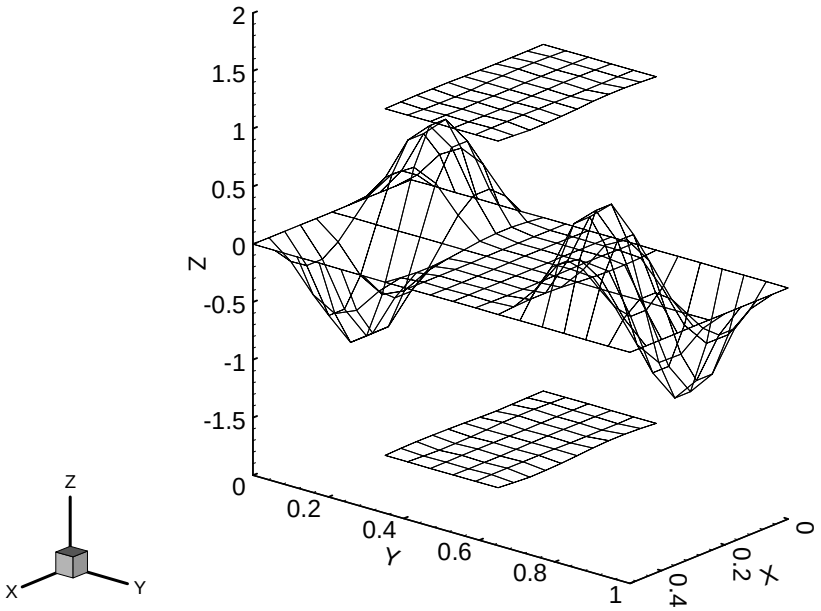
Figure 11.220. Mode Shapes and Natural Frequencies of "Centrally Doubly Stiffened, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Adhesive Layer Equations with $\frac{\partial \omega}{\partial x}$ *and* $\frac{\partial \omega}{\partial y}$ terms)

(Base Plate=Plate 1= Aluminum, Plate 2= Plate 3= Graphite-Epoxy)
((FFCCFF) B.C.'s in y-direction, "Hard" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m.)

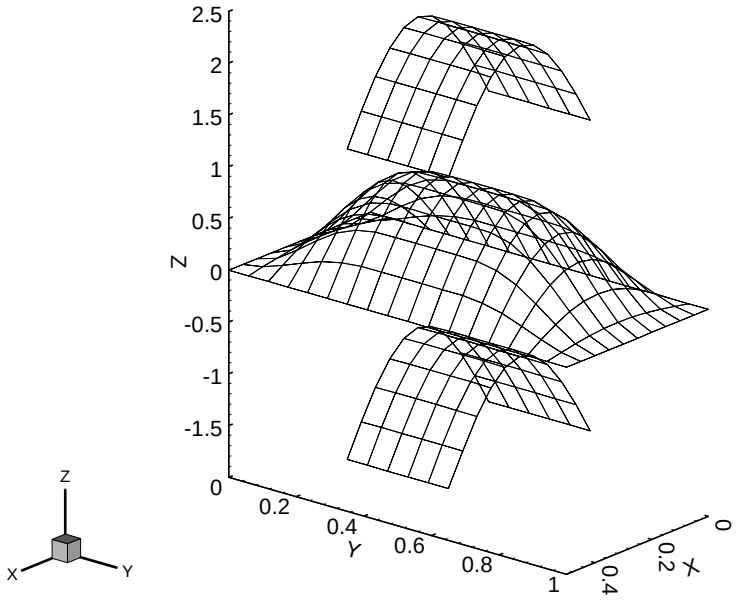


c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{21} = 423.333$

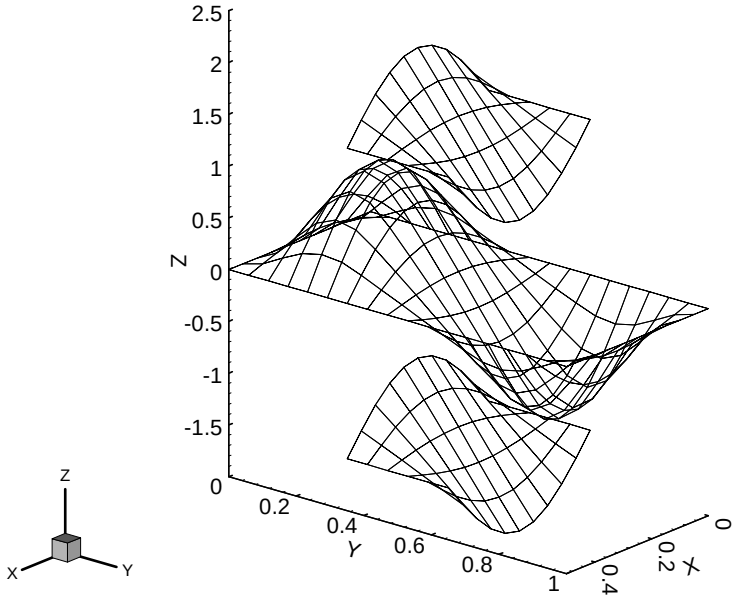


d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{22} = 434.054$

Figure 11.220. (continued)



a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 34.791$

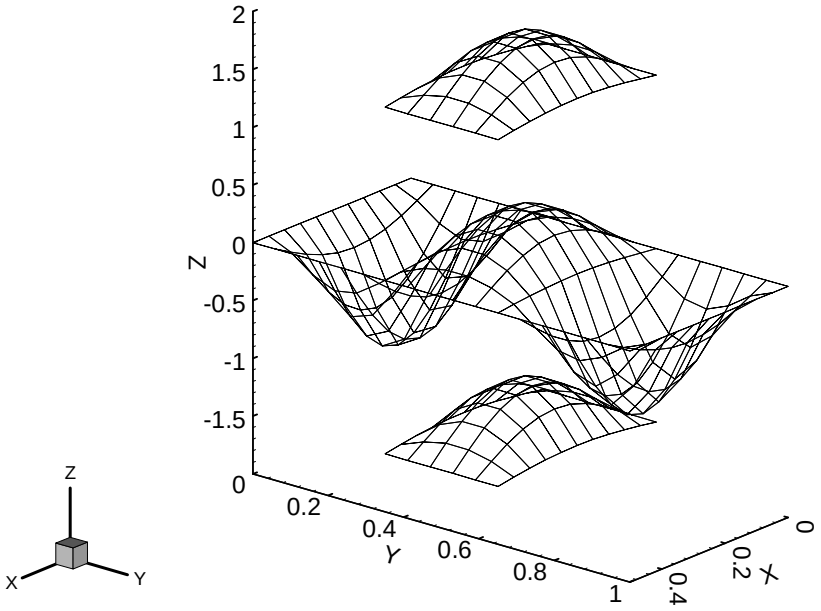


b) Second Mode with $\overline{\Omega}_2 = \overline{\omega}_{12} = 79.775$

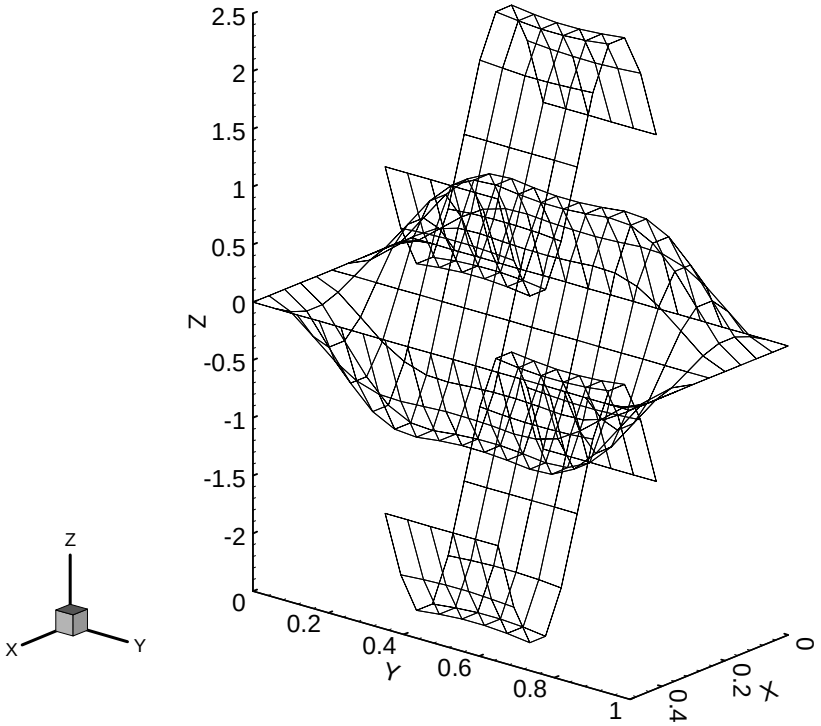
Figure 11.221 Mode Shapes and Natural Frequencies of "Centrally Doubly Stiffened, Isotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"

(Adhesive Layer Equations with $\frac{\partial \omega}{\partial x}$ *and* $\frac{\partial \omega}{\partial y}$ terms)

(Base Plate=Plate 1= Aluminum , Plate 2= Plate 3= Graphite-Epoxy)
((FFCCFF) B.C.'s in y-direction, "Soft" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m.)



c) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{13} = 169.085$



d) Fourth Mode with $\overline{\Omega}_4 = \overline{\omega}_{21} = 198.166$

Figure 11.221. (continued)

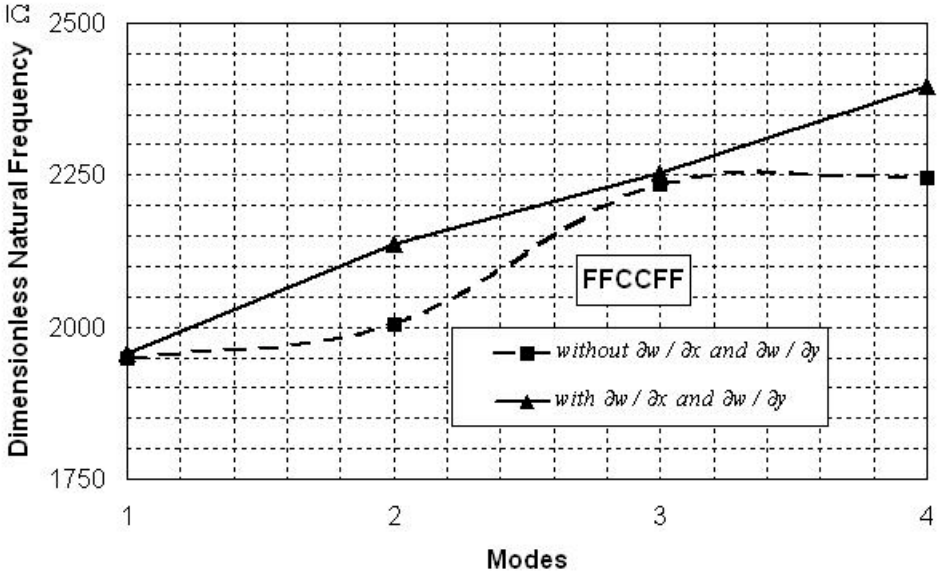


Figure 11.222. Effect of Adhesive Layer $\partial w/\partial x$ *and* $\partial w/\partial y$ Terms on Natural Frequencies in "Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFCCFF) B.C.'s, ''Hard" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

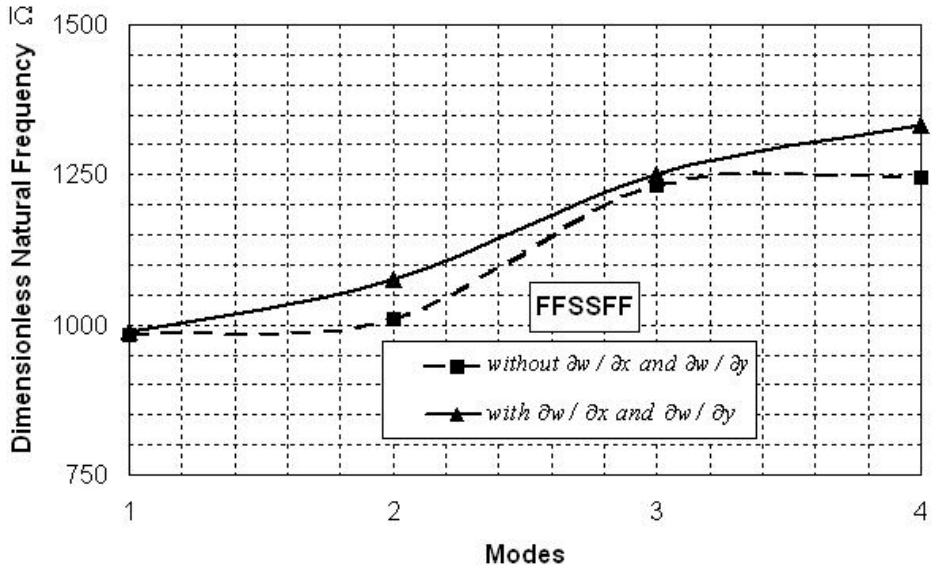


Figure 11.223. Effect of Adhesive Layer $\partial w/\partial x$ *and* $\partial w/\partial y$ Terms on Natural Frequencies in "Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFSSFF) B.C.'s, ''Hard" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

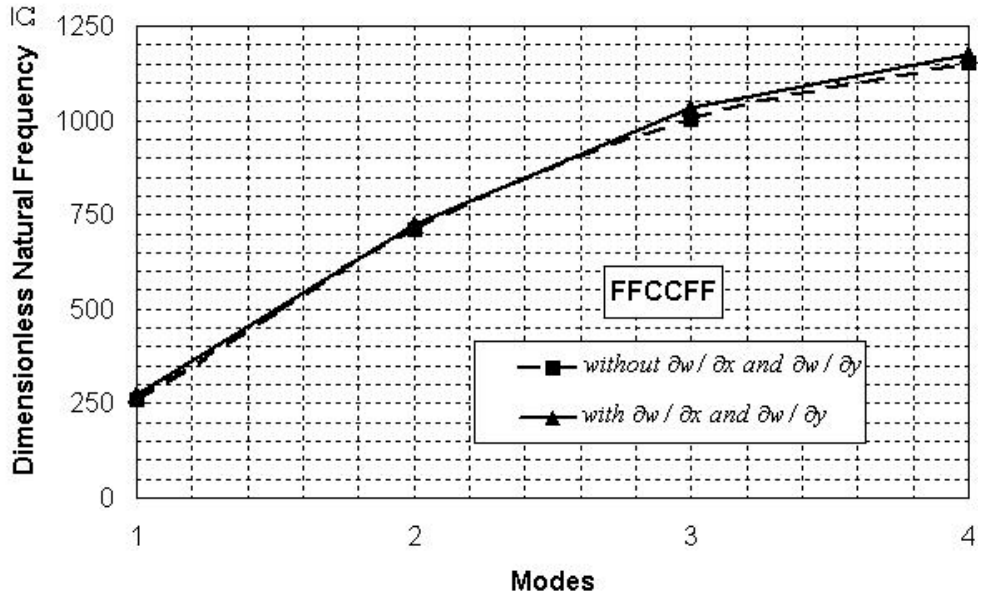


Figure 11.224. Effect of Adhesive Layer $\partial w/\partial x$ *and* $\partial w/\partial y$ Terms on Natural Frequencies in "Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFCCFF) B.C.'s, ''Soft" Adhesive, b_2=0.30 m, a=0.50 m, L=1.0 m)

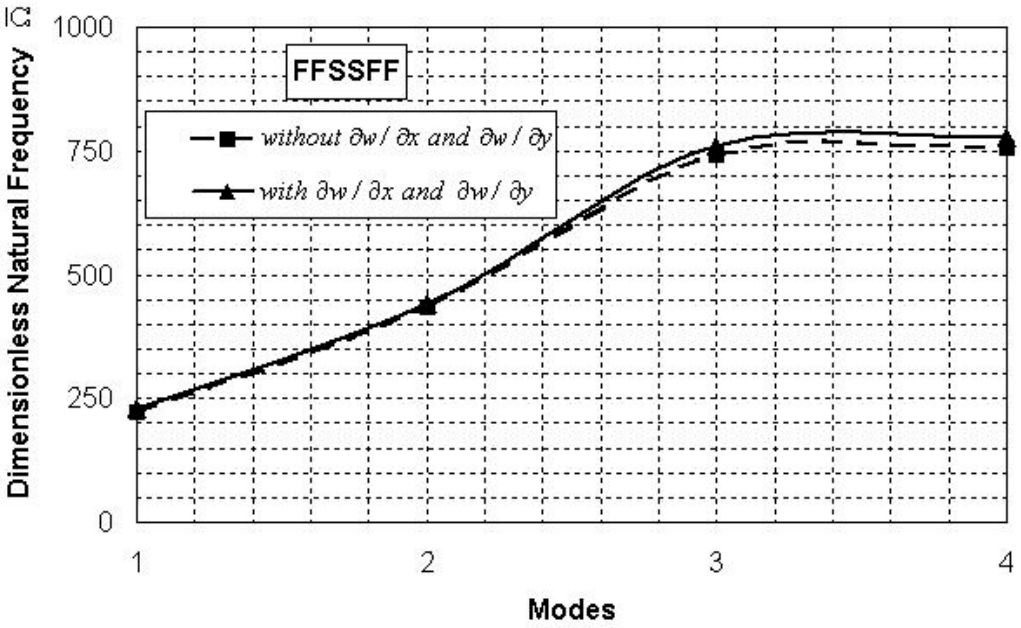


Figure 11.225. Effect of Adhesive Layer $\partial w/\partial x$ *and* $\partial w/\partial y$ Terms on Natural Frequencies in "Centrally Doubly Stiffened Composite Orthotropic Base Plate or Panel (with Upper and Lower Orthotropic Stiffening Plate Strips)"
(Base Plate=Plate 1= Kevlar-Epoxy , Plate 2= Plate 3= Graphite-Epoxy)
((FFSSFF) B.C.'s, ''Soft" Adhesive, b_2=0.30.m, a=0.50 m, L=1.0 m)

11.9 Some Remarks About the Accuracy of the Present Solution Technique

In the present Thesis, firstly, the "Governing System of Partial Differential Equations" for "Main PROBLEM I", "Main PROBLEM II" and "Main PROBLEM III" were obtained. Then, these equations were reduced to a set of the "Governing System of First Order Ordinary Differential Equations" which constitutes a "Two-Point Boundary Value Problem" in terms of the dependent or the "state vector" variables in the y-direction. Then, these equations are solved by means of the "Modified Transfer Matrix Method". The running time of the computer depends on some parameters in the ''Fortran'' solver code. The "number of discretizing points n_i (i=1 for Part I, i=2 for Part II and i=3 for Part III)" is the most important parameter which affects the running time. As the "number of discretizing points'' increases, the running time of the computer also increases. The reason for this is that the dimensions of the "Modified Transfer Matrices" increase. Thus, the inversion processes needed to obtain "Modified Transfer Matrices $[\dot{U}]$, $[\dot{V}]$ and $[\dot{W}]$" require more computer time as the "number of discretizing points'' increases. The transfer matrices also include the "degree of the approximating polynomials ndp_i" but its effect on the running time of the computer is much less than that of ''n_i''. Therefore, it is essential that the minimum appropriate "number of discretizing points n_i" and a suitable ''degree ndp_i of the approximating polynomials'' should be chosen for the sufficiently accurate results.

It is important to note here that the ''Boundary Conditions'' at the right hand-side support (or at the ''final end points'') for the base plate and the stiffening plate strips should be satisfied. In trying to satisfy the right hand-side support conditions, the chosen ''number of discretizing points n_i'' and the ''degree ndp_i of the approximating polynomials) are important factors.

It is observed that a large "number of discretizing points n_1" in the ''three layer region (i.e. Part I)'' is required in order to obtain accurate results especially for higher modes of vibration. Only 5 discretizing points can be enough to satisfy the "Boundary Conditions" at the "final end points" within an acceptable error (for

instance 0 (10^{-7}-10^{-8}) for zero quantities) for the first natural frequency. However, this number "n_1" increases up to 10 or 12 in order to obtain higher natural frequencies. Also, the error increases for the free edge boundary conditions, but this is also negligible. Although large number of discretizing points help to achieve more accurate results, it may also cause to increase the error after a certain value. As the "number of discretizing points" increase, the intervals or steps between intermediate stations become extremely small.

As mentioned above, the "degree ndp_i of approximating polynomials" also affects accuracy. But this effect is not as much as those of the "number of discretizing points n_i". The accuracy may not necessarily increase ss the ''degree of the approximating polynomial'' increases because of the numerical behavior of the solution method. Some numerical test runs in the computer are necessary in order to get used to and have a feeling for any particular problem.

By investigating the values or magnitudes of the dependent variables at the "final end points" (or at the right hand-side support conditions) and by observing the numerical results obtained, one can conclude the following,

1) In the "Centrally Doubly Stiffened, Composite Orthotropic and/or Isotropic Plates or Panels (with Upper and Lower Orthotropic Stiffening Plate Strips)" (i.e."Main PROBLEMS I.a and I.b"), the number of "discretizing points" $n_1=6$, $n_2=n_3=7$ (for each Part I, Part II and Part III, respectively) and the degrees of the "Approximating or the Interpolation Polynomial" $ndp_1=5$, $ndp_2=ndp_3=6$ (for each Part, respectively) were chosen as appropriate values in order to obtain accurate natural frequencies and the corresponding accurate and smooth mode shapes.

2) In the "Non-Centrally Doubly Stiffened, Composite Orthotropic and/or Isotropic Plates or Panels (with Upper and Lower Orthotropic Stiffening Plate Strips)" (i.e."Main PROBLEMS II.a and II.b"), the number of "discretizing points" $n_1=n_2=6$ and $n_3=16$ (for each Part I, Part II and Part III, respectively) and the degrees of the "Approximating or the

Interpolation Polynomial" $ndp_1=ndp_2=5$ and $ndp_3=6$ (for each Part, respectively) were employed as appropriate values in order to obtain accurate natural frequencies and the corresponding accurate and smooth mode shapes.

3) In the "Centrally Doubly Stiffened-at Support, Composite Orthotropic and/or Isotropic Plates or Panels (with Upper and Lower Orthotropic Stiffening Plate Strips)" (i.e."Main PROBLEMS III.a and III.b"), the number of "discretizing points" $n_1=6$ and $n_2=16$ (for Part I and Part II respectively) and the degrees of the "Approximating or the Interpolation Polynomial" $ndp_1= =5$ and $ndp_2=6$ (for each Part, respectively) were taken as appropriate values in order to obtain accurate natural frequencies and the corresponding accurate and smooth mode shapes.

4) Last but not the least, in all PROBLEMS, one has to plot the mode shapes too in order to see if they exhibit smooth and appropriate surfaces to the mode shapes they correspond.

There is no available study published in the open literature on the "Main PROBLEMS" considered here. Therefore, the comparisons of accuracy with other available solutions of the aforementioned "Main PROBLEMS" were not possible in the present Thesis. However, for a single layer orthotropic Mindlin Plate, the numerical results of the present method of solution and the analytical closed form solutions are compared in Toghi's Thesis [VIII.8] and also Özerciyes' Thesis [V.7]. The accuracy of the present solution technique was found to be exceptionally good in those cases. On the basis of present studies, one may conclude that the accuracy of the present solution technique in the case of "Main PROBLEMS" studied here should be sufficiently good.

It should be pointed out here, however, in Özerciyes' Thesis [V.7], there is only single stiffening orthotropic stiffening plate strip connected to a base plate. In the "Main PROBLEMS" involved in the present research work, the orthotropic and/or isotropic base plates are reinforced as "Doubly Stiffened Composite

Orthotropic and/or Isotropic Base Plates or Panels". The frequencies obtained in this study when compared with the Özerciyes' Thesis [V.7] are found to be slightly higher. This is, of course, due to the doubly stiffened nature of the composite plate or panel system of the present study. Therefore, frequencies will be higher as expected.

Also, for the ''Doubly Stiffened-at Support'' cases, both studies give almost the similar mode shapes. However, for the ''Centrally Stiffened'' cases, the mode shapes are similar but not the same because of the effect of the ''Modulus of Rigidities (G_a's)'' of the upper and lower adhesive layers in the present Thesis.

The numerical results presented in this Thesis may be compared approximately with the numerical results obtained for the same ''Main PROBLEMS'' by ''Finite Element Method''. This, of course, means a new study or thesis in terms of the commercially available "Finite Element Codes" or re-formulating the "Main PROBLEMS" considered here in the. "Finite Element Method" and then, writing an independent "Finite Element Code" which will require considerable programming effort.

It should, of course, be emphasized here that, the "Finite Element Method (F.E.M.)" itself is an approximate method. Therefore, the numerical results by "F.E.M." will not necessarily more exact or accurate. In the present study, the exact governing equations without any approximations are employed. The errors, if any, are the results of the numerical procedures associated with the present solution technique which is the "Modified Transfer Matrix Method (with Interpolation Polynomials and/or Chebyshev Polynomials)". Therefore, one may say that, by reasonable guessing, the numerical results of the present study are more accurate than those of the "F.E.M.".

CHAPTER 12

CONLUSIONS

12.1 Conclusions for "Main PROBLEM I.a" and "Main PROBLEM I.b"

The analytical modeling for the **''Free Flexural (or Bending) Vibrations of Adhesively Bonded, Centrally Doubly Stiffened, Composite Plate or Panel System (with Orthotropic or Isotropic Base Plate and Orthotropic Stiffening Plate Strips)'' (i.e. "Main PROBLEM I")** based on the Mindlin's Plate Theory is analyzed and a suitable semi-analytical solution procedure is developed. In previous chapters, the solution technique employed is the "Modified Transfer Matrix Method" which is a combination of the ''Classical Levy's Method", the "Integrating Matrix Method" and the "Transfer Matrix Method". Based on the numerical results given in the preceding chapters, one can reach the following conclusions:

1. The analytical formulation of the problem in the "state vector" form and the present solution procedure (i.e. "Modified Transfer Matrix Method") is fairly general and very efficient. The formulation and the solution technique used in this Thesis are general enough and can be extended to other stiffened composite plate systems.

2. "Classical Lévy's Solutions" were used in the formulation of the problem. Therefore, the only limitation in the solution technique is that the base plate and the stiffening strips are to be simply supported at two opposite edges, i.e. the edges at x=0 and x=a, while other edges, that is, the edges at y=0 and y=b_2, may have arbitrary boundary conditions.

3. The mode shapes and the corresponding natural frequencies of the composite plate or panel system are significantly effected by the “hardness” and the “softness” (i.e. elastic constants) of the upper and lower adhesive layers which connect the base plate and the central stiffening plate strips. In the “hard” adhesives case, the stiffened portion of the composite plate or panel system behaves as a relatively thick and very stiff plate in comparison with the rest of the system. Consequently, the stiffened portion stays almost stationary (except perhaps at relatively higher modes) while the remaining portions experience relatively large deflection shapes. In the “soft” adhesive layers case, the stiffening plate strips have the same or almost similar deflection shapes with the base plate in the central three-layer portion of the plate system. In other words, in the “soft” adhesives case, the central stiffening plate strip moves almost in step with the base plate in any mode, at least up to fourth mode.

4. For the ‘‘hard’’ adhesives case, the mode shapes obtained are not exactly symmetric or skew-symmetric in spite of the symmetry of the entire system (the geometry, the material and the B.C.’s) with respect to the mid-plane perpendicular to the base plate. This is because of large values of ‘‘Modulus of Rigidities (G_{a2} and G_{a3})’’ in the ‘‘hard’’ adhesive layers. The mode shapes tend to be symmetrical or skew-symmetrical when G_a’s practically go to zero.

5. As the constraining or the stiffenning effect of the boundary conditions in the y-direction increases so do the natural frequencies of the composite plate or panel system.

6. In the ‘‘hard’’ adhesives case, the dimensionless natural frequencies $\overline{\Omega}$ increase as the “aspect ratio b_2/a” increases and as the modes get higher so do the frequencies. Also, the general trend is that the natural frequency curves corresponding to each mode are close to each other for less than 1.0 value of the “Aspect Ratio b_2/a” as expected. After this value, they go up sharply and start to separate from each other. However, in the case of the ‘‘soft’’ adhesives, the natural frequency curves corresponding to each mode are separate from each other while they are close to each other in the ‘‘hard’’ adhesives case for less

than 1.0 value of the "Aspect Ratio b_2/a". These curves increase gradually without any sharp slopes. This is because of the "softness" of the adhesive layers and consequently of the relatively loose coupling between the base plate and the stiffening strips. The base plate and the stiffening strips don't interact with or influence each other as the adhesive layers become relatively "softer" and "softer". Eventually, in the limit, they will vibrate as three separate plates at their own independent natural frequencies uneffected by each other.

7. In the ''hard'' adhesives case, the natural frequency parameter $\overline{\Omega}$ decrease as the "Joint Length Ratio L/b_2" increases. In the ''soft'' adhesives case, as the "Joint Length Ratio L/b_2" increases, the influence of the support conditions decrease and the natural frequency curves get closer to each other and the natural frequency parameter $\overline{\Omega}$ decreases. In both cases, the natural frequency curves converge to the same value for the relatively large values of L/b_2 ratio.

8. As the "Bending Stiffness Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$" increases, the natural frequencies $\overline{\Omega}$ decrease almost linearly and then remain almost constant. It should be pointed out here, that, while the dimensionless natural frequency parameter decreases as the ''Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$'' increases, the real dimensional natural frequency increases as expected (see for instance, the expression (4.16) for the dimensionless natural frequency parameter).

9. As the "Bending Stiffness Ratio $D_{11}^{(2)}(=D_{11}^{(3)})/D_{11}^{(1)}$ or $(D_{11}^{(2)}\text{ or }D_{11}^{(3)})/D_{11}^{(1)}$" increases, the dimensionless natural frequency parameter $\overline{\Omega}$ almost remain constant in both of the ''hard'' and ''soft'' adhesive cases.

10. The dimensionless natural frequency curves increase linearly as the ''Bending Rigidity Ratio $D_{22}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$'' increases and these curves have almost the same slope and are very close to each other for the ''hard'' adhesives case but the dimensionless natural frequency curves increase gradually as the ''Bending

Rigidity Ratio $D_{22}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})$'' increases. These curves have different slopes and are separate from each other for the ''soft'' adhesives case.

11. As the ''Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3}) / B_{11}^{(1)}$ increases, the natural frequency parameter $\overline{\Omega}$ first slowly increases and then becomes almost constant. However, in the case of the "Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3}) / B_{11}^{(1)}$", there is a jump in frequencies in a narrow "transition region" (with sharp increases in natural frequencies) .After then, the natural frequencies become asymptotic to a certain value. This clearly indicates that the natural frequencies are very much dependent up to a certain value of the $G_{a2}(=G_{a3}) / B_{11}^{(1)}$ ratio.

12. The mode shapes exhibit similar trends whether the additional partial derivative terms $\frac{\partial w}{\partial x}$ *and* $\frac{\partial w}{\partial y}$ in the adhesive layer equations are taken into account or not. However, as expected, there are some increases in the natural frequencies if those additional terms are considered. Their effect on the first natural frequency is relatively small, however, in higher frequencies, their influence starts becoming significant. These conclusions are valid for the "hard" adhesives case. However, in the "soft" adhesives case, the effect of these $\frac{\partial w}{\partial x}$ *and* $\frac{\partial w}{\partial y}$ terms seems rather negligible.

12.2 Conclusions for "Main PROBLEM II.a" and "Main PROBLEM II.b"

The analytical modeling for the **''Free or Flexural Bending Vibrations of Adhesively Bonded, <u>Non-Centrally</u> Doubly Stiffened, Composite Plate or Panel System (with Orthotropic or Isotropic Base Plate and Orthotropic Stiffening Plate Strips)'' (i.e. "Main PROBLEM II")** based on the Mindlin's Plate Theory is analyzed and the appropriate solution procedure is developed by following a similar approach to that of the "Main PROBLEM I". The solution technique is the "Modified Transfer Matrix Method" which is, as pointed out before, a semi-

analytical and numerical method. On the basis of the numerical results in the preceding chapters, the following conclusions can be reached:

1. The “state vector” form of the formulation of the problem and the solution technique developed in this study is fairly general and also efficient numerical method. For certain class of plate vibration problems, here in this case too, the only limitation is that the two opposite edges of the orthotropic or isotropic composite plate or panel system have to be simply supported, while the supports at other opposite edges can be arbitrary.

2. The “hardness” and the “softness” of the adhesive layers influence significantly the natural frequencies and mode shapes of the “Main PROBLEM II”. The central stiffened part of the composite plate or panel system with the “hard” adhesive layers stays almost stationary in any mode (except perhaps at relatively higher modes). As in the “Main PROBLEM I”, the central stiffening strips connected by the “soft” adhesive layers to the orthotropic or isotropic base plate have the same or almost similar deflection shapes with the base plate in the central three-layer portion of the plate system. Thus, the central stiffening plate strips move almost in step with the base plate in any mode, at least up to fourth.

3. As the constraining or the stiffenning effect of the boundary conditions in the y-direction increases so do the natural frequencies of the composite plate or panel system.

4. In the “hard” adhesives case, the natural frequencies increase with sharper slopes compared to the ones in the ‘‘soft’’ adhesives case as the “Aspect Ratio b_2/a” increases.

5. The dimensionless natural frequencies $\overline{\Omega}$ of each mode decrease almost linearly and then remain almost constant for both ‘‘hard’’ and ‘‘soft’’ adhesive cases as the “Bending Rigidity or Stiffness Ratio $D_{11}^{(1)} / D_{11}^{(2)} (= D_{11}^{(3)})$” increases.

6. As the "Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$" increases, the natural frequencies for the first four modes increase slowly at first and then remain almost constant. However, for the "Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3})/B_{11}^{(1)}$", there is also a jump in frequencies in a narrow "transition region" and then frequencies remain almost constant.

12.3 Conclusions for "Main PROBLEM III.a" and "Main PROBLEM III.b"

The analytical formulation for the **''Free or Flexural Bending Vibrations of Adhesively Bonded, Doubly Stiffened-at Support, Composite Plate or Panel System (with Orthotropic or Isotropic Base Plate and Orthotropic Stiffening Plate Strips)'' (i.e. "Main PROBLEM III")** has been developed on the basis of the Mindlin's Plate Theory and the solution procedure follows the similar approaches to those of "Main PROBLEM I" and "Main PROBLEM II". The present solution technique is again the "Modified Transfer Matrix Method". Based on the numerical results given in the preceding chapter, the following conclusions can be drawn:

1. The present formulation of the problem in a "state vector" form and the solution procedure is fairly general and very efficient.

2. The only limitation in the solution technique is that the stiffening strips and the base plate have to be simply supported at two opposite edges while for the other edges, arbitrary boundary conditions may be prescribed.

3. The "Elastic Constants" (or the "hardness" and the "softness") of the in-between adhesive layers drastically influences the mode shapes and the corresponding natural frequencies of the composite plate or panel system. In the case of the "hard" adhesive, the stiffened portion of the composite plate or panel system stays almost stationary (except perhaps at relatively higher modes). In mode shapes, in the case of the "soft" adhesive, the stiffening strips and the base plate experience different deflection shapes in the same mode.

4. As the constraining or the stiffenning effect of the boundary conditions in the y-direction increases so do the natural frequencies of the composite plate or panel system.

5. In ‘‘hard’’ adhesives case, the natural frequencies increase as the ‘‘Aspect Ratio b_2/a” increases. When the ‘‘Aspect Ratio b_2/a” takes 2.0 value (i.e. the stiffening plate strips become almost equal to the base plate in the y-direction), the natural frequencies for the same modes but for different boundary conditions converge to almost same values regardless of the boundary conditions. In the ‘‘soft’’ adhesives case, the natural frequency curves remain more or less constant or they go up gradually without any sharp slopes or increases. This is, of course, due to the “softness” of the adhesive layers and to the relatively loose coupling (because of the “soft” adhesives) between the base plate and the stiffening plate strips.

6. For both ‘‘hard’’ and ‘‘soft’’ adhesive cases, the dimensionless natural frequency curves decrease almost linearly and then remain almost constant as the ‘‘Bending Rigidity Ratio $D_{11}^{(1)}/D_{11}^{(2)}(=D_{11}^{(3)})$’’ increases.

7. As the ‘‘Adhesive Elastic Modulus Ratio $E_{a2}(=E_{a3})/B_{11}^{(1)}$ increases, the natural frequency parameter $\overline{\Omega}$ first very slowly increases and then becomes almost constant. However, for the “Adhesive Shear Modulus Ratio $G_{a2}(=G_{a3})/B_{11}^{(1)}$”, there is again a jump in frequencies in a narrow “transition region”.

CHAPTER 13

RECOMMENDATIONS FOR FUTURE STUDY

Some recommendations for future research may be suggested. In this section, the recommendations will be considered in two groups, namely 1 and 2:

(1) Recommendations for the furher extension of the present solution technique to some other related problems which do not require considerable modification in the present method of solution:

(2) Recommendations for the extension of the present solution technique to the problems which require considerable modification in the present method of solution:

Recommendations for (1):

- The damping effects in the plate system may be considered in terms of complex damping by employing some modifications in the present method of solution and the numerical procedures.

- The elastic support conditions in terms of mechanical springs, that is, torsional and/or extensional springs, can be considered in the boundary conditions in the y-direction.

- The stiffening plate strips may be replaced by beams of regular rectangular cross-sections. This case may be handled with relative ease.

- The upper and lower adhesive layers may be assumed to be of dissimilar materials. Then, the present solution technique can easily be used.

Recommendations for (2):

- Considering the supports in the y-direction stationary, one can obtain higher order "state vectors", and consequently increase the number of governing system of equations by adding u and v displacements and the membrane stress resultants. In this case, the solution procedure needs some modifications.

- The base plate in the centrally, non-centrally or at-support doubly stiffened plate systems may be replaced by a multi-layer composite plate. Then, the present solution technique will need considerable changes.

- The stiffening plate strips may be composed of multi-layer plates or plate strips. Again the solution method does need some significant changes.

- Several stiffening plate strips separated by equal or unequal spans may be used to reinforce the base plate. In this case, the present solution technique will be suitable, too. However, increased number of numerical calculations will be needed.

- Slightly curved plates or shallow shell panels may be considered instead of rectangular plates.

- The non-linear governing system of equations by using non-linear ''strain-dislacement'' and ''strain-stress'' relations (or Hooke's Law) may be developed and investigated.

- The stiffening strips and the base plate may have arbitrary directions of orthotropy.

- The adhesive layers between the base plate and the stiffening strips may be considered as viscoelastic layers with damping characteristics

- The extension of problems here into forced vibrations, flutter problems, etc., may be considered.
- The extension into problems with stiffeners as thin walled hat or thin walled open or closed cross-section beams may be considered.

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