AN INVESTIGATION ON THESES AND DISSERTATIONS ON MATHEMATICAL MODELING IN TURKEY IN THE LAST TWO DECADES

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ABSTRACT

AN INVESTIGATION ON THESES AND DISSERTATIONS ON MATHEMATICAL MODELING IN TURKEY IN THE LAST TWO DECADES.

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In line with the developing science and technology, mathematical modeling which links up real life and mathematics has become increasingly popular, and the number of studies conducted on mathematical modeling has increased recently. This study aimed to investigate master thesis (MS) and doctoral dissertations (Ph.D.) completed over the last two decades in Turkey to understand the portrait of mathematical modeling, by particularly focusing on the Models-and-Modeling Perspective (MMP).

The features and nature of modeling research that this study focused on involves level of the graduate studies (i.e., MS thesis and Ph.D. dissertations), the completion years, the university contributions, the employed research methods, the participants, the amount of the modeling research involving mathematical modeling activity, and the extent of which mathematical modeling activities reflecting the six design principles of Models-and-Modeling Perspective.

Data set of the study involved 70 modeling studies (MS/Ph.D.) that were published between 2000 –2019 on mathematics education field and that were gathered from the database of Higher Education Council. Document analysis and content analysis, the two qualitative data analysis methods, were utilized to investigate the features and

the nature of these modeling studies. The data analysis was deepened by investigating 42 modeling studies (among 70) comprising of at least one modeling activity through a concept-driven coding frame.

Consequently, it was revealed that the type of modeling studies published in the mathematics education field were mostly master thesis. The first modeling study was published in 2005 as a master thesis. The number of modeling MS and Ph.D. studies was gradually increased from 2000 to 2020, and there was a rapid increase in conducted modeling studies after 2013. Twenty-four different universities in Turkey contributed to the academic work about modeling in this field. The most used research method in modeling studies was qualitative research method which was followed by mixed and quantitative research methods, respectively. The participants of modeling MS and Ph.D. studies differed from 4^{th} grade to 9^{th} grade students along with pre-service teachers and in-service teachers; the major focus appeared to be on pre-service teachers. 19 out of 42 modeling MS and Ph.D. studies, almost half of them, included 3-6 modeling activities whilst the rest of 23 modeling MS and Ph.D. studies involved various number of modeling activities. With this study, it is expected to provide a portrait of the modeling studies that was conducted as MS thesis and Ph.D. dissertations in Turkey, which is hoped to be informative for researchers in mathematical modeling.

Keywords: Modeling, Mathematical Modeling, Mathematics Education, Models-and-Modeling Perspective, Model-Eliciting Activities

ÖZ

MATEMATİKSEL MODELLEME ÜZERİNE YAZILMIŞ SON YİRMİ YILDA TAMAMLANAN YÜKSEK LİSANS VE DOKTORA TEZ ÇALIŞMALARININ İNCELEMESİ

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Bilim ve teknolojinin de gelişmesiyle, gerçek hayat ve matematik arasında bağlantı kuran matematiksel modelleme, gün geçtikçe popülerliğini arttırmış, buna bağlı olarak da matematiksel modelleme çalışmaları artmıştır. Bu çalışmanın amacı son yirmi yılda Türkiye ‘de matematik eğitimi alanında yapılan yüksek lisans ve doktora tezlerini özellikle model ve modelleme perspektifi açısından inceleyerek, Türkiye’de yapılan modelleme çalışmalarının genel durumunu ortaya koymaktır.

Modelleme çalışmalarının özellikleri ve yapısı incelenirken çalışmaların türleri (yüksek lisans/doktora), tamamlanma yılları, hangi üniversitelerde tamamlandığı, çalışmalarda hangi araştırma yöntemlerinin kullanıldığı, katılımcılarının kimler olduğu, çalışmalarda kullanılan modelleme etkinliği sayıları ve modelleme aktivitelerinin hangi ölçüde model ve modelleme perspektifinin altı prensibini karşıladıkları göz önünde bulundurulmuştur.

Bu çalışmanın veri seti Yüksek Öğretim Kurulu tez veri tabanında 2000 - 2019 yılları arasında matematik eğitimi alanında yayınlanmış olan 70 modelleme çalışmasını (yüksek lisans ve doktora tezleri) içermektedir. Nitel araştırma yöntemlerinden doküman analizi ve içerik analizi kullanılarak modelleme çalışmalarının özellikleri ve yapısı incelenmiştir. Bu 70 modelleme çalışması içinden modelleme etkinliği içeren 42 çalışmanın veri analizi model ve modelleme yaklaşımının altı tasarım prensibi temelinde kavramsal kodlama yöntemiyle incelenmiştir.

Sonuç olarak, matematik eğitimi alanında yayınlanan modelleme tezlerinin daha çok yüksek lisans tezlerinden oluştuğu görülmüştür. Bu alandaki ilk modelleme çalışması 2005 yılında tamamlanan bir yüksek lisans tezidir. Modelleme konusunda yapılan yüksek lisans ve doktora tezlerinin 2000' den 2019 yılına doğru giderek arttığı tespit edilmiştir. Türkiye'de 24 farklı üniversite matematik eğitimi alanının modelleme konusu kapsamında akademiye katkı sağlamıştır. Bu çalışmalarda araştırma yöntemlerinden en çok nitel araştırma yöntemi kullanılmış ve bunu sırasıyla karma desen yöntemi ve nicel araştırma yöntemi takip etmiştir. Bu 42 modelleme çalışmasının 19'unda, yaklaşık yarısında, kullanılan modelleme etkinliği sayısı 3 ile 6 arasında değişirken, diğer 23 çalışmada çeşitli sayılarda aktivite dağılımı görülmüştür. Bu çalışma ile Türkiye'de yüksek lisans ve doktora çalışması kapsamındaki modelleme tezlerinin genel durumu anlaşılmaya çalışılmış ve bu anlamda modelleme alanında çalışan araştırmacılara yol gösterici olması beklenmektedir.

Anahtar Kelimeler: Modelleme, Matematiksel Modelleme, Matematik Eğitimi, Model ve Modelleme Perspektifi, Model Oluşturma Etkinliği

To myself

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ABBREVIATIONS

HEC	Higher Education Council
MEA	Model Eliciting Activity
MMP	Models and Modeling Perspective
MoNE	Ministry of National Education
NCTM	National Council of Teachers of Mathematics
OECD	Organization for Economic Cooperation and Development
STEM	Science, Technology, Engineering and Mathematics

CHAPTER 1

INTRODUCTION

The changes that occurred with the opportunities of science and technology have been reflected in the lives of individuals. In order to adapt these reflections, the changes were adopted to the curriculum. In line with these developments in education, there have been great innovations in mathematics teaching. The innovations in mathematics teaching are aimed to develop the mathematical skills of students by contributing to their problem solving and analyzing skills. In this regard, mathematical modeling helps individuals to gain mathematical skills by overcoming the difficulties they faced during problem-solving. Since the innovations in mathematics teaching and its' significance in individuals' lives have increased recently, mathematical modeling studies have been increased collaterally.

The National Council of Teachers of Mathematics (NCTM) has been the driving force for changes in the teaching of mathematics. Findings in mathematics teaching revealed that solving traditional mathematics problems, which are oriented to find a direct solution, is not sufficient to enrich students' mathematical learning. According to NTCM (2000), learning mathematics reaches its' highest level when teachers prioritize mathematical thinking and reasoning in mathematics classes. Therefore, it began to seek ways to teach students mathematics in different ways by reorganizing mathematics teaching with consideration to mathematical modeling.

Emerging methods stem from the fact that current trends and methods are not sufficient to train a new generation already skilled in the field of Science, Technology, Engineering, and Mathematics (STEM) (Bulgar, 2008).

This is why STEM training is needed in order to acquire and develop new skills for a new generation. Pitt (2009) states that:

> Some people define any activity that involves any of science, technology, engineering or mathematics as a STEM activity; others argue that intrinsic to the concept is some linking of two or more of the component areas of learning and that real STEM must be more than the sum of its parts. (p. 41)

Modeling is considered as a fundamental aspect of STEM instruction because modeling may construct a potential bridge between science, technology, and mathematics education (Gilbert, Boulter & Elmer, 2000). The integration of STEM education in the mathematics curriculum revealed two perspectives: context integration and content integration (Kertil & Gürel, 2016). In context integration, one subject area is chosen as a basis, and it is taught by integrating connected contexts from different subject areas. Differently in content integration, different subject areas are combined in a flexible and organized way to use in a mathematics curriculum. Models-and-modeling perspective, which is an effective educational perspective based on theory-based sociocultural and constructivist theories, allows students to understand and interpret real-world problems or situations by using their critical thinking skills. Correspondingly, mathematical modeling helps students to understand the real world better by interpreting, generalizing, sharing, and revising similar situations in the process of developing models (Kertil & Gürel, 2016).

Since mathematical modeling helps students develop their problem solving and analytical thinking skills, it is one of the most appropriate learning processes to be applied in school mathematics (Lesh & Doerr, 2003). Mathematical modeling also allows students to internalize underlying mathematical concepts of problems in real life (NCTM, 2000). There are different modeling perspectives, and the models-and-modeling perspective is one of them, and it has a specific form of modeling activities called Model-Eliciting Activities (MEAs).

MEAs are problem-solving activities involving conceptual tools that are sharable, changeable, and reusable to construct, explain, estimate, and control mathematically significant systems (Lesh & Doerr, 2003). It has been shown in many studies that MEAs provide great facilities for students to learn analytical thinking, problem-solving, analyzing, and conceptual learning of basic mathematical ideas in real-life problems (Chamberlin & Moon, 2005; Lesh & Zawojewski, 2007). Students intuitively explore mathematical ideas of real-life problems and construct models in MEAs (Erbaş, Çetinkaya, Alacalı, Kertil, Çakıroğlu & Baş, 2014). Implementation of well-constructed MEAs in classes takes a proportionally short period of time so that teachers can manage MEAs in the classroom for a few hours (Erbaş et al., 2014; Kertil & Gürel, 2016). The effectiveness of MEAs utilization in classes shows that MEAs are applicable for the mathematics curriculum (Moore, Doerr, Glancy & Ntow, 2015).

The implementation of MEAs as an instructional tool was recommended to use with STEM training as a context integration approach (English, 2017; Hamilton & Brilleslyper, 2008; Magiera, 2013). By incorporating MEAs into the school curriculum, students can engage in both collaborative mathematical thinking and productive engineering design processes. During this process, mathematics, physics, and other STEM concepts can be taught effectively by qualified teachers who know how to implement MEAs in the classroom. Researchers are investigating various competencies that trainers must follow to implement MEAs effectively. Some of the competencies are: (i) knowing how to manage and organize the classroom in a MEAs process, (ii) giving useful and motivating responses to students, and (iii) finding quick solutions to deal with unpredicted situations (Schorr & Richard, 2003; Doerr & English, 2006).

Teachers also need to be aware of the difficulties that may be encountered during the implementation of MEAs in the classroom. Some of the difficulties that teachers faced are; (i) the insisted behaviors of students for feedback, (ii) the desire of students to get approval for their strategies (Moore et al., 2015), (iii) the resistance of students in working together (Eraslan & Kant, 2015), (iv) the desire of students for a quick solution without detailed thinking, and (v) the desire of students to ask for an evaluation of their answers (Zawojewski, Lesh & English, 2003).

On the other hand, students also encounter difficulties while solving MEAs for the first time. Some difficulties they face are stated as follows: (i) not understanding the problem, and (ii) not developing a well-constructed or sufficient model (Eraslan & Kant, 2015). Although researchers described the difficulties faced by trainers and students during the implementation of MEAs, there are not enough resources focused on how to overcome these difficulties.

Mathematical modeling appears to be a method of facilitating the adaptation of mathematics to daily life. The Organization for Economic Cooperation and Development (OECD) works on mathematical modeling since it predicts that there can be a better education method for a better future generation: “Capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to engage in mathematics, in ways that meet the needs of that individual’s life as a constructive, concerned, and reflective citizen” (OECD, 2000, p. 50). With the PISA exam that students in OECD countries undertake, the importance of the use of mathematics in various contexts is mentioned in order to overcome the real-life difficulties of people with mathematical knowledge. The knowledge of the students’ learning mathematics was measured, and the importance of mathematical modeling started to be discussed (Turner, 2007).

In Turkey, modeling research also became popular in recent decades. Therefore, I intended to examine modeling research depicted by the theses and dissertations on mathematical modeling that were listed in the Higher Education Council (HEC) of Turkey thesis and dissertation database, in order to understand the portrayal of modeling research in Turkey. In addition to this portrayal, I also analyzed the modeling activities involved in these theses and dissertations to understand to what extent they reflected the six design principles of MEAs of Models-and-Modeling Perspective.

1.1. Purpose of the Study

The purpose of this study was to examine modeling theses and dissertations, conducted in Turkey between 2000-2019, concerning the features and the nature of the studies and the modeling activities they involved.

1.2. Research Questions of the Study

The main purpose of the study was to understand modeling research in Turkey. Particularly, the following research questions were addressed:

1. What are the features of modeling research depicted by the theses and dissertations on mathematical modeling that were listed on the HEC thesis and dissertation database?
    a. What is the distribution of the modeling research in Turkey in terms of the type of work (i.e., master's thesis or doctoral dissertation)?
    b. What is the distribution of the modeling research in Turkey across the years?
    c. What is the distribution of the completed modeling research in Turkey across universities?

2. What is the nature of modeling research depicted by the theses and dissertations on mathematical modeling that were listed on the HEC thesis and dissertation database, and that involves at least one mathematical modeling activity?
    a. What is the distribution of the modeling research involving at least one mathematical modeling activity in terms of the research methods employed by the researchers?
    b. What is the distribution of the modeling research involving at least one mathematical modeling activity in terms of the participants that they focused on?
    c. To what extent do the mathematical modeling activities involved in the theses and dissertations that were listed on the HEC thesis and dissertation database indicate the six design principles of the Models-and-Modeling Perspective?

1.3. Significance of the Study

For 37 years, mathematical modeling has been included in several congresses arranged by the International Commission on Mathematical Instruction (ICMI) and International Community of Teachers of Mathematical Modeling and Applications (ICTMA), which indicates an increase in the research of mathematical modeling (Blum, 2002). Recently, many researchers are conducting mathematical modeling studies with students at different grade levels and in different subjects (Blum & Borromeo Ferri, 2009; Kaiser, 2006).

In this study, I analyzed completed master theses (MS) and doctoral dissertations (Ph.D.) on modeling published in the last two decades in the field of mathematics education in Turkey. Firstly, I reflected on the general features of the modeling studies in this field. Secondly, I investigated the nature of the modeling activities with their alignment to the models-and-modeling perspective. It was determined to reflect both the quantity and the quality of modeling studies by this way. In other words, the quantity of the studies was presented by indicating the general portrayal of modeling studies in the feature analysis, and the quality of the studies was considered by linking between the theory (i.e. modeling perspective and six design principles) and the practice (i.e. implemented modeling activities) in the nature analysis of this study.

This study aims to present the current status of modeling research showing the researchers' direction of the investigation of this topic in Turkey. Furthermore, this study also presents the gaps in the modeling research in Turkey. It is important to conduct such a document analysis in order to see the scope and type of modeling research that was carried out and to reveal the issues that require more investigation from a different perspective or the research that need to investigate particular participants more than others. In this sense, this study is expected to shed light for the researchers who want to study modeling in Turkey, to broaden the researchers' point of view, and to provide acceleration to the studies in mathematics.

Modeling studies, conducted in Turkey, generally focus on MEA implementation in classes. There was only one study that was conducted on the descriptive content analysis of mathematical modeling research in Turkey, similar to this study to some extent (Albayrak, 2017). In this study, 38 articles, and 28 theses and dissertations were utilized to compare descriptive features of the articles and theses/dissertations (Albayrak, 2017). However, a larger data set consisting of 70 modeling theses and dissertations were utilized in the current study and so provided a broader overview of modeling research conducted in Turkey. Moreover in this study, besides the descriptive analyses, the nature of 42 modeling theses and dissertations, which involved modeling activities, were investigated by considering the six design principles of models-and-modeling perspective. From this aspect, this study serves as an extensive study with a different perspective on the field of modeling in mathematics education.

1.4. Definition of Terms

Model is a reflection of conceptual systems containing rules that are regulating interactions that are defined with external notation systems and used for constructing, describing, explaining, or predicting other systems' actions. Besides rules, a model consists of elements, operations, and relations, as well (Lesh & Doerr, 2003).

Mathematical Modeling can be defined as a process that includes a prediction of relationships, observation of a phenomenon, implementation of mathematical analyzes, handling of mathematical results, and re-explaining the model (Swetz & Hartzler, 1991).

Models-and-Modeling Perspective (MMP) is an educational perspective, which uses mathematical thinking effectively when it is required to be used in creating, revising, or producing new conceptual systems/models (Lesh & Zawojewski, 2007). Models-and-modeling perspective is also a problem-solving perspective in which the problems are not stereotypical textbook problems; they are thought-revealing problems called model-eliciting activities.

Model-Eliciting Activities (MEAs) are particular tools proposed by models and modeling perspectives (Moore et al., 2015). These activities do not involve traditional problems; on the contrary, these activities allow non-routine, challenging, and open problems. These activities include a mathematical modeling process and require creating a product called a model (Wessels, 2014).

The Six Design Principles of MEAs are principles which contribute to the production design of MEAs that include: model construction principles, real-life principles, self-assessment principle, construct documentation principle, construct shareability and reusability principle, and the effective prototype principle (Lesh, Hoover, Hole, Kelly & Post, 2000).

CHAPTER 2

LITERATURE REVIEW

The purpose of this study was to analyze the features and the nature of completed modeling theses and dissertations in the field of mathematics education by focusing on modeling perspectives. In line with the purpose, this chapter presents the related literature on modeling in mathematics education and modeling perspectives in mathematics education. The chapter is divided into two parts. The first part presents modeling as the theoretical framework of the study. In the second part, previous studies on MEAs are described.

2.1. Theoretical Framework

In this section, first of all, model, modeling, and mathematical modeling concepts are introduced, then different modeling perspectives and MEAs in the models-and-modeling perspective are explained. Lastly, the six design principles are described in detail.

2.1.1. Modeling in Mathematics Education

Mathematics education aims to provide the necessary mathematical knowledge to individuals and to gain mathematical abilities while teaching problem solving and analyzing situations. However, students see mathematics as a tedious lesson that needs to be memorized in order to pass the examinations and perceive it as a lesson independent of real-life (Baki, 2006). These prejudices that students feel about mathematics naturally pose general failures (Soylu & Soylu, 2005).

However, mathematics is a system of ideas where modeling is used to overcome the problems we face in our daily lives (Durmuş & Karakırık, 2006). The importance of mathematics is needed to be emphasized while solving real-life problems in mathematics classes (Kaiser & Schwarz, 2006).

In order to establish a strong relationship between mathematics and real life, the definitions of mathematical models and mathematical modeling needs to be visited. A model is a complex map or simplified representation of an object. This model can be an object created as a result of the visualization of an idea (Gilbert, Boulter & Elmer, 2000). In this sense, it is the external representations of structures that exist in the mind about real-life situations.

Many researchers have classified the models according to their different characteristics. Although a general definition of the model could not be identified, they described the model by addressing the common features of several definitions. According to Güneş, Gülçiçek, and Bağcı (2004), a model has the following properties:

a) It is associated with the goal or objective. The goal can be an object, a system, a process, or a phenomenon.
b) It is a research tool used to obtain information about a target that cannot be directly measured or observed. Therefore, a spectrum or photograph is not considered a model.
c) It is developed after the consecutive interacting processes, not one step after the goal is determined.
d) It provides researchers with the opportunity to create testable hypotheses about the intended concept by making appropriate analogies.

While creating a model, it is better to take into consideration the similarities and differences between the model and the goal in order to make predictions about what the model represents (Lesh & Yoon, 2004).

There are many similarities between the models that children develop and the ones that the scientists develop. Children develop models in order to understand the structurally interesting systems they encounter in their daily experiences. Scientists develop models to define and explain the behavior of the complex systems as they try to perceive or explain it (Sağırlı, 2010). It is also possible to encounter models in business and social life. Many companies use models to save money, since using real-designs instead of models can cause severe economic losses for companies. To prevent these losses, scientists have been experimenting and developing computer-based simulations (models). Models are used effectively in every kind of science, in every field of working life, and many situations encountered by individuals in their daily lives (Doruk, 2010).

Models can explain a situation, describe it mathematically, and interpret it thoroughly (Lesh & Doerr, 2003). In this sense, mathematical models focus on functional principles and structural characteristics of real-life situations (Lehrer & Schauble, 2003). Mathematical models contain a range of operations, representations, and relations that help to make sense of real-life situations.

Mathematics was often seen as a lesson that was done only in schools, and it was discriminated from in real-life. However, in today's world, this perception has changed in that people give importance to meaningful information, which is based on previous experiences. Today's information society connects mathematics with daily life, and it is seen that mathematics establishes a relationship between real-life situations and our understanding of them. One of the most important methods used to embody mathematics is modeling. Modeling is the mental process of interpreting (defining, explaining, or creating) events and problems; organizing, coordinating,

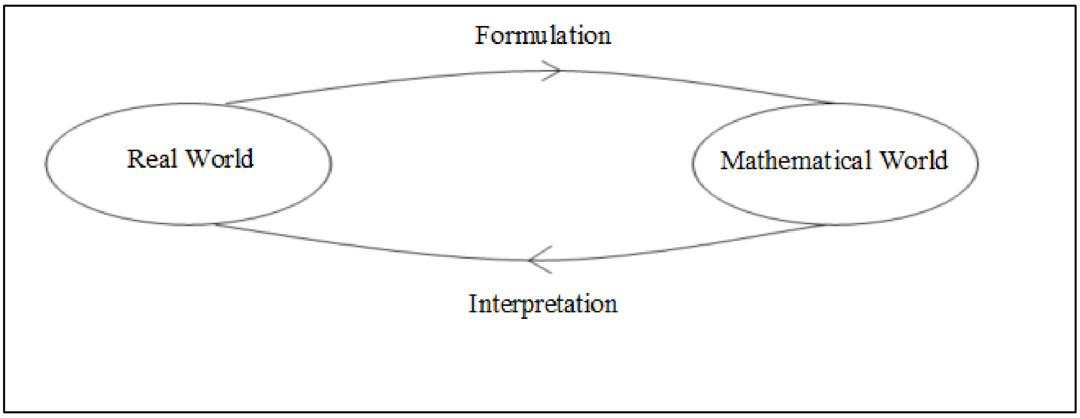
systematizing, and finding a pattern; using different schemes and creating new ones. According to Gravemeijer and Stephan (2002), models emerge as a result of the non-formal activities of students in the classroom environment.
In these informal activities, mathematics appeared to be a systematic way of thinking that produces solutions to real-world problems through modeling (Kaiser & Sriraman, 2006).

There is no consensus on the integration of meaning, purpose, use, or application of mathematical modeling into the mathematics curriculum in classroom settings (Gravemeijer & Stephan, 2002). However, in most of these descriptions, it is mentioned that real-life situations are translated into mathematical language and interpreted mathematically, which is referred to as mathematical modeling.

According to Swetz & Hartzler (1991, p. 3), "[*m*]athematical modeling is a mathematical process that involves observing a phenomenon, predicting relationships, applying mathematical analyzes, obtaining mathematical results, and reinterpreting the model". Blum and Niss (1991) explain mathematical modeling as a procedure for explaining real-life situations. In the process of mathematical modeling, a subject is discussed in detail and expressed mathematically. In this sense, mathematical modeling is a multifaceted problem-solving process (Blum and Niss, 1991). Niss (2003) also defined mathematical modeling as a combination of one or more mathematical constructs that were selected to represent some of the real-world situations, their expectations, and the relationships between them. According to Verschaffel, Greer, and De Corte (2002), the most general definition of mathematical modeling is that it is a process of defining nonmathematical circumstances or phenomena mathematically, expressing the relations between events, and revealing mathematical patterns within those events and phenomena.

Berry and Houston (1995) state that mathematical modeling can be expressed simply as an interaction between the real world and the mathematical world, which is shown in Figure 2.1 below.

Figure 2.1. A simple representation of mathematical modeling (Berry & Houston, 1995, p. 24)



According to this loop, a problem in the real world is converted to a mathematical problem by formulating, and the mathematical results of this problem deliver a solution to the real-world problem by interpreting them in the real world. In other words, the real-world problem is expressed in the mathematical language that is demonstrated as a formulation. Then this formulation is tested and interpreted in the real world.

Similarly, in the models-and-modeling perspective, Lesh and Doerr (2003) argue that a student in the modeling process passes through a continuous cycle with certain steps. According to Lesh and Doerr (2003), the modeling cycle follows the four steps shown in Figure 2.2 below.

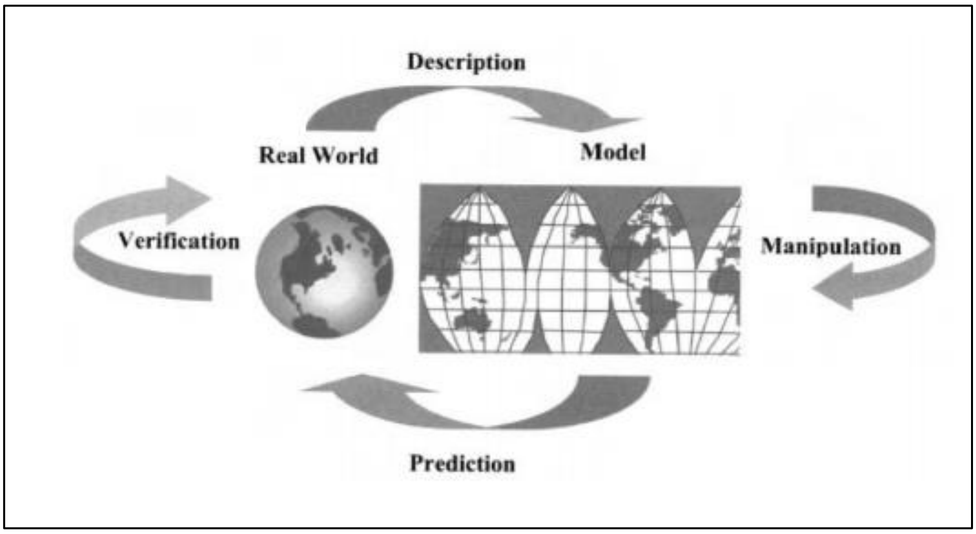


Figure 2.2. Four steps in a modeling cycle (Lesh & Doerr, 2003, p. 17)

As seen in the figure, the first step of the modeling cycle is "description", which is the process of transferring the given daily life situation from real life to the model. In the second step, "manipulation", operations related to the problem situation is carried out, and assumptions are made about the model based on the description process. The third step is the "prediction" step, which involves estimation and transfer. At this stage, the most effective solution to the problem is chosen after evaluating the possible consequences and applied to real life. In the last stage verification, the accuracy and usefulness of the model are examined. The mathematical modeling cycle (see Figure 2.2) can be repeated multiple times during the modeling process.

Unlike solving traditional word problems in textbooks, the process in mathematical modeling is not linear (Lesh & Harel, 2003). During repeated cycles in the process, students can make multiple interpretations of the problem situation, produce different ways of thinking, and decide on the most appropriate model. In other words, students who are in the process of modeling cycle shape findings with necessary modifications, test their suitability, and revise them to find the most appropriate result. In this process, students learn to evaluate their ideas, discuss their opinions with peers, and to communicate with each other appropriately.

The cyclic structure is emphasized in most of the definitions of the mathematical modeling procedure. While Lesh & Doerr (2003) mentioned four basic steps (see Figure 2.2) during the mathematical modeling process, Blum & Leiß (2007) used a six-step modeling cycle while explaining the modeling process under a cognitive perspective (see Figure 2.3).

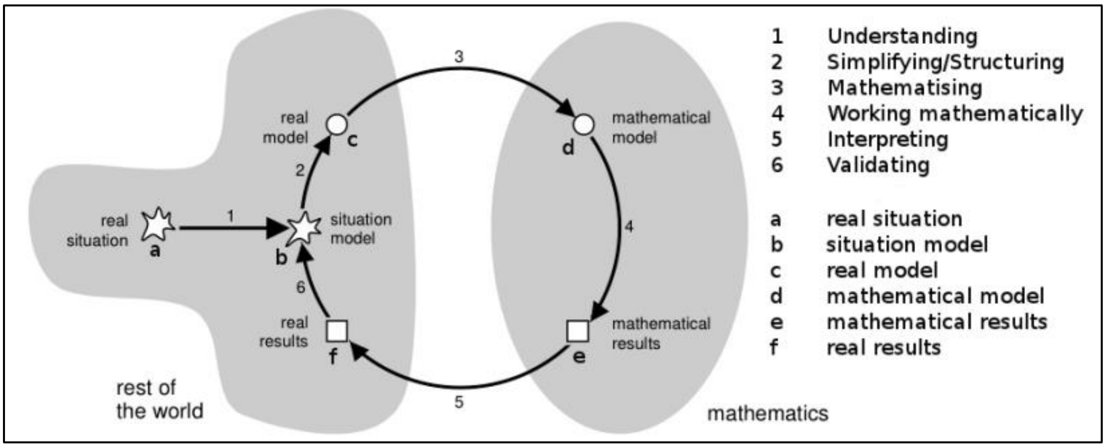


Figure 2.3. Mathematical modeling cycle (Blum & Leiß, 2007, p. 225)

In Figure 2.3, “a” represents a real-life situation. In the process of “1”, this real-life situation is understood, and a situation or conceptual model is constructed, named “b”. During the second process “2”, necessary simplification is made by eliminating unnecessary variables and circumstances from the essence. Then the real model appeared, named “c”. In the process of “3”, the real model is denoted through mathematizing, then a mathematical model “d” is distinguished as an expression of the key variables. The process “4” designated the mathematical work for a meaningful result, and mathematical results “e” are achieved at the end of it. Acquired mathematical results were evaluated and interpreted to obtain real results in the procedure “5”, and real results appeared as “f”. Lastly, the validation process of the real results against the situation model occurred and is called “6”. It is important to note that the modeling process is as important as the product reached at the end of this cycle. Therefore, all of the steps involved in a modeling cycle have a significant impact on students' learning. To understand different perspectives on the utilization of modeling in mathematics education, different modeling perspectives were reviewed and presented below.

2.1.2. Different Modeling Perspectives

There are different approaches to the use of modeling in mathematics education within the framework of different theoretical backgrounds, and no common understanding has yet emerged in international studies (Kaiser& Sriraman, 2006). While some researchers (Lesh & Doerr, 2003a, 2003b) adopt modeling as a paradigm beyond constructivism in mathematics education, as a new approach in interpreting education and teaching, some researchers think that mathematical modeling as a way of expressing real-life situations in mathematical language, and real-life applications of prepared mathematical structures, models, and formulas (Haines & Crouch, 2007). Scientific studies analyzing the differences in understanding mathematical modeling in a detailed and systematic way is not sufficient (Kaiser, Blomhoj & Sriraman, 2006). Therefore, it is not yet possible to talk about a theory that is accepted all over the world about the teaching and learning of mathematical modeling (Kaiser et al., 2006).

In the congresses organized by the ICMI and ICTMA, Kaiser and Sriraman (2006) attempted to make a classification that provided a useful point of view, considering the general objectives and theoretical frameworks of modeling studies. By understanding the importance of modeling, Kaiser and Sriraman (2006) distinguished the following perspectives:

- A pragmatic perspective is focusing on utilitarian or pragmatic goals by applying mathematics to solve practical problems.
- A scientific-humanistic perspective of mathematics is an ideal of science and humanitarian education, with a focus on the students' ability to create relationships between mathematics and reality.

- An integrative perspective requires applications and modeling to serve purposes at different levels, namely scientific, mathematical, and pragmatic purposes, but in a coherent relationship. This perspective is not limited to specific objectives and derives its power from a wide range of objectives and discussions.

These perspectives were identified as the three main streams. Other approaches were also explained as follows:

1. The realistic or applied modeling approach aims to develop students' problem solving and modeling skills. In this approach, students are given problem situations from engineering and other disciplines, and they apply the mathematical knowledge they have learned in different contexts.
2. The educational modeling approach can be considered as a kind of mix of the realistic and contextual modeling approach. In this approach, mathematical modeling aims to teach students the concepts by creating appropriate learning environments and processes.
3. The epistemological or theoretical modeling approach focuses on the relationships between mathematical concepts and the students that affect the process of each step problem-solving. According to this approach, the realistic context in modeling activities have less importance, and the effort of mathematizing is accepted as a modeling process.
4. The cognitive modeling approach focuses on analyzing the cognitive and metacognitive thinking processes of the students during the modeling process. According to this approach, modeling activities provide a guiding environment for teachers to understand and support students' thinking processes.
5. The contextual modeling approach justifies the philosophy that knowledge is organized around experience, at least as much as abstractions and the ways of thinking necessary to make realistic complex decision-making situations. Almost always, there is a need to combine more than one discipline or subject matter or major theory.

Incomprehensible situations are not given to students as a real-life situation in a problem context. Thus, it is assumed that students can learn mathematical concepts more meaningfully by experiencing them in appropriate contexts. These mathematical concepts are understood by model-eliciting activities, which indicate another perspective, Models-and-Modeling perspective (MMP), which will be explained further in detail in the following section.

2.1.3. Model-Eliciting Activities in the MMP and Six Design Principles

Lesh and Doerr (2003) developed modeling activities to improve students' mathematical and high-level thinking skills during the modeling process called Model-Eliciting Activities (MEAs). Model-eliciting activities can be defined as problem-solving activities that are created by using special principles in which students make inferences from meaningful real-life situations, invent and expand their mathematical structures, and review and organize them (Lesh & Doerr, 2003). The difference between traditional and model-eliciting problems is while traditional problems use a given specific procedure to solve the problem, the model-eliciting problem prioritizes the process itself, and it considers the modeling process as a part of the solution.

Model-eliciting activities, which have different entry points in the real-world context, are complex non-routine problems. MEAs are formed for students to emphasize a deeper and conceptual understanding is expected to be developed as a team while creating models (Lesh & Doerr, 2003).
Students are expected to develop higher conceptual models by differentiating or deepening the usual conceptual structures and systems. To achieve this, students need to see, understand, organize, and adapt their initial mathematical interpretations as a whole (Lesh & Yoon, 2004).

The MMP rejects the idea that only a few exceptionally bright students can develop important mathematical concepts unless they are provided with continuous guidance from a teacher. The literature includes studies that contain transcripts of model-eliciting activities in which the underlying structures or conceptual systems of models (and conceptual tools) that students developed to make sense of situations (Lesh & Doerr, 2003). There are two main reasons for the use of MEAs. The first, students develop the mathematical problems in the context of real-life given their knowledge of mathematics and have the opportunity to reach new information. The other is that teachers can see and examine the mathematical thoughts of their students during this period (Chamberlin & Moon, 2005).

Modeling activities, as the name implies, are problem-solving activities that reveal a model. That is, their solutions require students to express their current thinking in many tested and refined forms. Therefore, final solutions include conceptual systems formed by models. The principles for designing model-eliciting activities include:

- Students are expected to be able to engage in problem-solving activities in which they recognize the need to review or improve existing ways of thinking about the situation,
- Students are expected to be able to express their current understanding in ways that they can express, test, and revise multiple times,
- The conceptual tools developed by the students are expected to be shared and reused beyond the specific situations in which they have been developed (Lesh, Hoover, Hole, Kelly & Post, 2000).

A productive MEA is described with the six design principles proposed by Lesh et al.(2000) as follows:

1. Model Construction Principle

The first principle, model construction principle, shows the need to create a model for the solution of the problem in the MEA. In the model construction principle, the following questions are tried to be answered:

- Does the given situation require students to create a model by interpreting the givens, objectives, and possible solution procedures? (Lesh, et al., 2000; Lesh & Caylor, 2007).

In the most basic sense, this principle states that the problem situation in the MEA requires modeling (Chamberlin & Moon, 2005; English, 2009; Lesh et al., 2000; Lesh & Caylor, 2007). This is because the aim of the MEA is not only to make a decision but also to develop a suitable tool that enables making decisions (Lesh et al., 2000). Students need to know in which situations they develop, correct, expand, and review models. Situations required for the development of models are:

- Expectations about real events, reconstructing past circumstances, or adapting events that cannot be reached to make predictions by considering proper relationships.
- To define the patterns and relationships involving lots of data (Lesh et al., 2000).

It is important to note that these situations mentioned above involve realistic context because of the relationship of the problem with real-life enables adoption to the problem. Therefore, it is necessary to look at the second principle, which is the reality principle.

2. Reality Principle

The main purpose of the reality principle is to enable students to establish the connection between the problem and real life. The most precise way to determine whether an MEA provides this principle is to try to answer the question:

- Can this situation happen in the real life of the student or any person that the student is familiar to? (Lesh & Caylor, 2007; Lesh et al., 2000).

A situation that is meaningful in the real life of students does not have to be meaningful for adults (Lesh & Caylor, 2007). Furthermore, students can have different lifestyles dependent on a social environment, families, and socio-economic situations that affect the real-life perception of students.

For example, the lifestyles and school situations of a student who lives in a big city and whose family works in management positions are quite different from a family who lives in a village and whose family works in farming. This situation causes real-life experiences to be different. At this point, in order to meet the reality principle, the question content can be chosen carefully by considering students' real-life conditions to provide them a connection with their life. Therefore, students can interpret situations based on their real-life knowledge and experience more comfortably (Lesh & Caylor, 2007).

MEAs possess this principle to keep the student's interest alive and attract attention. With this principle, it is expected that students are able to make a connection with their daily lives and to be motivated to solve a problem when they feel the problem has occurred from a necessity. The topics in an individual's life prepare the ground for the person to understand the problem more easily. Another important point in the solution of real-life modeling is the processes of self-evaluation. This is the third principle of the six design principles.

3. The Self-Assessment Principle

The self-assessment principle refers to the students' self-assessment during the problem-solving phase. This principle indicates that students are able to self-assess the suitability and usefulness of their solutions, without teacher support or approval (Chamberlin & Moon 2005).

In order to find out whether the MEA reflected the self-assessment principle, the questions below are tried to be answered:

- Does the problem situation require appropriate criteria to evaluate alternative solutions?
- Is the purpose of the problem situation clear?
- Will students be able to evaluate themselves when their answers need to be improved?
- Will the students realize that they have completed the solution to the problem, or will they need to ask their teacher if they need to continue to solve the problem? (Lesh & Caylor 2007; Lesh et al., 2000).

During the MEAs, students are expected to evaluate whether their solutions need to be revised and in which direction they can proceed, and choose the ones that are useful from many alternative solutions to achieve the given purpose (Lesh et al., 2000). Also, Doerr and English (2006) stated that teachers are expected to find teaching strategies in MEA implementation without informing about the correctness of the students' solution approaches and to enable students to make self-evaluations.

Lesh et al. (2000) emphasized that the self-assessment principle was particularly important because acceptable solutions for MEA’s required multiple modeling cycles and the students’ effort to find solutions in groups. While reaching the solution in the group work, it is important to consider the following issues:

- Students compensate for deficiencies in their current thinking patterns.
- Students evaluate each of the alternative ideas and select the ideas that work and eliminate the rest of the ideas that do not work.
- Students integrate the strengths of alternative ways of thinking while minimizing their weaknesses.

- Students reorganize and correct the most appropriate interpretations to solve the problem.
- Students evaluate the new adaptations made (Lesh et al., 2000).

With the contribution of the self-assessment principle, a model constructer learns to find suitable ways to deal with a problem. Besides that, identifying the proper tools which would enable them to reach the solution and the tools themselves are both important. Thus, it is necessary to look at the fourth principle, which is the construct documentation principle.

4. The Construct Documentation Principle

In this principle, students develop their own ways of thinking and documentation for the solutions during the process of MEAs (Chamberlin & Moon, 2005).

The main question in the construct documentation principle is:

– Do responses of students to a problem reveal how they think about the situation? (Lesh & Caylor, 2007; Lesh et al., 2000).

In this principle, teachers are expected to encourage students to understand and interpret mathematical IDEAS and to use these notations to communicate (Doerr & English, 2006). English (2009) emphasized the importance of this principle by saying that the notations used by students while creating their models are also needed to contain explanations revealing the thinking behind.

When the solutions to a problem situation in the MEA needed to be created, the model constructor is expected to present the model understandably by detailing their thinking as much as possible. Basically, this principle explains what MEAs produce and the students' way of thinking (Chamberlin & Moon, 2005). In other words, the existence of this principle sheds light on the researchers when MEAs are used in research to reveal what students think (Lesh & Caylor, 2007).

Model documentation helps students' self-assessment because when they express their way of thinking, they can see the missing aspects or the weaknesses of their models. Also, one way to ensure the students' documentation of thinking is to work with a group exchanging ideas (Lesh et al., 2000).

Another important feature of modeling is to reveal the generalizable properties of the solutions to use in different problems. Therefore, ways for solutions need to be documented. The construct documentation principle not only provides documentation of the solutions but also contributes to the construct shareability and reusability principle, the next principle.

5. The Construct Shareability and Reusability Principle

This principle is also known as the model generalization principle that can be adapted to similar situations comfortably (Lesh et al., 2000).

The main question in the construct shareability and reusability principle is:

- Is the developed model useful only for the person who developed it, or does it provide a way of thinking that can be shared, transformed, easily adapted, and reused in different situations? (Lesh & Caylor 2007; Lesh et al., 2000).

In line with this principle, it is emphasized that a model can be used several times for different purposes rather than using a specific situation or purpose (Lesh & Caylor, 2007). According to Chamberlin and Moon (2005), the solution can be considered successful if the developed model is generalized to different situations that require a similar model. Being able to adapt to other problems provides us convenience for different problems. A generalizable model is expected to be easy to understand and to be applied to another situation, which indicates the efficiency in the model and modeling process.

6. The Effective Prototype Principle

The effective prototype principle searches for an answer to those questions:

- Does the developed model create a prototype useful for other structurally similar situations?
- Will students be able to consider the previous problem in structurally similar situations long after the problem is solved? (Lesh & Caylor, 2007; Lesh et al., 2000).

It is expected from students to create models that are mathematically equipped and easily understandable for complicated problems in a simple way. Lesh & Caylor (2007) state that this principle ensures the solution can be remembered by the students. In context, this principle seems to be similar to the construct shareability and reusability principle.

The difference is that students are expected to use the model which they constructed due to similar but not parallel situations (Chamberlin and Moon, 2005). According to Tekin (2012), a model is needed to be remembered and usable when a different situation occurs after a while to ensure an effective prototype principle.

2.2. Review of the Related Literature

In this part of the thesis, the studies designed with the modeling perspective of international and national sources were reviewed under two subsections, respectively.

2.2.1. International Studies

Matson (2018) examined how teachers learned mathematical modeling and the change they had after learning it by sharing their point of view. The teachers chose to attend a professional development initiative and apply mathematical models in their classrooms to study mathematical modeling throughout the learning process.

The results have pointed the following inferences. First of all, teachers had the chance of being free while choosing, and of experiencing the mathematical models as students do, these opportunities helped them to learn the approach. Secondly, the teachers saw that their instructional practices had inadequacies, and it needed to be modified. In addition to these, the teachers' perspective and emotions about mathematics and mathematical modeling were enhanced positively by learning, using, and teaching activities.

In another study, Jung (2015) attempted to understand the strategies two middle school teachers applied to their students throughout MEAs. A researcher and two eighth grade teachers taught together advanced modeling lessons with the MEAs during a semester. These two teachers applied three kinds of MEAs in their classes over eleven weeks. The data sources of this study were audiotapes, including interviews and discussions of the teachers, and written documents of their students. The improvement of the students about the modeling process was handled by the strategies chosen by the researcher. There were six principles of the MEAs that the strategies were classified under because these principles need to be satisfied for productive models.

As a conclusion, Jung (2015) pointed out the followings:

- Concerning the reality principle, a teacher can ask students questions to check whether they understand the assignment based on their own real-life experiences;
- Concerning the effective prototype principle, a teacher can guide students by questioning them to produce a useful model;
- Concerning the model generalizability principle, a teacher can provide students information to remember so that they can build up a generalizable and reusable model;
- Concerning the model documentation principle, a teacher can ask students to record their process in a written document;

- Concerning the model construction principle, a teacher can supply sources of presentations and discussions that students can see another alternative solution; and
- Concerning the self-evaluation principle, a teacher can request students assess their responses by using peer-review forms.

In addition, Sol, Giménez, and Rosich (2011) revealed the modeling behavior of twelve to sixteen years old students based on written studies in realistic mathematical modeling projects. They conducted a four-week study on modeling activities with groups of two or four students aged between 12–16. During these activities, a new modeling process framework was created by combining the stages of the modeling process defined by different researchers to determine the behavior of the students in the modeling process. The modeling behavior of students was observed by using a model-eliciting activity (MEA) named Nadal Castle. It was seen that students had difficulty in constructing models during the modeling process. The difficulties students had were (i) understanding the problem while modeling, (ii) using variables, (iii) discovering mathematical relations, (iv) ensuring the validity of the model, and (v) communicating during the modeling process.
The conclusion of this study emphasized that the reasons for these difficulties could be related to the deficiency of the instructional program of primary school and the inadequacy in expressing the processes correctly. In this sense, this study unfolded the difficulties students came across in initial steps and the difficulty in the verification of the modeling process.

Caron and Belair (2007) conducted a study which aimed at enhancing the ability of modeling of Mathematics Science's senior students by applying open-ended modeling projects in mathematical modeling classes. Nine Mathematics Science senior students of Montreal University attended the research voluntarily. A ten paper-survey was conducted about the students' educational life, their perceived level of mathematics, modeling, implementations, and technology. Students' attitudes toward mathematical modeling and their abilities at different stages were determined by evaluating their answers about mathematical modeling. At the end of the research, it was deemed that there was a statistically significant difference between the students' mathematical modeling ability and their attitudes. Furthermore, it was seen that the students' evaluation skills of real-life circumstances were related to understanding the aim of the modeling.

Blum and Leiß (2007) examined the behaviors of primary school students and teachers as they work on modeling problems. The conclusion discovered that students had difficulties in understanding problems and in expressing mathematical models properly. The reason for this was shown as not controlling the validation of the mathematical model. Regarding the students' performances on modeling, Maaß (2006) approached from a different and soughtfor an answer to the question of "What are the modeling skills?" in his experimental study. At a seventh grade level, five modeling activities that lasted 12 lessons of 45 minutes were applied to two parallel classes of 42 students. Besides the tests that determined the mathematical capacity of the students and modeling, tests were used as a data collection tool, written class tests, homework, notion maps to investigate the metacognitive proficiency of students, interviews of students, dairies of students, and surveys were also used. It was seen that most of the students became skilled in mathematical modeling activities (MEA), and low-degree students participated in the process. Thus, it was concluded that mathematical models were effective on not only high-degree students but also low-degree students; low-degree students could improve their mathematical modeling abilities. Even if students could not demonstrate all of their sub-modeling competencies, at least they were included in the introduction stage of the modeling process. In this study, the factors affecting the modeling competencies of students were determined, and the modeling abilities of students were explicated at the end of the process. Moreover, most of the students were able to configure appropriate metacognitive modeling competencies.

Lastly, Lesh and Harel (2003) performed a study to examine the similarities and differences of modeling cycles that were used by students at the implementation stage of MEAs. In the study, 60-90 minutes of group work was studied for modeling activities called the Sears Catalog Problem, the Big Foot Problem, and the Quilt Problem with three eighth grade students. The studies were recorded on video, and the results were analyzed. The students were introduced to powerful representation systems to express the related structures in the problem, and they were encouraged to think about these structures. Eventually, it was seen that students could develop a variety of different models that served as powerful solution to the problems.

2.2.2. National Studies

Zengin (2019) presented an approach on how to assess students while interpreting the modeling processes by incorporating the teachers in the students' modeling processes. Nine students from eighth grade participated in the research. During the study, written works and audio records of the students were used as data sources, and assessment rubrics were used as an assessment instrument. According to the results of the study, the following statements were deduced:

- Teachers were not expected to evaluate MEA with just one type of evaluation method; and
- It was needed to consider process-based evaluation besides evaluating result-based.

Furthermore, it was stated that teachers need to attend seminars to apply MEAs properly. On a side note, it was observed that motivating the speeches of teachers had a positive effect on students during MEAs.

Sağıroğlu (2018) researched mathematics teachers' creating activities oriented to mathematical modeling methods, and the examination of the implementation process. This research was a qualitative case study. Five mathematics teachers working at a high school were assigned to this study. These teachers received a training process for four weeks. In this process, teachers were informed about the mathematical modeling method, the properties of modeling activities, the creation of activities, and the implementation of processes. At the end of the training, the mathematics teachers were expected to create and apply mathematical modeling activities. The results obtained showed that the mathematical modeling knowledge levels of the mathematics teachers were not sufficient before the study. In the process of creating modeling activity, it was determined that the mathematics teachers' competencies at creating modeling activities were inadequate. Some of the teachers had difficulty in time management during the creation of the mathematical modeling process. Moreover, it was revealed that most of the teachers could not provide an environment for their students to follow the steps of mathematical modeling activities. Also, it was observed that teachers either intervened students' modeling process too much or did not help students at all.

Similar to Sağıroğlu's study (2018) mentioned above, Deniz (2014) conducted a study to observe the competencies of secondary school teachers in creating activities pertinent to mathematical modeling and the implementation of the activities created. Thirteen mathematics teachers working in different high school types participated in this study. Teachers were asked to create at least three activities based on the six design principles of MEAs and to implement these activities in their classes. As a data source, observation forms, pre-post interview forms, and teachers' activities were used. When the study was completed, the collected results showed that the activities performed by teachers in their classes were in accordant with the Reality Principle and Construct Shareability and Reusability Principle. On the other hand, the activities were partially in conformity with the Self-Assessment Principle. Furthermore, only some activities partially reflected the Model Construction and Construct Documentation Principles. Conveniences of activities concerning the Effective Prototype Principle were not examined in this study. In the interviews with the teachers, it was concluded that model-eliciting activities were different from the foreknown traditional activities, the MEAs developed students' reasoning skills, and daily use of mathematics was well understood with the help of MEAs. Also, some teachers stated that they would use the modeling activities only if the degree of the students' were good enough in classes.

In another study, Dedebaş (2017) examined the behaviors of fifth-grade students that emerged during Model-Eliciting Activities (MEAs) and how these behaviors changed in the process. The research was a qualitative case study. There were 31 fifth grade students in the study. Three different MEAs were implemented for five weeks. Video and audio recordings, written works of students, and field notes were used as data sources. At the end of the research, the continuous implementation of MEAs proved to reduce the difficulties that students were struggling with gradually. Another result was pointed out that the continuous implementation of MEAs was crucial for teachers who want to use MEAs in their classes.

In addition, Çelikkol (2016) wanted to examine the mathematical modeling steps that the seventh-grade students reached in mathematical modeling activities and their mathematical modeling competencies. Another reason for conducting this study was to observe the effect of model-eliciting activities (MEA) on students' success in the process of solving algebraic verbal problems. The study was a mixed study and action research. As a consequence of the study, it was seen that the students' success in solving algebraic verbal problems strongly depended on using MEAs' steps. Moreover, the success of students in solving algebraic verbal problems was increased with MEAs implementation.

From a teacher education point of view, Zeytun (2013) conducted a study to understand how teacher candidates created models while working on modeling activities. The opinions of prospective teachers about what factors might affect the modeling processes were investigated. The study was performed with six teacher candidates by implementing five modeling activities for 14 weeks. The study was a qualitative case study. The result of the study revealed that teacher candidates' deficiencies regarding the lack of experience in mathematical modeling, the inadequacy of notion perceptiveness, limited time, and anxiety caused a failure on implementation of the MEAs steps.

In another modeling research with pre-service teachers, Eraslan (2011) examined the opinions of mathematics teacher candidates about the level of participation in mathematical modeling activities, and the opinions about the effects mathematical modeling has on mathematics education. Modeling activities and videotapes were preferred as data sources in the research. According to the results obtained from the research, teacher candidates acknowledged the uncertainty of model-eliciting activities. On the other hand, they also stated that MEAs were positively contributed to mathematics learning, and MEAs were able to be used effectively in elementary education and other levels of mathematics teaching. As a consequence, the benefits of the MEAs were revealed while the boundedness and difficulties of the MEAs were identified at the same time.

Doruk and Umay (2011) investigated the effect of mathematical modeling on transferring mathematics to daily life. The study was implemented with 116 students in sixth and seventh grades. According to the pretest, posttest, and interview results, it was determined that the groups using mathematical modeling activities at both grade levels were more successful at transferring mathematics to daily life, rather than the groups for which mathematical modeling activities were not used. The reason for this difference was explained as follows; mathematical models presented mathematical reasoning of daily life situations, mathematical modeling provided students' improvement with their social life, and mathematical modeling had a greater impact on the critical thinking skills of the students.

Besides the studies about modeling activities with MEA that were provided above, a thesis was conducted by Albayrak (2017) in Turkey, which had similarities to this current study. Albayrak (2017) aimed to determine the tendency of the mathematical modeling researches, which was published on mathematics, and made suggestions under the light of this tendency to the community of mathematics researchers. 28 theses and dissertations, which were carried out at 14 different universities, and 38 articles were examined. Research findings indicated that the studies on mathematical modeling in Turkey had a history of nearly ten years, and it has been increasingly continued. Furthermore, theses about mathematical modeling were concentrated on the postgraduate/master level. These studies' subjects were focused on the education of teachers and how mathematical modeling could be used as a method in the learning environment. The study also showed that MS theses and Ph.D. dissertations about mathematical modeling involved modeling activities that addressed a variety of mathematics topics. In other words, instead of focusing on a single topic, they used multiple modeling activities related to multiple mathematical big ideas to enrich the learning environment. When the research methods that mathematics educators frequently use in mathematical models and modeling studies were investigated, it was seen that the most frequently used pattern was a case study. The majority of the studies consisted of qualitative studies

To sum up, modeling studies conducted in mathematics education were mostly focused on the modeling implementation and the process evaluation of the studies. These studies in this chapter showed, mathematical modeling had a significant role in learning mathematics. I encountered only one study that descriptively investigated the modeling research. I want to note that I have performed my thesis about the portrait of the modeling research, which indicated not only the quantity but also quality of the research, in mathematics education in Turkey.

At the end of the search, all accessible modeling studies were downloaded. The initial analysis was done by taking into consideration their titles and modeling activity inclusivity. Document analysis was employed in three steps.
In the first step, the documents were divided into two categories: the title including “model” and “modeling” keywords, and the title not including “model” and “modeling” keywords.

In the second step, the documents that had a title including “model” and “modeling” keywords were analyzed by considering whether they involved at least one mathematical modeling activity or not. After the analysis, the documents that had a title including “model” and “modeling” keywords were categorized under two headings: the ones involving at least one mathematical modeling activity, and the ones not involving at least one mathematical modeling activity.

In the third step, the documents that had titles not including “model” and “modeling” keywords were investigated similarly by considering whether they involved at least one mathematical modeling activity or not. After the investigation, the documents that had a title not including “model” and “modeling” keywords were categorized under two headings: the ones involving at least one mathematical modeling activity, and the ones not involving at least one mathematical modeling activity.

The search in the HEC thesis database resulted in 70 modeling studies. Through these three steps, the 70 modeling studies were categorized as follows:

- In the first step, 14 studies were founded that included the keywords of “model” and “modeling” in their titles, and the rest of the 56 studies did not include “model” and “modeling” keywords in their titles.
- In the second step, it was observed that among 14 studies, which included “model” and “modeling” keywords in their titles, 5 of them involved modeling activities, and the rest of the 9 studies did not involve modeling activities.

- In the third step, among 56 studies that did not include “model” and “modeling” keywords in their titles, it was found that 37 of them involved modeling activities, and the rest of the 19 studies did not involve modeling activities. The distribution of these studies, regarding the two inclusion criteria following these three steps, is shown in Table 3.1 below.

Table 3.1. The Data Set of Modeling Studies Gathered From HEC

<table>
<tr><th>Modeling theses and dissertations</th><th>Involving at least one mathematical modeling activity</th><th>Not involving at least one mathematical modeling activity</th></tr>
<tr><td>Title including the keywords of “model” and “modeling”.</td><td>5</td><td>9</td></tr>
<tr><td>Title not including the keywords of “model” and “modeling”.</td><td>37</td><td>19</td></tr>
<tr><td rowspan="2">TOTAL</td><td>42</td><td>28</td></tr>
<tr><td colspan="2">70</td></tr>
</table>

As a result, I continued my study with 42 modeling theses and dissertations that included at least one mathematical modeling activity to investigate the second research question, the nature of the modeling theses and dissertations, and the modeling activities involved.

Essentially, this study involves two document sets, and they can be classified as follows:

- Document Set 1: 70 theses and dissertations on mathematical modeling that were listed on the HEC thesis and dissertation database and completed between 2000-2019.
- Document Set 2: 42 theses and dissertations on mathematical modeling that were listed on the HEC thesis and dissertation database, completed between 2000-2019, and involve at least one mathematical modeling activity.

The data analysis procedure of these two document sets are presented in detail below.

3.3. Data Analysis

The two document sets mentioned in the data set section above were analyzed with different approaches. Content analysis was used as a data analysis method to analyze the document sets. It is a qualitative research method that has become broadly used in mathematics education studies. Hsieh & Shannon (2005) stated that content analysis is a method for the personal interpretation of the content of text data. Basically, it can be said that content is a message, and analysis is the significance of this message. According to Bowen (2009), content analysis can be summarized as a 'first-pass document review' that can allow the researcher to recognize meaningful and correlated passages.

In the analysis of the second document set, I coded the texts using the six design principles as a coding frame, because one of the aims of this study investigated the extent of which the modeling activities are linked with the theory; that is, the models-and-modeling perspective. Coding can be defined as tagging or labeling allocated units a meaning to the descriptive or inferential information assembled through a study (Miles & Huberman, 1994). Coding can be classified into two forms: concept-driven coding frame and data-driven coding frame. According to Ryan and Bernard (2003), while codes can evolve a priori from existing theory in a concept-driven frame, they can come out from raw data in a data-driven frame. Boyatzis (1998) states that creating concept-driven and data-driven codes have a different course of action.

Evolving concept-driven codes requires three steps:

1. Generating the code based on a theory or a compile of related research.
2. Revising the code in the context of data.
3. Determining the reliability of the code and coders.

On the other hand, data-driven codes involve five steps:

1. Reducing raw information
2. Identifying subsample themes
3. Comparing themes across subsamples
4. Generating codes
5. Determining the reliability of codes

Table 3.2 below shows the research questions, the data set that was analyzed to answer the research question, and the data analysis method used.

Table 3.2. Overview of the Document Sets and the Analysis Methods of the Study

Research Questions	Document Sets	Data Analysis Method
1. What are the features of the modeling research depicted by the theses and dissertations on mathematical modeling that were listed on the HEC thesis and dissertation database in the last two decades?	Document Set 1: 70 theses and dissertations on mathematical modeling that were listed on the HEC thesis and dissertation database and completed between 2000 -2019.	Document analysis incorporating content analysis
2. What is the nature of modeling research depicted by the theses and dissertations on mathematical modeling that were listed on the HEC thesis and dissertation database, and that involves at least one mathematical modeling activity?	Document Set 2: 42 theses and dissertations on mathematical modeling that were listed on the HEC thesis and dissertation database, completed between 2000 -2019, and involved at least one mathematical modeling activity.	Document analysis incorporating content analysis

As seen in Table 3.2, to answer the first research question, I utilized 70 modeling theses and dissertations (i.e., document set 1). I analyzed this document set through content analysis following the steps below.

1. I gathered all of the modeling theses and dissertations from HEC,
2. 70 modeling studies gathered and downloaded, and I used all of them to classify their features,
3. These features comprise of the distribution of modeling theses and dissertations in terms of the type of the work (i.e., master, doctorate), years (i.e., 2000 – 2019), and universities where the studies were conducted.

To answer the second research question, I used 42 modeling theses and dissertations, which contained at least one mathematical modeling activity (i.e., document set 2). I analyzed this document set through content analysis in which I used a concept-driven coding frame and followed the steps below.

1. Forty-two modeling theses and dissertations were elected among 70 modeling studies by considering the modeling activity involvement.
2. Forty-two modeling study parts were analyzed concerning references to the MMP and MEA in an abstract, literature review, and methodology sections.
3. 42 modeling documents were examined concerning the six design principles using a concept-driven coding frame strategy, and proper codes were assigned for each rule as follows

Table 3.3. Applied Concept Driven Coding Frame of Six Design Principles

Six Principles	Attended Question(s)	Codes Driven
Model Construction Principle	-Does the given situation require students to create a model by interpreting the givens, objectives, and possible solution procedures?	-Optimization (more than one option like economic, safe and practical) -Mathematical reasoning -Requirement of solution development based on various information
Reality Principle	-Can this situation happen in the real life of the student or any person that the student is familiar with?	-Real-life situations -Hypothetical situations -Meaningful situations

Table 3.3. (cont'd)

Six Principles	Attended Question(s)	Codes Driven
Self-Assessment Principle	-Does the problem situation require appropriate criteria to evaluate alternative solutions? -Is the purpose of the problem situation clear? -Will students be able to evaluate themselves when their answers need to be improved? -Will the students realize that they have completed the solution to the problem? -Will they need to ask the teacher if they need to continue to solve the problem?	-Self-Assessment criteria -Optimization criteria (most economic, most comfortable, i.e.)
Construct Documentation Principle	-Do responses of students to a problem reveal how they think about the situation?	-Type of solution expression (letter, table, graph, i.e.) -Request for an expression/explanation (partially meets the principle)
Construct Shareability and Reusability Principle	-Is the developed model useful only for the person who developed it, or does it provide a way of thinking that can be shared, transformed, easily adapted, and reused in different situations?	-Model construction -Documentation -Potential to become a prototype -Ways of shareability and reusability
Effective Prototype Principle	-Does the developed model create a prototype useful for other structurally similar situations? -Will students be able to consider the previous problem in structurally similar situations long after the problem is solved?	-Recalling the realistic situation -Shareability and reusability

It is noted that some principles provide the requirement of other principles based on the theoretical premises of the models-and-modeling perspective. For example, as seen in Table 3.3, the construct shareability and reusability principle depended on two other principles, namely, the model construction and documentation principles. To share and reuse a model, first, it needs to be constructed and documented.

Another principle is the effective prototype principle, which also depends on two other principles, namely, the reality and construct shareability and reusability principles. Because of being able to utilize a model for structurally similar situations effectively, the model needs to be meaningful in real life; besides, it is expected to be shareable and reusable for different situations.

3.4. Credibility and Trustworthiness

Validity and reliability issues are indispensable parts of any research. Therefore, they are needed to be considered at each stage of the study, particularly when collecting, examining, and evaluating data and presenting the findings (Merriam, 2009). Briefly, while validity is described as the accuracy of findings by applying certain methods, reliability is described as the consistency of the results (Creswell, 2009). Validity and reliability concepts carry a major significance for quantitative studies; however, qualitative studies use different terminology instead of these concepts.

Internal validity, external validity, and reliability concepts correspond to credibility, transferability, and consistency (dependability) concepts, respectively, in qualitative studies (Lincoln & Guba, 1985).

It is significant for researchers to provide enough evidence and detailed explanations to meet trustworthiness in their studies (Creswell, 2009). Trustworthiness involves establishing credibility and consistency. Credibility is defined as confidence in the 'truth' of the findings (Lincoln & Guba, 1985). Denzin (1978) and Patton (1999) propound four types of triangulation methods to provide credibility for researches: (1) triangulation with multiple data sources, (2) triangulation with multiple data types, (3) triangulation with multiple researchers, and (4) triangulation with the theory or perspective.

Multiple data source triangulation: The collection of data from a variety of data sources (i.e., a different type of people, groups, families, multiple perspectives).
Multiple data type triangulation: The use of multiple data types to provide information about the research process (i.e., video, audio, or written data).

Multiple researcher triangulation: Different researchers or evaluators represent their ideas in the research process.

Theory/perspective triangulation: The use of multiple perspectives to examine a single set of data. Perspectives can belong to just one researcher/individual or more than one researcher/person.

Since the first two ways of triangulation were not appropriate to the design of the study, that is, document analysis, I used coding the data with multiple researchers and checking for the consistency between them to ensure credibility and consistency. First, the code list that was compiled from theory (i.e., six design principles of MMP) and indicators of each design principle was discussed.

Later, I analyzed some of the documents together with my advisor to decide whether each of the indicators was clearly identifiable. After reaching a consensus about the indicators of each design principle, I coded the entire document set. For the documents and design principles that I was not sure about, I met with my advisor several times to work on the two sets of data that I had selected. After we agreed on the presence or the absence of the codes (i.e., indicators of the design principles), I continued the analysis myself.

3.5. Researcher Role

In a qualitative research study, the researcher role has some complexities, and the researcher specifies the whole process of data collection. In this study, as a researcher, my role can be summarized in three steps. Firstly, I searched sources to gather data from an appropriate website because the researcher is responsible for recognizing relevant documents and obtaining significant information for the validity of data (Denzin, 1978; Lincoln & Guba, 1985). To provide relevant and reliable data, I chose the site of the Council of Higher Education and obtained the necessary information by limiting my search criteria.

I set the criteria by considering the research questions. I specified the criteria by choosing ‘modeling’ and ‘mathematics education’ keywords to find related data from the database because I wanted to analyze modeling studies in the field of mathematics education. The years of investigated modeling studies were assigned between 2000 and 2019, according to the research questions again. Secondly, during the analysis of data set 1, the common descriptive features of the data constituted clear outputs such as year, participants, and the universities; therefore, they relied on the expressions from the authors of the documents. However, the document analysis of modeling activities concerning the six design principles in a modeling perspective involved the interpretation of the researcher. However, I reaffirmed the codes that I had used to analyze documents several times and utilized the triangulation of multiple researchers to provide the trustworthiness of the study.

Moreover, during the analysis, I sent an electronic version of the outcomes (i.e., long Excel tables showing the matrix of the six principles and the modeling activities in the theses and dissertations) to the advisor via e-mail when it was necessary. Meaningful advisor feedback helped the study to re-dispose necessary parts of it. Lastly, in the conclusion part of the analysis, I evaluated the obtained data outcomes based on the result of analyses. I read several document analyzes researches to enrich the evaluation talent of myself. After I interpreted the results, I consulted my findings with my advisor to reach a consensus. If this study was investigated by a different researcher, they could use different data sets and coding frame to analyze modeling studies in the field of mathematics education. However, I believe that the results would be similar to this study in some senses, particularly if the theoretical lens was chosen as the six design principles of MEA.

Table 4.1. Distribution of the modeling studies in terms of the years

YEARS	MS Thesis	Ph.D. Dissertations	MS Thesis + Ph.D. Dissertations
2000			
2001			
2002			
2003			
2004			
2005	1		1
2006			
2007			
2008	1		1
2009		1	1
2010		1	1
2011	2	1	3
2012	2		2
2013	2	2	4
2014	4	5	9
2015	5	3	8
2016	7	4	11
2017	10	1	11
2018	7	2	9
2019	8	1	9
TOTAL:	49	21	70

The number of completed master theses and Ph.D. dissertations reached a peak as 11 studies were published in both 2016 and 2017. As for Ph.D. studies, Gazi University was the leading university concerning the first completion of modeling dissertations in 2009. The most popular year in terms of the number of completed dissertations was 2014, issuing 5 Ph.D. studies. In general, Table 4.1 indicated a trend in MS and Ph.D. modeling studies between 2014-2019. After analyzing the distribution of modeling studies in terms of graduate programs and year, I observed university contributions to modeling studies. Table 4.2 shows that 24 different universities in Turkey contributed academic work under the topic of model and modeling in this field.

Table 4.2. University Contributions to Modeling Studies in Mathematics Education

UNIVERSITIES	MS Thesis	Ph.D. Dissertations	MS Thesis + Ph.D. Dissertations
ABANT İZZET BAYSAL U.	2		2
ADIYAMAN U.		1	1
ANADOLU U.		2	2
ATATÜRK U.	5	3	8
BALIKESİR U.		2	2
BAŞKENT U.	2		2
BİLKENT U.	1		1
ÇUKUROVA U.	1		1
DİCLE U.	1		1
DOKUZ EYLÜL U.	5	2	7
ESKİŞEHİR OSMANGAZİ U.	4		4
ERCİYES U.	1		1
FIRAT U.	3		3
GAZİ U.	5	3	8
GAZİOSMANPAŞA U.	1		1
HACETTEPE U.		2	2
KOCAELİ U.	1		1
MARMARA U.	5		5
MERSİN U.	2		2
NECMETTİN ERBAKAN U.	2	2	4
ONDOKUZ MAYIS U.	4		4
ORTA DOĞU TEKNİK U.	2	4	6
RECEP TAYYİP U.	1		1
VAN YÜZÜNCÜ YIL U.	1		1
TOPLAM	49	21	70

As seen in Table 4.2, Gazi University, Ataturk University, Dokuz Eylül University, and Middle East Technical University were outstanding universities in which graduate students published more academic theses and dissertations with modeling perspectives in mathematics education. Specifically, Gazi University and Ataturk University were issued eight studies, Dokuz Eylül University published seven studies, and Middle East Technical University has published six studies on this topic. Furthermore, it is notable that Middle East Technical University was leading in the number of published Ph.D. dissertations.

As seen in Table 4.5, 73 out of the 281 modeling activities reflected all six design principles, either completely or partially. There were 169 modeling activities involving more than three design principles either partially or completely, which constituted 60% of the total number of activities. On the contrary, one modeling activity did not meet any of the six design principles. Below, I provided sample modeling activities in each category along with detailed examination of the design principles that they indicated either partially or completely.

4.2.1.1. Modeling Activities Completely or Partially Indicating All the Six Design Principles

There were 73 activities that indicated all six design principles where some of them were observed partially in some activities. While 54 activities involved all the six design principles completely, 19 of them indicated three of these principles partially. The Summer Job problem was one of the activities that meet all of the design principles, along with the other 54 activities. While the Summer Job problem was originally developed by Lesh and her colleagues in one of the leading modeling projects in the United States, the version that was adapted to Turkish context and used in one of the dissertations in the data set was given in Figure 4.4 as an example of the activities in this category.

Yaz İşi Problemi

Levent, geçen yaz Gençlik Parkı'nda bir iş aldı. O'nun çalıştırdığı seyyar satıcılar, park içerisinde dolaşarak patlamış mısır ve içecek satışı yapıyorlar. Levent' in gelecek yaz hangi elemanlarını tekrar çalıştırmaya karar vermek için yardımınıza ihtiyacı var. Geçen yaz Levent' in 9 satıcısı vardı. Bu yaz ise üçü tam gün, üçü yarım gün olmak üzere 6 satıcı çalıştırabilecek. Levent geçen yaz çalıştırdığı elemanlardan kendisine en çok gelir getirecek olanları tekrar işe almak istiyor. Fakat onları nasıl karşılaştırabileceğini bilmiyor. Çünkü geçen yılki kayıtlara göre satıcıların günlük çalışma saatleri farklı. Bunun yanında parkın yoğunluk durumu da satışta önemli etkiye sahip. Örneğin kalabalık bir Cuma gecesi satış yapmak yağmurlu bir öğleden sonraya göre çok daha kolaydır. Levent geçen yılki kayıtları inceleyerek, parkın yoğunluk durumuna göre her satıcının çalışma süresini ve topladığı para miktarını belirledi. (Tablo 1–2) Satıcıların geçen yılki performanslarını inceleyiniz ve üç tane tam gün üç tane de yarım gün çalışmak üzere satıcı belirleyiniz. Levent' e sonuçlarınızı bir mektupla bildiriniz. Teklifinizin kullanışlı olup olmadığına karar verebilmesi için mektupta satıcıları nasıl değerlendirip seçtiğinizi ayrıntılı bir şekilde açıklayınız.

TABLO 1: GEÇEN YAZ ÇALIŞMA SÜRELERİ (SAAT- AYDA)

	HAZİRAN			TEMMUZ			AĞUSTOS		
Yoğunluk	Çok	Orta	Düşük	Çok	Orta	Düşük	Çok	Orta	Düşük
GİZEM	12.5	15	9	10	14	17.5	12.5	33.5	35
KAAN	5.5	22	15.5	53.5	40	15.5	50	14	23.5
TARIK	12	17	14.5	20	25	21.5	19.5	20.5	24.5
JALE	19.5	30.5	34	20	31	14	22	19.5	36
CAN	19.5	26	0	36	15.5	27	30	24	4.5
CANAN	13	4.5	12	33.5	37.5	6.5	16	24	16.5
RIZA	26.5	43.5	27	67	26	3	41.5	58	5.5
ALİ	7.5	16	25	16	45.5	51	7.5	42	84
AYTEN	0	3	4.5	38	17.5	39	37	22	12

TABLO2: GEÇEN YAZ TOPLANAN PARA (TL)

	HAZİRAN			TEMMUZ			AĞUSTOS		
Yoğunluk	Çok	Orta	Düşük	Çok	Orta	Düşük	Çok	Orta	Düşük
GİZEM	690	780	452	699	758	835	788	1732	1462
KAAN	474	874	406	4612	2032	477	4500	834	712
TARIK	1047	667	284	1389	804	450	1062	806	491
JALE	1263	1188	765	1584	1668	449	1822	1276	1358
CAN	1264	1172	0	2477	681	548	1923	1130	89
CANAN	1115	278	574	2972	2399	231	1322	1594	577
RIZA	2253	1702	610	4470	993	75	2754	2327	87
ALİ	550	903	928	1296	2360	2610	615	2184	2518
AYTEN	0	125	64	3073	767	768	3005	1253	253

Not: Bu etkinlik Johnson ve Lesh (2003)'in çalışmasından uyarlanmıştır.

Figure 4.4. Summer Job Problem – sample modeling activity meeting all of the six design principles

In this problem, there is an employer; whose name is Levent, who wants to hire hawkers and employ them in Gençlik Park. The criterion for the hired individuals is to maximize income. He has the data of nine employees from the previous year that includes the information on working hours and corresponding collected money concerning the crowdedness rate of the park. Levent will hire six workers by considering the information of data from last year. He will employ half of them for full time and the other half for part-time. Levent wants to hire six workers from nine; therefore, he needs help for this election. Thus, in the problem, it was asked to pick three full-time and three part-time workers by considering the last year's data and to inform them about the decision by a letter.

When the activity was examined, each principle was observed as follows:

- *Reality Principle:* The situation of the problem was presented in a real and meaningful way that students might encounter this kind of situation when they want to work in the summertime. Thus, the real-life principle was observed.
- *Model Construction Principle:* The data of nine employees from the previous year was provided statistically. Students who would solve this question needed to re-organize and optimize the information to make it meaningful and understandable. There was no guidance for the rearrangement of the data, and it was left to the mathematical reasoning of the problem solver. Therefore, the problem solver may construct their model by optimizing last year’s working hours. Thus, the model construction principle was observed.
- *Self-Assessment Principle:* The criterion for hiring for the job was increasing the income. Therefore, the problem solver may organize the information by considering this criterion. Thus, the self-assessment principle was observed.
- *Construct Documentation Principle:* Models created as an answer to this question were expected to be delivered by a letter. The letter was identified as a documentation tool. Thus, the construct documentation principle was met.

- *Construct Shareability and Reusability Principle:* This principle is dependent on two other design principles, namely, model construction and documentation. To share and to reuse a model, first, a model needed to be constructed and documented.

  If a created model provided a way of thinking to be shaped, transformed, easily adapted, or reused in different situations, it ensures the construct shareability and reusability principle. However, it is not easy to analyze these factors before implementing them to problem solvers. Therefore, we can say that if a problem ensures model construction and construct documentation principles, it has the potential to provide construct shareability and reusability principles. Thus, this problem has the potential to provide this principle because it carries model construction and construct documentation principles well.
- *Effective Prototype Principle:* This principle is also based on two other design principles, namely, the reality and shareability and reusability principles. To use a model as a prototype, it needed to be attached to a realistic and meaningful situation and had the potential to be reusable in different situations. This problem possessed reality and shareability and reusability principles; therefore, it has the potential to be an effective prototype for structurally similar situations; to illustrate, a factory manager is expected to use this prototype effectively for employee shifts to increase overall productivity.

Similar to the Summer Job problem, 54 modeling activities in the analyzed MS thesis and Ph.D. dissertations reflected six design principles.

On the other hand, 19 of the 73 modeling activities addressed some of the six design principles partially. To put it more explicitly, these 19 modeling activities were provided three of the six design principles, which were the reality principle, model construction principle, and self-assessment principle. The other three of the six design principles were observed partially. The partially observed principles were the construct documentation principle, construct shareability and reusability principle, and the effective prototype principle. The Travelling Problem was given in Figure 4.5 as an example of similar activities in this category.

SEYAHAT PROBLEMİ*

Gül ailesi bir haftalık bir tatil için araç kiralayarak Ankara' dan Antalya' ya tatile gideceklerdir. Gül ailesi bir yandan rahat bir yolculuk yapmak isterken bir yandan da ekonomik bir seçim yapmak istemektedir. Gül ailesine otomobil seçimi için yardımcı olmanız istenmektedir. Nasıl yardımcı olunabilir açıklayınız.

Not: Ankara – Antalya arası yaklaşık 550 km olarak belirlenmiştir.
* Bu etkinlik Sandalcı (2013)'nın çalışmasından uyarlanmıştır.

				
ARACIN MODELİ	A ARACI	B ARACI	C ARACI	D ARACI
YAKIT TÜRÜ	BENZİNLİ	DİZEL	BENZİNLİ	DİZEL
KLİMA	YOK	VAR	YOK	VAR
İÇ HACİM	DAR	DAR	GENİŞ	GENİŞ
1 KM DE HARCADIĞI YAKITIN FİYATI	0,28 TL	0,23TL	0,30 TL	0,26 TL
GÜNLÜK KİRALAMA BEDELİ	70 TL	90 TL	80 TL	100 TL

Figure 4.5 Travelling Problem - sample modeling activity addressing some of the six design principles partially

In this problem, the Gül family wants to rent a car to travel from Ankara to Antalya for a holiday. A list of different car brands and their features are given in terms of vehicles fuel type, the air conditioning feature, internal volume type, fuel expense per km, and the daily rental price. They want to make their journey as comfortable and as economical as possible. Thus, in the problem, it was asked to help the Gül family choose the most appropriate car by considering the economic and comfortable issues of car brands. An explanation was requested for the chosen vehicle.

When the activity was examined, each principle was observed as follows:

- *Reality Principle:* The situation of the problem was presented in a real and meaningful situation in that it is usual to see families traveling long distances. Thus, the real-life principle was observed.
- *Model Construction Principle:* The data of four different cars concerning their features are demonstrated in the problem. Students who would solve this problem needed to re-organize and optimize the information to choose the appropriate car for the Gül family. There was no guidance involved for rearranging the data, therefore it was left to the mathematical reasoning of the problem solver. Thus, the model construction principle was noticed.
- *Self-Assessment Principle:* The criteria for choosing an appropriate car were its' being comfortable and economical. Students might optimize the data by considering these criteria. Thus, the self-assessment principle was observed.
- *Construct Documentation Principle:* The problem asked students to clarify their answers with an explanation. It was not requested that any of the documentation tools such as letters, graphs, tables, models, or formulas reveal how students thought about the question clearly. Thus, the construct documentation principle was observed partially.
- *Construct Shareability and Reusability Principle:* Two principles which are model construction and construct documentation principles are required to be observed to meet this principle.

  The previous analyses on principles showed that a model construction can be observed to choose the most appropriate car for the Gül family; however, the documentation of the model was requested with an explanation rather than a documentation tool which provided the construct documentation principle only partially. Therefore, this problem has the potential to provide the construct shareability & reusability principle partially.

- *Effective Prototype Principle:* Two other principles, which are the reality and construct shareability & reusability principles, are required to be observed to meet this principle. The previous analyses presented that although the situation in the problem was meaningful and realistic, a constructed model for the representation of the problem-solution had the potential to be partially reusable in different circumstances. Therefore, the problem has a potential to be a partially effective prototype for structurally similar situations. To illustrate, a private jet pilot can use this prototype effectively while choosing an appropriate jet for customer satisfaction with the condition that the model was documented or expressed well and reused in different situations.

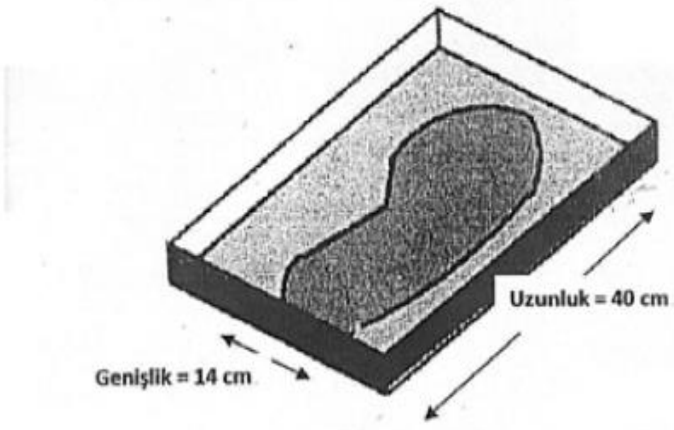
19 out of the 73 modeling activities provided half of the six design principles, namely, the reality principle, the model construction principle, and the self-assessment principle. These 18 modeling activities also indicated another half of the six design principles partially, namely, the construct documentation principle, the construct shareability and reusability principle, and the effective prototype principle, and the traveling problem was one of such modeling activities.

4.2.1.2. Modeling Activities Completely or Partially Indicating Five Design Principles

There were 83 modeling activities indicating five of the design principles either partially or completely. More specifically, among these modeling activities, 50 different modeling activities indicated five of the six design principles completely. Interestingly, all of these 50 activities possessed five principles except the same principle, that is, self-assessment. That means these activities did not provide self-assessment criteria. Big Foot Problem (see Figure 4.6) was an example of this category, which provided five design principles except for the self-assessment principle. It was the most prominent activity, which was included in seven different studies (i.e., MS thesis and Ph.D. dissertations), adapted from Lesh and Doerr's study (2003).

Büyük Ayak Problemi

Bir kış günü sabah okula gelen öğrenciler hiç de beklemedikleri bir durumla karşılaşırlar. Okulun bahçesinde polis ve olay yeri inceleme ekibinin bulunduğunu görürler. Polis, dün gece bazı insanların okulun bahçesine çok sayıda kitap bıraktığını belirtmiştir. Okul yönetimi ve öğrenciler bunu yapan insanlara teşekkür etmek isterler fakat hiç kimse bunu kimin yaptığını görmemiştir. Polis olay yerinde birçok ayak izine rastlar. Ayak izlerinin birisi aşağıda görülmektedir. Bu kişiyi ve arkadaşlarını bulmak için bu ayak izinin sahibinin boyunu belirlemeniz faydalı olabilir. Sizin göreviniz polise ayak izi bulunan kişinin boyunun uzunluğunu belirlemede kullanmak üzere bir araç geliştirmek ve bir mektupla bu aracın nasıl geliştirildiğini ve kullanıldığını polise anlatmak. Unutmayınız ki geliştirdiğiniz bu araç buradaki ayak izi için işe yaradığı gibi diğer ayak izleri için de işe yaramalıdır?



Not: Bu etkinlik Lesh ve Doerr (2003)'in çalışmasından uyarlanmıştır.

Figure 4.6. Big Foot Problem - sample modeling activity indicating five design principles

The Big Foot Problem mentions a circumstance that occurred in a school garden where students found numerous books on a winter day. School management and students wanted to thank these people who left the books in the garden. However, people living around the school did not see anyone leave the books. Afterward, the police went to the locale and determined many footprints. One of the footprints was given as a draft in Figure 4.5 that had a length of 40 cm and a width of 14 cm. To find the footprint owner and his friends, developing a tool that calculates the height of a person following their feet measurements could be useful. The question asked problem solvers to develop a tool and explain the improvement and utilization process of it in detail by a letter. It was noted that this tool was expected to be used as a model in different similar situations.

When the activity was examined, each principle was observed as follows:

- *Reality Principle:* When the dimensions of the footprint given in the draft (see Figure 4.5) were analyzed, finding a person who has this foot size may be hard in today's world. However, when the ancient ages or big size boots are considered, this situation can come true. Thus, the reality principle was observed hypothetically.
- *Model Construction Principle:* Footprint measurements were given numerically. Students were expected to construct a tool by discriminating important and unimportant variables to find the right height of the person who fits the given footprint. To develop a tool, students needed to use their mathematical reasoning because there is not one or a certain solution to solving this question. Thus, the model construction principle was observed.
- *Self-Assessment Principle:* It was asked in the problem to find the height of the footprint owner by developing a tool or model. However, there were not any criteria given in the problem to develop this tool. All the criteria settings were dependent on the problem solver. Thus, the self-assessment principle was not observed.

- *Construct Documentation Principle:* Models or tools developed to find the height of the footprint owner was expected to be delivered by a letter. The letter was determined as a documentation tool. Thus, the construct documentation principle was met.

- *Construct Shareability and Reusability Principle:* The validity of this principle is dependent on the model construction and documentation principles. As was observed from the previous analysis of the problem, a model can be constructed for this problem, and the documentation principle was provided with a letter requested through the question. Therefore, this problem has the potential to ensure the construct shareability & reusability principle.
- *Effective Prototype Principle:* The validity of this principle is also dependent on reality and shareability and reusability principles. The reality principle was observed hypothetically because of the vast size of the footprint. Furthermore, a model or tool constructed for the solution to this problem had the potential to be reusable in different circumstances.

  This problem held the reality and the shareability & reusability principles. Therefore, it carries the potential to be an effective prototype to be used in structurally similar situations such as a biologist is expected to use the same model or tool to find the wing lengths of flying animals by measuring their wing size.

Similar to the Big Foot Problem, 50 modeling activities reflected five of the six design principles except for the self-assessment principle.

In addition, there were 33 modeling studies that indicated two of the design principles completely but three design principles partially. One of the activities was the Camping Problem (see Figure 4.7).

KAMPA GİDİYORUZ

Amasya İzcilik ve Spor Kulübü, yaz tatili için 8. Sınıf öğrencilerine yönelik bir kamp düzenlemeye karar vermiştir. Kamp için 30 öğrenci kayıt yaptırmıştır. Kulüp her çadırda 3 öğrenci kalacak biçimde kare piramit şeklinde kamp çadırları kurulacaktır.

Çadırların tabanının her bir kenarının 4 m, yüksekliğinin ise 2 m olmasına karar verilmiştir. İzcilik Kulübü'nün kamp çadırlarının tamamının maliyetini belirleyebilmesi için tüm kamp çadırları için gereken kumaş miktarını tespiti gerekmektedir.

Kulüp yöneticilerine toplam çadır maliyetini belirlemek için nasıl yardımcı olursun, detaylıca açıklayabilir misin?

Figure 4.7. Camping Problem - sample modeling activity indicating five design principles, three of which were partially observed

In this problem, Amasya Tracking and Sport Club decided to arrange a camp activity for 8th grades in summer holiday. 30 students were registered for the camp. The shape of tents for the camp would be square pyramid, and three students would be able to stay in each tent. Each side of the base of tents would be 4 m, and the height will be 2 m. Amasya Tracking and Sport Club needed to know the necessary amount of tent fabric to determine the cost of all tents. Thus, the problem asked for a help to determine the cost of all tents. An explanation was requested for the cost details. When the activity was examined, each principle was observed as follows:

- *Reality Principle:* The situation of the problem was presented in a real and meaningful situation in that students can go for a camping in summer holidays. Thus, the real-life principle was observed.
- *Model Construction Principle:* The measurements of tents were demonstrated in the problem. Students who would solve this problem needed to identify how many tents would be required, the amount of fabrics for each tent and the total cost of all tents. To determine the total cost, students needed to use their

mathematical reasoning because there was not a single solution to solving this question. Thus, the model construction principle was observed.

- *Self-Assessment Principle:* The students were asked to find the total cost of tents in which they would stay. However, there was not any criteria given in the problem to consider in evaluating the model developed to determine the total cost of tents. Thus, the self-assessment principle was not observed.
- *Construct Documentation Principle:* The problem asked students to enlighten their answers with an explanation. However, there was no documentation method such as letters, graphs, tables, models, or formulas identified for students to document their models. Thus, the construct documentation principle was met partially.
- *Construct Shareability and Reusability Principle:* Two principles, model construction and construct documentation principles, were required to be observed to provide this principle. Since the latter principle was partially observed, construct shareability and reusability was considered to reflect a potential existence but partially. More specifically, since the documentation of the model was requested with an explanation rather than a documentation tool, this problem has the potential to provide the construct shareability & reusability principle partially.
- *Effective Prototype Principle:* This principle was based on two other design principles, namely, the reality and shareability & reusability principles. The previous analyses showed that despite the situation in the problem was meaningful, a constructed model for the representation of the problem-solution had the potential to be partially reusable in different circumstances. Since the reusability principle was partially indicated in the problem, the problem had a potential to be a partially effective prototype for structurally similar situations. To illustrate a situation where this problem can serve as a prototype, an architect can use the model developed for this problem while constructing small bungalows for three people.

Thus, as seen in the Camping Problem, while reality and model construction principles were fully observed, model documentation, construct shareability and reusability, and effective prototype were partially observed. The self-assessment was not observed in this problem because there was no quality evaluation criteria given in the problem.

4.2.1.3. Modeling Activities Completely or Partially Indicating Four Design Principles

13 modeling activities in this category provided three of the six design principles completely which were the reality principle, the model construction principle, and the self-assessment principle, and one principle, the effective prototype principle, partially.

Two of the design principles were not observed, which were the construct documentation principle and the construct shareability and reusability principle in these 13 activities. A Shelter Problem was given in Figure 4.8 as an example of similar activities in this category.

SIĞINAK

Teknolojik gelişmelere paralel olarak günümüzde silah ve silah sistemlerinin güç ve yetenekleri de artmış bulunmaktadır. Bunun doğal sonucu olarak cephe ve sınır gibi kavramlar bütünü ile ortadan kalkmış, tüm yurt ve sivil halk saldırı alanı içine girmiştir. Bu büyük tehlikelerden Silahlı Kuvvetlerin alacağı önlemlerin ve diğer tedbirlerin yanında sığınak yapımına önem verilerek kurtulabilinir.

44 dairelik bir apartmanın bodrum katına sığınak planlanmaktadır. Ancak mimar plana baktığında sığınak için planda çok yer kalmadığını görmüştür. Mimar sığınak için en az kaç m^2 lik bir alan planlamalıdır hesaplayınız.

Figure 4.8. Shelter Problem - sample modeling activity indicating four design principles, one of which was partially observed

In this problem, foreknowledge about shelters was presented with these words: Parallel to the technological developments, the power of the weapon industry is evolving expeditiously. As a natural consequence of this progress, front-line and border-line concepts disappeared entirely and civil society entered the attack area. To avoid from this great danger, the civil society can give an importance in building shelters in addition to the precautions of the military. In the problem above, building a shelter in the cellar of a 44-floor apartment was planned. However, when the architect reviewed the plan, they realized that there was insufficient space for the construction of a shelter. Thus, students were asked to calculate the minimum area required for a shelter to help the architect.

When the activity was examined, each principle was observed as follows:

- *Reality Principle:* A real and meaningful situation was presented in this problem that people might need shelters when an extraordinary situation was to appear. Thus, the real-life principle was met.
- *Model Construction Principle:* The minimum area requirement that may be constructed under the 44-floor apartment building was asked with no extra information provided. Students could construct models by investigating other shelter samples, or they could follow their strategy to find the minimum area required for the shelter, it was left to the mathematical reasoning of the problem solver. Therefore, the model construction principle was observed.
- *Self-Assessment Principle:* The criterion for building the shelter was choosing the smallest area for the construction. Therefore, the problem solver may organize the information by considering this criterion. Thus, the self-assessment principle was observed.
- *Construct Documentation Principle:* The problem did not request or make apparent the use of any tools, documents, or models to justify or to reveal the solution of the problem. Thus, the construct documentation principle was not observed clearly.
- *Construct Shareability and Reusability Principle:* This principle is dependent on two other design principles; namely, the model construction and the construct documentation principles. As was observed from the previous analysis of principles, a model (shelter) can be constructed for this problem; however, the problem did not request or specify the requirement for any documentation tool to reflect the model.

Therefore, although model construction was observed, the model does not have the potential to be shared or be reused for different situations because documentation was not ensured or captured. Thus, the construct shareability & reusability principle was not observed.

- *Effective Prototype Principle:* The reality and the construct shareability & reusability principles are needed to enable use of this principle. To use a model as a prototype, it is required to be conjoined to a meaningful and realistic circumstance and has the potential to be reusable in various situations. This problem had attached to a realistic situation along with it; however, it did not provide the potential to be reused in different situations. Thus, the effective prototype principle was observed only partially. If the model were documented, it would be used for structurally similar situations; for example, an architect could use the potential prototype of the shelter for the construction of home-offices, which comprised of the minimum area.

Similar to the shelter problem, 13 out of the 281 different modeling activities reflected three of the six design principles completely, which were the reality principle, model construction principle, and the self-assessment principle. The lack of documentation has also impacted the shareability and reusability of a model negatively. Therefore, both of the principles, which were the documentation principle and construct shareability and reusability principle, were not observed in these 13 modeling activities. As for the effective prototype principle, these 13 modeling activities indicated it only partially. The Traveling Problem was one of them.

4.2.1.4. Modeling Activities Completely or Partially Indicating Three Design Principles

66 modeling activities were indicating three design principles, either partially or completely. The Tray Pastry problem was one of them. More clearly, this problem provided two of the six design principles completely, namely, the reality principle

and self-assessment principle. It also reflected one of the six design principles partially which was the effective prototype principle. The other principles were not observed in this activity. The tray pastry problem was different from the other activities in this category since it was the only activity that reflected the self-assessment principle without the essence of the model construction principle. The Tray Pastry Problem was given below in Figure 4.9 as an example.

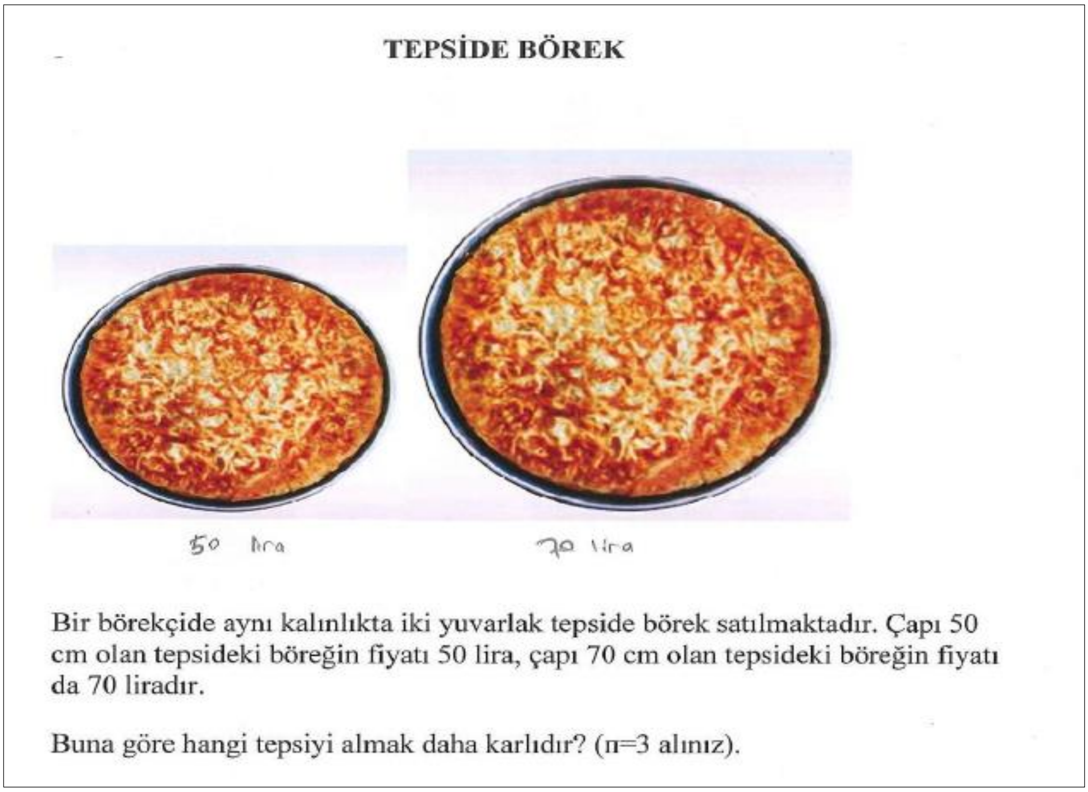


Figure 4.9. Tray Pastry Problem - sample modeling activity indicating three design principles, one of which was partially observed

In this problem, there are two round tray pastries, which have the same thickness but with different diameters. The price of the one which has a diameter of 50 is 50 TL, the price of the other one which has a diameter of 70 is 70TL. The question asked was which tray pastry is more profitable. (π=3)

When the activity was examined, each principle was observed as follows:

- *Reality Principle:* The situation of this problem was existent and meaningful as students might encounter this kind of a situation in bakeries or other shops with several different forms of items which were available for sale. Thus, the real-life principle was observed.
- *Model Construction Principle:* The prices of the two tray pastries with different diameters were given and to find the more profitable one was asked. To answer the problem, students might not be needed to optimize information or use their mathematical reasoning, because the information presented in the problem itself was clear. The solution of the problem may be understandable after only a few calculations. Thus, the model construction principle was not observed.
- *Self-Assessment Principle:* The problem asked to identify the more profitable tray pastry. This enabled a criterion for the students. Therefore, students may solve this problem by considering this criterion. Thus, the self-assessment principle was observed.
- *Construct Documentation Principle:* This problem did not request any explanation or documentation tools (i.e., letter, graph, table, formula) for the solution. Thus, the construct documentation principle was not met.
- *Construct Shareability and Reusability Principle:* This principle is dependent on two other design principles, namely, the model construction and the documentation principles. To share a model and to reuse it, model construction and its' documentation were essential.

  However, this problem did not possess both of these principles. Thus, the shareability & reusability principle was not observed.
- *Effective Prototype Principle:* The validity of this principle is also dependent on two other principles, namely, the reality and the shareability & reusability principles. According to the previous principle analysis, this problem was ensured a realistic situation; however, the potential of being shareable and

reusable for different situations was not provided. Therefore, the effective prototype principle was observed only partially.

This problem was different from the other modeling activities in this category because the tray pastry problem was the only activity in which the self-assessment principle was observed although it did not indicate model construction principle. However, as it was generally observed in the modeling activities, the self-assessment principle was observed in the situations when the model construction principle was provided. The tray pastry problem reflected two of the six design principles which were the reality principle and the self-assessment principle. It reflected one of them only partially, which was the effective prototype principle. Three of the six design principles were not observed which were the model construction principle, the documentation principle and the construct shareability & reusability principle in this problem.

4.2.1.5. Modeling Activities Completely or Partially Indicating Two Design Principle

45 modeling activities were indicating two design principle where one of them was partially observed in these activities. More specifically; these modeling activities provided only the reality principle, and another principle partially, which was the effective prototype principle. Four of the six design principles were not observed in these 45 activities; they were the model construction principle, self-assessment principle, documentation principle, and the construct shareability & reusability principle. The Bus Problem is given in Figure 4.10 as an example of similar activities in this category.

Figure 4.10. Bus Problem - sample modeling activity addressing two design principles, one of which was partially observed

In this problem, there are two busses. One is from Erzurum and the other is from Erzincan, departing at the same time to arrive in İstanbul. The distance from Erzurum to İstanbul is 1260 km and the distance from Erzincan to İstanbul is 1065 km. The average velocity of the Erzurum-bus is 95 km/h and the average velocity of the Erzincan-bus is 80 km/h. These two buses use the same route to arrive in İstanbul.

It was asked in the problem whether these two buses can come side by side with each other along the way. If they do, what would be the difference in the time it took? Which bus would arrive in İstanbul earlier?

When the activity was examined, each principle was observed as follows:

- *Reality Principle:* A real and meaningful situation was observed in the problem that students might encounter this kind of situation while they are traveling long distances. Thus, the real-life principle was observed.
- *Model Construction Principle:* The conditions of the two different busses from different cities were explained in the problem. Students who would solve this question might not need to re-organize or optimize the information because it was given quite clearly and directly. In parallel to this situation, students might not use their mathematical reasoning to solve the question as it doesn’t seem challenging either. Thus, the model construction principle was not observed.
- *Self-Assessment Principle:* This problem consisted of three sub-questions, which required direct answers. No criterion was set to find answers. Therefore, the self-assessment principle was not observed.
- *Construct Documentation Principle:* The question did not request any explanation or documentation tool to provide solutions. Thus, the construct documentation principle was not met.
- *Construct Shareability and Reusability Principle:* To share and reuse a model or a tool, first, it is needed to be constructed and, then it is expected to be documented. In this problem, both the construction and documentation processes were not observed. Thus, the construct shareability and reusability principle was not observed.
- *Effective Prototype Principle:* Two design principles, namely, the reality and the construct shareability and reusability principles are foundations for this principle. The problem reflected a meaningful and realistic situation that might be encountered while traveling by intercity busses.

  However, an indication of shareability & reusability issues of the problem was not provided. Thus, the effective prototype principle was observed only partially. In the cases where the question had provided more comprehensive data to construct a model and documentation tools to share and reuse it for different situations, this question has a potential to be an effective prototype for structurally similar situations. For example, ship captains are expected to use this model while on route.

This question is very similar to the questions which were involved in classic mathematics books. This type of question generally reflects on some common issues that are usually not challenging for students and not requiring them to use the mathematical reasoning. These questions also do not require an explanation or a model to reflect the possible solutions of problems. In models-and-modeling perspective, this kind of a problem may provide a reality principle, and considering this principle, the partial observation of effective prototype principle may be provided.

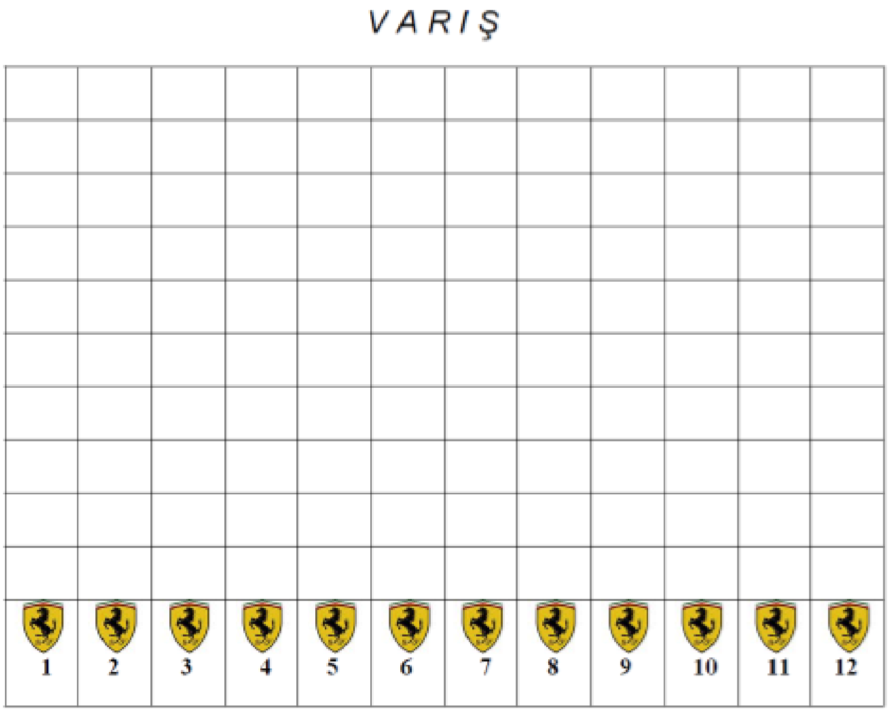
Similar to the bus problem, 45 modeling activities reflected only one of the six design principles completely, which was the reality principle. Being dependent to the reality principle, these 45 modeling activities also indicated effective prototype principle partially. The rest of the six design principles, which were the model construction principle, self-assessment principle, documentation principle, and the construct shareability & reusability principle were not observed. In addition, in the graduate modeling studies on modeling in mathematics education, there was no modeling activity that possessed only one of the design principle completely without an additional principle that was partially met.

4.2.1.6. Modeling Activities Indicating None of the Design Principles

The Horse-Racing problem was the only activity that did not meet any of the six design principles. This activity was used in two different graduate studies on modeling in mathematics education. The horse-racing problem is given in Figure 4.11.

BÜYÜK AT YARIŞI OYUNU*

Arkadaşımız yarın okulunda aşağıda kuralları verilen oyunun oynandığı bir yarışmaya katılacak. Ona bir mektup yazarak hangi numaralı atı seçmesinin daha iyi olacağını nedenleriyle birlikte açıklayınız.



Kurallar:
1) Atları başlama pozisyonuna 1 den 12 ye kadar yerleştir.
2) Her oyuncu farklı bir at seçsin
3) İki zar yuvarla ve çıkan sayıları topla.
4) Bulunan numaradaki atı bir kare ilerlet.
5) Bitişe varan ilk at kazanır.

*Bu etkinlik Doruk (2010)'un çalışmasından alınmıştır.

Figure 4.11. Horse-Racing Problem - the modeling activity indicating none of the six design principles

In this problem, there was a competition in a school. It expected students to write a letter in detail outlining reasons to explain which horse was to be chosen from 1 to 12 to reach the opposite side first. The rules were given below:

1. Arrange the horses in their starting positions from 1 to 12.

2. Each player selects a different horse.
3. Throw two dices and sum the total of the dices up.
4. Place the horse, which represents the sum of dices, one step forward.
5. The horse first reaching the opposite side wins.

When the activity was examined, each principle was observed as follows:

- *Reality Principle:* The rules of the game were given in the problem and they were not a reflection of a meaningful situation in real life, because the arrival of the first horse to the opposite side depends on the probability of dices. To find a consistent pattern with the dices which can work in every game of horse-racing is not probable, therefore cannot be used in every situation. Since the rules of the game were not realistic and meaningful, the reality principle was not observed.
- *Model Construction Principle:* The rules of the horse-racing game were given enumerated. Students who would solve this question might find the probabilities of the upside sums of dice; this might be helpful for them to decide which one step forward of the horses; however, it is hard for students to know the remaining 11 steps forward which are conditionally dependent on each other to win the game. Therefore, reaching a meaningful solution by organizing and optimizing the probabilities and also using mathematical reasoning does not seem to be an efficient way to win the game. Each horse-racing game has its own probability. Thus, the model construction principle was not observed.

- *Self-Assessment Principle:* Reaching the opposite side first seemed to be a criterion for the self-assessment at first glance. However, the results are dependent on the conditional probability of dices. Thus, the self-assessment principle was not provided.

- *Construct Documentation Principle:* The activity asked for a letter as a documentation tool to outline the reasons for the selected horse to reach the opposite side first. However, a meaningful situation was not observed to be documented in this problem. Thus, the documentation principle was not directly provided.
- *Construct Shareability and Reusability Principle:* When a created model provides a way of thinking to be shared, transformed, easily adapted, or reused in different settings, it ensures the construct shareability & reusability principle. However, in this problem, when the previous principle analyses were considered, it was not observed as a meaningful situation to create a model and the documentation ultimately was not observed. Therefore, this problem was not providing the construct shareability & reusability principle because it does not carry the model construction and documentation principles.
- *Effective Prototype Principle:* This principle is based on two other design principles, namely, reality and shareability and reusability principles. Since the rules of the horse-racing game were not meant to construct a model, and it did not have the potential to be reusable in different situations, this problem did not possess the reality and shareability and reusability principles. Thus, the effective prototype principle was not observed.

When the studies, which used the horse-racing activity were considered, the researchers aimed to observe the competences of students in the way of solving and creating probability problems. This activity was adapted from the study of Doruk (2010). The researchers used this activity as a model eliciting activity to analyze the related competences of students. However, when the rules of the horse-racing problem are considered, this problem does not involve a real-life situation. Considering that the general premise of modeling perspectives (not only the MMP but other modeling perspectives as well) is transforming real-life situations to a mathematical way of expression, this problem does not indicate a modeling problem since it violates the main premise of modeling. I tried to observe for the other principles, but not surprisingly, other design principles were not identified in this problem. The horse-racing problem was the only activity in the data set 2 that did not address any of the six design principles.

To conclude, in this part of the study, the nature of 281 different modeling activities were analyzed in detail by considering the six design principles of the MMP. Among 281 modeling activities, the activities that were presented in this chapter were the ones that best represented each category.

To sum up, the type of modeling studies were mostly comprised of master thesis. The first modeling study was conducted by Dokuz Eylül University in 2005, as a master thesis. It was notable that Middle East Technical University was the leading university when the number of published Ph.D. dissertations was considered. Modeling studies mostly involved the number of modeling activities changing between 3 and 6. When six design principles of MMP was considered in modeling activities, it was observed that many of modeling studies were provided six design principles completely while some of them provided these principles only partially. There was only one activity which did not ensure any of six design principles. Indeed, the number of modeling activities which met more than three of six design principles was 169 (60%), while the number of modeling activities provided less than three of six principles (either completely or partially) was 112 (40%). Thus, it is hard to underestimate the amount of modeling activities which involve less than three of six design principles, correspond to 40% of modeling activities analyzed in this study.

CHAPTER 5

CONCLUSION AND DISCUSSION

This study analyzed modeling MS and Ph.D. studies conducted in Turkey in the field of mathematics education during the last two decades. Modeling studies were investigated concerning their features and characteristics of their nature. Foremost, 70 modeling studies, found from the HEC, were analyzed for their general features. These features comprised of the distribution of modeling studies with respect to study-type, years of completion, and among universities. Furthermore, 42 of the modeling studies, which involve modeling activities, were investigated for the reflection of the modeling studies' nature, particularly their alignment with the six design principles of the Models-and-Modeling Perspective (MMP). The nature of modeling studies was analyzed by taking account of their methodology types, participants, number of activity inclusions, and the number of modeling activity distributions. Additionally, seven different modeling activities involved in 42 modeling studies were analyzed, serving as examples to show to what extent the six design principles of MMP were reflected in the activities.

Similar to this study, Albayrak (2017) investigated the descriptive features of mathematical modeling studies in Turkey. 28 MS theses and Ph.D. dissertations in the field of mathematics education were analyzed in her study. It was the only study that resembled this study concerning a descriptive analysis conducted in the mathematics education field in Turkey. Therefore, while reflecting descriptive features of modeling activities, the results of both studies were compared to some extent in this part.

On commencement, the general features of the 70 modeling theses and dissertations were analyzed. According to the results (see Figure 4.1), 49 (70%) of the modeling studies conducted in the field of mathematics education were master theses (MS), while 21 (30%) of them were doctoral dissertations (Ph.D.). In the study of Albayrak (2017), the distribution of modeling studies were 57% master thesis (MS) and 43% of them were doctoral dissertations (Ph.D.). In both of the studies, the results showed that the amount of MS theses was more than the Ph.D. dissertations and the percentage was increased in the favor of master theses in the progress of time. Considering the extent and the duration of doctoral studies, this distribution is not surprising. In addition, since the scope of the Ph.D. studies is often broader and the data analysis procedure is deeper than MS thesis, it takes longer time to complete a doctoral study, which might have influenced the smaller percentage appeared in this study.

The data consisted of years of modeling studies completed over the last two decades (see Table 4.1.) indicated that the number of conducted modeling studies increased gradually from 2000 to 2019, particularly in the master thesis. However, a rigid increase was observed in the number of modeling studies from 2013 to 2014 particularly. The reason for more than the double fold from 2013 to 2014 may be the revise of the mathematics curriculum in 2013. In the objectives of the renewed mathematics curriculum, MONE (2013) emphasized the significance of modeling in the teaching of mathematics. This could be a reason for this rigid increase in the number of modeling studies conducted in the mathematics education field after 2013.

When Table 4.1 was analyzed closely, it was seen that the first modeling study in the mathematics education field was published by Dokuz Eylül University as a type of master thesis in 2005. On the other hand, the first Ph.D. dissertation was published in 2009, which was pioneered by Gazi University. That the first Ph.D. dissertation was published later than the first MS thesis in modeling is not surprising considering the longer time needed to complete a Ph.D. and the broader scope of a Ph.D. dissertation compared to an MS thesis. The most popular years of published modeling studies were 2016 and 2017 when the numbers were taken into account.

Modeling theses and dissertations, written in the field of mathematics education, were completed at 24 different universities (see Table 4.2). The outstanding universities where the maximum number of modeling studies was published were Gazi University and Ataturk University, which were followed by Dokuz Eylül University and the Middle East Technical University. It draws attention that these universities are public universities. According to the study of Albayrak (2017), the highest number of modeling studies in the mathematics education field was published at Atatürk University, and it was followed by Balıkesir University and Middle East Technical University. It seems that in the last three years, Gazi and Dokuz Eylül Universities have increased the number of studies on modeling conducted in the subject of mathematics education. Moreover, the highest number of Ph.D. dissertations was published at the Middle East Technical University, which indicated that this university highly contributed to the modeling literature in mathematics education through extensive doctoral research projects.

The nature of the 42 modeling studies included modeling activities, and their alignment with the MMP was also investigated. As a first step, the methodology types of modeling studies were analyzed (see Figure 4.2), and it was found that the qualitative research method was the most used in modeling studies. A similar result was obtained in the study of Hart et al. (2009) that research studies in the field of mathematics education, although not particularly modeling studies, mostly utilized qualitative research methods between years 1995 and 2000. It was also indicated in the study of Albayrak (2017) that the qualitative research method was the most preferred type of research method in modeling studies. Furthermore, in the last two decades, the majority of qualitative modeling studies that were analyzed in this study were case studies.

A case study based on the philosophy of the hermeneutic paradigm is a qualitative study that investigates and interprets cases or situations in detail (Cohen, Manion & Morrison, 2000). This study also compares the possible results to generalize them analytically. Following the qualitative research method, modeling researchers preferred the mixed research method, and a few of them (only 4 of them) used quantitative research method in their graduate studies. The reasoning behind this might be that qualitative research methods such as interviews or observations could provide the modeling procedure more clearly and in detail. Thus, this may indicate that researchers mostly needed qualitative data collection methods to understand the modeling process of the participants.

When the participants of modeling studies were investigated (see Table 4.3), the analysis revealed that pre-service mathematics teachers were mostly involved in modeling studies. One of the possible explanations to this tendency may be that modeling studies might be conducted by researchers who were working or studying in universities and so, it might have been easier for them to reach and communicate with pre-service teachers. Furthermore, when the most used research method (qualitative research method) was considered, these researchers might have possibly used convenient sampling method while selecting their participants as pre-service mathematics teachers for their modeling studies. Another possible explanation to this tendency is that researchers may have had difficulty to get permission from elementary or high schools to implement modeling activities. Therefore, they may decide to work with pre-service teachers. Table 4.3 pointed out that there were a couple of modeling studies which worked with 4^{th} and 5^{th} grades. This indicated that researchers might have a perception that modeling activities required advanced mathematics knowledge and therefore could be more applicable in middle and high schools.

Differently from this, it was not observed that any MS or Ph.D. modeling study worked with 10^{th}, 11^{th}, and 12^{th}-grade students. In this case, researchers may have preferred not to work with these grades by considering that the implementation period of the modeling activities may be long and not manageable in these grades, particularly if the class sizes are large and group work is not suitable.

Besides these, almost half of the modeling studies included modeling activities that were changing in the range of 3–6 which is the number of activities involved (see Table 4.4 or Figure 4.3). Considering the duration of an MEA implementation, which usually took four or six weeks, it is considered a moderate number of activities (i.e., 3–6). When the distribution of modeling activities was analyzed by taking into account MS theses and Ph.D. dissertations separately, it was observed that the number of used modeling activities in MS theses was mostly less than six. In contrast, in Ph.D. dissertations, it was equal and more than three. Considering the duration and the scope of MS and Ph.D. studies, this distribution is not surprising. Since Ph.D. studies require an extensive data collection which was possible in a relatively longer time, it is expected to see that Ph.D. studies involved implementation of more modeling activities than the MS thesis studies.

As a result of the modeling activity analysis, 288 modeling activities were observed in 42 modeling theses and dissertations. Since one of these activities (i.e., the Big Foot Problem) was used in seven graduate studies and another one (i.e., Horse Racing Problem) was used in two graduate studies, there were 281 different modeling activities. These 281 modeling activities were analyzed by speculating to what extent the six design principles of MMP were observed (see Table 4.5). According to the results, all of the six design principles of MMP were fully observed in 54 of the modeling activities. The rest of the 227 modeling activities satisfied some of the design principles either partially or completely however some did not fulfill the conditions of the principles.

As the findings of this study showed, although many of the six design principles were not observed in the modeling activities, they were used as modeling activities in the graduate studies. One of the possible explanations could be that these activities might have adopted other modeling approaches although they referred to the models-and-modeling perspective and the MEAs in literature and methods sections of these thesis and dissertations. Another explanation could be that researchers might think these activities as modeling activities, but not necessarily the MEA because they 227 of them did not possess all the six design principles of MEAs. Another possible explanation could be that researchers may prepare modeling activities by imitating existing ones without considering each of the six design principles one by one. During the analysis, it drew attention that the self-assessment principle was mostly observed in the situation when the model construction principle was also observable, except for one modeling activity. The reverse of this situation was not notable. The reason may be that if a problem has a potential to be constructed as a model for a situation, it is possible to set criteria to evaluate the quality of the model. These criteria can then be used by students whilst self-assessing. On the other hand, setting criteria may not be a reason for constructing a model, as seen in the findings of this study.

Another observation was that when the problem situation was not meaningful or realistic (or hypothetically observable), depending on the lack of the reality principle, the model construction, documentation, self-assessment, shareability and reusability and effective prototype principles were not observed. Considering the general premise of the modeling perspectives which forge a link between real life situations and mathematical expressions, the reason may be the violation of this foundation.

5.1. Limitations and Recommendations

This study was limited in respect to the number of modeling graduate studies (MS and Ph.D.) on mathematics education conducted in Turkey. Therefore, it is recommended to include the other modeling studies, particularly journal articles, completed in Turkey into this investigation, which would increase the content validity of the findings and present the situation of modeling research in Turkey more extensively.

Above and beyond, the Model-and-Modeling Perspective (MMP) and the coding frame of the six design principles assisted in forming the basis of this study. In other words, six design principles of the MMP set the concept-driven coding frame. To this respect, this study suggests other modeling researchers to consider another modeling perspective to build the concept-driven coding frame and examine the modeling studies from a different modeling perspective. In this sense, it is expected to have this study as a starting point of more extensive and broader document analysis that would present the portrait of mathematics education in Turkey.

In addition, this study is limited with the metadata and modeling activities of the MS thesis and Ph.D. dissertations. More specifically, the student/modeler responses to the modeling activities were not included in the content analysis of the documents. In this sense, this study can be extended by considering the student/modeler responses; that is, data excerpts shared in the modeling studies. As seen in the findings of the study, some of the design principles were difficult to observe on paper; however we could identify their potential existence considering their relation with other principles. This was one of the challenges that I experienced in the data analysis, which influenced the validity of the findings. To this vein, I suggest other researchers who would use six design principles as a methodological framework conducting observations during the implementations of the modeling activities.

Furthermore, this study was limited to the MS theses and Ph.D. dissertations, completed until 2020, which were loaded onto HEC's website. These studies were accessed by using advance research section of the HEC's website with the keywords modeling and mathematics education. There may be modeling studies conducted in mathematics education but not indexed with both of these keywords in the HEC thesis database. Therefore, I may not be able to reach those studies to include this investigation. The graduate studies completed in 2020 were not included because the year has not been completed yet, and including the studies completed in the first couple months of 2020 would be misleading.

As a consequence, this study may carry importance for stakeholders such as the Ministry of National Education (MoNE), researchers, and teachers. MoNE can integrate MEAs into the mathematics curriculum by considering the analysis of this study concerning the six design principles of the MMP. Also, this study may take researchers' attention to evaluate the quality of modeling activities before the implementation of them. Besides, this study provides MoNE and researchers with the current portrayal of modeling studies completed in Turkey. In this sense, this study shows the gaps in the modeling research in Turkey such as not having many studies that used modeling in elementary school and upper elementary school levels. The study also showed the contribution of universities to this area, which may inform HEC and MoNE stakeholders and curriculum developer about the experienced modeling researchers and which universities carried out modeling graduate studies the most. This information may be useful when they were seeking for the most experienced and qualified researchers in this area who may be asked for teacher seminars. Such information is also crucial for researchers to fulfill these gaps to progress modeling studies on mathematics education.

Furthermore, teachers can use MEAs in their classes by adopting a model and modeling approach instead of traditional teaching methods; however they need to know how to choose the appropriate modeling activities for their classes. In this regard, this study may provide them a perspective to evaluate their modeling activities.

Apart from these, I appreciate very much to the researchers and the HEC theses dissertation database that enabled sharing of their modeling studies that were used in this study. Without this online platform and the endeavors of researchers on mathematical modeling, this current study would not be possible.

To sum up, this academic study may be viewed a building block for the decision of further modeling studies in Turkey. Moreover, academics and researchers can conduct similar modeling studies in different countries. They can also use existing modeling studies that have been conducted to compare the features and the nature of modeling studies managed in different countries. Hence, this study is expected to broaden the way for researchers who are on the path of modeling in mathematics education particularly in Turkey but hopefully around the world.

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APPENDIX

Permission Paper

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Konu: Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi: İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Dr. Öğretim Üyesi Şerife SEVİNÇ

Danışmanlığını yaptığınız Dilek KOÇ'un **"Türkiye'de Matematik Eğitiminde Modelleme Üzerine Yazılmış Yükseklisans ve Doktora Tezlerinin İçerik Analizi"** başlıklı araştırması İnsan Araştırmaları Etik Kurulu tarafından uygun görülmüş ve **2018-EGT-209** protokol numarası ile onaylanmıştır.

Saygılarımla bilgilerinize sunarım.

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