LOCATION–ROUTING AND SYNCHRONIZATION PROBLEMS IN CITY LOGISTICS

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MOHAMMAD SALEH FARHAM

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submitted by MOHAMMAD SALEH FARHAM in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Industrial Engineering Department, Middle East Technical University by,

Prof. Dr. Halil Kalıpcılar
Dean, Graduate School of Natural and Applied Sciences

Prof. Dr. Yasemin Serin
Head of Department, Industrial Engineering

Prof. Dr. Haldun Süral
Supervisor, Industrial Engineering, METU

Assoc. Prof. Dr. Cem Iyigun
Co-supervisor, Industrial Engineering, METU

Examine Committee Members:

Prof. Dr. Ömer Kırca
Industrial Engineering, METU

Prof. Dr. Haldun Süral
Industrial Engineering, METU

Prof. Dr. Bahar Yetis Kara
Industrial Engineering, Bilkent University

Prof. Dr. Sinan Gürel
Industrial Engineering, METU

Prof. Dr. Ferda Can Çetinkaya
Industrial Engineering, Çankaya University

Date: 08/05/2020
I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Surname:  Mohammad Saleh Farham

Signature:  

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ABSTRACT

LOCATION–ROUTING AND SYNCHRONIZATION PROBLEMS IN CITY LOGISTICS

Farham, Mohammad Saleh
Ph.D., Department of Industrial Engineering
Supervisor: Prof. Dr. Haldun Süral
Co-Supervisor: Assoc. Prof. Dr. Cem Iyigun

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City logistics aims to improve urban freight transportation by considering the costs and benefits of public and private sectors, consolidating segmented freight shipments, and integrating the individual actors in a collaborative environment. This thesis studies network design problems in city logistics systems to address managerial challenges in urban freight transportation. We consider two network design schemes, namely the one and the two-echelon distribution networks, to formulate strategic and tactical level problems in urban freight transportation. In the single-echelon systems, freight is distributed from consolidation centers located on city boundaries to the customers inside the city. In the two-echelon systems, goods are unloaded at the intermediate facilities, called satellites, and consolidated into smaller vehicles suitable for the last-mile delivery in city centers. From an operational level perspective, we highlight the importance of synchronizing first and second echelon vehicles at the satellite locations and discuss the relation of the satellite synchronization problem to the network design problems. We propose mathematical programming formulations for the introduced strategic, tactical, and operational level problems in city logistics,
and develop exact and heuristic solution approaches. The exact approaches use column generation to find optimal delivery routes efficiently. The heuristics are based on the hierarchical decomposition of the original problem into its basic decisions. Extensive computational studies in this thesis provide new insights into designing and implementing a practical city logistics system in real-world.

Keywords: Urban freight transportation systems, Location-routing problem with time windows, Decomposition, Branch-and-price, Constrained clustering
ÖZ

ŞEHİR LOJİSTİĞİNDE YER SEÇİMİ–ROATALAMA VE SENKRONİZASYON PROBLEMLERİ

Farham, Mohammad Saleh
Doktora, Endüstri Mühendisliği Bölümü
Tez Yöneticisi: Prof. Dr. Haldun Süral
Ortak Tez Yöneticisi: Cem İyigün

Mayıs 2020 , 201 sayfa

Şehir lojistik, kamu ve özel sektörlerin maliyet ve faydalarını göz önünde bulundurarak, bölümlere ayrılmış nakliye sevkiyatlarını birleştirerek ve bireysel paydaşları işbirlikçi bir ortama entegre ederek kentsel yük taşımacılığını geliştirmeyi amaçmaktadır. Bu tez, kentsel yük taşımacılığında yönetimsel zorlukları ele almak için şehir lojistik sistemlerinde ağ tasarım problemlerini inceler. Kentsel yük taşımacılığında stratejik ve taktik seviye problemlerini formüle etmek için iki ağ tasarım şeması, yanı bir ve iki seviyeli dağıtım ağlarını göz önüne alıyoruz. Tek seviyeli sistemlerde, yük şehir sınırları içinde bulunan konsolidasyon merkezlerinden şehir içindeki müşterilere dağıtılan. Iki seviyeli sistemlerde, mallar uydu olarak adlandırılan ara tesislerde boşaltılır ve şehir merkezlerinde son kilometrelık teslimat için uygun olan daha küçük araçlarla birleştirilir. Operasyonel bakış açısından, birinci ve ikinci seviyeli araçların uydu konumlarında senkronize edilmesinin önemini vurgular ve uydu senkronizasyon probleminin ağ tasarım problemlerini ile ilişkisini tartışıyoruz. Şehir lojistikinde tanımlanan stratejik, taktik ve operasyonel seviye problemleri için matematiksel program-
lama formülasyonları öneriyor, kesin ve sezgisel çözüm yaklaşımları geliştiriyoruz. Kesin çözüm yaklaşımlar, optimum dağıtım rotalarını verimli bir şekilde bulmak için kolon üretim yöntemini kullanır. Sezgisel yöntemler, orijinal problemin temel kararlarını hiyerarşik olarak ayrımsına dayanır. Bu tezdeki kapsamlı bilgisayarsal deneyleri, gerçek dünyada pratik bir şehir lojistik sisteminin tasarlanması ve uygulanmasına yeni bakış açıları sunmaktadır.

Anahtar Kelimeler: Kentsel yük taşma sistemleri, Zaman pencereleyyle yer seçimi-rotalama problemi, Dal-ve-fiyat, Kısıtlı kümeleme
To my family.
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CHAPTER 1

INTRODUCTION

City logistics aims to improve freight transportation by considering the costs and benefits of schemes to the public and private sectors, coordinating the individual actors, and managing segmented components of the urban freight transportation in a cooperative environment. In this chapter, we study city logistics systems and address urban freight transportation problems from an operations research/industrial engineering (OR/IE) perspective. Our modeling and solution approaches to the city logistics problems are introduced, and the contribution of the thesis is highlighted.

1.1 Motivation and Problem Definition

The increasing trend of urbanization (United Nations Population Division 2018) leads to high demand for goods and services in cities. The cities turn into places of production and consumption with large number of origins (i.e., supply points) and destinations (i.e., demand points), various segmented freight transportation modes (e.g., trains, trucks, and electric cars), and high volume of polluting traffic. From the operational point of view, freight transportation is a commercial activity with complex segmented supplier-customer relations and involves sorting, storage and distribution costs, location, transportation and carbon emission related constraints, and pickup/delivery time and service quality considerations. Therefore, providing a cheap, reliable, fast, and sustainable freight movement in today’s growing cities is a challenging issue that concerns the ones who are involved in or affected by the freight transportation activities. The major stakeholders of the urban freight transportation are:

- *Shippers* whose main interest is delivery/pickup of goods at maximum level of
service and minimum cost to satisfy demands. Manufacturers, wholesalers, and retailers can be considered as shippers.

- **Freight carriers** or transport operators who seek a low-cost but high-quality transport operation that satisfies the interests of shippers and receivers.

- **Customers** or receivers who expect to receive their goods on time at low prices with minimum traffic congestion, noise, and air pollution and without accidents around their residential or retail areas. Shopkeepers and store owners are considered as receivers.

- **Public administrators** including local municipalities and the government. The local administrators try to make the city an attractive and safe place for inhabitants and visitors with a sustainable, effective, and efficient freight transportation system, while the main concern of the national government is to minimize external effects of the transportation activities in order to enhance the socio-economic development of the city, and to increase the employment opportunities.

Different kinds of needs and often conflicting interests among these actors create economic and environmental inefficiencies if they act individually and follow their own goals without any collaboration. For this reason, **city logistics** (CL) is developed to address planning, designing, implementing, operating, and controlling transportation activities in order to provide efficient, effective, and seamless flow of freight and related information from supply points to consumption points within the entire city.

The interaction among different stakeholders in cities is shown in Figure 1.1. Shippers, carriers, and customers have high level of information exchange. While administrators usually do not interfere at operational levels, their regulations and requirements constrain the urban freight transportation network.

CL considers the costs and benefits to the citizens and public and private sectors, integrates the individual actors and resources, and converts segmented components of the urban freight transportation to a cooperative environment (Taniguchi et al. 2001, Taniguchi & Thompson 2002, Crainic et al. 2004, Crainic 2008, Crainic et al. 2009). Therefore, key elements of CL are **coordination** of shippers, carriers, customers, and
administrators and consolidation of freight in a collaborative and integrated transport system, where stakeholders can cooperate in making decisions, exchanging data, and dealing with transactions by means of an advanced information system (Crainic et al. 2004, Crainic 2008, Gonzalez-Feliu & Salanova 2012). The components of a collaborative city logistics system (CLS) is illustrated in Figure 1.2.

Planning and decision-making activities in CL are performed at strategic, tactical, and operational levels (Crainic & Laporte 1997, Crainic 2008). Network design and modal choice, location of facilities and consolidation centers, and capital investment planning are the major long-term decisional activities in strategic planning. In this stage, the decision maker evaluates the performance of the proposed system and analyses its relation to the general transportation system of the city (Crainic 2008).

Tactical planning involves mid-term to short-term interrelated decisions about allo-
cating demands and managing limited available resources for consolidating, shipping, and quality control activities. The decision makers aim to construct an efficient transportation through frequency and flow assignments and to perform an effective resource utilization in order to satisfy customer demands on time (Mancini et al. 2014). The main concerns at this level are fleet size and composition, vehicle departure times, routes, and loads, and, if applicable, the utilization of the facilities and distribution of work among them. Tactical planning is also important for evaluating CLSs from initial proposals to deployment scenarios and operation policies (Crainic 2008).

At the operational stage, short-term or daily operations, e.g. carbon emission of vehicles, daily distribution routes, synchronization of in and out flows, work schedule of drivers and terminal personnel, and dynamic adjustment of vehicles and terminal operations are considered.

Understanding the structure and evolution of cities is crucial for CL practitioners and decision makers. Spatial organization of the city and its centers and sub-centers, distribution of suppliers and customers, and underlying infrastructure network are important factors in designing and deploying an effective CLS. Historically, before the appearance of public transport inside the cities, high cost of communication and transportation led to concentration of businesses and manufacturers near the city center that resulted a monocentric city with high density of population in its center. Monocentric structure is still observed in small cities with limited social and commercial activities. The rise of subways and production of streetcars allowed people to live around stations that are further away from the city center. Availability of cars increased the mobility of individuals and the low-value non-accessible zones rapidly became important residential areas. Hence, sub-centers were formed and grown as the population in the city increased (Barthelemy 2016).

The dynamics of the cities impact the management of the CLSs in all three levels of decision-makings. For example, customers might be located in different zones subjected to different policies (such as time-windows and vehicle load restriction), which challenges vehicle fleet composition and routing decisions. Another example is where the land use regulations or the available infrastructure vary from one sun-center to another. Therefore, facility location and consolidation center selection decisions
have to be made accordingly.

Consolidation activities usually take place at physical facilities called *urban consolidation center* (UCC) or *city distribution center* (CDC). CDCs are common facilities for freight shippers and carriers who try to deliver goods to the customers. They enable an inter-modal transportation systems where the link between CL service providers and end-users is established (Song et al. 2008). Goods from different shippers arrive at CDCs and are consolidated such that the same vehicle delivers demands of customers in the same inner-city area. The delivery can be done using environmental-friendly vehicles that meet the city transport regulations. Therefore, an efficient transshipment is obtained compared to segmented deliveries by individual carriers. With a reduced number of vehicle movements in populated city areas and an improved vehicle load factor, a CLS can help to reduce emission and noise, increase safety, and decrease barrier effects (Benjelloun et al. 2010, Crainic et al. 2004). Hence, the consolidation centers are the key element for sustainability of CLSs in terms of economical, environmental, and social dimensions (Filippi et al. 2010, Allen et al. 2012b, Björklund & Gustafsson 2015, van Heeswijk 2017).

Even though fixed and variable costs of a CDC might be high, it can provide the possibility of offering value-added logistics and retail services such as sorting, stocking, unpacking, and labeling/pricing that can reduce cost for shippers and carriers (van Rooijen & Quak 2010, Allen et al. 2012b). Estrada & Roca-Riu (2017) analyze costs and benefits of carriers and CDC operators and identify conditions under which a CLS becomes beneficial for its involved actors. For customers, poor accessibility by large trucks, lower receiving costs, reduced number of shipments due to consolidated deliveries, and the offer of other services such as waste collection and reverse logistics are main motivations to support a CLS. Other than receivers and carriers, local administrators also affect CL practices by providing supports or setting regulations, such as zoning and parking limits in place. As CDCs involve multiple actors of CL, their utilization is an important step toward a better organization of urban freight transportation and is instrumental in most CLSs (Browne et al. 2005, 2007).

From a practical standpoint, CDCs can be classified into three types based on the type of operation and the geographical area they serve (Allen et al. 2012b). The most
common kind is the one that serve all or a part of an urban area. Such systems are usually initiated or supported by the local authorities to mitigate negative impacts of the freight transportation on the city environment. The other type of CDCs are employed to serve large sites with a single proprietor (e.g. supplying hospital services or shopping center products). CDCs for construction projects are in the last category. They are used for consolidating construction materials for major building projects and exist until the project ends. The last two types of CLSs are usually started by private sectors. As CDCs require public/private capital investment and their location can affect tactical and operational decisions, infrastructure development and network design are among the most important decisions in CLSs.

From a network design point of view, CLSs are either single or multi-echelon systems. In one-echelon (1E) systems, freight is delivered from suppliers to the CDCs in large batches. The delivery is performed through different transportation modes such as road, sea, railroad, etc. Infrastructure and location of CDCs are determined accordingly. In CDCs, goods are consolidated and loaded into urban trucks to be delivered to the customers. Single-echelon systems are studied vastly in the literature. Such systems are useful for small cities with a limited number of carriers and shippers and small public involvements (Crainic 2008, Crainic et al. 2009) and are commonly studied as cost-minimization problems.

Multi-echelon CLSs are more applicable for large cities with big population and high social and cultural activities in minor and major city centers. Multi-echelon CLSs involve modal change in their transportation and are first implemented by cargo and mail delivery companies in 2007 and 2009 (Crainic et al. 2009). The presence of intermediate facilities in the multi-echelon CLSs strongly affects the cost of the system and the service quality (Guastaroba et al. 2016) and can lead to sustainable solutions for urban freight transportation (Soysal et al. 2015). Two-echelon (2E) CLSs are the most common type of multi-echelon systems. In the first echelon, urban trucks carry goods from CDCs to the satellite platforms. These trucks, called primary vehicles, may not be allowed to enter the city center due to the city regulations or limitations. CDCs are located on city borders that are close to the geographical area that they serve. Satellites, on the other hand, are small inner-city locations where usually no warehousing activities like cross-ducks are allowed. At the satellite locations, goods
are consolidated and loaded into smaller vehicles, called secondary vehicles, for final delivery. The type of secondary vehicles depends on the city characteristics and transports options. They are designed to be environmental-friendly and meet city regulations. Therefore, 2E systems allow using different transportation mode in each echelon.

Freight distribution networks aim to “optimize” the flow of goods and “improve” the network by choosing the best design configuration and transportation modes (Ambrosino & Scutellà 2005). This study introduces one and two-echelon urban freight distribution network design problems for CLSs, proposes mathematical formulations to reflect real-world applications of the CLSs, and develops practical solution approaches. In the next section, we provide more details on the problem formulations and solution approaches.

1.2 Proposed Problem Formulations and Solution Methods

In this thesis, we concentrate on the main network design problems of the CLSs and study the synchronization problem in the intermediate facilities of the multi-echelon systems. First, the optimization challenges of the 1E systems are highlighted and the corresponding freight distribution problem is investigated from an OR/IE perspective. Next, a 2E urban freight transportation solution is proposed that addresses managerial issues of the 2E-CLSs. The 2E system extend the 1E one by incorporating more decisions, constraints, and coordination issues in a larger network. Finally, an important operational level decision-making problem is formulated to solve the synchronization problem at the satellite platforms and improve the solution of the 2E problems.

1.2.1 Single–Echelon Urban Freight Transportation

In the 1E urban freight transportation, a set of customers has to be served from a set of distribution centers. The customers are considered as the demand points located in urban areas. The supply points are city distribution centers that are located outside the city centers but close to the demand areas they serve. Since the majority of the transportation activities take place in city environment, the problem is formulated un-
der constraints imposed by local authorities (such as access time windows and vehicle size regulations). We define the 1E urban freight transportation problem under suppliers, carriers, customers, and administrators’ interests and constraints as: satisfying all customer demands from selected CDCs such that customer due dates, access time window, vehicle size, and CDC capacity restrictions are satisfied with minimum total costs. The total cost is considered as the sum of facility selection (opening), vehicle utilization, and traveling costs.

We propose two mixed-integer linear programming (MIP) formulations for the problem, namely the arc-based formulation and the path based formulation, and develop an exact branch-and-price solution algorithm based on the proposed path-based formulation. In the branch-and-price approach, we use dynamic programming techniques to generate vehicle paths. In order to solve the problem more efficiently, a two-stage heuristic, called top-to-bottom approach, is proposed. In the first stage, strategic decisions of the problem are fixed, while in the second stage tactical decisions are made according to the decided strategic plan. Our computational study show the performance of the proposed solution methods on a large set of instances with different characteristics. We also show the benefit of using the proposed heuristic approach to find an upper bound or a solution as the starting point for the exact approach.

1.2.2 Two–Echelon Urban Freight Transportation

The 2E urban freight distribution problem involves inter-modal transportation by allowing another layer of consolidation at the intermediate facilities (i.e., satellites). The vehicle fleet used at each echelon can vary according to the characteristics of the network in that echelon. In the first echelon, larger vehicles are used to transport the consolidated freight to the satellites. Using larger trucks avoids numerous travels to the urban areas, where satellites are located, and reduces vehicle utilization and drivers’ costs. The second echelon vehicles, on the other hand, are usually small vehicles that can traverse the streets of inhabited areas of the city more easily. The secondary vehicles meet vehicle size restrictions and carbon emission standards and have less impact on the environment and traffic. The 2E urban freight distri-
bution problem decides on: (i) The number and location of CDCs, (ii) the number and location of satellites, (iii) assignment of the selected satellites to the open CDCs, (iv) customer assignment to the selected satellites, (v) the number of vehicles in each echelon, and (vi) vehicle routing and scheduling in each echelon. The problem seeks minimum total facility location and transportation costs such that all customers are served under the following conditions: (i) Customer due dates and access time windows are not violated, (ii) CDC and satellite capacities are respected, (iii) all vehicle routes are finished before satellite’s closing time, and (iv) vehicle capacities (or size limits) are not exceeded. The transportation cost is equal to the sum vehicle utilization and traveling costs in each echelon.

We propose MIP formulations for the 2E urban freight transportation problem. The problem is solved using an exact approach based on branch-and-price framework, and two heuristic approaches based on the hierarchical decomposition of the problem’s decisions. The first heuristic is the modified top-to-bottom approach, which is proposed for the 1E problem. The second heuristic, called bottom-to-top approach, starts by determining the domain of the complicated tactical/operational level decisions, and fixes the remaining decisions later. In this heuristic, we design and implement a novel constrained clustering technique to consolidate customers into second echelon vehicle routes. Our numerical results highlight the success of our exact approach in solving the problem instances with up to 100 customers and imply that the bottom-to-top approach saves a significant amount of time to solve the problem instances without sacrificing much of the solution quality.

1.2.3 Satellite Synchronization

The strategic/tactical 2E urban freight transportation problem does not concern the operational activities performed at the intermediate facilities. Satellites are commonly not physical facilities but places with other usages such as parks, parking lots, bus stations, etc., located in urban areas. Therefore, they have limited space and can only be accessed during specific times. It is also highly undesirable to handle inventory or to form vehicle queues at these locations. Therefore, satellites are used as rendezvous points, where goods are transferred from one vehicle to another without stocking.
Considering the above limitations, we define the satellite synchronization problem to determine the exact arrival and departure times of the primary and secondary vehicles assigned to that location. Given the strategic network design solution, the objective of the satellite synchronization problem is to minimize the congestion confronted in the satellite location by scheduling the vehicles such that all secondary vehicles are served before they miss their customer due dates. As a result, no inventory handling is required and vehicle queues are diminished. We provide different MIP formulations for the problem and point out the advantage/disadvantage of each formulation when dealing with the problem instances with different characteristics. In order to simulate real-life situations, we solve the satellite synchronization under vehicle arrival time and service time delays and provide managerial insights that can be considered to improve the current network design solution.

1.3 Our Contribution

This thesis fills the current gap in the city logistics literature by providing the means of modeling and solving strategic to operational level decision-making problems arising in urban freight distribution. The main contribution of our study can be concisely summarized as follows.

- We propose mathematical formulations for the 1E and 2E-CLSs. The formulations incorporate strategic and tactical level decisions in CLSs considering the consolidation aspect of city logistics.

- Efficient exact solution approaches are presented based on the decomposition of the introduced formulations.

- Novel heuristic approaches are developed to solve large-size problem instances efficiently, and provide an upper bound.

- We introduce the satellite synchronization problem to address a challenging coordination issue at the operational level planning of the 2E-CLSs. Effective mathematical formulations are provided that can be used to solve problem instances with reasonable sizes.
A simulation-based numerical study is conducted to provide managerial insights about the affect of operational decisions on the 2E network design solution.

1.4 The Outline of the Thesis

The remainder of this thesis is organized as follows. In Chapter 2, we review the related city logistics literature and established the links between the defined optimization problems in single and 2E the city logistics and the classical facility location and vehicle routing problems. The challenges of modeling and solving urban freight transportation problems are explained and the current gap in the literature is identified. Chapter 3 studies the 1E urban freight distribution problem, provides mathematical formulations, and proposes applicable exact and heuristic solution algorithms. The 2E urban freight distribution problem is extensively studied in Chapter 4. In this chapter, we explain the characteristics of the problem and its optimization challenges. A path-based mathematical formulation is provided, which is solved using an exact branch-and-price algorithm. Two efficient heuristics are also proposed that find high quality and efficient solutions to large-size problem instances. Chapter 5 introduces the satellite synchronization problem, presents MIP formulations for the problem, and provides managerial insights based on a comprehensive computational experiment. Chapter 6 concludes the thesis and outlines significant future research areas.
CHAPTER 2

LITERATURE REVIEW

In this chapter, strategic, tactical, and operational level decision-making in city logistics are highlighted and the related optimization problems in the operations research/industrial engineering (OR/IE) literature are addressed for each planning level. We concentrate on network design models for city logistics systems and highlight the challenges of an effective consolidated and coordinated city logistics system. We show how well various conventional OR/IE problems in the literature can help to formulate city logistics network design problems by identifying the advantages and disadvantages of these models in coping with real-life situations.

2.1 OR/IE Applications in City Logistics

Although OR and management science have been applied in freight transportation, including oceanic, rail, air and inter-modal freight transportation (Gorman et al. 2014), the literature does not contain many studies that apply OR/IE techniques in dealing with city logistics (CL) problems. Most of the available CL studies in the literature focus on characteristics of the targeted city logistics systems (CLs), evaluate the existing real-life projects, or aim to measure the effects of CL policies. We believe that CL practitioners can also benefit from OR/IE literature for designing, planning and controlling the transportation system in order to develop a balance between economic, environmental, and social objectives and to deal with coordination and consolidation directions of CL. Therefore, we reviewed the literature for the state-of-the art optimization problems in this area. Our aim is to find out how review studies address optimization problems and OR/IE tools in the CL context. The sources used for our
survey consist of refereed journals and conference proceedings in the field of city logistics. Publications in languages other than English are excluded. The keyword combination used for our search is as follows. Either ‘city logistics’ or ‘urban logistics’ keywords should exits along with any of the following keywords: ‘review’, ‘survey’, ‘overview’, or ‘challenges’. The time interval is taken as 2000-2018. Table 2.1 presents the result of this search in chronological order. For each publication, we identify its area of the focus in terms of consolidation and coordination aspects of CL. Consolidation addresses planning and operation of CDCs and vehicle routes. Studies considering physical urban transportation network links and the service area as well as those focusing on utilizing eco-friendly equipment in the transportation process fall into this category. Coordination regards administration and the interaction between actors. The studies concerning policy evaluation and development (such as traffic regulations, time windows, load factors, traffic limits, loading/unloading zones, and road pricing) and the ones referring to the actors’ knowledge in research, learning, and training to improving their collaboration belong to this category. We also demonstrate whether the review papers provide a solution approach perspective for any network design issue. The solution methods are categorized into three groups: (i) mathematical models (i.e. formulating an optimization problem using logic and mathematics); (ii) simulation models (i.e. creating and analyzing an artificial prototype of a physical model to predict its performance in the real world); (iii) conceptual models (i.e. explaining and integrating underlying relationships between components of the systems and/or its concepts) or empirical analysis (i.e. observing and analyzing the real world cases). The review papers on the consolidation field suggest considering more real-life issues (e.g. time windows, multi-commodity, and synchronization) in CL models hence show the need of integrating multiple segments of the system to improve technical aspects of the existing solutions or proposing new approaches. Research on the coordination field emphasize the importance of stakeholders’ involvement and their interaction in designing and developing CLSs and argue that high majority of modeling efforts are carried out from an administrator point of view and ignore the interests of other stakeholders.

There are very limited number of studies that review both consolidation and coordination aspects in CL and have a solution approach related to network design issues.
from an OR/IE perspective. Savelsbergh & van Woensel (2016) highlight trending issues in CL and point out growing managerial challenges that make CLSs a complex area of research. Due to this complexity, simulation methods are broadly studied and used as an OR tool to provide models and support manage transportation activities and support decision-making processes. Crainic et al. (2018) conclude that the current simulation literature lacks a system-wide vision of the problem setting including the decision makers, the actors involved, and their interests. The authors suggest integrating simulation and multi-criteria analysis for the urban inter-modal transportation system and more studies on sustainable policies in this context. Nuzzolo et al. (2018) argue that although the effect of a broad set of city logistics measures (such as CDC usage and load consolidation) can be studied by simulation, the current applications only consider small systems and a few agents with no or very limited interaction. Letnik et al. (2018) review the policies and measures for real-life CL cases and identifies the effect of implemented policies on the performance of the CL based on defined measures.

Considering the studies in Table 2.1, extensive research is needed on real-life features of city logistics in a highly dynamic and competitive decision-making environment. There is no study specifically on the assessment of OR/IE techniques in formulating and solving underlying problems of CLSs. Many are focused on managerial issues in CLSs and CL modeling based on social and economic aspects of the system (Anand et al. 2012). Although there are discussions on CL modeling from an OR/IE perspective (Crainic & Laporte 1997, Crainic et al. 2009, Mancini 2013, Akeb et al. 2018), the available models either do not consider the network design issues of an integrated system in CL or provide conceptual models without solving the problems and complexity analysis.

With the existing gap in the literature, we believe that this study will provide an in-depth understanding of underlying optimization problems in urban freight transportation networks. The aim of this chapter is summarized as follows: (i) Considering strategic, tactical, and operational level decisions in CL, we present the related network design models in the literature and discuss advantages/disadvantages of the models in terms of covering different aspects of CLSs. (ii) By addressing related well-known problems in the literature to the CLSs, we identify the existing gap in the
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<tr>
<th>Article</th>
<th>Problem perspective</th>
<th>Solution perspective</th>
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<td>Consolidation</td>
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<td>van Duin &amp; Quak (2007)</td>
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† Conference paper.

literature in terms of exact and heuristic solution methods. (iii) Coordination issues of CLSs are addressed by considering stakeholders’ interests as well as consolidation and synchronization problems in the underlying transportation network. The links between synchronization problems in CL with other related OR/IE problems are established.

The rest of this chapter is organized as follows. In Section 2.2, we explain the components of CLSs. In Section 2.3 and Section 2.4, we present network design models.
for the most common city logistics systems, namely one-echelon and two-echelon systems, respectively. The relations of well-known problems in the literature to the CLS models, and consolidation challenges of CLSs are also presented in Section 2.3 and Section 2.4. We discuss coordination aspects of city logistics in Section 2.5. Section 2.6 summarizes our findings in this chapter.

2.2 City Logistics Network Design Models

Freight distribution network design problems involve strategic decisions which influence tactical and operational decisions (Crainic & Laporte 1997). In classical distribution networks, strategic planning involves inventory decisions in addition to facility location and transportation decisions in order to reflect the distribution cost and the quality of the customer service level (Ambrosino & Scutellà 2005). In urban freight distribution context, however, we exclude inventory decisions or warehousing and focus on the facility location and transportation network as the core problems.

The CL networks are categorized into single or multi-echelon systems. In one-echelon (1E) CLSs, freight is delivered from city distribution centers (CDCs) to the final customers on a single transportation mode without any intermediate activities. CDCs have different types in terms of their operation, the demand area they serve, and the parties involved in consolidation activities (Allen et al. 2012b). Multi-echelon systems, on the other hand, employ a modal change at the intermediate facilities, called satellites. In the two-echelon (2E) systems, consolidated freight are delivered from CDCs to satellites using large trucks (also known as primary vehicles). At the satellite locations, goods are unloaded, sorted, and loaded into smaller vehicles (also known as secondary vehicles) for the last-mile delivery. Discussion and reviews of multi-echelon urban freight transportation problems are available in Gonzalez-Feliu (2012, 2013), Mancini (2013).

Whether a CLS is designed as a 1E or 2E structure, it has at least one CDC facility as its core element in consolidation and distribution. Therefore, facility location and customer-to-facility assignment remains as a strategic decision in any CLS whereas vehicle routing and scheduling decisions are considered at tactical level (Mancini
Table 2.2: Relation of the CLSs decisions with OR/IE problems.

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<tr>
<th>CLS planning level</th>
<th>Decisions</th>
<th>Related OR/IE problem</th>
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<tr>
<td>Strategic</td>
<td>Network design, location of facilities and consolidation centers</td>
<td>Facility location problem</td>
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<tr>
<td>Tactical</td>
<td>Fleet size, vehicle routes (and schedules), spatial synchronization</td>
<td>Vehicle routing problem (with time windows)</td>
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<tr>
<td>Operational</td>
<td>Worker schedules, daily vehicle operations, temporal synchronization</td>
<td>Scheduling problems</td>
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et al. 2014). In Table 2.2, we list main decisions at three levels of a CLS planning, and address a (well-studied) OR/IE problem related to that level. We could barely find a comprehensive study in the literature that considers time and space synchronization problem observed in tactical/operational CL planing. In Section 2.4, we discuss synchronization issues in more details. Below, we focus on city logistics system design problem from an OR/IE perspective by excluding inventory and replenishment aspects.

2.3 Single–Echelon Systems

The related problems to the single-echelon systems fall into three categories: (a) facility location problems (FLPs), (b) vehicle routing problems (VRPs), and (c) location-routing problems (LRPs) depicted in Figure 2.1. Below, we provide a brief discussion on each problem in order to identify their role in CLSs.

2.3.1 The Facility Location Problem

Facility location is one of the core strategic problems arising in distribution networks and is a well-established research area in OR/IE. The problem is to locate a set of new facilities (CDCs) such that the facility operating cost and transportation or service cost from facilities to customers are minimized. A major variant assuming capacitated facilities is denoted as the CFLP. Large-size FLP and CFLP instances can be solved
fast by today’s mixed-integer linear programming (MIP) solvers. If it is required that each customer is satisfied from exactly one facility, the problem is called the CFLP with single-sourcing (CFLPSS). Single-sourcing constraints make the problem a pure integer problem, which is much harder to solve. Gadegaard et al. (2017) propose an exact cutting-plane solution approach to solve CFLPSS instances with up to 60 candidate facility locations and 300 customers. CFLPSSs can be solved using exact methods like branch-and-price, branch-and-cut, or cut-and-solve (Ceselli et al. 2008, Gadegaard et al. 2017). Time related restrictions at customer locations might also be considered. The reader is referred to Klose & Drexel (2005) for the FLP survey, modeling, and applications. Melo et al. (2009) review the literature of facility location models in the context of supply chain distribution networks and discuss how this problem is related to strategic supply chain planning.

Strategic decisions about CDC locations and CDC-to-customer allocations can be modeled as a FLP. These decisions have long-lasting effects on the efficiency of a
CLS. The city characteristics strongly impact the number and locations of CDCs. Veličković et al. (2017) show the effect of strategic decisions on CDC number and locations on the CL external costs. In a CLS, one also needs to consider the objectives of different stakeholders (i.e. shippers, carriers, and customers) and their multiple and conflicting criteria when making such decisions. Among the most important criteria are accessibility by public and private transport modes, connectivity to multi-modal transport, environmental impacts, quality of service, facility/vehicle utilization costs, and resource availability. One can also deal with the problem under stochastic travel times or demands in order to reflect uncertainty (see Kunter 2015). For a short review on FLPs in city logistics see Agrebi et al. (2015).

2.3.2 The Vehicle Routing Problem

One of the basic decisions in CLSs is to determine vehicle routes, especially for the last-mile delivery. To this end, the optimization methods developed for the vehicle routing problem (VRP) and its variants, including the multi-depot VRP (MDVRP), the capacitated VRP (CVRP), the multi-trip VRP, the VRP with soft or hard time windows, the VRP with pickup and deliveries, and the dynamic VRP, can be used.

VRPs contain single or multiple CDC (or depot) facilities with known location(s) and the objective is to determine routes and schedules of vehicles with minimum transportation cost. Compared to the FLP, the VRP can improve overall transportation process by reducing the number of vehicle trips into the cities. However, long-term decisions about where to locate CDCs and how many to open are still to be made. The literature of the VRP and its variants is well developed in OR/IE. The reader is referred to Caceres-Cruz et al. (2014), Lahyani et al. (2015), Braekers et al. (2016), and Cattaruzza et al. (2018) for recent taxonomy and review of the VRP.

The VRP can cover tactical and operational level planning in CL by incorporating decisions about vehicle fleet size and routes. In this problem, customer demands are consolidated and loaded to the vehicle that serves them. If multiple distribution centers are available, the MDVRP models can be used to determine customer-to-facility allocations as well. Zissis et al. (2018) show how CVRP can be applied in a real-life urban freight distribution system. The VRP in CL differs from the conventional city
logistics in terms of stakeholders involved and the mutual relationship among their objectives. Kim et al. (2015) and Cattaruzza et al. (2017) provide literature reviews on the City VRP by considering the objective(s) of one or several stakeholders. The main challenges for routing in CL are time-related constraints, city dynamics, traffic regulations, air and noise pollution, and fast response to customers. The authors show that the articles in this area focus on benefits of carriers and consider time windows and/or air pollution as their preferred characteristics to study. Ehmke et al. (2018) argue that minimizing time and distance (as the conventional carriers’ objectives) are poor substitutes for minimizing route costs in urban areas. The authors suggest alternative objective functions to minimize time-dependent total costs (including driver and fuel costs, considering individual speed values and load at every connection in the course of the route) and time-dependent fuel consumption.

Exact solutions of the CVRP can be obtained for instances with up to 275 customers. When time window constraints are added (i.e. VRPTW), the most recent approaches can solve instances with up to 200 customers. Branch-and-price and branch-and-cut-and-price algorithms are the most successful solution methods (see Ropke & Cordeau 2009, Azi et al. 2010, Baldacci et al. 2012, Dabia et al. 2013, Pecin et al. 2014, Contardo et al. 2015, Uchoa et al. 2017, Pecin et al. 2017). In Section 2.5.1, we discuss the difference between time restrictions in CLSs and conventional VRPs with time windows. More studies are needed to reflect objectives and benefits of all stakeholders in problem formulation, incorporate city characteristics, and develop efficient solution methods (Kim et al. 2015).

2.3.3 The Location-Routing Problem

VRPs are strongly NP-hard as they generalize the well-known traveling salesman problem. The 2E-LRPTW adds more complexity to the classical VRP by incorporating more decisions (e.g. facility location) in addition to the routing decisions. The aim is to open a set of CDCs among candidate locations in order to meet customer demands at minimum operation cost.

In addition to identifying the number and location of the facilities, the LRP determines allocation of customers to the facilities, allocation of customers to the routes, and the
order of visiting customers in the routes.

Although facility location and vehicle routing decisions have been traditionally considered separately at strategic and tactical planning levels, it is shown that the integrated approach for the LRP reduces overall cost in the long planning horizon (Salhi & Nagy 1999, Nagy & Salhi 2007). LRPs have been successfully applied in food and drink distribution, waste collection, blood bank location, and parcel delivery (Hassanzadeh et al. 2009). Lopes et al. (2013), Prodhon & Prins (2014), and Drexl & Schneider (2015) provide recent reviews of the LRP literature, its taxonomy, and the problem characteristics.

The emerging number of articles in the literature considering the LRP as a part of supply chain and providing solution approaches (Berger et al. 2007, Akca et al. 2009, Belenguer et al. 2011, Baldacci et al. 2011b, Contardo et al. 2013, 2014) indicate the importance of this problem in CL area, as well. Largest LRP instances with capacity constraints (CLRPs) contain 14 candidate facilities and 199 customers which are solved by column-and-cut generation approaches (Baldacci et al. 2011b, Contardo et al. 2014). For a more detailed survey on the exact and heuristic solution approaches for the (C)LRP see Schneider & Drexl (2017).

Among the above three basic problems, the capacitated LRP with time windows (LRPTW) adopts most of the features of single-echelon CLSs in real-life by considering strategic to operational level decisions. Although time window restrictions have been considered for the VRP for a long time, it is not well-studied in the context of LRPs (Drexl & Schneider 2015). There are a few studies in the literature that propose solution approaches for the LRPTW. Koç (2016) develops a unified adaptive large neighborhood search (ALNS) to tackle different classes of the periodic LRP including the LRPTW with heterogeneous fleet. The author solves test instances with 10 candidate locations and 100 customers over a three-period time horizon. Considering heterogeneous vehicles, Koç et al. (2016) formulate a fleet size and mix LRPTW and propose a hybrid evolutionary search algorithm to solve the problem. The largest instance contains 10 candidate locations and 100 customers. To the best of our knowledge, there is only one study proposing an exact method for the LRPTW. Ponboon et al. (2016) provide an MIP formulation for the problem and develop a

Table 2.3: Recent exact solution methods for the single-echelon problems.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Problem</th>
<th>Solution approach</th>
<th>Problem size†</th>
</tr>
</thead>
<tbody>
<tr>
<td>Díaz &amp; Fernández (2002)</td>
<td>CFLPSS</td>
<td>Branch &amp; price</td>
<td>30-90</td>
</tr>
<tr>
<td>Gadegaard et al. (2017)</td>
<td>CFLPSS</td>
<td>Cut &amp; solve</td>
<td>60-300</td>
</tr>
<tr>
<td>Uchoa et al. (2017)</td>
<td>CVRP</td>
<td>Branch &amp; cut &amp; price</td>
<td>1-275</td>
</tr>
<tr>
<td>Contardo et al. (2015)</td>
<td>VRPTW</td>
<td>Branch &amp; cut &amp; price</td>
<td>1-100</td>
</tr>
<tr>
<td>Pecin et al. (2017)</td>
<td>VRPTW</td>
<td>Branch &amp; cut &amp; price</td>
<td>1-200</td>
</tr>
<tr>
<td>Akca et al. (2009)</td>
<td>CLRP</td>
<td>Branch &amp; price</td>
<td>5-40</td>
</tr>
<tr>
<td>Belenguer et al. (2011)</td>
<td>CLRP</td>
<td>Branch &amp; cut</td>
<td>5-50</td>
</tr>
<tr>
<td>Baldacci et al. (2011b)</td>
<td>CLRP</td>
<td>Branch &amp; cut &amp; price</td>
<td>14-199</td>
</tr>
<tr>
<td>Contardo et al. (2013)</td>
<td>CLRP</td>
<td>Branch &amp; cut</td>
<td>5-50</td>
</tr>
<tr>
<td>Contardo et al. (2014)</td>
<td>CLRP</td>
<td>Branch &amp; price</td>
<td>14-199</td>
</tr>
<tr>
<td>Ponboon et al. (2016)</td>
<td>LRPTW</td>
<td>Branch &amp; price</td>
<td>3-40</td>
</tr>
<tr>
<td>Farham et al. (2018)</td>
<td>LRPTW</td>
<td>Branch &amp; price, two-stage heuristic</td>
<td>5-50</td>
</tr>
</tbody>
</table>

† The problem instance size shows the number of (candidate) facility locations followed by the number of customers.

Table 2.3 summarizes the recent work that proposes exact approaches for the single echelon problems and lists the largest test instances that could be solved by each approach. Considering only location-allocation decisions as in the CFLPSS, the problem instances with up to 60 candidate facilities and 300 customers are solved. If the problem contains routing from a single depot, instances with as many as 275 customers can be solved. This is expected as the CVRP is more challenging problem than the CFLPSS, even though it does not deal with location decisions.

Adding time window restrictions makes the problem even more challenging. VRPTW instances can be solved exactly when the number of customers drop to 200. If location-allocation and vehicle routing decisions are combined as in the CLRP, the
largest solvable instance size drops to 14-199. Note that the VRP is a special case of
the LRP when there exists only one distribution center. Restricting the CLRP with
time windows yields a much harder problem as demonstrated in Table 2.3. There
are only a few studies focusing on solution approaches for the LRPTW and optimal
solutions in the literature are only found for the problem instances smaller than real
size CL networks.

2.4 Two–echelon systems

There is a limited number of articles in the literature that study the city logistics from
a network design point of view, and even less is dedicated to the multi-echelon city lo-
gistics systems. The two-echelon freight distribution systems are the most commonly
studied problems related to the multi-echelon CLSs (Gonzalez-Feliu 2013, Mancini
2013). Although comprehensive planning, modeling, and evaluation studies of the
2E-CLS started since 2004 (Crainic et al. 2004, Crainic 2008), the models for 2E-
CLs appeared five years later. Crainic et al. (2009) is the first study that includes
well-known aspects in the CLS. The authors conceptually considered tactical plan-
ing of the two-echelon, synchronized, scheduled, multi-depot, multiple-tour, and
heterogeneous VRPTW, which is applicable in the real-life situations, even though
the facility location decisions are still needed to be considered. These models, how-
ever, are either generic (Crainic et al. 2009) or based on strong assumptions (Crainic
& Sgalambro 2014).

The important problems related to CL in which multi-echelon structures are consid-
ered are the two-echelon facility location problem, the two-echelon vehicle routing
problem, the two-echelon location-routing problem, and the truck and trailer routing
problem (TTRP) (Mancini 2013, Cuda et al. 2015). The TTRP is also a variant of the
two-echelon VRP in which each vehicle is composed of a truck and a trailer. Some
customers may be served by a truck pulling a trailer, while the others may only be
served by a single truck. Different configurations of the two-echelon problems are
illustrated in Figure 2.2. Decisions about location, allocation, and/or routing should
be made for each echelon having its own characteristics. Restrictions such as satellite
area availability, vehicle load, and time windows may exist at the second echelon.
(a) Two-echelon FLP.

(b) Two-echelon VRP.

(c) Two-echelon LRP.

(d) Two-echelon location with first-echelon routing problem.

(e) Two-echelon location with last-echelon routing problem.

Figure 2.2: Related problems to the two-echelon CLSs. White (gray) shapes refer to candidate (open) locations.
2.4.1 The Two–Echelon Facility Location Problem

Strategic and long-term decisions in 2E-CLSs include facility location selection, CDC-to-satellite allocations, and customer-to-satellite assignments. These decisions have a crucial impact on the efficiency of a CLS. Therefore, the two-echelon capacitated facility location problem (2E-FLP) can be considered as the underlying strategic level problem in the 2E-CLS. Figure 2.2a illustrates a 2E-FLP. Squares and triangles represent candidate CDC and satellite locations, respectively. Fixed customer locations are shown by filled circles. Given a set of candidate CDC locations (in the first echelon), a set of candidate satellite locations, and a set of customer nodes to be served (in the second echelon), the 2E-FLP addresses the following major decisions: which CDC and satellite facilities to open, how to allocate open satellites to open CDCs, and how to assign customers to the open satellites. Decisions are made simultaneously under facility capacity and, if applicable, satellite and customer time windows. The objective is to minimize total facility opening cost and CDC-to-satellite and satellite-to-customer (direct) transportation costs. This problem is extensively studied in the literature of (hierarchical) facility location problem. Addis et al. (2012, 2013) and Wu et al. (2015) provide problem formulation and solution approaches for the 2E-FLP with single-sourcing constraints (2E-FLPSS). For further details on the problem characteristics and mathematical modeling of multi-echelon FLPs see Klose & Drexl (2005), Şahin & Süral (2007), Farahani et al. (2014), and Ortiz-Astorquiza et al. (2018).

2.4.2 The Two–Echelon Vehicle Routing Problem

The two-echelon capacitated VRP (2E-VRP) is a relatively new problem appearing in multi-echelon freight transportation systems in large cities. In 2E-VRPs, the location of available first and second echelon facilities are given, hence no location decisions are made. The delivery from the given CDC(s) to the customers is managed by rerouting and consolidating the freight through intermediate satellites. The first echelon addresses CDC(s)-to-satellites routing problem, while satellites-to-customers delivery routes are decided in the second echelon (see Figure 2.2b). The aim is to ensure an efficient and on-time freight delivery while minimizing overall transportation cost.
Restrictions on the maximum capacity of vehicles and the intermediate facilities are usually considered. Crainic et al. (2010) consider the 2E-VRP in CL and analyzed the effect of customer distribution, depot and satellite location, number and accessibility of satellites, and associated distribution cost on the transportation cost through computational experiments. The results indicate that the 2E-VRP approach leads to lower overall cost compared to the classical VRP in most cases, in particular, when the CDC is located externally with respect to the customer area and a certain number of satellites are located close to the demand points. Perboli et al. (2011) provide an MIP formulation and valid inequalities for the 2E-VRP with one CDC. The authors also propose math-based heuristics to solve large problem instances more efficiently. Baldacci et al. (2013) propose an exact method for the similar problem based on decomposition of the 2E-VRP into several multi-depot capacitated VRP with side constraints. Wang et al. (2017) study the 2E-VRP by considering fuel consumption and emission of vehicles to reduce air pollution and ensure environment-friendly transportation. The authors present mathematical formulation of the problem and a metaheuristic to solve the problem. Having multiple depots (CDCs) in the first echelon and a pickup and delivery setting, Zhou et al. (2018) propose a hybrid genetic algorithm to solve a 2E-VRP for the last-mile distribution. The literature contains other exact (Jepsen et al. 2013, Baldacci et al. 2013, Santos et al. 2013, 2015) and inexact (Crainic et al. 2011, Perboli et al. 2011, Hemmelmayr et al. 2012, Crainic et al. 2013, Breunig et al. 2016) approaches to solve the 2E-VRP.

Largest 2E-VRP instances solved in the literature by exact and heuristic approaches contain 1 CDC, 5 satellites, and 50 customers and 1 CDC, 20 satellites, and 200 customers, respectively Cuda et al. (2015). To the best of our knowledge, Dellaert et al. (2019) is the only study that proposes an exact solution methods for the 2E-VRP with both capacity and time window constraints. The authors introduce two different path-based formulations for the problem and develop branch-and-price algorithms to solve instances with up to 3 depots, 5 satellites and 100 customers.
2.4.3 The Two–Echelon Location-Routing Problem

The two-echelon capacitated LRP (2E-LRP) combines decisions and constraints of the 2E-FLP and the 2E-VRP as the core problems. In other words, the 2E-LRP determines location of facilities and vehicle routes (and schedules) for each echelon simultaneously (see Figure 2.2c). With capacity and time window constraints, the 2E-LRP can reflect many real-life features of a 2E-CLS. However, achieving decision makers’ objectives under the environmental and temporal constraints is a challenging issue in urban freight transportation problems. The inherent complexity of the 2E-LRP with time windows (2E-LRPTW) makes it a hard optimization problems when dealing real-world cases.

The two-echelon location-routing problem in the literature is often studied under simplified settings such as assuming only one CDC or fixed CDC locations or determining fleet size and composition without explicitly finding vehicle routes. Winkenbach et al. (2016a,b), and Zhao et al. (2018) highlight the application of the 2E-LRP models in parcel delivery networks in urban areas. Winkenbach et al. (2016a) develop a closed-form approximation for optimal routing cost that takes maximum service time constraint and different physical and economic vehicle characteristics into account. The authors construct a 2E-LRP model that yields high-quality approximations within a reasonable time. Winkenbach et al. (2016b) extend the strategic model of Winkenbach et al. (2016a) by allowing direct or indirect deliveries from CDCs. Zhao et al. (2018) present an optimization model to determine a cost-optimal urban logistics network and fleet composition for joint delivery alliances under vehicle capacity, working hours, and traffic restrictions. To solve the problem instances efficiently, the authors propose a cooperative approximation heuristic algorithm and test it on well-known benchmark instances. Pichka et al. (2018) study a variant of the 2E-LRP, called the two-echelon open location-routing problem, where vehicle routes do not return to CDCs in the first echelon and do not return to satellites in the second echelon due to the presence of individual contractors and third party logistics providers. Considering a single CDC in the network, the authors propose MIP formulations for the problem and a hybrid heuristic solution approach.

A comprehensive overview on 2E-LRPs is provided by Prodhon & Prins (2014),
Drexel & Schneider (2015). There are some research focusing on the 2E-LRP where time restrictions are not considered. Boccia et al. (2010) decompose the 2E-LRP into a capacitated facility location problem and a multi-depot VRP in each echelon. The authors develop a tabu search heuristic to solve problem instances with up to 200 customers and 20 satellites. Contardo et al. (2012) consider the 2E-LRP with several capacitated first and second echelon facilities and develop an exact branch-and-cut algorithm as well as an ALNS heuristic method inspired by the ALNS algorithm of Hemmelmayr et al. (2012). The authors decompose the 2E-LRP into two different CLRPs, one at each echelon, connected through the satellite nodes. Nikbakhsh & Zegordi (2010) study the 2E-LRP with soft time windows, where a cost is incurred based on the amount of time window violation. In their study, a bound is calculated by relaxing a set of constraints that link the location with routing aspects. The authors present a sequential location-allocation-routing heuristic to solve the instances with 10 depots, 50 intermediate facilities, and 100 customers. Govindan et al. (2014) study the 2E-LRP with soft time windows under two objectives. One objective seeks the minimum facility opening and variable routing costs and the other minimizes environmental impact of the system, measured by carbon emissions. They introduce a hybrid multi-objective heuristic based on particle swarm optimization and adaptive multi objective variable neighborhood search. Wang et al. (2018) introduce a bi-objective model for the 2E-LRPTW incorporating vehicle routes in both echelons. In addition to conventional cost minimization objective, their model seeks for maximum customer satisfaction measured by customers’ demand and delivery times. The authors propose a three-step heuristic where a $k$-means clustering technique is used at the initial step to group customers based on their preferences. In the second step, the generated clusters are used to locate facilities. The last step applies a genetic algorithm to find vehicle routes stemmed from a located facility and serving the customers assigned to that facility. The literature of the 2E-LRP also contains other heuristic solution methods such as greedy randomize adaptive search (Nguyen et al. 2010, 2012b), iterative local search (Nguyen et al. 2012a), adaptive large neighborhood search (Hemmelmayr et al. 2012), variable neighborhood search (Schwengerer et al. 2012), and route construction methods based on customer clustering (Rahmani et al. 2016). Largest 2E-LRP instances that could be solved in reasonable computing time by exact and heuristic approaches contain 1 CDCs, 10 satellites, and 50...
customers, and 5 CDCs, 20 satellites, and 200 customers, respectively Cuda et al. (2015).

The 2E-LRP with capacity and time window constraints can reflect many features of a 2E-CLS as it covers both strategic-level decisions (i.e. facility locations) and tactical-level planning (i.e. vehicle routing and scheduling). This problem is quite new to the literature and, due to its challenges, very limited number of approaches are proposed to solve the problem. In Appendix A, we provide formulations for different 2E-CLSs and discuss the effect of time window constraints on the solution.

2.4.4 The Two–Echelon Facility Location with First Echelon Routing Problem

This problem is observed in pickup/delivery networks. Pickup points are locations where customers visit to pick up or deliver their commodities. Locker boxes, post boxes, and fuel stations are examples of such pickup/delivery points. Another application of the two-echelon facility location with first echelon routing problem (2E-LR1P) is in cargo and food delivery networks where either in-time deliveries to customers are required or the capacity of the delivery vehicles (called city freighters) are only limited to a single customer or a small customer zone. Cargo bikes and tricycles are examples of city freighters used for urban freight transport (Schliwa et al. 2015). The 2E-LR1P, illustrated in Figure 2.2d, can mitigate the negative effects of door-to-door deliveries in the direct-to-customer networks by reducing the congestion and environmental pollution generated by urban freight trips. The 2E-LR1P is an special case of the 2E-LRP where routing decisions are only made for the first echelon. A related problem in the literature is the ring-star problem appeared in telecommunication networks (Labbé et al. 2004). In the multi-depot ring-star problem (Sundar & Rathinam 2017), a set of rings (vehicle routes), each starting from a depot and passing through a set of intermediate nodes, is found. Each customer is assigned to one of the visited nodes. The objective is to minimize the sum of routing and assignment costs. The literature contains exact algorithms based on branch-and-cut to solve the ring-star problem (see Labbé et al. 2004, Baldacci et al. 2007, Sundar & Rathinam 2017).

The 2E-LR1P differs from the other 2E-CLS problems in terms of characteristics of
their intermediate facilities. Pickup/delivery points are usually predefined locations inside the city that their structures are less restrictive than satellite locations. The location choices in this problem are based on the points frequently visited by the potential customers, such as railway stations. Therefore, once location decisions are made, they rarely change. On the contrary, satellite locations are subject to frequent changes due to dynamic nature of the second echelon transportation network or land-use restrictions. Lastly, unlike satellites, pickup/delivery points can have storage. Existence of inventory, even for a small amount, eliminates the need of synchronization between first and second echelon flows. This can ease the problem and allow decomposition-based solution approaches. Despite the positive effects of using pickup/delivery networks, relatively little research has been done to focus this problem (Savelsbergh & van Woensel 2016).

2.4.5 The Two–Echelon Facility Location with Last Echelon Routing Problem

The first echelon transportation phase of the most two-echelon systems takes place far from the city center. Usually, it requires limited number of CDCs and satellites to be considered. The second echelon transportation phase, however, takes place inside inner city areas with lots of customers, each having a time window. Besides, traffic flow on the road network, limited parking space, congestion, and environmental concerns make it crucial to run an “optimization” based transportation system inside the city. Therefore, while facility location and assignment costs are important factors in the first echelon decisions, the detailed routing and scheduling of the vehicles are more essential in the second echelon than in the first echelon. As a result, many CL service providers focus on the second echelon routing while they only concern about the long-term facility locations in the first echelon. The two-echelon facility location with last echelon routing problem (2E-LR2P) is a variant of the 2E-LRP where the location-routing decisions in the first echelon is replaced by location-allocation decisions (see Figure 2.2e). Therefore, the resulting problem has more emphasis on the tactical and operational level planning in the second echelon.

Nikbakhsh & Zegordi (2010) study the 2E-LR2P with soft time windows, where a cost is incurred based on the amount of time window violation. In their study, a
bound is calculated by relaxing a set of constraints that link the location with routing aspects. The authors presented a sequential location-allocation-routing heuristic to solve the instances with 10 depots, 50 intermediate facilities, and 100 customers. Gündüz (2015) formulate the 2E-LR2P with time windows (2E-LR2PTW) where the location of CDCs are known. The author presented a tabu search algorithm and compared its result with a sequential location-allocation-routing approach on a set of instances containing up to 4 depots, 50 candidate intermediate facilities, and 400 customers. This study is the first that proposes an exact solution approach for the 2E-LR2PTW. We present mathematical formulations for the problem and propose an exact method to solve the path-based formulation of the 2E-LRP2PTW. In this approach, a column generation technique is implemented to generate vehicle routes in the second echelon. We also develop heuristic methods that find high quality and efficient solutions to the problem. In Chapter 4, we provide more details on the 2E-LR2PTW.

Table 2.4 lists the recent (in)exact solution methods proposed for various two-echelon freight transportation problems and shows the largest instances solved.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Problem</th>
<th>Solution approach</th>
<th>Problem size $^\dagger$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addis et al. (2012)</td>
<td>2E-FLPSS</td>
<td>Branch &amp; price</td>
<td>30-30-200</td>
</tr>
<tr>
<td>Addis et al. (2013)</td>
<td>2E-FLPSS</td>
<td>Two-phase heuristic</td>
<td>100-100-1000</td>
</tr>
<tr>
<td>Baldacci et al. (2013)</td>
<td>2E-VRP</td>
<td>Decomposition-based exact method</td>
<td>1-6-100</td>
</tr>
<tr>
<td>Breunig et al. (2016)</td>
<td>2E-VRP</td>
<td>LNS metaheuristic</td>
<td>1-10-200</td>
</tr>
<tr>
<td>Dellaert et al. (2019)</td>
<td>2E-VRPTW</td>
<td>Branch &amp; price</td>
<td>3-5-100</td>
</tr>
<tr>
<td>Contardo et al. (2012)</td>
<td>2E-LRP</td>
<td>Branch &amp; cut</td>
<td>1-10-50</td>
</tr>
<tr>
<td>Contardo et al. (2012)</td>
<td>2E-LRP</td>
<td>ALNS heuristic</td>
<td>5-20-200</td>
</tr>
<tr>
<td>Nikbakhsh &amp; Zegordi (2010)</td>
<td>2E-LR2PTW</td>
<td>Location-allocation heuristic</td>
<td>10-50-100</td>
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<td>Hybrid heuristic</td>
<td>12-18-30</td>
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<td>Gündüz (2015)</td>
<td>2E-LR2PTW</td>
<td>Tabu search heuristic</td>
<td>4-50-400</td>
</tr>
<tr>
<td>This study</td>
<td>2E-LR2PTW</td>
<td>Branch &amp; price, two-stage heuristics</td>
<td>3-5-100</td>
</tr>
</tbody>
</table>

$^\dagger$ The problem instance size shows the number of (candidate) CDC locations followed by the number of (candidate) satellites and the number of customers.
The multi-echelon related problems in the literature lack several important aspects of city logistics. Majority of the studies consider only one CDC and eliminate location decisions in the first echelon. Usually, temporal aspects are ignored and synchronization issues are limited to the balance of product flows at intermediate facilities. Therefore, time restriction and synchronization concerns need to be considered explicitly in order to improve the overall performance of CLSs. In the following section, we discuss coordination aspects regarding synchronization issue in multi-echelon urban freight distribution networks.

2.5 Coordination Issues in City Logistics

A successful CL needs to take the cooperation between the stakeholders as well as involved third-party companies into account. The literature involves case studies where CL projects fail due to poor coordination and information exchange among the actors. Schoemaker (2002) provides an example where a CL initiative in Europe failed due to following reasons: (i) reluctance of carriers as the believed that transshipment involves extra costs, risks and delays; (ii) insurance companies did not allow shipment or changes in transport modalities for valuable products; (iii) the competitions between carriers encouraged them to initiate their own consolidation centers and benefit from municipality’s support; (iv) the CDC was located too far from the highway and city center; and (v) the supporting policy measures, i.e. time windows and vehicle restrictions, resulted in opposition against the CDC. Other studies also promote effective coordination and consolidation in a public-private collaboration environment and show how uncoordinated local policies and regulations fail to improve fright transportation inside the city (for example, see Taniguchi et al. 2007, Quak & de Koster 2009, Taniguchi & Thompson 2014, van Heeswijk 2017). Moreover, unbalanced cost and profit distribution may lead to a disjoint system (Estrada & Roca-Riu 2017).

In this section, we investigate two important aspects of coordination in CLSs. The most basic one is information collaboration that concerns the mutual exchange of information among different stakeholders (Gonzalez-Feliu et al. 2018). Information dissemination become more critical in large CLSs with large amount of information such as multi-echelon ones and e-commerce logistics (de Souza et al. 2014). How-
ever, it is unclear how to achieve a cooperated urban freight transportation. Primary steps are identifying relevant stakeholders, prioritizing them and determining the factors that motivate actors to participate in CL initiatives (Rubini & Lucia 2018). In order to obtain a reliable result, one needs to perform surveys and collect sufficient data on urban freight transport (Lindawati et al. 2014). Freight data is commonly collected by many companies from both public and private sectors. These data collection efforts are not coordinated, and the results obtained from different data sources and data sets that vary widely in quality and methodology, makes comparisons and combinations of them difficult or impossible (Allen et al. 2014). Furthermore, data acquisition is critical, costly, and time-consuming process and is one of the biggest challenges of evaluating CLSs (Leonardi et al. 2014). It is critical since stakeholders are usually unwilling to share information that might have a strategically relevant commercial value. It is costly since high-quality data usually require face-to-face interviews (Gatta & Marcucci 2016, Allen et al. 2012a). Data collection and classification is usually done using empirical methods and data analysis tools. Hence, we see studies addressing OR/IE techniques for this purpose in the literature.

The other aspect of coordination is decisional collaboration where decisions on transport planning and management (e.g. resource sharing) are made (Gonzalez-Feliu et al. 2018). The primary requirement to achieve decisional collaboration is to perceive the objectives of different stakeholders. The level of pollution, the diffusion of e-commerce, and high gross domestic product are shown to be important objectives of a CL deployment (de Marco et al. 2018). Most of the classical transportation problems define a single objective (typically minimization of travel time/cost) or multi-objectives of a single actor (see Section 2.3). However, the actors involved in CLSs might have different perception about urban goods distribution (de Oliveira & de Oliveira 2017) which yields different and usually divergent objectives. For example, one can notice that fragmented and frequent deliveries in areas with large e-commerce market can negatively affect the performance of the system. Besides, concerns about confidentiality of customers’ information, responsibilities emerging from of physical, financial, and information flows, and asymmetric partition of benefits and losses may arise among stakeholders.

A solution based on only one actor’s objective might sacrifice the other actors’ ob-
jective. As a result, it is difficult to develop a sustainable CLS where all actors are willing to commit (Bektaş et al. 2017) and the vast majority of CL schemes fail after a short time (Browne et al. 2005). Hence, the relation between actors’ objectives need to be carefully studied as well.

The next requirement is to understand and formulate the interaction between actors. One of the analytical ways is to use game theory (Reyes 2005, Hollander & Prashker 2006, Yang & Odani 2007, Allen et al. 2017). However, game theory fails when it comes to analyzing CL schemes due to the following reasons: (i) perfect information about the impact of one player’s action on the others does not exist; (ii) large number of payoff impacts can emerge from the interaction between many actors; and (iii) building a model to cover all aspects of such a complex environment can be time consuming and inefficient to solve (van Heeswijk 2017).

Simulation-based methods are another way of studying the interaction between CL actors. Dynamic simulation and agent-based methods are widely used in this area (see Barceló et al. 2007, Boussier et al. 2011, Teo et al. 2014, Anand et al. 2016, Marcucci et al. 2017, for further information about simulation applications in urban freight transportation). Taniguchi et al. (2007) presented a multi-agent approach in urban freight transport systems by considering the behavior of multiple stakeholders. They propose a solution where carriers use a routing algorithm that dynamically considers the real-time travel information on the road network. The authors showed that the multi-agent simulation generated good performance in terms of increasing profits for freight carriers and decreasing costs for shippers, while emissions are also reduced. van Heeswijk (2017) develop an agent-based simulation framework to evaluate the effectiveness of CL schemes that include both governmental policies and company-driven initiatives. To define company-driven initiatives, the author considers the collaboration between carriers as well as interactions between the actors and CDC managers. Based on the numerical experiments, van Heeswijk (2017) identifies two main solution concepts that are financially viable and yield environmental benefits: (i) collaboration between carriers allows consolidating orders already upstream and (ii) outsourcing last-mile distribution to the CDC operators allows consolidating orders downstream. The author argues that the consolidation centers only function well when being substantially supported by effective local regulations (such as road
Agent-based approaches need to be more developed and consider different angels of the complex CLSs in order to reflect agent behaviors better and provide accurate results (Crainic et al. 2018, Nuzzolo et al. 2018).

Coordination between actors does not constitute a mature body of the literature, especially when it comes to modeling network design problems (see Crainic et al. 2018, for a review). Almost all the studies that fall into OR/IE use simulation methods and mostly concern business interactions among the actors, policy evaluation, and environmental impacts. The coordinated inter-modal urban freight transportation systems are rarely studied in the current literature. More analytical studies on the stakeholders’ partnership in CLSs (such as Estrada & Roca-Riu 2017) is required.

Below, we highlight two components of a CLS that highlight the importance of coordination and collaboration between actors.

### 2.5.1 Stakeholders and Policies

Institutional and public authorities play an important role (as initiators, enablers, and customers) in CLSs (Björklund et al. 2017). They are interested in the improvement of freight distribution within urban areas and reducing its environmental impacts. Therefore, authorities are directly involved in CLSs in two ways: (i) Providing financial supports for CL projects during their investment, trial, and operation that is crucial for viability and feasibility of these projects (Veličković et al. 2017, Paddeu et al. 2018). (ii) Imposing different policy, enforcement, and/or promotion as solutions to improve transportation systems. Such solutions include licensing, regulations (such as land-use planning, road pricing, setting low-emission zones, and forcing off-peak deliveries), and supporting green transportation, which are done to improve quality of life in cities and support better practices. The major restrictions imposed by the local government are time windows for delivery operations, limitations on vehicle speed and load, and traffic/parking regulations. Time windows restrict the urban freight vehicle access time to intervals in which fewer residents in the delivery areas feel the impact of transportation. Vehicle restrictions are imposed to improve livability and reduce environmental effects.
In conventional transportation problems such as the VRPTW, time windows are referred to demand due dates of the customers or their opening and closing times. Hence, time windows are assigned individually for each customer. As a result, the solution to the problem instances of VRP or LRP with time windows contains either short vehicle routes with long waiting times or long routes with crosses creating a congested region. This behavior is a result of an uncoordinated system considering the objective of only one actor (i.e. customers) and can harm environmental sustainability in city logistics. In CLSs, on the other hand, time window regulations are imposed by the municipalities over a city zone or district rather than the individual customers (Nuzzolo et al. 2016, Akyol & de Koster 2013). Therefore, customers in the same area of the city have the same or overlapping time windows. Time window policies depend on the characteristics of the whole city (such as size, population, city structure, and transportation modes) and the area of interest (e.g. the amount of traffic congestion during different times of the day, historical background, or existence of tourist attractions).

While effective policy management create a balance between transportation cost and environmental objectives in CLSs (Nuzzolo et al. 2016), there are a few studies in the literature that investigate the effect of different policies on overall performance of the CLS. Quak & de Koster (2009) argue that local policies might not only increase global and local pollution but also retailers’ costs. Besides, the impacts of time windows and vehicle restrictions on improving social sustainability may negatively affect environmental performance. Taniguchi et al. (2007) and Quak & van Duin (2010) examine the effect of the road pricing policy imposed by the government on urban logistics. The authors claimed that such policies increase the carrier costs and may fail to achieve expected results due to the short-term behavioral reactions of carriers and long-term logistic changes due to the proposed policies. This is due to the fact that national governments and urban authorities often neglect to involve interests of shippers and carriers in their decisions (Browne et al. 2005). Carriers may respond to accessibility and parking restrictions by dividing the city into sectors and serving each sector by a vehicle. However, Franceschetti et al. (2017) conclude that this approach can lead to an increase in the total number of vehicles and short tours. Boussier et al. (2011) studied parking policies and proposed a prototype tool for local authorities
to optimize the sharing of parking places between different carriers. van Heeswijk (2017) claim that while heavy vehicle ban is favorable for receivers, it could reduce the profit of carriers. Therefore, a combination of a pricing policy (e.g., road pricing) and subsidizing the carriers is advised as a more effective solution. Akyol & de Koster (2013) show that time window policies become efficient if neighboring areas have (partially) overlapping time windows and larger areas have tight time windows that are aligned with the time windows preferred by the existing customers. Using simulation, Marcucci et al. (2017) indicated that interaction among stakeholders help to achieve convergent opinions and provide a policy ranking based on the maximization of consensus building and the minimization of utility losses. Akyol & de Koster (2018) develop analytical models to determine time windows by considering cooperation between the stakeholders. The authors showed that coordinated time windows offer advantages to both customers and retailers. Gatta & Marcucci (2014) show how an agent-specific modeling approach can increase policy-makers’ awareness and help taking better decisions. Gatta & Marcucci (2016) suggest stakeholder-specific data acquisition strategy for agent-based approaches. Appendix A offers numerical examples to show how different city characteristics and time window policies affects the solution in a CLSs.

Most of the reviews on CL policy practices have one conclusion in common: local authorities and policy-makers need in-depth evaluation and detailed analysis of CL projects and other actors’ behavior in order to identify potential improvements and opportunities (van Duin & Quak 2007, Papadimitirou et al. 2011, Maggi & Vallino 2016, de Marco et al. 2018). Suggested policies and regulations can become successful if the relationship between local authorities, carriers, and customers is perceived and their preferences are recognized in a balanced way (Gonzalez-Feliu & Salanova 2012, Stathopoulos et al. 2012, Ballantyne et al. 2013, van Heeswijk 2017). Therefore, one needs to first define CLS problems that are more realistic in terms of policy requirement and actors’ behavior; and second, develop or use an applicable solution approach considering the problem difficulty and limitations of computing technology (Farahani et al. 2013).
2.5.2 Satellite Synchronization

Improving the flows in multi-echelon systems and reducing the overall transportation cost require a careful coordination between shippers and carriers. Management of day-to-day operations in satellite locations is essential for achieving these objectives, as unsynchronized satellites can diminish benefits of consolidation in CLSs. Recall that satellites can be only accessed during a limited duration and have limited space. Hence, the number of vehicles being simultaneously present at a satellite location for loading/unloading activities is restricted. Besides, no storage or inventory handling is possible. Therefore, trans-docking is the only feasible way of transferring goods from one vehicle to another. Figure 2.3 illustrates trans-dock operations in a satellite facility. The inbound trucks bring freight from CDCs and unload them at the unloading doors of the satellite. Then, goods are sorted, consolidated, and loaded into the outbound trucks waiting at the loading doors. The outbound trucks leave the satellite to make the final delivery.

Such satellite characteristics bring many challenges to planning operational freight distribution in two-echelon CLSs. An important issue is to schedule vehicles such that unloading the goods from the primary vehicles (inbound trucks) and loading them into the secondary ones (outbound trucks) are done with short delay. In order to have minimum congestion, secondary vehicles should be ready in the satellites when primary vehicles arrive so that transferring the freight between these vehicles is done with no or minimum delay, long incoming/outgoing vehicle queues have to

![Figure 2.3: Trans-dock operations in a satellite facility.](image-url)
be prevented, and vehicles may only wait for a short duration or they will miss their customer due dates. The synchronization problem becomes more complicated when supply and demand consist of multiple products. End customers may demand one or several products and a CDC may have one or a limited number of product types available. Therefore, the time and load synchronization between primary and secondary vehicles are crucial in order to make seamless flow of goods and avoid vehicle congestion and waiting times in city-centers.

Synchronization constraints have been recently studied in the VRP literature. Drexl (2012) defines the VRP with multiple synchronization constraints as “a vehicle routing problem in which more than one vehicle may or must be used to fulfill a task”. The author classifies synchronization into five categories as follows:

- **Task synchronization**: A task is a duty that may consist of picking up a load and/or delivering load to a location. Vehicle capacity is considered in performing tasks. Decisions on which vehicle(s) should fulfill each task are made. For example, in the classical VRP, each customer demand should be satisfied once by exactly one vehicle. Therefore, the task synchronization problem states that each task must be performed exactly once by one vehicle.

- **Operation synchronization**: An operation is something that is performed at a transfer location in order to make fulfilling a task possible (e.g. unloading/loading vehicles). The aim is to synchronize operations of different vehicles at the same or different locations by considering the time at which vehicles perform their operations at the respective location(s). Therefore, this type of synchronization decides on spatial and temporal aspects of tasks. In an urban freight transportation network, for example, transferring the goods from a primary vehicle to a secondary one is possible if both vehicles are present in the same satellite location at the same time. Operation synchronization requires that the elapsed time between the start of execution of an operation by a suitable vehicle at a certain location and the start of execution of another operation by another compatible vehicle lies within a specific interval.

- **Movement synchronization**: Movements of at least two vehicles must be synchronized with respect to time and space. It determines which vehicles should
join to traverse a path together. Movement synchronization is required when a vehicle (e.g. a trailer) has to be pulled by another type of vehicles (e.g. a lorry).

- **Load synchronization**: The vehicle capacities must be considered to fulfill a feasible task. It must be ensured that the correct amount of load is collected, delivered, or transshipped. Load synchronization controls the amount unloaded from a vehicle, partitioned and loaded into other vehicles in a transshipment operation.

- **Resource synchronization**: It determines the use of common resources by vehicles. For example, if a satellite has a limited space for the vehicles to perform their operations, the total space occupied by primary and secondary vehicles should not be exceeded the available space. Resource synchronization requires that the total utilization of a specific resource by all vehicles cannot exceed the limit.

Based on this classification, synchronization at the satellites requires *task, operation, load, and resource synchronization* in order to make seamless flow of goods and avoid vehicle congestion and waiting times in city center areas. The available research on the two-echelon routing problems with spatial, temporal, and load synchronization required at transfers are very limited. The literature mostly contains pure spatial operation synchronization in the context of the multi-echelon VRPs, which ignore the temporal dimension (Cattaruzza et al. 2017). There are only a few papers considering the temporal aspect. Grangier et al. (2016) formulated a 2E-VRPTW under time synchronization constraints. In this problem, second echelon vehicles can perform multiple trips and must be synchronized with the first echelon vehicles every time they start their service from a satellite location. There is no limit on the number of vehicles that can be processed at the same time in the satellite. The authors propose a heuristic to solve the problem and show that time windows have a significant impact on the solution cost in this problem. Li et al. (2016) formulate a 2E-VRPTW where vehicle routes on different echelons are interacted by time constraints. The synchronization is handled by keeping track of arrival of the vehicles to the intermediate facility as well as their waiting time in these locations. A second echelon vehicle cannot leave the intermediate facility before a first echelon vehicle delivers the required product.
The waiting times, along with the transportation costs, are minimized in the objective function. The authors present a two-stage heuristic that incorporates a savings-based procedure followed by a local search phase. Anderluh et al. (2017) consider a 2E-VRP, inspired by a pharmacy wholesaler in Vienna, Austria, in which the inner-city delivery on the second echelon is performed by cargo bikes. After loading in a satellite location, the cargo bikes perform their delivery and when they have to reload, they move again to a satellite. In this problem, first and second echelon vehicles must meet in a synchronized way at the same time at the same physical satellite, while their waiting times are minimized. The authors develop a greedy randomized adaptive search heuristic to solve the problem and consider three distribution policies to evaluate the performance of their city logistics concept: direct delivery, two-echelon deliveries with satellites outside the inner-city environment, two-echelon deliveries with inner-city small satellites that require exact synchronization between the echelons. They conclude that synchronization is costly, but can reduce emissions. Bala et al. (2017) study a 2E-LRPTW arising in delivering perishable goods. They consider a production schedule system where availability of products at facility locations affects origin and departure time of the routes. The authors propose a heuristic solution approach and solve instances with up to 2000 customers and four products.

Since none of these studies consider capacity and congestion constraints for satellites, their problem settings are only suitable for small transportation systems with limited number of vehicles and trips. Although it is not crucial to incorporate such limitations while dealing with cargo bikes, it is important to consider satellite restrictions when the two-echelon vehicles are larger motor vehicles. Ignoring satellite capacity and congestion costs can lead to an unsustainable transportation system in the long-run. Due to lack of extensive studies on satellite synchronization under capacity and scheduling constraints, we discuss similar freight distribution problems where one or more synchronization constraints are applied, and we establish the link between this literature and the satellite synchronization problem in CL. One of the well-known structures in the freight distribution literature, which exhibit similar behavior as satellites in CLSs, are cross-dock facilities (CDs). Cross-docking aims to reduce inventory (staging) costs while improving flow of goods and shipping cycle. In a CD facility, freight is unloaded from the incoming trucks (arriving to the inbound
doors) and reloaded to the outbound trucks (waiting at the outbound doors). Intermediate activities such as sorting, labeling, and storing for a short duration are usually done between unloading and loading activities. van Belle et al. (2012) present a state-of-the-art of cross-docking that addresses a large range of the CD problems. Buijs et al. (2014) define synchronization as “coordination of the local and network-wide operations” in the CD networks and provide a classification of synchronization problems in the CD networks. Ladier & Alpan (2016) extend the works of van Belle et al. (2012) and Buijs et al. (2014) and provide a review and classification of cross-dock studies in the literature. In this classification, a truck sequencing problem is defined when the time dimension represents the order of customers visited by the trucks. If the temporal dimension is modeled explicitly by determining the vehicle arrival/departure times to/from the facility, the problem becomes a truck scheduling problem. It does not take the spatial dimension (i.e., exact locations of the doors) into account. Track-to-door assignment combined with timing decisions lead to the truck-to-door sequencing problem if the decision relates to truck sequence, and lead to the truck-to-door scheduling problem if the time is considered explicitly. Vehicle schedules enable us to deal with the truck processing time deviation, which is an important performance measure under restricted times.

Yu & Egbelu (2008) consider inbound and outbound truck scheduling problems in CDs in a multi-commodity environment. The problem minimizes the total operation time where the product-to-trucks assignment and the docking sequences of inbound and outbound trucks are simultaneously determined. However, they do not include truck-to-door assignments at CDs. It is assumed that there is a temporary storage in front of the shipping dock. The buffer stores the product arriving at the dock but not intended for loading into the outbound truck. They develop an enumeration procedure and a heuristic that minimizes the total number of product passing through the shipping buffer. Chen & Lee (2009) and Chen & Song (2009) study the vehicle scheduling problem in CDs to minimize make-span (i.e. finishing time of processing all vehicles) and show that the problem is strongly NP-hard even if there is only one inbound and one outbound dock doors.

Chmielewski et al. (2009) consider the CD with limited temporary storage and the problem of determining the schedules of inbound and outbound trucks to minimize
the internal distance of material handling equipment and waiting time of inbound vehicles. The problem is transformed into a network where the time aspect is discretized into periods with equal lengths. They suggest an approximate column generation procedure and a meta-heuristic. Considering no temporary storage at the CDs and the limited number of inbound and outbound doors, Boysen (2010) formulates the truck-scheduling problem and suggests three different objectives to minimize: flow time, processing time, or tardiness of outbound trucks. It is assumed that the travel times inside CDs are negligible. Since there is neither intermediate storage nor transfer time, the outbound truck should be docked as soon as an inbound truck is scheduled at a door. The author presents a dynamic program and a meta-heuristic to solve the problem under a discretize time period. van Belle et al. (2013) formulate the truck-to-door scheduling problem considering predefined arrival and departure times of the vehicles and propose a meta-heuristic. The problem seeks scheduling of inbound and outbound vehicles within their time limits in order to minimize the weighted sum of the total travel time and the total tardiness of trucks with respect to their departure times. Boysen et al. (2013) study the inbound truck-scheduling problem in the CD where the outbound trucks are scheduled beforehand and their departure times are fixed. They present its MIP formulation in order to minimize the lost profit. The profit is lost whenever a shipment is not unloaded, transshipped to the outbound gate, and loaded onto the designated outbound truck before its departure. Two heuristics are developed to solve the problem. Bodnar et al. (2017) consider scheduling of inbound and outbound trucks subject to time windows at a multi-door cross-dock, where dock doors can either be dedicated to inbound or outbound trucks or be capable of handling both truck types. An MIP formulation is presented to minimize the total operation costs composed of handling loads in temporary storage and tardiness caused by processing outbound trucks after their respective due times. A meta-heuristic is proposed to solve the problem.

The literature of transportation with cross-docking is the most related area to the satellite synchronization in CL. However, CDs are different from CL transportation systems in several ways. The vehicle routing and facility location problems together with synchronization issues, as other parts of the transportation system, are not considered explicitly in the cross-docking. Satellites and CD facilities have different character-
istics (e.g. in terms of available capacity and allowed inventory) leading to different problem structure. Besides, due to difficulties arising in the truck-to-door scheduling problem, most of the works in the literature use heuristic solution approaches. A few studies consider dynamic programming or other enumeration procedures to find the optimal solutions of small size test instances. However, as the problem size increases, these methods become computationally impractical. This study is the first that formulates the satellite synchronization problem to schedule primary and secondary vehicles assigned to a satellite location by defining a congestion minimizing objective function. Chapter 5 provides more details about the satellite synchronization problem and its formulations.

2.6 Summary

City logistics has been getting attention since 1990s. Although it has potentials to enhance freight transportation in cities, there has not been extensive research on its impacts on costs and benefits of involved actors. A sustainable CLS can be obtained if (i) high level of information exchange exists between its stakeholders, (ii) coordination and collaboration of actors are maintained, and (iii) an effective and efficient consolidation is conducted in the freight transportation process.

Through an in-depth literature review, we identified major decisions of a CLS and addressed related OR/IE problems in the literature to its strategic, tactical, and operational level planning. Pros and cons of each model in the context of city logistics are discussed, the available solution approaches are listed, and the solvable problem sizes are mentioned. Considering the current gaps in terms of formulation CL networks and providing efficient solution approaches, we study both 1E and 2E urban freight transportation problem and develop efficient exact and heuristic solution methods.

Coordination challenges of CL are explored and the shortcoming of conventional optimization models when it comes to coordinating CL actors are discussed. In this regard, we define and formulate the satellite synchronization problem to address operational decision-making issues in the 2E urban freight distribution problems.
CHAPTER 3

SINGLE–ECHELON FREIGHT DISTRIBUTION SYSTEMS IN CITY LOGISTICS

In the single–echelon city logistics systems, goods are delivered from distribution centers to the customers using a single transportation mode. Demands of different customers are consolidated in city distribution centers and then distributed using vehicles that are suitable for urban environment. In this chapter, we formulate the location-routing problem with time window constraints (LRPTW) to address the main strategic and tactical level decisions of the single-echelon (1E) urban freight transportation. In order to find the exact solution to the LRPTW, we propose a branch-and-price algorithm based on a set-partitioning approach, where the subproblem is solved using dynamic programming. We introduce several strategies to improve the lower and upper bounds as well as acceleration techniques to enhance our column generation. Computational results show the higher performance of the proposed method on a set of small and medium size instances in the literature and demonstrate its efficiency in solving generated large size instances. 1

3.1 Introduction

Known as one of the integrated problems in logistics, the location-routing problem (LRP) consists of two basic decisions to be made, each of which known as a hard combinatorial optimization problem: decisions on the location of facilities (such as city distribution centers, depots, warehouses, etc.) and decisions on the routing of vehicles. Although these two types of problems have been traditionally considered

1 The research done in this chapter has been published in *Computers & Operations Research* (see Farham et al. 2018).
separately at different planning levels, it is shown that the integrated approach of the LRP reduces overall cost in the long-run (Salhi & Nagy 1999, Nagy & Salhi 2007). Urban freight transportation, food and drink distribution, waste collection, parcel delivery, and blood bank location are some examples of LRP applications.

In the LRP, a set of potential facility locations with known capacity, a set of demand points, and a fleet of delivery vehicles with limited size are given. We assume fixed costs for opening the facilities and using vehicles. The problem is to decide which facilities to open and which vehicle routes to use such that all customers are served with optimal total cost. Facility opening cost, vehicle usage cost, and traveling cost constitute the total cost to be minimized.


In this chapter, we consider the LRP where serving a customer has to be started during a predefined time interval specified to that customer. This problem, called the location-routing problem with time windows (LRPTW), addresses the strategic and tactical decisions in 1E city logistics systems as it considers vehicle weight and customer access time restrictions. Although time window constraints have been considered for the vehicle routing problem (namely, VRPTW) for a long time, it is not well-studied in the context of LRP. Ponboon et al. (2016) is the only one study on the LRPTW that proposes an exact solution algorithm. Their approach is based on
the branch-and-price (BP) framework implemented to solve the set-partitioning (or path-based) formulation of the problem. BP embeds column generation (CG) in the branch-and-bound method to solve hard combinatorial optimization problems and is shown to be effective in the context of vehicle routing problems (Desaulniers et al. 2005, Contardo et al. 2015). It decomposes the original problem into two problems, namely the master problem and the subproblem. The master problem is a path-based formulation of the problem, which is initialized with a restricted set of paths. The subproblem determines new path variables (columns) to enter to the master problem. The new variables are priced out with respect to the current dual solution of the master problem. If the new variable potentially improves the objective function, it is added to the master problem, and the master problem is re-solved to obtain new dual variables for generating a new column. This process is repeated until no further improvement on the objective function is obtained. Ponboon et al. (2016) generate new set of instances by modifying the LRP and VRPTW benchmark instances in the literature.

The major difficulty of BP is solving the subproblem, i.e., generating new columns to be added to the master problem. The efficiency of CG approaches depends on the quality of the bound obtained at each node of the search tree and the computational time needed to achieve this bound. In vehicle routing problems, the subproblem is an elementary shortest path problem with resource constraints, which is \( NP \)-hard. Desaulniers et al. (2005), Lübbecke & Desrosiers (2005), Contardo et al. (2015) review the challenges of column generation techniques for the restricted routing problems.

We contribute to the LRP literature by presenting an exact solution approach for the LRPTW based on branch-and-price that larger problem sizes more efficiently. The difference with Ponboon et al. (2016) is that our column generation framework is enhanced by various techniques inspired from the VRPTW and the LRP literature for improving lower and upper bounds on the objective function value of the LRPTW. The performance of the algorithm is tested on a large set of instances with different characteristics. For the instances in Ponboon et al. (2016), the numerical results show that the computational efforts are significantly reduced when the enhancement techniques are applied on the proposed algorithm. Our approach finds the optimal solution for the instances with up to 5 candidate CDCs and 50 customers, which are solved for
the first time in the literature. We also introduce a two-stage heuristic, where the first stage fixes the CDC locations under time window constraints and the second stage allocates customers to the open CDCs through routing with time windows. The computational experiments indicate that the proposed heuristic saves significant amount of time to solve the problem without sacrificing the solution quality.

This chapter is organized as follows. In Section 3.2, we provide the arc-flow MIP formulation of the LRPTW by addressing its strategic and tactical levels decisions. We also present the decomposed formulation of problem and show how the proposed techniques are adopted to improve the lower bound and solve the problem more efficiently in the subsequent sections. Section 3.3 introduces the branch-and-price solution algorithm and the column generation technique. We develop a heuristic method for finding efficient solutions in Section 3.4. Section 3.5 presents the problem test instances and the experimental results of the proposed solution approaches. We make our concluding remarks in Section 3.6.

3.2 Problem Formulation

In this section, we provide the arc-flow formulation as well as the path-based formulation (also known as set-partitioning formulation) for the LRPTW. We first provide the MIP formulation of the facility location problem with time windows (FLPW) to address the strategic decisions of the LRPTW. Then, we extend the FLPTW to incorporate lower-level decisions that are addressed in the LRPTW. Figure 3.1 illustrates FLP and LRP as the 1E freight distribution systems.

3.2.1 Arc–Flow Formulation

The strategic planning of the 1E systems can be formulated as an FLP (see Figure 3.1a). FLPs seek location of the open CDC(s), and allocation of customers to the located CDCs, such that the total cost of facility location and customer allocation (considered as direct shipment routes) is minimized. The FLPTW includes additional constraints to satisfy customer time windows.
Consider a transportation network consisting of two sets of nodes. The first set, $\mathcal{I}$, represents candidate CDC locations, while the second set, $\mathcal{K}$, denotes the customer or demand nodes. Each customer $k \in \mathcal{K}$ is characterized by a demand $D_k$ and a time window $[A_k, B_k]$. Like most VRPTW studies, we assume that a vehicle is free to arrive at a customer location before its time window begins. However, serving customer $k$ cannot be started earlier than $A_k$ or later than $B_k$. Each CDC $i \in \mathcal{I}$ has an opening fixed cost $F_i$ and a capacity $Q_i$, and is available during working hours $[0, B_i]$. Assume that a sufficient number of homogeneous vehicles with fixed usage cost $F'$ and capacity $Q'$ is available. Also assume that customer demands are all less than the vehicle capacity and cannot be split. We define the directed network $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N} = \mathcal{I} \cup \mathcal{K}$ is the set of all nodes, and $\mathcal{E} = \{(m, n) : m \neq n, \forall m, n \in \mathcal{N}\}$ is the set of arcs. We do not include CDC-to-CDC arcs in $\mathcal{E}$. Define $T_{mn}$ as the sum of service time at node $m$ and traveling time from node $n$ to node $m$, $\forall m, n \in \mathcal{N}$. Consider $C_{mn}$ as the cost of traveling on arc $(m, n) \in \mathcal{E}$. The problem is formulated using the following decision variables: binary variable $z_i$ to denote whether CDC $i$ is opened, and binary variable $r_{ik}$ indicating assignment of customer $k$ to CDC $i$.

The FLPTW is formulated below.

\begin{align}
(\text{FLPTW}) & \quad \text{Minimize} & & \sum_{i \in \mathcal{I}} F_i z_i + \sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}} (F' + C_{ik} + C_{ki}) r_{ik} \\
& \quad \text{subject to:} & & \sum_{i \in \mathcal{I}} r_{ik} = 1, \quad \forall k \in \mathcal{K} \quad (3.2) \\
& & & \sum_{k \in \mathcal{K}} D_k r_{ik} \leq Q_i z_i, \quad \forall i \in \mathcal{I} \quad (3.3) \\
& & & (T_{ik} - B_k) r_{ik} \leq 0, \quad \forall i \in \mathcal{I}, k \in \mathcal{K} \quad (3.4)
\end{align}
\[
[\max(T_{ik}, A_k) + T_{ki} - B_i] r_{ik} \leq 0, \quad \forall i \in I, k \in K
\]  
\[z_i \in \{0, 1\}, \quad \forall i \in I\]  
\[r_{ik} \in \{0, 1\}, \quad \forall i \in I, k \in K\]  

Objective function (3.1) minimizes total location and allocation cost. The allocation cost of a customer to a CDC is considered as the sum of fixed vehicle cost and traveling cost of CDC–customer–CDC routes. Constraint (3.2) ensures that all customers are served. The capacity of an open CDC is satisfied by (3.3). Constraints (3.4) and (3.5) satisfy time windows of customers and CDCs, respectively. The time window constraints make sure that a \(r_{ik}\) variable is zero if a vehicle cannot reach customer \(k\) from CDC \(i\) before \(B_k\), or the vehicle cannot go back to the CDC \(i\) before \(B_i\). (3.6) and (3.7) are variable domain constraints.

The LRPTW, illustrated in Figure 3.1b, adds routing decisions to the above formulation. Therefore, we define additional variables as follows. Let \(x_{mn}\) be a binary variable that takes a value if and only if a vehicle traverses on arc \((m, n) \in E\). Define \(t_n\) as the arrival time of a vehicle to node \(n\). Let \(q_k\) be the load on a vehicle upon arrival to customer \(k\). Therefore, the arc-flow formulation of the LRPTW is as follows.

\[
(\text{LRPTW}) \quad \text{Minimize} \quad \sum_{i \in I} F_i z_i + \sum_{i \in I} \sum_{k \in K} F' x_{ik} + \sum_{(m,n) \in E} C_{mn} x_{mn}
\]
subject to: (3.2), (3.3), (3.6), (3.7),

\[\sum_{n \in N} x_{nk} = 1, \quad \forall k \in K\]  
\[\sum_{m \in N} x_{nm} - \sum_{m \in N} x_{mn} = 0, \quad \forall n \in N\]  
\[x_{ik} \leq r_{ik}, \quad \forall i \in I, k \in K\]  
\[r_{ik} + x_{kl} \leq 1 + r_{il}, \quad \forall i \in I, k, l \in K\]  
\[q_k - D_k - q_l \leq Q'(1 - x_{kl}), \quad \forall k, l \in K\]  
\[T_{ik} - t_k \leq B_{ik}(1 - x_{ik}), \quad \forall i \in I, k \in K\]  
\[t_k + T_{kn} - t_n \leq B_{kn}(1 - x_{kn}), \quad \forall k \in K, n \in N\]  
\[D_k \leq q_k \leq Q', \quad \forall k \in K\]  
\[A_n \leq t_n \leq B_n, \quad \forall n \in N\]  
\[x_{mn} \in \{0, 1\}, \quad \forall (m, n) \in E\]  

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where, \( B_{mn} \) is a constant equal to \( \max (B_m + T_{mn} - A_n, 0) \), \( \forall (m,n) \in \mathcal{E} \). The objective function (3.8) minimizes the total CDC opening cost, vehicle fixed cost, and traveling cost. Constraint (3.9) adds the required constraints from the FLPTW model. Constraint (3.10) ensures that exactly one incoming arc is selected for each customer node. The flow balance at each node is satisfied by constraint (3.11). Constraints (3.12) and (3.13) assign customer \( k \) to CDC \( i \) if there is a route from node \( i \) that passes through node \( k \). Constraint (3.14) is the Miller–Tucker–Zemlin constraint that controls the load of a vehicle in its route and eliminates subtours. If \( x_{kl} = 0 \), this constraint is redundant. Otherwise, \( q_l \geq q_k - D_k \) is held. Constraints (3.15) and (3.16) control vehicle arrival times. The load on a vehicle needs to be at least equal to a customer’s demand when the vehicle arrives at the customer’s location, and at most equals to the vehicle’s capacity. In addition, the vehicle has to start serving a customer only during the allowed time window. These constraints are satisfied by (3.17) and (3.18), respectively. Finally, (3.19) meets the binary requirement for arc-flow variables.

The arc-flow LRPTW formulation (3.8)–(3.19) contains a large number of variables and constraints even for small-size instances. The number of binary variables is in the order of \( |\mathcal{N}|^2 \) and the number of constraints is in the order of \( |\mathcal{K}| \times |\mathcal{N}| \). Therefore, realistic problem sizes become intractable to solve with standard MIP solvers.

### 3.2.2 Path–Based Formulation

The proposed column generation approach (CG) is based on Dantzig-Wolfe decomposition of the original formulation (3.8)–(3.19) into two problems: the master problem and the subproblem (Dantzig & Wolfe 1960). The aim of this reformulation is to provide better bound when linear relaxation of the problem is solved. Let \( \mathcal{P}_i \) be the set of all feasible vehicle routes originated at CDC \( i \). A route in \( \mathcal{P}_i \) is performed by a single vehicle that leaves CDC \( i \) with a load no more than \( Q' \), serves a set of customers within their time windows, and returns to CDC \( i \) before it closes. A vehicle path \( p \) is a set of nodes visited by the vehicle. Therefore, a cost \( C_p \) is associated with path \( p \) to address the vehicle fixed cost and the traveling costs of all arcs traversed in the path. Let \( H_{pk} \) indicate the number of times customer \( k \) is visited in path \( p \). Define \( \lambda_p \) as a binary decision variable that takes value 1 if path \( p \) is selected, and 0
The master problem is formulated as the following path-based model.

\[
\text{Minimize } \sum_{i \in I} F_i z_i + \sum_{j \in J} \sum_{p \in \mathcal{P}_j} C_p \lambda_p \\
\text{subject to: } \sum_{i \in I} \sum_{p \in \mathcal{P}_i} H_{pk} \lambda_p = 1, \quad \forall k \in \mathcal{K} \tag{3.21}
\]

\[
\sum_{p \in \mathcal{P}_i} H_{pk} D_k \lambda_p \leq Q_i z_i, \quad \forall i \in I \tag{3.22}
\]

\[
z_i \in \{0, 1\}, \quad \forall i \in I \tag{3.23}
\]

\[
\lambda_p \in \{0, 1\}, \quad \forall p \in \mathcal{P}, i \in I. \tag{3.24}
\]

The objective function (3.20) minimizes total CDC opening and route operating costs. By set-partitioning constraint (3.21), each customer is visited exactly once. Constraint (3.22) ensures that the total customer demand served from an open CDC does not exceed its capacity. Constraints (3.23) and (3.24) hold binary conditions.

### 3.2.2.1 Valid Inequalities

In order to make the master problem stronger, two valid inequalities are introduced. The first inequality provides a lower bound for the number of open CDCs by defining \(Z\) as the minimum number of CDCs required to serve customer demands. The second inequality sets a lower bound on the total number of vehicles required to serve all demands denoted by \(V\). Let \(v_i\) be a nonnegative integer decision variables indicating the number of vehicle routes from CDC \(i\). Then, (3.25) and (3.26) are valid for formulation (3.20)–(3.24). Note that defining \(v_i\) requires additional constraints (3.27) and (3.28).

\[
\sum_{i \in I} z_i, \geq Z \tag{3.25}
\]

\[
\sum_{i \in I} v_i, \geq V \tag{3.26}
\]

\[
v_i = \sum_{p \in \mathcal{P}_i} \lambda_p, \quad \forall i \in I \tag{3.27}
\]

\[
v_i \in \{0, 1, 2, \ldots\}, \quad \forall i \in I. \tag{3.28}
\]

We denote the new master problem (3.20)–(3.28) as MP. This problem can be solved efficiently using standard MIP solvers if \(\mathcal{P}_i\) sets are small. However, it is impractical to generate and add all possible paths in realistic situations. Therefore, instead of
enumerating over all path variables (columns), a column generation approach is used. In the next section, we propose an exact approach to solve the MP.

### 3.3 Branch-and-Price Algorithm

The LRPTW (3.20)–(3.28) can be solved efficiently using standard MIP solvers if \( \mathcal{P}_i \) sets are small. However, it is impractical to generate and add all the possible paths in realistic situations. We propose a branch-and-price algorithm (BP) for solving the LRPTW. The idea is to initiate the original problem with a limited number of columns and generate new columns as needed instead of enumerating over all path variables. The MP (3.20)–(3.28) with only a subset of path variables is called the restricted master problem (RMP). In each iteration, BP solves the relaxation of the RMP and finds the dual solution. Next, a number of subproblems, also called as pricing problems, are solved to price out new path variables and extend \( \mathcal{P}_i \) sets. If a column with negative reduced cost is found, it is added to the RMP and the relaxed RMP is resolved. Otherwise, the algorithm checks the current solution of the RMP against integrality constraints. If any fractional integer variable exists, a branching rule is applied and the algorithm solves a new RMP. Otherwise, it stops by returning the optimal solution. The outline of the proposed BP is given in Figure 3.2. In the first step of the algorithm, the RMP is constructed with initial columns. One way is to obtain trivial initial columns by solving the FLPTW (3.1)–(3.7) and provide the resulting CDC–customer–CDC routes as the initial columns for the RMP.

#### 3.3.1 The Subproblems

In order to generate new path variables for set \( \mathcal{P}_i \), a subproblem \( \text{SP}_i \) is defined for all \( i \in \mathcal{I} \). Let \( \mathcal{E}_i \) be the set of arcs for \( \text{SP}_i \). \( \mathcal{E}_i \) includes all arcs in \( \mathcal{E} \) excepts the ones starting/ending at any CDC other than \( i \). Let \( \alpha_k \), \( \beta_i \), and \( \gamma_i \) be the dual values associated with constraints (3.21), (3.22), and (3.27), respectively. Then, the reduced cost of path variable \( \lambda_p \) for \( p \in \mathcal{P}_i \), indicated as \( \tilde{C}_p \), is calculated by (3.29).

\[
\tilde{C}_p = C_p - \sum_{k \in p} \alpha_k - \sum_{k \in p} D_k \beta_i - \gamma_i
\]  

(3.29)
Start

Construct the RMP with an initial set of columns

Solve the relaxation of the RMP and calculate dual prices

For each CDC, solve the corresponding subproblem

Any column with negative reduced cost?

Add such column(s) to the RMP

Solution integral?

Branch

no

yes

Stop

Figure 3.2: Branch-and-price algorithm for the LRPTW.

The subproblem $SP_i$ for CDC $i$ is the problem of finding a path with minimum reduced cost from the source node $i$ to the sink node $i'$ (which is a duplicate of $i$) in a graph consisting of the customer nodes $K$ and arcs $E_i$, such that vehicle capacity is satisfied and a customer is visited at most once during its time window. This corresponds to an elementary shortest path problem with resource constraints (ESPPRC), where the capacity and time window restrictions are reflected as limited resources (Irnich & Desaulniers 2005). To the ESPPRC, one can use equation (3.29) to find the reduced cost of all arcs $(m, n) \in E_i$, denoted by $\tilde{C}_{inn}$, as follows:

$$
\tilde{C}_{inn} = \begin{cases} 
F' + C_{mn} - \gamma_i, & \text{if } m = i, \\
C_{mn} - \alpha_m - \beta_i D_m, & \text{otherwise.}
\end{cases}
$$

(3.30)

Therefore, $SP_i$ is given by the following formulation.
The objective function (3.31) minimizes the cost of selected arcs. Constraint (3.32) initiates one path from CDC \(i\). (3.33) is the flow conservation constraint. (3.32) and (3.33) make up the elementary condition. Constraint (3.34) ensures that the accumulated demand in a path does not exceed vehicle capacity. By constraints (3.35) and (3.36), vehicle arrival times are set with respect to the order of nodes visited in the path. These constraints also eliminate sub-tours in a solution. Constraint (3.37) limits arrival times to the time windows. Constraint (3.38) meets the binary requirement of the arc-flow variables.

If the optimal objective function value of this problem is negative, a column \(\lambda_p\) is generated for the RMP based on the selected arcs in the optimal solution of the SP\(_i\). In the following sections, we provide the details on how the introduced subproblems are solved.

### 3.3.2 Solving a Subproblem

The SP\(_i\) (3.31)–(3.38) is shown to be strongly NP-hard (Dror 1994). In this section, we use a dynamic programming approach, called labeling algorithm (LA), to solve an SP\(_i\). LA extends the Bellman–Ford shortest path algorithm by taking the resource constraints into account (Feillet et al. 2004). In this algorithm, a label present a partial path on the graph starting from the source node. LA starts with an initial label at the
origin node and creates new labels by extending it to the reachable nodes on the graph using resource extension functions. A node is reachable from a partial path \( p \) if it can be appended to \( p \) without making it infeasible. Label extension procedure terminates when all partial paths are complete, i.e. all paths are extended to the sink node. However, this process may produce an exponential number of labels and become inefficient even with tight resource constraints. To overcome this issue, we can keep only Pareto-optimal labels and discard the ones that cannot yield an optimal path. To decide which labels to discard, a dominance rule is applied. Below, we define labels, extension functions, and dominance rules.

A label \( L \) is represented by a tuple with the following resource elements: reduced cost of the path \( \tilde{C}(L) \), the total load delivered by the path \( D(L) \), the total time of the path \( T(L) \), the set of unreachable (or forbidden) nodes \( U(L) \), the last node of the path \( \text{last}(L) \), and the predecessor node of the last node \( \text{pre}(L) \). Visiting a node is forbidden for a label if any of the following conditions occur: (i) the node is already visited in the path, (ii) it is not possible to reach that node before its due date or closing time, (iii) vehicle capacity is exceeded if the node is included in the path.

To solve the SP for any CDC \( i \), we set the initial label to \( L^0 \) with the following attributes: \( \tilde{C}(L^0) = 0, D(L^0) = 0, T(L^0) = 0, U(L^0) = \{k \in K : T_{ik} > B_k\}, \text{last}(L^0) = i, \text{and pre}(L^0) = \text{N/A} \), where N/A indicates that no predecessor node exists for \( L^0 \). LA extend a label \( L \) ending at node \( m \) along an arc \( (m, n) \in E_i \) to create a new label \( L^{\text{new}} \) with the following components:

\[
\begin{align*}
\text{last}(L^{\text{new}}) &= n, \quad (3.39) \\
\tilde{C}(L^{\text{new}}) &= \tilde{C}(L) + \tilde{C}_{imn}, \quad (3.40) \\
D(L^{\text{new}}) &= D(L) + D_n, \quad (3.41) \\
T(L^{\text{new}}) &= \max\{A_n, T(L) + T_{mn}\}, \quad (3.42) \\
U(L^{\text{new}}) &= U(L) \cup \{k \in N : k = n, T(L) + T_{nk} > B_k, \text{or} D(L) + D_k > Q'\}, \quad (3.43) \\
\text{pre}(L^{\text{new}}) &= m. \quad (3.44)
\end{align*}
\]

For each adjacent node of \( m \), a new label is created. However, to maintain feasibility, we do not extend \( L \) to a node \( n \) if \( n \in U(L) \).

In order to identify and remove non-Pareto-optimal labels, we apply the following
dominance rule. For any two labels $L^1$ and $L^2$, we say that $L^1$ dominates $L^2$ if all the following conditions hold:

\[
\text{last}(L^1) = \text{last}(L^2),
\]

(3.45)

\[
\tilde{C}(L^1) \leq \tilde{C}(L^2),
\]

(3.46)

\[
D(L^1) \leq D(L^2),
\]

(3.47)

\[
T(L^1) \leq T(L^2),
\]

(3.48)

\[
U(L^1) \subseteq U(L^2).
\]

(3.49)

Conditions (3.45)–(3.49) ensure that for any two labels $L^1$ and $L^2$ with the same end node, none of the feasible extension of $L^2$ will yield a better (partial) path than the ones produced by feasible extensions of $L^1$. Therefore, $L^2$ is not a Pareto-optimal path and can be discarded.

Finally, the algorithm stops whenever no label can be extended and returns the non-dominated labels corresponding to the complete paths (i.e. paths that have reached CDC $i$). For more information on labeling algorithm and its applications, see Feillet et al. (2004), Irnich & Desaulniers (2005). One of the advantages of using LA to solve the subproblem is that it can produce more than one non-dominated path with negative reduced cost. Therefore, one can add multiple columns with negative reduced cost to the RMP at once in order to speed up the computational time of the proposed BP.

### 3.3.3 Column Generation Enhancements

The performance of the proposed CG is highly dependent on the lower bound quality obtained at each node of the branch-and-bound tree and the computational time required to achieve this bound. In this section, we introduce the tools to increase the efficiency of the CG procedure.

#### 3.3.3.1 Reduced–Size Network

In order to find a path faster, LA can be run on a smaller graph. Desaulniers et al. (2008) suggest constructing a graph by removing arcs with high reduced costs. There-
fore, given a parameter $\Phi_1$ in LA, we extend a path along only $\Phi_1$ number of arcs with most negative reduced costs.

### 3.3.3.2 Eliminating Non-improving Arcs and Vertices

As a preprocessing to LA, one can reduce the size of the problem by removing the arcs that cannot appear in the final optimal path. We use arc elimination technique of Rousseau et al. (2004) as follows: (i) Remove arc $(m, n)$ if for any customer $k$ that is reachable from $n$, it is always cheaper (in terms of reduced costs) to go directly from $m$ to $k$ than to travel through $n$. (ii) Remove arc $(m, n)$ if for any customer $k$ that can reach $m$, it is always cheaper (in terms of reduced costs) to go directly from $k$ to $n$ than to travel through $m$.

After removing unnecessary arcs, Tarjan’s algorithm (Tarjan 1972) is applied to find strongly connected components of the resulting graph. Then, the components disconnected from the CDC node are eliminated from the graph.

### 3.3.3.3 Relaxing Elementary Condition

When a shortest path graph contains arcs with a negative cost, it is possible that a non-elementary shortest path algorithm stuck in a cycle. In LA, we maintain the elementary conditioning by keeping track of visited nodes in set $\mathcal{U}$. This condition can be relaxed to allow paths with cycles (i.e. paths in which one or several customers are visited more than once). Removing elementary restriction leads to the pure shortest path problem with resource constraints (SPPRC). Note that limited capacity and time resources always prevent infinite cycles. Changing extension function (3.43) to (3.50) yields less restrictive dominance rule, enabling LA to dominate more labels and find a path faster.

\[
\mathcal{U}(L_{\text{new}}) = \mathcal{U}(L) \cup \{k \in \mathcal{N} : T(L) + T_{nk} > B_k \text{ or } D(L) + D_k > Q'\}.
\]  

(3.50)

Note that the set-partitioning constraint (3.21) and binary requirements forbid cyclic columns to be a part of an integer solution. Therefore, extending $\mathcal{P}_t$ sets with cyclic path does not result in an infeasible solution. Below, we describe three techniques to
reach the elementary lower bound faster when the cycles are allowed.

**ng-paths.** Baldacci et al. (2011a) introduced ng-path relaxation for the VRPTW. The idea is to allow certain cycles in a path based on a neighborhood definition. The neighborhood of a customer node \( k \in \mathcal{K} \), denoted by \( Nbr(k) \), is a set of adjacent customer nodes with size \( \Phi_2 \), where \( \Phi_2 \) is a parameter. A cycle \( n \rightarrow k_1 \rightarrow k_2 \rightarrow \cdots \rightarrow k_{\Phi_2} \rightarrow n \) is allowed only if there exists a \( k' \in \{k_1, k_2, \cdots, k_{\Phi_2}\} \) such that \( n \notin Nbr(k') \). Note that if \( Nbr(n) = \mathcal{N} \) for all \( n \in \mathcal{N} \), no cycles are allowed, and we obtain an ESPPRC. On the other hand, setting \( Nbr(n) = \{n\}, \forall n \in \mathcal{N} \), yields a SPPRC.

For example, suppose that the set of customers is given by \( \mathcal{K} = \{1, \cdots, 5\} \) and the ng neighborhoods are as follows: \( Nbr(1) = \{1, 2, 3\}, Nbr(2) = \{1, 2, 5\}, Nbr(3) = \{1, 3, 4\}, Nbr(4) = \{3, 4, 5\} \), and \( Nbr(5) = \{2, 4, 5\} \). Now, the path \( i \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 1 \rightarrow 5 \rightarrow i \) for a given CDC \( i \) is not a feasible ng-path since node 1, which is revisited after node 3, is in \( Nbr(3) \). However, a path \( i \rightarrow 1 \rightarrow 3 \rightarrow 5 \rightarrow 3 \rightarrow 4 \rightarrow i \) is allowed since even though node 3 is revisited after node 5, it is not in \( Nbr(5) \).

When using this method, set \( \mathcal{U} \) in LA is replaced by a new set \( \hat{\mathcal{U}} \) which is initially set to \( \mathcal{U}(L^0) \) and is extended along arc \((m, n) \in \mathcal{E}_i\) using the following resource extension function instead of (3.50):

\[
\hat{\mathcal{U}}(L^{\text{new}}) = \{\hat{\mathcal{U}}(L) \cap Nbr(n)\} \cup \{k \in \mathcal{N} : T(L) + T_{nk} > B_k \text{ or } D(L) + D_k > Q'\}.
\]

(3.51)

Therefore, the dominance condition (3.49) is replaced by \( \hat{\mathcal{U}}(L^1) \subseteq \hat{\mathcal{U}}(L^2) \), which is less restrictive than (3.49). Since some cyclic paths are forbidden to enter the master problem, better lower bound than the pure SPPRC case is obtained.

**2-cycle Elimination.** Since the ng-paths may include 2-cycles, i.e. the paths with \( \cdots \rightarrow k \rightarrow k' \rightarrow k \rightarrow \cdots \) cycles, one can prevent extending a label \( L \) with \( \text{pre}(L) = k \) to node \( k \) to strengthen SPPRC and ng-paths relaxation and obtain a better elementary lower bound. Irnich & Desaulniers (2005) discuss how the LA and dominance rule are modified when 2-cycles elimination is applied.
**Strong Degree Constraints.** Elementary condition imposed by resource $\mathcal{U}$ is applied to all nodes, regardless of the chance that a node is revisited in a path. However, when paths with cycles are allowed, it can be observed that the routes in the optimal linear relaxation solution often cycle on some nodes more than the others. For example, customers with wide time windows or small demands might be visited more than once, whereas the ones with tight time windows or high demands might not appear repeatedly in a path. Based on this observation, Contardo et al. (2014) introduced strong degree constraints (SDCs) given below.

$$\sum_{i \in I} \sum_{p \in P_i} \hat{H}_{pk} \lambda_p \geq 1 \quad \forall k \in \mathcal{K},$$  \hspace{1cm} (3.52)

where, $\hat{H}_{pk}$ is 1 if the customer $k$ is visited at least once in path $p$, and is 0 otherwise. The authors showed that adding the SDC for customer $k$ to the master problem imposes elementary condition for $k$ in a way that all columns associated with paths cycling on $k$ vanish in the optimal linear relaxation solution.

Therefore, when the linear relaxation solution revisits a customer in a path, we add the corresponding SDC(s) to the master problem. Let $\mathcal{K}' \subseteq \mathcal{K}$ be a subset of customers for which SDCs are added. Let $\mu_k$ be the dual variable associated with SDC $k \in \mathcal{K}'$. Then, when a label $L$ is extended along arc $(m,n)$ for SP$_i$, the following reduced cost calculation is used.

$$\tilde{C}(L^{\text{new}}) = \begin{cases} 
\tilde{C}(L) + \tilde{C}_{imn} - \mu_n & \text{if } n \in \mathcal{K}' \setminus \mathcal{U}(L), \\
\tilde{C}(L) + \tilde{C}_{imn} & \text{otherwise}. 
\end{cases}$$  \hspace{1cm} (3.53)

The dominance rule is modified such that conditions (3.46) and (3.49) are replaced by (3.54).

$$\tilde{C}(L^1) + \sum_{k \in \mathcal{K}^{1,2}} \mu_k \leq \tilde{C}(L^2),$$  \hspace{1cm} (3.54)

where $\mathcal{K}^{1,2}$ is the set of customers in $\mathcal{K}'$ that are in $\mathcal{U}(L^1)$ but not in $\mathcal{U}(L^2)$. Therefore, we use the combination of ng-paths and SDCs with 2-cycle elimination to improve the bound obtained by solving the linear relaxation of the problem.
### 3.3.3.4 Column Generation Procedure

Algorithm 3.1 presents the steps of solving any SP<sub>i</sub>. We only run the exact CG approach (i.e. solving the ESPPRC) if the approaches with the relaxed elementary condition fail to find a promising column.

### 3.3.4 Branching

Since the solution to the linear relaxation of the master problem is often fractional, a branching rule is applied to obtain an integer solution. Branching on the route variables is inefficient and results in an unbalanced problem. \( \lambda_p \geq 1 \) is a strong branch, whereas \( \lambda_p \leq 0 \) is a very weak branch and destroys the subproblem structure as \( p \) must not be generated again. Therefore, the following branching strategies are implemented to ensure an integer solution. First, we prioritize more strategic variables over the other ones in the following order and select the ones with a fraction closest to 0.5: (i) fractional \( z_i \) variables, \( \forall i \in \mathcal{I} \), (ii) fractional \( v_i \) variables, \( \forall i \in \mathcal{I} \), and (ii) fractional \((m,n)\) arcs, \( \forall (m,n) \in \mathcal{E} \). Then, we treat the resulting branches as follows. For any CDC \( i \), if \( z_i = 0 \) or \( v_i = 0 \) holds in a branch, we simply ignore solving SP<sub>i</sub> in that branch. For any fractional arc \((m,n) \in \mathcal{E} \) a binary branching is performed. In the first branch, we remove arc \((m,n)\) from \( \mathcal{E}_i \) for any subproblem \( i \) to be solved.

**Algorithm 3.1: Column generation procedure**

**Step 1.** Construct the reduced graph and eliminate unnecessary arcs and nodes (see §3.3.3.1 and §3.3.3.2).

**Step 2.** Run LA to solve the ng-SPPRC by applying 2-cycle elimination (see §3.3.3.3). If any path with negative reduced cost is found, go to Step 5, otherwise go to Step 3.

**Step 3.** Construct the full graph and eliminate unnecessary arcs and nodes.

**Step 4.** Run LA to solve the ng-SPPRC by applying 2-cycle elimination (see §3.3.3.3). If any path with negative reduced cost is found, go to Step 5, otherwise go to Step 6.

**Step 5.** Add the generated columns to the RMP. If there are cycles in a column, add the required SDCs (see §3.3.3.3) to the RMP (if they are not added before). **Stop.**

**Step 6.** Run LA to solve the ESPPRC. **Stop.**
in that branch. In the other branch, we remove all outgoing arcs from node $m$ except arc $(m, n)$ as well as all incoming arcs to node $n$ except arc $(m, n)$ in order to force arc $(m, n)$ to appear in the solution of the corresponding subproblems. Note that branching on the arc-flow variables guarantees integer $\lambda_p$ values.

### 3.4 A Two-Stage Heuristic

We introduce a simple two-stage heuristic, called top-to-bottom heuristic (T-B), to solve medium and large size problems more efficiently. In the first stage, the problem FLPTW (3.1)–(3.7) is solved to find the strategic level (top) decisions. This problem considers direct shipments to the customers with CDC–customer–CDC routes satisfying customer and CDC time windows. In the second stage, we solve the problem for the tactical level (bottom) decisions. In this stage, we first fix the CDC locations in the master problem (3.20)–(3.28) based on the optimal locations of the FLPTW. Next, the LRPTW with fixed CDC locations is solved to obtain vehicle routes. This problem is a multi-depot VRPTW and can be solved by the proposed BP. To accelerate the heuristic at this stage, T-B only solves $ng$-SPPRC on the reduced-size network (see Section 3.3.3.1). Note that the time window and capacity constraints in

![Figure 3.3: Top-to-bottom approach for the LRPTW.](image-url)
Algorithm 3.2: Column generation procedure in T>B

**Step 1.** Construct the reduced graph and eliminate unnecessary arcs and nodes (see §3.3.3.1 and §3.3.3.2).

**Step 2.** Run LA to solve the $ng$-SPPRC by applying 2-cycle elimination (see §3.3.3.3).

**Step 3.** Add the generated columns with a negative reduced cost to the RMP. If there are cycles in a column, add the required SDCs (see §3.3.3.3) to the RMP (if they not added already). Stop.

the FLPTW ensure that a feasible route can always be generated in the subproblems corresponding to open CDCs.

The outline of T>B algorithm is shown in Figure 3.3. The procedure of generating a column in T>B is given in Algorithm 3.2, where the steps regarding construction of the full graph and solving the ESPPRC supproblem are eliminated from the CG procedure.

### 3.5 Computational Study

In this section, we implement the proposed BP and T>B on a set of LRPTW instances and present computational results. The purpose of the computational experiments is as follows: (i) assessment of the proposed algorithms on the LRPTW, (ii) testing the effect of problem instance characteristics (such as problem size and CDC/customer point distribution on the plane) on the performance of the algorithms, and (iii) examining the effect of acceleration and enhancement techniques (see Section 3.3.3) on generating elementary routes. To this end, a set of problem test instances are taken from the literature. Since the available problem instances in the literature are very limited, we generate new test instances with different characteristics. Section 3.5.1 explains the problem instance characteristics and how test instances are generated for our experiments. Main experimental results are provided in Section 3.5.3. In this section, the numerical results of BP are reported in details, the performance of T>B is analyzed, and an upper-bounding scheme is proposed.
3.5.1 Test Instances

We use four sets of instances, which are all based on R1 and C1 type Solomon’s VRPTW benchmark instances introduced in Solomon (1987). There are twelve R1 type instances with randomly distributed customers and nine C1 type instances with clustered customers. We indicate instance sizes by \#_1-\#_2 notation, where \#_1 and \#_2 indicate the number of candidate CDC locations and the number of customers, respectively. The first set contains R1 type instances used by Ponboon et al. (2016). The largest instance size in this set is 3-40. The second set includes R1 instances with 50 customers and 2, 3, 4, and 5 candidate CDC locations. Since the VRPTW instances only include one depot location, we use \(k\)-medoids approach of Park & Jun (2009) to place candidate CDC locations as follows. First, 5 clusters of R1 customers are found. Then, the centroid of each cluster is selected as a candidate CDC location for the 5-50 instances (similar strategy is used by Ponboon et al. (2016)). For \#-50 instances with \# = 2, 3, or 4, we arbitrarily remove 3, 2, and 1 CDC(s), respectively, from the corresponding 5-50 instances.

Set 3 and Set 4 contain C1 type instances with 25 and 50 clustered customers and 3 or 5 candidate CDC locations. We select the cluster centroids as the candidate CDC locations in Set 3. To generate Set 4, the candidate CDC locations are selected on the midpoint of the line connecting the center of all points and the cluster centers. Similar to Set 2, the CDC locations of 3-50 instances are obtained by removing two CDCs in the corresponding 5-50 instances. We assume that the generated CDC locations have the same time windows as the original depot in the VRPTW instance. Distribution of the points in Set 2, Set 3, and Set 4 instances are illustrated in Figure 3.4, Figure 3.5, and Figure 3.6, respectively. In total, we generate and solve 138 problem test instances. All instance files are available in https://gitlab.com/pharham/test-instances.

3.5.2 Implementation Details

The proposed BP is coded using SCIP optimization suite v3.2 (Gamrath et al. 2016) linked with ILOG CPLEX v12.6.3 as the MIP solver. The experiments are done on
a Linux workstation with Intel® Xeon 4 × 3.20GHz processors and 16GB memory. In order to find $\bar{Z}$ and $\bar{V}$ values defined in Section 3.2.2.1, one can set $\bar{Z}$ equal to $\lceil \sum_{k \in K} D_k / \max_{j \in J} Q_j \rceil$ and $\bar{V}$ equal to $\lceil \sum_{k \in K} D_k / Q'' \rceil$, where $\lceil \cdot \rceil$ is the ceiling function. We set $\Phi_1$ parameter to 5 (see Section 3.3.3.1) and $\Phi_2$ parameter to the smallest integer greater or equal to $0.1 \times |K|$ (see Section 3.3.3.3) as these values produced acceptable results in our preliminary experiments. Time limit ($TL$) is set to six hours.

### 3.5.3 Experimental Results

Before solving the LRPTW instances, we solve a few benchmark LRP instances from the literature in order to validate the proposed BP. We use three LRP instances available in Barreto et al. (2007) with a similar size as the medium size instances in Set 3 and Set 4. Since the original instances do not consider time windows, we set time-windows with nonrestrictive lengths (starting from time 0 to infinity) for CDCs and customers. The selected instances show a good trade-off between instance size and the algorithm efficiency.

The results are provided in Table 3.1. Here, $Inst$ refers to the name of the instance we solved. $O^*$ is the optimal objective function value. $CDC$, $Veh$ indicate the number of open CDCs and the total number of vehicles used in the optimal solution. $PCall$ shows the number of times a pricing problem is called to generate columns in BP. $DBound$ is the LP solution in the root node (i.e. first dual bound) of the branch-and-bound tree. $BB$ nodes is the total number of branching nodes explored in this tree. $Time$ corresponds to computational time in the format of (MM:SS) or (H:MM:SS).

All solutions given in Table 3.1 are identical to those reported in the literature (see Table 3.1: Numerical results for the LRP instances with infinite-length time windows

<table>
<thead>
<tr>
<th>Inst</th>
<th>Size</th>
<th>$O^*$</th>
<th>CDC, Veh</th>
<th>PCall</th>
<th>$DBound$</th>
<th>BB nodes</th>
<th>Time</th>
</tr>
</thead>
<tbody>
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<td>5-21</td>
<td>424.90</td>
<td>2, 4</td>
<td>91</td>
<td>405.17</td>
<td>19</td>
<td>0:31</td>
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<tr>
<td>Gaskell67-22x5</td>
<td>5-22</td>
<td>585.11</td>
<td>1, 3</td>
<td>1050</td>
<td>539.28</td>
<td>81</td>
<td>16:18</td>
</tr>
<tr>
<td>Min92-27x5</td>
<td>5-27</td>
<td>3062.02</td>
<td>2, 4</td>
<td>136</td>
<td>2685.01</td>
<td>15</td>
<td>10:21</td>
</tr>
</tbody>
</table>
Belenguer et al. 2011). In the following sections, we present the results of the algorithms on the LRPTW instances of Ponboon et al. (2016) as well as newly generated ones.

3.5.3.1 Small and Medium Size Test Instances

The small instances are the ones with 10 or 25 customers, i.e. R1 type instances with size 3-10 and 3-25 in Set 1 and C1 type instances with size 3-25 in Set 3 and Set 4. The computational results of BP are provided in Table 3.2 (1\textsuperscript{st} and 2\textsuperscript{nd} panel), Table 3.4 (1\textsuperscript{st} panel), and Table 3.5 (1\textsuperscript{st} panel). The algorithm solved all small instances to optimality in short time. The longest computational time is 16 minutes for R108 (size 3-25), which is a difficult instance due to its very wide customer time windows.

Medium size instances include 40 customers. R1 type instances with size 3-40 are considered to be in this group. As the number of customers increases from 25 to 40, the problem becomes very difficult to solve. *Time* of the instances which could not be solved within time limit is denoted as TL. The percentage of relative MIP gaps are provided under column \%Gap. 7 out of 12 instances listed in Table 3.2 (3\textsuperscript{rd} panel), could not be solved to optimality in 6 hours. For three of these instances, i.e. R104, R108, and R111, the algorithm failed to find a feasible integer solution within the allowable time. In these cases, we solve the problems by providing initial columns that are the final integer solutions found for the instance with tighter time windows. Therefore, the vehicle routes obtained by solving R103, R107, and R110 are provided as the initial columns to solve R104, R108, and R111 instances, respectively. The original instances are generated in a way that the only differences between the pairs of instances mentioned above are the time windows. Customers in R103 instance have time windows that are sub-intervals of those in R104 instance and so on (see Solomon 1987, for more information about the problem instances). Therefore, the provided initial solutions are always feasible for the new problem instance. In this way, we were able to find a near optimal solution for R104 and a feasible solution for R108 and R111 instances (indicated by symbol † in Table 3.2). The resulted gaps are all below 7\% for the small and medium size instances.

Ponboon et al. (2016) also solved the small size instances and three medium size
instances (namely R101, R102, and R105) in Set 1 and reported the results. For some instances, marked by a and b superscripts in Table 3.2, we found different solution than theirs. We believe that the differences are due to precision settings or some instance characteristics Ponboon et al. (2016).

3.5.3.2 Large Size Test Instances

Problem instances with 50 customers (i.e., R1 instances in Set 2 and C1 instances with size #-50 in Set 3 and Set 4) are large size instances. The numerical results for these instances are given in Table 3.3 and Table 3.4 to Table 3.5 (2nd and 3rd panel). When the number of candidate CDC location increases, the number of subproblems increases, hence the problems become more challenging. The algorithm could not find a feasible solution to R108 (size 2-50 and larger) and R104 and R107 with size 5-50. Therefore, as for medium size instances, we solve these problems using initial columns which are the final solutions found for an instance with tighter time windows. These instances are indicated by † in Table 3.3. Among these six instances, we were able to obtain an upper bound for R108 with size 2-50 and R107 with size 5-50. The largest gap for R1 type large instances is around 5%.

All C1 type instances with size #-50 are either solved to optimality or an upper bound is found in the available time. The largest gap observed for these instances is higher than R1 type instances with 50 customers. The algorithm spends more time in the root node for solving R1 instances compared to C1 ones. BP yields small average gap value for the instances that are not solved during time limit. The algorithm finds an integer solution faster but converges slower when solving C1 type instances. For some cases, the optimal solution is found at the first node of the tree.

Figure 3.4 illustrates distribution of points and the solution to problem instance R102 with 50 customers and 2 to 5 CDC locations from Set 2. Optimal solutions to problem instance C101 with 50 customers and 3 and 5 CDCs from Set 3 and Set 4 are illustrated in Figure 3.5 and Figure Figure 3.6, respectively.
Table 3.2: Numerical results for small and medium size R1 type instances in Set 1.

<table>
<thead>
<tr>
<th>Inst</th>
<th>Size</th>
<th>$O^*$</th>
<th>CDC, Veh</th>
<th>%Gap</th>
<th>PCall</th>
<th>DBound</th>
<th>BB ndoes</th>
<th>Time</th>
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<td>2358.00</td>
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<td>2358.00</td>
<td>5</td>
<td>0:01</td>
</tr>
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<td>2137.60</td>
<td>3</td>
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<td>1, 2</td>
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<td>2144.00</td>
<td>3</td>
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(continued)
Table 3.2: (Continued.)

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<th>%Gap</th>
<th>PCall</th>
<th>DBound</th>
<th>BB ndoes</th>
<th>Time</th>
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<td>153  TL</td>
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</tbody>
</table>

\(a\) The \(O^*\) values reported in Ponboon et al. (2016) for these instances are 2691, 2434, 2434, 2162, 2438, 2194, 2194, 2162, 2429, 2184, and 2159, in order.

\(b\) The \(O^*\) values reported in Ponboon et al. (2016) for these instances are 4587, 4438, 4077, and 4250, in order.

\(c\) The \(O^*\) value reported in Ponboon et al. (2016) for these instances are 7150 and 6919, in order.

\(\dagger\) The results are obtained when initial columns are provided.

Table 3.3: Numerical results for large size R1 type instances in Set 2.

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† The results are obtained when initial columns are provided.
‡ Starting with initial columns, the following instances cannot be solved during the available time: R108 (size 3-50 and larger), R104 (size 5-50).

Table 3.4: Numerical results for small and large size C1 type instances in Set 3.

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### Table 3.5: Numerical results for small and large size C1 type instances in Set 4.

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<td>C102</td>
<td>3-50</td>
<td>5879</td>
<td>3, 5</td>
<td>0.00</td>
<td>151</td>
<td>5877.94</td>
<td>5</td>
<td>5:20</td>
</tr>
<tr>
<td>C103</td>
<td>3-50</td>
<td>5870</td>
<td>3, 5</td>
<td>0.04</td>
<td>225</td>
<td>5865.60</td>
<td>11</td>
<td>TL</td>
</tr>
<tr>
<td>C104</td>
<td>3-50</td>
<td>5885</td>
<td>3, 5</td>
<td>0.00</td>
<td>29</td>
<td>5885.00</td>
<td>1</td>
<td>0:12</td>
</tr>
<tr>
<td>C105</td>
<td>3-50</td>
<td>5885</td>
<td>3, 5</td>
<td>0.00</td>
<td>23</td>
<td>5885.00</td>
<td>1</td>
<td>0:06</td>
</tr>
<tr>
<td>C106</td>
<td>3-50</td>
<td>5884</td>
<td>3, 5</td>
<td>0.00</td>
<td>51</td>
<td>5884.00</td>
<td>1</td>
<td>0:26</td>
</tr>
<tr>
<td>C107</td>
<td>3-50</td>
<td>5884</td>
<td>3, 5</td>
<td>0.00</td>
<td>63</td>
<td>5884.00</td>
<td>1</td>
<td>1:04</td>
</tr>
<tr>
<td>C108</td>
<td>5-50</td>
<td>7600</td>
<td>4, 6</td>
<td>0.00</td>
<td>1886</td>
<td>7352.00</td>
<td>204</td>
<td>40:46</td>
</tr>
<tr>
<td>C109</td>
<td>5-50</td>
<td>7596</td>
<td>4, 6</td>
<td>0.00</td>
<td>4238</td>
<td>7348.00</td>
<td>376</td>
<td>4:26:33</td>
</tr>
<tr>
<td>C110</td>
<td>5-50</td>
<td>8842</td>
<td>5, 5</td>
<td>20.36</td>
<td>497</td>
<td>7342.00</td>
<td>16</td>
<td>TL</td>
</tr>
<tr>
<td>C111</td>
<td>5-50</td>
<td>8835</td>
<td>5, 5</td>
<td>20.45</td>
<td>155</td>
<td>7335.00</td>
<td>2</td>
<td>TL</td>
</tr>
<tr>
<td>C112</td>
<td>5-50</td>
<td>7600</td>
<td>4, 6</td>
<td>0.00</td>
<td>2141</td>
<td>7352.00</td>
<td>235</td>
<td>53:29</td>
</tr>
<tr>
<td>C113</td>
<td>5-50</td>
<td>7600</td>
<td>4, 6</td>
<td>0.00</td>
<td>2532</td>
<td>7352.00</td>
<td>282</td>
<td>57:42</td>
</tr>
<tr>
<td>C114</td>
<td>5-50</td>
<td>7600</td>
<td>4, 6</td>
<td>0.00</td>
<td>5330</td>
<td>7352.00</td>
<td>741</td>
<td>2:56:04</td>
</tr>
<tr>
<td>C115</td>
<td>5-50</td>
<td>7598</td>
<td>4, 6</td>
<td>0.00</td>
<td>9005</td>
<td>7349.00</td>
<td>1076</td>
<td>4:38:18</td>
</tr>
<tr>
<td>C116</td>
<td>5-50</td>
<td>7624</td>
<td>4, 6</td>
<td>3.47</td>
<td>1294</td>
<td>7348.00</td>
<td>99</td>
<td>TL</td>
</tr>
</tbody>
</table>

3.5.3.3 Overall Results and Observations

We summarize the performance of BP in Table 3.6 showing the total number of instances (#Inst) for each size, the instances which could be solved to optimality in available time (#Opt), the number of instances for which a feasible integer solution is obtained (#Subopt), the average percentage of gap for such instances (Av %gap), and the number of instances for which no feasible solution is found (#Unsolved). This approach is able to solve 72% of the instances optimally, and report an average MIP gap of 4.73% for the ones that are not solved in the available time.

Instances with tight time windows are solved faster than those with wider time windows. Tighter time windows impose more restrictions in LA. Therefore, less number
of paths are extended, which results in less number of labels. More vehicles are used when the time windows are tight. In other words, vehicle load factor is lower in this case compared to the instances with wider time windows, which allow more space to design vehicle routes. No significant difference in terms of objective function value or computational time is observed between Set 3 and Set 4 instances. On average, more routing cost is incurred when the CDCs are located toward the middle of the instance (as in Set 4) compared to the case when they are located in the center of customer clusters (as in Set 3).

![Diagram](attachment:image.png)

Figure 3.4: Illustration of the solution for the problem instance R102 with 50 customers (Set 2). □ shows unused CDCs.
Figure 3.5: Illustration of the solution for the problem instance C101 with 50 customers (Set 3).

Figure 3.6: Illustration of the solution for the problem instance C101 with 50 customers (Set 4).
Table 3.6: Summary of the BP performance.

<table>
<thead>
<tr>
<th>Type</th>
<th>Size (#Inst)</th>
<th>#Opt</th>
<th>#Subopt</th>
<th>Av %gap</th>
<th>#Unsolved</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>Small (24)</td>
<td>24</td>
<td>0</td>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td>R1</td>
<td>Medium (12)</td>
<td>5</td>
<td>7</td>
<td>2.09</td>
<td>0</td>
</tr>
<tr>
<td>R1</td>
<td>Large (48)</td>
<td>28</td>
<td>16</td>
<td>2.23</td>
<td>4</td>
</tr>
<tr>
<td>C1</td>
<td>Small (18)</td>
<td>18</td>
<td>0</td>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td>C1</td>
<td>Large (36)</td>
<td>24</td>
<td>12</td>
<td>9.62</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3.7: Comparison of the exact approaches.

<table>
<thead>
<tr>
<th>Size (#Inst)</th>
<th>Av BB ndoes</th>
<th>Av Time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BP</td>
<td>PonBP</td>
</tr>
<tr>
<td>3-10 (12)</td>
<td>9.67</td>
<td>14.33</td>
</tr>
<tr>
<td>3-25 (12)</td>
<td>26.42</td>
<td>123.67</td>
</tr>
<tr>
<td>3-40 (3)</td>
<td>267.33</td>
<td>3194.67</td>
</tr>
</tbody>
</table>

Table 3.8: Performance of different CG frameworks in solving R102 with size 4-50 (Set 2).

<table>
<thead>
<tr>
<th>Method</th>
<th>Runs</th>
<th>SuccRun</th>
<th>Col</th>
<th>Time (s)</th>
<th>Av Col/Run</th>
<th>Av Time/Run</th>
</tr>
</thead>
<tbody>
<tr>
<td>ng-SPPRC-red</td>
<td>3400</td>
<td>727</td>
<td>6252</td>
<td>46.30</td>
<td>1.84</td>
<td>0.01</td>
</tr>
<tr>
<td>ng-SPPRC</td>
<td>2673</td>
<td>978</td>
<td>4299</td>
<td>367.38</td>
<td>1.61</td>
<td>0.14</td>
</tr>
<tr>
<td>ESPPRC</td>
<td>1695</td>
<td>251</td>
<td>668</td>
<td>306.73</td>
<td>0.39</td>
<td>0.18</td>
</tr>
</tbody>
</table>

We compare the performance of our BP to the exact approach of Ponboon et al. (2016) over the R1 type instances they solved. Ponboon et al. (2016) ran their algorithm (which we call PonBP) on a computer with Intel® Core i7 3.20GHz processor and 32GB RAM with Gurobi Optimizer 5.6 as the LP solver. Table 3.7 shows the comparison of BP and PonBP in terms of the size of branch-and-bound tree and computational time. Av BB ndoes and Av Time respectively indicate the average number of nodes of the resulting tree and the average computational time over the instances in each size group. Comparing to PonBP, the proposed BP explores less branch-and-bound nodes and finds the integer solution faster. This shows the efficiency of BP and
the positive effect of using enhancement techniques and branching rules introduced in Sections 3.3.

In order to demonstrate the performance of the proposed CG framework, we provide the numerical results obtained for each of the steps while solving problem instance R102 with size 4-50 in Table 3.8. \textit{ng-SPPRC-red} correspond to the case when \textit{ng-SPPRC} is applied on the reduced size graph (see Section 3.3.3.1). \textit{Runs} shows the total number of times the method is applied. A method may or may not find an improving column in its current run. \textit{SuccRun} shows the total number of runs in which the method was successful to found at least one improving column to be added to the master problem. For each method, \textit{Col} is the total columns added to the master problem. \textit{Av Col/Run} and \textit{Av Time/Run} show the average number of columns the methods have found per run and the computational time spent in a run, respectively. The results are summarized in Figure 3.7 and Figure 3.8. For this particular instance, more than 55\% of the total columns are found by \textit{ng-SPPRC-red} which only took 6\% of total time spent in solving the pricing problem, whereas the exact method consumed 43\% of the time to produce 6\% of the total columns (see Figure 3.7). By applying \textit{ng-SPPRC-red} and \textit{ng-SPPRC}, we prevent solving the ESPPRC in every pricing iteration. Since each ESPPRC run consumes a time which is approximately 14 times more than \textit{ng-SPPRC-red} and 1.3 times more than \textit{ng-SPPRC} (see Figure 3.8), this approach can save a lot of time while solving the subproblem.

![Figure 3.7: Comparisons of the total number of routes found and the total computational time spent by the proposed methods for the problem instance R102 with size 4-50.](image-url)
Figure 3.8: Comparisons of average number of routes found and average computational time per route by the proposed methods for the problem instance R102 with size 4-50.

3.5.3.4 Results of the Heuristic Approach

T>B is implemented on all problem instances with 25 or more customers. The algorithm is terminated whenever the relative MIP gap falls below a certain value. Our preliminary experiments show that setting this value to 0.05% gives a good trade of between solution quality and computational time.

Performance of T>B is compared to BP by calculating the relative gap percent of the objective function value ($O$), number of open CDCs, number of vehicle routes, number of times pricing problems are solved, and the size of the branch-and-bound tree, as reported Table 3.9. The average percentage of relative gap values over the instances sizes are provided in Table 3.9. The relative gap percent values are calculated as $100 \times (\text{heuristic value} - \text{best value}) / \text{best value}$, where best value refers to the corresponding value reported in Table 3.2 to Table 3.5. Time Imp(%) shows the average percentage improvement on the computational times obtained by using T>B over BP.

Compared to BP, T>B explores more nodes of branch-and-bound tree by solving the subproblem in much shorter time. On average, T>B saved almost 80% of the time in the expense of only 0.47% increase in the objective function value.
### Table 3.9: Numerical results of T>B.

<table>
<thead>
<tr>
<th>Set</th>
<th>Size (#Inst)</th>
<th>O gap(%)</th>
<th>CDC, Veh gap(%)</th>
<th>PCall gap(%)</th>
<th>BB ndoes gap(%)</th>
<th>Time Imp(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3-25 (12)</td>
<td>0.12</td>
<td>0.00, 0.00</td>
<td>−72.51</td>
<td>−61.45</td>
<td>83.12</td>
</tr>
<tr>
<td></td>
<td>3-40 (12)</td>
<td>−0.03</td>
<td>0.00, 1.67</td>
<td>43.04</td>
<td>157.64</td>
<td>72.65</td>
</tr>
<tr>
<td>2</td>
<td>2-50 (12)</td>
<td>0.35</td>
<td>0.00, 0.00</td>
<td>−8.53</td>
<td>91.31</td>
<td>82.61</td>
</tr>
<tr>
<td></td>
<td>3-50 (12)</td>
<td>0.15</td>
<td>0.00, 0.00</td>
<td>−15.70</td>
<td>127.67</td>
<td>93.04</td>
</tr>
<tr>
<td></td>
<td>4-50 (12)</td>
<td>0.09</td>
<td>0.00, 0.00</td>
<td>80.00</td>
<td>369.71</td>
<td>88.87</td>
</tr>
<tr>
<td></td>
<td>5-50 (12)</td>
<td>−0.15</td>
<td>0.00, −1.43</td>
<td>221.57</td>
<td>241.17</td>
<td>83.08</td>
</tr>
<tr>
<td>3</td>
<td>3-25 (9)</td>
<td>0.04</td>
<td>0.00, 0.00</td>
<td>−75.85</td>
<td>−83.81</td>
<td>79.51</td>
</tr>
<tr>
<td></td>
<td>3-50 (9)</td>
<td>0.04</td>
<td>0.00, 0.00</td>
<td>−42.23</td>
<td>−20.83</td>
<td>80.99</td>
</tr>
<tr>
<td></td>
<td>5-50 (9)</td>
<td>−4.30</td>
<td>−6.67, 6.67</td>
<td>1662.88</td>
<td>12931.56</td>
<td>50.90</td>
</tr>
<tr>
<td>4</td>
<td>3-25 (9)</td>
<td>0.10</td>
<td>0.00, 0.00</td>
<td>−53.08</td>
<td>−63.06</td>
<td>72.60</td>
</tr>
<tr>
<td></td>
<td>3-50 (9)</td>
<td>0.04</td>
<td>0.00, 0.00</td>
<td>−30.67</td>
<td>−18.99</td>
<td>73.42</td>
</tr>
<tr>
<td></td>
<td>5-50 (9)</td>
<td>−3.04</td>
<td>−4.44, 4.44</td>
<td>271.96</td>
<td>4006.76</td>
<td>89.38</td>
</tr>
<tr>
<td>Average</td>
<td>−0.47</td>
<td>−0.82, 0.87</td>
<td>148.07</td>
<td>1318.83</td>
<td>79.62</td>
<td></td>
</tr>
</tbody>
</table>

### 3.5.3.5 Upper-Bounding Effect

Numerical experiments in Section 3.5.3.4 show that high quality heuristic solutions can be obtained in relatively short computational times. Therefore, we can use the T>B solution as an upper bound for BP. Since BP already solves small problems efficiently, we only test upper-bounding approach (U-BP) for medium and large size problem instances. Table 3.10 presents numerical results of this experiment. \#Opt is the number of problem instances solved to optimally by U-BP. \#New is the number of new instances solved to optimality by U-BP. O gap(%) shows the average relative gap between objective functions of BP and U-BP calculated as $100 \times \left( \frac{O^{U-BP} - O^{BP}}{O^{U-BP}} \right)$ where $O^{U-BP}$ ($O^{BP}$) corresponds to the objective function value obtained by implementing U-BP (BP). O gap(%) is only calculated for the instances for which the two methods find different solutions due to time limit. Time Imp(%) shows the percentage improvement in computational time obtained by using the U-BP. Negative values indicate that on average, providing upper bound resulted in longer computational time.
According to Table 3.10, the positive effect of providing upper bound on the final solution is more significant for large size C1 type instances comparing with large size R1 type instances. Upper-bounding also saves computational time for solving medium size R1 type instances.

Table 3.10: Summary of the BP performance with upper-bounding.

<table>
<thead>
<tr>
<th>Type</th>
<th>Size (#Inst)</th>
<th>#Opt (#New)</th>
<th>#Subopt</th>
<th>Av gap(%)</th>
<th>O gap(%)</th>
<th>Time Imp(%)</th>
<th>#Unsolved</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>Medium (12)</td>
<td>7 (2)</td>
<td>5</td>
<td>1.61</td>
<td>−0.59</td>
<td>21.35</td>
<td>0</td>
</tr>
<tr>
<td>R1</td>
<td>Large (48)</td>
<td>29 (1)</td>
<td>19†</td>
<td>2.02</td>
<td>−0.39</td>
<td>−3.67</td>
<td>0</td>
</tr>
<tr>
<td>C1</td>
<td>Large (36)</td>
<td>26 (2)</td>
<td>9</td>
<td>3.04</td>
<td>−6.32</td>
<td>−50.85</td>
<td>1†</td>
</tr>
</tbody>
</table>

† Starting with upper bound, the algorithm hits TL while solving problem instance R111 (size 2-50).
‡ Starting with upper bound, the algorithm cannot solve problem instance C104 (size 3-50) from Set 4. Therefore, it is omitted in calculating average values in the table.

Table 3.11: New optimal LRPTW solutions and solutions to unsolved instances found by U-BP.

<table>
<thead>
<tr>
<th>Inst (set)</th>
<th>Size</th>
<th>O⁺</th>
<th>CDC, Veh</th>
<th>%Gap</th>
<th>PCall</th>
<th>DBound</th>
<th>BB ndoes</th>
<th>Time (T&gt;B+BP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R106 (1)</td>
<td>3-40</td>
<td>6552</td>
<td>3, 6</td>
<td>0.00</td>
<td>4556</td>
<td>6464.4</td>
<td>942</td>
<td>7:58+1:08:13</td>
</tr>
<tr>
<td>R110 (1)</td>
<td>3-40</td>
<td>6380</td>
<td>3, 6</td>
<td>0.00</td>
<td>14907</td>
<td>6205.99</td>
<td>4103</td>
<td>1:17:46+4:08:00</td>
</tr>
<tr>
<td>R108 (2)</td>
<td>3-50</td>
<td>4926</td>
<td>2, 6</td>
<td>4.63</td>
<td>382</td>
<td>4646.76</td>
<td>21</td>
<td>19:16+TL</td>
</tr>
<tr>
<td>R108 (2)</td>
<td>4-50</td>
<td>6247</td>
<td>2, 6</td>
<td>1.83</td>
<td>194</td>
<td>6129.05</td>
<td>19</td>
<td>TL+TL</td>
</tr>
<tr>
<td>R104 (2)</td>
<td>5-50</td>
<td>7873</td>
<td>2, 6</td>
<td>2.35</td>
<td>215</td>
<td>7690.6</td>
<td>20</td>
<td>26:10+TL</td>
</tr>
<tr>
<td>R108 (2)</td>
<td>5-50</td>
<td>7931</td>
<td>2, 6</td>
<td>4.14</td>
<td>118</td>
<td>7610.35</td>
<td>11</td>
<td>TL+TL</td>
</tr>
<tr>
<td>C105 (3)</td>
<td>5-50</td>
<td>7589</td>
<td>4, 6</td>
<td>0.00</td>
<td>4867</td>
<td>7303</td>
<td>745</td>
<td>18:07+1:32:08</td>
</tr>
<tr>
<td>C107 (3)</td>
<td>5-50</td>
<td>7589</td>
<td>4, 6</td>
<td>0.00</td>
<td>6294</td>
<td>7303</td>
<td>875</td>
<td>25:35+2:54:52</td>
</tr>
</tbody>
</table>

There are rare cases in which U-BP consumed more time than BP to find the optimal solution (in 5 out of 92 instances) or returned a slightly inferior solution when both reach TL (in 8 out of 92 instances). The reason is that providing an upper bound changes the way that branch-and-bound tree is explored. Overall, the upper-bounding techniques helped us to find more optimal solution and reduce the average optimality gap of the unsolved instances to 1.96%. In Table 3.11 we list numerical results for instances for which BP failed to find a solution, but U-BP returned either the optimal
or a feasible solution. Under Time, we provide computational time needed by T>B to find a solution used for upper bound plus the time spent by the U-BP to solve the problem.

3.6 Concluding Remarks

The capacitated location-routing problem with time windows addresses the core decisions in 1E urban freight distribution systems. In this chapter, we present an exact as well as a heuristic solution approach for this problem. The exact approach implements column generation for the path-based formulation of the LRPTW. The column generation framework is enhanced by using and modifying several techniques in the literature, such as reduced size graph or non-elementary path generations. Optimal solutions for large size instances with up to 5 candidate facility locations and 50 customers are found for the first time in the literature. The heuristic method decomposes the problem based on its strategic and tactical level decisions. It shows a promising performance by producing good quality solutions in limited times. For larger problem instances or the ones with wide customer time windows that are difficult to solve, the proposed heuristic method can provide an upper-bounding for the exact algorithm and improve its performance. We believe that the proposed modeling framework and solution approaches can be effectively used in more complex CL problems as well. In Chapter 4, we study the two-echelon urban freight transportation problem.
CHAPTER 4

TWO–ECHELON FREIGHT DISTRIBUTION SYSTEMS IN CITY LOGISTICS

The two-echelon city logistics systems consist of delivering freight from city distribution centers (CDCs) to intermediate facilities, called satellites, in large batches, which builds up the first echelon. In the second echelon, goods are consolidated into smaller vehicles to be delivered to the customers inside the city. In this study, we consider the two-echelon facility location and last echelon routing problem with time windows to address the strategic and tactical decisions of the two echelon systems. Given a set of candidate CDC locations, a set of candidate satellite locations, and a set of customers, the problem seeks the minimum total transportation cost consisting of CDC and satellite opening costs as well as first and second echelon vehicle routing costs such that all customer demands are satisfied. The problem is constrained by CDC, satellite, and vehicle capacities as well as customer time windows. We provide a path-based formulation for the problem and propose an exact solution approach based on branch-and-price. To tackle larger problems, we also develop two heuristics inspired by the hierarchical structure of the problem. The heuristics benefit from a novel constrained clustering approach for generating feasible routes considering time and capacity limitations. A comprehensive computational study is conducted to assess the performance of the exact and heuristic approaches on solving a large set of problem instances with different sizes and characteristics.
4.1 Introduction

In the single-echelon urban transportation systems, freight is delivered from the selected city distribution centers (CDCs) to the customers without any intermediate activities. Such systems are used in small cities with limited number of carriers and shippers (Crainic et al. 2009). Taniguchi & Thompson (2002) and Crainic et al. (2004) introduce two-echelon (2E) distribution systems as a solution to simultaneously reduce pollution, traffic congestion and operating cost of the freight transportation in large cities. In the first echelon, freight is delivered from CDCs, located on the outskirts of the city, to the intermediate facilities called satellites. Satellites are small inner-city locations where no inventory or staging is possible. In the second echelon, goods are sorted and consolidated into environmental-friendly vehicles for the last-mile delivery. Vehicles in the second echelon start their route from a satellite location, visit a set of customers, and end their trip by returning to the same satellite location. To reduce the negative effect of the transportation on citizens’ quality of life, Local authorities usually impose restrictive regulations on customer access times or on the weight of delivery vehicles.

In this chapter, we consider the two-echelon location with last echelon routing problem with capacity and time windows constraints (2E-LR2PTW) as a core problem arising in designing and planning urban freight transportation systems for large cities. We assume that the first echelon vehicles perform direct shipments from open CDCs to selected satellites forming CDC–satellite–CDC routes. This is a valid assumption in urban areas where the first-echelon network consists of one or two CDCs, a limited number of satellites, and high capacity roads far from city centers (e.g. ring roads). While considering the strategic-level decisions in the first echelon, the introduced 2E-LR2PTW incorporates the tactical/operational level planning in the second echelon where high concerns about transportation cost, time window feasibility, vehicle utilization, and environmental impacts exist. The second echelon consists of satellites and customers nodes. The 2E-LR2PTW aims to minimize the total transportation cost consisting of facility opening, vehicle utilization, and vehicle traveling costs such that all customer demands are satisfied. It decides on the number and location of CDC and satellite platforms, the number of vehicles used in each echelon, and the vehicle routes.
and schedules. The problem is defined under facility and vehicle capacity constraints as well as hard time windows, where serving a customer is only possible during a specific time interval.

Although access time window and vehicle capacity constraints are important in planning urban freight transportation, there is no study on formulating and solving the 2E-LR2PTW to optimality. This study is the first that presents a path-based MIP formulation of the 2E-LR2PTW and develops an exact method to solve the problem. The exact approach is based on the branch-and-price (BP) algorithm that is one of the most successful solution approaches for the constrained routing problems in the literature (see Baldacci et al. 2012, Dabia et al. 2013, Contardo et al. 2015, Pecin et al. 2017). BP decomposes the original problem into two: the master problem and the subproblem. The master problem of consists of the first-echelon decisions, i.e. facility location and CDC-to-satellite vehicle routes, as well as routing decision on the second echelon routes. In order to generate candidate routes in the second echelon, a number of subproblems is solved. A subproblem corresponds to a constrained shortest path problem which is \( NP \)-hard. Different enhancement techniques are proposed to improve the overall performance of the proposed BP.

In order to find the solution of the large-size 2E-LR2PTW instances, we propose two heuristics. Both heuristics benefit from decomposing the problem based on its strategic and tactical level decisions. The first heuristic, called top-to-bottom approach, solves an optimization problem to determine the strategic-level decisions first, and then executes the proposed BP on the reduced problem to find the vehicle routes in both echelons. The second heuristic, on the other hand, starts by determining the domain of the complicated tactical/operational-level decisions, and fixes the remaining decisions later. In this heuristic, called bottom-to-top approach, we design and implement a novel constrained clustering technique to group the customers that a second echelon vehicle might visit. We form a one-to-one relation between a cluster and a feasible route. Therefore, time window and capacity restrictions are satisfied while shaping the clusters. Once candidate second echelon routes are generated, a mixed-integer linear program is solved to determine facility locations and vehicle routes.

We conduct extensive computational experiments to assess the performance of the
proposed solution approaches and analyze the effect of different instance characteristics on the solution of the 2E-LR2PTW. The numerical results indicate that the proposed heuristics save a significant amount of time to solve the problem instances without sacrificing much of the solution quality.

The reminder of this chapter is organized as follows. Section 4.2 presents mathematical formulation of the problem. In Section 4.3, we introduce the exact approach to solve the formulated 2E-LR2PTW. We propose our heuristic solution algorithms in Section 4.4. Problem test instances and the computational study of the proposed solution approaches are provided in Section 4.5. We conclude the chapter in Section 4.6.

4.2 Problem Formulation

In Appendix A, we formulated the 2E facility location problem with time windows (2E-FLPTW). The 2E-FLPTW focuses on the strategic level decisions of the system and is used as a basis for the 2E-LR2PTW. In addition to the decisions of the 2E-FLPTW (see Section A.2), the 2E-LR2PTW covers tactical level decisions about the vehicle routes and schedules in the second echelon. Figure 4.1 illustrates 2E-FLP and 2E-LRP as the 2E freight distribution systems.

The underlying transportation network consists of three sets of nodes: set $\mathcal{I}$ indicating candidate CDC locations, set $\mathcal{J}$ consisting of candidate satellite locations, and set $\mathcal{K}$ denoting customer nodes. Let $\mathcal{M} = \mathcal{I} \cup \mathcal{J}$ and $\mathcal{N} = \mathcal{J} \cup \mathcal{K}$ be the set of first and second echelon nodes, respectively. Each customer $k \in \mathcal{K}$ is characterized by a demand $D_k$, a time window $[A_k, B_k]$, and a nonnegative service time. No time window is considered for CDCs, but a satellite $j \in \mathcal{J}$ can only be accessed during time interval $[0, B_j]$. We assume that secondary vehicles are available at satellite locations at time 0. If a vehicle arrives to a customer location earlier than the time window, it should wait until the time window starts. A facility $m \in \mathcal{M}$ has opening fixed cost $F_m$ and capacity $Q_m$. A fixed cost $F'$ ($F''$) and a capacity $Q'$ ($Q''$) are associated to each first echelon (second echelon) vehicle. We assume that a customer is served by exactly one satellite. However, multiple CDCs can ship freight to one satellite location. For any two nodes $m, n$ of the same echelon, define $C_{mn}$ as the
Figure 4.1: Two-echelon freight distribution systems.

cost of traveling on arc \((m, n)\). The value of \(C_{mn}\) depends on the distance, time, or energy consumption of reaching node \(n\) from node \(m\), as well as the type of vehicle in use. Let \(T_{mn}\) be the sum of setup or service time at node \(m\) and traveling time on arc \((m, n)\). We assume (i) a single commodity in the system, (ii) unsplittable customer demands that are all less than or equal to \(Q''\), (iii) no direct service from a CDC to customers, and (iv) unrestricted number of homogeneous vehicles in each echelon. Traveling times and costs can be asymmetric, but both satisfy triangle inequality. Let \(C'_{ij} = F' + C_{ij} + C_{ji}\) be the cost of a first echelon route starting from CDC \(i\), visiting satellite \(j\) and returning to CDC \(i\). In order to determine open facilities, a binary decision variable \(z_m\) is defined that takes value 1 if facility \(m\) is used. Let \(y_{ij}\) be a non-negative integer decision variable that determines the number of first echelon vehicles traveling from CDC \(i\) to satellite \(j\). Non-negative decision variable \(w_{ij}\) represents the amount of flow from CDC \(i\) to satellite \(j\).

We provide the arc-flow formulation of the 2E-LR2PTW in Appendix A (see Section A.3). Note that adding vehicle routing and scheduling decisions leads to a large number of variables and constraints, which makes the problem intractable for...
standard MIP solvers. The set-partitioning models, on the other hand, have significantly less number of constraints and allow column generation approaches to be used. Therefore, they have been commonly studied in vehicle routing literature (see Baldacci et al. 2012, Farham et al. 2018). The solution approaches proposed in this chapter are also based on the set-partitioning (path-based) formulation of the 2E-LR2PTW. Therefore, we provide the path-based formulation of the 2E-LR2PTW in the remainder of this section.

Let $\mathcal{P}_j$ be the set of all feasible second echelon vehicle paths originating and ending at satellite $j$. A second echelon route is feasible if all the following route feasibility conditions (RFCs) hold:

\[
\text{RFCs: } \begin{cases} 
(i) \text{ route starts and ends at the same satellite node,} \\
(ii) \text{ each customer is visited exactly once,} \\
(iii) \text{ serving a customer is started during its time window,} \\
(iv) \text{ the route is completed before satellite closing time, and} \\
(v) \text{ vehicle capacity is not exceeded.}
\end{cases} \tag{4.1}
\]

Define second echelon arc set $\mathcal{E} = \{(m, n) \in \mathcal{N} \times \mathcal{N}\}$. We exclude satellite-to-satellite arcs from $\mathcal{E}$. Let $C_p$ be the cost of second-echelon path $p$ given by the sum of $F''$ and the traveling costs of all arcs traversed in the path. Let $H_{pk}$ indicate the number of times customer $k$ is visited in path $p$. Define $\lambda_p$ as a binary variable that takes value 1 if and only if path $p$ is selected. The path-based formulation of the 2E-LR2PTW is given below.

\begin{align*}
\text{(2E-LR2PTW)} \quad & \text{Minimize} \quad \sum_{m \in \mathcal{M}} F_m z_m + \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} C'_{ij} y_{ij} + \sum_{j \in \mathcal{J}} \sum_{p \in \mathcal{P}_j} C_p \lambda_p \\
& \text{subject to} \quad \sum_{j \in \mathcal{J}} \sum_{p \in \mathcal{P}_j} H_{pk} \lambda_p = 1, \quad \forall k \in \mathcal{K} \tag{4.3} \\
& \quad \sum_{p \in \mathcal{P}_j} H_{pk} D_k \lambda_p \leq \sum_{i \in \mathcal{I}} w_{ij}, \quad \forall j \in \mathcal{J} \tag{4.4} \\
& \quad \sum_{j \in \mathcal{J}} w_{ij} \leq Q_i z_i, \quad \forall i \in \mathcal{I} \tag{4.5} \\
& \quad \sum_{j \in \mathcal{J}} w_{ij} \leq Q_j z_j, \quad \forall j \in \mathcal{J} \tag{4.6} \\
& \quad 0 \leq w_{ij} \leq Q' y_{ij}, \quad \forall i \in \mathcal{I}, j \in \mathcal{J} \tag{4.7} \\
& \quad (T_{ij} - B_j) y_{ij} \leq 0, \quad \forall i \in \mathcal{I}, j \in \mathcal{J} \tag{4.8}
\end{align*}
Objective function (4.2) minimizes the total transportation cost consisting of CDC and satellite location costs as well as first and second echelon vehicle routing costs. Constraint (4.3) guarantees that each customer is visited exactly once. Constraint (4.4) ensures that total incoming flow to a satellite location is not less than total customer demands it serves. Capacity limit of open CDCs and satellites are satisfied by (4.5) and (4.6), respectively. (4.7) holds the lower bounds on the flow variables and sets the correct number of first echelon vehicles with respect to their capacity. Closing time of satellites is satisfied by (4.8). (4.9)–(4.11) are variable domain constraints.

4.2.1 Valid Inequalities

In Chapter 3, we suggest two valid inequalities for the path-flow formulation of the LRPTW. The first one sets a lower bound on the number of open satellites whereas the second one sets a lower bound on the total number of vehicles required to serve all demands. Define a nonnegative integer variable $v_j$ to indicate the number of vehicle routes from satellite $j$. Then, we can use valid inequalities (4.12) and (4.13), where $\bar{Z}$ ($\bar{V}$) is the minimum number of satellites (second echelon vehicles) required to serve all customers demands. Defining $v_j$ requires additional constraints (4.14) and (4.15).

$$\sum_{j \in \mathcal{J}} z_j \geq \bar{Z}$$

$$\sum_{j \in \mathcal{J}} v_j \geq \bar{V}$$

$$v_j = \sum_{p \in \mathcal{P}_j} \lambda_p, \quad \forall j \in \mathcal{J}$$

$$v_j \in \{0, 1, 2, \cdots\}, \quad \forall j \in \mathcal{J}.$$
4.3 Exact Algorithm

We propose a branch-and-price algorithm (BP) for the 2E-LR2PTW inspired by the exact algorithm proposed for the (one-echelon) LRPTW in Chapter 3. The idea is to initiate the original problem with a limited number of columns and generate new columns as needed. We call the path-based formulation (4.2)–(4.15) with only a subset of path variables as the restricted master problem (RMP). In each iteration, BP solves the relaxation of the RMP and finds the dual solution. Next, a number of sub-problems, also called as pricing problems, are solved to price out new path variables and extend \( P_j \) sets. If a column with negative reduced cost is found, it is added to the RMP and the relaxed RMP is resolved. Otherwise, the algorithm checks the current solution of the RMP against integrality constraints. If any fractional integer variable exists, a branching rule is applied and the algorithm solves a new RMP. Otherwise, it stops by returning the optimal solution.

![Branch-and-price algorithm for the 2E-LR2PTW.](image)

Figure 4.2: Branch-and-price algorithm for the 2E-LR2PTW.
In the first step of the proposed BP, the RMP is constructed with initial columns. One way is to obtain trivial columns by solving the 2E-FLPTW (see Appendix A) and provide the resulting satellite–customer–satellite routes as the initial columns for the RMP. Another way is to solve the original problem by a fast heuristic to find a solution for the 2E-LR2PTW. Any feasible solution can be used to provide initial columns and an upper-bound for the exact algorithm. Providing a good upper-bound for the algorithm in the beginning helps to prune more nodes in the underlying branch-and-bound tree and converge to the optimal solution faster (see Chapter 3). The outline of the proposed BP for the 2E-LR2PTW is given in Figure 4.2. Below, we describe the main stages of the proposed BP in details.

4.3.1 The Subproblems

For any satellite \( j \in \mathcal{J} \), we define a subproblem \( SP_j \) to find a new path variable for entering set \( \mathcal{P}_j \). Let \( \mathcal{E}_j \) be the set of arcs for \( SP_j \). \( \mathcal{E}_j \) includes all arcs in \( \mathcal{E} \) except the arcs starting/ending at any satellite other than \( j \). Then, the \( SP_j \) needs to find a feasible path \( p \) with the most negative reduced cost \( \tilde{C}_p \), starting from satellite \( j \), visiting a set of customer nodes, and returning to the same satellite. Let \( \alpha_k, \beta_j, \) and \( \gamma_j \) be the dual values associated with constraints (4.3), (4.4), and (4.14), respectively. Then, the reduced cost of a path \( p \) for satellite \( j \) is calculated as:

\[
\tilde{C}_p = C_p - \sum_{k \in p} \alpha_k - \sum_{k \in p} D_k \beta_j - \gamma_j. \tag{4.16}
\]

As any path is composed of a set of arcs, one can also calculate the reduced cost of any arc \((m,n) \in \mathcal{E}_j\) for \( SP_j \), denoted by \( \tilde{C}_{jmn} \), as follows:

\[
\tilde{C}_{jmn} = \begin{cases} 
F'' + C_{mn} - \gamma_j, & \text{if } m = j, \\
C_{mn} - \alpha_m - \beta_j D_m, & \text{otherwise.}
\end{cases} \tag{4.17}
\]

Define a binary variable \( x_{mn} \) on all second echelon arcs to indicate whether arc \((m,n) \in \mathcal{E}\) is traversed by a vehicle. Define \( t_k \) as the arrival time of a vehicle to node \( n \). Therefore, the \( SP_j \) is formulated as follows.
(SP$_j$) Minimize $\sum_{(m,n) \in E_j} \tilde{C}_{jmn} x_{mn}$ \hspace{1cm} (4.18)

subject to $\sum_{k \in K} x_{jk} = 1$, \hspace{1cm} (4.19)

$\sum_{m:(m,n) \in E_j} x_{mn} = \sum_{m:(n,m) \in E_j} x_{nm}, \quad \forall n \in N$ \hspace{1cm} (4.20)

$\sum_{(m,n) \in E_j} D_n x_{nm} \leq Q''$, \hspace{1cm} (4.21)

$T_{jk} - t_k \leq B_{jk} (1 - x_{jk}), \quad \forall k \in K$ \hspace{1cm} (4.22)

$t_k + T_{kn} - t_n \leq B_{kn} (1 - x_{kn}), \quad \forall k \in K, n \in N : (k, n) \in E_j$ \hspace{1cm} (4.23)

$A_n \leq t_n \leq B_n, \quad \forall n \in N$ \hspace{1cm} (4.24)

$x_{mn} \in \{0, 1\}, \quad \forall (m,n) \in E_j$. \hspace{1cm} (4.25)

The objective function (4.18) minimizes the cost of selected arcs. Constraint (4.19) initiates one path from satellite $j$. (4.20) is the flow conservation constraint. Constraint (4.21) ensures that the accumulated demand in a path does not exceed second echelon vehicle capacity. By constraints (4.22) and (4.23), vehicle arrival times are set with respect to the order of nodes visited in the path. Here, $B_{mn}$ is a sufficiently large number equal to $\max(B_m + T_{mn} - A_n, 0)$. (4.22) and (4.23) also eliminate subtours in a solution. Constraint (4.24) limits the arrival times to the time windows and constraint (4.25) meet the binary requirement of the arc-flow variables.

If the optimal objective function value of an SP is negative, column $\lambda_p$ is generated and added to the RMP based on the selected arcs in the optimal solution of the subproblem. In Section 4.3.2, we provide the details on how the introduced subproblems are solved.

### 4.3.2 Solving a Subproblem

Similar to the subproblems of the LRPTW (see Section 3.3.1), the SP$_j$ (4.18)–(4.25) is an NP-hard problem called the elementary shortest path problem with resource constraints (ESPPRRC). This problem is commonly solved using Labeling algorithm (Feillet et al. 2004, Irnich & Desaulniers 2005). More recently, Lozano et al. (2016) pro-
posed an alternative approach, called Pulse algorithm (PA), to solve the ESPPRC arising in vehicle routing problems. The authors show that the algorithm is competitive with Labeling algorithm and can improve solution time when solving benchmark test instances. In this section, we show how PA can be implemented in our approach instead of Labeling algorithm.

PA is an enumeration-based algorithm that comprises two main stages: (i) a bounding scheme to narrow the solution space by finding a lower bound on the objective function value, and (ii) a recursive exploration procedure that finds the optimal solution based on an implicit enumeration of the solution space.

To solve an SP\(_j\), PA initiates a partial path from the starting node \(j \in J\). An elementary forward path \(p\) is characterized by the following attributes: (i) The set of visited nodes \(\mathcal{N}(p)\), (ii) the cumulative reduced cost of the path \(\bar{C}(p)\), (iii) the total delivered load \(D(p)\), (iv) the cumulative traveling time \(T(p)\), and (v) the last and the second last visited nodes on path \(p\), i.e. last\((p)\) and pre\((p)\), respectively. A partial path \(p\) for SP\(_j\) is initialized with \(\mathcal{N}(p) = \{j\}\), \(\bar{C}(p) = 0\), \(D(p) = 0\), \(T(p) = 0\), last\((p) = j\), and pre\((p) = \text{N/A}\). PA recursively extends the current partial path by propagating throughout the outgoing arcs of last\((p)\). We use different pruning strategies to prevent exploring the inferior search space. When a partial path \(p\) ending at node \(m\) is extended along an arc \((m, n) \in \mathcal{E}_j\), a new path \(p^{\text{new}}\) is formed with the following attributes:

\[
\begin{align*}
\mathcal{N}(p^{\text{new}}) &= \mathcal{N}(p) \cup \{n\} \\
\bar{C}(p^{\text{new}}) &= \bar{C}(p) + \bar{C}_{jmn} \\
D(p^{\text{new}}) &= D(p) + D_n \\
T(p^{\text{new}}) &= T(p) + T_{mn} \\
\text{last}(p^{\text{new}}) &= n \\
\text{pre}(p^{\text{new}}) &= \text{last}(p) = m.
\end{align*}
\]

In order to satisfy capacity and time window constraints, we discard a partial path \(p^{\text{new}}\) if the extension leads to any of the following situations: \(D(p^{\text{new}}) > Q''\), \(T(p^{\text{new}}) > B_n\), or \(T(p^{\text{new}}) + T_{nj} > B_j\). Therefore, once a partial path reaches satellite \(j\), its feasibility is ensured. The algorithm also forbids cost-dominated extensions based on the triangle inequality. Therefore, path \(p^{\text{new}}\) is also discarded if \(\bar{C}_{j,\text{pre}(p),m} + \bar{C}_{jmn} > \bar{C}_{j,\text{pre}(p),n}\).
A key procedure in PA is lower-bounding. It is applied to prune the search space by forbidding extension of unpromising paths based on their reduced cost and time consumption. In the preprocessing step of PA, we calculate a lower bound $\bar{C}(n, T)$ for each node $n \in \mathcal{N}$ and any value $T \in \{B_j - \Phi_1, B_j - 2\Phi_1, \cdots, 0\}$, for a given time step $\Phi_1$. $\bar{C}(n, T)$ denotes the minimum reduced cost that can be achieved by any partial path $p$ that reaches node $n$ with $T(p) \geq T$. Therefore, the algorithm checks whether a possible extension on a path $p^\text{new}$ can improve an upper-bound $\tilde{C}^*$:

$$\tilde{C}(p^\text{new}) + \bar{C}(n, T(p^\text{new})) < \tilde{C}^*.$$ (4.26)

Here, we initially set $\tilde{C}^*$ to 0 and update it with $\min(\tilde{C}^*, \tilde{C}(p))$ as soon as any path $p$ is completed (i.e. reaches the satellite node). If (4.26) does not hold, partial path $p^\text{new}$ is discarded. When PA terminates, it returns a path with the most negative reduced cost, if such a path exists.

### 4.3.3 Column Generation Enhancements

Since PA enumerates over all outgoing arcs of the current node in a path, it can be time-consuming in initial stages of BP. Hence, heuristic approaches are commonly used in the literature of vehicle routing problems to find path columns more efficiently (see, for example, Contardo et al. 2015, Lozano et al. 2016, Farham et al. 2018).

We propose two techniques to reduce the search space of PA and improve its runtime. The first approach modifies the underlying graph. The set $\mathcal{E}_j$ used to solve $\text{SP}_j$ is reduced to contain only a fixed number of outgoing arcs (denoted by $\Phi_2$) with smallest reduced costs from each node. This method is also used in the branch-and-price approach for the LRPTW in Section 3.3.3.

In the second approach, we reduce the search space of PA in a more greedy fashion to explore paths with larger negative reduced costs. To this end, we replace (4.26) with the following condition.

$$\tilde{C}(p^\text{new}) + \bar{C}(n, T(p^\text{new})) < \Phi_3 \times \tilde{C}^*,$$ (4.27)

where the right-hand-side of (4.26) is scaled using a parameter $\Phi_3 > 1$. In this way, the paths with no significant effect on the best bound are discarded.
Algorithm 4.1: Column generation procedure

**Step 1.** Construct the reduced graph and run PA. If columns with negative reduced costs are found, **Stop**. Otherwise, go to Step 2.

**Step 2.** Construct the full graph and run PA with bound scaling in (4.27). If columns with negative reduced costs are found, **Stop**. Otherwise, go to Step 3.

**Step 3.** Run PA with original bounding in (4.26). **Stop.**

PA in Lozano et al. (2016) only returns the best route it finds, i.e. the one with the most negative reduced cost. However, it is possible to keep track of all paths that update \( \tilde{C}^* \) and use them as new columns. In this study, we allocate a memory to store such paths and return them when PA terminates. Having more routes provides more information about the solution space and it can improve the convergence of BP.

Algorithm 4.1 presents the steps of the proposed CG procedure. We only run the exact PA if the heuristic approaches fail to find a promising column.

### 4.3.4 Column Generation Stabilization

Although column generation is very effective in solving hard combinatorial problems, it has its own drawbacks (Irnich & Desaulniers 2005) such as: (i) slow convergence or tailing-off effect, (ii) producing poor columns in early iterations due to lack of dual information, (iii) degeneracy in the primal resulting in multiple optimal dual solutions, and (iv) instability in the dual solutions that oscillate from one value to another. Non-smooth convergence of dual prices has been regarded as a major efficiency issue that has attained many attentions in the literature (Lübbecke & Desrosiers 2005).

In this study, we implement a dual variable smoothing technique inspired by the work of Neame (1999) and Pessoa et al. (2013). The arc reduced costs in iteration \( \tau \geq 2 \) of BP, indicated as \( \tilde{C}_{jmn}^\tau \) used for \( SP_j \) is corrected based on the best reduced cost \( \tilde{C}_{jmn}^* \) found so far:

\[
\tilde{C}_{jmn}^\tau \leftarrow \Phi_4 \tilde{C}_{jmn}^* + (1 - \Phi_4) \tilde{C}_{jmn}^\tau \quad \forall (m, n) \in J, \tag{4.28}
\]

where \( 0 \leq \Phi_4 < 1 \) parameterizes the level of smoothing. In other words, the pricing problem is solved using the arc reduced cost obtained by taking a step size of \( (1 - \Phi_4) \)
from the current dual prices towards the best dual prices found so far. It is possible that the pricing problem fails to find a solution over the smoothed dual prices, while there exists a solution when real dual values are used. This is a sequence of mis-pricing. In this case, the $\Phi^4$ value is reduced iteratively and the pricing problem is resolved until a solution is found or $\Phi^4$ converges to 0 (Pessoa et al. 2013). However, since our pricing problem is a difficult problem to solve, we set $\Phi^4 = 0$ after a mis-pricing in order to solve the pricing problem at most twice in one iteration.

### 4.3.5 Branching

BP applies a branch-and-bound method to ensure the solution to the master problem is integral. Similar to the branching strategy used for solving the LRPTW, we prioritize more strategic variables over the other ones. The variables are selected for branching in the following order: (i) Fractional $z_i$ variables, $\forall i \in I$, (ii) fractional $z_j$ variables, $\forall j \in J$, (iii) fractional $y_{ij}$ variables, $\forall i \in I, j \in J$, (iv) fractional $v_j$ variables, $\forall j \in J$, and (v) fractional $(m,n)$ arcs, $\forall (m,n) \in E$.

For any satellite $j$, if $z_j = 0$ or $v_j = 0$ holds in a branch, we simply ignore solving SP$_j$ in that branch. We perform a binary branching on arc $(m,n) \in E$ that has the closest value to 0.5. In the first branch, we remove arc $(m,n)$ from $E_j$ for any subproblem $j$ under that branch. In the other branch, we remove all outgoing arcs from node $m$ except arc $(m,n)$ as well as all incoming arcs to node $n$ except arc $(m,n)$ in order to force arc $(m,n)$ to appear in the solution of the corresponding subproblems. Note that branching on the arc-flow variables guarantees integer solution.

### 4.4 Heuristic Algorithms

In the 2E-LR2PTW, the strategic (top-level) decisions constitute CDC and satellite locations whereas the tactical (bottom-level) decisions involve first echelon allocations and second echelon vehicle routes and schedules. The proposed exact solution approach in this study deals with all decisions simultaneously. However, this can be computationally expensive when solving large-size 2E-LR2PTW instances. In this section, we proposed two heuristics based on the hierarchical decomposition of the
problem’s decisions. The idea is to fix the decisions at one level and solve the (reduced) problem to determine the decisions of the other level.

### 4.4.1 Top-to-Bottom Approach

The first heuristic is inspired by the two-stage heuristic proposed in Section 3.4 for the LRPTW. It consists of two main stages. In the first stage, we reduce the problem by fixing CDC and satellite location decisions. Based on these decisions, first echelon allocations and second echelon vehicle routes are determined in the second stage. This method is called top-to-bottom approach, denoted by T>B. It starts from the strategic resolution and makes tactical decisions later.

T>B starts by solving a 2E-FLPTW (see Appendix A) to find the location decisions by ignoring any routing decision. In the next stage, it constructs the 2E-LR2PTW (see (4.2)–(4.15)) by fixing all CDC and satellite location variables to their optimal value obtained in the first stage. Next, the reduced 2E-LR2PTW is solved by the proposed BP to determine the remaining variables. The main stages of T>B is given in Figure 4.3.

T>B is expected to run faster than BP since no branching is required on the location variables. However, solving large problems by T>B can still be time-consuming as the complicated decisions (i.e. vehicle routes) are determined by the exact CG in the algorithm.

### 4.4.2 Bottom-to-Top Approach

The 2E-LR2PTW can be solved efficiently with off-the-shelf solvers when route sets $P_j, \forall j \in \mathcal{J}$, are not very large. Branch-and-price-based approaches for the routing problems in the literature start by a small set of routes, commonly containing trivial facility–customer–facility routes, and generate new routes (columns) iteratively until no better route can be found. Different from BP, we may obtain a solution by generating a “good” set of routes first, and then solve the original problem once to find the optimal solution over the generated routes. Therefore, we introduce a two-stage ap-
Solve the 2E-FLPTW.

Let $z^*_m$ be the optimal value of location variable $z_m$, $\forall m \in M$.

Stage 1

Construct the RMP by setting $z_m = z^*_m$, $\forall m \in M$.

Solve the problem by the proposed BP.

Stage 2

Stop

Figure 4.3: Top-to-bottom approach for the 2E-LR2PTW.

For each satellite $j$, generate $P'_j$, a set of vehicle routes found by the proposed clustering approach.

Stage 1

Construct the 2E-LR2PTW over generated set $P'_j$, $\forall j \in J$.

Solve the 2E-LR2PTW formulation using an MIP solver.

Stage 2

Stop

Figure 4.4: Bottom-to-top approach for the 2E-LR2PTW.

proach, called bottom-to-top approach, that starts with the tactical level decisions (i.e. second echelon vehicle routes) and next, determines the strategic decisions (i.e. facility locations). Bottom-to-top heuristic, indicated by B>T, is outlined in Figure 4.4.

The solution quality of B>T highly depends on the quality of the routes generated
in its first stage. A similar approach is used by Ryan et al. (1993) to solve the capacitated VRP. The authors used a construction-based heuristic to find vehicle routes and solve a set-partitioning formulation for optimal selection of the generated routes. In this study, we generate second echelon routes for each satellite $j$ by proposing a novel clustering technique that takes both capacity and time window constraints into account. In the following sections, we explain the clustering technique applied in the first stage of B>>T.

4.4.2.1 Cluster Analysis of the Second Echelon Nodes

The aim of clustering is to divide a given set of data points into a number of groups such that the points in the same group are more similar to each other than to those in other groups. In other words, clustering segregates data points with similar attributes and assigns them into clusters based on a distance measure. Clustering-based heuristics used to solve vehicle routing problems can be categorized as follows (Laporte & Semet 2002).

- **Cluster-first, route-second**: This method is used in three distinct ways. In the first approach, the original problem is decomposed into smaller problems, each dealing with a subset (a cluster) of customers. Next, a VRP is solved for each cluster as a part of the whole problem. The second approach clusters customers into a given number of groups equal to the number of available vehicles. Next, either a traveling salesman problem is solved or a construction heuristic is used to find a vehicle route within each cluster. The last approach aggregates customers into small clusters to make smaller number of nodes, called macro nodes, and finds routes to visit the macro nodes. Then, each macro node is disaggregated and vehicle routes are modified accordingly.

- **Route-first, cluster-second**: This method starts with a giant vehicle route, disregarding the side constraints. Then, this route is iteratively discomposed into smaller routes until all constraints are satisfied.

In the vehicle routing problems, customers can be viewed as data points. They have different attributes such as their location, time window, and demand. There is only
a limited number of studies in the literature that cluster customers under time window restrictions. Dondo & Cerdá (2007) and Pugacs (2014) propose clustering approaches to aggregate customers into macro nodes and then find vehicle routes to visit customers in each node. Qi et al. (2012) uses a different cluster-first, route-second approach where customers are clustered using spatiotemporal distances. Then, a VRP with soft time windows is solved to find vehicle routes for each cluster. Spatiotemporal distances between two customers consider two factors: the spatial distance (i.e. the Euclidean distance) and the temporal distance based on their time windows. The temporal distance between customer $k$ and customer $l$ is a function of the time at which $l$ is reached from $k$. This distance increases if the arrival time to $l$ falls outside its time windows. Therefore, in the problems with soft time windows, an additional cost is added to the objective function based on the amount of time window violation. Unlike soft time windows, hard time windows affect route feasibility.

As the time windows in the 2E-LR2PTW are hard and the capacity of vehicles cannot be violated, we propose a constrained clustering technique. A cluster represents a set that accepts a feasible vehicle route starting from a satellite $j$, visiting a set of customers $K' \subseteq K$, and returning to satellite $j$ such that vehicle capacity constraint and all customer time windows are satisfied. The following terminologies are used in our clustering approach.

The Distance Measure. The distance (or dissimilarity) between any two nodes $m, n \in N$ is calculated as:

$$\text{dist}(m, n) = \begin{cases} \|(n, m)\|, & \text{if arcs } (m, n) \text{ and } (n, m) \text{ are both feasible}, \\ \Phi_5 \times \|(n, m)\|, & \text{otherwise}, \end{cases}$$

(4.29)

where, $\|(n, m)\|$ is the length of arc $(n, m) \in \mathcal{E}$ and $\Phi_5$ is a given parameter to penalize the distance between two nodes that are unreachable from each other. An arc $(m, n)$ is called feasible if it satisfies all the following arc feasibility conditions (AFCs):

$$\text{AFCs:} \begin{cases} (i) \quad D_m + D_n \leq Q'' \\
(ii) \quad T_{mn} \leq B_n \\
(iii) \quad \text{if } m, n \in \mathcal{K}, \text{ then } \exists j \in \mathcal{J} : \text{route } j-m-n-j \text{ satisfies RFCs (4.1)}. \end{cases}$$

(4.30)
Condition (i) ensures that the demand of nodes \( m \) and \( n \) can be delivered by one vehicle. Conditions (ii) checks whether a vehicle can reach node \( n \) from node \( m \) before the closing time of node \( n \). Finally, for any two customers \( m \) and \( n \), condition (iii) ensures that there exists at least one feasible route that traverses arc \((m, n)\).

**Route Construction and Validity of Clusters.** A cluster containing satellite \( j \) and customer set \( \mathcal{K}' \) is denoted by \( \text{Cluster}(j, \mathcal{K}') \). \( \text{Cluster}(j, \mathcal{K}') \) is called a valid cluster if there exists a vehicle route that starts from \( j \) and visits all nodes in \( \mathcal{K}' \) by satisfying RFCs (4.1). In order to construct such a route, we use sequential insertion heuristics, called \( \text{I1} \), proposed by Solomon (1987). \( \text{I1} \) is shown to produce good results for the VRPTW (Bräysy & Gendreau 2005). Given a depot location, \( \text{I1} \) initializes a route with a seed customer and the remaining unvisited customers are added into this route while it yields a feasible route. If any customer remains unvisited, the initialization and insertion procedures are repeated until all customers are served. The quality of routes found by \( \text{I1} \) depends on its seeds. The seed customers are commonly selected by finding either the geographically farthest unvisited customer to the depot or the one with the earliest closing time \( B_k \). Given a satellite node \( j \) and customer set \( \mathcal{K}' \), we construct the routes of \( \text{Cluster}(j, \mathcal{K}') \) by applying \( \text{I1} \) over \( \mathcal{K}' \) considering satellite \( j \) as the depot. Since \( \mathcal{K}' \) usually contains a small subset of customers, \( \text{I1} \) heuristic can be executed efficiently. Hence, we repeat the insertion heuristic for each of the customers as the seed and select the best route found by all seeds. At the end, if a customer in set \( \mathcal{K}' \) remains unvisited, we conclude that no feasible route can be found under the given settings and \( \text{Cluster}(j, \mathcal{K}') \) is called as invalid.

The proposed constrained clustering is different from the ones in the literature that are mentioned earlier. Here, the clustering and route construction phases are done simultaneously to generate feasible vehicle routes. In the literature, however, the clustering and the routing phases are done separately in a sequential manner. We propose three different clustering methods to form clusters, namely agglomerative route clustering (ARC), divisive route clustering (DRC), and greedy route clustering (GRC). First two of the proposed approaches generate the clusters recursively in an hierarchical order. The routes obtained by the clustering approaches are provided as \( \mathcal{P}_j' \subset \mathcal{P}_j \) sets, \( \forall j \in \mathcal{J} \), that represent \( \lambda_p \) columns in the 2E-LR2PTW (4.2)–(4.15).
4.4.2.2 Agglomerative Route Clustering

ARC treats each node as a singleton cluster initially, and then successively merges (or agglomerates) pairs of clusters until all clusters have been merged into a single cluster or a stopping criterion is met. The ARC in our study starts with a given set of clusters and applies its merging procedure until no further merging is possible. For a given satellite location, two clusters \( S_1 \) and \( S_2 \) are merged if a feasible vehicle route starting at \( j \) and visiting all customers in \( S_1 \) and \( S_2 \) can be constructed. In this way, the termination point of ARC is naturally determined by the algorithm and there is no need for an external stopping criterion. The pseudo-code of the proposed ARC is given in Algorithm 4.2.

ARC algorithm may start with any given initial set of clusters denoted by \( \Sigma^0 \). If the initial set is not provided, it can be formed by generating singleton clusters. A singleton cluster contains a satellite node \( j \) and a customer \( k \) such that the trivial route \( j - k - j \) is feasible.

In order to merge two clusters \( S_1 \) and \( S_2 \) in ARC, we use single-linkage distance measure defined as:

\[
\text{dist}(S_1, S_2) = \min_{m \in S_1, n \in S_2: m, n \in K} \text{dist}(m, n).
\] (4.31)

The algorithm keeps pair-wise distances in a two-dimensional matrix \( \Delta \) and update the corresponding elements of the matrix whenever two clusters are merged. If merging the two clusters results an invalid cluster, the distance between them is set to \( \infty \) in order to prevent them from merging in the future. Figure 4.5 illustrates three different steps of the ARC. In the first step, initial singleton clusters are provided. In the next step, two closest clusters are merged to form a new (valid) cluster. The final step returns the clusters that cannot be merged anymore (see Figure 4.5c). Figure 4.5d illustrates the vehicle routes provided by the final clusters.

4.4.2.3 Divisive Route Clustering

In contrast to ARC, where smaller clusters are merged into larger clusters, DRC is based on the idea of splitting larger clusters into smaller ones. Therefore, DRC starts
Algorithm 4.2: Agglomerative route clustering procedure

Procedure $\text{ARC}(j, \Sigma^0)$

input : A satellite $j$, a set of initial clusters $\Sigma^0$
output: A set of clusters with routed customers

1. construct the distance matrix $\Delta$ where $[\Delta]_{uu'} = \text{dist}(S_u, S_u')$, $\forall S_u, S_u' \in \Sigma^0$, $u \neq u'$, using equation (4.31)

// merging procedure

2. while minimum of $\Delta < \infty$ do

3. let $S_u$ and $S_{u'}$ be the two closest clusters

4. let $K^{\text{new}}$ be the set of all customers in $S_u$ and $S_{u'}$

5. let $S^{\text{new}} \leftarrow \text{Cluster}(j, K^{\text{new}})$

6. if $S^{\text{new}}$ is valid then

7. $S_u \leftarrow S^{\text{new}}$

8. remove row/column of $\Delta$ corresponding to $S_{u'}$

9. update row/column of $\Delta$ corresponding to $S_u$

10. else $[\Delta]_{uu'} \leftarrow \infty$

11. end

12. let $\Sigma$ be the set of clusters corresponding to the remaining rows (or columns) of $\Delta$

end

with a large cluster containing all data points. Then the cluster is split recursively until a stopping criterion is met.

The algorithm runs over a given satellite $j$ and a given set of customers $\mathcal{K}^0 \subseteq \mathcal{K}$. $\mathcal{K}^0$ is initially equal to the set of all customers $k \in \mathcal{K}$ that can form feasible $j$--$k$--$j$ routes. At the beginning, DRC creates a cluster to cover all given customers. If such a cluster is valid, it is returned and the algorithm terminates. Otherwise, it splits the current set of customers by finding the customer that has the largest average distance to the other customers. Next, this customer, say $k^{\text{far}}$, is removed from $\mathcal{K}^0$, to form a new set $\mathcal{K}^{\text{new}} = \{k^{\text{far}}\}$. Then, $\mathcal{K}^0$ and $\mathcal{K}^{\text{new}}$ sets are balanced by moving customers from the larger set to the smaller one. A customer is moved from $\mathcal{K}^0$ to $\mathcal{K}^{\text{new}}$ if its average distance to the customers in $\mathcal{K}^{\text{new}}$ is smaller than its average distance to the customers in $\mathcal{K}^0$. At the end, DRC is recursively applied on both sets $\mathcal{K}^0$ and $\mathcal{K}^{\text{new}}$. When the current set of customers forms a valid cluster, the recursive procedure terminates. Therefore, validity of the generated clusters determines termination point for DRC.
Figure 4.5: Different steps of ARC. ▲ shows the satellite node and ● is a customer point.

The steps of the proposed DRC are presented in Algorithm 4.3.

4.4.2.4 Greedy Route Clustering

In addition to ARC and DRC introduced above, we also propose a simple greedy clustering method to find customer clusters. Given a satellite node $j$ and a set of customers $\mathcal{K}^0 \subseteq \mathcal{K}$, the GRC starts a cluster containing only satellite $j$. Then, the algorithm repeatedly adds the closest customer to the current cluster as long as the resulting cluster is valid. If the next candidate customer cannot be added to the current cluster, the current cluster is closed, and a new cluster containing satellite $j$ is initialized. Then, the algorithm tries to add the remaining customers to the new cluster. This procedure is repeated until all customers are clustered. The distance between an
Algorithm 4.3: Divisive route clustering procedure

\begin{algorithm}
\textbf{Procedure} $\text{DRC}(j, K^0)$
\begin{algorithmic}[1]
\State \textbf{input}: A satellites $j$, a set of customers $K^0$
\State \textbf{output}: A set of clusters with routed customers $\Sigma$
\State let $S \leftarrow \text{Cluster}(j, K^0)$
\If{$S$ is valid}
\State let $\Sigma \leftarrow \{S\}$
\EndIf
\Else
\State // split the set
\State let $k_{\text{far}} \leftarrow$ the customer in $K^0$ with the largest average distance to the other
\State customers with respect to the distance function (4.29)
\State remove $k_{\text{far}}$ from $K^0$
\State let $K^{\text{new}} \leftarrow \{k_{\text{far}}\}$
\State // balance the two sets
\For{	ext{each customer} $k \in K^0$}
\If{customer $k$ has smaller average distance to the customers in $K^{\text{new}}$ than to the
\text{other customers in} $K^0$}
\State move $k$ from $K^0$ to $K^{\text{new}}$
\EndIf
\EndFor
\State let $\Sigma_1 \leftarrow \text{DRC}(j, K^0)$
\State let $\Sigma_2 \leftarrow \text{DRC}(j, K^{\text{new}})$
\State let $\Sigma \leftarrow \Sigma_1 \cup \Sigma_2$
\EndIf
\end{algorithmic}
\end{algorithm}

An unassigned customer $k$ and a cluster $S$ is calculated as:

$$\text{dist}(k, S) = \min_{n \in S} \text{dist}(k, n).$$

(4.32)

The proposed GRC is outlined in Algorithm 4.4.

4.4.2.5 The Main Clustering Procedure

Algorithm 4.5 presents the main clustering procedure used to find candidate second
echelon vehicle routes in the first stage of B$\tilde{\text{B}}$T (see Figure 4.4). Given a satellite node $j$ and the set of customer nodes $K$, the algorithm generates a set of second echelon
Algorithm 4.4: Greedy route clustering procedure

Procedure $\text{GRC}(j, K^0)$

input: A satellite $j$, a set of customers $K^0$
output: A set of clusters with routed customers $\Sigma$

1. let $K^\text{unassigned} \leftarrow K^0$
2. let $\Sigma \leftarrow \emptyset$
3. repeat
4. let $K^\text{current} \leftarrow \emptyset$
5. let $S \leftarrow \text{Cluster}(j, K^\text{current})$
6. loop
7. let $k \leftarrow$ closest customer in $K^\text{unassigned}$ to $S$ according to equation (4.32)
8. let $S^\text{new} \leftarrow \text{Cluster}(j, K^\text{current} \cup \{k\})$
9. if $S^\text{new}$ is valid then
10. move $k$ from $K^\text{unassigned}$ to $K^\text{current}$
11. $S \leftarrow S^\text{new}$
12. else
13. add $S$ to set $\Sigma$
14. break loop
15. end loop
16. until $K^\text{unassigned}$ is empty
17. end

vehicle routes originating at satellite $j$ and visiting customers in $K$. First, we find the valid singleton clusters for satellite $j$. If a customer $k$ cannot form a feasible $j$–$k$–$j$ route then it cannot be part of any other route for $j$. Hence, we exclude $k$ from being processed in the clustering stage. The clustering stage takes the advantage of all the clustering methods presented above. It consists of the following steps: (i) Run the ARC over $j$ and the set of initial (i.e. singleton) clusters (line 9), (ii) run the DRC over $j$ and the set of valid customers (line 10), (iii) run the GRC over $j$ and the set of valid customers (line 11), and (iv) run ARC over $j$ and the clusters obtained from DRC (line 12).

In line 12, ARC is rerun with a different initial cluster set, i.e. clusters provided by DRC. This enables us to merge clusters in order to find new ones which may not have
Algorithm 4.5: Main clustering procedure

Procedure \texttt{Clustering}(j, \mathcal{K})
\begin{enumerate}
\item \textbf{input}: A satellite \(j\), set of customers \(\mathcal{K}\)
\item \textbf{output}: \(\mathcal{P}'_j\), a set of vehicle routes for satellite \(j\)
\end{enumerate}

\begin{algorithmic}
\\// create the valid customer set and initial clusters
1 \texttt{let } \mathcal{K}^0 \gets \emptyset \text{ and } \Sigma^0 \gets \emptyset
2 \texttt{foreach customer } k \text{ in } \mathcal{K} \text{ do}
3 \quad \texttt{let } S \gets \texttt{Cluster}(j, \{k\})
4 \quad \textbf{if } S \text{ is valid then}
5 \quad \quad \texttt{add } S \text{ to } \Sigma^0
6 \quad \quad \texttt{add } k \text{ to } \mathcal{K}^0
7 \quad \textbf{end}
8 \texttt{end}
9 \texttt{let } \Sigma_1 \gets \texttt{ARC}(j, \Sigma^0)
10 \texttt{let } \Sigma_2 \gets \texttt{DRC}(j, \mathcal{K}^0)
11 \texttt{let } \Sigma_3 \gets \texttt{GRC}(j, \mathcal{K}^0)
12 \texttt{let } \Sigma_4 \gets \texttt{ARC}(j, \Sigma_2)
13 \texttt{return } \mathcal{P}'_j \text{ as the set of all the vehicle routes represented by the cluster set}
\end{algorithmic}

\Sigma_1 \cup \Sigma_2 \cup \Sigma_3 \cup \Sigma_4

be been generated by other methods.

4.4.2.6 Linking clustering with the master problem

The aim of clustering in B\(\triangledown\)T is not to optimize a clustering objective, but to produce a reliable set of second-echelon vehicle routes for each satellite. In this section, we introduce two methods that potentially improve the quality of B\(\triangledown\)T solutions. In the first method, we explain how \(\mathcal{P}_j\) sets generated during the clustering procedure can be extended to include more routes. Larger \(\mathcal{P}_j\) sets provide more alternatives for the MIP solver, which enables it to obtain a better composition of the second-echelon routes. In our approach, it is possible to keep not only the final clusters in ARC and GRC, but also the history of all valid clusters already generated through the iterations. In ARC, set \(\Sigma\) can be extended as soon as a new valid cluster is formed. Therefore,
Algorithm 4.6: Post–clustering procedure

<table>
<thead>
<tr>
<th>Procedure PostClustering($P'_j$)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>input</strong>: A set of vehicle routes $P'_j$, a stopping criterion</td>
</tr>
<tr>
<td><strong>output</strong>: $P^*_j$, the extended set with improved vehicle routes</td>
</tr>
<tr>
<td>let $P^*_j \leftarrow \emptyset$</td>
</tr>
<tr>
<td><strong>foreach</strong> route $p \in P'_j$ <strong>do</strong></td>
</tr>
<tr>
<td>let $p^* \leftarrow \text{Improve1}(p)$ // intra-route improvement</td>
</tr>
<tr>
<td>add $p^<em>$ to $P^</em>_j$</td>
</tr>
<tr>
<td><strong>end</strong></td>
</tr>
<tr>
<td><strong>repeat</strong></td>
</tr>
<tr>
<td>select a random pair of routes $p_1, p_2$ from $P'_j$</td>
</tr>
<tr>
<td>let ${p_1^<em>, p_2^</em>} \leftarrow \text{Improve2}(p_1, p_2)$ // inter-route improvement</td>
</tr>
<tr>
<td>add ${p_1^<em>, p_2^</em>}$ to $P^*_j$</td>
</tr>
<tr>
<td><strong>until</strong> the stopping criterion is met</td>
</tr>
<tr>
<td><strong>end</strong></td>
</tr>
</tbody>
</table>

one can add a copy of $S_r$ to $\Sigma$ after it is updated in line 7 of Algorithm 4.2. In GRC, we can add the current cluster to set $\Sigma$ once it is expanded to cover a new customer. Therefore, one can add a copy of $S$ to $\Sigma$ after it is updated in line 11 of Algorithm 4.4.

Since the solution obtained in the second stage of B$\triangleright$T depends on the quality of the routes generated in the first stage, we can also benefit from a post-processing procedure as the second clustering enhancement method. The final routes in $P_j$ sets can be improved by using two conventional techniques from the literature (see Bräysy & Gendreau 2005): The first function $\text{Improve1}(p)$ applies two *intra*-route improvement operators, namely 2-Opt and Or-Opt, on a given route $p$. This function returns the same route if no improvement is obtained. Otherwise, it returns the improved route. The second function $\text{Improve2}(p_1, p_2)$ applies 2-Opt*, Relocate, and Exchange operators on a given pair of routes $p_1$ and $p_2$. These operators are called *inter*-route improvement operators. This function returns the same pair of routes if no improvement is obtained. Otherwise, it returns the improved routes. The post-clustering procedure is outlined in Algorithm 4.6. Once this procedure is applied over $P'_j$ (generated by Algorithm 4.5), new sets of vehicle routes $P^*_j$ are obtained. Hence, we can provide $P^*_j$ instead of $P'_j$ in the first stage of B$\triangleright$T.
The clustering method presented in Algorithm 4.5 is implemented for each satellite \( j \) independent of other satellites. Therefore, in order to boost the run-time of \( B\theta T \), Algorithm 4.5 can be executed in parallel with respect to \( j \).

### 4.5 Computational Experiments

In this section, we implement the proposed exact and heuristic approaches on a set of 2E-LR2PTW test instances and present extensive computational results. The numerical study presented in this section enables us to assess the proposed exact and heuristic algorithms on solving 2E-LR2PTWs and analyze the effect of instance characteristics (such as problem size and facility/customer distributions) on the performance of the algorithms and final solutions. Since there are no benchmark instances for the 2E-LR2PTW in the literature, we generate new sets of tests instances based on a set of well-known VRPTW instances in the literature (see Section 4.5.1). Section 4.5.2 explains a number of preprocessing steps, based on the problem instance characteristics, that can be used to enhance the solution procedure. Parameter adjustments are presented in Section 4.5.3. In Section 4.5.4, we analyze the effect of a good upper bound on the exact approach (i.e. BP) and \( T\theta B \) algorithm. Finally, the computational studies of the proposed exact and heuristic approaches are provided in Section 4.5.5 and Section 4.5.6, respectively.

#### 4.5.1 Problem Instances

We use two sets of problem instances in this study. The first set, named Set 1, is based on the benchmark instances of Solomon (1987) that are also used to generate LRPTW instances in page 47. We modified these instances to include candidate CDC and satellite points. The test instances in Set 1 are classified into three groups based on the distribution of customers on the plane: clustered (indicated by C), random (R), and a mix of random and clustered (RC). Solomon test instances are of two types: they have either tight time windows and low vehicle capacity or wide time windows and high vehicle capacity. In Set 1, we consider the former type of instances with 2 candidate CDC nodes, 2 to 4 candidate satellite nodes, and 15, 20, 25, or 30 customer
nodes.

The test instances of Set 2 are based on Dellaert et al. (2019) instances generated for the 2E-VRPTW. Although the original instances contain CDC and satellite facilities, they do not incorporate facility capacity and opening costs. Therefore, we modify these instances by assigning capacity and fixed costs to CDCs and satellites as the potential facility locations. The instances in Set 2 are categorized into four groups based on customer time windows and customer demands. Group-\(a\) and Group-\(b\) instances have tight time windows but Group-\(b\) has more diverse demand distribution. Group-\(c\) and Group-\(d\) instances have similar demand distribution to Group-\(a\) instances but have time windows with larger starting times. Group-\(c\) instances have wider time windows than Group-\(d\). Each group contains test instances with 2, 3, or 6 candidate CDC nodes, and 3 to 5 candidate satellite nodes, and 15, 30, 50, or 100 customers. An instance size is indicated by three numbers ordered as \#1-\#2-\#3 denoting the number of candidate CDC locations, the number of candidate satellite locations, and the number of customers, respectively. All instance data files are available in https://gitlab.com/pharham/test-instances.

4.5.2 Preprocessing

Desrochers et al. (1992) suggest tightening customer time windows based on travel times. For each customer node \(k \in K\), the time window width is reduced using (4.33). The first two terms, adjust the beginning of customer \(k\)’s time window \(A_k\), by calculating the minimal arrival time from predecessors and minimal arrival time to successors, respectively. The last two terms of (4.33) fix the end of customer \(k\)’s time window \(B_k\), based on the maximal departure time from predecessors and maximal departure time to successors, respectively. Tighter time windows apply more restrictions on search space of PA, hence the algorithm can run faster.

\[
\begin{align*}
A_k &\leftarrow \max (A_k, \min (B_k, \min_{m \in N} A_m + T_{mk})) \\
A_k &\leftarrow \max (A_k, \min (B_k, \min_{n \in N} A_n - T_{kn})) \\
B_k &\leftarrow \min (B_k, \max (A_k, \max_{m \in N} B_m + T_{mk})) \\
B_k &\leftarrow \min (B_k, \max (A_k, \max_{n \in N} B_n - T_{kn})) .
\end{align*}
\]
Table 4.1: Parameter settings.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi_1$</td>
<td>Time step in the lower-bounding procedure of PA</td>
<td>$0.05B_j$</td>
</tr>
<tr>
<td>$\Phi_2$</td>
<td>No. of outgoing arcs in the reduced graph</td>
<td>5</td>
</tr>
<tr>
<td>$\Phi_3$</td>
<td>Bound scaling multiplier in pulse algorithm</td>
<td>2.0</td>
</tr>
<tr>
<td>$\Phi_4$</td>
<td>Dual price smoothing coefficient</td>
<td>0.5</td>
</tr>
<tr>
<td>$\Phi_5$</td>
<td>Distance penalty for unreachable nodes</td>
<td>1.75</td>
</tr>
</tbody>
</table>

4.5.3 Implementation Details

The experiments are done on a Linux workstation with Intel® Xeon $4 \times 3.20$GHz processors and 16GB memory. All algorithms are coded in C++ compiled with GCC v7.3 using SCIP optimization suite v6.0 (Gleixner et al. 2018) linked to CPLEX v12.8 (IBM 2018) as the linear programming solver. We use single-thread computing in our experiments.

In order to determine the value of our parameters, we conducted preliminary experiments on a small set of instances and report the selected parameter values for the exact and heuristic approaches in Table 4.1.

In our experiments, we terminate T$\triangleright$B algorithm whenever the relative MIP gap reaches 0.5% or less. This allows us to approximate the solution faster without sacrificing much of its quality. B$\triangleright$T and exact approaches are allowed to run until this gap closes. All algorithms are run in a time limit of 4 hours.

We can set $Z$ and $V$ values similar to the ones set for the LRPTW (see Section 3.5). However, these bounds can be improved by solving small problems during the preprocessing stage. To calculate $Z$, a bin-packing problem is solved where item sizes are customer demands and each bin represents a satellite with the given capacity. $V$ is found similarly, except bin capacities are all equal to the capacity of second-echelon vehicles ($Q''$) and there can be as many bins as the number of customers. Since the number of customers is not very large in 2E-LR2PTW instances, these bin-packing problems can be solved very efficiently with today’s MIP solvers.
Using the above settings, we provide detailed numerical results of solving the test instances by the proposed BP and heuristic algorithms in the remainder of this section.

### 4.5.4 The Upper-Bounding Effect

BP and $T\triangleright B$ approaches can use the solution of the 2E-FLPTW as a starting point (an upper-bound) to search for the final solution. In BP, no decision is fixed in the beginning, and the algorithm is allowed to investigate decisions other than the ones provided by the solution of the 2E-FLPTW. In $T\triangleright B$, however, we use the 2E-FLPTW solution to fix the locations of CDCs and satellites. Once these decisions are fixed, they will never change in the later steps. Therefore, if the locations are decided poorly, high overall cost might be incurred as a facility location affects the cost of vehicle route originated at that location. Since the facility location decisions in the 2E-FLPTW are made according to customer-to-satellite assignments without considering routing decisions in the second echelon, they can lead to undesirable results when establishing vehicle routes in the next step. This potential drawback can be avoided if our perception of the network design solution is improved. Here, we take the advantage of information available in the solution of $B\triangleright T$ to make better location decisions in the $T\triangleright B$ approach. Since the location decisions in the $B\triangleright T$ approach are made according to nontrivial approximated second echelon routes, they are more reliable compared to the ones made in the 2E-FLPTW approach and can provide more precise information about the final solution.

In order to see the initialization effect on the $T\triangleright B$ solutions, we initialize this algorithm in two different ways: (i) fixing the facility locations based on the 2E-FLPTW solution, and (ii) based on the $B\triangleright T$ solution. If the latter case is used, we call the algorithm $T^*\triangleright B$. Figures 4.6a and 4.6b illustrate the effect of initial solution in $T\triangleright B$. In Figure 4.6a, facility locations are fixed according to the 2E-FLPTW solution. The 2E-FLPTW solution suggests opening CDC 0 and satellites 3, 4, and 5. However, even though its opening cost is incurred, satellite 5 is not used in any second echelon routes of the final solution. In Figure 4.6b, facility locations are fixed according to the $B\triangleright T$ solution. This time, CDC 1 is used and satellite 3 is kept closed. Such decisions resulted in a dominating solution with much less objective function value.
(a) T>B solution (objective function value: 1649.2).

(b) T*B solution (objective function value: 1494.8). This solution is identical to the BP solution.

Figure 4.6: Comparison of T>B and T*B solutions for the test instance R102 with size 2-4-25.

Figure 4.7: Amount of time spent in different stages of the exact approach for the test instance C102 with size 2-3-20 (Total time: 507.1s).
Here, starting $T\triangleright B$ with wrong location decisions led to a high objective function value, even though the routing decisions are optimal. On the other hand, using the location decisions provided by the $B\triangleright T$ solution led us to the optimal solution for this instance. Section 4.5.6 provides a more detailed comparison of $T\triangleright B$ and $T\triangleright B*$ performances.

We can also use the $B\triangleright T$ as an initialization step for BP. Therefore, before solving any problem instance by BP, we first solve the problem by $B\triangleright T$ and then use the objective function value of the $B\triangleright T$ solution as an upper-bound and its final second-echelon vehicle routes as initial columns in BP (see the first step in Figure 4.4). Numerical results presented in Section 4.5.6 show that $B\triangleright T$ can provide efficient and high-quality vehicle routes. Figure 4.7 shows the contribution of finding initial columns in the exact approach in total computational time for a particular test instance. Here, finding initial columns corresponds to solving the problem with $B\triangleright T$ and providing its solution as the starting point for the BP. As it is shown in Figure 4.7, this step is done very fast compared to the total computational time of BP.

### 4.5.5 Numerical Results of the Exact Approach

This section presents the extensive computational results of implementing BP to solve our 2E-LR2PTW problem test instances. We group the instances based on their type and size, and consider the following measures in evaluating the proposed BP for each group. $\#opt$ shows the number of optimal solutions found. $Av\ obj\ val$ indicates the average objective function value. $Av\ subopt\ gap$ is the average percentage MIP gap reported for the instances that are not solved to optimality during the given computing time (4 hours). $Av\ #pricer\ calls$ denotes the average number of times the algorithm called the pricing problems. $Av\ #BB\ nodes$ shows average number of nodes processed in the branch-and-bound tree. $Av\ #CDC-sat$ denotes the average number of open CDCs and satellites in the final solution. $Av\ #veh1-veh2$ is the average number of vehicles used in the fist and second echelon. $Av\ time$ is the average solution time in seconds.

Table 4.2 shows the computational details of solving Set 1 instances by BP. The instances are grouped according to the customer distribution type (i.e. C, R, and RC).
There are more than one instances having the same size in each group that are different in terms of customer time windows and/or demands. \#Inst shows the number of instances in each size group. The summary of each type group is provided in the last row of the group.

In total, BP finds the optimal solution of 281 out of 348 test instances in Set 1. The average MIP gap for unsolved instances is 3.22%. When the number of candidate satellite locations and customers increase, the problem gets more difficult to solve. More branching nodes are explored and, consequently, the pricing problems are called more frequently. This leads to higher computational times. Among the three instance type groups, \(R\) is the most challenging one to solve. When customers are distributed randomly, more BB nodes are processed and more time is spent to solve the pricing problems. The reason is that the number of alternative second echelon routes increases for randomly distributed customers. Therefore, ensuring the optimal solution requires enumerating more BB nodes and vehicle route options. This is also shown by \#veh2 value. The average \#veh2 increases when moving from \(C\) instance group to \(RC\), and from \(RC\) to \(R\). On average, BP consumes 1 hour to solve an instance in Set 1. The optimality hit rate for this algorithm is around 80% for each type group. Customer distribution does not affect the performance of BP in searching for the optimal solution.

Numerical results for Set 2 instances are provided in Table 4.3. BP finds the exact solution of 195 instances out of 240 in Set 2. Considering the number of customers served, the exact algorithm is able to find the optimal solution for all instances with 15 and 30 customers in Set 2. Only two (out of 60) instances with 50 customers yield nonzero MIP gaps (around 0.35%). Among 60 instances with 100 customers, 17 are optimally solved by BP. For the instances that the optimal solution is not guaranteed during the time limit, the average MIP gap is 4.81%. The amount of optimaly solved instances for Group-\(a\) and Group-\(d\) is 88% and 86%, respectively, that is more than the other groups. Solution times are also smaller for Group-\(a\) and Group-\(d\) test instances. This shows that the problem is easier to solve when the customers have tight time windows and the demand distribution is more uniform. The most difficult group of Set 2 instances is Group-\(c\), where wide customer time windows are considered. When time windows are wide, search space of PA increases and the algorithm needs
more time to generate promising columns. BP is able to reach optimality for 73% of the instances in Group-c. The results in Table 4.3 indicate that while the overall instance size affects the performance of BP, the algorithm is more sensitive to the number of customers than to the number of candidate satellite locations, and more sensitive to the number of satellite locations than the number of candidate CDC locations. It is expected as most of the complications arises by the routing problem in the second echelon. On average, when the size of the second echelon network gets larger and/or customer time windows get tighter, more second echelon vehicles are used. On average, BP results in small MIP gap values and a computational time which is less than 1 hour.

Our experiments demonstrated the effect of the CG enhancements introduced in Section 4.3.3. We observed that the reduced graph and bound scaling steps of Algorithm 4.1 significantly contributed to CG by eliminating the need for running exact PA in many cases. Since the exact PA is more time-consuming than the other strategies, running it less frequently, can boost the performance of the algorithm. Figure 4.8 illustrates the contribution of different strategies in generating columns of the master problem for a particular test instance. The results indicate that implementing PA on reduced graph and on the complete graph with bound scaling can help to find a significant number of columns without a need to run exact PA on the complete graph.

Table 4.2: Numerical results of BP for Set 1 instances.

<table>
<thead>
<tr>
<th>Inst type/size</th>
<th>#Inst (#opt)</th>
<th>Av obj val</th>
<th>Av Av subopt gap (%)</th>
<th>Av #pricer calls</th>
<th>Av #BB nodes</th>
<th>Av #CDC-sat</th>
<th>Av #veh1-veh2</th>
<th>Av time (s)</th>
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<tbody>
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<td>9 (9)</td>
<td>1452.63</td>
<td>365.7 3.8</td>
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<td>9 (8)</td>
<td>1519.29</td>
<td>1.86 1230.4 53.3</td>
<td>1 - 2</td>
<td>2 - 2.6</td>
<td>3331.3</td>
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</tr>
<tr>
<td>C 2-2-25</td>
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<td>1577.08</td>
<td>4.7 1757.1 77.3</td>
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<td>2 - 4.0</td>
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Table 4.2: (Continued.)

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<th>Inst type/size</th>
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<th>Av obj val</th>
<th>Av subopt gap (%)</th>
<th>Av #pricer calls</th>
<th>Av #BB nodes</th>
<th>Av #CDC-sat</th>
<th>#veh1-veh2</th>
<th>Av time (s)</th>
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<td>1 - 2</td>
<td>2 - 4.1</td>
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<td>2.62</td>
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Table 4.3: Numerical results of BP for Set 2 instances.

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<th>Av #pricer calls</th>
<th>Av #BB nodes</th>
<th>Av #CDC-sat</th>
<th>#veh1-veh2</th>
<th>Av time (s)</th>
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<td>2.0 - 5.8</td>
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<tr>
<td>a 2-3-30</td>
<td>5 (5)</td>
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<td>1008.6</td>
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<td>1 - 2</td>
<td>4.0 - 10.0</td>
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(continued)
### Table 4.3: (Continued.)

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<th>Av #pricer calls</th>
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<td>3.4 - 9.6</td>
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<tr>
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<tr>
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<td>1472.6</td>
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<td>7.8 - 30.4</td>
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<tr>
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<td>(46)</td>
<td>2604.29</td>
<td>2472.3</td>
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<td>1617.39</td>
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<td>4.0 - 15.2</td>
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<td>739</td>
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<td>2538.09</td>
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<td>4.0 - 16.0</td>
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<td>4037.24</td>
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<td>8.0 - 31.8</td>
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<td>3323.6</td>
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<tr>
<td>c 60</td>
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<td>4.4 - 15.5</td>
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<td>d 2-3-15</td>
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<td>86</td>
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<td>2.5</td>
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</tr>
<tr>
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<td>5 (5)</td>
<td>2082.91</td>
<td>60.6</td>
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<tr>
<td>d 2-3-50</td>
<td>5 (5)</td>
<td>2611.14</td>
<td>644.6</td>
<td>1 - 2.0</td>
<td>4.0 - 15.8</td>
<td>210.7</td>
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</table>

(continued)
Table 4.3: (Continued.)

<table>
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<tr>
<th>Inst type/size</th>
<th>#Inst (#opt)</th>
<th>Av obj (val)</th>
<th>Av subopt gap (%)</th>
<th>Av #prier calls</th>
<th>Av #BB nodes</th>
<th>Av #CDC-sat</th>
<th>Av #veh1-veh2</th>
<th>Av time (s)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>5 (3)</td>
<td>4052.20</td>
<td>3.12</td>
<td>9969.6</td>
<td>4503</td>
<td>1 - 2.0</td>
<td>8.0 - 30.2</td>
<td>8626.8</td>
</tr>
<tr>
<td>d 3-5-15</td>
<td>5 (5)</td>
<td>1607.05</td>
<td>806</td>
<td>396.4</td>
<td>414.8</td>
<td>1 - 2.0</td>
<td>2.0 - 5.6</td>
<td>10.1</td>
</tr>
<tr>
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<td>2149.84</td>
<td>1725.6</td>
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<td>4.0 - 10.2</td>
<td>46.8</td>
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<td>2542.38</td>
<td>8696.8</td>
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<td>4.0 - 16.0</td>
<td>872.3</td>
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<td>d 3-5-100</td>
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<td>3986.58</td>
<td>12273.4</td>
<td>4400.6</td>
<td>1 - 2.0</td>
<td>8.0 - 30.0</td>
<td>13357.6</td>
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</tr>
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<td>1607.54</td>
<td>801.2</td>
<td>440</td>
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<td>2.0 - 5.6</td>
<td>8.3</td>
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</tr>
<tr>
<td>d 6-4-30</td>
<td>5 (5)</td>
<td>2149.15</td>
<td>1572.6</td>
<td>476.2</td>
<td>1 - 2.0</td>
<td>4.0 - 10.2</td>
<td>48.0</td>
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</tr>
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<td>d 6-4-50</td>
<td>5 (5)</td>
<td>2562.51</td>
<td>4344</td>
<td>1252.4</td>
<td>1 - 2.0</td>
<td>4.0 - 16.0</td>
<td>359.9</td>
<td></td>
</tr>
<tr>
<td>d 6-4-100</td>
<td>5 (3)</td>
<td>4017.73</td>
<td>13442.4</td>
<td>5670.2</td>
<td>1 - 2.2</td>
<td>7.8 - 30.8</td>
<td>10600.1</td>
<td></td>
</tr>
<tr>
<td>d 60 (52)</td>
<td>2583.18</td>
<td>2.45</td>
<td>4753.1</td>
<td>1879.8</td>
<td>1 - 2.0</td>
<td>4.5 - 15.4</td>
<td>2846.1</td>
<td></td>
</tr>
<tr>
<td>Grand total</td>
<td>240 (195)</td>
<td>2586.46</td>
<td>4.81</td>
<td>5556.4</td>
<td>2163.6</td>
<td>1 - 2.0</td>
<td>4.5 - 15.5</td>
<td>3470.2</td>
</tr>
</tbody>
</table>

Figure 4.8: Amount of columns found by different subproblem solvers for the test instance C102 with size 2-3-20 (Total columns: 5241).

4.5.6 Numerical Results of the Heuristic Approaches

In this section, we run the proposed heuristic approaches to solve Set 1 and Set 2 problem instances and analyze the results. Table 4.4 provides numerical results of the three heuristics, namely $T^B$, $T^*$, and $B_T$, over Set 1 instances, and their comparisons to the exact (i.e. BP) approach. #best shows the total number of best solutions found by an algorithm considering the solution of BP or any of the heuristics. Av %dev from best indicates the average percentage deviation of the objective function value found by an algorithm from the best value reported by any of the algorithms.
Out of 348 instances in Set 1, BP returns 322 best solution hits and 281 optimality hits, which are the highest hit rates of all algorithms. Among the heuristics, $T^\ast \triangleright B$ is the most successful one in finding best and optimal solutions (294 and 241, respectively). It also outperforms $T \triangleright B$ in terms of solution quality and computational time. On average, $T^\ast \triangleright B$ gives 2% lower deviation from the best solutions, explores 42% less BB nodes, and uses 9% less computational time compared to $T \triangleright B$. This shows the advantage of using high quality upper-bounds, good location decisions, and nontrivial candidate columns provided by $B \triangleright T$ in the initial step of $T^\ast \triangleright B$ algorithm. Compared to BP (see Table 4.2), $T^\ast \triangleright B$ makes less computational effort, on average, as it explores less number of BB nodes (1459.9 versus 1671.3) and uses less time to find the solution (3156.1 sec. versus 3787.5 sec.). On average, $T^\ast \triangleright B$ deviates 0.04% from the best available solution. Note that around 82% of the best solutions of $T^\ast \triangleright B$ are proved to be optimal by BP. Therefore, one can conclude that $T^\ast \triangleright B$ produces solutions that are compatible with BP in terms of quality.

Considering the solution quality, $B \triangleright T$ cannot find as many best or optimal solutions as other approaches, however it is able to find solutions that deviate from the best ones by only 0.43%. In terms of solution time, $B \triangleright T$ is significantly faster than the other approaches. The average computational time of $B \triangleright T$ is 4.5 seconds and almost all $B \triangleright T$ solutions are found in the root node of the branch-and-bound tree.

Experimental results over Set 2 instances are given in Table 4.5. We report only $T^\ast \triangleright B$ over $T \triangleright B$ as it outperforms $T \triangleright B$ in most cases (see Table 4.4). Out of 240 instances in Set 2, BP finds the highest number of best and optimal solutions (207 and 195, respectively) and, on average, deviates 0.2% from the best solution. $T^\ast \triangleright B$ is able to find the best available solution for 139 instances. 75% of the best solutions found by this algorithm are optimal. Although the optimality hit rate of $T^\ast \triangleright B$ is less than that of BP, this algorithm is able to find a better solution for many cases where BP does not close the optimality gap in the given time. On average, $T^\ast \triangleright B$ solutions deviate only 0.09% from the best available solutions.

In terms of computational effort, $T^\ast \triangleright B$ proves to be more efficient than BP. It explores 80% less BB nodes, requires 60% less pricing calls, and reports 60% less solution times compared to BP. Table 4.5 also presents $Av \ root \ gap(\%)$ values to show the
average percentage of MIP gap obtained after solving the root node by BP and T*B. T*B benefits from small initial MIP gaps by fixing binary location variables. Therefore, if good location decisions are provided to this algorithm, it can find a reliable solution fast.

B*T showed a promising performance by generating high quality solutions in short computational times. For small size instances, it was able to find high number of optimal solutions, and for larger ones, its solutions are not far from the best ones. Around 26% of the instances in Set 2 were solved to optimality by B*T. The overall $\text{Av \%dev from best}$ value for B*T is 1.09. Note that 81% of the best solutions are found to be optimal by BP. On average, B*T required 13.5 sec. to return a solution.

The above observations indicate: (i) BP is successful to find the optimal solution of around 80% of the instances in Set 1 and Set 2. For the instances for which the exact solution is not guaranteed, BP reports small MIP gaps, on average. (ii) T*B shows a notable performance in terms of both solution quality and computational effort. (iii) B*T solutions are close to the optimal solutions or the best ones found by our algorithms. This algorithm uses significantly less computational time than the others. Therefore, one can rely on B*T to find a fast but good approximation of the 2E-LR2PTW solutions. B*T solutions also provide a high quality information that can be used as a starting point for other approaches that seek more precise solutions. The advantage of using this information is demonstrated in T*B results.

Experimental results demonstrated how different clustering approaches in Algorithm 4.5 are involved in finding second echelon vehicle routes in B*T. As an example, we show the contribution of ARC, DRC, and GRC in generating initial columns for a particular test instance in Figure 4.9. All approaches were able to find new routes, however the contribution of ARC and GRC approaches is more significant.

4.5.6.1 Bottom-to-Top Approach for the LRPTW

Our computational experiments show that B*T is an efficient algorithm that can find acceptable solutions in very short times. This algorithm is also flexible and can be used to solve the routing problems for which a path-based formulation is available.
Table 4.4: Numerical results of the proposed heuristics for Set 1 instances.

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<tr>
<th>Inst</th>
<th>Type/Size</th>
<th>BP</th>
<th>T</th>
<th>Inst #best/#opt</th>
<th>Av %dev from best</th>
<th>Av #prier calls</th>
<th>Av #BB nodes</th>
<th>Av time (s)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>9(9) 9(9) 8(8) 2(2)</td>
<td>0</td>
<td>0.02</td>
<td>0.52</td>
<td>171.8</td>
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<td>1.02</td>
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<td>0.25</td>
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<td>72.1</td>
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<td>9(6) 3(3) 8(5) 3(0)</td>
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<td>0.03</td>
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<td>0.24</td>
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<td>1.0</td>
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<td>0.26</td>
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<td>55.3</td>
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<tr>
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<td>104(86) 76(72) 89(71) 33(19)</td>
<td>0.01</td>
<td>0.23</td>
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<td>2407.7</td>
<td>87.3</td>
<td>82.2</td>
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</tr>
<tr>
<td>R 2-2-15</td>
<td>12(12) 12(12) 12(11) 11(11)</td>
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<td>0</td>
<td>0.86</td>
<td>784.7</td>
<td>705.1</td>
<td>160.3</td>
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</tr>
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<tr>
<td>R 2-3-25</td>
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<td>0.17</td>
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<td>5303.8</td>
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</tr>
<tr>
<td>R</td>
<td>129(114) 48(46) 117(96) 50(48)</td>
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<td>0.62</td>
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Grand total: 322(281) 200(183) 294(241) 142(122)
Table 4.5: Numerical results of the proposed heuristics for Set 2 instances.

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<th>Av #picer calls</th>
<th>Av #BB nodes</th>
<th>Av root gap (%)</th>
<th>Av time (s)</th>
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Grand total  207(195)  139(104)  64(62)  0.20  0.09  1.09  2209.6  434.4  4.5  9.85  2.41  1382.9  13.5

123
If needed, small modifications can be made to adopt different route feasibility conditions and constraints in the clustering step and implement B≠T to tackle a wide range of LRP and VRPs. In this section, we implement this algorithm to solve the LRPTW introduced in Chapter 3, and investigate its performance.

Since the LRPTW only contains a single echelon, the master problem has to be modified in order to contain satellite facilities and not CDCs. Therefore, the variable and constraints regarding the first echelon are removed. However, the procedure to find candidate variables in $P_j, \forall j \in J$, remains the same. B≠T approach for the LRPTW uses the clustering method to find vehicle routes and then, solves the master problem of the LRPTW (see Section 3.2) over the generated routes.

The LRPTW test instances consist of four sets. The first set includes 36 test instances containing up to 3 candidate depot locations and 40 randomly distributed customers. The second set has larger instances with up to 5 candidate depot locations and 50 randomly distributed customers. The other two sets contain at most 5 candidate CDCs and 50 clustered customers. All LRPTW instances have nonzero facility location and vehicle fixed costs. We call the exact solution algorithm proposed for the (one-echelon) LRPTW in Chapter 3 as BP(1E). For each instance set, Table 4.6 shows (i) $Obj\ val\ (Min,\ Max\ and\ Av\ %dev)$: the minimum, maximum, and average percent deviation of the objective function value found by B≠T from BP(1E), (ii) $Total\ #CDCs$ and $Total\ #vehicles$: the total number of open CDCs and the total number of vehicle routes found by each algorithm, and (iii) $Av\ time$: the average computational time reported by BP(1E) and B≠T (in seconds). BP(1E) was provided 6 hours of time limit and was run on the same computer.
B>T yields an average deviation of 2.09% from BP(1E) solutions. Since facility locations are considered as strategic decisions that make up a large portion of the total cost in the LRPTW instances, it is important to make a correct decision about the number and the location of open facilities. Table 4.6 shows that B>T is successful in keeping the number of open CDC close to the ones reported in the literature, while it uses 0.52 more vehicles per instance (72 more vehicles are used in total of 138 instances). Negative deviation values indicate that our algorithm is able to improve the solutions of BP(1E) for some instances. When the performance of B>T is analyzed according to the instance types, it finds high quality solutions for C type instances, some of which dominate the ones reported by BP(1E). The most challenging set for B>T to solve is the one corresponding to the second row of Table 4.6. It contains instances with many randomly distributed customers. For 48 test instances in this set, the average deviation is returned as 5.62%. Higher deviations are expected for a clustering-based heuristic when both locations and time windows of customers are randomly gener-

Table 4.6: Numerical results of B>T for LRPTW test instances.

<table>
<thead>
<tr>
<th>Inst set (type)</th>
<th># Insts</th>
<th>Obj val</th>
<th>Total #CDCs</th>
<th>Total #vehicles</th>
<th>Av time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Min %dev</td>
<td>Max %dev</td>
<td>Av %dev</td>
<td>BP(1E)</td>
</tr>
<tr>
<td>1 (R)</td>
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</tr>
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<td>5.62</td>
<td>127</td>
</tr>
<tr>
<td>3 (C)</td>
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<td>-1.07</td>
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</tr>
<tr>
<td>4 (C)</td>
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<td>5.43</td>
<td>-0.38</td>
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</tr>
<tr>
<td>Grand Total</td>
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<td>2.09</td>
<td></td>
<td></td>
<td></td>
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</table>

Table 4.7: Improved solutions for the LRPTW test instances.

<table>
<thead>
<tr>
<th>Inst set</th>
<th>Inst</th>
<th>Inst size</th>
<th>Obj val</th>
<th>#CDC</th>
<th>#vehicles</th>
<th>time (s)</th>
</tr>
</thead>
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<td>5.5</td>
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<td>7615.2</td>
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<tr>
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<td>0-5-50</td>
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<td>45.2</td>
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<td>7636.6</td>
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<td>58.8</td>
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<tr>
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<td>94.7</td>
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<td>4</td>
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<td>66.5</td>
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</tbody>
</table>
ated. However, if the computational times matter, B>T can provide efficient solutions in any instance type. The results indicate that B>T takes only 0.23% of the BP(1E) reported times to find the solution. The average time to solve an LRPTW test instance in Table 4.6 is 17 sec. We list the solution details for which B>T found a better result than BP(1E) in Table 4.7.

4.6 Concluding Remarks

In this chapter, we consider the two-echelon location with last echelon routing problem under capacity and time window constraint (2E-LR2PTW) to address main strategic and tactical-level decisions in urban freight transportation systems. Despite the important application of this problem in both CL and other inter-modal freight transportation systems, a very limited amount of research has been done that provide closed-form formulation and suggest effective solution approaches. We present a path-based formulation for the problem that is solved by an exact branch-and-price-based algorithm. Different enhancement techniques are proposed and the optimal solutions are found for the instances with up to 3 candidate CDC locations, 5 candidate satellite locations, and 100 customers. In order to solve large problems more efficiently, we develop two heuristics that reduce the original problem based on its decisions. One approach is to make the facility location decisions first, and solve for routing decisions next. However, as problems with routing decisions are shown to be difficult combinatorial problems, this stage becomes computationally expensive. Another approach is to estimate routing decisions first, and find the optimal facility location and select the best routes in the next step. Once a candidate set of the most detailed decisions (i.e. vehicle routes) is determined, solving the problem becomes more straightforward. We show that customers can be clustered into vehicle routes based on not only their spatial characteristic but also their temporal attribute (i.e. time window). The experimental results indicate that the latter approach is highly successful in solving problem instances with different size and characteristics. Therefore, we highlight the importance of taking tactical level decisions into account while making strategic level decisions in such a complex system. The effective clustering method proposed in this study can be used to find an initial solution and/or a tight upper bound
for the problem in a relatively short computational time.

Satellites are critical points in the two-echelon freight distribution systems as they enable changing the transportation mode. They are usually places with limited space and no possibility inventory handling. Therefore, in order to maintain a seamless flow of goods, and avoid congestion and vehicle queues inside the city, it is important that the goods are transferred between the vehicles without a delay. In Chapter 5, we formulate synchronization problem at satellite locations to address these issues.
CHAPTER 5

SATELLITE SYNCHRONIZATION IN THE TWO-ECHelon URBAN FREIGHT DISTRIBUTION SYSTEMS

As freight deliveries in city logistics take place in populated areas and interact with city environment, it is crucial to conduct a sustainable and responsive transportation with lowest possible cost and minimum impact on the inhabitants’ quality of life. One of the major problems in two-echelon systems is synchronizing first echelon and second echelon vehicles at their meeting point, i.e. satellite location. Satellites are located in urban areas, close to the customers they serve, and have limited space and tight time windows. They can only be accessed during a specific time, usually in off-peak hours, and do not allow inventory or staging due to space limitations. Therefore, the time and load synchronization between primary and secondary vehicles are needed to make seamless flow of goods, and avoid vehicle congestion and waiting times in city center areas. In this chapter, we formulate satellite synchronization problem (SSP) in two-echelon urban freight distribution systems to address the synchronization challenges arising in the operational level planning of such systems. The problem consists of scheduling vehicles from both echelons such that minimum congestion is encountered at the satellite location. The SSP solution can be used to measure the performance of the deployed transportation system and provide valuable feedback on the feasibility of the proposed strategic configuration under different circumstances at the operational environment.
5.1 Introduction

In recent years, two-echelon freight transportation systems have gained growing attentions among city logistics (CL) researchers and practitioners. In the first echelon, urban trucks, called primary vehicles, carry the consolidated freight from city distribution centers (CDCs) to intermediate facilities called satellite platforms. While CDCs are generally located on the outskirts of the city, satellites are small inner-city locations where usually no warehousing or staging activities are possible. In the second echelon, goods are sorted and loaded into smaller vehicles, called secondary vehicles, to be distributed to customers in city-center areas. These vehicles meet city regulations, have less negative impacts on urban environment (such as gas emission and noise), and can access customers in any street of the city. Examples of two-echelon CL applications are: e-commerce and home delivery services, newspaper and press distribution, and cargo/parcel deliveries.

Satellites are commonly not referred to physical facilities but temporary places with other usages such as parking lots, bus stations, etc., located in urban zones. Therefore, they have limited availability in terms of time and space. That is, the number of vehicles and their traffic that can be simultaneously present at a satellite location is limited to the available space, there is no storage capacity, and the time required for loading/unloading activities is limited to a tight hard time window. As a result, satellites are generally rendezvous points, where goods are transferred from one vehicle to another in a trans-dock fashion (Crainic 2008, Crainic et al. 2009). Figure 5.1 illustrates trans-dock operations in a satellite facility. The inbound trucks bring freight from CDCs and unload them at the satellite. Then, goods are sorted, consolidated, and loaded into the outbound trucks waiting at the satellite. Finally, the outbound trucks leave the satellite to distribute their load to their assigned customers.

While the literature contains studies on the two-echelon freight distribution systems, the inter-dependency of different echelons in the two-echelon freight distribution systems are commonly ignored. Although the location and routing decisions in CL systems are considered as strategic/tactical level decisions and are at the top priority when dealing with the network design problems, ignoring the loading/unloading operations at satellite locations can result in a suboptimal or even infeasible solution at
the operational level. Satellite synchronization problem deals with the management of day-to-day trans-docking operations, which is seen as one of the core managerial problems in CL Cleophas et al. (2019). Satellite characteristics, however, bring many challenges to planning operational level freight distribution in two-echelon CL systems. The most important issue is scheduling vehicles so that trans-docking is performed during the available time with minimum congestion and short delays. In order to have minimum congestion, secondary vehicles should be ready in the satellites when primary vehicles arrive so that transferring the freight between these vehicles is done with no or minimum delay. There cannot be long incoming/outgoing vehicle queues due to limited space. Finally, vehicles may only wait for a short duration in order to start their tour on time and serve customers in their time windows. The synchronization problem becomes more complicated when supply and demand consist of multiple products. An end customer may demand one or several products and a CDC may have one or a limited number of product types available.

In this chapter, we formulate the satellite synchronization problem (SSP) in the two-echelon freight distribution systems that finds the best rendezvous schedules between the first and second echelon vehicles such that minimum “congestion” is observed at the satellite location. Two mathematical formulations, namely the arc-flow formulation and the time-indexed formulation are provided in Section 5.2 and their performance are compared in Section 5.3. We also study the problem under possible vehicle arrival time and service time delays and provide managerial insights in Section 5.3. Section 5.4 concludes the chapter.
5.2 Problem Formulation

Considering that the network design problem has a higher priority in decision-making than the synchronization problem, we assume that the strategic problem about the transportation network is already solved. That is, we know about the open satellites as well as primary and secondary vehicle routes. Hence, the number of primary and secondary vehicles that are going to meet at a satellite location and the due dates for the secondary vehicles are known. The due date of a secondary vehicle, i.e. the latest times by which it has to leave the satellite, is determined by its routing plan in the second echelon. This information can be obtained by solving the two-echelon location-routing problem introduced earlier in Chapter 4.

The satellite synchronization problem (SSP) aims to assign secondary vehicles to the primary vehicles and determine arrival and departure times of both types of vehicles at a given satellite location such that minimum congestion is observed. We define congestion as the amount of time that two vehicles are simultaneously present at the satellite location. In this problem, all secondary vehicles need to be served and leave the satellite location before their due dates. We assume that a primary vehicle can serve multiple secondary vehicle therefore the load of a secondary vehicle is not split between primary vehicles. We also assume that a primary vehicle can serve secondary vehicles one at a time. This is a reasonable assumption due to limited space and human resources available at the satellite location. Besides, primary vehicles are medium-sized trucks (and not trailers); hence, the limited room and space behind them does not allow serving multiple vehicles at a time. Since no waiting time is favorable, once a primary vehicle arrives at a satellite location, it should start serving secondary vehicles without a delay, and leave the satellite as soon as possible. Since primary vehicles serve only one secondary vehicles at a time, for each primary vehicle there can be at most one secondary vehicle at the satellite location. A service done by a primary vehicle consists of the following operations in order: (i) The vehicle arrives to the satellite location, (ii) the freight is unloaded from the primary vehicle and loaded into a set of secondary vehicles (the trans-docking operation), (iii) the primary vehicle leaves the satellite. We call the place where a service is done as dock. In other words, a dock is an area where a primary vehicle is parked to serve one or
more secondary vehicles. Note that there can be at most two vehicles at each dock at a time: one primary vehicle unloading the freight and one secondary vehicle being loaded. In order to minimize the congestion at the satellite, we minimize the time during which two docks are simultaneously used. More precisely, we can minimize the time during which two services overlap. By scheduling the exact arrival and departure times of the vehicles, the SSP solution prevents forming vehicle queue in front of a satellite location.

We provide two different formulations for the SSP. The first formulation uses arc-flow variables, where we consider continuous time space to schedule vehicles. The second formulation, on the other hand, discretizes time dimension into a finite number of time slots with equal lengths.

5.2.1 Arc–Flow Formulation

Let $U$ and $I$ be the set of primary and secondary vehicles, respectively. Let $N_1 = |U|$ and $N_2 = |I|$ be the number of primary and secondary vehicles assigned to the satellite location, respectively. Primary vehicles are assumed to be homogeneous with a capacity equal to a given value $Q$. Denote $D_i$ and $T_i$ as the total load to be delivered by vehicle $i \in I$ and the time required to serve (i.e. to load) this vehicle, respectively. Let $L_i$ be the due date of vehicle $i \in I$, i.e., the latest time by which it needs to leave the satellite. Let $0$ ($0'$) be an auxiliary index denoting the start (end) of a service. Consider auxiliary parameters $D_0 = D_0' = 0$, $T_0 = T_0' = 0$, and $L_0 = L_0' = \infty$.

For any secondary vehicle $i \in I$, define $A^-(i) = \{j \in I \cup \{0\} : j \neq i, T_j \leq L_i, D_j + D_i \leq Q\}$ as the set of vehicles that can be served before vehicle $i$. Similarly, define $A^+(i) = \{j \in I \cup \{0'\} : j \neq i, T_i \leq L_j, D_i + D_j \leq Q\}$ as the set of vehicles that can be served after vehicle $i$. Let $A^+(0) = A^-(0') = I$. The following decision variables are defined. Let $x_{uij}$ be a binary decision variable that equals to 1 if vehicle $u \in U$ serves vehicle $i \in I$ immediately after vehicle $j$, $\forall j \in A^+(i)$, and 0 otherwise. Therefore, variable $x_{u0i}$ ($x_{ui0'}$) takes a value if and only if vehicle $i \in I$ is the first (last) vehicle served by vehicle $u$. Let $t_{ui}$ be a non-negative variable equal to the time at which vehicle $u \in U$ starts serving vehicle $i \in I$. $t_{u0}$ ($t_{0i}$) denotes the arrival (departure) time of vehicle $u$ to (from) the satellite location. Let $s_{uv}$ be a non-negative
variable equal to the amount of service time overlap between two primary $u, v \in \mathcal{U}$. We consider $u < v$ as $s_{uv} = s_{vu}$ by definition. Figure 5.2 illustrates a service time overlap between two primary vehicles.

![Figure 5.2: Illustration of a service time overlap of two primary vehicles $u$ and $v.$](image)

The arc-flow formulation (AFF) of the SSP is given below.

\[
\min \sum_{u \in \mathcal{U}} \sum_{v \in \mathcal{U}: u < v} s_{uv} \tag{5.1}
\]

subject to

\[
\sum_{u \in \mathcal{U}} \sum_{j \in A^+(i)} x_{u ji} = 1, \quad \forall i \in \mathcal{I} \tag{5.2}
\]

\[
\sum_{i \in \mathcal{I}} x_{u i 0} = \sum_{i \in \mathcal{I}} x_{u i 0'} = 1, \quad \forall u \in \mathcal{U} \tag{5.3}
\]

\[
\sum_{j \in A^-(i)} x_{u ji} - \sum_{j \in A^+(i)} x_{uj} = 0, \quad \forall u \in \mathcal{U}, i \in \mathcal{I} \tag{5.4}
\]

\[
\sum_{i \in \mathcal{I}} \sum_{j \in A^+(i)} D_i x_{uij} \leq Q, \quad \forall u \in \mathcal{U} \tag{5.5}
\]

\[
x_{uij} (t_{ui} + T_i - t_{uj}) \leq 0, \quad \forall u \in \mathcal{U}, i \in \mathcal{I} \cup \{0\}, j \in A^+(i) \tag{5.6}
\]

\[
\min \{t_{u 0'}, t_{v 0'}\} - \max \{t_{u 0}, t_{v 0}\} \leq s_{uv}, \quad \forall u, v \in \mathcal{U}: u < v \tag{5.7}
\]

\[
s_{uv} \geq 0, \quad \forall u, v \in \mathcal{U}: u < v \tag{5.8}
\]

\[
0 \leq t_{ui} \leq L_i - T_i, \quad \forall u \in \mathcal{U}, i \in \mathcal{I} \cup \{0, 0'\} \tag{5.9}
\]

\[
x_{uij} \in \{0, 1\}, \quad \forall u \in \mathcal{U}, i \in \mathcal{I} \cup \{0\}, j \in A^+(i). \tag{5.10}
\]

Objective function (5.1) minimizes the sum of service time overlaps between all pairs of primary vehicles. Constraint (5.2) guarantees that all secondary vehicles are served. Constraint (5.3) ensures that all primary vehicles arrive to and depart from the satellite location. Constraint (5.4) equates the number of predecessors of a secondary vehicle in service to the number of its successors. Capacity of primary vehicles are satisfied by constraint (5.5). Constraint (5.6) schedules the vehicles based on their or-
der and required service time. Constraints (5.7) and (5.8) provide a lower bound for service time overlap between any two primary vehicles. Constraints (5.9) and (5.10) meet variable domains.

Constraint (5.6) can be replaced by a linear constraint (5.11).

\[ t_{ui} + L_i x_{ui'} - t_{ui'} \leq L_i - T_i, \quad \forall u \in U, i \in I \cup \{0\}. \]  

(5.11)

Note that the term \( \gamma = \min\{\alpha, \beta\} \) for any set of variables \( \alpha, \beta, \gamma \) can be represented by the following set of linear constraints:

\[ \alpha - Mb \leq \gamma \leq \alpha, \]  

(5.12)

\[ \beta - M (1 - b) \leq \gamma \leq \beta, \]  

(5.13)

where, \( M \) is a sufficiently large number and \( b \) is a binary variable. Similarly, \( \gamma = \max\{\alpha, \beta\} \) is formulated as follows:

\[ \alpha \leq \gamma \leq \alpha + Mb, \]  

(5.14)

\[ \beta \leq \gamma \leq \beta + M (1 - b). \]  

(5.15)

Therefore, constraint (5.7) can be represented as a set of linear constraints based on (5.12)–(5.15). The \( M \) value in these constraints can be set to \( \max_{i \in I} \{T_i\} \).

The AFF (5.1)–(5.10) is a difficult optimization problem due to large number of binary variables, elementary routing constraints, capacity restrictions, big-M values, and symmetry. One can reduce symmetry in the AFF, by adding the following constraint to sort the primary vehicle departure times.

\[ t_{u0'} \leq t_{v0'}, \quad \forall u, v \in U : v = u + 1. \]  

(5.16)

### 5.2.2 Time–Indexed Formulation

The time-indexed formulation (TIF) discretizes the time dimension into a finite number of time intervals. TIFs has been shown to provide good bounds when solving single machine scheduling problems (see, for example, Dyer & Wolsey 1990, van den Akker et al. 2000, Edis et al. 2013). In this section, we propose a TIF for the SSP considering congestion minimization objective function and capacity and due date restrictions. Define \( H = \{0, 1, \cdots, B\} \) as the set of time indices, where \( B \) is the index
corresponding the time interval after which the satellite is closed. We assume all time
intervals have a length equal to \(\Delta\). Let \(P_i = \lceil T_i / \Delta \rceil\) be the number of time intervals
required to load vehicle \(i \in I\). Let \(H_i\) be the index of the time interval by which vehicle \(i \in I\) has to leave the satellite location. Define \(y_{tui}\) as a binary decision variable indicating whether or not vehicle \(u \in U\) starts serving vehicle \(i \in I\) at time \(t \in H\).

Let binary decision variable \(z_{tu}\) equal to 1 if and only if vehicle \(u \in U\) presents at
the satellite location at time \(t \in H\). Define \(z_{tu}^-\) and \(z_{tu}^+\) as binary variables to indicate whether there is a secondary vehicle assigned to primary vehicle \(u\) before and after
time \(t\), respectively. Let binary variable \(w_{tuv}\) equal to 1 if primary vehicles \(u\) and \(v\)
are both present at the satellite location at time \(t \in H, u, v \in U\), and 0 otherwise.

The time-indexed formulation (TIF) of the SSP is given below.

\[
\begin{align*}
\text{Min} & \quad \sum_{t \in H} \sum_{u \in U} \sum_{v \in U: u < v} \Delta w_{tuv} \\
\text{s.t.} & \quad \sum_{t \in H} \sum_{i \in I} D_i y_{tui} \leq Q, \quad \forall u \in U \quad (5.18) \\
& \quad y_{tui} + \sum_{r \in H: r \leq t + P_i} y_{ruj} \leq 1, \forall t \in H, u \in U, i \in I, j \in I \setminus \{i\} \quad (5.19) \\
& \quad \sum_{t \in H} \sum_{u \in U} y_{tui} = 1, \quad \forall i \in I \quad (5.20) \\
& \quad y_{rui} \leq z_{tu}, \quad \forall t \in H, u \in U, i \in I, r \in H : t - P_i < r \leq t \quad (5.21) \\
& \quad y_{rui} \leq z_{tu}^-, \quad \forall t \in H, u \in U, i \in I, r \in H : r < t \quad (5.22) \\
& \quad y_{rui} \leq z_{tu}^+, \quad \forall t \in H, u \in U, i \in I, r \in H : r \geq t \quad (5.23) \\
& \quad z_{tu}^- + z_{tu}^+ - z_{tu} \leq 1, \quad \forall t \in H, u \in U \quad (5.24) \\
& \quad w_{tuv} \leq z_{tu}, \quad \forall t \in H, u, v \in U : u < v \quad (5.25) \\
& \quad w_{tuv} \leq z_{tv}, \quad \forall t \in H, u, v \in U : u < v \quad (5.26) \\
& \quad z_{tu} + z_{tv} - w_{tuv} \leq 1, \quad \forall t \in H, u, v \in U : u < v \quad (5.27) \\
& \quad y_{tui} \leq 0, \quad \forall u \in U, i \in I, t \in H : t > H_i - P_i \quad (5.28) \\
& \quad y_{tui} \in \{0, 1\}, \quad \forall t \in H, u \in U, i \in I \quad (5.29) \\
& \quad z_{tu}, z_{tu}^-, z_{tu}^+ \in \{0, 1\}, \quad \forall t \in H, u \in U \quad (5.30) \\
& \quad w_{tuv} \in \{0, 1\}, \quad \forall t \in H, u, v \in U : u < v. \quad (5.31)
\end{align*}
\]

Objective function (5.17) minimizes the sum of service time overlaps between primary vehicles. Constraint (5.18) satisfies the capacity of the primary vehicles. By
(5.19), we forbid assigning a new secondary vehicle to a primary vehicle if there is already a secondary vehicle being served. Constraint (5.20) ensures that a secondary vehicle is assigned to exactly one primary vehicle at exactly one time slot. Constraints (5.21)–(5.24) link primary and secondary vehicle variables. They ensure that a primary vehicle remains present in the satellite location at time $t$ when either it is serving a secondary vehicle at this time, or it has served a secondary vehicle by this time and is waiting to serve another one. Constraints (5.25)–(5.27) force the service time overlap variable $w_{tuv}$ to take a value if both primary vehicles $u$ and $v$ are present at the satellite location at time $t$. Constraint (5.28) satisfies the due date of secondary vehicles. Finally, constraints (5.29)–(5.31) define variable domains. Note that we can relax binary requirements of $w_{tuv}$ variables as it is implicitly induced by objective function (5.17) and constraints (5.25)–(5.27). Therefore, constraint (5.31) can be replaced by the following constraint.

$$w_{tuv} \geq 0, \quad \forall t \in \mathcal{H}, u, v \in \mathcal{U} : u < v. \quad (5.32)$$

One advantage of TIF is that it does not require any big-$M$ values. However, it contains a large number of constraints. TIF is also symmetric over $u$ indices since all first echelon vehicles are identical. This can make the problem difficult to solve as $N_1$ becomes large. In the reminder of this section, we propose an alternative formulation to the TIF (5.17)–(5.31), denoted by TIF$, that can overcome some of the above disadvantages.

The symmetry of TIF can be reduced by replacing constraint (5.20) by the following constraints.

$$\sum_{t \in \mathcal{H}} \sum_{u \in \mathcal{U}, w \leq \min\{i, N_1\}} y_{tui} = 1, \quad \forall i \in \mathcal{I} \quad (5.33)$$

$$y_{tui} \leq 0, \quad \forall t \in \mathcal{H}, i \in \mathcal{I}, u \in \mathcal{U} : u > \min\{i, N_1\}. \quad (5.34)$$

Constraints (5.33) and (5.34) imply that the $1^{st}$ secondary vehicle is always assigned to the $1^{st}$ primary vehicle, the $2^{nd}$ secondary vehicle is assigned to either the $1^{st}$ or the $2^{nd}$ primary vehicle, and so on. If the index of the secondary vehicle is greater than $N_1$, constraint (5.33) behaves like (5.20) and (5.34) is not applied.

We can also apply variable aggregation on constraint sets (5.21)–(5.23) and reduce
the number of constraints as follows:

\[
\sum_{i \in I} \sum_{r \in H: t - p_i \leq t} y_{rui} \leq z_{tu}, \quad \forall t \in H, u \in U
\]  
(5.35)

\[
\sum_{i \in I} \sum_{r \in H: r < t} y_{rui} \leq M z_{tu}^-, \quad \forall t \in H, u \in U
\]  
(5.36)

\[
\sum_{i \in I} \sum_{r \in H: r \geq t} y_{rui} \leq M z_{tu}^+, \quad \forall t \in H, u \in U
\]  
(5.37)

where, \( M \) is set to \( N_2 \). We refer problem formulation (5.17)–(5.19) and (5.24)–(5.37) as TIF*.

All three formulations, namely, the AFF, the TIF, and the TIF*, can be solved using standard MIP solvers. In the next section, we provide computational results of solving a set of problem instances using the above formulations and discuss their advantages and disadvantages.

5.3 Computational Experiments

This section presents a computational study on the satellite synchronization problem introduced in this chapter. Our computational study consists of three parts. In Section 5.3.2, we compare the performance of the formulations proposed in Section 5.2 on a set of problem test instances. Section 5.3.3 investigates the effect of problem instance characteristics, such as due dates and size, on the final solution. In Section 5.3.4, we consider the SSP under a more realistic situation to answer the following question through a simulation study: How does the system cope with the delays in arrival times and/or service time of the second echelon vehicles?

5.3.1 Problem Instances

We generate three sets of problem test instances as follows. In each set, random service time and due dates are assigned to the second echelon vehicles. The due dates are either early or late, both from a uniform distribution. Late due date (LD) values have larger mean than early due date (ED) values, providing more freedom to serve the second echelon vehicles. Test instances have a size indicated by \( N_1 - N_2 \).
where $N_1 \in \{2, 3\}$ and $N_2 \in \{9, 15, 20\}$ or $N_1 \in \{4, 5\}$ and $N_2 \in \{15, 20, 24\}$. Our three sets contain test instances with similar sizes but different service time and due date values. The capacity of the primary vehicles are set to $Q = \lceil \sum_{i \in I} D_i / K \rceil$, where $K$ is a given parameter. We consider $K \in \{N_1, N_1 - 1\}$ in order to investigate the effect of $Q$ on the final solution. Smaller $K$ value yield larger primary vehicle capacities. We assume that one unit of product takes one unit of time to be unloaded from a primary vehicle and loaded into a secondary vehicle, i.e., $D_i = T_i, \forall i \in I$. All constant values, i.e. vehicle service times, loads, and due dates are generated as multiples of 10. Therefore, we can set the time period length as $\Delta = 10$. In total, 108 instances are generated in three sets. All instance data files are available in https://gitlab.com/pharham/test-instances.

All computations are done on a Linux workstation with Intel® Xeon 4 × 3.20GHz processors and 16GB memory. We used ILOG CPLEX 12.10 as the MIP solver. A time limit of 2.5 hours is provided to solve each problem test instance.

5.3.2 Comparisons of Different Problem Formulations

In this section, we present numerical results of solving SSP instances using the AFF, the TIF, and the TIF*. We select 12 instances in each set with different characteristics and group them based on their due date type, size, and capacity parameter ($K$). For each group, the following performance measures are used: (i) $\#opt$: number of optimal solutions found (out of three); (ii) $Av\ %dev$ from best: average percentage deviation of the objective function value from the best value reported by any of the approaches; (iii) $Av\ gap\ (%)$: average percent MIP gap reported by the solver; (iv) $av\ #BB\ nodes$: average number of processed nodes in the branch-and-bound tree; (v) $av\ time\ (s)$: average computational time (in seconds) spent by the solver to return a solution. The results are provided in Table 5.1. Note that each row represents the results over three instances (one from each data set). The overall results for ED and LD instances are provided in the last row of each group. The results indicate that although AFF returned a solution identical to the optimal one in the majority of cases, it fails to close the MIP gap for many medium and large size problem instances. AFF requires less computational time, explores less branch-and-bound nodes, and reports
Table 5.1: Performance comparison of the three problem formulations.

<table>
<thead>
<tr>
<th>Due Size</th>
<th>#opt</th>
<th>Av %dev from best</th>
<th>Av gap (%)</th>
<th>Av #BB nodes</th>
<th>Av time (s)</th>
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</thead>
<tbody>
<tr>
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<td>TIF</td>
<td>TIF*</td>
<td>AFF</td>
<td>TIF</td>
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<tr>
<td>2-9 (2)</td>
<td>3</td>
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<tr>
<td>3-15 (3)</td>
<td>2</td>
<td>3</td>
<td>11.11</td>
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<td>4-15 (3)</td>
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<td>LD total</td>
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<tr>
<td>Grand total</td>
<td>33</td>
<td>36</td>
<td>36</td>
<td>3.03</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

smaller MIP gap values when solving LD instances, compared to ED ones. When due dates are early, the feasible region of the AFF becomes more restrictive and the solver needs to investigate more options before proving optimality. In contrast to the AFF, both TIF and TIF* require more computational effort in solving LD instances. LD instances lead to wider time horizons, hence more time intervals for the time-indexed formulations are considered. This increases the number of variables and constraints in these formulations, leading to a higher computational complexity. TIF shows better performance than AFF by reporting smaller MIP gap values, exploring less #BB nodes, and finding the solutions faster, on average. However, it fails to close the MIP gap for two LD instances with size 4-15. TIF*, on the other hand, solves all instances to optimality and finds solutions faster than TIF.

The above experiment indicates that the time-index formulations introduced in Section 5.2.2 can provide more efficient solutions than the AFF presented in Section 5.2.1. The results also verify that the TIF is improved by using symmetry reduction and constraint aggregation applied through constraints (5.33)–(5.37) in the TIF*.
5.3.3 Numerical Results of the TIF*

In this section, we solve all problem instances by using the TIF* as it shows superior performance over both AFF and TIF. Our numerical experiments are conducted to analyze the effect of different problem instance characteristics on the final solution and the computational time. More specifically, we seek the answer to the following questions: (i) How does the congestion at a satellite location is affected if more vehicles are assigned to that satellite? (ii) What is the trade-off of using smaller vehicles versus larger vehicles in the first echelon? (iii) How the space at the satellite (i.e. the number of docks) has to be managed in order to minimize the congestion under early and late due date settings?

The problem instances in this section are grouped based on the due date type, the number of primary vehicles, and the capacity parameter of the primary vehicles ($K$). Table 5.2 and Table 5.3 show the results of solving ED and LD problem instances, respectively. Each row of the table presents the result of three test instances (one from each set). $Av \ obj \ val$ shows the average objective function value and $Av \ #docks$ ($Max \ #docks$) indicates the average (maximum) number of docks that are simultaneously open (i.e. the average number of services being done at the same time).

The results indicate that for a given number of primary vehicles ($N_1$), when the number of secondary vehicles assigned to a satellite ($N_2$) increases, both congestion and $#docks$ increase. It is expected since the primary vehicles need to spend more time in the satellite location to serve the additional number of secondary vehicles. The effect of primary vehicle capacity is more significant for ED instances. On average, when $Q$ decreases (i.e. $K$ increases), more congestion is observed and more docks are needed to perform trans-docking operations. Smaller primary vehicles can serve less number of secondary vehicles. Therefore, more of them need to be present simultaneously at the satellite location in order to serve all secondary vehicles on time. On average, instances with larger $Q$ value are less restrictive and are solved faster. When due dates are late, there is more freedom to schedule secondary vehicles. Therefore, compared to ED instances, LD ones lead to less congestion and less number of docks on average. However, solving LD is more difficult as more time intervals exist.

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The above experiment concludes that in the following cases the decision maker should consider larger satellites with more available docks: (i) When due dates are early, (ii) when there are more vehicles assigned to a satellite location, (iii) when the primary vehicles have small capacity. If the available space at the satellite location is limited, the decision maker needs to assign fewer customers to the current satellite, open a new satellite, or relocate some secondary vehicles to the satellites that emit less congestion.
Table 5.3: Numerical result of LD instances solved by TIF*.

<table>
<thead>
<tr>
<th>$N_1$ ($K$)</th>
<th>$N_2$</th>
<th>Av obj val</th>
<th>Av #docks</th>
<th>Max #docks</th>
<th>Av gap (%)</th>
<th>Av #BB nodes</th>
<th>Av time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 (2)</td>
<td>15</td>
<td>1.83</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0.3</td>
<td>2.3</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>73.33</td>
<td>1.33</td>
<td>2</td>
<td>0</td>
<td>1.3</td>
<td>2.3</td>
</tr>
<tr>
<td>2 (2) total</td>
<td></td>
<td>27.78</td>
<td>1.22</td>
<td>2</td>
<td>0</td>
<td>1.3</td>
<td></td>
</tr>
<tr>
<td>3 (2)</td>
<td>15</td>
<td>1.83</td>
<td>2</td>
<td>0</td>
<td>3.67</td>
<td>2.9</td>
<td>6.8</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>73.33</td>
<td>1.33</td>
<td>2</td>
<td>0</td>
<td>9.67</td>
<td>6.8</td>
</tr>
<tr>
<td>3 (2) total</td>
<td></td>
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<td>1.22</td>
<td>2</td>
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<td>4.44</td>
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<td>75.22</td>
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<td>2</td>
<td>0</td>
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<td>1,606.6</td>
<td></td>
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<tr>
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<td>1.33</td>
<td>2</td>
<td>0</td>
<td>1,108</td>
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</tr>
<tr>
<td></td>
<td>24</td>
<td>166.67</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>17,024.33</td>
<td>2,608.5</td>
</tr>
<tr>
<td>4 (3) total</td>
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<td>83.33</td>
<td>1.56</td>
<td>2</td>
<td>0</td>
<td>17,164.44</td>
<td>1,411.5</td>
</tr>
<tr>
<td>4 (4)</td>
<td>15</td>
<td>1.83</td>
<td>2</td>
<td>0</td>
<td>5.700</td>
<td>256.3</td>
<td></td>
</tr>
<tr>
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<td>20</td>
<td>73.33</td>
<td>1.33</td>
<td>2</td>
<td>0</td>
<td>27.33</td>
<td>11.7</td>
</tr>
<tr>
<td></td>
<td>24</td>
<td>166.67</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>2,032.33</td>
<td>80.4</td>
</tr>
<tr>
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<td>2</td>
<td>0</td>
<td>2,586.56</td>
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</tr>
<tr>
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<td>2</td>
<td>0</td>
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<td>3,003.0</td>
</tr>
<tr>
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<td>73.33</td>
<td>1.33</td>
<td>2</td>
<td>0</td>
<td>2,359</td>
<td>85.8</td>
</tr>
<tr>
<td></td>
<td>24</td>
<td>166.67</td>
<td>2</td>
<td>2</td>
<td>66.67</td>
<td>44,005</td>
<td>6,037.7</td>
</tr>
<tr>
<td>5 (5) total</td>
<td></td>
<td>83.33</td>
<td>1.56</td>
<td>2</td>
<td>0</td>
<td>29,759.33</td>
<td>3,042.2</td>
</tr>
<tr>
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<td></td>
<td>55.56</td>
<td>1.39</td>
<td>2</td>
<td>0</td>
<td>8,265</td>
<td>763.2</td>
</tr>
</tbody>
</table>

5.3.4 The SSP Under Arrival Time and Service Time Delays

The SSP defined in this chapter assumes that the due dates and service times of the secondary vehicles are determined in advance. Based on these data, the problem finds the assignments of the secondary vehicles to the primary vehicles and schedules arrival and departure times of the vehicles from/to the satellite location. However, reaching a high service quality level in real-life situations is a challenging issue in the SSP. First, adjusting the service times only based on vehicle loads might not reflect the actual service times observed in the system. Since serving a second echelon vehicle requires physical activities and human interactions, it is natural to think of uncertain delays in service times. Such delays might be the result of human errors, unavailability of workers, equipment failures, etc. Second, the drivers might not fol-
low the exact arrival times proposed by the SSP solution. Reaching an inner-city satellite location on time is highly dependent on the dynamic and stochastic nature of urban traffic network. Therefore, one should also consider the inevitable delays in secondary vehicle arrival times while solving the problem.

In this section, we investigate the effect of delays on the SSP solution. We first show how the delays discussed above can be both interpreted as a service time delay, and then, present a simulation based study for this purpose. Figure 5.3 illustrates all possible cases where the arrival time and service time of a secondary vehicle is delayed. Case 0 is the deterministic case where a specific secondary vehicle arrives at and leaves the satellite at the scheduled times. In this case, the vehicle arrives at time 2, waits for 4 time units to be served, and leaves the satellite at time 6. In Case 1, there is a delay of 1 time unit in the vehicle’s arrival time. Since no service time delay is observed in this case, the departure time of the vehicle is shifted to the right by 1 unit. In Case 2, the vehicle arrives on time, but it takes 1 time unit more than it was expected to serve it. As a result, the vehicle departs the satellite at time 7 instead of 6. Case 3 illustrates the situation where both arrival time and service time of the vehicle are delayed. In this case, the vehicle departure time is delayed by the sum of arrival time and service time delays, i.e. 2 time units.

Note that both types of the delays affect only the departure time. Therefore, we represent both as a single delay type and assume that only the service time is delayed. That is, any delay observed for secondary vehicle \( i \) is reflected as an increment in \( T_i \). Therefore, it is sufficient to modify the original instance data by changing the service times and generate new data to reflect different delay scenarios.

We assume that the delays follow a truncated normal distribution, \( \text{Norm}(\alpha, \beta, \mu, \sigma) \), in interval \([\alpha, \beta]\) with mean \( \mu \) and standard deviation \( \sigma \). Two levels for each delay type is considered. Table 5.4 shows the distribution and Figure 5.4 illustrates the probability density function of the delays under each level. Let AD (SD) be the amount of arrival time (service time) delay of a secondary vehicle \( i \) obtained from the truncated normal distribution given in Table 5.4. Then, the anticipated amount of delay is set to the nearest integer to \( \Delta \times (\text{AD} + \text{SD}) \) and is added to \( T_i \). The rounded value is used in order to keep \( \Delta \) as a valid time interval length.
We consider two sets of instances with different sizes (see Section 5.3.1). For each instance, we assume that $ND$ percent of the total secondary vehicles are delayed, $ND \in \{10, 25\}$. For example, if $N_2 = 15$ and $ND = 10$, we select $\lceil \frac{10}{100} \times 15 \rceil = 2$ of the second echelon vehicles randomly, and assign a delay value to each. The delay amount is calculated by generating $AD$ and $SD$ from level set $\{H, L, O\}$, where level $O$ denotes zero delay. For example, the delay configuration $LO$ indicates that $AD$ is generated from the truncated normal distribution with $L$ attributes (see Table 5.4), whereas no delay is assigned to the service time. Therefore, the delayed time is equal to the nearest integer to $10 \times AD + 0$ for the selected secondary vehicle.

Table 5.4: Distribution of delays at each level.

<table>
<thead>
<tr>
<th></th>
<th>Low ($L$)</th>
<th>High ($H$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arrival time delay</td>
<td>$Norm(3, 5, 4, 1)$</td>
<td>$Norm(5, 8, 6, 1.5)$</td>
</tr>
<tr>
<td>Service time delay</td>
<td>$Norm(0, 1, 0.75, 0.5)$</td>
<td>$Norm(1, 2, 1.75, 0.5)$</td>
</tr>
</tbody>
</table>
Figure 5.4: Distribution of the delays at each level.

Table 5.5: Number of unsolved ED instances under different delay scenarios.

<table>
<thead>
<tr>
<th>Size</th>
<th>Delays</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ND (K)</td>
</tr>
<tr>
<td>10</td>
<td>2-15 (2)</td>
</tr>
<tr>
<td>10</td>
<td>3-15 (2)</td>
</tr>
<tr>
<td>10</td>
<td>3-20 (2)</td>
</tr>
<tr>
<td>10</td>
<td>3-20 (3)</td>
</tr>
<tr>
<td>10</td>
<td>4-20 (4)</td>
</tr>
<tr>
<td>10 total</td>
<td>5</td>
</tr>
<tr>
<td>25</td>
<td>2-15 (2)</td>
</tr>
<tr>
<td>25</td>
<td>3-15 (2)</td>
</tr>
<tr>
<td>25</td>
<td>3-20 (2)</td>
</tr>
<tr>
<td>25</td>
<td>3-20 (3)</td>
</tr>
<tr>
<td>25</td>
<td>4-20 (4)</td>
</tr>
<tr>
<td>25 total</td>
<td>5</td>
</tr>
<tr>
<td>Grand total</td>
<td>5</td>
</tr>
</tbody>
</table>

OO represents the base instance with no delays. Since vehicle selection, AD, and SD are random, we replicate our experiment as follows. First, two replication of random vehicle selections are done. Then, for each vehicle three replications of a delay configuration is performed. Therefore, for each combination of due dates, $N_1$.  

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$N_2$, $ND$, and a delay configuration, six instances are solved in each set.

High amount of delays can make the problem infeasible as the delayed vehicles might not be able to reach their customers on time. The delays can harm the system more when we have early due dates. In our experiments, all delayed LD instances are solved, whereas some of the ED instances became infeasible under delays. Table 5.5 shows the number of unsolved ED instances under different delay configurations. Since there are two instance groups, we have 12 instances, in total, under each delay configuration. As expected, more infeasibility issues are encountered for the higher $ND$ value. AD values affect the solution more than SD values, as the arrival delays are larger than the service time delays. When there are less number of primary vehicles, it is more likely that we encounter infeasible solutions under delays. The reason is that the maximum number of secondary vehicles that are served simultaneously at the satellite location is smaller. Hence, the available time for primary vehicles are more limited and more secondary vehicle schedules are postponed. If the number of secondary vehicles are considerably more than the number of primary vehicles, such deferral may cause some vehicles to miss their due dates. Table 5.5 also shows that for a fixed number of primary vehicles, more infeasible instances are observed if the capacity of primary vehicles are smaller.

The next experiment compares the affect of different delay configurations on congestion and on the required number of docks. Here, we only consider the problem instances that are feasible under all delay configurations. Table 5.6 and Table 5.7 present the numerical results of solving the instances in the first and second set, respectively. For a group of instances with a given size, a capacity parameter, and a percentage of delayed vehicles, these tables present the average objective function value and the average number of docks found by the TIF*. #Inst shows the number of instances (out of 6 replications) solved in each group (row).

The results indicate that the average amount of congestion and the average number of docks do not decrease when the delay times increase, i.e. when we move from left to right in Table 5.6 and Table 5.7 in each row. In order to avoid service time overlaps, the solutions keep the average #docks as low as possible and close to the original case even if the congestion increases. However, if the amount of delays are large enough,
additional docks are required to serve the secondary vehicles on time. For example, for the instance group 3-20 (2) with early due dates and ND = 25, the average #docks is kept as 2 for all delay times ranging from OO to LL, even though the congestion increases in this range. However, when the delays are LH or larger, the optimal number of docks (i.e. 2) is not sufficient and the solutions suggest more docks. In many cases that report no congestion under the OO case, nonzero congestion is perceived when more delays are detected. The average #docks also increases significantly when high amount of delays take place. Therefore, solving the problem under the OO configuration might not provide an affordable solution for the time that delays are anticipated. The results show that for a given delay configuration, the amount of congestion and #docks increase when more secondary vehicles are assigned to the satellite, more vehicles are delayed, or the primary vehicles are smaller. Note that although smaller Q values can yield more congestion, the congestion increase can be avoided if more primary vehicles are used. For example, consider rows 4 and 5 in Table 5.6. The average objective function value decreases when the number of primary vehicles increases by 1, even though Q is less.

The above numerical experiments suggest the following ways to keep the congestion at a satellite location low and avoid allocating space for new docks when delays are expected: (i) Use more primary vehicles to deliver freight to the satellite location; (ii) use larger primary vehicles with more capacity; (iii) if the satellite can not be expanded to provide more docks under medium or high delays, open a new satellite and/or reassign secondary vehicles to the satellites with available space.

5.4 Concluding Remarks

Despite the critical role of synchronization problem in the multi-echelon freight distribution systems in the operational-level planing, the satellite synchronization problem has not received much attention in the literature. In this chapter, we define the SSP to deal with trans-docking operations in the intermediate facilities. Two formulations, namely the arc-flow formulation and the time-indexed formulation, each with a different approach to the problem, are proposed for the SSP. We showed that the time-indexed formulation is stronger and finds a solution more efficiently compared
Table 5.6: Objective function values of Set 1 instances under arrival and service time delays.

<table>
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<th>Size</th>
<th># Av obj val (#blocks)</th>
<th>Inst</th>
<th>Curve function ND (#K)</th>
<th>Date</th>
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<th>3-15 (2)</th>
<th>3-20 (2)</th>
<th>3-20 (3)</th>
<th>4-20 (4)</th>
</tr>
</thead>
<tbody>
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<td></td>
<td></td>
<td></td>
<td></td>
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<td>0.00 (1.00)</td>
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<td>80.00 (2.00)</td>
<td>80.00 (2.00)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>53.33 (2.00)</td>
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<td>173.33 (2.00)</td>
<td>163.33 (2.00)</td>
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<td></td>
<td></td>
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<td></td>
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</table>

To the arc-flow formulation. SSP solutions can provide a feedback to the upper-level decision makers by considering the operational level environment. In order to reflect realistic situations, we conducted simulation-based analysis to show how the arrival time and service time delays affect the SSP solution. As delays are inevitable in (multi-modal) transportation networks, such analysis provides valuable information that can be used to expand the satellite capacity, open new satellites, or reassign vehicles to the satellite locations in the upper-level strategic problem when needed. We believe that the hierarchical combination of the 2E-LRPTW and SSP provides a reliable approach to deal with the realistic two-echelon freight transportation problems in city logistics.
Table 5.7: Objective function values of Set 2 instances under arrival and service time delays.

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</thead>
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<td>O0</td>
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<td>376.67 (2.00)</td>
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<td>376.67 (2.00)</td>
</tr>
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<td>30.00 (2.00)</td>
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<td>226.67 (2.00)</td>
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<tr>
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<td>6</td>
<td>220.00 (2.00)</td>
<td>226.67 (2.00)</td>
</tr>
<tr>
<td>4-20 (4)</td>
<td>6</td>
<td>220.00 (2.00)</td>
<td>226.67 (2.00)</td>
</tr>
<tr>
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<td>326.67 (2.20)</td>
</tr>
<tr>
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<td>354.33 (2.64)</td>
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<tr>
<td>LD total</td>
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<td>174.00 (2.00)</td>
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<tr>
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<td>228.81 (2.12)</td>
</tr>
</tbody>
</table>
City logistics (CL) has received growing attention due to its potential to improve freight transportation in complex and highly dynamic city environments. From a network design point of view, CL systems include distribution and consolidation centers, customers, and a transportation network through which goods are moved. In order to provide a sustainable solution for the urban freight transport, city logistics practitioners need to consider the interests and constraints of the involved stakeholders and take the city characteristics and environmental impacts of the transportation into account. Through a comprehensive literature review, we identified the gaps in CL literature in terms of formulating and solving freight distribution problems. We presented mathematical models for one and two-echelon urban freight transportation problems in CL, which not only consider carrier’s cost minimization objective, but also customer due dates as well as regulations imposed by the government (such as access time windows, and vehicle load restrictions). At the operational level resolution, CL systems need careful management of flows at the intermediate facilities, where the goods need to be transferred from primary vehicles to secondary vehicles without stocking. We introduced satellite synchronization problem that minimizes the congestion (hence, vehicle queues) confronted at a satellite location.

6.1 Major Findings

The one-echelon urban freight transportation system is formulated as a capacitated location-routing problem with time windows (LRPTW). The LRPTW involves strategic and tactical level decisions, namely the number and the location of consolidation
centers (i.e., CDCs), customer-to-CDC assignments, the number of vehicles needed for freight delivery, and vehicle routes and schedules. The problem is constrained by customer due dates and access time windows, CDC capacities, and vehicle load limits. The objective is to minimize total transportation cost as the sum of facility opening, vehicle utilization, and traveling costs such that all customer demands are met. The LRPTW is \(\text{NP}\)-hard as it generalizes the traveling salesman problem. We proposed an exact solution approach that implements branch-and-price over the path-based formulations of the problems. The algorithm prices out vehicle paths found by a column generation procedure and branches over the fractional variables to reach the optimal integer solution. We used dynamic programming enhanced by various improvement techniques to generate columns efficiently. Our exact approach was able to find optimal solutions to the problem instances with up to 5 candidate CDC locations and 50 customers. In total, we were able to solve 72\% of our 138 test instances optimally and report an average MIP gap of 4.73\% for the ones that are not solved in the available time. In order to solve the problem instances more efficiently, we proposed a two-stage heuristic approach. In the first stage, the strategic decisions of the LRPTW (i.e., CDC locations) are fixed by solving a facility location problem. In the second stage, the vehicle routes are found by the proposed branch-and-price technique with respect to the fixed decisions. This method, called as top-to-bottom approach, improved the solution times by almost 80\% and deviated only 0.47\% from the objective function values found by the exact method. As top-to-bottom is fast, it can be used to provide an upper bound for the exact approach. Doing this, helped us to find more optimal solution and reduce the average optimality gap of the unsolved instances to 1.96\%.

To formulate the two-echelon urban freight transportation systems, we introduced the two-echelon facility location with last echelon routing problem under capacity and time window constraints. This problem, denoted by the 2E-LR2PTW, extends the one-echelon problem to incorporate larger networks with additional intermediate facilities and different transportation modes. In the first echelon, the 2E-LR2PTW finds the number and location of the facilities (i.e., CDCs and satellites), the allocation of open CDCs to the selected satellites, and the number of primary vehicles performing direct shipments from CDCs to satellites. In the second echelon, the problem decides
on the customer-to-satellite assignments and secondary vehicle routes and schedules. All decisions are made simultaneously under given facility capacities, vehicle size limits, and customer due dates and time windows. The 2E-LR2PTW minimizes total transportation cost consisting of facility opening costs, primary and secondary vehicle utilization costs, and the traveling costs in both echelons. We presented a path-based formulation for the problem and decomposed it into two: (i) The master problem containing the first echelon decisions as well as selection of the best secondary vehicle paths over a restricted set. (ii) The subproblem where promising secondary vehicle paths (or columns) for the master problem are generated. The 2E-LR2PTW is solved by a branch-and-price framework where the vehicle paths are generated using an enumeration-base method. In order to solve the 2E-LR2PTW instances more efficiently, two heuristics are proposed. The first one is a top-to-bottom approach modified for the two-echelon settings. The second heuristic, called bottom-to-top approach, implements a novel constrained clustering technique to generate a set of vehicle paths for the master problem. Then, the master problem is solved over the generated paths to find the 2E-LRP2TW solution. The computational study indicates that the bottom-to-top approach is highly successful in solving problem instances with different sizes and characteristics. Therefore, we used the proposed clustering method to find an initial solution and an upper bound for exact solution approach. In this way, optimal solutions are found for the instances with up to 3 candidate CDC locations, 5 candidate satellite locations, and 100 customers. The exact approach was able to solve over 80% of the 588 test instances to optimality and provide an average MIP gap of 3.86% for the ones that are not solved during the time limit. We also analyzed the effect of different problem instance characteristics (such as demand point distribution, time window policies, facility capacities, and vehicle load restrictions) on the LRPTW and the 2E-LR2PTW solutions.

In the two-echelon city logistics systems, satellites are used as rendezvous point where primary and secondary vehicles meet to transfer the goods from one to another. Since neither inventory handling nor queuing the vehicles at a satellite location is feasible, it is crucial to plan timely schedules for the vehicles beforehand. Given the network design solution from the strategic/tactical level planning, the satellite synchronization problem (SSP) is formulated to make secondary-to-primary vehicle as-
signments and schedule arrival and departure times of the vehicles that are planned to meet at the satellite location such that all secondary vehicles are able to deliver their load on time. The objective function of the SSP minimizes the congestion cost at the satellite location. The defined objective implicitly minimizes the number of docks required for performing the transfers, i.e. the number of vehicles being simultaneously present in a satellite locations. We proposed two mathematical formulations for the SSP. First formulation considers scheduling vehicles in a continuous time space, while the second formulation discretizes the time dimension into a finite number of time intervals. We showed that the second formulation becomes stronger with constraint aggregation and performs significantly better than the first formulation when the number of time intervals is not very large.

As the majority of the transportation activities in CL are carried out in urban streets with dynamic traffic, it is inevitable to consider possible vehicle arrival time delays. We simulated the solution of the SSP under anticipated arrival time and service time delays, and performed a sensitivity analysis. Our computational study on the operational level decisions in SSP provides valuable information about the feasibility of the current strategic network design setting. For example, the trade-off between congestion and satellite capacity or congestion and the number of vehicles arriving to the satellite location can be studied to refine decisions about the satellite location, size, vehicle size, and vehicle-to-satellite assignments made at the strategic/tactical level planning. Feasibility check under arrival time or service time delays also provides feedback for the authorities who make time window policies.

6.2 Future Research Directions

One of the important objectives of CL is to provide an eco-friendly freight distribution solution. Therefore, developments in freight transportation modeling need to not only consider conventional economic costs, but also evaluate environmental, ecological, and social impacts explicitly in order to maintain a sustainable system in the long run. The literature contains studies on facility location and vehicle routing problems under environmental concerns (e.g. the green Weber problem, pollution-routing problems, and energy-minimizing VRPs) and/or using alternatives to fossil fuel vehicles (such
as electric or unmanned aerial vehicles) in the last-mile delivery (see, for example, Demir et al. 2014, Lin et al. 2014, Pelletier et al. 2016, Khoei et al. 2017, Goodchild & Toy 2018). Therefore, the literature related to green location and/or routing problems can be used to propose a more attractive urban freight transportation solution for its stakeholders.

This study focused on the deterministic network design cases in which all information is known at the time of the planning. In most real-life applications, however, stochastic and/or dynamic information has to be accounted for in the decision-making process. Therefore, considering nondeterministic factors such as demand, service time, and/or traveling time uncertainty is also important for deploying a more sustainable CL system. In this context, integrating the research about dynamic and stochastic VRP (see, Pillac et al. 2013, for a review) into urban freight transportation problems can provide more reliable results. Kunter (2015) and Crainic et al. (2016) are recent studies in the extremely narrow area of research where CLSs are targeted under uncertainty.

In this thesis, we considered conventional cost minimization objective functions for one and two-echelon freight distribution systems, where the sum of facility location, vehicle utilization, and traveling costs are minimized. An alternative approach is to include the cost of vehicle waiting times inside the city. We assumed that vehicles can wait at a customer location if they arrive before the beginning of the customer’s time windows. Therefore, it is possible that some solutions yield high waiting times in favor of a lower traveling cost. However, such solutions might be undesirable for local citizens as the vehicles occupy available parking lots or cause congestion by blocking the streets. Therefore, objective functions with route duration considerations can also be considered (see, for example, Dabia et al. 2013).

In city logistics, local authorities impose access time windows in certain urban areas to prevent deliveries in peak hours. Hence, we modeled our problems under hard time window constraints, where violating the delivery time intervals are not permitted. One can consider soft time windows if no such restrictions exist or the customer due dates are flexible. Soft time window settings allow serving customers before and after their time windows with some penalty (Qureshi et al. 2009, Liberatore et al. 2011, Taş et al.
Although the transportation problems formulated in this study are motivated by commercial logistics systems, they can also be employed in the area of humanitarian logistics. Humanitarian relief chains aim to minimize human casualties and death by efficiently and effectively allocating emergency supplies and scarce resources in disaster areas. Facility location models provide an approach for dealing with strategic decisions about locating warehouses, shelters, distribution centers, medical units, or waste/recycling sites (Balcik & Beamon 2008, Boonmee et al. 2017). The one- and two-echelon facility location problems with time windows formulations in this thesis can be used in strategic planning for both preparation and recovery phases of a disaster relief. The travel times in our formulation denote the times needed to reach and serve the demand points. Time window constraints can reflect time limits or priority defined for the demand sites where it is crucial to deliver emergency supplies as quickly as possible. The 2E-FLPTW is a starting point to model the situations where two layers of facilities, such as central warehouses or suppliers and local distribution centers or transitional nodes, need to be located (Döyen et al. 2012, Tofighi et al. 2016). The location-routing approaches in this study can also be utilized in the post-disaster planning. In this case, traveling times in our model represent length, reliability, or security of the arcs and can include the time required to remove the debris from a blocked edge in a damaged transportation network. Therefore, the decisions about field hospital locations and ambulance routes can be made simultaneously.

In terms of coordination, there is still a need for better understanding of CL systems operations, their components, and the objectives of the involved actors and their behavior in a business environment. There is no comprehensive research in the literature on modeling CLSs by considering both coordination and consolidation aspects. The available studies either focus on urban transportation networks with consolidation and try to solve the underlying problem using solution techniques from classical OR/IE literature, or try to conduct surveys to construct agent-based models and deal with coordination issues without explicitly considering network design and vehicle routing optimization. There is also very little attempt in providing analytical models and frameworks considering the effect of spatial and environmental city characteristics on CLSs. Besides, while solution approaches for traditional routing problems may be
used to obtain a primary solution, they do not guarantee a stable result when dealing with multiple decision makers. Coordination and consolidation are equally important for a viable CL solution and need to be considered simultaneously while formulating a CL problem.

The satellite synchronization problem introduced in this thesis provides new insights into modeling an important coordination and consolidation problem in the inter-modal CL systems. The SSP formulation can be extended by taking the routing schedules into account and scheduling the secondary vehicles such that they spend less waiting time to serve their first customer after leaving the satellite. Stochastic programming techniques can also be employed to model the delays and/or travel time uncertainties encountered in trans-docking and delivery operations. An alternative SSP can be formulated by considering the number of candidate satellite locations and their dock capacity, vehicle fleet size, and secondary vehicle routes as the given information provided by a two-echelon freight distribution system. Then, the new SSP chooses the satellites to use, assigns primary and secondary vehicles to the selected satellites, and schedules the satellite operations to provide a synchronized freight transportation solution for the whole system. The Synchronization problem in multi-echelon freight distribution systems is a relatively new optimization problem. It requires coordinating carriers and administrators to effectively manage transfer operations in the intermediate facilities. More research is needed to provide a unified framework to achieve rich solutions for CL and inter-modal freight distribution systems.
REFERENCES


de Oliveira, G. F., & de Oliveira, L. K. (2017). Stakeholder’s perception about urban goods distribution solution: Exploratory study in Belo Horizonte (Brazil). *Trans-


In this appendix, we provide mathematical formulations of three major problems arising in the two-echelon freight transportation systems in city logistics, namely the two-echelon facility location problem, the two-echelon facility location with last echelon routing problem, and the two-echelon location-routing problem under facility and vehicle capacity constraints. Our formulations also incorporate time windows constraints where customers are required to be served during a fixed time interval. The computational challenges are investigated and demonstrated by some numerical examples. At the end, a framework is introduced to generate problem test instances reflecting spatial and temporal characteristics of the city.

A.1 Mathematical Notations

The notations used to formulate our problems are provided in this section. Table A.1 lists the parameters (i.e., sets, indices, and constants) and Table A.2 presents decision variables.

A.2 The Two–Echelon Capacitated Facility Location Problem

In the two-echelon capacitated facility location problem (2E-FLP), a satellite can be served by multiple primary vehicles coming from different CDCs, whereas a customer is assigned to exactly one vehicle. The 2E-FLP seeks (i) location of the open
### Table A.1: Problem parameters.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I)</td>
<td>Set of candidate CDC nodes</td>
<td></td>
</tr>
<tr>
<td>(J)</td>
<td>Set of candidate satellite nodes</td>
<td></td>
</tr>
<tr>
<td>(K)</td>
<td>Set of customer nodes</td>
<td></td>
</tr>
<tr>
<td>(\mathcal{M} = I \cup J)</td>
<td>Set of first echelon nodes (i.e., facility nodes)</td>
<td></td>
</tr>
<tr>
<td>(\mathcal{N} = J \cup K)</td>
<td>Set of second echelon nodes</td>
<td></td>
</tr>
<tr>
<td>(\mathcal{E}', \mathcal{E}'')</td>
<td>Set of first and second echelon arcs, respectively</td>
<td></td>
</tr>
<tr>
<td>(F_m)</td>
<td>Location cost of facility (m \in \mathcal{M})</td>
<td></td>
</tr>
<tr>
<td>(Q_m)</td>
<td>Capacity of facility (m \in \mathcal{M})</td>
<td></td>
</tr>
<tr>
<td>(F', F'')</td>
<td>Primary and secondary vehicle utilization costs, respectively</td>
<td></td>
</tr>
<tr>
<td>(Q', Q'')</td>
<td>Primary and secondary vehicle capacity, respectively</td>
<td></td>
</tr>
<tr>
<td>(D_k)</td>
<td>Demand of customer (k \in K)</td>
<td></td>
</tr>
<tr>
<td>([A_n, B_n])</td>
<td>Time window of node (n \in \mathcal{N})</td>
<td></td>
</tr>
<tr>
<td>(C_{mn})</td>
<td>Traveling cost of edge ((m, n) \in \mathcal{E}' \cup \mathcal{E}'')</td>
<td></td>
</tr>
<tr>
<td>(T_{mn})</td>
<td>Traveling time of edge ((m, n) \in \mathcal{E}' \cup \mathcal{E}'') including the setup/service time at node (m)</td>
<td></td>
</tr>
<tr>
<td>(C'_{ij})</td>
<td>Total cost of the direct first echelon route (i \rightarrow j \rightarrow i) performed by a primary vehicle, (C'<em>{ij} = F' + C</em>{ij} + C_{ji}, \forall i \in I, j \in J)</td>
<td></td>
</tr>
<tr>
<td>(C''_{jk})</td>
<td>Total cost of the direct second echelon route (j \rightarrow k \rightarrow j) performed by a secondary vehicle, (C''<em>{jk} = F'' + C</em>{jk} + C_{kj}, \forall j \in J, k \in K)</td>
<td></td>
</tr>
<tr>
<td>(B_{mn})</td>
<td>A constant equal to (\max{B_m + T_{mn} - A_n, 0}), (\forall m, n \in \mathcal{N})</td>
<td></td>
</tr>
</tbody>
</table>

### Table A.2: Decision variables.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>(z_m)</td>
<td>Whether or not facility (m \in \mathcal{M}) is selected</td>
<td>({0, 1})</td>
</tr>
<tr>
<td>(w_{ij})</td>
<td>Amount of freight sent from CDC (i \in I) to satellite (j \in J)</td>
<td>(\mathbb{R}^+)</td>
</tr>
<tr>
<td>(y_{ij})</td>
<td>Number of vehicles traveling on arc ((i, j) \in \mathcal{E}')</td>
<td>(\mathbb{Z}^+)</td>
</tr>
<tr>
<td>(r_{jk})</td>
<td>Whether or not customer (k \in K) is assigned to satellite (j \in J)</td>
<td>({0, 1})</td>
</tr>
<tr>
<td>(x_{jk})</td>
<td>Whether or not arc ((j, k) \in \mathcal{E}'') is traversed by a vehicle</td>
<td>({0, 1})</td>
</tr>
<tr>
<td>(q_k)</td>
<td>Amount of load on a vehicle upon arrival to node (k \in K)</td>
<td>(\mathbb{R}^+)</td>
</tr>
<tr>
<td>(t_n)</td>
<td>Arrival time of a vehicle to node (n \in \mathcal{N})</td>
<td>(\mathbb{R}^+)</td>
</tr>
</tbody>
</table>
CDC(s), (ii) location of the open satellite(s), (iii) the flow from open CDCs to open satellites, and (iv) assignment of customers to open satellites, such that the total cost of facility location and freight distribution is minimized. Note that the assignments are considered as origin–destination–origin (direct) routes. The 2E-FLP is formulated as follows.

\[ \text{(2E-FLP)} \quad \text{Min } \sum_{m \in M} F_m z_m + \sum_{i \in I} \sum_{j \in J} C'_{ij} y_{ij} + \sum_{j \in J} \sum_{k \in K} C''_{jk} r_{jk} \]  

s.t.: \[ \sum_{j \in J} r_{jk} = 1, \quad \forall k \in K \]  

\[ \sum_{k \in K} D_k r_{jk} \leq \sum_{i \in I} w_{ij}, \quad \forall j \in J \]  

\[ \sum_{j \in J} w_{ij} \leq Q_i z_i, \quad \forall i \in I \]  

\[ \sum_{i \in I} w_{ij} \leq Q_j z_j, \quad \forall j \in J \]  

\[ 0 \leq w_{ij} \leq Q' y_{ij}, \quad \forall i \in I, j \in J \]  

\[ z_m \in \{0, 1\}, \quad \forall m \in M \]  

\[ r_{jk} \in \{0, 1\}, \quad \forall j \in J, k \in K \]  

\[ y_{ij} \in \{0, 1, 2, \cdots\}, \quad \forall i \in I, j \in J. \]  

Objective function (A.1) minimizes total transportation cost consisting of CDC and satellite location costs, first echelon CDC-satellite-CDC route costs, and second echelon satellite-customer-satellite route costs. Constraint (A.2) guarantees that each customer is assigned to exactly one satellite. Constraint (A.3) ensures that total incoming flow to a satellite location is not less than the total customer demands it serves. Capacity limit of open CDCs and satellites are satisfied by (A.4) and (A.5), respectively. Constraint (A.6) holds the lower bounds on the flow variables and sets the correct number of first echelon vehicles with respect to their capacity. (A.7)–(A.9) are variable domain constraints.
A.2.1 Time Window Constraints

When time restrictions are imposed, we add the following constraints to the 2E-FLP (A.1)–(A.9) to formulate the 2E-FLP with time windows (2E-FLPTW).

\[(T_{jk} - B_k)r_{jk} \leq 0, \quad \forall j \in J, k \in K \quad (A.10)\]
\[\left[\max (T_{jk}, A_k) + T_{kj} - B_j\right] r_{jk} \leq 0, \quad \forall j \in J, k \in K \quad (A.11)\]
\[T_{ij} - B_j y_{ij} \leq 0, \quad \forall i \in I, j \in J \quad (A.12)\]

Constraint (A.10) satisfies the time window of customers. Constraints (A.11) and (A.12) ensure that the closing time of a satellite is not violated. By the above time window constraints, an arc between two nodes is used only if the destination node is reached no later than its allowable time.

The 2E-FLPTW (A.1)–(A.12) contains polynomial number of variables and constraints and it can be solved efficiently with common MIP solvers.

A.3 The Two–Echelon Capacitated Facility Location with Last Echelon Routing Problem

The two-echelon capacitated facility location with last echelon routing problem (2E-LR2P) considers vehicle routes in the second echelon instead of direct shipments. Here, we formulate the 2E-LR2P as a two-index vehicle-flow formulation given below.

\[(2E-LR2P) \quad \text{Min} \sum_{m \in M} F_m z_m + \sum_{i \in I} \sum_{j \in J} C'_{ij} y_{ij} \]
\[+ \sum_{j \in J} \sum_{k \in K} F'' x_{jk} + \sum_{(m,n) \in \mathcal{E}''} C_{mn} x_{mn} \quad (A.13)\]
\[\text{s.t.: } (A.2)–(A.9), \quad (A.14)\]
\[\sum_{n \in \mathcal{N}} x_{nk} = 1, \quad \forall k \in \mathcal{K} \quad (A.15)\]
\[\sum_{m \in \mathcal{N}} x_{nm} - \sum_{m \in \mathcal{N}} x_{mn} = 0, \quad \forall n \in \mathcal{N} \quad (A.16)\]
\[x_{jk} \leq r_{jk}, \quad \forall j \in \mathcal{J}, k \in \mathcal{K} \quad (A.17)\]
\[r_{jk} + x_{kl} \leq 1 + r_{jl}, \quad \forall j \in \mathcal{J}, k, l \in \mathcal{K} \quad (A.18)\]
Objective function (A.13) minimizes total transportation cost consisting of CDC and satellite location costs, first echelon CDC-satellite-CDC routes cost, and the cost of second echelon vehicle routes. Constraint (A.14) adds required constraints from the 2E-FLP model. (A.15) ensures that exactly one incoming arc is selected for each customer node. The flow balance at each second echelon node is satisfied by constraint (A.16). Constraints (A.17) and (A.18) assign customer $k$ to satellite $j$ if there is a route from node $j$ that passes through node $k$. Constraint (A.19) is the Miller–Tucker–Zemlin constraint that controls the load of a vehicle in its route and eliminates subtours. If $x_{kl} = 0$, (A.19) is redundant. Otherwise, $q_l \geq q_k - D_k$ is satisfied by this constraint. Constraint (A.20) implies that the load on a vehicle is at least equal to a customer’s demand when the vehicle arrives at the customer’s location, and at most equals to the vehicle’s capacity. Finally, (A.21) meets the binary requirement for arc-flow variables.

### A.3.1 Time Window Constraints

The two-echelon capacitated facility location with last echelon routing problem with time windows (2E-LR2PTW) adds (A.12) and the following constraints to the 2E-LR2P (A.13)–(A.21).

\[
T_{jk} - t_k \leq B_{jk}(1 - x_{jk}), \quad \forall j \in J, k \in K \tag{A.22}
\]

\[
t_k + T_{kn} - t_n \leq B_{kn}(1 - x_{kn}), \quad \forall k \in K, n \in N \tag{A.23}
\]

\[
A_n \leq t_n \leq B_n, \quad \forall n \in N. \tag{A.24}
\]

Constraints (A.22) and (A.23) control vehicle arrival times based on the order of visited nodes on the route. Constraint (A.24) limits the arrival time to a node to its time window.
A.4 The Two–Echelon Capacitated Location–Routing Problem

The two-echelon capacitated location-routing problem (2E-LRP) contains routing in both echelons. As the first echelon involves multi sourcing, i.e., a satellite can be served by multiple vehicles from multiple CDCs, we redefine decision variables by adding a vehicle index. Let $\mathcal{V}$ be the set of primary vehicles. Let decision variable $y^v_{mn}$ take value 1 if vehicle $v \in \mathcal{V}$ traverses on arc $(m, n) \in \mathcal{E}'$, and 0 otherwise. Define $w^v_{ij}$ as the total weight delivered from CDC $i \in \mathcal{I}$ to satellite $j \in \mathcal{J}$ by vehicle $v \in \mathcal{V}$. Define $q^v_j$ as the load on vehicle $v \in \mathcal{V}$ upon arrival to satellite $j \in \mathcal{J}$. Then, the 2E-LRP is formulated as follows.

\[
\text{(2E-LRP) Min } \sum_{m \in \mathcal{M}} F_m z_m + \sum_{v \in \mathcal{V}} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} F^u_{ij} y^v_{ij} + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} F^\eta_{jk} x^v_{jk} \tag{A.25}
\]

s.t.: (A.2), (A.7), (A.8), (A.15)–(A.21), (A.26),

\[
\sum_{k \in \mathcal{K}} D_{kj} r_{jk} \leq \sum_{v \in \mathcal{V}} \sum_{i \in \mathcal{I}} w^v_{ij}, \quad \forall j \in \mathcal{J} \tag{A.27}
\]

\[
\sum_{v \in \mathcal{V}} \sum_{i \in \mathcal{I}} w^v_{ij} \leq Q_i z_i, \quad \forall i \in \mathcal{I} \tag{A.28}
\]

\[
\sum_{v \in \mathcal{V}} \sum_{j \in \mathcal{J}} y^v_{ij} \leq 1, \quad \forall v \in \mathcal{V} \tag{A.29}
\]

\[
\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} y^u_{ij} \leq 1, \quad \forall v \in \mathcal{V} \tag{A.30}
\]

\[
\sum_{n \in \mathcal{M}} y^u_{mn} - \sum_{n \in \mathcal{M}} y^v_{nm} = 0, \quad \forall v \in \mathcal{V}, m \in \mathcal{M} \tag{A.31}
\]

\[
q^v_j - \sum_{v \in \mathcal{V}} w^v_{ij} - q^l_j \leq Q' \left(1 - y^v_{ji}\right), \quad \forall v \in \mathcal{V}, j, l \in \mathcal{J} \tag{A.32}
\]

\[
0 \leq w^v_{ij} \leq Q', \quad \forall v \in \mathcal{V}, i \in \mathcal{I}, j \in \mathcal{J} \tag{A.33}
\]

\[
\sum_{i} w^v_{ij} \leq q^v_j \leq Q', \quad \forall v \in \mathcal{V}, j \in \mathcal{J} \tag{A.34}
\]

\[
y^v_{ij} \in \{0, 1\}, \quad \forall v \in \mathcal{V}, i \in \mathcal{I}, j \in \mathcal{J}. \tag{A.35}
\]

Objective function (A.25) minimizes the sum of CDC and satellite location costs and first and second echelon routing costs. (A.26) adds required constraint from the 2E-FLP and the 2E-LR2P models. Constraint (A.27) ensures that total incoming flow to a satellite location is not less than the total customer demands it serves. Capacity limit of open CDCs and satellites are satisfied by constraints (A.28) and (A.29), re-
respectively. By (A.30), each primary vehicle is used at most once. Constraint (A.31) preserves the flow at the first echelon nodes. Vehicle loads are set by constraint (A.32) based on the nodes visited on the vehicle route. Constraint (A.33) holds the bounds on the flow variables. By (A.34), the load on a vehicle upon visiting a satellite remains between the amount assigned to that vehicle and the vehicle capacity. (A.35) is the integrality constraint.

A.4.1 Time Window Constraints

The two-echelon location-routing problem with time windows (2E-LRPTW) requires additional variables and constraints. Define decision variable $t_{vj}$ to indicate the arrival time of vehicle $v \in V$ to satellite $j \in J$. Then, time window constraints in the first echelon can be met by the following constraint:

$$T_{ij} - t_{vj} \leq T_{ij} \left(1 - y_{vij}\right), \quad \forall v \in V, i \in I, j \in J \quad (A.36)$$

$$t_{vj} + T_{jm} - t_{vm} \leq (B_j + T_{jm}) \left(1 - y_{vjl}\right), \quad \forall v \in V, j \in J, m \in M \quad (A.37)$$

$$0 \leq t_{vm} \leq B_m, \quad \forall v \in V, m \in M, \quad (A.38)$$

where, $B_i, \forall i \in I$, is considered as a large number. Similar to the 2E-LR2PTW, the time windows in the second echelon of the 2E-LRPTW can be satisfied by adding constraints (A.22)–(A.24).

A.5 Numerical Examples

This section presents examples to illustrate how different configurations of a city logistics system (CLS) affect the result of the freight distribution decision problems.

A.5.1 Example 1: Different Distribution Schemes

Consider a two-echelon city logistics problem instance with 1 CDC, 3 candidate satellite locations, and 10 customers. All customers demand one unit of a single product with a negligible service time. The instance is generated according to the circular city pattern (see A.6). Point coordinates on the plane are given in Table A.3. We solve the
Table A.3: CC-1-3-10 instance data.

<table>
<thead>
<tr>
<th>Point type</th>
<th>X coordinate</th>
<th>Y coordinate</th>
<th>Due date</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDC</td>
<td>27</td>
<td>32</td>
<td></td>
</tr>
<tr>
<td>Satellite 1</td>
<td>38</td>
<td>124</td>
<td></td>
</tr>
<tr>
<td>Satellite 2</td>
<td>113</td>
<td>165</td>
<td></td>
</tr>
<tr>
<td>Satellite 3</td>
<td>82</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>Customer 1</td>
<td>80</td>
<td>113</td>
<td>100</td>
</tr>
<tr>
<td>Customer 2</td>
<td>102</td>
<td>82</td>
<td>100</td>
</tr>
<tr>
<td>Customer 3</td>
<td>115</td>
<td>117</td>
<td>100</td>
</tr>
<tr>
<td>Customer 4</td>
<td>79</td>
<td>91</td>
<td>100</td>
</tr>
<tr>
<td>Customer 5</td>
<td>121</td>
<td>86</td>
<td>100</td>
</tr>
<tr>
<td>Customer 6</td>
<td>96</td>
<td>117</td>
<td>100</td>
</tr>
<tr>
<td>Customer 7</td>
<td>143</td>
<td>93</td>
<td>250</td>
</tr>
<tr>
<td>Customer 8</td>
<td>53</td>
<td>111</td>
<td>250</td>
</tr>
<tr>
<td>Customer 9</td>
<td>83</td>
<td>49</td>
<td>250</td>
</tr>
<tr>
<td>Customer 10</td>
<td>90</td>
<td>19</td>
<td>300</td>
</tr>
</tbody>
</table>

The problem with and without time restrictions. The beginnings of time windows are considered zero. The end of time windows (due dates) are listed in Table A.3. No time restriction is considered for satellites. Therefore, vehicle schedules are done only for second echelon in the instances with time windows. Customers in central zone have tighter time windows, whereas those in farther locations have wider time windows. All satellites have capacities of eight units and fixed costs of 300 monetary units. One unit distance between any two locations corresponds to one unit travel time and all distance values are rounded to their closest integer numbers. Each unit travel time in the second (first) echelon costs one (two) monetary unit(s). This test instance is called `CC-1-3-10` indicating that it has circular pattern with circular zones including one CDC, three candidate satellites, and ten customers.

The problems are formulated according to the models provided earlier in this appendix. We first consider the 2E-FLP on this instance. Since there is one CDC in the first echelon, we only need to decide on satellite locations and their allocations to the customers. As the facility location problem concerns strategic level decisions, we do not consider time restrictions in this stage. The 2E-FLP is solved on a Linux workstation with Intel® Xeon 4 × 3.20GHz processors and 16GB memory using CPLEX.
Table A.4: Numerical results for CC-1-3-10.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Objective value</th>
<th>Solution time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2E-FLP</td>
<td>2150</td>
<td>0.01</td>
</tr>
<tr>
<td>2E-LR2P</td>
<td>1570</td>
<td>58.54</td>
</tr>
<tr>
<td>2E-LR2PTW</td>
<td>1583</td>
<td>309.21</td>
</tr>
<tr>
<td>2E-LRP</td>
<td>1224</td>
<td>74.09</td>
</tr>
<tr>
<td>2E-LRPTW</td>
<td>1237</td>
<td>382.28</td>
</tr>
</tbody>
</table>

12.6 under its default settings. The optimal objective function value and solution time are given in Table A.4. The solution is illustrated in Figure A.1a.

To see the effect of including more tactical decisions into the problem, we consider routing in the second echelon. Secondary vehicle capacities are equal to six and no cost is associated with initiating a route. The results for the 2E-LR2P and the 2E-LR2PTW problem are presented in Table A.4. The optimal solutions to these problems are illustrated in Figures A.1b and A.1c.

We also investigate 2E-LRP settings considering routing in both echelons. For this example, a single vehicle is available to deliver goods from the CDC to open satellites in a route. The first echelon vehicle capacity is large enough for this instance. No cost is associated with using the vehicles. For simplicity, suppose that goods will be available at satellites in the beginning of the time horizon. Therefore, no synchronization is considered between first echelon and second echelon vehicles. The 2E-LRP is solved with and without time windows and the results are presented in Table A.4. Figures A.1d and A.1e illustrate solutions to the 2E-LRP and 2E-LRPTW, respectively.

The optimal solution to the strategic problem, the 2E-FLP, indicates that Satellite 1 and 3 are open and Satellite 2 is closed. The problem is solved fast and the total location and allocation cost is 2150. When routing decisions are added to the problem (consider the 2E-LR2P), more computational effort is required to find the solution. However, the objective function is improved due to less amount of distance traveled by the vehicles. When no time restriction is considered, any route is feasible as long
as vehicle capacity is not exceeded. Time windows add another restriction to the
problem that makes it more difficult to solve for optimal satellite locations and routes. The routing is changed under time restriction (see Figure A.1c) due to limitations on vehicle arrival times. Comparing to the 2E-LR2P, both computational time and objective cost increased in the 2E-LR2PTW.

The 2E-LRP is more complex as more decisions are made. However, it can improve the cost by using the information from customer demands and satellite locations to perform the routes in the first echelon. Demands of different customers are consolidated in the CDC location(s) and distributed to open satellite platforms for final delivery. First echelon deliveries can be done via vehicle routes if certain level of coordination is developed among shippers and carriers. In the example, the 2E-LRP could improve the overall cost by almost 43% comparing to the 2E-FLP and by 22% comparing the 2E-LR2P. Time restrictions had similar effect on this problem.

**A.5.2 Example 2: Effect of Time Window Policies**

We consider a single-echelon system with one CDC and 20 customers to investigate the effect of different access time windows policies in a circular city. All customers demand one unit of a single product with negligible service time. There are unlimited number of homogeneous vehicle with capacity of five units. All the time windows are generated according to ringed and sectored discrete patterns. In both cases, there are three districts. Four different time window scenarios are considered (see Figure A.2). District 1 in the ringed case corresponds to the central districts while in the sectored case, it is an arbitrary sector.

Districts in Scenario 1 have tight and non-overlapping time windows with equal lengths. In Scenario 2, neighboring districts have overlapping time windows and the time windows are wider compared to Scenario 1. Time windows of all districts in Scenario 3 overlap. Scenario 3 has the widest time windows. In Scenario 4, districts are assigned tight time windows where neighboring districts have overlapping time windows with equal lengths. It is assumed that customers should be served between 4:00PM and 7:00PM in Scenario 1, 2, and 3, and between 4:00PM and 6:00PM in Scenario 4. Vehicles are not allowed to arrive at a customer site later than the end
of its time window. However, we assume that a vehicle can wait at a customer site before its time window starts.
The problem is formulated as a LRPTW with one fixed CDC and is solved by branch-and-price algorithm introduced in Chapter 3 on the same machine as in Example 1. We generated five instances for each time window setting (ringed/sectored) and applied the scenarios on each of them. There are 40 instances in total. No feasible solution for two instances with sectored time windows under Scenario 4 is found due to very tight time windows. Ringed district instances are all solved and the results are
presented in Table A.5. This table shows the relative average results for the objective function value, waiting time, number of vehicles used in the optimal solution, and the time spent to solve the instances. Waiting time corresponds to the time the vehicles have to wait at a customer location until its time window starts. Under Scenario 3, we obtain the minimum objective function value, waiting time, number of vehicles, and solution times for the ringed districts. The results in Table A.5 show that the more time windows overlap, the lower values are obtained for the transportation cost, waiting time, and the number of vehicles used to deliver products. Under tight time windows and short service horizon (Scenario 4), we have to use more vehicles to perform on-time deliveries and the transportation cost increases. However, compared to Scenario 1, we still have less waiting times due to overlapping time windows.

Table A.5: Relative average results for five ringed (R) instances under four scenarios.

<table>
<thead>
<tr>
<th>District</th>
<th>Scenario</th>
<th>Objective value</th>
<th>Waiting time</th>
<th>No. of vehicles</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>1</td>
<td>1.13</td>
<td>1.95</td>
<td>1.20</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.03</td>
<td>1.32</td>
<td>1.04</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1.42</td>
<td>1.67</td>
<td>1.56</td>
<td>3</td>
</tr>
</tbody>
</table>

Table A.6: Relative average results for three ringed (R) and sectored (S) instances under four scenarios.

<table>
<thead>
<tr>
<th>District</th>
<th>Scenario</th>
<th>Objective value</th>
<th>Waiting time</th>
<th>No. of vehicles</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>1</td>
<td>1.15</td>
<td>1.43</td>
<td>1.13</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.02</td>
<td>1.19</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1.46</td>
<td>1.24</td>
<td>1.47</td>
<td>1</td>
</tr>
<tr>
<td>S</td>
<td>1</td>
<td>1.53</td>
<td>1.70</td>
<td>1.53</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.30</td>
<td>1.37</td>
<td>1.33</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.18</td>
<td>1.19</td>
<td>1.20</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1.62</td>
<td>1.65</td>
<td>1.73</td>
<td>1</td>
</tr>
</tbody>
</table>

In order to compare results for the ringed and sectored time window settings, we re-
moved the two instances that were infeasible under Scenario 4. Objective function values, waiting times, number of vehicles used, and solution time results of the remaining three instances are compared. The relative average values are presented in Table A.6. Again, the lowest values are obtained when the time windows overlap most (Scenario 3). On average, sectored city districts resulted in higher costs. The main reason is that when the time windows are assigned to sectored districts, a tight time window may be assigned to a far customer. Therefore, in order to make on-time delivery to that customer we have to dispatch a vehicle to reach that customer without spending time to visit other customers on the way.

The experimental results for this example show that having ringed districts with overlapping time windows yields less transportation cost, lower waiting times, and less number of vehicles needed to deliver goods.

A.6 A Framework to Generate City Logistics Test Instances

Although there are test instances in the VRP and LRP literature (see Drexl & Schneider 2015, for a review), benchmark test instances for city logistics problems are not available. Here, we present a scheme for generating problem test instances involving real-life problem features in Riegler (2013).

The instances are generated based on the following city patterns which are observed in large cities around the world.

- Circular city pattern with one center: The city has co-centric circular zones where the central zone (city center) has the most number of customers. The farther the zones are from the city center, the less number of customers they cover. Satellites are located on a zone boundary not very far from the city center. CDCs are located on the border of the city.

- Rectangular city pattern with multiple centers: Satellites are located on rectangle, centered according to the location of city centers. The city border is a larger rectangle on which CDCs are located.

- Rectangular city with no center: CDCs are located on the rectangular border of
the city inside which customers and satellites are located uniformly.

Different city patterns are illustrated in Figure A.3. These city instances include 3 candidate CDC locations, 10 candidate satellite locations, and 250 customers.

![Figures A.3](image)

(a) A circular city with one center  
(b) A rectangular city with two centers  
(c) A rectangular city with no center

Figure A.3: Illustration of instance patterns.

![Figures A.4](image)

(a) Ringed city districts  
(b) Sectored city districts  
(c) Divided city districts

Figure A.4: Illustration of time window patterns.

In city logistics, time windows are either given by the customers themselves (i.e. due dates) or imposed by the local authorities. As for the former case, it is common that the customers demanding same products, have similar or overlapping time windows. For example, almost all flower shops want their ordered flowers to be delivered early in the morning. Such kind of customers have similar time windows but their locations might be spread across the city. The latter case is observed for the customers in historical zones or central areas with high volume of traffic and commercial and social
activities. In this situation, customers are located close to one another and usually have tight time windows. Therefore, instead of having individual or random time windows for each customer, customers from the same area are assigned similar access time windows. We consider two time window patterns for the circular city and another pattern for rectangular cities:

- **Ringed city districts (for circular cities):** Districts are aligned with the circular zones. Customers in each zone have similar (not necessarily the same) time restrictions. The customers in city center may have tighter time windows. Most cities with dense city centers can be considered in this class.

- **Sectored city districts (for circular cities):** City is divided into sectors (slices) with certain central angles. City pattern of Paris is an example of such a structure.

- **Divided city entities (for rectangular cities):** The city is divided by a river or highway where each district has at least one city center. Frankfurt, for example, exhibits such a pattern.

The time window patterns are illustrated in Figure A.4 (the instances have the same point distributions as shown in Figure A.3). For these examples, we used equal angles of $\pi/6$ radian for the circular city sectors. In the rectangular city, the district border is perpendicular to the line connecting two city centers and passes through their midpoint. The artificial squares used for locating city centers, satellites, and CDCs are also shown in Figure A.4c. Note that a combination of circular city districts and city sectors can also be used. For the single echelon patterns, satellite locations can be removed from the instances. Instances with higher number of echelons can be generated with similar patterns.
CURRICULUM VITAE

PERSONAL INFORMATION

Surname, Name: Farham, Mohammad Saleh
Nationality: Iran
Date and Place of Birth: 1987-08-31, Esfahan, Iran
Marital Status: Single
Electronic Mail: pharham@tuta.io

EDUCATION

• Master of Science, Middle East Technical University, Ankara, Turkey, January 2013
• Bachelor of Science, Middle East Technical University, Ankara, Turkey, September 2010

RESEARCH ACTIVITIES

• Research member of Consolidation and Coordination in Urban Areas, Joint Programming Initiative – Urban Europe (Grant No: TÜBİTAK 113M121)
• Research member of Hub Location under Congestion and Capacity Considerations (Grant No: TÜBİTAK 218M520)
• Participated in EURO PhD School on Routing and Logistics, Brescia, Italy, June 2015
• Participated in School on Column Generation, Paris, France, March 2014
• Participated in ASBU Transportation & Logistics Workshop, Ankara, Turkey, March 2019 and January 2020
• Participated in National Operations Research and Industrial Engineering Society – Doctoral Colloquium, Istanbul, Turkey, April 2016 and Eskişehir, Turkey, April 2019
RESEARCH INTERESTS

- Facility location problems
- Hub location problems
- Vehicle routing and scheduling problems
- Freight distribution networks
- Transportation networks

PUBLICATIONS

Journal Articles


International Conference Presentations


