NEW PRODUCT INTRODUCTION INCENTIVES FOR SUPPLIERS AND A COMMON RETAILER

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ABSTRACT

NEW PRODUCT INTRODUCTION INCENTIVES FOR SUPPLIERS AND A COMMON RETAILER

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In this thesis, we investigate the effect of the new product introduction in a supply chain of two suppliers and a common retailer. We study two settings. In this first, we find the two-product equilibrium in which each supplier produces one product and sells his product through a two echelon supply chain. In the first stage, suppliers announce their wholesale price simultaneously. Then, the retailer sets the retail price that maimizes her profit. In the second setting, one of the suppliers considers introducing to a category that consists of two incumbent products. Then, similar to first setting, we consider the wholesale price and the consequent retail price decision of the new product. In our study, all products are partially substitutable and have price-dependent linear demand. In addition, we investigate the effect of the slotting fee on the new product entry decisions and we compare the decentralized supply chain decisions with those of the centralized system.

Keywords: Product Introduction, Supplier Competition, Game Theory, Slotting Fee

ÖZ

TEDARİKÇİLER VE ORTAK PERAKENDECİLERİ İÇİN YENİ ÜRÜN SÜRÜMÜNÜN GETİRİSİ

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Bu çalışmada, iki üretici ve bir perakendeciden oluşan bir tedarik zincirine yeni bir ürün dahil etmenin etkisi incelenmektedir. İki ayrı kurgu oluşturulmuştur. İlk kurguda her üreticinin birer ürün üreterek, ürünlerini iki aşamalı bir tedarik zinciri yoluyla sattığı bir denge çözülmüştür. Oyunun birinci asamasinda üreticiler eş zamanlı olarak ürün toptan fiyatını açıklarlar, bu fiyatları gören perakendeci de kendi karını en çoklayacak şekilde perakende fiyatlarını belirler. İkinci kurguda ise üreticilerden bir tanesi bu markete yeni bir ürün sürmeyi değerlendirmektedir. Yeni ürünün fiyatlandırma kararları da birinci kurgudaki gibi önce üreticinin toptan fiyatı belirlemesi, sonra ise perakendecinin perakende fiyatına karar vermesi şeklinde çalışılmıştır. Bu çalışmada tüm ürünler birbirleri yerine ikame edilebilen ve fiyata bağlı doğrusal talebe sahip ürünlerdir. Buna ek olarak raf bedelinin yeni ürün kararlarına etkisi incelenmiş, ve tedarik zincirinin performansı merkezi çözümle karşılaştırılmıştır.

Anahtar Kelimeler: Ürün Sürümü, Üretici Rekabeti, Oyun Teorisi, Raf Bedeli

To my wife, to my son and to my family

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CHAPTER 1

INTRODUCTION

In today's compatitive marketplace, both product variety and consumption rate of each product category are steadily increasing. Manufacturers continuously develop new products and introduce them to the market in order to increase their profit and satisfy the changing customer preferences even if they are already active in the market. According to Mintel's GNPD (Global New Product Database), the number of new product introduction in CPG (Consumer Packaged Goods) market for 1998-2016 can be seen in Figure 1.1 (United States Department of Agriculture, 2019).

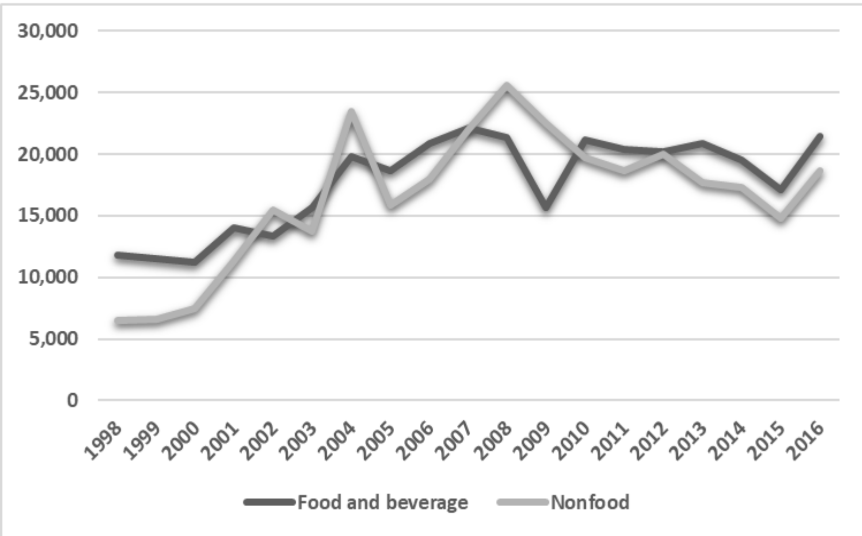


Figure 1.1: New Product Introductions of Consumer Packaged Goods, 1998-2016 (Source: Calculated by ERS, USDA using data from Mintel's Global New Product Database)

Although there is a decrease in the number of new product introduction in some years, many new products enter the market every year. By introducing new products, manufacturers increase the awareness of their product category. In addition, they enlarge their market with the new features of their new products. Then, their competitors respond with similar products and try to increase their demand share. Thus, variety in each product category increases day after day.

A big portion of newly introduced products are close substitutes to products that already exist in the market, or other new entrants. For example, Eti and Ülker are active in the snack category with their salty snacks and their products are close substitutes. Then, they produce new kind of snacks and introduce them to the market. "Crax Baharatlı" and "Krispi Baharatlı" can be example for these new products. By introduction of the new product, they expect to increase their demand share in the snacks category. In addition, they expand the snacks market with the new features of their new product by drawing attention of customers.

Coca Cola and Pepsi can be another example for this situation. They have already been in the soft drink category with their cokes. However, new products keep entering the market. Cola Cola introduced "Coca Cola Zero" into the market although "Coca Cola Light" is active in the market. Similary, Pepsi introduced "Pepsi Max" into the market when "Pepsi Light" is already available on shelves. With these new products, they expect to dominate the competition, and attract new customers who were buying neither the "Light" nor the original coke.

As variety of product increases, competition in the market also increases. In this competitive environment, the profitability of a new product is affected by the demand potential, prices and substitutability levels with other products.

Competing manufacturers aside, retailers must consider all products in the category in their pricing decisions. In addition, retailers want to control new product introductions since they have to manage all products of a category on a limited shelf space. If the new product has no significant demand potential, the retailer may want to deter/reject the entry of the new product. For these reasons, retailers use some control mechanisms in new product introduction.

If a manufacturer wants to introduce a new product into the market, the retailer may want an upfront payment for the new product. In the supply chain literature this is called as a "slotting fee" or a "slotting allowance". Manufacturers pay this one time fee to the retailer in order to reserve shelf space from their stores. Thus, manufacturers offer a new product only if they can justify the slotting fee in their profit increase. Otherwise, the new product is not introduced. Because of this, manufacturers want to develop and introduce new products with high demand and profit potential.

In this study, we analyze a two-echelon supply chain which consists of two suppliers and one retailer. Each supplier produces his products and sells them to customers through a common retailer. Products of suppliers are partially substitutable and each product has a price-based linear demand. In addition to its own retail price, demand of each product is affected by the retail price of the competitor's products. Suppliers and the retailer play a two stage dynamic game for their pricing decisions. In the first stage, suppliers simultaneously determine the wholesale prices of their products. Then, the retailer sets the retail price for each product in order to maximize her profit by considering her total cost of shelf space allocated.

In order to reveal the effect of the new product, we study two settings. In the first setting, each manufacturer produces one product. Then, in the second setting, one of the suppliers considers introducing a new partially substitute product into the supply chain when the two-product setting is already in equilibrium. One of the suppliers (Supplier 2) determines the wholesale price of the new product, followed by the retailer deciding its retail price. Upon the third product's introduction, the retailer may implicitly reject the product by setting a high price for it, or accept a marginal addition to the category only to increase the demand of the incumbent products.

We compare the profit of each firm under the two-product equilibrium and the new product introduction setting in order to analyze the effect of the new product. In addition, we also study the centralized system as a benchmark.

Our main objectives in this study are;

- to investigate the new product introduction incentives for the suppliers as well as the retailer
- to compare the new product introduction decision in the decentralized chain

with the centralized system decisions

- to investigate the effect of the slotting fee on firms' price and consequently new product introduction decisions, profit changes, and total chain profitability

We find that, how profit of each firm is affected from the new product introduction depends on the demand potential of the new product relative to those of the existing products. When the new product has too low demand potential relative to incumbent products, in cannot enter the market. On the other hand, all firms benefit from the new product introduction when it has significantly large demand potential. If the new product's relative demand potential is moderate-to-high, it can enter the market. However, profit of the supplier who does not offer the new product (Supplier 1) decreases, since the new product captures demand from the existing products with its more competitive retail price. In addition to this, the supply chain profit may also decrease with the entry of the new product if Suppler 1's loss is greater than the total gain of the retailer and Supplier 2.

The retailer can deter the introduction of a new product that may hurt the supply chain profit by means of a slotting fee. However, if she wants a high slotting fee, a new product cannot enter the market unless it has significant demand potential. Thus, high slotting fees may also deter the entry of a new product that could increase the total supply chain profit.

The rest of the thesis is organized as follows: we review the related literature in the next chapter. In Chapter 3, we explain the details of our model. In Chapter 4, centralized form of our model is examined. Then, in Chapter 5 we conduct numerical experiments in order to analyze our model. Finally, our study is concluded in Chapter 6.

CHAPTER 2

LITERATURE REVIEW

In this part, we discuss the literature related to this thesis. We review the related studies in three groups: category management decisions, manufacturer competition, assortment decisions and slotting fees.

2.1 Category Management Decisions

There are some studies that study the category management decisions in the supply chain.

Kurtuluş and Toktay (2011) study a supply chain that consists of one retailer and two manufacturers, and compare the two forms of category management decisions: Retailer Category Management (RCM) or Category Captainship (CC). In the first setting, which is RCM, firstly the retailer determines the shelf space for the category. Then, the manufacturers simultaneously determine their wholesale prices. Finally, the retailer sets the retail prices in order to maximize total category profit. In CC, the retailer chooses one of the manufacturers as "category captain." The retailer determines the shelf space for this product category. Then, the manufacturer who is not category captain determines his wholesale price. Finally, the category captain determines the retail price of all products to maximize the total profit of the alliance which is composed of the category captain and the retailer. At the end of their study, they state that CC may not influence the non-captain manufacturer negatively if their products are sufficiently differentiated. In this study, each product has price-based linear demand as in our study. This work shares the same setting and model assumptions with our study. However, we differ in the main research questions targeted.

Kurtuluş and Nakkaş (2011) analyze the effect of category captainship mechanism on the retail assortment. They consider a game theoretic model with multiple manufacturers and a retailer. Each manufacturer produces one product and sells his product to the end-customer through a common retailer. As in Kurtuluş and Toktay (2011) , the category management decisions RCM and CC are analyzed in this study. In RCM, the retailer decides how many items to include in the retail assortment. In the category captainship, the retailer chooses one of the manufacturers as category captain and this manufacturer makes the assortment selection decision as the category captain. They find that not only the category captain and the retailer, but also non-captain manufacturer may benefit from the catergory captainship under some conditions. This thesis shares the same basic problem setting with this paper, and they both ultimately investigate the assortment size, and hence product proliferation in supply chains. However, our study differs in model details and assumes that the retailer is the one that determines the assortment through prices, but in a sequential manner.

2.2 Manufacturer Competition

In the literature, there are many papers that study a supply chain of competing manufacturers and a single retailer. Below, we provide some examples, which we find relevant for this thesis.

Martinez de Albeniz and Roels (2010) study shelf space competition in a multi-supplier retail point. They model the shelf space competition as a game where first the suppliers decide their wholesale prices and then the retailer allocates her shelf space among the suppliers. They consider both the endogenous and exogenous retail price settings. They use these settings in order to compare the loss of efficiency due to suboptimal shelf space allocation and the loss of efficiency associated with the double marginalization. As a result of their study, they reveal that the loss of efficiency due to the shelf space competition is no larger than 6% whereas the loss of efficiency due to the double marginalization can rise to 27%. The multiple supplier - single retailer setting is we only consider implicitly consider shelf space decision, with no space constraints.

Baron et. al. (2010) study the joint shelf space allocation and inventory decisions for multiple items when demand depends on both of these factors. They use two heuristics in order to analyze the effect of shelf space allocation and inventory level decisions of the retailer. In the first heuristic, they ignore the effect of inventory level on demand and focus on shelf space allocation. In the second heuristic, they ignore the influence of the shelf space and analyze the effect of inventory level. Then, they compare the results of these two heuristics and find that ignoring the effect of the shelf space may be less harmful than ignoring the effect of inventory level. They focus on the retailer's shelf space and inventory level decisions whereas we focus on the retailer's prices, and the consequent new product decision in the supply chain.

Herran et. al. (2006) study a supply chain that consists of two manufacturers and a common retailer. Brands of manufacturers are close substitutes and have deterministic demand. They assume that the total shelf space of the category is already known. In this environment, manufacturers simultaneously determine their wholesale prices. Then, the retailer determines the cost- based retail price of each product and allocation of the shelf space to each brand. Their study shows that there is a strong relationship between manufacturers' wholesale prices and shelf spaces. They obtain two results at the end of this study. First result is that, the higher shelf-space elasticity, the lower wholesale price set to that brand and the lower profit all channel members. The second result is that, the lower unit cost and the higher price elasticity, the greater shelf space allocated to that brand. Their supply chain is similar to our model but we do not have an explicit constraint on the retailer's shelf space.

Cachon and Kök (2010) study a retail supply chain where two competing manufacturers sell their products through a single retailer. In this setting, suppliers simultaneously offer one type of contract to the retailer. The contract types included in this study are: wholesale price contract, quantity discount contract and two part tariff. The quantity discount contract and two part tariff are categorized as sophisticated contracts. Then the retailer chooses prices, which determine the products' demand rates to maximize her profit. They investigate the effect of the three contract types when manufacturers compete in the supply chain. Their supply chain is similar to our model but we only consider wholesale price contract and focus on the new product introduction decision in our study. The authors find that the manufacturers are influenced from the sophisticated contract negatively when there are close substitutes to their products. On the other hand, quantity discount and two part tariff can be beneficial for a single manufacturer case.

Shang, Ha and Tong (2015) analyze the effect of information sharing in a supply chain in which there are two competing manufacturers and a common retailer. The retailer offers either concurrent or sequential information sharing contract to the manufacturers before there is no demand signal. When the retailer observes a demand signal, she shares it with the manufacturers according to their contracts. Then, firstly, manufacturers simultaneously determine their wholesale prices based on the shared demand information. Secondly, the retailer determines the retail price for both products. Each product has a deterministic demand that is determined by the prices and competition intensity in the market. Additionally, Shang et al. study scenarios of production economy and diseconomy. The production economy occurs when marginal cost of manufacturer is decreasing with the production quantity whereas production diseconomy occurs when the marginal cost of the manufacturer is increasing with the production quantity. They find that, information sharing is always beneficial for the retailer when the manufacturers facing either production economy or diseconomy. However, we do not consider information sharing in our model.

Ha, Shang and Wang (2017) study manufacturer rebate competition in a supply chain which includes two competing manufacturers and a retailer. Manufacturers sell substitutable products through the retailer. In this study, firstly, manufacturers simultaneously determine their wholesale prices and rebate values which is not considered in our study. After that, the retailer determines the retail prices for their product. If a manufacturer offers a rebate, he has to pay a fixed cost for launching a rebate promotion, advertising, and distribution and processing fees. As a result of their study, they state that manufacturers offer lower rebate or stop offering rebate entirely when there is intense competition. Morevoer, if retailer subsidizes manufacturers to offer a rebate, it always benefits her.

Choi (1991) studies price competition in a retail supply chain with two manufacturers and a retailer. He considers a linear demand and a nonlinear demand model. In addition, they compare their model in which there is a common retailer with the exlusive dealership model in the literature. The results of the study changes depending on the demand model. For the linear demand model, both prices and profits increase as substitutability of products increases contrary to many other findings in the literature. Under the nonlinear demand model, a manufacturer who sells his products through a common retailer obtains less profit whereas the one who sells his products through an exclusive retailer obtains more profit as differentiation between the products increases. The linear demand model with a common retailer is similar to our two-product equilibrium setting. However, Choi (1991) also compares results of his model with the models with exlusive dealership in the literature this is not considered in our study.

2.3 Entry Deterrence

Although we do not study an entry deterrence problem, we analyze under what kind of conditions the new product is introduced and when it stays out of the market. In this respect, we share the same perspective as the entry deterrence papers.

Karaer and Erhun (2015) study the use of quality as a deterrent in a new product scenario. They study a monopoly and a duopoly setting where quality and price are decision variables. The demand of a product is affected by its own price and quality, as well as the competitor's price and quality. In this respect, the incumbent firm's invested quality determines the profitability of a potential entrant.As a result of their study, they state that incumbent may prefer both over invesment in order to deter the product entry and under investment in order to accomodate the product entry according to the characteristics of the market. In our study we study a new product entry scenario as by an upstream partner as well, but in our model, the competitor does not have a strategy that he can use for deterrence.

Xiao and Qi (2010) study o strategic wholesale pricing of a supplier with a potential entrant. They analyze a two-tier supply chain with an incumbent supplier, a retailer and a potential entrant who produces a partially substitute product. The potential entrant also sells his product through the same retailer. In order to find the best wholesale price strategy for the incumbent supplier they study three alternative models.

These are full deterrence, partial deterrence and full allowance strategies. Their study shows that the existing supplier frequently chooses the fully deterrence strategy while making his wholesale price decision. In our study, the other supplier cannot foresee a potential entry and determines his wholesale price based on the two-product competition only.

2.4 Assortment Decisions and Slotting Fees

Assortment decisions are extensively studied in the literature. Below, we discuss a few example papers relevant to our work.

Guo and Heese (2017) study the effect of manufacturer's distribution strategy on his product variety decision. They consider manufacturer's variety decision in two scenarios: directly to the end-consumers or through a retailer. As a result of this study, they find out that the optimal product variety may be higher in either scenario: based on balance of power between manufacturer and channel intermediaries and effect of the product variety on consumer demand in the way of both scale and variability. They identify a manufacturer's incentives to provide product variety for both two scenarios. If effect of the product variety on demand scale is greater than its effect on demand variability, manufacturer may prefer to offer higher variety when he sells his products through a retailer. On the other hand, if effect of the product variety on demand variability is higher, he should offer higher variety in centralized scenario. In our study, we determine the optimal prices of products rather than product variety.

Aydın and Hausman (2009) investigates the effect of slotting fees on the retailer's product assortment decisions. They study a supply chain with one retailer and one manufacturer. The retailer determines the assortment of products offered to the customers. The authors consider a contract that requires the manufacturer to pay a slotting fee to the retailer for every product if the retailer increases the assortment level to over a certain level. The authors show that the retailer offers the supply chain optimal assortment with this contract when the wholesale of products is lower than a threshold level. With this contract, both the manufacturer and the retailer are better off when the wholesale price is lower than this threshold level. In our study, we also analyze the effect of the slotting fee on new product introduction, and hence the total assortment.

Lariviere and Padmanabhan (1997) the effect of slotting allowances in new product introduction as well. In their setting, a manufacturer determines the wholesale price and the slotting allowance for his new product and announces them to the retailer. Then, the retailer decides whether to allocate shelf space to the new product or not. The retailer does not accept the new product introduction unless summation of its profit and slotting allowance is greater than her stocking costs. They state that, there is no need for a slotting allowance when the demand of the product is known by both the retailer and the manufacturer. A reduction in wholesale price is the most suitable way to compansate the stocking costs. However, if only the manufacturer knows the demand of the new product, the decision of slotting allowance depends on the size of retailer's costs. If her costs are low, again there is no role for slotting allowance because these low costs can be covered by the sales of the product. If her costs are high, however, the reduction in the wholesale price will not be enough to compansate these costs. Thus, the manufacturer offers slotting allowances in order to cover them. We also analyze the slotting fee as the new product introduction control mechanism implemented by the retailer. However, we take the slotting fee as exogenous, and demand level as known. The retailer also determines the shelf space to allocate, implicitly, through the product's retail price.

Desiraju (2001) also studies how to determine the value of slotting fees in a new product introduction scenario. There is a manufacturer and a retailer in this study. The author works on two methods in order to determine the magnitude of slotting fee. In the first setting, the retailer demands a different slotting fee for every brand introduced. In the other setting, she wants an uniform slotting fee for every new product introduction. Both the old and the new product has price-dependent linear demand. In addition to their own prices, each product's demand is affected from its substitute product's price. The author finds that the retailer obtains higher allowances with the brand-to-brand slotting fee whereas the retailer enjoys the success of every reasonable new product introduction with the uniform slotting fee. However, we analyze under which conditions slotting fee is meaningful rather than the magnitude of a slotting fee.

CHAPTER 3

MODEL DECSRIPTION

In this chapter, we will introduce the details of our model. We study a two echelon supply chain with one retailer (R) and two suppliers $(S_1$ and $S_2)$ in a single-period setting. The suppliers produce partially substitutable products and sell them to the retailer. Both suppliers and the retailer are profit maximizers. Firstly, suppliers announce the wholesale prices for their products. Then, the retailer sets the retail price for each product taking into account the announced wholesale prices.

In our study, we assume there is no limit on the shelf space the retailer can allocate to the products. However, the retailer faces a financial burden that increases with the number of products on shelves. This may be attributed to handling, and inventory-related costs or an opportunity cost due to the competing categories for the same real estate. We model this cost tradeoff by defining a shelf space cost that increases exponentially with the total shelf space allocated to the whole category (of two products). We refer to this total shelf space allocated as S. By this way, the total allocated shelf space is equal to the total demand of all products.

In our environment each product has a price-dependent deterministic demand. In addition to own-price, each product's demand is also affected by the price of the substitute product, through the cross price sensitivity parameter. Unit production cost of products are equal each other and for simplicity we assume this cost is equal to zero.

We first study the "Two Product Competition" in Section 3.1 as a baseline for further analysis. Later, we study a potential introduction of a third product by one of the suppliers and how this decision affects the chain in Section 3.2. We refer to this part as "Introduction of a Third Product".

3.1 Two-Product Competition

In the Two Product Competition model each supplier produces one type of product and sells them to the retailer. As mentioned above, in this setting, suppliers simultaneously announce their wholesale prices (w_1 and w_2). After that, the retailer determines the retail prices for their products (p_1 and p_2) in order to maximize her profit. By setting the product prices, the retailer implicitly determines the demand quantities for each product.

The notation we use in this section is provided in Table 3.1.

Table 3.1: Notation of Two Product Competition

Decision Variables	Description
p_1	Retail price of product 1
p_2	Retail price of product 2
w_1	Wholesale price of product 1
w_2	Wholesale price of product 2
Parameters	Description
a	Intrinsic market potential of product 1 and product 2
k	Shelf space cost parameter
θ	Cross price sensitivity parameter
S	Total shelf space allocated for products

Price-dependent demand functions of each product are given below:

$$\begin{aligned} q_1^{(2)}(p_1, p_2) &= (a - p_1 + (p_2 - p_1)\theta)^+ \\ q_2^{(2)}(p_1, p_2) &= (a - p_2 + (p_1 - p_2)\theta)^+ \end{aligned}$$

Products have identical base market potentials, a. Demand quantities always take nonnegative values. The cross price sensitivity parameter, $\theta \in (0, 1)$, shows the sensitivity of a product's demand to the price of both the focal and the substitute products. A product's demand decreases by its own price with a linear effect $(1+\theta)$, and increases with the substitute product's price with a linear effect of θ.

The products produced by Supplier 1 and Supplier 2 are symmetric products. Thus, we use a single substitution rate θ for both two products.

We will use backward induction in order to find the equilibrium of the two-product setting and steps of our solution can be listed as:

1. Find the retailer's best response to wholesale prices w_1 and w_2 for retail price decisions.
2. Find the suppliers' profit maximizing wholesale price decisions w_1 and w_2, considering the retailer's best response.

We will analyze each step separately in the following sections.

3.1.1 Second Stage: The Retailer's Pricing Decision

In the second stage, the retailer determines the retail price for each product for given wholesale prices, w_1 and w_2, in order to maximize her own profit. With the price decisions, the retailer implicitly determines the demand for each product, and hence the total shelf space allocated to the category. As a first step, we first structurally characterize the retailer's unconstrained profit function with respect to p_1 and p_2.

Unconstrained profit function of the retailer can be expressed as:

$$\pi_R \;=\; \hat{q_1}^{(2)}(p_1-w_1)+\hat{q_2}^{(2)}(p_2-w_2)-kS^2 \; where \; S=\hat{q_1}^{(2)}+\hat{q_2}^{(2)}$$

where $\hat{q_1}^{(2)}$ and $\hat{q_2}^{(2)}$ are the counterparts of $q_1^{(2)}$ and $q_2^{(2)}$ without the nonnegativity constraint, respectively. Then, we obtain the full form of the retailer's relaxed profit function as below:

$$\pi_R \;=\; (a-p_1+\theta(p_2-p_1))(p_1-w_1)+(a-p_2+\theta(p_1-p_2))(p_2-w_2)-k(2a-p_1-p_2)^2$$

Proposition 3.1.1 below details the retailer's optimal price decisions that maximizes the unconstrained profit function. For this relaxed problem, we can find the optimal retail price for each product using the first order conditions.

Proposition 3.1.1 *Considering* $\hat{q_1}^{(2)}(p_1, p_2)$ *and* $\hat{q_2}^{(2)}(p_1, p_2)$*, the retailer's best response* $p_1^*(w_1, w_2)$*,* $p_2^*(w_1, w_2)$ *to a given* (w_1, w_2) *is:*

$$p_i^*(w_1, w_2) = \frac{a + 4ak + (k+1)w_i - kw_j}{2(1+2k)} \quad where \ \ i, j = 1, 2 \ \ and \ \ i \neq j \qquad (3.1)$$

Proof of Proposition 3.1.1:We must first show the joint concavity of the retailer's profit function in p_1 and p_2. For this, two conditions must be satisfied: the second order partial derivative must be negative and the determinant of the Hessian matrix must be greater than zero.

First and second derivative of retailer's profit function with respect to p_1 and p_2 are as follows:

$$\begin{aligned}
\frac{\partial \pi_R}{\partial p_1} &= a + 4ak - 2kp_2 + w_1 + (2p_2 + w_1 - w_2)\theta - 2p_1(1 + k + \theta) \\
\frac{\partial^2 \pi_R}{\partial p_1^2} &= -2k + 2(-1 - \theta) < 0 \ (since \ k > 0, \theta > 0) \\
\frac{\partial \pi_R}{\partial p_2} &= a + 4ak - 2p_2 - 2k(p_1 + p_2) + w_2 + (2p_1 - 2p_2 - w_1 + w_2)\theta \\
\frac{\partial^2 \pi_R}{\partial p_2^2} &= -2k + 2(-1 - \theta) < 0 \ (since \ k > 0, \theta > 0) \\
\frac{\partial^2 \pi_R}{\partial p_2 \partial p_1} &= -2k + 2\theta \\
\frac{\partial^2 \pi_R}{\partial p_1 \partial p_2} &= -2k + 2\theta
\end{aligned}$$

The first condition is trivially satisfied since $k > 0$ and $\theta > 0$. When we calculate the determinant of the Hessian Matrix, we find:

$$\frac{\partial^2 \pi_R}{\partial p_1^2}\frac{\partial^2 \pi_R}{\partial p_2^2} - (\frac{\partial^2 \pi_R}{\partial p_1 \partial p_2})^2 = 4(1 + 2k)(1 + 2\theta) > 0 \ \ since \ \ k > 0 \ \ and \ \ 0 < \theta < 1$$

Hence, π_R is jointly concave in p_1 and p_2. Then, first order conditions will be sufficient to find the optimal price decisions.

$$\frac{\partial \pi_R}{\partial p_1} = a + 4ak - 2kp_2 + w_1 + (2p_2 + w_1 - w_2)\theta - 2p_1(1 + k + \theta) = 0$$

$$p_1^*(w_1, w_2, p_2) \ = \ \frac{a + 4ak - 2kp_2 + w_1 + (2p_2 + w_1 - w_2)\theta}{2(1 + k + \theta)}$$

$$\frac{\partial \pi_R}{\partial p_2} = a + 4ak - 2kp_1 + w_2 + (2p_1 - w_1 + w_2)\theta - 2p_2(1 + k + \theta) = 0$$

$$p_2^*(w_1, w_2, p_1) = \frac{a + 4ak - 2kp_1 + w_2 + (2p_1 - w_1 + w_2)\theta}{2(1 + k + \theta)}$$

Solving the two equations above together, we get

$$p_i^*(w_1, w_2) = \frac{a + 4ak + (k+1)w_i - kw_j}{2(1 + 2k)} \quad where \ i, j = 1, 2 \ and \ i \neq j \ \blacksquare$$

The retail price of a product increases in its intrinsic market potential and its wholesale price, because the profit margin $p_i - w_i$ is the critical factor for the profit maximizing retailer. Note that, the impact of the wholesale price gets even stronger with a higher shelf space cost since it also affects the profitability of the product. Moreover, if cost of shelf space increases, the retailer will increase the retail price in order to compensate the cost and reduce the space allocated.

A product's retail price decreases in its substitute product's wholesale price. Note that, the retail price of the substitute product also increases when its wholesale price increases. The retailer partially compensates for this drop in the substitute product's demand by decreasing the retail price of the primary product. In other words, she shifts some of the demand from the product with the increased wholesale price to its competitor.

Using the retail price response given in Proposition 3.1.1, the difference between the retail prices can be expressed as:

$$p_i^*(w_i, w_j) - p_j^*(w_i, w_j) = \frac{w_i - w_j}{2}$$

In our model we have two symmetric products. Therefore, market potential and cost of shelf space do not create a difference between the retail price decisions. The only difference is due to the wholesale prices.

The corresponding demand quantities of products, $\hat{q}_i(w_1, w_2)$, according to retail prices set by the retailer, $p_i^*(w_1, w_2)$ can be expressed as;

$$\hat{q}_i^{(2)}(w_1, w_2) = \frac{a + w_j(k + \theta + 2k\theta) - w_i(1 + k + \theta + 2k\theta)}{2(1 + 2k)} \quad i, j = 1, 2 \ and \ i \neq j$$

If the wholesale price of a product increases, its order quantity $\hat{q}_i$ decreases and the other product's quantity increases. Note that sensitivity of quantity levels to wholesale prices increases with cross-sensitivity parameter θ. As products become more substitutable (and thus competitive), the retailer's response to an increased wholesale price in one gets stronger. As a product becomes more expensive, the retailer prefers to increase the demand for the other product and generate the bulk of the sale from it.

Difference between the quantity levels can be simplified as a function of the wholesale prices and the cross-price sensitivity parameter θ as follows:

$$q_i^{(2)} - q_j^{(2)} = (w_j - w_i)(\frac{1}{2} + \theta) \tag{3.2}$$

3.1.2 First Stage: The Suppliers' Pricing Decisions

In this stage, foreseeing the retailer's price response, each supplier simultaneously determines the wholesale price for its own product. Supplier i's unconstrained profit function can be expressed as:

$$\pi_{S_i}(w_1, w_2) = \hat{q_i}^{(2)}(w_1, w_2)w_i \qquad i = 1, 2$$

Proposition 3.1.2 *Assuming the retailer price decisions will follow as in Equation 3.1, Supplier i's best response to the substitute product j's wholesale price is:*

$$w_i^*(w_j) = \frac{a + w_j(k + \theta + 2k\theta)}{2(1 + k + \theta + 2k\theta)} \qquad i, j = 1, 2 \qquad and \qquad i \neq j$$

Proof of Proposition 3.1.2: Before finding the best response of the suppliers' for wholesale price decision, we should check the concavity of the supplier's profit function in its wholesale price. The second-order derivative of profit function of Supplier i:

$$\frac{\partial^2 \pi_{S_i}}{\partial w_i^2} = -\frac{1 + k + \theta + 2k\theta}{1 + 2k} < 0 \quad since \quad k > 0 \quad and \quad 0 < \theta < 1$$

Hence, Supplier i's profit function is concave in w_i. Thus, first order condition will be sufficient for optimality:

$$\frac{\partial \pi_{S_1}}{\partial w_1} = \frac{a + w_2(k + \theta + 2k\theta) - w_1(1 + k + \theta + 2k\theta)}{2 + 4k} = 0$$

$$\rightarrow w_1^*(w_2) = \frac{1}{2}(\frac{a + w_2(k + \theta + 2k\theta)}{1 + k + \theta + 2k\theta})$$

The derivation for Supplier 2 will follow similarly to that for Supplier 1. ■

Similar to the retailer's pricing decision, the supplier's wholesale price decision increases with the product's intrinsic market potential. If one of the suppliers decreases his wholesale price, his competitor will also decrease his wholesale price as a response. Otherwise, the primary supplier seizes a portion of demand from his competitor based on the price differences and the degree of substitution between the products. As mentioned in the second stage, if cost of shelf space increases, the retailer will increase her retail price. Afterwards, the demand quantities will decrease due to this retail price increase. As a result of this, suppliers decrease the wholesale price of their products as shelf space cost increases, to compensate for consequent decrease in demand.

At this point we will solve these best response functions of suppliers simultaneously and find the Nash equilibrium wholesale prices based on Proposition 3.1.1 and 3.1.2.

Proposition 3.1.3 *Considering the* $\hat{q_1}^{(2)}(p_1, p_2)$ *and* $\hat{q_2}^{(2)}(p_1, p_2)$ *and suppliers' equilibrium prices will be as follows:*

$$w_i^* = \frac{a}{(2 + k + \theta + 2k\theta)} \quad i = 1, 2$$

Proof of Proposition 3.1.3: In Proposition 3.1.2, we found each supplier's best wholesale price to that of the competitor's. If we solve $w_2^*(w_1)$ and $w_1^*(w_2)$ together we can find the equilibrium wholesale prices as follows:

$$\begin{aligned} w_1^*(w_2^*(w_1)) &= w_1^* \\ \rightarrow w_1^* &= \frac{a}{(2 + k + \theta + 2k\theta)} \end{aligned}$$

The derivation for Supplier 2's wholesale price follows similarly. ■

Based on Proposition 3.1.3 the equilibrium prices, corresponding demand quantities and profits in the Two Product setting are available in Table 3.2.

Table 3.2: Equilibrium Prices, Corresponding Demand Quantities and Profits

Expression	**Value**
p_1^*	$\frac{a+4ak+\frac{a}{2+k+\theta+2k\theta}}{2(1+2k)}$
p_2^*	$\frac{a+4ak+\frac{a}{2+k+\theta+2k\theta}}{2(1+2k)}$
w_1^*	$\frac{a}{(2+k+\theta+2k\theta)}$
w_2^*	$\frac{a}{(2+k+\theta+2k\theta)}$
$q_1^{(2)}(w_1^*, w_2^*, p_1^*, p_2^*)$	$\frac{a(1+k+\theta+2k\theta)}{2(1+2k)(2+k+\theta+2k\theta)}$
$q_2^{(2)}(w_1^*, w_2^*, p_1^*, p_2^*)$	$\frac{a(1+k+\theta+2k\theta)}{2(1+2k)(2+k+\theta+2k\theta)}$
$\pi_R^{(2)}(w_1^*, w_2^*, p_1^*, p_2^*)$	$\frac{a^2(1+k+\theta+2k\theta)^2}{2(1+2k)(2+k+\theta+2k\theta)^2}$
$\pi_{S_1}^{(2)}(w_1^*, w_2^*, p_1^*, p_2^*)$	$\frac{a^2(1+k+\theta+2k\theta)}{2(1+2k)(2+k+\theta+2k\theta)^2}$
$\pi_{S_2}^{(2)}(w_1^*, w_2^*, p_1^*, p_2^*)$	$\frac{a^2(1+k+\theta+2k\theta)}{2(1+2k)(2+k+\theta+2k\theta)^2}$

In this part, we found the two product setting equilibrium by relaxing the demand nonnegativity restriction. The resulting quantity values, wholesale and retail prices here are nonnegative by our initial assumptions $\theta > 0$ and $k > 0$, and hence feasible in the original problem. Thus, these expressions are in fact the equilibrium values of our original problem (with the nonnegativity constraint).

3.2 Introduction of a Third Product

In the second part of our study, Supplier 2 considers introducing a new product which is a partial substitute to both products 1 and 2. Given the existing two products, he determines the wholesale price of the new product. Then the retailer determines the retail price of this third product. By determining its retail price, the retailer can practically reject the new product, producing zero demand and allocating zero shelf space for it.

If the retailer accepts the third product, demand of each product will be expressed as follows:

$$\begin{aligned} q_1^{(3)}(p_1,p_2,p_3) &= (a-p_1+(p_2-p_1)\theta+(p_3-p_1)\alpha)^+ \\ q_2^{(3)}(p_1,p_2,p_3) &= (a-p_2+(p_1-p_2)\theta+(p_3-p_2)\alpha)^+ \\ q_3^{(3)}(p_1,p_2,p_3) &= (b-p_3+(p_1-p_3)\alpha+(p_2-p_3)\alpha)^+ \end{aligned}$$

The new product is a new variant of our existing products. We use the same substitution level, α, between the new product and the existing products. After introducing the third product due to the substitution effect, demand quantities of product 1 and 2 may change according to cross price sensitivity parameter, market potential and retail price of the third product. The new product may draw the attention of customers, who do not buy a product from this category, by means of its new features. In addition, these "new" customers may buy one of the existing products depending on the prices at the time of purchase. Thus, with the new product introduction, demand potential of this product category may increase and each product may take a share from this new demand potential. Based on the retail price of the third product, demand quantities of product 1 and 2 may also decrease.

The notation we use in this section is provided in Table 3.3.

Table 3.3: Notation used in Introduction of Third Product Setting

Decision Variables	Description
p_3	Retail price of product 3
w_3	Wholesale price of product 3
Parameters	**Description**
a	Market potential of product 1 and 2
b	Market potential of product 3
p_1	Retail price of product 1
p_2	Retail price of product 2
w_1	Wholesale price of product 1
w_2	Wholesale price of product 2
k	Shelf space cost parameter
θ	Cross price sensitivity parameter between product 1-2
α	Cross price sensitivity parameter between product 3 and others
S	Total shelf space allocated

We use backward induction to find the optimal retail and wholesale price decisions for the new product as follows:

1. The retailer determines the retail price response for product 3 for a given wholesale price of product 3.

2. Foreseeing the retailer's best response, Supplier 2 determines his wholesale price for product 3.

Each step of this model will be analyzed separately in the following sections.

3.2.1 Second Stage: Retailer's Pricing Decision For The New Product

The retailer earns profit from all 3 products when the third product is active in the market. Otherwise, profit of the retailer formulated in the two-product setting will be valid. Thus, the retailer's profit function for the new product introduction case can be described as follows:

$$\pi_R = \begin{cases} \pi_R^{(3)} = q_1^{(3)}(p_1 - w_1) + q_2^{(3)}(p_2 - w_2) \\ \qquad\qquad + q_3^{(3)}(p_3 - w_3) - k(S^{(3)})^2, & \text{if } q_3^{(3)} > 0 \\ \\ \pi_R^{(2)} = q_1^{(2)}(p_1 - w_1) + q_2^{(2)}(p_2 - w_2) - k(S^{(2)})^2, & \text{if } q_3^{(3)} = 0 \text{ (Is Not Active)} \end{cases} \tag{3.3}$$

The retailer determines the best retail price response for the new product for given wholesale price. By determining the retail price, the retailer implicitly decides whether to accept the new product on her shelves, or not.

In our study, we define $\bar{p}_3$ as the lowest retail price where demand quantity of the third product is equal to zero. If the retail price of the new product is lower than $\bar{p}_3$, the third product will be active in the market with a positive demand quantity.

Proposition 3.2.1 *Given $(p_1, w_1), (p_2, w_2)$ and w_3, the retailer's best response retail price that maximizes her profit is as follows:*

$$p_3^* = \begin{cases} p_3^0 = \frac{b+2k(2a+b)+2(p_1+p_2)(\alpha-k)}{2(1+k+2\alpha)} \\ \qquad + \frac{-\alpha(w_1+w_2)+w_3(1+2\alpha)}{2(1+k+2\alpha)}, & \textit{if } w_3 \leq \bar{w}_3, \pi_R^{(3)}(p_3^0(w_3)) \geq \pi_R^{(2)} (Case\ 1) \\ \\ \bar{p}_3 = \frac{b+\alpha(p_1+p_2)}{1+2\alpha}, & \textit{if } w_3 > \bar{w}_3, \pi_R^{(3)}(\bar{p}_3) \geq \pi_R^{(2)} \quad (Case\ 2) \\ \\ (\bar{p}_3, \infty)(third\ product\ is\ not\ active), & \textit{if } w_3 \leq \bar{w}_3, \pi_R^{(3)}(p_3^0(w_3)) < \pi_R^{(2)} (Case\ 3) \\ & \textit{or if } w_3 > \bar{w}_3, \pi_R^{(3)}(\bar{p}_3) < \pi_R^{(2)} \quad (Case\ 4) \end{cases}$$

where $\bar{w}_3 = \frac{2k(p_1+p_2)(1+3\alpha)}{(1+2\alpha)^2} + \frac{\alpha(w_1+w_2)}{(1+2\alpha)} + \frac{b-4ak}{(1+2\alpha)} - \frac{4bk\alpha}{(1+2\alpha)^2}$. $\pi_R^{(2)}$ and $\pi_R^{(3)}$ are defined in Equation 3.3 above. The open form of the conditions $\pi_R^{(3)}(p_3^0(w_3)) \geq \pi_R^{(2)}$ and $\pi_R^{(3)}(\bar{p}_3) \geq \pi_R^{(2)}$ are available in Table 0.1 in Appendix D .

Proof of Proposition 3.2.1: Firstly, we need to analyze the retailer's profit function with three products; $\pi_R^{(3)}$.

Second order derivative of $\pi_R^{(3)}$ with respect to p_3 is

$\frac{\partial^2 \pi_R^{(3)}}{\partial p_3^2} = -2(2\alpha + k + 1) < 0 \ \ since \ \ 0 < \alpha < 1 \ \ and \ \ k > 0$. Hence, $\pi_R^{(3)}$ is concave in p_3.

Then, we can find the maximizing price of $\pi_R^{(3)}$ from FOC as below:

$$\begin{aligned}\frac{\partial \pi_R^{(3)}}{\partial p_3} &= b + 4ak + 2bk - 2p_3 - 2k(p_1 + p_2 + p_3) + w_3 + (2p_1 + 2p_2 - 4p_3 \\ &\quad -w_1 - w_2 + 2w_3)\alpha = 0\end{aligned}$$

and we define $p_3^0(w_3)$ as the unconstrained maimizer from FOC:

$p_3^0(w_3) := \frac{b+2k(2a+b)+2(p_1+p_2)(\alpha-k)-\alpha(w_1+w_2)+w_3(1+2\alpha)}{2(1+k+2\alpha)}$

For feasibility, the retail price of the third product must be between zero and $\bar{p}_3$. Thus, we can reformulate the retailer's profit function wrt p_3 as follows:

$$\pi_R = \begin{cases} q_1^{(3)}(p_1 - w_1) + q_2^{(3)}(p_2 - w_2) + q_3^{(3)}(p_3 - w_3) - k(S^{(3)})^2, & \text{if } p_3 < \bar{p}_3 \\ q_1^{(2)}(p_1 - w_1) + q_2^{(2)}(p_2 - w_2) - k(S^{(2)})^2, & \text{if } p_3 \geq \bar{p}_3 \text{ (Is Not Active)} \end{cases}$$

From the demand function of the third product, $\bar{p}_3$ is expressed as:

$$\bar{p}_3 = \frac{b + \alpha(p_1 + p_2)}{1 + 2\alpha}$$

There may be cases where p_3 is very close but not equal to $\bar{p}_3$ (i.e., $\bar{p}_3 - \varepsilon$), which would produce practically zero demand for product 3 but move the existing products into a 3-product market. To reflect this, we will reformulate π_R as follows:

$$\pi_R = \begin{cases} \pi_R^{(3)}, & \text{if } p_3 < \bar{p}_3 \\ \pi_R^{(3)}, & \text{if } p_3 = \bar{p}_3 \\ \pi_R^{(2)}, & \text{if } p_3 > \bar{p}_3 \text{ (Is Not Active)} \end{cases}$$

Note that, π_R above esentially has two pieces. For [0, $\bar{p}_3$], we observe $\pi_R^{(3)}$ which is concave in p_3, and for ($\bar{p}_3$, ∞) we observe $\pi_R^{(2)}$ which does not change with p_3. Note that, π_R is not necessarily a continuous function in p_3, mainly because of changing market potentials between the two and three-product settings, and hence changing values between $\pi_R^{(3)}$ and $\pi_R^{(2)}$. Thus, we can describe the best retail price response of the retailer as follows:

$$p_3^* = \begin{cases} p_3^0 = \frac{b+2k(2a+b)+2(p_1+p_2)(\alpha-k)-\alpha(w_1+w_2)+w_3(1+2\alpha)}{2(1+k+2\alpha)}, & \text{if } p_3^0 \le \bar{p}_3, \pi_R^{(3)}(p_3^0) \ge \pi_R^{(2)} \\ \bar{p}_3 = \frac{b+\alpha(p_1+p_2)}{1+2\alpha}, & \text{if } p_3^0 > \bar{p}_3, \pi_R^{(3)}(\bar{p}_3) \ge \pi_R^{(2)} \\ (\bar{p}_3, \infty)(third\, product\; is\; not\; active), & \text{if } p_3^0 \le \bar{p}_3, \pi_R^{(3)}(p_3^0) < \pi_R^{(2)} \\ & \text{or if } p_3^0 > \bar{p}_3, \pi_R^{(3)}(\bar{p}_3) < \pi_R^{(2)} \end{cases}$$

When we check the the first derivative of the p_3^0 with respect to w_3, we can find that

$$\frac{\partial p_3^0}{\partial w_3} = \frac{1+2\alpha}{2(1+k+2\alpha)} > 0 \;\; where \;\; 0 < \alpha < 1 \;\; and \;\; k > 0$$

So, $p_3^0(w_3)$ is a monotonically increasing function of w_3. Thus, we can find the cutoff w_3 which makes $p_3^0(w_3)$ equal to $\bar{p}_3$ as follows:

$$\begin{aligned} p_3^0(\bar{w}_3) &= \bar{p}_3 \\ \bar{w}_3 &= \frac{2k(p_1+p_2)(1+3\alpha)}{(1+2\alpha)^2} + \frac{\alpha(w_1+w_2)}{(1+2\alpha)} + \frac{b-4ak}{(1+2\alpha)} - \frac{4bk\alpha}{(1+2\alpha)^2} \end{aligned}$$

If the wholesale price of the third product is less than or equal to $\bar{w}_3$, p_3^0 will also be less than or equal to $\bar{p}_3$. Otherwise, $p_3^0 > \bar{p}_3$. Thus, we can reformulate the retailer's optimal p_3^* decision WRT w_3 as follows:

$$p_3^* = \begin{cases} p_3^0 = \frac{b+2k(2a+b)+2(p_1+p_2)(\alpha-k)-\alpha(w_1+w_2)+w_3(1+2\alpha)}{2(1+k+2\alpha)}, & \text{if } w_3 \le \bar{w}_3, \pi_R^{(3)}(p_3^0(w_3)) \ge \pi_R^{(2)} \\ \bar{p}_3 = \frac{b+\alpha(p_1+p_2)}{1+2\alpha}, & \text{if } w_3 > \bar{w}_3, \pi_R^{(3)}(\bar{p}_3) \ge \pi_R^{(2)} \\ (\bar{p}_3, \infty)(third\, product\; is\; not\; active), & \text{if } w_3 \le \bar{w}_3, \pi_R^{(3)}(p_3^0(w_3)) < \pi_R^{(2)} \\ & \text{or if } w_3 > \bar{w}_3, \pi_R^{(3)}(\bar{p}_3) < \pi_R^{(2)} \end{cases}$$

■

Here, $\bar{w}_3$ is the wholesale price of the product 3 where $p_3^0(\bar{w}_3) = \bar{p}_3$.

When $Case\ 1$ conditions hold, the retailer chooses to set a price so that the third product is active because her profit from the three-product market is higher than that with two products only. In this case, the third product's retail price is increasing in its intrinsic market potential b and its own wholesale price w_3, and decreasing in the wholesale prices w_1 and w_2. How the retail prices of the currently active two products influence p_3 depends on the third product's substitutability with the other two products (cross-price sensitivity parameter α) vs. the shelf space cost. If the third product is a close substitute with a high α (relative to k), then the retailer's p_3 increases with p_1 and p_2. In this case, a third product with a high p_3 creates awareness for the whole category and enhances the demand of the two incumbent products. On the other hand, if the third product is introduced with a low α, p_3 will decrease with p_1 and p_2. Under this condition, the third product does not have much of an effect on the other two products, and thus decreasing p_3 when p_1 and/or p_2 increases is preferable for the retailer to avoid a big fall in the total quantity ordered.

If $Case$ 2 conditions hold, the retailer sets the retail price of the third product to $\bar{p}_3$. In this case, the demand of the new product will practically be zero. The retailer includes the third product in the category to increase the total awareness and demand potential of the incumbent products, but will set its retail price so that the third product has practically zero sales and thus profit. This condition may arise only if the Supplier 2's suggested wholesale price w_3 is above a certain threshold. In this case, the retail price of the new product is not sensitive to w_3, the retailer sets the retail price to $\bar{p}_3$ for any w_3 as long as $w_3 > \bar{w}_3$. The retail price of the new product depends on only the market potential b, retail prices of the incumbent products p_1 and p_2 and substitution level of the new product α. If the retail prices of the existing products increase, the third product's retail price will also increase. How the optimal price is affected from substitution level α depends on the size of the market potential b. If the demand potential of the new product is approximately equal or gretaer than market potential of the existing products, the optimal price decreases as α increases. On the other hand, when the new product has significantly lower demand potential, the optimal retail price may increase as α increases. Other market parameters affect only the threshold level of wholesale price, $\bar{w}_3$, in this case.

The retailer may prefer not to allocate any shelf space to the new product when either $w_3 \leq \bar{w}_3$ or $w_3 > \bar{w}_3$ holds, unless it increases her profit compared to the two-product equilibrium setting. These correspond to $Case$ 3 and $Case$ 4 in the proposition, respectively.

3.2.2 First Stage: Supplier 2's Pricing Decision For The New Product

In this stage, Supplier 2 determines the wholesale price of the third product in order to maximize his profit foreseeing the retailer's best response. If the new product is introduced, Supplier 2 will earn a total profit from both products 2 and 3. Based on the retailer's best response, we can formulate Supplier 2's profit function as follows:

$$\pi_{S_2}(w_3, p_3^*(w_3)) = \begin{cases} q_2^{(3)}(p_3^0(w_3))w_2 \\ \qquad +q_3^{(3)}(p_3^0(w_3))w_3, & \text{if } w_3 \leq \bar{w}_3, \pi_R^{(3)}(p_3^0(w_3)) \geq \pi_R^{(2)} \ (Case\ \ 1) \\ q_2^{(3)}(\bar{p}_3)w_2 + 0, & \text{if } w_3 > \bar{w}_3, \pi_R^{(3)}(\bar{p}_3) \geq \pi_R^{(2)} \qquad (Case\ \ 2) \\ \pi_{S_2}^{(2)} = q_2^{(2)}w_2, & \text{if } w_3 \leq \bar{w}_3, \pi_R^{(3)}(p_3^0(w_3)) < \pi_R^{(2)} \ (Case\ \ 3) \\ & \text{or if } w_3 > \bar{w}_3, \pi_R^{(3)}(\bar{p}_3) < \pi_R^{(2)} \qquad (Case\ \ 4) \end{cases}$$

In $Case$ 2, the retail price of the new product is equal to $\bar{p}_3$ and this retail price does not depend on w_3. The retailer accepts the new product, but its demand is practically equal to zero. In $Case$ 4, the new product will not be active in the market. If the wholesale price of the new product is higher than $\bar{w}_3$, we will observe either $Case$ 2 or $Case$ 4. Which of these two cases occurs depends on whether the condition $\pi_R^{(3)}(\bar{p}_3) \geq \pi_R^{(2)}$ holds or not. If $\pi_R^{(3)}(\bar{p}_3) \geq \pi_R^{(2)}$, we will observe $Case$ 2. Otherwise, we will observe $Case$ 4. Note that, the comparison of $\pi_R^{(3)}$ and $\pi_R^{(2)}$ do not depend on w_3 and are driven by the market parameters and the existing product prices (w_1, p_1), (w_2, p_2) only. [1]

For $w_3 \leq \bar{w}_3$, we will observe either $Case$ 1 or $Case$ 3. To understand the transition between these cases with respect to w_3, we will analyze the change in $\pi_R^{(3)}(p_3^0(w_3))$ with respect to w_3.

Lemma 3.2.2 $\pi_R^{(3)}(p_3^0(w_3))$ *is a convex function in* w_3.

Proof of Lemma 3.2.2

$$\frac{\partial^2 \pi_R^{(3)}(p_3^0(w_3))}{\partial w_3^2} \quad = \quad \frac{(1+2\alpha)^2}{2(1+k+2\alpha)} > 0 \quad where \quad 0 \ < \alpha < \ 1 \quad and \quad k > 0$$

[1]The condition $\pi_R^{(3)}(\bar{p}_3) \geq \pi_R^{(2)}$ is available in Table 0.1 in Appendix D

Therefore, $\pi_R^{(3)}(p_3^0(w_3))$ is a convex function in w_3. ■

Due to convexity, $\pi_R^{(3)}(p_3^0(w_3))$ has a minimum value at a unique w_3. We define this level of w_3 as w_3'.

Lemma 3.2.3 *The wholesale price, w_3', which minimizes $\pi_R^{(3)}(p_3^0(w_3))$ is equal to $\bar{w}_3$.*

Proof of Lemma 3.2.3: We can find w_3' by using first order condition.

$$\frac{\partial \pi_R(p_3^0)}{\partial w_3} = 0$$

$$w_3' = \frac{b + 2k(-2a + p_1 + p_2) + b(2 - 4k)\alpha + \alpha(-8ak + 6k(p_1 + p_2) + (w_1 + w_2)(1 + 2\alpha))}{(1 + 2\alpha)^2}$$

compare with

$$\bar{w}_3 = \frac{b + 2k(-2a + p_1 + p_2) + b(2 - 4k)\alpha + \alpha(-8ak + 6k(p_1 + p_2) + (w_1 + w_2)(1 + 2\alpha))}{(1 + 2\alpha)^2}$$

$$w_3' - \bar{w}_3 = 0$$

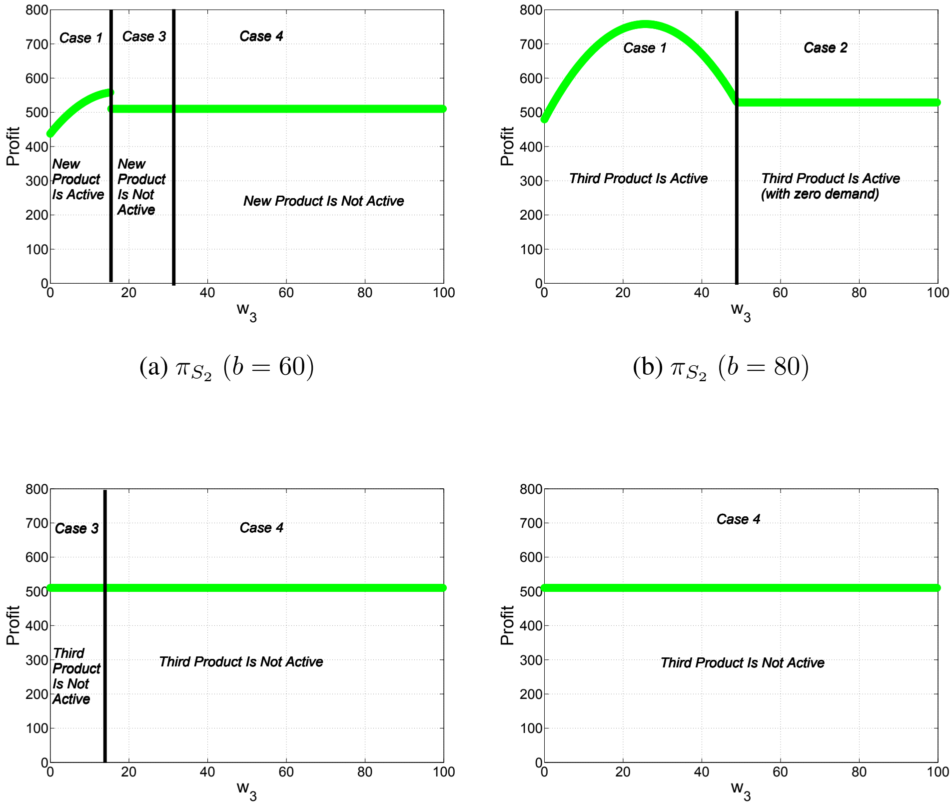
■

Due to convexity of $\pi_R^{(3)}(p_3^0(w_3))$ and equality of w_3' and $\bar{w}_3$, we may observe first Case 1 then Case 3, only 1 or only 3 when $w_3 \leq \bar{w}_3$. In addition, we may observe $Case\ 2$ or $Case\ 4$ and the profit of the retailer does not change wrt w_3 when $w_3 > \bar{w}_3$. Thus, we may observe six different configurations with the combination of these possible cases.

The profit function of Supplier 2 with respect to w_3 and the transition between the cases can be observed in the numerical examples in Figure 3.1.

In Figure 3.1a, we observe $Case\ 1$ and $Case\ 3$ when $w_3 \leq \bar{w}_3$. In addition, we observe $Case\ 4$ when $w_3 > \bar{w}_3$. In Figure 3.1b, we observe $Case\ 1$ up to $\bar{w}_3$ and $Case\ 2$ beyond $\bar{w}_3$. On the other hand, the retailer rejects the new product for any level of w_3 and we observe $Case\ 3$ and $Case\ 4$ in Figure 3.1c. And finally, in Figure 3.1d we observe only $Case\ 4$ when $\bar{w}_3 < 0$ and every nonnegative w_3 is greater than $\bar{w}_3$.

We will describe the wholesale price decision of Supplier 2 by considering the transition between the cases.



(a) π_{S_2} $(b = 60)$ (b) π_{S_2} $(b = 80)$

(c) π_{S_2} $(b = 20)$ (d) π_{S_2} $(b = 1)$

Figure 3.1: π_{S_2} wrt w_3(a=100, θ=0.5, α=0.1, k=0.3)

Lemma 3.2.4 *When $\bar{w}_3 \geq 0$, if $\pi_R^{(3)}(\bar{w}_3)$ is greater than or equal to zero, $\pi_R^{(3)}(w_3 = 0)$ will also be greater than zero.*

Proof of Lemma 3.2.4: Follows from the definition of $\bar{w}_3$. ■

Lemma 3.2.5 *When $\bar{w}_3 \geq 0$, if $\pi_R^{(3)}(w_3 = 0)$ is lower than zero, $\pi_R^{(3)}(\bar{w}_3)$ will also be lower than zero.*

Proof of Lemma 3.2.5: Follows from the definition of $\bar{w}_3$. ■

Proposition 3.2.6 *Supplier 2's optimal wholesale price decision for the new product is:*

(i) When $\bar{w}_3 \geq 0$, $C_1 \geq 0$ and $C_2 < 0$ hold:

$$w_3^* = \begin{cases} w_3^0, & \text{if } 0 < w_3^0 \leq w_3'', \pi_{S_2}^{(3)}(w_3^0) \geq \pi_{S_2}^{(2)} \quad (\text{Case 2.1}) \\ w_3'', & \text{if } w_3^0 > w_3'', \pi_{S_2}^{(3)}(w_3'') \geq \pi_{S_2}^{(2)} \quad (\text{Case 2.2}) \\ 0, & \text{if } w_3^0 \leq 0, \pi_{S_2}^{(3)}(w_3 = 0) \geq \pi_{S_2}^{(2)} \quad (\text{Case 2.3}) \\ (w_3'', \infty), & \text{if } 0 < w_3^0 \leq w_3'', \pi_{S_2}^{(3)}(w_3^0) < \pi_{S_2}^{(2)} \quad (\text{Case 2.4}) \\ & \text{or if } w_3^0 > w_3'', \pi_{S_2}^{(3)}(w_3'') < \pi_{S_2}^{(2)} \quad (\text{Case 2.5}) \\ & \text{or if } w_3^0 \leq 0, \pi_{S_2}^{(3)}(w_3 = 0) < \pi_{S_2}^{(2)} \quad (\text{Case 2.6}) \end{cases}$$

(ii) When $\bar{w}_3 \geq 0$ and $C_2 \geq 0$ hold:

$$w_3^* = \begin{cases} w_3^0, & \text{if } 0 < w_3^0 \leq \bar{w}_3 \quad (\text{Case 2.7}) \\ (\bar{w}_3, \infty), & \text{if } w_3^0 > \bar{w}_3 \quad (\text{Case 2.8}) \\ 0, & \text{if } w_3^0 \leq 0 \quad (\text{Case 2.9}) \end{cases}$$

(iii) When $\bar{w}_3 \geq 0$ and $C_1 < 0$ hold, $w_3^* \in [0, \infty)$ but the third product will never be active in the market. $(Case\ 2.10)$

(iv) When $\bar{w}_3 < 0$ and $C_2 < 0$ hold, $w_3^* \in [0, \infty)$ but the third product will never be active in the market. $(Case\ 2.11)$

where

$$w_3^0 = \frac{k(p_1+p_2)(1+3\alpha)}{(1+2\alpha)^2} + \frac{\alpha(w_1+2w_2)}{2(1+2\alpha)} - \frac{2bk\alpha}{(1+2\alpha)^2} + \frac{b-4ak}{2(1+2\alpha)}$$

$$\bar{w}_3 = \frac{2k(p_1+p_2)(1+3\alpha)}{(1+2\alpha)^2} + \frac{\alpha(w_1+w_2)}{(1+2\alpha)} + \frac{b-4ak}{(1+2\alpha)} - \frac{4bk\alpha}{(1+2\alpha)^2}$$

$$\begin{aligned} w_3'' = \; & \frac{1}{(1+2\alpha)^2}\Big(b + b(2-4k)\alpha + w_1\alpha(1+2\alpha) + w_2\alpha(1+2\alpha) + 2k(p_1+p_2+3(p_1+p_2)\alpha - 2a(1+2\alpha)) \\ & - 2(1+k+2\alpha)\Big\{\frac{1}{1+k+2\alpha}(\alpha(p_1^2 + 2kp_1^2 + 4kp_1p_2 + p_2^2 + 2kp_2^2 - p_1w_1 - p_2w_2 + 4b^2k\alpha \\ & + ((3+5k)p_1^2 + p_2((3+5k)p_2 + w_1 - 3w_2) + p_1(2(-1+5k)p_2 - 3w_1 + w_2))\alpha \\ & + 2(p_1-p_2)(p_1-p_2-w_1+w_2)\alpha^2 + 4ak(2b-p_1-p_2)(1+2\alpha) \\ & + b(w_1+w_2+2(w_1+w_2)\alpha - p_1(1+2\alpha+4k(1+3\alpha)) - p_2(1+2\alpha+4k(1+3\alpha)))))\Big\}^{1/2}\Big) \end{aligned}$$

$\pi_{S_2}^{(3)}(w_3^0) = q_2^{(3)}(p_3^0(w_3))w_2 + q_3^{(3)}(p_3^0(w_3))w_3^0$; $\pi_{S_2}^{(3)}(\bar{p}_3) = q_2^{(3)}(\bar{p}_3)w_2$; $\pi_{S_2}^{(2)} = q_2^{(2)}w_2$

$C_1 := \pi_R^{(3)}(p_3^0(w_3 = 0)) - \pi_R^{(2)}$, $C_2 := \pi_R^{(3)}(p_3^0(\bar{w}_3)) - \pi_R^{(2)}$ The open form of C_1 and C_2 are available in Table 0.2 in Appendix F.

Proof of Proposition 3.2.6: We have already established in Lemma 3.2.3 that $\pi_R^{(3)}(p_3^0(w_3))$ takes its minimum value at $w_3\prime = \bar{w}_3$.

Then, for $0 \leq w_3 \leq \bar{w}_3$:

We will observe first $Case$ 1 and then $Case$ 3 if $\pi_R^{(3)}(p_3^0(w_3 = 0)) \geq \pi_R^{(2)}$ and $\pi_R^{(3)}(p_3^0(\bar{w}_3)) < \pi_R^{(2)}$; only $Case$ 1 if if $\pi_R^{(3)}(p_3^0(w_3 = 0)) \geq \pi_R^{(2)}$ and $\pi_R^{(3)}(p_3^0(\bar{w}_3)) \geq \pi_R^{(2)}$; and only $Case$ 3 if if $\pi_R^{(3)}(p_3^0(w_3 = 0)) < \pi_R^{(2)}$.

For $w_3 > \bar{w}_3$:

We will observe $Case$ 2 if $\pi_R^{(3)}(\bar{p}_3) \geq \pi_R^{(2)}$, and $Case$ 4 otherwise. Since $\bar{p}_3$ by definition equal to $p_3^0(\bar{w}_3)$; we can say $Case$ 2 occurs iff $C_2 \geq 0$ and $Case$ 4 occurs iff $C_2 < 0$.

Thus we will observe Supplier 2's profit function in the form of five possible configurations below:

(i) $Case$ 1 then 3, followed by 4 (if $\bar{w}_3 \geq 0$, $C_1 \geq 0$ and $C_2 < 0$);

(ii) $Case$ 1 followed by $Case$ 2 (if $\bar{w}_3 \geq 0$, $C_1 \geq 0$ and $C_2 \geq 0$) and

(iii) $Case$ 3, followed by $Case$ 4 (if $\bar{w}_3 \geq 0$ and $C_1 < 0$ which guarantees $C_2 < 0$ as well)

(iv) Only $Case$ 4 (if $\bar{w}_3 < 0$ and $C_2 < 0$)

(v) Only $Case$ 2 (if $\bar{w}_3 < 0$ and $C_2 \geq 0$)

We will analyze each of five options above separately in order to find the best wholesale price decision of Supplier 2.

$Option$ (i):

When $\bar{w}_3 \geq 0$, $\pi_R^{(3)}(p_3^0(w_3 = 0)) \geq \pi_R^{(2)}$ and $\pi_R^{(3)}(p_3^0(\bar{w}_3)) < \pi_R^{(2)}$, we will observe $Case$ 1, $Case$ 3, and then $Case$ 4 as w_3 increases.

Here, we define w_3'' as the w_3 value that satisfies $\pi_R^{(3)}(p_3^0(w_3)) = \pi_R^{(2)}$. Then

$$\begin{aligned}
w_3'' : \quad = \quad & \frac{1}{(1+2\alpha)^2}\Bigg(b + b(2-4k)\alpha + w_1\alpha(1+2\alpha) + w_2\alpha(1+2\alpha) + 2k(p_1+p_2+3(p_1+p_2)\alpha \\
& -2a(1+2\alpha)) - 2(1+k+2\alpha)\Bigg\{\frac{1}{1+k+2\alpha}(\alpha(p_1^2+2kp_1^2+4kp_1p_2+p_2^2+2kp_2^2-p_1w_1-p_2w_2 \\
& +4b^2k\alpha + ((3+5k)p_1^2 + p_2((3+5k)p_2+w_1-3w_2) + p_1(2(-1+5k)p_2-3w_1+w_2))\alpha \\
& +2(p_1-p_2)(p_1-p_2-w_1+w_2)\alpha^2 + 4ak(2b-p_1-p_2)(1+2\alpha) \\
& +b(w_1+w_2+2(w_1+w_2)\alpha - p_1(1+2\alpha+4k(1+3\alpha)) - p_2(1+2\alpha+4k(1+3\alpha)))))\Bigg\}^{1/2}\Bigg)
\end{aligned}$$

We observe $Case$ 1 when $0 \leq w_3 \leq w_3''$ and profit of Supplier 2 is equal to $\pi_{S_2}^{(3)}(w_3, p_3^0(w_3))$. $Case$ 3 arises when $w_3'' < w_3 \leq \bar{w}_3$ and then Supplier 2's profit is equal to $\pi_{S_2}^{(2)}$. $Case$ 4 arises when $w_3 > \bar{w}_3$ and then Supplier 2's profit is equal to $\pi_{S_2}^{(2)}$. Thus we observe $\pi_{S_2}^{(3)}(w_3, p_3^0(w_3))$ for $w_3 \leq w_3''$, and $\pi_{S_2}^{(2)}$ for $w_3 > w_3''$.

We first check the concavity of $\pi_{S_2}^{(3)}(w_3, p_3^0(w_3))$.

$$\frac{\partial^2 \pi_{S_2}^{(3)}(w_3, p_3^0(w_3))}{\partial w_3^2} = -\frac{(1+2\alpha)^2}{1+k+2\alpha} < 0 \quad since \quad 0 \leq \alpha \leq 1 \;\; and \;\; k > 0$$

So, $\pi_{S_2}^{(3)}(w_3, p_3^0(w_3))$ is concave in w_3. We can find the unconstrained profit maximizer wholesale price of $\pi_{S_2}^{(3)}(w_3, p_3^0(w_3))$ by using the first order condition.

$$\begin{aligned}
\frac{\partial \pi_{S_2}^{(3)}(w_3, p_3^0(w_3))}{\partial w_3} &= 0 \\
\rightarrow w_3^0 &= \frac{k(p_1+p_2)(1+3\alpha)}{(1+2\alpha)^2} + \frac{\alpha(w_1+2w_2)}{2(1+2\alpha)} - \frac{2bk\alpha}{(1+2\alpha)^2} + \frac{b-4ak}{2(1+2\alpha)}
\end{aligned}$$

w_3^0 is the unconstrained maximizer of $\pi_{S_2}^{(3)}(w_3, p_3^0(w_3))$. We need to consider its feasible range $[0, w_3'']$ while determining the maximizer of the constrained problem. If $0 < w_3^0 \leq w_3''$, $\pi_{S_2}^{(3)}(w_3)$ has its maximum value when $w_3 = w_3^0$. If $w_3^0 > w_3''$, $\pi_{S_2}^{(3)}(w_3)$ has its maximum value when $w_3 = w_3''$. If $w_3^0 \leq 0$, $\pi_{S_2}^{(3)}(w_3)$ has its maximum value when $w_3 = 0$.

Therefore, we can express the best wholesale price decision of Supplier 2 for $Option$ (i) as:

$$w_3^* = \begin{cases} w_3^0, & \text{if } 0 < w_3^0 \le w_3'', \pi_{S_2}^{(3)}(w_3^0) \ge \pi_{S_2}^{(2)} \\ \\ w_3'', & \text{if } w_3^0 > w_3'', \pi_{S_2}^{(3)}(w_3'') \ge \pi_{S_2}^{(2)} \\ \\ 0, & \text{if } w_3^0 \le 0, \pi_{S_2}^{(3)}(w_3 = 0) \ge \pi_{S_2}^{(2)} \\ \\ (w_3'', \infty), & \text{if } 0 < w_3^0 \le w_3'', \pi_{S_2}^{(3)}(w_3^0) < \pi_{S_2}^{(2)} \\ & \text{or if } w_3^0 > w_3'', \pi_{S_2}^{(3)}(w_3'') < \pi_{S_2}^{(2)} \\ & \text{or if } w_3^0 \le 0, \pi_{S_2}^{(3)}(w_3 = 0) < \pi_{S_2}^{(2)} \end{cases}$$

$Option\ (ii)$:

If $\bar{w}_3 \ge 0$ and $\pi_R^{(3)}(p_3^0(\bar{w}_3)) \ge \pi_R^{(2)}$, we will observe first $Case$ 1 then $Case$ 2 as w_3 increases. $Case$ 1 arises when $w_3 \le \bar{w}_3$ and Supplier 2's profit is $\pi_{S_2}^{(3)}(w_3, p_3^0(w_3))$, $Case$ 2 arises when $w_3 > \bar{w}_3$ and then Supplier 2's profit is equal to $\pi_{S_2}^{(3)}(w_3, \bar{p}_3)$.

We need to compare $\pi_{S_2}^{(3)}(w_3, \bar{p}_3)$ and the maximum value of $\pi_{S_2}^{(3)}(w_3, p_3^0(w_3))$ in order to find optimal wholesale price decision. If we analyze these two functions when $w_3 = \bar{w}_3$:

$$\begin{aligned} \pi_{S_2}^{(3)}(\bar{w}_3, p_3^0(\bar{w}_3)) =& [a - p_2 + (p_1 - p_2)\theta + (p_3^0(\bar{w}_3) - p_2)\alpha]w_2 \\ &+ [b - p_3^0(\bar{w}_3) + (p_1 - p_3^0(\bar{w}_3))\alpha + (p_2 - p_3^0(\bar{w}_3))\alpha]\bar{w}_3 \\ =& [a - p_2 + (p_1 - p_2)\theta + (p_3^0(\bar{w}_3) - p_2)\alpha]w_2 \\ \pi_{S_2}^{(3)}(w_3, \bar{p}_3) =& [a - p_2 + (p_1 - p_2)\theta + (\bar{p}_3 - p_2)\alpha]w_2 \end{aligned}$$

As it can be seen in Equations given above and by definition of $\bar{w}_3$ such that $p_3^0(\bar{w}_3) = \bar{p}_3$ Supplier 2's profit is continuous at $\bar{w}_3$ in $Option\ (ii)$. By this continuity and concavity of $\pi_{S_2}^{(3)}(w_3, p_3^0(w_3))$, we can characterize the optimal wholesale price w_3^* as follows:

$$w_3^* = \begin{cases} w_3^0, & \text{if } 0 < w_3^0 \leq \bar{w}_3 \\ \\ (\bar{w}_3, \infty), & \text{if } w_3^0 > \bar{w}_3 \\ \\ 0, & \text{if } w_3^0 \leq 0 \\ \end{cases}$$

$Option\ (iii)$:

When $\bar{w}_3 \geq 0$ and $\pi_R^{(3)}(p_3^0(w_3 = 0)) < \pi_R^{(2)}$, we observe $Case$ 3 and then $Case$ 4 as w_3 increases. Thus, the retailer always finds $\pi_R^{(2)} > \pi_R^{(3)}(w_3, p_3^*(w_3))$ and $w_3 \in [0, \infty)$.

$Option\ (iv)$:

When $\bar{w}_3 < 0$ and $\pi_R^{(3)}(p_3^0(\bar{w}_3)) < \pi_R^{(2)}$, we will observe only $Case$ 4 for any nonnegative w_3. By this way, $\pi_R^{(2)} > \pi_R^{(3)}(w_3, p_3^*(w_3))$ always holds and $w_3 \in [0, \infty)$.

$Option\ (v)$:

When $\bar{w}_3 < 0$ and $\pi_R^{(3)}(p_3^0(\bar{w}_3)) \geq \pi_R^{(2)}$, we will observe only $Case$ 2 for any nonnegative w_3. By this way, $\pi_R^{(3)}(p_3^0(\bar{w}_3)) \geq \pi_R^{(2)}$ always holds and $w_3 \in [0, \infty)$. But, we never observe this option in profit of Supplier 2.

■

Proposition 3.2.6 (i) portrays the case where, the retailer will accept the new product and Supplier 2's profit will be $\pi_{S_2}^{(3)}$, if the wholesale price of the third product is lower than or equal to w_3''. Otherwise, the retailer rejects the third product and Supplier 2's profit is equal to $\pi_{S_2}^{(2)}$. Thus, Supplier 2 compares these two profits in order to determine the optimal wholesale price for the third product. If his profit with three products is higher than in the two-product setting, he will set the wholesale price to a value which maximize $\pi_{S_2}^{(3)}$ between 0 and w_3''. Otherwise, he sets the wholesale price high enough so that the retailer rejects the new product, i.e., $w_3 \in (w_3'', \infty)$.

Under the conditions of Propositon 3.2.6 (ii), the new product will always be active in the market with either positive or practically zero demand. So, Supplier 2 compares the profit of these two scenarios in order to determine the best wholesale price of the new product. If his profit is higher when the third product has a positive demand, its optimal wholesale price will be set to a value which maximizes $\pi_{S_2}^{(3)}$ between 0 and $\bar{w}_3$. Otherwise, Supplier 2 sets a wholesale price to any value greater than $\bar{w}_3$, and the retailer will prefer to accept the third product but set its retail price so that is has practically zero demand.

Under the conditions of Proposition 3.2.6 (iii) and (iv), the new product does not increase the profit of the retailer for any w_3. Thus, the third product will be rejected by the retailer and be kept out of the market.

When $Case$ 2.4 or $Case$ 2.5 or $Case$ 2.6 conditions hold, the new product does not increase the profit of Supplier 2. The demand potential of the third product is not high enough to provide higher profit to its supplier. Thus, Supplier 2 sets the wholesale price high enough so that the retailer will not accept the product, i.e., $w_3 \in (w_3'', \infty)$.

When $Case$ 2.8 occurs, Supplier 2 sets the wholesale price of the new product to any value between $\bar{w}_3$ and ∞. Then, the retail price of the new product will be very close to $\bar{p}_3$ (i.e., $\bar{p}_3 - \varepsilon$). By this way, third product will be active in the market but its corresponding demand quantity will be very close to zero. However, the new product increases awareness of the whole market despite its practically zero demand. By this way, Supplier 2's profit increases with the new product introduction. In this case, the retail price of the third product is not sensitive to w_3.

When $Case$ 2.2 arises, the new product increases Supplier 2's profit and the optimal wholesale price of the new product is equal to w_3''. By this way, the third product will be active with positive demand in the market. Based on the definition of w_3'', the introduction of the new product does not increase the retailer's profit since the retailer's profit with three product is equal to her profit in two-product equilibrium when $w_3 = w_3''$.

If $Case$ 2.1 or $Case$ 2.7 conditions hold, the new product increases the profit of Supplier 2 and the optimal wholesale price of the third product (w_3^0) produces positive demand for the third product and increases with the wholesale prices of the two existing products. Here, Supplier 2's own product, product 2 is twice as effective as the opponent's price. Note that this effect is increasing in the substitutability of product

3 with the other two products. How the market potential of the new product effects its wholesale price depends on the shelf space cost. If the shelf space cost is low, the retailer can tolerate higher prices for product 3. So, Supplier 2 can increase the wholesale price as its market potential increases. However, if the shelf space cost is high, the retailer will demand a low wholesale price for the new product in order to compansate the cost of the shelf space. Thus, the supplier decreases the wholesale price in order to entice the retailer to buy this new product.

If $Case$ 2.3 or $Case$ 2.9 conditions hold, the new product increases the profit of Supplier 2 but the optimal wholesale price of the new product is equal to zero. The corresponding retail price of the third product will be equal to p_3^0 at $w_3 = 0$. In this case, Supplier 2 can not obtain benefit directly from the third product. The demand quantity of product 2 increases with the introduction of the new product. By this way, Supplier 2 increases his profit compared to the two-product equilibrium setting.

Table 3.4: Summary of Equilibrium Cases

Cases	**Active or Not**	**Demand**	w_3^*
$Case$ 2.1	Active	Positive	w_3^0
$Case$ 2.2	Active	Positive	w_3''
$Case$ 2.3	Active	Positive	0
$Case$ 2.4	Not Active	-	(w_3'', ∞)
$Case$ 2.5	Not Active	-	(w_3'', ∞)
$Case$ 2.6	Not Active	-	(w_3'', ∞)
$Case$ 2.7	Active	Positive	w_3^0
$Case$ 2.8	Active	Practically Zero	$(\bar{w}_3, \infty)$
$Case$ 2.9	Active	Positive	0
$Case$ 2.10	Not Active	-	$[0, \infty)$
$Case$ 2.11	Not Active	-	$[0, \infty)$

Summary of all equilibrium cases can be seen in Table 3.4.

3.2.3 New Product Introduction and the Slotting Fee

Nowadays, retailers want to control the new product introductions into their market through simple mechanisms. Slotting fee or slotting allowance is one of them. In this mechanism, if a manufacturer wants to introduce a new product into the market, the retailer may want an upfront payment for the new product. Manufacturers have to pay this one time and fixed fee to the retailer in order to get into the shelves of the retailer. We use the symbol of F for the slotting fee in our study. If we incorporate this fee into our model, the retailer's profit function will be as follows:

$$\pi_R = \begin{cases} q_1^{(3)}(p_1 - w_1) + q_2^{(3)}(p_2 - w_2) \\ \qquad\qquad + q_3^{(3)}(p_3 - w_3) - k(S^{(3)})^2 + F, & \text{if } q_3^{(3)} > 0 \\ \\ q_1^{(2)}(p_1 - w_1) + q_2^{(2)}(p_2 - w_2) - k(S^{(2)})^2, & \text{if } q_3^{(3)} = 0 \text{ (Is Not Active)} \end{cases}$$

In order to reflect the cases where p_3 is very close but not equal to $\bar{p}_3$ (i.e., $\bar{p}_3 - \varepsilon$), we can reformulate π_R as follows:

$$\pi_R = \begin{cases} \pi_R^{(3)} + F, & \text{if } p_3 < \bar{p}_3 \\ \pi_R^{(3)} + F, & \text{if } p_3 = \bar{p}_3 \\ \pi_R^{(2)}, & \text{if } p_3 > \bar{p}_3 \text{ (Is Not Active)} \end{cases}$$

We can derive the retailer's best response retail price decision in a similar way to that in Proposition 3.2.7.

Corollary 3.2.7 *Given $(p_1, w_1), (p_2, w_2)$, w_3 and F, the retailer's best response retail price that maximizes her profit is as follows:*

$$p_3^* = \begin{cases} p_3^0 = \frac{b+2k(2a+b)+2(p_1+p_2)(\alpha-k)}{2(1+k+2\alpha)} \\ \qquad + \frac{-\alpha(w_1+w_2)+w_3(1+2\alpha)}{2(1+k+2\alpha)}, & \text{if } w_3 \le \bar{w}_3, \pi_R^{(3)}(p_3^0(w_3)) + F \ge \pi_R^{(2)} (Case\ 1) \\ \\ \bar{p}_3 = \frac{b+\alpha(p_1+p_2)}{1+2\alpha}, & \text{if } w_3 > \bar{w}_3, \pi_R^{(3)}(\bar{p}_3) + F \ge \pi_R^{(2)} \quad (Case\ 2) \\ \\ Third\ product\ is\ not\ active, & \text{if } w_3 \le \bar{w}_3, \pi_R^{(3)}(p_3^0(w_3)) + F < \pi_R^{(2)} (Case\ 3) \\ & \text{or if } w_3 > \bar{w}_3, \pi_R^{(3)}(\bar{p}_3) + F < \pi_R^{(2)} \quad (Case\ 4) \end{cases}$$

where $\bar{w}_3 = \frac{2k(p_1+p_2)(1+3\alpha)}{(1+2\alpha)^2} + \frac{\alpha(w_1+w_2)}{(1+2\alpha)} + \frac{b-4ak}{(1+2\alpha)} - \frac{4bk\alpha}{(1+2\alpha)^2}$

Proof of Corollary 3.2.7: Follows similarly to the proof of Proposition 3.2.1.

Using the best retail price response of the retailer, Supplier 2's profit can be formulated as:

$$\pi_{S_2}(w_3) = \begin{cases} q_2^{(3)}(p_3^0(w_3))w_2 + q_3^{(3)}(p_3^0(w_3))w_3 - F, & \text{if } w_3 \leq \bar{w}_3, \pi_R^{(3)}(p_3^0(w_3)) + F \geq \pi_R^{(2)} \\ q_2^{(3)}(\bar{p}_3)w_2 + 0 - F, & \text{if } w_3 > \bar{w}_3, \pi_R^{(3)}(\bar{p}_3) + F \geq \pi_R^{(2)} \\ \pi_{S_2}^{(2)} = q_2^{(2)}w_2, & \text{if } w_3 \leq \bar{w}_3, \pi_R^{(3)}(p_3^0(w_3)) + F < \pi_R^{(2)} \\ & \text{or if } w_3 > \bar{w}_3, \pi_R^{(3)}(\bar{p}_3) + F < \pi_R^{(2)} \end{cases}$$

■

Then, in similar way to our analysis in Proposition 3.2.6, we can derive Supplier 2's optimal wholesale price decision as follows:

Corollary 3.2.8 *Supplier 2's optimal wholesale price decision for the new product is:*

(i) When $\bar{w}_3 \geq 0$, $C_1 + F \geq 0$ and $C_2 + F < 0$ hold:

$$w_3^* = \begin{cases} w_3^0, & \textit{if } 0 < w_3^0 \leq \hat{w}_3, \pi_{S_2}^{(3)}(w_3^0) - F \geq \pi_{S_2}^{(2)} & (\textit{Case 2.1}) \\ \hat{w}_3 & \textit{if } w_3^0 > \hat{w}_3, \pi_{S_2}^{(3)}(\hat{w}_3) - F \geq \pi_{S_2}^{(2)} & (\textit{Case 2.2}) \\ 0, & \textit{if } w_3^0 \leq 0, \pi_{S_2}^{(3)}(w_3 = 0) - F \geq \pi_{S_2}^{(2)} & (\textit{Case 2.3}) \\ (\hat{w}_3, \infty), & \textit{if } 0 < w_3^0 \leq \hat{w}_3, \pi_{S_2}^{(3)}(w_3^0) - F < \pi_{S_2}^{(2)} & (\textit{Case 2.4}) \\ & \textit{or if } w_3^0 > \hat{w}_3, \pi_{S_2}^{(3)}(\hat{w}_3) - F < \pi_{S_2}^{(2)} & (\textit{Case 2.5}) \\ & \textit{or if } w_3^0 \leq 0, \pi_{S_2}^{(3)}(w_3 = 0) - F < \pi_{S_2}^{(2)} & (\textit{Case 2.6}) \end{cases}$$

(ii) When $\bar{w}_3 \geq 0$ and $C_2 + F \geq 0$ hold:

$$w_3^* = \begin{cases} w_3^0, & \text{if } 0 < w_3^0 \leq \bar{w}_3 \qquad (\textit{Case 2.7}) \\ (\bar{w}_3, \infty), & \text{if } w_3^0 > \bar{w}_3 \qquad (\textit{Case 2.8}) \\ 0, & \text{if } w_3^0 \leq 0 \qquad (\textit{Case 2.9}) \end{cases}$$

(iii) When $\bar{w}_3 \geq 0$ and $C_1 + F < 0$ hold:

$w_3^* \in [0, \infty)$ *but the third product will never be active in the market.* ($Case$ 2.10)

(iv) When $\bar{w}_3 < 0$ and $C_2 + F < 0$ hold:

$w_3^* \in [0, \infty)$ *but the third product will never be active in the market.* ($Case$ 2.11)

where

$$w_3^0 = \frac{k(p_1+p_2)(1+3\alpha)}{(1+2\alpha)^2} + \frac{\alpha(w_1+2w_2)}{2(1+2\alpha)} - \frac{2bk\alpha}{(1+2\alpha)^2} + \frac{b-4ak}{2(1+2\alpha)}$$

$$\bar{w}_3 = \frac{2k(p_1+p_2)(1+3\alpha)}{(1+2\alpha)^2} + \frac{\alpha(w_1+w_2)}{(1+2\alpha)} + \frac{b-4ak}{(1+2\alpha)} - \frac{4bk\alpha}{(1+2\alpha)^2}$$

$$\begin{aligned} \hat{w}_3 = \; & \frac{1}{(1+2\alpha)^2}\Big(w_2 + 4w_2\alpha(1+\alpha) - w_2(1+2\alpha) + 2w_1\alpha(1+2\alpha) + 4k(p_1+p_2)(1+3\alpha) \\ & + b(2+(4-8k)\alpha) - 8a(k+2k\alpha) - 4(1+k+2\alpha)\Big\{\frac{1}{1+k+2\alpha}(-F(1+2\alpha)^2 + \alpha(p_1^2 + 2kp_1^2 \\ & + 4kp_1p_2 + p_2^2 + 2kp_2^2 - p_1w_1 - p_2w_2 + 4b^2k\alpha + ((3+5k)p_1^2 + p_2((3+5k)p_2 + w_1 - 3w_2) \\ & + p_1(2(-1+5k)p_2 - 3w_1 + w_2))\alpha + 2(p_1-p_2)(p_1-p_2-w_1+w_2)\alpha^2 + 4ak(2b-p_1-p_2)(1+2\alpha) \\ & + b(w_1+w_2+2(w_1+w_2)\alpha - p_1(1+2\alpha+4k(1+3\alpha)) - p_2(1+2\alpha+4k(1+3\alpha)))))\Big\}^{1/2}\Big) \end{aligned}$$

$\pi_{S_2}^{(3)}(w_3^0) = q_2^{(3)}(p_3^0(w_3))w_2 + q_3^{(3)}(p_3^0(w_3))w_3^0$; $\pi_{S_2}^{(3)}(\bar{p}_3) = q_2^{(3)}(\bar{p}_3)w_2$; $\pi_{S_2}^{(2)} = q_2^{(2)}w_2$

$C_1 := \pi_R^{(3)}(p_3^0(w_3 = 0)) - \pi_R^{(2)}$, $C_2 := \pi_R^{(3)}(p_3^0(\bar{w}_3)) - \pi_R^{(2)}$ *The open form of C_1 and C_2 are available in Table 0.2 in Appendix F.*

Proof of Corollary 3.2.8: Follows similarly to the proof of Proposition 3.2.6.

When conditions of $Case$ 2.4 or $Case$ 2.5 or $Case$ 2.6 hold, either Supplier 2's profit decreases with the entry of the new product or his gain can not compansate the slotting fee set by the retailer. Thus, Supplier 2 will set the wholesale price of the new product high enough so that the retailer rejects it.

If conditions of $Case$ 2.1 or $Case$ 2.2 or $Case$ 2.3 hold, Supplier 2's gain is higher than the slotting fee. So, he accepts to pay this fee to the retailer and the new product will be active in the market with positive demand quantity.

The presence of a slotting fee is especially pronounced in $Case$ 2.7, 2.8 and 2.9. Here, the retailer never rejects the new product and the product will always be active in the market (with positive or practically zero demand). Without a slotting fee, the supplier never suffered a loss in any of these cases. He could always increase the wholesale price to $\bar{w}_3$ and result in zero demand for the new product and a positive market effect on the existing products. However, with a slotting fee, Supplier 2 may suffer from a loss because of the entry; his profit increase may or may not compensate the slotting fee. Thus, Supplier 2 faces a risk of loss with the new product entry and should be especially cautious with new product decisions.

In order to develop more insights on the equilibrium behavior and the effect of slotting fees on it, we will continue with numerical experiments in Chapter 5.

CHAPTER 4

CENTRALIZED MODEL

In this chapter, we analyze the centralized system decisions. Firstly, we find the optimal prices in the two-product setting. Then, we consider to introduce a new product to the centralized system. We find the best retail price decision for the third product. If the third product increases the profit of the centralized system, the new product will be active in the market. Otherwise, the new product will be rejected by the centralized system. The notation used in this chapter is given in Table 4.1.

Table 4.1: Notation of the Centralized Model

Expression	Description
p_{i_C}	Retail price of product i
$q_{i_C}^{(j)}$	Demand quantities of product i for case with j products
$\pi_C^{(j)}$	Profit of supply chain for case with j products
$S^{(j)}$	Total shelf space allocated for case with j products

In the centralized system, there is only retail price decisions to determine, to maximize the total profit below:

$$\begin{aligned}\pi_C^{(2)} &= q_{1_C}^{(2)}p_{1_C} + q_{2_C}^{(2)}p_{2_C} - k(S^{(2)})^2 \quad where \quad S^{(2)} = q_{1_C}^{(2)} + q_{2_C}^{(2)} \\ \pi_C^{(2)} &= (a - p_{1_C} + (p_{2_C} - p_{1_C})\theta)^+ p_{1_C} + (a - p_{2_C} + (p_{1_C} - p_{2_C})\theta)^+ p_{2_C} - k(S^{(2)})^2\end{aligned}$$

Proposition 4.0.1 *The optimal retail price and corresponding demand quantities of each product and total centralized profit for two product equilibrium setting are:*

$$p_{i_C}^* = \tfrac{a+4ak}{2(1+2k)}, \qquad q_{i_C}^{(2)} = \tfrac{a}{2(1+2k)}, \qquad \pi_C^{(2)} = \tfrac{a^2}{2(1+2k)}, \quad i = 1, 2$$

Proof of Proposition 4.0.1: Below, we first prove the joint concavity of the total supply chain profit function.

First and second derivative of the profit function with respect to p_1 and p_2 are as follows:

$$\begin{aligned}
\frac{\partial \pi_C^{(2)}}{\partial p_{1_C}} &= a + 4ak - 2(p_{2_C}(k-\theta) + p_{1_C}(1+k+\theta)) \\
\frac{\partial^2 \pi_C^{(2)}}{\partial p_{1_C}^2} &= -2(1+k+\theta) < 0 \ \ (since \ \ k > 0, \ \ \theta > 0) \\
\frac{\partial \pi_C^{(2)}}{\partial p_2} &= a + 4ak - 2(p_{1_C}(k-\theta) + p_{2_C}(1+k+\theta)) \\
\frac{\partial^2 \pi_C^{(2)}}{\partial p_{2_C}^2} &= -2(1+k+\theta) < 0 \ \ (since \ \ k > 0, \ \ \theta > 0) \\
\frac{\partial^2 \pi_C^{(2)}}{\partial p_{2_C} p_{1_C}} &= -2(k-\theta) \\
\frac{\partial^2 \pi_C^{(2)}}{\partial p_{1_C} p_{2_C}} &= -2(k-\theta)
\end{aligned}$$

The second order derivative is negative since $k > 0$ and $\theta > 0$. The determinant of the Hessian Matrix

$$\frac{\partial^2 \pi_C^{(2)}}{\partial p_{1_C}^2}\frac{\partial^2 \pi_C^{(2)}}{\partial p_{2_C}^2} - (\frac{\partial^2 \pi_C^{(2)}}{\partial p_{1_C} p_{2_C}})^2 = 4(1+2k)(1+2\theta) > 0 \quad since \quad k > 0 \ and \ 0 < \theta < 1$$

Hence, $\pi_C^{(2)}$ is jointly concave in p_{1_C} and p_{2_C}. Then, the first order conditions will be sufficient for optimality.

$$\frac{\partial \pi_C^{(2)}}{\partial p_{1_C}} = a + 4ak - 2(p_{2_C}(k-\theta) + p_{1_C}(1+k+\theta)) = 0$$

$$\frac{\partial \pi_C^{(2)}}{\partial p_{2_C}} = a + 4ak - 2(p_{1_C}(k-\theta) + p_{2_C}(1+k+\theta)) = 0$$

Solving the two equations above together, we get:

$$p_{i_C}^* = \frac{a+4ak}{2(1+2k)} \ \ where \ \ i = 1, 2$$

Then, corresponding demand quantity of each product is $q_{i_C}^{(2)} = \frac{a}{2(1+2k)}$ and the profit of the centralized supply chain is $\pi_C^{(2)} = \frac{a^2}{2(1+2k)}$ ■

We then analyze the new product introduction setting in the centralized system. There will only the retail price decision for product 3 in the new product introduction setting. Supply chain profit function of the centralized system is expressed as:

$$\pi_C = \begin{cases} \pi_C^{(3)} = q_{1_C}^{(3)} p_{1_C} + q_{2_C}^{(3)} p_{2_C} + q_{3_C}^{(3)} p_{3_C} - k(S^{(3)})^2, & \text{if } q_{3_C}^{(3)} > 0 \\ \\ \pi_C^{(2)} = q_{1_C}^{(2)} p_{1_C} + q_{2_C}^{(2)} p_{2_C} - k(S^{(2)})^2, & \text{if } q_{3_C}^{(3)} = 0 \text{ (Is Not Active)} \end{cases}$$

Similar to the decentralized model, if the retail price of the new product is lower than $\bar{p}_{3_C}$, the new product will be active in the market and its demand quantity is greater than zero. Otherwise, demand quantity of the new product is equal to zero when $p_{3_C} \geq \bar{p}_{3_C}$. Thus, we can express the profit of the centralized system as follows:

$$\pi_C = \begin{cases} \pi_C^{(3)} = q_{1_C}^{(3)} p_{1_C} + q_{2_C}^{(3)} p_{2_C} + q_{3_C}^{(3)} p_{3_C} - k(S^{(3)})^2, & \text{if } p_{3_C} < \bar{p}_{3_C} \\ \\ \pi_C^{(2)} = q_{1_C}^{(2)} p_{1_C} + q_{2_C}^{(2)} p_{2_C} - k(S^{(2)})^2, & \text{if } p_{3_C} \geq \bar{p}_{3_C} \text{ (Is Not Active)} \end{cases}$$

Proposition 4.0.2 *In the centralized system, for a given (p_{1_C}, p_{2_C}), the new product's price is set as follows:*

$$p_{3_C}^* = \begin{cases} p_{3_C}^0 = \frac{b+4ak+2bk+2(p_{1_C}+p_{2_C})(\alpha-k)}{2(1+k+2\alpha)}, & \textit{if } p_{3_C}^0 \leq \bar{p}_{3_C}, \pi_C^{(3)}(p_{3_C}^0) \geq \pi_C^{(2)} \\ \\ \bar{p}_{3_C} = \frac{b+\alpha(p_{1_C}+p_{2_C})}{1+2\alpha}, & \textit{if } p_{3_C}^0 > \bar{p}_{3_C}, \pi_C^{(3)}(\bar{p}_{3_C}) \geq \pi_C^{(2)} \\ \\ Third\ product\ is\ not\ active, & \textit{if } p_{3_C}^0 \leq \bar{p}_{3_C}, \pi_C^{(3)}(p_{3_C}^0) < \pi_C^{(2)}\ \ or \\ & \textit{if } p_{3_C}^0 > \bar{p}_{3_C}, \pi_C^{(3)}(\bar{p}_{3_C}) < \pi_C^{(2)} \end{cases}$$

Proof of Proposition 4.0.2: Firstly, we need to analyze the centralized profit function with three products; $\pi_C^{(3)}$.

Second order derivative of $\pi_C^{(3)}$ with respect to p_3 is:

$$\frac{\partial^2 \pi_C^{(3)}}{\partial p_{3_C}^2} = -2(1+k+2\alpha) < 0 \quad since\ \ k > 0\ \ and\ \ 0 < \alpha < 1$$

Hence, $\pi_C^{(3)}$ is a concave in p_{3_C}. Then, we can use the first order condition to find the optimal price of the unconstrained function, which refer to as $p_{3_C}^0$.

$$\begin{aligned} \frac{\partial \pi_C^{(3)}}{\partial p_{3_C}} &= b + 4ak + 2bk - 2kp_{1_C} - 2kp_{2_C} + 2p_{1_C}\alpha + 2p_{2_C}\alpha - 2p_{3_C}(1+k+2\alpha) = 0 \\ p_{3_C}^0 &= \frac{b + 4ak + 2bk + 2(p_{1_C} + p_{2_C})(\alpha - k)}{2(1+k+2\alpha)} \end{aligned}$$

In order to obtain nonnegative $q_{3_C}^{(3)}$, the retail price of the third product must be between zero and $\bar{p}_{3_C}$. Thus, we can reformulate the centralized profit function wrt p_{3_C} as follows:

$$\pi_C = \begin{cases} q_{1_C}^{(3)} p_{1_C} + q_{2_C}^{(3)} p_{2_C} + q_{3_C}^{(3)} p_{3_C} - k(S^{(3)})^2, & \text{if } p_{3_C} < \bar{p}_{3_C} \\ \\ q_{1_C}^{(2)} p_{1_C} + q_{2_C}^{(2)} p_{2_C} - k(S^{(2)})^2, & \text{if } p_{3_C} \geq \bar{p}_{3_C} \text{ (Is Not Active)} \end{cases}$$

$\bar{p}_{3_C}$ can be defined as the retail price of the third product that makes its demand zero. Thus, $\bar{p}_{3_C} = \frac{b+(p_{1_C}+p_{2_C})\alpha}{1+2\alpha}$

Similar to the decentralized system, there may be cases where p_{3_C} is very close to but not equal to $\bar{p}_{3_C}$ (i.e., $\bar{p}_{3_C} - \varepsilon$), which would produce practically zero demand for product 3 but move the existing products into a 3-product market. To reflect this, we reformulate the centralized profit function as follows:

$$\pi_C = \begin{cases} \pi_C^{(3)}, & \text{if } p_{3_C} < \bar{p}_{3_C} \\ \pi_C^{(3)}, & \text{if } p_{3_C} = \bar{p}_{3_C} \\ \pi_C^{(2)}, & \text{if } p_{3_C} > \bar{p}_{3_C} \text{ (Is Not Active)} \end{cases}$$

Using the concavity of $\pi_C^{(3)}$, we can characterize the optimal price $p_{3_C}^*$ as follows:

$$p_{3_C}^* = \begin{cases} p_{3_C}^0 = \frac{b+4ak+2bk+2(p_{1_C}+p_{2_C})(\alpha-k)}{2(1+k+2\alpha)}, & \text{if } p_{3_C}^0 \leq \bar{p}_{3C}, \pi_C^{(3)}(p_{3_C}^0) \geq \pi_C^{(2)} \\ \\ \bar{p}_{3_C} = \frac{b+\alpha(p_{1_C}+p_{2_C})}{1+2\alpha}, & \text{if } p_{3_C}^0 > \bar{p}_{3C}, \pi_C^{(3)}(\bar{p}_{3_C}) \geq \pi_C^{(2)} \\ \\ Third\ product\ is\ not\ active, & \text{if } p_{3_C}^0 \leq \bar{p}_{3C}, \pi_C^{(3)}(p_{3_C}^0) < \pi_C^{(2)}\ or \\ & \text{if } p_{3_C}^0 > \bar{p}_{3C}, \pi_C^{(3)}(\bar{p}_{3_C}) < \pi_C^{(2)} \end{cases}$$

■

In the centralized system, the new product is active in the market unless it decreases the total centralized profit. If the third product decreases the total centralized profit, it will be rejected by the centralized system. Thus, profit of the centralized system never decreases with the new product offer.

If the third product is priced so that it has positive demand on shelf; i.e., when $p_{3_C}^0 \leq \bar{p}_{3_C}$ and $\pi_C^{(3)} \geq \pi_C^{(2)}$, its resulting price and the consequent demand and profit values are as follows:

$$\begin{aligned}
p^*_{3_C} &= \frac{b(1+2k)^2+2a(k+\alpha+4k\alpha)}{2(1+2k)(1+k+2\alpha)}\\
q^{(3)}_{1_C} &= \frac{b(1+2k)^2\alpha+a(1+k+\alpha-k(3+4k)\alpha)}{2(1+2k)(1+k+2\alpha)}\\
q^{(3)}_{2_C} &= \frac{b(1+2k)^2\alpha+a(1+k+\alpha-k(3+4k)\alpha)}{2(1+2k)(1+k+2\alpha)}\\
q^{(3)}_{3_C} &= \frac{b(1+2k+2\alpha-8k^2\alpha)+2ak(-1+(-1+4k)\alpha)}{2(1+2k)(1+k+2\alpha)}\\
\pi^{(3)}_{C} &= \frac{(b+2bk)^2(1-8k\alpha)-2a^2(1+k(3+4k))(-1+(-1+4k)\alpha)}{4(1+2k)^2(1+k+2\alpha)}\\
&\quad+\frac{4ab(1+2k)(\alpha+k(-1+(2+8k)\alpha))}{4(1+2k)^2(1+k+2\alpha)}
\end{aligned}$$

If the third product is priced so that it is active but has essentially zero demand; i.e., when $p^0_{3_C} > \bar{p}_{3_C}$ and $\pi^{(3)}_C(\bar{p}_{3_C}) \geq \pi^{(2)}_C$, its resulting retail price and the consequent demand and profit values are as follows:

$$\begin{aligned}
p^*_{3_C} &= \frac{b+(\frac{a(1+4k)\alpha}{(1+2k)})}{(1+2\alpha)}\\
q^{(3)}_{1_C} &= \frac{a+a\alpha+2b\alpha-4ak\alpha+4bk\alpha}{2+4k+4\alpha+8k\alpha}\\
q^{(3)}_{2_C} &= \frac{a+a\alpha+2b\alpha-4ak\alpha+4bk\alpha}{2+4k+4\alpha+8k\alpha}\\
q^{(3)}_{3_C} &= 0\\
\pi^{(3)}_{C} &= \frac{(a(1+2\alpha+2k(1+(3+4k)\alpha))-4bk(1+2k)\alpha)(a+(a+2b-4ak+4bk)\alpha)}{2(1+2k)^2(1+2\alpha)^2}
\end{aligned}$$

CHAPTER 5

NUMERICAL ANALYSIS

In this chapter, we analyze the three-product setting through numerical examples. We assume that two-product setting is in equilibrium. Thus, the equilibrium wholesale and retail prices of the incumbent products are used as given for the three-product setting. We then study the retail and wholesale price decisions given for the third product. With these decisions we analyze the change in the each firm's profit and reveal the effect of the new product introduction. We conduct our analysis by changing one parameter at a time, with the rest of the parameters as given in Table 5.1.

Table 5.1: Base Model Parameters

Parameter	a	b	θ	α	k
Value	100	100	0.5	0.5	0.1

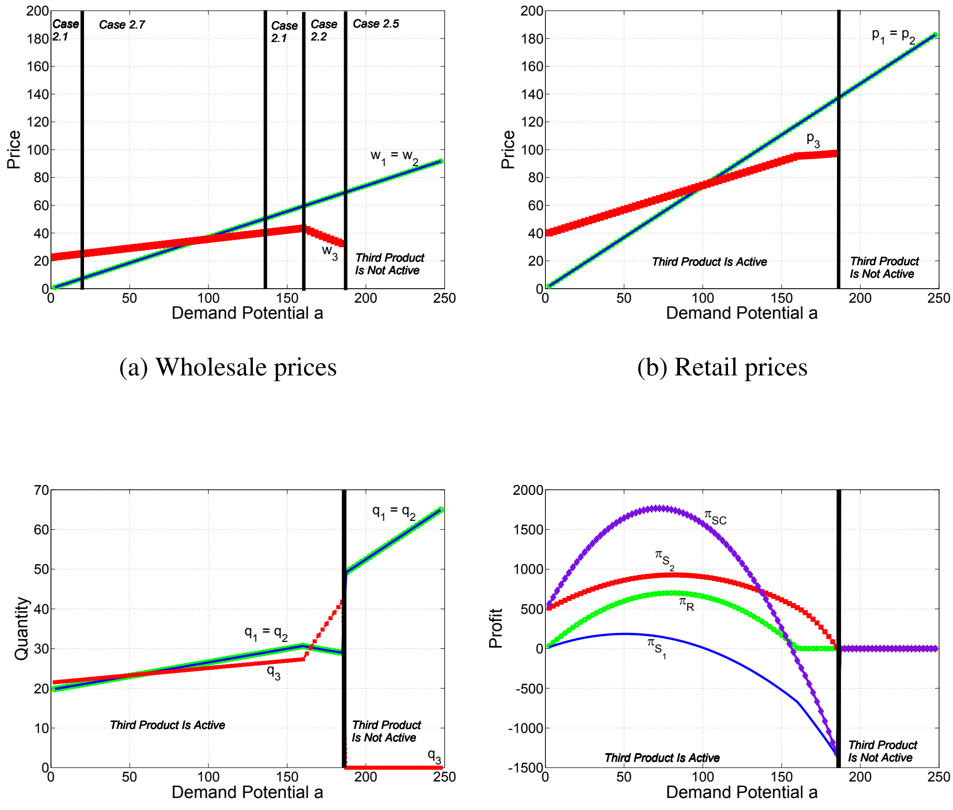
5.1 Analysis for Market Potential a

In this section, we study the effect of the market potential a which represents the base market potential of the two incumbent products. We evaluate the results by changing its value from 2.5 to 250 with an increment size of 2.5. All other parameters are equal to their base value given in Table 5.1.

5.1.1 Three Product Market with respect to a

The size of the market potential a affects the extent of competition in the market, as well as the relative power of the third product when it is active in the market. Figures

5.1a and 5.1b portray the change in the third product's price with respect to a, in comparison with the two-product equilibrium setting[1]. Up to a level of a the third product will be active in the market whereas beyond this level the new product will not be active in the market.



(a) Wholesale prices (b) Retail prices

(c) Demand quantities (d) Change in the profit of each firm

Figure 5.1: The Third Product Introduction Decisions WRT Incumbent Product Market Potential, a

As market potential a increases we observe that third product starts with the wholesale price w_3^0, changes to w_3'', and eventually becomes non-active. Specifically, we observe Cases 2.1, 2.7, 2.2 and 2.5. At the beginning, when the market potential of the incumbent products increase, both suppliers and the retailer can afford to set higher prices for them. The third product that is introduced takes a higher price ac-

[1]Note that, the prices of two-product setting are derived from the equilibrium levels, and thus they take equal values

cordingly. So, all prices increase as a increases. Whether the third product will take a higher or lower price compared to the incumbent products depends on the relative size of market potential a and b. When the third product has a bigger market potential, Supplier 2 can induce the retailer to give shelf space for his new product even if he sets a higher wholesale price for it. When a equals b, wholesale price of the third product is already lower than that of the incumbent products. Thus, the third product's wholesale price, determined after observing the two-product equilibrium, is set more aggressively compared to the two incumbent products. The trend in the retail price follows the same pattern as the wholesale prices. However, the retailer matches the retail price of the incumbent products after a surpasses b $(a > b)$. Here, the retailer prefers to sustain a higher margin from the third product and pressures Supplier 2 to offer a lower wholesale price. Up to this point, we observed $Case$ 2.1 and $Case$ 2.7.[2] As a continues to increase, we observe $Case$ 2.2. In this case, the retailer's presure for lower wholesale price is stronger since the awareness effect of the new product disappears as a increases. Thus, Supplier 2 decreases the wholesale price of the new product in order to induce the retailer to give shelf space for the new product. When we look at the retail prices, we see that the retailer continues to increase the third product's price to sustain its margin. However, the increase in the retail price of the new product in $Case$ 2.2 is lower than the that of in $Case$ 2.1 and $Case$ 2.7. Finally, in $Case$ 2.5, Supplier 2 cannot afford to set a low wholesale price to entice the retailer any more. Thus, he sets an arbitrarily high price and lets the third product be kept out of the market.

When the demand potential of the new product is too high compared to existing products, its demand level is also higher despite its high retail price. As a increases, demand of each incumbent product increases as well. The new product's demand also increases as a increases as its retail price also increases and substitution demand between the products changes accordingly. As a increases, first the third product's price approaches the incumbents' prices capturing the substituted demand back, and at even higher levels, the gap between the prices changes direction and widens, increasing the substituted demand to the new product. However, with a's increase the increase in the demand level of incumbent products is higher than that of the third

[2] Due to our profit function structure we observe $Case$ 2.1 when a is too low although relative demand potential of the new product is too high

product. Then, when we observe $Case$ 2.2, and the increase in the third product's retail price is much slower. By this way, the demand captured by the new product from the incumbent products increases. Thus, demand of the existing products decrease whereas the demand of the new product increases. After a certain level we observe $Case$ 2.5 and the third product is not active any more. With this transition, the incumbent products' demand quantities show up and continue to increase with a. The change in demand quantity of each product is available in Figure 5.1c.

5.1.2 Effect of the Third Product

In order to reveal the effect of the new product introduction on profit of each firm, we calculate change in profits for each party in the supply chain. When the demand potential of the incumbent products is too low compared to the new product, the new product increases the awareness of the whole product category with its high intrinsic demand potential. Thus, the new product introduction produces an extra profit for each firm through substitution and market expansion. As a increases, the incremental contribution of the third product starts to decrease. Consequently, the gain of each firm starts to decrease and zeroes out at a point. Then, firstly Supplier 1's gain, secondly, the retailer's gain starts to decrease, followed by that of Supplier 2. As a continues to increase, in the region of $Case$ 2.2, the new product does not provide extra profit to the retailer, her profit will be equal to two-product equilibrium setting. In addition, the decrease in Supplier 2's profit is faster due to the widening retail price difference. Only Supplier 2 captures extra profit from the new product introduction in this case. Thus, total supply chain profit also decreases compared to the two-product equilibrium setting. When a is too high compared to b, the third product will be kept out of the market. Therefore, profit of all firms calculated in the two-product equilibrium setting will be valid. The change in the profit of each firm and the total supply chain with respect to the market potential a can be seen in Figure 5.1d.

5.2 Analysis for Market Potential b

We study effect of the market potential b in this section. We conduct numerical experiments by changing the value of market parameter b from 2.5 to 250 with an increment size of 2.5. All other parameters are equal to their base level given in Table 5.1.

5.2.1 Three Product Market with respect to b

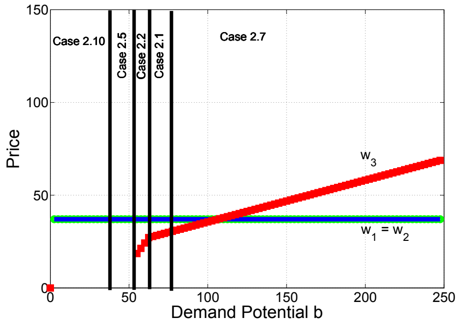
We assume the two-product market is already in equilibrium before the third product is introduced and take the incumbent product prices as given. Therefore, the retail and wholesale prices of these products are not sensitive to b.

Similar to a, the level of b also determines the competition intensity in the market and the relative power of the third product compared to incumbent products. Thus, as we observe in the case for a, there are two main factors that drive the price (and hence entry) decisions: market expansion and substitution. When the market potential of the new product is too low, it cannot be active in the market since it will only hurt the demand and profit of the existing products. Only when b reaches a certain level, the third product becomes active in the market. Up to this level of b we observe $Case$ 2.10 and $Case$ 2.5. This threshold for b is achieved at a lower value than those of the incumbent product market potentials; i.e.a. When b is beyond this level we observe $Case$ 2.2, $Case$ 2.1 and $Case$ 2.7, respectively, as b increases. At first, Supplier 2 and the retailer price the third product at lower levels than the incumbent products, to make up for its inferior market potential. As b increases, the third product is eventually priced at higher wholesale and retail prices compared to the incumbent products. As the market potential b reaches a, first the retailer then Supplier 2 match, and consequently surpass the price of the existing products. Note that, this increase in price is faster first (in the region of $Case$ 2.2), and then slows down (in the regions of $Case$ 2.1 and 2.7). As we also observed in Section 5.1, we see that the retailer captures a higher unit margin from the third product; the second supplier sets a lower wholesale price for the third product when b is exactly equal to a, and the margin difference is sustained as b continues to increase. The change in the wholesale and retail prices of all products with respect to demand potential b are shown in Figures

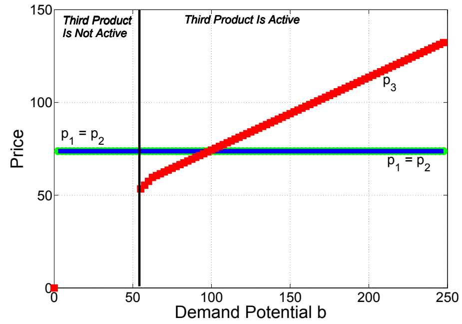
5.2a and 5.2b, respectively.

Up to threshold level of b where the third product turns active in the market, demand quantities do not change. When the new product is active in the market, the market potential of the third product will affect all demand quantities, altough their prices remain fixed. When b is low, the third product is priced aggressively to capture substitution demand from the two incumbent products. As b increases, its price approaches the others'. At first, demand of the new product decreases due to rapidly increasing retail price in $Case\ 2.2$, although its demand potential increases. The existing products, however, enjoy increased demand. Then, the increase in the new product's price slows down and demand of both the new and existing products increase as b increases. At a high b, Supplier 2, followed by the retailer, intentionally sets a less competitive price for the new product, sharing its demand potential with the others. This is linked with the fact that the existing products are priced for a more competitive market, compared to the three-product setting. The changes in the demand quantities with respect to the market potential b can be seen in Figure 5.2c.[3]

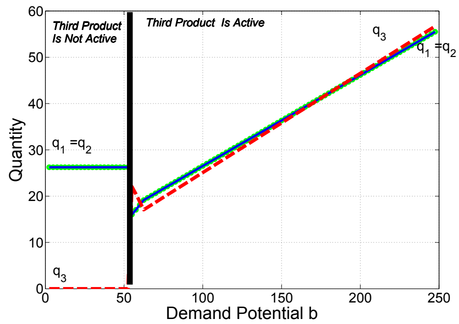
[3]When $a = 100$, both the retailer and Supplier 2 start to have a profit increase at $b = 55$, whereas Supplier 1 enjoys an increase at $b = 100$.



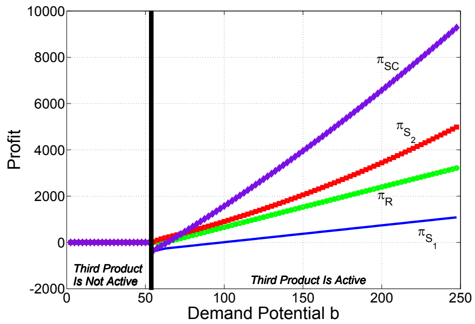
(a) Wholesale prices



(b) Retail prices



(c) Demand quantities



(d) Change in the profit of each firm

Figure 5.2: The Third Product Introduction Decisions WRT Incumbent Product Market Potential, b

5.2.2 Effect of the Third Product

When the market potential of the new product is too low, it will not be able to enter the market, and thus profits of all parties in the chain remain unchanged. However, when the new product's demand potential becomes high enough to become active in the market, Supplier 1's profit decreases at first. Because, demand potential of the third product is still lower than that of incumbent products and the third product's retail price is lower than those of the incumbent products. By this way, the third product captures demand from the existing products. At the beginning, only Supplier 2 increases profit with the new product introduction whereas Supplier 1's profit decreases and the retailer's profit remains unchanged (in the zone of $Case$ 2.2). In this case, Supplier 2 is smaller than Supplier 1's loss. Thus, the total supply chain profit decreases with the introduction of the third product. As b continues to increase, the new product's market expansion effect strengthens and the substitution effect weakens. Therefore, profit of each firm and total supply chain keep increasing as b increases. The change in the profit of each firm can be seen in Figure 5.2d.

5.3 Analysis for Substitution Level θ

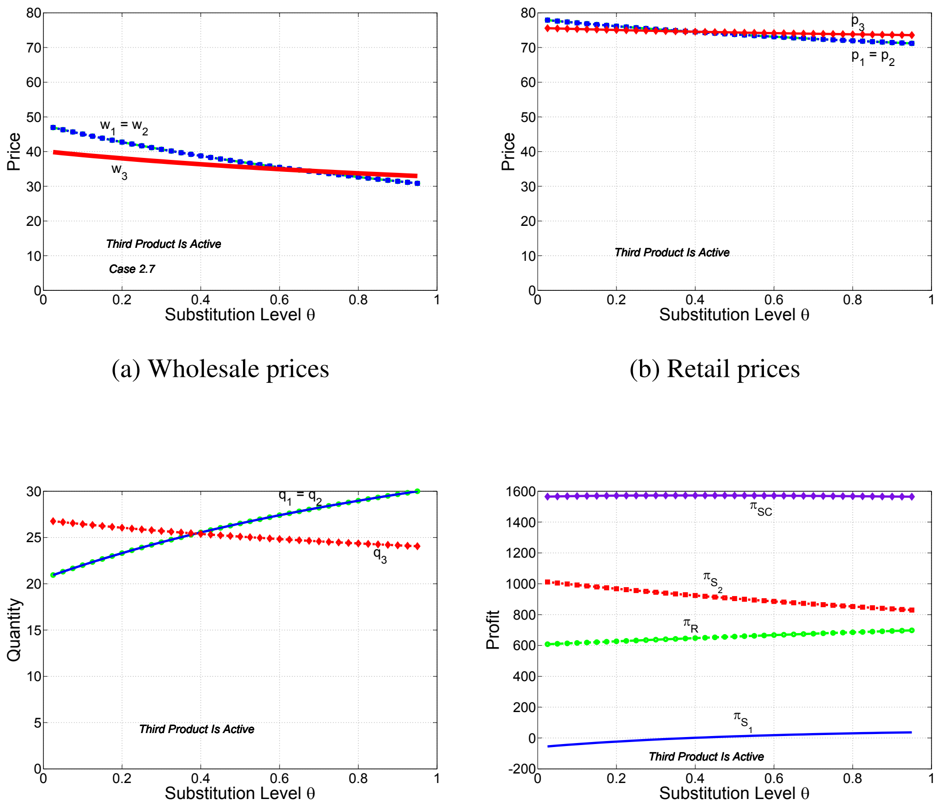
In this section, we study the effect of the substitution paramter θ. θ shows the degree of substitution between the two existing products. We find the results by changing the level of θ from 0.025 to 0.975 with an increment size 0.025. Other parameters are equal to their base value given in Table 5.1.

5.3.1 Three Product Market with respect to θ

In our parameter set, since we take the demand potential of the incumbent (a) and the third product (b) as all equal to 100, in the base case, the retailer always accepts the new product and allocates shelf space to it. This result is independent of the substitution effect between the existing products. Thus, for all values of θ, the third product is always active in the market we observe only $Case$ 2.7 in this section.

When θ is low, the competition between the incumbent products is not intense. Therefore, the third product becomes an effective tool for Supplier 2 to capture more market with a higher substitution effect α. Thus, he sets a lower price for the third product. As θ increases, the competition intensity between the two products increases. Therefore, wholesale prices of the existing products decrease. Consequently, the new product's wholesale price also decrease as θ increases, since it will compete with the incumbent products when introduced. The retail prices follow same pattern as the respective wholesale prices. However, the retailer wants to sustain a higher margin and pressures Supplier 2 to offer lower a wholesale price for the new product. Because of this, the third product's retail price achieves equality with that of the incumbent products' at lower substitution levels (when $\theta = 0.375$) when compared to when its wholesale becomes equal to those ($\theta = 0.65$). The changes in both the wholesale and the retail prices with respect to substitution level θ are shown in Figures 5.3a and 5.3b.

As θ increases, the demand level of the incumbent product increases whereas the demand of the new product decreases. Because the new product cannot sustain the price difference and starts to capture lower demand from the incumbent products. As θ reaches even higher levels, incumbent products capture demand from the new product due to their lower retail prices. The change in the demand levels of each product with respect to substitution level θ can be seen in Figure 5.3c.



(a) Wholesale prices

(b) Retail prices

(c) Demand quantities

(d) Change in the profit of each firm

Figure 5.3: The Third Product Introduction Decisions WRT Substitution Level, θ

5.3.2 Effect of the Third Product

Supplier 2 easily captures demand from Supplier 1 by inducing a lower retail price for the new product when θ is too low. Thus, Supplier 1's profit decreases with the entry of the new product. Supplier 2's profit gain is at its highest when θ is low. As θ increases, firstly the loss of Supplier 1 decreases and then he starts to obtain an extra profit due to his product's more competitive prices. In addition, the gain of Supplier 2 decreases due to the same effect. Consequently, as θ increases. Supplier 2's gain decreases but he is always better off with the new product introduction; and Supplier 1's profit change increases; starting at negative values and eventually achieving profit gains. The retailer benefits from the new product introduction for any value of θ. In fact, the the retailer's gain increases as θ increases. Besides, the total supply chain profit increases for any level of substitution level θ. The change in profit of each firm and profit of total supply change with respect to θ can be seen in Figure 5.3d.

5.4 Analysis for Substitution Level α

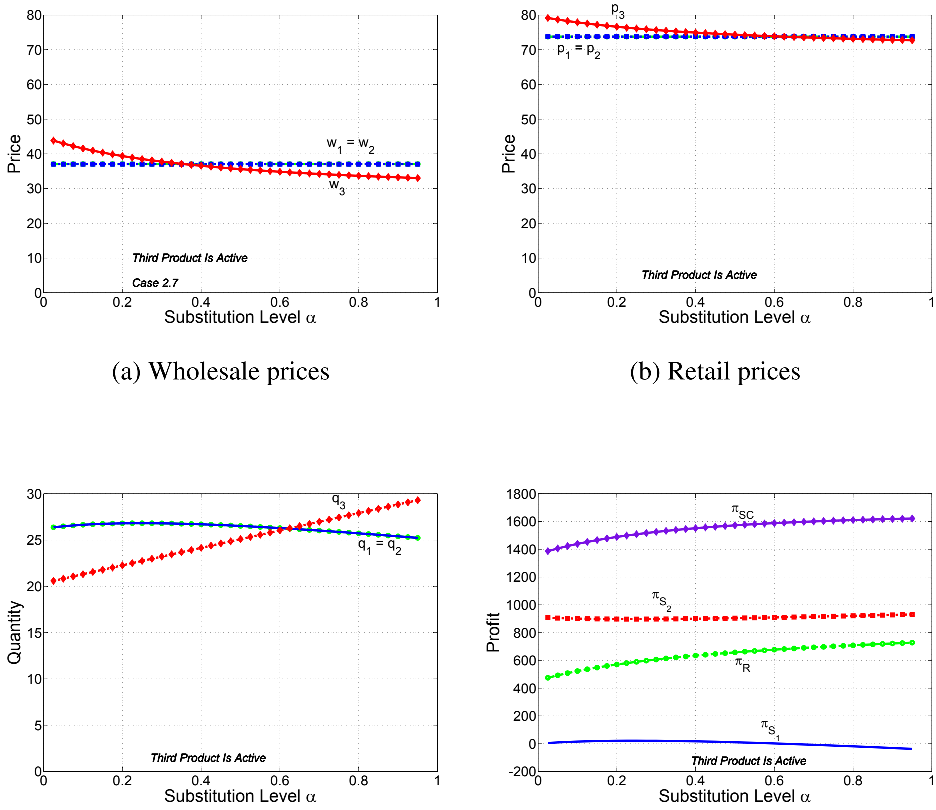
In this section, we analyze the effect of the subsitutability level α, which determines the competitive intensity between the new product and incumbent products. We evaluate the results by changing the value of α from 0.025 to 0.975 with an increment size 0.025. The base value set is used for all the other parameters.

5.4.1 Three Product Market with respect to α

As also observed in the case of θ, at base parameter values, the third product always enters the market and we observe $Case$ 2.7 for any value of α. In addition, both the wholesale and retail price of the incumbent products are not sensitive to α, since their prices in the two-product equilibrium setting are treated as given in the third product analysis.

The substitution level, α, determines the extent of the existing market that can be captured by the third product. Thus, Supplier 2 and the retailer decrease prices of the third product and become more aggressive as α increases. When α is low, both the retail and wholesale price of the incumbent products is low compared to the new product due to relatively high level of θ. As α increases, comptitiveness between the new product and the incumbent products increases. So, both Supplier 2 and the retailer decrease the wholesale and retail price of the third product, respectively. However, Supplier 2 is more agressive on pricing compared to the retailer since he obtains profit from only product 2 and product 3. Thus, wholesale price of all products reach equality when α is lower than θ whereas the retailer sets equal retail prices only when α substantially surpasses θ. Compared to Supplier 2, the retailer is less inclined to capture demand from the first supplier. Thus, she requires a higher margin from the new product to make up for her loss from the first. This translates into a delayed response to the decreasing wholesale price of the new product. The change in the wholesale and retail price of each product with respect to α can be seen in Figures 5.4a and 5.4b, respectively.

Demand of the incumbent products are higher than that of the new product when α is low since low-to-moderate competitive power of the third product produces higher retail prices for it. As α increases, demand of incumbent products increases although difference between the retail prices decreases. When α equals to 0.25, existing products reach their maximum demand level and show a decreasing trend. After this point, the increase in α cannot compansate the drop in the retail price difference. Then, the new product will capture demand from the incumbent products due to a more competitive retail price as α continues to increase. Thus, demand of the third product always increases as α increases. The effect of α on the demand quantities of each product can be seen in Figure 5.4c.



(a) Wholesale prices

(b) Retail prices

(c) Demand quantities

(d) Change in the profit of each firm

Figure 5.4: The Third Product Introduction Decisions WRT Substitution Level, α

5.4.2 Effect of the Third Product

When α is low, the incumbent products capture demand from the new product due to their more competitive retail prices. Consequently, Supplier 1's profit increases with the new product entry. Supplier 2's profit also increases mainly because of the market expansion through product 3 and indirectly through product 2, despite the substitution demand captured by product 1. As α increases beyond a certain threshold, the third product takes even more competitive prices and eventually takes lower values than incumbent prices. In fact, when α is higher than 0.65, Supplier 1's profit starts to decrease. We observe the reverse behavior in Supplier 2's profit. Up to a certain threshold, his profit change decreases with α, and then it keeps increasing as α increases. Note that, Supplier 2 always gains with the introduction of the new product.

The retailer, as the price-setter of the third product, is also better off with the new product entry. Her gain monotonously increases as α increases. In addition, total supply chain profit always increases for each level of α when other parameters at their base value. The effect of α on the profit change of each firm and the total supply chain can be seen in Figure 5.4d.

5.5 Analysis for Shelf Space Cost Parameter k

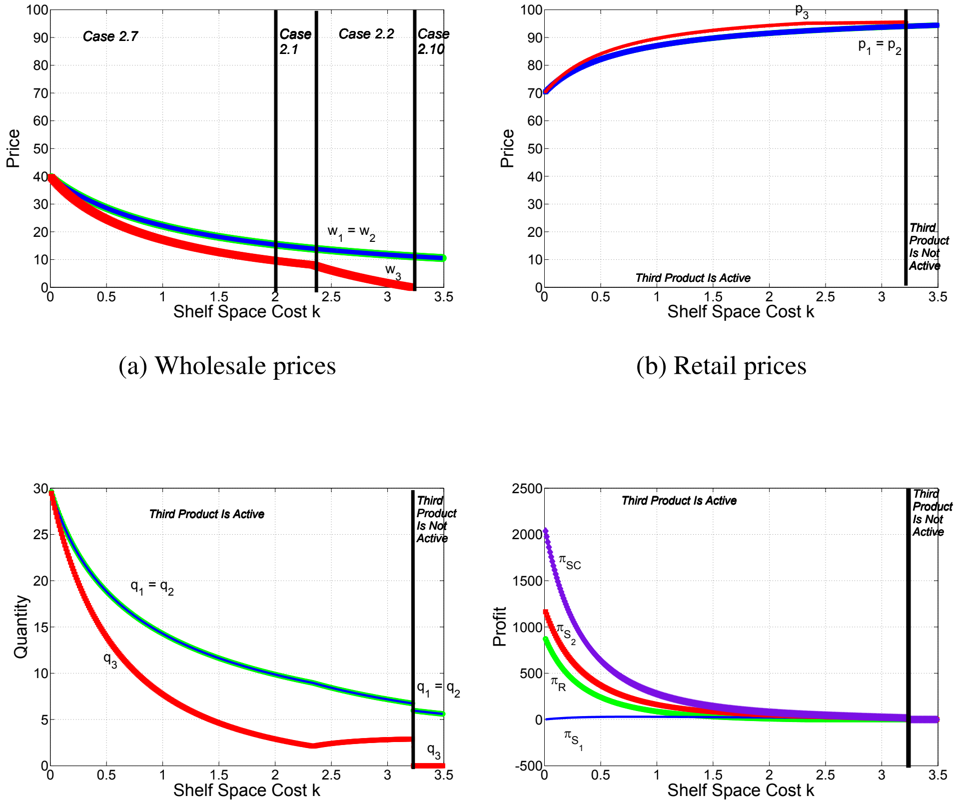
In this section, we analyze the effect of the shelf space cost parameter k on prices, demand quantities and profits. We evaluate the results by changing the value of k from 0.01 to 3.5 with an increment size 0.01. The base value set is used for all the other parameters.

5.5.1 Three Product Market with respect to k

The shelf space cost parameter k, based on the opportunity cost of shelf space and storage, determines the cost charged to the retailer for total units sold in that category. Thus, it discourages the retailer from decreasing prices and increasing the sales volume. As cost of shelf space increases, suppliers decrease their wholesale prices. Otherwise, they can not induce the retailer to allocate shelf space to their products. In addition, the retailer increases the retail price of each product as k increases. Both decrease in the wholesale price and increase in the retail price increases the retailer's profit margin. By this way, she compansates the high shelf space cost.

As we observed previously, in the base case, the new product has a lower wholesale price but has a higher retail price than the incumbent products. As k increases, wholesale prices drop while retail prices increase. Interestingly, the new product's wholesale price decreases even more rapidly than the other two products, to offer a high margin to the retailer, so that she can justify the shelf-space cost of the new product. Similarly, the retail price of the third product increases more rapidly compared to that of the incumbent products. Up to a certain threshold of k, the third product will be active in the market and we observe $Case$ 2.7, $Case$ 2.1 and $Case$ 2.2. The decrease in the wholesale price of the new product becomes even more rapid in the zone of $Case$ 2.2. As k continues to increase, shelf-space becomes too expensive for the retailer to expand the category with a third product. In this situation, we observe $Case$ 2.10 and the new product will not be active in the market. The change in the wholesale and retail prices of each product with respect to k can be seen in Figures 5.5a and 5.5b, respectively.

When the shelf space cost is too low, demand quantities of each product are approximately equal to each other due to their almost equal retail prices. As k increases, demand quantity of each product decreases since the retailer prefers to focus on margin instead of volume, and increases the retail prices. With a rapidly increasing retail price, the new product's demand falls substantially. Interestingly, we observe a small increase in the third product's demand in the region of $Case$ 2.2. This can be attributed to a steeply decreasing wholesale price under $Case$ 2.2, followed by a retail price that is lower and closer to that of the incumbent products. As k increases further, the new product will not be active in the market. However, the demand level of the incumbent products continues to decrease, as k increases. The change in demand quantities of each firm with respect to k, can be seen in Figure 5.5c.



(a) Wholesale prices

(b) Retail prices

(c) Demand quantities

(d) Change in the profit of each firm

Figure 5.5: The Third Product Introduction Decisions WRT Shelf Space Cost Parameter, k

5.5.2 Effect of the Third Product

When shelf space cost parameter k is too low, both the retail and wholesale prices of the third product are almost equal to those of the existing products. Thus, Supplier 1's profit does not change much. However, both the retailer and Supplier 2 gain due to the market expansion effect of the new product. As k increases, both the equilibrium wholesale price and the demand quantities of incumbent products decreases. But, the existing products capture demand from the new product due to their lower retail price. Thus, Supplier 1 enjoys a slight gain. Moreover, the substitution demand lost to product 1 increases due to widening difference between the retail prices. Therefore, Supplier 2's gain decreases as k increases. The increase in the unit profit margin of the retailer can not compansate the decrease in demand quantity and the increase in shelf space cost. Therefore, the retailer's gain decreases, as k increases. Note that, no party in the chain including Supplier 1, transitions from a loss to a gain or vice versa. With the current parameter set, we see that all parties either enjoy a profit increase or break-even with the introduction of the new product. Thus, we can conclude that whether Supplier 1 will suffer from the new product entry or not, is independent of the shelf space cost k, and is driven by other market characteristics. The change in the profit of each firm and the change in the total supply chain profit wih respect to k can be seen in Figure 5.5d.

5.6 Effect of the Slotting Fee on the New Product Introduction

Slotting fees are widely used in the industry. Yet, one wonders if they are effective and/or necessary in all cases. It may deter a new product entry that would benefit all parties in the chain, or it may fall short of deterring a new product that eventually hurts the retailer and/or the chain.

In this section, we evaluate the effect of slotting fees on new product entry. We analyze the profit change of Supplier 1, Supplier 2, the retailer and the supply chain in order to characterize cases where the slotting fee is meaningful. In this analysis, we change the demand potential (b) and substitution level of the new product (α) when shelf space cost parameter k, the demand potential a and the substitution level θ are kept as given in Table 5.1. As a baseline for our study, we first analyze the case without a slotting fee, i.e., $F = 0$. In our analysis, we observe that eight different cases may arise based on the change in profit of each firm with the new product introduction.

The profit changes of each firm in these regions are summarized below:

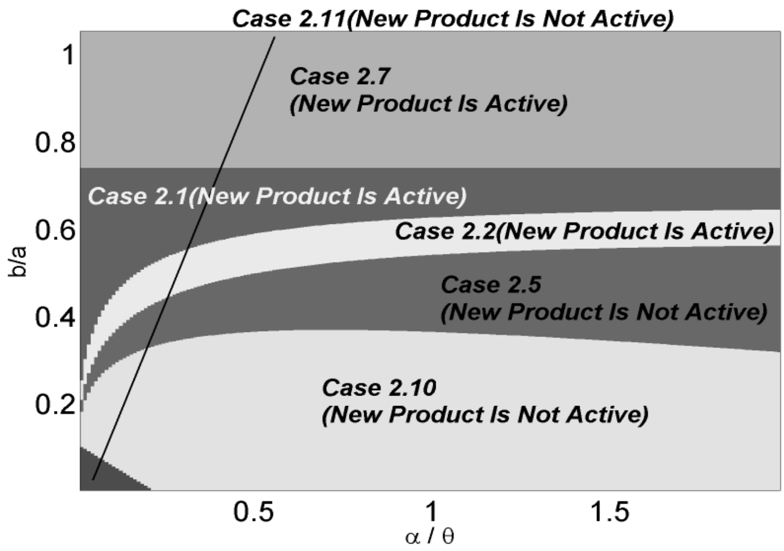
- A_1: $\Delta\pi_R > 0$, $\Delta\pi_{S_1} > 0$, $\Delta\pi_{S_2} > 0$, $\Delta\pi_{SC} > 0$
- A_2: $\Delta\pi_R > 0$, $\Delta\pi_{S_1} < 0$, $\Delta\pi_{S_2} > 0$, $\Delta\pi_{SC} > 0$
- A_3: $\Delta\pi_R > 0$, $\Delta\pi_{S_1} < 0$, $\Delta\pi_{S_2} < 0$, $\Delta\pi_{SC} > 0$
- A_4: $\Delta\pi_R > 0$, $\Delta\pi_{S_1} < 0$, $\Delta\pi_{S_2} > 0$, $\Delta\pi_{SC} < 0$

- A_5: $\Delta\pi_R > 0$, $\Delta\pi_{S_1} < 0$, $\Delta\pi_{S_2} < 0$, $\Delta\pi_{SC} < 0$
- A_6: $\Delta\pi_R = 0$, $\Delta\pi_{S_1} < 0$, $\Delta\pi_{S_2} > 0$, $\Delta\pi_{SC} > 0$
- A_7: $\Delta\pi_R = 0$, $\Delta\pi_{S_1} < 0$, $\Delta\pi_{S_2} > 0$, $\Delta\pi_{SC} < 0$
- A_8: $\Delta\pi_R = 0$, $\Delta\pi_{S_1} = 0$, $\Delta\pi_{S_2} = 0$, $\Delta\pi_{SC} = 0$

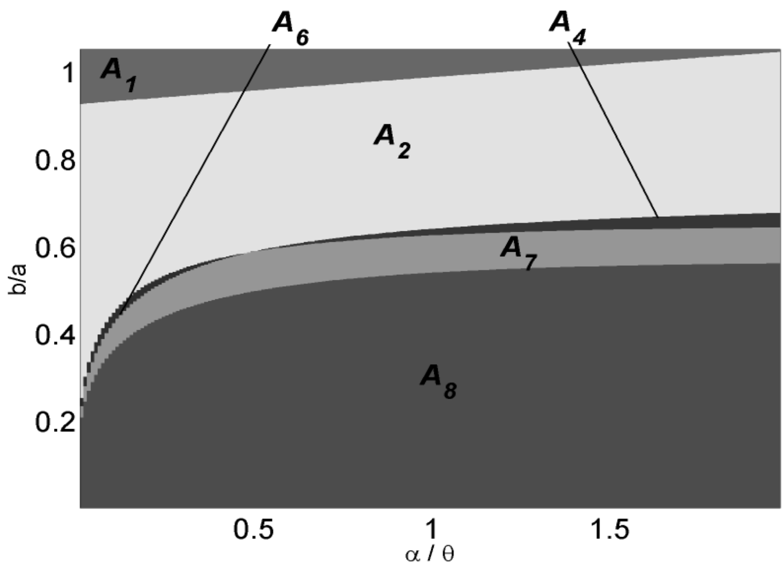
For the base model ($F = 0$), the equilibrium cases and the profit changes are available in Figures 5.6a and 5.6b, respectively. In region A_1 the new product's demand potential is close to demand potential of the incumbent products. So, each firm in the supply chain benefits from this market expansion and their profits increase. In regions A_2 and A_4, the retailer and Supplier 2 benefit from the new product entry at the expense of Supplier 1. However, in A_2 total gain of the retailer and Supplier 2 is greater than the loss of Supplier 1 and the supply chain profit increases. In A_4, however, Supplier 1's loss is higher than total gain of others and the supply chain profit decreases. In all these three regions (A_1, A_2 and A_4), the retailer and Supplier 2 enjoy a profit increase, so the new product will be active in the system. A_1, A_2 and A_4 cases occur when the equilibrium is either in $Case$ 2.1 or $Case$ 2.7, i.e., Supplier 2 sets a wholesale price w_3^0 for the new product and induces positive demand for it.

In region A_6 and A_7, entry of the new product increases Supplier 2's profit whereas the retailer's profit remains uchanged and Supplier 1's profit decreases. These two cases arise at moderate levels of b. In region A_6, the gain of Supplier 2 is greater than the loss of Supplier 1 and the supply chain profit increases. A_6 occurs only when the substitution level between the new product and the others is low. In region A_7, Supplier 2's gain can not compansate the loss of Supplier 1 and the supply chain profit decreases. In these regions, we observe $Case$ 2.2 where Supplier 2 sets a wholesale price w_3'' and the new product is active in the market with positive demand.

If demand potential of the new product is at moderate levels or less, the new product will not be active in the market unless substitution level α is low enough. We show this region as A_8 in Figure 5.6b. This region arises as a result of three different equilibrium cases. In $Case$ 2.5 which can be seen in Figure 5.6a, the retailer's profit may increase with the new product introduction based on the wholesale price of it. But Supplier 2 does not obtain extra profit for any level of w_3 in this case. Thus, Supplier



(a) Equilibrium Cases



(b) Change in Profits

Figure 5.6: The New Product Introduction Decisions without Slotting Fee
($a = 100$, $\theta = 0.5$, and $k = 0.1$)

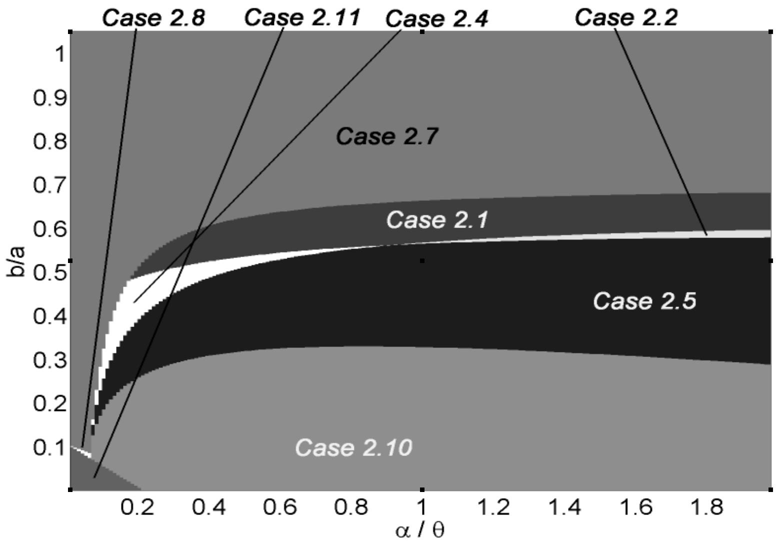
2 sets the wholesale price of the new product high enough so that the retailer rejects it, i.e., $w_3 \in (w_3'', \infty)$. By this way, the new product cannot enter the market. When the new product's demand potential is at lower levels, the new product does not pro-

duce extra profit for both the retailer and Supplier 2. So, we observe either $Case$ 2.10 or $Case$ 2.11 and the new product will not be active in the market. Therefore, even without a slotting fee the new product is not introduced in region A_8.

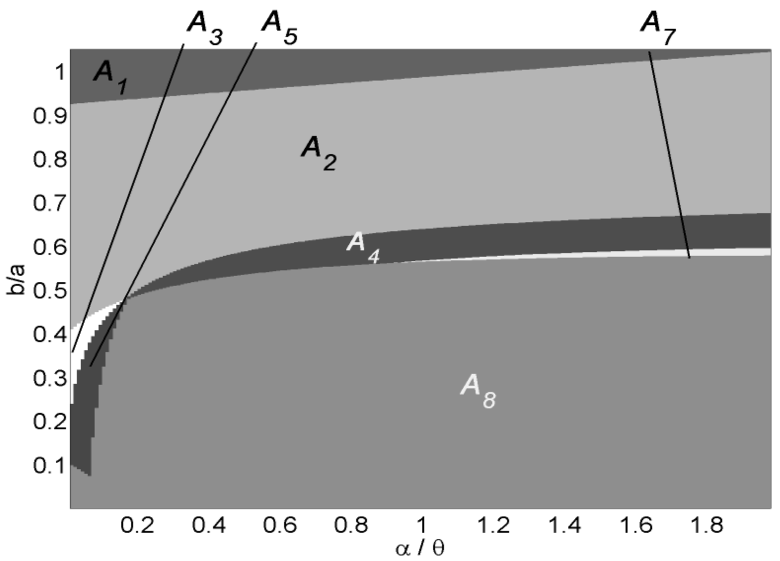
As we mentioned before, the new product enters the market in regions A_4 and A_7 although total supply chain profit decreases. Then, if the retailer wants to deter the new product introduction in order to protect the supply chain profit, she must set a slotting fee that is high enough so that its supplier gives up the new product introduction idea. In order to deter the entry of the new product, the slotting fee must be greater than Supplier 2's gain, otherwise he accepts to pay the slotting fee and the supply chain profit can not be protected.

After the analysis of the base case without any slotting fees, we will study three different levels for slotting fee (F), to see its effect on the equilibrium decisions and the eventual product introduction. Firstly, we analyze our model when $F = 100$. Then, we also make analysis for $F = 250$ and $F = 500$.

Figure 5.7 contains the equilibria and the changes in profits for $F = 100$. Atfer the implementation of the slotting fee, the retailer also accepts the new product when her loss is less than the slotting fee. By this way, the total area of the regions where the third product is accepted by the retailer widens. Differently from the base case, we observe $Case$ 2.4 and $Case$ 2.8 when $F = 100$. In $Case$ 2.4, the entry of the new product does not benefit its supplier. So, Supplier 2 sets the wholesale price of the third product high enough so that the retailer rejects it, i.e., $w_3 \in (w_3'', \infty)$. By this way, the new product is not active in the market. This case is included in region A_8 in Figure 5.7b.



(a) Equilibrium Cases



(b) Change in Profits

Figure 5.7: The New Product Introduction Decisions when $F = 100$
($a = 100$, $\theta = 0.5$, and $k = 0.1$)

Moreover, $Case$ 2.8 occurs when the demand potential and substitution level of the new product are low as it can be seen in Figure 5.7a. In this case, the wholesale price of the new product is set to any value in $(\bar{w}_3, \infty)$ and it is active in the market with practically zero demand. The retailer does not obtain profit directly from the third product but slotting fee compansates her loss and her profit increases. Thus, the retailer accepts the third product in order to take the slotting fee provided that Supplier 2 offers. However, in regions A_3 and A_5 which also includes the region satisfied the conditions of $Case$ 2.8, Supplier 2's gain cannot compansate the slotting fee. Because of this, Supplier 2's profit decreases if the new product is introduced. Therefore, Supplier 2 should refrain from offering a new product and the new product entry must be avoided in this region.

With the new product introduction, profit of both the retailer and Supplier 2 increase in region A_4 whereas profit of Supplier 2 increases and the retailer's profit remains unchanged in region A_7 when $F = 100$. However, total gain of the retailer and Supplier 2 is lower than Supplier 1's loss and the supply chain profit decreases in these regions. Thus, the slotting fee falls short and the entry occurs with a decrease in the total chain profit.

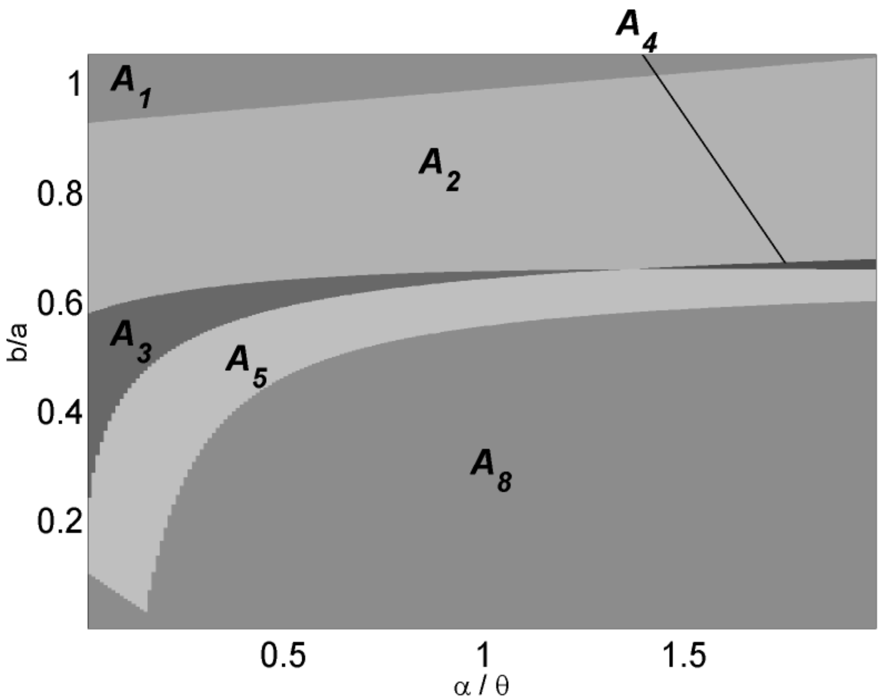


Figure 5.8: The Profit Change Regions when $F = 250$
($a = 100$, $\theta = 0.5$, and $k = 0.1$)

If the retailer raises the slotting fee to 250, we observe $Case$ 2.8 in a larger area since a larger slotting fee can compansate a higher retailer loss. However, the regions A_3 and A_5, which also includes the condition of $Case$ 2.8, widen in this situation since the gain of Supplier 2 can not recover this high slotting fee. In addition to this, the new product is active in the market at the expense of the supply chain in region A_4. Thus, the slotting fee still falls short and the retailer can not protect the supply chain profit in this region when $F = 250$. The profit change regions of case with $F = 250$ can be seen in Figure 5.8.

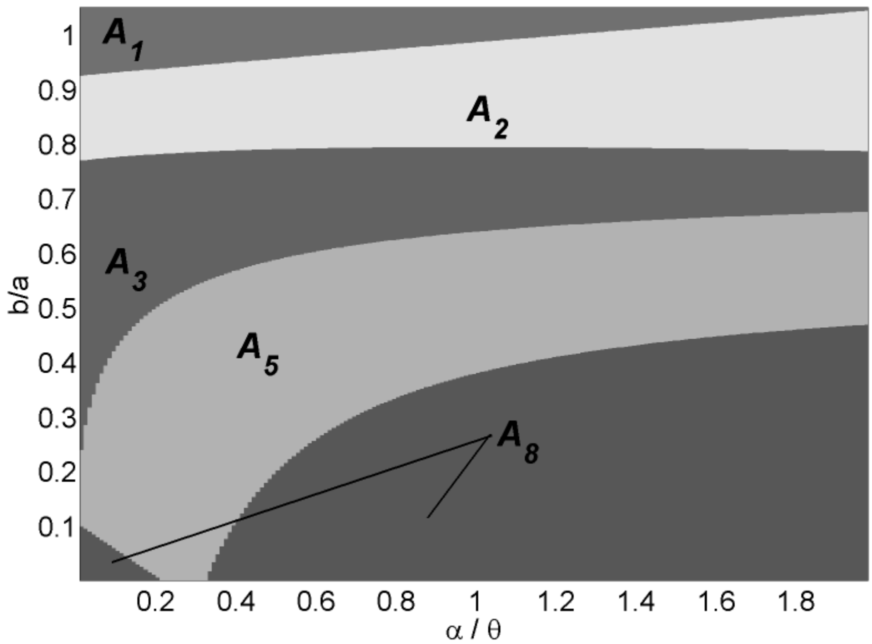


Figure 5.9: The Profit Change Regions when $F = 500$
($a = 100$, $\theta = 0.5$, and $k = 0.1$)

Finally, if the retailer raises the slotting fee to 500, she accepts the third product at the expense of supply chain in only region A_5 as we can see in Figure 5.9. However, Supplier 2's profit also decreases and he should give up the new product introduction idea in this region. Thus, the retailer protects the supply chain profit and deters the entry of the new product which decreases the supply chain profit.

As a result of our base case analysis, we state that whether the new product will be active in the market or not mainly depends on its relative demand potential. If its demand potential is moderate-to-high, it will be active in the market for any level of

α. On the other hand, if its demand potential is below the moderate level, the new product may be active in the market only when the substitution level of it is low. Thus, entrance probability of the new product increases as demand potential b increases or substitutiton level α decreases.

When a new product is introduced, the retailer can accept the new product or not, based on her profit change. If the new product increases her profit, she will accept the new product. Otherwise, she will make the new product inactive by determining a high retail price. Therefore, her profit never decreases and thus does not need protection through a mechanism like a slotting fee. However, the retailer may use the slotting fee in order to protect the supply chain profit. If the introduction of the new product decreases the supply chain profit, the retailer may set a slotting fee which is high enough so that its supplier gives up the new product introduction idea. By this way, the new product entry is deterred and it is kept out of the market.

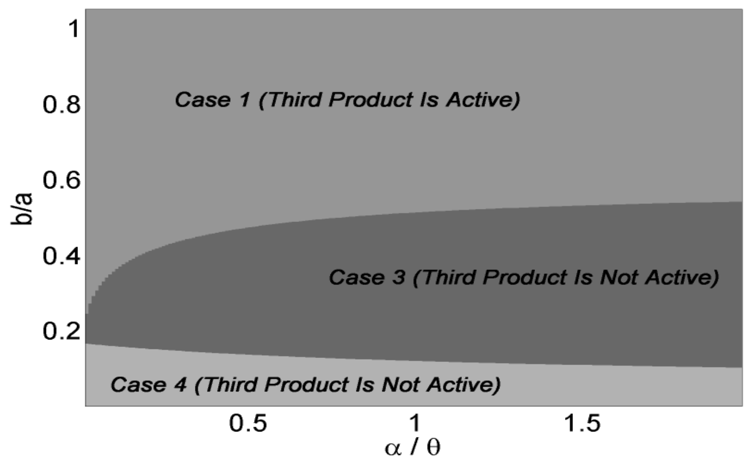
Without a slotting fee, a new product can enter the market provided that entry of it provides extra profit for both the retailer and Supplier 2. Entry decision does not depend on the change in Supplier 1's profit. Thus, a new product entry may benefit both Supplier 2 and the supply chain as we observe in regions A_1, A_2 and A_6, or benefit Supplier 2 but hurt the supply chain profit as we observe in regions A_4 and A_7.

If the retailer wants a high slotting fee for new product introductions in order to protect the supply chain profit, region A_4 and A_7 may vanish. However, Supplier 2 faces the risk of an unprofitable entry due to this high slotting fee. If his gain can not compansate the slotting fee Supplier 2 does not offer the new product to the retailer. Thus, slotting fee may shrink or remove regions A_4 and A_7, but also may hinder new product introductions: entries that benefit the chain become less likely as well. This is especially visible in the reduced regions of A_1 and A_2 when $F = 500$. In these cases, a new product enters the market only if its demand potential is significant.

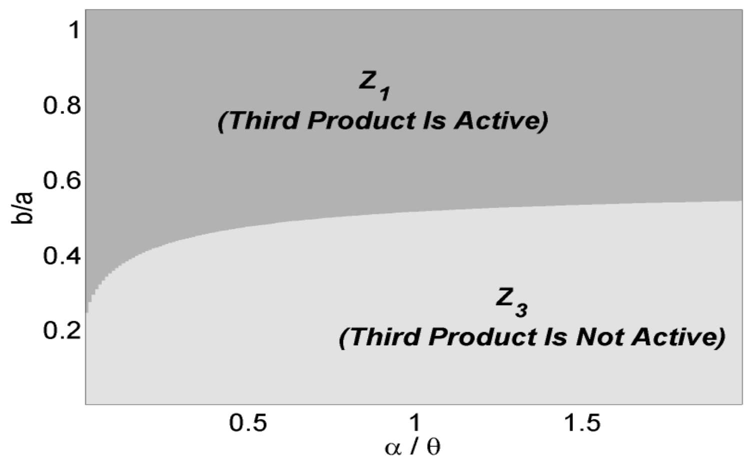
Therefore, slotting fees succesfully deter the entries that may hurt the supply chain profit and create a pressure on suppliers to achieve a succesful product introduction. However, high slotting fees may overprotect the market deter introductions that could boost the supply chain profit.

5.7 Analysis of Centralized Model

As we mentioned in Chapter 4, if the centralized system profit increases with the introduction of the new product, it is accepted by the system. Otherwise, the third product is not active in the market. So, the profit of the centralized system never decreases with the new product offer.



(a) Equilibrium Cases



(b) Profit Changes

Figure 5.10: The New Product Introduction Decision in the Centralized System ($a = 100$, $\theta = 0.5$, and $k = 0.1$)

The cases observed and the changes in centralized system profit can be seen in Figures 5.10a and 5.10b, respectively. If the demand potential of the third product is about half of each existing product or lower, the new product is rejected unless its substitutability level is low. Thus, we observe either $Case$ 3 or $Case$ 4 and the new product is not active in the centralized system in region Z_3. Thus, centralized system profit does not change. If its demand potential is higher than this threshold, however, the new product is always active for any substitutiton level α. So, we observe $Case$ 1 and the centralized system profit increases with the new product entry in region Z_1.

5.8 Effect of the Slotting Fee on Supply Chain Profit

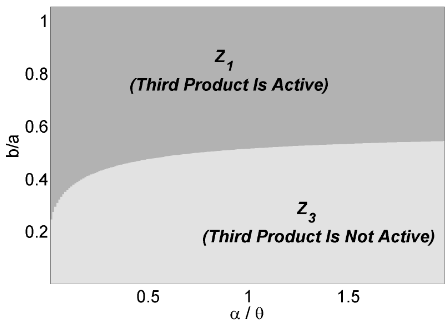
In the decentralized supply chain, the introduction of the new product does not depend on the supply chain profit. The new product enters the market provided that both the retailer and its supplier benefits from this entry. On the other hand, centralized system decision is based on total chain profit.

In this section, we analyze the effect of the slotting fee on the protection of the supply chain profit through numerical experiments. In addition, we investigate whether the slotting fee helps to decentralized system to approach centralized system solution or not.

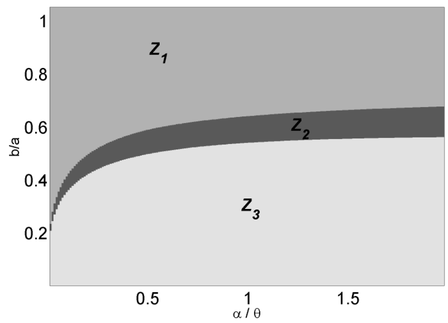
The change in the centralized system profit can be seen in Figure 5.11a. In addition to this, changes in the total supply chain profit of decentralized system can be seen in Figures 5.11b , 5.11c, 5.11d and 5.11e for the cases $F = 0$, $F = 100$, $F = 250$ and $F = 500$, respectively.

In these figures, the supply chain profit increases with the new product introduction in region Z_1. In region Z_2, both the retailer and Supplier 2 benefit from the new product introduction but their total gain can not recover the Supplier 1's loss. Thus, the new product is active in the market at the expense of the supply chain. In region Z_3, the new product is not active in the market and the supply chain profit remains unchanged.

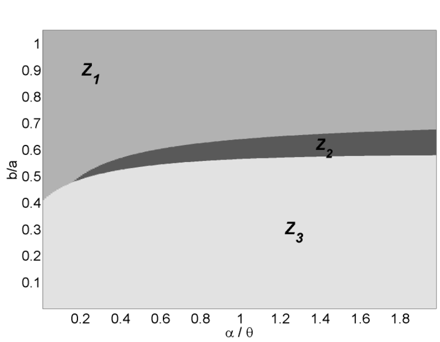
In the base case, we observe region Z_2 and the new product is active in the market at



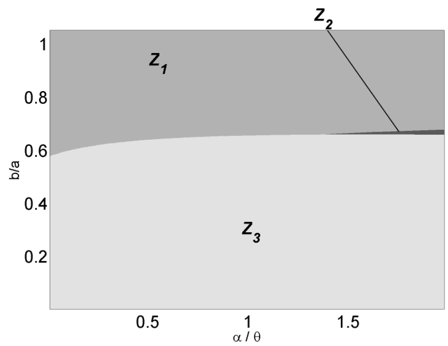
(a) Centralized System



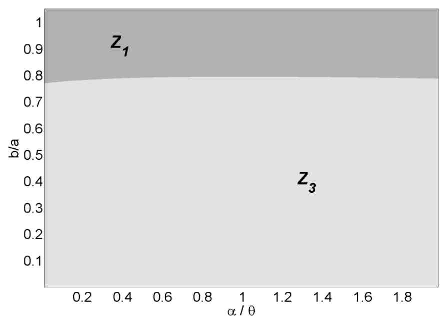
(b) Decentralized System where $F = 0$



(c) Decentralized System where $F = 100$



(d) Decentralized System where $F = 250$



(e) Decentralized System where $F = 500$

Figure 5.11: Profit Change Regions ($a = 100$, $\theta = 0.5$, and $k = 0.1$)

the expense of the supply chain when there is no slotting fee. As the retailer increases the slotting fee, region of Z_2 shrinks as it can be seen in Figure 5.11. If the retailer raises the slotting fee to 500, region Z_2 will vanish. By this way, entry of the new product which decreases the supply chain profit is deterred. Moreover, region Z_1 where the supply chain profit increases with entry of the new product also shrinks as the slotting fee increases. Since Supplier 2's profit can not compansate the high slotting fee and he does not offer the new product to the retailer.

As it can be seen in Figure 5.11a, the new product can enters the centralized market if its demand potential at moderate levels or higher. If we compare the centralized system and the decentralized system without slotting fee, we observe similar demand potential thresholds for entrance of the new product to the market. In addition, we do not observe significant change in this threshold of decentralized system when the slotting fee is low. However, it increases, as the slotting fee increases. By this way, region Z_1 of decentralized system will shrink as the slotting fee increases. Thus, a new product can enter the decentralized market only if it has significant demand potential when the slotting fee is too high. In this situation, slotting fees can succesfully protect the supply chain profit but it may provide overprotection for the market and hinder the entry of the new product which could increase the supply chain profit. Therefore, especially high slotting fees do not help the decentralized system to approach the centralized system solution in new product introduction.

CHAPTER 6

CONCLUSION

In this thesis, we analyze the effect of the new product introduction on each firm in the supply chain. At the beginning, we have a decentralized supply chain in which there are two suppliers and a retailer. Each supplier sells his product to customers through a common retailer. Their products are partially substitute and they have price-based deterministic demand. However, the retailer has a shelf space cost which increases with the total shelf space allocated to the category. Then in the new product introduction setting, Supplier 2 considers to introduce a new product into the supply chain which is also a partially substitute with the existing products. The third product also has price-based deterministic demand. In the new product introduction setting, we assumed that the retail and the wholesale prices of the existing products are taken as in the two-product equilibrium. We analyze both the two-product equilibrium and the introduction of a new product settings in our study. In each setting, we solve a two stage dynamic game. In the first stage, we will analyze the suppliers' best wholesale price decisions foreseeing the the retailer's best response. In the second stage, we find out the best retail price response of the retailer. Then, we compared two settings in order to find out the effects of the new product introduction on each firm in the market. In addition, we investigate the effect of the slotting fee on new product introduction decisions.

Because of the our too complicated equations, we conduct numerical experiments to analyze the effect of new product introduction. In addition to this, we also make numerical analysis in order to find out effect of different levels of slotting fee. Besides, we analyze the centralized supply chain system as a benchmark.

As a result of our analysis, we reveal the following results. The best wholesale and retail price of suppliers and the retailer are based on the relative size of the demand potentials a and b. If the new product has higher demand potential, it increases awareness of the whole product category. Its supplier shares the demand potential of the new product with the incumbent products with higher retail prices. By this way, each firm in the supply chain benefits from the new product introduction. On the other hand, when the demand potential of the new product is moderate-to-high, it will be active in the market and compete with the existing products with its lower retail price. Then, the new product seizes demand from the incumbent products due to spillover effect. Therefore, profit of the supplier which is not introduce the new product decreases. However, if substitution level between the existing products is high, the loss of the supplier who does not introduce the new product will be lower. Because, prices of the existing products are set as more competitive and the new product can seize lower demand from them. In addition, if the demand potential of the third product is moderate-to-low, the new product does not provide more profit for its supplier. Then, he sets the wholesale price of it high enough so that the retailer rejects it. By this way, it will not be active in the market. Finally, introduction of the third product does not increase profit of any firm when its demand potential is too low and it does not enter to the market.

Besides, the retailer has power to reject the new product if her profit decreases in our supply chain. So, there is no need for slotting fee in order to protect the retailer's profit. However, she may implement slotting fee for the new product introductions in order to protect the supply chain profit. When the retailer wants a slotting fee, the gain of its supplier must compansate this fee. If his gain is greater than the slotting fee, Supplier 2 accepts to pay the slotting fee and the third product enters to the market. Otherwise, Supplier 2 gives up the new product introduction idea. By this way, the entry of the new product whose demand potential is not high enough to compansate the slotting fee is deterred. As a result of this, only the products which have a high enough demand potential can enter the market. So, the supply chain profit can be protected with the implementation of slotting fee. However, high slotting fees may overprotect the market and hinder the entry of a new product which could increase the total supply chain profit.

Morevoer, we also analyze the centralized system as a benchmark. The new product is accepted only if it provides higher profit to the centralized system. Otherwise, the new product is rejected and the centralized supply chain profit remains unchanged. Thus, profit of the centralized system never decreases with the new product offer.

We can do extend this study with multiple different settings. First, the other supplier may consider a new product entry threat by his competitor while setting his wholesale price in the two-product setting. Second, only the retailer may consider a new production entry possibility while determining the retail prices of the two products. Third, we could add one more period upon the new product entry, and study the price response of the other supplier to a third product in the market.

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APPENDICES

A. The Derivatives of the Retail Price of the Products in Two-Product Setting

The retail price of the products in the two-product equilibrium setting is found in Proposition 3.1.1 in terms of wholesale prices. Then, we find the derivates of retail price with respect to parameters of our model in order to find out the effects of parameters.

$$\frac{\partial p_i}{\partial a} = \frac{1+4k}{2(1+2k)}$$

$$\frac{\partial p_i}{\partial k} = -\frac{-2a+w_1+w_2}{2(1+2k)^2}$$

$$\frac{\partial p_i}{\partial w_i} = \frac{1+k}{2(1+2k)}$$

$$\frac{\partial p_i}{\partial w_j} = -\frac{k}{2(1+2k)}$$

B. The Derivatives of the Wholesale Price of the Products in Two-Product Setting

The supplier's best wholesale price response to other supplier's wholesale price is found in Proposition 3.1.2. The derivatives of the wholesale price of the products are expressed as:

$$\frac{\partial w_i}{\partial a} = \frac{1}{2(1+k+\theta+2k\theta)}$$

$$\frac{\partial w_i}{\partial k} = \frac{(-a+w_j)(1+2\theta)}{2(1+k+\theta+2k\theta)^2}$$

$$\frac{\partial w_i}{\partial \theta} = \frac{(1+2k)(-a+w_j)}{2(1+k+\theta+2k\theta)^2}$$

$$\frac{\partial w_i}{\partial w_j} = \frac{1}{2} - \frac{1}{2(1+k+\theta+2k\theta)}$$

C. The Derivatives of the Equilibrium Wholesale Prices of the Products in Two-Product Setting

The equilibrium wholesale prices of the products in thw two-product equilibrium setting is found in Proposition 3.1.3. The derivatives of the this price with respect to parameters of our model are given below:

$$\frac{\partial w_i}{\partial a} = \frac{1}{2+k+\theta+2k\theta}$$

$$\frac{\partial w_i}{\partial k} = \frac{a(1+2\theta)}{(2+k+\theta+2k\theta)^2}$$

$$\frac{\partial w_i}{\partial \theta} = \frac{a(1+2k)}{(2+k+\theta+2k\theta)^2}$$

D. Conditions of the Retail Price Decision of the New Product

Table 0.1: Conditions of the Retail Price Decision of the New Product

Conditions	Expression
$\pi_R^{(3)}(p_3^0(w_3)) - \pi_R^{(2)} \geq 0$	$\Big((b+2k(-2a+p_1+p_2)-w_3)^2 - 2(4b^2k+2((1+3k)p_1^2$ $-p_1(w_1+k(-4p_2+2w_1+w_2))+p_2(p_2+3kp_2-w_2-k(w_1+2w_2)))$ $+4ak(4b-2p_1-2p_2+w_1+w_2-2w_3)+(6k(p_1+p_2)+w_1+w_2)w_3-2w_3^2$ $+b(-2(1+6k)p_1+w_1+w_2-2(p_2+6kp_2-(w_1+w_2-2w_3))+2w_3))\alpha$ $-(4p_1^2+4p_2^2+4p_2(w_1-w_2)-4p_1(2p_2+w_1-w_2)$ $+(3(w_1+w_2)-2w_3)(w_1+w_2+2w_3))\alpha^2\Big)$
$\pi_R^{(3)}(\bar{p}_3) - \pi_R^{(2)} \geq 0$	$\Big(\alpha(-p_1^2-2kp_1^2-4kp_1p_2-p_2^2-2kp_2^2+p_1w_1+p_2w_2-4b^2k\alpha$ $-((3+5k)p_1^2+p_2(3p_2+5kp_2+w_1-3w_2)$ $+p_1(-2p_2+10kp_2-3w_1+w_2))\alpha$ $-2(p_1-p_2)(p_1-p_2-w_1+w_2)\alpha^2-4ak(2b-p_1-p_2)(1+2\alpha)$ $+b(p_1+4kp_1+p_2+4kp_2-w_1-w_2$ $+2(p_1+6kp_1+p_2+6kp_2-w_1-w_2)\alpha))\Big)$

E. The Derivatives of the Retail Price of p_3^0

The best retail price of the retailer for the third product is found in Proposition 3.2.1. The derivatives of it with respect to parameters are given below:

$$\frac{\partial p_3^0}{\partial a} = \frac{2k}{1+k+2\alpha}$$

$$\frac{\partial p_3^0}{\partial b} = \frac{1+2k}{2+2k+4\alpha}$$

$$\frac{\partial p_3^0}{\partial p_1} = \frac{-k+\alpha}{1+k+2\alpha}$$

$$\frac{\partial p_3^0}{\partial p_2} = \frac{-k+\alpha}{1+k+2\alpha}$$

$$\frac{\partial p_3^0}{\partial \alpha} = -\frac{b(2+4k)-2p_1-2p_2+w_1+w_2+k(8a-6p_1-6p_2+w_1+w_2-2w_3)}{2(1+k+2\alpha)^2}$$

F. Conditions of the Wholesale Price Decision of the New Product

Table 0.2: Conditions of the Wholesale Price Decision of the New Product

Conditions	Expression
$C_1 = \pi_R^{(3)}(p_3^0(w_3=0)) - \pi_R^{(2)}$	$\Big((b+2k(-2a+p_1+p_2))^2 - 2(4b^2k + 4ak(4b-2p_1-2p_2+w_1+w_2)$ $+b(-2(1+6k)p_1 - 2(1+6k)p_2 + w_1 + w_2 + 2k(w_1+w_2))$ $+2((1+3k)p_1^2 - p_1(w_1 + k(-4p_2 + 2w_1 + w_2))$ $+p_2(p_2 + 3kp_2 - w_2 - k(w_1 + 2w_2))))\alpha$ $+(-4p_1^2 - 4p_2^2 + 4p_1(2p_2 + w_1 - w_2) + 4p_2(-w_1 + w_2) + (w_1+w_2)^2)\alpha\Big)$
$C_2 = \pi_R^{(3)}(p_3^0(\bar{w}_3)) - \pi_R^{(2)}$	$\Big(\alpha(-p_1^2 - 2kp_1^2 - 4kp_1p_2 - p_2^2 - 2kp_2^2 + p_1w_1 + p_2w_2 - 4b^2k\alpha$ $-((3+5k)p_1^2 + p_2(3p_2 + 5kp_2 + w_1 - 3w_2)$ $+p_1(-2p_2 + 10kp_2 - 3w_1 + w_2))\alpha$ $-2(p_1-p_2)(p_1-p_2-w_1+w_2)\alpha^2 - 4ak(2b-p_1-p_2)(1+2\alpha)$ $+b(p_1 + 4kp_1 + p_2 + 4kp_2 - w_1 - w_2$ $+2(p_1 + 6kp_1 + p_2 + 6kp_2 - w_1 - w_2)\alpha))\Big)$