IMPROVING DATA FRESHNESS IN RANDOM ACCESS CHANNELS

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ABSTRACT

IMPROVING DATA FRESHNESS IN RANDOM ACCESS CHANNELS

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The conventional network performance metrics such as throughput and delay do not accurately reflect the needs of some applications. Age of information (AoI) is a newly proposed metric that indicates the freshness of information from the receiver's perspective. In this work, a network of multiple transmitter devices continuously updating a central station over an error-free multiaccess channel is studied. The average AoI expressions are derived for Round-Robin, Slotted ALOHA, and a proposed random access strategy called LAZY. Optimal transmission rates are found for the two source network case with different threshold parameters under LAZY. It is proven that for the two source network, LAZY performs significantly better in terms of data freshness compared to the Slotted ALOHA due to the utilization of the instantaneous AoI knowledge of the sources.

Keywords: Age of information, random access, slotted ALOHA, age minimization

ÖZ

RASTGELE ERİŞİM KANALLARINDA BİLGİ TAZELİĞİNİN İYİLEŞTİRİLMESİ

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Verimlilik ve gecikme gibi geleneksel ağ performansı ölçütleri, bazı uygulamaların ihtiyaçlarını doğru şekilde yansıtmamaktadır. Bilgi yaşı, alıcı tarafın bakış açısından bilgi tazeliğini gösteren yeni bir ölçüttür. Bu çalışmada, hatasız bir çoklu erişim kanalı üzerinden tek bir merkezi sürekli olarak güncelleyen, çok sayıda verici içeren bir haberleşme ağı incelenmiştir. Değişmez Zaman Paylaşım Sistemi, Dilimli ALOHA ve önerilen LAZY isimli rastgele erişim stratejisi altında, bu ağ için ortalama bilgi yaşı elde edilmiştir. Farklı eşik değerlerindeki LAZY stratejisi altında, iki vericili ağlarda bu değeri en düşük seviyede tutmak için gereken gönderim sıklıkları bulunmuştur. Anlık bilgi yaşının kullanımı sayesinde LAZY stratejisinin veri tazeliği açısından Dilimli ALOHA'ya göre önemli ölçüde daha iyi performans sergilediği gösterilmiştir.

Anahtar Kelimeler: Bilgi yaşı, rastgele erişim kanalı, dilimli ALOHA, bilgi yaşı enküçültme

To my family

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AoI	Age of Information
AAoI	Average Age of Information
FCFS	First Come First Served
LAZY	Lazy Policy
MC	Markov Chain
RR	Round-Robin
SA	Slotted ALOHA

CHAPTER 1

INTRODUCTION

1.1 Motivation

Many real-world applications require continuous and timely delivery of information from remote devices to a central station, over a wireless network, for sensing, monitoring, positioning, or various other purposes. The conventional performance metrics such as throughput and delay do not always reflect the requirements of such applications. The timeliness of the updates is a more relevant criterion. Age of information (AoI) is a newly proposed, relatively unstudied network performance metric that indicates the freshness of the knowledge a system has about a remotely observed process [1]. The primary focus of this thesis is the analysis and improvement of time-average AoI in a random access network, where there are multiple sensor nodes set to continually send updates to a single monitor node over a noiseless multiaccess channel.

1.2 Contributions

The concept of AoI has been studied for various types of networks. This work builds up on the findings of [2], where AoI is studied for an unreliable multiaccess network operating under the classical Slotted ALOHA protocol by providing an alternative, generally applicable method to derive the average AoI for unbuffered sources, and proposing "LAZY", to significantly improve the average AoI performance of a network, compared to Slotted ALOHA, by utilizing the instantaneous AoI knowledge.

1.3 The Outline of the Thesis

The thesis contains 3 chapters excluding the introduction and the conclusion.

In Chapter 2, some background information on Age of Information is given to familiarize readers with the concept.

Chapter 3 starts with definitions of the network model. The problem of interest is stated and the average AoI of the network is obtained under LAZY by modeling the AoI state of the sources as a Markov chain and applying steady-state analysis. Slotted ALOHA is studied as a special case of LAZY. The average AoI under Round-Robin policy is found and the policy is shown to be AAoI-optimal. Using the explained method, the closed-form average AoI expressions are derived for the 2-source case under LAZY with different threshold parameters and optimal attempt rates are found.

Chapter 4 consists of simulation results and discussions. The AoI performances of classical Slotted ALOHA, LAZY, and Round-Robin are presented and compared for various cases. The effects of each system parameter on the performances are shown.

CHAPTER 2

AGE OF INFORMATION

2.1 Literature Review

Age of information (AoI) is a concept for quantizing the freshness of the knowledge a system has about the status of a remote process [1]. It was first introduced in [3], where a network of vehicles exchanging information using an IEEE 802.11 based CSMA mechanism for channel access is studied. These works were followed by [4] and [5], which characterized AoI for simple queuing systems and provided remarkable results on how neither maximizing throughput nor minimizing delay meant maximizing data freshness.

AoI then quickly attracted a growing interest due to its novelty and applicability as a performance metric for a wide range of communication networks. There have since been many works studying the AoI concept under various system models. Under FIFO service, seminal analyses on AoI have been done for M/M/1, M/D/1, and D/M/1 queues in [5]. In [6], it was shown that a throughput and delay optimal zero-wait policy is not always age-optimal for a "generate-at-will" model. Effect of packet errors and packet drops on AoI were studied in [7] and utilizing packet management to reduce AoI was studied in [8] and [9]. In [10, 11], AoI optimization problem was studied under energy harvesting constraints.

Age-based scheduling problems in wireless networks have also been extensively studied. It was shown in [12] that scheduling finitely many update packets under physical interference constraints is an NP-hard problem. Age optimal scheduling in broadcast wireless networks was studied in [13] and [14]. In [14], both offline and online optimal scheduling algorithms based on a sequence of finite-state approximate MDPs are

found and the finite-state MDP approximations are shown to converge to the infinite-state MDP model. Some of the recent works include [15], [16], and [17], where age optimal scheduling in wireless networks under general interference constraints and time-varying channels are studied.

Although the focus of the literature, regarding age-based scheduling, has mainly been on centralized scheduling, distributed scheduling schemes have also received attention. In [18], the proposed Round-Robin policy with single packet buffering is shown to be optimal as the number of devices in the network tends to infinity. In [19], distributed scheduling under pairwise interference constraints, where each link is activated with a certain probability, is studied. For each link, a relation between its optimal attempt probability and the optimal AoI of it and its neighboring links are obtained.

This thesis focuses on the study of AoI for a network consisting of multiple nodes transmitting updates over a multiaccess channel using random access policies. The work is inspired mainly by the system model in [2], where AoI performance has been analyzed and age-optimal transmission rates are found for an unreliable multiaccess channel operating under the Slotted ALOHA random access protocol. This work studies the same model with a different approach on the formulation of the problem and contributes by proposing the "lazy policy", which simply introduces an idle period after each successful transmission. The proposed policy is shown to perform significantly better than the classical Slotted ALOHA due to the utilization of AoI knowledge.

2.2 Age of Information: Definition and Analysis

This section serves as an introduction to the concept of AoI and how it is analyzed as a performance metric in FCFS systems [5].

Consider a real-time status update system where a source node samples the current state of a variable (position, temperature, etc.) and transmits the samples to a sink node in the form of data packets. The transmitted data packets experience random delays. The successful reception of a sample by the sink is called an “update”.

Age of information (AoI) is the "staleness" of the most up-to-date information regarding the state of the observed system, at any point where the information is processed. Therefore, from a network perspective, it is a metric relevant at the destination (i.e. the sink), not the source.

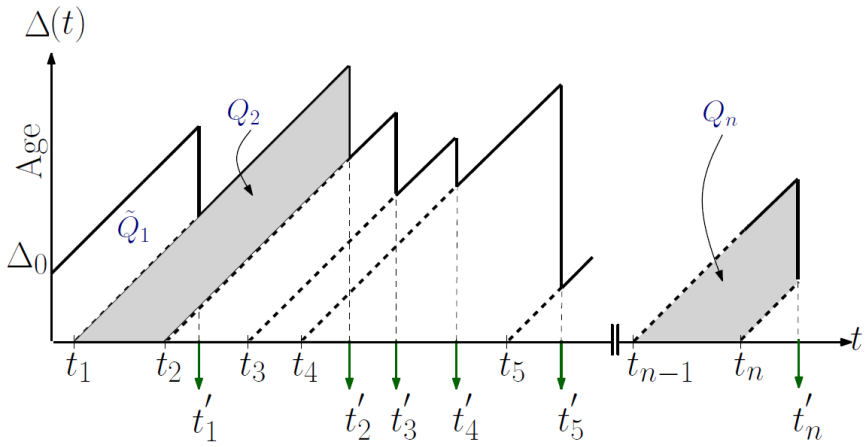

Figure 2.1: A sample variation of age in time.

Fig. 2.1 illustrates the evolution of age $\Delta(t)$ in time for a single source-sink pair, over a finite time interval $[0, T]$. Without loss of generality, it is assumed that $\Delta(0) = \Delta_0$ at the beginning of observation. The ith sample of the variable is taken at time t_i and successfully received by the sink at time t'_i. Upon successful reception of the ith sample (the ith update), $\Delta(t)$ resets to $t'_i - t_i$, and increases linearly with time otherwise. Thus, $\Delta(t)$ is defined as

$$\Delta(t) = t - max\{t_i | t'_i < t\}, \quad t > t'_1. \tag{2.1}$$

There are various proposed performance metrics utilizing AoI, such as average age, peak average age [20, 21], peak age violation probability [22, 23, 24], etc. The focus of this thesis is on average age (AAoI), which is defined as the average value of $\Delta(t)$ over time defined as

$$\langle \Delta \rangle_T = \frac{1}{T} \int_0^T \Delta(t) dt. \tag{2.2}$$

For simplicity, T is chosen to be the time instant t'_n where the nth update occurs.

The total area under the age curve, defined by the integral in (2.2) can be written as the sum of the areas of the polygon $\tilde{Q}_1$, the trapezoids Q_i for $i \geq 2$, and the isosceles

triangle over the time interval (t_n, t'_n) as shown in Fig. 2.1. Let $N(T) = max\{n|t_n \leq T\}$ denote the number of samples taken in the interval $[0, T]$. Then, defining

$$\begin{aligned} X_i &= t_i - t_{i-1}, \\ Y_i &= t'_i - t_i, \end{aligned} \tag{2.3}$$

the average age over interval $[0, T]$ equals

$$\langle \Delta \rangle_T = \frac{\tilde{Q}_1 + Y_n^2/2 + \sum_{i=2}^{N(T)} Q_i}{T}, \tag{2.4}$$

where the trapezoidal areas Q_i can be calculated as the area difference of two isosceles triangles as

$$Q_i = \frac{1}{2}(X_i + Y_i)^2 - \frac{1}{2}Y_i^2 = X_i Y_i + \frac{X_i^2}{2}. \tag{2.5}$$

Substituting (2.5) into (2.4) gives

$$\begin{aligned} \langle \Delta \rangle_T &= \frac{\tilde{Q}_1 + Y_n^2/2 + \sum_{i=2}^{N(T)} (X_i Y_i + X_i^2/2)}{T} \\ &= \frac{\tilde{Q}_1 + Y_n^2/2}{T} + \frac{N(T) - 1}{T} \frac{\sum_{i=2}^{N(T)} (X_i Y_i + X_i^2/2)}{N(T) - 1}, \end{aligned} \tag{2.6}$$

where the term $(\tilde{Q}_1 + Y_n^2/2)$ is finite and vanishes as T goes to infinity.

Let

$$\lambda = \lim_{T \to \infty} \frac{N(T)}{T} \tag{2.7}$$

be the ergodic rate at which the samples are taken and transmitted. Then, the average AoI of the system is obtained as

$$\Delta = \lim_{T \to \infty} \langle \Delta \rangle_T = \lambda (E[XY] + \frac{E[X^2]}{2}), \tag{2.8}$$

where X and Y are random variables corresponding to inter-sampling time and transmission delay respectively, and $E[.]$ is the expectation operator.

The result in Eqn. (2.8) is valid for a broad class of networks where the transmitted packets are processed in a FCFS manner.

2.3 AoI in Slotted ALOHA

In this section, the findings of [2] regarding the analysis and optimization of AoI for a network operating under the Slotted ALOHA protocol is presented.

Consider a more specific version of the system model defined in Section 2.2 , where M source nodes, each one observing a different variable, send updates to a single sink node over a common channel. The channel access is done randomly under the Slotted ALOHA protocol. All nodes in the network are assumed to be synchronized to the same slot timings. The sequence of events inside a time slot is illustrated in Fig. 2.2.

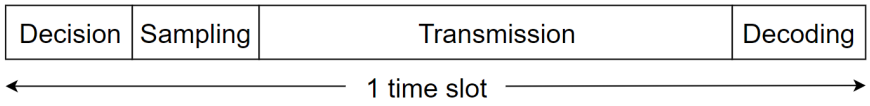

Figure 2.2: Sequence of events in a time slot.

At the beginning of a time slot, each source i independently decides to transmit a packet with a fixed probability. If a source decides to transmit, it generates a data packet containing information on the current state of the variable it is observing. Successfully transmitted packets are decoded by the sink at the end of the time slot.

If two or more sources transmit at the same time, a collision occurs and all transmitted packets are lost. A packet from source i that has reached the sink without any collision is correctly decoded with probability p_i. Incorrect decoding might happen due to channel impairments. Incorrectly decoded packets are not retransmitted. It is assumed that p_i are known by each source in the network.

Let t_{ij} be the time instant when the jth update by source i occurs. The time between the jth and the $(j+1)^{th}$ update is the inter-update time $Z_{ij} = t_{i(j+1)} - t_{ij}$. The Z_{ij} of source i are assumed identically distributed as Z_i. The instantaneous age of information of source i is defined as

$$\Delta_i(t) = t - \max_j\{t_{ij}|t_{ij} \leq t\} + 1, \quad t > t_{i1}. \tag{2.9}$$

$\Delta_i(t)$ increases linearly in time between updates. A successful update by source i resets $\Delta_i(t)$ to 1, because of the 1 slot transmission delay. Decision, sampling, and decoding times are assumed negligible. The evolution of $\Delta_i(t)$ with time is shown in Fig.2.3.

Recall from Eqn. (2.8) that the AoI of any source i, time-averaged over a sufficiently long interval is

$$\Delta_i = \lim_{T\to\infty} \langle \Delta_i \rangle_T = \lambda_i(E[X_iY_i] + \frac{E[X_i^2]}{2}), \tag{2.10}$$

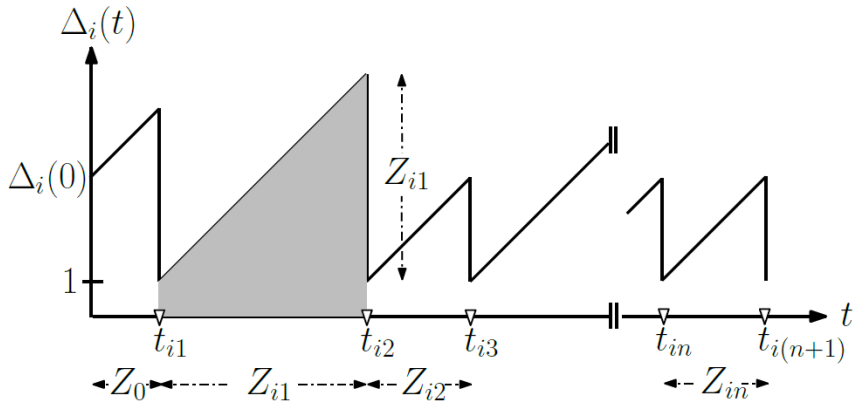

Figure 2.3: Evolution of age in time for a single source i in a Slotted ALOHA network.

where X_i and Y_i are random variables corresponding to source i's inter-sampling time and transmission delay respectively, and λ_i is the rate at which the samples from source i are generated.

It is assumed that if a source decides to transmit in a time slot, it generates a sample immediately at the beginning of that time slot. It takes 1 slot for each packet (containing the sample) to reach the sink. The transmission delays are therefore deterministic and equal to 1 slot for all update packets. Hence, the transmission delay Y_i in Eqn. (2.10) can be substituted with 1 to yield

$$\Delta_i = \lambda_i(E[X_i] + \frac{E[X_i^2]}{2}), \tag{2.11}$$

In this network, packet losses may occur. Hence, the inter-sampling time X_i here is redefined as the time between the generation instants of consequent "successfully" transmitted packets of source i as

$$X_i = (t_{i(j+1)} - 1) - (t_{ij} - 1) = Z_i, \tag{2.12}$$

where Z_i was previously defined as the random variable representing the inter-update time of source i.

Let τ_i be the transmission attempt probability of source i in any time slot. A successful update by source i occurs if and only if source i is the only sender in the time slot and the packet is correctly decoded at the sink with probability p_i. The probability of

source i successfully updating in a time slot is thus

$$\gamma_i = \tau_i p_i \prod_{j \neq i} (1 - \tau_j). \tag{2.13}$$

The inter-update time Z_i is a geometric random variable with mean $E[Z_i] = 1/\gamma_i$ and second moment $E[Z_i^2] = 2/\gamma_i^2 - 1/\gamma_i$. Furthermore, since λ_i is the rate at which successfully transmitted packets of source i are generated,

$$\lambda_i = \gamma_i. \tag{2.14}$$

Substituting these into (2.11),

$$\begin{aligned} \Delta_i &= \lambda_i (E[Z_i] + \frac{E[Z_i^2]}{2}) \\ &= \gamma_i \left(\frac{1}{\gamma_i} + \frac{\frac{2}{\gamma_i^2} - \frac{1}{\gamma_i}}{2} \right) \\ &= \frac{1}{2} + \frac{1}{\gamma_i}, \end{aligned} \tag{2.15}$$

Overall AoI of the network is

$$\Delta \triangleq \frac{1}{M} \sum_{i=1}^{M} \Delta_i = \frac{1}{2} + \frac{1}{M} \sum_{i=1}^{M} \frac{1}{\gamma_i}, \tag{2.16}$$

where M is the number of sources in the network.

The problem of interest is finding a set of τ_i such that Δ is minimized. Differentiating (2.16) with respect to τ_i, the first order optimality conditions are found as

$$\frac{1 - \tau_i}{p_i \tau_i^2} = \sum_{j=1}^{M} \frac{1 - \tau_j}{p_j \tau_j}, \quad 1 \leq i \leq M. \tag{2.17}$$

For a network of $M = 2$ sources, AoI-optimal attempt probability for source i is obtained as

$$\tau_i^* = \frac{1}{1 + \sqrt[3]{\frac{p_i}{p_j}}}, \quad i, j \in \{1, 2\}, i \neq j. \tag{2.18}$$

It is difficult to derive an exact closed form solution for $M > 2$ and to the best of our knowledge, there is no study in the related literature that has done it. However, for large M, AoI-optimizing attempt probability for source i is approximated as

$$\tau_i^* \approx \frac{1/\sqrt{p_i}}{\sum_{j=1}^{M} (1/\sqrt{p_j})}, \quad 1 \leq i \leq M. \tag{2.19}$$

For the proof of this approximation, the reader is referred to Appendix C of [2].

Despite being inefficient in terms of both throughput and AoI compared to many other scheduling policies, Slotted ALOHA's greatest advantage is its low design complexity. In the next chapter, we propose another low complexity random access policy, LAZY, where the sources in the network take channel access decisions according to their own instantaneous AoIs. Due to the utilization of instantaneous AoI information, LAZY performs significantly better than Slotted ALOHA in terms of average AoI. However, one major assumption of LAZY is that each source in the network is aware of its own AoI at all times. For channels where the effect of noise is negligible, the only additional requirement of LAZY is, therefore, collision detection at the source side. For noisy channels, however, the sources would require feedback from the destination for every successful transmission, which would increase design complexity. In Chapter 3, the AAoI performance of LAZY in a network where the sources are communicating over a noiseless multiaccess channel is studied and compared with the performances of Slotted ALOHA and the AAoI-optimal Round-Robin policy for this specific network model.

CHAPTER 3

AVERAGE AOI IN MULTIACCESS CHANNELS

In this chapter, the system model under study is defined and a low-cost random access policy to minimize the average AoI of the system is proposed. Theoretical analyses are presented for three different access policies. Numerical evaluations are done for the 2-source case and the average AoI performances of the policies are compared.

3.1 System Model

We consider a network of M remote terminals and a central monitoring station. The terminals, referred to as the *sources*, continuously acquire and transmit their readings (position, temperature etc.) to the central monitor through a shared channel, with the aim of keeping the monitor as up to date as possible. The monitor, referred to as the *destination*, processes the readings, possibly for a certain situational awareness application. The monitor thus requires sufficiently timely status updates from each source.

A status update comprises a single data packet. The time is slotted and all nodes in the network are synchronized to the same slot timing. The total time it takes to transmit a packet is exactly one slot for each source. Sampling and processing times are assumed negligible.

The sources will send update packets to the destination according to a distributed random access strategy. At the beginning of a time slot, each source decides independently whether or not to transmit an update. Accordingly, packets are prone to collisions, that is, if two or more sources transmit at the same slot, then all pack-

ets are lost. An update by source i is successful if and only if it is the only source transmitting in the slot. There are no retransmission attempts for the lost packets. Whenever a source decides to transmit, it generates a new packet at the beginning of the time slot as in the "generate-at-will" model in [6].

The channel is assumed to be lossless, i.e. if no collision has occurred, then the probability of a packet being lost during transmission is negligible.

When a packet is generated at a source, the moment of observation is included as the timestamp. This way, the destination knows the freshness of the received information. To quantize information freshness, we use the Age of Information (AoI) metric, or simply the age. Let $\Delta_i(t)$ represent the age of source i at time t, then

$$\Delta_i(t) \triangleq t - u_i(t), \tag{3.1}$$

where $u_i(t)$ is the timestamp of the last received packet from source i at the destination. $\Delta_i(t)$ is thus defined as the time elapsed since the generation of the last received packet from source i, at time t.

It is worth emphasizing that, AoI is a metric relevant from the receiver's perspective. At the receiver side, the most recent information gets older as time progresses. Upon successful reception of a more recently generated information packet, most recent information is refreshed, i.e. AoI at the receiver drops down to the age of the last received packet.

Because the time is slotted and transmission decisions are taken at the beginning of each slot, we adopt a discrete-time model for age. $A_i[k]$ indicates the value of $\Delta_i(t)$ at the beginning of time slot k . Let ζ be the duration of one time slot, then

$$A_i[k] \triangleq \Delta_i(k\zeta)\ ,\ k \in \mathbb{Z}\ . \tag{3.2}$$

For simplicity, we consider $\zeta = 1$ in our analyses.

The sources in the network are unbuffered, i.e. if source i decides to transmit in a time slot, it immediately generates a new packet with a fresh timestamp before transmission. Knowing this, the evolution of $A_i[k]$ can be defined as

$$A_i[k+1] = \begin{cases} 1, & \text{source } i \text{ updates successfully at time slot } k, \\ A_i[k]+1, & \text{otherwise.} \end{cases} \tag{3.3}$$

$A_i[k]$ is incremented by one at each time slot between successful updates. Upon a successful update, it is reset to 1 because of the 1 slot transmission delay after the acquisition of the sample at the beginning of the slot.

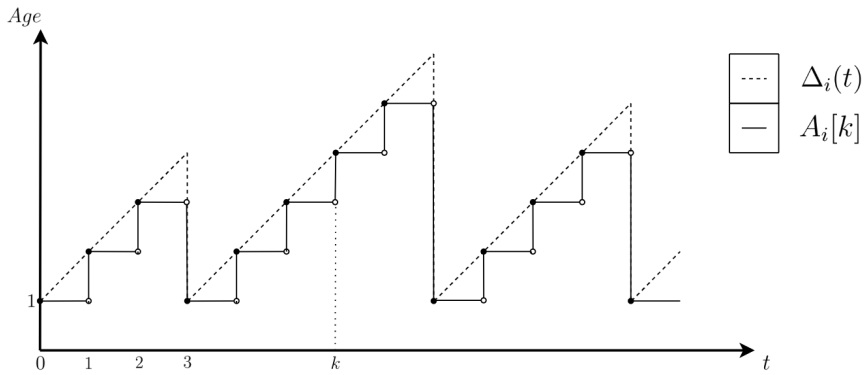

Figure 3.1: Sample evolution curves for $\Delta_i[k]$ and $A_i[k]$ in time.

Fig.3.1 shows the evolution of $\Delta_i(t)$ and $A_i[k]$ in time.

The quantity of interest for us is the time-average age of information (AAoI) of each source i, defined as

$$\Delta_i = \lim_{t' \to \infty} \frac{1}{t'} \int_0^{t'} \Delta_i(t) dt. \tag{3.4}$$

Then, the AAoI of the network can be found as

$$\Delta = \frac{1}{M} \sum_{i=1}^{M} \Delta_i, \tag{3.5}$$

where M is the number of sources in the network.

3.2 Derivation of AAoI

3.2.1 Lazy Policy

We define the Lazy Policy ("LAZY") as the following: each successful transmission by source i is followed by an idle period for i. Accordingly, the source makes its next transmission attempt, with probability τ, only after its age $A_i[k]$ exceeds a threshold

θ as

$$\tau_i[k] \triangleq \begin{cases} \tau, & A_i[k] \geq \theta \\ 0, & A_i[k] < \theta \end{cases}, \ \theta \in \mathbb{Z}^+, \ i \in \{1, ..., M\}, \ 0 < \tau < 1. \tag{3.6}$$

In LAZY, the sources are assumed to be aware whether their transmissions are successful or not through collision detection. If a collision occurs, all the transmitting sources are aware of it at the end of the slot.

In this work, we focus on the problem of finding the transmission rate τ to minimize the AAoI of the network Δ for a given (M, θ) pair as

$$\tau^* = \arg\min_{\tau} \ \Delta. \tag{3.7}$$

The AAoI expression stated in (3.4) can be rewritten in terms of $A_i[k]$ as

$$\begin{aligned} \Delta_i &= \lim_{K\to\infty} \frac{1}{K} \sum_{k=0}^{K-1} \left(A_i[k] + \frac{1}{2} \right) \\ &= \frac{1}{2} + \lim_{K\to\infty} \frac{1}{K} \sum_{k=0}^{K-1} A_i[k] \end{aligned} \tag{3.8}$$

where the summation term $\sum_{k=0}^{K-1} A_i[k]$ equals the total area under the $A_i[k]$ curve observed over K slots and the term $\frac{1}{2}$ is due to $\Delta_i(t)$ increasing continuously inside each slot.

Let the observation interval $[0, K]$ start and end at the time instants where an update by source i occurs. In this time interval, we decompose the area under $A_i[k]$ into rectangular areas $Q_i[k]$, as shown in Fig. 3.2. Then, we redefine the total area under the curve as their sum

$$\sum_{k=0}^{K-1} A_i[k] = \sum_{k=0}^{K-1} Q_i[k]. \tag{3.9}$$

Rectangular areas $Q_i[k]$ are of unit height, but differ in length. Let $X_i[k]$ be defined as the time left until the next update by source i, at the beginning of time slot k

$$X_i[k] \triangleq \min\{\tilde{k} \ : A_i[\tilde{k}] = 1, \ \tilde{k} > k, \ \tilde{k} \in \mathbb{Z}^+\} - k. \tag{3.10}$$

Then,

$$Q_i[k] = X_i[k]. \tag{3.11}$$

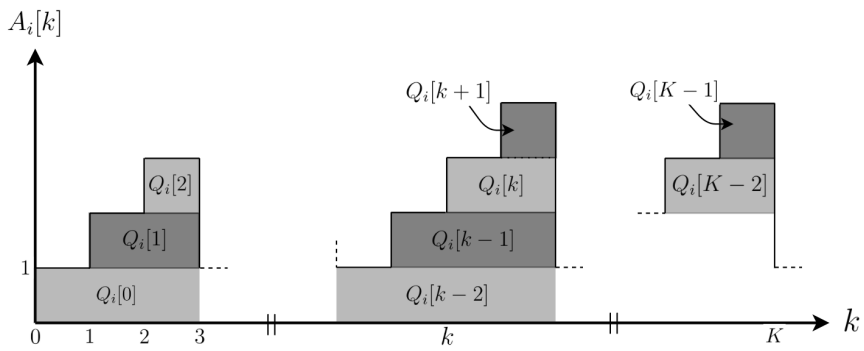

Figure 3.2: Decomposition of the area under the $A_i[k]$ curve.

Substituting (3.9) and (3.11) into (3.8),

$$\begin{aligned}\Delta_i &= \frac{1}{2} + \lim_{K\to\infty} \frac{1}{K}\left(\sum_{k=0}^{K-1} X_i[k]\right) \\ &= \frac{1}{2} + E\{X_i[k]\},\end{aligned} \tag{3.12}$$

where $E\{.\}$ is the expectation operator over k assuming $X_i[k]$ is ergodic.

Under LAZY, the sources transmit according to the rule stated in (3.6). A source can successfully update in a time slot, if and only if it is the only source transmitting and its packet is correctly received by the destination. We use $\gamma_i[k]$ to indicate the successful update probability of source i in time slot k under LAZY. Let

$$\mathbb{1}\left(x\right) \triangleq \begin{cases} 1, & x \text{ is true,} \\ 0, & x \text{ is false.} \end{cases} \tag{3.13}$$

Then,

$$\gamma_i[k] = \tau\, \mathbb{1}\left(A_i[k] \geq \theta\right) \prod_{j\neq i} [1 - \tau \mathbb{1}\left(A_j[k] \geq \theta\right)]. \tag{3.14}$$

The probability distribution of $X_i[k]$ is

$$P\{X_i[k] = x\} = \begin{cases} \gamma_i[k], & x = 1, \\ \gamma_i[k+x-1] \prod_{x'=0}^{x-2}(1-\gamma_i[k+x']), & x > 1,\ x \in \mathbb{Z}, \\ 0, & \text{otherwise.} \end{cases} \tag{3.15}$$

Notice that $X_i[k]$ are not identically distributed over time slots because of $\gamma_i[k]$'s dependence on the AoI of each source in the network. Therefore, obtaining $E\{X_i[k]\}$

is no trivial task. This problem is addressed by partitioning $X_i[k]$ into components that are identically distributed over time slots where the AoI state of the system is the same.

The AoI state of the system can be represented as the vector

$$\mathbf{A} \triangleq \langle\ A_1[k] \quad A_2[k] \quad ... \quad A_M[k]\ \rangle. \tag{3.16}$$

However, this representation constitutes an infinite state space and impractical for our calculations. Without loss of information on AAoI, the AoI state of the system can be defined such that the state space is finite, by exploiting the fact that the attempt probability of each source i is constant for all AoI above the threshold θ.

Let $\mathbf{A}^\theta[k]$ be the vector representing the AoI state of the network at time k,

$$\mathbf{A}^\theta[k] \triangleq \langle\ A_1^\theta[k] \quad A_2^\theta[k] \quad ... \quad A_M^\theta[k]\ \rangle, \tag{3.17}$$

where $A_i^\theta[k]$ is the AoI of source i truncated at θ, at time slot k, which evolves in time as

$$A_i^\theta[k+1] = \begin{cases} 1, & \text{source } i \text{ updates successfully at time slot } k, \\ \min\{A_i^\theta[k]+1,\ \ \theta\}, & \text{otherwise.} \end{cases} \tag{3.18}$$

Proposition 1. $\mathbf{A}^\theta[k]$ *is a Markov process under LAZY.*

Proof: As stated in (3.14), the successful update probability of source i at time slot k is

$$\begin{aligned} \gamma_i[k] &= p_i\tau_i\ \mathbb{1}\left(A_i[k] \geq \theta\right) \prod_{j\neq i}[1-\tau_j \mathbb{1}\left(A_j[k] \geq \theta\right)] \quad \forall i, \\ &= p_i\tau_i\ \mathbb{1}\left(A_i^\theta[k] = \theta\right) \prod_{j\neq i}[1-\tau_j \mathbb{1}\left(A_j^\theta[k] = \theta\right)] \quad \forall i, \end{aligned} \tag{3.19}$$

From the lack of memory terms in (3.19), it is clear that

$$\begin{aligned} &P\{\mathbf{A}^\theta[k+1] = \bar{a} \mid \mathbf{A}^\theta[k] = \bar{a}_k\} \\ &= P\{\mathbf{A}^\theta[k+1] = \bar{a} \mid \mathbf{A}^\theta[k] = \bar{a}_k,\ \mathbf{A}^\theta[k-1] = \bar{a}_{k-1},\ ...\ ,\ \mathbf{A}^\theta[0] = \bar{a}_0\}\ \ \forall i. \end{aligned} \tag{3.20}$$

□

The defined state space constitutes a Markov chain with θ^M possible states and it is indicated from here on as $\mathbb{A}_{M,\theta}$.

Proposition 2. *$\mathbb{A}_{M,\theta}$ has a unique steady-state distribution.*

Proof: The components of the AoI state vector $\mathbf{A}[k]$ evolve according to the rule stated in (3.18) as

$$A_i^\theta[k+1] = \begin{cases} 1, & \text{source i updates successfully,} \\ \min\{A_i^\theta[k]+1,\ \ \theta\}, & \text{otherwise.} \end{cases} \tag{3.21}$$

We define Θ as the AoI state in $\mathbb{A}_{M,\theta}$ such that $\Theta = \langle \theta\ \ \theta\ \ ...\ \ \theta \rangle$.

Let

$$u_k = \theta + 1 - \min_i A_i[k], \tag{3.22}$$

then,

$$\begin{aligned} P\{\mathbf{A}[k+u_k] = \Theta \mid \mathbf{A}[k] = \bar{a}\} &= \prod_{\tilde{k}=0}^{u_k-1} \left(1 - \sum_{i=1}^{M} \gamma_i[k+\tilde{k}]\right), \quad \forall \bar{a} \in \mathbb{A}_{M,\theta} \\ &\triangleq \alpha[k], \end{aligned} \tag{3.23}$$

where $\gamma_i[k]$ is the successful update probability of source i at time slot k as defined in (3.14).

Simply stated, (3.23) implies that regardless of which state the MC is at in time slot k, the probability of reaching state Θ at time slot $k+u_k$ is equal to $\alpha[k]$, which is the probability that no successful update occurs for u_k consecutive time slots.

Since we know that

$$\begin{aligned} 1 \leq u_k \leq \theta \\ 0 \leq \gamma_i[k] < 1 \end{aligned} \quad \forall i \in \{1, ..., M\},\ k \in \mathbb{Z},\ \ 0 < \tau < 1, \tag{3.24}$$

we can see that

$$0 < \alpha[k] \leq 1, \quad \forall k \in \mathbb{Z}. \tag{3.25}$$

This result shows that starting from any state $\bar{a} \in \mathbb{A}_{M,\theta}$, there is a non-zero probability of reaching state Θ, i.e. state Θ is accessible from all states in $\mathbb{A}_{M,\theta}$. This indicates

that there is exactly one recurrent class in $\mathbb{A}_{M,\theta}$ and it contains state Θ. Let $P_{\Theta\Theta}$ represent the transition probability from state Θ to itself, then

$$P_{\Theta\Theta} = \tau \prod_{i=1}^{M-1} (1-\tau) > 0, \quad 0 < \tau < 1. \tag{3.26}$$

Inequalities obtained in (3.25) and (3.26) prove that $\mathbb{A}_{M,\theta}$ contains a single recurrent class that is also aperiodic. Hence, we conclude that $\mathbb{A}_{M,\theta}$ has a unique steady-state distribution [25].

□

From law of total expectation, we reformulate

$$E\{X_i[k]\} = \sum_{\bar{a} \in \mathbb{A}^{\theta}_{(M,\theta)}} E\{X_i[k] \mid \mathbf{A}[k] = \bar{a}\} P\{\mathbf{A}[k] = \bar{a}\}. \tag{3.27}$$

Since there exists a unique steady-state distribution, we can apply steady-state analysis. We define the expected time until next update by source i at a given AoI state $\bar{a}$

$$Y_i(\bar{a}) \triangleq E\{X_i[k] \mid \mathbf{A}[k] = \bar{a}\}. \tag{3.28}$$

In our Markov chain, an update by source i is represented as a transition from any state to a state where the AoI of source i is equal to 1. Let

$$\bar{a} \triangleq \langle a_1 \;\; a_2 \;\; ... \;\; a_M \rangle, \tag{3.29}$$

where a_i represents the AoI of source i. The set of update states of source i is then defined as

$$\mathbb{A}_{up}(i) \triangleq \{\bar{a} : a_i = 1, \; \bar{a} \in \mathbb{A}_{M,\theta}\}. \tag{3.30}$$

$Y_i(\bar{a})$ is the mean first passage time through a state in the set $\mathbb{A}_{up}(i)$ starting from state $\bar{a}$. Let $P_{\bar{a}\bar{a}'}$ indicate the transition probability from state $\bar{a}$ to state $\bar{a}'$, the mean first passage times starting from each state $\bar{a}$ can be obtained by using one-step analysis i.e. solving the system of equations

$$Y_i(\bar{a}) = \begin{cases} 0, & \bar{a} \in \mathbb{A}_{up}(i), \\ 1 + \sum_{\bar{a}' \in \mathbb{A}^{\theta}_{(M,\theta)}} P_{\bar{a}\bar{a}'} Y_i(\bar{a}'), & \text{else}, \end{cases} \tag{3.31}$$

for $Y_i(\bar{a})$, where $\bar{a} \notin \mathbb{A}_{up}(i)$.

Then, we use the solution of this system to obtain the mean first return times to the update set for the states in the update set $\mathbb{A}_{up}(i)$ as

$$Y_i(\bar{a}) = 1 + \sum_{\bar{a}' \in \mathbb{A}^{\theta}_{(M,\theta)}} P_{\bar{a}\bar{a}'} Y_i(\bar{a}'), \quad \bar{a} \in \mathbb{A}_{up}(i). \tag{3.32}$$

The next step in obtaining $E\{X_i[k]\}$ is finding the probability distribution $P\{\mathbf{A}[k]\}$ over all time slots k, which is equal to the steady-state distribution of the MC as

$$\pi(\bar{a}) = P\{\mathbf{A}[k] = \bar{a}\}. \tag{3.33}$$

The steady-state distribution can be obtained by solving the system of equations

$$\begin{aligned} \pi(\bar{a}) &= \sum_{\bar{a}' \in \mathbb{A}_{M,\theta}} \pi(\bar{a}') P_{\bar{a}'\bar{a}}, \qquad \forall \bar{a} \in \mathbb{A}_{M,\theta}, \\ \sum_{\bar{a} \in \mathbb{A}_{M,\theta}} \pi(\bar{a}) &= 1. \end{aligned} \tag{3.34}$$

Combining the solutions of (3.31), (3.32), and (3.34) and then substituting them into (3.27), we get

$$E\{X_i[k]\} = \sum_{\bar{a} \in \mathbb{A}^{\theta}_{(M,\theta)}} Y_i(\bar{a}) \pi(\bar{a}), \tag{3.35}$$

which in turn is substituted into (3.12) to obtain

$$\Delta_i = \frac{1}{2} + \sum_{\bar{a} \in \mathbb{A}^{\theta}_{(M,\theta)}} Y_i(\bar{a}) \pi(\bar{a}). \tag{3.36}$$

For the system model under study, the resulting AAoI expression is a function of attempt rate τ and it is unique for each (M, θ) pair. Obtaining a general closed-form solution for AAoI as a function of all three parameters is difficult and not provided in this work, however empirical findings regarding the effects of (M, θ) on AAoI performance are presented in Chapter 4. Also note that for large M or θ values, the proposed method AAoI derivation method suffers greatly in terms of computational complexity, due to dimensionality.

In the following subsection, AAoI performance of Slotted ALOHA is analyzed using the proposed method.

3.2.2 A Special Case: Slotted ALOHA

Recall from (3.6) that the transmission rule in LAZY is

$$\tau_i[k] \triangleq \begin{cases} \tau, & A_i[k] \geq \theta \\ 0, & A_i[k] < \theta \end{cases}, \quad \theta \in \mathbb{Z}^+ \ \ \forall i. \tag{3.37}$$

Slotted ALOHA, in which sources transmit with a fixed probability at every time slot, can be considered a special case of LAZY with $\theta = 1$. For our network model, we had derived the AAoI of a single source i, in (3.12) as

$$\Delta_i = \frac{1}{2} + E\{X_i[k]\}. \tag{3.38}$$

where $X_i[k]$ is the time left until the next update by source i at time slot k.

Because all sources in the network transmit with a fixed probability at every slot, the successful update probability of source i in each time slot is

$$\gamma_{sa,i}[k] = \gamma_{sa} = \tau(1-\tau)^{M-1}, \quad \forall k, i \tag{3.39}$$

where M is the number of sources in the network.

Substituting γ_{sa} into (3.15), we can rewrite the probability distribution of $X_i[k]$

$$P\{X_i[k] = x\} = \begin{cases} \gamma_{sa}(1-\gamma_{sa})^{x-1}, & x \in \mathbb{Z}^+, \\ 0, & \text{otherwise.} \end{cases} \tag{3.40}$$

We see in (3.40) that $X_i[k]$ are geometrically distributed and i.i.d. for all sources i and time slots k. Thus

$$E\{X_i[k]\} = \frac{1}{\gamma_{sa}}. \tag{3.41}$$

The AAoI is then obtained as

$$\Delta_i = \frac{1}{2} + E\{X_i[k]\} = \frac{1}{2} + \frac{1}{\gamma_{sa}}. \tag{3.42}$$

The result is consistent with (2.15) which is the AAoI expression found in [2] for a network operating under Slotted ALOHA.

3.2.3 Ideal Case: Round-Robin

By definition, the network model under study has a channel capacity of 1 packet per slot. This capacity can only be achieved when the channel is fully utilized i.e. a successful update occurs in every time slot. However, this is not possible under random access constraints because channel access decisions are taken independently by sources, therefore, collisions are bound to occur.

In this subsection, we consider an ideal network of M sources in which the sources are coordinated by a centralized scheduler. The scheduling is done in a Round-Robin fashion, where the sources take turns in updating the monitor. Each source transmits in its dedicated time slot and idles for exactly $M-1$ slots for other sources to transmit. Since every source knows exactly when to transmit, there are no collisions or unnecessary idling periods. A successful update occurs in each time slot.

Proposition 3. *Round-Robin policy is AAoI-optimal.*

Proof: Consider a network of M sources that transmit according to the decision taken by a centralized scheduler at the beginning of each time slot. The scheduler knows the AoI of each source at all times. Let $A_i[k]$ be the AoI of source i at the beginning of time slot k.

We define the AoI averaged over all sources in time slot k as

$$A[k] = \frac{1}{M}\sum_{i=1}^{M} A_i[k] \tag{3.43}$$

Then, the AAoI of the entire network observed over the time interval $[0, K]$ is

$$\Delta = \frac{1}{2} + \frac{1}{K}\sum_{k=0}^{K-1} A[k] \tag{3.44}$$

The scheduler has $M+1$ possible actions at any time slot k, that is, to either schedule one of the M sources or schedule none. Let the event indicating that source i is scheduled in time slot k be defined as

$$\mathbb{S}_i[k] \triangleq \begin{cases} 1, & \text{source } i \text{ transmits at time slot } k, \\ 0, & \text{else.} \end{cases} \tag{3.45}$$

According to the scheduling decision at time k, the AoIs are going to evolve as

$$A_i[k+1] = A_i[k] - A_i[k]\mathbb{S}_i[k] + 1, \quad \forall i, \tag{3.46}$$

hence

$$\begin{aligned} A[k+1] &= \frac{1}{M}\sum_{i=1}^{M} A_i[k+1] \\ &= \frac{1}{M}\sum_{i=1}^{M} (A_i[k] - A_i[k]\mathbb{S}_i[k] + 1) \\ &= A[k] + 1 - \frac{1}{M}\sum_{i=1}^{M} A_i[k]\mathbb{S}_i[k]. \end{aligned} \tag{3.47}$$

Moreover, at any time slot k' in the interval $[0, K]$,

$$A[k'] = A[0] + k' - \frac{1}{M}\sum_{k=0}^{k'}\sum_{i=1}^{M} A_i[k]\mathbb{S}_i[k]. \tag{3.48}$$

Then the AAoI over $[0, K]$ can be expressed as

$$\begin{aligned} \Delta &= \frac{1}{2} + \frac{1}{K}\sum_{k'=0}^{K-1} A[k'] \\ &= \frac{1}{2} + \frac{1}{K}\sum_{k'=0}^{K-1}\left(A[0] + k' - \frac{1}{M}\sum_{k=0}^{k'}\sum_{i=1}^{M} A_i[k]\mathbb{S}_i[k]\right) \\ &= \frac{1}{2} + A[0] + \frac{1}{K}\sum_{k'=0}^{K-1} k' - \frac{1}{MK}\sum_{k'=0}^{K-1}\sum_{k=0}^{k'}\sum_{i=1}^{M} A_i[k]\mathbb{S}_i[k]. \end{aligned} \tag{3.49}$$

Let

$$\beta[k] = \sum_{i=1}^{M} A_i[k]\mathbb{S}_i[k]. \tag{3.50}$$

Since all other terms are constants, in order to minimize AAoI, we need to maximize

$$\sum_{k'=0}^{K-1}\sum_{k=0}^{k'} \beta[k] = K\beta[0] + (K-1)\beta[1] + ... + \beta[K-1]. \tag{3.51}$$

We see in (3.51) that as time progresses the weight of $\beta[k]$ decreases linearly, therefore at each time slot k, the scheduler must try to maximize the corresponding $\beta[k]$. Hence, the optimal scheduling decision in each slot k is

$$\mathbb{S}_i^*[k] = \begin{cases} 1, & i = \min\{j : A_j[k] \geq A_l[k], \ \forall\, l \neq j\}, \\ 0, & \text{else.} \end{cases} \tag{3.52}$$

Minimum AAoI is achieved when the source with the highest AoI is scheduled at each time slot. If more than one source has the highest AoI, any one of the sources can be prioritized. For simplicity of exposition, we set the decision rule in (3.52) such that the source with the smallest index is prioritized. Regardless of the initial AoIs, after at most M time slots, the same sequence of sources are going to be scheduled, with each source transmitting once every M time slots, which is, by definition, the Round-Robin policy.

□

Recall from (2.11) that AAoI is

$$\Delta_i = \frac{E[Z_i^2]}{2E[Z_i]} + 1, \tag{3.53}$$

where Z_i is the time between updates of source i. The time between updates under Round-Robin is deterministic and equal to M, the number of sources in the network.

The AAoI under Round-Robin is then

$$\begin{aligned}
\Delta_{RR} &= \frac{1}{M}\sum_{i=1}^{M} \Delta_{RR,i} \\
&= \frac{1}{M}\sum_{i=1}^{M} \left(\frac{E[Z_i^2]}{2E[Z_i]} + 1\right) \\
&= 1 + \frac{1}{M}\sum_{i=1}^{M} \frac{M^2}{2M} \\
&= 1 + \frac{M}{2}.
\end{aligned} \tag{3.54}$$

Although Round-Robin is not a random access policy, it provides an AAoI lower bound due to its universal optimality. Its performance can therefore be used as a benchmark for any policy in the studied network model.

3.3 Throughput Considerations in LAZY

Let $N_i(K)$ be the number of successful updates by source i in the time interval $[0, K]$. We know from (3.18) that upon a successful update by source i, $A_i[k]$ resets to 1 in

the next slot. Hence, $N_i(K)$ can be stated as

$$N_i(K) = \sum_{k=1}^{K+1} \mathbb{1}(A_i^{\theta}[k] = 1), \tag{3.55}$$

where $\mathbb{1}(.)$ is the event indicator function defined in (3.13) as

$$\mathbb{1}\left(x\right) \triangleq \begin{cases} 1, & x \text{ is true,} \\ 0, & x \text{ is false.} \end{cases} \tag{3.56}$$

The throughput of source i is

$$\begin{aligned} R_i &\triangleq \lim_{K\to\infty} \frac{1}{K} N_i(K) \\ &= \lim_{K\to\infty} \frac{1}{K} \sum_{k=1}^{K+1} \mathbb{1}(A_i[k] = 1) \\ &= P\{A_i^{\theta}[k] = 1\} \\ &= \sum_{\bar{a}\in\mathbb{A}_{up}(i)} \pi(\bar{a}), \end{aligned} \tag{3.57}$$

where $\mathbb{A}_{up}(i)$ is the set of update states of source i defined in (3.30) as

$$\mathbb{A}_{up}(i) = \{\bar{a} : a_i = 1,\ \bar{a} \in \mathbb{A}_{M,\theta}\}. \tag{3.58}$$

Let

$$\mathbb{A}_{up}(i,j) \triangleq \mathbb{A}_{up}\left(i\right) \cap \mathbb{A}_{up}\left(j\right),\ \ \forall i,j. \tag{3.59}$$

Then,

$$P_{\bar{a}\bar{a}'} = 0, \qquad \forall \bar{a} \in \mathbb{A}_{M,\theta},\ \ \bar{a}' \in \mathbb{A}_{up}(i,j),\ i \neq j, \tag{3.60}$$

where $P_{\bar{a}\bar{a}'}$ is the transition probability from state $\bar{a}$ to $\bar{a}'$. This is a statement of the fact that no two sources can update at the same time slot. Hence,

$$\pi(\bar{a}') = \sum_{\bar{a}\in\mathbb{A}_{M,\theta}} \pi(\bar{a}) P_{\bar{a}\bar{a}'} = 0, \qquad \forall \bar{a}' \in \mathbb{A}_{up}(i,j),\ i \neq j. \tag{3.61}$$

Thus, the total throughput of the network consisting of M sources can be obtained as

$$R = \sum_{i=1}^{M} R_i = \sum_{i=1}^{M} \sum_{\bar{a}\in\mathbb{A}_{up}(i)} \pi(\bar{a}). \tag{3.62}$$

3.4 2-Source Network Performance Analysis

In this section, we obtain the closed-form AAoI expressions for the $M = 2$ source case under LAZY with $\theta = 2$ and find the AAoI-optimal rate τ that minimizes its AAoI. We also provide the AAoI expressions and AAoI-optimal rates for $\theta = 3, 4$ and present a performance comparison with Slotted ALOHA and Round-Robin policies.

We define AoI state vector in the $(M, \theta) = (2, 2)$ case as

$$\mathbf{A}^2[k] = \langle\ A_1^2[k] \quad A_2^2[k]\ \rangle, \tag{3.63}$$

where

$$A_i^2[k] = \begin{cases} 1, & A_i[k] < 2, \\ 2, & A_i[k] \geq 2. \end{cases} \tag{3.64}$$

Then the state space becomes

$$\mathbb{A}_{2,2} = \{\langle 1\ 1\rangle, \langle 1\ 2\rangle, \langle 2\ 1\rangle, \langle 2\ 2\rangle\}. \tag{3.65}$$

The transition probability matrix is

$$\mathcal{P} = \begin{array}{c} \\ \langle 1\ 1\rangle \\ \langle 1\ 2\rangle \\ \langle 2\ 1\rangle \\ \langle 2\ 2\rangle \end{array} \begin{array}{c} \begin{array}{cccc} \langle 1\ 1\rangle & \langle 1\ 2\rangle & \langle 2\ 1\rangle & \langle 2\ 2\rangle \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & \tau & 1-\tau \\ 0 & \tau & 0 & 1-\tau \\ 0 & \tau(1-\tau) & \tau(1-\tau) & 1-2\tau(1-\tau) \end{bmatrix} \end{array} \tag{3.66}$$

The state transition diagram for the Markov chain is shown in Fig.3.3.

The set of update states of source 1 is

$$\mathbb{A}_{up}(1) = \{\langle 1\ 1\rangle, \langle 1\ 2\rangle\}. \tag{3.67}$$

As in (3.31), we first find the mean first passage times, $Y_1\left(\langle 2\ 1\rangle\right)$ and $Y_1\left(\langle 2\ 2\rangle\right)$ by

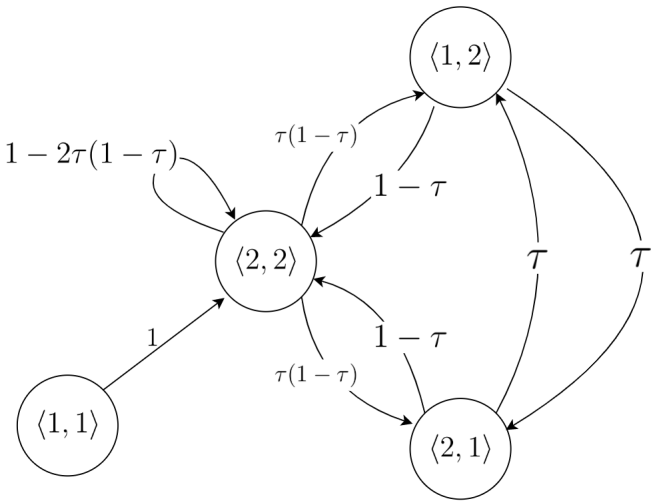

Figure 3.3: AoI State Transition Diagram under LAZY for $M = 2$ and $\theta = 2$.

solving

$$
\begin{aligned}
&Y_1\left(\langle 1\ 1\rangle\right) = 0,\\
&Y_1\left(\langle 1\ 2\rangle\right) = 0,\\
&Y_1\left(\langle 2\ 1\rangle\right) = 1 + \tau Y_1\left(\langle 1\ 2\rangle\right) + (1-\tau) Y_1\left(\langle 2\ 2\rangle\right),\\
&Y_1\left(\langle 2\ 2\rangle\right) = 1 + \tau(1-\tau) Y_1\left(\langle 1\ 2\rangle\right) + \tau(1-\tau) Y_1\left(\langle 2\ 1\rangle\right) + 1 - 2\tau(1-\tau) Y_1\left(\langle 2\ 2\rangle\right).
\end{aligned}
\tag{3.68}
$$

Then, we find the mean first return times to the update set, $\{Y_1\left(\langle 1\ 1\rangle\right), Y_1\left(\langle 1\ 2\rangle\right)\}$ by substituting the solutions of (3.68) into

$$
\begin{aligned}
&Y_1\left(\langle 1\ 1\rangle\right) = 1 + Y_1\left(\langle 2\ 2\rangle\right),\\
&Y_1\left(\langle 1\ 2\rangle\right) = 1 + \tau Y_1\left(\langle 2\ 1\rangle\right) + (1-\tau) Y_1\left(\langle 2\ 2\rangle\right),
\end{aligned}
\tag{3.69}
$$

yielding

$$
\begin{aligned}
&Y_1\left(\langle 1\ 1\rangle\right) = 1 + \frac{1+\tau-\tau^2}{\tau(1-\tau^2)},\\
&Y_1\left(\langle 1\ 2\rangle\right) = 2 + \frac{1}{\tau},\\
&Y_1\left(\langle 2\ 1\rangle\right) = \frac{1+2\tau}{\tau(1+\tau)},\\
&Y_1\left(\langle 2\ 2\rangle\right) = \frac{1+\tau-\tau^2}{\tau(1-\tau^2)}.
\end{aligned}
\tag{3.70}
$$

The steady-state distribution $\pi = [\pi\left(\langle 1\ 1\rangle\right)\ \ \pi\left(\langle 1\ 2\rangle\right)\ \ \pi\left(\langle 2\ 1\rangle\right)\ \ \pi\left(\langle 2\ 2\rangle\right)]$ is found

by solving the matrix equation

$$\pi \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & \tau & 1-\tau \\ 0 & \tau & 0 & 1-\tau \\ 0 & \tau(1-\tau) & \tau(1-\tau) & 1-2\tau(1-\tau) \end{bmatrix} = \pi, \tag{3.71}$$

yielding

$$\pi = \begin{bmatrix} 0 & \frac{\tau}{1+2\tau} & \frac{\tau}{1+2\tau} & \frac{1}{1+2\tau} \end{bmatrix}. \tag{3.72}$$

The AAoI of source 1 for $M = 2$ and $\theta = 2$ case is

$$\begin{aligned} \Delta_1 &= \frac{1}{2} + E\{X_1[k]\} \\ &= \frac{1}{2} + \sum_{\bar{a}\in\mathbb{A}_{2,2}} E\{X_1[k] \mid A_1^2[k] = \bar{a}\} P\{A_1^2[k] = \bar{a}\} \\ &= \frac{1}{2} + \sum_{\bar{a}\in\mathbb{A}_{2,2}} Y_1(\bar{a})\pi(\bar{a}) \\ &= \frac{1}{2} + \frac{2+7\tau+6\tau^2-7\tau^3-6\tau^4}{2\tau\left(1+2\tau-\tau^2-2\tau^3\right)} \\ &= \frac{3}{2} + \frac{1+2\tau-2\tau^3}{\tau\left(1+2\tau-\tau^2-2\tau^3\right)}. \end{aligned} \tag{3.73}$$

Since the parameters M, θ and τ are identical for all sources in the network, AAoI of both sources are equal. Therefore the network AAoI

$$\begin{aligned} \Delta &= (\Delta_1 + \Delta_2)/2 \\ &= \Delta_1 \\ &= \frac{3}{2} + \frac{1+2\tau-2\tau^3}{\tau(1+2\tau-\tau^2-2\tau^3)}. \end{aligned} \tag{3.74}$$

The optimal attempt rate τ^* to minimize Δ for $(M, \theta) = (2, 2)$ is

$$\tau^* = \arg\min_{\tau} \Delta \approx 0.707. \tag{3.75}$$

The details of optimization are included in Appendix A.

Optimal AAoI and AAoI-minimizing attempt rates can be found using this method for any (M, θ) pair. Table 3.1 shows the closed-form AAoI expressions and optimal attempt rates under LAZY for $\theta = 1, 2, 3, 4$.

Table 3.1: AAoI expressions and optimal attempt rates for $\theta = 1, 2, 3, 4$ under LAZY.

AoI Threshold θ	Network AAoI Δ	Optimal Attempt Rate τ^*	Optimal AAoI Δ^*
1	$\frac{1}{2} + \frac{1}{\tau(1-\tau)}$	0.5	4.5
2	$\frac{3}{2} + \frac{1+2\tau-2\tau^3}{\tau(1+2\tau-\tau^2-2\tau^3)}$	0.707	3.5
3	$\frac{3}{2} + \frac{1+5\tau+5\tau^2-7\tau^3-4\tau^4+\tau^6}{\tau(1+5\tau+3\tau^2-9\tau^3-2\tau^4+2\tau^5)}$	0.783	3.383
4	$\frac{3}{2} + \frac{2+17\tau+40\tau^2-4\tau^3-58\tau^4-25\tau^5+48\tau^6-18\tau^7}{2\tau(1+8\tau+12\tau^2-20\tau^3-13\tau^4+18\tau^5-6\tau^6)}$	0.823	3.601

Fig.3.4 shows the AAoI vs attempt rate graphs for a $M = 2$ source network under LAZY ($\theta = 1, 2, 3, 4$) compared to the Round-Robin policy, which is AAoI-optimal. The dashed lines in the figure indicate the AAoI of the network under Round-Robin policy that is $\Delta = 1 + \frac{M}{2} = 2$. The optimal AAoI performance of Slotted ALOHA

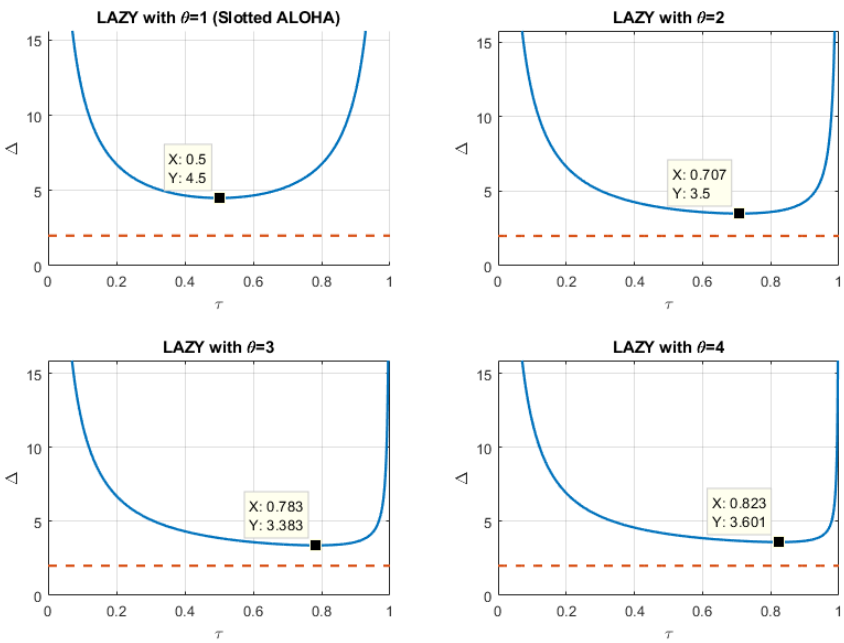

Figure 3.4: AAoI vs Attempt Rate for $M = 2$ under LAZY ($\theta = 1, 2, 3, 4$).

is worse than Round-Robin by a factor of

$$\frac{\Delta^*_{SA}}{\Delta_{RR}} = \frac{4.5}{2} = 2.25, \tag{3.76}$$

whereas for the optimal performance of LAZY ($\theta = 3$ and $\tau = 0.783$), this factor is

$$\frac{\Delta^*_{LAZY}}{\Delta_{RR}} = \frac{3.383}{2} = 1.6915. \tag{3.77}$$

The results are further discussed in Chapter 4.

CHAPTER 4

SIMULATION RESULTS AND DISCUSSIONS

In this chapter, simulation results are presented for the studied network model operating under LAZY. The simulations are performed using the Monte Carlo method and run for 10^6 time slots for any given parameter set. There are 3 parameters that affect the AAoI performance in LAZY:

- Attempt rate (or probability) τ,
- AoI threshold θ,
- Number of sources in the network, M.

The effect of each system parameter on AAoI performance is discussed in a seperate section. Please note that the discussions in this chapter regarding the effects of θ and M are mostly empirical, and ongoing research subjects.

4.1 Attempt Rate

For the $M = 2$ source network operating under LAZY ($\theta = 1, 2, 3, 4$), the simulation results representing how AAoI and throughput behave with respect to the attempt rate are given in Fig.4.1 and Fig.4.2 respectively. The simulation results are consistent with our findings previously shown in Fig.3.4.

The Δ vs. τ curves appear to be convex as expected. Decreasing the attempt rate too much causes the sources to be idle for excessive periods of time, which increases the AAoI. On the other hand, increasing the attempt rate has more complex effects on the AAoI. Although the throughput increases monotonically with τ, the AAoI

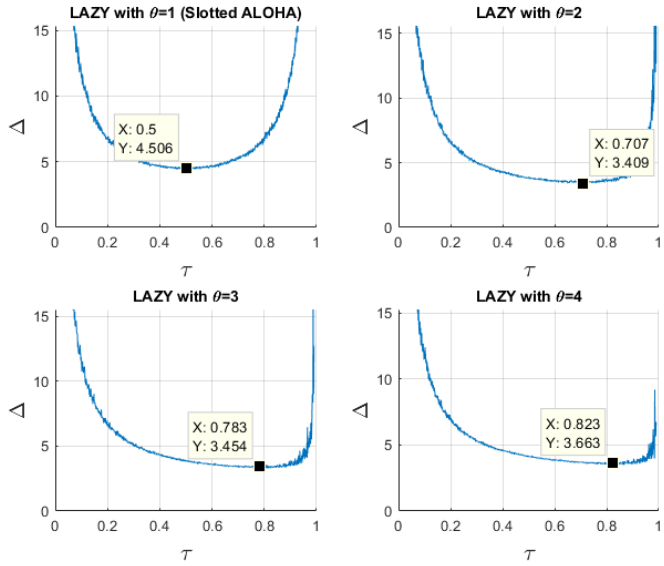

Figure 4.1: AAoI vs. Attempt Rate simulated for $M = 2$ under LAZY ($\theta = 1, 2, 3, 4$).

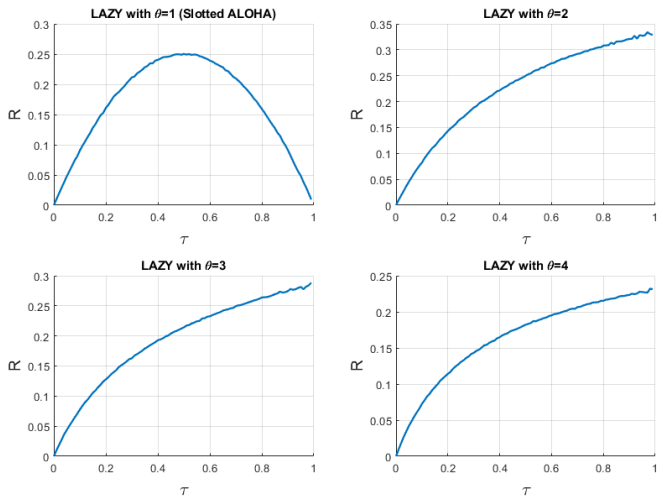

Figure 4.2: Throughput vs. Attempt Rate simulated for $M = 2$ under LAZY ($\theta = 1, 2, 3, 4$).

performance degrades for very high values of τ due to the burstiness of the updates. The network alternates between long durations of updating without interference and being locked in contention. This is not good for AAoI because of its dependence on the second moment of the inter-update time, i.e. the longer inter-update times have a greater weight on the average compared to the shorter inter-update times.

Fig.4.3 and Fig.4.4 illustrate the simulation results of AAoI and throughput vs. τ

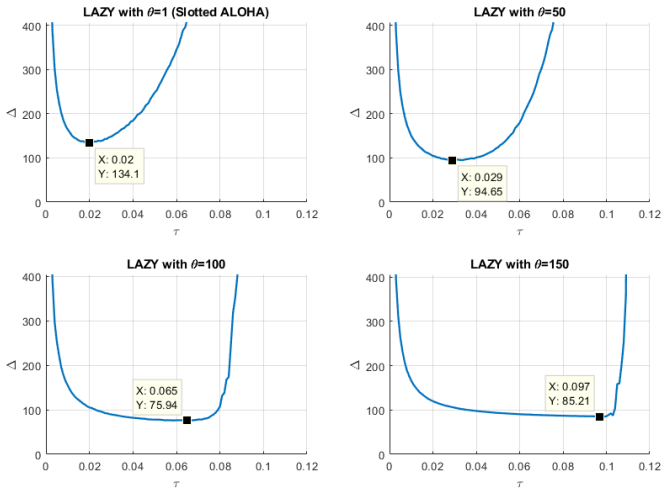

Figure 4.3: AAoI vs. Attempt Rate simulated for $M = 50$ under LAZY ($\theta = 1, 50, 100, 150$).

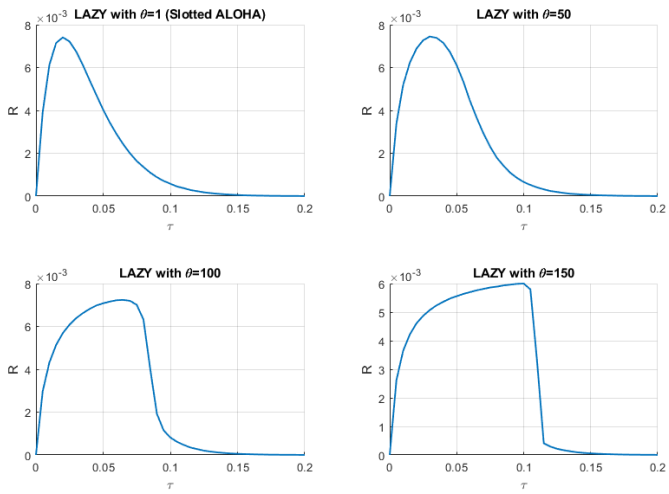

Figure 4.4: Throughput vs. Attempt Rate simulated for $M = 50$ under LAZY ($\theta = 1, 50, 100, 150$).

curves respectively, for $M = 50$ under LAZY ($\theta = 1, 50, 100, 150$). The AAoI behavior is similar to the $M = 2$ case, in the sense that for higher threshold values θ, the optimal attempt rates are higher. However, note that the increase in AAoI for excessively high τ are not caused by burstiness in this case. It is caused by the drop in throughput. Unlike the $M = 2$ case, the throughput is not monotonically increasing because the effect of interference is dominant, i.e. it is much more difficult for one of the sources to break free of a possible contention. One of the key points to remark

here is that LAZY does not provide an increase in throughput (in fact reduces it), but it significantly reduces AAoI due to prioritizing the transmission sources with higher AoI.

4.2 AoI Threshold

Fig.4.5 illustrates how the choice of θ affects the optimal AAoI value Δ^* for a network of $M = 50$ sources. The optimal AAoI value for each θ is found through a brute force method where the simulation is run for $0 < \tau < 1$ in 10^{-3} increments.

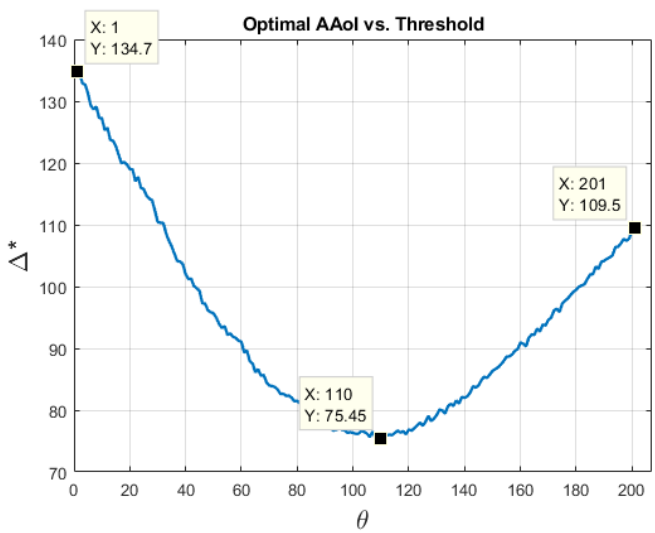

Figure 4.5: Optimal AAoI vs. AoI threshold simulated for $M = 50$ under LAZY.

We see in Fig.4.5 that there exists an AAoI-minimizing choice for θ at roughly $\theta = 110$ for $M = 50$. Increasing θ from 1 up to the AAoI-optimal point improves the AAoI performance because the average number of sources contending for the channel (sources with an AoI above θ) decreases as seen in the probability distribution graph in Fig.4.6.

When θ is increased further beyond the optimal point, the effect of unnecessary idling starts to be dominant which leads to a linear increase in AAoI with respect to θ.

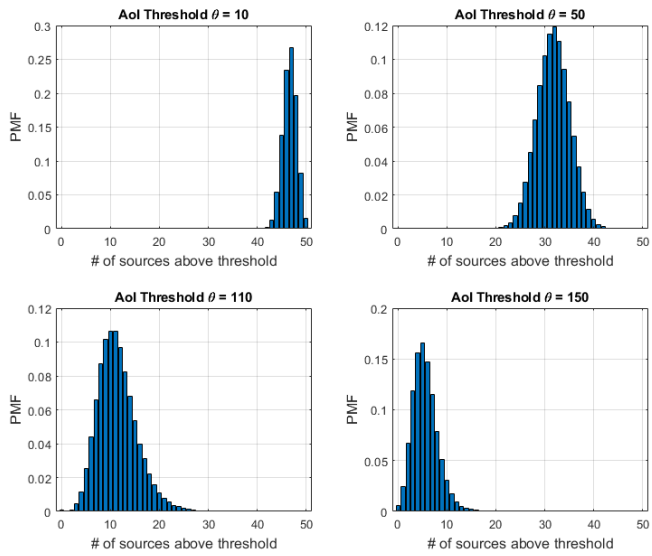

Figure 4.6: Probability mass function of the number of sources with an AoI above the threshold simulated for $M = 50$ under LAZY ($\theta = 1, 50, 110, 150$).

4.3 Number of Sources

In Fig.4.7, optimal AAoI performances under three different channel access policies are presented. The optimal AAoI value under LAZY for each M is found through a brute force method where the simulation is run for $0 < \tau < 1$ in 10^{-3} increments, for each θ starting from $\theta = 1$ until a minimum is detected.

The simulation results show that for $M > 10$, under each policy, optimal AAoI Δ^* increases almost linearly with M in $10 < M < 100$ as

$$\begin{aligned} \Delta^*_{SA} &\approx 2.6M, \\ \Delta^*_{LAZY} &\approx 1.4M, \end{aligned} \tag{4.1}$$

whereas from (3.54), we know that

$$\Delta^*_{RR} = \frac{M}{2} + 1. \tag{4.2}$$

Fig.4.7 shows the AAoI-optimizing thresholds θ^* for different M under LAZY. We observe that, similar to Δ^*_{LAZY}, θ^* increases almost linearly with M as

$$\theta^* \approx 2.2M \ . \tag{4.3}$$

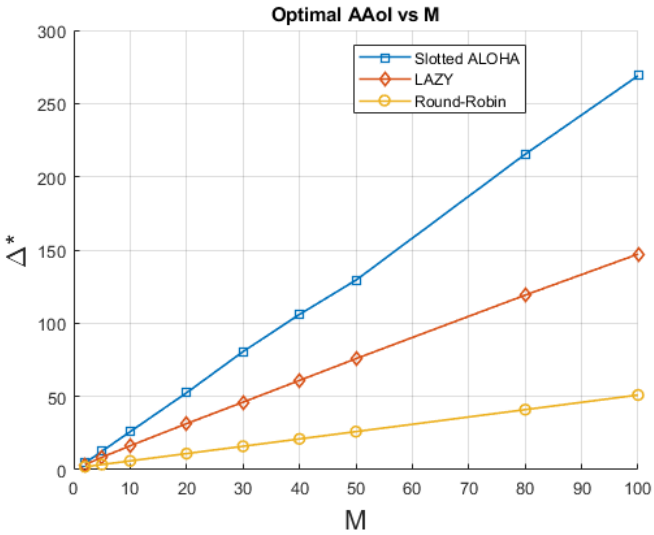

Figure 4.7: Optimal AAoI vs. Number of Sources in the Network under Slotted ALOHA, LAZY and Round-Robin.

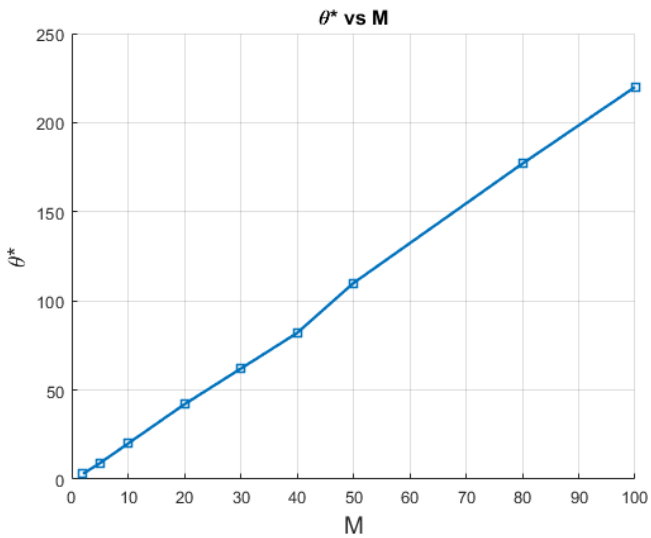

Figure 4.8: Optimal AoI Threshold vs. Number of Sources in the Network under LAZY.

CHAPTER 5

CONCLUSIONS

In this thesis, a network of multiple remote terminals sending status updates to a single receiver over a random access channel is studied. The channel is assumed noiseless. A random access strategy called LAZY is proposed to reduce the average age of information (AAoI), which is a newly proposed network performance metric quantizing the timeliness of the updates continuously sent by each source in the network. An alternative method for the derivation of AAoI is explained and the closed form AAoI expressions for a 2-source network operating under LAZY are found for different threshold parameters. The AAoI performance of LAZY is compared with the classical Slotted ALOHA and the AAoI optimal Round-Robin policy. It is shown that for the 2-source network case, Slotted ALOHA achieves an AAoI performance worse than Round-Robin by a factor 2.25, whereas for LAZY this factor is ≈ 1.7.

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APPENDIX A

OPTIMIZATION OF AAOI FOR THE 2-SOURCE NETWORK

Average AAoI expression under LAZY for $(M, \theta) = (2, 2)$ is obtained in (3.74) as

$$\Delta = \frac{3}{2} + \frac{1 + 2\tau - 2\tau^3}{\tau(1 + 2\tau - \tau^2 - 2\tau^3)}.$$

The aim is to find optimal τ^* such that

$$\begin{aligned}
\tau^* &= \arg\min_{0<\tau<1} \Delta \\
&= \arg\min_{0<\tau<1} \left(\frac{3}{2} + \frac{1 + 2\tau - 2\tau^3}{\tau(1 + 2\tau - \tau^2 - 2\tau^3)} \right) \\
&= \frac{3}{2} + \arg\min_{0<\tau<1} \left(\frac{1 + 2\tau - 2\tau^3}{\tau(1 + 2\tau - \tau^2 - 2\tau^3)} \right) \\
&\triangleq \frac{3}{2} + \arg\min_{0<\tau<1} C(\tau).
\end{aligned} \tag{A.1}$$

The necessary conditions to satisfy for the global minimum of $C(\tau)$ are

$$\begin{aligned}
C'(\tau) &= \frac{dC(\tau)}{d\tau} = 0, \\
C''(\tau) &= \frac{d^2C(\tau)}{d\tau^2} > 0,
\end{aligned} \qquad 0 < \tau < 1. \tag{A.2}$$

The first-order derivative of $C(\tau)$ equals

$$C'(\tau) = \frac{(\tau - 1.6906)(\tau - 0.7071)(\tau^2 + 1.3679\tau + 0.9166)(\tau + 0.7071)(\tau + 0.3227)}{(\tau^2)(\tau - 1)^2(\tau + 1)^2(\tau + 0.5)^2}. \tag{A.3}$$

The only critical point in the interval $0 < \tau < 1$ is at $\tau \approx 0.7071$, hence if the point is a minimum, it is also the global minimum in the interval. In order to verify if this

point is a minimum, we check the second derivative at $\tau = 0.7071$.

$$C''(\tau) = \frac{N_1 N_2 N_3 N_4 N_5}{\tau^3(\tau-1)^3(\tau+1)^3(\tau+0.5)^3}$$

$$\begin{aligned} N_1 &= \tau^2 + 1.5\tau + 0.544 \\ N_2 &= \tau^2 + 0.62\tau + 0.114 \\ N_3 &= \tau - 2.239 \\ N_4 &= \tau^2 + 1.539\tau + 1.68 \\ N_5 &= \tau^2 - 1.37\tau + 0.5349. \end{aligned} \tag{A.4}$$

Then for $\tau = 0.7071$,

$$C''(\tau)|_{\tau=0.7071} \approx 18.73 > 0. \tag{A.5}$$

Hence,

$$\tau^* = \underset{0<\tau<1}{\arg\min}\, \Delta \approx 0.7071. \tag{A.6}$$