SYMMETRY ANALYSIS FROM HUMAN PERSPECTIVE

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ABSTRACT

SYMMETRY ANALYSIS FROM HUMAN PERSPECTIVE

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Ornaments are repetitive patterns. They are created by repeating a base unit using four primitive geometric operations: translation, reflection, glide reflection and rotation. Using combinations of these primitive operations one can fill the plane in 17 different ways, which are known as 17 Wallpaper groups. In recent studies, an automated method is presented which can detect the symmetry group of given ornament. While automated methods aim to capture theoretical representation of the symmetry, they lack the ability to understand how the symmetries are perceived by human. In this study we focused on understanding human perception of symmetry and symmetry groups. We used an ornament that is challenging to classify for stimulating human perception and conducted an experiment to understand which symmetries people see in it and which wallpaper groups they tend to match the ornament. The results show that current groupings are not adequate to fully cover symmetry.

Keywords: symmetry, ornament, wallpaper groups, human perspective

ÖZ

İNSAN PERSPEKTİFİNDEN SİMETRİ ANALİZİ

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Bezemeler tekrarlayan modellerdir. Bir temel ünitenin dört ilkel geometrik işlem kullanılarak tekrarlanması ile yaratılırlar: Taşıma, yansıma, yansıma sonrası taşıma ve döndürme. Bu temel işlemlerin kombinasyonlarını kullanarak bir düzlem 17 duvar kağıdı grubu olarak bilinen 17 farklı şekilde doldurulabilir. Yapılan son araştırmalarda, verilen süslemenin simetri grubunu tespit edebilen otomatik bir yöntem sunulmaktadır. Otomatik yöntemler simetrinin teorik temsilini yakalamayı amaçlarken, simetrilerin insan tarafından nasıl algılandığını anlama yeteneğinden yoksundurlar. Bu çalışmada insanların simetri ve simetri gruplarını algısını anlamaya odaklandık. İnsan algısını uyarmak için sınıflandırması zor olan bir bezeme kullandık ve insanların bu bezeme içinde hangi simetrileri ve hangi duvar kağıdı gruplarını görebildiklerini anlamak için bir deney yaptık. Sonuçlar, mevcut gruplandırmaların simetriyi tam olarak karşılayacak düzeyde olmadığını göstermektedir.

Anahtar Kelimeler: simetri, bezeme, simetri grupları, insan perspektifi

To my dear family...

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INTRODUCTION

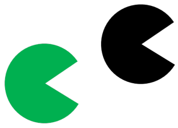
Repetition is a part of our lives. It is present in all living and nonliving components of nature, from the most basic building blocks of our being to the entire cosmos. The objects, systems, and works of art we create consist of repeating pieces. When analyzing any formation, finding repetitive fragments enhances our understanding. Its ubiquity makes repetitive structures one of the most important topics of scientific studies.

Everything that repeats is, by nature, an indicator of an order. However, the order of repetition may be at different levels. Some repetitive structures can only consist of randomly placed repetitions of an idea or concept. On the other hand, some may consist of a basic unit that repeats itself in a very proportional and strict order.

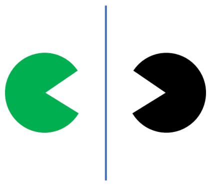
Figure 1.1: Hands, by Abidin Dino

Figure 1.1 shows one of Abidin Dino's hand drawings. In this figure, repetitive fingers stand out. The fingers have aesthetic value and repeat in harmony. However, they have no proportional arrangement. In this context, the repetition of the fingers in this figure is indicative of a low order.

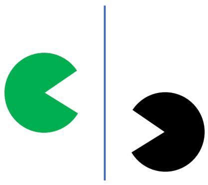
We can introduce more regularity to repetitive patterns by repeating the exact motif by only changing its position and not its size and shape. We achieve these repetitions by using Euclidean isometries: Translations, reflections, and rotations. A translation shifts each point of a shape in the same direction by the same distance. A reflection mirrors a shape across a line, whereas a glide reflection is a translation of the mirror of the shape by a distance. A rotation turns a shape around a fixed point by a certain angle. Concept of symmetry begins to show itself in shapes that are repeated using isometries. A pattern or a shape is symmetric if it is invariant under any transformation. Figure 1.2 shows simple examples of translation, reflection, glide reflection, and rotation symmetry.



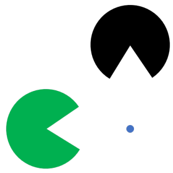
(a) Translation



(b) Reflection



(c) Glide Reflection

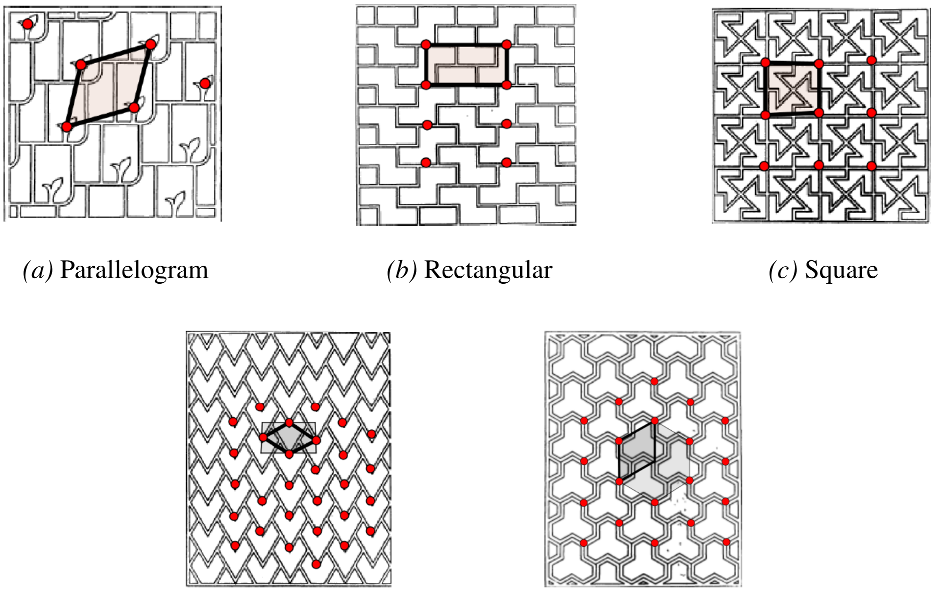


(d) Rotation

Figure 1.2: Isometries

Another instrument of regularity in repetitive patterns are tilings. A tiling is the repetition of a shape, or a tile, infinitely many times in two directions. Tiles of a tiling do not overlap, and they do not leave any gaps. Using periodic tilings, we can create planar patterns that are symmetrical and regular infinitely. Grid of these tiles is known as lattices. Five types of lattices allow translational symmetry: Parallelogram, rectangular, rhombic, square, and hexagonal lattices. Square, rectangular, and parallelogram lattices consist of units of the same name. The unit of a hexagonal lattice has the shape of a rhombus that has angles of 60° and 120°. Any other rhombus unit belongs to a rhombic lattice. Figure 1.3 shows different lattice types on simple tilings. Planar periodic tilings are also known as ornaments.

Inherently, all tilings have translational symmetry. Periodic tilings can also have reflection, glide reflection, and certain rotation symmetries without breaking its global symmetry. Angles of the rotations are restricted by crystallographic restriction the-



(a) Parallelogram *(b)* Rectangular *(c)* Square

(d) Rhombic *(e)* Hexagonal

Figure 1.3: Tilings with different lattice types (Tilings are taken from [1], lattice illustrations are taken from [2])

orem, which proves that a pattern repeating in two dimensions can only contain rotations by 180°, 120°, 90°, and 60°. These rotations are also known as two-fold, three-fold, four-fold, and six-fold rotations. Possible rotation types are visible in Figure 1.4.

There are three essential terms to explain the building blocks of a periodic tiling. The smallest translational element of a periodic tiling is known as the *unit cell*. The smallest element of a periodic tiling that is enough to regenerate the entire tiling by applying isometries is the *fundamental domain*. The group of isometries that is applied to the fundamental domain to regenerate the tilings is the *symmetry group* of the tilings. Figure 1.5 shows the unit cell and the fundamental domain of a tiling that has a six-fold rotation symmetry.

We can classify the various planar ornaments by their symmetry groups that leave them invariant. An analysis of these groups [3, 4, 5] shows that there are precisely seventeen different symmetry groups. The seventeen groups are known as Wallpaper Groups. Naming of the groups are according to the crystallographic notation [6].

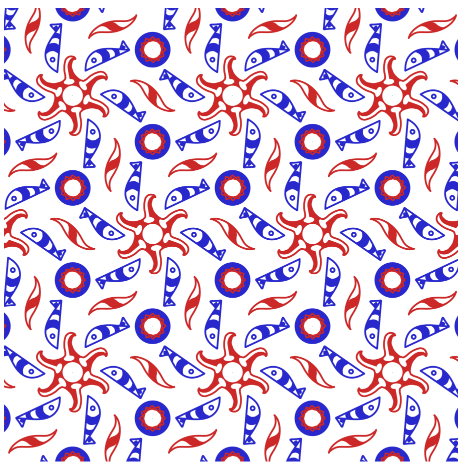
(a) two-fold rotational symmetry

(b) three-fold rotational symmetry

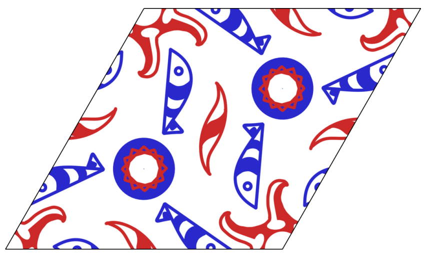
(c) four-fold rotational symmetry

(d) six-fold rotational symmetry

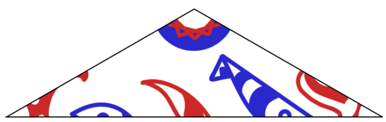
Figure 1.4: Types of rotational symmetry



(a) Ornament with six-fold rotational symmetry



(b) Unit cell



(c) Fundamental domain

Figure 1.5: Unit cell and fundamental domain of a tiling

The first letter can be *p* or *c*: The letter *p* stands for the primitive unit cell that has the centers of the highest order of rotation are in the vertices of the cell. The letter *c* stands for the centered unit cell that has reflection axes on one or both diagonals of the cell. The second character denotes the highest order of rotation, and it can take *1*, *2*, *3*, *4*, or *6* due to the crystallographic restriction. The third character is for reflection; it can take *m* for mirror reflection, *g* for glide reflection, or *1* for not symmetry axis. We can symbolize a second reflection that is not parallel to the first one by using the fourth character the same way as the third character. Figure 1.6 shows 17 wallpaper groups with their unit cells and symmetrical structures.

The geometric rules and connections behind repetitive patterns put them in strict categories through group theory. Usually, studies on ornaments are carried out on this modeling. In this approach, to detect the mathematical group of an ornament, the basic repetitive element and its repetition structure must be found and then these attributes are matched with a predefined group. There are directive guidelines (Figure 1.7) and computer implementations [7] to help detecting the symmetry group of an ornament for the group theory approach. However, as subjects in the world of these patterns who live around and experience them, humans may not use the same thresholds to categorize ornaments. Samples that show very similar characteristics according to human perception can be included in different groups if they are only classified by their geometric attributes. These groupings that categorize ornaments as a set of technical rules fail to explain the connections and order that may have different meanings in people's perception.

In [8], an experiment was carried out where the participants sorted ornaments from different symmetry groups according to their similarity. The findings showed that while different samples from the same symmetry group were matched more by the participants, there were clear indicators that the classification was not relied only on its wallpaper group, but a collection of visual properties, which is not represented by a mathematical model.

The goal of this thesis is to understand more about the human perception of symmetry and analyze how their perception matches the current classification of the ornaments. To achieve this goal, we created and conducted an experiment that will shed light

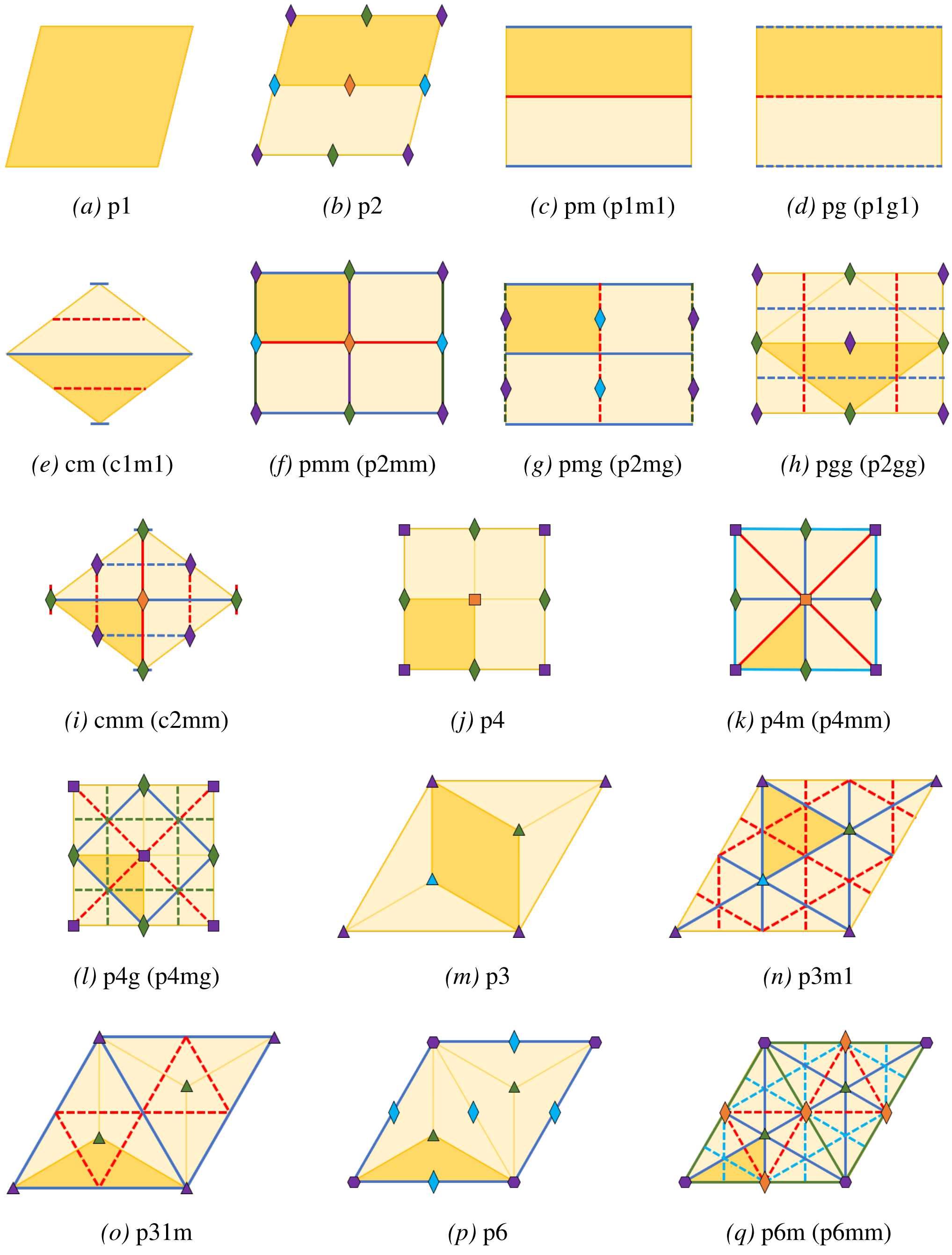


Figure 1.6: Unit cells of seventeen wallpaper groups. Full names are in parenthesis. Yellow areas reveal the fundamental domain. (◇: 2-fold rotation centres, △: 3-fold rotation centres, □: 4-fold rotation centres, ⬡: 6-fold rotation centres, ——: Axis of reflection, - - -: Axis of glide reflection)

on what people see in repetitive patterns. When the ornament samples around the world are examined closely, it can be noticed that some challenging cases can mislead the group-finding guidelines or computational applications designed according to mathematical models. We assumed that ornaments that are challenging to detect its symmetry group are more stimulative for human perception. According to this assumption, we developed the experiments around a single challenging ornament.

In the thesis, first, we thoroughly analyzed what characteristics define this ornament as challenging to classify. Then we designed a survey in which we asked participants to select the most similar ornament between multiple options that represent different properties of repetitive patterns in the ornament. To implement this experiment setting, we built a data set consisting of new ornaments that are created to exemplify some or all of the key properties of the main challenging ornament. After we gathered the results of the first experiment, we designed another experiment to try our new findings in a different setting. Analysis on the results of both surveys shows that mathematical groupings are not adequate to categorize and study the repetitive patterns, there is more information in ornaments to see for humans.

The rest of the thesis is organized as follows. In Chapter 2, we review works that are related to our work. In Chapter 3, we talk about the challenging ornaments which are ornaments difficult to categorize. We introduce a challenging ornament, the Moroccan, which is essential in our experiments. We explain hypersymmetric features. In Chapter 4, we describe in detail the design and implementation of two consecutive experiments on human perception of the Moroccan. The data set created for the experiments are presented and the results of the experiments are visualized. In Chapter 5 we conclude our work with a summary and a discussion on the results of the experiments.

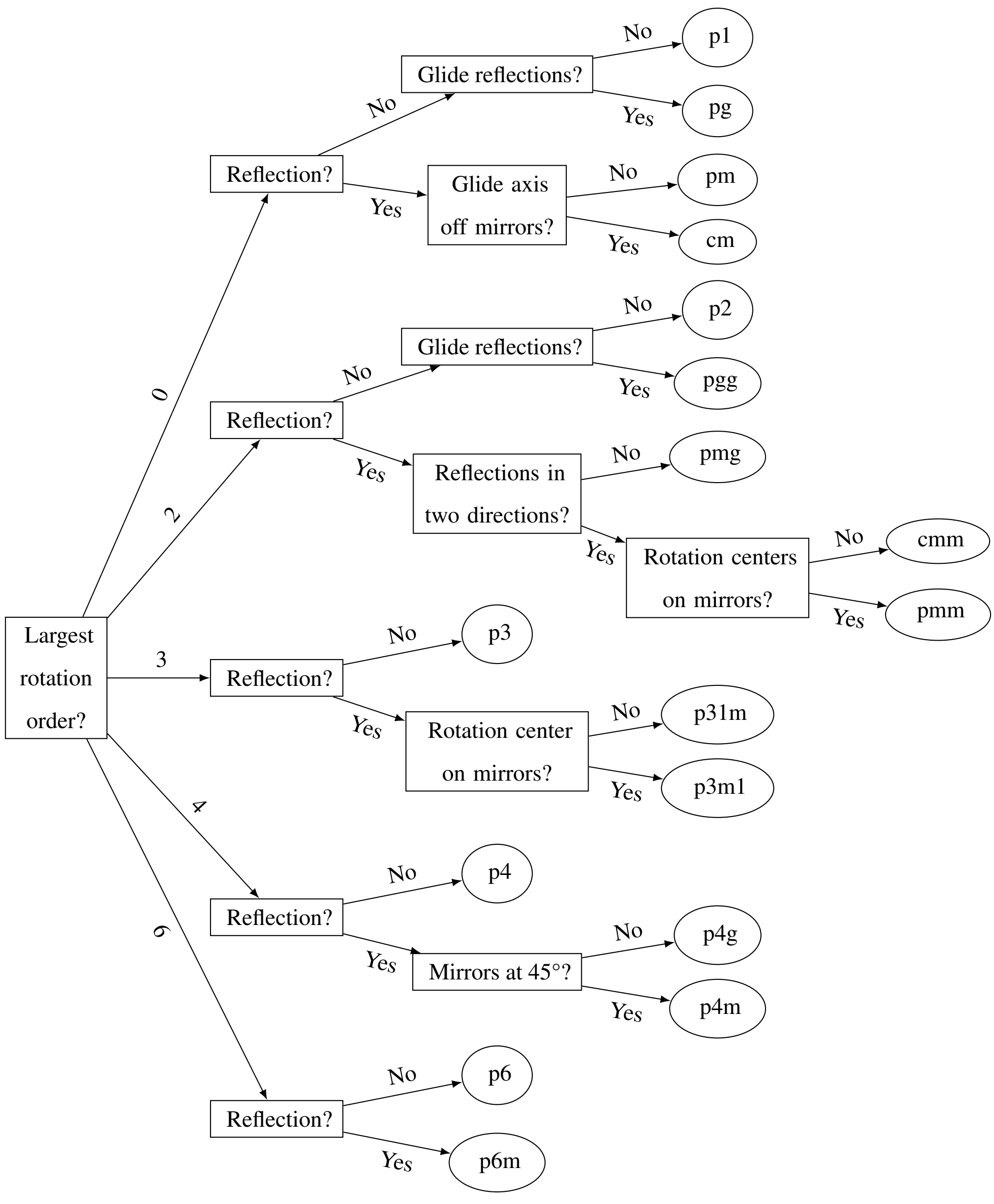


Figure 1.7: Wallpaper group decision process

CHAPTER 2

RELATED WORK

In order to make the studies on symmetry meaningful, there is a need to define and classify symmetry first. Galois [9] was the first in the field to incorporate group theory into symmetry studies. Following the footsteps of this study, Niggli [3, 5], and Polya [4] created a system that classifies symmetries. While these studies examine symmetry in the field of crystallography, Spieser's research [10] was the first known work to interpret this classification on the symmetries created by humans. The study classified the motifs found in the tombs in Egypt according to their symmetrical properties, made important discoveries about the beginnings of higher mathematics, and contributed to the shift of cultural studies into the field of symmetry. Among the researchers who follow this path, the work of Brainerd [11] and Shephard [12] pursues the history of symmetry by examining the motifs found in archaeological findings. Müller's research [13] gives a detailed mathematical analysis of the designs in the walls of the Alhambra in Granada, Spain. These designs were samples of Islamic art, and they were following a tradition of geometric expression. Even though they contain highly symmetrical objects, non-mathematicians created these ornaments. (Figure 2.1)

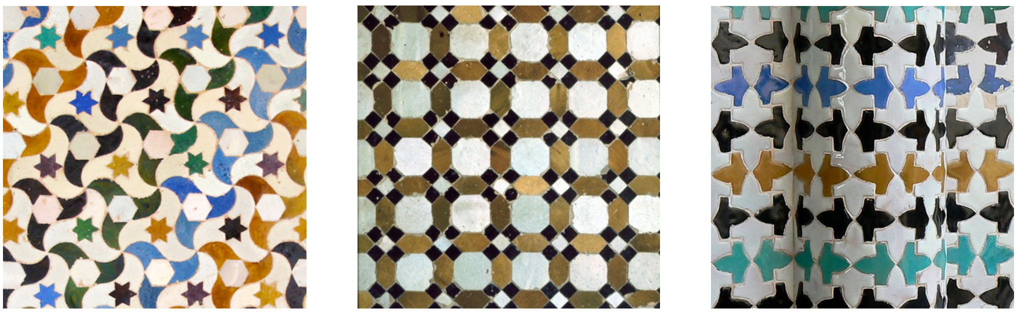


Figure 2.1: Symmetrical structures in Alhambra

Another person who worked in detail on the motifs in the Alhambra in the same period was M. C. Escher. He took notes by examining the motifs covering Islamic architecture and created his classification system [14]. He improved on symmetries by studying the examinations of those working in the field, such as Müller. His symmetrical drawings are considered to be some of the highest examples of art with symmetrical elements. (Figure 2.2)

Figure 2.2: Symmetrical structures in the works of M. C. Escher

2.1 Human Perception of Symmetry

Previous studies on human perception on symmetries tested the possibility of a preference in humans among different properties of symmetry. In Chapter 1 we mentioned four types of isometries (translation, rotation, reflection, and glide reflection) as properties of symmetry. However, in related studies, it was claimed that participants were inclined to match mirror reflection with symmetry in general. There are different explanations for this result. In [15], authors explained that subjects could recognize a 180° rotation better than a 45° or 90° rotation. In other words, it makes the vertically oriented axis of symmetry easy to recognize by people. Experimental study of Corballis and Roldan [16] supports this explanation. They reported that, according to the subjects, vertical and oblique axes in symmetry are more comfortable to grasp than horizontal symmetry. An examination of their findings is in [17], and the authors approved that vertical symmetry is the least complicated symmetry type that can be detected by participants. These two studies explained later by Shepard and Metzler's study [18]. The subjects can recognize vertical symmetry faster because people rotate the images in their minds until they set the symmetry vertically.

In recent studies, there are some other indicators of how humans understand symmetry. Authors of [8] conducted experiments where the participants had to sort the ornaments according to their symmetry groups. Its results showed that humans could classify the same wallpaper groups together more often than they would classify them by chance. Another study [19] proves that humans can reliably distinguish all but one pairing (p4m and pmm) of wallpaper groups. In these studies, the authors interconnected human classification of symmetry with other features of symmetry besides wallpaper groups, such as the number of axes.

CHAPTER 3

CHALLENGING ORNAMENTS

In the previous chapters, we explained a system that classifies symmetrical properties of planar repetitive patterns. However, some patterns can be difficult to classify using the current classification system. These patterns usually appear more symmetric than they are. In this chapter, we analyze the symmetrical properties of challenging ornament examples to figure out what makes planar patterns challenging to classify.

3.1 The Moroccan Ornament

One of the challenging cases is an ornament located in Morocco (Figure 3.1). Throughout the thesis, we will call this ornament as the Moroccan ornament. The ornament has many symmetrical properties. There is a three-fold rotation center in the middle of the black diamond figures and four-fold and six-fold rotation centers in the middle of the blue twelve-pointed star and the yellow octagram. Below, this rotation centers will be examined to reveal the wallpaper group of the Moroccan ornament and to understand what makes it challenging.

At the first look, by looking at the blue twelve-pointed stars and the three-fold diamond-shaped rotations, one may assume that this ornament has a six-fold rotation and a mirrored symmetry. These properties belong to *p6m*-type ornaments. To examine this claim, lattice points are placed in the rotation centres of the six fold rotations (Figure 3.2(a)) and a unit cell is extracted (Figure 3.2(b)). Instead of the photograph of the ornament, a vectorized figure is used to smooth out the wear and noise. After that by using the template in Figure 1.6, we can reach to its fundamental domain (Figure 3.2(c)). At this point, it is apparent that the ornament does not have one single

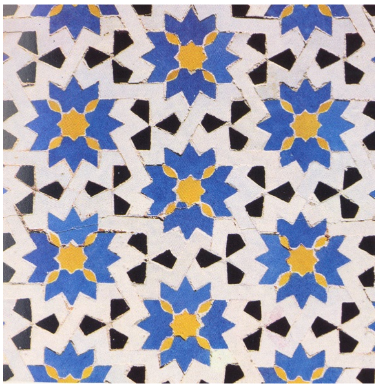


Figure 3.1: The Moroccan ornament

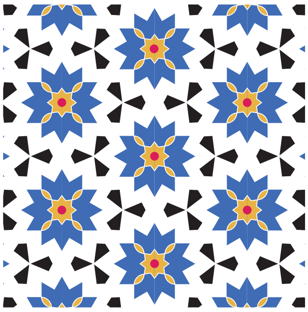
fundamental domain but two, which results in two different unit cell (Figure 3.2(d) and 3.2(e)). Thus it can be seen that the Moroccon ornament is not a *p6m*.

In the second examination, yellow octagrams are our focus. They seem to have a four-fold rotation, and we know from the lattice types in Figure 1.6 that to have a four-fold rotation, it must have a square lattice. However, when we place lattice points on the four-fold rotation centers, we see that they do not form a square lattice pattern. Thus even though there are 90° rotations in this ornament, it does not belong to a group that has a four-fold rotation symmetry.

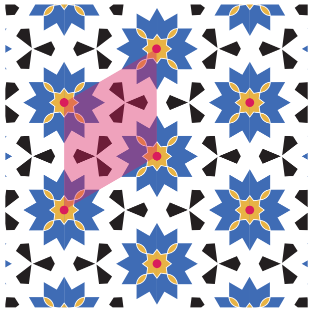
Visualization of our regeneration efforts is in Figure 3.3. It is visible that rotating the ornament by 120° around the three-fold rotation center, by 90° around the four-fold rotation center or by 60° around the six-fold rotation center does not reproduce the same ornament.

A revaluation of the lattice points shows that there is a possibility for another type of unit cell: A centered cell. Only two types of wallpaper groups have these kinds of cells: *cm* and *cmm*. When we position this rhombic lattice unit on the Moroccan ornament in Figures 3.4(a) and 3.4(b), we see that it has two-fold rotation centers on the corners and the midpoints of its edges. These properties belong to *cmm*-type ornaments. Its extracted unit cell (Figure 3.4(c)) has a single fundamental domain (Figure 3.4(d)). Figure 3.4(e) shows successful regeneration, therefore wallpaper group of the Moroccan ornament is *cmm*.

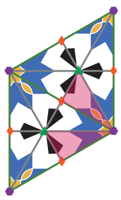
We examined what makes the Moroccan ornament challenging to detect its wallpaper



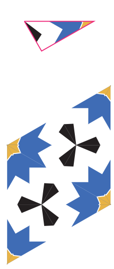
(a) Lattice points



(b) Unit cell



(c) Extracted unit cell

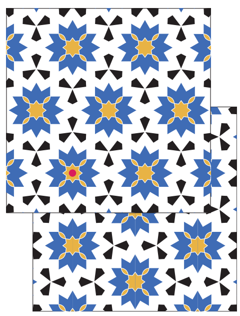


(d) First fundamental domain and reconstruction of unit cell

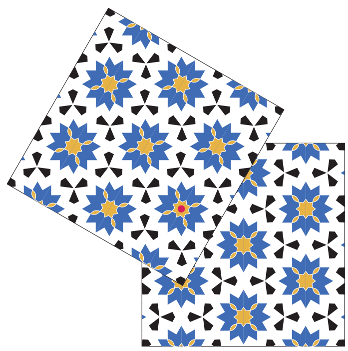
(e) Second fundamental domain and reconstruction of unit cell

Figure 3.2: p6m examination of the Moroccan ornament

(a) 120° rotation

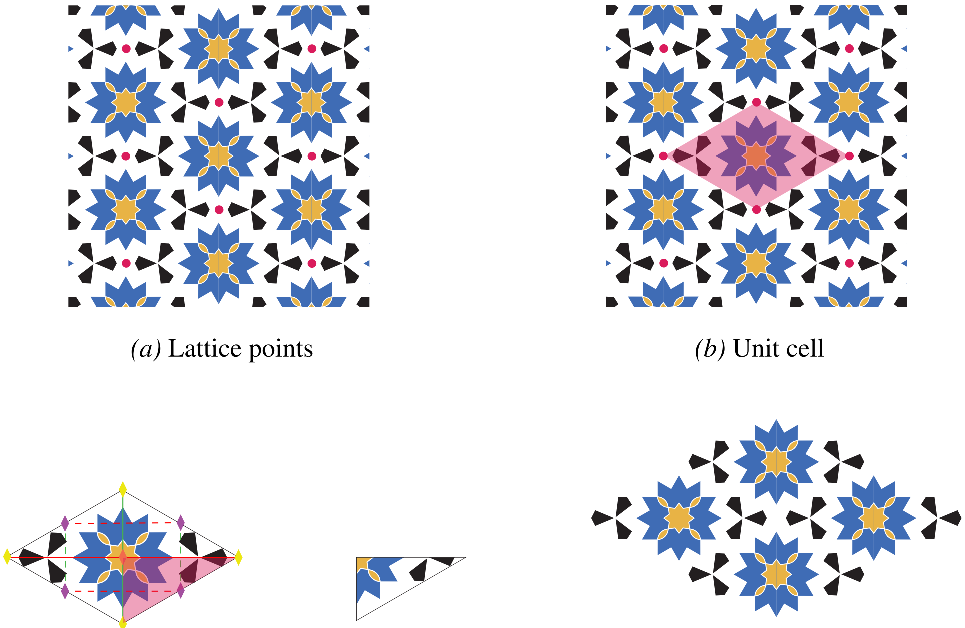


(b) 90° rotation



(c) 60° rotation

Figure 3.3: Rotations of the Moroccan ornament around fixed rotation centres



(a) Lattice points

(b) Unit cell

(c) Extracted unit cell

(d) Fundamental Domain

(e) Regenerated ornament

Figure 3.4: cmm examination of the Moroccan ornament

group. There may be similar challenges for computational methods. In the study by Adanova and Tari [7], a method for extracting the fundamental domain for 13 wallpaper groups (only *pm*, *cm*, *pmm*, and *pmg* groups are excluded) is presented. The method detects the fundamental domain even in ornament samples that do not contain more than one repeating unit. The method focuses primarily on identifying planar arrangements of asymmetric interlocking shapes rather than focusing on global features such as the translational lattice of the ornament or repetitive shapes on the ornament. However, when we examine the Moroccan ornament with this method, it is seen that the wallpaper group cannot be detected correctly. In Figure 3.5 it can be seen that the method detects four-fold rotation centers instead of two-fold rotation centers of *cmm*.

What makes Moroccan ornament unique and challenging is the yellow motif inside the blue star. If we remove the yellow octagram, the order of rotation of the ornament will increase from two to six. We examined the ornament further using different shapes with different symmetrical features instead of the yellow octagram. In Figure

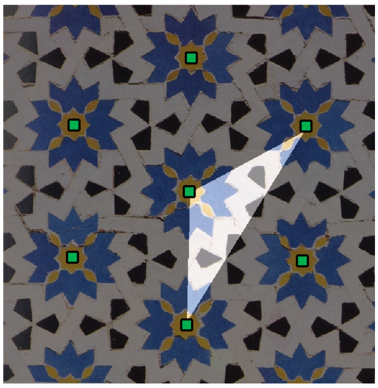


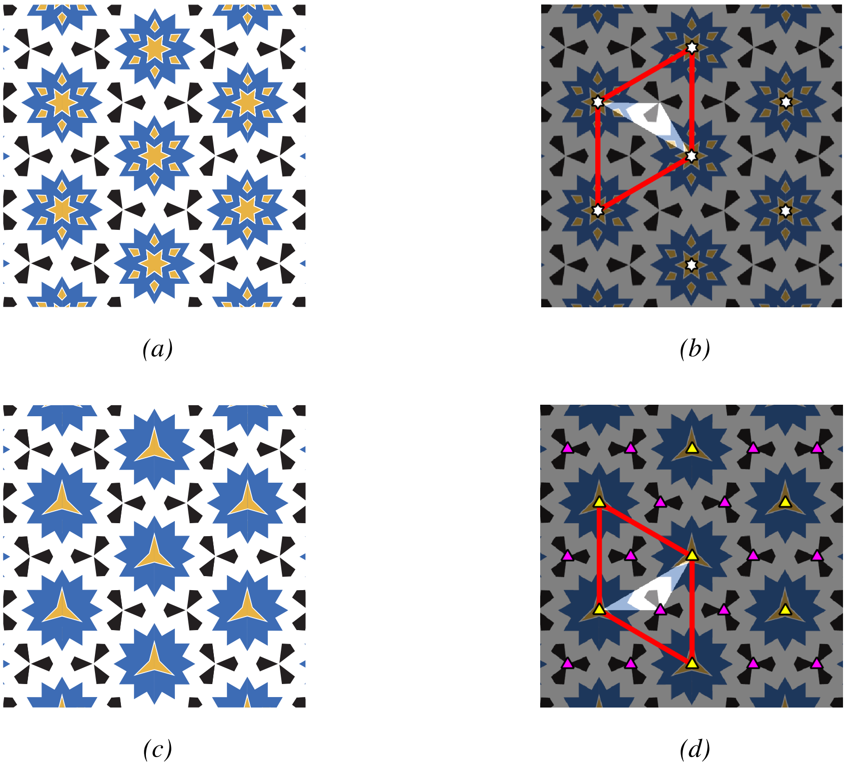
Figure 3.5: Output of the computational symmetry detection system

3.6(a) we replaced it with a motif that has six-fold rotational symmetry and in Figure 3.6(c) we used a three-fold symmetrical shape. First alteration belongs to the p6m group, and the second one is a p31m-type. Their outputs by the computational method are visible in Figures 3.6(b) and 3.6(d) respectively. In the first alteration, the computational method detects the six-fold rotation centers successfully, but it misses the reflection axes. However, in the second alteration, the method detects the wallpaper group correctly.

Outputs show that the computational method may detect the wallpaper group of the input incorrectly when challenged by ornaments that appear more symmetric than they are. This may be due to the constraints considered for the 13 wallpaper groups. However, the method uses local features of an ornament to decide on its output. Similar local features make an ornament challenging to detect its wallpaper group or make it hypersymmetric. Therefore the method may be modified to evaluate extreme examples such as the Moroccan ornament correctly.

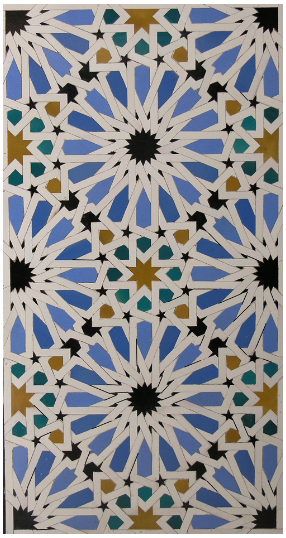
3.2 Hypersymmetry in Planar Patterns

The Moroccan ornament reveals that some of the symmetrical properties of this ornament are not evident on the entire tiling; they are only there apparently. Such tiles are called hypersymmetric. A tile is hypersymmetric if it contains additional symmetry that is not present in any tiling by the tile. In other words, its local symmetrical features do not manifest themselves on a global scale. Islamic geometric patterns

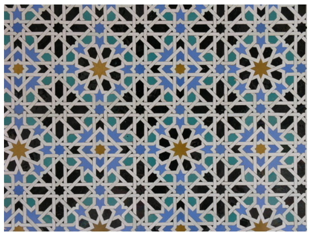


(a) (b) (c) (d)

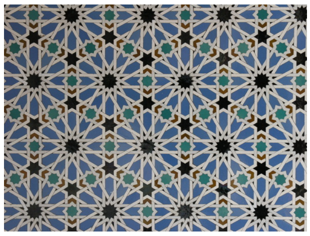
Figure 3.6: Alterations on the Moroccan ornament and their outputs by computational symmetry detection system



(a)



(b)



(c)

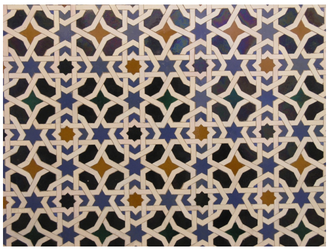
Figure 3.7: Hypersymmetric ornament samples

have many samples of hypersymmetric ornaments. Examples in Figure 3.7 shows local symmetrical features that represents a higher rotational order or contains more reflection axes than their wallpaper group (*p4m*, *p4m*, and *p6m* respectively).

There are many other ornaments with similar challenges around the world. In figure 3.8, there are three ornament samples from Spain: one from Alhambra located in Granada and two from The Alcázar of Seville. At first glance, it is visible that these ornaments contain many kinds of symmetrical features. Especially the star figures give the impression that the ornament belongs to a symmetry group which contains some form of rotation symmetry. However, in further examinations, it is understood that these ornaments have details that are not global features of the whole ornament.

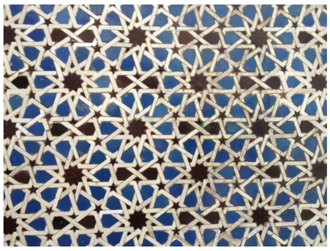
In Figure 3.8(a), there are four, six, and eight-pointed stars visible at first glance. The ornament appears to contain a four-fold or six-fold rotation symmetry and diagonal reflectional axes. However, when we examine the ornament, it is seen that there is no rotation symmetry. There are only mirror reflection axes that run over the hexagrams vertically and glide reflection axes that run over the hexagrams horizontally. These symmetries reveal that the ornament is in the *pmg* group.

Figure 3.8(b) has noticeable three-fold and six-fold rotation symmetry. These sym-



(a) An ornament from Alcazar

(b) An ornament from Alhambra



(c) Another ornament from Alcazar

Figure 3.8: Challenging hypersymmetric ornaments

metries may create the illusion that the ornament's symmetry group includes three-fold or six-fold rotation. However, after a detailed examination, it is seen that the ornament only has perpendicular reflection axes and two-fold rotation centers at the intersection points of these axes. The fundamental domain of the ornament is in the form of a rectangular and can form a unit cell after two mirror reflection symmetries. This fact puts the ornament into *pmm* category instead of groups that has a higher order of rotation symmetry such as *p3*, *p6*, etc.

In Figure 3.8(c), crosses, and five and ten-pointed stars are the most striking visual hints for symmetries. The ornament appears to have rotational symmetry and many different axes of reflectional symmetry. However, examination shows that there are only two reflection axes, and the fundamental domain has a right triangular shape. Under the circumstances, the ornament belongs to the *cmm* group.

These ornaments are examples of challenging cases for the detection of a wallpaper group. There are rotation centers and hypersymmetric features that are easy to see, but acting on these points may misguide researchers or computational methods. Their fundamental domain is hard to notice, and it takes a detailed examination to detect the correct symmetry group. As a result of their complicated structure, these ornaments are exceptional patterns to analyze what observers see when they look at an ornament and how they decide on a wallpaper group. In the next chapter, we present two experiments that are designed around the Moroccan ornament to find out which symmetries people see in it.

CHAPTER 4

EXPERIMENTS

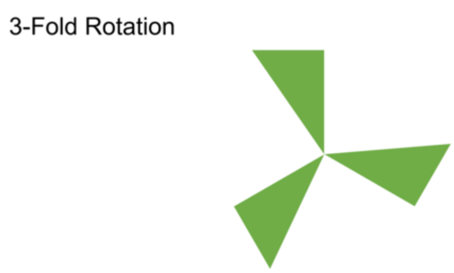
In Chapter 3, the Moroccan ornament and its complex symmetry structure is explained. It makes the Moroccan ornament a great example to understand which symmetries people see in an ornament and with which wallpaper group they match these symmetries.

In this chapter, two surveys and their results are discussed. We designed the surveys to find out about people's perception of the Moroccan ornament to see better if the systematic way for studying ornaments are in line with the perception of people in challenging ornaments. Surveys were built around the Moroccan ornament to understand which symmetries people see in it and with which groups they tend to match the ornament. The first section summarizes the first survey and its results. In the second section, the questions left unanswered in the first survey are summarized and the work on the second survey and its results are shown.

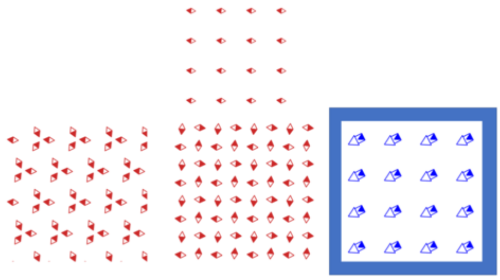
4.1 The First Survey

In the first survey, we asked participants to choose one of three different ornaments according to their similarity to the Moroccan ornament. This survey was designed to be conducted as a presentation to groups of people. To record the choices of the participants, a form was distributed to the participants before the presentation and they were asked to only write their answers to the questions asked.

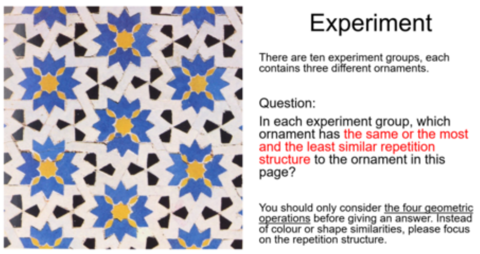
The survey comprises three main sections. The first section was planned as a course to educate the participants about the symmetries and the ornaments. In this section,



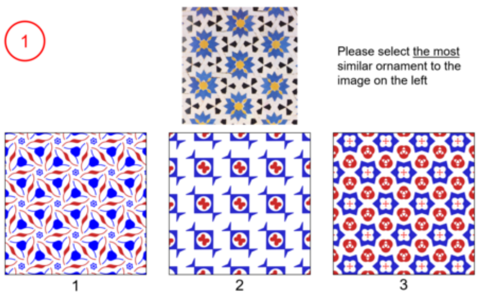
(a) A slide from the course on ornaments



(b) Warm-up example



(c) Survey question

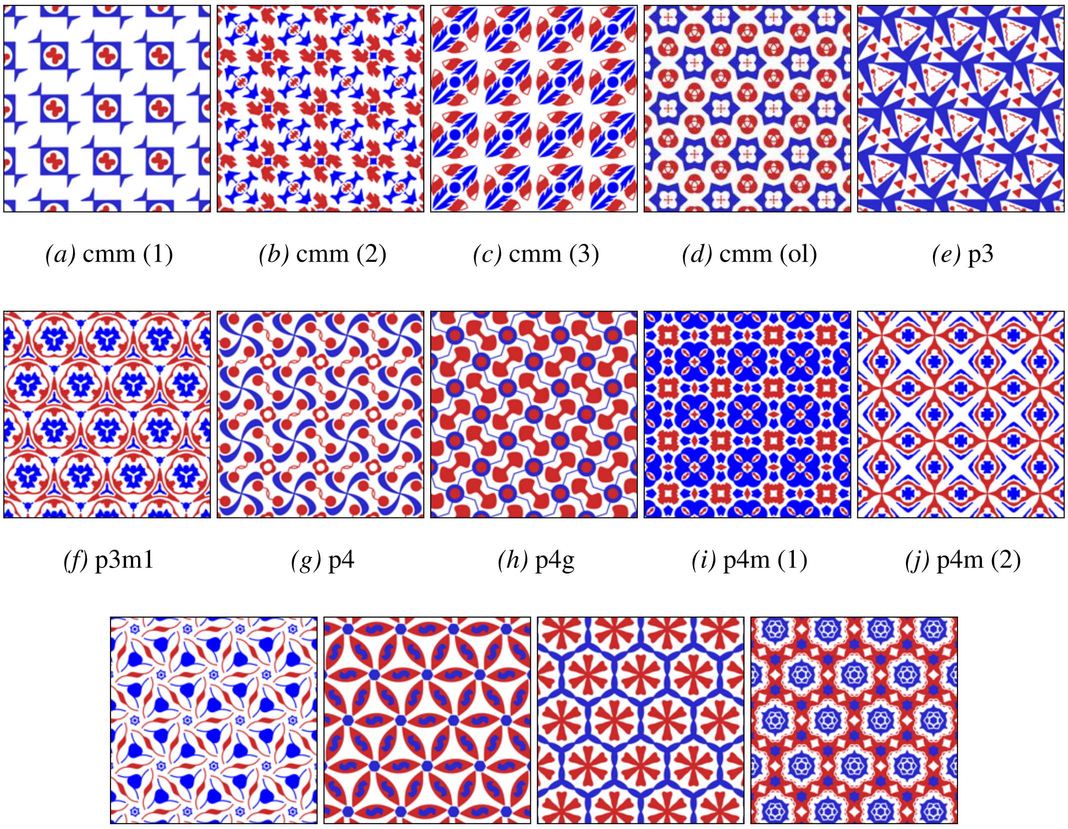


(d) One of the data groups

Figure 4.1: Slides from the first survey

ornaments are defined and animations of four types of isometry are presented to give information about the concept of symmetry to the participants. After that, another animation which displayed several symmetries applied on a fundamental domain to create a p6 ornament was presented to the participants to reinforce more on the isometries. A slide from the first section of the presentation is in Figure 4.1(a). The second section was planned as a warm-up example, only a simple question (Figure 4.1(b)) was asked to the participants to emphasize the difference between what we are looking for and what we are not. In a three-choice question, we asked which one of the three options has the same repetition structure as the query ornament. Two of the three options were created to have the same basic motif as query ornament, but they did not have the same repetition structure. The third option did not have the same motif or the same colors as the other two but had the same repetition structure as the query ornament. After asking the question, we revealed the answer and showed the symmetry structure in each ornament.

In the third section, a total of ten question sets consisting of the Moroccan ornament as the query ornament and three ornament options were created (Figure 4.1(c)). Using the same question sets, two types of questions - twenty in total - were asked. For the first ten questions, we asked which one has the same or the most similar repetition structure to the Moroccan ornament, and for the last ten questions, we asked for the ornaments with the least similar repetition structure to the Moroccan ornament. The reason why we did not ask these two types of questions for each ornament set one after the other was to help the participants focus their attention on finding the most similar ornament for the first ten questions and the least similar ornament for the last ten questions.



(a) cmm (1) *(b)* cmm (2) *(c)* cmm (3) *(d)* cmm (ol) *(e)* p3

(f) p3m1 *(g)* p4 *(h)* p4g *(i)* p4m (1) *(j)* p4m (2)

(k) p6 (1) *(l)* p6 (2) *(m)* p6m (1) *(n)* p6m (2)

Figure 4.2: Ornaments used in the first survey

To use in the first survey, four ornaments for the warm-up question (Figure 4.1(b)) and fourteen ornaments for the survey (Figure 4.2), a total of eighteen ornaments were created using the iOrnament tool [20]. Each ornament was drawn using only red, blue and white. This was a deliberate choice to make the questions more about the ornaments' repetition structure and less about their colors. The dataset contained four samples from the *cmm* group, two samples from *p4m*, *p6* and *p6m* groups, and one sample from *p3*, *p3m1*, *p4*, and *p4g* groups. These samples were labeled according to their wallpaper groups. Ornaments containing only three-fold, four-fold or six-fold symmetries or belonging to the *cmm* group were added to the data set. The reason behind this decision was to create a similarity to the Moroccan ornament that seems to have a rotational symmetry but instead belongs to the *cmm* group. From the ornaments, the one in Figure 4.2(d) (*cmm(ol)*) was created by overlapping two symmetry groups to make it similar to the Moroccan ornament. First, a *p6m* and a *p4m* ornament were created and then the *p4m* ornament was placed onto the *p6m* ornament by fixing their rotation centers at the same point. Using this method, we broke the symmetry of the ornaments of *p6m* and *p4m* groups and created a *cmm* ornament in a similar way to the Moroccan ornament.

Ten different data groups were assembled using these new ornaments. (Figure 4.3) Each of these groups were formed to make participants choose between different symmetry types in the Moroccan ornament. In questions 1, 2, 3, 4, 11, 12, 13 and 14 we inserted a *cmm* type, a rotation type and *cmm (ol)* as answers. In questions 5, 6, 8, 9, 15, 16, 18 and 19 we asked the participants to choose between a *cmm* type and two different rotation types. Finally, in questions 7, 10, 17 and 20 we gave the participants three different rotation options. The group names of these ornaments were not given to participants, they only had the prior information described in the first section of the survey. All of the questions were asked to find out what people see in the Moroccan and with which types of ornaments they make a connection.

The first survey was conducted with 30 participants: 22 students from the Department of Computer Engineering at METU and eight students from the Department of Computer Engineering at TED University. They were given thirty seconds to answer each question. This time interval was chosen with the idea that thirty seconds were too short to perform a detailed analysis of the ornaments but longer than the time required to just give an intuitive answer. In this way, the participants were able to combine what they saw in the options with the symmetry training they received in the first section of the survey but to respond without a long evaluation. The results of their answers are presented in Table 4.1. For each option, the number of partici-

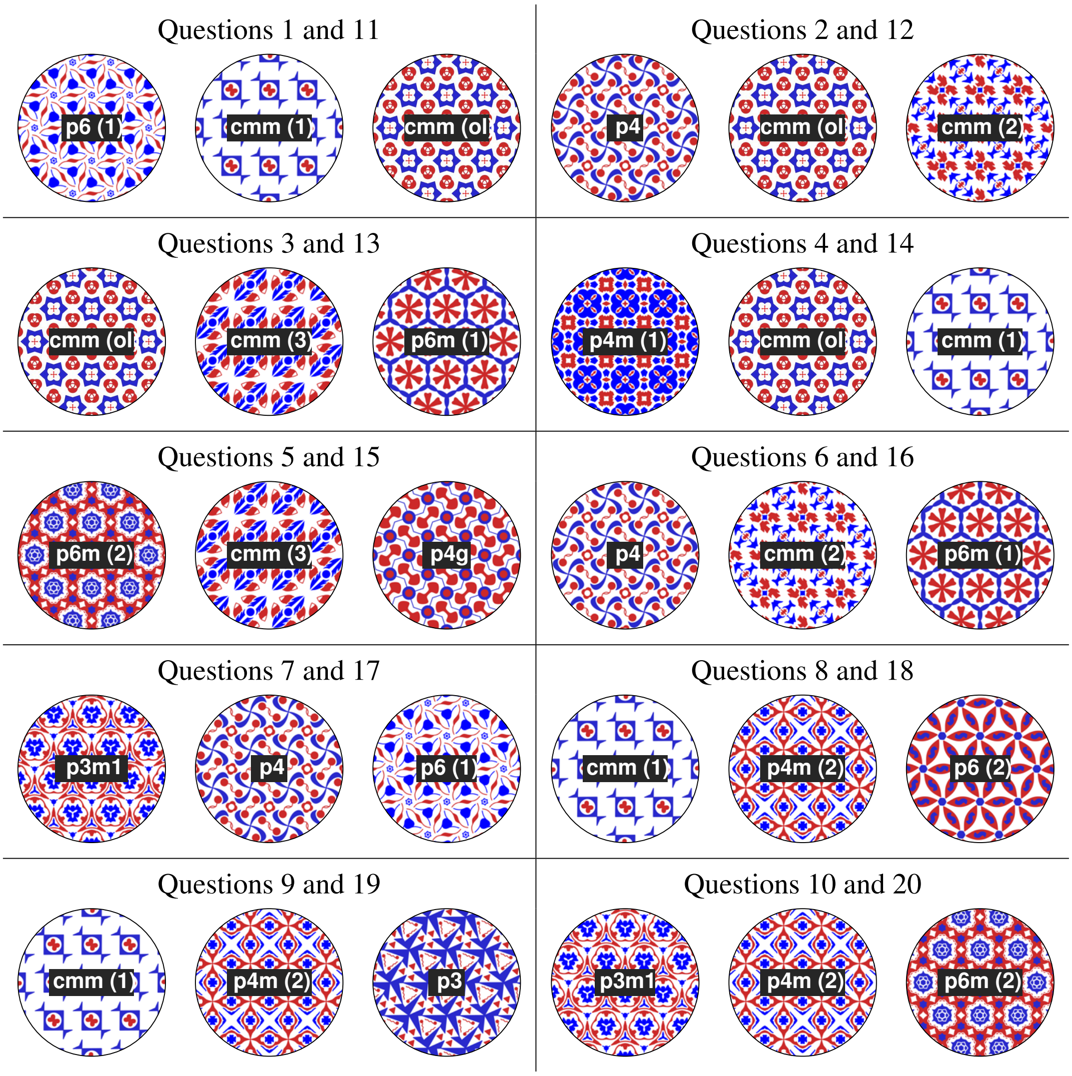


Figure 4.3: Ornament sets used in the first survey

Table 4.1: *Results of the first survey collected from 30 participants (Their percentages are given in parenthesis)*

Questions	Option 1			Option 2			Option 3		
	Group	Most	Least	Group	Most	Least	Group	Most	Least
Q1 + Q11	p6 (1)	7 (23%)	2 (7%)	cmm (1)	2 (7%)	28 (93%)	cmm (ol)	21 (70%)	0 (0%)
Q2 + Q12	p4	6 (20%)	16 (53%)	cmm (ol)	20 (67%)	2 (7%)	cmm (2)	4 (13%)	12 (40%)
Q3 + Q13	cmm (ol)	12 (40%)	2 (7%)	cmm (3)	1 (3%)	27 (90%)	p6m (1)	17 (57%)	1 (3%)
Q4 + Q14	p4m (1)	12 (40%)	5 (17%)	cmm (ol)	18 (60%)	1 (3%)	cmm (1)	0 (0%)	24 (80%)
Q5 + Q15	p6m (2)	24 (80%)	2 (7%)	cmm (3)	0 (0%)	17 (57%)	p4g	6 (20%)	11 (37%)
Q6 + Q16	p4	8 (27%)	14 (47%)	cmm (2)	6 (20%)	13 (43%)	p6m (1)	16 (53%)	3 (10%)
Q7 + Q17	p3m1	13 (43%)	6 (20%)	p4	6 (20%)	17 (57%)	p6 (1)	11 (37%)	7 (23%)
Q8 + Q18	cmm (1)	1 (3%)	24 (80%)	p4m (2)	9 (30%)	3 (10%)	p6 (2)	20 (67%)	3 (10%)
Q9 + Q19	cmm (1)	0 (0%)	20 (67%)	p4m (2)	9 (30%)	4 (13%)	p3	21 (70%)	6 (20%)
Q10 + Q20	p3m1	5 (17%)	8 (27%)	p4m (2)	5 (17%)	19 (63%)	p6m (2)	20 (67%)	3 (10%)

pants who selected that option as the most similar and as the least similar were listed separately. For instance; in the first and eleventh questions, the first option (*p6 (1)*) was selected as the most similar by seven participants and the least similar by two participants. The second option (*cmm (1)*) was selected as the most similar by two participants and the least similar by 28 participants. Finally, the last option (*cmm (ol)*) was selected as the most similar ornament by 21 participants and no participant chose it as the least similar ornament. These results show the third option (*cmm (ol)*) as the most similar ornament to the Moroccan ornament among the three options, while the second option (*cmm (1)*) places as the least similar ornament in the order.

Using participants' answers, distance matrices were calculated for each of the ten ornament sets to examine the consistency of the participants between each other. The distance between the answers of two participants was measured using Kendall tau distance [21], which is used to calculate the disagreements between two ranking lists. To be able to utilize Kendall tau distance, the answers were reorganized for each ornament set as 1 for the most similar, 3 for the least similar and 2 for the unselected ornament. Thirteen participants selected the same ornament in at least one group for both the most similar and the least similar questions. These answers caused inconsistent input records for the sorting algorithm, so they were discarded. The remaining

Table 4.2: *Results of the first survey of 17 participants who gave consistent answers (Their percentages are given in parenthesis)*

Questions	Option 1			Option 2			Option 3		
	Group	Most	Least	Group	Most	Least	Group	Most	Least
Q1+Q11	p6(1)	3 (18%)	1 (6%)	cmm(1)	0 (0%)	16 (94%)	cmm(ol)	14 (82%)	0 (0%)
Q2+Q12	p4	3 (18%)	9 (53%)	cmm(ol)	12 (71%)	1 (6%)	cmm(2)	2 (12%)	7 (41%)
Q3+Q13	cmm(ol)	7 (41%)	1 (6%)	cmm(3)	0 (0%)	16 (94%)	p6m(1)	10 (59%)	0 (0%)
Q4+Q14	p4m(1)	5 (29%)	1 (6%)	cmm(ol)	12 (71%)	1 (6%)	cmm(1)	0 (0%)	15 (88%)
Q5+Q15	p6m (2)	15 (88%)	1 (6%)	cmm(3)	0 (0%)	10 (59%)	p4g	2 (12%)	6 (35%)
Q6+Q16	p4	3 (18%)	7 (41%)	cmm(2)	2 (12%)	8 (47%)	p6m(1)	12 (71%)	2 (12%)
Q7+Q17	p3m1	6 (35%)	4 (24%)	p4	4 (24%)	10 (59%)	p6(1)	7 (41%)	3 (18%)
Q8+Q18	cmm(1)	1 (6%)	13 (76%)	p4m(2)	5 (29%)	2 (12%)	p6(2)	11 (65%)	2 (12%)
Q9+Q19	cmm(1)	0 (0%)	12 (71%)	p4m(2)	6 (35%)	2 (12%)	p3	11 (65%)	3 (18%)
Q10+Q20	p3m1	3 (18%)	5 (29%)	p4m(2)	2 (12%)	12 (71%)	p6m (2)	12 (71%)	0 (0%)

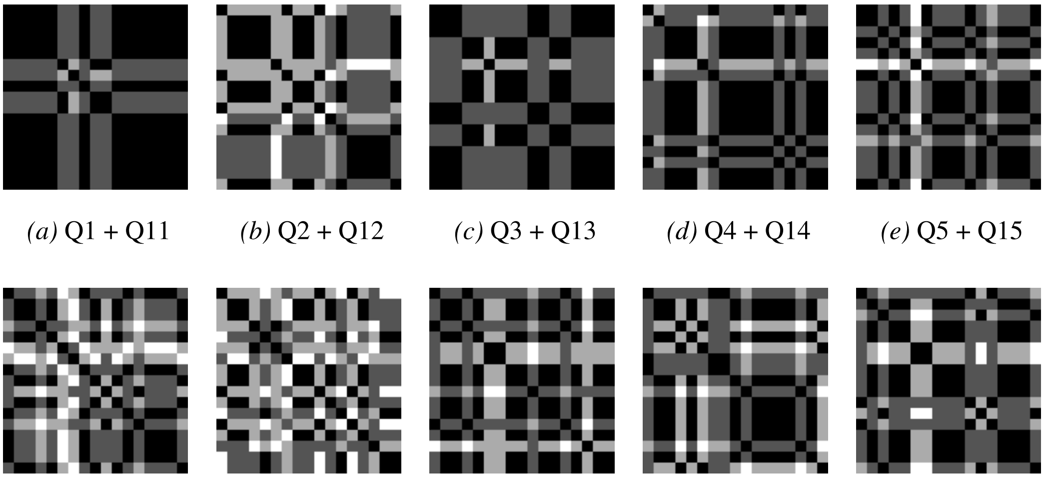
17 participants' answers can be seen in Table 4.2. These answers were used while calculating the distance matrices.

Kendall tau distance between two ranking lists is defined in Equation 4.1. In this equation, P represents the set of pairs of distinct elements in ranking lists τ_1 and τ_2, and $\bar{K}_{i,j}\left(\tau_1, \tau_2\right)$ is the penalty function for i and j, which is 0 for if they are in the same order and 1 for if they are in the opposite order. For the distance matrices, distance values are normalized as in Equation 4.2, where n is the number of ornaments in each set.

$$K(\tau_1, \tau_2) = \sum_{i,j \in P} \bar{K}_{i,j}\left(\tau_1, \tau_2\right) \tag{4.1}$$

$$\frac{K(\tau_1, \tau_2)}{\frac{n(n-1)}{2}} \tag{4.2}$$

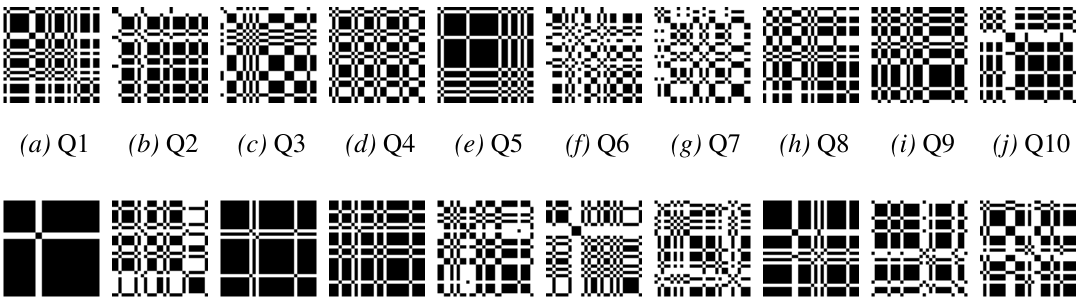
Calculated distance matrices for the first survey are shown in Figure 4.4. It can be interpreted from the matrices that the participants' answers were mostly the same for the first, third and fourth ornament sets. There was no consensus for the sixth, seventh and eighth questions.



(a) Q1 + Q11 *(b)* Q2 + Q12 *(c)* Q3 + Q13 *(d)* Q4 + Q14 *(e)* Q5 + Q15

(f) Q6 + Q16 *(g)* Q7 + Q17 *(h)* Q8 + Q18 *(i)* Q9 + Q19 *(j)* Q10 + Q20

Figure 4.4: Distance matrices of the first survey (For each ornament set)



(a) Q1 *(b)* Q2 *(c)* Q3 *(d)* Q4 *(e)* Q5 *(f)* Q6 *(g)* Q7 *(h)* Q8 *(i)* Q9 *(j)* Q10

(k) Q11 *(l)* Q12 *(m)* Q13 *(n)* Q14 *(o)* Q15 *(p)* Q16 *(q)* Q17 *(r)* Q18 *(s)* Q19 *(t)* Q20

Figure 4.5: Distance matrices of the first survey (For each question)

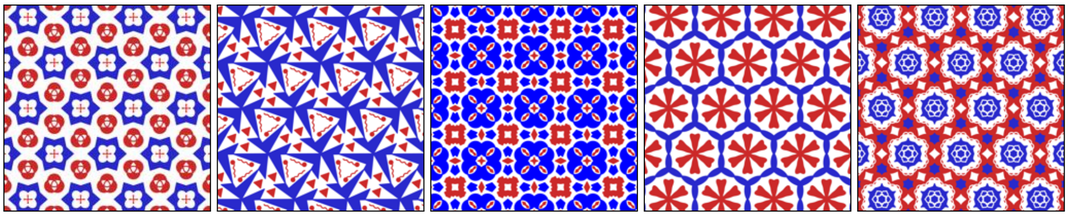
Figure 4.4 shows the results for each question set. To better evaluate or interpret the results, another set of distance matrices are calculated for each of the 20 questions (Figure 4.5). In this case, there was no inconsistency because the two types of questions were not evaluated together, so the answers of all participants were evaluated. In these matrices, identical answers are scored as 0 (black) and different answers are scored as 1 (white). These sets of matrices are calculated using all of the thirty participants' answers. It can be seen that for questions 5, 11 and 13 the answers were mostly the same and in questions 6, 7 and 16 the participants did not agree on an answer.

By combining the outcomes of the two types of distance matrices, we can say that all *cmm* types except for *cmm (ol)* were selected as the least similar to the Moroccan ornament. *cmm (ol)* was selected as the most similar ornament in three out of four questions, when it was against *cmm*, *p6*, *p4*, and *p4m* types. The only case that *cmm (ol)* was not selected as the most similar ornament is in questions 3 and 13, where a p6m type was selected by 17 out of 20 participants instead. In questions that contain a *cmm* option other than the *cmm (ol)*, the ornaments which has a six-fold or a three-fold rotation were selected as the most similar over four-fold rotation types. This superiority shows itself again in questions that have three different rotation types as options. In these questions, four-fold rotations were always selected as the least similar ornament to the Moroccan. For other options, we can say it is clear that *p6m* ornament was chosen over *p3m1* one but in questions 7 and 17 participants were indecisive between *p3m1* and *p6* types.

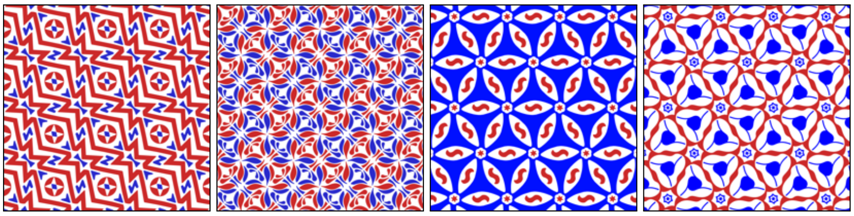
In summation, the participants did not recognize the Moroccan ornament by its characteristics that make it a *cmm* type, but its rotation symmetries. The only *cmm* type that was selected as the most similar, *cmm (ol)*, was also the only one in *cmm*-type ornaments that seem to have a rotation symmetry. Among the groups that have a rotation symmetry, participants mostly selected six-fold rotations as the most similar and four-fold rotations as the least similar ornaments to the Moroccan ornament.

4.2 The Second Survey

In light of the results of the first survey, a second survey is prepared. In the first survey it was understood that participants did not select *cmm* type ornaments as the most similar ornament except for the *cmm (ol)*. Therefore we decided to remove *cmm (1)*, *cmm (2)* and *cmm (3)* from the dataset. It was also seen in the first survey that ornaments that contain a four-fold or a three-fold rotation are underperformed against ornaments that have a six-fold rotation symmetry. As a result of this circumstance, it was decided to use one sample from three-fold and four-fold rotation groups in the second survey. *p3* from groups that have three-fold rotational symmetry and *p4m (1)* from groups that have four-fold rotational symmetry were selected considering they were only asked to the participants once and there were possible comparisons left



(a) cmm (ol) *(b)* p3 *(c)* p4m (1) *(d)* p6m (1) *(e)* p6m (2)



(f) cmm (4) *(g)* p4g/cmm *(h)* p6 (3) *(i)* p6 (5)

Figure 4.6: Ornaments used in the second survey

for these two ornaments. It was also decided to make alterations on two ornaments. First, we created *p6 (5)* by thickening the lines of *p6 (1)* to make the symmetries more visible in the presentation. Then we created *p6 (3)* by changing the colors of *p6 (2)* so that the S-shaped detail that removes the possibility of reflection symmetry becomes more visible. Our last modification to the data set was the addition of two more *cmm* type ornaments with one of them being a color symmetry group (*p4g/cmm*). In these ornaments, we tried to create interlocking motifs by reducing the amount of white spaces used in the base motif. The aim was to increase the chances of *cmm* ornaments to be selected as the most similar one to the Moroccan ornament. Whole data set for the second survey can be seen in Figure 4.6

In this survey, the second section is improved both to strengthen the domain knowledge of the participants and to create a way to check the success of the informative first section of the presentation. Instead of one, five warm-up questions are asked (Figure 4.7). Just like in the first survey, warm-up questions asked which option has the same repetition structure as the query ornament and unlike the survey questions, all warm-up questions had a correct answer in its possible options. This time, the responses of the participants were recorded to understand the level of understanding of the subjects described in the first section. In the first three warm-up questions, after

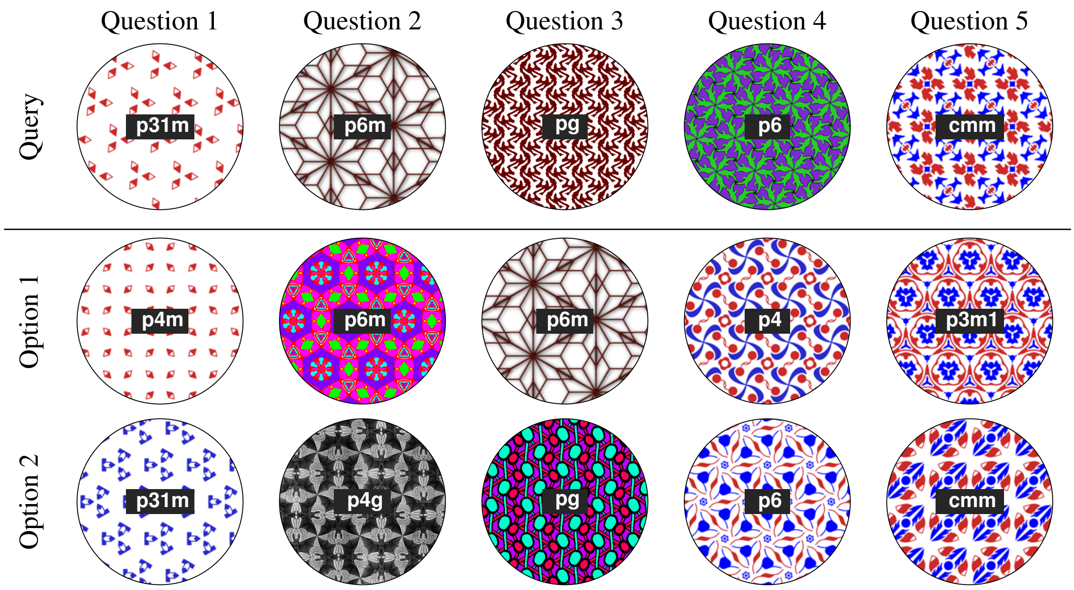


Figure 4.7: Warm-up questions used in the second survey

participants wrote their answers down, the correct answer was revealed to help them understand the concept. In the last two questions, the answers were not disclosed to ensure that they had not changed their answers.

In the third section of the survey, sixteen different questions were asked. This time we asked the participants to choose from two options instead of three, and we asked only to choose the option which has the same or the most similar repetition structure as the Moroccan ornament. Ornament sets for the sixteen questions are in Figure 4.8. Additionally, Question 1 was repeated in fifteenth place to find out how participants' choices are affected throughout the survey.

20 participants took part in the second survey, seventeen of them had also participated in the first survey. The results can be seen in Table 4.3. The first five rows in the table represent the warm-up questions and other rows contain the answers for the survey questions. For warm-up questions, the correct answer is italicized. It can be seen that almost all of the participants gave the correct answer for the first three warm-up questions by using the information given in the first section. For the fourth and fifth warm-up questions, 16 out of 20 participants gave correct answers. In total, 91% of the answers given to the warm-up questions were correct. It confirms that participants grasped the concept of symmetry before answering the survey questions in the third

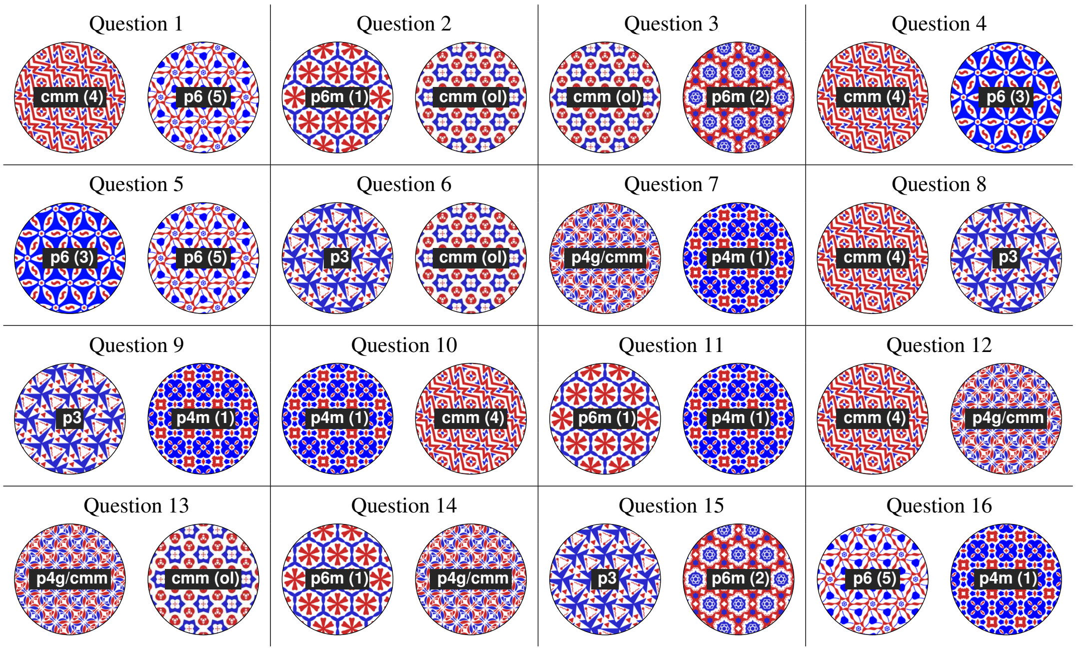


Figure 4.8: Questions in the second survey

section.

The other rows of Table 4.3 shows the participants' answers for the survey. To interpret the results easily, the existing nine ornament types are divided into seven triple groups. These triple groups include ornaments that are compared with each other in three different questions. Seven triple groups are numbered and their results are compared in Table 4.4. It can be seen in the results that *cmm (4)* was never selected as the most similar ornament to the Moroccan and *p4g/cmm* only came first once it was against *cmm (4)*. On the other hand, *cmm (ol)* was selected as more similar than three, four and six-fold rotation groups. The only case that ended in a tie was between *cmm (ol)* and *p6m (1)*. However, *cmm (ol)* was selected more than *p6m (2)*in a different question. When we look at the rotation groups, we can see the same results of the first survey repeating: Six-fold rotations are always found more similar than the three or four-fold rotations. *p4m* is only selected as the most against *cmm* (4) and *p4g/cmm* groups. The only three-fold rotation in the data set is chosen by the participants as more similar to the Moroccan than *cmm (4)* and four-fold rotation groups but less similar than *cmm (ol)* and six-fold rotation groups. When the repeated question was first asked to the participants, 19 participants had selected *p6 (5)* over

Table 4.3: *Results of the second survey (Correct answers of warm up questions are italicized. The most selected option for each question is in bold.)*

Questions	Option 1		Option 2		Questions	Option 1		Option 2	
	Group	Most	Group	Most		Group	Most	Group	Most
W1	w-p4m	1	***w-p31m***	19	↓				
W2	***w-p6m***	20	w-p4g	0	Q7	p4g/cmm	7	**p4m (1)**	13
W3	w-p6m	0	***w-pg***	20	Q8	cmm (4)	5	**p3**	15
W4	w-p4	4	***w-p6***	16	Q9	**p3**	12	p4m (1)	8
W5	w-p3m1	4	***w-cmm***	16	Q10	**p4m (1)**	14	cmm (4)	6
Q1	cmm (4)	1	**p6 (5)**	19	Q11	**p6m (1)**	16	p4m (1)	4
Q2	**p6m (1)**	10	**cmm (ol)**	10	Q12	cmm (4)	8	**p4g/cmm**	12
Q3	**cmm (ol)**	14	p6m (2)	6	Q13	p4g/cmm	2	**cmm (ol)**	18
Q4	cmm (4)	3	**p6 (3)**	17	Q14	**p6m (1)**	20	p4g/cmm	0
Q5	p6 (3)	8	**p6 (5)**	12	Q1 (2^{nd})	cmm (4)	3	**p6 (5)**	17
Q6	p3	2	**cmm (ol)**	18	Q15	p3	3	**p6m (2)**	17
				→	Q16	**p6 (5)**	15	p4m (1)	5

cmm (4). However, when they encountered the question again in the latter part of the survey, 17 participants selected the *p6 (5)* over *cmm (4)*. It can be apprehended that the results did not change dramatically over the time of the survey and participants' answers were consistent when they faced the same question again.

In conclusion, the first and second survey has similar results. *cmm (ol)* ornament is added to the dataset to resemble the Moroccan ornament in both appearance and symmetry groups. However, in some comparisons ornaments with a six-fold symmetry surpassed it: *p6m (1)* come up more similar than the *cmm (ol)* in the first survey and they tied in the second survey.

Although the Moroccan ornament was proven to be *cmm* in Chapter 3, participants always chose *cmm* as the least similar one. The sole exception was *cmm (ol)*, which was created specifically to resemble the Moroccan ornament.

In other words, participants always selected ornaments that have, or in *cmm (ol)*'s case seems to have, rotational symmetry as the most similar ornament. We introduced the ornament *p4g/cmm* which is a *cmm* type if the color is considered and a *p4g* type

Table 4.4: *Triple groups of ornaments in the second survey*

No	Group 1				Group 2				Group 3			
	Option 1		Option 2		Option 1		Option 2		Option 1		Option 2	
	Group	Most	Group	Most	Group	Most	Group	Most	Group	Most	Group	Most
1	cmm (4)	3	**p6 (3)**	17	cmm (4)	1/3	**p6 (5)**	19/17	p6 (3)	8	**p6 (5)**	12
2	cmm (4)	5	**p3**	15	cmm (4)	6	**p4m (1)**	14	**p3**	12	p4m (1)	8
3	cmm (4)	6	**p4m (1)**	14	cmm (4)	1/3	**p6 (5)**	19/17	p4m (1)	5	**p6 (5)**	15
4	cmm (4)	8	**p4g/cmm**	12	cmm (4)	6	**p4m (1)**	14	p4g/cmm	7	**p4m (1)**	13
5	**cmm (ol)**	18	p4g/cmm	2	**cmm (ol)**	10	**p6m (1)**	10	p4g/cmm	0	**p6m (1)**	20
6	**cmm (ol)**	18	p3	2	**cmm (ol)**	14	p6m (2)	6	p3	3	**p6m (2)**	17
7	p4g/cmm	7	**p4m (1)**	13	p4g/cmm	0	**p6m (1)**	20	p4m (1)	4	**p6m (1)**	16

otherwise, to challenge this disposition. However, participants always selected it as the least similar one against the ornaments that have a rotational symmetry. If we compare the rotation types there is a definitive ranking in which the six-fold rotations are the most similar and the four-fold rotations are the least similar ornaments to the Moroccan. Three-fold rotations were always resulted in between. Evidently, there is a distinction in participants' answers between other isometry types as well. From the ornaments that already have a rotation symmetry, the ones that also have mirror reflection were chosen slightly more than the ones with no mirror reflection or glide reflection.

CHAPTER 5

CONCLUSION

In this thesis, we approached the problem of classifying symmetrical properties of ornament. First, we explained how the classification process is made and explained the situations that make the classification of ornaments difficult. Then we talked about challenging ornaments; we gave examples and elaborately analyzed the Moroccan ornament. Based on this ornament, we set up survey experiments. In these experiments, we asked the participants in multiple-choice questions to select the ornament that has the most similar repetitive structure to the Moroccan ornament. We created a new dataset for these surveys. Our dataset contained 18 ornaments that have similar symmetrical properties to the Moroccan ornament. We created six ornaments that belong to the cmm group as the Moroccan ornament. In one of these ornaments, we replicated the exact repetitive structure.

The results showed that between six cmm-type ornaments, the participants only found the ornament that has the same repetitive structure similar to the Moroccan. Besides, they found the other five cmm-type ornament to be the least similar between the 18 possible options. Also, the participants identified ornaments that have high-order rotation symmetries more similar than groups that have low-order rotation symmetries. In other words, when the local symmetries did not match with the global symmetry, participants could not detect the symmetry group.

The group-theoretical framework classifies symmetrical patterns by their global symmetry. The results of the experiment suggest that human perception of symmetry also depends on local symmetries. In challenging cases, the human perspective seems to detect local properties over global symmetry and perform comparison between local properties of the ornaments rather than considering the distinctions between wallpaper groups.

We conclude that what human see is beyond this strict 17 symmetry groups. Wallpaper group of an ornament does not contain information about local symmetries. However, the human perspective of repetitive structures depends on these local properties.

Exciting research directions as future works are listed below:

Hypersymmetric ornaments have higher local symmetries than their global wallpaper group. We did not notably include these ornaments and their characteristic properties in the design process of our surveys. In our future work, we plan to explore hypersymmetry to understand the human perspective of local symmetrical properties better.

Another research direction is to detect the whole repetitive structure of an ornament computationally. Current methods can be improved by not considering the restrictions of strict wallpaper groups. Studies on computational methods can help understand the differences between symmetrical properties of ornaments, even when they belonged to the same wallpaper group.

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