ESTIMATION OF EFFECTIVE PARAMETERS AND OPERATIONAL RESPONSE OF TUNED VIBRATION ABSORBERS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

HASAN CAN ÖZDEN

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
MECHANICAL ENGINEERING

SEPTEMBER 2019

Approval of the thesis:

ESTIMATION OF EFFECTIVE PARAMETERS AND OPERATIONAL RESPONSE OF TUNED VIBRATION ABSORBERS

submitted by **HASAN CAN ÖZDEN** in partial fulfillment of the requirements for the degree of **Master of Science in Mechanical Engineering Department, Middle East Technical University** by,

Prof. Dr. Halil Kalıpçılar
Dean, Graduate School of **Natural and Applied Sciences** ______________

Prof. Dr. M.A. Sahir Arıkan
Head of Department, **Mechanical Engineering** ______________

Assist. Prof. Dr. Gökhan O. Özgen
Supervisor, **Mechanical Engineering, METU** ______________

Examining Committee Members:

Assoc. Prof. Dr. Bülent Özer
Mechanical Engineering, METU ______________

Assist. Prof. Dr. Gökhan O. Özgen
Mechanical Engineering, METU ______________

Assoc. Prof. Dr. Yiğit Yazıcıoğlu
Mechanical Engineering, METU ______________

Prof. Dr. Ender Ciğeroğlu
Mechanical Engineering, METU ______________

Assoc. Prof. Dr. Çağlar Başlamışlı
Mechanical Engineering, Hacettepe University ______________

Date: 03.09.2019

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Surname: Hasan Can Özden

Signature :

ABSTRACT

ESTIMATION OF EFFECTIVE PARAMETERS AND OPERATIONAL RESPONSE OF TUNED VIBRATION ABSORBERS

Özden, Hasan Can
M.S., Department of Mechanical Engineering
Supervisor: Assist. Prof. Dr. Gökhan O. Özgen

September 2019, 113 pages

In a certain frequency vibration induced problems, tuned vibration absorbers (TVA) are widely used in order to decrease vibration amplitudes of structures at tuned frequency. If the tuned frequency can be changed by altering effective parameters such as stiffness and damping of the TVA by passive or active methods, this type of TVA can be referred as adjustable tuned vibration absorber (ATVA). In the first part of this thesis study a previously designed variable stiffness adjustable frequency TVA has been investigated in the regard of estimating tuning frequency. Firstly, FE model based tuning frequency estimation has been done for different stiffness values obtained by different configurations of the device. Later, experiments has been carried out in order verify FE model of the TVA used to estimate tuning frequency. Lastly, a relationship has been developed for estimating tuned frequency of the device. Motivation for the second part of this thesis study is based on performance evaluation of this previously designed TVA in regard of fatigue life. However this evaluation can be made only if 3D model or mathematical model of a host structure to be attached by TVA, is available as well as with the forcing acting on it. Therefore, simulations

can be carried out obtained by numerical solution of host structure attached by TVA using analytical methods or FE analysis. However, in practical applications, generally dynamic properties of the host structure and excitation source are not available readily. Therefore using FEM methods or mathematical models are not suitable in order to predict modified structure response so that performance of the TVA could be evaluated. In the second part of the thesis study development of a method is given where response of the host structure modified by TVA at TVA attachment DOF and response of the attached TVA in the operational conditions are estimated without using dynamic model of host structure and actual excitations acting on it. Developed method uses only operational response of the host structure at related DOF as well as direct point FRF of this DOF along with TVA effective parameters. Lastly a case study is presented in order to verify developed method on a real life application.

Keywords: vibration control, tuned vibration absorber, identification, system modification, fatigue, response estimation

ÖZ

AYARLANMIŞ TİTREŞİM EMİCİLERİN ETKİLİ PARAMETRELERİNİN BELİRLENMESİ VE OPERASYONEL CEVAPLARININ TAHMİNİ

Özden, Hasan Can
Yüksek Lisans, Makina Mühendisliği Bölümü
Tez Yöneticisi: Dr. Öğr. Üyesi. Gökhan O. Özgen

Eylül 2019 , 113 sayfa

Belirli titreşim kaynaklı problemlerde, ayarlanmış titreşim emiciler (TVA), bağlanacakları yapının ayarlandıkları frekanstaki titreşim genliklerini azaltmak için yaygın olarak kullanılırlar. Ayarlanan frekans pasif veya aktif yöntemlerle TVA'nın katılık ve sönümleme parametreleri ile ayarlanarak değiştirilebiliyorsa, bu TVA tipleri ayarlanabilir titreşim emiciler (ATVA) olarak adlandırılabilir. Bu tez çalışmasının birinci bölümünde, önceden tasarlanan değişken katılıklı ayarlanabilir frekanslı bir TVA'nın ayarlanan frekansının tahmin edilmesi için çalışılmıştır. İlk olarak, cihazın farklı konfigürasyonları ile elde edilen farklı katılık değerleri için sonlu elemanlar modeli kullanılarak ayarlanan frekans tahmini yapılmıştır. Daha sonra, ayarlama frekansını tahmin etmek için kullanılan TVA'nın FE modelini doğrulamak için deneyler yürütülmüştür. Son olarak, cihazın ayarlanmış frekansını tahmin etmek için matematiksel bir ilişki geliştirilmiştir. Bu tez çalışmasının ikinci bölümünde yer alan motivasyon, daha önce tasarlanan bu TVA'nın yorgunluk ömrü açısından performans değerlendirmesinin yapılmasına dayanmaktadır. Ancak, bu değerlendirme TVA tarafından modifiye edilen yapının 3B modeli veya matematiksel modeli yanı

sıra, ona etki eden tahrikler biliniyorsa yapılabilir. Böylelikle, analitik yöntemler kullanılarak ya da sonlu elemanlar analizi yapılarak simülasyonlar yürütülebilir. Ancak, pratik uygulamalarda, genellikle konakçı yapının dinamik özellikleri ve ona etki eden tahrik kaynakları hali hazırda temin edilememektedir. Bu nedenle sonlu elemanlar yöntemleri veya matematiksel modeller, TVA'nın performansının değerlendirilebilmesi için değiştirilmiş yapının ve TVA'nın tepkisini öngörmek için uygun değildir. Tez çalışmasının ikinci bölümünde, bu konuk yapının dinamik modeli ve ona etki eden gerçek tahrik kaynakları kullanılmadan, TVA'nın ve TVA tarafından modifiye edilmiş bu konak yapının işletim koşullarında yanıtlarının tahmin edildiği bir yöntem geliştirmiştir. Geliştirilen bu yöntem, TVA etkin parametrelerinin yanı sıra, ilgili serbestlik derecesindeki operasyonel cevabı ve bu serbestlik derecesindeki direk nokta frekans tepkisini kullanmaktadır. Son olarak, gerçek bir uygulamada geliştirilen bu yöntem kullanılarak doğrulanmıştır.

Anahtar Kelimeler: titreşim kontrolü, ayarlanmış titreşim sönümleyici, karakterizyon, sistem modifikasyon, yorulma, cevap tahmin etme

Sevgili aileme.

ACKNOWLEDGMENTS

The author wants to thank his supervisor Asst. Prof. Dr. Gökhan Özgen for his invaluable advices, critics and suggestions throughout this thesis study.

The author also wishes to thank his colleague Yunus Yılmaz and Barkan Ulubalcı because of their substantial support and assistance during thesis study.

Finally author wants to express his very profound gratitude to his parents and to his wife for providing him with unfailing support and continuous encouragement throughout years of study and through the process of researching and writing this thesis.

For some of the experiments conducted in this thesis, some of the equipment (shaker and shaker control system) and test setups acquired through TÜBİTAK 1001 grant number 214M065 were used.

The prototype of adjustable tuned vibration absorber tested and modeled in this thesis was designed and fabricated by a team of senior year undergraduate students comprised of Barkan Ulubalcı, Boğaç Can Yıldırım, Mehmet Efe Tiryaki, Pelin Yılmazer, Yunus Yılmaz and the author of this thesis, as part of a term project assignment of ME 407 Mechanical Engineering Design in the Department of Mechanical Engineering, Middle East Technical University titled "Design of Frequency Adjustable Tuned Vibration Absorber".

TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

LIST OF ABBREVIATIONS

ABBREVIATIONS	
3D	3 Dimensional
TVA	Tuned Vibration Absorber
ATVA	Adjustable Tuned Vibration Absorber
SDOF	Single Degree of Freedom
MDOF	Multi Degree of Freedom
TMD	Tuned Mass Damper
DOF	Degree of Freedom
FRF	Frequency Response Function
PSD	Power Spectral Density
FE	Finite Element
FEA	Finite Element Analysis
RMS	Root Mean Square

CHAPTER 1

INTRODUCTION

Tuned vibration absorbers (also known as tuned dynamic absorbers), are frequently used devices for vibration problems in order to absorb oscillations at particular forcing frequency or damp vibration amplitudes at resonant frequency of the structure. TVAs are generally consist of an inertia element, a stiffness element and an energy dissipation element. If stiffness or damping parameters can be altered via passive or active methods, these type of TVA can be referred as adjustable tuned vibration absorbers (ATVA). This kind of TVAs works in a frequency range obtained by stiffness and/or damping parameter change. Therefore, if host structure troublesome vibration frequency or frequency band shifts due to change of loading characteristics or structural dynamic, by adjusting absorber parameter, this type of devices can adapt altered conditions passively or actively for absorption of oscillations.

An adjustable TVA with stiffness variable element has been developed previously in the scope of design project of METU ME of which author of this thesis was the member of the design team. Design of this ATVA consist of a mass element, stiffness element and magnetic damping element. For stiffness element of the design, a circular beam is used where the beam's ends are constrained with clamped boundary condition. This stiffness element is actually a beam constrained at both ends formed in a circular geometry instead of a straight one. At very center of this circular beam, a bulk cylindrical copper mass element is located. Magnets have been used surrounding this copper mass in order to dissipate energy by means of resilient magnetic forces. Copper mass constrained by a linear bearing and shaft couple therefore only transverse movement of beam center is allowed. Stiffness variation of the design is achieved by changing the location of the clamped boundary conditions implemented on the circular beam stiffness element. By changing the location of the clamped boundary conditions, active length of beam referring to part of the beam that can be elastically deformed, changes that is resulting in a different stiffness values.

During the project studies, especially three design requirements have been challenging that lead to this type of stiffness element and stiffness variation method. The two of them were volume occupation and aspect ratio of the design. Volume of the design limited to a certain value by design criteria, additionally aspect ratio of the design is expected to be as much as close to one. These two requirements make it unavailable to be construct the design as a lengthy, slender structure. Third challenging requirement was to cover a frequency range that ATVA is going to be operating in. However in earliest times of the project proposed design ideas for stiffness element in order to cover a certain frequency range occupy much space and make the aspect ratio overall design of ATVA far from one. In order to cover this frequency range while occupying less volume with aspect ratio of one, circular beam has been developed and used for stiffness element that met the requirements.

During project, several of studies have been conducted in order to decide crucial design parameters and verify them. One of the important study was determination of the working frequency range of the absorber by FEA. In this FEA, modal analyses of stiffness element is carried out for at two extreme boundary conditions. By help of this study max and min tunable frequency of ATVA is obtained. In order met the frequency range (max and min. frequency) requirement design parameters are revised (material, dimensions geometry etc.). Through experimental studies conducted at these two extreme conditions, frequency range of ATVA has been verified as well. Another expectation of this ATVA was to act as an SDOF system in a particular frequency range. Therefore, in this certain frequency range there shouldn't be resonant frequency of ATVA device other than its tuned frequency. In order to verify this requirement, modal analysis of the ATVA have been done whether to observe its closest modal frequency to its tuned frequency is in the prescribed range. For this analysis, FEM is constructed using all structural elements of the device as well as its stiffness and mass elements. Another example of studies was experimental determination of damping coefficient of ATVA with respect to different magnet distances to copper mass. In this study main objective was construction of a look up table for varying damping coefficients obtained via changed positions of magnets. Evaluation of designed ATVA in other aspects other than mentioned studies considered as essential to improve design parameters and to put forward the features of the device. However during the project some of the critical features of the devices couldn't be investigated and evaluated.

In this thesis study investigation of the two essential feature of the previously designed ATVA has been done. First study is focused on tuning law of the ATVA where relationship of the tuned frequency of the device with boundary condition locations applied on its stiffness element has been established. For this purpose, absorber FEM is constructed for each configuration sample (different boundary conditions). For each of them modal analysis is carried out. Results of the FEA are then verified by experimental studies. As a result of this study, a linear relationship has been identified that relates tuned frequency of device to a physical parameter of stiffness element for tuning law of the absorber.

Second study of this thesis study based on investigation of absorber fatigue life. Vibration induced fatigue failure is well known phenomena in the literature. Vibration absorber are auxiliary vibrating unit attached to host structure, in fact, their primary task is to vibrate rather that structure by means of vibration energy transfer from host structure to absorber. Therefore investigation of the fatigue life of a vibration absorber presents great importance. In order to apply fatigue life prediction methods on this absorber, one should first obtain stress response of its stiffness element during operation. If host structure to be attached by TVA dynamic model is known together with excitation sources acting on it, stress response of the stiffness element can be calculated by FEM, numerical or analytical methods. However, in practical applications, generally host structure dynamic properties or 3D model does not exist or too complex to model and measure. Moreover, excitations sources are unknown or not practical to measure. Under these overwhelming conditions where absorber modification is intended, evaluation of in-situ TVA performance (when attached to host structure in operational conditions) is troublesome. There is no study presented on focused on finding response in these conditions. Therefore, in this thesis study, a method is proposed where estimation of TVA and host structure operational response is estimated using a practical procedure and generate necessary information obtained to evaluate the absorber in regard of its fatigue behavior. Then, method is validated on case study using a basic, tuned to single frequency vibration absorber rather than using previously designed ATVA. By this way, uncertainties are reduced that can affect experimental results. Moreover, the experiments were carried out in a more practical way.

In first chapter literature survey consisting of a brief theory section about tuned vibration absorbers followed by various examples of vibration absorbers. Moreover, adjustable vibration absorbers theory and several types of stiffness variable tuned vibration examples are explained regarding their frequency coverage and stiffness variation methods. TVA evaluation methods are given based different type of performance parameters. Fatigue life estimation studies for elastic components of vibration isolations systems briefly explained. Since TVA is type of a structural modification, system modifications techniques are investigated.

In the second chapter, investigation of the tuning law of absorber is presented. Firstly, an introductory part is given where the TVA design is explained. Later, FEA of the absorber for different configurations of boundary conditions are shared. In following part of the chapter experimental studies that used to validate the results of the FEA are presented. In the discussion part of the chapter, evaluation of the presented studies and suggestion for further investigation on the absorber features are given.

In the third chapter, developed method for the operation response estimation of the TVA and host structure in-situ conditions is presented. Method is firstly described on a process tree, later, analytical relationship has been put forward on an example 2DOF host structure attached with 1DOF TVA. Lastly a numerical example problem is solved by proposed method and using actual system dynamic information and acting forces. Then results are compared with each other.

In the fourth chapter, a case study is presented where developed method presented in chapter 3, is employed. In order to apply and verify developed method an example vibration problem is created in laboratory environment where a cantilever beam element is excited by a dynamic shaker. Tip of the beam taken into consideration in regard of vibration response. In order to suppress oscillations around a particular frequency a prototype of small TVA is designed and manufactured. Before physical implementation of prototype of TVA on to the host structure, TVA response and host structure response are firstly estimated by developed method. Then, experiments have been conducted where TVA physical integration on the host structure is realized. During the experiments host structure is excited again with the same forcing and TVA mass response and host structure responses are acquired. Actual results obtained via experiments are then compared with responses acquired by developed method. Lastly stress PSD of a selected element on the stiffness component of the TVA is obtained using developed method.

In the final chapter of this thesis, main results of the studies are summarized and evaluated. General inferences in the aspect of the previously designed ATVA is presented together with suggestions about future work to characterize and improve the design. Additionally results obtained using developed method are summarized and evaluated.

CHAPTER 2

LITERATURE SURVEY

Vibration is harmonic or random motion of particles of material depending on forcing characteristics, existing around equilibrium point of material because of the elasticity of material. In engineering world, vibration is the one of the main reasons that causes machines, electronic devices etc. to fail or even break downs because of damage on mechanical components or structural elements. Moreover,vibration can affect performance of devices caused from unwanted oscillations. Because of this high potential of vibration leading problematic results, engineering world also academic society focus on reducing vibration levels existing on structural elements, mechanical components etc. There are several methods to diminish vibration induced problems on structure, however, reducing or controlling vibration amplitudes is suitable method when vibration source cannot be eliminated and/or modifying system is only option left to solve vibration problem. TVAs are frequently used to reducing vibration levels for particular frequency of frequency band by modifying the structure.

2.1 Tuned Vibration Absorber Theory

Most of the mechanical systems are subjected harmonic or random excitation due to moving mechanical components of systems. Moreover, structures may have been built on vibrating base that could excite structure in by means of harmonic or random manner. These examples can be increased in numbers. Yet, all could result in unwanted vibration levels on mechanical components or structural elements around particular frequencies either due to forcing frequency or cause of resonant characteristic of structure. When modifying the structure is an available option, TVA usually selected method to reduce and controlling vibration amplitudes around selected frequency mainly referred as tuned frequency. TVA is invented by Frahm in 1909 [1], since then TVA has vast usage area in vibration engineering field because of its high potential for solving vibrational problem by its simple form. To generalize, TVAs are used under two particular methods that lead two application of use for different problems [2]:

- Absorption of vibration at resonant frequency due to modal characteristics of structure: In this case TVA is tuned slightly lower frequency than natural frequency of structure that is interested. In this case damping should have been added to TVA in order to suppress over range of frequencies. This type of TVA also can be referred as Tuned Mass Dampers (TMDs).

- TVA can be tuned to a specific frequency where vibration amplitudes are critical level due to excitation frequency. In this application TVA is tuned to problematic frequency and suppress vibration over a narrow band around tuned frequency. When damping is not persisting on TVA, total absorption of vibration at tuned frequency is achieved.

In application TVA basically comprise of mass, spring and damping element. To better understand the theory of the TVAs, consider a SDOF system consisting a spring, damping and a mass element [3];

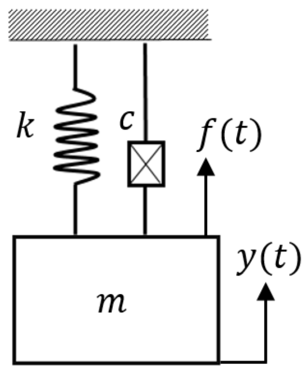

Figure 2.1: SDOF System

Differential equation of motion of SDOF system is given by below;

$$\ddot{y} + c\dot{y} + ky = f(t) \tag{2.1}$$

Where m is the mass, k is the stiffness and c is the viscous damping coefficient. The natural frequency (ω_n) and the fraction of critical damping (ζ) are defined as;

$$\omega_n^2 = \frac{k}{m} \tag{2.2}$$

$$2\zeta\omega_n = \frac{c}{m} \tag{2.3}$$

The equation of motion is rewritten (if the system is lightly damped $\zeta << 1$);

$$\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = \frac{f(t)}{m} \tag{2.4}$$

Solving the differential equation one gets transfer function between excitation and the absolute response as;

$$H(\omega) = \frac{Y(\omega)}{F(\omega)} = \left(\frac{1}{m}\right)\left(\frac{1}{(\omega_n^2 - \omega^2) + 2i\zeta\omega\omega_n}\right) \tag{2.5}$$

Frequency response of the SDOF given in graphical form in Figure 2.2

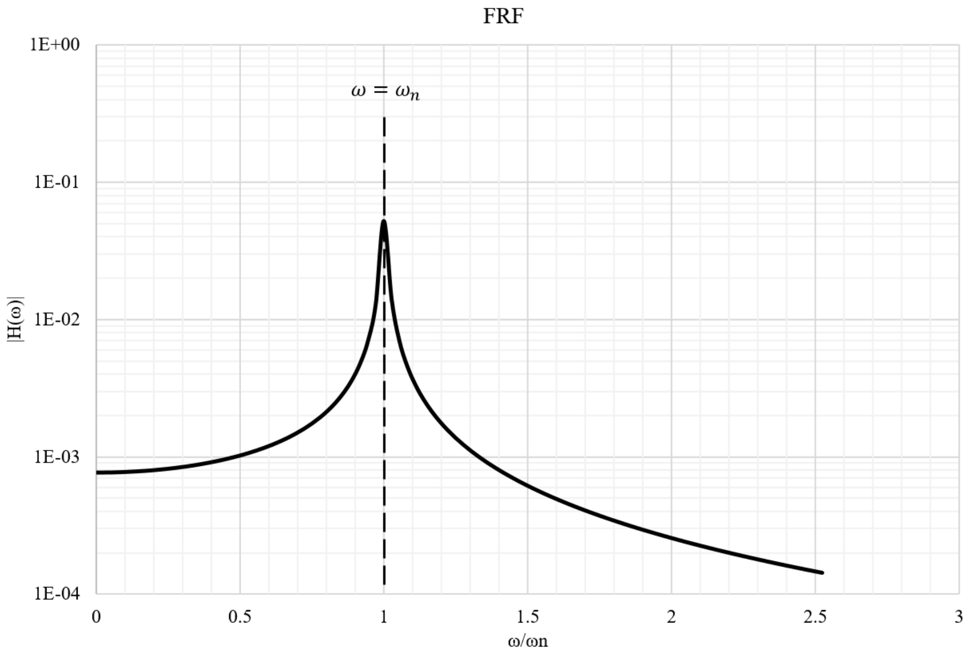

Figure 2.2: SDOF Represantative FRF Graph

A TVA can modify SDOF system as such that at $\omega = \omega_n$ system response is reduced drastically. See Figure 2.3 for modified system. (Host structure damping is neglected in the following section)

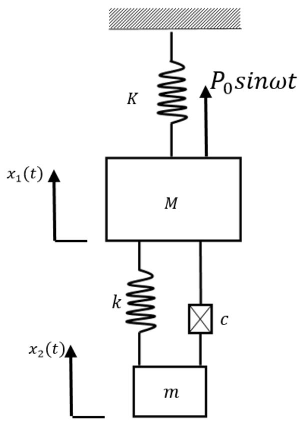

Figure 2.3: Modified system with TVA

In the Figure 2.3 TVA mass, damping and spring constant are defined by m, c, k respectively.

Force equilibrium is written for Mass M and small mass m [4]

$$M(\ddot{x_1}) + Kx_1 + k(x_1 - x_2) + c((\dot{x_1}) - (\dot{x_2})) = P_0 \sin \omega t \tag{2.6}$$

$$m(\ddot{x_2}) + k(x_2 - x_1) + c((\dot{x_2}) - (\dot{x_1})) = 0 \tag{2.7}$$

Since only steady state response is considered for the time being both equations (2.6 and 2.7) are rewritten as given below;

$$-M\omega^2 x_1 + Kx_1 + k(x_1 - x_2) + i\omega c(x_1 - x_2) = P_0 \tag{2.8}$$

$$-m\omega^2 x_2 + k(x_2 - x_1) + i\omega c(x_2 - x_1) = 0 \tag{2.9}$$

Using equation 2.8 and 2.9 one can solve for x_1;

$$\frac{x_1}{P_0} = H(\omega) \tag{2.10}$$

$$H(\omega) = \sqrt{\frac{(k - m\omega^2)^2 + \omega^2 c^2}{((-M\omega^2 + K)(-m\omega^2 + k) - m\omega^2 k)^2 + \omega^2 c^2 (-M\omega^2 + K - m\omega^2)^2}} \tag{2.11}$$

As it seen from the eqn. 2.11 numerator , as frequency approaches the natural frequency of the TVA ($\omega = \sqrt{\frac{k}{m}}$) alternatively tuned frequency, response of the main mass approaches the zero. This deduction clearly shows that TVAs are suitable devices to eliminate vibration at problematic frequencies by adjusting parameters of the TVA as it is mention.As doing such referring to Figure 2.2; setting the absorber parameter equal to the natural frequency of SDOF system ω_n given in Figure 2.1 below FRF of Mass M ($\frac{x_1}{P_0}$) is obtained and given in the Figure 2.4.

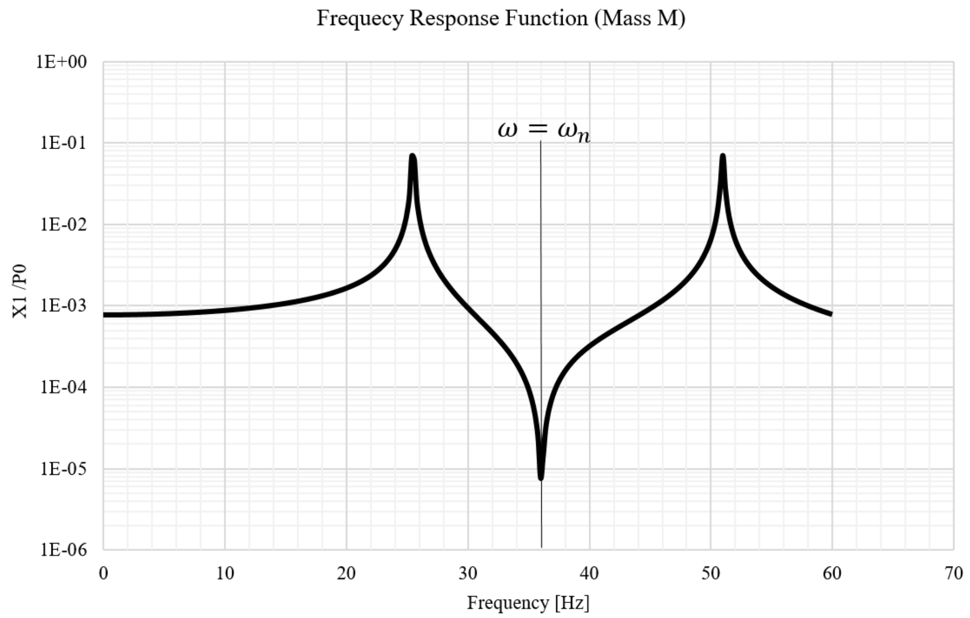

Figure 2.4: Modified system with TVA: Mass M (x_1) FRF

As it is seen from Figure 2.4 at desired frequency ($\omega = \omega_n$) vibration amplitudes are dropped drastically. Referring to the eqn. 2.11 vibration characteristics can be also controlled around this frequency by help of setting mass ratio (m/M), and damping (c).

In studies of Özgüven's and Çandır's [5], they used two vibration absorber consisting of spring, mass and damper elements, to suppress the first and second resonance of the cantilever beam. Absorber are attached to end of the beam where the displacements occurring is most. By doing so maximum performance is aimed. Moreover parameter optimization (tuning frequency and damping ratio) is done by solving max-min problems. J. C. Snowdan [6], in his work, designed a cruciform dynamic absorber consisting of two free-free beams where mass are located and the free ends of the beams. Two beams are connected each other centrally at right angles at which it is supposed to attached to host structure. Purpose of this type of absorber is suppressing more than one troublesome frequency on host structure.

M.R. Jolly [7] presented theoretical study presented where SDOF TVAs attached to a vibrating simply supported panels in order to reduce sound radiation. Absorbers are tuned to critical modes of panel in order to increase effectiveness. It is also emphasized that when TVAs are tuned to a mode other than critical modes, effectiveness drops drastically. S.M. Hashemi [8] studied a TVA application designed for suppression of rest tremor in Parkinson's disease. In the application, human arm is modeled as two degree of freedom host structure in order to develop tremor attenuation strategy. Firstly numerical study carried out followed by experimental work where optimum parameters are decided that lead to $\%80$ vibration reduction.

2.1.1 Adjustable Tuned Vibration Absorbers

Adjustable TVAs can be referred as Semi-Active vibration absorbers in literature. This reference implies that there is no active system that needs energy to perform. On contrary, semi-active systems are passive but has tunable characteristics. Alternatively, absorber parameters can be altered in order to adjust the absorber so that transfer vibration energy to absorber at the interested frequency or frequency range thus vibration amplitudes are reduced and controlled [9].

Mechanical systems operating under wide band load and frequency range requires more complicated treatment when it comes to controlling vibration. Otherwise it would lead insufficient or even damaging results trying to impose single choice of stiffness and damping. De-tuning (mistuning) is an example of the damaging results since it could cause performance losses even, host structures response could be amplified. This problem far more important when considering absorbers without damping mechanism [2]. In order to prevent from such adversities, adjustable TVAs are introduced. As it is described before, this kind of TVA's parameters can be changed, in other words, stiffness values of spring element as well as damping coefficient of the damping element can be tunable. Hence absorber tuned frequency can be changed by help of this phenomena. Considering below two degree of freedom system where above system denotes absorber and below represents host structure.

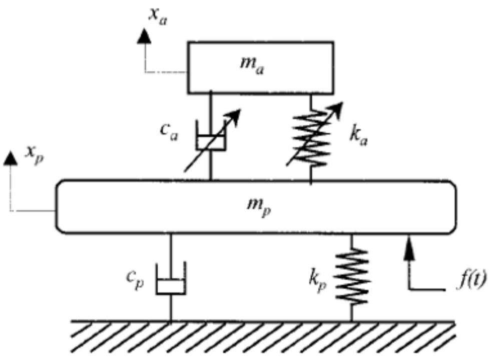

Figure 2.5: Host Structure with Absorber which has tunable damping and spring constant c_a and k_a

Transfer function that relating $X_p(s)$ and $F(s)$ given in the eqn. 2.12.

$$TF(s) = \frac{X_p(s)}{F(s)} = \frac{m_a s^2 + c_a s + k_a}{H(s)} \tag{2.12}$$

Where;

$$H(s) = m_p s^2 + (c_p + c_a)s + k_p + k_a(m_a s^2 + c_a s + k_a) - (c_a s + k_a)^2 \tag{2.13}$$

$x_p(t)$ host structure displacement, $f(t)$ external forcing, k_a and c_a adjustable stiffness and damping $X_p(s)$ and $F(s)$ laplace transformations of $x_p(t)$ and $f(t)$, respectively. Steady state response of host structure given in the eqn. 2.14.

$$\left|\frac{X_p(\omega)}{F(\omega)}\right| = \left|\frac{k_a - m_a\omega^2 + jc_a\omega}{H(\omega)}\right| \tag{2.14}$$

When subjected to harmonic excitation and there is no damping on absorber, it's possible to make response of host structure equal to zero.

$$k_a = m_a\omega^2, c_a = 0 \tag{2.15}$$

As seen from equation 2.15 even though troublesome frequency changes, by changing the stiffness value of the absorber (k_a) vibration can be taken under control. Also this flexibility make it possible to do tuning optimization of absorber easily and offers more convenient methods.

2.2 Various Types of Stiffness Variable Tuned Vibration Absorbers

In literature there are various types of stiffness variable TVAs using different types of stiffness elements with adjusting mechanism. Their stiffness variation methods are investigated and range of frequency coverage are evaluated. One of the early variable stiffness absorber proposed by Longbottom et al. in 1990 [10]. Pneumatic rubbers are used as stiffness elements and mass is located between these rubbers. Stiffness is adjusted by means of air pressure inside of the rubbers. One of the major disadvantage of the device was high amount of damping resulting from rubber structure itself. One of the recent method introduced by Rustighi et al. in 2005 [11]. They introduced beam-like TVA where beam-like structure is used as spring element. They integrated shape memory alloy conductor (SMA) wire forming double cantilever beam. Two beams connected centrally where the host structure supposed to attach. Young-modules of the beam-like element changes when temperature variation occurred via SMA. Hence stiffness of the spring element alters, thereby tunable frequency is achieved. Adjustment is done by controlling the current on the SMA wire. This device offers 20% frequency change. One of the main disadvantage of this device is that it took 2 min. with 9A current to change the tuned frequency up to 20% Another recent study by Bonello et al. in 2005 was consisting mass-spring TVA with two parallel curved beam [12]. Curved beams are used in longitudinal compression as a spring element of this design. Stiffness is adjusted controlling the curvature of the beams. Curvature control is achieved by using piezo-ceramic actuators embedded on beams. This idea is tested with prototype that can be adjusted in frequency range of 36-54 Hz (by 56% change). They tested TVA on shaker by random excitation given on the base of the prototype. Then transmissibility frequency response is measured relating base acceleration to that of the TVA mass. Excitation is given in two levels. Even though the frequency tracking abilities are rapid, inertia of the beams limited the application for low frequencies. Another beam-like TVA design proposed by Carneal et. al. in 2004 [13]. They developed adjustable TVA for minimization of the sound radiation from plates. The design (called V-type TVA) consist of two parallel shafts one adjustment screw positioned between shafts. There are two supporting ends at which shafts are guided through. At the center of the supporting ends threaded holes exists where adjustment screw is attached. Stepper motor is connected to adjustment screw so that supporting ends moves as the stepper motor operates. In this way double ended shafts boundary conditions are changes, hence tuning frequency could be adjustable. They tested V-type TVA under different distances of supporting ends. Results are showing that tunable frequency range is between 95-310 Hz. Theoretical study of another beam-like TVA has been done by Brennan in 2000 [14]. The conceptual design involves two beams where actuator used for pushing the apart two constituent beams at center. By doing so, beam cross-section changes so that effective stiffness. The experimental study of this design carried out by Kidner et al. in 2002 [15]. They built four beam-like neutralizers with the different configurations where the attachment type of constituent beams differ from each other. (Parallel, triangular, pinned and clamped type). They measured input force given to absorber by shaker and acceleration of the beam tip. As a result of this frequency response functions are obtained. Also for different separation values tuned frequencies are compared with that of analytical predictions. Clamped type and pinned type configurations showed good agreement between experimental results and analytical models. For these configurations frequency range test for absorber carried out and max change of 35% is acquired (192-142 Hz). In the study of Gorka Aquirre et al. in 2013, they designed, built and tested an adjustable vibration absorber for machine tools in order prevent chattering to occur [16]. In order to adjust the tuned frequency, a rotary element with variable stiffness is used. (Rotary stiffness element has unsymmetrical characteristics, therefore angular orientation results in different stiffness values.) A stepper motor controls the angular orientation of this element, thus stiffness changes as well as tuned frequency. Their test results indicates total change of frequency in the range of 66 to 105 Hz. An example study on TMD (tuned mass damper) using leaf spring is carried out by Daniel Gsell et. al. in 2007 [17]. Their main purpose was suppress the vibration mitigation on lightweight bridges due to high live load on them. Since this high load is subject to change depending on what kind of transportation vehicle crossing on, natural frequency of the bridge also alters continuously. This requirement suggest using an adjustable vibration absorber on bridge. They used two parallel leaf springs where the absorber mass is located centers of the springs. Stiff steel frame is used in order to support the ends of the leaf-springs. In order to stress the beams and alter the stiffness, they utilized PZT stack actuators on leaf -springs ends that has free displacement up to 30 micro meter. By help of these actuators up to 2.1 kN force is applied axially along the leaf-springs, so that the springs stiffness values change. They tested their prototype by experimental methods. They excite absorber mass with white noise signal forcing. By help of the accelerometer they measure the acceleration response of the mass under different values of stress level existing on leaf-springs. At the zero pre-stress level natural frequency of the absorber is found as 13.8 Hz whereas at the maximum pre-stress level absorber could be tuned up to 20.5 Hz. (max. change is up to 48.5%).

2.3 Evaluation of Tuned Vibration Absorbers

Regardless of type of vibration absorber or system at which absorber is going to be used, there is an iterative designing process of TVA where performance of it is evaluated on different type of aspects and followed by optimization procedure at which TVA parameters are set accordingly. In the following studies TVA is optimized on different aspects. In Tigli,O [18] study, focus is on optimizing dynamic vibrations attached to linear damped systems under the random loading. Optimizing criteria is set as minimizing the variance of the displacement, velocity and acceleration of main mass. 2DOF system where primary mass is host structure and secondary mass is TVA mass. System is solved with excitation that is assumed as white-noise input spectrum. In Asami, T. [19] work, dynamic absorber is studied attached to linear system under stationary random excitation. Optimization performance criteria is defined by mean square acceleration value of response of host structure to unit input acceleration. In this paper its presented that when TVA design parameters is set as optimum values, host structure vibration amplitudes caused from random excitation are reduced efficiently. Kwok, K., and Samali, B. [20] worked on performance of tuned mass dampers installed on tall structures to reduce vibration amplitudes induced from wind loads. Parametric study is carried out showing relationship between damping ratio of TMD, frequency tuning ratio and effective damping as well as, damping ratio of TMD, frequency tuning ratio and relative movement of TMD. For measuring effectiveness of the systems installed, full-scale measurements programs are utilized in this paper.

2.3.1 Evaluation on Fatigue Life of Tuned Vibration Absorber

Another performance criterion can be set as fatigue behavior of TVAs. Since TVAs are auxiliary vibrating unit attached to a host structure, vibration induced fatigue failure could occur on elastic components (stiffness element of an absorber) of an absorber. However, there is no study focused on vibration induced fatigue failure identification of elastic component of an absorber. Yet, in vibration isolation systems elastic components are also used, such as helical springs, leaf springs, viscoelastic materials, i.e. to isolate interested structural elements from vibration sources. In order to prevent early failures caused by vibration isolation system there are studies presented regarding fatigue life prediction of elastic components used in vibration isolation systems. In study of R.K. Luo [21], anti-vibration rubber springs used in industrial equipment investigated in the aspect of fatigue life. They carried out both FE analysis and regarding tests to ensure that rubber spring withstand 1,200,000 cycles of specified loading without visual failure. Another study regarding elastic components used in vibration isolator systems is presented by W.B. Shangguan [22]. In their paper a typical powertrain rubber vibration isolator is studied as an example. Fatigue life of the rubber isolator is predicted using three different models and results are compared with experimental ones. Xu, Y [23] study is another example of fatigue property investigation of a vibration isolator. In this paper, fatigue property of steel rubber vibration isolator for offshore jacket platform in cold environment is studied experimentally. Fatigue property of the components are evaluated by shear stiffness and percent reduction changes of shear stiffness after certain number of loading cycles.

2.3.1.1 Fatigue Analysis Under Random Loading

In many practical applications, mechanical and structural components are exposed to random loading instead of harmonic ones. Therefore, absorbers attached to these components are also under loading behaves in random nature. In order to study on modeling the fatigue property of a TVA, analysis of elastic components of the absorbers run with random loading cases. There are many tools to estimate fatigue property of structures in random vibration environment. In the study presented by Bishop [24] emphasizes frequency-based fatigue calculations in random loading environments because of the difficulty of representation of the random behavior of loadings in time signals. In the frequency domain, loadings and responses are defined in power spectral density functions. In related analysis to set fatigue property of structure, PSD of stress responses are utilized. One of the example is presented by E. Cigeroglu [25] where a cantilever beam element fatigue analysis is carried out using different fatigue theories. In the study, power spectral density functions of stress are utilized to fatigue life predictions using different theories. Later these results are compared with the ones acquired experimental methods. Chen, M [26] studied on statics on von Misses stresses occurring on structure subjected to random loading. Simulation is done using an aluminum shaker table subjected to loading defined in power spectral density. Root mean square values of von-misses stresses at selected nodes are utilized in algorithm they developed in order to do statistical analysis.

2.4 Operational Response Estimation of Tuned Vibration Absorber

In order construct performance characteristics of a TVA, as it is mentioned in the previous parts, response of TVA as well as host structure response should be utilized. For example, as it is given in previous part in the fatigue life prediction of elastic components, it is stated that power spectrum responses (displacements, stresses, strains) are used for investigation on fatigue properties. For each design step of TVA for solving a particular vibration problem, responses should be estimated in order to evaluate performance and to be used in iterative design process to optimize design parameters. For well-defined vibration problems where dynamic properties of the host structure as well as excitation source in operational conditions are known responses can be computed via using analytical and numerical methods or with FE modelling. However, for complex structures and large scale applications re-calculation times are longer in order to carry out iterative design process for performance evaluation and optimization. Therefore, frequency response functions based system modifications techniques are developed. A theoretical study was given by H.N. Ozguven [27]. In his work, an analytical relationship that is giving modified structure receptance matrix derived by using two sub-structure receptance matrixes.

In addition to this, he presented numerical examples in one of which 1DOF TVA is attached to a 3DOF host structure. In this example, modified (TVA attached) structure FRFs is calculated using the analytical relationship that relates receptance of modified structure with host and TVA receptance matrixes (receptance coupling). Another theoretical study presented by Nad, M [28]in which structural dynamic modification is studied in two different numerical cases. In the first one structural modification of cantilever beam is studied. Cantilever beam is partially constrained by viscoelastic layers. Change of Eigen values and Eigen functions presented with respect to length of viscoelastic layers using system modifications techniques. In the second example, change of natural frequency of circular disk with partially deformed zone with respect to parameters of deformation (deformed area distance to center and deformed area radius). An application based study is given by Ertürk, A., Özgüven, H., and Budak, E. [29] where tool point FRF is predicted using receptance coupling method. Firstly an reliable analytical model is constructed that can be used in receptance coupling method. Later tool point FRF is calculated and verified by results obtained by finite element software. Wang and Zhu [30] , in their studies utilized structural modification method in order to find modified system in-situ frequency functions using structural modification method without knowledge of excitation information during operation. Basic assumption in their theory is that excitation source remains unchanged regardless of modification. They used operational response of the original system to estimate forcing using inverse transfer functions to be used in calculation of response of modified system. Later on, with knowledge of dynamic properties of original system and modification using with estimation of forcing, modified system response is acquired under operation. This technique allows rapid calculation for iterative processes and optimization problems.

CHAPTER 3

EXPERIMENTAL VALIDATION OF THE FE MODEL OF ADJUSTABLE FREQUENCY TVA USED TO ESTIMATE TUNING FREQUENCY

The prototype of variable stiffness adjustable frequency TVA is METU ME design project product where main objective of project is set as frequency coverage over a range whereas aspect ratio and volume of design limited to a certain value. Therefore a circular beam stiffness element has been utilized in this design.This feature of the design make the TVA differs from its counterparts. During project studies, design team one of which was the author of this thesis worked on several aspects of the TVA for evaluation. Very first study was determination of the working frequency range of the TVA. Using FEM, model analysis of the stiffness element of this device has been done at two extreme conditions in order to determine upper and lower limit of frequency. Another example of the studies was SDOF behavior of the device. In order to ensure that absorber acts as and SDOF for a certain frequency range, modal analysis of the whole structure is carried out. However some of the essential studies can not be completed during project. The first of these studies was investigation on frequency change behavior of its stiffness element. In this chapter, frequency change behavior of the device has been investigated. This investigation of the device was important in order to set tuning law of the TVA.

Firstly, a brief introduction part is given where design is explained in detail. Afterwards, objective of study is will be presented. Lastly, analysis and experimental results will be shared.

3.1 Description of Previously Designed Variable Stiffness Adjustable Frequency Tuned Vibration Absorber

TVA design consist of one stiffness element, one effective mass element along with structural elements. Therefore, TVA design can be considered as a SDOF system for a certain frequency range. This SDOF system stiffness value can be adjusted, so that changing k in the equation $\omega_n = \sqrt{\frac{k}{m}}$ results in change of ω_n. Hence, system natural frequency changes which alternatively means that vibration absorber is tuned to this ω_n value.

In this design a cantilever beam type stiffness element is used that can be considered as a preferable choice when designing a vibration absorber as it's mentioned in literature survey chapter with examples. When it comes to changing stiffness value of the cantilever beam considering uniform cross section, most practical option is changing active length of beam. This objective can be achieved by changing boundary condition location on the beam as it is described in Figure 3.1.

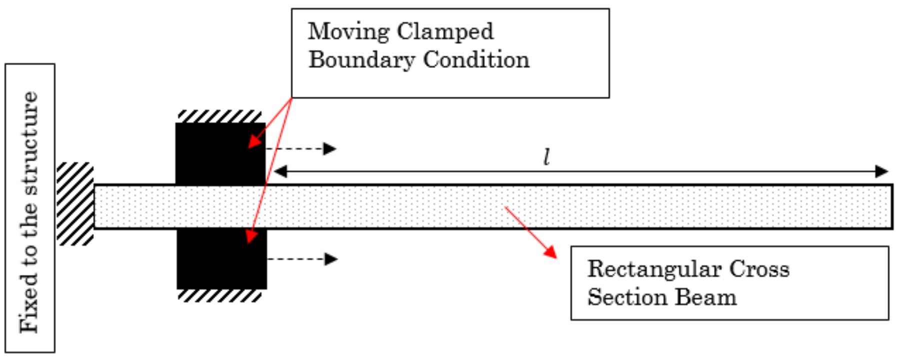

Figure 3.1: Cantilever Beam with Moving Boundary Condition.

Considering the DOF at the tip of the beam stiffness value can be expressed as given in the eqn. 3.1. (Required force to obtain unit amplitude displacement at this DOF.)

$$k = \frac{3EI}{l^3} \tag{3.1}$$

Where k, E, I and l stands for stiffness value, elastic modules, moment of area of cross section and active length of beam respectively. Using the phenomena described in the Figure 3.1 and relationship given in the equation, one can change the stiffness value of the cantilever beam by changing active length so that vibration absorber tuned frequency that uses this type of stiffness element. Consider that k_1 and k_2 are stiffness values corresponding active length of beam l_1 and l_2 respectively. In order to change frequency of the absorber up to $\%100$ that utilizes a this type of stiffness element and changing mechanism;

$$k_1 = \frac{3EI}{l_1^3}, k_2 = \frac{3EI}{l_2^3} \tag{3.2}$$

$$\omega_1 = \sqrt{\frac{k_1}{m}}, \omega_2 = \sqrt{\frac{k_2}{m}} \tag{3.3}$$

$$\omega_2 = 2\omega_1 \tag{3.4}$$

$$4\omega_1^2 = k_2/m \tag{3.5}$$

$$\omega_1^2 = k_1/m \tag{3.6}$$

$$4k_1 = k_2 \tag{3.7}$$

$$\frac{12EI}{l_1^3} = \frac{3EI}{l_2^3} \tag{3.8}$$

$$l_1 = 1.59 l_2 \tag{3.9}$$

In order to cover a frequency range where change of frequency is $\%100$, active length of the beam impose a change up to 1.6 times. When frequency change is set as $\%300$ this value becomes 2 times. When designing a tunable vibration absorber device, using cantilever beam type stiffness element limits the design over frequency coverage considering length of beam element versus aspect ratio of the absorber design and volume occupation.

This new design of stiffness element uses same methodology as it is described for straight cantilever beam however, in order to overcome physical limitations imposed by straight beam, this design basically a curved beam shown at Figure 3.2.

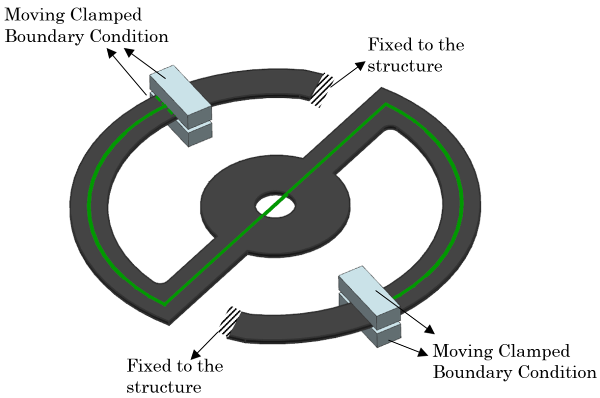

Figure 3.2: Curved Cantilever Beam.

In the Figure 3.2, it can be seen that configuration is very similar to presented at Figure 3.1. (Green line represents active length of the beam.) Cantilever beam has free end presented in the Figure 3.1. This new design circular beam can be considered as two cantilever beam formed in circular geometry bonded by their free ends together.

At the Figure 3.3 3D model of absorber is presented with other structural elements along with stiffness and mass elements.

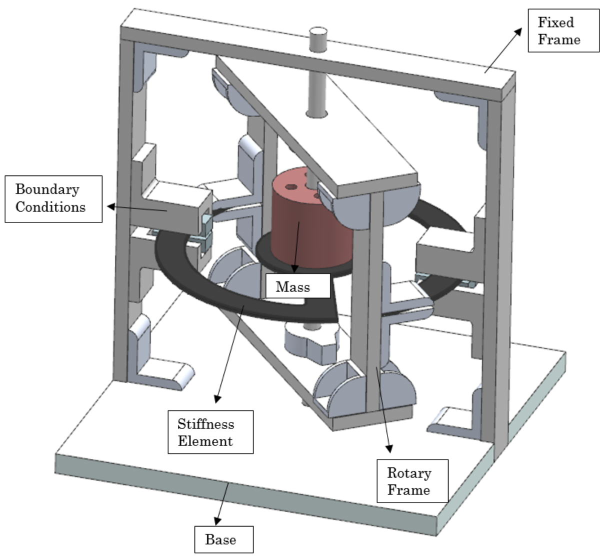

Figure 3.3: 3D Model of TVA Device

Mass is the copper cylindrical component assembled on the center of the circular beam acting as inertia element of the TVA. Mass is constrained by linear bearing embedded into it guided on the stainless steel shaft. Therefore only transverse displacement is allowed at the center of the circular beam. Stiffness element is the Al circular beam constrained at 3 different locations. First constraint is center of the beam where the only transverse movement of the center location is allowed that is obtained by linear bearing in the mass. Second constraint is applied on beam at boundary conditions locations as clamped. At the boundary conditions cast polyamid material is used embedded in to Al case. Last constraint is applied on ends of the beam to fix them on the device structure by rotary frame. Rotary frame is made of Al plates that can rotate about center of the circular beam guided by shaft. Therefore,when rotary frame rotates, circular beam rotates too. Since boundary conditions locations are fixed to the fixed frame, when the circular beam rotates, boundary conditions relatively changes that results in different stiffness values. Lastly, a base made of bulky Al plate is used to where fixed frame and shaft is assembled on.

Manufactured prototype of TVA device is given at Figure 3.4.

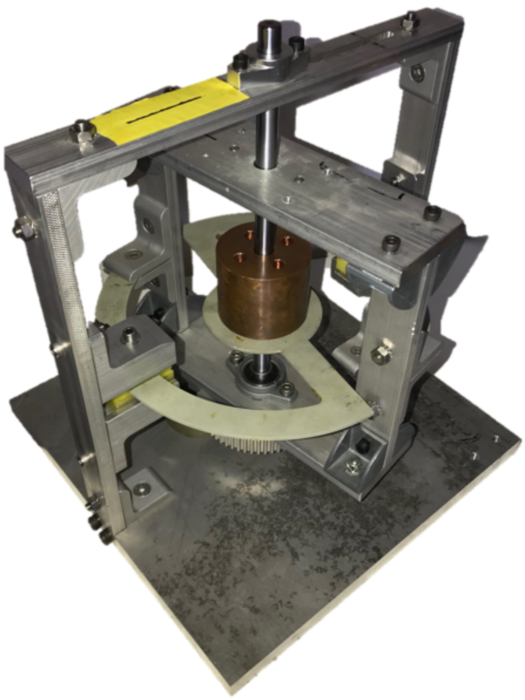

Figure 3.4: Manufactured and Assembled Prototype

Crucial design features of the device is summarized at Table 3.1.

Table 3.1: Desing Features

Desing Feature	Parameter	Material
Stiffness Element	Thickness:2.5mm Width: 10mm Outer Diameter: 220mm	Al Alloy (AL6061 T6)
Mass Element	Mass: 1kg	Copper
Boundary Condition	Thickness: 10 mm Width: 15 mm	Cast Polyamid (PA6 G)
Structural Elements	-	Steel and Al Alloys

3.2 Tuned Frequency Estimation of TVA

In this section studies related with tuned frequency estimation of the TVA has been presented. Firstly a FE model is constructed on ANSYS Workbench. After for each configuration of TVA where each configuration have different boundary conditions, modal analyses have been carried in order to obtain natural frequency of device. For the SDOF behavior of the device for a certain frequnecy range, this natural frequency is actually tuned frequency of TVA. FEA are later verified by experimental studies consisting of impact hammer tests.

3.2.1 Modal Analysis of Tuned Vibration Absorber

In this part studies related to modal analysis of TVA based on 3D model given in the previous part, will be presented. Main objective for this study is acquiring first natural frequency of this device (tuned frequency of the vibration absorber) for different stiffness values of curved stiffness element. Alternatively objective can be expressed as; for different boundary condition locations, investigation of the tuned frequency in order to set a tuning law. For analysis, ANSYS Workbench Modal Analysis module has been used. Firstly convergence analysis has been made to ensure analysis results.

For different element sizes of stiffness element and other structural elements first natural frequency of the model is observed.

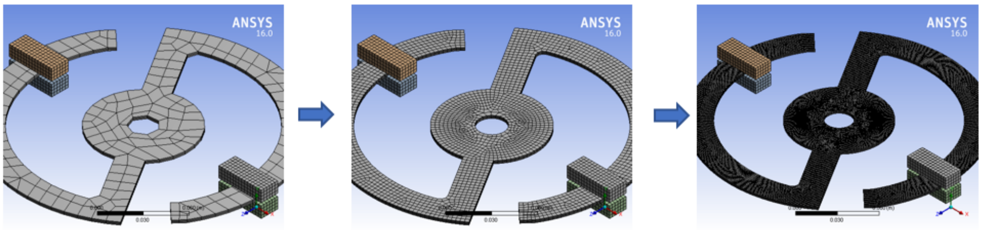

Figure 3.5: Meshing done on stiffness element with different element size, 0.01m, 0.003m and 0.0008m (Trilinear Hexahedral Elements -HEX8)

As result of convergence analysis, plot given in Figure 3.6 is acquired, where first natural frequency of the absorber is observed as the element size decreases.

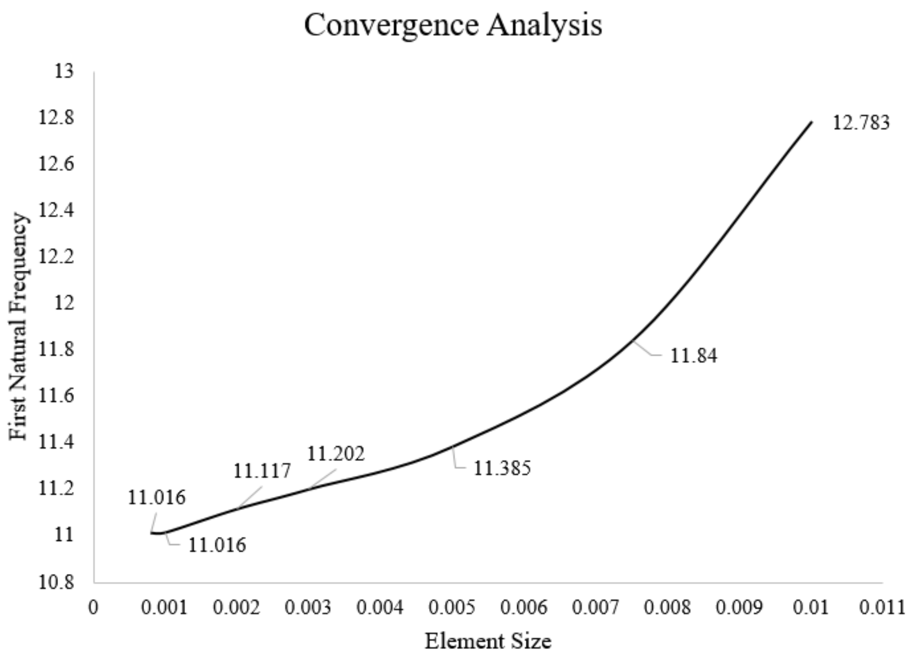

Figure 3.6: First natural frequency of the absorber vs Element size on Stiffness Element

In Figure 3.6 first natural frequency of the device is acquired for different meshing sizes. Beyond below of element size of 0.001m, natural frequency converged to certain value. Therefore analysis are carried out using 0.001m element size for stiffness element.

At the Figure 3.7 first mode shape is presented of the absorber for first configuration of boundary condition.(using 0.001m element size on stiffness element) For modal analysis where the result of first configuration is presented at Figure 3.7, all parts are bonded together except copper mass and shaft couple. Joint defined between shaft and copper mass that only allows center of beam move along the axis of the shaft.

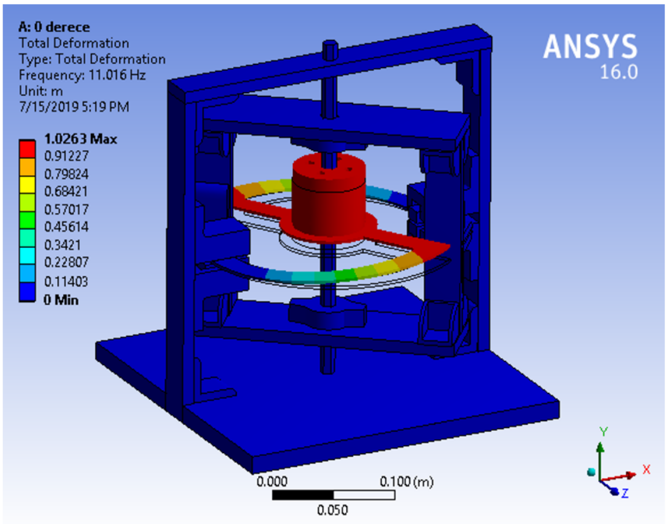

Figure 3.7: First Mode Shape of the Absorber at 11.016 Hz

For other configurations of TVA a reference parameter is defined. At the Figure 3.8 reference parameter for different boundary locations is showed at 3D model which is angle between rotary frame and fixed frame defined in previous part.

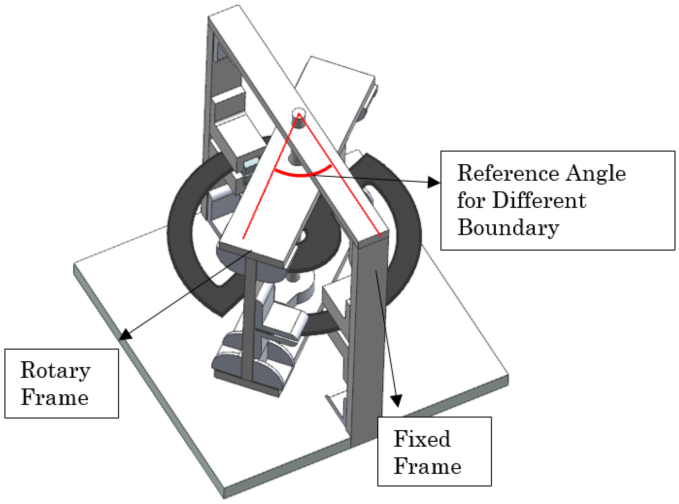

Figure 3.8: Reference Angle for Different Boundary Conditions

Using different boundary locations (for different stiffness values of absorber) modal analysis have been carried and tuned frequency Table 3.2 is obtanied as given.

Table 3.2: Tuning Frequency Table

Reference Angle Value [Degrees]	Absorber Tuned Frequency [Hz]
39	11.01
49	11.98
59	13.14
69	14.52
79	16.16
89	18.12
99	20.43
109	23.12
119	26.26
129	29.86
139	33.99

3.2.2 Validation of Analysis Studies by Experiments

In order to verify results obtained by modal analysis for different configurations of TVA, experimental work has been done for setting tuning law of the of TVA. Firstly, experimental procedure, used equipment and interfaces are explained. Then, experimental results are presented in this section.

3.2.2.1 Experimental Setup

TVA threated as an SDOF system, as it is described in previous sections. Therefore, measurements during experimental study focuses on main mass element acceleration. System is considered as a structure consisting of one mass element which is cylindrical bulky copper, whereas its stiffness element is circular beam. Hence, modal frequency (vibration absorber tuned frequency) is obtained of this basic SDOF system by measuring mass element acceleration, to impact induced on this element by modal testing hammer (Hammer test). Experimental setup is presented at below Figure 3.9.

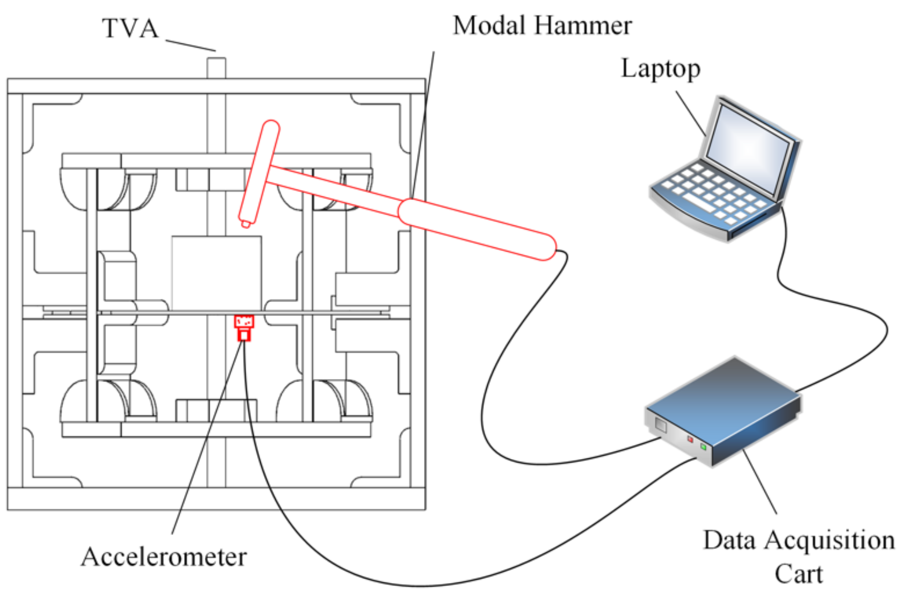

Figure 3.9: Experimental Setup

Table 3.3: Experimental Setup Equipment Table

Equipment	**Remarks**
TVA	Designed TVA
Accelerometer	KISTLER 8704B50
Impact Hammer	Modal Hammer (Meggitt- Model 02302-5)
Data Acquisition Cart	National Instruments (NI USB 4421)
Laptop	Computer- NI express software is installed
NI Express (Software)	Data Review Interface

3.2.2.2 Experiments

Using experimental setup presented in the previous section, test facilities are carried out. Measurements are based on input, output quantities where input is impact applied by modal hammer in Newtons, and output is acceleration of copper mass in m/s^2. Experimental procedure starts by adjusting angle between fixed and rotary frame shown at Figure 3.8 as it is described in previous before. This angle is basically changes active length of circular beam, therefore, stiffness value of the beam. After adjusting the angle (from corresponding analysis value given in the Table 3.2), copper mass is excited by means of impact hammer. Acceleration data is collected and processed in NI Express (Lab view) environment. In the software interface, coherence, FRF, input spectrum, and out spectrum are present. After checking the validity of the input (clear impact without double hit etc. checked from input spectrum). Second and third hit to copper mass is given. Averaging three results obtained by three impacts is then stored for first configuration of TVA where angle between rotary and fixed frame is 38.9 degrees. Results for first configuration are shared in Figures 3.11, 3.12, 3.13, 3.14.

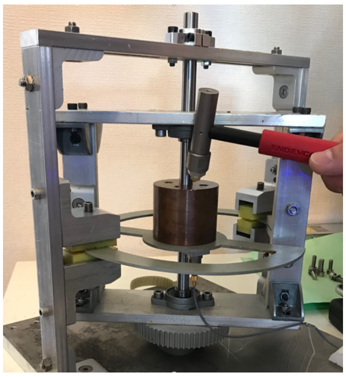

Figure 3.10: Experiment

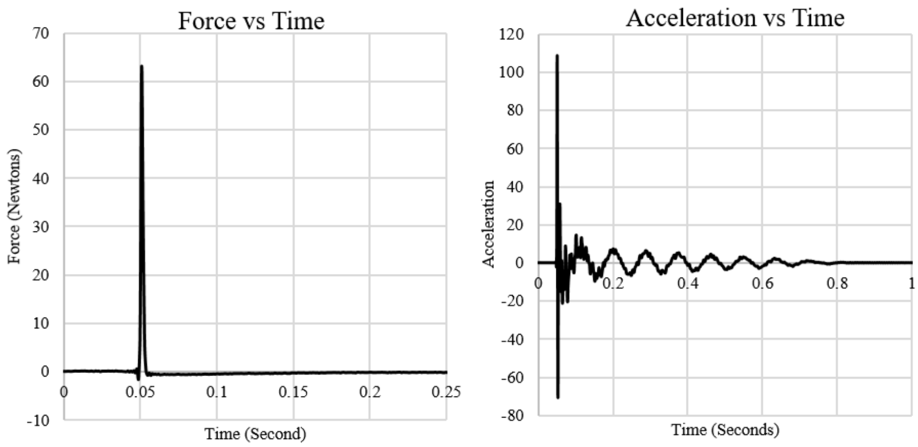

Figure 3.11: Time Signals

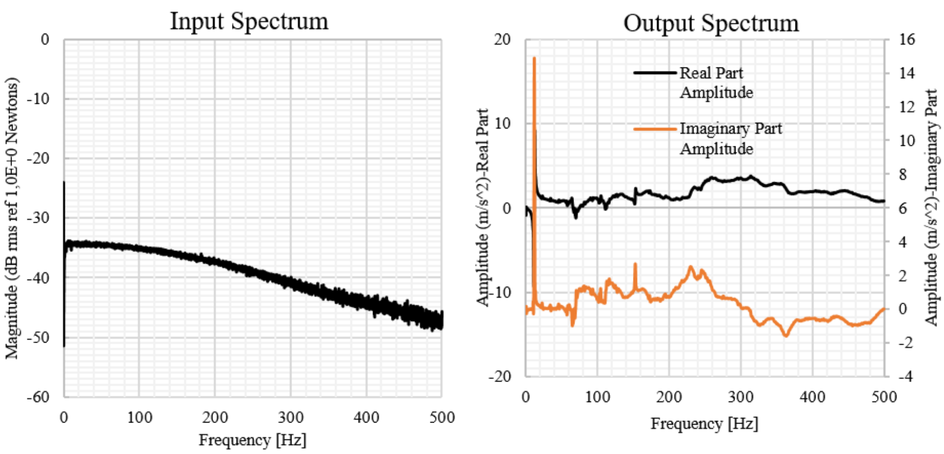

Figure 3.12: Spectrum Of Signals

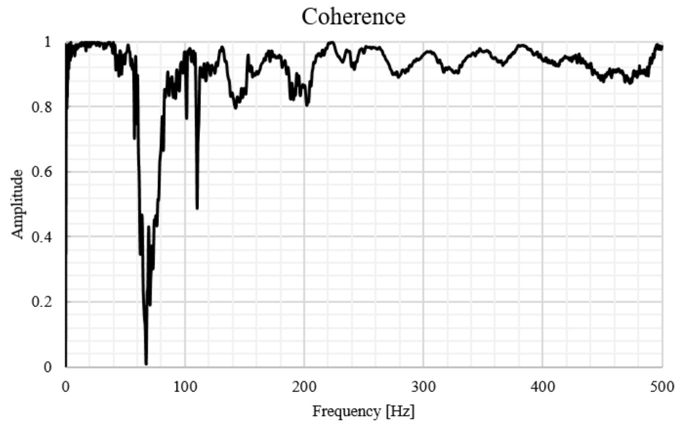

Figure 3.13: Coherence

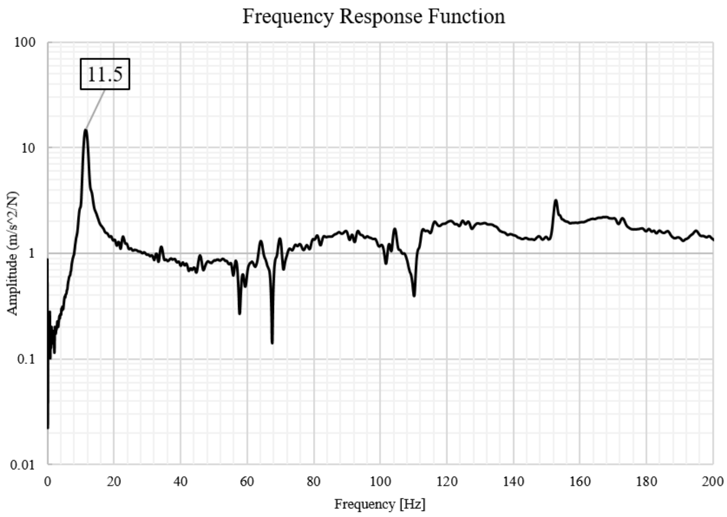

Figure 3.14: Frequency Response Function

Hence, for the first configuration where the angle is 38.9 degrees, according to modal testing tuned frequency is experimentally acquired as 11.5 Hz. For other configurations of TVA, experimental procedure is followed as it is described. Results for other configurations are presented in the Table 3.4.

Table 3.4: Experimental Results Table

Angle [Degrees]	**Experimental Result [Hz]**
38.9	11.5
48.9	12.25
58.9	13.125
68.9	14.375
78.9	15.875
88.9	17.5
98.9	20.5
108.9	22.25
118.9	27
128.9	30.25
138.9	35.125

3.3 Discussion

In this section results obtained via analysis and modal test will be presented and compared with each other. In order to set the tuning law for this TVA, a meaningful outcome will be seek from this comparison. Results graph and table for different boundary conditions locations for circular beam resulting in altering stiffness values, versus tuned frequency is presented below for both analysis and experimental study in Table 3.5 and Figure 3.15.

Table 3.5: Experimental and Analysis Results Table

Angle [Degrees]	Exp. Result [Hz]	Analysis Results	Difference %
38.9	11.5	11.016	4.2
48.9	12.25	11.986	2.2
58.9	13.125	13.141	0.1
68.9	14.375	14.521	1
78.9	15.875	16.168	1.8
88.9	17.5	18.124	3.6
98.9	20.5	20.430	0.3
108.9	22.25	23.129	4
118.9	27	26.261	2.7
128.9	30.25	29.868	1.3
138.9	35.125	33.991	3.2

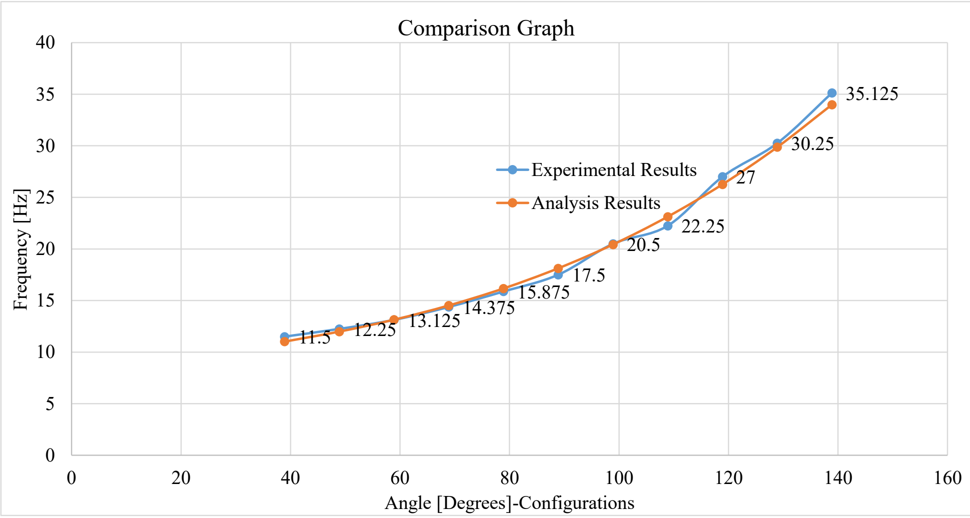

Figure 3.15: Analysis and Experimental Results Comparison Graph

In the above Figure 3.15, it is clearly showed that, analysis results fits experimental results very well, this deduction verify both experiments and analysis together. To set tuning law for this stiffness element, Figure 3.16 is used where x axis represents square root of cubic of one over active length whereas y axis represents tuned frequency. Trend line fitted curve in Figure 3.16 actually represents the tuning law of the absorber relating active length of beam. Since TVa considered as SDOF system, natural frequency of TVA can be expressed as in given in the equation 3.10.

$$\omega_n = \sqrt{\frac{k}{m}} \tag{3.10}$$

Considering TVA stiffness element as a cantilever beam type, natural frequency of the absorber can be related with active length of the beam using eqn. 3.1 presented in the below relationship.

$$\omega_n \propto \sqrt{\left(\frac{1}{L^3}\right)} \tag{3.11}$$

Using the relationship presented in the eqn. 3.11, Figure 3.16 is obtained where linear relationship is observed for tuning law of the device relating tuned frequency to active length of the beam.

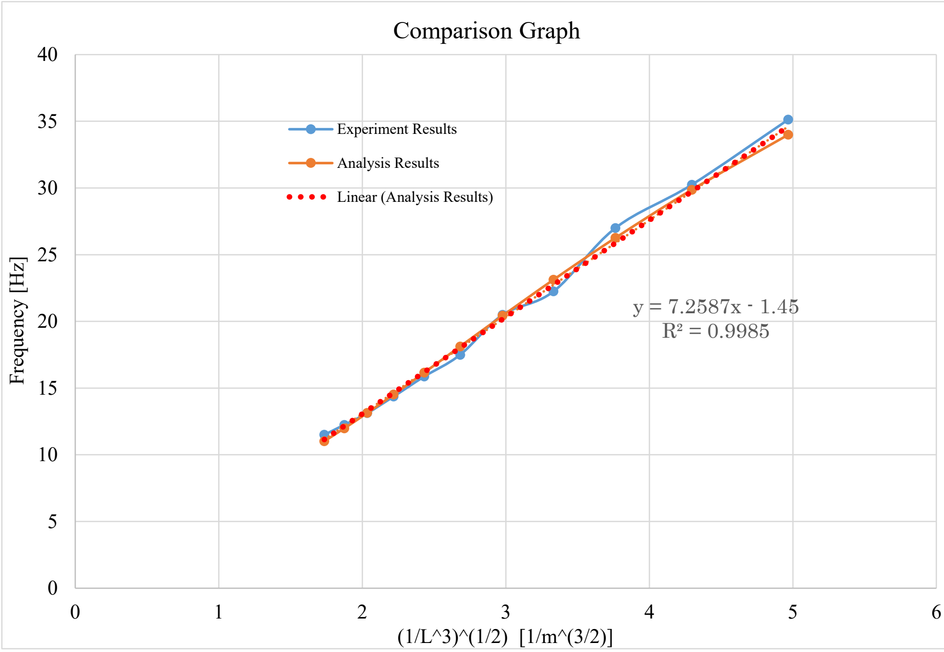

Figure 3.16: Tuning Law Graph

Based on the fitted trendline to curve presented in the Figure 3.16 tuning law of the absorber is presented in the eqn. 3.12.

$$\omega_{tuned} = 7.26\sqrt{\frac{1}{L^3}} - 1.45 \tag{3.12}$$

Using same elastic modulus (E) and cross sectional area moment of inertia (I) value for straight cantilever beam as it is used in the design of circular beam, tuning law for straight cantilever beam can be obtained using relationship presented in the eqn 3.1 as given in the eqn. 3.13 and eqn. 3.14.

$$\omega_{cantilever} = \sqrt{\frac{3EI}{L^3}} \tag{3.13}$$

$$\omega_{cantilever} = 1.64\sqrt{\frac{1}{L^3}} \tag{3.14}$$

As it observed from eqn. 3.12 and 3.14 circular beam more sensitive to change of active length in aspect of tuned frequency. Therefore, it is convenient to express that circular beam covers wider frequency band compared to cantilever beam for given same physical parameters while circular beam keeps the aspect ratio of the design closes to one and occupy less volume because of it geometry.

CHAPTER 4

OPERATIONAL RESPONSE ESTIMATION METHOD FOR TUNED VIBRATION ABSORBER

In this chapter, response estimation method is presented where response of TVA and host structure modified by TVA is obtained without modeling the host structure or excitation sources acting on it. Firstly, theoretical development of method is given. Later analytical verification of method is done. Lastly a example problem is solved by proposed method and numerical method. Results are compared acquired by both methods.

4.1 Theory

Theory of the proposed method is firstly described in the process tree (Figure 4.1) briefly then explained in detail on a 2 DOF host structure given in the Figure 4.2.

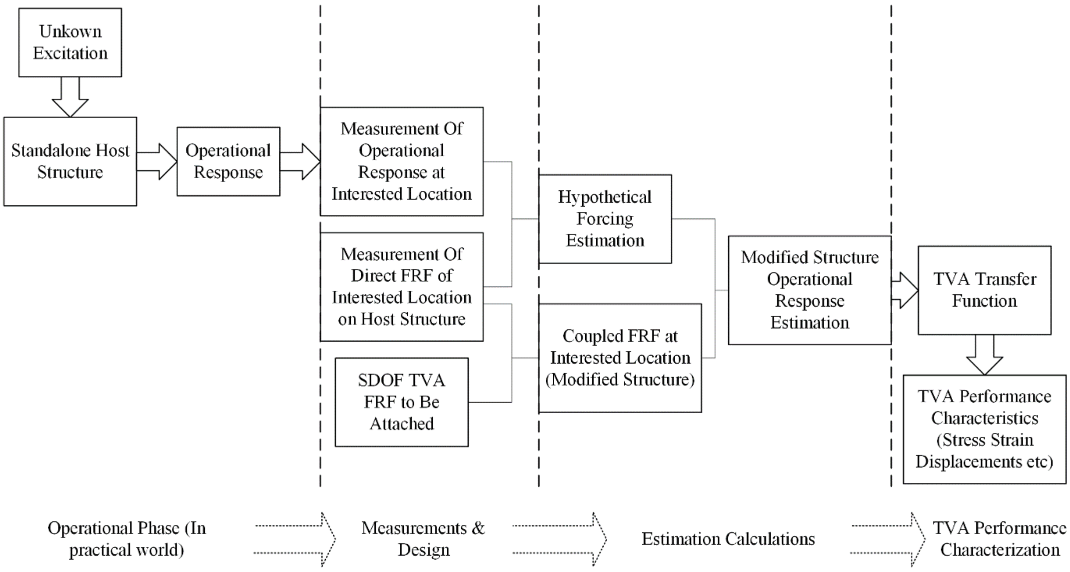

Figure 4.1: Process Chart.

Response estimation method for TVA attached to a host structure is basically formed in four main step. In the first step, vibration problem is introduced where a host structure is under loading with an unknown excitation source acting on it resulting in unwanted vibration levels on interested DOF. Second step of the method consisting of design of TVA and measurements taken on the host structure. In this step, operational response and direct point FRF of interested DOF on host structure is measured. Also designed TVA base point direct FRF is calculated (where it is going to be attached to host structure) in this step as well. In the third step estimation calculations are done. In this step a hypothetical forcing estimation is done using two measurement result obtained on host structure. Hypothetical forcing is a fictional forcing that is assumed to be applied on this point, however it simulates all excitation sources acting on this structure that results same reaction on interested DOF in operational conditions. Also using a well know coupling method, TVA base FRF and host structure FRF on interested DOF is coupled. Therefore, problem is reduced to point on host structure where all excitation is assumed to be acting on this point. Also modified FRF is calculated for this point as well. Therefore, operational response of modified point can be computed. Last step of the method consisting of calculations of TVA performance parameters. In this step attached TVA is considered as standalone structure that is excited by the loading found in step three. Basically, response of the modified host structure at interested DOF is base input for the TVA that is the TVA is going to be exposed in the service. After FEA and related simulations TVA performance characteristics can be computed.

Response estimation of TVA modified structure is given under two main assumptions; system is linear and regardless of the modification, excitation source remain same after the TVA attachment. Considering two conditions, 2DOF linear system with complex springs and masses are given at Figure 4.2 below. Random excitation (considered as random stationary process) defined by power spectral density (PSD) function $G_f^1(\omega)$ is applied at DOF-1.

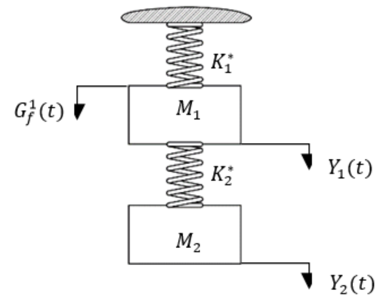

Figure 4.2: 2 DOF Structure.

$Y_1(\omega)$ and $Y_2(\omega)$ displacement responses of masses whereas $G_f^1(\omega)$ forcing acting on DOF-1 are given in the form of PSD (Power spectral density of $Y_1(t), Y_2(t)$ and$G_f^1(t)$ respectively). There a input-output relationship persist between $G_f^1(\omega)$ and $Y_1(\omega)$, $Y_2(\omega)$ due to linear system dynamics of the 2DOF structure. Considering practical applications, it is going to be assumed as there is no information about neither excitation source nor dynamic properties of structure. $Y_1(\omega)$ or $Y_2(\omega)$ can be measurable whichever is pointed out as problematic point by means of vibrational considerations. From this point on $Y_2(\omega)$ is threated as known property as a result of measurement. As a result of evaluation of measurement data in this case it is selected arbitrarily as $Y_2(\omega)$, for vibrational suppression at a certain frequency band, TVA is going to be attached to structure via 2nd DOF, consisting of 1DOF system with mass M_t and K_t (Figure 4.3).

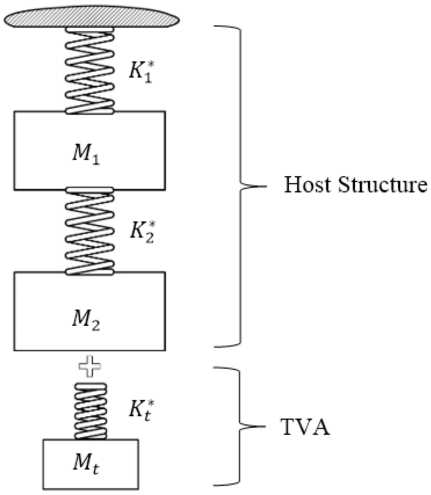

Figure 4.3: 3 DOF Structure.

System matrixes of TVA alone:

$$[M] = \begin{bmatrix} M_t & 0 \\ 0 & 0 \end{bmatrix} [K_t] = \begin{bmatrix} K_t^* & -K_t^* \\ -K_t^* & K_t^* \end{bmatrix} \tag{4.1}$$

For system matrix of TVA

$$[H^t] = [-\omega^2[M] + [K_t]]^{-1} \tag{4.2}$$

$$[H^t] = \begin{bmatrix} H_{1,1}^t & H_{1,2}^t \\ H_{2,1}^t & H_{2,2}^t \end{bmatrix} \tag{4.3}$$

$$[H^t] = \begin{bmatrix} \frac{1}{(-\omega^2 M_t)} & \frac{1}{(-\omega^2 M_t)} \\ \frac{1}{(-\omega^2 M_t)} & \frac{-\omega^2 M_t + K_t)}{(-\omega^2 M_t)} \end{bmatrix} \tag{4.4}$$

H_{22}^o (2^{nd} DOF direct FRF of host structure) is going to be assumed as known property as result of measurement output of host structure. Now it is possible to calculate modified structure direct FRF at connecting DOF by impedance coupling method using H_{22}^o and H_{22}^t. From compability equations at connecting DOF [31]. One can calculate the FRF of modified 2nd DOF as follows:

$$[\widetilde{H_{22}^m}] = \frac{H_{22}^o(\omega) H_{22}^t(\omega)}{H_{22}^o(\omega) + H_{22}^t(\omega)} \tag{4.5}$$

Where $[\widetilde{H}_{22}^m]$ stands for modified structure 2nd DOF direct FRF.

Actual systems generally have forcing acting on several DOFs. In this method forcing on the other DOFs will be simulated as if there is only one source of forcing that is acting on the attachment point, in this case that's 2^{nd} DOF. Under mentioned considerations, below set of equations are formed (Random vibration input-output relationship for random stationary process [3].)

$$[G] = [H^o][G_f][H^o]^H \tag{4.6}$$

Where $[G]$,$[H^o]$ and $[G_f]$ are refer to system response, system matrix and forcing matrix respectively. Forcing is assumed to be acting only second DOF therefore forcing matrix is represented by eqn. 4.7

$$[G_f] = \begin{bmatrix} 0 & 0 \\ 0 & G_f^h \end{bmatrix} \tag{4.7}$$

$$\begin{bmatrix} G_{11}^Y & G_{12}^Y \\ G_{21}^Y & G_{22}^Y \end{bmatrix} = \begin{bmatrix} H_{11}^o & H_{12}^o \\ H_{21}^o & H_{22}^o \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & G_f^h \end{bmatrix} \begin{bmatrix} H_{11}^o & H_{12}^o \\ H_{21}^o & H_{22}^o \end{bmatrix}^H \tag{4.8}$$

$$\begin{bmatrix} G_{11}^Y & G_{12}^Y \\ G_{21}^Y & G_{22}^Y \end{bmatrix} = \begin{bmatrix} |H_{12}^o|^2 G_f^h & \bar{H}_{21}^o H_{11}^o G_f^h \\ \bar{H}_{11}^o H_{21}^o G_f^h & |H_{22}^o|^2 G_f^h \end{bmatrix} \tag{4.9}$$

Where $G_f^h(\omega)$, $G_{11}^Y(\omega)$, $G_{22}^Y(\omega)$, G_{12}^Y and G_{21}^Y stands for hypothetical forcing PSD function assumed to be acting on 2nd DOF, PSD response of DOF 1,PSD response of DOF 2, coherences of responses at DOF 1 and DOF 2 respectively.(Superscript H denotes the Hermitian matrix.)

Hence, hypothetical forcing found as,

$$G_f^h(\omega) = \frac{G_{22}^Y(\omega)}{|H_{22}^o(\omega))|^2} \tag{4.10}$$

It is observed that hypothetical forcing is only function of direct FRF of interested DOF and operational response of same DOF as well. Therefore eqn. 4.10 can generalized for higher degrees of freedom linear system as given in the eqn. 4.11.

$$G_f^h(\omega) = \frac{G_{nn}^Y(\omega)}{|H_{nn}^o(\omega))|^2} \tag{4.11}$$

Hypothetical forcing $(G_f^h(\omega))$ acting on n-th DOF, computed using spectral response measurement on n-th DOF $G_{nn}^Y(\omega)$ and direct FRF measurement on n-th DOF $(H_{nn}^o(\omega))$.

Hypothetical forcing assumption enable us to predict modified point response after attachment of the TVA with the help of modified point direct FRF that is found in eqn 4.5. Using inverse relationship that is given in eqn 4.11 one can find modified point response considering host structure is SDOF system composed of only interested DOF. Therefore that point response can be calculated using FRF and forcing function acting that point.

$$\widetilde{G_{22}^{Y}}^{m}(\omega) = |\widetilde{H}_{22}^{m}(\omega)|^{2} G_{f}^{H}(\omega) \tag{4.12}$$

Where $\widetilde{G_{22}^{Y}}^{m}(\omega)$ stands for nodified point (2nd DOF) response estimation.

Open form of the above equation is given at below;

$$\widetilde{G_{22}^{Y}}^{m}(\omega) = \left| \frac{H_{22}^{o}(\omega) H_{22}^{t}(\omega)}{H_{22}^{o}(\omega) + H_{22}^{t}(\omega)} \right|^{2} \frac{G_{22}^{Y}(\omega)}{|H_{22}^{o}(\omega)|^{2}} \tag{4.13}$$

$$\widetilde{G_{22}^{Y}}^{m}(\omega) = \left| \frac{H_{22}^{o}(\omega) H_{22}^{t}(\omega)}{H_{22}^{o}(\omega) + H_{22}^{t}(\omega)} \right|^{2} G_{22}^{Y}(\omega) \tag{4.14}$$

For sake of checking the validity of proposed method, full solution of the original and modified structure has been made and given in the following steps. 2 DOF original structure mass and stiffness matrixes are formed:

$$[M_s] = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} [K_s] = \begin{bmatrix} K_1^* + K_2^* & -K_2^* \\ -K_2^* & K_2^* \end{bmatrix} \tag{4.15}$$

$$H_s(\omega) = [-\omega^2 [M_s] + [K_s^*]]^{-1} \tag{4.16}$$

Where H_s denotes ("ω" omitted from both sides for the sake of simplicity);

$$[H_s] = \begin{bmatrix} H_{11}^{o} & H_{12}^{o} \\ H_{21}^{o} & H_{22}^{o} \end{bmatrix} \tag{4.17}$$

*Where superscript 'o' denotes original system. Since PSD function of forcing input is random stationary process, response of the 2 DOF system under random excitation [3] can be found as;

$$\begin{bmatrix} G_{11}^{Y} & G_{12}^{Y} \\ G_{21}^{Y} & G_{22}^{Y} \end{bmatrix} = \begin{bmatrix} H_{11}^{o} & H_{12}^{o} \\ H_{21}^{o} & H_{22}^{o} \end{bmatrix} \begin{bmatrix} G_{f}^{1} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} H_{11}^{o} & H_{12}^{o} \\ H_{21}^{o} & H_{22}^{o} \end{bmatrix}^{H} \tag{4.18}$$

Where G_f^1, G_{11}^Y, G_{22}^Y, G_{12}^Y and G_{21}^Y refer to PSD of input force, PSD response of DOF 1, PSD response of DOF 2, coherence of responses at DOF 1 and DOF 2.

$$\begin{bmatrix} G_{11}^Y & G_{12}^Y \\ G_{21}^Y & G_{22}^Y \end{bmatrix} = \begin{bmatrix} |H_{11}^o|^2 G_f^1 & \bar{H}_{21}^o H_{11}^o G_f^1 \\ \bar{H}_{11}^o H_{21}^o G_f^1 & |H_{21}^o|^2 G_f^1 \end{bmatrix} \tag{4.19}$$

Hence second DOF response in PSD form is given at below;

$$G_{22}^Y = |H_{21}^o|^2 G_f^1 \tag{4.20}$$

Inserting the eqn.4.20 into eqn.4.14, estimation as a function of input forcing as given below,

$$\widetilde{G_{22}^Y}^m = |H_{21}^o|^2 G_f^1 |\frac{H_{22}^t}{H_{22}^o + H_{22}^t}|^2 \tag{4.21}$$

Modified structure (3DOF) system matrixes are defined at below,

$$[M_s^m] = \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_t \end{bmatrix} \tag{4.22}$$

$$[K_s^m] = \begin{bmatrix} K_1^* + K_2^* & -K_2^* & 0 \\ -K_2^* & K_2^* + K_t & -K_t \\ 0 & -K_t & K_t \end{bmatrix} \tag{4.23}$$

$$H_s^m(\omega) = [-\omega^2[M_s^m] + [K_s^{*m}]]^{-1} \tag{4.24}$$

$$[H_s^m] = \begin{bmatrix} H_{11}^m & H_{21}^m & H_{31}^m \\ H_{21}^m & H_{22}^m & H_{23}^m \\ H_{31}^m & H_{32}^m & H_{33}^m \end{bmatrix} \tag{4.25}$$

Where $[H_s^m]$ and $[G^{Y^m}]$ refer to modified system FRF matrix and modified system response matrix.

$$[G^{Y^m}] = \begin{bmatrix} H_{11}^m & H_{21}^m & H_{31}^m \\ H_{21}^m & H_{22}^m & H_{23}^m \\ H_{31}^m & H_{32}^m & H_{33}^m \end{bmatrix} \begin{bmatrix} G_f^1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} H_{11}^m & H_{21}^m & H_{31}^m \\ H_{21}^m & H_{22}^m & H_{23}^m \\ H_{31}^m & H_{32}^m & H_{33}^m \end{bmatrix}^H \tag{4.26}$$

$$[G^{Y^m}] = \begin{bmatrix} G_{11}^{Y\,m} & G_{12}^{Y\,m} & G_{13}^{Y\,m} \\ G_{21}^{Y\,m} & G_{22}^{Y\,m} & G_{23}^{Y\,m} \\ G_{31}^{Y\,m} & G_{32}^{Y\,m} & G_{33}^{Y\,m} \end{bmatrix} = \begin{bmatrix} |H_{11}^m|^2 G_f^1 & \overline{H}_{21} H_1 1 G_f^1 & \overline{H}_{31} H_{11} G_f^1 \\ \overline{H}_{11} H_{21} G_f^1 & |H_{21}^m|^2 G_f^1 & \overline{H}_{31} H_{21} G_f^1 \\ \overline{H}_{11} H_{31} G_f^1 & \overline{H}_{21} H_{31} G_f^1 & |H_{31}^m|^2 G_f^1 \end{bmatrix} \tag{4.27}$$

Exact result of the modified system response is given above in eqn. 4.27. Second DOF exact reaction is denoted in the above matrix as;

$$G_{22}^{Y\,m} = |H_{21}^{m}|^2 G_f^1 \tag{4.28}$$

Below eqn.4.29 is expected to be hold, thus, in order to check analytical relationship between exact results of the second DOF reaction with the estimation, following steps are followed;

$$G_{22}^{Y\,m} = \widetilde{G_{22}^{Y\,m}} \tag{4.29}$$

Both sides of the equation is found as a function of input forcing recall from eqn.4.28 and eqn.4.22;

$$|H_{21}^{m}|^2 G_f^1 = |H_{21}^{o}|^2 G_f^1 \left| \frac{(H_{22}^{t})}{(H_{22}^{o} + H_{22}^{t})} \right|^2 \tag{4.30}$$

With omitting input forcing from both sides, above equation reduced to;

$$|H_{21}^{m}| = \left| \frac{H_{21}^{o} H_{22}^{t}}{H_{22}^{o} + H_{22}^{t}} \right| \tag{4.31}$$

In order to check above identity one should calculate modified structure cross point FRF(H_{21}^{m}) in terms of original structure FRFs (H_{21}^{o}, H_{22}^{o}) and TVA FRF (H_{22}^{t}).

In order to check the validity of above eqn.4.31, consider the general case for attachment of TVA to host structure (Figure 4.4);

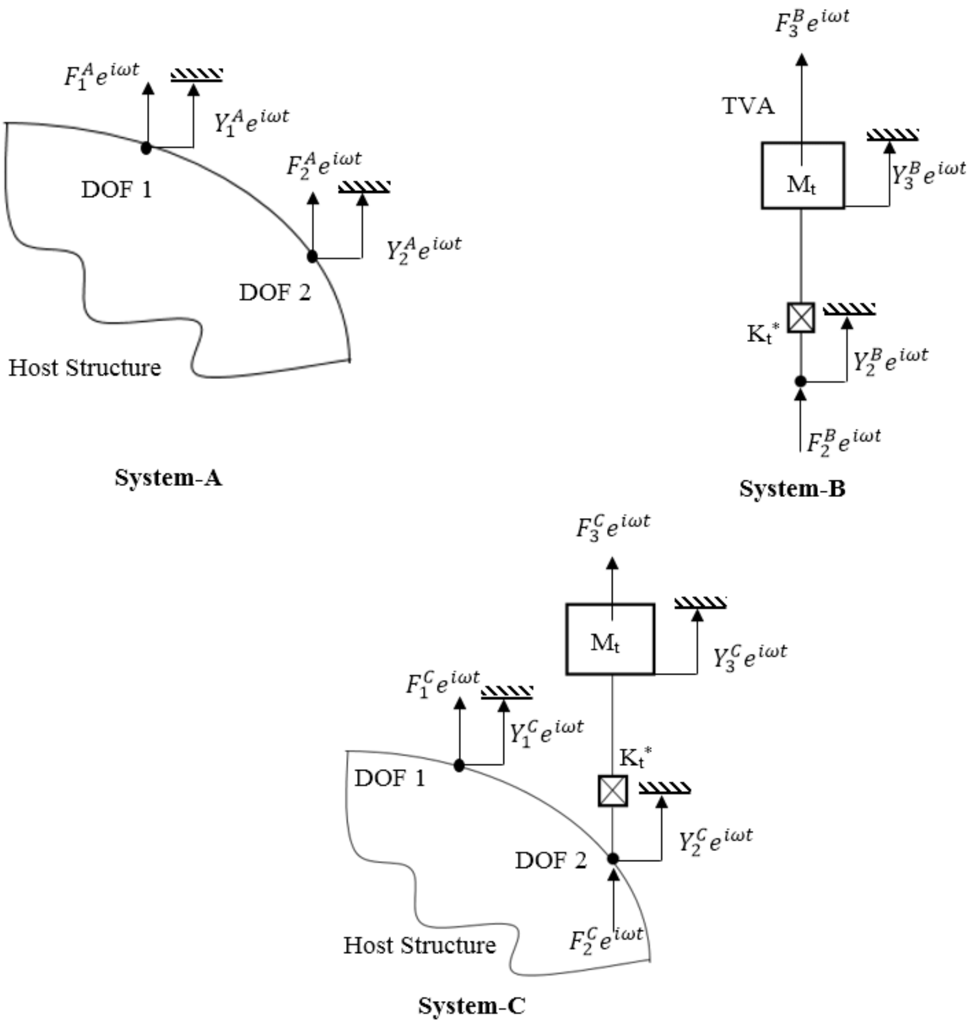

Figure 4.4: General Case for TVA Application

In Figure 4.4, a host structure and TVA are given where the TVA is attached to host structure with connecting DOF ‘2’.

Considering the system B, displacements are defined in terms of forcing acting on the DOFs;

$$Y_3^B = H_{32}^t F_2^B \tag{4.32}$$

$$Y_2^B = H_{22}^t F_2^B \tag{4.33}$$

$$F_2^B = \frac{Y_2^B}{H_{22}^t} \tag{4.34}$$

In the above set of equations $Y_3^B, Y_2^B, F_2^B, H_{32}^t$ and H_{22}^t stands for harmonic amplitudes of displacement at DOF 3 on system B, displacement at DOF 2 on system B, forcing acting on DOF 2 on system B, cross point FRF between DOF-3 and DOF-2 on system B and direct point FRF of DOF-2 on system B respectively.

Considering the system-A, displacements are defined in terms of forcing acting on the DOFs;

$$Y_2^A = H_{22}^o F_2^A \tag{4.35}$$

$$F_2^A = \frac{Y_2^A}{H_{22}^o} \tag{4.36}$$

$$Y_1^A = H_{12}^o F_2^A \tag{4.37}$$

$$F_2^A = \frac{Y_1^A}{H_{12}^o} \tag{4.38}$$

In the above set of equations$Y_2^A, Y_1^A, F_2^A, H_{22}^o$ and H_{12}^o stands for harmonic amplitudes of displacement at DOF-2 on system A, displacement at DOF-1 on system-A, forcing acting on DOF-2 on system-A, direct point FRF of DOF-2 on system-A and cross point FRF between DOF-1 and DOF-2 on system-A respectively.

Considering the system-C, displacements are defined in terms of forcing acting on the DOFs;

$$Y_2^C = H_{22}^m F_2^C \tag{4.39}$$

$$Y_1^C = H_{12}^m F_2^C \tag{4.40}$$

$$F_2^C = \frac{Y_1^C}{H_{12}^m} \tag{4.41}$$

In the above set of equations$Y_1^C, Y_2^C, F_2^C, H_1 2^m$ andH_{22}^m stands for harmonic amplitudes of displacement at DOF-1 on system-C, displacement at DOF-2 on system-C, forcing acting on DOF-2 on system-C, cross point FRF between DOF-1 and DOF-2 on system-C and direct point FRF of DOF-2 on system-C respectively.

Compability equations;

$$Y_1^A = Y_1^C \tag{4.42}$$

$$Y_2^A = Y_2^B = Y_2^C \tag{4.43}$$

Force equilibrium;

$$F_2^B + F_2^A = F_2^C \tag{4.44}$$

Substituting 4.43 into 4.38

$$F_2^A = \frac{Y_1^C}{H_{12}^o} \tag{4.45}$$

Substituting 4.43 into 4.34

$$F_2^B = Y_2^C / H_{22}^t \tag{4.46}$$

Substituting 4.45 and 4.46 into eqn. 4.44;

$$F_2^C = (Y_1^C)/(H_{12}^o) + (Y_2^C)/(H_{22}^t) \tag{4.47}$$

Using eqn.4.40 one can write 4.47 as;

$$F_2^C = (Y_1^C)/(H_{12}^o) + (H_{22}^m F_2^C)/(H_{22}^t) \tag{4.48}$$

After some manipulation;

$$H_{12}^o\left(1 - \frac{H_{22}^m}{H_{22}^t}\right) = \frac{Y_1^C}{F_2^C} = H_{12}^m = H_{21}^m \tag{4.49}$$

Recall that;$[H_{22}^m] = \frac{H_{22}^o H_{22}^t}{H_{22}^o + H_{22}^t}$ (impedance coupling) from eqn.4.5.

$$H_{12}^o\left(1 - \frac{\frac{H_{22}^o H_{22}^t}{H_{22}^o + H_{22}^t}}{H_{22}^t}\right) = H_{21}^m \tag{4.50}$$

After some manipulation;

$$H_{21}^m = \frac{H_{22}^t H_{12}^o}{H_{22}^o + H_{22}^t} \tag{4.51}$$

As it is seen from eqn.(4.51), it is obvious that eqn.(4.31) holds so does eqn.(4.29), which clearly indicates that it's possible to estimate modified point response as given below with only two measurement data on connecting DOF and TVA parameters.

$$\widetilde{G_{22}^{Y}}^m(\omega) = G_{22}^Y(\omega)\left|\frac{H_{22}^t(\omega)}{H_{22}^o(\omega) + H_{22}^t(\omega)}\right| \tag{4.52}$$

As it is mentioned in the introduction part, purpose of this study based on evaluation of TVA performance. Hence, TVA mass response has to be found in order to make analysis regarding TVA performance. Recall eqn. 4.27;

$$[G^{Ym}] = \begin{bmatrix} G_{11}^{Y\,m} & G_{12}^{Y\,m} & G_{13}^{Y\,m} \\ G_{21}^{Y\,m} & G_{22}^{Y\,m} & G_{23}^{Y\,m} \\ G_{31}^{Y\,m} & G_{32}^{Y\,m} & G_{33}^{Y\,m} \end{bmatrix} = \begin{bmatrix} |H_{11}^m|^2 G_f^1 & \overline{H}_{21} H_1 1 G_f^1 & \overline{H}_{31} H_{11} G_f^1 \\ \overline{H}_{11} H_{21} G_f^1 & |H_{21}^m|^2 G_f^1 & \overline{H}_{31} H_{21} G_f^1 \\ \overline{H}_{11} H_{31} G_f^1 & \overline{H}_{21} H_{31} G_f^1 & |H_{31}^m|^2 G_f^1 \end{bmatrix} \tag{4.53}$$

In the above equation solving 3 DOF system we have found system masses' responses. Also, by help of estimation method, we have found modified second DOF response as well. Defining a transfer function $T(\omega)$ between 2nd DOF response $G_{22}^{Y\,m}$ with 3rd DOF Response (TVA mass) $G_{33}^{Y\,m}$;

$$G_{33}^{Y\,m}(\omega) = G_{22}^{Y\,m}(\omega)|T(\omega)|^2 \tag{4.54}$$

$$|H_{31}^m|^2 G_f^1 = |H_{21}^m|^2 G_f^1 |T|^2 \tag{4.55}$$

In a simpler form;

$$|H_{31}^m|^2 = |H_{21}^m|^2 |T|^2 \tag{4.56}$$

Returning to system described in Figure 4.3 system mass and stiffness matrices (Modified system – 3DOF)

$$[M_s^m] = \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_t \end{bmatrix} \tag{4.57}$$

$$[K_s^m] = \begin{bmatrix} K_1^* + K_2^* & -K_2^* & 0 \\ -K_2^* & K_2^* + K_t & -K_t \\ 0 & -K_t & K_t \end{bmatrix} \tag{4.58}$$

$$H_s^m(\omega) = \Big[-\omega^2 [M_s^m] + [K_s^{*m}] \Big]^{-1} \tag{4.59}$$

$$H_{21}^m(\omega) = \frac{K_2^*(-\omega^2 M_t + K_t^*)}{det\Big(\big[-\omega^2[M^m] + [K^{m*}]\big]\Big)} \tag{4.60}$$

$$H_{31}^m(\omega) = \frac{K_2^* K_t^*}{det\Big(\big[-\omega^2[M^m] + [K^{m*}]\big]\Big)} \tag{4.61}$$

Inserting eqn.(4.61) and eqn.(eqn:53) into eqn.(4.56);

$$T(\omega) = \frac{H_{21}^m(\omega)}{H_{31}^m(\omega)} = \frac{\frac{K_2^*(-\omega^2 M_t + K_t^*)}{det\left(\left[-\omega^2[M^m]+[K^{m*}]\right]\right)}}{\frac{K_2^* K_t^*}{det\left(\left[-\omega^2[M^m]+[K^{m*}]\right]\right)}} \tag{4.62}$$

With some simplifications;

$$T(\omega) = \frac{K_t^*}{-M_t\omega^2 + K_t^*} \tag{4.63}$$

As it is seen from eqn.(4.63) transfer function $T(\omega)$ is only function of TVA parameters. Recall eqn.(4.56);

$$|H_{31}^m|^2 = |H_{21}^m|^2|T|^2 \tag{4.64}$$

Since left-hand side represents TVA response under operation, from above identity it can be deduced that TVA mass response can be found using transfer function $T(\omega)$ and second DOF response ($|H_{21}^m|^2G_f^1$ -reaction at connecting DOF) which is estimated using eqn.(4.14). Hence TVA mass response can be shown in a simpler form as it is given at below:

$$G_{33}^{Y\,m} = \widetilde{G_{22}^{Y}}^{m}\left(\frac{K_t^*}{-M_t\omega^2 + K_t^*}\right)^2 \tag{4.65}$$

Equation 4.65 can be expressed as in terms of tuned frequency of TVA (ω_n^t).

$$G_{33}^{Y\,m} = \widetilde{G_{22}^{Y}}^{m}\left(\frac{(\omega_n^t)^2}{\omega^2 + (\omega_n^t)^2}\right)^2 \tag{4.66}$$

4.2 Numerical Example

Operational response estimation of TVA modified structure theory is demonstrated in a case study where host structure consist of 2 DOF system with masses and complex springs given at below. Random forcing (Figure 4.6) generated using MATLAB is applied 1st DOF (mass-1) of the structure as in the Figure 4.5. Power spectral density of the forcing is calculated and system response is gathered in frequency domain as PSD outputs: $Y_1(\omega)$ first DOF response and $Y_2(\omega)$ second DOF response.

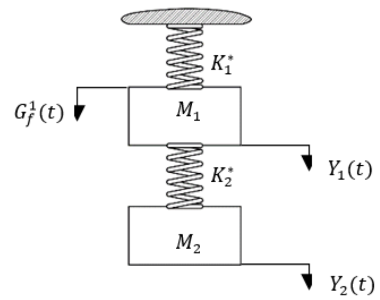

Figure 4.5: 2 DOF Host Structure.

2 DOF host structure properties are presented in the Table 4.1.

Table 4.1: Property Table

Property	Value
Mass-1 (M_1)	2 kg
Mass-2 (M_2)	3 kg
Spring-1 Stiffness (K_1^*)	20000(1+0.01i) N/m
Spring-2 Stiffness (K_2^*)	15000(1+0.01i) N/m

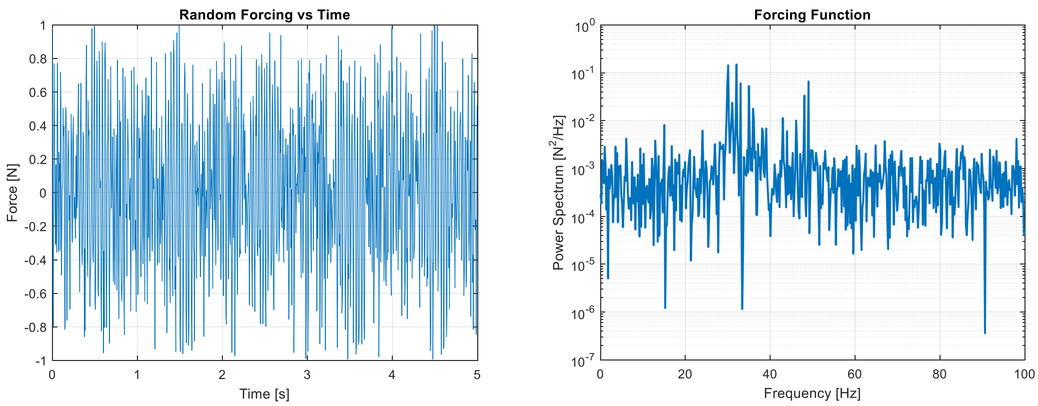

Figure 4.6: Random forcing given in both time domain and spectral domain

Using system parameters are defined in the Table 4.1, and forcing function given in Figure 4.6 system solution has been done. As a result of that response of the 2nd DOF $(Y_2(\omega))$ is found as given Figure 4.7.

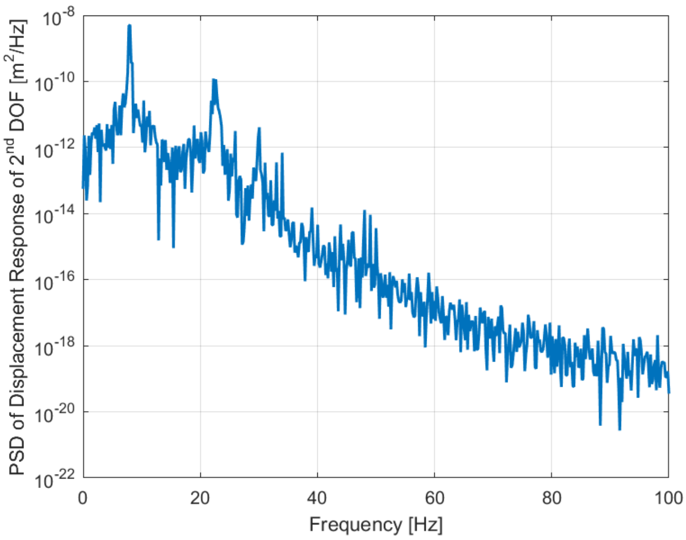

Figure 4.7: Response of 2nd DOF

SDOF TVA just as described in previous section, is attached to second DOF to suppress vibration around 22 Hz. TVA parameters are given in Table 4.2.

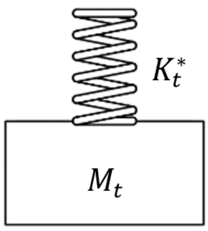

Figure 4.8: SDOF TVA

Table 4.2: TVA Property Table

Property	Value
TVA Mass (M_t)	1 kg
Spring-1 Stiffness (K_t)	19915(1+0.01i) N/m

Since TVA attachment point is selected as 2nd DOF of host structure, in order to accomplish coupling of the FRFs for this point, TVA direct FRF for connecting DOF and host structure 2nd DOF direct FRF are obtained and then coupled given in the

Figure 4.9.

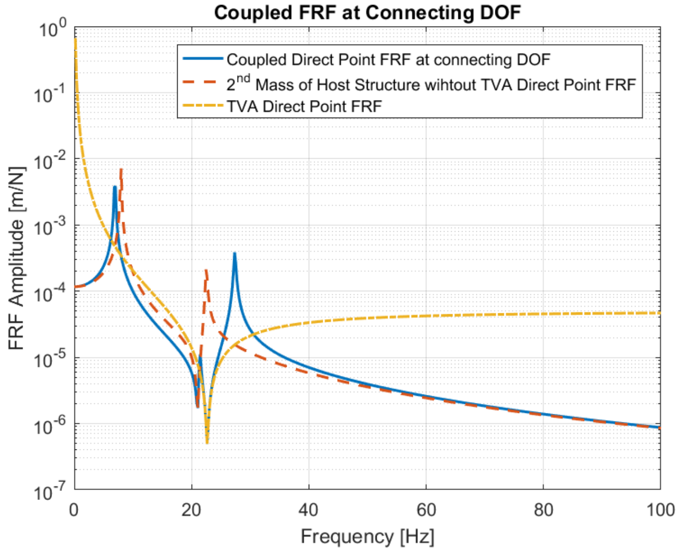

Figure 4.9: Receptance Coupling

Hypothetical forcing is estimated using 2nd DOF direct FRF and operational response:

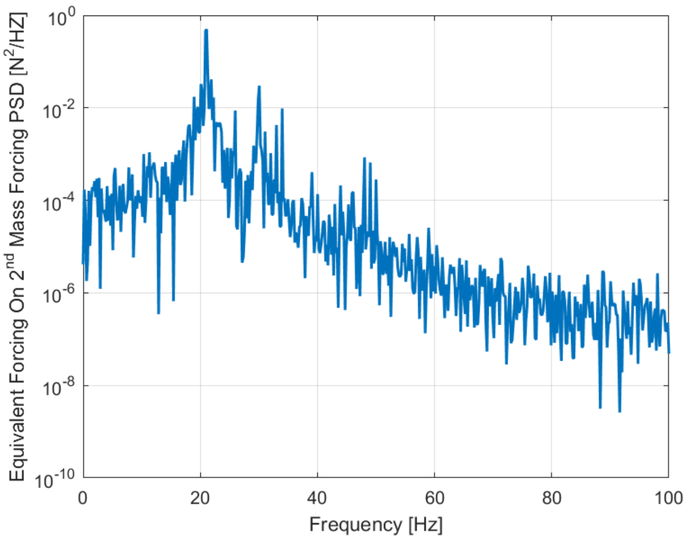

Figure 4.10: Hypothetical Forcing Function

2nd mass and TVA mass displacement response under operational conditions is calculated both 3DOF system numerical solution and response estimation method after attachment of TVA given at the below.

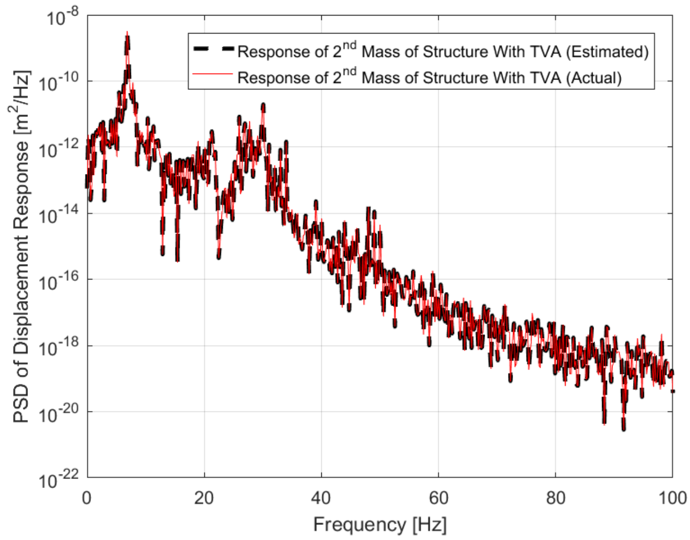

Figure 4.11: Estimation and Actual Response Comparison Host Structure

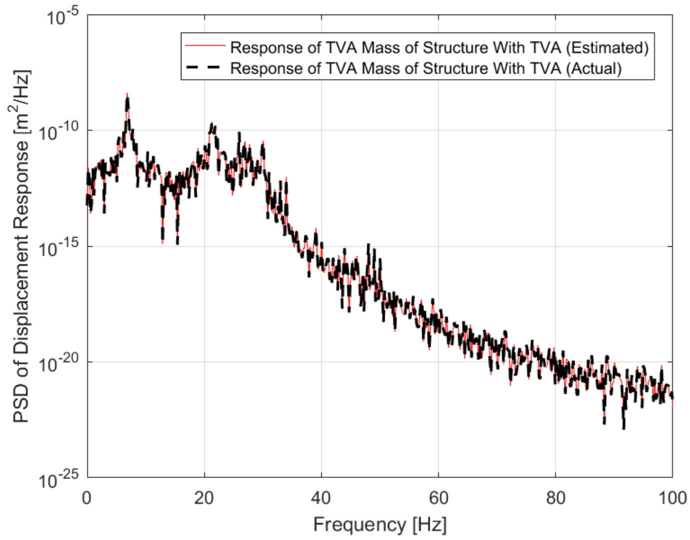

Figure 4.12: Estimation and Actual Response Comparison TVA Mass

As it can be observable from above graph, estimations for both TVA mass and 2nd DOF operational response fitted actual results very well.

CHAPTER 5

CASE STUDY ON OPERATIONAL RESPONSE ESTIMATION METHOD FOR TVA

In order to verify response estimation method presented in previous chapter, procedure is applied on a case study using both FE modeling and experimental methods. In the case study an aluminum beam element is used as a host structure assumed to be a structural element under random forcing. A custom TVA is designed and manufactured for response reduction near first natural frequency of that beam element due to vibration induced from random loading. TVA attached beam response is acquired by FE modeling, experimental methods and following procedure of response estimation method presented in this chapter. Verification process based on two approach to obtain frequency response of modified beam and TVA. In the first approach Al beam model and random forcing acting on it integrated into both experimental procedure and analysis. Response results gathered by both methods are compared each other for verification and consistency. In the second approach frequency response estimation method is applied where neither Al beam model nor forcing acting on it utilized. Procedure is followed described in chapter 3, where Al beam tip response to forcing (where TVA is going to be attached) and Al beam tip direct FRF are used along with TVA parameters to estimate TVA attached beam tip response. Later results acquired using two approach are compared whether to observe frequency response estimation method gives an accurate estimate on a case study. Lastly stress PSDs on elastic components of TVA are constructed as a performance parameter that can be utilize to estimate fatigue life of these elements. Stress PSDs calculation is done using ANSYS Classic and based on three different approach. In the first two approach TVA is base excited with different PSD displacement functions whereas, in the third approach, stress PSD obtained via using TVA attached beam model is used with actual forcing acting on it.

5.1 Problem Definiton

In this case study, an aluminum beam element is used as a source of problem, assumed to be a structural element of a system, under excitation resulting a vibrational problem on the tip of beam. This vibration problem actually created in the controlled lab environment using dynamic shaker and related equipment exciting Al beam at a particular location on beam. Therefore tip of the beam reacts, leading assumed to be unwanted vibration. Al beam used in the case study is given in the Figure 5.1.Al beam (20x20x500mm) is constrained on one end with clamped boundary condition. The other end of the beam is free referred as tip of beam. Beam is excited at 320mm far from tip.

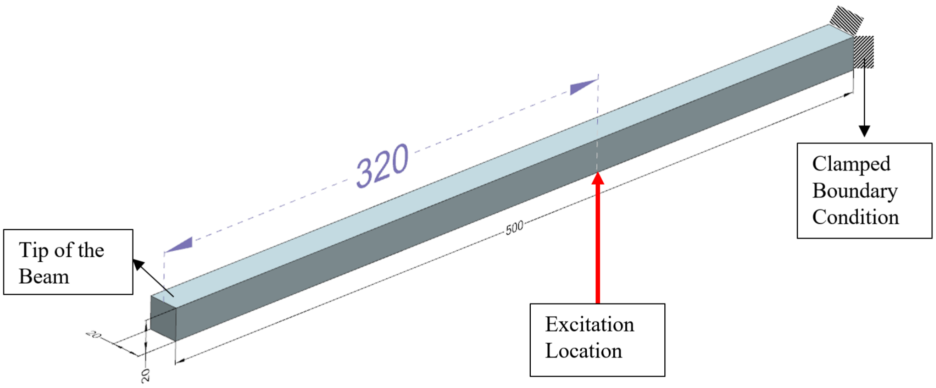

Figure 5.1: Problem Definition

Since, in many application faced in industry, excitations and responses are defined random signal in time domain, in this case study excitation given to the beam is in random manner. Therefore, excitation is defined with power spectral density (PSD) function. Profile shape of the PSD is generic one, used in military standards etc. Level of the PSD is decided based on observation in experiments where visual demonstration and frequency response of the tip response of the beam satisfactory enough to be classified as a vibration problem to be suppressed via TVA.

5.2 TVA Design

TVA design started with identification of frequency of vibration problem. For this purpose firstly, modal analysis of host structure (in this case study host structure is Al beam) described in problem definition part carried out. First natural frequency of the beam where the transverse vibration is observed, is selected tuning frequency of the vibration absorber. For modal analysis, Ansys Workbench Modal analysis module has been used. In the Figure 5.2 aluminum beam FE model used in analysis is presented.

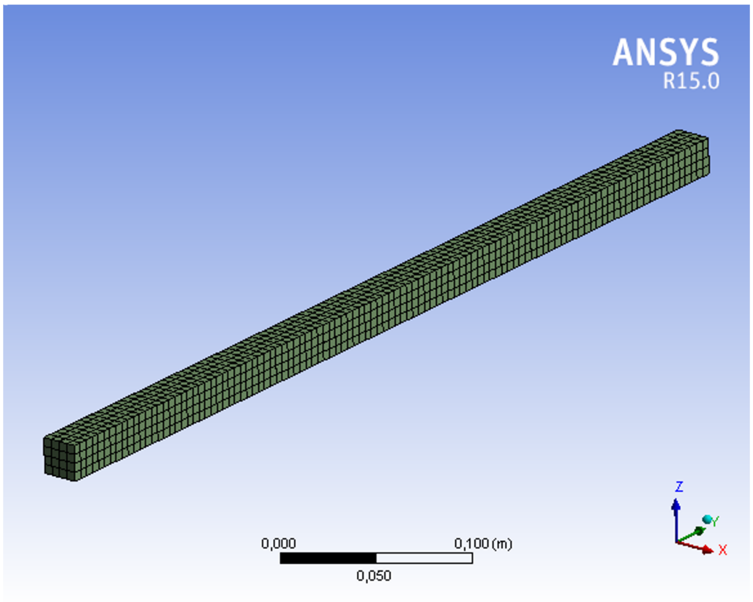

Figure 5.2: FE Model of Beam

FE model is constructed with 5mm edge length hex elements. Also clamped boundary condition is applied at nodes given in the Figure 5.3.

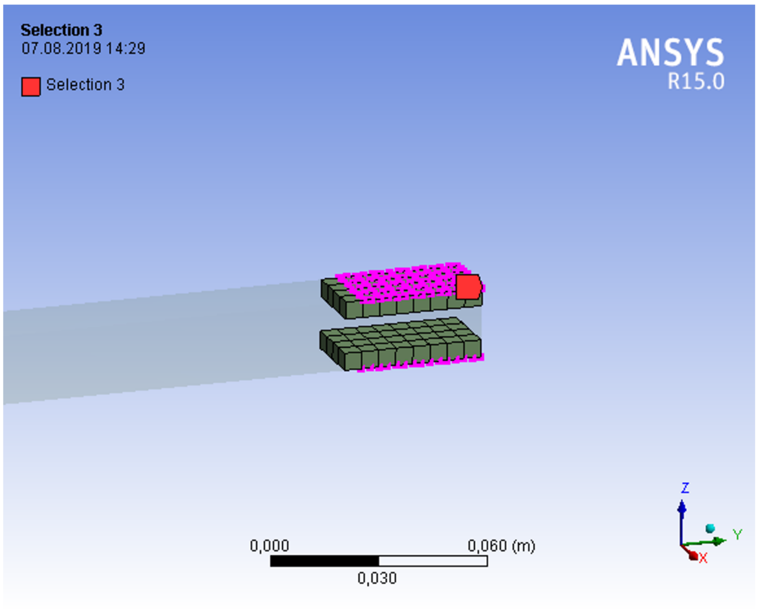

Figure 5.3: Clamped Boundary Condition of Beam

Modal properties for related modes and mode shape of beam for first natural frequency in transverse direction is given in the Figure 5.4 and 5.5.

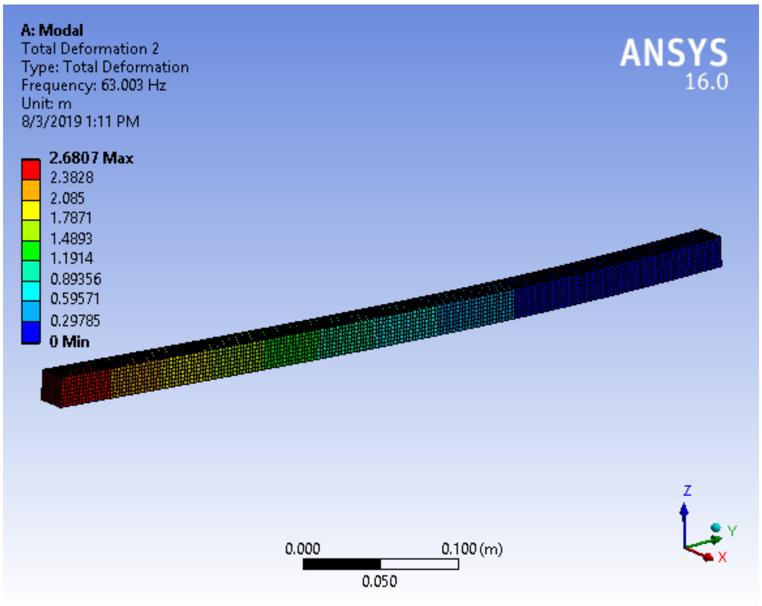

Figure 5.4: Mode Shape of Beam

***** PARTICIPATION FACTOR CALCULATION ***** Z DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION	RATIO EFF.MASS TO TOTAL MASS
1	62.3178	0.16047E-01	-0.48245E-06	0.000001	0.232756E-12	0.482856E-12	0.392652E-12
2	63.0035	0.15872E-01	0.58392	1.000000	0.340968	0.707343	0.575202
3	387.641	0.25797E-02	0.0000	0.000000	0.00000	0.707343	0.00000
4	392.193	0.25498E-02	-0.32398	0.554824	0.104960	0.925085	0.177064
5	1072.93	0.93203E-03	0.0000	0.000000	0.00000	0.925085	0.00000
6	1086.08	0.92074E-03	0.19003	0.325439	0.361122E-01	1.00000	0.609201E-01
sum					0.482041		0.813187

Figure 5.5: Modal Properties of Beam

TVA target frequency is selected as 63 Hz. Design construction of TVA based on SDOF system consisting of one stiffness element and one mass element. Since target frequency is selected, in order to set the tuning frequency, mass and stiffness values should be decided. TVA main mass (effective mass) is set based on modal mass of host structure at related resonance conditions. Therefore, TVA effective mass is decided as %10 of modal mass of beam at related mode. However, even if the initial design limit for TVA mass set as 0.031kg (0.341kgx0.1), in order to prevent technical difficulties for realization of TVA, this value increased up to 0.060kg in final design.

Second important step for the design of TVA is the stiffness elements. Since tuning frequency and effective mass is decided, stiffness value can be computed from simple equation that relates natural frequency (ω_n) of SDOF system with mass (m) and stiffness (k).

$$\omega_n = \sqrt{\left(\frac{k}{m}\right)} \tag{5.1}$$

Where $\omega_n = 63 \times 2 \times \pi$ (in radians), $m = 0.060kg$ and resulting stiffness is $k = 9500N/m$ for TVA design.

After parametric study has been done for TVA, structural design for mass, stiffness and foundation elements started. For stiffness element, helical springs are used in design, since they are frequently used in many applications and reliable elements and they are easy to implement into system. In the design, two helical springs are utilized in compression mode where mass of the absorber is located between in those two compressed springs. Neglecting the springs masses this main mass located between springs is actually effective mass of SDOF system. In the Figure 5.6 3D model of the design is presented where mass, stiffness and structural elements are shown.

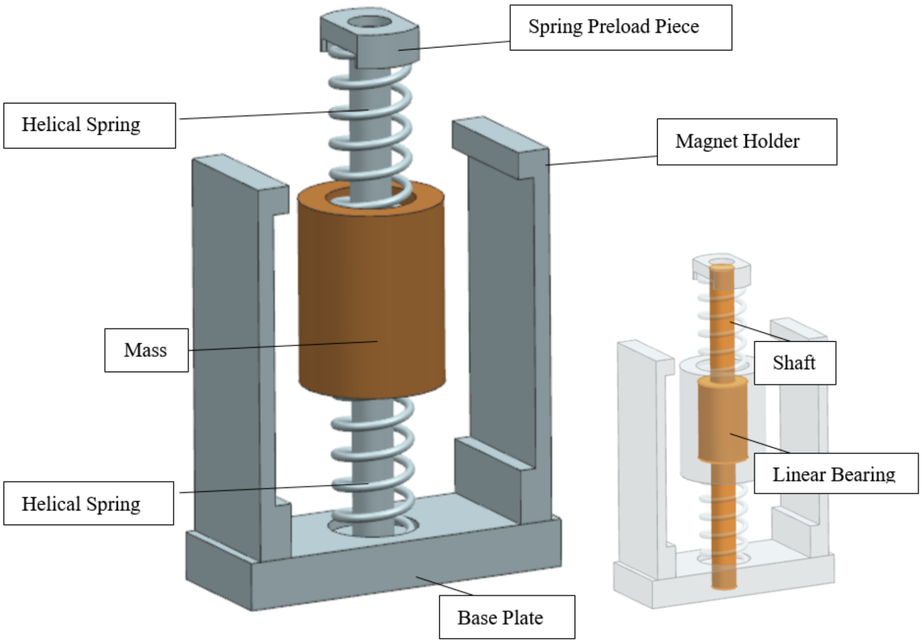

Figure 5.6: 3D Model of Designed TVA

Mass: Mass is constrained with cylindrical type joint where linear bearing is used in the middle axis of the cylindrical mass. Mass value of this element determined in the previous part, implement in to design by setting outer diameter and height. Material for the mass element is decided as copper since copper is dense enough to keep aspect ratio of the design as much as close to one. Moreover, copper has low conductive resistance. Hence, adding Eddy-Current damping into design in the future work is going to be easy without changing either mass element or structural elements.

Stiffness: Springs configurations are parallel since they deflect equal amount when the mass moves. Therefore, assuming equal springs are going to be used at each side of mass, for single helical spring stiffness (k_s) value should be equal to $k_s = k/2$. In the previous part k is determined as $9500N/m$ based on the relation with tuned frequency. However, tuned frequency is revised due to design of TVA. Referring to Figure 5.6, TVA consist of passive elements (other than mass and stiffness elements) that adds up as mass on host structure such as Base Plate, Magnet Holders and Shaft. This addition slightly modifies host structure modal properties. Therefore, another modal analysis have been carried out to identify new target frequency. Structural elements modeled with beam and modal analysis repeated as given in the Figure

Parameter	Value
D (Mean Diameter of Spring)	0.0106 m
n (Number of active coils)	6
d (Coil Diameter)	0.0013 m
G (Shear Modulus Of Material)	76.9 GPa

Table 5.1: Spring Parameter Table

5.7.

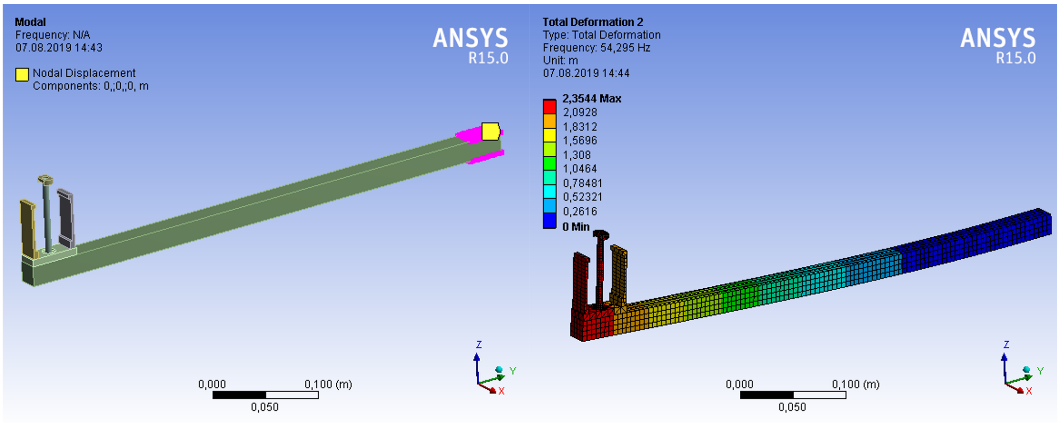

Figure 5.7: Revised Modal Analysis

New stiffness value becomes ($k_s = 3500N/m$) when the revised tuned frequency is set as 54 Hz using the eqn5.1. Stiffness value of the helical spring is related with shear modules of material, coil diameter, mean diameter of spring number of spring coils represented by G, d, D and n in the eqn 5.2.

$$k_s = \frac{Gd^4}{8D^3n} \tag{5.2}$$

Determined parameters for springs are given in the Table 5.1 After manufacturing process actual springs constants are measured by simple test where deflection and corresponding forces are determined by caliper and force gage device. In the below graphs actual spring parameters are shown acquired by test where slope of the trendline fitted to test results represents the stiffness constants of the helical springs.

Due to errors faced during manufacture process and imperfections due to material, stiffness values slight diverge from desired value

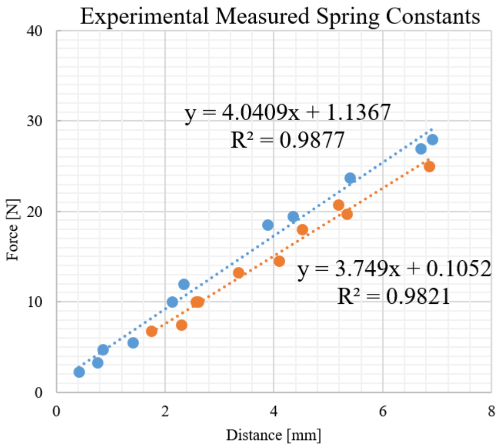

Figure 5.8: Experimentally Measured Spring Constants

Structural elements: Linear bearing is used to constrain the mass element guided on stainless steel shaft. Base plate, spring preload piece and magnet holders are made of Aluminum. Final product after manufacturing is shown in the Figure 5.9. Mass values are measured after manufacturing for verification given in the Figure 5.10. Tuned frequency and other features are validated through tests given in the experimental study section.

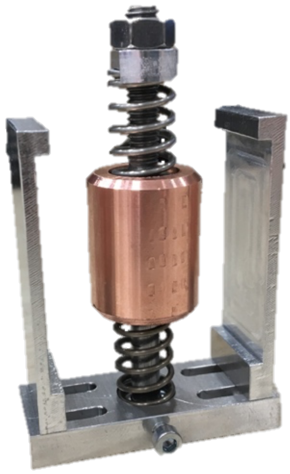

Figure 5.9: Manufactued Prototype of TVA

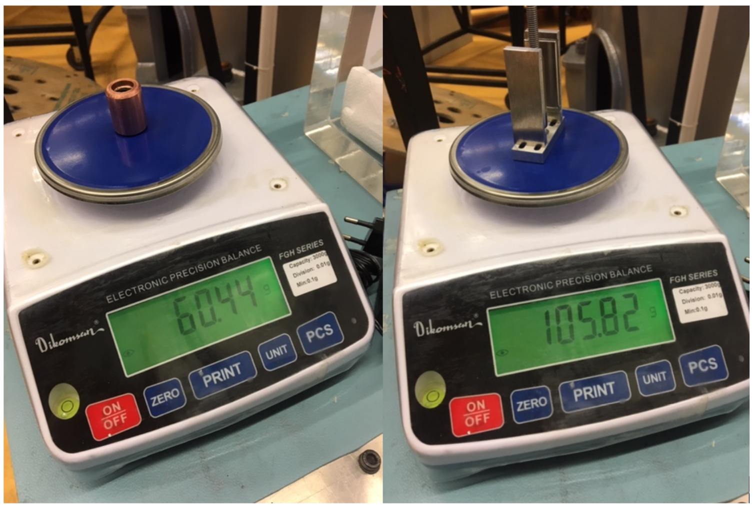

Figure 5.10: Mass values of the components

5.3 Analysis Studies

FE analysis have been taken place in this case study, in order to verify TVA design as well as merged system (TVA attached to host structure) beforehand prototpye testing. For verifications, frequency response functions to unit amplitude harmonic loading are obtained for both configurations and presented in this section. In addition to verification of the models, frequency response functions under loading applied at beam element as it is described in the problem definition part, acquired using FEA. All analysis results are later compared with ones obtained by experiments.

5.3.1 Frequency Response Functions of TVA

For designed TVA, two analysis are carried out to be used in both verification of design parameters, as well as computation for response estimation of TVA in-operation method described in chapter 3. First analysis is transmissibility of TVA and second one is FRF of TVA base, to unit amplitude loading applied at base of TVA. (Direct point FRF)

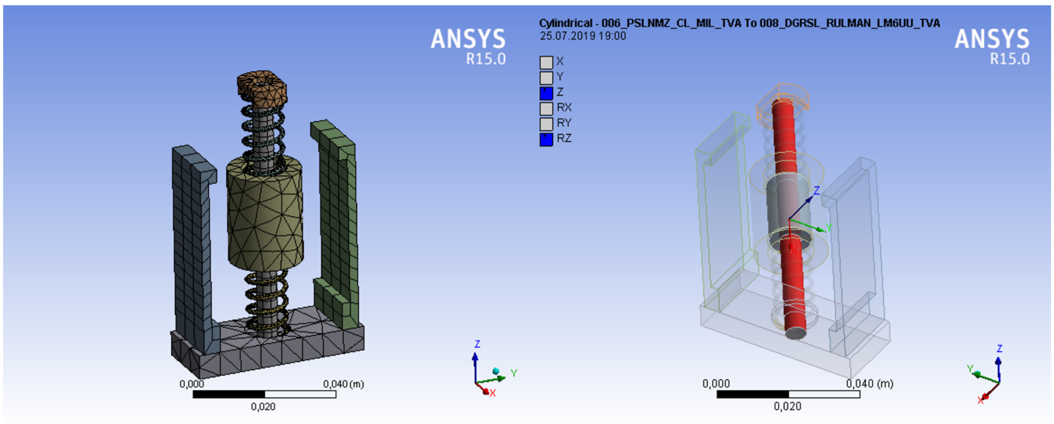

Figure 5.11: FE model of TVA and Contact between Shaft and linear bearing

In the Figure 5.11 FE model of the TVA and joint representation between shaft and linear bearing embedded in cylindrical copper as cylindrical joint contact type. Other contacts between structural elements are defined as bonded for related contact areas.

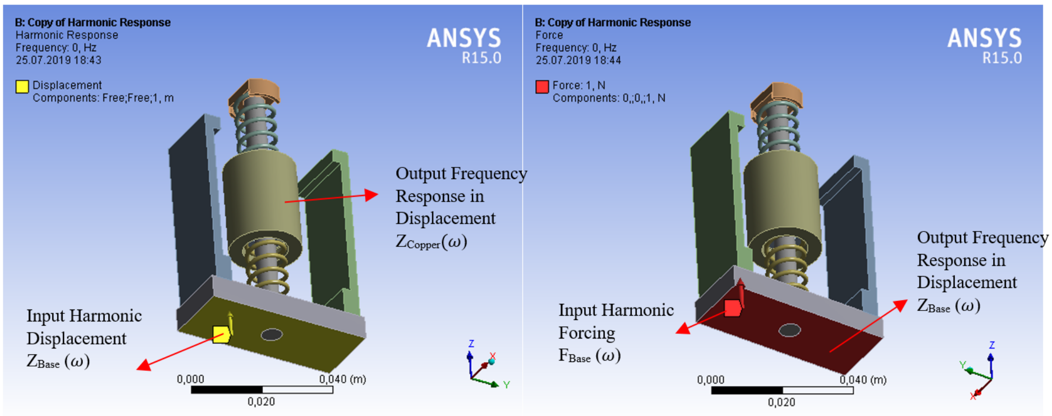

Figure 5.12: Input and Output Locations

In the figure 5.12 locations are presented where the loads are applied and responses are obtanied. Frequency response functions for related analysis are presented in the Figure 5.13. In the first Figure transmissibility plot is given. In the second one direct point FRF where excitation location and response location is the base of TVA.

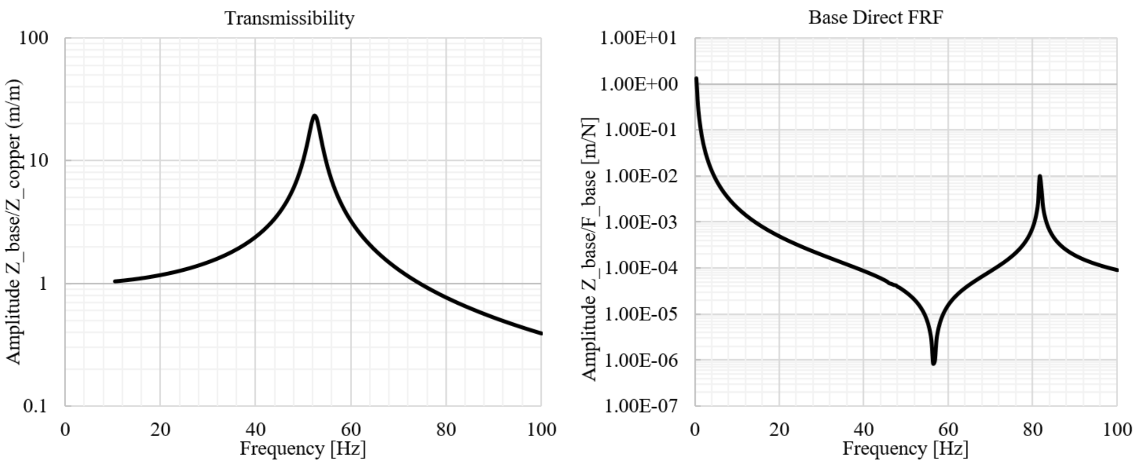

Figure 5.13: TVA Transmissibility Plot and Base Direct Point FRF

5.3.2 Frequency Response Functions of Beam

Frequency response functions of beam are obtained via FEA before and after attachment of TVA. Firstly standalone (before attachment of TVA) beam FE model is constructed as it is given in the Figure 5.14. Clamped boundary condition is applied on beam as showed at the Figure 5.14.

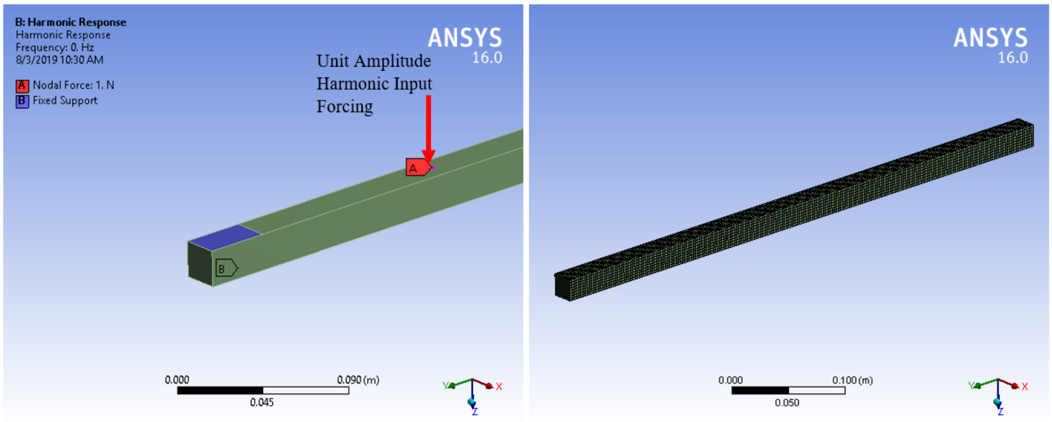

Figure 5.14: FE model of Beam, Beam's Clamped Boundary Condition

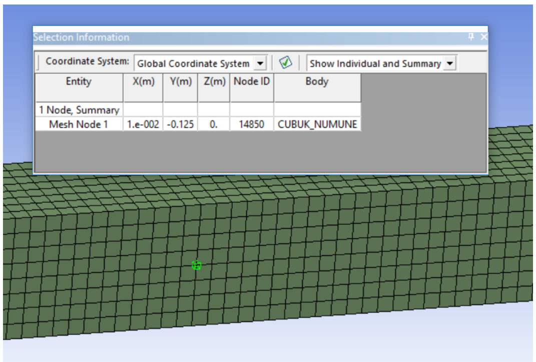

Figure 5.15: Forcing Input Location Nodal Information

For FRF computation input and output locations are selected for standalone beam. For input location, excitation point is selected as it is defined in problem definition section where the random forcing assumed to be applied for the vibration problem. For the output (response) location, tip point of the beam is selected where the problem is observed as it is explained in the problem definition part as well. For the solver of the FE model, ANSYS Harmonic Response module has been used for beam as well. For input, unit amplitude harmonic forcing is applied and response of the tip point is obtained as given in the Figure 5.16.

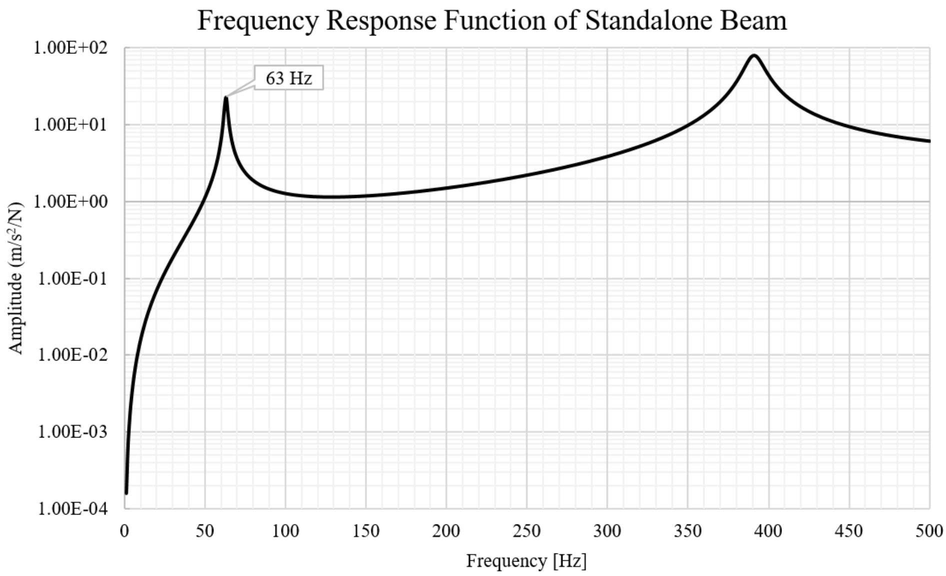

Figure 5.16: Frequency Response Function of Standalone Beam

Frequency response of TVA attached beam is acquired from FEA as well. In the Figure 5.17. FE model of TVA attached beam is shared.

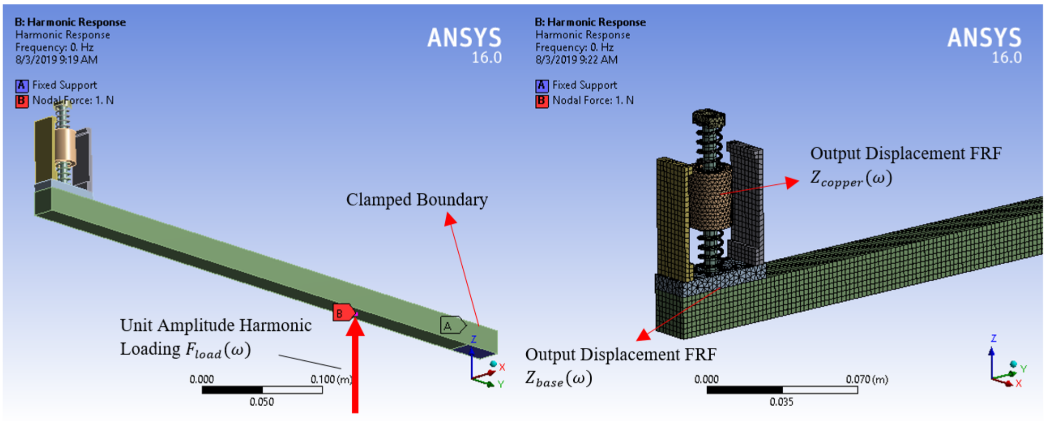

Figure 5.17: FE mode of TVA attached beam, input and output locations

Unit harmonic forcing is applied at the location where the random forcing is applied in the problem definition part. Responses are acquired at the TVA mass (cylindrical copper piece) and TVA base where is actually referred as tip point of beam in the problem definition part.

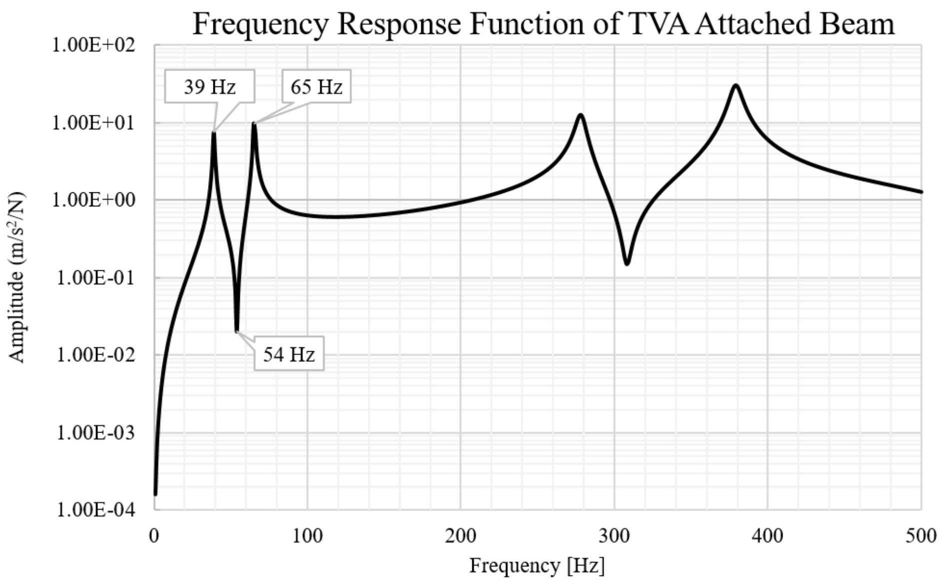

Figure 5.18: Frequency Response Function of TVA attached Beam

5.3.3 Forced Vibration Analysis

In addition to FRF generation of TVA, beam and TVA attached beam, forced vibration analysis have been carried out in order to compare results obtained via experiments. For forced vibration analysis two configuration of FEM have been used as it is previously done for FRF generation for beam. FEA for forced vibration have been done on ANSYS Classic where PSD forcing functions are applied at beam as nodal excitation used in PSD spectrum analysis. For standalone beam; FE model, PSD forcing input location, input PSD function and response PSD of tip of beam is given in the Figures 5.19,5.20,5.21,5.22.

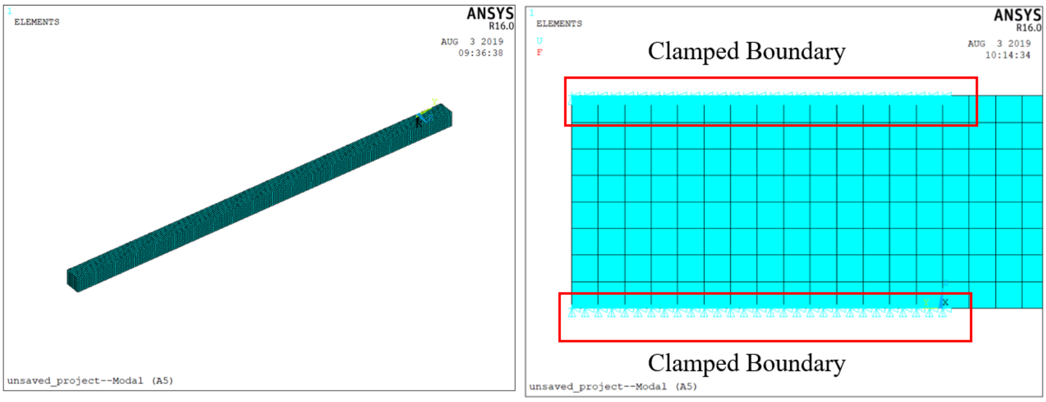

Figure 5.19: FE model of beam and applied clamped boundary condition

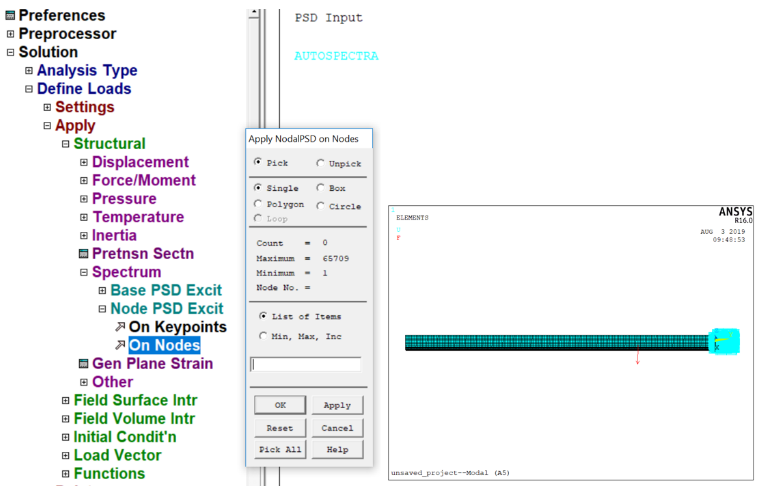

Figure 5.20: Nodal Excitation Features

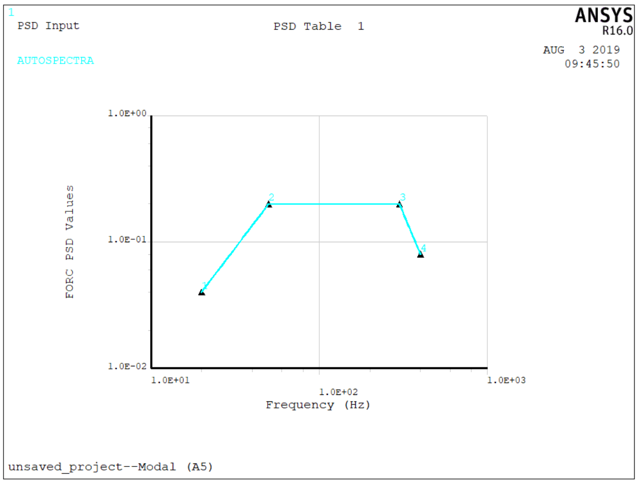

Figure 5.21: Input Random Forcing PSD Function used in Analysis $[N^2/Hz]$ vs Hz

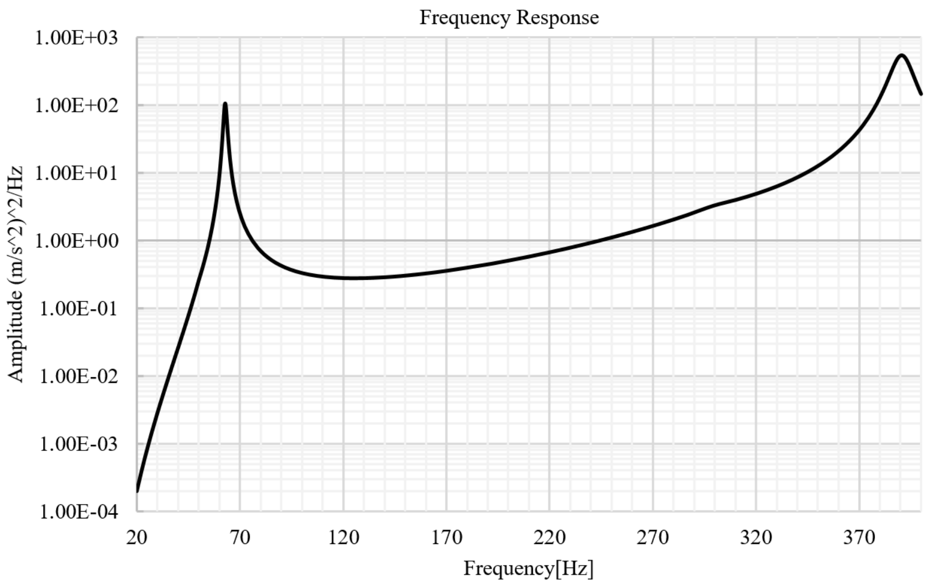

Figure 5.22: Beam Tip Acceleration Frequency Response to Random Loading

In the figure 5.22 beam tip response is obtained. This response is considered as operational reponse of host structure. Similar analysis applied on TVA attached beam as well as it is done for standalone beam. In the Figures 5.23,5.24; FE model of TVA attached beam, PSD forcing input location, response location and response PSD of beam and TVA is given. PSD function applied on TVA attached beam is exactly same used in standalone beam forced vibration analysis.

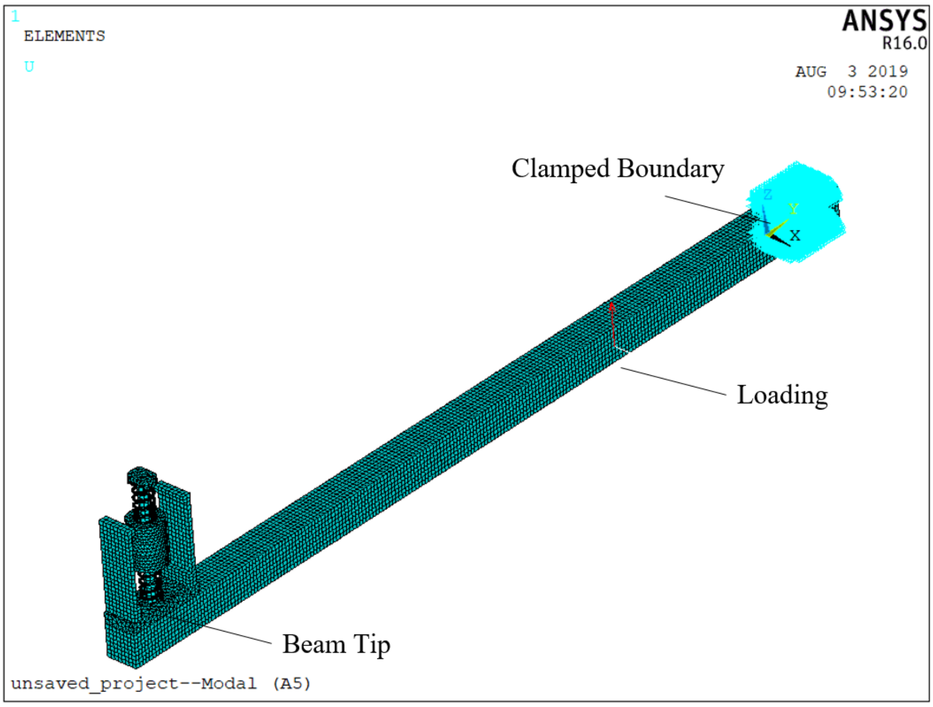

Figure 5.23: FE model of TVA attached beam constructed on ANSYS Classic

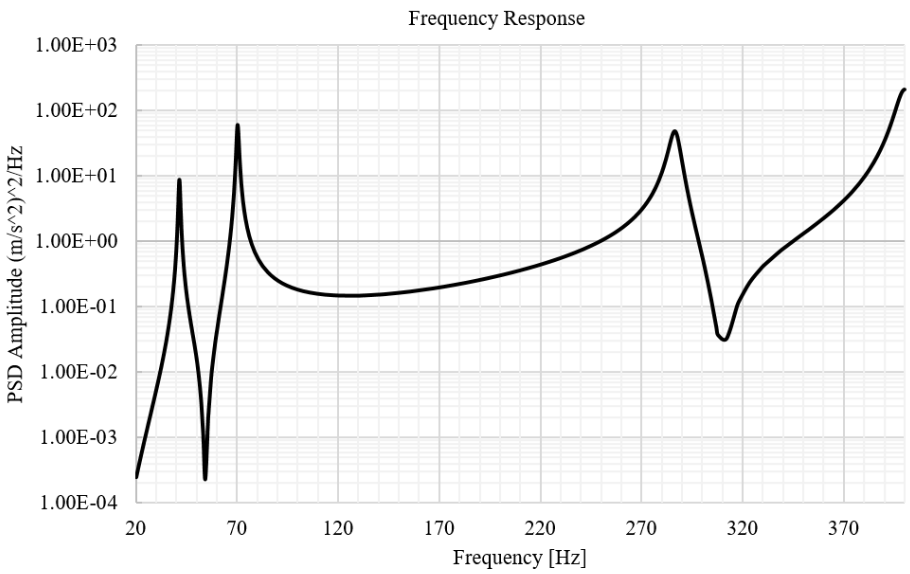

Figure 5.24: Frequency Response of Beam Tip

In the Figure 5.24 frequency response of beam modified by TVA is presented. These analysis studies are later by experimental results presented in the following section.

5.4 Experimental Studies

Experimental studies are composed of three main set of measurements. The first set consist of measuring frequency response functions of standalone beam, TVA attached beam and transmissibility of TVA. These measurements are required for confirmation of beam and TVA FRFs by comparing results with the ones obtained by FEA and mathematical models. After this verification, experimental studies are continued with second set of measurements. In the second experimental step, firstly vibration problem is created as it is given in the problem definition part where beam is excited at particular location and direction with prescribed PSD forcing function. As a result of the loading, frequency response of the tip of beam is acquired. Then, manufactured TVA is integrated on tip of the beam to suppress the vibration amplitudes at tuned frequency defined at TVA design section. This new configuration (modified beam-TVA attached beam) is then excited with the very same loading as it applied on standalone beam. Responses of TVA mass and beam acquired are required to be used in the verification of the frequency response estimation method presented in chapter 3 by comparing the results obtained by actual physical implementation of TVA on to the host structure and ones estimated by frequency response estimation method. Afterwards, third set of the measurements are carried out where standalone Al beam tip (where TVA is attached) direct point FRF is measured using impact hammer. This measurement is required for calculations used in frequency response estimation method.

5.4.1 Experimental Setup

Test setup is consisting of below components;

1. Aluminum Beam (20mmx20mmx500mm)

2. Designed TVA

3. Accelerometers (3)

4. Force transducer (1)

5. Stringer (1)

6. Dynamic Shaker and Control Unit

7. Data Acquisition Device (DEWESOFT)

8. Laptop

9. Polyamide Tip Impact Hammer

Test configurations are presented in the Figures 5.25,5.26,5.27,5.28,5.29.

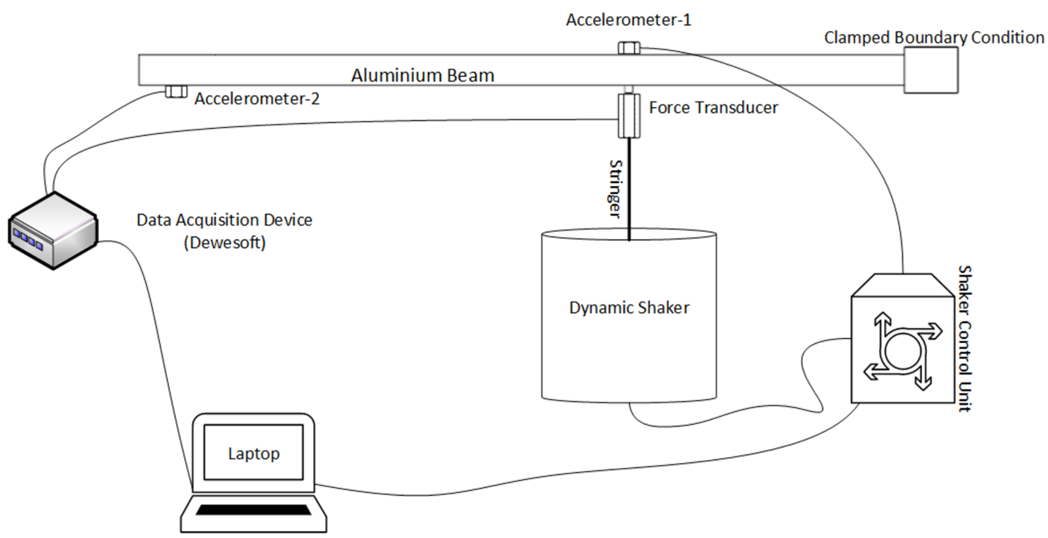

Figure 5.25: Experimental Setup Configuration-1 (Closed Loop Shaker Contol, Beam Without TVA)

In the Figure 5.25, experimental setup configuration-1 is presented. In this configuration dynamic shaker is controlled in closed loop method where accelerometer-1 is driven. Input forcing is measured via force transducer whereas output as acceleration response of tip of the beam.

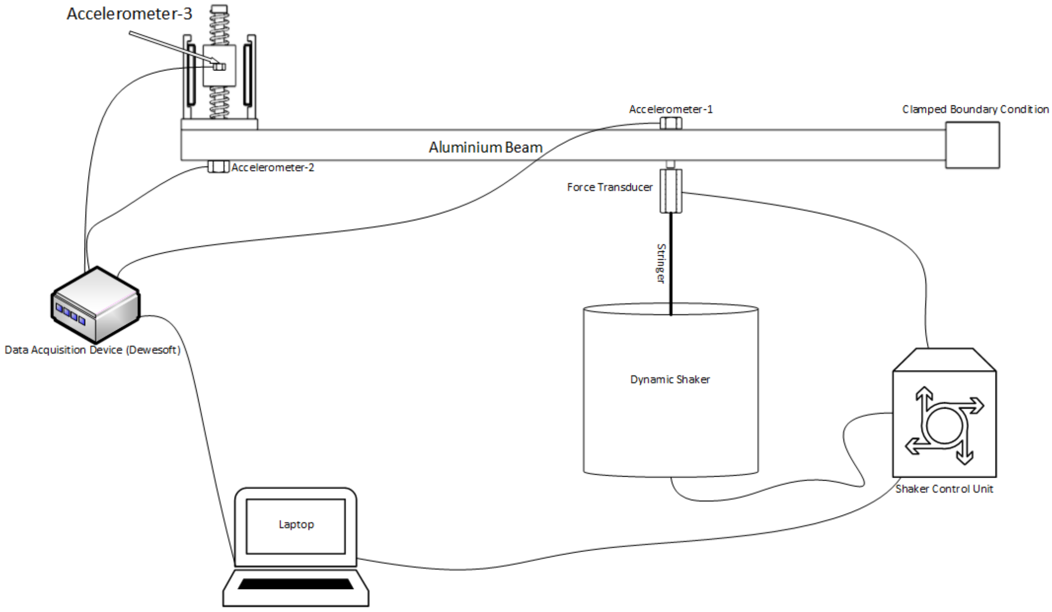

Figure 5.26: Experimental Setup Configuration-2 (Open Loop Shaker Control, With TVA)

In the Figure 5.26, experimental setup configuration-2 is presented. In this configuration dynamic shaker is controlled in open loop control methodology. Forcing is measured along with acceleration responses of TVA mass and tip of the beam.

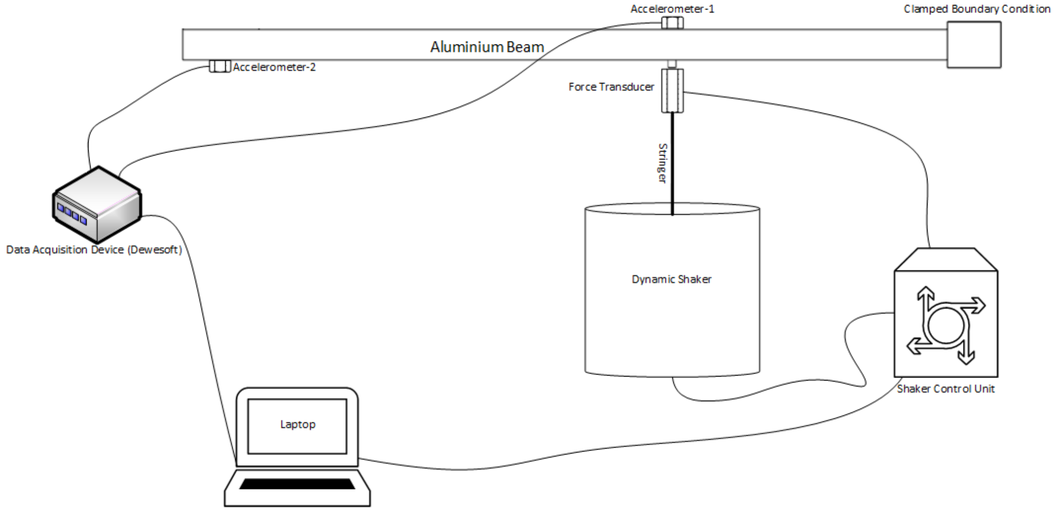

Figure 5.27: Experimental Setup Configuration-3 Forcing is controlled, Loading is applied on beam

In the Figure 5.27 experimental setup configuration-3 is presented. In this configuration vibration problem is created by applied random forcing via dynamic shaker. Response of the tip of the beam is measured via accelerometer-2.

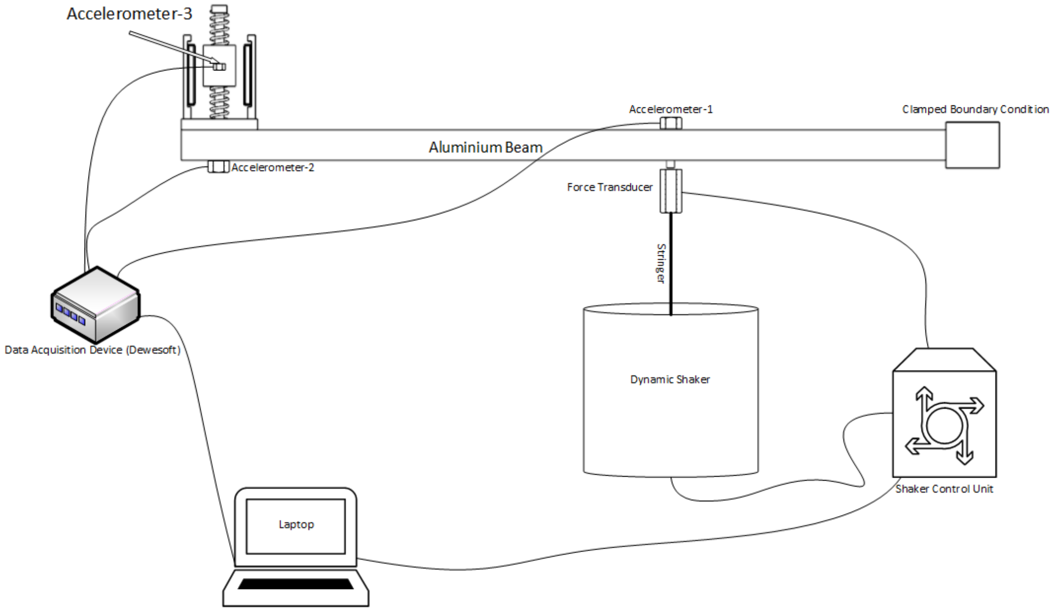

Figure 5.28: Experimental Setup Configuration-4 Forcing is controlled TVA attached beam is excited with random loading

In the Figure 5.28 experimental configuration-4 is presented. In this configuration vibration problem is solved by TVA attachment. In order to obtain TVA mass and beam tip response in this modified case, accelerometer-3 and accelerometer-2 is used.

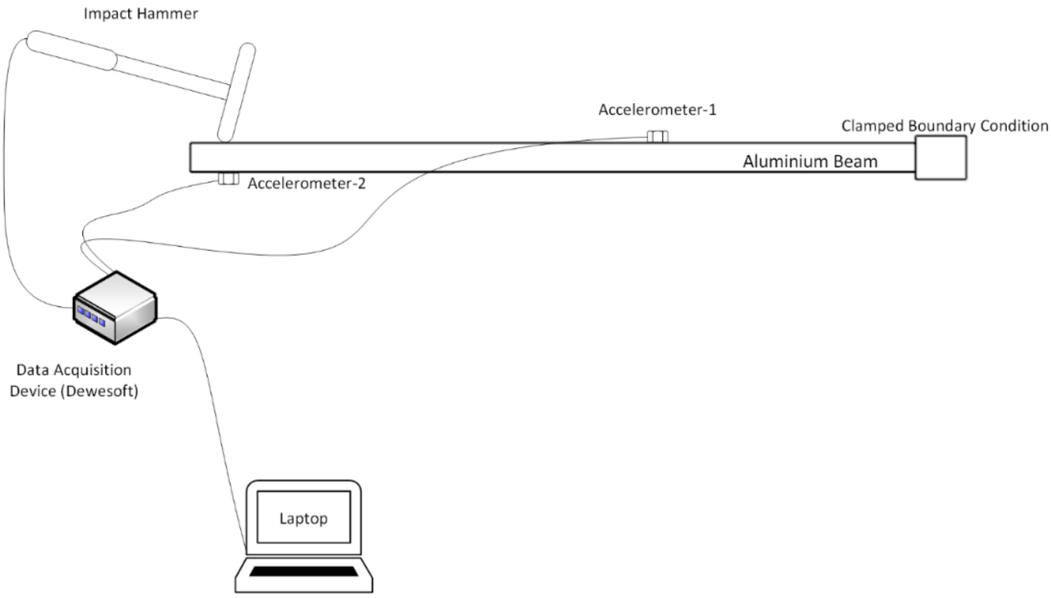

Figure 5.29: Experimental Setup Configuration-5 Impact Hammer Test

In the Figure 5.29, experimental setup configuration-5 is presented. In this configuration beam tip direct FRF is measured.

5.4.2 Experiments

Actual components used in experimental setup is presented in the following Figures. In the Figure 5.30 data acquisition device and accelerometer-2 is given. In the next Figure 5.31 images of impact hammer, accelerometer-1 ,accelerometer-3 and force transducer are shared.

Figure 5.30: Data Acquisiton Device and Accelerometer-2

Figure 5.31: Impact Hammer,Accelerometer-1,Accelerometer-3, and Dynamic Shaker Control

5.4.2.1 FRF and Transmissibility Measurements

For validation of FRF of host structure, in our experimental case this structure is beam, FRF measurements are done using experimental setup configuration 1 and 2. First FRF measurement has taken place using testing configuration-1. In this configuration given in the Figure 5.32, measurements are taken using accelerometer-2 at the end of the beam where TVA is going to be attached, and dynamic input location using force transducer.

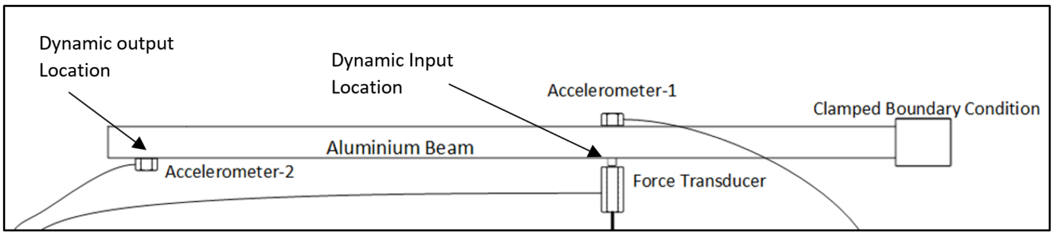

Figure 5.32: Testing Configuration-1

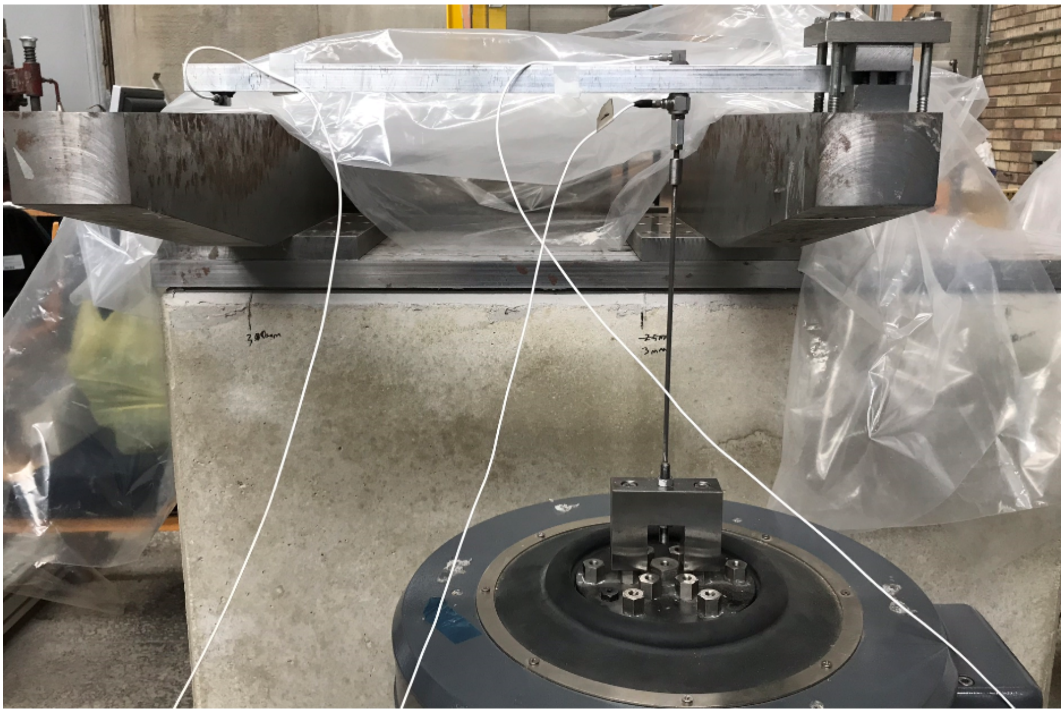

Figure 5.33: Testing Configuration-1 Setup Used in Lab Environment

In this experimental configuration dynamic shaker is controlled in closed loop control methodology where dynamic input corrected in a way in which the measurement taken from accelerometer-1 is adjusted to desired levels. Desired levels to be observed at accelerometer-1 location is set using dynamic shaker control unit interface as given in the Figure 5.34.

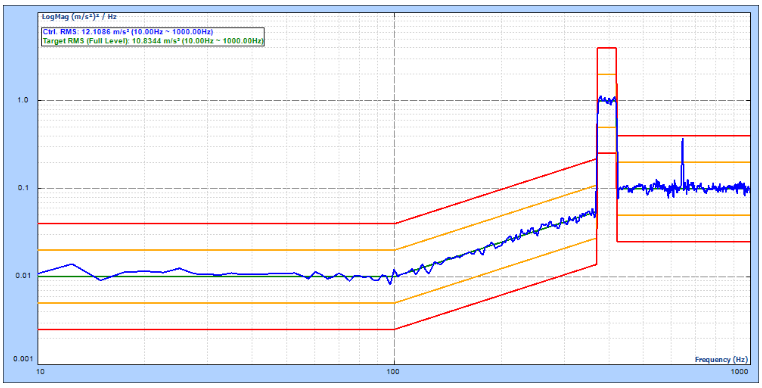

Figure 5.34: PSD Vibration Profile Applied via Dynamic Shaker

Using data acquisition device modal testing interface forcing input coming from measurement via force transducer and acceleration measurement at the end of the beam (accelerometer-2) is utilized in order to construct of frequency response function of beam tip given in the Figure 5.35.

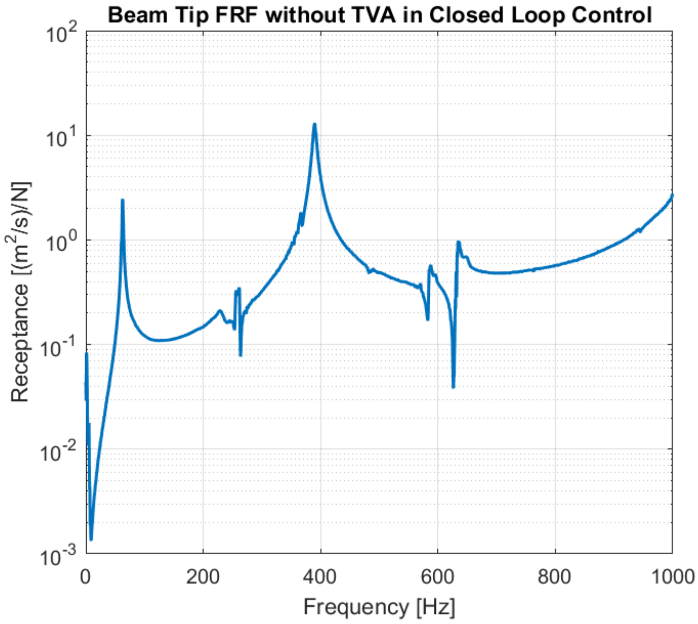

Figure 5.35: Standalone Beam Tip FRF

In experimental setup configuration-2 beam is attached with TVA where accelerometer-2 is located given in the Figure 5.36.

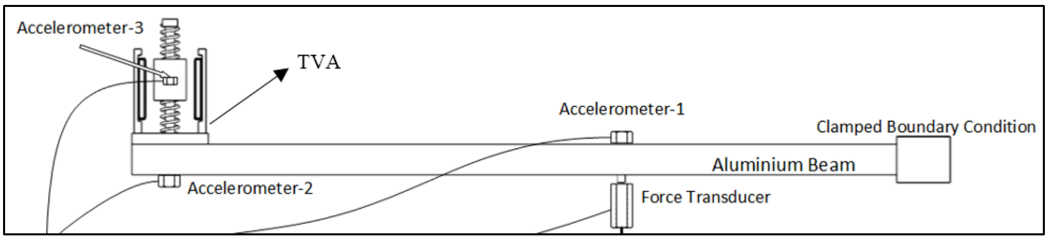

Figure 5.36: Testing Configuration-2

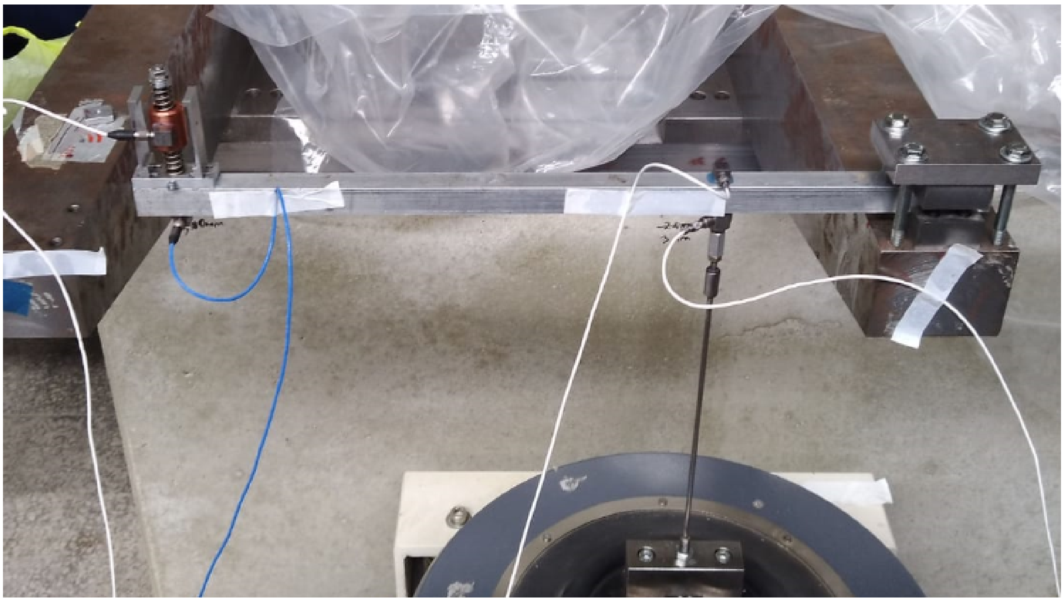

Figure 5.37: Testing Configuration-2 in Lab. Environment

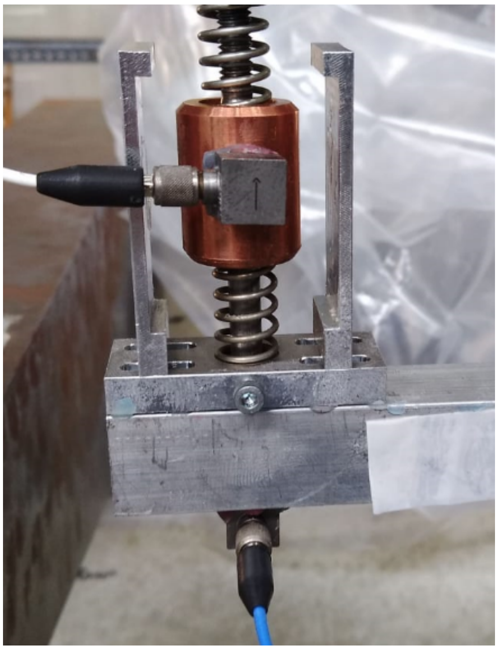

Figure 5.38: TVA attachment to beam

Dynamic shaker is controlled in open loop control methodology. FRFs are generated using measurements taken from force transducer and accelerometer-2 and accelerometer-3. Modified structure (TVA attached beam) and TVA mass frequency response functions are given in the Figures 5.39,5.40.

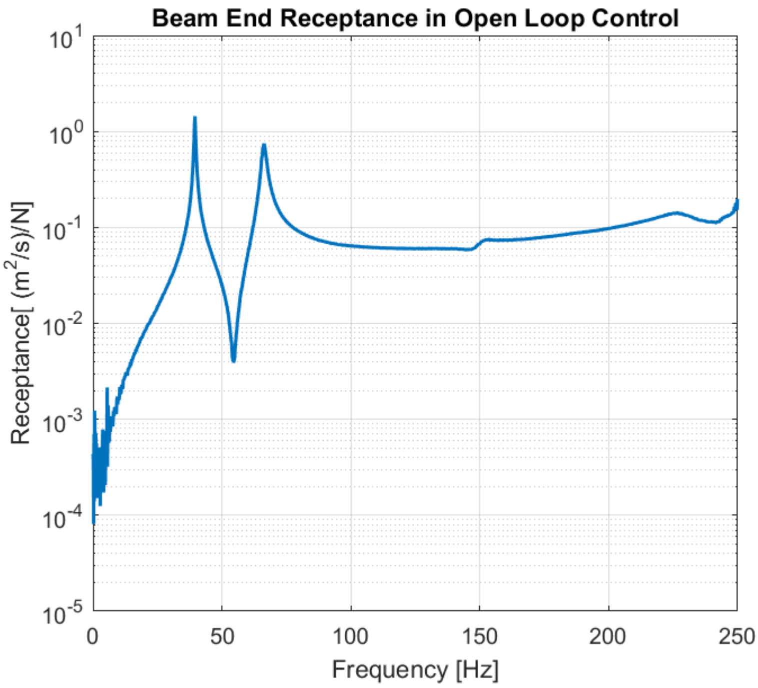

Figure 5.39: TVA Attached Beam Tip FRF

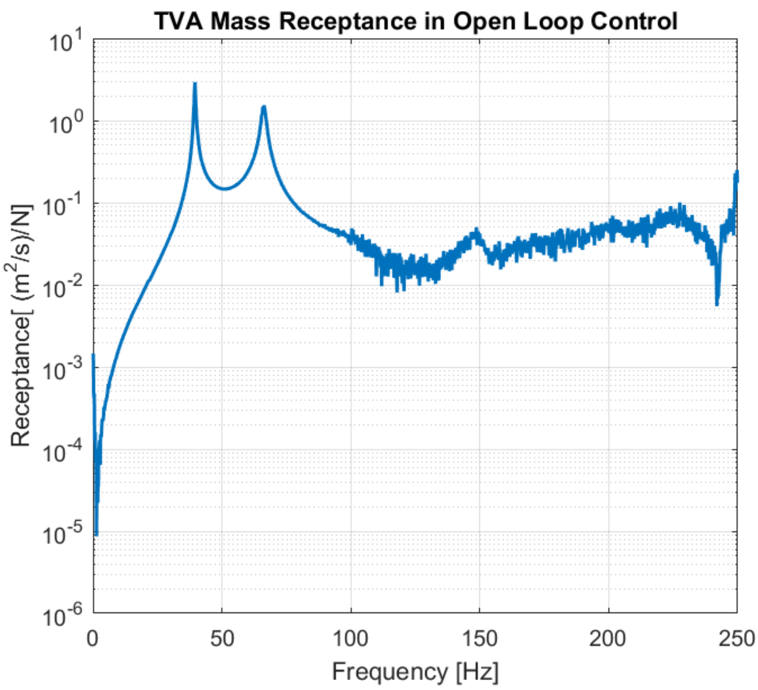

Figure 5.40: TVA Mass FRF

Transmissibility measurement is done in-situ conditions where TVA mass response is divided by base input. Recall Figure 5.39 and Figure 5.40, TVA mass response acquired by accelerometer-3 for modal identification this can be considered as output whereas input set as data obtained via accelerometer-2 that is actually base of the TVA. Therefore input-output relation has been realized and result is obtained for transmissibility for TVA presented in the Figure 5.41.

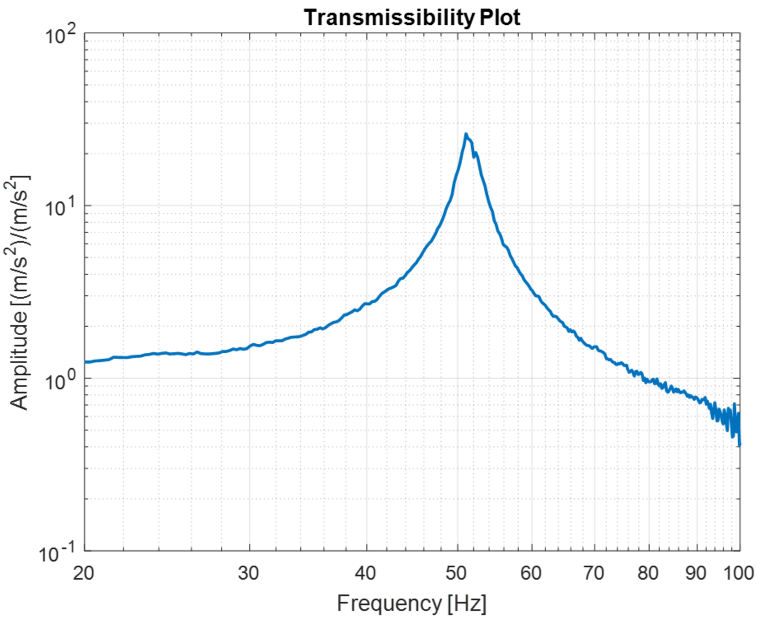

Figure 5.41: TVA Transmissibility Plot

5.4.2.2 Random Excitation Forced Vibration Test

In this experimental configurations (Configuration-3 and Configuration-4) standalone beam and TVA attached beam is excited using random forcing generated through shaker control unit at pre-described vibration profiles given Figure 5.42. Dynamic shaker is controlled using force transducer given in the configuration schematic-3 and 4. Excitation forcing is applied where the force transducer is located shown in the Figure 5.43.

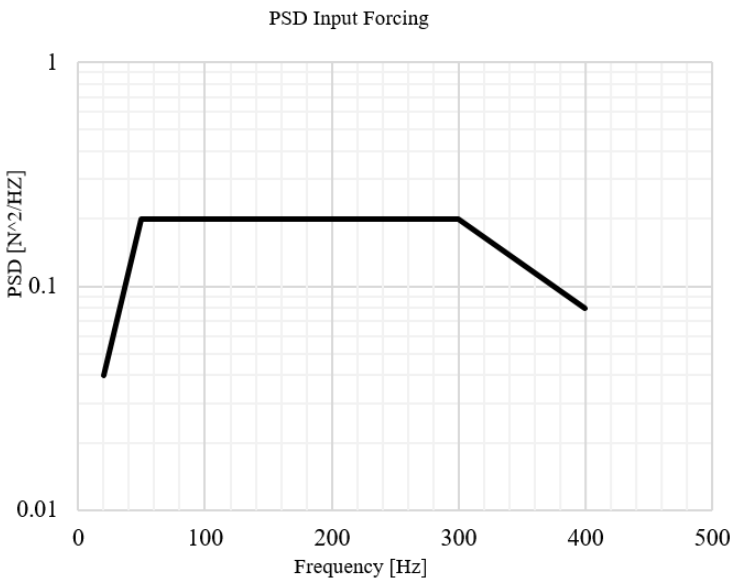

Figure 5.42: PSD Input Forcing

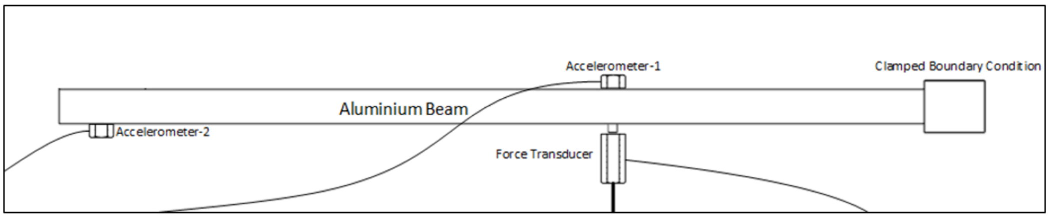

Figure 5.43: Testing Configuration-3

As a result of this forcing applied on beam, tip response is acquired given in the PSD form in the below graph.

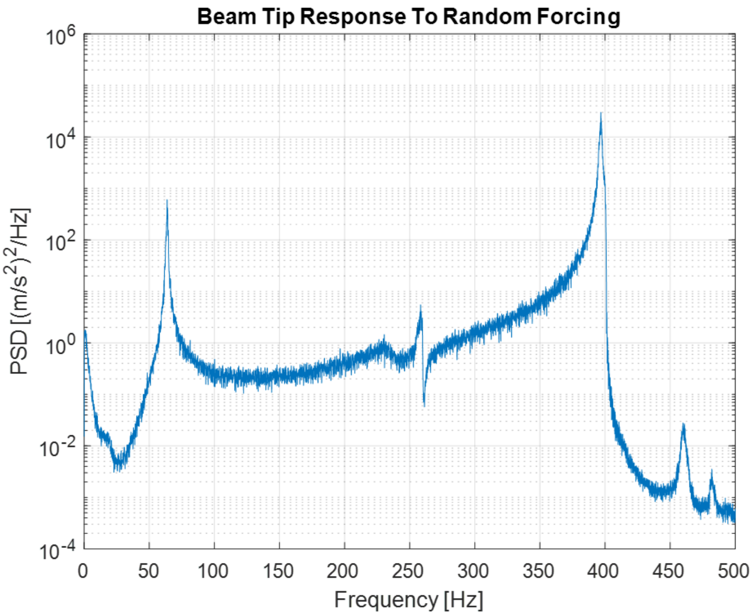

Figure 5.44: Beam Tip Response

In this configuration of experiment TVA attached beam is exited through where the force transducer is located as given in the Figure 5.45. Excitation vibration profile (random forcing) is same as applied to standalone beam given in the Figure 5.42.

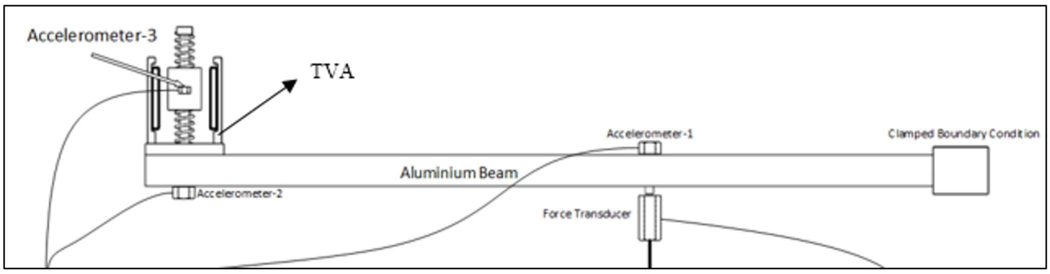

Figure 5.45: Testing Configuration-4

As a result of this forcing applied on beam, tip response of beam and TVA Mass response are acquired given in the PSD form in the Figure 5.46.

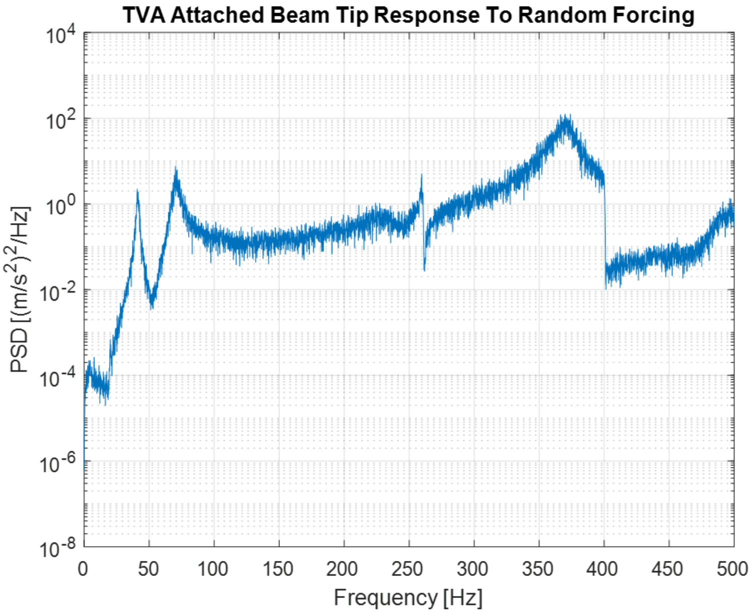

Figure 5.46: TVA Attached Beam Tip Response

5.4.2.3 Impact Hammer Test

Impact hammer test is carried on in order to estimate tip point direct FRF of the beam. Configuration is presented in the Figure 5.47.

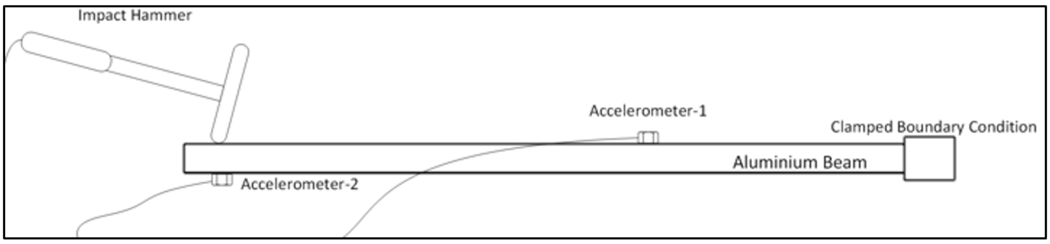

Figure 5.47: Impact Hammer Test Configuration

FRFs are given in the Figures 5.48,5.49 using the data acquired by Accelerometer-1 and Accelerometer-2.

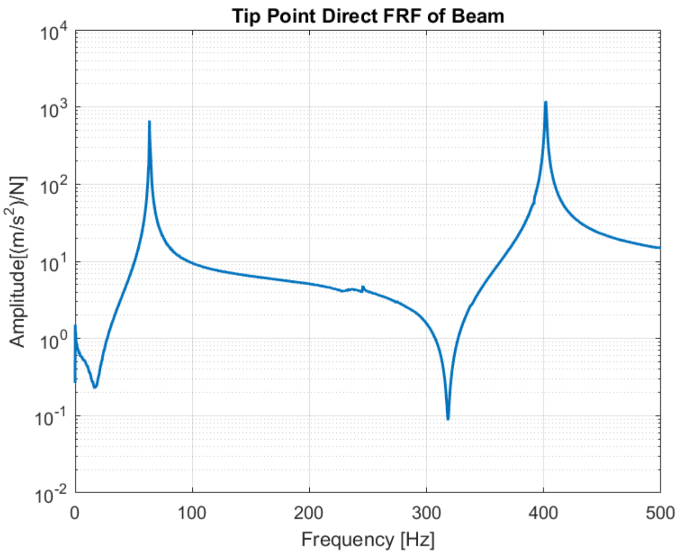

Figure 5.48: Tip Point Direct FRF

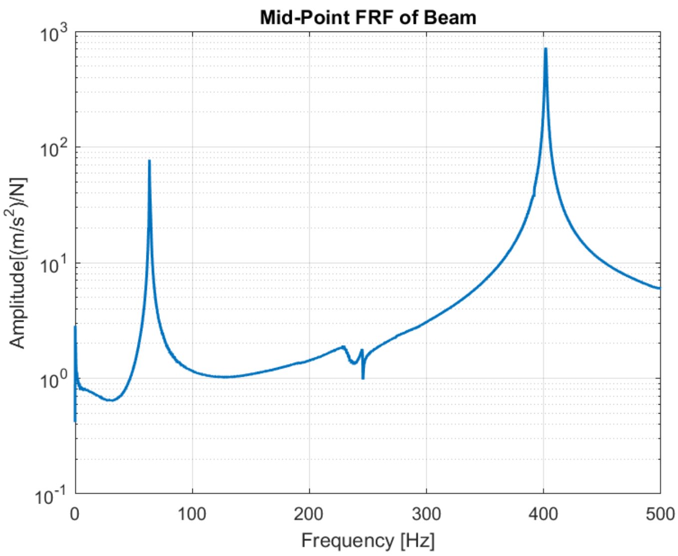

Figure 5.49: Mid Point FRF

5.5 Results and Discussion

This section compromises of two main subsection. In the first one host structure and TVA frequency response functions obtained by FEA and experimental methods are compared with each other and verified. In the second one, firstly frequency responses to random loading of TVA attached beam and TVA obtained via response estimation method which is explained in detail in chapter 3 will be presented. Then verification of the method applied on this case study is carried out by comparing frequency responses to random loading of these structures obtained via response estimation method with the ones acquired by experiments and FEA that are shared in the previous sections.

5.5.1 Frequency Response Functions Comparisons

In the proceeding part, results of TVA transmissibility, FRF of tip point of standalone beam, FRF of tip point of TVA attached beam and lastly FRF of TVA mass obtained via FEA and experiments presented together.

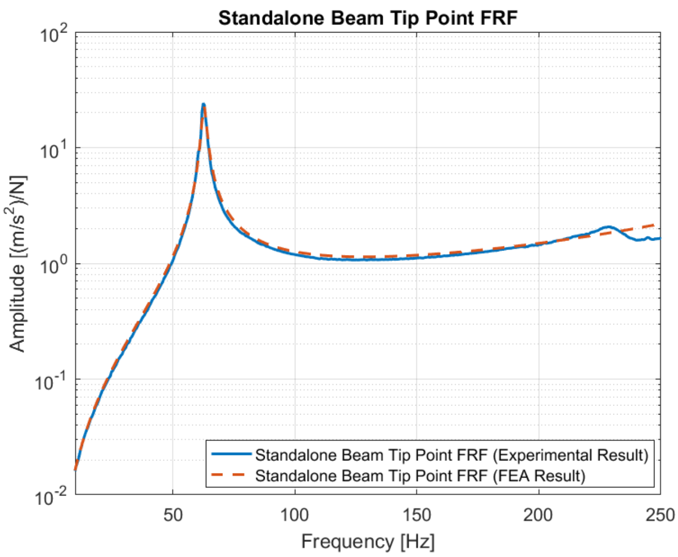

Figure 5.50: Standalone Beam Tip Point FRF Experimental and FEA Results Comparison

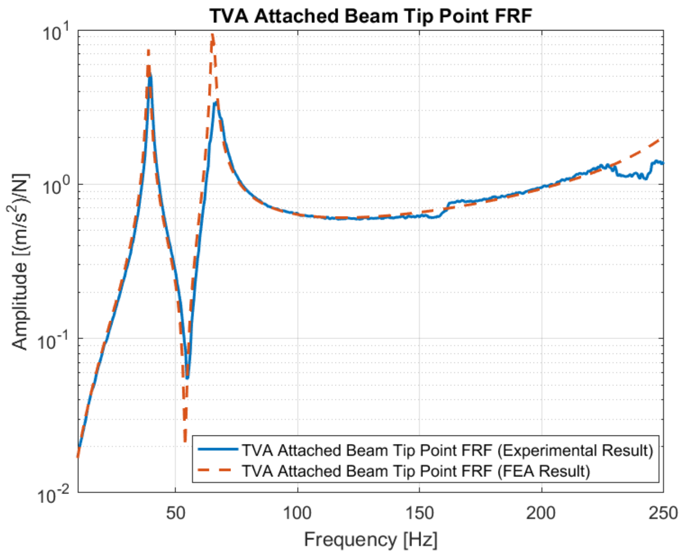

Figure 5.51: TVA Attached Beam Tip Point FRF Experimental and FEA Results Comparison

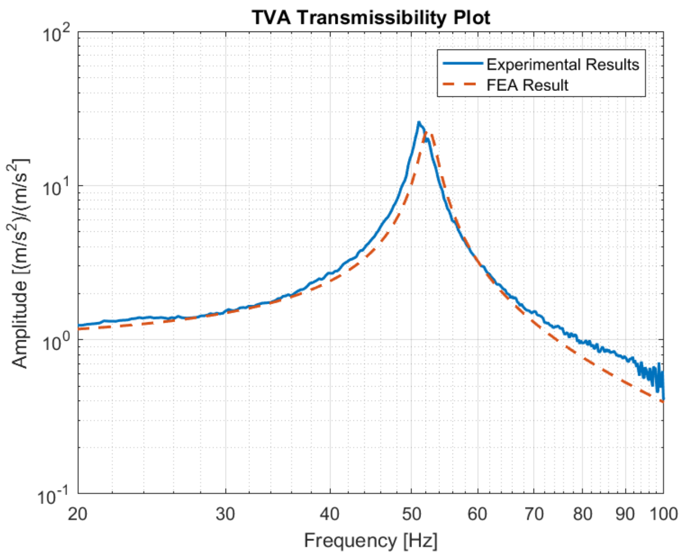

Figure 5.52: TVA Transmissibility Plot Comparison

5.5.2 Random loading Results

In this part firstly results of frequency response to random loading of standalone beam and TVA attached beam presented obtained by FEA and experiments for sake of checking the consistency of results for verification.

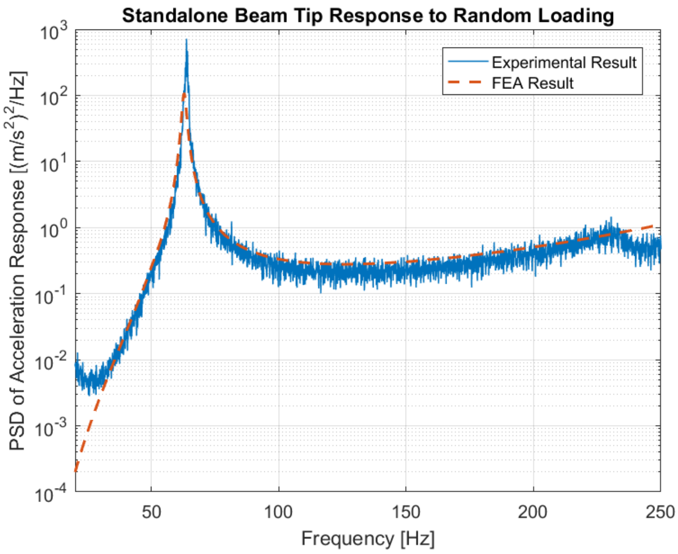

Figure 5.53: Standalone Beam Response to Loading

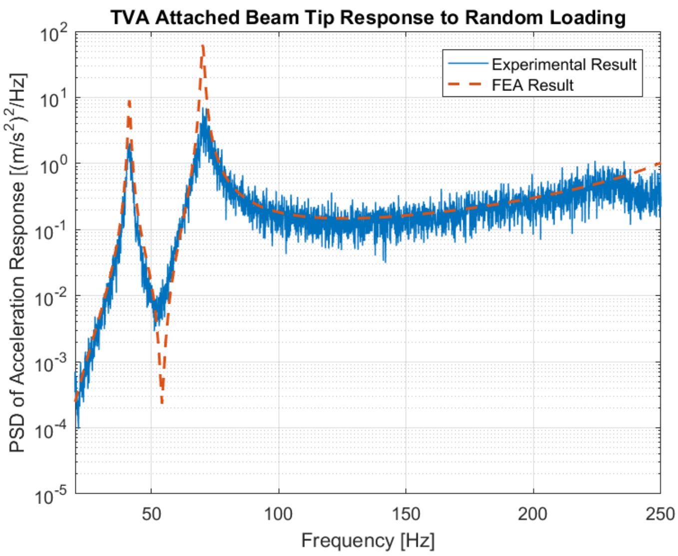

Figure 5.54: TVA Attached Beam Response to Loading

As seen from Figure 5.53 and Figure 5.54, forced vibration results obtained via FEA and experiments are consistent with each other except at peak values. This deduction observed at peak especially in the Figure 5.54, could be eliminated by suitable damping estimation to be used in analysis studies.

5.5.3 Applying Response Estimation Method

In this section response estimation method is applied on this case study step by step referring to process tree presented in the Figure 4.1.

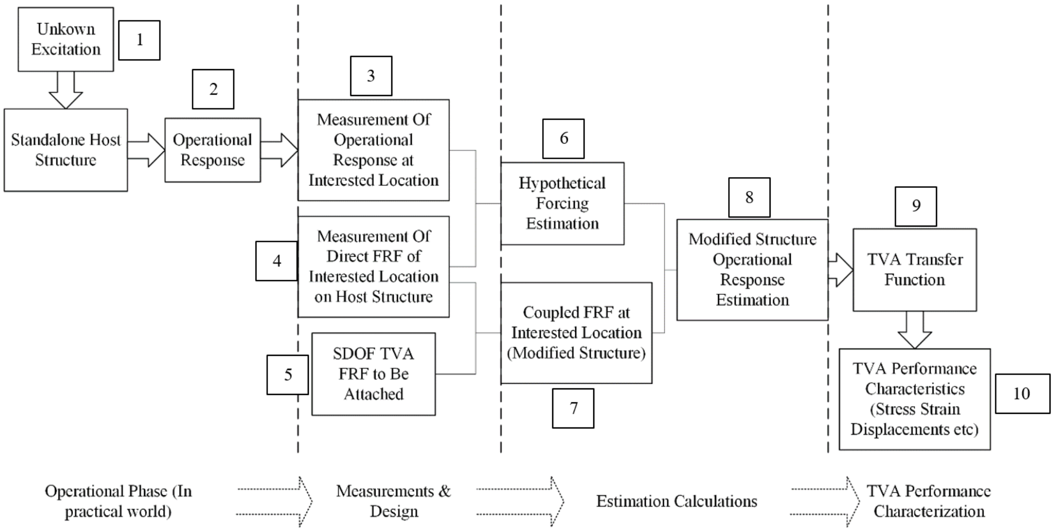

Figure 5.55: Response Estimation Procedure

For our case study unknown excitation (1) is actually known and controlled that is applied via dynamic shaker on standalone beam structure. (Even if the excitation is known for this case study, it will not contribute in calculations while following the procedure.) Therefore, operational response is acquired with both experiments and FEA (2). Measurements are taken at the tip of the Al beam in operational conditions (3). Impact hammer test is conducted at the tip of the beam in order to acquire tip point direct point FRF (4). TVA is designed and its base direct point FRF is acquired via FEA (5). So far every data is collected or computed in order to estimate TVA and host structure response in operational conditions (8). Therefore, utilizing the measurement of operational response of Al beam when subjected to random loading with direct point FRF at TVA attachment point (tip of Al beam) hypothetical forcing is estimated (6) presented in the Figure 5.56. Coupled FRF (7) is computed using TVA base direct FRF and Al beam tip point direct FRF (Figure 5.57). Modified structure response to random loading is estimated (8) as given in the Figure 5.58. (9) TVA transfer function is nothing but transmissibility obtained by FEA verified by experiments. Using transfer functions TVA mass response (10) is also obtained and presented in the Figure 5.59.

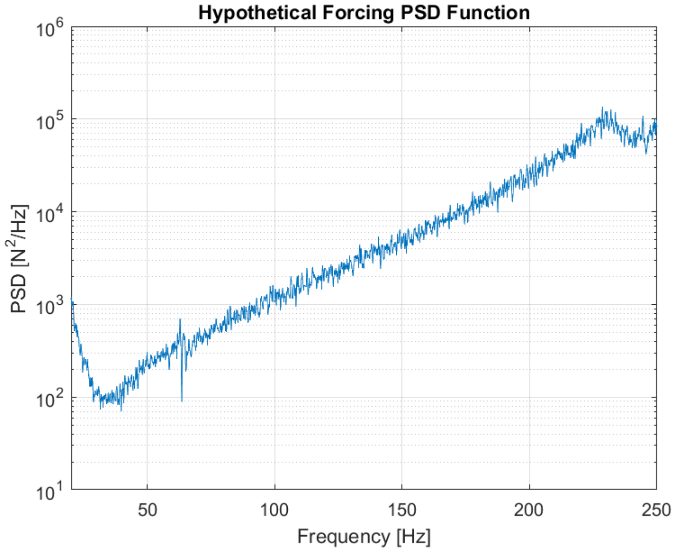

Figure 5.56: Hypothetical Forcing

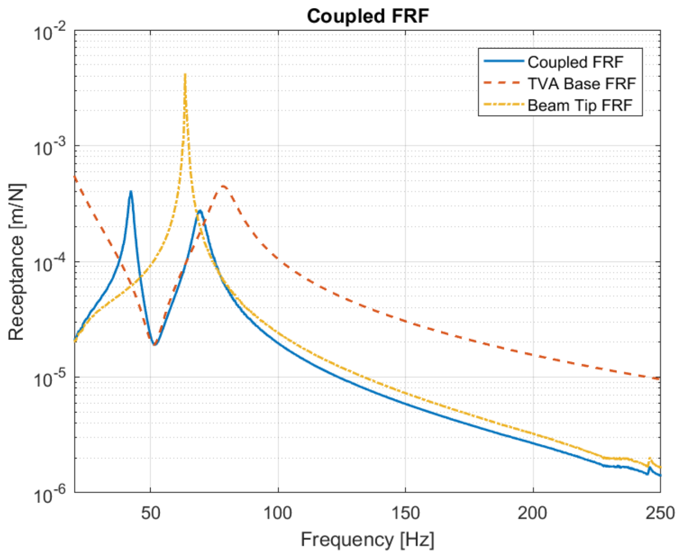

Figure 5.57: Coupled FRF

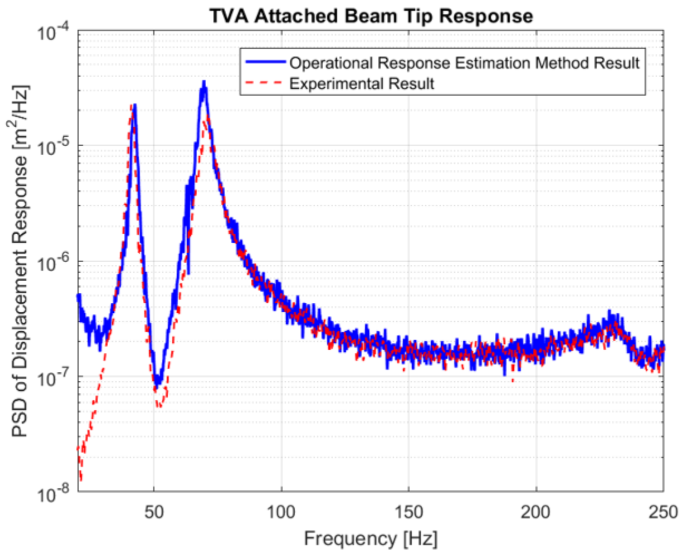

Figure 5.58: Modified Beam Response Experimental and Response Estimation Method Results Comparison

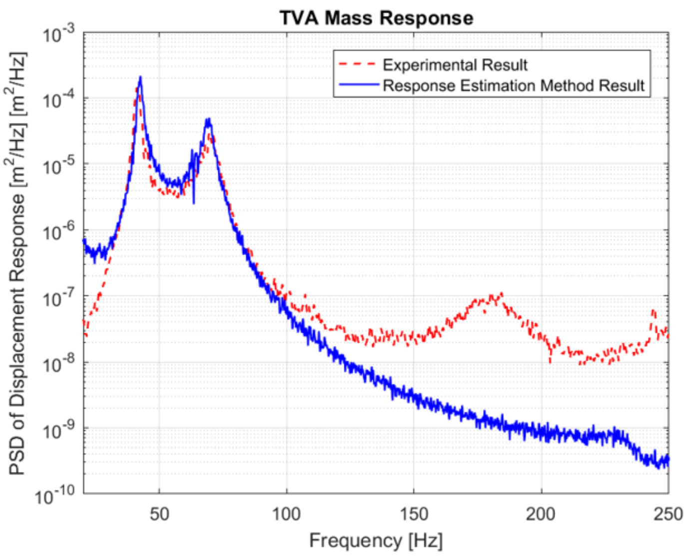

Figure 5.59: TVA Mass Response Experimental and Response Estimation Method Results Comparison

In Figure 5.58 and 5.59 estimations results and experimental results are shared together. In the Figure 5.58 a good agreement has been observed between estimation and experimental result therefore it is convenient to express that response estimation theory gives a valid prediction about modified system response. Even if a good agreement has been identified there is still difference between results. This deviation is related with non-ideal experimental setup. Dynamic shaker random excitation defined by PSD function is not ideal. Even if the same PSD functions are used in different configuration of experiments, there will be still differences in the time domain signal coming from the nature of random behavior. Lastly, response estimation method connect 1 DOF at interested location as it is described in the chapter 4, however in real applications several of DOFs are connected during attachment of TVA, therefore for an real applications response estimation method gives approximate result. In Figure 5.59 estimation and experimental results are shared for TVA mass response. Even if a good agreement has been identified between approximately 30Hz and 100Hz, outside of this region there is a difference between results. Response estimation method uses beam tip response and transmissibility of TVA that is obtained via mathematical model, in order to estimate TVA mass response. In mathematical model transmissibility calculation is based on rigid connection between beam and TVA however, in the experiments this rigid connection cannot be realized outside of this frequency region. In order to comprehensive comparison between results, RMS (root mean square) values are obtained from results presented in Figure 5.58 and 5.59. RMS of displacements (σ_{rms}) of tip of the beam and TVA mass is nothing but square root of area under of displacement PSD functions.

	Experimental Result	**Response Estimation Result**
$\sigma_{rms}^{BeamTip}$	0.015 m	0.017 m
$\sigma_{rms}^{TVAMass}$	0.026 m	0.029 m

Table 5.2: RMS Displacement Comparison Table

Response estimation method predicts of responses of TVA mass and beam tip, approximately with %12 error according to rms values of responses (Table 5.2) compared with experimental results.

5.6 TVA Stress PSD Reaction Estimation Using Response Estimation Method

In this case study, displacement response estimation of TVA mass and modified host structure is done using response estimation method. Also obtained results are compared and verified with the ones acquired by FEA and experiments. In this section an example study will be presented regarding the performance of attached TVA. Fatigue failure due to vibration can be considered as destructive for TVAs as well especially for lightly damped systems. Therefore, fatigue life estimation on elastic components of the TVA possess great importance. In this case study, random loading applied to structure therefore responses (displacements, stresses, strains) display random behavior. For calculation of fatigue life for structures under random loading, stress PSD are used.

Designed TVA consist of two helical spring and copper mass along with structural elements. Elastic component that are exposed to alternating stresses are helical springs. Therefore, analysis regarding stresses PSDs are focused on helical springs. Using FE models (ANSYS Classic Random Vibration tool) stress PSDs are constructed using three different approach. In all three approach, TVA is base excited with different PSD function. In the first method TVA attached beam FE is constructed and beam is excited with actual loading defined in problem definition section. Stress PSD at regarding element of helical spring is obtained. This stress PSD is considered as an actual result since its FEA consists of all physical structures used in the case study also it utilizes actual loading applied in the experiments.

In the second method, only TVA FE model is constructed and its base excited with frequency response obtained by response estimation method. Result obtained using this method make it enable to estimate stress PSD of TVA elastic component without using beam model and actual loading defined in problem definition, therefore, estimation would be made without using actual excitation sources or model of host structure as it is promised by response estimation method in chapter 3. In the third method TVA base excited in frequency domain with non-modified operational response of the beam. Non-modified operational response refers to frequency response of standalone beam acquired at the location where TVA attachment is intended. This analysis carried out in order emphasizes on importance of using modified response in order to obtain stress PSD. This deduction is made by pointing out the difference in the results obtained by three different methods.

5.6.1 Stress PSD Analysis

Spectrum analysis have been carried out for all three configuration of FE model in order to obtain stress PSD functions on an element of a helical spring. In the first analysis TVA attached beam FE model is constructed and its boundary condition is specified as given in the following Figures 5.60, 5.61.

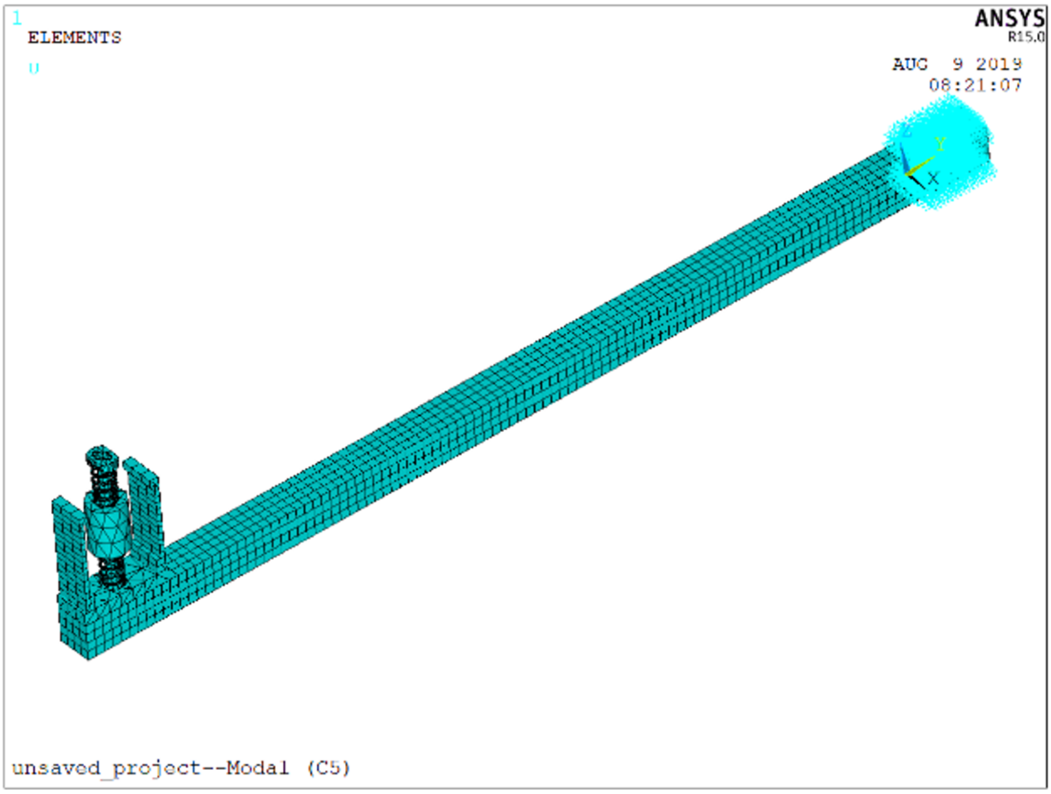

Figure 5.60: FE Model of TVA Attached Beam

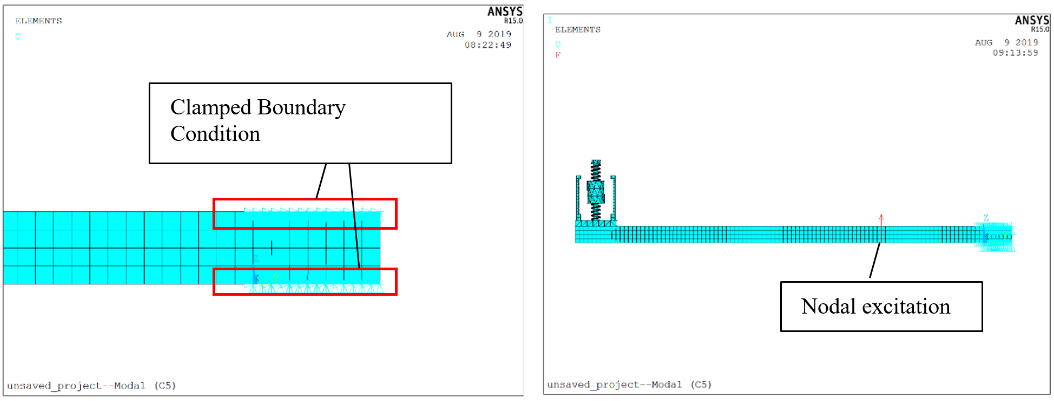

Figure 5.61: Applied Loads and Constraints on FE Model

Nodal excitation is applied on node number “1988” which is at the middle of the beam and 125 mm far from boundary condition given in the Figure 5.62. This excitations simulating forcing applied on beam as it is described in problem definition.

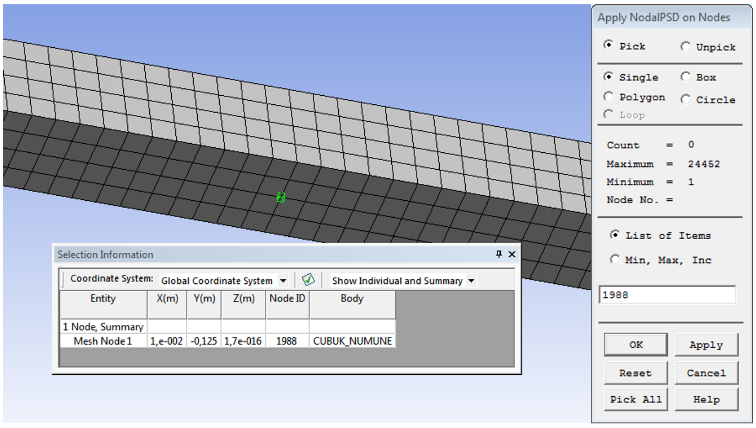

Figure 5.62: Nodal Point of Applied Excitation

Defined excitation PSD is given as shared in the Figure 5.63.

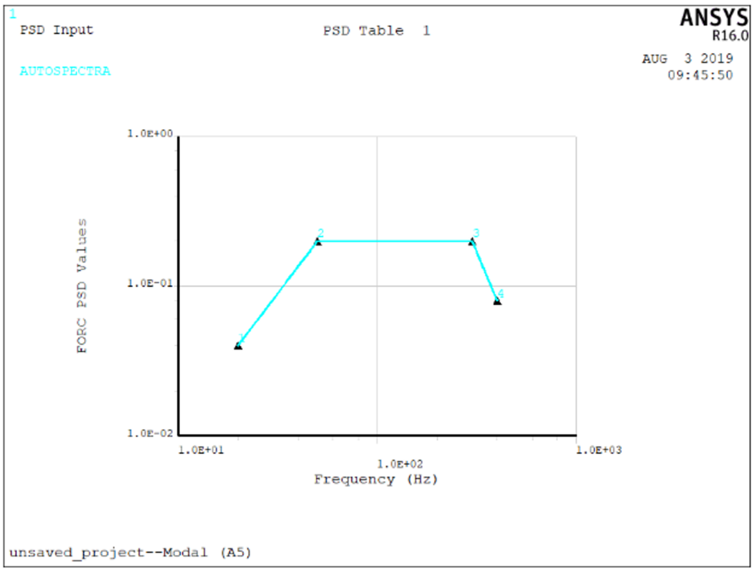

Figure 5.63: PSD Excitation

1σ stress distribution results on whole structure is given in the Figure 5.64. Also for a selected element on a helical spring especially on inner ring of the spring where max strain is observed, stress PSD function (equivalent von-misses Stress) result is presented in Figure 5.64 as well.

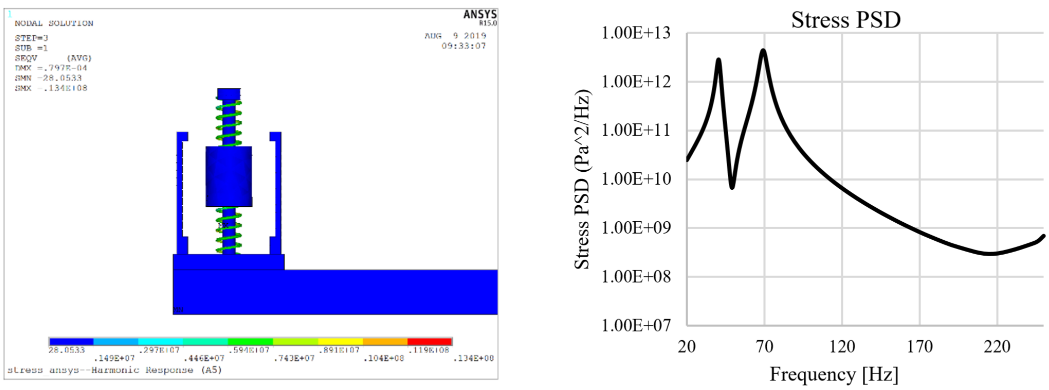

Figure 5.64: First Analysis Results

For second analysis where only TVA FE model constructed and loading is defined with base excitation as given in the Figure 5.65 and 5.66. In this analysis base PSD function is actually the one obtained via response estimation method given in the Figure 5.58. On the other hand, response of the modified beam is utilized in this analysis applying it to base of TVA.

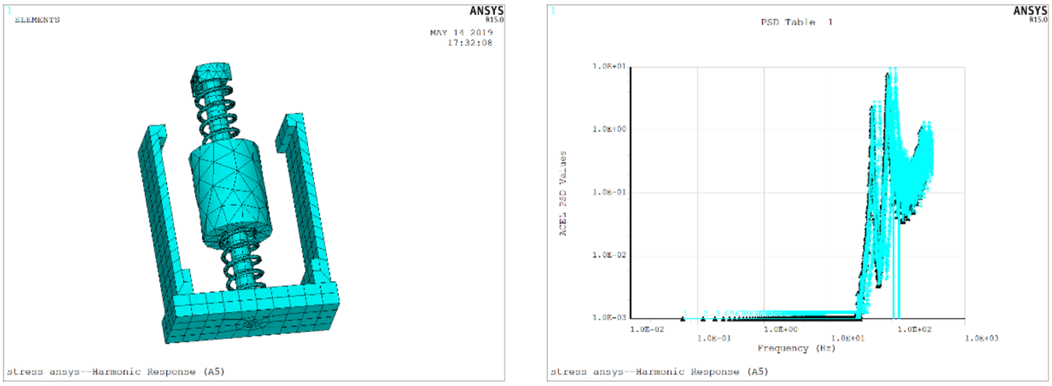

Figure 5.65: FE Model of TVA Alone and Applied Base PSD

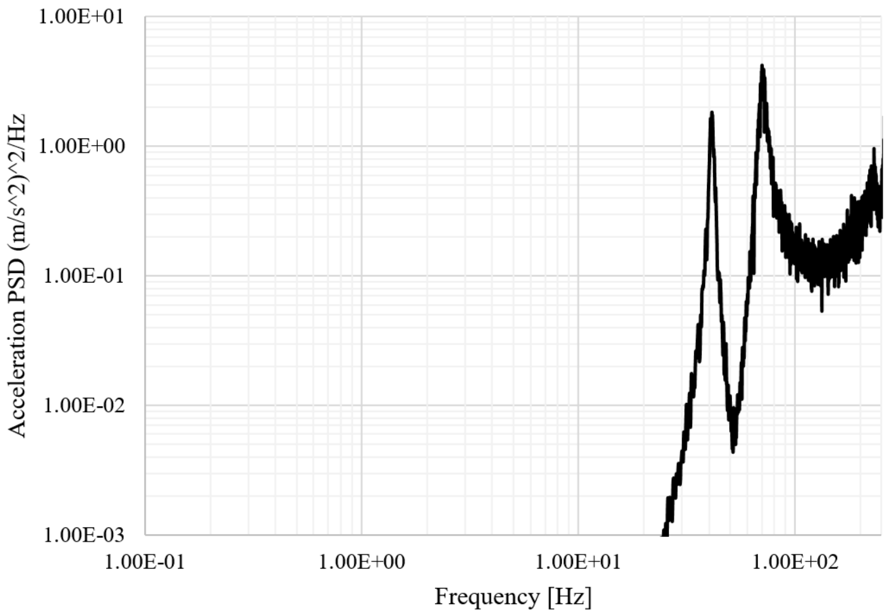

Figure 5.66: Applied Base PSD

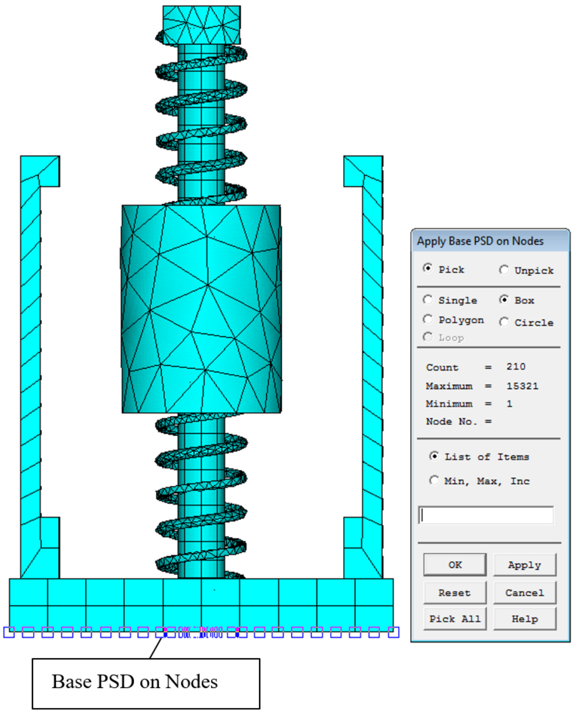

Figure 5.67: Base PSD Applied on TVA

Result is obtained and presented in the Figure 5.68 for the element where the highest strain is observed in order to make meaningful comparison between acquired results for different analysis.

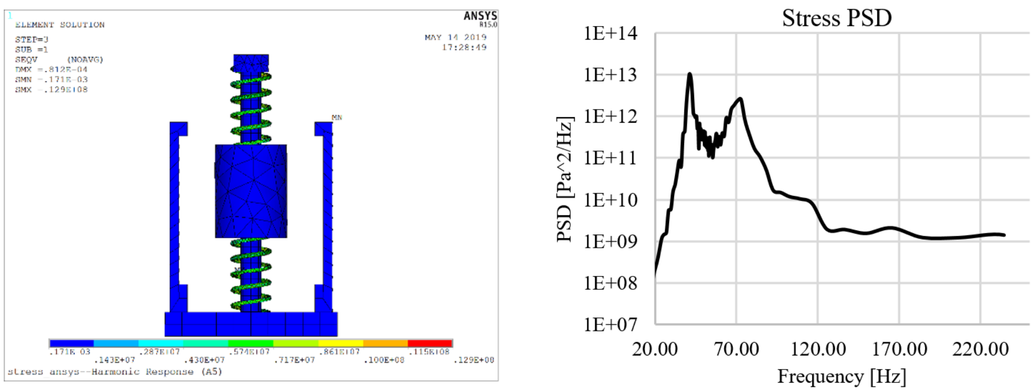

Figure 5.68: Second Analysis Results

In the last analysis, only TVA FE model is constructed as it is done for second analysis given previously. However, base excitation is replaced with non-modified operational response that is acquired at tip of the standalone beam during loading as given in the Figure 5.69 and 5.70.

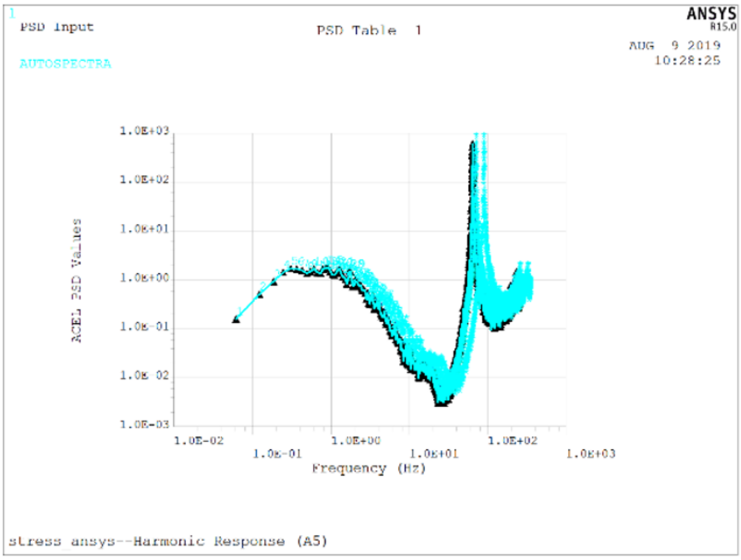

Figure 5.69: Applied PSD on Base of TVA in Third Analysis

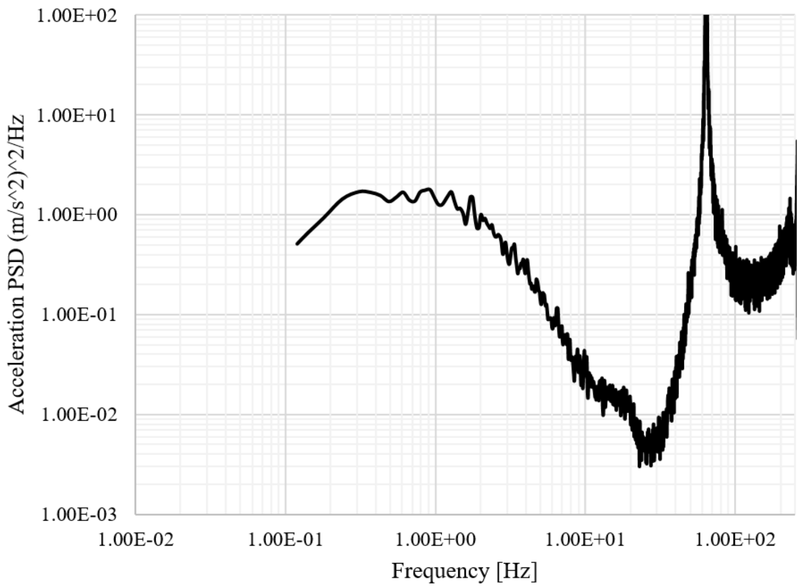

Figure 5.70: Applied Base PSD

Acquired result is shared in the Figure 5.71. PSD stress is calculated for the element on the helical spring especially on the inner ring of the spring where maximum strain is observed as it's done for all analysis.

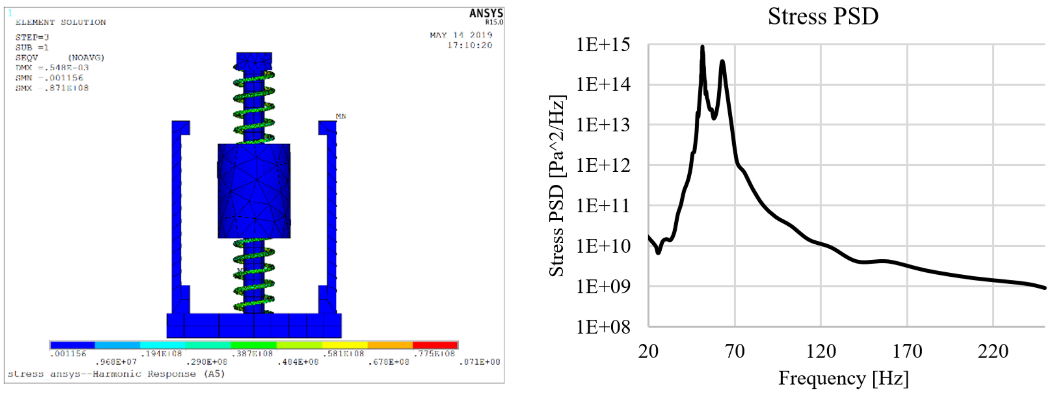

Figure 5.71: Third Analysis Results

5.6.2 Results and Discussion

In this section PSD stress results obtained via FEA for 3 different cases are compared with each other. Also 1 σ stress occurrence on an element of helical spring is calculated using these stress PSD graph by taking square root of under the area of graph. In the Figure 5.72 stress PSD results obtained using different methods are presented together.

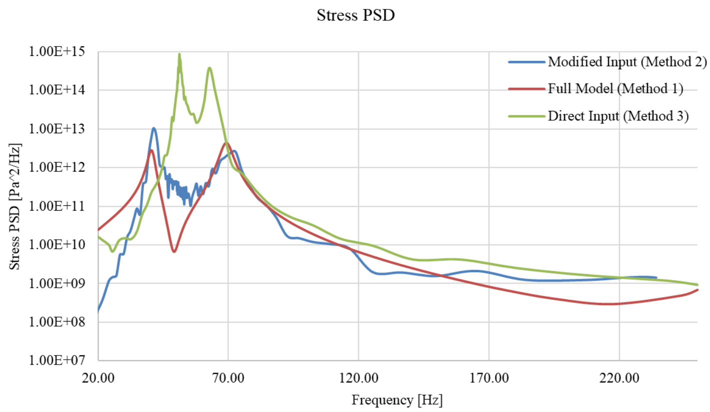

Figure 5.72: Analysis Results

In the Figure 5.72, 1. Method result can be considered as actual one whereas others as estimations. Considering 2.method result where modified excitation is applied on a base of TVA, it is seen that, even if the plot follows the same trend as 1. Method graph did, deviations has been observed at the peaks of the plot. Whereas plot obtained via 3. Method where the non-modified base excitation is employed on FEA of TVA, neither follows a trends nor its peaks consistent with actual result. Moreover, in order to make a comprehensive comparison RMS stress values are obtained for each analysis and presented in the Table 5.3.

Analysis Type	**RMS Stress Value Result**
1.Method (Actual Model FEA)	6.51 MPa
2.Method (With modified Excitation)	7.69 MPa
3.Method (With non-modified Excitation)	48.49 MPa

Table 5.3: 1σ Stress Results

Same comment can be done according the RMS stress values obtained via different methods. 2. Method gives an estimation on RMS stress occurrence on helical spring as 7.69 Mpa whereas 1.method this value is 6.51 Mpa. Therefore, 2. Method gives an estimation with error approximately %18 whereas 3 method gives with error approximately %650.

To summarize, in this section evaluation of TVA design in aspect of fatigue life is made as an example of performance characterization of design. This evaluation is based on a stress PSD results that utilized in fatigue life predictions. Stress PSD of element on helical spring is acquired by FEA using three different method. Response estimation method used to predict base excitation (response of the tip of the beam) that TVA is going to experience during operation. By FEA this base excitation is applied on TVA and stress PSD is obtained. In order to verify result, FEA of beam attached beam with actual loading is also carried out. Results are compared. Moreover, another TVA FEM constructed in order to acquire stress PSD. However in this last analysis TVA base is excited with non-modified operational response of beam. Result of last FEA showed that utilizing modified response in TVA design evaluation process possess great importance. By this study it showed that approximate performance characterization can be done of a TVA using response estimation method.

CHAPTER 6

CONCLUSIONS

In this thesis, two main subject has been studied in the scope of evaluation of previously designed TVA. In the first study designed TVA with adjustable stiffness element has been introduced. This design consist of a cantilever beam type stiffness element formed in curved geometry. By changing boundary condition of beam element, stiffness variation is realized. Study is focused on tuning law of TVA that is adjusted by this stiffness variation.

Tuning law of the absorber is firstly acquired by FEA. Then experiments has been conducted by impact hammer tests. Modal frequency change of the absorber is identified by FEA and experiments. Results obtained by both methods are then compared and it is observed that analysis and experimental results yield a good agreement. Parametric study has been conducted in order to set linear relationship between tuned frequency and a physical parameter of stiffness element. This parameter has been found as active length of the curved beam. Stiffness value of a straight cantilever beam has a linear relationship with its active length. It is observed that same relation persist for the curved beam as well, however curved beam stiffness value is more sensitive to active length of beam. Therefore, curved is superior compared to straight cantilever beam when evaluated for stiffness variation by changing active length. In the last part of this study curved beam design is evaluated by its frequency coverage comparing it with the equivalent length straight cantilever beam. Based on these evaluations it is convenient to consider this curved beam as novel design for variable stiffness element. Study is successfully completed by setting a tuning law for this novel design. Tuning law of this TVA can be used effectively when this type of a TVA is integrated on a host structure.

Main object of the second study presented on this thesis is developing a method in order to evaluate TVA in regard of fatigue life by simulations even though it is not possible to model host structure or excitation sources acting on it. Method is based on obtaining modified host structure response at the DOF where TVA is going to attach. At this DOF, two main measurements are needed in order to employ developed method. First measurement is to obtain operational response of host structure at the interested DOF whereas second measurement is to acquire direct point FRF of this DOF. TVA base (assuming TVA is SDOF system) direct point FRF is then coupled with this DOF direct point FRF. By doing so frequency response of the modified system is obtained at the interested DOF. In this step, a hypothetical forcing assumption is made. Hypothetical forcing is referring to an assumed forcing function applied on interested DOF that is going to simulate all excitations on host structure. For interested DOF of modified system operational response can be calculated with frequency response and acting hypothetical forcing function on this point. Analytical relationship has been developed to prove that obtained result of modified response by proposed method is equal to one obtained by solving using host structure model and actual excitation acting on it. Later, simulation of TVA is carried out by modeling of standalone TVA where TVA base is excited by this modified operational response. A numerical example is presented where host structure consist of 2DOF system attached with 1DOF TVA. Proposed method is applied on this numerical example. It is showed that results obtained by proposed method fit to actual results very well.

Response estimation method is applied on a case study where an aluminum beam used as a host structure excited by random loading defined by particular PSD function. A TVA is designed and manufactured in order to suppress vibration amplitudes results from loading at first natural frequency of this beam. Firstly FEA carried out to verify beam model and TVA design. Later, experimental studies has been conducted. During experiments three main objectives are completed successfully. First one was to verify FE models of beam, TVA and TVA attached beam. Second objective was acquiring related measurements of beam in order to be utilized in response estimation method. Last one was take measurement of actual response of modified beam and TVA mass response when the beam is subjected loading that is defined by PSD function previously. In order to verify proposed method applied on this case study, modified beam operational frequency response obtained by experiments where physical implementation of TVA on beam has been done, is compared with one obtained by response estimation method. A good agreement has been observed between results. Later, same comparison has been made for TVA mass response as well. Even though some deviations has been observed between graphical results, RMS displacements of beam and TVA mass is predicted by proposed method with approximately %10 error.

At the last step of this case study, an example performance characterization of TVA has been done. In order to estimate the fatigue life of elastic components of TVA, stress PSD of an element on the helical springs of TVA are obtained by three different method. In the first method, TVA and beam are modeled using FE and PSD excitation is applied as it is done on case study. Results obtained via this method is considered as actual ones. In the second method, TVA is modelled as standalone and its base is excited with PSD of modified beam operational response acquired by response estimation method. In the third method, TVA is again modelled as standalone but its base excited by non-modified response of beam. Results are then compared with each other and it is observed that result obtained by response estimation method gives good prediction about RMS stress value with the error %18 whereas this value is estimated by %650 error by third method. This comparison has been made in order to point out importance of using modified response in evaluation of performance of a TVA.

REFERENCES

[1] E. Frahzm, "Device for damping vibrations of bodies," 1909.

[2] A. H. Vonflotow, A. Beard, and D. Bailey, "Adaptive tuned vibration absorbers: Tuning laws, tracking agility, sizing, and physical implementations," in *Noise Con 1994: Proceedings of the 1994 National Conference on Noise Control Engineering* (J. M. Cuschieri, S. A. L. Glegg, and D. M. Yeager, eds.), 1994.

[3] A. Preumont, *Random Vibration and Spectral Analysis*. 1994.

[4] J. P. D. Hartog, *Mechanical vibrations*. McGraw-Hill, 1940.

[5] H. N. Özgüven and B. Çandir, "Suppressing the first and second resonances of beams by dynamic vibration absorbers," *Topics in Catalysis*, 1986.

[6] J. C. Snowdon, A. A. Wolfe, and R. L. Kerlin, "The cruciform dynamic vibration absorber," *The Journal of the Acoustical Society of America*, 2005.

[7] M. R. Jolly and J. Q. Sun, "Passive tuned vibration absorbers for sound radiation reduction from vibrating panels," 1996.

[8] S. M. Hashemi, M. F. Golnaraghi, and A. E. Patla, "Tuned vibration absorber for suppression of rest tremor in Parkinson's disease," *Medical and Biological Engineering and Computing*, 2004.

[9] N. Jalili, "A Comparative Study and Analysis of Semi-Active Vibration-Control Systems," *Journal of Vibration and Acoustics*, 2002.

[10] E. Longbottom, C.J.; Day M.J. & Rider, "A self tuning vibration absorber," 1990.

[11] E. Rustighi, M. J. Brennan, and B. R. Mace, "A shape memory alloy adaptive tuned vibration absorber: Design and implementation," *Smart Materials and Structures*, 2005.

[12] P. Bonello, M. J. Brennan, S. J. Elliott, J. F. Vincent, and G. Jeronimidis, "Designs for an adaptive tuned vibration absorber with variable shape stiffness element," *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 2005.

[13] J. P. Carneal, F. Charette, and C. R. Fuller, "Minimization of sound radiation from plates using adaptive tuned vibration absorbers," *Journal of Sound and Vibration*, 2004.

[14] M. Brennan, "Actuators for active vibration control - tunable resonant devices," *International Journal of Applied Mechanics and Engineering*, vol. 5, no. 1, pp. 63–74, 2000.

[15] M. R. F. Kidner and M. J. Brennan, "Varying the Stiffness of a Beam-Like Neutralizer Under Fuzzy Logic Control," *Journal of Vibration and Acoustics*, 2002.

[16] G. Aguirre, M. Gorostiaga, T. Porchez, and J. Munoa, "Self-tuning dynamic vibration absorber for machine tool chatter suppression," *Proceedings of the 28th Annual Meeting of the American Society for Precision Engineering, ASPE 2013*, 05 2013.

[17] D. Gsell, G. Feltrin, and M. Motavalli, "Adaptive tuned mass damper based on pre-stressable leaf-springs," *EMPA Activities*, 2007.

[18] O. F. Tigli, "Optimum vibration absorber (tuned mass damper) design for linear damped systems subjected to random loads," *Journal of Sound and Vibration*, 2012.

[19] T. Asami, T. Wakasono, K. Kameoka, M. Hasegawa, and H. Sekiguchi, "Optimum design of dynamic absorbers for a system subjected to random excitation.," *JSME international journal. Ser. 3, Vibration, control engineering, engineering for industry*, vol. 34, pp. 218–226, 06 1991.

[20] K. C. Kwok and B. Samali, "Performance of tuned mass dampers under wind loads," *Engineering Structures*, 1995.

[21] R. K. Luo and W. X. Wu, "Fatigue failure analysis of anti-vibration rubber spring," *Engineering Failure Analysis*, 2006.

[22] W. B. Shangguan, T. K. Liu, X. L. Wang, C. Xu, and B. Yu, “A method for modelling of fatigue life for rubbers and rubber isolators,” *Fatigue and Fracture of Engineering Materials and Structures*, vol. 37, no. 6, pp. 623–636, 2014.

[23] Y. Xu, Y. Liu, C. Kan, Z. Shen, and Z. Shi, “Experimental research on fatigue property of steel rubber vibration isolator for offshore jacket platform in cold environment,” *Ocean Engineering*, 2009.

[24] N. Bishop and A. Caserio, “Vibration Fatigue Analysis in the Finite Element Environment,” *Americas Users’ Conference*, 1998.

[25] Y. Eldoğan and E. Cigeroglu, “Vibration Fatigue Analysis of a Cantilever Beam Using Different Fatigue Theories,” in *Conference Proceedings of the Society for Experimental Mechanics Series*, 2014.

[26] M. T. Chen and R. Harichandran, “Statistics of the von Mises stress response for structures subjected to random excitations,” *Shock and Vibration*, 1998.

[27] H. Nevzat Özgüven, “Structural modifications using frequency response functions,” *Mechanical Systems and Signal Processing*, 1990.

[28] M. Nad, “Structural dynamic modification of vibrating systems,” *Applied and Computational Mechanics 1*, 2007.

[29] A. Ertürk, H. N. Özgüven, and E. Budak, “Analytical modeling of spindle-tool dynamics on machine tools using Timoshenko beam model and receptance coupling for the prediction of tool point FRF,” *International Journal of Machine Tools and Manufacture*, 2006.

[30] Z. Wang and P. Zhu, “Response prediction for modified mechanical systems based on in-situ frequency response functions: Theoretical and numerical studies,” *Journal of Sound and Vibration*, 2017.

[31] D. J. Ewins and H. Saunders, “Modal Testing: Theory and Practice,” *Journal of Vibration Acoustics Stress and Reliability in Design*, 2011.