NUMERICALLY AIDED DESIGN PROCESS FOR PIANO KEY WEIRS

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I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

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ABSTRACT

NUMERICALLY AIDED DESIGN PROCESS FOR PIANO KEY WEIRS

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Piano Key weirs are recently developed hydraulic structures and provide valuable benefits in discharge efficiency and dam safety. This new type of labyrinth weir is an excellent alternative for traditional labyrinth weirs with its relatively small footprint. Labyrinth weirs were developed for increasing the discharge capacities of existing dams. However, their footprint does not allow them to be placed into dam crests easily. Nevertheless, Piano Key weirs can be directly implemented on existing spillways and increases the unit discharge capacity higher than the traditional weirs for similar heads and spillway widths. Piano Key weirs reduce the construction costs with their simple and replicable shapes. With all of these, this new type of structure becomes an interesting solution for dam rehabilitation. However, design methods of a Piano Key weir are still insufficient. The flow behavior of a Piano Key weir cannot be predicted accurately. Although there have been many investigations carried out, current data are not enough to allow a general design procedure because of the complex hydraulic behavior of the Piano Key weir. In this study, a comprehensive three-dimensional numerical investigation will be performed in order to support a reliable design process to obtain a generalized design procedure for the discharge capacity of Piano Key weirs by using computational fluid dynamics CFD-Software Flow-3D®.

Keywords: Piano Key Weir, Labyrinth Weir, Weir Efficiency, Discharge Capacity, Dam Rehabilitation

ÖZ

PİYANO TUŞLU SAVAKLARIN TASARIMININ NUMERİK DESTEKLİ SÜRECİ

Ademoğlu, Serhan
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Piyano Tuşlu savaklar, yakın zamanda geliştirilen ve baraj güvenliği ile birlikte savaktan geçen debinin verimliliğini artırarak fayda sağlayan hidrolik yapılardır. Bu yeni labirent savağı türü, diğer türdeki labirent savaklara göre nispeten daha az yer kaplamasından dolayı mükemmel bir alternatiftir. Labirent savaklar, mevcut barajların debi kapasitelerini artırmak için geliştirilmiştir. Ancak, geometrik yapılarından dolayı baraj üzerine yerleştirilmeleri bazen mümkün olmamaktadır. Bununla birlikte, Piyano Tuşlu savaklar mevcut baraj üzerine doğrudan yerleştirilebilir ve aynı su yüksekliği ve savak genişliğinde diğer labirent savaklara göre suyun boşalma kapasitesini arttırır. Piyano Tuşlu savaklar, yapım maliyetlerini geometrik yapılarının basit ve tekrarlanabilir şekillerde olmasından dolayı da azaltmaktadır. Bütün bunlarla birlikte, bu yeni yapı türü baraj rehabilitasyonu için iyi bir alternatif haline gelmiştir. Ancak, Piyano Tuşlu savakların tasarım yöntemleri hala yetersizdir. Bir Piyano Tuşlu savağın üzerinden geçen akımın akış davranışları doğru bir şekilde henüz tahmin edilememektedir. Yapılan birçok araştırma olmasına rağmen mevcut veriler Piyano Tuşlu savakların karmaşık hidrolik davranışı sebebiyle genel bir tasarım yöntemini hala gerçekleştirememiştir. Bu çalışmanın amacı, Piyano Tuşlu savaklar için güvenilir bir genel tasarım yöntemi oluşturmak için hesaplamalı akışkanlar dinamiği yazılımı olan Flow-3D® kullanılarak Piyano Tuşlu savakların deşarj kapasitesi için genelleştirilmiş bir tasarım prosedürü oluşturmaktır.

Anahtar Kelimeler: Piyano Tuşlu Savak, Labirent Savak, Savak Verimliliği, Debi Kapasitesi, Baraj Rehabilitasyonu

To my family,

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CHAPTER 1

INTRODUCTION

1.1. Spillways and Spillway Improvements

Dams are important structures owing to serving a lot of purposeful actions by storing water. Domestic and industrial use of water, irrigation and power generation can primarily be listed for such these actions. With all of these, dam safety becomes the most important subject to be considered to prevent loss of life and property and to ensure the sustainability of such needs. Therefore, spillways are one of the main structures of dams and serves quite important features. They are constructed to discharge excess water from upstream floods to prevent overtopping that may cause a dam breach and a possible dam failure. Most of the dam failures have been caused by design failures. For this reason, the essentials and responsibilities must be fulfilled before designing spillways.

Today, dam rehabilitation is an interesting subject for many researches because of the increasing number of floods caused by the changing climate conditions. Prone to more severe floods due to the climate change, spillways are being rehabilitated all around the world to increase their discharge capacity. It should also be kept in mind that, spillways constitute a significant part of the total cost of an entire dam. For these reasons, implementation of new type of structures with low cost solutions has been considered for the upgrade. Therefore, in order to represent a reliable, efficient, preferable and economic solution, many investigations have been carried out to challenge with the problems to fulfill the demands and ensure dam safety for present dams.

1.2. Problem Statement and Research Objectives

In recent years, a new type of nonlinear weir has been introduced. Lempérière and Ouamane (2003) presented an excellent solution for extending the weir length to increase the discharge capacity by developing piano shaped spillways. A Piano Key weir is one of the best developments of those recent studies which serve clear advantages. They are modified nonlinear spillways that can discharge large amounts of water and show great results in both hydraulics and cost compared to the conventional spillways in the same restrictions. The most important function of a Piano Key weir is the evacuation of important discharges up to 100 $m^3/s/m$ which can be at least 4 or 5 times higher than a traditional weir with a limited footprint area for their implementation (Lempérière and Ouamane 2003). This advantage allows reservoir water level to increase dam safety. They can be constructed easily with their simple and replicable shapes which also decreases the construction costs because of the reduced amount of concrete volumes. Their utilization and maintenance are also favorable.

Although there are many advantages of the usage of Piano Key weirs, the hydraulic behavior of them is still not well understood. Accordingly, a generalized design process for the Piano Key weirs is still a problem to be studied. Although, the design processes for the existing Piano Key weirs was studied, it was not possible to obtain a generalized procedure. Pfister and Schleiss (2013b) showed that first design equations conducted experimentally do not give identical results because of many reasons such as scale effects and the difference of their application ranges. Additionally, a comprehensive research for a design procedure for a local construction of a Piano Key weir can be expensive and takes too much time to be completed. Thus, a generalized design procedure that can help us to develop this new type of hydraulic structures to replace the existing spillways can only be obtained through numerical investigation.

The main objective of this study is to develop a generalized design procedure for a Piano Key weir by numerical investigations using a computational fluid dynamics

CFD-Software Flow-3D®. Design processes of a Piano Key weir have mostly been based on experimental studies with scale models. All these studies have been modified to investigate specific cases and enabled to feature some important points of geometrical parameters of Piano Key weirs. However, it is not possible to study all the geometric parameters in an experimental study. Moreover, experimental models are prone to scale models. In other words, experimental studies do not give comprehensive results within the limitations of their scale and the number of parameters being investigated. Therefore, the way of investigation should be flexible to create various models with ease and should not include scale effects so that numerical investigation is preferred within this study.

The research objectives for this study can be divided into four parts.

First, literature will be reviewed. The principal aim for the review of literature for the present study is to find the main hydraulic and geometrical parameters that affect the discharge efficiency of a Piano Key weir. There are many hydraulic and geometrical parameters that influence the discharge capacity of a Piano Key weir but some of them have a greater influence on the discharge capacity. Therefore, numerical investigations will be aimed to understand the effects of the parameters that have major influence on the discharge capacity independently. After the review of the major dimensionless parameters, models which will be investigated by numerical investigations will be created with reasonable mesh sizes to get practical results. There are several Piano Key weirs constructed worldwide and their size distribution can help us to devise these models to be investigated. Therefore, another aim for literature review is to find real sizes of geometrical components of the existing Piano Key weirs.

Second, numerical model is generated to be used in the CFD-Software Flow-3D®. Numerical investigation method and mesh size distribution are essential to get reasonable results. Therefore, grid resolution is assumed in a numerical study.

Third part is the review of results of each investigation. Figures related with the head-discharge relations for each study are generated. The reason for generation of these tables are to find effects of each parameter on the discharge capacity for generating a design process.

And the last part of the research objectives for the present study is to develop a generalized design process for Piano Key weirs.

CHAPTER 2

LITERATURE REVIEW

2.1. Piano Key Weir

Piano Key weir is a type of labyrinth weir composed of Piano Key weir units which is defined as the smallest structure of a Piano Key weir. A unit of Piano Key weir consists of an entire inlet key, two halves of outlet keys on both sides and two sidewalls as it can be seen in Figure 1. Inlet key is the key opened on upstream region where fluid enters to Piano Key weir and outlet key is the key opened on the downstream region where fluid exits from Piano Key weir. Inlet and outlet keys consist of inclined aprons and they are limited with sidewalls and crest of the Piano Key weir.

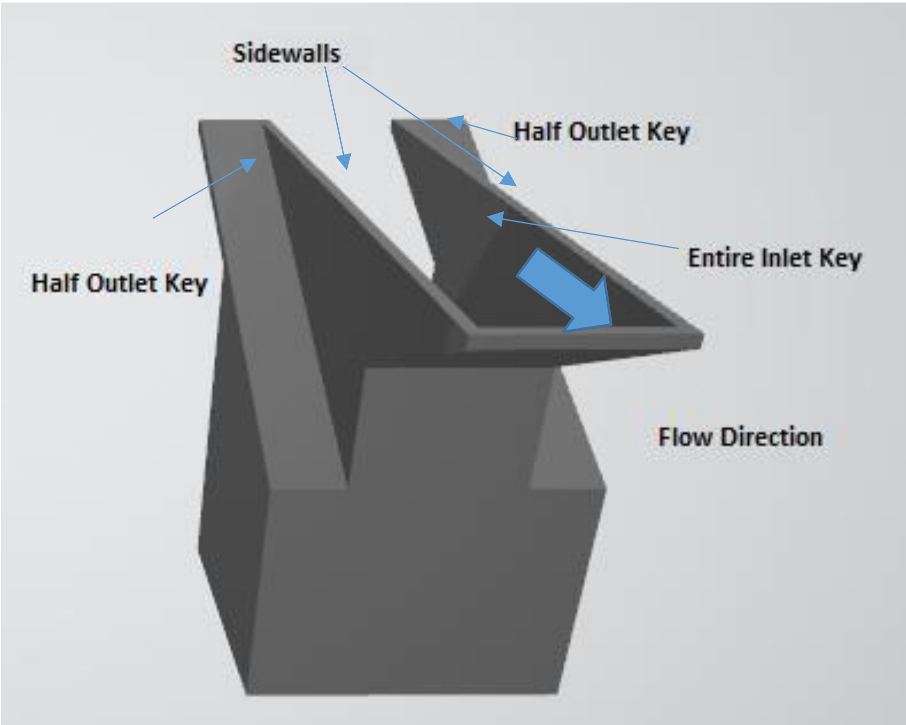

Figure 1: Components of a Piano Key Weir Unit

Piano Key weir is a complex structure with several geometrical components. A three-dimensional view and fundamental parameters of a Piano Key weir can be seen in Figure 2a, Figure 2b and Figure 2c. The main geometrical parameters of a Piano Key weir are represented according to the Pralong et al. (2011). These geometrical parameters are the number of Piano Key weir units N, the height of a Piano Key weir P, the total developed length along the crest of Piano Key weir L, the developed length along the crest of a Piano Key weir unit L_u, the total width of the Piano Key weir W, the unit width of Piano Key weir W_u, the inlet and outlet key widths of Piano Key weir W_i and W_o, the upstream and downstream length of Piano Key weir B, the upstream and downstream overhang lengths of Piano Key weir B_o and B_i, the sidewall thickness of Piano Key weir T_s and the dam height P_d. It is also important to note that the upstream flow head and the flow discharge are defined as h and Q in this study.

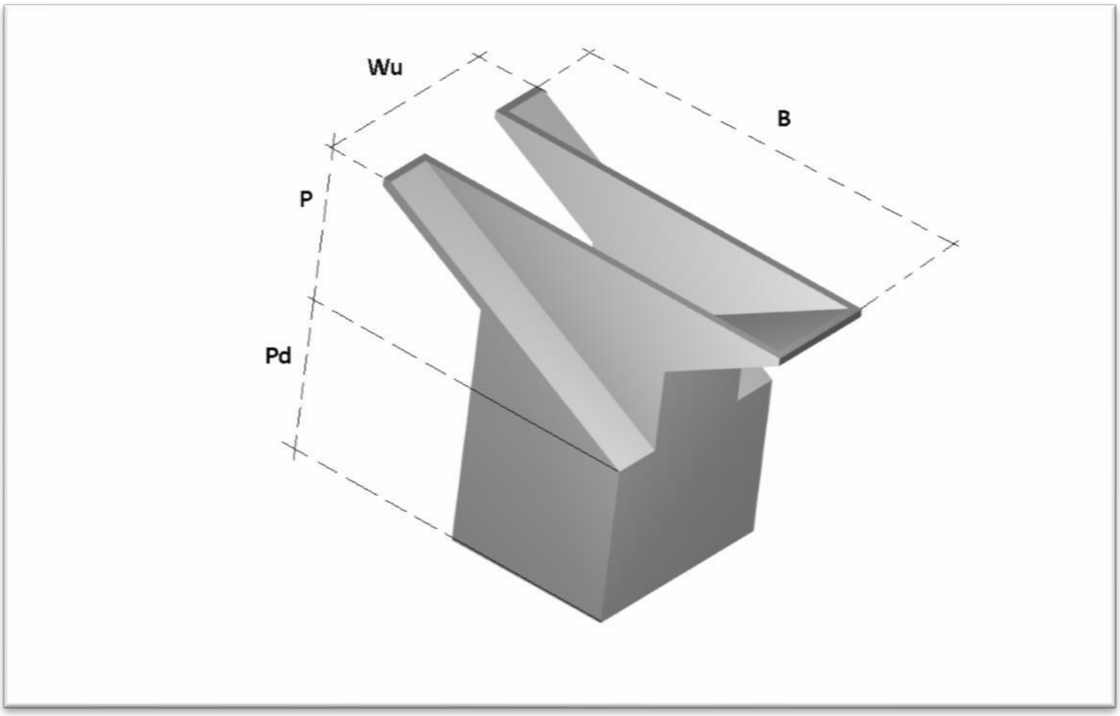

Figure 2a: Schematic Diagram of Piano Key Weir

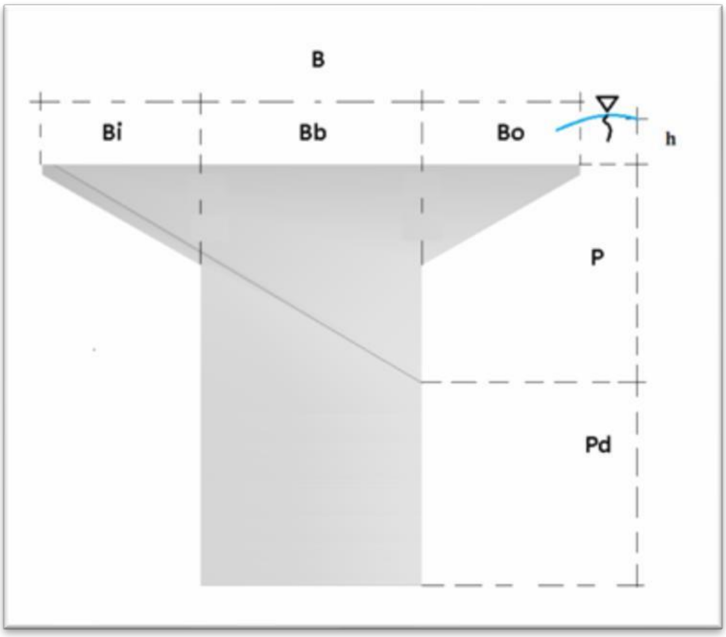

Figure 2b: Side View of a Piano Key Weir

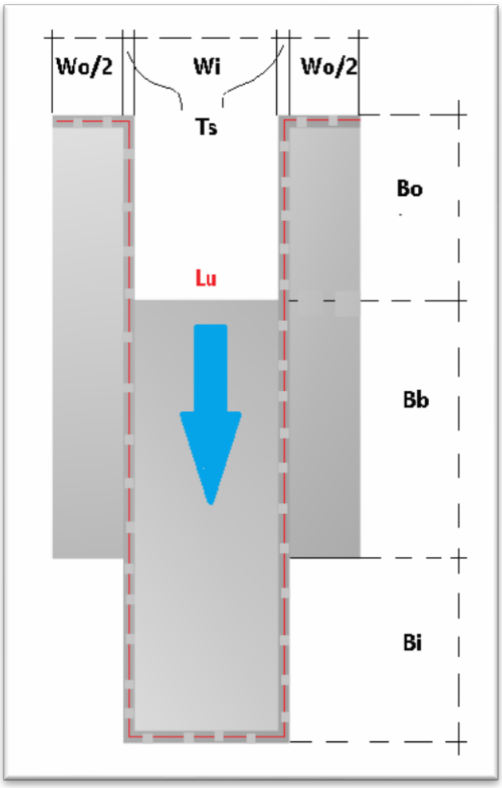

Figure 2c: Top View of a Piano Key Weir

2.2. Main Parameters Controlling the Discharge Capacity of a Piano Key Weir

The discharge efficiency of a Piano Key weir is affected by many geometrical parameters. However, it was investigated by several studies worldwide that some of these geometrical parameters are more effective than the other parameters. The ratio between the vertical and horizontal lengths (P/W_u), the rate of the increase of the developed length along the crest of Piano Key weir unit with respect to the unit weir width (L_u/W_u), the inlet and outlet widths (W_i/W_o) and the downstream and upstream overhang lengths (B_i/B_o) of the weir were appeared to have a great influence on the discharge efficiency of a Piano Key weir.

It is important to note that the water head, h, is also an important aspect to be considered before the design process. It was studied and found in many investigations that the water head influences the function of the geometrical parameters of a Piano Key weir. In this study, investigations and reviews of the influence of the water head , h, to the function of the most important geometrical parameters of a Piano Key weir were given in the titles of these key parameters below.

2.2.1 Effect of P/W_u on Discharge Capacity and Flow Conditions

The discharge capacity of a Piano Key weir is a function of several geometrical parameters. One of the most important parameters that affect the discharge capacity is the height of a Piano Key weir, P, which has a great impact on the entrance area of inlet and key slopes, S_i and S_o. The previous studies concluded that the effect of the weir height, P, is one of the most significant components to influence the discharge efficiency. Ouamane and Lempérière (2006) showed that the discharge efficiency is dependent on the ratio of P/W_u. It was found that as the weir height, P, is increased 20%, the increase of the discharge is between 5% to 10%. Lately, it was supported by another study by Ouamane and Lempérière (2008) with very similar conclusions that the height of the Piano Key weir, P, affects the performance clearly. Cicero et al. (2010) studied an experimental optimization with 4 Piano Key weir alternatives with varying weir heights, P, for increasing the spillway capacity of Malarce Dam. It was tested that the wall height, P, improves the discharge capacity. Another conclusion was made by Noui and Ouamane (2011) that discharge efficiency can be increased 25% for low heads and 5% for medium heads by increasing the weir height, P. It was also suggested that a Piano Key weir should be operated for low heads in order to have high performance. A sensitivity analysis made by Pralong et al. (2011) also supported the conclusion that the Piano Key weir efficiency can be increased by heightening the Piano Key weir. Another investigation for understanding the effect of the weir height, P, was studied by Lefebvre et al. (2013) by numerical simulations. It was highlighted that the cases which provides the increase of the discharge efficiency by increasing the weir height, P, are the decrease of the hydraulic head losses in the inlet section area, the increase of the flow evacuation because of the steepness of the outlet key and the decrease of the local submergence. Machiels et al. (2014) stated that the discharge capacity can be increased with the weir height, P, without the dependence of the upstream head for the values of the ratio of P/W_u lesser than 1 and conversely the weir height, P, has no effect on the discharge efficiency for the values of ratio of P/W_u greater than 1. The best hydraulic efficiency was gained with the value of ratio of P/W_u greater than 1. However, this ratio was suggested to be selected as close to 0.50 because of the construction and design constraints. Another investigation agrees with the study Machiels et al. (2014) was researched by Erpicum et al. (2014) and highlighted the importance of the weir height, P, and the function of a Piano Key weir with certain values of the upstream heads.

2.2.2. Effect of Lu/Wu on Discharge Capacity and Flow Conditions

The relation between the total crest length and the total width of the Piano Key weir is the main influential portion of the discharge efficiency of a Piano Key weir. It was studied by Ouamane and Lempérière (2006) that the dimensionless ratio of L_u/W_u is very efficient for low water heads ($h/P<0.60$) but the efficiency decreases as the water head increases. Lately, the effect of the ratio of the total crest length, L, and the total weir width, W, was investigated by changing the slope of aprons, P/B, of the models by Ouamane and Lempérière (2008). It was found that the increase of the discharge capacity by increasing the dimensionless ratio of L_u/W_u is dependent on the slope of aprons. It was revealed that the dimensionless ratio of L_u/W_u has no impact on the discharge efficiency for the values of slope of aprons lower than 0.50. However, for the values greater than 0.50, the influence of the ratio of L_u/W_u was found effective. It is important to note that this investigation was studied for the models without downstream overhangs, B_i. Ouamane (2011) stated that the discharge efficiency of a Piano Key weir increases continuously as the ratio of L_u/W_u increases. Also, it was noted that the discharge efficiency decreases with the head increase which is more significant for the weirs with large values of the ratio of the elongation of the crest. Therefore, an optimal ratio for a Piano Key weir was given between 5 and 6. Leite Ribeiro et al. (2012) also stated that the ratio between the vertical and horizontal lengths of the Piano Key weir should be considered as a crucial design parameter. It was found that the width of the inlet key, W_i, become more important than the elongation of the crest of the Piano Key weir, L_u, when the water head was normalized with the height of the Piano Key weir, P.

2.2.3. Effect of W_i/W_o on Discharge Capacity and Flow Conditions

Piano Key weirs are the hydraulic structures designed with inlet and outlet widths, W_i and W_o. Therefore, the impact of the geometrical characteristics and the relation between inlet and outlet widths, W_i and W_o is quite important. In literature, there are many studies investigating the impact of the key widths and it is agreed that one of the main parameters controlling the discharge capacity is the ratio of the inlet and outlet widths, W_i and W_o. Ouamane and Lempérière (2006) found that inlet alveolus, W_i, higher than that of outlet alveolus, W_o, makes Piano Key weir have a better efficiency for low heads. Lately, it was added by Ouamane and Lempérière (2008) that the choice of a relative width W_i/W_o equals to 1.20 allows to have a higher hydraulic efficiency about 5% regarding the other models. In these two studies, it is important to note that the cost estimation for designing a Piano Key weir with higher inlet alveolus width, W_i, than that of outlet alveolus width, W_o was also slightly highlighted. It was reported that without any additional expense, it is the same cost to increase the inlet alveolus width, W_i, and decrease the outlet alveolus width, W_o. It was also stated by Ouamane (2011) that the optimal ratio between the inlet and outlet alveolus width, W_i and W_o, is between 1.20 and 1.50. In 2011, a sensitivity analysis for geometrical parameters including the ratio of the inlet and outlet widths, W_i/W_o, was tested with a 3D numerical model developed at EDF on the CDF Software Flow-3D® by Pralong et al. (2011). The investigation was carried out with keeping total width, W, constant and changing the ratio of the inlet and outlet alveolus widths, W_i/W_o. It was concluded that the optimal ratio between the inlet and outlet alveolus widths, W_i/W_o, is dependent on the incoming head and always above 1. It was clarified that the optimal ratio is becoming lower for higher heads. On the other hand, it was also stated that for low incoming heads, the discharge is controlled by the developed length as no submergence effect happens. Another conclusion of the effect of the ratio of the alveolus widths, W_i/W_o, was made by Noui and Ouamane (2011). It was informed that the value of the optimal ratio where the best hydraulic performance was noticed along the tested values for low and medium incoming heads were noticed as 1.20 and 1.50. Leite Ribeiro et al. (2011b) also studied the effect of the alveolus widths, W_i and W_o, and agreed with the studies which have been mentioned above. It was found that the ratio between the inlet and outlet alveolus widths, W_i/W_o, show a higher efficiency when it is above 1. However, it was noted that the ratios W_i/W_o equals to 1.25, 1.60 and 2 do not show any differences in their investigations. Another study which is consistent with the findings about the effect of the ratio of the inlet and outlet alveolus widths, W_i/W_o, was studied by Anderson (2011). It was found that the ratios W_i/W_o equals to 1.25 and 1.50 produce the highest discharges and this the ratio of W_i/W_o was suggested to be in the range between 1.25 and 1.50. It was highlighted that the ratio W_i/W_o equals to 1.25 produces relatively higher discharges than W_i/W_o equals to 1.50 for higher incoming heads for h/P=0.60. On the other hand, for lower incoming heads, the ratio W_i/W_o equals to 1.50 reveals higher discharges than W_i/W_o equals to 1.25 for h/P=0.60. In this study, the effect of the ratio between the inlet and outlet alveolus width, W_i/W_o, was explained. It was clarified that as inlet alveolus width, W_i, increases, the total effect of the head loss related with flow entering inlet alveolus decreases which results the increase of the discharge capacity. It was also stated that the discharge capacity decreases with the decrease of the inlet alveolus width, W_i, which results with an increase of the local submergence. This phenomenon was also investigated by Kabiri-Samani and Javaheri (2012) and agreed with the conclusions made by Anderson (2011). Another investigation which clarified the effect of the outlet key was carried out by Vermeulen et al. (2011). It was stated that the outlet alveolus width, W_o, is the limiting factor for the selection of the inlet and outlet alveolus widths, W_i and W_o. Another investigation for understanding the effect of the ratio of the inlet and outlet alveolus widths, W_i/W_o, considering two Piano Key weir heights, P, was studied by Machiels et al. (2012d). It was stated that the optimal value of the ratio of the inlet and outlet alveolus width, W_i/W_o, is 1.25 and the ratio of W_i/W_o must be selected without decreasing the outlet alveolus width, W_o, too much. It was clarified that a too narrow outlet alveolus may be unable to evacuate the flow under supercritical conditions and result in the decrease of the discharge efficiency. However, it was noted that for projects with a high number of Piano Key weir units, a W_i/W_o ratio of 1 appears satisfactory for economic and hydraulic considerations. The functioning of the outlet key was also explained by Leite Ribeiro et al. (2012). It was stated that the efficiency of the Piano Key weir depends on the functioning of the outlet. It was explained that with the increasing head, the efficiency of the Piano Key weir decreases because of the occurrence of the lateral jet-overcrossing and flow drowning. However, for low heads, the functioning of the outlet becomes unimportant and it was recommended for low head that the ratio of W_i/W_o can be selected as 1.60. Another investigation for understanding the effect of the ratio of inlet and outlet alveolus widths, W_i and W_o, of Piano Key weir was also done by Lefebvre et al. (2013) by using the CFD-Software Flow-3D®. It was agreed with the studies mentioned above that the increasing the inlet alveolus width, W_i, increases discharge efficiency. It was also shown that the optimal value for the ratio W_i/W_o is lower for small incoming heads. Anderson and Tullis (2013) studied to understand the effects of Piano Key weir modifications by varying the ratio of the inlet and outlet alveolus width, W_i/W_o, and found that the optimal range lies between 1.25 and 1.50. Also, it was stated that the influence of the ratio of W_i/W_o decreases as water head increases. Another investigation studied by Machiels et al. (2014) for understanding the effect of the ratio of the inlet and outlet alveolus width, W_i/W_o, considering weir heights, P, revealed that the ratios of W_i/W_o between 1.29 and 1.57 provide the optimal discharge capacity whatever the weir height, P, for the ranges of h/W_u which is suitable to be selected for the dam rehabilitations in Europe and Asia. This investigation was agreed with Erpicum et al. (2014) which confirmed that whatever the Piano Key weir height, P, optimization of the ratio of the inlet and outlet alveolus widths, W_i/W_o, increases the weir efficiency by about 30% which remains far below the effect of the weir height, P, optimization.

2.2.4. Effect of Bi/Bo on Discharge Capacity and Flow Conditions

The discharge of a Piano Key weir is affected by many parameters because of the several geometrical components of it. It was clearly investigated by the previous studies that one of the main geometrical parameters which have an influence on the discharge efficiency of a Piano Key weir is the ratio between the inlet and the outlet overhang lengths, B_i/B_o. It was studied by Ouamane and Lempérière (2006) that the model without downstream overhangs showed higher discharge efficiency than the models with downstream overhangs. The increase was found nearly 12% for relative head of $h/P<0.40$. It was stated that the models without downstream overhangs may be great solutions for huge discharges, but it was also noted that symmetrical overhangs still represent economical solutions. The same conclusion was stated by Ouamane and Lempérière (2008) that the model with a single upstream overhang is the best solution because of allowing an increase of 10% for the discharge efficiency. The effect of the upstream overhang was also highlighted by Ouamane and Lempérière (2010) with the result that the increase of the discharge with upstream overhangs was found nearly 4 times higher than a standard weir and furthermore 3.50 times higher without upstream overhangs for low heads. The design without downstream overhangs were recommended for only high discharges by Ouamane (2011) that during the periods of when the level of sill is higher than the reservoir level. The length of the upstream overhang, B_o, was not recommended by Vermeulen et al. (2011) to be designed more than 5 meters because of the economic reasons despite its hydraulic advantageous. It was stated by Machiels et al. (2012d) that the hydraulic optimum for selecting the length of the upstream overhang, B_o, is the longest one without decreasing the efficiency of the crest of the sidewall, T_s. It was clarified that the discharge efficiency decreases as the length of the upstream overhang, B_o, exceed the optimal value because of the decrease in the slope and the resilience capacity of the outlet slope, S_o. Also, a design with symmetrical overhangs was recommended by in their study. It was stated that symmetrical overhangs make construction more stable and favors the use of the precast elements. A study for understanding the effect of the upstream and downstream overhangs was performed by Anderson and Tullis (2011) by comparing Piano Key weirs with rectangular labyrinth weirs. The efficiency of the overhangs of a Piano Key weir was related with the flow contractions and the energy loss. It was stated that as the length of the upstream overhang, B_o, increases, the area and wetted perimeter in the inlet key increases which results in an increase in the discharge capacity because of decreasing the flow velocities, flow contractions and energy loss. It was also stated that the length of the downstream overhang, B_i, can increase the area and the wetted perimeter of the outlet key which results the discharge to exit efficiently. A sensitivity analysis made by Pralong et al. (2011) showed that increasing the length of the upstream overhang, B_o, increases the discharge efficiency. However, it was stated that the ratio of the lengths of the upstream and downstream overhangs, B_i/B_o, is not a sensitive parameter because of the insignificant gains on discharge. Another investigation to improve the understanding of the efficiency of Piano Key weirs were performed by Machiels et al. (2012d) showed that the influence of the dimensionless ratio of the B_i/B_o equals to 0.33 results with 10% increase in the discharge efficiency for the values of P/W_u equals to 1.30. Also, it was found that the highest discharge efficiency was obtained with only upstream overhangs for low heads. However, it was noted that this efficiency decreases with the increasing water head and becomes negligible for the values of h/W_u equals to 0.30. All the tested models with the ratio of B_i/B_o lead to decrease in the discharge efficiency for the values of P/W_u equals to 0.50 and it was found that the model with symmetrical overhangs is the best option for this value. It was stated by Erpicum et al. (2014) that the influence of the overhang lengths on the discharge efficiency of a Piano Key weir is not a significant factor because of increasing the efficiency up to 20% and agrees with the statements made by Pralong et al. (2011).

2.3. Examples of the Applications of the Piano Key Weirs

Piano Key weir was first designed in 2001 and constructed in France in 2006 (Laugier, 2007). Now, there are several Piano Key weirs constructed worldwide. In this study, the geometrical components of existing Piano Key weirs are examined to create convenient models to derive a design process. Therefore, important geometrical parameters of some of the present Piano Key weirs are listed below in Table 1.

Table 1: Values of Geometrical Components of Existing Piano Key Weirs (Laugier et al. 2013)

Dams	Region	Year	P (m)	W_i (m)	W_o (m)	W_u (m)	B_i (m)	B_o (m)	B (m)	L (m)
Goulours	France	2006	3.10	2.70	1.50-1.80	4.75	1.50	3.35	9.30	59.00
Saint-Marc	France	2008	4.20	3.10	2.20	5.90	4.00	4.00	12.70	77.00
L'Etroit	France	2009	5.30	2.45-2.75	1.50	4.80	2.00	3.20	12.20	78.00
Les Gloriettes	France	2010	3.00	2.3-2.5	1.50	4.40	2.60	3.50	10.00	86.80
Escouloubre	France	2011	1.80	1.30	0.90	3.80	1.20	1.20	5.10	22.00
Malarce	France	2012	4.40	1.25-1.65	1.58	3.63	2.03	6.63	13.46	350.00
Gage	France	-	6.00	1.60	1.30	3.70	3.00	4.00	13.00	208.00
La Raviege	France	-	4.67	2.40	1.65	4.85	3.33	4.00	13.24	177.00
Charmines	France	2015	4.38	2.40	1.60	4.70	4.41	3.97	13.24	2*120.00
Campauleil	France	2014	5.35	1.55	1.40	3.65	2.80	4.90	13.10	115.00

The most important non-dimensional geometrical parameters for the existing Piano Key weirs were also generated by the values given in Table 1 and these values are given in Table 2 below to identify the effective range of those parameters.

Table 2: Values of Geometrical Parameters of Existing Piano Key Weirs

Dams	Region	Year	L_u/W_u	W_i/W_o	B_i/B_o	P/W_u
Goulours	France	2006	4.92	1.80	0.45	0.65
Saint-Marc	France	2008	4.28	1.41	1.00	1.71
L'Etroit	France	2009	4.17	1.83	0.63	1.01
Les Gloriettes	France	2010	4.69	1.67	0.74	0.68
Escouloubre	France	2011	4.58	1.44	1.00	0.47
Malarce	France	2012	8.23	1.04	0.31	1.21
Gage	France	-	7.82	1.23	0.75	1.62
La Raviege	France	-	6.86	1.45	0.83	0.96
Charmines	France	2015	5.21	1.50	1.11	0.93
Campauleil	France	2014	6.95	1.11	0.57	1.47

CHAPTER 3

NUMERICAL MODELING

3.1. Mesh Selections

Mesh size selection is a critical factor before starting numerical investigations. It is essential to determine the domain size and the mesh size to obtain accurate results. In this study, before selecting the convenient mesh size, domain is divided into blocks. Selected mesh size for each block within the flow domain is shown in Figure 3a and Figure 3b below.

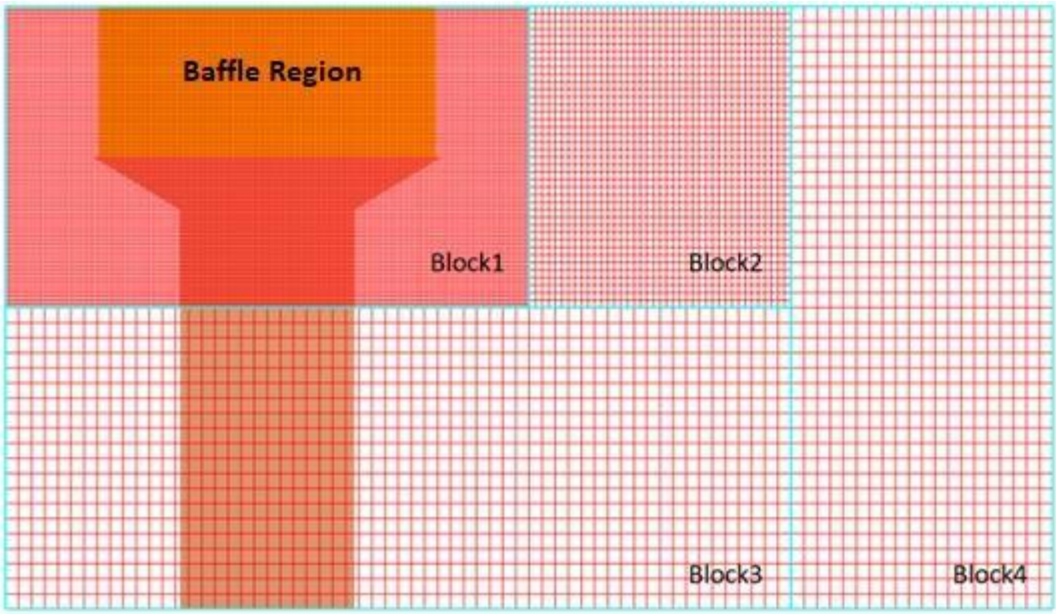

Figure 3a: Side View of the Flow Domain for Numerical Investigation

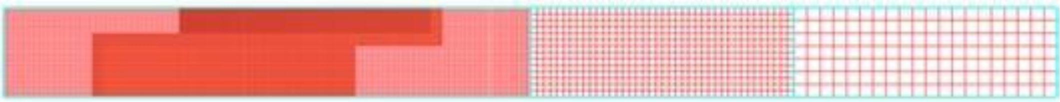

Figure 3b: Top View of the Flow Domain for Numerical Investigation

Mesh sizes are refined near the water surface and the crest level of the Piano Key weir. Also, baffle regions are defined on Piano Key weir crest to read the exact discharge values passing through the crest of the inlet, outlet and sidewall regions. Baffle regions are indicated as yellow on weir crest and can be also seen in Figure 3a.

The information of the mesh blocks can be seen in the Table 3. All the mesh blocks were designed in such a way that water level can be raised up to 4 meters. Therefore, the upper level of the mesh blocks was selected to be 5 meters more than the spillway crest considering the probable water level changes.

Table 3: Information of Mesh Blocks

Block	**Block Height (m)**	**Block Length (m)**	**Block Width (m)**
Block 1	P+5	1.5B	Wu/2
Block 2	P+5	0.75B	Wu/2
Block 3	2P	2.25B	Wu/2
Block 4	3P+5	0.75B	Wu/2

After the selection of the mesh blocks, a numerical investigation was performed to select convenient mesh sizes for each mesh block. Five alternative mesh sizes proportional to each other on each block were considered. The geometric details of all the mesh sizes that were tried on the same model are given below in Table 4. The numerical investigation was performed for maximum and minimum water levels where h_{max} equals to 4 meters and h_{min} equals to 1 meter to compare the results of the 5 mesh alternatives to select the most suitable one.

Table 4: Values of Geometrical Components of Selected Mesh Sizes

P (m)	P_d (m)	W_i (m)	W_u (m)	B_i (m)	B (m)	L_u/W_u	P/W_u	W_i/W_o	B_i/B_o
5.00	10.00	2.50	5.80	2.90	11.60	5.00	0.86	1.00	1.00

It can be seen in the Table 5 that as the total number of cells increases, the calculated value of the discharge surpassing over the spillway crest, Q_{pkw}, decreases. However, the results of Alternative 3, Alternative 4 and Alternative 5 do not diverge from each other for the minimum upstream water head as it can be seen in Figure 4. Also, the results of the Alternative 3 and Alternative 5 are nearly same for the investigation with the maximum upstream head as it can be seen in Figure 5.

Table 5: Discharge Values for Selection of Convenient Mesh Sizes

Mesh	**Total Cell**	**h (m)**	**Q_{pkw} (m^3/s)**
Alternative 1	194.254	4.00	167.24
	194.254	1.00	39.08
Alternative 2	324.762	4.00	167.73
	324.762	1.00	39.17
Alternative 3	458.263	4.00	162.46
	458.263	1.00	38.49
Alternative 4	623.652	4.00	165.54
	623.652	1.00	38.44
Alternative 5	892.402	4.00	162.67
	892.402	1.00	38.56

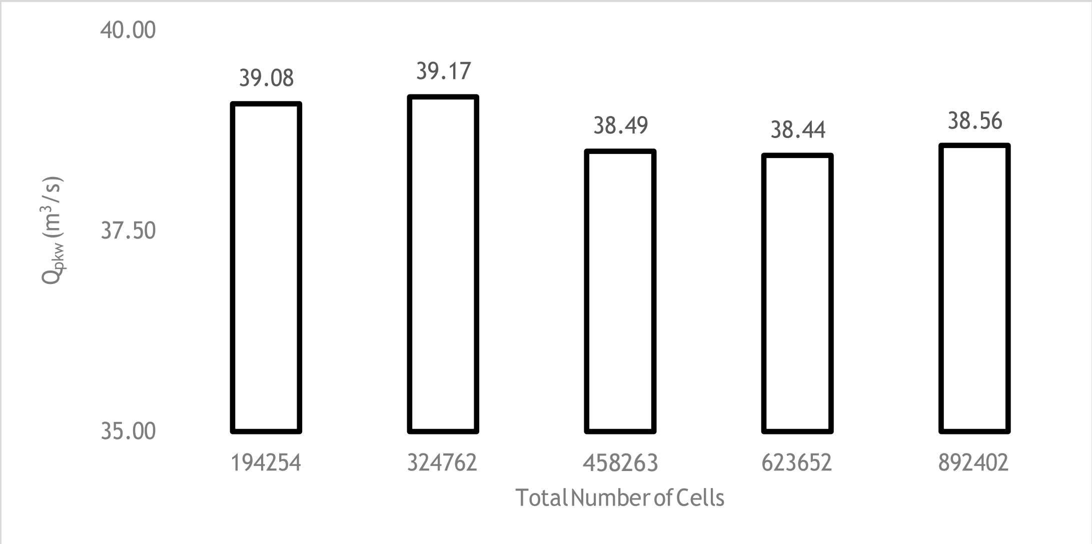

Figure 4: Model Studies for Mesh Size Selection for Head over Weir h=1m

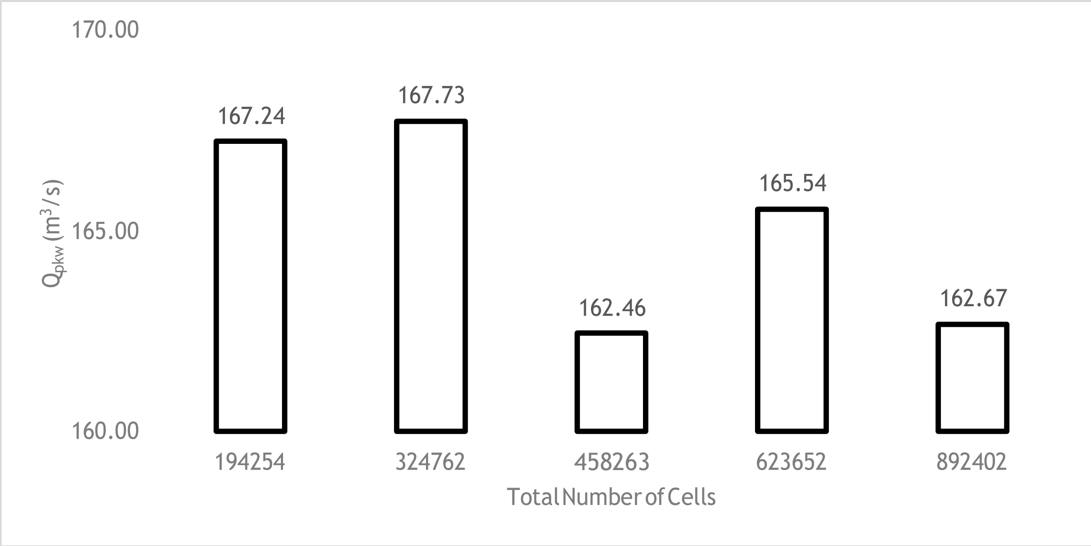

Figure 5: Model Studies for Mesh Size Selection for Head over Weir h=4m

It is important to note that as the total number of cells increases, the mesh sizes decrease, and this can decelerate the investigation time of the study. Therefore, Alternative 3 with convenient mesh sizes and minimum total number of cells was selected to be applied on all the models for the numerical investigations. The cell sizes on different blocks of Alternative 3 is given below in Table 6.

Table 6: Information of Mesh Sizes Selected for Numerical Investigations

Mesh Block	Coordinate	Cell Size (m)	Section	Aspect Ratio
Block1	X	0.109	x:y	1.05
	Y	0.104	y:z	0.83
	Z	0.125	z:x	1.15
Block2	X	0.218	x:y	1.05
	Y	0.207	y:z	0.83
	Z	0.250	z:x	1.15
Block3	X	0.435	x:y	1.05
	Y	0.414	y:z	0.83
	Z	0.500	z:x	1.15
Block4	X	0.435	x:y	1.05
	Y	0.414	y:z	0.83
	Z	0.500	z:x	1.15

3.2. Numerical Set-up

As it was mentioned earlier, models are considered with a basic structure of half of a Piano Key weir unit which has half of an inlet key and half of an outlet key to decrease the run time for all simulations. Therefore, the total number of cells differs for keeping the cell sizes constant in each model. As a result, being different for each model, run times were around 12-48 hours until steady-state solutions were obtained.

The boundary conditions for the numerical calculations can be seen in Figure 6 below. In the upstream, boundary type was selected as specified pressure (P) and water level was fixed corresponding to a given head. In the outflow, boundary type was specified as outflow (O). Boundary type was selected as wall (W) for the underside of the mesh blocks. Lastly, remaining boundaries of the mesh blocks were selected as symmetry boundary (S).

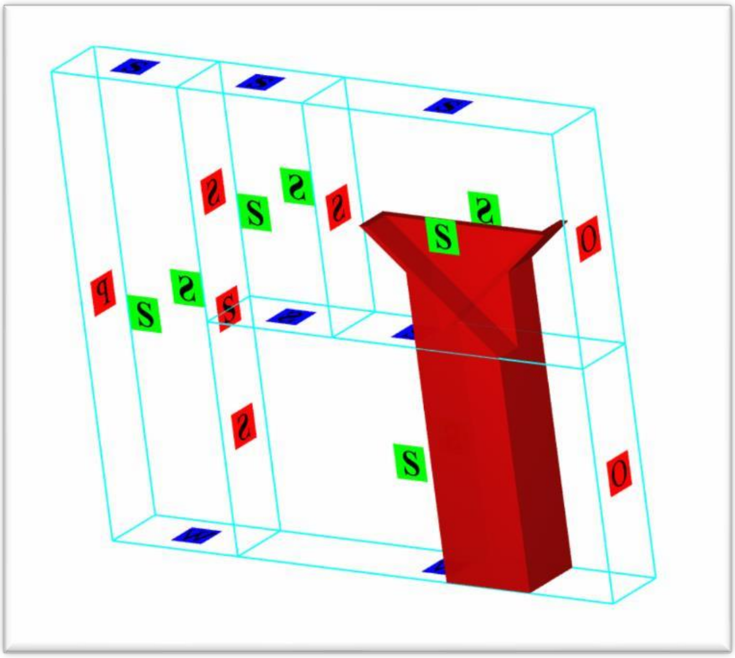

Figure 6: Selection of Boundary Conditions in Numerical Investigations

In the present study, numerical investigations were performed by using computational fluid dynamics CFD-Software Flow-3D®. Free surface flow conditions were simulated by using k-epsilon RNG turbulence model equations at least 40 seconds to ensure convergence. In Figure 7, velocity magnitude contours in one of the models can be seen.

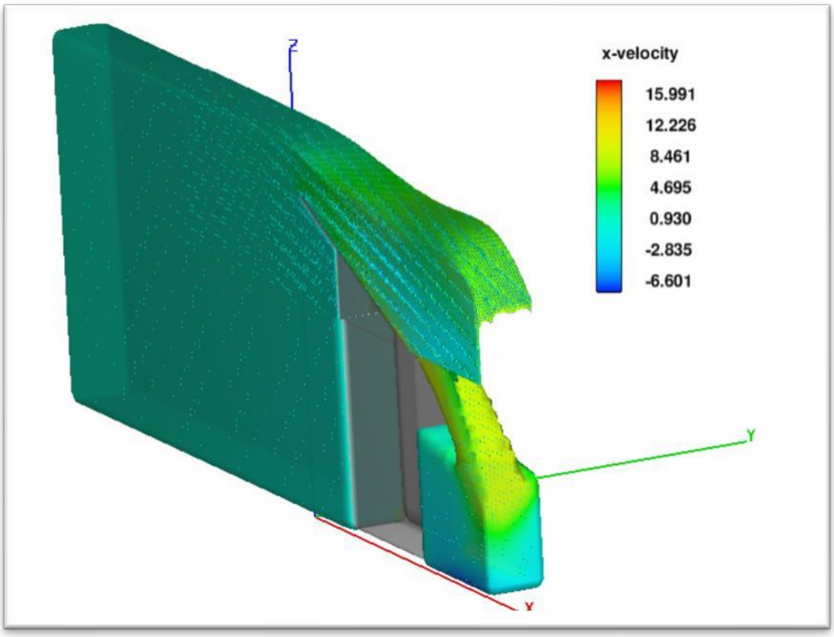

Figure 7: 3-D Analyze of a Given Numerical Simulation

3.3. Validation

In the present study, numerical results of the investigations by using CFD-Software Flow-3D® were validated with hydraulic model studies investigated by Aydın et al. (2017). In their study, the discharge values obtained from numerical results by using computational fluid dynamics CFD-Software Flow-3D® were in the experimental error margin.

3.4. Summary of the Numerical Models

In this study, a total number of 29 models were created to start numerical investigations to understand the effects of the dimensionless parameters of the Piano

Key weir on the influence on the discharge efficiency after the selection of the model. Each of the models were created with the same unit width, W_u, of 5.80 m and weir thickness, T_s, of 0.40 m. Investigation objectives and the information about the models are given below in Table 7. It can be seen in the table that the total number of cells differs from each other in order to be able to keep cell sizes constant.

Table 7: Objects and Information of the Models for Numerical Investigations

Model	Total Cell	Investigation Object	Brief Explanation
Model 1	344.513	The Effect of the Weir Height	P=2.50 m
Model 2	458.263	The Effect of the Weir Height	P=5.00 m
Model 3	572.013	The Effect of the Weir Height	P=7.50 m
Model 4	685.763	The Effect of the Weir Height	P=10.00 m
Model 5	799.513	The Effect of the Weir Height	P=12.50 m
Model 6	458.263	The Effect of the Overhang Lengths	P=5.00 m and Bi/Bo=0.50
Model 7	458.263	The Effect of the Overhang Lengths	P=5.00 m and Bi/Bo=0.33
Model 8	458.263	The Effect of the Overhang Lengths	P=5.00 m and Bi/Bo=0.00
Model 9	685.763	The Effect of the Overhang Lengths	P=10.00 m and Bi/Bo=0.50
Model 10	685.763	The Effect of the Overhang Lengths	P=10.00 m and Bi/Bo=0.33
Model 11	685.763	The Effect of the Overhang Lengths	P=10.00 m and Bi/Bo=0.00
Model 12	458.263	The Effect of the Weir Widths	Wi/Wo=1.10
Model 13	458.263	The Effect of the Weir Widths	Wi/Wo=1.20
Model 14	458.263	The Effect of the Weir Widths	Wi/Wo=1.30
Model 15	458.263	The Effect of the Weir Widths	Wi/Wo=1.40
Model 16	458.263	The Effect of the Weir Widths	Wi/Wo=1.50
Model 17	458.263	The Effect of the Weir Widths	Wi/Wo=1.60
Model 18	458.263	The Effect of the Weir Widths	Wi/Wo=1.70
Model 19	458.263	The Effect of the Weir Widths	Wi/Wo=1.80
Model 20	458.263	The Effect of the Weir Widths	Wi/Wo=1.90
Model 21	458.263	The Effect of the Weir Widths	Wi/Wo=2.00
Model 22	235.313	The Effect of the Crest Length	P=5.00 m and L/W=3.00
Model 23	346.788	The Effect of the Crest Length	P=5.00 m and L/W=4.00
Model 24	569.738	The Effect of the Crest Length	P=5.00 m and L/W=6.00
Model 25	681.213	The Effect of the Crest Length	P=5.00 m and L/W=7.00
Model 26	352.313	The Effect of the Crest Length	P=10.00 m and L/W=3.00
Model 27	519.038	The Effect of the Crest Length	P=10.00 m and L/W=4.00
Model 28	852.488	The Effect of the Crest Length	P=10.00 m and L/W=6.00
Model 29	1.019.213	The Effect of the Crest Length	P=10.00 m and L/W=7.00

3.4.1. Investigation of the Effect of the Dam Height

The effect of the dam height, P_d, is investigated before starting the investigation on the main parameters controlling the discharge capacity of the Piano Key weirs. The reason for this investigation is to define a constant dam height, P_d, for all the models in order to eliminate the possible effect of dam height, P_d, on the discharge. Therefore, three different dam heights, P_d, are investigated for the minimum and maximum water levels. All three models have constant ratios of $W_i/W_o=1$, $B_i/B_o=1$ and $L_u/W_u=5$.

The discharge values of the models of the models for the investigation of the effect of the dam height, P_d, which are calculated by numerical investigation are given in Table 8 below. Note that, the discharge passing through the crest of the Piano Key weir is represented as $Q_{pkw.}$

Table 8: Geometrical Values of the Models for the Investigation of the Effect of the Dam Height

Model	Pd/P	P (m)	h (m)	Q_{pkw} (m^3/s)
Pd1	1.00	5.00	4.00	162.61
		5.00	1.00	39.04
Pd2	2.00	5.00	4.00	162.46
		5.00	1.00	38.49
Pd3	3.00	5.00	4.00	163.82
		5.00	1.00	38.44

It can be seen in Figure 8 and Figure 9 that the effect of dam height, P_d, has negligible effect on the discharge capacity of Piano Key weir. It is also important to note that the time for the numerical solutions become longer as the dam height, P_d, increases because of the increase in the total mesh size. Therefore, the ratio of $P_d/P=2$ is selected for all the models.

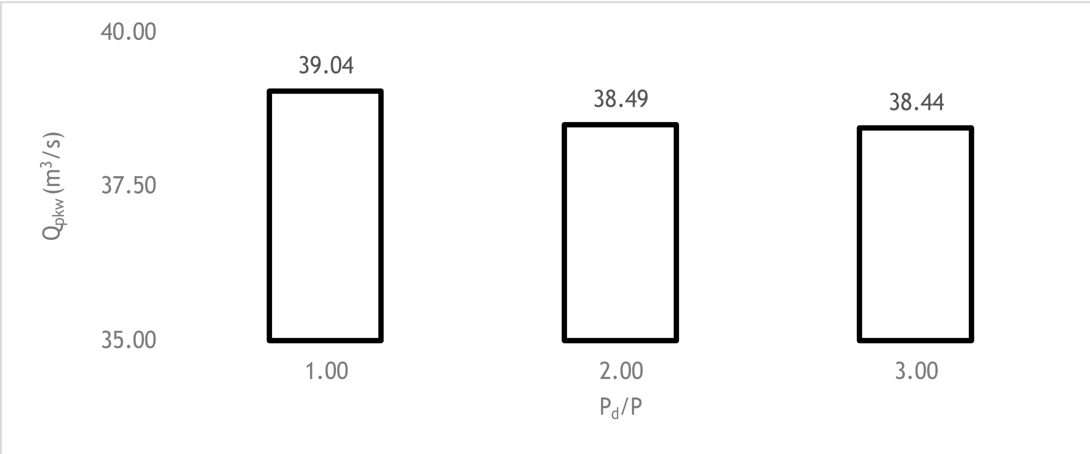

Figure 8: Model Studies for Dam Height Selection for Head over Weir h=1m

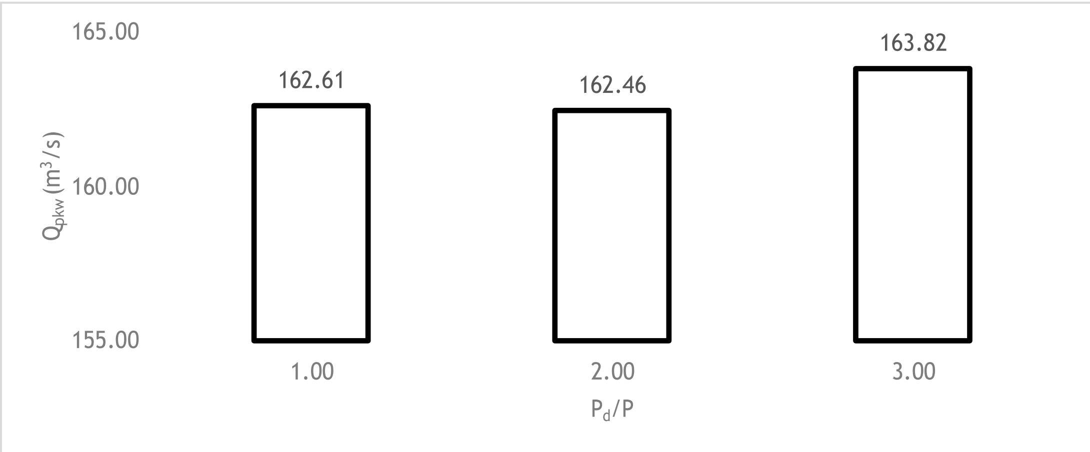

Figure 9: Model Studies for Dam Height Selection for Head over Weir h=4m

3.4.2. Investigation of the Effect of the Weir Height

The effect of the weir height, P, for the discharge efficiency of a Piano Key weir will be investigated in this subsection with 5 models. All these models have the same geometries with varying weir heights, P. All the important values and the parameters of the models for investigating the effect of the weir height are given below in Table 9.

Table 9: Geometrical Values of the Models for the Investigation of the Effect of the Weir Height

Model	P (m)	P/W_u	P/B	P/T_s	S_i	S_o	L_u/W_u	W_i/W_o	B_i/B_o	B_i/B
Model 1	2.50	0.43	0.22	6.25	0.30	0.30	5.00	1.00	1.00	0.25
Model 2	5.00	0.86	0.43	12.50	0.60	0.60	5.00	1.00	1.00	0.25
Model 3	7.50	1.29	0.65	18.75	0.90	0.90	5.00	1.00	1.00	0.25
Model 4	10.00	1.72	0.86	25.00	1.20	1.20	5.00	1.00	1.00	0.25
Model 5	12.50	2.16	1.08	31.25	1.51	1.51	5.00	1.00	1.00	0.25

The selection criteria for the values of the parameters of different models were based on the present Piano Key weirs. However, the weir heights, P, which are greater than the value of P=7.50 m for this study may not be realistic as it is not present in the present Piano Key weirs already built up to now (See Table 2). The reason for selecting such great values for the weir heights, P, is to understand the correlation between the weir height, P, and the discharge efficiency. Generally, the height of a Piano Key weir, P, which was designed or constructed does not exceed the value of P=6 m as it was shown previously in Table 1.

3.4.3. Investigation of the Effect of the Crest Length

The effect of the crest length on the discharge capacity of a Piano Key weir is investigated in this study with 10 models using a constant unit width, W_u, with varying L_u/W_u ratios of 3, 4, 5, 6 and 7. Each model is created with 2 different weir heights of P=5 m and P=10 m to see the possible effects of the weir heights, P, with varying unit developed crest length, L_u, on the discharge capacity. All the important values and the parameters of the models for investigating the effect of the crest length are given below in Table 10.

Table 10: Geometrical Values of the Models for the Investigation of the Effect of the Crest Length

Model	P (m)	W_i (m)	W_u (m)	B_i (m)	B (m)	L_u (m)	L_u/W_u	P/W_u	P/B
Model 22	5.00	2.50	5.80	1.45	5.80	17.40	3.00	0.86	0.86
Model 23	5.00	2.50	5.80	2.18	8.70	23.20	4.00	0.86	0.57
Model 2	5.00	2.50	5.80	2.90	11.60	29.00	5.00	0.86	0.43
Model 24	5.00	2.50	5.80	3.63	14.50	34.80	6.00	0.86	0.34
Model 25	5.00	2.50	5.80	4.35	17.40	40.60	7.00	0.86	0.29
Model 26	10.00	2.50	5.80	1.45	5.80	17.40	3.00	1.72	1.72
Model 27	10.00	2.50	5.80	2.18	8.70	23.20	4.00	1.72	1.15
Model 4	10.00	2.50	5.80	2.90	11.60	29.00	5.00	1.72	0.86
Model 28	10.00	2.50	5.80	3.63	14.50	34.80	6.00	1.72	0.69
Model 29	10.00	2.50	5.80	4.35	17.40	40.60	7.00	1.72	0.57

The selection criteria for the range of the values of the unit developed crest lengths of the models are also based on the existing Piano Key weirs as it is shown in Table 1.

3.4.4. Investigation of the Effect of the Weir Width

The effect of the weir width for the discharge efficiency of a Piano Key weir is investigated in this study with 11 models. All the models for the investigation have identical geometries with varying inlet and outlet weir widths, W_i and W_o, for a constant unit width, W_u. The weir height, P, for investigating the effect of the weir width for all models is selected as P=5 m. All the important values and parameters of the models for investigating the effect of the weir width are given below in Table 11.

Table 11: Geometrical Values of the Models for the Investigation of the Effect of the Weir Widths

Model	P (m)	T_s (m)	W_i (m)	W_o (m)	W_i/W_o	W_u (m)	L_u/W_u	B_i/B_o	B_i/B
Model 2	5.00	0.40	2.50	2.50	1.00	5.80	5.00	1.00	0.25
Model 12	5.00	0.40	2.62	2.38	1.10	5.80	5.00	1.00	0.25
Model 13	5.00	0.40	2.73	2.27	1.20	5.80	5.00	1.00	0.25
Model 14	5.00	0.40	2.83	2.17	1.30	5.80	5.00	1.00	0.25
Model 15	5.00	0.40	2.92	2.08	1.40	5.80	5.00	1.00	0.25
Model 16	5.00	0.40	3.00	2.00	1.50	5.80	5.00	1.00	0.25
Model 17	5.00	0.40	3.08	1.92	1.60	5.80	5.00	1.00	0.25
Model 18	5.00	0.40	3.15	1.85	1.70	5.80	5.00	1.00	0.25
Model 19	5.00	0.40	3.21	1.79	1.80	5.80	5.00	1.00	0.25
Model 20	5.00	0.40	3.28	1.72	1.90	5.80	5.00	1.00	0.25
Model 21	5.00	0.40	3.33	1.67	2.00	5.80	5.00	1.00	0.25

The selection criteria for the values of the weir widths of the models are based on the existing Piano Key weirs. The common unit weir widths, Wu, of the present Piano Key weirs are about 5 m as it is shown in Table 1.

3.4.5. Investigation of the Effect of the Overhang Length

The effect of the overhang lengths, B_i and B_o, for the discharge efficiency of a Piano Key weir is investigated in this study with 8 models under a constant unit width, W_u, and weir lengths, B, with varying B_i/B_o ratios of 1, 0.50, 0.33 and 0. Each model was created with 2 different weir heights, P, to see the possible effects of the weir heights, P. All the important values and parameters of the models for investigating the effect of the overhang lengths, B_i and B_o, are given below in Table 12.

Table 12: Geometrical Values of the Models for the Investigation of the Effect of the Overhang Lengths

Model	**P (m)**	**B_i (m)**	**B_o (m)**	**B (m)**	**B_i/B_o**	**B_i/B**	**B_o/B**	**S_i**	**S_o**	**P/B**	**L_u/W_u**	**W_i/W_o**
Model 2	5.00	2.90	2.90	11.60	1.00	0.25	0.25	0.60	0.60	0.43	5.00	1.00
Model 6	5.00	1.93	3.87	11.60	0.50	0.17	0.33	0.68	0.54	0.43	5.00	1.00
Model 7	5.00	1.45	4.35	11.60	0.33	0.13	0.38	0.73	0.51	0.43	5.00	1.00
Model 8	5.00	0.00	5.80	11.60	0.00	0.00	0.50	0.93	0.45	0.43	5.00	1.00
Model 4	10.00	2.90	2.90	11.60	1.00	0.25	0.25	1.20	1.20	0.86	5.00	1.00
Model 9	10.00	1.93	3.87	11.60	0.50	0.17	0.33	1.36	1.08	0.86	5.00	1.00
Model 10	10.00	1.45	4.35	11.60	0.33	0.13	0.38	1.46	1.03	0.86	5.00	1.00
Model 11	10.00	0.00	5.80	11.60	0.00	0.00	0.50	1.85	0.89	0.86	5.00	1.00

The selection criteria for the values of the geometries of these models to investigate the effect of the overhang lengths, B_i and B_o, are also based on the present Piano Key weirs. It is important to note that the overhang length of the existing Piano Key weirs does not exceed 5 m as it can be seen in Table 1.

CHAPTER 4

NUMERICAL RESULTS

4.1. Head-Discharge Relations

It was mentioned above that there were 29 models created in order to understand the effect of various geometrical parameters on the discharge capacity of Piano Key weir units. It is essential to obtain a head-discharge relation for a Piano Key weir with different head over weirs, h, to understand the effect of specific parameters on the discharge capacity. In order to achieve this, all these 29 models are investigated with different head over weirs within the range of 1 to 4 meters. In other words, a total number of 145 simulations are executed with different objectives.

In the present study, discharge values integrated from velocity fields for different head over weirs are given in tables for each investigation and it can be seen in Figure 10 that the discharge passing through the crest of the Piano Key weir is represented as Q_{pkw} and separated into Q_{in}, Q_{out} and Q_{side} to see the effects of the weir components on the discharge capacity.

In Figure 10, the discharge passing through the downstream of the inlet key is represented as Q_{in} and the crest line where Q_{in} passes through is marked with blue. Secondly, the discharge passing through the upstream of the outlet key is represented as Q_{out} and the crest line where Q_{out} passes through is marked with yellow line. Lastly, the discharge passing through the sidewall from the inlet key through the outlet key is represented as Q_{side} and the crest line where Q_{side} passes through is marked with red line.

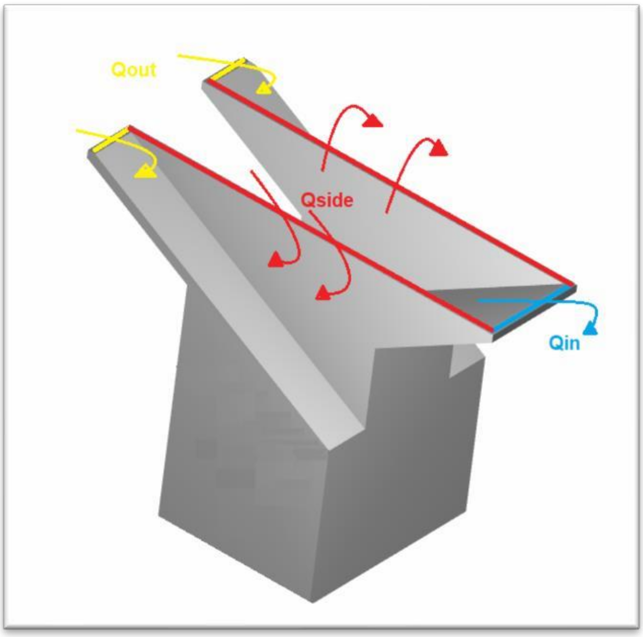

Figure 10: Discharge Passing Through the Crest of the Piano Key Weir

It is important to interpret discharge passing through the crest and therefore a parameter is needed to compare the discharge values obtained each investigation. In order to assess the efficiency of a Piano Key weir, the discharge values obtained here are compared with a sharp crested (linear) weir situated on the same width, W. Plan views of both weirs on the same weir width can be seen in Figure 11.

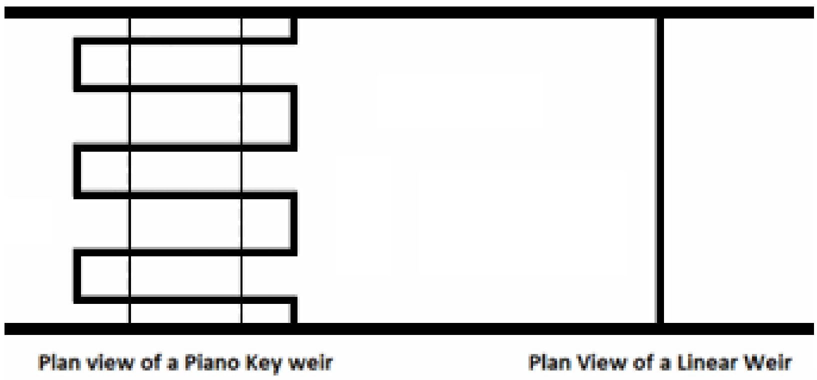

Figure 11: Plan Views of Piano Key and Linear Weirs

In the present study, the weir equations given in Gharahjeh et al. (2015) was used to calculate the discharge of linear weirs to define discharge efficiency parameter, r, for each investigation. Discharge efficiencies of each investigation for the present study can be calculated by using Eqn. 1 given below.

$$r = \frac{Q_{Piano\ Key\ weir}}{Q_{Sharp\ Crested\ weir}} \qquad Eqn. 1$$

The weir equations generated as function of geometry in Gharahjeh et al. (2015) are given in Eqn. 2 and Eqn. 3 below.

$$Q_{Sharp\ Crested\ weir} = V_w . W . h \qquad Eqn. 2$$

$$V_w = c\sqrt{2gh} \qquad Eqn. 3$$

In Eqn. 2, W represents the total width of the weir and h represents the head over weir. With all these, c presented in Eqn. 3 is the dimensionless constant for a given weir which is obtained from experimental data. For the present conditions, the c value can be obtained as 0.4744.

4.1.1. Investigation of the Effect of the Weir Height

The results of the investigation of the weir height, P, and its relationship with the unit width, W_u, showed that weir height, P, has a great impact on the discharge efficiency. 5 models have been generated with constant unit weir width (W_u=5.80 m), symmetrical overhang lengths (B_i=B_o=2.90 m), with a constant weir length (B=11.60 m) and consequently, a constant developed length along the crest of Piano Key weir unit (L_u=29 m). Discharge capacities obtained for different water heads from the results of 5 models for the investigation of the effect of the weir height, P, are shown in Table 13-17 below.

Table 13: Discharge Values for the Investigation of the Effect of the Weir Height for Model 1

Model	**h (m)**	**h/P**	**P (m)**	**P/W_u**	**P/B**	**S_i**	**S_o**	**Q_{in} (%)**	**Q_{out} (%)**	**Q_{side} (%)**	**Q_{pkw} (m^3/s)**
Model 1	4.00	1.60	2.50	0.43	0.22	0.30	0.30	21.72	31.07	47.21	113.33
	3.25	1.30	2.50	0.43	0.22	0.30	0.30	19.36	28.80	51.84	87.91
	2.50	1.00	2.50	0.43	0.22	0.30	0.30	17.64	25.82	56.54	64.16
	1.75	0.70	2.50	0.43	0.22	0.30	0.30	16.96	22.07	60.97	42.69
	1.00	0.40	2.50	0.43	0.22	0.30	0.30	15.54	17.58	66.88	23.19

Table 14: Discharge Values for the Investigation of the Effect of the Weir Height for Model 2

Model	**h (m)**	**h/P**	**P (m)**	**P/W_u**	**P/B**	**S_i**	**S_o**	**Q_{in} (%)**	**Q_{out} (%)**	**Q_{side} (%)**	**Q_{pkw} (m^3/s)**
Model 2	4.00	0.80	5.00	0.86	0.43	0.60	0.60	20.58	25.21	54.21	162.46
	3.25	0.65	5.00	0.86	0.43	0.60	0.60	19.83	23.13	57.04	132.10
	2.50	0.50	5.00	0.86	0.43	0.60	0.60	19.15	20.72	60.13	102.44
	1.75	0.35	5.00	0.86	0.43	0.60	0.60	16.11	17.46	66.43	71.75
	1.00	0.20	5.00	0.86	0.43	0.60	0.60	13.21	14.19	72.60	38.47

Table 15: Discharge Values for the Investigation of the Effect of the Weir Height for Model 3

Model	**h (m)**	**h/P**	**P (m)**	**P/W_u**	**P/B**	**S_i**	**S_o**	**Q_{in} (%)**	**Q_{out} (%)**	**Q_{side} (%)**	**Q_{pkw} (m^3/s)**
Model 3	4.00	0.53	7.50	1.29	0.65	0.90	0.90	22.38	23.98	53.63	182.76
	3.25	0.43	7.50	1.29	0.65	0.90	0.90	21.07	21.84	57.09	149.31
	2.50	0.33	7.50	1.29	0.65	0.90	0.90	19.09	19.58	61.33	116.23
	1.75	0.23	7.50	1.29	0.65	0.90	0.90	16.03	16.68	67.28	80.96
	1.00	0.13	7.50	1.29	0.65	0.90	0.90	12.71	13.31	73.98	42.68

Table 16: Discharge Values for the Investigation of the Effect of the Weir Height for Model 4

Model	**h (m)**	**h/P**	**P (m)**	**P/W_u**	**P/B**	**S_i**	**S_o**	**Q_{in} (%)**	**Q_{out} (%)**	**Q_{side} (%)**	**Q_{pkw} (m^3/s)**
Model 4	4.00	0.40	10.00	1.72	0.86	1.20	1.20	23.17	23.12	53.72	190.28
	3.25	0.33	10.00	1.72	0.86	1.20	1.20	22.05	20.92	57.02	156.01
	2.50	0.25	10.00	1.72	0.86	1.20	1.20	18.82	18.81	62.37	120.81
	1.75	0.18	10.00	1.72	0.86	1.20	1.20	15.63	16.08	68.29	84.69
	1.00	0.10	10.00	1.72	0.86	1.20	1.20	12.46	12.88	74.66	44.88

Table 17: Discharge Values for the Investigation of the Effect of the Weir Height for Model 5

Model	h (m)	h/P	P (m)	P/W_u	P/B	S_i	S_o	Q_{in} (%)	Q_{out} (%)	Q_{side} (%)	Q_{pkw} (m^3/s)
Model 5	4.00	0.32	12.50	2.16	1.08	1.51	1.51	23.45	22.63	53.92	194.04
	3.25	0.26	12.50	2.16	1.08	1.51	1.51	20.69	20.73	58.58	159.15
	2.50	0.20	12.50	2.16	1.08	1.51	1.51	18.01	18.65	63.34	124.25
	1.75	0.14	12.50	2.16	1.08	1.51	1.51	15.15	15.91	68.94	87.18
	1.00	0.08	12.50	2.16	1.08	1.51	1.51	12.23	12.80	74.97	46.07

For a constant water depth, as the weir height, P, increases; the discharge capacity of the Piano Key weir unit increases. However, the rate of this increase in the discharge capacity is larger for small weir heights and it decreases with increasing weir height. For example, for water depth of 2.50 m, the capacity increase by increasing the weir height from 10 m to 12.50 m is only 2.85%. This result agrees with the findings of the recent studies except the outcome that Machiels et al. (2014) stated. In their study, it was expressed that the weir height, P, has no effect on the discharge efficiency for the values of P/W_u greater than 1. It can be seen in the Figure 12 that discharge capacity slightly increases for the values of the ratio P/W_u equals to 1.29, 1.72 and 2.15.

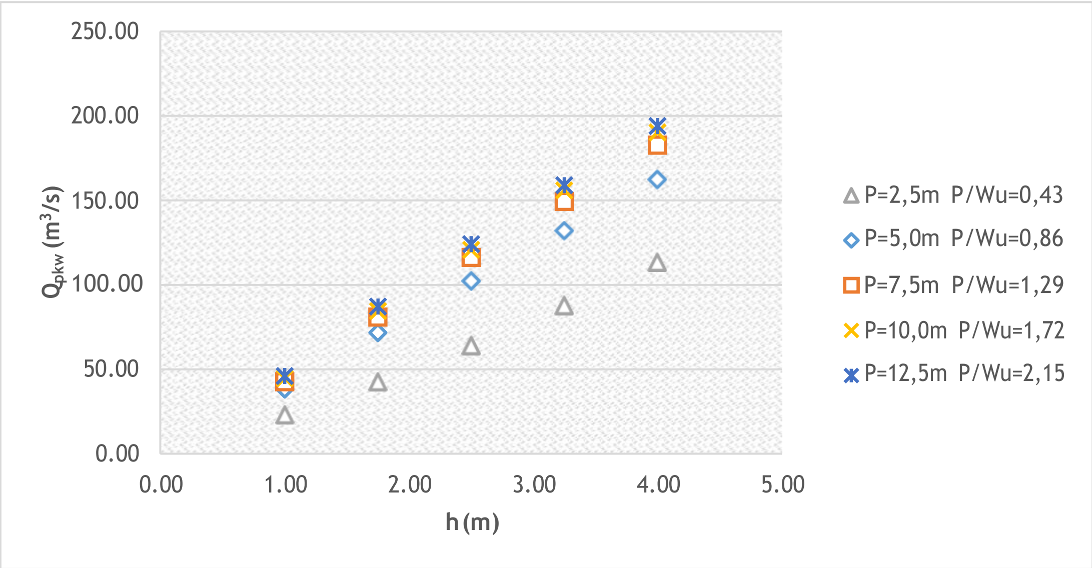

Figure 12: Head-Discharge Relation of the Models for the Investigation of the Effect of the Weir Height

The reason for the increase of the discharge capacity as the weir height, P, increases were expressed before by recent studies. In the present investigation, all the values of the discharge evacuated from the crest of the inlet, outlet and sidewall were highlighted and percentages of the distributions of the discharge evacuation of these parts are given in the Table 13-17 above. In all the cases investigated the percentage of the total discharge passing through the front of the inlet and outlet keys increases as the water depth increases. On the other hand, the percentage of the total discharge passing through the sides decreases with the increase in the water depth. The contributions from the back of the inlet, front of the outlet and sides are not sensitive to the weir height, P. This result can be explained with the statements of Lefebvre et al. (2013) and Machiels et al. (2014). It can be said that the discharge efficiency can be increased by increasing the weir height, P, in order to increase the inlet section area to decrease head losses and the height of the outlet key to decrease the occurrence of the submergence.

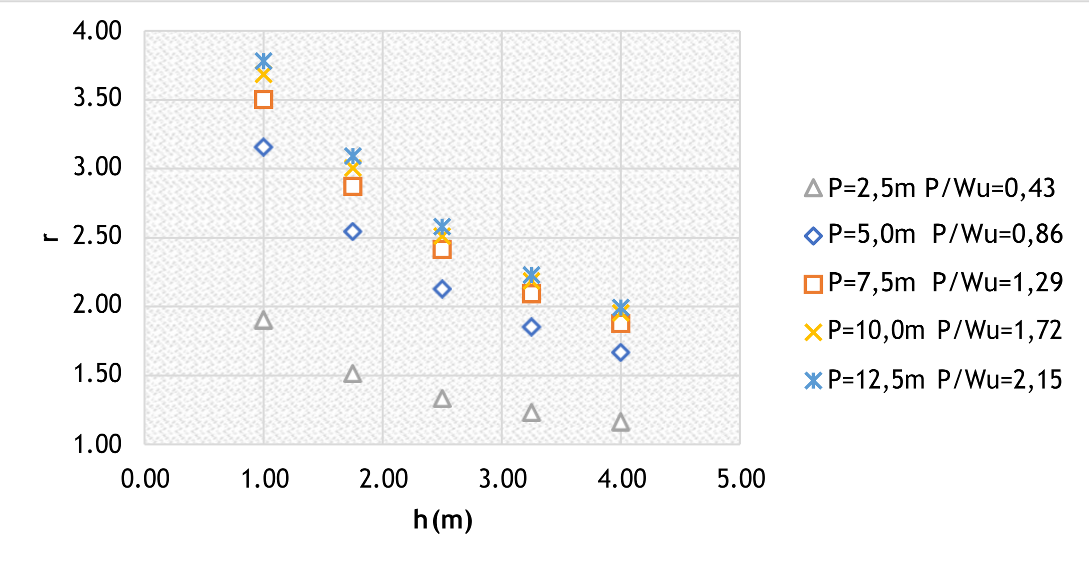

Figure 13: Head-Discharge Efficiency Relation of the Models for the Investigation of the Effect of the Weir Height

In all the models as the water depth increases, efficiency decreases. This means that for larger flow depths, Piano Key weir units will work more like a sharp crested weir as it can be seen in Figure 13. Another important conclusion obtained here is the increase of the discharge efficiencies as the weir height, P, increases. However, the rate of this increase decreases with the increasing weir height, P.

The design criteria for selecting the weir height of a Piano Key weir, P, is very important for both hydraulic and economic purposes. It was mentioned before by many researchers that as the weir height, P, increases, the efficiency decreases, and construction cost increases rapidly. So that, an optimal ratio for the ratio of P/W_u should be selected in the design process. As it was stated by Erpicum et al. (2014), the height of a Piano Key weir, P, is a function of absolute values of the upstream head. It can be seen in Figure 14 that the model with the ratio of P/W_u equals to 0.43 (P=2.50 m) has nearly the same discharge efficiency with all the other models for the ratio of h/P equals to 0.40. In other words, depending on the upstream water depth, a smaller Piano Key weir unit may be a better choice as it will give more or less the same efficiency with the large Piano Key weir units with a much smaller cost.

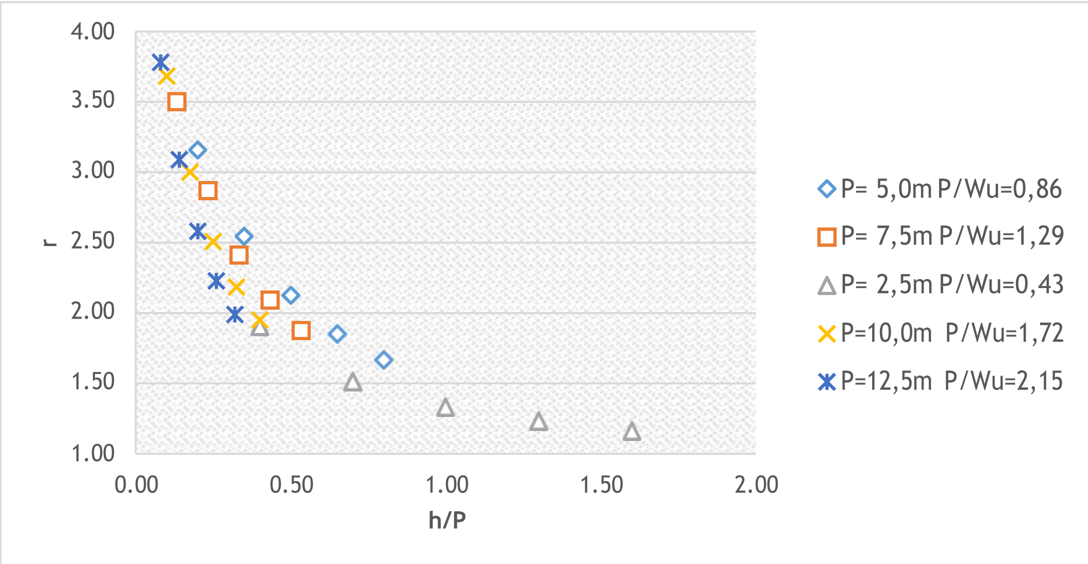

Figure 14: Dimensionless ratio of h/P-Discharge Efficiency Relation of the Models for the Investigation of the Effect of the Weir Height

4.1.2. Investigation of the Effect of the Crest Length

The effect of the crest length on the discharge capacity of a Piano Key weir is investigated with 10 models with constant unit widths (W_u=5.80 m) with varying L/W or L_u/W_u ratios of 3, 4, 5, 6 and 7 as it can be seen in Table 7. Each model is created with 2 different weir heights of P=5 m and P=10 m to see the possible effect of the weir height, P, on varying developed crest length along the crest of Piano Key weir, L_u, for the discharge capacity. Discharge capacities obtained for different water heads from the results of 10 models for the investigation of the effect of the crest length are shown in Tables 18-27 below.

Table 18: Discharge Values for the Investigation of the Effect of the Crest Length for Model 22

Model	h	h/P	P (m)	Wu (m)	B (m)	Lu (m)	Lu/Wu	Q in (%)	Q out (%)	Q side (%)	Q pkw (m3/s)
Model 22	**4.00**	**0.80**	**5.00**	**5.80**	**5.80**	**17.40**	**3.00**	**31.65**	**32.52**	**35.83**	**130.90**
	3.25	**0.65**	**5.00**	**5.80**	**5.80**	**17.40**	**3.00**	**29.24**	**30.94**	**39.82**	**103.59**
	2.50	**0.50**	**5.00**	**5.80**	**5.80**	**17.40**	**3.00**	**27.16**	**28.78**	**44.07**	**77.48**
	1.75	**0.35**	**5.00**	**5.80**	**5.80**	**17.40**	**3.00**	**24.95**	**25.65**	**49.39**	**51.84**
	1.00	**0.20**	**5.00**	**5.80**	**5.80**	**17.40**	**3.00**	**21.08**	**21.72**	**57.20**	**25.97**

Table 19: Discharge Values for the Investigation of the Effect of the Crest Length for Model 23

Model	h	h/P	P (m)	W_u (m)	B (m)	L_u (m)	L_u/W_u	Q_{in} (%)	Q_{out} (%)	Q_{side} (%)	Q_{pkw} (m^3/s)
Model 23	4.00	0.80	5.00	5.80	8.70	23.20	4.00	24.82	27.98	47.20	154.01
	3.25	0.65	5.00	5.80	8.70	23.20	4.00	23.62	26.20	50.19	124.06
	2.50	0.50	5.00	5.80	8.70	23.20	4.00	22.07	23.79	54.14	92.95
	1.75	0.35	5.00	5.80	8.70	23.20	4.00	19.95	20.65	59.40	64.83
	1.00	0.20	5.00	5.80	8.70	23.20	4.00	15.89	16.66	67.45	33.49

Table 20: Discharge Values for the Investigation of the Effect of the Crest Length for Model 2

Model	h	h/P	P (m)	W_u (m)	B (m)	L_u (m)	L_u/W_u	Q_{in} (%)	Q_{out} (%)	Q_{side} (%)	Q_{pkw} (m^3/s)
Model 2	4.00	0.80	5.00	5.80	11.60	29.00	5.00	20.58	25.21	54.21	162.46
	3.25	0.65	5.00	5.80	11.60	29.00	5.00	19.83	23.13	57.04	132.10
	2.50	0.50	5.00	5.80	11.60	29.00	5.00	19.15	20.72	60.13	102.44
	1.75	0.35	5.00	5.80	11.60	29.00	5.00	16.11	17.46	66.43	71.75
	1.00	0.20	5.00	5.80	11.60	29.00	5.00	13.21	14.19	72.60	38.47

Table 21: Discharge Values for the Investigation of the Effect of the Crest Length for Model 24

Model	h	h/P	P (m)	W_u (m)	B (m)	L_u (m)	L_u/W_u	Q_{in} (%)	Q_{out} (%)	Q_{side} (%)	Q_{pkw} (m^3/s)
Model 24	4.00	0.80	5.00	5.80	14.50	34.80	6.00	17.61	22.81	59.58	169.36
	3.25	0.65	5.00	5.80	14.50	34.80	6.00	17.05	20.65	62.30	139.38
	2.50	0.50	5.00	5.80	14.50	34.80	6.00	15.78	17.98	66.24	109.04
	1.75	0.35	5.00	5.80	14.50	34.80	6.00	13.15	15.00	71.85	78.20
	1.00	0.20	5.00	5.80	14.50	34.80	6.00	10.83	11.79	77.38	43.62

Table 22: Discharge Values for the Investigation of the Effect of the Crest Length for Model 25

Model	h	h/P	P (m)	W_u (m)	B (m)	L_u (m)	L_u/W_u	Q_{in} (%)	Q_{out} (%)	Q_{side} (%)	Q_{pkw} (m^3/s)
Model 25	4.00	0.80	5.00	5.80	17.40	40.60	7.00	15.54	21.37	63.09	174.33
	3.25	0.65	5.00	5.80	17.40	40.60	7.00	14.85	19.18	65.97	143.64
	2.50	0.50	5.00	5.80	17.40	40.60	7.00	13.82	16.29	69.89	114.05
	1.75	0.35	5.00	5.80	17.40	40.60	7.00	12.05	13.25	74.69	83.16
	1.00	0.20	5.00	5.80	17.40	40.60	7.00	9.90	10.32	79.78	47.73

Table 23: Discharge Values for the Investigation of the Effect of the Crest Length for Model 26

Model	h	h/P	P (m)	W_u (m)	B (m)	L_u (m)	L_u/W_u	Q_{in} (%)	Q_{out} (%)	Q_{side} (%)	Q_{pkw} (m^3/s)
Model 26	4.00	0.40	10.00	5.80	5.80	17.40	3.00	31.39	31.17	37.44	129.52
	3.25	0.33	10.00	5.80	5.80	17.40	3.00	28.86	29.31	41.83	103.37
	2.50	0.25	10.00	5.80	5.80	17.40	3.00	26.48	26.95	46.57	77.77
	1.75	0.18	10.00	5.80	5.80	17.40	3.00	23.45	24.39	52.15	53.40
	1.00	0.10	10.00	5.80	5.80	17.40	3.00	20.02	21.10	58.88	27.08

Table 24: Discharge Values for the Investigation of the Effect of the Crest Length for Model 27

Model	h	h/P	P (m)	W_u (m)	B (m)	L_u (m)	L_u/W_u	Q_{in} (%)	Q_{out} (%)	Q_{side} (%)	Q_{pkw} (m^3/s)
Model 27	4.00	0.40	10.00	5.80	8.70	23.20	4.00	25.56	26.48	47.96	164.56
	3.25	0.33	10.00	5.80	8.70	23.20	4.00	23.85	24.33	51.82	133.31
	2.50	0.25	10.00	5.80	8.70	23.20	4.00	21.72	22.21	56.07	103.12
	1.75	0.18	10.00	5.80	8.70	23.20	4.00	18.12	19.36	62.51	71.40
	1.00	0.10	10.00	5.80	8.70	23.20	4.00	15.19	15.76	69.05	36.76

Table 25: Discharge Values for the Investigation of the Effect of the Crest Length for Model 4

Model	h	h/P	P (m)	W_u (m)	B (m)	L_u (m)	L_u/W_u	Q_{in} (%)	Q_{out} (%)	Q_{side} (%)	Q_{pkw} (m^3/s)
Model 4	4.00	0.40	10.00	5.80	11.60	29.00	5.00	23.17	23.12	53.72	190.28
	3.25	0.33	10.00	5.80	11.60	29.00	5.00	22.05	20.92	57.02	156.01
	2.50	0.25	10.00	5.80	11.60	29.00	5.00	18.82	18.81	62.37	120.81
	1.75	0.18	10.00	5.80	11.60	29.00	5.00	15.63	16.08	68.29	84.69
	1.00	0.10	10.00	5.80	11.60	29.00	5.00	12.46	12.88	74.66	44.88

Table 26: Discharge Values for the Investigation of the Effect of the Crest Length for Model 28

Model	h	h/P	P (m)	W_u (m)	B (m)	L_u (m)	L_u/W_u	Q_{in} (%)	Q_{out} (%)	Q_{side} (%)	Q_{pkw} (m^3/s)
Model 28	4.00	0.40	10.00	5.80	14.50	34.80	6.00	20.56	20.11	59.34	215.02
	3.25	0.33	10.00	5.80	14.50	34.80	6.00	19.28	18.26	62.46	177.94
	2.50	0.25	10.00	5.80	14.50	34.80	6.00	15.79	16.29	67.92	139.10
	1.75	0.18	10.00	5.80	14.50	34.80	6.00	13.01	13.75	73.24	98.63
	1.00	0.10	10.00	5.80	14.50	34.80	6.00	10.15	10.75	79.10	53.00

Table 27: Discharge Values for the Investigation of the Effect of the Crest Length for Model 29

Model	h	h/P	P (m)	W_u (m)	B (m)	L_u (m)	L_u/W_u	Q_{in} (%)	Q_{out} (%)	Q_{side} (%)	Q_{pkw} (m^3/s)
Model 29	4.00	0.40	10.00	5.80	17.40	40.60	7.00	18.31	18.36	63.33	239.33
	3.25	0.33	10.00	5.80	17.40	40.60	7.00	16.63	16.52	66.85	198.35
	2.50	0.25	10.00	5.80	17.40	40.60	7.00	13.53	14.43	72.04	156.43
	1.75	0.18	10.00	5.80	17.40	40.60	7.00	10.94	11.98	77.08	111.61
	1.00	0.10	10.00	5.80	17.40	40.60	7.00	8.59	9.29	82.13	60.38

As expected, the results of the investigation of the developed crest length along the crest of Piano Key weir, L_u, and its relationship with the unit width, W_u, has a great impact on the discharge efficiency. It can be seen in the Figure 15 that as the ratio of L_u/W_u increases, discharge capacity also increases.

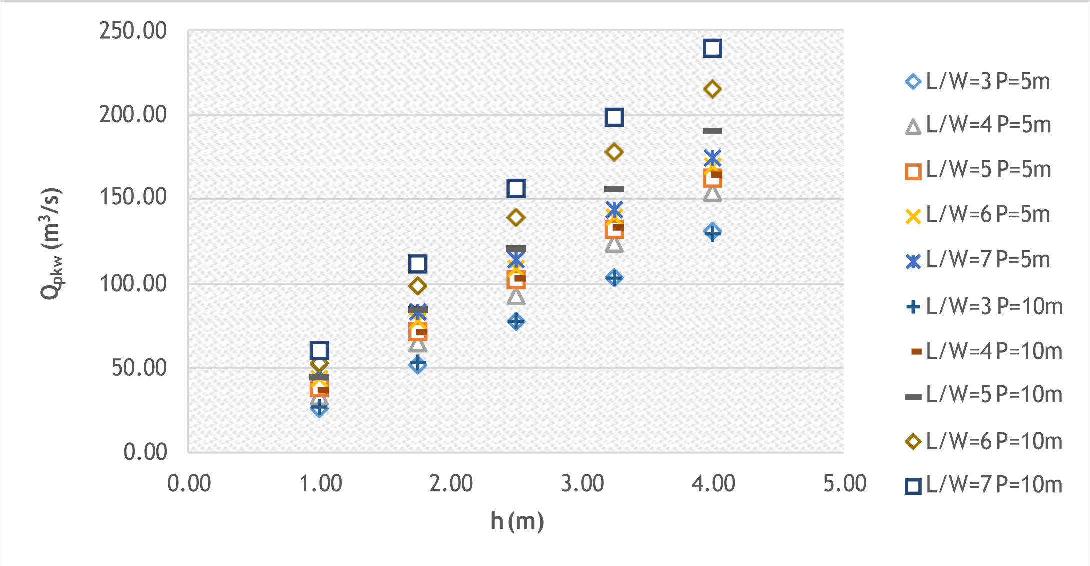

Figure 15: Head-Discharge Relation of the Models for the Investigation of the Effect of the Crest Length

It can be concluded from Figure 16 that for a given weir height, P, as the L_u/W_u increases, the discharge efficiency of a Piano Key weir increases. However, the discharge efficiency decreases with the increase of head over weir. It was stated by Ouamane (2011) that this phenomenon is more significant for the weirs with large values of the ratio of the elongation of the crest, L_u.

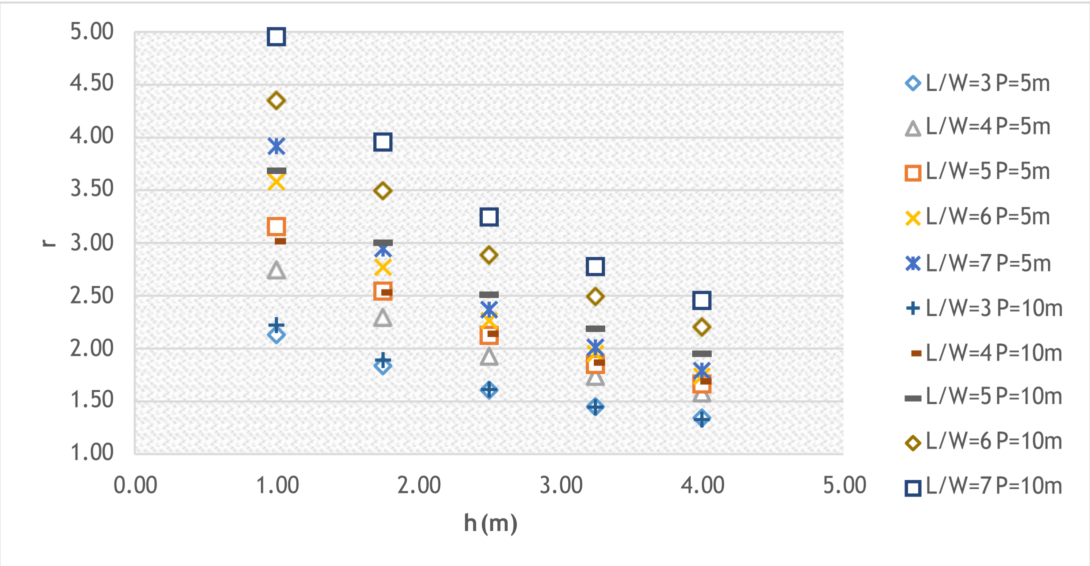

Figure 16: Head-Discharge Efficiency Relation of the Models for the Investigation of the Effect of the Crest Length

It was also found that the rate of increase of discharge values are not same for the models with the same L_u/W_u ratios for different Piano Key weir heights, P. For example, the rate of the increase of the discharge values of L_u/W_u =3 and L/W=4 for the models with weir height P=5 m is in the range between 17% and 29% for all head over weirs. On the other hand, these values are in the range between 27% and 36% for the other model with weir height P=10 m. The increase of the rate of the discharge values decreases as the ratio of L_u/W_u increases, however, the increase of the rate of the discharge values of the models with P=10 m is still greater than the models with P=5 m. It should be noted that at small water depths, increasing L_u/W_u ratio results in a larger percent of increase in the discharge. On the other hand, at larger flow depths, increase percentage is smaller.

4.1.3. Investigation of the Effect of the Weir Width

An investigation is performed to understand the effect of the relation of the inlet and outlet key widths, W_i and W_o. 11 models have been developed with constant weir height (P=5 m), unit weir width (W_u=5.80 m), symmetrical overhang lengths (B_i=B_o=2.90 m), constant weir length (B=11.60 m) and consequently constant developed length along the crest of unit Piano Key weir (L_u=29 m). All the results of 11 models for investigating the effect of the ratio of inlet and outlet key widths, W_i/W_o, are shown in Tables 28-38 below.

Table 28: Discharge Values for the Investigation of the Effect of the Weir Width for Model 2

Model	h (m)	h/P	W_i (m)	W_o (m)	W_i/W_o	Q_{in} %	Q_{out} %	Q_{side} %	Q_{pkw} (m^3/s)
Model 2	4.00	0.80	2.50	2.50	1.00	20.58	25.21	54.21	162.46
	3.25	0.65	2.50	2.50	1.00	19.83	23.13	57.04	132.10
	2.50	0.50	2.50	2.50	1.00	19.15	20.72	60.13	102.44
	1.75	0.35	2.50	2.50	1.00	16.11	17.46	66.43	71.75
	1.00	0.20	2.50	2.50	1.00	13.21	14.19	72.60	38.47

Table 29: Discharge Values for the Investigation of the Effect of the Weir Width for Model 12

Model	**h (m)**	**h/P**	**W_i (m)**	**W_o (m)**	**W_i/W_o**	**Q_{in} %**	**Q_{out} %**	**Q_{side} %**	**Q_{pkw} (m^3/s)**
Model 12	4.00	0.80	2.62	2.38	1.10	21.88	23.21	54.91	163.01
	3.25	0.65	2.62	2.38	1.10	20.50	21.25	58.25	131.59
	2.50	0.50	2.62	2.38	1.10	19.27	18.89	61.84	102.18
	1.75	0.35	2.62	2.38	1.10	17.02	16.08	66.90	72.22
	1.00	0.20	2.62	2.38	1.10	13.75	11.70	74.55	39.26

Table 30: Discharge Values for the Investigation of the Effect of the Weir Width for Model 13

Model	**h (m)**	**h/P**	**W_i (m)**	**W_o (m)**	**W_i/W_o**	**Q_{in} %**	**Q_{out} %**	**Q_{side} %**	**Q_{pkw} (m^3/s)**
Model 13	4.00	0.80	2.73	2.27	1.20	19.86	22.86	57.28	169.29
	3.25	0.65	2.73	2.27	1.20	19.35	20.75	59.91	136.90
	2.50	0.50	2.73	2.27	1.20	19.16	18.40	62.44	106.14
	1.75	0.35	2.73	2.27	1.20	17.02	15.59	67.39	74.26
	1.00	0.20	2.73	2.27	1.20	13.95	12.52	73.52	39.74

Table 31: Discharge Values for the Investigation of the Effect of the Weir Width for Model 14

Model	**h (m)**	**h/P**	**W_i (m)**	**W_o (m)**	**W_i/W_o**	**Q_{in} %**	**Q_{out} %**	**Q_{side} %**	**Q_{pkw} (m^3/s)**
Model 14	4.00	0.80	2.83	2.17	1.30	21.11	20.86	58.03	166.93
	3.25	0.65	2.83	2.17	1.30	22.57	19.07	58.37	133.07
	2.50	0.50	2.83	2.17	1.30	20.91	16.90	62.20	103.91
	1.75	0.35	2.83	2.17	1.30	18.08	14.26	67.66	73.89
	1.00	0.20	2.83	2.17	1.30	14.60	11.52	73.88	39.91

Table 32: Discharge Values for the Investigation of the Effect of the Weir Width for Model 15

Model	**h (m)**	**h/P**	**W_i (m)**	**W_o (m)**	**W_i/W_o**	**Q_{in} %**	**Q_{out} %**	**Q_{side} %**	**Q_{pkw} (m^3/s)**
Model 15	4.00	0.80	2.92	2.08	1.40	26.28	20.57	53.15	163.18
	3.25	0.65	2.92	2.08	1.40	24.06	18.76	57.18	133.66
	2.50	0.50	2.92	2.08	1.40	21.99	16.56	61.45	104.80
	1.75	0.35	2.92	2.08	1.40	18.29	13.91	67.80	74.60
	1.00	0.20	2.92	2.08	1.40	14.69	11.19	74.13	40.36

Table 33: Discharge Values for the Investigation of the Effect of the Weir Width for Model 16

Model	h (m)	h/P	W_i (m)	W_o (m)	W_i/W_o	Q_{in} %	Q_{out} %	Q_{side} %	Q_{pkw} (m^3/s)
Model 16	4.00	0.80	3.00	2.00	1.50	23.10	20.47	56.42	166.61
	3.25	0.65	3.00	2.00	1.50	21.93	18.56	59.50	135.53
	2.50	0.50	3.00	2.00	1.50	21.12	16.42	62.46	105.12
	1.75	0.35	3.00	2.00	1.50	18.41	13.87	67.72	75.56
	1.00	0.20	3.00	2.00	1.50	14.67	10.96	74.36	40.82

Table 34: Discharge Values for the Investigation of the Effect of the Weir Width for Model 17

Model	h (m)	h/P	W_i (m)	W_o (m)	W_i/W_o	Q_{in} %	Q_{out} %	Q_{side} %	Q_{pkw} (m^3/s)
Model 17	4.00	0.80	3.08	1.92	1.60	28.86	18.82	52.33	160.63
	3.25	0.65	3.08	1.92	1.60	26.15	17.11	56.74	131.21
	2.50	0.50	3.08	1.92	1.60	23.08	15.05	61.87	103.12
	1.75	0.35	3.08	1.92	1.60	18.14	12.55	69.31	75.28
	1.00	0.20	3.08	1.92	1.60	15.40	9.97	74.63	40.99

Table 35: Discharge Values for the Investigation of the Effect of the Weir Width for Model 18

Model	h (m)	h/P	W_i (m)	W_o (m)	W_i/W_o	Q_{in} %	Q_{out} %	Q_{side} %	Q_{pkw} (m^3/s)
Model 18	4.00	0.80	3.15	1.85	1.70	26.53	18.48	54.99	165.80
	3.25	0.65	3.15	1.85	1.70	23.12	16.82	60.06	137.44
	2.50	0.50	3.15	1.85	1.70	21.83	14.71	63.46	107.05
	1.75	0.35	3.15	1.85	1.70	19.18	12.53	68.29	76.73
	1.00	0.20	3.15	1.85	1.70	15.53	9.91	74.56	40.86

Table 36: Discharge Values for the Investigation of the Effect of the Weir Width for Model 19

Model	h (m)	h/P	W_i (m)	W_o (m)	W_i/W_o	Q_{in} %	Q_{out} %	Q_{side} %	Q_{pkw} (m^3/s)
Model 19	4.00	0.80	3.21	1.79	1.80	27.12	18.49	54.39	165.42
	3.25	0.65	3.21	1.79	1.80	23.56	16.66	59.78	135.32
	2.50	0.50	3.21	1.79	1.80	23.37	14.72	61.91	105.82
	1.75	0.35	3.21	1.79	1.80	19.80	12.30	67.90	75.39
	1.00	0.20	3.21	1.79	1.80	15.52	9.66	74.82	41.14

Table 37: Discharge Values for the Investigation of the Effect of the Weir Width for Model 20

Model	h (m)	h/P	W_i (m)	W_o (m)	W_i/W_o	Q_{in} %	Q_{out} %	Q_{side} %	Q_{pkw} (m^3/s)
Model 20	4.00	0.80	3.28	1.72	1.90	31.48	16.83	51.68	161.66
	3.25	0.65	3.28	1.72	1.90	29.70	15.31	54.99	130.64
	2.50	0.50	3.28	1.72	1.90	25.59	13.35	61.06	102.50
	1.75	0.35	3.28	1.72	1.90	21.05	11.12	67.82	73.85
	1.00	0.20	3.28	1.72	1.90	16.37	8.77	74.87	41.37

Table 38: Discharge Values for the Investigation of the Effect of the Weir Width for Model 21

Model	h (m)	h/P	W_i (m)	W_o (m)	W_i/W_o	Q_{in} %	Q_{out} %	Q_{side} %	Q_{pkw} (m^3/s)
Model 21	4.00	0.80	3.33	1.67	2.00	34.18	16.51	49.31	157.97
	3.25	0.65	3.33	1.67	2.00	30.65	14.97	54.38	129.68
	2.50	0.50	3.33	1.67	2.00	26.39	13.11	60.50	102.15
	1.75	0.35	3.33	1.67	2.00	21.52	11.00	67.47	74.37
	1.00	0.20	3.33	1.67	2.00	16.53	8.61	74.86	40.79

Head-discharge relation for different weir width ratios, W_i/W_o, can be seen in Figure 17. It can be said that the effect of the weir width ratio W_i/W_o on the total discharge changes with respect to the incoming water depth. For small incoming water depths W_i/W_o ratio has nearly no effect on the discharge capacity. Lefebvre et al. (2013) also stated that the optimal value for the ratio W_i/W_o is lower for small incoming heads. It was also found that discharge capacity increases for large water depths, as W_i/W_o decreases into 1. This phenomenon agrees with the findings of Pralong et al. (2011). It was before mentioned in their study that the optimal ratio becomes lower for higher heads.

It is important to note that for the design of Piano Key weirs, it is the same cost to increase the inlet width, W_i and decrease the outlet width, W_o. However, it was clarified by Machiels et al. (2012d) that too narrow outlets may be unable to evacuate the flow under supercritical conditions and result in the decrease of the discharge efficiency so that selection of the ratio of Wi/Wo as 1 appears good because of playing minor roles on the discharge capacity considering economic and hydraulic conditions.

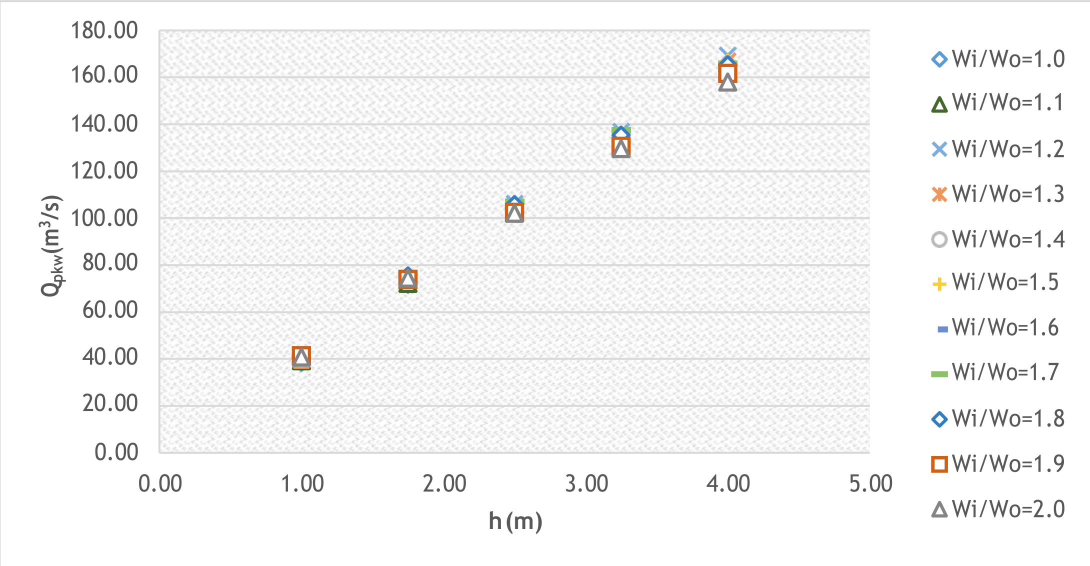

Figure 17: Head-Discharge Relation of the Models for the Investigation of the Effect of the Weir Width

Water depth-discharge efficiency curve for various W_i/W_o ratios are also given Figure 18. As it can be seen from this figure, there is no correlation in between discharge efficiency and W_i/W_o ratio.

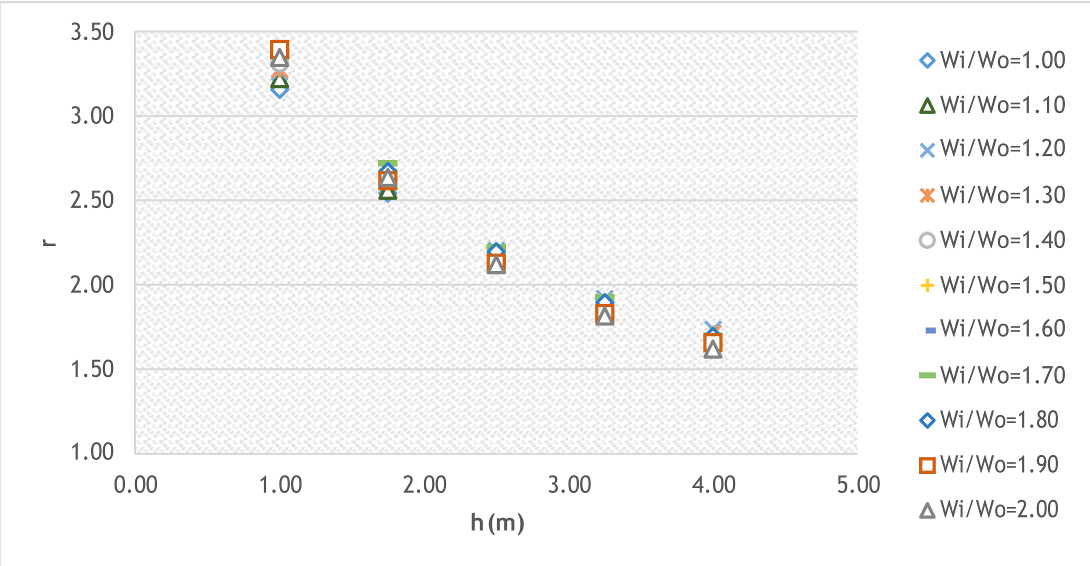

Figure 18: Head-Discharge Efficiency Relation of the Models for the Investigation of the Effect of the Weir Width

4.1.4. Investigation of the Effect of the Overhang Lengths

The results of the investigation of the overhang lengths, B_i and B_o and their effect on the discharge capacity of the Piano Key weir is investigated with 8 models with varying weir heights, P, as it can be seen in Table 7. Models have been generated with constant unit weir width (W_u=5.80 m), constant weir length (B=11.60 m) and consequently constant developed length along the crest of Piano Key weir unit (L_u=29 m). Discharge capacities obtained for different water heads from the results of 8 models for the investigation of the effect of the overhang lengths, B_i and B_o, are shown in Tables 39-46 below.

Table 39: Discharge Values for the Investigation of the Effect of the Overhang Lengths for Model 2

Model	h	h/P	B_i (m)	B_o (m)	B_i/B_o	B (m)	Q_{in} (%)	Q_{out} (%)	Q_{side} (%)	Q_{pkw} (m^3/s)
Model 2	4.00	0.80	2.90	2.90	1.00	11.60	20.58	25.21	54.21	162.46
	3.25	0.65	2.90	2.90	1.00	11.60	19.83	23.13	57.04	132.10
	2.50	0.50	2.90	2.90	1.00	11.60	19.15	20.72	60.13	102.44
	1.75	0.35	2.90	2.90	1.00	11.60	16.11	17.46	66.43	71.75
	1.00	0.20	2.90	2.90	1.00	11.60	13.21	14.19	72.60	38.47

Table 40: Discharge Values for the Investigation of the Effect of the Overhang Lengths for Model 6

Model	h	h/P	B_i (m)	B_o (m)	B_i/B_o	B (m)	Q_{in} (%)	Q_{out} (%)	Q_{side} (%)	Q_{pkw} (m^3/s)
Model 6	4.00	0.80	1.93	3.87	0.50	11.60	21.77	23.84	54.39	166.61
	3.25	0.65	1.93	3.87	0.50	11.60	20.50	21.74	57.76	136.61
	2.50	0.50	1.93	3.87	0.50	11.60	19.15	19.14	61.71	106.30
	1.75	0.35	1.93	3.87	0.50	11.60	16.38	16.13	67.49	74.89
	1.00	0.20	1.93	3.87	0.50	11.60	12.93	12.95	74.12	40.36

Table 41: Discharge Values for the Investigation of the Effect of the Overhang Lengths for Model 7

Model	h	h/P	B_i (m)	B_o (m)	B_i/B_o	B (m)	Q_{in} (%)	Q_{out} (%)	Q_{side} (%)	Q_{pkw} (m^3/s)
Model 7	4.00	0.80	1.45	4.35	0.33	11.60	22.58	23.14	54.28	168.38
	3.25	0.65	1.45	4.35	0.33	11.60	20.81	20.95	58.24	137.88
	2.50	0.50	1.45	4.35	0.33	11.60	18.97	18.32	62.71	107.56
	1.75	0.35	1.45	4.35	0.33	11.60	16.45	15.52	68.02	76.96
	1.00	0.20	1.45	4.35	0.33	11.60	12.63	12.36	75.00	41.33

Table 42: Discharge Values for the Investigation of the Effect of the Overhang Lengths for Model 8

Model	h	h/P	B_i (m)	B_o (m)	B_i/B_o	B (m)	Q_{in} (%)	Q_{out} (%)	Q_{side} (%)	Q_{pkw} (m^3/s)
Model 8	4.00	0.80	0.00	5.80	0.00	11.60	24.77	21.46	53.76	169.37
	3.25	0.65	0.00	5.80	0.00	11.60	22.36	19.30	58.34	138.78
	2.50	0.50	0.00	5.80	0.00	11.60	19.82	16.85	63.33	108.65
	1.75	0.35	0.00	5.80	0.00	11.60	16.81	14.02	69.17	77.29
	1.00	0.20	0.00	5.80	0.00	11.60	12.68	11.32	76.00	42.39

Table 43: Discharge Values for the Investigation of the Effect of the Overhang Lengths for Model 4

Model	h	h/P	B_i (m)	B_o (m)	B_i/B_o	B (m)	Q_{in} (%)	Q_{out} (%)	Q_{side} (%)	Q_{pkw} (m^3/s)
Model 4	4.00	0.40	2.90	2.90	1.00	11.60	23.17	23.12	53.72	190.28
	3.25	0.33	2.90	2.90	1.00	11.60	22.05	20.92	57.02	156.01
	2.50	0.25	2.90	2.90	1.00	11.60	18.82	18.81	62.37	120.81
	1.75	0.18	2.90	2.90	1.00	11.60	15.63	16.08	68.29	84.69
	1.00	0.10	2.90	2.90	1.00	11.60	12.46	12.88	74.66	44.88

Table 44: Discharge Values for the Investigation of the Effect of the Overhang Lengths for Model 9

Model	h	h/P	B_i (m)	B_o (m)	B_i/B_o	B (m)	Q_{in} (%)	Q_{out} (%)	Q_{side} (%)	Q_{pkw} (m^3/s)
Model 9	4.00	0.40	1.93	3.87	0.50	11.60	23.20	22.44	54.35	193.81
	3.25	0.33	1.93	3.87	0.50	11.60	21.69	20.35	57.96	159.38
	2.50	0.25	1.93	3.87	0.50	11.60	17.90	18.52	63.58	124.80
	1.75	0.18	1.93	3.87	0.50	11.60	15.15	15.65	69.20	87.41
	1.00	0.10	1.93	3.87	0.50	11.60	12.29	12.51	75.20	45.85

Table 45: Discharge Values for the Investigation of the Effect of the Overhang Lengths for Model 10

Model	h	h/P	B_i (m)	B_o (m)	B_i/B_o	B (m)	Q_{in} (%)	Q_{out} (%)	Q_{side} (%)	Q_{pkw} (m^3/s)
Model 10	4.00	0.40	1.45	4.35	0.33	11.60	23.28	22.08	54.64	195.92
	3.25	0.33	1.45	4.35	0.33	11.60	21.25	20.22	58.53	161.90
	2.50	0.25	1.45	4.35	0.33	11.60	17.66	18.11	64.23	126.39
	1.75	0.18	1.45	4.35	0.33	11.60	14.89	15.62	69.49	88.77
	1.00	0.10	1.45	4.35	0.33	11.60	12.11	12.36	75.53	46.60

Table 46: Discharge Values for the Investigation of the Effect of the Overhang Lengths for Model 11

Model	h	h/P	B_i (m)	B_o (m)	B_i/B_o	B (m)	Q in (%)	Q out (%)	Q side (%)	Q pkw (m^3/s)
Model 11	4.00	0.40	0.00	5.80	0.00	11.60	23.00	21.20	55.80	199.90
	3.25	0.33	0.00	5.80	0.00	11.60	19.42	19.39	61.19	164.47
	2.50	0.25	0.00	5.80	0.00	11.60	16.85	17.25	65.90	129.53
	1.75	0.18	0.00	5.80	0.00	11.60	14.21	14.60	71.19	91.42
	1.00	0.10	0.00	5.80	0.00	11.60	11.90	11.92	76.18	47.65

The results show that for a given weir height, P, and flow depth as the ratio of B_i/B_o decreases, discharge capacity of a Piano Key weir increases. It is shown in Figure 19 that highest discharge values are obtained as the ratio of B_i/B_o becomes zero, regardless of the Piano Key weir height, P. This conclusion agrees with the findings that Lempérière and Ouamane (2006). In their study, it was found that the model without downstream overhangs, B_i, showed higher discharge efficiency than the models with downstream overhangs, B_i. This phenomenon can be explained with flow contractions and energy loss. As the length of the upstream overhang, B_o, increases, the area and wetted perimeter in the inlet key increases which results in an increase in the discharge capacity because of decreasing the flow velocities, flow contractions and energy loss.

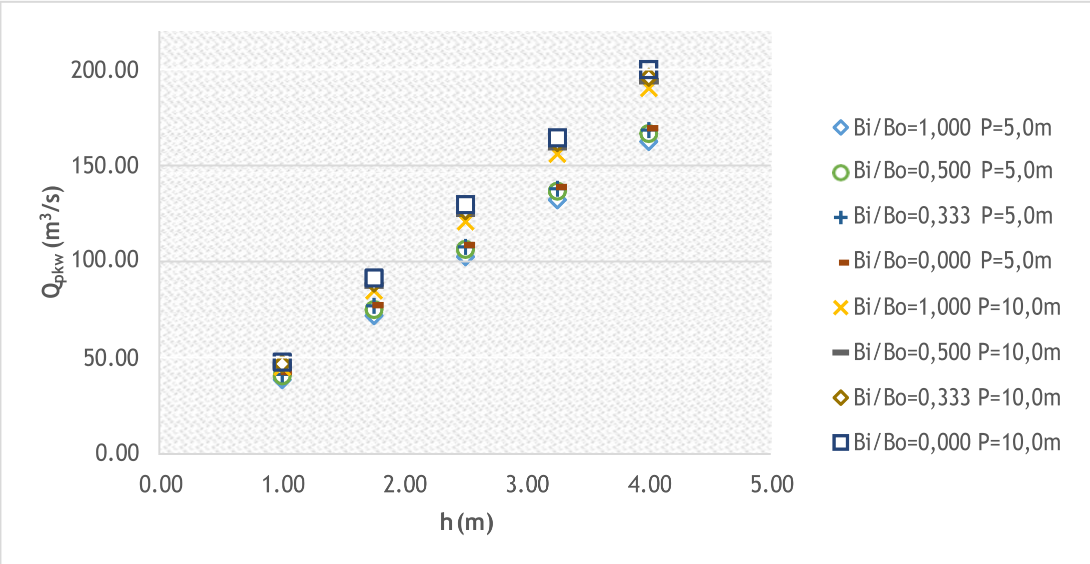

Figure 19: Head-Discharge Relation of the Models for the Investigation of the Effect of the Overhang Lengths

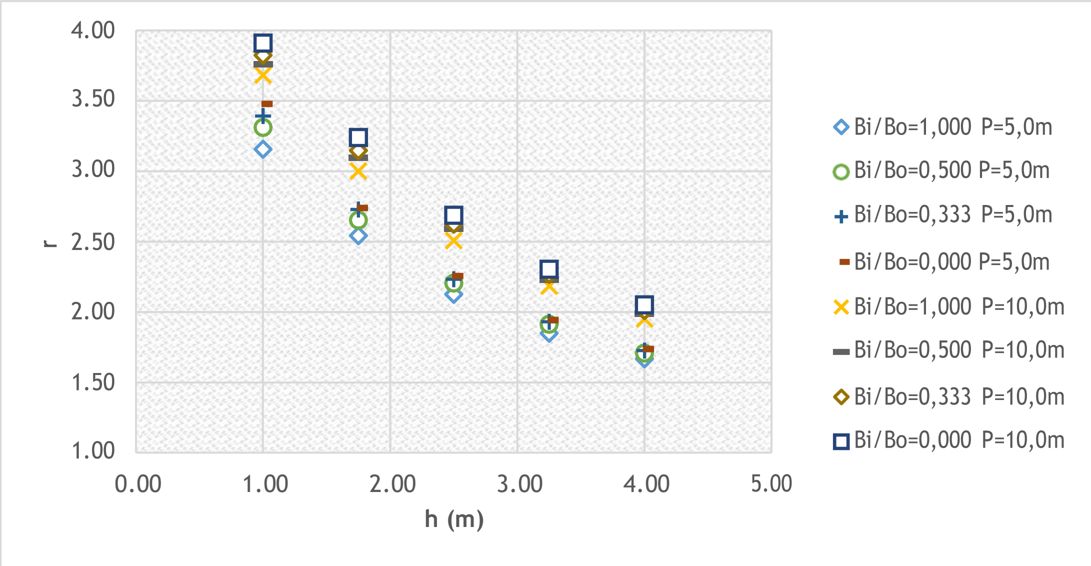

Figure 20: Head-Discharge Efficiency Relation of the Models for the Investigation of the Effect of the Overhang Lengths

For a given weir height, P, the rate of change between the maximum and minimum discharge values for different B_i/B_o ratios ranges between 5% to 10%. The figures for the relation of discharge efficiencies and water heads for all the models can be seen in Figures 20 above. It can be said that for a given weir height, P, as the B_i/B_o ratio decreases, discharge efficiency slightly increases. It was before mentioned by Pralong et al (2011) that the ratio of the lengths of the upstream and downstream overhangs, B_o and B_i are not sensitive parameter because of the insignificant gains on the discharge.

CHAPTER 5

DESIGN PROCESS

5.1. Design of the Piano Key Weirs

In the present study, important parameters that have an influence on the discharge capacity of Piano Key weirs have been investigated separately to develop a design process for Piano Key weirs. It was concluded that the weir height, P, and the developed length along the crest of a Piano Key weir unit, L_u, are the major parameters on the discharge capacity. At the same time, the effects of the ratio of the inlet and outlet key widths, W_i/W_o, and the ratio of the inlet and outlet overhang lengths, B_i/B_o, were found to play minor roles on the discharge capacity so that these parameters can be considered as insignificant on the design phase.

In the design stage, the head over weir, h, must be fixed as an operational condition. The discharge expected to surpass over the crest of a Piano Key weir, Q_{pkw} is known from the hydrological analysis. The width of the spillway on which the Piano Key weir units will be installed, W, is also known. Therefore, the discharge passing through the spillway can be calculated by considering a standard sharp crested weir.

In the present study, relations of discharge efficiency with the weir height, P, and the developed length along the crest of a Piano Key weir unit, L_u, were generated for dimensionless head over weir, h/P and h/L_u, and these relations can be observed in Figure 21 and Figure 22. These figures were obtained by excluding Model 1, Model 26 and Model 29 from the total 29 models previously calculated. The reason for excluding these 3 models is that they represent extreme conditions which are not practical. Model 1 was investigated with a very short weir height, P, and subjected to a large head over weir relative to its height. Model 26 was investigated with a large ratio of P/B and Model 29 was investigated with the high ratios of L_u/W_u and P/W_u.

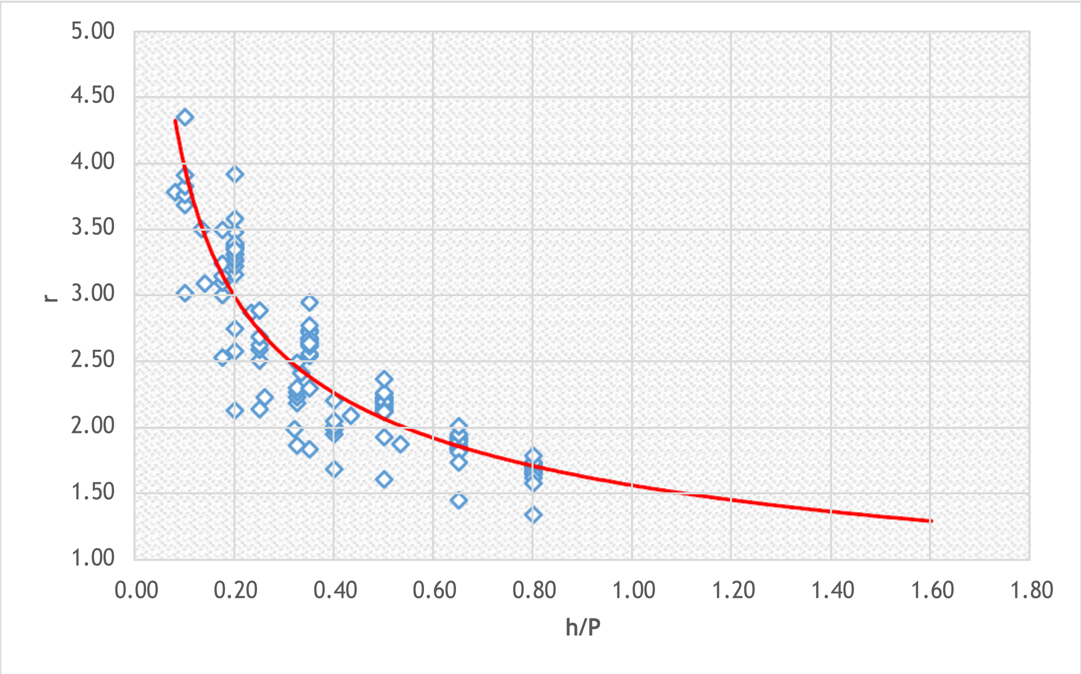

Figure 21: Design Phase for the Selection of the Height of a Piano Key Weir

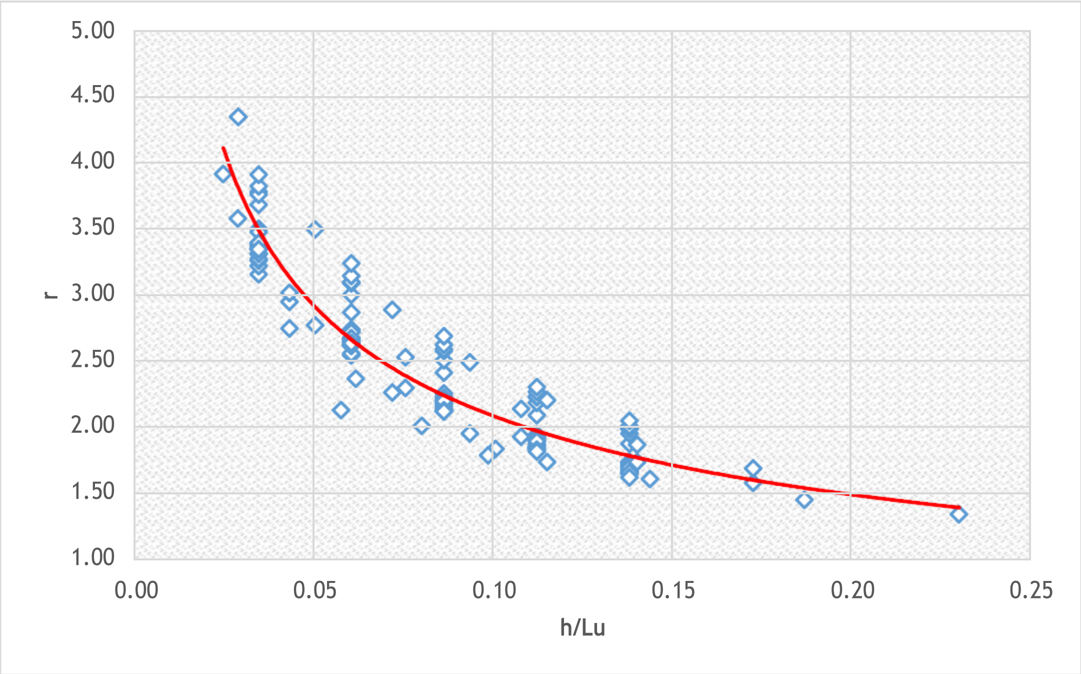

Figure 22: Design Phase for the Selection of the Developed Length along the Crest of a Piano Key Weir Unit

As it can be seen in Figure 21 and Figure 22 above, there is an exponential relationship for both h/P and h/L_u values with the values of discharge efficiency. The discharge efficiency can be directly calculated as an input value by obtaining the ratio of the discharge expected to surpass over the weir crest of Piano Key weir and the discharge of a standard sharp crested weir with the same total width, W. Then, the weir height of a Piano Key weir, P, and the developed length along the crest of a Piano Key weir unit, L_u, can be obtained from Figure 21 and Figure 22 above with the knowledge of the head over weir, h. After obtaining both weir height, P, and the developed length along the crest of a Piano Key weir, L_u, the total number of unit Piano Key weir, N, is assumed to calculate the unit width of weir, W_u. The decision for selecting the total number of unit Piano Key weir, N, effects the Piano Key sizes and the total concrete volume of Piano Key weirs. With the same total weir width, W, and unit crest length along the crest, L_u, the length of the Piano Key weir, B, differs as the unit weir width changes, W_u. So that, the total number of Piano Key weir units, N, is also an input for this design process. Finally, all the other geometrical values can be calculated by deciding the ratios of W_i/W_o and B_i/B_o.

Table 47: Example of the Design Process for Piano Key Weirs

h (m)	W (m)	Qpkw (m^3/s)	Qscw (m^3/s)	r	Lu (m)	P (m)	Nu	Concrete Volume (m^3)	Q com (m^3/s)	Difference (%)
1.80	150.00	1500.00	761.19	1.97	15.99	3.20	24	597.82	1201.06	-19.93
							30	695.90	1318.12	-12.13
							36	777.31	1374.48	-8.37
1.50	150.00	1500.00	579.06	2.59	23.42	5.26	24	1628.86	1395.02	-7.00
							30	1829.33	1407.74	-6.15
							36	2002.39	1462.10	-2.53
1.20	150.00	1500.00	414.34	3.62	37.35	9.68	24	5223.16	1773.12	18.21
							30	5726.89	1932.34	28.82
							36	6180.19	2054.16	36.94

An example investigation of the design process for the present study was done. It can be seen in Table 47 that 3 different cases were studied for testing the design process generated in the present study. The design discharge value was represented as Q_{pkw}, the sharp crested weir discharge value as Q_{scw}, and the discharge value found from numerical solution was represented as Q_{com}. The total width, W, and the design discharge value for Piano Key weirs, Q_{pkw}, for the design process were selected as 150 m and 1500 m^3/s for these 3 cases with head over weirs, h, 1.80 m, 1.50 m and 1.20 m. The discharge value for a sharp crested weir, Q_{scw}, is calculated from Gharahjeh et al. (2015) and the expected discharge efficiency value, r, can be obtained. Then, the values of the weir height, P, and the developed crest length along the crest of unit Piano Key weir, L_u, can be calculated from Figure 21 and Figure 22 above. Thereafter, the total number of Piano Key weir units, N, is decided to define the unit weir width, W_u, and the length of Piano Key Weir, B. It can be concluded from Table 47 that as the total number of Piano Key weir unit, N, increases, the total concrete volume of Piano Key weir increases. However, it can be also concluded that discharge values of the computations, Q_{com}, increases as the number of Piano Key weir units, N, increases. This phenomenon is because of the increase of the total developed crest length along the crest, L, as the total number of Piano Key weir units increase.

A design process for Piano Key weir were also studied by Karaeren and Bozkuş (2015). In their study, different total heads and Wi/Wo ratios were studied to see the possible alterations in the discharge capacity.

CHAPTER 6

CONCLUSION

6.1. Summary and Conclusions

The main goal of this study is to investigate the main parameters controlling the discharge capacity of Piano Key weirs to generate a generalized design procedure for this new type of labyrinth weirs. Therefore, a sum of 29 models were generated to investigate the major dimensionless parameters which were found in the previous studies that have influence on the discharge capacity of Piano Key weirs. These major parameters are the dimensionless ratios of L_u/W_u, P/W_u, W_i/W_o and B_i/B_o.

It was confirmed that the ratios L_u/W_u and P/W_u have a strong influence on the discharge capacity of Piano Key weir. However, the ratios of W_i/W_o and B_i/B_o were found as parameters which have minor influences on the discharge capacity.

The dimensionless ratio of P/W_u was studied with 5 models. It was confirmed that as the weir height, P, increases, the discharge capacity increases. However, it was found that the rate of the increase of the discharge values decreases as the weir height, P, increases. Also, it was noted that as the weir height, P, increases, the total cost of an entire Piano Key weir unit increases with the same rate. Therefore, it is important to select the most appropriate Piano Key weir height, P, to minimize the construction cost.

The study for investigating the effect of the dimensionless ratio of L_u/W_u was studied with 10 models with varying weir heights, P. It was confirmed that the developed crest length along the Piano Key weir, L, has a strong influence on the discharge capacity of the Piano Key weir. It was found that as the ratio of L_u/W_u increases, the discharge capacity of Piano Key weir increases rapidly as expected. One important conclusion is related to the increase in the rate of the discharge for different Piano Key weir heights, P. It was seen that the increase in the rate of the discharge capacity for models with greater Piano Key weir height, P, is greater than the models with less Piano Key weir height, P, with the same ratio of L_u/W_u.

The ratio of W_i/W_o was investigated with 11 models. It was found that the relation between the discharge efficiency, r, and the ratio of W_i/W_o has no correlation at all. It was concluded that the effect of the ratio of W_i/W_o has a minor influence on the discharge capacity of Piano Key weir. It was seen that as the ratio W_i/W_o decreases, the discharge efficiency, r, increases slightly for large head over weirs, h, relatively 4%.

The investigation for understanding the effect of the ratio of B_i/B_o was studied with 8 models with varying Piano Key weir heights, P. It was confirmed that the models without downstream overhangs, B_i, showed higher discharge efficiency than the models with downstream overhangs, B_i. However, the effect of the ratio of B_i/B_o on the discharge capacity of Piano Key weirs was also found as playing minor roles.

The design process for the Piano Key weirs were studied after the investigation of the models created according to the dimensionless parameters summarized above. The head over weir, h, and the weir width of a linear weir, W, are known data. With the knowledge of the values of the head over weir, h, and the weir width, W, the discharge capacity of a linear weir intended to be replaced by a Piano Key weir can be calculated. It is also known that the value of the design discharge of the Piano Key weir is obtained by a hydrological analysis. Therefore, the discharge efficiency, r, for a certain design process can be calculated. It was found that there is an exponential relationship for the relation of the discharge efficiency, r, and the ratios of h/P and h/L_u. Therefore, Figure 21 and Figure 22 were formed to define the two of the most important geometrical components, the Piano Key weir height, P, and the developed length along the crest of unit Piano Key weir, L_u. Then, the total number of Piano Key weir units, N, can be selected to define all the geometrical parameters of a Piano Key weir to complete the design process. The design process generalized in the present study was investigated for verification of the results. 9 models with the same weir widths, W, and varying head over weir, h, were created to be investigated. It was calculated that the range of the differences between the calculated discharge values and the design discharge values for the design processes were found between 18% to 37% for the chosen efficiency of 3.67. In the same way, the differences were found between -9% to -20% for the chosen efficiency which of 1.97. Nevertheless, the difference between the discharges were found in the range between 2% to 8% for the discharge efficiency of 2.59. It was observed that for the efficiency values less than or equal to 2.59, as the number of Piano Key weir units, N, increases, error decreases and for larger efficiency value of 3.12, as the number of Piano Key weir units, N, increases, error increases.

As a conclusion, this example study reveals the design procedure for an assumed design discharge, head over weir, crest length of spillway. It is important to note that this study investigated the main parameters that have major influences on the discharge capacity. For future investigations, the effect of the ratio of L_u/W_u can be investigated with more models with varying weir heights and the other parameters such as parapet walls, noses built under upstream overhangs, etc.

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