AN ANALYSIS OF THE OPPORTUNITIES TO LEARN AFFORDED BY THE TASKS IN A NINTH GRADE TURKISH MATHEMATICS TEXTBOOK

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ABSTRACT

AN ANALYSIS OF THE OPPORTUNITIES TO LEARN AFFORDED BY THE TASKS IN A NINTH GRADE TURKISH MATHEMATICS TEXTBOOK

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The purpose of this study was to investigate the learning opportunities afforded in the tasks related to the number and quantity in a ninth-grade mathematics textbook used throughout Turkey by the majority of the students. The analysis of the tasks was guided by the PISA 2012 Mathematics Framework in terms of the process capabilities needed to solve tasks, contextualization, and proficiency levels required to solve the tasks. Analysis of the 400 tasks related to the number and quantity revealed that while about 54% of them included the process of employing mathematical concepts, about 44% of the tasks were about formulating situations mathematically. Tasks about the process of interpreting mathematical outcomes were found to be rare, only about 3%. The results also revealed that about 75% of the tasks were decontextualized, which means that they included mathematical structures only without depicting a real-life situation. The quantity questions were also analyzed according to the six proficiency levels described in the PISA 2012 Mathematics Framework. The questions were mostly level 2 and level 3 types implying that students were expected to make direct inferences based on familiar contexts, carry out procedures and calculations that were clearly described in the problem context, and select and apply problem-solving strategies that were not needed to insight to choose the problem-solving strategy. The findings of the study revealed about Turkish students' learning opportunities afforded by the mathematics textbooks that have important implications for teachers and textbook writers for the use of different types of tasks in order to provide better learning opportunities in mathematics for the student.

Keywords: Opportunity to Learn, Textbook Analysis, Task Analysis, Mathematical Processes, Context, Proficiency Levels, Number and Quantity, Programme for International Student Assessment (PISA)

ÖZ

TÜRKİYE 9. SINIF MATEMATİK DERS KİTABINDAKİ SORULARIN SAĞLADIĞI ÖĞRENME FIRSATLARININ BİR ANALİZİ

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Bu çalışmanın amacı Türkiye'de 9. sınıf öğrencileri tarafından kullanılan bir matematik ders kitabındaki sayı ve nicelik konularında yer verilen soruların sağladığı öğrenme fırsatlarını araştırmaktır. Kitaptaki sorular analiz edilirken PISA 2012 Matematik Çerçevesi kullanılarak soruların bağlamsallaştırılması, soruları çözmek için gerekli süreç becerileri ve yeterlilik seviyeleri dikkate alınmıştır. Analiz sonuçlarına göre sayılar ve nicelik konularında kitapta yer verilen 400 sorunun yüzde 54'ü matematiksel kavramları uygulama sürecini, yüzde 44 kadarı ise matematiksel durumları formüle etme sürecini içermektedir. Ayrıca, matematiksel sonuçları yorumlama sürecini içeren soruların sayısının çok az (tüm soruların %3 kadarı) olduğu, soruların yüzde 75'inin bağlamsallaştırılmamış yani sadece matematiksel yapıları içerip herhangi bir gerçek yaşam durumunu temsil etmeyen sorular olduğu bulunmuştur. PISA 2012 Matematik Çerçevesi'nde açıklanan altı yeterlilik düzeyi göz önüne alınarak yapılan analiz sonucunda ise sayı ve nicelik konularındaki soruların genellikle öğrencilerden bilinen durumlarla ilgili doğrudan çıkarımlar yapması ve problem içeresinde açıkça tanımlanmış prosedürler ve hesaplamaları yapması beklenen, öngörü gerektirmeyen problem çözme stratejilerini seçmesi ve uygulaması istenen seviye 2 ve seviye 3 türünde sorular olduğu görüldü. Çalışmanın bulguları

genel olarak Türkiye'deki ders kitaplarının öğrencilere sunduğu öğrenme fırsatları ile ilgili bilgi vermektedir. Bulgular öğretmenlerin ve ders kitabı yazarlarının ders kitaplarındaki sorularda sağlanan öğrenme fırsatları konusunda daha dikkatli olmaları gerektiğini işaret etmektedir.

Anahtar Kelimeler: Öğrenme Fırsatları, Ders Kitapları, Kitap Analizi, Soru Analizi, Süreç, Bağlam, Yeterlilik Seviyesi, Sayı ve Nicelik, PISA

To my family…

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ABBREVIATIONS

TIMMS: Trends in International Mathematics and Science Study

PIRLS: Progress in International Reading Literacy Study

PISA: Programme for International Student Assessment

OTL: Opportunity to Learn

MoNE: Ministry of National Education

FIMMS: First International Mathematics Study

MEG: Mathematics Expert Group

CR: Change and Relations

SS: Space and Shape

UD: Uncertainty and Data

Q: Quantity

F: Frequeny

CHAPTER 1

INTRODUCTION

For the last few decades, international large-scale assessments such as Trends in International Mathematics and Science Study (TIMSS), the Progress in International Reading Literacy Study (PIRLS), and the Programme for International Student Assessment (PISA) have engaged the attention of policymakers, academics and educators (Singer, Braun, & Chudowsky, 2018). The results of these assessments help researchers to examine their education systems and make comparisons among different countries education systems. PISA is one of those international assessments that was held by the participation of thousands of students with diverse backgrounds around the world. The aim of developers of PISA is not only assessing the knowledge of students in specific branches, but also applying different surveys in a variety of contexts that helps to examine students correlated with the society and education system (She, Stacey, & Schmidt, 2018). The test has been applied since 2000 triennially and has three major areas –reading, mathematics, and science – even one of these areas is the major area in each test (PISA, 2012). Besides these major areas, each test has a focus on different aspects of education such as learning opportunities of students or measuring the adolescent well-being (OECD, 2016). The last two tests, applied in 2012 and 2015, chose mathematics and science as the main areas and 'opportunity to learn (OTL)' as one of the focus points (OECD, 2016b). Even though OTL was used by PISA in 2012, the concept was introduced in 1959 with the idea of explaining differences of students' achievements with differences of curricula (Foshay, 1962; Suter, 2017). The first studies were unable to discover a positive relationship between student achievement and opportunity to learn. However, later international comparative studies found a significant relationship between students performances and learning opportunities, which made OTL one of the focus of interests in mathematics education (Schmidt et al., 2008; Törnroos, 2005). The concept, OTL, was used by PISA to clarify how well students from different backgrounds were familiar with the items placed in PISA. This focus led the researchers to work on different national curriculums since it was excepted widely that studying on competencies of curricula across nations could elicit explanations about the variability of students mathematics scores (Stacey & Turner, 2015).

Understanding and explaining the variability of students' scores is a challenging point for many countries and researchers (She et al., 2018) including Turkey (Aydoğdu, Erkan, & Serbest, 2013). Turkey had been attended PISA since 2003, and the results across years were both inconsistent and inadequate (Çelen, Çelik, & Seferoğlu, 2011). Mathematics scores of Turkish students increased from 423 to 448 between the years 2003 and 2012 and decreased to 420 in 2015 (MoNE, 2016). Despite the ascent or descent, none of these variations were statistically meaningful comparing to the OECD average (Anıl, Özkan, & Demir, 2016). Starting from 2004, PISA scores alerted Turkey to reevaluate the education system and curriculum (MoNE, 2005). Mathematics curriculums and textbooks have been revised for a few times to provide better learning opportunities (Urhan & Dost, 2018).Despite the revisions done by taking PISA scores and national education targets into consideration, the results of PISA 2012 and 2015 still alert educators and policymakers about reexamining educational materials, national exams, curriculums, and textbooks, in-depth concerning international standards.

According to 2012 PISA Mathematics Framework, the items in the mathematics test were developed into four main areas which were change and relationships, uncertainty and data, space and shape, and quantity (OECD, 2013). The scores of Turkish students concerning each area were summarized in Table 1.1.

Table 1.1. Scores of Turkey Regarding the Content Areas in PISA 2012 (OECD, 2016a)

Content Areas	Change and Relations	Quantity	Space and Shape	Uncertainty and Data
OECD Average	493	495	490	493
Turkey	448	442	443	447

Looking at Turkey's scores among these content areas, the highest score belongs to change and relations with 448 and the lowest score belongs to quantity with 442. The scores concerning content areas were not given in each PISA assessment. Hence, the detailed scores of PISA 2012 were expected to enlighten researchers until 2021, the year that mathematics will be the focal area. All of the four areas' scores were under the OECD average, but examining the OECD scores, the highest score was in quantity with a score of 495. In PISA Mathematical and Analytical Framework, the quantity was shortly described as the area related to number sense and number operations which is central and the most essential part within others (OECD, 2013). Since the quantity area was the most fundamental area within others (Stacey & Turner, 2015), and the area that Turkish students struggled most in a large-scale international assessment, in this study, the focal point will be 'quantity' questions placed in Turkish mathematics textbooks.

1.1. Problem Statement and Rationale for the Study

Recent researchers (e.g., Lafontaine, Baye, Vieluf, & Monseur, 2015; She et al., 2018) agree that some countries have high scores in mathematics since the students in that countries were more likely to learn contents in a way that it was available in international tests. This agreement refers that students' learning opportunities might have an effect on their achievement levels in large-scale international assessments. Majority of Turkish students participated in PISA is from ninth and tenth grade. For example, about 21% and 73% of the Turkish students participated in PISA 2015 were in 9th and 10th grades respectively (Özgürlük, Ozarkan, Arıcı, & Taş, 2017). This implies that learning opportunities provided in 9th grade might be crucial in mathematical preparation of students for life.

Textbooks are one of the most important sources in education (Vincent & Stacey, 2008), and accepted as the indicators of opportunity-to-learn (Shield & Dole, 2013). Teachers and students rely on textbooks to follow intended curriculums, and they use the tasks of textbooks that have the potential to direct the students' way of thinking (Jäder, Lithner & Sidenvall, 2019). To clarify the learning opportunities of students, analysis of textbooks assumed to be one of the best ways (Törnroos, 2005). Despite the existence of studies that evaluated the opportunities to learn afforded by textbooks in Turkey (e.g., Bayrakdar, Deniz, Akgün, & Işleyen, 2011; Iskenderoglu & Baki, 2011; Kablan, 2011; Bayazit, 2013), studies analyzing the Turkish mathematics textbooks regarding multi-dimensions (e.g., process, content, context) of an international survey's framework were not found. In Turkey, the mathematics curriculum has been revised in 2018. Therefore, the studies based on the analysis of textbooks written by using the revised mathematics curriculum were also needed. Thus, an analysis of the learning opportunities provided to Turkish students in the 9th mathematics textbook from a lens of PISA's Mathematics Framework would be of great importance for teachers, textbook writers and policy makers to better prepare students mathematically. The following research question guided this study:

Research Question: What are the learning opportunities provided in Turkish mathematics textbooks in the numbers and quantity area concerning the process and context domains and proficiency levels in PISA 2012 Mathematics Framework?

1.2. Definition of Terms

Worked-out example: The questions that were presented with an accompanying solution and answer (Delil, 2006). In this study, the questions under the heading of 'example' refer the worked-out examples.

To-be-solved question: The questions that were given for the practice of students and has no accompanying solutions (Delil, 2006). In this study, the questions under the headings of ‘exercise’, ‘it is your turn’, and ‘the evaluation of unit’ refer the to-be-solved questions.

CHAPTER 2

REVIEW OF THE RELATED LITERATURE

In this chapter, I described the theories and ideas that underlie behind this study. I also reviewed the previous studies related to the conceptual framework. The chapter covers the studies and ideas from the aspects of PISA, opportunity to learn (OTL), and textbooks.

2.1. Educational Studies Related to PISA

The Programme for International Student Assessment (PISA) is an international survey held by Organization for Economic Co-operation and Development (OECD) since 2000 triennially. The goal of the survey is to evaluate different countries' education systems. PISA survey evaluates not only students' knowledge in particular areas but also how students can use that knowledge in daily life (Törnroos, 2005).

This survey is conducted for 15 years old students (OECD, 2013). The survey includes three major areas –reading, science, and mathematics. The weight of the major areas is changing every year, so every focal major has been tested in 9 years sequence. For example, in 2012, the focal area was mathematics, so 9 years later, in 2021, mathematics will be the focal major area in the survey again. According to survey results, revisions including new educational reforms and enhancements on the current system can be considered by policymakers and educators. Also, this nine-year timeline allows policymakers and educators to evaluate effectiveness of revisions.

Starting from the first examination, PISA is one of the most attracted assessments by researchers because of broad participation from all around the world. This allows educators and policy makers to make comparisons among different countries' education systems and curricula (Alacaci & Erbaş, 2010). Being one of the most comprehensive international surveys in the world and having comprehensive sample (e.g., in 2015 about 540 000 students attended representing 72 countries and 29 million of the world's population) provide more generalizable results. This helps researchers to make interpretations about particular areas among various school systems (OECD, 2016; Berberoglu, Celebi, Ozdemir, Uysal, &Yayan, 2003). PISA surveys have extensive structure and focus on issues in various content areas and education system. That is why examining and classifying PISA studies is difficult (Stacey & Turner, 2015). Although classifying PISA studies is challenging, Stacey & Turner (2015) succeeded to contribute classification of some of the PISA studies by providing a list of ways that the PISA can affect educational researches as follows:

> The influence is of many types, including as a call to action from poor results, as a stimulus for new teaching and learning practices and curriculum review, as a model for new assessment practices and provoking deeper education debates more generally and the creation of new educational standards. The underlying themes of the part are first of inspiration from PISA (both the need for change and possible directions for a change), but second, of adaptation of PISA resources, ideas and methods to meet the needs of very different educational environments (p.217).

As highlighted by Stacey and Turner (2015), Nortvedt (2018) made comments on the poor results of international comparative studies and stated that those poor results are alert and a "shock" for Norway education system. Also, Nortvedt (2018) described concrete steps taken by the government and policymakers after the PISA 2000 survey. Then she mentioned some applications still needs to be reevaluated to have a better national education policy. Although Norwegian policymakers believed that Norway has one of the best education systems in the world (Nortvedt, 2018), results of the PISA did not support that claim that caused a "shock". That is why in Norway, national exams and curriculums had been revised by considering the PISA questions.

Similar to Norway education system, according to Arzarello, Garuti, and Ricci (2015), PISA scores affected Italian national education system as well. The study of Arzarello et al. (2015) begins with the unexpected PISA scores of Italia and continues with the developments completed to improve their education system. As mentioned in the study, 2003 PISA scores convinced the Ministry of Education in Italia to make enhancements on policies of learning and teaching mathematics. One of the most important impacts was leading the changes in teacher education programs.

As well as those international research studies, PISA items were used as an efficient assessment tool to evaluate Turkish mathematics teachers' solving methods of PISA 2003 items (Bayrakdar, Deniz, Akgün, & Işleyen, 2011). They believed that the tasks that the teachers had difficulty might give clues about the unexpected scores of students in PISA. The results of the study revealed that teacher candidates successfully solved familiar survey questions related to traditional curriculum; however, they could not apply strategies and formulas for PISA type questions.

Lemke et al. (2004) examined reasons of U.S. students' failure in PISA 2003 for some specific areas and suggested some revisions that were needed in their handbook of International Outcomes of Learning in Mathematics Literacy and Problem Solving: PISA 2003 Results from the U.S. Perspective.

Providing a wide-ranging data that a researcher may be unable to reach individually, data of PISA is an important source for both the researchers and policymakers of countries. Some of the studies that used the PISA data were listed above. Both independent researchers and policymakers had used the data of PISA. In this study, the results of unexpected PISA scores of Turkey were one of the motivating factors for me to start this study. The suggestions for the national education strategy are given at the end of the study.

2.2. The Concept of Opportunity to Learn

Finding measurement methods that would explain the differences in students performances is one of the most attracted subjects of educational researchers (Stacey & Turner, 2015; Suter, 2017). Starting from the 1960s, different concepts and ideas had been offered to explain these variability one of which was 'opportunity to learn' abbreviated as OTL (Törnroos, 2005). The term was used firstly by John Carroll in 1963, and described as 'the time advocated to learn a concept' (Carroll, 1963). Following Carroll's study, in 1967, Husen was also one of the first researchers focusing on the opportunity to learn to explain differences in students' scores with his report prepared for First International Mathematics Study (FIMMS). Husen (1967) described OTL simply as *"... is whether or not the students have had an opportunity to study a particular topic or learn how to solve a particular type of problem (p.162)"*

Accepting OTL as the '*time advocated to learn a concept'* lead researchers to study on issues related to time such as 'the time a student spent on learning a subject' or 'the time a teacher spent to teach a subject' (Törnroos, 2005). On the other hand, accepting the definition of Husen (1967) lead researchers to study on educational materials such as curriculums and textbooks (Floden, 2002; Mullis, Martin, Gonzalez, & Chrostowski, 2003). Husen's definition explains the OTL as '*the opportunities that a student has to study a particular subject',* and textbooks and curriculums have been seen as the most common standard sources that all students can reach to study a subject.

2.3. Opportunity to Learn and International Comparative Studies

The concept of OTL grew from the efforts of explaining the differences of achievement levels between students; hence, the concept attracted the attention of international large-scale studies. As mentioned in the article of Suter (2017), OTL affected the educators and policymakers way of thinking (p.175). It became a focal point in international assessments such as TIMMS (Trends in International Mathematics and Science Study) and PISA. Between the years 1995 and 2011, TIMMS focused on learning opportunities of students from all over the world, and connected the three-level curriculum model, *intended-implemented-attained curriculum*, to the concept of OTL (Suter, 2017; Törnroos, 2005). In this model, the intended curriculum was designed by the curriculum developers and included the goals of the national education system. Implemented curriculum refers to the applied curriculum by schools, and attained curriculum refers to the curriculum related to the results of intended and implemented curriculum (Mullis et al., 2003; Törnroos, 2005). These three curricula are related to each other, and affect students. Within these curricula, the implemented curriculum is related to the applications in class which means depends on the school-teacher-student interactions. Hence, analyzing implemented curriculum necessitates analyzing different elements of education. Instead of working on implemented curriculum, intended curriculums were seen as more standard sources valid for all students (Schmidt, McKnight, Cogan, Jakwerth, & Houang, 2017).

Similar to the TIMMS, OTL concept attracted the attention of developers of PISA. In 2012, PISA focused on the concept of OTL differently. TIMMS studies were mostly interested in including subjects that cover the intended traditional curriculums of different countries (Mullis et al., 2003). In addition to this, the PISA survey included a survey that asks students whether they are familiar with the subject or not. The survey also includes parts related to the time that had spent to teach a subject, the time had spent outside of school, familiarity with the types of questions, frequency of experiencing mathematical tasks in a lesson, and exposure to different types of mathematics tasks during the time in school (OECD, 2016a). Including these parts reversed the situation in some ways, and allowed the countries to question their students' familiarity with the subjects in international assessments (OECD, 2013; Suter, 2017). To question the curriculums one of the best ways was seen as analyzing textbooks since textbooks reflect the intended curriculums, and textbooks are one of the best sources to analyze the learning opportunities provided by the curricula (Stacey & Turner, 2015; Van Zanten & Van Den Heuvel-Panhuizen, 2018).

2.4. Opportunity to Learn and Textbooks

Textbooks are educational materials that were designed to support the process of teaching and learning (Schmidt, 2012). In many countries including Turkey, teachers follow prescribed textbooks to plan their lessons and implement the current mathematics curriculum (Thomson & Fleming, 2004; Kablan, 2011). Therefore, textbooks have an invaluable role in teaching which is being the link between the intended curriculum and implemented curriculum (Shield and Dole, 2013). Textbooks are written regarding different cultures, curriculum, content, design, and item types. This implies that each textbook provides different opportunities to learn the subjects (Schmidt, 2012). The studies of Barnard-Brak, Lan, and Yang (2018) shows that different learning opportunities affect students' achievement level as well as the quality of textbooks (Van den Ham & Heinze, 2018). Even though setting a direct relation between the textbooks and achievement levels of students is not possible, evaluating textbooks as one of the most important sources of opportunity-to-learn bring the studies of textbook analysis front.

As a result of increased awareness of the importance of textbooks, there has been a variety of studies related to the review and evaluations of textbooks. In their studies, Johansson (2005), Bahru (2005), Vincent and Stacey (2008), and Kablan (2011) have indicated the critical role of the textbooks. Concurrent to the studies related to the importance of textbooks, studies related to the methods of assessing textbooks exist in the literature. Straesser (2009) has developed a framework to assess textbooks, which was based on the use of artifacts and representations. In the same year, Rezat (2009) has developed a model to assess the textbooks, but he claimed that textbooks should be evaluated with their relations to students, teachers and mathematical knowledge. Taking these four components as corners, he developed the tetrahedron model of textbook use (p.1261). Similar to Straesser (2009) and Rezat (2009), Shield & Dole (2013) worked on analyzing textbooks and developing a method for assessing textbooks. Throughout the study, the researchers analyzed five textbooks series, and provided a method comprises of five indicators to assess the textbooks. The indicators mentioned in the study are the use of real-life situations, identifications of multiplicative structures, meaningful symbolic representations, related fraction ideas explicitly connected and effective use of a range of representations (Shield and Dole, 2013).

As an alternative way to these frameworks, the frameworks of international surveys were also used to evaluate the learning opportunities provided in textbooks.

Törnros (2005) had used the TIMSS 1999 results and framework to assess Finish students' opportunity to learn the topics covered in TIMSS. To assess these opportunities, he experienced different methods, but in the end, Törnroos (2005) decided that analyzing textbooks was one of the best ways to assess what he wanted (Törnroos, 2005). Similarly, in their study, Iskenderoglu and Baki (2011) used the PISA framework to evaluate learning opportunities provided in 8th grade Turkish mathematics textbooks and analyzed 444 questions from the textbook. The results of the study revealed the discrepancies between PISA standards and Turkish mathematics textbooks.

Besides the studies that compare textbooks with the cognitive domains of international assessment frameworks, there are also cross-national comparisons of textbook studies. For example, Erbas, Alacaci, & Bulut (2012), compared 6th grade Turkish, Singapore and American mathematics textbooks in terms of certain features, and the comparison brought a variety of differences among these features such as numbers of contents, use of visual design or weight of curriculum strands. Hong & Choi (2018) compares textbooks series from Korea and the United States to get clues about the similarities and differences of learning opportunities provided by these textbooks. The results of the study interpret that US textbook that was analyzed provides more learning opportunities related to the high-level questions than Korean textbooks. Besides, the variety of questions in the US textbook was greater than the Korean textbook. Hence, the study alerts the US stakeholders about teacher education and Korean stakeholders about the opportunities to learn provided by the textbooks.

As mentioned by Gronmo et.al (2016), the evaluation and analysis processes are more important for the students at 8th grade and later since the students are starting to develop mathematical skills and understanding within this interval. At the end of the 8th grade, developments of students are still being in progress and feedbacks are being more invaluable. Besides the researchers, countries are also aware of the importance of developing and revising textbooks to provide better learning opportunities for their students. In Turkey, the developments and revisions were accelerated after the 2004 and studies that evaluate the textbooks had potential to improve opportunities to learn afforded by these textbooks (Iskenderoglu & Baki, 2011; Kablan, 2011; Erbaş, Alacaci, &Bulut, 2012).

2.5. Theoretical Framework of the Study

Bransford, Brown, and Cocking (2000) explains why it is important to organize the knowledge by saying "organized knowledge enables us to solve original problems and remember more relevant information than if we have only memorized isolated mathematical facts or procedures". The developers of the PISA organize the knowledge by considering the skills and competencies that students should gain through their education (OECD, 2016a). PISA survey has been developed by the members of Mathematics Expert Group (MEG) since 1999 (Stacey & Turner, 2015). The focus of the group members was developing instruments to assess the use of mathematics into real-life contexts which is one of the much-debated areas of mathematics education since the 1980s (de Lange, 1987; Stacey & Turner, 2015). As mentioned in the framework of the survey, three questions led the developers of the PISA assessment, which are:

- What processes do individuals engage in when solving contextual mathematical problems, and what capabilities do we expect individuals to be able to demonstrate as their mathematical literacy grows?
- What mathematical content knowledge can we expect of individuals and 15-year-old students in particular?
- In what contexts are mathematical literacy able to be observed and assessed? (OECD, 2003, 2013).

Under the direction of these questions, in 2012, Mathematical Framework containing three dimensions, process, content, and context, had been prepared. The components of the framework are interwoven and one question could be categorized into different dimensions separately. A general picture of the three dimensions and proficiency levels were given in Table 2.1.

Table 2.1. The Categorization of Questions in PISA 2012 (OECD, 2013)

Proficiency Levels and Three Dimensions of Mathematical Literacy		The Subsets of Dimensions
Proficiency Levels (6-5-4-3-2-1)	Process	-Formulate -Employ -Interpret
	Content	-Space and Shape -Quantity -Change and Relations -Uncertainty and Data
	Context	- Societal -Personal -Occupational -Scientific

2.5.1. The Process Domain

According to the functional model of Rico (2006), mathematical learning should have three elements, which are, contextualized tasks, conceptual tools, and cognitive subjects (as cited in Sáenz, 2009). Contextualized tasks refer to the situations that the problems placed in. Conceptual tools are mathematical contents, and the cognitive subject is the subject that is necessary for cognitive competencies and processes. The PISA framework contains these three elements, however, it cannot be said that the PISA framework flexibly fits in all national curricula of countries (Sáenz, 2009). Instead, PISA framework can be used to evaluate curricula from different perspectives such as content analysis, task analysis, process analysis, or analysis of extracurricular activities (OECD), 2016a; Stacey & Turner, 2015).

Mathematics was defined as the 'act of generating algorithms and relationships based on the set of procedures and facts' by Wheatley (1991). As we can see from the definition, the construction of mathematical knowledge requires a process and procedures. According to PISA, if the mathematical knowledge was constructed adequately, a person should be able to *formulate, employ, and interpret mathematics in a variety of contexts'* (OECD, 2013). These three words, formulate, employ and interpret, emphasize the three subsets of the process domain. The definitions of these subsets were as follows in the framework:

> The word "formulate" in the mathematical literacy definition refers to individuals being able to recognize and identify opportunities to use mathematics and then provide mathematical structure to a problem presented in some contextualized form. The word "employ" in the mathematical literacy definition refers to individuals being able to apply mathematical concepts, facts, procedures, and reasoning to solve mathematically formulated problems to obtain mathematical conclusions. The word "interpret" used in the mathematical literacy definition focuses on the abilities of individuals to reflect upon mathematical solutions, results, or conclusions and interpret them in the context of real-life problems (OECD, 2013, p.28).

In 2012, for the first time, PISA released the scores of students based on each subset separately. While preparing and scoring the questions, the desired distributions of scores for each category were shown in Table 2.2.

Table 2.2. Desired Distribution of Score Points by the Mathematical Process (OECD, 2013)

Process category	Percentage of score points
Formulating situations mathematically	Approximately 25
Employing mathematical concepts, facts, procedures, and reasoning	Approximately 50
Interpreting, applying and evaluating mathematical outcomes	Approximately 25
TOTAL	100

2.5.2. The Context Domain

According to De Lange (1987), students should engage in solving problems from different situations if we want them to transfer their knowledge between areas. The developers of the PISA surveys care about using these situations in a wide variety (OECD, 2013). These real-life situations were categorized into four subsets and the desired distribution of scores was approximately 25%, which refers to an equal distribution over each category. None of the subsets were used for a higher or lower-level question, instead, they refer that the distribution was done to represent the real-life challenges of a student. Each context was explained in the framework as follows:

> Problems classified in the personal context category focus on activities of one’s self, one's family or one's peer group…Problems classified in the occupational context category are centered on the world of work...Problems classified in the societal context category focus on one's community…Problems classified in the scientific category relate to the application of mathematics to the natural world and issues and topics related to science and technology. (OECD, 2013, p.37).

The results were released separately because each process category emphasizes a different view of mathematical knowledge. OECD (2013) explained ‘what does each category implies for the educators and policymakers' as follows:

> It is important for both policymakers and those engaged more closely in the day-to-day education of students to know how effectively students can engage in each of these processes. The results of the PISA survey for the formulating process indicate how effectively students can recognize and identify opportunities to use mathematics in problem situations. The results of the PISA survey for the employing process indicate how well students can perform computations and manipulations and apply the concepts and facts that they know to arrive at a mathematical solution to a problem formulated mathematically. The results of the PISA survey for the interpreting process indicate how effectively students can reflect upon mathematical solutions or conclusions, interpret them in the context of a real-world problem, and determine whether the results or conclusions are reasonable (p.28).

The process domain has three subsets, which are formulate, employ, and interpret. The activities that were related to each subset were also identified clearly in the framework. A summary of the activities was interpreted in Table 2.3.

Table 2.3. The Subsets of Mathematical Process and Activities Related to Each Subset (OECD, 2013)

The Subsets of the Mathematical Process	Activities Related to Subsets
Formulating situations mathematically	• Identifying the mathematical aspects of a problem • recognizing mathematical structure in problems or situations • simplifying a situation or problem • identifying constraints and assumptions behind any mathematical modeling •representing a situation mathematically • representing a problem in a different way • understanding and explaining the relationships between the context-specific language of a problem and the symbolic and formal language • translating a problem into mathematical language or a representation • recognizing aspects of a problem that correspond with known problems or mathematical concepts •using technology (such as a spreadsheet or the list facility on a graphing calculator) to represent a situation
Employing mathematical concepts, facts, procedures, and reasoning	• devising and implementing strategies for finding mathematical solutions; • using mathematical tools, including technology, to help find exact solutions; • applying mathematical facts, rules, algorithms, and structures when finding solutions • manipulating numbers, graphical and statistical data and information, algebraic expressions and equations, and geometric representations • making mathematical diagrams, graphs, and constructions and extracting mathematical information from them; • using and switching between different representations in the process of solutions; • making generalizations based on the results • reflecting on mathematical arguments and explaining and justifying mathematical results.
Interpreting, applying and evaluating	• Interpreting a mathematical result back into the real world context; • evaluating the reasonableness of mathematical solutions • understanding how the real world impacts the outcomes of a mathematical procedure • understanding the extent and limits of mathematical concepts and mathematical solutions • critiquing and identifying the limits of the model used to solve a problem.

As mentioned by Blömeke and Kaiser (2012), engaging in different contexts while learning mathematics affected students achievement as it was supported by researchers such as the study of Klieme and Baumert (2001). In some cases, the use of real-life context may affect students' achievement level negatively, since dealing with context-based problems requires high verbal abilities. However, the researches support that solving real-life problems could elicit different behaviors in students (Hickendorff, 2013). Since one of the main purposes of the MoNE is preparing students to the mathematical challenges in real life (MoNE, 2018a), the use of real-life contexts becomes a necessity. According to Hickendorff (2013), the use of real-life problems can motivate students to study mathematics; students can develop different behaviors based on the problems chosen, and real-life problems allow constructing knowledge about the use of mathematics in daily life.

PISA 2012 Mathematics Framework divides real-life contexts into four categories, which are personal, social, occupational, and scientific. These categories do not determine a difficulty level but they ensure to provide wide-ranged learning environments for students.

2.5.3. The Proficiency Levels

Engaging students in tasks from different cognitive levels provide them opportunities to think and learn effectively (Simon & Tzur, 2004). Smith and Stein (1998) supports this idea by their findings ‘high-level cognitive tasks support individuals to develop reasoning, thinking and problem-solving skills (p.344)’. Selecting or designing cognitive tasks appropriate to different levels is a challenging task for many teachers; instead, teachers choose and use questions from the prescribed textbooks and digital sources (Thomson & Fleming, 2004). Hence, the textbooks have an important responsibility such as providing OTL for tasks from different cognitive levels. From this point of view, in this study to analyze the variability of cognitive levels in Turkish Mathematics Textbook, the proficiency scale of PISA 2012 Framework was used. In 2000, the developers of PISA prepared questions based on three competency levels, but this has changed since 2003. In the years 2003, 2006, 2009 and 2012, the questions were prepared and the results were announced concerning six proficiency levels. The proficiency levels and their descriptions were explained in the Figure 2.1 as follows:

Levels	What a student can do at this level?
6	At Level 6 students can conceptualize, generalize and utilize information based on their investigations and modeling of complex problem situations. They can link different information sources and representations and flexibly translate among them. Students at this level are capable of advanced mathematical thinking and reasoning. These students can apply their insight and understandings along with a mastery of symbolic and formal mathematical operations and relationships to develop new approaches and strategies for attacking novel situations. Students at this level can formulate and precisely communicate their actions and reflections regarding their findings, interpretations, arguments and the appropriateness of these to the original situations.
5	At Level 5 students can develop and work with models for complex situations, identifying constraints and specifying assumptions. They can select, compare and evaluate appropriate problem-solving strategies for dealing with complex problems related to these models. Students at this level can work strategically using broad, well-developed thinking and reasoning skills, appropriate linked representations, symbolic and formal characterizations and insight pertaining to these situations. They can reflect on their actions and formulate and communicate their interpretations and reasoning.
4	At Level 4 students can work effectively with explicit models for complex concrete situations that may involve constraints or call for making assumptions. They can select and integrate different representations, including symbolic, linking them directly to aspects of real-world situations. Students at this level can utilize well-developed skills and reason flexibly, with some insight, in these contexts. They can construct and communicate explanations and arguments based on their interpretations, arguments, and actions.
3	At Level 3 students can execute clearly described procedures, including those that require sequential decisions. They can select and apply simple problem-solving strategies. Students at this level can interpret and use representations based on different information sources and reason directly from them. They can develop short communications when reporting their interpretations, results, and reasoning.
2	At Level 2 students can interpret and recognize situations in contexts that require no more than direct inference. They can extract relevant information from a single source and make use of a single representational mode. Students at this level can employ basic algorithms, formulae, procedures, or conventions. They are capable of direct reasoning and making literal interpretations of the results.
1	At Level 1 students can answer questions involving familiar contexts where all relevant information is present and the questions are clearly defined. They are able to identify information and to carry out routine procedures according to direct instructions in explicit situations. They can perform actions that are obvious and follow immediately from the given stimuli.

Figure 2.1. Proficiency Scale Descriptions of PISA 2012 Mathematics Framework (OECD, 2013)

2.6. Sample Questions Coded Concerning the PISA Assessment

After each survey, one-third of PISA items were released to public access (OECD, 2016b). Some of the items released by PISA were given to give clues about the categorizations of questions.

Mei-Ling from Singapore was preparing to go to South Africa for 3 months as an exchange student. She needed to change some Singapore dollars (SGD) into South African rand (ZAR).

Mei-Ling found out that the exchange rate between Singapore dollars and South African rand was:

1 SGD = 4.2 ZAR

Mei-Ling changed 3000 Singapore dollars into South African rand at this exchange rate.

How much money in South African rand did Mei-Ling get?

Answer: ....

Figure 2.2. The Exchange of ZAR Item from the PISA 2012 Assessment (NCBR, 2015)

As mentioned by National Centre Research and Development Department (2015), the question in Figure 2.2 belongs to the ***quantity*** area. The context category of the question is ***societal*** being related to the knowledge of citizenship. Since the question requires the only application of a standard algorithm, it belongs to ***Level 1***.

During these 3 months the exchange rate had changed from 4.2 to 4.0 ZAR per SGD.

Was it in Mei-Ling's favour that the exchange rate now was 4.0 ZAR instead of 4.2 ZAR, when she changed her South African rand back to Singapore dollars? Give an explanation to support your answer.

Answer: ....

Figure 2.3. The Exchange of ZAR-2 Item from the PISA 2012 Assessment (NCBR, 2015)

The exchange of ZAR-2 item is highly related to the item mentioned in Figure 2.2. This is still categorized as a ***quantity*** questions set in a ***societal*** context. Being different from the first item, the question asks for a reflection on students' answer. As mentioned by National Centre Research and Development Department (2015), the combination of familiar context, complex situation, non-routine problem and the need for reasoning places the item at ***Level 4*** (p.45).

Rock Concert

For a rock concert a rectangular field of size 100 m by 50 m was reserved for the audience. The concert was completely sold out and the field was full with all the fans standing.

Which one of the following is likely to be the best estimate of the total number of people attending the concert?

A. 2 000

B. 5 000

C. 20 000

D. 50 000

E. 1000 000

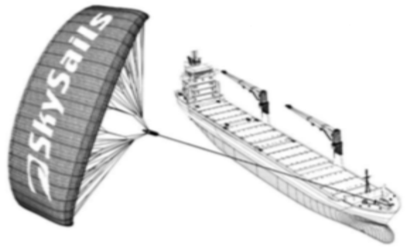
Figure 2.4. The Rock Concert Item from the PISA 2012 Assessment (OECD, 2013)

According to the PISA 2012 framework, the question of Rock Concert is placed in ***quantity*** questions, which sets in the ***societal*** context. As mentioned in the framework (2013), only 28% of the students answered the question correctly, which makes it ***moderately difficult***. The item attributes ***all three levels of process category*** but mainly belongs to '***formulate'*** category since student needs to transform the question into an adequate mathematical representation.

MATHEMATICS UNIT 2: Sailing ships

Ninety-five percent of world trade is moved by sea, by roughly 50 000 tankers, bulk carriers and container ships. Most of these ships use diesel fuel.

Engineers are planning to develop wind power support for ships. Their proposal is to attach kite sails to ships and use the wind's power to help reduce diesel consumption and the fuel's impact on the environment.



Sailing Ships – Question 1

One advantage of using a kite sail is that it flies at a height of 150 m. There, the wind speed is approximately 25% higher than down on the deck of the ship.

At what approximate speed does the wind blow into a kite sail when a wind speed of 24 km/h is measured on the deck of the ship?

A 6 km/h
B 18 km/h
C 25 km/h
D 30 km/h
E 49 km/h

Figure 2.5. The Sailing Ship Item from the PISA 2012 Assessment (OECD, 2013)

The sailing ship item, placed in PISA 2012 survey belongs to ***quantity*** area and ***scientific*** context. The question asks for a certain answer, and students need to apply a strategy to solve real-life questions. Hence, the item is categorized as an ***employ*** question. In the question, the situation is clearly described and student needs to select a strategy and apply it. This makes the item belongs to ***Level 3***.

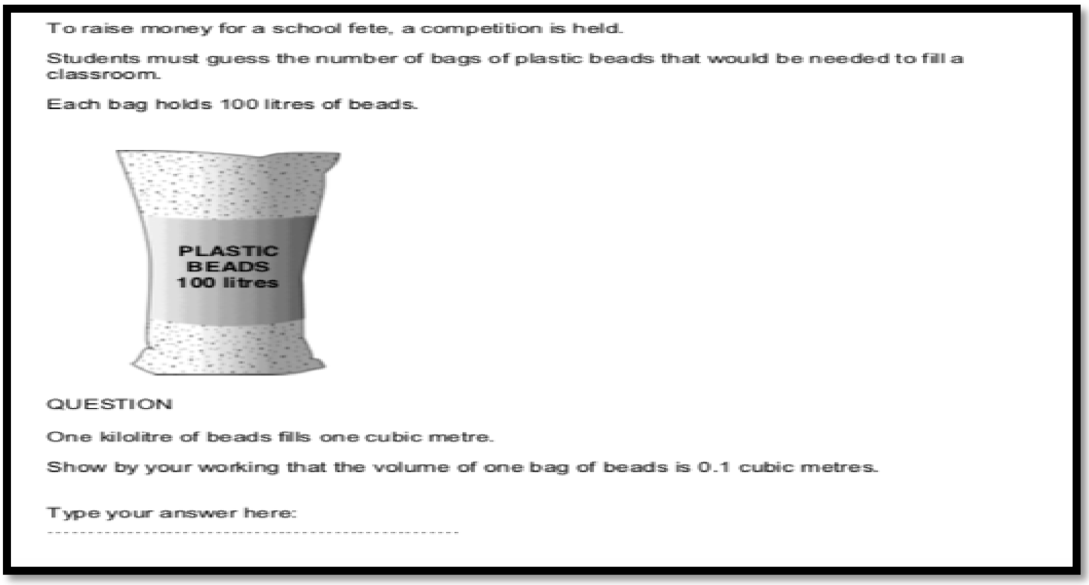


Figure 2.6. The Plastic Beads Item from PISA 2012 Assessment (OECD, 2013)

The question about Plastic Beads is categorized as a highly difficult question, ***Level 6***, according to OECD (2013). This level requires linking different sources of information and representations flexibly. Involving measurement and different representations of numbers make the question a ***quantity*** question that sets in a ***personal*** context. To complete the task, students need to apply mathematical procedures and get a conclusion. Hence, the process category of the question is ***employ***.

2.7. Summary of Literature Review

The results of international comparative studies alert researchers, policymakers and educators to evaluate their education systems from different points of views (İskenderoğlu et al., 2013; Bayrakdar et al., 2011; Lemke et al., 2004; Nortvedt, 2018; Törnroos, 2005; Urhan & Dost, 2018). One of the best ways to search for reasons of failure in international comparative studies is analyzing textbooks (Törnroos, 2005) since textbooks are the prescribed and standard sources that provide OTL to students. There exist different frameworks and method to assess OTL provided in textbooks (Stacey & Turner, 2015), one of which was using the framework of international large scale assessments. In the literature, there exist many studies related to the analyzing textbooks, but studies related to the multidimensional analysis of revised Turkish mathematics textbooks were found rarely.

CHAPTER 3

METHODOLOGY

3.1. Research Design

The purpose of this study was drawing a picture of the categorizations of the learning opportunities provided by the Turkish Mathematics Textbooks. In this qualitative study, I used content analysis as the method. Content analysis is defined as ‘the systematic set of procedures for the rigorous analysis, examination, and verification of the contents of written data’ (Freankel & Wallen, 2006). To get information about the learning opportunities of textbooks, the textbook written by Karataş (2018) was analyzed concerning the domains of the PISA 2012 Mathematics Framework. The chapter will be given in the following order. In the beginning, I explained the selection of the content area, the matching process between the units of textbook and content areas of framework, and the selection of the textbooks. Then, I covered the reliability and validity issues of the study. I explained the coding scheme used in the study. Finally, I described the coding procedure, and gave some examples of coding from the study.

3.2. Selection of the Content Area

In the study, the numbers and quantity was chosen as the focus area within four areas described in PISA Framework since the quantity was the most fundamental area of these four areas, and Turkish students struggled most in PISA assessment (PISA, 2012; Stacey & Turner, 2015). The PISA Framework divides mathematics tests into four main areas– change and relationships, space and shape, quantity, uncertainty, and data-. The four content areas were designed to correspond to the areas of traditional curriculum even though the distinction between areas is not too strict. As mentioned by Stacey & Turner (2015), the quantity area is closely related to the problems under the '*Number and Measurement'* category in traditional curriculums. Similarly, '*Space and Shape*' covers topics on '*Geometry'*, 'Uncertainty and Data' on '*Probability and Statistics*' and 'Change and Relationships' on '*Algebra and Functions'* (p.19).

3.3. Matching Quantity Area with subjects in Turkish Curriculum

Turkish Mathematics Curriculum was designed based on three main content areas, "Numbers and Algebra", "Geometry", and "Data, Probability & Statistics". These three areas were placed in each grade from 9^{th} to 12^{th} comprising different subjects. Distributions of mathematical subjects concerning three main content areas and grade levels were given in Table 3.1.

In Table 3.1, to match the content areas of PISA to the units described in the curriculum of MoNE (2018), I assigned the subjects related to the algebra and functions as change and relations (CR), the subjects of geometry topics as space and shape (SS), the subjects of data, probability and statistics as uncertainty and data (UD), and the subject of equations and inequalities as quantity (Q). The learning outcomes of the quantity (OECD, 2015) and the unit of equations and inequalities (MoNE, 2018a) were compared in Table 3.2. The consistency between the learning outcomes of areas was my reason for working on questions under the heading of equations and inequalities in Turkish Mathematics Textbook.

Table 3.1. Distributions of Mathematical Subjects Regarding the Content Areas and Grade Levels

	9th Grade	10th Grade	11th Grade	12th Grade
Numbers and Algebra	-Logic and Sets (CR) -Equations and Inequities (Q)	-Functions (CR) -Polynomials & Equations (CR)	-Applications of Functions (SS, CR) -Applications of Equations and Inequalities (CR, SS, Q)	-Exponential and Logarithmic Functions (CR) -Differentiation (CR) -Integration (CR)
Geometry	Triangles (SS)	-Quadrilaterals (SS) -Polygons (SS) -Solids (SS)	Trigonometry (SS, CR) -Analytical Geometry (SS)	-Trigonometry (SS, CR) -Transformations on Coordinate Plane (SS)
Data, Probability, and Statistics	-Data (UD)	Introduction to the Probability (UD)	-Probability (UD)	---

Table 3.2. Learning Outcomes of Quantity Area and Equations and Inequalities

Quantity area described in PISA Framework	Learning Outcomes of Equations and Inequalities in Turkish Mathematics Curriculum
• The quantification of attributes of objects, relationships, situations and entities in the world • Understanding various representations of quantifications, and judging interpretations and arguments based on quantity. • Understanding measurements, counts, magnitudes, units, indicators, relative size, and numerical trends and patterns (OECD, 2015)	• Recognizing and relating different sets of quantities • Solving the problems related to the integers • Solving the problems related to repetitive situations • Explaining the intervals of real numbers • Carrying out procedures to solve equations • Solving problems related to the exponential numbers • Solving problems related to the root numbers • Solving real-life problems including the different representations of numbers (MoNE, 2018a)

3.4. Selection of the Textbooks

According to the National Report of PISA 2015, the sample of PISA survey includes students from 7th grade to 12th grade with different ratios (MoNE, 2016, p.7). The distributions of students across grade levels were as follows: 0.6% from 7th grade, 2.6% from 8th grade, 20.7% from 9th grade, 72.9% from 10th grade, 3.0% from 11th grade, and 0.1% from the 12th grade (MoNE, 2016, p.7). These statistics imply that most of the students having the PISA survey belong to 9th grade and 10th grade. Hence, working on the 9th and 10th grade high school curriculum seems logical to get a clue about the learning opportunities of Turkish students who are also in the target sample of the PISA survey. By taking the statue of quantity questions in Turkish Mathematics Curriculum into consideration, I chose 9th grade textbooks in this study.

The national curriculum currently in use in 2019 has been revised in 2018 by considering the results of international assessments, academic studies, surveys related to the use of textbooks of teachers' and students', and national educational targets determined by the Regulations and MoNE (MoNE, 2018a, p.11). The textbooks that were released for the use of formal-informal schools were defined in the MoNE Course Books and Educational Tools Regulations in Turkey and approved by Board of Education (Urhan & Dost, 2018). Both MoNE and private publications prepared the textbooks being compatible with the related regulations and Turkish national curriculum. In 2018, MoNE approved two textbooks for the use of ninth graders. For the 2018-2019 academic year, there were two official mathematics textbooks approved by MoNE to be send to schools at the ninth grade level. While one of them published by MoNE was for the Science High Schools, the other (Karataş, 2018) published by a private textbook publisher for all the other type of schools including Anatolian High School, Vocational and Technical High School, and Anatolian Imam Hatip (Religious) High School. The students in (public and private) Science High Schools constitute less than three percent of all the students in high schools in Turkey (MoNE, 2018). Thus, the textbook by the private publisher (Karataş, 2018) was selected for analysis, since it is used by the majority of the students in ninth grade.

3.5. The Unit of Analysis

The 9th-grade textbook had 384 pages and consisted of 5 chapters. The questions in the book were given under the headings of "example", "it is your turn", "exercises", and "evaluation of the unit". The questions given under the "example" sections were consists of worked out questions where the questions of "it is your turn", "exercises", and "evaluation of the unit" consists of to-be-solved questions. The "equations and inequalities" section was placed at the 2nd unit of the textbook. The pages between 83 and 204 were devoted to the 'equations and inequalities' part in the textbook, which implies 31.77% of the whole pages of the textbook. The unit has five subtitles, which are, number sets, division rules, equations and inequalities, exponential numbers, and applications of equations and inequalities. Being related to the change and relations category, the page ranges between the 122 and 154 under the subtitle of 'equations and inequalities' were not included in this study. Hence, I categorized 400 questions from five subtitles concerning the domains and proficiency levels of the PISA framework. The distributions of questions were given in Table 3.3.

Table 3.3. The Distributions of Questions Regarding the Question Types and Sections of the Unit

	Number Sets	Division Rules	Equations and Inequalities	Exponential Numbers	Applications of Equations and Inequalities
Example (174)	17	46	10	59	42
It is your turn (21)	3	4	2	5	7
Exercises (140)	12	35	26	42	25
Evaluation of unit (65)	0	0	0	0	65

3.6. Validity

According to Fraenkel and Wallen (1990), the validity was described as "the appropriateness, meaningfulness, and the usefulness of the specific inferences researchers make based on the data they collect" (as cited in Delil, 2006, p.41). Hence, to indicate the validity of the current study, I explained the appropriateness of the framework to the study, the meaningfulness of the study, and usefulness of the results of the study in this part. The study aims to examine the learning opportunities of

Turkish students provided by the Turkish Mathematics Textbooks. In the PISA 2012 Mathematics Framework, the relation between OTL and items in PISA were explained as follows:

> PISA 2012 aims to identify the country (and probably school) level profiles in learning opportunities. Students will be confronted with carefully crafted mathematics tasks – some representing mathematical abilities and content categories as mentioned in the PISA mathematics framework, some representing more traditional tasks asking for procedural and declarative knowledge. Following each of those items, students are asked to judge whether and how often they have seen similar tasks in their mathematics lessons and previous assessments. Thus, it is possible – aggregated at the country, but possibly also at the school level – to measure learning opportunities in a way that allows for differentiation between types of problems and content (p.187).

The consistency between the aim of the PISA assessment and the aim of the study implies that the framework is appropriate for this study. The framework is developed to assess the skills of 15-year-old students, which indicates that using the framework to assess the 9^{th}-grade students' textbook is also appropriate. The meaningfulness of the study is related to the aim of the study. This study examines the learning opportunities of a textbook by using a framework of an international comparative survey. As mentioned by Törnroos (2005), analyzing textbooks is one of the best ways of measuring the OTL of students. Hence, conducting the study is meaningful and even necessary.

The results of the study reveal some of the characteristics of a textbook that is used by the majority of high school students in Turkey. Hence, both the textbook writers and researchers who would conduct researches related to the analysis of textbooks can benefit from the study. This implies that the results of the study are useful from different perspectives. Therefore, the study provides the properties of the validity of a content analysis study.

3.7. Reliability

Two independent coders have worked on 9^{th} grade textbook of Karataş (2018) to code the textbook according to the coding scheme described in Table 3.4. One of the coders is a researcher and the other coder is a mathematics educator. Before starting the coding, two coders came together to understand the coding scheme clearly. Two researchers worked on some of the questions released by the OECD and coded by the experts and developers of PISA (OECD, 2013). Two researchers coded the 50 questions separately concerning three coding categories. Hence, each coder made 150 coding (approximately 12.50% of the whole coding). Two researchers came together to compare the results of the coding. Among 150 coding, 25 contradictions were found, which refers to an agreement level of 83.33%. To eliminate the contradictions, two researchers discussed questions and an expert's opinion was also taken to reach the agreement. After the discussions, the agreement level reached to 95.33%, and disagreement on 18 coding was eliminated. Since the agreement level was 95.33%, inter-rater reliability was provided. The contradictions were mostly related to the following issues:

- Some questions include both *formulating* and *employing* processes, and this was a complexity for two of the coders. To handle the complexity, figure 6 was created.
- To determine the difference between *Level1* and *Level2,* the terms *'direct inference'* and *'direct instruction'* in the framework were used. If the item includes *'direct instruction'* the item was coded as Level1. If the item includes *'direct inference',* it was coded as *Level2.*
- To determine the difference between *Level3* and *Level4*, the necessity of insight strategy application was used as a criterion. If the strategy of the question was clear and the question includes only one problem situation, it was coded as *Level3.* If the student should have some insight to decide the strategy of the question, and the question includes more than one situation, it was coded as *Level4.*

3.8. Coding Scheme

In this study, PISA 2012 Mathematics Framework were used since the learning opportunities provided by Turkish Mathematics textbooks concerning PISA domains was the curiosity that led this research. The framework has three domains–content, context, and process. The framework also defines six proficiency levels interwoven to each domain (OECD, 2013). However, I used the required proficiency levels as a separate domain in this study to examine questions in the textbook of Karataş (2018). These three domains, proficiency levels, and subsets of the domains were given in Table 2.1. In this study, the content domain was already determined as 'quantity'. Hence coding scheme will include three domains, which are process, context, and proficiency levels.

3.8.1. The Domains of Coding Scheme

Table 3.4. The Coding Scheme of the Study (OECD, 2013)

Domains of Mathematical Literacy	Subsets of the Domains	Key Words /Sentences for the Domains
Process	**Formulate**	• Recall/recognize a definition or property • Simplify/represent a problem • Use symbolic language • Notice similar or equivalent representation of an expression • Use technology to represent a problem or situation
	Employ	• Implement strategies for mathematical solutions • Use any tool for exact solutions • Carry out rules, algorithms, and structures when finding solutions • Extract mathematical information from different solutions • Make generalizations based on the results • Reflecting on mathematical arguments • Explain and justifying mathematical results.
	Interpret	• Relate a solution to the real world • Evaluate the reasonableness of solutions • Understand how the real-world impacts the outcomes of a mathematical procedure • Evaluate the limits of the model used to solve a problem
Context	**Personal**	• Food preparation/ shopping/ games /personal health /personal transportation/ sports/travel/personal scheduling/ personal finance.
	Occupational	• Measuring/costing/ordering materials for building/ payroll/accounting/qualitycontrol/scheduling/inventory/design/architecture/job-related decision making
	Societal	• Voting systems/public transport/government/public policies/demographics/advertising/ national statistics/economics.

Table 3.4. The Coding Scheme of the Study (OECD, 2013), Continued

	Scientific	• Weather/climate/ecology/medicine/space science/genetics/measurement/the world of mathematics itself.
	Decontextualized	• No real-life context was used
Proficiency Levels (What students can do at these levels?)	**Level 6**	• conceptualize, generalize and utilize information based on *their investigations* and modeling of *complex problem situations* • link different information sources and representations, flexibly translate them • apply *their* insight and understandings along with a *mastery* of symbolic and formal mathematical operations and relationships • *develop new approaches and strategies* for attacking novel situations • formulate and precisely communicate *their* actions and reflections regarding students' own findings, interpretations, arguments and the appropriateness of these to the original situations
	Level 5	• *develop* and work with models for complex situations, identifying and specifying assumptions • select, compare and evaluate appropriate problem-solving strategies for dealing with complex problems • work strategically using broad, well-developed thinking and reasoning skills, appropriate linked representations, symbolic and formal characterizations to these situations • reflect on actions and formulate and communicate interpretations and reasoning
	Level 4	• work with *explicit models* for complex concrete situations • *select and integrate* different representations • utilize well-developed skills and reason flexibly • *construct and communicate explanations* and arguments based on interpretations, arguments, and actions.
	Level 3	• execute *clearly described procedures* • select and apply *simple* problem-solving strategies • interpret and use representations based on different information sources • develop short communications when reporting results
	Level 2	• interpret and recognize situations that require *direct inference* • extract relevant information from a single source • *use single representations* • employ *basic algorithms, formulae, procedures* • make reasoning and making literal interpretations of the results
	Level 1	• answer questions involving *familiar contexts* • carry out routine procedures according to *direct instructions* • perform actions that are *obvious* and follow from the given *stimuli*

3.9. Data Analysis

The coding scheme (Table 3.4) was prepared with respect to PISA 2012 Mathematics framework. Besides one category was added, decontextualized, considering the nature of textbooks in Turkey. Regarding the coding scheme (Table 3.4.), I analyzed the questions placed in the 'Equations and Inequalities' unit of textbook of Karataş (2018). The questions were firstly coded concerning the process category. To decide the process category, the following question was taken into consideration: “what process should students engage in when solving this problem”. The questions of PISA were prepared in a way that they would fit into one of the categories in the framework. However, the questions in the Karataş (2018) textbook were fitting in more than one category in some cases. Especially, the ‘employ’ and ‘formulate’ processes were used together in many questions where ‘interpret’ questions were obvious in the textbook. I assigned and evaluated these cases separately, but still, I decided one of the categories as the dominant category. Figure 3.1 helped to handle the confusion. Examples of coding were given in Figure 3.2 and Figure 3.3.

The process categories included in the question	**The criteria for the selection of dominant process category**	**The dominant process category**
Formulate +Employ	If the question requires a special formulation or representation, but students can carry out the algorithms after the formulation the question was categorized as formulate.	Formulate+.
Employ +Formulate	If the formulization of the question was obvious or unique, but the important part is to apply the solving strategy or carry out the algorithms, the question was coded as employ	Employ+

Figure 3.1. The Coding Process for the Questions Including More Than One Process Category

Example 3, Page 107:

Hülya Hanım will divide their garden that is located in front of their apartment into equal squared pieces as big as possible. The garden has the 4-meter of height and 5-meter of width. She is planning to plant a different flower for each square. How many flowers can she plant at most?

Coding

Context Category: Personal

Process Category: Employ+Formulate=Employ+

Proficiency Level: Level 3

Figure 3.2. Sample Coding of Question Fitting in More Than One Process Category

The question is related to a personal hobby of a person, thus the context category of the question was signed as ***personal.*** This question was one of the questions that were fitting in more than one process category. To solve the question; students need to represent the question in an appropriate form, implement a mathematical strategy to find the solution, and carry out calculations. Hence the question provides an opportunity for formulating and employing the strategy. Applying strategy and carrying out the calculations seemed more challenging in the questions, and this question was coded as an ***employ*** question dominantly.

Exercises 7, Page 203: Ekrem will divide one-half of an iron bar into 10 equal pieces and the other half of the iron bar into 15 equal pieces. Each small iron piece is shorter for 4 cm than each big iron piece. Calculate the length of the initial iron bar. 
Coding: Context Category: Personal Process Category: Formulate+Employ=Formulate+ Proficiency Level: Level 3

Figure 3.3. Sample Coding of Question Fitting in More Than One Process Category

The question is contextualized concerning one's activity and coded within the ***personal*** context. To solve this question, students should formulate and represent the situation in an appropriate mathematical form, and carry out the calculations based on the formulated mathematical form. Hence the question was coded as a ***formulate*** question dominantly emphasizing that the more important process in this question was formulization process.

After coding questions concerning the process category, I analyzed questions regarding the context categories. To code the questions, I used five different sub-categories were used which are personal, societal, occupational, scientific, and *decontextualized.* PISA 2012 Mathematics Framework has four context categories, personal, social, occupational, and scientific, for the questions. In the PISA, there exist a few questions that include mathematical structure, and the developers of the framework included these questions into the scientific category. However, in the textbook of Karataş (2018), the questions that include only mathematical structure and no real-life context were a large amount. Hence, to analyze the amount of these questions, I used a fifth category, *decontextualized*, in this study. An example of a *decontextualized* question was given in Figure 3.4. The question includes only recognizing the mathematical structure and carrying out calculations without a real-life context. Hence, the question was coded as *decontextualized.*

Example 1, Page 165 The following representation belongs to a real number: $\sqrt{a-2} + \sqrt[4]{6-a}$ Find the integers that could be equal to “a”?
Coding: Context Category: Decontextualized Process Category: Formulate+Employ=Formulate+ Proficiency Level: Level 2

Figure 3.4. Sample Coding of a Decontextualized Question

Finally, I coded the questions for proficiency levels of PISA Mathematics Framework. In the framework, the questions were prepared considering six different difficulty levels. To decide the levels of questions, I applied the following criteria:

- If the question is clearly described and the students would only apply basic calculations directly, classify the item as Level 1. Use of *direct instruction* is a clue to code the questions as Level 1.
- If the question asks for making a *direct inference*, the solution method is obvious, and students should apply it; classify the question as Level 2.
- If the question includes following sequential situations, but the sequence of situations was clearly described in the question; classify the item as Level 3. The questions at this Level should allow students to choose a strategy within options.
- If the question includes complex mathematical situations that include more than one representation, and students should be able to know the limitations and the assumptions of the situation; classify the question as Level 4.
- If the question requires modeling a complex mathematical situation from the real-life, applying a strategy to model and solve the question and reflect on the solution and solving strategy of the problem; classify the question as Level 5.
- If the question requires generalizing and utilizing information based on students' work and solutions, relate more than one sources of information to solve the question, and making real-life inferences based on their findings; classify the question as Level 6.

3.10. Sample Coding from the Study

Exercises 8, Page 105: Find the remainder of the following numbers when divided by 8? a) 1184 b) 27564 c) 10004 d) 4527 e) 61716
Coding: Context Category: Decontextualized Process Category: Employ Proficiency Level: Level 1

Figure 3.5. Sample Coding of the Problem Located in Page 105 of Textbook of Karataş (2018)

The item, placed in textbooks of Karataş (2018) belongs to ***the quantity*** area and has no real-life context. Hence it was coded as ***decontextualized.*** The question asks for carrying out calculations directly. For this reason, the item was categorized as an ***employ*** question. In the question, the instructions are given directly, and the only thing that students should do is carry out routine procedures, which makes item belong to ***Level 1***.

Exercises 2, Page 204: Represent $\sqrt{7}$ in the real line.
Coding: Context Category: Decontextualized Process Category: Formulate Proficiency Level: Level 2

Figure 3.6. Sample Coding of Problem Located in Page 204 of Textbook of Karataş (2018)

This item is a ***decontextualized*** question from page 204 of textbook of Karataş (2018). The question asks directly to represent a number into another representation which makes question belongs to ***formulate*** process. In the question, the instruction was not given directly but the method is obvious. Students need to determine the place of the number and represent it. Hence, the students should be able to understand the direct inference in the question. Therefore, the question illustrates ***Level 2.***

Example 1, Page 104: Let A be a natural number. The remainder of A, when divided by 2, is 1, and the remainder is 6 when divided by 9. Let's find the remainder of A when divided by 18.
Coding: Context Category: Decontextualized Process Category: Employ Proficiency Level: Level 4

Figure 3.7. Sample Coding of Problem Located in Page 104 of Textbook of Karataş (2018)

The question that was located on page 104 of textbooks of Karataş (2018) belongs to ***the quantity*** area and has no real-life context, which makes it coded as ***decontextualized.*** To solve the question, students should link two different symbolic representations and should have some insight to solve the question. Hence the item was categorized as a ***Level 4*** question. The situation is clearly described, but students need to choose a strategy and apply it to solve the question, which makes the question to belong to ***employ*** category.

Exercises 3, Page 112: Oils in 8, 12 and 24 L cans will be filled into equal volume bottles to sell so that they do not overflow and do not mix in between. Find how many bottles are required.
Coding: Context Category: Occupational Process Category: Employ Proficiency Level: Level 3

Figure 3.8.Sample Coding of Problem Located in Page 112 of Textbook of Karataş (2018)

The question was located on page 112 of textbooks of Karataş (2018) and belonged to ***the quantity*** area. The question is related to a physical activity related to selling oil, and the question was coded as occupational. The students are familiar to that kind of context, and they should be able to apply a strategy to solve that question which makes the question belong to ***employ*** category. Since the situation is clearly described and the only thing that students should do is applying sequential procedure, and this makes question fit in Level 3.

Example 1, Page 98: Let ABCDE be a five-digit natural number. Show your work for the corollary that says if CDE is divisible by 8 without remainder than ABCDE is also divisible by 8.
Coding: Context Category: Decontextualized Process Category: Interpret Proficiency Level: Level 5

Figure 3.9. Sample Coding of Problem Located in Page 98 of Textbook of Karataş (2018)

Since the item has no real-life context, it was a ***decontextualized*** question. The question does not ask to carry out some procedures and get a result. Instead, it asks to work for the proof of a corollary. The students need to analyze the given information and communicate their arguments. Therefore, the item is illustrating ***Level 5.*** Students were asked to make a generalization based on the information given. Hence the required process of the item was ***interpret.***

Exercises 8, Page 203:
3 runners were racing at a constant speed on a track of certain length. In this race, when the first runner finishes the race, the second runner had 1050 m to finish the race, and the third runner had 2100. When the second racer finishes the race, the third racer had 1200 m to finish the race. Accordingly, calculate the length of the track. 
Coding: Context Category: Social Process Category: Formulate+Employ=Formulate Proficiency Level: Level 6

Figure 3.10. Sample Coding of Problem Located in Page 203 of Textbook of Karataş (2018)

The question placed on page 203 of the textbook of Karataş (2018) was a ***quantity*** question that belongs to ***social*** context since the question is related to a race. Solving the question requires relating different sources of information such as velocity, distance, and time within two different situations. This makes the item belongs to ***Level 6.*** Before solving the question, students need to formulate and represent the situation. Otherwise, they cannot carry out calculations. Hence, the most important process for solving the question is ***formulate.***

Example 2, Page 113:
Three friends take turns visiting the senior home every 3, 4 and 8 days. Since they visited for the first time on Saturday, March 18 (Seniors Week, March 18-24), let's find out which day they will visit for the second time together.
Coding: Context Category: Social Process Category: Employ Proficiency Level: Level 3

Figure 3.11. Sample Coding of Problem Located in Page 113 of Textbook of Karataş (2018)

The question is related to the social activity of a group of friends. Hence, this question belongs to ***social*** context. To solve the question, students need to choose a strategy and apply calculations sequentially. This implies that the question is an ***employ*** question. Since the question does not include a complex situation, the students should select and apply simple problem-solving strategies, the question interprets ***Level 3.***

Example 1, Page 106: Mehmet Bey will renew the tiles of his bathroom. The base of the bath is square and has an edge length of 2 m. He liked a tile from a shop selling construction materials. The tiles that he wanted were 40x50 cm, and one box has 20 tiles. Accordingly, how many boxes should he buy? Explain the strategies that you could use to solve the problem. Give reasoning to your strategy.	
Coding: Context Category: Personal Process Category: Formulate+Employ=Formulate Proficiency Level: Level 5	

Figure 3.12. Sample Coding of Problem Located in Page 106 of Textbook of Karataş (2018)

This item is a *quantity* question and belongs to ***personal*** context since one's activity and the choice was described in the situation. The situation is clearly described but students were not asked to solve the question only. Instead, they were asked to give reasoning about choosing and applying a strategy, which makes the item more difficult than others, a ***Level 5*** question. Before solving the question, students need to represent the situation in another form. Formulating situation makes the question easier, and student can explain the strategy better. Hence, the question was categorized as ***formulate*** even though carrying out the calculations was also necessary.

CHAPTER 4

RESULTS

In this chapter, I provide the detailed results of the content analysis of the textbook written by Karataş (2018). I analyzed the data in a structure that would answer the research question “What are the learning opportunities provided by Turkish mathematics textbooks in the quantity area concerning the cognitive domains and proficiency levels in PISA 2012 Mathematics Framework? I provide the results in the following order: the analysis of textbook considering the categories of process domain, the analysis of textbook considering categories of context domain, the analysis of textbook considering the proficiency levels, and analysis of textbook considering the sections of the unit individually. Figure 4.1 illustrates the organization of the chapter. For each domain, process, context and proficiency levels, I gave an overall analysis at first. Then, the analysis of questions considering the separation of worked-out example and to-be-solved questions were given. Finally, to provide the opportunity to see the categorizations of questions of each section separately, I conducted the fourth part of the analysis. At this part, each section of the unit was evaluated in terms of process, context, proficiency levels, and worked-out questions and to-be-solved questions.

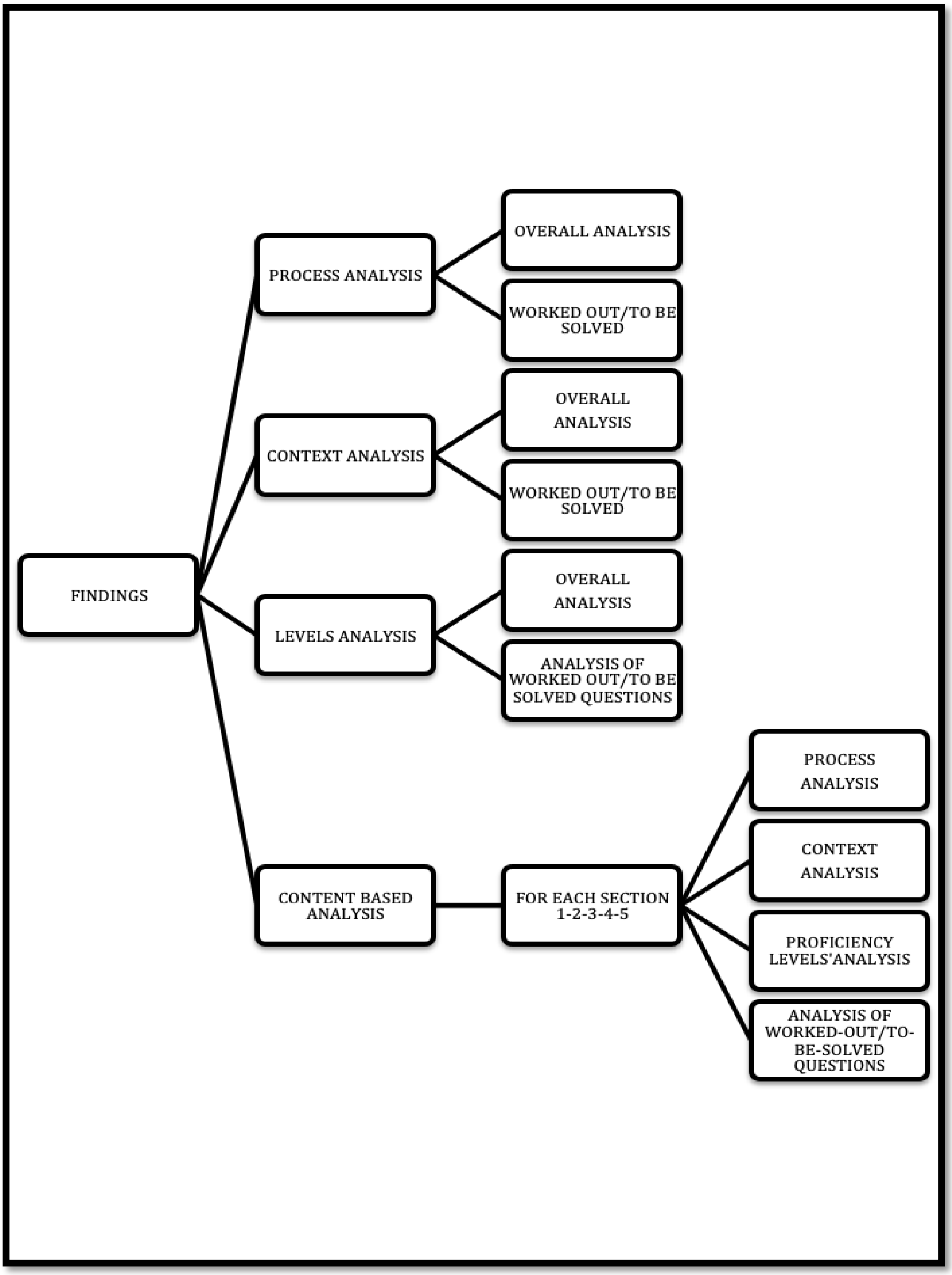


Figure 4.1. The Organization of Analysis of Findings

4.1. Results Regarding the Process Domain

4.1.1. Overall Results Regarding the Process Domain

I analyzed 400 questions from the textbook of Karataş (2018) in the study. I coded the 400 questions considering the three categories: process, context, and proficiency levels (see Table 3.4). Figure 4.2 illustrates the frequencies of each process categories generated by descriptive statistics provided by using Microsoft Excel. In Figure 4.2, 'formulating questions' and 'employing questions' were given into two subdomains. The findings were given into two-fold: a general picture of the distribution of each question type by process categories, and process categories for worked-out questions and to-be-solved questions.

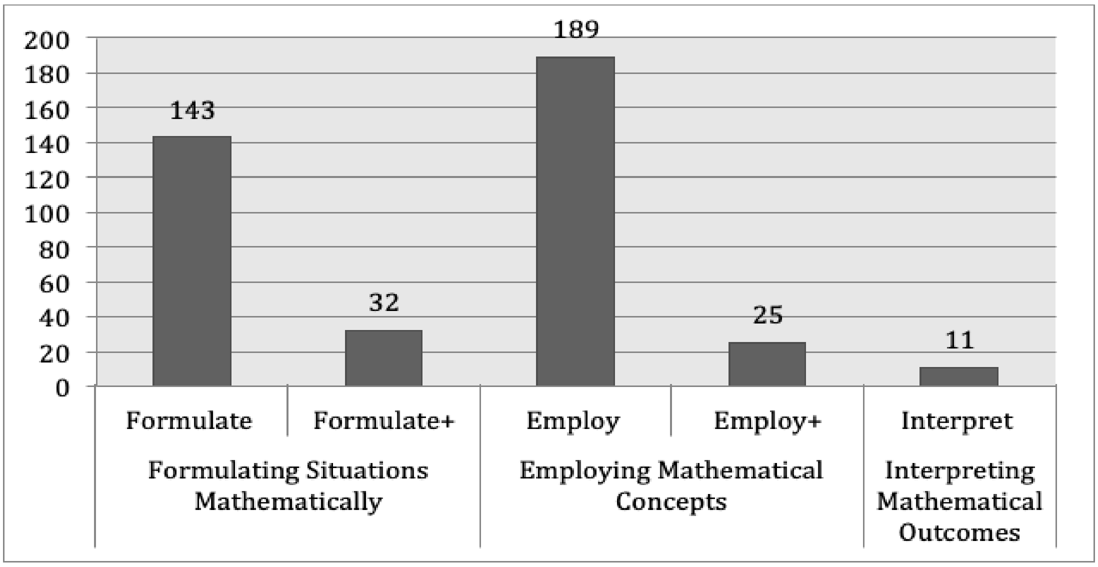


Figure 4.2. Frequencies of Questions Regarding Process Categories of the Coding Scheme

As mentioned by OECD (2013) in the PISA 2012 Mathematics Framework, 'the employing process indicates how well students can perform computations and manipulations and apply the concepts and facts that they know to arrive at a mathematical solution to a problem formulated mathematically (p.28)'. More than the half of the questions (53.5%) placed in textbook of Karataş (2018) are providing students opportunity to learn the questions including the process of employing mathematical concepts, facts, procedures, and reasoning (see Figure 4.2). Among the 214 questions coded as "employing" question, 25 questions included both employing and formulating processes. An example of an employing question were given in Figure 4.3. Working on these questions, students would be able to understand and formulate a situation, choose a strategy to solve the question and apply calculations to conclude the solving process. Hence, including these questions can evoke more problems solving behavior of students.

Exercise 11, Page 203:

Place the numbers 1, 2, 3, 4, 5, and 6 for the circles on the right side so that the sum of the numbers remaining between each corner and these corners will be equal.

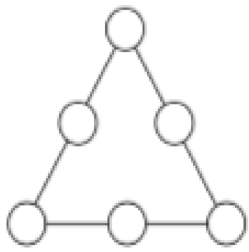


Figure 4.3. The Question Coded As Employing Question from the Page 203 of Karataş (2018) Textbook

The questions that provide the opportunity to formulate situations mathematically emerge about less than half of the questions (43.75%) placed in the quantity questions of the textbook. According to OECD (2013), 'the formulating process indicates how effectively students are able to recognize and identify opportunities to use mathematics in problem situations (p.28)'. In the textbook, 32 of 172 formulating questions contain both formulating and employing questions. The dominant process category of these questions was chosen as formulate since the solution strategy of these questions would be obvious if the students can formulate the given situation. Hence, these questions provide the opportunity for both formulating a given situation mathematically and carrying out calculations to conclude with an exact solution. Besides, by working on these questions, students would be able to recognize a solution strategy for the familiar problem contexts that they can face in their lessons and real life. Similarly, the questions coded directly as ‘formulating’ questions provide the opportunity for students to see different representations of numbers and situations, which allow them to switch between the representation of a problem and the solving strategies.

Example 1, Page 188:

“A person shares some amount of money as b TL per person. If the number of people increases by c and they share the same amount of money, how much TL per person decreases?”

Please write the algebraic expression of the situation described above.

Figure 4.4. The Question Coded As Formulating Question from the Page 188 of Karataş (2018) Textbook

According to OECD (2013), *‘integrating* process indicated how effectively students are able to reflect upon mathematical solutions or conclusions, interpret them in the context of a real-world problem, and determine whether the results or conclusions are reasonable (p.28)’. As illustrated in the figure, the questions coded as ‘*interpreting’* questions has a small amount (2.75%) within 400 quantity questions placed in the textbook of Karataş (2018). These questions provide the opportunity to reason on the solutions and results of the problem, and this process set a bridge between the mathematical results and real-life situations. Hence to be able to catch the targets of national curriculums that aims to raise students who have the capacity to handle problems in real life by using mathematical knowledge (MoNE, 2018a), the questions including the ‘interpreting, applying and evaluating mathematical outcomes' process were necessary. The frequencies of questions including interpreting process were low–only 11 within 400 questions, in Karatas (2008) textbook.

4.1.2. Results Regarding the Question Types and Process Domain

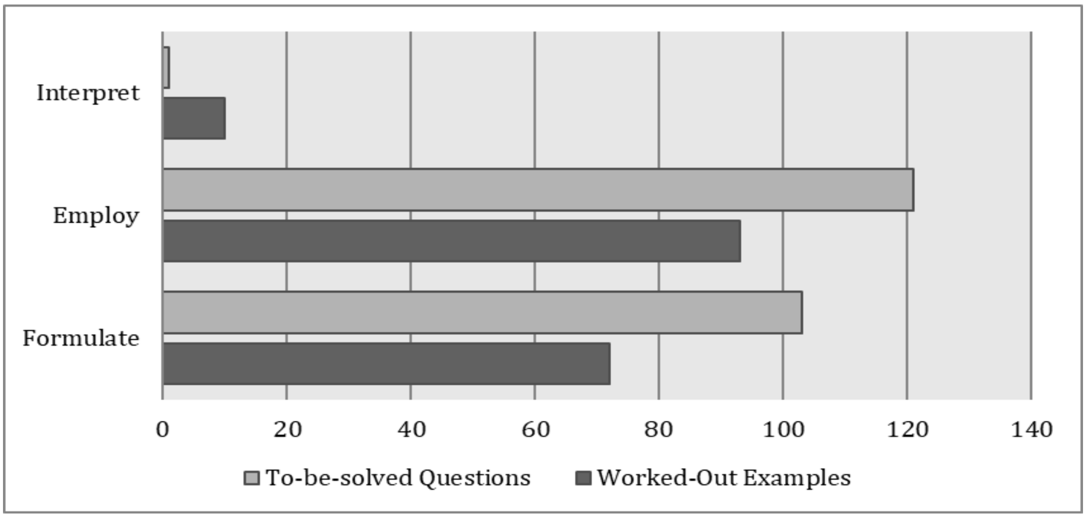


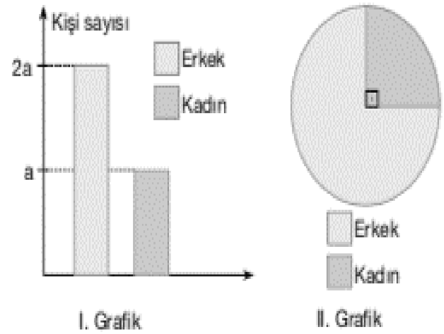
Figure 4.5. Overall Distribution of Questions Regarding Question Types and the Process Domain

The distribution of different kinds of questions by process categories also gave clues about the learning opportunities provided by these questions. Karataş (2018) provided questions under four different titles, which are 'example', 'exercises', 'it is your turn', and 'evaluation of the unit'. The questions under the heading of 'example' belong to worked-out examples where the questions of 'exercise', 'it is your turn', and 'evaluation of unit' belongs to to-be-solved questions. The worked-out examples referred to the questions given with an accompanying solution. However, to-be-solved questions are the questions that have no solution and given for the practice of students (Delil, 2006). The studies supported that in a well-structured lesson, worked out examples might be more efficient for the students who are unable to initiate problem-solving by themselves and need a guide to develop problem-solving skills (Renkl, Stark, Gruber, & Mandl, 1998). Nevertheless, it should be kept in mind that the problem-solving strategy given by example has a controlling effect on students and might be restricting for the students who could develop an individual problem-solving strategy. Especially, giving the problem-solving strategy of a problem, and providing similar questions in to-be-solved questions might limit the students' way of thinking. They may not develop insight to choose a problem-solving strategy.

In Figure 4.5, the distribution of question types by process categories was given. Karataş (2018) offered the opportunity for formulating process in less than half of the questions in the textbook. Within these formulating questions, about half of them were the worked-out example and the other half was to-be-solved questions Formulate-worked out questions allow students to see the solution of representing and formulating a mathematical situation, where formulate-to-be-solved questions allow for practicing on formulating process. Similarly, more than half of the questions, 214 out of 400, require *employing* process. Within all these *employ* questions, about half of them offer a solution for the students (employ-worked-out questions), and the other questions (employ-to-be-solved questions) expect to develop a problem-solving strategy and apply it to get a result. These questions are suitable for the students who are comfortable with selecting and applying problem-solving strategies, where employ-worked-out questions help students to learn these strategies from the textbook. An example of worked-out employ question was shown in the Figure 4.6. In the textbook of Karataş (2018), a very small amount of the quantity questions, 11 questions, include interpreting process. 10 of these interpreting questions (interpreting-worked-out questions) have an accompanying solution and only one question (ınterpreting-to-be-solved) was asked students to practice on an interpreting question. Findings revealed that students who taught with only this textbook would not have a chance to work on situations that require making interpretations for real-life and generalization through the structures given in the curriculum.

Example 2, Page 186:

The first graph on the right side shows the number of men and women at a wedding. 17 male, 2 female guests attend to the wedding later. The final distribution of men and women in the wedding hall is shown by graph 2.
Let us find out how many people are in the wedding hall.



Solution:

Initially, it is understood from the first chart that there are 2a male and a female in the hall. The number of males was 2a + 17, since 17 males come to the hall; and the number of women was a + 2, due to the 2 females who attended to the wedding later. The circle has 270 ° for men and 90 ° for women as we can see from the second chart. Hence, the number of men is 3 times the number of women. In that case;

$2a + 17 = 3 \cdot (a + 2)$

$2a + 17 = 3a + 6$

$3a - 2a = 17 - 6$

$a = 11$

Here the number of people in the hall in the last case;

$2a + 17 + a + 2 = 3a + 19$

$= 3 \cdot 11 + 19$

$= 52.$

Figure 4.6. Sample Coding of a Worked-Out Employing Question from the Page 186 of Karataş (2018) Textbook

4.2. Results Regarding the Context Domain

4.2.1. Overall Results Regarding the Context Domain

I analyzed the data came from quantity-typed questions of Karataş (2018) textbook by using five categories—personal, social, occupational, scientific, and decontextualized. The frequency analysis was conducted to understand how Karataş (2018)'s textbook provides an opportunity for students to deal with contextualized mathematical problems.

Table 4.1. Frequency of Questions Regarding Context Categories

The Context Categories Provided by Coding Scheme	Personal	Social	Occupational	Scientific	Decontextualized
Frequencies	28 (7%)	20 (5%)	18 (4.5%)	42 (10.5%)	292 (73%)

In the PISA Assessment, the four context categories were used equally (about 25%). However, no evidence supports these ratios are helpful or necessary for the design of a textbook or assessment. According to Hickendorff (2013), the use of real-life context can make the question easier to solve compared to numerical questions, but this is the case for non-complex situations. If the problem includes a complex situation, the language becomes a key factor to solve the question. In the textbook of Karataş (2018), about three-quarters of the questions were decontextualized, and one-quarter of the questions have a real-life context.

Although there are no criteria to decide the proper ratio of real-life questions in a textbook, I would tell that including merely 27% contextualized questions is low considering MoNE (2018) emphasis on the importance of real-life context questions in a textbook. According to Table 4.1, contextualized questions' ratio was ranging from 4.5% to 10.5% where the scientific category was the highest. Since Karataş (2018) did not aim to have an equal distribution for categories of contextualization, percentages of categories within contextualized problems in the textbook varies which is not surprising (see Figure 4.7).

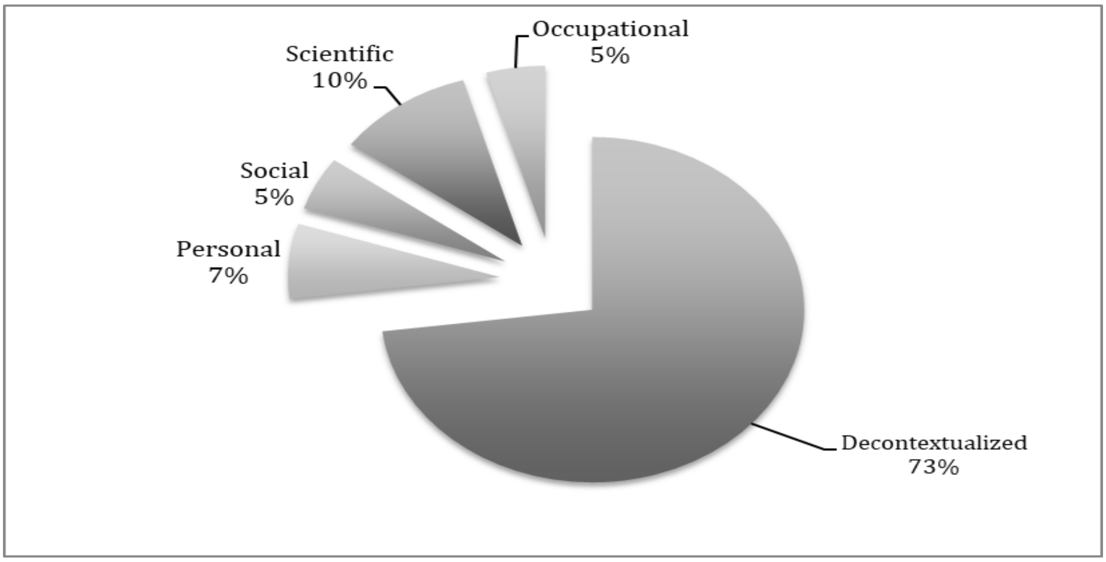


Figure 4.7. The Distribution of Questions Regarding the Contextualization

4.2.2. Results Regarding the Question Types and Context Domain

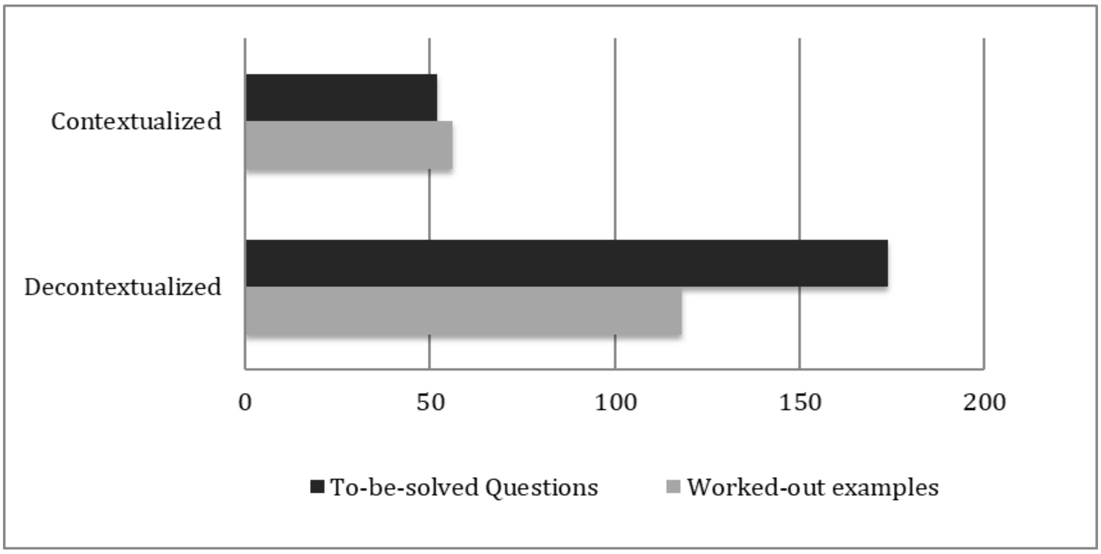


Figure 4.8. Overall Distribution of Worked-Out Examples and To-Be-Solved Questions Regarding the Contextualization

The distribution of question types by contextualization was given in the Figure 4.8. Each cell in the Figure 4.8 attributes to a different type of question. The decontextualized-worked out questions had a great ratio within all questions. These types of questions allow students to see the solution to a problem related to the mathematical structures only. The students could learn the rules and properties of concepts, carrying out the sequential procedures and limits of the mathematical concepts. Besides, decontextualized to-be-solved questions support students for practicing these skills that they learned through the worked out questions. As illustrated in Figure 4.9, the students should apply rules and computation to get the result of the question. The decontextualized to-be-solved questions had the greatest ratio in the textbook of Karataş (2018). In the textbook, I coded 56 questions within 400 questions as contextualized worked-out questions. These questions important since the students learn about the use of problem-solving strategies placed in a real-life context. To provide the target of MoNE (2018), that is rising students who can face the mathematical challenges of daily life, one of the most important questions belong to the category of contextualized to-be-solved questions. Solving these questions, students develop and apply problem-solving strategies for real-life situations, gather insight for the solutions of problems, and learn how to relate the mathematical structures into daily life. However, in the textbook of Karataş (2018) students rarely had the opportunity to learn these skills.

Exercise 4, Page 164:

Find the result of the expression given as folows:

$$\frac{3^3 \cdot 9^2 \cdot 27^2}{9 \cdot 27 \cdot 81}$$

Figure 4.9. Sample Coding of a Decontextualized To-Be-Solved Question from the Page 164 of Karataş (2018) Textbook

Exercise 45, Page 209:

Two racers move from point A at the same time and in the opposite direction on a circular track with speeds of 12 and 8 m / min. Since the circumference of the circular track is 400 m, how many minutes does is take the slowest one to reach point A, after their meeting?

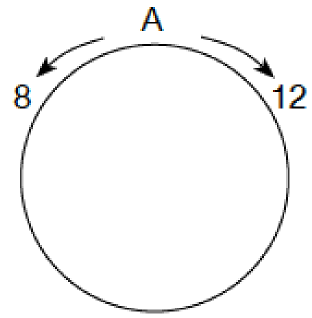


Figure 4.10. Sample Coding of a Contextualized To-Be-Solved Question from the Page 209 of Karataş (2018) Textbook

4.3. Results Regarding the Proficiency Levels

4.3.1. Overall Results Regarding Proficiency Levels

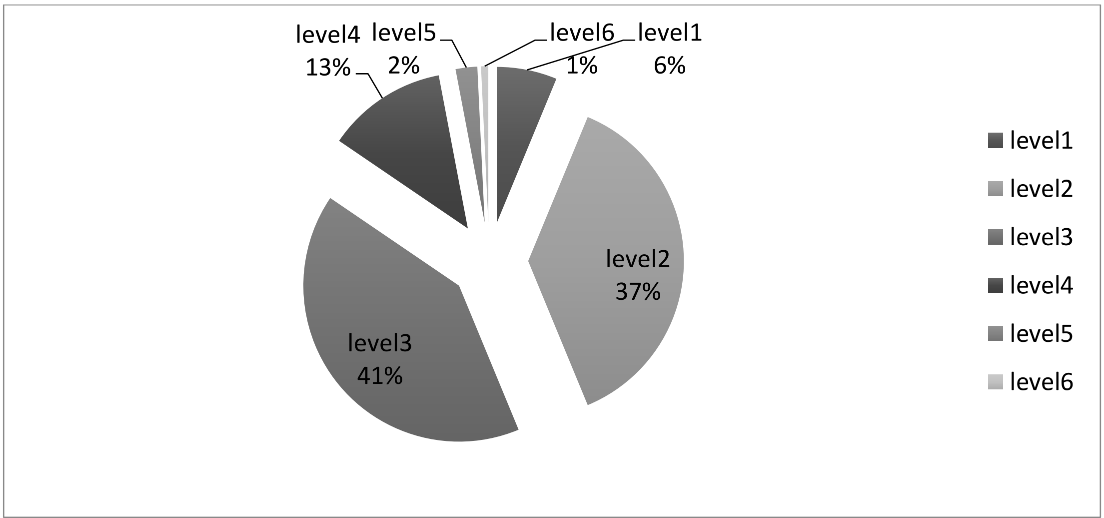


Figure 4.11. Overall Distribution of Questions Regarding the Proficiency Levels

The proficiency levels refer to different difficulty levels for questions. In PISA Assessment, the questions were prepared considering different proficiency levels to provide the opportunity for students at different levels. In the framework, each level was described considering what a student can do at that level. However, in this study, the questions were coded considering proficiency levels. The questions were coded according to which level and above of students were able to solve the question by using the indicators described in the coding scheme. According to descriptive statistics provided by MS Excel, the highest amount of questions belong to level2 and level3. At these levels, the questions provide the opportunity for making direct inferences based on the context, carrying out calculations, and selecting and applying simple problem-solving strategies. In the textbook of Karataş (2018), about 80% of the questions provide and evoke opportunities for those capabilities. Level 4 questions are about 13% of all quantity questions. The questions that belong to level 4 requires working on formulating complex problem situations, having the insight to select and apply problem-solving strategies for those complex problems and making reasoning on their solutions. Level1 questions require following direct instructions and carrying out calculations based on those instructions. Hence, they are suitable for recognizing and learning the fundamental concepts of a unit. In the textbook of Karataş (2018), these questions emerged about 6% of whole questions related to quantity area. The least frequencies belong to level5 (2%) and level6 (3%) in the textbook of Karataş (2018). The questions at level5 and level6 provide the opportunity to work on high-level complex mathematical situations. Working on these questions, students gather the skills of developing individual models for complex mathematical situations, linking different sources of information to get a solution, generalizing the results of solutions and connecting these results with the real-life situations. Looking at the Figure 4.11, the findings reveal that the students working with this textbook do not have the opportunity to gather these skills.

4.3.2. Results Regarding the Question Types and Proficiency Levels

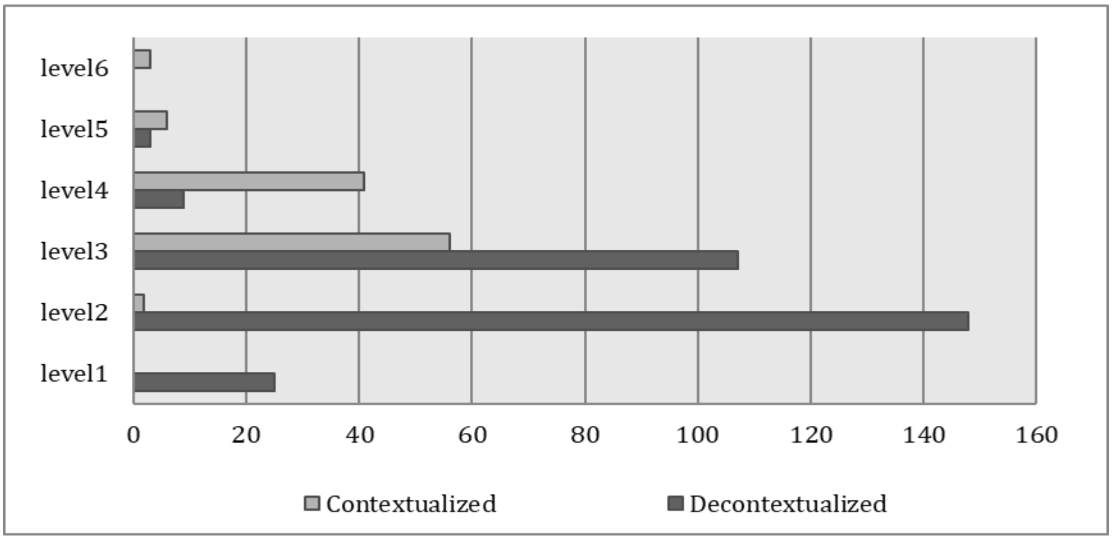


Figure 4.12. Overall Distribution of Worked-Out Examples and To-Be-Solved Questions Regarding the Proficiency Levels

Figure 4.12 illustrates the distributions of worked-out examples and to-be-solved questions according to proficiency levels. According to findings in Figure 4.11, the amount of to-be-solved questions was more than the worked-out questions for level 1-2-3. This finding showed that students would practice on these questions individually. However, this is not the case for the higher levels since Karatas (2018) provided solutions with worked-out questions mostly for students. This fact showed that in the textbook students individual working opportunities were hindered. The findings also revealed that Karatas (2018) did not provide adequate opportunities for higher-order problem-solving skills for students. However, worked-out questions with solutions might be helpful for some students who cannot initiate developing individual models for complex problems if the textbook provides more follow-up practice questions for students.

4.4. Results Regarding the Sections of the Unit

In the textbook of Karataş (2018) , I analyzed 400 questions being related to the quantity area described in PISA 2012 Mathematics Framework (OECD, 2013). These questions were separated into 5 different sections under the headings of ‘numbers sets’, ‘division rules’, ‘equations and inequalities’, ‘exponential numbers’ and ‘applications of equations and inequalities’. In this section, I gave the analysis of questions of each section considering the process and context categories, and proficiency levels of questions. The general picture of the distributions of questions for the sections of unit and categories of coding scheme were given in Figure 4.13. The analysis of units were given after the general picture of the distribution.

Table 4.2 The Distribution of Questions Regarding the Sections of the Unit

		Number Sets	Division Rules	Equations and Inequalities	Exponential Numbers	Application of Equations and Inequalities
PROCESS	Formulate	30	27	37	33	48
	Employ	2	53	-	73	87
	Interpret	-	5	1	-	4
CONTEXT	Scientific	-	6	2	1	33
	Personal	-	5	-	-	22
	Occupational	-	8	1	-	9
	Social	-	6	1	-	14
	Decontextualized	32	60	34	105	61
PROFICIENCY LEVELS	Level 1	8	1	3	4	9
	Level 2	21	22	23	53	31
	Level 3	3	41	11	47	61
	Level 4	-	15	-	2	33
	Level 5	-	4	1	-	4
	Level 6	-	2	-	-	1

4.4.1. Number Sets

In the textbook, Karataş (2018) defined number sets, natural numbers, integers, rational and irrational numbers, and real numbers under the subtitle of 'number sets'. Operations and different representations related to these numbers sets were given to the students. This subtitle is an introduction to number sense for students. Under this subtitle, I coded 32 questions. The questions in this part provide opportunities for formulating and representing numbers. Almost all the questions were formulating questions where employing questions were only two and there were no interpreting questions. The formulating questions provide an equal opportunity both for learning from the worked-out examples and practice on to-be-solved questions. The subtitle is an introduction and finding formulating questions was natural. However, employing and interpreting questions could help students to work on the number sets and relate these number sets with real-life situations.

Example 1, Page 88:

Define the place of $\sqrt{2}$ on the real line.

Figure 4.13. Sample Decontextualized Formulating Question from the Page 88 of Karataş (2018) Textbook

Exercises 5, Page 92:
Let a, b, and c be integers.
$a.b = 12$
$b.c = 16$
Calculate the smallest value for $a + b + c$.

Figure 4.14. Sample Decontextualized Employing Question from the Page 92 of Karataş (2018) Textbook

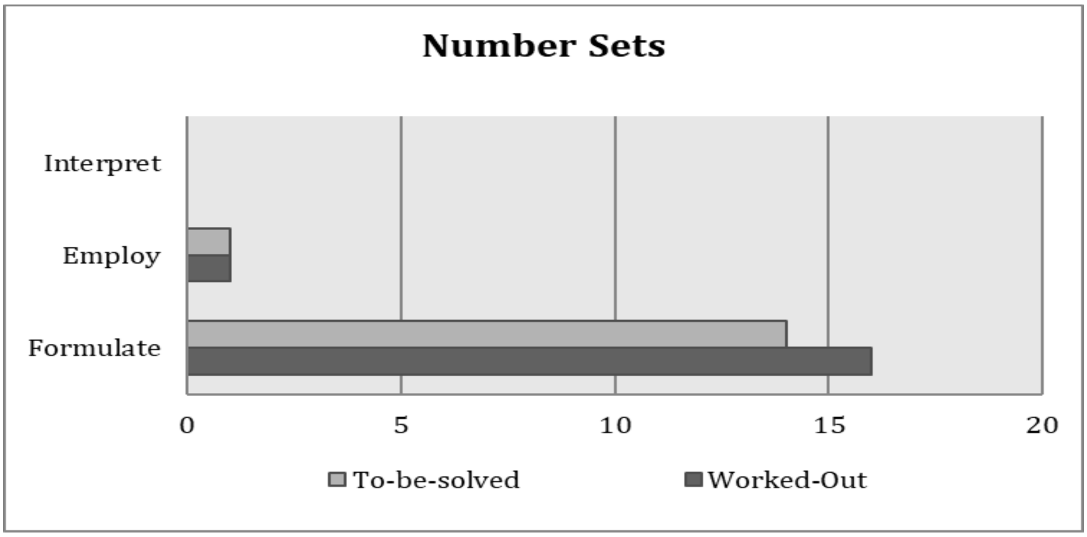


Figure 4.15. The Distribution of Worked-Out Examples and To-Be-Solved Questions of the Number Sets Section Regarding the Process Domain

As interpreted in Figure 4.16, none of the questions was contextualized in number sets section, which means the students who worked with Karataş (2018) textbook, did not have the opportunity to see the number sets in a real-life context.

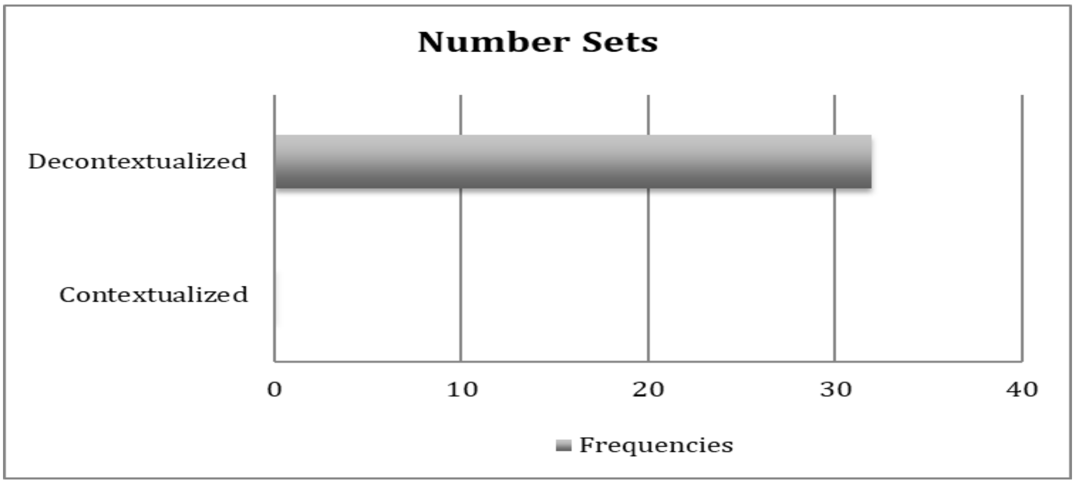


Figure 4.16. The Distribution of Questions of the Number Sets Section Regarding the Contextualization

The proficiency levels of the questions varied from level 1 to level 3 where level 2 questions comprise 66% of the questions under this subtitle. I did not find any higher-level question under this subtitle that belongs to level 4-5-6. This result reveals that this part provided an opportunity for working on familiar contexts, making direct inferences and following sequential procedures and calculations, which are explained clearly and directly in the problem context.

It is your turn 1, Page 87:

For a and b,

$$a = \frac{b+5}{b-4}$$

Find the values of a for which b cannot be calculated.

Figure 4.17. Sample Level 3 Question from the Page 87 of Karataş (2018) Textbook

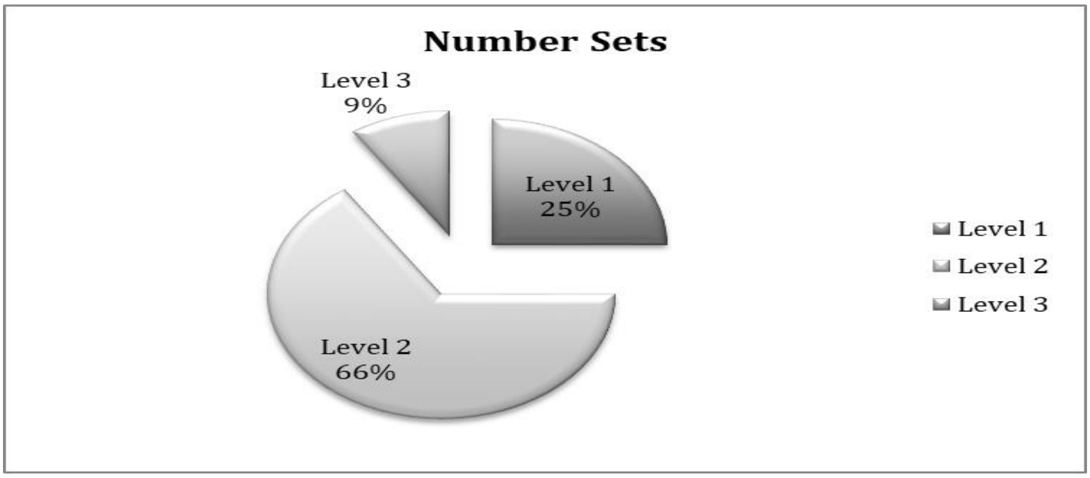


Figure 4.18. The Distribution of Questions of the Number Sets Section Regarding the Proficiency Levels

Within 32 questions that I coded belonging to number sets, half of the questions were worked-out questions and the other half was to-be-solved questions, which imply that students could both learn from the solutions and practice on the questions, which are adequate both for the students who need support and for the students who can work individually.

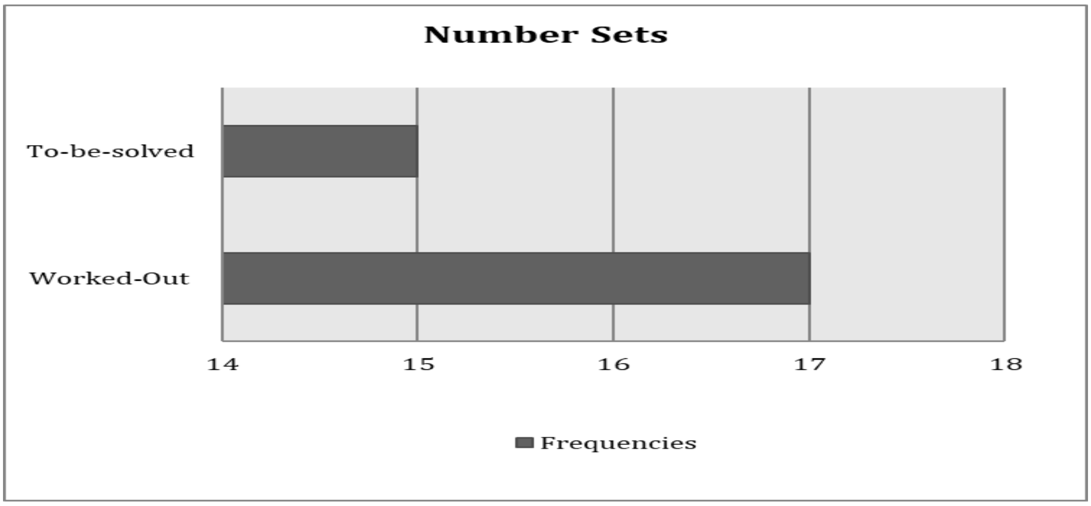


Figure 4.19. The Distribution of Worked-Out Examples and To-Be-Solved Questions of the Number Sets Section

4.4.2. Division Rules

The second subtitle of the unit belongs to division rules in the textbook of Karataş (2018). Under this subtitle, students will work on division rules related to integers, properties of the greatest common divisor and least common multiple, and repetitive situations from real life (MoNE, 2018a). Working on the textbook of Karataş (2018), I evaluated 85 questions related to division rules concerning domains described in the coding scheme. The questions under this subtitle have formulating, employing, and interpreting processes, which make the section having heterogeneous structure compared to other subtitles. The questions under this title were mostly (about 60%) employ questions, which imply that students would learn and practice about selecting a problem-solving strategy, carrying out calculations related to division rules and conclude the problem to get an exact result. The questions were about equally distributed over worked-out questions and to-be-solved questions. This shows that students have an equal chance to follow solutions of Karataş (2018) and practice on the questions individually. 27 questions (about 30% of 85 questions) include formulating process in this part. These questions allow students making representations and formulating situations about the division rules with an equal opportunity to see both worked-out examples and to-be-solved questions.

Example 1, Page 95:

Let xyz be a three-digit natural number. Show that if $x + y + z = 3k$ ($k \in Z+$), then xyz is divisible by 3.

Figure 4.20. Sample Interpreting Question from the Page 95 of Karataş (2018) Textbook

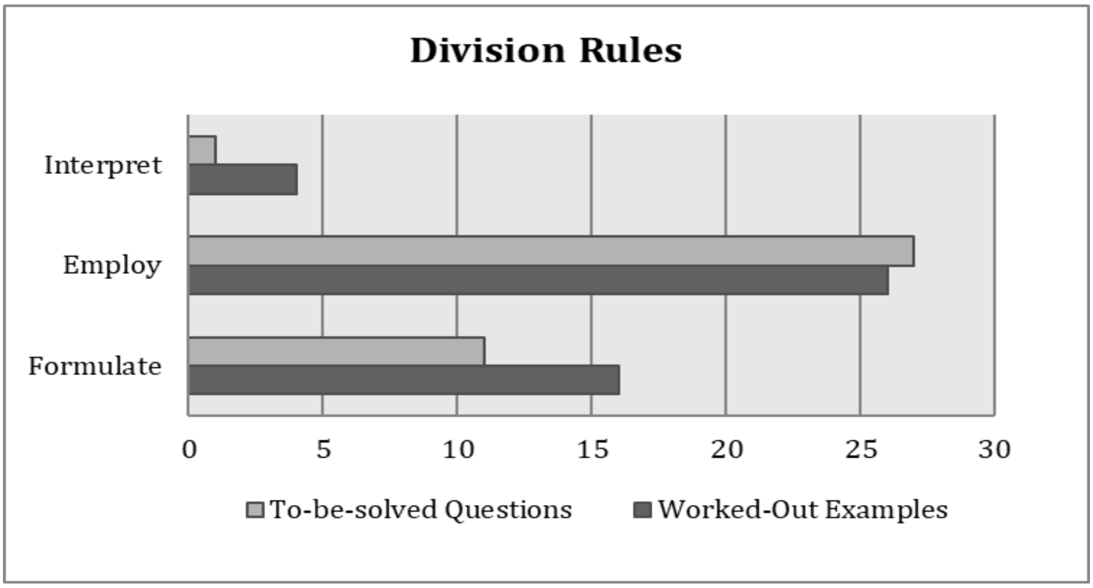


Figure 4.21. The Distribution of Worked-Out Examples and To-Be-Solved Questions of the Division Rules Section Regarding the Process Domain

The division rules section includes both contextualized and decontextualized questions, but decontextualized questions have a much greater ratio, about 70% of the 85 questions. This section has a target for the use of real-life situations related to the division (MoNE, 2018a). Considering this, the ratio of contextualized questions was inadequate for the division rules section.

Example 1, Page 109:

Elif counts her clasps as group of three, five, and six, and two clasps remain for each time. Assuming that Elif has more than 14 clasps, find at least how many clasps does Elif has.

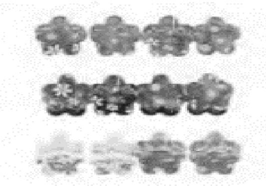


Figure 4.22. Sample Contextualized Employing Question from the Page 109 of Karataş (2018) Textbook

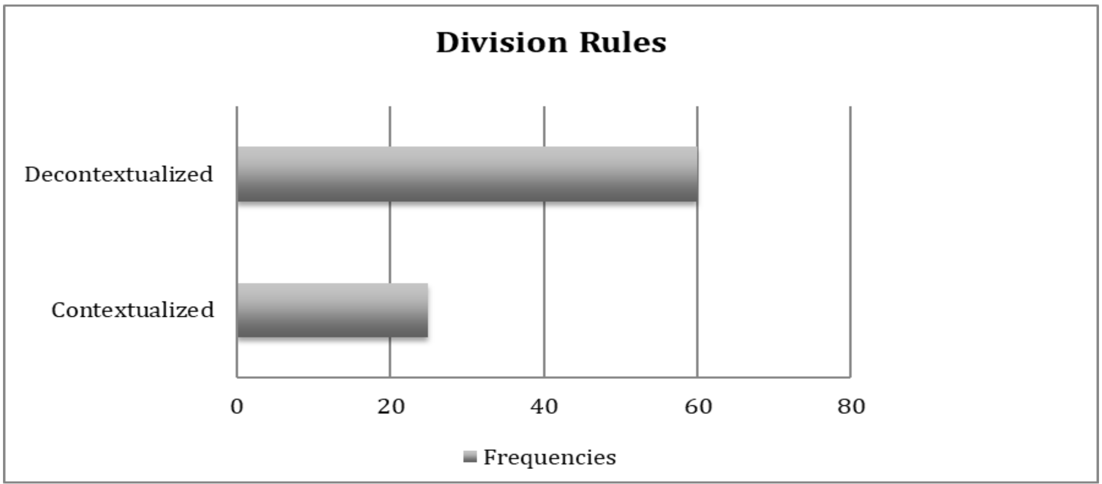


Figure 4.23. The Distribution of Questions of the Division Rules Section Regarding the Contextualization

The proficiency levels of the questions in this section were also heterogeneous compared to other sections. The proficiency levels vary from level 1 to level 6. The biggest amount belongs to level 3. Level 3 questions provide opportunity on working simple problem situations, following sequential calculations, and selecting and applying strategies for problems where the strategy of the problem does not require insight. Hence, the students working with this textbook would work on non-complex division problems mostly. In the section, I found level 5 and level 6 questions rarely. Therefore, the students would have a little chance to work on high-level questions of division rules.

Example 1, Page 115:

The 100th anniversary of the Çanakkale Victory was celebrated on Wednesday, March 18, 2015. Let us find out which date and day the 50th anniversary is celebrated.

Figure 4.24. Sample Contextualized Level 6 Question from the Page 115 of Karataş (2018) Textbook

As Figure 4.26 shows, more than half of the questions under this subtitle were worked-out questions. Even though the process categories and proficiency levels of the questions were heterogeneous, the students would have opportunity to learn the division rules by the solution of Karataş (2018) mostly, and they would have less chance to develop models and work on them individually.

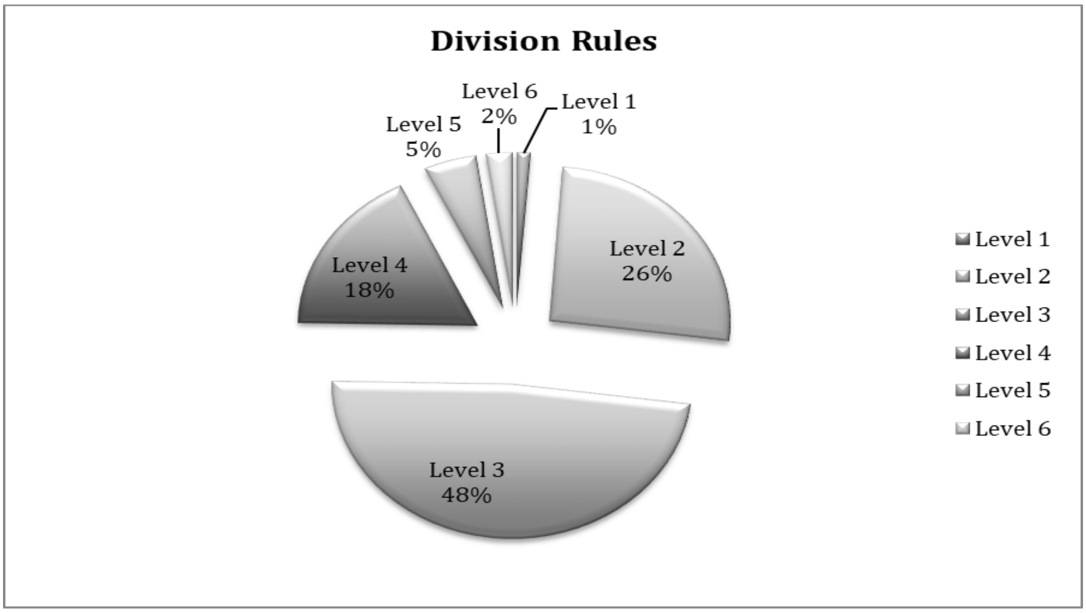


Figure 4.25. The Distribution of Questions of the Division Rules Section Regarding the Proficiency Levels

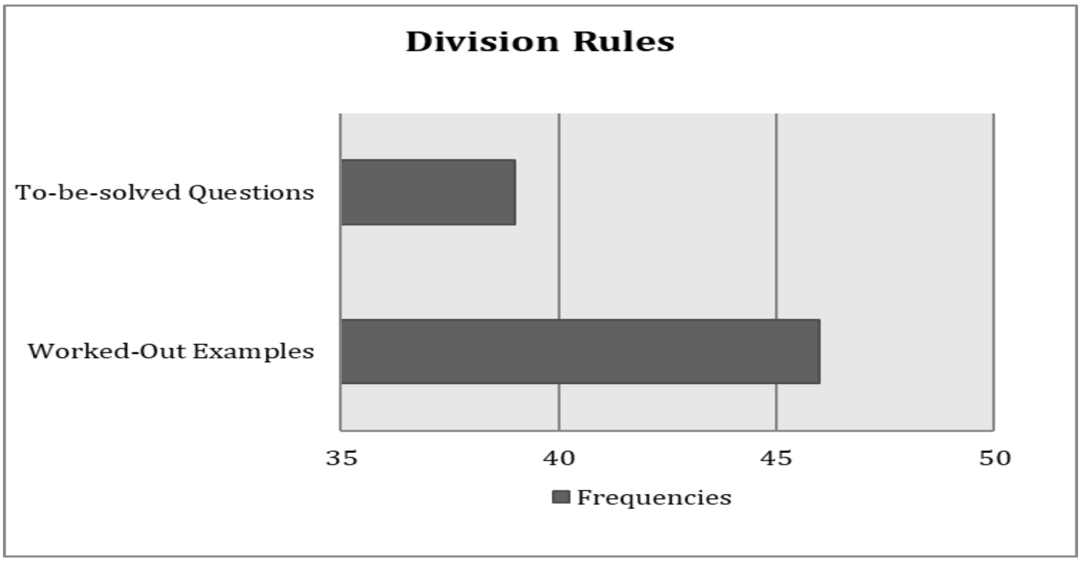


Figure 4.26. The Distribution of Worked Out Examples and To-Be-Solved Questions of the Division Rules Section

4.4.3. Equations and Inequalities

In the textbook, Karataş (2018) explained the interval concept, absolute value for real numbers and integers, the idea of less and greater, and the concept of variable and unknown under the subtitle of ‘equations and inequalities'. Since the concept of ‘variable' is related to the change and relation area described in PISA Framework, I did not include this subtitle into this study. Within this section, I coded 38 questions. Due to the learning outcomes of the section, most of the questions were formulating questions. The students were mostly asked to represent the interval into another form. The questions similar to the form ‘represent the interval (-1, 7) and (2, 5) in the real line' (Karataş, 2018, p.118)' was very common in this section. However, I found no employ question in this section. The students were never asked to develop or work a strategy related to the use of intervals or absolute value. There exist one interpreting question in this section. The question was describing the events that a construction worker had during a work accident. The question form was as follows; " Describe and evaluate this story considering the concept of inequality" (Karataş, 2018, p.117). The concept of the interval is very adequate to set relations with real life. Therefore, the use of more employ and interpret questions was necessary for this section. The broad use of to-be-solved questions provides students the opportunity to practice more.

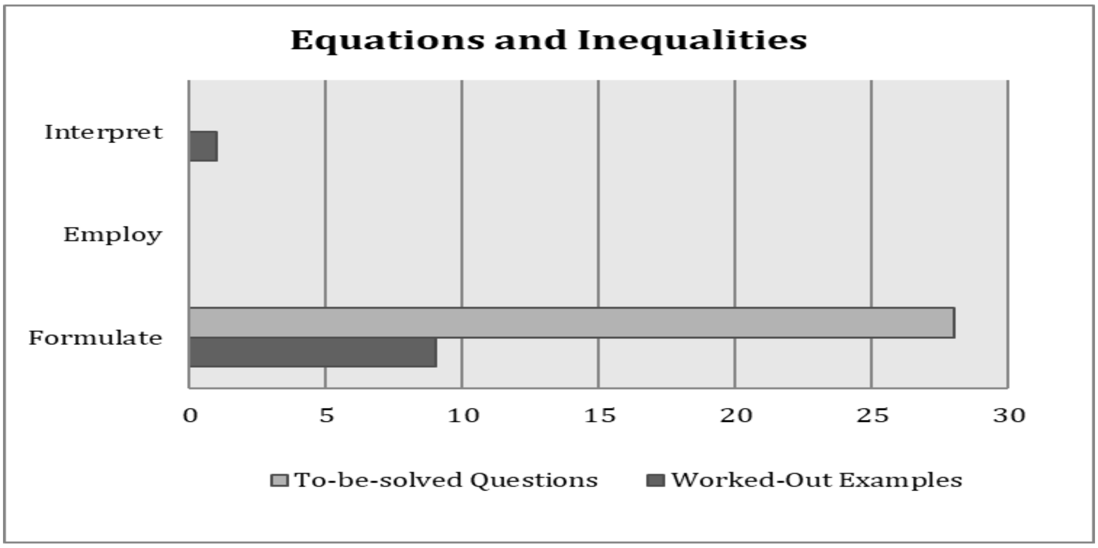


Figure 4.27. The Distribution of Worked-Out Examples and To-Be-Solved Questions of the Equations and Inequalities Section Regarding the Process Domain

At this section, the questions were mostly decontextualized, and I found only four contextualized questions. Since the structure and the concepts of the section was adequate to be used in real-life contexts, this ratio was inadequate. The students should have more chance to see these concepts in real-life contexts.

Example 1, Page 119
Let's find the intervals for the following expressions, and show them in number line.

a) Haktan's height was 52 cm when he was born. Now his height is less than or equal to 160 cm.
b) In Tokat, the temprature is greater than -11°C, and less than 16°C.
c) Zeynep picks a number in her mind, and tells that the number is greater than 5.

Figure 4.28. Sample Contextualized Question from the Page 119 of Karataş (2018) Textbook

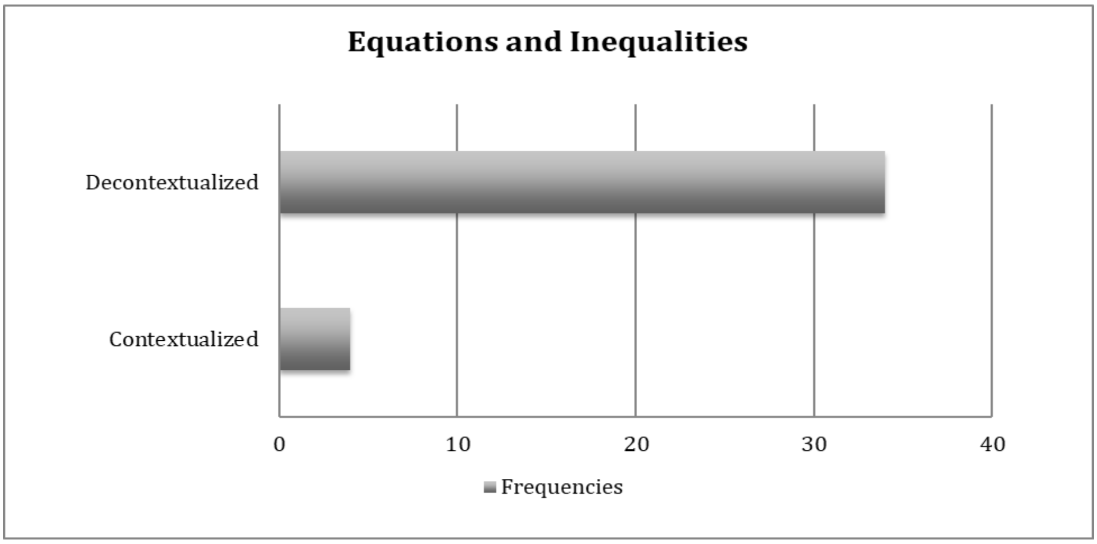


Figure 4.29. The Distribution of Questions of the Equations and Inequalities Section Regarding the Contextualization

Under this section, I found questions from level1-2-3 and level5. 60% of the questions were from level2, and 30% from level3. This result reveals that the students would have the opportunity to work on the familiar contexts, make direct inferences that were given directly, and solve the simple problems by following sequential procedures.

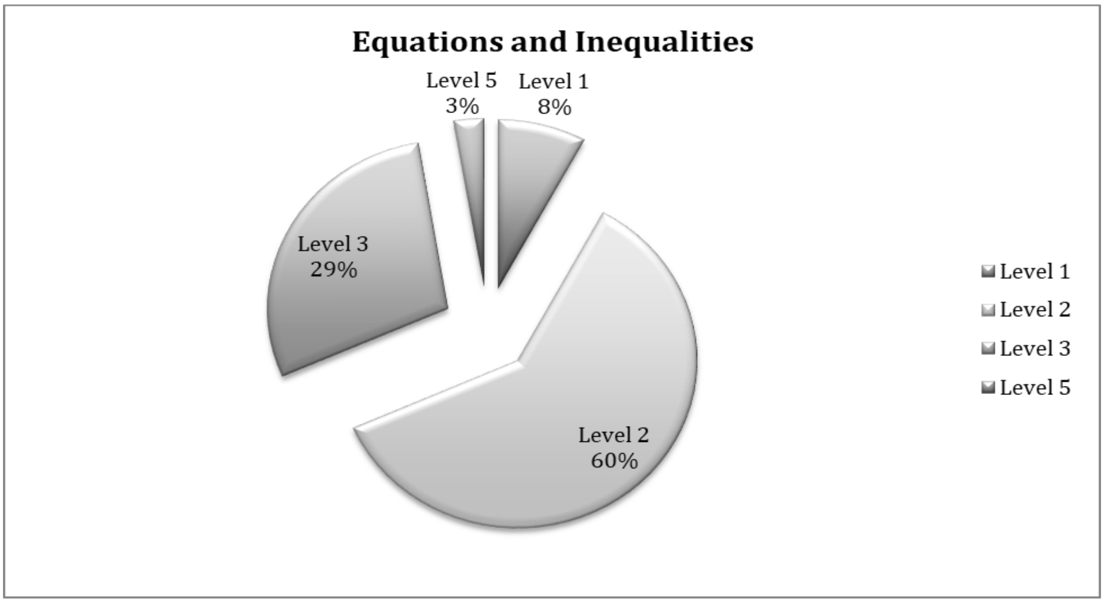


Figure 4.30. The Distribution of Questions of the Equations and Inequalities Section Regarding the Proficiency Levels

The questions were mostly to-be-solved questions under the subtitle of 'equations and inequalities'. Hence, the students would work on the questions by themselves which could have a positive effect on the learning the applications of the concepts.

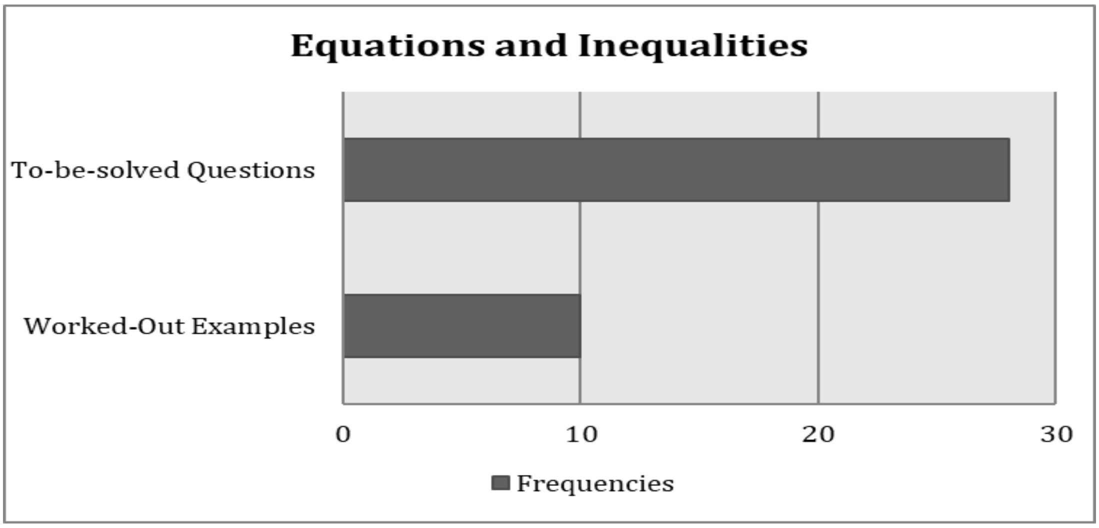


Figure 4.31. The Distribution of Worked-Out Examples and To-Be-Solved Questions of the Equations and Inequalities Section

4.4.4. Exponential Numbers

Under the subtitle of ‘exponential numbers’ of Karataş (2018), the students would learn the properties and representation of exponential numbers and root numbers. I coded 106 questions within this section. The questions were mostly employ questions where I also found formulate questions. Using both formulating questions and employing questions in a section helps students both to formulate and represents situations related to the concepts, and solve the problems placed in a real-life context. Karataş (2018) did not use any interpreting questions. Even though relating the root numbers with real-life and including situations that students will face in their real-life could be difficult, exponential growth could be more familiar for the students. Forcing students to think about the use of these contexts in real life might evoke high-level thinking skills and could change their habits of mind, but Karataş (2018) did not use this strategy in his textbook.

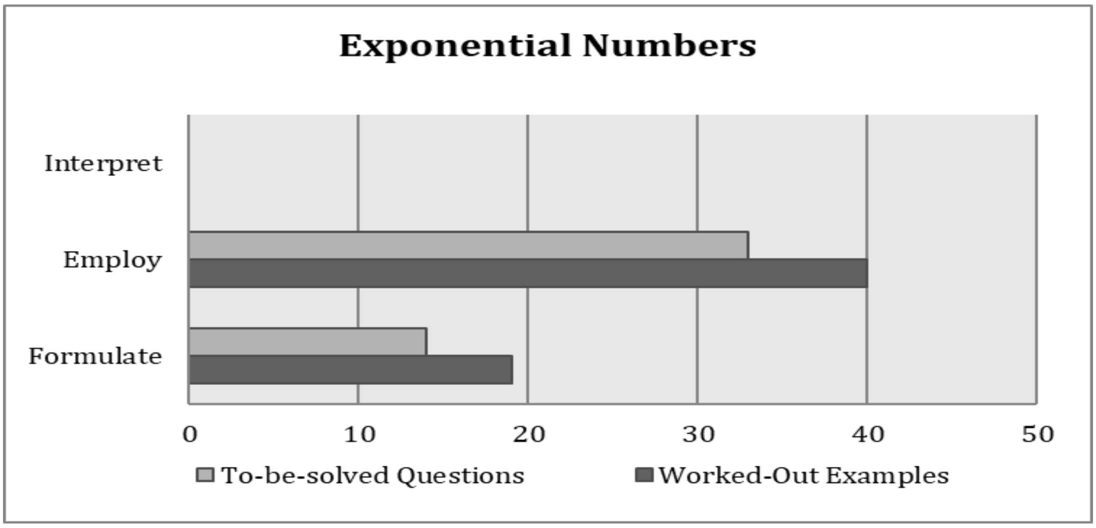


Figure 4.32. The Distribution of Worked-Out Examples and To-Be-Solved Questions of the Exponential Numbers Section Regarding the Process Domain

Evaluating 106 questions related to the exponential numbers I found only 1 contextualized question which hindered students chance to see these concepts in real life. The only contextualized questions coded within the scientific context was given in the Figure 4.33.

Example 1, Page 162:

One bacterial species divides every hour and doubles. It was found that 3 bacteria that were allowed to grow under favorable conditions were 768 after a certain time. Let's find out how many hours were passed since the bacteria were released.

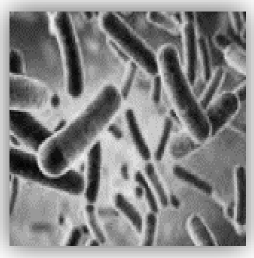


Figure 4.33. Sample Contextualized Question from the Page 162 of Karataş (2018) Textbook

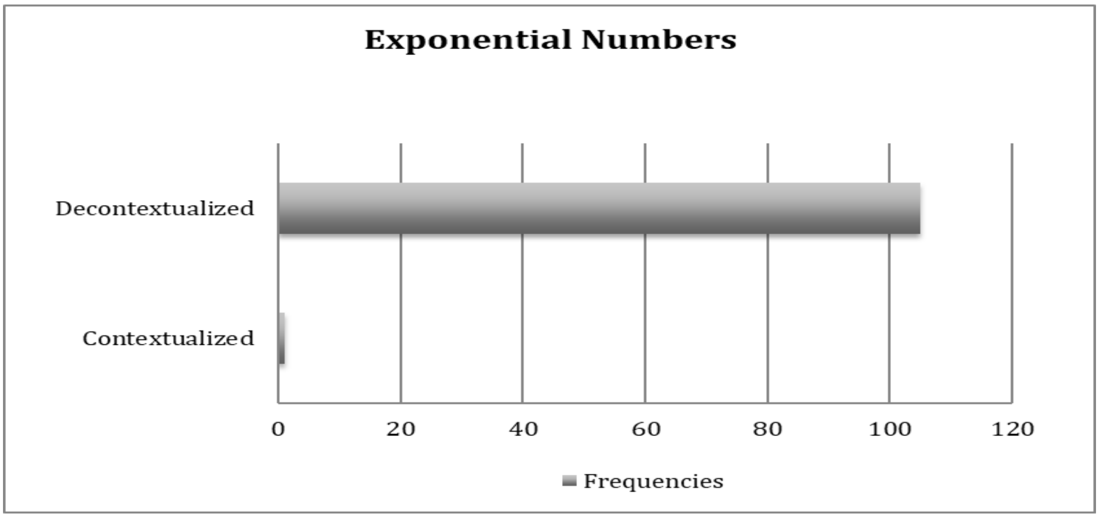


Figure 4.34. The Distribution of Questions of the Exponential Numbers Section Regarding the Contextualization

The exponential and root number questions were illustrating level 2 and level 3 for the 94% of the 106 questions. I found very few questions that belong to level 4, which shows students were not challenged about solving high-level exponential numbers questions. The use of worked-out examples and to-be-solved questions was almost equal. Including more to-be-solved questions could help students to practice what they have learned. The only contextualized question was also a worked out questions, so the students never supported for the practice of a contextualized exponential number question.

It is your turn 1, Page 175:

Find the value of x for the following equation:

$$\frac{3x-5}{\sqrt{4}-\sqrt{12}}=\sqrt{3}+1$$

Figure 4.35. Sample To-Be-Solved Question from the Page 175 of Karataş (2018) Textbook

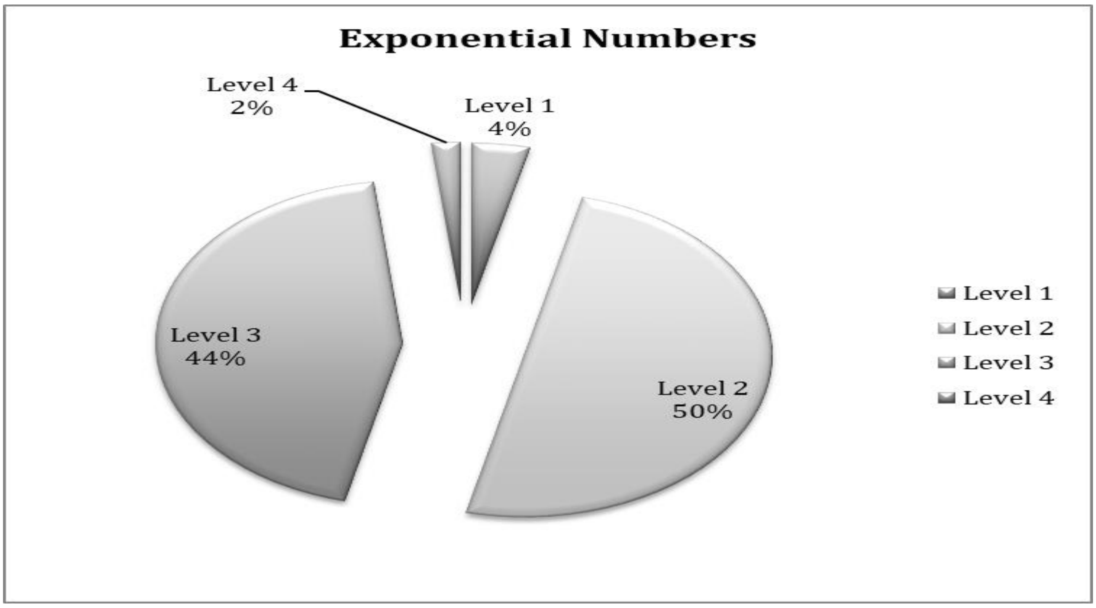


Figure 4.36. The Distribution of Questions of the Exponential Numbers Section Regarding the Proficiency Levels

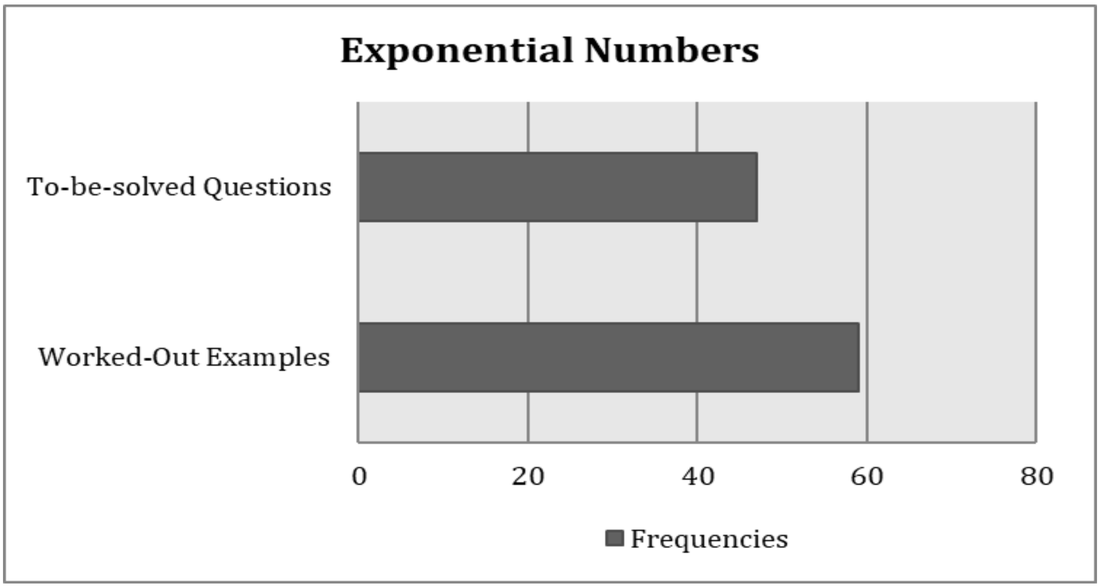


Figure 4.37. The Distribution of Worked-Out Questions and To-Be-Solved Questions of the Exponential Numbers Section

4.4.5. Applications of Equations and Inequalities

The last section under the equations and inequalities unit at the textbook of Karataş (2018) was 'the application of equations and inequalities'. This is one of the most important sections in the unit since students should have the chance of applying what they have learned about concepts. I coded 142 questions under this subtitle. Since the section is related to the application of concepts, employing questions were the majority compared to others. I found 87 questions that require employing process where most of them were to-be-solved questions. This shows that students would have the chance to work on these questions individually. They need to select and apply strategies for problems and carry out calculations to solve the questions. In addition, there exist 4 interpreting questions which help students to make generalizations related to the concepts and relate concepts to the real-life. However, the use of more interpreting questions could help students to develop these skills.

Example 1, Page 192:

For an electiricity fee, the payment configuration for the late fee is as follows:

- For the first 4 days, 5 TL in total.
- For the days between 5-14, $\frac{1}{400}$ of the total fee.
  After this period,
- For each subsequent 10-day period, a daily delay increase of twice the previous rate is taken.
  Therefore, if you would pay 200 TL electricity fee after 20 days, how much money would you pay in total.

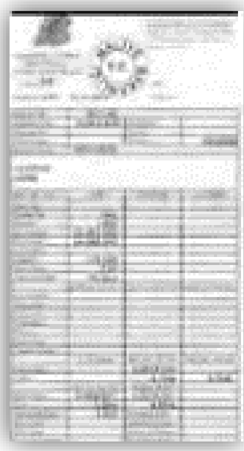


Figure 4.38. Sample To-Be-Solved Interpreting Question from the Page 192 of Karataş (2018) Textbook

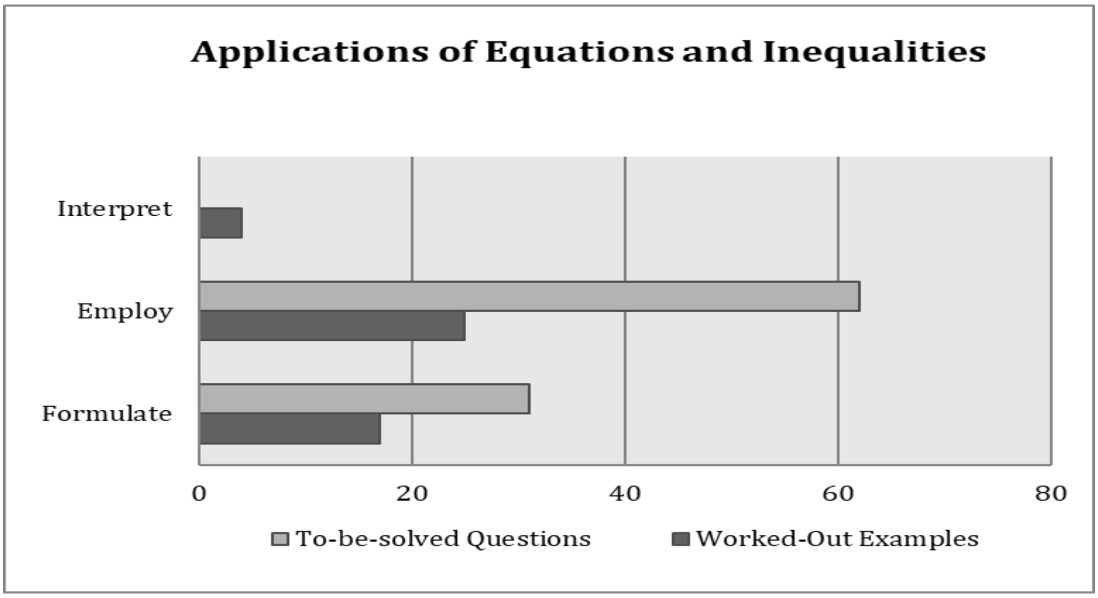


Figure 4.39. The Distribution of Worked-Out Examples and To-Be-Solved Questions of the Applications of Equations and Inequalities Section Regarding the Process Domain

More than half of the questions in this section were contextualized being adequate to the structure of the section. I found 81 questions that have a real-life context. With the help of these questions, students could have the opportunity to learn the mathematical structures presented in real-life situations. They may have experiences about the challenges that they will face in their real lives. Since this was one of the targets of MoNE (2018), the use of these questions is a necessity.

Example 2, Page 179:

In a chocolate cake, the amount of biscuits and sugar has inverse proportion. The amount of chocolate and biscuits has direct proportion. If 200 g biscuits was used, 30 g sugar and 120 g chocolate was used proportionally. If one uses 300 g biscuits and 45 g sugar, find the amount of chocolate by using the same proportions.

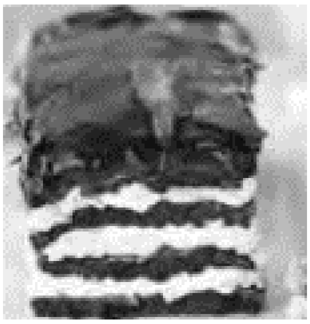


Figure 4.40. Sample Contextualized Questions from the Page 179 of Karataş (2018) Textbook

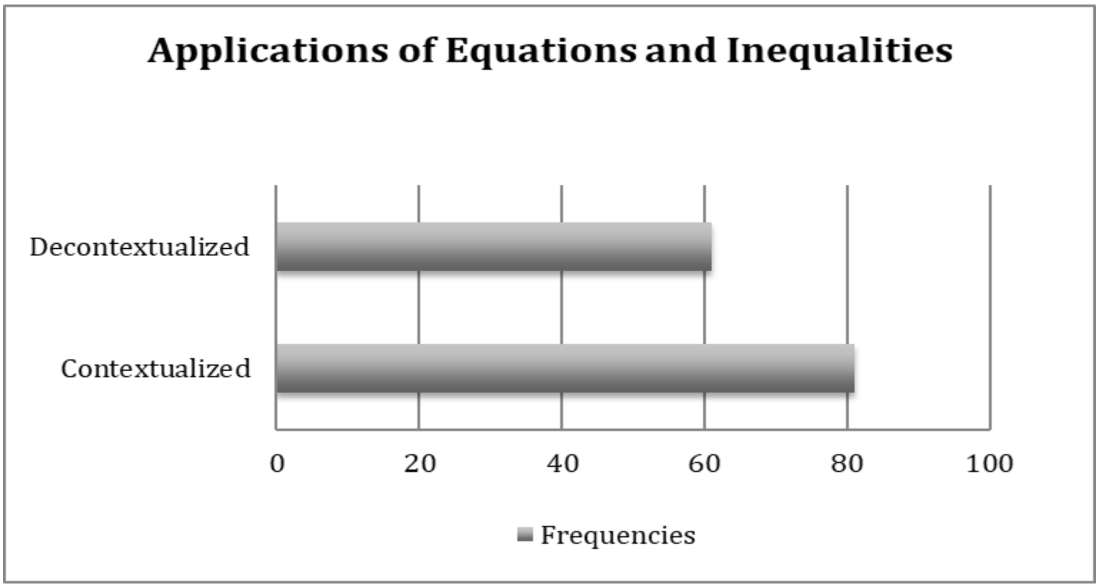


Figure 4.41. The Distribution of Questions of the Applications of Equations and Inequalities Section Regarding the Contextualization

The questions under this subtitle have various proficiency levels. The questions were mostly belonging to level 3, level 4, and level 2 sequentially. Level 4 questions were about 24% of which shows students have the opportunity to see complex problem situations that requires insight to choose a strategy for the solution of the problem. I found also level 5 and level 6 questions, but they were not used often.

It is your Turn 1, Page 200:

The graph on the right shows the change in the fuel in a tank of a vehicle over time. Since the vehicle's average speed per hour is 90 km / h and it does not take fuel until 12 liters of fuel remain, how many km can it take until it has fueled again?

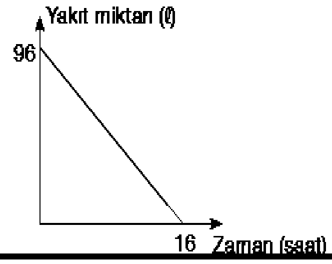


Figure 4.42. Sample Contextualized Questions Belong To Level 5 from the Page 179 of Karataş (2018) Textbook

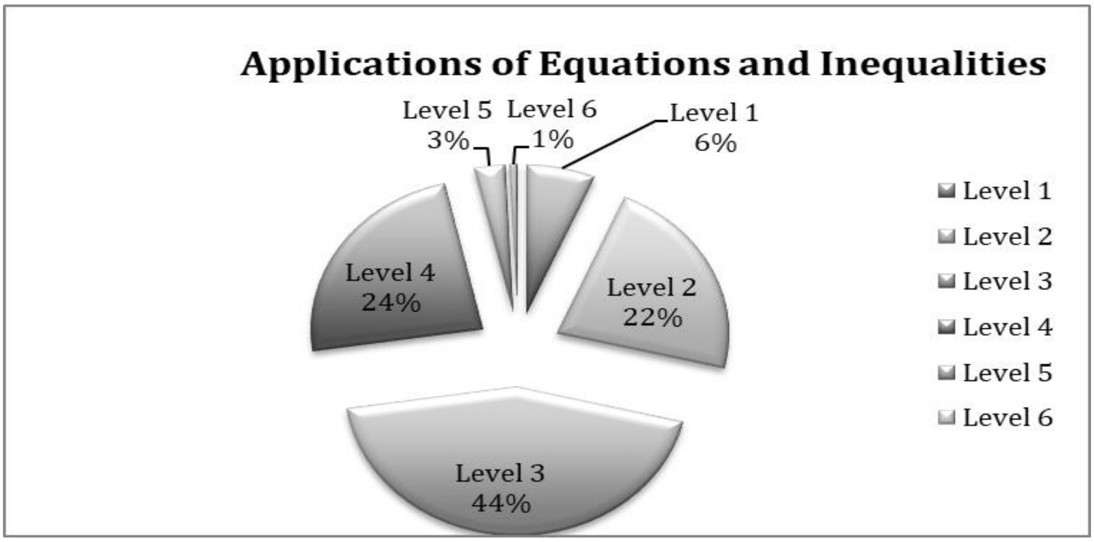


Figure 4.43. The Distribution of Questions of the Applications of Equations and Inequalities Section Regarding the Proficiency Levels

The applications of equations and inequalities section included to-be-solved questions more than the worked out questions. In this section, students were not expected to learn from the solution of the writer. Instead, they were encouraged to work on the problems by themselves.

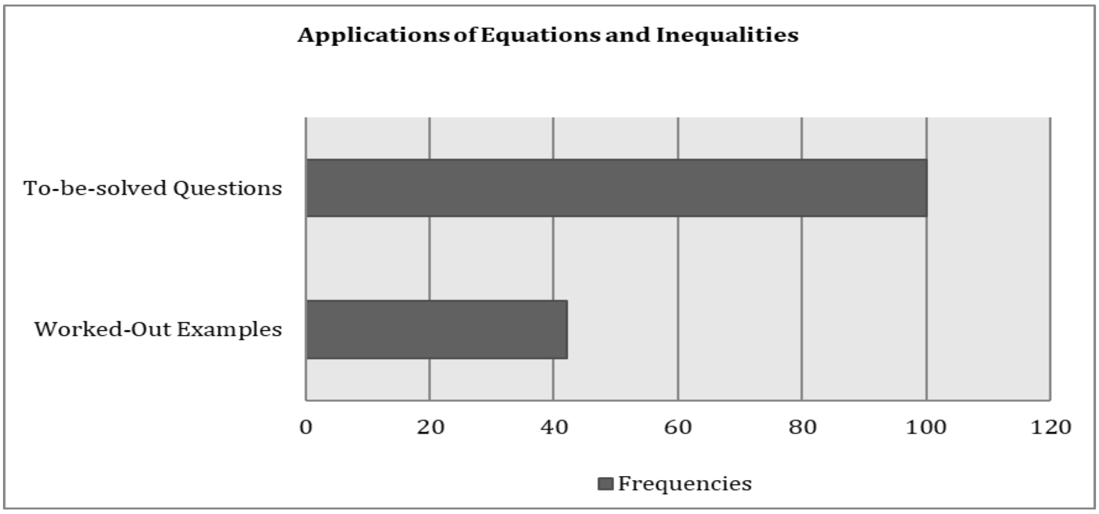


Figure 4.44. The Distribution of Worked-Out Examples and To-Be-Solved Questions of the Applications of Equations and Inequalities Section

CHAPTER 5

DISCUSSIONS

The current study analyzed the learning opportunities afforded through tasks related to numbers and quantity in the official ninth grade mathematics textbook for Turkish students. The results were drawn on: (a) process skills required to solve the questions, (b) contextualization of questions, and (c) proficiency levels of the questions. Overall analysis of the questions and individual analysis of sections were given together to provide both broad and in-depth perspective. Findings of this study not only discovered the characteristics of the textbook but also revealed the existence of gaps between the national mathematics curriculum and textbooks. The research does not only discover the characteristics of textbooks. Moreover, the study exhibits the gaps between the aims of the revised national curriculum (MoNE, 2018a) and textbooks, provides a holistic view for evaluating the textbooks, which makes the study more plausible, and presents implications for teachers, educators, curriculum developers, and policymakers.

In general, the findings of the study are consisted with those of previous studies that investigated the Turkish mathematics textbooks (e.g., Bayazit, 2013; Delil, 2006; İskenderoğlu & Baki, 2011; Özgeldi & Esen, 2010) and compared international textbooks (e.g., Hong & Choi, 2014; Jones & Tarr, 2007; Li, 2000). Even though, this study was not a replication study, the finding that students are provided opportunities mostly for solving low-level tasks is congruent with that reported by İskenderoğlu and Baki (2011). Congruently, Delil (2006), and Özgeldi and Esen (2010) claim that the textbooks they evaluated included mostly ‘routine' numerical questions that require applying facts and rules without connections. The combinations of the results of the current study and those of other studies highlight that similar problems related to the characteristics of learning opportunities provided in Turkish mathematics textbooks still exist, and policymakers and textbooks writers should be aware of those studies for self-evaluations.

Even though the findings are in congruent with similar others in the literature, the results of the current study provided more plausible analyses compared to other studies for analyzing the textbooks in a more holistic way (Bayazit, 2013) and providing analysis of textbooks concerning multi-dimensions (Hong & Choi, 2018; Carter et al., 1997). The current study is also valuable for evaluating a textbook that was written based on the latest Turkish mathematics curriculum for secondary schools (MoNE, 2018a). Li (2000) supports the use of multidimensional analysis of textbook in his study conducted to compare Chinese and American textbooks by claiming the idea that ‘a textbook analysis with multi dimensions tells us more compared to others that focus on one way of the textbooks, and eliminates the limitations of those one-way analyses'. The findings of this study verify that multidimensional analysis of questions such as a combination of process, context, and proficiency levels analysis provides a more powerful view for evaluating textbooks, and eliminates the limitations of previous studies. Moreover, the comparative analysis of different domains and question types helps to evaluate the characteristics of textbook as a whole, and provides a more holistic view (Bayazit, 2013), as it is in this study.

Overall, the textbook evaluated in the study did not propose a systematic distribution over the process categories and contextualization (see Figure 4.2 and Figure 4.8), has low-level questions dominantly (see Figure 4.11), did not have a strategy to develop the difficulty levels of questions such as from low to high level, and the questions were mostly representing pure mathematical structures (see Figure 4.8) which were called ‘routine' by Delil (2006) and ‘..representing the traditional view of applied facts and rules' by Bayazit (2013).

The analysis of the questions in terms of mathematical processes involved revealed that among 400 questions, 53.5% of them provide the opportunity of employing mathematical concepts, facts, and procedures. 43.75% of the questions allow for formulating situations mathematically. As mentioned by OECD (2013), formulating process is the bridge between the real-life situations and mathematical structures, and employing process is mostly related to applying a mathematical strategy to solve a problem and using facts and rules in mathematics. Students can understand a real-life situation and they might work on mathematical structures separately. However, unless students can connect their prior conceptual and procedural knowledge, problem-solving process might be incomplete. Being consistent to the interpretation of findings of this study, Ojose (2011) informs that setting these connections might be one of the most difficult processes within others, and representing situations mathematically is a big step to become mathematically literate. Seeing the process of interpreting, applying and evaluating mathematical outcomes was not very usual in the textbook evaluated. Only 2.75% of the whole quantity questions were including the process of interpreting. The process includes reasoning on mathematical solutions, making inferences for real-life situations and determining the validity of results for real-life challenges (OECD, 2013). The use of this process effectively is one of the aims that was placed in many national curriculums (De Lange, 1996; Stacey & Turner, 2015; MoNE, 2018)); however, many countries are unsuccessful about the applications of the process to the questions in textbooks (Hong & Choi, 2014), as confirmed by this study. De Lange (1996) argues that to be able to connect the real-life and mathematical concepts effectively, the students should get in a cycle which requires facing the real-life challenges repeatedly. This supports the finding that eleven questions placed in the textbook evaluated in this study will not be enough to help students to be successful in the interpreting process. This discussion highlights that, the opportunity provided in the textbook for making interpretations is not consistent with the national educational targets that aim to reach students who can deal with real-life problems (MoNE, 2018a).

Another crucial finding of the study was that the contextualization of questions does not follow an arrangement. In the PISA 2012 Mathematics Framework, the questions were categorized into four context categories such as scientific, personal, occupational and social (OECD, 2013). PISA is a context-based survey and equal distribution of questions considering these context categories is natural. In reality, there exist questions with no real-life contexts, which were coded as decontextualized questions in the study. The use of decontextualized questions is not necessarily a handicap particular to the textbook. In fact, Hickendorff (2013) supports that the use of two different problem types, contextualized-decontextualized, might evoke different problem-solving abilities. Decontextualized problems allow students to understand mathematical structures better and develop students' computational skills. However, in the textbook analyzed in the study, no harmony for the placement of contextualized- decontextualized questions was observed. The students have very little chance to see contextualized questions, and the questions were mostly (73% of the 400 questions) included mathematical structures only. Evaluating the findings related to the context categories, it was illustrated that most of these contextualized questions were worked-out examples. As mentioned by Delil (2006), due to the worked-out examples, the students would mostly see how the writer applied the mathematical structures and strategies into real-life challenges instead of working on them independently. Teachers and students would solve the rest of the questions in the textbook, to-be-solved questions, which is not enough for both in-class applications, and students' independent studies. These findings imply that this textbook does not directly provide an opportunity for students to carry the mathematical structures to their lives outside of the school. Therefore, the question of "how are we going to use this in our lives?" that comes from students will be left unanswered. This simple question originates from a much-debated area between mathematics educators (De Lange, 1996; Hickendorff, 2013; Schmidt, Zoido, & Cogan, 2014). In this study, the questions were evaluated either using a context that represents a situation or not. These researchers argue that also the power of representation of these contexts is important to be able to tell exactly whether the textbook offers the opportunity to handle real-life challenges. In other words, not only using a context guarantees to represent a real-life situation but also students should feel that situation is 'real', and has a place in their lives (De Lange, 1996). This debate might be taken into consideration to conduct further studies that would evaluate the quality of contextualized questions in textbooks.

The analysis of proficiency levels of questions in the textbook evaluated revealed students' chance of dealing with task from various proficiency levels. The results demonstrated that the textbook evaluated in this study did not have a systematic view of developing the question levels (such as spiral or linear) which could have provide better learning opportunities for students as claimed by Hong and Choi (2014) in their comparative textbook analysis. The results illustrated that the questions analyzed in this study mostly belonged to level 2 (37%) and level 3 (41%). 13% of the questions were belong to level, and upper-level questions were rarely found. These findings clearly demonstrated that, the students have opportunity to make direct inferences, following routines that were described in the problem context, and solving uncomplicated problems (OECD, 2013). However, such questions do not provide students with opportunity to work on high-level questions to develop insight and individual strategies for non-routine problems. Therefore, teachers using the textbook analyzed in this study (and other textbooks with similar task structures) should be aware of this fact and use additional tasks with the desired characteristics of high-level non-routine problems.

Up to this point, the evaluation of 400 quantity questions considering the process, context domains, and proficiency levels were introduced. Even though the overall analysis of questions draws a general picture of the textbook, it should be noted that individual analysis of sections could provide a deeper understanding. In the textbook (Karataş (2018) , the equations and inequalities unit had five different sections named as 'numbers sets', 'division rules', 'equations and inequalities', 'exponential numbers' and 'applications of equations and inequalities'. Some sections included particular question types due to the natural structures of the sections. Therefore, the results of the analysis of some parts were not necessarily surprising. In-line with the expectations, the number sets unit included low level formulating questions more than employing or interpreting questions since it was an introduction section. Similarly, the section of applications of the equations and inequalities has more contextualized questions than the decontextualized questions because the section is related to the use of facts and rules of equations and inequalities in real life and other mathematical structures. However, some results revealed the gap between the anticipated results of the analysis and the reality. Kilpatrick et al. (2001) emphasize that gathering number sense is one of the key factors for a better understanding of other concepts in mathematics, and understanding the notion of 'number' is the core idea of many curriculums of countries. Therefore, the sections related to the properties of numbers should have a systematic presentation of concepts and ideas. However, the findings of the study failed to find a systematic view of presenting the numbers in the textbook evaluated. The sections of number sets, equations and inequalities, and exponential numbers almost never include questions of interpreting type. Only a few contextualized questions are observed in these sections. Congruent to this finding, these sections include questions from the level 4-5-6 rarely, and high-level questions were in division rules and applications of equations and inequalities sections. This implies that in the textbook, high proficiency levels were applied for contextualized questions and contextualized questions were linked to specific sections. Another result of the analysis revealed that, for the number sets and exponential numbers sections, worked-out examples were more than the to-be-solved questions. Combining the results listed, it is obvious that that number sets and exponential numbers sections include decontextualized-low level-worked out examples which cannot help to students to foster reasoning through numerical structures. In their textbook analysis, Jones and Tarr (2007) support this idea that use of high- level questions has potential to provide a better understanding of the concept and to develop connections that would make the view of mathematics of students more meaningful.

As mentioned by Stacey and Turner (2015), understanding the numbers, using different representations of numbers, having the ability to perform with different number sets and gathering the number sense are the most fundamental skills to be able to gather mathematical literacy. Hence, the section of the number sets is one of the most important sections in the unit of equations and inequalities (Briand-newman, Wong, & Evans, 2012). Nonexistence of another section within other units and grades that covers the properties of numbers also make this part highly important. In a well-developed textbook, the students can be introduced to all the number sets deeply, be able to practice on formulating, employing and interpreting processes related to numbers, and should deal with various questions from low-levels to high levels. The sections should not be seen as a simple introduction, but the writers were suggested to look for methods to present the number sets deeper.

In the revised mathematics curriculum, it was highlighted that the results of academic studies and international surveys were taken into consideration while preparing the new mathematics curriculum (MoNE, 2018a). Hong and Choi (2018), Van Zanten and Van Den Heuvel-Panhuizen (2018), and Törnroos (2005) proposed that analyzing textbooks exhibits the characteristics of the textbook in terms of certain futures and reveals the gap between intended curriculum and implemented curriculum. Reading the revised mathematics curriculum and being aware of the idea that MoNE highlights the importance of considering the standards of international assessments; the consistency between the aims of MoNE and the characteristics of the textbooks has become a curiosity besides the main purpose of this study. Therefore, the study enlightened this curiosity for particular characteristics such as process categories, contextualization and proficiency levels described in the coding scheme. Usiskin (2013) informs that implemented curriculums are not always indicators of intended curriculums due to the lack of control systems between the curriculum developers and textbook writers. This might be a possible reason for the gap found in this study. However, further studies might clarify the reasons of gap between implemented and intended curriculums in Turkey better.

The study presents a theoretical framework for analyzing and preparing a textbook and open ways of conducting similar studies for other textbooks or units. This kind of analysis evaluates the opportunities that students have, appreciate for opportunities provided in textbooks, and criticize for missing aspects to provide better sources both for the teachers and for students. I hope that teachers, students, researchers, educators, and policymakers benefit from the study, and better learning opportunities were provided for students who could live successfully as a world citizen.

5.1. The Implications and Limitations of the Study

The results of the study have implications for curriculum developers, textbook writers, teachers, and researchers individually.

Studies on "opportunity to learn" claim that being familiar with the items asked in international studies have a positive impact on students' achievement (e.g., She, Stacey, & Schmidt, 2018). Therefore, textbook analysis research enlightens researchers and educators about the learning opportunities provided in textbooks related to a particular area and gives clues about some of the explanations of the results of international surveys. This study clarified that the mathematics results of PISA assessment comprise of different domains, and each domain has different subdomains and indicators, which evaluates different skills of students. To be able to interpret these results; teacher, researchers, textbook writers and curriculum designers should have a better understanding of these international assessments. Not only the ranking of the international surveys should be followed but also the structures of these assessments should be understood better. Evaluating PISA assessments, one should be able to think of the answer of 'what does the lowest and greatest scores of particular domains tell about students?'.

As illustrated in the MoNE's National Report on PISA 2015 results, more than half of the Turkish students were placed at low levels and only about two percent of the Turkish students were at level 5 and above which was a declined ratio compared to PISA 2012 (Özgürlük, Erbay, Arıcı, & Taş, 2016). Such results alarm that Turkish students need to practice high-level questions to get familiar with these PISA type questions and work flexibly on them. The Turkish curriculum developers claim that these results were taken into consideration while preparing the curriculum. However, this study implied that the textbook evaluated in the study do not seems to consider such an alarm. Therefore, textbook writers should be aware that there is a cycle between the results of the international assessments and students learning opportunities. The writers have a role in this cycle, which is being one of the designers of learning opportunities.

The study has pedagogical implications for educators and teachers. The questions and examples used in the textbooks and lessons have different characteristics and evoke different skills of students. Teachers and educators should be aware of this situation and choose questions consistent with the aim of the learning outcomes. While talking about the students' education, mentioning teacher education is necessary. Teacher candidates should be informed about the different questions types, their effects on students' learning, and the aim of the curriculums and national policies so that they could provide better learning opportunities for their students.

The concept of opportunity to learn is a comprehensive concept that includes the textbook prepared for students, digital educational materials provided by organizations and institutions such as MoNE, additional materials used by teachers, and students' opportunities outside of the school. In this study, the textbook analysis was chosen to clarify the learning opportunities of students since textbooks are a standard source that all students in Turkey have a chance to reach. This was a limitation of this study. To eliminate this limitation, the studies including the analysis of other learning opportunities such as in-class applications or students' opportunities provided outside of schools might be conducted after this study. As mentioned by Rezat (2009), the learning process of students includes different element such as teacher, student, and textbook, and different interactions of these elements bring different opportunities to learn. Hence, the nature of learning process also limits the study since the effective use of textbooks for each class is an unknown variable that effects the students' opportunities to learn, and eliminating this limitation is a challenging point.

The unit of analysis in the current study was determined as quantity questions in a ninth-grade textbook, which was another limitation of the study, but the theoretical framework of the study can be used to analyze other content areas of Turkish mathematics curriculum, which are change and relations, space and shape, and uncertainty and data. The results of the analysis of all content areas might provide a more holistic view of the learning opportunities of high school students in Turkey. Congruently, other textbook series and educational materials used in Turkey or other countries could be evaluated by using the theoretical framework of the study which provides a better understanding of the differences in students' PISA results across different countries.

In this study, the textbook that was written for the use of ninth grade science high schools was not evaluated. Therefore, further study can be conducted to analyze this textbook. The science high schools accept high- level students in Turkey, so the analysis of a textbook prepared for the use of science high school students can reveal different results from this study. Also, a further study that will evaluate the tenth grade textbooks provides useful information since the majority of students attending PISA assessment were tenth graders.

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