MIDDLE SCHOOL STUDENTS' USE OF PROBLEM-SOLVING STEPS AND THEIR STRATEGIES IN SOLVING WORD PROBLEMS

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ABSTRACT

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The aim of the present study was to investigate the middle school students' use of problem solving steps and their use of the strategies in the process of solution of Problem-Solving questions related to topics of numbers generally.

The data was gathered in the fall semester of the 2017-2018 academic year. The study sample included 116 (29 fifth graders, 29 sixth graders, 25 seventh graders, and 33 eighth graders) middle school students in Konya, Turkey. Different Problem Solving Achievement tests were used in the study for each grade level. In this study, basic qualitative research design was used to answer the questions "To what extend students use problem solving steps based on the identified framework?" and "What are the middle school students' strategies in the word problems?". Furthermore, descriptive statistics was used to describe the data.

The results of the study revealed that in some word problems, students showed their ability to use different strategies although the students generally preferred to use the arithmetic strategy. At the same time, the research findings showed that the most important deficiency of students was that they did not use real world knowledge and experience in their solutions. They did not take into account the real relationships between real-life contexts revealed by the problem statements and the operations they carried out in the problem solution. According to the results of the study, most of the students had difficulties in explaining their mathematical reasoning and making critics related to real life problems.

Keywords: Problem solving, middle school, problem solving strategies

ÖZ

ORTAOKUL ÖĞRENCİLERİNİN KELİME PROBLEMLERİNDE PROBLEM ÇÖZME ADIMLARINI KULLANIMLARI VE BU PROBLEMLERİN ÇÖZÜMLERİNDE KULLANDIKLARI STRATEJİLER

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Bu çalışmanın amacı, farklı sınıf seviyelerindeki (5., 6., 7. ve 8. sınıf) ortaokul öğrencilerinin genel olarak sayılarla ilgili problem çözme sorularının çözümü sürecinde problem çözme adımlarını kullanımları ve kullandıkları stratejileri gözlemlemektir.

Çalışmanın verileri 2017-2018 akademik yılı güz döneminde toplanmıştır. Çalışma örneklemini Konya ilindeki 116 (29 beşinci sınıf, 29 altıncı sınıf, 25 yedinci sınıf ve 33 sekizinci sınıf) ortaokul öğrencileri oluşturmaktadır. Her bir sınıf seviyesi için farklı Problem Çözme Başarı testleri kullanılmıştır. Bu çalışmada araştırma sorularına cevap vermek için temel nitel araştırma yöntemi kullanılmıştır. "Ortaokul öğrencileri ne dereceye kadar, belirlenen çerçeveye dayalı olarak problem çözme adımlarını kullanıyor?" ve "Ortaokul öğrencilerinin kelime problemlerinde kullandıkları stratejiler nelerdir?" sorularına cevap vermek için içerik analizi yapılmıştır. Ayrıca verileri tanımlamak için tanımlayıcı istatistikler kullanılmıştır.

Bazı kelime problemlerinde, öğrenciler genel olarak aritmetik stratejiyi kullanmayı tercih etmelerine rağmen, farklı stratejiler kullanabilme yeteneklerini gösterdiler. Aynı zamanda, araştırma bulguları öğrencilerin en önemli eksikliğinin çözümlerinde gerçek yaşam bilgi ve deneyimini kullanamadıklarını göstermiştir. Problem ifadelerinin ortaya koyduğu gerçek yaşam bağlamları ile problem çözümünde gerçekleştirdikleri işlemler arasındaki gerçek ilişkileri göz önünde bulundurmamışlardır. Çalışmadan elde edilen sonuca göre çoğu öğrencinin gerçek yaşam problemlerindeki matematiksel akıl yürütme becerilerini kullanmada zorlandıkları gözlemlenmiştir.

Anahtar Kelimeler: Problem çözme, ortaokul, problem çözme stratejileri

To the memories of my father

&

My mother, sister and wife

Ayşe, Serpil and Mesude ALKAN

who support and love me unconditionally throughout my life.

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LIST OF ABBREVIATIONS

MoNE	Ministry of National Education
NCTM	National Council of Teachers of Mathematics
SPSS	Statistical Package for the Social Sciences
TIMSS	Trends in International Mathematics and Science Study
PISA	Programme for International Student Assessment

CHAPTER 1

INTRODUCTON

Many people think that mathematics is one of the subjects that makes life difficult and unbearable (Arslan, Yavuz & Deringol-Karatas, 2014). Yet, mathematics is one of the ways of understanding and loving life, and similar to many other situations, loving something requires understanding it. In other words, we love what we understand (Sertöz, 2011). Moreover, it is mentioned by Hagaman (1964) that when students understand the meaning of what they are learning, they learn best as it is also known by teachers through their own experiences and interests. We generally exhibit negative thoughts and behaviors against situations we do not understand. Many people have negative attitudes towards mathematics since they are not thoroughly able to understand it. One of the most important reasons underlying student' dislike for mathematics is about self-confidence in issues related to problem solving skills (Arslan, Yavuz & Deringol-Karatas, 2014). Mathematical problem solving has great importance in school curriculum, and schools need to make mathematics more understandable with the help of innovative instructional methods such as problem solving based instruction. Currently, one of the essential goals of mathematics education is the development of the ability of solving mathematical problems (Shiakalli & Zacharos, 2014). For this reason, school programs focus on students' problem solving skills (Zakaria, Haron, & Daud, 2010). Creative problem solving is important to be successful in various areas of life (NCTM, 2000). Many countries renewed their instructional programs to use the problem solving method. This is also valid for all grade school programs in Turkey; problem solving has become a more significant element of mathematics education (Yıldızlar, 2001). In this context, one of the main aims of Mathematics curriculum in Turkey is stated as "Mathematics education should help individuals gain a language and systematic approach so as to analyze, explain, estimate some experiences, and solve a problem" (MoNE, 2009).

As Gagne stated, “The central point of the education is to teach people to think, to use their rational powers, to become better problem solvers” (1980, p.85). Parallel to Gagne, Jonassen (2000) also state that because all people regularly encounter problems in their daily life and professional life, most psychologists and educators consider problem solving as the most prominent learning outcome for life. People who are better problem solvers are rewarded in their professional life unlike the people memorizing information and completing examinations. Jonassen (2000) emphasized that the curriculum does not encourage students to solve meaningful problems.

> The few problems that students do encounter are normally well-structured (story) problems, which are inconsistent with the nature of the problems they will need to learn to solve in their everyday lives ("How can I get so and- so to pay attention to me?"), professional lives ("What kind of marketing approach is appropriate for this new product line?"), or even their school lives ("Should I spend the next two hours studying for my math exam or go outside and play ball with my friends?") (Jonassen, 2000, p.63).

A major reason for this is the lack of comprehension of the breadth of problem solving activities adequately to engage and encourage learners in them. Children often try to remember a rule that is often used when they encounter a problem (Gür & Hangül, 2015). The main goal of a teacher must be to teach the systematic of solving problems, the problem solving strategies, and how to use them. The strategies used in the process of solving problems have an important place in the literature (e.g. Montague & Applegate, 2000; Che &Wiegert &Threlkeld, 2012; Neimark &Lewis, 1967; Durmaz & Altun, 2014; Gür & Hangül, 2015).

Some studies revealed that Problem Solving Based Instruction is important because it is one of the most effective methods in mathematics education (e.g. Hembre, 1992; Ayaz & Aydoğdu, 2009; Özsoy, 2005). Problems can be seen in general sense as a way to build a bridge between the real world and mathematics (Van Dooren, Lem, De Wortelaer & Verschaffel, 2019). Stacey (2005) emphasizes that students should perform mathematical investigations by themselves and identify where the mathematics they have learned is applicable in real life. This should be the main aim of teaching. Hence, problem solving is one of the major goals of teaching mathematics, but at the same time it is one of the most elusive concepts of mathematics (Stacey, 2005).

Furthermore, many studies have shown that there is a significantly positive correlation between problem solving skills and student's mathematics achievement scores (Özsoy, 2005; Karaoğlan, 2009; Özalkan, 2010). Van de Walle (2007) emphasizes the importance of teaching with problems. For him, there are good reasons for teaching with problems: Problem solving places the focus of the students' on ideas and sense making; develops the belief in students that they are capable of mathematics and that mathematics makes sense; provides ongoing assessment data that can be used to make instructional decisions, help students succeed, and inform parents; allows various students an entry point; engages students so that there are fewer discipline problems; develops "mathematical power"; and finally it is fun for all students. In addition, in NCTM standards documents (1989) it is pointed out that "Solving problems is not only a goal of learning mathematics but also a major means of doing so…. Problem solving is an integral part of all mathematics learning, and so it should not be an isolated part of mathematics curriculum" (p.52).

It becomes more important to study on problem solving processes and strategies as the emphasis on problem solving in mathematics education increases (Gür & Hangül, 2015). The development of critical and creative thinking, the choice and use of strategies, improving unique approaches and methods and using them, and the application of real-life word problems in different settings by adapting knowledge are provided with appropriate learning and teaching environments (NCTM, 1991; Brown, 2001). In her study Yazgan (2007) investigated the fourth and fifth grade students' strategies to solve the word problems and Yazgan and Bintaş (2005) observed fourth and fifth grade students' level of using problem solving strategies. In their study, it is revealed that the 4^{th} and 5^{th} grade students could use some problem solving strategies unconsciously even though they did not have any education on this strategies. Furthermore, Sulak (2010) conducted an experimental study to investigate the effect of problem solving strategies on problem solving achievement in primary school mathematics. At the end of the study, she found that the experimental group in which students have been trained about problem solving strategies was significantly more successful in using strategies of making a drawing-diagram, making a table, etc. (Sulak, 2010). Moreover, there are also studies on preservice elementary mathematics teachers' problem solving strategies preferences (Özyıldırım Gümüş, 2015).

Furthermore, Gür and Hangül (2015) worked on secondary school students' problem solving strategies to determine 6th grade students' use of problem solving strategies and difficulties that they had in this process. Moreover, Beyazıt (2013) conducted a study to investigate the capability of 7th and 8th grade students in using realistic considerations, problem solving strategies and mathematical models.

In addition to national studies on this subject, there are many international studies. A study conducted by Che, Wiegert and Threlkeld (2012) examined patterns in written problem solving strategies of middle grade boys and girls for a mathematical task related with proportional reasoning. Moreover, Csikos, Szitanyi and Kelemen (2012) conducted a study that aimed to develop the knowledge of students about word problem solving strategies by emphasizing the role of visual representations in mathematical modeling. The feasibility, possibility and importance of learning about visual representations in mathematical word problem solving in grade 3 were pointed out in the study. In their study, using a large set of initial mathematics achievement scores, mathematical problem solving strategies, and math attitudes, Ramirez et al. (2016) revealed that children's math anxiety (i.e., a fear or anxiety related to mathematics) was negatively associated with their use of more advanced problem solving strategies (Ramirez, Chang, Maloney, Levine & Beilock, 2016). In another study, students' problem solving strategies in a problem solving mathematics classroom were investigated using an open approach including four phases as the teaching approach (Intaros, Inprasitha & Srisawadi, 2014). Even though many successful and related studies are conducted around the world, there is still considerable need for a study to analyze how middle school students use problem solving steps and which strategies are mostly preferred by middle school students from different grade levels.

1.1. Significance of the Study

A student has to improve his/her ability to transform problem into mathematical equations as s/he progresses in mathematics. On the other hand, this skill is unnecessary for their problem-solving skills since most people in the world will not use mathematical models to the extent that they are difficult (Durmaz & Altun, 2014).

Henderson and Pingry (1953, p.263) recommend that;

> "Students should become aware that the process of solution is very important. A student should develop the habit of trying several solutions. This will help him avoid the mechanized approach of solving a problem by a formula-applying, step-by-step procedure, as well as give the student a good check of his answer. Teachers should give proper recognition and reward to the student who does try several solutions or has searched until he has found an interesting or neat solution."

Hence, there is a great need to understand and use solving strategies. It can be said that the results and findings obtained from the current study will contribute to the clarification of the level of achievement of the students regarding the application of problem solving and mathematical knowledge to daily life in the new curriculum revised in 2018, and in this context, a better understanding of the present situation. This study is also important for stakeholders such as the Ministry of National Education (MoNE) and teachers. The theory of problem solving is substantially sterile when it is taught separately from the implications and consequences (Henderson & Pingry, 1953). That is, the essential theory of problem solving that is derived from studies should be comprehended by teachers, and teachers should clearly notice the implications of this theory for methods and procedures in the classroom.

This study was devised considering students' use of problem solving strategies. Recently, some research studies have been conducted on students' beliefs about problem solving (Stylianides, A. J., & Stylianides, G. J., 2014; Prendergast, Bray, Faulkner, Carroll, Quinn & Carr, 2018), on problem based learning and its impact on students' achievement (Azer, 2009; Guven & Cabakcor, 2013), and on students' problem solving and problem posing skills (Rosli, Goldsby & Capraro, 2013; Cai & Hwang, 2019). As we can see, there are many studies on problem solving, but there is a gap in the knowledge about students' adequacy of the choice of appropriate strategy and about the strategies used mostly in solving problems by the students from different grade levels. Put differently, there are limited studies on students' adequacy of deciding on and using the appropriate strategy in solving problems. As a result of this study, stakeholders may gain awareness into students' preferences about problem solving strategies. Correspondingly, teachers can better design their lessons, determine the problems and strategies to be used, and implement problem based instruction successfully in the regular elementary classes. Also, as it is stated by Lester (2013), both the cognitive and metacognitive knowledge of teachers and students, as well as their beliefs throughout the lesson will positively affect both the effectiveness and nature of instruction. Santos-Trigo (1998) states that Schoenfeld (1994) spent a lot of time to select, formulate, and redesign the problems to be used in mathematics classes. Mathematics class should provide an environment in which students are continually asked to make explanations and communicate their ideas to other students (Santos-Trigo, 1998). The questions of "What is true? What do the examples we look at suggest? How can it be done?" etc. encourage students to express what they are thinking, organize their ideas, and ensure persuasive arguments to defend their predictions.

It is believed that in the light of the results of the current study, questions on teachers' mind about which problem solving strategy should be emphasized for each grade level would be eliminated. Indeed, research might provide useful information for teachers on what type of strategies the students prefer and draw on to solve problems. In addition, this study might also offer good practices for mathematics curriculum in Turkey. To develop students' problem solving and analytical thinking abilities, the MoNE can integrate problems in accordance with students' need and level into school curriculum to create better classroom environments. The MoNE can examine the findings of this study on students' use of problem solving steps when they face the word problems in the school curriculum, and in this way, it can implement the problem based learning in classroom setting better and more efficiently. In this study, the content area was determined as "Numbers". The study investigates the choice of problem solving strategies of students from different grade levels and observes their competence in using these strategies. The sample included 5th, 6th, 7th and 8th grade students from a public middle school, and data was gathered in the first semester of the 2017-2018 academic year.

1.2. Aim of the Study

The aim of this study was to investigate the middle school students' use of problem solving steps and use of the strategies in the process of solution of Problem-Solving questions related to topics of numbers.

1.3. Problem of the Study

Two main questions are addressed in the study are:

RQ1: "To what extend students use problem solving steps based on the identified framework?"

RQ2: "What are the middle school students' strategies in the word problems?"

1.4. Definition of Important Terms

Problem: The ability to convert the word into numerical information and being able to comprehend the relationship between these numbers (Montague, 2003). In general terms, in a problem, the solution path is previously unknown, and the solution is not considered obviously (MoNE, 2009).

Problem-Solving: refers to applying the solution method to a task that is not clearly visible (NCTM, 2000).

Problem-Solving Achievement Scores: It refers to the scores obtained from Problem Solving Achievement Tests. Problem Solving Achievement Tests include six real-life situation word problems related to the concepts of numbers.

Use of Problem-Solving steps: This statement refers to the students' use of problem solving steps based on the identified framework. Five sub-questions were given to the students under each word problems in the Problem-Solving Achievement Tests to observe how they can use the problem solving steps which are understanding the problem, making a plan, implementing the plan, and control of the solution.

CHAPTER 2

LITERATURE REVIEW

This chapter aims to review the relevant literature related to national elementary mathematics curriculum, what problem and problem solving is, the importance of problem solving, the problem solving strategies, and the research studies on problem solving.

2.1. What Does "Problem" Mean?

The definition of problem can vary from person to person. For this reason, there are many different definitions about what a problem is. All of us have a problem, but a situation that is a problem for one person may not be a problem for another person. Hence, a task assigned to a student may or may not be a problem for that student.

First, in some definitions of problem, the expression 'uncertainty of the situations' is emphasized. According to the Shergıll (2012) "problem is a situation in which there is a discrepancy between one's current state and one's desired goal state, with no clear way of getting from to the other" (p.296). In addition to this, Booker and Bond (2008) define problem as a task or situation for which there is no immediate and obvious solution. In parallel with them, Posamentier and Krulik define problem as "a situation that confronts a person, that requires resolution, and for which the path to the solution is not immediately (underlined by the researcher) unknown" (1998, p.1). Similarly, Polya (1962) also defines problem as "to search consciously for some action appropriate to attain a clearly conceived, but not immediately attainable, aim". In these definitions, the main focus is that a task can be defined as a problem if it has no immediate solutions and it has an uncertain situation. That is, these given definitions point out that when solution is known in a mathematical task, it will not be a problem already.

Another aspect of problem is that uncertainty is not the only characteristic for a situation to be called as a ‘problem’. At the same time, student must accept the unknown situations as a question to be solved (Herlihy, 1964). These are also the types of problems that will be focused on in this study. It can be said that when the solution of the "problem" is made a goal for students, then "problem" becomes really a problem for them. In this respect, Henderson and Pingry (1953) argue that not every suggested question to be solved is a problem. Henderson (1953) points out that a problem exists for a particular individual when three essential conditions are satisfied:

1) having a clearly defined goal which someone is consciously aware of,
2) being insufficient for removing the block of the path toward the goal, and for fixing patterns of behavior or habitual responses,
3) becoming aware of the problem, being able to define it more or less clearly, identify various possible solutions, and testing these for feasibility.

There is a problem for an individual unless he has an appropriate answer form habit, or he sees relationships in the conditions. Especially for a gifted student, in every mathematics class, just few "problems" are really problems in an assignment. Indeed, certain types of tasks have some non-routine aspects, while they have highly routine aspects as well. A given task may be a routine procedure to a person, while another person may not have a routine operation for that task; even when working on algorithms, a student may behave in an “automatic” manner, whereas the other may think at each stage about why that stage is needed (Mamona-Downs & Downs, 2005). Briefly, a problem may really be a problem for a student, while that the same problem may be just an exercise for another student, and it may be a frustration for the third student (Henderson & Pingry, 1953).

Jonassen (2000) points to two critical attributes of a problem: “First, a problem is an unknown entity in some situations (difference between a goal state and current state), and second, finding or solving the unknown must have some social, cultural, or intellectual value” (p.65). Word problems do not usually follow the typical language and structure associated with textbook vocabulary problems (Grischenko, 2009). That is, especially in social and cultural matters, a situation is supposed to be feel a necessity to detect an unknown to be considered as a problem. Also, in mathematics, to consider a question as a problem, the person to solve the question must feel the need to solve the problem.

From these different aspects of problems, we can conclude that if a situation, which can vary from algorithmic mathematics problems to social problems we face in our daily life, challenges students and it is worth to solve, then it can be called as a problem. In mathematics, number operations that require simple calculations such as $4 + 5 = 9$ is different from problem. Questions should encourage students to think and form relations between the topic and method by using prerequisite knowledge. Hence, in solving a problem, just having necessary knowledge is not sufficient. To be aware of the knowledge's applicability in the activity of problem solving, a problem solver must also possess "meta-knowledge" which has two main components: integrating the knowledge by making special formulations of the knowledge which will foster the identification of particular kinds of applications and improving executive control while solving to direct the argument to a situation where an application can be made (Mamona-Downs & Downs, 2005). Related with that point, Zeitz (1999) highlights the difference between mathematics problems and exercises that can be solved easily and immediately with simple calculations and that do not need any puzzling about what techniques to use. In other words, a problem requires thinking critically and resourcefulness before finding out the appropriate approaches.

Besides all these definitions, Grischenko makes a more superficial and general definition. According to Grischenko (2009), most word problems in textbooks are verbal translations of symbolic exercises that can be easily solved without much effort. In other words, word problems are any math exercises where significant background information is presented as text rather than in mathematical notation (Boonen, Schoot, Wesel, Vries & Jolles, 2013).

All these types of problems require the use of problem solving strategies. During the process of solving problems in this study, students are required to use the problem solving strategies and the four-step approach to problem solving as defined by George Polya (1957). The four-step approach and problem solving strategies are described in more detail after the definition of problem solving. Finding the unknown, i.e. obtaining the solution, is the problem-solving process. As we can understand, even though there are various definitions and statements about what a problem is, there is no one way and absolute solution to solve a problem. As Lester (1994) states, problem solving is even more than the reminiscence of the facts or the appeal of well-learned procedures.

2.2. What is "Problem Solving"?

Problem solving has an important place in teaching of mathematics (Posamentier, Smith & Stepelman, 2006). There are various answers to the question of "What is problem solving?", since it is related with personal interest and philosophy (Mamona-Downs & Downs, 2005). As in defining the problem, problem solving can also be defined differently depending on the research and how the researcher will use it. According to the NCTM (2000), problem solving is "engaging in a task for which the solution method is not known in advance, so, students must draw on their knowledge in order to find a solution, and through this process, they will often develop new mathematical understanding" (p.52). Therefore, students who are learning mathematics also learn how to investigate and explore various types of problems. Shergill (2012) states that problem solving is overcoming a difficulty by planning a strategy and implementing it to achieve the goal. In addition, very similar to this definition, problem solving is defined by Martinez (1998) as a process of moving toward a goal when the path to that goal is uncertain.

When these definitions are examined, it is seen that the process of reaching a solution is highlighted in defining problem solving. As it is also stated by the NCTM (2000), "The essence of problem solving is knowing what to do when confronted with unfamiliar problems" (p.259). In other words, problem solving takes place if a solution strategy is not immediately clear and apparent (DeVault, 1981). In solving a problem, we can reach the solution in more than one possible way. Shiakalli and Zacharos (2014) mentioned that at the same time, a number of answers to a problem should be accepted and opportunities to check these answers should be given to students since seeking new solutions to a mathematical problem should bring about the development of students' autonomy: defending their selections, discussion of concerning strategies developed and used, and classification of solutions and assessment of the strategy used. Neither the repeated use of school practices nor the memorization of rules and methods is the basis for solving a problem (Shiakalli & Zacharos, 2014). Jonassen points out two aspects of problem solving: "(a) problem solving requires mental representation of the situation in the world, and (b) problem solving requires some activity-based manipulation of the problem space". Turkish Ministry of Education emphasizes that problem solving involves the skills necessary to solve the problems that will be encountered by students in their lives (MoNE, 2005).

Moreover, Krulik and Rudnick (1987) define problem solving as "the means by which an individual use previously acquired knowledge, skills and understanding to satisfy the demands of an unfamiliar situation" (p. 4). Someone should know the related subject well to solve a problem. Students are usually very good in tests requiring direct knowledge, but they exhibit very poor performance in problem solving tests, and this situation shows that knowing how to apply knowledge is at least as important as having knowledge itself (Mamona-Downs & Downs, 2005). Anderson (1980) argues that problem solving is the series of any target-focused cognitive process. For this reason, solving a problem necessitates the mental representation of the case. Like Anderson (1980), Jonassen (2000) also states that mental construction of the problem space is the most critical for problem solving.

In problem solving based lessons, teachers need to guide students during the problem-solving processes (Van de Walle, 2010). Polya, Bransford and Stein, and Van De Walle (2010) analyzed Polya's four problem solving steps which are stated in his famous book "How to solve it":

1. Understanding the problem
2. Making a plan
3. Implementing the plan
4. Looking back and extending the problem

Understanding the problem involves being aware of what is given and what is to be found out in the problem after reading the problem carefully. It is stated by Mamona-Downs and and Downs (2005) that the reading of mathematics texts is a significant element in problem solving. After the comprehension of the problem, what is given is organized, and an appropriate strategy is chosen to solve the problem. Then, the process continues with the implementation of the plan (doing the operations). And finally, in the fourth step, the solution is checked to ensure its correctness.

After analyzing the problem solving model of Mevarech and Kramarski (1997), the model of Bransford and Stein (1984), and the framework of Garofalo and Lester

(1985), they were synthesized and compared, and categories for coding the data were decided. Each model or framework was presented, described, and organized with regard to main headings and was synthesized and merged by the researcher in order to select the categories. Figure 2.1 presents the Mevarech and Kramarski's meta-cognitive instructional model of problem solving (2006). Figure 2.2 is the presentation of the problem solving model of Bransford and Stein (1984). The Cognitive and Metacognitive Framework of Garofalo and Lester (1985) is given in Figure 2.3. In the domain of mathematics, meta-cognitive refers to a student's awareness of his or her ability of what s/he is thinking about and selecting a helpful thought process. It helps students to analyze their way of thinking, whether they have high self-awareness, and whether they can control their ideas and select the appropriate strategy for the given task.

Mevarech and Kramarski's problem solving model (1997) helps teachers examine students' process through the task and strategy knowledge so that problem conditions, selection and organization of strategies, actions and progress, evaluations of plans, review of plans, and control of results can be observed. The main headings of all the problem solving models and the framework used in this study are based on the work of Polya (1957).

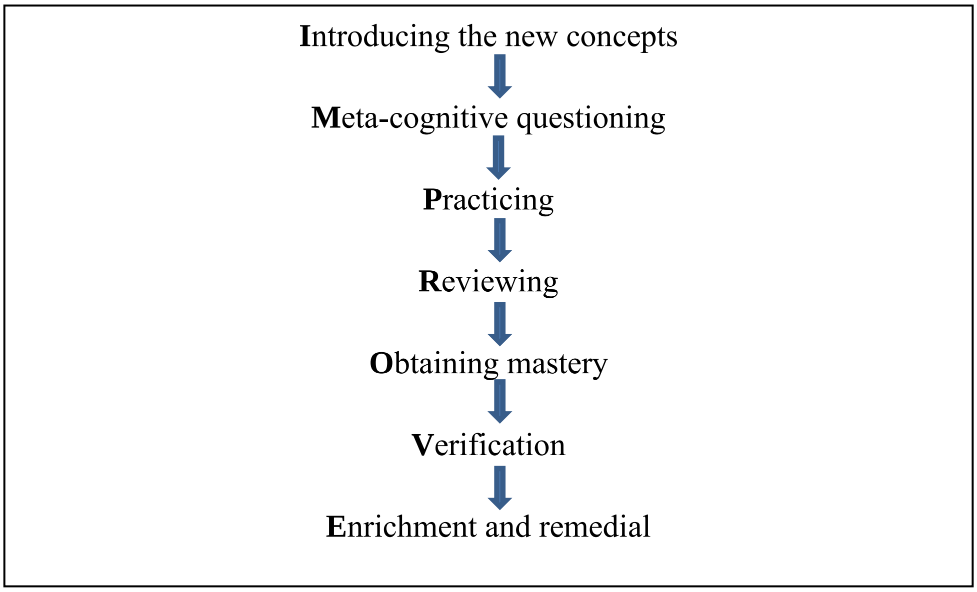


Figure 2.1. Meta-Cognitive Instructional Problem Solving Model of Mevarech and Kramarski (1997).

The problem solving model of Mevarech and Kramarski (1997) is less procedural. In the meta-cognitive instructional model, new concepts are introduced to class and students are provided to work in small groups. In that progress, students alternate with asking and answering three various metacognitive questions which are comprehension questions, strategic questions, and connection questions. With the comprehension questions, the main ideas in the problem are tried to be understood. What is given and what is asked in the problem is tried to be determined and the definition of terms in the problem is discussed. Appropriate strategies for solving the problem are decided with the strategic questions. Last, connection questions aim to establish connection with the previously solved problems. In the practice part, the problem is solved in accordance with the solution plan and the problem solving strategies determined in the Meta-cognitive questioning previously. In the review section, whether the solution plan was implemented correctly, and the accuracy of arithmetic operations are evaluated. After the review part, the subject is fully conceptually mastered in the section of obtaining mastery. The verification section corresponds to checking/controlling the solution. In this section, students are expected to prove the correctness of their solutions and explain why their answer was correct. Finally, Enrichment and Remedial section has the same purpose as Polya's problem posing phase. Students are expected to pose new problems related to the problem solved. In addition, if there are deficiencies and disruptions in the problem solving process, it is tried to be revised and improved in the enrichment and remedial section. This model is based on peer interaction, and students' mathematical achievement and thinking abilities can be improved as a result of the feedback and corrections they give to each other. A study conducted by Mevarech and Fridkin (2006) examined the effects of the meta-cognitive instructional model "IMPROVE", and results indicated that IMPROVE has significantly positive effects on students' mathematical achievement.

Figure 2.2. Problem Solving Model of Bransford and Stein (1984).

In this model, problem solving is thought as a uniform process of *I*dentifying potential problems, *D*efining and representing the problem, *E*xploration of the suitable strategy, *A*cting on those strategies and *L*ooking back. This instructional problem solving model is similar to the work of Polya when it is compared with other model and the framework. The work of Bransford and Stein (1984) is more procedural when it is compared with the other two frameworks.

ORIENTATION: Strategic behavior to assess and understand a problem
A. Comprehension strategies
B. Analysis of information and conditions
C. Assessment of familiarity with task
D. Initial and subsequent representation
E. Assessment of level of difficulty and chances of success
ORGANIZATION: Planning of behavior and choice of actions
A. Identification of goals and sub goals
B. Global planning
C. Local planning (to implement global plans)
EXECUTION: Regulation of behavior to conform to plans
A. Performance of local actions
B. Monitoring of progress of local and global plans

C. Trade-off decisions (e.g., speed vs. accuracy, degree of elegance)
VERIFICATION: Evaluation of decisions made and of outcomes of executed plans
A. Evaluation of orientation and organization
1. Adequacy of representation
2. Adequacy of organizational decisions
3. Consistency of local plans with global plans
4. Consistency of global plans with goals
B. Evaluation of execution
1. Adequacy of performance of actions
2. Consistency of actions with plans
3. Consistency of local results with plans and problem conditions
4. Consistency of final results with problem conditions

Figure 2.3. Cognitive-Metacognitive Framework of Garofalo and Lester (1985).

Garofalo and Lester's (1985) framework differs from others in that their framework is closely in the same line with Polya's work, in which the instructional technique is a step-by-step logical procedure. When Garofalo and Lester's framework has been examined, it has been seen that four main problem solving steps which are orientation, organization, execution and verification have been pointed out. However, when compared with the four problem solving steps of Polya, it could be observed that the aims and objectives of problem solving steps in that framework were developed and defined more clearly. In each step, the process was examined in more detail and a more conceptual and procedural learning environment was established for students. For example, in the verification step, unlike the corresponding control and check step of Polya, the evaluation of decisions made was conducted alongside the outcomes of the executed solution plans. The adequacy of representation, adequacy of organizational decisions, consistency of local plans with global plans, and consistency of global plans with goals are involved in the evaluation of orientation and organization, and the evaluation of execution involved the adequacy of performance of actions, consistency of actions with plans, consistency of local results with plans and problem conditions, and consistency of final results with problem conditions. Pieces of this framework and previously mentioned problem solving models serve as a conceptual framework in this study for analyzing cognitive processes evoked during the mathematical problem solving process.

One of the important points of the problem solving process is to decide on the appropriate problem solving strategy at the stage of planning to reach a solution after understanding the problem. In the next section, the necessity of problem solving strategies is discussed in detail and different problem solving strategies are introduced.

2.3. Problem Solving Strategies

Skills such as interpreting information, planning and working methodically, checking solutions and trying alternative strategies are required in solving problems (Muir, Beswick & Williamson, 2008). There are also strategies in solving problems. As Posamentier and Krulik state, it is rare to use all the strategies and not solve a problem. Also, it is equally rare that in solving a problem, a single strategy is used (1998). Fadlelmula (2010) maintains that an effective problem-solving process involves identifying components of the problem, understanding what information is missing, developing an effective strategy for solution of the problem, implementing the chosen strategy, knowing when and how to try an alternative strategy, and evaluating whether the results and the decisions taken are relevant or not. That is to say, he points out that choosing an appropriate strategy and using it have an important place for an effective problem solving process. Indeed, “Learning mathematics goes beyond studying rules, procedures or algorithms, it involves the use of both heuristics and metacognitive strategies to solve problems, the use of various presentations to make sense of information, and the search of mathematical connections or applications in different contexts” (Santos-Trigo, 1998, p.631).

Hohn and Frey (2002) conducted a study in which 223 elementary students participated. The study revealed that students benefited from a simple heuristic strategy, and the use of it resulted in improved problem solving skills. In addition, it was concluded from the study that the acquisition of the heuristic approach compared to the traditional textbook approach resulted in a more superior learning rate. Therefore, instead of using pre-printed knowledge, instruction should put emphasis on the discussion of various strategies in the process of solution of the problem to improve unsuccessful approach. Besides, becoming familiar with various problem solving strategies and practicing using them enable students to solve mathematical problems related to everyday life situations by introducing problem solving strategies in both mathematical and real life situations (Posimentier & Krulik, 2008). How students approach these relationships and how they use this discovery effectively when they face unfamiliar situations determine their problem solving ability (Herlihy, 1964).

Teachers who prefer direct instruction think that teaching is a simple change in students' long-term memories, while teachers who are part of constructivist teaching think about how they can help students choose and use the right path and strategy in the process of problem solving (Gresalfi & Lester, 2009). For this reason, it is crucial for teachers to give prior attention to encourage students to use variety of problem solving approaches and techniques (Herlihy, 1964). One of the vital roles of mathematics teachers is helping the students find out numerous problem solving strategies to use in solving mathematical problems which contain relationships with real life situations. Choosing the appropriate problem solving strategy is definitely one of the important components of being successful in problem solving (Ersoy, Güner, 2015).

Furthermore, based on his experiences and knowledge, Lester (2013) emphasizes some skills that teachers must have in teaching problem solving. One of these is to pay attention to and be familiar with the methods and strategies students use to solve problems. Posamentier and Krulik (1998) describe ten problem-solving strategies as illustrated in Figure 2.4.

1. *Working backwards:* In this strategy, solution starts from the last step and moves toward a beginning point (Posamentier & Krulik, 1998).
2. *Finding patterns:* In this strategy, a pattern is found by using the given series of numbers (Posamentier & Krulik, 1998).
3. *Adopting a different point of view:* Looking at the problem from a different aspect when solution could not be obtained through the way that can be seen easily (Krulik & Rudnick, 1987).
4. *Solving a simpler, analogous problem:* Finding out the solution by using solution of a simpler similar problem (Krulik & Rudnick, 1987).
5. *Considering extreme cases:* In this strategy, the problem solver considers the extreme values of the known problem (Krulik & Rudnick, 1987).
6. *Making a drawing:* In this strategy, the problem solver pictures the known by using schemes, tables, charts etc. to reach the solution (Krulik & Rudnick, 1987).
7. *Intelligent guessing and testing:* In this strategy, a value is guessed to find out the solution, and the value is tested to understand whether it is right or not (Krulik & Rudnick, 1987).
8. *Accounting for all possibilities:* In solving a problem, the problem solver considers all possibilities of the problem situation (Krulik & Rudnick, 1987).
9. *Organizing data:* In this strategy, the problem solver organizes all the given values to reach the solution (Krulik & Rudnick, 1987).
10. *Logical reasoning:* In this strategy, to find out the solution relation of the knowns and unknown was analyzed (Krulik & Rudnick, 1987).

Figure 2.4. Problem-Solving Strategies described by Posamentier and Krulik (1998).

As understood from the figure above, there is hardly just one unique way of solving a problem. There are various studies about the effectiveness of problem solving strategies and students’ use of problem solving strategies during the class (Durmaz &Altun, 2014; Gür & Hangül, 2015; Jiang, Hwang & Jai, 2014). In the study conducted by Durmaz and Altun (2014), the 6^{th}, 7^{th} and 8^{th} grade students' level of use of non-routine problem solving strategies and whether there is a relationship between the scores obtained by using these strategies or not without any instruction about problem solving strategies are explored. For this purpose, 118 secondary school students did a problem-solving test that was composed of one problem appropriate for each strategy. Also, it was investigated whether there is a difference between the grades in terms of the level of use of strategy. The results of the study indicated that while the making a table, elimination and drawing a diagram strategy has the lowest percentage of use, extraordinary division problem and looking for a pattern had the highest percentage among the strategies. Another study about problem solving conducted by Gür and Hangül (2015) aimed to determine 6^{th} grade students' use of problem solving strategies and the difficulties they face in the process of solving problems. Twelve 6^{th} grade students from a public school participated in the study. The problem test included 7 problems which were taken from some mathematics websites and PISA as the data collection tools. The study revealed that all the participants correctly solved the problems using the strategies "look for a pattern, start at the end, use an equation and make an organized list". Two students could not solve the problems using "draw a diagram and divide and conquer" strategies. The problems including "guess and check" strategy could not be solved by three students. In addition, the research has shown that students had some difficulties when they used the "guess and check" strategy, and they spent a lot of time on the problem using the "divide and conquer" strategy.

Another study conducted by Jiang, Hwang and Jai (2014) examined the use of problem solving strategies in solving 14 speed problems and evaluated the performance of a total of 706 sixth grade students (361 Chinese and 345 Singaporean students). To provide a useful perspective on the differences between these groups from two distinct countries in East Asia that have high performance, students' preparations and problem-solving strategies were focused on. In other words, the study aimed to reveal the thought processes and the use of strategies of Chinese and Singaporean students on the topic of speed during the problem solving process. The analysis of the strategies showed that even though a limited variety of strategies were used by Chinese sample, algebraic strategies were used more frequently and more successfully by the Chinese sample than the Singaporean sample. Moreover, the use of the model drawing strategy of the Singaporean sample led to a performance advantage in one problem about converting multiplication/ division of fractions into multiplication/division of whole numbers.

Another study conducted by Swanson, Lussier and Orosco (2013) investigated the role of strategy instruction and cognitive abilities in the accuracy of solving word problems in students with math difficulties. 120 third grade students participated in the study. Students were randomly assigned to four conditions which are general heuristics, visual-schematic, general heuristics + visual schematic, and untreated control group. As a result, the study showed that the posttest performance on measures of problem solving accuracy, calculation, and identification of components of problem solving of students with mathematics difficulties was promoted. In addition, a study conducted by Santos-Trigo (1998) focused on the identification of the problems' qualities used to corroborate the improvement of student strategies and values that reflect mathematical practice in the classroom. In other words, the study aimed to discuss the aspects concerned with the implementation of problem solving activities in the classroom. The study revealed that teacher should create a classroom environment where students are coherently asked to (a) work on tasks which propound diverse challenges; (b) debate the significance of using diverse types of strategies including the metacognitive strategies; (c) participate in small and whole group discussions; (d) return on feedback and challenges that come along from interactions with the instructor and other students; (e) communicate their ideas by writing and speaking; and (f) search for connections and extensions of the problems. So far, the concepts of problem and problem solving have been defined and problem solving strategies have been explored. In the next section, the importance of problem solving is discussed.

2.4. Importance of Problem Solving

"Fortunately - or unfortunately depending upon one's point of view - life is not simple and unchanging. Rather, it is changing so rapidly that all we can predict is things will be different in the future. In such a world, the ability to adjust and to solve one's problems is of paramount importance" (Henderson & Pingry, 1953, p.233). Martinez (1998) thinks of problem solving ability as the cognitive passport to the future. In other words, need for the ability of problem solving increased considerably in our real life, jobs, even in our daily life, since the world in which we live is going more complex day by day. When students graduate and/or get a job, they will clearly encounter problems at work or in their further education life. When we look from this aspect, in the field of mathematics, most teachers spend great effort to teach how to solve problems. For students, it is important to be able to succeed in this changing world and be ready for new and unusual situations as well as making the familiar processes fast and effective. We know that students must face with problems to become a better problem solver since studying processes is the only way to learn how to solve problems for students. Booker and Bond (2008) state that students should see mathematics as a way of thinking instead of as a means of providing true or wrong answer to be judged by a teacher. Moreover, word problems in mathematics classrooms motivate students and help them to assess their intelligence and mathematical skills, to improve their creative and heuristic reasoning, and to develop new mathematical concepts and skills. For these reasons, word problems should be included in the elementary school mathematics curriculum and students should develop the problem solving skill to know when and how to use mathematics in real life situation (Van Dooren, Lem, De Wortelaer & Verschaffel, 2019).

It is commonly argued by teachers whether mathematics courses should contain more problems or not. Some teachers want their students to experience and solve as many problems as possible while some teachers want their students to be confronted with few real-life word problems. Considering the first argumentation, Henderson and Pingry (1953) state that students will learn generalizations which will enable them to transfer their ability of solving problems to new problems if they study the process of solving problems as an end in itself. Focusing on problem solving in lessons will lead to the development of high thinking level of students (Ersoy, 2016). Therefore, students should be provided to perform self-learning in mathematics lessons with problem solving process since most of learning comes forward as a result of problem solving process. Problem solving has a crucial place in mathematics education. Moreover, it is said that both pre-formulated ‘problems’ and real-life ‘problems’ have their place in mathematics education. Verbal problems can become real ‘problems’ if the teacher chooses the ‘problems’ carefully with respect to students’ level of learning and s/he can ensure that students can identify themselves with these problems (Henderson & Pingry, 1953). Verbal problems are good practice materials on which students can apply the principles and processes of solving problem that they have learned. Also, word problems help students to improve problem solving abilities (Jiang, Hwang & Cai 2014).

Hagaman (1964) examines two kinds of meanings in arithmetic which are the intrinsic meaning of the quantitative relationships which emphasize the abstract mathematical thinking and the functional meaning connected with students' experiences. He believes that word problems that emphasize the functional meaning of arithmetic are used to provide additional motivation or a familiar setting for the better understanding of certain operations. Another important point is that students' contextual knowledge can be advanced with story-related (real life situation) problems, instead of presenting problems symbolically using only mathematics symbols, numerals, variables (Rittle-Johnson & Koedinger, 2005). Verbal problems aim to teach generalizations relative to method and the process of solving problems, while exercises including fundamental operations and the practice of theorems are for teaching the basic mathematical concepts. Word problems can help students to improve their problem solving skills and creativity, and they motivate students to realize the significance of mathematical concepts by providing practice with real life problems (Chapman, 2006). Muir, Beswick and Williamson (2008) also stated that problem solving is an important mechanism for enhancing the skills of thinking and communicating and for instilling deep understanding in students.

Needless to say, teachers must also understand problem solving before they teach it. On the grounds of considerable evidence obtained from studies, it can be said that many mathematics teachers have lack of understanding of problem solving. A list of problems such as 'velocity problems', 'age problems', 'interest problems', 'profit – loss problems' are given to the students to show them how to solve these types of problems, and they generally use memorized techniques instead of performing the problem solving process (Henderson & Pingry, 1953). Students' choice of the appropriate strategy, development of the use of strategy, and generating different solutions can be improved by problem solving and by teachers who know about the problem solving process (Ersoy, 2016).

Problem solving is described as the main goal or as a way of achieving a broader goal of achievement in school mathematics in the official curriculum documents of Australia, the UK, the USA and Singapore (Stacey, 2005). Turkey is also taking part in these reform movements and making changes in the nature of elementary school curriculum. When the results of formerly carried out major international studies such as TIMSS (1999), PISA (2003) and PISA (2006) are examined, it is seen that the revision in the content of the elementary school curriculum is inevitable. Those international studies revealed the deficiency of quality in mathematics and science education at the elementary level. After the analysis of these formerly carried out international exams, at the elementary school and secondary school levels, sometimes considerable and sometimes superficial changes have been made over the last fifteen years. The changes in Turkish education system are like those in other countries such as the US, the UK, Singapore, Ireland and Holland (Babadoğan & Olkun, 2006).

Mathematical competence is the development and application of mathematical thinking to solve a number of problems in everyday life. The process built on sound arithmetic skills emphasizes the activity and knowledge. The ability and willingness to use different degrees of mathematical thinking modes (logical and spatial thinking) and presentation (models, graphs and tables) indicate mathematical competence (MoNE, 2018). In this respect, the reviewed mathematics curriculum aims that students can learn by exploring, become better problem solvers, improve analytical thinking and use ability of problem solving in their daily life. It can obviously be observed that more importance is given to problem solving skills in the recent (2003, 2009, 2013, 2017 and 2018) renewed mathematics curricula. Ministry of National Education (MoNE) explains the importance of problem solving as follows: "Mathematical knowledge and skills will be more meaningful, and it will be easier to apply this knowledge to different situations. Word problems should be included in the mathematics course. These problems can be solved by using more than one strategy or different types of results are obtained" (MoNE, 2009). One of the main objectives of mathematics education is to improve students' problem solving skills. From this point of view, problem solving has an important place in the secondary school curriculum, and it is considered as a basic skill that should be developed for each subject within the curriculum. It is also suggested that problem solving should sometimes be considered as a teaching approach or as a learning tool (MoNE, 2013, p.3). In the Elementary School curriculum of the Ministry of National Education (2013, p.8), the principles of teaching approaches in the program were listed. Some of those principles of teaching approaches in the program were using problem-based learning environment and ensuring the active participation of students in the course, meaningful learning, and realistic learning environments. When these principles are briefly reviewed, it can be understood that they can be met by the problem-based instructional method.

CHAPTER 3

METHODOLOGY

The main aim of this chapter is to give information about research design and procedures used in the study in seven main parts which are population and sample, data collection instruments, data collection procedure, data analysis, assumptions and limitations, and internal and external validity.

3.1. Research Design of the Study

The study aims to investigate the middle school students' use of problem solving steps and the use of strategies to solve the word problems related to numbers. Understanding how people make sense of their experiences, how they construct their world, and what meaning they attribute to their experiences were the purposes of a basic qualitative research (Merriam, 2009). In addition, data is collected via observations, documents and interviews in a basic qualitative research (Merriam, 2009). In this study, basic qualitative research design was used to answer the questions "To what extend students use problem solving steps based on the identified framework?" and "What are the middle school students' strategies in the word problems?". To investigate students' use of problem solving steps during the problem solving processes and students' tendency to use problem solving strategies, students' worksheet were examined in detail. Content analysis of the written documents obtained from the students was carried out. Furthermore, descriptive statistics was used to describe the data. Descriptive statistics helps researchers to summarize the overall tendencies in the data and to provide an understanding of how varied scores might be (Creswell, 2002). Descriptive statistic (means, standards deviations) was calculated for the independent variable "grade level". The Problem Solving Achievement Tests were assessed using a rubric which was developed by the researcher and two mathematics teachers after the review of the related literature.

Scoring for correctness was done using a 0-1-2-3 scale. If a student gave a completely correct solution, 3 points were given. 2 points was given to solutions with an almost correct answer with only minor errors in computation. Answers that solved part of the problem were given one point. Zero point was given to answers if they were completely wrong and in cases when no solution was provided.

3.2. Sampling

The study sample includes 116 (29 fifth graders, 29 sixth graders, 25 seventh graders, and 33 eighth graders) middle school students in Konya, Turkey. The lived in Konya and they were students in a public middle school in Konya. The age of the students changed between 11 and 13.

Table 3.1. Distributions of Students from Each Grade Level

Grade Level	Number Students
5	29
6	29
7	25
8	33

Additionally, the socio-economic status of the families of students who participated in the study was not too high, but it can be said that it was average considering the Turkish standards. As for the educational status, a large part of the students' families completed primary school or high school, while few families had bachelor's degree.

In addition, in this study, convenience sampling was used as the method of sampling the participants. Convenience sampling enables researches to save time, money and energy (Fraenkel, Wallen &Hyun, 2011). In this study, convenience sampling was chosen because of the school's familiarity to the researcher and the easy accessibility of the school, the students, and the necessary permissions from the school administrations. Additionally, the researcher has information about the participants' backgrounds, personalities and mathematics ability and achievement. The researcher was a teacher in the school where data was gathered. Due to the familiarity of the researcher with the school and the students, the data collected by the researcher could be interpreted more accurately by the researcher, which increases the reliability of the study. In other words, Problem Solving Achievement Tests was carried out by the researcher during the data collection procedure.

Kulu, a district of Konya, was chosen to conduct the study. Students generally have lower achievement levels in Kulu, Konya compared with the students from other districts. However, the school where the research was conducted is the most successful school in the district and students are divided with regard to their achievement level in this school. The data was gathered from the students from the class with higher achievement level. Hence, students who participated in the study had average and above average mathematics achievement, so they generally had no difficulty in using the necessary mathematics knowledge during the mathematics problem solving achievement test. Consequently, necessary and adequate information was gathered in order to answer the research question properly.

3.3. Data Collection Instruments

As stated above, the aim of the study was to investigate the middle school students' use of problem solving steps and use of the strategies to solve the word problems related to numbers.

In each grade level, different Problem Solving Achievement tests were used to reveal the way of thinking and the strategies used in the problem-solving questions. Therefore, the results obtained from the students at different grade levels were not compared and the relationship between the achievement scores of these students was not examined. The test was developed by the researcher. The participants' grade levels were considered by the researcher while choosing mathematics problems. Also, problems were selected among the mathematics topics covered by the 5^{th}, 6^{th}, 7^{th} and 8^{th} grade students so far. As it was mentioned before, the topics chosen were numbers. Six word problems were included in each Problem Solving Achievement Test.

Table 3.2. Distribution of Objectives in the 5th, 6th, 7th and 8th grade Problem Solving Achievement Test

Grade Level	**Problems**	**Objective**	**Mathematics Content**
5th Grade	Item 1 Item 3	Calculate the desired number of simple fractions of a plurality and whole of a plurality given proportion of simple fraction by taking into account the unit fraction	Sets and Whole Numbers
	Item 2 Item 4 Item 5 Item 6	Solve problems involving four processes with whole numbers	Fractions
6th Grade	Item 1 Item 6	Solve problems that require four operations with natural numbers	Sets and Whole Numbers
	Item 2 Item 4	Solve problems that require four operations with decimal express ions	Decimal Numbers Operation
	Item 3 Item 5	Solve problems that require processing with fractions	Fractions
7th Grade	Item 1 Item 2 Item 3 Item 5	Solve problems that require doing operations with integers	Sets and Whole Numbers
	Item 4 Item 6	Solves problems that require doing operations with rational numbers	Sets and Whole Numbers
8th Grade	Item 1 Item 2	Calculate the greatest common factor and the least common multiple of the two natural numbers; solve related problems	
	Item 3	Calculate the probability of a simple event	Statistic

Table 3.2. continued

	Item 4	Understand the basic rules related to the exponential expressions, and create equivalent expressions	Sets and Whole Numbers
	Item 5 Item 6	Do multiplication and division operations with square root expressions	Sets and Whole Numbers

Problems in the Problem Solving Achievement Tests were adapted from the TIMSS, PISA and mathematics course books. The pilot study of all the Problem Solving Achievement Tests was done in a classroom. Since some of the problems were in English, they were translated to Turkish by the researcher. On the other hand, students' grade level, development level, and socio-cultural status were considered in the process of developing and revising both the English and Turkish problems.

Reliability and validity issues of problems were needed to be addressed because problems were adapted by the researcher. For this purpose, an English translator, a faculty member who was interested in Mathematical Problem Solving, and two experienced mathematics teachers who have enough knowledge about mathematics curriculum in each grade level were determined as experts. First, the format and content of the Problem Solving Achievement Tests were checked by these experts in the field for validity issue. For the consistency of problems with the purpose of the study and the participants, the questions were updated in accordance with feedback from these experts. Then, Problem Solving Achievement Tests were piloted before the actual study. Essential revisions in the problems were carried out in compliance with the results of the pilot study.

The tests consisted of six open ended problems which enable students to solve the questions by using Polya's five steps, which are understanding the problem, planning, carrying out the plan, checking the answer, and extension. A holistic rubric was prepared by the researcher. The scoring in the rubric was prepared with respect to the five steps of problem solving. Moreover, the problems proposed to students during the study were assumed to help students think of different questions which necessitate mathematical resources and strategies. Students were exposed to new challenges that could encourage them to discuss different methods of solutions carefully, connect to other situations and extend to more general cases. In addition, the process of solution of problems in the study was assumed to enable students to discuss strategies that included working backwards, making a drawing, trying special cases, logical reasoning and so on. After the pilot study and literature review six problem solving strategies were specified to be included in the current study.

3.4. Pilot Study

Pilot study is defined as "small-scale study administered before conducting an actual study in order to reveal defects in the research plans" (Fraenkel, Wallen & Hyun, 2011). Creswell (2003) emphasizes the importance of pilot study stating that it establishes the instrument's construct validity, meaning whether the items measure the intended construct, and ensures that the questions, format and items are clear. The pilot study was done at the end of 2016-2017 academic year with twenty volunteer students from each grade in the school where the main study was also intended to be done. Twenty students were selected among the students who would not participate in the main study. Ten problems were used for each grade level in the pilot study to select six problems to be used in the main study.

The pilot study helped the researcher to finalize the problem solving achievement tests to be used in the main study. The pilot study aimed to answer the following questions:

- Are the problems easily understood by the students?
- Do the problems require strategy?
- How long does it take to solve problems?
- Is the level of problems suitable for students?

After the pilot study, the students' use of the problem solving steps, the time spent on each problem question, and the intelligibility of the problems were analyzed, and as a result of this analysis, the problem-solving achievement tests were finalized for use in the main study.

Before the pilot study, it was assumed that 6-item problem solving achievement test to be used in the actual study would be completed in one class hour. Hence, two lessons were allocated for the 10-item test in the pilot study, but the test could not be completed within two hours. Therefore, by considering the results of the pilot study, some questions were removed and some of them were revised, and testing time for the main study was planned as two hours. In addition, where necessary, grammar errors were corrected with the help of Turkish teachers in order to make the problems more accurate and easier to understand.

Furthermore, this study covers the strategies mentioned above: working backwards, making a drawing, intelligent guessing and testing, organizing data and logical reasoning. There are different reasons why these strategies were placed in the study. Firstly, most of the strategies were covered by different problems, but after the pilot study, some questions were eliminated, so there were no questions to use some strategies. Moreover, after the literature review, some strategies were eliminated because they were too difficult considering the students' levels.

As a conclusion, the problem statements in the problem solving achievement tests were reviewed and simplified taking into account the feedback from the participants in the pilot study. Moreover, by taking into consideration the field notes and observations of the researcher, the most convenient and most feasible procedure for in class practice of problem solving achievement tests was decided for the main study. Instead of given problems to students in a copy at once, problems were given on two separate papers. It was decided that the participants would be given the second paper once they were done with the first paper. The data collection procedure was explained in detail below.

3.5. Context of the Study

After the pilot study was carried out and the necessary corrections and reviews were made, a problem solving based course were given to the students in five lesson hours. The main aim of these five hour-problem based instruction was to reinforce students' problem solving processes (Polya's four steps) and to review and to remind problem solving strategies. As mentioned earlier, the researcher is also the mathematics teacher of the students who participated to the study. Hence, the lessons were taught by the researcher. A total of 20 hours of lessons were given to fourth grade levels. For these lessons for students from all grade levels, lesson plans were prepared (Appendix C). These problem-based lessons were conducted in students' own classes during a week. The lessons were student-based as in all mathematics courses throughout the semester.

The first lesson began with an activity called “I am solving a problem”. Before starting with activity, first, the students were asked to answer the question “What does problem mean for you?”. Students mostly expressed their feelings and thoughts about problem solving in mathematics and they talked about doing operations in mathematics. Additionally, students mentioned real life problems in some classes. If there was no mention of everyday life problems in the classroom, the teacher asked some questions to students to get ideas on everyday life problems such as “What about daily life problems?, “Don’t we ever have problems in our daily life?, “What kind of problems do we encounter in our daily life?, and continued by asking the question “Do you think there is a relationship between the problems we encounter in daily life and the problems we solve in mathematics class?”. The teacher provided an environment to argue on and talk about these questions. During this process, the teacher tried to guide students by asking some questions such as “How do we solve everyday life problems?”, and “Do you think that learning mathematics and how to solve math problems help us to overcome the problems we face in our daily life?”. Moreover, the teacher asked a question “5^{th} grade students of our school will organize a trip to Antalya. How do you think they can go?”. The teacher asked the students some questions like “Is this a math problem or a daily life problem?”, “What information do we need to solve this problem?”. After the discussions on what we need to solve this problem, the students were asked to express this problem mathematically and the lesson continued with activity. The students filled out the figure shown in Figure 3.1 together.

1. <u>Daily Life Problem</u>
5th grade students of our school will organize a trip to Antalya. How do you think they can go?

2. <u>Problem's Mathematical Expression</u>
5th grade students of our school will organize a trip to Antalya. The capacity of each bus is 40 students. How many buses are needed for 240 students?

3. <u>Mathematical Problem's Solution</u>
$240 \div 40 = 6$

4. <u>Daily Life Problem's Solution</u>
6 buses are needed.

Figure 3.1. Daily life problem's mathematical expression

Next, the students were asked to identify different daily life problems in a completely independent manner and to solve that problem after expressing the daily life problems mathematically. In this way, it was aimed to help students to see the relationship between the problems we encounter in daily life and the problems we solve in mathematics course. Also, it was aimed to make students realize that the knowledge and experience we gain from solving mathematical problems are also important in solving the problems we face in daily life.

In the next one hour of the lesson, Polya's four steps of problem solving were introduced to the students. Each problem solving step, namely understanding the problem, making a solution plan, implementing the solution plan, and checking/controlling of the solution, was discussed in detail. Furthermore, in the last three hours of mathematics lesson, problem solving strategies were discussed, and the lesson continued with the activity called "problem-solving" which includes six mathematics problems for which they need to use different problem solving strategies. In this way, the students were expected to gain in-depth understanding of the use of the different strategies. Besides, Polya's four problem solving steps were given just below each problem and the students were guided about what they were expected to do at each step in a template in the "Problem-Solving" worksheet.

Firstly, in the step "understanding the problem", the students were asked to determine and write down what is given and what is wanted in problem statement after they read the problem statement carefully. In the second step "make a plan", the students were asked to make a solution plan and decide on the appropriate solution plan taking into consideration what is given. During this step, the students were guided to be aware that there are strategies to reach the solution of the problem. Some students were asked to share their ideas with the whole class about how they planned to solve the problem and which strategy they decided to use. As mentioned before, six problem solving strategies were determined after a pilot study and literature review. In these five hours lessons, the six problem solving strategies were specifically emphasized and it was aimed for the students to adopt, recognize and internalize these strategies. Most of the time, some of the students were not aware of the strategy they used in problem solving and one of the aims of these problem solving classes was to raise awareness about the strategies they use in problem solving and to teach them other strategies they did not know. The name of the strategy that students used was defined by the teacher to help the students. The main purpose of this worksheet was to introduce and teach how to use problem solving strategies such as algebraic strategy, arithmetic strategy, making a drawing strategy, guess-and-check strategy, working backward strategy and organizing data strategy, which were defined by Possamentier and Krulik (2008).

Problem: Uncle Mehmet feeds goats and chickens on his farm. Uncle Mehmet feeds 28 goats on the farm, and the total number of the animals is 104. According to this, how many chickens does Uncle Mehmet have on his farm?

1. Rewrite the above problem with your own sentences in the same sense.

2. What is given in the problem? What is unknown? Explain in your own sentences.

   Given: Unknown:

3. Make a plan on how to solve the problem, taking into account the given and desired information you specified in step 2. Explain your plan in your own words.

4. Solve the problem considering the plan you made in step 3.

5. Are you sure that the result you obtained in step 4 is correct? Explain if you are sure.

Figure 3.2. An Example Problem from the "Problem-Solving" Worksheet in 5th Grade Lesson Plan

The problem given in Figure 3.2 was tried to be solved by students using different strategies. In the solution process of this problem, students preferred to use different problem solving strategies such as guess-and-check, finding a logical reasoning, and arithmetic strategy.

3.6. Data Collection Procedure

The data was gathered in the fall semester of the 2017-2018 academic year. The data were collected from 5^{th}, 6^{th}, 7^{th} and 8^{th} grade students of a public school in Konya. Initially, the required permissions were received from the school, the Ethic Committee of METU, and from the other official committees (Appendix B). The pilot study and the main study were conducted after getting necessary permissions. First, the pilot study was applied during a week at the end of the spring semester of the 2016-2017 school year. Secondly, a problem solving based course were given to the students in five lesson hours after the pilot study was carried out and the necessary corrections and revisions were done. The main aim of these five hour-problem based instruction was to reinforce students' problem solving processes (Polya's four steps) and to review and to remind problem solving strategies. For this five-hour problem based instruction, lessons plans were prepared for all grade levels (Appendix C). These problem based lessons were conducted in students' own classrooms during a week. Lessons were student-based as in all mathematics courses throughout the semester. The researcher provided students with an environment where students could discuss freely and ask all the questions they have to the teacher and to each other. Then, the problem solving achievement test was applied to 116 elementary school students in the fall semester of 2017-2018 school year by the researcher. Each test was carried out in different weeks for four weeks. Problem Solving Achievement Tests were implemented to students from all grade levels in their own classes during the mathematics course hours. Detailed time schedule is available in Table 3.3 below.

Table 3.3. Time Schedule of the Study

Date	Event
February 2017-April 2017	Choosing and development of Problem Solving Achievement Tests

Table 3.3. continued

May 9, 2017	Pilot study 5^{th} and 6^{th} grade Problem Solving Achievement Tests
May 12, 2017	Pilot study 7^{th} and 8^{th} grade Problem Solving Achievement Tests
May 2017-November 2017	Revising and improving the Tests and procedures of the study
October 2, 2017- October 21, 2017	Problem-Based Instruction
December 18, 2017	Implementation of 5^{th} grade Problem Solving Achievement Tests
December 25, 2017	Implementation of 6^{th} grade Problem Solving Achievement Tests
January 4, 2018	Implementation of 7^{th} grade Problem Solving Achievement Tests
January 11, 2018	Implementation of 8^{th} grade Problem Solving Achievement Tests
April 2018-October 2018	Data Analysis

The same procedure was followed during all test implementations by the researcher. As it was mentioned above, procedures were determined according to the results of the pilot study. In the study, there was one class from each of the fifth, sixth, seventh and eighth grade levels, and two mathematics teachers. The researcher is fifth and seventh grade level students' mathematics teacher, and the other teacher was responsible for the sixth and eighth grade level students during the whole academic year. However, the problem solving lessons were conducted by the researcher for one week, and the data was collected by the researcher from all grade levels. As it was mentioned in the previous section, problem solving classes lasted 5 lesson hours, and the data collection procedure lasted 2 lesson hours on the same day in each grade level. In total, 7 lesson hours of application was implemented in all grade levels by the researcher. In the implementation of Problem Solving Achievement Tests in all grade levels, all six problems in each test were not given to students at once. Six problems in each test were separated into three. The students were given the first two problems and then they were given the remaining two problems, followed by two more questions. Moreover, Problem Solving Achievement Tests were administered to the students in their own classes. In class, problem papers which included two problems each were given to the students one by one by the researcher. Each student who solved the two problems was given the other problem paper containing two problems. As the data collection process was carried out in the examination atmosphere, the students did not ask any questions about the content of the problem. In the solution of the questions, students were asked not to erase anything they wrote, but to only cross it out where they thought it was wrong. Still, some students asked questions about this issue or similar situations.

The consent forms were prepared to be signed by the parents of the students. The students and their parents were given information about the study. In addition, the students were said that there will be no grading for their participation. All the students answered the problems independently, carefully, and seriously. All the students participated in the study were volunteer students, and their names were confidential, and the students were not harmed psychologically or physically.

Lastly, the approval of the Ministry of National Education (MoNE) was taken to gather data from a public middle school in Konya. The current study was decided to be conducted in this specific public middle school since the researcher is a mathematics teacher in that school. The subject of “numbers” were chosen for the study because they are in the 5^{th}, 6^{th}, and 7^{th} grade curricula. These topics were taught through problem solving to the students in accordance with the yearly plan, and they are taught in the first semester of the academic year. For this reason, this study was conducted in the first semester of the 2017-2018 academic year.

3.7. Data Analysis

Students’ responses to problems in the problem solving achievement test were analyzed in two steps. Firstly, descriptive statistics was used to describe the data. Descriptive statistics helps researchers to summarize the overall tendencies in the data and to provide an understanding of how varied scores might be (Creswell, 2002). Descriptive statistic (means, standards deviations) was calculated for the independent variable “grade level”.

The Problem Solving Achievement Tests were assessed using a rubric which was developed by the researcher and two mathematics teachers after the review of the related literature. Scoring for correctness was done using a 0-1-2-3 scale. If a student gave a completely correct solution, 3 points were given. 2 points was given to solutions with an almost correct answer with only minor errors in computation. Answers that solved part of the problem were given one point. Zero point was given to answers if they were completely wrong and in cases when no solution was provided. The detailed rubric for scoring the problems in the problem solving achievement test is given in Table 3.4.

Table 3.4. Problem Solving Skills Rubric

Score	Understanding the Problem	Developing a Plan to Solve the Problem	Carrying out the Plan and Interpreting Findings
3	Stating the problem clearly and identifying the underlying issues	Developing a clear and concise plan to solve the problem with alternative strategies, and following the plan to the conclusion	Providing a logical interpretation of the findings and solving the problem clearly.
2	Defining the problem adequately	Developing an adequate plan and following it to the conclusion	Providing an adequate interpretation of findings and solving the problem.
1	Failing to define the problem adequately	Developing a marginal plan, and not following it to conclusion	Providing an inadequate interpretation of the findings and not deriving a

Table 3.4. continued

			logical solution to the problem
0	The problem is not identified	Could not develop a coherent plan to solve the problem	Could not interpret the findings and could not reach a conclusion.

In addition, basic qualitative research design was used to answer the questions:

- To what extend students use problem solving steps based on the identified framework?
- What are the middle school students' strategies in the word problems?

To investigate students' use of problem solving steps during the problem solving processes and students' tendency to use problem solving strategies, students' worksheet were examined in detail. Hence, content analysis of the written documents obtained from the students was carried out. Content analysis is the application of an objective coding scheme to notes or data that is not suitable for analysis until the information they transmit is concentrated and systematically comparable, such as interviews, field notes, and various inconspicuous data (L Berg, 2001). Content analysis allows researchers to identify patterns within and between sources (L Berg, 2001). As it was mentioned before, ten problem solving strategies were defined by Posamentier and Krulik (1998). First, six specific categories of problem solving strategies used by students for solving problems in the problem solving achievement test were identified after documenting the achievement scores of problem solving achievement test. Indeed, these six problems solving strategies were determined considering the pilot study and based on the review of related literature review. Also, the contents of problems and level of students' understanding were considered while choosing six strategies. These strategies are arithmetic, algebraic, making a drawing, working backward, guess and check, and organizing data. Also, when the students did not use any strategy, a no-strategy category was used. The percentages of students using each strategy in each group for each item were obtained. The strategies were described in Table 3.5.

Table 3.5. Description of problem-solving strategies

Code	Strategy	Description
ART	Arithmetic strategy	The student writes down a mathematical statement involving one or more arithmetic operations on the numbers given in the problem.
ALG	Algebraic strategy	The student chooses one or more unknowns as variables and sets up one or more equations.
MD	Making a Drawing	The solution is suggested or the solution follows a model or a diagram, table and graphic.
GC	Guess-and-check	The student uses the following processes: (a) Make a guess of an answer or the unknown in the problem based on an estimation; (b) Check if the constraints given in the question or implied from some of the question statements are satisfied. If all the constraints are satisfied, the guess is correct; the answer has been obtained or can be worked out. All the processes will end at this point. If the constraints are not satisfied, the guess will be refined or adjusted, followed by another round of guess-and-check.
WB	Working Backward	Starting the solution from the last step and moving to the beginning point
ORD	Organizing Data	The student organizes the data to reach a solution.
NOS	No Strategy	Absence of a written response, or only the pieces of information taken from the question is written down without any further work.

Each item was examined and coded considering the response of each student from all grade levels and using the framework as the analysis tool to determine the middle school students' cognitive process in terms of use of the problem solving steps. The theoretical framework that was used in this study included five categories:

1. Understanding the problem,

2. Making a solution plan,
3. Carrying out the solution plan,
4. Looking back,
5. Progression through all five categories.

Indeed, it can be concluded that five categories comprise the Polya's four problem solving steps. It was aimed to determine whether the students progressed through all four steps or not. The codes were obtained from the abbreviations of name of the Polya's problem solving steps by the researcher. Table 3.6 shows the list of codes and descriptions.

Table 3.6. Problem -Solving Steps Codes and Descriptions

Problem-Solving Steps	Code	Description
Seeks understanding the problem	SUP	Student understands the verbal information presented in the item.
Making a plan/organization	MPO	Student provides an indication of solution plan to solve the item.
Implementing the plan	IP	Student is able to generate a solution plan and also s/he is able to successfully carry it out to solve the item.
Control, assessment of situation	CAS	Student provides an explanation for why his thinking is correct.
Progression through all categories	PTA	Student demonstrates progression through the five processes.

Based on Table 3.6, it is clear that problem-solving steps consist of five main categories: seeking understanding the problem (SUP), making a plan/organization (MPO), implementing the plan (IP), control, assessment of situation (CAS), and progression through all categories (PTA). After the analysis of the written work obtained from the Problem Solving Achievement Tests, the sub-categories of SUP, MPO, IP, CAS and PTA were defined by the researcher.

In this study, each problem in the Problem Solving Achievement Tests includes five sub-questions. First two questions of the five sub-questions of each problem were related to SUP. In the first sub-question, the students were asked to rewrite the given problem with their own sentences in a way to give the same meaning. In the second sub-question, the students were asked to answer the following questions with their own sentences: "What is given in the problem?" and "What is requested in the problem?".

After the analysis of these two questions from all problems through all grade levels, it was decided that the "SUP" category included five sub-categories:

- **SUP1:** The student re-writes the problem in his own words and he writes properly what is given and what is wanted in the problem to understand the verbal information presented in the item.
- **SUP2:** The student is unable to re-write the problem statement in his own words and to understand the verbal information presented in the item, but he writes what is given and what is wanted in the problem.
- **SUP3:** The student is able to write what is given in the problem, while he is unable to determine what is wanted in the problem.
- **SUP4:** The student did not understand the verbal information presented in the item. The student fails in the progresses of determining what is given and what is wanted in the problem and re-writing the problem statement in his own words.
- **SUP5:** The student re-writes the problem in his own words, but he is unable to state what is given and what is wanted in the problem clearly.

Problem-Solving Steps:	Seeks Understanding the Problem (SUP)
Sub-Categories of Problem-Solving Steps:	SUP1

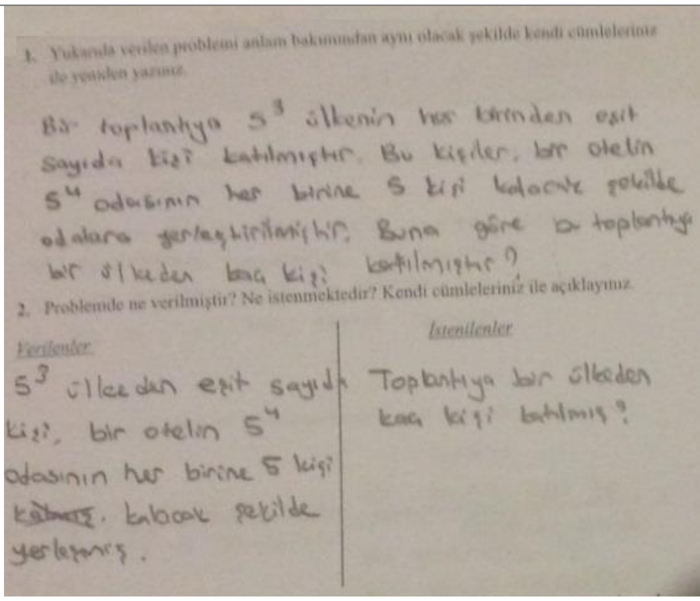


Figure 3.3. A sample item from eighth grade problem solving achievement test (Item 4)

Figure 3.3 was given as an example for the sub-category "SUP1". Problem 4 in eight grade Problem Solving Achievement Test was "An equal number of people from each of the 5^3 countries participated in a meeting. These people were placed in each of the 5^4 rooms of a hotel as 5 persons. According to this, how many people have participated in this meeting?". In this problem, the student rewrites the problem in his own words without changing the meaning of the problem, and it can be concluded from Figure 3.1 that the student can determine what is given and wanted in the problem. Hence, it was categorized as SUP1.

Secondly, the category MPO is also important to understand the problem during the process of solution of the problem. This category included four sub-categories. Like in category SUP, sub-categories of MPO were determined taking into consideration of the third sub-questions of the problems. In the third sub-question of each problem, the students were asked to "make a plan on how to solve the problem by taking into account the information in step two (in second sub-question) and explain their plan using their own sentences". Afterwards, students' written answers were analyzed in order to determine the sub-categories of MPO. In this study, the components of MPO were framed as:

- **MPO1:** The student makes an appropriate plan by organizing what is given to reach what is wanted and an appropriate strategy is chosen to solve the problem.
- **MPO2:** The student does not state his plan clearly.
- **MPO3:** The student is not able to organize and relate what is given to reach what is asked.
- **MPO4:** The student is unable to make a plan thoroughly – no organization of what is given, no use of strategy.

Problem-Solving Steps:	Making a Plan/Organization (MPO)
Sub-Categories of Problem-Solving Steps:	MPO3

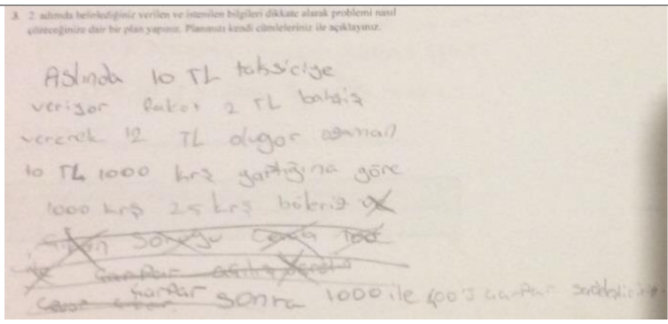


Figure 3.4. A sample item from sixth grade problem solving achievement test (Item 2)

As seen in Figure 3.4, one of the sixth-graders' written work exemplifies how a written work was categorized as MPO3. In item two of the sixth grade problem solving achievement test, the students were asked to answer the following problem: "Yaşar who missed the school bus gets into a taxi in order not to be late for work. The starting fee of the taximeter is 3.50 TL. The price for every 100 meters is 25 Krş. Yaşar gave 2 TL tip to the taxi driver and in total he paid 12 TL which also included 2TL tip. What is the distance between Yaşar's house and his workplace in kilometers?".

As seen in the figure, the student' planned to solve the problem as follows: "Yaşar gave 10 TL for the trip since 2 TL of 12 TL was the tip to taxi driver. Then, 1000 is divided by 25 since 10 TL is equal to 1000 Krş. Finally, 1000 is multiplied by 400." This answer was evaluated as MPO 3 since the student is not able to organize what is given to reach the result.

IP is another category in this study. To obtain the sub-categories of IP, the students' written answers to the fourth sub-question of the problems were evaluated. In step 4 (sub-question 4) under each problem, the students were asked to solve the problem by considering the plan which they made in step 3. In the study, after the analysis of the students' written work in step 4, IP was categorized into four:

- **IP1:** The student successfully solves the item.
- **IP2:** The student is successful in the implementation of the plan, but his plan is not appropriate for the correct solution.
- **IP3:** The student is not able to carry out the solution plan.

➢ **IP4:** The student does not possess or is not able to recall the necessary knowledge to solve the item.

Problem-Solving Steps:	Implementing Plan (IP)
Sub-Categories of Problem-Solving Steps:	IP1

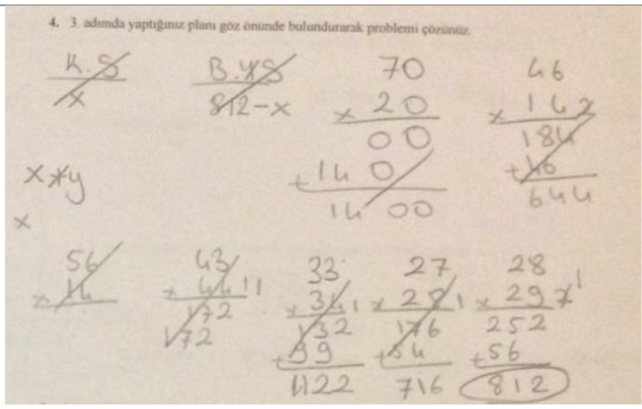


Figure 3.5. A sample item from the seventh grade problem solving achievement test (Item 1)

One solution of the problem included in the seventh grade problem solving achievement test is illustrated in Figure 3.5. In the problem, the students were asked to find two numbers whose multiplication is equal to 812. The students used "guess and check" strategy to solve the problem. As seen in Figure 3.3, the student first chose 70 and 20 and multiplied them. Then, she chose the numbers 46 and 14, 56 and 14, 43 and 44, 33 and 34, 27 and 28, respectively until she obtained the result 812. Finally, she found the numbers 28 and 29.

CAS is another category in this study. This category was divided into five sub-categories. To determine the sub-categories, the analysis of students' written work from step 5 (fifth sub-question) were used. In step 5, the students were asked: "Are you sure that the result you obtained in step 4 is correct? Explain whether you are sure or not?". In the light of the analysis of students' written work, the components of CAS can be presented as:

➢ **CAS1:** The student provides an explanation for why his way of thinking is correct.

➢ **CAS2:** The student just states "I am sure" without clear explanation or without any explanation.

- **CAS3:** The student has no control for whether his way of thinking is correct or not.
- **CAS4:** The student solves the problem by using different strategies to ensure whether his answer is correct or not.
- **CAS5:** The student just re-states his solution carried out in the implementation of the solution plan.

Problem-Solving Steps:	Control/Assessment of Situation
Sub-Categories of Problem-Solving Steps:	CAS1

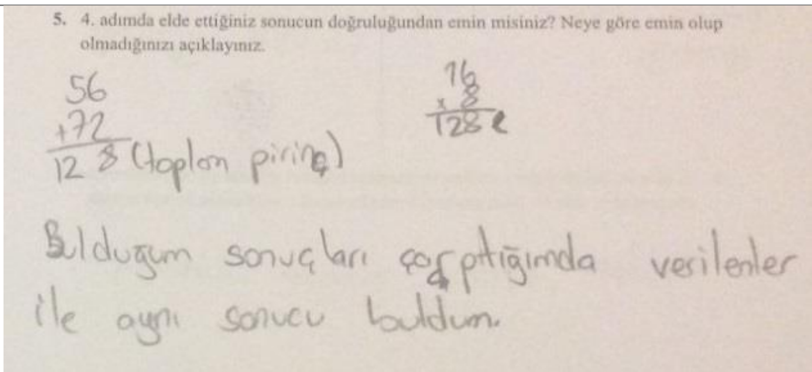


Figure 3.6. A sample item from the eighth grade problem solving achievement test (Item 2)

An example for CAS1 is given in Figure 3.6.

Problem: *56 kg and 72 kg bags of two types of rice in the bag will be put into bags with largest size without being mixed with each other.*

According to this;

a. *How many kilograms of rice will be put in a bag?*
b. *How many bags are required for this process?*

The student first found the greatest common divisor of 56 and 72 as 8, and then, she divided 56 and 72 with 8. She found that 7 bags are needed for 56 kg of rice, and 9 bags are needed for 72 kg of rice. In the control step of the problem, she found the total numbers of bags as 16 and multiplied 16 with 8 and she observed that the result of the multiplication is equal to the total amount of rice.

PTA is the last category in this study. PTA refers to the demonstration of progressions through the five processes – understanding the problem, making a plan, carrying out the plan, and checking the solution. The attitude of the students towards the problem is generally similar to their attitude toward a simple process. Instead of trying to read and understand the problem first, the students started to solve the problem. The main aim of this category was to see how the students at different grade levels adopted the problem solving process. The components of PTA are:

- **PTA1:** Student demonstrated progression through the five processes successfully.
- **PTA2:** Student could not progress at least one of the five processes successfully.

3.8. Reliability and Validity Issues

Frankel, Wallen and Hyun (2004) defines validity as the appropriateness, meaningfulness, and usefulness of the inferences made based specifically on the data by researchers, while reliability was defined as the consistency of these inferences over time, places and circumstances. The concepts of reliability and validity are very important in both qualitative and quantitative research (Frankel, Wallen & Hyun, 2011). As opposed to quantitative studies that discuss internal validity, external validity, and reliability, in qualitative studies, credibility encompasses the terms *instrument validity, internal validity and reliability* (Frankel, Wallen & Hyun, 2011). In this study, the process of coding the items with respect to the students' use of problem solving steps and the use of strategy was conducted by two researchers. These two researchers were graduate students in mathematics education. In the previous part, detailed explanation of frameworks was given in order to categorize the items before beginning the coding of the items. Afterwards, two researchers came together to analyze the some randomly selected items from each examination of different grade levels. When the coders made different categorizations of items related to explanations in the frameworks, or when the coders taught that changes were needed on these categorizations, they came together and discussed the situation. Next, some of the items were coded individually by the coders and they came together again in order to match their categorizations. If there was an agreement on the categorization of the items, the coders continued to categorize individually. However, if there was a disagreement on the categorization of an item, this item was discussed until they reached an agreement about the coding of the item. All the items in the problem solving achievement test were coded in this manner by two coders.

3.9. Limitations of the Study

There are some limitations in the study. The first of these is having only written resources for analysis. Since only written sources were available in the study, what the students thought and how they selected and used strategies to solve the problems were not fully understood and could not be determined. Some of the written work by the students was ambiguous, and thus, it was difficult to interpret the explanation of the students. Furthermore, especially in some parts, students did not give any idea about the ideas they determined. An example of a student who said “Yes, I am sure” to the question related to the checking of the solution was shown in Figure 5. It appears that the students were asked to answer the problem’s sub-question (step 5) – “Are you sure that the result you obtained is correct? Explain”. It can be seen that the student just wrote “Yes, I am sure” with no explanations.

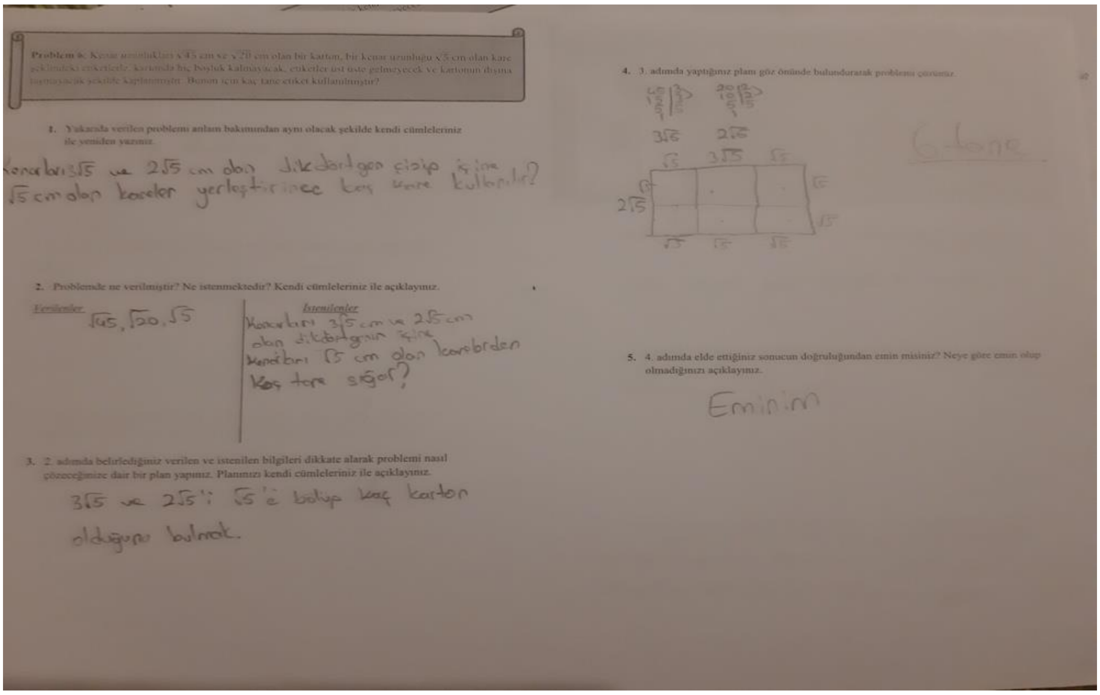


Figure 3.7. Ambiguous student solution

Another limitation is that the implementation process was limited. First, the lessons prepared for the introduction of strategies to the students were not enough. Therefore, the students could not fully comprehend the problem solving strategies. The problems in the five hour problem solving lessons and some problems in the problem solving achievement test were similar to each other. This might have partially affected the findings obtained in the current study. Also, the implementation of the problem-solving achievement test was very limited.

CHAPTER 4

RESULTS

This chapter has two sections. In the first section, the results of the descriptive statistics which is used to describe 5th, 6th, 7th, and 8th grade students' achievement scores obtained from the problem solving achievement test are presented. In the second section, results obtained after the content analysis of the students' written documents are explained. Students' strategy use and use of problem solving steps during the process of problem solving were analyzed using content analysis. The purpose of this study was to investigate the middle school students' use of problem solving steps and use of the strategies in Problem-Solving questions related to numbers gathered throughout the semester. Furthermore, 5th, 6th, 7th and 8th grade students' problem solving achievement scores obtained from problem solving achievement tests were presented. The findings that are discussed in this chapter address the following research questions:

- "To what extend students use problem solving steps based on the identified framework?"
- "What are the middle school students' strategies in the word problems?"

4.1. Descriptive Statistics of Problem Solving Achievement Scores

Descriptive statistics concerning the 5th, 6th, 7th, and 8th grade students' problem solving achievement scores is presented in this section. Problem solving achievement scores' mean, standard deviation, minimum-maximum scores, and skewness and kurtosis values were calculated and used to describe the data. As mentioned before, the achievement scores of the students were obtained from the problem solving achievement tests applied to each grade level. Therefore, descriptive statistics for problem solving achievement scores of each grade level was summarized separately.

A total of 116 students (29 fifth grade, 29 sixth grade, 25 seventh grade and 33 eighth grade) were applied the problem solving achievement tests. Table 4.1 summarizes the mean, standard deviation, minimum-maximum scores, and skewness and kurtosis values of achievement scores for each grade level separately.

Table 4.1. Descriptive Statistics for Scores Obtained from the Problem Solving Achievement Tests at Each Grade Level

	5th Grade	6th Grade	7th Grade	8th Grade
N	29	29	25	33
Mean	40,66	38,83	35,84	40,30
Std. Deviation	6,96	11,59	13,29	11,77
Minimum	28	16	10	12
Maximum	54	54	54	54
Skewness	-0,16	-0,62	-0,61	-0,57
Kurtosis	-0,79	-0,83	-0,96	-0,36

As given in Table 4.1, the maximum scores taken from problem solving achievement tests at each grade level are the same (*max* = 54), but the minimum scores change from grade level to grade level. Therefore, the biggest difference between the minimum and maximum scores taken from test was seen in the 7th grade since the minimum score obtained from test is 10. Considering the 5th grade problem solving achievement score, the maximum score is 54 and the minimum score is 28, with a mean of 40.66 (SD = 6.96). Based on the 6th grade problem solving achievement score, the maximum score is 54 and the minimum score is 16, with a mean of 38.83 (SD = 11.59). When the 7th grade achievement scores are examined, it is seen that the maximum score is 54 and minimum score is 10, with a mean of 35.84 (SD = 13.29). As for the 8th grade achievement score, the maximum score is 54 and the minimum score 12, with a mean of 40.30 (SD = 11.77). When the mean scores of the problem solving achievement tests are examined at different grade levels, it is observed that there is a difference of 5 points between the biggest and the smallest mean scores. The problem solving achievement tests includes 6 problems, with 9 points each. Since the difference of 5 points on mean score does not even correspond to one question, the difference can be considered as insignificant. It is seen that the mean scores obtained from the problem solving achievement tests of fifth and eighth grade students are almost equal. It might be because the achievement level of fifth and eighth grade students was the same, or another reason might be that the difficulty level of the problem solving achievement tests of the fifth and eighth grade students was parallel to each other.

Furthermore, the skewness and kurtosis values of problem solving achievement scores are presented in Table 4.1. The achievement scores of the fifth grade students with skewness of -0.16 and kurtosis of -0.79, of sixth grade students with skewness of -0.62 and kurtosis of -0.83, of seventh grade students with skewness of -0.61 and kurtosis of -0.96, and of eight grade students with skewness of -0.57 and kurtosis of -0.36 were normally distributed. Briefly, information about the nature of the distribution of the scores are provided with these values. According to these values, at each grade level, the problem solving achievement scores are normally distributed.

4.2. A Basic Qualitative Analysis: Students' Use of Problem-Solving Steps and Strategy Use

Results are reported with respect to the middle school students' conceptions about mathematical problem solving. Specifically, this section describes 5th, 6th, 7th and 8th grade students' strategy use, and use of problem solving steps in the word problems, and demonstration of meaningful understanding, and students' errors. In other words, a description of middle school students' use of problem solving steps and strategy use, and demonstration of meaningful and procedural understanding and errors are presented in this section. An overall table showing the results of all items is given at the beginning of the evaluation each grade level. In the evaluation of the items in the Problem Solving Achievement Test at each grade level, the items were grouped as binary or triple according to the objectives of subjects. Specifically, problems in the tests of each grade level were assessed by grouping objectives related to numbers. A description of students' use of problem solving steps, demonstration of meaningful learning and procedural understanding, errors, and misconceptions were presented. Under each group of items, there is also the description of students' strategy use. Special cases were illustrated with solution of students who did not reach a correct solution but showed positive behavior in understanding of the concepts in the problem evaluated.

4.2.1 Fifth Grade Problems

Table 4.2. Frequency of problem solving steps codes of all problems in the fifth grade

		ITEMS					
		ITEM 1	ITEM 2	ITEM 3	ITEM 4	ITEM 5	ITEM 6
		Count (%)	Count (%)	Count (%)	Count (%)	Count (%)	Count (%)
Seeks to Understand the Problem	SUP1	22 (75,9%)	23 (79,3%)	26 (89,7%)	22 (75,9%)	25 (86,2%)	22 (75,9%))
	SUP2	2 (6,9%)	0 (0,0%)	0 (0,0%)	1 (3,4%)	2 (6,9%)	3 (10,3%)
	SUP4	3 (10,3%)	1 (3,4%)	0 (0,0%)	1 (3,4%)	1 (3,4%)	1 (3,4%)
	SUP5	2 (6,9%)	5 (17,2%)	3 (10,3%)	5 (17,2%)	1 (3,4%)	3 (10,3%)
Making a Plan/Organization	MPO1	11 (37,9%)	9 (31,0%)	23 (79,3%)	9 (31,0%)	9 (31,0%)	24 (82,8%)
	MPO2	5 (17,2%)	1 (3,4%)	1 (3,4%)	3 (10,3%)	3 (10,3%)	2 (6,9%)
	MPO3	12 (41,4%)	19 (65,5%)	5 (17,2%)	15 (51,7%)	14 (48,3%)	3 (10,3%)
	MPO4	1 (3,4%)	0 (0,0%)	0 (0,0%)	2 (6,9%)	3 (10,3%)	0 (0,0%)
Implementing the Plan	IP1	16 (55,2%)	9 (31,0%)	24 (82,8%)	10 (34,5%)	9 (31,0%)	26 (89,7%)
	IP2	11 (37,9%)	20 (69,0%)	5 (17,2%)	17 (58,6%)	15 (51,7%)	3 (10,3%)
	IP3	0 (0,0 %)	0 (0,0 %)	0 (0,0 %)	0 (0,0 %)	2 (6,9%)	0 (0,0 %)
	IP4	2 (6,9%)	0 (0,0%)	0 (0,0%)	2 (6,9%)	3 (10,3%)	0 (0,0%)
Control/Assessment of Situation	CAS1	7 (24,1%)	10 (34,5%)	6 (20,7%)	6 (20,7%)	5 (17,2%)	6 (20,7%)
	CAS2	18 (62,1%)	16 (55,2%)	17 (58,6%)	18 (62,1%)	12 (41,4%)	20 (69,0%)
	CAS3	3 (10,3%)	2 (6,9%)	3 (10,3%)	4 (13,8%)	11 (37,9%)	2 (6,9%)
	CAS5	1 (3,4%)	1 (3,4%)	3 (10,3%)	1 (3,4%)	1 (3,4%)	1 (3,4%)
Progression Through All	PTA1	3 (10,3%)	5 (17,2%)	4 (13,8%)	4 (13,8%)	4 (13,8%)	4 (13,8%)
	PTA2	26 (89,7%)	24 (82,8%)	25 (86,2%)	25 (86,2%)	25 (86,2%)	25 (86,2%)

Students' understanding of the concept of a fraction as whole-to-a part relationship is assessed in the 1st Problem and 3rd Problem. In the problems, ability to find the desired fraction of a given whole is aimed to be assessed. In addition, these problems aim to examine students' ability to use the "making a drawing strategy" as well as the arithmetic strategy. Therefore, these two problems were analyzed together.

Table 4.3. Frequency of problem solving steps codes with respect to Problems 1&3 in the fifth grade

		ITEMS	
		ITEM 1	ITEM 3
		Count (%)	Count (%)
Seeks to Understand the Problem	SUP1	22 (75,9%)	26 (89,7%)
	SUP2	2 (6,9%)	0 (0,0%)
	SUP4	3 (10,3%)	0 (0,0%)
	SUP5	2 (6,9%)	3 (10,3%)
Making a Plan/an Organization	MPO1	11 (37,9%)	23 (79,3%)
	MPO2	5 (17,2%)	1 (3,4%)
	MPO3	12 (41,4%)	5 (17,2%)
	MPO4	1 (3,4%)	0 (0,0%)
Implementing the Plan	IP1	16 (55,2%)	24 (82,8%)
	IP2	11 (37,9%)	5 (17,2%)
	IP4	2 (6,9%)	0 (0,0%)
Control, Assessment of Situation	CAS1	7 (24,1%)	6 (20,7%)
	CAS2	18 (62,1%)	17 (58,6%)
	CAS3	3 (10,3%)	3 (10,3%)
	CAS5	1 (3,4%)	3 (10,3%)
Progression Through All	PTA1	3 (10,3%)	4 (13,8%)
	PTA2	26 (89,7%)	25 (86,2%)

As seen in the 1st problem, *Buse uses $\frac{7}{24}$ of her 24 crayons, and Esra uses $\frac{1}{3}$ of Buse's crayons.* The students were asked to calculate *how many crayons Buse and Esra use in total.* Twenty-nine students completed the 1st Problem. Sixteen students were successful, while eleven students were unsuccessful in this problem, and two students had no correct or incorrect answer. As can be seen in Table 4.3, twenty-two students were able to re-write the problem statement with their own words and write properly what is given and what is wanted (SUP1), which implies that they demonstrated an understanding of the verbal information presented in the 1st Problem, but five students appeared not to understand the verbal information as three of them failed in the progress of deciding what is given and what is wanted in the problem and re-writing the problem statement in their own words, and two students were unable to state what is given and what is wanted in the problem clearly even though they were able to re-state the problem statement. A solution plan was not indicated clearly by five students in this problem (MPO2). Eleven students made an appropriate plan by organizing what is given to reach what is wanted and they chose an appropriate strategy to solve the problem (MPO1), but twelve students were unable to organize and relate what is given to reach what is wanted (MPO3). As illustrated in the Figure below, the main reason for this is students' misconceptions about the subject of fractions. In other words, it seems that students did not clearly understand the fraction concept which represents a part of the whole.

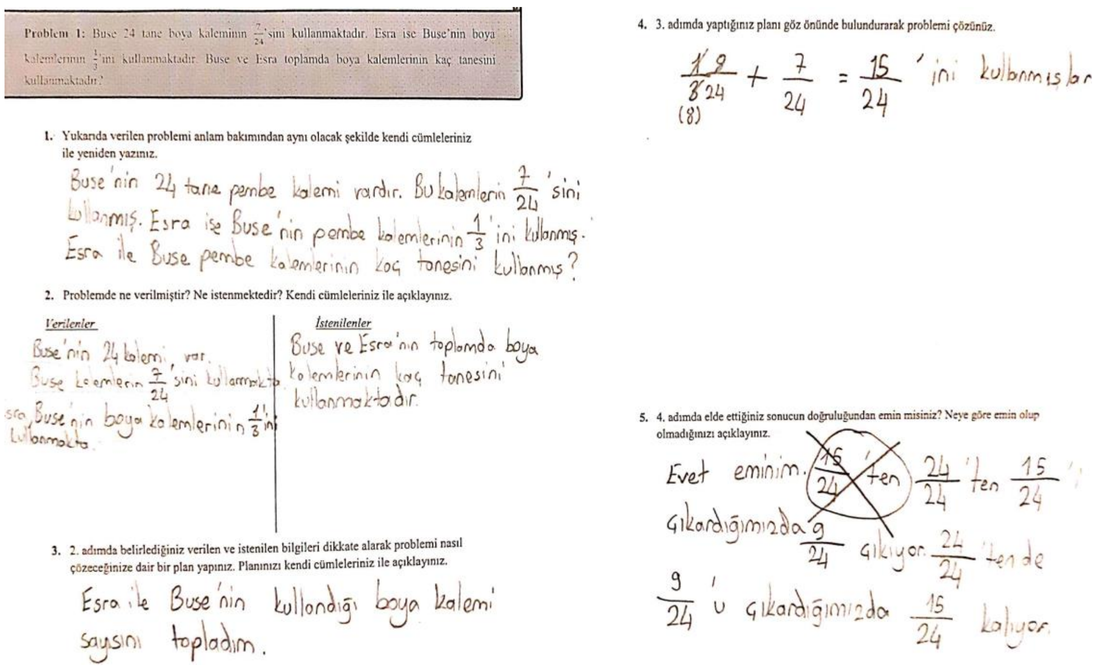
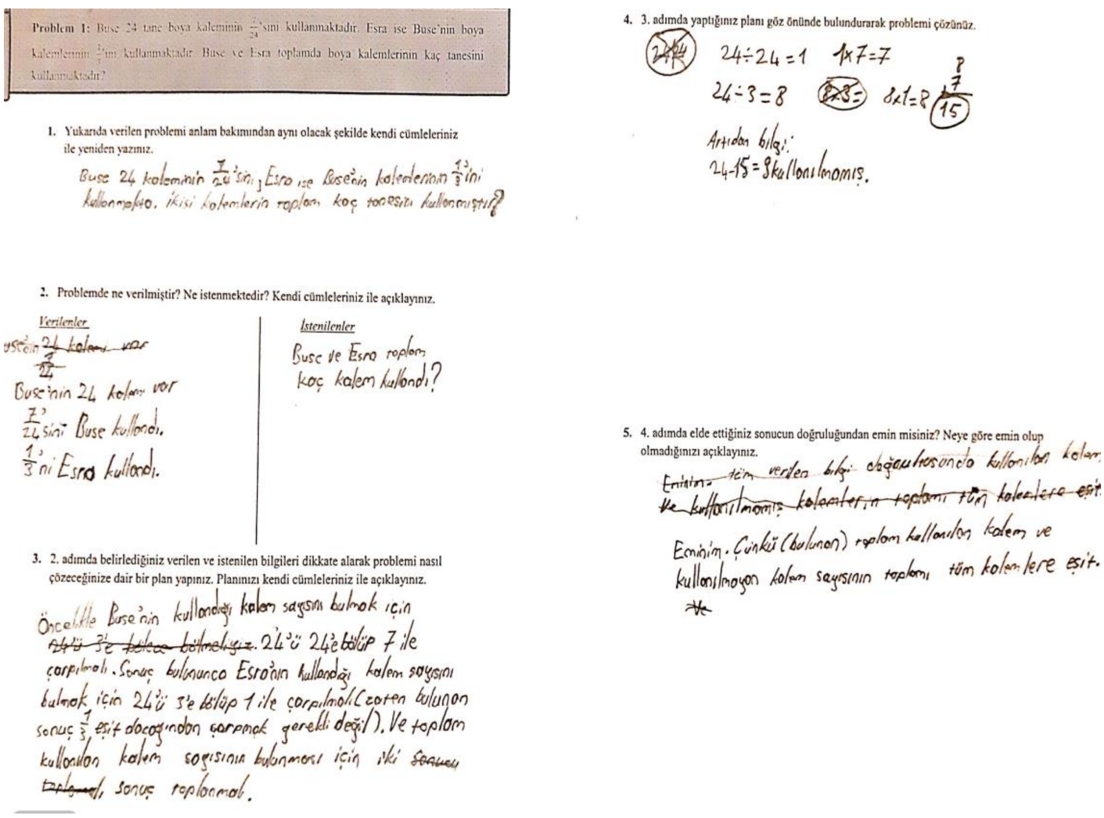


Figure 4.1. An example worksheet from a fifth grade student for the 1st problem (coded as SUP1, MPO3, and IP2)

As illustrated in Figure 4.1, a student appropriately determines what is given and what is asked in the problem statement, but in making a plan and implementation of the solution plan step, given fractions were considered as the number of crayons. Hence, the student should first calculate the desired fraction of the quantities ($\frac{7}{24}$ *of 24 and* $\frac{1}{3}$ *of Buse's crayons)* and then add them instead of adding the fraction expressions directly. On the contrary, in the fourth step of the solution of the problem, it is seen that the student added up $\frac{1}{3}$ and $\frac{7}{24}$ and s/he found $\frac{15}{24}$ as the answer.

Sixteen students appeared to carry out solution plan successfully (IP1). Eleven students also appeared to implement their plan successfully, but their plan was not appropriate to reach the correct answer (IP2). The students implemented their solution plan accurately, but their solution plan was not true. An example for IP2 in this problem was shown in Figure 4.1. In the making a plan and implementing the plan steps, it can be observed that students mostly used arithmetic calculations instead of using visual representations and mathematical modeling. It was understood from Table 4.3 that two students did not possess the necessary knowledge to solve the problem.


Problem 1: Buse 24 tane boya kaleminin $\frac{7}{24}$'sını kullanmaktadır. Esra ise Buse'nin boya kalemlerinin $\frac{1}{3}$'ini kullanmaktadır. Buse ve Esra toplamda boya kalemlerinin kaç tanesini kullanmaktadır?

1. Yukarıda verilen problemi anlam bakımından aynı olacak şekilde kendi cümleleriniz ile yeniden yazınız.

Buse 24 kaleminin $\frac{7}{24}$'sini, Esra ise Buse'nin kalemlerinin $\frac{1}{3}$'ini kullanmakta. İkisi kalemlerin toplam kaç tanesini kullanmıştır?

2. Problemde ne verilmiştir? Ne istenmektedir? Kendi cümleleriniz ile açıklayınız.

Verilenler	İstenilenler
Buse'nin 24 kalemi var $\frac{7}{24}$'sini Buse kullandı. $\frac{1}{3}$'ni Esra kullandı.	Buse ve Esra toplam kaç kalem kullandı?

3. 2. adımda belirlediğiniz verilen ve istenilen bilgileri dikkate alarak problemi nasıl çözeceğinize dair bir plan yapınız. Planınızı kendi cümleleriniz ile açıklayınız.

Öncelikle Buse'nin kullandığı kalem sayısını bulmak için ~~24'ü 3'e bölmeliyiz~~. 24'ü 24'e bölüp 7 ile çarpılmalı. Sonuç bulununca Esra'nın kullandığı kalem sayısını bulmak için 24'ü 3'e bölüp 1 ile çarpılmalı (zaten bulunan sonuç $\frac{1}{3}$ eşit olduğundan çarpmak gerekli değil). Ve toplam kullanılan kalem sayısının bulunması için iki ~~sonucu toplamalı~~, sonuç toplanmalı.

4. 3. adımda yaptığınız planı göz önünde bulundurarak problemi çözünüz.

$24 \div 24 = 1$ $1 \times 7 = 7$

$24 \div 3 = 8$ $8 \times 1 = 8$ $8 + 7 = 15$

Artıdan bilgi:
24-15 = 9 kullanılmamış.

5. 4. adımda elde ettiğiniz sonucun doğruluğundan emin misiniz? Neye göre emin olup olmadığınızı açıklayınız.

Eminim. Çünkü (bulunan) toplam kullanılan kalem ve kullanılmayan kalem sayısının toplamı tüm kalemlere eşit.

Figure 4.2. An example worksheet from a fifth grade student for the 1st problem (coded as SUP1, MPO1, IP1, and CAS5)

As can be seen in Figure 4.2, the student re-stated the problem statement with his own words and wrote what is given and what is wanted in the 1st Problem. Thus, the category of ‘seek understanding the problem” was coded as SUP1. Then, the student made an appropriate solution plan for the 1st Problem. He preferred to use the arithmetic strategy and made an appropriate solution plan, so ‘making a plan’ category was coded as MPO1. His solution plan was as follows: “*I find* $\frac{7}{24}$ *of 24 to find the number of crayons that Buse uses and I will find* $\frac{1}{3}$ *of 24 to find the number of crayons Esra uses. Then, I add up the results”.* He implemented his solution plan (IP1) accurately. The category of ‘control and assessment of the situation’ was coded as CAS5 since he re-stated his solution in this phase.

In the control, and assessment of solution phase of the 1st Problem, seven students were able to provide an explanation for why their thinking was correct (CAS1), while eighteen students only said “I am sure” without any explanation (CAS2). Four students were not able to articulate why their solution was correct. An example for code CAS1 is given below.

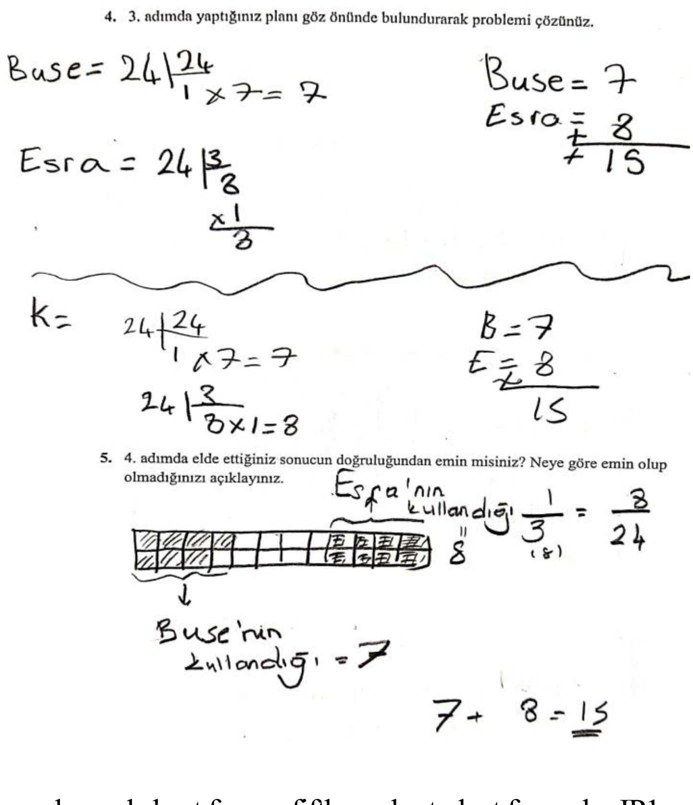
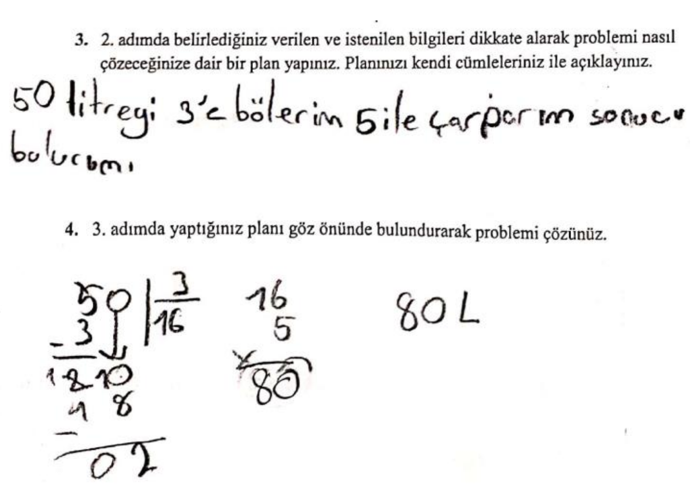


Figure 4.3. An example worksheet from a fifth grade student for codes IP1 and CAS1

As can be seen in Figure 4.3, the student solved the problem using the arithmetic strategy similar to the solution of the student in Figure 4.2. Here, the student solved the 1st problem by using the making a drawing strategy in the control and assessment of situation phase, that is, in the fifth step of the problem solution, while he used the arithmetic strategy in the fourth step of the problem solution. Hence, the category of 'control and assessment of the situation' was coded as CAS1.

As it was mentioned previously, the 3rd problem also includes the concept of fraction. The 3rd Problem is as follows: *"Onur, who went on holiday with his own car, reached his destination with* $\frac{3}{5}$ *of the 50 liter tank. Onur filled the car's 50-liter tank completely before starting the journey. How many liters of gasoline were left in the tank of his car when Onur arrived his destination?".*

Problem 3 was completed by twenty-nine students. Twenty-four students were successful, and five students were unsuccessful. As can be seen in Table 4.3, twenty-six students understood the given verbal information, but five students were unable to state what is given and what is wanted in the problem, while they were able to re-write the problem in their own words. Twenty-four students clearly indicated and carried out a solution plan, but five of them were unsuccessful in organizing what is given in order to reach the correct solution. The reason for this might be that the students who preferred to use the arithmetic strategy had a misconception about fractions.


3. 2. adımda belirlediğiniz verilen ve istenilen bilgileri dikkate alarak problemi nasıl çözeceğinize dair bir plan yapınız. Planınızı kendi cümleleriniz ile açıklayınız.

50 litregi 3'e bölerim 5 ile çarparım sonucu bulurum

4. 3. adımda yaptığınız planı göz önünde bulundurarak problemi çözünüz.

50 | 3 → 16; 16 × 5 = 80

80 L

Figure 4.4. An example worksheet from a fifth grade student (Problem 3)

As shown in Figure 4.4, they stated in “making a plan step” that “$\frac{3}{5}$ of 50 must be calculated firtly and then the result of this should be substracted from 50”, but in the calculation, they devided 50 by 3 instead of 5. The students who used the “making a drawing strategy” reached the correct solution and made fewer mistakes in that situation compared to the students who used the arithmetic strategy.

In the 3rd Problem, seventeen students were also unable to articulate why their solution was correct as they just said “I am sure” without giving any explanation. Three students had any control for their solution. Six students solved the problem in a different way and checked their solution appropriately.

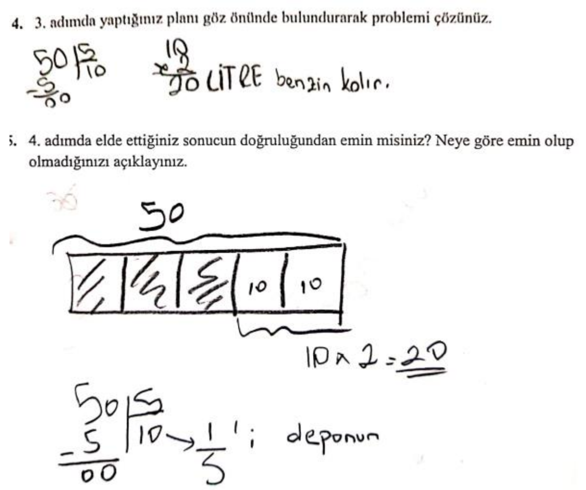


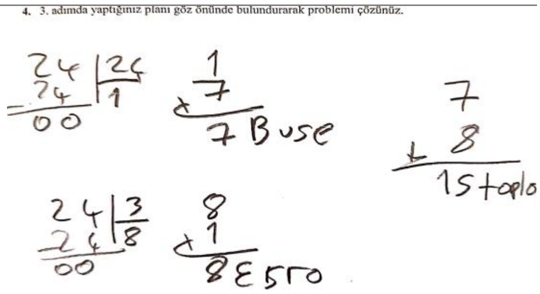
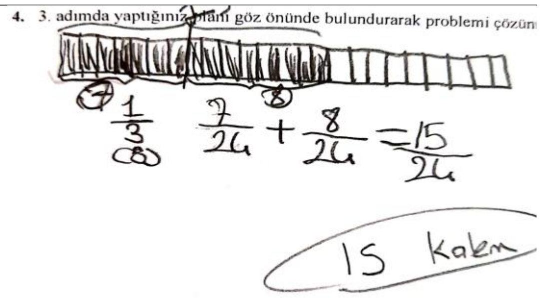
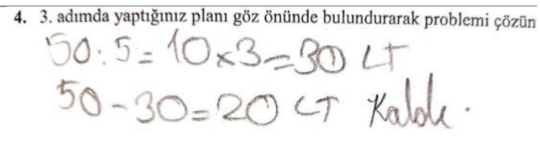
Figure 4.5. An example worksheet from a fifth grade student for codes IP1 and CAS1

As illustrated in Figure 4.5, the student used the arithmetic strategy successfully in the implementation of the solution plan step. In the control and assessment of the situation phase, that is, the fifth step of the problem solution, the student solved the problem in a different way. He solved the problem by using the making a drawing strategy and obtained the same result as the one he obtained in the previous step.

Table 4.4. Frequency of the strategies used in Problems 1 and 3 in the fifth grade

		ITEMS	
		ITEM 1	ITEM 3
		Count (%)	Count (%)
Use of Strategy	ART	27 (93,1%)	28 (96,6%)
	MD	2 (6,9%)	1 (3,4%)

As it can be concluded from the Table 4.4 that twenty-seven students in Problem 1 and twenty-eight students in Problem 2 used the arithmetic strategy (ART) demonstrating procedural understanding, and two students in Problem 1 and one student in Problem 2 used the making a drawing strategy demonstrating meaningful understanding. When students' written works were analyzed, it was seen that many students modeled the questions and used the making a drawing strategy in their minds, but the students saw the arithmetic part when they wrote down their plan.

Explanation of Students' Use of Strategy	Example of Students' Worksheet
Arithmetic Strategy (Problem 1) Firstly, the student calculated the $\frac{7}{24}$ of 24, and he found $\frac{1}{3}$ of 24 so that the number of Buse's and Esra's crayons could be founded. Then, he added up the results he found (7 + 8).	
Making a Drawing Strategy (Problem 1) The student firstly divided a whole into 24 equal parts as it was given in the problem statement that Buse uses $\frac{7}{24}$ of her 24 crayons, and Esra uses $\frac{1}{3}$ of 24 crayons. So, the student marked 7 parts and 8 parts respectively. Finally, the student saw that 15 parts that corresponded to 15 crayons were used.	
Arithmetic Strategy (Problem 3) As it is seen, the student first calculated $\frac{3}{5}$ of 50 and found 30. Then, he subtracted 30 out of 50 to find the remaining amount of gasoline.	

Making a Drawing Strategy (Problem 3)

At first, a whole was divided into 5 equal parts. $\frac{1}{5}$ of the whole was calculated by dividing 50 by 5 and the result was found as 10. Finally, by multiplying 10 by 2, the remaining two parts ($\frac{2}{5}$) were found to be equal to 20.

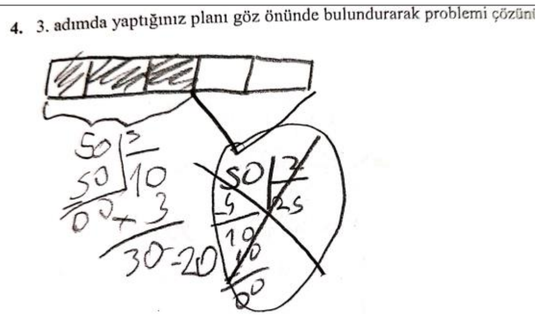


Figure 4.6. Use of strategies of fifth grade students in Problem 1 and Problem 3

The 2nd and the 4th Problem assessed students' understanding of Sets and Whole numbers. Additionally, the fifth grade students' ability to use of the arithmetic strategy and the working backward strategy was aimed to be examined with these two problems. Therefore, these two problems were grouped and analyzed together.

Table 4.5. Frequency of problem solving steps codes with respect to Problems 2 and 4 in the fifth grade

		ITEMS	
		ITEM 2	ITEM 4
		Count (%)	Count (%)
Seeks to Understand the Problem	SUP1	23 (79,3%)	22 (75,9%)
	SUP2	0 (0,0%)	1 (3,4%)
	SUP4	1 (3,4%)	1 (3,4%)
	SUP5	5 (17,2%)	5 (17,2%)
Making a Plan/Organization	MPO1	9 (31,0%)	9 (31,0%)
	MPO2	1 (3,4%)	3 (10,3%)
	MPO3	19 (65,5%)	15 (51,7%)
	MPO4	0 (0,0%)	2 (6,9%)
Implementing the Plan	IP1	9 (31,0%)	10 (34,5%)
	IP2	20 (69,0%)	17 (58,6%)
	IP4	0 (0,0%)	2 (6,9%)
Control, Assessment of Situation	CAS1	10 (34,5%)	6 (20,7%)
	CAS2	16 (55,2%)	18 (62,1%)
	CAS3	2 (6,9%)	4 (13,8%)
	CAS5	1 (3,4%)	1 (3,4%)
Progression Through All	PTA1	5 (17,2%)	4 (13,8%)
	PTA2	24 (82,8%)	25 (86,2%)

In the 2nd Problem, the following problem statement was given: "*Anıl receives six new marbles and gives 13 of his marbles to his friend. If Anil has 41 marbles at the end,* *how many marbles did he have at first?".* In the 4th Problem, which is also related to subject of Sets and Whole Numbers, the following problem statement was given: "*Burak gives his brother 11 game cards in exchange for six game cards, and Burak buys 15 new game cards and he has a total of 94 game cards. How many game cards did Burak initially have?".* Nine students were successful in solving 2nd Problem, while twenty students were unsuccessful. 4th Problem was solved successfully by ten students and seventeen students could not solve it. Furthermore, two students did not have a correct or incorrect answer. As can be seen in Table 4.5, twenty-two students appeared to demonstrate an understanding of the verbal information given in the 2nd and 4th Problems, but five students seemed to not understand what is given and what is wanted. Nine students made a solution plan successfully, while nineteen students in the 2nd Problem and fifteen students in the 4th Problem were unable to make an appropriate solution plan. When the students' solutions were examined in general, it was seen that the students could not properly interpret the values given in the problem statements. An example of a fifth grade students' solution is given below.

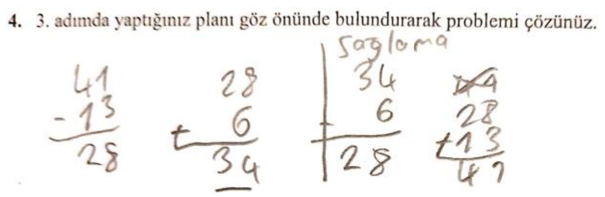


Figure 4.7. An example worksheet from a fifth grade student for codes MPO3 (Problem 2)

In the solution of the 2nd Problem given above, the students were supposed to subtract 6 from 41 and add up 13. As seen in Figure 4.7, in contrast, the students added 6 to 41 and then subtracted 13.

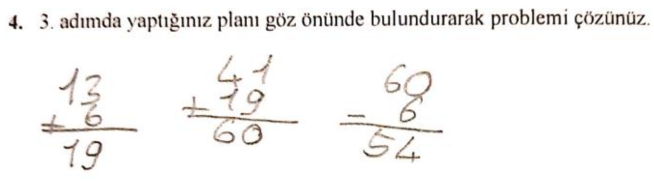


Figure 4.8. An example worksheet from a fifth grade student for codes MPO3 (Problem 2)

Another example was given in Figure 4.8. It illustrates why students could not make a solution plan properly and failed to solve problems 2 and 4 which are related to Sets and Whole Numbers. Another common error was that many students could not make sense of the values given in the problem statement, and tried to reach the result by adding 6 and 13 to 41 as illustrated in Figure 4.8.

In addition, it was concluded from the Table 4.5 that in the 4^{th} Problem, two students appeared not to have the necessary knowledge about the concept. Like ten students in the 2^{nd} Problem, six students provided an explanation for why their thinking was correct in Problem 4. In these problems, students also just said "Yes, I am sure" in the process of checking the problem. In this problem, students were given the chance to practice the working backward strategy. It might be concluded that the students who chose the working backward strategy generally solved the problem easily. The students who used the arithmetic strategy summed up all the numbers given in the problem without considering that some of them should be subtracted and some of them should be added.

Table 4.6. Frequency of the strategies used in Problems 2 and 4 in the fifth grade

		ITEMS	
		ITEM 2	ITEM 4
		Count (%)	Count (%)
Use of Strategy	ART	22 (75,9%)	20 (69,0%)
	NOS	0 (0,0%)	2 (6,9%)
	WB	7 (24,1%)	7 (24,1%)

The students again mostly (twenty-two students in Problem 2 and twenty students in Problem 4) tended to use the arithmetic strategy in the solution process, and seven students appeared to use the working backwards strategy in the solution process of the problems.

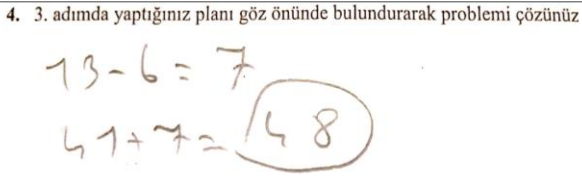
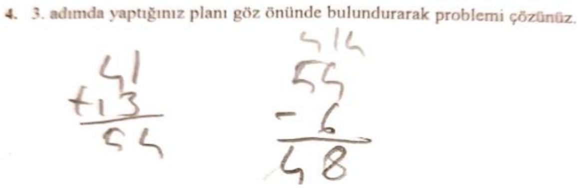
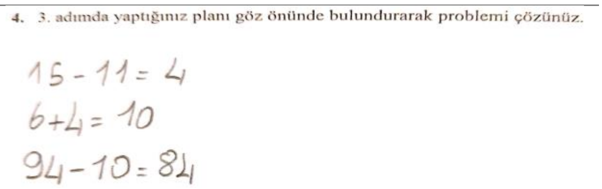
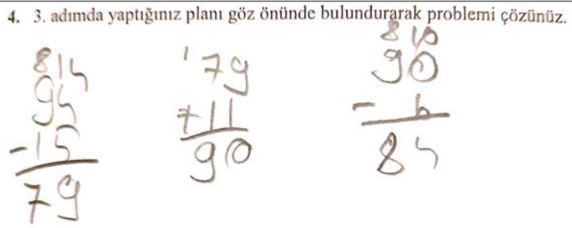
Explanation of Students' Use of Strategy	Example of Students' Worksheet
<u>Arithmetic Strategy (Problem 2)</u> The student first subtracted 6 from 13 because Anıl gives his friend 13 marbles and receives 6 new marbles, and the student found 7 as the result. Then, he added up 41 and 7.	
<u>Working Backward Strategy (Problem 2)</u> The student firstly added 41 marbles to 13 marbles that Anıl gives to his friends, and then subtracted 6 new marbles from the 54 marbles.	
<u>Arithmetic Strategy (Problem 4)</u> The student first subtracted 11 from 15 because Burak buys 15 new game cards and receives 11 new game cards from his friends. Then, the student added the results 4 and 6 because Burak gave his friends 6 game cards. Lastly, the student subtracted 10 from 94.	
<u>Working Backward Strategy (Problem 4)</u> By using the backward strategy, the student firstly subtracted the 15 newly purchased game cards from 94 game cards. Then, he added 11 game cards, which Burak gives to his friend, to 79 playing cards. Finally, the student added 6 game cards, which Burak received from his brother, to 90 game cards.	

Figure 4.9. Use of strategies of fifth grade students in Problem 2 and Problem 4

Like the 2nd and 4th Problem, the 5th Problem and the 6th Problem also assessed students' understanding of Sets and Whole Numbers. Furthermore, students' ability to use the arithmetic strategy, the guess and check strategy, the making a drawing strategy, and the organizing data strategy were examined in these problems. In order to make the tables simpler and more understandable, 1st and 2nd problems and 4th and 5th problems are grouped separately. Therefore, the 5th and the 6th Problems were analyzed together. In the 5th problem, that the following situation was given: "*Tolga* *wants to build a wooden car for his little brother, and Tolga spent a total of 50 TL for the boards and for the wheels. How much did he pay for the boards if the boards are 2 TL more expensive than the wheels?".* In the 6th problem, the students were asked the following problem: *"Selin takes piano lessons every weekday for 30 minutes. Selin also takes piano lessons for 60 minutes on Saturdays and Sundays. Find how many minutes of piano lessons she takes for five days from Monday to Friday".*

Table 4.7. Frequency of problem solving steps codes with respect to Problems 5 and 6 in the fifth grade

		ITEMS	
		ITEM 5	ITEM 6
		Count (%)	Count (%)
Seeks to Understand the Problem	SUP1	25 (86,2%)	22 (75,9%))
	SUP2	2 (6,9%)	3 (10,3%)
	SUP4	1 (3,4%)	1 (3,4%)
	SUP5	1 (3,4%)	3 (10,3%)
Making a Plan/Organization	MPO1	9 (31,0%)	24 (82,8%)
	MPO2	3 (10,3%)	2 (6,9%)
	MPO3	14 (48,3%)	3 (10,3%)
	MPO4	3 (10,3%)	0 (0,0%)
Implementing the Plan	IP1	9 (31,0%)	26 (89,7%)
	IP2	15 (51,7%)	3 (10,3%)
	IP3	2 (6,9%)	0 (0,0%)
	IP4	3 (10,3%)	0 (0,0%)
Control/Assessment of Situation	CAS1	5 (17,2%)	6 (20,7%)
	CAS2	12 (41,4%)	20 (69,0%)
	CAS3	11 (37,9%)	2 (6,9%)
	CAS5	1 (3,4%)	1 (3,4%)
Progression Through All	PTA1	4 (13,8%)	4 (13,8%)
	PTA2	25 (86,2%)	25 (86,2%)

Problem 5 was solved correctly by nine students, while Problem 6 was solved correctly by twenty-six students. Seventeen students answered Problem 5 incorrectly, and three students could not solve it in any way. Moreover, three students failed in the solution of Problem 6. The reason why students were more successful in the 6th Problem than in the 5th problem could be that the 6th Problem is easier to understand than the 5th

Problem, and the solution of Problem 6 requires only one operation. Another reason might be that the 5th Problem requires conceptual knowledge of algebra, while the 6th problem is more procedural. As can be seen in Table 4.7, twenty-seven students appeared to demonstrate an understanding of the verbal information in 5th Problem, while two students appeared not to have understood the problem. Many of the students (25 students) also clearly stated what is given and what is wanted in 6th Problem, but four students were unable to understand the problem. In the 5th Problem, fourteen students could not make a plan to solve the problem, and three students seemed not to have the necessary knowledge to solve the problem. Only nine students made an appropriate solution plan for the 5th problem and implemented their solution plan accurately. Briefly, in this problem, mathematically "two numbers with a difference equal to 2 and the sum equal to 50" were asked to be found. Many students had difficulty in understanding this statement because they thought more procedural. Furthermore, this might be due to the fact that the students were not accustomed to solving such problems in their math class or the students may lack conceptual knowledge of algebra and they were not familiar with using the guess and check strategy. The five of nine students using the guess and check strategy, given as an example in Figure 4.10 below, found two numbers with a total of 50 and a difference of 2.

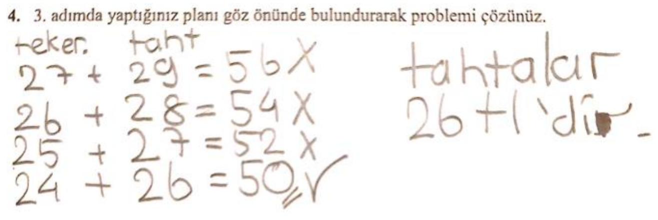


Figure 4.10. An example worksheet from a fifth grade student for codes IP1 (Problem 5)

As seen in Figure 4.10, the student selected two numbers with a difference of 2 and added them up until he obtained 50 as the result. In the fourth attempt, the student found the numbers 24 and 26. At the same time, this method of solution (using the guess and check strategy) made the student's work much easier because he did not need to use the concept of algebra, and knowing the addition operation was enough to solve the problem. In Figure 4.11, the most common mistake in solving the fifth problem are exemplified below.

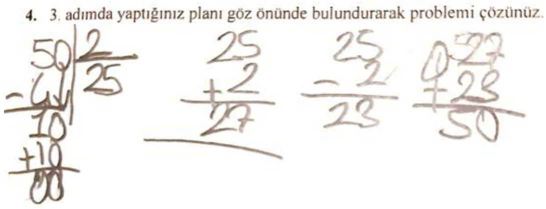


Figure 4.11. An example worksheet from a fifth grade student for codes IP2 (Problem 5)

As it can be concluded from Figure 4.11, the student firstly assumed that two numbers are equal, so he divided 50 by 2 and obtained 25. Since the difference between these two number was 2, he must have added 1 to 25, and subtracted 1 from 25 to get the numbers 24 and 26. However, since the student misinterpreted the expression in the question, he added 2 to 25, and subtracted 2 from 25 and obtained the numbers 23 and 27, whose sum is 50 but the difference is 4.

Moreover, Table 4.7 shows that unlike the 5th Problem, twenty-four students appeared to organize what is given to make an appropriate plan, and those students with two students who did not clearly indicate their solution plan carried out their solution plan successfully. Only three students were unable to solve the problem successfully. As mentioned previously, the following problem statement was given: "*Selin takes piano lessons every weekday for 30 minutes, and Selin also takes piano lessons for 60 minutes on Saturdays and Sundays. How many minutes of piano lessons does she take for five days from Monday to Friday?"* The problem is quite easy and requires only one operation. In this problem, the level of attention of the students was wanted to be observed by giving them extra information that would not be used in solving problem. When the number of successful students is considered, it can be concluded that the students' attention level is high. Only three students solved the problem by considering the whole week although the students were asked how many minutes of piano lessons were taken during a weekday.

In addition, it is seen in Table 4.7 that twelve students in 5th Problem and twenty students in 6th Problem only wrote "I am sure" in "controlling the answer" part as in the previous problems. Eleven students did not give any explanations why their solution was correct. On the other hand, six students were able to articulate why their solution was correct in both problems.

Table 4.8. Frequency of the strategies used in Problems 5 and 6 in the fifth grade

		ITEMS	
		ITEM 5	ITEM 6
		Count (%)	Count (%)
Use of Strategy	ALG	1 (3,4%)	0 (0,0%)
	ART	18 (62,1%)	23 (79,3%)
	GC	5 (17,2%)	0 (0,0%)
	MD	0 (0,0%)	6 (20,7%)
	NOS	4 (13,8%)	0 (0,0%)
	ORD	1 (3,4%)	0 (0,0%)

At least 3 different strategies were used in the previous problems. Unlike the previous problems, 5 different strategies were used in the solution process of Problem 5. One student used the algebraic strategy; eighteen students used the arithmetic strategy; five students used the guess and check strategy; and one student preferred to use the organizing the data strategy in this problem, and four students could not use a strategy to solve the problem. In the 6th Problem, the students again mostly (23 students) used the arithmetic strategy in the solution process, while six students preferred to use the making a drawing strategy demonstrating complete meaningful learning and in-depth understanding.

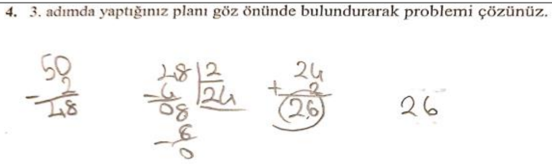
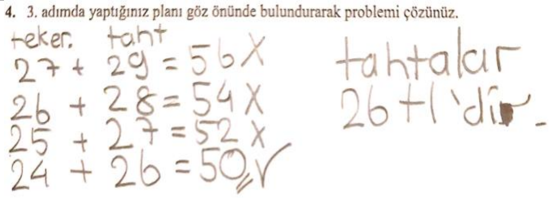
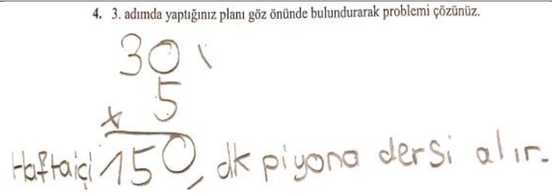
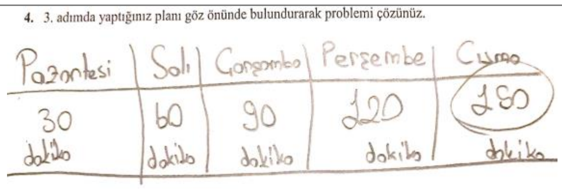
<table>
<tr><th>Explanation of Students’ Use of Strategy</th><th>Example of Students’ Worksheet</th></tr>
<tr><td><u>Arithmetic Strategy (Problem 5)</u>
Firstly, the student subtracted 2 from 50 since one of the numbers is 2 more than the other. Then, he divided 48 by 2, and he found 24. Finally, he added 2 to 24 and found the</td><td>
</td></tr>
<tr><td><u>Guess and Check Strategy (Problem 5)</u>
The student tried to find the numbers whose total is 50, and the difference is 2.</td><td>
</td></tr>
<tr><td><u>Arithmetic Strategy (Problem 6)</u>
The student multiplied 5 by 30 because weekday lesson time was 30 minutes per day.</td><td>
</td></tr>
<tr><td><u>Making a Drawing (Problem 6)</u>
The student organized the data by using the table.</td><td>
</td></tr>
</table>

Figure 4.12. Use of strategies of fifth grade students in Problem 5 and Problem 6

As talked above, first and third problems were about the concept of fractions and other four problems were about the concept of sets and whole numbers. In general, students have been successful in the step of understanding the problem in all problems. Otherwise, when compared with the natural number problems, it is seen that the students proceeded more successfully in the problem solving steps of fractions problems. It is seen that students are more successful in planning and implementing plan in fraction problems. When we look at the students’ use of strategy in solving problems, a significant number of students used the arithmetic strategy while just 3 students preferred to use the making a drawing strategy in fraction problems. Students’ use of strategies is more diverse in whole number problems. In addition to the arithmetic strategy, the students used the working backward, the guess and check, and organizing data strategies in whole number problems.

4.2.2. Sixth Grade Problems

Table 4.9. Frequency of problem solving steps codes all problems in the sixth grade

		ITEMS					
		ITEM 1	ITEM 2	ITEM 3	ITEM 4	ITEM 5	ITEM 6
		Count (%)	Count (%)	Count (%)	Count (%)	Count (%)	Count (%)
Seeks to Understand the Problem	SUP1	27 (93,1%)	25 (86,2%)	22 (75,9%)	22 (75,9%)	21 (69,0%)	22 (75,9%)
	SUP2	0 (0,0%)	0 (0,0%)	0 (0,0%)	1 (3,4%)	1 (3,4%)	1 (3,4%)
	SUP4	1 (3,4%)	1 (3,4%)	3 (10,3%)	4 (13,8%)	3 (10,3%)	2 (6,9%)
	SUP5	1 (3,4%)	3 (10,3%)	4 (13,8%)	2 (6,9%)	4 (17,2%)	4 (13,8%)
Making a Plan/Organization	MPO1	25 (86,2%)	12 (41,4%)	17 (58,6%)	10 (34,5%)	7 (24,1%)	17 (58,6%)
	MPO2	0 (0,0%)	1 (3,4%)	1 (3,4%)	5 (17,2%)	2 (6,9%)	0 (0,0%)
	MPO3	4 (13,8%)	11 (37,9%)	10 (34,5%)	9 (31,0%)	12 (41,4%)	5 (17,2%)
	MPO4	0 (0,0%)	5 (17,2%)	1 (3,4%)	5 (17,2%)	8 (27,6%)	7 (24,1%)
Implementing the Plan	IP1	25 (86,2%)	8 (27,6%)	15 (51,7%)	6 (20,7%)	9 (31,0%)	17 (58,6%)
	IP2	4 (13,8%)	13 (44,8%)	11 (37,9%)	9 (31,0%)	12 (41,4%)	4 (13,8%)
	IP3	0 (0,0%)	4 (13,8%)	2 (6,9%)	0 (0,0%)	0 (0,0%)	0 (0,0%)
	IP4	0 (0,0%)	4 (13,8%)	1 (3,4%)	14 (48,3%)	8 (27,6%)	8 (27,6%)
Control/Assessment of Situation	CAS1	12 (41,4%)	4 (13,8%)	1 (3,4%)	0 (0,0%)	1 (3,4%)	0 (0,0%)
	CAS2	8 (27,6%)	5 (17,2%)	20 (69,0%)	10 (34,5%)	13 (44,8%)	19 (65,5%)
	CAS3	3 (10,3%)	18 (62,1%)	8 (27,6%)	19 (65,5%)	15 (51,7%)	9 (31,0%)
	CAS5	6 (20,7%)	2 (6,9%)	0 (0,0%)	0 (0,0%)	0 (0,0%)	1 (3,4%)
Progression Through All	PTA1	13 (44,8%)	3 (10,3%)	1 (3,4%)	0 (0,0%)	1 (3,4%)	0 (0,0%)
	PTA2	16 (55,2%)	26 (89,7%)	28 (96,6%)	29 (100,0%)	28 (96,6%)	29 (100,0%)

Problem 1 and Problem 6 assessed students' understanding of the Sets and Whole Numbers concept. Here, the students were asked to solve the problems that required four operations with natural numbers. These problems are not very difficult for students in terms of difficulty level. They mostly analyze the attention and concentration of students while solving problems. In the 1st problem, students were given extra information that was not needed to use in solving the problem. The 6th Problem was adapted from a PISA problem. This problem tries to measure the level of students' ability to interpret and understand the detailed information, values and the table given. Hence, these two problems were preferred to be analyzed together. At the same time, these problems encourage students to use the making a drawing, guess and check and organizing data strategies as well as the arithmetic strategy. In the first problem, the students were asked the following problem: *Atakan has 17 marbles and 6 toy cars. The number of Sinan's marbles is 5 minus the 4 times of the number of Atakan's marbles. How many marbles does Sinan have?*

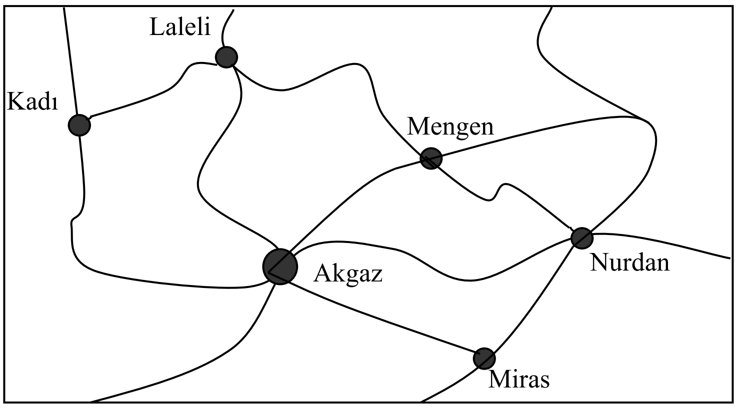
Table 4.10. Frequency of problem solving steps codes with respect to Problems 1 & 6 in the sixth grade

		ITEMS	
		ITEM 1	ITEM 6
		Count (%)	Count (%)
Seeks to Understand the Problem	SUP1	27 (93,1%)	22 (75,9%)
	SUP2	0 (0,0%)	1 (3,4%)
	SUP4	1 (3,4%)	2 (6,9%)
	SUP5	1 (3,4%)	4 (13,8%)
Making a Plan/Organization	MPO1	25 (86,2%)	17 (58,6%)
	MPO3	4 (13,8%)	5 (17,2%)
	MPO4	0 (0,0%)	7 (24,1%)
Implementing the Plan	IP1	25 (86,2%)	17 (58,6%)
	IP2	4 (13,8%)	4 (13,8%)
	IP4	0 (0,0%)	8 (27,6%)
Control/Assessment of Situation	CAS1	12 (41,4%)	0 (0,0%)
	CAS2	8 (27,6%)	19 (65,5%)
	CAS3	3 (10,3%)	9 (31,0%)
	CAS5	6 (20,7%)	1 (3,4%)
Progression Through All	PTA1	13 (44,8%)	0 (0,0%)
	PTA2	16 (55,2%)	29 (100,0%)

Twenty-nine students solved these problems in the sixth grade. In the solution of Problem 1, twenty-five students were successful, while four students were unsuccessful. As seen in Table 4.10, twenty-seven students appeared to re-write the problem statement in their own words and to decide what is given and what is wanted in the problem, but two students were unable to understand the verbal information presented in the problem. Twenty-five students were able to make an appropriate solution plan after understanding the problem and to carry out their solution plan successfully, but four students could not establish a relationship between what is given and what is wanted. It can be concluded that the students were able to recognize the extra given information in the problem statement, and so they solved the problem accurately. Moreover, twelve students gave an explanation for why their answer was correct, while nine students had no explanations or just re-stated their solution. Eight students just said "I am sure" without a clear explanation in the control part of the problem. The reason for this is that the students were not accustomed to control and check the solution of the problem.

The other problem is the 6th Problem about choosing the best and the most comfortable way to go on holiday. The students were asked the following problem*: Figure 1 shows the map of the region, and Figure 2 shows the distance between the towns.*

Figure 1: The map of the region.



Akgaz						
Kadı	*550*					
Laleli	*500*	*300*				
Mengen	*300*	*850*	*550*			
Nurdan	*500*		*1000*	*450*		
Miras	*300*	*850*	*800*	*600*	*250*	
	Akgaz	*Kadı*	*Laleli*	*Mengen*	*Nurdan*	*Miras*

Figure 2. The distance between the towns

Calculate the shortest distance between Nurdan and Kadı.

In the 6th Problem, seventeen students were successful, whereas four students were unsuccessful. Eight students could not solve this problem. Twenty-three students appeared to demonstrate an understanding of the verbal information presented in 6th Problem, whereas six students seemed not to have understood the problem situation in any way. Seventeen students carried out their solution plan successfully, but seven students seemed not to have the necessary knowledge to solve the problem. The reason is not that students' lack procedural knowledge since this problem only requires knowing addition after understanding the story of the problem (problem situation) and the values given in the problem statement. The reason might be the fact that the values did not make sense to students and students had difficulty in interpreting the table given in the problem.

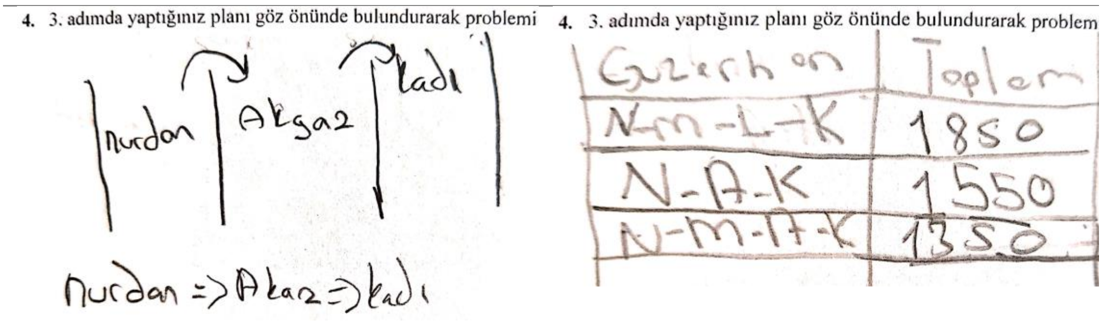


Figure 4.13. An example worksheet from a sixth grade student for codes MPO3 (Problem 6)

As seen in Figure 4.13 and the solution on the left, the student determined the route, but he could not determine the distances between the towns because he could not interpret the table in which the distance between the towns was given. As seen in the solution given on the right, the student could not reach the right solution because he chooses the possible routes incomplete.

In addition, in the 6th Problem, nineteen students only said "I am sure" without a clear explanation about why their solution was correct, and nine students did not give any explanations about the correctness of their answers. The reason might be that the students could not find an alternative way of solution.

Table 4.11. Frequency of the strategies used in Problems 1 and 6 in the sixth grade

		ITEMS	
		ITEM 1	ITEM 6
		Count (%)	Count (%)
Use of Strategy	ART	28 (96,6%)	10 (34,5%)
	GC	0 (0,0%)	1 (3,4%)
	MD	1 (3,4%)	10 (34,5%)
	NOS	0 (0,0%)	7 (24,1%)
	ORD	0 (0,0%)	1 (3,4%)

Twenty-eight students used the arithmetic strategy, while one student used the making a drawing strategy. As it can be concluded above, the students mostly had difficulty in solving Problem 6. Five different problem solving strategies were used by the students in solving this problem. Ten students preferred to use the arithmetic strategy, and ten students used the making a drawing strategy in the solution process of Problem 6. Additionally, one student used the 'guess and check strategy' and one student used the 'organizing the data strategy'.

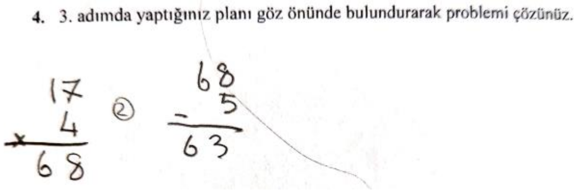
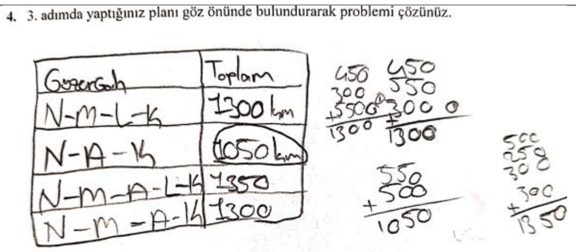
Explanation of Students' Use of Strategy	Example of Students' Worksheet
<u>Arithmetic Strategy (Problem 1)</u> The student first multiplied 17 by 4 since it was given in the problem statement that Sinan's marbles were four times of Atakan's marbles. Then, he subtracted 5 from 68 and found 63 as the answer.	
<u>Guess and Check Strategy and Making a Drawing (Problem 6)</u> The student drew a table to organize the given information and reached the solution by checking all possible ways.	

Figure 4.14. Use of strategies of sixth grade students in Problem 1 and Problem 6

Students' knowledge of the concept of Decimal Numbers was assessed in Problem 2 and Problem 4 in the sixth grade. Students were asked to solve the problems that require four operations with decimal expressions. For this reason, these two problems were analyzed together. In the second problem, the students were given that the following problem statement: *"Yaşar, who missed the service bus, gets into a taxi in order not to be late for work and the opening fee of the taximeter is 3.50 TL,* and *the price of every 100 meters is 25 Krş. Yaşar gave 2 TL tip to the taxi driver and in total he gave 12 TL, which included 2TL tip. What is the distance between Yaşar's house and workplace in kilometers?"*

Eight students gave the correct answer to Problem 2 and seventeen students answered it incorrectly. Four students did not have any solution to Problem 2. The reason why students mostly became unsuccessful in this problem might be that the story of the problem is long and the problem requires interdependent number operations. This reason might be offered because the students' procedural knowledge about the subject of decimal numbers seems to be generally good.

Table 4.12. Frequency of problem solving steps codes with respect to Problems 2& 4 in the sixth grade

		ITEMS	
		ITEM 2	ITEM 4
		Count (%)	Count (%)
Seeks to Understand the Problem	SUP1	25 (86,2%)	22 (75,9%)
	SUP2	0 (0,0%)	1 (3,4%)
	SUP4	1 (3,4%)	4 (13,8%)
	SUP5	3 (10,3%)	2 (6,9%)
Making a Plan/Organization	MPO1	12 (41,4%)	10 (34,5%)
	MPO2	1 (3,4%)	5 (17,2%)
	MPO3	11 (37,9%)	9 (31,0%)
	MPO4	5 (17,2%)	5 (17,2%)
Implementing the Plan	IP1	8 (27,6%)	6 (20,7%)
	IP2	13 (44,8%)	9 (31,0%)
	IP3	4 (13,8%)	0 (0,0%)
	IP4	4 (13,8%)	14 (48,3%)
Control/Assessment of Situation	CAS1	4 (13,8%)	0 (0,0%)
	CAS2	5 (17,2%)	10 (34,5%)
	CAS3	18 (62,1%)	19 (65,5%)
	CAS5	2 (6,9%)	0 (0,0%)
Progression Through All	PTA1	3 (10,3%)	0 (0,0%)
	PTA2	26 (89,7%)	29 (100,0%)

Twenty-five students determined what is given and what is asked, but four students appeared not to have understood the verbal information presented in Problem 2. Twelve students were able to make a plan to solve the problem, while five students were unable to make a plan in any way. Eleven students had an inappropriate solution plan since they could not organize what is given to reach what is asked in the problem. Twenty-one students implemented their solution plan correctly, but eight of them had a correct solution plan. Four students seemed not to have the essential knowledge to solve the problem. This might be due to the fact that many students could not make sense of the problem situation, the story of the problem, and the values given in the problem statement.

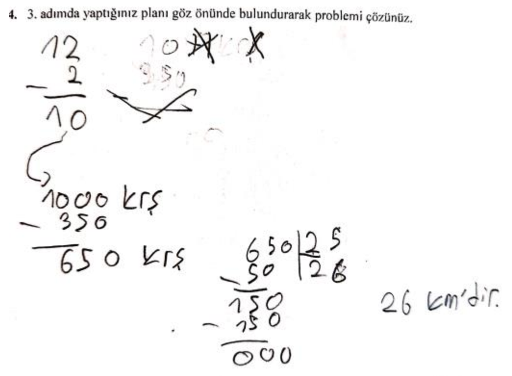


Figure 4.15. An example worksheet from a sixth grade student for codes MPO3 (Problem 2)

The example given in Figure 4.15 shows that the student made the number operations correctly. However, he made a mistake in the last part of the solution. After he found 26, he should have multiplied it by 100 since number 26 refers to the numbers of 100 meters. Moreover, the reason why some other students could not make any solution plan or they had an inappropriate solution plan might be that they could not make sense of the values given in the problem. Furthermore, in this problem, eighteen students had no control for whether their thinking was correct or not. Four students provided an explanation for why their thinking was correct, and five students just said "I am sure" in controlling the problem. Based on the high numbers of students who became unsuccessful in the solution of this problem, it can be concluded that students generally have no control of their solutions.

In the 4th problem, the students were asked the following problem: *Ahmet works in the cafeteria of a company. Monthly membership fee is 8 TL in this cafeteria.*

As shown in the table below, the price of a meal for members is lower than for non-members.

Price of a meal for Non-members	*Price of a meal for members*
2,8 TL	*2,3 TL*

Onur was the member of the cafeteria last month. Last month, including the membership fee, he spent 56.3 TL in total. If Onur had not been a member, but had eaten the same meals, how much would he have spent in TL?

This problem is different from the exercises or practice problems that students are familiar with and frequently encounter in mathematics classes. The 4th Problem as well as the 2nd Problem require the ability of analytical thinking and the ability to analyze and interpret the values in the problem statement by understanding the problem situation. Six students were able to solve Problem 4 successfully, while nine students could not solve it. Fourteen students don't have a solution. As exemplified in Figure 4.16, one of the reasons why students also mostly failed in the 4th Problem is that students were unable to make division with two decimal numbers.

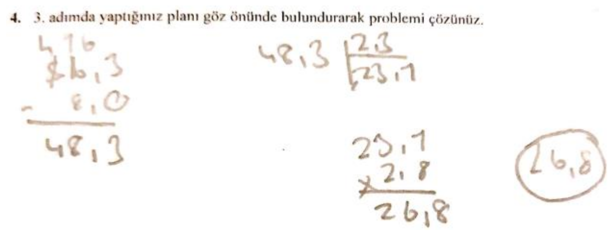


Figure 4.16. An example worksheet from a sixth grade student for codes MPO3 (Problem 4)

As it is seen in the example worksheet of a fifth grade student, the student was able to do the necessary subtraction operation, but he was unable to divide 48.3 by 2.3.

As concluded from the Table 4.12, twenty-three students appeared to demonstrate understanding of the verbal information presented in Problem 4, but six students appeared not to have understood the problem. Ten students were able to demonstrate a solution plan clearly, but nine students could not make a correct solution plan. The students mostly understood the problem situation, but the lack of students' ability to use mathematical reasoning could be another reason for failure. While six students who had the right solution plan applied their plans correctly, the solution of the nine students was not enough to reach the correct result. Additionally, in the control phase of the solution of the problem, ten students said "I am sure" without any explanation, and nineteen students did not make any explanation about the correctness of the solution in this problem.

Table 4.13. Frequency of the strategies used in Problems 2& 4 in the sixth grade

		ITEMS	
		ITEM 2	ITEM 4
		Count (%)	Count (%)
Use of Strategy	ART	17 (58,6%)	23 (79,3%)
	MD	1 (3,4%)	1 (3,4%)
	NOS	4 (13,8%)	4 (13,8%)
	ORD	1 (3,4%)	1 (3,4%)
	WB	6 (20,7%)	0 (0,0%)

Seventeen students used the arithmetic strategy, while six students used the 'working backward strategy'. One student used the 'making a drawing strategy' and one student used the 'organizing the data strategy' in Problem 2. When the data were analyzed, it can be concluded that the students who chose to use the working backward strategy were more successful in their solutions. However, it is seen that students mostly tended to use the arithmetic strategy. This might be because of that the students were not accustomed to use such strategies for meaningful learning and in-depth understanding. In problem 4, twenty-three students preferred to use the 'arithmetic strategy, but the 'making a drawing strategy' and 'organizing the data strategy' were used by one student each.

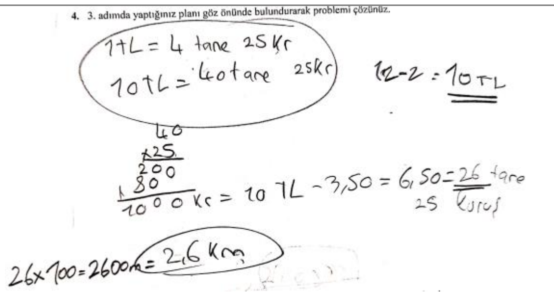
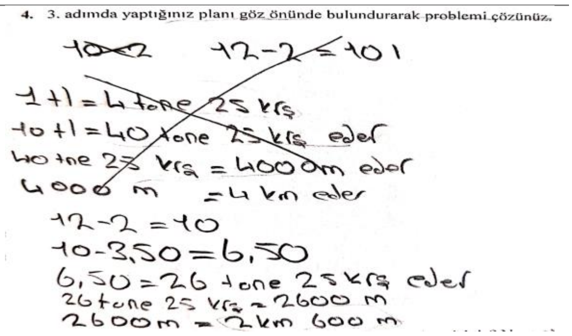
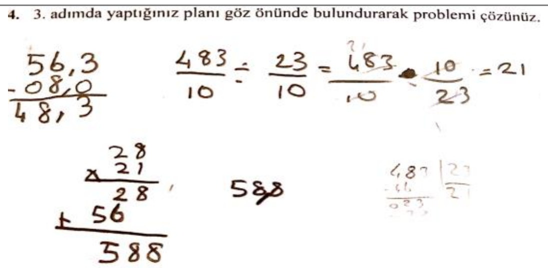
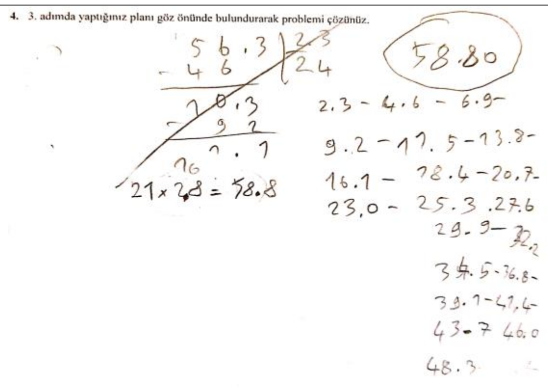
Explanation of Students' Use of Strategy	Example of Students' Worksheet
<u>Arithmetic Strategy (Problem 2)</u> Firstly, the student subtracted 2 from 12. Then, the student calculated how many 25 Krş there are in 1 TL and 10 TL. Next, he subtracted the opening fee of 3.5 TL from 10 TL, and he found the amount paid for the route traveled as 6.5 TL. He calculated the number of 25 krş for 6.5 TL and found 26. Finally, he multiplied 26 by 100 and found 2600 m (2 km) since the charge was 25 krş for 100 m.	
<u>Working Backward Strategy (Problem 2)</u> The student used the working backward strategy. First, he subtracted the tip 2 TL, and then subtracted the opening fee 3.5 TL from 12, respectively. He calculated the number of 25 krş in 6.5 TL and found 26. Finally, he multiplied 26 by 100 and obtained 2600 m (2.6 km).	
<u>Arithmetic Strategy (Problem 4)</u> Firstly, the student calculated the amount paid for food by subtracting 8 from 56.6 and obtained 48.3. Then, he divided 48.3 by 2.3 to find how many times Ahmet Usta ate, and obtained 21. Lastly, he found the solution by multiplying 2.8 by 21, and found 58.8 TL.	
<u>Organizing the Data Strategy (Problem 4)</u> The student first calculated how many times Ahmet Usta went to cafeteria, and he found 21. Then, he counted 21 times 2.3 rhythmically.	

Figure 4.17. Use of strategies of sixth grade students in Problem 2 and Problem 4

Students' understanding of the concept of a fraction as whole-to-a part relationship was assessed in Problem 3 and Problem 5. The students solved the problems that require processing with fractions. Students' tendency to use the making a drawing strategy was also wanted to be evaluated with these problems. Thus, the 3rd and the 5th Problem were analyzed together. In the 3rd Problem, the students were given the following problem statement: *A patisserie owner bought 240 eggs, and he used* $\frac{3}{8}$ *of these eggs to make baklava, while the* $\frac{4}{5}$ *of the rest of the eggs were used for a flaky, savory pastry.* How *many eggs are left?*

In the 5th Problem related to the concept of fraction, the students were asked the following problem: "*A ball which was dropped from a certain height rises up to* $\frac{2}{5}$ *of its previous height after its first hit. If the ball increased by 24 cm after its second hit to the ground, what was the height in cm when the ball was first dropped?*

Fifteen students solved Problem 3 successfully, while thirteen students were unsuccessful. One student did not have a solution to this problem. In the 3rd problem, the students who were unsuccessful in solving the problem lacked attention in general. An example was given in Figure 4.18 below. When the student's work is analyzed, it is seen that the student missed the statement "…$\frac{4}{5}$ of the rest of the eggs…" given in the problem. Therefore, after he calculated the $\frac{3}{8}$ of 240, which is equal to 90, he found $\frac{4}{5}$ of 90 instead of taking $\frac{4}{5}$ of 150. That is the general reason why some students were unsuccessful in solving the 3rd Problem.

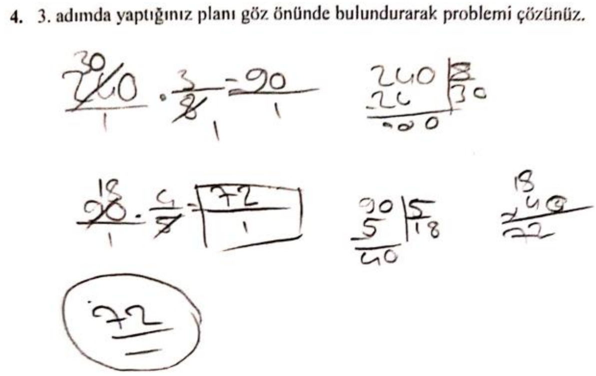


Figure 4.18. An example worksheet from a sixth grade student for codes MPO3 (Problem 3)

Problem 5 was solved by nine students successfully and twelve students were unsuccessful in this problem. Eight students did not have a solution to this problem. Similar to the 2nd and 4th Problem, the reason why students also generally failed in the

5^{th} Problem is not the lack of procedural knowledge about the concept of decimal number operation, but the lack of students' ability to use mathematical reasoning.

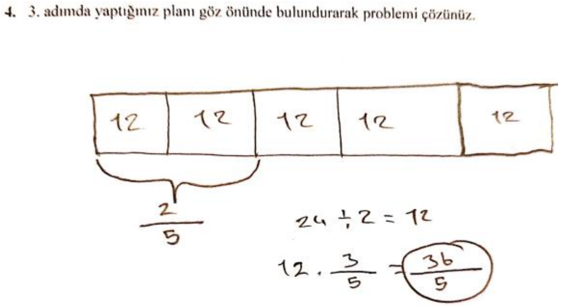


Figure 4.19. An example worksheet from a sixth grade student for codes MPO3 (Problem 5)

An example worksheet was given in Figure 4.19. In the 5^{th} Problem, it was given that a ball rises up to $\frac{2}{5}$ of its previous height after its first hit. The students were asked *the height in cm when the ball was first dropped, if the ball increased by 24 cm after its second hit to the ground*. Mathematically, the students were first asked to find out which number's $\frac{2}{5}$ is equal to 24. Hence, they should have divided 24 by 2 and then multiplied the result by 5. However, as it can be seen, the student tried to find $\frac{3}{5}$ of 24, and so he multiplied 24 by $\frac{2}{5}$.

Table 4.14. Frequency of problem solving steps codes with respect to Problem 3 and 5 in the sixth grade

		ITEM	
		ITEM 3	ITEM 5
		Count (%)	Count (%)
Seeks to Understand the Problem	SUP1	22 (75,9%)	21 (69,0%)
	SUP2	0 (0,0%)	1 (3,4%)
	SUP4	3 (10,3%)	3 (10,3%)
	SUP5	4 (13,8%)	4 (17,2%)
Making a Plan/Organization	MPO1	17 (58,6%)	7 (24,1%)
	MPO2	1 (3,4%)	2 (6,9%)
	MPO3	10 (34,5%)	12 (41,4%)
	MPO4	1 (3,4%)	8 (27,6%)
Implementing the Plan	IP1	15 (51,7%)	9 (31,0%)
	IP2	11 (37,9%)	12 (41,4%)
	IP3	2 (6,9%)	0 (0,0%)

Table 4.14. continued

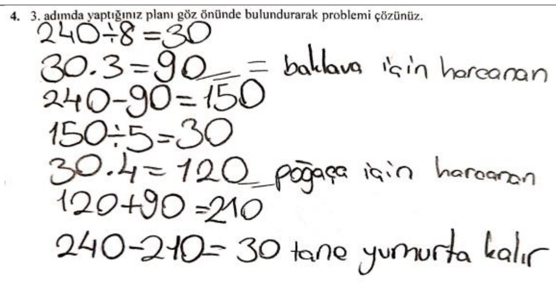
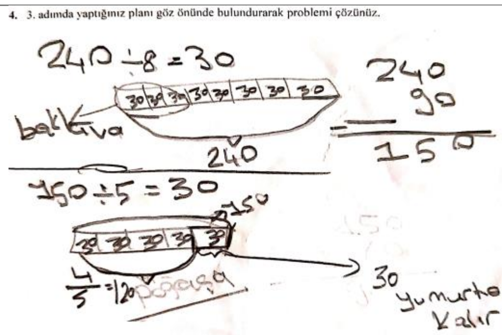
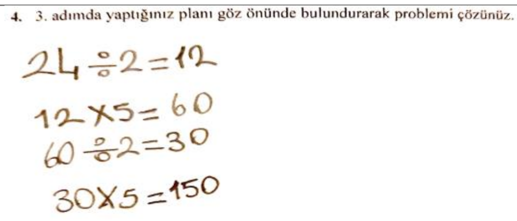
	IP4	1 (3,4%)	8 (27,6%)
Control/Assessment of Situation	CAS1	1 (3,4%)	1 (3,4%)
	CAS2	20 (69,0%)	13 (44,8%)
	CAS3	8 (27,6%)	15 (51,7%)
Progression Through All	PTA1	1 (3,4%)	1 (3,4%)
	PTA2	28 (96,6%)	28 (96,6%)

As given in Table 4.14, twenty-two students seemed to demonstrate an understanding of the verbal information, but seven students failed to comprehend the verbal information in these problems. Seventeen students could make an appropriate plan by organizing what is given to reach what is wanted and chose the appropriate strategy to solve the problem in Problem 3, while only seven students succeeded in this phase in Problem 5. This might be due to the fact that, as illustrated in Figure 4.19, the students lack meaningful learning and they failed to comprehend the problem situation in the 5th Problem. Therefore, they were unable to choose and implement the correct calculations. Ten students in Problem 3 and twelve students in Problem 5 had a solution plan, but it was not enough to reach the right result. Problem 3 was solved by fifteen students, and Problem 5 was solved by nine students correctly. It was seen that eight students did not possess the necessary knowledge to solve Problem 5. In the control phase of the solution, twenty students only said "I am sure", and eight students made no attempt in Problem 3. In Problem 5, fifteen students made no attempt in the control phase of the solution, and thirteen students only said "I am sure" without giving any explanation.

Table 4.15. Frequency of the strategies used in Problems 3 and 5 in the sixth grade

		ITEMS	
		ITEM 3	ITEM 5
		Count (%)	Count (%)
Use of Strategy	ART	24 (82,8%)	17 (58,6%)
	MD	4 (13,8%)	4 (13,8%)
	NOS	1 (3,4%)	8 (27,6%)

As seen in Table 4.15, twenty-four students in Problem 3 and seventeen students in Problem 5 seemed to use the 'arithmetic strategy' in the solution process. In each problem, four students tended to use the making a drawing strategy. In the sixth-grade problem solving achievement test, when the students' solution process is considered, it is seen that students generally preferred to use the arithmetic strategy as the fifth graders. The reason for this might be that they are used to the exam marathon, and they perceive using other strategies as a waste of time. In addition, it can be said that the students are very familiar with arithmetic strategies and they do not encounter other strategies.

Explanation of Students' Use of Strategy	Example of Students' Worksheet
<u>Arithmetic Strategy (Problem 3)</u> The student calculated $\frac{3}{8}$ of 240 and found 90. Then, he subtracted 90 from 240 to find the rest of the eggs and found 150. Next, he calculated $\frac{4}{5}$ of 150 to find numbers of eggs used for flaky. He added 90 with 120 to find the total number of eggs used. Finally, he subtracted 210 from 240 and found 30.	 4. 3. adımda yaptığınız planı göz önünde bulundurarak problemi çözünüz. 240÷8=30 30.3=90 = baklava için harcanan 240-90=150 150÷5=30 30.4=120 poğaça için harcanan 120+90=210 240-210=30 tane yumurta kalır
<u>Making a Drawing Strategy (Problem 3)</u> In this solution, the student made the calculations by using the making a drawing strategy, and he found the number of eggs left as 30.	 4. 3. adımda yaptığınız planı göz önünde bulundurarak problemi çözünüz.
<u>Arithmetic Strategy (Problem 5)</u> The student first divided 24 by 2 and multiplied the result by 5 to find the number that $\frac{2}{5}$ of it equals to 24, and he obtained 60. Then, he divided 60 by 2 and multiplied the result by 5 to find the number that $\frac{2}{5}$ of it equals to 60, and he found the result as 150.	 4. 3. adımda yaptığınız planı göz önünde bulundurarak problemi çözünüz. 24÷2=12 12×5=60 60÷2=30 30×5=150

Making a Drawing Strategy (Problem 5)

In this solution, the student first used the mathematical model to understand the problem and determined the calculations. Then, he made his calculations and found the result as 150.

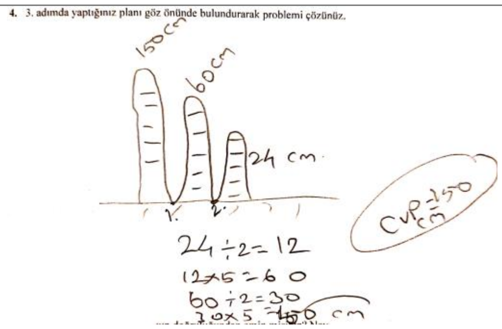


Figure 4.20. Use of strategies of sixth grade students in Problem 3 and Problem 5

In the sixth grade problem solving achievement test, the first and sixth problems are whole numbers problems, the second and fourth problems are decimal number problems, and third and fifth problems are fraction problems. It is seen that the problems in which sixth grade students progress best in problem solving steps are whole number problems, and the problems that they fail most in problem solving steps are decimal number problems. Although they succeeded in the step of understanding the problem in decimal number problems, they failed to form an appropriate solution plan and implement the plan. While the sixth grade students used different strategies in whole number problems, the students could not go beyond to use the arithmetic strategy in decimal number problems. In the fraction problems, the sixth grade students used the making a drawing strategy as well as arithmetic strategy more than fifth grade students.

4.2.3. Seventh Grade Problems

Table 4.16. Frequency of problem solving steps codes all problems in the seventh grade

		ITEMS					
		ITEM 1	ITEM 2	ITEM 3	ITEM 4	ITEM 5	ITEM 6
		Count (%)	Count (%)	Count (%)	Count (%)	Count (%)	Count (%)
Seeks to Understand the Problem	SUP1	21 (84,0%)	15 (60,0%)	16 (64,0%)	17 (68,0%)	19 (76,0%)	18 (72,0%)
	SUP2	2 (8,0%)	1 (4,0%)	0 (0,0%)	1 (4,0%)	2 (8,0%)	2 (8,0%)
	SUP4	1 (4,0%)	4 (16,0%)	9 (36,0%)	5 (20,0%)	1 (4,0%)	4 (16,0%)
	SUP5	1 (4,0%)	5 (20,0%)	0 (0,0%)	2 (8,0%)	3 (12,0%)	1 (4,0%)
Making a Plan/Organization	MPO1	14 (56,0%)	11 (44,0%)	8 (32,0%)	12 (48,0%)	12 (48,0%)	15 (60,0%)
	MPO2	3 (12,0%)	6 (24,0%)	4 (16,0%)	4 (16,0%)	4 (16,0%)	5 (20,0%)
	MPO3	5 (20,0%)	3 (12,0%)	1 (4,0%)	1 (4,0%)	5 (20,0%)	1 (4,0%)
	MPO4	3 (12,0%)	5 (20,0%)	12 (48,0%)	8 (32,0%)	4 (16,0%)	4 (16,0%)
Implementing the Plan	IP1	15 (60,0%)	2 (8,0%)	11 (44,0%)	8 (32,0%)	13 (52,0%)	14 (56,0%)
	IP2	5 (20,0%)	9 (36,0%)	1 (4,0%)	1 (4,0%)	6 (24,0%)	5 (20,0%)
	IP3	0 (0,0%)	8 (32,0%)	1 (4,0%)	8 (32,0%)	3 (12,0%)	1 (4,0%)
	IP4	5 (20,0%)	6 (24,0%)	12 (48,0%)	8 (32,0%)	3 (12,0%)	5 (20,0%)
Control/Assessment of Situation	CAS1	0 (0,0%)	0 (0,0%)	2 (8,0%)	0 (0,0%)	1 (4,0%)	7 (28,0%)
	CAS2	16 (64,0%)	14 (56,0%)	7 (28,0%)	14 (56,0%)	19 (76,0%)	11 (44,0%)
	CAS3	8 (32,0%)	11 (44,0%)	16 (64,0%)	11 (44,0%)	5 (20,0%)	7 (28,0%)
	CAS5	1 (4,0%)	0 (0,0%)	0 (0,0%)	0 (0,0%)	0 (0,0%)	0 (0,0%)
Progression Through All	PTA1	0 (0,0%)	0 (0,0%)	2 (8,0%)	0 (0,0%)	0 (0,0%)	7 (28,0%)
	PTA2	25 (100,0%)	25 (100,0%)	23 (92,0%)	25 (100,0%)	25 (100,0%)	18 (72,0%)

Problem 1, Problem 2 and Problem 3 were analyzed together because in these problems, students' understanding of the whole numbers concept was assessed. These problems which require four operations with natural numbers were solved by twenty-five seventh grade students. In the 1st Problem, the students were asked the following problem: "*Mathematics teacher Sinan is writing a new test book. One person changes two pages facing each other in the book. The product of these two numbers is 812. What are these two numbers?".* Fifteen students completed Problem 1 successfully, while five students were unsuccessful. Five students did not make any attempt to solve this problem. This problem is actually a problem that the students are not very familiar with, and that cannot be solved by using equations and direct arithmetic operations. For this reason, the 1st Problem requires using mathematical reasoning before directly making calculations. The students who realized that the guess and check strategy should be used were able to reach a solution in some way. However, the others were unable to solve the problem.

Table 4.17. Frequency of problem solving steps codes with respect to Problems 1, 2 and 3 in the seventh grade

		ITEMS		
		ITEM 1	ITEM 2	ITEM 3
		Count (%)	Count (%)	Count (%)
Seeks to Understand the Problem	SUP1	21 (84,0%)	15 (60,0%)	16 (64,0%)
	SUP2	2 (8,0%)	1 (4,0%)	0 (0,0%)
	SUP4	1 (4,0%)	4 (16,0%)	9 (36,0%)
	SUP5	1 (4,0%)	5 (20,0%)	0 (0,0%)
Making a Plan/Organization	MPO1	14 (56,0%)	11 (44,0%)	8 (32,0%)
	MPO2	3 (12,0%)	6 (24,0%)	4 (16,0%)
	MPO3	5 (20,0%)	3 (12,0%)	1 (4,0%)
	MPO4	3 (12,0%)	5 (20,0%)	12 (48,0%)
Implementing the Plan	IP1	15 (60,0%)	2 (8,0%)	11 (44,0%)
	IP2	5 (20,0%)	9 (36,0%)	1 (4,0%)
	IP3	0 (0,0%)	8 (32,0%)	1 (4,0%)
	IP4	5 (20,0%)	6 (24,0%)	12 (48,0%)
Control/Assessment of Situation	CAS1	0 (0,0%)	0 (0,0%)	2 (8,0%)
	CAS2	16 (64,0%)	14 (56,0%)	7 (28,0%)
	CAS3	8 (32,0%)	11 (44,0%)	16 (64,0%)
	CAS5	1 (4,0%)	0 (0,0%)	0 (0,0%)

Table 4.17. continued

Progression Through All	PTA1	0 (0,0%)	0 (0,0%)	2 (8,0%)
	PTA2	25 (100,0%)	25 (100,0%)	23 (92,0%)

As can be concluded from Table 4.17, twenty-three students seemed to demonstrate an understanding of the verbal information presented in Problem 1, but two students were unable to determine what is given and what is wanted in the problem. Fourteen students indicated an appropriate solution plan, and three students' solution plan is not clear. However, five students had a plan in which what is given in the problem could not be organized to generate an appropriate plan to reach a solution. Fifteen students seem to implement his solution plan and solve the problem successfully. Five students seemed not to have the essential knowledge to solve the problem. The five students who did not have any solution and the five students who had the wrong solution did not lack the knowledge of content, because the solution of the problem requires only the knowledge of multiplication. The reason might be that students could not consider using the guess and check strategy. None of the students provided an explanation for why their answer was correct. Sixteen students just said "I am sure" and eight students re-stated the solution they applied in the solution plan in the control phase of the solution process. The reason for this is that there is no alternative solution other than providing checksum.

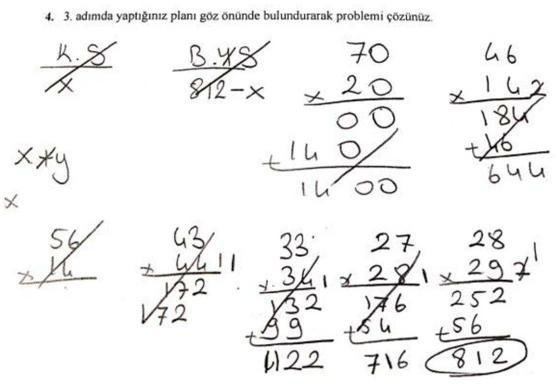


Figure 4.21. An example worksheet from a seventh grade student for codes IP1 (Problem 1)

As shown in the example above, the student chose two different numbers and multiplied these numbers to obtain the result 812.

The 2nd Problem is also related to the whole numbers concept. In this problem, the ability of students' comprehension of problem situation and the use of working backward strategy was intended to be analyzed. In the 2nd Problem, the students were given the following problem statement: "*There are some candies in Emir's bag. As he travels home with a bag of candies, he meets his friend Onur and gives half of his candies to him and one more candy to another friend. While walking home, he also encounters Selin and gives her half of the remaining candies and one more candy in his bag. As he moves along the way, he sees a child crying and gives him half of the remaining candies in his bag and another candy. As soon as Emir arrives his home, he opens his bag and sees that 5 candies were left. How many candies did Emir have at first?"*. Problem 2, in which the seventh-grade students failed most, was solved only by two students successfully. Seventeen students solved this problem incorrectly and six students could not answer it in any way. Thus, in this problem, the rate of students' success is quite low. As you can recall, the fifth and the sixth grade problem solving achievement tests included problems similar to this problem. The success rate of the fifth and sixth grade students in such problems was also not high there, though not as low as the seventh grade students' success. The problem asked to the 7th grade students, the story of the problem is longer and requires a little more attention to make sense of the values given. There are only two students who got exactly the right result, but the others' results are not entirely wrong. These students often made small mistakes and missed some values because they were a bit careless during the solution.

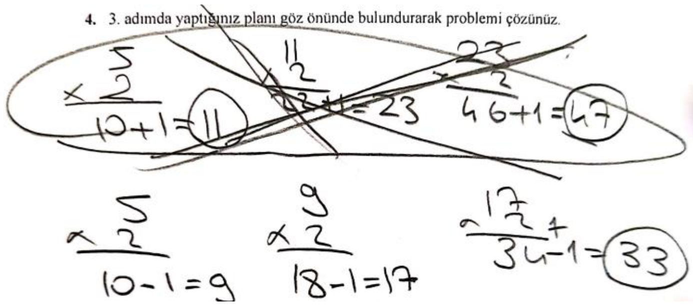


Figure 4.22. An example worksheet from a seventh grade student for codes IP1 (Problem 2)

When the example given in Figure 4.22 is examined, it is seen that the student added 1 after multiplying the number of remaining candies by 2 in the first solution attempt.

However, he had to multiply by 2 after adding 1 to the remaining candies in his bag since the statement was "*…he sees a child crying and gives him half of the remained candies and one more candy…"* in the problem. He noticed his mistake and canceled this solution and provided another solution. In his second solution, he subtracted a candy he gave to his friend every time instead of adding it back.

As seen in Table 4.17, sixteen students understood what is given and what is wanted in the 2nd problem and re-wrote the problem statement in their own words, but nine students were unable to understand the verbal information presented in the problem. Four of the nine students failed in the process of deciding what is given and what is wanted in the problem and re-writing the problem statement in their own words, and five of them were able to re-write the problem in their own words, while they were unable to determine what is given and what is asked in the problem. When compared with other problems in the test, this problem was seen as the most difficult problem to understand by the students. One of the reasons for this is that, compared with the other problems, this problem may be thought to include more verbal expressions and a longer story. Eleven students indicated their solution plans clearly, but six students did not show their right or wrong solution plans. Three students made a wrong solution plan, and five students were unable to make a plan thoroughly as there was no organization of what is given and no use of strategy. While only two of the eleven students with the right solution plan applied the solution plan correctly and reached the correct result, nine students could not carry out the solution plan properly. Although six students who did not have an accurate solution plan, and three students who had a completely wrong solution plan could not reach the correct result, they were able to implement their solution plan. Five students seemed not to have the necessary knowledge to solve the problem. In the control phase of problem solution, eleven students gave no opinion about whether their answer was correct or not.

Another problem about the whole number is the 3rd Problem. In the 3rd Problem, students were given that the following statement: *"To make a library, a carpenter needs the following parts: 4 long wooden boards, 6 short wooden boards, 12 small nails, 2 large nails, and 14 screws. There are 26 long wooden boards, 33 short wooden boards, 200 small nails, 20 large nails and 510 screws in carpenter's warehouse. How many libraries can this carpenter make?* Eleven students were successful in solving the problem, while two students were unsuccessful. Exactly eleven of the students could not give a right or wrong answer in any way. In fact, this problem is an easy problem in terms of the operations it contains. However, unlike questions of exercise or practice that the students are accustomed to, one of the reasons that make it difficult for students to understand the problem and cause them to be unsuccessful might be that the problem is a word problem.

As can be concluded from Table 4.17, sixteen students seemed to have an understanding of the verbal information presented in the problem, but nine students were unable to understand the problem statement. Eight students were able to indicate their solution plan clearly, while four students did not state their plan clearly. Eleven of the twelve students solved the problem accurately. One student had a plan, but he was completely unable to organize what is given to reach the correct result. Twelve students appeared not to possess the necessary knowledge to solve the problem since they could not make a solution plan and carry out the solution plan. As mentioned briefly above, this might be due to the students' inability to understand the problem situation and the story of the problem. For this reason, the students who gave the wrong answer also generally gave unrealistic answers.

Correct solution	Wrong solution

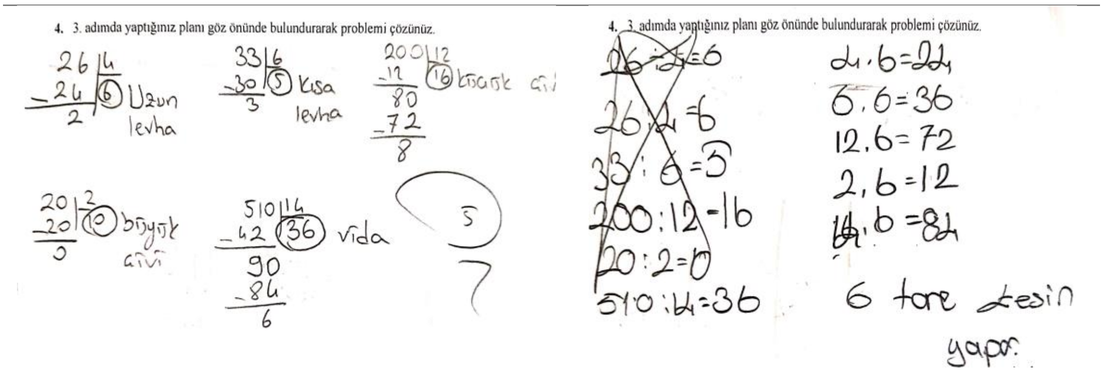


Figure 4.23. An example worksheet from a seventh grade student (Problem 3)

As with the solution on the left given in Figure 4.23, the student is expected to divide the number of materials the carpenter has to use to make the library by the number of materials required to make a library. Then, the smallest of the result can be generalized for the number of libraries that can be made. In the correct solution, the student calculated the long wooden boards enough to make 6 libraries, short wooden boards enough to make 5 libraries, small nails enough to make 16 libraries and so on. As a result, the student found that the carpenter can make 5 libraries with the materials he has. In the wrong solution, the students answered the problem as 6 libraries. However, to make 6 libraries, 36 short wooden boards are needed, but the carpenter has 33 short wooden boards.

Two students demonstrated an explanation for why their answer was correct, and seven students just said “I am sure” without any explanation in the control phase of the problem. Sixteen students gave no explanation for the accuracy of their answer. The reason for this might be that the students who answered the problem correctly could not find an alternative solution and the number of unsuccessful students was high.

Table 4.18. Frequency of the strategies used in Problems 1, 2 and 3 in the seventh grade

		ITEMS		
		ITEM 1	ITEM 2	ITEM 3
		Count (%)	Count (%)	Count (%)
Use of Strategy	ALG	2 (8,0%)	2 (8,0%)	0 (0,0%)
	ART	2 (8,0%)	3 (12,0%)	13 (52,0%)
	GC	16 (64,0%)	0 (0,0%)	0 (0,0%)
	NOS	5 (20,0%)	7 (28,0%)	12 (48,0%)
	WB	0 (0,0%)	13 (52,0%)	0 (0,0%)

As seen in the table 4.18, the majority of the fifth and sixth grade students preferred the arithmetic strategy in the process of problem solving. However, in the seventh grade, this situation is different. While two students in Problem 1 and three students in Problem 2 used the arithmetic strategy, two students used the algebraic strategy in these problems. Sixteen students used the ‘guess and check strategy’ in Problem 1, and thirteen students preferred to use the ‘working backward strategy’ in Problem 2. All the students took a short course to be familiar with the problem solving strategies before doing the problem solving achievement test. Nonetheless, the main reason for this situation might be the fact that the researcher was familiar with the seventh grade students and that they were together for longer periods in the mathematics classes than in the fifth and sixth grade students. In Problem 3, thirteen students used the ‘arithmetic strategy’, and twelve students were unable to use a strategy to solve the problem.

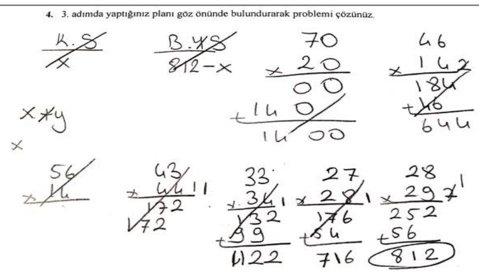
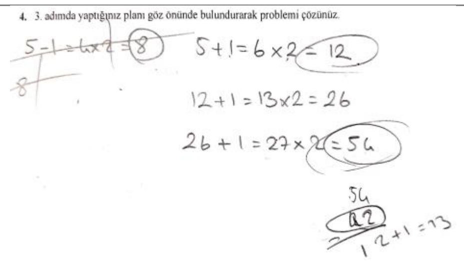
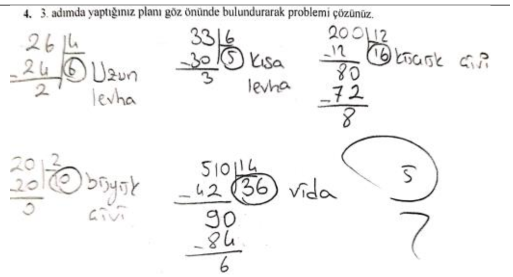
Explanation of Students' Use of Strategy	Example of Students' Worksheet
Guess and Check Strategy (Problem 1) The student chose two numbers and multiplied them until he obtained 812 as a result.	
Working Backward (Problem 2) As the student used the working backward strategy in solving this problem. First, he added 1 to 5 and multiplied the result by 2. Then, he added 1 to 12 and multiplied by 2. Finally, he added 1 to 26 and multiplied by 2, and he obtained 54 as a result.	
Arithmetic Strategy (Problem 3) The student divided the number of materials the carpenter had by the number of the materials that are required to make a library. Then, he generalized the smallest results to find the answer.	

Figure 4.24. Use of strategies of seventh grade students in Problem 1, Problem 2 and Problem 3

Problem 4 and Problem 6 assessed students' understanding and ability of solving problems with rational numbers. Problem 4 and Problem 6 were completed by twenty-five students. In the 4th Problem, the students were given the following problem statement: *Each time a ball falls, it rises up to 4/10 of its previous height. If the ball rises 16 cm above the ground in its third fall, how many meters was the first drop of this ball?*

Table 4.19. Frequency of problem solving steps codes with respect to Problem 4 and 6 in the seventh grade

		ITEMS	
		ITEM 4	ITEM 6
		Count (%)	Count (%)
Seeks to Understand the Problem	SUP1	17 (68,0%)	18 (72,0%)
	SUP2	1 (4,0%)	2 (8,0%)
	SUP4	5 (20,0%)	4 (16,0%)

Table 4.19. continued

	SUP5	2 (8,0%)	1 (4,0%)
Making a Plan/Organization	MPO1	12 (48,0%)	15 (60,0%)
	MPO2	4 (16,0%)	5 (20,0%)
	MPO3	1 (4,0%)	1 (4,0%)
	MPO4	8 (32,0%)	4 (16,0%)
Implementing the Plan	IP1	8 (32,0%)	14 (56,0%)
	IP2	1 (4,0%)	5 (20,0%)
	IP3	8 (32,0%)	1 (4,0%)
	IP4	8 (32,0%)	5 (20,0%)
Control/Assessment of Situation	CAS1	0 (0,0%)	7 (28,0%)
	CAS2	14 (56,0%)	11 (44,0%)
	CAS3	11 (44,0%)	7 (28,0%)
Progression Through All	PTA1	0 (0,0%)	7 (28,0%)
	PTA2	25 (100,0%)	18 (72,0%)

In solving the 4th Problem, eight students became successful and nine of them were unsuccessful. The number of the students who did not have any solution was eight. This Problem is similar to the fifth problem in the fifth grade problem solving achievement test. It seems that most of the 6th grade students also could not make sense of the values given in the problem. As seen in Table 4.19, eighteen students appeared to have understood the verbal information given in Problem 4, but seven students were unable to determine what is given and what is asked in the problem. Twelve students were able to make a solution plan accurately, and four students did not explain their solution plan clearly. One student could not organize and relate between what is given to reach what is asked. In this problem, eight students appeared not to have organized what is given and not to have used a strategy, so they could not make a plan thoroughly. The reason why 8 students were unable to understand the problem statement and to develop a solution plan might be that the students are accustomed to exercise and practice-oriented questions rather than to the questions that need mathematical reasoning. Also, they seemed not to have the necessary knowledge to solve the problem. While implementing the solution plan, eight students were successful, but nine students were unable to carry out their solution plan correctly. The students who could not fully implement the solution plan preferred to use the arithmetic strategy. The method of solution followed by these nine students was correct but not complete. Fourteen students said “I am sure” without a clear explanation about why their answer was correct, and eleven students had no control for whether their answer was correct or not.

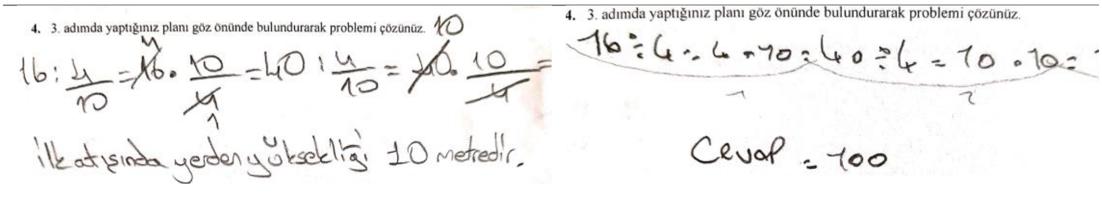


Figure 4.25. An example worksheet from two seventh grade students (Problem 4)

As you remember, in the 4th Problem, the students were given that each time a ball falls, it rises up to $\frac{4}{10}$ of its previous height, and the students were asked to find out from how many meters the ball was dropped if the ball rises 16 cm above the ground in its third fall. In the solutions given in the example in Figure 4.25, the solutions of the students would have been correct if the ball had risen 16 cm above the ground in its second hit. Thus, the students have to make another calculation like this. When the data was analyzed, in general, it was seen that the students who modelled the problem did not make this mistake. It might be concluded that using the making a drawing strategy facilitated students' meaningful learning and logical thinking and reduced mistakes.

Another rational number related problem was the Problem 6. In the 6th Problem, the students were given the following problem statement: *After the 8/15 of a road has been taken, 5/7 of the rest of the road is gone. If the remaining road is 50 km, how many kilometers is the whole road?* Fourteen students solved this problem correctly and six students were successful in solving this problem. In addition, five students had no solution in any way. The reason is that these students did not have enough and necessary knowledge about the subject. Moreover, it can be concluded that the students were more successful in the 6th Problem than the 4th Problem related to the same subject.

Twenty students seemed to have demonstrated an understanding of the verbal information presented in the problem, while five students were unable to understand the problem. Fifteen students had a clear solution plan, and five students did not show their solution plan completely. One student was not able to make a solution plan correctly, and four students had no correct or incorrect solution plan. Nineteen students solved the problem in accordance with their solution plan. Fourteen of them were able to reach the correct answer, but five students' solution was not correct because of the inappropriate solution plan. One student was unable to implement his correct solution plan accurately, and five students seemed not to have the necessary knowledge to solve the problem. In the control phase of the problem, seven students clearly explained why their answer was correct, and eleven students just said "I am sure" without any explanation. Seven students did not have any control of whether their thinking was correct or not.

Table 4.20. Frequency of the strategies used in Problems 4 and 6 in the seventh grade

		ITEMS	
		ITEM 4	ITEM 6
		Count (%)	Count (%)
Use of Strategy	ART	3 (12,0%)	10 (40,0%)
	MD	5 (20,0%)	10 (40,0%)
	NOS	8 (32,0%)	5 (20,0%)
	WB	9 (36,0%)	0 (0,0%)

In the seventh grade, three students used the arithmetic strategy, while five students used the making a drawing strategy and nine students used the working backward strategy in the solution process of Problem 4, while eight students were unable to use any problem solving strategy. In solving Problem 6, ten students used the 'arithmetic strategy'' and ten students used the 'making a drawing' strategy, but five students were not able to use any problem solving strategy. In the 5^{th} and 6^{th} grade problem solving achievement test, similar problems related to fractions were asked. The ability of the students to use the making a drawing strategy can be observed in the fractional problems in the fifth and sixth grade tests and the rational number problems in the seventh grade test. It can be observed that the students at higher grade levels preferred to use the making a drawing strategy more.

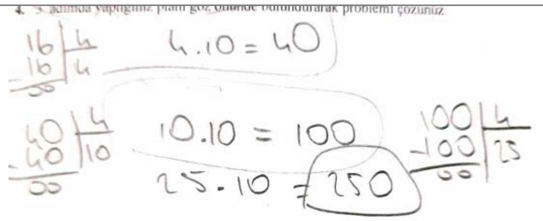
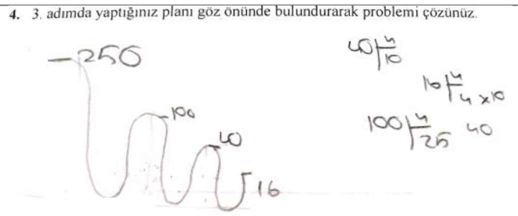
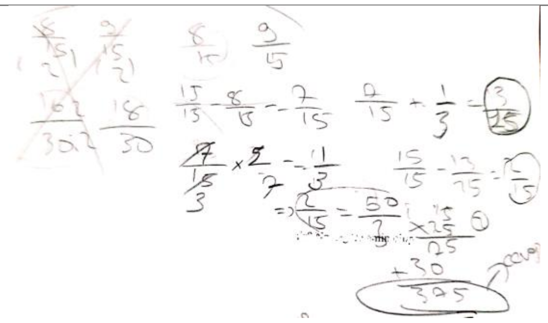
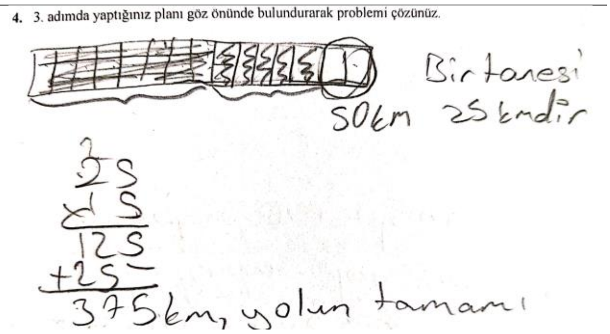
Explanation of Students' Use of Strategy	Example of Students' Worksheet
<u>Arithmetic Strategy (Problem 4)</u> The student first calculated the $\frac{4}{10}$ of 16 and found the ball's height before the 3rd hit to ground as 40 cm. Then, he calculated the $\frac{4}{10}$ of 40 and found the ball's height before the 2nd hit to ground as 100 cm. Finally, he calculated the $\frac{4}{10}$ of 100 and found the ball's height before the 1st hit to ground as 250 cm which equals to 2.5 m.	
<u>Making a Drawing Strategy (Problem 4)</u> The student reached the answer by modelling the problem and making the necessary calculations.	
<u>Arithmetic Strategy (Problem 6)</u> The student first subtracted $\frac{8}{15}$ from $\frac{15}{15}$ to find the remaining road. Then, he multiplied $\frac{7}{15}$ by $\frac{5}{7}$ to find $\frac{5}{7}$ of the remaining road. Next, he subtracted $\frac{1}{3}$ from $\frac{7}{15}$ to find the remaining road in the last situation. It is given that the remaining $\frac{2}{15}$ of the road is 50 km, so the whole road is equal to 375 km.	
<u>Making a Drawing Strategy (Problem 6)</u> The student first divided a whole into 15 equal parts and scans 8 of them. Then, he scanned 5 of the remaining 7 parts and the last 2 parts remained. Since the remaining two parts ($\frac{2}{15}$) were equal to 50, $\frac{1}{15}$ of the road was equal to 25. Therefore, the entire road was the result of the multiplication of 15 and 25, which is 375 km.	

Figure 4.26. Use of strategies of seventh grade students in Problem 4 and Problem 6

In the 5th Problem, students' knowledge of subject of integers was assessed. The problem situation to use of the arithmetic strategy and the organizing data strategy was provided in this problem. In the 5th Problem, the problem statement was as follows: *An electric heater is switched on in a room where the air temperature is 12* ℃*. The heater increases the temperature of the room every 4 minutes by 4* ℃*. How many minutes does it take for the temperature of the room to reach 24* ℃*?*

Table 4.21. Frequency of problem solving steps codes with respect to Problem 5 in the seventh grade

		ITEM
		ITEM 5
		Count (%)
Seeks to Understand the Problem	SUP1	19 (76,0%)
	SUP2	2 (8,0%)
	SUP4	1 (4,0%)
	SUP5	3 (12,0%)
Making a Plan/Organization	MPO1	12 (48,0%)
	MPO2	4 (16,0%)
	MPO3	5 (20,0%)
	MPO4	4 (16,0%)
Implementing the Plan	IP1	13 (52,0%)
	IP2	6 (24,0%)
	IP3	3 (12,0%)
	IP4	3 (12,0%)
Control/Assessment of Situation	CAS1	1 (4,0%)
	CAS2	19 (76,0%)
	CAS3	5 (20,0%)
Progression Through All	PTA2	25 (100,0%)

This problem was completed by thirteen students successfully, and nine students were unsuccessful. The reason why the students were unsuccessful in solving the problem might be that they were unable to understand the problem situation and the values given in the problem situation. Moreover, three of the students did not solve the problem. These three students seemed not to have the necessary knowledge to solve the problem. Twenty-one students wrote what is given and what is asked in the problem statement, but four students were unable to understand the problem. Twelve students made a proper solution plan, and four students also had a solution plan, but they could not show it clearly. Nine students did not have a proper solution plan. Five of those nine students were unable to correctly organize what is given to reach what is asked, and four of them did not have any correct or wrong solution plan. Thirteen students successfully solved the problem, but three students were unable to carry out the solution plan correctly. Three students seemed not to have the necessary knowledge to solve the problem. The example worksheets of two of the students who were unsuccessful in solving the problem indicate that these students mostly misinterpreted the values given in the problem statement. One student explained why his answer was correct, but five students could not give an explanation about the correctness of their solutions. Nineteen students just said “I am sure” without any explanation.

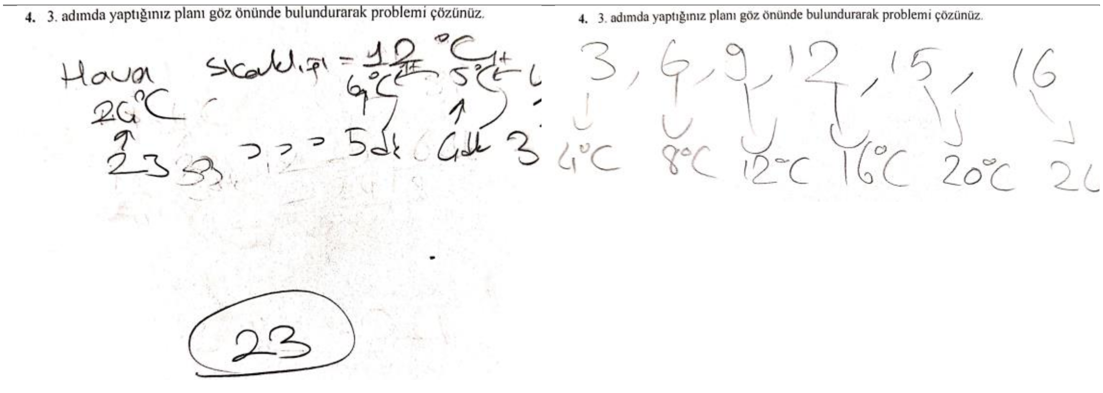


Figure 4.27. An example worksheet from two seventh grade students (Problem 5)

In the Problem, it was given that the temputure is 12 °C, and the students were asked how many minutes later it can be 24 °C if the temperature increases to 4 °C in 3 minutes. In the solution on the left, the temperature increased by 4 °C in the first 3 minutes, and then, it increased by one every minute until 24 °C. Hence, the student found the answer as 23. In the other solution on the right side, the student increased the temperature by 4 °C in 3 minutes, but he did not notice that the temperature was 12 degrees at the beginning, so the temperature was needed to be increased by 12 °C instead of 24 °C.

Table 4.22. Frequency of the strategies used in Problem 5 in the seventh grade

		ITEM
		ITEM 5
		Count (%)
Use of Strategy	ART	14 (56,0%)
	NOS	4 (16,0%)
	ORD	7 (28,0%)

As seen in Table 4.22, in Problem 5, fourteen students preferred to use the arithmetic strategy, while seven students used the ‘organizing the data’ strategy. It can also be seen in this problem that seventh grade students seemed to be inclined to use a variety of problem solving strategies.

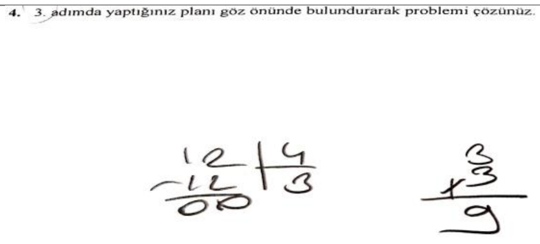
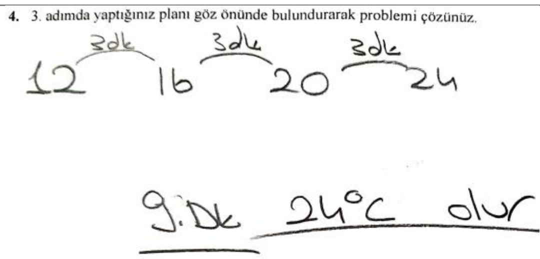
Explanation of Students’ Use of Strategy	Example of Students’ Worksheet
<u>Arithmetic Strategy (Problem 5)</u> The temperature rose by 4 °C for 3 minutes. The student divided 12 by 4 to find out how many times the temperature should be increased, and he found 3. Then, he multiplied 3 by 3, and he found the answer as 9 minutes.	
<u>Organizing Data Strategy (Problem 5)</u> The student found the answer as 9 by organizing the data considering what was given.	

Figure 4.28. Use of strategies of seventh grade students in Problem 5

In the seventh grade problem solving achievement test, the first, second and third problems are whole number problems, the fourth and sixth problems are rational number problems and the fifth problem is integer problem. Seventh grade students, like fifth and sixth grade students, seem to be successful in the step of understanding the problem in the solution of all problems. Seventh grade students seem to be more successful in problem solving steps in integer problem. However, they were unable to proceed in the problem solving steps of whole number problems -especially in the second problem- than the fifth and sixth grade students. Although 11 students made an appropriate solution plan for the second problem, only two students were able to implement the solution plan correctly. In addition, fifth and sixth grade students usually solved the problem by using the traditional method of arithmetic strategy. On the other hand, seventh grade students used the other problem solving strategies such as the guess and check, the working backward strategy, the making a drawing strategy, the algebraic strategy in all problems. For example, in the solution of the firth problem, 16 students used the guess and check strategy, in the solution of the second problem 13 students used the working backward strategy and in the solution of the fourth problem 9 students used the working backward strategy.

4.2.4. Eighth Grade Problems

Table 4.23. Frequency of problem solving steps codes of all problems in the eighth grade

		ITEMS					
		ITEM 1	ITEM 2	ITEM 3	ITEM 4	ITEM 5	ITEM 6
		Count (%)	Count (%)	Count (%)	Count (%)	Count (%)	Count (%)
Seeks to Understand the Problem	SUP1	29 (87,9%)	28 (84,8%)	29 (87,9%)	24 (72,7%)	18 (54,5%)	21 (63,6%)
	SUP2	3 (9,1%)	3 (9,1%)	2 (6,1%)	2 (6,1%)	1 (3,0%)	2 (6,1%)
	SUP4	0 (0,0%)	1 (3,0%)	2 (6,1%)	2 (6,1%)	12 (36,4%)	6 (18,2%)
	SUP5	1 (3,0%)	1 (3,0%)	0 (0,0%)	5 (15,2%)	2 (6,1%)	4 (12,1%)
Making a Plan/Organization	MPO1	32 (97,0%)	27 (81,8%)	26 (78,8%)	12 (36,4%)	14 (42,4%)	17 (51,5%)
	MPO2	0 (0,0%)	4 (12,1%)	3 (9,1%)	3 (9,1%)	0 (0,0%)	1 (3,0%)
	MPO3	1 (3,0%)	1 (3,0%)	1 (3,0%)	7 (21,2%)	4 (12,1%)	6 (18,2%)
	MPO4	0 (0,0%)	1 (3,0%)	3 (9,1%)	11 (33,3%)	15 (45,5%)	9 (27,3%)
Implementing the Plan	IP1	31 (93,9%)	26 (78,8%)	28 (84,8%)	11 (33,3%)	14 (42,4%)	17 (51,5%)
	IP2	0 (0,0%)	4 (12,1%)	1 (3,0%)	11 (33,3%)	2 (6,1%)	3 (9,1%)
	IP3	1 (3,0%)	2 (6,1%)	0 (0,0%)	1 (3,0%)	0 (0,0%)	0 (0,0%)
	IP4	1 (3,0%)	1 (3,0%)	4 (12,1%)	10 (30,3%)	13 (39,4%)	13 (39,4%)
Control/Assessment of Situation	CAS1	13 (39,4%)	8 (24,2%)	8 (24,2%)	6 (18,2%)	5 (15,2%)	5 (15,2%)
	CAS2	14 (42,4%)	21 (63,6%)	17 (51,5%)	10 (30,3%)	15 (45,5%)	15 (45,5%)
	CAS3	5 (15,2%)	4 (12,1%)	6 (18,2%)	17 (51,5%)	13 (39,4%)	13 (39,4%)
	CAS5	1 (3,0%)	0 (0,0%)	2 (6,1%)	0 (0,0%)	0 (0,0%)	0 (0,0%)
Progression Through All	PTA1	12 (36,4%)	8 (24,2%)	8 (24,2%)	5 (15,2%)	3 (9,1%)	3 (9,1%)
	PTA2	21 (63,6%)	25 (75,8%)	25 (75,8%)	28 (84,8%)	30 (90,9%)	30 (90,9%)

Problem 1 and Problem 2 assessed students' understanding of the concept of Whole Numbers as least common multiple, greatest common factor. All these items in the 8^{th}

grade problem solving achievement test were completed by thirty-three 8th grade students. In the 1st Problem, the students were given the following problem statement: *Blue and red bulbs flash in the window of a workplace. The blue light bulbs are lit every 6 seconds and the red-light bulbs are lit every 9 seconds. How many seconds after these two bulbs are lit together will they light up again?* In Problem 1 and Problem 2, 8th grade students were mostly highly successful. Thirty-one students solved Problem 1 successfully, and just one student was unsuccessful in solving this problem. Moreover, Problem 1 was not solved in any way by one student correctly or incorrectly. The success rate might be high because the students might be familiar with such problems and they solved such problems in the mathematics courses.

Table 4.24. Frequency of problem solving steps codes with respect to Problems 1 and 2 in the eighth grade

		ITEMS	
		ITEM 1	ITEM 2
		Count (%)	Count (%)
Seeks to Understand the Problem	SUP1	29 (87,9%)	28 (84,8%)
	SUP2	3 (9,1%)	3 (9,1%)
	SUP4	0 (0,0%)	1 (3,0%)
	SUP5	1 (3,0%)	1 (3,0%)
Making a Plan/Organization	MPO1	32 (97,0%)	27 (81,8%)
	MPO2	0 (0,0%)	4 (12,1%)
	MPO3	1 (3,0%)	1 (3,0%)
	MPO4	0 (0,0%)	1 (3,0%)
Implementing the Plan	IP1	31 (93,9%)	26 (78,8%)
	IP2	0 (0,0%)	4 (12,1%)
	IP3	1 (3,0%)	2 (6,1%)
	IP4	1 (3,0%)	1 (3,0%)
Control/Assessment of Situation	CAS1	13 (39,4%)	8 (24,2%)
	CAS2	14 (42,4%)	21 (63,6%)
	CAS3	5 (15,2%)	4 (12,1%)
	CAS5	1 (3,0%)	0 (0,0%)
Progression Through All	PTA1	12 (36,4%)	8 (24,2%)
	PTA2	21 (63,6%)	25 (75,8%)

As can be concluded from Table 4.24, in the 1st Problem, thirty-two students seemed to have demonstrated an understanding of the verbal information and made a solution plan, while only one student was unable to understand the problem and to make a solution plan. Thirty-one students implemented the solution plan successfully, but one student was unable to carry out the solution plan appropriately. One student seemed not to have the necessary knowledge to solve the problem. Thirteen students were able to provide an explanation for why their answer was correct, and fourteen students only said "I am sure" without an explanation. Six students did not give any explanation in the control phase of this problem.

In the second problem, the students were asked the following problem: "*56 kg and 72 kg bags of two types of rice will be put into bags with the largest size without being mixed with each other.*

According to this;

a. *How many kilograms of rice will be put in a bag?*
b. *How many bags are required for this process?"*

Twenty-six students had correct solution, whereas six students could not reach the correct solution in Problem 2. Furthermore, only one student did not have any answer to the problem. As seen in Table 4.24, like Problem 1, students were quite successful in understanding the verbal information in Problem 2. Thirty-one students seemed to have understood the verbal information, but two students were not able to show what is given and what is asked in the problem. Twenty-seven of thirty-one students made a solution plan, but four of them did not show their solution plan clearly enough. One student was not able to organize and relate between what is given and what is asked which he determined in the understanding the problem phase, and one student seemed not to have the essential knowledge to solve the problem. Twenty-seven students implemented their solution plans successfully, but two students were not able to apply the accurate solution plan correctly. In the control phase of the problem, eight students gave an explanation for why their solution was correct, and twenty-one students only said "I am sure" without an explanation. Four students could not give an explanation about their solutions. As in the fifth, sixth and seventh grade students, the eighth grade students were also generally unable to do what they wanted in the control of the problem section because they could not produce an alternative solution or they were not familiar with the control of the problem solution phase.

Table 4.25. Frequency of the strategies used in Problems 1 and 2 in the eighth grade

		ITEMS	
		ITEM 1	ITEM 2
		Count (%)	Count (%)
Use of Strategy	ART	31 (93,9%)	31 (93,9%)
	NOS	1 (3,0%)	2 (6,1%)
	ORD	1 (3,0%)	0 (0,0%)

In both Problem 1 and Problem 2, the students preferred to use the arithmetic strategy. Only one student used the organizing data strategy in the solution of the 1st Problem. This might be due to the fact that the students were accustomed to solving such problems in this way.

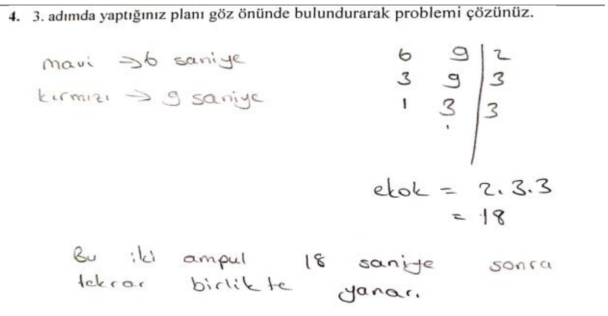
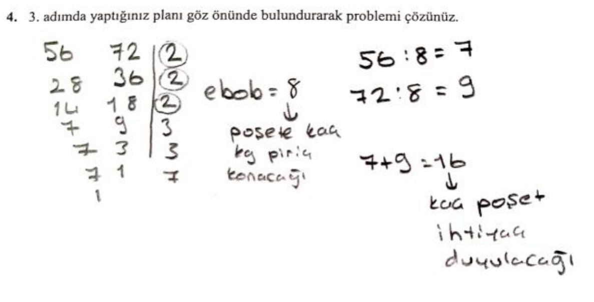
Explanation of Students' Use of Strategy	Example of Students' Worksheet
Arithmetic Strategy (Problem 1) Firstly, the student found the least common multiple of 6 and 9. The result is also the answer since the students were asked how many seconds after these two bulbs are lit together they will light up again.	
Arithmetic Strategy (Problem 2) The student first found the greatest common factor of the 56 and 72 to find how many kilograms of rice will be put in a bag. Then, he divided 56 and 72 by 8 to find the number of bags needed. Finally, he added 7 and 9 to find the total number of bags needed.	

Figure 4.29. Use of strategies of eighth grade students in Problem 1 and Problem 2

Problem 3 is related to the concept of probability. In the third problem, the students were asked the following problem: "*There are 36 colored beads in a container, which are all the same size. Some of these beads are blue, some are green, some are red, and* *the rest is yellow. The possibility of drawing a blue bead from the container is* $\frac{4}{9}$*. How many blue beads are there in the container?".*

Table 4.26. Frequency of problem solving steps codes with respect to Problems 3 and 4 in the eighth grade

		ITEMS	
		ITEM 3	ITEM 4
		Count (%)	Count (%)
Seeks to Understand the Problem	SUP1	29 (87,9%)	24 (72,7%)
	SUP2	2 (6,1%)	2 (6,1%)
	SUP4	2 (6,1%)	2 (6,1%)
	SUP5	0 (0,0%)	5 (15,2%)
Making a Plan/Organization	MPO1	26 (78,8%)	12 (36,4%)
	MPO2	3 (9,1%)	3 (9,1%)
	MPO3	1 (3,0%)	7 (21,2%)
	MPO4	3 (9,1%)	11 (33,3%)
Implementing the Plan	IP1	28 (84,8%)	11 (33,3%)
	IP2	1 (3,0%)	11 (33,3%)
	IP3	0 (0,0%)	1 (3,0%)
	IP4	4 (12,1%)	10 (30,3%)
Control/Assessment of Situation	CAS1	8 (24,2%)	6 (18,2%)
	CAS2	17 (51,5%)	10 (30,3%)
	CAS3	6 (18,2%)	17 (51,5%)
	CAS5	2 (6,1%)	0 (0,0%)
Progression Through All	PTA1	8 (24,2%)	5 (15,2%)
	PTA2	25 (75,8%)	28 (84,8%)

Like Problem 1 and Problem 2, a great majority of students were successful in solving Problem 3. Twenty-eight students were successful and just one student was unsuccessful in solving Problem 3. As the students were preparing for the high school entrance exam, it could be observed that the students were familiar with this problem as in the first and second problems. The students used the algorithm they used to solve the problems which are similar to this problem. Also, four students could not answer Problem 3 in any way. As seen in Table 4.26, thirty-one students seemed to have demonstrated an understanding of the verbal information presented in Problem 3, but two students were unable to understand it. Twenty-six students made a solution plan by accurately organizing what is given, while four students were not able to make an appropriate solution plan. Three students did not show their solution plan clearly. Twenty-eight students seemed to have carried out their solution plans correctly, but four students seemed not to have the necessary knowledge to solve the problem. In the control phase, eight students had a clear explanation for why their answer was correct, and seventeen students only said “I am sure” without giving an explanation. Eight students did not have an explanation about whether their answer was correct or not.

Problem 4 is related to exponential numbers. In the 4th Problem, the students were given the following problem statement: *An equal number of people from each of the* 5^3 *countries attended a meeting. These people were placed in each of the* 5^4 *rooms of a hotel so as to have 5 people in a room. According to this, how many people have participated in this meeting from one country?* Eleven eighth grade students gave the correct answer to Problem 4, and twelve students were unsuccessful in solving this problem. Ten students had no correct or wrong answers to this problem. As it can be seen in Table 4.26, in the 4th Problem, twenty-six students understood the problem statement, while seven students seemed not to have understood what is given and what is asked. While only twelve students among twenty-six students who understood the problem could make a proper solution plan, seven students could not establish the correct relationship between what is given and what is asked in the problem. Eleven students did not have a correct or incorrect solution plan. It can be concluded that these eleven students generally failed because they did not understand the problem in any way, not because they did not have enough knowledge to solve the problem. Three students’ solution plans were not wrong, but they were not clear. Eleven students successfully solved the problem, but one student did not solve the problem correctly despite the correct solution plan. Eleven students indicated a solution, but it was not correct. It can be observed that these eleven students had the necessary knowledge in the solution of the problem and that they could make calculations with exponential expressions correctly. However, as illustrated in Figure 4.25, they were mostly unable to make sense of the values given in the problem statement.

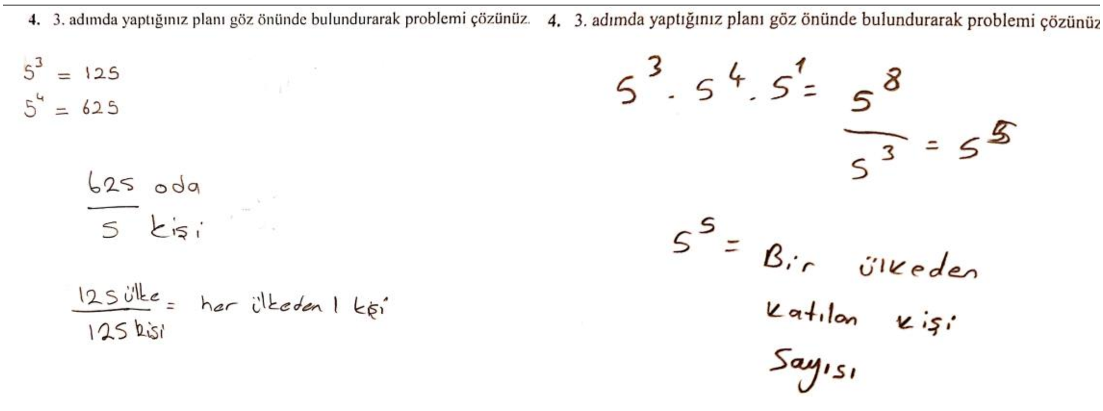


Figure 4.30. An example worksheet from two eighth grade students (Problem 5)

In the solution of the 4th Problem, firstly, the students were asked to find the total number of people by multiplying the total number of rooms by the number of people in each room. Then, he needed to divide the total number of people who participated in the meeting by the number of countries to find out how many people participated in the meeting from one country. In the solution given on the left in Figure 4.30, the student found the total number of people by dividing the total number of rooms by the total number of people in each room instead of multiplying, and so he found the total number of people as 125. In another solution given on the right side, the number of countries, the number of rooms and the number of people in each room were multiplied by each other by the student to find the total number of students, and he found the total number of participants as 5^8.

Ten students seemed not to have the necessary knowledge to solve the problem. In this problem, six students clearly explained why their answer was correct, and ten students only said “I am sure” without an explanation. Seventeen students did not express any opinion that their answer was correct.

Table 4.27. Frequency of the strategies used in Problems 3 and 4 in the eighth grade

		ITEMS	
		ITEM 3	ITEM 4
		Count (%)	Count (%)
Use of Strategy	ALG	5 (15,2%)	0 (0,0%)
	ART	23 (69,7%)	22 (66,7%)
	MD	1 (3,0%)	0 (0,0%)
	NOS	4 (12,1%)	11 (33,3%)

As seen in Table 4.27, five students used the algebraic strategy, while twenty-three students used the arithmetic strategy, and one student preferred to use the making a drawing strategy in the 3rd Problem. In the 4th Problem, twenty-two students used the arithmetic strategy in the solution process.

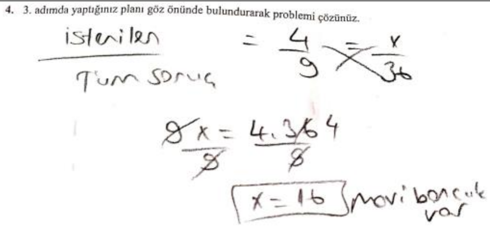
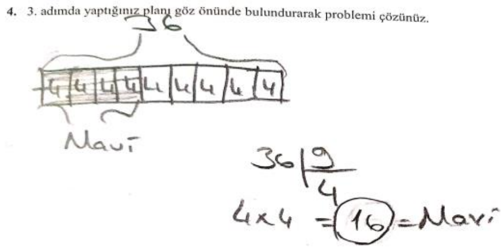
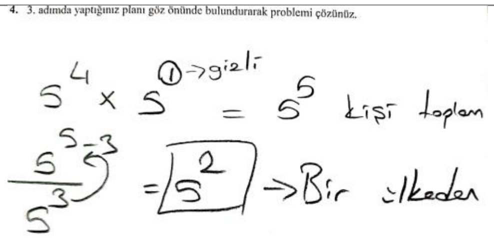
Explanation of Students' Use of Strategy	Example of Students' Worksheet
<u>Arithmetic Strategy (Problem 3)</u> Firstly, the student equated $\frac{4}{9}$ to $\frac{x}{36}$ and set the equation. Then, after he solved the equation, he found the answer as 16.	
<u>Making a Drawing Strategy (Problem 3)</u> He divided a whole into 9 equal parts, and he calculated $\frac{1}{9}$ of the whole by dividing 36 with 9. Then, he multiplied 4 by 4 to find the $\frac{4}{9}$ of the 36, and he found the answer as 16.	
<u>Arithmetic Strategy (Problem 4)</u> The student first multiplied 5^4 by 5 to find the total number of participants. Then, he divided 5^5 with 5^3 to find the number of participants from one country.	

Figure 4.31. Use of strategies of eighth grade students in Problem 3 and Problem 4

Students' understanding of the concept of square root was assessed in Problem 5 and Problem 6. These two problems were related to the concepts of square roots. In the 5th Problem, the students were given the following problem statement:

An archer shoots at a circular target board with a diameter of 1 meter as shown in the picture. The height of the target board is 3 meters. If the thrown arrow hits the target board, how many meters can the height of the point where the arrow hits be?

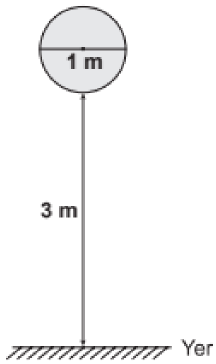


Table 4.28. Frequency of problem solving steps codes with respect to Problem 5 and 6 in the eighth grade

		ITEMS	
		ITEM 5	ITEM 6
		Count (%)	Count (%)
Seeks to Understand the Problem	SUP1	18 (54,5%)	21 (63,6%)
	SUP2	1 (3,0%)	2 (6,1%)
	SUP4	12 (36,4%)	6 (18,2%)
	SUP5	2 (6,1%)	4 (12,1%)
Making a Plan/Organization	MPO1	14 (42,4%)	17 (51,5%)
	MPO2	0 (0,0%)	1 (3,0%)
	MPO3	4 (12,1%)	6 (18,2%)
	MPO4	15 (45,5%)	9 (27,3%)
Implementing the Plan	IP1	14 (42,4%)	17 (51,5%)
	IP2	2 (6,1%)	3 (9,1%)
	IP4	17 (51,5%)	13 (39,4%)
Control/Assessment of Situation	CAS1	5 (15,2%)	5 (15,2%)
	CAS2	4 (12,1%)	15 (45,5%)
	CAS3	22 (66,7%)	13 (39,4%)
	CAS5	2 (6,1%)	0 (0,0%)
Progression Through All	PTA1	5 (15,2%)	3 (9,1%)
	PTA2	28 (84,8%)	30 (90,9%)

Most of the students could not give even a wrong answer. In other words, seventeen students could not give any correct or wrong answer to Problem 5. Fourteen students were successful in Problem 5 and two students had a wrong solution. As seen in Table 4.28, in the 5th Problem, nineteen students understood the problem statement as they were able to determine what is given and what is asked in the problem statement. However, fourteen students seemed not to have understood the values given in the problem statement. Fourteen students had an appropriate solution plan, while nineteen students could not make a proper solution plan since they could not establish the correct relationship between what is given and what is asked in the problem. This problem is a real-life word problem which is easier with respect to mathematical calculations then the previous problems, but it requires using mathematical reasoning. The reason for the high failure rate in this problem might be that, as with the 5th, 6th, 7th grades students, the 8th grade students also found it difficult to understand such real-life word problems that require mathematical reasoning. All fourteen students who made an appropriate solution plan solved the problem correctly, but two of the remaining nineteen students solved the problem incorrectly and seventeen students could not provide any solution. The main reason for this is not the lack of conceptual and procedural knowledge of the students about the concept of square root, but the students' inability to understand the problem situation.

In the 6th Problem, the students were given that the following problem statement: *A rectangular cardboard with side lengths of* $\sqrt{45}$ *cm and* $\sqrt{20}$ *cm will be covered with square-shaped labels with an edge length of* $\sqrt{5}$ *cm, and there will be no gaps in the cardboard, and the labels will not overlap. How many labels should be used for this?*

In Problem 6 related to the concept of square roots, seventeen students gave the correct answer, while three students gave a wrong answer. Thirteen students did not have even a correct or wrong solution to Problem 6. It can be concluded from Table 4.28 that twenty-three students were quite successful in understanding the verbal information given in the problem statement, but ten students were not able to show what is given and what is asked in the problem. Seventeen of twenty-three students who understood the problem statement made an appropriate solution plan, and one students' solution plan was not clear. Six students were not able to organize and relate between what is given and what is asked, and so they did not have a correct solution plan. Nine students seemed not to have the necessary knowledge to solve the problem, so they did not have any correct or wrong solution plan. One of the reasons might be that the students could not understand the problem situation as in the 5th Problem, or students might not have been able to devote enough time to this problem since the problem was the last problem of the test. Seventeen students solved the problem correctly, while thirteen students seemed not to have the essential knowledge to solve the problem because they did not have a wrong or correct answer. Only five students controlled their solution plan, but thirteen students did not have any control of the solution. Fifteen students only said "I am sure" without an explanation.

Table 4.29. Frequency of the strategies used in Problems 5 and 6 in the eighth grade

		ITEMS	
		ITEM 5	ITEM 6
		Count (%)	Count (%)
Use of Strategy	ART	12 (36,4%)	18 (54,5%)
	GC	3 (9,1%)	0 (0,0%)
	MD	1 (3,0%)	3 (9,1%)
	NOS	17 (51,5%)	12 (36,4%)

As shown in Table 4.29, in the 5th Problem, twelve students preferred to use the arithmetic strategy, while three students preferred to use the guess and check strategy, and one student preferred to use the making a drawing strategy. Seventeen students did not use a strategy in solving Problem 5. In the 6th Problem, eighteen students used the arithmetic strategy, while three students used the making a drawing strategy. Twelve students did not use any strategy in solving Problem 6.

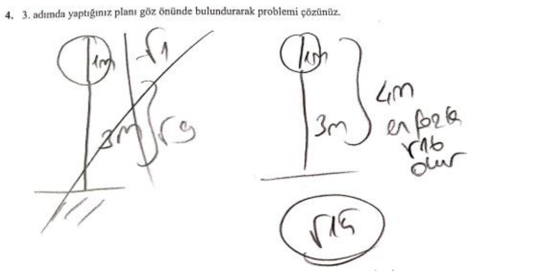
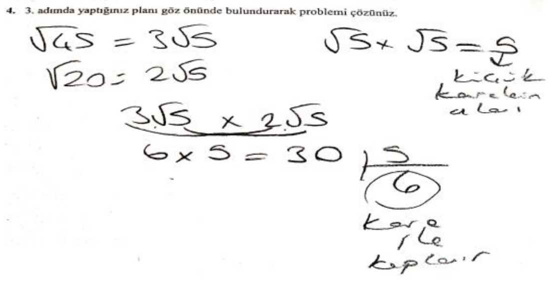
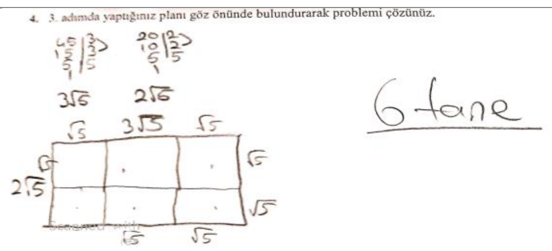
Explanation of Students' Use of Strategy	Example of Students' Worksheet
Guess and Check Strategy (Problem 5) The student solved the problem by trying out the possible results.	
Arithmetic Strategy (Problem 6) The student first found the area of cardboard by multiplying $\sqrt{45}$ by $\sqrt{20}$ as 30. Then, he found the area of label as 5. Finally, he divided 30 by 5 to find the number of the labels needed.	
Making a Drawing Strategy (Problem 6) As seen in the solution, the student modeled the problem by drawing and found the answer as 6.	

Figure 4.32. Use of strategies of eighth grade students in Problem 5 and Problem 6

In the eighth grade problem solving achievement test, the first and second problems are whole number problems, the third problem is probability problem, fourth problem is exponential expression problem, and fifth and sixth grade problems are square root problems. Eighth grade student, like fifth, sixth and seventh grade students, did not have any difficulty in understanding the problem step in the solution of the all problems. In general, eighth grade student have proceeded successfully in problem solving stages of problem except exponential expression problem. Most of the eighth grade students used the arithmetic strategy which is the traditional method in problem solving when compared with fifth, sixth and seventh grade students.

CHAPTER 5

CONCLUSION, DISCUSSION AND RECOMMENDATIONS

The aim of this study was to investigate the use of strategies in the process of problem solving of middle school students from different grade levels (5^{th}, 6^{th}, 7^{th} and 8^{th} grade) and their use of problem solving steps in the word problems related to numbers. In the previous chapter, the results of the statistical analysis of the study were explained. In this chapter, discussions and conclusions with respect to the findings will be shared. In addition to these, comparisons of the studies in the literature and implications and recommendations for practice and further studies are mentioned in this chapter.

5.1. Middle Schools Students' Use of Problem Solving Strategies and Use of Problem-Solving Steps in the Word Problems

As mentioned in the 'methodology' chapter, problem solving achievement scores in each grade level were obtained after performing the Problem Solving Achievement Tests which involved real-life word problems. Furthermore, students' written works which gave ideas about students' level of understanding of the problem, the ability of making and implementation of a plan, and the ability of control of their solution were obtained from the Problem Solving Achievement Tests. Then, students' mathematics achievement mean scores, and standard deviation and minimum-maximum scores were calculated based on the results of Problem Solving Achievement Tests related to numbers at each grade level by utilizing descriptive statistics. The results of the study revealed that the students were successful in problem solving achievement tests applied at each grade level. Additionally, in some word problems, students showed their ability to use different strategies although the students generally preferred to use the arithmetic strategy. Moreover, in general, the students from different grade levels showed that they could use their problem solving abilities successfully. In a study conducted by Maluleka (2013) it is revealed that adding meaning to statement given in the problem appeared to be missing skill, and the students could not link real life problems with mathematical content learned in class. Thus, this turned out to be crucial factor in the failure to make a solution plan of problems identified during the problem solving phase. In a similar manner, the research findings show also that the most important deficiency of students was that they did not use real world knowledge and experience in their solutions. On the other hand, problem solving is also the interaction of the demands of the task with a person's real-life experience (Martinez, 1998). Therefore, they mostly had more difficulty in understanding and solving real-life word problems. They did not take into account the real relationships between real-life contexts revealed by the problem statements and the operations they carried out in the problem solution. Most students are not critical in real life problems, and their mathematical reasoning skills are not very good. Since problem-based instruction is more effective than the traditional method to enhance critical thinking of the students in mathematic lesson (Cantürk-Günhan, & BAŞER, 2009) the reason for this might be that the students do not have enough problem solving habits in the classroom.

More specifically, the fifth grade students seemed to be successful in the phase of understanding of the problem statement in all content areas. In other words, they were mostly able to re-write the problem statement and determine what is given and what is asked in all problem statements. However, they had difficulty in making a solution plan in some problems. They could not form an appropriate solution plan in the first problem related to fractions. Depending on the data, the main reason for this might be students' misconceptions about the subject of fractions. As stated by Deringöl (2019) that the primary school students had difficulties most in solving problems related to fractions, in representing fractions by models and in reading and writing concepts expressing fractions. One of the students' deficiencies in meaningful understanding might be that the students chose the arithmetic strategy instead of other strategies (e.g. making a drawing strategy). Also, the fifth grade students had difficulties in the second, fourth and fifth problems related to whole numbers. When the students' solutions were examined in general, it was seen that the students could not properly interpret the values given in the problem statement. In the solution of this problem, the students mostly preferred to use the arithmetic strategy. It seems that the students who used strategies different from the arithmetic strategy such as the working backward strategy made an appropriate solution plan and implemented their solution plan more successfully. When we look at the students' use of strategy in solving problems, a significant number of students used the arithmetic strategy while just 3 students preferred to use the making a drawing strategy in fraction problems. Students' use of strategies is more diverse in whole number problems. In addition to the arithmetic strategy, the students used the working backward, the guess and check, and organizing data strategies in whole number problems.

Like the fifth grade students, the sixth grade students seemed to be quite successful in re-writing the problem statement in their own words and in determining what is given and what is asked in the problems. However, many sixth grade students had difficulty in making a plan and reaching a solution in the second and fourth problems related to decimal numbers. The reason why students mostly became unsuccessful in this problem might be that the story of the problem is long, and the problem requires many interdependent number operations because the students' procedural knowledge about the subject of decimal numbers was generally good. Furthermore, this might be due to the fact that many students could not make sense of the problem situation, the story of the problem, and the values given in the problem statement. The students mostly understood the problem situation, but the lack of students' ability to use mathematical reasoning could be another reason for failure. Also, in the fifth problem related to fractions, the sixth grade students were unable to make a plan and solve the problem. This is due to the fact that the students lacked meaningful understanding and they failed to comprehend problem situation in the 5^{th} Problem. Therefore, they were unable to determine correct calculations. It can be said that all of these are mostly due to the students' adherence and persistence to use only the arithmetic strategy, similar to the fifth grade students. As a result of students' inability to use various problem solving strategies, they cannot perform adequately despite their self-efficacy and beliefs about the problems they faced (Guven & Cabakcor, 2013). Sulak (2010) also states that problem solving strategies very effective in problem solving based on findings in her studies. While the sixth grade students used different strategies in whole number problems, the students could not go beyond to use the arithmetic strategy in decimal number problems. Furthermore, in the fraction problems, the sixth grade students used the making a drawing strategy as well as arithmetic strategy more than fifth grade students.

Seventh grade students, like fifth and sixth grade students, seem to be successful in the step of understanding the problem in the solution of all problems. Seventh grade students seem to be more successful in problem solving steps in integer problem. However, the rate of seventh grade students' success is quite low in the problems 1,2 and 3. In other words, they were unable to proceed in the problem solving steps of whole number problems -especially in the second problem- when compared with the fifth and sixth grade students. Designing a solution plan, which is a much more complex part of the problem solving, is a most crucial step of the problem solving (Maluleka, 2013). The inability of students to associate their calculations with what they planned in the previous stage, the inability to combine or associate the two stages, and not taking the word problem seriously are resulted in failure to get an appropriate solution plan and contributed to those errors (Raoano, 2016). Although 11 students made an appropriate solution plan for the second problem, only two students were able to implement the solution plan correctly. That is to say, the fifth and sixth grade problem solving achievement tests included problems similar to this problem. The success rate of the fifth and sixth grade students in such problem was also not high there, though not as low as the seventh grade students' success. The story of the problem that was asked to the 7th grade students is longer and requires a little more attention to make sense of the values given. In addition, fifth and sixth grade students usually solved the problem by using the traditional method of arithmetic strategy. On the other hand, seventh grade students used the other problem solving strategies such as the guess and check, the working backward strategy, the making a drawing strategy, the algebraic strategy in all problems. For example, in the solution of the firth problem, 16 students used the guess and check strategy, in the solution of the second problem 13 students used the working backward strategy and in the solution of the fourth problem 9 students used the working backward strategy. It might be because of the reason that the researcher entered the mathematics class of the seventh grade students for the longest time compared with other students. Strategies for solving mathematical word problems are very important and students need to be exposed to these strategies in order to apply these various strategies while solving problem (Raoano, 2016).

Eighth grade student, like fifth, sixth and seventh grade students, did not have any difficulty in understanding the problem step in the solution of the all problems. However, the eighth grade students mostly failed in solving the fourth problem. It can be concluded that these students who failed in solving this problem did not understand the problem in any way. The reason they could not solve the problem was not that they did not have enough content knowledge to solve the problem. In this respect, students need to be encouraged to read and often make sense of the texts and books so that students improve their comprehension skill and vocabulary, and will help them understand the problem statement given to them (Raoano, 2016). The eighth grade students also had difficulty in solving the fifth problem. The reason for the high failure rate in this problem might be that, as in the 5th, 6th, 7th grades students, the 8th grade students also found it difficult to understand such real-life word problems that require mathematical reasoning. Moreover, most of the eighth grade students used more the arithmetic strategy which is the traditional method in problem solving when compared with fifth, sixth and seventh grade students.

As result of the study carried out by Ersoy (2014), problem-based learning resulted in an increase in the points for the creative thinking skills of the students. Problem solving based instruction is important and ensures more permanent learning. As it is revealed in the studies (Özsoy, 2002; Guven, & Cabakcor, 2013), there is a significant relationship between students' mathematics achievement score and mathematical problem solving ability. That statement is also supported with the findings from a study conducted by Karaoğlan (2009), which found that there is a significant positive correlation between students' mathematics achievement scores and their problem solving achievement scores after completing problem solving based instruction. The findings of Yazgan and Bintaş's (2005) study showed that the 4th and 5th grade students can informally use problem solving strategies without any training, and the 4th and 5th grade students can learn the problem solving strategies, and training on problem solving strategies has a positive effect on the problem solving success of students. In addition, Bayazit (2013) revealed in his study that although students tended to use a variety of problem solving strategies, they mostly lacked ability to use alternative approaches and appropriate strategies. Findings of this study might be considered as consistent with most of the previous studies which are related to students' use of problem solving strategies during the class (Bayazit, 2013; Durmaz & Altun, 2014; Gür & Hangül, 2015; Hwang & Jai, 2014; Intaros, Inprasitha & Srisawadi, 2014; Erdoğan 2015; Yazgan & Bintaş, 2005). However, different from these studies, this study included all levels of the middle school (5^{th}, 6^{th}, 7^{th}, and 8^{th} grade students), and a short one-week problem based course was given in order to make students realize the relationship between mathematical word problem and daily life word problems before conducting the study. During the instruction, the students were able to develop and use different strategies for word problems. In his study, Erdoğan (2015) also revealed that word problem solving strategies were not very weak and flexible as a result of students' problem solving attempts for 5 weeks. Results of the current study was consistent with the findings of the Erdogan's study. When the results obtained from the students' written works were examined, it was seen that most of the students used the arithmetic strategy and there was little tendency towards different strategies. However, even if it is small, it can be seen that some students are prone to use different problem solving strategies at each grade level. For example, the fifth, sixth, seventh, and eighth grade students were mostly able to use at least two different strategies for each problem. Some of students from each grade level even used four different strategies in one of the problems. Therefore, the study revealed that students have flexible thinking. Middle school students could comprehend the strategies and use them in similar problems. Moreover, it can be concluded after analyzing the students written work that they mostly had more difficulty in understanding and solving real life word problems. They did not take into account the real relationships between real life contexts revealed by the problem statements and the operations they carried out in the problem solution. In general, the results also showed that most of the students had difficulties in explaining and making critics related to real life problems. Furthermore, the basic step to solve the problem correctly is to understand the problem statement. To determine the appropriate problem solving strategies and solve the problem correctly, problem statement should be understood well. Without determining the givens and unknown, it is difficult to solve a problem correctly. It is concluded that at least 76 % (22 students) of all students were able to re-write the problem in their own words and explain the given as well as the asked information in the problem. However, most of them had difficulties in organizing and relating the given information to reach the correct solution. They also had problems in implementing their plans appropriately. The results indicated that even though the students could rewrite the problems or find solution strategies, they often failed to find the correct answer. One of the reasons was students’ difficulties in planning to find the solution. Additionally, they had difficulties to apply the plan in correct order.

5.2. Implications and Recommendations

The 5^{th}, 6^{th}, 7^{th}, and 8^{th} grade students’ use of problem solving steps and use of strategies in solving word problem related to numbers were the main focus of this study. Some recommendations for further research could be offered depending on the analysis of the data.

As it was mentioned previously in Chapter 3, the convenience sampling method was used to select sample of this study. The sample included 116 middle school students (29 fifth grade, 29 sixth grade, 25 seventh grade, and 33 eighth grade students) of a public school in Konya. Therefore, the research findings cannot be generalized to a wide range of other students in Turkey. This study was conducted in a public school. There are no participants from a private school in the study. Thus, it is recommended to determine whether similar results will be obtained through the replication of the present study in not only public schools but also private schools. In addition, the study can be repeated by spreading it to a wider schedule. Students can be given more opportunities to receive more and more detailed problem solving based instruction since the students were provided only one-week problem based instruction in the current study. Furthermore, a longitudinal research method can be applied by starting from fifth grade students and continuing with them in the following years.

It might be important to give students opportunities and environment where they can practice word problems and planning. During the solution of these problems, teachers need to focus on not only the answers but also the process of the students' solution strategies. The findings in a study carried out by Shiakalli & Zacharos (2014) showed that the consistent participation of the students in the mathematical problem solving process enabled them to improve, apply and demonstrate their skills and in-depth understanding to solve the given mathematical word problems. Moreover, teachers need to ask students about their plan to reach the correct answer and also check whether they can apply the plan they wrote. As students have more opportunities to solve these problems, they will develop more flexible mathematical thinking and reasoning. Word problem activities should not only be given to students as class work or homework activities, but also strategies for how solve them needed to be taught to learners. Furthermore, class activities including problem solving strategies should be implemented since students' beliefs regarding mathematical problem solving affect their problem solving achievement positively (Higgins, 1997). Under each problem in the test students were guided to the problem solving steps. Students mostly endeavored to implement problem solving steps successfully. Therefore, problem solving steps can be given under each problem in the course books in order to make the habit of using problem solving steps and to be more successful. Problem-based instruction, which aims to facilitate teachers' enrichment of word problems used in mathematics teaching, has a positive effect not only on students' word problem solving performance but also on their beliefs about word problem solving structure (Pongsakdi, Laakkonen, Laine, Veermans, Hannula-Sormunen, & Lehtinen, E. (2019). In the written works of all students, it was generally observed that arithmetic strategies are used mostly in the process of problem solving. One of the major reasons for this is that the students could not reflect their ideas on paper. In other words, it can be observed that the students who used the arithmetic strategy thought about different strategies to reach the solution of the problems, but they could not implement their ideas and they preferred the arithmetic strategy as a practical way.

To conclude, in general, students' level of the selection and use of an appropriate strategy should be increased in order to ensure that student gain problem solving skills and use them effectively. Also, the problem solving steps and problem solving strategies facilitate teachers' job in teaching of problem solving. Since the skills of students to solve problem solving will take shape based on the problem solving approach and knowledge level of teacher, teachers who teach problem solving to students should be supported well. This is important in terms of increasing students skills of solving problems (Ersoy, Güner, 2015).

REFERENCES

Arslan, Ç., Yavuz, G., & Deringöl-Karataş, Y. (2014). Attitudes of elementary school students towards solving mathematics problems. *Procedia-Social and Behavioral Science 152,* (2014) 557-562.

Anderson, J.R. (1980). *Cognitive psychology and its implications.* New York: Freeman.

Ayaz M. F., Aydoğdu M. (2009). İlköğretim ikinci kademe öğrencilerinin problem çözme başarıları ile problem çözme aşamalarını kullanmaları arasındaki ilişki. *Matematikçiler Derneği*, 8. Matematik Sempozyumu

Azer, S. A. (2009). Problem-based learning in the fifth, sixth, and seventh grades: Assessment of students' perceptions. *Teaching and Teacher Education, 25*(8), 1033-1042.

Babadoğan, C., & Olkun, S. (2006). Program development models and reform in Turkish primary school mathematics curriculum. *International journal for mathematics teaching and learning, 1*(1), 1-6.

Bayazit, I. (2013). An Investigation of Problem Solving Approaches, Strategies, and Models Used by the 7th and 8th Grade Students When Solving Real-World Problems. *Educational sciences: Theory and practice, 13*(3), 1920-1927.

Booker, G., & Bond, D. J. (2008). *Problem-solving in mathematics.* RIC Publications.

Boonen, A. J., van der Schoot, M., van Wesel, F., de Vries, M. H., & Jolles, J. (2013). What underlies successful word problem solving? A path analysis in sixth grade students. *Contemporary Educational Psychology, 38*(3), 271-279.

Brown, S. (2001). *Reconstructing school mathematics: Problems with problems and the real world.* New York: Lang.

Cai, J., & Hwang, S. (2019). Learning to teach through mathematical problem posing: Theoretical considerations, methodology, and directions for future research. *International Journal of Educational Research.*

Cantürk-Günhan, B., & BAŞER, N. (2009). Probleme Dayali Öğrenmenin Öğrencilerin Eleştirel Düşünme Becerilerine Etkisi. *Türk Eğitim Bilimleri Dergisi, 7*(2), 451-482.

Chapman, O. (2006). Classroom practices for context of mathematics word problems. *Educational Studies in Mathematics, 62*(2), 211-230.

Che, M., Wiegert, E., & Threlkeld, K. (2012). Problem solving strategies of girls and boys in single-sex mathematics classrooms. *Educational studies in mathematics, 79*(2), 311-326.

Creswell, J. W. (2002). *Educational research: Planning, conducting, and evaluating quantitative* (pp. 146-166). Upper Saddle River, NJ: Prentice Hall.

Creswell, J. (2003). *Research design: Qualitative, quantitative and mixed methods approaches* (2nded.).Thousand Oaks, CA: SAGE Publications.

Csíkos, C., Szitányi, J., & Kelemen, R. (2012). The effects of using drawings in developing young children's mathematical word problem solving: A design experiment with third-grade Hungarian students. *Educational Studies in Mathematics, 81*(1), 47-65.

Deringöl, Y. (2019). Misconceptions of Primary School Students about the Subject of Fractions. *International Journal of Evaluation and Research in Education, 8*(1), 29-38.

DeVault, M. V. (1981). *Doing mathematics is problem solving. The Arithmetic Teacher*, 28(8), 40-43.

Durmaz B., Altun M. (2014). The usage of the problem solving strategies of the secondary students'. *Mehmet Akif Ersoy Üniversitesi Eğitim Fakultesi Dergisi,* Sayı 30, 73-94.

Erdogan, A. (2015). Turkish primary school students' strategies in solving a nonroutine mathematical problem and some implications for the curriculum design and implementation. *International Journal for Mathematics Teaching & Learning*.

Ersoy, E. (2014). The effects of problem-based learning method in higher education on creative thinking. *Procedia-Social and Behavioral Sciences, 116*, 3494-3498.

Ersoy, E. (2016). Problem solving and its teaching in mathematics. *The Online Journal of New Horizons in Education, 6*(2), 79.

Ersoy, E., & Guner, P. (2015). The Place of Problem Solving and Mathematical Thinking in The Mathematical Teaching. *The Online Journal of New Horizons in Education-January, 5*(1), 120-130.

Fadlelmula, F. K. (2010). Educational motivation and students' achievement goal orientations. *Procedia-Social and Behavioral Sciences, 2*(2), 859-863.

Fraenkel, J., N. Wallen, and H. Hyun. (2011). *How to design and evaluate research in education.* 8th ed. New York, NY: McGraw-Hill.

Gagne, R. M. (1980). Learnable aspects of problem solving. *Educational Psychologist, 15*(2), 84-92.

Garofalo, J., & Lester, F. K. (1985). Metacognition, cognitive monitoring, and mathematical performance. *Journal for research in mathematics education*.

Guven, B., & Cabakcor, B. O. (2013). Factors influencing mathematical problem-solving achievement of seventh grade Turkish students. *Learning and Individual Differences, 23,* 131-137.

Gür H., Hangül T. (2015). A study on secondary school students' problem solving strategies. *Pegem Eğitim ve Öğretim Dergisi, 5*(1), 95-112.

Gresalfi, M. S., & Lester, F. (2009). What's worth knowing in mathematics?. In *Constructivist Instruction* (pp. 276-302). Routledge.

Grishchenko, S. (. (2009). *A comparative analysis of word problems in selected united states and russian first grade textbooks* (Order No. 3385758). Available from ProQuest Dissertations & Theses Global. (304861466). Retrieved from https://search.proquest.com/docview/304861466?accountid=13014

Hagaman, A. P. (1964). Word problems in elementary mathematics. *The Arithmetic Teacher, 11*(1), 10-11.

Hembree, R. (1992). Experiments and Relational Studies in Problem Solving: A Meta-Analysis. *Journal for Research in Mathematics Education, 23*(3), 242-273. doi:10.2307/74912

Henderson, K. B., & Pingry, R. E. (1953). Problem solving in mathematics. *The learning of mathematics: Its theory and practice*, 228-270.

Herlihy, K. V. (1964). A look at problem solving in elementary school mathematics. *The Arithmetic Teacher, 11*(5), 308-311.

Higgins, K. M. (1997). The effect of year-long instruction in mathematical problem solving on middle-school students' attitudes, beliefs, and abilities. *The Journal of Experimental Education, 66*(1), 5-28.

Hohn, R. L., & Frey, B. (2002). Heuristic training and performance in elementary mathematical problem solving. *The Journal of Educational Research, 95*(6), 374-380.

Intaros, P., Inprasitha, M., & Srisawadi, N. (2014). Students' problem solving strategies in problem solving-mathematics classroom. *Procedia-Social and Behavioral Sciences, 116*, 4119-4123.

Jiang, C., Hwang, S., & Cai, J. (2014). Chinese and Singaporean sixth-grade students' strategies for solving problems about speed. *Educational Studies in Mathematics, 87*(1), 27-50.

Jonassen, D. H. (2000). Toward a design theory of problem solving. *Educational technology research and development, 48*(4), 63-85.

Krulik, S., & Posamentier, A. (1998). *Problem Solving Strategies for Effcient and Elegant Solutions*. Corwin Press Inc.

Krulik, S., & Rudnick, J. A. (1987). *Problem solving: A handbook for teachers*. Allyn and Bacon, Inc.

Karaoğlan D. (2009). The relationship between 6th grade students' problem solving achievement and mathematics achievement scores after completing instruction on problem solving. *Unpublished Master's Thesis, Middle East Technical University, Institute of Social Sciences, Ankara, Turkey*.

L BERG, B. R. U. C. E. (2001). *Qualitative research methods for the social sciences*.

Lester, F. K. (1994). Musings about mathematical problem-solving research: 1970-1994. *Journal for research in mathematics education, 25*, 660-660.

Lester Jr, F. K. (2013). Thoughts about research on mathematical problem-solving instruction. *The mathematics enthusiast, 10*(1), 245-278.

Maluleka, B. K. (2013). *Improving grade 9 learners' Mathematical processes of solving word problems* (Doctoral dissertation, University of Limpopo (Turfloop Campus)).

Mamona-Downs, J., & Downs, M. (2005). The identity of problem solving. *The Journal of Mathematical Behavior, 24*(3-4), 385-401.

Martinez, M. E. (1998). *What is problem solving?.* The Phi Delta Kappan, 79(8), 605-609.

Merriman, S. B. (2009). *Qualitative research: A guide to design and implementation*. San Francisco, JA: Jossey-Bass.

Mevarech, Z., & Fridkin, S. (2006). The effects of IMPROVE on mathematical knowledge, mathematical reasoning and meta-cognition. *Metacognition and learning, 1*(1), 85-97.

Mevarech, Z. R., & Kramarski, B. (1997). Improve: A Multidimensional Method For Teaching Mathematics in Heterogeneous Classrooms. *American Educational Research Journal, 34*(2), 365–394. https://doi.org/10.3102/00028312034002365

Ministry of National Eduction (MoNE, 2009). *Matematik (6-8. Sınıflar) Öğretim Programı*.

Montague, M. (2003). *Solve It!: A practical approach to teaching mathematical problem solving skills*. Reston, VA: Exceptional Innovations.

Montague, M., & Applegate, B. (2000). Middle School Students' Perceptions, Persistence, and Performance in Mathematical Problem Solving. *Learning Disability Quarterly, 23*(3), 215-227. doi:10.2307/1511165

Muir, T., Beswick, K., & Williamson, J. (2008). "I'm not very good at solving problems": An exploration of students' problem solving behaviors. *The Journal of Mathematical Behavior, 27*(3), 228-241

National Council of Teachers of Mathematics. (1989). *Principles and standards for school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics (NCTM). (1991). *Professional standards for teaching mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics (NCTM). (2000). *Professional standards for teaching mathematics.* Reston, VA: Author.

Neimark, E., & Lewis, N. (1967). The Development of Logical Problem-Solving Strategies. *Child Development, 38*(1), 107-117. doi:10.2307/1127132

Özalkan, B.E. (2010) The Effects of Problem Solving on the Topic of Functions on problem Solving Performance, Attitude towards problem solving and mathematics. *Master's thesis, Middle East Technical University*.

Özsoy, G. (2002). İlköğretim beşinci sınıfta matematik dersi genel başarısı ile problem çözme becerisi arasındaki ilişki. *Unpublished Master's Thesis, Gazi Üniversitesi, Ankara, Türkiye*.

Özsoy, G. (2005). The relationship between problem solving skills and mathematical achievement. GÜ, *Gazi EğitimFakültesi Dergisi, Cilt 25*, Sayı 3, 179-190.

Özyıldırım Gümüş F. (2015). Analyzing preservice elementary mathematics teachers' problem solving strategies preferences and mathematics self-efficacies. *Electronic Journal of Social Sciences, Volume:14*, Issue:52, 34-42.

Polya, G. (1957). *How to Solve It?* (2 nd ed.). Princeton, N.J.: Princeton University Press.

Polya, G. (1962). *Mathematical Discovery*, 1962. John Wiley & Sons.

Polya, G. (1973). *How to Solve it, a new aspect of mathematical Method.* Princeton University Press.

Pongsakdi, N., Laakkonen, E., Laine, T., Veermans, K., Hannula-Sormunen, M. M., & Lehtinen, E. (2019). The role of beliefs and motivational variables in enhancing word problem solving. *Scandinavian Journal of Educational Research, 63*(2), 179-197.

Posamentier A. S. & Krulik S. (1998). *Problem-solving strategies for efficient and elegant solutions: a resource for the mathematics teacher.* California: Corwin Press, Inc.

Posamentier, A. S., & Krulik, S. (2008). *Problem-solving strategies for efficient and elegant solutions, grades 6-12: a resource for the mathematics teacher*. Corwin press.

Posamentier, A. S., Smith, B. S., & Stepelman, J. (2006). *Teaching secondary mathematics: Techniques and enrichment units.* Upper Saddle River, NJ: Pearson Merrill Prentice Hall.

Prendergast, M., Breen, C., Bray, A., Faulkner, F., Carroll, B., Quinn, D., & Carr, M. (2018). Investigating secondary students beliefs about mathematical problem-solving. *International Journal of Mathematical Education in Science and Technology, 49*(8), 1203-1218.

Ramirez, G., Chang, H., Maloney, E. A., Levine, S. C., & Beilock, S. L. (2016). On the relationship between math anxiety and math achievement in early elementary school: The role of problem solving strategies. *Journal of experimental child psychology, 141*, 83-100.

Raoano, M. J. (2016). *Improving learners Mathematics problem solving skills and strategies in the intermediate phase: a case study of primary school in Lebopo Circuit* (Doctoral dissertation, University of Limpopo).

Riley, M. S. (1984). Development of children's problem-solving ability in arithmetic.

Rittle-Johnson, B., & Koedinger, K. R. (2005). Designing knowledge scaffolds to support mathematical problem solving. *Cognition and Instruction, 23*(3), 313-349.

Rosli, R., Goldsby, D., & Capraro, M. M. (2013). Assessing students' mathematical problem-solving and problem-posing skills. *Asian social science, 9*(16), 54.

Santos‐Trigo, M. (1998). Instructional qualities of a successful mathematical problem‐solving class. *International Journal of Mathematical Education in Science and Technology, 29*(5), 631-646.

Schoenfeld, A. H. (1994). What do we know about mathematics curricula?. *The Journal of Mathematical Behavior, 13*(1), 55-80.

Sertöz, S. (2011). *Matematiğin Aydınlık Dünyası*. Ankara: Tubitak Yayınları.

Shergill, H. K. (2012). *Experimental psychology.* PHI Learning Pvt. Ltd..

Shiakalli, M. A., & Zacharos, K. (2014). Building meaning through problem solving practices: the case of four-year olds. *The Journal of Mathematical Behavior, 35*, 58-73.

Stacey, K. (2005). The place of problem solving in contemporary mathematics curriculum documents. *The Journal of Mathematical Behavior, 24*(3-4), 341-350.

Stylianides, A. J., & Stylianides, G. J. (2014). Impacting positively on students' mathematical problem solving beliefs: An instructional intervention of short duration. *The Journal of Mathematical Behavior, 33*, 8-29.

Sulak, S. (2010). Effect of problem solving strategies on problem solving achievement in primary school mathematics. *Procedia-Social and Behavioral Sciences, 9*, 468-472.

Swanson, H. L., Lussier, C., & Orosco, M. (2013). Effects of cognitive strategy interventions and cognitive moderators on word problem solving in children at risk for problem solving difficulties. *Learning Disabilities Research & Practice, 28*(4), 170-183.

Van Dooren, W., Lem, S., De Wortelaer, H., & Verschaffel, L. (2019). Improving realistic word problem solving by using humor. *The Journal of Mathematical Behavior, 53*, 96-104.

Van de Walle, J. A. (2007). *Elementary and Middle School Mathematics Teaching Developmentally.* (6th ed.). Boston: Allyn and Bacon.

Van de Walle, J. A. (2010). *Elementary and Middle School Mathematics Teaching Developmentally.* (7th ed.). Boston: Allyn and Bacon.

Yazgan, Y. (2007). Dördüncü ve beşinci sınıf öğrencilerinin rutin olmayan problem çözme stratejileriyle ilgili gözlemler. *İlköğretim Online, 6*(2), 249-263.

Yazgan, Y. (2014). Observations about fourth and fifth grade students' strategies to solve non-routine problems. *İlköğretim Online, 6* (2), 249-263. Retrieved from http://dergipark.gov.tr/ilkonline/issue/8604/107172

Yazgan Y., Bintaş J. (2005). Fourth and fifth grade students' level of using problem solving strategies: a teaching experiment. *Hacettepe Üniversitesi Eğitim Fakultesi Dergisi 28*: 210-2018.

Yıldızlar, M. (2001). *Matematik Problemlerini Çözebilme Yöntemleri*. Ankara: Eylül Kitap Yayınları.

Zakaria E., Haron, Z. and Daud, Y. (2010). The reliability and construct validity of scores on the attitudes toward problem solving scale. *Journal of Science and Mathematics Education in Asia, 27* (2), 81-91.

Zeitz, P. (1999). *The art and craft of problem solving* (Vol. 85). New York: John Wiley.

APPENDICES

A. PERMISSION OBTAINED FROM METU APPLIED ETHICS RESEARCH CENTER

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

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15 ARALIK 2017

Konu: Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi: İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Yrd.Doç.Dr. Didem AKYÜZ ;

Danışmanlığını yaptığınız yüksek lisans öğrencisi Cafer Sinan ALKAN'ın **"Investiga The 5TH, 6TH, 7TH, and 8 TH Grade Students' Approaches to Problem-Solving Question and Relationship Between Their Problem-Solving Performance"** başlıklı araştırması İnsan Araştırmaları Etik Kurulu tarafından uygun görülerek gerekli onay **2017-EGT-187** protokol numarası ile **15.12.2017-30.09.2018** tarihleri arasında geçerli olmak üzere verilmiştir.

Bilgilerinize saygılarımla sunarım.

Prof. Dr. Ş. Halil TURAN
Başkan V

Prof. Dr. Ayhan SOL
Üye

Prof. Dr. Ayhan Gürbüz DEMİR
Üye

Doç. Dr. Yaşar KONDAKÇI
Üye

Doç. Dr. Zana ÇITAK
Üye

Yrd. Doç. Dr. Pınar KAYGAN
Üye

Yrd. Doç. Dr. Emre SELÇUK
Üye

B. FIFTH GRADE PROBLEM SOLVING ACHIEVEMENT TEST / BEŞİNCİ SINIF PROBLEM ÇÖZME BAŞARI TESTİ

Problem 1: Buse 24 tane boya kaleminin $\frac{7}{24}$'sini kullanmaktadır. Esra ise Buse'nin boya kalemlerinin $\frac{1}{3}$'ini kullanmaktadır. Buse ve Esra toplamda boya kalemlerinin kaç tanesini kullanmaktadır?

Problem 2: Anıl 6 tane yeni bilye alıyor ve kendi bilyelerinden 13 tanesini arkadaşına veriyor. Son durumda Anıl'ın 41 bilyesi olduğuna göre başlangıçta kaç tene bilyesi vardır?

Problem 3: Kendi arabasıyla tatile giden Onur 50 litrelik deponun $\frac{3}{5}$ ile gitmek istediği yere varmıştır. Onur yola çıkarken deposunu tamamen doldurduğuna göre vardığında arabanın deposunda geriye kaç litre benzin kalmıştır?

Problem 4: Burak, kardeşine 6 oyun kartı karşılığında 11 tane oyun kartını verir. Burak daha sonra 15 tane yeni oyun kartı satın alır ve toplamda 94 tane oyun kartı olur. Burak'ın başlangıçta kaç tane oyun kartı vardır?

Problem 5: Tolga küçük kardeşi için tahtadan bir araba yapmak istiyor. Tolga tahtalar için ve tekerlekler için toplam da 50 TL harcamıştır. Tahtalar tekerleklerden 2 TL daha pahalı olduğuna göre tahtalar için ne kadar ödemiştir?

Problem 6: Selin hafta içi her gün günde 30 dakika piyano dersi almaktadır. Selin aynı zamanda hafta sonları cumartesi ve pazar günü günde 60 dakika piyano dersi almaktadır. Selin pazartesi gününden cuma gününe kadar beş günde toplamda kaç dakika piyano dersi aldığını bulunuz.

C. SIXTH GRADE PROBLEM SOLVING ACHIEVEMENT TEST / ALTINCI SINIF PROBLEM ÇÖZME BAŞARI TESTİ

Problem 1: Atakan'ın 17 tane misketi, 6 tane de oyuncak arabası vardır. Sinan'ın misketlerinin sayısı ise Atakan'ın misketlerinin sayısının 4 katının 5 eksiği kadardır. Sinan'ın kaç tane misketi vardır?

Problem 2: Servisi kaçıran Yaşar Bey işe geç kalmamak için taksiye biner, taksimetrenin açılış ücreti 3,50 TL'dir. Taksimetre her 100 metrede 25 Krş yazar. Yaşar bey taksiciye 2 TL bahşiş bırakarak 12 TL verdiğine göre, Yaşar Bey'in evi ile iş yeri arası kaç Km'dir?

Problem 3: Bir pastane işletmecisi 240 yumurta almıştır. Bu yumurtaların $\frac{3}{8}$'ünü baklava yaparken, geri kalanının $\frac{4}{5}$'ünü de poğaça yaparken kullanmıştır. Geriye kaç yumurta kalmıştır?

Problem 4: Ahmet Usta bir şirketin kafeteryasında çalışmaktadır. Bu kafeteryada aylık üyelik ücreti 8 TL'dir.
Aşağıdaki tabloda gösterildiği gibi, üye olanlar için bir öğün yemek ücreti üye olmayanlara göre daha düşüktür.

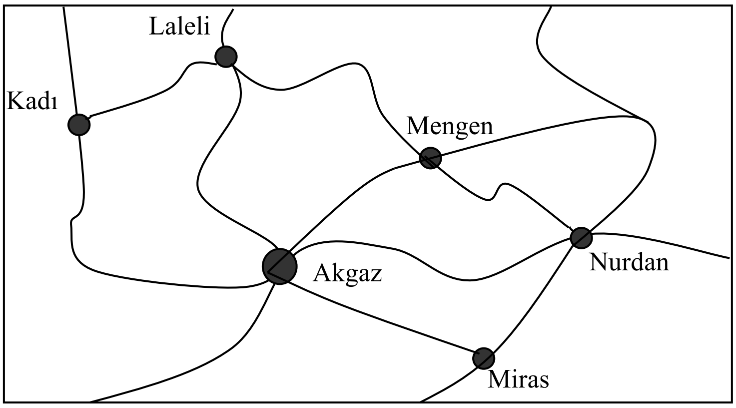
Üye olmayanların bir öğün yemek ücreti	Üye olanların bir öğün yemek ücreti
2,8 TL	2,3 TL

Onur geçen ay kafeteryanın bir üyesiydi. Geçen ay toplamda, üyelik ücreti de dahil, 56,3 TL harcadı. Eğer Onur üye olmasaydı, fakat aynı sayıda öğün yemek yeseydi, kaç TL harcayacaktı?

Problem 5: Belirli bir yükseklikten bırakılan bir top, yere ilk vuruşundan sonra bir önceki yüksekliğin $\frac{2}{5}$'si kadar yükselmektedir. Top, yere 2. Vuruşundan sonra 24 cm yükseldiğine göre kaç cm'den bırakılmıştır?

Problem 6: Bu problem, tatile giderken en iyi ve rahat olan yolun seçilmesiyle ilgilidir.
Şekil 1 bölgenin haritasını, Şekil 2 kasabalar arasındaki uzaklıkları göstermektedir.

Şekil 1: Kasabalar arasındaki yolların haritası.



Şekil 2: Kasabaların kilometre olarak birbirlerine uzaklıkları.

Akgaz						
Kadı	550					
Laleli	500	300				
Mengen	300	850	550			
Nurdan	500		1000	450		
Miras	300	850	800	600	250	
	Akgaz	Kadı	Laleli	Meng en	Nurda n	Miras

Nurdan ve Kadı arasındaki kara yolu ile en kısa uzaklığı hesaplayınız.

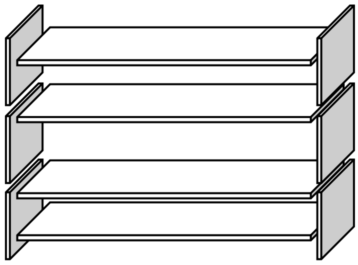
D. SEVENTH GRADE PROBLEM SOLVİNG ACHİEVEMENT TEST / YEDİNCİ SINIF PROBLEM ÇÖZME BAŞARI TESTİ

Problem 1: Matematik öğretmeni Sinan yeni bir test kitabını düzenlemektedir. Bir kişi kitabın birbirine bakan iki yüzündeki sayfaları değiştiriyor. Bu iki sayının çarpımı 812 ise. Bu iki sayı kaçtır?

Problem 2: Emir'in çantasında bir miktar şeker vardır. Bir çanta şekeriyle eve doğru giderken arkadaşı Onur ile karşılaşır ve çantasındaki şekerlerin yarısını ve bir tane daha fazla şekeri arkadaşına verir. Biraz daha yürüdüğünde Selin ile karşılaşır ve çantasında kalan şekerlerin yarısını ve bir tane daha fazla şekeri ona verir. Biraz daha ilerlediğinde ağlayan bir çocuk görür ve çantasında kalan şekerin yarısını ve bir tane daha şekeri ona verir. Emir eve vardığında çantasını açar bakar ve geriye 5 tane şekeri kaldığını görür. Emir'in başlangıçta kaç tane şekeri vardır?

Problem 3: Bir kitaplık yapmak için, bir marangoz aşağıdaki parçalara gereksinim duyar:

4 uzun tahta levha,
6 kısa tahta levha,
12 küçük çivi,
2 büyük çivi ve
14 vida.



Marangozun deposunda 26 uzun tahta levha, 33 kısa tahta levha, 200 küçük çivi, 20 büyük çivi ve 510 vida vardır.

Bu marangoz kaç tane kitaplık yapabilir?

Problem 4: Bir top her düştüğünde önceki yüksekliğinin $\frac{4}{10}$'ü kadar yükselmektedir. Top üçüncü düşüşünde yerden 16 cm yükseldiğine göre, bu topun ilk düşüşü yerden kaç metre yüksektedir?

Problem 5: Bir yolun önce $\frac{8}{15}$'i gidilmiş, daha sonra geri kalan yolun $\frac{5}{7}$'i gidilmiştir. Geriye kalan yol 50 km ise yolun tamamı kaç km'dir?

Problem 6: Hava sıcaklığının 12 ℃ olduğu bir odada elektrikli ısıtıcı çalıştırılıyor. Isıtıcı her 3 dakikada bir, odanın sıcaklığını 4 ℃ arttırıyor. Odanın sıcaklığı kaç dakika sonra 24 ℃ a ulaşır?

E. EIGHTH GRADE PROBLEM SOLVING ACHIEVEMENT TEST / SEKİZİNCİ SINIF PROBLEM ÇÖZME BAŞARI TESTİ

Problem 1: Bir iş yerinin vitrininde mavi ve kırmızı renkli ampuller yanıp sönmektedir. Mavi ampul her 6 saniyede bir, kırmızı ampuller ise her 9 saniyede bir yanmaktadır. Bu iki ampul birlikte yandıktan en az kaç saniye sonra tekrar birlikte yanar?

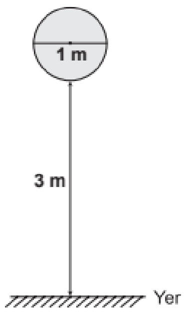
Problem 2: 56 kg ve 72 kg'lık çuvallarda bulunan iki cins pirinç birbirine karıştırılmadan hiç artmayacak şekilde en büyük ölçüdeki poşetlere konulacaktır. Buna göre;

a. Bir poşete kaç kilogram pirinç konulacaktır?
b. Bu iş için toplam kaç poşet gereklidir?

Problem 3: Bir kabın içinde, hepsi aynı büyüklükte olan 36 tane renkli boncuk vardır. Bu boncukların birazı mavi, birazı yeşil, birazı kırmızı ve geri kalanı da sarıdır. Kaptan, rengine bakılmadan bir boncuk çekildiğinde bu boncuğun mavi olması olasılığı $\frac{4}{9}$ dur. Kapta kaç tane mavi boncuk vardır?

Problem 4: bir toplantıya 5^3 ülkenin her birinden eşit sayıda kişi katılmıştır. Bu kişiler, bir otelin 5^4 odasının her birine 5 kişi kalacak biçimde odalara yerleştirilmiştir.
Buna göre bu toplantıya bir ülkeden kaç kişi katılmıştır?

Problem 5: Bir okçu, yanda gösterildiği gibi çapı 1 metre olan daire şeklindeki bir hedef tahtasına atış yapmaktadır. Hedef tahtasının yerden yüksekliği 3 metredir.
Atılan ok hedef tahtasına isabet ettiğine göre, saplandığı noktanın yerden
yüksekliği, metre cinsinden aşağıdakilerden hangisi olabilir?



Problem 6: Kenar uzunlukları $\sqrt{45}$ cm ve $\sqrt{20}$ cm olan bir karton, bir kenar uzunluğu $\sqrt{5}$ cm olan kare şeklindeki etiketlerle, kartonda hiç boşluk kalmayacak, etiketler üst üste gelmeyecek ve kartonun dışına taşmayacak şekilde kaplanmıştır. Bunun için kaç tane etiket kullanılmıştır?

F. RUBRIC FOR PROBLEM SOLVING ACHIEVEMENT TEST / PROBLEM ÇÖZME BAŞARI TESTİ DEĞERLENDİRME ÖLÇEĞİ

Score	Understanding the Problem	Developing a Plan to Solve the Problem	Carrying out the Plan and Interpreting Findings
3	Stating the problem clearly and identifying the underlying issues	Developing a clear and concise plan to solve the problem, with alternative strategies following plan to conclusion	Providing a logical interpretation of the findings and solving the problem clearly.
2	Defining the problem adequately	Developing an adequate plan and following it to conclusion	Providing an adequate interpretation of findings and solving the problem.
1	Failing to define the problem adequately	Developing a marginal plan, and not following it to conclusion	Providing an inadequate interpretation of the findings and does not derive a logical solution to the problem
0	The problem is not identified	Could not developing a coherent plan to solve the problem	Could not interpret the findings and could not reach a conclusion.

G. 5^{TH} GRADE PROBLEM SOLVING LESSON PLAN / 5. SINIF PROBLEM ÇÖZME DERS PLANI

Adı & Soyadı: Cafer Sinan Alkan
Konu: Problem Çözme
Sınıf Düzeyi: 5. Sınıf
Süre: 5 ders saati (bir hafta)
Öğrenme Alanı: Sayılar
Alt Öğrenme Alanı: Doğal Sayılar, Kesirlerle İşlemler,
Gerekli olan ön bilgiler:

- Doğal sayılarla dört işlem yapma

Kazanımlar:

- Doğal sayılarla dört işlem yapmayı gerektiren problemleri çözer
- Kesirlerle işlem yapmayı gerektiren problemleri çözer.

Gerekli Materyaller:

- "Problem Çözüyorum" adlı etkinlik kâğıdı

<u>Giriş</u>
Öğrencilere "problem deyince aklınıza ne geliyor?" sorusu sorulur.

Günlük yaşam problemler → Matematik problemler

- ➢ *"Sizce sınıfta çözdüğümüz matematik problemleri ile günlük yaşamda karşımıza çıkan problemler aynı mıdır?" sorusu öğrencilere sorulur ve bunun üzerine tartışılır.* Sorunun yanıtı öğrencilere verilmez, aşağıdaki etkinlik ile sonuca kendilerinin ulaşması sağlanır.

Okulumuzun 5. Sınıf öğrencileri Antalya'ya gezi düzenleyecektir. Sizce nasıl giderler?

- ➢ Bu bir günlük hayat problemi midir yoksa matematik problemi midir? Neden?
- ➢ Bu problemi matematiksel olarak nasıl ifade edersiniz?

Bu problemi çözmek için sizce hangi bilgilere ihtiyaç vardır?

- 5. Sınıf öğrencilerinin sayısı (150)
- Ne ile yolculuk yapacakları (Otobüs, Tren,….)
- Bir otobüsün kaç öğrenci taşıyabileceği (30 kişi veya 40 kişilik otobüs)

- ➢ *Bu bir matematik problemi olsaydı nasıl sorardık?*

1. <u>Günlük yaşam problemi</u>
   5. sınıf öğrencileri olarak Antalya'ya bir gezi düzenleyeceksiniz. Nasıl?

2. <u>Problemin matematiksel anlatımı</u>
   Okulumuzun 5. Sınıf öğrencileriyle Antalya'ya bir gezi düzenlenecektir. 150 öğrenci 30 yolcu taşıyabilen araçlarla yolculuk edeceklerdir. Bu gezi için kaç araç gereklidir?

3. <u>Matematiksel problemin çözümü</u>
   $150 \div 30 = 5$

4. <u>Günlük yaşam probleminin çözümü</u>
   5 araç gereklidir.

- *Yukarıdaki tablo ile öğrencilere matematik problemlerinin yakın çevrelerinde ve günlük yaşamda karşılaşılan durumlar olduğu öğrencilere fark ettirilir.*

Etkinlik:

- Sınıf 3'er kişilik gruplara ayrılır.
- Her gruba 1'er tane A3 kâğıdı verilir.
- Öğrencilerden önce A3 kağıdını ikiye katlamaları istenir.
- Daha sonra, bu bölümleri aşağıdaki gibi dörde bölmeleri istenir.
- Her gruptan iki adet gerçek yaşam problemleri kurmaları istenir.
- Çözümler sınıfta paylaşılır. Sınıfça seçilen 4 problem poster haline getirilmesi için hazırlayan öğrencilere ödev verilir.

1. <u>Gerçek yaşam problemleri</u>	3. <u>Problemin matematiksel anlatımı</u>
2. <u>Matematiksel problemin çözümü</u>	4. <u>Gerçek hayat probleminin çözümü</u>

- *Problem çözmenin önemi üzerinde durulur.*
- *Problem çözmek neden bu kadar önemli? Neden problem çözmeyi öğreniyoruz? Gibi sorular öğrencilere yöneltilir.*
- Problem çözme; ne yapılacağının bilinmediği durumlarda yapılması gerekeni bulmaktır.

Günlük yaşamda karşılaştığımız problemler kişisel de olabilir, tüm toplumu ilgilendiren bir problemde olabilir. Örneğin,

- Ankara’da su sıkıntısı var ve bu problem küresel ısınmanın da etkisiyle her geçen gün daha da artmakta. Su sıkıntısı günlük yaşam problemi ve tüm toplumu ilgilendiriyor.
- Bir adada tek başına kalan bir adam için nasıl hayatta kalacağı bir problem.

Karşımıza çıkan bütün problemleri matematik dersinde çözemeyiz ama problem çözmeyi öğrendiğimizde, derste öğrendiklerimizi günlük yaşamda, farklı alandaki problemleri çözmede kullanabiliriz. Bilgisayar oyunlarında basit bir oyunu oynamak için geliştirdiğimiz stratejileri daha zor ve karmaşık bir oyunu oynarken kullandığımız gibi

- *Problem çözme basamakları (Problemi Anlayalım, Plan Yapalım, Planı Uygulayalım, Kontrol Edelim, Problem Kuralım) üzerinde durulur.*

İlk olarak	PROBLEMİ ANLAMA
Problemi anlamalıyız ne?	Bilinmeyenler neler? Verilenler neler? Problem durumu
	Problem durumu ve verilenler, bilinmeyeni bulmak için yeterli mi? Ya da yetersiz mi? Ya da gereksiz mi? Ya da çelişkili mi? Problem durumu için bir figür çiz. Problem durumlarını parçalara ayır.
İkinci olarak	PLAN YAPMA
Verilenler ile	Problemi daha önce gördün mü? Ya da aynı problem
bilinmeyenler arasındaki	durumundan biraz farklı bir problem gördün mü?
bağlantıyı bul. Eğer hemen biliyor	Problem durumu ili ilgili ilişkili bir başka bir problem
bir bağlantı bulamazsan teorem	musun? Problem çözümünde faydalı olabilecek bir
yardımcı problemlere	biliyor musun?
göz önünde sahip	Bilinmeyene bak! Ve aynı veya benzer bilinmeyene
bulundurabilirsin.	Başka bir problem düşün.
En sonunda problemin problem.	İşte daha önce çözülmüş senin probleminle ilişkili bir
Çözümü için bir plan	Bu problemi kullanabilir misin? Bu problemin sonucunu
yapmalısın.	kullanabilir misin? Bu problemin çözüm metodunu
kullanabilir	misin?
	Eğer sana sorulan problemi çözemediysen öncelikle benzer başka bir problemi çözmeye çalış. Daha genel bir problem hayal edebilir misin? Daha özel? Daha kıyaslanabilir? Problemin bir parçasını çözebilir misin? Verilenlerden işe yarayacak bir şeyler türetebilir misin? Verilenleri veya bilinmeyeni ya da gerekirse ikisini de değiştirebilir misin? Bütün verilenleri kullandın mı? Bütün durumları göz ününde bulundurdun mu? Problem içinde verilen bütün gerekli kavramları dikkate aldın mı?

Üçüncü olarak Planı uygula Adımların	PLANI UYGULAMA Çözüm için planı uygula ve her bir adımı kontrol et. doğru olduğunu açık bir şekilde görebiliyor musun? Doğru olduğunu ispatlayabilir misin?
Dördüncü olarak Çözümünü kontrol et kısaca	ÇÖZÜMÜ KONTROL ET Çözümü kontrol edebilir misin? Çözümü farklı bir şekilde elde edebilir misin? Çözüme bir göz atabilir misin? Elde edilen sonucu veya problem çözümünde kullanılan metot başka bir problem çözümünde kullanılabilir mi?

- *Problem çözme stratejileri (örüntü arama, şekil çizme, tahmin ve kontrol etme, sayı cümleleri, geriye dönerek çalışma, akıl yürütme, benzer bir problemi çözme, denklem kurma, deneme yanılma, ekstra verilen bilgileri veya eksik verilen bilgileri fark etme) üzerinde durulur.*

- *"Problem Çözüyorum" etkinlik kâğıdı verilir ve öğrencilerle birlikte problemler problem çözme basamaklarına uygun olarak ve farklı stratejiler kullanılarak çözülür. Bu stratejiler sınıfça tartışılır.*

Problem Çözüyorum

PROBLEM 1: Mehmet Amca'nın çiftliğinde keçi ve tavuk beslemektedir. Mehmet Amca çiftlikte 28 tane hayvan beslemekte ve bu hayvanların toplam ayak sayısı 104'tür. Buna göre Mehmet Amca'nın çiftliğinde kaç tane tavuk vardır?

1) Yukarıda verilen problemi anlam bakımından aynı olacak şekilde kendi cümleleriniz ile yeniden yazınız.

2) Problemde ne verilmiştir? Ne istenmektedir? Kendi cümleleriniz ile açıklayınız.

Verilenler *İstenilenler*

3) 2. adımda belirlediğiniz verilen ve istenilen bilgileri dikkate alarak problemi nasıl çözeceğinize dair bir plan yapınız. Planınızı kendi cümleleriniz ile açıklayınız.

4) 3. adımda yaptığınız planı göz önünde bulundurarak problemi çözünüz.

5) 4. adımda elde ettiğiniz sonucun doğruluğundan emin misiniz? Neye göre emin olup olmadığınızı açıklayınız.

- *Problem 1 için kullanılabilecek stratejiler tartışılır ve problem çözme adımları da göz önünde bulundurularak, Problem 1 "Tahmin ve Kontrol", "Akıl Yürütme", "Deneme Yanılma", stratejileri kullanılarak öğrencilerle birlikte çözülür. Stratejiler üzerinde konuşulur ve tartışılır.*

PROBLEM 2: Ayşe 40 soruluk bir sınavda soruların $\frac{2}{5}$'sini doğru cevaplıyor. Ayşe'nin hiç boşu olmadığına göre yanlış cevapladığı soru sayısı kaçtır?

Problemi Anlayalım:
Verilenler: *İstenilenler:*

Plan Yapalım:

Planı Uygulayalım:

Kontrol Edelim:

- *Problem 2, "Şekil Çizme (Problemi Modelleme)" stratejisi kullanılarak öğrencilerle birlikte çözülür.*

PROBLEM 3: Buğra ve Tolga oyun oynarken, Buğra, Tolga'ya 13 tane oyuncak arabasının olduğunu söylüyor. Tolga ise Buğra'ya; onun oyuncak arabalarının sayısı kendi oyuncak arabası sayısının 2 katından 5 fazla olduğunu söylüyor. Buna göre Tolga'nın kaç tane oyuncak arabası vardır?

Problemi Anlayalım:
Verilenler: *İstenilenler:*

Plan Yapalım:

Planı Uygulayalım:

Kontrol Edelim:

- *Problem 3, “Denklem Kurma”, “Matematik cümlesi Oluşturma” ve “Uygun olan İşlemi Seçme” stratejilerini kullanarak öğrencilerle birlikte çözülür.*

PROBLEM 4: Barış Bey, işe geç kaldığı için evinin önünden taksiye biner, taksimetrenin açılış ücreti 2 TL’dir. Taksimetre her 1 kilometrede bir 1 TL yazar. Barış Bey, taksiciye 3 TL de bahşiş bırakarak toplam 10 TL verir. Barış Bey’in iş yeri evinden kaç km uzaklıktadır.

Problemi Anlayalım:
Verilenler: *İstenilenler:*

Plan Yapalım:

Planı Uygulayalım:

Kontrol Edelim:

- *Problem 4, “Geriye dönerek çalışma”, “Akıl yürütme” stratejilerini kullanarak öğrencilerle birlikte çözülür.*

PROBLEM 5: Ayşenur, aşağıdaki örüntüyü devam ettirmek için sizden yardım istemektedir. Ona yardımcı olur musunuz?

AA

BBBB

CCCCCC

ÇÇÇÇÇÇÇÇ

???

a) F satırında kaç harf vardır?
b) Örüntünün hangi satırında 22 harf vardır?

Problemi Anlayalım:
Verilenler: *İstenilenler:*

Plan Yapalım:

Planı Uygulayalım:

Kontrol Edelim:

- *Problem 5, "Örüntü Arama", "Akıl Yürütme" stratejilerini kullanarak öğrencilerle birlikte çözülür.*

PROBLEM 6: Kanal D hava durumu spikeri Cuma akşamı; "Cumartesi günü 6 ℃ olan hava sıcaklığı her gün iki derece artacak." demiştir. Buna göre hafta içi Perşembe günü tahmini hava sıcaklığı kaç derece olacaktır?

Problemi Anlayalım:
Verilenler: *İstenilenler:*

Plan Yapalım:

Planı Uygulayalım:

Kontrol Edelim:

- *Problem 6, "Bir Liste, Grafik ve Tablo Yapma" stratejisi kullanarak öğrencilerle birlikte çözülür.*

H. 6TH GRADE PROBLEM SOLVING LESSON PLAN / 6. SINIF PROBLEM ÇÖZME DERS PLANI

Adı & Soyadı: Cafer Sinan Alkan

Konu: Problem Çözme
Sınıf Düzeyi: 6. Sınıf
Süre: 5 ders saati (bir hafta)
Öğrenme Alanı: Sayılar
Alt Öğrenme Alanı: Doğal Sayılar, Kesirlerle İşlemler, Ondalık Gösterimli İfadelerle İşlemler
Gerekli olan ön bilgiler:

- Doğal sayılarla dört işlem yapma
- Ondalık gösterimli ifadelerle işlem yapma

Kazanımlar:

- Doğal sayılarla dört işlem yapmayı gerektiren problemleri çözer
- Ondalık ifadelerle dört işlem yapmayı gerektiren problemleri çözer.
- Kesirlerle işlem yapmayı gerektiren problemleri çözer.

Gerekli Materyaller:

- "Problem Çözüyorum" adlı etkinlik kâğıdı

<u>Giriş</u>
Öğrencilere "problem deyince aklınıza ne geliyor?" sorusu sorulur.

Günlük yaşam problemler → Matematik problemler

- ➢ *"Sizce sınıfta çözdüğümüz matematik problemleri ile günlük yaşamda karşımıza çıkan problemler aynı mıdır?" sorusu öğrencilere sorulur ve bunun üzerine tartışılır.* Sorunun yanıtı öğrencilere verilmez, aşağıdaki etkinlik ile sonuca kendilerinin ulaşması sağlanır.

Okulumuzun 6. Sınıf öğrencileri Antalya'ya gezi düzenleyecektir. Sizce nasıl giderler?

- ➢ Bu bir günlük hayat problemi midir yoksa matematik problemi midir? Neden?
- ➢ Bu problemi matematiksel olarak nasıl ifade edersiniz?

Bu problemi çözmek için sizce hangi bilgilere ihtiyaç vardır?

- 6. Sınıf öğrencilerinin sayısı (150)
- Ne ile yolculuk yapacakları (Otobüs, Tren,….)
- Bir otobüsün kaç öğrenci taşıyabileceği (30 kişi veya 40 kişilik otobüs)

- ➢ *Bu bir matematik problemi olsaydı nasıl sorardık?*

5. Günlük yaşam problemi
   6. sınıf öğrencileri olarak Antalya'ya bir gezi düzenleyeceksiniz. Nasıl?

6. Problemin matematiksel anlatımı
   Okulumuzun 6. Sınıf öğrencileriyle Antalya'ya bir gezi düzenlenecektir. 150 öğrenci 30 yolcu taşıyabilen araçlarla yolculuk edeceklerdir. Bu gezi için kaç araç gereklidir?

7. Matematiksel problemin çözümü
   $150 \div 30 = 5$

8. Günlük yaşam probleminin çözümü
   5 araç gereklidir.

➢ *Yukarıdaki tablo ile öğrencilere matematik problemlerinin yakın çevrelerinde ve günlük yaşamda karşılaşılan durumlar olduğu öğrencilere fark ettirilir.*

Etkinlik:

- Sınıf 3'er kişilik gruplara ayrılır.
- Her gruba 1'er tane A3 kâğıdı verilir.
- Öğrencilerden önce A3 kağıdını ikiye katlamaları istenir.
- Daha sonra, bu bölümleri aşağıdaki gibi dörde bölmeleri istenir.
- Her gruptan iki adet gerçek yaşam problemleri kurmaları istenir.
- Çözümler sınıfta paylaşılır. Sınıfça seçilen 4 problem poster haline getirilmesi için hazırlayan öğrencilere ödev verilir.

3. Gerçek yaşam problemleri	3. Problemin matematiksel anlatımı
4. Matematiksel problemin çözümü	4. Gerçek hayat probleminin çözümü

➢ *Problem çözmenin önemi üzerinde durulur.*
➢ *Problem çözmek neden bu kadar önemli? Neden problem çözmeyi öğreniyoruz? Gibi sorular öğrencilere yöneltilir.*
➢ Problem çözme; ne yapılacağının bilinmediği durumlarda yapılması gerekeni bulmaktır.

Günlük yaşamda karşılaştığımız problemler kişisel de olabilir, tüm toplumu ilgilendiren bir problemde olabilir. Örneğin,

- Ankara'da su sıkıntısı var ve bu problem küresel ısınmanın da etkisiyle her geçen gün daha da artmakta. Su sıkıntısı günlük yaşam problemi ve tüm toplumu ilgilendiriyor.
- Bir adada tek başına kalan bir adam için nasıl hayatta kalacağı bir problem.

Karşımıza çıkan bütün problemleri matematik dersinde çözemeyiz ama problem çözmeyi öğrendiğimizde, derste öğrendiklerimizi günlük yaşamda, farklı alandaki problemleri çözmede kullanabiliriz. Bilgisayar oyunlarında basit bir oyunu oynamak için geliştirdiğimiz stratejileri daha zor ve karmaşık bir oyunu oynarken kullandığımız gibi

➢ *Problem çözme basamakları (Problemi Anlayalım, Plan Yapalım, Planı Uygulayalım, Kontrol Edelim, Problem Kuralım) üzerinde durulur.*

İlk olarak	PROBLEMİ ANLAMA
Problemi anlamalıyız	Bilinmeyenler neler? Verilenler neler? Problem durumu
ne?	
	Problem durumu ve verilenler, bilinmeyeni bulmak için yeterli mi? Ya da yetersiz mi? Ya da gereksiz mi? Ya da çelişkili mi?
	Problem durumu için bir figür çiz. Problem durumlarını parçalara ayır.
İkinci olarak	PLAN YAPMA
Verilenler ile	Problemi daha önce gördün mü? Ya da aynı problem
bilinmeyenler arasındaki	durumundan biraz farklı bir problem gördün mü?
bağlantıyı bul. Eğer hemen	Problem durumu ili ilgili ilişkili bir başka bir problem
biliyor	
bir bağlantı bulamazsan	musun? Problem çözümünde faydalı olabilecek bir
teorem	
yardımcı problemlere	biliyor musun?
göz önünde	Bilinmeyene bak! Ve aynı veya benzer bilinmeyene
sahip	
bulundurabilirsin.	Başka bir problem düşün.
En sonunda problemin	İşte daha önce çözülmüş senin probleminle ilişkili bir
problem.	
Çözümü için bir plan	Bu problemi kullanabilir misin? Bu problemin sonucunu
yapmalısın.	kullanabilir misin? Bu problemin çözüm metodunu
kullanabilir	misin?
	Eğer sana sorulan problemi çözemediysen öncelikle benzer başka bir problemi çözmeye çalış. Daha genel bir problem hayal edebilir misin? Daha özel? Daha kıyaslanabilir? Problemin bir parçasını çözebilir misin? Verilenlerden işe yarayacak bir şeyler türetebilir misin? Verilenleri veya bilinmeyeni ya da gerekirse ikisini de değiştirebilir misin? Bütün verilenleri kullandın mı?

	Bütün durumları göz ününde bulundurdun mu? Problem içinde verilen bütün gerekli kavramları dikkate aldın mı?
Üçüncü olarak Planı uygula Adımların	PLANI UYGULAMA Çözüm için planı uygula ve her bir adımı kontrol et. doğru olduğunu açık bir şekilde görebiliyor musun? Doğru olduğunu ispatlayabilir misin?
Dördüncü olarak Çözümünü kontrol et kısaca	ÇÖZÜMÜ KONTROL ET Çözümü kontrol edebilir misin? Çözümü farklı bir şekilde elde edebilir misin? Çözüme bir göz atabilir misin? Elde edilen sonucu veya problem çözümünde kullanılan metot başka bir problem çözümünde kullanılabilir mi?

- *Problem çözme stratejileri (örüntü arama, şekil çizme, tahmin ve kontrol etme, sayı cümleleri, geriye dönerek çalışma, akıl yürütme, benzer bir problemi çözme, denklem kurma, deneme yanılma, ekstra verilen bilgileri veya eksik verilen bilgileri fark etme) üzerinde durulur.*

- *"Problem Çözüyorum" etkinlik kâğıdı verilir ve öğrencilerle birlikte problemler problem çözme basamaklarına uygun olarak ve farklı stratejiler kullanılarak çözülür. Bu stratejiler sınıfça tartışılır.*

Problem Çözüyorum

1. Mehmet Amca'nın çiftliğinde keçi ve tavuk beslemektedir. Mehmet Amca çiftlikte 28 tane hayvan beslemekte ve bu hayvanların toplam ayak sayısı 104'tür. Buna göre Mehmet Amca'nın çiftliğinde kaç tane tavuk vardır?

- *Problem 1 için kullanılabilecek stratejiler tartışılır ve problem çözme adımları da göz ününde bulundurularak, Problem 1 "Tahmin ve Kontrol", "Akıl Yürütme", "Deneme Yanılma", stratejileri kullanılarak öğrencilerle birlikte çözülür. Stratejiler üzerinde konuşulur ve tartışılır.*

2. Ayşe öğretmen matematik dersinde bir etkinlik yapacaktır. Bunun için büyük bir fon kartonunu 15 makas darbesiyle eş parçalara ayırır ve elindeki parçaları

hiç artmayacak şekilde öğrencilere dağıtır. Karton almayan öğrenci kalmadığına göre Ayşe öğretmenin sınıfında kaç öğrencisi vardır?

- *Problem 2, "Şekil Çizme (Problemi Modelleme)" stratejisi kullanılarak öğrencilerle birlikte çözülür.*

3. Buğra ve Tolga oyun oynarken, Buğra, Tolga'ya 13 tane oyuncak arabasının olduğunu söylüyor. Tolga ise Buğra'ya; onun oyuncak arabalarının sayısı kendi oyuncak arabası sayısının 2 katından 5 fazla olduğunu söylüyor. Buna göre Tolga'nın kaç tane oyuncak arabası vardır?

- *Problem 3, "Denklem Kurma", "Matematik cümlesi Oluşturma" ve "Uygun olan İşlemi Seçme" stratejilerini kullanarak öğrencilerle birlikte çözülür.*

4. Barış Bey, işe geç kaldığı için evinin önünden taksiye biner, taksimetrenin açılış ücreti 1,5 TL'dir. Taksimetre her 100 metrede bir 20 kuruş yazar. Barış Bey, taksiciye 2 TL de bahşiş bırakarak toplam 10 TL verir. Barış Bey'in iş yeri evinden kaç km uzaklıktadır.

- *Problem 4, "Geriye dönerek çalışma", "Akıl yürütme" stratejilerini kullanarak öğrencilerle birlikte çözülür.*

5. Ayşenur, aşağıdaki örüntüyü devam ettirmek için sizden yardım istemektedir. Ona yardımcı olur musunuz?

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c) F satırında kaç harf vardır?

d) Örüntünün hangi satırında 22 harf vardır?

- *Problem 5, "Örüntü Arama", "Akıl Yürütme" stratejilerini kullanarak öğrencilerle birlikte çözülür.*

6. Kanal D hava durumu spikeri Cuma akşamı; "Cumartesi günü 6 ℃ olan hava sıcaklığı her gün iki derece artacak." demiştir. Buna göre hafta içi Perşembe günü tahmini hava sıcaklığı kaç derece olacaktır?

- *Problem 6, "Bir Liste, Grafik ve Tablo Yapma" stratejisi kullanarak öğrencilerle birlikte çözülür.*

İ. 7^{TH} GRADE PROBLEM SOLVING LESSON PLAN / 7. SINIF PROBLEM ÇÖZME DERS PLANI

Adı & Soyadı: Cafer Sinan Alkan
Konu: Problem Çözme
Sınıf Düzeyi: 7. Sınıf
Süre: 5 ders saati (bir hafta)
Öğrenme Alanı: Sayılar
Alt Öğrenme Alanı: Doğal Sayılar, Rasyonel Sayılar, Tam Sayılar
Gerekli olan ön bilgiler:

- Doğal sayılarla dört işlem yapma

Kazanımlar:

- Tam sayılarla işlemler yapmayı gerektiren problemleri çözer.
- Rasyonel sayılarla işlem yapmayı gerektiren problemleri çözer
- Gerçek yaşam durumlarına uygun birinci dereceden bir bilinmeyenli denklemleri kurar.

Gerekli Materyaller:

- "Problem Çözüyorum" adlı etkinlik kâğıdı

<u>Giriş</u>
Öğrencilere "problem deyince aklınıza ne geliyor?" sorusu sorulur.

Günlük yaşam problemler → Matematik problemler

- *"Sizce sınıfta çözdüğümüz matematik problemleri ile günlük yaşamda karşımıza çıkan problemler aynı mıdır?" sorusu öğrencilere sorulur ve bunun üzerine tartışılır.* Sorunun yanıtı öğrencilere verilmez, aşağıdaki etkinlik ile sonuca kendilerinin ulaşması sağlanır.

Okulumuzun 7. Sınıf öğrencileri Antalya'ya gezi düzenleyecektir. Sizce nasıl giderler?

- Bu bir günlük hayat problemi midir yoksa matematik problemi midir? Neden?
- Bu problemi matematiksel olarak nasıl ifade edersiniz?

Bu problemi çözmek için sizce hangi bilgilere ihtiyaç vardır?

- 7. Sınıf öğrencilerinin sayısı (150)
- Ne ile yolculuk yapacakları (Otobüs, Tren,….)
- Bir otobüsün kaç öğrenci taşıyabileceği (30 kişi veya 40 kişilik otobüs)

- *Bu bir matematik problemi olsaydı nasıl sorardık?*

9. Günlük yaşam problemi
   7. sınıf öğrencileri olarak Antalya'ya bir gezi düzenleyeceksiniz. Nasıl?

10. Problemin matematiksel anlatımı
   Okulumuzun 7. Sınıf öğrencileriyle Antalya'ya bir gezi düzenlenecektir. 150 öğrenci 30 yolcu taşıyabilen araçlarla yolculuk edeceklerdir. Bu gezi için kaç araç gereklidir?

11. Matematiksel problemin çözümü
   $150 \div 30 = 5$

12. Günlük yaşam probleminin çözümü
   5 araç gereklidir.

- *Yukarıdaki tablo ile öğrencilere matematik problemlerinin yakın çevrelerinde ve günlük yaşamda karşılaşılan durumlar olduğu öğrencilere fark ettirilir.*

Etkinlik:

- Sınıf 3'er kişilik gruplara ayrılır.
- Her gruba 1'er tane A3 kâğıdı verilir.
- Öğrencilerden önce A3 kağıdını ikiye katlamaları istenir.
- Daha sonra, bu bölümleri aşağıdaki gibi dörde bölmeleri istenir.
- Her gruptan iki adet gerçek yaşam problemleri kurmaları istenir.
- Çözümler sınıfta paylaşılır. Sınıfça seçilen 4 problem poster haline getirilmesi için hazırlayan öğrencilere ödev verilir.

5. Gerçek yaşam problemleri	3. Problemin matematiksel anlatımı
6. Matematiksel problemin çözümü	4. Gerçek hayat probleminin çözümü

- *Problem çözmenin önemi üzerinde durulur.*
- *Problem çözmek neden bu kadar önemli? Neden problem çözmeyi öğreniyoruz? Gibi sorular öğrencilere yöneltilir.*

- Problem çözme; ne yapılacağının bilinmediği durumlarda yapılması gerekeni bulmaktır.

Günlük yaşamda karşılaştığımız problemler kişisel de olabilir, tüm toplumu ilgilendiren bir problemde olabilir. Örneğin,

- Ankara'da su sıkıntısı var ve bu problem küresel ısınmanın da etkisiyle her geçen gün daha da artmakta. Su sıkıntısı günlük yaşam problemi ve tüm toplumu ilgilendiriyor.
- Bir adada tek başına kalan bir adam için nasıl hayatta kalacağı bir problem.

Karşımıza çıkan bütün problemleri matematik dersinde çözemeyiz ama problem çözmeyi öğrendiğimizde, derste öğrendiklerimizi günlük yaşamda, farklı alandaki problemleri çözmede kullanabiliriz. Bilgisayar oyunlarında basit bir oyunu oynamak için geliştirdiğimiz stratejileri daha zor ve karmaşık bir oyunu oynarken kullandığımız gibi

- *Problem çözme basamakları (Problemi Anlayalım, Plan Yapalım, Planı Uygulayalım, Kontrol Edelim, Problem Kuralım) üzerinde durulur.*

İlk olarak	PROBLEMİ ANLAMA
Problemi anlamalıyız ne?	Bilinmeyenler neler? Verilenler neler? Problem durumu
	Problem durumu ve verilenler, bilinmeyeni bulmak için yeterli mi? Ya da yetersiz mi? Ya da gereksiz mi? Ya da çelişkili mi? Problem durumu için bir figür çiz. Problem durumlarını parçalara ayır.
İkinci olarak	PLAN YAPMA
Verilenler ile	Problemi daha önce gördün mü? Ya da aynı problem
bilinmeyenler arasındaki	durumundan biraz farklı bir problem gördün mü?
bağlantıyı bul. Eğer hemen biliyor	Problem durumu ili ilgili ilişkili bir başka bir problem
bir bağlantı bulamazsan teorem	musun? Problem çözümünde faydalı olabilecek bir
yardımcı problemlere	biliyor musun?
göz önünde sahip	Bilinmeyene bak! Ve aynı veya benzer bilinmeyene
bulundurabilirsin.	Başka bir problem düşün.
En sonunda problemin problem.	İşte daha önce çözülmüş senin probleminle ilişkili bir
Çözümü için bir plan	Bu problemi kullanabilir misin? Bu problemin sonucunu
yapmalısın. kullanabilir	kullanabilir misin? Bu problemin çözüm metodunu misin?
	Eğer sana sorulan problemi çözemediysen öncelikle benzer başka bir problemi çözmeye çalış. Daha genel bir problem hayal edebilir misin? Daha özel? Daha kıyaslanabilir? Problemin bir parçasını çözebilir misin?

	Verilenlerden işe yarayacak bir şeyler türetebilir misin? Verilenleri veya bilinmeyeni ya da gerekirse ikisini de değiştirebilir misin? Bütün verilenleri kullandın mı? Bütün durumları göz ününde bulundurdun mu? Problem içinde verilen bütün gerekli kavramları dikkate aldın mı?
Üçüncü olarak Planı uygula Adımların	PLANI UYGULAMA Çözüm için planı uygula ve her bir adımı kontrol et. doğru olduğunu açık bir şekilde görebiliyor musun? Doğru olduğunu ispatlayabilir misin?
Dördüncü olarak Çözümünü kontrol et kısaca	ÇÖZÜMÜ KONTROL ET Çözümü kontrol edebilir misin? Çözümü farklı bir şekilde elde edebilir misin? Çözüme bir göz atabilir misin? Elde edilen sonucu veya problem çözümünde kullanılan metot başka bir problem çözümünde kullanılabilir mi?

- *Problem çözme stratejileri (örüntü arama, şekil çizme, tahmin ve kontrol etme, sayı cümleleri, geriye dönerek çalışma, akıl yürütme, benzer bir problemi çözme, denklem kurma, deneme yanılma, ekstra verilen bilgileri veya eksik verilen bilgileri fark etme) üzerinde durulur.*

- *"Problem Çözüyorum" etkinlik kâğıdı verilir ve öğrencilerle birlikte problemler problem çözme basamaklarına uygun olarak ve farklı stratejiler kullanılarak çözülür. Bu stratejiler sınıfça tartışılır.*

Problem Çözüyorum

1. Mehmet Amca'nın çiftliğinde keçi ve tavuk beslemektedir. Mehmet Amca çiftlikte 28 tane hayvan beslemekte ve bu hayvanların toplam ayak sayısı 104'tür. Buna göre Mehmet Amca'nın çiftliğinde kaç tane tavuk vardır?

- *Problem 1 için kullanılabilecek stratejiler tartışılır ve problem çözme adımları da göz ününde bulundurularak, Problem 1 "Tahmin ve Kontrol", "Akıl Yürütme", "Deneme Yanılma", stratejileri kullanılarak öğrencilerle birlikte çözülür. Stratejiler üzerinde konuşulur ve tartışılır.*

2. 30 öğrencinin bulunduğu bir sınıfta öğrencilerin $\frac{5}{6}$'i gözlüklüdür. Gözlüklü öğrencilerin $\frac{3}{5}$'ü erkektir.
   Buna göre sınıfta kaç tane gözlüklü erkek öğrenci vardır?

- *Problem 2, "Şekil Çizme (Problemi Modelleme)" stratejisi kullanılarak öğrencilerle birlikte çözülür.*

3. 22 TL si bulunan Özlem günde 2 TL, 10 TL'si bulunan Melih günde 4 TL biriktirmektedir.
   Kaç gün sonra Özlem ve Melih'in paraları eşit olur?

- *Problem 3, "Denklem Kurma", "Matematik cümlesi Oluşturma" ve "Uygun olan İşlemi Seçme" stratejilerini kullanarak öğrencilerle birlikte çözülür.*

4. Barış Bey, işe geç kaldığı için evinin önünden taksiye biner, taksimetrenin açılış ücreti 1,5 TL'dir. Taksimetre her 100 metrede bir 20 kuruş yazar. Barış Bey, taksiciye 2 TL de bahşiş bırakarak toplam 10 TL verir. Barış Bey'in iş yeri evinden kaç km uzaklıktadır.

- *Problem 4, "Geriye dönerek çalışma", "Akıl yürütme" stratejilerini kullanarak öğrencilerle birlikte çözülür.*

5. Ayşenur, aşağıdaki örüntüyü devam ettirmek için sizden yardım istemektedir. Ona yardımcı olur musunuz?

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e) F satırında kaç harf vardır?
f)Örüntünün hangi satırında 22 harf vardır?

- *Problem 5, "Örüntü Arama", "Akıl Yürütme" stratejilerini kullanarak öğrencilerle birlikte çözülür.*

6. Kanal D hava durumu spikeri Cuma akşamı; "Cumartesi günü 6 ℃ olan hava sıcaklığı her gün iki derece azalacaktır." demiştir. Buna göre hafta içi Perşembe günü tahmini hava sıcaklığı kaç derece olacaktır?

Problem 6, "Bir Liste, Grafik ve Tablo Yapma" stratejisi kullanarak öğrencilerle birlikte çözülür.

J. 8TH GRADE PROBLEM SOLVING LESSON PLAN / 8. SINIF PROBLEM ÇÖZME DERS PLANI

Adı & Soyadı: Cafer Sinan Alkan
Konu: Problem Çözme
Sınıf Düzeyi: 8. Sınıf
Süre: 5 ders saati (bir hafta)
Öğrenme Alanı: Sayılar
Alt Öğrenme Alanı: Doğal Sayılar, Rasyonel Sayılar, Tam Sayılar
Gerekli olan ön bilgiler:

- Doğal sayılar, Rasyonel Sayılar ve Tam Sayılarla dört işlem yapma

Kazanımlar:

- İki doğal sayının en büyük ortak bölenini (EBOB) ve en küçük ortak katını (EKOK) hesaplar; ilgili problemleri çözer.
- Üslü ifadelerle ilgili temel kuralları anlar, birbirine denk ifadeler oluşturur.
- Kareköklü ifadelerde çarpma ve bölme işlemlerini yapar.
- Kareköklü ifadelerde toplama ve çıkarma işlemlerini yapar.
- Basit olayların olma olasılığını hesaplar.

Gerekli Materyaller:

- "Problem Çözüyorum" adlı etkinlik kâğıdı

<u>Giriş</u>
Öğrencilere "problem deyince aklınıza ne geliyor?" sorusu sorulur.

Günlük yaşam problemler → Matematik problemler

- *"Sizce sınıfta çözdüğümüz matematik problemleri ile günlük yaşamda karşımıza çıkan problemler aynı mıdır?" sorusu öğrencilere sorulur ve bunun üzerine tartışılır.* Sorunun yanıtı öğrencilere verilmez, aşağıdaki etkinlik ile sonuca kendilerinin ulaşması sağlanır.

Okulumuzun 8. Sınıf öğrencileri Antalya'ya gezi düzenleyecektir. Sizce nasıl giderler?

- Bu bir günlük hayat problemi midir yoksa matematik problemi midir? Neden?
- Bu problemi matematiksel olarak nasıl ifade edersiniz?

Bu problemi çözmek için sizce hangi bilgilere ihtiyaç vardır?

- 8. Sınıf öğrencilerinin sayısı (150)
- Ne ile yolculuk yapacakları (Otobüs, Tren,….)
- Bir otobüsün kaç öğrenci taşıyabileceği (30 kişi veya 40 kişilik otobüs)

➢ *Bu bir matematik problemi olsaydı nasıl sorardık?*

13. Günlük yaşam problemi
    8. sınıf öğrencileri olarak Antalya'ya bir gezi düzenleyeceksiniz. Nasıl?

14. Problemin matematiksel anlatımı
    Okulumuzun 8. Sınıf öğrencileriyle Antalya'ya bir gezi düzenlenecektir. 150 öğrenci 30 yolcu taşıyabilen araçlarla yolculuk edeceklerdir. Bu gezi için kaç araç gereklidir?

15. Matematiksel problemin çözümü
    $150 \div 30 = 5$

16. Günlük yaşam probleminin çözümü
    5 araç gereklidir.

➢ *Yukarıdaki tablo ile öğrencilere matematik problemlerinin yakın çevrelerinde ve günlük yaşamda karşılaşılan durumlar olduğu öğrencilere fark ettirilir.*

Etkinlik:

- Sınıf 3'er kişilik gruplara ayrılır.
- Her gruba 1'er tane A3 kâğıdı verilir.
- Öğrencilerden önce A3 kağıdını ikiye katlamaları istenir.
- Daha sonra, bu bölümleri aşağıdaki gibi dörde bölmeleri istenir.
- Her gruptan iki adet gerçek yaşam problemleri kurmaları istenir.
- Çözümler sınıfta paylaşılır. Sınıfça seçilen 4 problem poster haline getirilmesi için hazırlayan öğrencilere ödev verilir.

7. Gerçek yaşam problemleri	3. Problemin matematiksel anlatımı
8. Matematiksel problemin çözümü	4. Gerçek hayat probleminin çözümü

- ➢ *Problem çözmenin önemi üzerinde durulur.*
- ➢ *Problem çözmek neden bu kadar önemli? Neden problem çözmeyi öğreniyoruz? Gibi sorular öğrencilere yöneltilir.*
- ➢ Problem çözme; ne yapılacağının bilinmediği durumlarda yapılması gerekeni bulmaktır.

Günlük yaşamda karşılaştığımız problemler kişisel de olabilir, tüm toplumu ilgilendiren bir problemde olabilir. Örneğin,

- Ankara'da su sıkıntısı var ve bu problem küresel ısınmanın da etkisiyle her geçen gün daha da artmakta. Su sıkıntısı günlük yaşam problemi ve tüm toplumu ilgilendiriyor.
- Bir adada tek başına kalan bir adam için nasıl hayatta kalacağı bir problem.

Karşımıza çıkan bütün problemleri matematik dersinde çözemeyiz ama problem çözmeyi öğrendiğimizde, derste öğrendiklerimizi günlük yaşamda, farklı alandaki problemleri çözmede kullanabiliriz. Bilgisayar oyunlarında basit bir oyunu oynamak için geliştirdiğimiz stratejileri daha zor ve karmaşık bir oyunu oynarken kullandığımız gibi

- ➢ *Problem çözme basamakları (Problemi Anlayalım, Plan Yapalım, Planı Uygulayalım, Kontrol Edelim, Problem Kuralım) üzerinde durulur.*

İlk olarak	PROBLEMİ ANLAMA
Problemi anlamalıyız	Bilinmeyenler neler? Verilenler neler? Problem durumu
ne?	
	Problem durumu ve verilenler, bilinmeyeni bulmak için yeterli mi? Ya da yetersiz mi? Ya da gereksiz mi? Ya da çelişkili mi?
	Problem durumu için bir figür çiz. Problem durumlarını parçalara ayır.
İkinci olarak	PLAN YAPMA
Verilenler ile	Problemi daha önce gördün mü? Ya da aynı problem
bilinmeyenler arasındaki	durumundan biraz farklı bir problem gördün mü?
bağlantıyı bul. Eğer hemen	Problem durumu ili ilgili ilişkili bir başka bir problem
biliyor	
bir bağlantı bulamazsan	musun? Problem çözümünde faydalı olabilecek bir
teorem	
yardımcı problemlere	biliyor musun?
göz önünde	Bilinmeyene bak! Ve aynı veya benzer bilinmeyene
sahip	
bulundurabilirsin.	Başka bir problem düşün.
En sonunda problemin	İşte daha önce çözülmüş senin probleminle ilişkili bir
problem.	
Çözümü için bir plan	Bu problemi kullanabilir misin? Bu problemin sonucunu
yapmalısın.	kullanabilir misin? Bu problemin çözüm metodunu
kullanabilir	misin?

	Eğer sana sorulan problemi çözemediysen öncelikle benzer başka bir problemi çözmeye çalış. Daha genel bir problem hayal edebilir misin? Daha özel? Daha kıyaslanabilir? Problemin bir parçasını çözebilir misin? Verilenlerden işe yarayacak bir şeyler türetebilir misin? Verilenleri veya bilinmeyeni ya da gerekirse ikisini de değiştirebilir misin? Bütün verilenleri kullandın mı? Bütün durumları göz ününde bulundurdun mu? Problem içinde verilen bütün gerekli kavramları dikkate aldın mı?
Üçüncü olarak Planı uygula Adımların	PLANI UYGULAMA Çözüm için planı uygula ve her bir adımı kontrol et. doğru olduğunu açık bir şekilde görebiliyor musun? Doğru olduğunu ispatlayabilir misin?
Dördüncü olarak Çözümünü kontrol et kısaca	ÇÖZÜMÜ KONTROL ET Çözümü kontrol edebilir misin? Çözümü farklı bir şekilde elde edebilir misin? Çözüme bir göz atabilir misin? Elde edilen sonucu veya problem çözümünde kullanılan metot başka bir problem çözümünde kullanılabilir mi?

- *Problem çözme stratejileri (örüntü arama, şekil çizme, tahmin ve kontrol etme, sayı cümleleri, geriye dönerek çalışma, akıl yürütme, benzer bir problemi çözme, denklem kurma, deneme yanılma, ekstra verilen bilgileri veya eksik verilen bilgileri fark etme) üzerinde durulur.*

- *"Problem Çözüyorum" etkinlik kâğıdı verilir ve öğrencilerle birlikte problemler problem çözme basamaklarına uygun olarak ve farklı stratejiler kullanılarak çözülür. Bu stratejiler sınıfça tartışılır.*

Problem Çözüyorum

1. Boyu 120 m, eni 80 m olan dikdörtgen şeklindeki bir bahçenin kenarına eşit aralıklarla (köşelere de konulmak üzere) fidan dikilecektir. Bu iş için en az kaç tane kavak ağacı gereklidir?

- *Problem 1, "Şekil Çizme (Problemi Modelleme)" stratejisi kullanılarak öğrencilerle birlikte çözülür.*

2. Bir kutudaki bilyeler dörderli ve altışarlı sayıldığında her seferinde 3 bilye artıyor. Bu kutudaki bilye sayısının 400'den fazla olduğu bilindiğine göre bilye sayısı en az kaçtır?

Problem 2 için kullanılabilecek stratejiler tartışılır ve problem çözme adımları da göz önünde bulundurularak, Problem 2 "Tahmin ve Kontrol", "Akıl Yürütme", "Deneme Yanılma", stratejileri kullanılarak öğrencilerle birlikte çözülür. Stratejiler üzerinde konuşulur ve tartışılır.

3. Uzunlukları 120 cm ve 165 cm olan iki tahta parçası bir kesme makinesi ile santimetre cinsinden tam sayı olan eşit uzunluktaki parçalara ayrılacaktır.

Bu makine ile bir kesme işlemi 15 saniye sürdüğüne göre işin tamamı en az ne kadar zaman alacaktır?

- *Problem 3, "Denklem Kurma", "Matematik cümlesi Oluşturma" ve "Uygun olan İşlemi Seçme" stratejilerini kullanarak öğrencilerle birlikte çözülür.*

4. Barış Bey, işe geç kaldığı için evinin önünden taksiye biner, taksimetrenin açılış ücreti 1,5 TL'dir. Taksimetre her 100 metrede bir 20 kuruş yazar. Barış Bey, taksiciye 2 TL de bahşiş bırakarak toplam 10 TL verir. Barış Bey'in iş yeri evinden kaç km uzaklıktadır.

- *Problem 4, "Geriye dönerek çalışma", "Akıl yürütme" stratejilerini kullanarak öğrencilerle birlikte çözülür.*

5. Ayşenur, aşağıdaki örüntüyü devam ettirmek için sizden yardım istemektedir. Ona yardımcı olur musunuz?

AA
BBBB
CCCCCC
ÇÇÇÇÇÇÇÇ
???

g) F satırında kaç harf vardır?
h) Örüntünün hangi satırında 22 harf vardır?

- *Problem 5, “Örüntü Arama”, “Akıl Yürütme” stratejilerini kullanarak öğrencilerle birlikte çözülür.*

6. Aslı hemşire 9 günde bir, Elif hemşire ise 12 günde bir nöbet tutmaktadır. İkisi birlikte nöbet tuttuktan en az kaç gün sonra tekrar birlikte nöbet tutarlar?

Problem 6, “Bir Liste, Grafik ve Tablo Yapma” stratejisi kullanarak öğrencilerle birlikte çözülür.

K. TURKISH SUMMARY

ORTAOKUL ÖĞRENCİLERİNİN KELİME PROBLEMLERİNDE PROBLEM ÇÖZME ADIMLARINI KULLANIMLARI VE BU PROBLEMLERİN ÇÖZÜMLERİNDE KULLANDIKLARI STRATEJİLER

GİRİŞ

Birçok insan matematiğin hayatı zor ve dayanılmaz kılan konulardan biri olduğunu düşünmektedir (Arslan, Yavuz ve Deringol-Karataş, 2014). Oysa matematik, hayatı anlama ve sevme yollarından biridir ve diğer birçok durumda olduğu gibi bir şeyi sevmek, onu anlamayı gerektirir. Başka bir deyişle, bir şeyi anlayabildiğimizde sevebiliriz (Sertöz, 2011). Dahası, Hagaman (1964), öğretmenler tarafından da kendi deneyimlerine dayanarak bilindiği gibi, öğrencilerin en iyi öğrendiklerini anlamış olduklarında öğrendiklerini belirtiyor. Genel olarak anlamadığımız durumlara karşı olumsuz düşünce ve davranışlar sergileriz. Pek çok insan tam olarak anlamadığı için matematiğe karşı olumsuz tutumlara sahiptir. Öğrencinin matematikten hoşlanmamasının altında yatan en önemli nedenlerden biri, problem çözme becerileri ile ilgili konularda yaşadıkları özgüven eksikliği ile ilgilidir (Arslan, Yavuz ve Deringol-Karataş, 2014). Bu yüzden, matematiksel problem çözme okul müfredatında büyük önem taşır ve okulların problem çözme temelli öğretim gibi yenilikçi öğretim yöntemleriyle matematiği daha anlaşılır hale getirmesi gerekir. Günümüzde matematik eğitiminin temel amaçlarından biri matematik problemlerini çözme yeteneğinin geliştirilmesidir (Shiakalli ve Zacharos, 2014). Bu nedenle okul programları, öğrencilerin problem çözme becerilerine odaklanmaktadır (Zakaria, Haron ve Daud, 2010). Yaratıcı problem çözme, yaşamın çeşitli alanlarında başarılı olmak için önemlidir (NCTM, 2000). Bu sebeple birçok ülke problem çözme yöntemini daha etkili kullanmak için öğretim programlarını yeniledi. Bu, Türkiye'deki tüm ilkokul programları için de geçerlidir; problem çözme matematik eğitiminin daha önemli bir unsuru haline gelmiştir (Yıldızlar, 2001). Matematik eğitiminde problem çözme vurgusu arttıkça problem çözme süreçleri ve stratejileri üzerinde çalışmak daha da önem kazanmaktadır (Gür ve Hangül, 2015). Eleştirel ve yaratıcı düşüncenin gelişimi, stratejilerin seçimi ve kullanımı, özgün yaklaşım ve yöntemlerin iyileştirilmesi ve kullanılması ve bilginin uyarlanması ile gerçek hayattaki kelime problemlerinin farklı ortamlarda uygulanması uygun öğrenme ve öğretme ortamlarını sağlamak ile mümkündür (NCTM, 1991; Brown, 2001). Bu nedenle, problem çözme stratejilerini anlamaya ve kullanmaya büyük bir ihtiyaç vardır. Tüm dünyada bu konu ile ilgili başarılı pek çok çalışma yapılsa da ortaokul öğrencilerinin problem çözme adımlarını nasıl kullandıklarını ve farklı sınıf seviyelerindeki ortaokul öğrencileri tarafından hangi stratejilerin daha çok tercih edildiğini analiz etmeye yönelik bir çalışmaya halen ihtiyaç duyulmaktadır. Bu çalışmanın amacı ortaokul öğrencilerinin problem çözme adımlarını ne derece kullandıkları ve problem çözme sorularının çözümünde hangi stratejileri kullandıklarını incelemektir. Bu çalışmanın sonuçları, öğretmenlerin her sınıf düzeyinde hangi problem çözme stratejisinin üzerinde durulması gerektiğine ilişkin sorularına ışık tutabilir. Aslında, araştırma öğretmenlere öğrencilerin hangi tür stratejileri tercih ettikleri ve problemleri çözmek için hangi stratejileri kullandıkları konusunda yararlı bilgiler sağlayabilir. Ayrıca, bu çalışma Türkiye'deki matematik müfredatı için iyi uygulamalar sunabilir. Öğrencilerin problem çözme ve analitik düşünme yeteneklerini geliştirmek için, MEB problem sorularını daha iyi sınıf ortamları oluşturmak için öğrencilerin ihtiyaçlarına ve seviyelerine göre okul müfredatına dahil edebilir.

Araştırma Soruları

Bu çalışmanın amacı ortaokul öğrencilerinin problem çözme adımlarını ne derece kullandıkları ve problem çözme sorularının çözümünde hangi stratejileri kullandıklarını incelemektir.

Bu kapsamda aşağıdaki araştırma soruları cevaplanacaktır:

1. Öğrencilerin belirlenen çerçeveye dayanarak problem çözme adımlarını ne derece kullanıyor?
2. Öğrenciler kelime problemlerinin çözümünde hangi stratejileri kullanıyorlar?

LİTERATÜR TARAMASI

Problemin ne anlama geldiği hakkında birçok farklı tanım vardır. Hepimizin bir problemi var ama bir kişinin problemi olan bir durum başka bir kişi için problem olmayabilir. İlk olarak, bazı problem tanımlarında “durumların belirsizliği” ifadesi vurgulanmaktadır. Shergıll'a göre (2012) “problem, birinin şu anki durumu ile istediği hedef durumu arasında, diğerinden elde etmenin net bir yolu olmadan bir belirsizliğin ve tutarsızlığın olduğu durumdur” (s.296). Buna ek olarak, Booker ve Bond (2008) problemi, acil ve açık bir çözümü olmayan bir görev veya durum olarak tanımlamaktadır. Bunlara paralel olarak, Posamentier ve Krulik, problemi “bir kişinin karşı karşıya kaldığı, çözüm gerektiren ve çözüme giden yolun derhal olmadığı (araştırmacı tarafından altı çizilen) bilinmeyen bir durum” olarak tanımlamaktadır (1998, s.1). Bazı tanımlarda ise problemin tanımımı içim, belirsizliğin bir durumun “problem” olarak adlandırılabilmesi için tek özellik olmadığı belirtilmektedir. Aynı zamanda, bir durumun problem olarak tanımlanabilmesi için öğrenci bilinmeyen durumları çözülmesi gereken bir soru olarak kabul etmeli ve çözmeye ihtiyaç duymalıdır (Herlihy, 1964). Örneğin, özellikle yetenekli bir öğrenci için, her matematik dersinde, sadece birkaç "problem" gerçekten problemler özelliği taşımaktadır. Bu tanımlardan yola çıkarak, algoritmik matematik problemlerinden günlük hayatımızda karşılaştığımız sosyal problemlere kadar her bir problemin, değişebilen bir durum içerdiği, öğrencilere meydan okuduğu ve çözüme değer olduğu takdirde, problem olarak adlandırılabileceği sonucuna varabiliriz.

Kişisel ilgi ve felsefe ile ilgili olduğu için “Problem çözme nedir?” Sorusuna çeşitli cevaplar vardır (Mamona-Downs & Downs, 2005). NCTM'ye (2000) göre, problem çözme “çözüm yönteminin önceden bilinmediği bir işe girmedir, bu nedenle, öğrenciler bir çözüm bulmak için kendi bilgileri üzerine çekmelidirler ve bu süreç boyunca sık sık yeni matematiksel anlayış geliştirmelidir” (s.52). Bu nedenle, matematik öğrenen öğrenciler, çeşitli problemleri nasıl araştırıp keşfedeceklerini de öğrenirler. Shergill (2012), problem çözmenin, hedefe ulaşmak için bir strateji belirleyerek çözüm planı hazırlayıp zorluğun üstesinden gelmek için uygulamak olduğunu belirtmektedir. Ek olarak, bu tanımlamaya çok benzer şekilde, problem çözme Martinez (1998) tarafından, çözüm yolunun belirsiz olduğu bir hedefe doğru hareket etme süreci olarak tanımlanmaktadır. Problem çözme temelli derslerde öğretmenlerin öğrencileri problem çözme sürecinde yönlendirmeleri gerekir (Van de Walle, 2010). Polya, Bransford ve Stein ve Van De Walle (2010) Polya’nın “Nasıl çözülür” adlı ünlü kitabında belirtilen dört problem çözme adımını analiz etti:

1. Problemi anlama
2. Çözüm planı yapma
3. Planı uygulama
4. Çözümü kontrol etme

Problemi anlamak, problemi dikkatlice okuduktan sonra neyin verildiğini ve problemde neler bulunacağının farkında olmaktır. Mamona-Downs ve Downs (2005) tarafından matematik metinlerinin okunmasının problem çözmede önemli bir unsur olduğu belirtilmektedir. Sorunun anlaşılmasından sonra verilenler düzenlenir ve sorunu çözmek için uygun bir strateji seçilerek plan yapılır. Ardından süreç, planın uygulanmasıyla (işlemlerin yapılmasıyla) devam eder. Ve son olarak, dördüncü adımda, doğruluğunu sağlamak için çözüm kontrol edilir.

Bilgiyi yorumlama, metodik olarak planlama ve çalışma, çözümleri kontrol etme ve alternatif stratejileri denemek gibi beceriler problem çözmede gereklidir (Muir, Beswick ve Williamson, 2008). Ayrıca problem çözmede birçok problem çözme stratejileri var. Posamentier ve Krulik'in belirttiği gibi, tüm stratejileri kullanmak ve bir sorunu çözmemek nadirdir. Ayrıca, bir problemin çözümünde tek bir stratejinin kullanılması da eşit derecede nadirdir (1998). Fadlelmula (2010) etkili bir problem çözme sürecinin, problemin bileşenlerini tanımlamayı, hangi bilginin eksik olduğunu anlama, problemin çözümü için etkili bir strateji geliştirme, seçilen stratejiyi uygulama, alternatif bir stratejinin ne zaman ve nasıl deneneceğini bilmeyi ve alınan sonuçların ve alınan kararların uygun olup olmadığının değerlendirilmesi gerektirdiğini belirtmektedir. Martinez (1998), geleceğe yönelik bilişsel pasaport olarak problem çözme yeteneğini düşünüyor. Başka bir deyişle, problem çözme yeteneğine olan ihtiyaç, yaşamımızda, işlerimizde, günlük yaşamımızda bile önemli ölçüde artmıştır, çünkü yaşadığımız dünya gün geçtikçe daha karmaşık hale gelmektedir. Bu yüzden matematik eğitiminin temel amaçlarından biri öğrencilerin problem çözme becerilerini geliştirmektir.

YÖNTEM

Bu çalışmada araştırma sorularını cevaplamak için temel nitel araştırma tasarımı kullanılmıştır. Bu kapsamda öğrencilerden elde edilen yazılı belgelerin içerik analizi yapılmıştır. Ayrıca, verileri tanımlamak için tanımlayıcı istatistikler kullanılmıştır.

Katılımcılar

Çalışma örneklemini Konya'da (Türkiye) 116 (29 beşinci sınıf öğrencisi, 29 altıncı sınıf öğrencisi, 25 yedinci sınıf öğrencisi ve 33 sekizinci sınıf öğrencisi) oluşturmaktadır. Öğrenciler Konya'da yaşıyorlar ve Konya'da bir devlet ortaokuluna gidiyorlar. Öğrencilerin yaşı 11 ile 13 arasında değişmektedir.

Ayrıca, bu çalışmada örnekleme yöntemi olarak uygun örnekleme kullanılmıştır. Bu çalışmada, araştırmacının okula ve öğrencilere aşina olması ve okul idaresinden gereken izinlere kolay erişilebilirliği nedeniyle uygun örnekleme seçilmiştir. Araştırmacının okula ve öğrencilere aşina olması nedeniyle, araştırmacının topladığı veriler araştırmacının daha doğru bir şekilde yorumlanmasıyla araştırmanın güvenilirliğini arttırmaktadır.

Veri Toplama Araçları

Her sınıf düzeyinde, düşünme biçimini ve problem çözme sorularında kullanılan stratejileri ortaya çıkarmak için farklı Problem Çözme Başarı testleri kullanılmıştır. Bu nedenle, farklı sınıf seviyelerindeki öğrencilerden elde edilen sonuçlar karşılaştırılmamış ve bu öğrencilerin başarı puanları arasındaki ilişki incelenmemiştir. Test araştırmacı tarafından geliştirilmiştir. Katılımcıların sınıf seviyeleri matematik problemlerini seçerken araştırmacı tarafından göz önünde bulunduruldu. Ayrıca, problemler bugüne kadar 5., 6., 7. ve 8. sınıf öğrencilerinin kapsadığı matematik konuları arasından seçildi. Daha önce de belirtildiği gibi, seçilen konular genellikle Kümeler ve Tam Sayılar, Kesirler ve Ondalık Sayılar İşlemini kapsamaktadır. Her bir Problem Çözme Başarı Testi altı kelime problemi içermektedir. Problem Çözme Başarı Testlerindeki Problemler TIMSS, PISA ve matematik ders kitaplarından uyarlanmıştır.

Veri Toplama Süreci

Başlangıçta, okuldan, ODTÜ Etik Kurulundan ve diğer resmi komitelerden gerekli izinler alınmıştır. Gerekli izinleri aldıktan sonra pilot çalışma ve ana çalışma yapılmıştır. İlk olarak, pilot çalışma 2016-2017 öğretim yılının bahar dönemi sonunda bir hafta boyunca uygulanmıştır. Pilot çalışma yapıldıktan sonra beş ders saati öğrencilere problem çözme temelli bir kurs verildi ve elde edilen sonuçların değerlendirilmesinden sonra gerekli düzeltmeler ve revizyonlar yapıldı. Bu beş saatlik probleme dayalı öğretimin asıl amacı, öğrencilerin problem çözme süreçlerini (Polya'nın dört aşaması) güçlendirmek ve problem çözme stratejilerini gözden geçirmek ve hatırlatmaktı. Bu beş saatlik probleme dayalı öğretim için, tüm sınıf seviyeleri için ders planları hazırlandı. Veriler Konya'da bir devlet okulunun 5., 6., 7. ve 8. sınıf öğrencilerinden 2017-2018 akademik yılının güz döneminde toplanmıştır. Bu süreçte araştırmacı tarafından 116 ilköğretim öğrencisine problem çözme başarı testi kendi sınıflarında uygulanmıştır.

Veri Analizi

Öğrencilerin problem çözme başarı testindeki problemlere verdiği cevaplar iki adımda analiz edilmiştir. İlk olarak, verileri tanımlamak için tanımlayıcı istatistikler kullanılmıştır. Tanımlayıcı istatistik (ortalamalar, standart sapmalar), bağımsız "sınıf seviyesi" değişkeni için hesaplandı. Problem Çözme Başarı Testleri, ilgili literatür taramasından sonra araştırmacı ve iki matematik öğretmeni tarafından geliştirilen bir değerlendirme listesi kullanılarak değerlendirildi. Bir öğrenci tamamen doğru bir çözüm verdi ise, 3 puan verildi. Hesaplamada sadece küçük hatalarla neredeyse doğru cevaplı çözümlere 2 puan verildi. Sorunun bir bölümünü çözen cevaplara bir puan verildi. Verilen cevapların tamamen yanlış olduğu ve çözüm bulunmadığı durumlarda sıfır puan verildi.

Ayrıca araştırma sorularına cevap vermek için temel nitel araştırma yöntemi kullanılmıştır. Öğrencilerin problem çözme süreçlerinde problem çözme adımlarını kullanmalarını ve öğrencilerin problem çözme stratejilerini kullanma eğilimini araştırmak için öğrencilerin çalışma sayfası içerik analizi yapılarak ayrıntılı olarak incelenmiştir. Daha sonra içerik analizi için her bir madde, her sınıfın tüm seviyelerden gelen tepkisi dikkate alınarak ve ortaokul öğrencilerinin bilişsel süreçlerini problem çözme adımlarının kullanımı açısından belirlemek için içerik analiz aracı olarak beş kategoriden oluşan teorik çerçeve oluşturulmuştur.

İlk kategori problemi anlama adımının incelendiği kategori "SUP" kategorisidir ve beş alt kategori içermektedir.

- SUP1: Öğrenci problemi kendi sözcükleriyle tekrar yazar ve maddede verilen sözlü bilgiyi anlamak için problemde ne verildiğini ve ne istendiğini doğru bir şekilde yazar.

- SUP2: Öğrenci, problem ifadesini kendi cümleleriyle tekrar yazamaz ve maddede sunulan sözlü bilgiyi anlayamaz, fakat problemde ne verildiğini ve ne istendiğini yazar.
- SUP3: Öğrenci, problemde ne istendiğini belirleyemez fakat problemde ne verildiğini yazabilir.
- SUP4: Öğrenci, maddede sunulan sözlü bilgiyi anlamadı. Öğrenci, neyin verildiğini ve neyin istendiğini belirleme ve problem ifadesini kendi sözcükleriyle tekrar yazmada başarısız olmuştur.
- SUP5: Öğrenci problemi kendi sözcükleriyle yeniden yazar, fakat problemde ne verildiğini ve ne istendiğini açıkça söyleyemez.

İkinci kategori plan yapma adımının incelendiği "MPO" kategorisidir ve dört alt kategori içermektedir.

- MPO1: Öğrenci, istenenlere ulaşmak için verilenleri düzenleyerek uygun bir plan yapar ve sorunu çözmek için uygun bir strateji seçer.
- MPO2: Öğrenci planını açıkça belirtmiyor.
- MPO3: Öğrenci, çözüme ulaşmak için verilenleri organize edemez ve ilişkilendiremez.
- MPO4: Öğrenci ayrıntılı bir plan yapamıyor - verilenler hakkında bir organizasyon yok, strateji kullanmıyor.

Üçüncü kategori planı uygulama adımının incelendiği "IP" kategorisidir ve dört alt kategoriden oluşmaktadır.

- IP1: Öğrenci problemi başarıyla çözer.
- IP2: Öğrenci planın uygulanmasında başarılı ancak çözüm planı doğru çözüm için uygun değil.
- IP3: Öğrenci, çözüm planını gerçekleştiremez.
- IP4: Öğrenci, problemi çözmek için gerekli bilgiyi sahip değil veya hatırlayamıyor.

Dördüncü adım ise çözümün kontrolü adımının incelendiği "CAS" kategorisidir. Bu kategori beş alt kategoriden oluşmaktadır.

- CAS1: Öğrenci, çözümünün neden doğru olduğu konusunda bir açıklama sunar.
- CAS2: Öğrenci sadece net bir açıklama yapmadan veya bir açıklama yapmadan “eminim” şeklinde belirtir.
- CAS3: Öğrencinin çözümünün doğru olup olmadığına dair kontrolü yoktur.
- CAS4: Öğrenci, cevabının doğru olup olmadığından emin olmak için farklı stratejiler kullanarak sorunu çözer.
- CAS5: Öğrenci, çözüm planının uygulanmasında yürüttüğü çözümü yeniden ifade eder.

Son kategori ise öğrencinin bütün problem adımlarını ne derece kullandığının değerlendirildiği “PTA” kategorisidir ve iki alt kategori içermektedir.

- PTA1: Öğrenci, beş süreçte başarı ile ilerlediğini gösterdi.
- PTA2: Öğrenci, beş işlemden en az birini başarıyla ilerleyemedi.

Varsayımlar ve sınırlılıklar

Çalışmada bazı sınırlamalar var. Bunlardan ilki analiz için sadece yazılı kaynaklara sahip olmaktır. Araştırmada sadece yazılı kaynaklar bulunabildiğinden, öğrencilerin ne düşündükleri ve problemleri çözmek için stratejileri nasıl seçtikleri ve kullandıkları tam olarak anlaşılmamıştır ve tespit edilememiştir. Öğrencilerin yazdıkları ifadelerin bazıları belirsizdi ve bu nedenle öğrencilerin açıklamalarını yorumlamak zordu. Ayrıca, özellikle bazı bölümlerde öğrenciler belirledikleri fikirleri tam olarak ifade edemedirler.

Diğer bir sınırlama, uygulama sürecinin sınırlı olmasıdır. İlk olarak, öğrencilere stratejileri hatırlaması ve öğrenmesi için hazırlanan dersler yeterli değildi. Bu nedenle öğrenciler problem çözme stratejilerini tam olarak anlamadılar. Beş saatlik problem çözme dersindeki problemler ve problem çözme başarı testindeki bazı problemler büyük benzerlik göstermektedir. Bu, bu çalışmada elde edilen bulguları kısmen etkilemiş olabilir. Ayrıca, problem çözme başarı testinin uygulanması çok sınırlıydı.

BULGULAR

Problem Çözme Başarı Puanlarının Tanımlayıcı İstatistikleri

Tablo 1, her not seviyesi için ortalama, standart sapma, minimum-maksimum puanlar, başarı puanlarının çarpıklık ve kurtosis değerlerini ayrı ayrı özetlemektedir.

Tablo 1 Problem çözme başarı testi puanına göre betimleyici istatistikler

	5^{th} Sınıf	6^{th} Sınıf	7^{th} Sınıf	8^{th} Sınıf
N	29	29	25	33
Ortalama	40,66	38,83	35,84	40,30
Std. Sapma	6,96	11,59	13,29	11,77
Minimum	28	16	10	12
Maximum	54	54	54	54
Çarpıklık	-0,16	-0,62	-0,61	-0,57
Kurtosis	-0,79	-0,83	-0,96	-0,36

Tablo 1'de verildiği gibi, problem çözme başarı testlerinden alınan her seviyesindeki maksimum puanlar aynıdır (en fazla = 54), ancak minimum puanlar sınıf seviyesinden sınıf seviyesine değişmektedir. Bu nedenle, testten alınan minimum ve maksimum puanlar arasındaki en büyük fark, testten elde edilen minimum puan 10 olduğu için 7. sınıflarda görülmüştür. 5. sınıf problem çözme başarı puanına bakıldığında, maksimum puan 54, minimum puan 28'dir, ortalama olarak 40.66 (SD = 6.96). 6. sınıf problem çözme başarı puanına dayanarak maksimum puan 54, minimum puan 16, ortalama 38.83 (SD = 11.59) şeklindedir. 7. sınıf başarı puanları incelendiğinde, en yüksek puan 54, en düşük puan 10 ve ortalamanın 35.84 (SD = 13.29) olduğu görülmektedir. 8. sınıf başarı notuna gelince, en yüksek puan 54, en düşük puan 12, ortalama 40.30'dur (SD = 11.77).

Temel Nitel Analiz: Öğrencilerin Problem Çözme Adımlarını ve Stratejileri Kullanımı

Beşinci Sınıf Sonuçları

Beşinci sınıf öğrencileri, tüm problemlerde problem ifadesinin anlaşılması aşamasında başarılı görünmektedir. Ancak, kesirler probleminde uygun bir çözüm planı oluşturamadılar. Bunun temel nedeni, öğrencilerin kesirlerle ilgili kavram yanılgıları olabilir. Ayrıca, beşinci sınıf öğrencileri sayılarla ilgili olan ikinci, dördüncü ve beşinci problemlerde zorlandıkları görülmektedir. Öğrencilerin bu problemlerdeki çözümleri incelendiğinde, öğrencilerin genel olarak problem cümlesinde verilen değerleri doğru şekilde yorumlayamadıkları ve anlamlandıramadıkları görülmüştür. Öğrencilerin problem çözmede stratejilerini kullandığımızda, kayda değer sayıda öğrenci aritmetik stratejiyi kullanırken, sadece 3 öğrenci kesirli problemlerde çizim stratejisi kullanmayı tercih etti.

Altıncı Sınıf Sonuçları

Beşinci sınıf öğrencilerinde olduğu gibi, altıncı sınıf öğrencileri de tüm problemlerde verilen ifadeyi anlama konusunda başarılı gözükmektedir. Ancak, genellikle ondalık gösterim problemlerinde ve kesir probleminde uygun bir çözüm planı yapmakta güçlük çektiler. Öğrencilerin bu problemde çoğunlukla başarısız olmasının nedeni, problemin hikayesinin uzun olması ve problemin birbiriyle bağlantılı birçok işlem gerektirmesi olabilir. Ayrıca, bu durum birçok öğrencinin problem durumunu, problemin hikayesini ve problem ifadesinde verilen değerleri anlamamasından kaynaklanıyor olabilir. Öğrencilerin matematiksel akıl yürütme yeteneklerini kullanamamaları başarısızlığın bir başka nedeni olabilir. Öğrencilerin problem çözme stratejilerini kullandığımızda, birçok öğrenci aritmetik stratejisini, sadece 3 öğrenci kesirli problemlerde çizim yapma stratejisini tercih etti.

Yedinci Sınıf Sonuçları

Yedinci sınıf öğrencileri, beşinci ve altıncı sınıf öğrencileri gibi, tüm problemlerin çözümünde problemi anlama adımında başarılı görünmektedir. Bununla birlikte, yedinci sınıf öğrencilerinin başarı oranı 1, 2 ve 3 numaralı problemlerde oldukça düşüktür. 11 öğrenci ikinci problem için uygun bir çözüm planı yapmış olsa da sadece iki öğrenci çözüm planını doğru şekilde uygulayabilmiştir. 7. sınıf öğrencilerine sorulan problem cümleleri daha uzun ve problemler verilen değerleri anlamak için biraz daha dikkat gerektirmektedir. Diğer taraftan, yedinci sınıf öğrencileri genel olarak tahmin ve kontrol, geriye dönük çalışma stratejisi, çizim stratejisi ve cebirsel strateji gibi diğer problem çözme stratejilerini daha başarılı ve yaygın kullanmışlardır.

Sekizinci Sınıf Sonuçları

Sekizinci sınıf öğrencisi, beşinci, altıncı ve yedinci sınıf öğrencileri gibi, tüm problemlerin çözümünde problemi anlamada zorluk çekmedi. Ancak, sekizinci sınıf öğrencileri çoğunlukla dördüncü problemi çözmede başarısız oldular. Bu problemi çözmede başarısız olan öğrencilerin problemi hiçbir şekilde anlamadıkları sonucuna varılabilir. Yeterli konu bilgisine sahip olmadıkları için değil, uzun problem cümlelerini anlayamadıkları için problemi çözemediler. Bu bağlamda, öğrencilerin metinleri ve kitapları okumaları ve genellikle anlamaları için teşvik edilmeleri gerekir; böylece öğrencilerin anlama becerilerini ve kelimelerini geliştirebilirler ve kendilerine verilen problem ifadesini anlamalarına yardımcı olurlar (Raoano, 2016).

TARTIŞMA

Araştırmanın sonuçları, öğrencilerin genel olarak her sınıfta uygulanan problem çözme başarı testlerinde başarılı olduklarını ortaya koymuştur. Ek olarak, bazı kelime problemlerinde, öğrenciler genel olarak aritmetik stratejiyi kullanmayı tercih etmelerine rağmen, farklı stratejiler kullanma yeteneklerini göstermişlerdir. Ayrıca, genel olarak, farklı sınıf seviyelerindeki öğrenciler problem çözme yeteneklerini başarıyla kullanabileceklerini göstermiştir. Dolayısıyla bu, problem çözme aşamasında tanımlanan problemlerin çözüm planının yapılmamasında çok önemli bir faktör olduğu gözükmektedir. Benzer şekilde, araştırma bulguları, öğrencilerin en önemli eksikliğinin, çözümlerinde gerçek dünya bilgi ve deneyimini kullanamadıklarını göstermektedir. Bu nedenle, gerçek hayattaki kelime problemlerini anlama ve çözmede daha çok zorluk çektiler. Problem ifadelerinin ortaya koydukları gerçek yaşam bağlamları ile problem çözümünde gerçekleştirdikleri işlemler arasındaki gerçek ilişkileri göz önünde bulundurmamışlardır. Çoğu öğrenci gerçek hayat problemlerini anlamlandıramamakta ve matematiksel akıl yürütme becerilerini çok iyi kullanamamaktadır.

Öğrencilerin yazılı çalışmalarından elde edilen sonuçlar incelendiğinde, öğrencilerin çoğunun aritmetik stratejiyi kullandığı ve farklı stratejilere çok az eğilim olduğu görülmüştür. Ancak, az da olsa bazı öğrencilerin her sınıf düzeyinde farklı problem çözme stratejileri kullanmaya yatkın oldukları görülmüştür. Örneğin, beşinci, altıncı, yedinci ve sekizinci sınıf öğrencileri çoğunlukla her problem için en az iki farklı strateji kullanabilmişlerdir. Her sınıf seviyesinden bazı öğrenciler, problemlerden birinde dört farklı strateji bile kullandılar. Bu nedenle, öğrencilerin esnek düşünceye sahip olduğu söylenebilir. Ortaokul öğrencileri stratejileri kavrayabilir ve benzer problemlerde kullanabilirler.

Uygulamalar ve Tavsiyeler

Öğrencilere, kelime problemlerini ve planlamayı uygulayabilecekleri olanaklar ve ortam sağlamak oldukça önemlidir. Problemlerin çözümü sırasında, öğretmenlerin sadece cevaplara değil, aynı zamanda öğrencilerin çözüm stratejileri sürecine de odaklanmaları gerekir. Ayrıca, öğretmenlerin öğrencilere doğru cevaba ulaşma konusundaki planlarını sormaları ve yazdıkları planı uygulayıp uygulayamadıklarını kontrol etmeleri gerekir. Bu şekilde öğrenciler problem çözmek için daha fazla fırsata sahip olduklarından, daha esnek matematiksel düşünme ve akıl yürütme geliştirebileceklerdir. Kelime problemi aktiviteleri sadece öğrencilere sınıf içi çalışma veya ev ödevi aktiviteleri olarak verilmemeli, aynı zamanda öğrencilere öğretilmeleri için ihtiyaç duydukları çözüme yönelik stratejiler de verilmelidir.

Sonuç olarak, öğrencilerin problem çözme becerileri kazanabilmesi ve bunları etkin bir şekilde kullanabilmesi için problem sorularına problem çözme stratejileri ile birlikte ağırlık verilmektedir. Ayrıca, problem çözme adımları ve problem çözme stratejileri öğretmenlerin problem çözmeyi öğretme görevini kolaylaştırmaktadır. Öğrencilerin problem çözme becerisi ve problem çözme yaklaşımı öğretmenin bilgi düzeyine göre şekilleneceğinden, öğrencilere problem çözmeyi öğreten öğretmenlerin iyi desteklenmesi gerekmektedir.

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