AN INVESTIGATION ON SEVENTH GRADE STUDENTS' UNDERSTANDING OF NEGATIVE INTEGERS VIA MATHEMATICS HISTORY-BASED MODEL-ELICITING ACTIVITIES

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Prof. Dr. Yaşar KONDAKÇI
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

Assoc. Prof. Dr. Elvan ŞAHİN
Head of Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

Assist. Prof. Dr. Şerife SEVİNÇ
Supervisor

Examining Committee Members

Assist. Prof. Dr. Zeynep Sonay AY (Hacettepe Uni., MSE) _____

Assist. Prof. Dr. Şerife SEVİNÇ (METU, MSE) _____

Assoc. Prof. Dr. Didem AKYÜZ (METU, MSE) _____

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name : BÜŞRA AY

Signature :

ABSTRACT

AN INVESTIGATION OF SEVENTH GRADE STUDENTS' UNDERSTANDING OF NEGATIVE INTEGERS VIA MATHEMATICS HISTORY-BASED MODEL-ELICITING ACTIVITIES

AY, Büşra
M.S., Department of Elementary Science and Mathematics Education
Supervisor: Assist. Prof. Dr. Şerife SEVİNÇ

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The main purpose of the study was to investigate the role of mathematics history-based model-eliciting activities (MEAs) on seventh grade students' understanding of negative integers. The study was conducted in one public middle school in Istanbul, in the Fall semester of 2018-2019 academic year. A qualitative educational case study design was employed with 29 seventh grade students. A set of integer questions were implemented twice to the participant students as an initial and a follow-up assessment. After the initial assessment, data collection procedure was continued by implementing three mathematics history-based MEAs. The data sources for this study were written worksheets of each group from the implementation of mathematics history-based MEAs, initial and follow-up assessments of each student, video and audio records of the implementation of activities. The data were analyzed by content analysis. The findings of this study indicated that mathematics history-based MEAs helped students to better make sense of negative integers, improved students' understanding of negative integers and mathematical reasoning and justification capabilities in operations with integers. In the implementation process of activities, it was observed that most of the students are more productive and

concentrated while they are studying as groups on a real-life related mathematical model. With the light of these findings, it is suggested that integration of the mathematics history into modeling perspective could be considered as an alternative teaching method for different mathematics topics in different grade levels. This study also suggests that the mathematics history-based MEAs could also be used to improve professional development of mathematics teachers to understand the role of such activities on students' understanding and to enhance teachers' teaching portfolio.

Keywords: History of Mathematics, Model-Eliciting Activities, Mathematical Modeling, Negative Integers

ÖZ

MATEMATİK TARİHİ TABANLI MODELLEME ETKİNLİKLERİ İLE 7. SINIF ÖĞRENCİLERİNİN NEGATİF TAM SAYILAR KONUSUNDAKİ ANLAMALARININ İNCELENMESİ

AY, Büşra

Yüksek Lisans, İlköğretim Fen ve Matematik Eğitimi Alanları Eğitimi Bölümü
Tez Yöneticisi: Assist. Prof. Dr. Şerife SEVİNÇ

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Bu çalışmanın amacı, matematik tarihi tabanlı modelleme etkinlikleri ile yedinci sınıf öğrencilerin negatif tam sayılar konusuna ilişkin anlamalarını incelemektir. Çalışma, 2018-2019 eğitim öğretim yılının sonbahar döneminde, İstanbul ilinde bulunan bir devlet ortaokulunda yapılmıştır. Bu çalışmada nitel örnek olay araştırma yöntemi 29 yedinci sınıf öğrencisinin katılımı ile uygulanmıştır. Tam sayılar ile ilgili açık uçlu sorulardan oluşan bir değerlendirme aracı, ilk ve son değerlendirme amacıyla öğrencilere iki kez uygulanmıştır. İlk değerlendirme sonrası, tam sayılar konusu, araştırmacı tarafından hazırlanan üç adet matematik tarihi tabanlı modelleme etkinlikleri kullanılarak işlenmiştir. Bu çalışmanın verileri, matematik tarihi tabanlı modelleme etkinlikleri sürecindeki öğrencilerin yazılı çalışmaları, her bir öğrencinin ilk ve son değerlendirmede sorulan açık uçlu sorulara verdikleri yazılı cevaplar, aktivitelerin uygulanması sırasında alınan video ve ses kayıtları ile toplanmıştır. Bu çalışmada toplanan veriler nitel içerik analizi yöntemi ile analiz edilmiştir. Çalışmanın bulgularına göre, matematik tarihi tabanlı modelleme etkinlikleri ile yedinci sınıf öğrencilerin negatif tam sayılar konusundaki anlamaları ve negatif tam

sayıları anlamlandırmaları gelişme göstermiştir. Matematik tarihi tabanlı modelleme etkinliklerinin uygulanması sürecinde, öğrencilerin gerçek hayat durumları ile ilgili modelleme etkinlikleri üzerinde grup olarak çalışırken daha üretken ve derse karşı daha ilgili oldukları gözlemlenmiştir. Çalışmanın bulguları doğrultusunda, matematik tarihi tabanlı modelleme etkinliklerinin bir öğretim yöntemi olarak farklı matematik konularının farklı düzeyde öğrencilere öğretilmesi sürecinde kullanılması önerilir. Ayrıca, matematik tarihi tabanlı modelleme etkinliklerinin öğrencilerin konuyu anlamalarındaki rolünü anlamaları için ve öğretim yöntemlerini geliştirmeleri için matematik öğretmeni eğitiminde kullanılması önerilir.
Anahtar kelimeler: Matematik Tarihi, Matematiksel Modelleme, Negatif Tam Sayılar

To Mustafa Kemal ATATÜRK

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LIST OF ABBREVIATIONS

MEA: Model Eliciting Activity

MoNE: Ministry of National Education

NCTM: National Council of Teachers of Mathematics

CHAPTER 1

INTRODUCTION

1.1. Introduction

Negative integers have always been an interesting topic specifically from the point of view of didactics of mathematics (Thomaidis, 1993). The necessity of negative integers and human need for negative integers come into view both in the everyday world of measurement and in the mathematical world of number (Galbraith, 1974). Researches have shown that students learn operational procedures with integers to solve mathematics problems, but they do not understand and make sense of negative integers (Bolyard, 2005; Ferguson, 1993; Lyte, 1994; Shore, 2005; Steiner, 2009; Wilkins, 1996).

Students count objects while learning about numbers and they adjust with concrete meaning of numbers and real-life. On the other hand, working with negative integers and focusing on operational procedures lead to contemplating without focusing on concrete meaning of situations in real-life problems (Gallardo, 2002). As Goldin (2003) pointed out that people's understanding and employing ideas are affected by representation of mathematical ideas. Understanding of a topic and reasoning in mathematics are related to concrete experiences, observations, making connections with different situations in real life (English, 1997). The ability of representing different mathematical concepts and accommodating between these different representations improve students' understanding (NCTM, 2000). Concrete experiences and representations contribute to students' understanding and development of well-grounded mathematical ideas (Stein & Bovalino, 2001). With this object in mind, history of negative integers which were integrated through

Model-Eliciting Activities (MEAs) and conjectured to provide students with a deeper understanding and making sense of negative integers.

The integration of history of mathematics into mathematics classes has been investigated for more than a century (Fried, 2001). There are several ways to include historical origins of mathematical concepts in teaching such as using ancient problems and texts, giving historical information about the topic, role playing about the historical development of the topic with the help of primary or secondary mathematics sources (Tzanakis & Arcavi, 2000).

Reasons and integration methods for the use of history of mathematics in mathematics education were analyzed and mainly distinguished by Jankvist (2009) in terms of reasons (the whys) and integration ways (the hows) of history of mathematics in mathematics teaching and learning. The two main reasons of integration of history of mathematics are (i) to assist mathematics instruction and (ii) to learn the history of subject (Jankvist, 2009). The first reason focuses on improving students' understanding in terms of cognitive and affective aspects of mathematics learning with the help of history of mathematics. The second addresses that history of mathematics encourages students to considering about the evolution of mathematics and role of humanity on the development of mathematics (Jankvist, 2009).

National Council of Teachers of Mathematics (NCTM) pointed out that mathematics is affected by different cultures and inherited to humanity, and students should be allowed to notice and perceive worldwide human effect on the field of mathematics (NCTM, 2000). With this in mind, Jankvist (2009) stated three basic approaches to include history of mathematics in mathematics education: (i) the modules refer to the integration of history of mathematics into a range of mathematics lessons related to topic, (ii) the history based approach in which mathematics lessons are fully arranged taking the history and evolution of mathematics into account, (iii) the illumination refers to include some historical facts and information in mathematics lessons.

The place of history of mathematics in the formal textbooks of mathematics lesson mostly appear as the illumination approach of the Jankvist (2009) and contains solely some limited historical information and direct historical facts about mathematical concepts rather than learning activities (Alpaslan, 2011). For this reason, in this study the modules approach has been taken into consideration since mathematics history-based MEAs foster students to make criticism and comments on topic while experiencing a deep understanding of concept. The modules approach supports both affective and cognitive development of students while the illumination approach mostly serves for affective purposes with pure historical information. Therefore, in the current study, history of negative integers were integrated into mathematics lessons through MEAs.

When the mathematical concepts appear to be disconnected from real-life, students tend to have difficulties in understanding mathematical concepts, and in this vein the integration of history of mathematics enables students to understand the need for the concept (Gulikers & Blom, 2001). Likewise, a distinctive feature of modeling is the emphasis on MEA(s) which are implementations of real-life problem-solving examples by eliciting students' assessments on understanding (Lesh, Hoover, Hole, Kelly, & Post, 2000). The models have been implemented in mathematics education to analyze the nature and progress of problem solvers' cognitive development throughout a wide range of study areas (Lesh, 2006). It is important in modeling with problem solving that students should improve their understanding through real-life mathematics problems (Bonotto, 2007). The MEAs resemble to real life situations in which students make meaningful mathematical explanations (Doerr & Lesh, 2003). In this present study, MEAs have been prepared taking the situations from the history of mathematics into consideration. I, as the researcher of this study, have put forward a qualitative educational case study based on mathematics history-based MEAs to improve students' understanding of negative integers through real-life mathematics problems.

1.2. Statement of the Purpose and the Research Question

The main purpose of this study was to investigate the role of mathematics history-based model-eliciting activities (MEAs) on seventh grade students' understanding of negative integers. Specifically, this research was conducted to answer the following research questions:

1. What understandings do 7th grade students develop negative integers as they engage in mathematics history-based model-eliciting activities in small groups?

2. How do 7th grade students' individual understandings change about negative integers after their engagement in mathematics history-based model-eliciting activities?

To serve this aim, mathematics history-based MEAs were prepared and implemented based on Turkish Middle Grades Mathematics Teaching Program (MoNE, 2018). The initial and follow-up assessment questions were implemented to investigate the role of mathematics history-based MEAs on students' understanding and making sense of negative integers. The responses of students both in the mathematics history-based MEAs and in assessment questions were analyzed deeply.

1.3. Significance of the Study

There are a number of researches conducted in Turkey addressing the problems that students encountered while solving questions involving operations with integers (Altun, 2006; Aydın Ünal & İpek, 2009; Durmaz & Avcı, 2011; Hativa & Cohen, 1995; Işıksal-Bostan, 2009; Yenilmez & Bağdat, 2014). The students encountered challenges specifically on the concept of integers and operations with integers. The main reason of students' having difficulties in understanding integers is students' not having sufficient opportunity to notice and perceive the need for negative integers (Altun, 2006). On the other hand, the related research is limited in the accessible

literature. There is no study particularly involved mathematics history-based model-eliciting activities and their roles on students' understanding of negative integers. In the present study, the history of negative integers were integrated into teaching process with the help of model-eliciting activities.

In the accessible literature, there were many studies that investigated the role of history of mathematics integration on teacher education and prospective mathematics teachers (Alpaslan, 2011; Biber, İspir, & Ay, 2015; Clark, 2012; Fenaroli, Furinghetti, & Somaglia 2014; Furinghetti, 2007; Philippou & Christou, 1998). Moreover, recent studies have focused on affective aspects and investigated the effects of history of mathematics on students' motivation, self-efficacy, attitudes towards mathematics and academic achievement (Albayrak, 2011; Bayam, 2012; Ersoy 2015). There are quantitative and quasi-experimental design studies investigated different grade level students in Turkey and concluded that using history of mathematics in mathematics lessons have increased the academic achievement (Bayam, 2012; Ersoy, 2015; İdikut, 2007; Özcan, 2014). On the other hand, a focus on the role of mathematics history in terms of cognitive aspects such as understanding and making sense of a content is rare in the accessible literature. Besides, there is no study in the literature bringing mathematics history and model-eliciting activities together. For this reason, the present study is expected to contribute to the literature that focused on the use of mathematics history and modeling in mathematics education via mathematics history-based model-eliciting activities.

Furthermore, students begin to learn numbers with counting, and make connections between each number and object. When they encounter with negative integers, they no longer make connections with objects, and in that case, models provide students to interpret and understand negative integers (Steiner, 2009). The most common two models used in teaching of integers are number line and neutralization models. The "number line model does not have any compelling inner logic, instead it assumes familiarity with underlying representational conventions, which are to some extent

arbitrary" (Ernest, 1985, p. 418). In the neutralization model, when students work with two- color counters, they get confused about the representation and meaning of zero while they observe and perceive some number of counters (Steiner, 2009). Students need to experience both the number line model and the neutralization model to understand integers since the neutralization model focuses on cardinal understanding of integers while the number line model emphasizes ordinal understanding of integers (Van de Walle, 2004). Besides, these two models are lacking in terms of letting students to experience the real meaning of an integer since they do not have necessary relations with real life, which results in students' not differentiating between "negativeness" and "positiveness" of an everyday life situation (Bell, 1983; Hackbarth, 2000; Werner, 1973). Therefore, model-eliciting activities are useful teaching materials to help students making connection with school learnings and everyday contexts. With these studies in mind, the present study has aimed to investigate mathematics history-based MEAs on students' understanding of negative integers by combining real-life situations in history and presenting uses of negative integers in daily life through model-eliciting activities.

1.4. Definitions of Important Terms

The term *history* is defined as "the whole series of past events connected with a particular person or thing" and *mathematics* as "the abstract science of number, quantity, and space, either as abstract concepts (pure mathematics), or as applied to other disciplines such as physics and engineering (applied mathematics)" in the Oxford Dictionaries (2019a, 2019b). With these definitions in mind, *history of mathematics* is defined as "the linked developments from the very past to the modern times related to the scientific branch of numbers, quantities, and space with its pure and applied versions" (Alpaslan, 2011, p.14).

Tzanakis & Arcavi (2000) indicated three main ways for the aim of *the use of history of mathematics in mathematics education* as an alternative mathematics teaching approach: (i) "learning history: to provide direct historical information" (p.208)

refers to aim of learning history, (ii) "learning mathematical topics: to implement a teaching approach inspired by history" (p.208) refers to aim of learning mathematics and (iii) "developing deeper awareness: to focus on mathematics as a discipline and the cultural and social context in which it has been evolving" (p.208) emphasizing evolutionary and epistemological characteristics of mathematics in which related with other disciplines and affected by various cultures and humans.

In this study the term *model* refers to mathematical models that are built, defined, emphasized mathematically significant products, processes and mathematical reasoning implemented by the 7th grade level students (Doerr & Lesh, 2003). A distinctive feature of modeling appears as *model-eliciting activities* "which so called because the products that students produce go beyond short answers to narrowly specified questions involving sharable, manipulatable, modifiable, and reusable models for constructing, describing, explaining, manipulating, predicting, or controlling mathematically significant simulations of real life" (Doerr & Lesh, 2003, p.3).

1.5. My Motivation to Conduct the Study

When I was an undergraduate student and a prospective mathematics teacher, I had a chance to be a voluntary participant for the PhD dissertation of Mustafa Alpaslan who was a teaching assistant in Middle East Technical University studying on the use of history of mathematics in mathematics education. I got inspired by his study which gave me an opportunity to improve myself on history of mathematics literature and to experience the use history of mathematics in mathematics education during my teaching practices. After I used history of mathematics in a real classroom environment and observed the effect of the activities on students' motivation and attitude towards mathematics class, I decided to study on the use of history of mathematics in mathematics teaching during my graduate education. My implementation of history of mathematics activities in a real classroom environment made me curious about the role of history of mathematics on the students'

understanding. I decided to focus on the cognitive aspects of using history of mathematics as a mathematics teaching method. Afterwards, with the encouragement of my supervisor, I decided to investigate by means of this study whether students' understanding of negative integers could be improved via mathematics history-based model-eliciting activities.

CHAPTER 2

THEORETICAL FRAMEWORK

The main purpose of this study was to investigate the role of mathematics history-based MEAs on seventh grade students' understanding of negative integers. For this reason, studies investigating history of mathematics in mathematics education and the integration of mathematics history in teaching particularly in Turkey have been reviewed. Another face of this study is models and modeling perspective based on which mathematics history-based MEAs were prepared. Lastly, studies investigating students' understanding of integers have been reviewed.

2.1. History of Mathematics in Mathematics Education

The role of mathematics throughout the history of science is crucial (Fauvel, 1991). As a result of that significant role, integrating history of mathematics into teaching process provides students with a meaningful learning of mathematics. Integration of history of mathematics into mathematics teaching as a teaching strategy is not a new idea. History of mathematics has been accepted as a new discipline by the end of 1900s (Furinghetti & Radford, 2002), and many studies have been carried out that field considering different aspects up. Since then society has changed, and lots of innovations have emerged in technology but the idea of the integration of history of mathematics into mathematics education in schools is still on the agenda of many researchers (Fenaroli, Furinghetti, & Somaglia, 2014).

The initial purpose of integration of history of mathematics was to contribute to teachers' own mathematical knowledge with a different pedagogical orientation (Furinghetti, 2004). Thus, many studies have investigated the role of history of mathematics integration on teacher education and prospective teachers (Clark, 2012; Fenaroli, Furinghetti & Somaglia 2014; Furinghetti, 2007; Philippou & Christou, 1998). A considerable number of studies has focused on affective aspects and

investigated the effects of history of mathematics on students' motivation, self-efficacy, attitudes towards mathematics and academic achievement (Bayam 2012; Ersoy 2015; Albayrak 2011). However, a focus on the contribution of using mathematics history in mathematics courses in terms of cognitive aspects such as students' understanding and making sense of a topic is rare. It is reported that some problems regarding integration of history of mathematics into mathematics education are still open to investigate (Tzanakis & Thomaidis, 2011). In that sense, the present study has been carried out with the aim of investigating how the integration of history of mathematics as a tool affects students' understanding, hoping that the results of the study will support the role of the history of mathematics in mathematics education in terms of cognitive aspects.

"Whys" of integrating history of mathematics into mathematics lessons classified within two major approaches: *use of history as a tool* and *use of history as a goal* by Jankvist (2009). The use of history as a tool aims to use the history of mathematics to develop students' motivation within an affective aspect besides strengthening mathematical cognition and understanding of mathematics topics within a cognitive aspect (Jankvist, 2009). As mentioned before, many studies implemented on students, mathematics teachers and prospective mathematics teachers to investigate effectiveness of using history of mathematics focusing on both teaching and learning processes. The history of mathematics can be used to develop a deep understanding of mathematics for middle grade students. There are lots of quantitative and quasi-experimental design studies have been implemented on students at different grade levels in Turkey. The researchers have come up with the result that the use of history of mathematics in mathematics lessons have increased the academic achievement (Bayam, 2012; Ersoy, 2015; İdikut, 2007; Özcan, 2014). Furthermore, Lit, Siu, and Wong (2001) have implemented a quasi-experiment in Hong Kong with 8th grade students, and similarly they have concluded that experience of the historical process of development of the Pythagorean theorem has a positive influence on students' attitudes towards topic and mathematics.

On the other hand, history as a goal refers to teaching mathematics topics within the

historical development and evolution of mathematics by taking the existence of mathematics, effects and contributions of people and cultures on the development of mathematics into consideration (Jankvist, 2009). An interdisciplinary project containing a role play about the life of a mathematician in four scenes with the 7th grade students was carried out by Ponza (1998). According to the Ponza, the process of discovering and acting out the life of a mathematician in the history provides students with an experience of humanistic and real-life aspects in mathematics and science. At the end, the project has indicated that "many changes were observed in the attitude of pupils towards mathematics: their rejection of the subject decreased, and they experienced a surge of interest" (Ponza, 1998, p.10). These studies support that making connection between real-life experiences in history and school learnings increase the students' interest in studying mathematics and their motivation towards mathematics. Students' interest and motivation towards studying mathematics is significant to make sense with the mathematical contents.

Fried (2001) has stated three main themes to represent the reasons of the use of history of mathematics in mathematics education considering the fifteen reasons formed by Fauvel (1991) earlier. Fried (2001) indicated that history of mathematics (i) enables students to experience and sense mathematics as human made and human product, (ii) makes mathematics more interesting, understandable, and approachable for students, (iii) helps to perceive mathematical concepts, problems and their solutions. Students discover multicultural side of mathematics while discovering historical role models and associate mathematics with human needs, motivation and emotions via the first theme. The second theme claims that the integration of history of mathematics varies in teaching activities and helps to overcome the fear of and bias to mathematics and notice the place of mathematics in society. The third theme supports the Jankvist's (2009) history as a tool classification and the argument that "To really learn and master mathematics, one's mind must go through the same stages that mathematics has gone through during its evolution" (p.239). Historical development of a subject provides a parallel path to learn this subject within context revealing relationships between ideas, definitions, and applications. Gulikers and Blom (2001) conducted a survey on the use and value of history of geometry in

mathematics education focusing on the literature and the arguments of the researchers about the reasons of using history.

Why the integration of history of mathematics has been categorized under three main themes by Gulikers and Blom (2001) involve (i) conceptual, (ii) (multi) cultural and (iii) motivational aspects. The first aspect means the use of history of mathematics to improve students' mathematics learning in terms of understanding of contents. The multicultural aspect of the development of mathematics motivates students while discovering the effect of people on the field of mathematics. Some of the researches have resulted that mathematics is a human activity and product, which means it is a dynamic field affected by social and cultural factors (Esteve, 2008; Furinghetti, 2007; Lingard, 2001; Liu, 2003; Tzanakis & Arcavi, 2000; Tzanakis & Thomaidis, 2011). With the help of the integration of mathematics history into mathematics lessons, students notice that mathematics has developed with the role of different cultures and people. Thus, they give necessary importance to study mathematics. In addition, studying on the mathematics problems from the past, experiencing different solution methods for the problems and various teaching activities including history of mathematics motivate students while overcoming the fear of mathematics (Fauvel, 1991; Liu, 2003; Swetz, 1997; Tzanakis & Arcavi, 2000).

Tzanakis and Arcavi (2000), stated five reasons to explain the importance of using history of mathematics in mathematics education: (i) history of mathematics facilitates learning activities, (ii) improve the viewpoints of students towards mathematics and the nature of mathematical activities, (iii) develops teachers' teaching portfolio, (iv) affects students' affective dispositions towards mathematics positively, and (v) encourages students to value mathematics as cultural and human product. Similarly, Liu (2003) pointed out the necessity the use of history of mathematics in four items and underlined that (i) history of mathematics motivates students and provides positive attitude towards learning of mathematics, (ii) historical problems helps to improve mathematical thinking ability, (iii) past reveals the humanistic aspects of the mathematics, (iv) historical problems guide teachers for the learning and teaching activities while asserting that the difficulties

mathematicians encountered in the past helps teachers to identify and prevent the problems of students of today. Both studies expressed the affective impacts of using history of mathematics in mathematics education such as increasing motivation, improving positive attitude towards mathematics with the help of discovering humanistic aspects of mathematics science. Moreover, these studies stated that the integration of mathematics history into lessons enriches teachers' teaching repertoire with a different teaching material which helps teachers understand students' difficulties better.

Although investigating the use of history of mathematics and its effects in mathematics teaching had a vast background worldwide, it started to be studied in mathematics education in Turkey since the beginning of the 2000s (Alpaslan, 2011). Furthermore, throughout the history, mathematics researches concentrated on the improvement of teacher education (Alpaslan, 2011; Biber, İspir & Ay, 2015). In the latest revised mathematics teaching program (MoNE, 2018), one of the main purposes of the mathematics education is students' being aware of mathematics as a common inheritance and wealth of humanity. For these reasons, such studies about the use of history of mathematics on mathematics learning and teaching need to be conducted since there is an obvious need for the research in that area in Turkey.

The mathematics education must be attached to real human circumstances, social and intellectual situations, real human beings and thinking to humanize and experience that aspect of mathematics (Fried, 2001). In modern times, when some mathematical concepts appear to be disconnected from real-life and raising difficulties in understanding mathematics topics, history enables students to understand the need for the development in mathematics (Gulikers & Blom, 2001).

As Savizi (2007, p.46) stated: "For students, issues of past real world are more tangible and understandable than today's problems or solving problems from real life by using human approaches may work better than application of complicated methods or offering high amount of information." That also improves students' self-confidence and encourages them to believe in their own abilities as human beings

(Savizi, 2007). Moreover, recent studies and researches on this field have indicated that students experiencing mathematical concepts within a meaningful historical context will develop more positive attitudes towards concepts (Lim & Chapman, 2010).

NCTM has recommended integration of history of mathematics into classroom discussions of mathematical topics improve students' both affective and cognitive outcomes towards mathematics (NCTM, 2000). The activities with meaningful context make students more willing to learn about the subject while they understand the importance of mathematics and real-life relevance of the concept (Lim &Chapman, 2010). Lingard (2001) stated that students usually consider mathematics as a dead field of the study, and there is not much to improve because of oversimplification of mathematical contents in lessons. That problem has been identified by Schwartz, Michal and Myles (1985, p.1) as "there is something odd about the way we teach mathematics in our schools. We make little or no provision for students to play an active and generative role in learning mathematics and we teach mathematics as if we expected that students will never have occasion to invent new mathematics". For meaningful learning in schools, students need to see the human side of mathematics and they need to be aware how they can play a role in the development of mathematical knowledge, and the value of mathematics in their lives (Ernest, 1994). As a body of knowledge, mathematics is developing and continuously evolving to respond social problems and as people change so does the mathematics to satisfy the needs of human beings (Panagiotou, 2010). In the light of the research that have been done, mathematics history-based mathematics education provides students with discovering why human needs mathematics in real life, how mathematics evolves from past to today, how social and cultural changes in human history affect the mathematics, and thus learning and studying mathematics may become more meaningful for the students.

2.2. Studies Investigating Integration of Mathematics History in Mathematics Teaching in Turkey

The use of mathematics history in mathematics education have been investigated by mathematics education researchers in terms of different aspects (Baki & Güven; 2009; Bütüner, 2008; Gürsoy, 2010; İdikut, 2007; İlhan, 2011; Kar & İpek, 2009; Karakuş, 2009; Oprukçu-Gönülateş, 2004; Tözlüyurt, 2008). The review of the accessible literature on the integration of mathematics history presented that most of the Turkish studies focused on teachers' education. There are also several studies have taken the teaching of mathematics in elementary grades into consideration and investigated the integration of mathematics history into mathematics education (Bayam, 2012; Bütüner, 2008; Hacıömeroğlu & Apaydın, 2009; İdikut, 2007; İlhan, 2011; Kar & İpek, 2009; Karakuş, 2009).

İdikut (2007) investigated the effect of using mathematics history in mathematics education on the seventh-grade level students' attitudes towards mathematics, achievement and permanence of students' learnings by employing an experimental research design. In this experimental study, the mathematicians such as Carl Friederich Gauss, Leonardo Fibonacci, Omar Khayyam, and Pierre de Fermat were placed in teaching materials, and their studies on mathematics were introduced with their voices. The use of studies of different mathematicians enabled students to experience how mathematics developed with the impact of many different cultures while discovering the place of mathematics in real life. In the result of the İdikut's study, the mathematics history integration has a positive effect on the students' achievement. However, this positive effect didn't reveal in students' attitudes towards the subject and permanence of their learnings.

Similarly, one of the studies employing an experimental design investigated the effect of the use of mathematics history on sixth grade students' achievement and attitudes (Bayam, 2012). In this study, the anecdotes of the mathematicians who played a role in the development of numbers, geometry, algebra and probability areas and historical anecdotes were given to students as performance project. The teaching

process was supported with the materials including mathematics history. The result of the study indicated that the use of mathematics history increased the achievement of students in addition to students' positive opinions about teaching process. Furthermore, a quasi-experimental designed study has been conducted to investigate the effect of using mathematics history on the academic success, retention and motivation of fourth grade level students about the decimals (Ersoy, 2015). The study presented that students' achievement, motivation and retention levels on decimals increased with the help of the use of mathematics history.

The review of accessible literature of Turkish studies showed that the studies involving students mostly investigated the effect of the use of mathematics history on the achievement of students. These experimental, mixed and quasi-experimental designed studies resulted with the positive effect on students' academic success. Moreover, mathematics history was used as a tool in these researches to investigate its' affective impacts such as students' motivation and attitudes towards mathematics. However, the investigation of cognitive aspects of the students' learning such as understanding by employing a qualitative designed study is rare in the accessible literature. Therefore, the present study investigates the role of mathematics history-based model-eliciting activities on the students' understanding by employing a qualitative design to contribute to mathematics education research literature. Since model-eliciting activities are a type of teaching materials used in mathematical problem solving called modeling, in the following section I will present modeling perspective as an approach for problem solving.

2.3. Models and Modeling Perspective

In this study, the term "model" refers to mathematical models that are built, defined, emphasized mathematically significant products, processes and mathematical reasoning (Doerr & Lesh, 2003).

The term "problem solving" has had different "multiple and often contradictory meanings through the years" (Schoenfeld, 1992, p.337). Lester (1983) defined problem solving as an activity that involves studying process to find a solution for

the problem which can be solved by individually or with a group, and there is not a one certain method that ensures the solution.

Problem solving strategies used in traditional mathematics teaching mostly based on "drawing picture, using a simpler version of problem, determining givens and goals of problem" (Lesh & Zawojewski, 2003, p. 317), and main aim of these strategies is to help students overcome their difficulties while solving problems by making connection with prior knowledge. On the other hand, MEAs provide students with extending, refining or modifying interpretations or explanations of the current problem (Lesh & Zawojewski, 2003). In MEAs, involving sharable, manipulatable, modifiable and reusable models, the aim is not to get a simple product or answer but to focus on a process of producing models (Doerr & Lesh, 2003). In the 21st century, machines mostly handle with procedural and computational works creating a need for human beings to contemplate mathematical models to solve new and unfamiliar problems (English, Lesh, & Zawojewski, 2003). For this reason, MEAs emphasize the teamwork and communication that required in using new technologies (Lesh & Zawojewski, 2003). In MEAs, it is provided that not only computing is an essential process, but also constructing, describing and mathematical reasoning are significant for understanding of a mathematics topic (Doerr & Lesh, 2003).

The model-eliciting activities aimed to satisfy need for social functions such as "recreating explicit constructions, descriptions, explanations, justified predictions, assessed situations and generalized procedures and plans" (English, Lesh, & Zawojewski, 2003, p. 338). Besides that, considering the construction and generalizability principles of the modeling, a mathematical model should be shareable and reusable to foster students' productivity in terms of more scientific and professional work (English, Lesh, & Zawojewski, 2003). MEAs contribute to students' understanding through iterative problem-solving processes and produce mathematical ideas in real-life problems (Lesh & Zawojewski, 2003). To illustrate, instead of focusing on symbolically described word problems and computational skills, MEAs emphasize on the meaning of symbolic representations of integers and operations (Doerr & Lesh, 2003). In modeling classrooms, teachers focus on

students' understanding and processes of constructing, expressing, reasoning abilities while solving mathematically word problems rather than solely arithmetic computations (Lesh, Cramer, Behr, & Post, 1998; Lehrer & Schauble, 2000). Since MEAs encourage students to question and discover while building mathematically significant models, these activities foster students to create models in order to solve real-life related, open-ended mathematics problems. The six essential principles of MEAs are (1) model-construction, (2) model-documentation, (3) reality (meaningfulness), (4) self-assessment, (5) model shareability and reusability, and (6) effective prototype (Lesh et al., 2000). These six essential principles of MEAs help students to improve their mathematical thinking and reasoning. Therefore, with the implementation of an MEA, students produce mathematically significant, shareable and reusable model related to real-life experiences. Since MEAs are thought-revealing activities bringing out students' thinking process while they are studying in a group and allowing them to assess themselves. For these reasons, MEAs have a beneficial role in eliciting students' understanding.

In the study of Doerr and Lesh (2003), it is emphasized that MEAs resemble to real life situations in which students make meaningful mathematical explanations whereas in traditional textbooks word problems necessitate students' meaning symbolically defined circumstances. In the mentioned study, a middle school version of "The Big Foot Problem," an example of real-life problem-solving situation, was implemented to students. Their research has shown that students invent models that are much more effective than traditional learning since the conceptual systems that students construct via MEA are mathematically effective sense-making products involving mathematical reasoning (Lesh & Doerr, 2003).

In addition, to identify mathematical knowledge two aspects took into consideration: "doing pure mathematics" and "doing applied mathematics" (Lesh, Cramer, Doerr, Post, & Zawojewski, 2003, p.36). Doing pure mathematics refers to construct and explore mathematical knowledge for mathematics' own sake (Steen, 1987). Besides, doing applied mathematics necessitates using models to construct, discuss, justify, predict and accommodate mathematical knowledge (Lesh & Doerr, 1998). It is

indicated that while students studying in teams during MEA(s), and they interact not only with concrete materials but also with peers (Lesh et al.2003). Therefore, MEA(s) support both Vygotsky's social perspective and Piaget's cognitive perspective. Similarly, the research of Slavin (1989) on the cooperative learning literature indicated that cooperative learning has a positive influence on students' understanding. Moreover, creating a mathematically significant model to solve a real-world mathematics problem by working in groups rather than solving a traditional class question individually increases the interest and motivation of students (English, Lesh, & Zawojewski 2003). With that in mind, this study investigated the role of mathematics history-based MEAs on 7th grade students' understanding of negative integers while studying in small groups.

2.4. Students' Understanding of Integers

Negative integers mostly described in terms of minus sign that attached to in front of the integer. The negative integers are represented on a horizontal number line as the integers which are on the left from zero and on the opposite direction of positive integers or on the bottom of zero on a vertical number line. These positions refer to negative integers are less than zero (Altıparmak & Özdogan, 2010). Moreover, in terms of everyday life contexts, negative integers are represented with different concepts such as debt and loss (Akyuz, Dixon & Stephan, 2012). In addition to these definitions, three dimensions for the negative integers and meaning of minus sign are expressed as (Altıparmak & Özdogan, 2010): (i) the negative integer case referring to meaning of a negative integer, direction of integers and quantity of numbers in the numerical system, (ii) the use of number line focusing on arithmetic operations and the meaning of calculations, (iii) the explanations and interpretations of how students understand the negative integers within contexts and associate negative integers and contexts. With keeping these definitions in mind, in the present study, negative integers are expressed with a minus sign in front of the integer that means these integers are less than zero. Besides, students were expected to reflect their understanding and making sense of negative integers and how they are associating negative integers with everyday life contexts.

The capability of representing different mathematical concepts and accommodating between these different representations improve students' understanding (NCTM, 2000). There are many studies investigating how to advance students' understanding of integers by emphasizing mainly on neutralization and number line models (Lyte, 1994). The neutralization model includes physical objects such as two-colored math counters to represent negative and positive integers and operations (Lyte, 1994). In the study of Lyte (1994), it was indicated that students had difficulty in subtraction problems after implementation of neutralization model. The line model focuses on operation with integers considering the position and distance of integers by the direction of movement on the number line (Lyte, 1994). Moreover, it is concluded that the instruction didn't improve operational skills of students in long term when comparing the neutralization and the number line models (Hayes, 1999). There are several researches indicated that most students have difficulty in understanding negative integers (Bolyard, 2005; Ferguson, 1993; Lyte, 1994; Shore, 2005; Wilkins, 1996) since students focus on operational procedures rather than meaning of operations.

Furthermore, natural numbers are more intuitive for students since students firstly study on natural numbers with counting and make connections between each number and object (Whitacre et al., 2017). The concept of negative integers which commonly described as being less than zero, making sense and being aware of the use of negative integers in real-life is more difficult for students (Whitacre et al., 2017). There are several studies supporting that students have difficulty in understanding negative integers as they try to accommodate their prior knowledge about natural numbers for integers (Gallardo, 2002; Gallardo & Romero, 1999; Mukhopadhyay & Resnick, 1990; Whitacre et al., 2017). This transition between natural numbers and integers cause difficulties in terms of number sense and making sense of the negative integers. In that case, models provide students to interpret and making sense of negative integers (Steiner, 2009).

In addition, the sense of negative integers and the idea of a number less than zero seems absurd for most of the everyday contexts from the viewpoints of students

(Whitacre et al., 2017). Therefore, taking mathematics teaching into consideration through children's thinking perspective is essential for understanding of a context since the nature of students' mathematics is different from adults' thinking of mathematics in interesting and significant ways (Steffe, 1991). Students' prior knowledge and everyday experiences they bring into classroom environment affect their mathematical thinking perspective and understanding. Although students meet negative integers in their everyday life before instruction of integers in school, after they encounter and focus on operational procedures in school, they do not make connection between before school learnings and school instruction (Steiner, 2009). Therefore, school teaching has a negative role but the separation of everyday experiences and school learnings could be prevented to support students' understanding of integers. For this reason, emphasizing and appreciating different solution ways for problems by mathematics teachers leads to overcome disconnection between everyday experiences and school learnings (Steiner, 2009). Similarly, there are several studies emphasizing that concrete models and real-world problems support students' understanding of integers and operations with integers (Ball, 1993; Stephan & Akyuz, 2012). The real-life contexts, word problems and models including incomes and expenses, assets and debts, elevators, weather temperatures support students' understanding and reasoning about integers (Ball, 1993; Hayes & Stacey, 1998; Pettis & Glancy, 2015; Stephan & Akyuz, 2012). Likewise, Lesh Translation Model (Lesh & Doerr, 2003) supports the idea that students' capability to represent mathematical concepts within different ways and making meaningful connections between these ways improves students' understanding.

With these studies in mind, in this study students are expected to come up with a mathematical model considering given everyday life mathematics problems or real-life mathematics word problems from the history such as number system and trade habits. The study of Cramer (2003) showed that if students aren't able to achieve accommodating between different representations, they also encounter difficulties in acquiring deeper understanding. Similarly, it is stated that students may not be able to comprehend situations involving opposites such as incomes and expenses, weather

temperatures and elevators (Pettis & Glancy, 2015). Thus, it is important that students be encouraged within this context to improve their understanding of integers. In addition, the understanding and learning of a mathematics content is accepted as an active process that students should be allowed and encouraged to be active participants rather than being passive listeners. Working on mathematical problems as a group is more effective in terms of both social and individual development while engaging in mathematical conversation (Cobb & McClain, 2001). Besides, students might get confused about determining the right solution way of a problem because of focusing on operational procedures and trying to decide on which way to follow while dealing with integer problems (Bolyard, 2005; Ferguson, 1993). Therefore, it is significant to foster students about being aware of that the solution process of a problem is important not the result and the best solution way for a problem depends on the sense-making of individuals (Steiner, 2009). For these reasons, the scope of this study is solely students' understanding and making sense of negative integers through model-eliciting activities.

CHAPTER 3

METHODOLOGY

The purpose of this study is to investigate 7th grade students' understanding of negative integers through mathematics history-based model-eliciting activities. The following research questions have guided the current study:

1. What understandings do 7th grade students develop negative integers as they engage in mathematics history-based model-eliciting activities in small groups?

2. How do 7th grade students' individual understandings change about negative integers after their engagement in mathematics history-based model-eliciting activities?

The focus of this chapter is to describe methodology used to conduct present study. This chapter provides information about the research design, participants and their major characteristics, data collection instruments, data collection procedures and analysis. The role of the researcher and trustworthiness issues are also addressed.

3.1. Design of the Study

The main purpose of this research is to investigate 7th grade students' understanding of negative integers. Specifically, the research questions aimed at analyzing the role of mathematics history-based activities, on students' understanding and sense making through modeling; therefore, qualitative educational case study design was employed. The case boundaries set for each research question differs, and so the following cases are focused in this study:

- For investigating the Research Question 1, *eight groups of seventh grade students* who were engaged in mathematics history-based model-eliciting activities in small groups were considered as the case. Thus, *students' group work data* were analyzed for this research question.
- For investigating the Research Question 2, *29 seventh grade students* who attended in mathematics history-based model-eliciting activities were considered as the case. Thus, *students' individual performances* on the initial and follow-up assessment were analyzed and compared for this research question.

3.2. Participants

The participants involved in this case study were voluntary 29 seventh grade students and convenient to implement present study. The participants of this study are 7th grade students (15 male and 14 female) of a public middle school class in Istanbul, one of the metropolitan cities in Turkey. The students randomly assigned to groups by the researcher who was also the mathematics teacher of the classroom. Ten groups were formed in the classroom but only eight of them were included in the data set. These eight groups' work were video recorded during data collection process. These eight groups contained 3-4 students per group; 29 students in total. The data set involves their written work in activity sheets and video records of their work during the implementation of activities. The researcher also implemented both initial and follow-up assessments to these 29 students. The researcher didn't collect or use any data from remaining 7 students in the classroom whose parents did not provide a consent, but they were involved in the lesson and implementation of the activities.

This class is one of the project classes which are formed concerning students' grade point averages, and these students were determined based on their achievement. The project class students take more English lessons compared to other 7th grade classes; however, there is no difference in other lessons or curriculum. A general view of mathematics teacher for this class is that most of the students' prior knowledge and

mathematics backgrounds are similar to each other as they have attended the same class from the beginning of the primary and middle school.

3.3. Data Collection Procedure

The data collection process began after the necessary permissions were gotten from the University Ethics Committee (see Appendix A for the Ethics Committee Permission), Istanbul Provincial Directorate for National Education (See Appendix B) and students' parents (See Appendix C).

A set of integer questions were implemented to participant students as initial and follow-up assessments. The initial assessment aimed to evaluate the current status of student's understandings of negative integers. The same set of questions were implemented as follow-up assessment to understand students' understanding and making sense of negative integers and the role of the mathematics history-based model-eliciting activities (MEAs) on students' understanding. Each student answered the mathematics questions of each assessment individually in one-class hour period.

Before the initial assessment and the implementation of mathematics history-based MEAs, any formal instruction and prior teaching about negative integers did not provided. Therefore, initial assessment showed the students' initial state about negative integers. Students began to study on negative integers with the implementation of mathematics history-based MEAs. The mathematics history-based MEAs implemented with the aim of guiding students to achieve related objectives of middle school mathematics teaching program. The activities covered three dimensions of students' understanding and making sense of negative integers in terms of: (i) why negative integers were needed in mathematics, (ii) how to identify positive and negative integers, and (iii) how to use negative integers in real life contexts.

After the initial assessment, I continued data collection procedure by implementing three MEAs in six mathematics lessons. Each activity was completed in two class

hours. In the implementation process of activities, students studied in groups that they were assigned beforehand. In each part of the activities, students firstly discussed and answered problems in groups, and then they shared their findings as a whole class discussion. The teacher guided students during their group works and whole class discussion when they needed suggestion or asked for help. The implementation of MEAs were video-recorded and students' group discussions were audio-recorded.

Two days after the implementation of mathematics history-based MEAs, follow up assessment involving the same mathematics questions with initial assessment were given to students and their individual performances were recorded in their written responses.

3.4. Data Collection Tools

In educational case study approach, researchers rely upon "multiple sources of evidence" (Yin, 2003, p.14) and theoretical propositions conduce selection of data sources (Yin, 2003). Hence, the use of several sources of evidences support the effectiveness and trustworthiness of case studies (Yin, 2003). In the present study, the data set was gathered from multiple sources that initial and follow-up assessments were implemented before and after the implementation of mathematics history-based MEAs. In addition to these written initial and follow up assessments, students' performances during MEA implementations were recorded by a video camera capturing both students' and teacher's reactions, by audio recorder capturing the conversation of small groups, and by activity sheets recording small groups' written works. Hence, the data for this study were collected through written works of all groups via the mathematics history-based MEAs, video recording of each group, and initial and follow-up assessments to understand individual student's understanding of negative integers. To provide more detail, the list of all the data sources and data they provided for the purpose of this study were explained in the following section.

3.5. Development of Questions for Assessing Negative Integers

The researcher prepared a set of questions aiming to obtain students' prior knowledge about integers and to understand the role of the implementation of mathematics history-based MEAs on students' understanding. These questions were examined by another researcher who an expert in mathematics education field is and revised based on the feedbacks gathered. Furthermore, the questions were shown to a 7^{th} grade student who was not one of the participants of this study and checked whether the statements were understandable.

The set of questions involved essay type items constructed by the researcher according to the literature and objectives in Turkish Middle Grades Mathematics Teaching Program (MoNE, 2018) (See Appendix D for the assessment questions). The 6^{th} grade objectives in the middle school mathematics teaching program related to integers which were covered to activate prior knowledge of students and the 7^{th} grade objectives related to integers which were also covered in the assessment tool are given in Table 3.1.

The initial and follow-up assessment questions were composed of six main questions, three of which involved sub-questions. These questions implemented before and after the implementation of the mathematics history-based MEAs, allowing 40 minutes.

Table 3. 1 Objectives related to integers in mathematics teaching program addressed in the assessment tool

Grade Level	**Objectives related to Integers**
6	1. Students should be able to know integers. a. Students should be able to notice the need for integers. b. Students should be able to notice positive and negative integers are used for different values.
7	1. Students should be able to solve problems requiring operations with positive and negative integers.

Question 1 and 2, given in Figure 3.1 below, were prepared by the researcher to measure whether students have the knowledge to select positive or negative integers from a given set of integers which have plus, minus or no sign in front of the number. The researcher aimed to determine whether students are able to identify negative integers considering minus sign in front of the integer.

1. Aşağıda verilen tam sayılardan pozitif olanlarını yuvarlak içine alınız.

- 305	**1**	**- 10**	**+ 12**	**27**
- 36	**+ 143**	**15**	**32**	**- 4**

2. Aşağıda verilen tam sayılardan negatif olanlarını yuvarlak içine alınız.

127	**+ 34**	**- 71**	**12**	**- 43**
- 36	**+ 324**	**- 56**	**3**	**- 1**

Figure 3.1 1st and 2nd Questions of Initial and Follow-up Assessment Questions

As seen in the Figure 3.1, the first two questions of the assessment tool were covered the objective that students should be able to know integers.

Question 3, given in the Figure 3.2 below, in the assessment tool was prepared to understand how 7th grade level students explain a negative integer to another person. For this aim, students firstly asked to write a negative integer, and then they were expected to explain in the sub-question why this example is a negative integer.

3. Aşağıya bir negatif tam sayı örneği yazınız.

Bu sayının negatif tam sayı olduğunu bir başkasına nasıl açıklarsınız.

Figure 3.2 3rd Question of Initial and Follow-up Assessment Questions

As shown in the Figure 3.2, the third question of the assessment tool focused on the objective that students should be able to know integers while noticing the need for negative integers.

Question 4 covered addition with integers in three sub-questions. The students were given an addition operation with an unknown addend and they were firstly expected to determine whether the unknown is a negative or positive integer. The second sub-question asked the reason of first sub-question, and students were expected to explain their reasons. The third sub-question asked the exact value of the unknown integer and the explanations of students to find the value of unknown. Question 4 and sub-questions are given in Figure 3.3.

4. **3 + ☐ = 7**

Yukarıda verilen işlemde kutucuk yerine pozitif bir tam sayı mı yoksa negatif bir tam sayı mı gelmelidir?

Sebebini açıklayınız.

Kutucuk yerine gelmesi gereken tam sayı kaçtır? İşlemlerinizi açıklayarak gösteriniz.

Figure 3.3 4th question of Initial and Follow-up Assessment Questions

As seen in the Figure 3.3, the fourth question of the assessment tool was covered the objective that students should be able to solve problems requiring operations with integers. In this question, students were expected to reflect their understanding of negative integers in terms of their mathematical reasoning for their solutions rather than the operational procedures.

Similarly, Question 5 covered addition with integers. Different from the fourth question, the sum of the operation was less than the addends. The students were given an addition operation with an unknown addend, and firstly they were expected to determine whether the unknown is a negative or positive integer. The second sub-question asked the reason of first sub-question and students were expected to explain their reasons. The third sub-question asked the exact value of the unknown integer and the explanations of students to find the value of unknown. Question 5 and sub-questions are given in Figure 3.4.

5. **3 +** ☐ **= 1**

Yukarıda verilen işlemde kutucuk yerine pozitif bir tam sayı mı yoksa negatif bir tam sayı mı gelmelidir?

Sebebini açıklayınız.

Kutucuk yerine gelmesi gereken tam sayı kaçtır? İşlemlerinizi açıklayarak gösteriniz.

Figure 3.4 5th question of Initial and Follow-up Assessment Questions

As shown in the Figure 3.4, the fifth question of the assessment tool was also covered the objective that students should be able to solve problems requiring operations with negative integers. Similar to the fourth question, students were expected to reflect their understanding of negative integers in terms of their mathematical reasoning for their solutions rather than the operational procedures.

Question 6, given in Figure 3.5, presented eight real life situation and asked students to express a given everyday life situation with integers.

6. Aşağıda verilen durumları tam sayı olarak ifade ediniz.

Asansör girişin 5 kat üstündeki kata geldiğinde:

Asansör girişin 3 kat altındaki kata geldiğinde:

Hava durumu sıfırın altında 7 derece gösteriyor:

Hava durumu sıfırın üstünde 10 derece gösteriyor:

Elçin'in Onur'dan 55 TL alacağı vardır:

Bu ay evin gideri 500 TL'dir:

Sattığım karpuzdan 300 TL kar elde ettim:

Bir şirketin sattığı bilgisayarsan 1000 TL zarar etmesi:

Figure 3.5 6th Question of Initial and Follow-up Assessment Questions.

Moreover, as seen in the Figure 3.5, the sixth question of the assessment tool focused on the objective that students should be able to notice positive and negative integers are used for different values.

3.6. Development of Mathematics History-Based MEAs

The MEAs that incorporates mathematics history were developed by the researcher considering the framework of six design principles of models-and-modeling perspective: (1) model construction, (2) reality (meaningfulness), (3) self-assessment, (4) model documentation, (5) model share-ability and reusability, and (6) effective prototype. Table 3.2 below presents the modeling activities, their purpose and the model asked in each activity.

Table 3.2 Mathematics history-based model-eliciting activities

	Activity	Purpose	Model
MEA 1	Yaşasın Matematik	Understanding why we need number sense and negative integers	Development of a rationale for using negative integers and a solution for problems involving integers
MEA 2	Arda'nın Youtube Kanalı	Understanding number sense and how to identify positive and negative integers	Construction of a number system
MEA 3	Konuğumuz Muhasebeci	Understanding the use of negative integers in real life	Organization of monthly income and expenses on a bank statement using integers

As seen in Table 3.2, I, as the researcher, implemented three activities that aim students' (i) understanding of why negative integers were needed in mathematics, (ii) understanding and making sense of how to identify positive and negative integers, and (iii) understanding of how to use negative integers in real life context. Accordingly, the models that students developed in these activities involved a letter explaining their rationale for why mathematicians needed negative integers, a unique

number system differentiating positive and negative integers by writing a letter to explain their own number system, a bank statement showing income and expenses with positive and negative integers. Each mathematics history-based MEA implemented in a one lesson hour. Before the implementation of activities, any prior teaching about integers did not provided. Students began to study on integers with the implementation of mathematics history-based MEAs. In the implementation process of activities, students were expected to reflect their understanding and making sense of negative integers. The essential principles followed during the implementation of these activities are: (i) students should study as small groups and interact with each other, (ii) after they finished their studies, students should be encouraged to share their works and opinions with the guidance of teacher during whole class discussion, (iii) teacher should guide students when they needed without providing any right answer for the questions of the activities, and (iv) students should be allowed to reveal and reflect their own experiences by making connection with everyday life contexts. One example for the small group work of students is shown in the Figure 3.6.

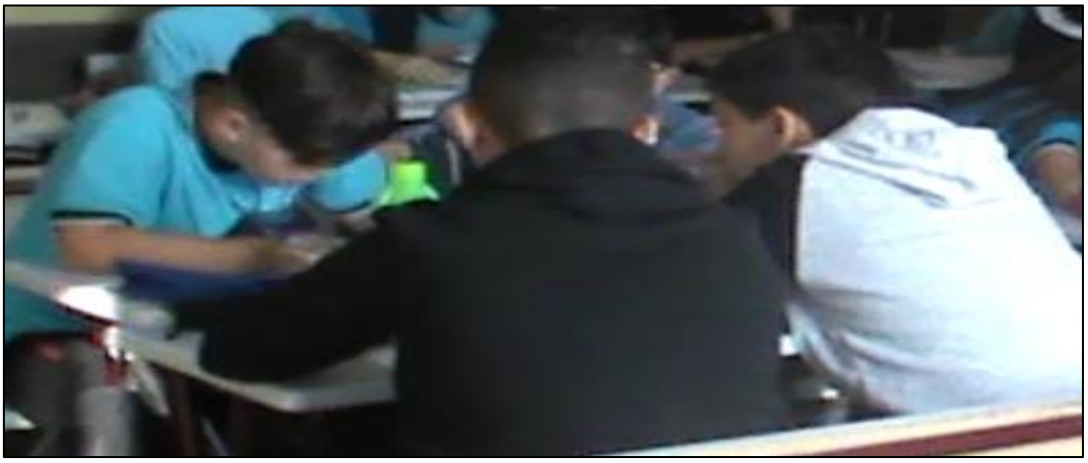


Figure 3.6 One of the small groups

In the following section, I explained each MEA in the light of six design principles of model-eliciting activities. The implemented version of these MEAs were given in Appendix E, F and G.

3.6.1. The Mathematics History-Based MEA 1: Yaşasın Matematik

Mathematics history-based-based MEA 1 was prepared considering the objective in Turkish Middle Grades Mathematics Teaching Program (MoNE, 2018) that students be able to understand integers and notice the need for integers. For this reason, in this activity the researcher aimed to attract students' attention to the origins of mathematics field, numbers and especially on negative integers. First part of the activity emphasized how people use the negative integers in daily life and why people needed negative integers in the history. The researcher intended to help students to consider and question necessity and need for negative integers not only for mathematical operations but also in everyday problems. The second part of the activity contains information about Diophantus, a mathematician, and is followed by a problem which is called as "absurd" by the Diophantus because of its' negative solution. Students were expected to try solving the problem with their group members and find a reason for why Diophantus called the solution as absurd via using a letter to represent the unknown. Thus, the researcher brings history of mathematics and modeling together. The last question of the activity aimed to foster students' consideration about how mathematics as a field has been developed over the years and affected by studies of different mathematicians.

The mathematics history-based MEA 1 was designed by taking students' learning levels into consideration and focusing on not only computational procedures but also understanding and making sense of topic (i.e., effective prototype principle of MEA). Since it is significant to provide a useful leaning material that band history of mathematics and modeling together. In this activity, students are expected to write a letter about a given problem related to the mathematics history (i.e., the reality/meaningfulness principle of MEA). This activity was a thought revealing activity that students tried to solve the given mathematics problem by writing a letter in detail to describe and explain their solutions (i.e., model documentation principle of MEA). In their solutions, students made predictions about the reason of this problem called as absurd by Diophantus and provided justification (i.e., model construction principle of MEA). The students were asked to write a letter to a peer

student who will benefit and use their answers on his school magazine. With this real-life related aim, students firstly studied as groups to find out the solution of the problem by discussing with their groupmates (i.e., the reality/meaningfulness principle of MEA). After that, each group shared their letters and solutions in a whole class discussion. Studying as groups and having real-life connected common aim provided students experience an alternative solution way for the problem while extending and refining their solutions with groupmates (i.e., model shareability and reusability principle of MEA). With the help of whole class discussion and sharing their letters with others, they elaborated their answers by explaining their ways of thinking for self-reflection and assessing their thinking process (i.e., self-assessment principle). Besides, they had a chance to compare their findings with alternatives to assess, improve and elaborate through joining in a whole class discussion. Below, I present the mathematics history-based MEA 1 in the Figure 3.7a and Figure 3.7b and the implemented version of the activity in Appendix E.

Figure 3.7a Mathematics History-Based MEA 1-Part 1

Tatilde okuduğum bir matematik dergisinde yazanları size anlatacağım. M.Ö. 2. yüzyıl ve 4. yüzyıl arasında Diophantus adında İskenderiyeli Yunan bir matematikçi yaşamış ve bizim g6. Sınıfta öğrendiğimiz cebirsel ifadeler konusunda çalışmış ve 130 problemden oluşan Arithmetica adlı bir kitap yazmış. Diophantus "cebirin yaratıcısı" olarak biliniyormuş. Aslında cebir çok daha önceki zamanlarda bile kullanılıyormuş. Merak edip araştırdım Rhind Papirüsü de M.Ö. 1650 yılında Mısırlı Ahmes tarafından yazılmış ve çarpma, bölme, cebirsel ifadeler gibi konuları içeriyormuş. Birçok ileri matematik konusunun gelişimine katkıda bulunan eski uygarlıkların ve matematikçilerin negatif tam sayıları anlamaları aslında çokta kolay olmamış ve çeşitli zorluklar yaşamışlar. Tıpkı bizim gibi!

Arkadaşlar dergide Diophantus'un bir problemin çözümünde karşısına çıkan

$$4 \times ? + 20 = 4$$

ifadesinin çözümü için "absürt (saçma)" ifadesini kullandığını okudum.

1. Acaba Diophantus neden bu problemin çözümü için saçma demiş olabilir?
2. Arkadaşlar soru işareti yerine gelecek olan tamsayıyı bulmama yardım eder misiniz?
3. Diophantus'un kendinden önceki cebir çalışmalarını bir araya toplayarak geliştirmesi bize matematik bilimi hakkında ne fikir verebilir?

Cevaplarınızı ve açıklamalarınızı bana mektup olarak gönderirseniz okul dergisindeki yazımda paylaşacağım. Mektuplarınızı dört gözle bekliyorum.

Sevgili Ali,

……

……

Figure 3.7b Mathematics History-Based MEA 1-Part 2

3.6.2. The Mathematic History-Based MEA 2: Arda'nın Youtube Kanalı

Mathematics history-based MEA 2 was prepared considering the objective in Turkish Middle Grades Mathematics Teaching Program (MoNE, 2018) that students be able to notice that positive and negative integers are used for different values and directions. In this activity, firstly the Chinese number system and its properties were introduced to students that Chinese people used sticks to demonstrate numbers and they distinguish the positive and negative integers depending on the color of the sticks. Afterwards, students were asked to make connections between Chinese number system and our number system. For this aim, some examples from Chinese number system were given on an activity sheet and students were expected to convert and write them with our number system considering colors of sticks and positive-negative integer difference. In the second part of the activity, students were expected to form their own number system with their group members considering positive and negative integers and write a letter in detail about characteristics of their own number system. Hence, students have a chance to study modeling with the help of mathematics history.

In the mathematics history-based MEA 2, firstly Chinese numeral system was introduced to students and then students were expected to construct their own number system by writing a letter to a peer student to support the content of his Youtube channel (i.e., model construction principle of MEA). Therefore, mathematics history-based MEA aimed to make connections between students' school and everyday life to attract their attention (i.e., the reality principle of MEA). In the first two parts of the activity, students tried to rewrite Chinese numerals by using present numerals and after they compared their answers with other groups' answers in a whole class discussion and assessed themselves (i.e., self-assessment principle of MEA). In the last part of the activity, the question did not have a single right answer, and students formed their own number system in their groups by drawing and explaining in detail in the letter and then they shared their number systems as a whole class (i.e., model shareability and reusability principle, effective prototype principle of MEA). The whole class discussion provided students with observing and examining how other

groups formed their unique number system considering demonstration of numbers, difference between positive and negative integers and digit value of a number (i.e., model documentation principle of MEA). Figure 3.8a and Figure 3.8b present mathematics history-based MEA 2 and implemented version of the activity in Appendix F.

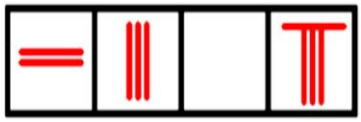
Figure 3.8a Mathematics History-Based MEA 2-Part 1

Arkadaşlar, günümüzden yaklaşık olarak 3000 yıl öncesinde Çinliler kendilerine özgü bir sayma sistemi geliştirmişlerdi. Hesaplama yapan insanlar büyük bir dama tahtası üzerinde kırmızı veya siyah renkteki çubukları kullanarak sayıları ifade ediyorlardı. Bu sistemde dama tahtasındaki her bir karede tek bir rakam bulunurdur. Her bir sütun da birler, yüzler, binler gibi farklı bir basamağı gösteriyordu.

Size az önce gösterdiğim (yukarıdaki) tabloda üst satırdaki çubuk düzeni birler, yüzler ve on binler basamakları vb. için, alt satırdaki çubuk düzeni ise onlar ve binler basamağı vb. için kullanılmaktaydı.

Ayrıca kırmızı çubuklar pozitif sayıları, siyah çubuklar ise negatif sayıları göstermek için tercih ediliyordu. M.Ö. 200lü yıllarda kullandıkları bu çubuk sistemi ile eski Çin uygarlığı negatif sayıları ilk kullanan insanlar arasında yer almaktadır.

Arkadaşlar şimdi sıra sizde! Açıklamaları ve aşağıda verilen örneği inceleyerek verilen sayıları bugün kullandığımız şekilde rakamlarla göstermeyi deneyelim.



2 3 0 8

....

Evet arkadaşlar Çinlilerin sayı sistemini tanıyarak negatif ve pozitif tam sayıları nasıl ayırt ettiklerini sizlerle paylaşmış oldum. Ve hatta ben de kendi sayı sistemimi oluşturmaya karar verdim.

Sizlerde kendi sayı sisteminizi oluşturarak benimle paylaşır mısınız? Bir sonraki videoda sizlerden gelen sayı sistemlerini anlatacağım.

Sevgili Arda,

.....

.....

.....

Figure 3.8b Mathematics History-Based MEA 2-Part 2

3.6.3. The Mathematics History-Based MEA 3: Konuğumuz Muhasebeci

Mathematics history-based MEA 3 aimed to clarify usage of integers in real life and through that students should be able to solve problems requiring operations with integers. The first part of the activity related to accounting, knowledge about it, considering relationship between accounting and trade activities of people. Moreover, this part contains trade examples and how Chinese people benefited from negative and positive integers to solve their daily problems about trade such as interchange, tax, rate and construction of buildings. For this aim, firstly The Sign Rule of Liu Hui and how positive and negative integers were used to represent income and loss depending the colors of the sticks which are from Chinese number system. Then, students were expected to compare this system with our number system. In the second part of the activity, a bank statement of a person for a month was given and students were expected to interpret expenses and incomes of this person. After that, students prepared a bank statement with their group members depending on given expenses and incomes for another month.

The mathematics history-based MEA 3, firstly introduce students how and why negative integers have been used in everyday life by a guest accountant. Afterwards, students were reminded how Chinese people differentiated between negative and positive integers, and then students were asked to make comparison between present number system and Chinese number system. In the second part of the activity students firstly examined a bank statement and then commented on expenses and incomes considering given plus and minus signs in front of the transaction amount on a given bank statement. In the third part of the activity, students formed a bank statement model, and they tried to give advises to the owner of this bank statement considering given expenses and incomes (i.e., model construction principle of MEA). The bank statement model of this history-based MEA aimed to provide students with justifying, predicting, and describing their studies while making connections with everyday life usage of integers (i.e., the reality principle of MEA). Moreover, after students discussed with their groupmates, they shared as a whole class discussion

their comments and advises about bank statement and transaction amounts (i.e., self-assessment principle, model shareability and reusability principle of MEA). The form of a bank statement is a proper and effective model documentation to make connections between everyday life and the usage of integers in the present and the past (i.e., model documentation principle and effective prototype principle of MEA). Figure 3.9a, Figure 3.9b and Figure 3.9c present third activity, and the implemented version of this activity is in Appendix G.

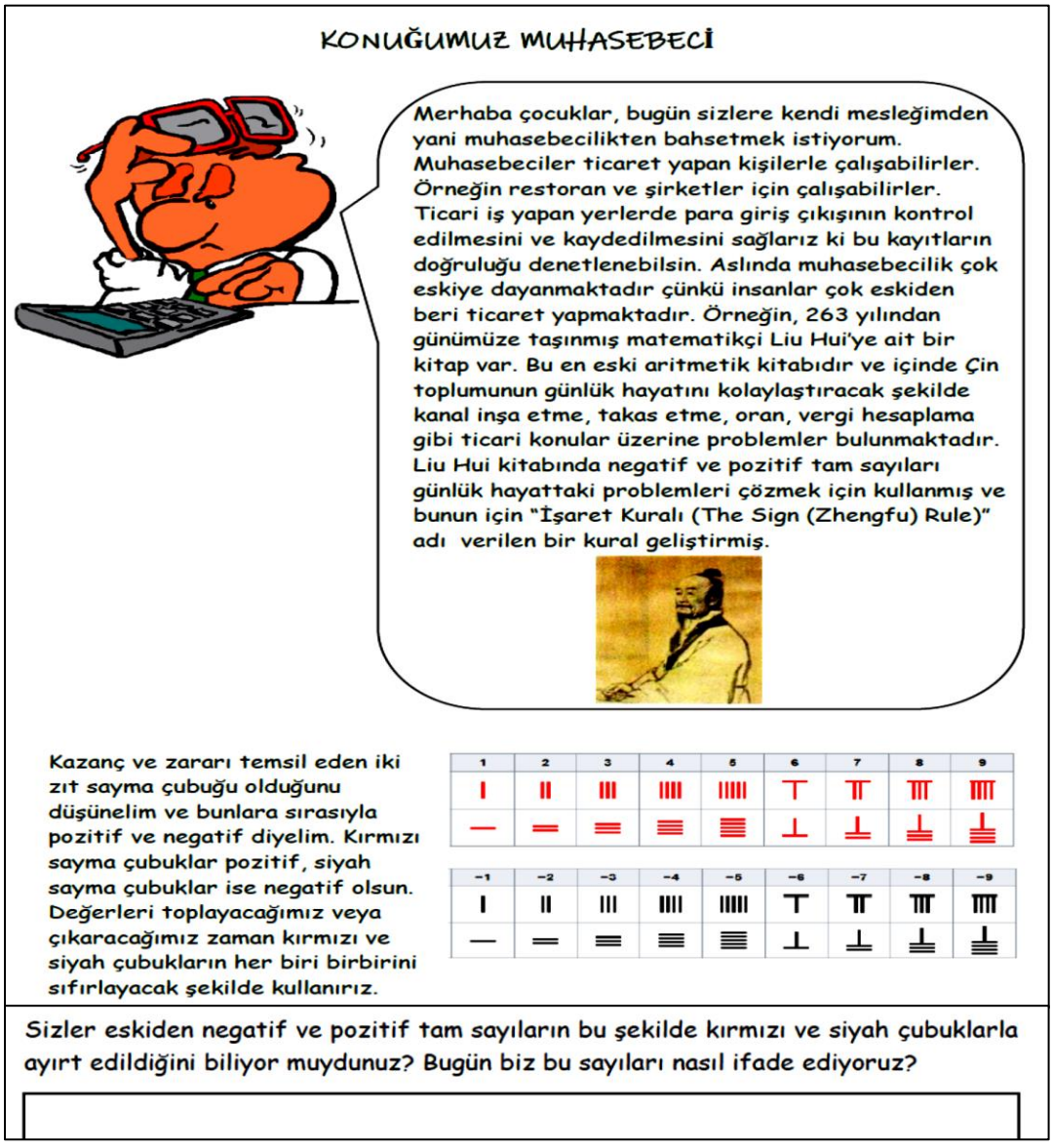


Figure 3.9a Mathematics History-Based MEA 3-Part 1

Çocuklar şimdi sizlerle Ayşe Hanım'ın mayıs ayı hesap özetine bakalım ve harcamalarını ve kazançlarını birlikte yorumlayalım.

Hesap tipi : VADESİZ HESAP
Döviz Cinsi : TRY
Dönem (Tarih Aralığı) :26.05.2018-26.06.2018

İşlem Tarihi	İşlem Tutarı	Açıklama
26.05.2018	- 26	Alışveriş/İpek Market
27.05.2018	- 29	Alışveriş/İpek Market
27.05.2018	- 15	Alışveriş/Gökçe Fırın
27.05.2018	- 170	Alışveriş/Petshop
29.05.2018	- 68	Alışveriş/Kırtasiye
29.05.2018	+ 250	Para yatırma/Alacak
29.05.2018	- 16	Alışveriş/İpek Market
29.05.2018	- 180	Alışveriş/Spor Salonu
29.05.2018	- 48	Alışveriş/Spor Mağazası
30.05.2018	- 100	Alışveriş/Resim Kursu
31.05.2018	- 1200	Kira
31.05.2018	+ 4100	Maaş

Bu hesap özetine dair yorumlarımız:

Figure 3.9b Mathematics History-Based MEA 3-Part 2

Çocuklar şimdi de sizlerden bana haziran ayının ilk on beş gününde sırasıyla her bir gün yaptığım harcamalarım, gelir ve giderlerim için hesap özeti oluşturmamda yardım etmenizi istiyorum. Her bir gün için yaptığım harcamalarım şöyle;

220 lira ayakkabı, 80 lira market alışverişi, 200 lira spor salonu ücreti, 130 lira kedi maması ve kedi kumu, 125 lira kedilerimin aşısı, 75 lira kitapçı, 1300 lira ev kirası, 250 lira benzin, 300 lira fatura ücreti için harcadım. Ayrıca geçen ay internet üzerinden alıp geri gönderdiğim kitaplığın 280 lira olan ücreti 10 Haziran'da hesabıma geri geçti. 11 Haziran'da 5 taksitle aldığım 2500 liralık buzdolabının ilk taksitini ödedim. Ayrıca daha sonra 1200 lira kredi kartı borcumu ödedim. Altın hesabımdan 1500 liralık altın bozdurarak hesabıma aktardım. 4300 lira maaşımı aldım ve arkadaşıma verdiğim 420 lira borcu geri aldım.

HAZİRAN AYI HESAP ÖZETİ

Hesap tipi : VADESİZ HESAP		
Döviz Cinsi : TRY		
Dönem (Tarih Aralığı) :01.06.2018-01.07.2018		
İşlem Tarihi	İşlem Tutarı	Açıklama
01.06.2018		
02.06.2018		
03.06.2018		
04.06.2018		
05.06.2018		
06.06.2018		
07.06.2018		
08.06.2018		
09.06.2018		
10.06.2018		
11.06.2018		
12.06.2018		
13.06.2018		
14.06.2018		
15.06.2018		

Haziran ayı hesap özetime göre, hesabımdaki miktar kaç TL'dir? Buna göre, gelecek ay için yapacağım harcamalar konusunda bana ne önerirsiniz?

Figure 3.9c Mathematics History-Based MEA 3-Part 3

3.7. Data Analysis Procedure

The questions in the initial and follow up assessments were first evaluated as correct and incorrect responses. This preliminary examination indicated that students' responses mostly to Questions 3, 4, and 5 varied. Therefore, these responses were analyzed by content analysis. Content analysis is often used to interpret well-organized, transparent, qualitative analysis of the data including codes and categories (Neuendorf, 2002).

Students' performances during the MEA implementation were recorded in written form on activity sheets and as video and audio records. These written data were also coded through two cycles: (1) initial coding and (2) descriptive coding (Saldana, 2009). In the first cycle, the written data of each group first examined holistically, and then open codes were identified to make sense of students' conceptions. In the second cycle, these open codes were revised to create categories that were more descriptive of students' conceptions. Afterwards, the resulted codes were checked with the video and audio data. Specifically, audio and video records were not coded separately but used to make sure about students' conceptions written in the activity sheets. The code list emerged from this analysis is presented in Appendix H.

3.8. Trustworthiness and Credibility

The codes were formed taking written works of students, video and audio records into consideration and checked by two researchers for interrater reliability of research (Lincoln & Guba, 1985). Multiple sources of data helped to triangulate the findings. In addition, the researcher kept a research journal during both data collection and data analysis. Writing each step of the study transparently contributed to the credibility of the interpretations based on findings (Lincoln & Guba, 1985). Lastly, since I, the researcher, was also the teacher of the classroom, my longitudinal engagement with the students already established a good rapport with participant students, helped me to identify the useful data instances during data collection, and increased trustworthiness (Lincoln & Guba, 1985).

3.9. Researcher Role

The role of the researcher in a qualitative research is crucial since the researcher collects and analyze the data and so become a part of the data (Creswell, 2007). Furthermore, researchers of most of the qualitative studies were primary instrument of the data collection, and so researchers' being aware of their roles in data collection is important in qualitative research to improve the credibility (Denzin & Lincoln, 2003).

My roles as a researcher were preparing of data collection instruments, coding and analyzing the data collected via activity sheets, initial and follow-up assessments, video and audio records to uncover the revealing patterns and concepts. I kept a personal research journal during the data collection and analysis process to document my actions. The video records and the process of checking with another researcher were also helpful to control the researcher bias.

Besides, as a mathematics teacher, my role was implementing mathematics history-based MEAs. Although there is a potential bias on my role as a teacher, I achieved the balance through being transparent and nonjudgmental in my actions. In the implementation process of activity sheets, students were active participants and source of data, worked and discussed with their groupmates, and shared their findings in a whole class discussion. As a mathematics teacher, I solely guided students at the beginning of each implementation to make clear the organization of lesson and during whole class discussion with peers after they finished working with groupmates. Nonetheless, being a teacher have aided me in understanding data collection process and revealing students' understanding while analyzing the data since having experience is important and necessary to scaffold students properly instead of letting them work alone and solely observing them as they work. Being a mathematics teacher have also aided me in the preparation of data collection instruments since I had a teaching experience on students' difficulties in understanding and making sense of negative integers. Still, as a teacher and researcher, it was challenging to integrate mathematics history into modeling activities. This challenge was overcame through regular meetings with an expert in mathematics education and modeling.

CHAPTER 4

FINDINGS

This study investigates the role of mathematics history-based MEAs on the 7th grade students' understanding of negative integers. In this chapter, the findings of the initial and follow-up assessments of students' understanding and making sense of integers and their engagement in mathematics history-based MEAs are presented.

4.1. Initial Assessment of Students' Understanding of Integers

Students began to know integers in the 6th grade but in the 7th grade, before the implementation of activities, any formal instruction and prior teaching about integers did not provided. Students began to study on negative integers with the implementation of mathematics history-based MEAs. Therefore, initial assessment showed the students' initial state about negative integers and how they make sense of negative integers.

Questions 1, 2, and 6 in the initial assessment were evaluated based on correctness. All students selected positive integers which have plus sign or no sign in front of the integer. All students selected negative integers which have minus sign in front of the number. All students expressed a real-life situation with integers correctly.

Questions 3, 4, and 5 asked students to explain their reasoning; therefore, their responses were categorized qualitatively. Table 4.1, Table 4.2 and Table 4.3 presents students' response categories and their frequencies for the Questions 3, 4 and 5 respectively.

Table 4. 1 Students' response categories and frequencies for the Question 3

Questions	Response Categories	Frequencies
3.1. Write a negative integer.	Minus sign in front of the number	16
	Less than zero	12
	Debt	6
3.2. How you explain this integer is a negative integer to another person?	On the left of the zero on number line	3
	The flat below ground floor	2
	Subtraction operation	2
	Temperature less than zero	1
	Below sea level	1
	Loss	1

As seen in Table 4.1 in the question three, students were expected to write how they explain a negative integer to another person. Most of the students (16 students) focused on symbolic representation and described a negative integer with the minus sign in front of the number. 12 of the students indicated that negative integers are less than zero, but they did not provide any other explanation to support their understanding of why negative integers are less than zero.

In the question four, students firstly were asked to determine whether the unknown addend should be a positive or a negative integer by explaining their reasoning. All students determined the unknown addend as a positive integer for the operation of 3+__=7. 18 students indicated that addition of two positive integers is equal to a positive integer by focusing on signs of the result and the given addend (See Table 4.2).

Table 4. 2 Students' response categories and frequencies for the Question 4

Questions	**Response Categories**	**Frequencies**
4.1. Determine whether the unknown addend should be a positive or negative integer. 4.2. Explain your reasoning.	Positive integer. The result and the given addend are positive integers. The difference of seven and three is equal to four. If we add four to three, the result is seven. Four is a positive integer.	10
	Positive integer. If the unknown addend were a negative integer, the result would be a negative integer and less than three, but the result is greater than three.	9
	Positive integer. Addition of two positive integers is equal to a positive integer.	8
	Positive integer. The negative integers neutralize the positive integers.	1
4.3. What is the value of unknown integer? Explain your operations.	Addition of three and four is equal to seven.	13
	The difference of seven and three is four.	12
	What should we add to three to get seven? Four.	3
	When we move on four units from three on number line, we find seven.	2
	If we have three oranges, we should add four more oranges to have seven oranges.	1

As seen in the table above, ten of these students found the difference of the result and the given addend by focusing on the result of the operation. Moreover, less than half of the students (eight students) explained that if the unknown addend had been a negative integer, the result would have been a negative integer and less than three, but the result is greater than three. After this part, students were asked to find the value of the unknown addend. 13 students took the result of the operation into consideration and they solely showed that the addition of three and four is equal to

seven. 12 students found the value of unknown addend by subtracting the given addend from the result (i.e., 7-3=4).

In the question five, likewise question four, students firstly were asked to determine whether the unknown addend should be a positive or a negative integer in the question of 3+__=1 by explaining their reasoning. The unknown addend should be a negative integer in question five, and as seen in Table 4.3 below all students answered accordingly.

Table 4. 3 Students' response categories and frequencies for the Question 5

Questions	**Response Categories**	**Frequencies**
5.1. Determine whether the unknown addend should be a positive or negative integer.	Negative integer. Addition of three and a positive integer cannot be equal to one.	15
	Negative integer. The difference of three and minus two is equal to one.	6
	Negative integer. The result (one) is less than three.	6
5.2. Explain your reasoning.	Negative integer. The negative and positive integers neutralize each other, and we subtract.	2
	Negative integer. The addition of three and minus two is equal to one.	1
5.3. What is the value of unknown integer? Explain your operations.	Addition of three and minus two is equal to one.	12
	The subtraction of three from one is equal to minus two.	6
	The difference of three and minus two is equal to one.	5
	The negative and positive integers neutralize each other.	3

Table 4.3 (continued)

	We subtract one from three, then we write minus sign in front of the two.	2
	When we move on two units left from three on number line, we find one.	2
	Addition of three and minus four is one, the unknown is minus four.	2

As seen in the Table 4.3, more than half of the students (15 students) explained their reasoning by focusing on the result of the operation. They stated that addition of three and a positive integer cannot be equal to one. Six students stated that the difference of three and minus two is equal to one. Moreover, six students expressed that the result (one) is less than three that's why the unknown addend is a negative integer. After this part, students were expected to find the value of the unknown addend. Most of the students (12 students) focused on the result of the operation and they expressed that addition of three and minus two is equal to one. Six students found the value of unknown addend by subtracting three from one. On the other hand, five students stated that the difference of three and minus two is equal to one.

4.2. Students' Engagement in Mathematics History-based MEAs

In this section, I present the findings related to students' engagement in math-history-based MEAs during class time. As mentioned before, students worked in groups of 3-4 and recorded their work on worksheets. I present students' work in each MEA separately.

4.2.1. MEA 1: Students' Making Sense of Negative Integers

In the first activity, students firstly introduced negative integers relating integers to real life situations and showing how and why integers work in solving mathematics problems from the past to present. In the implementation of this activity, the students firstly discussed with their groupmates, and they decided on their answers and

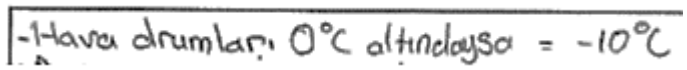
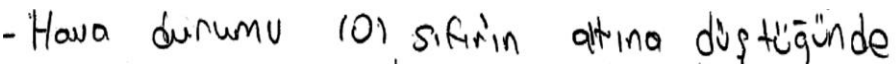
examples together. Afterwards, they shared their answers with other groups in a whole class discussion.

The first two questions of the activity aimed to discover 7th graders thoughts about the need for negative integers. For this aim, students were firstly asked to give examples how and in what situations people use the negative integers in daily life. Their answers and examples reflected following four aspects:

- to represent weather temperature,
- to represent debt and loss,
- to compare with a reference point,
- to show flats in elevators.

Below, I present two statements of the groups who illustrated negative integers in the context of the temperature. Table 4.4 shows the answers of these groups.

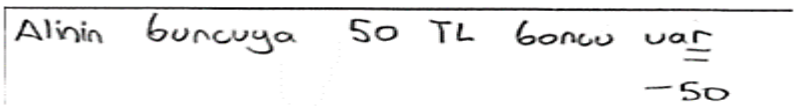
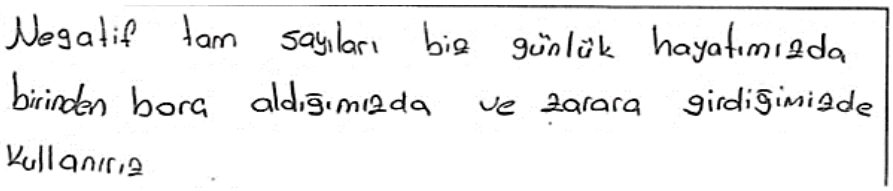
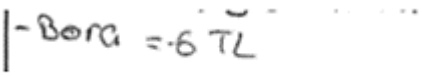
Table 4. 4 The answers of students for the first question in the context of temperature

Group 7	
Group 8	

As seen in these examples, students used negative integers to communicate about cold weathers; that is below the Zero Celsius.

Moreover, some of the groups approached to using negative integers in daily life in terms of the context of debt and loss. Below, in the Table 4.5, the answers of these groups are presented.

Table 4. 5 The answers of students for the first question in the context of debt and loss

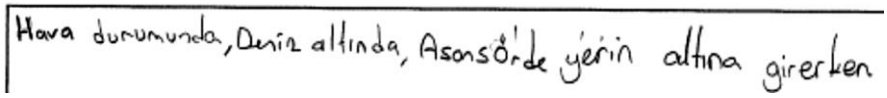
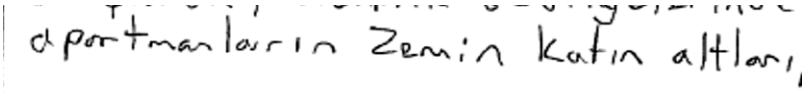
Group 3	
Group 4	
Group 7	

These groups used negative integers to explain the money a person loses or an amount of money that is owed. To illustrate, one of the students from Group 3 stated that "*for instance, when we took money from a friend as debt, we demonstrate this money by using negative integers*." Group 4 explained their answers solely with the words in a sentence while Group 3 and Group 7 showed their answers by writing a negative integer.

In addition to temperature and debt and loss contexts; some of the groups explained the use of negative integers in daily life by making comparison with a reference point and focusing on ordinality.

In the Table 4.6, I present the statements of groups who illustrated negative integers in the context of below sea level and the apartment flat that is under the ground floor.

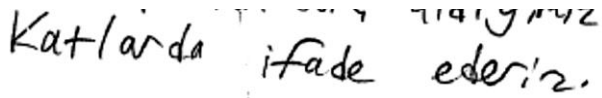
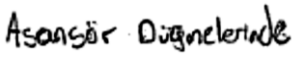
Table 4. 6 The answers of the students for the first question in the context of making comparison with a reference point

Group 1	
Group 5	

As seen in the Table 4.6, for Group 1 the sea level is a reference point (zero), and below sea level is represented with the negative integers. Similarly, Group 5 accepted the ground floor as a reference point (zero), and they represented the flats under the ground floor with negative integers.

Furthermore, I present two statements of the groups below who illustrated negative integers to show flats in elevators. Table 4.7 presents the statements of these groups.

Table 4. 7 The answers of students for the first question in the context of elevators

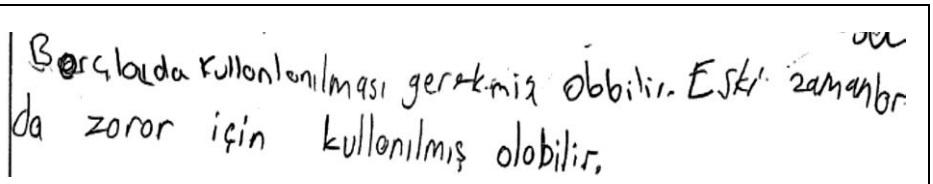
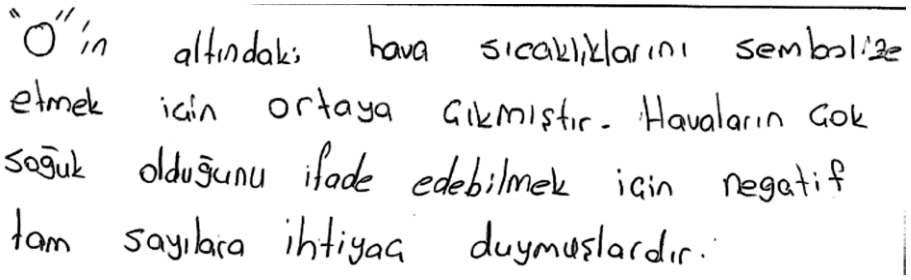
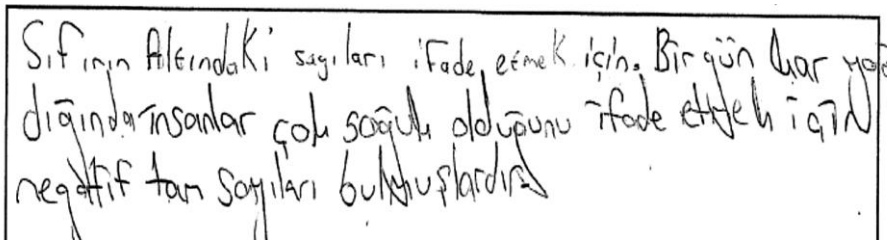
Group 2	
Group 6	

As shown in Table 4.7., Group 2 and Group 6 took symbolic representations of negative integers into consideration and they gave examples in terms of their daily life observations.

In the second question, students were asked what they think about the origins and arrival of negative integers and why people need these integers. Their answers and examples reflected following aspects: (1) Contexts, and (2) People. Most of the groups stated contextual reasons about the use of negative integers and why people need these integers. Below, I present the three statements of groups which explained

the use of negative integers and why people need these integers in terms of contexts. Table 4.8 shows the answers of these groups.

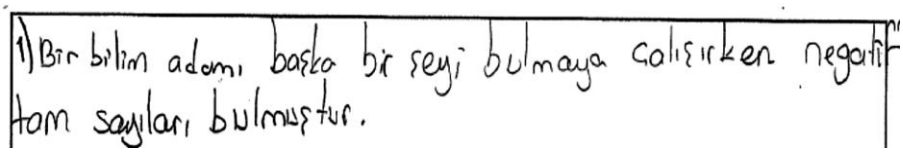
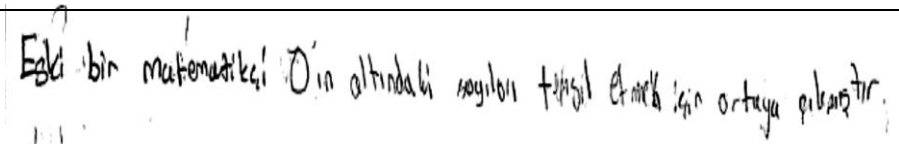
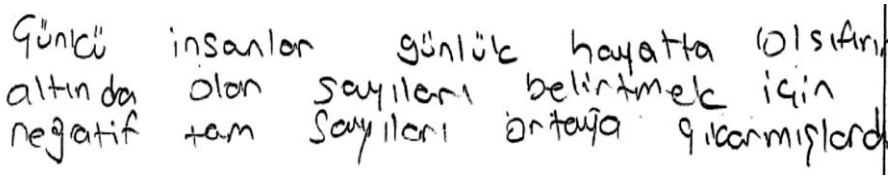
Table 4. 8 The statements of groups which explained the use of negative integers and why people need these integers in described contexts

Group 2	
Group 4	
Group 5	

As seen in Table 4.8, most of the students took contexts such as very cold weather, and debt and loss into consideration to explain the use of negative integers and why people need these integers. These students stated that people needed negative integers for their daily life requirements. For instance, one of the students from Group 5 stated that "*one day, when the weather was too cold and snowy, people used negative integers to express the very cold weather*."

In addition to contextual aspect, some of the groups indicated that people such as mathematicians, scientists, folks needed and invented negative integers to illustrate the values less than zero. In the Table 4.9, I present the statements of these groups.

Table 4. 9 The statements of groups explaining the use of negative integers and why people need integers

Group 1	 1) Bir bilim adamı başka bir şeyi bulmaya çalışırken negatif tam sayıları bulmuştur.
Group 6	 Eski bir matematikçi Ö'in altındaki sayıları tanımlamak için ortaya çıkmıştır.
Group 8	 Günlü insanlar günlük hayatta 0'ın altında olan sayıları belirtmek için negatif tam sayıları ortaya çıkarmışlardır.

As seen in these statements, some students explained the use and need for negative integers in terms of people and their studies on mathematics. They stated that people needed negative integers to represent the integers less than zero.

The second part of the MEA1 aimed to reveal 7th graders thoughts about the use of negative integers in a mathematics problem. In this part, groups were expected to write their answers as a letter to help a peer student's school magazine. Firstly, students were given a text about Diophantus on their worksheets and he was introduced to students. After, they have examined a mathematics problem which was called as "absurd" by Diophantus. They were expected to consider on the possible reasons why this problem was called as absurd while trying to find a solution for this problem.

All groups' common thought about why a given problem was called as absurd is that because of the lack of knowledge of negative integers. They had two different aspects:

- A question may be called as absurd if they don't have the knowledge of negative integers,
- A question may be called as absurd if one can't find a solution for this mathematics question.

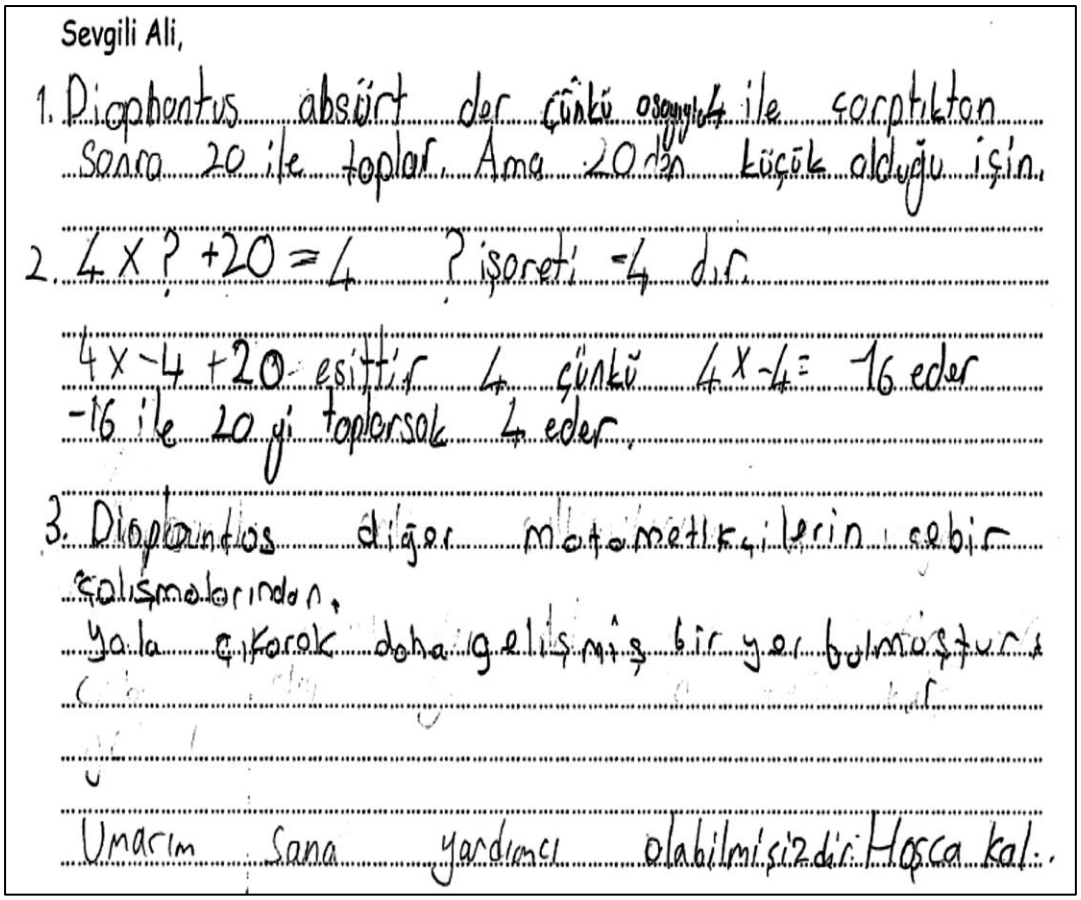
After that, students were asked to find the value of unknown which was demonstrated with a question mark on the given problem that is 4 x ? + 20 = 4. All groups inserted that they used trial and error method. The students followed two different ways while using trial and error method:

- Some of the groups stated that the unknown number can't be a positive integer, so they accepted the unknown as a negative integer. They tried negative integers respectively to find the value of unknown.
- Some of the groups firstly tried zero and positive integers respectively, but they didn't find the correct result. Therefore, they tried negative integers to find the proper value for the unknown.

Lastly, groups discussed and wrote their interpretations about the field of mathematics considering the given information that Diophantus gathered the algebra studies before his era and developed these studies. The answers of the groups for this question were shaped considering three aspects:

- mathematics is a continuously evolving field,
- a single math idea was developed by contribution of many peoples thinking and studies, and
- many different mathematicians and cultures contributed to mathematics science.

Below, in the Figure 4.1 and Figure 4.2, I present the answers and letters of two groups having ideas related to given aspects above.


Sevgili Ali,

1. Diophantus absürt der çünkü soruyu 4 ile çarptıktan sonra 20 ile toplar. Ama 20'den küçük aldığı için.

2. 4X ? +20 = 4 ? işareti -4 dür.

4X-4 +20 eşittir 4 çünkü 4X-4 = 16 eder -16 ile 20 yi toplarsak 4 eder.

3. Diophantos diğer matematikçilerin cebir çalışmalarından yola çıkarak daha gelişmiş bir yer bulmuştur.

Umarım Sana yardımcı olabilmişizdir Hoşca kal..

Figure 4.1. The answers and letter of Group 2 in MEA 1

As seen in the Figure 4.1, students of Group 2 stated that the problem was called as absurd since the unknown number multiplied by 4 and added by 20 but the result is less than 20. They found the unknown integer as -4 via trial and error method. Moreover, they expressed that Diophantus developed his studies with the contribution of other mathematician's studies on algebra.

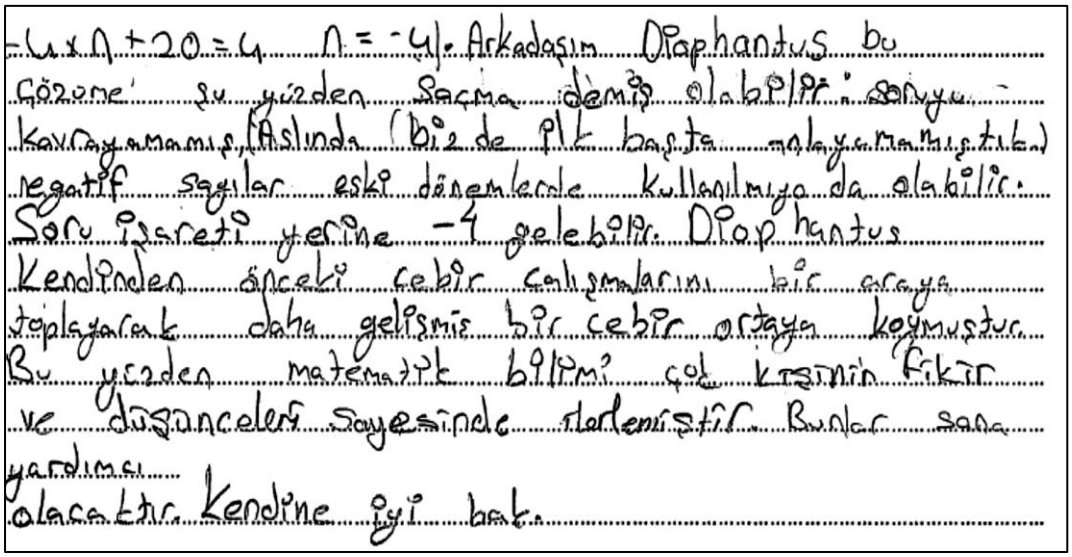


Figure 4.2 The answers and the letter of Group 7 in MEA 1

As shown in Figure 4.2, students of Group 7 expressed that negative integers may not be used, and known in the past, and so the question does not make sense; that's why the given problem was called as absurd. They also found the unknown integer as -4. Group 7 stated that mathematics developed with the help of many people's opinions and Diophantus collected and improved the algebra studies from the past.

Lastly, as mentioned and shown in the Table 3.2 in the previous chapter, the purpose of mathematics history-based MEA1 was improving students' understanding of why people need number sense and negative integers. The implementation of mathematics history-based MEA1 helped students to notice the need for negative integers via their own examples about the use of negative integers in daily life. Students had chance to reveal and improve their understanding of negative integers with the help of group work and whole class discussion. They created a model addressing the need of using of negative integers both in real life and in mathematics problems through modeling process. With the help of model-eliciting activity, they developed a more meaningful rationale about the historical development of mathematical contents such as the use of negative integers.

4.2.2. MEA 2: Students' Identifying Positive and Negative Integers

The second activity firstly introduced Chinese numeral system in history to 7th graders through the introduction of YouTube channel of a hypothetical peer student. A number system was given including red and black sticks representing Chinese numerals. The students were expected to compare our base-ten number system and past Chinese number system while interpreting the meanings of sticks for each number. Their interpretation reflected following aspects:

- The Chinese numerals which were demonstrated with sticks look like roman numerals.
- The learning and use of the Chinese numerals is difficult.
- The similarities of given number system with our present number system are that (i) the quantity of numbers increase one by one, and (ii) each number has a different demonstration.

Below, in the Table 4.10, I present the two statements of the groups interpreted these aspects.

Table 4. 10 The interpretations of the two groups about the Chinese number system

Group 4	Biz bunu romen rakamlarına benzettik. Biz bu sayıları öğrenmenin zor olduğunu düşünüyoruz. Bizce bu sayıları öğrenmek bizim sayılarımızı öğrenmekten daha zor.
Group 1	Kırmızı çubuklar çinlilerin sayı birimi bunlar artıkça şekilleri değişir. Aynı bizim sayılarımız gibi.

As seen in the Table 4.10, the students associated the Chinese numerals with the roman numerals and our present number system in terms of demonstration of the

numbers and increment of the numbers one by one. They stated that the Chinese numerals including red and black sticks is more complicated than our number system. After this part, students were expected to write the given Chinese numerals via our number system while they were discovering that red sticks represent positive integers and the black sticks represent negative integers. The zero was represented with an empty digit in Chinese number system. All students correctly wrote the given Chinese numerals with our numerals. Below, I present the answer of one of the groups for this part.

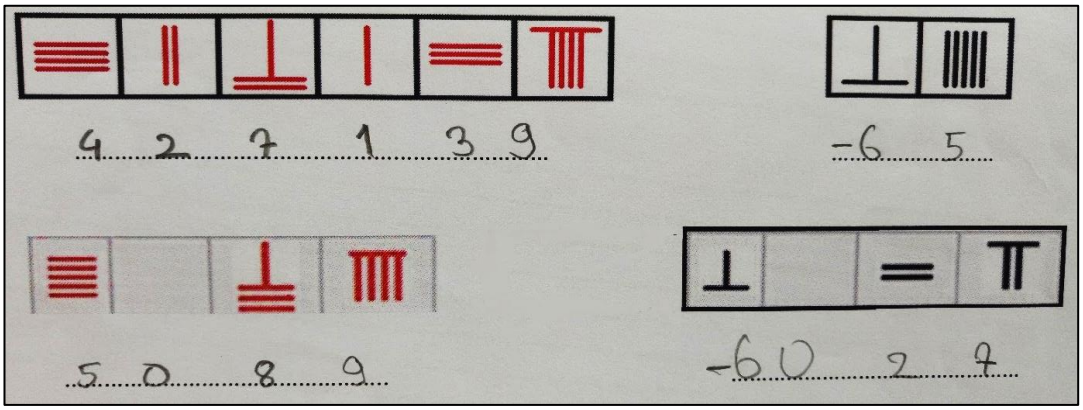


Figure 4.3 The writings of the Chinese numerals with present numerals by students of Group 6 in MEA *2*

The Figure 4.3 shows that, with second part of MEA 2, students discovered an old number system. They had a chance to analyze how negative integers, positive integers, zero, and digit values of numbers were differentiated and demonstrated in a different number system.

In the last part of the activity, students created their own number system via group work. They explained their number system by writing a letter in detail. Students used their creativity and knowledge of integers while forming a new number system in terms of following three aspects: (i) demonstration of numbers, (ii) representation of positive and negative integers, and (iii) representing zero.

Below in the Table 4.11, I present how students' approaches to given three aspects and they illustrated a new number system in their letters.

Table 4. 11 The characteristics of number systems which were constructed by students and explained in their letters

Demonstration of Numbers	**Representation of Positive and Negative integers**	**Representation of Zero**
• Regular geometric shapes and regular convex polygons. • Bullions, emeralds and coal, inspired by a computer game. • Symmetrical demonstration of numerals. • Horizontal, vertical, inclined sticks and operational symbols. • Interbedded circles. Number of circles increase one by one for each number. • Regular geometric shapes, regular convex polygons, regular star polygon and heart. • YouTube play button, sticks and rectangles. • Flowers, leaves and fruits.	• Using different colors. • Using shapes such as star for positive integers and moon for negative integers. • Using symbols such as plus and minus. • Using solely a dot before negative numbers. • Using fruits for negative numbers and the number of fruits increase respectively for each number.	• Interbedded circles. • A circular coal. • A circular region. • A circle. • A tree with no fruit.

Below, in the Figure 4.4, I present the number system and letter of Group 2 which demonstrated the numbers by regular geometric shapes and regular convex polygons.

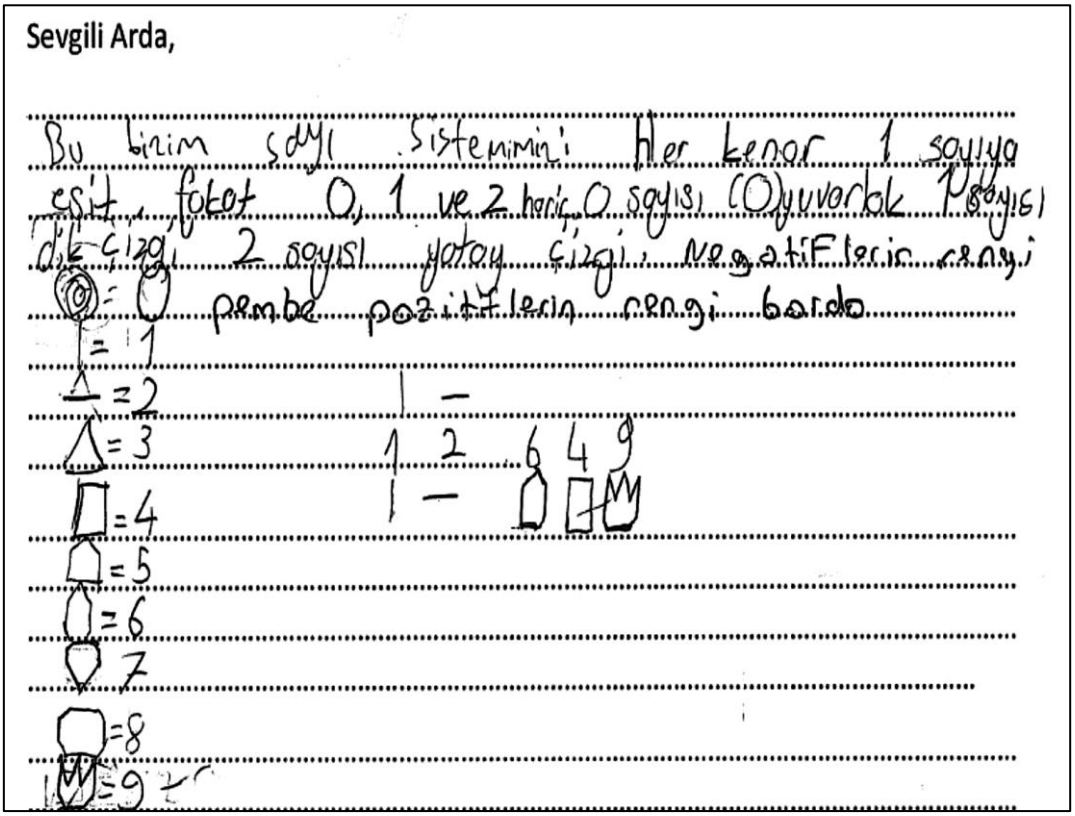


Figure 4.4 The number system created by Group 2 in MEA 2

As seen in the Table 4.11 and Figure 4.4, students discovered the characteristics of a number system while analyzing a different number system in the history that was presented in the MEA. They constructed a new number system and identified demonstration of numbers, differentiation of positive integers, negative integers and zero in their own number system. The students also illustrated a number using their own number system in which Group 2 used a vertical line representing 1, a horizontal line representing 2 and a circle representing 0. In this number system, after number 2, other numbers were associated to the number of sides of a polygon so, students used a triangle representing 3, a square representing 4, a pentagon for 5, and so on. They differentiated positive and negative integers by using different colors in which they used pink representing negative integers and claret red representing positive

integers. One of students from Group 2 commented on this characteristic of number system as "*they used different colors to demonstrate negative and positive integers like Chinese used black and red sticks."*

Below, I also presented the number systems of Groups 1, 3 and 6 respectively in the Figure 4.5, Figure 4.6 and Figure 4.7.

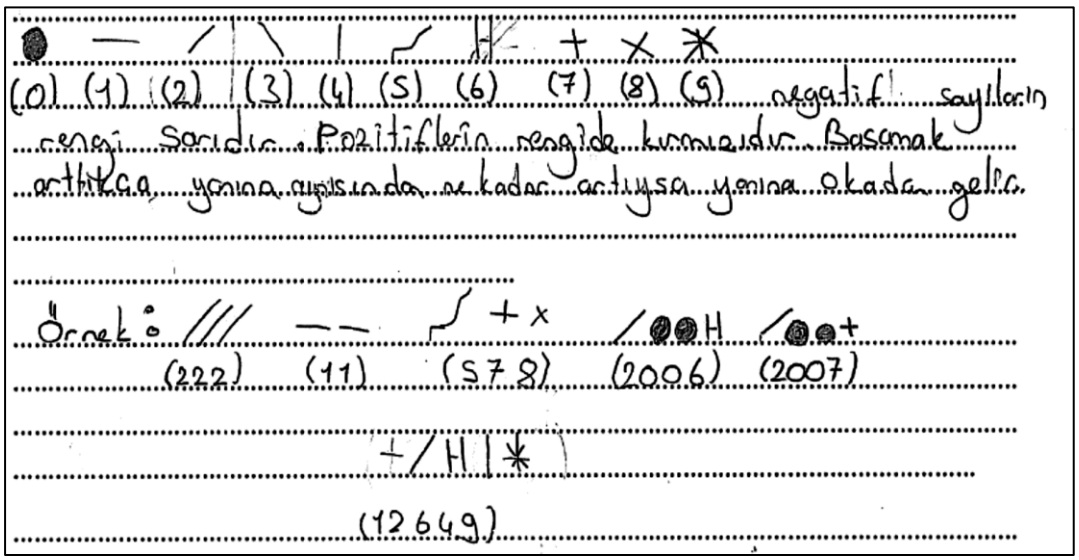


Figure 4.5 The number system created by Group 1 in MEA 2

As seen in the Figure 4.5, students of Group 1 used horizontal, vertical, inclined sticks and operational symbols in their number system to show numerals as Chinese showed the numerals by using sticks in the past. These students represented zero by painting the inside of 0. Likewise Group 2, they used a horizontal stick representing 1, inclined sticks representing 2 and 3, a vertical stick representing 4. After number 4, they used different demonstrations including two vertical lines representing 6 and plus sign representing 7. They also used two different colors to differentiate positive and negative integers in which yellow representing negative integers and red representing positive integers.

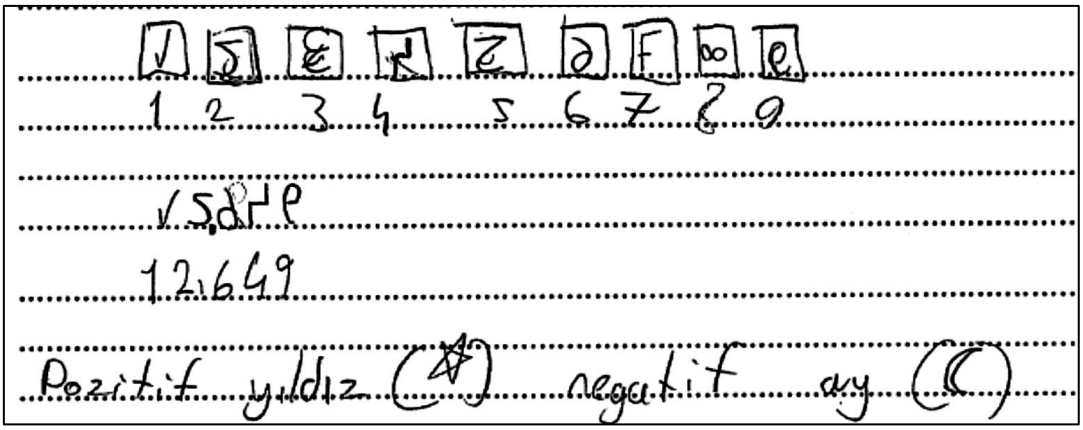


Figure 4.6 The number system created by Group 3 in MEA 2

As seen in the Figure 4.6, Group 3 demonstrated numbers taking the symmetrical demonstration of numbers into consideration. To illustrate, they used mirror symmetry in the representation of 3, 6 and 9 by using their science lesson learnings. Besides, they used distinct demonstrations to show other numbers instead of using mirror symmetry. For instance, they used symbol of infinite representing 8. As distinct from Group 1 and 2, these students used two different symbols to differentiate negative and positive integers. They used a star representing positive integers and a moon representing negative integers.

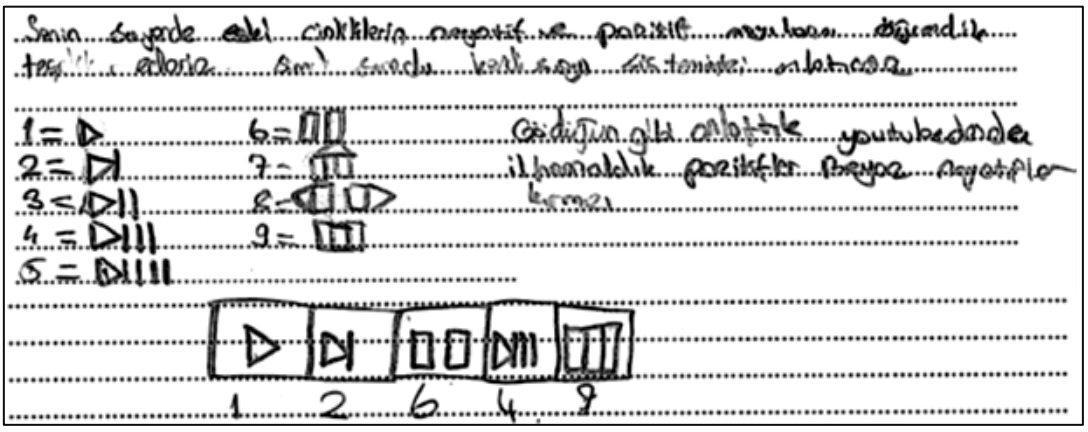


Figure 4.7 The number system created by Group 6 in MEA 2

As seen in the Figure 4.7, Group 7 created a number system by inspiring Youtube play button, so they showed numbers by using triangles, sticks and rectangles. They used a triangle representing 1, a triangle and a line representing 2, a triangle and two

lines representing 3, and so on. They increased the number of lines one by one until representation of 6. They used two rectangles to represent 6. After number 6, they used triangles and rectangles to show the numbers. To illustrate, they used three rectangles representing 9. Likewise Group 1 and 2, this groups used two different colors in which white color representing positive integers and red color representing negative integers.

As shown in the Figures 4.4, 4.5 and 4.7, Group 1, 2 and 6 differentiated positive and negative integers by using two different colors as the Chinese differentiated negative and positive integers with colors. Besides, as shown in the Figure 4.6, students of Group 3 differentiated positive and negative integers by using shapes which are star for positive integers and moon for negative integers as we showed positive and negative integers with plus and minus signs in front of the numbers. Moreover, students of Group 1 and 2 included the zero in their number systems while Group 3 and 6 didn't show and provide any explanation for the zero. The digit concept was mentioned solely in the number system of Group 1, but they didn't provide any detail explanation.

To summarize, the purpose of mathematics history-based MEA2 was improving students' understanding of number sense and making sense of how to identify positive and negative integers. With the help of this model-eliciting activity, students constructed their own number system and they had a concrete experience on characteristics of a number system while identifying integers. This experiences and modeling process allowed them to consider the way in which positive and negative integers could be distinguished and the role of zero in constructing negative integers.

4.2.3. MEA 3: Students' Understanding of the Use of Negative Integers in Real-Life

In the first part of the MEA3, information about accountancy was given when making connection with the history via expression of a guest accountant. The accountancy, trade activities of people and the use of integers to solve problems related to account and trade activities in the past were mentioned through arithmetic book of Liu Hui.

The Sign (Zhengfu) Rule was introduced to students from the arithmetic book of Liu Hui and they were expected to make connection with the previous activity. Chinese people demonstrated income and loss with black and red sticks to differentiate negative and positive integers. Red sticks represent positive integers and incomes while black sticks are representing negative integers and loss. The red and black sticks neutralize each other in the operations with integers.

After having whole class discussion on given explanations, students were expected to make connection and comparison with how we identify and differentiate integers and operate with them.

Below, I present statements of two groups in the Table 4.12.

Table 4. 12 The statements of students about The Sign (Zhengfu) Rule

Group 6	Evet Biliyorduk geçen derste öğrendik Biz tam sayıları O aşağısı negatif rakamları O yukarısını Pozitif rakamlar diyoruz Rakam olarak ifade ediyoruz
Group 8	Eskiden negatif ve pozitif tam sayılarının kırmızı ve siyah çubuklarla ifade edildiğini bilmiyorduk. Biz şimdi ise negatif ve pozitif tam sayıları (+) artı ve (-) eksi işaretleriyle gösteriyoruz. Negatifleri (-) eksi pozitifleri ise (+) artı ile ifade ediyoruz.

As seen in the Table 4.12, students made connection with the previous activity while discovering how Chinese represented income and loss with black and red sticks. In the second part of the mathematics history-based MEA 3, a bank statement was given in the activity sheet. Students were expected to make interpretations with their groupmates depending on given bank statement. The answers for this question showed how students interpreted the positive and negative integers within contextual

representations of minus and plus signs. Their interpretations reflected following five aspects:

- Minus sign represents expenses and plus sign represents incomes.
- Minus sign represents expenses and plus sign represents debt owed.
- Minus sign represents debt and plus sign represents income.
- Minus sign represents expenses and plus sign represents profit.
- Minus sign represents cost and plus sign represents the money got from the bank.

Below, in the Figure 4.8, I present the statement of one of the four groups who stated that "the minus sign represents expenses and the plus sign represents incomes".

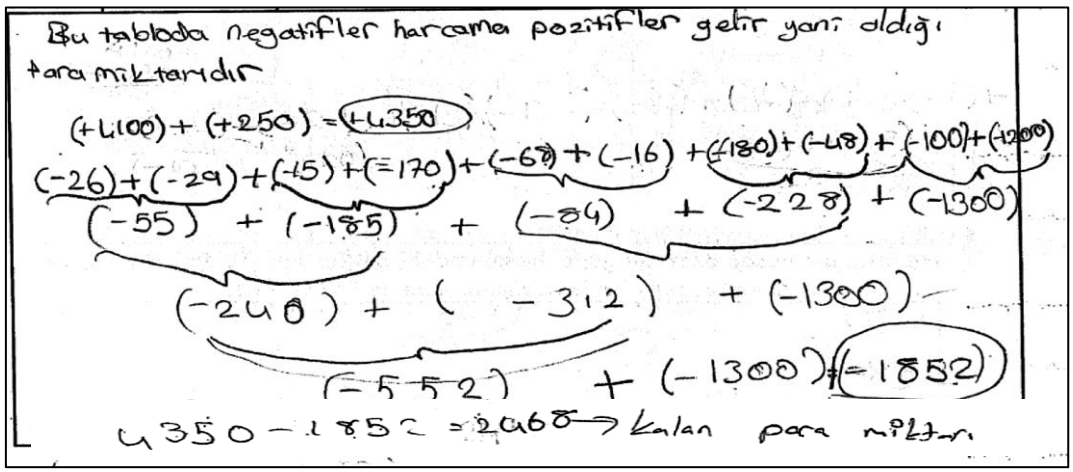


Figure 4.8 The interpretation of the Group 7 in MEA 3

As seen in the Figure 4.8, students stated the negative integers in the bank statement as expenses and the positive integers as incomes, in fact half of the participant students shared this idea. These groups followed four steps while interpreting the bank statement: (i) they firstly added incomes and (ii) secondly added expenses separately, (iii) thirdly they found the difference of incomes and expenses, (iv) they interpreted the difference as the remainder of the money. Thus, they operated with

income and expenses separately. As stated in the introduction chapter, the scope of current study was not understanding operations with integers, it solely was students' understanding and making sense of negative integers. Therefore, the operational signs in the operations did not focused.

Moreover, students in Group 3 interpreted that "minus sign represents expenses and plus sign represents debt owed". Below, I present the comments of Group 3 in the Figure 4.9.

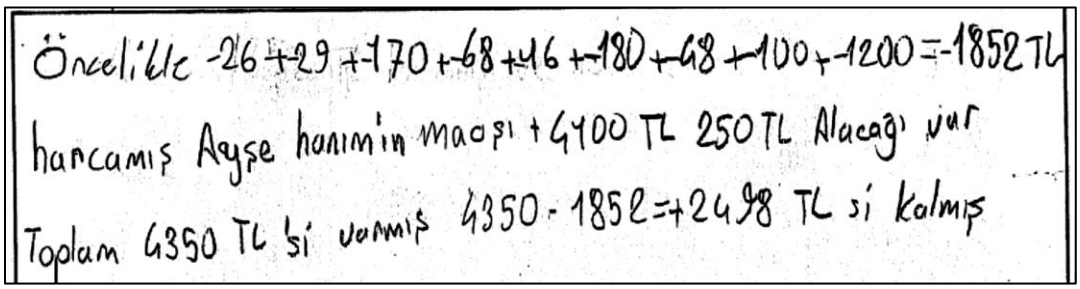


Figure 4.9 The solution of Group 3 in MEA 3

As shown in the Figure 4.9, students of Group 3 firstly added the negative integers on the bank statement and interpreted the result as expense. Afterwards, they added the positive integers and interpreted the result of addition as debt owed. Lastly, they found the difference of debt owed and expenses and interpreted the result as the remainder of the money. Although they represented expenses with negative integers, they operated with both income and expenses as positive integers and reflected the decrease on the balance as subtraction of the expenses from the income.

In addition, as can be seen in the Figure 4.10 below, some of the students interpreted minus sign as debt and plus sign as income.

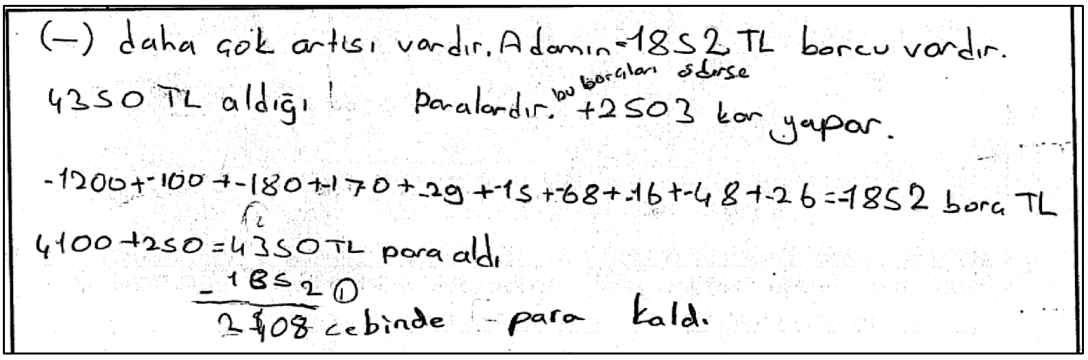


Figure 4.10 The solution of the Group 1 in MEA 3

As shown in the Figure 4.10, students of Group 1 interpreted negative integers as debt and added negative integers to find the total debt. Also, they considered positive integers as income and added these integers to find total incoming money. They showed total debt with minus sign and total income with plus sign. Then, they subtracted total debt from total income and stated the result as the remaining money. They specified that if the owner of bank statement pays debts, the remainder of the money will be profit. As expected, students were reflected how they associate the negative integers with everyday life contexts such as debt through modeling.

Furthermore, students of Group 2 interpreted minus sign as debt and plus sign as profit. Below, I present the comments and operations of these students in the Figure 4.11.

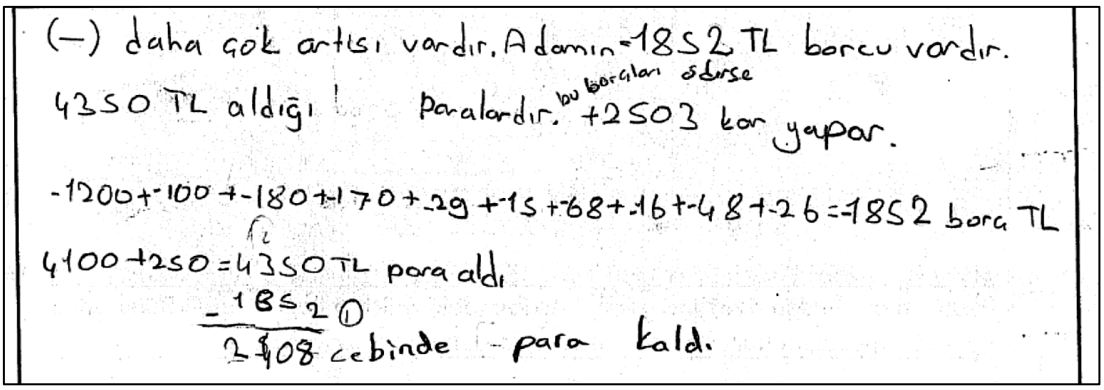


Figure 4.11 The interpretations of Group 2 in MEA 3

As seen in the Figure 4.11, these students stated negative integers as expenses and the positive integers as profit. Similar to Group 1 and 3, they added the negative and positive integers separately and then they found the difference of expenses and incomes. The addition of negative and positive integers separately is a commonly used strategy (Akyuz, Dixon & Stephan, 2012), but this situation might also be resulted because of the students' experiences on how Chinese showed these integers with two different colors. Besides, they interpreted that the owner of bank statement spent most of the income for paying the rent of her home and the least for market shopping. Moreover, as expected students were reflected how they associate the negative integers with everyday life contexts such as debt through modeling.

Moreover, students of Group 8 interpreted minus sign as cost and plus sign as the money got from the bank. Below, in the Figure 4.12, I present the interpretations and operations of Group 8.

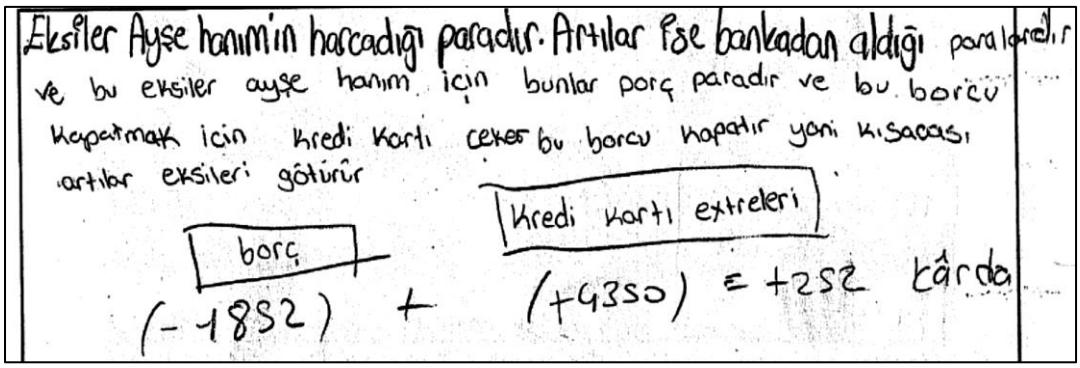


Figure 4.12 The interpretations of Group 8 in MEA 3

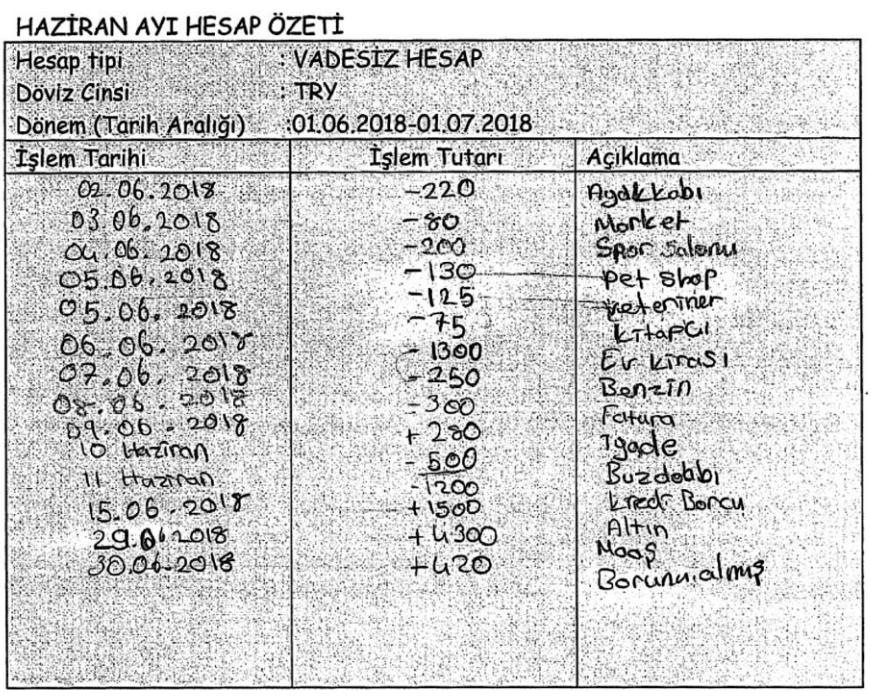
The Figure 4.12 shows that students of Group 8 stated the negative integers as expense, but they accepted the positive integers as the money which is gotten from the bank; that is, a kind of debt according to them. Like other groups they added the negative and positive integers separately. On the other hand, this group followed a different way to find the difference of total expenses and money. They took the signs of integers into consideration and stated that negative and positive integers neutralize each other in addition operation. At the end, they interpreted the result as profit. Moreover, as mentioned in the introduction chapter, the scope of current study was not understanding operations with integers, it solely was students' understanding and

making sense of negative integers. Therefore, the operational signs in the operations did not focused.

In the second part of the mathematics history-based MEA 3, most of the groups (except one group) added positive and negative integers correctly. On the other hand, solely two groups found the remainder money correctly since most students made a mistake in their final operation which is the subtraction of expenses from incomes. In the third part of this activity, students were expected to form bank statement of a person by taking given monthly incomes and expenses into consideration and give advice to this person in terms of her expenses for the next month.

Below, I present the bank statements formed by Group 7 and Group 8 in the Table 4.13.

Table 4.13 The bank statements of two groups and their representations of given incomes and expenses with integers

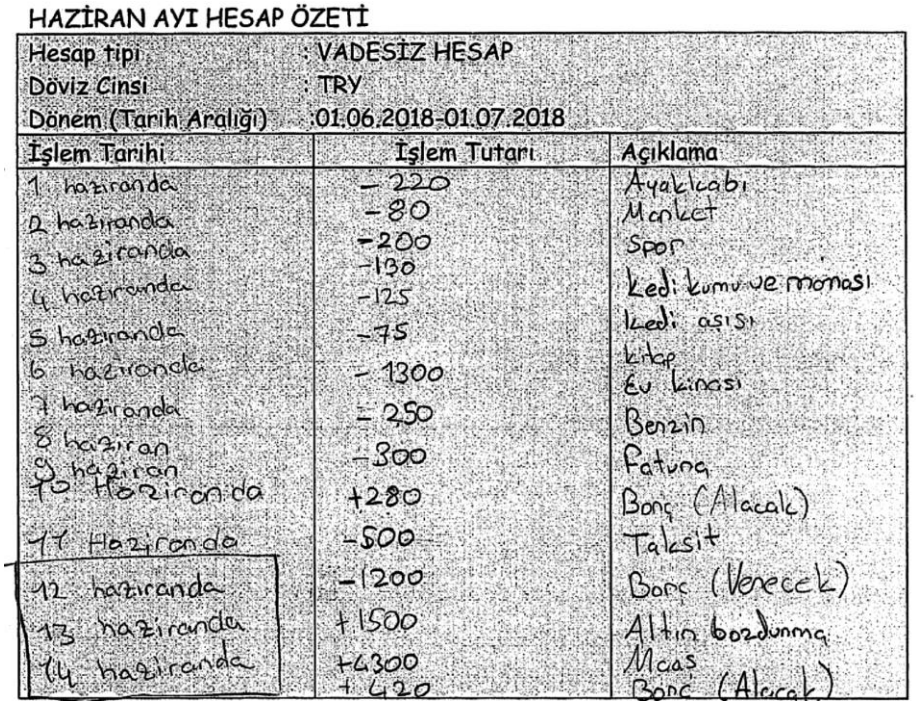
Group 7	

HAZİRAN AYI HESAP ÖZETİ

Hesap tipi : VADESİZ HESAP
Döviz Cinsi : TRY
Dönem (Tarih Aralığı) :01.06.2018-01.07.2018

İşlem Tarihi	İşlem Tutarı	Açıklama
02.06.2018	-220	Ayakkabı
03.06.2018	-80	Market
04.06.2018	-200	Spor Salonu
05.06.2018	-130	Pet shop
05.06.2018	-125	veteriner
06.06.2018	-75	Kitapçı
07.06.2018	-1300	Ev kirası
08.06.2018	-250	Benzin
09.06.2018	-300	Fatura
10 Haziran	+280	İyade
11 Haziran	-500	Buzdolabı
15.06.2018	-1200	kredi Borcu
29.06.2018	+1500	Altın
30.06.2018	+4300	Maaş
	+420	Borunu almış

Table 4.13 (continued)

Group 8	

HAZİRAN AYI HESAP ÖZETİ

Hesap tipi : VADESİZ HESAP
Döviz Cinsi : TRY
Dönem (Tarih Aralığı) : 01.06.2018-01.07.2018

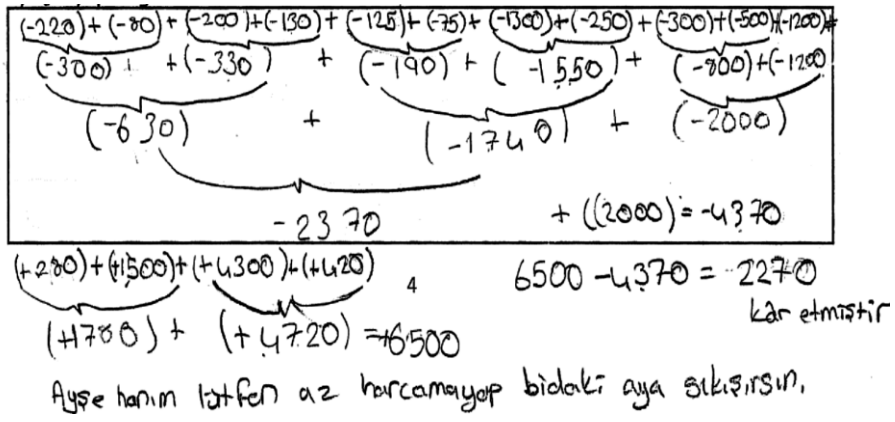
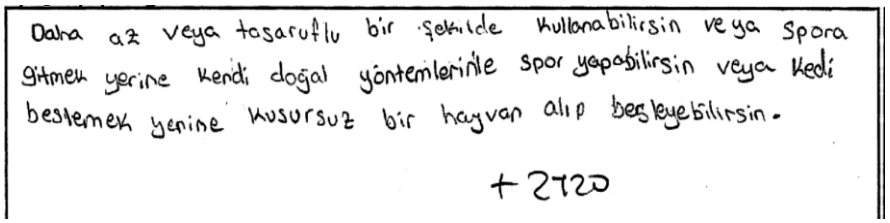
İşlem Tarihi	İşlem Tutarı	Açıklama
1 haziranda	-220	Ayakkabı
2 haziranda	-80	Market
3 haziranda	-200	Spor
4 haziranda	-130	Kedi kumu ve monası
5 haziranda	-125	Kedi aşısı
6 haziranda	-75	Kitap
7 haziranda	-1300	Ev kirası
8 haziran	-250	Benzin
9 haziran	-300	Fatura
10 Haziranda	+280	Borç (Alacak)
11 Haziranda	-500	Taksit
12 haziranda	-1200	Borç (Verecek)
13 haziranda	+1500	Altın bozdurma
14 haziranda	+4300	Maaş
	+420	Borç (Alacak)

As shown in the table 4.13, these two groups, in fact all groups, correctly illustrated given expenses and incomes with integers. After students formed bank statements, all groups followed the same steps that they did in the second part of this MEA. They firstly added negative integers and positive integers separately stating these results as incomes and expenses. After, they found the difference of incomes and expenses to find the remainder of her incomes. The students interpreted this result while giving advice about her expenses for the next month. Their suggestions reflected following two aspects:

- suggestions rely on mathematical computations, and
- suggestions rely on everyday life thinking.

Below, I present the operations and suggestions of two groups rely on bank statements. The table 4.14 shows the statements of Group 7 and Group 8.

Table 4.14 The operations and suggestions of Group 7 and Group 8 in terms of bank statements

Group 7	
Group 8	

As can be seen in the Table 4.14, some of the groups followed the steps they implemented in the second part, and they interpreted the remainder of the incomes as profit. They took their computations into consideration, and they suggested the owner bank statement to reduce her outgoings. Moreover, some groups took their everyday life experiences into consideration while giving advice on her outgoings. To illustrate, one of the students from Group 6 stated that "*she can change her sport habit and she can do sport at home rather than going to gym*." Hence, as I presented in the Table 3.2 in the previous chapter, the purpose of mathematics history-based MEA3 was to improve students' understanding of the use of negative integers in real life in terms of everyday life use of negative integers. All students successfully formed expected model while organizing of monthly income and expenses on a bank statement using integers through this model-eliciting activity. Moreover, as expressed in the introduction chapter, the scope of current study was not understanding operations with integers, it solely was students' understanding and

making sense of negative integers. Therefore, the operational signs in the operations did not focused.

4.3. Follow-up Assessment of Students' Understanding of Integers

Similar to the initial assessment analysis, questions 1, 2, and 6 in the follow-up assessment were evaluated based on correctness. All students selected positive integers which have plus sign or no sign placed before the integer. All students selected negative integers which have minus sign placed before the integer. All students expressed a real-life situation with integers correctly.

Questions 3, 4, and 5 asked students to explain their reasoning; therefore, their responses were categorized qualitatively. Table 4.15, Table 4.16 and Table 4.17 presents students' response categories and their frequencies in comparison with those in the initial assessment.

Table 4.15 Students' response categories and frequencies for the Question 3 in the follow-up assessment in comparison with the initial assessment

Questions	**Response Categories**	**Frequencies in Initial Assessment**	**Frequencies in Follow-up Assessment**
3.1. Write a negative integer.	Minus sign in front of the number	16	19
	Less than zero	12	11
3.2. How you explain this integer is a negative integer to another person?	Temperature less than zero	1	4
	Debt	6	3
	On the left of the zero on number line	3	2
	Below sea level	1	2
	Expenses on a bank statement	-	2

Table 4.15 (continued)

	Giving example from Chinese number system	-	2
	Loss	1	1
	The flat below ground floor	2	-
	Subtraction operation	2	-

In the third question, students were expected to write how they explain a negative integer to another person. Most of the students (19 students) stated that a negative integer has minus sign in front of the number while supporting their descriptions with contextual examples from real-life such as temperature, debt, expenses and loss while 16 students solely addressing use of minus sign in the initial assessment. Some of these students also expressed that negative integers are less than zero. As expected, students were reflected successfully how they associate the negative integers with everyday life contexts such as debt and expenses through modeling. Moreover, eight of the students solely focused on symbolic representations of integers and minus sign while explaining a negative integer. In comparison with initial assessment, the number of students who merely considered symbolic representation of negative integers while defining a negative integer decreased. Moreover, the use of "expenses on a bank statement" and "giving example from Chinese number system" illustrations while identifying a negative integer only revealed in the follow-up assessment. Likewise, "the flat below ground floor" and "with subtraction operation" illustrations only revealed in the initial assessment.

In the question 4, students were firstly asked to determine whether the unknown addend should be a positive or a negative integer at the equation of 3+__=7 by explaining their reasoning. As seen in the table 4.16, all students determined the unknown addend as a positive integer.

Table 4.16 Students' response categories and frequencies for the Questions 4 in the follow-up assessment in comparison with the initial assessment

Questions	**Response Categories**	**Frequencies in Initial Assessment**	**Frequencies in Follow-up Assessment**
4.1. Determine whether the unknown addend should be a positive or negative integer. 4.2. Explain your reasoning.	Positive integer. The result and the given addend are positive integers. The difference of seven and three is equal to four. If we add four to three, the result is seven. Four is a positive integer.	10	9
	Positive integer. If the unknown addend were a negative integer, the result would be a negative integer and less than three, but the result is greater than three.	9	16
	Positive integer. Addition of two positive integers is equal to a positive integer.	8	8
	Positive integer. The negative integers neutralize the positive integers.	1	-
4.3. What is the value of unknown integer? Explain your operations.	Addition of three and four is equal to seven.	13	18
	The difference of seven and three is four.	12	9
	What should we add to three to get seven? Four.	3	2
	When we move on four units from three on number line, we find seven.	2	-
	If we have three oranges, we should add four more oranges to have seven oranges.	1	1

In comparison with the initial assessment nine students improved their reasoning, and in the follow-up assessment most of the students (16 students) explained that if the unknown addend were a negative integer, the result would be a negative integer and less than three, but the result is greater than three. Nine of the students found the difference of the result and the given addend by focusing on the result of the operation. Eight students solely focused on signs of the result and the given addend, they indicated that addition of two positive integers is equal to a positive integer. In comparison with the initial assessment, solely one student took the neutralization model into consideration in the initial assessment and stated that the negative integers neutralize the positive integers. After this part, students were asked to find the value of the unknown addend. 18 students showed that the addition of three and four is equal to seven and the unknown integer is four. Nine of these students found the value of unknown addend by subtracting the given addend from the result. In comparison with the initial assessment, two students took the number line model into consideration while finding the value of unknown addend but neither of them didn't use this illustration in the follow-up assessment.

In the question 5, similar to question 4, students were firstly asked to determine whether the unknown addend should be a positive or a negative integer by explaining their reasoning. On the other hand, the unknown addend should be a negative integer in question 5, 3+__=1, and all students answered in this manner (See Table 4.17).

Table 4.17 Students' response categories and frequencies for the Questions 5 in the follow-up assessment in comparison with the initial assessment

Questions	**Response Categories**	**Frequencies in Initial Assessment**	**Frequencies in Follow-up Assessment**
5.1. Determine whether the unknown addend should be a positive or negative integer.	Negative integer. Addition of three and a positive integer cannot be equal to one.	15	8

Table 4.17 (continued)

5.2. Explain your reasoning.	Negative integer. The result (one) is less than three.	6	18
	Negative integer. The negative and positive integers neutralize each other, and we subtract.	2	2
	Negative integer. The addition of three and minus two is equal to one.	1	1
	Negative integer. The difference of three and minus two is equal to 1.	6	-
5.3. What is the value of unknown integer? Explain your operations.	Addition of three and minus two is equal to one.	12	22
	The negative and positive integers neutralize each other.	3	5
	The subtraction of three from one is equal to minus two.	6	3
	The difference of three and minus two is equal to one.	5	-
	Addition of three and minus four is one, the unknown is minus four.	2	2
	We subtract one from three, then we write minus sign in front of two.	2	-
	When we move on two units left from three on number line, we find one.	2	-

In comparison with the initial assessment (6 students), most of the students (18 students) expressed that the result (one) is less than three that's why the unknown addend is a negative integer. Eight students explained their reasoning by focusing on the result of the operation. They stated that addition of three and a positive integer cannot be equal to one. Moreover, solely in the initial assessment, six students answered incorrectly and stated that the difference of three and minus two is equal to 1. However, in the follow-up assessment none of them didn't answer incorrectly. After this part, students were expected to find the value of the unknown addend. Most of the students (22 students) expressed that addition of three and minus two is equal to one while finding the unknown addend. Five of these students stated in their solutions that the negative and positive integers neutralize each other. Three of these students found the value of unknown addend by subtracting three from one. Furthermore, in comparison with the initial assessment, two students subtracted one from three, then they wrote minus sign in front of two, and two students took the number line model into consideration while finding the value of unknown addend, but neither of them didn't provide these explanations in the follow-up assessment.

To sum up, in the initial assessment, most of the students solely used the knowledge of symbolic representation of negative integers, and they had difficulties in making sense of negative integers and connection with the use of these integers in real life, and contextual situations. They used their memorizations without any mathematical reasoning to solve mathematics problems including operations with integers. Most of them followed solely operational procedures to find the unknown addends in the Questions 4 and 5. On the other hand, in the follow-up assessment, in addition to symbolic representation of a negative integer, most of the students provided contextual examples while identifying a negative integer such as debt, expenses, loss and temperature with the help of model-eliciting activities. Furthermore, comparing to initial assessment, students' mathematical reasoning for the Question 4 and 5 improved. Instead of finding the unknown addend by using solely procedural knowledge and rules about operations with integers, they provided more mathematical explanations by making meaningful connection with the result and the given addend.

CHAPTER 5

CONCLUSIONS AND DISCUSSION

The purpose of the present study is to investigate the role of mathematics history-based model-eliciting activities (MEAs) on seventh grade students' understanding of negative integers. In this chapter, conclusions of the study are summarized and discussed. In addition, limitations of the study, recommendations for the future studies and implications for educational practices are presented.

5.1. The Role of Mathematics History-based MEA on Students' Understanding of Negative Integers

The initial and follow-up assessments were compared taking the mathematics history-based model eliciting activities into consideration. The conclusions of the study were drawn in terms of students' making sense of negative integers, identifying positive and negative integers, and understanding the use of negative integers in real life to reveal students' understanding.

The results of the initial assessments showed that most of the students solely had the knowledge of symbolic representation of negative integers rather than making sense of the meaning of negative integers. All students had knowledge to select and distinguish negative and positive integers considering the signs placed before the numbers. On the other hand, students had difficulties in making sense of negative integers and connection with the use of these integers in real life, contextual situations. These findings of the initial assessment confirm that making sense and being aware of the use of negative integers in real-life is more difficult for students (Whitacre et al., 2017). The initial assessment revealed that students used their memorizations without any mathematical reasoning to solve mathematics problems including operations with integers. To illustrate, in the Question 4, most of the

students tried to find the unknown integer via a rule stating that addition of two positive integers is equal to a positive integer. The students solely used this rule without providing a mathematical reasoning or explanation about it. Besides, when they encountered the sum of a positive and negative integer, most of them used trial and error method to find the unknown addend without any explanation about their solution ways. Most of them followed solely operational procedures to find the unknown addends in the Questions 4 and 5. The initial assessment supported that students focused on symbolic representations of integers and used solely their procedural knowledge to solve problems including negative integers. Therefore, they had difficulties while making connections with real-life, providing mathematical reasoning and contextual explanations about their solutions. These findings of the initial assessment support several studies which indicated that most of the students have difficulty in understanding negative integers since students focus on operational procedures rather than meaning of operations (Bolyard, 2005; Ferguson, 1993; Lyte, 1994; Shore, 2005; Steiner, 2009; Wilkins, 1996).

However, in the follow-up assessment, most of the students used contextual examples while identifying a negative integer such as debt, expenses, loss and temperature. Considering the design of the study, I conjectured that this conclusion might be resulted from the mathematics history-based MEAs since students questioned the use of negative integers in real life while they were discovering how people needed negative integers in history through model-eliciting activities. The group work and creating models through model-eliciting activities provided students with opportunities that they could reveal their opinions while they were engaging other students' and groups' opinions. Although they used symbolic representation of a negative integer in their explanations, they also supported their statements with the contextual illustrations. Students used their learnings form the mathematics history-based MEAs in their contextual explanations. Furthermore, the follow-up assessment showed that students' mathematical reasoning for the Question 4 and 5 improved when compared to initial assessment.

Most of the students did not find the unknown addend by using solely their procedural knowledge and rules about operations with integers. Instead, they provided more meaningful explanations. To illustrate, most of the students commented on unknown addend by making meaningful connection with the result and the given addend. They stated that if the result of addition is less than the given addend, the unknown addend is a negative integer, or if the result of addition is greater than the given addend, the unknown addend is a positive integer. The findings of the follow-up assessment confirm the results of the studies which indicated that the real-life contexts, word problems and models including incomes and expenses, assets and debts, elevators, weather temperatures support students' understanding and reasoning about integers (Ball, 1993; Stephan & Akyuz, 2012; Pettis & Glancy, 2015). The mathematics history-based MEAs helped students improve their mathematical reasoning while discussing on solutions of given problems. Furthermore, these findings are supported by several studies because increasing motivation have also been observed (e.g., English, Lesh & Zawojewski, 2003). The interests and motivation of students increase while creating a mathematical significant model to solve a real-world mathematics problem working in a group rather than solving a traditional class question individually (English, Lesh & Zawojewski, 2003). Studying on the everyday mathematics problems, discussing on different solution methods for the problems and using various teaching activities including history of mathematics motivate students (Fauvel, 1991; Liu, 2003; Swetz, 1997; Tzanakis & Arcavi, 2000). Although students didn't experience a similar lesson environment or classroom arrangement before the one which was implemented in this study, all students willingly and actively participated in the implementation of the activities and they studied with their groupmates collaboratively.

One of the major contributions of this study is integrating mathematics history to modeling perspective, which has not been present in the related literature yet. This integration increased the motivational and attitudinal effect of MEAs as Savizi (2007, p.46) stated: "For students, issues of past real world are more tangible and understandable than today's problems or solving problems from real life by using

human approaches may work better than application of complicated methods or offering high amount of information." To illustrate, in the second mathematics history-based MEA, students studied on a problem which is called as "absurd" by the Diophantus because of its' negative solution, and they had chance to notice that different people encountered with similar difficulties. In this study, the mathematics history-based model eliciting activities brought real-life related mathematics problems from the past and the present together. Although students didn't study on mathematics history-based MEAs before, all groups actively answered the questions as they were expected. After the implementation of all mathematics history-based MEAs, most of the students stated their wishes to continue mathematics lessons by working on similar model-eliciting activities, confirming the claims of the studies in the literature that studying on the mathematics problems real-world including history of mathematics motivates students while overcoming the fear of mathematics (Fauvel, 1991; Liu, 2003; Swetz, 1997; Tzanakis & Arcavi, 2000).

The observations during implementation process of mathematics history-based MEAs showed that students were more willingly participated the lesson comparing to other traditional teacher centered lessons. Considering the design of the study, this conclusion might be resulted from the group work and real-life related mathematics history-based MEAs. Studying on real-life related questions through model-eliciting activities and discovering the connection of these questions with history motivated students. The students overcame their fear of participating the mathematics lesson since they showed a great enthusiasm in sharing their opinions and findings freely. These observations also support and extend the findings of several studies. It is concluded in the quasi-experimental study of Lit, Siu, and Wong (2001) that studying on historical process of development of the Pythagorean theorem positively affected eight-grade students' attitudes towards topic and mathematics. Moreover, an interdisciplinary project of Ponza (1998) including a role play about the life of a mathematician showed that the process of discovering and acting out the life of a mathematician in the history increased the seventh-grade level students' interest to subject.

In addition, as mentioned before, as a teacher-researcher, I followed some essential principles during the implementation of these activities. Students studied as small groups and interacted with each other. After they finished their studies, students were encouraged to share their works and opinions as a whole class discussion. Students solely were guided as scaffolding when they needed without providing any right answer for the questions of the activities. They were provided enough time and encouraged to reveal and reflect their own experiences by making connection with everyday life contexts. This implementation process and modeling might be effective on students' understanding and making sense of negative integers. Since they also had chance to benefit from each other learnings with the help of sharing their models as a whole class discussion. Moreover, that kind of implementation of model-eliciting activities might be reason for students' active participation and increasing motivation. The implementation process of mathematics-history based MEAs showed that most of the students are more productive and concentrated while they are working as a group. Working with their peers encouraged students to share their opinions and findings without the fear of making mistake. Although the students didn't experience a similar lesson environment before the one which is implemented in this study, they worked with their groupmates collaboratively. Some of the students were more passive than the other groupmates and this problem might be resulted because of effects of students' regular practices on traditional teacher centered teaching.

Similarly, an experimental design study was conducted to investigate the effect of the use of mathematics history on sixth grade students' achievement and attitudes (Bayam, 2012), and students' positive opinions about teaching process and increase in students' achievement were concluded. Furthermore, the effect of using mathematics history on the academic success, retention and motivation of fourth grade level students about the decimals were investigated by employing a quasi-experimental design study (Ersoy, 2015). The result of this study indicated that the use of mathematics history through model-eliciting activities increased students' achievement, motivation and retention levels on decimals. The findings of the

present study in which mathematics history was integrated into modeling perspective both supported and extended the claims of these research.

Since for meaningful learning in schools, students need to see the human side of mathematics and they need to be aware the value of mathematics in their lives (Ernest, 1994) and the use of mathematics history-based MEAs served that aim. In this sense, this study improved students' understanding of negative integers. Furthermore, there are several studies investigated the effect of integration of mathematics history into mathematics teaching on achievement of students. These studies concluded that using history of mathematics in mathematics lessons have increased the academic achievement (Ersoy, 2015; Özcan, 2014; Bayam, 2012; İdikut, 2007). The present study extends the findings of these studies by focusing on understanding and making sense of the topic rather than achievement. This study also allowed to make comparison of initial and follow-up assessment and showed that students' capability to make connection with real-life by providing contextual examples improved after the implementation of mathematics history-based MEAs.

Particularly, the findings of initial assessment showed that students had difficulties in explaining and justifying their reasons for a solution. They focused on operational procedures and solely explained their operations without providing sufficient explanation with their solution methods. The lack of capability of mathematical reasoning and justification might be resulted from students' regular practices on memorization of the rules and operational procedures. The teaching and problem-solving process without making meaningful connection with real-life, reasoning and justification might be the reason for preventing the understanding of subjects. In particular, the difficulties in making sense of negative integers reveals not only because of epistemological characteristics of the subject but also teaching methods and materials and students' engagement process with the topic which was implemented as model-eliciting activities that incorporates mathematics history. In this manner, present study supports and extends the findings of several studies on understanding of integers. To illustrate, line and neutralization models are most common two models using in the teaching of integers but these two models are

lacking in terms of letting students to experience the real meaning of an integer since they do not have necessary relations with real life (Bell, 1983; Hackbarth, 2000; Werner, 1973). Besides, these models focus on operation with integers (Lyte, 1994). Therefore, concrete models and real-world problems such as creating a number system or bank statement improve students' understanding of integers (Ball, 1993; Stephan & Akyuz, 2012). The real-life contexts, word problems and models that include incomes and expenses, assets and debts, elevators, weather temperatures support students' understanding and reasoning about integers (Ball, 1993; Stephan & Akyuz, 2012; Pettis & Glancy, 2015). With keeping these studies in mind, the mathematics history-based MEAs were beneficial tools to create a meaningful and real-life related learning environment as Doerr and Lesh (2003) stated that modeling is significant not only for computing, but also for constructing, describing, mathematical reasoning and understanding.

The students' work on mathematics history-based MEAs and the follow-up assessment support the findings that students may not be able to comprehend situations involving opposites such as incomes and expenses, weather temperatures and elevators (Pettis & Glancy, 2015). It is important to encourage students within this context to improve their understanding of integers since making sense and being aware of the use of negative integers in real-life is difficult for students (Whitacre et al., 2017). With keeping these findings in the mind, the conclusions of this study supported the necessity of solving real-life related mathematics problems via creating mathematically meaningful models. Since students' studies on real-life related model-eliciting activities provided them to reveal their contextual every-day experiences. The contextual and real-life related experiences of students with making connection with school learnings supported their making sense of negative integers.

When compared to initial assessment, in the Question 3 of the follow-up assessment, students defined a negative integer within contextual relationship with the use of negative integers in real life. Moreover, their mathematical reasoning and justifications for the Question 4 and 5 improved. I conjectured that these conclusions might be resulted from their studies on mathematics history-based MEAs. Taking

students' thinking perspective into account is essential for understanding of a mathematics topic because the nature of students' mathematics is different from adults in interesting and significant ways (Steffe, 1991). In this sense, the present study aimed to reveal students' understanding of negative integers through modeling. The implementation of model-eliciting activities including concrete experiences, representations, making connections with different situations in real life contribute to students' understanding and development of well-grounded mathematical ideas (Bonotto, 2007; Doerr & Lesh, 2003; English, 1997; Gulikers & Blom, 2001; Stein & Bovalino, 2001). The model eliciting activities resemble to real life situations in which students make meaningful mathematical explanations (Doerr & Lesh, 2003), and the use of history of mathematics to improve students' mathematics learning in terms of understanding and making sense of the content (Gulikers & Blom, 2001).

This observation supports the findings of Steiner (2009) and Gallardo (2002) who stated that students meet negative integers in their everyday life before instruction of integers in school, after they encounter and focus on operational procedures in school, they do not make connection between before school learnings and school instruction. Focusing on operational procedures while working with negative integers leads to contemplating without focusing on concrete meaning of situations in real-life problems (Gallardo, 2002). Therefore, school teaching has a negative role, but the separation of everyday experiences, and school learnings could hinder students' understanding of integers (Steiner, 2009). In contrary, the current study showed that with the help of mathematics history-based MEAs, students learned negative integers in historical situations. Likewise, working in a group to create real-life related mathematically significant models and sharing the findings in a whole class discussion supports the idea of Lesh Translation Model (Lesh & Doerr, 2003) that students' capability to represent mathematical concepts within different ways, and to make connections between these ways of thinking, which improves students' understanding.

5.2. Limitations & Recommendations

The present study has been conducted to reveal the role of mathematics history-based model eliciting activities on seventh grade students' understanding of negative integers. In the previous chapter the findings of the present study were presented in detail, but there are some limitations of this qualitative study. In this part, limitations of the study, recommendations for the future studies and implications for educational practices are presented**.**

This study involved three mathematics history-based model eliciting activities because of the limited amount of time. The implementation of the activities was arranged considering the middle school mathematics teaching program and annual plan of mathematics lessons. More meaningful data about the students' understanding of negative integers might reveal if more time was spent and more activities were implemented.

In addition, this study solely investigated the students' understanding of negative integers, but more studies could be conducted about different mathematics contents by using mathematics history-based model eliciting activities. This study clearly shows that such activities helped students improve their understanding and make sense of negative integers. The integration of real-life problems from the history and connect it with present real-life situations enabled students to make connections with mathematical contents and their everyday life while modeling their understanding of negative integers. For this reason, this study suggests as a future research that different mathematical topics for different grade levels could be considered for integration of the mathematics history into modeling perspective. Although the present study investigated the role of mathematics history-based MEAs on students' understanding, these activities could also be used to improve mathematics teachers' education for their teaching repertoire. Thus, this study also suggests a professional development aiming to train teachers how mathematics history and modeling perspective could be used to enhance students' mathematical understanding.

The participant students of this research were from a project classroom in a public middle school. Although they follow the same middle school mathematics teaching program, their overall school achievement was relatively higher compared to the students in regular classrooms of the same school, but they were not gifted or better than students in other public schools. Therefore, the results of this study were limited with the capability of students who were good at expressing their thinking and better in mathematics problem solving. Therefore, I suggest other researchers interested in investigating the same or similar phenomenon working with students at the average level in terms of school success.

As a suggestion for educational implication, textbooks and other mathematics teaching materials could include mathematics history-based MEAs to help students make connections between school learnings and everyday experiences. Furthermore, teachers may arrange their lessons by preparing activities in the light of this study since integrating history of mathematics and solving real life mathematics problems by creating mathematical models improved students' understanding of negative integers.

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APPENDICES

A. METU HUMAN SUBJECTS ETHICS COMMITTEE APPROVAL

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

DUMLUPINAR BULVARI 06800
ÇANKAYA ANKARA/TURKEY
T: +90 312 210 22 91
F: +90 312 210 79 59
ueam@metu.edu.tr
www.ueam.metu.edu.tr

Sayı: 28620816 / 470

12 EYLÜL 2018

Konu: Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi: İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Dr.Öğretim Üyesi Şerife SEVİNÇ

Danışmanlığını yaptığınız; yüksek lisans öğrencisi Büşra AY SUSUZ'un **"Matematik Tarihi ile Modelleme: 7. Sınıf Öğrencilerin Negatif Tam Sayılar Konusuna İlişkin Kavramsal Anlamalarının ve Tutumlarının (Motivasyonlarının) İncelenmesi"** başlıklı araştırması İnsan Araştırmaları Etik Kurulu tarafından uygun görülerek gerekli onay **2018-EGT-124** protokol numarası ile **12.09.2018 - 30.06.2019** tarihleri arasında geçerli olmak üzere verilmiştir.

Bilgilerinize saygılarımla sunarım.

Prof. Dr. Ş. Halil TURAN
Başkan V

Prof. Dr. Ayhan SOL
Üye

Prof. Dr. Ayhan Gürbüz DEMİR
Üye

Doç. Dr. Yaşar KONDAKÇI
Üye

Doç. Dr. Zana ÇITAK
Üye

Doç. Dr. Emre SELÇUK
Üye

Dr. Öğr. Üyesi Pınar KAYGAN
Üye

B. PERMISSION OBTAINED FROM MINISTRY OF EDUCATION

T.C.
İSTANBUL VALİLİĞİ
İl Millî Eğitim Müdürlüğü

Sayı : 59090411-20-E.22035779
Konu : Anket ve Araştırma İzin Talebi

19/11/2018

VALİLİK MAKAMINA

İlgi: a) Orta Doğu Teknik Üniversitesinin 16.10.2018 tarih ve 42 sayılı yazısı.
b) MEB. Yen. ve Eğ. Tk. Gn. Md. 22.08.2017 tarih ve 12607291/ 2017/25 No'lu Gen.
c) Milli Eğitim Araştırma ve Anket Komisyonunun 08.11.2018 tarihli tutanağı.

Orta Doğu Teknik Üniversitesi Eğitim Bilimleri Enstitüsü yüksek lisans öğrencisi Büşra AY SUSUZ'un **"Matematik Tarihi ile Modelleme: 7. Sınıf Öğrencilerin Negatif Tam Sayılar Konusuna İlişkin Kavramsal Anlamlarının ve Tutumlarının (Motivasyonlarının) İncelenmesi"** konulu tezi kapsamında, ilimiz Kartal ilçesinde bulunan Kartal Cevizlik Ortaokulunda öğrenim gören 7. sınıf öğrencilerine; negatif tam sayılara ilişkin tutum ölçeği ve negatif tam sayılar testini uygulama istemi hakkındaki ilgi (a) yazı ve ekleri Müdürlüğümüzce incelenmiştir.

Araştırmacının söz konusu talebi; bilimsel amaç dışında kullanılmaması, **uygulama sırasında bir örneği müdürlüğümüzde muhafaza edilen mühürlü ve imzalı veri toplama araçlarının kurumlarımıza araştırmacı tarafından ulaştırılarak uygulanılması, katılımcıların gönüllülük esasına göre seçilmesi, araştırma sonuç raporunun müdürlüğümüzden izin alınmadan kamuoyuyla paylaşılmaması koşuluyla, okul idarelerinin denetim, gözetim ve sorumluluğunda, eğitim-öğretimi aksatmayacak şekilde ilgi (b) Bakanlık emri esasları dâhilinde uygulanması, sonuçtan Müdürlüğümüze rapor halinde (CD formatında) bilgi verilmesi kaydıyla** Müdürlüğümüzce uygun görülmektedir.

Makamlarınızca da uygun görülmesi halinde olurlarınıza arz ederim.

Levent YAZICI
İl Milli Eğitim Müdürü

Ek:
1- Genelge.
2- Komisyon Tutanağı.

OLUR
19/11/2018

Ahmet Hamdi USTA
Vali a.
Vali Yardımcısı

Milli Eğitim Müdürlüğü Binbirdirek M. İmran Öktem Cad.
No:1 Eski Adliye Binası Sultanahmet Fatih/İstanbul
E-Posta: sgb34@meb.gov.tr

A. BALTA VHKİ
Tel: (0 212) 455 04 00-239
Faks: (0 212)455 06 52

C. PARENT CONSENT FORM

Veli Onay Formu

Sevgili Anne/Baba,
Bu çalışma Orta Doğu Teknik Üniversitesi yüksek lisans öğrencisi Büşra AY tarafından danışmanı olan Doç. Dr. Şerife SEVİNÇ ile sürdürdüğü yüksek lisans tezi kapsamında yürütülmektedir.
Bu çalışmanın amacı nedir? Araştırmanın amacı 7.sınıflarda matematik dersine modelleme yaklaşımı yardımıyla matematik tarihinin entegre edilmesi ve matematik tarihi bağlamında sunulan modelleme etkinlikleriyle öğrencilerin negatif tamsayılar konusundaki kavramsal anlamalarını ve tutumlarını incelemektir.
Çocuğunuzun katılımcı olarak ne yapmasını istiyoruz? Bu amaç doğrultusunda, araştırmacı tarafından MEB matematik öğretim programına uygun olarak hazırlanmış ders içi matematik etkinliklerine katılım sağlayacaklar. Yaklaşık üç ders sürecek olan bu etkinlikler süresince öğrencilerin anlamaları ile tutumlarını incelemek için video ve ses kaydı alınacaktır. Ayrıca, çocuğunuzdan uygulanacak olan testteki 6 soru ile tutum ölçeğindeki 30 soruyu matematik dersi sırasında cevaplamasını isteyeceğiz ve cevaplarını yazılı biçiminde toplayacağız. Sizden çocuğunuzun katılımcı olmasıyla ilgili izin istediğimiz gibi, çalışmaya başlamadan çocuğunuzdan da sözlü olarak katılımıyla ilgili rızası mutlaka alınacak.
Çocuğunuzdan alınan bilgiler ne amaçla ve nasıl kullanılacak? Çocuğunuzdan alacağımız cevaplar ve görüntüler tamamen gizli tutulacak ve sadece araştırmacı tarafından değerlendirilecektir. Elde edilecek bilgiler sadece bilimsel amaçla kullanılacak, çocuğunuzun ya da sizin ismi ve kimlik bilgileriniz, hiçbir şekilde kimseyle paylaşılmayacaktır.
Çocuğunuz ya da siz çalışmayı yarıda kesmek isterseniz ne yapmalısınız? Katılım sırasında sorulan sorulardan ya da herhangi bir uygulama ile ilgili başka bir nedenden ötürü çocuğunuz kendisini rahatsız hissettiğini belirtirse, ya da kendi belirtmese de araştırmacı çocuğun rahatsız olduğunu öngörürse, çalışmaya sorular tamamlanmadan ve derhal son verilecektir.
Bu çalışmayla ilgili daha fazla bilgi almak isterseniz: Çalışmaya katılımınızın sonrasında, bu çalışmayla ilgili sorularınız yazılı biçimde cevaplandırılacaktır. Çalışma hakkında daha fazla bilgi almak için Matematik ve Fen Bilimleri Eğitimi yüksek lisans öğrencisi Büşra Ay (e-posta: busra.ay_07@metu.edu.tr) ile iletişim kurabilirsiniz. Bu çalışmaya katılımınız için şimdiden teşekkür ederiz.
Yukarıdaki bilgileri okudum ve çocuğumun bu çalışmada yer almasını onaylıyorum **(Lütfen alttaki iki seçenekten birini işaretleyiniz.**
Evet onaylıyorum **___** ***Hayır, onaylamıyorum*** **___**
Annenin adı-soyadı: _____ Bugünün Tarihi:_____
Çocuğun adı soyadı ve doğum tarihi:_____
(Formu doldurup imzaladıktan sonra araştırmacıya ulaştırınız).

D. THE INITIAL AND FOLLOW-UP ASSESSMENT TOOL

NEGATİF SAYILARA DAİR KENDİMİZİ TEST EDELİM!

1. Aşağıda verilen tam sayılardan pozitif olanlarını yuvarlak içine alınız.

- 305	**1**	**- 10**	**+ 12**	**27**
- 36	**+ 143**	**15**	**32**	**- 4**

2. Aşağıda verilen tam sayılardan negatif olanlarını yuvarlak içine alınız.

127	**+ 34**	**- 71**	**12**	**- 43**
- 36	**+ 324**	**- 56**	**3**	**- 1**

3. Aşağıya bir negatif tam sayı örneği yazınız.

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Bu sayının negatif tam sayı olduğunu bir başkasına nasıl açıklarsınız.

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4. **3 + □ = 7**

Yukarıda verilen işlemde kutucuk yerine pozitif bir tam sayı mı yoksa negatif bir tam sayı mı gelmelidir?

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Sebebini açıklayınız.

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Kutucuk yerine gelmesi gereken tam sayı kaçtır? İşlemlerinizi açıklayarak gösteriniz.

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5. **3 + □ = 1**

Yukarıda verilen işlemde kutucuk yerine pozitif bir tam sayı mı yoksa negatif bir tam sayı mı gelmelidir?

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Sebebini açıklayınız.

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Kutucuk yerine gelmesi gereken tam sayı kaçtır? İşlemlerinizi açıklayarak gösteriniz.

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6. Aşağıda verilen durumları tam sayı olarak ifade ediniz.

- Asansör girişin 5 kat üstündeki kata geldiğinde:

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- Asansör girişin 3 kat altındaki kata geldiğinde:

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- Hava durumu sıfırın altında 7 derece gösteriyor:

  ……

- Hava durumu sıfırın üstünde 10 derece gösteriyor:

  ……

- Elçin'in Onur'dan 55 TL alacağı vardır:

  ……

- Bu ay evin gideri 500 TL'dir:

  ……

- Sattığım karpuzdan 300 TL kar elde ettim:

  ……

- Bir şirketin sattığı bilgisayarsan 1000 TL zarar etmesi:

  ……..

E. MATHEMATICS HISTORY-BASED MODEL-ELICITING

ACTIVITY 1

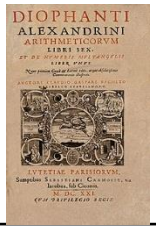
YAŞASIN MATEMATİK

Merhaba arkadaşlar ben Ali. Yaz tatilinde ben de sizler gibi gezdim, oyunlar oynadım, dinlendim ve kitaplar, dergiler okudum. Özellikle matematikle ilgili olan dergileri çok seviyorum. Matematik aslında çok ilginç. Ben, öğrendiğimiz matematik konularının ne zaman, nasıl, neden ortaya çıktığını hep merak etmişimdir. 6. sınıfta negatif tamsayıları öğrenmiştik. Negatif tam sayılarla ilgili merak ettiklerimi şimdi size soracağım. Bana yardım eder misiniz?

Negatif tamsayıları biz günlük hayatımızda nasıl ve nerelerde kullanıyoruz, bana örnek verir misiniz?

Sizce negatif tamsayılar nasıl ortaya çıkmıştır? İnsanlar neden negatif tam sayılara ihtiyaç duymuştur?

Tatilde okuduğum bir matematik dergisinde yazanları size anlatacağım. M.Ö. 2. yüzyıl ve 4. yüzyıl arasında Diophantus adında İskenderiyeli Yunan bir matematikçi yaşamış ve bizim g6. Sınıfta öğrendiğimiz cebirsel ifadeler konusunda çalışmış ve 130 problemden oluşan Arithmetica adlı bir kitap yazmış. Diophantus "cebirin yaratıcısı" olarak biliniyormuş. Aslında cebir çok daha önceki zamanlarda bile kullanılıyormuş. Merak edip araştırdım Rhind Papirüsü de M.Ö. 1650 yılında Mısırlı Ahmes tarafından yazılmış ve çarpma, bölme, cebirsel ifadeler gibi konuları içeriyormuş. Birçok ileri matematik konusunun gelişimine katkıda bulunan eski uygarlıkların ve matematikçilerin negatif tam sayıları anlamaları aslında çokta kolay olmamış ve çeşitli zorluklar yaşamışlar. Tıpkı bizim gibi!



Arkadaşlar dergide Diophantus'un bir problemin çözümünde karşısına çıkan

$$4 \times ? + 20 = 4$$

ifadesinin çözümü için "absürt (saçma)" ifadesini kullandığını okudum.

1. Acaba Diophantus neden bu problemin çözümü için saçma demiş olabilir?
2. Arkadaşlar soru işareti yerine gelecek olan tamsayıyı bulmama yardım eder misiniz?
3. Diophantus'un kendinden önceki cebir çalışmalarını bir araya toplayarak geliştirmesi bize matematik bilimi hakkında ne fikir verebilir?

Cevaplarınızı ve açıklamalarınızı bana mektup olarak gönderirseniz okul dergisindeki yazımda paylaşacağım. Mektuplarınızı dört gözle bekliyorum.

Sevgili Ali,

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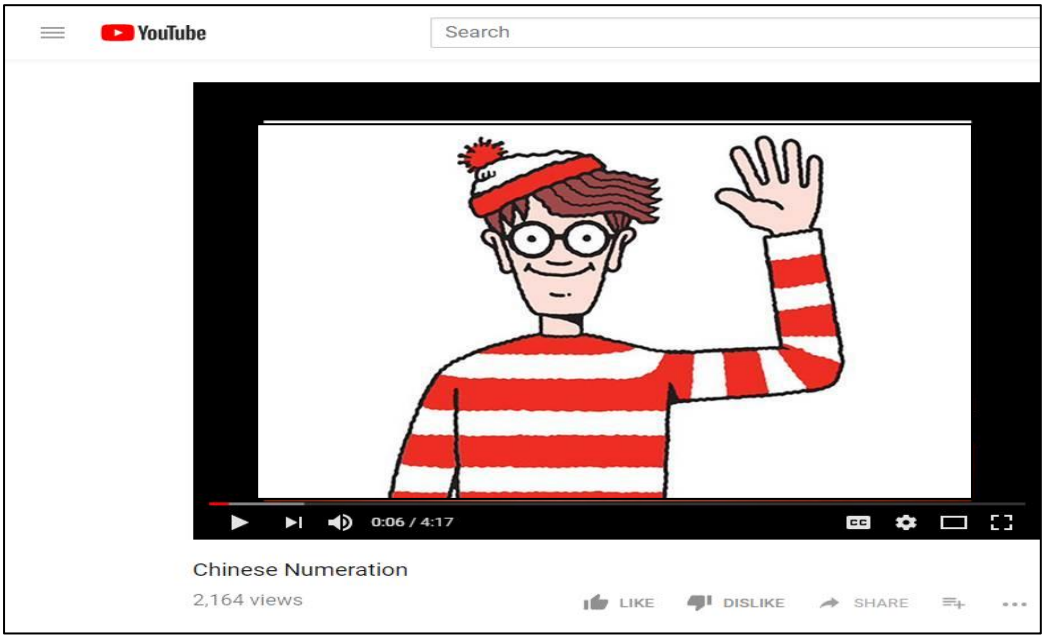
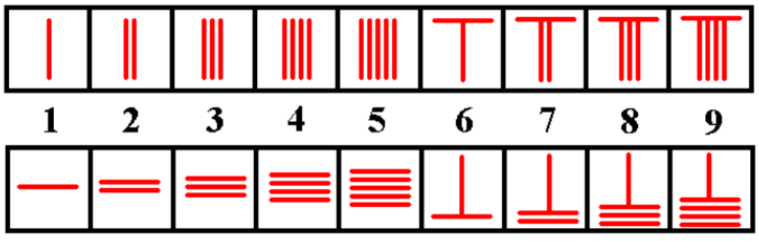
F. MATHEMATICS HISTORY-BASED MODEL-ELICITING ACTIVITY 2

ARDA'NIN YOUTUBE KANALI



Herkese merhaba arkadaşlar, kanalıma hoş geldiniz, ben Arda. İstanbul'da yaşıyorum ve 7. Sınıf öğrencisiyim. Matematik ve matematiğe dair her şeyle ilgileniyorum ve araştırmalarımı kanalımda sizlerle paylaşıyorum. Umarım seversiniz! Bu videomda sizlere Çinlilerin kullandığı sayı sistemini tanıtacağım. Öncelikle bu tabloya bakarak rakamlar ve çubuklar ile ilgili ne söyleyebilirsiniz?



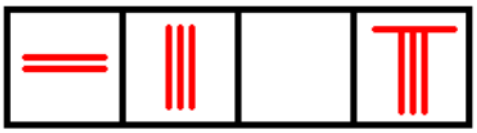
Arkadaşlar, günümüzden yaklaşık olarak 3000 yıl öncesinde Çinliler kendilerine özgü bir sayma sistemi geliştirmişlerdi.

Hesaplama yapan insanlar büyük bir dama tahtası üzerinde kırmızı veya siyah renkteki çubukları kullanarak sayıları ifade ediyorlardı. Bu sistemde dama tahtasındaki her bir karede tek bir rakam bulunurdur. Her bir sütun da birler, yüzler, binler gibi farklı bir basamağı gösteriyordu.

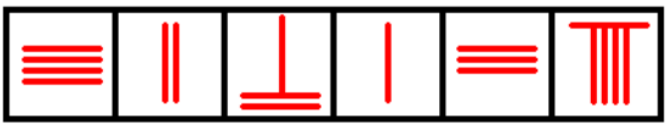
Size az önce gösterdiğim (yukarıdaki) tabloda üst satırdaki çubuk düzeni birler, yüzler ve on binler basamakları vb. için, alt satırdaki çubuk düzeni ise onlar ve binler basamağı vb. için kullanılmaktaydı.

Ayrıca kırmızı çubuklar pozitif sayıları, siyah çubuklar ise negatif sayıları göstermek için tercih ediliyordu. M.Ö. 200lü yıllarda kullandıkları bu çubuk sistemi ile eski Çin uygarlığı negatif sayıları ilk kullanan insanlar arasında yer almaktadır.

Arkadaşlar şimdi sıra sizde! Açıklamaları ve aşağıda verilen örneği inceleyerek verilen sayıları bugün kullandığımız şekilde rakamlarla göstermeyi deneyelim.



2 3 0 8



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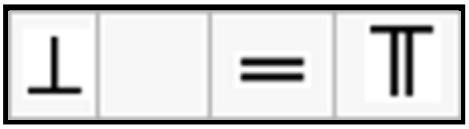

…… ……

Evet arkadaşlar Çinlilerin sayı sistemini tanıyarak negatif ve pozitif tam sayıları nasıl ayırt ettiklerini sizlerle paylaşmış oldum. Ve hatta ben de kendi sayı sistemimi oluşturmaya karar verdim.

Sizlerde kendi sayı sisteminizi oluşturarak benimle paylaşır mısınız? Bir sonraki videoda sizlerden gelen sayı sistemlerini anlatacağım.

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G. MATHEMATICS HISTORY-BASED MODEL-ELICITING ACTIVITY 3

KONUĞUMUZ MUHASEBECİ

Merhaba çocuklar, bugün sizlere kendi mesleğimden yani muhasebecilikten bahsetmek istiyorum. Muhasebeciler ticaret yapan kişilerle çalışabilirler. Örneğin restoran ve şirketler için çalışabilirler. Ticari iş yapan yerlerde para giriş çıkışının kontrol edilmesini ve kaydedilmesini sağlarız ki bu kayıtların doğruluğu denetlenebilsin. Aslında muhasebecilik çok eskiye dayanmaktadır çünkü insanlar çok eskiden beri ticaret yapmaktadır. Örneğin, 263 yılından günümüze taşınmış matematikçi Liu Hui'ye ait bir kitap var. Bu en eski aritmetik kitabıdır ve içinde Çin toplumunun günlük hayatını kolaylaştıracak şekilde kanal inşa etme, takas etme, oran, vergi hesaplama gibi ticari konular üzerine problemler bulunmaktadır.

Liu Hui kitabında negatif ve pozitif tam sayıları günlük hayattaki problemleri çözmek için kullanmış ve bunun için "İşaret Kuralı (The Sign (Zhengfu) Rule)" adı verilen bir kural geliştirmiş.

Kazanç ve zararı temsil eden iki zıt sayma çubuğu olduğunu düşünelim ve bunlara sırasıyla pozitif ve negatif diyelim. Kırmızı sayma çubuklar pozitif, siyah sayma çubuklar ise negatif olsun. Değerleri toplayacağımız veya çıkaracağımız zaman kırmızı ve siyah çubukların her biri birbirini sıfırlayacak şekilde kullanırız.

Sizler eskiden negatif ve pozitif tam sayıların bu şekilde kırmızı ve siyah çubuklarla ayırt edildiğini biliyor muydunuz? Bugün biz bu sayıları nasıl ifade ediyoruz?

Çocuklar şimdi sizlerle Ayşe Hanım'ın mayıs ayı hesap özetine bakalım ve harcamalarını ve kazançlarını birlikte yorumlayalım.

Hesap tipi : VADESİZ HESAP Döviz Cinsi : TRY Dönem (Tarih Aralığı) :26.05.2018-26.06.2018		
İşlem Tarihi	İşlem Tutarı	Açıklama
26.05.2018	- 26	Alışveriş/İpek Market
27.05.2018	- 29	Alışveriş/İpek Market
27.05.2018	- 15	Alışveriş/Gökçe Fırın
27.05.2018	- 170	Alışveriş/Petshop
29.05.2018	- 68	Alışveriş/Kırtasiye
29.05.2018	+ 250	Para yatırma/Alacak
29.05.2018	- 16	Alışveriş/İpek Market
29.05.2018	- 180	Alışveriş/Spor Salonu
29.05.2018	- 48	Alışveriş/Spor
30.05.2018	- 100	Mağazası
31.05.2018	- 1200	Alışveriş/Resim Kursu
31.05.2018	+ 4100	Kira
		Maaş

Bu hesap özetine dair yorumlarımız:

Çocuklar şimdi de sizlerden bana haziran ayının ilk on beş gününde sırasıyla her bir gün yaptığım harcamalarım, gelir ve giderlerim için hesap özeti oluşturmamda yardım etmenizi istiyorum. Her bir gün için yaptığım harcamalarım şöyle;

220 lira ayakkabı, 80 lira market alışverişi, 200 lira spor salonu ücreti, 130 lira kedi maması ve kedi kumu, 125 lira kedilerimin aşısı, 75 lira kitapçı, 1300 lira ev kirası, 250 lira benzin, 300 lira fatura ücreti için harcadım. Ayrıca geçen ay internet üzerinden alıp geri gönderdiğim kitaplığın 280 lira olan ücreti 10 Haziran'da hesabıma geri geçti. 11 Haziran'da 5 taksitle aldığım 2500 liralık buzdolabının ilk taksitini ödedim. Ayrıca daha sonra 1200 lira kredi kartı borcumu ödedim. Altın hesabımdan 1500 liralık altın bozdurarak hesabıma aktardım. 4300 lira maaşımı aldım ve arkadaşıma verdiğim 420 lira borcu geri aldım.

HAZİRAN AYI HESAP ÖZETİ

Hesap tipi : VADESİZ HESAP Döviz Cinsi : TRY Dönem (Tarih Aralığı) :01.06.2018-01.07.2018		
İşlem Tarihi	İşlem Tutarı	Açıklama
01.06.2018 02.06.2018 03.06.2018 04.06.2018 05.06.2018 06.06.2018 07.06.2018 08.06.2018 09.06.2018 10.06.2018 11.06.2018 12.06.2018 13.06.2018 14.06.2018 15.06.2018		

Haziran ayı hesap özetime göre, hesabımdaki miktar kaç TL'dir? Buna göre, gelecek ay için yapacağım harcamalar konusunda bana ne önerirsiniz?

H. THE CODE LIST

Codes Emerged in Students' Work on Mathematics History-Based MEA 1

Questions and students' answers after they discussed with group members:

1. How and where people use the negative integers in daily life? Give examples.

1. To represent weather temperatures.

2. To represent debt and loss.

3. In elevators.

3a. Symbolic representation. For instance, in elevator buttons.

3b. Ordinality. For instance, place of a person that is under the entrance.

4. Elevation.

4a. Apartment flat.

4b. Below sea level.

5. Comparing with a reference point.

2. What do you think about the origins and arrival of negative integers? Why did people need negative integers?

1. Symbolic use for practicality as a shortcut. For instance, people might have preferred to use minus sign in front of numbers rather than writing minus five with letters.

2. Contexts. For instance, to represent very cold weathers and the level below the water level.

3. People.

3a. Mathematicians.

3b. Scientists.

3c. Folks.

3. What might be the reason for the solution of this problem was called as absurd by Diophantus?

1. Knowledge of negative integers.

1a. We can't find the solution if we don't have the knowledge of negative integers.

1b. A question may be called as absurd if we can't find a solution for this question.

4. Can you find the value of unknown that is demonstrated with question mark on this problem?

1. Trial and error.

1a. This unknown can't be a positive integer and we tried negative integers respectively.

1b. We tried positive integers, zero and negative integers respectively.

5. What can we interpret about mathematics science considering that Diophantus gather the algebra studies before his era and developed?

1. Many mathematicians contributed to mathematics.

2. A single math idea was developed by contribution of many people's thinking.

3. Mathematics is continuously evolving.

Codes Emerged in Students' Work on Mathematics History-Based MEA 2

Questions and students' answers after they discussed with group members:

1. What can you interpret about sticks which represent Chinese number system and the numbers given in the table?

1. Knowledge of roman numerals

2. Similarity with our number system.

2a. Numbers' value increase one by one.

2b. Each number has a different demonstration.

3. Learning and use of this number system is difficult.

2. Write the given Chinese numerals with our number system.

1. Red sticks represent positive integers and black sticks represent negative integers.

2. The zero was represented with an empty digit in Chinese number system.

3.Create your own number system with your group members and describe it by writing a letter.

1.Demonstration of numbers.

1a. Regular geometric shapes and regular convex polygons.

1b. Bullions, emeralds and coal. They were inspired from Minecraft game.

1c. Symmetrical demonstration of our numbers.

1d. Horizontal, vertical, inclined sticks and operational symbols.

1e. Interbedded circles. Number of circles increase one by one for each number.

1f. Regular geometric shapes, regular convex polygons, regular star polygon and heart.

1g. YouTube play button, sticks and rectangles.

1h. Flowers, leaves and fruits.

2. Difference of positive and negative integers.

2a. Using different colors.

2b. Using shapes such as before positive numbers and moon before negatives.

2c. Using symbols such as plus and minus.

2d. Using solely a dot before negative numbers.

2e. Using fruits for negative numbers and the number of fruits increase respectively for each number.

3. Representation of zero.

3a. Interbedded circles.

3b. A circle shaped coal.

3c. A circular region.

3d. A circle.

3e. A tree with no fruit.

Codes Emerged in Students' Work on Mathematics History-Based MEA 3

Questions and students' answers after they discussed with group members:

1. Did you know that positive and negative integers differentiate with red and black sticks in the past by Chinese people? How do we represent and differentiate these numbers at the present days?

1. Activation of prior knowledge. Remembering previous activity.

2. Symbolic representation with plus and minus signs before numbers.

3. Accepting zero as a reference point.

2. Examine the given abstract account. What can we interpret depending on given example?

1.Meanings of plus and minus signs.

1a. Minus sign represents expenses and plus sign represents incomes.

1b. Minus sign represents expenses and plus sign represents debt owed.

1c. Minus sign represents debt and plus sign represents income.

1d. Minus sign represents expenses and plus sign represents profit.

1e. Minus sign represents cost and plus sign represents the money got from the bank.

2. How to make sense of incomes and expenses?

1. Computational comments.

1a. Firstly add incomes. Secondly add expenses. Thirdly find the difference of

incomes and expenses. Fourthly interpret the result.

1b. Add integers in the order that they appeared on the bank statement. Then, interpret the result.

4. Examine the given incomes and expenses and form an abstract account. What do you suggest this person for the next month considering expenses?

1. **Suggestions rely on mathematics in terms of computations.**
2. **Suggestions rely on everyday life thinking.**

I. TURKISH SUMMARY/TÜRKÇE ÖZET

MATEMATİK TARİHİ TABANLI MODELLEME ETKİNLİKLERİ İLE 7. SINIF ÖĞRENCİLERİNİN NEGATİF TAM SAYILAR KONUSUNDAKİ ANLAMALARININ İNCELENMESİ

Giriş

Negatif tam sayılar ve negatif tam sayıların öğretiminde kullanılabilecek yöntemler, öğrencilerin negatif tam sayılarla ilgili karşılaştıkları zorluklar ve bu zorlukların nasıl üstesinden gelineceği, matematik eğitimi araştırmacılarının ilgisini çeken konular arasındadır. Özellikle negatif tam sayıların öğretiminin pedagojik yönden ele alınması ve incelenmesi her zaman ilginç bir araştırma konusu olmuştur (Thomaidis, 1993). Negatif tam sayıların gerçek hayattaki ve matematikteki yeri negatif tam sayıların gerekliliği, insanların negatif tam sayılara duyduğu ihtiyacı ortaya çıkarmıştır (Galbraith, 1974).

Negatif tam sayılarla ilgili yapılan araştırmalar öğrencilerin negatif tam sayılarla ilgili problemleri çözerken kullandıkları işlemsel süreçleri öğrendiklerini ancak negatif tam sayıları anlamada ve kavramada zorlandıklarını ortaya koymuştur (Bolyard, 2005; Ferguson, 1993; Lyte, 1994; Shore, 2005; Steiner, 2009; Wilkins, 1996). Öğrenciler sayıları öğrenirken nesneleri sayarlar ve sayıların gerçek hayattaki somut karşılıkları ile ilişki kurarlar. Negatif tam sayıları öğrenmeye başladıklarında doğal sayılarla ilgili eski öğrenmeleri ile ilişki kurmaya ve somutlaştırmaya çalıştıklarında zorlanırlar. Bu nedenle, negatif tam sayıları öğrenirken işlemsel süreçlere odaklanılması, öğrencilerin negatif tam sayıların günlük hayattaki yeri ile ilgili bağlantı kurmalarını zorlaştırır (Gallardo, 2002). Çünkü öğrencilerin matematiksel konuları anlamlandırmaları bu konuların nasıl temsil edildiğinden ve ortaya konulduğundan etkilenir (Goldin, 2003). Bir matematik konusunun anlamlandırılması, bu konu ile ilgili somut tecrübeler ile gözlemler ve gerçek

hayattaki kullanımı ile bağlantı kurmakla ilişkilidir (English, 1997). Farklı matematik konuları ve farklı gösterimler arasında bağlantı kurmak öğrencilerin konuyu anlamlandırmalarını geliştirir (NCTM, 2000). Bu bağlamda matematik tarihi tabanlı modelleme etkinlikleri öğrencilerin farklı gösterimler arasında ve gerçek hayat ile ilişki kurmasını sağlar.

Matematik eğitiminde matematik tarihi kullanılması ve matematik tarihi ile matematik eğitiminin bütünleştirilmesi bir yüzyıldan daha fazla zamandır araştırılmaktadır (Fried, 2001). Çünkü bilim tarihinde matematiğin çok önemli bir yeri vardır ve matematik bilimi çok zengin bir tarihe sahiptir (Fauvel, 1991). Matematik eğitiminin, matematiğin bu zengin tarihinden ayrı tutulmadan öğretilmesi daha anlamlı bir öğrenme ortamı oluştururken öğrencilerin matematik konuları ile günlük hayat arasında ilişki kurmalarını sağlar. Matematik tarihinin matematik öğretimine bir öğretim yöntemi olarak dahil edilmesi yeni bir fikir değildir, 1900lerin sonundan itibaren matematik tarihi yeni bir disiplin olarak kabul edilmiştir (Furinghetti & Radford, 2002) ve günümüze kadar farklı açılardan bu konuya yaklaşan birçok çalışma yapılmıştır. Bugüne kadar toplumlar değişmiş, teknolojide birçok yenilik meydana gelmiştir ancak matematik tarihinin okullarda matematik eğitimine dahil edilmesi güncel bir çalışma ve tartışma konusudur (Fenaroli, Furinghetti, & Somaglia, 2014). Matematiğin bu önemli rolü nedeniyle öğretim süreci, öğrencilerin matematik çalışmaya değer verdiği, öğrenmeye istekli olduğu ve matematiğe karşı pozitif bir bakış açısı geliştirmesini sağlayacak şekilde düzenlenmelidir ve matematik tarihi bu amaca hizmet eder. Matematik tarihi matematik kaynaklarını analiz etmeyi teşvik eden, geçmişte yaşamış matematikçilerin karşılaştığı problemler ile bu matematikçilerin problemleri çözmek için geliştirdikleri bilgileri araştırarak düşündüren çok geniş bir çalışma alanıdır.

Matematik tarihinin matematik öğretimine entegre edilmesinin başlangıç amacı farklı bir pedagojik yöntem ile öğretmenlerin kendi matematiksel bilgilerine katkı sağlamaktı (Furinghetti, 2004). Bu nedenle birçok araştırmacı öğretmenler ile öğretmen adayları üzerindeki etkisi ile ilgili çalışarak matematik tarihinin öğretmen eğitimindeki rolünü araştırmıştır (Clark 2012; Fenaroli, Furinghetti, & Somaglia,

2014; Furinghetti 2007; Philippou & Christou 1998). Bunun yanında matematik tarihinin öğrencilerin motivasyonuna, öz yeterliliğine, matematiğe karşı tutumuna ve akademik başarılarına olan etkilerini araştıran çalışmalar da bulunmaktadır (Albayrak 2011; Bayam 2012; Ersoy 2015). Türkiye'de farklı yaş düzeyinden öğrenciler üzerine yapılan çeşitli nicel ve yarı deneysel çalışmalar matematik tarihinin matematik öğretimine dahil edilmesinin öğrencilerin akademik başarısını arttırdığı sonucuna ulaşılmıştır (Bayam 2012; Ersoy 2015; İdikut 2007; Özcan 2014).

Jankvist (2009) matematik tarihinin matematik derslerine entegre edilme sebeplerini "matematik tarihini araç olarak kullanmak" ve "matematik tarihini amaç olarak kullanmak" olarak iki ana yaklaşım altında sınıflandırmıştır. Matematik tarihini bir araç olarak kullanılması yaklaşımı, duyuşsal yönden matematik tarihinin öğrencinin motivasyonunu attırmasını, bilişsel yönden ise öğrencinin matematikle ilgili anlamalarının güçlenmesini amaçlamaktadır (Jankvist, 2009). Diğer yandan matematik tarihini bir amaç olarak kullanmak yaklaşımı ise matematik konularının öğretiminin tarihsel süreci ile matematiğin zaman içerisinde gelişimlerini ve değişimlerini, insanların ve farklı kültürlerin matematiğin gelişimine olan katkılarını ve etkilerini ele alarak öğretmeyi amaçlamaktadır (Jankvist, 2009).

Ayrıca, Fried (2001); matematik tarihinin matematik öğretimine entegre edilmesinin on beş sebebinden yola çıkarak, matematik tarihinin matematik öğretimine dahil edilme sebeplerini üç ana başlık altında toplamıştır (Fauvel, 1991). Bunlar; (i) matematik tarihi matematiğin gelişimindeki insan etkisini hissetmeye ve matematiği insan ürünü olarak tecrübe etmeye olanak sağlar, (ii) matematiği daha ilginç, anlaşılabilir ve ulaşılabilir yapar, (iii) matematiksel konuları, problemleri ve çözümlerini kavramaya yardım eder. Birinci sebebe göre, öğrenciler matematik tarihinin matematik öğretimine entegre edilmesi ile tarihteki rol modelleri keşfederken insanların matematiğe olan ihtiyacını, matematik çalışmak için olan motivasyonlarını ve duygularını ilişkilendirerek matematiğin çok kültürlü yönünün fark edebilirler. İkinci sebep ise matematik tarihi ile matematik öğretim aktivitelerinin zenginleşip çeşitlenebildiğini, öğrencilerin matematiğe karşı olan korku ve önyargılarının üstesinden gelerek matematiğin toplum içerisindeki yerini

fark edebileceklerini belirtmektedir. Üçüncü görüş, Jankvist (2009) tarafından yapılan matematik tarihinin araç olarak kullanılması sınıflandırması ile örtüşerek bir matematik konusu öğrenmenin bu konunun tarihsel gelişimine paralel olduğunu ve bu sayede matematik problemleri ile ilgili farklı çözüm yolları sağlarken fikirler, tanımlar ve uygulamalar arasındaki ilişkilerin fark edilmesini kolaylaştıracağını savunmaktadır. Tzanakis ve Arcavi (2002) de çalışmalarında, matematik eğitiminde matematik tarihi kullanmanın önemine dair beş temel sebep sunmuşlardır; (i) matematik tarihi öğretim aktivitelerini zenginleştirir, (ii) matematik etkinliklerinin yapısı gereği öğrencilerin matematiğe bakış açılarını olumlu yönde geliştirir, (iii) öğretmenlerin ders içi etkinliklerini zenginleştirir, (iv) öğrencilerin matematikle ilgili duyuşsal eğilimlerini pozitif olarak etkiler ve (v) öğrencilerin matematiği kültürel yönü ile insan ürünü bir bilim olarak değerlendirmelerini sağlar. Yukarıda bahsedilen her iki çalışmada matematik tarihinin matematik eğitiminde kullanılmasının ve matematiğin gelişimindeki insan etkisini keşfetmenin, öğrencilerin motivasyonunu arttırması ve matematiğe karşı olumlu tutum geliştirmeleri gibi duyuşsal etkilerini ifade etmişlerdir. Ayrıca, bu çalışmalar matematik eğitimine matematik tarihi entegre edilmesinin öğretmenlerin öğretim materyallerine katkı sağlarken öğretmenlerin öğrencilerin zorluklarını anlamalarına da fayda sağlayacağını belirtmişlerdir.

İlgili literatüre göre, matematik, insan hayatından durumlarla, sosyal ve düşünsel bir iklimle, gerçek hayattan insan uğraşları ile ilişkili bir şekilde düşünülmelidir (Fried, 2001) ve günümüzde matematik tarihi öğrencilerin matematiğin gelişimine neden ihtiyaç duyulduğunu anlamalarına olanak sağlar. Çünkü öğrenciler özellikle gerçek hayat ile bağlantısı olmayan matematik konularını kavramada zorluklarla karşılaşmaktadırlar (Gulikers & Blom, 2001). Öğrenciler için günümüzde derste kullanılan matematik problemlerinden ziyade geçmiş hayatta günlük olarak karşılaşılmış ve çözümünde matematik kullanılmış konular daha somut ve anlaşılması kolaydır (Savizi, 2007). Çünkü öğrencilerin matematiğin gerçek hayatla ilgili problemlerde basit araçlar ve tekniklerle nasıl kullanıldığını tecrübe etmeleri karmaşık yöntemlerden ve bir yığın matematiksel bilgi sunmaktan daha faydalı bir yöntemdir (Savizi, 2007). Ayrıca bu şekilde bir matematik eğitimi öğrencilerin

özgüvenini geliştirerek bir insan olarak kendi yeteneklerine olan inançlarını arttırmaları için onları destekler (Savizi, 2007). Matematik konularının gerçek hayatla ilişki kurulmadan öğretilmesi öğrencilerin bu konuları anlamlandırmalarını zorlaştırır (Guliker & Blom, 2001). Benzer şekilde, modelleme etkinlikleri de öğrencilerin matematik konuları ve gerçek hayattaki kullanımları arasında ilişki kurmalarını sağlar ve konuyu anlamalarını destekler (Lesh, Hoover, Hole, Kelly, & Post, 2000). Modelleme ile problem çözme süreci ve gerçek yaşam durumları ile ilgili matematik problemleri üzerinde çalışmak öğrencilerin ilgili konuyu anlamalarını destekler (Bonotto, 2007). Modelleme etkinlikleri sayesinde gerçek yaşamla bağlantı kuran öğrenciler daha mantıklı ve anlamlı matematiksel gerekçelendirmeler yapabilir (Doerr & Lesh, 2003).

Matematik tarihinin matematik dersine entegre edilmesi ve matematik öğretimine olan etkileri dünya genelinde geniş bir araştırma geçmişine sahip olsa da Türkiye'de bu alanda yapılan çalışmalar 2000lerin başından itibarendir (Alpaslan, 2011). Oysaki günümüz ilkokul ve ortaokul matematik öğretim programında belirlenen özel amaçlardan biri, öğrencilerin, matematiğin insanlığın ortak bir değeri olduğunun bilincinde olarak matematiğe değer vermesine yöneliktir (MoNE, 2018). Türkiye'de matematik tarihi üzerine yapılan çalışmaların ise birçoğu matematik tarihi ile öğretmen eğitimini geliştirmeye yöneliktir (Alpaslan, 2011). Öğrenciler ile yapılan çalışmalarda ise çoğunlukla motivasyon, öz yeterlilik, özgüven, matematiğe karşı tutum gibi duyuşsal eğilimlere odaklanılmış ve buna ek olarak akademik başarıya olan etkileri incelenmiştir. Ancak, erişilebilen alan yazılarında matematik tarihinin öğrencilerin konuları anlamlandırmalarına ve anlamalarına olan etkilerini ele alarak bilişsel yönden inceleyen araştırma sayısı azdır.

Yukarıda anlatılan çalışmalar ışığında ve açıklanan sebeplerden dolayı, bu çalışmanın amacı matematik tarihi tabanlı modelleme etkinlikleri ile yedinci sınıf öğrencilerin negatif tam sayıları anlamalarını incelenmesi olarak belirlenmiştir.

Çalışmanın Amacı

Bu çalışmanın temel amacı matematik tarihi tabanlı modelleme etkinlikleri ile

yedinci sınıf öğrencilerin negatif tam sayılar konusunu anlamalarını incelemektir. Bu bağlamda, bu çalışma özellikle aşağıda belirtilen araştırma sorularına cevap aramak için yapılanmıştır:

1. Matematik tarihi tabanlı modelleme etkinlikleri ile küçük gruplar halinde çalışan 7. Sınıf öğrencilerinin negatif tam sayıları anlamalarında nasıl bir gelişim olur?

2. Matematik tarihi tabanlı modelleme etkinlikleri ile çalışan 7. Sınıf öğrencilerinin negatif tamsayıları bireysel anlamaları nasıl değişir?

Bu amaçla, ilkokul ve ortaokul matematik öğretim programı göz önünde bulundurularak matematik tarihi tabanlı modelleme etkinlikleri hazırlanmış ve uygulanmıştır. Aynı açık uçlu sorulardan oluşan ilk ve son değerlendirme testi öğrencilerin negatif tam sayılara dair anlamalarını ortaya çıkarmak için uygulanmıştır. Matematik tarihi modelleme etkinlikleri ile ilk ve son değerlendirme aracından elde edilen veriler derinlemesine analiz edilmiştir.

Önemli Terimlerin Tanımları

Matematik tarihi: *Tarih* terimi "bir kişi veya herhangi bir şey ile ilgili geçmişte olan olaylar serisi" olarak ve *matematik* terimi "sayılar, miktar ve uzaya dair soyut bir bilim yani soyut matematik veya fizik ve mühendislik gibi farklı disiplinler ile ilişkili uygulamalı matematik" olarak tanımlanmışlardır (Oxford Dictionaries, 2019). Bu tanımlardan yola çıkarak *matematik tarihi* "sayılar, miktar ve uzayla ilgili soyut veya uygulamalı matematik alanlarında geçmişten günümüze bağlantılı olarak meydana gelen gelişmeler" olarak tanımlanmıştır (Alpaslan, 2011, s.14).

Matematik tarihinin matematik eğitiminde kullanılması: Matematik tarihinin matematik eğitiminde bir öğretim yöntemi olarak kullanılması üç ana yönden ele alınarak: (i) "tarihi öğrenmek; doğrudan tarihi bilgiler sağlamak" (s. 208) yani tarihi öğrenmenin amaçlanması, (ii) "matematik konularını öğrenmek; tarihten etkilenerek bir öğretim yaklaşımı uygulamak" (s.208) yani matematiksel öğrenmenin amaçlanması ve (iii) "matematiğe dair daha derin bir farkındalık geliştirmek;

matematiğin kültürel ve sosyal etkiler altında gelişimine odaklanmak" (s.208) yani matematiğin bir bilim olarak gelişimini, diğer bilimler ve insanlar ile olan ilişkisini vurgulamak şeklinde tanımlanmıştır (Tzanakis & Arcavi, 2000).

Model: Bu çalışmada *model* terimi yedinci sınıf öğrenciler tarafından inşa edilen, oluşturulan, tanımlanan ve vurgulanan matematiksel olarak anlamlı ürünler, süreçler ve matematiksel gerekçelendirmeleri içeren, günlük hayatla ilişkili olarak paylaşılabilen, geliştirilebilen ve tekrar kullanılabilen modelleri ifade etmektedir (Doerr & Lesh, 2003).

Yöntem

Çalışma Deseni

Bu çalışmanın araştırma sorusu, matematik tarihi tabanlı modelleme etkinliklerinin yedinci sınıf öğrencilerin negatif tam sayıları anlamaları üzerindeki rolünü analiz etmeyi amaçlamaktadır. Bu nedenle, nitel örnek olay araştırma deseni çerçevesinde uygulanmıştır. Bu bağlamda hazırlanmış olan matematik tarihi tabanlı modelleme etkinlikleri ile açık uçlu sorulardan oluşan ilk ve son değerlendirme testleri ile elde edilen veriler derinlemesine incelenmiştir.

Katılımcılar

Bu çalışmada örneklem uygun örnekleme yöntemi kullanılarak belirlenmiştir. Bu bağlamda, çalışmanın örneklemini 2018-2019 öğretim yılında İstanbul'un Kartal ilçesinde bulunan bir devlet okulunda öğrenim gören 29 (14 kız ve 15 erkek) 7. sınıf öğrencisi oluşturmaktadır.

Veri Toplama Aracı

Eğitim alanında yapılan nitel örnek olay araştırma deseninde, araştırmacılar çoklu veri toplama araçlarını tercih etmektedirler çünkü birden fazla veri kaynağı araştırmanın geçerliliğini ve güvenilirliğini desteklemektedir (Yin, 2003). Bu bağlamda, bu çalışmada üç tane matematik tarihi tabanlı modelleme etkinliği ile öğrencilerin grup olarak hazırladıkları yazılı cevapları toplanmıştır. Ayrıca, bu

etkinlikler uygulanmadan önce ve sonrasında aynı değerlendirme aracı ilk ve son değerlendirme amacı ile her bir öğrenciye uygulanmıştır. Bu değerlendirme aracı altı adet açık uçlu tam sayı sorusundan oluşmaktadır. Aktivitelerin uygulanması sırasında video ve ses kaydı ile toplanan veriler, yazılı verileri desteklemek ve kontrol etmek amacıyla kullanılmıştır.

Verilerin Analizi

Bu çalışmanın amacı matematik tarihi tabanlı modelleme etkinlikleri ile yedinci sınıf öğrencilerin negatif tam sayıları anlamalarını incelemektir. Bu amaçla üç tane etkinlik ve ayrıca aynı soruları içeren, ilk ve son değerlendirmede kullanılan bir değerlendirme testi hazırlanmıştır. Bu teste öğrencilerin bireysel olarak verdikleri cevaplar, matematik tarihi tabanlı modelleme etkinliklerindeki öğrencilerin yazılı çalışmaları göz önünde bulundurularak karşılaştırılmıştır.

İlk ve son değerlendirme testinde bulunan açık uçlu sorular ilk olarak doğru ve yanlış cevaplar olarak ele alınmıştır. Bu ilk inceleme sonucunda bütün öğrencilerin değerlendirme testinin 1., 2. ve 6. sorularına doğru cevap verdikleri gözlemlenmiştir. Bu nedenle öğrenci cevaplarının çeşitlilik ve değişim gösterdiği 3., 4. ve 5. sorular içerik analizi yöntemiyle derinlemesine incelenmiştir. İçerik analizi yöntemi öğrenci cevaplarının kodlanması ve kategorize edilmesi ile verilerin düzenli, tarafsız ve nitel olarak analiz edilmesi için kullanılmaktadır (Neuendorf, 2002). Ayrıca, öğrencilerin matematik tarihi tabanlı modelleme etkinliklerine grup olarak verdikleri cevaplar ile aktivitelerin uygulanması sırasındaki performansları video ve ses kaydı ile kayıt altına alınmıştır. Öğrencilerin aktivitelere verdikleri yazılı cevaplar ilk kodlama ve betimsel kodlama olmak üzere iki aşamada kodlanmıştır (Saldana, 2009). İlk kodlamada öğrencilerin cevapları bütünsel olarak ele alınmış ve bu kodlamalar üzerinden öğrencilerin genel kavramaları incelenmiştir. Daha sonra betimsel kodlama ile öğrencilerin kavramaları ve anlamalarını incelemek için öğrenci cevapları kategorize edilmiştir. Bu iki aşamalı kodlamalardan elde edilen veriler ayrıca video ve ses kayıtları ile kontrol edilerek desteklendi.

Bulgular ve Tartışma

Matematik Tarihi Tabanlı Modelleme Etkinliklerinin Öğrencilerin Negatif Tam Sayıları Anlamaları Üzerindeki Rolü

Bu çalışmada matematik tarihi tabanlı modelleme etkinliklerinin yedinci sınıf öğrencilerinin negatif tam sayıları anlamaları üzerindeki rolü incelenmiştir. Bu bağlamda, çalışmanın bulguları ele alınırken öğrencilerin matematik tarihi tabanlı modelleme etkinliklerine verdikleri cevaplar göz önünde bulundurularak ilk ve son değerlendirme testindeki sorulara verdikleri cevaplar karşılaştırılmıştır. Çalışmada elde edilen bulgular öğrencilerin anlamalarını ortaya çıkarmak için negatif tam sayıları anlamlandırmaları, pozitif ve negatif tam sayıları tanımlamaları ve negatif tam sayıların günlük hayattaki kullanımı ile ilişki kurmaları yönünden incelenmiştir.

İlk değerlendirme testindeki öğrenci cevapları incelendiğinde, öğrencilerin genel olarak sadece negatif tam sayıların sembolik gösterimi bilgisine sahip oldukları ancak negatif tam sayıları anlamlandıramadıkları gözlemlenmiştir. Bütün öğrencilerin tam sayıların önünde bulunan artı ve eksi işaretlerinden yola çıkarak negatif ve pozitif tam sayıları seçme ve ayırt etme bilgisine sahip oldukları belirlenmiştir. Buna rağmen, öğrenciler negatif tam sayıları anlamlandırırken ve bu sayıların günlük hayattaki kullanımı ile ilişki kurarken zorluk yaşamışlardır. Çalışmanın bu bulguları öğrencilerin negatif tam sayılar anlamlandırmada ve günlük hayattaki kullanımlarının bilincinde olma konusunda zorluklar yaşadıklarını desteklemiştir (Whitacre et al.,2017). Ayrıca, ilk değerlendirme aracından elde edilen bulgular, öğrencilerin tam sayılarla işlemleri içeren matematik problemlerini çözerken matematiksel mantık yürütme ve gerekçelendirme yapmadan ezber bilgilerini kullanarak cevapladıklarını ortaya koymuştur. Örneğin, öğrencilerin birçoğu değerlendirme testinin 4. sorusunda bilinmeyen tam sayının değerini bulmak için "iki pozitif tam sayının toplamı yine bir pozitif tam sayıya eşittir" şeklinde ezberledikleri bir kuralı kullanmışlardır. Bu kuralı kullanırken matematiksel bir gerekçelendirme yapamadıkları gözlemlenmiştir. Bunun yanında, değerlendirme testindeki 4. ve 5. sorulara cevap verirken işlemsel süreçleri takip eden öğrencilerin

birçoğunun, bir pozitif ile bir negatif tam sayının toplama işlemi ile karşılaştıklarında deneme-yanılma yöntemini kullandıkları ve yaptıkları işlemleri açıklayamadıkları gözlemlenmiştir. Bu nedenle öğrencilerin problemlerin çözümlerine matematiksel açıklamalar yapmakta ve çözümlerini gerekçelendirmekte zorlandıkları, negatif tam sayıların günlük hayattaki kullanımı ile ilişki kuramadıkları belirlenmiştir. İlk değerlendirmenin bu bulguları, öğrencilerin işlemsel süreçlere odaklanırken tam sayıları anlamlandırmada eksik kaldıkları için negatif tam sayıları anlamada zorluk yaşadıklarını belirten çalışmaları desteklemektedir (Bolyard, 2005; Ferguson, 1993; Lyte, 1994; Shore, 2005; Wilkins, 1996).

Buna rağmen, son değerlendirme testinin 3. sorusunda öğrencilerin birçoğunun bir negatif tam sayıyı tanımlarken günlük hayat ile ilişki kurarak borç, gider, zarar ve sıcaklık gibi örnekler vererek tanımlarını destekledikleri belirlenmiştir. Bu çalışmanın amacı ve düzenlenmesi göz önünde bulundurularak, öğrencilerin negatif bir tam sayıyı tanımlamada gösterdikleri gelişimin matematik tarihi tabanlı modelleme etkinliklerinden kaynaklandığı düşünülmektedir. Çünkü hazırlanan günlük hayatla ilişkili etkinlikler ile öğrencilerin grup çalışması yapmaları, kendi fikirlerini ve bilgilerini arttırarak anlamalarını geliştirmeleri için uygun bir öğrenim ortamı hazırlamıştır. Öğrenciler son değerlendirme testinde negatif bir tam sayıyı tanımlarken sembolik gösteriminden faydalanmış olsalar da ilk değerlendirmeden farklı olarak bu tanımlamalarını farklı örneklerle ilişkilendirerek desteklemişlerdir. Bu nedenle matematik tabanlı modelleme etkinlilerinin öğrencilerin negatif bir tam sayıyı anlamlandırmaları üzerinde olumlu bir rolü olduğu sonucuna ulaşılabilir. Ayrıca, değerlendirme testinin 4. ve 5. sorularında, ilk değerlendirmede işlemsel süreçlere ve işlemin sonucuna odaklanarak bilinmeyen tam sayının değerini bulan öğrenciler, son değerlendirmede kendi çözüm yolları için matematiksel gerekçelendirmeler yapabilmişlerdir. Çünkü değerlendirme testinin 4. ve 5. sorularında, öğrencilerin birçoğu, toplama işlemlerinde verilen sonuç ile toplanan arasında matematiksel bir mantık yürütme yaparak bilinmeyen tam sayının çeşidine doğru karar verip değerini bulmuşlardır. Örneğin, toplama işleminde sonucun, soruda verilen toplanan tam sayıdan büyük olması durumunda bilinmeyen tam sayının pozitif bir tam sayı olacağını veya toplama işleminin sonucu verilen

toplanandan küçük ise bilinmeyen toplananın negatif bir tam sayı olacağını belirtmişlerdir. Bu bulgular göz önünde bulundurularak matematik tarihi tabanlı modelleme etkinliklerinin ve günlük hayatla ilişki modelleme etkinlikleri üzerinde grup olarak çalışmalarının, öğrencilerin matematiksel gerekçelendirme ve dolayısı ile anlamalarını geliştirdiği söylenebilir. Çalışmanın bu bulguları, tam sayılar üzerine yapılan farklı çalışmaların, öğrencilerin kazanç, borç, sıcaklık, gider gibi günlük hayatla ilişkili modelleme ve problem çözme çalışmalarının tam sayıları anlamlandırma ve kavramalarını geliştirdiğini belirten bulgularını desteklemektedir (Ball, 1993; Stephan & Akyuz, 2012; Pettis & Glancy, 2015). Benzer şekilde, günlük yaşamla ilişkili matematik problemlerini üzerinde çalışmanın, farklı çözüm yolları üzerinde grup olarak tartışmanın ve matematik tarihi içeren öğretim aktivitelerinin öğrencileri motive ettiği ve matematiğe karşı olan korkularını yenmelerini sağladığını belirten çalışmaların bulgularını da desteklemektedir (Fauvel, 1991; Liu, 2003; Swetz, 1997; Tzanakis & Arcavi, 2000). Çünkü, aktivitelerin uygulanması sürecinde alınan kayıtlara ve araştırmacının notlarına göre öğrenciler bütün matematik derslerinin bu çalışma için düzenlenen eğitim ortamında olduğu gibi olmasını istediklerini ifade etmişlerdir.

Bu çalışmanın ilgili literatüre bir diğer önemli katkısı ise matematik tarihinin modelleme etkinliklerine entegre etmesidir. Çalışmanın bu yönü, modelleme etkinliklerinin, öğrencilerin motivasyonu ve matematiğe karşı olan tutumları üzerindeki rolünü daha etkili kılmıştır. Çünkü, geçmişte günlük hayatta karşılaşılan problemler, günümüz matematik problemlerinde sunulan durumlara göre öğrenciler için daha somut ve anlaşılır olabilir ve karmaşık kuralların uygulandığı problem çözme sürecine göre insanlarla ve gerçek yaşamla ilişkilendirerek daha anlaşılır bir çözüm üzerinde çalışmak öğrenciler için daha motive edicidir (Savizi, 2007). Bu bağlamda, bu çalışmada matematik tarihi tabanlı modelleme etkinlikleri ile geçmişten ve günümüzden gerçek yaşamla ilişkili matematik problemleri ortaya konulmuştur. Öğrenciler daha önce bu çalışmada sağlandığı gibi bir sınıf ve öğretim ortamında çalışmamalarına rağmen, aktiviteler üzerinde yaptıkları grup çalışmaları sırasında aktif katılım göstermişledir. Aktivitelerin uygulanması sürecinde yapılan gözlemler, öğrencilerin geleneksel olarak öğretmen merkezli işlenen derslere göre

daha istekli bir şekilde derse katıldıklarını göstermiştir. Bu çalışmada uygulanan aktiviteler ve öğrencilerin çalıştıkları öğretim ortamı düşünüldüğünde matematik tarihi tabanlı modelleme etkinlikleri ile günlük hayatla ilişkili problemler üzerinde grup olarak çalışmanın bu sonuca neden olduğu söylenebilir. Çünkü bu çalışma bağlamında sunulan etkinlikler ve öğrenim ortamı, öğretmen merkezli bir ders ortamında bireysel olarak problem çözmeye kıyasla, öğrencilerin matematik dersine daha aktif bir şekilde katılmalarını, fikirlerini ve çözümlerini korkmadan ifade edebilmelerini sağlamış olabilir. Literatürde de belirtildiği gibi gerçek yaşamla ilişkili bir problemi, grup çalışması ile matematiksel olarak anlamlı bir model oluşturarak çözmek, geleneksel bir eğitim ortamında bireysel olarak çalışmaya kıyasla öğrencilerin ilgilerini ve motivasyonlarını arttırır (English, Lesh, & Zawojewski, 2003). Bu bağlamda, çalışmanın bu bulguları matematik tarihi içeren ve günlük hayatla ilişkili olan matematik problemlerinin öğrencileri motive ederek matematiğe karşı olan korkularını yenmelerini sağladığını belirten çalışmaların bulgularını desteklemektedir (Fauvel, 1991; Liu, 2003; Swetz, 1997; Tzanakis & Arcavi, 2000).

Öneriler

Bu çalışma matematik tarihi tabanlı modelleme etkinliklerinin yedinci sınıf öğrencilerin negatif tam sayıları anlamaları ve anlamlandırmaları üzerindeki rolünün incelenmesine odaklanmıştır. Çalışmanın bulguları göz önünde bulundurularak gelecek araştırmalar için bazı önerilerde bulunulabilir.

Bu çalışmada matematik tarihi tabanlı modelleme etkinliklerinin uygulanması ortaokul matematik öğretim programı ve matematik dersi yıllık planı göz önünde bulundurularak uygulanmıştır. Bu nedenle belirtilen aktivitelerden üç tane hazırlanarak sınırlı bir zaman dilimi içerisinde uygulanmıştır. Daha fazla sayıda aktivite ile öğrencilerin daha uzun bir zaman dilimi süresince gözlenmesi bu öğrencilerin negatif tam sayıları anlamaları üzerine daha fazla yorumlanabilir nitel veri elde edilmesini sağlayabilir.

Ayrıca, bu çalışmada yalnızca öğrencilerin negatif tam sayılar konusunu anlamaları

ele alınmıştır. Matematik tarihi tabanlı modelleme etkinlikleri farklı konular ile ilgili olarak farklı yaş gruplarındaki öğrencilere de uygulanabilir ve bu öğrencilerin anlamaları, matematiksel yorumlama ve gerekçelendirme becerileri üzerindeki etkileri incelenebilir.

Bunlara ek olarak, matematik tarihi tabanlı modelleme etkinlikleri ve bu çalışmada elde edilen bulgular ele alındığında bir öğretim yöntemi olarak matematik öğretmenlerinin gelişimine de katkıda bulunabilir. Bu etkinlikler öğretmenlerin öğretim materyallerine katkıda bulunurken aynı zamanda öğrencilerin anlamalarındaki rolünü anlamaları için kullanılabilir.

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YAZARIN / AUTHOR

Soyadı / Surname : AY
Adı / Name : BÜŞRA
Bölümü / Department : THE DEPARTMEN OF ELEMENTARY MATHEMATICS AND SCIENCE EDUCATION

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