TWO ESSAYS ON AMBIGUITY AND ASSET PRICING

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF SOCIAL SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

BAKİ CEM ŞAHİN

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY
IN
THE DEPARTMENT OF BUSINESS ADMINISTRATION

SEPTEMBER 2019

Approval of the Graduate School of Social Sciences

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ABSTRACT

TWO ESSAYS ON AMBIGUITY AND ASSET PRICING

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September 2019, 169 pages

This thesis consists of two essays on the impact of ambiguity on asset pricing. In the first essay, we provide a detailed review of theoretical models incorporating ambiguity into both decision-making and asset pricing models. In the framework of these discussions, we derive ambiguity indices and we provide both a comparison among themselves and an analysis showing the impact of ambiguity on asset pricing for Turkey. Our results confirm the existence of impact of ambiguity on asset returns even it is not strong. Second essay extents the analysis on the relationship between ambiguity and asset pricing by focusing on portfolio and stock level returns. The analysis incorporating other risk factors used commonly in the literature show that ambiguity is a factor priced in stock returns in Turkey.

Keywords: Ambiguity, Asset Pricing, Ambiguity Index, Turkey

ÖZ

BELİRSİZLİK VE VARLIK FİYATLAMASINA İLİŞKİN İKİ MAKALE

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Tez Yöneticisi: Doç. Dr. Seza Danışoğlu

Eylül 2019, 169 sayfa

Bu çalışma belirsizliğin varlık fiyatlaması üzerine etkisi üzerine yazılan iki makaleden oluşmaktadır. İlk makalede belirsizliği dahil eden karar ve varlık fiyatlaması teorik modelleri üzerine detaylı bir inceleme sunulmaktadır. Bu tartışmalar çerçevesinde farklı belirsizlik endeksleri elde edilmiş ve bu endeksler arasındaki ve bu endekslerin hisse senedi endeksi ile arasındaki ilişkiyi Türkiye için gösteren bir analiz sunulmuştur. Sonuçlar belirsizliğin hisse senedi getirisi ürerindeki etkisini güçlü olmasa da doğrulamaktadır. İkinci makale belirsizlik ve varlık fiyatlaması arasındaki ilişkiyi portföy ve hisse senedi bazında getirilere odaklanarak genişletmektedir. Yazında sıklıkla kullanılan diğer risk faktörlerinin de yer aldığı analizlerdeki sonuçlar belirsizliğin Türkiye'de hisse senedi getirisinde bir faktör olarak fiyatlandığını göstermektedir.

Anahtar Kelimeler: Belirsizlik, Varlık Fiyatlaması, Belirsizlik Endeksi, Türkiye

To My Daughter DEFNE ŞAHİN

ACKNOWLEDGMENTS

I would like to express my deepest gratitude to my supervisor Assoc. Prof. Dr. Seza Danışoğlu for her continuous support and encouragements, guidance and insight during the research. I would also thank to my thesis committee members, Prof. Dr. Nuray Güner, Prof. Dr. Aslıhan Salih, Assoc. Prof. Dr. Semih Akçomak and Assist. Prof. Işıl Sevilay Yılmaz for their review, comments and discussions.

I would like to thank Prof. Dr. Nuray Güner, Assoc. Prof. Dr. Seza Danışoğlu, Assoc. Prof. Dr. Engin Küçükkaya, Assoc. Prof. Dr. Adil Oran, Dr. Hande Ayaydin Haciömeroğlu, Dr. İlkay Şendeniz Yüncü, Prof. Dr. Uğur Soytaş and Prof. Dr. Ramazan Sarı for their excellent guidance during PhD period.

I thank to my mother Havva Şahin and father Ali Şahin for their unconditional love, support, encouragement and motivation. I also thank to my sisters Nilay Keçeli and Nilgün Şahin who have always been next to me when I need help.

Lastly, I thank my daughter Defne Şahin to be in my life and make life meaningful. I also thank my wife Emel Şahin to be next to me.

TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

LIST OF ABBREVIATIONS

APT	Arbitrage Pricing Theory
ARCH	Autoregressive Conditional Heteroskedasticity
ATM	At The Money
BIST	Borsa Istanbul
CAPM	Capital Asset Pricing Model
CBOE	Chicago Board Options Exchange
CCAPM	Consumption-Based Capital Asset Pricing Model
CDS	Credit Default Swap
CPI	Consumer Price Index
EMBI	Emerging Markets Bond Index
ETF	Exchange-Traded Fund
FED	The Federal Reserve
GDP	Gross Domestic Product
GMM	Generalized Method of Moments
ICAPM	Intertemporal Capital Asset Pricing Model
MEU	Maxmin Expected Utility
REIT	Real Estate Investment Trust
SEU	Subjective Expected Utility
SPF	Survey of Professional Forecasters
TL	Turkish Lira
USD	United States Dollar
USDTRY	US Dollar Turkish Lira Exchange Rate
VIOP	Borsa Istanbul Derivatives Market
VIX	CBOE Volatility Index
VNM	von Neumann-Morgenstern
VRP	Variance Risk Premium

CHAPTER 1

INTRODUCTION

Finance literature has discussed the risk-return relationship extensively and early discussions goes back to portfolio theory of Markowitz (1952) and Capital Asset Pricing Model (CAPM) of Sharpe (1964). Although CAPM successfully establishes risk-return relationship and has been at the center of asset pricing literature, weak empirical support ((Black, Jensen, and Scholes (1972), Fama and Macbeth (1973) and Fama and French (1992, 1993))) makes the model open to criticism. Early empirical evidences (Basu (1977), Reinganum (1981), Banz (1981) and Basu (1983)) show that variables like the earnings to price ratio (E/P) ratio and size have additional explanatory power on the cross-sectional stock returns in addition to risk definition of market beta. Later, Fama and French (1996) combine the existing empirical evidence on other risk factors by introducing the concept of "asset pricing factors" and test CAPM where the market, size (SMB) and value (HML) factors are shown to be relevant risk factors. Successively, literature added up new asset pricing factors that Carhart's (1997) momentum, Amihud (2002), Pastor and Stambaugh (2003) and Acharya and Pedersen's (2005) liquidity, Harvey and Siddique's (2000) coskewness, Xing (2008), Hou, Xue, and Zhang (2015) and Fama and French's (2015) investment and profitability are some these risk factors.

Another widely-discussed point of CAPM has been the restrictive assumptions of the model diverging from real investment decision making. Black's (1972) two-factor model, Merton's (1973) multi-period CAPM (ICAPM), Ross's (1976) Arbitrage Pricing Theory (APT), Breeden's (1979) consumption CAPM are theoretical models aiming to relax some of these restrictive assumptions to reach a more realistic model setup. However, assumptions in CAPM are not independent from the assumptions in other fields of science. For example, assumptions of asset pricing theory is closely related with the assumptions in decision-making theory. Decision-making in earlier finance theories is based on the setup that agents make investment decisions over unique probability distributions (Von Neumann-Morgenstern (1944) and Savage (1954)) and this single probability distribution assumption is closely related with perfect information and homogenous expectations assumptions of CAPM contradicting with real world investment decision-making as inferred from the experimental results of Ellsberg (1961).

Relaxing the single distribution assumption requires to modify decision-making theory and introduces ambiguity over the probability distribution having implications for the definition of risk in conventional asset pricing models. The literature has spent serious effort in integrating ambiguity into the decision-making process (Gilboa and Schmeidler (1989), Schmeidler (1989), Epstein and Wang (1994), Chen and Epstein (2002), Hansen and Sargent (2001) and Klibano, Marinacci and Mukerji (2005)) and into the asset pricing theories (Chen and Epstein (2002), Kogan and Wang (2003), Bansal and Yaron (2004), Boyle et al. (2009) and Anderson, Ghysels, and Juergens (2009)). The theoretical models successfully differentiate the role of risk and ambiguity on asset pricing but empirical applications (Bollerslev, Tauchen, and Zhou (2009), Bekaert, Engstrom, and Xing (2009), Andreou et al. (2014), and Brenner and Izhakian (2018)) remain limited due to the difficulty in measuring ambiguity compared to risk.

In this study, we aim to investigate the impact of ambiguity on cross-sectional asset return in Turkey first by deriving an ambiguity index and then by incorporating this ambiguity index into asset pricing model. Since the literature shows risk factors other than ambiguity to have significant effect on stock returns, ambiguity is added to the asset pricing models alongside these other factors.

Accordingly, this study contributes to the literature in a number of ways by providing two essays. First essay brings together a complete review of the development of the relationship between ambiguity and asset pricing, and contributes to the empirical literature on ambiguity by constructing an index that will be the first of its kind for Turkey and first of a limited number studies for other countries. Also, first essay provides an initial evidence on the impact of ambiguity on asset return even it is not strong. Second essay, first, provides a detailed review of asset pricing models, testing methodologies and a review of risk factors. Subsequently, risk factors are calculated and a detailed and up-to-date analysis on the relationship between the risk factors and stock returns is provided for Turkish stock market. Finally, ambiguity is tested as an additional risk factor for the Turkish stock market and the results confirm that ambiguity is a factor affecting cross-sectional stock returns.

CHAPTER 2

ASSET PRICING, AMBIGUITY AND MEASURING AMBIGUITY IN TURKEY

2.1 Introduction

In finance literature, the risk-return relationship has been discussed extensively. As one of the pioneers in the field, Markowitz (1952) defined the risk of a portfolio as the variance of returns. Later, Sharpe (1964) moved one step further and re-defined the risk for an individual asset as its contribution to the overall riskiness of the portfolio that includes the asset. Sharp's model was the Capital Asset Pricing Model (CAPM) and its major innovation was the way it defined risk, leading to an evolutionary change in the finance literature. Although the model is one of the cornerstones in the asset pricing literature, its restrictive assumptions often contradict real life applications, and, therefore, the model has weak empirical support. Subsequent studies relaxed these assumptions with the purpose of constructing a setup that is closer to reality. Nonetheless, most of these models still carry assumptions that highly limit the ability of the model in explaining investor behavior.

Model assumptions used in the asset pricing literature are not independent from earlier theories that were developed in other fields of science. In earlier economics and finance theories, agents were assumed to be rational, and, consistent with the decision theory, to make their choices in a way to maximize their utility. In this setting, agents first form their preferences and then make investment decisions over objective probability distributions conditional on a list of outcomes and they do not make systematic errors. As a matter of fact, these assumptions are the main building blocks of Von Neumann-Morgenstern's expected utility theory (1944). Although

Savage (1954) replaced Von Neumann-Morgenstern's objective probability with a subjective probability distribution, he still assumed rational investors to behave consistently with Bayes' law and update their beliefs following the arrival of new information; as a result, the probability distribution for each event is unique in Savage's model.

This unique probability distribution assumption is very closely related with the other assumptions of CAPM. For example, the assumptions of perfect information and homogenous expectations both imply a unique probability distribution. Such assumptions remove the uncertainty regarding the return distribution, making it possible to establish the risk-return relationship. However, the assumption of a single probability distribution contradicts with investor behavior because perfect information and homogenous expectations are only partially realistic. Agents, especially financially unsophisticated agents, may not have a concrete idea about the return distribution of an individual asset or a portfolio. Such a realization conveys new insights about the decision-making process in asset pricing and leads to less demanding prerequisites and assumptions. One such change is to drop the assumption that the probability distribution of returns is known and unique. Removing the single distribution assumption introduces a vagueness over the return distribution that in turn leads to uncertainty or ambiguity. Ambiguity over the distribution of returns has the potential to affect the investment and consumption decisions of agents and also makes it necessary to make modifications to the asset pricing models.

The existing literature has spent serious effort in integrating ambiguity into the decision-making process (Gilboa and Schmeidler (1989), Schmeidler (1989), Epstein and Wang (1994), Chen and Epstein (2002), Hansen and Sargent (2001) and Klibano, Marinacci and Mukerji (2005)). Such new decision rules also affect the asset pricing theories and several new asset pricing models that address ambiguity have been proposed recently (Chen and Epstein (2002), Kogan and Wang (2003), Bansal and Yaron (2004), Boyle et al. (2009) and Anderson, Ghysels, and Juergens (2009)). They successfully differentiate between the impact of ambiguity and the impact of risk on asset pricing. The increasing number of theoretical studies showing the impact of ambiguity on asset pricing also created an interest in quantifying this impact. To date, empirical applications lag behind theory and remain limited due to the difficulty in measuring ambiguity. Although risk can be calculated by simple metrics like the variance of return or the market beta, the definition of ambiguity makes its measurement a complicated task.

In this study, the purpose is to introduce an ambiguity index for Turkey that (i) is consistent with the definition of ambiguity in the literature, (ii) has the longest possible time coverage and (iii) allows studying the impact of ambiguity on asset pricing. The study contributes to the empirical literature on ambiguity by constructing an index that will be the first of its kind for Turkey and first of a limited number studies on emerging markets. To this end, the study first makes a detailed literature review on ambiguity, its impact on decision-making and asset pricing models. Although there are separate review studies covering theoretical models and empirical studies, this study also contributes to the literature by bringing together a complete review of the development of the relationship between asset pricing and ambiguity. The next section of the study reviews ambiguity and its impact on decision-making. The following part of the study summarizes asset pricing models incorporating ambiguity and compares them with the conventional models. The fourth part of the study reviews the empirical studies on the measurement of ambiguity and their application within the asset pricing framework. In the last section, alternative ambiguity indices for Turkey from different computation methods are introduced and an initial analysis of the impact of ambiguity on stock returns is presented.

2.2 Ambiguity, Ambiguity Aversion and Decision-Making

2.2.1 Ambiguity and Ambiguity Aversion

Ambiguity is important in asset pricing due to its potential impact on investment and consumption decisions. Although risk and uncertainty, or ambiguity, are used interchangeably, they are not alternative expressions for each other because they each refer to variability reflecting different information sets. Risk is a measure defined in a world with known probabilities of outcomes whereas ambiguity (uncertainty)[1] refers to a measure that needs to be calculated in a world where available information is not precise enough to represent the outcomes with complete probabilities. Epstein and Wang (1994) propose that probabilities represent only the relative likelihoods of events but they do not provide any hint regarding the reliability of the information that was used in extracting those likelihoods. The introduction of uncertainty on probability distribution in decision-making and finance leads to a new discussion in the literature where risk is redefined in terms of its conventional representation as well as its newly recognized component of ambiguity.

When we investigate the historical development of the discussion, Knight (1921) was the first study to differentiate between risk and uncertainty which is the reason why uncertainty is usually called the "Knightian uncertainty" in the literature. According to Knight, risk exists when an investment's future results are unknown but unique probabilities can be assigned to each possible outcome. Contrarily, uncertainty exists when investment results are unknown and also unique probabilities cannot be assigned to each possible outcome. Although Knight's separate definitions of risk versus uncertainty emphasizes the absence of objective probabilities, later studies focus more on the uncertainty over subjective probabilities.

Ellsberg (1961) is one of the first experimental studies demonstrating the impact of ambiguity in decision-making. The author questions whether Knight's distinction between measurable uncertainty (risk) and unmeasurable uncertainty (ambiguity) has a significant impact on decision-making. The latter should be more relevant in cases when economic agents are uninformed about the probability distribution and

[1] In the literature, the terms "uncertainty" and "ambiguity" are used interchangeably. In this study, we use the term "ambiguity."

tend to behave as if they have priors when in fact those priors are only a representation of their beliefs.

The experiments by Ellsberg (1961) show that in some cases agents do not behave in a way that is described by the Savage axioms. In the experimental setup, there are two urns and each contains 100 balls in red and black. For each urn, subjects were asked to play a gamble offering $100 if the ball randomly drawn from the urn is red and zero otherwise. The subjects were also informed that the first urn contains 100 red and black balls with unknown proportions and the second urn includes 50 red and 50 black balls. Afterwards, participants were asked to determine which of the cases would be more likely in the following questions:

i) Drawing a red ball or a black ball in urn 1? Or, are they equally likely?
ii) Drawing a red ball or a black ball in urn 2? Or, are they equally likely?
iii) Drawing a red ball from urn 1 or urn 2? Or, are they are equally likely?
iv) Drawing a black ball from urn 1 or urn 2? Or, are they are equally likely?

The results show that participants assign equal probabilities to red and black balls in the first two questions. In the third and fourth questions, they choose urn 2 as the one where it would be more likely to draw a red or a black ball. This result presents a contradiction. If, in question 3, it is more likely to draw a red ball from urn 2, then it should follow that it is less likely to draw a black ball from urn 2. However, in question 4, the participants choose urn 2 as the one where it is more likely to draw a black ball. This result is known as the “two-urn-paradox” and contradicts with Savage’s Subjective Expected Utility (SEU) model which describes decision-making based on additive probabilities. Moreover, choosing urn 2 in the third and fourth questions confirms that participants prefer known unknowns to unknown unknowns (ambiguity).

Another experiment in Ellsberg (1961) presents further evidence in support of the ambiguity aversion. In this setup, participants were informed that there is an urn that contains 30 red balls and 60 black and yellow balls with unknown proportions.

In the first game, one ball was drawn from the urn and participants were asked to bet on red or black to win $100 if they are correct and receive nothing otherwise. In the second game, participants were asked to choose among two alternatives: i) red or yellow, and, ii) black or yellow. The results show that participants pick the red ball in the first game and choose the alternative of the black or yellow balls in the second game. These results suggest that in the first game, since the participants pick red over yellow, they must estimate the probability of picking a red to be higher than that of picking a yellow ball. This implies that there must be fewer than 30 yellow balls, which further implies that there must be more than 30 black balls. However, contrary to this implication, the participants were equally likely to choose a black or a yellow ball in the second game, which should only be the case if they estimate that there is an equal number of black and yellow balls in the urn. The contradictory result of these games is known as the “Three-Color Ellsberg Paradox” and confirms two important characteristics of decision-making. First, participants behave differently when faced with ambiguity and they do not base their decisions on additive probabilities. Second, the behavior of participants is consistent with ambiguity aversion which manifests itself through participants avoiding events with unknown probability distributions. All these results point out the importance of considering ambiguity aversion apart from risk aversion in decision-making.

Aversion to ambiguity, in addition to risk aversion, also has important implications in asset pricing. Finance theory suggests that asset returns should compensate higher risk due to risk aversion. Following this logic, since ambiguity aversion is distinct from risk aversion, agents should also ask for additional compensation for ambiguity. In this framework, Ellsberg’s findings open a new discussion about asset pricing and suggests replacing the investor behavior in Savage’s SEU theory with another one that has less demanding prerequisites. Before going over the models incorporating ambiguity, we first give some background information about decision-making under uncertainty and notation from fundamental theorems.

2.2.2 Decision-Making

Modeling the decision-making process of investors is a complicated task and previous studies developed rather complex and somewhat representative models. In simple terms, decision-making can be considered as a selection process based on beliefs regarding different choices. Furthermore, agents form a preference by accessing to different information sets for most of the time. For example, agents may not be fully informed about the distribution of future realizations so they have to make a decision under ambiguity. Indeed, this is a realistic scenario considering the complex nature of financial markets; therefore, decision-making under ambiguity may be considered as a more relevant framework for modeling real world investment processes. It should be noted that earlier theoretical studies on decision-making have simplified the process in a way that agents have access to accurate information about the next period's state of the nature and thereby can assign probabilities to outcomes. Risk (measurable uncertainty) or a known probability distribution is at the center of the expected utility theory.

2.2.2.1 Decision-Making Under Risk (Measurable Uncertainty)

Fundamentals of decision-making under uncertainty go back to the famous study of Bernoulli (1738). This study on risk measurement has been a cornerstone in finance and economics due to its contributions to the development of utility theory and the theory of decision-making under uncertainty. Bernoulli (1738) proposed that agents make decisions by calculating the expected value of an uncertain event but they take also into account the utility of possible outcomes (moral expectations) instead of basing their decision on purely the mathematical values of outcomes (mathematical expectations). Hence, preferences based on expected utilities may differ from preferences derived from mathematical expectations. Bernoulli's other important contribution to the utility theory was his definition of the diminishing marginal utility. He argued that the utility of gain is lower than the utility of loss for the same monetary amount and at the same level of wealth. These results laid the groundwork for the concave utility function of wealth and risk aversion. The next section provides a review of this theoretical framework.

2.2.2.1.1 Objective Expected Utility

Objective expected utility theory by Von Neumann and Morgenstern (1944) is another building block of the discussion on decision-making under risk. According to the theory, decision-making can be formulized as a function of the expected utility of an event provided that an agent's preferences are consistent with certain axioms[2]. In this framework, preferences over uncertain events with objective probabilities are mapped on utilities and agents aim to maximize their utility that is quantified by an expected utility function. The expected utility of an uncertain event is calculated through Equation (2.1) where $V(\cdot)$ is the expected value of utility and is written as an ordinal preference function. $U(x_i)$ is a utility function of outcomes (x_i) and p_i is the objective probability distribution of outcomes. In the objective expected utility theory, while the probability distribution of the event is objective, the utility function is subjective. In other words, while agents have homogeneous expectations, they each attach different utilities to different outcomes; hence, the expected utility is subjective.

$$V(x_1,p_1;\ldots;x_n,p_n)=\sum_{i=1}^{n}U(x_i)\cdot p_i \tag{2.1}$$

2.2.2.1.2 Subjective Expected Utility

In the objective utility theory, the outcomes of an event are uncertain but their probabilities are objectively determined. For example, probabilities of the outcomes of tossing a fair coin or spinning a roulette wheel are objectively determined and

[2] The details of the axioms are beyond the scope of this study. Karni (2014) and Machina and Siniscalchi (2014) provide complete review of axioms in both objective and subjective expected utility theories.

known. On the other hand, the outcomes in a horse race or a football game cannot be determined objectively; instead, subjective probabilities have to be assigned to the possible outcomes.

Savage (1954) introduced the subjective expected utility (SEU) theory in which probabilities of events are determined subjectively. Similar to the objective expected utility theory, SEU has axioms and shows that a preference structure under uncertainty consistent with these axioms obtains the unique utility and probability distribution, and is equivalent to a preference structure that maximizes expected utility conditional on a set of outcomes and their associated probabilities. In SEU, the decision-making process has three main components: i) state space (Ω); ii) outcome space ($\mathcal{F}$), and iii) preference. The elements (s) of state space (Ω) are called states of nature and sets of states of nature are called events (E). The elements of state space are given and represent all relevant possible futures so that states are a complete description of the world. The outcome space contains random outcomes of decisions but all outcomes of every action are also known. Preferences are revealed via the mappings from Ω to $\mathcal{F}$; these mappings are called as acts (f(s)) and they combine states of nature (s) with outcomes.

In summary, the SEU theory has two important components. First, decision-making should be defined as a process consisting of two sequential steps: i) defining the possible outcomes of an event, and, ii) the assessment of their probabilities. Second, these probabilities and the utilities at each outcome should be quantifiable. Intuitively, probabilities and utilities can be considered analogous to beliefs and tastes, respectively. Hence, expected utility ($\mathrm{W}(\cdot)$) is a function of beliefs and tastes (Equations (2.2) and (2.3)). According to the model, beliefs about the likelihoods of different states are quantifiable by a subjective finite and additive probability measure $\mu(\mathrm{s})$ and tastes are quantifiable by the von Neumann-Morgenstern utility function (U(f(s))) which also reflects the decision maker's risk aversion attitude.

$$W(x_1,E_1;\ldots;x_n,E_n)=\sum_{i=1}^{n}U(x_i)\cdot\mu(E_i) \tag{2.2}$$

$$W(f(s))=\int_{s}U(f(s))\cdot d\mu(s) \tag{2.3}$$

2.2.2.2 Decision-Making Under Ambiguity (Unmeasurable Uncertainty)

Empirical evidence suggests that the actual choices of agents contradict with the theoretical formulation of the expected utility theory. Earlier models are based on the assumption of precise information and exact subjective beliefs, in other words, measurable uncertainty. In reality, information is not always precise and, in general, beliefs cannot be identified specifically. Furthermore, experimental evidence by Ellsberg (1961) contradicts with other assumptions such as Savage's sure-thing axiom[3]. Hence, the expected utility theory, regardless of whether it includes objective or subjective probabilities, has difficulties in approximating the real world due to its restrictive assumptions.

Since information is incomplete and a single additive probability measure is unrealistic, later studies focus on the theoretical features of behavioral attitudes toward ambiguity and attempt to adjust the SEU model to incorporate ambiguity aversion in addition to risk aversion. Although there is almost no vagueness about the definition of risk and risk aversion, there is no agreement on the definitions of ambiguity and ambiguity aversion. In this framework, ambiguity aversion has become a particular field of interest, and there are different models and definitions in the literature. In general, beliefs under ambiguity are represented by imprecise beliefs or a set of probability distributions. While there is only one expected utility in SEU that depends on unique priors, more than one prior proposes that agents may have different expected utilities depending on the respective probability

[3] The sure-thing axiom in SEU refers to the case where preferences are independent from the source of risk.

distributions. In the next section, we review the models incorporating ambiguity and their implications on asset pricing.

2.2.2.2.1 Maxmin Expected Utility

Gilboa and Schmeidler (1989) introduced one of the popular models known as the maxmin expected utility (MEU) or, the multiple priors model, and provided the axiomatic foundations of the difference between risk and ambiguity. Accordingly, agents are not capable of a precise assessment of the event probabilities and this leads to incomplete preferences. This means that agents employ multiple probability distributions in order to calculate expected utilities and to make a decision. In the end, agents maximize their utility according to the probabilities in the worst case scenario, implying that agents act in a precautious manner. In Equation (2.4) below, $W(f(s))$ is the expected utility level in a standard SEU setup. On the right hand side of the equation, $U(f(s))$ is the utility level at each state and μ is the probability measure but it is the worst case probability distribution chosen among a probability set of C.

$$W(f(s))=\min_{\mu\in C}\int_s U(f(s))\cdot d\mu(s) \tag{2.4}$$

In this regard, MEU assumes that agents are extremely pessimistic, so they are ambiguity averse at the highest level. Ghirardato, Maccheroni, and Marinacci (2004) introduced α-MEU model that they relaxed the extreme ambiguity aversion and instead adopted heterogeneous levels of ambiguity aversion. In the α-MEU model presented in Equation (2.5) below, α represents the strength of ambiguity aversion. In the α-MEU model, α could be instrumental in comparing ambiguity aversion levels among agents provided that they have identical utility functions.

$$W(f(s))=\alpha\min_{\mu\in C}\int_s U(f(s))\cdot d\mu(s)+(1-\alpha)\max_{\mu\in C}\int_s U(f(s))\cdot d\mu(s) \tag{2.5}$$

In a similar fashion, Epstein and Wang (1994) constructed a model in which asset returns are derived by using the most pessimistic beliefs in an intertemporal setup with Knightian uncertainty. In the model, beliefs are not certain and cannot be represented by a single prior; instead, they are represented by a multi-valued probability function and exhibit the rule-based evolution of a Markov chain. Chen and Epstein (2002) also criticized utility models with a unique probability measure of beliefs and claimed that a unique probability measure is possible only if agents have probabilistic sophistication. They have extended the intertemporal MEU model of Epstein and Wang (1994) by adding a dynamic and continuous time component but their study differs from Epstein and Wang (1994) in that they decompose excess returns into risk and ambiguity premiums.

2.2.2.2.2 Choquet Expected Utility

Schmeidler (1989) introduced the Choquet expected utility model (rank-dependent model) which also incorporates ambiguity aversion. The model describes beliefs by capacity and not by subjective probabilities as in SEU. Capacity is a single non-additive[4] probability measure and captures the ambiguity aversion of agents. Contrary to the additive probabilities used by objective and subjective expected utility models, non-additive probabilities make expected utility models more flexible to address different behaviors like ambiguity aversion. Accordingly, an agent makes a decision by first choosing and assigning capacity to the lowest possible outcome. Next, the agent defines the other outcomes as increments to the first one and assigns capacities by weighing these increments depending on the personal beliefs regarding the occurrence of increments. The model is given in

[4] Non-additivity implies that the probabilities of occurrence of two mutually exclusive events do not equal to the sum of the probabilities of occurrence of each individual event. Dow and Werlang (1992) clarify the issue by citing the following example. An agent believes that an asset's value will be either high or low and assigns equal probabilities to each outcome (p(high) = p(low) = 1/2). If there is no ambiguity and the agent only considers risk, then the agent weighs the high and low outcomes with probabilities summing up to 1 (p(high) + p(low) = 1). Non-additivity would imply that the agent may weigh the high and low outcomes with a probability lower than 1 (p(high) + p(low) < 1). In this framework, incorporating non-additive probabilities in expected utilities implies that expected utility includes not only risk aversion but also ambiguity aversion.

Equation (2.6) where $U(f(s))$ is a von Neumann-Morgenstern type utility function, $v(s)$ is capacity and the non-additive probabilities are integrated through the Choquet integral.

$$W(f(s)) = \int_S U(f(s)) \cdot dv(s) \tag{2.6}$$

2.2.2.2.3 Robust Control Model

Hansen and Sargent (2001) used a robust control model in order to present the role of model uncertainty in asset pricing. Their framework is consistent with Ellsberg's urn experiment and differentiates between payoff uncertainty and model uncertainty. The robust control theory provides a "good" decision when the model approximates the correct decision whereas the standard control theory provides the "optimal" decision when the model is correct. Hence, the robust control theory is a good approximation of ambiguity in asset pricing since the decision-making agent does not know whether the model used in pricing an asset is correct or not. Relative entropy in the model, which is a measure of the distance between two probability distributions, is used as the measure of ambiguity. Similar to Gilboa and Schmeidler (1989), in the robust control model agents also try to maximize utility by optimizing consumption under the worst reasonable case scenario in an intertemporal setting. In the model setup presented in Equation (2.7), μ^* is the reference prior but the agent also considers other possible probability distributions such as μ in ranking acts of $f(s)$. The relative likelihood of the distribution μ compared to μ^* is given by the relative entropy of μ with respect to μ^* represented by $R(\mu \| \mu^*)$. Relative entropy is weighted by the assessment of agent (θ) about whether the distribution μ^* is correct or not.

$$W(f(s)) = \min_{\mu \in \Delta(s)} \int_S U(f(s)) \cdot d\mu(s) + \theta \cdot R(\mu \| \mu^*) \tag{2.7}$$

2.2.2.2.4 Smooth Ambiguity Model

Klibanoff, Marinacci, and Mukerji (2009) introduced the smooth ambiguity model which is composed of two stages presented in Equation (2.8) where $u(f(s))$ is the VNM type utility function and embodies the utility of the act $f(s)$ in state s. Since the probability distribution is not certain, there are more than one probability distributions for states represented by $\mu(s)$ as a prior. The term within the parentheses represents the expected utilities of acts depending on different priors denoted by $\Delta(s)$. In a similar setup, the maxmin expected utility model of Gilboa and Schmeidler (1989) took the minimum expected, or the most pessimistic, utility among the alternative expected utilities. Likewise, in the smooth ambiguity model, agents evaluate expected utilities based on the second order prior of M because they are uncertain about the correct probability distribution of the event. In Equation (2.8) below, the $\phi(\cdot)$ function is the second-order utility function and the expected utilities are derived from this function. The shape of the $\phi(\cdot)$ function represents the agent's attitude towards ambiguity. If the agent gives more weight to the expected utility in the pessimistic case, then the shape of the function will be concave and this would imply that the agent is ambiguity averse. Similarly, the convexity of $\phi(\cdot)$ would imply a preference for ambiguity and the linear form of $\phi(\cdot)$ would imply neutrality towards ambiguity. Hence, the model is set up to make it possible to evaluate ambiguity and ambiguity aversion separately.

$$W(f(s))=\int_{\Delta(s)} \phi\left(\int_{s} u(f(s)) \cdot d\mu(s)\right) dM(\mu(s)) \tag{2.8}$$

2.3 Ambiguity and Its Implications for Asset Pricing

In asset pricing models, decision-making has a center role in model building. Conventional asset pricing models like CAPM are based on a framework of decision-making under measurable uncertainty. It follows that any modification to decision-making rules, such as the incorporation of ambiguity, also should have implications for asset pricing. The previous section of the study summarizes the theoretical literature on the issue in order to emphasize how important it is to take ambiguity into account while modeling decision-making for building models that resemble real life decision-making more closely. In a similar fashion, an increasing number of asset pricing models also are being modified to incorporate ambiguity. However, before we go over the asset pricing models that take ambiguity into account, we present a short summary of the conventional asset pricing models in order to form a basis for comparison with the models including ambiguity. These so-called conventional asset pricing models do not incorporate ambiguity and model asset pricing under risk (measurable uncertainty).

2.3.1 Modeling Asset Pricing Under Risk (Measurable Uncertainty)

2.3.1.1 Static Asset Pricing Models

The 1952 study of Markowitz is a pioneer in asset pricing where the tradeoff between risk and return is quantified in a mean-variance framework and the concept and importance of portfolio diversification is demonstrated for the first time. The tradeoff between risk and return suggests that there is a rate at which the agents would bear higher risk for higher return, or, would be willing to accept lower return for lower risk. In Markowitz's model, agents make portfolio selection decisions at the current time based on the expected return of the portfolio in the next period. The investors are assumed to be risk averse and make decisions that are mean-variance efficient, implying that the selected portfolio either has the minimum variance for a given level of expected returns, or, it has the maximum expected returns for a given level of variance. Following Markowitz, Arrow and Debreu (1954) underlined the benefits of diversification in reducing uncertainty. They developed a fundamental concept in the finance literature known as a "complete market" where investors are able to eliminate uncertainty and insure their portfolios against losses.

The Markowitz and Arrow-Debreu concepts lead to the development of the Capital Asset Pricing Model (CAPM) in a series of three studies by Sharpe (1964), Lintner

(1965) and Mossin (1966). CAPM explains the determinants of expected returns and establishes a framework for the relationship between return and risk. The model redefined risk relevant in portfolio selection as the "systematic" risk which is the contribution of an individual asset to the riskiness of a portfolio. This definition is different from that of total risk which is measured by return volatility. This insight regarding the difference between systematic and total risk has been at the center of the asset pricing discussion and underpins further extension of the literature. CAPM formulation is given in Equation (2.9) where excess return ($E(R_i)$-R_f) is a function of the market price of risk ($E(R_M)$-R_f) and asset's systematic risk represented by the market beta (β_i). CAPM has been discussed extensively in the literature in terms of its assumptions, model buildup and implications. Following these discussions, alternative models such as the Arbitrage Pricing Theory (APT) and other factor models emerged to relax some of the restrictive assumptions of CAPM and incorporated other risks beside market risk in asset pricing. A detailed discussion of these models is beyond the scope of the study. Nevertheless, even with its original premise, CAPM provides the basis upon which all subsequent models were built in the literature.

$$E(R_i)-R_f= \beta_i(E(R_M)-R_f) \tag{2.9}$$

2.3.1.2 Dynamic Asset Pricing Models

2.3.1.2.1 Intertemporal Capital Asset Pricing Model (ICAPM)

As investment decision is typically a multi-period decision, it was necessary to transform the static, one-period, decision-making framework of CAPM into a dynamic process. The discussion on the relationship between equity premiums and risk for longer than a single period goes back to Merton's (1973) Intertemporal Capital Asset Pricing Model (ICAPM). The model derives the relationship between an asset's expected return and the market return's volatility in continuous time. In the model setup, agents not only consider portfolio returns at the end of the current period as in conventional CAPM but they also take into account the end-of-period investment opportunities. This implies that agents are concerned about the return variability of the current as well as the future investment opportunities.

Merton (1973) obtains a three-fund separation in which an agent separates the investment decision into two parts by forming three mutual funds: i) n risky assets, ii) the n^{th} asset whose return is perfectly negatively correlated with risk free rate, and, iii) a risk free asset. In this framework, Merton (1973) derives the excess return on the market (α_M-r) as the sum of two terms. As presented in Equation (2.10), the first term is the proportional impact of the market return's variability ($M/A\,\sigma_M^2$)[5]. The second term is the risk of a shift in the investment opportunity set ($[Hg/A\,\sigma_n]/\sigma_{M,n}$). Equation (2.11) suggests that the excess return for an individual asset is not only a function of its beta coefficient and the excess market return, similar to the conventional CAPM, but also the excess return on the state as well as the risk of an unfavorable shift in the investment opportunity set. Hence, the model allows for risks from multiple sources to be priced in the market such as future consumption, investment etc.

$$\alpha_M - r = \frac{M}{A}\sigma_M^2 + \frac{Hg}{A\sigma_n}\sigma_{M,n} \qquad (2.10)$$

$$\alpha_i - r = \frac{M}{A}\sigma_{i,M} + \frac{Hg}{A\sigma_n}\sigma_{i,n} \qquad (2.11)$$

[5] In the ICAPM framework, M is the equilibrium value of all assets, A is a function of agents' absolute risk aversion, H represents the demand for the asset to hedge against unfavorable shifts in the investment opportunity set and g is the standard deviation of the change in risk free interest rate r. n is the n^{th} risky asset with a correlation of -1 with the risk free interest rate.

2.3.1.2.2 Consumption-Based Capital Asset Pricing Model (CCAPM)

In Merton's ICAPM, the state variables were not identified and this complicates the applicability of the model. The Consumption-Based CAPM[6] (CCAPM) put forward by Lucas (1978) and Breeden (1979) improved the applicability of ICAPM and associated the risk with a shift in the state variable of consumption. In this model, total utility is formulated as the sum of (i) utility from current consumption, and, (ii) discounted utility from future consumption[7] which is random due to unknown future wealth. CCAPM differs from Markowitz's portfolio theory and the classical CAPM since in CCAPM utility is related directly with the consumption level whereas, in the other two models, utility is defined as a function of the mean and variance of the portfolio return.

The two period model in Equation (2.12) shows that total utility is the sum of utility from current consumption (C_t) and discounted expected utility from future consumption (C_{t+1}). The model captures the intertemporal substitution and risk aversion by discounting the future consumption utility by β, the subjective discount factor. The utility function depends on the consumption level and it is modeled as a power utility function[8] where γ is the parameter of risk aversion (Equation (2.13)).

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[6] A complete review of dynamic asset pricing models and CCAPM are presented in Cochrane (2005).

[7] The multi-year representation of utility assumes time separable utility of consumption.

[8] A power utility function defines the utility as an increasing function of consumption and it is concave in consumption. The functional form of power utility implies utility increases at a decreasing rate with higher consumption. The power utility function has advantages for empirical research because it allows relaxing the restrictive assumption that return distributions and risk premiums do not change over time. Also, power utility functions make it possible to represent different agents by a single preference structure even though they may differ in terms of their wealth. On the other hand, power utility functions come with some disadvantages. One major disadvantage is that it lacks the capability of separating the elasticity of intertemporal substitution and the relative risk aversion. Indeed, the elasticity of intertemporal substitution was defined as the inverse of risk aversion.

$$U(C_t, C_{t+1})=u(C_t)+\beta E_t[u(C_{t+1})] \tag{2.12}$$

$$u(C_t)=\frac{1}{1-\gamma}C_t^{1-\gamma} \tag{2.13}$$

Solving the optimization problem of maximizing consumption subject to current and future consumption as a function of endowments, investment quantity and asset prices[9] reveals the fundamental asset pricing in Equation (2.14). Asset pricing formulation suggests that price of an asset is defined as a function of expected payoff (x_{t+1}) and expected stochastic discount factor ($\beta U'(C_{t+1})/U'(C_t)$) as a function of consumption. Stochastic discount factor is also called marginal rate of substitution that agent prefer to substitute future consumption for current consumption.

$$p_t=E_t\left[\beta\frac{U'(C_{t+1})}{U'(C_t)}x_{t+1}\right] \tag{2.14}$$

After making adjustments[10], price equation is redesigned in Equation (2.15). Accordingly expected excess return of an asset ($E_t(r_{i,t+1}-r_{f,t+1})$) is proportional to covariance between consumption growth and asset return, and the proportion depends on the level of relative risk aversion (γ). Since agents prefer a smooth

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[9] As presented by the optimization problem below, the objective of the agent is defined as maximizing total utility subject to current and future consumption where current (C_t) and future (C_{t+1}) consumption depends on initial endowments (e) and future payoff of investment (x_{t+1}). In this regard, today consumption is defined as the difference between current endowment (e_t) and investment as a function of asset price (p_t) and quantity of investment (ξ).

$$\max_{\xi} u(C_t)+\beta E_t[u(C_{t+1})] \text{ s.t.}$$

$$C_t=e_t-p_t\xi$$

$$C_{t+1}=e_{t+1}+x_{t+1}\xi$$

[10] Stochastic discount factor and asset returns are assumed to be lognormally distributed so the returns and consumption turned into logarithmic form such that $r_{i,t}=\log(1+R_{i,t})$ and $c_t=\log(C_t)$.

consumption stream over time and across states, positive covariance between return and consumption growth implies more volatile consumption. Hence, agents ask for higher return to hold assets generating return covarying positively with consumption. In comparison with portfolio theory and CAPM, this infers that extra return of an asset is not determined by the covariance of the return with market return, rather, it is a function of covariance with consumption growth.

$$E_t\left(r_{i,t+1}-r_{f,t+1}\right) \approx \gamma \times Cov_t\left(r_{i,t+1}, \frac{c_{t+1}}{c_t}\right) \tag{2.15}$$

2.3.2 Modeling Asset Pricing Under Ambiguity (Unmeasurable Uncertainty)[11]

The main assumption of the fundamental models in economics and finance is that an agent has complete preferences and follows this preference structure systematically. Although these assumptions increase the explanatory capabilities of the theoretical models, they fit neither the observed agent behavior nor the characteristics of the economic environment. Ambiguity is one of the factors that distort the assumption of complete preferences; therefore, agents are hypothesized to behave differently in the presence of ambiguity. This makes behavior under ambiguity and its impact on asset pricing valuable to investigate, especially when it is considered together with the empirical failure of the classical asset pricing models with risk as the only determinant of expected returns. The observed change in investment behavior under ambiguity suggests that ambiguity, in addition to risk, may be a determinant of the investment decision as well as the required return.

In one of the first empirical studies on the subject, Olsen and Troughton (2000) investigate the role of ambiguity in the risk assessment of investors through a survey and reveal that ambiguity has a stronger impact on decision-making compared to

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[11] Hereafter, we use ambiguity and uncertainty interchangeably to keep the original descriptions used in the studies that we cite.

the classic risk indicators. Survey results point out that investors consider large losses as the most important component of risk. The second most important dimension of risk was reported as the uncertainty over the distribution of returns, in other words ambiguity. Interestingly, the traditional risk indicators of standard deviation and beta were placed low on a list of factors related with risk.

In a theoretical study, Chen and Epstein (2002) demonstrate the effect of ambiguity on individual asset returns. Referring to equity premium puzzle[12], Chen and Epstein document that the observed equity premium may be comprised of an ambiguity as well as a risk premium. This implies that investors do not base their investment decisions only on the realized risk but they also consider the vagueness over future values of risk, which can be defined as ambiguity. Chen and Epstein propose a continuous-time asset pricing model with a recursive utility function[13]. Their model also introduces multiple priors due to the inability of agents to consolidate multiple priors to a single prior through learning. The model results are consistent with

[12] Mehra and Prescott (1985) call high excess return on equities as equity risk premium puzzle. They document that high equity premium is not plausible to be explained by transactions costs, liquidity constraints and other frictions. Insufficiency of risk aversion in explaining excess return necessitates to adapt other determinants of preference into the models to explain equity premium puzzle. In this framework, elasticity of intertemporal substitution is added as another determinant of consumption and asset return. Elasticity of intertemporal substitution embodies the smoothing choice on consumption throughout time while risk aversion represents the choice among different states. Therefore, inputting elasticity of intertemporal substitution is valuable to understand excess return dynamics coherently. In addition, inverse relationship between risk aversion and elasticity of intertemporal substitution in additive utility models is counterintuitive in the sense that agents prefer smooth consumption patterns so the relation between two indicators should be positive.

[13] Epstein and Zin (1989) introduce recursive utility functions enabling the separation of risk aversion and elasticity of intertemporal substitution, and the model holds power utility function. Recursive utility function in equation below incorporates a nonlinear aggregator to add up present and future values of utility that total utility equals to current consumption and risk adjusted utility of future consumption. In Epstein-Zin utility function, β is subjective discount rate, γ is risk aversion and ψ is elasticity of intertemporal substitution. The term θ is the ratio of $(1-\gamma)/(1-1/\psi)$ and Epstein-Zin utility function drops to power utility function if θ equals 1 or inverse of elasticity of intertemporal substitution is equal to risk aversion.

$$U_t=\left\{(1-\beta)C_t^{\frac{1-\gamma}{\theta}}+\beta\left(E_t\left(U_{t+1}^{1-\gamma}\right)\right)^{\frac{1}{\theta}}\right\}^{\frac{\theta}{1-\gamma}}$$

previous studies in the sense that, compared to the mean-variance results, ambiguity aversion changes the distribution of weights in the portfolio. The excess return of an asset is defined in Equation (2.16) as the sum of the risk premium in the first term and the ambiguity premium in the second term. Risk premium is defined in its conventional form and it is a function of the degree of risk aversion (γ) and the covariance between consumption growth (s^c) and asset returns (s^i). The second term showing the ambiguity premium is a function of ambiguity aversion (κ) and the covariance between asset returns and the sign function of consumption growth. Hence, the model modifies excess returns in a way to separate the premium for risk from the premium for ambiguity.

$$E(R^e_{i,t})=\gamma\times s^i_t\cdot s^c_t+\kappa\left(s^i_t\otimes \mathrm{sign}(s^c_t)\right) \tag{2.16}$$

In a later study, Bansal and Yaron (2004) separate economic uncertainty from the long term and predictable part of consumption growth and investigate the impact of economic uncertainty, defined as a conditional variance of consumption growth, on the distribution of market returns. The model is based on the Epstein-Zin preferences separating the elasticity of intertemporal substitution from risk aversion, and, in addition, decomposes consumption and dividend growth rates into their long-run predictable and economic uncertainty components. The model results show that economic uncertainty and volatility in consumption raises the equity premium while long-run growth prospects increase equity prices.

Compared to previous models with multi-period setup, Kogan and Wang (2003) investigate the impact of ambiguity on cross-sectional stock return distributions. Ambiguity, or model uncertainty, implies that there is imperfect information about the probability rule that generates the realized states. The return distributions were formulized as a combination of two factors: (i) the contribution of the asset to the riskiness of the portfolio, and, (ii) the uncertainty of the portfolio return. Kogan and Wang use a single period CAPM with a riskless asset and n risky assets but they remove the unique probability distribution assumption. In the model, mean asset returns are unknown but the variance-covariance structure of returns is known. As shown in Equation (2.17) below, they derive a two-factor representation of the return-generating function. The first part is composed of the market risk premium (λ_r) and the market risk (β_r) similar to CAPM. The second part is a multiplication of the ambiguity risk premium (λ_u) and the exposure to market ambiguity (β_u). This implies that ambiguity could be diversifiable to some extent but the undiversifiable portion of it should be compensated by a higher return.

$$\mu\text{-}r=\lambda_r\beta_r+\lambda_u\beta_u \tag{2.17}$$

In a similar single-period framework, Boyle et al. (2009) develop a static and discrete-time model of portfolio choice for analyzing the impact of ambiguity in asset pricing through emphasizing the difference between Markowitz and Keynes in portfolio formation. Although Markowitz favors diversification of investment among alternative assets, Keynes supports the view that assets that one is less familiar with should be avoided. In order to analyze the difference between the two approaches, Boyle et al. (2009) introduce ambiguity into the return distributions and ambiguity aversion into the preferences. Similar to other studies in integrating ambiguity into asset pricing models, this study also concentrates on the mean return for measuring ambiguity. The model has different implications for different familiarity assumptions. If agents have limited information about a particular asset, they prefer to diversify their portfolio. If they are familiar with a particular asset, they tend to invest more in this asset but they still diversify the remaining part of the portfolio. If they are familiar with a particular asset but other assets are ambiguous, then they invest in the familiar asset. Finally, agents do not participate in the market at all if all assets are ambiguous. The results suggest that agents tend to invest more in familiar assets at the expense of lower diversification and undertake higher risk when ambiguity is introduced to the models. According to the specification in Equation (2.18) below, the excess return (μ) is derived as a function of two terms if the number of risky asset is high enough. The first term is the risk premium and it is a function of the degree of risk aversion (γ) and a systematic component of the asset's return variance (σ_S^2). The second term is the ambiguity premium and it depends on the estimated standard deviation of the expected return ($\hat{\sigma}$) as well as the ambiguity common among other assets (α_{-1}).

$$\mu=\gamma\times\sigma_S^2+\hat{\sigma}\alpha_{-1} \tag{2.18}$$

Anderson, Ghysels, and Juergens (2009) investigate the impact of ambiguity via a continuous time infinite horizon model similar to Merton (1973) in Equation (2.19). In Equation (2.20), the excess market return is defined as a function of volatility (V_t) and uncertainty (M_t) that are scaled by the parameters of risk aversion (γ) and ambiguity aversion (θ). The agent considers a state vector of x_t (Equation (2.21)) that follows continuous time diffusion process where B_t is an independent Brownian motion, $a_t = a(x_t)$ and $\Lambda_t= \Lambda(x_t)$ are functions of the current state. The price of the asset follows a similar process of x_t and, in Equation (2.22), $\alpha_{kt} = \alpha_k(x_t)$ is the conditional mean return of asset k and a scalar in vector of α_t. ζ_{kt} is the conditional variance of the k^{th} asset return and it is in the k^{th} row of the matrix σ_t. Agents can invest in n risky assets and a risk free asset of $\rho_t = \rho(x_t)$. The dynamics of wealth (y_t) is defined in Equation (2.23) where $\lambda_t= \alpha_t-\rho_t$ is the vector of excess returns over the risk free rate. ψ_t is a vector of asset weights in the portfolio in which the k^{th} element represents the weight of the k^{th} asset in the portfolio.

$$E_t r_{Mt+1}=\gamma V_t \tag{2.19}$$

$$E_t r_{Mt+1}=\gamma V_t+ \theta M_t \tag{2.20}$$

$$dx_t=a_t dt+ \Lambda_t dB_t \tag{2.21}$$

$$dP_{kt}=\alpha_{kt}P_{kt}dt+ \zeta_{kt}P_{kt}dB_t \tag{2.22}$$

$$dy_t=\left(\psi_t'\lambda_t y_t+\rho_t y_t-c_t\right)dt+ \psi_t'\zeta_t y_t dB_t \tag{2.23}$$

Anderson, Ghysels, and Juergens (2009) incorporate uncertainty into the model by assuming that agents have limited information about the mean return but they are highly informed about the return volatility. Thus, in this framework, ambiguity is about the mean return of an asset. The asset price and wealth generating functions are given in Equations (2.25) and (2.26) below. The agent thinks that the conditional mean of asset returns is $d_t-\eta_t g_t$ rather than d_t which would imply that the agents have no information about g_t. Here, η_t captures the level of confidence agents have in their expectations regarding conditional means. Anderson, Ghysels, and Juergens solve the robust control problem to obtain g_t as a function of the exogenous state and wealth variables. Since agents consider the worst case specification for g_t, the model includes the penalty term $(g_t^{'}g_t)/2\phi_t$ where ϕ_t is a function of exogenous events and wealth (ϕ_t (x,y)) and as a result the model converges to the reference model.

$$dx_t=(a_t-\Delta g_t)dt+\Lambda_t dB_t \qquad (2.24)$$

$$dP_{kt}=(d_{kt}-\eta_{kt}g_t)P_{kt}dt+\zeta_{kt}P_{kt}dB_t \qquad (2.25)$$

$$dy_t=\left(\psi_t^{'}\lambda_t y_t-\psi_t^{'}y_t\eta_t g_t+\rho_t y_t-c_t\right)dt+\psi_t^{'}\zeta_t y_t dB_t \qquad (2.26)$$

$$E_0\int_0^{\infty}\exp(-\delta t)\left[\frac{c_t^{1-\gamma}}{1-\gamma}+\frac{1}{2\phi_t}g_t^{'}g_t\right]dt \qquad (2.27)$$

According to the model setup presented above, the agent tries to maximize the objective function in Equation (2.27) consisting of expected utility and a penalty term ($(g_t^{'}g_t)/2\phi_t$) subject to the constraints in Equations (2.24), (2.25) and (2.26). Utility is represented by a power function in an infinite–horizon, continuous-time dynamic equilibrium model and the asset return is specified as a misspecification of the return generated by Brownian motions. E_0 represents the expectation conditional on current information, δ is the time discount rate and γ denotes risk aversion. At equilibrium, the excess return model given in Equation (2.28) below is optimized. Accordingly, the excess return of an asset (λ) is a function of its covariance with the market return (ς) and the covariance of its uncertainty with the market's uncertainty (ϱ) under some restrictive assumptions.

$$\lambda=\gamma\varsigma+\theta\varrho \tag{2.28}$$

Afterwards, Anderson, Ghysels, and Juergens (2009) transformed the excess return definition in continuous time to a discrete time format in order to make the model more applicable to available data. The discrete time formulation in Equation (2.29) suggests that the expected excess return of the market portfolio ($E_t r_{Mt+1}$) is a function of its conditional variance ($V_t = \varsigma_{Mt}$) and the conditional uncertainty of the market ($M_t = \varrho_{Mt}$). In other words, if an agent does not solely invest in a riskless and unambiguous portfolio, s/he has to bear both risk and ambiguity. Similarly, the excess return on asset k ($E_t r_{kt+1}$) in Equation (2.30) is defined as a function of its conditional covariance with the market risk (ς_{kt}) and uncertainty (ϱ_{kt}) but these conditional covariances are converted into beta measures such that $\beta_{vk} = \varsigma_{kt}/V_t$ and $\beta_{uk} = \varrho_{kt}/V_t$ representing the sensitivity of the k^{th} asset's return with respect to market volatility and uncertainty, respectively. In this framework, an individual asset's return is defined as a function of the sensitivity of asset k to market volatility (β_{vk}) and uncertainty (β_{uk}), risk aversion (γ), ambiguity aversion (θ), market volatility (V_t) and uncertainty (M_t).

$$E_t r_{Mt+1}=\gamma V_t+\theta M_t \tag{2.29}$$

$$E_t r_{kt+1}=\beta_{vk}\gamma V_t+\beta_{uk}\theta M_t \tag{2.30}$$

2.3.3 Ambiguity and Related Issues in Asset Pricing

In the previous section, we reviewed the theoretical studies integrating ambiguity and ambiguity aversion into asset pricing models. In addition to the theoretical discussion, there is a wide literature that addresses the empirical factors that may proxy ambiguity in asset pricing tests. Although there is no formal classification, in this section, we gathered and classified studies under three broad headings according to how the literature discusses the relationship between ambiguity and asset pricing.

2.3.3.1 Ambiguity and Information

Traditional asset pricing models assume perfect information and rational investor behavior. However, during extraordinary periods such as the Global Financial Crisis, we observe that agents' behavior may diverge from rationality as information in the market becomes less perfect. One important transmission channel from ambiguity to asset returns is the information quality. Low information quality or heterogeneous information among agents distorts the formation of expectations on asset returns and distorted expectations make it impossible for agents to eliminate multiple priors and form a unique prior.

Zhang (2006) identifies low information quality and volatility in firm fundamentals as the two main sources of ambiguity. The results of his study demonstrate that behavioral biases such as under or overreaction are more obvious under low information quality implying that information quality is closely related with return anomalies. Zhang uses six separate indicators as a proxy for ambiguity: firm size, firm age, analyst coverage, dispersion in analyst forecasts, return volatility, and cash flow volatility. The results show that stock prices underreact to new information arrival in case of higher ambiguity. Zhang concludes that market reaction to new information is complete for low uncertainty stocks but incomplete for high uncertainty stocks.

Epstein and Schneider (2008) separate information into its tangible and intangible components. Tangible information is certain information such as dividend announcements while intangible information refers to uncertain information such as earnings forecasts. The signal quality of intangible information could be low and noisy in the case of incomplete knowledge. Agents evaluate this noisy signal as ambiguous and do not update their beliefs according to Bayesian rule. Rather, they behave consistently with the maxmin expected utility theory and make decision based on the outcomes of the worst-case scenario. Epstein and Schneider claim that this behavioral motivation implies asymmetrical response to information releases and agents are relatively more responsive to bad news because they evaluate bad news more reliable than good news. Higher sensitivity to bad news coincides with lower expected excess return during periods with low information quality. Since the arrival of low quality information implies ambiguity and lower expected returns, ambiguity-averse agents ask for extra return for holding assets that have a probability of low quality information releases.

In a later study, Epstein and Schneider (2010) support the intuition that agents make their decision based on multiple probability distributions if information is limited to form a single probability measure. In this regards, low information quality leads agents to have a wider interval of possible mean returns and results in ambiguity. In a related study, Illeditsch (2011) emphasizes the role of investor capability in interpreting the new information rather than the arriving information's quality. If agents lack data and experience in processing the new information, then they do not have complete information about the asset's return distribution and have multiple priors. Illeditsch shows that the arrival of new information to the market motivates the investors to hedge against the ambiguity regarding the return distribution and results in portfolio inertia and excess return volatility.

2.3.3.2 Ambiguity and Anomalies

Chen and Epstein (2002) associate excess return anomalies observed in the market with an ambiguity premium. Anomaly in asset return refers to the cases where asset returns cannot be explained by a naïve version of the CAPM. For example, the literature associated the equity premium puzzle (Mehra and Prescott (1985)) with ambiguity aversion. The annualized return difference between the stock market return and the risk free rate is around 6% for the period of between 1889 and 1978 in US. According to the general equilibrium models, historical averages of the equity premium are not consistent with the representative agent; that is, equity premium is too high and the risk aversion among investors should be very high to account for such a high level of the premium. At this point, the existence of an ambiguity premium in addition to the conventional risk premium may provide a plausible explanation for the famous equity premium puzzle.

Another anomaly closely related with equity premium is the home bias[14] (French and Poterba (1991)). Barberis and Thaler (2003) state that home bias leads to under-diversified portfolios with unnecessary risk and agents demanding a higher premium for bearing the higher risk level. Once again, ambiguity and ambiguity aversion may be the reason behind the distorted relationship between risk and return leading to home bias and portfolio under-diversification (Boyle et al. (2009)).

Company size effect is another return anomaly discussed extensively in the literature. The size of the company may affect the information quality which is generally accepted to be lower for small firms. As a result, agents ask for a higher premium for holding small and low information quality equities. Antoniou, Galariotis, and Read (2014) support the relationship between size and ambiguity and conclude that market sentiment affects the premium on the return of small stocks through ambiguity and the return premium on small stocks is higher when the market sentiment is worse and ambiguity is high. Olsen and Troughton (2000) also supported this view in an earlier study and hypothesized that part of the abnormal excess return of stocks explained by size could represent the ambiguity premium.

Momentum is another return anomaly that is documented by Jegadeesh and Titman (1993). Zhang (2006) associates the momentum anomaly with ambiguity aversion and concludes that if there is ambiguity, an agent becomes pessimistic and

[14] Home bias is a Keynesian concept and refers to the reluctance of agents to invest in assets with which they do not have familiarity. Hence, agents with a home bias behave inconsistently with the theoretical relationship between risk and return by investing in the assets of the country of their residence and avoiding foreign assets.

undervalues new signals in the market. According to Zhang, agents can overweigh their own information and underreact to new information such as a better earning announcement. Caskey (2009) examines the impact of ambiguity aversion on the mispricing of assets. In his model, ambiguity averse investors consider aggregate information and do not fully incorporate publicly available information. Inadequate incorporation of publicly available information into prices leads to lagged and continuous price adjustments towards the correct price level and this adjustment mechanism in turn leads to the momentum effect. In the Caskey setup, the characteristic of the revealed information determines the direction of the reaction and whether it would be an underreaction or an overreaction. If the information content about the firm is ambiguous, agents prefer to put higher weight on the available information and overreact to released information. Similarly, Daniel, Hirshleifer, and Subrahmanyam (1998) support the overreaction in asset prices but, according to their study, the reason for the overreaction is agents overweighing the private signal relative to the publicly available information.

2.3.3.3 Ambiguity, Limited Market Participation and Liquidity

Ambiguity aversion also has been associated with market liquidity. Pastor and Stambaugh (2003) confirm that liquidity is priced in asset returns as a systematic risk factor. The impact of market liquidity can turn out to be worse than predicted by models as was witnessed during the Global Financial Crisis. An earlier study by Dow and Werlang (1992) sheds light into the relationship between ambiguity and market liquidity. Using a non-additive subjective probability distribution of returns, Dow and Werlang show that agents neither buy nor short sell within a specific price interval under uncertainty. Similarly, Routledge and Zin (2009) use the model of Epstein and Wang (1994) and show the role of tumbled liquidity and trade in the market. Uncertainty limits reallocation of portfolios and limited reallocation prevents the market maker from hedging its positions. Unable to hedge against new positions, the market maker widens the bid/ask spreads and Routledge and Zin conclude that uncertainty increases the bid/ask spread, decreases liquidity and forms a potential source of freeze in the market.

Ozsoylev and Werner (2011) study the transmission of information into asset prices when the quality of information is unknown or ambiguous and establish the link between ambiguity, liquidity and information. Their model assumes that agents have ambiguous beliefs so they have a set of probability distribution on asset return. Following the arrival of new information, informed agents receive private information about the return distribution and remove ambiguity while uninformed arbitrageurs' beliefs remain ambiguous because they do not observe the signal and extract information from prices. If information is ambiguous, arbitrageurs cannot identify the correct signal and do not trade in the market. Since arbitrageurs are liquidity providers in the market, their absence in the market reduces liquidity. In this context, market depth and trading volume decreases as ambiguity increases.

Similarly, Easley and O'Hara (2010) use the model in Bewley (2002) and investigate the role of incomplete preferences due to ambiguity in lower liquidity. Incomplete preferences mean that agents have their beliefs but they cannot sort portfolios due to ambiguity. Agents can only choose a portfolio if the portfolio provides higher utility for every belief in the set. In such a market with uncertainty, quoted prices have biases of the best and worst case outcomes, unlike the prices under normal market conditions. Hence, inertia in beliefs due to ambiguity limits market participation and liquidity.

2.4 Measuring Ambiguity

The summary of the theoretical and empirical literature presented above shows that ambiguity has an influence on asset prices that is not captured by risk alone and, as such, it needs to be measured and incorporated into asset pricing models separately. Since ambiguity is not directly measurable like return or risk, the number of empirical studies that propose ambiguity measures remains limited. Most of the existing studies attempt to measure ambiguity by building proxies and compare their estimating power in stock return with conventional risk metric of realized volatility. On the other hand, there is no study providing a comparison between the proposed proxies in the literature to the best of our knowledge.

Typically, the proposed ambiguity measures are built around the concept of evaluating whether information is interpreted homogenously among the market participants. The heterogeneity in the market is consistent with the intuition of theoretical models on ambiguity, which assume that there are multiple beliefs and priors in the market. Although there are alternative estimators for ambiguity, there is still ambiguity over the true ambiguity measure. In this section, the methodologies in measuring ambiguity are described under two categories: (i) market-based measures, and, (ii) sentiment-based measures.

2.4.1 Market-Based Measures

2.4.1.1 Options

Among the market-based measures, market turnover and trading volume are often used to proxy the heterogeneity in information evaluation and the distribution of perceptions among market participants but these measures are shown to be inadequate in quantifying the differences among subjective expectations. These traditional market indicators concentrate more on the current period and they typically do not represent future expectations or the distribution of future expectations.

Since ambiguity is shown to be more relevant for future expected returns, it is necessary to proxy it with a variable or a measure that captures the uncertainty regarding the future. For this purpose, options emerge as a good candidate to proxy ambiguity since these contracts contain information about the variability of the expected return's probability distributions. For instance, Andreou et al. (2014) use the S&P500 index options where the dispersions of quoted strike prices and volumes provide a basis for measuring the dispersion of opinions. Higher dispersion of trading volume at different strike prices means investors have diverse subjective beliefs about future prices, namely lower consensus on expected return and higher ambiguity. Andreou et al. focus on explaning capability of ambiguity on stock return so they investigated the power of ambiguity in estimating expected return and compared the power of ambiguity with other explanatory variables such as variance risk premium, tail risk etc. In this context, they didn’t go over any analysis to quantify representetive power of their ambiguity measure with any alternative one but the rationale and information content of ambiguity measure is very close to theoretical definition of ambiguity in the literature.

2.4.1.2 Variance Risk Premium

The variance risk premium (VRP) is another ambiguity measure and is defined as the difference between the future and realized value of volatility; so, it is the risk premium paid by risk-averse investors to hedge against the jump of variance in future consumption growth. Since agents do not prefer higher volatility and prefer smooth consumption paths, VRP measures the premium an agent is willing to pay against the deviation in future volatilities. Hence, as the agents’ willingness to hedge against future volatility due to higher uncertainty increases, the premium that they are ready to pay gets higher.

Bollerslev, Tauchen, and Zhou (2009) use VRP in analyzing equity premium but not associate VRP directly with uncertainty or ambiguity. Following Bansal and Yaron (2004) who show that variability of consumption growth is a determinant of the equity premium, Bollerslev, Tauchen, and Zhou investigate the relationship between the equity premium and VRP and decompose the consumption growth volatility into its expected and unexpected components. They eliminate the expected part by taking the difference between the risk-neutralized expected return variation[15] and the realized return variation, empirically, it is derived by subtracting realized variance of S&P500 from the squared VIX. Positive VRP infers that investors are risk averse and do not like jumps in volatility. The remaining measures

[15] The derivation of the implied volatility of a stock index is “model-free” and it differs from the estimation of implied volatility within the Black–Scholes-Merton model. Realized variance is based on the summation of the 78 within-day five-minute squared returns during the normal trading for the S&P 500.

the unexpected part about the consumption growth volatility, intuitively uncertainty.

Drechsler (2013) emphasizes different role of risk and uncertainty on investor decision and argued that options are sensitive to uncertainty perception of investors and variation in uncertainty impacts on options premia. Drechsler built a general equilibrium model with Knightian uncertainty where jump shocks and variance of future cash flow are identified as the main sources of model uncertainty and claims that VRP successfully captures the variability in uncertainty in the model. In making decision, the agent has a tendency to choose the worst-case scenario in order to make the investment decisions robust to alternative models and is ready to pay higher in order to hedge against higher variance. Hence, the premium paid to hedge against volatility increases as the concerns over model uncertainty increases, consequently, higher variance premiums will affect portfolio formation as well as the required rate of return on equity.

Bekaert, Hoerova, and Lo Duca (2013) investigate the impact of looser monetary policy on risk aversion and uncertainty which is derived by decomposing the VIX index. They partition the VIX index into its risk aversion and uncertainty components and proxy uncertainty with expected realized volatility. Expected realized volatility, in turn, is estimated by regressing realized volatility over past values of squared VIX and realized variance. The estimated conditional variance is called the uncertainty measure. VRP as the difference between the squared VIX and the conditional variance is dedicated as the risk aversion.

2.4.1.3 Other Measures

Bekaert, Engstrom, and Xing (2009) combine two main approaches in the literature in explaining equity returns and test uncertainty as a factor in asset pricing. They use changes in the conditional variance of fundamentals such as consumption growth and dividend growth or changes in risk aversion as uncertainty measures. Their model introduces a stochastic risk aversion and time varying uncertainty in the fundamentals where uncertainty is instrumented by the conditional volatility of dividend growth. The results confirm that uncertainty has explanatory power for the equity premium but also underline the critical role of risk aversion.

In another study, Brenner and Izhakian (2018) construct a monthly ambiguity index by using the S&P500 ETF data where ambiguity is proxied by the variability of daily return distribution within month. The results confirm that ambiguity aversion increases along with the expected probability of favorable returns, and ambiguity loving increases along with the expected probability of unfavorable returns.

2.4.2 Sentiment-Based Measures

Disagreement among the forecasts of survey participants is an alternative approach to measure ambiguity. As the dispersion of information and beliefs among agents are sources of ambiguity, investor sentiment and its distribution also may be a candidate in measuring ambiguity. Although they didn't explicitly intent to measure the impact of ambiguity on stock return, earlier studies from Diether, Malloy, and Scherbina (2002), and Johnson (2004) investigate the relationship between analyst forecast dispersion and cross-sectional stock return. Both of these studies use the standard deviation of earning forecasts as dispersion of beliefs and suggest that stocks with higher dispersion in analyst earnings forecasts generate lower future returns. Diether, Malloy, and Scherbina (2002) show that this effect is stronger in small sized stocks and those that perform worse in the last year, and claim that dispersion in analyst forecasts is not a proxy for risk. Johnson's (2004) study also supports the Diether, Malloy, and Scherbina findings. The author decomposes the risk of an asset's return into two components: (i) fundamental risk measured by the variability in the economy, and, (ii) parameter risk measured by the information setting. Johnson uses the dispersion of analyst forecasts as a proxy for unpriced information risk. Although it contradicts with the efficient markets hypothesis that agents pay a premium for uncertainty rather than ask for a premium, the negative relationship between uncertainty and stock returns is associated with the option-pricing results which state that expected returns decrease along with higher idiosyncratic risk for a levered firm.

Zhang (2006) use dispersion in analyst forecasts of stock prices as one of the ambiguity measure in addition to the other ambiguity proxies such as firm size, firm age, analyst coverage, return volatility, and cash flow volatility. Although the indicators are different from each other, they share a commonality since all indicators measure the level of homogenous perceptions of information.

Anderson, Ghysels, and Juergens (2009) also proxy ambiguity with the divergence of analyst opinions on corporate profits and analyze the impact of ambiguity on stock return. The analyst opinions are obtained from the Survey of Professional Forecasters (SPF) carried out by the Federal Reserve and risk is measured by realized volatility. In order to quantify the impact of ambiguity, they form portfolios according to the ambiguity level of stocks. The results suggest that the annual return of the high ambiguity portfolio is higher compared to that of the lower ambiguity portfolio and ambiguity has explanatory power over and above the Fama-French three factors and the momentum factor. Although analyst forecasts seem like a good ambiguity proxy, since the forecasts are collected on a monthly and quarterly basis, their representativeness is questionable.

Bali, Brown, and Tang (2014) investigated the impact of uncertainty on cross-section of stock returns. They proxied uncertainty by the dispersion of the expectations of forecasters from SPF. They collected the cross-sectional distribution of quarterly forecasts for US real GDP level and growth, nominal GDP level and growth, GDP price index level and growth and unemployment rate. The dispersion for each series is calculated by the interquartile range. However, they didn’t use raw dispersion measures as uncertainty indicator. Rather, they used AR(1) model and generated the standardized residuals. In this context, they defined the subcomponents of uncertainty as the innovations to dispersion indicators. Average of the standardized residuals for these seven dispersion measures represent uncertainty and measures the shock over economy.

Jurado, Ludvigson, and Ng (2015) define uncertainty as the conditional volatility of a disturbance that could not be forecasted by agents but the approach is different than other studies mentioned in this section in the sense that forecasts don't come from surveys, instead from econometric estimation. They use the difference between the estimates and realizations of macroeconomic variables and company based data[16]. The deviations between realizations and estimates are aggregated through equal weighting and alternatively by their first principle component. Jurado, Ludvigson, and Ng compared macroeconomic uncertainty index with other uncertainty indices referenced commonly such as volatility of stock market returns, the cross-sectional dispersion of firm profits and survey-based forecasts. The comparison reveals that the number of uncertainty episodes is lower and more persistent compared to other uncertainty indices. However, there are some disadvantages of using macroeconomic variables in deriving an uncertainty indicator. One of the disadvantages is that the frequencies are typically lower so the indicator's tractability at higher frequency is not possible. In addition, the representativeness of the survey participants' responses in terms of the typical investor in the market may be another concern regarding the use of this indicator as an uncertainty proxy.

Baker, Bloom, and Davis (2016) also use survey results and develop an uncertainty indicator capturing three aspects of economic policy uncertainty in the US: i) the frequency of policy-related economic uncertainty, ii) the number and revenue impact of federal tax code provisions to expire within 10 years, iii) the degree of disagreement among forecasters over future government purchases and future

[16] Macroeconomic time series used in the study are as follows: real output and income, employment and hours, real retail, manufacturing and trade sales, consumer spending, housing starts, inventories and inventory sales ratios, orders and unfilled orders, compensation and labor costs, capacity utilization measures, price indexes, bond and stock market indexes, and foreign exchange measures. Company variables used in the study are as follows: dividend-price ratio and earnings-price ratio, growth rates of aggregate dividends and prices, default and term spreads, yields on corporate bonds of different ratings, yields on Treasuries and yield spreads, and a broad cross-section of industry, size, book-to-market, and momentum portfolio equity returns.

inflation.[17] Baker, Bloom, and Davis examine the capacity of economic policy uncertainty index in reflecting uncertainty through comparing it with other indicators closely related with uncertainty. One of them is VIX Index with that a separate uncertainty index constructed for equity market from news was compared. Another indicator to compare economic policy uncertainty index was derived from word counts of "uncertainty" in the Beige Book released by FED. Both comparisons confirm all uncertainty indices move very close to each other.

2.5 Constructing an Ambiguity Index for Turkey

In this part, we aim to derive alternative measures for ambiguity for the Turkish financial markets with the ultimate objective of comparing the measures to determine the one that is the most representative, has the longest coverage and most relevant for studying the impact of ambiguity on asset pricing. Since the ambiguity index will be used as a factor in asset pricing and our interest is to form an index with an explanatory capacity for expected returns, we focus on financial series and aim to construct an index that is representative of ambiguity regarding the future. Such an index can be constructed by using data from a number of different financial instruments from the spot and derivatives markets that have the potential of providing information about ambiguity.

Although volatility of stock returns and exchange rates, CDS rates, and EMBI spreads are examples of the well-known variables that are used widely to proxy uncertainty or ambiguity, they suffer from an important specification error: an ambiguity proxy should include unmeasurable uncertainty, as discussed in Section 3, but these variables include uncertainty that is quantifiable. Also, their historical

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[17] The first component is based on scaled monthly counts of articles published in the 10 leading US newspapers containing the following words: uncertain, uncertainty, economic, economy and one or more policy relevant terms: regulation, Federal Reserve, deficit, congress, legislation, and white house. The index related with tax cuts is calculated as the discounted sum of projected revenue effects related with the expiring tax code provisions. The disagreement among economic forecasters consolidates interquartile ranges for quarterly forecasts of federal, state, and local government expenditures and 1-year CPI from the Survey of Professional Forecasters.

coverage remains limited for analyzing long-term asset pricing dynamics. Keeping these issues in mind, we use two Turkish financial indicator series with the longest possible time series coverage. These series are the stock market index and the Turkish Lira exchange rate. We utilize the methodologies reviewed in the literature to extract an ambiguity index for Turkey while we take into account the applicability of methodologies.

Although our aim is to get a representative ambiguity index with the longest possible historical coverage, we produce more than one index, each with a different methodology and time series coverage in order to perform robustness checks. Our contribution to the literature is twofold. First, our index is the first Turkish market ambiguity index constructed with a long historical coverage; therefore, its characteristics come close to the ambiguity definition in the literature. Second, the ambiguity indices produced for comparison purposes also are constructed for the first time using the Turkish market data and they are among a very limited number of indices constructed for emerging markets. Due to data availability, different indices are calculated over different periods and, as a result, a one-to-one comparison is not possible among the indices.

2.5.1 Alternative Ambiguity Indices for Turkey

Most of the previous studies conducted for the Turkish market attempt to measure uncertainty or ambiguity at the macroeconomic level. Although it is not clear how to distinguish macroeconomic uncertainty from the uncertainty in financial markets, our measures are more relevant for proxying the latter. In this section, existing literature is reviewed in terms of proxy construction and the appropriateness of the proxy in measuring ambiguity for Turkey. In one of the earliest studies, Arslan et al. (2015) derive an uncertainty index from firm level data extracted from the Central Bank of Turkey’s monthly survey. More specifically, they use the answers given to two of the survey questions: (i) expectations for the next three month’s production level, and, (ii) realizations for the last three month’s production. They compared the expectations at time t with the realizations at time t + 3 in order to understand how successful firms are in projecting future production levels. The more successful the firms, the less uncertainty is said to exist regarding the future. The analysis is conducted at the firm level and results for each firm are aggregated to build up an index at time t. Next, the authors decomposed this dispersion index into its idiosyncratic and macro level uncertainty components in order to develop an index representing uncertainty at the macro level. The index covers period between 1987 and 2010. Although the index captures the periods with uncertainty very well and is also close to the ambiguity definition in the literature since it gives information about the divergence of probability distributions for the future production realizations, there are some well-known drawbacks of using survey data such as reliability, consistency and differences in understanding and interpretation, participants’ homogeneity and continuity.

Another economic uncertainty index is developed by Coşar and Şahinöz (2018). The authors derive the economic uncertainty index from different sub-indices of a financial uncertainty index, forecasters’ uncertainty index, firms' uncertainty index, consumers' uncertainty index and economic policy uncertainty index. Sub-indices were constructed from monthly data except for the financial uncertainty index, which was calculated from daily data on financial indicators that are commonly used in many financial distress indexes. The financial indicators are volatility of stock exchange return (BIST-100 all shares index), VIX, implied volatility of USD/TL exchange rate, EMBI Turkey, realized interest rate volatility and CDS rates. Although these types of indicators are successful in reflecting stress and uncertainty in financial markets, historical data availability is a major concern for emerging markets. Hence, it is not possible to generate a long series of uncertainty by using these indicators. The indicators under the financial uncertainty index carry information about the uncertainty about the future but they have fundamental differences from the ambiguity definition in the literature. First, historical volatility series may only partly carry information about the ambiguity level because they contain limited information about the future prospects and the divergence between probability distributions. Second, other financial indicators like implied volatility, CDS etc. contain information about the future uncertainty but these financial instruments are still based on uncertainty measures that are quantifiable. For example, the implied volatility of the USDTRY exchange rate is based on the at-the-money (ATM) option contracts so the uncertainty measure incorporates quantified uncertainty.

Although the uncertainty indices constructed by Arslan et al. (2015) and Cosar and Sahinoz (2018) perform well in capturing the economy-wide uncertainty during periods of high stress, they are not appropriate to use in our research due to two main reasons. First and most importantly, these indices do not closely match the ambiguity definition put forward in the literature. Second, since we investigate the impact of ambiguity on asset pricing, we need to construct an ambiguity series that reflects uncertainty specifically in financial markets and has the longest possible historical coverage.

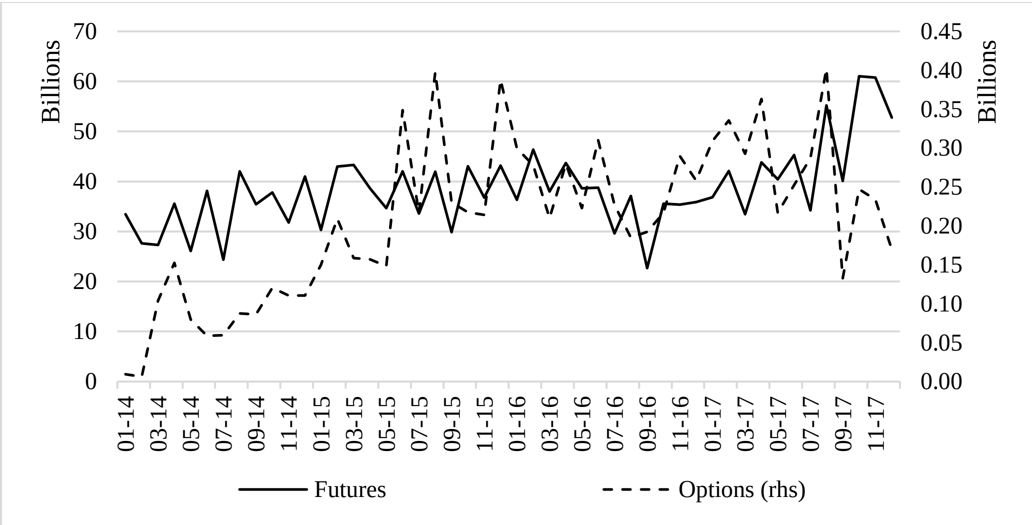
2.5.1.1 A Derivatives-Based Ambiguity Index

We calculated our first ambiguity index from the derivatives in Turkey to make the index to capture forward looking uncertainty which is more relevant in asset pricing. Among alternative derivative instruments, we use futures contracts rather than options due to two main reasons. First, options in Turkey started to trade in 2013 so the time dimension of the series is too short to use the index in asset pricing tests. Second, trading volume of options compared to the volume of futures contracts is quite low; so, liquidity of the contracts and the accuracy of information could be limited (Figure 2.1).

In Turkey, futures contracts and options are traded electronically in the organized market of Borsa Istanbul Derivatives Market (VIOP). We obtained high frequency intraday transaction data for the BIST30 futures contracts[18] from Borsa Istanbul's database for the period between January 2006 and December 2017. For robustness

[18] The BIST30 futures contracts are written on the BIST30 Index which is a composite index of Borsa Istanbul companies including 30 stocks with the highest market value. The index value is calculated as the average of prices weighted by market capitalization.

checks and comparisons, we also gathered data for the USDTRY futures contracts to derive an alternative index by using the same methodology. Comovement of two indices may be considered as an indication of the consistency of the methodology.



Source: BIST Datastore

Figure 2.1: Comparison of Trading Volume of Futures Contracts and Options on Stock Exchange: The figure compares monthly trading volume of futures contracts and options on BIST30 Index in Borsa Istanbul over 2014-2017. All values are in billions of Turkish lira.

In measuring ambiguity from futures contracts, we are inspired by the formulation in Andreou et al. (2014) where the ambiguity measure is based on the dispersion of the trading volume at different strikes of the S&P500 index options[19]. On the other

[19] Distribution of option strikes is considered as a proxy for how much information is evaluated homogenously in the market. As each strike contains an underlying distribution and signals the evaluation of information, a wider dispersion of strikes implies more disagreement about the probability distribution of future prices and thereby heterogeneity over the evaluation of current information. Heterogeneity among distribution assessments fits well to the arguments put forward in the theoretical models on ambiguity. In the following formula by Andreou et al. (2014), ambiguity is calculated as the weighted sum of dispersion of strikes ($X_{t,j}$) from daily weighted average of strikes ($\sum_{j=1}^{K} w_{t,j} X_{t,j}$).

$$\text{Ambiguity}_t = \sum_{j=1}^{K} w_{t,j} \left| X_{t,j} - \sum_{j=1}^{K} w_{t,j} X_{t,j} \right|$$

hand, we modified the formula in a number of ways. Since we have chosen futures contracts to estimate uncertainty, we have no strikes. Even the pricing of futures contracts and options have fundamental differences and they have different information content, so they could not be comparable one to one, we adapt our dataset to the formulation in Andreou et al. (2014) to proxy the dispersion of expectations on price distribution. First, similar to specifying the strikes and the volumes at each strike price to derive the dispersion, we ranked intraday transaction data of the nearest maturity BIST30 Index futures contract according to its trading price to construct price bins and determine volumes within each bin[20]. Following the derivation of bins for each day in the sample, we calculated the weighted average trading price and total trading volume for each bin for each day throughout the sample period. Next, for each day, we calculated the dispersion of the weighted average of trading prices among bins. Dispersion of average prices among the bins carries information about the divergence of assessment about the probability distribution, which is an indication of how homogeneously the information is interpreted by the market participants.

We also modified the formula by adding the weighted average price on day t to the denominator in order to normalize the dispersion measure by removing the scale effects since the futures prices on the BIST30 Index and USDTRY contracts both have a unit root. If we preserve the original formula without the term for normalization, the dispersion measure proxying ambiguity would increase along with the increase in the series through time.

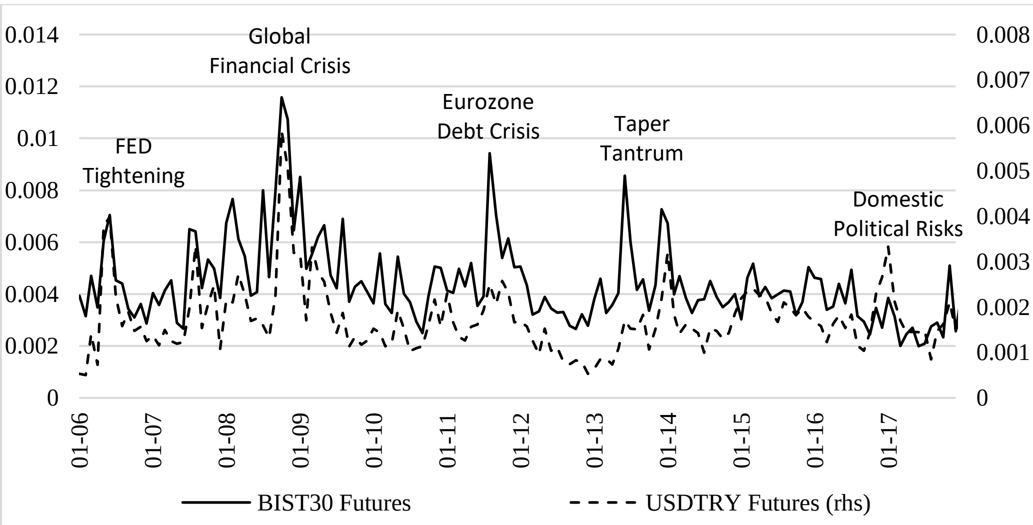
As a result, the formulation in Equation (2.31) measures the weighted dispersion of futures prices in different bins. In the equation, $w_{t,j}$ is the weight of the bin which is the ratio of the j^{th} bin transaction volume to the transaction volume for all bins in day t and $X_{t,j}$ is the weighted average of transaction prices for the j^{th} bin in day t. Hence, the numerator in the absolute value operator shows the dispersion of

[20] The number of bins is chosen as 30 in order to capture the dispersion of futures prices. We tried alternative bin counts, such as 60, and the results were qualitatively the same.

transaction prices in the jth bin from the weighted average of all transaction prices for that day. Next, this dispersion is normalized by the daily weighted average price. Finally, each normalized dispersion value for each bin is summed after multiplying by a weight that is equal to the ratio of the trading volume in the bin to the total trading volume for that day ($w_{t,j}$). Using high frequency data enables to construct the ambiguity index on a daily basis, a frequency that is not possible when macroeconomic variables are used for the same purpose.

$$\text{Ambiguity}_t = \sum_{j=1}^{K} w_{t,j} \left| \frac{X_{t,j} - \sum_{j=1}^{K} w_{t,j} X_{t,j}}{\sum_{j=1}^{K} w_{t,j} X_{t,j}} \right| \quad (2.31)$$

Figure 2.2 presents the two ambiguity indices we derived from futures contracts written on the BIST30 Index and the USDTRY exchange rate for the period between January 2006 and December 2017. There are two points standing out in Figure 2.2. First, the parallel movements of two indices reflects that the methodology is consistent in producing ambiguity indicator. Second, highs and lows of the indices overlaps with the stress level in financial markets so indices are successful in capturing stress periods and the ambiguity well.



Source: BIST Datastore, Own calculations

Figure 2.2: Ambiguity Indices from Futures Contracts of Stock Exchange and Exchange Rate: The figure shows monthly ambiguity indices computed by BIST30 futures contracts on the left hand side and USDTRY futures contracts on the right hand side for the period of 2006-2017.

2.5.1.2 A Variance Risk Premium-Based Ambiguity Index

Keeping in mind the lack of fit between an implied volatility measure and dispersion of options[21], we derive the variance risk premium (VRP) for the BIST30 Index by using implied and realized volatilities[22]. The data on the implied and realized volatility of the BIST30 Index[23] is obtained from Bloomberg for the period between 2013 and 2017. Instead, we calculate the VRP as the difference between the implied and realized volatility of the BIST30 Index by using the formulation in Equation (2.32) where IV_t is the implied volatility and RV_t is the realized volatility at date t.

$$VRP_t = IV_t - RV_t \tag{2.32}$$

The VRP for the BIST30 Index is shown in Figure 2.3. The index increases during periods of high economic uncertainty. Although the VRP index only goes back to 2013, we compare the VRP with the ambiguity index that is derived from the BIST30 futures contracts for the limited sample period. The two indices seem not to move close to each other and the correlation between them is rather at 0.03. This result is consistent with Andreou et al. (2014) who argue that the low correlation observed for the US market implies that the two ambiguity indices have different

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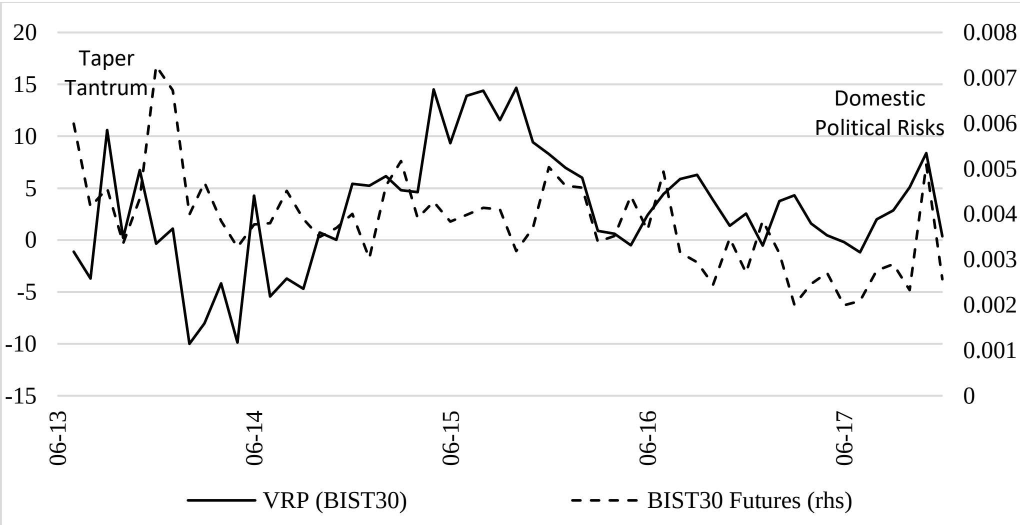
[21] In Turkey, there is no implied volatility index produced by Borsa Istanbul similar to the model free VIX index of the CBOE. However, there are implied volatility indices calculated by data vendors such as Bloomberg and Thomson Reuters. These indices are calculated based on the Black–Scholes-Merton option pricing formula so it is not possible to say that the VIX index and the implied volatility for BIST30 are equivalent in terms of both methodology and information content. The major difference between the two indices in terms of information coverage is that the VIX index is model free so it represents the dispersion of opinions. In addition, feasibility of calculating the model free implied volatility for Turkey is questionable because the number of available strikes is limited to make a calculation similar to the VIX Index.

[22] Implied volatility is derived from Black–Scholes-Merton formula and it is a model-based measure of the probability distribution for the nearest at-the-money option contract. Realized volatility is calculated by annualizing the monthly standard deviation of daily logarithmic return.

[23] We also calculate VRP from USDTRY by using a similar approach and results remained same. Both implied volatility and realized volatilities are collected from Bloomberg. There is no active option market for USDTRY before 2013 but Bloomberg calculates implied volatility from options trading over the counter market.

information contents. While the index based on the futures contracts reflects ambiguity regarding expected mean return in the stock market, the index based on the variance risk premium reflects ambiguity regarding expected volatility.



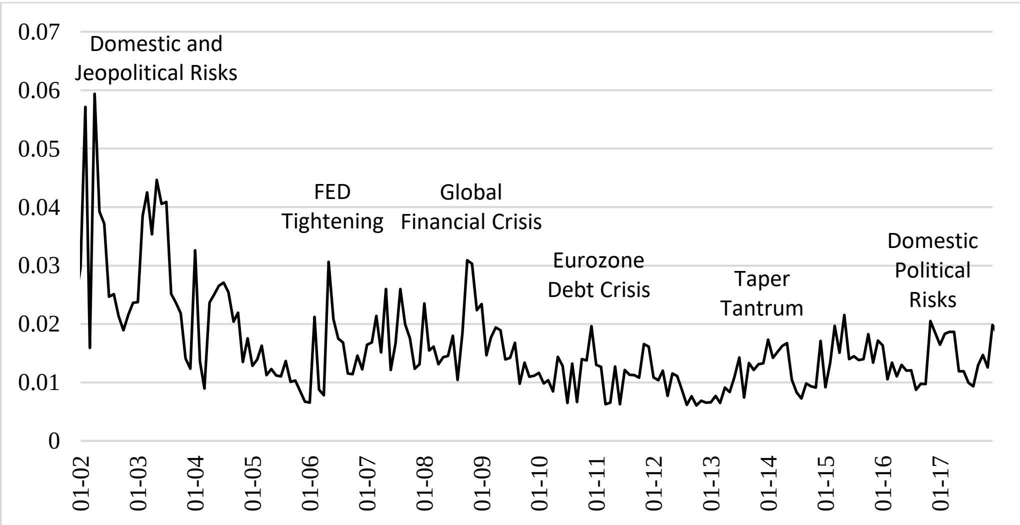
Source: Bloomberg, BIST Datastore, Own calculations

Figure 2.3: Ambiguity Index and Variance Risk Premium (VRP): The figure shows monthly ambiguity index and VRP for the period of June 2013-December2017. VRP on the left hand side of the graph is calculated as the difference between implied and realized volatility of BIST30 Index. Ambiguity index on the right hand side of the graph is derived from BIST30 futures contracts.

2.5.1.3 An Expectations-Based Ambiguity Index

In Turkey, the Central Bank of Turkey publishes a Survey of Expectations to monitor the real and financial sector expectations about macroeconomic indicators such as the inflation rate, exchange rates, interest rates, current account balance and GDP growth rate. Hence, it is possible to construct an ambiguity index from survey data similar to studies cited in the previous sections. For this purpose, we first consider data availability since the survey does not collect data on all variables across an equal period. In addition, we also consider the comparability of the survey data with our ambiguity indices and the representativeness of the indicators regarding the decision-making process in financial markets.

Among the alternatives such as the GDP, CPI, overnight interest rates and others, the USDTRY exchange rate is selected considering its popularity in evaluating financial conditions. We collect the survey's results for the forecasts on current month exchange rate between 2002 and 2017. We calculated the uncertainty indicator as the ratio of the standard deviation of the forecasts to the modified mean[24] of the forecasts for the current month USDTRY exchange rate. As seen in Figure 2.4, the uncertainty measure goes back to 2002 and captures both the domestic and international stress periods well. However, the index shows dramatic changes in the early periods and this limits reliability of the index. This volatile behavior of the index could be associated with the number of participants in the early years of survey.



Source: CBRT

Figure 2.4: Mean-Adjusted Standard Deviation of 1-Month Ahead USDTRY Expectations: The figure shows the ratio of standard deviation to mean of 1-month ahead USDTRY expectations in Survey of Expectations conducted by CBRT for the period of 2002-2017.

[24] The modified mean is calculated in order to eliminate the observations with extreme values that make it difficult to derive a central tendency indicator. CBRT specifies the modified mean according to following rules as: i) arithmetic mean if distribution is close to normal distribution, ii) arithmetic mean after excluding outliers if there are outliers, iii) median if skewness is high and iv) mode if kurtosis is relatively high.

2.5.1.4 An Intraday Returns-Based Ambiguity Index

The three uncertainty indices calculated above seem to have different sensitivities to domestic and external events and the longest of the series goes back to early 2000. For testing the role of ambiguity in asset pricing, another index that has the ability to represent the decision-making process in financial markets and can be calculated over a longer history is needed. Exchange rates and stock market index are two indicators satisfying these specifications. Also, both markets generate enough trading volume and liquidity that their indicators can be used as measures of investor sentiment. In addition, by using a methodology similar to the one used for deriving the ambiguity index from the futures contracts, it will be possible to calculate ambiguity by using trading price and volume data. However, contrary to price data, historical intraday trading volume is not available for exchange rate and stock exchange.

Brenner and Izhakian (2018) propose a novel approach to derive ambiguity from intraday price data. The authors define ambiguity as the uncertainty over the probability distribution of returns. They derive an ambiguity indicator for the US by using intraday data on the S&P 500 exchange traded funds (ETF) and they prefer to use the ETF instead of the S&P 500 Index itself because the ETF's variance is lower due to lower bid/ask spread and liquidity is higher compared to the S&P 500 Index which includes illiquid stocks as well. The methodology calls for determining how much the probability distribution of intraday returns varies within a month and higher variability of the probability distribution implies higher ambiguity consistent with the literature.

We follow the methodology of Brenner and Izhakian (2018) to produce a similar ambiguity index from the BIST30 Index, which includes the 30 largest market capitalization stocks on Borsa Istanbul, eliminating any concerns over illiquidity or low trading volume. Our dataset consists of intraday tick data from the BIST Datastore and spans the period between January 1998 and December 2017.

Since calculating returns from tick data has some problems due to market microstructure effects, we convert the tick data into 5-minute index data by taking the simple average of index values over the 5-minute intervals and calculate 5-minute log returns. Lastly, we derive the probability densities for each day from this return series. In order to extract information about the variability of the probability distribution on a daily basis, first, we partition the probability densities into 60 bins[25] to observe the variability in each bin. Next, we aggregate the measure of variability from each bin within a month across all bins.

Brenner and Izhakian (2018) use the formulation in Equation (2.33) to calculate the ambiguity measure. The core of the ambiguity measure is in the parentheses and it consists of three parts. The first and third terms are the average volatilities of probabilities at the tails of the distribution and the second term is the average volatilities of probabilities weighted by expected value within each bin. More specifically, the second term with the sigma notation is the sum of the products of expected value and variance of probabilities within each bin and it is called the weighted average volatility of probabilities. Summing up the average volatilities of probabilities at the tails and weighted average volatilities of probabilities for the bins gives the total average volatility of probabilities in a month. If the probabilities of bins or the probability distribution at the aggregate level is not predictable, ambiguity about the distribution of return is higher. Finally, the term just in front of the parentheses is a scaling parameter of the value where w is the width of the bin range. Brenner and Izhakian (2018) suggest that this form of the scaling parameter works better compared to alternatives and lessens the sensitivity of the weighted average volatility of probabilities to the size of the bin.

$$\begin{aligned}\Omega^2[r]= \frac{1}{w(1\text{-}w)} \times\Big(&E[\Phi(r_0;\mu, \sigma)]\times Var[\Phi(r_0;\mu, \sigma)] \\ &+ \sum_{i=1}^{60} E[\Phi(r_i;\mu, \sigma)\text{-}\Phi(r_{i\text{-}1};\mu, \sigma)] \times Var[\Phi(r_i;\mu, \sigma)\text{-}\Phi(r_{i\text{-}1};\mu, \sigma)] \\ &+ E[1\text{-}\Phi(r_{60};\mu, \sigma)]\times Var[1\text{-}\Phi(r_{60};\mu, \sigma)]\Big)\end{aligned} \tag{2.33}$$

[25] We replicate the analysis for 30 bins and the results are qualitatively the same.

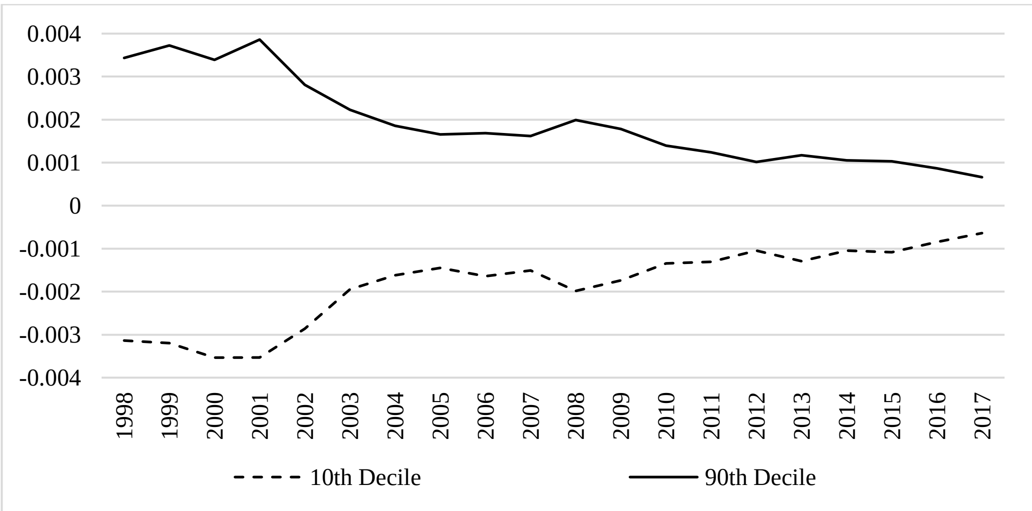
Although we adhere to the main methodology presented in Brenner and Izhakian (2018), our implementation differs in a number of ways. First, the range of returns that we divide into bins varies through time. Second, we have a different assumption about the distribution of intraday returns. Brenner and Izhakian (2018) define the lower and upper values of the daily return range for the S&P 500 ETF as -6% and 6%, and these values are fixed throughout the sample period. Rather than directly adapting this methodology, we first analyze the distribution of intraday return. Daily return distributions show a significant shift during the sample period. This shift could be associated with many factors such as the volume of trade, number of participants, composition of participants etc. Since the range of returns changes significantly, we define the lower and upper boundaries as the 10^{th} and 90^{th} percentiles[26], respectively.

Following the determination of lower and upper boundaries at the 10^{th} and 90^{th} percentiles, we update the ranges on a yearly basis by taking into account the shift in the distribution. A more frequent update of the ranges would contradict with the rationale of using the variability of probability distributions as an ambiguity measure. On the other hand, choosing the update frequency as two or three years may be too long to capture the changing dynamics and also may distort the comparison of stress levels. Figure 2.5 shows the yearly average returns at the 10^{th} and 90^{th} percentiles during the sample period. The range has narrowed down in more recent years and it was substantially larger in the earlier years of the sample.

After we determine the yearly upper and lower values for the range, we first calculate the probabilities for each bin on each day. Considering the leptokurtic distribution of the returns in financial markets, we assume a Student's t-distribution

[26] The boundary selection has the potential to affect the index values since choosing the tails at higher values would push up the index during high stress periods and smooth out the index during low stress periods. Conversely, choosing the boundaries at lower levels would put more weight to the tails in the index and makes the index highly volatile. Since the goal is to capture investor behavior in the face ambiguity and since investors do not only respond to tail events or continuously reevaluate their ambiguity perceptions, the 10th and 90th percentiles are considered to capture both high and low stress periods successfully.

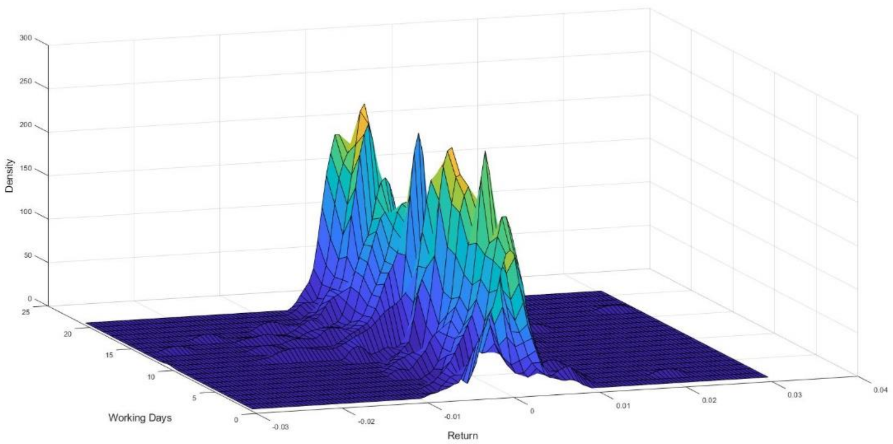
in calculating the probabilities at each bin while Brenner and Izhakian (2018) assume a normal distribution for the S&P 500 ETF. Afterwards, we derive the probability densities for each bin on each day and use these probabilities to obtain the ambiguity measure for each month following a few steps. First, as we calculate the uncertainty measure monthly, we calculate the mean and variance of probability densities for each bin within a month. For example, considering the number of business days, we have approximately 20 values of the probability for the 1^{st} bin in the 1^{st} month and so on. If the probability of this bin is stable and variation is low, ambiguity about the bin is low because we can form an expectation about the probability for the return interval. While we can use this information to make an inference about a specific range of returns, we can also use this information to make inferences on the probability distributions by aggregating the variability in all bins. Similarly, if the aggregated variability of all bins is low, it could be inferred that the probability distribution of returns is stable and the ambiguity is low.



Source: BIST Datastore, Own calculations

Figure 2.5: 10^{th} and 90^{th} Percentiles of BIST30 Daily Return Distributions: The figure plots the 10^{th} and 90^{th} percentiles of daily return distribution for each year between 1998 and 2017.

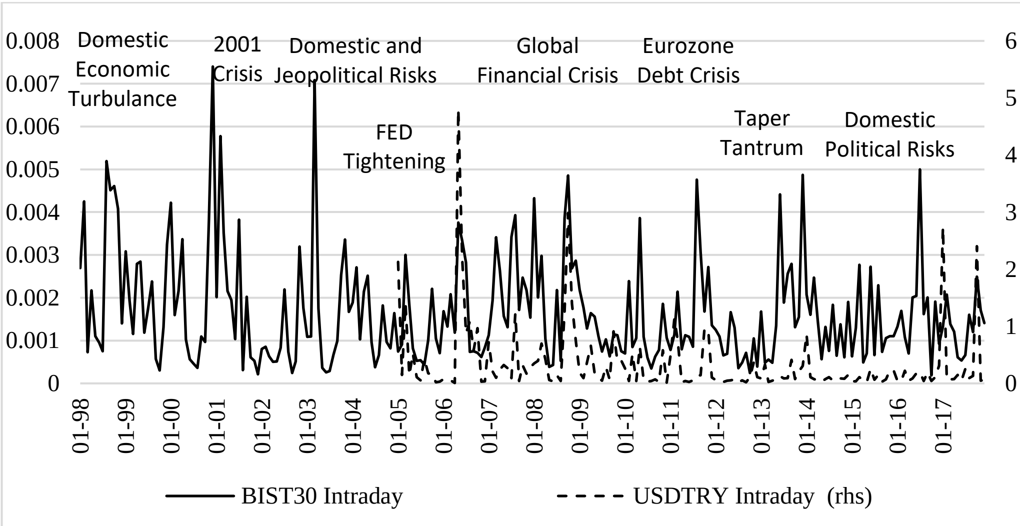
In order to demonstrate the rationale of the calculated ambiguity measure, in Figure 2.6 we present the 5-minute return distributions of BIST30 Index for each day during March 2003. We choose a month with a high ambiguity level to make the intuition behind the ambiguity calculations more obvious. As seen from the graph, the mean return is fluctuating from one day to another and the return distribution is wider on some days as captured by the variance in the ambiguity formulation.



Source: Own calculations

Figure 2.6: Daily Probability Densities of BIST30 Index Return in March 2003: The figure presents 5-minute return distributions of BIST30 Index for each working day during March 2003. x, y and z axes show log return, working days and kernel density respectively.

The ambiguity index calculated for the entire sample period is presented in Figure 2.7. The index seems to capture the stress periods quite well. In addition to the BIST30 Index, we use the intraday USDTRY exchange rate data and recalculate the ambiguity index by employing the same methodology described above. We collect high frequency data for the USDTRY exchange rate from Datascope for the period between January 2005 and December 2017. As can be seen from the Figure 2.7, the two ambiguity indices move very close to each other but the exchange rate-based ambiguity index is more sensitive to shocks and also is smoother during other periods. This asymmetric movement could be associated with the disparity between the depth levels and investor motives in the two markets. Stock market trading volume is lower compared to trading volume in fx market and agents trading in fx market do not only trade for speculative motive but also to meet fx demand for other purposes such as international trade obligations. Therefore, volatility in stock index is higher in normal times but the response in fx market is stronger during high stress times. Another explanation could be related with the fact that we kept bin range fixed during the sample period for USDTRY exchange rate.

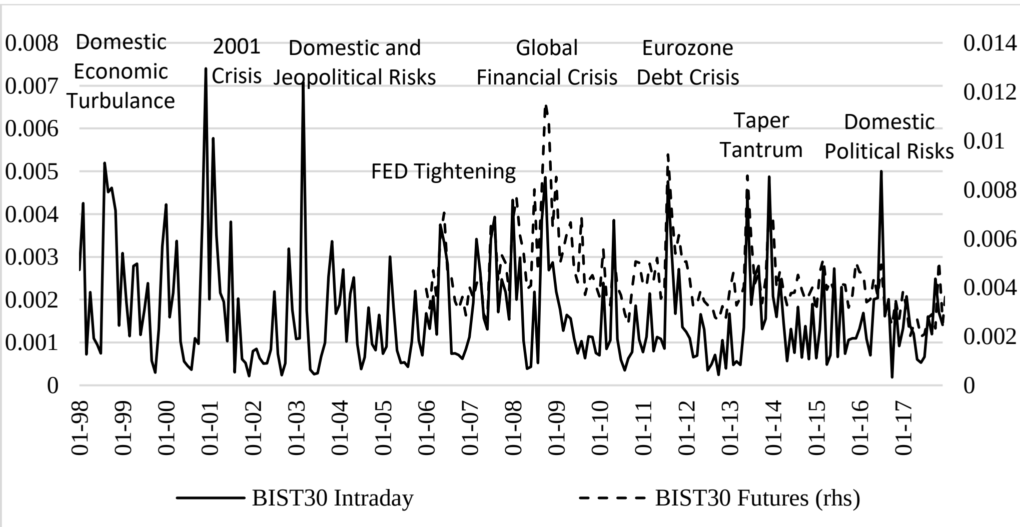


Source: Own calculations

Figure 2.7: Ambiguity Indices from BIST30 Index and USDTRY Intraday Data: The figure shows monthly ambiguity indices from BIST30 Index intraday data on the left hand side and USDTRY intraday data on the right hand side of the graph for the period of 1998-2017. USDTRY intraday data is available since 2005.

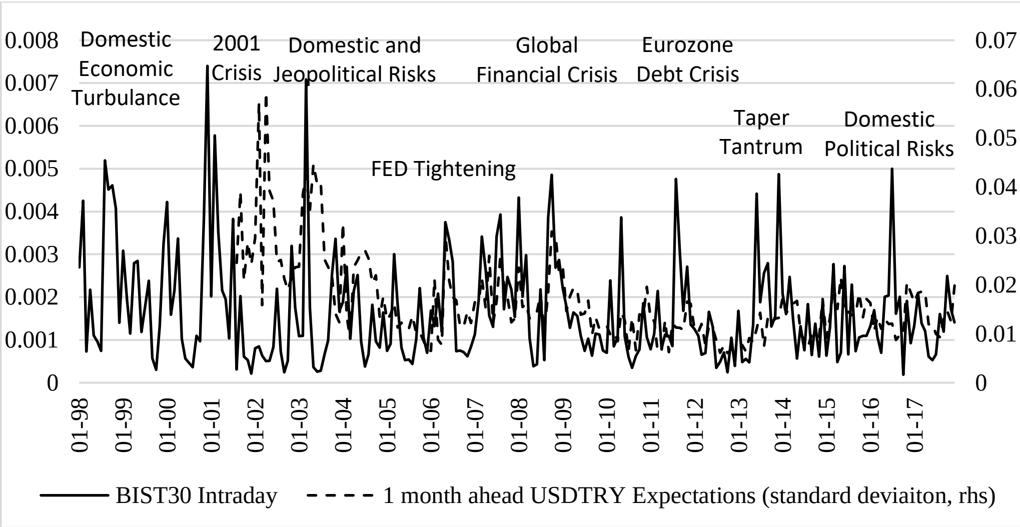
2.5.2 Comparison of Alternative Ambiguity Indices, Historical Volatility and Returns

Among the alternative ambiguity indices calculated, the index based on the BIST30 intraday probability distributions is the most appropriate one for using in asset pricing tests due to its longer historical coverage. In the following figures, we compare the ambiguity index based on the BIST30 intraday prices against the index based on the BIST30 futures contracts (Figure 2.8), the index based on the Central Bank survey that calculates the ratio of standard deviation to mean of the 1-month ahead USDTRY exchange rate expectations (Figure 2.9), and the index based on the variance risk premiums extracted from the BIST30 Index's implied and realized volatilities. As can be seen in Figure 2.8, indices based on the BIST30 futures and intraday prices move very close to each other. Also, Figure 2.9 shows that the ambiguity index based on the survey data is quite successful in reflecting the impact of high stress as higher ambiguity and move close to the index based BIST30 Intraday data. On the other hand, even the coverage of the index is very limited to make a concrete conclusion, the VRP based index in Figure 2.10 does not seem to move together with the ambiguity index that is derived from the BIST30 intraday data, and the difference between two indices could be attributable to the fact that VRP is based on ambiguity over the return volatility while ambiguity from BIST30 intraday data is based ambiguity over mean return.



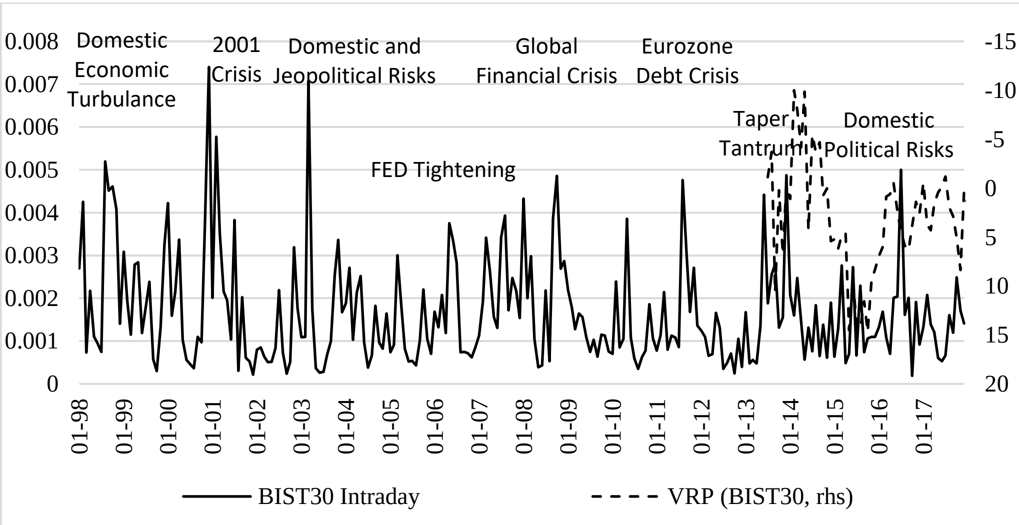
Source: Own calculations

Figure 2.8: Ambiguity Indices from BIST30 Futures Contracts and BIST30 Intraday Data: The figure shows monthly ambiguity indices from BIST30 Index intraday data on the left hand side and BIST30 Index futures contracts data on the right hand side of the graph for the period of 1998-2017. BIST30 Index futures contracts data is available since 2006.



Source: Own calculations

Figure 2.9: Ambiguity Indices from BIST30 Intraday Data and USDTRY Expectation: The figure shows monthly ambiguity indices from BIST30 Index intraday data on the left hand side and the ratio of standard deviation to mean of 1-month ahead USDTRY expectations in Survey of Expectations conducted by CBRT on the right hand side of the graph for the period of 1998-2017. Survey of Expectations data is available for the period of 2002-2017.



Source: Own calculations

Figure 2.10: Ambiguity Indices from BIST30 Intraday Data and VRP: The figure shows monthly ambiguity indices from BIST30 Index intraday data on the left hand side and VRP from BIST30 Index on the right hand side of the graph for the period of 1998-2017. VRP data is available since June 2013.

Following the graphical analysis, we present some descriptive information regarding the ambiguity indices and other variables such as the risk free rate, stock market return, stock market excess return and historical volatilities. The risk free rate is monthly average of discounted Turkish Treasury auctions interest rate. The stock market return is the BIST30 Index return and the stock market excess return is the difference between the BIST30 Index return and the risk free interest rate. Historical volatility is equal to the mean of the squared daily returns for each month during the sample period.

The descriptive statistics presented in Table 2.1 show that almost all ambiguity and volatility indicators are right skewed and the kurtosis is greater than 3, indicating leptokurtic distributions with fat tails. These are expected results since by definition the volatility indices are positive and have extreme values during stress periods. Although the standard deviation of the indices derived from BIST30 and USDTRY are close to each other, kurtosis values are higher for the USDTRY indices. Higher kurtosis tells that ambiguity index derived from intraday USDTRY data has more extreme values and higher proportion on tails of distribution. This could be associated with higher trading volume in fx market and more aggressive movements during high stress periods compared to non-stress periods.

Next, in Table 2.2, we check the correlation between the ambiguity indices and monthly returns on BIST30 and USDTRY for both the current month and the one month ahead. Since indices do not start at the same date and our aim is to make comparison on the success of indices in capturing ambiguity, we choose the period between January 2006 and December 2017 as a common period for comparison. All ambiguity measures except for the VRP are highly correlated with historical volatility of the BIST30 Index. This evidence suggests that realized volatility is closely associated with ambiguity and they are not mutually exclusive.

We also investigate the relationship between the ambiguity indices and the stock market return for both the current month and the one month ahead. Correlations between historical volatility and stock index returns for both the current and next period are negative but the correlation is lower for the one month ahead. When we look at the correlation between the ambiguity measures versus the absolute stock index return and the excess stock index return, we see that the highest correlation is with the ambiguity measure based on the BIST30 intraday returns.

Next, consistent with the approach in literature, we test the argument that ambiguity is different from risk and it is an additional factor in explaining asset returns. For this purpose, we estimate the model presented in Equation (2.34) and we regress the one month ahead stock index excess return ($R_{M,t}$-$R_{f,t}$) on historical volatility ($historicalvolatility_{t-1}$) and ambiguity index ($ambiguity_{t-1}$). The ambiguity index is either one of the four indices calculated above. Since the indices do not start at the same date, we choose the period between January 2006 and December 2017 for analysis excluding VRP by BIST30 Index because it is available since June 2013. We also added implied volatility of BIST30 Index to make a comparison with a ready-made index directly obtainable from data vendors.

$$R_{M,t}-R_{f,t}= \alpha+\beta_1 ambiguity_{t-1}+\beta_2 historicalvolatility_{t-1}+\varepsilon_t \qquad (2.34)$$

The results of the analysis are presented in Table 2.3. Since the variables used in the regression model are not normally distributed as shown in Table 2.3, we estimate the equation by GMM with Newey-West standard errors. The regression results below show that the historical volatility has a negative and statistically significant effect on stock returns. We added alternative ambiguity indices but the ambiguity indices based on the BIST30 futures contracts and intraday prices are not significant while the indices based on implied volatility of exchange rate as well as the VRP are significant. When we relax the period restriction and estimate the regression for the ambiguity index based on BIST30 intraday data for the period from January 1998 to December 2017, the results in the last column of the table show that historical volatility loses its significance and the coefficient of the ambiguity index becomes statistically significant. The negative coefficient value is consistent with the results of Andreou et al. (2014). Although bearing ambiguity should lead investors to ask higher return in conventional wisdom, negative relationship could be explained by the argument that higher ambiguity increases hedging demand against consumption volatility and decreases required return. Considering the significance in explaining BIST30 Index return, longer historical coverage and consistency with the theoretical framework, we conclude that the index we derived by the methodology in Brenner and Izhakian (2018) represents well the ambiguity in Turkey.

2.6 Conclusion

Since ambiguity differs from risk due to its information content, ambiguity changes the rules both in decision-making and asset pricing. Epstein and Wang (1994) describe the difference between ambiguity and risk very clearly and propose that probability measures represent only the relative likelihoods of events but they do not represent the reliability of the information used for extracting those likelihoods. Complicated events in real life lead to decision-making under ambiguity, not just under risk.

Knight (1921) is the first study differentiating risk and uncertainty (ambiguity) from each other so uncertainty is usually called the “Knightian uncertainty” in the literature. Afterwards, Ellsberg (1961) provides an experimental study showing the impact of ambiguity in decision-making and providing evidence that in some cases agents do not behave in a way that is consistent with the Savage axioms. Many of the previous theoretical studies in the literature attempted to adjust the subjective expected utility model to incorporate ambiguity aversion in addition to risk aversion (Gilboa and Schmeidler (1989), Schmeidler (1989), Epstein and Wang (1994), Chen and Epstein (2002), Hansen and Sargent (2001) and Klibano, Marinacci and Mukerji (2005)).

As decision-making has been at the center of asset pricing models, modifications in decision-making models also have reflections on asset pricing models. Chen and Epstein (2002), Kogan and Wang (2003), Bansal and Yaron (2004), Boyle et al. (2009) and Anderson, Ghysels, and Juergens (2009) develop theoretical models for

Table 2.1: Descriptive Statistics of Ambiguity Indices and Returns on BIST30 Index, USDTRY and Risk Free Rate

This table summarizes descriptive statistics of ambiguity indices, implied volatility indices, volatility indices and return data on BIST30 Index, USDTRY and risk free rate.

	N	Mean	Median	St.Dev	Skewness	Kurtosis
Ambiguity Index from BIST30 Intraday Data	240	0.002	0.001	0.001	1.591	6.04
Ambiguity Index from BIST30 Futures Contracts	153	0.004	0.004	0.002	1.606	6.602
Ambiguity Index from BIST30 Index Implied Volatility	64	23.617	23.557	4.806	0.46	3.345
Ambiguity Index from VRP of BIST30	63	3.236	2.873	5.586	-0.009	3.056
BIST30 Historical Volatility	250	0.001	0	0.001	4.031	25.632
BIST30 Index Return	249	1.847	1.58	9.823	0.988	7.813
Risk Free Rate	246	0.021	0.012	0.02	1.739	5.732
Excess BIST30 Index Return	246	-0.002	0.002	0.1	0.406	7.217
Ambiguity Index from USDTRY Intraday Data	156	0.339	0.108	0.617	4.029	23.467
Ambiguity Index from USDTRY Futures Contracts	153	0.002	0.002	0.001	2.277	10.453
Ambiguity Index from USDTRY Implied Volatility	161	12.798	11.977	4.333	1.763	8.771
Ambiguity Index from VRP of USDTRY	162	-1.41	-0.476	4.792	-3.779	30.237
Ambiguity Index from Survey*	206	0.017	0.014	0.01	2.095	8.74
USDTRY Return	249	1.498	0.91	4.56	2.234	13.228

**1 month ahead USDTRY Expectation*

Table 2.2: Correlations Between Ambiguity Indices, Returns on BIST30 Index and USDTRY, and Risk Free Rate

This table presents pairwise correlations between monthly ambiguity indices, implied volatility indices, volatility indices, return data on BIST30 Index and USDTRY, and risk free rate.

Variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
(1) BIST30 Hist. Vol.	1															
(2) Ambiguity Index from BIST30 Intraday	0.75*	1														
(3) Ambiguity Index from BIST30 Futures	0.86*	0.67*	1													
(4) Ambiguity Index from VRP of BIST30	0.11	0.01	0.04	1												
(5) Ambiguity Index from BIST Imp. Vol.	0.41*	0.23*	0.48*	0.55*	1											
(6) Ambiguity Index from USDTRY Intraday	0.43*	0.39*	0.36*	-0.02	-0.06	1										
(7) Ambiguity Index from USDTRY Imp. Vol.	0.60*	0.42*	0.64*	0.39*	0.15	0.48*	1									
(8) Ambiguity Index from USDTRY Futures	0.63*	0.51*	0.69*	0.22	0.23*	0.61*	0.89*	1								
(9) Ambiguity Index from VRP of USDTRY	-0.55*	-0.33*	-0.40*	0.09	0.05	-0.28*	-0.41*	-0.38*	1							
(10) Ambiguity Index from Survey	0.48*	0.44*	0.48*	0.22	0.15	0.48*	0.69*	0.70*	-0.27*	1						
(11) Risk Free Rate	0.35*	0.28*	0.25*	0.04	-0.47*	0.31*	0.47*	0.38*	-0.27*	0.54*	1					
(12) BIST30 Return	-0.52*	-0.48*	-0.44*	-0.14	-0.14	-0.30*	-0.36*	-0.43*	0.28*	-0.27*	-0.19*	1				
(13) Excess BIST30 Return	-0.53*	-0.49*	-0.45*	-0.15	-0.13	-0.32*	-0.37*	-0.44*	0.29*	-0.29*	-0.23*	1.00*	1			
(14) USDTRY Return	0.45*	0.38*	0.42*	0.19	0.14	0.47*	0.45*	0.59*	-0.34*	0.29*	0.04	-0.68*	-0.68*	1		
(15) BIST30 Return (1 month ahead)	-0.23*	-0.18*	-0.19*	-0.24*	-0.27*	-0.18*	0	-0.13	0.04	-0.07	-0.12	0.23*	0.23*	-0.18*	1	
(16) Excess BIST30 Return (1 month ahead)	-0.24*	-0.19*	-0.20*	-0.25*	-0.25*	-0.19*	-0.02	-0.14*	0.05	-0.1	-0.17*	0.24*	0.24*	-0.18*	1.00*	1

** shows significance at the .1 level*

Table 2.3: Regression Estimations for One-month Ahead BIST30 Index Return Predictability

This table summarizes regression results in which monthly excess return on BIST30 Index is regressed over different lagged ambiguity indices in each column. In the first and second columns, regressions are run for the period of January 2006 – December 2017. Regressions in the third and fourth columns are run for July 2013-December 2017. In the fifth column, regression is run for the period of January 1998 – December 2017. In the equation below, $R_{M,t}$-$R_{f,t}$ is BIST30 Index excess return, $ambiguity_{t-1}$ is lagged ambiguity index and $historicalvolatility_{t-1}$ is lagged historical volatility which is calculated as the mean of squared daily returns of stock index within month. We estimate following equation by GMM with Newey-West standard errors.

$R_{M,t}-R_{f,t}= \alpha+\beta_1 ambiguity_{t-1}+\beta_2 historicalvolatility_{t-1}+\varepsilon_t$

	(1)	(2)	(3)	(4)	(5)
BIST30 Historical Volatility	-45.35***	-61.16*	-46.60***	-19.74*	5.831
	(-4.73)	(-2.32)	(-4.10)	(-2.01)	(0.79)
Ambiguity Index from BIST30 Intraday	-5.048				-13.69**
	(-1.12)				(-3.27)
Ambiguity Index from BIST30 Futures		0.911			
		(0.22)			
Ambiguity Index from VRP of BIST30 Index			-0.00183***		
			(-7.97)		
Ambiguity Index from BIST30 Imp. Vol.				-0.00220***	
				(-6.69)	
Intercept	0.0228**	0.0127	0.0150***	0.0555***	0.0183***
	(2.92)	(1.23)	(3.72)	(7.34)	(4.61)
N	144	144	54	54	240
R^2	6.4%	5.8%	7.5%	6.7%	2%

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

incorporating ambiguity into asset pricing, and separating risk and ambiguity from each other. Since the definition of ambiguity is ambiguous itself, measuring ambiguity is a difficult task and there are only a limited number of diverse approaches in measuring ambiguity (Andreou et al. (2014), Bollerslev, Tauchen, and Zhou (2009), Bekaert, Engstrom, and Xing (2009) and Brenner and Izhakian (2018)). Studies by Olsen and Troughton (2000), Zhang (2006), Epstein and Schneider (2008), Epstein and Schneider (2010), Illeditsch (2011) and Ozsoylev and Werner (2009) contribute to the literature by providing empirical evidence on the impact of ambiguity on asset returns.

This study constructs an ambiguity index for Turkey. For this purpose, first, we provide a comprehensive and detailed review of the impact of ambiguity on both the theory of decision-making and its repercussions on the asset pricing theory along with a comparative analysis with the conventional asset pricing models. Although there are separate studies reviewing theoretical models and empirical studies on the relationship between ambiguity and decision-making and ambiguity and asset pricing, this study contributes to the literature by compiling a complete review on the development of the relationship between ambiguity and asset pricing. Afterwards, we introduce alternative ambiguity indices for Turkey using the methodologies existing in the literature and that are consistent with the theoretical framework. Comparison of alternative ambiguity indices suggests that indices provide similar information but they do not have similar explanatory power for stock returns. Analyses show that the intraday price based index calculated using the Brenner and Izhakian (2018) methodology is the most appropriate index to use in asset pricing tests. This ambiguity index for Turkey (i) satisfies the definition of ambiguity put forward in the literature, (ii) has the longest possible historical coverage, and, (iii) is relevant for studying the impact of ambiguity on asset pricing. This index is a first for Turkey considering the definition of ambiguity in the literature and one of first among a limited number of studies on emerging markets. The initial results confirm that ambiguity affects excess returns in a negative manner.

CHAPTER 3

THE IMPACT OF AMBIGUITY ON CROSS-SECTIONAL STOCK RETURN IN TURKEY

In finance literature, the risk-return relationship has been discussed extensively. The Capital Asset Pricing Model (CAPM) proposed by Sharpe (1964), Lintner (1965) and Mossin (1966) represented an evolutionary change in finance theory and introduced a solid theoretical framework for defining the risk-return relationship for an individual asset while also providing a benchmark for asset pricing models that are to be developed in later years. Empirical testing over the years showed that the restrictive assumptions of the model sometimes contradict with the actual asset pricing observed in the market. Subsequent studies aimed to relax some of these restrictive assumptions in order to reach a model setup that is closer to real life. For instance, Black's (1972) two-factor model, Merton's (1973) multi-period CAPM (ICAPM), Ross's (1976) Arbitrage Pricing Theory (APT), Breeden's (1979) consumption CAPM are theoretical models that were proposed by relaxing one or more of the assumptions with the purpose of improving model validity.

On the empirical side, studies testing the validity of CAPM use either a cross-sectional or a time series framework. In earlier tests, Basu (1977), Reinganum (1981), Banz (1981) and Basu (1983) show that market beta is not adequate to explain the cross-sectional stock returns and variables like earnings to price (E/P) ratio and size have additional explanatory power. In addition, findings by Black, Jensen, and Scholes (1972), Fama and Macbeth (1973) and Fama and French (1992, 1993) do not verify the empirical implications of CAPM. In light of these studies, Fama and French (1996) attempted to bring together the existing empirical evidence by introducing the concept of "asset pricing factors" and tested a version of the

CAPM where the market, size (SMB) and value (HML) factors are shown to be the three risk factors that seem to be priced by investors in the market.

As part of the attempts to improve CAPM's validity, the behavioral finance literature interpreted the empirical results of CAPM tests in a different light. More specifically, studies by De Bondt and Thaler (1985) and Jegadeesh and Titman (1993) argue that the observed return reversals or momentum are a result of behavioral biases and not necessarily an indication of the lack of empirical support for the CAPM. Following the Fama-French three-factor model and the De Bondt and Thaler and Jegadeesh and Titman arguments, several new asset pricing factors were proposed in the literature. Some examples of these factors are Carhart's (1997) momentum, Amihud (2002), Pastor and Stambaugh (2003) and Acharya and Pedersen's (2005) liquidity, Harvey and Siddique's (2000) coskewness, Xing (2008), Hou, Xue, and Zhang (2015) and Fama and French's (2015) investment and profitability27. Thus, these models focus on identifying factors that represent the risks priced in the market but not captured by the market beta from the original CAPM.

Another strand of literature investigates CAPM's single probability distribution assumption in decision-making and its implications for the definition of risk in asset pricing. In the original version of the CAPM, perfect information and homogenous expectations imply a unique probability distribution since any uncertainty over the return distribution is removed. Such a setting allows the risk-return relationship to be established as well. However, experimental evidences show that such a single probability distribution assumption contradicts with the characteristics of actual decision-making. As a matter of fact, the observed vagueness over the return distribution may turn out to be another factor that is priced in the market. The literature on the topic refers to this vagueness as "uncertainty" or "ambiguity" and argues that it has the potential to affect investment and consumption decisions.

[27] For a complete review of the literature on asset pricing factors, please see Harvey, Liu, and Zhu (2016).

The literature on the subject mainly focuses on integrating ambiguity into the decision-making and asset pricing models. Contrary to the increasing number of theoretical models (Chen and Epstein (2002), Kogan and Wang (2003), Bansal and Yaron (2004), Boyle et al. (2009) and Anderson, Ghysels, and Juergens (2009)), empirical studies testing the impact of ambiguity on asset pricing have remained limited because there is no consensus on the definition and measure of ambiguity (Bollerslev, Tauchen, and Zhou (2009), Bekaert, Engstrom, and Xing (2009), Andreou et al. (2014), and Brenner and Izhakian (2018)). As a contribution to the empirical literature, this study tests whether adding a sensitivity to ambiguity measure to the asset pricing model increases the model's explanatory capacity for the cross-section of stock returns in Turkey. From this perspective, this study is the first to test the relationship for the Turkish stock market and one of the very few ones that are conducted on emerging markets. Since the literature shows risk factors other than ambiguity to have a significant effect on stock returns, ambiguity is added to the asset pricing models alongside these other factors. This way, in addition to the tests of ambiguity as a risk factor in asset pricing, the study also provides up-to-date evidence on the significance of other risk factors for the Turkish stock market.

In this context, in the first part of the study, a detailed review of asset pricing models, testing methodologies and a review of risk factors are presented. In the second part, these risk factors are calculated and a detailed analysis on the relationship between the factors and stock returns is provided. Subsequently, ambiguity is incorporated into the analysis and tested as an additional risk factor to find out whether it is priced in the Turkish stock market.

3.1 Asset Pricing

3.1.1 Introduction

The asset pricing theory focuses on explaining the relationship between asset returns and risks that are considered to be basis for the returns generated. Since there are diverse factors in asset pricing due to complexity of the investors' decision-making process, to this day, modelling investor decisions and valuing assets remains one complicated puzzle. Despite its difficulties, finance theory continuously seeks to solve this puzzle and enhances the existing theories in a way to develop a theory that closely tracks the return-generating process actually observed in the market. For the generic asset pricing argument, economic agents are assumed to decide about purchasing an asset based on the savings versus investment tradeoff and this decision process is simplified as the process of giving up consumption today in exchange for earning returns and increasing consumption in the future. The decision to give up today's consumption is based on whether the expected return from the asset is larger than or equal to the investor's required return that is comprised of premiums for risks such as the time value of money, intertemporal consumption preference, risk aversion as well as the future payoff of an asset.

Although the Markowitz Portfolio Theory and the CAPM are the two famous theories that first come to mind, the roots of asset pricing go back as early as Bernoulli's (1738) famous study on expected utility. Bernoulli objected to the idea that agents consider mathematical expectations in decision-making and argued that wealth is a function of utility. Utility has distinct futures compared to mathematical expectations in that it increases at a decreasing rate with higher amounts of wealth. This is known as the diminishing marginal utility and has been instrumental in explaining the tradeoff between expected wealth and risk. This insight, along with the introduction of expected utility theory by von Neumann Morgenstern (1944) and subjective expected utility theory by Savage (1954), has played a principal role in modeling decisions made under risk. The Portfolio Theory by Markowitz (1952) and CAPM by Sharpe (1964), Lintner (1965) and Mossin (1966) followed in the footsteps of these earlier studies and modeled the complex pricing behavior in financial markets.

Markowitz's (1952) portfolio theory also known as the mean-variance theory has been the benchmark in finance theory for the tradeoff between return and risk. In the portfolio selection decision, agents are assumed to be risk averse and to make decisions at current time for the expected return of the next period by taking into account the tradeoff between risk and return. Accordingly, agents agree to higher investment risks only in exchange for higher returns or prefer to lower their risk by accepting lower returns. In other words, agents attempt to make a selection from among a set of efficient portfolios (the efficient frontier) by either i) choosing the portfolio with the minimum variance of return for a given expected return, or, ii) choosing the portfolio with the maximum return for a given level of variance. Another important contribution of the Markowitz (1952) study is to show how portfolio diversification allows the investors to achieve a lower variability of return without having to sacrifice from the portfolio’s expected return.

Later, Arrow and Debreu (1954) support the benefits of diversification in reducing uncertainty by developing the idea of complete markets where investors are able to eliminate uncertainty and insure themselves against loss. Tobin (1958) also extends the Markowitz results by introducing risk-free lending into the investment decisions. According to the Markowitz model, agents make their mean-variance efficient portfolio decisions based on either a quadratic utility function or asset returns with a normal distribution. With Tobin’s introduction of risk-free lending, Markowitz’s efficient frontier is transformed into a straight line that is drawn as a tangent between the risk-free rate of return and the original efficient frontier. Tobin’s study has two important implications for asset pricing: i) the investor’s risk aversion level determines the allocation of the budget between cash and non-cash assets, and, ii) the composition of non-cash assets is independent from the investor’s risk attitude and each investor ends up holding the market portfolio consisting of all assets. Hence, the investor’s degree of risk aversion is independent from the share of a particular security in the market portfolio and this is known as Tobin’s Separation Theorem. Later, Sharpe’s (1964) study provided support for the separation theorem by introducing risk-free borrowing in addition to risk-free lending and the relationship that is established between return and risk for individual assets in his study came to be known as the Capital Asset Pricing Model (CAPM).

3.1.2 Capital Asset Pricing Model (CAPM)

Initially, three studies following each other Sharpe (1964), Lintner (1965) and Mossin (1966) developed CAPM as an equilibrium model to establish a framework for the relationship between risk and return. The risk definition in CAPM diverges from earlier risk definitions of return volatility. In the CAPM framework, risk is defined as the contribution of an individual asset to the riskiness of a portfolio. Similar to Tobin (1958), the CAPM also assumes that economic agents allocate their budget between the risk-free asset and the market portfolio where the market portfolio is unique and the same for all players in the market. In this framework, an investor's risk attitude is revealed through the share of total wealth that is invested in the risk-free asset where more risk averse investors are expected to invest a larger share of their portfolio in the risk-free asset.

In Equation (3.1), CAPM establishes a linear and positive relationship between the expected return of a stock ($E(R_i)$) and the market risk (β_i). Market risk measures the sensitivity of the stock's return to the excess market return which is defined as the difference between the market portfolio's total return $E(R_M)$ and the risk-free rate (R_f). There are two implications of the model: i) the excess return of a stock should be proportional to its market beta, and, ii) market beta is the only factor that explains the differences among expected stock returns. The intercept term in the model represents the risk-free rate that is assumed to be uncorrelated with the market portfolio return. The linear relationship between asset returns and the market beta is also called the Security Market Line.

$$E(R_i) = R_f + \beta_i(E(R_M)\text{-}R_f) \tag{3.1}$$

The market beta in the model represents the systematic or non-diversifiable risk of the asset's return. The other component of an asset's total risk is unsystematic (idiosyncratic) and the investors is able to get rid of this risk by holding well-diversified portfolios. As a result, economic agents do not have to worry about an asset's total return volatility but only consider the systematic portion of risk. Hence, risk averse and rational investors ask for higher compensation (risk premium) for higher levels of systematic risk (market beta) and always strive to hold well-diversified portfolios. The implied positive relationship between expected asset return and systematic risk leads to an upward-sloping security market line.

Although CAPM has a solid theoretical background, its restrictive assumptions are criticized extensively in the literature. The assumptions of the model are i) agents are risk-averse and aim to maximize the expected utility of wealth, ii) agents are price takers and have homogeneous expectations on asset returns, iii) asset returns are normally distributed, iv) agents can borrow and lend as much as needed at the risk-free rate, v) all assets have a fixed quantity and are marketable and perfectly divisible, vi) markets are frictionless, and, vii) there are no market imperfections (Copeland, Weston, and Shastri (2005)). In addition to its strong assumptions, CAPM's validity is questioned in light of the failure to find empirical support and the difficulty of testing several of its testable implications.

There are several studies that relax one or more of the CAPM assumptions. For instance, Black (1972) modifies CAPM by removing the risk-free borrowing and lending assumption and augments the model by replacing the risk-free asset with a zero beta portfolio, and unlimited borrowing at the risk-free rate with unlimited short sales. The zero beta portfolio is a portfolio that has zero covariance with the market portfolio and it is on the efficient frontier so the portfolio is also called as the minimum-variance zero-beta portfolio. In Black's study, the CAPM equation is rewritten as in Equation (3.2) that the expected return of an asset is modeled as a linear combination of the market portfolio return ($E(R_M)$) and the minimum-variance zero-beta portfolio ($E(R_z)$) return. Since the expected stock return is defined as a function of the zero-beta portfolio and market portfolio returns, Black (1972) describes the model as a two-factor model.

$$E(R_i)=E(R_z)+\beta_i[E(R_M)-E(R_z)] \tag{3.2}$$

3.1.3 Extensions to CAPM

As stated in the previous section, although CAPM is a powerful model to explain the dynamics in asset pricing, the assumptions of the model are restrictive so there are problems in matching theory with reality. Furthermore, empirical tests of CAPM portrayed the weaknesses and deficiencies of the model and motivated researchers to develop alternative models and modify CAPM in a way to be more consistent with asset pricing dynamics. The initial steps have been to ease the model assumptions that were deemed unrealistic. In this regard, the modification of CAPM by Black (1972) could be considered as a first step in extending the risk factors included in the CAPM from a single market factor to the two factors of market portfolio and minimum-variance zero-beta portfolio. Following models ease some of other assumptions.

3.1.3.1 Intertemporal Capital Asset Pricing Model (ICAPM)

Merton (1973) relaxed the one-period decision-making assumption in CAPM and modified it by extending into a multi-period setup and adding one more factor to the model in addition to market factor. Merton criticizes CAPM's assumption of non-stochastic investment alternatives and argues that interest rates as a component in the investment set are stochastic. Stochastic investment opportunity contradicts with the implications of CAPM that a single market beta is enough to capture the risk of an asset. Hence, the model developed by Merton takes into account the intertemporal change in investment opportunities and it is called the Intertemporal CAPM (ICAPM). According to ICAPM, agents consider both current period and end of period investment opportunities so agents are not concerned only for the return variability but also they consider the variability of future investment opportunities in order to smooth the intertemporal consumption.

Merton (1973) obtains a three-fund separation in which an agent separates the investment decision into two parts by forming three mutual funds: i) n risky assets, ii) n^{th} asset with the highest correlation with the changing state variable, and, iii) a risk-free asset. In this framework, Merton derives the excess return on the market (α_M-r) as the sum of two terms presented in Equation (3.3). The first term is the proportional impact of the variability of the market return ($M/A\,\sigma_M^2$)[28] and the second term is the risk of shift in investment opportunity set ($[Hg/A\,\sigma_n]/\sigma_{M,n}$). The individual asset return is derived in a similar vein and Equation (3.4) suggests that the excess return for an individual asset is a function of its market beta nd the excess market return -as in conventional CAPM- but also it is a function of the excess return on the state and the risk of unfavorable shifts in the investment opportunity set ($\sigma_{i,n}$). In this setup, the uncertainty related to multiple sources, such as future consumption and investment, is priced in the asset's return. As shifts in the future opportunity set is not diversifiable and associated with future states, agents ask for extra return as a compensation for this risk.

$$\alpha_M\text{-}r=\frac{M}{A}\sigma_M^2+\frac{Hg}{A\sigma_n}\sigma_{M,n} \tag{3.3}$$

$$\alpha_i\text{-}r=\frac{M}{A}\sigma_{i,M}+\frac{Hg}{A\sigma_n}\sigma_{i,n} \tag{3.4}$$

3.1.3.2 Arbitrage Pricing Theory (APT)

Ross (1976) developed an alternative model with fewer assumptions compared to CAPM but in a similar spirit that an asset's return is explained by non-diversifiable systematic risks in the economy. Non-diversifiable systematic risks are the covariances of common risk factors with stock returns and constitute the determinants of the average returns similar to the risk of unexpected shifts in the investment set in ICAPM. APT is based on an arbitrage relationship and differs from multi-factor asset pricing models in that it has no equilibrium condition. Also,

[28] In the ICAPM framework, M is the equilibrium value of all assets, A is a function of the agent's absolute risk aversion, H represents the demand for the asset to hedge against unfavorable shifts in the investment opportunity set, g is the standard deviation of the change in the interest rate r, and, n is the n^{th} asset with a correlation of -1 with the interest rate.

APT drops the assumptions of a quadratic utility function and the normality of return distributions. Instead, APT replaces the assumption that agents make investment decisions based on the mean-variance framework with the assumption of the linear return-generating function presented in Equation (3.5). In this context, the role of a mean-variance efficient portfolio in CAPM is replaced with an arbitrage portfolio in APT. APT also makes some additional assumptions: i) perfectly competitive and frictionless capital markets, ii) homogeneous beliefs among agents, iii) number of assets being larger than the number of risk factors priced in the market, and, iv) unsystematic risk being uncorrelated with the systematic risk factors.

$$R_i=E(R_i)+b_{i1}F_1+...+b_{ik}F_k+\varepsilon_i \tag{3.5}$$

According to the APT formulation in Equation (3.5), an asset's return is defined as a function of its expected return and risk where risk is associated with two groups of factors: (i) common factors for all stocks (systematic risk, F1…Fk), and, (ii) factors specific for individual stocks (unsystematic risk, ε). Common factors could be anything related with the economy and they are assumed to have a zero mean and a finite variance. The model build-up starts with forming arbitrage portfolios as weighted sums of individual asset generating functions as in Equation (3.6). Arbitrage portfolios do not lead to a change in the investor's wealth and risk so arbitrage portfolios are formed on buying assets in exchange for selling other assets such that the sum of portfolio weights for each share should be equal to zero ($\sum_{i=1}^{n} w_i = 0$). Arbitrage portfolios are riskless implying that both systematic and unsystematic risks are eliminated. Unsystematic risk is diversified away and systematic risk is eliminated by choosing the weights such that the sum of weighted risk sensitivities is equal to zero for each factor ($\sum_i w_i b_{ik}=0 \ \forall\ k$).

$$R_p= \sum_{i=1}^{n} w_i R_i = \sum_i w_i E(R_i) + \sum_i w_i b_{i1} F_1 +...+ \sum_i w_i b_{ik} F_k + \sum_i w_i \varepsilon_i \tag{3.6}$$

Assuming that error terms in individual asset return generating functions are not correlated and the number of assets are large enough, $\sum_i w_i \varepsilon_i$ converges to zero. In addition, weights were selected in a way to eliminate all systematic risk and arbitrage portfolio become weighted average of expected values of assets and the equation doesn't include any randomness. Since arbitrage portfolio doesn't own any risk, it doesn't generate return and the return on arbitrage portfolio equals to zero ($R_p = \sum_{i=1}^{n} w_i E(R_i) = 0$). Three orthogonality conditions ($\sum_i w_i b_{ik} = 0$, $\sum_i w_i = 0$, $\sum_{i=1}^{n} w_i E(R_i) = 0$) bring the algebraic consequence that expected returns could be defined as a linear combination of constants and coefficients. In Equation (3.7), λ_0 equals to risk free rate, λ_k is risk premium for factor k and b_{ik} represents the sensitivity of asset i to the risk factor k.

$$E(R_i) = \lambda_0 + \lambda_1 b_{i1} + \ldots + \lambda_k b_{ik} \tag{3.7}$$

APT is based on arbitrage condition that agents take position in the asset until the price changes and arbitrage opportunities disappear. Since arbitrage portfolios could be formed on limited number of any asset under the assumption of no-arbitrage condition, APT drops the role of market portfolio in deriving asset pricing formula. This makes APT free of the criticisms on testability of CAPM because to form an arbitrage portfolio it is not necessary to identify all assets and incorporate into empirical study. In this regard, APT was argued to be testable with a subset of market portfolio. In this regard, empirical tests of CAPM by incorporating only stock market index could be considered as single factor APT tests. Although APT has advantages with less restrictive assumptions and being testability, it has some drawbacks. One is that the factors in asset pricing could vary from portfolio to portfolio and a valid model for a particular portfolio may not be valid for another set of assets, that is, APT formulates asset pricing model for only interested assets and it doesn't comprise all asset universe. This makes the assessment and testability of APT questionable.

Roll and Ross (1980) tested APT model. In order to make ex ante APT model testable with ex post asset returns, they introduced the assumption of homogeneity of expectations. Accordingly, agents agree on expected returns and factor coefficients. The results show that there are four factors explaining the stock return distribution in US stock market. In order to determine the number of factors from individual stock returns, they used factor analysis having both advantages and disadvantages. The main advantage of the model is that the factors are determined by data itself but the main drawback of the methodology is that the factors may not have a meaningful economic interpretation. However, associating risk factors with economic variables is important to make factors to be more understandable and meaningful. Chen, Roll, and Ross (1986) suggested to use macroeconomic variables having direct relationship with the main determinants of asset prices of future cash flow and discount rate. They concluded five factors having statistically significant relationship with US stock return. The list includes change in industrial production, the yield spread between short-term and long-term government bonds, the yield spread between low and high grade bonds, changes in expected and unexpected inflation. Interestingly, market index has no statistically meaningful impact on stock returns.

APT was argued to overcome some of the theoretical and empirical shortcomings of CAPM but APT is not free of debates. Shanken (1982) disagreed with the assumptions and implications of the model. Shanken (1982) argued that not rejecting APT doesn't infers rejection of another model, that is, CAPM. Furthermore, Shanken[29] showed that a return structure could be described by different factor structures. This raises the uncertainty on correct factor structure similar to uncertainty about detecting market portfolio in CAPM criticized by Roll (1977).

[29] Dybvig and Ross (1985) criticize Shanken (1982) for contradicting with the assumption of the bounded variance in making wrong inference on APT.

3.1.3.3 Consumption-Based Capital Asset Pricing Model (CCAPM)

Breeden (1979) introduced consumption CAPM (CCAPM) in a continuous time setting similar to intertemporal structure of ICAPM by Merton (1973) but Breeden criticizes the lack of empirical testability of ICAPM that the model doesn't define state variables. CCAPM resembles CAPM in that single factor explains asset return but excess return in CCAPM is proportional to beta with respect to aggregate consumption compared to excess market return in CAPM. In CCAPM, decision on utility encompasses not only current utility but also future utility of consumption and multi-year representation of utility assumes time separable utility of consumption. In this direction, total utility is mathematically formulated as sum of utility from current consumption and discounted utility from future consumption which is random because it is a function of future wealth. In this framework, CCAPM differs from portfolio theory and classical CAPM model in that the utility is directly related with consumption level whereas, in portfolio theory and CAPM, utility is defined as a function of the mean and variance of portfolio return.

A two period model setup in Equations (3.8) and (3.9) shows that total utility is the sum of utility from current consumption (C_t) and discounted expected utility from future consumption (C_{t+1}). The model captures the intertemporal substitution and risk aversion by discounting future consumption utility by β called also as the subjective discount factor. The utility function depends on consumption level and it is defined as power utility function[30] where γ is parameter of risk aversion.

$$U(C_t, C_{t+1})=u(C_t)+\beta E_t[u(C_{t+1})] \tag{3.8}$$

$$u(C_t)=\frac{1}{1-\gamma}C_t^{1-\gamma} \tag{3.9}$$

[30] Power utility function defines the utility as an increasing function of consumption and it is also concave. The functional form of power utility implies increasing utility at a decelerating rate with higher consumption.

Objective of the agent is maximizing total utility subject to current and future consumption where current (C_t) and future (C_{t+1}) consumption depends on initial endowments (e) and future payoff of investment (x_{t+1}). In this regard, today consumption is defined as the difference between current endowment (e_t) and investment as a function of asset price (p_t) and quantity of investment (ξ). Maximizing consumption subject to current and future consumption as function of endowments, investment quantity and asset prices is given in the optimization problem in (3.10). Solving the problem gives the fundamental asset pricing formula in Equation (3.11) Accordingly, the decision on consumption determines asset prices and asset pricing formulation suggests that price of an asset is defined as a function of expected payoff (x_{t+1}) and expected stochastic discount factor ($\beta U'(C_{t+1})/U'(C_t)$), which is also called marginal rate of substitution, that agent prefer to substitute future consumption for current consumption. Intuitively, the pricing formula could be rephrased as marginal utility of consumption loss to purchase the asset at a price of p_t equals discounted expected marginal utility of consumption at an amount of future payoff.

$$\max_{\xi} u(C_t)+\beta E_t[u(C_{t+1})] \text{ s.t.} \tag{3.10}$$

$$C_t=e_t-p_t\xi$$

$$C_{t+1}=e_{t+1}+x_{t+1}\xi$$

$$p_t=E_t\left[\beta \frac{U'(C_{t+1})}{U'(C_t)} x_{t+1}\right] \tag{3.11}$$

After making additional assumptions and functional transformations such that stochastic discount factor and asset returns are assumed to be lognormally distributed, and returns and consumption are transformed into logarithmic form as $r_{i,t}=\log(1+R_{i,t})$ and $c_t=\log(C_t)$, price equation is redesigned in a way to define expected excess return of an individual asset. Accordingly, expected excess return of an asset $\left(E_t\left(r_{i,t+1}\text{-}r_{f,t+1}\right)\right)$ is proportional to covariance between consumption growth and asset return as shown in Equation (3.12) and the proportion depends on the level of relative risk aversion (γ). Since agents prefer a smooth consumption stream over time and across states, and positive covariance between return and consumption growth implies more volatile consumption, agents do not prefer positive covariance between consumption and asset return. Therefore, they ask for higher return to hold assets generating return covarying positively with consumption. In comparison with portfolio theory and CAPM, this infers that extra return of an asset is not determined by the covariance of the return with market return, rather, it is a function of covariance with consumption growth.

$$E_t\left(r_{i,t+1}\text{-}r_{f,t+1}\right)\approx\gamma\times Cov_t\left(r_{i,t+1},\ \frac{c_{t+1}}{c_t}\right) \tag{3.12}$$

3.1.4 Empirical Anomalies

In asset pricing literature, an anomaly refers to a pattern in stock returns that cannot be explained by the predictions of CAPM known as traditional or naïve model. The return patterns have been explored empirically in general by portfolio approach as in Equation (3.13). Accordingly, first, stocks are sorted and allocated into portfolios by stock characteristic and these characteristics are either related with stock financials or price scaled financials even there is no theoretical model extracting these variables. If these generic portfolios create return higher than market risk adjusted return evidenced by statistically significant intercept term (α_p), CAPM is considered to be inadequate to explain return differential among stocks and statistical support to the relationship between return and market beta is considered to be weak.

$$R_{pt}\text{-}\ R_{ft}=\alpha_p+\beta_p(R_{Mt}\text{-}\ R_{ft})+\varepsilon_{pt} \tag{3.13}$$

Following the discussions and extensions on CAPM, literature tested and suggested a number of anomalies. Basu (1977) provides the evidence of E/P ratio as a factor in explaining cross-sectional return differential. Accordingly, Basu ranked stocks based on E/P ratio and formed five portfolios, and portfolios containing higher E/P ratio generate higher risk adjusted return. Banz (1981) points out another anomaly and concludes that there is a negative relationship between market value and return of a stock. However, Banz shows that statistical significance of size effect decreases throughout time similar to market beta as in Black, Jensen, and Scholes (1972).

In a similar way, Reinganum (1981) reveals results contradicting with CAPM. Portfolios formed on size and E/P ratios generate return over market risk adjusted return. Contrary to Basu (1977), size effect is the dominant one and E/P effect lost its significance after controlling for size. Basu (1983) extended previous study by including E/P ratio and the market value in testing CAPM. In forming portfolios, portfolios having different E/P ratios are designed in a way to have similar market values and, in a similar fashion, market value portfolios are constructed in a way to differentiate market values among portfolios but keep similar E/P within portfolios. Afterwards, risk return relationship is tested in a multivariate setting. The results contradict with the implications of CAPM and confirm the intercept terms of portfolios formed on E/P are different from zero and high E/P stocks generate higher risk adjusted return compared to lower ones. In addition, size effect lost its significance after controlling for E/P.

Alternatively, Bhandari (1988) test debt to equity ratio as another factor having explanatory power on asset returns besides market beta and size. Cross-sectional regressions on sorted portfolios propose that market beta is insignificant but size has negative and leverage has positive affect on average returns. Anomalies on asset pricing are not limited to size, E/P and leverage and the list is long enough that it is out of the scope of this study. On the other hand, there are many review studies on factors of asset pricing in the literature, for example, Harvey, Liu, and Zhu (2016) and Hou, Xue, and Zhang (2017) provide a recent and complete review.

Fama and French (2004) summarize lack of empirical support to CAPM by multiple dimension of its theoretical basis, assumptions and empirical implementation difficulties. There are a couple of explanations about anomalies: i) investors are rational and they make decision based on more than one factors, ii) consistent with the arguments of behaviorist that irrational investment behavior of investors leads to return anomalies, iii) spurious relations stemmed mainly from the factors such as survivorship bias, data snooping, poor proxies for market portfolio.

3.1.5 CAPM Testing

Theoretical modifications, empirical failure and restrictive assumptions of CAPM motivate testing CAPM validity. The studies testing the validity of CAPM concentrate on two main approaches of cross-sectional and time series tests. In cross-sectional tests, the significance of market beta is tested after market beta for each stock is estimated from time series regressions. This methodology of combining both time and cross-sectional regressions is also called two pass methodology. Time series tests investigates whether market beta is statistically significant and explains the variation of asset return throughout time.

Before going into details of cross-sectional and time series tests, there are a couple of points that should be mentioned to clarify the methodology of CAPM testing. Since CAPM establishes the relationship between expected return and market beta, testability of the model is only possible by observing expected return. However, expected returns are not available and empirical studies overcome this problem by substituting expected return with realized return to make CAPM testable. The substitution is based on the assumption that realized return is an unbiased estimator of expected return. In other words, expected return is reformulated into fair game form as in Equation (3.14) in which R_i is ex post stock return and δ_M is excess market return (R_M-$E(R_M)$). Substituting $E(R_i)$ with CAPM formulation turns CAPM in ex-ante form of Equation (3.14) into testable CAPM in ex-post form (3.15). Meanwhile, this makes applicability and testability of CAPM to be linked to additional assumptions beside model assumptions.

$$R_i = E(R_i) + \beta_i \delta_M + \varepsilon_i \tag{3.14}$$

where

$$\delta_M = R_M - E(R_M)$$

$$R_i = R_f + [E(R_M) - R_f]\beta_i + [R_M - E(R_M)]\beta_i + \varepsilon_i$$

$$R_i = R_f + [R_M - R_f]\beta_i + \varepsilon_i \tag{3.15}$$

In testing CAPM with real data, literature has generally utilize returns on stock portfolios rather than individual stock returns even CAPM is a theoretical model on individual assets. The main problem in utilizing individual market betas as risk factor in testing CAPM is that betas are not stable throughout time and the estimates contain error in variables problem[31]. High variability of market betas infers low statistical power and may lead to make wrong inference. Furthermore, portfolio formation reduces the computational complexities related with high number of stocks. Although forming portfolios alleviates the problems of instability and measurement error, it moderates the dispersion of return and questioned the statistical power of the results for individual stocks.

3.1.5.1 Cross-Sectional Testing

Black, Jensen, and Scholes (1972) utilize two-pass procedure in testing CAPM for US data. First, market beta of each stock is estimated from time series regression of stock return over market premium for 4 years. Next, stocks are ranked according to market betas and 10 portfolios are formed. For each of these portfolios, average excess return is calculated for next 12 months. Afterwards, portfolio betas are estimated from the regression of the average excess return of portfolios over excess

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[31] Roll (1981) argues that infrequent trading is one factor of error in variables problem.

market return. Finally, average return for the whole period for each portfolio is plotted with estimated betas. This exercise aims to test two things: i) intercept term equals to risk free rate, ii) market premium is positive. Cross-sectional regression results challenge with CAPM in a way that intercept term doesn't equal to zero and coefficient of estimated market beta is smaller than historical market premium. Therefore, the results suggest that portfolios with higher beta have lower return and portfolios with lower beta generate higher return compared to the theoretical formulation of CAPM.

In testing CAPM, Fama and MacBeth (1973) modify the methodology of Black, Jensen, and Scholes (1972) and it differs by running cross-sectional regression for each month rather than for a single estimate at the end of the sample period. The steps could be ordered in the following way: i) market betas are estimated for the last 4 years from the regression of excess return of individual stocks over excess market return and stocks are allocated into 20 portfolios according to this sorting, ii) market betas for each stock are estimated again for the next 5 years on a rolling windows, iii) at the end of 5-year period, market betas and average return for each month are calculated for next 4 years based on past 5 years rolling regressions for each portfolio formed according to the market beta from the initial 4-year period, iv) average excess return of portfolios are regressed cross-sectionally over estimated betas in each month, v) in the final step, intercept terms and beta coefficients are collected for each month from cross-sectional regressions. The statistical significance of the coefficient of market beta is tested by using the mean and the standard deviation of coefficients from monthly cross-sectional regressions conducted throughout the sample period. This methodology is instrumental in allowing betas to be time varying and taking into account cross-sectional correlation that could distort making statistical inference in time series regressions.

Fama and MacBeth (1973) tested the validity of CAPM for US in different dimensions and the empirical findings confirm CAPM in great extent but not fully. The regression in Equation (3.16) represents the functional form of CAPM testing. In the equation, $R_{p,t}$ is return of portfolio p at time t, $\hat{\beta}_{p,t-1}$ is the average portfolio beta derived from time series regression of excess portfolio return on excess market return in the last 5 years. $\hat{\beta}^2_{p,t-1}$ is the square of market beta and $\bar{s}_{p,t-1}(\hat{\epsilon}_i)$ is the average of idiosyncratic risk of stocks in the portfolio. The significance of the coefficients ($\hat{\gamma}_{it}$ for i=1,2,3) is tested by t statistics which was calculated as the ratio of mean of coefficient to standard deviation of coefficient during the sample period as given in Equation (3.17).

$$R_{p,t}=\hat{\gamma}_{0t}+\hat{\gamma}_{1t}\hat{\beta}_{p,t-1}+\hat{\gamma}_{2t}\hat{\beta}^2_{p,t-1}+\hat{\gamma}_{3t}\bar{s}_{p,t-1}(\hat{\epsilon}_i)+\hat{\eta}_{p,t} \tag{3.16}$$

$$t\left(\bar{\hat{\gamma}}_j\right)=\frac{\bar{\hat{\gamma}}_j}{s(\hat{\gamma}_j)/\sqrt{n}} \tag{3.17}$$

The hypothesis of tests could be summarized as follows:

1. First, they test the linearity of risk-return relationship by adding the square of market beta ($\hat{\beta}^2_{p,t-1}$). The coefficient of the square of market beta ($\hat{\gamma}_{2t}$) is not statistically significant so the linear relationship between return and market beta could not be rejected.
2. Second, the standard deviation of the idiosyncratic residuals ($\bar{s}_{p,t-1}(\hat{\epsilon}_i)$) is included to test whether market beta is the single factor capturing risk. The coefficient of idiosyncratic risk is not statistically different from zero consistent with the hypothesis that market beta is the sole measure of risk.
3. Third, they test whether market premium is positive ($\hat{\gamma}_{1t}$) that is also the test of positive relationship between return and market beta. The results confirms positive market premium but the estimated market risk premium ($\hat{\gamma}_{1t}$) is lower than the historical average market risk premium (R_M-R_f).
4. Lastly, equality of intercept term and risk free rate ($E(\hat{\gamma}_{0t})=R_f$)) is tested and the test result suggests that intercept term is statistically different from average risk free rate.

In sum, the results are mostly consistent with CAPM in that agents ask for higher return against higher systematic risk and market portfolio is on the minimum variance frontier. Although portfolio formation in two-pass methodology by Black, Jensen, and Scholes (1972) and Fama and MacBeth (1973) aim to lessen the impact of error in variables problem in estimating betas, the methodology still suffers from and not free from error in variables problem. Gibbons (1982) introduced an alternative model to alleviate the errors in variables problem and proposes maximum likelihood estimation to estimate beta and the risk premium at the same time without a need of sequential estimation of market beta and market risk premium. However, Gibbons' results contradict with Fama and MacBeth (1973) and reject the implications of CAPM.

Fama and French (1992) is another key and comprehensive study presenting empirical evidence on the failures of CAPM by using a wide set of stocks from NYSE, Amex, and NASDAQ stocks. First, they form portfolios by sorting stocks according to the factors dedicated as empirical anomalies in earlier studies such as size, E/P, leverage and book to market (B/M) ratio and investigate the difference between average returns on upper and lower ranking portfolios. Portfolios formed on size reveal that average return decreases and market beta of portfolio increases as size increases. This is consistent with CAPM intuition in the sense that there is positive relation between market beta and portfolio return. On the other hand, if portfolios are double-sorted on size and market beta, the relationship between size and average return persists but the relationship between average return and market beta weakens. A similar double sorting of portfolios for B/M ratio and market beta suggests a positive relationship between average return and both B/M ratio and market beta but the relation between average return and market beta lost its significance after controlling for B/M ratio.

In order to confirm the implications of portfolio level results, they conduct cross-sectional regressions of stock returns on beta and other factors. Since accounting ratios are not estimated rather they are realized values, they are free from error in variables problem. However, market beta in regressions is estimated and still suffer from error in variables so the authors assigned portfolio betas to the stocks in the portfolio to smooth out error in variables. In measuring beta, they followed multiple steps: i) market beta for each stock is estimated on 60 months with a minimum required period of 24 months, ii) 100 portfolios are formed based on pre-ranking beta and size, and average return is calculated for 12 months, iii) beta of each portfolio is calculated as the sum of beta estimates from regression of average portfolio returns over current and lagged market return, iv) portfolio betas are assigned to stocks that they are in at the end of each June.

In return generating function, return on asset is regressed on market beta, logarithm of market value, B/M ratio, E/P ratio and leverage ratios of asset to book value (A/B) and asset to market value (A/M) in different setups. Fama-Macbeth regressions shows that the factors have explanatory power on expected stock return but market beta lost its significance. Among all these factors, size and B/M ratio are the most significant factors and captures the variability of stock returns adequately. Moreover, results show that small sized stock returns outperform large sized stock return and high B/M stocks outperform low B/M stocks. The coefficients of A/B and A/M are in opposite sign and very close to each other in absolute terms. Fama and French suggest that B/M captures the effects of leverage ratios of A/B and A/M ratios, and size captures the effect of E/P. The parsimonious form of return generating function portrays in the following multiple linear regression of Equation (3.18) as a function of market beta, size and B/M. Fama-Macbeth regressions conclude that market beta is statistically insignificant but size and B/M has explanatory power on cross-sectional asset returns contradicting with CAPM.

$$R_{it}=a+b_{1t}\beta_{it}+b_{2t}\ln(ME_{it})+b_{3t}\ln(B/M)+e_{it} \tag{3.18}$$

3.1.5.2 Time-Series Testing

Although CAPM was formulated to establish a cross-sectional relationship between risk and return for individual assets, there have been also studies utilizing time series testing alternative to cross-sectional tests. Since the main intuition of CAPM is that market beta is the only determinant in explaining asset return differential, the testable hypothesis in times series models is whether the intercept term in return generating regression is different than zero. Although it was not a test specific to individual stocks but to mutual funds, Jensen (1968) has been cited as first to introduce time-series regression to test the relationship between expected return and market beta.

Black, Jensen, and Scholes (1972) test CAPM by time series regressions as well. In time series regression setup, they argue that the intercept term in market model given below should not be statistically different from zero if the assumptions of CAPM are valid. Since stock prices mostly comove with economic conditions, stocks are correlated and residuals from market model are also correlated. Therefore, they form portfolios in order to avoid cross-sectional correlation between error terms and making wrong inference. After forming 10 portfolios based on individual stock beta and calculating average return series for each portfolio, they run the market model in Equation (3.19) for each portfolio and test the significance of intercept terms. Most of the intercept terms are not statistically different from zero and the results support CAPM.

$$R_{it}=a_i+\beta_i R_{Mt}+\epsilon_{it} \tag{3.19}$$

Fama and French (1993) extended the cross-sectional regression analysis in Fama and French (1992) by time series regressions. As evidenced in Fama and French (1992), size and B/M are two common factors explaining cross-sectional stock return differential. Building on this evidence, Fama and French (1993) form two fcator mimicking portfolios from size and B/M as risk factors in addition to market factor. Market factor is the difference between market return and risk free rate and risk factors are derived as zero investment portfolios. In the regression of Equation (3.20), market return (R_{Mt}-R_{ft}), size factor (SMB) and value factor (HML) are regressed on 25 double sorted portfolios formed on size and B/M. The results show that the intercept terms are statistically not different from zero and confirm that size and B/M factors explain most of the variation among portfolio returns in addition to market factor.

$$R_{pt}-R_{ft}=a_p+b_p(R_{Mt}-R_{ft})+s_pSMB_t+h_pHML_t+\varepsilon_{it} \tag{3.20}$$

Similarly, Fama and French (1996) use time series regression similar to Fama and French (1993) and investigate the anomalies similar to previous attempts that they formed portfolios based on single and double sorting according to E/P, cash flow to price (C/P) ratio and sales growth. Average portfolio returns are regressed over Fama-French three factors and significance of intercept terms are tested by F-test introduced by Gibbons et al. (1989) (GRS test) with better small sample properties. The intercepts are not different from zero inferring that three factor model is adequate to explain return distribution but CAPM is not.

3.1.5.3 Criticisms on CAPM Testing

Following the empirical tests and theoretical criticisms on CAPM, Roll (1977) opened a new window of discussions by questioning the testability of CAPM and changed the interpretation of CAPM tests. Roll’s critics concentrates on two points. First, even market portfolio in CAPM consists of all asset universe, market indices like S&P 500 and others have limited representativeness because it only includes stocks but it doesn’t include other assets like bonds, housing, labor etc. representing total wealth. Stambaugh (1982), for example, overcomes this obstacle by extending the asset universe by including bonds, real estate, and consumer durables in addition to common stocks. Secondly, the test of CAPM only tests whether market portfolio is mean variance efficient. One can find any mean variance efficient portfolio and use it in testing CAPM but this type of tests could not confirm the validity of CAPM because true market portfolio could be not on mean variance efficient set. Similarly, the rejection of mean variance efficiency of market proxy doesn’t mean the rejection of CAPM because it may be the result of choosing false market proxy. Hence, Roll argued that CAPM is not testable and previous empirical studies could not confirm and reject the validity of CAPM.

3.1.6 Multifactor Asset Pricing Models

The empirical shortcomings of CAPM has triggered search for alternative theoretical formulations and risk factors in asset pricing models. Literature has aimed to capture different dimensions of asset prices by introducing factors besides market factor and calls the models with more than one factor as multifactor asset pricing models. ICAPM and APT are two early examples of multifactor asset pricing models. However, these models have a major drawback that they don't define risk factors explicitly. In explaining return differential among stocks, one dominant approach has been to utilize price scaled financial ratios to form risk factors by tradable portfolios representing return differential between highest and lowest level of stock characteristics. For example, size characteristic is added by size mimicking portfolio as the return difference between small sized and large sized firms. Although the number of factors in explaining cross-sectional stock return and the efforts to relate these factors with CAPM anomalies have increased rapidly, there are alternative methods like applying pure statistical models. Kozak, Nagel, and Santosh (2018) argue that statistical factor models are better in capturing cross-sectional distribution of stock returns compared to Fama-French three factor model.

3.1.6.1 Three-Factor Model

Fama and French (1993) extend cross-sectional regression analysis in Fama and French (1992) to time series regression. In time series regression setup, Fama and French (1993) form two factor mimicking portfolios from size and B/M as risk factors in addition to market factor. Market factor is derived as the difference between market return and risk free rate and risk factors are derived as zero investment portfolios, that is, taking long position on one side and short position on opposite side. Zero investment portfolios are not risk factors by themselves but they are unknown state variables and proxy non-diversifiable risk not captured by market risk.

In forming portfolios, Fama and French (1993) divided stocks into two groups according to median level of size. Stock below the median level of size form portfolio of small sized stocks (S) and above the median level of size form portfolio of big sized stocks (B). Similarly, stocks are grouped according to B/M ratio where stocks owning highest 30% of B/M ratio form high B/M portfolio (H), lowest 30% of B/M ratio form low B/M portfolio (L) and middle 40% of stocks form middle B/M portfolio (M). Afterwards, six portfolios are constructed as double sorted (2x3) from size and B/M portfolios. These portfolios are restructured in a way to capture systematic risks i) related with size represented by the return difference between the portfolio of small sized stocks and the portfolio of large sized stocks (SMB) and ii) related with B/M ratio represented by the return difference between the portfolio of high B/M ratio (value) stocks and the portfolio of low B/M ratio (growth) stocks (HML).

Afterwards, Fama and French (1993) form 25 portfolios formed on size and B/M ratio to test whether these two factors have explanatory power. The results from the regression in Equation (3.21) show that the intercept terms (a_p) are statistically not different from zero and confirm that SMB and HML factors explain most of the variation of portfolio returns in addition to market factor. This confirms the test results for cross-sectional returns in Fama and French (1992) but also pointed out that SMB and HML are not enough to explain the variability of returns but market factor should also be inputted into model to capture a higher portion of portfolio return variability. The authors also questioned whether any other accounting ratios has an explanatory power beside size and B/M ratio. In this direction, they investigate the impact of E/P and D/P (dividend to price) ratios by forming portfolios. Returns of E/P and D/P portfolios posit a U-shape average return structure. The intercept terms from the regressions of average return over market factor reveal that market factor is not adequate to explain the return variation of portfolios formed on E/P and D/P. However, adding SMB and HML factors removes the significance of intercept terms. In this line, SMB and HML with market factor capture also the effects of E/P and D/P on average portfolio return.

$$R_{pt}\text{-}R_{ft}=a_p+b_p(R_{Mt}\text{-}R_{ft})+s_pSMB_t+h_pHML_t+\varepsilon_{it} \qquad (3.21)$$

Fama and French (1996) confirm the significance of size and B/M ratio as factors and extend Fama and French (1993) through adding other anomalies of E/P ratio, C/P ratio, sales growth (for five years) and momentum. In empirical analysis, first, they form decile portfolios for B/M, E/P, C/P and sales growth. The average return on deciles portfolios demonstrate that higher B/M, E/P and C/P, and lower sales growth are associated positively with average excess returns. In addition, three factor model is successful in explaining anomalies listed in literature but it is short of explanation for momentum effect. Instead, portfolios having low past return tend have positive loading on SMB and HML factors, therefore, the model estimates reversal in return contrary to momentum.

In sum, apart from market factor, SMB and HML are two factors successful in explaining stock returns that cannot be fully explained by CAPM but three factor model is based solely on empirical results, and SMB and HML remain as unidentified state variables producing undiversifiable risks and the model is inadequate in explaining the momentum factor. It is also important to note that SMB and HML factors generate excess return and they are zero investment portfolios but they are not arbitrage portfolios and don't provide a basis for arbitrage opportunity because the return volatility of factors are not low so SMB or HML portfolios are not riskless.

3.1.6.2 Reversal and Momentum

Fama–French three factor model has been commonly used in empirical asset pricing and the model has been accepted as successful in explaining return differential among portfolios but the model has a major soft spot that state variables representing SMB and HML has remained as unknown. Lack of explanation of state variables motivated further work in explaining the systematic risk apart from the market factor. Some studies have associated additional risk factors with behavioral biases discussed extensively in behavioral finance literature.

Behavioral finance puts forward behavioral biases, deviation from rational behavior, in explaining inadequacy of naïve asset pricing model of CAPM. In behavioral finance, there are two main explanations for behavioral biases and return anomalies: i) limits to arbitrage and ii) beliefs and preferences. Although behavioral finance contributes to literature in explaining return anomalies, it has been considered as a complement rather than replacement.

De Bondt and Thaler (1985) investigate the impact of overreaction of human behavior on asset pricing and claim that overreaction leads return reversal dedicated as anomaly in asset pricing. De Bondt and Thaler questioned the reason of higher market risk adjusted return for the stocks having lower price to earnings ratio (P/E). In this direction, they form portfolios based on the rankings of NYSE stock excess returns in the last three years. 35 stocks generated highest cumulative return form winner portfolio and 35 worst performing stock constitute the loser portfolio. Portfolio returns are calculated for subsequent three years period. The loser portfolio overperform the winner portfolio inferring reversal of return. However, the reversal of winner portfolio is weaker and there is asymmetries in overreaction towards either side. In this context, their analysis confirms the overreaction hypothesis and reversal as an anomaly that could not be explained by CAPM.

De Bondt and Thaler (1987) associate the explanatory power of B/M with overreaction and underreaction hypotheses, and the correction of over and underreaction of prices. In this framework, high returns for value stocks and low returns for growth stocks reflect the underreaction of prices and reversal of overreaction respectively. This correction has been associated with the risk factor of HML in Fama-French three factor model. Hence, behavioral finance approach supports the view that CAPM is the correct model for asset pricing but behavioral biases leads anomalies and deviations from CAPM.

Although there is a raising interest and attention about contrarian strategy based on return reversals, relative strength strategies formed on buying past winners and selling past losers is the other side of the coin and evaluated as another return anomaly. The empirical analyses by Jegadeesh and Titman (1993) reveal that portfolios formed on past return generate higher return for the next 3 to 12 month based on US stock market data. Similar to De Bondt and Thaler (1985), the returns of decile portfolios are formed on each month and called relative strength portfolios. Portfolio returns during pre and post portfolio formation are compared and the results suggest that highest decile portfolio in last 6 months overperform the lowest decile portfolio in the next 12 months despite lost half of it in 24 months. Jegadeesh and Titman check whether momentum effect captures other systematic risks of market beta and size but return on relative strength portfolios are similar among different size and beta levels. Rather, momentum[32] is associated with strategy of investors that they buy winner stocks and sell loser stocks.

Carhart (1997) focus on equity mutual funds and confirms the momentum effect. Carhart form momentum factor monthly as the difference between winner and loser portfolios based on one month lagged past 11-month return so it is a zero investment factor mimicking portfolio for momentum. Since the model includes momentum and Fama-French three factors as in Equation (3.22) consisting of market factor (RMRF), SMB, HML and momentum (PR1YR), the model has been called as four factor model.

$$r_{it}=\alpha_{iT}+b_{iT}RMRF_t+s_{iT}SMB_t+h_{iT}HML_t+p_{iT}PR1YR_t+e_{it} \tag{3.22}$$

3.1.6.3 Liquidity

Liquidity is defined as the ability to trade at low transaction cost with insignificant price effect so investors prefer liquid assets and are expected ask for compensation

[32] An alternative explanation of momentum effect is that agents underreact to information releases like financial statements data so price adjustment leads to continuation of return and momentum.

for holding illiquid assets. As liquidity has potential source of return variation among stocks, some studies also tested liquidity as an anomaly and a factor not captured by CAPM. Amihud and Mendelson (1986) measure illiquidity by the cost of immediate execution and argue that quoted ask price contains the premium for immediate buying and quoted bid price reflects discount on price for immediate sale. The difference between bid-ask price reflects the sum of premium on buying and discount on selling so this difference represent the illiquidity for a stock. They included liquidity in asset pricing equation in addition to market beta and market size to test 49 market beta-illiquidity double sorted portfolios. According to the regressing setup in Equation (3.23), dependent variable is portfolio return and explanatory variables are portfolio beta (β_{pn}), mean adjusted spread (S'_{pn}), portfolio-group dummy variables (DP_{ij}) and year dummy variables (DY_n). The results suggest that expected return is an increasing function of bid-ask spread and liquidity has explanatory power in addition to market beta. Amihud and Mendelson also underline that illiquidity doesn't capture size effect because the coefficient of liquidity stay significant after size is added.

$$R^e_{pn}=a_0+a_1\beta_{pn}+\sum_{i=1}^{7} b_i\, S'_{pn}+\sum_{i=1}^{7}\sum_{j=1}^{7} c_{ij}\, DP_{ij}+\sum_{n=1}^{19} d_n\, DY_n+\varepsilon_{pn} \qquad (3.23)$$

Amihud (2002) also defines liquidity as the ability to trade without any significant pressure on returns. If change in stock price is high even at high trading volumes, the liquidity of stock is low because the traders are not keen to exchange the stock. Hence, Amihud introduces liquidity measure as yearly average of the ratio of daily price change to trading volume as in Equation (3.24) where D_{iy} is the number of days for which data is available for stock i in year y, R_{iyd} and $VOLD_{ivyd}$ are the return and volume on stock i on day d of year y respectively. Amihud confirms positive cross-sectional relation between return and illiquidity for stocks in NYSE by Fama-Macbeth regressions. Amihud (2002) also compares the impact of size and illiquidity on stock return and concludes that small sized firms are more open to the risk of illiquidity so capital flights to quality, big sized stocks, in the case of distressed financial conditions.

$$\text{ILLIQ}_{iy}=1/\text{D}_{iy}\sum_{t=1}^{\text{Diy}}\left|\text{R}_{iyd}\right|/\text{VOLD}_{ivyd} \tag{3.24}$$

Pastor and Stambaugh (2003) argue that liquidity could be a state variable in asset pricing considering the important role of market liquidity on investment and macroeconomic conditions. Pastor and Stambaugh introduce another liquidity factor for US gauging the size of return reversal upon trading volume shocks. In detail, lower liquidity on day t is signaled by return and order flow in the opposite direction on day t+1 compared to day t. The results suggest that sensitivity of stock returns to market liquidity measured by liquidity betas has a role in asset pricing and stocks with higher liquidity betas generate higher expected returns. The return differential between the portfolios at the top and bottom deciles of predicted liquidity betas generates abnormal return compared to four factors of market, size, value and momentum.

Acharya and Pedersen (2005) also incorporate liquidity into asset pricing model and their model captures different dimensions of liquidity by different liquidity risk measures. The results show that stock return is a function of covariance of return and market return, covariance of return and stock’s own liquidity, covariance of market return and stock’s own liquidity, and covariance of stock liquidity and market liquidity. A negative shock to stock liquidity leads lower contemporaneous returns but higher future returns. Although the model explains the return differential among portfolios sorted on liquidity and size, the model has lack the capacity to explain return differential among B/M portfolios.

Similarly, Liu (2006) points out the importance of multidimensional liquidity definition containing trading speed, trading quantity, trading cost and price change. Liquidity is defined as the standardized turnover-adjusted number of zero daily trading volumes over the prior 12 months to represent the trading speed and continuity. This liquidity measure identify illiquid stocks as small, value, low-turnover, high bid-ask spread, and high return-to-volume stocks. The model including market factor and liquidity factor explains return differential well among the portfolios sorted on size and B/M and infers that liquidity factor capture the anomalies of size and B/M simultaneously.

3.1.6.4 Coskewness

Portfolio theory and CAPM build on the assumption that agents make investment decision by taking into account only two parameters of return distribution: mean and variance. Implicitly, market portfolio variance is assumed to be an adequate measure of risk and agents don't consider higher moments in investment decision. This assumption is valid under two conditions: i) quadratic utility function or ii) normal distribution of stock return. On the other hand, return distributions of stocks don't have desired properties and own fatter tails compared to normal distribution. Mandelbrot (1963) points out outlier problem and leptokurtosis in return distributions and introduces the stock return distribution of stable Paretian with four parameters: location, scale, skewness and extreme tail height.

Furthermore, there are evidence from behavioral finance supporting to consider higher moment of return such as skewness and kurtosis. Kahneman and Tversky (1979) argue that agents are more averse to large losses and give higher weight to loss in wealth compared to same amount of wealth gain in prospect theory inferring the importance skewness and kurtosis. While skewness separates extreme returns from one tail to other tail and measures the asymmetry, kurtosis is a measure of extreme values in both tails. Agents avoid kurtosis and prefer right skewed portfolios so skewness and kurtosis should be considered by agents in investment decision. In this framework, higher moments of distribution may carry information on asset return.

Harvey and Siddique (2000) propose to include higher moments of return distribution compared to CAPM and investigate return differential by the model called three-moment CAPM in which skewness is argued to capture the effects of size and momentum effects. The model is closely related with Kraus and Litzenberger (1976) in which unconditional skewness helps to explain cross-sectional asset returns in addition to market beta. Coskewness of a stock is measured as the contribution of individual stock to market skewness. Hence, negative coskewness means negative contribution to market skewness and infers higher required return. In order to investigate the impact of coskewness, they form value weighted hedging portfolios. As an initial step, Harvey and Siddique derive skewness of individual stocks and market portfolio. Afterwards, they form portfolio as the difference between most negatively skewed stocks and most positively skewed stocks. Coskewness of individual stocks are measured twofold: i) beta from the regression over the difference of returns between negative and positive skewness portfolios, ii) beta from the regression over excess returns of negative skewness portfolio. Empirical results reveal that agents consider market beta but also consider coskewness and they give up some part of required return in exchange for positive skewness. As in other higher moment CAPM models, the limitation of three moment CAPM model by Harvey and Siddique (2000) is that coskewness is an estimated variable so it suffers from error in variables problem.

3.1.6.5 Five-Factor Model

Fama and French (2015) extend three factor model by adding investment and profitability similar to the studies Xing (2007) and Hou, Xue, and Zhang (2015) and call extended version of the model five-factor model. As a similar model setup, Hou, Xue, and Zhang (2015) introduce q-factor[33] model and the model has similarities with Fama and French (1996) in a way that it includes market and size factors in addition to investment and profitability factors where they are proxied by asset growth and return on equity respectively. The relationship between investment

[33] Hou, Xue, and Zhang (2015) defines q as net present value of future cash flows generated from an additional unit of assets. q-factor model is based on investment-based asset pricing theory (q-theory) of Cochrane (1991) and Cochrane (1996). q-theory tells that the expected return of a stock equals to the ratio of expected marginal benefit of investment to marginal cost of investment.

and expected return is associated with the expected cash flow such that higher expected return implies lower net present value of cash flows from investment. Conversely, stock return increases as profitability increases for a given level of investment so there is positive relation between profitability and expected return[34]. The results show that q-factor model outperforms both Fama-French three factor model and Carhart four factor model.

Fama and French (2015) explain the validity of investment and profitability by dividend discount model of Modigliani and Miller (1961). According to the model, market value of a stock is equal to discounted values of expected dividends per share. Price per share (m_t), in Equation (3.25), equals sum of expected dividends ($E(d_{t+\tau})$) discounted at internal rate of return (r) and total market value, in Equation (3.26), equals the difference between total earning ($Y_{t+\tau}$) and change in book value ($dB_{t+\tau}$). Dividing each side by book value drives Equation (3.27) and the equation infers that expected return depends on expected earnings change and change in book value (investment). Higher expected earnings increases expected stock return and higher book value, or investment, decreases expected stock return. In order to capture these effects, profitability and investment are two factors to proxy expected earnings and change in book value. The results confirm the validity of five factors: market, size, value, profitability and investment.

$$m_t=\sum_{\tau=1}^{\infty} E(d_{t+\tau})/(1+r)^{\tau} \tag{3.25}$$

$$M_t=\sum_{\tau=1}^{\infty} E(Y_{t+\tau}-dB_{t+\tau})/(1+r)^{\tau} \tag{3.26}$$

$$\frac{M_t}{B_t}=\frac{\sum_{\tau=1}^{\infty} E(Y_{t+\tau}-dB_{t+\tau})/(1+r)^{\tau}}{B_t} \tag{3.27}$$

[34] Alternatively, positive relationship between profitability and stock return has also been explained by behavioral biases. Wang and Yu (2013) argue that behavioral biases like conservatism may result in positive relationship due to correction of mispricing.

3.2 Ambiguity as a Factor in Asset Pricing: An Empirical Analysis for Turkey

3.2.1 Introduction

In this section, we test whether adding an ambiguity sensitivity measure to the asset pricing model increases the model's explanatory capacity for the cross-section of stock returns in Turkey. Since the literature shows risk factors other than ambiguity to have a significant effect on stock returns, ambiguity is added to the asset pricing models alongside these other factors. Among these other factors, we include the market factor, size (SMB), value (HML), profitability (RMW) and investment (CMA) from Fama and French (2015), momentum (WML) from Jegadeesh and Titman (1993), coskewness (COSKW) from Harvey and Siddique (2000) and illiquidity (ILLIQ) from Amihud (2002). This way, in addition to the tests of ambiguity as a risk factor in asset pricing, the study also provides up-to-date evidence on the significance of other risk factors for the Turkish stock market. Our sample includes stocks listed on Borsa Istanbul between January 1990 and December 2017 but exclude stocks from the financial sector, closed-end mutual funds, REITs, sports clubs and stocks with a negative book to market ratio due to the unique nature of their balance sheet structures and different price dynamics.

In the first part of the analysis, we uncover the return dynamics in Borsa Istanbul throughout the sample period. Second, we provide descriptive statistics of stock characteristics necessary to calculate the risk factors and present the relationship between excess returns and stock characteristics for single- and double-sorted portfolios. Third, we derive the risk factors from stock characteristics, and discuss in detail their statistical properties and the validity of these factors for the Turkish context. Finally, we incorporate the risk factors along with measures of ambiguity sensitivity by using time series and cross-sectional testing methodologies.

3.2.2 Descriptive Statistics for Borsa Istanbul

Descriptive statistics of individual stock returns are given in Table 3.1. Accordingly, the number of stocks to be included in the sample is 321 as of year-end 2017 but the number is only 78 in the early years of trading in Borsa Istanbul[35]. Although data frequency in this study is monthly, we first summarize daily stock return for each year to show how distribution of stock return changed throughout the history of Borsa Istanbul. Daily stock returns are calculated as the daily percentage change in share prices as in Equation (3.28) where price_{it} is the price of stock i at the end of day t and return_{it} equals the daily percentage change in price of stock i at the end of day t. Since return data may show extreme movements on some days, daily returns are trimmed at the bottom and top 0.1 percentage. Descriptive statistics of trimmed daily returns show that daily stock returns are positive in general and average standard deviation is higher in earlier years. Also, the average standard deviation of returns increased during the crisis years of 1994, 1998, 2001 and 2008. Higher moments of returns reveal that distributions are right-skewed and have fatter tails (are leptokurtic) compared to a normal distribution, implying that stocks have more extreme positive returns compared to extreme negative returns and higher extreme returns in general are probable compared to the normal distribution.

$$\text{return}_{it} = \left(\text{price}_{it}/\text{price}_{i,t-1}\ -1\right) \times 100 \qquad (3.28)$$

[35] Trading in Borsa Istanbul started in 1988 but we prefer to start our sample from 1990 in order to work with a sufficient number of stocks, have access to financial tables and test asset pricing models in a relatively more liquid market environment.

Table 3.1 Descriptive Statistics for Daily Stock Returns in Borsa Istanbul

Table summarizes the statistics of daily return of individual stocks excluding stocks in financial sector, closed-end mutual funds, REITs, sports clubs and having negative book to market ratio. Year column include all the years this study incorporates. Number of stocks shows total number of stocks traded within a year and stay in the market as of December. Number of observations' column counts total number of daily returns for all stocks. Daily return of series is calculated by daily percentage change as shown in equation below where price_{it} shows the price of stock i on day t and return_{it} shows the percentage return of stock i on day t.

$$\text{return}_{it}=\left(\text{price}_{it}/\text{price}_{i,t-1}-1\right)\times 100$$

Daily returns are trimmed at bottom and top 0.1 percent. The summary statistics of trimmed daily returns for each year is given in the following columns of mean, standard deviation, skewness, kurtosis and t-value.

Year	Number of Stocks	Number of Obs.	Mean	Standard Dev.	Skewness	Kurtosis	t-value
1990	78	16211	0.20	4.93	0.12	2.76	5.76
1991	98	21727	0.25	5.09	0.21	3.23	7.22
1992	109	25234	0.10	4.09	0.33	4.24	3.89
1993	122	28379	0.88	4.91	0.22	3.07	33.45
1994	140	33700	0.30	6.12	0.21	3.12	9.37
1995	164	39864	0.25	4.80	0.67	5.19	10.47
1996	187	43580	0.41	4.17	0.96	5.90	22.44
1997	211	50397	0.42	4.24	0.60	5.21	25.34
1998	225	54548	0.01	5.12	0.29	4.79	0.62
1999	222	52622	0.66	4.76	0.53	4.91	36.24
2000	234	57690	-0.02	5.06	0.70	5.93	-1.65
2001	223	58614	0.28	5.06	0.29	5.02	14.48
2002	209	59375	0.03	3.83	0.63	5.78	1.64
2003	213	57539	0.22	3.36	0.47	8.06	17.95
2004	229	59040	0.15	2.90	1.46	10.89	15.28
2005	237	61494	0.20	2.71	0.95	7.74	21.86
2006	249	63112	-0.01	2.98	0.51	7.40	-0.89
2007	246	64251	0.10	2.74	0.84	9.05	11.30
2008	242	63780	-0.26	3.70	0.44	6.34	-19.71
2009	241	63192	0.35	3.13	1.12	8.52	32.86
2010	257	65041	0.18	2.82	1.19	10.47	17.52
2011	280	71473	-0.03	3.05	0.50	8.61	-4.00
2012	318	78631	0.06	2.46	0.48	14.53	9.17
2013	334	84283	-0.03	2.87	0.34	9.83	-3.67
2014	338	84977	0.12	2.64	0.90	11.04	14.39
2015	337	87319	0.01	2.98	0.70	11.31	1.33
2016	326	85205	0.06	2.82	1.23	14.22	7.93
2017	321	83737	0.20	2.82	1.79	16.13	22.69

3.2.3 Portfolio Level Analysis of Stock Returns

3.2.3.1 Stock Characteristics

Before forming risk factors that we are going to use in the modeling of stock returns, we first analyze the portfolios formed on stock characteristics that are in turn used to form the risk factors. Stock characteristics that we use are market value, book-to-market ratio, investment, operating profitability, momentum, illiquidity, coskewness, market beta and ambiguity beta. Stocks are allocated to portfolios based on the breakpoints of the different sorts and portfolios are rebalanced at the beginning of July in each year by observing the stock characteristics of book-to-market ratio, investment and operating profitability at the end of December in the preceding year and at the end of June for market value. Stock characteristics are observed with a six-month lag in order to make the assumption that investors had sufficient time to gather information about these accounting variables before they make investment decisions. For the other stock characteristics (momentum, illiquidity, coskewness, market beta and ambiguity beta), portfolios are formed based on the values of the current month. Since portfolios are re-balanced annually or monthly depending on stock characteristics, portfolio membership of a stock in each characteristic may differ throughout time in different frequencies due to changing stock characteristic.

Before going into the details of how we form the monthly dataset from stock characteristics, it is essential to give some definitions that will be used in the remainder of the study. $R_{i,t}$ represents monthly percentage return of stock i in month t. $R_{M,t}$ is market portfolio return in month t where the market portfolio is proxied by the BIST100 Index consisting of 100 largest market capitalization stocks traded on Borsa Istanbul. $R_{f,t}$ represents risk-free interest rate proxied by the Turkish T-bill rate for month t. The difference between the monthly stock return and risk-free interest rate generates the excess return ($r_{i,t} = R_{i,t} - R_{f,t}$). Hence, $r_{i,t}$ and $r_{M,t}$ are excess returns for stock i and market respectively in month t. Next, we derive the stock characteristics as follows:

i. *Market value*[36] *(MV):* Market value equals the stock price per share times the number of shares outstanding. Market value of a stock in a given month from July in year t to June in year t+1 equals its natural logarithm of market value in June in year t.

[36] Size and market value are used interchangeably in this study.

ii. *Book to market ratio (B/M):* B/M ratio of a stock in a given month from July in year t to June in year t+1 equals to the ratio of book value to market value in December of year t-1.

iii. *Investment (INV):* Investment measures percentage increase in asset value. Investment of a stock in a given month from July in year t to June in year t+1 equals percentage change of total asset between December in year t-1 and December in year t-2.

iv. *Operating profitability (OP):* Operating profitability of a stock in a given month from July in year t to June in year t+1 equals the ratio of operating profit to book equity in December of year t-1.

v. *Momentum (MOM):* We follow Jegadeesh and Titman (1993) in calculating Momentum. Momentum of a stock in month t equals the percentage change in stock price between month t-1 and month t-12 as in Equation (3.29).

$$MOM_{i,t} = (price_{i,t-1}/price_{i,t-12}) \cdot 100 - 100 \tag{3.29}$$

vi. *Illiquidity (ILLIQA):* The illiquidity level during the past one year is proxied by the daily illiquidity measure introduced by Amihud (2002). ILLIQD equals the ratio of the absolute value of daily stock returns to the daily trading volume in USD[37] multiplied by 1000. In Equation (3.30), $return_{d,i,t}$ is the daily percentage return of stock i on day d in month t, $volume_{d,i,t}$ is daily trading volume of stock i on day d in month t. Illiquidity of a stock in month t is calculated in multiple steps. First, the daily illiquidity measure is transformed to a monthly measure (ILLIQM) as the simple average of ILLIQDs for each stock provided that a stock has illiquidity data for at least 10 days within a month (Equation (3.31)). Next, the ILLIQM measure is averaged over the past 12 months on a rolling basis if at least 6 months of illiquidity data are available in order to smooth out the volatility in liquidity and, finally, ILLIQA represents the annualized illiquidity of a stock in month t (Equation (3.32)).

$$ILLIQD_{d,i,t} = |return_{d,i,t}|/volume_{d,i,t} \cdot 10^3 \tag{3.30}$$

[37] The daily trading volume is normalized by the daily closing value of USDTRY exchange rate.

$$\mathrm{ILLIQM}_{i,t} = 1/k \cdot \sum_{d=1}^{k} \mathrm{ILLIQD}_{d,i,t} \tag{3.31}$$

$$\mathrm{ILLIQA}_{i,t} = 1/12 \cdot \sum_{k=t-11}^{t} \mathrm{ILLIQM}_{i,k} \tag{3.32}$$

vii. *Coskewness (COSKEW):* Coskewness of a stock in month t is derived from the coefficient of excess squared market return (γ) in the regression of excess stock return on the excess market return and excess squared market return as in Equation (3.33) following Adesi, Gagliardini, and Urga (2004). The coskewness coefficient is estimated each month on a rolling basis for the past 48 months provided that there are at least 24 observations.

$$r_{i,t}=\alpha_{i,t}+ \beta_{i,t} r_{M,t}+\gamma_{i,t}\left(R_{M,t}^2-R_{f,t}\right) \tag{3.33}$$

viii. *Market beta (β^M):* Market beta of stock i in month t ($\beta_{i,t}^M$) is derived from the coefficient of excess market return in the regression of excess stock return on the excess market return as in Equation (3.34). Market beta is estimated each month on a rolling basis for the past 48 months provided that there are at least 24 observations.

$$r_{i,t} =\alpha_{i,t}+\beta_{i,t}^M r_{M,t} \tag{3.34}$$

ix. *Ambiguity beta (β^A):* Ambiguity beta of a stock in month t is derived from the coefficient of expected ambiguity index in the regression of excess stock return on the estimated ambiguity index as in Equation (3.35). Ambiguity beta ($\beta_{i,t}^A$) is estimated each month on a rolling basis for the past 48 months provided that there are at least 24 observations. The derivation of the estimated ambiguity index (AMB_t^E) from the original ambiguity index is presented in the following section.

$$r_{i,t} = \alpha_{i,t}+\beta_{i,t}^A(\mathrm{AMB}_t^E) \tag{3.35}$$

3.2.3.1.1 Estimated Ambiguity Index

Agents make investment decisions and change their demand for a particular asset by taking into account the developments that impact the future value of their investments. In this study, we propose to investigate the impact of ambiguity on expected stock returns so we should incorporate the estimated value of ambiguity in the asset pricing models in order to understand the impact of ambiguity on expected returns. For this purpose, we derive an expected ambiguity index from the ambiguity index[38] by time series forecasting. First, we test the stationarity of the ambiguity index and the existence of a unit root by using the ADF test in Panel A of Table 3.2. Test results presented in Panel A of Table 3.2 reject the existence of a unit root in the series and imply that the ambiguity series is stationary. Furthermore, we test for the conditional heteroscedasticity in order to avoid inferential errors. Engle's autoregressive conditional heteroscedasticity (ARCH) Lagrange multiplier (LM) test does not reject the null hypothesis that there are no ARCH effects (Panel B of Table 3.2).

Table 3.2: Test of Ambiguity Index for Stationary and ARCH Effect

Table gives the results of ADF and ARCH LM test statistics for ambiguity index. In Panel A, the null hypothesis of ADF test is that the series has unit root. In Panel B, the null hypothesis of ARCH LM test is that there is no ARCH effect in the series.

Panel A: Augmented Dickey-Fuller test for unit root

	Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-11.256	-3.464	-2.881	-2.571

MacKinnon approximate p-value for Z(t) = 0.0001

Panel B: LM test for autoregressive conditional heteroskedasticity (ARCH)

lags(p)	chi2	df	Prob > chi2
1	0.677	1	0.4105

H0: no ARCH effects vs. H1: ARCH(p) disturbance

After confirming that the series are stationary and there are no ARCH effects, we follow the Box-Jenkings methodology to construct an estimated ambiguity index. We estimate autocorrelations, partial autocorrelations, Ljung–Box test statistics and their significance level, and plot the correlogram of the autocorrelation and partial autocorrelation functions (Table 3.3). The correlogram suggest that only the first

[38] We use our own ambiguity index derived from the monthly variability of the daily probability densities of the stock index's (BIST30 Index) return.

autocorrelation coefficient is statistically significant and an AR(1) model is appropriate for modelling the ambiguity index. Next, we estimate Equation (3.36) where $ambiguity_t$ is the ambiguity index in month t. Estimation results for the AR(1) model is given in Table 3.4 where the coefficient of AR(1) term is shown to be positive and statistically significant. As a result, we forecast the expected ambiguity index by using the AR(1) structure we constructed.

$$ambiguity_t = \alpha + \phi \cdot ambiguity_{t-1} + \varepsilon_t \tag{3.36}$$

3.2.3.2 Single-Sorted Portfolios

After we calculate the stock characteristics, we form portfolios by sorting and grouping stocks based on these characteristics. We prefer to divide stocks into five portfolios rather than a higher number of portfolios because the number of stocks traded on Borsa Istanbul was rather low in the earlier years and even more recent numbers remain low compared to the total number of stocks in markets of other countries. It is important to have a reasonably high number of stocks within portfolios for higher diversification and the validity of portfolio level analysis. We allocate stocks into five quantiles in a way that each portfolio contains an equal number of stocks depending on the relevant stock characteristics, i.e., portfolio 1 includes stocks within the lowest 20% of the relevant criterion's values, portfolio 2 includes stocks within the second 20% of the relevant criterion's values and so on. In the following section, first, we present the descriptive statistics of stock characteristics for five portfolios. Afterwards, we calculate the average excess return for each portfolio formed on each stock characteristics in order to conduct an initial assessment of the relationship between stock returns and stock characteristics.

Table 3.3: Estimated Autocorrelations, Partial Autocorrelations and Ljung–Box Test Statistics of Ambiguity Index

Table includes autocorrelations, partial autocorrelations, Ljung–Box test statistics and its significance level, and the correlogram of the autocorrelation and partial autocorrelation functions.

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocorrelation]
1	0.3046	0.3046	22.541	0	|--	|--
2	0.1723	0.0898	29.788	0	|-	|
3	0.117	0.045	33.143	0	|	|
4	-0.0085	-0.0757	33.161	0	|	|
5	-0.0732	-0.0776	34.485	0	|	|
6	-0.0475	-0.0032	35.044	0	|	|
7	0.0197	0.0671	35.14	0	|	|
8	0.0412	0.0485	35.565	0	|	|
9	-0.0117	-0.0446	35.6	0	|	|
10	-0.0534	-0.0724	36.32	0.0001	|	|
11	-0.0063	0.0218	36.331	0.0001	|	|
12	-0.0535	-0.0364	37.059	0.0002	|	|

Table 3.4: Modeling of Estimated Ambiguity Index

Table summarizes the estimation results of AR(1) model for ambiguity index. The model is given in equation below. ambiguity_t represents ambiguity index in month t.

$\text{ambiguity}_t = \alpha + \phi \cdot \text{ambiguity}_{t-1} + \varepsilon_t$

Ambiguity_t	Coef.		St.Err	t-value	p-value
Intercept	0.002***		0.000	10.61	0.000
Ambiguity_{t-1}	0.304***		0.058	5.29	0.000
Mean dependent var		0.002	SD dependent var		0.001
Number of obs		240	Chi-square		27.952
			Akaike crit. (AIC)		-2545.291

*** *p<0.01*, ** *p<0.05*, * *p<0.1*

3.2.3.2.1 Descriptive Statistics for Stock Characteristics

Table 3.5 summarizes the time series statistics of portfolios formed on stock characteristics by using monthly data. In each panel of the table, portfolio averages for each stock characteristic are calculated both on equal-weighted and value-weighted terms where value-based weights are calculated by using each stock's market value. Panel A includes statistics of market value[39] for five portfolios and market value for each portfolio is calculated as the simple arithmetic average. Portfolio means increase monotonically by construction. Standard deviation of portfolios increase as the size of stocks increases. Skewness and kurtosis values imply normality in each portfolio. In Panel B, summary statistics of B/M show that the average B/M in the fifth portfolio is greater than 1 suggesting that, on average, the market value of stocks is lower than their book value in this group. High skewness and kurtosis values in the fifth portfolio also suggest that this group includes stocks with extreme B/M values. Similar B/M values in value-weighted and equal-weighted portfolios imply that size is homogenous in each B/M portfolio. Panel C includes summary statistics of the investment characteristic for the five portfolios. Investment value in the fifth portfolio is significantly higher compared to other four portfolios. In the first four portfolios, the average investment between value- and equal-weighted portfolios are close to each other and exhibit characteristics close to normality. However, skewness and kurtosis in the fifth portfolio suggest the existence extreme values. In addition, higher equal-weighted investment average compared to the value-weighted average implies that smaller stocks have higher investment levels.

Panel D summarizes statistics of operating profitability and only the first portfolio has negative operating profitability. We observe that there is no size effect in the first portfolio when we compare value-weighted and equal-weighted portfolios. In the fifth portfolio, operating profitability is considerably higher compared to the

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[39] Market value is normalized by USDTRY exchange rate only for Table 3.5 to neutralize the scale effect of depreciation of Turkish Lira throughout sample.

other portfolios and value-weighted average is smaller than the equal-weighted average implying that smaller stocks are more profitable compare to larger stocks. Panel E includes summary statistics of momentum. Momentum of portfolios in each of the five groups shows no significant difference between value-weighted and equal-weighted averages implying that momentum is not affected from size. In Panel F, statistics for illiquidity show that illiquidity figures for the first four portfolios are close to each other but the fifth portfolio exhibits illiquidity that is significantly higher than the others. Skewness and kurtosis in each portfolio suggest the existence of extreme values and comparison of the value-weighted and equal-weighted portfolio values shows a substantial difference only for the fifth portfolio.

Panel G summarizes the statistics of coskewness and no notable difference is observed for the portfolios that are either value- or equal-weighted. Panel H presents the summary statistics of market beta. Interestingly, the average market beta for all portfolios is below 1.0. This might be a result of the market index proxy used in the study that stocks from the financial sector, closed-end mutual funds, REITs, sports clubs and stocks with a negative book-to-market ratio are excluded from our sample. On the other hand, stocks from the financial sector have a significant weight in the BIST100 index that is utilized as a proxy for the market portfolio. As a result, there is a slight misrepresentation bias in our analyses since a market-wide index that excludes the same type of stocks is not available for our sample period. Market beta averages for the value-weighted and equal-weighted portfolios show no notable differences implying that size is not associated with the market beta within each sorted portfolio. Finally, Panel I summarizes the statistics for the ambiguity beta. In all but both of the fifth portfolios, the average ambiguity betas are negative. The negative betas with the large absolute values imply that the stocks included in these portfolios are highly sensitive to ambiguity and their returns change in the opposite direction whenever ambiguity changes in the market. The exceptions to this finding are the fifth portfolios whose average betas are positive and relatively smaller in magnitude. This means that the stocks in this portfolio have higher return on average in the face of higher ambiguity and they are the only stocks that seem to provide a hedge against higher ambiguity in the market.

Table 3.5 Descriptive Statistics of Portfolios Formed on Stock Characteristics

This table summarizes the descriptive statistics of portfolios formed on the stock characteristics of market value, book to market (B/M) ratio, investment, operating profitability, momentum, illiquidity, coskewness, market beta and ambiguity beta. The sample includes stocks listed in Borsa Istanbul from January 1990 to December 2017 excluding stocks in financial sector, closed-end mutual funds, REITs, sports clubs and having negative B/M ratio. Portfolios are formed by grouping stocks into five quantiles in a way that each portfolio contains equal number of stocks depending on relevant stock characteristic. Monthly stock characteristics of portfolios are measured by averaging the values of stocks in two folds: weighted by market value (value weighted) and simple arithmetic average (equal weighted). Descriptive statistics includes number of months (N), mean, standard deviation (St.Dev), skewness and kurtosis of monthly stock characteristics for the period of January 1990 – December 2017.

Panel A includes statistics of monthly market value for five portfolios. Market value of a stock is natural logarithm of the ratio of market value to USDTRY exchange rate where market value of a stock in a given month from July in year t to June in year t+1 equals the market value of June in year t, and USDTRY represents monthly average of exchange rate of Turkish Lira against 1 US Dollar. Market value for each portfolio calculated as simple arithmetic average.

Panel B includes summary statistics of B/M ratio for five portfolios. B/M ratio of a stock in a given month from July in year t to June in year t+1 equals to the ratio of book value to market value in December of year t-1.

Panel C includes summary statistics of investment for five portfolios. Investment of a stock in a given month from July in year t to June in year t+1 equals to percentage change of total asset between December in year t-1 and December in year t-2.

Portfolio	1	2	3	4	5	1	2	3	4	5
Panel A					*Market Value*					
			Value-weighted					Equal-weighted		
N						336	336	336	336	336
Mean						15.50	16.66	17.55	18.48	20.57
St.Dev						0.51	0.50	0.54	0.63	0.82
Skewness						-0.48	-0.11	-0.11	-0.03	-0.17
Kurtosis						2.80	2.18	2.13	1.85	1.72
Panel B					*Book to Market Ratio*					
			Value-weighted					Equal-weighted		
N	336	336	336	336	336	336	336	336	336	336
Mean	0.24	0.47	0.70	0.98	2.18	0.25	0.48	0.70	1.00	2.61
St.Dev	0.11	0.19	0.29	0.40	2.40	0.10	0.20	0.29	0.40	2.54
Skewness	1.79	1.31	1.08	0.92	4.27	2.00	1.10	0.97	0.67	3.77
Kurtosis	8.37	5.80	4.98	4.63	21.36	9.48	4.89	4.58	3.65	18.11
Panel C					*Investment*					
			Value-weighted					Equal-weighted		
N	330	330	330	330	330	336	336	336	336	336
Mean	0.12	0.32	0.45	0.65	2.70	0.08	0.32	0.45	0.65	6.03
St.Dev	0.26	0.31	0.36	0.43	4.46	0.24	0.31	0.36	0.43	14.68
Skewness	0.45	0.50	0.52	0.51	3.11	0.55	0.53	0.51	0.50	3.98
Kurtosis	2.12	1.92	2.02	2.14	12.35	2.01	2.01	2.01	2.17	18.61

Table 3.5

(continued)

Panel D includes summary statistics of operating profitability for five portfolios. Operating profitability of a stock in a given month from July in year t to June in year t+1 equals to the ratio of operating profit to book equity in December of year t-1.

Panel E includes summary statistics of momentum for 5 portfolios. Following Jegadeesh and Titman (1993), momentum of a stock in month t equals to percentage change in stock price between month t-1 and month t-12 ($MOM_{i,t} = (price_{i,t-1}/price_{i,t-12}) \cdot 100 - 100$).

Panel F includes summary statistics of illiquidity for five portfolios. Daily illiquidity measure is calculated through Amihud (2002) measure in which illiquidity equals to the ratio of absolute value of daily stock return to daily trading volume in USD multiplied by 1000 ($ILLIQD_{d,i,t} = |return_{d,i,t}|/volume_{d,i,t}$). Illiquidity of a stock in month t is calculated in multiple steps: i) daily illiquidity measure is turned to monthly measure as simple average for each stock conditional on that a stock has illiquidity data at least 10 days within a month ($ILLIQM_{i,t} = 1/k \cdot \sum_{d=1}^{k} ILLIQD_{d,i,t}$), ii) ILLIQM measure is averaged on a rolling basis for the past 12 months conditional on at least 6 months of illiquidity data is available to smooth out volatility in liquidity and ILLIQA represents yearly illiquidity of a stock in month t.

Portfolio	1	2	3	4	5	1	2	3	4	5
Panel D	*Operating Profitability*									
	Value-weighted					Equal-weighted				
N	336	336	336	336	336	336	336	336	336	336
Mean	-0.24	0.11	0.24	0.39	0.75	-0.24	0.11	0.24	0.38	1.04
St.Dev	0.27	0.12	0.17	0.22	0.39	0.18	0.12	0.17	0.23	0.65
Skewness	-1.91	0.75	0.5	0.42	0.53	-0.74	0.63	0.51	0.45	1.12
Kurtosis	6	2.07	1.6	1.53	2.33	3.17	1.84	1.61	1.56	3.99
Panel E	*Momentum*									
	Value-weighted					Equal-weighted				
N	336	336	336	336	336	336	336	336	336	336
Mean	-0.09	0.23	0.51	0.91	2.12	-0.13	0.22	0.51	0.91	2.3
St.Dev	0.61	0.87	1.09	1.42	2.78	0.59	0.86	1.08	1.42	2.89
Skewness	3.48	3.51	3.21	2.92	3.07	3.49	3.43	3.17	2.93	3.17
Kurtosis	18.37	18.58	15.48	13.22	14.32	18.85	17.71	15.26	13.27	14.52
Panel F	*Illiquidity*									
	Value-weighted					Equal-weighted				
N	336	336	336	336	336	336	336	336	336	336
Mean	0.004	0.025	0.061	0.194	2.918	0.002	0.01	0.032	0.113	8.894
St.Dev	0.01	0.07	0.14	0.39	5.35	0.00	0.02	0.05	0.21	20.23
Skewness	6.80	6.78	5.50	4.73	3.33	3.14	2.94	3.15	3.04	3.64
Kurtosis	59.56	55.15	40.25	32.39	14.57	14.55	12.60	14.42	13.07	17.97

Table 3.5
(continued)
Panel G includes summary statistics of coskewness for five portfolios. Coskewness of a stock in month t is derived from the coefficient of excess squared market return (γ) in the regression of excess return over market excess return and excess squared market return ($r_{i,t}=\alpha_{i,t}+\beta_{i,t}r_{M,t}+\gamma_{i,t}(R^2_{M,t}-R_{f,t})$) following Adesi, Gagliardini, and Urga (2004). Coskewness coefficient is estimated in each month on rolling basis for past 48 months conditional on that there are at least 24 observations.
Panel H includes summary statistics of market beta for five portfolios. Market beta of a stock in month t is derived from the coefficient of excess market return (β^M) in the regression of excess return over market excess return ($r_{i,t}=\alpha_{i,t}+\beta^M_{i,t}r_{M,t}$). Market beta is estimated in each month on rolling basis for past 48 months conditional on that there are at least 24 observations.
Panel I includes summary statistics of ambiguity beta for five portfolios. Ambiguity beta of a stock in month t is derived from the coefficient of excess market return (β^A) in the regression of excess return over estimated ambiguity ($r_{i,t}=\alpha_{i,t}+\beta^A_{i,t}(AMB^E_t)$). Ambiguity beta is estimated in each month on rolling basis for past 48 months conditional on that there are at least 24 observations.

Portfolio	1	2	3	4	5	1	2	3	4	5
Panel G						*Coskewness*				
			Value-weighted					Equal-weighted		
N	325	325	325	325	325	325	325	325	325	325
Mean	0.66	1.17	1.42	1.62	1.92	0.68	1.18	1.42	1.62	1.98
St.Dev	1.19	1.10	1.11	1.14	1.19	1.17	1.10	1.11	1.14	1.22
Skewness	-0.78	-0.45	-0.20	0.04	0.19	-0.83	-0.41	-0.19	0.03	0.13
Kurtosis	2.69	3.92	4.51	4.58	4.33	2.98	3.99	4.54	4.70	3.98
Panel H						Market Beta				
			Value-weighted							
N	325	325	325	325	325	325	325	325	325	325
Mean	0.22	0.38	0.48	0.58	0.76	0.17	0.38	0.48	0.58	0.76
St.Dev	0.14	0.11	0.10	0.09	0.11	0.15	0.11	0.10	0.09	0.11
Skewness	-0.18	-0.03	0.14	0.30	0.61	-0.80	-0.10	0.12	0.30	0.63
Kurtosis	2.95	2.53	2.27	2.46	3.25	3.72	2.50	2.25	2.42	3.36
Panel I						Ambiguity Beta				
			Value-weighted							
N	217	217	217	217	217	217	217	217	217	217
Mean	-0.93	-0.55	-0.32	-0.09	0.45	-0.99	-0.55	-0.32	-0.07	0.51
St.Dev	0.48	0.43	0.40	0.38	0.36	0.48	0.43	0.39	0.38	0.43
Skewness	-0.86	-0.83	-0.64	-0.59	0.34	-0.81	-0.82	-0.70	-0.57	-0.26
Kurtosis	3.35	3.37	2.91	2.53	2.73	3.31	3.38	3.08	2.55	1.95

3.2.3.2.2 Descriptive Statistics of Excess Return in Single-Sorted Stock Characteristics Portfolios

In this part of the study, we focus on the return structure among the characteristics portfolios reviewed in the previous section. Table 3.6 presents the descriptive statistics of monthly excess return of stocks in five sorted-portfolios. Stocks in the portfolios are determined by the methodology described in the previous section. Excess return for each stock is calculated monthly and equals the difference between monthly stock return and the risk-free interest rate. Next, the monthly excess return of each portfolio is calculated by averaging the excess returns of stocks in the portfolio either on an equal- or value-weighted basis.

Panel A of Table 3.6 displays statistics of monthly excess returns for five portfolios sorted on market value. Statistics shows that excess return increases monotonically as market value decreases, consistent with size effect in the literature. This negative relation is more visible among the value-weighted portfolios and supports the argument that size is an important determinant of stock returns. Average excess returns for portfolios sorted on B/M are presented in Panel B. Value-weighted excess returns do not reveal a clear pattern to draw any conclusions regarding the impact of B/M on stock returns. On the other hand, equal-weighted excess returns explicitly show that portfolio return is positively correlated with B/M, consistent with findings in the literature. High B/M ratio means higher book value compared to market value and is generally associated with an underpriced stock compared to a stock with a low B/M ratio. A stock may be underpriced for a number of different reasons but an adverse risk perception against the stock is typically the reason cited in the literature. In this regard, a high B/M ratio means higher risk and it is associated with a higher required return. Panel C presents the average excess return of portfolios sorted on investment. Similar to the B/M sorts, average excess returns do not follow a pattern across the value-weighted portfolios but the average excess return decreases with higher investment in the equal-weighted portfolios, implying that for the smaller firms higher investment might mean lower dividends which would then lead to lower returns within the dividend discount valuation model's framework.

Panel D shows the return structure for operating profitability. The equal-weighted average excess returns do not exhibit a linear pattern across the five portfolios. For the value-weighted portfolios, a nonlinear relationship between operating profitability and excess returns may exist since the first and the fifth portfolios have the highest excess returns, implying a u-shaped relationship. Panel E presents the average excess return for portfolios sorted on momentum. Excess returns seem to increase with higher momentum and smaller firm size since equal-weighted portfolios exhibit slightly higher returns compared to value-weighted portfolios. Panel F summarizes the excess return statistics for portfolios sorted on illiquidity. For both types of returns are consistent with previous findings in the literature, excess return increases as the illiquidity of a portfolio increases. Also, the difference between value- and equal-weighted average excess return for the fifth portfolio suggests that the required return is higher for smaller and more illiquid stocks.

Panel G provides statistics of monthly excess returns portfolios formed on coskewness. There is no linear relationship between excess return and coskewness. However, excess return in the highest coskewness group is higher compared to the group in the lowest group and this contradicts with the findings in the literature. Harvey and Siddique (2000) argue that there should be negative relationship between excess return and coskewness. Since positive skewness and higher contribution to market skewness is desirable, agents are supposed to have higher demand for stocks with higher positive skewness. Therefore, investors should have a lower required return for the portfolio with a higher coskewness. In Panel H, statistics of monthly excess returns for portfolios sorted on market beta are presented. The returns do not exhibit a linear relationship with the market beta and this result is inconsistent with the implications of CAPM. Since the returns are not normally distributed, it may not be surprising to find that the excess returns do not seem to increase monotonously with higher market beta values. Finally, Panel I summarizes statistics of monthly excess returns for portfolios formed on ambiguity beta. For equal-weighted portfolios, there is a monotonous and negative relationship between ambiguity betas and excess returns. Based on the finding in Table 3.5, the average ambiguity betas change from a negative value for the first portfolio to the only positive beta for the fifth portfolio. Combining the results from the two tables, excess returns are higher for negative betas that are larger in magnitude (0.84% return for an ambiguity beta of -0.99) and lower for the positive beta that is smaller in magnitude (0.03% return for an ambiguity beta of 0.51). These results are consistent with expectations since the required return from stocks that are more sensitive to ambiguity should be higher. Likewise, for the fifth portfolio with the positive but relatively smaller magnitude beta, the excess return is lower since this portfolio includes stocks that are less sensitive to ambiguity. With a lower sensitivity to ambiguity (smaller ambiguity beta), these stocks already provide a hedge against increased ambiguity in the market so their required returns are lower because investors would prefer such safer assets in case of higher ambiguity.

In summary, average excess returns among portfolios sorted on stock characteristics supports the existence of a relationship between average excess returns and the stock characteristics of market value, book-to-market ratio, investment, momentum, illiquidity and ambiguity beta. In addition, portfolio level analysis does not provide evidence of a relationship between average excess returns and the stock characteristics of operating profitability, coskewness and market beta. In the following section, we extend our analysis by investigating the relationship between average excess return and stock characteristics in a different setup by forming double sorted portfolios.

Table 3.6: Descriptive Statistics of Excess Return in Single Sorted Stock Characteristics Portfolios

This table summarizes descriptive statistics of stocks' monthly excess returns in the portfolios formed on market value, book to market (B/M) ratio, investment, operating profitability, momentum, illiquidity, coskewness, market beta and ambiguity beta. Monthly return of each stock is calculated as percentage change in average price in current month compared to average price in previous month ($R_{i,t}=(price_{i,t}/price_{i,t-1})\cdot 100-100$). Excess return of each stock is calculated monthly and equals to the difference between stock return calculated from monthly average price and risk free interest rate where risk free interest rate is discounted Turkish Treasury auctions interest rate for the relevant month ($r_{i,t}=R_{i,t}-R_{f,t}$). The sample includes stocks listed in Borsa Istanbul from January 1990 to December 2017 excluding stocks in financial sector, closed-end mutual funds, REITs, sports clubs and having negative B/M. Portfolios are formed by grouping stocks into five quantiles in a way that each portfolio contains equal number of stocks. Monthly excess return of portfolios are measured by averaging the excess returns of stocks in portfolios formed on relevant stock characteristics in two folds: weighted by market value (value weighted) and simple arithmetic average (equal weighted). Descriptive statistics includes number of months (N), mean, standard deviation (St.Dev), skewness and kurtosis of monthly stock returns for the period of January 1990 – December 2017.

Panel A includes statistics of monthly excess returns for five portfolios formed on market value. Market value of a stock in a given month from July in year t to June in year t+1 equals the logarithm of market value of June in year t.

Panel B includes statistics of monthly excess returns for five portfolios formed on B/M ratio. B/M ratio of a stock in a given month from July in year t to June in year t+1 equals to the ratio of book value to market value in December of year t-1.

Panel C includes statistics of monthly excess returns for five portfolios formed on Investment. Investment of a stock in a given month from July in year t to June in year t+1 equals to percentage change of total asset between December in year t-1 and December in year t-2.

Portfolio	1	2	3	4	5	1	2	3	4	5
Panel A	*Market Value*									
	Value-weighted					Equal-weighted				
N	336	336	336	336	336	336	336	336	336	336
Mean	1.11	0.62	0.45	0.30	0.18	1.48	0.62	0.36	0.35	0.33
St.Dev	14.17	12.92	12.76	12.15	11.53	15.14	13.07	12.72	12.21	12.01
Skewness	1.59	1.01	0.85	0.98	0.68	2.49	1.13	0.88	0.99	0.83
Kurtosis	10.44	9.15	7.76	7.64	6.50	18.05	9.46	7.95	7.75	7.41
Panel B	Book to Market Ratio									
	Value-weighted									
N	336	336	336	336	336	336	336	336	336	336
Mean	-0.03	0.50	0.31	0.79	0.74	0.09	0.44	0.51	0.97	1.16
St.Dev	11.18	12.53	13.34	12.85	14.17	12.27	12.09	12.80	12.80	14.41
Skewness	0.69	1.01	1.39	1.03	1.01	1.84	0.84	1.12	1.10	1.61
Kurtosis	6.32	7.79	11.43	7.66	8.02	14.58	7.39	10.17	8.85	11.90
Panel C	Investment									
	Value-weighted									
N	330	330	330	330	330	333	330	330	330	330
Mean	-0.24	0.39	0.12	-0.23	-0.14	0.66	0.38	0.33	0.04	-0.07
St.Dev	12.03	11.83	12.32	11.45	11.66	12.64	11.64	12.03	11.25	12.20
Skewness	0.39	0.58	1.08	0.27	0.85	1.01	0.65	0.77	0.71	0.82
Kurtosis	6.08	5.69	8.69	4.69	7.62	8.81	7.17	8.79	7.08	8.74

Table 3.6
(continued)
Panel D includes statistics of monthly excess returns for five portfolios formed on operating profitability. Operating profitability of a stock in a given month from July in year t to June in year t+1 equals to the ratio of operating profit to book equity in December of year t-1.
Panel E includes statistics of monthly excess returns for 5 portfolios formed on momentum. Following Jegadeesh and Titman (1993), momentum of a stock in month t equals to percentage change in stock price between month t-1 and month t-12 ($MOM_{i,t} = (price_{i,t-1}/price_{i,t-12})\cdot 100-100$).
Panel F includes statistics of monthly excess returns for five portfolios formed on illiquidity. Daily illiquidity measure is calculated through Amihud (2002) measure in which illiquidity equals the ratio of absolute value of daily stock return to daily trading volume in USD multiplied by 10^3 ($ILLIQD_{d,i,t} = |return_{d,i,t}|/volume_{d,i,t}$). Illiquidity of a stock in month t is calculated in multiple steps: i) daily illiquidity measure is turned to monthly measure as simple average for each stock conditional on that a stock has illiquidity data at least 10 days within a month ($ILLIQM_{i,t} = 1/k \cdot \sum_{d=1}^{k} ILLIQD_{d,i,t}$), ii) ILLIQM measure is averaged on for the past 12 months on a rolling basis conditional on at least 6 months of illiquidity data is available to smooth out volatility in liquidity and ILLIQA represents illiquidity of a stock in month t.

Portfolio	1	2	3	4	5	1	2	3	4	5
Panel D					*Operating Profitability*					
			Value-weighted					Equal-weighted		
N	336	336	336	336	336	336	336	336	336	336
Mean	0.74	0.40	-0.14	0.02	0.24	0.64	0.60	0.54	0.47	0.55
St.Dev	13.70	13.44	11.68	11.49	11.96	13.19	12.84	12.50	12.25	12.37
Skewness	1.14	1.57	0.71	0.56	0.60	1.21	1.17	1.63	0.96	0.96
Kurtosis	7.98	11.27	6.34	6.54	6.62	9.46	9.03	11.98	8.47	9.18
Panel E					Momentum					
			Value-weighted							
N	336	336	336	336	336	336	336	336	336	336
Mean	-0.87	-0.47	0.33	0.47	1.02	-0.17	0.18	0.66	0.93	1.55
St.Dev	13.70	12.99	12.12	12.17	12.58	14.00	12.87	12.41	11.97	12.42
Skewness	1.29	1.03	0.78	0.80	0.62	1.52	1.34	0.88	0.85	0.85
Kurtosis	7.98	7.69	5.69	8.11	7.37	9.53	9.90	7.85	8.64	9.21
Panel F					Illiquidity					
			Value-weighted							
N	336	336	336	336	336	336	336	336	336	336
Mean	0.20	0.13	0.27	0.92	0.62	0.20	0.10	0.33	0.85	1.31
St.Dev	11.72	12.65	12.26	13.33	12.58	12.57	12.76	12.67	13.12	12.86
Skewness	0.69	0.94	0.78	1.67	1.31	0.88	0.95	0.96	1.48	1.31
Kurtosis	5.91	7.59	7.37	12.60	10.14	7.15	7.98	8.40	12.09	10.79

Table 3.6
(continued)
Panel G includes statistics of monthly excess returns for five portfolios formed on coskewness. Coskewness of a stock in month t is derived from the coefficient of excess squared market return (γ) in the regression of excess return over market excess return and excess squared market return ($r_{i,t}=\alpha_{i,t}+\beta_{i,t}r_{M,t}+\gamma_{i,t}(R^2_{M,t}-R_{f,t})$ following Adesi, Gagliardini, and Urga (2004). Coskewness coefficient is estimated each month on rolling basis for past 48 months conditional on that there are at least 24 observations.
Panel H includes statistics of monthly excess returns for five portfolios formed on market beta. Market beta of a stock in month t is derived from the coefficient of excess market return (β^M) in the regression of excess return over market excess return ($r_{i,t}=\alpha_{i,t}+\beta^M_{i,t}r_{M,t}$). Market beta is estimated each month on rolling basis for past 48 months conditional on that there are at least 24 observations.
Panel I includes statistics of monthly excess returns for five portfolios formed on ambiguity beta. Ambiguity beta of a stock in month t is derived from the coefficient of excess market return (β^A) in the regression of excess return over estimated ambiguity ($r_{i,t}=\alpha_{i,t}+\beta^A_{i,t}(AMB^E_t)$). Estimated ambiguity is derived by AR(1) model. Ambiguity beta is estimated each month on rolling basis for past 48 months conditional on that there are at least 24 observations.

Portfolio	1	2	3	4	5	1	2	3	4	5
Panel G					*Coskewness*					
			Value-weighted					Equal-weighted		
N	325	325	325	325	325	325	325	325	325	325
Mean	-0.43	0.33	0.44	0.07	0.49	-0.23	0.33	0.95	0.53	0.8
St.Dev	9.13	11.29	12.4	12.36	13.88	9.55	11.32	12.56	12.77	14.52
Skewness	0.15	0.63	0.69	0.37	0.96	0.28	0.65	0.94	0.75	1.39
Kurtosis	5.53	5.42	5.85	5.25	8.54	7.33	7.74	7.51	7.78	10.96
Panel H					*Market Beta*					
			Value-weighted					Equal-weighted		
N	324	324	324	324	324	324	324	324	324	324
Mean	-0.28	0.37	0.49	0.21	0.34	0.37	0.57	0.77	0.59	0.23
St.Dev	10.27	11.15	11.6	12.46	12.71	10.73	11.93	12.26	12.28	13.41
Skewness	0.41	1.04	0.83	0.67	0.45	0.48	1.07	0.83	0.74	0.95
Kurtosis	6.04	8.07	7.16	6.63	4.69	7.49	9.38	8.22	7.58	8.15
Panel I					*Ambiguity Beta*					
			Value-weighted					Equal-weighted		
N	217	217	217	217	217	217	217	217	217	217
Mean	0.49	0.55	0.18	-0.18	-0.45	0.84	0.58	0.51	0.21	0.03
St.Dev	11.56	10.14	9.60	8.25	8.52	10.60	10.16	9.84	9.46	9.64
Skewness	1.38	1.41	1.64	0.87	1.52	0.82	1.59	2.17	1.67	1.59
Kurtosis	14.14	13.39	13.13	9.5	12.73	10.99	15.6	19.66	16.04	14.8

3.2.3.3 Double-Sorted Portfolios

The evidence based on single-sorted portfolios suggests that average excess return differs between equal- and value-weighted portfolios and there is typically a monotonic relationship between average excess return and the firm's size or market value. In this section of the study, in order to analyze the effect of size further, we use two-dimensional sorts to form portfolios where market value is always the first dimension and a second stock characteristic is added for classifying stocks into the "double-sorted" portfolios. More specifically, first, portfolios are formed by grouping stocks into big and small groups where the median market value is used as the cutoff. Next, for the big and small groups separately, stocks are classified again into three groups as the lowest 30%, medium 40% and highest 30% according to the second stock characteristic used. Portfolios are rebalanced monthly in a similar fashion described in the previous section and value-weighted returns are calculated for the portfolios.

In Panel A of Table 3.7, the average excess returns for the size-B/M double-sorted portfolios are presented. The lowest B/M group is referred to as the "growth" stocks and the highest B/M groups is referred to as the "value" stocks in the literature. Excess returns increase as the B/M ratio increases suggesting that value firms may be perceived to be more risky and command a higher return in the market. Also, excess returns in the small group are lower except for the small-high classification for which the returns are larger compared to the big-high classification. In Panel B, stocks are double-sorted on size and investment. Firms with high investment are classified as "aggressive" and low investment stocks are classified as "conservative" in the literature. There are no observable patterns between the investment groups or the big versus small stocks. This is a similar result to the single-sort portfolios and the investment factor does not seem to be a determinant of returns in the market. In Panel C, highest and lowest operating profitability stocks are classified as "robust" and "weak" respectively within big and small groups. Unlike the single-sorted portfolios with no apparent relationship between excess return and operating profitability, in the double-sort portfolios, for small firms, excess return is higher for stocks with robust operating profitability and for big firms, excess return is higher for firms with weak operating profitability. Since the return relationship is not monotonous across the groups, it is not possible to draw a conclusion about the effect of profitability on small and big stock returns.

In Panel D, the size-momentum portfolio results show that the momentum effect is relevant for small stocks and a possible contrarian effect is observed for the big stocks. Small stock returns change from positive to negative between winner and loser portfolios. Big stock returns are all negative, suggesting that a contrarian investment strategy would generate profits. Panel E summarizes the results for the size-illiquidity double-sorted portfolios. The increasing impact of illiquidity on excess returns is observed for both small and big stocks but for big firms the difference in excess returns of liquid and illiquid stocks is smaller compared to smaller stocks. This is consistent with the single-sort results and suggests that illiquidity is a more important concern for small stocks. In Panel F, coskewness and average excess returns exhibit a positive relationship, especially for small stocks. This pattern was not observable with the single-sort portfolios and suggests that the possibility of more extreme returns in comparison to the market's average return is perceived to be a more relevant risk for the small stocks. In Panel G, excess returns for portfolios sorted on size and market beta exhibit no apparent pattern, consistent with the results from the single-sort portfolios. Once again, deviations from normality of returns may be the reason why market beta does not seem to have a relationship with excess returns. Finally, in Panel H, portfolios sorted on size and ambiguity betas confirm that the negative relationship between ambiguity beta and excess return is observable, similar to the single-sort portfolios. The stocks in the low ambiguity beta portfolio (A^-) have negative ambiguity betas that are relatively large in magnitude. Therefore, these are the stocks that have the highest sensitivity to changes in ambiguity. As a result, excess returns are higher for these stocks in both the small and large stock groups.

In summary, the results from the double-sorted portfolios reveal that only the book-to-market ratio and ambiguity beta have monotonic relationship with average excess returns. Disappearance of the relationship between excess returns and other stock characteristics implies that these stock characteristics capture the impact market value in the analysis of single-sorted portfolios.

3.2.4 Testing the Significance of Ambiguity

In testing the significance of ambiguity, we follow Bali, Brown, and Tang (2017). Similar to other studies in the literature, they use both time series and cross-sectional regression analyses in testing the explanatory power of stock characteristics on stock returns. In time series analysis, returns of portfolios sorted on different stock characteristic are regressed on returns of factor-mimicking portfolios derived from the relevant stock characteristics. Significance of the intercept term is evaluated as evidence of unique information that may be provided by the stock characteristic, in our case ambiguity sensitivity that is left out of the model. In cross-sectional testing, individual stock returns are regressed on lagged stock characteristics and the statistical significance of the regression coefficients provide evidence regarding the power of the stock characteristics in explaining the cross-section of stock returns. Moreover, the return structure from single- and double-sorted portfolios demonstrate that stock characteristics are associated with cross-sectional stock return distributions at different levels. This implies that there are factors explaining stock returns other than ambiguity. Therefore, the significance of ambiguity sensitivity in estimating stock returns needs to be tested by controlling for other stock characteristics simultaneously. In the following analyses, the Fama-French 5 factors, momentum, illiquidity and coskewness are included in the models alongside the ambiguity sensitivity.

Table 3.7: Descriptive Statistics of Excess Return on Double-Sorted Portfolios

This table summarizes the information of monthly stocks' excess returns in 2x3 form double sorted portfolios where size is fixed and other factor varies from panel to panel. Changing factors are book to market (B/M) ratio, investment, operating profitability, momentum, illiquidity, coskewness, market beta and ambiguity beta. Monthly return of each stock is calculated as percentage change in average price in current month compared to average price in previous month ($R_{i,t}$ =($price_{i,t}/price_{i,t-1}$)·100-100). Excess return of each stock is calculated monthly and equals to the difference between monthly average stock return and risk free interest rate where risk free interest rate is discounted Turkish Treasury auctions interest rate for the relevant month ($r_{i,t} = R_{i,t} - R_{f,t}$). The sample includes stocks listed in Borsa Istanbul from January 1990 to December 2017 excluding stocks in financial sector, closed-end mutual funds, REITs, sports clubs and having negative B/M ratio. Portfolios are formed by first grouping stocks into 2 quantiles according to size as big and small. Stocks in small and big are divided in two 3 groups as first 30%, medium 40% and last 30% for B/M ratio, investment, operating profitability, momentum, illiquidity, coskewness, market beta and ambiguity beta. Monthly excess return of portfolios are calculated by averaging stock excess returns weighted by market value of stocks. Descriptive statistics includes number of months (N), mean, standard deviation (St.Dev), skewness and kurtosis of monthly stock returns for the period of January 1990 – December 2017. Market value of a stock in a given month from July in year t to June in year t+1 equals the market value of June in year t.

Panel A includes statistics of monthly excess returns for six portfolios formed on market value and B/M. B/M of a stock in a given month from July in year t to June in year t+1 equals to the ratio of book value to market value in December of year t-1.

Panel B includes statistics of monthly excess returns for six portfolios formed on market value and investment. Investment of a stock in a given month from July in year t to June in year t+1 equals to percentage change of total asset between December in year t-1 and December in year t-2.

Panel C includes statistics of monthly excess returns for six portfolios formed on market value and investment. Operating profitability of a stock in a given month from July in year t to June in year t+1 equals to the ratio of operating profit to book equity in December of year t-1.

	Small			Big		
Panel A	Book to Market Value (B/M)					
	Low	Medium	High	Low	Medium	High
N	336	336	336	336	336	336
Mean	-0.02	0.23	0.98	0.15	0.35	0.69
St.Dev	12.72	12.61	14.04	11.29	12.92	14.18
Skewness	1.29	0.6	1.39	0.75	1.07	1.37
Kurtosis	9.32	7.08	11.01	6.66	8.37	10.42
Panel B	Investment					
	Aggressive	Medium	Conservative	Aggressive	Medium	Conservative
N	330	330	330	330	330	330
Mean	-0.21	0.43	0.2	-0.12	-0.01	-0.07
St.Dev	12.36	12.48	12.47	11.34	11.44	11.57
Skewness	0.5	0.85	0.86	0.57	0.47	0.46
Kurtosis	7.1	8.06	8.99	6.12	5.43	6.55
Panel C	Operating Profitability					
	Robust	Medium	Weak	Robust	Medium	Weak
N	336	336	336	336	336	336
Mean	0.68	0.44	0.36	0.17	-0.12	0.76
St.Dev	12.74	12.54	13.88	11.45	11.41	13.51
Skewness	0.65	0.89	1.38	0.63	0.67	1.07
Kurtosis	7.73	7.89	9.41	6.34	6.49	7.54

Table 3.7
(continued)
Panel D includes statistics of monthly excess returns for six portfolios formed on market value and Momentum. Momentum of a stock in month t equals to percentage change in stock price between month t-1 and month t-12 ($MOM_{i,t} = (price_{i,t-1}/price_{i,t-12}) \cdot 100-100$).
Panel E includes statistics of monthly excess returns for six portfolios formed on market value and illiquidity. Illiquidity of a stock in month t is calculated from daily illiquidity measure as simple average if a month contains at least 10 days illiquidity data. Daily illiquidity measure is calculated through Amihud (2002) measure in which illiquidity equals the ratio of absolute value of daily stock return to daily trading volume in USD multiplied by 10^3 ($ILLIQ_{i,t} = 1/k \cdot \sum_{d=1}^{k} return_{d,i,t}/volume_{d,i,t}$).
Panel F includes statistics of monthly excess returns for six portfolios formed on market value and coskewness. Coskewness of a stock in month t is derived from the coefficient of excess squared market return (γ) in the regression of excess return over excess market return and excess squared market return ($r_{i,t}=\alpha_{i,t}+\beta_{i,t}r_{M,t}+\gamma_{i,t}(R_{M,t}^2-R_{f,t})$) following Adesi, Gagliardini, and Urga (2004). Coskewness coefficient is estimated each month on rolling basis for past 48 months conditional on that there are at least 24 observations.
Panel G includes statistics of monthly excess returns for six portfolios formed on market value and market beta. Market beta of a stock in month t is derived from the coefficient of excess market return (β^M) in the regression of excess return over excess market return ($r_{i,t}=\alpha_{i,t}+\beta_{i,t}^M r_{M,t}$). Market beta is estimated each month on rolling basis for past 48 months conditional on that there are at least 24 observations.

	Small			Big		
Panel D			Momentum			
	Winner	Medium	Loser	Winner	Medium	Loser
N	335	333	336	334	333	331
Mean	0.1	0.03	-0.61	-0.32	-1.11	-0.3
St.Dev	13.55	14.32	15.6	13.48	13.55	17.43
Skewness	1.14	1.06	1.77	1.05	1	3.1
Kurtosis	8.35	7.24	11.86	7.69	7.3	22.39
Panel E			Illiquidity			
	Liquid	Medium	Illiquid	Liquid	Medium	Illiquid
N	334	336	336	336	336	334
Mean	-0.67	0.35	0.95	0.26	0.24	0.7
St.Dev	14.86	12.87	12.8	11.81	11.98	13.53
Skewness	1.61	0.8	1.05	0.65	0.94	2.23
Kurtosis	11.3	7.44	10.29	5.82	8.4	17.21
Panel F			Coskewness			
	S^-	Medium	S^+	S^-	Medium	S^+
N	325	325	325	325	325	325
Mean	-0.39	0.69	0.81	-0.22	0.31	0.26
St.Dev	10.81	12.72	15.49	9.44	11.79	13.15
Skewness	0.26	0.88	1.32	0.3	0.66	0.82
Kurtosis	7.01	7.76	9.85	5.16	5.6	7.7
Panel G			Market Beta			
	Low	Medium	High	Low	Medium	High
N	324	324	324	324	324	324
Mean	0.32	0.66	0.18	0.03	0.3	0.18
St.Dev	11.97	12.95	13.93	10.43	11.21	12.26
Skewness	0.93	0.8	0.98	0.74	0.65	0.47
Kurtosis	8.68	7.9	9.09	6.5	6.09	5.12

Table 3.7
(continued)
Panel H includes statistics of monthly excess returns for 6 portfolios formed on market value and ambiguity beta. Ambiguity beta of a stock in month t is derived from the coefficient of excess market return (β^A) in the regression of excess return over estimated ambiguity ($r_{i,t}=\alpha_{i,t}+\beta^A_{i,t}(AMB^E_t)$). Estimated ambiguity is derived by AR(1) model. Ambiguity beta is estimated each month on rolling basis for past 48 months conditional on that there are at least 24 observations.

	Small			Big		
Panel H	Ambiguity Beta					
	A^-	Medium	A^+	A^-	Medium	A^+
N	217	217	217	217	217	217
Mean	0.84	0.43	-0.07	0.55	0.02	-0.19
St.Dev	10.9	10.5	9.95	10.86	9.17	8.15
Skewness	1.33	1.8	1.11	1.17	1.38	1.53
Kurtosis	12.43	17.17	12.49	11.87	12.5	12.67

3.2.4.1 Time Series Testing

In time series testing, we need the returns from the factor-mimicking portfolios, in other words “risk factors”, and we follow Fama and French (2015) in deriving these series. More specifically, risk factors are the average stock returns from the factor mimicking portfolios that formed by going long in a portfolio containing stocks with high values of the stock characteristic in question and short in a portfolio containing stocks with low values of the same stock characteristic. Since factor mimicking portfolios contain both long and short positions, they are also called “zero investment portfolios”.

3.2.4.1.1 Factor Formation

We form zero investment risk factors for size, B/M ratio, investment, operating profitability, momentum, illiquidity, coskewness and ambiguity beta. Following Fama and French (2015), we separate the impact of size and other factors from each other by using double-sorted portfolios. As described in the previous section, double-sorted portfolios are formed by first classifying stocks based on size into big (B) and small (S) groups and then re-classifying each size group into three groups based on the first 30%, the next 40% and the top 30% of the second stock characteristic’s value. The formation of each factor-mimicking portfolio is described below (Panel A of Table 3.8):

i. SMB (Small Minus Big): The SMB factor proxies the risk related with firm size and is equal to the difference between the average excess return of small stocks (S) versus big stocks (B). As the calculation steps shown in Panel A of Table 3.8, SMB is the average of multiple SMB factors calculated by isolating the effects of other stock characteristics[40]. For example, SMB_{BM} is the return on the factor mimicking portfolio for size but it also separates the impact of B/M by taking the difference between the simple averages of small and big portfolios with about the same weighted-average B/M as follows: SMB_{BM} = ((SH + SM + SL)/3 - (BH + BM + BL)/3). In this calculation, BH, BM, BL, SH, SM and SL are six size-B/M double-sorted portfolios where H stands for high B/M, M for medium B/M and L for low B/M. Fama and French (1993) argue that this methodology helps to focus on the impact of size and isolate the impact of the B/M ratio in forming the risk factor. SMB for other stock characteristics[41] are calculated in a similar way as shown in Panel A of Table 3.8. Finally, the SMB factor is calculated as the simple arithmetic average of all the sub-category SMBs.

ii. HML (High Minus Low): The HML factor proxies the risk related with perceived growth opportunities and is calculated as the difference in the excess returns of the portfolios consisting stocks with high and low B/M ratios within the small and big size groups: HML = (SH + BH)/2 - (SL + BL)/2. In this calculation, the big (B) and small (S) as well as the high (H) and low (L) break points are the same as the previous section.

iii. RMW (Robust Minus Weak): The RMW factor proxies the risk related with operating profitability and it is calculated as the difference in excess returns between the portfolios of stocks with robust and weak operating profitability

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[40] Since we follow Fama and French (2015), we compute SMB by isolating from other stock characteristics and isolating other risk factors from size. We do not repeat the same procedure for the other characteristics, for instance we do not isolate the HML factor from INV, MOM or from other stock characteristics since such additional classifications are not feasible due to the limited number of stocks in our sample.

[41] Although we also have ambiguity sensitivity as an additional risk factor, we do not include it in deriving the SMB factor because ambiguity sensitivity is available only for a sub-period of our sample.

within the small and big size groups: RMW = (SR + BR)/2 - (SW + BW)/2. In this calculation, the big (B) and small (S) as well as the robust (R) and weak (W) break points are the same as the previous section.

iv. CMA (Conservative Minus Aggressive): The CMA factor proxies the risk related with the propensity of the firm to make investments and it is calculated as the difference in excess returns between the portfolios of stocks with conservative and aggressive investments within the small and big size groups: CMA = (SC + BC)/2 - (SA + BA)/2. In this calculation, the big (B) and small (S) as well as the conservative (C) and aggressive (A) break points are the same as the previous section.

v. WML (Winner Minus Loser): The WML factor proxies the risk related with the return momentum that implies past winners to be the future winners and the past losers to stay as the future losers. The returns for this factor are calculated as the difference in excess returns between the portfolios of stocks with high momentum (winner) and low momentum (loser) within the small and big size groups: WML = (SWinner + BWinner)/2 - (SLoser + BLoser)/2. In this calculation, the big (B) and small (S) as well as the winner (Winner) and loser (Loser) break points are the same as the previous section.

vi. COSKW (S^+ Minus S^-): The COSKW factor measures the risk associated with a stock's contribution to the skewness of the market return. In the literature, investors are expected to prefer a higher coskewness since such stocks will have the potential of generating higher returns. Results from the double-sort analysis are not consistent with these expectations so we calculate COSKW as the difference in excess returns between portfolios containing stocks with high and low coskewness within the small and big size groups: COSKW = $(SS^+ + BS^+)/2 - (SS^- + BS^-)/2$. In this calculation, the big (B) and small (S) as well as the high coskewness (S^+) and low coskewness (S^-) break points are the same as the previous section.

vii. ILLIQ (Illiquid Minus Liquid): The ILLIQ factor proxies the risk related with illiquidity and is calculated as the difference in excess returns between the portfolios of stocks with high illiquidity (low liquidity) and low illiquidity (high liquidity) within the small and big size groups: ILLIQ

=(SIlliq + BIlliq)/2 - (SLiq + BLiq)/2. In this calculation, the big (B) and small (S) as well as the illiquid (Illiq) and liquid (Liq) break points are the same as the previous section.

viii. AMBG (A^+ Minus A^-): The AMBG factor proxies the risk related with a stock's sensitivity to ambiguity and is calculated as the difference in excess returns between the portfolio of stocks with high ambiguity betas and low ambiguity betas within the small and big size groups: AMBG =(SA^+ + BA^+)/2 - (SA^- + BA^-)/2. In this calculation, the big (B) and small (S) as well as the high ambiguity sensitivity (A^+) and low ambiguity sensitivity (A^-) break points are the same as the previous section.

Panel B of Table 3.8 presents the correlations between the risk factors that are calculated as described above and correlations that are significant at the 1% level are marked with a star. Market factor has a positive correlation with HML and COSKW, and a negative correlation with RMW, ILLIQ and AMBG. Since the size and other factors are calculating by separating the impact of size on individual factors and other factors on size, correlations between SMB and other factors are small, reflecting the impact of the calculation methodology. Negative correlations between the market factor and HML is consistent with Cakici, Fabozzi, and Tan (2013) implying that value firms are less sensitive to market movements. On the other hand, positive correlation between SMB and HML contradicts with the results of studies on both developed and emerging markets (Fama and French (2015), Cakici, Fabozzi, and Tan (2013), Danışoğlu (2017)). Highest correlation among the factors is between the market factor and COSKW.

In Panel C of Table 3.8, descriptive statistics for the risk factors are presented. Statistics show that the average return for all risk factors are positive except for the RMW. HML, COSKW and ILLIQ have the highest mean return as well as the highest standard deviation and kurtosis. In fact, all risk factors have high kurtosis implying that the distribution of returns on these portfolios are leptokurtic. Excluding RMW, CMA and WML, distributions of all risk factors are positively skewed. In the last three rows of Panel C, the results for normality, stationarity and autoregressive conditional heteroscedasticity (ARCH) effects are presented for the factor series. The Shapiro-Wilk test rejects normality for all the risk factors. The Augmented Dickey Fuller test rejects the null hypothesis of a unit root for all risk factors, implying that all series are stationary. Since stock return series demonstrate changing volatility through time, risk factors are also tested for changing variance, or the ARCH effect. The LM test rejects the null of no ARCH effect for all risk factors, except for WML, implying that heteroscedasticity needs to be addressed when the risk factors are used in estimations.

3.2.4.1.2 Spanning Tests of Risk Factors

Before going into the time series tests of ambiguity in explaining stock returns, we first analyze the significance of risk factors by testing whether each risk factor can be explained or spanned by the remaining risk factors. Such a preliminary test is useful because the contribution of a risk factor in the asset pricing model is closely related with the amount of marginal information each factor provides. We follow Fama and French (2017) and test the distinctive information content of risk factors by regressing each risk factor on the remaining risk factors and test the statistical significance of the intercept term in the regressions. Since risk factors diverge from normality and show ARCH effects, regressions are estimated by GMM and t statistics are corrected by Newey-West at one lag. In Equation (3.37), $\text{factor}_{i,t}$ represents the return for the i^{th} factor in month t, α_i and $\beta_{i,j}$ represent intercept term of i^{th} factor and the sensitivity of i^{th} factor to j^{th} factor, respectively, where factors are market factor, SMB, HML, RMW, CMA, WML, COSKW, ILLIQ and AMBG.

$$\text{factor}_{i,t}=\alpha_i+\sum_{j=1,\, j\neq i}^{k}\beta_{i,j}\text{factor}_{j,t}+\varepsilon_{i,t} \tag{3.37}$$

Table 3.9 summarizes the coefficients and their t-values from the regressions for each factors. Each row of the Table 3.9 shows separate regressions and each column includes the coefficients and t-values for the intercept and the regressors consisting of all factors excluding the one chosen as the dependent variable. Significance of the intercept term implies that the risk factor has information content that is not spanned by the other factors. The intercept terms for all factors but the SMB and CMA are significant implying that the individual factors have information that is not covered by the remaining factors in the regression. It should be noted that different regression setups may generate different results so it is important not to rely solely on the intercept term in making decisions about which factors carry distinctive information. In addition, Fama and French (2017) argue that redundancy or significance of one factor in one sample period does not mean the factor continues to keep the same status in other periods. Therefore, spanning test results are valid for the sample at hand and should be evaluated carefully. Spanning test results in this study support the validity of HML, RMW, WML, COSKW, ILLIQ and AMBG as risk factors to be used in asset pricing models.

3.2.4.1.3 Competing Models on Double-Sorted Portfolios

After we test for the significance of distinct information included in the risk factors, we extend our analysis to testing the significance of risk factor combinations in explaining stock returns at the portfolio level. For this purpose, we use the double-sorted portfolios formed in the previous section. In addition, as a robustness check, we re-construct the portfolios and generate double-sorted portfolios by classifying stocks into three categories based on all the stock characteristics (except for ambiguity sensitivity) and re-sorting each class based on the previously determined three categories of the second stock characteristic to generate excess return series for nine portfolios.

Following Fama and French (2015), we test the success of specific factor combinations in explaining return differential by testing jointly the statistical significance of intercepts using the GRS (Gibbons, Ross, and Shanken) test statistic from the regression of double-sorted portfolio returns on different factor models. Equation (3.38) presents the calculation of the GRS test statistic where T is the number of observations, N is the number of portfolios, L is the number of factors

Table 3.8: Factor Formation and Descriptive Statistics of Risk Factors

Panel A shows formation of factor portfolios. Stocks are assigned into 2x3 double and independently sorted portfolios, that is, two size groups and three groups of either of book to market (B/M) ratio, investment (INV), operating profitability (OP), momentum (MOM), illiquidity (ILLIQA), coskewness (COSKEW) and ambiguity beta (β^A) groups. Abbreviations should be read as follows: First letter represents 2 size group and S is for small and B is for big and following letters represent 3 groups depending on sorting factor in a way that first group consists of first 30%, second group consists of medium 40% and third group consists of highest 30%. In B/M grouping H is for high B/M, M is for medium B/M and L is for low; in OP grouping, R is for robust OP, M is for medium OP and W is for weak OP; in INV grouping, C is for conservative INV, M is for medium INV and A is for aggressive INV; in WML grouping Winner is for winner MOM, M is for medium MOM and Loser for loser MOM; in COSKW grouping S^+ is for high COSKEW, M is for medium COSKEW and S^- is for low COSKEW; in ILLIQ grouping Illiq is for high ILLIQA, M is for medium ILLIQA and Liq is for low ILLIQA; in β^A grouping A^+ is for high ambiguity beta and A^- is for low ambiguity beta.

The risk factors are as follows: r_M (excess market return), SMB (small minus big), HML (high minus low B/M), RMW (robust minus weak OP), CMA (conservative minus aggressive INV), WML (winners minus losers MOM), COSKW (S^+ minus S^- COSKEW), ILLIQ (high minus low ILLIQA), AMBG (A^+ minus A^-).

Panel B shows correlations between risk factors of excess market return (r_M), SMB, HML, RMW, CMA, WML, COSKW, ILLIQ and AMBG for the period of January 1990 – December 2017.

Panel C summarizes descriptive statistics of risk factors (r_M, SMB, HML, RMW, CMA, WML, COSKW, ILLIQ and AMBG). Descriptive statistics includes number of months (N), mean, standard deviation (St.Dev), skewness, kurtosis, and significance levels from Shapiro-Wilk normality test, ADF test and ARCH LM test of monthly return premiums of risk factors for the period of January 1990 – December 2017.

Panel A	*Breakpoints*	*Factors and Their Components*
	MV: median	$SMB_{B/M}$=(SH + SM + SL)/3 - (BH + BM + BL)/3
		SMB_{OP}=(SR + SM + SW)/3 - (BR + BM + BW)/3
		SMB_{INV}=(SC + SM + SA)/3 - (BC + BM + BA)/3
		SMB_{MOM}=(SWin + SM + SLos)/3 - (BWin + BM + BLos)/3
		SMB_{COSKW} =(SS^+ + SM + SS^-)/3 - (BS^+ + BM + BS^-)/3
		SMB_{ILLIQ} =(SIlliq + SM + SLiq)/3 - (BIlliq + BM + BLiq)/3
		SMB=($SMB_{B/M}$+SMB_{OP}+SMB_{INV}+SMB_{MOM}+SMB_{COSKW}+ SMB_{ILLIQ})/6
	B/M: 30th and 70th percentiles	HML=(SH + BH)/2 - (SL + BL)/2
	OP: 30th and 70th percentiles	RMW=(SR + BR)/2 - (SW + BW)/2
	INV: 30th and 70th percentiles	CMA=(SC + BC)/2 - (SA + BA)/2
	MOM: 30th and 70th percentiles	WML=(SWinner + BWinner)/2 - (SLoser + BLoser)/2
	COSKEW: 30th and 70th percentiles	COSKW=(SS^+ + BS^+)/2 - (SS^- + BS^-)/2
	ILLIQA: 30th and 70th percentiles	ILLIQ=(SIlliq + BIlliq)/2 - (SLiq + BLiq)/2
	β^A: 30th and 70th percentiles	AMBG=(SA^+ + BA^+)/2 - (SA^- + BA^-)/2

Panel B	*Correlations Among Risk Factors*								
	r_M	SMB	HML	RMW	CMA	WML	COSKW	ILLIQ	AMBG
r_M	1								
SMB	-0.12	1							
HML	0.22*	0.17*	1						
RMW	-0.23*	0.07	-0.12	1					
CMA	0.06	0.07	0.28*	-0.15*	1				
WML	-0.11	-0.14	-0.27*	0.12	-0.08	1			
COSKW	0.47*	0.07	0.32*	-0.32*	0.17*	-0.27*	1		
ILLIQ	-0.28*	0.22*	0.26*	0.08	0.11	0.02	-0.22*	1	
AMBG	-0.26*	-0.07	-0.05	0.29*	0.03	0.05	-0.36*	0.24*	1
Panel C	*Descriptive Statistics of Risk Factors*								
N	336	336	336	336	330	329	325	332	217
Mean	0.26	0.18	0.77	-0.14	0.23	0.25	0.84	0.80	-0.82
St.Dev	13.86	5.30	5.99	5.09	4.14	7.12	6.40	7.01	5.26
skewness	1.05	0.70	0.17	-1.30	-0.51	-2.15	1.56	0.39	0.02
kurtosis	7.56	6.23	8.47	7.01	8.35	13.48	11.10	8.95	6.04
Shapiro-Wilk	0	0	0	0	0	0	0	0	0
ADF	0	0	0	0	0	0	0	0	0
ARCH LM	0.13	0.001	0	0	0	0.17	0	0.02	0.34

in the regression, $\hat{a}$ is the vector of intercept terms, $\hat{\Sigma}$ and $\hat{\Omega}$ are variance-covariance matrices of residuals and risk factors respectively. The GRS statistic has an F distribution and tests the null hypothesis that the intercepts from multiple regressions of a given dependent variable are jointly equal to zero.

$$\mathrm{GRS}= \left(\frac{T}{N}\right)\left(\frac{T-N-L}{T-L-1}\right)\left[\frac{\hat{a}'\hat{\Sigma}^{-1}\hat{a}}{1+\bar{\mu}'\hat{\Omega}^{-1}\bar{\mu}}\right] \tag{3.38}$$

Table 3.10 displays the average of absolute values of the intercept terms, the associated GRS test statistics and their significance levels. Each panel of the table presents the results of double-sorted portfolios constructed by a different combination of stock characteristics. We estimate Equation (3.39) for each row in the table depending on the factors of model specification. Since the distribution of risk factors do not satisfy the assumptions of an OLS regression, we estimate the regressions by GMM and the t statistics are corrected by Newey-West at one lag. In the equation, $r_{p,t}$ is the excess return of portfolio p and equals the difference between the return of the double-sorted portfolio p and the risk-free interest rate ($R_{p,t}$-$R_{f,t}$) and $r_{M,t}$ is the excess return of the market portfolio. $\mathrm{factor}_{i,t}$ represents return for the i^{th} risk factor in month t and $\beta_{p,j}$ is the coefficient of the j^{th} risk factor representing the sensitivity of the excess return of portfolio p to the j^{th} risk factor where factors are market factor, SMB, HML, RMW, CMA, WML, COSKW, ILLIQ and AMBG. Since there are nine[42] double-sorted portfolios for each couple of the stock characteristics, each regression model (one factor, three-factor, etc) is estimated nine times, once for each of the nine portfolios. The GRS test statistic gives the joint significance of all intercepts (α_p) from these nine regressions (H_0: $\alpha_p=0$, $\forall$ p=1,..,9).

$$r_{p,t}= \alpha_p+ \beta_{p,1} r_{M,t} + \sum_{j=2}^{k} \beta_{p,j}\mathrm{factor}_{j,t} + \varepsilon_{p,t} \tag{3.39}$$

[42] The number of double-sorted portfolios is six if size is included in one of the double sorting.

Table 3.9: Spanning Test Results Of Risk Factors

Table shows the spanning test results for the period of January 1990 – December 2017. In each row, one factor is regressed over remaining factors. First column includes the dependent variables in each regression. Remaining columns include the coefficients and t statistics in parenthesis for the intercept and the risk factors as independent variables in the regressions. Regressions are estimated by equation below in which $factor_{i,t}$ represents return for the i[th] factor in month t and $\beta_{i,j}$ is the coefficient of j[th] factor representing the sensitivity of i[th] factor to j[th] factor. Factors are market factor, SMB, HML, RMW, CMA, WML, COSKW, ILLIQ and AMBG. Regressions are estimated by GMM and t statistics are corrected by Newey-West methodology with 1-lag. Final column contains R-squares of the regressions.

$factor_{i,t}=\alpha_i+\sum_{j=1,\, j\neq i}^{k}\beta_{i,j}factor_{j,t}+\varepsilon_{i,t}$

	Int.	r_M	SMB	HML	RMW	CMA	WML	COSKW	ILLIQ	AMBG	R^2
r_M	-0.870		-0.516	0.578	-0.074	-0.127	0.043	-0.447	0.739	-0.150	0.36
	(-2.62)		(-6.44)	(4.79)	(-0.97)	(-2.35)	(0.45)	(-7.00)	(6.37)	(-3.36)	
SMB	-0.048	-0.086		0.004	-0.168	0.107	-0.091	0.054	0.115	-0.031	0.13
	(-0.21)	(-6.80)		(0.03)	(-1.81)	(4.04)	(-3.34)	(1.02)	(2.96)	(-1.29)	
HML	0.971	0.072	0.003		0.028	0.128	-0.091	0.112	0.128	0.027	0.18
	(8.52)	(12.07)	(0.03)		(0.32)	(5.40)	(-5.32)	(2.91)	(4.97)	(1.34)	
RMW	0.409	-0.008	-0.110	0.025		-0.229	-0.025	0.065	-0.245	0.089	0.32
	(4.99)	(-1.03)	(-1.57)	(0.32)		(-15.30)	(-0.82)	(3.10)	(-8.14)	(4.96)	
CMA	0.058	-0.011	0.057	0.093	-0.189		0.040	0.035	0.045	0.060	0.10
	(0.84)	(-2.31)	(4.14)	(5.46)	(-18.79)		(1.21)	(2.68)	(2.43)	(2.54)	
WML	1.326	0.013	-0.165	-0.225	-0.069	0.136		-0.043	-0.538	-0.139	0.28
	(15.06)	(0.46)	(-4.28)	(-6.73)	(-0.82)	(1.21)		(-1.36)	(-4.51)	(-3.95)	
COSKW	1.025	-0.120	0.085	0.242	0.158	0.104	-0.038		-0.259	0.069	0.25
	(5.88)	(-8.82)	(1.28)	(3.21)	(3.62)	(3.54)	(-1.34)		(-2.50)	(0.86)	
ILLIQ	0.979	0.125	0.117	0.175	-0.380	0.084	-0.299	-0.164		-0.174	0.56
	(6.98)	(5.95)	(5.08)	(4.92)	(-13.82)	(2.34)	(-7.32)	(-2.24)		(-4.44)	
AMBG	-0.672	-0.039	-0.048	0.056	0.212	0.174	-0.119	0.067	-0.267		0.18
	(-4.87)	(-3.09)	(-1.36)	(1.26)	(5.50)	(2.49)	(-4.29)	(0.96)	(-3.49)		

t values in parenthesis.

In the literature, some set of factor structures are given special names and we also follow these conventions to name the models so that our results are easily comparable to those of the existing studies. The single factor model (1factor) includes only the market factor. Following Fama and French (1993, 1996), the model that includes the market factor, SMB and HML is called the three factor model (3factor). Carhart (1997) adds momentum to the three-factor model and the model is called the four-factor model (4factor). Fama and French (2015) introduce the five-factor model (5factor (FF)) by adding RMW and CMA to Fama and French (1996) three–factor model. In addition, we test an alternative five-factor model which includes the market factor, SMB, HML, WML and ILLIQ (5factor). The final model includes all factors: market factor, SMB, HML, RMW, CMA, WML, COSKW, ILLIQ and AMBG (9factor).

In Panel A of Table 3.10, excess return of portfolios sorted on size and other factors are regressed over different sets of risk factors in each row depending on the model specification. Significance levels smaller than 10 percent for the GRS test statistic imply that the intercepts from the regressions are statistically different from zero and the factor model is not adequate to explain the cross-section of stock returns. For example, the first row of Panel A in Table 3.10 shows that the 1factor model is not adequate to explain the return variation of the six double-sorted portfolios formed on size and B/M. On the other hand, GRS test results from all the other models are insignificant, implying that models other than 1factor model are adequate in capturing the variation of the cross–section of portfolio returns formed on size and B/M.

In Table 3.10, different portfolio sorts reveal mixed results on the adequacy of models in explaining return differential among portfolios according to GRS test statistics. In the next step, as spanning tests provide mixed results and there is no specific factor combination to explain return differential between sorted portfolios, all risk factors are included in the model while testing the significance of ambiguity sensitivity in explaining the cross-section of stock returns.

Table 3.10: GRS Test Statistics of Double Sorted Portfolios

Table shows average absolute value of intercept terms (A|α_i|) from the regressions of double and independently sorted portfolios over different set of risk factors, F statistics of Gibbons, Ross, and Shanken test (GRS) and significance level of GRS test (p(GRS)). In each Panel of the Table, first column includes stock characteristic for first sorting and other columns include stock characteristic for second sorting where stock characteristics are book to market (B/M) ratio, investment (INV), operating profitability (OP), momentum (MOM), illiquidity (ILLIQA), coskewness (COSKEW). All double sorted portfolios are constructed by 3x3 except the ones including size sorting which are constructed as 2x3. Each row in Panels of the Table contains alternative model specifications. First one is single factor model (1factor) and the factor set only includes market factor. Second model is three factor model (3factor) containing market factor, SMB and HML. Four factors model includes market factor, SMB, HML and WML. Five factor model (5 factor (FF)) includes market factor, SMB, HML, RMW and CMA. Alternative five factor model contains market factor, SMB, HML, WML and ILLIQ (5 factor). Final model (9factor) contains market factor, SMB, HML, RMW, CMA, WML, COSKW, ILLIQ and AMBG. In each row of Panels, following regression is estimated by GMM methodology with Newey-West corrected standard errors for the period of January 1990 – December 2017 where $r_{p,t}$ is the excess return of portfolio p and equals to the difference between return of double sorted portfolio p and risk free interest rate ($R_{p,t}$-$R_{f,t}$) and $r_{M,t}$ is the excess return of market portfolio. $factor_{i,t}$ represents return for the i[th] risk factor in month t and $\beta_{p,j}$ is the coefficient of j[th] risk factor representing the sensitivity of excess return of portfolio p to j[th] risk factor where factors are market factor, SMB, HML, RMW, CMA, WML, ILLIQ, COSKW and AMBG.

$r_{p,t}= \alpha_p+\beta_{p,1}r_M+ \sum_{j=2}^{k}\beta_{p,j}factor_{j,t} + \varepsilon_{p,t}$

Panel A, B and C show average absolute value of intercept terms (A|α_i|), F statistics of Gibbons, Ross, and Shanken test (GRS) and significance level of GRS test (p(GRS)) from double sorted portfolios of i) SIZE and other stock characteristics (2x3), ii) B/M and other stock characteristics (3x3), and iii) OP and other stock characteristics (3x3) respectively.

Panel A		B/M			OP			INV			MOM			ILLIQA			COSKEW	
SIZE	A|α_i|	GRS	p(GRS)	A|α_i|	GRS	p(GRS)	A|α_i|	GRS	p(GRS)	A|α_i|	GRS	p(GRS)	A|α_i|	GRS	p(GRS)	A|α_i|	GRS	p(GRS)
1factor	0.30	11.15	0.08	0.37	10.12	0.12	0.20	5.76	0.45	0.54	13.95	0.03	0.47	13.68	0.03	0.42	21.32	0.00
3factor	0.16	4.24	0.64	0.22	8.59	0.20	0.35	6.64	0.36	1.03	15.48	0.02	0.34	17.63	0.01	0.34	19.30	0.00
4factor	0.20	6.08	0.41	0.25	7.96	0.24	0.27	7.82	0.25	0.98	11.77	0.07	0.35	11.36	0.08	0.27	19.85	0.00
5factor (FF)	0.29	6.70	0.35	0.32	8.70	0.19	0.29	6.90	0.33	1.12	13.65	0.03	0.39	12.02	0.06	0.27	20.98	0.00
5factor	0.18	7.11	0.31	0.22	7.28	0.30	0.20	6.55	0.36	1.01	12.55	0.05	0.23	7.30	0.29	0.32	21.80	0.00
9factor	0.17	9.13	0.17	0.15	4.51	0.61	0.24	11.36	0.08	0.77	12.80	0.05	0.16	6.31	0.39	0.28	15.84	0.01
Panel B		B/M			OP			INV			MOM			ILLIQA			COSKEW	
B/M	A|α_i|	GRS	p(GRS)	A|α_i|	GRS	p(GRS)	A|α_i|	GRS	p(GRS)	A|α_i|	GRS	p(GRS)	A|α_i|	GRS	p(GRS)	A|α_i|	GRS	p(GRS)
1factor				0.46	20.72	0.01	0.25	7.11	0.63	0.65	37.65	0.00	0.56	10.42	0.32	0.49	21.95	0.01
3factor				0.38	17.59	0.04	0.37	3.82	0.92	1.12	30.08	0.00	0.22	7.63	0.57	0.38	13.66	0.13
4factor				0.38	16.94	0.05	0.30	3.21	0.96	1.04	33.64	0.00	0.20	7.41	0.59	0.32	16.19	0.06
5factor (FF)				0.39	14.98	0.09	0.32	4.77	0.85	1.17	31.06	0.00	0.27	9.06	0.43	0.28	14.37	0.11
5factor				0.38	18.33	0.03	0.19	2.55	0.98	1.03	32.86	0.00	0.21	6.14	0.73	0.25	16.51	0.06
9factor				0.31	5.74	0.77	0.30	11.47	0.25	1.04	35.91	0.00	0.31	16.39	0.06	0.36	29.24	0.00
Panel C		B/M			OP			INV			MOM			ILLIQA			COSKEW	
OP	A|α_i|	GRS	p(GRS)	A|α_i|	GRS	p(GRS)	A|α_i|	GRS	p(GRS)	A|α_i|	GRS	p(GRS)	A|α_i|	GRS	p(GRS)	A|α_i|	GRS	p(GRS)
1factor	0.46	20.72	0.01				0.18	8.26	0.51	0.49	8.71	0.46	0.46	8.80	0.46	0.29	8.44	0.49
3factor	0.38	17.59	0.04				0.44	9.28	0.41	1.02	9.95	0.35	0.23	11.20	0.26	0.43	11.20	0.26
4factor	0.38	16.94	0.05				0.37	8.94	0.44	0.94	11.00	0.28	0.25	12.11	0.21	0.35	10.45	0.32
5factor (FF)	0.39	14.98	0.09				0.40	9.87	0.36	1.07	9.80	0.37	0.29	11.35	0.25	0.31	9.89	0.36
5factor	0.38	18.33	0.03				0.29	10.08	0.34	0.93	10.36	0.32	0.29	10.31	0.33	0.31	10.46	0.31
9factor	0.31	5.74	0.77				0.35	11.68	0.23	0.96	11.89	0.22	0.29	12.71	0.18	0.32	33.49	0.00

Table 3.10
(continued)
Panel D, E, F and G show average absolute value of intercept terms (A|α_i|), F statistics of Gibbons, Ross, and Shanken test (GRS) and significance level of GRS test (p(GRS)) from double sorted portfolios of i) INV and other stock characteristics (3x3), ii) MOM and other stock characteristics (3x3), iii) ILLIQA and other stock characteristics (3x3), and iii) COSKEW and other characteristics (3x3) respectively.

Panel D		B/M			OP			INV			MOM			ILLIQA			COSKEW	
INV	A|αi|	GRS	p(GRS)	A|αi|	GRS	p(GRS)	A|αi|	GRS	p(GRS)	A|αi|	GRS	p(GRS)	A|αi|	GRS	p(GRS)	A|αi|	GRS	p(GRS)
1factor	0.25	7.11	0.63	0.18	8.26	0.51				0.77	12.17	0.20	0.19	5.06	0.83	0.41	31.12	0.00
3factor	0.37	3.82	0.92	0.44	9.28	0.41				1.20	12.08	0.21	0.32	6.40	0.70	0.53	28.44	0.00
4factor	0.30	3.21	0.96	0.37	8.94	0.44				1.04	11.99	0.21	0.29	5.82	0.76	0.49	29.37	0.00
5factor (FF)	0.32	4.77	0.85	0.40	9.87	0.36				1.11	11.13	0.27	0.31	13.55	0.14	0.46	27.44	0.00
5factor	0.19	2.55	0.98	0.29	10.08	0.34				0.95	10.49	0.31	0.28	5.42	0.80	0.46	31.03	0.00
9factor	0.30	11.47	0.25	0.35	11.68	0.23				0.91	21.34	0.01	0.27	7.95	0.54	0.34	14.17	0.12
Panel E		B/M			OP			INV			MOM			ILLIQA			COSKEW	
MOM	A|αi|	GRS	p(GRS)	A|αi|	GRS	p(GRS)	A|αi|	GRS	p(GRS)	A|αi|	GRS	p(GRS)	A|αi|	GRS	p(GRS)	A|αi|	GRS	p(GRS)
1factor	0.65	37.65	0.00	0.49	8.71	0.46	0.77	12.17	0.20				0.55	17.77	0.04	0.62	15.58	0.08
3factor	1.12	30.08	0.00	1.02	9.95	0.35	1.20	12.08	0.21				1.10	22.50	0.01	1.18	18.05	0.03
4factor	1.04	33.64	0.00	0.94	11.00	0.28	1.04	11.99	0.21				0.91	24.44	0.00	1.00	15.71	0.07
5factor (FF)	1.17	31.06	0.00	1.07	9.80	0.37	1.11	11.13	0.27				1.18	19.77	0.02	1.00	18.27	0.03
5factor	1.03	32.86	0.00	0.93	10.36	0.32	0.95	10.49	0.31				0.93	24.23	0.00	0.88	13.51	0.14
9factor	1.04	35.91	0.00	0.96	11.89	0.22	0.91	21.34	0.01				0.74	15.89	0.07	1.12	25.12	0.00
Panel F		B/M			OP			INV			MOM			ILLIQA			COSKEW	
ILLIQA	A|αi|	GRS	p(GRS)	A|αi|	GRS	p(GRS)	A|αi|	GRS	p(GRS)	A|αi|	GRS	p(GRS)	A|αi|	GRS	p(GRS)	A|αi|	GRS	p(GRS)
1factor	0.56	10.42	0.32	0.46	8.80	0.46	0.19	5.06	0.83	0.55	17.77	0.04				0.42	16.34	0.06
3factor	0.22	7.63	0.57	0.23	11.20	0.26	0.32	6.40	0.70	1.10	22.50	0.01				0.35	21.14	0.01
4factor	0.20	7.41	0.59	0.25	12.11	0.21	0.29	5.82	0.76	0.91	24.44	0.00				0.32	18.12	0.03
5factor (FF)	0.27	9.06	0.43	0.29	11.35	0.25	0.31	13.55	0.14	1.18	19.77	0.02				0.33	19.85	0.02
5factor	0.21	6.14	0.73	0.29	10.31	0.33	0.28	5.42	0.80	0.93	24.23	0.00				0.29	18.05	0.03
9factor	0.31	16.39	0.06	0.29	12.71	0.18	0.27	7.95	0.54	0.74	15.89	0.07				0.32	24.40	0.00
Panel G		B/M			OP			INV			MOM			ILLIQA			COSKEW	
COSKEW	A|αi|	GRS	p(GRS)	A|αi|	GRS	p(GRS)	A|αi|	GRS	p(GRS)	A|αi|	GRS	p(GRS)	A|αi|	GRS	p(GRS)	A|αi|	GRS	p(GRS)
1factor	0.49	21.95	0.01	0.29	8.44	0.49	0.41	31.12	0.00	0.62	15.58	0.08	0.42	16.34	0.06			
3factor	0.38	13.66	0.13	0.43	11.20	0.26	0.53	28.44	0.00	1.18	18.05	0.03	0.35	21.14	0.01			
4factor	0.32	16.19	0.06	0.35	10.45	0.32	0.49	29.37	0.00	1.00	15.71	0.07	0.32	18.12	0.03			
5factor (FF)	0.28	14.37	0.11	0.31	9.89	0.36	0.46	27.44	0.00	1.00	18.27	0.03	0.33	19.85	0.02			
5factor	0.25	16.51	0.06	0.31	10.46	0.31	0.46	31.03	0.00	0.88	13.51	0.14	0.29	18.05	0.03			
9factor	0.36	29.24	0.00	0.32	33.49	0.00	0.34	14.17	0.12	1.12	25.12	0.00	0.32	24.40	0.00			

3.2.4.1.4 Time Series Testing of Ambiguity

In time series testing, we use the previous section's value-weighted return series of the portfolios formed on the ambiguity beta. Portfolio 1 includes stocks with the lowest ambiguity betas and Portfolio 5 includes stocks with the highest ambiguity betas. Previous analysis suggests that, on average, there is a negative relationship between excess returns and ambiguity beta. It is also possible that the negative relationship between stock returns and ambiguity beta is a reflection of the relationship between ambiguity beta and other risk factors. Hence, we control for the other risk factors in the models that test the significance of ambiguity sensitivity.

We regress the excess returns of portfolios sorted on ambiguity beta on different sets of risk factors in the same model specifications that were used in the previous section[43]. As demonstrated in the previous sections, the distributional properties of risk factors are not consistent with the OLS framework and therefore the regressions are estimated via GMM. The results are presented in Table 3.11 and each Panel in Table 3.11 provides the estimation results for different asset pricing model setups. Each row within the panels shows the coefficients and t statistics. The last row show the difference between the intercepts from the regressions of Portfolio 5 and Portfolio 1 and the t statistic testing the equality of the two intercepts.

Among the models estimated, a model would be successful in explaining the variability in the portfolio excess returns if the model intercepts, in other words the pricing errors, are all insignificant. Such a result would imply that the risk factors included in the model span the risk of ambiguity sensitivity sufficiently well. When Table 3.11 is analyzed, it is seen that for all models, except for the eight-factor model in Panel F, have significant intercepts for some or all of the five ambiguity-sorted portfolios and the differences between the returns of highest and lowest ambiguity beta portfolios are statistically significant. The eight-factor model, on the

[43] The last model is now an eight-factor model since ambiguity is used to sort the portfolios whose returns are used as the dependent variable.

other hand, generates all insignificant intercepts. In addition, the difference between the intercept terms of the returns on highest ambiguity and lowest ambiguity portfolios is not statistically significant either. These results imply that at the portfolio-level analysis, the ambiguity risk can be captured only by the model with the broadest set of risk factors and simple combination of risk factors are not adequate to capture ambiguity sensitivity so return related with ambiguity could be achieved by complex investment rules.

3.2.4.2 Cross-Sectional Testing

In cross-sectional testing of stock returns, we include all stock characteristics and we estimate the Fama-Macbeth regressions with Newey-West heteroscedasticity and autocorrelation consistent error terms. Similar to spanning tests in the time series analyses, we start by testing the significance of other stock characteristics in explaining the ambiguity beta.

3.2.4.2.1 Ambiguity Beta and Stock Characteristics

Before we go on to analyzing the impact of ambiguity beta on stock returns, we examine the relationship between ambiguity beta and other stock characteristics. For this purpose, we follow Bali, Brown, and Tang (2017) and use the Fama-Macbeth methodology and, for each month, estimate the cross-sectional regressions presented in Equation (3.40). After the cross-sectional estimations, we take the times-series averages of the estimated coefficients for the period between January 2000 and December 2017. These time-series averages show the sensitivity of the ambiguity beta (β^A) to stock characteristics (CHR) including SIZE, B/M, MOM, REV, ILLIQA, COSKEW, INV, OP and β^M. In the equation, $\beta^A_{i,t}$ is the ambiguity beta of stock i at month t and $CHR_{i,j,t}$ shows the j^{th} characteristic of the i^{th} stock in month t .

$$\beta^A_{i,t}=\delta_{0,t}+\delta_{1,t}CHR_{i,j,t}+\varepsilon_{i,t} \tag{3.40}$$

Table 3.11: Time Series Testing of Ambiguity

Table shows coefficients and their t statistics from the regression of excess returns from five ambiguity beta sorted portfolio over set of risk factors varying among different model specification. Each panel in the Table contains different model specifications that are 1factor, 3factor, 4factor, 5factor (FF), 5factor and 8factor models that the last one is same as 9factor excluding AMBG risk factor. Risk factors in the regressions are listed as follows: r_M (market factor), SMB, HML, RMW, CMA, WML, COSKW and ILLIQ. First column includes the portfolios ranked from lowest ambiguity sensitivity (Portfolio1) to highest ambiguity sensitivity (Portfolio5) and last row shows the difference between intercepts from the regression of Portfolio 5 and Portfolio 1 and t test statistics shows the test on the equality of intercepts from both regressions. Regressions are estimated by GMM for the period of January 2000-December 2017.

Panel A shows regression coefficients, difference between intercepts from the regression of Portfolio 5 and Portfolio 1 and t test statistics of equality of intercepts from both regressions over market factor.

Panel B shows regression coefficients, difference between intercepts from the regression of Portfolio 5 and Portfolio 1 and t test statistics of equality of intercepts from both regressions over market factor, SMB and HML.

Panel C shows regression coefficients, difference between intercepts from the regression of Portfolio 5 and Portfolio 1 and t test statistics of equality of intercepts from both regressions over market factor, SMB, HML and WML.

Panel A	1factor: $r_{p,t}=\alpha_p+\beta_{p,1}r_{M,t}+\varepsilon_{p,t}$																	
	Values									t-statistics								
	α	r_M	SMB	HML	RMW	CMA	MOM	ILLIQ	COSKW	α	r_M	SMB	HML	RMW	CMA	MOM	ILLIQ	COSKW
Portfolio1	0.39	0.62								2.61	23.77							
Portfolio2	0.46	0.58								3.85	28.21							
Portfolio3	0.10	0.51								0.86	30.07							
Portfolio4	-0.24	0.42								-0.95	41.77							
Portfolio5	-0.51	0.40								-2.93	26.78							
High-Low	-0.90									-21.43								
Panel B	3factor: $r_{p,t}=\alpha_p+\beta_{p,1}r_{M,t}+\beta_{p,2}SMB+\beta_{p,3}HML+\varepsilon_{p,t}$																	
	Values									t- statistics								
	α	r_M	SMB	HML	RMW	CMA	MOM	ILLIQ	COSKW	α	r_M	SMB	HML	RMW	CMA	MOM	ILLIQ	COSKW
Portfolio1	-0.25	0.58	0.23	0.54						-0.83	29.47	4.16	4.63					
Portfolio2	-0.14	0.54	0.05	0.51						-0.46	32.64	0.91	4.30					
Portfolio3	-0.34	0.48	0.09	0.38						-1.31	42.57	1.99	3.07					
Portfolio4	-0.62	0.40	0.14	0.31						-1.71	39.83	2.79	6.87					
Portfolio5	-0.85	0.38	0.16	0.28						-2.31	41.87	2.91	2.10					
High-Low	-0.59									-10.88								
Panel C	4factor: $r_{p,t}=\alpha_p+\beta_{p,1}r_{M,t}+\beta_{p,2}SMB+\beta_{p,3}HML+\beta_{p,6}WML+\varepsilon_{p,t}$																	
	Values									t- statistics								
	α	r_M	SMB	HML	RMW	CMA	MOM	ILLIQ	COSKW	α	r_M	SMB	HML	RMW	CMA	MOM	ILLIQ	COSKW
Portfolio1	0.18	0.55	0.12	0.39			-0.40			0.70	29.74	2.50	6.55			-4.65		
Portfolio2	0.24	0.51	-0.05	0.38			-0.36			0.88	27.97	-0.70	5.93			-4.54		
Portfolio3	0.13	0.45	-0.04	0.21			-0.44			0.71	25.34	-0.67	5.14			-3.31		
Portfolio4	-0.27	0.37	0.05	0.19			-0.33			-0.85	26.64	1.04	6.02			-3.95		
Portfolio5	-0.39	0.34	0.04	0.12			-0.42			-1.24	25.57	0.94	1.90			-4.37		
High-Low	-0.57									-11.02								

Table 3.11
(continued)

Panel D shows regression coefficients, difference between intercepts from the regression of Portfolio 5 and Portfolio 1 and t test statistics of equality of intercepts from both regressions over market factor, SMB, HML, RMW and CMA.

Panel E shows regression coefficients, difference between intercepts from the regression of Portfolio 5 and Portfolio 1 and t test statistics of equality of intercepts from both regressions over market factor, SMB, HML, WML and ILLIQ.

Panel F shows regression coefficients, difference between intercepts from the regression of Portfolio 5 and Portfolio 1 and t test statistics of equality of intercepts from both regressions over market factor, SMB, HML, RMW, CMA, WML, ILLIQ and COSKW.

Panel D	5factor (FF): $r_{p,t}=\alpha_p+\beta_{p,1}r_{M,t}+\beta_{p,2}SMB+\beta_{p,3}HML+\beta_{p,4}RMW+\beta_{p,5}CMA+\varepsilon_{p,t}$																	
	Values									t- statistics								
	α	r_M	SMB	HML	RMW	CMA	MOM	ILLIQ	COSKW	α	r_M	SMB	HML	RMW	CMA	MOM	ILLIQ	COSKW
Portfolio1	-0.06	0.51	0.10	0.56	-0.83	-0.48				-0.19	23.47	1.28	8.52	-6.51	-4.88			
Portfolio2	0.00	0.48	-0.05	0.52	-0.65	-0.33				0.01	19.73	-1.21	7.62	-4.45	-4.62			
Portfolio3	-0.22	0.44	0.01	0.39	-0.52	-0.32				-0.83	19.86	0.26	4.67	-3.14	-5.32			
Portfolio4	-0.57	0.38	0.10	0.31	-0.22	-0.07				-1.54	33.08	2.17	9.86	-4.90	-0.94			
Portfolio5	-0.78	0.35	0.09	0.27	-0.34	-0.05				-2.08	24.12	1.84	3.16	-3.95	-0.51			
High-Low	-0.72									-14.31								
Panel E	5factor: $r_{p,t}=\alpha_p+\beta_{p,1}r_{M,t}+\beta_{p,2}SMB+\beta_{p,3}HML+\beta_{p,6}WML+\beta_{p,7}ILLIQ+\varepsilon_{p,t}$																	
	Values									t- statistics								
	α	r_M	SMB	HML	RMW	CMA	MOM	ILLIQ	COSKW	α	r_M	SMB	HML	RMW	CMA	MOM	ILLIQ	COSKW
Portfolio1	0.57	0.45	0.12	0.50			-0.37	-0.52		3.58	13.16	2.88	7.89			-7.95	-5.32	
Portfolio2	0.62	0.41	-0.05	0.49			-0.33	-0.51		4.26	10.77	-1.22	6.07			-7.92	-4.66	
Portfolio3	0.51	0.35	-0.04	0.32			-0.41	-0.50		3.68	9.46	-1.40	5.37			-4.35	-5.03	
Portfolio4	0.02	0.29	0.05	0.28			-0.31	-0.38		0.08	9.20	1.29	10.15			-5.56	-4.02	
Portfolio5	-0.09	0.26	0.04	0.21			-0.40	-0.41		-0.43	8.40	1.00	2.68			-5.87	-4.21	
High-Low	-0.66									-13.13								
Panel F	8factor: $r_{p,t}=\alpha_p+\beta_{p,1}r_{M,t}+\beta_{p,2}SMB+\beta_{p,3}HML+\beta_{p,4}RMW+\beta_{p,5}CMA+\beta_{p,6}WML+\beta_{p,7}ILLIQ+\beta_{p,8}COSKW+\varepsilon_{p,t}$																	
	Values									t- statistics								
	α	r_M	SMB	HML	RMW	CMA	MOM	ILLIQ	COSKW	α	r_M	SMB	HML	RMW	CMA	MOM	ILLIQ	COSKW
Portfolio1	-0.18	0.31	-0.07	0.39	-0.34	-0.43	-0.14	-0.30	0.69	-0.43	5.01	-1.28	5.51	-4.78	-3.47	-2.64	-8.63	2.89
Portfolio2	0.04	0.31	-0.19	0.41	-0.23	-0.28	-0.15	-0.35	0.52	0.12	5.18	-3.19	8.62	-3.31	-3.28	-3.49	-12.31	2.95
Portfolio3	0.04	0.27	-0.13	0.26	-0.12	-0.25	-0.27	-0.38	0.42	0.18	4.58	-2.75	4.86	-1.87	-3.25	-5.33	-12.77	2.49
Portfolio4	-0.42	0.24	-0.01	0.21	0.11	-0.02	-0.19	-0.30	0.38	-1.05	4.87	-0.32	3.37	0.99	-0.35	-5.38	-5.99	2.39
Portfolio5	-0.37	0.22	-0.03	0.15	-0.05	0.01	-0.32	-0.34	0.25	-1.17	5.19	-0.73	3.54	-0.82	0.19	-8.17	-5.63	2.26
High-Low	-0.19									-0.76								

It should be noted that there is an additional stock characteristic, reversal or $REV_{i,t}$, that is included in the model. The reason for including this variable will be discussed later. $REV_{i,t}$ is the reversal of stock i in month t and equals the percentage change of stock price in the previous month, in other words, the lagged monthly return (Equation (3.41)).

$$REV_{i,t} = ((price_{i,t-1} / price_{i,t-2}) \times 100) - 100 \tag{3.41}$$

Table 3.12 summarizes the coefficients, t statistics and significance levels from the Fama-Macbeth regressions where ambiguity beta is the dependent variable and individual stock characteristics are the independent variables. First, each of the nine stock characteristics are included in the model on their own, and then, in the last model, all characteristic variables are included simultaneously. Model 1 results show a significant and negative coefficient for the market beta implying that when a stock's market risk is high, its ambiguity beta is lower and thus provides weaker hedging against ambiguity. Illiquidity and operating profitability do not seem to have a significant impact on a stock's ambiguity sensitivity. On the other hand, stocks with small size, low B/M, lower momentum, lower reversal and higher investment exhibit higher ambiguity beta implying that these stocks provide better hedging against ambiguity. In the nested regression (Model 10), the coefficients keep their signs but illiquidity becomes significant and reversal effect loses its significance. In addition, the intercept terms in all regressions are statistically significant, leading to the conclusion that the variation in the ambiguity beta cannot be fully explained by the other stock characteristics and the ambiguity beta itself conveys unique information over and above the other risk measures.

3.2.4.2.2 Cross-Sectional Testing of Ambiguity

In time series setup, we presented the significance of ambiguity beta in explaining excess return at the portfolio level. These analysis provide evidence of the impact of ambiguity beta on excess return but the results are not tested on individual stock basis. Although portfolio level analysis are capable of technical problems such as cross-sectional error correlation, portfolios omit a large amount of information by aggregation and it is difficult to control multiple factors simultaneously at portfolio level analysis.

For these reasons, we use the Fama-MacBeth methodology to test cross-sectional relationship between ambiguity betas and expected excess returns at the individual stock level. As mentioned earlier, coefficients from the Fama-MacBeth regressions are the time series averages from the monthly cross-sectional regressions. We estimate the regression model in Equation (3.42) where $r_{i,t+1}$ is the one-month ahead excess stock return for the i[th] stock. $\beta^{M}_{i,t}$ is the market beta of stock i in month t[44], $\beta^{A}_{i,t}$ is the ambiguity beta of stock i in month t and $CHR_{i,j,t}$ is characteristic j of stock i in month t. CHR includes the characteristics of SIZE, B/M, MOM, REV, ILLIQA, COSKEW, INV and OP. Table 3.13 summarizes the Fama-Macbeth regression results for the period between January 2000 and December 2017 for different model specifications. Coefficients are the time series averages of coefficients from cross-sectional regressions. Significance of the coefficients is tested based on the distribution of coefficients collected from the cross-sectional regressions and t statistics in the parentheses are corrected by Newey-West at lag one.

$$r_{i,t+1} = \delta_{0,t} + \delta_{1,t}\beta^{M}_{i,t} + \delta_{2,t}\beta^{A}_{i,t} + \sum_{j=3}^{k} \delta_{j,t} CHR_{i,j,t} + \varepsilon_{i,t} \tag{3.42}$$

---

[44] We use alternative definitions for market beta from Fama and French (1992) whose methodology was discussed in detail in the previous sections but the estimation results do not change. The first alternative is appointing portfolio betas obtained for the whole sample to the stocks in the portfolio and this methodology has multiple steps: i) For each stock, the market beta is estimated by regressing excess returns on the market return for the past 48 months provided that there are at least 24 months of data, ii) Next, stocks are sorted based on their market beta and 10 portfolios are formed in July of each year, iii) Excess return for each portfolio is calculated, iv) Portfolio betas are estimated for the sample period, and, v) portfolio beta is assigned to the stocks in the portfolio. With this methodology, the beta of a stock changes throughout the sample period if the beta ranking of the stock changes. The second alternative methodology is similar to first methodology but portfolio betas are estimated for 4 years and assigned as betas to the individual stocks in the portfolio. With this methodology, a stock's beta could change every 4 years if the ranking of its market beta changes.

Table 3.12: Ambiguity Beta and Stock Characteristics

This table summarizes the coefficients, t values and significance levels from Fama-Macbeth regressions. Regression coefficients show the sensitivity of ambiguity beta ($\beta^A_{i,t}$) to stock characteristics ($CHR_{i,j,t}$) on stock level. $CHR_{i,j,t}$ shows j^{th} characteristic of i^{th} stock in month t and CHR includes the characteristics of market value in logarithm (SIZE), book-to-market ratio (B/M), momentum (MOM), reversal (REV), illiquidity (ILLIQA), coskewness (COSKEW), investment (INV), operating profitability (OP) and market beta (β^M). Fama-Macbeth regressions are utilized through running monthly cross-sectional regressions by using following equation for the period of January 2000-December 2017.

$\beta^A_{i,t}=\delta_{0,t}+\delta_{1,t}CHR_{i,j,t}+\varepsilon_{i,t}$

First column shows the stock characteristics and the following columns show the regression results for each characteristics but final column includes the nested version. t statistics in the parenthesis are adjusted by Newey-West at lag one.

SIZE of a stock in a given month from July in year t to June in year t+1 equals the logarithm of market value of June in year t. B/M of a stock in a given month from July in year t to June in year t+1 equals to the ratio of book value to market value in December of year t-1. INV of a stock in a given month from July in year t to June in year t+1 equals to percentage change of total asset between December in year t-1 and December in year t-2. OP of a stock in a given month from July in year t to June in year t+1 equals to the ratio of operating profit to book equity in December of year t-1. MOM of a stock in month t equals to percentage change in stock price between month t-1 and month t-12 ($MOM_{i,t}=price_{i,t-1}/price_{i,t-12}\cdot 100-100$). ILLIQA is calculated through Amihud (2002) measure and it is equal to 12-month average of the ratio of absolute value of daily stock return to daily trading volume in USD multiplied by 1000 ($ILLIQD_{d,i,t}=|return_{d,i,t}|/volume_{d,i,t}$). COSKW of a stock in month t is derived from the coefficient of excess squared market return (γ) in the regression of excess return over market excess return and excess squared market return ($r_{i,t}=\alpha_{i,t}+\beta_{i,t}r_{M,t}+\gamma_{i,t}(R^2_{M,t}-R_{f,t})$ following Adesi, Gagliardini, and Urga (2004). Coskewness coefficient is estimated each month on rolling basis for past 48 months conditional on that there are at least 24 observations. Market beta of a stock in month t is derived from the coefficient of excess market return (β^M) in the regression of excess return over market excess return ($r_{i,t}=\alpha_{i,t}+\beta^M_{i,t}r_{M,t}$)). Market beta is estimated each month on rolling basis for past 48 months conditional on that there are at least 24 observations. Ambiguity beta of a stock in month t is derived from the coefficient of excess market return (β^A) in the regression of excess return over estimated ambiguity ($r_{i,t}=\alpha_{i,t}+\beta^A_{i,t}(AMB^F_t)$). Ambiguity beta is estimated each month on rolling basis for past 48 months conditional on that there are at least 24 observations.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
SIZE	-3.575***									-1.901***
	(-9.68)									(-5.28)
B/M		-1.097**								-0.660*
		(-3.18)								(-2.36)
MOM			-4.308*							-4.162*
			(-2.14)							(-2.18)
REV				-0.187*						-0.0711
				(-2.39)						(-1.00)
ILLIQA					5.558					-5.833*
					(1.69)					(-2.22)
COSKEW						-21.92***				-12.07*
						(-4.35)				(-2.38)
INV							1.022**			3.315***
							(2.75)			(4.77)
OP								-2.58		0.0351
								(-1.92)		(0.05)
β^M									-48.74***	-41.67***
									(-15.66)	(-12.16)
Intercept	35.85***	-30.62***	-28.10***	-30.02***	-31.83***	-13.67**	-32.12***	-30.60***	-9.019***	29.87***
	(3.80)	(-8.06)	(-7.62)	(-7.92)	(-8.18)	(-3.12)	(-8.14)	(-7.56)	(-3.41)	(4.07)
Adj. R^2	3.2%	1%	4.8%	3.3%	0.5%	3.6%	0.5%	1.3%	5.5%	19.1%

*t statistics in parentheses, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$*

First column shows that there is a negative and statistically significant relationship between excess returns and ambiguity beta. In the second column, market beta is added into the analysis. Ambiguity beta continues to be significant but the market beta is not statistically significant[45]. In the third regression, size is added and the coefficients of both ambiguity beta and size are statistically significant but market beta is still insignificant. The coefficient of size is negative, consistent with the literature that higher market value decreases excess returns. In the fourth regression, the B/M ratio is added to the model and it has a statistically significant and positive coefficient confirming that a higher B/M (indicative of a value stock) is associated with higher risk, and thus higher return. Fama and French (2012, 2017) and Cakici, Tang, and Yan (2016) confirm the value effect for other emerging markets. With the inclusion of B/M, size loses its significance. Similar weak evidence for the size effect is documented for other emerging markets in Cakici, Tang, and Yan (2016).

In the fifth model, momentum is included and it has a significant and negative coefficient implying a reversal effect. Therefore, in order to test the effect of short-term return reversals, we add the REV variable. As stated earlier, following Jegadeesh and Titman (1993), REV is the stock's one-month lagged return. The REV variable has a significant and positive coefficient, contrary to a negative coefficient that would be expected from a return reversal effect. These two coefficients imply that stocks traded on Borsa Istanbul have short term momentum as represented by the positive coefficient of REV and long term reversal as represented by the negative coefficient of MOM. In addition, adding both momentum and reversal does not affect the significant and negative coefficient of the ambiguity beta. These results contradict with the results of Fama and French (2012) who provide evidence that emerging markets exhibit a momentum effect

[45] Market beta is not statistically significant in the regressions excluding ambiguity beta (Table 3.14). The results show that sign and significance of other stock characteristics' coefficients do not change compared to the coefficients in the regressions including both market beta and ambiguity beta (Table 3.13).

and are consistent with Cakici, Tang, and Yan (2016) who document a weak momentum effect for emerging markets.

In the sixth regression, the variables from the Fama-French five-factor model (market beta, size, B/M, OP, INV) are added to the model alongside the ambiguity beta. Aside from market beta and size, the coefficients of other stock characteristics are statistically significant. The negative coefficient of INV and positive coefficient of OP are consistent with expectations and implications of the Modigliani and Miller (1961) valuation formula as discussed in Fama and French (2015) and empirical results for emerging markets in Titman, Wei, and Xie (2013), Sun, Wei, and Xie (2014) and Fama and French (2017).

In the seventh regression, only illiquidity and coskewness characteristics are added to size and B/M but neither of them are statistically significant in explaining the variation in the cross-sectional returns. Similar results for liquidity were presented by Donadelli and Prosperi (2012) for emerging markets. Illiquidity and coskewness do not seem to have an effect on the excess returns at the individual stock level. The final column of the table presents the nested regression including all characteristics we listed. Results are consistent with individual regression in Table 3.13. Excluding size, illiquidity and coskewness, all characteristics are statistically significant with the same signs as before. The results also confirm the negative impact of ambiguity beta on excess stock returns.

After we identify the significance of ambiguity beta in stock return, we quantify the impact of choosing stocks with higher ambiguity. Following Bali, Brown, and Tang (2017), we calculate the economic significance of higher ambiguity by the monthly return differential in the case of shifting from the lowest ambiguity beta group to the highest ambiguity beta group. In Table 3.5, the difference between high and low average ambiguity betas is equal to 1.5 and the coefficient of ambiguity beta from the Fama-MacBeth regressions equals 0.434%. Hence, moving from the lowest ambiguity beta group to the highest ambiguity beta group would decrease excess returns by about 0.65% (0.434%×1.5) per month.

Table 3.13: Cross-Sectional Testing of Ambiguity Beta

This table summarizes the coefficients, t statistics and significance levels from Fama-Macbeth regressions with different specifications. Regression coefficients show the sensitivity of excess stock return on lagged ambiguity beta (β^A), market beta (β^M) and other stock characteristics ($CHR_{i,j,t}$). $CHR_{i,j,t}$ shows j^th^ characteristic of i^th^ stock in month t and CHR includes the characteristics of market value in logarithmic terms (SIZE), book-to-market ratio (B/M), momentum (MOM), reversal (REV), illiquidity (ILLIQA), coskewness (COSKEW), investment (INV), and operating profitability (OP). Fama-Macbeth regressions are utilized through running monthly cross sectional regressions by using following equation for the period of January 2000 and December 2017. $r_{i,t+1}$ is the excess return of stock i in month t+1.

$$r_{i,t+1} = \delta_{0,t} + \delta_{1,t}\beta^M_{i,t} + \delta_{2,t}\beta^A_{i,t} + \sum_{j=3}^{k} \gamma_{j,t} CHR_{i,j,t} + e_{i,t}$$

First column shows the stock characteristics and the following columns show the regression results for different specifications. t statistics in the parenthesis are adjusted by Newey-West.

SIZE of a stock in a given month from July in year t to June in year t+1 equals the logarithm of market value of June in year t. B/M of a stock in a given month from July in year t to June in year t+1 equals to the ratio of book value to market value in December of year t-1. INV of a stock in a given month from July in year t to June in year t+1 equals to percentage change of total asset between December in year t-1 and December in year t-2. OP of a stock in a given month from July in year t to June in year t+1 equals to the ratio of operating profit to book equity in December of year t-1. MOM of a stock in month t equals to percentage change in stock price between month t-1 and month t-12 ($MOM_{i,t} = price_{i,t-1}/price_{i,t-12} \cdot 100 - 100$). ILLIQA is calculated through Amihud (2002) measure and it is equal to 12-month average of the ratio of absolute value of daily stock return to daily trading volume in USD multiplied by 1000 ($ILLIQD_{d,i,t} = |return_{d,i,t}|/volume_{d,i,t}$). COSKEW of a stock in month t is derived from the coefficient of excess squared market return (γ) in the regression of excess return over market excess return and excess squared market return ($R_{i,t}-R_{f,t}=\alpha_{i,t}+\beta_{i,t}(R_{M,t}-R_{f,t})+\gamma_{i,t}(R^2_{M,t}-R_{f,t})$) following Adesi, Gagliardini, and Urga (2004). Coskewness coefficient is estimated each month on rolling basis for past 48 months conditional on that there are at least 24 observations. Market beta of a stock in month t is derived from the coefficient of excess market return (β^M) in the regression of excess return over market excess return ($R_{i,t}-R_{f,t}=\alpha_{i,t}+\beta^M_{i,t}(R_{M,t}-R_{f,t})$). Market beta is estimated each month on rolling basis for past 48 months conditional on that there are at least 24 observations. Ambiguity beta of a stock in month t is derived from the coefficient of excess market return (β^A) in the regression of excess return over estimated ambiguity ($R_{i,t}-R_{f,t}=\alpha_{i,t}+\beta^A_{i,t}(AMB^F_t)$). Ambiguity beta is estimated each month on rolling basis for past 48 months conditional on that there are at least 24 observations.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β^A	-0.00434**	-0.00465**	-0.00459***	-0.00438***	-0.00341**	-0.00383**	-0.00425**	-0.00334**
	(-2.90)	(-3.32)	(-3.50)	(-3.35)	(-2.89)	(-2.78)	(-3.28)	(-2.72)
β^M		-0.311	0.0297	-0.315	-0.670	-0.646	-0.329	-0.857
		(-0.54)	(0.05)	(-0.54)	(-1.58)	(-1.01)	(-0.57)	(-1.91)
SIZE			-0.158*	-0.0985	-0.0690	-0.100	-0.0678	-0.0351
			(-2.00)	(-1.26)	(-1.14)	(-1.09)	(-0.90)	(-0.51)
B/M				0.327***	0.269***	0.413***	0.380***	0.351***
				(4.01)	(3.47)	(3.93)	(4.29)	(3.55)
REV					0.244***			0.246***
					(21.63)			(21.39)
MOM					-0.518**			-0.562**
					(-2.65)			(-2.88)
INV						-0.324*		-0.266*
						(-2.09)		(-2.10)
OP						0.652***		0.450**
						(3.45)		(2.81)
ILLIQA							-1.700	-0.844
							(-0.56)	(-0.32)
COSKEW							0.0124	-0.0412
							(0.02)	(-0.08)
Intercept	0.0843	0.210	2.805	2.026	1.526	2.191	1.162	0.859
	(0.14)	(0.40)	(1.75)	(1.27)	(1.20)	(1.23)	(0.79)	(0.63)
Adj. R^2	0.9%	2.2%	3.8%	4.7%	13%	6.4%	6%	15.8%

*t statistics in parentheses, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$*

Table 3.14: Cross-Sectional Testing of Market Beta

This table summarizes the coefficients, t statistics and significance levels from Fama-Macbeth regressions with different specifications. Regression coefficients show the sensitivity of excess stock return on lagged market beta (β^M) and other stock characteristics ($CHR_{i,j,t}$). $CHR_{i,j,t}$ shows j^{th} characteristic of i^{th} stock in month t and CHR includes the characteristics of market value in logarithmic terms (SIZE), book-to-market ratio (B/M), momentum (MOM), reversal (REV), illiquidity (ILLIQA), coskewness (COSKEW), investment (INV), and operating profitability (OP). Fama-Macbeth regressions are utilized through running monthly cross-sectional regressions by using following equation for the period of January 2000 and December 2017. $r_{i,t+1}$ is the excess return of stock i in month t+1.

$$r_{i,t+1} = \delta_{0,t} + \delta_{1,t}\beta^M_{i,t} + \sum_{j=3}^{k} \gamma_{j,t} CHR_{i,j,t} + e_{i,t}$$

First column shows the stock characteristics and the following columns show the regression results for different specifications. t statistics in the parenthesis are adjusted by Newey-West.

SIZE of a stock in a given month from July in year t to June in year t+1 equals the logarithm of market value of June in year t. B/M of a stock in a given month from July in year t to June in year t+1 equals to the ratio of book value to market value in December of year t-1. INV of a stock in a given month from July in year t to June in year t+1 equals to percentage change of total asset between December in year t-1 and December in year t-2. OP of a stock in a given month from July in year t to June in year t+1 equals to the ratio of operating profit to book equity in December of year t-1. MOM of a stock in month t equals to percentage change in stock price between month t-1 and month t-12 ($MOM_{i,t} = price_{i,t-1}/price_{i,t-12} \cdot 100-100$). ILLIQA is calculated through Amihud (2002) measure and it is equal to 12-month average of the ratio of absolute value of daily stock return to daily trading volume in USD multiplied by 1000 ($ILLIQD_{d,i,t} = |return_{d,i,t}|/volume_{d,i,t}$). COSKEW of a stock in month t is derived from the coefficient of excess squared market return (γ) in the regression of excess return over market excess return and excess squared market return ($R_{i,t}-R_{f,t}=\alpha_{i,t}+\beta_{i,t}(R_{M,t}-R_{f,t})+\gamma_{i,t}(R^2_{M,t}-R_{f,t})$) following Adesi, Gagliardini, and Urga (2004). Coskewness coefficient is estimated each month on rolling basis for past 48 months conditional on that there are at least 24 observations. Market beta of a stock in month t is derived from the coefficient of excess market return (β^M) in the regression of excess return over market excess return ($R_{i,t}-R_{f,t}=\alpha_{i,t}+\beta^M_{i,t}(R_{M,t}-R_{f,t})$). Market beta is estimated each month on rolling basis for past 48 months conditional on that there are at least 24 observations.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
β^M	-0.0763	0.224	0.0380	-0.215	-0.236	0.820	0.477
	(-0.15)	(0.40)	(0.07)	(-0.50)	(-0.41)	(1.40)	(0.91)
SIZE		-0.148	-0.0736	-0.0569	-0.0843	-0.0575	-0.0424
		(-1.90)	(-0.96)	(-0.90)	(-0.95)	(-0.75)	(-0.59)
B/M			0.282*	0.277*	0.417*	0.368**	0.418**
			(2.07)	(2.09)	(2.53)	(2.61)	(2.67)
REV				0.227***			0.233***
				(23.58)			(23.55)
MOM				-0.338			-0.338
				(-1.86)			(-1.77)
INV					-0.403*		-0.343*
					(-2.53)		(-2.32)
OP					0.457*		0.283
					(2.57)		(1.75)
ILLIQA						-0.00110	-0.000548
						(-0.56)	(-0.32)
COSKEW						-0.943	-1.022
						(-1.71)	(-1.62)
Intercept	0.585	3.487	2.769	3.140*	3.567*	3.384*	4.490*
	(0.90)	(1.92)	(1.63)	(2.10)	(2.02)	(2.01)	(2.59)
Adj. R^2	1.6%	4.3%	5.6%	13.4%	7.6%	7.4%	17.7%

*t statistics in parentheses, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$*

3.2.5 Conclusion

Literature on asset pricing has extensively discussed the relationship between risk and return. Among alternative models, CAPM put forward a solid theoretical framework in defining the risk-return relationship for an individual asset. However, restrictive assumptions of the model and weak empirical support made it necessary to look for alternative model specifications. Subsequent studies by Black (1972), Merton (1973), Ross (1976) and Breeden (1979) introduced new theoretical models that aim to relax CAPM assumptions and improve model testability. Return anomalies documented by Basu (1977, 1983), Reinganum (1981), and Banz (1981) were addressed by either behavioral biases or by the introduction of different state variables in the multifactor asset pricing models. Fama and French (1996) introduced size (SMB) and value (HML) factors in addition to the market factor in CAPM. Carhart (1997) added the momentum factor. The quest for finding new risk factors continues to date with studies such as Amihud (2002) and Pastor and Stambaugh (2003) testing liquidity, Harvey and Siddique (2000) testing coskewness, and Fama and French (2015) testing investment and profitability as additional factors.

Factor models have focused on introducing the risk factor(s) that proxy risks not captured by the market beta. Some of recent studies investigated the role of the unique probability distribution assumption in decision-making and its reflections on risk-return relationship. If the stock returns do not all come from the same distribution, it is said that there is ambiguity in the market since risks cannot be quantified based on a known and unique distribution’s characteristics. The literature has spent serious effort in integrating ambiguity, in addition to risk, into the models of decision-making and asset pricing. Chen and Epstein (2002), Kogan and Wang (2003), Bansal and Yaron (2004), Boyle et al. (2009) and Anderson, Ghysels, and Juergens (2009)) developed theoretical models and Bollerslev, Tauchen, and Zhou (2009), Bekaert, Engstrom, and Xing (2009), Andreou et al. (2014), and Brenner and Izhakian (2018) empirically examined the impact of ambiguity on asset pricing.

In this study, we test whether sensitivity to ambiguity has explanatory capacity for the cross-sectional stock returns in Turkey. Our sample includes stocks listed on Borsa Istanbul from January 1990 to December 2017 but excludes financial sector stocks, closed-end mutual funds, REITs, sports clubs and stocks with a negative book-to-market ratio. Since it is not possible to test stock return sensitivity to ambiguity as a single factor without controlling for the impact of other risk factors on stock returns, we calculated and tested the market factor, SMB, HML, RMW and CMA from Fama and French (2015), WML from Jegadeesh and Titman (1993), COSKW from Harvey and Siddique (2000) and ILLIQ from Amihud (2002). We also derived and added an ambiguity factor into the analysis. This way, in addition to the tests of ambiguity as a risk factor in asset pricing, the study also provides up-to-date evidence on the significance of other risk factors for the Turkish stock market.

Before we formed the risk factors, we performed an in depth analysis of the relationship between returns on single- and double-sorted portfolios and stock characteristics. Stock characteristics that we included are market value, book-to-market ratio, investment, operating profitability, momentum, illiquidity, coskewness, market beta and ambiguity beta. The average excess returns calculated for portfolios formed on single-sorts of stock characteristics provide evidence of a relationship between returns and market value, book-to-market ratio, investment, momentum, illiquidity and ambiguity beta. Operating profitability, coskewness and market beta do not seem to have an impact on the returns generated by the single-sort portfolios. Returns on double-sorted portfolios show that only the book-to-market ratio and the ambiguity beta have monotonic relationship with average excess returns. Return structure from single- and double-sorted portfolios show that stock characteristics are associated with cross-sectional stock returns.

Based on the evidence regarding stock characteristics and excess returns, we decided to include all risk factors related with these stock characteristics in testing the impact of ambiguity on stock return. Spanning test results on risk factors reveal that all factors, except size and investment, provide discrete information. In testing the significance of ambiguity, we followed Bali, Brown, and Tang (2017) and used both time-series and cross-sectional regression methodologies. Time series regression results do not fully confirm the significance of ambiguity. On the other hand, evidence suggests that conventional models such as the three-factor or five-factor models cannot fully capture the risk associated with ambiguity sensitivity. In cross-sectional analysis, Fama-Macbeth regressions reveal that there is a negative and statistically significant relationship between excess returns and ambiguity betas, even in the presence of the other risk factors in the models.

All in all, our analysis on the Turkish stock market points to the importance of ambiguity in explaining stock returns and provides evidence of its separate role in addition to the other risk factors already addressed in the literature. In this regard, our results support the implications of theoretical models focusing on the relationship between asset pricing and ambiguity. Our results are also important in providing support to a limited number of empirical studies confirming the impact of ambiguity on cross-sectional stock returns.

CHAPTER 4

CONCLUSION

This thesis investigates empirically the impact of ambiguity as a risk factor on asset pricing. Risk in conventional terms and ambiguity differ from each other due to different information content and this difference has changed and modified the rules in decision-making and asset pricing. Epstein and Wang (1994) put forward the difference between ambiguity and risk in a way that probability distributions as a measure of risk represent the relative likelihoods of events but they lack the information on the reliability of those likelihoods. Ellsberg (1961) provides experimental evidence showing how lack of reliability on probability distribution (likelihood) differs the decision-making from conventional decision-making models like Savage (1954). In this framework, ambiguity in decision-making suggest that the agents have multiple probability distributions in making decision and the previous theoretical studies aimed to modify the subjective expected utility model to incorporate ambiguity in addition to risk (Gilboa and Schmeidler (1989), Schmeidler (1989), Epstein and Wang (1994), Chen and Epstein (2002), Hansen and Sargent (2001) and Klibano, Marinacci and Mukerji (2005)).

New model setups in decision-making leads also modifications in asset pricing models considering the central role of decision-making in asset pricing models (Chen and Epstein (2002), Kogan and Wang (2003), Bansal and Yaron (2004), Boyle et al. (2009) and Anderson, Ghysels, and Juergens (2009)). Although theoretical models are rich to explore the ambiguity-asset pricing relationship, empirical evidence remains limited due to difficulties in measuring ambiguity (Andreou et al. (2014), Bollerslev, Tauchen, and Zhou (2009), Bekaert, Engstrom, and Xing (2009) and Brenner and Izhakian (2018)).

In the first essay, this thesis provides alternative ambiguity indices for Turkey consistent with the theoretical framework reviewed comprehensively in this part, as well, along with a comparative analysis with the conventional asset pricing models. Comparison of alternative ambiguity indices infers that they contain similar information but their explanatory power on stock returns differ in different subsamples. Among alternative ones, index based on variability of probability distribution of BIST30 Index return is the most appropriate index to use in asset pricing tests. This ambiguity index is a first for Turkey and one of first among a limited number of studies that (i) satisfies the definition of ambiguity put forward in the literature, (ii) has the longest possible historical coverage, and, (iii) is relevant for studying the impact of ambiguity on asset pricing. Regarding the relationship between ambiguity and asset pricing, the initial results confirm that ambiguity affects excess returns in a negative manner.

In the second essay, we extent the initial analysis in the first essay by making portfolio and stock level analysis. We test whether sensitivity to ambiguity has explanatory capacity for the cross-sectional stock returns in Turkey for the stocks listed in Borsa Istanbul excluding financial sector stocks, closed-end mutual funds, REITs, sports clubs and stocks with a negative book-to-market ratio from January 1990 to December 2017. Since it is not possible to test stock return sensitivity to ambiguity as a single factor, we derived and added the market factor, SMB, HML, RMW and CMA from Fama and French (2015), WML from Jegadeesh and Titman (1993), COSKW from Harvey and Siddique (2000) and ILLIQ from Amihud (2002) into the analysis. This way, in addition to the tests of ambiguity as a risk factor in asset pricing, the study also provides up-to-date evidence on the relationship between cross-sectional return and stock characteristics, and the significance of other risk factors for the Turkish stock market beside ambiguity.

Spanning test results on risk factors reveal that all factors, except size and investment, provide discrete information. In testing the significance of ambiguity, we used both time-series and cross-sectional regression methodologies. Time series regression results mostly confirm the significance of ambiguity, and conventional models such as the three-factor or five-factor models cannot fully capture the risk associated with ambiguity sensitivity. Cross-sectional analysis reveals that there is a negative and statistically significant relationship between excess returns and ambiguity sensitivity after controlling for other risk factors.

These two essays on the relationship between ambiguity and equity return in Turkey point out the importance of ambiguity in explaining stock returns and provides evidence of its separate role in addition to the other risk factors already addressed in the literature. In this regard, our results support the implications of theoretical models focusing on ambiguity-asset pricing relationship. The results are also important in the means of being one among the limited number empirical studies confirming the impact of ambiguity on cross-sectional stock returns.

REFERENCES

Acharya, Viral, and Lasse Pedersen, 2005, Asset pricing with liquidity risk, *Journal of Financial Economics* 77, 375–410.

Adesi, Giovanni Barone, Patrick Gagliardini, and Giovanni Urga, 2004, Testing Asset Pricing Models with Coskewness, *Journal of Business & Economic Statistics* 22, 474–485.

Amihud, Yakov, 2002, Illiquidity and stock returns : cross-section and time-series effects $, 5, 31–56.

Amihud, Yakov, and Haim Mendelson, 1986, Asset pricing and the bid-ask spread, *Journal of Financial Economics* 17, 223–249.

Anderson, Evan W., Eric Ghysels, and Jennifer L. Juergens, 2009, The impact of risk and uncertainty on expected returns, *Journal of Financial Economics*.

Andreou, Panayiotis C., Anastasios Kagkadis, Paulo F. Maio, and Dennis Philip, 2014, Stock Market Ambiguity and the Equity Premium, *SSRN Electronic Journal*, 1–43.

Antoniou, Constantinos, Emilios C. Galariotis, and Daniel Read, 2014, Ambiguity Aversion, Company Size and the Pricing of Earnings Forecasts, *European Financial Management* 20, 633–651.

Arrow, K. J., and G. Debreu, 1954, Existence of an Equilibrium for a Competitive Economy, *Econometrica* 22, 265–290.

Arslan, Yavuz, Aslihan Atabek, Timur Hulagu, and Saygin Şahinöz, 2015, Expectation errors, uncertainty, and economic activity, *Oxford Economic Papers*.

Baker, Scott, Nicholas Bloom, and Steven Davis, 2016, Measuring Economic Policy Uncertainty, *The Quarterly Journal of Economics* 131, 1593–1636.

Bali, Turan G., Stephen J. Brown, and Yi Tang, 2017, Is economic uncertainty priced in the cross-section of stock returns?, *Journal of Financial Economics* 126, 471–489.

Bali, Turan G, Stephen Brown, and Yi Tang, 2014, Disagreement in Economic Forecasts and Expected Stock Returns *SSRN*.

Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481–1509.

Banz, Rolf W., 1981, The relationship between return and market value of common stocks, *Journal of Financial Economics* 9, 3–18.

Barberis, Nicholas, and Richard Thaler, 2003, Chapter 18 A survey of behavioral finance, *Handbook of the Economics of Finance* 1, 1053–1128.

Basu, S, 1977, Investment Performance of Common Stocks in Relation to Their Price-Earnings Ratios: A Test of the Efficient Market Hypothesis, *The Journal of Finance* 32, 663–682.

Basu, Sanjoy, 1983, The relationship between earnings' yield, market value and return for NYSE common stocks: Further evidence, *Journal of Financial Economics* 12, 129–156.

Bekaert, Geert, Eric Engstrom, and Yuhang Xing, 2009, Risk, uncertainty, and asset prices, *Journal of Financial Economics* 91, 59–82.

Bekaert, Geert, Marie Hoerova, and Marco Lo Duca, 2013, Risk, uncertainty and monetary policy, *Journal of Monetary Economics* 60, 771–788.

Bernoulli, Daniel, 1738, "Specimen theoriae novae de mensura sortis". In Commentarii Academiae Scientiarum Imperialis Petropolitanae, *Econometrica*.

Bewley, Truman F., 2002, Knightian decision theory. Part I, *Decisions in Economics and Finance* 25, 79–110.

Bhandari, Laxmi Chand, 1988, Debt/Equity Ratio and Expected Common Stock Returns: Empirical Evidence, *The Journal of Finance* 43, 507–528.

Black, Fischer, 1972, Capital Market Equilibrium with Restricted Borrowing, *The Journal of Business* 45, 444–455.

Black, Fischer, M.C. Jensen, and Myron Scholes, 1972, The capital asset pricing model: Some empirical tests, *Studies in the theory of capital markets*.

Bollerslev, Tim, George Tauchen, and Hao Zhou, 2009, Expected stock returns and variance risk premia, *Review of Financial Studies* 22, 4463–4492.

Boyle, Phelim P., Lorenzo Garlappi, Raman Uppal, and Tan Wang, 2009, Keynes Meets Markowitz: The Tradeoff Between Familiarity and Diversification, *Ssrn*.

Breeden, Douglas T., 1979, An intertemporal asset pricing model with stochastic consumption and investment opportunities, *Journal of Financial Economics* 7, 265–296.

Brenner, Menachem, and Yehuda Izhakian, 2018, Asset pricing and ambiguity: Empirical evidence∗, *Journal of Financial Economics* 130, 503–531.

Cakici, Nusret, Frank J Fabozzi, and Sinan Tan, 2013, Size , value , and momentum in emerging market stock returns, *Emerging Markets Review* 16, 46–65.

Cakici, Nusret, Yi Tang, and An Yan, 2016, Do the size, value, and momentum factors drive stock returns in emerging markets?, *Journal of International Money and Finance* 69, 179–204.

Carhart, Mark, 1997, On Persistence in Mutual Fund Performance, *Journal of Finance* 52, 57–82.

Caskey, Judson A., 2009, Information in equity markets with ambiguity-averse investors, *Review of Financial Studies* 22, 3595–3627.

Chen, Nai-Fu, Richard Roll, and Stephen Ross, 1986, Economic Forces and the Stock Market, *The Journal of Business* 59, 383–403.

Chen, Zengjing, and Larry Epstein, 2002, Ambiguity, risk, and asset returns, in continuous time, *Econometrica* 70, 1403–1443.

Cochrane, John, 1991, Production-Based Asset Pricing and the Link Between Stock Returns and Economic Fluctuations, *The Journal of Finance* 46, 209–237.

Cochrane, John, 1996, A Cross-Sectional Test of an Investment-Based Asset Pricing Model, *Journal of Political Economy* 104, 572–621.

Copeland, T E, J F Weston, and K Shastri, 2005, *Financial Theory and Corporate Policy*. The Addison-Wesley series in finance (Addison-Wesley).

Coşar, Evren Erdoğan, and Saygın Şahinöz, 2018, Quantifying Uncertainty and Identifying its Impacts on the Turkish Economy, *CBRT* Working Pa.

Daniel, Kent, David Hirshleifer, and Avanidhar Subrahmanyam, 1998, Investor psychology and security market under- and overreactions, *Journal of Finance* 53, 1839–1885.

De Bondt, Werner F M, and Richard Thaler, 1985, Does the Stock Market Overreact?, *Journal of Finance* 40, 793–805.

De Bondt, Werner F M, and Richard Thaler, 1987, Further Evidence on Investor Overreaction and Stock Market Seasonalit y, *Journal of Finance* 42, 557–581.

Diether, Karl B., Christopher J. Malloy, and Anna Scherbina, 2002, Differences of opinion and the cross-section of stock returns, *Journal of Finance* 57, 2113–2141.

Donadelli, Michael, and Lorenzo Prosperi, 2012, On the role of liquidity in emerging markets stock prices, *Research in Economics* 66, 320–348.

Dow, James, and Sérgio Ribeiro da Costa Werlang, 1992, Uncertainty Aversion, Risk Aversion, and the Optimal Choice of Portfolio, *Econometrica* 60, 197–204.

Drechsler, Itamar, 2013, Uncertainty, time-varying fear, and asset prices, *Journal of Finance* 68, 1843–1889.

Dybvig, Philip, and Stephen Ross, 1985, Yes, the APT Is Testable, *Journal of Finance* 40, 1173–1188.

Easley, David, and Maureen O'Hara, 2010, Liquidity and valuation in an uncertain world, *Journal of Financial Economics* 97, 1–11.

Ellsberg, Daniel, 1961, Risk, Ambiguity, and the Savage Axioms, *The Quarterly Journal of Economics* 75, 643.

Epstein, Larry G., and Martin Schneider, 2010, Ambiguity and Asset Markets, *Annual Review of Financial Economics*.

Epstein, Larry G, and Martin Schneider, 2008, Ambiguity , Information Quality , ABSTRACT, 63, 197–228.

Epstein, Larry G, and Tan Wang, 1994, Intertemporal Asset Pricing under Knightian Uncertainty, *Econometrica* 62, 283–322.

Epstein, Larry G, and Stanley E Zin, 1989, Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework, *Econometrica* 57, 937–969.

Fama, Eugene F., and Kenneth R. French, 1996, Multifactor explanations of asset pricing anomalies, *Journal of Finance* 51, 55–84.

Fama, Eugene F., and Kenneth R. French, 2015, A five-factor asset pricing model, *Journal of Financial Economics* 116, 1–22.

Fama, Eugene F., and Kenneth R. French, 2017, International tests of a five-factor asset pricing model, *Journal of Financial Economics* 123, 441–463.

Fama, Eugene F, and Kenneth R French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.

Fama, Eugene F, and Kenneth R French, 2004, The Capital Asset Pricing Model: Theory and Evidence, *Journal of Economic Perspectives* 18, 25–46.

Fama, Eugene F, and James D MacBeth, 1973, Risk, return, and equilibrium: Empirical tests, *Journal of Political Economy* 81, 607–636.

Fama, Eugene, and Kenneth French, 1992, The Cross-Section of Expected Stock Returns, *Journal of Finance* 47, 427–465.

French, Kenneth R, and James M Poterba, 1991, Investor Diversification and International Equity Markets, *Source: The American Economic Review*.

Ghirardato, Paolo, Fabio Maccheroni, and Massimo Marinacci, 2004, Differentiating ambiguity and ambiguity attitude, *Journal of Economic Theory* 118, 133–173.

Gibbons, Michael R, 1982, Multivariate tests of financial models: A new approach, *Journal of Financial Economics* 10, 3–27.

Gibbons, Michael R, Stephen A Ross, Jay Shanken, and Jay Shanken1, 1989, A Test of the Efficiency of a Given Portfolio, *Econometrica* 57, 1121–1152.

Gilboa, Itzhak, and David Schmeidler, 1989, Maxmin expected utility with non-unique prior, *Journal of Mathematical Economics* 18, 141–153.

Hansen, Lars Peter, and Thomas J Sargent, 2001, American Economic Association Robust Control and Model Uncertainty, *Source: The American Economic Review*.

Harvey, Campbell R., Yan Liu, and Heqing Zhu, 2016, ⋯ and the Cross-Section of Expected Returns, *Review of Financial Studies*.

Harvey, Campbell, and Akhtar Siddique, 2000, Conditional Skewness in Asset Pricing Tests, *Journal of Finance* 55, 1263–1295.

Hou, Kewei, Chen Xue, and Lu Zhang, 2015, Digesting anomalies: An investment approach, *Review of Financial Studies* 28, 650–705.

Hou, Kewei, Chen Xue, and Lu Zhang, 2017, A Comparison of New Factor Models, *Ssrn*.

Illeditsch, Philipp Karl, 2011, Ambiguous Information, Portfolio Inertia, and Excess Volatility, *Journal of Finance* 66, 2213–2247.

Jegadeesh, Narasimhan, and Sheridan Titman, 1993, Returns to Buying Winners and Selling Losers : Implications for Stock Market Efficiency, *The Journal of Finance* 48, 65–91.

Jensen, Michael, 1968, The Performance of Mutual Funds in the Period 1945–1964, *Journal of Finance* 23, 389–416.

Johnson, Timothy C., 2004, Forecast dispersion and the cross-section of expected returns, *Journal of Finance* 59, 1957–1978.

Jurado, Kyle, Sydney C Ludvigson, and Serena Ng, 2015, Measuring Uncertainty, *American Economic Review* 105, 1177–1216.

Kahneman, Daniel, and Amos Tversky, 1979, Prospect Theory: An Analysis of Decision under Risk, *Econometrica* 47, 263–291.

Karni, Edi, 2014, *Axiomatic Foundations of Expected Utility and Subjective Probability Handbook of the Economics of Risk and Uncertainty*. Vol. 1 (Elsevier B.V.).

Klibanoff, Peter, Massimo Marinacci, and Sujoy Mukerji, 2009, Recursive smooth ambiguity preferences, *Journal of Economic Theory* 144, 930–976.

Knight, F. H., 1921, Risk, Uncertainty and Profit, 1921, *Boston and New York*.

Kogan, Leonid, and Tan Wang, 2003, A Simple Theory of Asset Pricing under Model Uncertainty, *Unpublished working paper, MIT and The University of British Columbia.*

Kozak, Serhiy, Stefan Nagel, and Shrihari Santosh, 2018, Interpreting Factor Models, *Journal of Finance* 73, 1183–1223.

Kraus, Alan, and Robert H Litzenberger, 1976, Skewness Preference and the Valuation of Risk Assets, *Journal of Finance* 31, 1085–1100.

Lintner, John, 1965, The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets, *The Review of Economics and Statistics* 47, 13–37.

Liu, Weimin, 2006, A liquidity-augmented capital asset pricing model, *Journal of Financial Economics* 82, 631–671.

Machina, Mark J., and Marciano Siniscalchi, 2014, Ambiguity and ambiguity aversion, *Handbook of the Economics of Risk and Uncertainty* 1, 729–807.

Markowitz, Harry, 1952, Portfolio Selection, *The Journal of Finance* 7, 77–91.

Mehra, Rajnish, and Edward C Prescott, 1985, The equity premium, *Journal of Monetary Economics* 15, 145–161.

Merton, Robert C., 1973, An Intertemporal Capital Asset Pricing Model, *Econometrica* 41, 867.

Miller, Merton, and Franco Modigliani, 1961, Dividend Policy, Growth, and the Valuation of Shares, *The Journal of Business* 34.

Mossin, Jan, 1966, Equilibrium in a Capital Asset Market, *Econometrica* 34, 768–783.

Olsen, Robert A., and George H. Troughton, 2000, Are Risk Premium Anomalies Caused by Ambiguity?, *Financial Analysts Journal* 56, 24–31.

Ozsoylev, Han, and Jan Werner, 2011, Liquidity and asset prices in rational expectations equilibrium with ambiguous information, *Economic Theory* 48, 469–491.

Pastor, Lubos, and Robert F. Stambaugh, 2003, Liquidity Risk and Expected Stock Returns, *Journal of Political Economy* 111, 642–685.

Reinganum, Marc R, 1981, Misspecification of capital asset pricing: Empirical anomalies based on earnings' yields and market values, *Journal of Financial Economics* 9, 19–46.

Roll, Richard, 1977, A critique of the asset pricing theory's tests Part I: On past and potential testability of the theory, *Journal of Financial Economics* 4, 129–176.

Roll, Richard, and Stephen Ross, 1980, An Empirical Investigation of the Arbitrage Pricing Theory, *Journal of Finance* 35, 1073–1103.

Ross, Stephen A, 1976, The Arbitrage Theory of Asset Pricing, *Journal of Economic Theory* 13, 341–360.

Savage, Leonard Jimmie, 1954, *The Foundations of Statistics* (John Wiley, New York).

Schmeidler, David, 1989, Subjective Probability and Expected Utility without Additivity Published by : The, *Econometrica* 57, 571–587.

Shanken, Jay, 1982, The Arbitrage Pricing Theory: Is It Testable?, *Journal of Finance* 37, 1129–1140.

Sharpe, William F., 1964, Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk, *The Journal of Finance* 19, 425.

Stambaugh, Robert, 1982, On the exclusion of assets from tests of the two-parameter model: A sensitivity analysis, *Journal of Financial Economics* 10, 237–268.

Sun, Lei, K.C. John Wei, and Feixue Xie, 2014, On the Explanations for the Gross Profitability Effect: Insights from International Equity Markets, *Ssrn*.

Titman, Sheridan, K. C.John Wei, and Feixue Xie, 2013, Market development and the asset growth effect: International evidence, *Journal of Financial and Quantitative Analysis* 48, 1405–1432.

Tobin, James, 1958, Liquidity Preference as Behavior Towards Risk, *Review of Economic Studies* 25, 65–86.

Wang, Huijun, and Jianfeng Yu, 2013, *Dissecting the Profitability Premium Ssrn*.

Xing, Yuhang, 2007, Interpreting the Value Effect Through the Q-Theory: An Empirical Investigation, *The Review of Financial Studies* 21, 1767–1795.

Zhang, X Frank, 2006, Information uncertainty and stock returns, *Journal of Finance* 61, 105–137.

APPENDICES

A : CURRICULUM VITAE

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EDUCATION

Ph.D., Middle East Technical University, Finance, 2019.

M.Sc., University of Exeter, Economics and Econometrics, 2008.

B.S., Middle East Technical University, Economics, 2003.

FELLOWSHIPS AND AWARDS

METU Graduate Courses Performance Award: Most Successful Student in the PhD Program of the Department of Business Administration, 2016.

Distinction in MSc. in Economics and Econometrics, University of Exeter Business School, 2008.

Graduate Scholarship: Scholarship for MSc., Central Bank of the Republic of Turkey, 2007-2008.

PROFESSIONAL EXPERIENCE

Expert, Central Bank of the Republic of Turkey, 2004-present.

RESEARCH AND PUBLICATIONS

[1] "External Financial Stress and External Financing Vulnerability in Turkey" Central Bank Review 13(Special Issue-March), pp.65-74.

[2] "Para politikası kararlarının hisse senedi piyasası üzerine etkisi", CBRT, 2011.

[3] "The Impact of Monetary Policy on Asset Pricing", University Exeter, 2008.

COMPUTER SKILLS

Stata, R, EViews, Spss, Matlab.

B : TURKISH SUMMARY/TÜRKÇE ÖZET

BELİRSİZLİK VE VARLIK FİYATLAMASINA İLİŞKİN İKİ MAKALE

Bu tez belirsizliğin varlık fiyatlaması üzerine etkisi üzerine yazılan iki makaleden oluşmaktadır. İlk makalede belirsizliğin dahil edildiği karar verme ve varlık fiyatlaması teorik modelleri üzerine detaylı bir inceleme sunulurken, Türkiye için farklı metotlar kullanarak belirsizlik endeksleri elde edilmiş, bunlar karşılaştırılmış ve son olarak da belirsizliğin Borsa İstanbul (BIST30 Endeksi) üzerindeki etkisi analiz edilmiştir. İkinci makalede ise portföy ve hisse senedi seviyesinde belirsizliğin hisse senedi getirisi üzerinde etkili olup olmadığı incelenmiştir.

Finans yazını varlık getirisinin bir belirleyicisi olarak risk ile getiri arasındaki ilişkiyi detaylı olarak incelenmiştir. Risk, Markowitz (1952) tarafından portföy seviyesinde tanımlanırken Sharpe (1964) varlık bazında risk-getiri ilişkisini teorik olarak tanımlamıştır. Sharpe (1964), Lintner (1965) ve Mossin (1966) tarafından yapılan çalışmalar yazında CAPM olarak bilinen modelin doğuşunu beraberinde getirmiştir. CAPM, varlık fiyatlamasında anahtar role sahip olsa da katı varsayımları modelin gerçek hayatla bağları arasında önemli bir engel olmuştur. Modele ilişkin ampirik desteğin de zayıf olması bu eleştirileri desteklemiştir.

CAPM varsayımları finans teorisinin dayandığı diğer bilim dallarındaki varsayımlardan bağımsız düşünülemez. Ekonomi ve finans yazınındaki modeller, ajanların karar teorisi ile uyumlu olarak objektif veya sübjektif olasılık dağılımlarına göre karar verdiklerini varsaymaktadır. Von Neumann ve Morgenstern (1944) ile Savage (1954) belirli varsayımlar altında bu olasılık dağılımlarının tek olduğunu göstermişlerdir. Olasılık dağılımlarının tek olması CAPM'in ana varsayımlarından olan tam bilgi ve homojen beklentilerle yakından ilişkilidir. Bu varsayımlar varlık getirisi dağılımı üzerindeki belirsizliğin kalkmasını ve getiri dağılımının tek bir tane olmasını sağlayarak risk getiri arasındaki ilişkinin kurulmasını sağlamaktadır. Diğer yandan varlık fiyatlarındaki belirsizlikler varlık getirisinin açıklanmasında tek olasılık dağılımı varsayımı ile çelişmekle birlikte bu varsayım gerçek yatırımcı davranış ve karar verme mekanizması ile uyuşmamaktadır.

Bu doğrultuda, bir finansal varlığın muhtemel getiri dağılımı hakkında tam bir bilgi sahibi olunmaması varlık getirilerinde olasılık dağılımının tek olmadığı, olasılık dağılımının birden fazla olduğu yaklaşımlarını beraberinde getirmiştir. Yazında birden fazla olasılık dağılımının olması belirsizlik olarak tanımlanmaktadır. Böylelikle belirsizlik olasılık dağılımının tek bir tane olarak tanımlandığı risk kavramından farklılaşmaktadır. Teorik modeller belirsizliğin ajanların tüketim ve yatırım kararlarını etkilediğini gösterirken varlık fiyatlaması modellerine risk dışında getiriyi etkileyen ayrı bir faktör olarak dahil edilmiştir.

Belirsizliğin risk tanımından farklı olması hem karar teorisinde hem de varlık fiyatlamasında önemli değişikliklere neden olmuştur. Epstein ve Wang (1994) risk ve belirsizlik arasındaki farkı şu tespit ile netleştirmektedir: Risk tanımı kapsamında olasılık kavramı olayların göreli gerçekleşme ihtimalini gösterirken bu olasılık dağılımının güvenilirliği konusunda bir şey söylenmemesi risk ve belirsizlik arasındaki farkı ortaya koymaktadır. Gerçek hayatta olayların karmaşık yapısı karar sürecinde risk yerine belirsizliğin daha geçerli bir tanım olduğunu ima etmektedir.

Belirsizlik ve risk arasındaki farka dikkat çeken ilk çalışma yazında Knight (1921) olarak gösterilirken, Ellsberg (1961) de ajanların sübjektif beklenen fayda teorisi ile uyumlu davranmadığını göstermiş, belirsizliğin karar vermede önemli olduğuna ilişkin deneysel kanıtlar sunmuştur. Takip eden çalışmalar sübjektif beklenen fayda teorisini belirsizliği de içerecek şekilde değiştirmeyi hedeflemiştir. Gilboa ve Schmeidler (1989), Schmeidler (1989), Epstein ve Wang (1994), Chen ve Epstein (2002), Hansen ve Sargent (2001) ve Klibano, Marinacci ve Mukerji (2005) belirsizliği karar verme teorisinde dahil eden çalışmalar arasında yer almıştır.

Karar teorisinin varlık fiyatlaması teorilerindeki yeri dikkate alındığında karar teorisinde yapılan güncellemelerin varlık fiyatlaması modellerine yansıması da kaçınılmaz olmaktadır. Chen ve Epstein (2002), Kogan ve Wang (2003), Bansal ve Yaron (2004), Boyle ve diğerleri (2009) ile Anderson, Ghysels ve Juergens (2009) varlık fiyatlamasına belirsizliği dahil eden teorik modeller geliştirmişlerdir. Olsen ve Troughton (2000), Zhang (2006), Epstein ve Schneider (2008), Epstein ve Schneider (2010), Illeditsch (2011) ile Ozsoylev ve Werner (2009) yazına belirsizliğin varlık fiyatlaması üzerine etkisine ilişkin ampirik destek sağlamıştır. Andreou ve diğerleri (2014), Bollerslev, Tauchen, ve Zhou (2009), Bekaert, Engstrom, ve Xing (2009) ile Brenner ve Izhakian (2018) ise belirsizliğin ölçülmesinde farklı yaklaşımlar sağlamışlardır.

Yazın belirsizliğin varlık fiyatlarına etkisi üzerine farklı teorik modeller geliştirse de bu modellerin ampirik olarak testi sınırlı kalmıştır. Bu, belirsizliğin ölçümünde yaşanan sıkıntılardan kaynaklanmaktadır. Finansal yazında risk standart sapma, beta vb. değişkenlerle temsil edilebilirken belirsizlik hesaplaması içerdiği belirsizlik nedeniyle zorlukları da beraberinde getirmektedir. Bu çalışmada Türkiye için yazında yer alan belirsizlik tanımı ile uyumlu, tarihsel olarak mümkün olduğunca geniş kapsamlı ve varlık fiyatlamasında kullanılabilecek bir belirsizlik endeksinin hesaplanması amaçlanmıştır. Elde edilen belirsizlik endeksi yazındaki tanıma uyumluluk açısından Türkiye için ilk olurken gelişmekte olan ülkeler arasında da ilk endekslerden bir tanesi olmuştur. Diğer yandan yazında belirsizlik karar teorisi ve varlık fiyatlaması alanlarında teorik ve ampirik olarak ayrı ayrı irdelenirken bu çalışmada belirsizliğin hem karar teorisi hem de varlık fiyatlaması ile olan ilişkisi bütüncül bir çerçeve çizilerek ele alınmış ve konu hakkında detaylı ve güncel bir değerlendirme sunularak yazına katkı sağlanmıştır.

Bu doğrultuda yazında yer alan vadeli işlemler, varyans risk primi, beklentilerin dağılımı, gün içi getiri dağılımındaki dalgalanma gibi farklı yaklaşımlarla birden fazla belirsizlik endeksi elde edilmiş ve endekslerin birbirleriyle karşılaştırılması büyük oranda aynı bilgiyi içerdiklerini ortaya koymuştur. Diğer yandan belirsizliğin risk dışında BIST30 endeks getirisini tahmin etme gücü araştırılmış, belirsizlik endekslerinin geçmiş dalgalanma olarak hesaplanan risk dışında getiriyi tahmin etmede bir katkı sağlamadığı görülmüştür. Buna karşın uzun dönemli analiz imkanı tanıyan ve Brenner ve Izhakian (2018) tarafından sunulan metodoloji ile elde edilen belirsizlik endeksinin risk dışında bir açıklama gücü olduğu ve aşırı getiri ile arasında ters yönlü bir ilişki olduğu gösterilmiştir.

Endeks seviyesinde belirsizlik ve hisse senedi getirisi arasındaki ilişki bu ilişkinin portföy ve hisse senedi seviyesinde de varlığı hakkında soruları beraberinde getirmiştir. Bu doğrultuda tezin ikinci makalesinde yine Türkiye için belirsizliğin hisse senedi getirilerinde fiyatlanıp fiyatlanmadığı test edilmiştir. Ancak hisse senedi getirisini etkileyen faklı faktörlerin olması ve belirsizlik ve hisse senedi getirisi arasındaki ilişkide belirsizliğin diğer faktörlerin etkisini yakalamaması için yazındaki diğer risk faktörleri de dahil edilmiştir.

Varlık fiyatlaması yazını risk ve getiri arasındaki ilişkiyi detaylı olarak ele almıştır. Bu noktada CAPM risk ve getiri arasındaki ilişkiyi ortaya koymada yazında temel bir model olmuştur. Ancak modelin katı varsayımları ve ampirik olarak zayıf desteği CAPM'in zayıf noktaları olmuştur. Black (1972), Merton (1973), Ross (1976) ve Breeden (1979) CAPM'in varsayımlarını hafifletecek ve modelin test edilebilmesini sağlayacak yeni teorik modeller sunmuşlardır. Basu (1977, 1983), Reinganum (1981), ve Banz (1981) tarafından dile getirilen büyüklük, kazanç fiyat oranı gibi anomaliler daha sonra davranışsal ve çok faktörlü modeller tarafından geliştirilen farklı durum değişkenleri ile açıklanmaya çalışılmıştır. Fama ve French (1996), CAPM'deki piyasa faktörü dışında büyüklük (SMB) ve değer (HML) faktörlerini getiri üzerinde açıklayıcı faktörler olarak eklemiştir. Carhart (1997) ise momentum faktörünü eklemiştir. Sonrasında ise yazında eklenen faktör sayısında bir artış yaşanmış, Amihud (2002) likidite faktörünü, Harvey ve Siddique (2000) eşçarpıklık, Fama ve French (2015) yatırım ve karlılık faktörlerini eklemiştir.

Yazında yer alan faktörlerin sayısı ve çeşitliliği çok olsa da bu çalışmada en çok kullanılan faktörlerin dahil edilmesi uygun görülmüştür. Bu faktörler arasında Fama ve French (2015) çalışmasından piyasa faktörü, büyüklük, değer, yatırım ve karlılık, Jegadeesh ve Titman (1993) çalışmasından momentum, Harvey and Siddique (2000) çalışmasından eşçarpıklık ve Amihud (2002) çalışmasından likidite faktörleri dahil edilmiştir. Çalışmanın temel motivasyonu Türkiye için varlık fiyatlamasında yer alan risk faktörlerinin analizi olmasa da bu tez tüm risk faktörlerin aynı anda değerlendirildiği güncel bir çalışma olarak da öne çıkmaktadır. Bu çerçevede, çalışmadaki örneklem Borsa İstanbul'da işlem gören mali sektör, yatırım ortaklıkları, gayrimenkul yatırım ortaklıkları, sportif faaliyet gösteren ve eksi defter/piyasa değeri oranına sahip şirketler dışındaki hisse senetlerini kapsamaktadır. Aynı zamanda örneklem Ocak 1990 - Aralık 2017 dönemini içermektedir.

Çalışmada ilk olarak risk faktörlerini oluşturmak için kullandığımız hisse senedi karakteristiklerine göre oluşturduğumuz tek ve çift sıralanmış portföylerin getiri yapıları incelenmiştir. Tek sıralanmış portföy aşırı getirileri ile piyasa değeri, defter değeri/piyasa değeri oranı, yatırım, momentum, düşük likidite ve belirsizlik hassaslığı arasında ilişki olduğu saptanmış, karlılık, eşçarpıklık ve piyasa betası ile portföy aşırı getirileri arasında bir ilişki gözlemlenememiştir. Çift sıralanmış portföylerde ise aşırı getiri ile sadece defter değeri/piyasa değeri oranı ve belirsizlik hassaslığı arasında bir ilişki olduğu görülmektedir. Bu çerçevede hisse senedi aşırı getirileri ve farklı hisse karakteristikleri arasında farklı seviyelerde ve değişken yönlerde ilişki olduğu ortaya konmaktadır. Bu nedenle belirsizlik ve getiri arasında yapacağımız analizlere bu hisse karakteristiklerinden elde edilen tüm risk faktörlerinin dahil edilmesi kararlaştırılmıştır.

Elde edilen risk faktörleri için yapılan kapsama testi büyüklük ve yatırım dışındaki tüm risk faktörlerinin farklı bilgi sağladığını ortaya koymuştur. Ancak kapsama test sonuçlarının dönem ve model yapısına göre hassasiyet gösterebilme olasılığı dikkate alınarak belirsizliğin hisse senedi getirisi üzerindeki etkisinin testi için tüm risk faktörleri analizlere dahil edilmiştir.

Yapılan analizlerde Bali, Brown ve Tang (2017) takip edilmiş, yazında yer alan diğer çalışmalara da benzer olarak zaman serisi ve kesit regresyonlar analizlerde kullanılmıştır. Zaman serisi analizleri belirsizliğin hisse senedi getirileri üzerinde diğer faktörleri içeren modellere göre bir bilgi sağladığını gösterse de bu sonuçların kompleks modellerde zayıfladığı görülmüştür. Diğer yandan tek faktör, üç faktör veya beş faktör modelleri gibi klasik modellerin belirsizlikten kaynaklanan getiri farkını tahmin edemediği sonucuna varılmıştır. Bu da belirsizlikten kaynaklanan aşırı getirinin basit faktör kombinasyonları ile açıklanamadığını ortaya koymaktadır. Kesit regresyon analizlerinde kullanılan Fama-Macbeth regresyonları belirsizlik hassaslığı ve aşırı getiri arasında beklendiği üzere ters yönlü bir ilişki olduğunu göstermiştir. Belirsizlik hassasiyeti dışında hisse karakteristikleri arasında defter değeri/piyasa değeri, momentum, dönüş, yatırım ve operasyonel karlılık ile hisse aşırı getirisi arasında ilişki olduğu ancak düşük likidite ve eşçarpıklığın hisse senedi aşırı getirisi üzerinde etkili olmadığı görülmüştür.

Sonuç olarak bu iki makale Türkiye hisse senedi piyasasında belirsizliğin hisse senedi getirisini açıklamada önemli olduğunu ve riskten farklılaştığını göstermiştir. Böylelikle her iki makale de belirsizliğin varlık fiyatlaması üzerindeki etkisini ortaya koyan sınırlı ampirik çalışmanın arasında kendine yer bulmuş ve konu hakkındaki teorik çalışmalara da destek sağlamıştır.

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TEZİN ADI / TITLE OF THE THESIS (**İngilizce** / English) :
TWO ESSAYS ON AMBIGUITY AND ASSET PRICING

TEZİN TÜRÜ / DEGREE: **Yüksek Lisans** / Master ☐ **Doktora** / PhD ☐

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